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A Formal, Hierarchical Design and Validation Methodology for VLSI

Ph D Thesis
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Abstract

The high cost of fabricating VLSI circuits requires that they be validated, that is, shown to function correctly, before manufacture. The cost of design errors can be kept to a minimum if such validation occurs as early as possible; this is achieved by integrating validation into a hierarchical design procedure.

In this thesis, a hierarchical approach to design, in which validation is performed between each pair of adjacent levels in the hierarchy, is developed. In order to adopt such an approach, a language is required for the formal description of hardware behaviour and structure. Therefore an important aspect of the development of the methodology, and a major theme of the thesis, is the development of languages to support the methodology. An enhanced version of CIRCAL, which enables large and abstract devices to be described concisely and supports formal reasoning about the behaviour of constructed systems, is presented.

Specifications should accurately model the behaviour of real hardware and should be useful for design and validation; they should also be easy to write. In order to realise these goals, a number of specification techniques have been developed and a new language which enforces some of these techniques, thereby easing the specification task, is proposed.

Ways in which a language may assist design have been investigated. Language constructs which restrict a designer, thereby removing some design decisions, have been developed. A simple correctness-preserving transformation is presented, illustrating another way in which a designer may be assisted by a formal language.

Specification techniques play an important part in the validation task, as accurate and consistent modelling is vital in establishing the correctness of implementations. Techniques have also been developed which enable detailed implementations to be usefully compared with more abstract specifications. This is demonstrated in a large example, the specification, design and formal verification of a simple microprocessor.

Finally, the concept of contextual constraints, restrictions on the environment in which a device may be placed, is introduced. A method of specifying such constraints has been developed, and it is shown that their formal treatment can provide assistance in specification, design and verification.
# Table of Contents

1. Introduction  
   1.1 Rationale ............................................... 1  
   1.2 The Basis of the Methodology .......................... 3  
   1.3 Aims of the Thesis ................................... 7  

2. Hardware Description Languages  
   2.1 Describing Behaviour ...................................... 11  
   2.2 A Selection of Languages ................................. 14  
       2.2.1 VHDL — The VHSIC Hardware Description Language ................................................. 14  
       2.2.2 LTS ..................................................... 21  
       2.2.3 Higher Order Logic ................................. 25  
       2.2.4 Others Languages .................................. 28  
   2.3 CIRCAL .................................................... 31  
       2.3.1 Behavioural Operators .............................. 31  
       2.3.2 Describing Hardware Behaviour in CIRCAL ............. 34  
       2.3.3 Structural Operators .............................. 38  
       2.3.4 Limits of Pure CIRCAL ............................ 43  
   2.4 Summary .................................................. 45
4.1.1 Design for Validation .................. 89
4.1.2 Restricting Design ....................... 90
4.2 Transformation ............................ 94
4.2.1 A CIRCAL-based Transformation ......... 95
4.3 Automatic Design .......................... 97
4.3.1 Specification and Design Automation .... 98
4.3.2 Validation of Automated Designs ......... 99
4.4 Summary .................................. 100

5. Validation ................................. 102
5.1 Simulation ................................. 103
5.1.1 Input Specification ....................... 103
5.1.2 Conventional Simulation .................. 104
5.1.3 Simulation by Manipulation ............... 109
5.1.4 Problems ................................ 111
5.1.5 Proof by Simulation ....................... 115
5.2 Verification .............................. 115
5.2.1 A Simple Proof ........................... 116
5.2.2 Specification Techniques for Verification 120
5.2.3 Verification Without Implementation Assumptions 126
5.2.4 Separating Timing and Function ......... 129
5.2.5 Verification vs. Exhaustive Simulation .... 131
5.2.6 Other Approaches ......................... 132
5.2.7 Verification of Transformations .......... 134
5.3 Summary .................................. 135
A. Glossary

B. Grammar of Enhanced CIRCAL

C. Implementation Issues

C.1 Motivation

C.2 System Features

C.2.1 Input Syntax

C.2.2 Commands

C.3 The Approach Taken

C.3.1 Language Choice

C.3.2 Structure of the System

C.3.3 Data Structures

C.4 Lessons Learned

C.5 Ideas for Additional Features

C.5.1 Complete Expansion

C.5.2 Behaviour Matching

C.6 Conclusion

D. Function and Datatype Definitions for Computer Specification

Bibliography
Chapter 1

Introduction

1.1 Rationale

Before describing the precise meaning and details of a hierarchical, integrated design and validation methodology for VLSI it is appropriate to consider why such an entity is desirable. This question can be split into three parts: why validation is necessary, why design should be performed hierarchically, and why validation should be integrated into the hierarchical design process.

The importance of validating a large design is widely accepted and can hardly be overstated. It is clearly important that a chip functions as required in any application for which it will be used i.e. it must conform to some specification. One way to establish this is to test the chip extensively after it has been fully designed and fabricated. However, the high cost of fabricating VLSI chips, the time taken to put a design on silicon (or some other material), and the difficulties which may be encountered in testing a fabricated chip due to unobservable internal behaviour make this approach extremely unattractive. Thus the vast majority of designs are simulated in some way prior to fabrication in the hope that any design errors will be detected before incurring the high costs, in both time and money, of fabrication. A wide range of simulation techniques is available and these are discussed below.

The reasons for performing design hierarchically are fairly well-known and are applicable to many more areas than just VLSI design. The idea of reducing a problem into subproblems of a more manageable size, and further subdividing
the subproblems if necessary, is well established. When this problem reduction technique is used in a design situation it is referred to as hierarchical design. The levels in the hierarchy are often called levels of abstraction, as devices become more abstract at the higher levels. There is some debate, however, in the VLSI field as to whether hierarchical design should be approached 'top-down' (starting from an abstract, high-level specification and moving towards a more concrete description of the circuit) or 'bottom-up' (constructing successively more complex devices from simpler devices which already exist). In general a mixture of the two techniques may be most appropriate. This issue will be returned to in a later chapter.

The integration of validation into the hierarchical design process can be justified by considering the shortcomings of the currently more common approach, in which simulation takes place as a post-design exercise. The most common commercial simulators can model the behaviour of only very 'low-level' components such as transistors, capacitors, etc. at the circuit level [Nagel75] or switch level [Terman83, Bryant84, Bryant81] or gates, inverters etc. at the logic level [Hayes86] thus requiring that the design be completed to that level before it can be simulated. Because such simulators can only model the behaviour of devices at a low level of abstraction, any hierarchy which was present in the design is lost at this stage. The problem with post-design validation for VLSI is that the amount of work required to take a design to the stage where it can be simulated is extremely large; often it may be measured in man-years. Thus the cost of finding a design error at such a late stage can be very high as it may require the redesign of large sections of the chip. Ideally one would like to be able to detect errors at the earliest opportunity in order to minimise the amount of wasted design effort. This capability is offered by incorporating the validation task into the hierarchical design process. The way in which this is done is presented below.
1.2 The Basis of the Methodology

For the purposes of the following discussion it will be assumed that design is to take place top-down, although, as suggested above, it is common for design to be a mixture of top-down and bottom-up techniques. For the moment this is an acceptable simplification to make. It will be made clear later how some of the ideas are also applicable to bottom-up design.

The validation of a design is the process of establishing that it conforms to a specification. Thus the essential first step of a design methodology which aims to support validation must be to create a specification of the required behaviour of the system. (The precise meaning of ‘behaviour’ will be discussed later. At this stage a definition along the lines of ‘what the circuit does’, as distinct from how it does it or how it is constructed, will suffice.) It is common for top-level specifications to be fairly informal, perhaps using a mixture of representational forms such as English language sentences, timing diagrams, tables of timing data, etc. There are strong arguments for making top-level specifications more formal. Formal specifications can clearly be less ambiguous than those relying on conventional techniques. They also permit the comparison between the behaviour of the implemented device and its specification to be made more easily and rigorously.

By far the most common approach to formal specification is the use of hardware description languages (HDLs). These languages have been the subject of much attention in recent years and the ability to use the languages to describe behaviour has been of particular importance. The methodology proposed here will depend heavily on the use of hardware description languages with behavioural description capabilities.

Thus, the design process begins with the writing of a formal specification of the required behaviour of the chip using a hardware description language. Throughout this thesis, a specification will be assumed to be a description of a device’s behaviour which gives no direct information as to how the device might be constructed — it could also be called a ‘black-box’ description. This corresponds to
the fact that a pictorial representation of the circuit at this stage would just be an empty box, containing no information about its internal structure. This first specification has already been referred to as 'top-level'; this implies that the specification represents the highest (most abstract) level in the design hierarchy. Note that at any level in the hierarchy there may be both structural and behavioural (and possibly also geometrical) descriptions of the design. Thus a behavioural description is not intrinsically 'higher-level' than a structural one. Rather than referring to behaviour, structure and geometry as different levels of description, it is better to consider them as different descriptive domains.

In order to move down to the second level of hierarchy the designer must partition the top-level 'box' into smaller parts and ascribe a behaviour (either formally or informally stated) to each of these parts. In the traditional approach to hierarchical design these parts would each become a new black box. Each of them would then be treated as a system in its own right, which would be further partitioned, thus creating more and more levels in the hierarchy. At some point a bottom level is reached. For example, in VLSI design the bottom level is usually the level at which the circuit can be described in terms of interconnected wires, transistors and capacitors, while in a standard cell design system the bottom level might consist of gates, counters, latches etc. When all parts of the system have been designed down to this level, the design is considered complete.

It has already been mentioned that if validation is left until the design has progressed to the bottom level the cost of a design error may be very high. Thus it is proposed that validation should take place between every pair of adjacent levels in the design hierarchy. This is illustrated in Figure 1-1. In this thesis, a lower level in the hierarchy will be referred to as an implementation of the higher level. The behaviour of the higher level, whether it is the top one or not, is a specification. Thus the validation task consists of trying to show that the behaviour of an implementation satisfies a specification. In order to do this it is essential that the boxes which constitute the design at every level are formally described in some language. The language which is used to write the behavioural description must either be suitable for input to a simulator or support
A specification design validate design implementation design step -.

A

design validate

B C

D

level 1 → level 2

C

design validate

E F G

level 2 → level 3

H I

design validate

F level 3 → level 4 (primitive components)

Figure 1–1: Validation in hierarchical design

mathematical reasoning about the behaviour of the constructed device. In this way the behaviour of an implementation can be established and then compared with the specification behaviour. If at any point the implementation is found to be incorrect, i.e. does not satisfy the specification, then the designer must return to the preceding level in the hierarchy and repeat the process of partitioning the current box into parts and specifying their behaviours. Note that the amount of work wasted here is minimised, because the error can be detected before the design has been carried out down to the lowest level in the hierarchy.

If the step of moving from one level in the design hierarchy down to the next lowest level is examined more closely it can be seen to consist of a number of subtasks. These tasks are illustrated by Figure 1–2. This diagram represents an attempt to implement a box 'A' by partitioning it into three boxes 'B', 'C' and 'D'. Rectangles are used to represent structural entities while ovals represent the behavioural descriptions corresponding to these structures. The specification of 'A' must be written first if 'A' represents the whole system being designed. If however 'A' is just one of several boxes at a lower level in a design hierarchy its specification will already exist, having been written in the 'describe' phase at an
earlier step. Thus there is no real difference between the two tasks of specifying and describing other than the fact that the former applies to the higher level of abstraction and the latter applies to the lower level.

Figure 1-2: Subtasks of a design/validation step

The 'partition' task is a purely structural one which involves splitting the higher level box into smaller boxes and describing their interconnections. It may be considered rather artificial to separate this from the description task, since a designer will invariably have an informal idea of the behaviour of each lower level box when he performs the partitioning. If for example box 'C' was actually called 'register' then this would imply a certain behaviour for that box involving the storage of data, clocking and so forth. The reason for showing description as a separate task is that the formal description of the behaviour of each of the boxes 'B', 'C' and 'D' must follow the partitioning task. These two subtasks make up the 'design' step shown in Figure 1-1.

The validation step is also made up of two parts, the subtasks labelled 'compose' and 'compare' in Figure 1-2. Precisely what these tasks entail depends on
whether the validation technique used is simulation or formal, mathematical proof, which will hereafter be called verification. If simulation is chosen, the composition may be considered to be performed by the simulator, whose task is to reproduce the behaviour of the circuit which consists of boxes 'B', 'C' and 'D' wired together in a certain way. This behaviour would then be compared, either automatically or by inspection, with the behaviour specified for the box currently being designed, 'A'. If formal verification is to be used, then composition is performed mathematically to establish the behaviour labelled as 'IMP' in the diagram. The comparison step then becomes the task of demonstrating equivalence (suitably defined) between the two behavioural expressions 'SPEC' and 'IMP'.

1.3 Aims of the Thesis

The argument for the development of a hierarchical methodology which integrates validation with design has been outlined in Section 1.1. In the last Section the salient features of such a methodology were presented. The two main aims of this thesis will be:

- to strengthen the argument for the proposed methodology;
- to examine the consequences of adopting such a methodology.

It has already been shown that languages with the capacity for the formal description of hardware behaviour are of central importance to the proposed approach. The nature and use of such languages will therefore be a main theme of the following discussion. In seeking to achieve the above aims, the advantages of a language-based approach to design will be demonstrated, and the language features which the methodology requires will be examined. It will be shown how a suitably defined language may support a hierarchical design and validation methodology.

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1 It should be noted that the term 'validation' will be used to include both simulation and verification. The definitions of a large number of terms appear in Appendix A
Because of the importance to this methodology of languages for the description of hardware behaviour, the body of the thesis begins with an introduction to the field of behavioural hardware description. Several languages with behavioural description capabilities will be described along with examples of their use. Of particular interest is CIRCAL\[Milne83a,Milne84,Milne85a\], the language which will be used extensively in the thesis to illustrate the ideas of the methodology, its subtasks, and the role which a language may play in these subtasks.

CIRCAL differs in an important way from most other hardware description languages: it was developed to support formal reasoning about the behaviour of hardware systems rather than simply for the description of hardware. It is quite closely related to Milner's CCS\[Milner80,Milner83\], as both formalisms grew out of work by Milne and Milner on the modelling of concurrent systems\[Milne79\]. The fundamentals of CIRCAL and its use for describing hardware will be presented in Chapter 2.

The next three chapters deal with the three main tasks of the proposed methodology, specification, design and validation. In Chapter 3 the problems of writing accurate and concise specifications will be addressed. The importance of language features to assist in this task, and the need for specification techniques coupled with the language, will be demonstrated. Various enhancements of the CIRCAL language to increase its usefulness for specification will be proposed.

In Chapter 4 the design task, comprising the subtasks of partitioning and description, will be discussed. Ways in which a language may assist design, and design techniques in turn may assist validation, will be presented. Fully manual, transformational, and automated approaches to design will be discussed along with the requirements which each of these approaches place on the design language.

The final task to be discussed is validation. The two approaches of simulation and formal verification will be presented and their respective merits compared. As with design, these different approaches place different requirements on the description language and its associated specification techniques. Some of the techniques proposed in Chapter 3 will be justified and some further techniques will be developed.
Following this treatment of the subtasks of the methodology, the ideas which have been presented will be illustrated by applying them to a reasonably large example. This will consist of the specification of a simple computer, its design to a lower (but still quite abstract) level of hierarchy, and the validation of the design step. This Chapter will aim to demonstrate the effectiveness of the techniques for specification, design and validation which have been proposed in previous chapters.

The body of the thesis concludes with a discussion of a topic which affects all the subtasks of the methodology, constraints. After defining this much-used term, this Chapter will demonstrate how a language-based approach to design supports the formal treatment of some types of constraint and how such a treatment can provide assistance in each of the tasks of specification, design and validation.

A recurring theme of this thesis will be the inter-relationship between the tasks of specification, design and validation and the importance of languages in the execution of these tasks. It is hoped that it will make a worthwhile contribution to the effort to integrate these tasks in the development of more powerful and reliable methods for the design of VLSI.
Chapter 2

Hardware Description Languages

In the preceding Chapter, the usefulness of a methodology in which validation is performed between each adjacent pair of levels in a design hierarchy as an integral part of the design process was demonstrated. In order to carry out such a design methodology there must be a mechanism for comparing the behaviours of design components at different levels of description in the hierarchy. This comparison is made between a behavioural description of a 'black box' at the higher level of abstraction (the specification) and a behaviour (the implementation behaviour) which is constructed using knowledge of the interconnection of parts and their respective behaviours at the lower level. It is therefore necessary to be able to describe behaviour at any level of abstraction; it is also necessary to describe the interconnection of parts, i.e. the structure of the circuit. Structure and behaviour are referred to as descriptive domains; there is a third domain, geometry, which is usually described in special-purpose languages such as CIF[Mead80] and EDIF[Carlstedt-Duke86].

It is for the description of circuit behaviour and structure in a rigorous way that many hardware description languages (HDLs) have been developed in recent years. The motivations for producing these languages have been many and varied and this is reflected in the diversity of languages which have arisen. It has already been seen that behavioural description is essential for hierarchical validation; it has also been required for documentation purposes, as a medium for the assistance of designers[Morison85] (analogous to the use of circuit diagrams
to help designers visualise structures), as input to design automation tools such as Macpits[82], and to provide a medium for the transfer of design information between different design tools. In the following Section a number of the recently developed languages will be examined and their salient features will be described and compared. Particular attention will be given to CIRCAL as it will be used extensively throughout this thesis for investigations into specification, design, validation and the application of constraints to these tasks. Before focusing attention on any particular languages, however, it is appropriate to consider more closely the issue of behavioural description.

2.1 Describing Behaviour

Before discussing how to describe behaviour and thus how one might choose to define a language to do this, the meaning of behaviour in the context of VLSI or circuit design must be established. The way in which behaviour is perceived will have a great effect on the features that will be required for its description. It is helpful to consider some common pieces of hardware to see what the word behaviour might be taken to mean. The behaviour of a piece of combinational logic, for example, might be characterised by the logical function which it computes. This could be represented as a mathematical function whose arguments are the values on the input ports of the piece of logic and whose result is a set of values on the output ports. In a very general sense, then, behaviour might be considered the dependency of certain values (outputs) on certain other values (inputs). Key concepts to note here are the ideas of values and ports. Another piece of information which might be important to the behaviour of the piece of combinational logic is the length of time it takes to compute the function. This may be measured as the amount of time between a new set of values being presented on the inputs and the new computed value appearing on the output. This introduces two more concepts of behaviour - the idea of changes on ports (i.e. the arrival of 'new' values) and that of the passage of time. Time may be measured qualitatively (certain events happen after other events) or quantitatively.
Combinational logic represents a very simple class of circuits, in which the output is a time-independent function of the inputs at a single instant in time, requiring no knowledge of their past history. However, the concepts of changes of values on ports, the interdependency of these changes and the passage of time between changes are common to both more complicated devices, such as microprocessors, and simpler devices such as pass transistors. Describing the function of a microprocessor could be a very laborious task, but the idea of port values changing over time in response to other changing port values is still applicable.

While hardware description languages have been used to describe behaviour, structure and geometry, it is really only in the behavioural domain that they are recognised as the preferred way to describe design characteristics. It is certainly easier for a human to examine a colour plot of a piece of a VLSI design than to read the CIF file or a similar geometric description. Similarly, the majority of designers find that structure is much more naturally represented by circuit diagrams than by languages, although it may sometimes be necessary to use a linguistic description of structure as the input to a design tool (e.g. Chipsmith[12]). For behaviour, however, it is difficult to come up with a reasonable alternative to a language, with attempts to represent behaviour diagrammatically being generally unnatural or limited in the range of devices which can satisfactorily be described.

Having established to some extent the meaning of behaviour it is now possible to discuss the requirements of a language for its description. It has been indicated that the central concepts of behaviour are common to a wide range of devices. Furthermore, in the methodology proposed it is essential that behaviour can be described in such a way that comparison between behaviours at adjacent levels in the hierarchy can be made. This suggests that a language should provide capabilities for the description of behaviour at a wide range of levels of abstraction. Most languages are not designed for accurate behavioural description below the gate level, since at this point some of the modelling assumptions cease to be valid, e.g. the concept of a signal's strength, not just its value, becomes important, and it may be necessary to take analog effects into account. Relatively few languages have facilities to cope at this level of description. At the other end of the spectrum
the main problem is coping with descriptive complexity, and thus a language must have sufficiently powerful constructs to deal with this.

A key requirement for a hardware description language is some means of representing time. It is this feature which most clearly distinguishes HDLs from programming languages, the vast majority of which have no facility for the description of time. As will be discussed in the following Section when some particular languages are examined, there is a number of different ways in which time can be dealt with and these offer varying degrees of generality and ease of use. The type of timing phenomena which a designer may wish to describe depends heavily on the level of abstraction at which he is working. At the microprocessor level, for example, a designer may be interested simply in what happens in the period of execution of a single instruction, whereas at the gate level he may be interested in the time taken for a change of value to propagate across a device, or even the time taken for the value on a port to move from 'false' through 'undefined' to 'true'. If a language is to be used at all levels of abstraction, it must have sufficient generality to cope with this.

The concept of changes of values is also critical to hardware description. It is a familiar concept to circuit designers, for example in the description of an edge-triggered latch. Thus a language should in some way allow the writing of descriptions which refer to value changes. As might be expected, there is also a number of different approaches to this problem. These will be discussed in the following sections.

In summary, the most basic requirement for a behavioural description language to be used in the proposed methodology is that it be able to describe any type of hardware over a range of levels of abstraction. With this as a starting point, the key features of a hardware description language should be:

- a wide spectrum of description, ranging from gates (or lower) up to microprocessors or even more complex systems;
- a means of describing the interdependency of values on ports;
• the facility to refer to changes on ports;

• facilities for the description of timing, with sufficient generality to allow timing description at all levels of abstraction supported by the language.

In the following Section a number of languages will be presented which, although not necessarily designed with the express aim of supporting a hierarchical design methodology, satisfy the above requirements to a greater or lesser extent.

2.2 A Selection of Languages

The number of Hardware Description Languages which have appeared over the last few years is very large. This Section deals with just a few of them but aims to provide some insight into both the features which are common to most languages and the considerable diversity of languages which exist. All of the chosen languages fulfill the basic requirements outlined above, yet they have significant differences in terms of the type of environment in which they were developed (e.g. academic or commercial), main application(s) for which they were intended, and therefore the features beyond the basic ones which were considered sufficiently important to be included in each language. They also differ substantially in the way in which they provide these features.

2.2.1 VHDL — The VHSIC Hardware Description Language

In addition to the selection criteria listed above, this language is worthy of discussion simply because of its importance to industry and the number of man-years that have been invested in its development. Associated with the Very High Speed Integrated Circuit (VHSIC) programme, the main motivation behind its development was to provide a standard HDL for all hardware design projects within that programme [Dewey86]. The magnitude of the VHSIC programme is such that any
HDL which aims to serve it must be an extremely general purpose one. Providing a medium for accurate specification and documentation of contracts was a high priority for the language. The language was designed by a large committee and underwent many reviews in an attempt to ensure that all the features that might possibly be required were included. In an article by three contributors to the VHDL development contract[Aylor86] the necessary features are listed as:

- ability to describe a wide range of hardware
- facilities for design management
- timing description capabilities
- architectural description capabilities
- ability to describe a design's interface
- ability to describe a design's environment
- language extensibility
- an implementation of the language which allows many tools to be driven by it
- language semantics which are independent of any particular implementation
- ‘user-friendliness’
- programming language-like appearance for procedural parts of the language
- ease of extraction and insertion of documentation data

It is clear that these requirements go far beyond those that were initially proposed in the preceding Section. This is partly because the language is required to do more than support a hierarchical design and validation methodology, and partly because the developers of the language wanted it to be not just adequate but also easy to use. The result of this is that the language has many features which are of no great relevance in this thesis. The primary concern here is the description of behaviour in such a way that hierarchical design and validation may take place. As structural description is also necessary in order to establish implementation behaviours and thus perform validation, this will also be examined.
All devices described in VHDL must have an entity declaration. This describes the interface of the device, associating a list of typed ports with the named part. No information about the device's internal structure or behaviour is given. The following example shows an entity declaration for a RAM.

Example

entity RAM is
    port(
        DATA: inout tristate_vector;
        ADDR: in bit_vector;
        CS1,Cs2: in bit;
        RW: in bit);
end RAM;

Explanation

The capitalised words in the port declaration are the port names, and they are followed by pairs of words which describe the direction of the port (inout implies a bidirectional port) and its type. Note that vector types are used to enable the data and address ports, each of which physically consists of a number of wires, to be described as single ports.

Structural Description

In describing the structure of a device, its component parts must be declared and instantiated within an architecture declaration. The declarations of the components should match their own entity declarations. In order to specify the way in which these components are wired up, their ports are renamed in such a way that all ports which are intended to be connected to each other are given the same name. (A similar convention is used in many other languages, e.g. MODEL[Lattice85], CIRCAL, etc.) The names of the wires used for interconnections within the device must be declared as signals. Because all ports have a type, type-checking can
provide a rudimentary check for design errors at this stage. The ports which will be connected to the interface of the whole device are renamed to match the interface ports to which they will connect. The renaming operation is done simply by replacing names in the ordered lists of ports which are introduced by the keywords port map within each component instantiation statement. This is illustrated in the example presented below.

Example

Figure 2–1 shows the structure of a full adder, constructed from two half adders and an or gate. The following is a possible description of the structure:

architecture Structure_Desc of Full_Adder is
  component Half_Adder
    port(I1,I2: in bit;
         Carry: out bit;
         Sum: out bit);
  component Or_Gate
    port(I1,I2: in bit; 0: out bit);
  signal a,b,c: bit;
  begin
    H1: Half_Adder port map (X,Y,a,b);
    H2: Half_Adder port map (b,Cin,c,Sum);
    O1: Or_Gate port map (a,c,Cout);
  end Structure_Desc;
The instantiations appear between the keywords begin and end. H1 and H2 are two instances of a component called Half_Adder, which will be defined in some way in a library or by the designer. The ordering of port and signal names in the port map list for H1 implies that its inputs are to be connected to ports X and Y, its carry output to internal signal a, and its sum output to signal b. In this way, all the connection information which is contained in Figure 2-1 is conveyed by this description in the language. Note that there is considerable redundancy in the description, as the declarations of Half_Adder and Or_Gate convey no information that would not be contained in the definitions of those parts. This probably reflects the aim of the language designers to provide a language suitable for documentation purposes.

Behavioural Description

There is a number of ways of representing behaviour in VHDL which are explained in detail in the user's manual [USAF84]. The most general is the process statement, which can appear in an architectural body. Many common programming language constructs are provided for the description of algorithms and functions. The principal timing construct is a delay specified in physical units (e.g. nanoseconds) using the keyword after. The following example, a behaviour of the RAM chip whose interface was defined above, illustrates the basic principles.

```
architecture Behaviour_Desc of RAM is
process(CS1,CS2,DATA,ADDR,RW)
    type matrix_item is tristate_vector (7 downto 0);
    type matrix is array (0 to 1023) of matrix_item;
    variable memory: static matrix;
begin
    if CS1 and CS2 then
        case RW is
            when '0' => DATA <= memory(IntVal(ADDR)) after 70ns;
            when '1' => memory(IntVal(ADDR)) := DATA;
    end if;
end process;
```
Most of this description looks just like a conventional programming language. The main distinguishing feature is the \texttt{after} construct. Its interpretation in a statement like

\begin{verbatim}
DATA <= memory(IntVal(ADDR)) after 70ns;
\end{verbatim}

is as follows: if any of the variables on which this statement depends (i.e. CS1, CS2 or RW) change, then a new value must be calculated for DATA. This new value will appear on the DATA port 70 nanoseconds after the change which caused it. Thus the underlying model of behaviour used is event-based, relying on the concept of changes on ports for the description of timing properties.

In order to make explicit reference to changes of value, to describe an edge-triggered device for example, it is necessary to use a pre-defined attribute 'STABLE'. This is illustrated in the following example, an edge-triggered D-latch.

\begin{verbatim}
entity D_Latch is
  port(
    DATA, CLOCK: in bit;
    Q:    out bit);
end D_Latch;

architecture Edge_Behaviour of D_latch is
  process(DATA,CLOCK)
  begin
    if CLOCK and not CLOCK'STABLE then
      Q <= DATA after 15 ns;
  end process;
end Behaviour_Desc;
\end{verbatim}
end if
end process
end Edge_Behaviour;

The above description specifies that the value on the input DATA will be transferred to the output Q if a rising edge occurs on the CLOCK, with a delay of 15 nanoseconds.

Comments

From the brief introduction to the language given above, it can be seen that it exhibits the essential features which were proposed for behavioural description. The language has many more features than those illustrated here, a consequence of its designers' aim to create a language suitable for a wide variety of applications. The description of functional dependencies between the values on various ports is achieved with the use of common programming language constructs such as conditional statements and logical operators. Timing characteristics can also be described. Reference to changes of ports values is usually implicit, relying on the underlying event-based model of the language. With a small amount of extra effort, explicit reference to value changes can be made if required. However, the way in which this is done seems to suggest that the ability to refer to changes was added as an afterthought, rather than being considered a central feature of the language.

The provision of high-level programming constructs, such as the ability to define datatypes, contributes additional power to the language. For example, the ability to consider a bundle of wires as either separate wires, a vector, or a single integer greatly simplifies behavioural descriptions, especially for complicated devices at high levels of abstraction. The ability to define types may also be important at lower levels of abstraction, for example when describing tri-state devices.

Subjectively, descriptions in this language seem to be highly readable (again, a result of the use of programming-language-like syntax) but rather verbose. Both
these characteristics probably result from the desire of the language designers to provide a language for documentation purposes.

A final comment about this language regards the type of validation which it is intended to support. It is fully intended to be used as an input language for multi-level simulators, but it does not have the simple, well-defined semantics which would make it suitable for formal reasoning about behaviour and thus for verification. This is a common feature of HDLs: that the syntax is defined first, followed by attempts being made to attach semantic meaning to the syntax. This contrasts with CIRCAL and HOL[Gordon85], which begin with a framework supporting formal reasoning and then attempt to apply this to the description of hardware. These different approaches will be discussed in Sections 2.3 and 2.2.3 respectively.

2.2.2 LTS

A language which provides a sharp contrast to VHDL is LTS (Layout and Timing for Structures)[Babiker83,Babiker85]. The language was designed by a fairly small team with the aim of supporting a design transformation methodology[Milne86b], depending on correctness-preserving transformations such as those described in Section 4.2. Like VHDL, it uses a programming language-like syntax to enable the description of dependencies of outputs on inputs, although in this case the language is a functional one, similar to ML [Harper86]. This leads to a language which looks very different from VHDL.

Structural Description

The functional style of LTS leads to a very straightforward way of describing structure. Since all devices are described by functions which map inputs to outputs, the internal structure of a device can be represented by the appropriate composition of the functions which represent its component parts. The full adder of Figure 2-1 is again used to illustrate the use of the language.
Example

fulladder(X,Y,Cin) = (Sum,Cout)
where
  (a,b) = halfadder(X,Y),
  (c,Sum) = halfadder(b,Cin),
  Cout = or(a,c)
end

Explanation

This part description is very similar to a definition of a ML function, where the arguments of each function represent its inputs and the returned results correspond to outputs. Internal signals are represented by local value bindings, as in (a,b) = halfadder(X,Y) which labels the outputs of one of the half adders as a and b.

This type of description is quite intuitive and fairly succinct. It does not convey quite as much information as the equivalent description in VHDL, as the type information for the ports is not stated explicitly. Since the type of a port is essentially a behavioural rather than a structural feature, this is not unreasonable. Type checking can still be performed, as the type of any port can be inferred from the behavioural definition of the component to which it is attached. The description also contains none of the other redundant information that was present in the VHDL description of the same device.

An unusual feature of LTS is that it allows quite detailed geometrical information, such as the size of cells and the positioning of ports, to be attached to device descriptions. This facility is not, however, relevant to the discussion here, so these features will not be described in detail.

Behavioural Description

The basic entities for behavioural description in LTS are signals which map time to values. A behavioural description of a device is simply a function which maps
input signals to output signals. In order to describe timing phenomena, a single built-in function is required. This function is \( \text{last}(x) \) which returns a signal whose value at any point in time equals the value of the signal \( x \) at the previous instant in time. The concept of a previous instant implies that time is viewed as progressing in discrete steps rather than continuously. This approximation is frequently made in hardware description and is generally acceptable as long as the steps are sufficiently small for the application in question.

Functions defined in LTS need not always represent pieces of hardware. For example, it is useful to define more sophisticated timing functions in terms of the primitive \( \text{last} \). A couple of examples of useful functions are:

\[
\text{delay}(n)(x) = \\
\text{if } n \text{ is} \\
0 \text{ then } x, \\
\text{as } m:1..? \text{ then } \text{delay}(m-1)(\text{last}(x)) \\
\text{end}
\]

\[
\text{rise}(\text{clock}) = \\
\text{and}(\text{clock}, \text{not}(\text{last}(\text{clock})))
\]

**Explanation**

The first of these functions defines a delay of length \( n \). If \( n \) equals zero, then the output of the function simply equals the input; otherwise, the input signal is delayed by one unit using \( \text{last}() \) and the resulting signal is delayed by \( n-1 \) units by calling \( \text{delay} \) recursively. The second function is true if the clock has risen at the most recent instant. These functions are used in the next example.

**Example**

The device described below is a rising-edge-triggered D-type latch.
dlatch(data,clock) = q

where

q = if (rise(clock)) is
true then delay(5)(data),
false then last(q)
end
end

Explanation

The behaviour of the latch is defined as a function whose arguments are the input signals data and clock and which returns the output signal q. The behaviour of the output is defined so that if there is a rising edge on clock then the output will take the value which was held by the data port 5 time units previously. In any other circumstance, the output retains the value it held at the previous instant in time.

LTS provides access to the datatype definition facilities of ML, which are very powerful. This facilitates the writing of comprehensible and consistent descriptions and is particularly useful at very abstract levels. It also supports the use of ranges and arrays to assist in the description of the regular structures which are common in VLSI design.

Comments

In some ways LTS provides very similar facilities for the description of behaviour to those of VHDL. Whereas the latter relies on imperative-style processes to define behaviours, LTS uses a functional approach. Both languages provide a tool for tackling complexity at abstract levels of description with powerful type definition capabilities.

A main difference between the languages is the treatment of timing. The discrete-time, backward-looking model of LTS relies on a single primitive function, last(x), which can be used as a basis for the definition of any number of more
complex timing functions. It is noteworthy that the LTS model of behaviour makes no reference to changes of port values (although functions to detect them, such as \textit{rise}, can be readily defined), instead permitting behavioural description in terms of a history of values.

One appealing characteristic of the language is that it seems to have been designed with the aim of achieving the desired degree of descriptiveness with the minimum number of primitive constructs. This perhaps reflects the fact that it was designed for use with a restricted set of tools, rather than aiming to serve a very large community of users with widely varying requirements. The result is a fairly intuitive and easily comprehended language, which is nevertheless very powerful. This contrasts with VHDL, which provides a very large, or even unwieldy, set of constructs with little if any gain in descriptive power.

2.2.3 Higher Order Logic

Another approach to hardware description is to try to use an existing descriptive framework rather than to create a new one. This approach has been taken by a number of researchers using higher-order logic to describe and reason about hardware, and offers the advantage of access to the reasoning capabilities of that framework. The behaviour of a device is represented by a predicate whose arguments, representing the values on the ports of the device, are, in general, functions of time. This predicate defines the relationships which must exist between the current and past values on the ports of the device. As in LTS, time is assumed to proceed in discrete steps. This ability to write predicates whose arguments are functions of time is a key attraction of higher-order logic over first-order logic for the purpose of hardware description. For example, a nand-gate with a delay of one time step would be represented by the following predicate:

\[ \text{NAND}(i_1, i_2, o) \equiv \forall t. o(t + 1) = \neg(i_1(t) \land i_2(t)) \]

Readers unfamiliar with the notation of predicate calculus are referred to standard texts on logic, such as [Hatcher82], or the numerous papers on hardware description using higher-order logic [Gordon85,Gordon86,Hanna85].
In general, the behaviour of pieces of hardware can be described as predicates making use of the standard notation of predicate logic using conjunction, disjunction, negation, implication, etc. The following example shows how this notation can be used to describe the behaviour of a multiplier which takes an unspecified length of time to produce a result and raises the signal `done` when it has finished.

The specification begins with the definition of some useful temporal predicates.

Example

\[
\begin{align*}
\text{Stable}(t_1, t_2)(f) & \equiv \forall t. t_1 \leq t \land t \leq t_2 \implies (f(t) = f(t_1)) \\
\text{Next}(t_1, t_2)(f) & \equiv t_1 \leq t_2 \land f(t_2) \land (\forall t. t_1 \leq t \land t \leq t_2 \implies \neg f(t)) \\
\text{Mult}(i_1, i_2, o, done) & \equiv \text{done}(t_1) \land \\
& \quad \text{Next}(t_1, t_2)(done) \land \\
& \quad \text{Stable}(t_1, t_2)(i_1) \land \\
& \quad \text{Stable}(t_1, t_2)(i_2) \\
& \quad \implies \\
& \quad (o(t_2) = i_1(t_1) \times i_2(t_2))
\end{align*}
\]

Explanation

`Stable` is true if \( f \) is constant between times \( t_1 \) and \( t_2 \). `Next` is true if \( t_2 \) is the first time after \( t_1 \) at which \( f(t_2) \) is true. Thus the meaning of the specification of the multiplier is that if the signal `done` is high at some time \( t_1 \) and subsequently at \( t_2 \), and the inputs \( i_1 \) and \( i_2 \) are stable during the interval, then the output at the end of the interval should equal the product of the inputs at the start.

Structural Description

The representation of structure in higher-order logic is quite straightforward. A device is constructed of several component parts. Each of these parts is described
by a predicate which defines the relationship which must hold between the values on the ports of each part. The behaviour of the whole device must therefore be such that all of these predicates hold, i.e. it is the conjunction of the predicates of the component parts. This may be considered a structural description of the device: it tells how it is constructed. There may be some ports which are to be considered internal to the device. That is to say, the signals on these ports will not be communicated outside the device. In this case, the relationship between values on the external ports is all that matters, and this is represented by a predicate which is true for some values on the internal ports. The lack of a definite statement about the values on the internal ports is achieved by existential quantification. Thus, the structure of the full adder circuit of Figure 2-1 would be described as follows:

\[
\text{Fulladder}(x, y, \text{cin}, \text{sum}, \text{cout}) = 3abc.\text{Halfadder}(x, y, a, b) \land \\
\text{Halfadder}(b, \text{cin}, c, \text{sum}) \land \\
\text{Or}(a, c, \text{cout})
\]

An important point to note here is that this 'structural description' is really more than that — it is a constructed predicate which fully describes the behaviour of the whole device in terms of the behaviour of its component parts. This distinguishes higher-order logic from the other descriptive frameworks seen previously, in which the behaviour of a constructed device could only be established by simulation. Because higher-order logic allows behaviours to be constructed in this way it is a suitable framework for formal verification. This issue will be addressed in Section 5.2.6.

Comments

It should be pointed out that the application of higher-order logic to hardware has been undertaken by a number of people and the exact approach taken varies. The methods illustrated above are modelled on those of Gordon, whose approach has been adopted by a number of other researchers. Hanna and Daeche [Hanna85] have taken a somewhat different approach, particularly suited to fairly low levels
of abstraction, based on the idea of partial waveform specifications. The model of timing adopted in Gordon's approach is essentially the same as that of LTS, i.e. a backward-looking discrete time model. In a similar manner to that seen in LTS, predicates which describe complicated timing phenomena can be defined as required using the primitive constructs of the language.

Two characteristics of higher-order logic distinguish it from the majority of hardware description languages. The first is that it is not a purpose-built language, but a pre-existing framework which has been found to be suitable for the description of hardware behaviour and structure. A potential disadvantage of this approach is that the description of real hardware in this framework may be less intuitive than it is in the specially designed languages. However, the principal advantage of using logic to describe hardware is that the 'structural' operators, which describe the wiring up of parts to form a larger device, have a well-defined behavioural meaning, thus allowing the behaviour of the larger part to be established given the behaviours of its components. This is essential to formal verification, the subject of Section 5.2. Furthermore, by using the existing framework of logic, the rules of inference of that framework are immediately available for reasoning about hardware behaviour.

2.2.4 Others Languages

It would be an exhausting task to attempt to cover all the current Hardware Description Languages in any detail. Even to catalogue them would be almost impossible: the bibliography of HDLs compiled in 1983 [Nash84] contained over 100 references, and the number of languages has dramatically increased since that time. Furthermore, many of the principles of hardware description have already been made clear by the small but diverse selection of languages described in the preceding sections. This Section will therefore provide just a very brief introduction to a few of the more important languages which are currently in use.
ELLA

Like VHDL, ELLA[Morison85,Morison86] has been developed with the aim of serving a large community of users in a wide variety of applications. It also has some similarity to LTS, in that it is based on a strongly-typed, functional programming language. ELLA's model of time is similar to the sequence of discrete instants used in LTS, but a different set of primitive timing constructs is provided. These enable the explicit description of a number of types of delay phenomena which are common in hardware, and support the specification of ambiguous delay behaviour. An additional primitive is provided for the description of random access-memory; the aim of this appears to be to increase simulation efficiency. It is noteworthy that the design of the language has been influenced by the implementation of the simulator which it is intended to drive.

µFP

Like LTS[Babiker83], µFP[Sheeran83] is based on a functional programming language (in this case, FP[Backus78]) and hardware is described by functions over sequences of values. By contrast with LTS, however, the semantics of µFP have been formally defined so that correctness proofs may be carried out. An area in which µFP has been successfully applied is in reasoning about highly regular structures, which constitute an important class of VLSI circuits. In particular, µFP has been used to support correctness-preserving transformations, the subject of Section 4.2. In subsequent work by Sheeran[Sheeran86,Sheeran88], binary relations between inputs and outputs have taken the place of functions.

CSP

Communicating Sequential Processes (CSP) [Hoare78] is, like CIRCAL and CCS [Milner83], a formal, mathematical framework designed to support reasoning about concurrent systems, of which hardware may be considered a subset. An example of the application of this framework to the problem of designing hardware appears in
Many of the concepts of the language are similar to those of CIRCAL, which is described in the following Section.

CONLAN

The name of this language [Piloty82, Piloty85] comes from 'Consensus Language' which is indicative of the designers' intentions to provide a very general descriptive framework. Rather than defining a single language, they have defined the basis for the construction of a family of new languages from a common starting point, which is Base Con Lan (BCL). Such languages may be set up using CONLAN constructs depending on the application, tool, or level of abstraction for which the language is required. An example of such a derived language is WISLAN [Vaidya83b, Vaidya83a] which was used as the input language for a gate-array design system. CONLAN languages have also been used as a front-end for formal verification [Eveking85b].

Zeus

The language Zeus [Lieberherr83, Lieberherr84, German85] was designed with the goals of encouraging a systematic approach to design and of providing a suitable medium for input to design tools, particularly silicon compilers. It is based on the principles of procedural programming languages, in particular Modula-2 [Wirth82], of which it is an extension. Behavioural descriptions are written in that language, so there is no convenient way to specify timing information. The language's intended use as input to a compiler is evident from the features which enable behavioural concepts to be associated with pieces of hardware (e.g. states with registers, logical functions with gates, etc.).

In addition to the above languages, a number of existing frameworks which support formal reasoning have been applied to the description of hardware. HOL, described above, is an example of such an approach. A number of other frameworks will be discussed under the topic of verification in Section 5.2.6.
2.3 CIRCAL

One language has been used extensively in the research leading up to this thesis and will be the subject of much of the following discussion, thus warranting a more detailed examination than those presented above. This language is CIRCAL [Milne83a] which was developed by Milne following his work with Milner on concurrent processes [Milne79] and his subsequent work on the Dot Calculus [Milne80]. It bears some similarities to Milner's CCS [Milner80] and SCCS [Milner83], which also evolved from Milner and Milne's earlier work, and also to CSP [Hoare78]. All of these are frameworks which have been developed with the aim of enabling reasoning about the behaviour of concurrent systems, and are based on the idea of processes communicating by a handshaking mechanism. The distinguishing characteristic of CIRCAL among other calculi is that its features have been defined with the specific aim of providing the ability to describe the type of behaviour exhibited by hardware. These features, described in detail below, can be seen to reflect the nature of hardware behaviour as discussed at the start of this Chapter.

2.3.1 Behavioural Operators

In CIRCAL the behaviour of devices can be described by assigning expressions to them. This is done using the definition operator '\(', as in

\[
\text{GATE1} \leftarrow \left( \text{a CIRCAL expression} \right)
\]

The physical ports of a device are the pins or wires by which it may communicate with other devices. As will be seen below, the ports used in CIRCAL expressions are not always identical to the physical ports of the device. The set of ports to which the expression may refer is called the sort. When ports are referred to in the text, they will be identified by an italic typeface.
Events and Guarding

One of the key concepts of hardware behaviour is the notion of changes on ports. These are sometimes referred to as events (hence the event-driven model of VHDL). In CIRCAL events are an essential part of behavioural description. However, although events can be interpreted as changes on ports, this need not always be the case. Some other interpretations are discussed below. The immediately following discussion will make no assumptions about the physical interpretation that might be placed on an event. In general, an event is identified by the port on which it takes place. To differentiate between events and the ports on which they occur, events will be written in a typewriter font, and ports in italics.

A number of events may take place simultaneously. This is represented in CIRCAL by a set of events, which is called a guard. For example, if a device has two ports called \texttt{ina} and \texttt{inb}, and an event takes place on each of these ports at one time, the guard would be represented as

\[ \{ \texttt{ina, inb} \} \]

The \emph{guarding operator} in CIRCAL prefixes a guard to a behaviour. If, for example, \texttt{GATE1} is a behaviour, then the following is also a behaviour:

\[ \{ \texttt{ina, inb} \} \texttt{GATE1} \]

Behaviours in CIRCAL are most easily interpreted in terms of what will happen if the device's environment (i.e. those components to which it is connected, as pictured in Figure 2-2) tries to perform a certain event or set of events.

The interpretation of the above behaviour is therefore as follows: if the environment performs the events \texttt{ina} and \texttt{inb} simultaneously, they will take place and the subsequent behaviour of the device will be that of \texttt{GATE1}. The behaviour of \texttt{GATE1} will be defined by some other expression using the operators of CIRCAL.
Deterministic Choice

CIRCAL has two choice operators. The more commonly used one is the deterministic choice operator \( \cdot \) \( \cdot \). It describes alternative behaviours of a device which may be chosen by events occurring in its environment. For example, the behaviour described by the following expression

\[
\{ \text{ina}, \text{inb} \} \text{GATE}_1 + \{ \text{inc} \} \text{GATE}_0
\]

would be interpreted as follows: if the environment performs the two events \( \text{ina} \) and \( \text{inb} \), then the subsequent behaviour of the device will be that of \( \text{GATE}_1 \); if the environment performs the event \( \text{inc} \) then the subsequent behaviour of the device will be that of \( \text{GATE}_0 \). The operands of the choice operator are sometimes referred to as branches, reflecting the tree-like organisation of behavioural descriptions.

Nondeterministic Choice

This operator is not often required for the writing of specifications, but may be necessary to describe the behaviour of a constructed device. It is usually represented by \( \oplus \), but in order to facilitate its use in ordinary text, will be represented here
by $\theta$. Like its deterministic counterpart, it provides alternatives in a behaviour, but the difference is that the choice between alternatives is made by some unseen action within the device being described, and is therefore beyond the control of the environment. Thus, for example, if a device is described by the following expression:

$$B_1 \oplus B_2$$

where $B_1$ and $B_2$ are behaviours, then this means that the device can behave either as $B_1$ or $B_2$, and the environment has no way of determining which one beforehand. Thus in the following behaviour

$$(\text{ina, inb})\text{GATE}_1 \oplus (\text{inc})\text{GATE}_0$$

an attempt by the environment to perform the event inc may or may not succeed; the 'decision' is made within the device, unseen to the environment.

Deadlock

One special behaviour may be required for behavioural description. This behaviour, called the deadlock operator, is represented by $\Delta$, or $\wedge$ in ordinary text. It has a sort associated with it, and its behaviour is such that no events can take place on any port in its sort.

2.3.2 Describing Hardware Behaviour in Circal

With the operators presented above it is possible to describe the behaviour of some common pieces of hardware. First of all it is necessary to assign a physical meaning to events. One possible meaning has been mentioned already, a change of value on a port. Suppose there is a physical port called $j$ on a device, and that the port can take either of the values 1 or 0. This could be modelled as a pair of imaginary ports, called $j1$ and $j0$. Then the event $j1$ would be interpreted as 'the
value on the port \( j \) changes to 1. The following example shows how this approach can be used to describe a delayless inverter.

**Example**

\[
\begin{align*}
\text{INV0} & \Leftarrow \{\text{in0, out1}\}\text{INV1} \\
\text{INV1} & \Leftarrow \{\text{in1, out0}\}\text{INV0}
\end{align*}
\]

**Explanation**

The two lines of this description can be thought of as representing two states of the inverter. The first line describes the state in which the output holds the value 0. From this state, the device can move into state \( \text{INV1} \) (sometimes referred to as an *end state*) by performing (simultaneously) the events \( \text{in0} \) and \( \text{out1} \). The second line is the symmetric description for state \( \text{INV1} \). Note that there is no causality or directionality implied by this description. It simply states that an \( \text{in0} \) event can only happen if accompanied by an \( \text{out1} \) event. This ensures that the output of the inverter, the physical port \( \text{out} \), always holds the value which is the complement of that on the input, \( \text{in} \). The ability to describe simultaneous events seen in this example is a distinctive feature of CIRCAL, contrasting for example with CCS [Milner80].

This type of description is closely related to that used for finite state automata. The relationship between CIRCAL and finite state machine notation is described in [Davie86]. Figure 2-3 illustrates the finite state machine representation of the above inverter.

![Finite State Diagram of Inverter](image)

**Figure 2–3:** Finite State Diagram of Inverter

35
Delays

It is easy in CIRCAL to write descriptions in which output events follow input events by some amount of time, as the following example illustrates.

Example

The above description of an inverter can readily be modified to include an unspecified delay between input and output events. One way to do this is as follows:

\[
\begin{align*}
\text{INV01} & \iff (\text{in1})\text{INV11} \\
\text{INV11} & \iff (\text{out0})\text{INV10} + (\text{in0})\text{INV01} + (\text{in0}, \text{out0})\text{INV00} \\
\text{INV10} & \iff (\text{in0})\text{INV00} \\
\text{INV00} & \iff (\text{out1})\text{INV01} + (\text{in1})\text{INV10} + (\text{in1}, \text{out1})\text{INV11}
\end{align*}
\]

Explanation

The two digits at the end of each state name represent the values on the inverter's input and output ports. \text{INV01} and \text{INV10} are therefore the stable states; no output change is pending, so the only possibility is that a new value will be presented on the port in. In the other two states, there are three possibilities: the output will change, leading to a new stable state; the input change will be reversed, leading back to the original stable state; or both changes will happen at once, leaving the device in the other unstable state. Note that this description makes no statement about how long the delay between input and output events is, only that the complemented input value will appear on the output some time after the input event. Such descriptions are of limited usefulness, as will be demonstrated in Section 5.1.4.

Measuring Time

In order to introduce delays of specified length it is necessary to have some way of measuring time. The standard way of doing this, as described for example in
[Traub86], is to introduce a universal clock by which the passage of time can be measured. This is an abstract device (i.e. it would not be a part of the physical circuit) which simply generates a continuous stream of ticks. These are represented by the event \( \{ t \} \). The time between the ticks is assumed to be constant and would correspond to some physical length of time e.g. 5 nanoseconds. All timing properties are then measured in terms of this basic unit of time. Note that this approach bears some relationship to that taken in LTS, in which the concept of a sequence of 'instants' was central to the description of timing phenomena.

The convention normally adopted when writing timed CIRCAL descriptions is to assume that events can only take place simultaneously with a 'tick' event. Even though this may not be the case physically, it is a reasonable assumption for modelling purposes providing the time between ticks is sufficiently small. This approach is illustrated in the following description of an inverter with a unit delay (i.e. a delay corresponding to the time between two successive ticks).

**Example**

\[
\begin{align*}
\text{INV01} &\Leftarrow \{ \text{in1, t}\} \text{INV11} + \{ \text{t}\} \text{INV01} \\
\text{INV11} &\Leftarrow \{ \text{out0, t}\} \text{INV10} + \{ \text{in0, out0, t}\} \text{INV00} \\
\text{INV10} &\Leftarrow \{ \text{in0, t}\} \text{INV00} + \{ \text{t}\} \text{INV10} \\
\text{INV00} &\Leftarrow \{ \text{out1, t}\} \text{INV01} + \{ \text{in1, out1, t}\} \text{INV11}
\end{align*}
\]

**Explanation**

As before, the stable states are denoted by \( \text{INV01} \) and \( \text{INV10} \). In these states, the second term in the choice sum indicates the possibility of a tick happening without any accompanying physical event. In the unstable states, the pending output event is effectively forced to happen on the next tick, either before or at the same time as the next input event. This contrasts with the untimed description above in which there is no guarantee that the pending output event will ever occur.

The above examples serve to demonstrate very briefly the principles of behavioural description with CIRCAL for very simple devices. The techniques used
and problems encountered in describing more complicated devices are the subject of much of Chapter 3. Before that, however, the structural operators of the language will be described.

### 2.3.3 Structural Operators

**Composition**

This is the most important of the structural operators in CIRCAL, being central to the use of the calculus to reason about the behaviour of constructed devices. The composition of two devices corresponds to their being wired together, with the convention being that ports of the same name are connected together. Thus, for example, the two devices depicted in Figure 2-4 would be described simply as

\[ \text{DEVA} \ast \text{DEVB} \]

The symbol \( \ast \) is the composition operator, often written as \( \cdot \). The sort of DEVA is \( \{a, b, c\} \), the sort of DEVB is \( \{b, c, d\} \), so that the composition operator joins together ports \( b \) and \( c \) leaving \( a \) and \( d \) unconnected. The sort of the constructed device is simply the union of the two sorts, \( \{a, b, c, d\} \).

![Figure 2-4: Two devices wired together](image)

The 'wiring up' operation can readily be extended to more than two devices. In contrast to CCS[Milner80] there is no restriction on the number of devices which may be connected to one port, so the composition of several devices connects them together in just the way that one would intuitively expect.
One of the significant features of CIRCAL is that, like higher-order logic, the structural operators have a behavioural meaning thus allowing the behaviour of a constructed device to be established mathematically, rather than, say, by simulation. The formal definition of the composition operator is as follows:

Given two devices $A$ and $B$, with sorts $M$ and $N$ respectively, with behaviours defined as a deterministic choice of guarded terms

\[
A \Leftarrow \sum a_i A_i; \quad B \Leftarrow \sum b_j B_j,
\]

where $a_i$ and $b_j$ are guards and $A_i$ and $B_j$ are behavioural expressions, then the behaviour of the device obtained by wiring them together is defined by the composition of the two behaviours as follows:

\[
A*B \Leftarrow \sum_{a_i \cap M \neq \emptyset} a_i(A_i*B) + \sum_{b_j \cap N \neq \emptyset} b_j(A*B_j) + \sum_{(b_j \cap M) = (a_i \cap N) \neq \emptyset} (a_i \cup b_j)(A_i*B_j) + \sum_{(b_j \cap M) = (a_i \cap N) = \emptyset} (a_i \cup b_j)(A_i*B_j)
\]

The four lines which make up this definition can be readily explained in an intuitive way. The first line represents those parts of the resultant behaviour which arise from guards involving ports which are only in the sort of $A$. If such a set of events takes place, then $A$ will evolve into the state $A_i$, but $B$ remains in its original state as its ports were not involved. The second line is the symmetric case for events involving ports which are only in the sort of $B$.

The third line is really of most interest, as it describes the interaction between the two devices when events occur on ports which are common to them both. These synchronising events can only take place if the ports of the guard $a_i$ which are in the sort of $B$ and the ports of $b_j$ which are in the sort of $A$ are exactly the same. The resultant guard then is the union of the two guards, and the new state will be the composition of $A_i$ and $B_j$. Supposing the sort of $A$ is, as before,
{a, b, c} and that of B is {b, c, d}. Then, the following synchronisations are possible:

{a, b} with {b}
{a, b} with {b, d}
{b, c} with {b, c, d}

while the following are not:

{a, b} with {c}
{a, c} with {b, d}
{b, c} with {c}

The final case to be considered is the simultaneous occurrence of events which involve ports only in the sort of A with events which involve only the sort of B. These are not synchronising events, but independent events which happen to occur simultaneously, and it is these which are covered by the last sum in the above definition. The only possibility for such events which exists in the above example is the simultaneous occurrence of {a} and {d}.

Relabelling

In order to describe structure fully, some further operators are required. One of these is the relabelling operator, which simply replaces occurrences of a certain port name with a new name. The syntax for relabelling a port called old to new for a device called DEV is

DEV [new/old]

This enables the convention of wiring up ports of the same name to be followed. For example, an inverter may be defined to have ports in and out, and it may be required to connect two such inverters in series as in Figure 2-5. Without relabelling, the wiring up operation would connect them in parallel, joining together
the two inputs and the two outputs. To connect them in series, the output of one inverter and the input of the other are relabelled to a common name, and the desired connection is achieved as follows:

\[ \text{INV[mid/out]} \times \text{INV[mid/in]} \]

![Figure 2-5: Two inverters wired in series](image)

The behavioural meaning of the relabelling operator is quite simple. Any occurrences of a port name designated for relabelling in a behavioural definition are replaced by the new name. In the case where a number of abstract ports are used to represent a single physical port, as was done in the inverter description above, the relabelling can be applied to the physical port name only as a convenient shorthand. For example

\[ \text{INV11[mid/out]} \]

would have the following behaviour:

\[ \text{INV11} \leftarrow \{\text{mid}0, \, t\}\text{INV10} + \{\text{in}0, \, \text{mid}0, \, t\}\text{INV00} \]

**Abstraction**

It is sometimes required to internalise some of the ports of a constructed device. This removes some ports from the device's sort, making communication with the outside world through those ports impossible. The analogous situation in VHDL, for example, is to declare certain nodes as signals within an architectural body used to describe the structure of a component. In order to make ports \(b\) and \(c\) internal to the constructed device of Figure 2–4, the following CIRCAL expression would be used:
The sort of this constructed device would now be \{a,d\}.

In general, the effect of abstracting a port is to remove it from any guards in which it occurs in the behaviour of a device. However, it is possible to introduce non-deterministic behaviour through abstraction, as the following definition shows:

\[
\left[ \sum_{a_i A_i} - p \right] \equiv_{\Delta f} \left[ \sum_{p \in A_i \setminus \left\{ p \right\}} \left( a_i \setminus \{p\} \right)(A_i - p) + \sum_{p \notin A_i} a_i(A_i - p) + M \right] \oplus M
\]

where

\[
M \equiv \sum_{a_i \in \left\{ p \right\}} (A_i - p)
\]

The symbol \(\sum\) is a non-deterministic choice sum. The subtleties of the non-determinism introduced here are discussed in detail by Traub [Traub86]. It is sufficient to notice here that non-determinism arises when there are guards consisting of only a single event, on the port which is to be abstracted. These events may now take place unseen and uncontrolled by the device's environment, thus introducing non-determinism into its behaviour. In general, non-deterministic behaviour is something to be avoided. It should be noted that abstraction will not introduce non-determinism if it is not carried out on a port which appears by itself in a guard. In timed descriptions, described below, the only port in this category is the 'tick' port (conventionally \(t\)), and it is not usually necessary or appropriate to perform abstraction on that port. Under these circumstances, the definition of abstraction is simplified:

\[
\left[ \sum_{a_i A_i} - p \right] \equiv_{\Delta f} \sum_{p \in A_i \setminus \left\{ p \right\}} \left( a_i \setminus \{p\} \right)(A_i - p) + \sum_{p \notin A_i} a_i(A_i - p)
\]

The structural operators are really all that is needed for full structural description of any device. With composition and relabelling, it is possible to describe any desired wiring configuration for a number of devices. Abstraction provides a means
to 'black-box' a constructed device, by concealing internal details. All these operators have a behavioural meaning, which provides the means for establishing the behaviour of a constructed device in terms of the behaviours of its components, an essential first step in the formal verification process.

2.3.4 Limits of Pure CIRCAL

The operators described above form the core of CIRCAL. The language formed from these operators, which will be referred to as pure CIRCAL, can be seen to have severe limitations as a medium for the description of real hardware. A very slightly more complicated example than those seen so far serves to illustrate this.

Example

The following description is of a delayless, three-input nand-gate, with inputs a, b and c, and an output z, modelled on the assumption that port values may be either 0 or 1:

\[
\begin{align*}
nand000 &\leq \{a1\} nand100 + \{b1\} nand010 + \{c1\} nand001 \\
&\quad + \{a1, b1\} nand110 + \{a1, c1\} nand101 + \{c1, b1\} nand011 \\
&\quad + \{a1, b1, c1, z0\} nand111 \\
nand100 &\leq \{a0\} nand000 + \{b1\} nand110 + \{c1\} nand101 \\
&\quad + \{a0, b1\} nand010 + \{a0, c1\} nand001 + \{c1, b1, z0\} nand111 \\
&\quad + \{a0, b1, c1\} nand011 \\
nand010 &\leq \{a1\} nand110 + \{b0\} nand000 + \{c1\} nand011 \\
&\quad + \{a1, b0\} nand100 + \{a1, c1, z0\} nand111 + \{c1, b0\} nand001 \\
&\quad + \{a1, b0, c1\} nand101 \\
nand110 &\leq \{a0\} nand010 + \{b0\} nand100 + \{c1, z0\} nand111 \\
&\quad + \{a0, b0\} nand000 + \{a0, c1\} nand011 + \{c1, b0\} nand101 \\
&\quad + \{a0, b0, c1\} nand001 \\
nand001 &\leq \{a1\} nand101 + \{b1\} nand011 + \{c0\} nand000 \\
&\quad + \{a1, b1, z0\} nand111 + \{a1, c0\} nand100 + \{c0, b1\} nand010
\end{align*}
\]
The fundamental problem of pure CIRCAL which is illustrated by this example is the lack of any mechanism for handling variables. This means that the states of the device must be explicitly enumerated, leading to a lengthy description in this simple example, and to totally unworkable descriptions for more complex examples. There is no capacity to simplify the description by making use of its regular properties, such as the symmetry between the three inputs. Nor is there any facility to make use of the fact that this device has a very similar behaviour (in terms of the occurrence of input and output events) to many other three-input devices, distinguished only by the particular combinational logic function which it realises.

CIRCAL in its most basic form then is not really very useful as a practical design and description language. For the description of fairly simple devices, it is rather cumbersome; for devices of reasonable complexity the enumeration of states becomes totally impractical. In order to avoid these problems the basic language needs to be modified or enhanced in some rigorous way to cope with variables, while maintaining the attractive features of CIRCAL such as its support for formal reasoning. Some approaches to this enhancement are discussed in the next Chapter.

These shortcomings of pure CIRCAL, which are certainly surmountable, are not considered a good reason to avoid using it. One of its appealing qualities is that it provides just those features which were identified as being essential for a
behavioural description language at the start of this Chapter. This is in stark contrast to many other languages, which, in trying to satisfy a very broad range of criteria (e.g. those listed in Section 2.2.1) provide an unwieldy collection of constructs. The provision of a bare minimum of descriptive features in CIRCAL makes it a useful starting point for the development of a language to support a hierarchical design methodology. Furthermore, its simple semantics enable formal reasoning about hardware behaviour, a quality which makes it of particular interest in the validation phase.

2.4 Summary

This Chapter has begun to tackle the issue of hardware description in a hierarchical design and validation methodology. The formal description of behaviour and structure, at any level in the design hierarchy, is essential to such a methodology, regardless of the particular validation method which is to be used. It is this formal description which enables the comparison of specifications with implementations, whether by formal proof or simulation, to take place throughout the design process rather than afterwards. The most viable approach to the description of behaviour is with Hardware Description Languages (HDLs), which have been the subject of much recent attention. These languages are invariably also used for the description of structure, although graphical input of structural information may be desirable from a hardware designer's point of view.

The meaning of hardware behaviour was examined and the concepts identified here led to a set of required criteria for behavioural HDLs. Given the need to describe a variety of hardware over a wide range of levels of abstraction, the following features are required: the ability to describe the interdependency of values on ports; ways to describe various timing phenomena; and facilities to refer to changes occurring on ports.

A small but widely varied selection of languages which met these criteria was examined. VHDL is a language which has been designed to fulfill a great many
roles and this is reflected in the extensive range of features which have been incorporated in it during its development. The features of a procedural programming language enable the description of functions, and timing phenomena are described using a single construct which specifies the length of time between a change of an input and the resultant output change. Type definition capabilities provide some assistance in dealing with complexity at high levels of abstraction.

By contrast, LTS offers the features of a functional programming language much like ML and uses a backward-looking, discrete model of time. The values on ports are represented by signals, which map instants of time to values, and the behaviour of hardware devices is modelled by functions which map signals to signals. The one primitive timing function looks back to the previous instant of discrete time; more elaborate functions can be defined in terms of that one primitive.

Neither of these languages supports formal reasoning about hardware. Higher-order logic is a pre-existing framework which does support such reasoning and has been applied to the description of hardware by various people with considerable success. The use of predicates to represent hardware behaviour with a similar timing model to that of LTS was examined and this approach will be addressed again in the chapter on verification.

Finally CIRCAL, the language which will be most extensively used in this thesis, was introduced in some detail. This introduction was to the language in its most basic form, which just manages to meet the criteria established for behavioural languages. The well-defined semantics of CIRCAL enable it to be used for formal reasoning about the behaviour of constructed devices. However, its shortcomings for the description of non-trivial devices were illustrated; ways in which these may be overcome will be presented in the following Chapter.
Chapter 3

Specification

The main argument which has been made for the adoption of a hierarchical design and validation methodology is to enable validation to take place before the design is completed. It was shown in Section 1.2 that in order to do this formal specifications must be written at each level of abstraction. The specification task, therefore, is of central importance to such a methodology; without it, validation cannot be performed until the design reaches the lowest level of abstraction.

In the previous Chapter, some of the issues of behavioural description were addressed. From a discussion of the nature of hardware behaviour, a small set of criteria for behavioural description languages was established. In approaching the task of specification within a hierarchical design methodology, it is generally not sufficient to use a language which meets only those few criteria. For example, since specifications must be written of all devices at every level in the design hierarchy, it is important that the task of writing specifications be as easy and concise as possible.

It is also important that specifications be accurate, in two senses of the word. The first of these is that they must accurately reflect the real intentions of the designer. The attainment of this goal can never be guaranteed, as it depends on being able to read the mind of the designer. However, it can be assisted by language features which allow 'intuitive' descriptions and which save the designer from the need to specify those details which do not currently concern him.
The second sense in which specifications must be accurate is that they should as far as possible be an accurate description of real hardware. There are limits to the extent to which this goal can be met, since it is basically a requirement that all specifications be implementable. However, a language should definitely help rather than hinder the attainment of this goal.

The languages examined in the preceding Chapter did generally meet these added requirements for specification languages to some extent. The exception to this was CIRCAL which, in its basic form, offered no high-level aids to conciseness or intuitiveness such as function or datatype definition, and which yields cumbersome descriptions at even quite low-level descriptions of simple devices. The following Section contains a discussion of ways to enhance or build on CIRCAL to produce a suitable specification language.

The writing of concise, accurate specifications is not just a matter of using an appropriate language. There must also be certain specification techniques which will assist the designer to write specifications more easily and accurately. The details of these techniques will generally depend on the language in use. The central Section of this Chapter describes some of the techniques which have been developed for use with the enhanced CIRCAL language.

The Chapter concludes with some proposals for a language which has the dual aims of further reducing the cumbersomeness of writing specifications and of ensuring that the specifications are accurate (in the second of the two senses mentioned above). The second aim is achieved in the following way: descriptions in this 'high-level' language can be translated into the enhanced version of CIRCAL, while the features of the higher-level language serve to restrict the designer's access to CIRCAL in such a way that some of the specification techniques mentioned above are automatically enforced. It is intended that such a language could be used as a 'front-end' to a CIRCAL -based design system.

It should be emphasised that the specification of a device need not always precede its design. Thus, some of the behavioural characteristics which are described in this Chapter may not be often required in a design, but may nevertheless require description after a device has been designed. A standard cell in a library, for ex-
ample, might have quite complicated timing characteristics; if such a part is to be used in a design, it must be possible to describe fully those characteristics. These considerations have influenced the development of the specification languages and techniques.

3.1 Enhanced CIRCAL

3.1.1 Related Work

Numerous papers by Milne [Milne85a, Milne84, Milne88a] have added to pure CIRCAL. In the chapter of Traub's thesis [Traub86] introducing the CIRCAL language there is also a number of extensions to the core language. Some of the derived operators were introduced simply to ease the task of structural description, which is not a major concern in this work. Other, behavioural, operators, such as the 'Any Actions' operator, have not been found to be helpful in the experience of the author. Traub's treatment of conditionals has been found not to be sufficiently general, and so a somewhat different treatment has been presented below. More recently, Pezze has worked with Milne on the formalisation of some enhancements to CIRCAL [Pezze87, Milne88b]. The proposed language, called 'typed CIRCAL', has several features in common with the language presented below.

In the following Sections, enhancements to CIRCAL are proposed with emphasis on reducing the complexity of behavioural descriptions of real devices. The enhancements have been developed as a result of the author's attempts to describe significantly complicated pieces of hardware, such as the simple microprocessor [Gordon81a] which is described in Chapter 6. The features are introduced by examples and their meaning explained in terms of their relationship to the constructs of pure CIRCAL.
### 3.1.2 Parameterisation of States

The example of a 3 input nand gate in Section 2.3.4 illustrated the fundamental problem of pure CIRCAL, which is the exponential growth in the size of descriptions as the number of states of a device increases. In order to avoid lengthy state enumeration it is necessary to introduce some sort of parameterisation. This will lead to requirements for other features, as described below.

The nand gate description could clearly be shortened considerably if, instead of writing the behavioural descriptions of \texttt{nand000}, \texttt{nand100} etc. it was possible to write just the one parameterised state description i.e.

\[ \texttt{nand}(p,q,r) \leftarrow \ldots \]

This notation is just shorthand for the list of states which appears on the left of the nand gate description of Section 2.3. In order to map such a description back to pure CIRCAL it is necessary to replace the parameters with actual values 0 and 1. It is assumed that the right hand side will carry sufficient information for the type of the parameters to be inferred.

Having adopted this sort of notation to name states on the left hand side of behavioural expressions, it becomes necessary to have some similar way to name them on the right hand side. This could be done in a similar way to the invocation of functions with actual parameters in a programming language. These actual parameters might have literal values (0, 1, true, etc.), may simply be named parameters, or may be functions of other named parameters. Examples of the use of each of these types of parameter assignment appear below. The use of parameter assignment can be more clearly understood once the nature of events in this enhanced language is explained. This is done below.

### 3.1.3 Value Passing

In pure CIRCAL events were identified only by port names. In order to represent new values appearing on physical ports, imaginary ports were defined to correspond to the pairing of physical ports with values. If on a certain physical port,
say \( p \), a number of alternative events is possible, this would normally be represented as:

\[
\{p_{v1}\}S_1 + \{p_{v2}\}S_2 + \{p_{v3}\}S_3 + \ldots
\]

where the \( v_i \) are values and \( S_i \) are state names. A shorthand way of representing this would be

\[
(p^x)N(\text{some parameters})
\]

where \( x \) is a variable ranging over all the \( v_i \). This will only work if all the end states \( S_i \) are just different 'invocations' of one parameterised state \( N \). Fortunately this is often the case. Consider the following example which describes a device to input an integer, increment it, then output the incremented value at the next tick of the universal clock.

\[
\text{INPUT} \leftarrow \{\text{in}^\sim x, t\} \text{OUTPUT}(x+1) \\
\text{OUTPUT}(n) \leftarrow \{\text{out}^\sim n, t\} \text{INPUT}
\]

Note that the two uses of value passing in this example are different. In the first case, the variable \( x \) is bound at the occurrence of the event, and is then used to determine the parameter for the end state. In the second case, the variable is already bound in the start state and is simply presented on the port. Thus it is only in the first case that the shorthand for a choice of alternative events is really being used. In the second, the notation is shorthand for a list of state definitions, as in

\[
\text{OUTPUT}(0) \leftarrow \{\text{out}0, t\} \text{INPUT} \\
\text{OUTPUT}(1) \leftarrow \{\text{out}1, t\} \text{INPUT} \\
\text{OUTPUT}(2) \leftarrow \{\text{out}2, t\} \text{INPUT} \\
\text{etc.}
\]

This fundamental difference between the two types of value passing has important consequences for synchronisation, as will be seen below. In order to differentiate between the first case, in which a parameter is input, and the second where
it is output, different symbols are used. Arrowheads which indicate whether information is being transferred from port to parameter or vice versa were suggested by Traub[Traub86] and are used here. Thus the above example is now

\[
\text{INPUT} \leftarrow \{\text{in} > x, t\}\text{OUTPUT}(x+1) \\
\text{OUTPUT}(n) \leftarrow \{\text{out} < n, t\}\text{INPUT}
\]

Note also the use of a function (addition) to assign the parameter of OUTPUT when it appears as an end state. It may also be desirable to represent output values as functions of state parameters, as the following equivalent description shows:

\[
\text{INPUT} \leftarrow \{\text{in} > x, t\}\text{ADDONE}(x) \\
\text{ADDONE}(n) \leftarrow \{\text{out} < (n+1), t\}\text{INPUT}
\]

The appearance of a literal value on a port, which in pure CIRCAL was represented by events such as \(z1\) can be considered as a special case of the outputting of a parameter, in which the output 'function' returns a constant value. Such events are therefore represented in the same manner as outputs, i.e. \(z1\).

In summary, there are three basic categories of event. These are

- pulses. These are the basic events of pure CIRCAL and they are retained in the enhanced version mainly for the description of timing phenomena.

- inputs. Written as \(p > x\) where \(p\) is a port and \(x\) is a variable, these events represent a range of possible events and result in the binding of \(x\) to some value from which parameters of the end state can be derived.

- outputs. Written as \(p < y\) where \(y\) is in general a function of the parameters of the state whose behaviour is being defined. There are two special cases: the function is a constant, i.e. a literal value is passed, or the function is the identity, i.e. one of the state parameters is passed.
3.1.4 Conditionals

It is not difficult to see the need for conditionals among the enhancements to the language. Having condensed a behavioural description by parameterising the states, it may be required to assign quite different behaviours to certain states. Thus there is a need for a form of conditional which assigns behaviours to states if some predicate on the parameters of the state is satisfied. This is illustrated in the following example.

Example

A counter which counts from 0 up to 7 then resets to 0 can be described using conditionals as follows:

\[
\text{COUNTER}(x) \leftarrow \begin{cases} 
\text{if eqs}(x,7) \text{ then } \{\text{clk}, \text{out}<0\}\text{COUNTER}(0) \\
+ \text{if noteq}(x,7) \text{ then } \{\text{clk}, \text{out}<x+1\}\text{COUNTER}(x+1) 
\end{cases}
\]

Explanation

eqs() is a function which returns true if its arguments are equal; noteq() returns true if they are unequal. The interpretation of the description is that the behaviour of COUNTER for any given state parameters is given by the choice of all branches for which the predicate evaluates to true. In this case, only one branch can evaluate to true for a single value of \(x\), but in general there will be several. If for a certain set of state parameters the predicate evaluates to false, then the corresponding branch is removed from the choice sum. Thus, for example, if \(x\) is equal to 5, then the behaviour of the device is simply

\[
\{\text{clk}, \text{out}<6\}\text{COUNTER}(6)
\]

Although in this example an if...then...else construct would be slightly more concise than the if...then construct, the latter was chosen as being more flexible, since there are some situations in which it is not required to specify what happens when the predicate is false. When, as in this example, this is required,
it must be done explicitly by using the negated predicate. This is a result of the aim of avoiding the addition of more constructs than are absolutely necessary. For conciseness, a number of branches may be introduced by a single predicate. That is

\[
\text{if } p \text{ then } (b_1 + b_2 + b_3)
\]

is equivalent to

\[
\text{if } p \text{ then } b_1 \\
+ \text{if } p \text{ then } b_2 \\
+ \text{if } p \text{ then } b_3
\]

where \( p \) is some predicate and \( b_1, b_2 \) and \( b_3 \) are branches.

There is a further requirement for predicates, when a parameter is input. When the notation for value passing was introduced, it was mentioned that, in an event written as \( p\!x \), \( x \) is a variable which can range over some set of values. So far this set has been assumed to be the full range of a variable of the type of \( x \). There are many cases where this assumption is not valid, so there needs to be some way of defining a more restricted set. This is done by attaching a predicate to the parameter. Since this is an assertion to determine the range of possible passed values, rather than a conditional to determine valid branches in the behaviour, a new syntax is adopted:

\[
\text{portname > variable : predicate}
\]

The passed value may now be any value of the appropriate type for which the predicate evaluates to true. The following example, a two-input and gate, illustrates how this is used.
Example

\[ \text{AND}(p,q) = \{ a > x : \text{eqs}(\text{and}(x,q), \text{and}(p,q)) \} \text{AND}(x,q) \]

+ \{ b > x : \text{eqs}(\text{and}(p,x), \text{and}(p,q)) \} \text{AND}(p,x) 
+ \{ a > x, b > y : \text{eqs}(\text{and}(x,y), \text{and}(p,q)) \} \text{AND}(x,y) 
+ \{ a > x : \text{noteq}(\text{and}(x,q), \text{and}(p,q)) \} \text{AND}(x,q) 
+ \{ b > x : \text{noteq}(\text{and}(p,x), \text{and}(p,q)) \} \text{AND}(p,x) 
+ \{ a > x, b > y : \text{noteq}(\text{and}(x,y), \text{and}(p,q)) \} \text{AND}(x,y) 

Explanation

In each of the first three branches, the predicates on input parameters ensure that any value which is input will be such that the value on the output does not need to change. The last three branches contain predicates which ensure that any new input value will be such that a new output value is required, and so the guard also contains an event on the output port \( z \). The reason for writing a description in this way is to prevent the generation of 'fictitious' events, a problem which is described in more detail in Section 3.2.1.

The reader may consider that, in terms of complexity, this description is not a great step forward from the pure CirCal description of a nand gate in Section 2.3. This is true, but is a consequence of the fact that this is a simple device with only boolean ports. The reductions of length of description are much more noticeable for more complex devices. Some of the more sophisticated devices which are described below would be virtually impossible to describe without the notation just presented. In particular, the large example of Chapter 6 illustrates the savings in effort which enhanced CirCal offers for the description of complicated circuits.

3.1.5 Functions and Types

In the foregoing discussion, three situations which require the use of arbitrary functions have been seen. These are:

1. to assign parameters in end states;
2. to describe output values in terms of state and input parameters;

3. to define predicates for conditionals.

Some framework for the definition of such functions is clearly required.

It is also necessary to have some means of defining types. The examples chosen so far have only made use of boolean and integer ports and state parameters, but more elaborate types are essential for the definition of more complex devices. As a simple example, for the counter described in Section 3.1.4, it might be helpful to define a type to represent the subrange of integers from 0 to 7, and to define an increment operation on such integers. Or, to describe a memory, it is necessary to have a parameter to represent the state of the memory. One way of doing this would be to define a type mem which would support read and write operations and use a variable of this type for the state parameter.

So, in addition to the features for hardware description developed in this Section, there is a clear need for the common programming language features of function and type definition. Where such features are required in this thesis, the language Standard ML[Harper86] will be used, as it provides the necessary power and the language is sufficiently intuitive to be fairly understandable even to those not familiar with it.

The features described above will be used for examples throughout the remainder of this thesis. A formal specification of the syntax of the enhanced language appears in Appendix B. In the following Section, the use of this language in a more complex example is demonstrated.

### 3.1.6 An Example

The following example is of an 8 bit counter which counts either up or down depending on the polarity of an input called up. The device is positive edge-triggered i.e. it counts when a rising edge occurs on the clock line (the port clk). The first step is to write ML function definitions for use in the CIRCAL behavioural description.
fun noteq(x,y) = if x = y then false else true;
fun not(x) = if x then false else true;
fun and(a,b) = if a then b else false;
fun incr(x) = if x = 255 then 0 else x+1;
fun decr(x) = if x = 0 then 255 else x-1;

The first function returns true if its arguments are not equal. The next two are standard boolean functions, and the last two are increment and decrement functions for integers in the range 0 to 255 (i.e. those which can be represented by 8 bits). The CIRCAL description is now:

\[
\text{UDCOUNT}(x,c,u) ::= \begin{cases} 
\text{if and}(u,\neg(c)) \text{ then} & 
\{\text{clk}\text{true}, \text{out}\text{incr}(x)\}\text{UDCOUNT}(\text{incr}(x),\text{true},u) \\
\text{if and}(\neg(u),\neg(c)) \text{ then} & 
\{\text{clk}\text{true}, \text{out}\text{decr}(x)\}\text{UDCOUNT}(\text{decr}(x),\text{true},u) \\
\text{if c then} & 
\{\text{clk}\text{false}\}\text{UDCOUNT}(x,\text{false},u) \\
\{\text{up}>z:\text{noteq}(z,u)\}\text{UDCOUNT}(x,c,z) 
\end{cases}
\]

Explanation

The state UDCOUNT has parameters x, c and u representing the values on the ports out, clk and up respectively. Thus the first branch states that if the value on the clock port is false, and the value on up is true, then a rising edge on the clock will cause the value on the output to be incremented. If u is false, then a rising edge on clk will cause the output to be decremented. The last two lines describe events which produce no output change — a falling edge on the clock, and the changing of the value on up.

This example exhibits most of the features introduced in the preceding paragraphs and demonstrates the increased power of the enhanced language over pure CIRCAL. UDCOUNT is a parameterised state. On the right hand side of the expression, its parameters are assigned variously to other parameters, functions of parameters, or literal values. Various types of events appear in the example: passing of literal values, the output of functions of state parameters, and the input...
of parameters. Also, both types of conditional are used. In the last line of the description the predicate not eq ensures that the value appearing on the port up is different from the one there initially. The if... then clauses in the other three branches test whether an event on the clk port may involve a change to true or false, and in the former case, determine then whether the counter should count up or down.

3.1.7 Composition

One of the attractive features of CIRCAL is the fact that it enables the construction of behaviours by use of the composition operator. It is to be hoped that the introduction of additional descriptive power would not remove this capability. Fortunately, this turns out to be the case, although composition in this richer language is certainly more complicated.

In Section 2.3.3 the synchronisation of guards was discussed. The rules for this synchronisation depended on the assumption that events were represented only by port names, and that any two events on the same port could synchronise. In the enhanced language, however, two events on the same port will only synchronise in certain instances. For example, if a port p on one device is able to pass the value 'true', and the port p on another device is able to input a variable x, then the effect when the two devices are wired together will be to pass the value 'true', this value being input to the second device. However, an event such as p<true could not synchronise with the event p<false even though they involve the same port, just as in pure CIRCAL the events in0 and in1 could not synchronise. A systematic way of determining whether two events can synchronise is required.

There are actually four factors to consider when a pair of events synchronise: whether synchronisation can take place at all; what the resultant event will be; what binding of variables takes place; and what the conditional in front of the event should be (this last consideration is explained below). Four types of events need to be considered: pulses (the events of pure CIRCAL), inputs, outputs (of either functions or parameters), and passing of literal values. These can interact
in potentially ten different ways, although some are trivial. These interactions will now be dealt with in turn.

1. Pulse – pulse. This is the simple synchronisation of pure CIRCAL. Two pulse events will always synchronise to yield a pulse if they involve the same port. No variables are bound, no conditional need precede the resultant event.

2. Pulse – anything else. No synchronisation can occur.

3. Input – input. Synchronisation can occur if the predicates associated with the two input parameters are not mutually exclusive. The resultant event is an input, with a predicate on the parameter which is the conjunction of the two original predicates. The two input parameters should be bound to a single parameter. Note that this binding should be carried out not just for the input event, but for all occurrences of that parameter in other events in the guard (e.g. in {in>x, out<ef1(x)}) and in the new state which follows the guard. No preceding conditional is required.

4. Input – output. Synchronisation can always occur. The resultant event is identical to the output event. The input parameter should be bound to the output parameter or function. A preceding conditional should be added to ensure that the value which is output satisfies the predicate on the input parameter. For example, if the events which are to synchronise are p>x: g(x) and p<fy(a,b) then the preceding conditional should be g(f(y(a,b))). The use of this conditional is explained below.

5. Input – value. Synchronisation can only occur if the value satisfies the condition on the input parameter. The resultant event is the passing of the value, and this literal value should replace any occurrences of the input parameter. There is no leading conditional.

6. Output – output. Synchronisation can always occur, and the resultant event can be either of the two synchronising events. The leading conditional must be a test for equality between the two passed parameters or functions.
7. Output – value. Synchronisation can always occur, resulting in the passing of a value. The leading conditional is the test for equality between the output parameter or function and the passed value. No variables need be re-bound.

8. Value – value. Synchronisation can only occur if the values are equal, this being equivalent to the synchronisation of pulses. There is no need for a leading conditional or the re-binding of variables.

In general, synchronisation involves not just events but guards. The rules for the synchronisation of guards given in Section 2.3.3 can now be generalised to cover this richer language. Suppose a guard $a$ is associated with a behaviour of sort $M$, and that the set of ports involved in events in this guard is called $pa$. Similarly, the guard $b$ with associated sort $N$ involves events on a set of ports $pb$. Then in order for synchronisation to occur, the intersection of $pa$ with $N$ must equal the intersection of $pb$ with $M$, and the events in $a$ which take place on these ports must all be able to synchronise with the corresponding events in $b$ according to the rules outlined above. In front of the resultant guard should be a conditional which is the conjunction of the two conditionals which preceded the two original guards and of all the other conditionals generated by the synchronisation of events, as described above.

Examples

The following examples should serve to clarify the ways in which guards can synchronise in the enhanced language. Let $A$ be a behaviour with sort $\{a, b, c, d\}$ and $B$ a behaviour with sort $\{b, c, d, e\}$. In each of the following examples, the first-mentioned branch forms part of the definition of $A$, the second is part of the definition of $B$. $p_1$, $p_2$, $p_3$ are predicates, and $f_1$ is a function. $eqs$ is also a predicate, which evaluates to true if its two arguments are equal.

- $\{b>x:p_1(x)\}$ $A(x)$ can synchronise with $\{b<y\}B(y)$ to give if $p_1(y)$ then $\{b<y\}A(y)\ast B(y)$. (Rule 4)
• \{b\to x:p_1(x)\} A(x) can synchronise with \{b\to y:p_2(y)\} B(y) to give
\{b\to x: \text{and}(p_1(x),p_2(x))\} A(x)*B(x). \text{ (Rule 3)}

• \{a<c, b<5, c>x:p_3(x)\} A(x) can synchronise with \{b<y, c<7\} B(y) to give if \text{eqs}(y,5) then \{a<c, b<5, c<7\} A(7)*B(5) only if p_3(7) evaluates to true. Otherwise the branches cannot synchronise. \text{ (Rules 7 and 5)}

• if p_1(z) then \{b<z\} A(z) can synchronise with \{b\text{cf}_1(y)\} B(y) to give if \text{and}(\text{eqs}(z,f_1(y)),p_1(z)) then \{b<\text{cf}_1(y)\} A(y)*B(y). \text{ (Rule 6)}

• \{b\to x:p_1(x), d<z\} A(x) cannot synchronise with \{b\to y\} B(y) simply because d is in the sort of B but does not appear in the second guard.

• \{b\to x:p_1(x), c<3\} A(x) cannot synchronise with \{b<y, c<5\} B(y) because the two events on the port c cannot synchronise. \text{ (Rule 8)}

The fact that synchronisation generally involves guards rather than just single events can sometimes cause problems. Consider the synchronisation between the following two guards:

\{\text{in}>x, \text{out}<\text{not}(x)\} \text{ and } \{\text{out}<y\}

Following the rules for synchronisation presented above, the resultant guard, given that \text{in} is not in the sort of the second device, should be

if \text{eqs}(\text{not}(x),y) \text{ then } \{\text{in}>x, \text{out}<\text{not}(x)\}

The trouble with this is that the leading conditional, which is supposed to depend only on state parameters, now depends on an input parameter as well. In order to turn this into a meaningful expression, the leading conditional should be attached instead to the input parameter to which it refers, as follows:

\{\text{in}>x:\text{eqs}(\text{not}(x),y), \text{out}<\text{not}(x)\}
The general rule for situations like this is that if a leading conditional is generated which refers to an input parameter, it should be moved to qualify the appropriate input parameter. There may be a choice of parameters to qualify, in which case the predicate should be attached to the one which appears last in the guard.

As a final example, consider the synchronisation between the following two guards:

\[ \{ \text{in}>x, \text{out}<f_1(x) \} \text{ and } \{ \text{in}>y, \text{out}<f_2(y) \} \]

Following the event synchronisation rules 3 and 6 gives

\[ \text{if } eqs(f_1(x), f_2(x)) \text{ then } \{ \text{in}>x, \text{out}<f_1(x) \} \]

with the input parameter \( y \) being bound to \( x \) throughout (including the leading conditional). By the argument of the previous paragraph, this becomes

\[ \{ \text{in}>x: eqs(f_1(x), f_2(x)), \text{out}<f_1(x) \} \]

That concludes the introduction to the enhanced CIRCAL language. The implementation of a system to manipulate expressions in this language is described in Appendix C. The following Section illustrates some of the techniques which assist the designer in using the language for the writing of specifications in a design and validation methodology.

### 3.2 Specification Techniques

The enhanced CIRCAL language has been introduced to provide more descriptive power and greater ease of description than is provided by pure CIRCAL. The task of writing specifications can be further eased by using certain techniques with the language. Certain techniques may also be required to improve the accuracy of descriptions. A number of such techniques have been developed through experience of using the language and are presented below. The motivation for easing the
specification task is not simply laziness: in a hierarchical design and validation methodology, specifications must be written at every level of hierarchy, and so their writing may constitute a significant portion of the design effort. Furthermore, a reduction in the effort of writing descriptions improves a designer's chances of correctly formalising his informal ideas about a device's behaviour.

Some of the following techniques have arisen from attempts to perform verification, as it is sometimes only when one tries to make practical use of a specification that its shortcomings become apparent. This issue is treated more fully in Chapter 5. Other techniques have been motivated by the need to use the CIRCAL modelling framework in a meaningful way, as is discussed in the next Section.

3.2.1 Specifying Valid Events

One of the first issues which arises when writing specifications in enhanced CIRCAL, which has already been seen in some of the foregoing examples, is the need to ensure that events really do represent changes in the values on ports. As was mentioned before, it is not really meaningful to allow an event such as \( p<3 \) if the value on the port \( p \) is already 3. For input events, the way to deal with this is to have a state parameter corresponding to the current value on the port, then attach a predicate to the input parameter to guarantee that it is different from the current value. The following example demonstrates this.

Example

\[
A(x) \leftarrow \{p \triangleright y : \text{noteq}(y, x)\} A(y)
\]

Explanation

The function \( \text{noteq}(x, y) \) was defined above to return true if its arguments differed. This device will therefore only accept events on the port \( p \) if the value differs from that already there, represented in this case by the state parameter \( x \).
For output events a fairly similar technique can be adopted. A state parameter is again required to retain the current value on the output port, and then some test of various state and input parameters must be applied to decide whether the output will change or not. The exact nature of this test depends on the type of device being described. A simple example will illustrate the technique.

Example

The device described below is intended to transfer input values to the output after a delay of two time units (ticks).

\[
\text{DEL}(x,y,z) \leq \text{if eqs}(y,z) \text{ then} \\
\{t, \text{in}\geq\text{p}:\text{noteq}(p,x)\}\text{DEL}(p,x,y) + \{t\}\text{DEL}(x,x,y) \\
\text{else if \text{noteq}(y,z) THEN} \\
\{t, \text{in}\geq\text{p}:\text{noteq}(p,x), \text{out}\geq\text{y}\}\text{DEL}(p,x,y) + \{t, \text{out}\text{y}\}\text{DEL}(x,x,y) \}
\]

Explanation

The parameter \(z\) represents the current value on the output port, and \(y\) represents the value which will appear there on the next tick. If these are equal, therefore, the next tick need not be accompanied by an output event, as indicated by the first two branches. If \(y\) differs from \(z\), as the predicate in the last two branches ensures, then an output event will be required.

The question arises of what would happen if a specification was written which did not ensure that events really did represent changes of value. Certainly if the descriptions were to be simulated, there would be a penalty in efficiency, as the number of events occurring could be greatly increased. The consequences for verification are a little more involved and will be discussed in Chapter 5.

3.2.2 Constructive Specification

Because the structural operators of CIRCAL have a well-defined behavioural meaning, it is possible to use them to construct a specification. This may be useful, for
example, if a device to be specified has a number of fairly independent functions. It is then possible to write the specification as if the device were constructed from a number of parts, each one performing one of the functions. The splitting of a specification in this way does not necessarily commit the designer to implementing the device in the same way. An example of a device which logically may be split into two parts appears below.

Example

The following constructive specification describes a counter with an asynchronous load capability. Its two functions, counting and loading, are described separately by the behaviours COUNT and LOAD.

\[
\text{LOAD}(\text{false}, n, m) \leftrightarrow \text{if } \text{noteq}(m, n) \text{ then }
\]
\[
( \{ \text{as1<true}, \text{int<n} \} \text{LOAD}(\text{true}, n, n)
+ \{ \text{as1<true}, \text{data>x:noteq(x,n)} \} \text{LOAD}(\text{true}, x, n) )
+ \text{if } \text{eqe}(m, n) \text{ then }
\]
\[
( \{ \text{as1<true} \} \text{LOAD}(\text{true}, n, n)
+ \{ \text{as1<true}, \text{data>x:noteq(x,n)} \} \text{LOAD}(\text{true}, x, n) )
+ \{ \text{as1<false} \} \text{LOAD}(\text{false}, n, m)
+ \{ \text{data>x:noteq(x,n)} \} \text{LOAD}(\text{true}, x, m)
+ \{ \text{as1<false}, \text{data>x:noteq(x,n)} \} \text{LOAD}(\text{false}, x, n)
\]

\[
\text{COUNT}(\text{false}, p) \leftrightarrow \{ \text{clk<true}, \text{out<incr(p)} \} \text{COUNT}(\text{true}, \text{incr(p)})
+ \{ \text{int>x:noteq(x,p)} \} \text{COUNT}(\text{false}, x)
+ \{ \text{clk<true}, \text{out<incr(p),int>x:noteq(x,p)} \} \text{COUNT}(\text{true}, x)
\]

\[
\text{COUNT}(\text{true}, p) \leftrightarrow \{ \text{clk<false} \} \text{COUNT}(\text{false}, p)
+ \{ \text{int>x:noteq(x,p)} \} \text{COUNT}(\text{true}, x)
+ \{ \text{clk<false}, \text{int>x:noteq(x,p)} \} \text{COUNT}(\text{false}, x)
\]

\[
\text{COUNTER} \leftrightarrow \text{LOAD} \cdot \text{COUNT} - \text{int}
\]
Explanation

The behaviour LOAD is parameterised over the values on the ports asl (asynchronous load), data, the port for input of new values, and int, the port for communicating these values to COUNT. Loading is triggered by a rising edge on asl, causing the value on the data port n to be transferred to int if it differs from the value there already. The parameters of COUNT are the value on the clk port and the stored count value p. A rising edge on clk will increment p, or it may be overwritten at any time by a new value being communicated on int.

Of course, the counter would probably not be implemented in two parts like this. However, writing the specification in two parts makes the whole task rather simpler because it removes the need for the designer to specify the possible interleaving of events on the asl and data ports with those on the clk port. This simplification improves the chances that the designer’s intentions will actually be captured by the specification.

The parts which make up a constructed specification need not even be physically realisable — their sole function is to simplify the writing of a specification. For example, it is common to write specifications in which only rising or falling edges of a signal are important. Such specifications can be somewhat simplified by the use of an edge detector box, as in the following example.

Example

An edge-triggered D latch may be described in the following way:

\[
\begin{align*}
DLAT &\leftarrow D + EDET - \text{out} \\
EDET(0) &\leftarrow (\text{clk} \triangleq 1, \text{out})\text{EDET}(1) \\
EDET(1) &\leftarrow (\text{clk} \triangleq 0)\text{EDET}(0) \\
D_{(x,z)} &\leftarrow \{\text{data} \triangleq y : \text{noteq}(y,x)\}D_{(y,z)} \\
&\text{+ if noteq}(x,z) \text{ then} \\
&\quad (\text{out}, \text{q} \triangleq x)D_{(x,x)} + \{\text{data} \triangleq y : \text{noteq}(y,x), \text{out}, \text{q} \triangleq x\}D_{(y,x)} \\
&\text{+ if eqs}(x,z) \text{ then} \\
&\quad (\text{out})D_{(x,z)} + \{\text{data} \triangleq y : \text{noteq}(y,x), \text{out}\}D_{(y,z)} \\
\end{align*}
\]
Explanation

The edge-detector box, \texttt{EDET}, produces a pulse on \texttt{out} only when a rising edge occurs on \texttt{clk}. The behaviour of \texttt{D} is parameterised over \texttt{x} and \texttt{z}, the values on the \texttt{data} (input) and \texttt{q} (output) ports respectively. A new value may always be placed on \texttt{data}. If \texttt{x} and \texttt{z} differ, then a rising edge on the clock, denoted by the \texttt{out} event, will cause \texttt{x} to be placed on the output; otherwise it causes no change.

The main advantage of this approach is that it saves the designer having to take account of all possible interleavings of rising and falling clock edges with other possible events. This fairly small saving in effort becomes considerable if a large number of edge-triggered devices are used, since the edge detector need only be specified once, and can then be used in all cases, suitably relabelled. Also, the amount of effort required to specify interleavings of events increases with the number of input ports, so the savings of this approach are more apparent.

Constructive specification is also used for the addition of constraints to a specification. In this situation, the composition operator can be seen to behave like conjunction in higher-order logic, which is also the 'wiring-up' operator in that framework. This aspect of constructive specification is discussed in Chapter 7.

3.2.3 Delays

The specification of timing phenomena has been seen to be one of the important aspects of hardware description, and in particular the specification of delays is frequently required. There are various types of delay: transport, where a change on an output follows a change on an input by a certain length of time; inertial, where a change on an output will only occur after an input has maintained its new value for a certain period of time; it may also be desired to specify delays which are of differing lengths depending on the direction of the input change or the port on which it occurs. CIRCAL has a number of ways of treating these types of delay, which are discussed below.
The simplest way to describe a delay in CIRCAL is illustrated in the following example, which is an inverter with a delay of two ticks.

**Example**

```plaintext
INV(x) <= \{t, in>y:noteq(y,x)\} \{t, out<not(y)}INV(y) 
  + \{t\} INV(x)
```

**Explanation**

The sequence of three guards here means that on the first tick a new input arrives, then another tick happens, then on the third tick a new output is presented. The problem with this description is that there is a period of time during which no further input events can occur. This is an example of an ‘inaccurate’ description, in the second sense mentioned at the start of this Chapter. It is contrary to the normal understanding of hardware behaviour, where inputs are assumed to be always able to accept inputs, even if no action is taken until some later time such as the occurrence of a clock. The consequences of this approach to specification can be serious when verification is attempted, as is discussed in Chapter 5.

A better approach, then, would always allow input events, but keep track of the pending output changes. This can be done by using some extra state parameters which act like a queue of values, from which the new output value eventually pops off the end. The description now becomes

```plaintext
INV(x,y,z) <= if eq(y,z) then ( \{t\}INV(x,not(x),y) 
  + \{t, in>p:noteq(x,p)}INV(p,not(x),y) )
+ if noteq(y,z) then ( \{t, out<y}INV(x,not(x),y) 
  + \{t, in>p:noteq(x,p), out<y}INV(p,not(x),y) )
```

**Explanation**

The parameter y represents the pending output value. If it equals z, the value on the output, then a tick will produce no change. However, a tick may
be accompanied by an input event on \textit{in}, which replaces the value on that port (represented by \textit{x}) with a new value, \textit{p}.

The above description is for transport delay. A little modification would provide inertial delay — this is explained below. For longer delays, extra state parameters are needed to provide a longer queue of pending outputs.

Yet another approach, and probably the best, makes use of the constructive specification technique. As with the edge detector example, the saving of effort becomes considerable when a number of similar but not identical devices are being described. The method here is to describe the gate as a delay-less device and compose it with a separate delay device, as in Figure 3-1(a). Thus the two-tick-delay inverter is now described as follows:

\begin{align*}
\text{INV} & \leftarrow \text{INVERT} \times \text{DEL2} - \text{z} \\
\text{INVERT}(x) & \leftarrow (\text{in} \not= y \not= q, z \not= \text{y}) \text{INV}(y) \\
\text{DEL2}(p,q,r) & \leftarrow \text{if eqs}(q,r) \text{ then } \{ t \} \text{DEL2}(p,p,q) \\
& \quad + \{ t, z \not= q \not= m \not= p \} \text{DEL2}(m,p,q) \\
& \quad + \text{if noteq}(q,r) \text{ then } \{ t, \text{out} \not= q \} \text{DEL2}(p,p,q) \\
& \quad + \{ t, z \not= q \not= m, \text{out} \not= q \} \text{DEL2}(m,p,q)
\end{align*}

The behaviour of \text{DEL2} is identical to that of the device \text{DEL} described in Section 3.2.1.

For inertial delay, this approach is equally simple. All that is required is to change the definition of \text{DEL2} to

\begin{align*}
\text{DEL2}(x,2) & \leftarrow \{ t \} \text{DEL2}(x,1) \times \{ t, z \not= y \not= m \not= x \} \text{DEL2}(y,2) \\
\text{DEL2}(x,1) & \leftarrow \{ t, \text{out} \not= x \} \text{DEL2}(x,0) \times \{ t, z \not= y \not= m \not= x \} \text{DEL2}(y,2) \\
\text{DEL2}(x,0) & \leftarrow \{ t \} \text{DEL2}(x,0) \times \{ t, z \not= y \not= m \not= x \} \text{DEL2}(y,2)
\end{align*}

Explanation

This type of delay box starts a count-down (the second state parameter) when a new input arrives, as the second branch of each line shows. The count is reset
if another new input arrives before the new output has propagated. Otherwise, after 2 ticks, the new value is placed on the output. This is a situation where it is important to ensure that events really do represent changes in values, as discussed in Section 3.2.1, since the count should only be reset for a genuine new input value.

Using these delay specification techniques, it is possible to combine inertial and transport delay, a situation which may sometimes be found in hardware, simply by cascading the appropriate delay boxes. The ease with which the delay characteristics of a device specification can be modified and the amount of repetitive description which is saved over a large design are key advantages of this technique. It also reduces the opportunity for errors, since once a delay box has correctly specified for one device it can be re-used for any number of others.

![Diagram](image)

**Figure 3-1:** Uses of delay boxes

In some situations, the above technique is not sufficiently general to describe a device's delays. If there is some asymmetry in the delay characteristics of a device which depends on the direction of input changes or, for multi-input devices, on the actual input port which causes the change, then the behaviour cannot be modelled by the simple attachment of a delay box to the output of an ideal device. What is required here is separate delay boxes on the input lines, as illustrated in Figure 3-1(b). By defining a 3 unit delay box as...
DEL3(n,p,q,r) <= if eqs(q,r) then ((t)DEL3(n,n,p,q)
    + (t, z m:noteq(p,m))DEL3(m,n,p,q) )
  + if noteq(q,r) then ( (t, out<q)DEL3(n,n,p,q)
    + (t, z m:noteq(p,m), out<q)DEL3(m,n,p,q) )

It would be possible to describe a 2-input gate which had a delay of 2 units on signals on the a input and a delay of 3 units on the b input as follows:

\[ \text{andgate} = \text{DEL2} [a/z] \ast \text{DEL3} [b/z] \ast \text{AND} [out/a][out1/b] \]

Propagation delays which depend on the direction of change of the input are quite common. For example, a delay of length 3 units on rising edges and of length 2 units on falling edges could be modelled by a box such as the following:

DEL2_3(n,p,q,r) <= if and(eqs(q,r),eqs(n,false)) then
  (t, z<true))DEL2_3(true,n,p,q)
  + if and(eqs(q,r),eqs(n,true)) then
    (t, z<false))DEL2_3(false,false,p,q)
  + if eqs(q,r) then (t)DEL2_3(n,n,p,q)
  + if and(noteq(q,r),eqs(n,false)) then
    (t, z<true, out<q)DEL2_3(1true,n,p,q)
  + if and(noteq(q,r),eqs(n,true)) then
    (t, z<false, out<q)DEL2_3(false,false,p,q)
  + if noteq(q,r) then (t, out<q)DEL2_3(n,n,p,q)

This device behaves very much like a 3 delay box except that when a falling edge occurs on its input, the value 'false' is placed in the first 2 spots in the queue, rather than just the first spot. This ensures that a falling edge reaches the output in just 2 ticks, while rising edges still take 3 ticks to propagate.
3.2.4 Other Timing Phenomena

Sometimes it is required to describe timing characteristics other than propagation delays. Examples of these are setup and hold times for clocked devices (the length of time before and after the clock edge for which the data inputs must be stable) and the rise and fall times of signals. These types of characteristics are not normally specified exactly, but as ranges of acceptable values. In order to deal with the imprecision of these situations another constructive specification technique can be used. In this case, an imaginary device which generates ‘ticks’ at the beginning and end of the required time period can be used. The following example illustrates how this works.

Example

Suppose a device has a clock input on a port clk and that rise time for signals on this port must be less than 4 ticks of the universal clock. Then a device is defined to generate two pulses, say ta and tb separated by 4 time units:

\[ TGEN \leftarrow \{ta, t\}(t)\{t\}(tb, t)TGEN + \{t\}TGEN \]

Explanation

The second branch of this behaviour allows TGEN to accept ticks passively; in the first branch, a pulse on ta must be followed by 3 ticks of the universal clock then a pulse on tb must accompany the fourth such tick.

Assuming that the clock line goes from low, through undefined, to high, then the behaviour of the device can now be described as follows:

```plaintext
datatype tristate = low | undef | high

DEV <= DEVA * TGEN - \{ta, tb\}
DEVA <= \{clk<low\}\{ta, clk<undef\}\{clk<high\}\{tb\}DEVA
```

72
By writing the specification in this way the clock line is only constrained to go high sometime less than 4 ticks after it becomes undefined. The ordering of events which is imposed by this description is shown in Figure 3-2. Similar techniques can be used to describe other timing characteristics which involve a range of timespans rather than a precise length of time. Some of these phenomena relate to the specification of constraints, which is discussed in Chapter 7.

3.3 SuperC

It has probably become apparent by now that, even in its enhanced form and using the techniques developed in the previous Section, CIRCAL does not offer the ease of description in many circumstances that might be desired in a hardware description language and which is generally provided by commercial languages. The ease of use of a language is not a purely cosmetic feature, since the language is the medium in which a designer attempts to formalise his ideas about hardware behaviours. If the language is difficult to use, the chances of capturing these ideas accurately is reduced. This difficulty of use arises mainly because of the event-driven model which underlies CIRCAL and which requires the designer to specify explicitly every possible event that can occur in every state of a device. Additional effort is required to test input and output events to ensure that they are 'real' events representing changes in port values.
Another shortcoming of CIRCAL is the ease with which unrealistic devices can be specified. For example, it was shown in Section 3.2.3 that a device can be specified which for certain periods cannot accept input events. Such a device really contradicts the normal understanding of an input. Because an input is not driven by the device to which it is attached, an attempt by another device to change its value cannot be resisted. Even clocked devices, which do not process inputs during certain periods, can still accept changes on their input ports during these times. Furthermore, such a specification may lead to erroneous conclusions when validation is attempted.

It would be quite difficult to overcome these problems by adding further features to enhanced CIRCAL. What is required instead is a way to restrict the manner in which CIRCAL is used. This can be achieved by defining a new language which is easily translatable into CIRCAL. The language should aim both to restrict the designer by preventing the specification of unrealistic devices and to reduce the effort of writing specifications. Such a language, which will be called SuperC, is proposed in this Section.

In Figure 3-3 the relationship between the three languages pure CIRCAL, enhanced CIRCAL and SuperC is illustrated. Whereas enhanced CIRCAL is an extension of pure CIRCAL which can be mapped into it, SuperC is a separate language giving access to only part of enhanced CIRCAL. In general, expressions in enhanced CIRCAL cannot be translated into SuperC. Therefore, a useful application of SuperC might be as a 'front-end' input language to a CIRCAL validation system such as that described in Appendix C; specifications could be more easily written in SuperC, then translated to CIRCAL for subsequent manipulation and comparison.

It may be asked whether yet another language is really necessary; might it not be possible to translate from an existing language into CIRCAL? Certainly it is possible to translate from subsets of some other languages to CIRCAL, but there are at least two advantages in defining a new language: the translation may be effected more easily, and some of the distinctive features of CIRCAL can be retained. In particular, the ability to specify constraints and constrained behaviours
is a feature which few other languages offer. This feature is of considerable importance and will be discussed in Chapter 7. In choosing the features of this new language, it was necessary to make some trade-offs between provision of a familiar, programming-language-like syntax and retention of the important features of CIRCAL. In general, the latter was given the higher priority. Perhaps if the language were to be used in a commercial design environment it would be necessary to improve its syntax; in the context of this thesis, however, the aim has been to illustrate some of the issues involved in developing a useful design language which offers access to some of the attractive features of CIRCAL.

3.3.1 Development of the Language

An initial observation which led to the development of this language was that many CIRCAL hardware specifications look very much alike. This similarity is a result of the fact that a large part of any specification is concerned only with the accepting of input events. In many circumstances, the sole effect of an input event is to put a new value on the appropriate port. A number of input events (often a relatively small number) also cause output events or changes of state parameters. So, the effort of writing descriptions could be reduced by defining the language in such a way that only those input events which produce 'interesting' results need

Figure 3-3: Relationship Between pure CIRCAL, enhanced CIRCAL and SuperC.
to be specified. All other input events are assumed to be possible and to have no
effect other than to change the value on the corresponding port. This approach
not only reduces the amount of effort required to write a description, it also makes
it impossible to describe a device in which an input is temporarily ‘locked’ i.e.
unable to accept changes.

Because input ports will be assigned a default behaviour, it is necessary in this
language to declare which ports are inputs and which are outputs, in contrast to the
approach of enhanced CIRCAL, in which the directionality of ports is determined
solely by the events which take place on them. It is then necessary to describe
in some way the input events which produce ‘interesting’ results i.e. changes of
output values or state parameters. One way to do this is to give a description for
each output or state parameter stating which input events will cause it to change.
The following example, a JK flip-flop, illustrates how this could be done.

Example

\[
\text{jkff}(j,k,\text{clk}) = q
\]

\[
\text{where } q =
\]

\[
\begin{align*}
\text{if } \text{and}(j,k) & \text{ then } \text{event}(\text{clk},\text{true}):\text{not}(q), \\
\text{if } \text{and}(j,\text{not}(k)) & \text{ then } \text{event}(\text{clk},\text{true}):\text{true}, \\
\text{if } \text{and}(\text{not}(j),k) & \text{ then } \text{event}(\text{clk},\text{true}):\text{false}, \\
\text{end;}
\end{align*}
\]

Explanation

In keeping with many other hardware description languages, the inputs \(j, k\)
and \(\text{clk}\) are declared as arguments of a function and the output(s) \(q\) as the
result(s). Then, in the body of the description, the combinations of port values
and input events which will result in output changes are listed. So, for example,
the third line above states that, if the values on the \(j\) and \(k\) ports are both true,
then a rising edge on the \(\text{clk}\) port, denoted by \text{event}(\text{clk},\text{true}), will be followed
by the output taking on a value which is the complement of its current value.
Before discussing how such a description could be translated into Circal, it is helpful to consider another example, in which the device being described has internal states, i.e. states not directly visible as values on ports. The behaviour of such states can be represented just as outputs are. The only need for different treatment is in the first line of the description, so that the state is declared in a way which distinguishes it from an input or output.

Example

A random access memory could be described as:

\[
\text{memory}(\text{memstate})(\text{read}, \text{write}, \text{data}, \text{address}) = \text{out}
\]

where

\[
\text{out} = \text{event}(\text{read}, \text{true}) : \text{fetch}(\text{memstate}, \text{address}),
\]

\[
\text{memstate} = \text{event}(\text{write}, \text{true}) : \text{store}(\text{memstate}, \text{address}, \text{data})
\]

end;

Explanation

The functions \text{fetch} and \text{store} would be defined using a suitable programming language such as ML, just as in enhanced Circal. In the above description, the line beginning 'out =' describes the behaviour of the output port: if the value on \text{read} becomes true, then the value on \text{out} becomes the value returned by \text{fetch}. The following line implies that the internal state variable \text{memstate} will be assigned a new value, given by the memory-updating function \text{store}, if the value on \text{write} becomes true.

3.3.2 Translation

Descriptions such as the two above can be easily and automatically translated into enhanced Circal for subsequent manipulation. The essential information could be extracted from the description by a parser and placed into suitable data struc-
tures to allow translation. A small ML program was written to do this translation.
The data structures are:

**partname** A string representing the name of the part (e.g. "jkff").

**inputport_list** A list of strings corresponding to the inputs (e.g. ["j", "k", "clk"]).

**outputport_list** A list of strings corresponding to the outputs (e.g. ["q"]).

**state_list** A list of the internal states (e.g. ["memstate"]).

**input_output_list** A list of the relationships defined between input events and output events. Each element of the list corresponds to one line of the description body and contains three components: an input event, a condition, and an output event. For example, the line if and(j,k) then event(clk,true):not(q) from the flip flop description would be represented by ("clk<true","and(j,k)","q<not(q)").

**input_state_list** Identical to the **input_output_list** except that the output events are replaced by state assignments. Assignment could be represented by the character ", so the penultimate line of the memory description would be represented by ("write<true","", "memstate"store(memstate, address, data)"). The null string indicating the absence of a conditional.

These data structures are input to the ML program, which produces, as a single string, a CIRCAL description of the device. The name of the device is unchanged, and its state parameters are the union of three sets: the values on the inputs, the values on the outputs, and the declared internal states. The names of the parameters representing the port values are the same as the corresponding port names.

The first steps of the translation add explicitly some implied information to each element of the **input_output_list** and **input_state_list**. For example, the event clk<true is only possible if the value on the port clk does not equal true
already. So, the conditional for the first element of the list must be modified to ensure this, becoming \( \text{and} (\text{noteq}(\text{clk}, \text{true}), \text{and}(j, k)) \). Input events involving parameters must be guaranteed to be valid in the sense defined in Section 3.2.1; this may require the addition of a predicate to an input parameter. For example, if an input event was written as \( \text{event}(\text{ina}, x) \), which becomes \( \text{ina} > x \), and \( (j, k) \)), Input events involving parameters must be guaranteed to be valid in the sense defined in Section 3.2.1; this may require the addition of a predicate to an input parameter. For example, if an input event was written as \( \text{event}(\text{ina}, x) \), which becomes \( \text{ina} > x \), and \( (j, k) \)), then a predicate to ensure that \( x \) differed from the current value on \( \text{ina} \) would be added to give “\( \text{ina} > x \):noteq(x, \text{ina}) \).

The next step is to ensure that output events are also valid. This involves adding another term to the conditionals to ensure that the new output value differs from the value currently on the port. For example, the second element in the \( \text{input_output_list} \) for the flip flop would be \( \text{"clk true", \"and(noteq(clk, true), and(j, not(k)))\"} \); the conditional would now become \( \text{and(noteq(q, true), and(noteq(clk, true), and(j, not(k))))} \), ensuring that the event \( q < \text{true} \) represents a genuine event.

Once the list of 'interesting' events is completely processed in the above way, it is necessary to compile a list of the other events, which cause no output changes and whose only effect on the state is to modify values on input ports. These can be divided into two classes: those which occur on inputs which are not mentioned in the body of the SuperC description (e.g. \( j \) and \( k \)), and those which occur on the ports which are mentioned, but cause no output or state change because of the value which is passed or the failure of a conditional to be satisfied. In this latter category for the JK flip flop are events such as \( \text{clk false} \), or \( \text{clk true} \) when none of the three conditionals is true. Events in the first category are simply of the form \( j > j1:noteq(j, j1) \).

In this way a complete list of all possible single input events and their effects on outputs or internal states can be compiled from the SuperC description. From this list can be constructed a list of all possible sets of input events which may occur simultaneously. This is done by constructing the set of all subsets of the original list, excluding those subsets in which different input events on a single port appear in two members of the subset.
From each element of this list of subsets a line of enhanced CIRCAL can be generated. The input events of all the members of the subset and any associated output events are formed into a guard. The conditionals of all the members of the subset are conjoined to form a single conditional. The effect on the states which represent port values can be easily deduced from the input and output events, while the effect on internal states is as specified in the original input_state_list data structure.

3.3.3 Combinational Logic

Devices whose outputs depend only on the values of the inputs at a single instant in time, i.e., combinational logic, can be described using the language features presented above. This approach is not as concise as might be desired, however, and the frequency with which such devices are used warrants some special treatment. A special static assignment operator could be used, as illustrated in the following example:

Example

```
andgate(a,b) = c
where c := and(a,b)
end;
```

where the function and would be suitably defined in the chosen programming language. Such a description would translate to an enhanced CIRCAL description such as that given for the and gate in Section 3.1.4. Automatic modification of the enhanced CIRCAL description for different functions or numbers of inputs would be quite straightforward.

3.3.4 Timing properties

In Section 3.2.3, considerable attention was given to the problem of specifying delays. Rather than forcing a designer to deal with such concepts as ticks and
queues of values, a language should allow him to work directly with the more familiar ideas of inertial and transport propagation delays. Section 3.2.3 showed a number of techniques for the description of these phenomena. These techniques can readily be incorporated into the SuperC language, as the following example illustrates:

Example

In order to specify the above and gate with a transport delay of 3 time units, the following notation could be used:

\[
\text{andgate}(a,b) = c \\
\quad \text{where } c := \text{and}(a,b) \text{ tdelay 3} \\
\text{end};
\]

Explanation

The output \( c \) takes the value calculated by applying the function \( \text{and} \) to the values on the ports \( a \) and \( b \) 3 time units after a change occurs on either of those inputs.

This description would translate to enhanced CIRCAL as

\[
\text{andgate} <= \text{and} \ast \text{de13} - z
\]

Where \( \text{de13} \) is a delay box as defined in Section 3.2.3, and \( \ast \) is the delayless and gate as before. Inertial delays could be described by using a different reserved word (\( \text{idelay} \)) and the translation would simply involve a suitably defined inertial delay box for \( \text{de13} \), such as was illustrated in Section 3.2.3.

The description of delays which differ depending on the input port at which the change occurs or the polarity of the change could not easily be incorporated into descriptions using the static assignment (\( := \)) operator. It would be necessary to use the full event-based notation presented at the start of this Section and to specify the various delays explicitly. Consider again the description of an and gate
which has a propagation delay of 2 units after changes on the a input and a delay of 3 units after changes on the b input. This could be described as follows:

\[
\text{andgate}(a,b) = c \\
\text{where } c = \begin{cases} 
\text{if } \text{eqs}(b, \text{true}) \text{ then event}(a,x):x \text{ tdelay } 2, \\
\text{if } \text{eqs}(a, \text{true}) \text{ then event}(b,x):x \text{ tdelay } 3 
\end{cases}
\]

Explanation

The second line states that the value on the output c will change to x, the value input on a, if the value on b is already true, and that this change will occur 2 time units after the change on a. The next line describes the case when a is already true and a new value x is input on b, the only difference being that the delay is now 3 units.

This is exactly the situation which was described in Section 3.2.3 and would be translated to give the same enhanced CIRCAL specification using a separate delay box for each of the input lines.

3.3.5 Sequential Behaviour

An assumption which has been made for all the descriptions so far is that an output event will always be triggered by an input event, possibly lagging behind it by some period of delay. There are, however, many devices for which this assumption is not valid, mainly those which have an internal clock. In such devices, output events are essentially triggered by the passage of time; therefore a way of representing the passage of time is required. A simple if slightly inelegant way to do this is to adopt the CIRCAL approach of a universal clock which generates regularly spaced ticks and to use these as the 'input event' to trigger the change in output. The following example illustrates this approach.
Example

A non-overlapping two-phase clock is described as:

twophase(onenext)() = (phione,phitwo)
where
    phione = if and(onenext,not(phione)) then tick: true,
            if phione then tick: false,
    phitwo = if and(not(onenext),not(phitwo)) then tick: true,
            if phitwo then tick: false,
    onenext = if phitwo then tick: true,
            if phione then tick: false
end;

Explanation

In the first line, the empty parentheses show that there are no inputs to this device. The outputs are \( \text{phione} \) and \( \text{phitwo} \). \( \text{onenext} \) is an internal state which determines which clock phase should rise next. The two lines after 'where' describe the behaviour of the first clock phase: if it is false, and the internal state is true, then a tick should cause it to rise; if it is true, then a tick should cause it to fall. The next two lines describe the second phase: it is identical, except that it rises when the internal state is false. Finally, the internal state's behaviour is defined: it becomes true when the value on \( \text{phitwo} \) is true, and false when the value on \( \text{phione} \) is true. It should be noted that all the output events occur not as a result of input events, as has been the case in other descriptions, but as the result of ticks.

3.3.6 Constrained Specifications

Another assumption which has been made throughout the discussion of this language is that all devices will be able to accept any input event at any time. Bearing in mind the fact that specifications must sometimes be written for devices which
have already been designed, such as the parts in a library, there may be situations in which this assumption is not valid. A simple example of this is the RS flip flop. A common implementation of this device will behave unpredictably if both inputs are true. Thus a specification of such a device might not allow the event r<true to occur if the value on the s port is true. This is an example of a constrained specification. Additional features are needed for the language to deal with such specifications. Indeed, the ability to deal with constraints has been given as one of the motivations for creating this new language instead of using an existing one. In Chapter 7, the topic of constraints and their description in SuperC will be discussed.

3.4 Summary

This Chapter has examined some of the issues associated with the specification task in a design and validation methodology. Specification is required to describe both the intended function of a device before it is implemented and the functions of the components which will be used to make an implementation. In either situation, a specification language should assist the designer by allowing concise, intuitive description, and it should if possible encourage the writing of descriptions which accurately reflect the behaviour of real hardware. A language must be sufficiently expressive to allow the description of any type of device in adequate detail, including its timing characteristics.

These issues were studied by considering the CIRCAL language. In its basic form, that language has several shortcomings, notably a lack of conciseness for many devices of only moderate complexity. A language, 'enhanced CIRCAL', was developed by considering these shortcomings and adding features to the language to overcome them while retaining the power of pure CIRCAL for manipulation of behavioural expressions. The meaning of the composition operator was defined for the enhanced language.
To write specifications effectively it is necessary to have not only a suitable language but also some associated specification techniques. A number of such techniques for use with enhanced CIRCAL were presented. Some of the techniques, such as constructive specification, reduce the effort of writing specifications, while others provide the means for describing a wide variety of timing phenomena. It is also important to ensure that descriptions fit in with the CIRCAL model of behaviour, which is event-based. To this end, techniques were developed which ensure that events which model changes of port values do in fact correspond to real changes of value. In Chapter 5, the importance of these techniques in validation will be demonstrated.

While enhanced CIRCAL provided a much more useful descriptive medium than pure CIRCAL, it was still somewhat difficult to use compared to many other languages. Thus, another language was presented which is intended to be more ‘user-friendly’ than even enhanced CIRCAL. The language, called SuperC, removes the designer to an extent from the modelling details of CIRCAL and incorporates some of the specification techniques described above, enabling him to concentrate on the function of the device being described. As well as removing some of the tedium of writing specifications, the new language prevents the designer from falling into some of the traps which are present in pure CIRCAL, such as describing devices so that their inputs are occasionally ‘locked’. The main shortcoming of the language is that, in order to carry out manipulation on the behavioural descriptions, they must first be translated into enhanced CIRCAL, and this translation is not generally reversible. However, the language would provide a useful front-end for a CIRCAL-based design system.
Chapter 4

Design

In the subtasks of Figure 1–2, specifying is followed by partitioning and describing. The latter two tasks are generally quite difficult to separate and will be discussed in this Chapter under the single heading of 'Design'. In an ideal world, partitioning might be considered the purely structural operation of splitting the box at the higher level into several smaller, interconnected boxes, and describing would consist of associating behaviours with these smaller boxes. A more realistic view of a human designer would be that partitioning consists of splitting the larger box into smaller boxes whose behaviour is only informally defined, probably in the designer's head; the describe phase then makes the behaviour of the smaller boxes formal.

This thesis is primarily concerned with the integration of design and validation within a hierarchical framework. In order to assist the integration of the two tasks, it is to be hoped that design techniques could be developed which assist the validation process. This Chapter will examine some possibilities for the achievement of this aim.

The other main theme of this thesis is the role of languages in supporting a hierarchical methodology. Since the design task consists largely of making choices, assistance in this task may be provided by reducing the number of choices available to a designer. The way in which a language may be used to represent this restriction will be presented.
A language may also assist the design phase by allowing a transformational approach to design to be adopted. In such an approach, the designer has a certain amount of freedom to make design decisions, but is forced to preserve the behaviour of the specification as he transforms it to an implementation, which may therefore be guaranteed to be correct. An example of a transformational approach to design is described in Section 4.2.1.

Of course, design is not always carried out purely by a human designer. A large number of design aids is available either to assist a human designer or to perform completely the design task in certain circumstances. Some of these automated design techniques and their role in a hierarchical methodology will also be discussed.

Without doubt the most general 'design system', however, is the human designer; all other tools and techniques are limited to specific areas of application (and may certainly outperform a human designer in these areas). Some of these tools, especially those referred to as 'expert systems', have been developed with the aim of mimicking the approach of a human designer. For these reasons it is appropriate to begin this chapter with an examination of the way in which a human designer might approach the design task in a hierarchical methodology.

4.1 Manual Design

It has been mentioned that design usually consists of partitioning and describing. For a human designer, partitioning often is effected by the drawing of circuit diagrams; these define the smaller boxes (components) from which the current box will be constructed and the interconnections between the components. That is, they provide an internal structural description of the device being implemented. In some design systems[Morison85,Lattice85], a designer is required to write this structural description linguistically. This is probably a less intuitive way to work, but is essentially equivalent, and it is not very difficult to translate between graphical and textual representations of circuit structure. Textual representations have
the advantage of being both human- and machine-readable, thus making them suitable for input to simulators, for example. A very large number of languages for the formal representation of structure exists [Nash84]; the features of a number of languages for the description of circuit structure were discussed in Section 2.2.

Some designers would contend that a circuit diagram contains behavioural as well as structural information. This is true in a sense, in the same way that a conventional picture of an inverter, drawn as a triangle with a circle at one vertex, conveys behavioural information. Certain words and shapes imply certain behaviours, so that if one sees a box with ‘RAM’ written on it, one has a reasonable idea of its function. Such pictures do not, however, provide the formal description of behaviour that is required for a formal design and validation methodology; the structural information which they convey, on the other hand, is entirely adequate.

Having defined the internal structure of a device, with possibly some informal behavioural information as well, the partitioning phase must be followed by the description phase in which the behaviours of the components are formally defined. Sometimes to associate a behaviour formally with a box may require the writing of its specification, a task which is essentially identical to the specification of a system at the top level. In other situations it may be possible to find the description of the required box in a library; in this case it would be expected that an implementation of that box also exists in the library. If the hierarchical design process is viewed as a tree, each node being a device and its children being the boxes from which it is constructed, then the design of a system is complete when all the leaves of the tree exist as implementations in the library.

Throughout this thesis, much of the discussion has assumed a rigorously top-down approach to design. In fact there are many situations where such an approach cannot or should not be taken. For example, attempts to implement a device may lead to a deliberate revision of its specification. Then, this revision may require that the devices to which the re-specified device is to be connected must also be re-specified; alternatively it may be necessary to formulate a new specification at a higher level in the hierarchy. Also, it is only by allowing some measure of bottom-up design that the available ‘leaves’ in the library (of standard cells or
whatever) can be most efficiently used; for example, a library of gates could be used bottom up to design flip flops, which in turn could be used to design counters, etc. A knowledge of the availability of such parts would clearly influence the design decisions made at higher levels in the design hierarchy.

When carried out by a human designer, design may be viewed as a creative process. Koomen attempts to model the designer's creative input in terms of information theory [Koomen85]; as the design progresses from an abstract specification to a concrete implementation, the amount of information must be increased with each step down the hierarchy, and this information is usually provided by the designer. Clearly a black box with an attached specification has less information that the same box after its internal structure has been defined and the behaviours of its component parts defined. The skill and experience of the designer will determine what information he chooses to add, i.e. how he chooses to partition a device into boxes and what behaviours he chooses to assign to them. It is this creative aspect of design which makes it so difficult to automate. In a later section some of the issues of the automation of design and how this fits in to a hierarchical design and validation methodology will be addressed.

While design and validation are often viewed as totally separate tasks, the case has already been made in previous chapters for the two tasks being integrated. This being the case, it is reasonable to ask whether the design task can be approached in such a way as to assist the verification task. The next Section addresses this question.

4.1.1 Design for Validation

One of the main arguments for hierarchical design is that, as a special case of the established technique of problem reduction, it enables a complex problem to be broken down into pieces of more manageable size. Within limits, a greater number of levels of hierarchy will result in a greater reduction in the complexity
of the problem. In the integrated design and validation methodology proposed in the preceding chapters, validation takes place between adjacent levels in the hierarchy as soon as they have been designed. One way in which design can assist validation, then, is to use closely-spaced levels in the hierarchy to reduce the complexity of individual validation steps.

In order to demonstrate this point effectively, the nature of validation needs to be more fully explained, a task which will be tackled in the next Chapter. For the moment, it is sufficient to say that validation consists of demonstrating that an implementation satisfies a specification, and that this generally involves matching a complex behaviour (the implementation) with a simpler behaviour (the specification). The greater the 'gap' between these two levels, the more difficult it becomes to establish their equivalence. In the example of Chapter 6, the way in which design can be carried out to reduce this gap, and the consequent reduction in verification effort, is demonstrated.

4.1.2 Restricting Design

The primary argument which has been used to justify a language-based approach to design so far is that such an approach is essential to the integration of validation into the hierarchical design process. In addition to this, it seems reasonable to suppose that a language may be able to assist in the design task itself by restricting the designer. This situation may be thought of as the hardware analogy to high-level programming languages; these languages restrict the programmer's access to the full function of the computer, but in so doing may improve his productivity and increase the program's likelihood of being correct.

Silicon compilers derive their name from the analogy to software compilation, so the input languages to these may be thought of as being analogous to high-

\[1\text{There is also a cost associated with increasing numbers of levels, as specifications of components and their interconnections have to be written at every level. This cost must be traded off against the gains obtained by using more levels.}]}
level programming languages. This approach to design is discussed in more detail below. However, a design language may be able to restrict a designer without forcing him to depend on a specific design automation tool with its attendant disadvantages. A design language could be formulated so that a designer using it is restricted in a helpful way, with the extent of the restriction depending on the designer's level of expertise. The following example will illustrate how this might be done.

![Figure 4-1: Latches in Combinational Logic Design](image)

A frequently adopted approach to the design of large circuits involving combinational logic requires that globally clocked latches are placed between 'lumps' of combinational logic of a certain size. A circuit of this type is pictured in Figure 4-1. By choosing a clock period which is longer than the delay across any one lump of logic, a system which will behave reliably can be constructed fairly easily. An experienced designer seeking to achieve maximum speed or minimum area might choose to ignore this design discipline, preferring to analyse the timing behaviour of each piece of logic in detail. A less experienced designer, however, would be considerably assisted by such a discipline, which ensures uniformity of delays across the circuit.

In order to provide assistance to a novice designer, high-level language constructs could be defined. In the above example, suitable high-level constructs would be a set of 'combinational wire-up' operators. For example, it might be decided to provide a 'wire-in-series' operator and a 'wire-in-parallel' operator. The arguments to the wire-in-parallel operator would just be gates chosen from some
library of available parts. For example, in order to describe the configuration of gates pictured in Figure 4-2, one could write:

\[
pblock1 = \text{wire-in-parallel}(\text{and}(a,b,c), \text{and}(e,f,c), \text{inv}(f,g))
\]

where the CircaI convention of wiring together similarly named ports is followed. This operator would raise an error flag if an input port on one gate had the same name as an output port on another gate, as this would mean that the gates were actually being wired in series. The above description could be translated into CircaI quite readily, as:

\[
pblock1 \leq \text{and} \ast \text{[a/b]} \ast \text{inv}
\]

assuming that the definitions of and and inv had sorts \{a, b, c\} and \{f, g\} respectively.

The 'wire-in-series' operator would be more involved, as its function includes ensuring that latches are inserted between appropriate numbers of gates. If, for example, the delays of the available gates are such that three gates in series can be guaranteed to have a delay less than the clock period, then the wire-in-series operator could be defined such that it must have exactly three arguments, which may be either gates or parallel blocks. As well as wiring its arguments together, the series operator would wire a latch onto the end of the new block. Thus blocks defined with this operator could be safely wired together without the designer
needing to be concerned about the delays of individual gates. An example circuit appears in Figure 4-3. Its description using the two high-level operators would be as follows:

\[\text{pblock0} = \text{wire-in-parallel}(\text{inv}(p,a), \text{nand}(x,y,e))\]
\[\text{pblock1} = \text{wire-in-parallel}(\text{and}(a,b,c), \text{and}(e,f,c), \text{inv}(f,g))\]
\[\text{sblock} = \text{wire-in-series}(\text{pblock0}, \text{pblock1}, \text{and}(c,g,z))\]

![Figure 4-3: Gates Wired in Series and Parallel](image)

The latch shown as a solid box is automatically attached by the wire-in-series operator; the latch drawn with dashes would be from the previous stage.

This example suggests the following approach to language-based design: techniques which lead to safe designs, such as the one presented above, may be encapsulated in the design language. A novice designer may be given a very restricted set of operators with which to work; these will limit the range of design options open to him, thus guiding him in the design task and improving his chances of producing correct designs. A more experienced designer is given access to a wider range of operators, giving him greater flexibility and the capacity to make fuller use of the available technology, at the cost of more difficult design decisions and increased chance of error. In this simple example, only two levels of operator were presented: a novice designer would be allowed to use only the wire-in-parallel and
wire-in-series operators, while an expert would be able to use the full range of CIRCAL structural operators. In a full design system, there would be perhaps several levels of operator corresponding to a variety of levels of expertise.

It is now clear that language features may directly assist the design task by restricting the designer in a useful way. A designer may also be restricted by the need to satisfy contextual constraints, which are the subject of Chapter 7. Another way in which a language may provide design assistance is by supporting design transformations. This topic is discussed below.

4.2 Transformation

In a sense all design is a process of transformation, in which abstract circuit descriptions are transformed into progressively more concrete ones. A human designer performs this transformation informally and manually; a design automation system performs it automatically according to some algorithm, although the algorithm may be defined only by the code of the system. Of particular interest in this Section are those transformations which are formally defined and which may be selectively applied by the designer in order to work towards a solution to the design task. Such transformations are often 'behaviour-preserving', meaning that the behaviour of the resultant implementation is equivalent (in some sense) to the original specification behaviour to which the transformation was applied.

This approach to design represents a middle path between fully manual and fully automated design. It may still offer the guarantee of correctness and some of the reduction of design effort which come from automated techniques, but retains some of the flexibility and creativity of manual design. For this reason it has been the subject of considerable recent attention. For example, a complete design system based on a transformational approach and using the LTS language described in Section 2.2.2 has been proposed and partially developed[Milne86b]. An example of a simple behaviour-preserving transformation using CIRCAL is presented below.
4.2.1 A CirCal-based Transformation

One way in which a correctness-preserving transformation could work is as follows: given a black box to be designed and its specification, the designer partitions the box into two components, using his own skill to decide what the sorts of the two components will be, and also to write a behavioural description of just one of the components. He may decide to extract that behaviour from a library. Then, an algorithm could be applied to determine the behaviour of the other component so that the two components wired together implement the specification. If the algorithm failed to find a suitable behaviour for the second component, the designer would need either to re-specify the other one or to choose a completely new partitioning. Thus the designer still has freedom in choosing the first component's behaviour, but is constrained by the algorithm to ensure that the implementation satisfies the specification. Such an approach represents a behaviour-preserving transformation which could be applied repeatedly until the specification was broken down into sufficiently small pieces.

In order to illustrate how this approach might be effected in CirCal, a slightly simplified problem will be tackled. Firstly, the implementation is restricted to have no hidden ports, i.e. the sort of the specification equals the intersection of the sorts of the two components which comprise the implementation. Secondly, all guards are required to include a tick of the universal clock; this will prevent the occurrence of events on one component which are completely independent of those on the other component.

The behaviours of the two components of the implementation can be written as

\[ A = \sum a_i A_i \quad \text{and} \quad B = \sum b_j B_j \]

and the specification as

\[ C = \sum c_k C_k \]

The composition operator was defined in Section 2.3.3. With the restrictions described above, only the third term in the expansion of A•B remains. This must
equal the specification, so

\[ \sum c_k C_k = \sum (a_i \cup b_j)(A_i \cdot B_j) \]

where \( M \) is the sort of \( A \) and \( N \) the sort of \( B \). Because of the restrictions in this simple example, the \( a_i \) and \( b_j \) can be easily found given the \( c_k \), which are simply the guards of the specification. Since each \( c_k \) corresponds to the union of some \( a_i \) and \( b_j \), the guards of the component behaviours can be found by removing those events which are not in their sorts. That is

\[ a_i = c_k \cap M \]
\[ b_j = c_k \cap N \]

It then follows that

\[ C_k = A_i \cdot B_j \]

so that the procedure for finding the behaviours \( A_i \) and \( B_j \) is exactly the same as that just demonstrated.

A simple example illustrates how this transformation could be used. A device is specified as

\[ C \leq (a, t)C1 + (b, t)C2 + (c, t)C3 + (a, b, t)C4 + (a, b, c, t)C5 \]
\[ C1 \leq \ldots \]

The designer decides to partition the box into a component \( A \) of sort \( \{a, b, t\} \) and a component \( B \) of sort \( \{b, c, t\} \). \( a_i = c_i \cap M = \{a, t\} \). \( b_i = c_i \cap N = \{t\} \). Continuing this procedure for all five guards results in

\[ A \leq (a, t)A1 + (b, t)A2 + (t)A3 + (a, b, t)A4 \]
\[ B \leq (t)B1 + (b, t)B2 + (c, t)B3 + (b, c, t)B4 \]

The behaviours \( A1-4 \) and \( B1-4 \) would need to be determined from the definitions of \( C1-5 \). It can easily be shown that the behaviour of \( A \cdot B \) can perform all the actions of \( C \). In this case, it can also perform several additional actions, but this
is acceptable, as will be shown in the following Chapter; such an implementation is said to satisfy its specification.

This particular transformation is really very limited in usefulness. It has not been defined for enhanced CIRCAL, the language in which most useful descriptions must be written. The restriction that guards must contain ticks is acceptable, but the requirement that the implementation have no hidden ports limits the application of this approach severely. However, it serves to illustrate some important points: that a language which supports formal reasoning about behaviour can also support a transformational approach to design; and that such an approach can leave a designer with some creative freedom while still giving him assistance in the design task and ensuring that design steps are correct.

A general point to be made here is that the more assistance a designer is given, the less flexibility he has. The approach that gives a designer the most assistance and limits his flexibility to the greatest extent is automated design, which is the subject of the next Section.

4.3 Automatic Design

The field of Design Automation (DA) is a vast one and this thesis cannot hope to cover it in any great detail. What is important here is to examine how design automation tools may be fitted in to the proposed methodology and how this affects the specification and validation tasks. It is also interesting to study whether the methodology, which has been motivated mainly by the desire for hierarchical validation, has anything to contribute in terms of design automation.

The ideal design automation system would accept some very abstract specification of the required device and from that would produce, without any human intervention, a complete implementation which was guaranteed to perform the required task correctly. Furthermore, the implementation would be at least as efficient as anything a human designer could produce. Such a system would completely remove the need for a design and validation methodology — all that would
be required to design perfect chips would be some means of extracting correct specifications to feed to the system. Unfortunately, such a system does not exist; current design automation tools are limited in their range of application, often produce less efficient designs than those produced manually, and are not always guaranteed to produce correct implementations. The fact that the systems are limited in scope means that they can only be used in some parts of the design process, and in order to preserve the guaranteed correctness which is the aim of the proposed methodology, it is necessary either to validate the output of DA tools against the input specifications or to validate the tools themselves.

4.3.1 Specification and Design Automation

If a design automation tool is to be used to design just part of a system which, as a whole, is being designed under the methodology proposed in Chapter 1, then the specification of that part has three important roles to play. Obviously it must convey sufficient information to the DA tool to enable it to produce an implementation. The specification must also be used to enable the validation of the preceding design step. And finally it may have to be used to permit the validation of the output of the design automation tool, if this tool has not itself been validated (i.e. if it has not been proven that the tool always produces correct implementations). The second of these roles dictates that the language which is accepted by the tool as input be the same specification language that is in use for the methodology. If the output of the tool is to be validated, then it is important that the output can at least be translated into a form which will permit this validation to take place. Preferably, the output should be in the same language as the input to the tool, but this may not always be possible, as discussed below.

One question which arises is whether these three roles for a specification being used for design automation place contradictory requirements on the specification language. Is it reasonable to expect, for example, that a single language should be suitable both as input to a PLA generator and as a specification language supporting formal reasoning about hardware behaviour? Certainly there have been
relatively few attempts to unify languages in this way, with many automated design
tools requiring input specifications in their own purpose-built language[Lattice85,
Siskind82]. However, in recent years with the numerous attempts to arrive at a
standard hardware description language[Morison85, USAF84, Piloty82] the need to
develop languages which are suitable for widely disparate tasks has been recog-
nised and catered to. The criteria by which languages were selected for study in
Chapter 2, especially the ability to describe a wide range of hardware, should
ensure that they are suitable for all of the above roles.

4.3.2 Validation of Automated Designs

The validation of the output of an automated design system is just like any other
validation if the output is produced in a form which supports validation. If the
output of the system is in some less convenient form, such as the mask information
which might be output by a PLA generator, then the validation task becomes a
little more complicated. If validation is to be performed by simulation, then the
solution is to use a simulator which can accept mask information, or at least a
circuit description extracted from the mask information, as input. The simulated
behaviour of the implementation can then be compared with the behaviour which
was specified, possibly by also simulating the specification with the appropriate
simulator. If formal methods are to be used, then a model of the implementation's
behaviour must be derived in the mathematical framework which is to be used for
the verification. This too may be done by circuit extraction and circuit level
simulation.

An alternative approach to the validation of devices produced by DA tools has
already been alluded to. This is to validate the tool itself, thus guaranteeing that
the implementations it produces will always satisfy the specifications which are fed
to it. A discussion of this approach will be postponed until the issues of validation
are discussed in more detail in Chapter 5.
4.4 Summary

In this Chapter, the design phase of the methodology, consisting of partitioning and description, has been examined. As the methodology aims to integrate design and validation, an important consideration is the way in which certain approaches to design affect the validation task. In particular, the importance of the distance between levels of hierarchy in determining the difficulty of validation was discussed, and this point will be returned to in Chapter 6.

Design is an essentially creative task, often relying heavily on a designer's skill. While it is widely accepted that hierarchical design reduces the difficulty of this task, this Chapter has gone some way to showing that a language-based approach to design can also provide the designer with considerable assistance.

By analogy to high-level programming, a language for hardware design may also be high-level in that it offers the designer access to more powerful constructs and prevents him from committing certain errors. Such a language may provide assistance to a designer by reducing the options open to him in a given situation. The idea of levels of language which restrict a designer according to his level of expertise was presented and illustrated by the well-known example of clocked latches placed between lumps of combinational logic.

A language which supports formal manipulation of behaviours may be able to provide design assistance by supporting a transformational approach to design. Transformations which preserve the behaviour of a specification, thus ensuring a correct implementation, while allowing a designer to make some creative input, were introduced. A simple example of such a 'behaviour-preserving' transformation using pure CIRCAL was presented.

There is one further way, not discussed in this Chapter, in which languages may assist design. It is only in language-based design that constraints may be reasoned about formally, and this may be of considerable importance. This topic is dealt with in Chapter 7.
Design cannot be fully discussed without some treatment of the area of design automation. Design automation tools, while suffering from some notable shortcomings, are often able to produce designs quickly and reliably. Such tools, when used in a hierarchical design methodology, place certain requirements on the specification language; fortunately these requirements are quite close to those which were outlined in Chapter 3. Either the output of the tools, or the tools themselves, need to be validated, as will be discussed in the next Chapter. These issues serve to highlight the interdependence of the features of a design language and the performance of the tasks of specification, design and validation.
Chapter 5

Validation

In Chapter 1 the motivation for a hierarchical design and validation methodology was given: in essence, this was to enable the detection of design errors at the earliest possible point in the design process, thus reducing the cost of such errors. The tasks in the methodology which have been described in the preceding chapters are essential to it, but it is on the validation task that the whole process hinges, for it is by performing this task that design errors can be detected, assuming that the design task has not been carried out in such a way as to guarantee their exclusion.

As in the previous Chapter, the task described here splits into two parts. In Figure 1–2 these were shown as ‘Compose’ and ‘Compare’. Because they are virtually never carried out in isolation, and the way in which one is performed strongly affects the other, they are discussed here under the single heading of ‘Validation’.

Broadly speaking, there are two approaches to the validation task. The more widely used of these is simulation: the attempt to establish the behaviour of an implementation by calculating its response to a selection of input stimuli. The first part of this Chapter will deal with this method of validation. The principal shortcoming of simulation, its failure to prove the absence of design errors except by the unacceptably costly technique of exhaustive simulation, will be discussed and a possible way to overcome this problem will be examined.

The other main approach to validation is referred to as verification. This word is often taken to mean different things by different people, but in this thesis will
always refer to the formal, mathematical proof that an implementation behaves as is required by its specification. This technique avoids the problem encountered in simulation, offering a guarantee that an implementation behaves correctly. The cost of this certainty is the complexity which is encountered in the use of formal methods. Ways of tackling this complexity will be discussed.

A theme throughout this Chapter will be the requirements which a particular validation technique places on the specification language, with emphasis on the suitability of CIRCAL for both simulation and formal verification. Also, by attempting to validate devices specified using the techniques of Chapter 3, the effectiveness of those techniques will be evaluated.

5.1 Simulation

The simulation of an implementation consists essentially of predicting or calculating the responses it will give to a range of input stimuli. In Sections 5.1.2 and 5.1.3, two different approaches to this task are presented. This is followed by some examples and a discussion of the problems which may arise in simulation, including difficulties caused by certain specification techniques. Finally a way of improving the confidence gained from simulation is discussed. Before performing a simulation, however, it is necessary to specify the input stimuli to be applied. This topic is discussed in the next Section.

5.1.1 Input Specification

If the behaviour of a device under certain input stimuli is to be established, then there must be some way of describing those input stimuli. It is common for a simulator to be equipped with a specialised language for this purpose, but in fact it should be possible to use a general-purpose hardware description language. In CIRCAL a sequence of input stimuli can be represented by a series of guards, as the following example illustrates.
Example

Suppose that in order to simulate a counter it was required to load it with the value 15, then apply 5 successive clock pulses. These input stimuli could be specified in CIRCAL as follows:

```
START <= {data<15}{asl<true}CLOCK(5)
CLOCK(n) <= if noteq(n,1) then {clk<true}{clk<false}CLOCK(n-1)
  + if eqs(n,1) then {clk<true}{clk<false} /
```

Explanation

The counter has an asynchronous load port (asl) which, when triggered by a rising edge, causes the value on the data port to be loaded. The stimulus generating device described above therefore begins by putting the value 15 on the input port data before providing the rising edge on asl. It then moves to state CLOCK(5), from which state pulses are generated on the clk port until its state variable is decremented to 1, at which time a final pulse is generated and the device terminates.

5.1.2 Conventional Simulation

To simulate a device which is specified behaviourally (i.e. with no reference to its internal structure) is not difficult. Whatever language it is described in, it should be possible to predict what outputs it will generate given a certain pattern of inputs. The event-based model of CIRCAL makes it quite a suitable language for this task, and the response of a device to input stimuli can be easily determined. It should be emphasised here that the simulation performed here does not depend on many of the unusual features of CIRCAL; virtually any hardware description language could be used in this role.
Example

The loadable counter mentioned above could be described in CIRCAL using only behavioural operators in the following way:

\[
\begin{align*}
\text{COUNT}(\text{false}, \text{false}, \text{n}, \text{p}) & \leftarrow \\
& \{\text{clk} < \text{true}, \text{out} < \text{incr(p)}\} \text{COUNT}(\text{true}, \text{false}, \text{n}, \text{incr(p)}) \\
& + \{\text{data} \times : \text{noteq}(\text{x}, \text{p})\} \text{COUNT}(\text{false}, \text{false}, \text{x}, \text{p}) \\
& + \{\text{asl} < \text{true}, \text{out} < \text{n}\} \text{COUNT}(\text{false}, \text{true}, \text{n}, \text{n}) \\
\text{COUNT}(\text{false}, \text{true}, \text{n}, \text{p}) & \leftarrow \\
& \{\text{clk} < \text{true}, \text{out} < \text{incr(p)}\} \text{COUNT}(\text{true}, \text{true}, \text{n}, \text{incr(p)}) \\
& + \{\text{data} \times : \text{noteq}(\text{x}, \text{p})\} \text{COUNT}(\text{false}, \text{true}, \text{x}, \text{p}) \\
& + \{\text{asl} < \text{false}\} \text{COUNT}(\text{false}, \text{false}, \text{n}, \text{p}) \\
\text{COUNT}(\text{true}, \text{false}, \text{n}, \text{p}) & \leftarrow \\
& \{\text{clk} < \text{false}\} \text{COUNT}(\text{false}, \text{false}, \text{n}, \text{p}) \\
& + \{\text{data} \times : \text{noteq}(\text{x}, \text{p})\} \text{COUNT}(\text{true}, \text{false}, \text{x}, \text{p}) \\
& + \{\text{asl} < \text{true}, \text{out} < \text{n}\} \text{COUNT}(\text{true}, \text{true}, \text{n}, \text{n}) \\
\text{COUNT}(\text{true}, \text{true}, \text{n}, \text{p}) & \leftarrow \\
& \{\text{clk} < \text{false}\} \text{COUNT}(\text{false}, \text{true}, \text{n}, \text{p}) \\
& + \{\text{data} \times : \text{noteq}(\text{x}, \text{p})\} \text{COUNT}(\text{true}, \text{true}, \text{x}, \text{p}) \\
& + \{\text{asl} < \text{false}\} \text{COUNT}(\text{true}, \text{false}, \text{n}, \text{p})
\end{align*}
\]

Explanation

The four state parameters of this device represent the values on the ports \texttt{clk} (the clock), \texttt{asl} (asynchronous load), \texttt{data} (input for loading a value) and \texttt{out} (the output port). The first two are of type boolean, the second two are \text{n}-bit integers. The function \texttt{incr(p)} should be defined to add 1 to \text{p} modulo 2\text{n}. In the states where the value on \texttt{clk} is false (the first two) an event \texttt{clk<true} causes the output to increment. In the states where the value on \texttt{asl} is false (the first and third) an event \texttt{asl<true} causes the output to be set to the value currently on the data input port \texttt{data}. All other events which may occur simply cause new values to be placed on input ports.
In order to carry out a simulation, a start state needs to be calculated or chosen. Taking this to be the first of the above four states, then it is quite clear that from this state the events generated by the input device described above, \{data<15\} \{asl<true\} will lead the counter into state COUNT(false, true, 15, 15) and that the following 5 rising edges on the clk line will cause the expected incrementing of the output.

Simulation of a purely behavioural description in this way could be useful in checking that a large specification correctly reflects the intentions of the designer. It could also be used to provide a useful description of the behaviour of a specification against which the simulation results for the implementation could be compared. The obtaining of these results requires that a description which is not purely behavioural be simulated, and this is a less straightforward task.

In many simulators, the approach which is taken to establish the behaviour of a constructed device is to carry out simulations of each of the component devices concurrently, treating each component in a manner similar to that just described. The following simple example will illustrate how this might be done.

![Diagram of 3 unit buffer]

**Figure 5-1:** Construction of 3 unit buffer

Example

The constructed device to be simulated, pictured in Figure 5-1, is a 3 time-unit delay buffer, constructed from three single unit delay buffers. The behaviour of a unit delay buffer is
BUF_1(p,q) <=
  if eqs(p,q) then ( (t)BUF_1(p,p)
      + (t, in>x:noteq(x,p))BUF_1(x,p) )
  + if noteq(p,q) then ( (t, int1<p)BUF_1(p,p)
      + (t, in>x:noteq(x,p), int1<p)BUF_1(x,p) )

and the other two buffers have identical behaviours, suitably relabelled, as follows:

BUF_1' <= BUF_1 [int2/int1][int1/in]
BUF_1'' <= BUF_1 [out/int1][int2/in]

Explanation

The buffer has a value p stored on the input in, and a value q on the output int1. If these are equal, then a tick of the universal clock will produce no change on the output. If they are unequal, then the value p will appear on the output when a tick occurs. In either case, the tick may also be accompanied by the arrival of a new value on the input port.

Now suppose that the input stimulus is described by the following device:

STIM <= {in<5, t}{t}{t}{t}{t}{in<2, t}{in<7, t}{t}{t}{t}/

Explanation

This device places the value 5 on the input port, then waits for 3 ticks, then places the values 2 and 7 on the input on successive ticks, then waits another 3 ticks before terminating.

Again, it is necessary to choose start states for the simulated devices; in this example, they are all chosen to be BUF_1(0,0). This immediately reduces the initial behaviour of each device to

BUF_1(0,0) <= (t)BUF_1(0,0)
      + (t, in>x:noteq(x,0))BUF_1(x,0)
To establish the events which will take place it is helpful to tabulate the possible events for each device.

<table>
<thead>
<tr>
<th>Simulation Step</th>
<th>Device</th>
<th>Device</th>
<th>Device</th>
<th>Device</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>STIM</td>
<td>BUF_1</td>
<td>BUF_1'</td>
<td>BUF_1&quot;</td>
</tr>
<tr>
<td></td>
<td>{inc5, t}</td>
<td>{t}</td>
<td>{t}</td>
<td>{t}</td>
</tr>
<tr>
<td></td>
<td>{int1}</td>
<td>{int1}</td>
<td>{int2}</td>
<td>{int2}</td>
</tr>
</tbody>
</table>

Note that the predicates on input parameters have been omitted for reasons of space only.

Given that an event can only take place on a port if all devices connected to that port are able to accept an event on it, then it can readily be established from the above table that only one guard is possible, i.e. \{inc5, t\}. This is because all devices can accept the event \textit{t} and both STIM and BUF_1 can accept \textit{inc5}. The event \textit{int1\} cannot be accepted by BUF_1.

With the occurrence of \{inc5, t\}, BUF_1 moves into the state BUF_1(5,0). The table can now be updated and the next event calculated, and so on until the simulation ends or is terminated. The result of this process is tabulated in Table 5–1.

From the above it is quite clear that events on the output lag those on the input by three time units. For a more complicated device, it would be preferable also to simulate a specification of a three unit delay buffer and compare the results of that simulation with those obtained from the simulation of the constructed device.

It is noteworthy that, in this method of simulation, no use was made of the feature of CIRCAL which enables behaviours of constructed devices to be established by manipulation of the behavioural expressions corresponding to its component parts. This method of simulation would work equally well if the language used to describe the components had no such feature. If a single device described in the language can be simulated as the loadable counter was simulated at the start of this Section, then a circuit constructed from devices described in that language can also be simulated. It is for this reason that many hardware description languages do not offer facilities to construct behaviours mathematically, since simulation can be performed without that capability.

108
Table 5-1: Possible and resultant events for BUF.3

The manner in which behavioural descriptions of components may be manipulated in CIRCAL offers a rather different approach to the simulation task from that described above. This approach, which was proposed by Milne [Milne85b] and described more fully by Traub [Traub86], warrants some discussion here.

5.1.3 Simulation by Manipulation

In the example just described, the constructed system could be described structurally as

109
BUF_3 <=

BUF_1 * BUF_1[int2/int1][int1/in] * BUF_2[out/int1][int2/in]

The composition (*) and relabelling ([ / ]) operators convey essentially structural information, but have a formal behavioural interpretation. If the start states for the three buffers are as before then an expansion of the above expression using the definition of the composition operator yields

BUF_3 <= \{t\}(BUF_1(0,0) * BUF_1'(0,0) * BUF_1''(0,0))
+ \{t, in>x:noteq(x,0)\}(BUF_1(x,0) * BUF_1'(0,0) * BUF_1''(0,0))

and subsequent composition of this with the expression for STIM yields

BUF_3 * STIM <= \{t\}(BUF_1(0,0) * BUF_1'(0,0) * BUF_1''(0,0))
+ \{t, in>x:noteq(x,0)\}(BUF_1(x,0) * BUF_1'(0,0) * BUF_1''(0,0))
    * \{in<5, t\}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}/
    \{in<5, t\} (BUF_1(x,0) * BUF_1'(0,0) * BUF_1''(0,0)) *
    \{t\}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}{t}/

Hence, the composition operator provides the result that the only possible event is \{in<5, t\}, just as before. Repeated applications of the composition operator could be used to establish the sequence of events which would follow. The advantage of using CIRCAL as the input specification language is clearly seen here, as this enables the simulation to be carried out entirely by expansion of the composition operator.

It may be thought that the simulation method just described is not greatly different from the method of the preceding Section. In a sense this is true, as the use of the composition operator really just formalises the process of establishing the resultant event when each device has a number of options open. The key difference, however, is that the first method could be used with a language which did not have the formal semantics which enable construction of behaviours to be performed, but this feature is crucial to the second method. It is also noteworthy that in attempting to implement a simulator of CIRCAL expressions, Johnson[Johnson86] found that this second method was preferable for ease of implementation.
5.1.4 Problems

Section 3.2 introduced a number of specification techniques for use with the CIRCULAR language which either eased the specification task or encouraged the writing of more useful or accurate specifications. Many of these techniques evolved from the process of trying to make use of specifications for simulation and verification. This Section discusses some of the problems which may arise from unsuitable specification techniques and how these may be avoided.

Unforced Outputs

In Section 2.3.2 a description similar to the following was given for an inverter with delay:

\[
\begin{align*}
\text{INV}(0,1) & \leq \{\text{in}<1\}\text{INV}(1,1) \\
\text{INV}(1,1) & \leq \{\text{out}<0\}\text{INV}(1,0) + \{\text{in}<0, \text{out}<0\}\text{INV}(0,0) \\
& \quad + \{\text{in}<0\}\text{INV}(0,1) \\
\text{INV}(1,0) & \leq \{\text{in}<0\}\text{INV}(0,0) \\
\text{INV}(0,0) & \leq \{\text{out}<1\}\text{INV}(0,1) + \{\text{in}<1, \text{out}<1\}\text{INV}(1,1) \\
& \quad + \{\text{in}<1\}\text{INV}(1,0)
\end{align*}
\]

If this device is simulated under the following input stimuli

\[
\text{STIM} \leq \{\text{in}<1\}\{\text{in}<0\}\{\text{in}<1\}/
\]

then the first event to take place, given that the start state is \(\text{INV}(0,1)\), may be determined by an expansion of the composition operator as follows:

\[
\text{STIM} * \text{INV}(0,1) \leq \\
\left((\{\text{in}<1\}\{\text{in}<0\}\{\text{in}<1\}/) * \{\text{in}<1\}\text{INV}(1,1)\right) \\
= \{\text{in}<1\} \left(\{\text{in}<0\}\{\text{in}<1\}/\right) * \text{INV}(1,1)
\]

So, the first event to take place is \(\text{in}<1\). To determine the next event, the required expansion is:
Rather than revealing what the next events will be, this has given a choice of three guards. This is not really what is expected of a simulator, which should describe the output behaviour for a given pattern of inputs. The fault here is not in the simulator but in the technique used to specify the inverter delay. The ability to specify a device such that it had a non-zero delay, without needing to specify the length of the delay, may seem quite attractive, but this example shows the problems which such an approach can cause.

If it is really desired to specify a device with non-zero but otherwise unspecified delay, then a more useful approach would be to use the method involving ticks of an external clock but leave the time interval between the ticks unspecified. The inverter could now be written as

\[
\begin{align*}
\text{INV}(0,1) & \Leftarrow \{\text{in} < 1, t\} \text{INV}(1,1) + \{t\} \text{INV}(0,1) \\
\text{INV}(1,0) & \Leftarrow \{\text{out} < 0, t\} \text{INV}(1,0) + \{t, \text{in} < 0, \text{out} < 0\} \text{INV}(0,0) + \{\text{in} < 0\} \text{INV}(0,1) \\
\text{INV}(1,1) & \Leftarrow \{\text{in} < 1, t\} \text{INV}(0,0) + \{t\} \text{INV}(1,0) \\
\text{INV}(0,0) & \Leftarrow \{\text{out} < 1, t\} \text{INV}(0,1) + \{t, \text{in} < 1, \text{out} < 1\} \text{INV}(1,1) + \{\text{in} < 1\} \text{INV}(1,0)
\end{align*}
\]

This specification differs slightly from some seen before in that it allows input events to occur other than simultaneously with a tick when an output is pending. This means that input pulses can be arbitrarily short.

In order to overcome the problems described above, the input stimulus generator STIM must also be written with reference to the ticks. The device described below will test the effect of changes on the input which are separated by less than \(T\) (the time between ticks) and also of changes separated by more than \(T\).
Repeated expansion of the composition operator on \( \text{STIM} \circ \text{INV}(0,1) \) now gives a unique sequence of output events:

\[
\text{STIM} \circ \text{INV}(0,1) = \{\text{in} < 1, t\} \{\text{in} < 0\} \{t\} \{\text{in} < 1, t\} \{\text{out} < 0, t\} /
\]

which reveals that a short (i.e. \(< T\)) pulse on the input produces no effect on the output, while a change on the input which is held for a period of longer than \(T\) results in the appearance of the complemented value on the output, delayed by \(T\).

This technique has potentially quite a wide range of application. It enables a specification to be written in such a way that the value output by a device is specified without an exact specification of how long it will take to calculate that value. This may be useful, for example, in the high-level specification of an arithmetic unit, when what is important is that it performs the appropriate functions, rather than the time taken to perform these functions.

Fictitious Events

One of the specification techniques which was proposed in Section 3.2 was designed to ensure that only 'genuine' events could occur. That is, if an event is being used to model a change of value on a port, then only those events which represent actual changes should appear in a device description. An event such as \(\text{out} < 3\) should not occur if the value on the port \(\text{out}\) is already 3. The consequences of writing specifications in which such fictitious events occur can involve a serious penalty in simulation efficiency, as the following example demonstrates.

Example

The single delay buffer of the previous Section could be described more concisely without regard for the validity of events as follows:

\[
\text{BUF}_1(p) \leftarrow \{t, \text{int} < p\} \text{BUF}_1(p)
+ \{t, \text{in} > x, \text{int} < p\} \text{BUF}_1(x)
\]
Explanation

This device outputs the stored value \( p \) on the port int1 with every tick of the universal clock, regardless of whether it is different from that already there. A value \( x \) may also be placed on the input simultaneously with a tick, and this value may or may not differ from the current value \( p \).

The table to determine events would then be

<table>
<thead>
<tr>
<th>Step</th>
<th>STIM</th>
<th>Device</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((a, 0, t))</td>
<td>((t, \text{int} &lt; 0))</td>
<td>((t, \text{out} &lt; 0))</td>
</tr>
<tr>
<td></td>
<td>((a, 0, t, \text{int} &lt; 0))</td>
<td>((\text{int} 1 \text{xs}, t, \text{int} 2 &lt; 0))</td>
<td>((\text{int} 2 &gt; 0, \text{int} 5 &lt; 0, \text{out} &lt; 0))</td>
</tr>
</tbody>
</table>

Note that the resultant guard now involves an event on every port, compared with just two events in the previous case. In a complicated design, this could lead to a proliferation of events throughout the circuit, drastically increasing the time required to calculate the events taking place at each instant in time. It is for this reason that true event-driven simulation is a popular approach, as adopted for example in the VHDL project [Gilman86, Lowenstein86].

The aspect of simulation which is widely acknowledged as its most serious shortcoming is its failure to guarantee the absence of design errors. Errors can be shown to be present by simulation, and confidence in a design's correctness can be increased when numerous simulations have failed to reveal any errors. However, only if all possible combinations of device states and input stimuli are considered can the correctness of the device be guaranteed. This approach, called exhaustive simulation, is far too time-consuming to be considered seriously in the majority of cases, and so other methods of providing greater confidence in a design have been developed. The main one of these, formal verification, is the subject of the second part of this Chapter; the following Section discusses some approaches which lie between verification and conventional simulation.
5.1.5 Proof by Simulation

The task of verifying a $k$-definite system by simulation has been addressed by Bryant[86]. Such a system has a behaviour such that any sequence of inputs of length $k$ will place the system in a unique state. This class of systems includes finite state machines, but excludes some simple circuits such as registers. Such a circuit can be proven correct by setting all internal states to an undefined value ‘X’, then calculating the output after all possible input sequences of length $k$ have been applied. This is simply exhaustive simulation. However, the unacceptable complexity of the task can be reduced if some values in the input sequences are unimportant to the output (such as the last $n$ inputs to an $n+1$ bit long shift register). In such a circumstance, values in the input sequences can be replaced by the undefined value ‘X’, thereby greatly reducing the number of sequences which must be tested to prove the circuit’s correctness. There is a significant number of situations in which the savings offered by this approach are considerable.

A further simulation technique which may lead to guaranteed correctness is symbolic simulation[Carter79,Darringer79], in which the input stimuli are represented by variables rather than constant values such as true and false. Such a technique can readily be adopted using CIRCAL, as was done by Traub[86], as it is easy to specify an input stimulus generator which supplies variables of any type, and the techniques of Section 5.1.3 may then be applied.

5.2 Verification

This Section discusses some of the issues involved in mathematical proof of circuit correctness or verification. Most of the discussion will be of verification using CIRCAL, although some other approaches to the task will also be briefly introduced. As in the previous Section, the use of certain specification techniques and the avoidance of others will be justified by considering the consequences of applying them to the solution of actual problems. Some of the advantages and disadvantages of using CIRCAL for verification will be discussed, along with the advantages
of verification over simulation. In order to illustrate how a simple verification proof might proceed a small example will first be presented.

5.2.1 A Simple Proof

The device to be constructed and verified is a buffer with a delay of two ticks. Its specification is as follows:

\[
\text{BUF}(w,x,y) \leq \\
\quad \text{if } \text{eqs}(x,y) \text{ then } (\{t\}\text{BUF}(w,w,x) \\
\qquad + \{t, \text{in}\not\equiv q:\text{noteq}(w,q)\}\text{BUF}(q,w,x) ) \\
\quad + \text{if } \text{noteq}(x,y) \text{ then } (\{t, \text{out}\lt x\}\text{BUF}(w,w,x) \\
\qquad + \{t, \text{in}\not\equiv q:\text{noteq}(w,q), \text{out}\lt x\}\text{BUF}(q,w,x) )
\]

Explanation

To model the delay of two ticks, a queue of values represented by the parameters \(w, x\) and \(y\) is used. If \(x\) and \(y\) are equal, then a tick produces no output change, but the values move one place along the queue. If \(x\) and \(y\) differ, then \(x\) is placed on the output port when the tick occurs. In either case the tick may be accompanied by the placing of a new value \(q\) on the input.

The buffer is to be implemented by wiring two inverters in series. This implementation is structurally described as

\[
\text{IMP} \leq \text{INV}[\text{mid/out}] \ast \text{INV}[\text{mid/in}] - \text{mid}
\]

where the behavioural description of \(\text{INV}\) is

\[
\text{INV}(m,n) \leq \text{if } \text{noteq}(m,n) \text{ then } (\{t\}\text{INV}(m,n) \\
\qquad + \{t, \text{in}\not\equiv p:\text{noteq}(p,m)\}\text{INV}(p,m) ) \\
\quad + \text{if } \text{eqs}(m,n) \text{ then } (\{t, \text{out}\not\equiv m\}\text{INV}(m,\text{not}(m)) \\
\qquad + \{t, \text{in}\not\equiv p:\text{noteq}(p,m), \text{out}\not\equiv m\}\text{INV}(p,\text{not}(m)) )
\]

116
### Explanation

The parameter \( m \) represents the value on the input port, while \( n \) represents the value on the output. If these are unequal, the inverter is stable and the occurrence of a tick will cause no output change, but may be accompanied by an input event. If they are equal, the output must change on the next tick to the complement of its current value.

The verification proof consists of attempting to show that IMP 'is equivalent to' BUF. The definition of equivalence is discussed in more detail below. To begin the proof, it is helpful to use the fact that both INV devices are connected to the port mid. So, if the parameters of the second device are changed to \( c \) and \( d \), then \( c \) must equal \( n \). This assumption is not essential to the proof, but reduces the amount of manipulation required here. The proof can now proceed by applying the definition of the composition operator, given in Section 2.3.3, to obtain the following:

\[
\text{IMP} \leq \text{INV[mid/out]} \cdot \text{INV [mid/in]} - \text{mid}
\]

\[
= ( \text{if noteq(m,n) then (} (t)\text{INV(m,n)} \\
+ (t, \text{inp:noteq(p,m)})\text{INV(p,n)}) \\
+ \text{if eqs(m,n) then (} (t, \text{mid<not(m)})\text{INV(m,not(m))} \\
+ (t, \text{inp:noteq(p,m), mid<not(m)})\text{INV(p,not(m))})) \\
* ( \text{if noteq(n,d) then (} (t)\text{INV(n,d)} \\
+ (t, \text{mid>not(p)})\text{INV(p,d)}) \\
+ \text{if eqs(n,d) then (} (t, \text{out<not(n)})\text{INV(n,not(n))} \\
+ (t, \text{mid>not(p), out<not(n)})\text{INV(p,not(n))})) \\
- \text{mid})
\]

\[
= \text{if and(noteq(m,n),noteq(n,d)) then (} (t)\text{INV(m,n)}*\text{INV(n,d)} \\
+ (t, \text{inp:noteq(p,m)})\text{INV(p,n)}*\text{INV(n,d)}) \\
* \text{if and(noteq(m,n),eqs(n,d)) then (} \\
\text{INV(m,n)}*\text{INV(n,not(n))} \\
+ (t, \text{inp:noteq(p,m), out<not(n)})\text{INV(p,n)}*\text{INV(n,not(n))} \\
* \text{if and(noteq(not(m),n),eqs(m,n),noteq(n,d)) then (}
\]

117
\[ \{t, \text{mid}<\text{not}(\text{n})\} \text{INV}(\text{m},\text{not}(\text{n})) \ast \text{INV}(\text{not}(\text{m}),\text{d}) \]

\[ + \{t, \text{in}>p:\text{noteq}(\text{p},\text{m}), \text{mid}<\text{not}(\text{m})\} \text{INV}(\text{p},\text{not}(\text{m})) \ast \text{INV}(\text{not}(\text{m}),\text{d}) \]

\[ + \text{if and(\text{noteq}(\text{not}(\text{m}),\text{n}), and(\text{eqs}(\text{n},\text{d}), \text{eqs}(\text{m},\text{n}))) \text{ then (} \]

\[ \{t, \text{mid}<\text{not}(\text{m}), \text{out}<\text{not}(\text{n})\} \text{INV}(\text{m},\text{not}(\text{m})) \ast \text{INV}(\text{not}(\text{m}),\text{not}(\text{n})) \]

\[ + \{t, \text{in}>p:\text{noteq}(\text{p},\text{m}), \text{mid}<\text{not}(\text{m}), \text{out}<\text{not}(\text{n})\} \]

\[ \text{INV}(\text{p},\text{not}(\text{m})) \ast \text{INV}(\text{not}(\text{m}),\text{not}(\text{n})) \text{ ) \]

\[ = \text{mid} \]

Predicates can be removed or simplified; for example and(\text{noteq}(\text{not}(\text{m}), \text{n}),
and(\text{eqs}(\text{m},\text{n}),\text{noteq}(\text{n},\text{d}))) \] becomes and(\text{eqs}(\text{m},\text{n}),\text{noteq}(\text{n},\text{d})). The expression \text{IMP}(\text{m},\text{n},\text{d}) can be used to represent \text{INV}(\text{m},\text{n}) \ast \text{INV}(\text{not}(\text{m}),\text{d}). Removing events on the port \text{mid} and using the expression \text{IMP}(\text{m},\text{n},\text{d}) to represent \text{INV}(\text{m},\text{n}) \ast \text{INV}(\text{not}(\text{m}),\text{d}) leads to

\[
\text{IMP}(\text{m},\text{n},\text{d}) \Leftarrow \\
\text{if and(\text{noteq}(\text{m},\text{n}),\text{noteq}(\text{n},\text{d})) then (} \{t\}\text{IMP}(\text{m},\text{n},\text{d}) \\
\quad + \{t, \text{in}>p:\text{noteq}(\text{p},\text{m})\}\text{IMP}(\text{p},\text{n},\text{d}) \)
\]

\[ + \text{if and(\text{eqs}(\text{m},\text{n}),\text{noteq}(\text{n},\text{d})) then (} \{t\}\text{IMP}(\text{m},\text{not}(\text{m}),\text{d}) \\
\quad + \{t, \text{in}>p:\text{noteq}(\text{p},\text{m})\}\text{IMP}(\text{p},\text{not}(\text{m}),\text{d}) \)
\]

\[ + \text{if and(\text{noteq}(\text{n},\text{d}),\text{eqs}(\text{m},\text{n})) then (} \{t, \text{out}<\text{not}(\text{n})\}\text{IMP}(\text{m},\text{not}(\text{m}),\text{not}(\text{n})) \\
\quad + \{t, \text{in}>p:\text{noteq}(\text{p},\text{m})\}\text{IMP}(\text{p},\text{not}(\text{m}),\text{not}(\text{n})) \)
\]

This can be further simplified, using the fact that whenever \text{eqs}(\text{m},\text{n}) is true, the second of \text{IMP}'s parameters becomes \text{not}(\text{m}), whereas when \text{noteq}(\text{m},\text{n}) is true, it becomes \text{n}, which must also equal \text{not}(\text{m}). This is the only effect of these predicates, so they can be removed, reducing the expression to:

\[
\text{IMP}(\text{m},\text{n},\text{d}) \Leftarrow \\
\text{if \text{noteq}(\text{n},\text{d}) then (} \{t\}\text{IMP}(\text{m},\text{not}(\text{m}),\text{d}) \\
\]

118
\[ + \{ t, \text{in}\!: \text{noteq}(p,m) \}\text{IMP}(p,\text{not}(m),d) \]
\[ + \text{if eqs}(n,d) \text{ then } (\{ t, \text{out}\!: \text{not}(n) \}\text{IMP}(m,\text{not}(m),\text{not}(n)) \]
\[ + \{ t, \text{in}\!: \text{noteq}(p,m), \text{out}\!: \text{not}(n) \}\text{IMP}(p,\text{not}(m),\text{not}(n)) \]  

This will be able to perform the same initial actions as BUF if \( m = w \), \( \text{not}(n) = x \) and \( d = y \). After performing the initial actions, IMP will be able to perform the same actions as BUF if these same relationships hold between the new parameters of BUF and those of IMP. That is, \( m = w \), \( \text{not}(\text{not}(m)) = w \), \( \text{not}(n) = x \) and \( d = x \). The first three equations follow from the initial conditions on the parameters; the last equation needs to be true only if \( \text{eqs}(x,y) \) is true, in which case it follows from the initial assumption that \( d = y \). It therefore follows that, if IMP and BUF commence in an equivalent state, they will always be able to perform the same actions.

It should be noted that in carrying out this proof, two types of reasoning were used. First, the laws of CIRCAL were used to develop a behavioural expression for IMP from its structural description and the behavioural specifications of its component parts. Then, laws of boolean logic were required to simplify predicates, to allow branches to be combined, and to show that the passed values and end states in the two behaviours IMP and BUF were equivalent. In general, a proof will involve reasoning about both the occurrence of actions and the equivalence of values or functions; these are referred to respectively as the temporal and functional aspects of a proof. It is only in the temporal aspect that CIRCAL provides a mechanism for carrying out the proof. In this example, the manipulations required to deal with the functional aspect were fairly straightforward; in more complicated example, the assistance of some sort of theorem prover might be desired. The use of such tools for verification is described in Section 5.2.6. However, dealing with the temporal aspect is both important and difficult, and it is apparent that CIRCAL is useful here. Whereas in this example the two aspects of the proof have been intermingled, ways in which they may be separated have been investigated, and are discussed in Section 5.2.4.

In this example, the implementation could be shown to be equivalent to specification in the fullest sense of the word, since the actions which can be performed
by the constructed device are exactly those which can be performed by the specification. A more useful definition of equivalence in many circumstances is that of 'strong satisfaction'[Milne85b]. Informally, this definition states that an implementation satisfies a specification if it can perform all the actions of the specification; it is also allowed to perform additional actions. This means that the implementation's behaviour need not be identical to the specification, as long as it includes all the behaviour of the specification. For deterministic devices (which includes the vast majority of hardware), the key requirement for satisfaction is that

\[ \text{SPEC} \ast \text{IMP} = \text{SPEC} \]

where '\(=\)' means 'can perform the same actions as'. The composition of \(\text{SPEC}\) with \(\text{IMP}\) ensures that any extra actions in \(\text{IMP}\) are removed before comparison with \(\text{SPEC}\) is attempted. Examples of the use of this approach to verification appear below.

### 5.2.2 Specification Techniques for Verification

It was shown in Section 5.1 that the development of specification languages and techniques in isolation can lead to unexpected problems when they are put to use. It is much more appropriate to develop a language by attempting to make use of it in some suitable application. One of the main applications of \textsc{CircaL} is for verification, and it is through attempting to use it for this purpose that many of the enhancements to the basic language and the techniques for using it have been developed and tested.

**Fictitious Events**

The effects of events which do not represent real changes of value (e.g., an event \(\{\text{out<true}\}\) when the value on the port \(\text{out}\) is already \text{true}) have already been discussed in the context of simulation, and it was shown that descriptions of this type could dramatically reduce the efficiency of a simulation. In verification, effi-
ciency may be less important, but this type of description can still cause problems, as the following example shows.

Example

A nand-gate can be specified in such a way that it generates an output event every time it receives an input event, as follows:

\[
\text{nandgate}(x,y) \leftarrow \{\text{ina}>p, \text{out}<\text{nand}(p,y)\}\text{nandgate}(p,y) \\
+ \{\text{inb}>p, \text{out}<\text{nand}(x,p)\}\text{nandgate}(x,p) \\
+ \{\text{ina}>p, \text{inb}>q, \text{out}<\text{nand}(p,q)\}\text{nandgate}(p,q)
\]

Explanation

The two state parameters represent the values on the two input ports \text{ina} and \text{inb} respectively. If a new value is input on either port, then the nand of this value and the value on the other port is output, regardless of whether it differs from the value currently on the port \text{out}.

This device is to be connected to the clock line of a positive-edge-triggered counter. Here it is vital that only true rising edges are accepted so that the counter really counts. Thus the counter would be specified as

\[
\text{counter}(c,n) \leftarrow \text{if not}(c) \text{ then} \\
\quad \{\text{clk}<\text{true}, \text{data}<\text{incr}(n)\}\text{counter}(\text{true},\text{incr}(n)), \\
+ \text{if } c \text{ then } \{\text{clk}<\text{false}\}\text{counter}(\text{false},n)
\]

Explanation

The value on the clock port is represented by \text{c} and the counter's stored value by \text{n}. True rising edges can only occur if \text{c} is false. In this state, the event \text{clk}<\text{true} may occur, causing \text{n} to be incremented and placed on the output. A falling edge on the clock (\text{clk}<\text{false}) has no effect on the stored value.
The effect of composing the nand-gate, which can generate the fictitious events, with this counter which cannot accept them, might be expected to lead to problems. In fact, expansion of the expression which describes their interconnection leads to the following:

\[
\begin{align*}
\text{nandgate}(x, y)[\text{clk/out}] & \cdot \text{counter}(c, n) \Leftarrow \\
& \{\text{clk}\lt\text{true, data}\lt\text{incr}(n), \text{ina}\lt\text{p}\lt\text{and}(\text{not}(c), \text{eqs}(\text{nand}(p, y), \text{true}))\} \\
& \text{nandgate}(p, y) \cdot \text{counter}(\text{true, incr}(n)) \\
+ & \{\text{clk}\lt\text{true, data}\lt\text{incr}(n), \text{inb}\lt\text{p}\lt\text{and}(\text{not}(c), \text{eqs}(\text{nand}(x, p), \text{true}))\} \\
& \text{nandgate}(x, p) \cdot \text{counter}(\text{true, incr}(n)) \\
+ & \{\text{clk}\lt\text{true, data}\lt\text{incr}(n), \text{ina}\lt\text{p, inb}\lt\text{q}\lt\text{and}(\text{not}(c), \text{eqs}(\text{nand}(p, q), \text{true}))\} \\
& \text{nandgate}(p, q) \cdot \text{counter}(\text{true, incr}(n)) \\
+ & \{\text{clk}\lt\text{false, ina}\lt\text{p}\lt\text{and}(c, \text{eqs}(\text{nand}(p, y), \text{false}))\} \\
& \text{nandgate}(p, y) \cdot \text{counter}(\text{false, n}) \\
+ & \{\text{clk}\lt\text{false, inb}\lt\text{p}\lt\text{and}(c, \text{eqs}(\text{nand}(x, p), \text{false}))\} \\
& \text{nandgate}(x, p) \cdot \text{counter}(\text{false, n}) \\
+ & \{\text{clk}\lt\text{false, ina}\lt\text{p, inb}\lt\text{q}\lt\text{and}(c, \text{eqs}(\text{nand}(p, q), \text{false}))\} \\
& \text{nandgate}(p, q) \cdot \text{counter}(\text{false, n})
\end{align*}
\]

Now consider what would happen if \(c\) were true. The only possible actions are the last three, since the others all contain input events where the parameter is restricted by a predicate which demands that \(c\) is false. The predicates in the last three actions all demand that any input events on the nand-gate be such that the new output of the nand-gate is false. Now suppose that the values on the ports \(x\) and \(n\) are both false. The output value of the nand-gate, and thus also \(c\), will be true. An event on only one of \(\text{ina}\) or \(\text{inb}\) would not make the nand-gate's output become false. These events, therefore, cannot take place. If any device connected to the inputs of the gate attempted to perform those events, deadlock would result.

This example illustrates the danger of using an inconsistent approach to modelling. One device was specified, of necessity, in such a way that only genuine
events (i.e. changes of value) could be accepted on its inputs. Another was specified, more conveniently, to accept any events and also to generate events which may not correspond to changes. The effect of wiring these two devices together was to introduce the possibility of deadlock in the model, even though the actual hardware which the two device descriptions are supposed to represent would not be able to deadlock. The solution is straightforward: the modelling philosophy which is adopted must be consistent; since there are situations such as the counter where non-genuine events are not acceptable, they must not be used in any device's specification.

A more general point to make here is that the language does not necessarily protect a user from writing incorrect specifications. It is to be hoped, however, that language features may be designed to reduce the opportunities for error. This was one of the main motivations for developing the language SuperC, described in Section 3.3.

Locked inputs

Some of the techniques presented in Section 3.2 were concerned with the prevention of 'locked' inputs. The justification given for this was simply that it contravenes the normal understanding of an input's behaviour in real hardware. In fact, these techniques arose from attempts to verify devices which were constructed from components whose inputs were occasionally locked. It had been hoped that, provided inputs were not required to change 'too quickly', then the specifications could be safely used. This turned out not to be the case, as the following example illustrates.

Example

A counter is to be constructed from cells, each of which controls one bit of the n-bit output and generates a carry signal for the next most significant cell. The cells can be specified constructively, with one part to control the output bit and
another to generate the carry. Both parts are to produce an output change one time unit after an input change. The first part is described as follows:

\[
\text{output}(c,d) \Leftarrow \\
\begin{align*}
&\text{if eqs}(c,\text{true}) \text{ then } \{\text{clk}, \text{t}\}\{\text{data<not}(d), \text{t}\}\text{output}(c,\text{not}(d)) \\
&\quad \quad \quad + \text{ if eqs}(c,\text{false}) \text{ then } \{\text{clk},\text{t}\}\text{output}(c,d) \\
&\quad \quad \quad + \{\text{cin>x:noteq}(x,c), \text{t}\}\text{output}(x,d) \\
&\quad \quad \quad + \{\text{t}\}\text{output}(c,d)
\end{align*}
\]

**Explanation**

The parameters \(c\) and \(d\) represent the values on the carry input port \(\text{cin}\) and the output bit, \(\text{data}\). For simplicity, the clock is assumed to be a pulse. The first branch indicates that if \(c\) is true then a clock event coinciding with a tick will cause the output value to change on the following tick. If \(c\) is false, a clock pulse has no effect. Other possible events are a new value being placed on \(\text{cin}\), and an unaccompanied tick of the universal clock.

The carry generation part is described by the following expression:

\[
\text{cgen}(c,d,z) \Leftarrow \text{if eqs}(d,\text{true}) \text{ then } \\
\begin{align*}
&\{\text{cin>x:noteq}(x,c), \text{t}\}\{\text{cout<x}, \text{t}\}\text{cgen}(x,d,x) \\
&\quad \quad \quad + \text{ if eqs}(d,\text{false}) \text{ then } \{\text{cin>x:noteq}(x,c), \text{t}\}\text{cgen}(x,d,z) \\
&\quad \quad \quad + \text{ if eqs}(c,\text{true}) \text{ then } \\
&\quad \quad \quad \quad \{\text{data>x:noteq}(x,d), \text{t}\}\{\text{cout<x}, \text{t}\}\text{cgen}(c,x,x) \\
&\quad \quad \quad + \text{ if eqs}(c,\text{false}) \text{ then } \{\text{data>x:noteq}(x,c), \text{t}\}\text{cgen}(c,x,z) \\
&\quad \quad \quad + \{\text{t}\}\text{cgen}(c,d,z)
\end{align*}
\]

**Explanation**

The values on the inputs \(\text{cin}\) and \(\text{data}\) are represented by the parameters \(c\) and \(d\) respectively, and the value on the carry output port \(\text{cout}\) by \(z\). The most important aspect of the behaviour here is that changes on \(\text{cout}\) occur on the tick following the input changes which caused them, as indicated in the first and third
branches, and that no other input events may take place until these output events have propagated.

Then the whole cell is described by

\[ \text{CELL} \leftarrow \text{cgen} \ast \text{output} \]

n of these cells must be connected together to construct an n-bit counter. Figure 5–2 shows the connection of two cells, with some relabelling to enable correct wiring up of ports.

Now consider what happens if the values on the ports cin, data0 and cout0 are true when a clock event occurs. On the first tick after the clock pulse, the values on both data0 and data1 are complemented. This now means that the carry generation parts of both cells are committed to producing an event on their carry output ports, cout0 and cout1 at the next tick. (This follows from the third branch of the description of cgen.) However, since the second cell is also connected to cout0, and is unable to accept an event on that port at the next tick, the system will reach a deadlock. Thus the conclusion to be drawn here is that the implementation is incorrect, since deadlock would certainly not be included in the counter's specification. In fact, if correctly described, this implementation would function correctly; therefore it is the specification technique which is at fault.

There are two ways in which this problem can be resolved. The obvious one is to use one of the techniques presented in Section 3.2 for the specification of devices
with delays. Another approach would be to attach a constraint to the specification to ensure that the assumptions under which it was written (i.e. that inputs do not change 'too quickly') are actually satisfied. This approach is discussed in Chapter 7.

5.2.3 Verification Without Implementation Assumptions

In the verification of the two time-unit delay buffer at the beginning of Section 5.2.1, some assumptions were made about the implementation before writing the specification. Because of the simplicity of the example, the timing properties which would be exhibited by the implementation could be easily deduced, and the specification was tailored accordingly. This approach is not really satisfactory in real-world situations; the specification of a complicated system will be written to meet some set of requirements without any knowledge of how the system will be implemented. The specification may need to be revised after attempts have been made to implement it, but it is not generally possible to start off with a specification which relies on information about the implementation.

The question to be asked, then, is whether the verification procedure outlined above can succeed when the specification is written without knowledge of how it will be implemented. The following example, a re-working of the buffer example of Section 5.2.1, demonstrates that it can.

Example

Rather than assuming that the delay across the buffer will be two ticks of the universal clock, another clock is introduced. The delay of the buffer will be one tick of this 'local' clock. The relationship between the local clock (whose ticks are designated by s) and the universal clock is not specified firmly; the only statement made about the two clocks is that the time between s ticks will be a non-zero multiple of the ticks of the universal clock. This relationship is described by the abstract device DEV as follows:

\[
\text{DEV} \Leftarrow \{t\text{DEV} + \{s,t\text{DEV}}
\]
This description ensures that ticks of the local clock, denoted by $s$, can only occur simultaneously with ticks of the universal clock, and that any number of $t$ events (including zero) may take place between a pair of $s$ events.

The buffer's specification is similar to that presented in Section 5.1.2:

$$\text{BUF}(x) \leq \{s\} \text{BUF}(x)$$
$$+ \{s$, in$>q:\text{noteq}(x,q)\} \text{BUF}(q)$$

$$\text{BUF}_1(x) \leq \{s$, out<x$\} \text{BUF}(x)$$
$$+ \{s$, in$>q:\text{noteq}(x,q), $out<x$\} \text{BUF}(q)$$

Explanation

$\text{BUF}(x)$ is the stable state, with $x$ being the current value on the input. A tick of the local clock in this state produces no output change. State $\text{BUF}_1(x)$ is unstable, and so an $s$ event will be accompanied by the placing of a new value on the output.

The proof now consists of trying to show that $\text{IMP}_1$ satisfies $\text{SPEC}$, where those two behaviours are defined by

$$\text{IMP}_1 \leq (\text{INV}[\text{mid/out}] \cdot \text{INV}[\text{mid/in}] - \text{mid}) \cdot \text{DEV}$$

$$\text{SPEC} \leq \text{BUF} \cdot \text{DEV}$$

Note that both the buffer and the inverters must be connected to $\text{DEV}$ to ensure that $t$ and $s$ are in the sorts of both the specification and the implementation.

The first part of $\text{IMP}_1$ was calculated in Section 5.2.1 and given the name $\text{IMP}$. This is composed with $\text{DEV}$ to give:

$$\text{IMP}_1(m,n,d) \leq \text{IMP} \cdot \text{DEV}$$
$$= \text{if eqs(not}(n),d) \text{ then } \{t\} \text{IMP}_1(m,m,\text{not}(n))$$
$$+ \{t, in>p:\text{noteq}(p,m)\} \text{IMP}_1(p,m,\text{not}(n))$$
$$+ \{s, t, in>p:\text{noteq}(p,m)\} \text{IMP}_1(p,m,\text{not}(n))$$
$$+ \{s, t\} \text{IMP}_1(m,m,\text{not}(n)) \}$$
if \text{noteq}(\text{not}(n),d) \text{ then } (\{t,\text{out}<n\}\text{IMP1}(m,m,\text{not}(n))
+ \{t,\text{in}>p:\text{noteq}(p,m),\text{out}<n\}\text{IMP1}(p,m,\text{not}(n))
+ \{s,t,\text{in}>p:\text{noteq}(p,m),\text{out}<n\}\text{IMP1}(p,m,\text{not}(n))
+ \{s,t,\text{out}<n\}\text{IMP1}(m,m,\text{not}(n)) )

The next step in the proof is to show that \text{IMP1} \ast \text{SPEC} is equal to \text{SPEC}. The expansion of \text{SPEC} gives

\text{SPEC} \leq \text{BUF} \ast \text{DEV}
- \{t\}(\text{BUF}(x) \ast \text{DEV}) + \{s,t\}(\text{BUF}(x) \ast \text{DEV})
+ \{s,t,\text{in}>q:\text{noteq}(x,q)\}(\text{BUF1}(q) \ast \text{DEV})

Therefore

\text{IMP1} \ast \text{SPEC} \leq \text{if } \text{eqs}(\text{not}(n),d) \text{ then } (\{t\}(\text{IMP1}(m,m,\text{not}(n)) \ast \text{BUF}(x) \ast \text{DEV})
+ \{s,t\}(\text{IMP1}(m,m,\text{not}(n)) \ast \text{BUF}(x) \ast \text{DEV})
+ \{s,t,\text{in}>p:\text{noteq}(p,m)\}(\text{IMP1}(p,m,\text{not}(n)) \ast \text{BUF1}(p) \ast \text{DEV}) )

which can be seen to be equal to \text{SPEC}, in terms of the actions which can be performed at this first level, provided \text{eqs}(\text{not}(n),d) is true (which is simply a requirement that the second inverter is initially in a stable state). Further expansions of the composition operator show that the implementation satisfies the specification fully.

The significant point here is that the specification was written without knowledge of the implementation and yet it was nevertheless possible to perform a successful verification of the implementation which was subsequently produced. The only assumption made about the behaviour of the implementation when writing the specification was that its delay would be an exact number of ticks of the universal clock. This is a reasonable assumption to make if the delays across all components in the implementation are to be measured in terms of those ticks. In more complicated examples, it may be important to be able to write a specification without guessing how the implementation will behave; this is illustrated in Chapter 6.
Two general principles emerge from this example. The first is that two separate but related clocks may be used to describe delays in such a way that the length of the delay is established by the implementation rather than by the specification. This technique should be applicable to a variety of situations, whenever a designer is not concerned with the exact length of time required by a piece of hardware to perform its function. The second principle, which is a generalisation of the first, is that it is possible to write a specification with an amount of uncertainty, this uncertainty being removed when the implementation is designed and verified. It is interesting to note that an implementation may contain more information than a specification and yet be shown to satisfy the specification. This is an encouraging result, as it demonstrates the usefulness of CIRCAL in showing the equivalence of circuit descriptions which contain different amounts of information.

5.2.4 Separating Timing and Function

In the example of Section 5.2.1 two aspects of the proof process, temporal and functional, were identified. CIRCAL does not provide any direct means for carrying out the functional aspects of the proof. However, it would be useful to be able to split the verification problem into its two parts, using suitable reasoning frameworks to deal with each. An attempt to formalise this separation procedure is presented in this Section.

Whether it is required to prove equivalence or satisfaction between specification and implementation, it is ultimately necessary to show that two behavioural expressions are equivalent. In most circumstances, each behaviour will be a deterministic sum of branches. It may be possible to show at once that they are not equivalent, if there is a guard in one behaviour which cannot be matched with any guard in the other. In the introductory example, branches were combined until the guards in the two behaviours were identical. However, it is possible for non-identical guards to match. In general, two guards can be matched if they contain events on exactly the same ports and the events match. Any type of output event (value, parameter or function) matches any other output event (on the same
port), two input events match if there is some set of values which satisfy both the attached predicates, and two synchronisation events match.

Using these rules, a number of guards in one behaviour may match a single guard in the other (as was the case in Section 5.2.1 before the branches of IMP were combined). In order to show that a number of branches in one behaviour are equivalent to a single branch in the other, certain relationships between the predicates and functions in the two behaviours must be satisfied. The derivation of these relationships in the case where the guard consists of a single output event is presented below.

The two behaviours are

\[
A(x) \Leftarrow \text{if } f_1(x) \text{ then } \{p<g_1(x)\}A(h_1(x)) \\
+ \text{ other branches}
\]

\[
B(y) \Leftarrow \text{if } f_2(y) \text{ then } \{p<g_2(y)\}B(h_2(y)) \\
+ \text{if } f_3(y) \text{ then } \{p<g_3(y)\}B(h_3(y)) \\
+ \text{ other branches}
\]

In each case, the 'other branches' contain guards involving ports other than p. All the fs are predicates, the gs and hs are functions. Although each behaviour is parameterised over only one variable, the following argument easily extends to a number of parameters.

Assuming that A described the specification and B the implementation, the first requirement is that, for any value of x, there is some value of y such that the value passed on p will be the same for both behaviours. That is,

\[
\forall x \ (\exists y. \ (f_1(x) \land f_2(y) \supset g_1(x) = g_2(y)) \\
\land (f_1(x) \land f_3(y) \supset g_1(x) = g_3(y)) \\
\land (f_1(x) \equiv f_2(y) \lor f_3(y)))
\]

The third line ensures that there will be a valid branch in the behaviour B whenever there is a valid branch in A.

The above formulae, if satisfied, ensure that the initial actions of A and B are identical. To ensure that subsequent actions are identical, some relations need
to be established between the end states of the two behaviours. In short, the relationships defined above which must exist between $x$ and $y$ must also exist between the parameters of the end states. If the three lines following $\exists y$ above are represented by a single predicate $P(x, y)$, then the following relationship must also be satisfied:

$$ (f_1(x) \land f_2(y) \supset P(h_1(x), h_2(y))) $$

$$ \land (f_1(x) \land f_3(y) \supset P(h_1(x), h_3(y))) $$

The first line ensures that the second branch of $B$ leads to states in which the two behaviours can perform identical actions; the second line takes care of the second branch of $B$.

This result demonstrates that it is possible to show that two behavioural expressions are equivalent by proving a logical formula which contains no reference to time or the ordering of events. That is, the proof problem has been effectively divided into the temporal part, which is handled by CIRCAL, and the functional part, which may be proved either manually or with some mechanical assistance. The above example is far from being completely general, but the basic principles of matching guards and establishing that passed values are equivalent in various circumstances could be readily extended to other situations. Since the verification of timing properties is often a major constituent of a correctness proof, the ability of CIRCAL to deal with them may make a significant contribution to verification.

5.2.5 Verification vs. Exhaustive Simulation

Only exhaustive simulation, in which a device is simulated under all possible combinations of input patterns and internal states, can provide the same certainty of an implementation's correctness that can be achieved by formal verification. It is widely accepted that exhaustive simulation is not an acceptable validation technique, as it is prohibitively time-consuming for devices of realistic complexity. Having seen how much effort is required to perform a very simple verification
exercise, however, one might be tempted to draw the same conclusion about verification. Fortunately, this turns out not to be the case.

The development of the CIRCAL language as described in Chapter 3 has meant that specifications in the language need not grow exponentially in size with the number of states of a device. This has been a necessary first step in reducing the complexity of the verification task. As was discussed above, the proof techniques which are employed for devices specified in CIRCAL deal quite well with timing and sequencing aspects of behaviour; the remaining complexity is in the area of functional correctness. There are, however, many situations in which functional correctness can be established with less effort than would be required if exhaustive simulation was performed. The equivalent of exhaustive simulation in theorem proving is the technique of case analysis; there are, fortunately, many other proof techniques which may be used in certain circumstances and which are far more efficient, such as mathematical induction for example.

One main advantage of mathematical proof for validation is that once some theorem has been proved for one part of a system, it can be re-used any number of times in the rest of the system. Furthermore, many hardware devices have a regular structure which may be parameterised by the number of bits in the data words which are processed. Then, a single verification of a general n-bit device will suffice for any number of specific instances of the device in the design of a whole chip. Such savings of effort are not generally possible when validation is performed by simulation.

Some other verification efforts have concentrated on these problems of tackling complexity. A selection of these projects is presented and discussed below.

5.2.6 Other Approaches

Whereas CIRCAL is a framework created specifically for the purposes of hardware description and verification, a number of other research efforts have attempted to use existing frameworks to reason about hardware. One such framework in which verification has been attempted with considerable success is that of the
Boyer-Moore logic[Boyer81] (a form of first-order logic), notably by Hunt[Hunt86a, Hunt86b]. A mechanised theorem prover was already available for this logic and Hunt was able to find a way of applying it to the description and verification of hardware, the largest example to be tackled being a 16-bit microprocessor of quite considerable complexity. Some simplifying assumptions were necessary to achieve this result, in particular a fairly simplified model of timing and the use of bistate logic throughout the design. The bottom level of hierarchy in this design was the level of logic gates. This verification represented quite a landmark in terms of the complexity of the device verified.

Another noteworthy framework in which hardware verification is being investigated is higher-order logic. The use of this logic for the description of hardware has already been examined in Section 2.2.3. Hanna[Hanna85] was one of the first to use this approach, representing hardware devices as predicates over waveform specifications. The treatment of time in this work is very rigorous, and the verification which is performed relies on a minimum of assumptions about the implementation; in this way it is similar to the approach proposed in Section 5.2.3. The penalty of this rigorous approach however is that the complexity of the verification task becomes quite high. Consequently, the examples which have been tackled in this way so far have been at a fairly low level of abstraction.

Higher-order logic has also been applied to verification using the HOL system [Gordon85] by Gordon and numerous others[Camilleri86,Joyce86]. Examples of the descriptive technique used with this system appear in Section 2.2.3. The representation of time is neither as simplistic as Hunt's nor as detailed as that adopted by Hanna. Consequently, the complexity of devices verified with the HOL system is also somewhere between those verified using the other approaches. The largest example tackled with HOL to date is probably the VIPER microprocessor, which has been verified between two fairly high levels in the design hierarchy[Cohn87].

A rather different framework which has been applied to the description and verification of hardware is temporal logic[Moszkowski83,Moszkowski85]. Rather than introducing time as a variable (as in HOL, for example), temporal logic extends traditional logic by adding temporal operators. These allow statements
of the form 'always \( p \)' and 'sometimes \( q \)' where \( p \) and \( q \) are logical formulae. Such statements are related to intervals, which are sequences of instants.

The above research efforts represent only a fairly small cross-section of those being undertaken; more comprehensive surveys of the subject exist[Camurat87]. The success with which these attempts have met illustrates the fact that complexity need not be an insurmountable obstacle to verification, while the diversity of areas in which they have been applied demonstrates the extent to which the nature of languages influences the verification task.

5.2.7 Verification of Transformations

In Section 4.3.2 the issue of validating automated designs was discussed. While it is perfectly possible to validate the output of the automation tool just as if it had been built by a human designer, it is preferable to validate the tool itself; in this way all devices produced by the tool may be guaranteed correct, at a cost of only a single validation effort.

The formal verification of a very simple design automation tool is the subject of a paper by Milne[Milne83b]. The tool is described formally as a transformation which maps logical expressions to layout information. A simple language \( \text{NE} \) is used to represent the logical expressions, and another language \( \text{LL} \) to represent layout. The transformation is represented by a function \( \mathcal{L} \). Functions to map \( \text{NE} \) expressions and \( \text{LL} \) expressions to \( \text{CIRCAL} \) are defined as \( \mathcal{N} \) and \( \mathcal{S} \) respectively. The task of validating the transformation is therefore to establish that

\[
\forall n \in \text{NE}. \mathcal{S} \circ \mathcal{L}(n) = \mathcal{N}(n)
\]

Because of the simplicity of the two languages used, this task is fairly straightforward. It remains to be seen whether it could be applied to realistic silicon compilers or other tools. However, it certainly establishes the value of using languages with well-understood semantics as the input and output languages of DA tools, as it is only with such languages that this type of validation could be attempted.
5.3 Summary

In this Chapter the validation task, consisting of the composition of implementation behaviours and their comparison with specifications in an attempt to establish that a design step has been correctly performed, has been presented. Two approaches to the task, simulation and formal, mathematical verification, were introduced. The former approach is popular because it places fewer requirements on the specification language to be used, as well as simply being better established as a validation technique. Verification, however, offers the significant advantage over simulation of guaranteed circuit correctness.

Since simulation consists of establishing the response of a circuit to selected input stimuli, some way of specifying these stimuli is required. A case was made for using a standard behavioural hardware description language to do this. Two approaches to the simulation of constructed devices were discussed. The more common of these involves establishing the possible responses of each component at each simulation step and using this information to calculate the resultant action. The only requirement on the language in this case is that the responses of behaviourally specified components to input stimuli can be established. This requirement is satisfied by the majority of behavioural hardware description languages.

The fact that CIRCAL enables behaviours to be constructed mathematically, together with its event-based model of behaviour, make it suitable for a different approach to simulation. Expansion of CIRCAL’s composition operator leads to behavioural expressions from which the responses of constructed circuits to input stimuli can readily be calculated.

The ability to construct behaviours mathematically is essential to formal verification. Some small examples were used to show how proofs could be carried out using CIRCAL. Two aspects of verification, temporal and functional, were identified, and it was seen that CIRCAL is able to provide real assistance in only the first of these. However, it also assists in separating the two aspects in a way which
would facilitate the solution of the functional aspect by some other means. It was shown that CIRCAL could enable a specification with some uncertainty to be matched against an implementation in which the uncertainty was removed. This means that specifications can be written using a minimum of assumptions or prior knowledge about the implementation (as they would normally be in real design situations) without inhibiting the verification task.

One of the main concerns in the development of verification techniques is the complexity which is encountered. While this is a problem, it was seen to be less severe than the computational complexity of exhaustive simulation which prohibits its use in realistic situations. The development of languages for verification was seen as an essential first step in combating complexity. Some of the other approaches to verification which have dealt with the complexity problem to some extent were discussed.

The attempts to carry out both simulation and verification using descriptions written in CIRCAL illustrated the need to test and develop specification techniques by putting specifications to work. Some of the specification techniques of Chapter 3 which seemed intuitively correct were justified by examples which demonstrated the dangers of ignoring those techniques. In particular, it was seen that a verification of a quite reasonable implementation could fail if the device specifications were not sufficiently accurate. The general principle here seems to be that the modelling philosophy which is adopted must be consistent and as realistic as possible.
Chapter 6

Example: A Simple Computer

In the preceding three Chapters, the three main tasks of the proposed methodology (specification, design and validation) have been presented and discussed. The examples used to illustrate the points in these chapters have been necessarily quite simple. Furthermore, the examples have generally been chosen to illustrate a single point about just one of the tasks. It is the aim of this Chapter to illustrate the way in which these tasks fit together in the execution of a single design step by tackling a significantly larger example than those presented above.

The system which has been chosen for examination is based on a simple computer which was originally verified using the LCF/LSM system[Gordon81a]. The same implementation has subsequently been verified using HOL[Joyce86]. In this Chapter, a specification of the computer using enhanced CIRCAL will be developed. The verification of the computer as specified would be extremely involved and rather too lengthy to present here. Therefore a simplified version of the problem will be posed. A way of approaching the design which assists subsequent verification will be presented, followed by the verification itself. Before commencing formal treatment of the problem, however, it is necessary to examine the informal statement of it.
6.1 Informal Specification

The computer has a 13-bit program counter, a 16-bit accumulator and an \(8k \times 16\)-bit random access memory (RAM). There is a 16-bit input port for the loading of values into the registers, and a 4-position switch to determine the mode of operation of the computer. There is also a button, the use of which is dependent on the current mode and is described below. The values stored in each register may be observed at a pair of output ports, one 13 bits wide and the other 16. A single bit output port \(idle\) is also provided; its function is also described below. A ‘black-box’ diagram of the computer appears in Figure 6-1. Ports are illustrated in various widths to indicate the width of word (in bits) which can be passed along it. The ports on the left are inputs, those on the right are outputs.

\[\text{Figure 6-1: Black-box Diagram of Computer}\]

The 4-position switch may be considered as an input port of type \(\text{int2}\), this being a type containing all the integers which may be represented by 2 bits. The value \(n\) on this port when the button is pushed, provided \(idle\) is true, determines the operation of the computer as follows:

\(n = 0\): The program counter register is loaded with the value obtained by truncating the value on the 16-bit input port \(\text{datain}\) to 13 bits.

\(n = 1\): The accumulator is loaded with the value on the input.

\(n = 2\): The contents of the accumulator are loaded into the memory location whose address is in the program counter.
n = 3: The program stored in the memory is executed, beginning at the address which is in the program counter.

Execution of the program is the most involved part of the computer’s operation and is described below.

A 16-bit word is interpreted as an instruction by separating it into an opcode (the most significant 3 bits) and a 13 bit operand. The meanings of the 8 opcodes are explained in Table 6-1, with L being the operand, interpreted as an address.

<table>
<thead>
<tr>
<th>Opcode</th>
<th>Mnemonic</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>HALT</td>
<td>Stop execution of program</td>
</tr>
<tr>
<td>001</td>
<td>JMP L</td>
<td>Jump to L</td>
</tr>
<tr>
<td>010</td>
<td>JZRO L</td>
<td>Jump to L if acc. contains 0</td>
</tr>
<tr>
<td>011</td>
<td>ADD L</td>
<td>Add contents of L to acc.</td>
</tr>
<tr>
<td>100</td>
<td>SUB L</td>
<td>Subtract contents of L from acc.</td>
</tr>
<tr>
<td>101</td>
<td>LD L</td>
<td>Load contents of L into acc.</td>
</tr>
<tr>
<td>110</td>
<td>ST L</td>
<td>Store contents of acc. at L.</td>
</tr>
<tr>
<td>111</td>
<td>SKIP</td>
<td>Skip to next instruction</td>
</tr>
</tbody>
</table>

Table 6-1: Meanings of the Opcodes

The value on idle becomes false when execution begins and true when execution stops. Execution may be stopped either by a HALT instruction or by pushing the button, in which case execution of the current instruction will be completed before stopping.

That describes fully the operation of the computer. The following Section shows how this description can be captured as a formal specification in enhanced CIRCAL.
6.2 Formal Specification

Before writing the actual CIRCAL descriptions for the computer, some type and function definitions are necessary. The required types are as follows:

- `ipt2`, `int13`, and `int16`. These are defined as 2-, 13- and 16-bit integers respectively.
- `memory`. This type represents the 8k × 16-bit random access memory.
- `opcode`. A variable of this type may be any one of the 8 opcodes.

Functions required for the manipulation of objects of the above types are:

- **truncate**: `int16 → int13`. Truncate a 16-bit integer to 13 bits.
- **store**: `memory * int16 * int13 → memory`. Return a new memory state, obtained by storing a 16-bit integer at a 13-bit address.
- **fetch**: `memory * int13 → int16`. Obtain the value stored at an address in a memory.
- **getop**: `memory * int13 → opcode`. Extract the opcode from a 16-bit integer stored at a location in memory.
- **getadd**: `memory * int13 → int13`. Extract the address from the 16-bit integer stored at a location in memory.
- **getarg**: `memory * int13 → int16`. Extract the low 13 bits from a location in memory and fetch the 16-bit integer stored at the resultant address.
- **incr**: `int13 → int13`. Increment a 13-bit integer.
- **add16**: `int16 * int16 → int16`. Add two 16-bit integers.
- **sub16**: `int16 * int16 → int16`. Subtract a 16-bit integer from a 16-bit integer.
Formal definitions of these types and functions are not presented here for reasons of space, but may be found in Appendix D.

A formal specification of the computer may now be formulated in enhanced CIRCAL. In order to simplify this process, it is helpful to make some assumptions about the way in which the device will be used. The informal specification does not, for example, state what happens if the switch is moved to a new position and the button is pushed simultaneously. Therefore, it is not particularly helpful to include in the formal specification a statement of what would happen in this instance. While it was seen in Section 5.2.2 that the writing of specifications which disallow certain input events could cause problems in verification, it will be demonstrated in the following Chapter that the placing of suitable restrictions on a device's environment can overcome these problems. For the moment, the implicit assumption is that simultaneous input events will not occur. In the next Chapter the formalisation of such assumptions will be presented.

Because it is rising edges on the port button which are significant, the edge detector idea introduced in Section 3.2 can be used to reduce the number of states and the interleaving of events in the main part of the specification. The edge detector box could be described as follows:

\[
\begin{align*}
\text{EDET} & \triangleq (t)\text{EDET} + (\text{button}\langle\text{true}\rangle)\text{EDET1} \\
& \quad + (t, \text{button}\langle\text{true}, \text{but}\rangle)\text{EDET2} \\
\text{EDET1} & \triangleq (t, \text{but})\text{EDET2} + (\text{button}\langle\text{false}\rangle)\text{EDET} \\
& \quad + (t, \text{button}\langle\text{false}\rangle)\text{EDET} \\
\text{EDET2} & \triangleq (t)\text{EDET2} + (\text{button}\langle\text{false}\rangle)\text{EDET} \\
& \quad + (t, \text{button}\langle\text{false}\rangle)\text{EDET}
\end{align*}
\]

**Explanation**

This is a little more complicated than the box of Section 3.2, as it ensures that but events occur synchronously with ticks, even if rising edges on the button port do not. In the state EDET the button is yet to be pushed i.e. the value on button is false. Ticks may be passively accepted, or if the button is pushed between ticks
then the device moves into state EDET1. If the button is pushed at the same time as a tick, then a pulse is generated immediately on but. In state EDET1 the next tick causes the pulse on but to be produced, unless the button is released first, in which case the device simply returns to the initial state EDET. In the third state, EDET2, the device simply waits for the release of the button before returning to EDET.

If this aspect of the computer's behaviour were included in the main part of the description, there would be quite a large number of ways in which the various events on button could interleave with events on other ports. This would complicate the description of the whole computer significantly. Separating this part of the behaviour from the main description considerably reduces the effort of writing that description.

The main specification can now be formulated. The states denoted by computer are the 'idle' states, in which the value on idle is true and pushing the button will cause an action dependent on the position of the mode switch. The state computer1 represents the execution of instructions in memory; in this state, idle is false and pushing the button will halt execution. The state parameters p, ac and dat are used to represent the values on the ports pc (program counter), acc (the accumulator) and datain respectively, and an additional parameter of type memory represents the state of the RAM. The last parameter represents the position of the mode switch.

```plaintext
computer(mem, p, ac, dat, 0) <=
    {but, pc=truncate(dat), t}
    computer(mem, truncate(dat), ac, dat, 0)
+ {datain>x:noteq(x,dat), t}
    computer(mem, p, ac, x, 0)
+ {mode>x:noteq(x,0), t}
    computer(mem, p, ac, dat, x)
+ {t} computer(mem, p, ac, dat, 3)
```

142
computer(mem, p, ac, dat, 1) <=
{but, acc<dat, t} computer(mem, p, dat, dat, 1)
+ {datain>x:noteq(x,dat), t}
computer(mem, p, ac, x, 1)
+ {mode>x:noteq(x,1), t}
computer(mem, p, ac, dat, x)
+ {t} computer(mem, p, ac, dat, 3)

computer(mem, p, ac, dat, 2) <=
{but, t} computer(store(mem,ac,p), p, ac, dat, 2)
+ {datain>x:noteq(x,dat), t}
computer(mem, p, ac, x, 2)
+ {mode>x:noteq(x,2), t}
computer(mem, p, ac, dat, x)
+ {t} computer(mem, p, ac, dat, 3)

computer(mem, p, ac, dat, 3) <=
{but, t, idle<false} computeri(mem, pc, dat, dat, 3)
+ {datain>x:noteq(x,dat), t}
computer(mem, p, ac, x, 3)
+ {mode>x:noteq(x,3), t}
computer(mem, p, ac, dat, x)
+ {t} computer(mem, p, ac, dat, 3)

computeri(mem, p, ac, dat, m) <=
if eqs(getop(mem,p),HALT) then {t, idle<true}
computer(mem, p, ac, dat, m)
+ if eqs(getop(mem,p),JMP) then
{t, pc<getadd(mem,p)}
computeri(mem, getadd(mem,p), ac, dat, m)
+ if and(eqs(getop(mem,p),JZRO),eqs(ac,0)) then
{t, pc<getadd(mem,p)}
computer\textsubscript{i}(\text{mem}, \text{getadd(\text{mem}, \text{p})}, \text{ac}, \text{dat}, \text{m})
+ \text{if and}(\text{eqs(\text{getop(\text{mem}, \text{p})}, \text{JZRO}), noteq(\text{ac}, 0)}) \text{ then } \\
\{t, \text{pc}<\text{incr(p)}\}
\text{computer}\textsubscript{i}(\text{mem}, \text{incr(p)}, \text{ac}, \text{dat}, \text{m})
+ \text{if eqs(\text{getop(\text{mem}, \text{p})}, \text{ADD}) \text{ then } \\
\{t, \text{pc}<\text{incr(p)}, \text{acc}<\text{add16(\text{getarg(\text{mem}, \text{p})}, \text{ac})}\}
\text{computer\textsubscript{i}(\text{mem}, \text{incr(p)}, \text{add16(\text{getarg(\text{mem}, \text{p})}, \text{ac})}, \text{dat}, \text{m})}
+ \text{if eqs(\text{getop(\text{mem}, \text{p})}, \text{SUB}) \text{ then } \\
\{t, \text{pc}<\text{incr(p)}, \text{acc}<\text{sub16(\text{ac}, \text{getarg(\text{mem}, \text{p})})}\}
\text{computer\textsubscript{i}(\text{mem}, \text{incr(p)}, \text{sub16(\text{ac}, \text{getarg(\text{mem}, \text{p})})}, \text{dat}, \text{m})}
+ \text{if eqs(\text{getop(\text{mem}, \text{p})}, \text{LD}) \text{ then } \\
\{t, \text{pc}<\text{incr(p)}, \text{acc}<\text{getarg(\text{mem}, \text{p})}\}
\text{computer\textsubscript{i}(\text{mem}, \text{incr(p)}, \text{getarg(\text{mem}, \text{p})}, \text{dat}, \text{m})}
+ \text{if eqs(\text{getop(\text{mem}, \text{p})}, \text{ST}) \text{ then } \{t, \text{pc}<\text{incr(p)}\}}
\text{computer\textsubscript{i}(\text{store(\text{mem}, \text{ac}, \text{p})}, \text{incr(p)}, \text{ac}, \text{dat}, \text{m})}
+ \text{if eqs(\text{getop(\text{mem}, \text{p})}, \text{SKIP}) \text{ then } \{t, \text{pc}<\text{incr(p)}\}}
\text{computer\textsubscript{i}(\text{mem}, \text{incr(p)}, \text{ac}, \text{dat}, \text{m})}
+ \{\text{but}, \text{t, idle<true}\} \text{ computer(\text{mem, p}, \text{ac}, \text{dat}, \text{m})}

An interesting point to note here is that the time taken for the execution of each instruction is assumed to be the interval between two ticks of the universal clock. The implementation may be expected to take varying amounts of time to execute different instructions, so there will need to be some way of relating the two different time scales.

6.2.1 A Reduced Problem

The above specification illustrates how the descriptive power of enhanced CIRCAL is sufficient to cope with the quite complicated example of a small computer. In order to illustrate the design and especially the verification phases of the methodology, however, it is necessary to reduce the complexity of the example. One simplification is to concentrate on only the execution of stored programs, ignoring
the three modes of operation in which data is input to registers or memory using \textit{datain}. A further simplification would be to reduce the instruction set to 4 instructions, say \textsc{ld}, \textsc{add}, \textsc{sub} and \textsc{jzro}. (This could be just enough to execute some useful programs stored in Read-Only Memory (ROM).) Since the opcode field needs only 2 bits now, addresses could be increased to 14 bits, and the ML functions and datatypes described above could be modified accordingly. The state of memory, \texttt{mem}, cannot now be modified, so need not be included in the list of parameters for the computer. When it appears as an argument to a function, it may now be considered as a constant. As the only mode of operation of the computer is execution of programs, there is no need for a button to halt it. The new computer therefore has sort \{\texttt{pc}, \texttt{acc}\} and can be specified as follows:

\begin{verbatim}
newcomputer(p, ac) <=
    if eqs(getop(mem,p),ADD) then
        \{t, accadd16(ac,getarg(mem,p)), pc<incr(p)}
        newcomputer(incr(p), add16(ac,getarg(mem,p)))
    + if eqs(getop(mem,p),SUB) then
        \{t, accsub16(ac,getarg(mem,p)), pc<incr(p)}
        newcomputer(incr(p), sub16(ac,getarg(mem,p)))
    + if eqs(getop(mem,p),LD) then
        \{t, accgetarg(mem,p), pc<incr(p)}
        newcomputer(incr(p), getarg(mem,p))
    + if and(eqs(getop(mem,p),JZRO),eqs(ac,0)) then
        \{t, pc<getadd(mem,p)}
        newcomputer(getadd(mem,p), 0)
    + if and(eqs(getop(mem,p),JZRO),noteq(ac,0)) then
        \{t, pc<incr(p)}
        newcomputer(incr(p), ac)
\end{verbatim}

\textbf{Explanation}

This specification is very similar to that for the execution mode of the full computer. In each branch, the current instruction is obtained using getop and
compared with one of the possible opcodes. In the case of JZRO, the action which results also depends on the value in the accumulator, represented by the parameter ac. Following the conditional in each branch is a guard consisting of a tick, an event on pc (either incrementing its value or loading it with a new value from memory), and possibly an event on acc\(^1\). The design and verification of the computer specified in this way will be described in the following Sections.

6.3 Design

In the verification of the full computer using both LCF\_LSM[Gordon81a] and HOL [Joyce86] the design step was a large one. That is to say, the amount of structural information which was added at the lower level of the design hierarchy was quite large. This is apparent simply from consideration of the large number and relatively small size of components into which the black-box computer was partitioned in each of these exercises. The large gap between the specification level and the implementation level makes the verification task significantly more complicated. As proposed in Section 4.1.1, if design is to assist verification, then smaller design steps should be taken. The contrast between these two approaches is illustrated in Figure 6-2.

In this Section, the design of the simplified example, newcomputer, will be presented. This will be done using a small design step, the black box being partitioned into just two fairly large boxes. Even though the example being tackled here is simpler than the computer first described, the boxes with which it is to be implemented are still larger than those of the implementation level of Gordon's

\(^1\)It should be noted that in this specification, as in the full specification of Section 6.2, there is the potential to perform fictitious output events, for example if getarg(mem,p) is 0 during an ADD operation. The consequences of this omission, which has been made to simplify the presentation of the example, are made clear below.
In the following Section, the way in which this assists the verification task will be demonstrated.

The first phase of design is partitioning. The simplest partitioning which can be made is to split the computer into two boxes, a control part and a data part. The latter handles the transfer of data between memory and registers, including the fetching of instructions, while the former provides control signals for the data part depending on the current instruction and possibly the accumulator value. This suggests that there should be a port to carry control signals from cpart to dpart, and ports to convey the values of the current instruction and the accumulator from dpart to cpart. This structural information is conveyed by the following CIRCAL expression and is illustrated in Figure 6-3.

```
computermp <= cpart * dpart - cnt1 - cur
```
6.3.1 The Data Part

The partitioning phase out of the way, the next step is to describe the boxes of the implementation. In order to define the behaviour of dpart it is helpful to consider what functions it must perform. At the start of each instruction cycle it must fetch an opcode and feed this to the control part. It may then be called upon to fetch a value from memory and either load this into the accumulator or add it to or subtract it from the value currently in that register. Alternatively, it may be called upon to load a new value into the program counter, or simply to increment the value in that register.

A simple way to view this behaviour is to define a special type for the cntl port so that any of these operations can be requested by the passing of the appropriate value. A suitable definition would be

```
datatype cntltype = opfetch /* fetch an opcode */
| argfetch /* fetch an argument */
| argld /* load argument into acc */
| argadd /* add argument to acc */
| argsub /* subtract argument from acc */
| pcld /* load new value into pc */
| pcincr /* increment value in pc */
```

By defining the control port in this way, the decision of how to implement it physically (e.g. with 7 wires of which only one may be true at one time) is left
until later, when the internal structure of dpart is defined. This is an example of
the way in which finer design steps assist design by allowing the postponement of
design decisions until more information becomes available.

It is now quite straightforward to define the behaviour of dpart. In order to
enable its timing characteristics to be reconciled with those of the specification,
the technique of Section 5.2.3 is adopted, with delays at this level of abstraction
being related to a different clock whose ticks are denoted by s.

dpart(p,ac,arg,cu) <=
  if noteq(cu,getop(mem,p)) then {cntl<opfetch, s}
    {cur<getop(mem,p), s}dpart(p,ac, arg, getop(mem,p))
  + if eqs(cu,getop(mem,p)) then {cntl<opfetch, s}
    {s}dpart(p,ac, arg, getop(mem,p))
  + {cntl<argfetch, s}dpart(p,ac, getarg(mem,p),cu)
  + if noteq(ac, arg) then {cntl<argld, s}
    {acc<arg, s}dpart(p, arg, arg, cu)
  + if eqs(ac, arg) then {cntl<argld, s}dpart(p, arg, arg, cu)
  + if noteq(arg, 0) then ( {cntl<argadd, s}
    {acc<add16(acc,arg), s}dpart(p, add16(acc, arg), arg, cu)
    + {cntl<argsub, s}
    {acc<sub16(acc,arg), s}dpart(p, sub16(acc, arg), arg, cu) )
  + if eqs(arg, 0) then ( {cntl<argadd, s}dpart(p, acc, arg, cu)
    + {cntl<argsub, s}dpart(p, acc, arg, cu) )
  + if noteq(p,getadd(mem,p)) then {cntl<pcld, s}
    {pc<getadd(mem,p), s}dpart(getadd(mem,p), acc, arg, cu)
  + if eqs(p,getadd(mem,p)) then {cntl<pcld, s}
    {s}dpart(p, acc, arg, cu)
  + {cntl<pcincr, s}{pc<incr(p), s}dpart(incr(p), acc, arg, cu)
  + {s}dpart(p, ac, arg, cu)
Explanation

The parameters $p$ and $ac$ represent the program counter and accumulator as before, while $arg$ represents the fetched argument following an $argfetch$ signal, and $cu$ is the value on the current instruction port $cur$. In each branch except the third and last the arrival of a new control signal on one tick is followed by an output event on the next, unless the current value on the output port equals the new value which would be placed there. The third branch involves no output event but simply a change in the internal state parameter $arg$ and takes only one tick of the clock, while the last branch simply states that the device will remain in a steady state as time passes if no $cntl$ input is provided.

There are a few things to note in this description. One is that the rule of avoiding locked inputs has been ignored, since an event on the $cntl$ input is usually followed by some output event on the next tick. The justification for this is that the only device which will ever attempt to communicate on the $cntl$ port is the control part, so as long as it is designed with this limitation in mind, there should be no problem. In fact, this is an example of a situation in which a constraint is being applied to a device's environment — the formal treatment of such situations is discussed in the next Chapter. An additional assumption made about the behaviour of $cpart$ is that it will not generate fictitious events on the $cntl$ port, which again should be borne in mind when writing its specification.

6.3.2 The Control Part

The data part implements the various functions of the computer; it is the role of the control part to ensure that these functions occur at the appropriate time. That is, given inputs of the current instruction and accumulator value, it must generate the correct sequence of values on the control port. This may be specified formally as follows:

\[
cpart(cu, ac) \leftarrow \{\text{cntl} : \text{opfetch}, s\}cpart1(cu, ac)
\]

\[
cpart1(cu, ac) \leftarrow \{\text{cur} : \text{noteq}(x, cu), s\}cpart2(x, ac)
\]
+ {s}cpart2(cu,ac)
cpart2(cu,ac) <=
  if or(egs(cu,ADD),or(egs(cu,SUB),egs(cu,LD))) then
    {cntl<argfetch, s}cpart3(cu,ac)
  + if and(egs(cu, JZRO),eqs(ac,0)) then
    {cntl<pcincr, s}cpart6(JZRO,0)
  + if and(egs(cu, JZRO),noteq(ac,0)) then
    {cntl<pcincr, s}cpart6(JZRO,ac)
cpart3(cu,ac) <=
  if eqs(cu,ADD) then {cntl<argadd, s}cpart4(cu,ac)
  + if eqs(cu,SUB) then {cntl<argsub, s}cpart4(cu,ac)
  + if eqs(cu,LD) then {cntl<argld, s}cpart4(cu,ac)
cpart4(cu,ac) <= {s}cpart5(cu,ac)
  + {s,acc>x:noteq(x,ac)}cpart5(cu,x)
cpart5(cu,ac) <= {cntl<pcincr, s}cpart6(cu,ac)
cpart6(cu,ac) <= {s}cpart(cu,ac)

Explanation

In the initial state cpart, the control signal is given to fetch the next opcode. In cpart1, the opcode is read in, the second term of the choice sum allowing for the possibility that the new opcode will be the same as the last one. cpart2 is the instruction decode stage. For all instructions other than the jump, the first step is to fetch the argument from memory, so the necessary control signal is argfetch. If the accumulator contains 0, then the jump requires a new value to be loaded into pc; otherwise pc must just be incremented. In either case, the next state is cpart6 and, after a delay of one tick, the instruction is now complete and the control part returns to its initial state. For the other 3 instructions, a further decoding is required, this being done in state cpart3. The control signal to add, subtract or load the argument into the accumulator is generated, then in state cpart4 there is another delay of one tick. Since the operation may cause a new value to be placed into the accumulator, this possibility is also accounted for in this state. In state
The program counter is incremented and after one further tick the device returns to its initial state.

Having partitioned and described the implementation completely, it now needs to be shown that its behaviour satisfies that of the specification. This task is described in the following Section.

6.4 Verification

As in Section 5.2.3, the timing characteristics of the implementation are measured with respect to a different clock from those of the specification. Therefore, the first step of the verification proof is to define the relationship between the two clocks. The requirement is that the implementation will, after a certain number of $s$ ticks, perform the correct action (as laid down by the specification) at the same time as a $t$ tick. As before, it is assumed that there may be one or more $s$ ticks between the $t$ ticks. At the outset of the verification, it is not known exactly how many $s$ ticks there will be, so the device which defines the relationship between the two clocks can be described in such a way as to allow this uncertainty:

$$
\text{DEV}(n) \leq \begin{cases} 
\text{noteq}(n,1) & \text{then } \{s\}\text{DEV}(\text{decr}(n)) \\
+ \text{if } \text{eqs}(n,1) & \text{then } \{s, t\}\text{DEV}(M)
\end{cases}
$$

The initial value of $n$ is left unspecified, as is the value of $M$ to which the tick-counting parameter is assigned. This uncertainty will be removed as the verification proceeds.

In order for the implementation to satisfy the specification, it is required that the actions which may be performed by

$$
\text{IMP} \leq (\text{cpart} \times \text{dpart} - \text{cntl} - \text{cur}) \times \text{DEV} \times \text{newcomputer}
$$

are the same as the actions of

$$
\text{SPEC} \leq \text{DEV} \times \text{newcomputer}
$$
The behaviour of IMP is obtained by expansion of the composition and abstraction operators. The first step is to expand \( \text{cpart} \circ \text{dpart} \) using the synchronisation rules of Section 3.1.7:

\[
\text{cpart}(cu, ac) \circ \text{dpart}(p, ac, arg, cu) = \\
\text{if noteq}(cu, \text{getop}(mem, p)) \text{ then } \{ \text{cntl}<\text{opfetch}, s\} \\
\text{ (cpart1}(cu, ac) \circ \\
\text{ (cur}<\text{getop}(mem, p), s)\text{dpart}(p, ac, arg, \text{getop}(mem, p)) ) \\
+ \text{if eqs}(cu, \text{getop}(mem, p)) \text{ then } \{ \text{cntl}<\text{opfetch}, s\} \\
\text{ (cpart1}(cu, ac) \circ \{s\}\text{dpart}(p, ac, arg, \text{getop}(mem, p)) )
\]

Inserting the definition of \( \text{cpart1} \) and expanding again gives:

\[
\text{cpart}(cu, ac) \circ \text{dpart}(p, ac, arg, cu) = \\
\text{if noteq}(cu, \text{getop}(mem, p)) \text{ then } \{ \text{cntl}<\text{opfetch}, s\} \\
\text{ (cur}<\text{getop}(mem, p), s)\text{ (cpart2}(\text{getop}(mem, p), ac) \circ \\
\text{ dpart}(p, ac, arg, \text{getop}(mem, p)) ) \\
+ \text{if eqs}(cu, \text{getop}(mem, p)) \text{ then } \{ \text{cntl}<\text{opfetch}, s\} \\
\text{ (s)(cpart1}(cu, ac) \circ \text{dpart}(p, ac, arg, \text{getop}(mem, p)) )
\]

Abstraction on \( \text{cur} \) and \( \text{cntl} \) leads to:

\[
\text{cpart}(cu, ac) \circ \text{dpart}(p, ac, arg, cu) - \text{cntl} - \text{cur} = \\
\text{if noteq}(cu, \text{getop}(mem, p)) \text{ then } \{s\} \{s\} \\
\text{ (cpart2}(\text{getop}(mem, p), ac) \circ \text{dpart}(p, ac, arg, \text{getop}(mem, p)) ) \\
+ \text{if eqs}(cu, \text{getop}(mem, p)) \text{ then } \{s\} \{s\} \\
\text{ (cpart1}(cu, ac) \circ \text{dpart}(p, ac, arg, \text{getop}(mem, p)) )
\]

which simplifies to

\[
\{s\} \{s\} (\text{cpart2}(\text{getop}(mem, p), ac) \circ \text{dpart}(p, ac, arg, \text{getop}(mem, p)) )
\]

If IMP\( k(cu, p, ac, arg, n) \) is defined as
(cpartk(cu,ac) * dpart(p,ac,arg,cu) - cntl - cur)
* DEV(n) * newcomputer(p,ac)

then further expansion of the composition operator gives

IMP(cu,p,ac,arg,n) <= if and(noteq(n,1),noteq(decr(n),1)) then
{s}(s)IMP2(getop(mem,p),p,ac,arg,decr(decr(n)))

The behaviour of SPEC is given by

SPEC(p, ac, n) <= if eqs(n,1) then {
    if eqs(getop(mem,p),ADD) then
        {t, s, acc<add16(ac,getarg(mem,p)), pc<incr(p)}
        SPEC(incr(p), add16(ac,getarg(mem,p)), M)
    + if eqs(getop(mem,p),SUB) then
        {t, s, acc<sub16(ac,getarg(mem,p)), pc<incr(p)}
        SPEC(incr(p), sub16(ac,getarg(mem,p)), M)
    + if eqs(getop(mem,p),LD) then
        {t, s, acc<getarg(mem,p), pc<incr(p)}
        SPEC(incr(p), getarg(mem,p), M)
    + if and(eqs(getop(mem,p),JZRO),eqs(ac,0)) then
        {t, s, pc<getadd(mem,p)}
        SPEC(getadd(mem,p), 0, M)
    + if and(eqs(getop(mem,p),JZRO),noteq(ac,0)) then
        {t, s, pc<incr(p)} SPEC(incr(p), ac, M )
    + if noteq(n,1) then {s} SPEC(p, ac, decr(n))
}

Now it can be seen that the two behaviours are equivalent at the first level (capable of performing the same initial actions) if noteq(n,1) is true, since in this case both behaviours can simply accept an s event.

Expansion of the composition and abstraction operators for IMP2 gives

IMP2(cu,p,ac,arg,n) <=
if \( \text{and} \left( \text{or} \left( \text{eqs}(c_u, \text{ADD}), \text{or} \left( \text{eqs}(c_u, \text{SUB}), \text{eqs}(c_u, \text{LD}) \right) \right), \text{noteq}(n,1) \right) \) then \{s\}\text{IMP3}(c_u, p, ac, \text{getarg}(\text{mem}, p), \text{decr}(n))
+ if \( \text{and} \left( \text{and} \left( \text{eqs}(c_u, \text{JZRO}), \text{noteq}(n,1) \right) \right) \right) \) then
\( \right) \) then
if \( \text{noteq}(\text{getadd}(\text{mem}, p), p), \text{eqs}(ac,0) \) then
\{s\}\{pc<\text{getadd}(\text{mem}, p), s, t\}
\text{IMP}(\text{JZRO}, \text{getadd}(\text{mem}, p), ac, arg, M)
+ if \( \text{noteq}(ac,0) \) then \{s\}\{pc<\text{incr}(p), s, t\}
\text{IMP}(\text{JZRO}, \text{incr}(p), ac, arg, M) )

In the last two branches of this behaviour, events involving external ports other than \( s \) can at last be observed. The process of showing that the actions of IMP and SPEC are the same now becomes a little more involved. It was shown above that the behaviour of IMP was given by

\[
\text{IMP}(c_u, p, ac, arg, n) \Leftarrow \text{if} \left( \text{noteq}(n,1), \text{noteq}(\text{decr}(n),1) \right) \text{ then}
\{s\}\{s\}\text{IMP2}(\text{getop}(\text{mem}, p), p, ac, arg, \text{decr}(\text{decr}(n)))
\]

The behaviour of IMP2 can be inserted in this expression, with \( \text{getop}(\text{mem}, p) \) replacing \( c_u \) and \( \text{decr}(\text{decr}(n)) \) replacing \( n \). This gives:

\[
\text{IMP}(c_u, p, ac, arg, n) \Leftarrow \text{if} \left( \text{noteq}(n,1), \text{noteq}(\text{decr}(n),1) \right) \text{ then}
\left( \text{if} \left( \text{or} \left( \text{eqs}(\text{getop}(\text{mem}, p), \text{ADD}), \text{or} \left( \text{eqs}(\text{getop}(\text{mem}, p), \text{SUB}), \text{eqs}(\text{getop}(\text{mem}, p), \text{LD}) \right) \right), \text{noteq}(\text{decr}(\text{decr}(n)),1) \right) \right)
\{s\}\{s\}\{s\}\text{IMP3}(\text{getop}(\text{mem}, p), p, ac, \text{getarg}(\text{mem}, p), \text{decr}(\text{decr}(\text{decr}(n))))
+ if \( \text{and} \left( \text{and} \left( \text{eqs}(\text{getop}(\text{mem}, p), \text{JZRO}), \text{noteq}(\text{decr}(\text{decr}(n)),1), \text{eqs}(\text{decr}(\text{decr}(\text{decr}(n))),1) \right) \right) \) then
\left( \text{if} \left( \text{noteq}(\text{getadd}(\text{mem}, p), p), \text{eqs}(ac,0) \right) \right)
\{s\}\{s\}\{s\}\{pc<\text{getadd}(\text{mem}, p), s, t\}
\text{IMP}(\text{JZRO}, \text{getadd}(\text{mem}, p), ac, arg, M)
+ if \( \text{noteq}(ac,0) \) then \{s\}\{s\}\{pc<\text{incr}(p), s, t\}
\text{IMP}(\text{JZRO}, \text{incr}(p), ac, arg, M) )
\]

155
In every branch, the first 3 guards are just ticks. It is easy to see that this is the same behaviour which would be exhibited by SPEC if the initial value of \( n \) were greater than 3. The next action of SPEC would be \( \{ pc\gets \text{getadd}(mem,p), s, t \} \) if the conditional \( \text{and}(eqs(getop(mem,p),JZRO), eqs(ac,0)) \) were true and \( n \) (which has by this point been decremented 3 times) equal to 1, i.e. the initial value of \( n \) must be 4. If this is the case, then all the predicates in IMP's behaviour which involve \( n \) evaluate to true. Finally, if \( \text{and}(eqs(getop(mem,p),JZRO), eqs(ac,0)) \) is true and \( \text{noteq(getadd(mem,p),p)} \) is also true then the fourth action of IMP will also be \( \{ pc\gets \text{getadd}(mem,p), s, t \} \). This extra condition arises because the descriptions of components of the implementation were written to preclude fictitious events, using the techniques of Section 3.2, while the specification of newcomputer was not. The significance of this difference is discussed below. Aside from that, however, it has been successfully shown that the implementation behaves correctly with respect to the specification for the performance of the JZRO instruction when the accumulator contains 0. A similar approach to that just described could be adopted to establish that the implementation executes JZRO correctly when the accumulator does not contain 0, and also that it performs the other three instructions as required by the specification. It should be noted that the initial value of \( n \) may vary from one instruction to another, but that this does not invalidate the result of the verification.

6.5 Discussion

In this Chapter, the tasks of specification, design and validation by formal means have been illustrated with an example of realistic size. This final Section discusses some of the issues which were raised by the exercise.

First of all, it has been demonstrated that the descriptive power of enhanced CIRCAL is sufficient to allow quite complex devices to be specified. By allowing variables of any type to be used as state parameters and passed on ports, and by giving access to the function and type definition capabilities of a language such as
ML, it is possible to produce quite succinct and intelligible descriptions of devices at high levels of abstraction.

In the design task, the advantage of using small design steps (i.e. adding a small amount of structural information at each step) was demonstrated. Hierarchical design is the application of the well-established technique of problem reduction to design, and it follows that the benefits of this technique are lost if the distance between levels is too large. Furthermore, the ability of a language to support closely-spaced levels was illustrated by the use of the datatype cnt1type, which enabled the descriptions of cpart and dpart to be more abstract.

The validation task is also simplified by the use of fine design steps, because the differences between the behaviour of the specification and of the implementation are reduced. In this example, the main difference in behaviour which had to be reconciled was the different granularities of time used at the two levels of abstraction. This was shown to be a fairly straightforward task, demonstrating the suitability of CIRCAL for dealing with the analysis of timing characteristics.

One unsatisfactory aspect of the verification was the fact that an additional predicate was present in the behaviour of the implementation. This arose from the different modelling philosophy adopted at the implementation level, which ensured that fictitious events could not occur. Since such events had not been precluded in the specification, it came as no surprise to discover this discrepancy when the verification was attempted. In this example, it was easy to spot the cause of the discrepancy and dismiss it; in many situations, the reason for such a difference might be more obscure, and a designer may only be able to conclude that the implementation is incorrect. This illustrates the point, made in Chapter 3, that a consistent modelling philosophy should be adopted throughout a design.

Working through this example has shown that the tasks of specification, design and validation are not independent: the way in which each is approached affects the ease with which the others can be tackled, and all three tasks are influenced by the characteristics of the language which is used. This illustrates the importance of developing a language in the context of an integrated approach to design and validation.
Chapter 7

Constraints

In the foregoing discussion of the subtasks of a hierarchical design and validation methodology, there have been numerous occasions on which the word 'constraints' has been mentioned. This word is frequently used both by designers and researchers in the field of design and validation, but it is rarely defined. The investigation of the use of constraints in design and validation has shown that the word can be used to embody several different concepts but that there is a quite specific definition which is particularly useful. The reasoning which underlies the choice of this definition is explained below.

Constraints are generally quite extensively used by designers, as some of the following examples will illustrate. They are normally used, however, only in an informal way. Following the initial work of Davie and Milne[158, Davie88], it has become apparent that a formal treatment of constraints may make an important contribution in the tasks of specification, design and validation, and this area has subsequently become the subject of considerable interest[158, Subramanyam88, Herbert88, Langevin88, Milne88a]. In this Chapter, the ways in which formally specified constraints may provide assistance in each of the these tasks will be examined in detail. In addition, the effect of using constraints in one task on the execution of other tasks will be presented. The fact that constraints can only be formally used in a formal, language based methodology strengthens the argument for the approach proposed in the preceding chapters. The first step towards making
use of constraints is to formulate a precise definition; this is done by considering some common examples.

### 7.1 Introductory Examples and Definitions

In the most general sense, a constraint is a collection of statements which impose restrictions on something, i.e. place limits on what that ‘something’ can do. Some familiar concepts in circuit design and validation which fall within the scope of this definition are:

1. setup, hold and rise times for a D flip flop;

2. temporal relationships between clock phases and data input ports in a PLA controlled by a two phase clock;

3. the restriction on an RS flip flop that its two inputs may not be simultaneously high;

4. geometric design rules;

5. the choice of a certain design style (e.g. microprocessor / microcode engine / PLA), thereby restricting future design decisions;

6. the use of a certain clocking scheme.

It is worthwhile considering what is actually constrained, and in what way, in each of the above examples. In the first, the devices which provide the input signals to the flip flop are constrained; for example, the value on the clock port must change from false to true in a period which is less than the specified rise time. The setup and hold times restrict the times at which the values on the data port may change relative to changes on the clock port. Similarly in the second example it is the devices which are to be connected to the PLA which are constrained. The two clock phases must be generated without overlap and with suitably long pulse
widths and separations, and the changes on the input ports must occur only at certain times in the clock cycle. The third example differs slightly, in that it is the values on ports which are restricted rather than the times of changes, but it is still the devices connected to the flip-flop which are being constrained.

The common thread in these first three examples is that in each case the constraint associated with a device restricts the behaviour of other devices connected to it. These devices may be called the environment or context of the first device, and this type of constraint is therefore referred to as a contextual constraint. The two words 'environment' and 'context' will be used interchangeably to refer to the set of devices to which a device is connected. (The concept of a device’s environment was first introduced in Section 2.3.1; a typical environment is pictured in Figure 2-2.) A contextual constraint, which is associated with a single component, is therefore some set of restrictions on the behaviour of the devices which are connected to that component.

Usually these restrictions will relate to the times at which new values may be placed on the device's inputs by the environment, or the actual values which may be supplied. Thus it will often be the case that the only part of the environment which is of interest is the part which is connected to the inputs of a device. There are exceptions to this, however; for example, the acceptable time for providing a new input value may be dependent on the time at which the last output change occurred.

The remaining examples of constraints involve a different type of restriction. The geometric design rules constrain a designer; they constitute a set of restrictions on the ways in which he may arrange polygons in the various layers. Similarly, in selecting a particular design style at some point in the design process, the designer places restrictions on himself which reduce the range of design options open to him at subsequent stages of the design. The use of a particular clocking scheme also restricts the designer, who is then required to design all components such that they will function and interact correctly under that scheme.

These three constraints are thus seen to be in a different category from the first three, in that, rather than restricting the behaviour of a component or set
of components, they all restrict the choices of the designer. These constraints are very difficult to treat in a formal way\(^1\); of the examples above, only the geometric design rules can be easily expressed as an unambiguous set of rules. By contrast, the former type of constraint can readily be described in a formal way, as illustrated below. Thus, although both types of constraint may have a role to play in a design methodology, the remainder of this Chapter will concentrate on contextual constraints and the ways in which their formal description and use may serve the design and validation process.

A formal treatment of contextual constraints can be of assistance in each of the main tasks of the methodology; following sections will discuss the roles of constraints in the tasks of specification, design and verification and examples of constraints which can be used in each task will be provided. Before making use of constraints, however, it is necessary to have a formal way to describe them, and this is presented in the next Section.

### 7.2 Specification of Constraints

A contextual constraint was defined above as a restriction on the behaviour, in terms of the events which may take place and the times when they may occur, of a device's environment. The environment consists of a collection of components connected to the device with which the constraint is associated, so there is no reason not to consider it as just another device.

In previous discussion of CIRCAL it was noted that it is all too easy to write descriptions in which input events were not allowed at certain times. In the description of real hardware this was considered to be a disadvantage, but in the specification of constraints the ability to prohibit certain events from occurring is just what is required. It therefore seems reasonable to suppose that a formal

\(^1\)One approach to restricting the designer is described in Section 4.1.2.
specification of a constraint for a device could be achieved by writing a CIRCAL description of some abstract component representing the environment of the device. It was seen in Chapter 3 that CIRCAL specifications could be constructed from a number of parts, so that this description might be considered a partial specification of the environment. A few examples will serve to illustrate how constraints may be specified in this way.

Example

To specify the constraint on an RS flip flop that its two inputs may never simultaneously be set to the value true, an abstract box called CON, with ports r and s could be specified as follows:

\[ CON \leq \{r<true\} \{r<false\} CON + \{s<true\} \{s<false\} CON \]

Explanation

A true event on either port can only be followed by a false event on the same port; this ensures that the two ports are never both set to the value true simultaneously. It should also be noted that this constraint specification places no other restrictions on the behaviour at these ports.

Constraints often occur in the form of acceptable limits within which some behavioural parameter must lie. The setup time for a latch mentioned above is an example of this.

Example

For a setup time of \( n \) time units, the constraint is that a waveform must assume a steady value at least \( n \) units before the latching edge of the clock. This type of non-rigid description can be achieved in CIRCAL using a similar technique to that which was described in Section 3.2.4.
Explanation

This constraint uses a constructive specification technique, the constraint box \(CON\) being constructed from two simpler boxes. The box \(TGEN\) may either passively accept ticks, or on the occurrence of a pulse on \(t1\) will wait for 4 more ticks before generating a pulse on \(t2\). The box \(CONA\), after accepting an input event on \(data\), generates the pulse on \(t1\). No event may then happen on \(clk\) until the pulse on \(t2\) occurs. This ensures that a \(clk\) event cannot occur less than 4 ticks after an event on the \(data\) port. The relationship between the various events is illustrated in Figure 7-1.

Example

The third of the examples of contextual constraints mentioned above could be described in the following way:

\[ CON \leftarrow (\text{phione<true})(\text{phione<false})CON + (data>x)CON \]
Explanation

The sort of this abstract box is \{\textit{phone}, \textit{data}\}. The second branch of its description allows any events on the \textit{data} port. However, if the value on the input-latching clock \textit{phone} becomes true, the only event which can follow is \text{(phone=false)}, i.e. there may be no change on the \textit{data} port until the clock is false again.

From these examples it is clear that \textsc{circal} is particularly well-suited to the task of constraint description because of the ease with which behaviours in which certain events are disallowed can be written. In Section 7.4 the problem of describing constraints in the high level language SuperC will be discussed. The following Section shows how formally specified constraints may be put to use in all phases of the design and verification process.

7.3 The Uses of Constraints

Having seen a few examples of constraints which are commonly used, although only in an informal way by most designers, and having established a mechanism by which such contextual information can be formally represented, it remains to be shown:

(a) that there are more than a few isolated situations in which contextual constraints can be used; and

(b) that a formal treatment of such constraints will provide assistance in a design and validation methodology.

The following Sections will serve to demonstrate both these points.

7.3.1 Constraints in Specification

It seems reasonable to suppose that, when specifying the behaviour of some component or even a whole system, the behaviour of the environment in which that
system will be used might also be taken into account. There are many devices
which will only behave in a predictable and useful way if the inputs which are
provided by the environment conform to certain patterns. A microprocessor, for
example, generally needs to be provided with a clock, a sequence of meaningful
instructions and operands, etc. In fact, the formal specification of restrictions
on the environment's behaviour in the form of a contextual constraint can be of
considerable assistance in the specification task, as will become apparent in the
following discussion.

In previous chapters the problems of writing specifications which are suitable
for subsequent use in the validation task have been discussed. It has been shown
that some approaches to specification which appear to simplify the task can ac-
tually prevent the successful validation of a implementation which, had it been
correctly specified, would have been satisfactory. This sort of problem arose be-
cause the specification was written under some assumptions about the behaviour
of the environment of the device being specified; when these assumptions turned
out to be invalid, the validation of the design could not be carried out successfully.
This is an unsatisfactory situation, as it leads to the false conclusion that an im-
plementation is incorrect, when in fact it is the specification technique that is to
blame. Contextual constraints can resolve this problem, as the following example,
an inverter with delay, illustrates.

Example

In Section 5.2.2 a specification technique was used in which output events
lagged input changes by some time interval and no input changes were allowed
during this time. In writing a specification in this way, the implicit assumption is
that the environment of the specified device will not fire input changes at it 'too
rapidly', i.e. an input event will be followed by a sufficiently long delay to allow
an output event to occur. However, this assumption was never made explicitly,
and the result was that the verification failed.

The solution to this problem is to associate a constraint with the specification,
as follows:
INV(x) <= {in>y:noteq(y,x), t}{out<not(y), t}INV(y)
+ {t}INV(x)

CON <= {in>x, t}{t}CON + {t}CON

Explanation

The first line states that a new value y can be input on one tick and that the complement of that value will be output on the next tick of the universal clock. The abstract device CON represents the contextual constraint. The sort of this device is \{in, t\}. If an event occurs on the port \textit{in} on one tick, then the only possible guard to follow this is \{t\}. Thus, no input event can follow on the next tick.

Suppose now that the inverter is to be connected to a component which will generate events on the port \textit{in} on successive ticks of the universal clock, such as the following:

DEV <= {in>true, t}{in>false, t}DEV

In order to check whether the assumptions about the environment's behaviour were correct, it is necessary to find whether \textit{DEV satisfies} the constraint \textit{CON} in some sense. What is required is that \textit{DEV} does not try to perform any actions which are not permitted by \textit{CON}. Now if \textit{DEV} did contain some actions which were prohibited by \textit{CON}, then \textit{DEV \textasciitilde CON} would not contain these actions. Therefore, \textit{DEV \textasciitilde CON} would not be equal to \textit{DEV}. Provided that the sorts of \textit{DEV} and \textit{CON} are the same, \textit{DEV \textasciitilde CON} will certainly not have any more actions than \textit{DEV}. So, the constraint is satisfied if

\textit{DEV \textasciitilde CON} = \textit{DEV}

where \textasciitilde means 'can perform exactly the same actions as'. This may be contrasted with the satisfaction requirement for implementations described in Section 5.2.1[Milne85b]. An implementation satisfies its specification if it can perform
at least all the actions of the specification. An environment satisfies its constraint if it can perform no more actions than the constraint allows. This can readily be tested:

\[
\begin{align*}
\text{DEV} \ast \text{CON} &= ((\text{in}\text{true}, t)\{\text{inc}\text{false}, t\}\text{DEV} \ast ((\text{in}\text{>false}, t)\{t\}\text{CON}) \\
&= ((\text{in}\text{true}, t) \{\text{inc}\text{false}, t\}\text{DEV} \ast ((t)\text{CON}) \\
&= \{\text{in}\text{true}, t\} /\setminus
\end{align*}
\]

which is clearly not equal to \text{DEV}, which is a non-terminating behaviour. However, if the inverter is connected to a different device, say \text{DEV1}, which waits one tick between events on \text{in}, specified as

\[
\text{DEV1} = \{\text{in}\text{true}, t\}\{\text{inc}\text{false}, t\}\{t\}\text{DEV1}
\]

then a check on the satisfaction of the constraint proceeds as follows:

\[
\begin{align*}
\text{DEV1} \ast \text{CON} &= ((\text{in}\text{true}, t)\{t\}\{\text{inc}\text{false}, t\}\{t\}\text{DEV1} \ast ((\text{in}\text{>false}, t)\{t\}\text{CON}) \\
&= \{\text{in}\text{true}, t\} ((t)\{\text{inc}\text{false}, t\}\{t\}\text{DEV1} \ast ((t)\text{CON}) \\
&= \{\text{in}\text{true}, t\}\{t\}((\text{inc}\text{false}, t)\{t\}\text{DEV1} \ast \text{CON}) \\
&= \{\text{in}\text{true}, t\}\{t\}\{\text{inc}\text{false}, t\}\{t\} \text{DEV1} \ast \text{CON}
\end{align*}
\]

It can be seen that this behaviour is equivalent to \text{DEV1}, so \text{DEV1} satisfies the constraint \text{CON}.

**Full and Partial Specifications**

One question to be addressed here is this: when writing a specification, how does the designer know whether it is necessary to write a contextual constraint to ensure that the environment in which the part is to be used will not violate the assumptions under which it was specified? The key to the answer to this question lies in the concepts of full and partial specifications.
A specification may be classed as 'full' if, in any state, any combination of valid events on the input ports is possible. 'Valid' events are those which represent a genuine change in value, as discussed in Section 5.2.2. This means that for a device with \( n \) ports on which inputs can occur, there will be at least \( 2^n - 1 \) guards, to allow for any possible combination of input events. If an input event has a qualifying predicate which is more restrictive than one to ensure the validity of the event, then there must be another guard to take account of the cases in which that predicate is not satisfied. Furthermore, if a state is parameterised, there must be a full complement of input events for any possible set of state parameter values.

It should also be noted that a state need not be named explicitly; any part of a behavioural description which follows a guard can be considered a state. A full specification can accept any input event at any time. It therefore does not require a contextual constraint, since its environment cannot possibly attempt to perform an input event which it would not accept.

Anything which does not conform to the definition of a full specification is a partial specification. The following examples illustrate both classes.

**Example**

A full specification of an and gate with no delay could be written as:

\[
\text{AND}(p,q) \leq \begin{cases} 
\text{if } p \text{ then } \{b > x : \text{noteq}(x,q), \text{out} < x\} \text{AND}(p,x) \\
+ \text{if } q \text{ then } \{a > x : \text{noteq}(x,p), \text{out} < x\} \text{AND}(x,q) \\
+ \text{if } \text{eqs}(p,q) \text{ then } \{a > x : \text{noteq}(x,p), b < x, \text{out} < x\} \text{AND}(x,x) \\
+ \text{if } \text{not}(p) \text{ then } \{b > x : \text{noteq}(x,q)\}\text{AND}(p,x) \\
+ \text{if } \text{not}(q) \text{ then } \{a > x : \text{noteq}(x,p)\}\text{AND}(x,q) \\
+ \text{if } \text{noteq}(p,q) \text{ then } \{a > x : \text{noteq}(x,p), b > y : \text{noteq}(y,q)\}\text{AND}(x,y) 
\end{cases}
\]

**Explanation**

The first three lines of the description take account of input events which produce an output change, the leading conditional ensuring that a valid output event will result. Conversely, the last three lines take account of input events which
will produce no output change. The important point here is that for any values of \( p \) and \( q \), input events may take place on \( a, b \) or both. Thus, all possibilities are taken into account, and the specification is a full one.

Example

The following is a full specification of an RS flip flop:

\[
\text{RSFF}(x, y, q) \iff \begin{cases} 
\text{if and(not(x),q) then } \{ r < \text{true}, t \} \text{RSFF(true,y,q)} \\
+ \text{if and(not(x),not(q)) then } \{ r < \text{true}, t \} \text{RSFF(true,y,q)} \\
+ \text{if and(not(y),not(q)) then } \{ s < \text{true}, t \} \text{RSFF1(x,not(y),q)} \\
+ \text{if eqs(x,y) then } \{ r > p : \text{noteq}(x,p), s < p, t \} \text{RSFF(p,p,q)} \\
+ \text{ifnoteq(x,y) then } \{ r > p : \text{noteq}(x,p), s < \text{not}(p), t \} \text{RSFF1(p,not(p),q)} \\
+ \{ t \} \text{RSFF1(x,y,q)} 
\end{cases}
\]

\[
\text{RSFF1}(x, y, q) \iff \{ t, \text{out<not(q)} \} \text{RSFF}(x,y,\text{not(q)}) \\
+ \text{if and(x,not(y)) then } \\
\quad ( \{ r < \text{false}, t, \text{out<not(q)} \} \text{RSFF(false,y,not(q))} \\
+ \{ s < \text{true}, t, \text{out<not(q)} \} \text{RSFF(x,true,not(q))} ) \\
+ \text{if and(not(x),y) then } \\
\quad ( \{ s < \text{false}, t, \text{out<not(q)} \} \text{RSFF(x,false,not(q))} \\
+ \{ r < \text{true}, t, \text{out<not(q)} \} \text{RSFF(true,y,not(q))} ) \\
+ \text{ifnoteq(x,y) then } \\
\quad ( \{ r > p : \text{noteq}(x,p), s < \text{not}(p), t \} \text{RSFF(p,not(p),q)} \\
+ \text{if eqs(x,y) then } \\
\quad \quad ( \{ r > p : \text{noteq}(x,p), s < p, t \} \text{RSFF(p,p,q)} \\
+ \{ r > p : \text{noteq}(x,y), t \} \text{RSFF(x,y,q)} \\
+ \{ s > p : \text{noteq}(y,p), t \} \text{RSFF(x,p,q)} ) 
\]
Explanation

In both state definitions, the parameters \( x, y \) and \( q \) represent the values on the \( r \) (reset), \( s \) (set) and \( o\!ut \) (output) ports respectively. The state \( \text{RSFF}(x, y, q) \) represents the stable states, in which no output change is due to occur. This is evident from the last branch in the description. In certain circumstances, an input event leads to the unstable states denoted by \( \text{RSFF1}(x, y, q) \). In these states, if there is no new input on the next tick of the universal clock, then the value on the output is complemented, as the indicated by the first branch in the description.

In the unstable states, if the next tick is accompanied by another input change, then this may cancel the pending output event (by leading back to a stable state) or it may have no effect.

The assertion that this is a full specification, according to the above definition, can be tested. The specification is full if, in all possible states of the device, it is possible for any combination of valid input events to occur. In any state, there are three possible combinations of valid input events: a change on the reset port, a change on the set port, and a simultaneous change on both ports.

A change on the reset port can take place from a state \( \text{RSFF}(x, y, q) \) under the following circumstances:

- when \( \text{and}(\text{not}(x), q) \) is true;
- when \( \text{and}(\text{not}(x), \text{not}(q)) \) is true;
- when \( x \) is true.

Given that \( \text{and}(\text{not}(x), q) \land \text{and}(\text{not}(x), \text{not}(q)) \land x = \text{true} \), it can be concluded that a change on the reset port is always possible. Similar checks for changes on the set port only and on both ports at once show that, for any values of \( x, y \) and \( q \), all three combinations of input events are possible.

To ensure that the specification is full for the unstable states denoted by \( \text{RSFF1}(x, y, q) \) it is necessary to carry out the same checks for each of the possible
combinations of valid input events. Thus the same conclusion, that each of the three combinations can occur for any values of \( x, y \) and \( q \), is reached.

This process of checking, as presented above, seems fairly laborious. However, it would be reasonably straightforward to automate it, since it is a purely mechanical process.

Before discussing the problems encountered in formulating a full specification for this device, it is helpful to consider how a partial specification of the same device might be written. If it is assumed that \( s \) and \( r \) will never be simultaneously set to true, then the following partial specification can be used:

\[
\begin{align*}
\text{RSFF}(x, y, q) &\leftarrow \text{if and}(\text{not}(x), q) \text{ then } \{r<true, t\} \text{RSFF1}(true, y, q) \\
&\quad + \text{if and}(\text{not}(x), \text{not}(q)) \text{ then } \{r<true, t\} \text{RSFF}(true, y, q) \\
&\quad + \text{if and}(\text{not}(y), \text{not}(q)) \text{ then } \{s<true, t\} \text{RSFF1}(x, true, q) \\
&\quad + \text{if and}(\text{not}(y), q) \text{ then } \{s<true, t\} \text{RSFF}(x, true, q) \\
&\quad + \text{if } x \text{ then } \{r<false, t\} \text{RSFF}(false, y, q) \\
&\quad + \text{if } y \text{ then } \{s<false, t\} \text{RSFF}(x, false, q) \\
&\quad + \text{if noteq}(x, y) \text{ then} \\
&\quad \quad \{r>p:\text{noteq}(x, p), s<\text{not}(p), t\} \text{RSFF1}(p, \text{not}(p), q) \\
&\quad + \{t\} \text{RSFF}(x, y, q)
\end{align*}
\]

\[
\begin{align*}
\text{RSFF1}(x, y, q) &\leftarrow \{t, \text{out}<\text{not}(q)\} \text{RSFF}(x, y, \text{not}(q)) \\
&\quad + \text{if } x \text{ then } \{r<false, t, \text{out}<false\} \text{RSFF}(false, y, false) \\
&\quad + \text{if } y \text{ then } \{s<false, t, \text{out}<true\} \text{RSFF}(x, false, false) \\
&\quad + \{r>p:\text{noteq}(x, p), s<\text{not}(p), t\} \text{RSFF}(p, \text{not}(p), q)
\end{align*}
\]

**Explanation**

As before, \( \text{RSFF}(x, y, q) \) represents the stable states and \( \text{RSFF1}(x, y, q) \) the states in which an output is pending. In this description, however, the possibility of ports \( r \) and \( s \) being set to true simultaneously is not taken into account. The specification is therefore a partial one. For example, in a stable state, simultaneous
events cannot take place on the two inputs unless noteq(x,y) is true (from the penultimate branch of the definition of RSFF(x,y,q)).

By assuming that the two inputs will never both be set to true at the one time, the process of writing the specification becomes much simpler. In particular, the possible scenarios in the unstable states denoted by RSFF1(x,y,q) are much reduced in both number and complexity. For example, it can now be assumed that RSFF1(x,y,q) will only be reached if exactly one of x and y is true, so there is no need to write a specification which takes account of any other possibilities. The specification which results is not just shorter, it is much easier to formulate.

In summary, the disadvantages of full specifications are:

- The number of possible events which must be taken account of in each state is larger than it would be if a partial specification were written, the difference being quite significant in some situations;

- Making sure that all possible event combinations have been covered is essential and may be quite a laborious task, especially when conditionals and input parameter predicates are used;

- Deciding how the device should behave under certain circumstances can be difficult and needlessly time-consuming. For example, what should the behaviour of the RS flip flop be when a reset event is immediately followed by a set event?

The advantages of writing partial specifications which only describe that part of the device's behaviour which is of interest can thus be clearly seen. In writing a partial specification, however, one is assuming that certain patterns of inputs will not occur. In order to ensure that this assumption is justified, there must be a constraint on the environment of the partially specified device which prohibits those patterns of inputs. It may then be necessary to check that the environment meets the constraint, or to design it with the constraint information taken into account. The consequences of designing a device's environment without regard for
the assumptions under which that device was specified have already been demonstrated in Section 5.2.2. Thus the use of constraints in one task of the methodology starts to influence other tasks. The spread of constraints in this way is discussed in the following Section.

One issue which has been ignored so far is the question of whether a constraint is adequate to prevent events which a partially specified device cannot accept. Just as a full specification must accept all possible valid inputs, a partial specification must be able to accept all inputs which its associated constraint does not preclude. A way in which this can be guaranteed is presented in Section 7.4.

At the start of this Section the question was posed of whether contextual constraints could only be applied in a few isolated situations. This question can now be partially answered. The ability to write a partial specification is something that may be required in the description of almost any device. Contextual constraints must be written if partial specifications are to be safely used. The role of constraints in the specification task may therefore be quite significant and extensive.

### 7.3.2 Constraints in Design

It has been shown that specifications may be more easily written if a contextual constraint is attached to that specification. The design task consists of partitioning a box, which has some specification, into smaller cells, and assigning specifications to those cells. This suggests two ways in which constraints may play a part in the design task:

1. in the designing of a box whose specification, because it is partial, has an associated constraint;

2. in the specification of cells which constitute the environment of a device which has an associated constraint.
Designing a Partially Specified Device

To examine how the association of a constraint with a box’s specification might affect its design, it is worth returning to the example of the RS flip flop described above. When this device was specified fully and without a constraint, it was necessary to decide how it would behave under various unusual conditions such as the arrival of a set pulse when the value on the reset port was already true. Because some response had to be specified under all of these conditions, even though the designer may not actually care what happens, the problem of producing a suitable implementation becomes significantly harder. For the implementation must perform at least all the actions which the specification can perform; if some of the actions of the specification were specified needlessly and arbitrarily, a considerable amount of effort may be expended in constructing an implementation which performs the same arbitrary actions.

![Figure 7-2: Nor Gate Implementation of RSFF](image)

In the flip flop example, the specification is written in such a way that if the value on r is true, and the event s=true occurs, then the value on the output will be complemented on the next tick. Now, if the common cross-coupled nor gate implementation of the RS flip flop, pictured in Figure 7-2, is used, it will actually fail to meet the specification because it does not behave that way. The same implementation could, however, be used to implement the partial specification of the flip flop, because it behaves correctly under all the input stimuli which that specification allows. It will also accept other input stimuli than those which are
covered by the partial specification, but this does not make it an unsatisfactory implementation, it simply has a richer behaviour than the specification. Some further consequences of this difference in behaviour between implementation and specification are discussed in Section 7.3.3. The important point to note here is that a simple and quite acceptable implementation could actually be rejected because the specification was unnecessarily stringent.

Thus there is another argument for the use of partial specifications with associated constraints. Such specifications are not just easier to write. By specifying only those aspects of a behaviour which actually matter, the construction of a satisfactory implementation is also made easier. Therefore, it may be desirable to specify a device partially whenever it is known that its environment will only provide a limited set of stimuli, thus providing the designer with more flexibility in choosing an implementation. This particular use of contextual information is the subject of some current research [Subramanyam88].

**Designing Environments Under Constraints**

The second role of constraints in the design task is in the designing of environments for devices whose specifications have associated constraints. This use of constraints may be most easily explained by way of an example.

![Figure 7-3: A Box to be Designed Using Constraints](image)

**Figure 7-3: A Box to be Designed Using Constraints**
Figure 7-3 illustrates part of a typical design step. In Chapter 4, design was seen to consist of two tasks, partitioning and description. The box $A$ is to be partitioned into cells which must then be described. It will be recalled that the description phase may either be performed by the designer or may consist of the selection of a suitable part, with an associated specification, from a library of standard parts.

In this example, there is already some specification for $A$, which may for the purposes of the argument be either full or partial. It has been decided to partition $A$ into three cells $B$, $C$ and $D$. Suppose that the description phase begins with the writing of a specification of the cell $B$ and that for some reason this specification has an associated contextual constraint. This may be because $B$ has been partially specified for one of the reasons discussed above, or it may be that $B$ is a part taken from a library of standard parts, which has an attached constraint such as setup and hold time requirements. This constraint constitutes a statement about the allowed behaviour of the environment of $B$. The cells $C$ and $D$ are a part of the immediate environment of $B$ and must therefore be behaviourally described in such a way that they conform to the environmental constraint.

It has been shown in Section 3.2.2 that behavioural descriptions in CIRCAL may be built up from parts by the technique of constructive specification. This technique may be effectively used in this situation; the first part of the specification for each of the boxes $C$ and $D$ is simply the contextual constraint associated with $B$. If this constraint is denoted by $\text{CON}_B$, then a specification for $C$ may be written as

$$C \Leftarrow \text{CON}_B \ast \text{SPEC}_C$$

and $\text{SPEC}_C$ may be written without any concern for the constraint; constructed in this way, $C$ will be guaranteed to satisfy the constraint $\text{CON}_B$. If $C$ also has an associated contextual constraint, denoted by $\text{CON}_C$, then $D$ would be specified as

$$D \Leftarrow \text{CON}_B \ast \text{CON}_C \ast \text{SPEC}_D$$

176
In the situation just described, constraints from neighbouring parts were used to construct the specifications of devices which were unspecified at the time the constraint was written. That is, when CON_B was written, there was as yet no specification for the parts C and D. If such specifications had already existed, it would have been necessary to modify them to ensure that they would satisfy CON_B. If the specifications had been written by the designer, this would present no problem; it would be a simple matter of composing CON_B with SPEC_C and SPEC_D respectively as before. The one slightly more awkward situation which could arise is that the description of C or D might have been taken from a library. In this case, their specifications are already fixed, so that one is not at liberty to modify them by adding the constraint to them. The first step then is to ascertain whether the specifications satisfy CON_B in the sense described in Section 7.3.1. If they do, then the specifications may be left as they stand. If the specifications do not satisfy the constraint, then there are two possible courses of action:

- to choose different parts from the library which will satisfy the constraint; or

- to abandon the use of standard parts at this stage, so that a new part whose specification does satisfy the constraint can be designed.

If the second option is chosen, then the components C and D will be treated as the specifications which are to be implemented at the next level down in the design hierarchy.

In the preceding discussion, the environment of B was assumed to involve only the parts C and D. This is actually something of an oversimplification, as the environment of a part is actually the sum total of all parts which are immediately connected to it. If the ‘parent’ part A, which is to be implemented by parts B, C and D, is actually part of some larger system, then it is entirely possible that part of the environment of B will lie outside of A. This situation is illustrated in Figure 7-4, in which A is connected to another box, called X.
Figure 7-4: An Environment Extending Beyond the Parent Box

This situation is rather more difficult to deal with. In a top-down approach to design, all the parts to which A is connected would have been at least specified, and possibly implemented as well, before A is designed. Therefore, it is not generally possible to impose the constraints associated with B on those parts of its environment which lie outside A. The first step is to test whether the constraints associated with B are satisfied by those parts of its environment. If they are, there is no problem. If not, there are two options:

- to take steps to modify the constraint on B so that it will be satisfied by the environment as it stands. This may involve modification of the specification of B, or the use of a different part from the library;

- to modify the environment so that it satisfies the constraints.

Modification of the environment means that the design is no longer rigorously top-down, but iterative. In real design situations, iterative design is actually the more common approach.

It is noteworthy that the uses of constraints just described have not been motivated by a desire to ease the task at hand. Instead, constraints are used here because they have been generated at some other stage and it is now necessary to ensure that they are satisfied by the environments to which they apply. Thus, it is at this stage that constraints may actually make the design task more difficult rather than less. However, since the effect of constraints in this situation may be
to limit the choices available to the designer, this may actually be considered a design aid. Anyway, whether constraints hinder or aid the designer in this task of the methodology, their use has already been seen to be important in other tasks. One task, validation, remains to be discussed; this is done below.

7.3.3 Constraints in Verification

The formal treatment of constraints is of greater use and importance when the method of validation is also formal. This Section therefore concentrates on formal verification rather than simulation. There are two ways in which constraints are involved in the verification task. These are:

- the verification of a device may be simplified if it is only partially specified, requiring a constraint to be attached;

- when a partially specified device is verified, it may be necessary to apply an additional constraint to its environment to ensure that an implementation which satisfies the partial specification will function correctly.

These two roles of constraints in verification are discussed in detail below.

Constraints to Ease the Verification Task

It has already been seen that it may be more difficult to implement a system if its specification contains more information than is absolutely necessary. The use of partial specifications is therefore an aid to the design task. It is reasonable to suppose that the same may be true of the verification task, for the following reason: if a system is only partially specified, then the range of input stimuli which it can accept is reduced. Therefore, the amount of work which has to be done to show that the implementation responds to all possible input stimuli in the same way as the specification will hopefully also be reduced. The following example will show how this reduction in verification effort may be achieved.
Example

A piece of combinational logic is 'sandwiched' between a pair of latches as illustrated in Figure 7-5. A partial specification for such a device could be written under the assumptions that the two clock phases are non-overlapping and separated by a certain interval, and that the input lines will not be subject to changes during the time that $\text{phione}$ is true. In CIRCAL these constraints could be described as follows:

\[
\begin{align*}
\text{CON} & \leftarrow (\text{phione=true})\{\text{phione=false}\}\text{CON}A + (\text{din}>x)\text{CON} \\
\text{CON}A & \leftarrow (\text{phitwo=true})\text{CON}B + (\text{din}>x)\text{CON}A \\
\text{CON}B & \leftarrow (\text{phitwo=false})\text{CON} + (\text{din}>x)\text{CON}B
\end{align*}
\]

Explanation

In the initial state $\text{CON}$, a new data value may be input, or the current value may be latched by $\text{phione}$. Once $\text{phione}$ becomes true, the only thing which may happen next is that it may become false again; no changes on the input port $\text{din}$ or the other clock phase are possible. After $\text{phione}$ becomes false, $\text{phitwo}$ is re-enabled, as is $\text{din}$. A rising edge on $\text{phitwo}$ leads to state $\text{CON}B$, in which changes on the input are still possible, and the only possible clock event is for $\text{phitwo}$ to fall again. This completes a full clock cycle. These events are shown in Figure 7-6.
Under these constraints, the clocked logic may be described by the following partial specification:

\[
\begin{align*}
CCL(x, y) &< \{ din > p : \text{noteq}(p, x) \} CCL(p, y) \\
&+ \{ phione \text{true} \} \{ phione \text{false} \} CCL1(x, x, y) \\
CCL1(x, x, y) &< \text{if noteq}(\text{comfun}(z), y) \text{ then} \\
&\{ phitwo \text{true}, \text{dout} = \text{comfun}(z) \} CCL2(x, y) \\
&+ \text{if eqs}(\text{comfun}(z), y) \text{ then} \{ phitwo \text{true} \} CCL2(x, y) \\
&+ \{ din > p : \text{noteq}(p, x) \} CCL1(p, z, y) \\
CCL2(x, y) &< \{ din > p : \text{noteq}(p, x) \} CCL2(p, y) \\
&+ \{ phitwo \text{false} \} CCL(x, y)
\end{align*}
\]

Explanation

The first two lines describe the initial state \( CCL(x, y) \) in which a pulse on \( phione \) may occur to latch the current value on \( din \) into an internal state. In state \( CCL1(x, x, y) \) a new output value, computed by the function \( \text{comfun}(z) \) whose argument is the latched internal value, may be read out by a pulse on \( phitwo \). New values may be put on the input in this state without affecting the latched value. In the third state, a falling edge on \( phitwo \) is required to complete the two-phase clock cycle.

\[\text{Figure 7-6: Timing Diagram for 2-phase Clock Constraint}\]

Now the implementation needs to be described. The latches have the following behaviour:
L1(x,y,false) <=
    if noteq(x,y) then (phione<true,lin<x)L1(x,x,true)
    + if eqs(x,y) then (phione<true)L1(x,x,true)
    + (din>p:noteq(x,p))L1(p,y,false)
L1(x,y,true) <= (phione<false)L1(x,y,false)
    + (din>p:noteq(x,p))L1(p,y,true)

L2 <= L1 [lout/din] [dout/lin] [phitwo/phione]

Explanation

The first set of states of L1 are those in which the value on the clock line is false. If it becomes true, the value on the input port is transferred to the output, provided of course that it is a different value from that already present. Alternatively, a new value may be placed on the input port, din. When the value on the clock line is true (as described in the 4th and 5th lines), the only possibilities are that it will become false or that a new value will be placed on the input. L2 has identical behaviour with a suitable renaming of ports as Figure 7-5 indicates.

The combinational logic can be described as follows:

LOGIC <= LOG * DEL2 - int
LOG(m,n) <= {lin>p:and(noteq(p,m), eqs(comfun(p),n))}LOG(x,n)
    + {lin>p:and(noteq(p,m), noteq(comfun(p),n)),
       int<comfun(p))}LOG(x,n)

Explanation

This is an example of the constructive technique for specifying devices with delays. DEL2 is a generic 2 unit delay box of sort {int,lout}, such as was described in Section 3.2.3. LOG is defined to be delayless, producing an output event whenever a new value p is input such that the value of comfun(p) differs from the current output value. The two devices are connected on the port int which is abstracted out in the first line.
The implementation is structurally described by

\[ \text{IMP} \leq \text{L1} \ast \text{LOGIC} \ast \text{L2} - \text{lin} - \text{lout} \]

To verify that this implementation satisfies the specification CCL, it needs to be shown that CCL \ast IMP can perform the same actions as CCL. The expansion of \( \text{L1} \ast \text{LOGIC} \ast \text{L2} - \text{lin} - \text{lout} \) is too lengthy to present here, having some 50 branches in the choice sum. Composition with CCL removes most of these branches, leaving only 16. In order to verify the implementation, the first step is to prove that these 16 branches can be matched with the 2 branches of CCL, using the rules for matching which were described in Section 5.2.4.

Now suppose that CCL had been specified fully rather than partially. With 3 inputs \( \text{din}, \text{phione} \) and \( \text{phitwo} \), the full specification would need at least \( 2^3 - 1 = 7 \) branches, and in all probability quite a few more than this. Since CCL would now be specified in such a way that all combinations of input events are possible, the pruning effect of composing CCL with IMP would therefore have been much reduced. Both the specification and the implementation would, for example, contain branches whose guards involved events on \( \text{phione} \) and \( \text{phitwo} \) simultaneously. Thus the first step of proof, proving that the initial actions of the implementation and specification are equivalent, would now involve an attempt to match more than 7 branches with nearly 50, instead of matching 2 with 16 as was required above. Similarly, the task of establishing that subsequent actions of the specification and implementation are equivalent would be much more involved, because of the much wider range of input events that may take place (e.g. successive pulses on \( \text{phione} \)) and the greater number of states which may be reached. It is apparent, therefore, that by using a partial specification for a device, the task of verifying that an implementation meets that specification is greatly reduced in complexity.

The use of partial specifications coupled with constraints to ease the verification task has also been attempted in the HOL system[Herbert88]. Device descriptions are split into three parts: the function, a set of input constraints, and a set of output stability assertions. The description of the function is essentially a partial specification; the device is guaranteed to perform this function if its in-
put constraints are met, and this will result in certain stability conditions on the output (e.g. the output never changes during a certain part of the clock cycle). Verification of devices described in this way can then be simplified, as the functional behaviour is verified without regard for detailed timing phenomena; these are dealt with later, simply by ensuring that the output stability assertions of each device satisfy the input constraints of the devices to which they are connected. In effect, this work involves the separation of timing and function, as discussed in Section 5.2.4.

Constraints Arising From Verification

In the preceding Sections a number of reasons for using partial specifications have been proposed: they are easier to write than full specifications, partially specified devices may be easier to implement, and it may be easier to prove that an implementation satisfies a partial specification than a full one. It has been asserted throughout that a partial specification can only be legitimately used if a constraint is attached to it to ensure that events which are not accounted for in the partial specification are not generated by the environment. The benefits of partial specifications thus become the rationale for the use of constraints, even though the restrictions which they impose may require some additional effort in the design task, as was shown in Section 7.3.2.

The assertion that constraints are a necessary accompaniment to partial specifications has been largely based on intuitive arguments. The most powerful of these is the following: if a partially specified device is used in an environment which tries to perform an event which the partially specified device cannot accept, then the result may be a deadlock caused not by an incorrect implementation but simply by an inadequate specification. An example of this was first described in Section 3.2. By using a constraint in this situation, the behaviour of the environment could be checked before seeking to establish how the partially specified device would function in it; the fact that the environment did not satisfy the constraint could be detected, and either the environment would be re-designed or the device specified more fully.
There is a further argument for the need to attach constraints when partial specifications are used. This argument was developed by Traub [Traub86] in his work on verification using CIRCAL and concerns the need to generate a constraint on the environment of a device which has been specified partially, designed and verified. In general, this device will form one of the boxes which implement another device at a higher level in the design hierarchy. Figure 7-7 shows a tree-structure representation of this hierarchy, in which the device currently being discussed is called box 'B'.

Both B and C may have partial specifications. It will have been shown that these specifications interact in such a way as to implement the specified behaviour for A correctly. When B is designed, it will be shown that the behaviours of D and E interact in such a way as to satisfy the partial specification of B. Similarly, F and G will satisfy C. However, D * E will, in general, have additional possible actions to those of B; this is acceptable by the definition of satisfaction. Likewise, F * G will be able to perform more actions than C. These two sets of additional actions may interact in such a way that the combined behaviour of boxes B and C is no longer such that it correctly implements the specification A.

In order to deal with this problem, a contextual constraint can be defined which
ensures that the additional, unwanted interactions cannot happen. Traub defines the *weakest safe context* as the 'least restrictive' contextual constraint which does this. The derivation of this is fairly involved, but the end result is that the weakest safe context is described by a device which rejects those actions which can be performed by \( D \star E \) but not by \( B \). Traub shows that if the environment of \( B \) satisfies this constraint in the sense defined in Section 7.3.1, then no unwanted interactions will occur. Thus the final reason for attaching a contextual constraint to a specification is to ensure that the implementation of the device will not interact unpredictably with the environment. Constraints which are attached for this reason should be dealt with in the design process using the techniques described in Section 7.3.2.

### 7.4 Constraints and SuperC

Throughout this Chapter, the use of constraints has been closely tied in to the use of the enhanced CIRCAL language. The discussion of partial specifications has generally assumed that these are written in CIRCAL and the specification of constraints has been effected by the description of abstract devices in CIRCAL. This Section discusses the modification of the language SuperC to enable the writing of partial specifications in that language. This is achieved by explicit use of constraints in a manner which solves the problem of ensuring that a partial specification can accept all the stimuli which are allowed by its associated constraint.

One of the reasons for the development of SuperC was to prevent a designer from specifying devices whose inputs could not always accept new values. That is, SuperC enforced the writing of full specifications. In this way, the problems which arise from using partial specifications, which were illustrated in Section 5.2.2, could be totally avoided. However, it has been demonstrated in this Chapter that partial specifications can be safely used provided they are accompanied by a suitable environmental constraint; furthermore there are numerous reasons, in addition
to the reduction of effort of writing specifications, for using them. Therefore, if SuperC is to be useful as a design language, it is important that some way of writing partial specifications be built into it.

Partial specification differ from full specifications in that a partially specified device may not necessarily accept all possible valid input events at all times. This difference can be subdivided into two broad categories:

- there may be certain times at which a value on a particular input port is not allowed to change (e.g. setup time constraint); or
- there may be certain combinations of values which may not appear on a set of ports (e.g. restriction on RS flip flop inputs).

Thus, to enable specifications written in SuperC, which are full by default, to be made partial, it is necessary to have some way of describing these two types of restriction.

7.4.1 Temporal Restrictions

To describe the first class of restriction, a new keyword stable could be introduced. (A similar feature is found in VHDL[USA84].) This would enable the writing of descriptions in which values may not change at certain times, as illustrated by the following example, a D-type flip flop with a hold time of 3 time units.

Example

```plaintext
dff(data,clk) = q
  where
    q = event(clk,true):data
    with constraint
      event(clk,true) => stable(data,3)  
  end
```

187
Explanation

The line beginning 'q = ' describes the behaviour of the flip flop as if it was to be fully specified. A rising edge on the clock port will cause the value on the input port (data) to be transferred to the output. The keywords 'with constraint' introduce the restrictions which will turn this into a partial specification. The syntax for describing the restriction is of the general form

\[
event(port, value) \Rightarrow stable(port, duration)
\]

The interpretation of the restriction in this case is this: when a rising edge occurs on the clock port \(clk\), the value on the port \(data\) must remain stable for 3 ticks.

Having found a way to describe the restriction on a specification, there needs to be some formal way of translating this into enhanced CIRCAL. A fairly elegant way to do this is:

1. Translate the part of the description which precedes the constraint into a full specification, as described in Section 3.3.

2. Translate the constraint specification into CIRCAL.

3. Compose the constraint specification with the full specification, thus reducing it to a partial specification.

Ignoring for the moment the fine details of step 2, there is an important point to be made here. In contrast to CIRCAL, it is not difficult to write a full specification in SuperC. The motivation for producing a partial specification is to assist the design and validation tasks, or to describe a library part whose behaviour can only be guaranteed for a limited range of input stimuli. The partial specification is therefore not produced at the outset, but is generated by the constraining of a full specification. A full specification is constrained by being composed with an abstract device which will only allow certain patterns of input events i.e. a contextual constraint as defined at the start of this Chapter. The composition process removes certain branches from the full specification, thus reducing it to
a partial one. This procedure may be considered a particular instance of the constructive specification technique of Section 3.2.2, as it relies on the composition of two behavioural descriptions to produce the actual specification.

An attractive feature of this approach to partial specification is that the constraint and the specification are guaranteed to be correctly matched. That is, the constraint which is used to generate the partial specification will ensure that the only actions which the environment can perform are the exact ones which the partially specified device can accept. This follows from the fact that the actions which are 'pruned' from the full specification by composition with the constraint are the same actions which the constraint prohibits the environment from performing. This is in contrast to the approach described in Section 7.3.1, in which a partial specification is written at the outset and a constraint is then added in the hope of preventing actions which the specified device cannot accept.

Step 2 of the above procedure, the translation of the SuperC representation of a constraint into an abstract device described in CIRCAL, can be achieved fairly easily. The device must be such that it accepts all possible events until the trigger event occurs. It should then disable the port named after the keyword stable for the specified length of time. In the example presented above, the trigger event is clk<true, the port to be disabled is data and the length of time is 3 units. This can be described by a box such as the following:

\[
\text{CON} \leftarrow \{ t \} \text{CON} + \{ t, \text{clk}<false \} \text{CON} \\
+ \{ t, \text{data}>x \} \text{CON} \\
+ \{ t, \text{clk}<false, \text{data}>x \} \text{CON} \\
+ \{ t, \text{clk}<true \} \text{CON1}(3) \\
+ \{ t, \text{clk}<true, \text{data}>x \} \text{CON1}(3)
\]

\[
\text{CON1}(n) \leftarrow \begin{cases} 
\text{if \text{noteq}(n,1) then} \\
( \{ t \} \text{CON1}(n-1) + \{ t, \text{clk}>x \} \text{CON1}(n-1) )
\end{cases} \\
+ \begin{cases} 
\text{if \text{eqs}(n,1) then} \\
( \{ t \} \text{CON} + \{ t, \text{clk}>x \} \text{CON} )
\end{cases}
\]
CON is the initial state in which any event is possible. When \( \text{clk} \) becomes true, the device moves into state \( \text{CON1}(3) \). In this state, no events are possible on the data port, but the counter variable \( n \) is decremented at every tick. After 3 ticks, the device returns to the state \( \text{CON} \), thus re-enabling the data port. It is not difficult to see how this description could be modified to enforce stability periods of different lengths.

7.4.2 Port Value Restrictions

The second class of restrictions, those which relate to the values which sets of ports may hold, can be represented simply by predicates which are true if the values on the ports are acceptable. The following example illustrates this.

Example

A device which might be specified with a restriction on the values on its input ports is a bus, which may only be driven by one device at a time:

\[
\text{bus}(\text{ina},\text{inb}) = \text{out} \\
\text{where out} = \begin{cases} \text{if eqs}(\text{inb},\text{ZZ}) \text{ then event}(\text{ina},x):x, \\ \text{if eqs}(\text{ina},\text{ZZ}) \text{ then event}(\text{inb},x):x \\ \end{cases}
\text{with constraint} \\
\text{or(eqs}(\text{inb},\text{ZZ}),\text{eqs}(\text{ina},\text{ZZ}))
\]

Explanation

The variables \( \text{ina} \) and \( \text{inb} \) are of a type which includes a special value 'ZZ' which represents the high-impedance or non-driving state in tristate logic. The behaviour of the bus is simple: if the value on the input \( \text{inb} \) equals 'ZZ', then any change on the other input \( \text{ina} \) will be passed directly to the output. The
symmetric behaviour when ina is set to 'ZZ' is described by the third line. The
constraint statement implies that at least one of the inputs must be set to the
high-impedance value at any time, thus ensuring that the bus is never driven by
both inputs at once, a situation which would have unpredictable results.

The translation of this type of constraint specification into CIRCAL is a little
more difficult than translation of the first type as it relates to static values on ports
rather than events. What is required is a device which can accept any events on
the constrained ports except those which will lead to states prohibited by the
predicate. In this case, the prohibited states are those in which neither port has
the value ZZ. The following device will never enter such a state:

\[
\text{CON}(a,b) \leftarrow \begin{cases} 
\text{if and(eqs}(a,\text{ZZ}),\text{eqs}(b,\text{ZZ})) \text{ then} \\
\text{(ina}\geq\text{x})\text{CON}(x,b) + \text{(inb}\geq\text{x})\text{CON}(a,x) \\
+ \text{if and(eqs}(a,\text{ZZ}),\text{noteq}(b,\text{ZZ})) \text{ then} \\
\text{(inb}\geq\text{x})\text{CON}(a,x) \\
+ \text{if and(eqs}(b,\text{ZZ}),\text{noteq}(a,\text{ZZ})) \text{ then} \\
\text{(ina}\geq\text{x})\text{CON}(x,b)
\end{cases}
\]

Explanation

The first two lines describe the situation in which both ports are set to ZZ. In
this case, an event on either input, but not both, will be safe, leading to a state in
which exactly one port is set to high-impedance. The next two lines specify that
if the value on inb is not ZZ, then no events on ina (which would cause it to leave
the high impedance state) are possible. The final two lines describe the symmetric
situation with ina and inb reversed. This description could then be composed with
a full specification of the bus as before to yield the desired partial specification.

7.4.3 A Shortcoming of SuperC

While the methods just presented for the writing of partial specifications in SuperC
certainly cover a wide variety of situations, it has not been conclusively proved
that all conceivable partial specifications can be written using the language features provided. Indeed, it is not very difficult to contrive an example to show that in some circumstances the language would lack the required descriptive power.

There are several reasons why this potential weakness of SuperC is not a great cause for concern. Firstly, a very large percentage of the constraints that have been encountered in realistic design situations can be described perfectly well using the features described above. Secondly, since the language is intended to be used as a front-end to a system which actually deals with expressions in CIRCAL, it would be possible, although rather messy, to write a constraint specification directly in CIRCAL and compose this with a full specification generated from a SuperC description to produce the required partial specification.

Finally, the main point of the development of this language has not been to produce a completely water-tight language suitable for commercial use, but to demonstrate some of the issues involved in building a suitable design language which can provide access to the descriptive and manipulative power of CIRCAL without allowing the designer to fall into some of the traps which occur when writing specifications directly in CIRCAL. The description of the features of SuperC has demonstrated well a number of these issues, in particular the role of constraints in the construction of partial specifications.

7.5 Summary

This Chapter has discussed the concept of constraints and their use in design and validation. This concept is a familiar one to designers and the number of examples of constraints which appear frequently in real design situations is quite large. However, constraints usually only receive an informal treatment, and the word itself is rarely defined.

The intuitive definition of a constraint as something which imposes a restriction was proposed, and a number of examples which fit this definition were examined. This led to the conclusion that constraints could be divided into two types: restric-
tions on the behaviour of a device's environment, and restrictions on the choices open to a designer. The first type, being more amenable to formal treatment, is the subject of this Chapter.

The formal use of constraints requires first of all that they can be formally specified. A mechanism for doing this is to represent a constraint by an abstract device which can perform any actions except those which the constraint prohibits. This type of device is easily described in CIRCAL, since it enables the description of devices which cannot always perform all possible actions.

In order to establish that constraints are of widespread usefulness, their role in each of the three main subtasks of the proposed methodology was examined. The concepts of full and partial specifications were defined and it was shown that the use of the latter, in which not all combinations of valid input events are possible at all times, could reduce the effort required to write specifications. The use of such specifications, however, is only justified if a contextual constraint is written to ensure that the environment of a partially specified device will only attempt to perform those actions which the device will accept. A formal method of establishing that an environment satisfies a constraint was presented.

Partial specifications can also assist in the design task, and a simple example was used to show that the unnecessary detail which may be included in a full specification could result in the rejection of a simple and acceptable solution to a design problem. There is also a price to be paid in the design task for the use of constraints — environments must be designed so that they satisfy any contextual constraints imposed on them. A method of modifying undesigned environments to meet constraints was presented. If parts of the environment have been designed using parts from a library, it may be necessary to choose a different library part, or not to use a library part at all. In this situation, contextual constraints can be seen to guide the designer by eliminating certain options. It is also possible that, in order to satisfy a constraint, an already-designed device may have to be re-designed, so that top-down design is replaced by iterative design.

In the verification task, partial specifications were again seen to be of assistance. The reduced behaviour of a partially specified device enables the process of
demonstrating equivalence between specification and implementation to be simplified considerably. The need for contextual constraints to accompany partial specifications was again demonstrated, the motivation in this case being to prevent unexpected interactions between implementations whose behaviours are richer than those of their specifications.

Finally, a mechanism for expressing partial specifications in the high-level language SuperC was presented. In this language, partial specifications are represented as full specifications with restrictions; the restrictions may be on the times at which port values may change or on the values which may be placed on certain ports. It was shown that a full specification can be made partial by composing it with a constraint, this approach having the advantage that it also ensures that any environment which meets the constraint will perform actions which the partial specification can accept.

The main point made in this Chapter was that a formal treatment of constraints can provide assistance in all tasks of a design and validation methodology. Conversely, it is only through the adoption of a formal, language-based methodology that the assistance provided by constraints can be made available.
Chapter 8

Concluding Remarks

8.1 Summary of Arguments

The main argument of this thesis could be summarised as follows: that the most suitable medium for the representation of hardware during the design process is a formal language, thus enabling the use of a hierarchical, integrated approach to design and validation. The thesis has examined both the reasons behind this argument and its consequences.

The main reasons for adopting an integrated hierarchical approach to design and validation are to improve the likelihood of correctness of a design and to minimise the cost of design errors by enabling their detection at the earliest possible stage. Hierarchical validation can only be carried out during the design process if behaviour can be formally represented at all levels of hierarchy; the most suitable medium for this representation is a formally defined language, as it offers the necessary expressive power and the ability to reason about behaviour in a rigorous way.

Hardware Description Languages

Having made the case for languages as a design medium, one of the first issues to be considered is what the nature of such a design language should be. This issue was approached firstly from the abstract side by considering the nature of hardware behaviour itself and thus how this might be described in a useful way. Key aspects of hardware behaviour were identified as the interdependency of values
on ports, the changing of values (or events), and the timing and sequencing of the occurrence of events. Based on these observations about behaviour, it was proposed that a behavioural Hardware Description Language (HDL) should have facilities for the description of these aspects. An additional feature, the ability to describe behaviour over a wide range of levels of abstraction, was also proposed.

A more concrete view of hardware description was presented by examining a number of HDLs which exhibit the necessary features proposed above. These languages were seen to differ quite considerably in the additional features they provide and the way in which they are provided. This reflects the differing motivations for the production of the languages and the range of environments in which they were developed. Only a handful of languages support formal reasoning about behaviour, a factor which becomes important when the issue of verification is addressed. One such language, CIRCAL, which has evolved from work on the modelling of concurrent processes and the application of this work to hardware, was selected for particularly close examination. Much of the work in the rest of the thesis relied on CIRCAL as the language for behavioural description of hardware.

Subtasks

The proposed hierarchical design methodology consists of a number of subtasks which were identified as specification, design and validation. These three tasks were discussed in the following chapters.

The task which must be performed first is specification. The meaning of ‘specification’ was taken to include any form of behavioural description which does not imply how the specified device is or will be constructed. The ways in which the chosen language can help or hinder the specification task were presented. A language may help the designer in this task simply by facilitating the writing of concise descriptions, or by encouraging ‘accurate’ specifications. Two ways in which a specification should be accurate were identified: it should reflect the real intentions of the designer, and it should as far as possible describe realistic hardware. Both these goals are only partially attainable.

196
While displaying those features which were identified as essential for hardware description, CirCAL was found to be lacking as a specification language when examined in the light of the above discussion. In particular, descriptions in CirCAL were found to be quite cumbersome for even fairly simple devices such as logic gates, and unacceptably so for more complex devices. Thus the pure language was enhanced with a number of features, such as the ability to write descriptions which are parameterised over state variables, and a richer variety of events.

Certain techniques may be used with a language to increase its ease of use and accuracy. A number of such techniques were presented for enhanced CirCAL. In order to ensure that these techniques are followed, a 'higher-level' description language (by analogy to high-level programming languages) was developed. This offered restricted access to the features of CirCAL but improved the ease of writing and accuracy of the resultant specification.

The specification task is followed by design. This task was seen to consist of two parts, partitioning and description. Separation of the two parts is rather difficult, as description (associating a behaviour with a structural object) is often performed informally in the designer's mind at the same time as partitioning. Partitioning consists of splitting a box (which has a behavioural specification but no known internal structure) into a set of smaller, interconnected boxes. Description then consists of formally assigning behavioural descriptions to the boxes. This part of the task is thus actually specification — formalising the informal idea of behaviour which exists in the designer's mind.

The skill of the design task is in deciding into what smaller boxes a box will be partitioned. Some choices lead to implementations which are more efficient by some criteria; design decisions may be made which ease the verification task; some decisions may lead to incorrect implementations. Ways in which the designer may be assisted in reaching the goals of efficiency and correctness were presented. Of particular interest in this thesis is the way in which the language-based approach to design can offer some of this assistance. It was seen that correctness-preserving transformations and design automation tools both rely on behavioural description
languages. It was also demonstrated that a design language may assist a designer by restricting the options open to him, thus reducing his 'search space'.

The third task of the methodology, which completes a design step, is validation. This is the attempt to show that the behaviour of the implementation which was produced in the design task is, in some sense, equivalent to the specification. The two methods of approaching this task are simulation and verification. The former is the testing of an implementation's behaviour by calculating its response to a set of input stimuli. The latter is the proof by mathematical means that the implementation performs correctly with respect to its specification.

Examination of each method of validation revealed a number of requirements for the specification language. In particular, only simulation can be carried out without the specification language having the facility for the derivation of behaviours of constructed devices. The vast majority of HDLs are therefore suitable for simulation. Two methods of simulation of expressions written in CirCal were presented. Some of the specification techniques which had been proposed previously were justified by demonstration of the undesirable consequences of ignoring them.

The main shortcoming of simulation was seen to be its failure to guarantee the correctness of an implementation. The second approach to validation, formal verification, seeks to provide such a guarantee. The first requirement for a language for verification is that the structural operators of the language have a formal interpretation which allows the behaviour of a constructed device to be mathematically established from the behavioural description of its component parts.

Verification using CirCal was examined and, as with simulation, it was found that specification techniques have an important part to play. It was shown that correct implementations could be rejected if some of the techniques of Chapter 3 were not used. A new technique, which enables specifications to be written without detailed knowledge of how they will be implemented, was also presented. It was shown that verification could be effectively carried out with such specifications in spite of the greater amount of information present in the implementation.
It was observed that CIRCAL really only addresses a part of the verification problem, that of showing that the ordering and timing of events is correct. The other part, the demonstration of 'functional' correctness, requires facilities for reasoning which are not supported by the CIRCAL language. An advantage of CIRCAL is that it assists in the separation of these parts, so that functional correctness may be more easily established using some other proof framework.

**Contextual Constraints**

Having discussed the three main tasks of a design and validation methodology, a concept which can provide assistance in all three tasks was introduced. The concept of contextual constraints was defined as a restriction imposed by a device on the behaviour of its environment, i.e. the pieces of hardware to which it is connected. The use of such constraints provides further justification for a language-based approach to design, as the formal use of constraints requires that they be formally specified.

In order to support the specification of constraints, a language must be able to specify events which may not happen or situations which may not arise. CIRCAL was shown to be particularly suitable for this because it naturally allows the specification of a subset of all possible events, so that some events are disallowed.

It was demonstrated that contextual constraints, formally described, could provide assistance in each of the tasks of the methodology. In specification, constraints can be used to ensure that the assumptions under which a specification is written are actually satisfied by its environment. In this way, partial specifications, which do not take account of all possible combinations of input events, can be safely used, thus reducing the difficulty of writing specifications.

The design task may also be assisted if partial specifications are used. The implementation of a device which is fully specified may be unnecessarily complicated by the need to respond correctly to input stimuli which will in fact never be applied. A contextual constraint is required in this instance to ensure that the
environment generates only those input stimuli of which the specification takes account.

Finally the validation task may also be assisted by the use of partial specifications. The example of a system controlled by a two phase clock showed that the magnitude of the task of proving equivalence between specification and implementation could be greatly reduced if the verification was carried out under the assumption that only selected patterns of stimuli would be applied to the system. A further use of constraints in verification was also discussed, this being to ensure that implementations do not interact in unpredictable and undesirable ways when they are capable of more actions that the specifications which they satisfy.

There is a cost of using constraints, which is that they must be satisfied by the environments to which they apply. This factor must be taken into account during the description phase of the design task. The description of a box must satisfy all constraints which are applied to it by parts to which it is connected. If the box is not already described when these constraints are generated, they can easily be incorporated into its description. If it has already been described, however, then it may be necessary to modify its description. This may mean that design decisions which were made at higher levels in the design hierarchy need to be revised, so that top-down design is replaced by iterative design.

8.2 Main Contributions

In developing the arguments outlined above, a number of contributions have been made to the fields of hardware description, design and verification. In this Section the most important of these contributions are summarised.

While this thesis has attempted to discuss the issues of language-based design in a general way where possible, the CIRCAL language has been frequently used for experimentation. Consequently, several of the tangible results of this work have been in the development of the CIRCAL language. Attempts to describe real pieces of hardware and to model their behaviour accurately led to a set of language
constructs which retain the facility to perform formal manipulation on behavioural expressions. The implementation of a system to assist this manipulation assisted in the formal definition of the meaning of composition in the enhanced language.

In addition to the development of the language itself, a number of techniques for writing specifications were developed. Some of these served to ease the specification task, while others ensure that satisfactory implementations do not appear to be incorrect.

Another significant development was the language SuperC, which removes the designer from some of the descriptive complexity and the pitfalls of writing CIRCAL descriptions, while still providing access to the formal reasoning which that language supports. While SuperC itself is perhaps not as 'user-friendly' as might be desired in a commercial environment, it nevertheless represents a useful first step in providing a convenient front end to a formal reasoning system.

The design task is one of the most difficult in which to provide assistance, as it generally relies heavily on the designer's creativity and skill. Two methods of assisting design have been developed: a simple 'high-level' design language, which restricts the options open to a novice designer, and a correctness-preserving transformation, which allows the designer some freedom while ensuring that his design is correct by construction.

In the field of verification, a notable advance is the development of techniques which enable specifications to be written without prior knowledge of how the implementation will be constructed, yet still allowing the implementation to be shown correct. In particular, this type of technique has been applied to the verification of devices whose timing behaviour is quite loosely defined at the specification level but defined in more detail at the implementation level. The use of this technique enabled the verification of a simple microprocessor, probably the largest and most abstract device yet verified using CIRCAL.

Finally, significant progress was made in the field of constraints. A useful definition of contextual constraints has been formulated, and a formal way to describe them presented. Effective uses of formally specified constraints, and the
partial specifications which they permit, have been developed in each of the three phases of the methodology. Also, a way to ensure that design is carried out in such a way that constraints are satisfied has been presented.

8.3 Future Work

Having seen what has been demonstrated and achieved in the work leading up to this thesis, it is worthwhile now discussing how this work could be extended and added to in the future. Future implementation work is discussed in Appendix C; this Section addresses more conceptual issues.

8.3.1 Language Development

Probably the most important theme of this thesis has been the role of languages in a design and validation methodology. It has been shown that the features of a language strongly influence its usefulness in the various tasks and that the techniques of using the language are equally important. Thus one of the most important fields for future work is that of language development. While some languages in the past have been developed in isolation, there are strong arguments for developing a language with the specific aim of supporting a methodology. This may be considered too restrictive an approach, but it need not be so if the methodology is as general as has been proposed in this thesis.

The development of the language SuperC suggests a possible route which could be followed for the development of a language to support a methodology. By giving the designer access to only certain features of CIRCAL the design process is assisted: the designer is prevented from falling into certain traps and the difficulty of the task of capturing his ideas formally is reduced thus improving the chances that it is done correctly. So, while SuperC may fall well short of the 'ideal design language', it represents a first step down a promising avenue of language development. Two specific ways in which the language could be further developed are:
• a syntax which more closely resembles that of an established programming
language or hardware description language could make the language more
"user-friendly" and acceptable to designers;

• the features which are required could be more fully researched, as it has not
been demonstrated that those currently included are adequate for a wide
range of design situations.

8.3.2 Assistance for Design

Whereas it is easily shown that a language-based methodology is essential for
the validation of implementations, it has been less amply demonstrated that the
language-based approach assists the design task. There are certainly some indica-
tions that it can: contextual constraints, which have been shown to assist the
design task, can only be formally used in a language-based methodology; the idea
of restricting a designer's access to the design language, which was seen to aid
specification, may also be applied to design.

The development in the past of high-level programming languages was a con-
siderable aid to the design of software systems; by enabling the programmer to
work at more abstract levels using objects such as integers and lists rather than
bits and addresses, high-level languages have greatly eased the task of software
development and improved the chances of designing correct systems. It certainly
seems reasonable to suppose that the development of suitable languages for hard-
ware could lead to similar gains in the field of hardware design. The analogy
between software and hardware could even be extended to include compilation;
software compilers are now quite well understood, while compilers for hardware
are still at a quite embryonic stage. It is to be hoped that future work on design
languages and their use to direct the design process might lead to similar advances
to those which have been made for software.
8.3.3 Verification

While the last few years have seen large advances in the field of hardware verification, there are still many problems to be overcome in making this process less complex and more amenable to machine assistance. It has been shown that CIRCAL provides considerable assistance in some aspects of the verification task, notably those which involve temporal and sequential, rather than functional, behaviour. In dealing with this latter aspect, theorem provers such as HOL[1] and Boyer-Moore[2] show promise. The development of tools of this kind may well lead to greater success in verification in the future.

In conclusion, the way forward for VLSI design is believed to lie with a formal, language-based approach. In this way, validation can be integrated with hierarchical design to minimise the chances and cost of design errors. The development of languages to support this kind of approach, and of closely related techniques for the use of such languages, should lead to greatly improved efficiency and correctness in VLSI design.
Appendix A

Glossary

The following definitions should not be regarded as universal. They represent the definitions used in this thesis; many of the words are defined differently by other authors, although none of the definitions used here is totally at variance with accepted practice.

action A set of events which a device may perform.

behaviour The way in which the values on the ports of a device depend on each other and on the passage of time.

black-box A device whose internal structure is not known or specified.

branch A part of a CIRCAL behavioural description, consisting of an action and an end state which is reached after the performance of the action. A CIRCAL behaviour consists of a choice sum of branches.

composition The process of establishing the behaviour of a device constructed from several behaviourally described components.

constraint Any statement or rule which places a restriction on something.

context The set of devices which may communicate with another device constitute its context.
contextual constraint A restriction on the behaviour of a device's context, usually either in terms of the times at which events may occur or the values which may be placed on ports.

description Any statement conveying information about a device. The information may relate to the device's behaviour, structure or geometry.

design step The complete process of producing and validating an implementation of a device at the next step down in the hierarchy.

device Any piece of hardware, not necessarily physically realisable.

domain One of the three possible types of description, either behavioural, structural or geometrical.

enhanced CIRCAL A language derived from CIRCAL in which states may be parameterised over variables of any type and events may involve the passing of values, variables or functions.

end state In a CIRCAL description, the behaviour which follows a guard.

environment See context.

event An occurrence on a port. Usually this involves a change of the value which is present on the port, although in CIRCAL events may simply be synchronisation pulses in which no value is communicated.

guard A set of events.

implementation A representation of a device which contains information as to how it will be constructed, in terms of the interconnection of smaller devices whose behaviour is either known or specified.

levels (of abstraction) The layers in a design hierarchy at which more or less structural detail is available. At the top level, the only structural information is the ports of the device; at the next level the device is described as
the interconnection of smaller devices; these smaller devices are described structurally at the next level down and so on.

**partition** To split a device into a collection of smaller devices and specify their interconnection.

**pure CIRCAL** The original CIRCAL calculus [Milne83a] as presented in Chapter 2.

**port** A channel through which a device may communicate with other devices. A port which may actually be realised in hardware is a *physical* port.

**simulation** The testing of the behaviour of a device by establishing its response to a (normally limited) set of input stimuli. If the response to all possible sets of input stimuli is tested, the simulation is termed *exhaustive*.

**sort** The set of ports through which a device may communicate with other devices.

**specification** A description of the behaviour of a device which does not directly imply anything about its structure. A *constructive* specification uses structural operators for descriptive purposes without committing the designer to construct the device in the same way.

**structure** The way in which a device is constructed, described in terms of the interconnections of smaller devices.

**synchronisation** The occurrence of an event on a port which is shared between two or more devices.

**synchronisation event** A CIRCAL event which involves no passing of values, but a port name only. Also called a pulse.

**tick** An synchronisation event used to represent the passage of one unit of time.

**validation** Any attempt to establish that the behaviour of an implementation meets a specification, whether by simulation or formal, mathematical proof.
verification The attempt to prove formally that an implementation meets a specification under all (or most) conditions.
Appendix B

Grammar of Enhanced CircaL

The following is a specification, in yacc input format, of a context-free grammar for enhanced CircaL, allowing the passing of values of any type and the use of any user-defined function to specify conditionals, output values and state parameter assignments.

```
spec : defn
    | spec defn
    ;

defn : name params '<=' branches ';'
    ;

name : STRING
    ;

params : /*empty*/
    | '(' paramlist ')'
    ;

paramlist : par
    | paramlist ',' par
    ;

par : STRING
    | INT
    | BOOL
    ;
```
branches : branch
  | branches '++' branch
  ;

branch : cond guard name assignments
  | cond '(' branches ')'
  ;

cond : /*empty*/
  | IF STRING THEN
  | IF function args THEN
  ;

function : STRING
  ;

args : arg
  | '(' arglist ')' |
  ;

arglist : arg
  | arglist ',' arg
  |

arg : STRING
  | INT
  | BOOL
  ;

guard : '{' eventlist '}'
  ;

eventlist : event
  | eventlist ',' event
  ;

event : input
  | output
  | synch
  ;

input : portname '>' arg incond
  ;
incond : /* empty */
| ':' function args
;
output : portname '<' arg
| portname '<' function
;
synch : portname
;
portname : STRING
;
assignments : /* empty */
| '(' asslist ')'
;
asslist : assig
| asslist ',' assig
;
assig : function args
| args
;
Appendix C

Implementation Issues

This Appendix discusses the major piece of implementation which was undertaken in the research leading up to this thesis. The implemented system is an ML program which assists the process of establishing the effect of composing together behaviours expressed in enhanced CIRCAL. The features of the system are described in Section C.2. First of all, the reasons why it was considered necessary to implement this system will be discussed. In the final Section, the lessons which were learned from both the development and the use of the system, including ideas for alternative approaches to the problem, are discussed.

C.1 Motivation

In the work leading up to this thesis, it was frequently found to be necessary to expand expressions which contain the composition operator. In Section 2.3.3 a formula was given for performing this expansion on two behavioural expressions represented as deterministic choice sums of guarded terms, to yield a third behavioural expression of the same type. This formula was extended in Section 3.1.7 to deal with the more complex synchronisation rules for the various types of events in enhanced CIRCAL. Manual application of this formula was soon found to excessively clumsy and time-consuming, so mechanical assistance was required. Earlier systems implemented by Traub[Traub86,Traub83] and Johnson[Johnson86] had
facilities for the expansion of pure CIRCAL expressions but could not deal with many of the features of enhanced CIRCAL.

The main reason for wishing to perform expansion of expressions involving the composition operator is so that behaviours can be put into a standard form, thus facilitating comparison between them. This form is the deterministic choice of guarded terms. Expressions in this form may be described as purely behavioural. The conditionals of enhanced CIRCAL make the comparison of such expressions more difficult than it would be in pure CIRCAL, as two equivalent behaviours may be represented in a number of different ways depending on the conditionals which are used. This is really just an unavoidable consequence of the greater expressive power of enhanced CIRCAL, and it is still the most feasible form for comparing two behaviours. The mechanism for comparing behaviours is described in Section 5.2.4.

The situations in which a behavioural expression involving the composition operator needs to be compared with a deterministic sum are numerous. Verification is a particularly important case, in which implementations are described by the composition of a number of behavioural descriptions and must be compared with a specification. The specification itself may be the composition of a number of choice sum expressions if the constructive specification technique of Section 3.2 is used, so this too may require expansion to put it into choice sum form. In working with constraints also there is often a need to determine whether an environment ENV satisfies a constraint CON; satisfaction was defined as the condition where the actions of ENV \* CON are the same as ENV. Thus expansion of the composition operator is again required to compare the two sets of actions.

A system which can expand an expression of the form A \* B, where A and B are both deterministic choice sums of guarded terms, to produce a third deterministic choice sum was therefore essential to enable experimentation with the issues of verification and contextual constraints. The more detailed features required in such a system are presented below.
C.2 System Features

C.2.1 Input Syntax

It was decided at the outset that the most suitable syntax for the input of purely behavioural expressions (i.e. deterministic sums of guarded terms) would be simply the syntax of enhanced CIRCAL as described in Section 3.1. Because of some limitations of the system, there is one main restriction: the names of states which are to be defined may not involve literal parameter values. For example, \textsc{dev} and \textsc{counter}(x,y) are both acceptable, but \textsc{counter}(3,5) or \textsc{nand}(true,true) are not. Literal values are, however, acceptable on the right-hand sides of definitions. There is no loss of generality due to this restriction, although it may lead to slightly longer specifications. A complete BNF specification of the syntax for behavioural expressions appears in Appendix B. As in the body of the thesis, parameters and passed values may be of any type including user-defined types, and user-defined functions may be used for conditionals and parameter assignments. Type and function definitions are written in Standard ML[Harper86].

All other input to the system consists of commands, such as 'expand the composition of A and B', where A and B are behavioural expressions which have already been input using the syntax described above. The manner of implementation of the system determined the syntax for these commands, which are described below.

C.2.2 Commands

Because it was decided, for reasons discussed below, to implement the system as an ML program, the commands (except the first) take the form of ML functions. The following paragraphs describe each of the functions which were included in the system.
Binding

This feature is actually built in to ML, the syntax being

\[
\texttt{val variable = ML-expression}
\]

It was considered important that the system should have a feature of this type, as it had been omitted from some of the earlier systems. In short, it enables a behavioural expression, which may have been calculated by some involved series of operations, to be bound to a single variable name for future re-use, rather than requiring the re-application of the operations.

\texttt{see(behav)}

This function translates a behaviour from the internal representation used in the system (i.e. an instance of an ML datatype) to a string which is written using the syntax of enhanced CIRCAL. This enables the behaviours which are bound to variables to be read by the user.

\texttt{dot(behav1, behav2)}

This function composes two purely behavioural expressions to yield a third. Thus it is the central function of the system, performing the expansion of the composition operator as defined in Section 3.1.7. Like all the remaining functions, the result returned by this function is an internally represented behaviour, which may be bound to a variable name or examined using the \texttt{see} function.

\texttt{rel(old,new,behav)}

Relabel a port in a behaviour. \texttt{old} and \texttt{new} are strings representing the name of the port to be relabelled and the new name it is to be given. Like the next two functions, it enables a behavioural expression to be modified to generate a new instance of a previously specified device.
reb(old,new,behav)

Replace a parameter, identified by the string old, with a new parameter, wherever it occurs in behav.

rename(old,new,behav)

Behavioural expressions have both a left- and a right-hand side. This function changes the state name denoted by old to new, whichever side it appears on.

brem(n,behav)

Two further functions provide assistance in comparing behaviours which have conditionals in the branches. This function removes a branch completely, a course of action which is justifiable if the conditional can never be true. It is up to the user to prove that this is the case. The integer argument n determines which branch is to be removed, the first branch in the choice sum being number 1.

prem(n,behav)

Conversely, if a predicate is always true, then it need not be present in the branch at all. This function therefore removes just the predicate from the nth branch. Again, the user needs to prove that the predicate actually is always true to justify its removal.

C.3 The Approach Taken

Many details of the implementation were a consequence of the decision to implement it in ML. The factors which led to that decision will therefore be discussed first.
C.3.1 Language Choice

Three languages, representing a fairly wide cross-section of language types, were considered for the implementation. In addition to ML, which is a functional language, the imperative language C and the logic language Prolog were examined for suitability. Interestingly, the 2 previous systems of Traub were implemented in Lisp (another functional language) and Prolog, while Johnson chose to implement his system in Pascal, an imperative language.

The language of choice for efficiency of execution would almost certainly be C. However, efficiency was not a high-ranking consideration, because the problems which were to be handled by the system, while too complicated for manual solution, would still be fairly small in terms of execution time on a computer. Of greater importance was the time which would be required to implement the system. On this criterion, C was considered the least appropriate language. Prolog, by contrast, is widely acknowledged as a fast prototyping language, due to the fact that the programmer can often specify what he wants the system to do, rather than how to do it. This advantage was counterbalanced by the author's lack of familiarity with the language.

Prolog was chosen by Traub for his second system and was found to be quite suitable for the textual manipulation which that system performed. The main concern about using Prolog for manipulation of enhanced CIRCAL expressions was the capability to deal with uninstantiated variables. This feature, considered to be quite important, was absent from Traub's system and it was not apparent that it could be readily included in a Prolog implementation.

One of the main advantages of ML was its datatyping facilities. Preliminary work had shown that these were highly suitable for the representation of the tree-like structure of CIRCAL expressions, which is central to the system. Also, as a functional language ML treats functions as first-class data objects; since functions are a central feature of enhanced CIRCAL, this is of considerable assistance in the design of a data structure to represent enhanced CIRCAL behaviours. Having
weighed up the above factors, ML was chosen as the language which offered the most likely chance of a ready solution to the task.

C.3.2 Structure of the System

In order to allow input of CIRCAL expressions without any modification of the syntax proposed above, a translator was written to generate ML expressions from an input written in enhanced CIRCAL. The main body of the system consists of datatype definitions (discussed below) and function definitions, the most important of which are those described above. The system is used as follows: the functions and datatype definitions are read in to the ML interpreter; the output of the translator is then read in. This generates instances of the datatype which is used to represent a purely behavioural expression. The user may then apply the various functions described above to the behaviours which have been thus defined.

C.3.3 Data Structures

A single datatype, called typval, is used to represent all legitimate types for values which may be passed on ports. This includes integers and booleans by default, and may also include user-defined types, such as tri-valued logic or bit vectors for example.

Functions all accept a list of typvals as argument and return a typval as result. The datatype representation of a function also includes a list of the names of its arguments, and a string to hold the name of the function. Predicates are represented in just the same way except that they may only return boolean values.

Associated with each branch of a behaviour is a (possibly null) list of assignments for the parameters of the new state to which the branch will lead. Another composite datatype is used to represent these assignments, which may be to a literal value, another parameter, or a function of parameters.

Three types of event were identified in Section 3.1.3. Because there may be three types of output event (outputting a value, a function or a parameter), this
could be considered five event types. These too are represented by a composite
datatype in the system. An event of any type has a string representing the port
on which it occurs, and other fields which depend on the event type.

The most important datatype is that used to represent purely behavioural
expressions. The choice sum is represented as a list of branches. A branch is
represented by a 4-tuple consisting of a predicate (for the leading conditional), a
guard (which is a list of events), a behaviour to which the branch leads, and a list
of the parameter assignments for the new state. In addition to the choice sum,
the datatype has fields for the sort of the behaviour (a list of strings), the name
of the state being defined, and the parameters of the state.

That concludes the discussion of the systems features and their realisation.
The following Section discussed how the building of the system has affected the
work on CIRCAL and its use in design and validation.

C.4 Lessons Learned

Both the implementation of the system and its use have taught some useful lessons.
Where experimentation with the system has provided information relevant to the
thesis it has been included in the appropriate section. For example, the fact that
the use of constraints could reduce the effort required to perform verification,
presented in Section 7.3.3, was established by using the system to investigate
examples. In this Section, the information that was obtained simply by developing
the system will be presented.

At an early stage, the planning of an implemented system led to a thorough
investigation of the necessary features in enhanced CIRCAL. It was clearly neces-
sary that the language for writing expressions be finalised before trying to design
a system to manipulate the expressions. In order to decide upon the language
features which would be most important for time- and space-saving behavioural
description, a large number of example devices were described. While some of the
features to include were obvious, others were less immediately apparent. Notably,
this investigation led to the conclusion that the two types of conditional described in Chapter 3 (leading and input-qualifying) were required in the language to enable efficient description.

One of the most significant results of the implementation of the system was that it necessitated the formalisation of synchronisation rules for the different types of events in enhanced CIRCAL. While expansion of the composition operator was being performed manually, it was possible to use an intuitive idea of synchronisation, but this clearly was inadequate for a mechanised system. All possible interactions between the five different event types had to be considered and the intuitive ideas about these interactions were made formal. The rules for synchronisation presented in Section 3.1.7 were a direct result of this aspect of the implementation effort.

In addition to the generation of ideas about enhanced CIRCAL, the implementation of the system has opened the way for more sophisticated systems. The following Section discusses some additional features which could be added to the current system to increase its usefulness for design and verification.

C.5 Ideas for Additional Features

C.5.1 Complete Expansion

If the states which can be reached from a defined state are all just different 'instances' of that state (i.e. have the same name but different parameters) then that behaviour may be considered 'complete' — no further information is required to ascertain the response of the described device to any pattern of stimuli. However, if it is possible to reach another state, then the behavioural description can only be considered complete if there is also a behavioural description for that other state, and for all the states that it may lead to. Another way to put this is that a behavioural description is complete if the behaviour from all reachable states is
known. Incomplete behaviours are analogous to finite state diagrams whose arcs do not all lead to nodes, and are clearly of limited usefulness.

The concept of complete behaviour is relevant to the expansion of composed behaviours. Consider the two sets of behavioural expressions which follow:

\[ A \subseteq (a)A1 + (b)A \]
\[ A1 \subseteq (a)A + (b)A1 \]
\[ M \subseteq (a)M + (a, b)M + (b)M \]

The first two lines form a complete behaviour, as does the third. Now, if the composition of \( A * M \) is expanded, the result is:

\[ A * M \subseteq ((a)A1 + (b)A) * ((a)M + (a, b)M + (b)M) \]
\[ = (a)(A1 * M) + (b)(A * M) \]

This is not a complete behaviour, because the behaviour of the state represented by \( (A1 * M) \) is not known. This leads to the concept of complete expansion, which consists simply of carrying out the expansion of terms such as \( (A1 * M) \) until the behaviour which was first expanded is complete. In this case only one further expansion is necessary, as follows:

\[ A1 * M \subseteq ((a)A + (b)A1) * ((a)M + (a, b)M + (b)M) \]
\[ = (a)(A * M) + (b)(A1 * M) \]

Complete expansion can only be carried out on the implemented system with user intervention to specify those states which require expansion. It would be a useful addition to the system to be able to perform complete expansion without any assistance from the user.

**C.5.2 Behaviour Matching**

A major task in verification with which the current system provides very little assistance is that of comparing two behavioural expressions. The functions \( \text{brew} \)
and \texttt{prem} described above offer only a small amount of help in this. Without actually implementing a theorem-prover, the system could be enhanced considerably by a function which attempts to match two purely behavioural expressions. This is essentially the problem which was addressed in Section 5.2.4. In that Section, however, only a small class of behaviours were considered; it would be most helpful to be able generate the logical formulae which have to be proven for any two behaviours to be shown to be equivalent. This part of the proof, the functional part, could then be tackled either manually or with the aid of a theorem-prover.

\section*{C.6 Conclusion}

The issues involved in the implementation of a system to manipulate enhanced \textsc{CIRCAL} expressions with the aim of assisting design and validation have been examined. The motivation for the development of this system was primarily to facilitate experimentation by enabling behaviours written using the composition operator to be expanded into the standard form of a deterministic choice sum. The necessary features of such a system and the way in which these have been provided were discussed. Some additional features which could increase the usefulness of the manipulation system were also proposed.

In addition to enabling much of the experimentation which has underlain this thesis, the implementation of the system itself has led to the formalisation of some important concepts. In particular, the definition of the features of enhanced \textsc{CIRCAL} and the establishment of a complete set of rules for synchronisation of different event types in this language were a direct result of the need for rigorousness imposed by the implementation.
Appendix D

Function and Datatype Definitions for Computer Specification

(* all integer types are defined as int, their sizes being taken care of by the functions which operate on them *)

```plaintext
type int2 = int;
type int13 = int;
type int16 = int;
(* a memory is represented as an ordered binary tree *)
datatype memory = empty
  | leaf of int13*int16
  | node of int13*memory*memory;
(* an enumerated type is used for opcodes *)
datatype opcode = HALT
  | JMP
  | JRO
  | ADD
  | SUB
  | LD
  | ST
  | SKIP;
```

(* Define some auxiliary functions and constants *)

```plaintext
fun tworaise(n) = if n = 0 then 1 else 2*tworaise(n-1);
val maxint2 = 3;
val maxint13 = tworaise(13) -1;
val maxint16 = tworaise(16) -1;
val defval = 0;
(* Truncate a 16-bit integer to 13 bits *)
fun truncate(x) = x mod (maxint13 + 1);
```
(* Extract the opcode from a 16-bit integer *)
fun getop(x) = let val opbits = x div (maxint13 + 1) in
  if opbits = 0 then HALT else
  if opbits = 1 then JMP else
  if opbits = 2 then JZERO else
  if opbits = 3 then ADD else
  if opbits = 4 then SUB else
  if opbits = 5 then LD else
  if opbits = 6 then ST else SKIP end;

(* Store data at addr in memory *)
fun store(empty, data, addr) = leaf(data, addr)
  | store(node(a, m, leaf(a, data)), data, addr) =
    if addr > a then node(a, m, leaf(addr, data))
    else if addr = a then leaf(a, data)
    else node(addr, leaf(addr, data), m)
  | store(node(ad, x, y), data, addr) =
    if addr > ad then node(ad, x, store(y, data, addr))
    else node(addr, store(x, data, addr), y);

(* Retrieve the data stored at addr *)
fun fetch(empty, addr) = defval
  | fetch(leaf(a, d), addr) = if a = addr then d else defval
  | fetch(node(ad, x, y), addr) =
    if addr > ad then fetch(y, addr)
    else fetch(x, addr);

(* Add two 16-bit integers *)
fun add16(a, b) =
  let val sum = a + b in
  if sum > maxint16 then sum - maxint16 else sum end;

(* Subtract two 16-bit integers *)
fun sub16(a, b) =
  let val dif = a - b in
  if dif < 0 then 0 else dif end;
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