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Costly Signaling and Generous Behavior

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This thesis explores the use of generous behavior as a costly signal to convey information about an unobservable social characteristic to other individuals in one's social environment. Building on recent contributions in this spirit, I develop a theoretical framework that contrasts signaling activities without social benefits with activities that benefit the observers in situations in which individuals compete for access to a scarce social good. The objective of the first part of the thesis is to characterize the possible separating equilibria in each case. While one obtains a multiplicity of equilibria when the agents employ neutral signals to convey information, one can make a unique prediction with respect to the individuals' equilibrium behavior if they use beneficial signaling activities, the agents are of two discrete types, behave symmetrically within their respective types, and the observers of the signals adopt non-decreasing beliefs vis-à-vis the signalers' relative quality. In view of their sharp divergence, the second part of the thesis investigates experimentally the precision of these predictions. The results provide support for many elements of the theory. Among others, the behavior of individuals in the treatments with beneficent signals is much more closely in line with the theoretical predictions than expected given the complexity of their behavioral implications, especially when it comes to the similarity of the participants' behavior within them. Behavior in the treatment with neutral signals, in turn, is consistent with multiple equilibria. The final part of the thesis explores what kind of signaling activity individuals trying to communicate their intentions to potential interaction partners will use in various social settings if given a choice. To this end, the framework developed in the first part is extended to allow the signalers to choose endogenously a signal from a “menu” of signaling activities rather than exogenously prescribing a messaging tool. Besides revealing that the uniqueness result of the framework without choice no longer obtains, the results indicate that the players may, under some conditions, opt for inefficient signals.
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I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified.

(Sascha Janina Mohr)
To Cathy – for always believing.
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CHAPTER 1

INTRODUCTION

As far back in history as hunter-gatherer societies, humans have formed alliances and engaged in coalitions with non-relatives in groups that have grown steadily over the centuries. In modern societies, cooperative social interaction even extends to complete strangers in one-shot, anonymous settings. People, for instance, routinely give directions to strangers, vote, recycle, and participate in community goings-on such as blood drives, Neighborhood Watch programs, and clean-up campaigns. Indeed, generous behavior towards others is nowadays so common that one does not tend to notice it or question why someone would (ever) bother to incur a cost – be it in terms of time or money – to contribute to the welfare of another individual, or group of individuals, (s)he does not know and may never interact with again. Yet, even though ubiquitous, people do not interact cooperatively with everybody at all times, and may even vary in their willingness to do so when facing the same environmental conditions. Rather, cooperative social interaction tends to depend on countless factors, including institutional, cultural, and technological factors, individuals’ emotional predisposition, as well as their ability to communicate with each other prior to interacting with one another (e.g., Richerson et al. 2003).

While neoclassical economic theory works well in predicting behavior in a wide range of environments beyond traditional economic markets, including politics, law, and various household- and family-related matters, it tends to yield inadequate projections in the context of cooperation. One of the central challenges in this regard is individuals’ apparent departure from self-interested utility maximization. The

11 There is a subtle difference between cooperative behaviors, which involve the provision of a direct benefit to others at some personal cost (i.e., a generous deed), and an individual engaging in a pro-social activity, in which case (s)he incurs a cost to maintain an individually costly behavior (Henrich & Henrich 2006). As this distinction is not essential for the purposes of the work presented in this thesis, the terms will at times be used synonymously.
Chapter 1. Introduction

Theories of repeated games and conditional reciprocity go some way to explaining cooperation among kinfolk and in settings involving small groups whose members interact quite frequently. However, since generous behavior among unrelated individuals in one-shot encounters does not comply with the basic tenets of these approaches, the proximate cause(s) of this behavioral pattern remain(s) a puzzle. The work presented in this thesis strives to illuminate this aspect of human social behavior by investigating a mechanism that allows individuals to forestall exploitation, in effect, by sharing information before interacting.

Approach. The inspiration for the approach promoted in each of the subsequent chapters is that (one of) the main objective(s) of a rational individual becoming involved in a social group is the maximization of his/her access to or control over limited resources, which naturally entails both cooperative and competitive interaction between the group members. For, besides increasing the risk of exploitation, competition for resources, and exposure to diseases, the formation of alliances – the likely origin of today’s vast societies – yields a plethora of benefits related to the acquisition, production, processing, and access to resources (e.g., Boone 1992; Cosmides & Tooby 1992; van Vugt et al. 2007). In order to minimize the risks linked to the interaction with others, it is clearly of paramount importance to be able to identify reliable interaction partners and to advertise one’s own qualities as an associate so as to be able to gain access to the most lucrative alliances.

The fundamental issue from this perspective is how individuals resolve the informational asymmetry characterizing this environment. After all, individuals’ intentions once part of an alliance, their “social quality” so to say, is not observable to others without prior interaction. As preliminary knowledge about others is implausible in one-shot encounters with strangers, the question arises how people decipher each other’s intentions in this type of setting. The remedy explored in this thesis, which has a long history in both the social and natural sciences, is that individuals seeking to gain access to profitable alliances may engage in costly signaling in order to inform other individuals of their objectives.

A signal is an observable measure acquired by an individual with private information (e.g., regarding some unobservable trait) in order to transmit credibly said information to a less informed individual, or group of individuals. The idea is

12 Chapter 2 (Section 2.5) considers these and a number of alternative approaches to explaining cooperation among humans in social settings in some detail.
that a reliable signal must be costly to the signaler to the extent that it could not be afforded by an individual with less or none of the trait in question. Key is that the members of a social group vary in some unobservable trait, that accurate information about this variation is valuable, that successful deceit regarding the true nature of the attribute would benefit the signaler at the expense of the observers of the signal, and that the cost of the signal is negatively correlated with the signaler’s endowment of the trait. Typical examples of this kind of “handicap” include the tails of peacocks to advertise gene quality (Zahavi 1975), investment in schooling to signal ability (Spence 1973, 1974), and conspicuous consumption such as the purchase of jewelry and luxury cars to suggest wealth (Veblen 1899). The essence of the signaling mechanism is that both parties to the exchange gain from the signaler’s investment as the signals indicate qualities that make it advantageous for the observers to interact with the signaler. Importantly, the agents behave in a purely self-interested manner.

In the spirit of this concept and a recent contribution by Gintis, Smith, and Bowles (2001), the work underlying this thesis builds on two observations. For one, recent anthropological evidence suggests that generous behavior towards others may constitute a costly signal. Potlatches, for instance, may be used to signal physical condition, knowledge, and leadership ability (e.g., Smith & Bliege-Bird 2005). Second, variation in traits associated with successful cooperative interaction (e.g., reliability) within any given group necessarily entails that profitable social goods such as alliances, mating opportunities, or leadership positions constitute as limited a resource as any other. In particular, combining these notions, I explore the use of generous behavior as a costly signal to convey information about an unobservable social characteristic to others in an environment in which the individuals compete for access to a scarce social good.

**Contributions.** For insight into the usefulness of this unconventional messaging tool, Chapter 2 develops a theoretical framework that contrasts traditional signaling activities, which do not feature social benefits (termed “neutral signals”), with activities that provide a direct benefit to the observers, or receivers, of the signals (referred to as “beneficent signals”). The objective is to characterize the possible separating equilibria in each case (if any). The main feature of the model, besides individuals differentiated in their social attributes striving to alleviate the asymmetric information within their social environment using costly signaling, is

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13 The details of their work are discussed in Chapter 2 (Section 2.2).
that the receivers prefer to interact with signalers of high social quality, an
opportunity the signalers compete for with each other.

In keeping with the literature, one obtains a multiplicity of equilibria when the
agents employ neutral signals to convey information to the receivers. It should,
however, be noted that the present setup is virtually the exact opposite of the
canonical signaling model, i.e., it features competition among multiple signalers
facing (in effect) a single receiver. Quite on the contrary, one can make a unique
prediction with respect to the individuals’ equilibrium behavior if they employ
beneficial signaling activities, they are of two discrete types, behave symmetrically
within their respective types, and the observers of the signals adopt non-decreasing
beliefs vis-à-vis the signalers’ relative quality. To be precise, the signaler types
randomize continuously on separate but contiguous interval supports (comprising
strictly positive signaling magnitudes) and the receivers adopt appropriate threshold
beliefs. What is more, the result is quite general in that it does not depend on the
exact value of the signal-dependent benefit, so long as it is strictly positive, nor does
it fall victim to equilibrium refinements.

Not only is the setup, notionally, a natural extension of conventional models,
requiring mostly mild assumptions, but so is the intuition for the results. While use of
neutral signals allows for a plethora of “belief-based” equilibria, as soon as the
signaling activity yields even a small benefit to the receivers, the players’ interaction
transforms into an all-pay auction in the sense that the signalers effectively employ
the magnitude of their signals as a “bid” for an alliance with a receiver. The
uniqueness result, in effect, follows straightforwardly from the fact that the receivers
can retain (some of) the benefit associated with the beneficial signaling activity,
which induces them to seek the signaler sending the highest signaling magnitude,
thereby restricting the impact of their beliefs on the equilibrium outcome.

In view of the sharp divergence of the equilibrium predictions across the
signaling activities, Chapter 3 investigates experimentally their precision. Based on a
subject pool of 165 individuals in three treatments – one with neutral and two with
(different) beneficent signals – the experiment provides strong support for many
elements of the theory. Among others, the behavior of the individuals in the
treatments with beneficent signals is much more closely in line with the theoretical

---

1.4 That is to say, the receivers associate higher levels of the signal with a (weakly) higher probability
of the signal having been sent by a signaler of (the) high(er) type.
predictions than expected given the complexity of their behavioral implications, especially when it comes to the similarity of the participants’ behavior within them (comparative statics). Behavior in the treatment with neutral signals, as projected, is consistent with multiple equilibria. Adjustments over the course of the experiment furthermore suggest that experience and the adaptation of their beliefs to the events in earlier rounds shape the participants’ behavior.

While the experimental setup does not constitute an innovation within the field of experimental economics, it is original in seeking validation of the framework developed in Chapter 2 in an environment that does not make the mixed strategy outcome in the treatments with beneficent signals explicit. In other words, granted an explicit frame, it allows for only one decision per decision-round. It is, in this context, particularly remarkable that even the participants in the treatment with neutral signals coordinate on quite complex outcomes, namely, a threshold-like separating arrangement involving “pooling” on the part of the signaler types on distinct signaling magnitudes as well as a configuration very similar to that predicted for the treatments with beneficent signals. For, the latter is suggestive of the fact that the signal-dependent benefit could be used as an equilibrium selection device.

The final chapter, Chapter 4, is based on the notion that an individual trying to communicate his/her intentions to potential exchange partners can, in principle, choose among a variety of signaling activities. After all, the development of progressively larger and more complex social groups will have required appropriately adapted strategies and flexible responses to one’s environment so as to achieve maximal success in one’s various cooperative and competitive interactions with other individuals (e.g., Boone 1992). In this vein, the model developed in this chapter explores which kind of signal an individual is likely to use in a given social setting. To this end, the framework developed in Chapter 2 is expanded to allow the agents to choose endogenously a messaging tool from a collection of activities.

Besides highlighting the tradeoff between the signaling activities’ relative cost and the signal-dependent benefit(s), if any, to the receiver, the results indicate that the uniqueness result of the framework without choice no longer obtains. Not only may several types of equilibrium coincide, but some instances support continua of equilibria, just as traditional economic signaling models. Moreover, under some conditions, the signalers may – in equilibrium – opt for inefficient signaling activities. More intriguingly, depending on the parameterization of the game, there
need not exist a universal “best.” The main divergence between the setups with and
without choice to account for these findings is the option to separate such that the
signaler types use different signaling activities when given choice, which can be
supported as an equilibrium even if the cost of the signaling activities does not vary.
The driving element in this regard is that the receivers may adopt a more widespread
configuration of beliefs, especially off the equilibrium path.

Even though one of the most elementary themes of economics and a quite
intuitive aspect in the context of “communication,” choice of signaling activity (or
messaging tool, more generally) has not been considered within frameworks
amenable to costly signaling. Yet, the introduction of choice into signaling
frameworks allows for a more careful delineation of the type of signaling activity to
be expected in a particular situation, which is useful since the equilibria supported by
conventional signaling models can often be satisfied by different types of signaling
activities.\textsuperscript{5} Along similar lines, other than noting that signaling is fundamentally a
wasteful activity, the welfare implications of various signaling activities in terms of
identifying whether there exists such a thing as a “(unanimously) best” signal have
thus far not been resolved. Though constrained as the settings being considered
(only) differ according to the cost of the signaling activities and the signalers (only)
face a single audience (i.e., the receivers are assumed to be homogeneous), the
framework sheds some light on these matters.

**Concluding Remarks.** In sum, the work presented in this thesis merges two
issues thus far commonly considered distinct entities – human social interaction and
the theory of costly signaling. It does so by adopting a somewhat unusual perspective
on the efficacy of cooperation, defined as individually costly generous behavior
towards others. Namely, it is based on the notion that individuals operating in a
social group choose to do so out of self-interest, entailing that all interactions within
the group, including cooperative ones, ultimately have a competitive basis. Rooted in
this foundation, the two theoretical frameworks suggest how individuals may be able
to decipher each other’s intentions in this type of environment, which turns out to be
possible in a manner that can be validated empirically under laboratory conditions.
The critical element is that the signaling activity provides a direct benefit to the
receiver; if not, one does not arrive at a sharper prediction. If the signalers are given

\textsuperscript{5} By identifying conditions for equilibrium for each kind of signaling activity, Chapter 2 also
provides some insight into this issue.
a choice of activity, the precision of the unique prediction is diluted to the extent that more than one outcome is possible, including ones involving apparently inefficient choices of signal. Nevertheless, overall, the findings developed in the following entail that competition and signals comprising a direct benefit to their observers may play an important role in individuals’ attempts to form cooperative alliances with other individuals in their social environment.


CHAPTER 2

GENEROUS BEHAVIOR: AN UNCONVENTIONAL APPLICATION OF THE THEORY OF COSTLY SIGNALING

2.1 Introduction

One of the most distinctive features of human social behavior is that humans, in contrast to most other primate societies, are strongly interdependent and routinely coordinate their actions with individuals they are not related to, do not know, and may never interact with again. In doing so, they achieve a broad range of mutually beneficial outcomes. A pervasive, cross-cultural pattern in this context is individuals’ pursuance of generous acts towards others within their social environment. Examples include sharing resources or information with individuals beyond the immediate family, doing volunteer work in the community, helping strangers, and more broadly, the adherence to and enforcement of social norms. All of these activities (primarily) benefit others while involving a cost for the individual carrying out the act.

The activities are peculiar since they, in effect, correspond to individuals providing local “public goods” to others – be it an individual or an assortment of other individuals – without paying careful attention to the history and probability of reciprocation of the beneficiaries of their generous acts. On the contrary, in apparent contradiction to their self-interest, the actions do not balance accounts with others but appear to involve an asymmetric or even entirely one-directional flow of good deeds. As a result, although rich, explanations relying on notions of reciprocity (Trivers 1971) do not provide sufficient insight into the forces driving individuals’ conduct in these instances. For in contrast to two-person interactions, many of which
meet the criteria of some form of reciprocity, generous behavior within a broad social arena cannot be made contingent on reciprocation.¹

Yet, generosity is not extended universally or randomly. Rather, generous behavior seems to be employed strategically. Susceptible contexts as well as the characteristics of the involved providers and recipients tend to be constrained and patterned (e.g., Smith & Bliege-Bird 2000, 2005). Likewise, even the members of the most closely related group of cooperative individuals face conflicts of interest, which tend to pose a barrier to mutually beneficial interaction. The question thus arises how providing a benefit to individuals not known to return the good deed could be worthwhile, or equivalently, how potential interactants discern each other’s intentions with any degree of reliability.

In keeping with recent work in biology and anthropology (e.g., Zahavi & Zahavi 1997; Smith & Bliege-Bird 2000; Bliege-Bird et al. 2001; Gintis et al. 2001), this chapter promotes the idea that generous behavior in a social context can be profitable if it has a sufficiently large impact on the outcome of one’s future interactions to offset its immediate cost. For instance, beneficent acts might enable individuals to advertise various socially important qualities in a reliable manner, thereby disclosing their value as future interactants. The provision of a beneficial resource or service would then not only benefit the recipients and/or observers, who acquire the resource or service as well as useful information about the provider, but also the contributor. The latter is the case not because (s)he expects future reciprocation (in kind), but because the provided information influences the observers’ optimal behavior in such a way that the contributor also gains, for example, by gaining access to contested social goods such as alliances, mating opportunities, or leadership positions. In other words, even though the nature of the activities is fundamentally different – i.e., they benefit the observer(s) – just as the famous idea that socially useless activities can convey information to prospective collaborators (Spence 1973, 1974), a beneficent act may represent a costly signal of some unobservable, heterogeneous social quality.

The condition underlying this perspective is that the amount of generosity an individual can, or is willing to, provide depends on his/her underlying social quality.

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¹ Conditional reciprocity is the most popular explanation for cooperative interaction within the field of economics. Note that (evolutionary) biologists consider a much broader range of mechanisms (West et al. 2007). Section 2.5 discusses a number of alternative explanations.
If so, generous behaviors can be used to transmit information about individual differences in social quality, which in turn can be used by others with respect to their choices when interacting with the relevant individual (Leimar 1997). It follows that if access to valuable social goods is restricted, say, because possible interaction partners can choose with whom they wish to interact, and generosity represents an avenue whereby individuals can gain access to one such good, individuals competing for the good would be challenged to be more generous than others or else forego the most valuable opportunities. The upshot would be what has been called “competitive altruism” (Roberts 1998; Lotem et al. 2002; Barclay & Willer 2007).

Note that it is not argued that costly signaling will eradicate the risk of defection (or exploitation) in social situations. Rather, the idea is that it could be a tool not only to help individuals advertise themselves as attractive exchange partners, but to enable them to predict which potential associates are not likely to defect on them. After all, the parties to an interaction have a mutual interest in sharing information about their relevant qualities even if their interests are strongly opposed.

Based on a recent contribution in this spirit (Gintis et al. 2001), I develop a model of individual behavior that contrasts signaling activities without social benefits with activities that benefit the observers in situations in which individuals compete for access to a scarce social good. The objective is to characterize the possible separating equilibria in each case. Paralleling traditional signaling models, the framework comprises a two-sided transaction within the context of which incompletely informed, self-interested agents are looking to interact with one another. The setting is framed in terms of the signalers striving to form coalitions with the receivers. In contrast to standard models, besides considering beneficent signals, the framework features competition among the signalers as opposed to the receivers. Conceptually, this implies that the intensity of the signal may affect the outcome of the agents’ interaction. The main result of this chapter is the derivation of a unique separating equilibrium in mixed strategies for a class of signaling games involving beneficent signals and symmetric signalers of two discrete types, subject only to a non-decreasing beliefs restriction on the part of the observers of the signals.

The remainder of this chapter is organized as follows. Section 2.2 outlines the theoretical underpinnings of the framework. Section 2.3 thereupon introduces and

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22 Although one could think of alternative backdrops, Section 2.4 highlights the relevance of this perspective in the context of human social interaction.
solves the model both with a restricted set of players and within a general \( n \)-player framework. Section 2.4 discusses the results. Section 2.5 reviews the related literature, followed by concluding remarks in Section 2.6.

### 2.2 Theoretical Foundation

Notwithstanding the fact that costly signaling has a long history in a number of disciplines, a first attempt to model the idea that individually costly behavior that yields a benefit to others can constitute a costly signal has been made only recently (Gintis et al. 2001). Building on anthropological fieldwork, the authors set out to provide an explanation for the evolution of group-beneficial behavior – the unconditional provision of a benefit to a collection of other individuals.\(^3\) The approach is founded on the notion that the fitness or material benefits derived from group-beneficial activities, if they result in advantageous alliances (e.g., mating opportunities or coalitions) for those using them as a costly signal, could account for the evolutionary proliferation of the relevant behaviors.

Their framework involves a series of one-period, multi-player public-goods games. As the players’ interaction is not repeated, without signaling, the unique equilibrium of the game prescribes universal defection as the players’ dominant strategy. When accounting for signaling benefits and focusing on separating equilibria in pure strategies, it can be shown that “signaling of underlying quality by providing a benefit to group members can be evolutionarily stable, and may proliferate [in a population] when [initially] rare” (p. 104). This result holds provided that individuals of high quality are not too common and the cost of signaling is sufficiently high for individuals of low quality.

Formally, every period features a group of \( n \) individuals of two discrete types, high or low quality; information on type is private. Prior to interacting with another group member, an individual can signal his/her type by executing a non-excludable, individually costly, group-beneficial action.\(^4\) If deemed adequate by a receiver, the provision of the benefit can result in an alliance with one or more receivers. Note that a signaler can ally with many receivers. A given receiver, in turn, is assumed to ally

\(^2\) Lotem et al. (2002) develop a model in a similar spirit, i.e., regarding the evolution of cooperation via costly signaling. Yet, in contrast to Gintis et al. (2001) and the present approach, their framework not only involves repeated interaction, but comprises cooperation even in the absence of signaling.

\(^3\) The cost of the activity is assumed to be strictly positive and lower for individuals of high quality.
with only one signaler. Every alliance yields a fixed positive benefit for the signalers, while the receivers obtain a type- and signal-dependent payoff. Hence, in its essence, the static model corresponds to a tournament among the signalers for alliances.

While my framework parallels the static elements of this approach, it differs in three essential respects. From a technical standpoint, rather than allowing for only two levels of the signal, I consider a continuous action space on the part of the signalers. An immediate implication of this extension is the relevance of a much more extensive array of configurations of the receivers’ beliefs, which are explicitly addressed. It turns out that these adaptations in conjunction with the assumptions of conventional economic models of costly signaling not only render my framework more general, but move the nature of the signal – i.e., neutral or beneficent – to the center of the analysis. This contrasts sharply with the framework by Gintis, Smith, and Bowles (2001; henceforth GSB), in whose case the nature of the signal is not decisive for the equilibrium: Neutral and even harmful signals can satisfy the same separating equilibrium as generous activities. Moreover, a wide range of parameter values gives rise to a non-signaling equilibrium.\(^5\) Although the main part of my analysis involves signals with strictly private benefits for the receiver once (s)he chooses his/her ally – one might think of the receiver having to return the benefits of the signalers (s)he does not ally with – it can easily be extended to encompass signals with non-excludable benefits to others (cf. Section 2.4).

From a conceptual point of view, my analytical angle differs from that of GSB in that I do not consider evolutionary mechanisms. Instead, I home in on the signaling issue. Given a link between the signal (neutral or beneficent) and the sender’s unobservable quality, I show that under a certain set of circumstances the signaling activity can be sustained in a one-shot environment. In effect, the underlying disparity is in the research questions. Whereas GSB investigate how a group-beneficial signal can arise in a framework that encourages defection, given standard assumptions, I try to determine what a separating equilibrium involving beneficent signals in conjunction with competition among the signalers would look like, if it exists, and how it contrasts with the equilibrium given neutral signals.

\(^{25}\) My model furthermore diverges from GSB’s framework in that I do not consider a cost to monitor signals. While requiring amendments to the receiver’s payoff function, for a sufficiently small cost, the results will qualitatively be the same as those derived in the following.
As such, the game developed in the following is a one-period, non-cooperative game of incomplete information involving two populations of risk-neutral individuals – signalers (he) and receivers (she). The signalers, who are differentiated in social quality, try to transfer information about their “type” to the homogenous receivers by simultaneously investing in a costly signal; the receivers are assumed worthwhile allies. I (exogenously) distinguish between neutral signals, which do not benefit the receivers, and beneficent signals, which confer a benefit (positive externality) on them. Observation of the intensity of a signal is presumed to facilitate detection of an individual’s willingness to invest in a collective good once part of an alliance, although the choices within the alliance are not explicitly modeled. Based on the informational content of the signals, the receivers choose to provide access to profitable alliances to some of the signalers. In particular, a signaler will gain access to an alliance if he is able to signal that he is most likely to be of high social quality.

By the nature of this game, the appropriate equilibrium concept is the perfect Bayesian Nash Equilibrium, i.e., establishing a strategy profile according to which the receivers’ beliefs about the signalers’ types, conditional on having observed their signals, and the signalers’ best-response functions are self-confirming. For transparency, the model is initially presented with a restricted set of players – two signalers and one receiver. Having established the main arguments of the analysis, the framework is generalized to \( n \geq 2 \) signalers and \( m \geq 1 \) receivers.

2.3 The Model

2.3.1 The Simplest Case: Two Signalers and One Receiver

Let \( s \in \mathbb{R}_+^n \) refer to the observable signal regarding some unobservable social characteristic, say, reliability. Attention is restricted to two types of individuals in terms of their social quality – high (H) and low (L). The amount, or value, of the social characteristic held by each type of signaler is denoted by \( \theta_i \in \{ \theta_L, \theta_H \} \), with \( \theta_H > \theta_L > 0 \). Let \( \lambda \in (0, 1) \) signify the prior probability that a signaler is of high type; it is assumed common knowledge. The cost for signaler \( i \) of sending a signal

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2.6 If the benefit was large enough to offset the signalers’ cost, the signals could be considered socially beneficial. The term “beneficence” is intended to indicate that they, in all cases, benefit the receivers.

2.7 For the purposes at hand, this assumption seems reasonable as the proportion of individuals of high and low quality is likely to be known within a given society (cf. Harsanyi 1967–68).
of level $s_i$ when endowed with amount $\theta_i$ of the social characteristic is given by $c(s_i, \theta_i)$. It is assumed to take the following linear form:

$$c(s_i, \theta_i) = \begin{cases} \gamma^H \cdot s_i & \text{if } \theta_i = \theta^H \\ \gamma^L \cdot s_i & \text{if } \theta_i = \theta^L, \end{cases}$$

where $\gamma^L > \gamma^H > 0$ represent the signaler types’ marginal cost of signaling. Note that the cost of not signaling is zero, the cost of signaling increases at a constant rate with the level of the signal, and the cost and marginal cost of signaling are assumed to be lower for signalers of high type.

The interaction of the two signalers and the receiver proceeds as follows:

1) Nature randomly and independently determines each signaler’s type – high (H) or low (L) – with probabilities $\lambda$ and $(1 - \lambda)$, respectively.
2) Each signaler is privately informed of his type.
3) The signalers simultaneously choose a signaling level $s_i \geq 0$ contingent on their type.
4) After observing both signals, the receiver decides which signaler to choose as her ally. She can form an alliance with exactly one signaler.
5) The payoffs are realized.

The players’ payoffs are as follows. If $d \in \{1, 2\}$ denotes the receiver’s choice of Signaler 1 or 2 conditional on having observed both of their signals ($s_1$ and $s_2$, respectively), signaler $i$’s payoff is given by:

$$u_i^R((s_1, s_2), d, (\theta_1, \theta_2)) = p \cdot [1 - (i - d)^2] - c(s_i, \theta_i).$$

Entry into an alliance yields a commonly known fixed positive “prize” of value $p > 0$ for the chosen signaler (first term on the right-hand side of the expression). It can be thought of as a material benefit such as economic resources, political power, status, or the like, acquired as part of the alliance or because of it. The specification captures that a signaler only nets a positive payoff if he is, in fact, chosen as the receiver’s ally, i.e., an alliance with the receiver represents an indivisible object of limited supply. As investing in a signal is costly, not being chosen as the receiver’s ally entails a loss. The receiver’s payoff, in turn, is given by:

$$u_R^R((s_1, s_2), d, (\theta_1, \theta_2)) = \theta_d + A \cdot s_d,$$

where $\theta_d$ signifies the type-dependent value of the chosen signaler, and $A \cdot s_d$ represents a possible signal-dependent benefit for the receiver, with parameter $A \geq 0$. Without loss of generality, an alliance yields the receiver a benefit in the form of the
chosen signaler’s type-dependent value \((\theta)\), which can be thought of as his efficacy or competence as an ally (e.g., in achieving a mutual benefit), and potentially a signal-dependent benefit proportional to the magnitude of the chosen signaler’s signal. Note that all this specification requires is that the signal benefits the receiver – it may or may not benefit other individuals in a signaler’s social environment.

The central payoff-relevant term for the purposes of the analysis is the receiver’s signal-dependent benefit \((A)\), which depending on its magnitude captures two distinct effects of the intensity of the signal on the receiver’s payoff. If \(A = 0\), the signalers are using a neutral signal, i.e., the receiver does not benefit from the activity beyond receiving information about the signalers’ types (an example would be conspicuous consumption). This setting is similar to Spence’s seminal signaling model (1973, 1974) in that the receiver is concerned about the type of individual she allies with given the signal she observes. Yet, whereas the signaler in Spence’s model receives a reward that depends on his perceived type, the signaler in this paper receives a fixed reward when “hired.” Moreover, as indicated, the present framework does not comprise competition among the receivers; only the signalers compete with each other. It is in this respect that the emphasis here is on the signalers’ behavior and competitive relationship, while Spence’s account pays particular attention to the receivers’ response to their uncertain environment.

If \(A > 0\), even if small in magnitude, the signalers are using a beneficent signal, i.e., the receiver benefits from the signaling activity itself. More precisely, the receiver obtains a private benefit proportional to the magnitude of the signal sent by the chosen signaler. A scenario illustrating this case might involve an individual with specialized knowledge relating to some project requiring the collaboration of several individuals, acquired at a cost (e.g., effort), sharing information fragments with other members of his social environment in order to signal his cooperative intent. In the instance of an alliance, the receiver might then privately be provided with all essential details. As she is only acquainted with parts of the information, she does not benefit from the information provided by the signaler she does not choose as her ally; though, as long as the receiver allies with the individual sending the highest signal, this assumption is effectively arbitrary (cf. Section 2.4).

---

28 Since standard signaling models assume competition among the receivers, any benefit of a signal (e.g., if education increases the productivity of workers, in which case the signal is not wasteful per se) is not recouped by the receivers as they compete it away; it is fully appropriated by the signaler.
The players’ equilibrium strategies clearly depend on the type of signal employed by the signalers as well as the configuration of the receiver’s beliefs about the signalers’ types when assessing the signals. The approach taken in the following is constructive: For each kind of signal, I construct symmetric separating equilibria in the sense that signalers of high type choose a higher level of a signal than signalers of low type, while the signalers behave symmetrically within their respective types. Both expositions pay careful attention to the configuration of the receiver’s beliefs and comprise statements relating to the uniqueness of the equilibrium profiles.

2.3.1.1 Optimal Behavior of the Receiver

By backward induction, consider first the receiver’s optimal strategy. Since $\theta^H > \theta^L > 0$ and (if applicable) anticipating a signal-dependent benefit, an alliance will occur even if the population is composed exclusively of signalers of low type. This implies that the receiver obtains a non-negative payoff regardless of the type of her ultimate ally. Yet, by the same token, she would strictly prefer to ally with a signaler of high type. The receiver will thus optimally want to behave as follows.

Observation 2.1 If $A = 0$, conditional on observing a signal from each signaler, the receiver will ally with the signaler who she assesses to have the highest probability of being of high type. If $A > 0$, and if the signalers appear to be of the same type, she will ally with the signaler who bestows the highest benefit on her. In case of a tie (i.e., the signalers send signals of the same magnitude and the receiver considers them to be of the same type), a natural strategy is the random selection of one of the individuals as her ally.

Her payoff is maximized in each case since she obtains $\theta^H > 0$ or $\theta^L > 0$ as well as the highest possible signal-dependent benefit, if any. Note that attention will be restricted to equilibria in which the receiver randomizes 50:50 when indifferent. As ties constitute zero-probability events, any other tie-breaking rule would also work (and be quite general); for, the randomization strategy does not (fundamentally) influence the signaler types’ expected payoffs.

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2.9 The foregoing is equivalent to noting that the receiver’s outside option is zero. This not only implies that she has an incentive to participate (always) in the interaction, but also that $p > 0$, to induce active participation by the signalers.

2.10 As ties constitute zero-probability events, any other tie-breaking rule would also work (and be quite general); for, the randomization strategy does not (fundamentally) influence the signaler types’ expected payoffs.
Chapter 2. Generous Behavior

2.3.1.2 Neutral Signals (A = 0)

Turn to the optimal behavior of the signalers and, as a point of reference, consider first the case when \( A = 0 \). That is, the signalers employ signaling activities that do not yield a signal-dependent benefit to the receiver, who therefore only receives her ally’s type-dependent intrinsic value as payoff. The receiver’s decision problem, correspondingly, amounts to:

\[
\max_{i \in \{1, 2\}} \left[ \Pr(\theta^H | s_i) \cdot \theta^H + \Pr(\theta^L | s_i) \cdot \theta^L \right],
\]

where \( \Pr(\theta^H | s_i) \) and \( \Pr(\theta^L | s_i) \) denote the receiver’s beliefs about the probability that signaler \( i \), having chosen \( s_i \), is of high or low type, respectively. Since \( \theta^H \) and \( \theta^L \) are constant, this problem is equivalent to the receiver maximizing the conditional probability of allying with a signaler of high type (cf. Spence 1973). Signaler \( i \)’s decision problem, in turn, is given by:

\[
\max_{s_i} \left[ E[W_i | s_i] - c(s_i, \theta_i) \right], \quad \text{where} \quad W_i = \begin{cases} 
1 + p & \text{if } \Pr(\theta^H | s_i) > \Pr(\theta^H | s_j) \\
\frac{1}{2} \cdot p & \text{if } \Pr(\theta^H | s_i) = \Pr(\theta^H | s_j) \\
0 & \text{if } \Pr(\theta^H | s_i) < \Pr(\theta^H | s_j) 
\end{cases}
\]

for \( i \neq j \in \{1, 2\} \).

 paralleling standard signaling games, the derivation of an equilibrium for this setting requires one to postulate a set of conditional probabilistic beliefs for the receiver and determine whether they are self-confirming – that is to say, establish that the beliefs feed back upon themselves in the form of the receiver’s after-the-fact observations not causing revisions in her original beliefs. To this end and in view of the restrictions on the equilibrium outcome (i.e., separation and symmetry within the signaler types), consider the following “threshold” beliefs:

For \( 0 \leq s_i < s^* \), \( \Pr(\theta^H | s_i) = 0 \),

for \( s_i \geq s^* \), \( \Pr(\theta^H | s_i) = 1 \).

A signaler contemplating a signal strictly below \( s^* \) will refrain from sending a non-zero signal as signaling is costly and he does not increase his probability of winning when raising it above zero. Likewise, a signal strictly above \( s^* \) is not sensible since he would incur additional costs without a corresponding benefit. Therefore, given the hypothesized beliefs, the signalers will either choose to set \( s = 0 \) or \( s = s^* \). If the receiver’s beliefs are to be confirmed, it must be that signalers of low type set \( s^L = 0 \)
and signalers of high type $s^H = s^*$. Namely, the signalers self-select according to their type.\textsuperscript{11}

Algebraically, the constraints on the signaler types’ behavior such that the receiver’s beliefs are confirmed can be derived by analyzing whether signaling at the “prescribed” level and/or imitation of the other type of signaler are profitable. As shown in Appendix 2.1.a (Section 2.A), the proposed symmetric separating equilibrium holds as long as:

$$\frac{p}{2 \cdot Y^L} \leq s^* \leq \frac{p}{2 \cdot Y^H}.$$

**Uniqueness.** Since the threshold value falls within a range of signaling magnitudes, paralleling standard signaling models, the proposed equilibrium is not unique.\textsuperscript{12} Rather, it depends inherently on the configuration of the receiver’s beliefs. One could thus construct a wide range of other symmetric separating equilibria as well as pooling (and hybrid) profiles – both in pure and mixed strategies. Note that this result can easily be generalized to non-linear cost functions.

**Observation 2.2** If $A = 0$, there exists a multiplicity of equilibria, even in pure strategies.

**Remark.** By restricting their model to discrete signals, GSB in effect assume threshold beliefs on the part of the receivers. In other words, an equilibrium featuring continuous neutral signals and threshold beliefs is, in principle, equivalent to restricting the analysis to two distinct levels of the signal.

### 2.3.1.3 Beneficent Signals ($A > 0$)

As soon as $A$ is taken to be even slightly positive, i.e., the signalers employ signals that yield a benefit to the receiver, the foregoing symmetric separating equilibrium with threshold beliefs on the part of the receiver breaks down. Suppose the receiver has threshold beliefs and the signalers send $s_1 = s_2 = s^*$. Since the

\textsuperscript{21} Under complete information, provided $A = 0$, both types of signaler would set $s = 0$ and the receiver would choose to ally with a signaler of high type, who would obtain a payoff of $p$ while a signaler of low type would receive 0. If both signalers are of the same type, the receiver might adopt one of several tie-breaking rules, including choosing randomly or selecting the signaler sending the highest signal, implying that there exist several possible outcomes. Note that the(se) outcomes depend on there being (at least) two types of signaler. With only one type and/or a positive level of $A$ (however small), all equilibria (if any) would be in mixed strategies as it would always be possible to deviate to a slightly higher signaling magnitude, thereby ensuring selection as an ally.

\textsuperscript{22} As detailed in Appendix 2.1.b, contrary to conventional signaling models, equilibrium refinement via the “intuitive criterion” (Cho & Kreps 1987) may not have bite in the present environment.
receiver’s payoff function now comprises a signal-dependent benefit, given that she believes both signalers to be of high type, she will choose to ally with the one sending the highest signal. Accordingly, either of the two signalers can gain from choosing a signaling intensity marginally greater than $s^*$, as such a choice *ceteris paribus* will guarantee that he wins the prize while only incurring a small increase in cost – a profitable deviation.

This reasoning contrasts sharply with the setting when $A = 0$, where signaling $s_i > s^*$ is dominated by $s_i = s^*$, since the probability of winning does not change when moving from the former to the latter signal while $c(s_i = s^*, \theta_i) < c(s_i > s^*, \theta_i)$. If $A > 0$, the signalers’ probability of winning the prize increases discontinuously with the magnitude of the signal they send, thereby bringing about the equilibrium’s collapse. Note that this argument holds for any non-decreasing configuration of beliefs, i.e., $\Pr(\theta^H|s_i)$ being non-decreasing in $s_i$. To be precise, if $A > 0$ and the receiver has non-decreasing beliefs, any pure-strategy separating equilibrium derived for the case when $A = 0$ will break down. This is true regardless of the fact that the specific equilibrium profile will differ if the receiver’s beliefs differ.

**Observation 2.3**  
Given non-decreasing beliefs on the part of the receiver, the transition from $A = 0$ to $A > 0$ corresponds to signals above $s^*$ being strictly preferred to signals at $s^*$.

Accordingly, given non-decreasing beliefs on the part of the receiver, the only payoff-relevant comparison for each signaler is between the magnitude of his own signal and that of his opponent. This suggests that if $A > 0$, the setting transforms into a contest of the form of a (perfectly discriminating) all-pay auction. That is to say, since raising his signal can increase a signaler’s probability of winning, the signal effectively corresponds to a costly “bid” for an indivisible object – the alliance with the receiver – for which several signalers compete.

In fact, with non-decreasing beliefs, the setting is fully equivalent to an incomplete-information all-pay auction with two discrete types of bidders who face asymmetric bidding costs and are competing for a prize $p$ by submitting the $s_i$’s as their bids. The reasoning is straightforward: Given $A > 0$, the receiver will optimally choose to ally with the highest “bidder,” while the cost of submitting a “bid” is sunk.

---

2.13 Given the nature of the game and restrictions on the equilibrium, decreasing beliefs seem unnatural – they would prescribe that higher signals are taken to correspond to lower social quality. The remainder of this chapter therefore only considers non-decreasing beliefs.
This shift in the structure of the interaction, again, is true for any non-decreasing configuration of beliefs, i.e., any equilibrium that satisfies non-decreasing beliefs must correspond to an equilibrium of the all-pay auction. Note, moreover, that neither of the preceding two arguments depends on the linearity of the cost function.

**Proposition 2.1** If \( A > 0 \), any equilibrium in the present game with non-decreasing beliefs on the part of the receiver is an equilibrium of the two-type incomplete-information all-pay auction with prize \( p \) in which the \( s_i \)’s are the bids, and where the types have asymmetric bidding costs corresponding to \( \gamma^L > \gamma^H > 0 \). Conversely, any equilibrium of the all-pay auction for prize \( p \) with two types of bidders and incomplete information that would be consistent with non-decreasing beliefs constitutes an equilibrium in the present game.

The correspondence to an all-pay contest is one of the main conceptual features of this model and, in some way, drives all of the arguments and derivations to follow.

With this background, turn to the players’ decision problems. Given \( A > 0 \) and non-decreasing beliefs, the receiver’s problem amounts to:

\[
\max_{\theta \in \{H, L\}} \left[ \Pr(\theta^H | s_i) \cdot \theta^H + \Pr(\theta^L | s_i) \cdot \theta^L + A \cdot s_i \right],
\]

while signaler \( i \)’s decision problem is given by:

\[
\max_{s_i} \left[ E[W_i | s_i] - c(s_i, \theta_i) \right], \text{ where } W_i = \begin{cases} p & \text{if } s_i > s_j \\ \frac{1}{2} \cdot p & \text{if } s_i = s_j \\ 0 & \text{if } s_i < s_j \\ \end{cases} \text{ for } i \neq j \in \{1, 2\}.
\]

Observe that if an equilibrium in line with the aforementioned restrictions (i.e., separation and symmetry) exists, it must be optimal for signalers of low type to choose signals below the support (of signal intensities) of signalers of high type. Signalers of low type must therefore be optimizing against the strategy of low types only; likewise, given the strategy of signalers of low type, signalers of high type

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\(^\text{2.14}\) Even with \( A = 0 \), strictly increasing beliefs imply an equivalence with an all-pay auction. This case, however, is unlikely to be consistent with any kind of equilibrium. Although increasing beliefs entail that the receiver allies with the highest “bidder,” under these circumstances, the receiver will be certain that the chosen ally is of high type (due to the signalers’ discrete types). In contrast, if \( A > 0 \), the signals are driven up on account of the signal-dependent benefit. The probability of allying with a high type therefore does not necessarily reach 1.

\(^\text{2.15}\) Even though the restrictions on the equilibrium imply that both types of signaler may, in equilibrium, end up with a negative payoff, presuming their outside option is zero and given the receiver’s optimal strategy as well as the signalers’ risk neutrality, participation in the contest nonetheless makes sense provided the expected equilibrium payoff is (at least) zero. If so, participation is profitable since each type of signaler has a strictly positive probability of winning the prize (i.e., \( \lambda \in (0, 1) \)).
must be optimizing against the strategy of high types only. This implies that part of
the probability of winning of a high type and the probability of losing of a low type
can be taken as exogenously fixed \((1 - \lambda)\) and \(\lambda\), respectively. A high type, for
instance, will win with probability \((1 - \lambda)\) plus \(\lambda\) times the probability of beating
another high type. The setting would thus be probabilistic only in the sense that each
type of signaler recognizes the existence of the other type, as per the rules of the
game, but otherwise plays as if he would be playing against his “own” type only.

This reasoning suggests that while the types’ supports inherently affect one
another, the equilibrium strategy of each type of signaler can be analyzed
independently. Specifically, from an intuitive point of view, the upper bound of
signalers of low type, in effect, constitutes a (fixed) lower boundary for the signals of
signalers of high type. In view of their uncertainty (ex ante) regarding their
opponent’s type, even if the actual opponent turns out not to be of low type, signalers
of high type would not want to extend their support to zero (to minimize cost). For,
they must base their strategy on their expectation of the other signaler’s type. To
maximize their probability of winning the prize (and guarantee victory against all
low types), exploiting the asymmetry in the marginal cost of signaling, signalers of
high type will thus always want to situate their support above the support of signalers
of low type. Along similar lines, the presence of signalers of high type will not
induce low types to signal more than the magnitude corresponding to their (expected)
break-even payoff. Given their marginal cost, the restrictions on the equilibrium
entail that the cost of surpassing their break-even signal exceeds the associated
benefit in terms of the extra mass of signalers beaten. Signalers of low type must
therefore expect to make a loss.\(^{16}\)

The analytical independence of the types’ equilibrium strategies entails that,
except for a fixed exogenous probability of losing, which effectively discounts the
prize for winning the contest, for signalers of low type the game is strategically
equivalent to a complete-information all-pay contest. This is convenient, as Baye et
al. (1990, 1996) have shown that all equilibria for this class of auctions are in mixed
strategies. The argument for a mixed-strategy outcome in the present game follows
from the fact that, if two signalers are perceived to be of the same type, it is always
profitable (within the bounds of the equilibrium supports) to deviate from a pure

\(^{16}\) The same reasoning applies to signals at and beyond the maximum “auction bid,” i.e., their
“valuation” of the alliance without uncertainty.
strategy by signaling slightly more than the other signaler, thereby guaranteeing oneself the prize. Note that this line of reasoning establishes mixing by both types.

**Observation 2.4** Given a fixed exogenous probability of winning or losing, each type of signaler effectively optimizes against his “own” type, as would be the case under complete information. The equilibrium if $A > 0$ will therefore be in mixed strategies.

Using these ideas, I shall now construct a feasible symmetric separating equilibrium. Let $G^T(s)$ denote the signalers’ symmetric equilibrium signal distribution (c.d.f.) when of type $\tau \in \{L, H\}$, i.e., the probability that the chosen signal is no greater than some level $s$ (Pr($s_i \leq s$)), and let $g^T(s)$ denote the density function associated with $G^T(s)$, if it exists. I shall derive an equilibrium in which $G^T(s)$ is (absolutely) continuous – meaning density $g^T(s)$ exists and the distribution does not contain any atoms – and the supports of the types’ mixed strategies are intervals. In particular, let the support of signalers of low type be defined as $[s^L, \bar{s}^L]$ and the support of signalers of high type be defined as $[s^H, \bar{s}^H]$, where $s^T$ and $\bar{s}^T$ represent the respective lower and upper bounds. Given this notation, in keeping with the aforesaid restrictions on the equilibrium, the supports must be such that $s^L \leq \bar{s}^H$.

These auxiliary assumptions entail the following expected payoffs for the signalers. Consider a signaler of low type sending a signal $s_i$. Given equilibrium behavior by the other players, with probability $\lambda$, he faces a signaler of high type and loses with certainty; with probability $(1 - \lambda)$, he faces another low type and incurs a strictly positive probability of entering into an alliance with the receiver. Therefore:

$$E[u^L_i(S_i)] = \Pr(\text{winning}) \cdot (\text{payoff as winner}) + \Pr(\text{losing}) \cdot (\text{payoff as loser})$$

$$= (1 - \lambda) \cdot [G^L(s_i) \cdot (p - \gamma^L \cdot s_i) + (1 - G^L(s_i)) \cdot (-\gamma^L \cdot s_i)] + \lambda \cdot (-\gamma^L \cdot s_i)$$

$$= (1 - \lambda) \cdot G^L(s_i) \cdot p - \gamma^L \cdot s_i.$$

Similarly, the expected payoff of a signaler of high type is given by:

$$E[u^H_i(S_i)] = (1 - \lambda) \cdot (p - \gamma^H \cdot s_i) + \lambda \cdot [G^H(s_i) \cdot (p - \gamma^H \cdot s_i) + (1 - G^H(s_i)) \cdot (-\gamma^H \cdot s_i)]$$

$$= [1 - \lambda \cdot (1 - G^H(s_i))] \cdot p - \gamma^H \cdot s_i.$$

In view of the correspondence of the present game to an all-pay contest, one approach to deriving an equilibrium would be to translate the setting into a complete-
information all-pay auction with players differing according to their “valuations” as opposed to their bidding costs.\footnote{To the best of my knowledge, the present variant of a (standard) all-pay auction has not yet been analyzed in the literature, i.e., incomplete information in conjunction with discrete types.} To do so, one would simply divide $E[u^\tau(s_i)]$, with $\tau \in \{L,H\}$, by the types’ respective marginal cost of “bidding” ($\gamma$). For reference, the types’ “auction valuations” of the alliance if achieved with certainty are given by $v^L = \frac{p}{\gamma^L}$ and $v^H = \frac{p}{\gamma^H}$. When maintaining the original formulation of the game, the mainspring of the signalers’ decisions is their expected break-even payoff given the strictly positive probability of encountering a competitor of the other type.

Consider first signalers of low type. The restrictions and auxiliary assumptions on the equilibrium entail that the lower bound of the support of this type’s equilibrium distribution is at zero. As I am looking for a continuous distribution function, at $s^L$, signaler $i$ will lose with certainty (all mass is above the lower bound), implying that a strictly positive lower bound will leave him with a negative payoff. By reducing his signal to zero, the signaler prevents a sure loss and guarantees himself a profit of zero – a profitable deviation. This argument (also) means that the expected equilibrium payoff of this type of signaler given the strategy of the competing signaler is zero. To be precise, since the support of the mixed strategy extends to zero, and the payoff at the lower bound is zero, the equilibrium payoff must be zero at every point along the support. The upper bound of this type’s equilibrium strategy, $\hat{s}^L$, is thus given by (Appendix 2.2.a):

$$\hat{s}^L = (1 - \lambda) \cdot \frac{p}{\gamma^L}.$$  

Note that $\hat{s}^L$ is essentially the probability-weighted auction valuation of this type of signaler. The intuition for the upper bound parallels that for the lower bound. Suppose $\hat{s}^L < \hat{s}^L$. Given continuous randomization, a signal at the upper bound guarantees victory against all other signalers of low type (all mass is below the upper bound). This, however, implies that a signal at $\hat{s}^L$ must yield a positive expected payoff, which contradicts the equilibrium payoff of zero. Similarly, an upper bound such that $\hat{s}^L > \hat{s}^L$ is not consistent with equilibrium, as it would imply a negative expected payoff. In short, an upper bound other than $\hat{s}^L$ contradicts randomization across the full support and can therefore not be an equilibrium strategy.

Turn now to the strategy of signalers of high type. Suppose that there is no gap between the types’ supports, i.e., $\hat{s}^L = \hat{s}^L$. I will show presently that there are, in fact,
no other equilibria. Since signalers of low type make zero profit at $s^L$, given that their marginal cost of signaling is lower, signalers of high type must make a positive profit as this point. Hence, as I am looking for a mixed strategy, this type of signaler must expect to make a positive profit at every point along his support. The upper bound of the support of the equilibrium strategy of signalers of high type, $s^H$, can correspondingly be derived by computing their payoff at $s^L$, where they win with probability $(1 - \lambda)$ and, using that the payoffs must be the same, determining the point at which they win with certainty. As shown in Appendix 2.2.a, it is given by:

$$s^H = (1 - \lambda) \cdot \frac{p}{y^L} + \lambda \cdot \frac{p}{y^H},$$

which, too, is a probability-weighted transformation of this type’s auction valuation.

Having established the equilibrium supports and payoffs, the signaler types’ equilibrium distribution functions follow straightforwardly from their respective expected payoff functions. They are given by (Appendix 2.2.b):

$$G^L(s_i) = \frac{y^L}{(1 - \lambda) \cdot p} \cdot s_i \quad \text{and} \quad G^H(s_i) = \frac{y^H}{\lambda \cdot p} \cdot \left(s_i - (1 - \lambda) \cdot \frac{p}{y^L}\right).$$

**Proposition 2.2** There exists an equilibrium according to which signalers of low type randomize continuously on $[0, s^L]$, signalers of high type randomize continuously on $[s^L, s^H]$, and the receiver adopts threshold beliefs, with the threshold set at $s^L$. The types’ equilibrium strategies are given by $G^L(s_i)$ and $G^H(s_i)$.

Existence can be verified by establishing (1) that the payoffs are constant along the signaler types’ supports and no higher elsewhere, which curtails imitation of the other type’s behavior, and (2) that the receiver’s beliefs conform to behavior. Appendix 2.2.c features the derivations verifying existence of the equilibrium for the signalers. Accordingly, given the prize for entry into an alliance and the cost of investing in the signal, in equilibrium, the signalers make rational signaling choices.

Consistency of the receiver’s beliefs with equilibrium behavior follows from the structure of the signalers’ payoffs and strategies. That is, if the receiver’s beliefs are to be confirmed, the signalers have to self-select according to their type in the sense that signalers of low type choose signaling levels between 0 and $s^L$, while signalers of high type choose magnitudes between $s^L$ and $s^H$. If so, the receiver’s beliefs are not disconfirmed by the observed signal intensities and subsequent experience having entered into an alliance with a signaler, which must entail that
they are “accurate,” or self-confirming. Threshold beliefs with the threshold set at \( \hat{s}^L \), such that signals beyond \( \hat{s}^H \) are (also) taken to indicate high quality, i.e., \( \Pr(\theta^H | s_i > \hat{s}^H) = 1 \), unambiguously satisfy this requirement. Note that, due to the analytical independence of the signalers’ equilibrium strategies, the definition of the receiver’s beliefs for signals at \( \hat{s}^L \) (exactly) does not affect the players’ payoffs.\(^{20}\)

**Intuition.** Figures 2.1 and 2.2 illustrate the intuition for the equilibrium. Figure 2.1 depicts the signaler types’ equilibrium densities for \( \lambda = \frac{1}{2} \), \( \gamma^L = 1 \), \( \gamma^H = \frac{1}{2} \), \( p = 6 \), and threshold beliefs on the part of the receiver. On account of the linearity of the cost function, both densities are uniform. Suppose a signaler of low type slightly increases the intensity of his signal, say, by \( \epsilon \). This step yields a marginal benefit (shaded region) – in terms of the probability mass of signals of the other signaler, provided that he is also of low type – of \( h^L \cdot \epsilon \), where \( h^L \) denotes the height of the density of this type of signaler. He is indifferent between the higher signal and the original one if this benefit exactly offsets his additional cost, \( \gamma^L \cdot \epsilon \). In the case at hand, this is precisely what happens for both types of signaler: A change in signal intensity results in the same change in the signaler’s probability of winning.

**Figure 2.1. A Sketch of the Signaler Types’ Equilibrium Densities and the Receiver’s Equilibrium Beliefs**

![Figure 2.1](image_url)

Figure 2.2 underlines this point by illustrating the signaler types’ expected payoff functions. Clearly, both types make a loss relative to their equilibrium payoff when exceeding the bounds of their equilibrium supports, which implies that neither type of signaler has an incentive to deviate from his equilibrium strategy. In the context of a given signaler type trespassing into the other type’s equilibrium support, this is true either because the marginal cost of the higher signal exceeds its marginal benefit (low type), or because the expected cost savings from reducing the signal fall

\(^{20}\) Appendix 2.2.d features further remarks relating to this issue.
short of the expected marginal loss in the signaler’s probability of winning (high type). Along similar lines, signalers of high type do not have an incentive to surpass the upper bound of their support as the probability of winning does not change (it remains one) while their cost of signaling increases.

**Figure 2.2. A Sketch of the Signaler Types’ Expected Equilibrium Payoffs**

Uniqueness. Having established the existence of the equilibrium described in Proposition 2.2, the question arises whether there are other equilibria that satisfy the aforesaid restrictions. To arrive at an answer to this question, consider the main assumptions underlying the derivations hitherto: continuity of the equilibrium distribution functions $G^T(s)$, for $\tau \in \{L, H\}$, and disjoint but contiguous supports. If atoms along, gaps within and between the supports, and overlaps of the supports can be ruled out, there do not exist any pooling equilibria nor separating equilibria of an alternative configuration, which – in conjunction with the arguments establishing existence – implies that there do not exist any other equilibria for the given class of games. For transparency, I shall initially focus on separating equilibria and then show that pooling equilibria are not sustainable.

Maintaining the assumptions of symmetric behavior within the signaler types and non-decreasing beliefs on the part of the receiver, consider first the existence of atoms along the types’ supports. Specifically, suppose that the strategy of signalers of low type features an atom at $0 \leq a \leq \bar{s}^L$. Moreover, suppose both signalers are

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21 I give an intuitive account of the relevant arguments. The technical details can be adapted from Baye et al. (1990, 1996). Even though the authors’ treatment only deals with a single type, it can straightforwardly be extended to the two-type case considered here.
Chapter 2. Generous Behavior

sending a signal of the magnitude of the atom. Can one of them gain by deviating? Yes. If one of the signalers considered increasing his signal by \( \varepsilon > 0 \) but small, his cost would increase by \( \gamma^L \cdot \varepsilon \). Yet, concurrently, his probability of winning would increase discontinuously since each signaler’s signal distribution function will comprise a discrete upward jump (equal to the size of the mass point) at the location of the atom. As a result, the marginal benefit of increasing his signal from mass point \( a \) to some nearby signal \( a + \varepsilon \) would exceed the small additional cost of sending the higher signal. This implies that the higher signal would indeed be profitable, and that an atom at \( 0 \leq a \leq \bar{\bar{a}} \) cannot be an equilibrium strategy. The same argument, with \( \bar{s}^L \leq a \leq \bar{s}^H \), also rules out mass points along the support of signalers of high type.

Gaps between the types’ supports can be ruled out as follows. Suppose \( \bar{s}^H > \bar{s}^L \). Irrespective of the receiver’s beliefs for signals between \( \bar{s}^L \) and \( \bar{s}^H \), signalers of high type can deviate by choosing a signal within the gap interval. For, this move will reduce their cost of signaling without affecting their probability of winning. Say a signaler of high type would signal \( \bar{s}^L \) with probability one, a pure strategy, as opposed to choosing \( \bar{s}^H \). He would continue to win against all signalers of low type but incur a strictly lower cost than at \( \bar{s}^H > \bar{s}^L \), which yields him a higher expected payoff.\(^{22}\) A lower bound at \( \bar{s}^H > \bar{s}^L \) can thus not be an equilibrium strategy. The same logic rules out gaps within the supports.

Finally, suppose the supports overlap – *ceteris paribus*. There must therefore exist two signals, say, \( x \) and \( y \) for \( x < y \), that belong to both types’ support. Between \( x \) and \( y \), the signaler types face the same marginal benefit of increasing their signal, yet a signaler of high type faces a strictly lower cost (since \( \gamma^L > \gamma^H \)). As the payoff of signalers of low type is constant between \( x \) and \( y \), by definition of an equilibrium in mixed strategies, signalers of high type must have a higher payoff at \( y \), which contradicts equilibrium. Note that this argument rules out all pooling and partial-pooling equilibria.\(^{23, 24}\) Hence, in contrast to the setting with neutral signals (\( A = 0 \)),

\(^{22}\) Note that this argument depends on there being no atoms (already established), because if there was one at \( \bar{s}^H \), deviation below that level would reduce the signaler’s probability of winning.

\(^{23}\) Alternating pieces of support, i.e., low-high-low-etc., can be ruled out immediately as they are inconsistent with non-decreasing beliefs on the part of the receiver.

\(^{24}\) The restrictions on the equilibrium, especially the one on the receiver’s beliefs, are critical for the foregoing line of reasoning. Without the specification of non-decreasing beliefs, there appear to exist other separating equilibria. For example, if the high types’ support comprises atoms at regular intervals and the receiver believes that signaling levels between the atoms correspond to low quality, randomization across the atoms would seem to be an equilibrium – presuming signalers of low type randomize continuously on \([0, \bar{s}^L] \).
the use of beneficent signals \((A > 0)\) gives rise to a near definitive prediction for the signalers’ equilibrium strategies.

**Proposition 2.3** The equilibrium characterized in Proposition 2.2 is unique amongst equilibria satisfying symmetry within the signaler types and non-decreasing beliefs on the part of the receiver.

**Remarks.** Observe that the equilibrium does not depend on the exact value of \(A\) – as long as it is strictly positive, the equilibrium characterized in Proposition 2.2 will hold. The linearity of the cost function, too, is not as restrictive as it may seem. For, as stated, the breakdown of the representative equilibrium derived for the setting with neutral signals and the all-pay auction structure of the interaction are independent of the nature of the cost function. The central requisite is that the signaler types face asymmetric marginal costs such that signaling is cheaper for high types. While non-linear (e.g., exponential) changes in the cost of signaling would clearly affect the structure of the payoff functions and thus the bounds of the equilibrium supports, the general pattern of the equilibrium with non-linear costs would qualitatively remain the same as the profile derived for a linear cost function. Likewise, it is to be expected that the results extend to any finite number of signaler types. The relevant equilibrium will tend to feature separate but contiguous strategies equal to the number of types being considered, ranked according to the relative size of the marginal cost of signaling of each type. As a final note, many of the results (especially with respect to the outcome’s uniqueness) are likely to hold in the presence of asymmetries among the signaler types in the sense that signalers of a given type do not necessarily behave the same.\(^{25}\)

### 2.3.2 The General Case

So far, the framework only considered a restricted set of players. It can, however, easily be generalized to comprise \(n \geq 2\) signalers and \(m \geq 1\) receivers. In fact, the extension is almost trivial. For, with the exception of the players’ expected payoff functions, none of the main arguments and derivations establishing the results for the simple case fundamentally relied on the number of signalers and receivers. In

\(^{25}\) As the framework is set in a world of non-transferrable utility, the receiver cannot make payments to the signalers, which renders welfare considerations somewhat problematic. In particular, policy interventions that reduce the level of (wasteful) costly signaling, e.g., taxation of signaling, are unlikely to bring about Pareto improvements because less signaling makes the receiver worse off (she obtains a lower benefit).
the setting with beneficent signals, for instance, all that mattered for a given signaler was the presence of a competitor for the indivisible alliance with the receiver.

Consider a social environment with \( n \geq 2 \) signalers and \( m \geq 1 \) homogeneous\(^{26}\) receivers, *ceteris paribus*. It is easy to see that even in the general case, there will be a multiplicity of equilibria if the signalers employ neutral signals (\( A = 0 \)). The case of beneficent signals (\( A > 0 \)) is similarly straightforward if one assumes that the signalers are not restricted with respect to the number of alliances they can enter, i.e., if, in the extreme, a single signaler could ally with all\(^{27}\) \( m \) receivers (cf. GSB). The receivers’ homogeneity in conjunction with the aforementioned restrictions on the equilibrium entail that, given the signalers’ behavior, each receiver will ultimately want to ally with the signaler sending the highest signal. In other words, all \( m \) receivers optimally behave the same.

To highlight the quantitative differences between the simple and general cases if \( A > 0 \), I shall briefly outline the main elements of the analysis with many players. If \( \delta_i \in \{1, \ldots, n\} \) denotes receiver \( i \)'s choice of signaler \( k \in \{1, \ldots, n\} \) conditional on having observed each of their signals (\( s_1 \) through \( s_n \), respectively), signaler \( i \)'s payoff is given by:

\[
u_i^S((s_1, \ldots, s_n), (\delta_1, \ldots, \delta_m), (\theta_1, \ldots, \theta_n)) = p \cdot \sum_{r=1}^{m} f(\delta_r, i) - c(s_i, \theta_i),
\]

where

\[
f(\delta_r, i) = \begin{cases} 
1 & \text{if } \delta_r = i \\
0 & \text{otherwise} 
\end{cases} \quad \text{for } r = 1, \ldots, m.
\]

Similarly, receiver \( i \)'s payoff is given by:

\[
u_i^R((s_1, \ldots, s_n), (\delta_1, \ldots, \delta_m), (\theta_1, \ldots, \theta_n)) = \theta_{\delta_i} + A \cdot s_{\delta_i}.
\]

Provided non-decreasing beliefs on the part of the receivers, the general setting, too, corresponds to a two-type incomplete-information all-pay contest about

\(^{26}\) Although this might seem a strong assumption, it is reasonable if one presumes that the average quality of the social goods the signalers are competing for is the same. Recall that the receivers are assumed to be worthwhile allies (also cf. Spence 1973, 1974).

\(^{27}\) For the purposes at hand, this assumption seems sensible. It implies that if \( m < n \), the signalers compete for a limited number of profitable alliances; if \( m = n \), it would be feasible that each signaler allies with one receiver, but given that a signaler obtains a prize for each alliance he enters into, the signalers will attempt to achieve as many alliances as possible. The latter continues to hold if \( m > n \). If, in turn, signalers were restricted to ally with a single receiver, consistency of the setting would require that \( m < n \). Otherwise, signaling would not be necessary as the number of receivers would correspond to or exceed the number of signalers. The result of this modification would be that, given non-decreasing beliefs on the part of the receivers, the \( m \) signalers sending the highest signals would achieve an alliance. As such, the game would be transformed into a matching contest.
prize $p$, in which the $s_i$’s are the players’ bids.\footnote{Note that the general setting could also be translated into a complete-information all-pay auction. For reference, the types’ “auction valuations” of the alliances are given by $\psi^L = \frac{mp}{\tau}$ and $\psi^H = \frac{mp}{\tau^2}$.} To write down the signaler types’ expected payoff functions and use them to construct the generalized version of the symmetric separating equilibrium characterized in Proposition 2.2, consider the following revised notation. Let $G^\tau(s)$ refer to the signalers’ (absolutely continuous) symmetric equilibrium signal distribution (c.d.f.) when of type $\tau \in \{L, H\}$, and let the associated density function be denoted by $g^\tau(s)$.

Now consider a signaler of low type sending a signal $s_i$. By separation, he only stands a chance of winning if all other signalers are also of low type, which occurs with probability $(1 - \lambda)^{n-1}$. If so, his probability of winning, presuming the receivers and all other signalers behave according to their respective equilibrium strategies, is given by the product of the equilibrium distribution functions of all of his competitors $j = 1, \ldots, n - 1$, which – by symmetry – amounts to $[G^L(s_i)]^{n-1}$. The expected payoff of signalers of low type is therefore given by:

$$E[u^L_i(s_i)] = (1 - \lambda)^{n-1} \cdot [G^L(s_i)]^{n-1} \cdot (m \cdot p - \gamma^L \cdot s_i) + (1 - [G^L(s_i)]^{n-1}) \cdot (-\gamma^L \cdot s_i)$$

$$= (1 - \lambda)^{n-1} \cdot [G^L(s_i)]^{n-1} \cdot m \cdot p - \gamma^L \cdot s_i.$$ 

The expected payoff of a signaler of high type follows from a similar logic. The main difference is that the intensity of his signal relative to that of other signalers depends on how many equally potent signalers $j = 0, \ldots, n - 1$ exist in his social environment. The expected payoff of this type of signaler is thus given by:

$$E[u^H_i(s_i)] = \sum_{j=0}^{n-1} \binom{n-1}{j} \lambda^j \cdot (1 - \lambda)^{n-1-j} \cdot \{[G^H(s_i)]^j \cdot (m \cdot p - \gamma^H \cdot s_i) + (1 - [G^H(s_i)]^j \cdot (-\gamma^H \cdot s_i),$$

where the term following the summation sign describes all possible combinations (without repetition) of high and low types in the population, i.e., $\binom{n-1}{j} = \frac{(n-1)!}{j!(n-1-j)!}$. Manipulation yields the following simplified expression:

$$E[u^H_i(s_i)] = m \cdot p \cdot \left( \sum_{j=0}^{n-1} \binom{n-1}{j} \lambda^j \cdot (1 - \lambda)^{n-1-j} \cdot [G^H(s_i)]^j \right) - \gamma^H \cdot s_i.$$ 

It is not hard to verify that the supports of the types’ equilibrium mixed strategies are on $[0, \hat{\psi}^L]$ for signalers of low type and on $[\hat{\psi}^L, \hat{\psi}^H]$ for those of high
type, where \( \hat{\psi}^L = (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^L} \) and \( \hat{\psi}^H = (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^H} + (1 - (1 - \lambda)^{n-1}) \cdot \frac{m \cdot p}{\gamma^H} \) (Appendix 2.3.a). Note that the upper bounds once again constitute probability-weighted transformations of each type’s auction valuation (cf. Footnote 2.28).

Having established the equilibrium supports and payoffs, the signaler types’ equilibrium distribution functions can be derived to be (Appendix 2.3.b):

\[
G^L(s_i) = \left( \frac{\gamma^L \cdot s_i}{m \cdot p \cdot (1 - \lambda)^{n-1}} \right)^{\frac{1}{n-1}} \quad \text{and} \quad G^H(s_i) = \frac{1}{\lambda} \cdot \left[ \lambda - 1 + \left( \frac{\gamma^H}{m \cdot p} \cdot (s_i - \hat{\psi}^L) + (1 - \lambda)^{n-1} \right)^{\frac{1}{n-1}} \right].
\]

It is easy to check that, for \( n = 2 \) and \( m = 1 \), these expressions reduce to the distributions derived for the simple case. Moreover, arguments corresponding to those establishing the existence and uniqueness of the equilibrium for the simple case can be used to establish the following result; Appendix 2.3.c features the derivations verifying existence of the equilibrium for the signalers.

**Proposition 2.4** The unique symmetric non-decreasing beliefs equilibrium of the game with beneficent signals, \( n \geq 2 \) signalers, and \( m \geq 1 \) homogeneous receivers, has signalers of low type randomizing continuously on \([0, \hat{\psi}^L] \), signalers of high type randomizing continuously on \([\hat{\psi}^L, \hat{\psi}^H] \), and all \( m \) receivers adopting threshold beliefs, with the threshold set at \( \hat{\psi}^L \). The types’ equilibrium strategies are given by \( G^L(s_i) \) and \( G^H(s_i) \).

**Intuition.** Despite the correspondence of the main arguments and derivations, as the generalized densities are not uniform, the intuition for the result in the game with \( n \) signalers and \( m \) receivers is somewhat more involved. The gist of the difference is that, in the general case, the distribution of players sending higher signals than oneself moves to the center stage. That is, victory in the setting with many players requires sending the highest signal of the group. In order to be selected as the receivers’ ally, signaler \( i \) has to outperform the winner of the \( n - 1 \) other signalers. In this sense, he effectively competes against the highest signal of all other signalers (the second-highest order statistic) as opposed to all competing signals (the whole distribution). Since the simple case only comprised two signalers, this distinction of the competing signals was irrelevant, the result being a uniform distribution of the winning signal. Competition against the highest competing signal,
however, entails that the distribution of the winning signal is skewed towards the right (i.e., towards higher signals). In consequence, an increase in an individual’s signal yields a disproportionately higher gain in his probability of winning when the starting signal was high to begin with. In fact, a signaler’s chance of winning increases linearly as he increases his signal (as the cost of signaling is linear). Nevertheless, given the asymmetry in the types’ marginal cost of signaling, in this case, too, it is not profitable for a signaler of low type to imitate a signaler of high type as the marginal cost of increasing the signal would exceed the associated marginal benefit. Likewise, a signaler of high type would incur a loss when deviating to a signaling intensity below $\psi^L$ or exceeding $\psi^H$. 29

2.4 Discussion

In view of the essential simplicity of the setup, the main finding is quite striking. Provided a number of mostly mild restrictions are satisfied, use of signaling activities that provide a (private) benefit to their recipients in an environment featuring competition among the signalers gives rise to a unique symmetric separating equilibrium. 30 Signaler competition in conjunction with neutral signals, in turn, does not lead to a sharp prediction. To delineate further the intuition and scope of these results, I shall comment on the breadth of the analysis and findings and contrast them with those by GSB and the standard signaling literature.

Among the most notable aspects of the analysis is the intuitive merit of the results’ comparative static properties. Given, among others, common information about the composition of the social environment, the findings suggest that individuals will be no more pro-social than is necessary to achieve the desired social good. When using neutral signals, given a set of beliefs on the part of the observers of the signals, the main determinant of the signalers’ behavior is their cost of sending

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2.29 To complete the framework, rather than considering two separate populations of agents, or equivalently, a single population comprising two kinds of agents, it would be necessary also to consider the case when each individual takes both roles, i.e., sends and receives signals (cf. GSB). This augmentation clearly requires that $m = n$ (cf. Footnote 2.27). The main adjustment to the analysis and results would be that the signaler sending the highest signal cannot ally with himself, implying that the signaler sending the second-highest signal will achieve at least one alliance.

2.30 The equilibrium can be perceived in two ways – ex ante or ex post. Assuming, as is done throughout the analysis, that the distribution of types is known (ex post), the signaler types’ equilibrium strategies (in the simple case) are given by $G^L(s_i)$ and $G^H(s_i)$. Conversely, if the distribution of types is taken to be unknown (ex ante), one can think of each signaler as optimally playing according the weighted sum of their “ex-post” strategies: $(1 - \lambda) \cdot G^L(s_i) + \lambda \cdot G^H(s_i)$. 
a particular signal intensity, i.e., the higher the cost, the lower the signals. With beneficent signals in use, the composition of the population of competitors gains in importance (for the signalers’ strategy). If the group has a large proportion of high-quality individuals, entailing that high competing signals are likely, the signalers will tend to send high(er) signals themselves (i.e., the upper bound of the high types’ support increases). They will, however, evade a fixed level of the activity (pure strategy), thereby avoiding preventable over- or under-bidding. Paradoxically, the unique nature of the outcome suggests that competition between asymmetric (in terms of their quality) individuals may be a key factor in groups’ attempts to achieve collectively beneficial outcomes.

More unexpectedly, as long as it is strictly positive, the equilibrium profile for the setting with beneficent signals does not depend on the exact size the marginal benefit of the signal (A). One might thus make the case that, in the limit, the equilibrium could be used to select the corresponding equilibrium when signalers use neutral signals. To be precise, as $A$ tends to 0, the separating equilibrium characterized in Proposition 2.4 continues to hold, which may suggest that the same equilibrium should also be the favored (refined) profile when analyzing settings involving neutral signals.\(^{31}\)

Turning to the reach of the framework, the two main (in part implicit) features of the setting with $A > 0$ are that the receiver has to “return” the benefit of the signaler she does not choose as her ally and that she cannot (per se) reject an alliance. While it may seem restrictive that the receiver cannot retain the benefit of all signalers, it is easy to show that the analysis and equilibrium straightforwardly extend to this case. In particular, suppose the receiver’s payoff function takes the following form – *ceteris paribus*:

$$u^R((s_1, s_2), d, (\theta_1, \theta_2)) = \theta_d + A \cdot s_d + B \cdot s_k,$$

where $d \neq k$.

It is not hard to see that, as long as $A > B$, the equilibrium will not be affected, as the receiver continues to strive for an alliance with the signaler sending the highest signal (i.e., $s_d \geq s_k$). In short, if the benefit provided by the signaler not chosen as the receiver’s ally ($k$) is lower than that by the chosen signaler ($d$), the analysis goes through as presented. On the other hand, if $A = B$, one is effectively revisiting the

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\(^{2,31}\) If the receivers adopt the same decision rule as their counterparts in the case with beneficent signals (i.e., always selecting the signaler sending the highest signal), the equilibrium outcome may also arise in the setting with $A = 0$, although it is unlikely to be unique.
setting with individuals sending neutral signals \((A = 0)\). For, if all signalers provide the same level of benefit, rather than caring about the individuals’ signals, the receiver is exclusively concerned about the sum of the signals. Consequently, paralleling the relevant analysis, she will not (be able to) rely on the size of the signal-dependent benefit to guide her selection of an ally. Rather, the outcome(s) will depend entirely on her beliefs. Note that it does not matter whether other signalers benefit from the signals, i.e., there is no strategic effect. They will simply be able to spend more on their own signals, which does not qualitatively affect the equilibrium.

The assumption that the receiver cannot reject an alliance is equivalent to assuming that her outside option is zero or even negative. Bearing in mind the environment motivating this chapter, this is not as unreasonable as it may seem. After all, in its essence, the model explores a mechanism by which individuals can detect the intentions of potential exchange partners. It is, in this context (as well as more generally, cf. Spence 1974), not rational to ignore and/or not take into account the information available at any one time. Indeed, when social groups of non-kin were initially forming and progressively growing, engaging in (profitable) coalitions was paramount in order to cope with the pressures of surviving in the new environment (van Vugt et al. 2007). Having identified someone as a cooperative interactant, people are unlikely to have rejected interaction with them.

More generally, observe that – akin to GSB and most orthodox economic signaling models – none of the agents in the present framework is driven by any form of other-regarding preferences. Their behavior is a purely self-interested best response to the information available to them at the time of their decisions. This is an advantage of the costly signaling approach to explaining cooperative social interaction as it can cope with the most prominent criticisms of standard explanations while maintaining the basic tenets of classical economic theory (e.g., Smith 2003; van Vugt et al. 2007). That is, it embraces unreciprocated behavior by presuming that the cost of generous behavior, via the disclosure of information about an individual’s attractiveness as an exchange partner, is compensated by future benefits (captured by \(p\) in the present framework). Likewise, although it could be, the provision of benefits does not need to be sustained by a social norm or explicit enforcement by the members of a given population. As individuals can avoid interaction with those who are unlikely to reciprocate their good deed(s), the approach furthermore does not fall victim to a second-order free-rider problem. This
is not to say that costly signaling is (necessarily) the sole explanation for cooperative interaction within broad social contexts. The approach is an alternative to other angles. In fact, in practice, it is likely that a number of explanations operate in concert to give rise to the behavioral patterns we observe on a day-to-day basis.

In contrast to GSB, the equilibrium in the case of beneficent signals neither prescribes a specific composition of the population of individuals, nor can it be satisfied by neutral (or harmful) signals. Whilst the need for a specific composition of the population in part stems from the evolutionary components of GSB’s analysis, both divergences expand the applicability of the present results. Given (exogenously) a signal of a certain kind (neutral or beneficent), two types of signaler, an intuitive condition on the cost of signaling, and non-decreasing beliefs on the part of the receiver(s), this chapter provides conditions for equilibrium for each signal that can only be satisfied by the particular signal being investigated. If the signalers use neutral signals \( A = 0 \), irrespective of the exact configuration of the receiver’s beliefs, none of the equilibria is unique (\textit{cf.} Spence 1974; Mailath 1988a), whereas with beneficent signals \( A > 0 \), all equilibria but one can be ruled out.

The intuition for this disparity is rooted in the signalers’ cost-benefit ratios. If \( A = 0 \), the receiver’s objective, using the informational content of the signals she observes, is to maximize the probability of allying with a signaler of high type. Given threshold beliefs, the signalers will optimally signal just enough to be considered high types. Any signal beyond the threshold level entails a loss as the cost of doing so exceeds the associated benefit in terms of the extra mass of competitors beaten. In sharp contrast, although the receiver continues to seek an ally of high type, if \( A > 0 \) and provided that the receiver adopts non-decreasing beliefs, the signalers can gain from increasing their signals beyond the threshold level. On account of the signals’ intrinsic benefit, while the receiver did not consider signals above her belief threshold superior, striving to obtain the maximum possible signal-dependent benefit, she will want to ally with the signaler sending the highest signal. Therefore, up to a point, the benefit of increasing one’s signal in terms of the extra mass of competitors beaten always at least offsets the associated cost. In effect, this suggests that GSB’s equilibrium can be satisfied by more than one kind of signal since their discrete (“on-off”) message space precludes changes in the signal intensity.

The relevant comparison with Spence’s seminal work imposes a fixed number constraint on the quantity of available positions at the firms (Spence 1974, p. 84ff). In
this case, given a continuous type space, the multiplicity of equilibria is eliminated as the competing receivers will set the available number of positions such that all signalers with an ability above a certain threshold can obtain a position. Even though, in the present *discrete* environment with $A = 0$, multiplicity persists, the results are not incompatible. For, given appropriate separating beliefs on the part of the receiver, the signalers optimally send the “appropriate” separating signal. As this outcome, again (i.e., as in conventional models), depends intrinsically on the belief threshold, competition among the signalers only appears to matter to the extent that type matters. The fixed versus variable reward structure associated with the presence (or lack thereof in the present setup) of competition among the receivers seems to be of secondary importance; a fixed reward simply entails that the number of available positions is binding *ex ante*. The distinctive aspect of the setting with $A > 0$ is the attainment of uniqueness in a discrete setup with a binding constraint on the number of “positions” for which the signalers compete. It suggests a nontrivial role for the degree of dispersal of any benefits associated with the signaling activities between the signalers and the receivers (cf. Footnote 2.8).\textsuperscript{32}

On a final note, the auction characteristics of the framework do not undermine the fact that the interaction being analyzed is a signaling game. Among others, in accordance with this kind of sender-receiver game, the messages have a direct effect on the signalers’ and receivers’ payoff functions. As such, the game also unambiguously contrasts with cheap-talk games, which involve costless, non-binding messages. Further distinctive differences are that the analysis does not require a commonality in the players’ interests, nor do the sender types have to have different preferences over the receivers’ actions (Farrell & Rabin 1996; Crawford 1998).

2.5 Related Literature

In the spirit of recent research by economists studying cooperative interaction among unrelated individuals, which frequently incorporates ideas and techniques from other disciplines (see Dawes & Thaler (1988) for an overview of some of the early work), this chapter takes up a concept with a long history in both the social and

\textsuperscript{32} Unlike the case with neutral signals (Appendix 2.1.b), belief refinements such as the “intuitive criterion” (Cho & Kreps 1987) do not rule out the equilibrium when $A > 0$. While it might be possible to alter the model and analysis to be able to apply the refinement and arrive at the same equilibrium, I would argue that the assumption of non-decreasing beliefs is a more natural point of departure.
It explores the idea that individually costly activities can be used to mitigate informational asymmetry. What renders the present paper interesting is the combination of the central premises of both literatures within a theoretical framework while drawing attention to two more unconventional aspects – signaling activities that yield benefits to the observers and competition among the signalers for some end. To accentuate the broader context and contribution of this chapter, this section surveys the related literature in both the social and natural sciences.

A rich and well-studied class of (possible) explanations for a substantial proportion of the ever-growing body of evidence that individuals behave cooperatively even when they do not necessarily expect to deal with each other in the future invokes conditional reciprocity, especially direct reciprocity (which is rooted in personal experience with others). Relevant accounts are based on such concepts as reciprocal altruism, tit-for-tat, repeated play of the Prisoner’s Dilemma, and the like (Trivers 1971; Axelrod & Hamilton 1981; Kreps et al. 1982; Axelrod 1984). Relying intrinsically on recurring social encounters, the concept holds that individuals respond in-kind to the behavior of others. While certainly appropriate to explain a wide variety of social phenomena, in the context at hand, lack of contingency and attention to the recipient’s history and/or future probability of reciprocation imply that the fundamental conditions for this explanation are violated.

An alternative explanation, based on reputation and status benefits associated with generosity, promotes what is commonly referred to as “indirect reciprocity” (Alexander 1987). According to this concept, individuals acquire information about potential interaction partners by observing them or by gathering (reputational) information from third parties about their past behavior with others. In its purest form, it constitutes a complete bookkeeping strategy. When considered from a “reputation building” perspective, beneficent behavior effectively enables individuals to signal their quality as cooperators (or cooperative intent) to future associates using reputational effects (Henrich & Henrich 2006). Even if the initial recipient of a generous act does not reciprocate, as long as sufficiently many individuals within his/her social environment pay attention, the contributor can nonetheless recoup his original costs via the benefits (s)he receives from individuals who observed his/her generosity and now give to him/her.  

\[2.33\] Fehr & Gächter (2000) provide an overview of a number of relevant studies.

\[2.34\] This argument is similar to the “broadcast efficiency” idea outlined in Footnote 2.36.
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While strategies based on indirect reciprocity have been shown to be evolutionarily stable under certain conditions (Boyd & Richerson 1989), this is only likely in small groups, where it is possible to keep track of everyone’s actions and to target one’s cooperation at reciprocators. Note that this limitation also restricts the usefulness of direct conditional reciprocity. To overcome this problem, it has been proposed that reputation could be an indirect benefit of generous behaviors. The purpose of establishing a reputation by way of costly displays of generosity would be the facilitation of trust in subsequent pair-wise interactions (e.g., Roberts 1998). In its essence, this angle is clearly a costly signaling argument, although as is common in indirect-reciprocity models (Smith & Bliege-Bird 2005) the focus is on the potential benefits achieved during the cooperative interaction following the individually costly reputation-building display. Accounts based on indirect reciprocity also do not tend to consider the possibility that potential interaction partners may prefer not just good intrinsic qualities, but signaling displays that provide additional benefits simply as a function of the costly display.

Explanations based on costly signaling do not necessarily require the former and allow for the latter. The idea is that, in a signaling equilibrium, the recipients and/or observers of some generous act will confer benefits on the signalers because doing so is their best response given the information available to them (e.g., Gibbons 1992; Smith 2003). That is, the signals indicate qualities that make it advantageous to interact preferentially with the signalers. Even though, on the face of it, analogous to conditional reciprocity – “I’ll scratch your back if you scratch mine” – the underlying mechanism does not require any form of other-regarding preferences (or the like). Quite on the contrary, the agents behave in a purely self-interested manner.35

Another broadly related branch of literature concerns a behavioral propensity termed “strong reciprocity” (Gintis 2000; Fehr et al. 2002; Bowles & Gintis 2004). It builds on a growing body of experimental evidence indicating that people are both conditionally altruistic cooperators and conditionally altruistic punishers (Fehr & Henrich 2003). The essence of strong reciprocity as an explanation for human social behavior is that agents are willing to engage in costly actions to reward cooperative behavior and punish free-riding by other group members, even if the relevant actions provide no apparent present or future material rewards. The merit of this concept is fiercely contested (e.g. Fehr & Henrich 2003; Lehmann et al. 2007), however, it is beyond the scope of this chapter to explore the details of the debate. The concept is relevant in the present context as it involves behavioral outcomes individuals might employ as beneficent costly signals. Rather than anticipating reciprocation in-kind, the agents (potentially) derive benefits from future profitable interactions. From this perspective, the individuals’ actions essentially correspond to “gifts” that signal their intention to invest in a relationship (Camerer 1988).
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("tolerated theft;" Burton Jones 1984; Hawkes 1992). The key conditions (Zahavi 1975; Grafen 1990) to be satisfied in order for the present line of reasoning to apply are (1) that the signaling activity must convey information about variation in the quality being advertised, and (2) that the signal must impose a quality-dependent cost on the signaler(s). The latter condition clearly ensures the former condition, but only if both are satisfied can signaler and receiver benefit. Yet, the theory of costly signaling as an explanation for generous behavior also has a central weakness: the good or service provided may be incidental to the signaling equilibrium, i.e., anti-social behaviors could, in principle, perform the same function (Smith & Bliege-Bird 2000; Gintis et al. 2001; Smith 2003). Therefore, depending on the setting, while signaling may be a necessary component of the explanation of generous behavior, it is not sufficient. As discussed in the previous section, this limitation does not apply to the present paper as the signals are not chosen endogenously. Rather, given a link between signal and quality and the appropriate assumptions on the marginal cost of signaling, the framework extracts conditions for a separating signaling equilibrium for each type of signal (neutral and beneficent).

Before turning to the second branch of related literature, i.e., economic models of pro-social behavior and signaling, it is worth noting that interaction involving competition between “signalers” also has a quite extensive history in the natural sciences, ranging from work on food solicitation involving sibling competition to threat displays and sexual advertisement (for an overview, see Johnstone 1998, 1999). The central premise of these works is that the level, or intensity, of a signal affects the outcome of the interaction. Although Spence (1974) alludes to the possibility of competition among signalers, the matter does not seem to have been explored in much detail. In fact, economists have only recently become interested in related issues, focusing mainly on the matching aspect of the interaction (Hopkins

2. In many cases, this problem is not decisive, as the weakness can be mitigated by supplementing the equilibrium with an argument justifying why generous activities are preferred to other types of behaviors an individual could use to convey information to others. The two most intuitive alternatives to this end are (1) that the activity is integral to the quality being signaled, and (2) that the signal serves to attract an audience and may thus increase "broadcast efficiency" (Boone 1998; Smith & Bliege-Bird 2000; Hawkes et al. 2001; Smith 2003). The broadcast efficiency argument applies when beneficent signals attract a larger audience – as measured by observers per unit signal – than alternative signals of equivalent cost. A third option would be to invoke group selection among alternative evolutionarily stable equilibria (Boyd & Richerson 1990). The practicability of this mechanism is, however, still being debated (cf. Alcock 1998).
The economic work most closely related to the present chapter explores the possible motivation(s) for pro-social behavior. It is rooted in the ever-growing body of evidence that suggests that individuals are not solely interested in their material payoff and are more cooperative than assumed in the standard literature (e.g., Ledyard 1995), although there are also examples of individuals behaving according to their self-interest (e.g., Smith & Williams 1990; Güth et al. 1997). To render these observations coherent, a substantial amount of work has investigated the explanatory power of interdependent preferences. According to these models, individuals seek to maximize utility functions that depend on the consumption (or other characteristics) of others (for an overview, see Sobel 2005). In the context of seemingly altruistic behavior, in particular, one can distinguish two strands of work. One posits that individuals are “purely” altruistic (e.g., Olson 1965) – they receive utility from the total amount of public good created via donations. The alternative approach, pioneered by Becker (1974) and expanded by Andreoni (1989), holds that the act of giving itself affects a person’s utility function, i.e., giving to charity or some other public good provides a “warm glow” to the givers that is an increasing function of the amount given. The costly signaling angle contrasts with both of these ideas in that the signalers do not obtain an intrinsic utility from being generous, although it could be extended in this direction.

Models viewing contributions to public goods or non-monetary gifts as signals have, among others, been proposed by Camerer (1988), Glazer and Konrad (1996), and Bénabou and Tirole (2006). The framework most closely in spirit to the present approach is that by Camerer (1988), who explores the idea that gifts could signal an individual’s intention with respect to future investment in a relationship. However, paralleling GSB, his model is discrete; in contrast to GSB, he does not consider the possibility of competition among the signalers. Glazer and Konrad (1996) suggest that contributions to a public good can constitute signals of wealth to achieve social

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While this statement is – to the best of my knowledge – accurate in the context of cooperation (and the “pure” signaling literature), the issue of competition among the informed agents in an environment of asymmetric information has received some attention in industrial organization (e.g., Mailath 1988b, 1989; Matthews & Fertig 1990; Ippolito 1990; Hertzendorf & Overgaard 2001; Daughtey & Reinganum 2007, 2008a; Adriani & Deidda 2011). However, the relevant literature is only tangentially related to the present work in that the setups usually do not consider Spence-type signals, i.e., signals with differential costs (according to type), or differ in other respects from the setup at hand (e.g., sequential moves).
status. A player derives utility from (unobservable) private consumption and income status, defined by other individuals’ beliefs about his/her income net of his/her contribution to some public good. Their approach is similar to the present one in that a player signals wealth in order to interact with people of the same or higher social status. However, the main driver of their results is the other individuals’ beliefs about the contributor’s wealth – the signal itself is “neutral” (i.e., it does not benefit the receivers) and the signalers do not compete with each other for some end.

Bénabou and Tirole’s (2006) contribution is the most detailed exploration of the motivations shaping individuals’ social conduct. They suggest that individuals are heterogeneous in their attitude towards altruism and greed, care about their social reputation and self-image, and may be influenced by explicit incentives (rewards and punishment). In fact, these motives are likely to interact in order to shape behavior. Their approach combines the motives for pro-social behavior suggested by the aforementioned works – they consider an intrinsic motive, an extrinsic motive, and a (self-)reputational motive. Given a signal, the observers infer the contributor’s motivation. Set within a learning model, the main result of this multi-dimensional signaling analysis is the (informational) crowding out of pro-social behavior as explicit incentives spoil the reputational value of good deeds, thereby creating doubt as to an individual’s true motivation (“over-justification effect”). The basic idea of the present framework is certainly subsumed in their general model. However, the particular aspects being explored – the role of the benefit of a signal (versus none) in conjunction with signaler competition – are not considered. My framework could surely be extended to incorporate explicit incentives, although it is questionable whether they apply in the relevant contexts.

To sum up, though related to a number of deep literatures, this chapter promotes an unconventional angle to a century-old puzzle. Whereas the signals being explored can easily be interpreted as (local) public goods, the framework does not involve other-regarding preferences of any kind – the players engage in purely self-centered optimization of their utility. Likewise, although signaling could enhance the signalers’ reputation, the present results hold even without accounting (explicitly) for subsequent interaction. Hence, using a simple setup, this chapter illustrates that generous behavior towards others can emerge in a one-shot framework, deriving a prediction that is not diluted by the existence of a multitude of possible outcomes.
2.6 Concluding Remarks

The objective of this chapter was to shed light on the proximate causes of a seemingly irrational behavioral pattern – the recurrence of generous behavior towards others within a broad social context. It postulates that generosity towards others may constitute a signal of cooperative intent that benefits signalers not because they receive compensation in-kind, but because they gain access to profitable social goods. Presuming that access to these goods is restricted, the framework incorporates the notion that signalers may be rivals for the same endeavor, implying that they compete with each other by way of the magnitude of the signals they send. The main, in part implicit, premises of the analysis are that individuals are motivated by self-interest, face asymmetric information about the social characteristics of other members of their social environment, that variation in behavior is linked to variation in underlying social quality, and that individuals prefer to interact with individuals of high quality.

Even though the analysis gives rise to a variety of interesting results, it prompts a number of queries for future work. From a technical point of view, for instance, an informative extension would be to move beyond a fixed prize for signalers when achieving an alliance, say, by introducing a bargaining stage with respect to the appropriate prize level before entering into an alliance with the receiver. Similarly, rendering the receivers heterogeneous by assuming that some proportion of them has non-Bayesian beliefs, or by modeling their preferences more explicitly, would render the interaction more realistic. By accounting for the receiving side of the interaction, both of these angles would move the analysis closer to the standard signaling literature. Within the former, the ultimate prize is likely to approximate more closely a type-dependent reward structure, whereas within the latter, the receivers are likely to compete with each other in order to ally with the “best” signaler(s). More explicit modeling of the receivers’ utility functions would permit a more careful consideration of the notion of “future benefits” of generous behaviors, be it via discounting or reputational effects. In a similar spirit, the introduction of a second stage would provide further insight into the individuals’ interaction upon entry into an alliance. An alternative angle would be to introduce noise into the signals, which makes accurate inferences by the receivers more difficult. Repetition of the interaction could account for possible dynamics such as reputation formation via
active signaling in early periods, the benefits of which can be reaped in later periods during which signaling is no longer necessary. One could then also explore the effects of shocks to the players’ fortunes and changing circumstances.

More broadly, this chapter has individuals signal about one particular social characteristic. As it is in many cases unlikely that individuals are able to distinguish precisely their various social characteristics, an interesting extension would be a multi-dimensional signaling approach. Although technically challenging, it could clarify more precisely the interaction of the various motives for individuals’ social behavior (e.g., Bénabou & Tirole, 2006). Within such a framework, it would furthermore be possible to explore the issue of which underlying qualities individuals signal about (and, correspondingly, how they do so, e.g., via which kind of signal(s)), as well as who the intended audience is (and how it reacts to various signals). In this way, the conditions for signaling within a social environment could be defined more accurately, and it may be possible to determine within a theoretical context – although empirical work will be crucial to guide the analyses’ assumptions – how the various mechanisms to explaining cooperative behavior interact with one another.
2. A Appendices

Appendix 2.1. The Simplest Case – Neutral Signals ($A = 0$)

This appendix establishes the representative separating equilibrium if $A = 0$ and argues that the “intuitive criterion” may not have bite in the environment at hand.

(a) Equilibrium

In view of the postulated threshold beliefs and restrictions on the equilibrium, the expected payoff for each type of signaler is given by:

$$E[u_i^L(0)] = (1 - \lambda) \cdot \frac{1}{2} \cdot P,$$

and

$$E[u_i^H(s^*)] = \lambda \cdot \frac{1}{2} \cdot P + (1 - \lambda) \cdot P - \gamma^H \cdot s^*.$$

These expressions give rise to the following constraints:

- “Individual Rationality” Constraint (IR) – $E[u_i^\tau(s_i)] \geq 0$ for $\tau \in \{L, H\}$
  
  **Signalers of Low Type:**
  
  $$(1 - \lambda) \cdot \frac{1}{2} \cdot P \geq 0 \quad \Rightarrow \quad P \geq 0$$

  **Signalers of High Type:**
  
  $$\lambda \cdot \frac{1}{2} \cdot P + (1 - \lambda) \cdot P - \gamma^H \cdot s^* \geq 0$$

  $$\Rightarrow \quad s^* \leq \frac{P}{\gamma^H} \cdot \left(1 - \frac{\lambda}{2}\right)$$

- “Incentive Compatibility” Constraint (IC) – $E[u_i^\tau(s^\tau)] \geq E[u_i^\tau(s^{-\tau})]$ for $\tau \in \{L, H\}$
  
  **Signalers of Low Type:**
  
  $$(1 - \lambda) \cdot \frac{1}{2} \cdot P \geq \lambda \cdot \frac{1}{2} \cdot P + (1 - \lambda) \cdot P - \gamma^L \cdot s^*$$

  $$\Rightarrow \quad s^* \geq \frac{P}{2\gamma^L}$$

  **Signalers of High Type:**
  
  $$\lambda \cdot \frac{1}{2} \cdot P + (1 - \lambda) \cdot P - \gamma^H \cdot s^* \geq (1 - \lambda) \cdot \frac{1}{2} \cdot P$$

  $$\Rightarrow \quad s^* \leq \frac{P}{2\gamma^H}$$

Note that, in both cases, only IC is binding because with $\lambda \in (0,1)$, $P > 0$, and $\gamma^L > \gamma^H > 0$, the right-hand side of IC is always at least equal to zero, implying that if IC is satisfied IR is also (at least) satisfied. The ordering presented in the main text follows. It is easy to check that the expected equilibrium payoff for the highest feasible threshold level is positive (entailing that the same holds for lower values), and that neither type can increase his payoff by deviating to other signaling magnitudes. This establishes existence of the strategy profile.
(b) **Intuitive Criterion**

In conventional signaling games, the “intuitive criterion” (Cho & Kreps 1987) can be used to eliminate a given perfect Bayesian Nash Equilibrium if there is a signaler of type $\theta$ who is guaranteed a deviation payoff strictly greater than his equilibrium payoff for *any* belief of the receiver in response to the deviating message, as long as she does not assign a non-zero probability to the deviation having been made by a type $\theta$ for whom this action is equilibrium dominated. The upshot is the elimination of all equilibria except for the “least-cost” separating equilibrium (Riley 1979) – which in the present context would be the outcome where $s^L = 0$ and $s^H = s^* = \frac{p}{2\gamma^L}$ – from the relevant setups. Seeing as the present framework involves two (competing) signalers, the refinement does not apply straightforwardly. Therefore, in the following, I apply an extended version of the arguments that strives to maintain the spirit of the originals.

Fix a separating “threshold” equilibrium such that $s^L = 0$ and $s^H = s^*$ with $\frac{p}{2\gamma^L} \leq s^* \leq \frac{p}{2\gamma^H}$ (cf. main text), and turn to the issue of identifying signaling levels that are equilibrium dominated for signalers of low type *no matter what* the response of the receiver to them. Consider the best possible scenario for a signaler of low type deviating from his equilibrium strategy, i.e., he is taken to be of high type and chosen as the receiver’s ally with probability one even if the other signaler is of high type and sends $s^*$, implying that he would also be deemed of high type. Given these beliefs and letting $s^d$ denote the low type’s deviating signal, deviation will be profitable if $p - \gamma^L \cdot s^d \geq (1 - \lambda) \cdot \frac{p}{2\gamma^L}$, which will be true provided that $s^d \leq (1 + \lambda) \cdot \frac{p}{2\gamma^L}$. It follows that deviation will be profitable irrespective of the level of $s^*$ (in that $(1 + \lambda) \cdot \frac{p}{2\gamma^L} \geq \frac{p}{2\gamma^H}$) as long as $(1 + \lambda) \cdot \gamma^H \geq \gamma^L$. In short, in this case the “intuitive criterion” does not apply as the equilibrium-dominated signals lie above the highest possible value of $s^*$.

Supposing (instead) that some signaling levels are dominated for signalers of low type, i.e., $(1 + \lambda) \cdot \gamma^H < \gamma^L$, can a signaler of high type profitably deviate to them? Observe that any such interval will be situated between $\frac{p}{2\gamma^L}$ and $\frac{p}{2\gamma^H}$ and extend downward from $\frac{p}{2\gamma^H}$. Consider the worst possible scenario for a high type deviating from his equilibrium strategy $s^*$ to, say, $s^* - \epsilon$, for $\epsilon > 0$. Namely,
although the deviant is considered to be of high type, he will lose with certainty against another high type behaving according to his equilibrium strategy. Since the receiver judges both signalers to be of the same type, this decision rule is a valid best response as she may adopt any arbitrary rule if indifferent. If so, the signaler will have an incentive to deviate if the constant loss in his probability of winning is compensated by the linear reduction in his cost of signaling. To be precise, \( \lambda \cdot \frac{1}{2} \cdot p + (1 - \lambda) \cdot p - \gamma^H \cdot s^* \leq (1 - \lambda) \cdot p - \gamma^H \cdot (s^* - \varepsilon) \) obliges that:

\[
\varepsilon \geq \lambda \cdot \frac{p}{2 \cdot \gamma^H},
\]

where \( \varepsilon \) in effect denotes the required (fixed) minimum savings in cost.

A pessimistic (in the above sense) high type can, in general, profitably deviate from his equilibrium strategy into the interval of equilibrium-dominated signal intensities for signalers of low type if the pre-deviation magnitude exceeds the sum of the highest possible non-dominated signaling level for low types and the lowest necessary deviation (Figure 2.A.1); i.e.:

\[
s^* \geq s^{d}_{\text{max}} + \varepsilon_{\text{min}},
\]

where \( s^{d}_{\text{max}} = (1 + \lambda) \cdot \frac{p}{2 \cdot \gamma^L} \) and \( \varepsilon_{\text{min}} = \lambda \cdot \frac{p}{2 \cdot \gamma^H} \). Accordingly, there will exist a region of signaling levels between \( s^{d}_{\text{max}} + \varepsilon_{\text{min}} \) and \( \frac{p}{2 \cdot \gamma^H} \) that can be eliminated via the “intuitive criterion” if:

\[
s^{d}_{\text{max}} + \varepsilon_{\text{min}} \leq \frac{p}{2 \cdot \gamma^H},
\]

which substitution and straightforward manipulation reduce to:

\[
\lambda < \frac{\gamma^L - \gamma^H}{\gamma^L + \gamma^H}.
\]

Importantly, note that even if \( s^{d}_{\text{max}} \) is close to \( \frac{p}{2 \cdot \gamma^L} \) (e.g., if \( \lambda \) is small), it will not be possible to rule out the equilibria between \( s^{d}_{\text{max}} \) and \( s^{d}_{\text{max}} + \varepsilon_{\text{min}} \). For, unless \( s^{d}_{\text{max}} < \frac{p}{2 \cdot \gamma^L} \), which is inadmissible by construction, the discrete nature of the game – \( \varepsilon \) in particular – prevents the argument from capturing (and eliminating) all equilibria except for the “least cost” configuration at \( \frac{p}{2 \cdot \gamma^L} \). The equilibria between \( \frac{p}{2 \cdot \gamma^L} \) and \( s^{d}_{\text{max}} + \varepsilon_{\text{min}} \) will thus always survive the refinement.

In essence, what happens is that the presence of a second signaler brings into
Figure 2.A.1. Neutral Signals – Application of the “Intuitive Criterion”

play the receiver’s decision rule when indifferent between the two signalers. While it may be possible that the extended version of the “intuitive criterion” rules out more equilibria when based on a less pessimistic outlook on the part of signalers of high type, the refinement’s scope will always be limited. After all, any signal between 0 and \( \frac{p}{2 \gamma^L} \) could, in principle, profitably be sent by a signaler of low type if he is in with a chance of winning the prize.

Appendix 2.2. The Simplest Case – Beneficent Signals (A > 0)

This appendix proves Proposition 2.2.

(a) Sufficient Conditions. Bounds of the Equilibrium Supports

Signalers of Low Type

Given the lower bound (cf. main text), the upper bound of the low types’ support (\( \hat{s}^L \)) follows straightforwardly from the fact that players randomizing over a number of different strategies must be indifferent (in terms of their payoff) between them. A signaler of low type must therefore obtain the same payoff when signaling zero as when signaling at the upper bound of his equilibrium distribution, namely, zero. In fact, it must be that \( \mathbb{E}[u^L(s_t)] = 0 \) for all \( s_t \) along the equilibrium support. Furthermore, in keeping with the restrictions on the equilibrium, a signal at the upper bound of the equilibrium distribution entails that the individual wins against all other signalers of low type (all mass is below \( \hat{s}^L \), which occurs with probability \( 1 - \lambda \)), while he loses against all signalers of high type (all mass is above \( \hat{s}^L \), which occurs with probability \( \lambda \)). Note that, under these conditions, \( \mathbb{E}[u^L(s_t)] = \mathbb{E}[u^L(\hat{s}^L)] \). Accordingly:

\[
(1 - \lambda) \cdot G^L(\hat{s}^L) \cdot p - \gamma^L \cdot \hat{s}^L = 0,
\]

which – using that \( G^L(\hat{s}^L) = \Pr(s_t \leq \hat{s}^L) = 1 - \) – can be manipulated to yield:

\[
\hat{s}^L = (1 - \lambda) \cdot \frac{p}{\gamma^L}.
\]
Signalers of High Type

The upper bound of the high types’ support $\hat{s}^H$ can be derived using a similar line of reasoning. A signaler of high type must obtain the same payoff when signaling $\hat{s}^L$ as when signaling at the upper bound of his equilibrium distribution. Moreover, given the restrictions on the equilibrium, a signal at the lower bound entails that the individual wins against all individuals of low type (all mass is below $\hat{s}^L$, which occurs with probability $(1-\lambda)$), while he loses against all other individuals of high type (all mass is above $\hat{s}^L$, which occurs with probability $\lambda$). Accordingly, endpoint $\hat{s}^H$ can be constructed as follows.

$$E[\hat{s}^H] = (1-\lambda) \cdot \left( p - \gamma^H \cdot (1-\lambda) \cdot \frac{p}{\gamma^L} \right) + \lambda \cdot \left( -\gamma^H \cdot (1-\lambda) \cdot \frac{p}{\gamma^L} \right)$$

This payoff is clearly strictly positive and constant. Observe that a signal at the upper bound guarantees victory, i.e., $G^H(\hat{s}^H) = Pr(s_i \leq \hat{s}^H) = 1$. Under these conditions, $E[u_i^H(\hat{s}^L)] = E[u_i^H(\hat{s}^H)]$, which again must be true for all $s_i$ along the equilibrium support. This implies that:

$$p - \gamma^H \cdot \hat{s}^H = (1-\lambda) \cdot p - \gamma^H \cdot (1-\lambda) \cdot \frac{p}{\gamma^L},$$

which can be manipulated to yield:

$$\hat{s}^H = (1-\lambda) \cdot \frac{p}{\gamma^L} + \lambda \cdot \frac{p}{\gamma^H}.$$

(b) Necessary Conditions. Equilibrium Distributions

Given the bounds of the equilibrium supports, the equilibrium distribution functions follow from the signaler types’ expected payoff functions.

Signalers of Low Type

As the strategy’s support is conjectured to be on $[0,\hat{s}^L]$ and, given randomization, this type’s expected equilibrium payoff will be zero:

$$E[u_i^L(s_i)] = (1-\lambda) \cdot G^L(s_i) \cdot p - \gamma^L \cdot s_i = 0.$$

Manipulation then yields:

$$G^L(s_i) = \frac{\gamma^L}{(1-\lambda) \cdot p} \cdot s_i.$$
Signalers of High Type
As the strategy’s support is conjectured to be on \([s^L, s^H]\), by construction, this type’s expected equilibrium payoff is given by \((1 - \lambda) \cdot p - \gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^H}\):

\[
E[u^H_i(s_i)] = (1 - \lambda) \cdot (1 - G^H(s_i)) \cdot p - \gamma^H \cdot s_i = (1 - \lambda) \cdot p - \gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^H}.
\]

Manipulation then yields:

\[
G^H(s_i) = \frac{\gamma^H}{\lambda \cdot p} \cdot (s_i - (1 - \lambda) \cdot \frac{p}{\gamma^L}).
\]

Note that the equilibrium densities \(g^\tau(s_i)\) for \(\tau \in \{L, H\}\) are uniform and represent the ratio of the marginal cost of an extra unit of the signal and the marginal benefit of the (ensuing) extra probability of winning against a signaler of the “own” type.

(c) Existence
The following derivations establish that the signalers do not have an incentive to deviate from their conjectured equilibrium strategies by confirming (1) that the payoffs are constant along the support of each type’s equilibrium distribution function, and showing (2) that each type of signaler makes a loss when imitating a signaler of the other type. Fulfillment of both conditions in conjunction with beliefs conforming to behavior proves existence of the equilibrium profile.

(1) Are the payoffs constant along the support of the types’ equilibrium distribution functions?

Profits are constant if, after a small change in the signal intensity, the marginal benefit (the extra probability of winning) and the extra cost of the higher signal cancel out exactly.

Signalers of Low Type: Raise signal from \(\varepsilon\) to \(2 \cdot \varepsilon\), for \(\varepsilon > 0\) but small.

Marginal Benefit: \((1 - \lambda) \cdot p \cdot (2 \cdot \varepsilon - \varepsilon) \cdot \frac{\gamma^L}{(1 - \lambda) \cdot p}\)

Marginal Cost: \(\gamma^L \cdot \varepsilon\)

\(\Rightarrow\) Marginal Benefit = Marginal Cost
Signalers of High Type: Raise signal from \((\hat{s}^L + \varepsilon)\) to \((\hat{s}^L + 2 \cdot \varepsilon)\), for \(\varepsilon > 0\) but small.

Marginal Benefit: \[\lambda \cdot p \cdot \left[ (\hat{s}^L + 2 \cdot \varepsilon) - (\hat{s}^L + \varepsilon) \right] \cdot \frac{y^H}{\lambda p}\]

Marginal Cost: \(\gamma^H \cdot \varepsilon\)

\(\Rightarrow\) Marginal Benefit = Marginal Cost

These derivations can clearly be reproduced for all signal intensities along the conjectured equilibrium supports. As profits are constant, the signalers are indifferent between the signals along their respective distributions, which is consistent with continuous randomization.

(2) Can either type of signaler increase his payoff by imitating the other type?

Suppose a signaler of low type were to imitate a signaler of high type, i.e., consider \(s^L_t > \hat{s}^L\) such as \(s^L_t = \hat{s}^L + \varepsilon\), for \(\varepsilon > 0\) but not so big that the sum exceeds the support of signalers of high type. Note that deviation to \(s^L_t < 0\) is ruled out by definition. The signaler is certain to win against all other low types (with probability \((1 - \lambda)\)) as well as some high types (with probability \(G\)). His expected payoff from imitation is thus given by:

\[E[u_t(s^L_t)] = (1 - \lambda) \cdot (p - \gamma^L \cdot s^L_t) + \lambda \cdot \left[ G^H(s^L_t - \gamma^L \cdot s^L_t) + (1 - G^H(s^L_t)) \cdot (\gamma^L \cdot s^L_t) \right] = \lambda \cdot G^H(s^L_t) \cdot p - \gamma^L \cdot \varepsilon.\]

Consistency with equilibrium requires that \(E[u_t(s^L_t)] < 0\). Substitution of \(G^H(s^L_t)\) and subsequent manipulation reduce the expression to \(\gamma^L > \gamma^H\), which is satisfied by assumption. Accordingly, signalers of low type do not have an incentive to deviate.

Instead, consider a signaler of high type imitating a signaler of low type, i.e., suppose \(s^H_t < \hat{s}^L\) such as \(s^H_t = \hat{s}^L - \varepsilon\), for \(\varepsilon\) as above. Note that \(s^H_t > \hat{s}^H\) would not occur since victory is already guaranteed with a signal at \(\hat{s}^H\) (i.e., the cost of signaling would increase while the probability of winning would remain unchanged). The signaler is certain to lose against all other high types (with probability \(\lambda\)), yet, he will win against some low types (with probability \(G^L(s^H_t)\)). The expected payoff from imitation is thus given by:
\[ E[u_i^H(s_i^H)] = (1 - \lambda) \cdot \left[ G^L(s_i^H) \cdot (p - \gamma^H \cdot s_i^H) + (1 - G^L(s_i^H)) \cdot (-\gamma^H \cdot s_i^H) \right] \\
+ \lambda \cdot (-\gamma^H \cdot s_i^H) \\
= (1 - \lambda) \cdot G^L(s_i^H) \cdot p - \gamma^H \cdot s_i^H. \]

Consistency with equilibrium requires that \( E[u_i^H(s_i^H)] < (1 - \lambda) \cdot p - \gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} \). Substitution of \( G^L(s_i^H) \) and subsequent manipulation reduce the expression to \( \gamma^L > \gamma^H \), which again is satisfied by assumption. Signalers of high type therefore do not have an incentive to deviate either.

(d) Equilibrium Beliefs. Signals at \( s^L \)

Given the mixed nature of the signaler types’ strategies, Bayes’ Rule cannot straightforwardly be applied to obtain the receiver’s beliefs when observing a signal at \( s^L \) (exactly) as the probability of sending a signal at a single point is zero. That is to say, when arranged to fit the relevant case, Bayes’ Rule takes the following form:

\[ \Pr(\theta^H | \hat{s}^L) = \frac{\Pr(\hat{s}^L \mid \theta^H) \cdot \Pr(\theta^H)}{\Pr(\hat{s}^L)}. \]

Yet, recall that the proportion of high and low types in the population of signalers is common knowledge, as is the value of an alliance with the receiver and the signalers’ cost function, all of which are non-zero constants. The probability assignment for this signaling magnitude can therefore be derived via the signalers’ uniform densities, i.e., the constant height of the densities at each point along the equilibrium supports (both of which include \( \hat{s}^L \)). In particular, paralleling Bayes’ Rule, the beliefs at \( \hat{s}^L \) can be expressed as the ratio of the probability of the event that “a signaler of type \( \theta \) sends a signal at \( \hat{s}^L \)” divided by the probability that a signal at \( \hat{s}^L \) is sent. Namely:

\[ \Pr(\theta^H | \hat{s}^L) = \frac{\lambda \cdot g^H(\hat{s}^L)}{\lambda \cdot g^H(\hat{s}^L) + (1 - \lambda) \cdot g^L(\hat{s}^L)}. \]

It is easy to verify that the denominator is a non-zero constant and that the sum of the complementary probabilities is one.
Appendix 2.3. The General Case

This appendix proves Proposition 2.4.

(a) **Sufficient Conditions.** Bounds of the Equilibrium Supports

**Signalers of Low Type**

Given the lower bound (*cf.* main text), in the general case, too, the upper bound of the low types’ support \( \hat{\psi}^L \) follows from the fact that players randomizing over a number of different strategies must be indifferent (in terms of their payoff) between them. In fact, barring the revised notation, the relevant conditions and arguments are identical to those presented in Appendix 2.2.a. Accordingly, endpoint \( \hat{\psi}^L \) can be constructed as follows.

\[
(1 - \lambda)^{n-1} \cdot [G^L(\hat{\psi}^L)]^{n-1} \cdot m \cdot p - \gamma^L \cdot \hat{\psi}^L = 0,
\]

which – using that \( G^L(\hat{\psi}^L) = \Pr(s_i \leq \hat{\psi}^L) = 1 \) – can be manipulated to yield:

\[
\hat{\psi}^L = (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^L}.
\]

**Signalers of High Type**

The upper bound of the high types’ support \( \hat{\psi}^H \) can likewise be derived via an argument paralleling that outlined in Appendix 2.2.a. The main quantitative difference is that the probability of victory following a signal at the lower bound of the distribution is given by \( (1 - \lambda)^{n-1} \), while the probability of defeat is given by \( (1 - (1 - \lambda)^{n-1}) \). Accordingly, endpoint \( \hat{\psi}^H \) can be constructed as follows.

\[
E[u_i^H(\hat{\psi}^L)] = (1 - \lambda)^{n-1} \cdot \left( m \cdot p - \gamma^H \cdot (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^L} \right) + (1 - (1 - \lambda)^{n-1})
\]

\[
\cdot \left( -\gamma^H \cdot (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^L} \right)
\]

\[
= (1 - \lambda)^{n-1} \cdot m \cdot p - \gamma^H \cdot (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^L}.
\]

Using that \( G^H(\hat{\psi}^H) = \Pr(s_i \leq \hat{\psi}^H) = 1: \)

\[
m \cdot p - \gamma^H \cdot \hat{\psi}^H = (1 - \lambda)^{n-1} \cdot m \cdot p - \gamma^H \cdot (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^L},
\]

which can be manipulated to yield:

\[
\hat{\psi}^H = (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^L} + (1 - (1 - \lambda)^{n-1}) \cdot \frac{m \cdot p}{\gamma^H}.
\]
(b) **Necessary Conditions.** Equilibrium Distributions

Given the bounds of the equilibrium supports, the equilibrium distribution functions once more follow from the signaler types’ expected payoff functions.

**Signalers of Low Type**

As the strategy’s support is conjectured to be on \([0, \hat{y}^L]\) and, given randomization, this type’s expected equilibrium payoff will be zero:

\[ E[u_t^L(s_i)] = (1 - \lambda)^{n-1} \cdot [G^L(s_i)]^{n-1} \cdot m \cdot p - \gamma^L \cdot s_i = 0. \]

Manipulation then yields:

\[ G^L(s_i) = \left( \frac{\gamma^L \cdot s_i}{m \cdot p \cdot (1 - \lambda)^{n-1}} \right)^{\frac{1}{n-1}}. \]

**Signalers of High Type**

The derivation of the equilibrium distribution function for signalers of high type is slightly more complicated, as this type’s expected payoff function cannot easily be solved for \(G^H(s_i)\). However, as it must hold at each point along the support, the equilibrium distribution can also be derived by maximizing \(E[u_t^H(s_i)]\), although this approach depends critically on the assumption that the equilibrium distribution functions are continuous. The appropriate optimization problem is given by:

\[
\max_{s_i} \left[ m \cdot p \cdot \left\{ \sum_{j=0}^{n-1} \left( \frac{n - 1}{j} \cdot \lambda^j \cdot (1 - \lambda)^{n-1-j} \cdot [G^H(s_i)]^j \right) - \gamma^H \cdot s_i \right\} \right].
\]

The first-order condition takes the form of the following differential equation:

\[
\sum_{j=1}^{n-1} \left( \frac{n - 1}{j} \cdot \lambda^j \cdot (1 - \lambda)^{n-1-j} \cdot j \cdot [G^H(s_i)]^{j-1} \cdot g^H(s_i) = \frac{\gamma^H}{m \cdot p}. \right.
\]

Using the fact that \(G^H(\hat{y}_L) = 0\), i.e., the probability of winning when signaling at the lower bound of the equilibrium support is zero, its solution is given by:

\[ G^H(s_i) = \frac{1}{\lambda} \left[ \lambda - 1 + \left( \frac{\gamma^H}{m \cdot p} \cdot (s_i - \hat{y}^L) + (1 - \lambda)^{n-1} \right)^{\frac{1}{n-1}} \right]. \]
(c) **Existence**

The following derivations establish that the signalers in the generalized version of the game do not have an incentive to deviate from their conjectured equilibrium strategies either. As before, this is achieved by confirming (1) that the payoffs are constant along the support of each type’s equilibrium distribution function, and showing (2) that each type of signaler makes a loss when imitating a signaler of the other type. Fulfillment of both conditions in conjunction with beliefs conforming to behavior proves existence of the equilibrium profile.

(1) Are the payoffs constant along the support of the types’ equilibrium distribution functions?

Given the complexity of the equilibrium distributions, computing the marginal benefit and cost of a small increase in the signal intensity is not straightforward. Therefore, in this case, constancy of the expected payoffs is verified by comparing the payoff when choosing a given signaling magnitude with that when opting for a (slightly) higher intensity. Payoffs are constant if $E[u_{i}(s)] = E[u_{i}(s + \epsilon)]$ for $\tau \in \{L, H\}, \epsilon > 0$ but small, and $s$ as well as $s + \epsilon$ within the bounds of the types’ respective equilibrium supports.

**Signalers of Low Type**

\[
(1 - \lambda)^{n-1} \cdot [G^L(s)]^{n-1} \cdot m \cdot p - y^L \cdot s
\]

\[
= (1 - \lambda)^{n-1} \cdot [G^L(s + \epsilon)]^{n-1} \cdot m \cdot p - y^L \cdot (s + \epsilon)
\]

Substitution of $G^L(\cdot)$ and subsequent manipulation reduce this expression to $y^L \cdot \epsilon = y^L \cdot \epsilon$.

**Signalers of High Type**

\[
m \cdot p \cdot \sum_{j=0}^{n-1} \binom{n-1}{j} \cdot \lambda^j \cdot (1 - \lambda)^{n-1-j} \cdot [G^H(s)]^j - y^H \cdot s
\]

\[
= m \cdot p \cdot \sum_{j=0}^{n-1} \binom{n-1}{j} \cdot \lambda^j \cdot (1 - \lambda)^{n-1-j} \cdot [G^H(s + \epsilon)]^j - y^H \cdot (s + \epsilon)
\]

Using the binomial theorem, i.e., the fact that:

\[
(x + y)^b = \sum_{a=0}^{b} \binom{b}{a} \cdot x^{b-a} \cdot y^a,
\]
with \( x = (1 - \lambda) \), \( y = \lambda \cdot G^H(\cdot) \), \( a = j \), and \( b = n - 1 \), this expression can be rewritten as:

\[
(1 - \lambda + \lambda \cdot G^H(s))^{n-1} \cdot m \cdot p - \gamma^H \cdot s = (1 - \lambda + \lambda \cdot G^H(s + \varepsilon))^{n-1} \cdot m \cdot p - \gamma^H \cdot (s + \varepsilon).
\]

Substitution of \( G^H(\cdot) \) and subsequent manipulation reduce it to \( \gamma^H \cdot \varepsilon = \gamma^H \cdot \varepsilon \).

These derivations can clearly be reproduced for all signal intensities along the conjectured equilibrium supports. As profits are constant, the signalers are indifferent between the signals along their respective distributions, which (again) is consistent with continuous randomization.

(2) Can either type of signaler increase his payoff by imitating the other type?

Suppose a signaler of low type were to imitate a signaler of high type, i.e., consider \( s^l_1 > \bar{\psi}^l \) such as \( s^l_1 = \bar{\psi}^l + \varepsilon \), for \( \varepsilon > 0 \) but not so big that the sum exceeds the support of signalers of high type. Note that deviation to \( s^l_1 < 0 \) is ruled out by definition. The signaler is certain to win against all other low types (with probability \( (1 - \lambda)^{n-1} \); he will also win against some high types, although his (marginal) benefit depends on the number of equally potent competitors. His expected payoff from imitation is thus given by:

\[
E[u^l_1(s^l_1)] = \sum_{j=0}^{n-1} \binom{n-1}{j} \cdot \lambda^j \cdot (1 - \lambda)^{n-1-j} \cdot \left\{G^H(s^l_1)^j \cdot (m \cdot p - \gamma^l \cdot s^l_1) + (1 - [G^H(s^l_1)]^j) \cdot (-\gamma^l \cdot s^l_1)\right\} = m \cdot p \cdot \sum_{j=0}^{n-1} \binom{n-1}{j} \cdot \lambda^j \cdot (1 - \lambda)^{n-1-j} \cdot [G^H(s^l_1)]^j - \gamma^l \cdot s^l_1.
\]

Using the binomial theorem (see above), this expression can be rewritten as:

\[
E[u^l_1(s^l_1)] = (1 - \lambda + \lambda \cdot G^H(s^l_1))^{n-1} \cdot m \cdot p - \gamma^l \cdot s^l_1.
\]

Consistency with equilibrium requires that \( E[u^l_1(s^l_1)] < 0 \). Substitution of \( G^H(s^l_1) \) and subsequent manipulation reduce the expression to \( \gamma^l > \gamma^H \), which is satisfied by assumption. Accordingly, signalers of low type do not have an incentive to deviate.

Instead, consider a signaler of high type imitating a signaler of low type, i.e., suppose \( s^H_1 < \bar{\psi}^l \) such as \( s^H_1 = \bar{\psi}^l - \varepsilon \), for \( \varepsilon \) as above. Note that in this

\[
E[u^H_1(s^H_1)] = (1 - \lambda + \lambda \cdot G^H(s^H_1))^{n-1} \cdot m \cdot p - \gamma^H \cdot s^H_1.
\]
case, too, \( s_t^H > \hat{\psi}^H \) would not occur since victory is already guaranteed with a signal at \( \hat{\psi}^H \) (i.e., the cost of signaling would increase while the probability of winning would remain unchanged). The signaler is certain to lose against all other high types (with probability \( (1 - (1 - \lambda)^{n-1}) \)), but will win against some low types (with probability \( G^L(s_t^H) \)). The expected payoff from imitation is thus given by:

\[
E[u_t^H(s_t^H)] = (1 - \lambda)^{n-1} \cdot \left[ \left( G^L(s_t^H) \right)^{n-1} \cdot \left( m \cdot p - \gamma^H \cdot s_t^H \right) + \left( 1 - \left[ G^L(s_t^H) \right]^{n-1} \right) \cdot \left( -\gamma^H \cdot s_t^H \right) \right] \\
+ (1 - (1 - \lambda)^{n-1}) \cdot \left( -\gamma^H \cdot s_t^H \right) \\
= (1 - \lambda)^{n-1} \cdot \left[ G^L(s_t^H) \right]^{n-1} \cdot m \cdot p - \gamma^H \cdot s_t^H.
\]

Consistency with equilibrium requires that \( E[u_t^H(s_t^H)] < (1 - \lambda)^{n-1} \cdot m \cdot p - \gamma^H \cdot (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^L} \). Substitution of \( G^L(s_t^H) \) and subsequent manipulation reduce the expression to \( \gamma^L > \gamma^H \), which again is satisfied by assumption. Signalers of high type therefore do not have an incentive to deviate either.
3.1 Introduction

In seeming contradiction to their self-interest, many individuals voluntarily engage in activities that are costly to them and benefit one or more others within their social environment, oftentimes strangers. More often than not, the “recipients” benefit more from the acts, if not primarily, than the “contributors,” who do not appear to exhibit profound concern for the beneficiaries’ history and likelihood of reciprocation. In essence, they are providing a (local) public good. Examples of such generous behavior penetrate all walks of life. People do volunteer work in their communities, donate blood, solicit and make donations to charity, help strangers, in some cases thereby putting themselves at risk of injury and, more broadly, adhere to and enforce social norms.

A somewhat unusual approach\(^1\) to explaining the recurrent incidence of generous behavior within broad social contexts, which despite a long history in a number of disciplines has only recently been formally modeled (Gintis et al. 2001), invokes costly signaling (Spence 1973, 1974; Zahavi 1975; Grafen 1990) as a proximate explanation for the behavioral pattern. The idea is that generous acts towards others may allow individuals to signal reliably various socially important qualities, thereby benefitting themselves as well as potential future interactants (e.g.,

\(^1\) Over the years, quite a number of theories striving to resolve the apparent incompatibility of generous behavior with self-centered optimization of utility or fitness have emerged. The most widespread of these theories are direct reciprocity (Trivers 1971; Axelrod & Hamilton 1981), reputation-based explanations such as indirect reciprocity (Alexander 1987; Nowak & Sigmund 1998), coercion or tolerated theft (Blurton Jones 1984; Hawkes 1992), as well as various forms of pro-social preferences (cf. Fehr & Gächter 2000). Yet, although rich, they tend to yield insufficient insight when it comes to explaining generous behavior towards strangers (cf. Chapter 2, Section 2.5).
The essential requisite for this causal relation is a link between generosity and some unobservable social quality such that variation in generosity can be traced back to variations in the particular characteristic. Given this correlation, the recipients of the signal gain as they obtain the benefit inherent in the signaling activity as well as useful information about the signaler, while the individual performing the good deed benefits not because of reciprocation, but because the signal reveals qualities that make it advantageous for the observer(s) to interact preferentially with him/her. It may, for example, enable the receiver(s) to identify the signaler as a cooperator or defector. Note that the interaction need not be repeated – in a signaling equilibrium, the exchange of benefits represents each player’s best response given their information at the time (e.g., Gibbons 1992; Smith 2003).

In keeping with this line of reasoning, costly signaling using beneficial activities facilitates individuals’ task of discerning others’ intentions in social contexts. Being able to do so is important since cooperative social interaction carries great potential benefits and costs, entailing that individuals are likely to choose their interaction partners carefully (Cosmides & Tooby 1992; Smith & Bliege-Bird 2005). This “selectiveness,” however, clearly restricts the supply of valuable social goods such as alliances, mates, or leadership positions. Correspondingly, if generosity can be used to convey information about a desirable social quality, say, by signaling reliability or trustworthiness, generous individuals are likely to be preferred as exchange partners. One would then expect that individuals be induced to compete to be more generous than others to gain access to the most profitable social opportunities (Roberts 1998).

Combining the notions of signaler competition and signaling using beneficent signals in a one-period, non-cooperative, game-theoretic framework of incomplete information, one obtains equilibrium predictions that diverge substantially from those of standard economic signaling models, which comprise (socially) neutral signals and competition on the receivers’ side of the interaction (Chapter 2). In particular, the prediction for a class of signaling games involving beneficent signals, symmetric signalers of two discrete types, and non-decreasing beliefs on the part of the receiver is unique, separating, and in mixed strategies. In contrast, given neutral signals to convey information to the receivers (ceteris paribus), paralleling the literature, one cannot definitively predict the players’ equilibrium behavior, for there
exists a multiplicity of equilibria – even in pure strategies. In other words, signaler competition alone does not lead to a sharper prediction. The objective of this chapter is to explore experimentally the precision of these predictions and their divergence.

While signaling games are ubiquitous in the theoretical literature, empirical and experimental work on them is quite limited. Early experimental work focused mostly on establishing the empirical usefulness of equilibrium selection devices (or refinements) in predicting behavior in classical signaling frameworks. The results suggest that the subjects tend to grasp the information-transmitting properties of the available signaling activities: Signalers often make active use of the signaling activities at their disposal and receivers make inferences based on the signals they observe, although variability is substantial (e.g., Miller & Plott 1985; Potters & van Winden 1996). However, while subjects engage in some strategic reasoning, they do not tend to reason as deeply as theory assumes. Accordingly, only simple refinements such as sequentiality and the Cho-Kreps “intuitive criterion” have some predictive power (Banks et al. 1994). Behavior typically adjusts to equilibrium only slowly (if at all), with the subjects following straightforward history-dependent learning processes. In fact, one can construct games in which play converges to equilibria that violate even the simplest equilibrium refinements (Brandts & Holt 1992, 1993; Cooper et al. 1997 a, b).²

A small number of experiments (Wedekind & Milinski 2000; Milinski et al. 2002 a, b; Barclay 2004, 2006) have explored the line of reasoning motivating the signaling approach to explaining generous behavior (in some cases implicitly). They show that people who voluntarily incur costs to provide a benefit to others are, indeed, bestowed with higher rewards from group members in subsequent interactions.³ This finding holds for indirect benefits such as the punishment of free-riders as well as the provision of direct benefits to others, say, by making a high contribution to a public good. Though some of the studies touch on signaling as a possible medium driving the observed behavioral patterns, their focus is on whether a good reputation accrued in a first game is rewarded in successive games. The present experiment, in contrast, explores whether individuals comprehend the concept of

³² More recent work has explored aspects such as the impact of meaningful context on strategic behavior (Cooper & Kagel 2003), individual versus team play (Cooper & Kagel 2005), learning across signaling games (Cooper & Kagel 2008), noisy signaling (Jeitschko & Normann 2009; de Haan et al. 2011), and the effect of the order of moves on the behavioral outcomes (Kübler et al. 2008).

³³ In all of these experiments, the participants are provided with a full history of the events in the current and preceding round(s).
using a beneficial activity to convey information and make active use of it when trying to initiate interaction with others in a one-shot environment.

Notwithstanding its intimate ties to the signaling literature, the present experiment is most closely linked to the work on all-pay auctions. For the main conceptual feature of the theoretical framework with beneficent signals is the interaction’s correspondence to an incomplete-information all-pay contest with two types of bidders who face asymmetric bidding costs and are competing for a fixed prize. Even though the study of auctions has a long history in experimental economics (cf. Kagel 1995), only a few experiments have studied all-pay auctions. Paralleling work on other auctions, their results indicate that subjects tend to bid more aggressively than predicted. This finding obtains in a variety of setups, including assorted informational treatments, player characteristics, strategy spaces, prize dimensions, as well as multiple-unit designs (Millner & Pratt 1989; Potters et al. 1998; Barut et al. 2002; Rapoport & Amaldoss 2000, 2004; Gneezy & Smorodinsky 2006).

The present setup is most closely related to that of Noussair and Silver (2006), who study an incomplete-information, single-unit all-pay auction with symmetric players whose valuations are drawn from a uniform distribution defined on a large but discrete interval. The main result of their investigation is that bidders with low valuations bid close to, but usually below, the equilibrium level, while bidders with high valuations overshoot the equilibrium. The crucial difference to the current setup is that the auctioneer, as is common in auction experiments, is an automaton – the winner is always the highest bidder. In the current framework, although the equilibrium strategy of the player with the equivalent role, the receiver, corresponds to selecting the “highest bidder,” depending on a subject’s beliefs having observed a particular signal, this may or may not be what actually happens. In this sense, this paper examines the behavioral implications of the fundamental difference between an auction and a signaling setup: Whereas the auctioneer only cares about a bidder’s action, the receiver in a signaling game is mainly interested in a signaler’s unobservable type. Although the players’ objective in both settings is to generate the

\footnote{\textsuperscript{3,4} Even though the present setup can be converted to a framework where the bidders face the same cost of bidding and instead differ in their valuation of the prize, the bidders might be considered asymmetric in the sense that the signaler types’ equilibrium strategies differ.}

\footnote{\textsuperscript{5} (S)he may also care about the kind or level of signal sent, yet his/her main concern is the signalers’ type. Depending on the payoffs, this need not be the case, even though it is in the present experiment.}
highest possible payoff, signalers simultaneously strive to convey information to the receivers, which the latter need to interpret. Given this dual role of signals, one might observe what appears to be “irrational” behavior by both signalers and receivers – signalers using unusual activities to convey their information and receivers responding in unexpected ways.

The results of the present experiment provide strong support for many elements of the theory. The participants swiftly appear to recognize the information-transmitting attributes of the (signaling) activities at their disposal, with the receivers mostly making appropriate inferences from the signals they observe. The signalers’ behavior, too, at the aggregate as well as in almost all individual sessions, consistently – though with notable variability in its course – approximately converges to some coordinated outcome, even when using neutral signals. Indeed, despite the intricate nature of the projected behavior, the signalers’ behavior when using beneficent signals is surprisingly close to that predicted by the theory, including the comparative static properties. Behavior in the treatment with neutral signals, in turn, is consistent with multiple equilibria.

The remainder of this chapter is organized as follows. Section 3.2 presents the game, derives the theoretical predictions, and outlines a number of hypotheses to be tested. Section 3.3 discusses the design and protocol of the experiment. Section 3.4 analyzes the results, followed by a discussion in Section 3.5. Section 3.6 concludes.

### 3.2 Theoretical Framework and Hypotheses

#### 3.2.1 Theoretical Predictions

The signaling game studied in this chapter involves three players – two signalers (he) and one receiver (she). The signalers observe private information about their endowment of some unobservable social characteristic ($\theta$) drawn from a discrete type space and may choose an action ($s$) from a message space defined on a large, fine grid to convey information about their attribute to the receiver, who knows the distribution of the private information but not the information of a particular signaler. Given the signals, the receiver makes an inference regarding each signaler’s information, based on which she chooses an action ($d$) from a discrete

---

3.6 Despite the restriction to a discrete interval, the large number of strategies should – in the limit – be sufficiently close to a continuum to satisfy this prerequisite of the equilibrium analysis.
action space. The experiment investigates the following parameterization of this sequential interaction:

1) The signalers are randomly and independently assigned a type – high (H) or low (L) – with equal probabilities. If of high type, a signaler is endowed with \( \theta^H = 80 \) units of the social characteristic; otherwise, he is bestowed with \( \theta^L = 50 \) units.

2) Each signaler is privately informed of his type.

3) The signalers contemporaneously choose a signaling level \( s \in [0, 125] \), with a precision of up to two decimal places, contingent on their type.

4) After observing both signals, the receiver decides which signaler to choose as her exchange partner. She can form an alliance with exactly one signaler.

5) The payoffs are realized.

If \( d \in \{1, 2\} \) denotes the receiver’s choice of Signaler 1 or 2 conditional on having observed each of their signals \( s_1 \) and \( s_2 \), respectively, signaler \( i \)’s payoff is given by:

\[
u^S_i \left((s_1, s_2), d, (\theta_1, \theta_2)\right) = 500 \cdot \left[1 - (i - d)^2\right] - c(s_i, \theta_i),\]

where \( \theta_i \in \{\theta^H, \theta^L\} \) signifies the amount, or value, of the social characteristic held by signaler \( i \), and \( c(s_i, \theta_i) \) the cost for signaler \( i \) of sending a signal of level \( s_i \) when endowed with \( \theta_i \) of the social characteristic, which takes the following linear form:

\[
c(s_i, \theta_i) = \begin{cases} 
5 \cdot s_i & \text{if } \theta_i = \theta^H \\
25 \cdot s_i & \text{if } \theta_i = \theta^L.
\end{cases}
\]

Note that the marginal cost of signaling is assumed to be lower for signalers of high type. In other words, entry into an alliance yields a (commonly known) fixed positive “prize” of value 500 for the chosen signaler, yet a signaler only nets a positive payoff if he is, in fact, chosen as the receiver’s ally. For, only then does he receive the prize, rendering the first term on the right-hand side of the signalers’ payoff function non-zero.\(^7\) The receiver’s payoff, in turn, is given by:

\[
u^R \left((s_1, s_2), d, (\theta_1, \theta_2)\right) = \theta_d + A \cdot s_d,
\]

where \( \theta_d \) denotes the type-dependent value of the chosen signaler, and \( A \cdot s_d \) represents a possible signal-dependent benefit for the receiver, with (fixed) parameter \( A \in \{0, 0.1, 1\} \). If \( A = 0 \), the setting corresponds to the signalers using neutral signals.

\(^7\) Given the cost structure, to prevent subjects from forming a precise idea of the behavior the experimenter is looking for (demand effects), the signalers’ “choice set” exceeds 100, the “break-even” level for a signaler of high type if he obtains the prize.
in the sense that the receiver does not benefit from the signaling activity beyond obtaining information about the signalers’ social characteristic. On the other hand, if \( A > 0 \), the signal is beneficent – the receiver obtains a private benefit proportional to the magnitude of the signal sent by the chosen signaler. That is to say, the signal by the chosen signaler has a positive externality.\(^8\) Note that she does not receive a benefit from the individual she does not choose as her ally.

The appropriate solution concept for this game is the perfect Bayesian Nash Equilibrium. Using the parameter values, the theoretical predictions can be summarized as follows. All proofs can be found in Chapter 2.

**Neutral Signals (\( A = 0 \)).** If the signaling activity does not convey a benefit to the receiver, the problem is equivalent to one in which the receiver maximizes the conditional probability of allying with a signaler of high type (cf. Spence 1973, 1974).\(^9\) Accordingly, the equilibrium depends intrinsically on the receiver’s beliefs, enabling one to construct a wide variety of separating and pooling equilibria, both in pure and mixed strategies, as well as a broad range of hybrid equilibria. For instance, assuming “threshold” beliefs on the part of the receiver in that signals below some level \( s^* \) are associated with low type and those above said level with high type, one can construct the following representative range of symmetric separating equilibria:

- Signalers of low type set \( s^L = 0 \) and signalers of high type \( s^H = s^* \), where \( 10 \leq s^* \leq 50 \). The receiver adopts appropriate threshold beliefs.
- She chooses to ally with the signaler whom she assesses to have the highest probability of being of high type. In case of a tie (i.e., the signalers send signals of the same magnitude and the receiver considers them to be of the same type), she will randomly\(^10\) select one of the signalers as her ally.

In terms of payoffs, a signaler of low type expects to receive \( 0.5 \cdot 0.5 \cdot 500 = 125 \), a signaler of high type between \( 0.5 \cdot 0.5 \cdot 500 + 0.5 \cdot 500 - 5 \cdot 50 = 125 \) and \( 0.5 \cdot 0.5 \cdot 500 + 0.5 \cdot 500 - 5 \cdot 10 = 325 \), and the receiver anticipates a payoff of 50 or 80.

---

\(^8\) It is useful to distinguish between signals yielding a benefit to others and “socially beneficial” signals. A (costly) signal can only be considered socially beneficial if the associated benefit is large enough to offset completely the signalers’ cost. As this is not necessarily the case in the present scenario, for transparency, I shall use the term “beneficence” to indicate that nonetheless, in all cases, the signal has a positive externality on the receiver.

\(^9\) While the present setting is similar to Spence (1973, 1974), there is a fundamental difference in that I do not consider competition among the receivers. Instead, in my setup, and in further contrast to Spence, only the signalers compete. Moreover, the signaler in this paper receives a fixed reward when “hired,” whereas Spence’s signaler receives a reward that depends on his perceived type.

\(^10\) I focus on equilibria in which the receiver randomizes 50:50 when indifferent.
The outcome with \( s^* = 10 \), the least-cost configuration of the indicated range, will henceforth be referred to as “LCS outcome.”

**Beneficent Signals** \((A > 0)\). If the signal comprises a benefit and the receiver adopts non-decreasing beliefs, the interaction transforms into an incomplete-information all-pay auction with two discrete types of bidders, who face asymmetric bidding costs, competing for a fixed prize by submitting the \( s_i \)’s as their bids. It can be shown that there exists a unique symmetric separating equilibrium according to which signalers of high type choose a higher level of the signal than those of low type and the receiver adopts threshold beliefs. It takes the following form:

Signalers of low type randomize uniformly on \([0, 10]\), while signalers of high type randomize uniformly on \([10, 60]\). The receiver adopts threshold beliefs (with the threshold set at \( s = 10 \)), and if the signalers appear to be of the same type she will ally with the signaler who bestows the highest signal-dependent benefit on her, i.e., who sends the highest signal. In case of a tie, she will randomly select one of the signalers as her ally.

Note that this equilibrium holds regardless of the specific value of \( A \), so long as it is strictly positive.

The expected equilibrium payoff can easily be established to be 0 for signalers of low type and 200 for high types. For a low type, for instance, consider his payoff for a signaling level of zero, with which he is bound to lose, the ensuing payoff likewise being zero. For a signaler of high type, in turn, consider a signal of 60, where he is guaranteed to win, yielding him a payoff of \( 500 - 5 \cdot 60 = 200 \). Both types of signaler make the same payoff at the opposite ends of their equilibrium supports, where their probability of winning is 0.5. In fact, they must obtain these payoffs for any signal within their respective equilibrium supports. The receiver’s expected payoff depends on the value of \( A \). If it is equal to 1, she anticipates a payoff between \( 50 + 0 = 50 \) and \( 80 + 60 = 140 \), whereas a value of 0.1 entails a payoff between \( 50 + 0.1 \cdot 0 = 50 \) and \( 80 + 0.1 \cdot 60 = 86 \).

### 3.2.2 Experimental Hypotheses

Four properties of the equilibrium solutions are amenable for experimental investigation. It is clearly not reasonable to expect that behavior be precisely in line with the theoretical predictions; however, they ought to be prognostic in a qualitative sense. For one, the accuracy of both of the equilibrium solutions is testable. Hence,
considering the multitude of possible equilibrium solutions, in the case of neutral signals \((A = 0)\), one would expect that the participants find it difficult to coordinate on a specific, well-defined outcome.

**Hypothesis 1**  
If \(A = 0\), the participants struggle to converge to a definite outcome, if any.

With beneficent signals \((A > 0)\) in use, notwithstanding the relative complexity of the equilibrium solution, previous work suggests that one might expect to observe separation of the signaler types’ behavior and, at least at the aggregate level, randomization across the various signaling magnitudes along the equilibrium supports (cf. Camerer 2003).

**Hypothesis 2**  
If \(A > 0\), the signaler types separate and, at least at the aggregate level, randomize approximately in line with the theoretical predictions, while the receivers choose the signalers sending the highest signals as their allies.

As a corollary to these conjectures, one would expect an unambiguous divergence in aggregate behavior across signaling activities – ambiguity if \(A = 0\) and clear separation if \(A > 0\).

**Hypothesis 3**  
The aggregate behavior of the signalers (and potentially receivers) differs sharply across signaling activities \((A = 0 \text{ vs. } A > 0)\).

Finally, as both activities are intrinsically beneficial and the theoretical prediction is thus identical, one would not expect a drastic difference in aggregate behavior for signals with benefits of different (marginal) magnitudes.

**Hypothesis 4**  
The aggregate behavior of the signalers and receivers is independent of the level of the signal-dependent benefit \((A = 0.1 \text{ vs. } A = 1)\).

In view of the comparatively elaborate nature of the theoretical framework, to be able to test the hypotheses reliably against data, the experimental design comprised a number of substitutable elements. They are now presented and the necessity of their combined use is justified.
3.3 Experimental Design and Procedures

As one purpose of this study is to compare the behavior when individuals employ beneficent as opposed to neutral signals, the experiment involved three treatments, one for each value of the signal-dependent benefit (A = 0, 0.1, and 1). It was conducted in September and October 2009 at the Scottish Experimental Economics Laboratory (SEEL), involving 165 subjects drawn from the general student population of the University of Aberdeen (U.K.) in 16 sessions – broken down by treatment, 60 in 6 sessions for A = 0, 54 in 5 sessions for A = 0.1, and 51 in 5 sessions for A = 1. Recruitment took place via an online database (ExLab), which comprises a utility that regularly makes students aware of experiments for which they are eligible. All sessions were fully computerized and involved the same general modus operandi (summarized in Appendix 3.1, Section 3.A).

Upon arrival, the participants received written instructions (Appendix 3.2) and were randomly assigned to computer terminals. Before the start of the experiment, the instructions were read aloud to ensure that all information was common knowledge; any clarifying questions about the setup were answered in private. Owing to the intricacy of the decision scenario, the instructions were framed in terms of a procurement contest: A government official (receiver) was looking to contract out the design and production of a new helicopter fleet for the Maritime and Coastguard Agency to a supplier (signaler), who could either face high (low type) or low (high type) costs. Given the information about his/her cost, each supplier was to produce a prototype of the helicopter (signal) he would deliver to the government official if awarded the contract. The suppliers’ decision variable was the quality of their prototype (signal), which was captured by way of discrete “quality units,” the unit cost of which differed according to their type. The choice of frame was based on its

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3.11 Students were invited for a total of nine timeslots. Upon arrival, if feasible, the participants of a given timeslot were randomly partitioned into two subsets (via the random assignment of seats; see Appendix 3.1 for details). Once established, the subsets within each timeslot were kept completely separate (controlled on different servers), but played the same treatment. “Session” refers to a subset of participants.

3.12 A post-experimental questionnaire indicates that the participants had an average age of 26 years and about 43% were female. Few were formally trained in economics – only 8.5% were economics majors. Around 48% were social scientists, 37% natural scientists, and 15% humanities majors.

3.13 The system allows the experimenter to keep track of the registration, participation, and earnings of individual subjects as well as the registered subject pool as a whole. Upon arrival, the protocol prescribed cross-checking the individuals’ Student IDs with the name and student number recorded by the system. It was thus ensured that no subject participated in more than one session.

3.14 The experiment was conducted using the software tool kit z-Tree (Fischbacher 2007).
transparency, while safeguarding that it was sufficiently abstract in terms of the participants’ likely real-world experience so as not to generate demand effects. To make sure that all participants understood the instructions, which contained a numerical example and a summary table of all payoff-relevant information, each participant had to answer three control questions. The experiment did not proceed until all participants had answered all questions correctly.

The decision phase consisted of 36 rounds of play. The participants moved through each round in groups of three – two signalers and one receiver. Apart from the frame, decision-making corresponded to the timing presented in Section 3.2. No further communication was permitted. The procedure therefore ensured that the participants’ interaction was as close to anonymous as possible. Earnings were recorded in “experimental dollars” (denoted $), which were exchanged into pounds sterling at the commonly known exchange rate of £1 per $100.

The three main challenges from a design point of view were the facilitation of convergence given the complexity of the equilibrium solutions, the preservation of the game’s one-shot nature in each round and, in view of its all-pay contest characteristics, the assurance of positive overall earnings for each participant. To these ends, I applied the following four measures. At the beginning of each round, each participant was randomly (re)assigned a (new) role and randomly (re)assigned to a (new) group; at the end of each round, (s)he received extensive feedback, while payment was according to a lump sum and his/her earnings in only nine randomly selected decision rounds. Each of these design elements was publicly announced to the participants.

Previous work on learning in games with unique mixed-strategy equilibria indicates that subjects tend to struggle to converge to the solution when asked to make (only) one decision per round, even in conjunction with a sizeable number of repetitions to aid learning (e.g., Brown & Rosenthal 1990; Rapoport & Boebel 1992). One way to evade this obstacle would be to implement a variant of the strategy method, which makes the mixed-strategy outcome explicit (Ochs 1995). As this approach undermines the present objective, the design involved a single decision per round. The frame alongside frequent repetition and the said features were deemed

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\[\text{The example and summary table (among a number of other modifications) were introduced to the setup following the evaluation of two pilot sessions comprising 27 participants carried out in May and June 2009.}\]
sufficient to clarify the framework and expedite adjustment by illuminating the thought processes of others.

With this background, repetition was supplemented by role switching. While the effect has not yet been established conclusively (Binmore et al. 1985; Bolton 1991), it is widely believed, that having each participant play all possible roles within a game facilitates learning. The idea is that it helps one gain a better understanding of the scenario as one has to work through the decision problems of all parties to the interaction. The subjects were informed of their role at the beginning of each round and could remind themselves on every screen throughout that round. Although allocation was random, the roles were assigned such that each participant played the role of receiver in \( \frac{1}{3} \) of the rounds. This mode of implementation was chosen to maintain the underlying probabilistic structure of the game. It moreover guarantees that each participant receives a positive payoff in at least \( \frac{1}{3} \) of the rounds and reduces the potential for boredom on the part of the receivers. A second measure to facilitate coordination was the provision of extensive feedback at the end of each round. Each participant was not only shown his/her own payoff for the particular round, but provided with a summary of all decisions within his/her group as well as the distribution of decisions within the population\(^ {16} \) of signalers (broken down by type) within the given round, including whether or not a signaler was awarded a contract.\(^ {17} \)

Although, in principle, the random allocation of the participants’ roles in each round should be sufficient to create a single-period environment, the measure was supplemented with the participants’ random assignment to new groups of three at the outset of each round. This design feature has the added benefit of preventing the emergence of concerns for the welfare of other group members. For it has, both theoretically and experimentally, been shown (e.g., Kreps et al. 1982; Camerer 2003; Duffy & Ochs 2009) that repeated play within a particular group generates reputational concerns and possible team sentiments (e.g., social norms), both of which dilute the mechanism underlying the costly signaling angle to explaining generous behavior, which is based on pure self-interest.

The all-pay contest structure of the interaction in the treatments involving beneficent signals is mainly a concern if an individual continually makes irrational

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\(^{16}\) Given the partitioned setup within each timeslot, “population of signalers” refers to the signalers in a particular subset of participants.

\(^{17}\) This is another divergence from Noussair and Silver (2006), who provide their participants with the full history of decisions.
decisions. Otherwise, role switching guarantees positive overall earnings, as $\frac{2}{3}$ of the payoff-relevant rounds, in expectation, yield a non-negative payoff. To allow for irrational choices (as well as bad luck) in some of the rounds, each participant was endowed with an initial balance equivalent to the prize for procuring one contract ($500) and final payment was based on a random-round payment mechanism. In particular, for each participant, the payment mechanism randomly selected three individual receiver rounds, three rounds in which (s)he was a signaler of high type, and three rounds in which (s)he was a signaler of low type. At the end of a session, the participants were paid the sum of their initial endowment and the earnings in the nine randomly selected decision rounds. Since the participants did not know ex ante which rounds would be selected for payment, the mechanism also diminished any possible income effects. Despite these provisions, there remains a small, strictly positive residual probability that an individual ends up with an overall loss or near-zero payoff (taken to be a payoff of less than £3). To account for these possibilities, it was decided that the relevant participant(s) be compensated with a “show-up” fee of £3. This commonly known fail-safe was binding in a total of 14 cases (8.5%).

The experimental sessions lasted between 75 and 90 minutes, door to door. The participants earned an average of £13.57, with a minimum of £3.00 and a maximum of £31.00, for their involvement. Payment occurred individually and in cash in a separate room after the completion of a brief questionnaire. The subjects exited the laboratory separately immediately after payment. For the purposes of statistical analyses, a session constitutes a statistically independent observation, which entails a total sample size of 16 data points (with the aforesaid breakdown by treatment).

### 3.4 Experimental Results

Table 3.1 provides an overview of the signalers’ aggregate behavior. For each treatment, broken down by signaler type, it presents the mean, median, mode,
standard deviation, and range of signaling choices. For reference, the predicted means, medians, and modes for the treatment with neutral signals \((A = 0)\), presuming one of the representative separating outcomes is attained, are 0 for signalers of low type and a point-prediction between 10 and 50 for those of high type; for the treatments with beneficent signals \((A > 0)\), the respective predictions are 5 and 35.

**Table 3.1.** Descriptive Statistics on Signaling Activity, decomposed by Treatment and Signaler Type

<table>
<thead>
<tr>
<th>Statistical Measures</th>
<th>(A = 0)</th>
<th>(A = 1)</th>
<th>(A = 0.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Type</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>6.934</td>
<td>6.859</td>
<td>6.042</td>
</tr>
<tr>
<td>Median</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Mode</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>16.012</td>
<td>9.826</td>
<td>11.599</td>
</tr>
<tr>
<td>Range</td>
<td>125</td>
<td>78</td>
<td>125</td>
</tr>
<tr>
<td>High Type</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>20.288</td>
<td>32.358</td>
<td>37.211</td>
</tr>
<tr>
<td>Median</td>
<td>15</td>
<td>29</td>
<td>31</td>
</tr>
<tr>
<td>Mode</td>
<td>20</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>21.089</td>
<td>18.784</td>
<td>24.809</td>
</tr>
<tr>
<td>Range</td>
<td>125</td>
<td>100</td>
<td>125</td>
</tr>
</tbody>
</table>

One of the most obvious features of the data is that the signalers choose to employ the signals at their disposal actively and variably. In fact, in all treatments, the signaler types separate in the sense that signalers of high type generally choose signals of higher magnitude than those of low type. This not only suggests that (most of) the participants seem to have grasped the decision scenario, but also that they put some thought into their choices when of a particular signaler type. At the same time, however, a non-negligible number of participants (across treatments and signaler types) make apparently irrational signaling choices; given the parameterization of the game, signaling levels beyond 20 for low types and 100 for high types yield a negative payoff regardless of the choices of the other players within a particular group and are therefore dominated by zero. As discussed in more detail below, while some of these choices occurred in early rounds and might thus be considered “practice,” their incidence in later rounds cannot be reconciled with rational (and if not that, at least observant) thought about the choices’ necessary implications. A straightforward consequence is that the means will be skewed towards higher signal intensities. Hence, to arrive at a reliable depiction of the signalers’ behavior, it is important also to consider the median signaling levels. Note, in this context, that the

\[20\]

Figure 3.A.1.i. (Appendix 3.3) provides an alternative depiction of the distribution of signaling choices within each of the treatments.
medians and modes are integer values, which indicates that many signalers (though certainly not all; cf. Footnote 3.25) choose whole numbers as their signaling magnitudes. The relevant values furthermore suggest that the restriction of the choice set was not binding to most participants.

Table 3.2. Selection of the Signaler Sending the Lower Signal by the Receivers within each Treatment

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Choice of Low(er) Signal</th>
<th>Total Number of Decisions</th>
<th>% Choice of Low(er) Signal</th>
<th>Number of Tied Signals</th>
<th>% Choice excluding Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 0</td>
<td>121</td>
<td>720</td>
<td>16.81%</td>
<td>51</td>
<td>18.09%</td>
</tr>
<tr>
<td>A = 1</td>
<td>37</td>
<td>612</td>
<td>6.05%</td>
<td>33</td>
<td>6.39%</td>
</tr>
<tr>
<td>A = 0.1</td>
<td>72</td>
<td>648</td>
<td>11.11%</td>
<td>19</td>
<td>11.45%</td>
</tr>
</tbody>
</table>

Legend: The values in this table are based on group-level data from all 36 rounds of play. In the second column, the choice between tied signals, i.e., the case when both signalers within a particular group choose the same signaling level, is counted as the receiver choosing the signaller having sent the higher signal as her ally. The fifth column explicitly counts out the number of ties; the sixth column, correspondingly, represents the percentage of receivers choosing the signaler who sent the lower signal when ties are not included in the total decision count (given in the third column).

Table 3.2 presents an analogous descriptive account of the receivers’ aggregate behavior by supplying the raw and relative number of chosen signalers who sent the lower signal within a given group, as well as information on tied signaling choices. Gauged by the observation that more than 80% (when excluding ties) of the receivers choose the signaler who sent the higher signal as their interaction partner, they too make inferences consistent with careful thought about the decision scenario. In particular, the data suggest that— in line with the respective predictions—the receivers in the treatment with neutral signals are somewhat more likely to choose the signaler sending the lower signal, while their counterparts in the treatments with beneficent signals appear to take full advantage of the signal-dependent benefit.

Hence, the aggregate data imply that the participants not only comprehend the decision scenario, but actively account for the information conveyed by the signaling activities at their disposal when making their decisions (across roles and signaler types). Not unexpectedly, especially on the part of the signalers, behavior in all treatments is quite variable. This is not only in line with the literature, but may be indicative of the attainment of the predicted equilibrium profiles. Recall that two prescribe random choice within the bounds of specified intervals \((A > 0)\), while the
other allows for a plethora of possible outcomes \((A = 0)\), in which case notable variability is not surprising either.

The remainder of this section is organized as follows. Section 3.4.1 describes in detail the behavior observed in the treatment with neutral signals \((A = 0)\), followed by a similarly thorough survey for the treatments with beneficent signals \((A > 0)\) in Section 3.4.2. Section 3.4.3 examines the changes and adjustments in behavior “over time” by comparing and contrasting the outcomes observed in the first 18 to those in the last 18 and last 9 rounds of play, at which stage one would expect the participants to have gathered experience with the game. Section 3.4.4, in turn, is devoted to a comparison of the observed behavioral outcomes across the three treatments. Each subsection starts out with a description of the signalers’ behavior followed by an account of the receivers’ conduct.

### 3.4.1 Neutral Signals: Treatment with Parameter \(A = 0\)

**Signalers.** Bearing in mind the somewhat vague equilibrium prediction for this treatment, consider Figure 3.1, which graphically summarizes the key features of the signalers’ behavior in this treatment (for all sessions combined), starting with the round-by-round average signaling levels plus/minus one standard deviation in the top panel. The outcome displays a high degree of separation (one-sided Wilcoxon Signed-Ranks Test: \(n = 6, T^+ = 21, p = 0.016\)): The mean for signalers of low type oscillates around 5 and that of high types around 20, although the standard deviations in each round are substantial and continually overlap. The scatter plots in the second and third panel bear witness to the root cause for the observed deviations, highlighting the notable number of (extreme) outliers in virtually all rounds of play. Low types in particular repeatedly send signals of much higher magnitude than can rationally be explained, choosing up to 125 units. Nevertheless, during the course of the game, many of the participants appear to discover that high signals are inevitably unprofitable, correcting their choices downward (to choices below 20 for low types and 40 for high types). The upshot is that the “linear trend” – the simple linear regression of the raw signaling magnitudes on the rounds – for both signaler types in this treatment is slightly negative. The histograms in the bottom panels provide insight into the raw and cumulative frequency distributions of signaling choices for each type of signaler. Although both frequency distributions are uni-modal, the parti-
Figure 3.1. Parameter A = 0 — Graphical Overview of the Signalers’ Observed Behavior

Scatter Plots of the Observed Signaling Activity and Linear Trends

Histograms of the Observed Signaling Behavior

Legend: The class labels of the histograms denote the upper bounds of the given intervals, with all intervals except for “0” — which captures the number of signalers sending a signal magnitude of exactly zero — open at the lower end. “5,” for example, denotes the interval (0, 5].
cipants’ behavior unmistakably diverges from the representative range of separating outcomes in that their choices are quite a bit more dispersed. Hence, the data at the aggregate level are consistent with a separating outcome, yet not one according to which the types coordinate on particular (separate) signaling levels.

In order to obtain further insight into the participants’ behavior, it is at this stage informative to turn to the behavior observed in the individual sessions. To this end, Figure 3.A.2 (Appendix 3.3) presents the session-level round-by-round average signaling levels and standard deviations. At this level of (dis)aggregation, behavior can be sorted into distinct classes. Two sessions swiftly (by about Round 10) approach the least-cost separating (LCS) outcome (Sessions 1 and 2), two sessions tend towards a separating outcome quite similar to that predicted for the treatments with beneficent signals (Sessions 3 and 4), and two display no clear convergence to any kind of equilibrium (Sessions 5 and 6). In short, at the level of the individual sessions, in agreement with the theoretical prediction, the data are consistent with there existing multiple equilibria. This observation is plainly also in keeping with the substantial variability observed when aggregating the data.

Table 3.A.1 (Appendix 3.4) presents descriptive statistics on signaling behavior for each individual session. Consider each class of outcomes in turn, taking as given appropriate behavior on the part of the receivers. The means and medians in Sessions 1 and 2 are unmistakably in line with the LCS outcome. Indeed, in due course, they settle at values in very close proximity to the predicted levels. Moreover, contrary to the other sessions in this treatment, the standard deviations (across signaler types) are very low, which is clearly indicative of the signalers, within type, sending the same signals. The remarkably quick approximate convergence to this outcome suggests that, once the required depth of reasoning to establish the strategy configuration has been achieved, maintaining it is essentially straightforward. Yet, given the apparent lack of convergence in Sessions 5 and 6, attaining the relevant level of understanding (and conveying it to other participants) is not necessarily simple.

The approximate outcome for Sessions 3 (tentatively) and 4 is not quite as close to a theoretical equilibrium as Sessions 1 and 2, yet one can easily make out a tendency towards the separating outcome predicted for the treatments with beneficent signals. While not as obvious a solution as for beneficent signals, the outcome can – in principle – also arise with neutral signals in use. The prerequisite for this result is that the receivers’ behavior is consistent with them recurrently
choosing the signaler having sent the higher signal as their allies. Even with neutral signals, this rule gives rise to competitive pressures along the lines of an all-pay auction. Accordingly, if so, the observed signaling behavior is a completely reasonable best response to the receivers’ conduct.

The lack of convergence in Sessions 5 and 6 underlines the complexity of the setup and behavioral implications of the equilibrium predictions. Although Session 5 ultimately appears to converge to a separating outcome, which in time may (or may not) have approached the outcome predicted for beneficent signals, the continual overlap in choices observed in Session 6 cannot be reconciled with any kind of equilibrium, as overlapping supports are not sustainable given the cost structure of the game. Hence, in sum, the signaling choices observed in the individual sessions within this treatment are consistent with at least two distinct equilibrium outcomes, though the apparent lack of convergence in two sessions highlights the fundamental difficulty of coordinating on an outcome.

Receivers. In view of the central role of the receivers’ strategy in the ultimate configuration of the equilibrium in this treatment (predicted and otherwise), it is useful to spend some time trying to establish what the participants ended up doing when assigned this role. To this end, Figure 3.2 illustrates (for all sessions combined) the receivers’ choice of the signaler sending the high(er) or low(er) signal, having divided the signalers’ choices within any given group into three classes: both signaling less than 10, both signaling more than 10, and one signaling less and the other more than 10. Plainly, in each signal category, the receivers tend to choose the higher of the two signals they observe in any given group. In other words, in keeping with the aggregate picture (Table 3.2), the receivers’ behavior in this treatment is consistent with a decision rule of always choosing the signaler sending the higher signal. Figure 3.A.3 (Appendix 3.3), which depicts the receivers’ choice of signaler from a session-by-session perspective, further underpins this conjecture.

In view of this discussion, the signalers’ behavior in Sessions 3 and 4 is certainly not unreasonable. The figures moreover suggest that, in line with the relevant predictions, the receivers in Sessions 1 and 2 are slightly less likely to choose the signalers having sent the higher signal – particularly if both signals fall below 10 – than those in the session(s) approaching the outcome for signaling using beneficent signals (particularly Session 4). While the sample is too small for a definitive statement in this context, to get a better sense of the distribution of choices
Figure 3.2. Parameter $A = 0$ – Choice of Winning Signal(er) by Signal Category (Relative to the Total Number of Decisions within a Signal Category, excluding Ties), Aggregate Perspective

Legend: This figure is based on group-level data from all 36 rounds of play, namely, the percentage of choices in favor of the high(er) or low(er) signal (excluding tied signaling choices) relative to the total number of decisions within a particular signal category; correspondingly, the height of the two bars within each signal category sums to 1 (or, equivalently, the height of all bars in the figure sums to 3). In this particular instance, “high” and “low” refer to the receiver’s choice of the signaler having sent the high(er) or low(er) signal within a given decision scenario as his/her ally, not signaler type. $x$ and $y$, in turn, represent the signalers’ respective choices within a given group.

In favor of the lower signals, Figures 3.A.4 and 3.A.5 (Appendix 3.3) zoom in on the relevant decisions; for the moment, focus only on those relating to the present treatment (and set of rounds). They yield two main messages. For one, the propensity to decide in favor of the lower signal is higher for (very) small absolute differences between the observed signal intensities, but is effectively independent thereof for differences beyond five or so units. In other words, rather than occurring “erroneously,” choices in favor of the lower signal tend to be genuine decisions for the relevant signalers, conceivably because the difference between the signals is perceived not to be large enough. Focusing on signals across the projected belief threshold of 10, one finds that the receivers in Sessions 1 and 2 are, indeed, somewhat more likely to choose the signaler sending the lower signal than those in Sessions 3 and 4, yet much less so than the receivers in Sessions 5 and 6, which may explain the indistinct outcomes in these sessions.

In sum, the receivers’ behavior is broadly consistent with the signalers’ behavior in each of the individual sessions. While most receivers in this treatment appear to adopt the same decision rule as their counterparts in the treatments with beneficent signals, those in the sessions approaching the LCS outcome tend to be
slightly less likely to choose the signalers sending the higher signal as their allies. Nevertheless, overall, the decisions in favor of the lower signal tend to be largely independent of the difference in magnitude between the signals.

**Result 3.1** [Hypothesis 1] In line with expectation, the participants’ behavior in the treatment with neutral signals, at the aggregate, does not converge to a definite outcome, although the signaler types display a high degree of separation. Rather, behavior in the individual sessions is consistent with three distinct sets of outcomes – two sessions tend towards the LCS outcome, two towards the outcome predicted for the treatment with beneficent signals, and two fail to converge to a coordinated outcome.

### 3.4.2 Beneficent Signals: Treatments with Parameter $A > 0$

Whereas the participants’ behavior in the treatment with neutral signals appears to converge to outcomes approximating equilibria in some of the individual sessions, this section describes behavior that, in large parts, is in very close proximity to the theoretical predictions. This finding is remarkable, as the structure of the unique projected outcome for signaling using beneficent signals is behaviorally quite intricate. It involves the signalers randomizing uniformly on separate but contiguous interval supports, while the receiver adopts non-decreasing (namely, threshold) beliefs, which “in practice” comes down to her always choosing the signaler having sent the higher signal as her ally.

#### 3.4.2.1 Treatment with $A = 1$

**Signalers.** With this background, consider first the treatment comprising the higher benefit, i.e., $A = 1$, for one might expect the participants’ behavior to be driven more readily by a large than a small benefit, which would suggest more clear-cut results in this case. Figure 3.3 provides the corresponding overview of the main features of the signalers’ behavior. Again, start with the round-by-round average signaling levels plus/minus one standard deviation presented in the top panel.

In line with the predictions, the observed outcome is unmistakably separating (one-sided Wilcoxon Signed-Ranks Test: $n = 5$, $T^+ = 15$, $p = 0.031$), with the projected means of 5 and 35 being approached swiftly from above in the case of signalers of low type and below by those of high type, respectively. Once more, the standard deviations for both signaler types are non-negligible and especially in the
**Figure 3.3.** Parameter $A = 1$ – Graphical Overview of the Signalers’ Observed and Predicted Behavior

**Predicted and Observed Round-by-Round Average Signaling Levels and Standard Deviations**

*Scatter Plots of the Observed Signaling Activity and Linear Trends*

- **Low Type**
  - Observed Mean
  - Predicted Mean

- **High Type**
  - Observed Mean
  - Predicted Mean

**Histograms of the Observed Signaling Behavior and Predictions**

- **Low Type**
  - Frequency
  - Cumulative Frequency Distribution (%)
  - Predicted Cumulative Frequency Distribution (%)

- **High Type**
  - Frequency
  - Cumulative Frequency Distribution (%)
  - Predicted Cumulative Frequency Distribution (%)

**Legend:** The class labels of the histograms in this figure correspond to those in Figure 3.1.
first half of the game tend to overlap; in the latter part of play, most distinctly on the
part of signalers of low type, the size of the deviations declines and they cease to
intersect. The scatter plots in the second and third panel of the figure underline this
account by illustrating that this treatment comprises only few outliers. Except for one
signal at 100 by a signaler of high type in Round 5, signalers of this type tend to
choose signaling magnitudes below 90 and, at least in the second half of the game,
only a handful of signalers of low type signal more than 20. Particularly the former
contrasts sharply with both of the other treatments. Note furthermore that, in line
with the foregoing observations, the linear trend for signalers of low type tends
downward while that for high types tends upward. Hence, at the aggregate level,
signaling behavior appears to correspond quite well to the theory. In the individual
sessions, too, the participants’ conduct broadly concurs with the predictions, though
not surprisingly it is more variable than what is depicted in Figure 3.3.

The raw and cumulative frequency distributions presented in the histograms in
the bottom panels reveal several departures of the observed outcomes from the
predictions. Yet, although the (raw) frequencies of neither type of signaler can be
considered uniform with any degree of conviction, the majority of choices
unmistakably falls in the anticipated neighborhoods. The cumulative distributions,
too, are in close proximity to the projected shapes, despite signalers of low type
tending to overbid slightly towards the upper end of their support and those of high
type tending to underbid somewhat, particularly in the intermediate segment of their
support (between about 20 and 40).

Table 3.3 numerically corroborates both of the foregoing observations.
Although more than 80% of both types of signaler choose signaling magnitudes
within the projected supports, the right tails of the distributions are too thin for the
outcomes to be classified as uniform randomization across the given intervals.
Formal statistics draw an even more sharply delineated picture. They back up
randomness of choice for signalers of high type, but not so for low types (Runs Test
for Randomness; Low Type: 280 runs, \( z = -2.488, p = 0.013 \); High Type: 283 runs,
\( z = -0.523, p = 0.601 \)).\(^{21}\) Nor do appropriately fitted Kolmogorov-Smirnov Tests

\(^{21}\) The non-parametric One-Sample Runs Test of Randomness tests whether the order or sequence of
a sample of observations is random (relative to some threshold value; Siegel & Castellan 1988). The
test was carried out using the raw signaling levels from all sessions, distinguished by type of signaler,
within a particular treatment. The threshold was taken to be the predicted mean. The interpretation of
allow for the conclusion that the observed cumulative distribution of signaling levels
for either type of signaler is consistent with a uniform distribution (Low Type: 
\( n = 520, |d| = 0.287, p < 0.001 \); High Type: \( n = 492, |d| = 0.189, p < 0.001 \)).

Table 3.3. Parameter \( A = 1 \) – Observed Frequencies and Mixed-Strategy
Equilibrium (MSE) Probabilities for the Predicted Equilibrium Supports,
decomposed by Signaler Type

<table>
<thead>
<tr>
<th>Type &amp; Classes</th>
<th>Low Type</th>
<th>High Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed Frequency</td>
<td>0.62</td>
<td>0.22</td>
</tr>
<tr>
<td>MSE Probability</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Legend: The class labels denote the upper bounds of the given intervals. All intervals except for the
first one are open at the lower end. Note that for signalers of high type in the treatments with \( A > 0 \),
“20” denotes the interval [10, 20].

In sum, the behavior of both types of signaler (taking as given the receivers’
behavior) is mostly in line with the equilibrium predictions, and quite notably so. In
case of the more subtle behavioral implications, however, one can make out
unmistakable points of divergence. Nonetheless, bearing in mind that each
participant only made one decision per round, even these aspects do not diverge
dramatically from the equilibrium path.

Receivers. In contrast to the scenario with neutral signals, the receivers’
equilibrium strategy in this treatment is pinned down unambiguously – (always)
choose the high(er) signal. This is indeed what happens (cf. Table 3.2): Close to 95%
of the receivers choose the signaler having sent the higher signal in any given group.
It therefore seems reasonable to infer that the receivers’ behavior in this treatment,
too, is consistent with the signalers’ behavior.

Paralleling the treatment with neutral signals, the receivers’ propensity to
choose the lower signal in the few cases not in line with the equilibrium strategy is

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3 The Kolmogorov-Smirnov One-Sample Test is a non-parametric test of the goodness-of-fit of an
observed cumulative (relative) frequency distribution (measured on at least an ordinal scale) and some
specified theoretical distribution (Siegel & Castellan 1988). The assessment is based on the point of
greatest divergence between the two distributions. The present analysis was carried out using the raw
signaling levels from all sessions between the indicated minimum and maximum levels (i.e., the test
was carried out using a restricted dataset, the size of which is denoted by \( n \) in the text). The results
when using the unrestricted dataset do not differ qualitatively. The interpretation of the results is that a
significant outcome indicates that the signaling levels are not distributed uniformly between the
relevant bounds.
higher for (very) small absolute differences between the observed signaling magnitudes, but is essentially independent thereof for differences beyond three or so units (Figure 3.A.4, Appendix 3.3). Moreover, in this treatment less than 5% of the relevant receivers choose the lower signal when deciding between a signal falling below 10 and the other above 10 (Figure 3.A.5, Appendix 3.3). These findings suggest that once settled on the appropriate strategy configuration, the signal-dependent benefit does not play a significant role.

**Result 3.2 (a)**  
[Hypothesis 2, A = 1] In their essence, the behavioral outcomes (for both roles) in the treatment with A = 1 provide support for the theory. The signalers generally separate, with signalers of low types sending lower signals than signalers of high type, and the observed frequency distributions conform reasonably well to the projected outcome. The receivers, too, consistently choose the signaler having sent the higher signal.

### 3.4.2.2 Treatment with A = 0.1

In view of the results when the signal-dependent benefit is comparatively large, one might be curious whether the observed proximity to the theory is replicated when the signal-dependent benefit is substantially smaller, namely, 10% of the original level. In other words, in this case, the receiver collects only 10% – as opposed to the full level – of the signaling magnitude sent by the signaler she chooses as her ally as additional payoff beyond the intrinsic value of the chosen signaler. Recall that the theoretical prediction does not vary for the two values of the signal-dependent benefit. The obvious alternative would be the existence of a benefit “threshold,” entailing that the outcome in this case would tend to be more similar to the one in the treatment with no benefit to begin with. Once more, turn first to the participants’ behavior when assigned the role of signaler.

**Signalers.** Figure 3.4 depicts the essential ingredients to surveying the signaling activity in this treatment. In congruence with the theoretical projection, the round-by-round average signaling levels plus/minus one standard deviation depicted in the top panel reveal that the signaler types undeniably separate (one-sided Wilcoxon Signed-Ranks Test: \( n = 5, T^+ = 15, p = 0.031 \)). Moreover, both types appear to arrive quickly at signaling levels in close proximity to their respective predicted means (5 for low types and 35 for high types). In fact, towards the end of the game, signalers of low type tend to send signals strictly below their projected
Figure 3.4. Parameter A = 0.1 – Graphical Overview of the Signalers’ Observed and Predicted Behavior

Predicted and Observed Round-by-Round Average Signaling Levels and Standard Deviations

Scatter Plots of the Observed Signaling Activity and Linear Trends

Histograms of the Observed Signaling Behavior and Predictions

Legend: The class labels of the histograms in this figure correspond to those in Figure 3.1.
mean, with very little variability. The standard deviations for signalers of high type, in contrast, tend to be quite sizeable – more along the lines of those observed in the treatment with no signal-dependent benefit. Even so, in the second half of the game, one observes only few overlaps.

In contrast to its companion treatment, the scatter plots in the second and third panel indicate that a considerable number of signalers choose exceedingly high signal intensities, recurrently up to 125 units. In this case, however, nearly all of the signals one would classify as (extreme) outliers are sent by signalers of high type (as opposed to low types in the setting with neutral signals). Most low types tend to choose signaling levels below 15 and, in the second half of the game, below 10. Nonetheless, note that when ignoring signals beyond 80 by signalers of high type, the scatter plots look very similar to those when $A = 1$, also with regard to the linear trends, although the high types’ slope in this treatment is somewhat steeper. Hence, at the aggregate level, behavior again appears to correspond quite closely to that predicted by the theory. In the individual sessions, too, behavior tends to accord well with the projections. Indeed, two of the sessions (Sessions 2 and 5) turn out to be just about exactly “on target;” the outcome for signalers of high type in Session 1, on the other hand, differs notably from the predicted strategy configuration.

The histograms in the bottom panels place the comparatively extensive variability in signaling choices displayed by signalers of high type into perspective. For, remarkably, especially when compared to the corresponding graph in the companion treatment, the observed and predicted cumulative frequency distributions for this type of player hardly differ. Moreover, the (raw) frequencies in this treatment look quite a bit more uniform than the uni-modal appearance of its counterpart when $A = 1$. The behavior of signalers of low type is (again) also quite similar to the prediction. In other words, while one could easily detect over- (low types) and under-signaling (high types) in the treatment with $A = 1$, such discrepancies are much less evident in this case. The figures presented in Table 3.4 further back up these observations. Even though certainly not faultless, close to 85% of the signaling choices fall into the predicted supports. What is more, the probabilities for signalers

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23 Figure 3.A.6 (Appendix 3.3) provides descriptive statistics for Session 2. While not perfect, behavior in this treatment is very close to that predicted by the theory, especially on the part of signalers of high type.
of high type, including the tails, are in a much closer neighborhood of the projected probabilities than those in the companion treatment.

Table 3.4. Parameter $A = 0.1$ – Observed Frequencies and Mixed-Strategy Equilibrium (MSE) Probabilities for the Predicted Equilibrium Supports, decomposed by Signaler Type

<table>
<thead>
<tr>
<th>Type &amp; Classes</th>
<th>Low Type</th>
<th>High Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed Frequency</td>
<td>0.67</td>
<td>0.18</td>
</tr>
<tr>
<td>MSE Probability</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Legend: The class labels denote the upper bounds of the given intervals. All intervals except for the first one are open at the lower end. Note that for signalers of high type in the treatments with $A > 0$, “20” denotes the interval $[10, 20]$.

Do the statistics corroborate these impressions relating to the more subtle elements of the theory? Yes and no. The Runs Tests for Randomness of the chosen signaling levels indicate that the sequence of signaling choices of both signaler types can indeed be deemed random (Low Type: 284 runs, $z = -1.364$, $p = 0.173$; High Type: 313 runs, $z = -1.466$, $p = 0.143$). Yet, once again, the respective Kolmogorov-Smirnov tests indicate that the observed frequency distributions of signal choices cannot be considered uniform at any conventional level of significance for either type of signaler (Low Type: $n = 535$, $|d| = 0.346$, $p < 0.001$; High Type: $n = 555$, $|d| = 0.133$, $p < 0.001$).

In sum, large parts of the signalers’ behavior in this treatment (taking appropriate behavior on the part of the receivers as given) appears to coincide even more closely with the theoretical predictions for beneficent signaling than that observed in the companion treatment. Indeed, in two of the individual sessions, the equilibrium distributions are obtained almost to the point. Curiously, these findings suggest that the pressure to exploit the strategic opportunities inherent in the setup may be inversely related to the size of the signal-dependent benefit.

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24 A visual inspection of the individual sessions in the treatments with beneficent signals reveals that both comprise one session that diverges notably from the predicted outcome (in both cases on the part of signalers of high type), and both comprise sessions in very close proximity to the theory. The latter include the behavior of signalers of low type, which – at the aggregate level – appears to deviate to some extent from the prediction. The comparative statement made here is based on the observation that two sessions in the present treatment are almost perfectly in line with the prediction and the others seem to exhibit less divergence from the projections than those in the companion treatment.
Receivers. Representing a scenario with beneficent signals, the receivers’ unique optimal strategy in equilibrium is also (always) to choose the high(er) signal. As in the companion treatment, this is precisely what happens (cf. Table 3.2): Around 90% of the receivers choose the signaler having sent the higher signal as their allies. Interestingly, this level is slightly lower than in the treatment with $A = 1$; in fact, it comes in almost exactly between the other two treatments. Even so, the receivers’ behavior in this treatment appears to be very much consistent with the signalers’ behavior. As in the other treatments, the propensity to choose the lower signal in the instances receivers do select this strategy is higher for small absolute differences between the observed signaling magnitudes, but effectively independent thereof for differences beyond around four units (Figure 3.A.4, Appendix 3.3). In the present case, however, about 9% of the receivers choose the lower signal if one observed signal is less and the other more than 10, about twice as many as in the companion treatment (Figure 3.A.5, Appendix 3.3).

Result 3.2 (b) [Hypothesis 2, $A = 0.1$] Most attributes of the behavior observed in the treatment with $A = 0.1$ are closely in line with the theoretical predictions – perhaps more so than in the companion treatment with beneficent signals. Even though, at times, the outcome appears to be virtually in-between the other treatments, the present behavior differs distinctly from that with neutral signals, at the aggregate as well as within the individual sessions.

3.4.3 Changes in Behavior “over Time”

On account of the complexity of the framework and the predicted behavioral outcomes, it is to be expected that the participants require some time to work out their optimal course of action and learn the behavior of others. As such, the first half of the game may involve a variety of “practice” choices to become familiar with their likely consequences given the behavior of the other participants. In other words, experience with the game is likely to have a considerable impact on the participants’ conduct. To examine this, this section compares and contrasts the behavior observed in the first 18 (Rounds 1 – 18) rounds of play to that in the last 18 (Rounds 19 – 36) and last 9 (Rounds 28 – 36) rounds. Besides presenting descriptive statistics for the selected subsets of rounds, the focus will be on the scale and direction of change. A final theme will be a brief discussion of the changes in significances for the statistical tests reported for all 36 rounds of play (if any).
Signalers. Whilst the linear trends in the various scatter plots are indicative, the data in each treatment comprise outliers. To elucidate the resulting skew of the average signaling level, Figure 3.5 presents the mean and median signaling levels for each (sub)set of rounds (including, for reference, all rounds of play). In conjunction with Figures 3.A.1.i through iv (Appendix 3.3) and Table 3.A.2 (Appendix 3.4), which replicate the aforementioned descriptive statistics and box-plots for the first and last 18, as well as the last 9 rounds of play, there is considerable support for the importance of experience in shaping the observed outcomes. Not only do the means tend in the expected directions and are the standard deviations and ranges in most cases reduced, but so is the occurrence of (extreme) outliers. Indeed, in the last 18 rounds of play – as projected – signalers of high type in the treatment with \( A = 0.1 \) always send strictly positive signals. Still, even though many participants do seem to acquire a working understanding of the game, some make irrational choices through to the very end (cf. Figures 3.1, 3.3, and 3.4).

Figure 3.5. Graphical Comparison of the Mean and Median Signaling Levels “over Time,” decomposed by Treatment and Signaler Type

A statistical comparison of signaler behavior across the various subsets of rounds using the session-level mean signaling levels (Table 3.A.3, Appendix 3.4) indicates that the most notable adjustments “over time” take place in the treatment with \( A = 1 \) \((p \leq 0.063)\). The main message of the relevant results is two-fold. For one, the behavior in the treatments with beneficent signals in particular converges to
levels very close to the theoretical means. Second, even though several of the adjustments are statistically significant, most participants appear to settle on levels close to those predicted within the first half of the game. Observe in this regard also that the incidence of tied signaling choices, which may be considered to capture the participants’ grasp of the strategic elements of the game, tends to decline over time in all treatments (from less than 8% to less than 5%), although the adjustments are not statistically significant (Table 3.A.4, Appendix 3.4). While these changes suggest that the understanding of the participants is sharpened with experience, most appear to have a good grasp of the game and the significance of choosing (slightly) different signaling magnitudes than other signalers from the outset.\textsuperscript{25}

Table 3.5. Percentage of Signals within the Predicted Equilibrium Supports “over Time,” decomposed by Treatment and Signaler Type

<table>
<thead>
<tr>
<th>(Sub)Set of Rounds</th>
<th>( A = 0 )</th>
<th>( A = 1 )</th>
<th>( A = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Type</td>
<td>High Type</td>
<td>Low Type</td>
</tr>
<tr>
<td>All Rounds</td>
<td>12.31</td>
<td>8.14</td>
<td>83.74</td>
</tr>
<tr>
<td>First 18 Rounds</td>
<td>11.94</td>
<td>5.83</td>
<td>76.97</td>
</tr>
<tr>
<td>Last 18 Rounds</td>
<td>12.68</td>
<td>10.41</td>
<td>90.22</td>
</tr>
<tr>
<td>Last 9 Rounds</td>
<td>12.21</td>
<td>10.64</td>
<td>91.30</td>
</tr>
</tbody>
</table>

Legend: For the treatment with \( A = 0 \), the percentage of signals at 0 and 10 (LCS outcome) is given. Recall that the supports for the treatments with \( A > 0 \) are predicted to be on \([0, 10]\) for low types and \([10, 60]\) for high types.

Does progressive experience with the setup and the conduct of the other participants translate into the aggregate behavior’s proximity to the more subtle predictions of the theory on signaling using beneficent signals, i.e., uniform randomization across well-defined interval supports? Table 3.5 presents the percentage of signals falling within the projected equilibrium supports and/or (exact) “pooling” levels (LCS outcome). In keeping with the discussion so far, the behavior in the treatment with neutral signals, when aggregated, in not consistent with the LCS prediction (nor any of the other indicated separating outcomes), and does not move in its direction “over time” either. Within the treatments with beneficent signals, on the other hand, more than 80% of the signalers (overall) – and in the case

\textsuperscript{25} An interesting observation in the context of strategic signaling choices is that carefully calibrated decimal configurations tend to be more important for lower signaling magnitudes (i.e., below 40) and close to focal points such as 0, 20, and around 35. Competition with higher signal intensities, in turn, tends to take place at integer level.
of low types, starting in the second half of the game, more than 90% – send signals in their respective equilibrium intervals. For low types, the percentage is strictly increasing over time, as is the levels for high types in the treatment with \( A = 1 \); in the companion treatment with beneficent signals, the percentage for high types is roughly constant around 83%. A uniform distribution along the intervals would require all observations to fall within the given bounds. While this is not the case, the high values do suggest that the observed bounds are not far off the mark.

For inferences regarding the nature of the observed strategy, Table 3.A.5 (Appendix 3.4) provides the observed and predicted frequencies for a mixed strategy along the projected intervals for the first 18, last 18, and 9 rounds of play. Paralleling the findings for all rounds, for signalers of low type and in the treatment with \( A = 1 \) also for high types (although one can make out some adjustment in the appropriate direction over time), the observed frequencies tend to be skewed towards the lower ends of the intervals with fairly thin right tails. When \( A = 0.1 \), however, signalers of high type achieve distributions in a remarkably close neighborhood of the predicted mixing probabilities virtually from the outset of the game (bearing in mind that the participants only make one decision per round). Nevertheless, statistically speaking, none of the distributions can be considered uniform.

In fact, most of the statistical results reported for all rounds of play are not qualitatively changed when zooming in on the subsets of rounds. As may be expected when inspecting the vertical differences in Figure 3.5, the separation results are both qualitatively and quantitatively unchanged. The only significant result in the context of the tests for randomness of the signaling choices in the treatments with beneficent signals is the sequence by signalers of low type in the last 18 rounds when \( A = 1 \) \((p = 0.009)\). All other choice sequences can statistically be considered random. Hence, while the descriptive statistics suggest that the participants gradually tend to choose more similar signaling magnitudes, the distributions as a whole appear to be sufficiently dispersed to be approximately in line with the equilibrium predictions.

In sum, as the participants progressively gain experience with the setup and its behavioral implications, they appear to become more strategic in their signaling choices. Particularly in the treatments with beneficent signals, behavior appears to converge to levels in close proximity to the theoretical means, although most adjustments in the signaling levels in this regard occur in the treatment with the large signal-dependent benefit. What is more, in the last half and quarter of play, the vast
majority of signals falls within the projected bounds, with the observed frequencies not far from their predicted levels.

**Receivers.** Some of the most substantial adjustments over time occur on the part of the receivers (Figure 3.6 and Table 3.A.6, Appendix 3.4). Especially the treatments with $A = 0$ and $A = 0.1$ appear to exhibit a notable decline in the relative number of choices in favor of the lower of the two signals in a given group. Accordingly, the clarification of the strategic elements of the setup with experience also appears to affect considerably the receivers’ conduct. Table 3.A.7 (Appendix 3.4) provides statistical support for this idea by comparing the session-level percentage levels (excluding ties) of receivers having chosen the signalers sending the lower signal as their allies across the various subsets of rounds; the results do not differ when including tied signaling choices. As indicated by the figure, the most notable adjustments occur in the treatment with neutral signals and that with a small signal-dependent benefit ($p \leq 0.094$).

**Figure 3.6.** Graphical Comparison of the Percentage of Receivers Selecting the Signaler Sending the Lower Signal (excluding Tied Signaling Choices) “over Time,” decomposed by Treatment

Graphs analogous to Figures 3.A.4 and 3.A.5 (Appendix 3.3) further underpin these findings. In the context of the former, the curves depicting the receivers’ propensity to choose the signaler having sent the lower signal as their ally in the first 18 rounds are consistently much flatter than the corresponding curves for the last 18 and last 9 rounds of play, which tend to slope downward beyond (absolute) differences of three to four units. In other words, the receivers’ choices in the first
half of the game are considerably less sensitive to differences in the signaling magnitudes than in the latter part of play. In the last 9 rounds, the curves are, again, fairly flat beyond differences of about four or so units, although this is reasonable given that one would expect the participants to have worked out the optimal strategy by then. As is the case in Figure 3.A.5, the treatments with beneficent signals are almost indistinguishable in the last quarter of play. When focusing on decisions when one signal is below and the other above 10 (Figure 3.A.5), the results are effectively the same. In the first half of the game, the receivers have a notably higher tendency to choose the lower signal than in the last half and quarter of the game, with the most significant downward adjustments occurring in the treatments with $A = 0$ and $A = 0.1$. More generally, the figures also seem to suggest that in the treatment with the higher signal-dependent benefit, the receivers’ strategy may be more obvious from the outset. The behavior in the companion treatment ($A = 0.1$) initially seems more similar to that in the treatment with neutral signals, although over time it converges to the predicted equilibrium strategy. In view of the relative degree of adjustment, it is therefore conceivable that large parts of the changes in the behavior of the signalers occur in response to modifications in the receivers’ behavior.

**Result 3.3**

Experience with the setup is important in shaping the participants’ behavior across roles. The observed adjustments suggest that, over time, the strategic elements of the setup become progressively more apparent. As a result, the signaling choices in the treatments with beneficent signals approximately converge to the theoretical means and a notably lower proportion of receivers choose signalers having sent the lower of the two signals they observe in a given group as their allies.

### 3.4.4 Comparison of Behavior across Treatments

The discussion thus far was devoted to identifying the basic characteristics of the equilibrium predictions in each of the three treatments. Rooted in this foundation, this section seeks to gauge the accuracy of the theory’s comparative static properties. While the findings are qualitatively similar for all subsets of rounds, I focus on the results for the last 18 rounds of play, judging it most representative as the key behavioral adjustments will have taken place.

**Signalers.** Recall that the comparative static results of the theory are quite succinct. Whereas they projected a sharp contrast in behavior between the treatments
Table 3.6. Statistical Comparison of the Signaling Activity across Treatments, decomposed by Signaler Type

<table>
<thead>
<tr>
<th>Signaler Type</th>
<th>(Two-Sided) Comparison</th>
<th>Test(^{26})</th>
<th>Value of Test Statistic ((\bar{U}))</th>
<th>(p)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Type</td>
<td>A = 0 vs. A = 1</td>
<td>RRO</td>
<td>-0.884</td>
<td>&gt; 0.204</td>
</tr>
<tr>
<td></td>
<td>A = 0 vs. A = 0.1</td>
<td>RRO</td>
<td>-0.497</td>
<td>&gt; 0.204</td>
</tr>
<tr>
<td></td>
<td>A = 0.1 vs. A = 1</td>
<td>RRO</td>
<td>0.275</td>
<td>&gt; 0.206</td>
</tr>
<tr>
<td>High Type</td>
<td>A = 0 vs. A = 1</td>
<td>RRO</td>
<td>2.622</td>
<td>0.052**</td>
</tr>
<tr>
<td></td>
<td>A = 0 vs. A = 0.1</td>
<td>RRO</td>
<td>3.914</td>
<td>0.022**</td>
</tr>
<tr>
<td></td>
<td>A = 0.1 vs. A = 1</td>
<td>RRO</td>
<td>-0.853</td>
<td>&gt; 0.206</td>
</tr>
</tbody>
</table>

Legend: RRO stands for Robust Rank Order Test. The test was carried out in its two-sided form, comparing the session-level mean signaling levels derived from the last 18 rounds of play within a particular treatment. The alternative hypothesis is given by \(H_1: \bar{x}_{T}^i \neq \bar{x}_{T}^{i'}\), where \(\bar{x}_{T}^i\), for \(T \in \{0,0.1,1\}\) and \(i \in \{1,\ldots,5\text{ or }6\}\), signifies the mean within a particular session. The order of comparison was such that the smaller sample was compared to the larger sample (e.g., \(A = 1\) vs. \(A = 0\)). Hence, a positive value for the test statistic indicates that the values of the smaller sample exceed those of the larger sample. The (two-sided) \(p\)-values are the closest values derived by Feltovich (2005). * ≡ Significance at the 10% level; ** ≡ Significance at the 5% level; *** ≡ Significance at the 1% level.

with neutral and beneficent signals, no divergence should be observed when comparing the treatments with beneficent signals. Table 3.6 presents the relevant statistical results, decomposed by signaler type.\(^{27}\) Despite a slight divergence in the descriptive graphs, the behavior of signalers of low type does not differ statistically across the treatments, though in view of the multiplicity of outcomes in the treatment with neutral signals, this finding is not necessarily surprising. On the part of signalers of high type, in turn, the analysis indicates a significant difference between the participants’ behavior when using neutral signals and either variant involving a signal-dependent benefit. However, no difference is detectable when contrasting the treatments with beneficent signals. These results thus provide strong support for the theory, although experience turns out to be important in this regard. For the results

\(^{26}\) The Robust Rank-Order Test is a modification of the non-parametric Wilcoxon-Mann-Whitney U-Test designed to test for differences in central tendency between two independent samples from the same population without requiring assumptions on higher-order moments (Siegel & Castellan 1988; Feltovich 2003). Its second crucial advantage for the purposes at hand is that it is insensitive to small sample sizes. The most notable obstacle to using this test is that the available probability tables are quite coarse (cf. Feltovich 2005).

\(^{27}\) Exploratory Kruskal-Wallis three-sample tests indicated that whereas the behavior of signalers of low type does not tend to diverge across the treatments (\(\chi^2 = 0.757, p = 0.685\)), the conduct of signalers of high type does differ (\(\chi^2 = 6.665, p = 0.038\)), which prompted the two-sample comparisons reported in Table 3.6. For reference, the Kruskal-Wallis \(k\)-sample test is a non-parametric testing procedure to assess whether a number of independent samples (jointly) were drawn from the same population (Siegel & Castellan 1988). The present analysis was carried out using the session-level mean signaling levels within a particular treatment (decomposed by signaler type). The interpretation of the test is such that a significant result indicates that the samples were drawn from different populations.
are not quite as strong when based on the first 18 rounds of play (when comparing $A = 0$ vs. $A > 0$).

Shifting gears, a comparison across treatments also sheds light on the consistency of the participants’ behavior. The most straightforward measure on the part of the signalers in this context is the lack of difference in the occurrence of tied signaling choices (Kruskal-Wallis three-sample test: $\chi^2 = 0.559, p = 0.756$). This suggests that the participants’ understanding of the game and acquisition of experience during its the course is similar across the treatments, a finding buttressing the procedure of the experiment.

**Receivers.** The receivers, too, are consistent in their decisions across treatments. For instance, as indicated above, the propensity to choose the signaler having sent the lower of the two signals in any given group is largely independent of the absolute difference between the observed signaling magnitudes; the figure for the second half of play looks very similar to Figure 3.A.4 (Appendix 3.3), yet rather more condensed. Observe also that the ranking of the curves is precisely as one would predict. Namely, the treatment with a small signal-dependent benefit is “sandwiched” between the other treatments, with the receivers in the treatment with a large benefit being (comparatively) “most correct” in their choices. This suggests that the higher the benefit, the lower the probability that the signaler sending the lower signal is chosen as the receiver’s ally, which in turn implies that the participants do tend to realize that a higher benefit nets them greater direct benefits.\(^{28}\)

When focusing on the decision scenario where one observed signal is less and the other more than 10 (cf. Figure 3.A.5, Appendix 3.3), in line with the comparative static results on the part of the signalers, the receivers in the treatments with beneficent signals are – to all intents and purposes – equally likely to opt for the signaler sending the lower signal across the projected belief threshold of 10. Their counterparts in the treatment with neutral signals make about twice as many such decisions, although this outcome is consistent with the existence of multiple equilibria. The behavior in the individual sessions of said treatment ($A = 0$) is as discussed, with the receivers in the sessions approaching the LCS outcome somewhat less likely to choose the signaler sending the higher signal.

\(^{28}\) A further interesting observation in this regard is that the curve for the treatment with $A = 0.1$ is more similar in shape to that for the treatment with $A = 0$ than the curve for $A = 1$ (especially for smaller differences), although the frequency of choices is unambiguously more similar to the latter.
On a final note, as indicated in Section 3.3, the participants completed a brief post-experimental questionnaire, which included questions relating to their beliefs when assigned the role of receiver. Although not incentivized, two of the subjective assessments are suggestive. Given the choice between a signaler sending a signal of 10 and another sending a signal of 35, for instance, the vast majority of participants considered the former as having been sent by a signaler of low type and the latter by a high type. The levels are somewhat higher, beyond 90%, for individuals who participated in the treatments with beneficent signals. In the treatment with neutral signals, only about 78% of the participants assigned the lower signal to a low type whereas around 86% assigned the higher one to a high type. Interestingly, when focusing on the LCS sessions, 54% of the individuals assigned the signal of 10 as having been sent by a signaler of high type, which is clearly in line with the relevant strategy profile. Indeed, all of the assessments are consistent with the respective equilibria, as are the estimations on how many signalers of low and/or high type choose signals beyond the projected maximum levels (for $A = 0$, the questionnaire inquired about 0 and 10, whereas the levels were set at 10 and 60 for $A > 0$). As before, the responses diverge notably between the treatments. In the treatment with neutral signals, about 55% of the participants for low types and 62% for high types thought the relevant types would send more than the indicated magnitudes. In this case, too, the participants in the sessions approaching the LCS outcome were least likely to think so. In the treatments with beneficent signals, in turn, less than 25% of the participants deemed signals beyond the indicated levels likely.

**Result 3.4**  
[Hypotheses 3 & 4] The data lend strong support to the comparative static properties of the model. Comparisons of behavior across the treatments furthermore suggest that the participants’ understanding of the setup and propensity to learn the behavior of others are quite consistent.

**3.5 Discussion**

In its essence, the behavior observed throughout the experiment is not far off the predicted outcomes. The participants recognize that the signaling activities at their disposal convey information and, in most sessions, make choices consistent with separating outcomes in the sense that signalers of high type choose larger magnitudes than those of low type. The participants’ subjective assessments
accentuate this impression. Whereas, at the aggregate level, the observed separating outcome in the treatment with neutral signals does not appear to converge to a single coordinated outcome, the behavior in two-thirds of the individual sessions tends approximately towards feasible equilibria. Two sessions swiftly approach outcomes corresponding to the LCS outcome, two further sessions converge to outcomes roughly in line with the one projected for the treatments with beneficent signals, and the equilibration process in two appears not to have succeeded (or needed more rounds). Hence, when taken together, the behavior in this treatment is very much in accordance with the prediction of multiple equilibria. The choices in the treatments with beneficent signals, on the other hand, converge to outcomes very similar to those predicted by the underlying theory, both at the aggregate level and in the majority of the individual sessions, especially within the treatment with a small signal-dependent benefit. Indeed, as suggested by the observation that adjustments in the signaling choices over time mainly occur in the treatment with a large signal-dependent benefit, convergence in the other settings appears to be quick. Nonetheless, in all treatments, experience seems to highlight the strategic attributes of the game, thereby sharpening the participants’ behavior. The theory’s comparative static properties, too, are satisfied. The purpose of this section is to explore the implications of these findings with respect to the breadth of the underlying theory.

One of the most notable implications of the experiments’ main results, in this context, is their support for the theoretical finding that signaler competition alone does not lead to a (more) precise outcome. In the same spirit, and in line with the literature, the outcomes in the treatment with neutral signals emphasize that the coordination on a particular strategy configuration is not necessarily straightforward. Yet, at the same time, the occurrence of multiple equilibria in said treatment provides support for the conjecture advocated in Chapter 2 that the signal-dependent benefit can potentially be used as an equilibrium selection device. The reasoning underlying this claim is as follows. As two of the sessions with neutral signals also approximate the equilibrium predicted for the treatments with beneficent signals (on the part of both roles), though the outcomes in the treatments with beneficent signals display a higher degree of separation, one could argue that the presence of the signal-dependent benefit helps select the equilibrium from a large(r) set. Beyond that, however, the intrinsic value of the signal may not play as much of a role in the participants’ decision process as the competitive pressures among the signalers.
Does it matter with respect to the nature of the observed behavioral outcomes that the receiver is not an automaton? The fact that most receivers seem to be playing the same strategy as an automaton in an auction setup, i.e., selecting the highest “bidder,” might suggest that it does not. Nevertheless, I would argue that it does have an impact – for two main reasons. First, the present results tend to contrast with the findings of Noussair and Silver (2006), who find that bidders with low valuations usually bid close to or below the equilibrium level while bidders with high valuations frequently bid too much. Second, the propensity to choose the signaler sending the lower signal is somewhat larger for small differences between the observed signals but otherwise largely independent thereof, and a non-zero number of receivers select the relevant signalers through to the very end (even in instances when one signal is above and the other below the projected belief threshold of 10).

In the present case, signalers of low type in the treatments with beneficent signals tend to “underbid” while those of high type “underbid” in the intermediate portions of their support and “overbid” towards the upper end. Considering that the receiver in a signaling game does not select the highest “bid” with probability one, the observed divergence in behavior (compared to the pure auction setup) may not be unreasonable because the participants may want to make sure that they have the highest “bid” to guarantee that they are selected. This logic would not only explain why high types “overbid” and, in response, low types may be discouraged from sending high signals (and thus “underbid”), but also the “underbidding” in the intermediate intervals, the justification for the latter being that the relevant signalers place a higher (subjective) probability on encountering a low type as competitor than another high type.\(^{29}\) The tendency towards the lower “bid” for small absolute differences between the observed “bids,” and the occurrence of such choices throughout the game, further underline this argument, as this decision rule encourages high(er) signals.

The fact that some experience is necessary for the comparative static properties of the model to obtain has to do with the receivers’ behavior in the treatment involving a small signal-dependent benefit. For, in line with the threshold hypothesis, the behavior of the signalers and receivers in the first half of the game exhibits a

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\(^{29}\) An alternative would be the presence of a certain degree of risk aversion, although one would – if it were the central driving force – expect more “overbidding” by signalers of high type (cf. Noussair & Silver 2006).
certain amount of similarity with the treatment with neutral signals. During the second part of the game, however, the receivers’ conduct in particular tends strongly towards the appropriate equilibrium strategy, presumably inducing the signalers to adopt suitable best responses. In other words, although the treatment does comprise a signal-dependent benefit, the small size initially (and for small differences between the observed signaling magnitudes to some extent also during the second part of play) tends to give rise to behavior at the interface of the two treatments. Only with experience does the significance of the benefit seem to become apparent, guiding the participants’ behavior towards the equilibrium.

A final point concerns the proximity of the outcomes in either of the treatments with beneficent signals to the theoretical predictions, which stands in notable contrast to large parts of the literature on experiments involving games with unique mixed-strategy equilibria. After all, recall that each participant only made one decision per round (with varying roles during the course of the game). It is certainly true that the statistical tests do not support uniformity of the observed frequency distributions. At the same time, however, more and more individuals, over time, choose signaling levels in the theoretical equilibrium supports for their type (signalers) and make inferences in congruence with the appropriate belief structure (receivers). One might therefore speculate that the signaling mechanism in the context of initiating social interaction is not an entirely unfamiliar concept, as mixed-strategy-play commonly requires considerable experience or repeated exposure (if at all achievable).

In sum, in spite of its various simplifying assumptions and limitations, the theory overall appears to predict behavior remarkably well. The results, moreover, illuminate the depth of the predictions, providing support for the notion that the signal-dependent benefit may indeed constitute an equilibrium selection device. Even so, the observed outcomes are clearly not perfect, although the aggregate divergence of the behavior, particularly of signalers of low type, in the treatments with beneficent signals may to some extent be misleading – in several individual sessions, their conduct is very close to what it is projected to be. More generally, the robustness of the participants’ behavior in both roles underlines the consistency of the incentives across treatments and the similarity in the fundamental characteristics of the participants in the experiment (e.g., as regards their propensity to learn the behavior of others). As such, the present study provides valuable insight into the workings and scope of the framework of interest.
3.6 Concluding Remarks

The purpose of this chapter was to explore experimentally the precision of and sharp divergence in the predictions derived in its theoretical companion chapter (Chapter 2). Although one can discern some obvious deviations, the results provide strong support for the theory as the general behavioral patterns are very consistent – both from a static and comparative perspective. Yet, notwithstanding the apparent success of the model and experiment, one can think of a number of informative extensions to the underlying theory and thus testable hypotheses to arrive at a more refined picture of individuals’ behavior in the envisioned signaling environment.

Many of the most interesting extensions to the theory – e.g., a noise factor in the signal-transmission process, measures to introduce receiver competition, and the like – are described in some detail in Chapter 2. Their main implication(s) would be to capture more readily the intricacies of human nature and to render the setup closer to conventional signaling models to facilitate comparison. As to the present work and data, from an analytical angle, the data (as is) could be manipulated in a number of additional directions to obtain further insight into the participants’ abilities and the forces driving their behavior. One such tangent would be to use the time to complete the control questionnaire to gauge the participants’ understanding of the setup. In a similar spirit, one could draw on more advanced econometric techniques to model the participants’ learning process in order to quantify the impact of experience on the observed behavioral outcomes. Both of these aspects are left for future work.

From an experimentation perspective, an interesting augmentation to the setup so as to obtain a more in-depth understanding of the influence of the size of the signal-dependent benefit on the observed outcomes would be a treatment with an even smaller benefit (e.g., $A = 0.001$) to capture the existence of a benefit threshold, if any. Likewise, to test the achievability of the mixed-strategy outcomes more explicitly, it might be useful to construct a setup rendering the envisioned mixed strategies more obvious. A further avenue would be the incentivization of the receivers’ beliefs, i.e., it would be instructive to extract the receivers’ beliefs in as accurate a manner as possible using monetary rewards. The ensuing data could then be used to obtain a more accurate demarcation of the forces driving their behavior. Nevertheless, even as is, the present setup and analysis yielded a non-negligible number of interesting results.
3.A Appendices

Appendix 3.1. Summary of the Experimental Protocol

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**Arrival & Introduction**

- Students arrive at the laboratory between 0 and 15 minutes before the scheduled start of the experiment
- Distribution of consent forms and instructions, followed by random assignment to the computer terminals
- Brief (oral) greeting and introductory comments
- De-registration depending on show-up rate

  **Partitioning Procedure**
  - Two “sessions” (subsets of participants) were feasible – created via the random assignment of terminals and controlled on different servers – if there were a total of 18, 21, 24, or 27 participants; one “session” was feasible if fewer than 18 participant attended
  - De-registration was necessary if the number of attendees fell short of 9 individuals or was not divisible by three, in which case a show-up fee of £5 was paid out (against receipt) with the assurance of guaranteed participation in a different session

- Collection of signed consent forms
- Students are read the instructions aloud
- Any clarifying questions are answered (in private); if applicable/useful for the whole group, repetition of question(s) and answer(s) to all by the experimenter
- The students are asked to work through the computerized control questions (three questions relating to the game’s payoff structure)

  - The experiment does not proceed to the game phase until all participants have answered all questions correctly

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**Game Phase**

36 Decision Rounds

- At the beginning of each round, the computer randomly allocates each subject to a (new) group and assigns each a (new) role
- Each participant is informed of his/her role
- Signalers
  - After receiving information about his/her role, each signaler is informed of his/her type
  - Given this information, (s)he is asked to decide how many quality units (s)he would like to invest in his/her prototype
After confirming his/her choice, each signaler enters a waiting stage until the receiver in his/her group has made his/her decision.

- **Receivers**
  - After being informed about his/her role for the particular round, each receiver enters a waiting stage until the signalers in his/her respective group have made their decisions.
  - As soon as both signalers within a given group have confirmed their choices, they are displayed on the receiver’s screen along with a table indicating his/her possible payoff if (s)he selects either signaler (broken down by type).
  - Upon confirmation of the receiver’s decision, the participants of his/her group enter a waiting stage. Once all groups within a “session” have completed the decision stage, they enter the feedback stage, during which they are shown their own payoff, all decisions (including each signaler’s type) within their group during that particular round as well as a graphical summary of all of the signalers’ decisions (including their success in procuring a contract) during that particular round.
  - Note that the subsets ("sessions") within each timeslot are kept strictly separate, i.e., the participants only receive information about the behavior within a given round for their subset of participants.

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**End of the Experiment**

- At the end of Round 36, the participants are asked to work through a brief questionnaire, which comprises (unincentivized) questions about generic demographic information and the participants’ experiences during the course of the experiment.
- Once all participants have completed the questionnaire, cash payment in private (against receipt) in the laboratory’s server room; subjects are taken to the room one by one.
- Separate exit after payment.

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**Appendix 3.2. Instructions**

You are about to participate in an experiment investigating decision-making. During the experiment, you will be asked to make a series of decisions that will yield you “experimental dollars” (denoted $). Your earnings will depend on your decisions as well as the decisions made by other participants. At the end of the session, your accumulated experimental dollars will be converted into pounds sterling at an exchange rate of $1 = £0.01, or equivalently, $100 = £1. The money you earn will be paid to you in cash and in private at the end of the session.

**Decisions**

In every round, you will randomly be assigned one of two roles, supplier or government official. Moreover, in every round, you will randomly be assigned to a
group of three, consisting of two suppliers and one government official. You will not be told the identity of the other members of your group(s), nor will they be told your identity – even after the end of the session.

The experiment consists of 36 rounds. At the beginning of each round, you will randomly be assigned a new role and be allocated to a new group. Throughout the session, you will be assigned the role of government official in a total of 12 rounds. During the course of a particular round, the decisions you will be faced with depend on the role you have been assigned.

The setting and sequence of play in each round are as follows. The government official is looking to contract out the design and production of a new helicopter fleet for the Maritime and Coastguard Agency to one of the suppliers in his/her group.

1) Suppliers. A supplier can have low costs or high costs. At the beginning of each round, a random draw determines each supplier’s cost level (or “type”). The likelihood of being assigned either type is 50%. After the draw, each supplier is told his/her own type, but not the type of the other supplier in his/her group; the government official is not told the type of either supplier. Given this information, each supplier has to produce a prototype of the helicopter (s)he would deliver to the government official if awarded the contract. In particular, (s)he needs to decide with how much quality to endow the prototype. (S)he can invest in any amount between 0 and 125 “quality units” (inclusive, with a precision of up to two decimal places). The cost of investing in a quality unit depends on the supplier’s type: a low-cost supplier has to pay $5 for every quality unit (s)he chooses to invest in his/her prototype, while a high-cost supplier has to pay $25 for each unit.

2) Government Official. Once the suppliers in a group have made their decisions, the government official in that group is informed of the number of quality units each supplier invested into his/her prototype – but not the type of either supplier. Given this information, (s)he has to decide which supplier (s)he would like to award the contract for the design and production of the helicopter fleet.

Earnings
Your payoff in each round is determined as follows.

- **Suppliers**
  - If a supplier is awarded the contract by the government official, (s)he receives $500 minus the cost of his/her prototype.
  - If a supplier is not awarded the contract by the government official, (s)he nonetheless has to meet the cost incurred for constructing his/her prototype.

- **Government Official**
  (a) Treatment with $A = 0$:
    - If the government official awards the contract to a high-cost supplier, (s)he receives $50.
    - If the government official awards the contract to a low-cost supplier, (s)he receives $80.
(b) Treatment with $A = 1\ [A = 0.1]$:

- If the government official awards the contract to a high-cost supplier, (s)he receives $50 and gets to keep that supplier’s prototype, which yields additional experimental dollars equal to \([\text{one-tenth (10\%)}\ of]\) the number of quality units of the prototype (that is, (s)he benefits from the prototype’s quality).
- If the government official awards the contract to a low-cost supplier, (s)he receives $80 and gets to keep that supplier’s prototype, which yields additional experimental dollars equal to \([\text{one-tenth (10\%)}\ of]\) the number of quality units of the prototype (that is, (s)he benefits from the prototype’s quality).

At the beginning of the experiment, each participant is given an initial balance of $500. Your total earnings at the end of the experiment will be the sum of this balance and your payoff in nine rounds chosen randomly for each participant. The nine rounds will consist of three rounds in which you played the role of government official, three in which you were a high-cost supplier, and three in which you were a low-cost supplier. Each participant is guaranteed £3 for completing the session.

For your information, at the end of each round, you will be shown a screen with your payoff for that round, a summary of all decisions within your group, and the distribution of decisions of a representative set of suppliers in that round – as well as whether or not a particular supplier was awarded a contract.

**Example**

To clarify the setting, consider the following example. Suppose you have been assigned the role of supplier and have been told that you have high costs. You aren’t told the type of the other supplier, but you know that there is a 50:50 chance that (s)he faces low or high costs. You now have to decide with how much quality to endow your prototype. Say you decide to invest in 7 quality units, costing you $175 ($25 per unit × 7 units). If the government official decides to award the contract to you, your profit will be $500 – $175 = $325. If, instead, (s)he decides to award the contract to the other supplier, you will lose $175 (that is, your profit will be –$175).

Once you and the other supplier in your group have decided on the quality of your respective prototypes, the government official in your group will be informed about each of your choices. Given this information, (s)he has to decide which one of you to award the contract.

(a) **Treatment with $A = 0$**: If (s)he awards the contract to you, given that you are a high-cost supplier, (s)he will earn $50. If (s)he awards the contract to the other supplier in the group, his/her payoff will depend on that supplier’s type. For illustration, if the other supplier has low costs, the government official will receive $80.

(b) **Treatment with $A = 1\ [A = 0.1]$**: If (s)he awards the contract to you, given that you are a high-cost supplier and invested in 7 quality units, (s)he will earn $50 + $7 = $57 \([50 + (0.1 \times 7) = 50.70] \). If (s)he awards the contract to the other supplier in the group, his/her payoff will depend on that supplier’s type and investment. For illustration, if the other supplier has low costs and invested in 20 quality units, the government official will receive $80 + $20 = $100 \([80 + (0.1 \times 20) = 82] \).
Quiz
To make sure that everyone understands the instructions, before the start of the experiment, you will be asked to work through a short “Quiz” comprising three questions. The experiment will start once everyone has answered all questions correctly.

End of Experiment
Upon the completion of Round 36, the experiment ends. While the computer calculates your earnings, you are kindly asked to fill out a brief questionnaire inquiring about your experience during the session; please follow the instructions provided on the relevant screens. Once all participants have completed the experiment, each participant is paid his/her earnings – shown on the final screen of the experiment.

Summary Tables

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Cost</th>
<th>Payoff if production contract is procured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Costs</td>
<td>$5 per quality unit</td>
<td>$500 – ($5 per quality unit × x quality units)</td>
</tr>
<tr>
<td>High Costs</td>
<td>$25 per quality unit</td>
<td>$500 – ($25 per quality unit × x quality units)</td>
</tr>
</tbody>
</table>

Government Official

(a) Treatment with $A = 0$:

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Payoff if contract is awarded to a …</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Cost Supplier</td>
<td>$80</td>
</tr>
<tr>
<td>High-Cost Supplier</td>
<td>$50</td>
</tr>
</tbody>
</table>

(b) Treatment with $A = 1$ [$A = 0.1$]:

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Payoff if contract is awarded to a …</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Cost Supplier</td>
<td>$80 + [(0.1 × $ equal to the quality units of the chosen supplier’s prototype)]</td>
</tr>
<tr>
<td>High-Cost Supplier</td>
<td>$50 + [(0.1 × $ equal to the quality units of the chosen supplier’s prototype)]</td>
</tr>
</tbody>
</table>
Appendix 3.3. Supplementary Figures

Figure 3.A.1. Box-and-Whisker Plots of the Signaling Choices within each Treatment, decomposed by Signaler Type

(i) All Rounds of Play

(ii) First 18 Rounds of Play

(iii) Last 18 Rounds of Play

(iv) Last 9 Rounds of Play

Legend: The plots were created using the raw data on signaling levels from the indicated rounds of play. The boxes comprise the sample median (band towards the center of the box), the 25th and 75th percentile (lower and upper limit of the box, i.e., 50% of the sample lies within the bounds of the box). The whiskers represent 1.5-times the height of the box, or inter-quartile range. The circles and stars beyond the whiskers denote outliers and extreme outliers, respectively. Outliers are sample values that do not fall within the whiskers, while extreme outliers are taken to be all those values with a size of at least three times the height of the box.
Figure 3.A.2. Parameter $A = 0$ – Average Signaling Levels and Standard Deviations for each Session, decomposed by Signaler Type

Legend: $n$ denotes the number of subjects in a given session.
Figure 3.A.3. Parameter $A = 0$ – Choice of Winning Signal(er) by Signal Category (Relative to the Total Number of Decisions within a Signal Category; excluding Ties), Session Perspective

Legend: This figure is based on group-level data from all 36 rounds of play, namely, the percentage of choices in favor of the high(er) or low(er) signal (excluding tied signaling choices) relative to the total number of decisions within a particular signal category; correspondingly, the height of the two bars within each signal category within a particular session sum to 1 (or, equivalently, the height of the bars within each session sums to 3). In this particular figure, “high” and “low” refer to the receiver’s choice of the signaler having sent the high(er) or low(er) signal within a given decision scenario as his/her ally, not signaler type. $x$ and $y$, in turn, represent the signalers’ choices within a given group.

Figure 3.A.4. (Accumulative) Percentage of Receivers Choosing the Lower Signal Relative to All Signals within the Given Interval and (Cumulative) Total Number of Choices in Favor of the Lower Signal for Various Absolute Differences between the Observed Signaling Magnitudes (excluding Ties), decomposed by Treatment
Legend: The figures are based on group-level data from all 36 rounds of play (excluding tied signaling choices). For each increment of 0.1 units, the plots on the left-hand side depict the accumulated percentage of receivers choosing the signaler sending the lower signal relative to the total number of decisions in the particular interval of signaling magnitudes. The values on the horizontal axes denote the absolute difference between the signals being considered. For instance, 0.5 indicates the interval “smaller signal + 0.5 ≥ larger signal;” the percentage value plotted at 0.5, in turn, represents the sum of all decisions in favor of the lower signal for all difference levels up to 0.5 (inclusive) divided by the sum of all decisions in the relevant intervals. Therefore, a negative slope indicates that the “current” proportion of decisions in favor of the lower signal is lower than the average up to that point. The plots on the right-hand side depict the cumulative total number of choices in favor of the lower signal for the particular interval of signaling magnitudes.

Figure 3.A.5. Percentage of Receivers Choosing the Lower Signal if One Observed Signal is Less and the Other More Than 10 Relative to All Signals within the Category, decomposed by Treatment, Session (where appropriate), and (Sub)Set of Rounds.
Figure 3.A.6. Parameter $A = 0.1$ – Descriptive Statistics on Signaling Behavior in Session 2, decomposed by Signaler Type

Histograms of the Observed Signaling Behavior and Predictions

<table>
<thead>
<tr>
<th>Type &amp; Classes</th>
<th>Low Type</th>
<th>High Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ± StD.</td>
<td>9.475 ± 14.195</td>
<td>35.332 ± 17.973</td>
</tr>
<tr>
<td>Median</td>
<td>5</td>
<td>35</td>
</tr>
</tbody>
</table>

Observed Frequencies and Mixed-Strategy Equilibrium (MSE) Probabilities for the Predicted Equilibrium Supports

<table>
<thead>
<tr>
<th>Type &amp; Classes</th>
<th>Low Type</th>
<th>High Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed Frequency</td>
<td>0.53  0.22</td>
<td>0.16  0.21  0.18  0.24  0.12</td>
</tr>
<tr>
<td>MSE Probability</td>
<td>0.50</td>
<td>0.50  0.20  0.20  0.20  0.20  0.20</td>
</tr>
</tbody>
</table>

Appendix 3.4. Supplementary Tables

Table 3.A.1. Parameter $A = 0$ – Descriptive Statistics on Signaling Activity within each Session, decomposed by Signaler Type

<table>
<thead>
<tr>
<th>Session</th>
<th>Signaler Type</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>Low Type</td>
<td>3.298</td>
<td>1.2</td>
<td>0.430</td>
</tr>
<tr>
<td></td>
<td>High Type</td>
<td>11.717</td>
<td>10</td>
<td>1.032</td>
</tr>
<tr>
<td>Session 2</td>
<td>Low Type</td>
<td>2.062</td>
<td>1</td>
<td>0.267</td>
</tr>
<tr>
<td></td>
<td>High Type</td>
<td>8.534</td>
<td>5</td>
<td>1.065</td>
</tr>
<tr>
<td>Session 3</td>
<td>Low Type</td>
<td>10.543</td>
<td>4.5</td>
<td>17.962</td>
</tr>
<tr>
<td></td>
<td>High Type</td>
<td>36.645</td>
<td>31</td>
<td>27.197</td>
</tr>
<tr>
<td>Session 4</td>
<td>Low Type</td>
<td>6.702</td>
<td>7</td>
<td>5.666</td>
</tr>
<tr>
<td></td>
<td>High Type</td>
<td>31.543</td>
<td>25.8</td>
<td>19.504</td>
</tr>
<tr>
<td>Session 5</td>
<td>Low Type</td>
<td>7.212</td>
<td>5</td>
<td>10.831</td>
</tr>
<tr>
<td></td>
<td>High Type</td>
<td>20.232</td>
<td>17.5</td>
<td>16.425</td>
</tr>
<tr>
<td>Session 6</td>
<td>Low Type</td>
<td>14.242</td>
<td>1.05</td>
<td>32.405</td>
</tr>
<tr>
<td></td>
<td>High Type</td>
<td>19.028</td>
<td>18</td>
<td>21.146</td>
</tr>
</tbody>
</table>
Table 3.A.2. Descriptive Statistics on Signaling Activity “over Time,” decomposed by Treatment and by Signaler Type

(a) First 18 Rounds of Play

<table>
<thead>
<tr>
<th>Statistical Measures</th>
<th>( A = 0 )</th>
<th>( A = 1 )</th>
<th>( A = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Type</td>
<td>High Type</td>
<td>Low Type</td>
</tr>
<tr>
<td>Median</td>
<td>2</td>
<td>16</td>
<td>5</td>
</tr>
<tr>
<td>Mode</td>
<td>1</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>Range</td>
<td>125</td>
<td>125</td>
<td>78</td>
</tr>
</tbody>
</table>

(b) Last 18 Rounds of Play

<table>
<thead>
<tr>
<th>Statistical Measures</th>
<th>( A = 0 )</th>
<th>( A = 1 )</th>
<th>( A = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Type</td>
<td>High Type</td>
<td>Low Type</td>
</tr>
<tr>
<td>Mean</td>
<td>6.937</td>
<td>19.154</td>
<td>5.016</td>
</tr>
<tr>
<td>Median</td>
<td>1.2</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>Mode</td>
<td>1</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Range</td>
<td>125</td>
<td>125</td>
<td>45</td>
</tr>
</tbody>
</table>

(c) Last 9 Rounds of Play

<table>
<thead>
<tr>
<th>Statistical Measures</th>
<th>( A = 0 )</th>
<th>( A = 1 )</th>
<th>( A = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Type</td>
<td>High Type</td>
<td>Low Type</td>
</tr>
<tr>
<td>Mean</td>
<td>6.380</td>
<td>20.088</td>
<td>4.339</td>
</tr>
<tr>
<td>Median</td>
<td>1.35</td>
<td>16</td>
<td>2.27</td>
</tr>
<tr>
<td>Mode</td>
<td>1</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Range</td>
<td>125</td>
<td>90</td>
<td>30</td>
</tr>
</tbody>
</table>
**Table 3.A.3.** Comparison of the Signaling Activity across the Various Subsets of Rounds, decomposed by Signaler Type

(a)  **Low Type**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>(One-sided) Comparison</th>
<th>Test</th>
<th>Values of Test Statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A = 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last 18 vs. First 18</td>
<td>WSR</td>
<td>( z = 0.105; T^* = 11 )</td>
<td>0.500</td>
<td></td>
</tr>
<tr>
<td>First 18 vs. Last 9</td>
<td>WSR</td>
<td>( z = 0.105; T^* = 11 )</td>
<td>0.500</td>
<td></td>
</tr>
<tr>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>( z = 0.524; T^* = 13 )</td>
<td>0.344</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A = 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First 18 vs. Last 18</td>
<td>WSR</td>
<td>( z = 2.023; T^* = 15 )</td>
<td>0.031**</td>
<td></td>
</tr>
<tr>
<td>First 18 vs. Last 9</td>
<td>WSR</td>
<td>( z = 2.023; T^* = 15 )</td>
<td>0.031**</td>
<td></td>
</tr>
<tr>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>( z = 1.753; T^* = 14 )</td>
<td>0.063*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A = 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First 18 vs. Last 18</td>
<td>WSR</td>
<td>( z = 1.483; T^* = 13 )</td>
<td>0.094*</td>
<td></td>
</tr>
<tr>
<td>First 18 vs. Last 9</td>
<td>WSR</td>
<td>( z = 1.483; T^* = 13 )</td>
<td>0.094*</td>
<td></td>
</tr>
<tr>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>( z = 0.674; T^* = 10 )</td>
<td>0.313</td>
<td></td>
</tr>
</tbody>
</table>

(b)  **High Type**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>(One-sided) Comparison</th>
<th>Test</th>
<th>Values of Test Statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A = 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First 18 vs. Last 18</td>
<td>WSR</td>
<td>( z = 0.734; T^* = 14 )</td>
<td>0.281</td>
<td></td>
</tr>
<tr>
<td>First 18 vs. Last 9</td>
<td>WSR</td>
<td>( z = 0.943; T^* = 15 )</td>
<td>0.219</td>
<td></td>
</tr>
<tr>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>( z = 0.314; T^* = 12 )</td>
<td>0.422</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A = 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last 18 vs. First 18</td>
<td>WSR</td>
<td>( z = 0.944; T^* = 11 )</td>
<td>0.219</td>
<td></td>
</tr>
<tr>
<td>Last 9 vs. First 18</td>
<td>WSR</td>
<td>( z = 1.483; T^* = 13 )</td>
<td>0.094*</td>
<td></td>
</tr>
<tr>
<td>Last 9 vs. Last 18</td>
<td>WSR</td>
<td>( z = 2.023; T^* = 15 )</td>
<td>0.031**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A = 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Last 18 vs. First 18</td>
<td>WSR</td>
<td>( z = 0.674; T^* = 10 )</td>
<td>0.313</td>
<td></td>
</tr>
<tr>
<td>Last 9 vs. First 18</td>
<td>WSR</td>
<td>( z = 0.674; T^* = 10 )</td>
<td>0.313</td>
<td></td>
</tr>
<tr>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>( z = 0.944; T^* = 11 )</td>
<td>0.219</td>
<td></td>
</tr>
</tbody>
</table>

Legend: WSR stands for Wilcoxon Signed-Ranks Test. The test was carried out using the session-level mean signaling levels for the indicated subsets of rounds within a particular treatment. The order of comparison (and, correspondingly, one-sided alternative hypothesis) was as stated in the “Comparison” column. Hence, a positive value for the test statistic indicates that the signaling levels within the former subset of rounds exceed those within the latter subset of rounds. The (one-sided) \( p \)-values are the closest available values by Siegel and Castellan (1988, Table H). * ≡ Significance at the 10% level; ** ≡ Significance at the 5% level; *** ≡ Significance at the 1% level.
### Table 3.A.4. Comparison of the Incidence of Tied Signaling Choices across the Various Subsets of Rounds, decomposed by Treatment

<table>
<thead>
<tr>
<th>Treatment</th>
<th>(One-sided) Comparison</th>
<th>Test</th>
<th>Values of Test Statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 0</td>
<td>First 18 vs. Last 18</td>
<td>WSR</td>
<td>z = 0.841; T⁺ = 14.5</td>
<td>&gt; 0.219</td>
</tr>
<tr>
<td></td>
<td>First 18 vs. Last 9</td>
<td>WSR</td>
<td>z = 0.738; T⁺ = 14</td>
<td>0.281</td>
</tr>
<tr>
<td></td>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>z = 0.000; T⁺ = 9</td>
<td>&gt; 0.500</td>
</tr>
<tr>
<td>A = 1</td>
<td>Last 18 vs. First 18</td>
<td>WSR</td>
<td>z = 0.962; T⁺ = 11</td>
<td>0.219</td>
</tr>
<tr>
<td></td>
<td>Last 9 vs. First 18</td>
<td>WSR</td>
<td>z = 0.813; T⁺ = 10.5</td>
<td>&gt; 0.219</td>
</tr>
<tr>
<td></td>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>z = 0.566; T⁺ = 8</td>
<td>0.500</td>
</tr>
<tr>
<td>A = 0.1</td>
<td>Last 18 vs. First 18</td>
<td>WSR</td>
<td>z = 0.820; T⁺ = 10</td>
<td>0.313</td>
</tr>
<tr>
<td></td>
<td>Last 9 vs. First 18</td>
<td>WSR</td>
<td>z = 1.089; T⁺ = 11</td>
<td>0.219</td>
</tr>
<tr>
<td></td>
<td>Last 9 vs. Last 18</td>
<td>WSR</td>
<td>z = 0.283; T⁺ = 7</td>
<td>&gt; 0.500</td>
</tr>
</tbody>
</table>

**Legend:** WSR stands for Wilcoxon Signed-Ranks Test. The test was carried out using the session-level percentage of ties (relative to all decisions within the session) for the indicated subsets of rounds within a particular treatment. The order of comparison was as stated in the “Comparison” column. Hence, a positive value for the test statistic indicates that the incidence of ties within the former subset of rounds exceeds that within the latter subset of rounds. The (one-sided) p-values are the closest available values by Siegel and Castellan (1988, Table H). * ≡ Significance at the 10% level; ** ≡ Significance at the 5% level; *** ≡ Significance at the 1% level.

### Table 3.A.5. Observed Frequencies and Mixed-Strategy Equilibrium (MSE) Probabilities for the Predicted Equilibrium Supports “over Time,” decomposed by Treatment and Signaler Type

(a) **First 18 Rounds of Play**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Type &amp; Classes</th>
<th>Low Type</th>
<th>High Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Frequencies</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MSE Probability</td>
<td>0.50 0.20 0.20 0.29 0.16 0.10 0.05</td>
<td></td>
</tr>
<tr>
<td>A = 1</td>
<td>Obs. Frequency</td>
<td>0.55 0.22 0.20 0.29 0.16 0.13 0.11</td>
<td></td>
</tr>
<tr>
<td>A = 0.1</td>
<td>Obs. Frequency</td>
<td>0.58 0.20 0.22 0.22 0.16 0.13 0.11</td>
<td></td>
</tr>
</tbody>
</table>

(b) **Last 18 Rounds of Play**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Type &amp; Classes</th>
<th>Low Type</th>
<th>High Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Frequencies</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MSE Probability</td>
<td>0.50 0.20 0.20 0.20 0.20 0.20 0.20</td>
<td></td>
</tr>
<tr>
<td>A = 1</td>
<td>Obs. Frequency</td>
<td>0.67 0.23 0.15 0.33 0.16 0.13 0.07</td>
<td></td>
</tr>
<tr>
<td>A = 0.1</td>
<td>Obs. Frequency</td>
<td>0.77 0.16 0.23 0.22 0.13 0.11 0.14</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.A.6. Selection of the Signaler Sending the Lower Signal by the Receivers within each Treatment “over Time”

(a) First 18 Rounds of Play

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Choice of Low(er) Signal</th>
<th>Total Number of Decisions</th>
<th>% Choice of Low(er) Signal</th>
<th>Number of Tied Signals</th>
<th>% excluding Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 0</td>
<td>76</td>
<td>360</td>
<td>21.11%</td>
<td>30</td>
<td>23.03%</td>
</tr>
<tr>
<td>A = 1</td>
<td>20</td>
<td>306</td>
<td>6.54%</td>
<td>12</td>
<td>6.80%</td>
</tr>
<tr>
<td>A = 0.1</td>
<td>42</td>
<td>324</td>
<td>12.96%</td>
<td>8</td>
<td>13.29%</td>
</tr>
</tbody>
</table>

(b) Last 18 Rounds of Play

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Choice of Low(er) Signal</th>
<th>Total Number of Decisions</th>
<th>% Choice of Low(er) Signal</th>
<th>Number of Tied Signals</th>
<th>% excluding Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 0</td>
<td>45</td>
<td>360</td>
<td>12.50%</td>
<td>21</td>
<td>13.27%</td>
</tr>
<tr>
<td>A = 1</td>
<td>17</td>
<td>306</td>
<td>5.56%</td>
<td>21</td>
<td>5.97%</td>
</tr>
<tr>
<td>A = 0.1</td>
<td>30</td>
<td>324</td>
<td>9.26%</td>
<td>11</td>
<td>9.59%</td>
</tr>
</tbody>
</table>

(c) Last 9 Rounds of Play

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Choice of Low(er) Signal</th>
<th>Total Number of Decisions</th>
<th>% Choice of Low(er) Signal</th>
<th>Number of Tied Signals</th>
<th>% excluding Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 0</td>
<td>17</td>
<td>180</td>
<td>9.44%</td>
<td>10</td>
<td>10.00%</td>
</tr>
<tr>
<td>A = 1</td>
<td>4</td>
<td>153</td>
<td>2.61%</td>
<td>9</td>
<td>2.78%</td>
</tr>
<tr>
<td>A = 0.1</td>
<td>7</td>
<td>162</td>
<td>4.32%</td>
<td>6</td>
<td>4.49%</td>
</tr>
</tbody>
</table>

Legend: The content of these tables corresponds to the content of Table 3.2 in the main text.
Table 3.A.7. Comparison of the Receivers’ Behavior across the Various Subsets of Rounds, decomposed by Treatment

<table>
<thead>
<tr>
<th>Treatment</th>
<th>(One-sided) Comparison</th>
<th>Test</th>
<th>Values of Test Statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 0</td>
<td>First 18 vs. Last 18</td>
<td>WSR</td>
<td>$z = 2.201; T^+ = 21$</td>
<td>0.016**</td>
</tr>
<tr>
<td></td>
<td>First 18 vs. Last 9</td>
<td>WSR</td>
<td>$z = 2.201; T^+ = 21$</td>
<td>0.016**</td>
</tr>
<tr>
<td></td>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>$z = 2.108; T^+ = 20$</td>
<td>0.031**</td>
</tr>
<tr>
<td>A = 1</td>
<td>First 18 vs. Last 18</td>
<td>WSR</td>
<td>$z = 0.135; T^+ = 8$</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td>First 18 vs. Last 9</td>
<td>WSR</td>
<td>$z = 1.214; T^+ = 12$</td>
<td>0.156</td>
</tr>
<tr>
<td></td>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>$z = 1.905; T^+ = 14$</td>
<td>0.063*</td>
</tr>
<tr>
<td>A = 0.1</td>
<td>First 18 vs. Last 18</td>
<td>WSR</td>
<td>$z = 1.483; T^+ = 13$</td>
<td>0.094*</td>
</tr>
<tr>
<td></td>
<td>First 18 vs. Last 9</td>
<td>WSR</td>
<td>$z = 1.753; T^+ = 14$</td>
<td>0.063*</td>
</tr>
<tr>
<td></td>
<td>Last 18 vs. Last 9</td>
<td>WSR</td>
<td>$z = 2.023; T^+ = 15$</td>
<td>0.031**</td>
</tr>
</tbody>
</table>

Legend: WSR stands for Wilcoxon Signed-Ranks Test. The test was carried out using the session-level percentage of choices in favor of signalers sending the lower signals (excluding ties) for the indicated subsets of rounds within a particular treatment; the results when using the session-level percentages including ties do not differ qualitatively. The order of comparison was as stated in the “Comparison” column. Hence, a positive value for the test statistic indicates that the percentage of lower signals chosen within the former subset of rounds exceeds that within the latter subset of rounds. The (one-sided) $p$-values are the closest available values by Siegel and Castellan (1988, Table H). * = Significance at the 10% level; ** = Significance at the 5% level; *** = Significance at the 1% level.
CHAPTER 4

SIGNALING GIVEN CHOICE OF SIGNALING Activity

4.1 Introduction

While the interests of individuals operating in a (broad) social environment will overlap to some degree, they will rarely coincide perfectly, especially among strangers. Conflicts and competition are thus as predestined as is the individuals’ mutual interest in sharing and gathering information about each other’s intentions (e.g., Hawkes 1992; Smith & Bliege Bird 2005). For, in order to reap the benefits linked to cooperative social interaction, one must circumvent the hazard of exploitation by others. The ensuing care when it comes to choosing one’s interaction partners inevitably restricts the availability of profitable social exchanges. To resolve the informational asymmetry impeding the initiation of cooperative interaction, the individuals therefore clearly have an incentive to communicate with each other about their objectives so as to gain access to the most valuable opportunities.

One means to convey information about one’s unobservable characteristics to others, generally and in this sort of competitive setting (cf. Chapter 2), is costly signaling. The standard modeling approach in this regard is to impose exogenously some signaling activity (or activities) consistent with the setup and to derive conditions rendering the exchange of messages reliable. In practice, however, individuals trying to convey information to others tend to have a variety of activities at their disposal, often more than one at a time, including conspicuous activities such as purchasing a flashy car,\(^1\) generous behaviors such as donating favors to others, as well as harmful deeds such as the destruction of valuable assets. In fact, considering

\(^1\) This type of activity will henceforth be referred to as “(socially) neutral,” as it entails a cost to the signaler but no tangible effect on others in his/her social environment.
the range of options, one might expect that they tailor their signals to their particular circumstances – be it according to their audience, environment, or preferences.\textsuperscript{2}

Recent psychological evidence (Griskevicius \textit{et al.} 2007) does suggest just that – namely, people tend to opt for different signals and signal intensities in different circumstances.\textsuperscript{3} Rooted in these observations, the objective of this chapter is to examine what kind of signaling activity (or set of activities) is most likely to occur in a variety of social settings, and how the relevant outcome(s), if any, fare(s) from a welfare perspective. The settings being explored vary with respect to the extent of the signaling activities’ impact on the agents’ payoffs beyond the signalers’ expenditure on them. The analytical foundation to this end will be the signaling game developed in Chapter 2, extended to allow each individual signaler to choose \textit{endogenously} a signal from a collection of activities.

Even though the framework in Chapter 2 comprises competition among the signalers (only) and considers beneficent signals, i.e., activities that confer a benefit (positive externality) on the receivers, it satisfies the assumptions typifying traditional signaling models (Spence 1973, 1974). Namely, self-interested signalers differing along a one-dimensional type space can use an exogenously determined signaling activity, the cost of which is non-decreasing in the signal intensity (with a minimum of zero) but decreasing in a signaler’s type, to convey information about their type to incompletely-informed receivers. However, it does not give rise to the multiplicity of (Pareto-ranked) equilibria that may be satisfied by various types of signals as is common in standard models. Rather, it distinguishes the equilibrium configuration when signalers use neutral signals, in which case multiplicity persists, vis-à-vis use of beneficent signals, in which case the outcome is unique. The introduction of a choice set comprising more than one element into this framework permits more precise insight into what kind of signaling activity will arise in equilibrium in a given social setting by expressly pitting various signals against each other. At the same time, the augmentation expands the receivers’ decision process, strategy space, and beliefs, as the messages may now differ in the type of signal and magnitude being used, to either of which they may (now) attach significance.

\textsuperscript{42} One might, equivalently, think of different individuals excelling in different activities, implying that they may want to use the signal(s) that best suit them, provided their feasibility in a given context. As such, a particular setting might support several alternative (equilibrium) outcomes.

\textsuperscript{43} In their seminal work on explaining contributions to charity using a costly signaling framework, Glazer and Konrad (1996) also suggest that individuals may ultimately choose among various signals.
In the tradition of the literature on multi-dimensional signaling, the additional information on type receivers may derive from the extra component of a given signaler’s message might be considered a second signal. In its purest form, this literature explores whether, and under what conditions, individuals whose type varies in more than one dimension can achieve separation using multiple signals (Quinzii & Rochet 1985; Engers 1987). Besides the obvious divergences in the type space, nature of competition, and variety of signaling activities considered, it is only tangentially related to the present work since a given signaler can only (ever) use one signaling activity at a time. Despite the fact that the signalers’ messages ostensibly comprise multiple components, this implies that once they have settled on a particular activity, all that matters is (once more) the magnitude of their signal.

In this respect, the framework also contrasts with the extensive work involving multiple signals in industrial organization.\(^4\) Seeking to determine firms’ optimal strategies when attempting to communicate to their potential customers the quality of their products, frameworks in this literature have explored a variety of scenarios.\(^5\) Following the seminal papers by Milgrom and Roberts (1986) and Wilson (1985), the settings most commonly involve a monopolist with a one- or multi-dimensional type looking to convey information about the unobservable quality of its products to buyers using a combination of several signals – including price, expenditure on advertising, and warranty provisions. Distinctly, even with a one-dimensional type, the models consider more than one signal (of the neutral variety) at any one time, and tend to assume that the production cost is independent of the products’ quality.\(^6,7\)

\(^{4}\) Theoretical biologists, too, have considered multi-component (or multi-modal) signals (e.g., Johnstone 1995, 1996). The relevant literature contrasts with the present framework in that it does not consider choice or competition among the signalers, and tends to focus on neutral signaling activities.\(^{4}\) Signaling is only one approach explored by this literature. A further means, provided that quality is exogenous, is disclosure via direct (credible) claims. Among others, the relevant work differs from the signaling approach in that it assumes that the marginal cost of production is independent of the products’ quality, which rules out separation via signaling (e.g., Daughtey & Reinganum 2008b). Marketing research, too, has studied quality signals as means to resolve the uncertainty of buyers regarding the quality of the product(s) provided by sellers (see Kirmani & Rao (2000) for an overview). The focus of these works, however, tends to be on the specific effects of and conditions relating to the successful implementation of individual tools and their classification rather than their use in the information transmission process.

\(^{4}\) Ippolito (1990) argues that this assumption is not as incongruous as it may seem (in the sense that signaling models in the Spencian spirit depend on type-dependent costs), since firms in most markets can quite easily adjust the nature and quality of their products, implying that they can acquire many potential signals at equal cost.

\(^{4}\) As one of the most popular signals is price, any assumption on the production cost is effectively equivalent to making assumptions on the marginal cost of signaling in a signaling framework. Namely, no difference in the production cost according to the quality of the product would correspond to there being no difference in the marginal cost of signaling across the signaler types.
More recent contributions in this field have extended this basic framework to allow for competition among firms (signalers). Hertzendorf and Overgaard (2001) and Fluet and Garella (2002), for instance, develop very similar duopoly models in this regard. The firms in their frameworks differ in only one dimension and may use price and/or advertising expenditure to signal the quality of their products, although the message space being examined is set exogenously. While consumers are incompletely informed about the firms’ type, the competitors always differ in type and are aware of the quality of each other’s product. Moreover, for tractability, the unit cost of production either does not differ across the types (Hertzendorf & Overgaard 2001) or is set to zero for low-quality firms (Fluet & Garella 2002). Notwithstanding the restrictiveness of their assumptions, in keeping with the literature, the frameworks support a multitude of equilibria. The present framework, in some respect, expands these approaches by generalizing most assumptions and allowing the competitors to choose their signals.

Daughtey and Reinganum (2007, 2008a) study a more traditional signaling environment in that the firms are privately informed about the quality of their product and only compete in price. The distinguishing feature of their approach when compared to the present work is that the consumers (receivers) have a clear preference ranking over the firms’ products even if their quality and price do not differ (i.e., the products are horizontally differentiated). In line with all of the other works mentioned above, they do not consider choice of signaling activity. Hence, the setup developed in the following is distinctive by exploring choice of signaling activity in an environment in which individuals differ along (only) one dimension and face differential costs such that signaling is more expensive for individuals of a low(er) social quality. Although one might contend that individuals usually have complex characters that may not sufficiently be conveyed by way of a one-dimensional signal, the approach nonetheless has merit since some situations do not necessarily require communication about all possible aspects of one’s personality. Rather, within a given social setting (e.g., one involving the formation of mutually beneficial alliances for some specific purpose (cf. Chapter 2)), awareness of a specific attribute, or homogenous composite of attributes, may be fully adequate.

The results of this chapter indicate that, depending on the parameterization of the game, more than one equilibrium may arise, and may do so at the same time. This circumstance is independent of the setting being considered. It is moreover the case
that inefficient signals may arise in equilibrium, across scenarios. The paper therefore, in part, supports the notion that the agents’ skills and preferences may play a role with respect to the ultimate outcome. As one may not only obtain more than one type of equilibrium co-existing, but in some cases even a continuum of equilibria, from a technical perspective, the introduction of choice leads to the sharp uniqueness prediction obtained in the environment without choice being lost.

The remainder of this chapter is organized as follows. Section 4.2 briefly compares and contrasts the present framework with the model developed in Chapter 2. Section 4.3 thereupon introduces and solves the model with a restricted set of players and signals, generalizes the arguments to an \( n \)-player framework, and ponders alternative collections of signaling activities. Section 4.4 discusses the results, followed by concluding remarks in Section 4.5.

### 4.2 Preliminaries

Before delving into the technical details, it is worth noting that except for the fact that the signalers in the present framework can endogenously choose the signaling activity they wish to use as a messaging tool from a predetermined “menu” of activities, the signaling game explored in the following is effectively the same as the interaction analyzed in Chapter 2. This implies that many of the arguments developed for that framework also apply here. The modeling section will thus focus on the divergences between the setups and be limited to synopses of recurring derivations. The purpose of this section is to review briefly the setting of interest, set out the main assumptions and restrictions, and outline the central conceptual differences between the frameworks.

The environment to be explored in this paper once again involves two populations of risk-neutral players – signalers (he) and receivers (she). The signalers are differentiated in social quality along a discrete type space and try to transfer information about their type to the homogeneous receivers by simultaneously investing in exactly one of a collection of costly signaling activities along a continuous message space. Given the signal and signaling magnitude, the receivers make an inference regarding the signalers’ information, based on which they choose an action from a discrete action space. That is, each receiver decides to which one signaler she would like to provide access to a profitable alliance.
In the present case, the receivers are thus not only assumed to be worthwhile allies but to be able to observe and distinguish the various signaling activities. Attention is initially restricted to two beneficent signals (only), although the benefits associated with the activities differ in size. The exogenous nature of the elements within the signalers’ choice set can be thought of as the limited set of appropriate activities in a given situation; for not all viable activities are necessarily always suitable and/or feasible in all circumstances. The scenarios to be considered involve different configurations of the cost of the signaling activities. In the first setting, the cost of the signals is constant, implying that the signaling activity influences the interaction via the receivers’ payoffs only. The second setting explores choice between activities of distinct cost, ranked according to the level of the associated benefit, in which case the activity operates via the signalers’ payoffs as well.

Recall that the unique perfect Bayesian Nash equilibrium of the environment without choice of signal is symmetric, separating, in mixed strategies, and involves separate but adjoining interval supports such that signalers of high type choose higher signal intensities than signalers of low type, subject only to a non-decreasing beliefs restriction on the part of the receivers (Chapter 2). As the structure of the interaction can be shown to be equivalent to an incomplete-information all-pay auction with discrete types of bidders, who face asymmetric bidding costs, competing for a fixed prize, the main attribute determining the signalers’ behavior is the relative magnitude of their signals. The present scenario is more complex, because the signaler types can separate “within” a particular signaling activity as well as “across” signals, i.e., using different activities.

In view of the intuitive merit of the restrictions, the analytical focus within the present framework will again be on symmetric separating equilibria involving non-decreasing beliefs on the part of the receivers – besides an individual signaler being restricted to choosing exactly one signal. One might, accordingly, expect that the equilibrium profiles (if any) in the current setting will essentially be analogous to the outcome developed in Chapter 2, especially those involving only one of the signaling activities. For, if the signalers opt for the same signal, it will once again be the case that (only) the signal intensities determine the receivers’ decisions. On the other hand, if they coordinate on different activities, the total benefit bestowed on the

---

4.8 In other words, the receivers associate higher signaling magnitudes with a (weakly) higher probability of the signal having been sent by a signaler of (the) high(er) type.
receivers will move center stage (since the different signals will be of different intrinsic worth to the receivers), which may shift their attention away from the intrinsic value of their prospective allies. Nevertheless, in keeping with the restrictions on the equilibrium, even with both signals in use, the structure of the interaction will correspond to an all-pay contest, as the signaling costs are sunk and the signal-dependent benefits will induce the receivers to seek to ally with the signaler(s) conveying the highest (overall) benefit.

Beyond extending the signaler types’ options with regard to separating from one another, the differing benefits associated with the signals potentially affect their propensity to deviate. Given the receivers’ beliefs, in the present environment, departure from one’s prescribed strategy may alter the (expected) size of the intrinsic value as well as the signal-dependent benefit conveyed to them. A related key point regards the receivers’ beliefs about messages they do not expect to observe along the equilibrium path. Whereas, in Chapter 2, the beliefs for virtually every signaling magnitude the receivers could observe were pinned down by trivial application of Bayes’ Rule, the addition of one or more signals to the setting considerably broadens the array of configurations for their beliefs, especially off the equilibrium path (and thus the potential for deviation). For transparency, the model is initially presented with a restricted set of players – two signalers and one receiver – and a choice set comprising two beneficent signals. Having established the main arguments of the analysis, the framework is generalized to $n \geq 2$ signalers and $m \geq 1$ receivers.

### 4.3 The Model

#### 4.3.1 The Simplest Case: Two Signalers, One Receiver, and Two Signals

Let $m \in \{\underline{b}, \overline{b}\}$ refer to the set of observable signaling activities regarding some unobservable social quality. Signal $\underline{b}$ is assumed to be associated with a benefit (to the receiver) per unit of signal denoted by $\underline{A}$ and signal $\overline{b}$ with a benefit denoted by $\overline{A}$. Let $s_i(m_i) \in \mathbb{R}_0^+$ indicate signaler $i$’s signaling magnitude when using signal $m_i$. The pair $(m_i, s_i(m_i))$ will be considered signaler $i$’s “message” to the receiver. Attention is restricted to two types of individuals in terms of their social quality – high (H) and low (L). The amount, or value, of the social characteristic held by each type of signaler is denoted by $\theta_i \in \{\theta^L, \theta^H\}$, with $\theta^H > \theta^L > 0$. Let $\lambda \in (0, 1)$ signify the
commonly known prior probability that a signaler is of high type. The cost for signaler $i$ of sending message $(m_i, s_i(m_i))$ when endowed with amount $\theta_i$ of the social characteristic is given by $c(m_i, s_i(m_i), \theta_i)$; it is assumed to be linear in $s_i(m_i)$.

The interaction of the two signalers and the receiver proceeds as follows:

1) Nature randomly and independently determines each signaler’s type – high (H) or low (L) – with probabilities $\lambda$ and $(1 - \lambda)$, respectively.

2) Each signaler is privately informed of his type.

3) The signalers simultaneously choose a message, comprising a signal $m_i \in \{\underline{b}, \overline{b}\}$ and signaling magnitude $s_i(m_i) \geq 0$, contingent on their type.

4) After observing both messages, the receiver decides which signaler to choose as her ally. She can form an alliance with exactly one signaler.

5) The payoffs are realized.

The players’ payoffs are as follows. If $d \in \{1, 2\}$ denotes the receiver’s choice of Signaler 1 or 2 conditional on having observed both of their messages $((m_1, s_1(m_1)), (m_2, s_2(m_2)))$, respectively, signaler $i$’s payoff is given by:

$$u_i^S((m_1, m_2), (s_1(m_1), s_2(m_2)), d, (\theta_1, \theta_2)) = p \cdot [1 - (i - d)^2] - c(m_i, s_i(m_i), \theta_i).$$

As in the environment without choice, entry into an alliance yields a commonly known fixed positive “prize” of value $p > 0$ for the chosen signaler, and a signaler only nets a positive payoff if he is, in fact, selected to be the receiver’s ally. The receiver’s payoff, in turn, is given by:

$$u^R((m_1, m_2), (s_1(m_1), s_2(m_2)), d, (\theta_1, \theta_2)) = \theta_d + A(m_d) \cdot s_d(m_d),$$

where $\theta_d$ signifies the type-dependent value of the chosen signaler, assumed to be independent of the signal, and $A(m_d) \cdot s_d(m_d)$ represents the receiver’s signal-dependent benefit, with (fixed) parameter $A(m_d) \in \{A, \overline{A}\}$ such that $\overline{A} > A > 0$.

Namely, without loss of generality, an alliance yields the receiver a benefit in the form of the chosen signaler’s type-dependent value ($\theta$) as well as a (private) benefit subject in size to the magnitude and type of the chosen signaler’s signal. Observe that all this specification requires is once more that the signal benefits the receiver – it may or may not benefit other individuals in a signaler’s social environment. Likewise, the players’ equilibrium strategies again depend on the signaling activity and level employed by the signalers and the configuration of the receiver’s beliefs about their type when assessing the messages.
4.3.1.1 Optimal Behavior of the Receiver

By backward induction, turn first to the receiver and consider her decision problem, which takes the following form:

$$\max_{i \in \{1, 2\}} \left[ \Pr(\theta^H | (m_i, s_i(m_i))) \cdot \theta^H + \Pr(\theta^L | (m_i, s_i(m_i))) \cdot \theta^L + A(m_i) \cdot s_i(m_i) \right].$$

As was the case in the absence of choice, the receiver would ultimately like to ally with a signaler of high type and, if the signalers appear to be of the same type, she could base her decision on the size of the signal-dependent benefit. In the present environment, however, she can also incorporate the signaling activity being used by each signaler into her assessment. Considering the differing size of the benefits associated with the signaling activities, this additional means brings the total benefit conveyed to her by each of the signalers into play. As a result, her main focus is no longer necessarily (only) on the intrinsic value of her prospective ally. To establish the receiver’s optimal strategy, one must therefore consider three issues.

For one, one needs to evaluate the impact of the signaling magnitudes on the receiver’s decision process. As the objective is to identify symmetric separating equilibria involving non-decreasing beliefs on the part of the receiver, given use of the same signal \((m)\) on the part of the signalers, she will believe that if \(s_i(m) \geq s_j(m)\), \(\Pr(\theta^H | s_i(m)) \geq \Pr(\theta^H | s_j(m))\) for \(i \neq j \in \{1, 2\}\). This implies that the only payoff-relevant consideration for a signaler within a particular signal is the level of his signal relative to that of the other signaler (Chapter 2). With different signals in use, provided that she adopts non-decreasing beliefs within each activity, the receiver may associate either signal with either type of signaler. Her judgment in this case will be driven by the (total) payoff from each signaler’s message.\(^9\)

The second aspect is the instance of out-of-equilibrium signals, i.e., messages she does not expect to observe along the equilibrium path. Although alternative configurations are feasible, a natural configuration for her beliefs in this context is that unanticipated messages are always associated with low type. The third and final concern are ties, which can arise in one of two ways: (1) The signalers choose the same signaling activity and magnitude, and (2) the signalers choose different signals but convey the same (total) payoff to the receiver. Paralleling Chapter 2, attention will be restricted to equilibria in which the receiver randomizes 50:50 between the

---

\(^9\) If all the receiver cared about was her ally’s type, any equilibrium profile featuring both signaling activities would fail to exist as one could not formulate a self-confirming profile for her beliefs.
signalers when indifferent. In other words, the receiver will optimally want to behave as follows; in the following, this generic strategy profile will be specified as needed.

**Observation 4.1** If the signalers appear to be of the same type, the receiver will ally with the signaler who bestows the highest signal-dependent benefit on her. In general, she will base her decision on the (total) payoff conveyed to her by each signaler. In case of a tie, a natural strategy is the random selection of one of the individuals as her ally.

### 4.3.1.2 Equal Cost

Turn now to the optimal behavior of the signalers, starting (as a benchmark) with the scenario in which the cost of sending either signal does not vary between the signals. In particular, assume that the cost function takes the following form:

\[
c(m_i, s_i(m_i), \theta_i) = \begin{cases} 
\gamma^H \cdot s_i(m_i) & \text{if } \theta_i = \theta^H \\
\gamma^L \cdot s_i(m_i) & \text{if } \theta_i = \theta^L,
\end{cases}
\]

where \(\gamma^L > \gamma^H > 0\) denote the signaler types’ marginal cost of signaling, i.e., a given message is assumed to be more expensive for a signaler of low type. Besides the cost function satisfying the standard assumptions for signaling games, observe that the signaler types’ marginal cost does not depend on the type of signal used. Hence, the only way in which the signaling activity enters the framework is that one signal has more value to the receiver than the other. The issue of particular interest in this environment is whether, and under what conditions, an equilibrium can arise such that the less beneficent signal (\(s\)) is used.\(^10\)

To this end, consider signaler \(i\)’s decision problem, which is given by:

\[
\max_{m_i, s_i(m_i)} [E[W_i | (m_i, s_i(m_i))] - c(m_i, s_i(m_i), \theta_i)],
\]

where

\[
W_i = \begin{cases} 
p & \text{if } \theta_i + A(m_i) \cdot s_i(m_i) > \theta_j + A(m_j) \cdot s_j(m_j) \\
\frac{1}{2} \cdot p & \text{if } \theta_i + A(m_i) \cdot s_i(m_i) = \theta_j + A(m_j) \cdot s_j(m_j) \\
0 & \text{if } \theta_i + A(m_i) \cdot s_i(m_i) < \theta_j + A(m_j) \cdot s_j(m_j)
\end{cases}
\]

for \(i \neq j \in \{1, 2\} \).

Its defining feature is that, in case of a substantial difference between the signal-dependent benefits, the only payoff-relevant aspect is the signal in use. For, if the signalers employ different signals, the signaler choosing the less beneficial activity

\(^{10}\) One could, equivalently, ask whether one can construct a self-confirming set of beliefs that supports use (in equilibrium) of the less beneficial activity.
will not (necessarily) wish to choose a signal intensity sufficient to compensate for
the difference in the benefit provided to the receiver.

By arguments corresponding to those developed in Chapter 2, in keeping with
the restrictions on the equilibrium outcome(s) – separation, symmetry, one signal per
signaler, and non-decreasing beliefs – the setting is fully equivalent to an incomplete-
information all-pay auction with two discrete types of bidders, who face asymmetric
bidding costs, competing for a (fixed) prize $p$ by submitting the $s_i(m_i)$’s as their
bids. Likewise, the signaler types’ behavior can be analyzed independently. That is,
except for a fixed exogenous probability of winning (high type) or losing (low type),
each type of signaler effectively optimizes against the strategy of his own type only.
Therefore, having settled on an activity, the signalers follow a mixed strategy: If the
two signalers are perceived to be of the same type, it is always profitable (within the
bounds of the equilibrium supports) to deviate from a pure strategy by signaling
slightly more than the other signaler, thereby guaranteeing oneself the prize.
Importantly, this reasoning holds if the signalers choose the same or different signals.

For transparency, the approach in the following is two-tiered. In a first step, I
shall put forward a number of putative equilibria and argue (intuitively) why some
break down under the restrictions on the framework. Given these insights, in a
second step, I construct equilibrium profiles for the remaining outcomes, paying
careful attention to the configuration of the receiver’s beliefs. Note in this regard
that, from a signaler’s perspective, deviation from a particular magnitude using one
signal to the same magnitude using the other signal – a step that does not affect the
deviant’s cost of signaling but may alter his probability of winning – has two effects
on the receiver. She obtains a different signal-dependent benefit ($A$) and, subject to
her beliefs, expects to receive a different type-dependent value ($\theta$) from the deviating
signaler. Depending on the pre-deviation strategy, either effect may dominate, which
ultimately controls the profitability of the move.

In principle, the setup could support the following strategy combinations:
(1) both types of signaler separate within the less beneficent signal; (2) low types
choose the less beneficent and high types the more beneficent signal; (3) both types
of signaler separate within the more beneficent signal; and (4) low types choose the
more beneficent and high types the less beneficent signal. For purposes of argument,

---

4.11 The equivalence is trivially true if the signalers employ the same signal. With different signals in
use, the statement is also true on account of the aforesaid restrictions (cf. Footnote 4.16).
assume that each of these profiles constitutes an equilibrium (with the signalers adopting appropriate mixed strategies), and one-by-one reflect on whether either type of signaler can profitably deviate from the proposed outcomes.

The deduction that the strategies of the first putative profile – i.e., both types of signaler choose $b$ – will be analogous to those developed for the case without choice of signaling activity is straightforward. That is, in order for this profile to exist, the signaler types must randomize continuously on disjoint but contiguous interval supports (comprising strictly positive signaling magnitudes) such that high types choose strictly higher signal intensities than low types. One configuration of beliefs consistent with this outcome would be that the receiver has non-decreasing beliefs for signals of type $b$ and associates signals of type $\overline{b}$ with low type. Yet, (even) if so, a signaler of low type will have an incentive to deviate to some message using $\overline{b}$, as this move will effectively “out” him as a low type. He is considered to be a low type to begin with, implying that the deviation cannot make him worse but potentially better off since the deviating signal may provide an overall higher payoff to the receiver than a high type using $b$. As the receiver cares about her total payoff, this step must render him more attractive to her (despite his type). Markedly, this argument does not depend on the exact specification of the receiver’s beliefs off the equilibrium path. It goes through as given for all reasonable configurations of beliefs. The first strategy profile can thus clearly not constitute an equilibrium.

Suppose next that the signaler types separate such that low types choose $\underline{b}$ and high types $\overline{b}$, using mixed strategies on interval supports consistent with those in the scenario without choice, though in this case opting for different signaling activities. A set of beliefs in line with this profile assigns signals of type $\underline{b}$ as well as messages off the equilibrium path to low type, and signals of type $\overline{b}$ beyond some threshold to high type. The argument rendering this configuration not sustainable as an equilibrium is identical to the one just given. Namely, regardless of the receiver’s exact beliefs off the equilibrium path, a signaler of low type can profitably deviate to the more beneficent signal. In short, given parity in the signals’ cost, signalers of low type always prefer $\overline{b}$ to $\underline{b}$.

**Observation 4.2** If the cost of the signaling activities does not differ between the signals, there do not exist symmetric separating equilibria that involve low types choosing the less beneficent signal.
The remaining outcomes cannot be ruled out via the foregoing argument. Rather, as shown in the following, one can construct equilibria according to which signalers of high type send the highest total benefit to the receiver without signalers of low type being able to outdo them by deviating from their purported equilibrium strategy. To this end, consider the following auxiliary notation. Let $G^\tau_\kappa(s)$ denote the signalers’ symmetric equilibrium distribution (c.d.f.) of signal intensities using a particular signal in equilibrium $\kappa$ when of type $\tau \in \{L, H\}$, i.e., the probability that the chosen magnitude is no greater than some level $s$ ($\Pr(s_i(m_i) \leq s)$), and let $g^\tau_\kappa(s)$ denote the density function associated with $G^\tau_\kappa(s)$, if it exists. I shall construct equilibria in which $G^\tau_\kappa(s)$ is (absolutely) continuous – meaning density $g^\tau_\kappa(s)$ exists and the distribution does not contain any atoms – and the supports of the signaler types’ mixed strategies are intervals. In particular, let the support of signalers of low type be defined as $[\underline{s}_\kappa^L, \bar{s}_\kappa^L]$ and the support of signalers of high type be defined as $[\underline{s}_\kappa^H, \bar{s}_\kappa^H]$, where $\underline{s}_\kappa^\tau$ and $\bar{s}_\kappa^\tau$ represent the respective lower and upper bounds in equilibrium $\kappa$.\footnote{Paralleling the setting without choice, if $\underline{s}_\kappa^H = \underline{s}_\kappa^L$, then – for separation – both distributions must not contain an atom at this point.} Turn first to the scenario where both signaler types choose $\vec{b}$.

**Both Signaler Types Choose $\vec{b}$ ($\kappa = 1$).** Taking as given the signalers’ choice of the more beneficial activity, it is easy to see that this setting, too, is equivalent to the one analyzed in Chapter 2. Therefore, barring a reconsideration of the receiver’s beliefs off the equilibrium path, the equilibrium – henceforth (also) referred to as $E_\kappa$ – will be identical to the unique symmetric separating outcome characterized in the absence of choice. To be precise, if $\delta_1^L$ denotes the upper bound of signalers of low type and $\delta_1^H$ that of signalers of high type, low types randomize continuously on $[0, \delta_1^L]$, high types randomize continuously on $[\delta_1^L, \delta_1^H]$, where:

$$
\delta_1^L = (1 - \lambda) \cdot \frac{p}{y^L} \quad \text{and} \quad \delta_1^H = (1 - \lambda) \cdot \frac{p}{y^L} + \lambda \cdot \frac{p}{y^H},
$$

and the receiver adopts threshold beliefs with the threshold set at $\delta_1^L$. The signaler types’ equilibrium strategies, in turn, are given by:

$$
G^L_\kappa(s_i(\vec{b})) = \frac{y^L}{(1 - \lambda)} \cdot \frac{p}{s_i(\vec{b})} \quad \text{and} \quad G^H_\kappa(s_i(\vec{b})) = \frac{y^H}{\lambda} \cdot \frac{p}{s_i(\vec{b})} - (1 - \lambda) \cdot \frac{p}{y^L}.
$$

The most elementary configuration of the receiver’s beliefs for signals of type $b$ would be that they are deemed to indicate low type (Figure 4.1). If so, neither
type of signaler – especially one of low type – has an incentive to deviate to a message involving $b$ as doing so would strictly reduce the signal-dependent benefit conveyed to the receiver (Appendix 4.1.a, Section 4.A). In other words, $A > \bar{A}$ in conjunction with parity in the signals’ cost induces the signalers to send $\bar{b}$ rather than $b$. In fact, this line of reasoning holds *even* if the receiver’s beliefs are such that $b$ is associated with high type for all non-zero signaling levels. Note that the arguments verifying existence within the signal as well as those ruling out atoms, gaps, and overlaps carry over to the present framework as given in Chapter 2.

**Figure 4.1.** Equal Cost – The Equilibrium Supports and Beliefs for the Case When Both Signaler Types “Pool” on the More Beneficent Signal ($E_1$)

The intuition for this outcome, as opposed to the signaler types coordinating on the less beneficent signal, is rooted in the fact that signalers of low type are able to bestow on the receiver the higher benefit without an additional expenditure relative to high types. All that differs between the signaler types is the relative cost of signaling *within* a particular signal; low types are thus not necessarily disadvantaged by their low(er) intrinsic value. Only if both signaler types choose the activity with the highest associated benefit can low types not “out-maneuver” high types via the benefit linked to their deviating message, thereby enabling a balancing of the competitive pressures inherent in the setup along the lines of Chapter 2.

**Low Types Choose $\bar{b}$ and High Types $b$ ($\kappa = 2$).** Although the outcome involving signalers of low type choosing the more and those of high type the less beneficent signal – hereafter (also) denoted $E_2$ – might seem counter-intuitive as signalers of high type effectively “understate” their messages (or “counter-signal”), it can obtain provided that high types are able to convey at least the same payoff to the receiver using $b$ as do low types using $\bar{b}$, i.e.:
\[ \theta^H + A \cdot s_i(b) \geq \theta^L + \bar{A} \cdot s_i(b). \]

One would expect, however, that this will only be possible if the difference between the signal-dependent benefits is limited and/or the intrinsic value of signalers of high type is quite substantial. They will otherwise not be able to offset the (smaller) benefit they send by way of their signal. Note in this regard that, given the signalers’ choice of signaling activity, the most clear-cut configuration of the receiver’s beliefs to support this profile would be the arrangement depicted in Figure 4.2.

**Figure 4.2. Equal Cost – Interval Supports and Beliefs Consistent with the Outcome Where Low Types Choose the More and High Types the Less Beneficent Signal (E₂)**

In view of the close relationship between the equilibrium supports and the receiver’s payoff from each signaler, consider the equilibrium support of signalers of low type. A strictly positive lower bound cannot be an equilibrium strategy in this case either because, taking as given the receiver’s beliefs, a signaler could profitably deviate to zero thereby lowering his cost of signaling without affecting his probability of winning. Consistency with a mixed-strategy solution, correspondingly, obliges that the expected equilibrium payoff also be zero, which fixes the upper bound of the support (\(\hat{s}^L_2\)) at this type’s expected break-even payoff. As shown in Appendix 4.1.b, paralleling E₁ and the environment without choice, it is given by:

\[ \hat{s}^L_2 = (1 - \lambda) \cdot \frac{p}{\gamma^L}. \]

Key in the context of the bounds of the high types’ equilibrium support is that the upper bound of the support of signalers of low type is fixed. The reasoning is straightforward. Given their lower marginal cost of signaling, in order to be able to win the prize, a signaler of high type must be able to outperform all low types. He will therefore consider their upper bound a rigid lower boundary when deliberating the lower bound (\(\hat{s}^H_2\)) of his own support. More formally, to determine the position of
Chapter 4. Choice of Signaling Activity

At first glance, a gap may seem most intuitive as it suggests that signalers of high type will compensate for the smaller benefit associated with their signal by choosing strictly higher signaling magnitudes than signalers of low type. Hence, suppose that $s_2^H > s_2^L$. Despite the fact that the receiver associates all signals above $s_2^L$ and below $s_2^H$ with low type, high types have an incentive to deviate. In particular, when choosing a signal within the gap interval (e.g., $s_2^L$ using $b_1$), a signaler of high type can reduce his expenditure without affecting his probability of winning – he continues to lose against all (other) high types and wins against all low types. A lower bound such that $s_2^H > s_2^L$ can thus not be an equilibrium strategy.

Suppose instead that $s_2^H < s_2^L$. This scenario (too) is susceptible to deviation because signalers of low type can profitably mimic those of high type. For, rather than choosing a signal intensity along his own support (using $b_1$), he could send the same magnitude using $b_2$ without incurring a higher cost of signaling, yet thereby strictly increasing his probability of being chosen as the receiver’s ally since she would (now) consider him to be of high type. Accordingly, a lower bound such that $s_2^H < s_2^L$ cannot constitute an equilibrium strategy either. Observe that this argument also rules out all possible pooling and partial pooling equilibria.

Finally, consider $s_2^H = s_2^L$, i.e., the signaler types randomize on disjoint but contiguous supports. This is the only configuration not susceptible to deviation since – given the receiver’s beliefs – neither type of signaler can improve his situation without either affecting his cost of signaling or probability of winning. In short, as in the preceding equilibrium $(E_1)$ and the scenario without choice, there cannot exist gaps between or overlaps of the equilibrium supports.

On account of the difference in benefits associated with the signaling activities, existence of the equilibrium requires that a further condition be satisfied. To be precise, the contiguity of the signaler types’ equilibrium supports does not rule out the possibility that a low type choosing $s_2^L$ conveys a higher payoff to the receiver than a high type choosing $s_2^H$ or, in fact, any other magnitude along the high types’ equilibrium support. It must therefore in equilibrium also hold that:

---

4.14 This argument hinges on the fact that there is no atom at $s_2^H$ because, if so, deviation below that level would strictly reduce the signaler’s probability of winning. The non-existence of atoms throughout the supports can be established via arguments paralleling those developed in Chapter 2.

4.15 This argument subsumes gaps within the bounds of the supports.
\[ \theta^H + A \cdot \hat{s}_t \geq \theta^L + \bar{A} \cdot \hat{s}_t, \]

where \( \hat{s}_t \) denotes some signal intensity along the high types’ equilibrium support. Manipulation straightforwardly reduces this expression to:

\[ \theta^H \geq \theta^L + \hat{s}_t \cdot (\bar{A} - A). \]

The “critical” levels of \( \hat{s}_t \) are the bounds of the supports. Namely, in order to ensure existence, it must be the case that (1) the receiver’s payoff when allying with the lowest high type is at least as great as her payoff when allying with the highest low type, and that (2) the message of the highest high type is valued less by the receiver if it involves the more beneficial activity but is deemed to come from a low type than said message along his equilibrium support. Seeing as the latter is the stricter requirement, only it is binding. In particular, owing to the infeasibility of overlaps in equilibrium, if it is true that a high type’s message involving his upper bound (\( \hat{s}_2^H \)) using \( \hat{b} \) maps into the receiver’s payoff such that she prefers to ally with him rather than his alter-ego using \( \hat{b} \) (but deemed a low type), this preference ranking will hold for all magnitudes along the signaler’s equilibrium support (Figure 4.A.2, Appendix 4.1.b). Hence, the necessary and sufficient condition for the equilibrium where signalers of low type choose \( \hat{b} \) and those of high type \( \hat{b} \) to exist is given by:

\[ \theta^H \geq \theta^L + \hat{s}_2^H \cdot (\bar{A} - A). \] (4.1)

The algebraic form of the upper bound of the support then follows from the fact that a signaler adopting a mixed strategy must make the same payoff at all points along his equilibrium support. As shown in Appendix 4.1.b, it is given by:

\[ \hat{s}_2^H = (1 - \lambda) \cdot \frac{p}{y^L} + \lambda \cdot \frac{p}{y^H}. \]

Having established the signaler types’ equilibrium supports to be on \([0, \hat{s}_2^L]\) for low types and on \([\hat{s}_2^L, \hat{s}_2^H]\) for high types, their optimal strategies once again follow straightforwardly from their expected payoff functions (Appendix 4.1.b):

\[ G^L_2(s_i(\hat{b})) = \frac{y^L}{(1 - \lambda) \cdot p} \cdot s_i(\hat{b}) \quad \text{and} \quad G^H_2(s_i(\hat{b})) = \frac{y^H}{\lambda \cdot p} \cdot \left(s_i(\hat{b}) - (1 - \lambda) \cdot \frac{p}{y^L}\right). \]

The arguments ruling out deviation within each signal carry over to the present framework as given in Chapter 2.16 Note that the receiver’s beliefs beyond the upper bound of signalers of low type are crucial with respect to sustaining this outcome as

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16 In this case, too, competition between the signaler types transpires via the relative level of their signals, implying that the setting unambiguously meets the criteria of an all-pay auction.
an equilibrium. If she were to associate the relevant magnitudes with high type, signalers of high type could profitably deviate to this region for all values of $\theta^{H}$.

**Synthesis.** Not unexpectedly, the players’ equilibrium behavior is fundamentally the same in all cases considered so far, i.e., with and without choice. In fact, signalers of low type and the receiver behave the same whether both signals are in use ($E_2$) or the signaler types coordinate on the same activity ($E_1$). The only divergence in the strategy of signalers of high type is the signaling activity in use; the support is of the same dimension in each case. The essential intuitive difference between the two scenarios with choice relates to the positioning of the equilibrium bounds. Note in this regard that Equation 4.1 could also be expressed as $s^H_2 \leq \frac{\theta^H - \theta^L}{\bar{A} - \underline{A}}$, a positive constant. The relationship indicates that “counter-signaling” ($E_2$) can arise if and only if the high types’ support balances the extra benefit to the receiver when allying with a high type (via their intrinsic value) with the extra benefit obtained by way of the higher signal-dependent benefit when allying with a low type. In line with expectation, this may be achievable if the signal-dependent benefits do not differ too much while the intrinsic values diverge markedly. If so, the “counter-signaling” outcome will co-exist with the equilibrium involving coordination on the more beneficent signal, which exists for all values of $A$ and $\theta$. If not, the degenerate nature of this outcome is sensible.

**Proposition 4.1** The framework with equality in the cost of the signaling activities allows for two symmetric separating equilibria with non-decreasing beliefs on the part of the receiver and the signalers choosing exactly one signal.

i. For all $\theta^H > \theta^L > 0$ and $\bar{A} > \underline{A} > 0$, the signaler types will coordinate on the more beneficent signal.

ii. In addition, so long as (4.1) holds, the types can concurrently separate across the signaling activities such that signalers of low type choose the more beneficent and signalers of high type the less beneficent signal. Otherwise, only the former equilibrium exists.

**Choice and Welfare.** The final piece of the puzzle relates to the signalers’ optimal choice of signaling activity. Although one might expect their decision for one activity over the other to be driven by their expected payoff, since the activities’ cost does not differ and the structure of the outcomes entails the same behavior and therefore expected payoff for each type of signaler, the signalers are indifferent with
respect to the signal they end up using. As such, the key player in this setting is the receiver – both in terms of her beliefs and her expected payoff.

To establish her preferred outcome, consider the welfare implications of each equilibrium. As costly signaling is inherently wasteful, the matter of interest is one of constrained efficiency. Provided both outcomes exist, which one is most beneficial? Since $\bar{A} > A > 0$, the answer to this question is immediate. The receiver is always better off when both types of signaler opt for the more beneficent signal ($E_1$).

**Corollary 4.1**

Given parity in the cost of the signaling activities, the contest on the more beneficent signal is always preferable to the one involving both signaling activities.

**Inferences.** In sum, if the signaling activity in use only affects the agents’ interaction via the receiver’s payoff function, the most desirable outcome from a welfare perspective coincides with the outcome most likely to obtain in the sense that it can arise for the broadest set of parameter values. Nevertheless, as speculated, even in this comparatively simple environment, more than one outcome may arise, and if the conditions permit, may arise at the same time. As such, in some cases, the signalers will in equilibrium opt for the inefficient signal.

From a technical perspective, the central feature is that signalers of low type favor the more beneficial activity as it enables them to compensate, to some extent, for their lower intrinsic value, thereby augmenting their chances of entering into an alliance with the receiver. This narrows the scope of the equilibrium identified in Chapter 2; while it, in principle, holds for any non-zero level of the signal-dependent benefit, given endogenous choice, it will only obtain if the signaler types coordinate on the most beneficent signal. Even so, observe that the outcomes summarized in Proposition 4.1 constitute the complete set of equilibria consistent with the restrictions on the framework and equality in the cost of the signals. All alternative configurations can be ruled out by arguments akin to those developed in Chapter 2.

**Observation 4.3**

Proposition 4.1 captures all equilibria consistent with parity in the signals’ cost and the (other) restrictions on the framework.

**Remarks.** Although the choice set underlying the analysis only comprises two elements (of the beneficent kind), the results are quite general in that use of a larger menu will yield the same set of equilibria. To be precise, as long as all of the signals
are inherently beneficent, since each signaler can only ever use exactly one signal at a time, the lone change will be that a broader range of signals are being set against each other. Since this adjustment does not affect the signaler types’ incentives, the set of equilibria will remain unchanged. In a similar spirit, the linearity of the cost function is not as restrictive as it may seem (cf. Chapter 2). Provided that the signaler types face asymmetric marginal costs in that signaling is more expensive for low types, non-linearity in the cost of signaling would not qualitatively affect the pattern of the equilibrium outcomes.\(^17\)

### 4.3.1.3 Unequal Cost

A legitimate critique of the analysis up until now is that it is not necessarily intuitive that activities with different benefits to the receiver do not differ in their costliness to the signalers. One might, instead, expect that the more beneficial activity requires more effort – be it in terms of time or money. The objective of this section is precisely this modification. In particular, assume that the cost function now takes the following form, \(ceteris paribus\):

\[
c(m_{i}, s_{i}(m_{i}), \theta_{i}) = \begin{cases} 
\gamma^{H}(m_{i}) \cdot s_{i}(m_{i}) & \text{if } \theta_{i} = \theta^{H} \\
\gamma^{L}(m_{i}) \cdot s_{i}(m_{i}) & \text{if } \theta_{i} = \theta^{L},
\end{cases}
\]

with \(\gamma^{L}(m_{i}) > \gamma^{H}(m_{i}) > 0\) and \(\bar{\gamma}^{\tau} > \gamma^{\tau}\), where \(\bar{\gamma}^{\tau} = \gamma^{\tau}(b)\) and \(\gamma^{\tau} = \gamma^{\tau}(b)\) for \(\tau \in \{L,H\}\). Namely, the signaling activities now differ in cost not only in that a given activity is more expensive for signalers of low type, but also in that sending the more beneficent signal is more expensive (per se). Accordingly, the signal in use matters with respect to its value to the receiver as well as its cost to the signalers.

Since the underlying structure of the game (e.g., the sunk nature of the expenditures on signaling) is unchanged, the revision to the cost function does not affect the players’ decision problems, nor does it alter the equivalence of the interaction to an incomplete-information all-pay auction (via the \(s_{i}(m_{i})\)’s). As such, the signaler types will once more adopt mixed strategies. It should furthermore be obvious that the present setting could, fundamentally, support the same strategy combinations as its equal-cost counterpart, i.e., “pooling” on either of the signaling activities and/or “separation” such that the signaler types opt for different signals.

\(^{17}\) Depending on the parameterization of the game, the “intuitive criterion” (Cho & Kreps 1987) may eliminate the outcome featuring separation across the signals (\(E_{2}\); Appendix 4.1.c).
In view of the results obtained thus far, the issue of interest now is whether, and under what conditions, an equilibrium can arise such that the more expensive signal ($\bar{b}$) is chosen. As before, I shall begin the analysis by exploring (intuitively) the signaler types’ incentives for deviation from the putative equilibrium outcomes, whereupon I construct equilibrium profiles for the strategy combinations not susceptible to deviation. Hence, for purposes of argument, assume once more that each putative profile constitutes an equilibrium (with the signalers adopting appropriate mixed strategies), and consider whether either type of signaler can profitably deviate from his proposed strategy.

Turn first to the outcome involving both signaler types choosing $\bar{b}$. Since this prospective equilibrium involves separation within a particular activity, the strategy profile will take the same form as the separating outcome developed for the scenario without choice, i.e., the signaler types randomize continuously on disjoint but contiguous interval supports (comprising strictly positive signaling levels) such that high types defeat all low types and the receiver adopts non-decreasing beliefs for signals of type $\bar{b}$. The most basic configuration of beliefs for messages off the equilibrium path to support this outcome is that all relevant signals are taken to indicate low type (cf. Figure 4.1). In contrast to the equal-cost case, where the more beneficial activity constitutes the only activity that could arise in equilibrium for this type of signaler, a signaler of low type may now have an incentive to depart from his purported equilibrium strategy to a message involving the less beneficial activity. For, this move will (strictly) reduce his cost of signaling without affecting the receiver’s beliefs about him. The activity’s lower cost may moreover enable him to choose a higher magnitude, implying that the deviation need not adversely affect his probability of winning in this respect either. Key to this end is clearly that the less beneficial activity is notably less expensive than its counterpart, while the reduction in the signal-dependent benefit is not substantial. Note that this reasoning (too) holds for all reasonable configurations of beliefs off the equilibrium path. Thus, counter to the setting with parity in the signals’ costs, this profile may fail to be an equilibrium.

Suppose next that the signaler types separate such that low types choose $\bar{b}$ and high types $\underline{b}$, i.e., they “counter-signal” using mixed strategies on interval supports. A set of beliefs consistent with this profile assigns signals of type $\bar{b}$ as well as messages off the equilibrium path to low type, and signals of type $\underline{b}$ beyond some
threshold to high type (cf. Figure 4.2). Contrary to the equal-cost case, this profile too may break down by the forgoing argument. That is, regardless of the receiver’s exact beliefs off the equilibrium path, given a suitable asymmetry in the costs and benefits, a signaler of low type can potentially deviate to the less beneficent signal thereby reducing his cost without necessarily reducing his probability of winning.

The pivotal type of signaler in the context of the remaining profiles is the high type. To be precise, the primary reason the outcome with both signaler types choosing $b$, which except for the signaling activity in use will have the same structure as the outcome with both types choosing the more beneficial activity, may break down is not that signalers of low type deviate to the more expensive signal. Rather, if the difference in the signals’ cost is not too large and/or the asymmetry in the signal-dependent benefits is sizeable, signalers of high type may have an incentive to deviate to the more beneficent signal. While said move will increase their cost, the associated increase in the benefit conveyed to the receiver may be sufficient to compensate the added expenditure via the ensuing rise in the probability of being chosen as the receiver’s ally. The outcome involving low types choosing $b$ and high types choosing $\bar{b}$, i.e., the reverse of the aforesaid outcome involving both signals,$^{18}$ may break down on account of the converse of this line of reasoning. That is, if the difference in the cost of the signals is large and/or the asymmetry in the signal-dependent benefits small, signalers of high type may have an incentive to deviate to the less beneficent signal, thereby reducing their cost without inevitably reducing their probability of winning as they could afford a higher magnitude.

In short, depending on the (relative) asymmetry of the activities’ cost and the associated benefits, each of the individual strategy profiles may (or may not) arise as an equilibrium of the setting with disparity in the cost of the signals. To formalize the players’ strategies and the conditions supporting the existence of the various outcomes, I shall once again resort to the auxiliary notation introduced in Section 4.1.2 and attempt to establish symmetric separating equilibria with (absolutely) continuous mixed strategies on interval supports. Turn first to the outcomes involving coordination on the same signal.

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$^{18}$ That is to say, the signaler types adopt mixed strategies on interval support consistent with that outcome and the receiver’s beliefs are such that she ascribes signals of type $\bar{b}$ as well as messages off the equilibrium path to low type, and signals of type $\bar{b}$ beyond some threshold to high type,
Both Signaler Types Choose $b$ ($\kappa = 3$). Taking as given the signalers’ choice of signaling activity, the algebraic form of the bounds of the equilibrium supports and the equilibrium strategies for the set of profiles where both signaler types opt for the same signal does not require much of a prologue. Starting with the outcome where both signaler types choose $b$, henceforward (also) labeled $E_3$, if $\delta^L_3$ denotes the upper bound of the equilibrium support of signalers of low type and $\delta^H_3$ that of signalers of high type, it is easy to validate that low types randomize continuously on $[0, \delta^L_3]$, high types randomize continuously on $[\delta^L_3, \delta^H_3]$, where:

$$\delta^L_3 = (1 - \lambda) \cdot \frac{P}{Y^L} \quad \text{and} \quad \delta^H_3 = (1 - \lambda) \cdot \frac{P}{Y^L} + \lambda \cdot \frac{P}{Y^H},$$

the receiver adopts threshold beliefs with the threshold set at $\delta^L_3$, and the signaler types’ equilibrium strategies are given by:

$$G^L_3(s_i(b)) = \frac{Y^L}{(1 - \lambda) \cdot p} \cdot s_i(b) \quad \text{and} \quad G^H_3(s_i(b)) = \frac{Y^H}{\lambda \cdot p} \cdot \left(s_i(b) - (1 - \lambda) \cdot \frac{P}{Y^L}\right).$$

Paralleling the case with parity in the signals’ cost, the most obvious configuration of the receiver’s beliefs for signals of type $b$ would be the judgment that all relevant signals indicate low type (Figure 4.3). Likewise, the arguments ruling out atoms, gaps, and overlaps as well as deviations within the signaling activity (again) transfer straightforwardly to the present framework as given in Chapter 2.

Figure 4.3. Unequal Cost – The Equilibrium Supports and Beliefs for the Case When Both Signaler Types “Pool” on the Less Beneficent Signal ($E_3$)

The most interesting aspects of the present scenario are the conditions precluding deviation across the signaling activities. Observe, in this regard, that if deviation is profitable when the marginal cost of sending the deviating message is zero (or, equivalently, the cost of signaling following deviation is unchanged), deviation to the same signal intensity using the other signal will be, too
(Appendix 4.2.a.i). In this light, consider a signaler of low type contemplating deviation from some $s_i(b)$ along his equilibrium support to some $s_i^d(b)$ such that $\gamma^L \cdot s_i(b) = \gamma^L \cdot s_i^d(b)$. Given her beliefs, this move would induce the receiver to expect a payoff of $\theta^L + A \cdot s_i(b)$. Deviation is not profitable if this level is less than or equal to her pre-deviation payoff of $\theta^L + A \cdot s_i(b)$. Accounting for the restriction that $s_i^d(b) = \frac{\gamma^L}{\gamma^C} \cdot s_i(b)$, this is the case for all non-zero signaling magnitudes if:

$$\frac{\gamma^L}{\gamma^C} \geq \frac{A}{A}.$$  \hspace{1cm} (4.2)

Since deviation from a signaling level of zero using the less beneficent signal to zero using the more beneficent signal does not affect the signaler’s cost, the benefit conveyed to the receiver, or the receiver’s beliefs about the signaler’s type (and thus the deviant’s probability of winning), this move is not profitable either. Therefore, if Equation 4.2 is satisfied, signalers of low type do not have an incentive to deviate from any point along their equilibrium support. Indeed, Equation 4.2 completely determines this type’s behavior – it is necessary and sufficient. As such, if the condition is violated, only outcomes involving signalers of low type using the more beneficial activity will be sustainable as equilibria. The strict-inequality converse of Equation 4.2 will hereafter be referred to as Equation 4.2*.

**Observation 4.4** If the cost of the signaling activities differs, Equation 4.2 completely determines the behavior of signalers of low type.

The decisive signaling magnitude for signalers of high type is their upper bound. If it is not profitable to deviate from that level, they will not have an incentive to deviate from lower levels either (cf. Figure 4.A.2). The reasoning to derive the appropriate relationship between the costs and benefits is otherwise analogous to that for signalers of low type: Deviation from $\delta^H_3 \cdot s_i^d(b)$ to some $s_i^d(b)$ such that $\gamma^H \cdot \delta^H_3 = \gamma^H \cdot s_i^d(b)$) would induce the receiver to expect a payoff of $\theta^L + A \cdot s_i(b)$, which is to be compared to her pre-deviation payoff of $\theta^H + A \cdot s_i^d(b)$. Using that $s_i^d(b) = \frac{\gamma^H}{\gamma^C} \cdot \delta^H_3$, deviation is not profitable if:

$$\theta^H \geq \theta^L + \delta^H_3 \cdot \left( \frac{\gamma^H}{\gamma^C} \cdot \delta^H_3 - A \right).$$  \hspace{1cm} (4.3)
In other words, akin to $E_2$, in order to preclude deviation by signalers of high type it must be the case that the message of the highest high type is valued less by the receiver if it involves the more beneficial activity but is deemed to come from a low type than said message along his equilibrium support.\(^{19}\)

The focus on deviations from the upper bound can (furthermore) be justified by noting that Equation 4.3 is hardest to satisfy at the upper bound, especially if the term in brackets is positive. Yet, even if it is not positive, deviation does not pay because the condition rendering deviation unprofitable if the receiver were to believe that messages involving $\bar{b}$ beyond the low types’ upper bound indicate high type is given by \(\frac{\bar{y}_H}{\bar{b}} > \frac{\bar{b}}{\bar{b}}\), the condition rendering the bracket negative (Appendix 4.2.a.ii). Nevertheless, contrary to signalers of low type, since it is defined in terms of the upper bound of their support, the exact location of which depends on the cost parameters, Equation 4.3 only constitutes a necessary condition on the high types’ behavior in equilibria involving the less beneficent signal.

The intuition for this outcome is straightforward. The key element preventing deviation by signalers of low type to their “preferred,” more beneficial activity is its (absolutely) higher cost, which cannot be offset by a higher probability of winning as even maintenance of the pre-deviation cost of signaling entails transmission of a lower magnitude. Whereas all that mattered in the equal-cost case was the asymmetry in the signal-dependent benefits, in the present setting, a precise balance must be struck between the asymmetry in the activities’ marginal cost and the associated benefits. Similarly, unless the cost of the more beneficial activity can be compensated by a higher benefit to the receiver and the signalers’ intrinsic value (to compensate for the choice of a lower signaling magnitude of the deviating signal), signalers of high type do not have an incentive to deviate either. Hence, as conjectured, this outcome is only viable if the difference in the cost of the activities is large and/or the benefits associated with the signals hardly diverge.

**Both Signaler Types Choose $\bar{b}$ ($\kappa = 4$).** Without further ado, if $\delta_h^L$ denotes the upper bound of the equilibrium support of signalers of low type and $\delta_h^H$ that of signalers of high type, the algebraic form of the equilibrium involving “pooling” on the more beneficent signal, henceforth (also) referred to as $E_4$, takes the now familiar

\[^{19}\text{In fact, given a constant ratio of costs and barring the positioning of the upper bounds, Equation 4.3 immediately reduces to Equation 4.1.}\]
form. Signalers of low types randomize continuously on \([0, \hat{\delta}_4^L]\), signalers of high types randomize continuously on \([\hat{\delta}_4^L, \hat{\delta}_4^H]\), where:

\[
\hat{\delta}_4^L = (1 - \lambda) \cdot \frac{p}{Y} \quad \text{and} \quad \hat{\delta}_4^H = (1 - \lambda) \cdot \frac{p}{Y} + \lambda \cdot \frac{p}{Y},
\]

the receiver adopts threshold beliefs with the threshold set at \(\hat{\delta}_4^L\), and the signalers’ equilibrium strategies are given by:

\[
G_4^L(s_i(b)) = \frac{\hat{\gamma}_L}{1 - \lambda} \cdot p \cdot s_i(b) \quad \text{and} \quad G_4^H(s_i(b)) = \frac{\hat{\gamma}_H}{\lambda} \cdot p \left( s_i(b) - (1 - \lambda) \cdot \frac{p}{Y} \right).
\]

The most straightforward configuration of the receiver’s beliefs for signals of type \(b\) would, as indicated, be the assessment that all relevant signals indicate low type (cf. Figure 4.1). Note that, in line with the disparity in costs, the equilibrium supports in this case are more condensed than those of the preceding outcome (\(E_3\)).

It is easy to verify that the condition preventing deviation by signalers of low type is indeed given by Equation 4.2\(^c\) (Appendix 4.2.b). Likewise, deviation from a signal at the upper bound by a signaler of high type to some \(s_i^d(b)\) such that \(\hat{\gamma}_H \cdot \hat{\delta}_4^H = \gamma_H \cdot s_i^d(b)\) is not profitable if \(\theta^H + \bar{A} \cdot \hat{\delta}_4^H\) is greater than or equal to \(\theta^L + \bar{A} \cdot s_i^d(b)\), which – using that \(s_i^d(b) = \frac{\hat{\gamma}_H}{\hat{\gamma}_H} \cdot \delta_4^H\) – can be expressed as:

\[
\theta^H \geq \theta^L - \delta_4^H \cdot \left( A - \frac{\gamma_H}{\hat{\gamma}_H} \cdot \bar{A} \right).
\]

Not unexpectedly, Equation 4.4 is nearly identical to Equation 4.3. In fact, the brackets have the same sign. Hence, the conditions can hold simultaneously (even if the bracket is non-zero). They cannot, however, fail at the same time. Rather, if Equation 4.3 fails, which may be the case if its bracket is positive, Equation 4.4 will hold, as the relevant relationship between the signaler types’ intrinsic values make it comparatively easier to satisfy. Analogously, if Equation 4.4 fails, which may occur if its bracket is negative, Equation 4.3 will hold.\(^{20}\) The intuition for this outcome is thus essentially the reverse of that for \(E_3\), i.e., as speculated, it is likely to be sustainable if the difference in the signals’ cost is small and/or the benefits associated with the activities diverge substantially. In conjunction with the fact that the

\(^{20}\) Yet, even if the bracket of Equation 4.4 is negative, the condition is not easily broken, for if it is satisfied at the upper bound it will hold for all other points along the equilibrium support.
conditions determining the behavior of signalers of low type, which cannot hold, nor fail simultaneously, these observations entail that the equilibria involving coordination on the same signaling activity by the signaler types, may obtain at the same time. However, unless \( \frac{v^l}{v^h} = \frac{\alpha}{\lambda} \), in which case one or other of the equilibria must obtain, existence of an equilibrium for this setting is not guaranteed.

**Lemma 4.1** Depending on the degree of asymmetry in the cost of the signaling activities vis-à-vis the asymmetry in the benefits associated with them, the framework with disparity in the signals’ cost allows for two symmetric separating equilibria involving use of the same signaling activity by the signaler types and non-decreasing beliefs on the part of the receiver.  
i. If (4.2) and (4.3) obtain, the outcome where both types of signaler choose the less beneficent signal constitutes an equilibrium.  
ii. If (4.2c) and (4.4) hold, the outcome where both types of signaler choose the more beneficent signal constitutes an equilibrium.  
Depending on the parameter values, the outcomes may arise or fail simultaneously, unless (4.2) holds with equality, in which case one or other must obtain.

**Low Types Choose \( b \) and High Types \( \bar{b} \) (\( \kappa = 5 \)).** Having explored the outcomes involving only one signaling activity, consider now the outcomes involving both signals, starting with the scenario where low types send \( b \) and high types \( \bar{b} \), henceforward (also) denoted \( \mathcal{E}_5 \). Considering the cost structure, this outcome would seem the most natural profile involving both activities. Signalers of low type choose the less expensive signal, signalers of high type the more expensive signal, and the receiver adopts the beliefs depicted in Figure 4.4.

The equilibrium support and strategy of signalers of low type will unambiguously be identical to that derived for the case when both signaler types “pool” on the less beneficent signal (\( \mathcal{E}_3 \)), i.e., signalers of low type will randomize continuously on \([0, \delta^l_b = \delta^l_{\bar{b}}]\), where:

\[
\delta^l_b = (1 - \lambda) \cdot \frac{p}{\sqrt{\lambda}},
\]

\(^{421}\) The conditions cannot hold simultaneously unless they are satisfied with equality, in which case the ultimate outcome will depend on the conditions determining the behavior of signalers of high type, although it could still happen that both equilibria arise.
while their equilibrium strategy will be given by:

\[ G_L(s_i(b)) = \frac{y^L}{(1 - \lambda) \cdot p} \cdot s_i(b). \]

As established in the context of \( E_3 \), the necessary and sufficient condition precluding deviation by this type of signaler to the more beneficial activity is (again) given by Equation 4.2.

**Figure 4.4.** Unequal Cost – Interval Supports and Beliefs Consistent with the Case When Low Types Choose the Less and High Types the More Beneficent Signal (\( E_5 \))

The inference that the signaler types’ equilibrium supports are likely to feature an overlap such that the high types’ lower bound (\( \delta^H_5 \)) undercuts the low types’ upper bound, too, is intuitive. In view of the higher cost (and benefit) of the more beneficent signal, signalers of high type will want to position their lower bound at a signaling magnitude that is (just) too expensive for signalers of low type while concurrently ensuring that they convey at least the same payoff to the receiver as do their counterparts, and with the least possible expenditure. As their chosen activity is more beneficial and their intrinsic value to the receiver higher, the signal intensity to achieve these ends will be relatively lower than the equivalent magnitude of signalers of low type (their upper bound in particular). To formalize this intuition, note that it can be decomposed into three requisites (or “limits;” Figure 4.5); bear in mind that the receiver associates all signals below \( \delta^L_5 \) with low type.

First, to guarantee victory against all low types, the lower bound of signalers of high type must not fall short of the signaling magnitude at which the cost for a low type is the same when sending said level using \( \bar{b} \), call it “\( C \),” as when signaling at his equilibrium upper bound (using \( \bar{b} \)), call it “\( E \).” Note that \( C < E \), because the more beneficial activity is strictly more expensive. A signaler of high type would not want
Figure 4.5. Unequal Cost – The “Limits” on the Lower Bound of the Equilibrium Support of Signalers of High Type for the Outcome Where Low Types Choose the Less and High Types the More Beneficent Signal ($E_s$)

Legend: The dashed box features preference relations on the part of the receiver. Superscripts denote the type of signaler being considered.

to stray to intensities below $C$, since signalers of low type could afford to mimic him, thereby increasing their chances of being chosen as the receiver’s ally at his expense. In order for an equilibrium involving the present combination of signals to exist, it must therefore be satisfied that $\delta^H_s \geq C$, with $C := \left\{ \frac{1}{\gamma^L} \cdot C = \gamma^L \cdot E \right\}$.

Second, it must be the case in any equilibrium consistent with the restrictions on the framework (separating, symmetry, one signal per signaler, and non-decreasing beliefs) that the receiver’s benefit from a signaler of high type sending a message involving the more beneficent signal is no less than her benefit from a signaler of low type signaling at his upper bound. That is to say, if the receiver’s point of indifference between a high type using $b$ and a low type choosing $E$ is denoted “$J$” (i.e., $J := E^L \approx_R J^H$, where the superscripts indicate the type of signaler), it must in equilibrium be the case that $J^H \geq_R E^L$ or, equivalently, that $(\theta^H + A \cdot J) \geq (\theta^L + A \cdot E)$. Signalers of high type will not want to locate their lower bound at a magnitude below $J$ as the ensuing benefit conveyed to the receiver would fail to match that of the highest low type, implying that $\delta^H_s < J$ cannot constitute an equilibrium strategy. Even though Figure 4.5 depicts $J$ as greater in magnitude than $C$, it may well be the case that their order is reversed. Since high types will not want to send signaling magnitudes below either of these boundaries, the lowest magnitude their lower bound can take is $\max \{C, J\}$.

The final limit on the high types’ lower bound, the upper boundary so to say, is given by the point – call it $B$ – at which the receiver is indifferent between a low type signaling at his upper bound and him sending a message using $b$ that conveys the
same (total) benefit as the equilibrium upper bound; namely, \( B := B^L \approx_R E^L \) or, equivalently, \( (\theta^L + \bar{A} \cdot B) = (\theta^L + A \cdot E) \). Signalers of high type will consider \( B \) the upper boundary of the range of possible values for their lower bound because magnitudes beyond that level are susceptible to deviation. In particular, a high type choosing \( \hat{s}_5^H > B \) could strictly reduce his cost of signaling without affecting his probability of winning, despite the receiver (now) considering him to be a low type (i.e., he would still win against all low types and lose against all high types), by sending \( B \) rather than \( \hat{s}_5^H \) – a profitable deviation. A lower bound such that \( \hat{s}_5^H > B \) can thus not be an equilibrium strategy. Observe that Equation 4.2 and \( \bar{A} > A \), imply that \( E > B > C \). Since \( \theta^H > \theta^L \), it will furthermore be the case that \( B > J \), for the higher intrinsic value of a signaler of high type entails that the benefit he conveys at \( J \) will be at least as great as that of a low type choosing \( B \).

In short, any outcome with a lower bound between \( \max \{ C, J \} \) and \( B \) may arise as an equilibrium for the scenario involving low types choosing the less and high types the more beneficent signal, with the lowest possible equilibrium support starting at \( \hat{s}_{5(\min)}^H = \min \{ \max \{ C, J \}, B \} \). For illustration, consider a lower bound (\( s \)) in the interior of this range. While deviation below \( s \) would reduce his cost of signaling, a signaler of high type would not want to do so as this move would also reduce his probability of winning, and more so than his savings in cost. For, he would continue to lose against all other high types but the receiver would now consider him a low type, implying that she would expect his (total) benefit to fall short of that of some (other) low types.

While it will always be the case that \( \hat{s}_5^H < \hat{s}_5^L \) – all alternative configurations, i.e., contiguous and gapped supports, can be ruled out (Appendix 4.2.c.i) – the algebraic form of the lower (and upper) bound of signalers of high type depends on which of the boundaries obtains (Appendix 4.2.c.ii). Supposing \( C \) constitutes the binding limit, i.e., the lowest lower bound, the lower bound \( \hat{s}_{5(C)}^H \) follows from the equality of the low types’ cost at \( C \) and \( E \). It is given by:

\[
\hat{s}_{5(C)}^H = \frac{Y^L}{Y} \cdot \hat{s}_5^L = (1 - \lambda) \cdot \frac{P}{Y^L}.
\]

The upper bound of the support \( \hat{s}_{5(C)}^H \) then, once more, ensues from the fact that a mixed strategy entails that a signaler must expect to obtain the same payoff at all points along his support. It takes the following form:
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\[
\hat{s}^H_{5(c)} = (1 - \lambda) \cdot \frac{p}{Y^L} + \lambda \cdot \frac{p}{Y^H}.
\]

Naturally, the support is more condensed than its counterpart in the equilibrium involving the less expensive signal only (E3) – both bounds are clearly of a smaller magnitude. Indeed, as presented, it is identical to the outcome where both signaler types “pool” on the more beneficial activity (E4), although this link depends on the limit being considered. Even so, with \( C \) binding and the equilibrium support (thus) established to be on \([\hat{s}^H_{5(c)}, \hat{s}^H_{5(c)}]\), the optimal strategy of signalers of high type can again be derived via their expected payoff function, yielding:

\[
G^H_{5(c)}(s_i(\bar{b})) = \frac{Y^H}{\lambda \cdot p} \left( s_i(\bar{b}) - (1 - \lambda) \cdot \frac{p}{Y^H} \right).
\]

Atoms can be ruled out by the arguments developed in Chapter 2. While the current derivations focus on \( C \), the equilibrium supports and strategies for the other boundaries can be ranked in the same order as the limits (Appendix 4.2.c.ii).

In view of the discussion relating to Equation 4.4, the necessary condition to rule out deviation by signalers of high type to messages involving \( b \) in the context of each individual equilibrium will unambiguously take the same form, subject to the substitution of the appropriate upper bound. For instance, if \( C \) is binding, deviation is not profitable if:

\[
\theta^H \geq \theta^L - \hat{s}^H_{5(c)} \cdot \left( \frac{A}{\bar{Y}^H} - \frac{\bar{Y}^H}{\bar{Y}^H \cdot A} \right).
\]

It follows that the condition ensuring existence of any equilibrium of this type must involve the lowest equilibrium. To be precise, if the intrinsic value of signalers of high type is (strictly) sufficient to compensate for the lower signaling magnitude at the lowest possible outcome, it will do so for higher levels as well. Accordingly, if \( \hat{s}^H_{5(min)} \) denotes the lowest upper bound, the necessary condition is given by:

\[
\theta^H > \theta^L - \hat{s}^H_{5(min)} \cdot \left( \frac{A}{\bar{Y}^H} - \frac{\bar{Y}^H}{\bar{Y}^H \cdot A} \right). \tag{4.5}
\]

Note that the receiver’s beliefs for signals below the lower bound of signalers of high type carry some importance with respect to the derivation of the bounds of the equilibrium support. Along similar lines, if the receiver deemed signals beyond the
low types’ upper bound to correspond to high type, Equation 4.5 would reduce to 
\[ \frac{\bar{A}}{\bar{A}} > \frac{\bar{F}^H}{\bar{Y}^H}, \] 
although this condition is (inevitably) satisfied if its bracket is positive.

**Observation 4.5** If the cost of the signaling activities differs, there exists a continuum of equilibria for the case when low types choose the less and high types the more beneficent signal.

Overall, the defining features of \( E_S \) are the existence of an overlap of the equilibrium supports and the viability of a range of equilibria. Fortuitously, the intuition for both is quite simple in that the only sensible way to induce a signaler of high type to consider sending the more expensive signal is by keeping his expenditure as low as possible. As the more expensive signal is inherently more beneficial to the receiver than the alternative option and the intrinsic value of signalers of high type strictly exceeds that of low types, achieving this end is straightforward as a lower signaling magnitude need not entail a lower (total) benefit for the receiver. More than one equilibrium outcome is feasible as it may be the case that the receiver’s point of indifference between signalers of low and high type (in terms of the benefits conveyed by the highest low type and lowest high type) does not coincide with the maximum level of the more beneficial activity low types can afford. The present configuration is not sustainable as an equilibrium if the cost of the signaling activities is constant because signalers of low type can always profitably deviate to the more beneficent signal. Still, as surmised, essential in the setting with unequal cost is that the disparity in the signals’ cost is not too large and/or the asymmetry in the signal-dependent benefits substantial.

Low Types Choose \( \bar{b} \) and High Types \( b \) (\( \kappa = 6 \)). Given the derivations for the preceding outcome (\( E_S \)) and its counterpart in the environment with equal cost (\( E_2 \)), the formalization of the “counter-signaling” outcome when the cost of the signals differs (hereafter (also) labeled \( E_S \)) is straightforward. Key is once more that signalers of high type convey at least the same payoff to the receiver using the less beneficial activity as do signalers of low type using the more beneficial activity. Recall that beliefs consistent with this profile assign signals of type \( \bar{b} \) as well as messages off the equilibrium path to low type, and signals of type \( b \) beyond some threshold to high type (cf. Figure 4.2).
Paralleling $E_5$, the signaler type of particular interest is the high type. For, it is a trivial exercise to verify that signalers of low type will behave just like their counterparts in the case when both types “pool” on the more beneficent signal ($E_4$), i.e., they randomize continuously on $[0, \delta^L_6 = \delta^L_4]$, where:

$$\delta^L_6 = (1 - \lambda) \cdot \frac{p}{\sqrt{r}},$$

while their equilibrium strategy takes the following form:

$$G^L_6(s_i(\bar{b})) = \frac{\gamma^L}{(1 - \lambda) \cdot p} \cdot s_i(\bar{b}).$$

Moreover, as established in that context, the necessary and sufficient condition precluding deviation to messages involving the less beneficial activity is given by Equation 4.2$^C$.

In sharp contrast to all other outcomes, the present profile will feature a gap between the signaler types’ equilibrium supports. The reasoning is virtually identical to the argument justifying the existence of an overlap in the companion case with both signals in use ($E_5$). Specifically, signalers of high type will want to position their lower bound at a signaling magnitude that is (just) too expensive for low types while conveying at least the same benefit to the receiver as do they, at the smallest possible expense. Since the activity in use by signalers of low type is more beneficial and expensive, implying that a given expenditure procures a higher signal intensity of the cheaper signal, the magnitude to achieve these ends must be relatively higher than the equivalent level of signalers of low type (their upper bound in particular). To formalize this intuition, note that it too can be broken down into the three aforesaid “boundaries” (Figure 4.6); once again, bear in mind that the receiver ascribes all signals below the lower bound of signalers of high type ($\delta^H_6$) to low type.

In particular, let $\mathcal{C}$ denote the point of indifference of signalers of low type with respect to their cost of signaling at their equilibrium upper bound ($\mathcal{E}$) or $\mathcal{C}$, let $\mathcal{J}$ refer to the receiver’s point of indifference between a high type choosing $\mathcal{J}$ and the highest low type (at $\mathcal{E}$), and let $\mathcal{B}$ signify the receiver’s point of indifference between a low type opting for $\mathcal{B}$ and one choosing $\mathcal{E}$. It is easy to establish that $\mathcal{E} < \mathcal{C} < \mathcal{B}$ and $\mathcal{J} < \mathcal{B}$, while the position of $\mathcal{J}$ relative to $\mathcal{C}$ depends on the parameter values. By arguments paralleling those developed for $E_5$, any outcome with a lower bound between max $\{\mathcal{C}, \mathcal{J}\}$ and $\mathcal{B}$, with the lowest lower bound given by $\delta^H_6(\text{min}) = \min \{\max \{\mathcal{C}, \mathcal{J}\}, \mathcal{B}\}$, may arise as an equilibrium for the scenario at hand.
Figure 4.6. Unequal Cost – The “Limits” on the Lower Bound of the Equilibrium Support of Signalers of High Type for the Outcome Where Low Types Choose the More and High Types the Less Beneficent Signal ($E_6$)

Legend: The dashed box features preference relations on the part of the receiver. Superscripts denote the type of signaler being considered.

It will in this case always be true that $\delta_b^H > \delta_b^L$, although the algebraic form of the lower (and upper) bound once more depends on which of the boundaries obtains (Appendix 4.2.d.i). If $C$ constitutes the binding limit, the lower bound (as before) follows from the equality of the low types’ cost at $C$ and $E$, and is given by:

$$\delta_b^{C} = \frac{\gamma^L}{\gamma^L} \cdot \delta_b^L = (1 - \lambda) \cdot \frac{p}{\gamma^L}.$$  

By the now familiar arguments, the upper bound of the support ($\delta_b^{C}$) amounts to:

$$\delta_b^{C} = (1 - \lambda) \cdot \frac{p}{\gamma^L} + \lambda \cdot \frac{p}{\gamma^H}.$$  

As may be expected, the support is broader than its counterpart when both types coordinate on the more beneficent signal ($E_4$); in fact, as presented, it is identical to that in the setting where both types “pool” on the less beneficial activity ($E_3$), although this correlation depends on the binding limit. The comparison with the equal-cost case ($E_2$), in turn, depends on the relative size of the cost parameters.

With $C$ binding and the equilibrium support established to be on $[\delta_b^{C}, \delta_b^{C}]$, the equilibrium strategy for signalers of high type once more follows from their expected payoff function (Appendix 4.2.d):

$$G_b^{C}(s_h(b)) = \frac{\gamma^H}{\lambda \cdot p} \cdot \left( s_h(b) - (1 - \lambda) \cdot \frac{p}{\gamma^L} \right).$$  

Atoms can be ruled out by the arguments developed in Chapter 2. In view of the discussion relating to Equations 4.3 and 4.5, it is easy to corroborate that the necessary condition to ensure existence of any equilibrium of this type is given by:
\[ \theta^H > \theta^L + \delta_{6(\text{min})} \cdot \left( \frac{\gamma^H}{\gamma^H} \cdot \bar{A} - \underline{A} \right), \]  

(4.6)

where \( \delta_{6(\text{min})} \) denotes the upper bound of the lowest possible equilibrium profile. The same caveats relating to the receiver’s beliefs as for \( E_2 \) and \( E_5 \) apply.

Intuitively, the defining features of \( E_6 \) are the existence of a gap between the equilibrium supports, while said configuration was ruled out in the corresponding case with equal cost (\( E_2 \)), and the viability of a continuum of equilibria. In line with expectation, the intuition for the attainment of the “counter-signaling” outcome is effectively the reverse of the previous case (\( E_5 \)). As construed, it is crucial that the difference in the cost of the activities is not too excessive while the divergence in the benefits associated with the signals is quite substantial. The equal-cost case (\( E_2 \)) was deterministic in the sense that only one outcome could arise, because \( \mathcal{E} = \mathcal{C} = \mathcal{I} < \mathcal{B} \).

To be precise, with parity in the signals’ cost, the equal-cost condition (\( \mathcal{C} \)) is trivial as the cost of signaling depends exclusively on the signaling magnitude being chosen, while the condition ensuring existence effectively corresponds to the receiver’s point of indifference with respect to her payoff between the lowest high type and the highest low type (\( \mathcal{I} \)). The point at which the receiver is indifferent between a signaler of low type signaling at his upper bound and one conveying the equivalent payoff using the less beneficent signal (\( \mathcal{B} \)) is not significant in that context because a signaler of low type would need to make a substantial investment – since his intrinsic value is not sufficient to compensate for the difference in benefits – without a corresponding increase in his probability of winning, implying that deviation to his equilibrium upper bound would always be more profitable.

In line with the comparison of the necessary conditions supporting the equilibria involving use of only one of the signaling activities, in this case, too, Equations 4.5 and 4.6 are inversely related, implying that they may hold but can never fail at the same time. This argument is independent of the exact positioning of the upper bound; the bound does, however, influence the range of parameters for which both equilibria may hold simultaneously. In conjunction with the fact that the conditions determining the behavior of signalers of low type, which as before cannot hold nor fail simultaneously, these observations indicate that the equilibria involving coordination on different signaling activities (\( E_5 \) and \( E_6 \)) may obtain and fail at the same time, unless \( \frac{\gamma^L}{\gamma^L} = \bar{A} \), in which case one or other of the equilibria must obtain.
Lemma 4.2

Depending on the degree of asymmetry in the cost of the signaling activities vis-à-vis the asymmetry in the benefits associated with them, the framework with disparity in the signals’ cost allows for two symmetric separating equilibria involving use of different signaling activities by the signalers, with each signaler using exactly one signal, and non-decreasing beliefs on the part of the receiver.

i. If (4.2) and (4.5) hold, there exists a continuum of equilibria involving signalers of low type sending the less and those of high type the more beneficent signal.

ii. If, instead, \((4.2^C)\) and (4.6) obtain, there exists a continuum of equilibria involving signalers of low type using the more and those of high type the less beneficent signal.

Depending on the parameter values, they may arise or fail simultaneously, unless (4.2) holds with equality, in which case one or other must obtain.

Synthesis. In general, the conditions supporting the outcome where both signaler types coordinate on the less beneficent signal \((E_3)\) and those ensuring existence of the outcome where low types choose the less and high types the more beneficial activity \((E_5)\) are such that if the condition preventing deviation by signalers of high type fails in one case, the chances that the other outcome obtains are high – if not both. More precisely, in both cases, the condition preventing deviation by signalers of low type is given by Equation 4.2, while the conditions ruling out deviation by high types (Equations 4.3 and 4.5, respectively) are inversely related. The situation is identical in the context of the outcome involving both types “pooling” on the more beneficial activity \((E_4)\) and that involving low types choosing the more and high types the less beneficent signal \((E_6)\). Provided that Equation 4.2\(^C\) ensures that signalers of low type do not have an incentive to deviate, the inverse relationship of Equations 4.4 and 4.6 effectively guarantees that one or the other, if not both, of the equilibria arises. Note that, if Equation 4.2 holds with equality, all equilibria may arise at once. In other words, at least one of the equilibria must obtain, although more than one may obtain at the same time.

Proposition 4.2

In the framework with unequal cost, at least one of the equilibria characterized in Lemmata 4.1 and 4.2 must obtain, although depending on the parameter values, more than one – and, in the extreme, all four – may arise simultaneously.
**Choice and Welfare.** What about the signalers’ optimal choice of signaling activity? Contrary to the case with parity in cost, the signal in this setting does affect their payoff, implying that the signalers are no longer (necessarily) indifferent vis-à-vis the activity they use. To establish the most efficient outcome, it is thus essential to consider explicitly the welfare implications of the profiles for the receivers as well as the signalers. The point of interest in this regard, taken the case that all four outcomes exist simultaneously,\(^\text{22}\) is whether one of them is always most preferable in terms of being least costly and/or most beneficial. To this end, consider the expected utility of a given player when of either role, starting with his/her preference ranking (i.e., in terms of the outcomes’ Pareto-dominance) when appointed signaler.

While signalers of low type, by construction, expect a payoff of zero in each case, implying indifference with respect to the ultimate outcome, signalers of high type expect various positive payoffs, entailing that they do care about which equilibrium obtains. As such, the expected benefit of a player assigned to be of high type amounts to his/her expected payoff in a given equilibrium, weighted by the probability that (s)he is of high type (\(\lambda\)). As illustrated in Table 4.A.1 (Appendix 4.2.e), the player’s expected utility for a given equilibrium is then simply the sum of his/her expected benefit when of each type.\(^\text{23, 24}\)

The derivation of a player’s expected utility when allocated the role of receiver follows a similar line of reasoning. One must account for all possible combinations of signaler types the receiver may encounter in a given interaction, weighted by the events’ appropriate probabilities. Specifically, provided equilibrium behavior by all parties, one must determine the expected value of the highest signaling magnitude to be expected in each of three possible scenarios – for, she may either encounter two low types, one low type and one high type, or two high types.\(^\text{25}\) Table 4.A.4 (Appendix 4.2.e) summarizes the receiver’s expectation in each equilibrium.

Seeing as the expected utility from one outcome must be greater than that of another if each element of the former exceeds that of the latter, straightforward mani-

---

\(^{22}\) One would ideally want to construct a multi-dimensional space featuring “regions of existence” for each equilibrium, including all relevant overlaps (i.e., areas of joint existence), and use it to derive general conditions regarding the Pareto-dominance of the various outcomes. Due to the complexity of this task, it is left for future work. This section, instead, focuses on necessary conditions in only one possible instance, i.e., the situation in which all outcomes exist at the same time.

\(^{23}\) The focus in the cases with multiple equilibria is on limits \(C\) and \(B\) (only) for ease of tractability.

\(^{24}\) Table 4.A.2 (Appendix 4.2.e) provides necessary and sufficient conditions establishing various preference rankings from the perspective of the signaler(s).

\(^{25}\) Table 4.A.3 (Appendix 4.2.e) illustrates the derivations for \(E_3\).
### Table 4.1. Unequal Cost – The Conditions Determining a Player’s Preference over the Various Equilibria (Row > Column)

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>$E_3$</th>
<th>$E_4$</th>
<th>$E_5^C$ ($E_5^B$)</th>
<th>$E_6^C$ ($E_6^B$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_3$</td>
<td></td>
<td></td>
<td>$\frac{y^H}{y^π} \geq \frac{y^L}{y^π} \geq \frac{\bar{A}}{A}$</td>
<td>$\frac{\bar{A}}{A} &lt; \frac{\bar{y}^H}{\bar{y}^π}$</td>
</tr>
<tr>
<td>$E_4$</td>
<td>$\frac{\bar{A}}{A} &gt; \frac{\bar{y}^L}{\bar{y}^π} \geq \frac{\bar{y}^H}{\bar{y}^π}$</td>
<td></td>
<td>$\frac{\bar{A}}{A} &gt; \frac{\bar{y}^L}{\bar{y}^π}$ &amp; $\frac{\bar{A}}{A} &gt; \frac{\bar{y}^H}{\bar{y}^π}$ (and vice versa)</td>
<td></td>
</tr>
<tr>
<td>$E_5^C$ ($E_5^B$)</td>
<td>$\frac{\bar{A}}{A} &gt; \frac{\bar{y}^L}{\bar{y}^π} \geq \frac{\bar{y}^H}{\bar{y}^π}$</td>
<td>$\frac{\bar{A}}{A} &lt; \frac{\bar{y}^L}{\bar{y}^π}$ &amp; $\frac{\bar{A}}{A} &gt; \frac{\bar{y}^H}{\bar{y}^π}$ (and vice versa)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_6^C$ ($E_6^B$)</td>
<td>$\frac{\bar{A}}{A} &gt; \frac{\bar{y}^L}{\bar{y}^π}$ &amp; $\frac{\bar{A}}{A} &gt; \frac{\bar{y}^H}{\bar{y}^π}$ (and vice versa)</td>
<td>$\frac{\bar{A}}{A} &gt; \frac{\bar{y}^L}{\bar{y}^π}$ &amp; $\frac{\bar{A}}{A} &gt; \frac{\bar{y}^H}{\bar{y}^π}$ (and vice versa)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend: The superscripts on the equilibria supporting multiple outcomes denote the limit being considered. The two within-outcome comparisons compare the row outcome using limit $C$ to the column outcome using limit $B$, e.g., the value provided for $E_5$ gives $E_5^C > E_5^B$.

A brief examination of the preference rankings reveals that they are, in fact, quite intuitive. For instance, as one might expect, coordination on the more beneficent signal ($E_4$) is favored to coordination on the less beneficent one ($E_3$) if $\bar{A}$ is sizeable and use of the associated activity ($\bar{b}$) is more expensive for signalers of low type. This intuition also rationalizes $E_5 > E_3$ and $E_4 > E_6$; as the receiver is (by construction) in both cases indifferent about the signal in use by signalers of low type, with $B$ binding, all that population yields the necessary conditions in Table 4.1.26 A brief examination of the preference rankings reveals that they are, in fact, quite intuitive. For instance, as one might expect, coordination on the more beneficent signal ($E_4$) is favored to coordination on the less beneficent one ($E_3$) if $\bar{A}$ is sizeable and use of the associated activity ($\bar{b}$) is more expensive for signalers of low type. This intuition also rationalizes $E_5 > E_3$ and $E_4 > E_6$; as the receiver is (by construction) in both cases indifferent about the signal in use by signalers of low type, with $B$ binding, all that

\[\text{4.26 Extensive manipulation reveals that a further necessary condition for the comparison of } E_5^C \text{ and } E_6^C \text{ is given by } \frac{\bar{A}}{A} > \frac{\bar{y}^L}{\bar{y}^π} \text{ alongside } \lambda > 1 - \frac{1}{\sqrt{2}} (\approx 0.25), \text{i.e., in contrast to the other conditions, this ranking also depends on the frequency of high types in the population.}\]
matters is the relationship between the signal-dependent benefits and the marginal cost of signalers of high type.

The fact that the conditions for $E_3 \text{ vs. } E_6^B$, $E_4 \text{ vs. } E_5^B$, and those “within” $E_5$ and $E_6$ are mutually exclusive, implying that they cannot be ranked as signalers and receiver always prefer different outcomes, too, makes intuitive sense. Take $E_3 \text{ vs. } E_6^B$, for example; observe that signalers of high type behave the same in both cases. If $\mathcal{B}$ – the condition rendering the receiver indifferent between a signaler of low type signaling at his equilibrium upper bound and one choosing $\mathcal{B}$ – is binding, the signalers will prefer “pooling” on the less beneficent signal because the relevant support involves lower signaling magnitudes, while the receiver favors the higher signal-dependent benefit. Note that the non-rankability of the range-outcomes contrasts sharply with traditional signaling models. The reason for this divergence is that the receiver in the present environment retains the benefit associated with the message of the signaler she chooses as her ally, whereas in standard frameworks any signal-dependent benefit is fully appropriated by the signalers.

In general, contrary to the scenario with equal cost, all generic rankings depend inherently on the parameter values of the particular setting being considered. Suppose, for illustration, that $\frac{Y^L}{Y^H} > \frac{Y^H}{Y^L}$, a natural conjecture as it entails that the more beneficent signal is comparatively more expensive for signalers of low type than their counterparts of high type. If so, the outcome involving “pooling” on the more beneficent signal ($E_4$) or the one with signalers of low type choosing the less and those of high type the more beneficial activity ($E_5$) will be most preferable, though the ultimate solution depends on the relative size of the signal-dependent benefits.

**Corollary 4.2** Given disparity in the cost of the signaling activities, which of the equilibria will be most preferable is ex ante ambiguous.

**Inferences.** Akin to the environment with parity in the signals’ costs, more than one equilibrium may arise and, if the conditions permit, may do so at the same time. This is the case despite the fact that the signalers’ behavior in this case, too, is fundamentally quite simple – low types effectively choose between two strategies and high types between a total of four. The results given a difference in the signaling activities’ cost therefore further underscores the conclusion that the sharp uniqueness prediction of the environment without choice is lost if the signalers may choose the
conduit of their messages. For, besides several types of equilibrium potentially co-existing, one may in some cases even obtain a continuum of equilibria. Moreover, as the size of the expected utility of the various outcomes depends on the parameter values, not only can one in this case not definitively rank the outcomes (ex ante), but a different outcome may be Pareto-optimal at any one time, including (apparently) inefficient ones. Even so, since all alternative configurations can be ruled out, the equilibria constitute the complete set of outcomes consistent with the restrictions on the framework and disparity in the cost of the signaling activities.

Proposition 4.3 Propositions 4.1 and 4.2 characterize all equilibria for the class of games involving symmetry among the signaler types, choice of exactly one signaling activity by each signaler from a choice set involving exactly two elements of a beneficent nature, and non-decreasing beliefs on the part of the receiver.

Remark. Extension of the choice set to more than two beneficent signals or non-linearity in the cost function once again do not qualitatively affect the results.\(^{27}\)

4.3.2 A Generalization

In view of the links between the frameworks with and without choice, it will come as no surprise that the generalization of the game to comprise a broader set of players, \(n \geq 2\) signalers and \(m \geq 1\) receivers, to be exact, is again effectively trivial. Consider a social environment with \(n \geq 2\) signalers and \(m \geq 1\) homogeneous\(^{28}\) receivers, ceteris paribus, and assume that the signalers are not restricted with respect to the number of alliances they can enter, i.e., a given signaler could, in the extreme, ally with all\(^{29}\) \(m\) receivers. As before, these assumptions alongside the other restrictions on the framework imply that each receiver will ultimately want to ally with the same signaler (the one conveying the highest total benefit). To demonstrate that these adjustments to the setup do not qualitatively affect the results in either the equal or unequal cost setting, I shall briefly outline the main elements of the analysis

\(^{27}\) As detailed in Appendix 4.2.f, the “intuitive criterion” does not rule out either of the equilibria involving coordination on the same signaling activity on the part of the signaler types (i.e., \(E_3\) and \(E_4\)). In sharp contrast, in the cases featuring separating across the signals (i.e., \(E_3\) and \(E_6\)) it does eliminate all outcomes between \(\max\{\mathcal{C},\mathcal{J}\}\) and \(\mathcal{B}\), reducing the set of equilibria in both cases to a unique point-prediction at \(\max\{\mathcal{C},\mathcal{J}\}\).

\(^{28}\) This assumption is not unreasonable if one presumes that the average quality of the social goods the signalers are competing for is constant (cf. Chapter 2).

\(^{29}\) Refer to Footnote 2.27 (Chapter 2) for a consideration of the implications of this assumption.
with many players for the case when the cost of the signaling activities differs and
the signaler types coordinate on the less beneficent signal – call it $E_{3(G)}$.

If $\delta_i \in \{1, \ldots, n\}$ denotes receiver $i$’s choice of signaler $k \in \{1, \ldots, n\}$ conditional on having observed each of their messages $((m_1, s_1(m_1))$ through $(m_n, s_n(m_n))$, respectively), signaler $i$’s payoff is given by:

$$u_i^S \left( (m_1, \ldots, m_n), (s_1(m_1), \ldots, s_n(m_n)), (\delta_1, \ldots, \delta_m), (\theta_1, \ldots, \theta_n) \right) = p \cdot \sum_{r=1}^{m} f(\delta_r, i) - c(m_i, s_i(m_i), \theta_i),$$

where

$$f(\delta_r, i) = \begin{cases} 1 & \text{if } \delta_r = i, \text{ for } r = 1, \ldots, m. \\ 0 & \text{otherwise} \end{cases}$$

Receiver $i$’s payoff, congruently, takes the following form:

$$u_i^R \left( (m_1, \ldots, m_n), (s_1(m_1), \ldots, s_n(m_n)), (\delta_1, \ldots, \delta_m), (\theta_1, \ldots, \theta_n) \right) = \theta_{\delta_i} + A(m_{\delta_i}) \cdot s_{\delta_i}(m_{\delta_i}).$$

Provided non-decreasing beliefs on the part of the receivers, the correspondence of the setting to a two-type all-pay auction with incomplete information continues to hold. In order to derive the signalers’ equilibrium supports and strategies, let $G_{3(G)}(s)$ refer to the (absolutely continuous) symmetric equilibrium distribution (c.d.f.) of signaling magnitudes when of type $\tau \in \{L, H\}$, and let $g_{3(G)}(s)$ denote the associated density function. As the signalers’ choice set is limited to two activities, the generalized framework once more only supports the now familiar four strategy combinations, i.e., “pooling” on the same signal or “separation” across activities.

Paralleling the environment without choice, by separation and symmetry, a signaler of low type sending $s_L(b)$ will only stand a chance of being chosen as a receiver’s ally if all other signalers are also of low type (which occurs with probability $(1 - \lambda)^{n-1}$); if so, his probability of winning is given by $[G_{3(G)}(s)]^{n-1}$. Using these insights, the equilibrium support can be shown to be on $[0, G_{3(G)}^L(s)]$, where:

$$\hat{\delta}_{3(G)}^L = (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{\gamma^L},$$

while the equilibrium distribution function takes the following form:

$$G_{3(G)}^{L}(s_1(b)) = \left( \frac{\gamma^L}{m \cdot p \cdot (1 - \lambda)^{n-1} \cdot s_1(b)} \right)^{n-1}.$$
By a similar logic, the equilibrium support of signalers of high type will be on $[\hat{s}^H_{3(G)}, \hat{s}^H_{3(G)}]$, where:

$$
\hat{s}^H_{3(G)} = (1 - \lambda)^{n-1} \cdot \frac{m \cdot p}{Y^L} + (1 - (1 - \lambda)^{n-1}) \cdot \frac{m \cdot p}{Y^H},
$$

with the equilibrium distribution function given by:

$$
G^H_{3(G)}(s_i(b)) = \frac{1}{\lambda} \left[ \lambda - 1 + \left( \frac{Y^H}{m \cdot p} \cdot (s_i(b) - \hat{s}^L) + (1 - \lambda)^{n-1} \right)^{\frac{1}{n-1}} \right].
$$

Essential in this regard is that the intensity of the signal of a given high type relative to that of other signalers depends on the number of equally strong signalers in his social environment. It is easy to validate that the intuition for this outcome will mirror that for the setting without choice while, for $n = 2$ and $m = 1$, the foregoing expressions reduce to those derived for the simple case (i.e., $E_3$).

Arguments analogous to those developed in Chapter 2 can be used to verify the existence of the equilibrium within the activity. Given the receivers’ homogeneity, it furthermore follows (immediately) that the conditions ruling out deviation across the signaling activities are given by Equations 4.2 and 4.3, the latter adjusted with respect to the appropriate upper bound ($\hat{s}^H_{3(G)}$). The reasoning is straightforward. If one receiver prefers to ally with a given deviant, so will all others.

These computations could obviously be reproduced for all other outcomes derived for the simple case, which entails that the set of equilibria is again exhaustive. The welfare results, too, remain unchanged. Hence, there does not necessarily exist a universal “best” under this set of circumstances either.

**Proposition 4.4** The results characterized in Propositions 4.1 through 4.3 generalize straightforwardly to a setting with $n \geq 2$ signalers and $m \geq 1$ homogeneous receivers, *ceteris paribus*.

**Many Signals.** Crucial for the simplicity of the extension to many players is the compact nature of the signalers’ choice set – comprising only two signals (of beneficent nature). With many signals, the analysis would be considerably more complicated, although mainly with respect to the signaler types’ incentives for deviation. For, given symmetry within the signaler types and the restriction that a given signaler can use exactly one signal at a time, the equilibrium supports and strategies will not fundamentally be affected. As one needs to consider more than one potential “destination” for deviation, however, it will be necessary to establish
conditions ruling out deviation to every possible alternative signaling activity, i.e., if
the setting involved q signals, one would need to construct q − 1 conditions. While
the conditions will be identical to those developed for the two-signal case, each
would need to be compared to every other condition, which clearly renders the
analysis more intricate. Nonetheless, I would expect to be able to construct most of
the outcomes derived for the simplified settings in this environment as well.30

4.3.3 Alternative Menus of Signaling Activities

Besides questioning whether the results derived for the setting with two
signalers, one receiver, and two signals generalize to a broader set of players, one
may wonder whether they will continue to hold if the choice set comprises
alternative combinations of signals, particularly ones without benefits to the
receiver(s). Recall, in this regard, that even if one only considers separating
equilabilia with threshold beliefs, in the scenario without choice, use of neutral
signaling activities results in a multiplicity of “belief-based” equilibria. In
consequence, the analysis given choice of messaging tool will be more complex
because it is ex ante not clear which of the various outcomes will arise, which
renders the foregoing comparative approach not viable.31 As before, the main
complicating factor will be the signalers’ incentives for deviation. Rather than a
formal analysis of this extension, the purpose of this section is therefore to ponder
briefly the likely adjustments given the two main alternative compositions of the
choice set, presuming it only comprises two activities.

Consider first the case of the choice set featuring a neutral and a beneficial
activity, and suppose hypothetically that both cases (separately) yield the same
equilibrium.32 If so, the results would unambiguously match those with a choice set
of two beneficent signals. From a welfare perspective, however, the contest on the
beneficial activity would always be Pareto-dominant (on account of the non-zero
benefit to the receiver). Even with other outcomes in the context of the neutral signal
— say, low types pool on zero, high types pool on some s∗ > 0, and the receiver

4.30 For completeness, rather than considering two separate populations of agents, or equivalently, a
single population comprising two kinds of agents, it would again be necessary also to consider the
case when each individual takes both roles, i.e., sends and receives signals (cf. Chapter 2).
4.31 Hertzendorf and Overgaard (2001) develop (novel) customized equilibrium refinements to render
their multi-dimensional signaling approach involving neutral signals (only) tractable. I would expect
that a similar method be required in this case.
4.32 As revealed in Chapter 3, this is not only a theoretical possibility but may also arise in practice.
adopts appropriate threshold beliefs – one would expect equilibria to obtain, including ones involving both signals, although most are prone to be degenerate. Hence, all one can conclude for this putative state is that, regardless of the positioning of $s^*$ (i.e., above or below the equilibrium support of signalers of high type), any outcome involving the beneficent signal will be strictly preferable from a welfare perspective. More generally, one would expect a multiplicity of equilibria, exaggerated by the belief-based nature of outcomes involving neutral signals, with equilibria featuring the beneficial activity always considered Pareto-superior.

How about a choice set comprising neutral signals only? While one would certainly expect equilibria to exist (involving coordination on the same and different activities), without refinement to limit the number of possible outcomes, predictions are virtually impossible. If the signaler types “pool” on distinct magnitudes using different signals, the profile is likely to be sustainable by arguments paralleling those for the environment without choice. Nevertheless, as in that case, the receiver’s beliefs can take countless configurations, potentially leading to a plethora of outcomes. From a welfare perspective, given the lack of benefits to the receiver, the main aspect will once more be the costliness of the signaling activities (only).33

**Observation 4.6** Given a more general choice set, one would expect a multiplicity of equilibria, with those involving a beneficent signal deemed Pareto-superior.

### 4.4 Discussion

One of the most conspicuous findings of this chapter is that the sharp uniqueness prediction exemplifying the setting without choice is lost. Besides several types of equilibrium potentially co-occurring, some circumstances support continua of equilibria, as in Spence’s seminal model (1973, 1974). The main difference between the scenarios with and without choice to account for this divergence is the option to separate across the signaling activities when given choice, which can be supported – be it in a degenerate manner – even if the cost of the signals does not differ. Whereas all that matters with respect to the equilibrium if the signaler types separate within a particular signaling activity is the relative magnitude of their signals, in the cases with more than one activity in use, the overall benefit conveyed

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33 In view of the resemblance to standard models, the outcomes are likely to be Pareto-rankable.
to the receiver(s) by a given signaler becomes the driving element, which distorts the receiver’s attention away from her ultimate ally’s type. In other words, in these cases, the interactants may effectively “marry for money.”

An associated maxim is that inefficient signals may arise, and may do so in equilibrium, regardless of one’s assumption on the asymmetry in the signals’ costliness. Markedly, with disparity in their cost, it may under some conditions even be the case that an apparently inefficient outcome Pareto-dominates. In fact, depending on the parameterization of the game, different outcomes may Pareto-dominate at different times, entailing that there need not exist a universal “best.” Only with parity in the cost of the signaling activities, or a choice set comprising neutral and beneficent signals, can one reach a definitive conclusion on the welfare implications of the agents’ behavior. Namely, it is in those cases always preferable to opt for the most beneficent signal. The purpose of this section is to delineate further the intuition and scope of the results of this chapter.

In some respect, the multiplicity of outcomes and their non-rankability, in some cases, from a welfare perspective is neither unexpected nor unintuitive. The results in cases with well-defined equilibria, for instance, suggest that different situations give rise to distinct outcomes, tailored to the cost-benefit ratios associated with the signals. Likewise, in keeping with the literature, the signal intensities – while overlapping to some extent – tend to differ across the various profiles. The variability in outcomes might furthermore be seen as reflecting the signalers’ preferences, while the cases in which the signaler types coordinate on different activities can be perceived as (implicitly) capturing those of the receiver(s).

Even though much of the analysis revolved around beneficial signals, most results will tend to hold under broader circumstances as well (besides others, possibly). This once again entails that the signaling activity being modeled is not incidental to the equilibrium (cf. Chapter 2). Rather, given a link between each signal at the signalers’ disposal and some underlying social quality, the framework extracts conditions ensuring the achievement of separation of the signaler types for each combination of activities. In doing so, it demarcates the “boundaries” of each equilibrium, thereby shedding (further) light on the issue that the equilibrium profile(s) of many traditional signaling models can, in principle, be satisfied by several different types of signaling activities. The essential difference between the present approach and the environment without choice in this regard is that the latter
extracts conditions for individual types of signals (neutral versus beneficent), while the current set of results allows for their co-occurrence.

Considering the breadth of the analysis, can any of the outcomes be considered particularly likely? In the theory, of course, all outcomes are equally plausible, provided all relevant conditions are satisfied. From a limiting perspective, however, one might perceive the outcomes derived for the equal-cost case as most readily achievable – the one featuring coordination on the more beneficent signal (E₁) in particular. Since they can, under some conditions, also be obtained in the setting with disparity in the signals’ cost, and the one involving coordination on the more beneficial activity given parity in cost is not only Pareto-dominant but feasible under the broadest set of parameter values, one might expect said outcome to be the most readily conceivable result as the divergence in cost becomes negligible.

Resembling the environment without choice, the features that the receiver effectively has to “return” the benefit of the signaler she does not choose as her ally and that she cannot (per se) reject an alliance are once more not as stringent as they may seem. As discussed at some length in Chapter 2, the analysis and equilibria straightforwardly extend to the case when the receiver can retain the benefit of all signalers. Specifically, as long as the benefit provided by the signaler not chosen as the receiver’s ally is lower than that by the chosen signaler, the analysis goes through as given. Conversely, if the benefits are identical, the setting effectively corresponds to one with neutral signals in that it is driven entirely by the receivers’ beliefs. Note furthermore that there is no strategic effect to whether the signalers benefit from each other’s signals. They will simply be able to spend more on their signals, which does not qualitatively affect the analysis. As before, the assumption that the receiver cannot reject an alliance is equivalent to assuming that her outside option is zero or even negative, entailing that disregard of information is not reasonable.

As a final note, turn briefly to the relationship of several of the outcomes to findings in the related literature. Not least from an intuitive point of view, two of the outcomes (E₂ and E₆) involve “counter-signaling” in the sense that signalers of high type effectively “understate” their messages by using the less beneficial (and potentially expensive) signaling activity without inevitably diminishing their perceived social quality. In this point, the relevant strategy profiles are similar to the outcome developed by Feltovich et al. (2002). Yet, their analysis relies on the type space comprising three types and, more importantly, the signaler types being
stochastically separated via additional (exogenous) noisy information for the receivers, assumptions the present analysis does not require. Along the lines of the present results, they find that counter-signaling can only arise under fairly stringent conditions, as part of a multitude of equilibria, though in their case it may not survive equilibrium refinements. While the focus of the analyses differs, i.e., they are striving to explain the phenomenon that individuals of high type in some cases avoid signaling (to differentiate themselves from low types), whereas individuals of intermediate type seem particularly keen to send appropriate signals, the results harmonize in that additional information allows for equilibria that involve high types voluntarily choosing the “bad” signal, or as in their case not to signal at all.

The breakdown of the sharp contrast in the number equilibria with frameworks in the Spencian tradition even in the setting with equal costs derives from the fact that the additional “information” contained in the type of signal being used effectively strengthens the receiver’s position in terms of the configuration of her beliefs playing a more integral role than in the environment without choice. There, the transition to an all-pay contest essentially restricts the set of viable beliefs, whereas in the present environment, even non-decreasing beliefs can support a much wider range of outcomes, especially ones involving separation across the signaling activities. Multiplicity is thus not necessarily unexpected. Involvement of neutral signals will tend to expand the set of equilibria even further because the receiver’s beliefs (non-decreasing or otherwise) can take yet broader configurations. In this regard, the present work also resembles the literature on multi-dimensional signaling.

4.5 Concluding Remarks

The objective of this chapter was to shed light on what type of signaling activity, or set of activities, one might expect to arise under various sets of circumstances, and how the outcomes (if any) fare from a welfare perspective. To this end, the signalers were allowed to choose freely the vehicle(s) for their message(s) to the receivers from a predetermined collection of activities. Although the setup is fundamentally quite simple, the uniqueness prediction derived in the setting without choice is lost, and the agents may, under some conditions, opt for inefficient signals (in equilibrium). Even so, different settings give rise to different combinations of signaling activities and involve diverse signal intensities.
As ever, although the analysis’ tractability may suffer, one can think of a number of extensions to explore further the use of generous behavior to initiate cooperative social interaction in environments featuring endogenous choice of signaling activity. Besides rendering the receivers heterogeneous by dividing their population into subsets that adopt different beliefs (“multiple audiences”) and/or exploring some of the other modifications to the setup suggested in Chapter 2, a particularly interesting tangent in the present context would be the consideration of context-specific signals, potentially in conjunction with multiple audiences. The idea would be to investigate whether signaling in the manner explored in this paper (and its companion without choice) can also arise in non-social or even non-economic settings, or whether it is restricted to social contexts. It would, in this regard, also be interesting to explore the effects of additional variations in the cost of signaling, such as the instance that the level of one signal influences the cost of another. A similarly useful angle might be a test of the practicability of the predictions using experimental techniques. From a more fundamental perspective, the game could be modified to allow each signaler to use several signals at a time, i.e., to extend it in the direction of a multi-dimensional signaling framework, with some of the aforesaid amendments used to expand further its scope. For, on top of the equilibria derived in accordance with the present restrictions, equilibria involving randomization across the signaling activities might arise. In this fashion, the role and merit of choice in social settings amenable to costly signaling could be established more completely.
4.A Appendices

Appendix 4.1. The Simplest Case – Equal Cost

This appendix proves Proposition 4.1.

(a) Both Signaler Types Choose $\bar{b}$ ($\kappa = 1$)

In order to show that, given the receiver’s beliefs, the signalers do not have an incentive to deviate across the signaling activities, i.e., to signaling levels using the less beneficial activity, one merely needs to compare the receiver’s pre- and post-deviation payoff presuming a signaler deviates such that his cost of signaling remains the same. For, if a deviation to a signal intensity of the same cost is profitable, so will a deviation to a cheaper magnitude.

Signalers of Low Type

Signalers of low type do not have an incentive to deviate to some $s_l^d(b)$ if $\theta^l + \bar{A} \cdot s_l(\bar{b}) \geq \theta^l + A \cdot s_l^d(b)$ for $s_l(\bar{b}) = s_l^d(b)$, which reduces to $\bar{A} \geq A$ and is thus satisfied by assumption.

Signalers of High Type

Signalers of high type do not have an incentive to deviate to some $s_l^d(b)$ if $\theta^H + \bar{A} \cdot s_l(\bar{b}) \geq \theta^l + A \cdot s_l^d(b)$ for $s_l(\bar{b}) = s_l^d(b)$, which can be expressed as:

$$\theta^H \geq \theta^l - s_l(\bar{b}) \cdot (\bar{A} - A).$$

As long as $\theta^H > \theta^l$ and $\bar{A} > A$, both of which hold by assumption, this expression, too, will inevitably be satisfied for all $s_l(\bar{b}) > 0$.

Notably, the signaler types do not even have an incentive to deviate if the receiver were to believe that non-zero messages involving $\bar{b}$ indicate high type.

Signalers of Low Type

Given the alternative configuration of the receiver’s beliefs, signalers of low type do not have an incentive to deviate to some $s_l^d(b)$ if $\theta^l + \bar{A} \cdot s_l(\bar{b}) \geq \theta^H + \bar{A} \cdot s_l^d(b)$ for $s_l(\bar{b}) = s_l^d(b)$, which can be expressed as:

$$\theta^l \geq \theta^H - s_l(\bar{b}) \cdot (\bar{A} - A).$$

While it must ultimately be true that $\theta^H > \theta^l$ (by assumption), this requirement may be satisfied for some values of $s_l(\bar{b})$. The most restrictive
case in this regard is a signal at the upper bound ($\hat{s}^L$). It is easy to confirm that
the foregoing situation is fulfilled if $\hat{s}^L \geq \frac{\theta^H - \theta^L}{\bar{A} - \underline{A}}$, a positive constant.

**Signalers of High Type**

Similarly, signalers of high type do not have an incentive to deviate to some
$s^d_i(b)$ if $\theta^H + \bar{A} \cdot s_i(b) > \theta^H + \underline{A} \cdot s^d_i(b)$ for $s_i(b) = s^d_i(b)$, which reduces to
$\bar{A} \geq \underline{A}$ and is thus satisfied by assumption.

Note that the foregoing arguments only encompass non-zero signals because
signalers of low type would otherwise have an incentive to deviate to $b$.

In short, given the assumptions underlying the framework, the signaler types do
not have an incentive to deviate for a broad range of beliefs.

**(b) Low Types Choose $\bar{b}$ and High Types $b$ ($\kappa = 2$)**

i. **Disjoint but Contiguous Supports**

Figure 4.A.1 graphically depicts the (putative) disjoint but contiguous
equilibrium supports and consistent beliefs on the part of the receiver. The
sufficient conditions for the outcome can be stated as follows:

(1) Signalers of low type randomize continuously on $[0, \hat{s}^L]$ using $\bar{b}$.

(2) Signalers of high type randomize continuously on $[\hat{s}^H, \hat{s}^H]$ such that $\hat{s}^H = \hat{s}^L$
using $b$. To ensure existence, it must furthermore be satisfied that
$\theta^H \geq \theta^L + \hat{s}^H \cdot (\bar{A} - \underline{A})$ (labeled (4.1) in the main text).

**Figure 4.A.1.** Equal Cost – The Equilibrium Supports and Beliefs for the
Outcome Where Low Types Choose the More and High Types the Less
Beneficent Signal ($E_2$)
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(3) The receiver believes that all signals of type $\bar{b}$ as well as signals of type $b$ below $s^H_2$ indicate low type, while signals of type $b$ beyond $s^H_2$ are associated with high type.

The restrictions and auxiliary assumptions on the equilibrium entail the following expected payoffs for the signalers (cf. Chapter 2). Consider a signaler of low type sending some $s_i(\bar{b})$. Given equilibrium behavior by the other players, with probability $\lambda$, he faces a signaler of high type and loses with certainty; with probability $(1 - \lambda)$, he faces another low type and incurs a strictly positive probability of entering into an alliance with the receiver. Accordingly:

$$E[u^L_i(s_i(\bar{b}))] = \text{Pr(winning)} \cdot (\text{payoff as winner}) + \text{Pr(losing)} \cdot (\text{payoff as loser})$$

$$= (1 - \lambda) \cdot [G^L(s_i(\bar{b})) \cdot (p - \gamma^L \cdot s_i(\bar{b})) + (1 - G^L(s_i(\bar{b}))) \cdot (-\gamma^L \cdot s_i(\bar{b}))] + \lambda \cdot \left(-\gamma^L \cdot s_i(\bar{b})\right)$$

$$= (1 - \lambda) \cdot G^L(s_i(\bar{b})) \cdot p - \gamma^L \cdot s_i(\bar{b}).$$

Similarly, the expected payoff of a signaler of high type can be established to take the following form:

$$E[u^H_i(s_i(b))] = (1 - \lambda) \cdot \left(p - \gamma^H \cdot s_i(b)\right) + \lambda \cdot \left[G^H(s_i(b)) \cdot (p - \gamma^H \cdot s_i(b)) + (1 - G^H(s_i(b))) \cdot (-\gamma^H \cdot s_i(b))\right]$$

$$= \left[1 - \lambda \cdot \left(1 - G^H(s_i(b))\right)\right] \cdot p - \gamma^H \cdot s_i(b).$$

Given the expected payoffs, the bounds of the equilibrium supports follow straightforwardly from the fact that players randomizing over a number of different strategies must be indifferent (in terms of their payoff) between them. Consider first a signaler of low type. As discussed in the main text, the lower bound for this type of signaler must be at zero. He must therefore also obtain a payoff of zero when signaling at the upper bound ($\hat{s}^L_2$) of his equilibrium distribution. In fact, it must be that $E[u^L_i(s_i(\bar{b}))] = 0$ for all $s_i(\bar{b})$ along the equilibrium support. Furthermore, in keeping with the restrictions on the equilibrium, a signal at the upper bound of the equilibrium distribution entails that the individual wins against all other low types (all mass is below $\hat{s}^L_2$, which occurs with probability $(1 - \lambda)$), while he loses against all signalers of high type.
(all mass is above \(s_2\), which occurs with probability \(\lambda\)). Note that under these conditions \(E\left[u^L_i\left(s_i(\bar{b})\right)\right] = E[u^L(s_2)]\). Therefore:

\[
(1 - \lambda) \cdot G^L(s_2^L) \cdot p - \gamma^L \cdot s_2^L = 0,
\]

and, using that \(G^L(s_2^L) = \Pr(s_i \leq s_2) = 1:\)

\[
s_2^L = (1 - \lambda) \cdot \frac{p}{\gamma^L}.
\]

By a similar line of reasoning (cf. Chapter 2), it can be shown that the expected equilibrium payoff of signalers of high type amounts to:

\[
E[u^H_i(s_2^H)] = (1 - \lambda) \cdot \left(p - \gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^H}\right) + \lambda \cdot \left(-\gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^H}\right)
\]

\[
= (1 - \lambda) \cdot p - \gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L},
\]

which too must obtain at every point along the equilibrium support, implying that the upper bound of this type of signaler \((s_2^H)\) can be shown to be given by:

\[
s_2^H = (1 - \lambda) \cdot \frac{p}{\gamma^L} + \lambda \cdot \frac{p}{\gamma^H}.
\]

Having established the equilibrium supports, manipulation of the signaler types’ expected payoff functions (accounting for the expected equilibrium payoffs; cf. Chapter 2) yields the following expressions for their equilibrium distribution functions:

**Low Type.** \((1 - \lambda) \cdot G^L\left(s_i(\bar{b})\right) \cdot p - \gamma^L \cdot s_i(\bar{b}) = 0\)

\[
\Rightarrow \quad G^L\left(s_i(\bar{b})\right) = \frac{\gamma^L}{(1 - \lambda) \cdot p} \cdot s_i(\bar{b})
\]

**High Type.** \[
\left[1 - \lambda \cdot \left(1 - G^H\left(s_i(\bar{b})\right)\right)\right] \cdot p - \gamma^H \cdot s_i(\bar{b})
\]

\[
= (1 - \lambda) \cdot p - \gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L}
\]

\[
\Rightarrow \quad G^H\left(s_i(\bar{b})\right) = \frac{\gamma^H}{\lambda \cdot p} \cdot \left(s_i(\bar{b}) - (1 - \lambda) \cdot \frac{p}{\gamma^H}\right)
\]

The arguments ruling out atoms and deviations beyond the bounds of the equilibrium supports are identical (barring notational modifications) to those presented in Chapter 2 and will therefore not be repeated here. Deviations across the signals are treated in the main text; Figure 4.A.2 illustrates the reasoning underlying the necessary and sufficient condition for the equilibrium to exist.
ii. Gaps

Suppose that, rather than $\hat{s}_2^H = \hat{s}_2^L$, the equilibrium bounds feature a gap, i.e., $\hat{s}_2^H > \hat{s}_2^L$, *ceteris paribus*. As argued intuitively in the main text, this configuration cannot be sustained as an equilibrium because signalers of high type have an incentive to deviate. Formally, take a signaler of high type choosing $\hat{s}_2^H$ (using $\hat{b}$). Since all mass of the support of signalers of high type is above its lower bound, the signaler will lose with certainty against all (other) high types but will win against all low types, yielding him an expected payoff of $E[u_i^H(\hat{s}_2^H)] = (1 - \lambda) \cdot p - \gamma^H \cdot \hat{s}_2^H$. When choosing $\hat{s}_2^L$ (using $\hat{b}$), in turn, he expects to receive $E[u_i^H(\hat{s}_2^L)] = (1 - \lambda) \cdot p - \gamma^H \cdot \hat{s}_2^L$, as he will continue to lose against all high types and win against all low types. Since $\hat{s}_2^H > \hat{s}_2^L$, $E[u_i^H(\hat{s}_2^H)] < E[u_i^H(\hat{s}_2^L)]$; namely, a signaler of high type considering sending $\hat{s}_2^H$ has an incentive to deviate to $\hat{s}_2^L$, thereby reducing his cost of signaling without affecting his probability of winning.

iii. Overlaps

Overlaps of the equilibrium supports, too, have intuitively been shown not sustainable in equilibrium because signalers of low type choosing a signaling level in the overlapping region have an incentive to deviate. Formally, suppose that $\hat{s}_2^H < \hat{s}_2^L$, *ceteris paribus*. Let $\bar{s}_1$ be situated in the overlapping region and consider the expected payoff of a signaler of low type choosing this magnitude along his equilibrium support (i.e., using $\bar{b}$):
\[ E[u_e^i(s_i)] = (1 - \lambda) \cdot \left[ G^H_2(s_i) \cdot (p - y^L \cdot s_i) + \left( 1 - G^H_2(s_i) \right) \cdot (-y^L \cdot s_i) \right] + \lambda \cdot (-y^L \cdot s_i) \]

\[ = (1 - \lambda) \cdot G^H_2(s_i) \cdot p - y^L \cdot s_i, \]

where subscript \( e \) indicates that the signaler is behaving according to his equilibrium strategy. His expected payoff when imitating signalers of high type (in the overlapping region), i.e., choosing \( s_i \) using \( b \), in turn, is given by:

\[ E[u_d^i(s_i)] = (1 - \lambda) \cdot (p - y^L \cdot s_i) + \lambda \cdot \left[ G^H_2(s_i) \cdot (p - y^L \cdot s_i) + \left( 1 - G^H_2(s_i) \right) \cdot (-y^L \cdot s_i) \right] \]

\[ = \left[ 1 - \lambda \cdot (1 - G^H_2(s_i)) \right] \cdot p - y^L \cdot s_i, \]

where subscript \( d \) indicates that the signaler is deviating from his equilibrium strategy. Deviation is profitable if \( E[u_d^i(s_i)] > E[u_e^i(s_i)] \), i.e.:

\[ \left[ 1 - \lambda \cdot (1 - G^H_2(s_i)) \right] \cdot p - y^L \cdot s_i > (1 - \lambda) \cdot G^H_2(s_i) \cdot p - y^L \cdot s_i, \]

which manipulation reduces to:

\[ G^H_2(s_i) - \frac{\lambda}{1 - \lambda} \cdot G^H_2(s_i) < 1. \]

Substitution of the relevant distribution functions, which by the nature of the setup must hold for this configuration as well, then yields:

\[ s_i < (1 - \lambda) \cdot \frac{p}{y^L} = s^L_2, \]

which holds by assumption.

(c) Intuitive Criterion

As in the setting with neutral signals explored in Chapter 2, the “intuitive criterion” (Cho & Kreps 1987) may eliminate some outcomes in the present environment. Recall that the refinement can be used to eliminate a given perfect Bayesian Nash Equilibrium if there is a signaler of type \( \theta \) who is guaranteed a deviation payoff strictly greater than his equilibrium payoff for any belief of the receiver in response to the deviating message, as long as she does not assign a non-zero probability to the deviation having been made by a type \( \theta \) for whom this action is equilibrium dominated.\(^{34}\)

Consider first the case when both types of signaler coordinate on the more beneficent signal (\( E_1 \)). As discussed in the main text and established in

\(^{34}\) Since the signal-dependent benefit renders the receiver’s choice of ally deterministic even if she assesses the two signalers to be of the same type, the “intuitive criterion” can be applied in its original form despite the present framework once more comprising two (competing) signalers.
Appendix 4.1.a, this outcome holds for virtually any configuration of beliefs off the equilibrium path. It therefore easily survives the “intuitive criterion.”

The scenario when the signaler types separate across the signaling activities (E₂) is slightly more complicated since, depending on the parameterization of the game, there may exist a region of signaling magnitudes that is equilibrium dominated for signalers of low type regardless of the receiver’s beliefs. Fix a separating equilibrium such that $s^L(\tilde{b}) \in [0, s^L_2]$ and $s^H(\tilde{b}) \in [s^H_2, \tilde{s}^H_2]$, and ponder the incentives for deviation of a signaler of low type given various configurations of the receiver’s beliefs for messages off the equilibrium path. If, as discussed in the main text and depicted in Figure 4.2, the receiver associates messages beyond the upper bound of the low types’ equilibrium support with low type, neither type of signaler has an incentive to deviate as said move will adversely affect their probability of winning. If, on the other hand, she were to assign messages in this region to high type, a signaler of low type signaling at the upper bound could profitably deviate to, say, $s^L_2 + \varepsilon$, for $\varepsilon > 0$ but small. For, on account of the higher signal-dependent benefit associated with $\tilde{b}$, the benefit (to the receiver) of the deviating message would map to a signal intensity strictly beyond the lower bound of the high types’ support. The marginal benefit of the deviating message would thus clearly exceed the (small) additional cost. Yet, depending on the parameter values, there are bound to exist signaling magnitudes beyond the low types’ upper bound whose cost exceeds their (marginal) benefit even if the receiver were to associate them with high type. That is, they are equilibrium dominated for signalers of low type. Provided the relevant region lies within their equilibrium support (just involving the more beneficial activity), signalers of high type will indisputably profit from deviating to messages in this interval. In other words, the “intuitive criterion” may eliminate this equilibrium for some parameterizations of the game.

To establish formally the existence of signaling levels that are equilibrium-dominated for signalers of low type if the receiver deems messages beyond $s^L_2$ to have been sent by high types, consider the following notation. Let $\alpha$ denote the deviating message of a low type (using $\tilde{b}$), and let $\beta$ denote the point at which the payoff to the receiver associated with $\alpha$ maps into the high types’ support,
i.e., \( \beta = \frac{K}{A} \cdot \alpha \). The additional cost of \( \alpha \) relative to a signal at \( \delta_2^L \) is given by \( \gamma^L \cdot (\alpha - \delta_2^L) \), while the benefit of the deviating signal in terms of the additional mass of high types defeated amounts to \( \lambda \cdot p \cdot (\beta - \delta_2^L) \cdot \frac{\gamma^H}{\kappa p} \). \( \alpha \) is equilibrium-dominated for low types if \( \gamma^L \cdot (\alpha - \delta_2^L) > (\beta - \delta_2^L) \cdot \gamma^H \), or:

\[
\alpha > \delta_2^L \cdot \left( \frac{\gamma^L - \gamma^H}{\gamma^L - \frac{A}{\lambda} \cdot \gamma^H} \right).
\]

Now, in order for the refinement to have bite, it must be the case that \( \alpha < \delta_2^H \); otherwise, the equilibrium-dominated signals for signalers of low type lie above the upper bound of signalers of high type, rendering them unattractive for the latter. Substitution and simple manipulation then yield the following sufficient condition for the existence of a region of equilibrium-dominated signaling levels for low types:

\[
(1 - \lambda) \cdot \frac{\lambda^H}{\lambda^L} \cdot \left( \frac{A}{\lambda^H} - 1 \right) < \lambda \cdot \left( \frac{\lambda^L}{\lambda^H} - \frac{A}{\lambda^H} \right).
\]

Hence, the equilibrium profile is (only) likely to be eliminated by the “intuitive criterion” if the marginal cost for signalers of low type is high, that for high types small, and/or the ratio of the signal-dependent benefits is small.

**Appendix 4.2. The Simplest Case – Unequal Cost**

Parts (a) and (b) of this appendix establish Proposition 4.2, and parts (c) and (d) Proposition 4.3.

(a) **Both Signaler Types Choose \( b (\kappa = 3) \)**

i. **Incentives for Deviation When Maintaining the Signaling Magnitude**

The key signaler when it comes to deviation by signalers of low type such that the pre- and post-deviation signaling magnitudes are the same is the one sending the highest possible magnitude, i.e., \( \delta_2^L \). For, if deviation from the upper bound of the equilibrium support is profitable, deviation from lower magnitudes will be, too. It is similarly easy to deduce that deviation in this manner requires an increase in the signaler’s expenditure. When starting from the upper bound it will moreover be the case that the deviant will be able to defeat some signalers of high type, as the benefit (to the receiver) of the deviating message must map
to a magnitude along the high types’ equilibrium support. Using these observations, to rule out deviation of this nature, it must be the case that:

\[ \lambda \cdot p \cdot (b - \delta_3^H) \cdot \frac{y^H}{\lambda \cdot p} \leq \frac{y^L}{y^H} \cdot \delta_3^L - \frac{y^L}{y^H} \cdot \delta_3^L, \quad (4.A.1) \]

where \( b \) denotes the point at which the receiver’s (true) payoff from the deviating message (using \( \bar{b} \)) maps into the high types’ equilibrium support, i.e., \( b = \bar{x} \cdot \delta_3^L \). To be precise, the left-hand side of Equation 4.A.1 signifies the marginal benefit of the deviating message (i.e., the extra mass of high types defeated), while the right-hand side denotes the deviating move’s (positive) marginal cost. Manipulation yields the following simplified expression:

\[ \frac{y^L - y^L + y^H}{y^H} \geq \frac{A}{\bar{A}}. \]

Seeing as it can be shown that for all \( y^L > y^L > 0 \) and \( y^H > y^H > 0 \):

\[ \frac{y^L - y^L + y^H}{y^H} > \frac{y^L}{y^L}, \]

Equation 4.2 is necessary and sufficient.

ii. Incentives for Deviation Given a Variation of the Receiver’s Beliefs

Derivation of the condition ruling out deviation across the signaling activities for signalers of high type if the receiver were to believe that the deviating message indicates high type, once again simply involves comparing the receiver’s pre- and post-deviation expected payoff. In the case at hand, deviation from \( \delta_3^H \) to some \( s_i^d(\bar{b}) \) such that \( y^H \cdot \delta_3^H = y^H \cdot s_i^d(\bar{b}) \) is not profitable if:

\[ (\theta^H + \bar{A} \cdot \delta_3^H) \geq (\theta^H + \bar{A} \cdot s_i^d(\bar{b})), \]

which – using that \( s_i^d(\bar{b}) = \frac{y^H \cdot \delta_3^H}{y^H} \) – reduces to:

\[ \frac{y^H}{y^H} \geq \frac{A}{\bar{A}}. \]

\[ ^{35} \text{It is, in this regard, important to note that any condition (and/or argument) comprising } \theta^L < \theta^H \text{ constitutes a more stringent requirement than a condition involving } \theta^H, \text{ because the mapping comprising } \theta^L \text{ will be of a lower magnitude than the one involving } \theta^H. \]
For reference, the bracket of Equation 4.3 is negative if:

\[ \frac{\gamma^H}{\gamma^L} \cdot A - \Delta < 0 \Rightarrow \frac{\gamma^H}{\gamma^L} \geq \frac{A}{\Delta}. \]

(b) Both Signaler Types Choose \( \tilde{b} (\kappa = 4) \)

The purpose of this appendix is to show that deviation by signalers of low type are indeed ruled out by the strict-inequality converse of Equation 4.2, i.e., Equation 4.2\(^c\). To this end, consider a signaler of low type looking to deviate from some \( s_i(\tilde{b}) \) along his equilibrium support to some \( s_i^d(b) \) such that \( \tilde{\varphi}^L \cdot s_i(\tilde{b}) = \gamma^L \cdot s_i^d(b) \). Given the her beliefs, this move would lead the receiver to expect a payoff of \( \theta^L + \tilde{\varphi}^L \cdot s_i(\tilde{b}) \). Deviation is not profitable if this level is less than or equal to her pre-deviation payoff of \( \theta^L + \tilde{\varphi}^L \cdot s_i(\tilde{b}) \). Accounting for the restriction that \( s_i^d(b) = \frac{\tilde{\varphi}^L}{\gamma^L} \cdot s_i(\tilde{b}) \), this is the case if:

\[ \frac{\tilde{\varphi}^L}{\gamma^L} \leq \frac{\Delta}{A}, \]

which, indeed, has the opposite sign of Equation 4.2.

(c) Low Types Choose \( \bar{b} \) and High Types \( \tilde{b} (\kappa = 5) \)

i. Gaps and/or Disjoint but Contiguous Supports

Suppose that, rather than \( \tilde{\delta}_5^H < \delta_5^L \), the equilibrium bounds feature a gap, i.e., \( \delta_5^H > \delta_5^L \), ceteris paribus. As argued intuitively in the main text, this configuration cannot be sustained as an equilibrium because signalers of high type have an incentive to deviate. The formal argument is effectively the same as that given in Appendix 4.1.b.ii. Take a signaler of high type choosing \( \delta_5^H \) (using \( \bar{b} \)). Since all mass of the support of this type is above its lower bound, he will lose with certainty against all (other) high types but will win against all low types, yielding him an expected payoff of \( E[u_i^H(\delta_5^H)] = (1 - \lambda) \cdot p - \gamma^H \cdot \delta_5^H \).

When choosing \( \delta_5^L \) (using \( \tilde{b} \)), in turn, he expects to receive \( E[u_i^H(\delta_5^L)] = (1 - \lambda) \cdot p - \gamma^H \cdot \delta_5^L \), as he will continue to lose against all high types and win against all low types. Since \( \delta_5^H > \delta_5^L \), \( E[u_i^H(\delta_5^H)] < E[u_i^H(\delta_5^L)] \); namely, a signaler of high type considering sending \( \delta_5^H \) has an incentive to deviate to \( \delta_5^L \), thereby reducing his cost of signaling without affecting his probability of winning.
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Suppose, instead, that the equilibrium bounds are separate but adjoining rather than overlapping, i.e., $\delta_5^H = \delta_5^L$, ceteris paribus. The argument ruling out this configuration (also) is essentially analogous to the one just given. In particular, take a signaler of high type choosing $\delta_5^H$ (using $\bar{b}$), bearing in mind that all relevant limits on the positioning of the bounds of the equilibrium support are smaller in magnitude than $\delta_5^L$. He will lose with certainty against all (other) high types but will win against all low types, yielding him an expected payoff of $E[u_t^H(\delta_5^H)] = (1 - \lambda) \cdot p - \gamma^H \cdot \delta_5^H$. When choosing $s^d_t(\bar{b}) = \delta_5^L - \epsilon$, for $\epsilon$ sufficiently small so as not to exceed the binding limit on the lower bound of the support, he will expect to receive $E[u_t^H(s^d_t(\bar{b}))] = (1 - \lambda) \cdot p - \gamma^H \cdot s^d_t(\bar{b})$, as he will once again continue to lose against all high types and win against all low types. Since $\delta_5^H > s^d_t(\bar{b})$, $E[u_t^H(\delta_5^H)] < E[u_t^H(s^d_t(\bar{b}))]$. Hence, in this case, too, a signaler of high type contemplating $\delta_5^H$ has an incentive to deviate, thereby reducing his cost of signaling without affecting his probability of winning.

ii. **Overlapping Supports**

The sufficient conditions for the present outcome can be summarized as follows:

(1) Signalers of low type randomize continuously on $[0, \delta_5^L]$ using $\bar{b}$. Existence moreover requires that $\frac{\gamma^L}{\gamma^H} \geq \frac{\bar{A}}{\bar{A}}$ (labeled (4.2) in the main text).

(2) Signalers of high type randomize continuously on $[\delta_5^L, \delta_5^H]$ such that $\delta_5^H < \delta_5^L$ using $\bar{b}$. To ensure existence, it must furthermore be satisfied that $\theta^H > \theta^L - \delta_5^{H(min)} \cdot \left( -\frac{\bar{A}}{\gamma^H} \cdot \bar{A} \right)$ (labeled (4.5) in the main text).

(3) The receiver believes that all signals of type $\bar{b}$ as well as signals of type $\bar{b}$ below $\delta_5^H$ indicate low type, while signals of type $\bar{b}$ beyond $\delta_5^H$ are associated with high type.

Akin to the previous outcomes and the setting with equal cost, the derivations for the algebraic form of the equilibrium strategy of signalers of low type parallel those developed in Chapter 2. Indeed, with the exception of there existing a range of possible lower bounds, the derivations for signalers of high type, too, are identical to them. In particular, in keeping with the restrictions and
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auxiliary assumptions on the equilibrium, the expected payoff of a signaler of high type is given by:

$$E[u^H_t(s_i(b))] = (1 - \lambda) \cdot \left( p - \gamma^H \cdot s_i(b) \right) + \lambda \cdot \left[ G^H_L(s_i(b)) \cdot \left( 1 - G^H_L(s_i(b)) \right) \cdot \gamma^H \cdot s_i(b) \right]$$

$$= \left[ 1 - \lambda \cdot \left( 1 - G^H_L(s_i(b)) \right) \right] \cdot p - \gamma^H \cdot s_i(b),$$

where subscript $\mathcal{L}$ denotes the relevant “limit” on the lower bound of the equilibrium support. That is to say, given equilibrium behavior by the other players, with probability $\lambda$, he faces another high type, in which case he incurs a strictly positive probability of entering into an alliance with the receiver, whereas with probability $(1 - \lambda)$, he faces a low type and wins with certainty.

The algebraic form for the bounds of the equilibrium supports then follows straightforwardly from the fact that players randomizing over a number of different strategies must be indifferent (in terms of their payoff) between them. The distinctive feature in this case, in contrast to all other cases considered thus far, is that the lower bound is determined by three limits relating to the receiver’s points of indifference between the signaler types given their chosen signals and/or the low types’ cost of signaling (cf. main text). Accordingly, consider each of the boundaries in turn, starting with the alternative that $C$ is binding.

1) $C$ is binding

In view of the simplicity of this limit, all that is required to establish the algebraic form for the lower bound ($\delta^H_{5(C)}$) is a simple manipulation of the relevant cost-condition ($\gamma^L \cdot C = \gamma^L \cdot \mathcal{E}$) to yield:

$$C = \frac{\gamma^L}{\gamma} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L},$$

or equivalently:

$$\delta^H_{5(C)} = (1 - \lambda) \cdot \frac{p}{\gamma^L}.$$
upper bound ensures victory against all other signals. In this vein, by the
now familiar line of reasoning, the expected payoff of a signaler of high
type for a signal at his lower bound amounts to:

$$E[u_i^H(s_i^{H(\mathcal{C})})] = (1 - \lambda) \cdot \left( p - \bar{\gamma}^H \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}} \right) + \lambda \cdot \left( -\bar{\gamma}^H \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}} \right)$$

$$= (1 - \lambda) \cdot p - \bar{\gamma}^H \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}},$$

which must obtain at every point along the equilibrium support, implying
that the upper bound of this type of signaler is given by:

$$\bar{s}_i^{H(\mathcal{C})} = (1 - \lambda) \cdot \frac{p}{\bar{y}} + \lambda \cdot \frac{p}{\bar{y}^H}.$$ 

Given the equilibrium support, further manipulation of the expected payoff
function (accounting for the expected equilibrium payoff) then yields the
following expression for the equilibrium distribution function:

$$\left[ 1 - \lambda \cdot \left( 1 - G_i^{H(\mathcal{C})} \left( s_i(\bar{b}) \right) \right) \right] \cdot p - \bar{\gamma}^H \cdot s_i(\bar{b}) = (1 - \lambda) \cdot p - \bar{\gamma}^H \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}}$$

$$\Rightarrow G_i^{H(\mathcal{C})} \left( s_i(\bar{b}) \right) = \frac{\bar{\gamma}^H}{\lambda \cdot p} \cdot \left( s_i(\bar{b}) - (1 - \lambda) \cdot \frac{p}{\bar{y}} \right).$$

The arguments ruling out atoms and deviations beyond the bounds of the
equilibrium supports for this and both subsequent outcomes are again the
same (barring notational modifications) as those developed in Chapter 2 and
will therefore not be repeated here. Deviations across the signaling activities
are discussed in the main text.

2) $\mathcal{B}$ is binding

It should be obvious that the gist of the arguments to derive the bounds and
equilibrium distribution $\left( G_i^{H(\mathcal{B})} \left( s_i(\bar{b}) \right) \right)$ will be the same as the above if the
binding limit is $\mathcal{B}$. So as not to be repetitive, this section will therefore be
limited to the actual algebraic derivations.

Given the definition of $\mathcal{B}$, i.e., $(\theta^L + \bar{A} \cdot \mathcal{B}) = (\theta^L + \bar{A}) \cdot \mathcal{E}$, straightforward
manipulation yields the following expression for the lower bound ($\underline{s}_i^{H(\mathcal{B})}$):

$$\underline{s}_i^{H(\mathcal{B})} = \frac{A}{\bar{A}} \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}^L}.$$
It is easy to corroborate that the expected payoff at this point amounts to:

\[ E[u^H_i(\delta_{5(\mathcal{B})}^H)] = (1 - \lambda) \cdot \left( p - \gamma^H \cdot \left( \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} \right) \right) + \lambda \cdot \left( -\gamma^H \cdot \left( \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^H} \right) \right) \]

\[ = (1 - \lambda) \cdot p - \gamma^H \cdot \left( \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} \right), \]

implying that the upper bound (\( \delta_{5(\mathcal{B})}^H \)) takes the following form:

\[ \delta_{5(\mathcal{B})}^H = \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} + \lambda \cdot \frac{p}{\gamma^H}. \]

The equilibrium distribution function is then given by:

\[ \left[ 1 - \lambda \cdot \left( 1 - \sigma_{5(\mathcal{B})}^H \right) \left( s_i(\mathcal{B}) \right) \right] \cdot p - \gamma^H \cdot s_i(\mathcal{B}) = (1 - \lambda) \cdot p - \gamma^H \cdot \left( \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} \right) \]

\[ \Rightarrow \quad \sigma_{5(\mathcal{B})}^H \left( s_i(\mathcal{B}) \right) = \frac{\gamma^H}{\lambda \cdot p} \cdot \left( s_i(\mathcal{B}) - \left( \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} \right) \right). \]

3) \( \mathcal{J} \) is binding

The final alternative is the limit at \( \mathcal{J} \), where \( (\theta^H + A \cdot \mathcal{J}) = (\theta^L + A \cdot \mathcal{E}) \). As before, simple manipulation of this condition yields the expression for the lower bound (\( \delta_{5(\mathcal{J})}^H \)), namely:

\[ \delta_{5(\mathcal{J})}^H = \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} - \frac{1}{A} \cdot (\theta^H - \theta^L). \]

A signaler of high type’s expected payoff at this point amounts to:

\[ E[u^H_i(\delta_{5(\mathcal{J})}^H)] = (1 - \lambda) \cdot \left( p - \gamma^H \cdot \left( \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} - \frac{1}{A} \cdot (\theta^H - \theta^L) \right) \right) \]

\[ + \lambda \cdot \left( -\gamma^H \cdot \left( \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^H} - \frac{1}{A} \cdot (\theta^H - \theta^L) \right) \right) \]

\[ = (1 - \lambda) \cdot p - \gamma^H \cdot \left( \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} - \frac{1}{A} \cdot (\theta^H - \theta^L) \right), \]

entailing that the upper bound (\( \delta_{5(\mathcal{J})}^H \)) takes the following form:

\[ \delta_{5(\mathcal{J})}^H = \left( \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} - \frac{1}{A} \cdot (\theta^H - \theta^L) \right) + \lambda \cdot \frac{p}{\gamma^H}. \]
The equilibrium distribution function \( G_{5(c)}^{H}(s_i(\bar{b})) \) is then given by:

\[
G_{5(c)}^{H}(s_i(\bar{b})) = (1 - \lambda) \cdot p - \bar{y} \cdot \left( \frac{A}{A} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} - \frac{1}{A} \cdot (\Theta^H - \Theta^L) \right)
\]

\[
\Rightarrow G_{5(c)}^{H}(s_i(\bar{b})) = \frac{\bar{y}^H}{\lambda} \cdot p \cdot \left( s_i(\bar{b}) - \left( \frac{A}{A} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} - \frac{1}{A} \cdot (\Theta^H - \Theta^L) \right) \right).
\]

4) Comparison

Taking into account that \( \frac{\gamma^L}{\gamma} \geq \frac{A}{A} - \) or, equivalently, \( \frac{A}{A} \geq \frac{\gamma^L}{\gamma} - \) constitutes the necessary and sufficient condition on the behavior of signalers of low type, it is easy to verify that \( \gamma_{5(B)}^H > \gamma_{5(c)}^H \) and \( \gamma_{5(B)}^H > \gamma_{5(c)}^H \). That is, as one would expect, the support when the binding limit is \( B \) compared to the case when the limit is \( C \) is simply shifted towards higher signaling magnitudes but otherwise has the same dimensions. Since \( \frac{1}{A} \cdot (\Theta^H - \Theta^L) \) amounts to a positive constant, it will furthermore hold that \( \gamma_{5(B)}^H > \gamma_{5(c)}^H \) and \( \gamma_{5(B)}^H > \gamma_{5(c)}^H \), though the supports will again be of the same size. Likewise, as suggested in the main text, the position of the support when the binding limit is \( J \) relative to the position when \( C \) is the relevant boundary depends on the parameter values. In other words, whether:

\[
\frac{\gamma^L}{\gamma} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} \geq \frac{A}{A} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} - \frac{1}{A} \cdot (\Theta^H - \Theta^L)
\]

is determined by:

\[
(1 - \lambda) \cdot \frac{p}{\gamma^L} \cdot \left( \frac{\gamma^L}{\gamma} - \frac{A}{A} \right) \geq - \frac{1}{A} \cdot (\Theta^H - \Theta^L).
\]

Given the assumptions of the framework, the first two terms on the left-hand side and the second term on the right-hand side will always be positive while the remaining terms will be negative, although their relative size is \textit{ex ante} ambiguous.

A final issue is the size and positioning of the aforesaid (range of) supports relative to those of the other equilibria discussed in the main text. The
outcome of particular interest in this regard is the scenario where both types of signaler coordinate on the more beneficial activity \( (E_4) \). As one might anticipate, if \( C \) is binding, the equilibrium support of signalers of high type is exactly the same as that of \( E_4 \) – in terms of dimension and positioning. In the other cases, the support is “shifted” towards relatively higher signal intensities in the expected direction.

(d) **Low Types Choose \( \overline{b} \) and High Types \( b \) (\( \kappa = 6 \))**

i. **Gapped Supports**

The sufficient conditions for the outcome at hand are as follows:

1. Signalers of low type randomize continuously on \([0, \delta_6^L] \) using \( \overline{b} \). Existence moreover requires that \( \frac{\gamma^L}{\gamma^L} \leq \frac{\overline{A}}{\overline{A}} \) (labeled (4.2.1\( ^c \) in the main text).

2. Signalers of high type randomize continuously on \([\delta_6^H, \delta_6^H] \) such that \( \delta_6^L > \delta_6^L \) using \( b \). To ensure existence, it must furthermore be satisfied that \( \theta^H > \theta^L + \delta_6^{H (\text{min})} \cdot \left( \frac{\gamma^H}{\gamma^H} \cdot \overline{A} - \overline{A} \right) \) (labeled (4.6) in the main text).

3. The receiver believes that all signals of type \( \overline{b} \) as well as signals of type \( b \) below \( \delta_6^L \) indicate low type, while signals of type \( b \) beyond \( \delta_6^H \) are associated with high type.

Seeing as all relevant arguments have already been discussed in some detail elsewhere (in the context of \( E_5 \) in particular), this section only features an abridged version of the derivations, focusing on signalers of high type (only). In particular, akin to \( E_5 \), except for the existence of a continuum of possible lower bounds determined by three limits (two relating to the receiver’s points of indifference between the signaler types given their chosen signaling activities and one to the low types’ cost of signaling), the derivations for this type of signaler parallel those of the setting without choice (cf. main text). Hence, in keeping with the restrictions and auxiliary assumptions on the equilibrium, the expected payoff of a signaler of high type is given by:

\[
E \left[ u^H_i \left( s_i (b) \right) \right] = (1 - \lambda) \cdot \left( p - \gamma^H \cdot s_i (b) \right) + \lambda \cdot \left[ G^H_i \left( s_i (b) \right) \right] \cdot \left( p - \gamma^H \cdot s_i (b) \right) + \\
\left( 1 - G^H_i \left( s_i (b) \right) \right) \cdot \left( -\gamma^H \cdot s_i (b) \right)
\]
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= \left[ 1 - \lambda \cdot \left( 1 - G^H_\mathcal{L} \left( s_1(b) \right) \right) \right] \cdot p - \gamma^H \cdot s_1(b),

where subscript $\mathcal{L}$ once more signifies the relevant limit on the lower bound of the equilibrium support. Consider once again each of the boundaries on the lower bound of the equilibrium support.

1) $\mathcal{C}$ is binding

As before, with $\mathcal{C}$ binding, the algebraic form for the lower bound ($\hat{s}^H_6(\mathcal{C})$) only requires a simple manipulation of the relevant cost-condition ($\gamma^L \cdot \mathcal{C} = \gamma^L \cdot \mathcal{E}$) to yield:

$$\hat{s}^H_6(\mathcal{C}) = (1 - \lambda) \cdot \frac{p}{\gamma^L}.$$ 

The expected payoff at this point amounts to:

$$E[u^H_i(\hat{s}^H_6(\mathcal{C}))] = (1 - \lambda) \cdot \left( p - \gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} \right) + \lambda \cdot \left( -\gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L} \right)$$

$$= (1 - \lambda) \cdot p - \gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L},$$

implying that the upper bound ($\hat{s}^H_6(\mathcal{C})$) takes the following form:

$$\hat{s}^H_6(\mathcal{C}) = (1 - \lambda) \cdot \frac{p}{\gamma^L} + \lambda \cdot \frac{p}{\gamma^L}.$$ 

The equilibrium distribution function ($G^H_6(\mathcal{C}) \left( s_1(b) \right)$) is then given by:

$$\left[ 1 - \lambda \cdot \left( 1 - G^H_6(\mathcal{C}) \left( s_1(b) \right) \right) \right] \cdot p - \gamma^H \cdot s_1(b) = (1 - \lambda) \cdot p - \gamma^H \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L}$$

$$\Rightarrow G^H_6(\mathcal{C}) \left( s_1(b) \right) = \frac{\gamma^H}{\lambda \cdot p} \cdot \left( s_1(b) - (1 - \lambda) \cdot \frac{p}{\gamma^L} \right),$$

with the same provisions regarding atoms and deviations beyond the bounds of the equilibrium supports and/or across the signals as in the context of $E_5$.

2) $\mathcal{B}$ is binding

Given the definition of $\mathcal{B}$, i.e., ($\theta^L + A \cdot \mathcal{B} = \theta^L + A \cdot \mathcal{E}$), straightforward manipulation yields the following expression for the lower bound ($\hat{s}^H_6(\mathcal{B})$):

$$\hat{s}^H_6(\mathcal{B}) = \frac{A}{\lambda} \cdot (1 - \lambda) \cdot \frac{p}{\gamma^L}.$$
It is easy to verify that the expected payoff at this point amounts to:

\[
E[u^H(\delta^H_{(b)})] = (1 - \lambda) \cdot \left( p - \bar{Y}^H \cdot \left( \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} \right) \right) + \lambda \cdot \left( -\bar{Y}^H \cdot \left( \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} \right) \right)
\]

\[
= (1 - \lambda) \cdot p - \bar{Y}^H \cdot \left( \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} \right)
\]

entailing that the upper bound \(\delta^H_{(b)}\) takes the following form:

\[
\delta^H_{(b)} = \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} + \lambda \cdot \frac{p}{\bar{Y}^H}.
\]

The equilibrium distribution function \(G^H_{(b)}(s_i(b))\) is then given by:

\[
[1 - \lambda \cdot \left( 1 - G^H_{(b)}(s_i(b)) \right)] \cdot p - \bar{Y}^H \cdot s_i(b) = (1 - \lambda) \cdot p - \bar{Y}^H \cdot \left( \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} \right)
\]

\[
\Rightarrow G^H_{(b)}(s_i(b)) = \frac{\bar{Y}^H}{\lambda \cdot p} \cdot \left( s_i(b) - \left( \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} \right) \right).
\]

3) \(J\) is binding

The final alternative is the limit at \(J\), where \((\theta^H + A \cdot J) = (\theta^l + \bar{A} \cdot \mathcal{E})\).

Once again, manipulation of the condition yields the expression for the lower bound \(\delta^H_{(J)}\), namely:

\[
\delta^H_{(J)} = \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} - \frac{1}{\bar{A}} \cdot (\theta^H - \theta^l)
\]

A signaler of high type’s expected payoff at this point amounts to:

\[
E[u^H_i(\delta^H_{(J)})] = (1 - \lambda) \cdot \left( p - \bar{Y}^H \cdot \left( \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} - \frac{1}{\bar{A}} \cdot (\theta^H - \theta^l) \right) \right) + \lambda
\]

\[
\cdot \left( -\bar{Y}^H \cdot \left( \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} - \frac{1}{\bar{A}} \cdot (\theta^H - \theta^l) \right) \right)
\]

\[
= (1 - \lambda) \cdot p - \bar{Y}^H \cdot \left( \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} - \frac{1}{\bar{A}} \cdot (\theta^H - \theta^l) \right)
\]

implying that the upper bound \(\delta^H_{(J)}\) takes the following form:

\[
\delta^H_{(J)} = \left( \frac{\bar{A}}{A} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} - \frac{1}{\bar{A}} \cdot (\theta^H - \theta^l) \right) + \lambda \cdot \frac{p}{\bar{Y}^H}.
\]
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The equilibrium distribution function \( G^H_{6(7)}(s_i(b)) \) is then given by:

\[
\left[ 1 - \lambda \cdot \left( 1 - G^H_{6(7)}(s_i(b)) \right) \right] \cdot p - \frac{\gamma^H}{\lambda} \cdot s_i(b) = (1 - \lambda) \cdot p - \frac{\gamma^H}{\lambda} \cdot \left( \frac{\bar{A}}{\bar{A}} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} - \frac{1}{\bar{A}} \cdot (\theta^H - \theta^L) \right)
\]

\[
\Rightarrow G^H_{6(7)}(s_i(b)) = \frac{\gamma^H}{\lambda} \cdot \left( s_i(b) - \left( \frac{\bar{A}}{\bar{A}} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} - \frac{1}{\bar{A}} \cdot (\theta^H - \theta^L) \right) \right).
\]

4) Comparison

Given that, in this case, \( \frac{\bar{Y}^L}{\bar{Y}^L} \leq \frac{\bar{A}}{\bar{A}} \) is the necessary and sufficient condition on the behavior of signalers of low type, it is not hard to confirm that \( s_{6(B)}^H > s_{6(C)}^H \) and \( s_{6(B)}^H > s_{6(C)}^H \). Hence, the support when the binding limit is \( B \) compared to the setting when the limit is \( C \) is once more simply shifted towards higher signaling magnitudes but otherwise has the same dimensions. Since \( \frac{1}{\bar{A}} \cdot (\theta^H - \theta^L) \) constitutes a positive constant, it will furthermore hold that \( s_{6(B)}^H > s_{6(C)}^H \) and \( s_{6(B)}^H > s_{6(C)}^H \), with the supports again of the same size. Likewise, as before, the position of the support when the binding limit is \( J \) relative to the position when \( C \) is the relevant boundary depends on the parameter values. Namely, whether:

\[
\frac{\bar{Y}^L}{\bar{Y}^L} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} \geq \frac{\bar{A}}{\bar{A}} \cdot (1 - \lambda) \cdot \frac{p}{\bar{Y}} - \frac{1}{\bar{A}} \cdot (\theta^H - \theta^L)
\]

is determined by:

\[
(1 - \lambda) \cdot \frac{p}{\bar{Y}} \cdot \left( \frac{\bar{Y}^L}{\bar{Y}^L} - \frac{\bar{A}}{\bar{A}} \right) \geq - \frac{1}{\bar{A}} \cdot (\theta^H - \theta^L).
\]

In view of the assumptions of the framework, the first two terms on the left-hand side and the second term on the right-hand side will always be positive while the remaining terms will be negative. Their relative size, however, is \textit{ex ante} ambiguous.

Finally, consider the size and positioning of the aforesaid (range of) supports relative to those of the other equilibria discussed in the main text. The outcome of particular interest in this regard is the scenario where both
types of signaler coordinate on the less beneficial activity \((E_3)\). In line with expectation, if is \(C\) binding, the equilibrium support of signalers of high type is exactly the same as that of \(E_3\) – in terms of dimension and positioning. In the other cases, the support is “shifted” towards relatively higher signaling levels, in the appropriate direction. The comparison to the “counter-signaling” equilibrium when the cost of the activities is constant \((E_2)\), in turn, depends on the relative size of the cost parameters (e.g., \(\gamma^L\) vs. \(\gamma^H\)).

ii. Overlaps and/or Disjoint but Contiguous Supports

Overlaps of the equilibrium supports can be shown not sustainable in equilibrium because signalers of low type choosing a signaling magnitude in the overlapping region have an incentive to deviate. The formal argument is, in effect, identical to that given in Appendix 4.1.b.iii. Suppose that \(\delta_e^H < \delta_e^L\), \textit{ceteris paribus}. Let \(\bar{\xi}_i\) be situated in the overlapping region and consider the expected payoff of a signaler of low type choosing this magnitude along his equilibrium support (i.e., using \(\bar{b}\)):

\[
E[u^L_e(\bar{\xi}_i)] = (1 - \lambda) \cdot \left[ G^L_b(\bar{\xi}_i) \cdot (p - \gamma^L \cdot \bar{\xi}_i) + (1 - G^H_b(\bar{\xi}_i)) \cdot (-\gamma^L \cdot \bar{\xi}_i) \right] + \lambda \cdot (\gamma^L \cdot \bar{\xi}_i).
\]

His expected payoff when imitating signalers of high type (in the overlapping region), i.e., choosing \(\bar{\xi}_i\) using \(\bar{b}\), in turn, is given by:

\[
E[u^H_e(\bar{\xi}_i)] = (1 - \lambda) \cdot (p - \gamma^L \cdot \bar{\xi}_i) + \lambda \cdot \left[ G^H_b(\bar{\xi}_i) \cdot (p - \gamma^L \cdot \bar{\xi}_i) + (1 - G^H_b(\bar{\xi}_i)) \cdot (-\gamma^L \cdot \bar{\xi}_i) \right]
= \left[1 - \lambda \cdot (1 - G^H_b(\bar{\xi}_i))\right] \cdot p - \gamma^L \cdot \bar{\xi}_i.
\]

Deviation is profitable if \(E[u^L_e(\bar{\xi}_i)] > E[u^H_e(\bar{\xi}_i)]\), i.e.:

\[
\left[1 - \lambda \cdot (1 - G^H_b(\bar{\xi}_i))\right] \cdot p - \gamma^L \cdot \bar{\xi}_i > (1 - \lambda) \cdot G^L_b(\bar{\xi}_i) \cdot p - \gamma^L \cdot \bar{\xi}_i.
\]

Substitution of the distribution functions when the binding limit is \(C\) (tentatively the most restrictive criterion in this context) yields:

\[
\bar{\xi}_i < (1 - \lambda) \cdot \frac{p}{\gamma^L} = \delta^H_C.
\]

Since \(\delta^H_{C(e)} > \delta^L_b\), this condition holds by assumption. Substitution of either of the other distribution functions yields similar results.
Suppose, instead, that the equilibrium bounds are disjoint but contiguous rather than overlapping, i.e., $\hat{x}_0^H = \hat{x}_0^L$, ceteris paribus. The argument ruling out this configuration is similarly straightforward as the one just given. In particular, take a signaler of high type choosing $\hat{x}_0^H$ (using $b$), bearing in mind that all relevant limits on the positioning of the bounds of the equilibrium support are greater in magnitude than $\hat{x}_0^L$. Since the difference in the cost of the signaling activities implies that a signaler of low type can, in principle, afford a higher magnitude than his upper bound when choosing the less beneficial activity, the signaler faces a positive probability of losing against some low types. When choosing $\hat{x}_0^H > \hat{x}_0^L$, in turn, mimicry is not profitable for signalers of low type, implying that the signaler strictly increases his probability of winning – although he continues to lose against all signalers of high type, he will now win against all low types. Hence, in sharp contrast to the setting with par cost, separate but adjoining supports cannot be sustained in equilibrium.

(e) Welfare

As indicated in the main text, in order to determine a player’s welfare in a given equilibrium one must establish and compare his/her expected utility when assigned the role of signaler (for each type) and the expected utility when assigned the role of receiver.

Given the discussion relating to each type of signaler in the main text, it is not only easy to verify the expressions in Table 4.A.1, but also that the conditions summarized in Table 4.A.2, which can be shown to be necessary and sufficient, give rise to preference rankings (in the sense that one outcome Pareto-dominates the other) such that the equilibrium utility achieved via the row outcome is favored to that of the column outcome. In fact, they are quite intuitive.

For instance, coordination on the more beneficent signal ($E_4$) is favored to coordination on the less beneficent one ($E_3$) if using the former activity is relatively less expensive for a signaler of high type than one of low type. This intuition also justifies the ranking of $E_5$ and $E_6$. Similarly, when comparing $E_3$ to $E_5$, in which case signalers of low type behave identically, $E_3$ (which involves the less beneficial activity only) is favored if use of the more beneficent signal is relatively more expensive for signalers of high type. As one might expect, the
### Table 4.A.1. Unequal Cost – A Player’s Expected Utility from Each Equilibrium Where Assigned the Role of Signaler

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both types choose $b$ ($E_3$)</td>
<td>$\lambda \cdot \left[ (1 - \lambda) \cdot p - \bar{y}^H \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}} \right]$</td>
</tr>
<tr>
<td>Both types choose $\bar{b}$ ($E_4$)</td>
<td>$\lambda \cdot \left[ (1 - \lambda) \cdot p - \bar{y}^H \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}} \right]$</td>
</tr>
<tr>
<td>Low Types Choose $b$ and High Types $\bar{b}$ ($E_5$)</td>
<td>If $C$ is taken to be binding: $\lambda \cdot \left[ (1 - \lambda) \cdot p - \bar{y}^H \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}} \right]$</td>
</tr>
<tr>
<td></td>
<td>If $B$ is taken to be binding: $\lambda \cdot \left[ (1 - \lambda) \cdot p - \bar{y}^H \cdot \frac{A}{\bar{A}} \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}} \right]$</td>
</tr>
<tr>
<td>Low Types Choose $\bar{b}$ and High Types $b$ ($E_6$)</td>
<td>If $C$ is taken to be binding: $\lambda \cdot \left[ (1 - \lambda) \cdot p - \bar{y}^H \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}} \right]$</td>
</tr>
<tr>
<td></td>
<td>If $B$ is taken to be binding: $\lambda \cdot \left[ (1 - \lambda) \cdot p - \bar{y}^H \cdot \frac{A}{\bar{A}} \cdot (1 - \lambda) \cdot \frac{p}{\bar{y}} \right]$</td>
</tr>
</tbody>
</table>

The difference in the conditions across the limits highlights their relative “foci,” i.e., if $C$ is binding, the marginal cost of signalers of low type plays a role, whereas the condition when $B$ is binding involves the signal-dependent benefits to the receiver only. The intuition for the rankings for $E_4$ and $E_6$ is basically the same. Lastly, note that the conditions when ranking the equilibria “within” $E_5$ and $E_6$ emphasize the tradeoff between the conditions underlying the relevant limits. That is to say, in the context of $E_5$, if the cost ratio for signalers of low type is high in the sense that the more beneficial activity is quite a bit more expensive than its counterpart, the equilibrium with limit $C$ – the one involving a lower expenditure on the part of signalers of high type – is more preferable.

In general, however, it is not possible to rank the outcomes without auxiliary assumptions. Even the briefest examination of Table 4.A.2 suffices to conclude that all complete rankings depend on the parameter values. As an example (cf. main text), if $\frac{\bar{y}}{\bar{y}^L} > \frac{\bar{y}^H}{\bar{y}}$, $E_4$ or $E_5$ will be most preferable from the signalers’
Table 4.A.2. Unequal Cost – The Conditions Determining a Player’s Preference over the Various Equilibria When Assigned the Role of Signaler (Row > Column)

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>$E_3$</th>
<th>$E_4$</th>
<th>$E_3^c$ ($E_3^b$)</th>
<th>$E_4^c$ ($E_4^b$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_3$</td>
<td></td>
<td>$\frac{y^L}{y^L} &lt; \frac{y^H}{y^H}$</td>
<td>$\frac{\alpha}{\alpha} &lt; \frac{y^H}{y^H}$</td>
<td>Indifference</td>
</tr>
<tr>
<td>$E_4$</td>
<td>$\frac{y^L}{y^L} &gt; \frac{y^H}{y^H}$</td>
<td></td>
<td>$\frac{\alpha}{\alpha} &gt; \frac{y^H}{y^H}$</td>
<td>Indifference</td>
</tr>
<tr>
<td>$E_3^c$ ($E_3^b$)</td>
<td>$\frac{y^L}{y^L} &gt; \frac{y^H}{y^H}$</td>
<td>$\frac{\alpha}{\alpha} &gt; \frac{y^H}{y^H}$</td>
<td>Indifference</td>
<td></td>
</tr>
<tr>
<td>$E_4^c$ ($E_4^b$)</td>
<td>Indifference</td>
<td>$\frac{\alpha}{\alpha} &lt; \frac{y^H}{y^H}$</td>
<td>$\frac{\alpha}{\alpha} &gt; \frac{y^H}{y^H}$</td>
<td></td>
</tr>
</tbody>
</table>

Legend: The superscripts on the equilibria supporting multiple outcomes denote the limit being considered. The two within-outcome comparisons compare the row outcome using limit $C$ to the column outcome using limit $B$, e.g., the value provided for $E_3$ gives $E_3^c > E_3^b$.

The derivations for a player’s expected utility when assigned the role of receiver involve determining the expected value of the highest signaling magnitude in each of three possible scenarios (cf. main text). Table 4.A.3 demonstrates the relevant computations for the outcome where both signaler types coordinate on the less beneficial activity ($E_3$). Those for the other equilibria follow from a similar logic. Table 4.A.4 summarizes the results.

Granted that the expected utility of one outcome in Table 4.A.4 has to be greater than that of another if each factor of the former expression exceeds that of the latter, straightforward manipulation yields the necessary conditions presented in Table 4.A.5. Comparing the results in this table to those in Table 4.A.2, one finds that most receiver conditions subsume those when assigned the role of sig-
Table 4.A.3. Unequal Cost – Derivation of the Receiver’s Expected Utility for the Outcome Where Both Signaler Types Choose $b$ ($E_3$)

<table>
<thead>
<tr>
<th>Event</th>
<th>Preliminaries</th>
<th>Computations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two low types</td>
<td><strong>Objective</strong>: $E[\max(s^L_1, s^L_2)]$</td>
<td>$D = \Pr(s^L_1 \leq \varsigma$ and $s^L_2 \leq \varsigma) = G^L(\varsigma) \cdot G^L(\varsigma)$</td>
</tr>
<tr>
<td></td>
<td>$\Pr(LL) = (1-\lambda)^2$</td>
<td>$d = \Pr(\max(s^L_1, s^L_2)) = \frac{\partial D}{\partial \varsigma} = \frac{2 \cdot \varsigma}{(\delta^L_3)^2}$</td>
</tr>
<tr>
<td></td>
<td>$0 \leq \varsigma \leq \delta^L_3$</td>
<td>$E[\max(s^L_1, s^L_2)] = \int_{\delta^L_3}^{\delta^L_3} \varsigma \cdot d\varsigma = \frac{2}{3} \cdot \delta^L_3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>⇒ &quot;Benefit&quot; = $(1-\lambda)^2 \cdot [\theta^L + \Lambda \cdot \left(\frac{2}{3} \cdot \delta^L_3\right)]$</td>
</tr>
<tr>
<td>One low type, one high type</td>
<td><strong>Objective</strong>: $E[\max(s^H)]$</td>
<td>$D = \Pr(s^L \leq \varphi$ and $s^H \leq \varphi) = G^H(\varphi)$</td>
</tr>
<tr>
<td></td>
<td>$\Pr(LH$ and $HL)$</td>
<td>$d = \Pr(\max(s^H)) = \frac{\partial D}{\partial \varphi} = \varphi H(\varphi)$</td>
</tr>
<tr>
<td></td>
<td>$\delta^H_3 \leq \varphi \leq \delta^H_3$</td>
<td>$E[\max(s^H)] = \int_{\delta^H_3}^{\delta^H_3} \varphi \cdot d\varphi = \delta^H_3 + \frac{1}{2} \cdot \frac{\lambda \cdot p}{\varphi H}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>⇒ &quot;Benefit&quot; = $2 \cdot [\lambda \cdot (1-\lambda)] \cdot \left[\theta^H + \Lambda \cdot \left(\frac{2}{3} \cdot \delta^H_3 + \frac{1}{2} \cdot \frac{\lambda \cdot p}{\varphi H}\right)\right]$</td>
</tr>
<tr>
<td>Two high types</td>
<td><strong>Objective</strong>: $E[\max(s^H_1, s^H_2)]$</td>
<td>$D = \Pr(s^H_1 \leq \varphi$ and $s^H_2 \leq \varphi) = G^H(\varphi) \cdot G^H(\varphi)$</td>
</tr>
<tr>
<td></td>
<td>$\Pr(HH) = \lambda^2$</td>
<td>$d = \Pr(\max(s^H_1, s^H_2)) = \frac{\partial D}{\partial \varphi} = \frac{(\lambda \cdot p)^2}{\varphi H}$</td>
</tr>
<tr>
<td></td>
<td>$\delta^H_3 \leq \varphi \leq \delta^H_3$</td>
<td>$E[\max(s^H_1, s^H_2)] = \int_{\delta^H_3}^{\delta^H_3} \varphi \cdot d\varphi = \delta^H_3 + \frac{2}{3} \cdot \frac{\lambda \cdot p}{\varphi H}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>⇒ &quot;Benefit&quot; = $\lambda^2 \cdot \left[\theta^H + \Lambda \cdot \left(\frac{2}{3} \cdot \delta^H_3 + \frac{\lambda \cdot p}{\varphi H}\right)\right]$</td>
</tr>
</tbody>
</table>

⇒ Expected Utility of the Receiver ($E_3^R$):

$$E_3^R = (1-\lambda)^2 \cdot \left[\theta^L + \Lambda \cdot \left(\frac{2}{3} \cdot \delta^L_3\right)\right] + 2 \cdot [\lambda \cdot (1-\lambda)] \cdot \left[\theta^H + \Lambda \cdot \left(\delta^H_3 + \frac{1}{2} \cdot \frac{\lambda \cdot p}{\varphi H}\right)\right]$$

$$+ \lambda^2 \cdot \left[\theta^H + \Lambda \cdot \left(\delta^H_3 + 2 \cdot \frac{\lambda \cdot p}{\varphi H}\right)\right]$$
naler. Only $E_5$ and $E_6$ explicitly require the conditions for both roles. Note correspondingly, that the signaler and receiver conditions for $E_3$ vs. $E_6$, $E_4$ vs. $E_5$, and those “within” $E_5$ and $E_6$ are mutually exclusive, implying that they cannot be ranked as signalers and receivers always prefer different outcomes.

**Table 4.A.4.** Unequal Cost – A Player’s Expected Utility in Each Equilibrium When Assigned the Role of Receiver

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both types choose $b$ ($E_3$)</td>
<td>$(1 - \lambda)^2 \left[ \theta^L + \frac{\lambda^L}{3} \right] \left[ (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$+2 \cdot \lambda \cdot (1 - \lambda) \cdot \left[ \theta^H + \frac{\lambda}{2} \cdot (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$+\lambda^2 \cdot \left[ \theta^H + \frac{\lambda}{2} \cdot (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td>Both types choose $\bar{b}$ ($E_4$)</td>
<td>$(1 - \lambda)^2 \left[ \theta^L + \frac{\lambda^L}{3} \right] \left[ (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$+2 \cdot \lambda \cdot (1 - \lambda) \cdot \left[ \theta^H + \frac{\lambda}{2} \cdot (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$+\lambda^2 \cdot \left[ \theta^H + \frac{\lambda}{2} \cdot (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td>If $C$ is taken to be binding:</td>
<td></td>
</tr>
<tr>
<td>(Low Types Choose $b$ and High Types $\bar{b}$ ($E_5$))</td>
<td>$(1 - \lambda)^2 \left[ \theta^L + \frac{\lambda^L}{3} \right] \left[ (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$+2 \cdot \lambda \cdot (1 - \lambda) \cdot \left[ \theta^H + \frac{\lambda}{2} \cdot (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$+\lambda^2 \cdot \left[ \theta^H + \frac{\lambda}{2} \cdot (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td>If $B$ is taken to be binding:</td>
<td></td>
</tr>
<tr>
<td>(Low Types Choose $\bar{b}$ and High Types $\bar{b}$ ($E_6$))</td>
<td>$(1 - \lambda)^2 \left[ \theta^L + \frac{\lambda^L}{3} \right] \left[ (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$+2 \cdot \lambda \cdot (1 - \lambda) \cdot \left[ \theta^H + \frac{\lambda}{2} \cdot (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
<tr>
<td></td>
<td>$+\lambda^2 \cdot \left[ \theta^H + \frac{\lambda}{2} \cdot (1 - \lambda) \cdot \frac{p}{\gamma} \right]$</td>
</tr>
</tbody>
</table>
Chapter 4. Choice of Signaling Activity

Low Types Choose $B$ and High Types $b$ (E_b)

If $C$ is taken to be binding:

$$(1 - \lambda)^2 \cdot \left[ \theta^L + \bar{A} \cdot \frac{2}{3} \cdot (1 - \lambda) \cdot \frac{p}{V^L} \right]$$
$$+ 2 \cdot \lambda \cdot (1 - \lambda) \cdot \left[ \theta^H + \bar{A} \cdot \left( (1 - \lambda) \cdot \frac{p}{V^L} + \frac{1}{2} \cdot \frac{\lambda \cdot p}{V^H} \right) \right]$$
$$+ \lambda^2 \cdot \left[ \theta^H + \bar{A} \cdot \left( (1 - \lambda) \cdot \frac{p}{V^L} + \frac{2}{3} \cdot \frac{\lambda \cdot p}{V^H} \right) \right]$$

If $B$ is taken to be binding:

$$(1 - \lambda)^2 \cdot \left[ \theta^L + \bar{A} \cdot \frac{2}{3} \cdot (1 - \lambda) \cdot \frac{p}{V^L} \right]$$
$$+ 2 \cdot \lambda \cdot (1 - \lambda) \cdot \left[ \theta^H + \bar{A} \cdot \left( (1 - \lambda) \cdot \frac{p}{V^L} + \frac{1}{2} \cdot \frac{\lambda \cdot p}{V^H} \right) \right]$$
$$+ \lambda^2 \cdot \left[ \theta^H + \bar{A} \cdot \left( (1 - \lambda) \cdot \frac{p}{V^L} + \frac{2}{3} \cdot \frac{\lambda \cdot p}{V^H} \right) \right]$$

Nevertheless, from this perspective, too, the conditions make intuitive sense. In the cases involving coordination of the signaler types on the same signal, for instance, the outcome featuring the more beneficial activity will always be preferred (by all parties) if the signal-dependent benefit associated with it is relatively sizeable. Along similar lines, if $C$ is binding, the ranking of $E_s$ vs. $E_6$, as one would expect, depends intrinsically on the activities’ relative costliness for signalers of high type. Yet, again, generic rankings depend on the parameter values. In short, in contrast to the equal-cost case, the welfare implications differ subject to the setting being considered.

(f) Intuitive Criterion

i. Coordination on the Same Signaling Activity

Even though the equilibria involving coordination on the same signaling activity on the part of the signaler types may fail to exist on account of the parameter values, the “intuitive criterion,” cannot be used to eliminate either of them. Because even if it were possible to construct a region of signaling magnitudes that is equilibrium-dominated for signalers of low type regardless of the receiver’s beliefs, it would never be attractive to signalers of high type.
### Table 4.A.5. Unequal Cost – The Conditions Determining a Player’s Preference over the Various Equilibria When Assigned the Role of Receiver (Row > Column)

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>$E_3$</th>
<th>$E_4$</th>
<th>$E_5^C$ ($E_5^B$)</th>
<th>$E_6^C$ ($E_6^B$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_5^C$ ($E_5^B$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_6^C$ ($E_6^B$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend: The superscripts on the equilibria supporting multiple outcomes denote the limit being considered. The two within-outcome comparisons compare the row outcome using limit $C$ to the column outcome using limit $B$, e.g., the value provided for $E_5$ gives $E_5^C > E_5^B$.

#### ii. Coordination on Different Signaling Activities

The situation is different in the instances supporting a continuum of equilibria, in which case application of the “intuitive criterion” does eliminate a range of outcomes. Take the profiles where low types choose the less and high types the more beneficent signal ($E_5$). Recall that any outcome with a lower bound between $\max\{C, J\}$ and $B$ may arise as an equilibrium in this scenario, with $C$ denoting the point where $Y^H \cdot C = Y^L \cdot E$, $J$ the point at which $(\theta^H + A \cdot J) \geq (\theta^L + A \cdot E)$, and $B$ the point where $(\theta^L + A \cdot B) = (\theta^L + A \cdot E)$.

Now, consider an equilibrium with a lower bound for the high types’ equilibrium support between $J$ and $B$, say at point $x$. Figure 4.A.3 reproduces Figure 4.5 in the main text, augmenting it with the receiver’s optimal beliefs for the equilibrium starting at $x$. Given the belief structure drawn in the figure, the argu-
ment to sustain the equilibrium starting at $x$ is that a signaler of high type would not want to deviate below his lower bound as he would be assumed to be a low type and, since $x < B$, he would be considered inferior to his alter-ego behaving according to the equilibrium strategy. A signaler of low type, in turn, would not want to send a message (using $\hat{b}$) at or slightly below $x$ since it would entail a strictly higher cost of signaling without a corresponding increase in his probability of winning; in fact, he would no longer be able to defeat all (other) low types because the benefit to the receiver of the deviating message would fall short of the benefit at his equilibrium upper bound (as $x < B$). This line of reasoning entails that all messages on the interval $[\mathcal{C}, B]$ are equilibrium dominated for signalers of low type regardless of the receiver’s response to them. This, in turn, implies that the receiver should not assign a non-zero probability to deviating messages in that interval having been sent by signalers of low type. Yet, if she were to assign the relevant messages to high type, signalers of high type could profitably deviate to the relevant region thereby reducing their expenditure on signaling without affecting their probability of winning; they would continue to win against all low types and lose against all (other) high types.

Hence, in Figure 4.A.3 the refinement eliminates all equilibria on the interval $(\mathcal{I}, B]$. It should be obvious that the same argument can be used to eliminate all equilibria on the interval $(\mathcal{C}, B]$ if $\mathcal{I} < \mathcal{C}$, and extends straightforwardly to the profile where signalers of low type opt for the more and those of high type for the less beneficent signal ($E_6$). In other words, the “intuitive criterion” eliminates
all equilibria between \( \max \{ C, J \} \) and \( B \), reducing the set of equilibria in both cases to a unique point-prediction at \( \max \{ C, J \} \).


