

Appendix C: Convolution Extension

It may prove useful to extend the convolution technique of chapter 2 to cover the case of convolution of two combs, one of which does *not* have unit spacing. Although not required for the specific technique of spectrum computation just described, it is a related problem and is solved in a similar manner.

Consider the general case of convolution between combs of spacing N and $N+1$, where $N \geq 2$. The limitation on the size of N means that the highest frequency ($N+1$) is not a multiple of the lowest frequency (N), so that the previous convolution algorithms cannot be used. As before, an observation frequency where the spectrum is desired to be calculated is chosen and denoted by $\nu = \gamma$.

The $N+1$ comb sits atop each δ -function of the N -comb in turn and whenever one of the δ -functions of the $N+1$ comb is coincident with γ another term in the complex amplitude at γ is added. The δ -function of the N -comb upon which the $N+1$ combs then sits is denoted m_N , and m_{N+1} denotes the δ -function of the $N+1$ comb sitting at γ . If γ is specified by

$$\gamma = \alpha N + \beta \quad (1)$$

where αN specifies the even part of γ and β allows the formation of odd frequencies, α, β integer, then it can be shown

$$m_N = \alpha - \beta + k(N + 1) \quad (2)$$

$$m_{N+1} = \beta + kN \quad (3)$$

where k is integer, $-\infty < k < +\infty$. These equations are the result of comparison of a dozen or so tables such as table 2.1, the values of m_N and m_{N+1} determined by using two graduated paper strips representing each δ -function comb. Their validity has been verified in as much as comparison of the resulting spectrum with that as determined by Fast Fourier Transform has always shown excellent agreement and they are used in the proof below. The observation frequency need not be limited to those specified by either comb as the process of convolution, it will now be shown, *always* results in a final comb of *unit* spacing between its member δ -functions.

The convolution of a comb of spacing N with a comb of spacing $N+1$ units, $N \geq 2$, results in a comb of unit spacing independent of the integer value of N

Both m_N and m_{N+1} specify the δ -function frequency of observation γ by

$$\delta(\nu - [m_N N + m_{N+1} (N+1)]) \quad (4)$$

Let ν_1 be one point of observation determined by

$$\nu_1 = m_N N + m_{N+1} (N+1) \quad (5)$$

Let the location of a neighbouring frequency ν_2 be specified by

$$\nu_2 = (m_N + a)N + (m_{N+1} + b)(N+1) \quad (6)$$

where 'a' and 'b' are integers. This second frequency is not a point of observation and as such the premultipliers of frequencies N and $N+1$ may take any value. The separation of ν_1 and ν_2 is then

$$\Delta\nu = b + (a + b)N \quad (7)$$

The summation indices premultiplying N and $N+1$ range over $-\infty$ to $+\infty$, so $m_N, m_{N+1}, 'a'$ and $'b'$ also lie in this range. It is then legitimate to set

$$b = -a \quad (8)$$

so that the closest separation of the distinct frequencies ν_1 and ν_2 is

$$\Delta\nu = \pm 1 \quad (9)$$

which completes the proof. This proof may also be modified to prove the earlier statement that convolution of an N-comb with a *unit*-comb always results in unit spaced comb, if so required.

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