Appendix A: Information Processing

This appendix will introduce some basic concepts of optical image processing under coherent illumination . The terminology introduced finds application in the description of several classical phase visualisation techniques described in chapter one.

The 6-f Optical Bench

With the commercial availability of a coherent light source in the mid 1960's (the He-Ne laser), the study of optical information processing became widespread in the optical community.

Figure 1.1 illustrates what is often referred to as the 6-f optical processing bench; it consists of a monochromatic point source of coherent light placed in the front focal plane of a convex lens, L1. The parallel beam thus produced illuminates the object plane, situated in the rear focal plane of L1, with a coherent plane wave. Laser illumination provides a both source with a high degree of spatial *and* temporal coherence as well as high intensity and is therefore the preferred choice of illumination. A quasi-monochromatic light source may, however, be used if filtered through a small pinhole to make it spatially coherent.

Light is diffracted by the object, which may be a conventional photographic negative for example, and falls upon a second lens L2 whereupon it is imaged to the rear focal plane of the lens. It can be shown [17] that the amplitude distribution of this light is identical with the Fourier Transform of the object transmittance, suitably scaled in both spatial extent and amplitude. L2 is commonly referred to as the *transform* lens and this special plane is commonly termed the *Fourier* or *frequency* plane .

The light from the frequency plane is imaged by the third lens L3, commonly termed the *re-transform* lens, resulting in an inverted image of the object in the rear focal plane of L3. The inversion may be thought of as a consequence of a fundamental result, that the Fourier Transform (by lens L3) of a Fourier Transform (formed by lens L2) results in an image described by $f(-x,-y)$ where $f(x,y)$ describes the object transmittance. Each lens in this system has an identical focal length `f', giving rise to the name of a `6-f' bench.

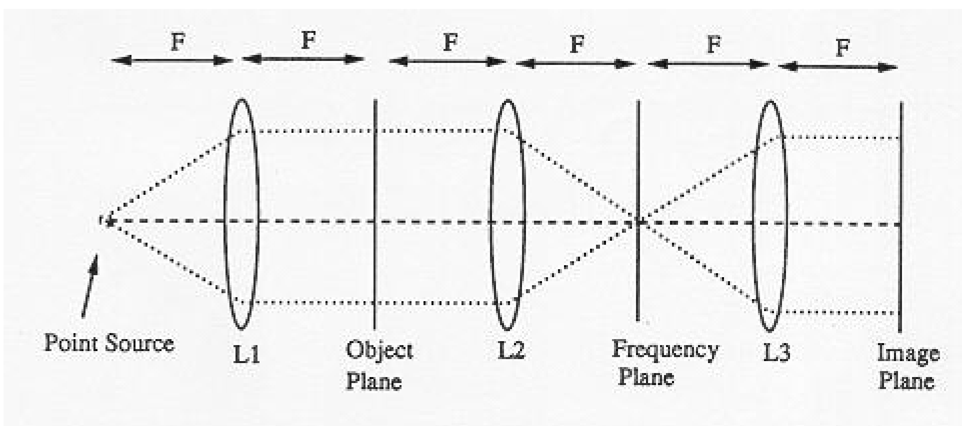

Figure A.1: 6–f Optical Processor

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If $f(x,y)$ describes the object transmittance, F_L is the focal length of the transform lens and λ is the wavelength of illumination used then the spatial frequency spectrum $F(u,v)$ is given by

$$F(u,v) = \int\int_{-\infty}^{+\infty} f(x,y)\, e^{-2\pi\,(ux + vy)}\, dx\, dy \qquad (1)$$

where $u=[(\lambda F_L)/(x_i)]$ and $v=[(\lambda F_L)/(y_i)]$. Here, x_i and y_i are spatial co-ordinates in the frequency plane. It will be observed that at the zero frequency position we obtain a pure integration of $f(x,y)$ and this is proportional to the mean value of $f(x,y)$. It is sometimes helpful to re-write the spectrum as

$$F(u,v) = |F(u,v)|\, e^{i\, \Phi(u,v)} \qquad (2)$$

which separates the amplitude and phase terms. Both the human eye and photo-sensitive detectors, such as CCD array cameras, are unable to detect the rapid oscillation of the electric field of a light wave. Consequently the phase of the light filed remains undetected and instead the time integrated *intensity* of the light field is observed. This is proportional to

$$I(u,v) = |F(u,v)|^2 \qquad (3)$$

A Physical Interpretation

It may be instructive to clarify what the Fourier Transform means in physical terms. If the amplitude transmittance of the object is described by

$$f(x) = c_0 + c_m \cos(2\, \pi\, m\, x - \Phi_m) \qquad (4)$$

then we have a sinusoidal variation in transmission with a *spatial wavelength* of λ_x=[1/m], biased with a constant transmission of c_0 (figure 1.2). The spatial offset of the sinusoid from the origin of the object plane is described by the term Φ_m. Alternatively, we may say this function has a *spatial frequency* of ν_x=m with units of mm^{-1} if the co-ordinate axes are to be measured in mm, for example.

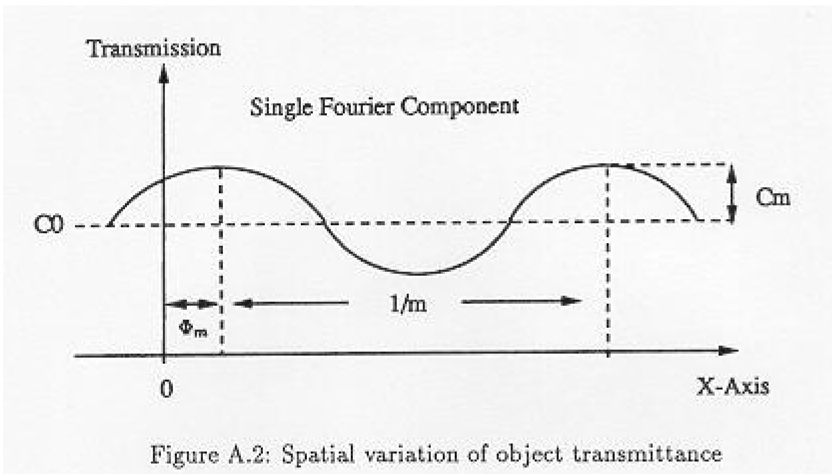

Figure A.2: Spatial variation of object transmittance

The constant, or `DC bias', term of the object transmittance c_0 causes no diffraction. Consequently, the plane wave illuminating the object is focussed to form an image of the point source of illumination in the centre of the Fourier plane. The sinusoidal component of object transmission *does* cause diffraction of the plane wave illumination, giving rise to two diffracted waves. The waves propagates at an equal but opposite angles to the optical axis of the system and each is focussed by the transform lens L2 to an image of the point illumination source in the Fourier plane as illustrated in figure 1.31. The field amplitude is proportional to $[(c_m)/2]$ and the distance from the frequency plane origin is proportional to the spatial frequency ν_x of the object function. This is what is meant by an *optical* Fourier Transform. Note that each point of the frequency plane is uniquely associated with one spatial frequency present in the object.

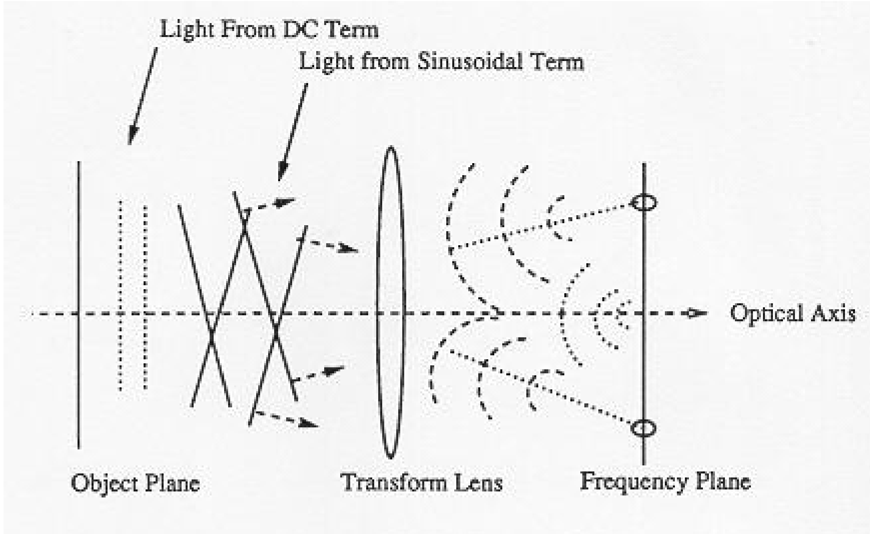

Figure A.3: Diffraction from a single spatial frequency

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A more realistic case concerns an object such as a simple ruled transmission grating. The transmission is described by a periodic square wave of fundamental frequency ν_0. We may synthesise such a function by an infinite series of sinusoidal functions whose frequencies are harmonics of ν_0 i.e. ν_0, $2\nu_0$, $3\nu_0$, etc. If the amplitude of each harmonic is denoted by c_m then each gives rise to the familiar positive (+m) and negative (-m) order spectrum in the frequency plane, the amplitude of which is none other than $[(c_m)/2]$.

In general the electric field at the ±m order position has a phase described by $e^{+i\Phi_m}$ and $e^{-i\Phi_m}$ respectively, the zero of phase being taken at the origin of the frequency plane.

The spatial offset Φ_m of the object transmission component is thus converted into a temporal (phase) delay of the electromagnetic field at the frequency of $\nu = \pm m$ in the Fourier Plane.

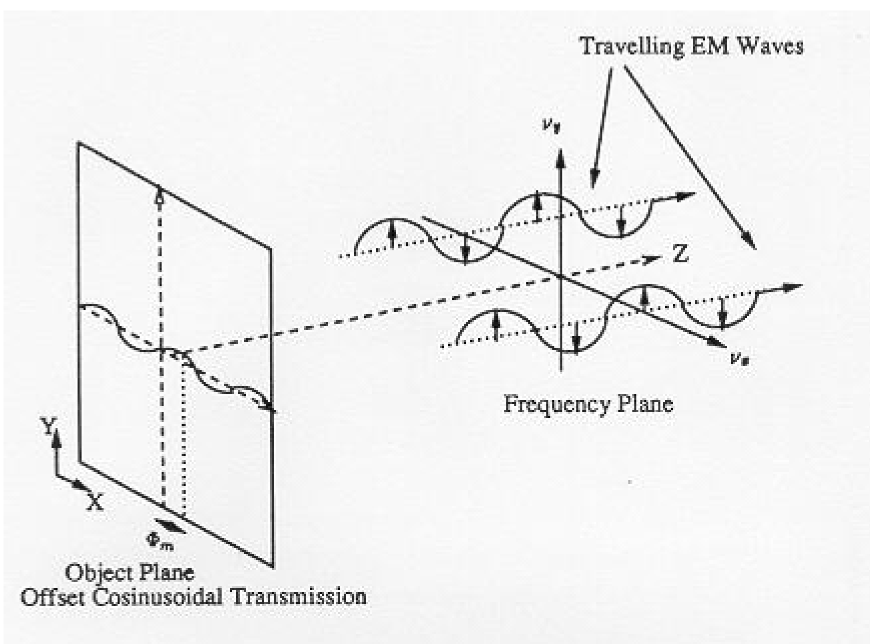

Figure A.4: Spatial offset conversion to temporal delay of light field

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As the image is none other than the Fourier Transform of the frequency plane, an appropriate phase delay applied to the m'th spectral order results in a lateral shift of that component in the image plane. We may, of course, remove that component altogether by simply blocking the desired order out altogether. These are the two fundamental operations of image processing, the operation of phase shifting forming the basis of which chapters six and seven are concerned.
