FRP RUPTURE STRAINS IN FRP WRAPPED COLUMNS

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A thesis
presented to the University of Edinburgh
in fulfilment of the
thesis requirement for the degree of
Doctor of Philosophy

Edinburgh, Scotland, United Kingdom, 2012
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Author's declaration


The research was solely the work of the author except where otherwise acknowledged in the text, and has not formed the basis of a submission for any other degrees. Publications based on this thesis include:


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January 2012
Abstract

Applying lateral confinement to concrete columns using fibre-reinforced polymer (FRP) composites is a very promising technique. FRP rupture is the typical failure mode of FRP wrapped columns under axial compression. Numerous experiments have shown that the FRP rupture strain in an FRP wrapped circular column is significantly lower than the FRP ultimate rupture strain determined from flat coupon test of FRP. Despite a large number of studies on the application of FRP confined columns, the mechanisms and level of lower-than-apparent FRP rupture strain still remain unclear. This thesis presents theoretical, numerical and experimental studies aiming at developing a deeper understanding of the fundamental mechanisms of this phenomenon.

A comprehensive literature review was presented providing the background on FRP confined columns, material properties of FRP composites as well as some factors which may lead to premature FRP rupture. A FE analysis was conducted to investigate the FRP hoop strains in the split-disk test, explaining for the first time that the fundamental mechanism of the lower FRP rupture strain in the split-disk test than in the flat coupon test is because strain localisation due to geometric discontinuities at the ends of the FRP and bending of the FRP ring at the gap due to change of curvature caused by the relative moment of the two half disks, as the FRP (as a brittle material) ruptures once the maximum strain at one of these locations reaches the FRP rupture strain.
A list of contributory factors affecting the apparent FRP rupture strain in FRP wrapped columns were next identified and classified. An analytical solution was developed to investigate the influence of the triaxial stress state on the FRP strain efficiency, this factor has been shown to have a potentially significant effect on the failure of the FRP wrap but considerable discrepancies exist between predictions using different failure criteria so further research has been identified in this area. FE models were developed to examine the effect of the geometrical discontinuities on the strain efficiency of FRP jackets in FRP wrapped concrete-filled circular steel tubes and FRP wrapped concrete columns. It is demonstrated that severe FRP hoop strain concentrations occur in very small zones near the ends of the FRP wrap in both types of FRP wrapped columns, leading to premature FRP rupture and thus lower strain efficiency.

The combined effects of end constraint and FRP overlap on the behaviour of FRP wrapped concrete columns was investigated using a three dimensional FE model considering one half of the length of an FRP-wrapped concrete cylinder. The results have shown that the friction between both ends of a column and the loading platens provides constraints to the ends of the column, but this constraint has little effect on the strain concentration caused by the geometrical discontinuities of the FRP overlap, though the ultimate axial strain of the FRP wrapped columns can be significantly overestimated if the end constraints are not considered.
Acknowledgements

Words cannot express my sincere thanks and gratitude to my supervisor, Dr Jian-Fei Chen, and the co-supervisor, Dr Luke A. Bisby, for their guidance, inspiration and constant support throughout my PhD study. The doors to their offices were always open to me when I had difficulties or questions in my research.

I would like to thank Dr Tim Stratford, Prof. Jin Ooi and Dr Jun Ai for their valuable advices and help. I am deeply indebted to Dr. Tao Yu for his help and teaching for the modelling work and thesis writing. Sincere thanks also go to Dr. Yueming Hu, for his help to provide his experimental data.

Many thanks are due to my other colleagues, Dr Xiaoqin Li, Dr Vijayabaskar Narayanamurthy, Mr. Yi Tao, Mr. Yin Wang and Dr Chong Zhou for their help and beneficial discussions. I am also grateful to the other members in the office, for their academic and emotional support.

For their assistance in data processing and providing experimental results, I would like to thank former MEng students, Noemi Cueva, Kate Crossling, Ben Duerden and Kirsty McLarty.

In addition, I would like to express my gratitude to Prof. Jin-Guang Teng, my host supervisor during my visit at Hong Kong Polytechnic University, for his supervision, advice and help.

Finally I would like to thank my family for their grateful love and supports, without which this study would not be possible. To them I dedicate this thesis.
# Table of contents

Author's declaration ............................................................................................................i  
Abstract ................................................................................................................................iii  
Acknowledgements ...........................................................................................................v  
Table of contents ...............................................................................................................vi  
List of figures .................................................................................................................... xiii  
List of tables ..................................................................................................................... xvii  
Notation ........................................................................................................................... xviii  

## CHAPTER 1 INTRODUCTION .......................................................................................1  
1.1 Background ..............................................................................................................1  
1.2 Statement of problem ...............................................................................................3  
1.3 Research scope and objectives ...............................................................................5  
1.4 Outline of thesis .......................................................................................................6  

## CHAPTER 2 LITERATURE REVIEW ..........................................................................10  
2.1 Introduction ............................................................................................................10  
2.2 FRP composites .....................................................................................................10  
2.3 Determination of FRP composites mechanical properties .......................................11  
   2.3.1 Introduction......................................................................................................11  
   2.3.2 Test methods ................................................................................................12  
   2.3.2.1 Flat coupon test .........................................................................................12  
   2.3.2.2 Split-disk test .............................................................................................13  
   2.3.2.3 Tubular specimen test ...............................................................................16  
   2.3.2.4 Other test methods ....................................................................................18  
2.3.3 Micromechanics of FRP composites ..................................................................18  
   2.3.3.1 Introduction...............................................................................................18  
   2.3.3.2 Mechanics of materials approach..............................................................19  
   2.3.3.3 The Halpin-Tsai method ...........................................................................21  
   2.3.3.4 Hahn equations ........................................................................................22  
2.4 FRP failure criteria ................................................................................................24
# TABLE OF CONTENTS

2.5 FRP confined columns ...........................................................................................................26
   2.5.1 FRP strengthened reinforced concrete columns .........................................................27
      2.5.1.1 Strength models ..................................................................................................28
      2.5.1.2 Stress-strain models .......................................................................................29
      2.5.1.3 FRP rupture strain .........................................................................................36
   2.5.2 FRP strengthened concrete-filled steel tubes ..............................................................37
   2.5.3 Concrete-filled FRP tubes ..........................................................................................37
   2.5.4 FRP repair of fire damaged concrete columns ..........................................................38
   2.5.5 FRP-concrete-steel hybrid tubular columns ..............................................................39
   2.6 FRP strain efficiency ......................................................................................................41
      2.6.1 Experimental observations .................................................................................41
      2.6.2 Proposed values of FRP strain efficiency ...............................................................43
      2.6.3 Reasons for the lower-than apparent rupture strain ..............................................45
   2.7 Conclusions ..................................................................................................................46

CHAPTER 3 FRP RUPTURE STRAINS IN THE SPLIT-DISK TEST ........................................48
   3.1 Introduction ..................................................................................................................48
   3.2 Geometry and materials ..............................................................................................49
      3.2.1 Geometry .............................................................................................................49
      3.2.2 Adhesive properties ............................................................................................51
      3.2.3 Properties of the FRP composite .........................................................................52
   3.3 Finite element modelling .............................................................................................52
      3.3.1 FE model .............................................................................................................52
      3.3.2 Mesh convergence ...............................................................................................55
      3.3.3 FRP thickness .....................................................................................................58
      3.3.4 Effect of FRP orthotropy .....................................................................................63
   3.4 Analysis of strains in fibre orientation in the FRP wrap ...............................................66
      3.4.1 Load versus peak strains .....................................................................................66
      3.4.2 Strains at locations A and B .................................................................................67
      3.4.3 Strains at Location C ..........................................................................................68
   3.5 Comparison of FE predictions with test results .........................................................70
3.5.1 Test data ...........................................................................................................70
3.5.2 Comparison with test results ............................................................................72
3.6 Parametric study .....................................................................................................74
  3.6.1 Effect of adhesive strength ..............................................................................75
  3.6.2 Effect of adhesive thickness ............................................................................76
  3.6.3 Effect of transition zone size ...........................................................................77
  3.6.4 Effect of FRP stiffness .....................................................................................78
  3.6.5 Effect of friction ..............................................................................................81
3.7 Conclusions ............................................................................................................82

CHAPTER 4 ON FACTORS AFFECTING THE ULTIMATE CONDITION OF FRP WRAPPED CONCRETE COLUMNS ............................................................................85

4.1 Introduction ............................................................................................................85
4.2 Failure modes .........................................................................................................86
  4.2.1 FRP rupture ......................................................................................................86
    4.2.1.1 Rupture near the outer end of FRP ............................................................87
    4.2.1.2 Rupture near the inner end of FRP ............................................................88
    4.2.1.3 Rupture outside the overlapping region ....................................................89
  4.2.2 Debonding failure at the overlap zone .............................................................90
  4.2.3 Other failure modes .........................................................................................90
4.3 Contributory factors ..............................................................................................91
  4.3.1 Geometrical factors ..........................................................................................95
    4.3.1.1 Geometrical discontinuities .......................................................................95
    4.3.1.2 FRP overlap region ...................................................................................98
    4.3.1.3 Geometrical imperfections ........................................................................99
    4.3.1.4 Curvature of the FRP jacket ....................................................................100
  4.3.2 FRP material factors ......................................................................................101
    4.3.2.1 Fibre orientation ......................................................................................101
    4.3.2.2 Misalignment and uneven tension of fibres ............................................103
    4.3.2.3 Damage of fibres .....................................................................................104
    4.3.2.4 Three dimensional stress state in the FRP ..............................................105
4.3.3 Concrete material factors ................................................................. 107
  4.3.3.1 Non-uniform deformation in the concrete ................................. 107
  4.3.3.2 Strain localisation in the concrete ............................................ 108
4.3.4 Adhesive material and geometry factors ........................................... 110
  4.3.4.1 Mechanical properties of the adhesive ....................................... 110
  4.3.4.2 Geometrical details of the adhesive .......................................... 112
  4.3.4.3 Non-uniform bonding and partial debonding of the FRP ............. 113
4.3.5 Loading factors .............................................................................. 114
  4.3.5.1 Loading eccentricity ................................................................. 114
  4.3.5.2 Non-uniform loading ................................................................. 115
  4.3.5.3 Frictional confinement at the supports .......................................... 116
  4.3.5.4 Stressing due to thermal deformation and creep ...................... 117
4.4 Future research .................................................................................... 118
4.5 Conclusions ......................................................................................... 118

CHAPTER 5 EFFECT OF TRIAXIAL STRESS ON FRP RUPTURE STRAIN IN FRP
CONFINED CONCRETE COLUMNS ................................................................. 121
  5.1 Introduction .......................................................................................... 121
  5.2 Stresses in FRP ...................................................................................... 122
    5.2.1 3-D stress state .............................................................................. 122
    5.2.2 Simplified 2-D stress states ............................................................ 126
  5.3 Failure criteria of FRP ......................................................................... 127
    5.3.1 Maximum stress theory ................................................................. 128
    5.3.2 Maximum strain theory ................................................................. 129
    5.3.3 Tsai-Hill theory ............................................................................ 130
    5.3.4 Tsai and Wu’s theory ................................................................. 131
  5.4 Comparison with results from existing analysis and test .................... 134
    5.4.1 Comparison with Fraldi et al’s analysis ........................................ 134
    5.4.2 Comparison with test results ......................................................... 135
      5.4.2.1 Experiments ............................................................................ 135
      5.4.2.2 Mechanical properties of CFRP composites ......................... 138
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.4.2.3</td>
<td>Comparison of different failure criteria</td>
<td>139</td>
</tr>
<tr>
<td>5.5</td>
<td>Parametric study</td>
<td>140</td>
</tr>
<tr>
<td>5.5.1</td>
<td>Strength in r-z plane</td>
<td>140</td>
</tr>
<tr>
<td>5.5.2</td>
<td>Concrete strength</td>
<td>141</td>
</tr>
<tr>
<td>5.5.3</td>
<td>The number of FRP layers</td>
<td>142</td>
</tr>
<tr>
<td>5.6</td>
<td>Conclusions</td>
<td>143</td>
</tr>
</tbody>
</table>

**CHAPTER 6 STRAIN EFFICIENCY OF FRP JACKETS IN FRP-CONFINED CONCRETE-FILLED CIRCULAR STEEL TUBES**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>145</td>
</tr>
<tr>
<td>6.2</td>
<td>Experiments</td>
<td>146</td>
</tr>
<tr>
<td>6.3</td>
<td>Geometry and materials</td>
<td>149</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Geometry</td>
<td>149</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Adhesive properties</td>
<td>150</td>
</tr>
<tr>
<td>6.3.3</td>
<td>FRP properties</td>
<td>151</td>
</tr>
<tr>
<td>6.4</td>
<td>Finite element modelling</td>
<td>152</td>
</tr>
<tr>
<td>6.4.1</td>
<td>Loading and boundary conditions</td>
<td>152</td>
</tr>
<tr>
<td>6.4.2</td>
<td>Mesh convergence</td>
<td>153</td>
</tr>
<tr>
<td>6.4.3</td>
<td>FRP strain distributions</td>
<td>156</td>
</tr>
<tr>
<td>6.5</td>
<td>Comparison between test results and FE predictions</td>
<td>161</td>
</tr>
<tr>
<td>6.6</td>
<td>Parametric study</td>
<td>164</td>
</tr>
<tr>
<td>6.6.1</td>
<td>FRP thickness</td>
<td>165</td>
</tr>
<tr>
<td>6.6.2</td>
<td>FRP membrane stiffness</td>
<td>166</td>
</tr>
<tr>
<td>6.6.3</td>
<td>FRP orthotropy</td>
<td>167</td>
</tr>
<tr>
<td>6.6.4</td>
<td>Adhesive yield stress and modulus</td>
<td>168</td>
</tr>
<tr>
<td>6.6.5</td>
<td>Adhesive thickness</td>
<td>169</td>
</tr>
<tr>
<td>6.6.6</td>
<td>Column size</td>
<td>170</td>
</tr>
<tr>
<td>6.7</td>
<td>Conclusions</td>
<td>170</td>
</tr>
</tbody>
</table>

**CHAPTER 7 FRP STRAIN EFFICIENCY IN FRP-WRAPPED CIRCULAR CONCRETE COLUMNS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>173</td>
</tr>
<tr>
<td>7.2 Test of FRP wrapped concrete columns</td>
<td>174</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------</td>
<td>-----</td>
<td></td>
</tr>
<tr>
<td>7.2.1 Test setup</td>
<td>175</td>
<td></td>
</tr>
<tr>
<td>7.2.2 Axial stress-strain curve</td>
<td>176</td>
<td></td>
</tr>
<tr>
<td>7.2.3 PIV analysis</td>
<td>178</td>
<td></td>
</tr>
<tr>
<td>7.2.4 Measurement of FRP thickness</td>
<td>181</td>
<td></td>
</tr>
<tr>
<td>7.3 FE modelling</td>
<td>184</td>
<td></td>
</tr>
<tr>
<td>7.3.1 Geometry</td>
<td>184</td>
<td></td>
</tr>
<tr>
<td>7.3.2 Concrete</td>
<td>186</td>
<td></td>
</tr>
<tr>
<td>7.3.3 Adhesive</td>
<td>187</td>
<td></td>
</tr>
<tr>
<td>7.3.4 FRP</td>
<td>187</td>
<td></td>
</tr>
<tr>
<td>7.3.5 Mesh convergence</td>
<td>187</td>
<td></td>
</tr>
<tr>
<td>7.3.6 Predicted FRP strain distributions</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>7.4 Verification of FE model</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>7.4.1 Comparison of stress-strain curves</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>7.4.2 Comparison of FE predictions with PIV results</td>
<td>193</td>
<td></td>
</tr>
<tr>
<td>7.5 Examination of FE results</td>
<td>196</td>
<td></td>
</tr>
<tr>
<td>7.5.1 FE virtual strain of FRP</td>
<td>196</td>
<td></td>
</tr>
<tr>
<td>7.5.2 3D versus 2D models</td>
<td>198</td>
<td></td>
</tr>
<tr>
<td>7.5.3 Concrete stress distribution</td>
<td>202</td>
<td></td>
</tr>
<tr>
<td>7.6 Conclusions</td>
<td>205</td>
<td></td>
</tr>
<tr>
<td>CHAPTER 8 COMBINED EFFECT OF END CONSTRAINT AND FRP OVERLAP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ON FRP WRAPPED CONCRETE COLUMNS</td>
<td>208</td>
<td></td>
</tr>
<tr>
<td>8.1 Introduction</td>
<td>208</td>
<td></td>
</tr>
<tr>
<td>8.2 Finite element modelling</td>
<td>208</td>
<td></td>
</tr>
<tr>
<td>8.2.1 Geometry</td>
<td>209</td>
<td></td>
</tr>
<tr>
<td>8.2.2 Materials</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>8.2.3 Boundary conditions</td>
<td>211</td>
<td></td>
</tr>
<tr>
<td>8.2.4 Mesh convergence</td>
<td>211</td>
<td></td>
</tr>
<tr>
<td>8.3 Axisymmetric model</td>
<td>215</td>
<td></td>
</tr>
<tr>
<td>8.3.1 Effect of friction</td>
<td>215</td>
<td></td>
</tr>
<tr>
<td>Chapter/Section</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>-------------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>8.3.2 Effect of concrete constitutive model</td>
<td>219</td>
<td></td>
</tr>
<tr>
<td>8.4 Verification of 3D model</td>
<td>222</td>
<td></td>
</tr>
<tr>
<td>8.5 Examination of 3D FE results</td>
<td>224</td>
<td></td>
</tr>
<tr>
<td>8.5.1 FRP Hoop strain</td>
<td>224</td>
<td></td>
</tr>
<tr>
<td>8.5.2 Concrete stress and strain distributions</td>
<td>227</td>
<td></td>
</tr>
<tr>
<td>8.5.3 Comparison of different FE models</td>
<td>230</td>
<td></td>
</tr>
<tr>
<td>8.6 Strain efficiency prediction using different failure criteria</td>
<td>232</td>
<td></td>
</tr>
<tr>
<td>8.7 Conclusions</td>
<td>235</td>
<td></td>
</tr>
<tr>
<td><strong>CHAPTER 9 CONCLUSIONS AND FUTURE RESEARCH</strong></td>
<td>237</td>
<td></td>
</tr>
<tr>
<td>9.1 Introduction</td>
<td>237</td>
<td></td>
</tr>
<tr>
<td>9.2 Overall conclusions</td>
<td>238</td>
<td></td>
</tr>
<tr>
<td>9.3 Detailed conclusions</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>9.3.1 FRP rupture strains in split-disk tests</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>9.3.2 Contribution factors for premature FRP rupture</td>
<td>241</td>
<td></td>
</tr>
<tr>
<td>9.3.3 Effect of triaxial stress state in FRP</td>
<td>242</td>
<td></td>
</tr>
<tr>
<td>9.3.4 Effect of geometrical discontinuities on FRP-wrapped concrete-filled</td>
<td>243</td>
<td></td>
</tr>
<tr>
<td>steel tubes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.3.5 Effect of geometrical discontinuities on FRP-wrapped concrete columns</td>
<td>244</td>
<td></td>
</tr>
<tr>
<td>9.3.6 Combined effect of end constraints and FRP overlap</td>
<td>245</td>
<td></td>
</tr>
<tr>
<td>9.4 Recommendation for future research</td>
<td>247</td>
<td></td>
</tr>
<tr>
<td>REFERENCES</td>
<td>249</td>
<td></td>
</tr>
</tbody>
</table>
List of figures

Figure 1.1 Strengthening of concrete columns using externally bonded FRPs applied in the hoop direction ................................................................. 3
Figure 1.2 Typical initiation of explosive failure mode of FRP wrapped columns ....... 4
Figure 2.1 Tensile flat coupon test of an FRP ................................................................ 12
Figure 2.2 Split-disk test .................................................................................................. 13
Figure 2.3 Tubular specimen .......................................................................................... 17
Figure 2.4 Comparison of four failure criteria for FRP composites (after Yu et al. 2009b) .................................................................................................................................. 25
Figure 2.5 Lam and Teng’s (2003a) design-oriented model (after Lam and Teng 2003a) .................................................................................................................................. 31
Figure 2.6 Typical sections of double-skin tubular columns (after Teng et al. 2007b) ... 40
Figure 3.1 Geometry of the split disk test .................................................................. 50
Figure 3.2 Distribution of FRP fibre direction strain on the ............................................ 54
Figure 3.3 Effect of mesh size on FRP fibre direction strain distributions ................. 57
Figure 3.4 mesh near locations A and B for a minimum mesh size of 0.2mm ............ 57
Figure 3.5 Effect of mesh size on the maximum FRP fibre direction strain ............... 58
Figure 3.6 Effect of thickness option on FRP fibre direction strain distributions ...... 60
Figure 3.7 Effect of thickness option on strain concentration ....................................... 61
Figure 3.8 Predicted FRP fibre direction strains using .................................................... 64
Figure 3.9 Comparison of FRP fibre direction strains between isotropic and orthotropic material models at locations A, B, and C ......................................................... 65
Figure 3.10 Load versus peak FRP strains at locations A, B and C ............................... 66
Figure 3.11 Effect of adhesive yield strength on the maximum FRP fibre direction strain ........................................................................................................ 76
Figure 3.12 Effect of adhesive thickness on peak FRP fibre direction strains ............ 77
Figure 3.13 Effect of transition angle on peak FRP fibre direction strains ............... 78
Figure 3.14 Effect of FRP thickness on peak fibre direction strain ......... 80
Figure 3.15 Effect of elastic modulus of FRP on peak fibre direction strain .......... 80
Figure 3.16 FRP fibre direction strain distribution with a friction coefficient of 0.3 .....82
Figure 3.17 Effect of friction between FRP and disks on .............................................82
Figure 4.1 A circular concrete column wrapped with two layers of FRP ...............87
Figure 4.2 FRP rupture initiating at the outer end of the FRP wrap ......................88
Figure 4.3 FRP rupture at the inner end of FRP wrap ..............................................89
Figure 4.4 FRP rupture outside to the FRP overlap zone .......................................89
Figure 4.5 Debonding failure of FRP .......................................................................90
Figure 4.6 Mixed FRP debonding and fracture failure ...........................................91
Figure 4.7 FRP circumferential strain distribution along .........................................97
Figure 4.8 Geometrical imperfection ...................................................................99
Figure 4.9 Curvature of the FRP jacket ..................................................................101
Figure 4.10 Effect of fibre orientation on strain efficiency ..................................103
Figure 4.11 Misalignment of fibres .........................................................................104
Figure 4.12 Damage of fibres ..............................................................................105
Figure 4.13 Diagram of three dimensional stress state in the FRP .......................107
Figure 4.14 FRP strain localisation due to non-uniform deformation of concrete ....108
Figure 4.15 FRP strain localisation due to concrete cracking ...............................109
Figure 4.16 Effect of adhesive properties ...............................................................111
Figure 4.17 Schematic representations of spew fillet configurations ......................112
Figure 4.18 Schematic diagram of non-uniform bonding and/or .............................113
Figure 4.19 FRP wrapped column subject to eccentric axial compression loading ...115
Figure 4.20 Schematic diagram of non-uniform support in FRP wrapped column ....115
Figure 4.21 FRP hoop strain variability (schematized) over the height .................116
Figure 4.22 Unintentional stressing in FRP due to differential thermal expansion
   between the FRP and the concrete ......................................................................117
Figure 5.1 Test results of FRP-wrapped concrete columns ....................................137
Figure 5.2 Effect of out-plane strength $f_p$ on FRP strain efficiency .....................141
Figure 5.3 Effect of concrete strength on FRP strain efficiency ..............................142
Figure 5.4 Effect of number of FRP layers on strain efficiency ..............................143
Figure 6.1 FRP rupture near the outer end of the FRP overlapping zone ..............148
Figure 6.2 Idealized cross-section of FRP-confined concrete-filled steel tube with a two-layer FRP jacket .......................................................................................................................... 149
Figure 6.3 Mesh near locations A and B for a minimum mesh size of 0.32mm ............... 155
Figure 6.4 Effect of element size on the predicted FRP strain efficiency ....................... 156
Figure 6.5 Distributions of hoop strains from an elastic-perfectly plastic adhesive model .............................................................................................................................. 159
Figure 6.6 Distributions of FRP hoop strains near locations A and B .......................... 160
Figure 6.7 Predicted versus test strain efficiency factors under uniform radial displacement (loading condition LB1) .......................................................... 163
Figure 6.8 Predicted versus test strain efficiency factors under internal pressure (loading condition LB4) ........................................................ 163
Figure 6.9 Effect of FRP thickness on FRP strain efficiency ...................................... 166
Figure 6.10 Effect of FRP membrane stiffness on FRP strain efficiency ..................... 167
Figure 6.11 Effect of FRP radial-to-circumferential modulus ratio on FRP strain efficiency .......................................................................................................................... 168
Figure 6.12 Effect of adhesive yield stress on FRP strain efficiency ......................... 169
Figure 6.13 Effect of adhesive thickness on FRP strain efficiency ............................ 170
Figure 7.1 3-dimensional model of FRP wrapped concrete column with thickness of 1mm .......................................................................................................................... 174
Figure 7.2 Details of test specimens, FRP wrap configuration, and cameras positioning with respect to the FRP overlap ........................................................ 176
Figure 7.3 Comparison of the stress-strain curve of FRP wrapped concrete columns . 178
Figure 7.4 Distance from the outer end of FRP wrap .................................................. 179
Figure 7.5 Effect of gauge length on PIV strain distribution (patch size =48×48) ....... 180
Figure 7.6 Effect of patch size on PIV strain distribution (gauge length = 5mm) .......... 181
Figure 7.7 FRP wraps in the microscope ...................................................................... 184
Figure 7.8 FRP-confined concrete column model ...................................................... 185
Figure 7.9 Effect of mesh on FRP strain efficiency ..................................................... 189
Figure 7.10 Distribution of hoop strains on the inner and outer surfaces of the FRP wrap ......................................................................................................................... 191
Figure 7.11 FRP Hoop strain distributions near Locations A and B .........................192
Figure 7.12 Hoop strain distribution near the outer end of the FRP wrap .............195
Figure 7.13 Deformation at the finishing end of FRP wrap: PIV measurement versus FE predictions ..............................................................................................................196
Figure 7.14 Virtual strain distribution from of FE predictions .................................................197
Figure 7.15 Predicted hoop strain distributions on the inner and outer surfaces of FRP wrap from different models.............................................................................................................199
Figure 7.16 FRP strain distribution near the ends of the FRP wrap .......................200
Figure 7.17 Predicted confining pressure and deformation from different FE models ....201
Figure 7.18 Stress distribution in concrete ........................................................................204
Figure 8.1 Assumed characteristics of model of FRP-confined concrete column ....209
Figure 8.2 3-dimensional model of half of a FRP wrapped concrete column .........213
Figure 8.3 Effect of different mesh sizes ...........................................................................214
Figure 8.4 Axis-symmetry model of FRP wrapped concrete columns ......................216
Figure 8.5 Predicted axial stress strain curve of axis-symmetry model .................218
Figure 8.6 Predicted axial strength-friction curve ..........................................................218
Figure 8.7 Predicted effect of friction on the hoop strain distribution in the vertical direction .........................................................................................................................219
Figure 8.8 Axial stress-strain curves using two concrete models .........................221
Figure 8.9 Predicted maximum principal strain contour of concrete ....................221
Figure 8.10 Comparison of FE and test results (Bisby et al. 2011) .........................224
Figure 8.11 Predicted inner and outer surfaces of FRP hoop strain distributions at mid-height .........................................................................................................................225
Figure 8.12 Predicted hoop strain distribution at the outer surface of the FRP at different heights .................................................................................................................................................226
Figure 8.13 Predicted hoop strain distribution along the column height ...............227
Figure 8.14 Strain distribution of concrete ......................................................................229
Figure 8.15 Predicted stress distributions in the concrete .............................................230
Figure 8.16 Comparison of various FE models’ results .............................................231
List of tables

Table 3.1 Comparison between test and FE predictions with different thickness options .................................................................62
Table 3.2 Split disk test specimens ...........................................................................................................................................71
Table 3.3 Properties of FRP constituents and composites ........................................................................................................71
Table 3.4 Comparison between test results and FE predictions ..................................................................................................72
Table 5.1 Comparison of failure criteria with Fraldi et al. (2008) .........................................................................................134
Table 5.2 Test results of Bisby et al. (2011) ............................................................................................................................136
Table 5.3 FRP Mechanical properties calculated using different methods.................................................................138
Table 5.4 Strain efficiency predicted from different failure criteria ......................................................................................140
Table 6.1 Details of test specimens and selected test results .................................................................................................147
Table 6.2 Loading schemes and boundary conditions ............................................................................................................153
Table 6.3 Strain efficiency factors for different loading conditions and adhesive models ..................................................164
Table 8.1 Comparison of failure criteria ...............................................................................................................................234
## Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>cross sectional area of the FRP</td>
</tr>
<tr>
<td>$c$</td>
<td>cohesion in Mohr-Coulomb model</td>
</tr>
<tr>
<td>$d$</td>
<td>plastic degradation variable</td>
</tr>
<tr>
<td>$d_a$</td>
<td>maximum aggregate size</td>
</tr>
<tr>
<td>$E_1$</td>
<td>Young’s modulus of the FRP lamina in fibre direction</td>
</tr>
<tr>
<td>$E_2, E_3$</td>
<td>Young’s modulus of the FRP lamina in transverse direction</td>
</tr>
<tr>
<td>$E_{2c}$</td>
<td>slope of the linear second portion of FRP confined concrete</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Young’s modulus of unconfined concrete</td>
</tr>
<tr>
<td>$E_f$</td>
<td>Young’s modulus of isotropic fibres</td>
</tr>
<tr>
<td>$E_{frp}$</td>
<td>Young’s modulus of FRP composites</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Young’s modulus of the matrix</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Young's modulus of FRP composites in transverse direction</td>
</tr>
<tr>
<td>$E_{sec}$</td>
<td>secant modulus at compressive strength of unconfined concrete</td>
</tr>
<tr>
<td>$E_{\theta}$</td>
<td>Young’s modulus of FRP composites in hoop direction</td>
</tr>
<tr>
<td>$f'_{cc}$</td>
<td>compressive strengths of the confined concrete</td>
</tr>
<tr>
<td>$f'_{cc}^*$</td>
<td>peak axial stress of concrete under a specific confining pressure</td>
</tr>
<tr>
<td>$f'_{co}$</td>
<td>compressive strengths of the unconfined concrete</td>
</tr>
</tbody>
</table>
\[ f_l \] lateral confining pressure

\[ f_{l,rup} \] confining pressure reached when the FRP ruptures

\[ f_o \] intercept of the stress axis by the linear second portion

\[ f_r, f_{\theta}, f_z \] radius, longitudinal and horizontal strength of the FRP

\[ F_{r\theta}, F_{rz}, F_{\theta\theta} \] parameters determined using biaxial tests.

\[ f_{r\theta}, f_{rz}, f_{\theta\theta} \] shear strengths in the three planes of symmetry

\[ f_t \] concrete uniaxial tensile strength

\[ G_{12} \] shear modulus of FRP lamina in fibre-transverse plane

\[ G_F \] fracture energy of concrete

\[ G_{p\theta} \] shear modulus of FRP in \( p-\theta \) plane

\[ k_t \] confinement effectiveness coefficient

\[ l \] final length of gauge

\[ L \] original length of gauge

\[ M \] FRP composite properties \( E_2, G_{12}, \) or \( v_{23} \)

\[ M_f \] fibre properties \( E_f, G_f, \) or \( v_f \)

\[ M_m \] matrix properties \( E_m, G_m, \) or \( v_m \)

\[ P \] FRP composite properties \( 1/G_{12}, 1/G_{23}, \) or \( 1/K_T \)

\[ R \] radius of disks in split-disk tests
\( R_o \)  
radius of the confined concrete

\( t_a \)  
thickness of adhesive layer between the FRP wraps

\( t_f \)  
actual thickness of FRP composites

\( t_{frp} \)  
nominal thickness of FRP composites

\( V_f \)  
volume fractions of fibres

\( V_m \)  
volume fractions of matrix

\( w_{cr} \)  
contact opening displacement at the complete loss of tensile stress

\( w_t \)  
contact opening displacement

\( \alpha \)  
angular length of an overlap zone

\( \beta \)  
angular length of transition zone

\( \rho \)  
strain reduction factor in split-disk tests

\( \Delta \phi \)  
maximum curvature change in the FRP

\( \sigma_t \)  
tensile stress normal to the crack direction

\( \varepsilon^*_{cc} \)  
corresponding axial strain at peak axial stress of concrete

\( \varepsilon_{cc} \)  
FRP hoop strain in columns subject to eccentric loading

\( \varepsilon_a \)  
FRP hoop strain in columns subject to concentric loading

\( \varepsilon_b \)  
local maximum bending strain

\( \varepsilon_c \)  
axial strain
\( \varepsilon_{co} \)  
axial strain at the compressive strength of unconfined concrete

\( \varepsilon_{cu} \)  
ultimate axial strain of confined concrete

\( \varepsilon_{frp} \)  
ultimate tensile strain of FRP composites

\( \varepsilon_{h, rup} \)  
hoop strain of FRP at the rupture of the jacket due to hoop tension

\( \varepsilon_l \)  
lateral strain of FRP composites

\( \varepsilon_r, \varepsilon_z, \varepsilon_\theta \)  
normal strain of FRP in \( r, z \) and \( \theta \), respectively

\( \varepsilon_r^u, \varepsilon_z^u, \varepsilon_\theta^u \)  
radius, longitudinal and horizontal ultimate strain of the FRP

\( \varepsilon_r^u, \varepsilon_z^u, \varepsilon_\theta^u \)  
radius, longitudinal and horizontal ultimate shear strain of FRP

\( \varepsilon_t \)  
strain of the parabolic first portion meets the linear second portion

\( \varepsilon_z, \varepsilon_\theta, \varepsilon_r \)  
shear strain of FRP in \( z-\theta, \theta-r \) and \( r-z \) plane, respectively

\( \phi \)  
fraction angle in Mohr-Coulomb model

\( \kappa \)  
FRP strain efficiency

\( \nu_{12} \)  
Poisson’s ratio of FRP lamina in fibre-transverse plane

\( \nu_{23} \)  
Poisson’s ratio of FRP lamina in transverse plane

\( \nu_f \)  
major Poisson’s ratio of fibres

\( \nu_m \)  
major Poisson’s ratio of the matrix

\( \nu_{h, \theta} \)  
Poisson’s ratio of FRP in hoop-transverse plane

\( \theta \)  
circumferential angular coordinate
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\sigma_c$</td>
<td>axial stress</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>current confining pressure of FRP with certain lateral strain</td>
</tr>
<tr>
<td>$\sigma_{l,eff}$</td>
<td>effective confining pressure</td>
</tr>
<tr>
<td>$\sigma_{fr}$, $\sigma_{fz}$, $\sigma_{f\theta}$</td>
<td>normal stress of FRP in r, z and $\theta$, respectively</td>
</tr>
<tr>
<td>$\tau_{z\theta}$, $\tau_{\theta r}$, $\tau_{rz}$</td>
<td>shear stress of FRP in $z-\theta$, $\theta-r$ and $r-z$ plane, respectively</td>
</tr>
<tr>
<td>$\bar{\xi}_M$</td>
<td>parameters ($\bar{\xi}_E$, $\bar{\xi}<em>G$, or $\bar{\xi}</em>\nu$) to be obtained from experiments</td>
</tr>
<tr>
<td>$\psi$</td>
<td>dilation angle in Mohr-Coulomb model</td>
</tr>
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</table>
CHAPTER 1
INTRODUCTION

1.1 Background

In recent years, fibre reinforced polymer (FRP) composites have received considerable interest in the civil engineering community, both in new construction and in the retrofit of existing structures. Due to the high strength-to-weight ratio, good corrosion resistance and ease of site handling, FRP composites are now widely used in a large variety of civil engineering applications. The continuous reduction in the material cost of FRP composites and growing confidence among engineers also leads to increasing of popularity of FRP composites (Teng et al. 2002).

There are many applications for using FRPs in structures, such as externally bonded reinforcement for the retrofit of existing concrete structures, internal reinforcement of concrete, all FRP bridge decks, concrete-filled FRP tubes and externally bonded FRP on steel beams (Teng et al. 2002). Many studies have been conducted in these areas, and it has been found that the applications of FRPs mentioned above have significant improvements or immediate promise on the behaviour of structures.

Among all the possibilities of FRP applications in structural engineering, the technique of applying confinement to concrete columns using FRP composites is considered among the most promising. This technique is developed based on a well-established fact
that the axial compressive strength and deformability of concrete can be considerably enhanced by applying lateral confinement to it. The traditional way of providing confinement to concrete columns is to install an external steel-jacket or to construct an additional Reinforced Concrete (RC) cage. However, these two methods are time consuming, costly, labour intensive and sometimes difficult to implement on site (Teng et al. 2002). Furthermore, corrosion of steel jackets is a serious concern when applying this method.

Several methods for the use of FRPs in columns have been developed and studied in recent years. The first of these is repair and strengthening of existing RC columns by externally wrapping the columns in the hoop direction with FRP sheets or straps, where the main fibres are generally horizontally oriented, as shown in Figure 1.1, to enhance the axial strength and ductility of RC columns. Many studies have been conducted to examine the benefits of this method, and this method is now widely used in strengthening and repair of RC columns and piers in buildings and bridges. Recently, this method has also been extended to repair or strengthen fire damaged concrete columns (Bisby et al. 2011), and concrete-filled steel tubes (Hu et al. 2011).

A second method which is also very promising is to use FRPs in seismic retrofit of RC columns. Similar to the above methods, the FRPs are bonded externally on columns, the fibres are mainly horizontally oriented, but can be vertically oriented in some circumstances in order to provide additional reinforcement to the flexural strength of the
columns. Therefore, the axial, shear and flexural capacity can be improved without an increase of the column stiffness, which could lead to the column attracting additional earthquake loads in RC columns (Teng et al. 2002).

![Figure 1.1 Strengthening of concrete columns using externally bonded FRPs applied in the hoop direction](image)

FRP is not only applicable in retrofit applications, but also can be used in new construction. Examples include concrete-filled FRP tubes (Becque et al. 2003), and Hybrid FRP-concrete-steel double-skin tubular columns (Yu 2007).

### 1.2 Statement of problem

Despite the large number of studies focusing on various types of FRP wrapped columns, some key aspects of their behaviour remain unclear. One of these is the details of the ultimate condition (i.e. conditions leading to failure) of FRP confined columns. The
typical failure mode of FRP wrapped columns under axial compression is FRP hoop rupture, as shown in Figure 1.2. Due to the nearly linear-elastic behaviour of FRP composites, the rupture of FRP composites is typically explosive and leads to a sudden release of confinement of the concrete or steel jacket inside the FRP jacket. The crushing of the concrete core or outward buckling of the steel jackets then takes place immediately after rupture of FRP jacket. Therefore, designers consequently need to accurately know the strain (or stress) at which rupture of the FRP will occur.

![Figure 1.2 Typical initiation of explosive failure mode of FRP wrapped columns](image)

As discussed in detail in Chapter 2, two types of material property tests are generally adopted to determine the tensile rupture strain of FRP composites for civil engineering applications: (1) flat coupon tests and (2) so-called ‘split-disk’ tests. However, numerous experiments have shown that the rupture strain of FRP jackets on FRP wrapped circular concrete columns is considerably lower than the FRP ultimate rupture strain determined
from tensile tests on flat coupons (e.g. Lam and Teng 2004; Xiao and Wu 2000). A few studies have suggested that the split-disk test provides a better estimate of the hoop rupture stain in FRP confined columns than the flat coupon test (i.e. Shahawy et al. 2000, Teng et al. 2002).

The ratio of hoop strain in the FRP jacket on a concrete column at rupture to the flat coupon ultimate tensile strain of the FRP is commonly referred to as the ‘strain efficiency’ of the FRP jacket. While the application of FRP composites to confine columns has been widely studied, and various models have been developed resulting in reasonably accurate design methods to predict the stress-strain relationship of axially-loaded circular columns, the fundamental mechanisms of the lower-than-expected hoop rupture strain of FRP, and the influencing factors of FRP strain efficiency, remain unclear.

1.3 Research scope and objectives

The aim of this study is to examine the mechanism and influencing factors of the lower-than-expected FRP rupture strain in FRP wrapped columns. To achieve this aim, exhaustive literature reviews have been conducted; and numerical models and analytical solutions have been developed. The main objectives of this research project are:

1) to investigate the relationship between FRP rupture strains in the split-disk test and those observed in the flat coupon test;
2) to examine all of the possible influencing factors of the lower-than-expected FRP rupture strain in FRP confined circular concrete columns;

3) to develop an analytical solution for the triaxial stress state in FRP wrapped circular concrete columns;

4) to numerically investigate the FRP strain efficiency in FRP wrapped circular concrete-filled steel tubes, as well as the influence of various parameters on the observed strain efficiency; and

5) to investigate, both numerically and experimentally, the FRP strain efficiency of FRP wrapped circular concrete columns.

1.4 Outline of thesis

This thesis is divided into nine chapters; a brief introduction of each chapter is summarized as follows.

Chapter 2 presents a detailed literature review of topics related to FRP confined concrete columns. The chapter begins with a discussion of existing research on FRP confined columns of various types, covering both experimental and theoretical investigations. The discussion then turns to issues associated with the lower-than-expected FRP rupture strain in FRP wrapped concrete columns, including influencing factors on this phenomenon and the existing suggested values for FRP strain efficiency quoted in the literature.
Chapter 3 presents an investigation using finite element (FE) analysis into the FRP hoop strains that develop in the split-disk test, with the aim of explaining possible reasons for the reduction of the FRP rupture strain in this test method. Particular interest is paid to the strain concentrations developed at the ends of the FRP overlapping region and due to circumferential bending of the FRP ring at the gap caused by the relative movement of the two half disks. The influence of various parameters, such as adhesive properties, FRP stiffness, geometry of the overlapping zone and FRP-to-split-disk friction are examined.

In Chapter 4, existing influencing factors of lower-than-expected FRP rupture strain are examined. The different failure modes of FRP wrapped concrete columns are also examined. Furthermore, a list of contributory factors affecting the apparent FRP rupture strain in FRP wrapped columns is identified and discussed in this chapter.

In Chapter 5, an analytical solution for the triaxial stress state of FRP wraps in FRP confined concrete columns is presented. Several failure criteria for composite laminates are used to examine the strain efficiency of FRP composites due to multi-axial stress states. Furthermore, the influence of parameters such as FRP strength, concrete strength and the number of FRP layers is examined.
Chapter 6 examines the effect of the geometrical discontinuities in the FRP wraps on the hoop rupture strain of FRP jackets in FRP-confined concrete-filled circular steel tubes. Detailed FE analyses are conducted using linear elastic and elastic-perfectly plastic adhesive constitutive models. Comparison between the FE predictions and available test results is given to validate and verify the FE models. The influence of parameters such as the FRP thickness, FRP orthotropy, FRP elastic modulus, adhesive yield strength, adhesive thickness and column size are examined.

Chapter 7 presents a three-dimensional FE analysis of FRP wrapped concrete columns representing one millimetre of column in the longitudinal axis direction at the middle height of an FRP wrapped concrete column. Strain concentrations at the ends of the FRP wraps is investigated using a particle image velocimetry (PIV) digital image correlation (DIC) technique. The FRP actual thickness, which has a considerable effect on the analysis results, is measured using an electronic microscope. Comparison between the predictions of the FE model and the test results is conducted to validate and verify the FE models.

The combined effect of end constraint on an FRP wrapped column and an FRP overlap is further examined in Chapter 8. A model of half the height of an FRP wrapped concrete column is developed. The effect of different FE modelling approaches is discussed. Different failure criteria for FRP composites are adopted to examine the
ultimate condition of FRP wrapped concrete columns under different assumptions based on stress and strain results from the FE models.

The thesis closes with Chapter 9, where conclusions from the previous chapters are given, along with a series of recommendations for further work in this area.
2.1 Introduction

This chapter presents a review of existing literature with regards to the application of fibre reinforced polymers (FRPs) in confining concrete columns. As pointed out in Chapter 1, FRPs have been applied to various types of columns, such as FRP strengthened concrete columns, FRP strengthened concrete-fill steel tubes, concrete-filled FRP tube and hybrid FRP-concrete-steel double-skin tubular column (DSTC), etc. Furthermore, the phenomenon of considerably lower-than-expected FRP rupture strain is widely observed in almost all of these applications (e.g. Xiao and Wu 2000; Lam and Teng 2004). Therefore, existing knowledge of these columns, including experimental and theoretical studies, are reviewed in this chapter. Previous research on the FRP strain efficiency in FRP confined concrete columns is also reviewed.

2.2 FRP composites

FRPs are a subgroup of composite materials, which are defined as materials consisting of two or more materials on a macroscopic scale, with enhanced physical or chemical properties compared to those of their constituents (Bisby 2003). FRP composites contain at least two materials: fibres and matrix. The material properties of an FRP mainly depend on its constituents, and also the directions and volume ratio of the fibres. The fibres provide the main part of strength and stiffness of an FRP. In most structural engineering applications, fibres have high stiffness and high ultimate strength along
fibre direction. The three most commonly used fibre types applied in columns are, carbon, glass, and aramid, which is less used compared to previous two (Rostasy 1993). The polymer matrix generally has poor mechanical properties but good adhesion characteristics. Therefore, the FRP composites which contain these two materials are orthotropic and depend on the fibre orientation.

FRPs were not introduced into civil engineering first; they have been used for many decades in aerospace, aircraft and automotive industries, where their high strength-to-weight ratio can be a great advantage. The limited use of FRPs in civil engineering applications was traditionally because of their high cost (Teng et al 2002; Teng et al 2003). However, their prices have been decreasing due to recent advances in knowledge, FRP technology and economies of scale, enabling their extensive application in various civil engineering projects around the world (Bisby 2003).

2.3 Determination of FRP composites mechanical properties

2.3.1 Introduction
In this section, the widely adopted material property test methods for FRPs used in confining columns are first reviewed. Following this, the constitutive relationships governing the calculation of FRP composites’ mechanical properties, and failure criteria used for predicting the response of FRP composites, are also reviewed.
2.3.2 Test methods

2.3.2.1 Flat coupon test

Flat coupon tests are widely used for testing FRPs’ tensile properties; this is due to its relative simplicity (Tsai 1977). The in-plane tensile strength, modulus of elasticity and ultimate elongation of an FRP composite can be obtained from this method (ASTM 1995; ACI 2008). Figure 2.1 shows a typical tensile flat coupon test of an FRP.

This test method focuses on the FRP composite itself, excluding the effect of the gripping tabs. Therefore failure and pullout at the tabs should not be considered, and test results should be based on the specimens’ failure in the gauge section.

Flat coupon tests are also adopted for testing angle-ply FRP laminates to determine their off-axis properties (Li et al. 2006). However, angle-ply coupon specimens are subjected to so-called free edge effects caused by the bending near the end constraint, which can substantially affect their ultimate tensile strain or strength (Pagano and Halpin 1968; Tsai 1968; Tsai 1977; Tsai and Wu 1971).
2.3.2.2 Split-disk test

Split-disk tests were first adopted to determine the tensile strength of plastic tubes or pipes (ASTM 1992). In recent years, this method has also been used to test the tensile strength of FRP tubes (Mirmiran and Shahawy 1997) or FRP wraps (Lam and Teng 2004) applied in circular concrete columns, due to the geometrical similarity of the test specimens and the FRP composites in column applications. Figure 2.2 shows a typical split-disk test of wet lay-up FRP composites.

![Diagram of split-disk test](image)

(a) Front elevation view of split-disk test of FRP wraps

(b) Reduced section specimen for split-disk test (after ASTM 1992)

**Figure 2.2 Split-disk test**
It should be noted that in a plastic pipe test, the specimens have one or two reduced areas (as shown in Figure 2.2b) in order to make sure the failure occurs in certain locations. However, the reduced section is seldom used in FRP composites. One of the main reasons for this is that the presence of defects has a significant influence on the ultimate strain of the FRP composites (Buarque and d'Almeida 2007), since the defects require cutting through the FRP composites, thus may cause damage to the fibres and the matrix.

Experiments have shown that the FRP rupture strain determined from the split-disk test is significantly lower than that from flat coupon tests or that provided by the manufacturer. The ratio of the tensile ultimate strain of split disk test to that of flat coupon tests ranges from 0.49 to 0.82 (Lam and Teng 2004; Mirmiran and Shahawy 1997; Tamuzs et al. 2006). Mirmiran and Shahawy (1997) determined the analytical strength and elastic modulus of GFRP tubes using micro-mechanics equations and the rule of mixtures. However, test results showed that both properties obtained from the analytical approach were about 25% higher than those obtained from the split-disk test (Mirmiran 1996).

Lam and Teng (2004) conducted both flat coupon and split-disk tests on FRP wraps and concluded that there are at least two contributing factors to which the reduction of ultimate strains of FRP in split-disk tests may be due: a) curvature of the FRP; and b)
circumferential bending of the FRP ring at the gap created by the relative movement of the two half disks.

Historically, an analytical study was carried out on anisotropic uniform rings (without an overlapping zone) stretched by two rigid half-disks (Partsevskii 1969). The equilibrium equations were solved by means of an expansion in Fourier series. However, the solutions given are complicated and cannot be easily applied in practice. Knight (1977) conducted an FE analysis together with a statistical strength theory to assess the influence of strain concentrations on the ultimate strength from split-disk tests of uniform composite rings, and concluded that thick rings may give test results with significant errors, even more than 20% in their cases. Shlitsa and Novikova (1983) presented split-disk test results of four types of FRP composites, with the effects of such parameters as the thickness and width of the ring specimen and the initial gap between the ring specimen and half-disks considered. They showed that a careful and correct setup of the experiment can minimise the detrimental effects of the split-disk method on the observed failure strain. Unfortunately, the details presented are insufficient for the purposes of comparative FE modelling. Buarque and d'Almeida (2007) evaluated the influence of cylindrical defects on the tensile strength of GFRP pipes using split-disk tests and FE analyses. The results showed that the presence of defects has a significant influence on the ultimate strain of the pipes, the ultimate strength of the defected specimens may by only 65% of that in non-defected specimens. The effects of resin type, fibre type, and winding angle on filament-wound FRP tubes’ hoop strength were
evaluated by Kaynak et al. (2005), by performing split-disk tests to determine their hoop tensile strength and modulus. However, none of these studies gives a comprehensive explanation on the differences in the mechanics between a split-disk test and a flat coupon test.

2.3.2.3 Tubular specimen test

As mentioned in Section 2.3.2.1, the effect of end constraints is commonly observed in off-axis tests using flat coupons, and the influence of this effect cannot be avoided in coupon specimens. Therefore, composite tube (Figure 2.3) ideal specimen geometry is proposed for testing composites under combined loading condition, or under a combined state of stress, such as a biaxial stress state (Al-Khalil et al. 1995; Cole and Pipes 1974; Tsai 1977), and the interaction between the transverse and shear stress components in composites (Al-Khalil et al. 1995; Tsai and Wu 1971).
In such a tubular specimen, not only tensile load parallel to the tube axis can be applied, but also compression and/or torsion. Furthermore, the specimens can be fabricated with different fibre orientations, without the so-called free edge effects during testing.

However, the available experimental results from tubular specimens are minimal compared to flat coupon tests; this is due to the high cost of fabrication and high quality testing of tubes (Tsai 1977).
2.3.2.4 Other test methods
Generally, the tensile behaviour of FRP composites in the fibre direction is the major concern in FRP confined columns. Less attention is typically paid to the compressive properties of FRPs. The most popular test method for the compressive properties is through shear loading tests (ASTM 2002). The compressive elastic modulus of FRP composites is usually smaller than the tensile modulus, the compressive modulus of elasticity is approximately 80%, 85% and 100% for GFRP, CFRP and AFRP of the tensile modulus of elasticity for the same product (ACI 2008). The in-plane shear response of FRP composites can be obtained by tensile test of a 45° laminate (ASTM 1994).

2.3.3 Micromechanics of FRP composites

2.3.3.1 Introduction
In the previous section, the ‘apparent’ properties of FRPs can be obtained from the tests by assuming FRP composites as a homogenous material. If a large enough piece of FRP composite has been considered, the fact that FRP is a two-component material cannot generally be detected (Jones 1999). However, it is rational to examine the properties of FRPs by treating them as heterogeneous composite material. Therefore, many micromechanical methods have been developed to predict FRP properties from the individual mechanical properties of the fibre and matrix based on micromechanics (e.g. Ekvall 1961; Hahn 1980; Halpin and Tsai 1967; Hashin and Rosen 1964). Unfortunately, many such methods have complicated equations and are therefore not suitable for civil
engineering practical application (Yu et al. 2009a). Thus, special attention is paid to three simple closed-formed methods in the following sections.

The following approaches assume that both the fibre and matrix are isotropic materials and that the FRP composite behaves as a transversely isotropic material, unless otherwise stated. Assuming the properties are the same in the 2-3 plane, and different in the 1 plane, which is normal to the 2-3 plane, the elastic properties involve only five engineering constants, namely, $E_1$, $E_2$, $\nu_{12}$, $\nu_{23}$, and $G_{12}$, where 1 is the fibre direction, $E_1$ and $E_2$ are the longitudinal modulus and transverse modulus of FRP respectively, $\nu_{12}$ and $\nu_{23}$ are the major Poisson’s ratio in 1-2 plane and the Poisson’s ratio in 2-3 plane, respectively, and $G_{12}$ is the shear modulus in 1-2 plane.

### 2.3.3.2 Mechanics of materials approach

Based on the rules of mixtures, and assuming that the fibres and the matrix deform compatibly when an FRP lamina is subjected to uniaxial loading in the fibre direction, the longitudinal modulus $E_1$ and the major Poisson’s ratio $\nu_{12}$ of FRP can be found from the following simple equations (Yu et al. 2009a):

$$
E_1 = E_f V_f + E_m V_m \tag{2.1}
$$

$$
\nu_{12} = \nu_f V_f + \nu_m V_m \tag{2.2}
$$
where \( E_f \) is the elastic modulus of the isotropic fibres; \( \nu_f \) is the major Poisson’s ratio of the fibres; \( E_m \) and \( \nu_m \) are the elastic modulus and the Poisson’s ratio of the matrix, respectively; and \( V_f \) and \( V_m \) are the volume fractions of fibres and matrix, respectively.

Assuming that the same transverse normal stress is applied to both the fibre and the matrix, the apparent transverse elastic modulus \( E_2 \) of an FRP composite can be found from

\[
E_2 = \frac{E_f E_m}{E_f V_m + E_m V_f}
\]  

(2.3)

Similarly, the in-plane shear modulus, by assuming the shear stresses on the fibre and matrix are the same, \( G_{12} \) can be determined by

\[
G_{12} = \frac{G_m G_f}{V_m G_f + V_f G_m}
\]  

(2.4)

where \( G_f \) and \( G_m \) are the shear modulus of the fibres and the matrix, respectively.

Unfortunately, \( \nu_{23} \) is not given by this method. The same value as \( \nu_{12} \) is assumed for \( \nu_{23} \) in this study. Therefore, \( G_{23} \) is given by

\[
G_{23} = \frac{E_2}{2(1 + \nu_{23})}
\]  

(2.5)
2.3.3.3 The Halpin-Tsai method

Halpin-Tsai method is a semi-empirical approach which is developed using an interpolation procedure to predict mechanical properties of FRP. $E_I$ and $\nu_{12}$ are calculated using Eqs. 2.1 and 2.2. The equations for other properties are given by:

$$\frac{M}{M_m} = \frac{1 + \xi_M \eta V_f}{1 - \eta V_f}$$  \hspace{1cm} (2.6)

where

$$\eta = \left( \frac{M_f}{M_m} \right)^{-1} \left( \frac{M_f}{M_m} + \xi \right)$$  \hspace{1cm} (2.7)

in which

$M = \text{FRP composite properties } E_2, G_{12}, \text{ or } v_{23}$

$M_f = \text{fibre properties } E_f, G_f, \text{ or } v_f$

$M_m = \text{matrix properties } E_m, G_m, \text{ or } v_m$

and $\xi_M$ is a parameter to be obtained from experiments, which can be $\xi_E$, $\xi_G$, or $\xi_v$. When $\xi_M = 0$, Eq. 2.6 reduces to Eqs 2.3 or 2.4. When $\xi_M = \infty$, Eq. 2.6 reduces to the rule of mixtures (Yu et al. 2009a).

Experimental studies have been conducted to determine the value of $\xi_M$. For circular fibres in a square array, which is commonly used in applications of FRP wrapped
columns, usually $\xi_E = 2$ and $\xi_G = 1$ (Yu et al. 2009a). Very little attention is paid to $\xi_v$, therefore no solid value of $\xi_v$ is available in Halpin and Tsai (1967). Similar to the previous method, $\nu_{23}$ is assumed to be the same as $\nu_{12}$ during the rest of this study if Halpin-Tsai equations are adopted, unless otherwise stated.

### 2.3.3.4 Hahn equations

Hahn (1980) developed an approach to predict the mechanical properties of FRPs by codified results for unidirectional fibres of circular cross section which are randomly distributed in a plane normal to the fibre direction (Vinson and Sierakowski 2002).

The rules of mixtures (e.g. Eqs. 2.1 and 2.2) are suggested by Hahn to predict $E_i$ and $\nu_{ij}$.

For the rest of the elastic constants Hahn states the following equations:

$$ P = \left( \frac{P_f V_f + \lambda P_m V_m}{V_f + \lambda V_m} \right) $$

in which

$P = $ FRP composite properties $1/G_{12}$, $1/G_{23}$, or $1/K_T$

$P_f = $ fibre properties $1/G_f$, or $1/K_f$

$P_m = $ matrix properties $1/G_m$, or $1/K_m$

in the above $K_T$ is the plane strain bulk modulus, $K_f = [E_f/2(1-\nu_f)]$ and $K_m = [E_m/2(1-\nu_m)]$.

The $\lambda$ values are given by:

when $P = 1/G_{12}$,
\[
\lambda_6 = \frac{1 + \frac{G_m}{G_f}}{2}
\]
when \( P = 1/G_{23} \)

\[
\lambda_4 = \frac{3 - 4v_m + \frac{G_m}{G_f}}{4(1 - v_m)}
\]

When \( P = 1/K_T \)

\[
\lambda_K = \frac{1 + \frac{G_m}{K_f}}{2(1 - v_m)}
\]

The transverse modulus of FRP, \( E_2 \), can be found from the following equation:

\[
E_2 = \frac{4K_T G_{23}}{K_T + mG_{23}}
\]

where

\[
m = 1 + \frac{4K_T v_{12}^2}{E_1}
\]

Hahn suggests that for most structural composites, \( G_m/G_f < 0.05 \), then \( \lambda_6 \approx 0.5 \); furthermore, since for most epoxies, \( v_m = 0.35 \), then \( \lambda_4 \approx 0.62 \) and \( \lambda_K \approx 0.77 \).

Finally, the Poisson’s ratio, \( v_{23} \), can be written as

\[
v_{23} = v_f V_f + v_m (1 - V_f) \left[ \frac{1 + v_m - v_{12}}{1 - v_m^2 + v_m v_{12}} \frac{E_m}{E_1} \right]
\]

(2.16)
It should be noted that fibres such as carbon fibre exhibit anisotropic behaviour, i.e. the elastic properties along and transverse direction to the fibre axis are significantly different (Chou 1992). Therefore, the assumption of isotropy of fibres may produce errors in prediction of mechanical properties of FRP composites, this issue is further examined in Chapter 5. The elastic properties of FRP composites composed of orthotropic fibres and in an isotropic matrix can be calculated by substituting the orthotropic properties of fibres into the above methods.

2.4 FRP failure criteria

Failure criteria for composites materials for a single lamina have been studied by many researchers (e.g. Daniel and Ishai 2006; Yu et al. 2009b; Jones 1999). Despite the popularity of micromechanics approaches in the prediction of mechanical properties of FRP, they have not been widely used for strength prediction, since accurate prediction of failure is often difficult using such approaches (Yu et al. 2009b). In this study, only macromechanics approaches are considered. At the macroscopic level, failures of FRP are categorized into two modes: (1) intralaminar failure, which occurs inside a lamina, and (2) interlaminar failure, which occurs between laminae (Yu et al. 2009b).

Various failure criteria are available for a lamina, such as maximum stress theory, maximum strain theory, Tsai and Wu theory and Tsai and Hill theory. The comparison of these four failure criteria in a plane stress condition is shown in Figure 2.4. The
prediction of failure in a lamina is believed to be fairly mature, and Tsai-Wu theory has been shown to be one of the best criteria in fitting the experimental results (Hinton et al. 2002; Hinton et al. 2004).

Figure 2.4 Comparison of four failure criteria for FRP composites (after Yu et al. 2009b)

The failure of FRP laminates is rather complex, although predictions can be made based on the failure criteria for a lamina and internal stress-strain analysis. Generally, the stresses inside laminae are determined using classical lamination theory, and interlaminar failure prediction is based on the calculation of interlaminar stresses, such as free-edge stresses (i.e. stresses that occur near the edges with no external forces). Unfortunately, the prediction of failure in laminates often leads to more than 50% errors (Hinton et al. 2002; Soden et al. 2004). Furthermore, some researchers do not believe failure theories for a lamina are sufficient to predict the failure of a laminate (Hinton and Soden 1998).
2.5 FRP confined columns

As mentioned in Chapter 1, FRPs are widely applied in columns to provide lateral confinement and are recognized as an effective material. The traditional way to laterally confine concrete is using steel jackets or steel stirrups. This method, however, has been found to be labour intensive and sometimes difficult to implement on site. Furthermore, the corrosion of steel is also a major concern when steel is applied in certain conditions, such as near to a seacoast. The significant increasing of column stiffness when applying steel jackets is also considered as a disadvantage for seismic retrofit of columns, since it may lead to the attracting additional earthquake forces during a seismic event (Teng et al. 2002).

Historically, the application of FRPs in columns was proposed and studies were first conducted in the early 1980s, when the behaviour of FRP-encased concrete columns was investigated by Fardis and Khalili (1981;1982). The compressive strength of concrete was found to be drastically improved due to external wrapping with FRPs. Since then, extensive studies focusing on the application of this technique in strengthening and seismic retrofit of RC columns have been performed. Several innovative forms of applications have also been developed, such as concrete-filled FRP tubes, FRP wrapped concrete-filled steel tubes, FRP repair of fire damaged concrete columns and hybrid FRP-concrete-steel double-skin tubular columns (Yu 2007), etc. It should be noted that FRPs are not only used for confining columns with circular cross-sections, but also
square, rectangular and elliptical sections (e.g. Lam and Teng 2003b). In this section, various types of FRP confined columns, together with existing analytical models, are presented. Only columns with circular cross-sections are considered, unless otherwise specified. Columns in circular solid section confined with an FRP jacket are simplified as FRP confined columns.

2.5.1 FRP strengthened reinforced concrete columns
As mentioned previously, since the 1980s, the technique of FRP strengthened concrete columns has become more and more popular, and it is now widely recognized for its effectiveness and speed and ease of application. In this technique, The FRP wraps are applied externally to RC columns, mainly in the circumferential direction, to provide confinement to concrete. This is well studied for its ability to substantially enhance concrete’s compressive strength and ductility (Ahmad and Shah 1982; Mander et al. 1988; Richart et al. 1928). The discussion in this section focuses on FRP strengthened RC columns.

Over the past decade, extensive research and field applications have been conducted on the behaviour of FRP wrapped concrete or RC columns. Numerous analytical models for the compressive strength and stress-strain behaviour of FRP-confined concrete have been proposed in the literature (e.g. Jiang and Teng 2007; Lam and Teng 2003a; Lignola et al. 2008a). Several design guidelines for FRP confined concrete or RC columns have been published in different countries (e.g. ACI 2008, Concrete-Society 2004).
From the large number of studies on FRP wrapped RC columns, many models for FRP confined concrete have been developed. These models can be classified into two categories: (1) strength models, and (2) stress-strain models. In the first category, only the compressive strength of FRP confined concrete are treated. In the second category, the stress-strain behaviour of FRP-confined concrete models is presented in either a closed-form expression or an incremental numerical procedure. In this section, these two types of models will be discussed.

2.5.1.1 Strength models
When an FRP-confined circular concrete column is subject to axial compression, the eventual failure mode is that the FRP ruptures when its hoop tensile strength is reached. Several models that predict the compressive strength of FRP-confined concrete columns are available in literature (Karbhari and Gao 1997; Lam and Teng 2002; Samaan et al. 1998; Wu and Wang 2009; Wu and Zhou 2010). Researchers have adopted different forms of the expression for the equation. Most of the existing strength models for FRP confined concrete take the following form:

\[
\frac{f'_{cc}}{f'_{co}} = 1 + k_1 \frac{f_l}{f'_{co}}
\]  

(2.17)

Where \(f'_{cc}\) and \(f'_{co}\) are the compressive strengths of the confined and the unconfined concrete, respectively, and \(k_1\) is the confinement effectiveness coefficient. In Lam and Teng (2002), \(k_1\) is suggested to be 2.0 for design use, and 2.15 for the ‘best fit’ model. \(f_l\) is the lateral confining pressure, given by
where $E_{frp}$, $t_{frp}$ and $\epsilon_{frp}$ are the Young’s modulus, nominal thickness and ultimate tensile strain of the FRP composite. $\epsilon_{frp}$ is determined from flat coupon tests. $R_o$ is the radius of the confined concrete. The FRP tensile strength should be determined according to ASTM (1995) or from flat coupon tests.

### 2.5.1.2 Stress-strain models

Several models that simulate the stress-strain behaviour of FRP-confined concrete columns are available in literature (e.g. Fardis and Khalili 1982; Jiang and Teng 2007; Karbhari and Gao 1997; Lam and Teng 2003; Mirmiran and Shahawy 1996; Samaan et al. 1998; Spoelstra and Monti 1999). Some of the models are presented in closed-form expressions (Fardis and Khalili 1982; Karbhari and Gao 1997; Lam and Teng 2003a; Samaan et al. 1998), while others are predicted using an incremental numerical procedure (Jiang and Teng 2007; Mirmiran 1996; Mirmiran and Shahawy 1997; Spoelstra and Monti 1999). The former approach is simple and familiar to engineers for determining the strength and ductility of FRP-confined RC columns, whereas the latter has advantages in accounting for the interaction between the concrete and FRP wraps, but is considerably more complex in its implementation and suitable only for incorporation in computer programmes (Yu 2007). Thus, in Lam and Teng (2003a), the former type of models were named as ‘design-oriented’ models; the latter type was named as ‘analysis-oriented’ models.

**Design-oriented models**
As mentioned earlier, several design-oriented models have been proposed in the literature (e.g. Karbhari and Gao 1997; Lam and Teng 2003; Samaan et al. 1998; Xiao and Wu 2000). Lam and Teng (2003a) have compared a number of existing design-oriented stress-strain models, and have suggested a new model verified by a large number of experimental results summarized in that paper. The new model has also been adopted by ACI (2008). The general equations of this model are briefly presented blow.

The basic assumptions of Lam and Teng (2003a)’s design-oriented stress-strain model are: (i) the stress-strain curve consists of two portions, the first portion is a parabolic curve and a second portion is linear (Figure 2.5); (ii) the initial slope of the parabola (slope at zero axial strain) is the same as the elastic modulus of unconfined concrete (Figure 2.5); (iii) the first portion is affected to some degree by the presence of an FRP jacket; (iv) the first portion meets the linear second portion smoothly (Figure 2.5); (v) the linear second portion terminates at a point where both the compressive strength and the ultimate axial strain of confined concrete are reached (Yu 2007).
Based on these assumptions, Lam and Teng’s (2003a) stress-strain model for FRP-confined concrete is described by the following expressions:

\[
\sigma_c = E_c \varepsilon_c - \frac{(E_c - E_{2c})^2}{4f_o} \varepsilon_c^2 \quad \text{for} \quad 0 \leq \varepsilon_c < \varepsilon_i 
\]  

(2.19)

and

\[
\sigma_c = f_o + E_{2c} \varepsilon_c \quad \text{for} \quad \varepsilon_i \leq \varepsilon_c < \varepsilon_{cu} 
\]  

(2.20)

where \( \sigma_c \) and \( \varepsilon_c \) are the axial stress and the axial strain, \( E_c \) is the elastic modulus of unconfined concrete, \( E_{2c} \) is the slope of the linear second portion, \( f_o \) is the intercept of
the stress axis by the linear second portion, and $\varepsilon_{cu}$ is the ultimate axial strain of confined concrete. The parabolic first portion meets the linear second portion with a smooth transition at $\varepsilon_t$, which is given by

$$\varepsilon_t = \frac{2f_o}{(E_c - E_{2c})}$$

(2.21)

The slope of the linear second portion $E_{2c}$ is given by

$$E_{2c} = \frac{f_{cc} - f_o}{\varepsilon_{cu}}$$

(2.22)

This model allows the use of test values or values specified by design codes for the elastic modulus of unconfined concrete $E_c$. Lam and Teng (2003a) suggested that $f_o$ be equal to the compressive strength of unconfined concrete $f_{co}'$, that Eq 2.23 be used to predict the ultimate axial strain $\varepsilon_{cu}$, and that Eqs 2.24 and 2.25 be used to predict the compressive strength of FRP-confined concrete $f_{cc}'$. They therefore suggest that

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1.75 + 12\left(\frac{E_{fpp,fpp}}{E_{secR_o}}\right)\left(\frac{\varepsilon_{h,rup}}{\varepsilon_{co}}\right)^{1.45}$$

(2.23)

$$\frac{f_{cc}'}{f_{co}'} = 1 + 3.3 \frac{f_l}{f_{co}'} \quad \text{when} \quad f_l / f_{co}' \geq 0.07$$

(2.24)

$$\frac{f_{cc}'}{f_{co}'} = 1 \quad \text{when} \quad f_l / f_{co}' < 0.07$$

(2.25)
where $\varepsilon_{h,\text{rup}}$ is the hoop strain of FRP at the rupture of the jacket due to hoop tension, $E_{\text{sec}}$ and $\varepsilon_{co}$ are the secant modulus and the axial strain at the compressive strength of unconfined concrete, with $E_{\text{sec}} = f'_{co}/\varepsilon_{co}$. The term $E_{frp}/E_{\text{sec}}R_o$ is the confinement stiffness ratio, representing the stiffness ratio between the FRP jacket and the concrete core. $\varepsilon_{h,\text{rup}}/\varepsilon_{co}$ is the strain ratio. The ratio $f_{l,\text{rup}}/f'_{c}$ should not be less than 0.07; this is the minimum level of confinement required to assure a non-descending second branch in the stress-strain response. With $f_{l,\text{rup}}/f'_{c} < 0.07$ no strength enhancement is assumed. $f_{l,\text{rup}}$ is the maximum confining pressure provided by an FRP jacket (the confining pressure reached when the FRP ruptures), which is defined by

$$f_{l,\text{rup}} = \frac{E_{frp}\varepsilon_{h,\text{rup}}}{R_o} \quad \text{(2.26)}$$

The ratio of maximum confining pressure to unconfined concrete strength $f_{l,\text{rup}}/f'_{co}$ has been commonly referred to as the confinement ratio.

**Analysis-oriented models**

Analysis-oriented models predict the behaviour of FRP-confined concrete by an explicit and iterative account of the interaction between the FRP jacket and the confined concrete core via radial displacement compatibility and equilibrium conditions. Compared with design-oriented models, analysis-oriented models are more accurate in general and are often the preferred choice in computer programmes and finite element models, particularly if the lateral strain of the confined concrete is required in the analysis.
While some of the models (e.g. Becque et al. 2003; Harmon et al. 1998) are based on alternative methods, most of the analysis-oriented models (e.g. Harries and Khare 2002; Mirmiran 1996; Spoelstra and Monti 1999; Teng et al. 2007a) are based on an active confinement model for steel-confined concrete.

Jiang and Teng (2007) and Teng and Lam (2004) assessed the accuracy of existing analysis-oriented stress-strain models for FRP-confined concrete, which showed that Teng et al. (2007a)’s model performs the best among all the analysis-oriented models mentioned above. This model provides accurate predictions of both the lateral-axial strain relationship and the ultimate condition, except for stress-strain curves with a descending branch. Jiang and Teng (2007) refined this model for weakly-confined concrete based on additional test data. Therefore, Teng et al. (2007a)’s model with the modification by Jiang and Teng (2007) is briefly presented below.

Eq. 2.27 was proposed by Teng et al. (2007a) for the relationship between the axial strain and the lateral strain.

\[
\Phi (\frac{\varepsilon_l}{\varepsilon_{co}}) = \frac{\varepsilon_c}{\varepsilon_{co}} / (1 + 8 \frac{\sigma_l}{f'_{co}}) = 0.85 \left[ \left( 1 + 0.75 \frac{\varepsilon_l}{\varepsilon_{co}} \right)^{0.7} - \exp \left[ -7 \left( \frac{\varepsilon_l}{\varepsilon_{co}} \right) \right] \right]
\]

(2.27)

where \(\sigma_l\) is the current confining pressure, which increases with the lateral strain \(\varepsilon_l\) and is given by \(\sigma_l = \frac{E_{fr} t \varepsilon_l}{R_o}\). \(\varepsilon_{co}\) is the axial strain of unconfined concrete at \(f'_{co}\). The axial
strain $\varepsilon_c$ and the lateral strain $\varepsilon_l$ are the only two variables in this equation, with their relationship being implicit.

With Eq. 2.27, the lateral strain-axial relationship is available, the axial stress-strain response of FRP-confined concrete can be predicted using the following equation, which was originally proposed for the active confinement model by Popovics (1973) and used by Mander et al. (1988).

$$\frac{\sigma_c}{f'_{cc}} = \frac{(\varepsilon_c / \varepsilon_{cc})r}{r - 1 + (\varepsilon_c / \varepsilon_{cc})^r}$$  \hspace{1cm} (2.28)

where $f'_{cc}$ and $\varepsilon_{cc}$ are the peak axial stress and the corresponding axial strain of concrete under a specific constant confining pressure, respectively. The constant $r$ in Eq. 2.28, approximately accounting for the brittleness of concrete, is defined in Carreira and Chu (1985) as

$$r = \frac{E_c}{E_c - f'_{cc} / \varepsilon_{cc}}$$  \hspace{1cm} (2.29)

Teng et al. (2007a) also proposed Eqs. 2.30 and 2.31 for the peak stress and strain.

$$\frac{f'_{cc}}{f'_{co}} = 1 + 3.5 \frac{\sigma_l}{f'_{co}}$$  \hspace{1cm} (2.30)

$$\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + 17.5 \frac{\sigma_l}{f'_{co}}$$  \hspace{1cm} (2.31)
Teng et al. (2007a) suggested that when test values of $\varepsilon_{co}$ are not available, a value of 0.0022 should be used in their model. Jiang and Teng (2007) refined Teng et al. (2007a)'s model based on additional weakly-confined concrete test data and proposed the following equation for the axial strain at peak axial stress $\varepsilon_{cc}^*$. 

$$\frac{\varepsilon_{cc}^*}{\varepsilon_{co}} = 1 + 17.5\left(\frac{f'_{co}}{f'_{co}}\right)^{1.2}$$ (2.32)

Teng et al. (2007a)'s model with Eq. 2.31 replaced by Eq. 2.32 is referred to as the refined model.

It is proposed in Jiang and Teng (2007) that when the refined model is used, the following equation for $\varepsilon_{co}$, proposed by Popovics (1973), should be used unless a test value is available.

$$\varepsilon_{co} = 0.000937\sqrt{f'_{co}} \quad (f'_{co} \text{ in MPa}).$$ (2.33)

### 2.5.1.3 FRP rupture strain

Extensive studies have show that the measured FRP hoop rupture strain in an FRP-confined circular concrete column at failure is considerably lower than the ultimate tensile strain of FRP material properties, i.e. rupture strain from a flat coupon tests or that provided by manufacturer (e.g. Bisby and Take 2009; Harries and Kharel 2002; Lam and Teng 2004; Shahawy et al. 2000; Xiao and Wu 2000). This issue is fundamental to the development of accurate and rational FRP confinement models.
Therefore, a lot of effort has been expended studying this phenomenon; however, the accuracy of prediction still remains relatively poor (Bisby and Take 2009).

2.5.2 FRP strengthened concrete-filled steel tubes
More recently, researchers (Hu et al. 2011; Xiao 2004; Xiao et al. 2005) have explored the benefits of using FRP jackets to provide additional confinement to concrete-filled steel tubes. In such an innovative system, the inward buckling deformation of the steel tube is prevented by the concrete core while its outward buckling deformation is restrained by the FRP jacket. As a result, FRP jacketing can increase both the strength and ductility of concrete-filled steel tubes (Xiao 2004).

The typical failure mode of FRP-confined concrete-filled steel tubes is tensile rupture of the FRP jacket in the hoop direction. It is commonly assumed that this rupture occurs when the hoop strain in the FRP jacket reaches its ultimate tensile strain, which is normally determined from flat coupon tests (ASTM 1995). However, existing experiments (Hu et al. 2011; Xiao et al. 2005) have shown that the jacket ruptures at a measured hoop strain considerably lower than the FRP ultimate tensile strain determined from flat coupon tests, very similar to the observation of premature rupture failure of FRP jackets in FRP-confined concrete columns.

2.5.3 Concrete-filled FRP tubes
Additional to the use of FRP jackets for retrofitting of concrete columns, concrete-filled FRP tubes are generally used in new construction of columns, such as cast-in-place and
precast columns (Saafi et al. 1999). There are a number of advantages of using FRP tubes. First of all, the FRP tube provides confinement to the concrete in compression, which significantly improves the strength and ductility, similar as the FRP retrofitted concrete columns. The contained concrete is also protected from severe environmental effects and moisture intrusion (Mirmiran and Shahawy 1997). Furthermore, the FRP tube acts as a stay-in-place formwork to contain the fresh concrete, which can save the costs of formwork and labour used by the cast-in-place or precast industries. Finally, the FRP tube acts as noncorrosive flexural and shear reinforcement for the concrete (Fam and Rizkalla 2001).

The FRP rupture strain results from concrete-filled FRP tubes are found to be much closer to material test results than the ones from FRP wrapped concrete columns (De Lorenzis and Tepfers 2003; Teng et al. 2002). This phenomenon is discussed further in this section.

2.5.4 FRP repair of fire damaged concrete columns

A potential additional application of FRP wraps is to repair fire damaged concrete columns, i.e. columns which have been exposed to fire or which have experienced a heat-induced reduction in concrete mechanical properties (Bisby et al. 2011). This possibility has been explored by the research group in the University of Edinburgh (Bisby et al. 2011), and the University of Manchester (Yaqub and Bailey 2011).
Concrete has two advantages in a fire. One is concrete does not combust, and hence will not contribute fuel to fire. The other is that concrete is a good insulating material possessing a low thermal diffusivity (e.g. when compared with steel). However, there are two problems of concrete in fire. These are: (1) deterioration in mechanical properties after heating; and (2) explosive spalling, which results in loss of material, reduction in section size and exposure of the reinforcing steel to excessive temperatures (Khoury 2000). Therefore, it is essential to repair reinforced concrete columns after fire damage in certain circumstances. Recently, the FRP wrapping technique has been extended to repair or strengthen fire damaged concrete columns (Bisby et al. 2011; Yaqub and Bailey 2011). FRP confinement has been found to be very effective for reinstating the strength of equivalent unheated concrete. However, FRP has little enhancing effect on the stiffness of the heat-damaged concrete within the typical service stress range. Similar to other FRP wrapped columns, the FRP rupture strain is measured to be only about 60% of the ultimate tensile strain, on average, provided by the manufacturer (Bisby et al. 2011).

2.5.5 FRP-concrete-steel hybrid tubular columns

Recently, a new form of hybrid column has been developed by Prof J.G. Teng’s group at the Hong Kong Polytechnic University. This new form of column consists of an inner steel tube inside an FRP tube and concrete placed in between the two tubes (Teng et al. 2007b; Wong et al. 2008). Figure 2.6 shows the typical sections of this new form of column, named FRP-concrete-steel hybrid double-skin tubular columns. The new columns combine the advantages of all the three materials, and the structural form of
double-skin tubular columns offers many advantages (Teng et al. 2007b; Yu 2007) such as: a) a more ductile response of concrete; b) the confinement of FRP tube does not buckle; c) less requirement for fire protection since the FRP tube contributes little to the loading-capacity of the columns under service loads; and d) less requirement for corrosion protection because the steel tube is protected by the concrete and the FRP. As expected, the average FRP rupture strain found in Teng et al. (2007b) is lower than the flat coupon tests; about 80% of the average value from flat coupon tests.

Figure 2.6 Typical sections of double-skin tubular columns (after Teng et al. 2007b)
2.6 FRP strain efficiency

As mentioned in Section 2.5, extensive studies have shown that the measured FRP hoop rupture strain in an FRP-confined column test is considerably lower than the ultimate tensile strain of FRP, i.e. rupture strain measured from a flat coupon test or that provided by manufacturer. The ratio of hoop strain in the FRP wrap at rupture to the FRP ultimate tensile strain from material property tests has been defined firstly by Pessiki et al. (2001) as the ‘strain efficiency’, and has since been investigated by a number of researchers, most of whom have based their studies on experimental investigation (e.g. De Lorenzis and Tepfers 2003; J. De Caso y Basalo et al. 2011; Lam and Teng 2004). The following section reviews the studies on the strain efficiency, including the suggested value and the possible reasons for the lower-than-expected FRP rupture strain.

2.6.1 Experimental observations

Numerous experiments have shown that the FRP jacket on an FRP wrapped circular column or cylinder fails by rupture of the FRP in the hoop direction, at a hoop strain which is observed to be on average significantly lower than the mean FRP ultimate rupture strain determined from tensile tests on flat coupons (e.g. Bisby and Take 2009; Harries and Kharel 2002; Lam and Teng 2004; Shahawy et al. 2000; Xiao and Wu 2000). However, the reduction of FRP ultimate strain on FRP wrapped columns has a large scatter. For instance, the recorded FRP strains at failure noted by Xiao and Wu (2000) were about 0.50 to 0.80 of the rupture strain obtained from flat coupon tests, and in the range of 0.58 to 0.91 observed by Lam and Teng (2004). Experiments conducted by Lam and Teng (2004) also showed that the FRP rupture strains in specimens are
strikingly close to the ultimate strains obtained from split-disk tests, however much lower than those obtained from flat coupon tests.

De Lorenzis and Tepfers (2003) showed that, based on a global survey of experiments on FRP wrapped concrete columns (Harmon and Slattery 1992; Picher et al. 1996; Watanabe et al. 1997; Miyauchi et al. 1997; Kono et al. 1998; Toutanji 1999; Matthys et al. 1999; Shahawy et al. 2000; Rochette and Labossière 2000; Micelli et al. 2001; Rousakis 2001), the recorded FRP strains at failure measured in FRP wrapped concrete columns ranged from more than 1.0 to less than 0.1 of the mean rupture strain obtained from flat coupon tests or from the FRP suppliers (note that FRP material property tests from suppliers invariably use flat coupon tests). Significant strain variation has also been observed at the ultimate condition of FRP wrapped columns using a digital image correlation strain measurement technique (Bisby et al. 2007); this is a further indication that point measurement of hoop strains using foil gauges cannot be expected to give accurate and consistent experimental hoop strain data.

Similar to FRP confined concrete columns, the typical failure mode of FRP-confined concrete-filled steel tubes is tensile rupture of the FRP jacket in the hoop direction. Existing experiments also have shown that the jacket ruptured at a hoop strain considerably lower than the FRP ultimate tensile strain determined from flat coupon tests (e.g. Hu et al. 2011; Xiao et al. 2005); about 80% of the flat coupon tests on average.
Despite the fact that the general stress strain behaviour of concrete-filled FRP tubes is very similar to the FRP wrapped concrete columns, the FRP strain efficiency result from concrete-filled FRP tubes is found to be higher than the one from FRP wrapped concrete columns (De Lorenzis and Tepfers 2003; Teng et al. 2002). Statistics summarized by De Lorenzis and Tepfers (2003) and Teng et al. (2002) support the conclusion from De Lorenzis and Tepfers (2003) that concrete-filled FRP tubes do not display ‘premature’ tensile failure of FRP. The test data collected in their study shows that the ratio of FRP rupture strain to the ultimate strain from property tests is about 1.073 on average, with a standard deviation of 0.195, based on experiments of FRP-encased cylinders available in literature (Ahmad et al. 1991; Nanni and Bradford 1995; Saafi et al. 1999; Mirmiran and Shahawy 1997; La Tegola and Manni 1999; Fam and Rizkalla 2000).

2.6.2 Proposed values of FRP strain efficiency

Empirical strain efficiency coefficients are applied in various design guidelines to take this apparent ultimate strain reduction for FRP confined concrete into account. For instance, Xiao and Wu (2000) suggest a value of 0.5 based on their own test results. Jin et al (2003), however, suggest a value of 0.96. Lam and Teng (2003a) suggest that the actual FRP hoop rupture strain measured in tests should be used, rather than using results from flat coupon tests. The reduction of FRP rupture strain is also considered in some standards. ACI (2008) suggests a strain efficiency of 0.55 for circular FRP wrapped concrete columns. CNR-DT 200/2004 (2004) suggests the reduced FRP design strain should be no more than 0.004 (although this reflects a desire to ensure the shear
integrity of columns). Jiang and Teng (2006) have suggested a strain efficiency of 0.5 for CFRP wraps and 0.7 for GFRP wraps in design; this value is included in the Chinese code for application of FRP composites in construction (GB-50608 2010).

As mentioned previously, the observed FRP rupture strain of concrete-filled FRP tubes is much closer to the material properties tests compared with that of FRP wrapped concrete columns. One reason of this phenomenon may be attributed to the different manufacturing procedure of these two FRP jackets. De Lorenzis and Tepfers (2003) suggested that the quality of applying FRP using hand-lay up procedure is lower than the one using automated and may cause local stress concentrations in FRP. The former one is used in FRP wrapped columns, and the latter in concrete-filled FRP tubes. Furthermore, the geometrical discontinuities of FRP due to the overlap, which may cause strain concentrations as discussed later in this thesis, usually do not exist in FRP tubes. It is also worth pointing out that most of the FRP tubes were tested using split-disk test, since the FRP tube has the same geometry as the application in circular columns. Shahawy et al. (2000) found out that split-disk tests of annular coupons provides a better estimate of the rupture strain than flat coupon tests, but the differences with the actual rupture strain may still be considerable, the ratio of the ultimate tensile strain from split-disk test to that from FRP-filled steel tubes are ranged from 0.73 to 1.36 based on the database in De Lorenzis and Tepfers (2003).
2.6.3 Reasons for the lower-than apparent rupture strain

There can be a variety of possible causes for the variability of measured FRP hoop strains in FRP wrapped concrete columns. Lam and Teng (2004) concluded that there are at least three factors: (a) the curvature of FRP jacket which results in a reduced strain capacity; (b) the deformation non-uniformity of cracked concrete which leads to a non-uniform strain distribution in the FRP; and (c) the existence of an overlapping zone in which the measured strains are much lower than strains elsewhere.

The effect of the geometrical discontinuities at the ends of the overlapping zone of the FRP wrap was investigated by Chen et al. (2010), in which a circular column wrapped with several layers of FRP sheets was considered. The result of the FE analysis shows that stress concentration occurred a) on the outer surface of the inner layer of FRP adjacent to the outer end of the wrap; b) on the inner surface of the outer layer of FRP adjacent to the inner end of the wrap. Büyüköztürk and Yu (2009) used a radar non-destructive testing technique to detect the defects and artificial debonding of FRP from the concrete, and found that this technique is effective to detect these defects. The effect of failure localisation within the confined concrete was investigated by Tabbara and Karam (2008) using FE model, who found that the failure localisation in the concrete can cause non-uniform strain distribution of FRP hoop strain. Fraldi et al. (2008) have investigated the ultimate condition of FRP composites using Tsai-Hill failure criterion. A two-dimensional stress state has been considered. By ignoring the vertical stress $\sigma_z$, the results show that the ultimate condition depends on the radial strength of FRP. Lu et
al. (2006) presented a model, also based on Tsai-Hill failure criterion with a two-dimensional stress state considered, by ignoring the radial stress $\sigma_r$. The result shows that this model can predict the ultimate strain for their own experiments. Lignola et al. (2009) used Tsai and Wu theory to account for the triaxial state of stress in the FRP wrapping. A closed form solution for the confining jacket effective strain coefficient, i.e. FRP strain efficiency, was proposed. However, the relationship between the axial strain and lateral strain of FRP used in Lignola et al. (2009) was found to be inappropriate since the equation used in this paper was proposed by Teng et al. (2007a) in order to obtain the axial strain at the peak stress of actively confined concrete. The effects of some factors, such as geometric imperfections of the column and non-uniform bonding between the FRP and the concrete, have not been investigated to the best knowledge of the author.

### 2.7 Conclusions

This chapter has provided a review of existing research relevant to the lower-than-expected rupture strain measured at ultimate in FRP confined circular columns. The material properties of FRP composites are presented, including property test methods for FRP in columns, constitutive relationships and failure criteria. Following this, various applications of FRP composites in columns are discussed, and it is found that the phenomenon of lower-than-expected FRP hoop rupture strains exist in almost all kinds of FRP wrapped columns. The term strain efficiency is defined as the ratio of hoop strain in the FRP wrap at rupture to the FRP ultimate tensile strain from material property tests.
The value of strain efficiency and the reasons for the lower-than-apparent rupture strain available in literature were examined.

From the discussions above it is clear that large interest is paid to the applications of FRPs in columns, and that the understanding of the behaviour of FRP-wrapped columns has been improved significantly through the past decades. However, it is also clear that the phenomenon of lower-than-expected FRP rupture strain is far from being well understood, and that engineers and researchers continue to study this important problem; from both the design and fundamental mechanism point of view. It is essential to carry out theoretical as well as experimental investigations into this phenomenon, and to develop rational and accurate modelling techniques, theoretical solutions and design approaches.
3.1 Introduction
As described in Chapter 2, the ultimate FRP rupture strain is typically determined from the flat coupon test (ASTM 1995) (as shown in Figure 2.1) or the split-disk test (ASTM 1992) (as shown in Figure 2.2), or sometimes supplied by the FRP manufacturer (Lam and Teng 2002).

Some researchers suggested that the split-disk test provides a better estimate of the hoop rupture strain in FRP confined columns than the flat coupon test (Shahawy et al. 2000). Experiments have shown that the FRP rupture strain determined from the split-disk test is significantly lower than that from the flat coupon test or that provided by the manufacturer (Lam and Teng, 2004; Mirmiran and Shahawy, 1997; Tamuzs et al., 2006).

This chapter presents an FE analysis of strain distributions in the split-disk test of FRP wraps, with the aim of helping to explain the observed reductions of ultimate FRP rupture strains from the split-disk tests as compared with those observed from flat coupon tests. FE modelling issues are first discussed, with particular attention paid to the modelling of orthotropic FRP and the adhesive. Comparisons between the predictions of the model and test results are given to demonstrate the validity of the FE model approach. The effects of key test parameters are also examined.
3.2 Geometry and materials

3.2.1 Geometry

Figure 3.1 shows a typical test with two assembled half disks and one layer of FRP wrapped around their circumference. The disks have a radius $R$, and the FRP sheet has a thickness $t_f$. There is an adhesive layer of uniform thickness $t_a$ between the FRP wraps in the overlapping (splice) region. A polar coordinate system is used to describe position (Figure 3.1). The FRP is assumed to start at a circumferential angle $\theta = 90^\circ - \alpha/2$ at the inner end of the wrap and finishes at $\theta = 450^\circ + \alpha/2$ at the outer free end, giving an overlap zone of angular length $\alpha$ (Figure 3.1), the details of the geometry of the FRP wrap is similar as Chen et al. (2010). The actual starting position of FRP varies from one test to another but it shall not have any significant effect on the predicted peak strains unless one of or both FRP ends is very close to the gap between the two half disks, or when the friction between the FRP and disks is non-negligible.

The change in FRP wrap radius necessary for the outer layer of FRP to overlap the inner layer occurs within a transition zone of angular length $\beta$. The shape of the transition is assumed to be sinusoidal, similar as Chen et al. (2010), so that the radii of the inner and outer surfaces of the FRP within the $n^{th}$ layer ($n = 1$ to $N$) of the transition, $r_i$ and $r_o$, are defined by:

$$
r_i = R + \left(t_a + t_f\right) \times (n-1) + \frac{1}{2} \left(t_a + t_f\right) \left(1 - \cos \left(\frac{180}{\beta} \left(\theta + \beta + \frac{\alpha}{2} - 90 - 360n\right)\right)\right)
$$

(3.1)
\[ r_o = r_i + t_f \quad (3.2) \]

Reference values of \( R = 75\text{mm}, \alpha = 114.6^\circ \) (which are the dimensions of specimens tested by Lam and Teng (2004)), and \( \beta = 30^\circ \) are used in this study.

Figure 3.1 Geometry of the split disk test

This system contains three important interfaces (Figure 3.1): (a) between the disks and the FRP; (b) between the inner surface of the FRP and the adhesive in the transition zone; and (c) between the outer surface of the FRP and the adhesive in the transition zone. For such a system under split-disk tension, strain concentrations are expected to occur on the outer surface of the FRP at \( \theta = \alpha/2 + 90^\circ \), adjacent to the outer end of the FRP (Location A in Figure 3.1) and on the inner surface of the layer of the FRP at \( \theta = 450^\circ - \alpha/2 \), adjacent to the inner end of the FRP (Location B in Figure 3.1). Furthermore, local bending is expected to occur in the FRP at the gap created by the relative movement of the two half disks (Location C in Figure 3.1).
3.2.2 Adhesive properties

Un-toughened, thermosetting adhesives are generally brittle materials that fail at relatively small strains by the initiation and propagation of a crack. Most commercial adhesives are however rubber toughened and they can sustain relatively large strains (>5%) (Dean and Crocker 2001). Modern adhesives, particularly those such as the rubber-modified epoxies that are used in FRP strengthening applications with steel or concrete, have a relatively large plastic strain to failure (Adams and Wake 1984). The nonlinear behaviour of adhesives is not often included in the analysis of adhesively bonded joints because of the increased complexity of mathematical formulations (da Silva et al. 2009). The elastic modulus and strength of the adhesive reported in Mirmiran and Shahawy (1997) were 4.3GPa and 72 MPa respectively. The properties of adhesive are not reported in split-disk test studies reviewed by the authors (e.g. Lam and Teng 2004, Tamuzs et al. 2006).

In this study, the adhesive was treated as an isotropic linear elastic-perfectly-plastic material (Adams and Wake 1984), obeying the von Mises yield criteria coupled with an isotropic work hardening. A linear elastic material can be treated as a special case with an infinite yield stress. The elastic modulus and the Poisson’s ratio for both the matrix and the adhesive were chosen as 3 GPa and 0.35 (Dean and Crocker, 2001). A yield strength of 30MPa was used as the reference value but its effect is further examined.
3.2.3 Properties of the FRP composite

Common FRP composites used in structural strengthening are usually orthotropic. Their mechanical properties depend on the fibre orientation and distribution and the relative proportions of fibre and matrix. Their macro properties may be estimated from the fibre and adhesive properties using the law of mixtures (Vinson and Sierakowski, 2002). In this study, the matrix was assumed to have the same properties as the adhesive. Thus, the properties of the FRP can be deduced from the fibre and adhesive properties and fibre volume ratio.

3.3 Finite element modelling

3.3.1 FE model

All the FE analyses in this study were conducted using the FE analysis package ANSYS Version 11.0 (2007). The split-disk test was modelled as a plane stress problem using the eight-node planar elements. The two steel half disks were assumed to be rigid and modelled as rigid boundaries in the analysis. The FRP inner surface was defined as a deformable boundary, which was in initial contact with the disks. Surface-to-surface contact elements were used to describe these two surfaces, with the disk surface modelled by target elements and the FRP surface modelled by contact elements. The split of the disks was defined as a displacement of the disk surface. Each rigid disk surface was associated with a “pilot node”, whose motion governed the motion of the entire target surface (Ansys 2007). In the FE model, the lower half disk was fixed and a vertical load was applied to the upper half. All the test specimens investigated in this study were loaded up to the failure load observed in the respective tests. The Newton-
Raphson method was adopted. For specimens with test failure loads not directly reported, load was calculated from the split-disk test strength and the nominal cross-sectional area of the FRP wrap.

The contact surface between FRP and the disks were treated as having zero friction in all the analyses unless otherwise stated, since in a real split-disk test of an FRP composite lubricants are usually applied between the FRP ring and the disks to reduce friction. However, the effect of friction is investigated later in the chapter.

Before finalizing the FE model, three issues were examined by comparing the FE predictions with the test results for a reference specimen: element size, FRP thickness, and FRP orthotropy. Test specimen Lam-1 reported by Lam and Teng (2004) was selected as the reference specimen because both flat coupon tests and split-disk tests are reported, and the actual thickness of the FRP is given. The diameter of the disks was 150mm and the nominal width of the CFRP rings was 25mm, with an overlap length of 150mm. The FRP ring had a nominal thickness of 0.165mm, a Young’s modulus of 258.81GPa based on the nominal thickness of FRP, and an actual thickness of 1.2mm.

Figure 3.2 shows the predicted FRP fibre direction strain distributions on the inner and outer surfaces of the FRP for the reference case. Note that fibre direction is the same as the circumferential or hoop direction in the FRP wrap except within the transition zone,
where the two are slightly different due to the change of radius. Strain concentrations occur at three locations in the FRP: (a) on the outer surface of the inner layer of FRP adjacent to the outer end of the wrap (Location A in Figure 3.1); (b) on the inner surface of the outer layer of FRP adjacent to the inner end of the wrap (Location B); and (c) on the inner surface of the FRP at the gap created by the relative movement of the two half disks (Location C). The strains at the two ends of the gap created by the relative movement of the two half disks are almost the same due to the fact that the structure is almost symmetrical. Therefore only one end of the gap, denoted as Location C in Figure 3.1, is examined.

![Figure 3.2 Distribution of FRP fibre direction strain on the inner and outer surfaces of FRP](image_url)

**Figure 3.2 Distribution of FRP fibre direction strain on the inner and outer surfaces of FRP**
3.3.2 Mesh convergence

The existence of strain concentrations presented a challenge for the choice of a suitable finite element mesh. A mesh convergence study was conducted to determine a suitable element size. Figure 3.3a, b and c show the effect of the circumferential element size respectively on the variation of FRP fibre direction strains on the FRP outer surface near Location A and on the inner surface near locations B and C, where local bending and strain concentrations occur. Corresponding to the four minimum element widths of 0.2, 0.4, 0.8, and 1.6mm, the number of elements used through the FRP thickness was, respectively, 6, 5, 3, and 1. The adhesive layer was divided into two elements in the radial direction. To reduce the total number of elements, the circumferential element length was gradually increased with distance from the smallest element using a bias factor of 1.03. Figure 3.4 shows details of the finest finite element mesh considered near locations A and B.

Figure 3.5 shows the convergent trends of the maximum FRP fibre direction strains at the three locations from the four meshes. A coarse mesh clearly under-estimates the peak strains. A minimum element size of 0.2mm, with 6 elements through the thickness of an FRP layer, was deemed sufficiently accurate and was used for all the analyses in the rest of the section.
(a) Outer surface near location A

(b) Inner surface near location B
Figure 3.3 Effect of mesh size on FRP fibre direction strain distributions

(c) Inner surface near location C

Figure 3.4 mesh near locations A and B for a minimum mesh size of 0.2mm
3.3.3 FRP thickness

There are two main types of FRP strengthening systems for concrete structures: adhesively bonded prefabricated plates and wet lay-up systems. In the former the plate and the bonding adhesive layer can be clearly separated, although the exact thickness of the adhesive layer is difficult to control precisely in practice. In the latter case, which is the more widely used method for concrete confinement applications, the FRP sheet is formed from the impregnation of the fibres with resin (Teng et al. 2002). In modelling a wet lay-up FRP wrap, two options were used in this study: (a) a similar situation as a prefabricated wrap, with a well-defined adhesive layer; and (b) a wrap with its thickness being equal to the actual thickness of the laminate including the adhesive, with the fibres...
evenly distributed across the plate thickness. The circumferential stiffness in both cases was assumed to be the same.

The material properties of the CFRP in option (a) were the fibre properties, which in option (b) were the properties of the CFRP composite. An adhesive thickness of 1mm was used in option (a), but 0.1mm was used in option (b) because the interface between the inner and outer layers of FRP is expected to be pure adhesive, as discussed earlier, despite the fact that most of the adhesive thickness was assumed to be included in the actual thickness of the FRP composite in this case.

Figure 3.6 compares the predicted FRP fibre direction strain along the inner and outer surfaces of the FRP layer using both the nominal and actual thickness of the FRP. Clearly the strains within the overlapping zone are substantially lower: approximately 50% of the value outside the overlapping zone as expected since twice the FRP thickness is present in this region.
Figure 3.6 Effect of thickness option on FRP fibre direction strain distributions

(a) Location A
Figure 3.7 Effect of thickness option on strain concentration
Table 3.1 Comparison between test and FE predictions with different thickness options

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Test results</th>
<th>FEA results</th>
<th>Reduction factor $\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Split disk (%)</td>
<td>Flat coupon (%)</td>
<td>Thickness outside overlap zone (%)</td>
</tr>
<tr>
<td>Lam-1</td>
<td>1.170</td>
<td>1.511</td>
<td>0.77</td>
</tr>
</tbody>
</table>

The values of the maximum strains predicted using the actual thickness are substantially higher than those predicted using the nominal thickness at all three locations (Figure 3.7). This can be attributed to the smaller FRP thickness associated with the nominal thickness. Table 3.1 lists the FE predicted strains from both thicknesses at all three locations for the reference specimen Lam-1 together with the test values. The test strain reduction factor was obtained as the ratio of the ultimate FRP strain from the split-disk test to that from a flat coupon test; and the reduction factor in the FE was defined as the ratio of the FRP hoop strain outside the overlapping zone to the maximum fibre direction strain among those at the three critical locations: A, B and C (Figure 3.1). The reduction factor obtained from the model using the nominal thickness of the FRP is substantially larger (i.e. a less severe reduction due to strain concentrations) than either the test result or the FE model using the actual thickness. The reduction factor obtained from the model using the actual thickness is much closer to the test results. All FE results presented hereafter were obtained using the actual thickness.
Note that the FE predicted maximum strain at Location B is slightly smaller than the strain away from the strain concentrations when the nominal thickness is used. This is because local bending is insignificant in this case, and the existence of the adhesive at that location reduces slightly the circumferential strain in the FRP.

### 3.3.4 Effect of FRP orthotropy

Both isotropic and orthotropic material models were used to model the FRP wrap. The differences in the predicted FRP fibre direction strain distributions from the two models are discernable (Figure 3.8) at locations A through C, while they are the same remote from these locations. Figure 3.9a, b and c show the strain at the inner and outer surfaces of the FRP near locations A, B and C, respectively. Compared with the isotropic model, the maximum fibre direction strain predicted using the orthotropic model is about 10% larger at locations A and B but about 9% lower at Location C. Since the effect of FRP orthotropy on the predicted maximum strain appears to be important, the orthotropic model, which is physically more realistic, was used in all the subsequent modelling.
Figure 3.8 Predicted FRP fibre direction strains using isotropic and orthotropic material models

(a) Location A
Figure 3.9 Comparison of FRP fibre direction strains between isotropic and orthotropic material models at locations A, B, and C
3.4 Analysis of strains in fibre orientation in the FRP wrap

The strain distribution around the locations where strain concentrations are experienced (i.e. locations A, B, and C) is further analysed in more detail here for the reference case (Lam-1).

3.4.1 Load versus peak strains

![Graph showing load versus peak FRP strains at locations A, B and C.]

Figure 3.10 shows the maximum FRP fibre direction strain at locations A to C under increasing normalized load, where the normalized load is the applied load divided by the absolute ultimate load. The results from the model using two sets of adhesive material properties are shown. The maximum strain at Location C is clearly linear with the applied load and not affected by the adhesive properties. On the contrary, the curves of Locations A and B are nonlinear after the adhesive first yields. The yielding of the...
adhesive reduces the maximum FRP fibre direction strain at Location A, but increases it at Location B.

### 3.4.2 Strains at locations A and B

Near Location A, a strain concentration occurs within the inner layer of the FRP adjacent to the outer end of the wrap (Figure 3.7a). The outer end of the FRP tends to peel away from the wrap and thus to pull the inner layer of FRP outwards, resulting in outward bending. At the predicted ultimate state, a maximum hoop strain of 1.37% occurs on the outer surface of the FRP at Location A, compared to the reference strain of 1.18% away from the overlapping zone. The strain on the inner surface of the FRP is about 1.12%, thus the strain concentration involves an increase in tensile membrane strain to 1.25% at Location A (a 6% increase over the reference strain); plus bending strains of \( \pm 0.12\% \) (10.2% of the reference strain).

By contrast, at Location B, the inner end of the FRP tends to pull the outer layer of the FRP inwards, resulting in inward bending strains within the outer layer of the FRP adjacent to the inner end of the wrap. The FE prediction of FRP fibre direction strain near Location B is shown in Figure 3.7b. A maximum fibre direction strain of 1.40% occurs on the inner surface of the FRP at this location, compared to the reference strain of 1.18%. The strain on the inner surface of the FRP is 0.92%, thus the strain concentration involves a bending strain of \( \pm 0.24\% \) (20.3% of the reference strain). The
tensile membrane strain is 1.16%, very slightly smaller than the reference strain because of the existence of pure adhesive zones in the transition region.

As shown in Figure 3.7a and b, the bending strains reverse a short distance to either side of locations A and B, although these reversals are much smaller in magnitude.

### 3.4.3 Strains at Location C

Local bending occurs at the gap created by the relative moment of the two rigid half disks, resulting in local maximum tensile strain on the inner surface of the FRP wrap. As shown in Figure 3.7c, this bending is highly localised, and the state of stress near the gap is distinctly non-homogeneous as previously noted by Knight (1977). At the ultimate state, a maximum tensile strain of 1.48% occurs on the inner surface of the FRP with a corresponding strain of 0.88% on the outer surface at the same location. Compared with the reference strain of 1.18%, the strain concentration leads to bending strains of ±0.3% (25.4% of the reference strain), with no change in the membrane strain.

The local maximum bending strain in Location C is the product of the maximum curvature change in the FRP $\Delta \phi$ and one half of the FRP thickness $t/2$:

$$
\varepsilon_s = \Delta \phi \frac{t}{2}
$$

(3.3)

Under a test load $F$, the tensile membrane strain of the FRP is
where $A$ is the cross sectional area of the FRP.

The sum of the bending and membrane strains gives the maximum FRP hoop strain at Location C

$$\varepsilon_{\text{max}} = \Delta \phi \frac{t}{2} + \frac{F}{2EA}$$  \hspace{1cm} (3.5)

During a split-disk test, the initial curvature of the FRP wrap is $1/R$ assuming that the two half disks are perfectly closed and the FRP wrap is made to perfectly fit the disks. As the load is applied, the two half disks are continuously separated creating a gap between them, straightening the FRP and reducing the curvature there. It is possible that the curvature there may eventually become negative, depending on the geometrical and material properties of the FRP wrap and the friction between it and the disks. An obvious state for estimation would be when the FRP becomes perfectly straight so the curvature is reduced to zero at the gap. In this case the change of the FRP curvature is $1/R$, so that the local maximum FRP strain from Eq. 3.5 is:

$$\varepsilon_{\text{max}} = \frac{t}{2R} + \frac{F}{2EA}$$  \hspace{1cm} (3.6)
The predictions of Eqs 3.4 and 3.6 are shown in Figure 3.10 for comparison. The FE predicted peak hoop strain at Location C lies between the predictions of Eq. 3.4 and Eq. 3.6. An estimation of the strain reduction factor may be obtained from:

\[
\rho = \frac{\varepsilon_{rup}}{\varepsilon_{rup} + \frac{t}{2R}}
\]  

where \( \varepsilon_{rup} \) is the rupture strain obtained from the flat coupon test. The estimation from Eq. 3.7 will be compared with test data in the next section.

3.5 Comparison of FE predictions with test results

3.5.1 Test data

All specimens listed in Table 3.2 were simulated with the FE model presented above. The material properties of the FRP composites calculated using the law of mixtures (Vinson and Sierakowski, 2002) are listed in Table 3.3.

The elastic moduli obtained from flat coupon tests for the specimens in Lam and Teng (2004) were used in the FE model because those obtained from split-disk test are less reliable due to the existence of the overlapping zone and strain concentrations. Furthermore, the ultimate strains from test results were deduced from the FRP ultimate strength divided by elastic modulus of the FRP, since the strain gauges are less reliable when they are located at the gap created by the relative movement of the two half disks (Lam and Teng, 2004). No flat coupon test data were reported in Mirmiran and Shahawy.
(1997) or Tamuzs et al. (2006), so the technical data from the supplier are adopted for specimens 3 to 5 in Table 3.4; the ultimate tensile strain calculated using micro-mechanics equations and the law of mixture is used for specimens 6 to 8 (Mirmiran and Shahawy 1997).

### Table 3.2 Split disk test specimens

<table>
<thead>
<tr>
<th>No.</th>
<th>Specimen name</th>
<th>FRP type</th>
<th>Nominal thickness (mm)</th>
<th>Actual thickness (mm)</th>
<th>Number of layers</th>
<th>Width (mm)</th>
<th>Length of overlap (mm)</th>
<th>Diameter (mm)</th>
<th>Composites elastic modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lam-1</td>
<td>CFRP</td>
<td>0.165</td>
<td>1.20</td>
<td>1</td>
<td>25</td>
<td>150</td>
<td>150</td>
<td>258.81</td>
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<tr>
<td>2</td>
<td>Lam-2</td>
<td>GFRP</td>
<td>1.27</td>
<td>1.24</td>
<td>1</td>
<td>23</td>
<td>150</td>
<td>150</td>
<td>22.455</td>
</tr>
<tr>
<td>3</td>
<td>Tamuzs-1</td>
<td>CFRP</td>
<td>0.17</td>
<td>1.20*</td>
<td>1</td>
<td>20</td>
<td>150</td>
<td>150</td>
<td>200.5</td>
</tr>
<tr>
<td>4</td>
<td>Tamuzs-2</td>
<td>CFRP</td>
<td>0.17</td>
<td>1.20*</td>
<td>2</td>
<td>20</td>
<td>150</td>
<td>150</td>
<td>230.5</td>
</tr>
<tr>
<td>5</td>
<td>Tamuzs-3</td>
<td>CFRP</td>
<td>0.17</td>
<td>1.20*</td>
<td>3</td>
<td>20</td>
<td>150</td>
<td>150</td>
<td>236</td>
</tr>
<tr>
<td>6</td>
<td>Mirmian-1</td>
<td>GFRP</td>
<td>0.216</td>
<td>0.24</td>
<td>6</td>
<td>27.3</td>
<td>-</td>
<td>145</td>
<td>37.233</td>
</tr>
<tr>
<td>7</td>
<td>Mirmian-2</td>
<td>GFRP</td>
<td>0.216</td>
<td>0.22</td>
<td>10</td>
<td>25.7</td>
<td>-</td>
<td>145</td>
<td>40.336</td>
</tr>
<tr>
<td>8</td>
<td>Mirmian-3</td>
<td>GFRP</td>
<td>0.216</td>
<td>0.21</td>
<td>14</td>
<td>26.2</td>
<td>-</td>
<td>145</td>
<td>40.749</td>
</tr>
</tbody>
</table>

Data sources: 1-2: (Lam and Teng, 2004); 3-5: (Tamuzs et al., 2006); 6-8: (Mirmiran and Shahawy, 1997).

* Estimated value as data not provided in the source.

### Table 3.3 Properties of FRP constituents and composites

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>$E_f$ (GPa)</th>
<th>$v_f$</th>
<th>Fibre volume ratio</th>
<th>Derived properties of FRP composites</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lam-1</td>
<td>240.02</td>
<td>0.2</td>
<td>13.75%</td>
<td>$E_1$ (GPa)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$E_2 = E_3$ (GPa)</td>
</tr>
<tr>
<td>Lam-2</td>
<td>35.43</td>
<td>0.2</td>
<td>60%</td>
<td>35.59</td>
</tr>
<tr>
<td>Tamuzs-1</td>
<td>182.3</td>
<td>0.2</td>
<td>14.17%</td>
<td>32.65</td>
</tr>
<tr>
<td>Tamuzs-2</td>
<td>212.2</td>
<td>0.2</td>
<td>14.17%</td>
<td>31.75</td>
</tr>
<tr>
<td>Tamuzs-3</td>
<td>217.8</td>
<td>0.2</td>
<td>14.17%</td>
<td>33.43</td>
</tr>
<tr>
<td>Mirmian-1</td>
<td>93.30</td>
<td>0.2</td>
<td>37%*</td>
<td>73.23</td>
</tr>
<tr>
<td>Mirmian-2</td>
<td>86.21</td>
<td>0.2</td>
<td>44%*</td>
<td>40.34</td>
</tr>
<tr>
<td>Mirmian-3</td>
<td>87.14</td>
<td>0.2</td>
<td>44%*</td>
<td>40.75</td>
</tr>
</tbody>
</table>

Note: (a) Direction 1 is the fibre direction, directions 2 and 3 are perpendicular to the fibre direction; (b) Other Poisson’s ratios can be derived from $E_i/E_j = v_{ij}/v_{ji}$, where $i, j = 1, 2, 3$.

* Provided by Mirmiran (1996).
Table 3.4 Comparison between test results and FE predictions

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Test</th>
<th>Split disk (%)</th>
<th>Flat coupon or supplier (%)</th>
<th>Strain reduction factor $\rho$</th>
<th>Strain outside overlap zone (%)</th>
<th>Max. strain in Loc. A (%)</th>
<th>Max. strain in Loc. B (%)</th>
<th>Max. strain in Loc. C (%)</th>
<th>Strain reduction factor $\rho$ estimate from Eq. 3.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lam-1</td>
<td>1.170</td>
<td>1.511</td>
<td>0.77</td>
<td>1.179</td>
<td>1.374</td>
<td>1.395</td>
<td>1.482</td>
<td>0.80</td>
<td>0.65</td>
</tr>
<tr>
<td>Lam-2</td>
<td>1.902</td>
<td>2.325</td>
<td>0.82</td>
<td>1.967</td>
<td>2.083</td>
<td>2.274</td>
<td>2.492</td>
<td>0.79</td>
<td>0.74</td>
</tr>
<tr>
<td>Mirmian-1</td>
<td>1.41</td>
<td>1.89</td>
<td>0.75</td>
<td>1.418</td>
<td>-</td>
<td>-</td>
<td>1.890</td>
<td>0.75</td>
<td>0.66</td>
</tr>
<tr>
<td>Mirmian-2</td>
<td>1.44</td>
<td>1.92</td>
<td>0.75</td>
<td>1.445</td>
<td>-</td>
<td>-</td>
<td>2.014</td>
<td>0.72</td>
<td>0.56</td>
</tr>
<tr>
<td>Mirmian-3</td>
<td>1.57</td>
<td>1.92</td>
<td>0.82</td>
<td>1.584</td>
<td>-</td>
<td>-</td>
<td>2.269</td>
<td>0.70</td>
<td>0.49</td>
</tr>
<tr>
<td>Tamuzs-1</td>
<td>0.95</td>
<td>0.49</td>
<td>0.49</td>
<td>0.956</td>
<td>1.225</td>
<td>1.108</td>
<td>1.221</td>
<td>0.78</td>
<td>0.71</td>
</tr>
<tr>
<td>Tamuzs-2</td>
<td>1.04</td>
<td>1.92</td>
<td>0.54</td>
<td>1.038</td>
<td>1.302</td>
<td>1.301</td>
<td>1.420</td>
<td>0.73</td>
<td>0.55</td>
</tr>
<tr>
<td>Tamuzs-3</td>
<td>1.13</td>
<td>1.92</td>
<td>0.59</td>
<td>1.128</td>
<td>1.392</td>
<td>1.422</td>
<td>1.617</td>
<td>0.70</td>
<td>0.44</td>
</tr>
<tr>
<td>Tamuzs-4</td>
<td>0.95</td>
<td>0.49</td>
<td>0.49</td>
<td>0.956</td>
<td>1.551</td>
<td>1.01</td>
<td>1.221</td>
<td>0.62</td>
<td>0.71</td>
</tr>
<tr>
<td>Tamuzs-5</td>
<td>1.04</td>
<td>1.92</td>
<td>0.54</td>
<td>1.067</td>
<td>1.638</td>
<td>1.297</td>
<td>1.418</td>
<td>0.65</td>
<td>0.55</td>
</tr>
<tr>
<td>Tamuzs-6</td>
<td>1.13</td>
<td>1.92</td>
<td>0.59</td>
<td>1.127</td>
<td>1.733</td>
<td>1.540</td>
<td>1.729</td>
<td>0.65</td>
<td>0.44</td>
</tr>
<tr>
<td>Tamuzs-7</td>
<td>0.95</td>
<td>0.49</td>
<td>0.49</td>
<td>0.956</td>
<td>1.861</td>
<td>1.004</td>
<td>1.222</td>
<td>0.51</td>
<td>0.71</td>
</tr>
<tr>
<td>Tamuzs-8</td>
<td>1.04</td>
<td>1.92</td>
<td>0.54</td>
<td>1.038</td>
<td>2.258</td>
<td>1.303</td>
<td>1.430</td>
<td>0.46</td>
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<tr>
<td>Tamuzs-9</td>
<td>1.13</td>
<td>1.92</td>
<td>0.59</td>
<td>1.126</td>
<td>2.505</td>
<td>1.553</td>
<td>1.615</td>
<td>0.45</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Adhesive yield strength = 30 MPa

Adhesive yield strength = 60 MPa

Elastic adhesive property

3.5.2 Comparison with test results

The peak FRP fibre direction strains at the ultimate state predicted by the FE model are compared with data obtained from tests or provided by the manufacturers in Table 3.4. There is a good agreement between the predicted fibre direction strains outside the overlapping zone and the ultimate strains obtained from the split-disk test for all the specimens, with differences of less than 3%, except experiments from Tamuzs et al. (2006), these tests are further discussed in this section. This indicates that the FE model described above accurately predicts FRP hoop strains for the split-disk tests on the basis of available test data.
It should be noted that there was no overlapping zone in the specimens reported by Mirmiran and Shahawy (1997) since these were cut from filament wound FRP tubes. Therefore, a strain concentration occurred only at Location C in these tests. The maximum strain among the peak values at Locations A, B and C for Lam and Teng (2004)’s tests, and the peak strain at Location C for Mirmiran and Shahawy (1997)’s test are clearly very close to the ultimate tensile strain of the FRP obtained from flat coupon tests or provided by the supplier, demonstrating good agreement between the tests and FE predicted strain reduction factors.

From the above it may be concluded that in split-disk tests the FRP ruptures once one of the peak strains at locations A, B, or C reaches the FRP flat coupon rupture strain. This leads to a lower apparent tensile strength than that determined from flat coupon tests.

For the set of three tests reported by Tamuzs et al. (2006), because the actual FRP thickness was not reported, the actual thickness of the CFRP of Lam and Teng (2004)’s specimens was used since the nominal thickness of the CFRP materials in the two studies is similar. Also, since the adhesive properties were not reported by Tamuzs et al. (2006), three different adhesive models were adopted to examine the FRP hoop strains: yield strength = infinite (linear elastic), 60MPa and 30MPa. The test reduction factors are smaller than the FE predictions using adhesive strength of 30 MPa and 60 MPa. In contrast, two of the three test values are larger than the FE predictions using the linear
elastic adhesive model. Thus, an adhesive yield strength slightly larger than 60 MPa gives a good match between the FE predictions and the test data.

The predicted strain reduction factor from Eq. 3.7 is also listed in Table 3.4. It is seen that Eq. 3.7 provides a reasonably good lower estimation for the strain reduction factor: the ratio of the prediction of Eq. 3.7 to the test value has an average value of 0.90 with a coefficient of variation of 28% for the eight specimens listed in the table.

### 3.6 Parametric study

A parametric study was undertaken to investigate the effects of the following parameters on the circumferential strain distribution within the FRP in a split-disk test:

- adhesive strength;
- adhesive thickness;
- geometry of the transition zone;
- FRP stiffness; and
- friction between FRP and split disks.

The reference case in the parametric study is again Specimen Lam-1 described earlier. Each of the parameters of interest was varied sequentially with all other parameters having the reference values as given in Table 3.2. The focus of the parametric study was the peak FRP strain in the direction of the fibres, which occurs at one of the locations A,
B, or C. The strain in the fibre direction is slightly different to the circumferential strain within the transition zone resulting from the change in fibre radius (Figure 3.1), but is identical to the circumferential strain outside the transition zone. The normalized FRP fibre direction strain is adopted in the parametric study for convenience of discussion; the reference strain is the FRP membrane strain calculated from Eq. 3.4, which is 1.17%.

3.6.1 Effect of adhesive strength

Figure 3.11 shows the effect of the adhesive yield strength on the maximum FRP strain near the three important locations. The comparison shows that the maximum strains remain almost unchanged at Location C but increase at Location A in an almost linear fashion and decrease slightly at Location B with an increase of the adhesive yield strength. An increase of the adhesive strength increases the strain concentration at Location B because it increases the pull-off force from the outer end of the FRP locally to the inner layer. In contrast, the strain concentration at Location B is reduced with a higher yielding adhesive as it transfers more of the pull-off force from the inner end to the larger zone of FRP in the transition zone. The phenomenon at Location C is expected because there is no adhesive at this location and the effects of strain concentration at Locations A and B are local according to Saint-Venant’s principle so they should not affect Location C.
3.6.2 Effect of adhesive thickness

All the underlying mechanisms as discussed above for the effects of increasing the adhesive strength on strain concentrations at locations B and C also apply for increasing the adhesive thickness, although a thicker adhesive is more effective than a stronger one in reducing the strain concentration at location B (Figure 3.12). At Location A, an increase of the adhesive thickness on the one hand increases the distance between the pull-off force at the outer end of the FRP and the layer below it, increasing the local bending effect, but on the other hand increases the spread of the force there by reducing the strain concentration. This may explain why the strain concentration increases initially with the adhesive thickness but then reduces slightly as the adhesive thickness further increases, as shown in Figure 3.12.
3.6.3 Effect of transition zone size

Figure 3.13 shows the effect of the transition angle on the maximum FRP fibre direction strain. The transition zone angle was varied from $\beta = 5^\circ$ to $30^\circ$ ($\beta = 30^\circ$ for the reference case). Results for an adhesive with a yield strength of 30 MPa and for an elastic adhesive are compared. The results show that there are essentially no changes for the peak FRP hoop strains at locations A and C for both types adhesives, but that the peak FRP fibre direction strain at Location B reduces significantly with an increase of the transition angle when the angle is small (i.e. less than 15 degrees when using an adhesive with a yield strength of 30 MPa, or less than 10 degrees when using an elastic adhesive). This can be attributed to the sharp change of radius in the transition zone. Note that the inner
end of the FRP may be not flush in reality as assumed in this study, which probably represents the worst case.

![Graph showing the effect of transition angle on peak FRP fibre direction strains.](image)

**Figure 3.13 Effect of transition angle on peak FRP fibre direction strains**

### 3.6.4 Effect of FRP stiffness

The effect of the FRP stiffness (i.e. elastic modulus $E_f$ and thickness $t_f$) on the peak FRP strain at locations A and B is dependent on three different effects, similar to those in an FRP wrapped column (Chen et al. 2009):

- the FRP bending stiffness increases with its thickness, resulting in reduced bending strains in the FRP. For a given bending moment the maximum bending strain is proportional to $1/E_f$ and $1/t_f^2$;

- an increase in axial stiffness of the FRP due to a change in either $E_f$ or $t_f$ leads to an increase in the strain concentration near both ends of the wrap; and
• an increase in $\eta$ increases the distance (and thus eccentricity) between the centroid of both ends of FRP section and the adhesive interface, leading to increased local bending.

Figure 3.14 shows that the first of the above factors is dominant when the FRP thickness is large (>0.4 mm in this example). The last two factors become dominant when the FRP thickness is small (<0.4 mm in this example). The figure also shows that the results from Eq. 3.6 are larger than the FE predictions at Location C when the FRP is thicker than 0.1 mm for this example. Both Eq. 3.6 and the FE model predict that the normalized FRP hoop strain at failure increases with an increase of the FRP thickness. Eq. 3.6 appears to give a very conservative estimate of the maximum strain at Location C.

For a constant FRP thickness, the eccentricity is also constant, so the third effect listed above does not apply. Figure 3.15 shows that, in terms of the local peak FRP bending strain, the first effect is dominant so that the maximum fibre strain decreases as the elastic modulus is increased near Location A. For Location B, the maximum FRP strain increases as the elastic modulus of the FRP is increased. Therefore, the second factor above is dominant over the range of parameters studied. The result from Eq. 3.6 remains constant with varying FRP elastic modulus; it is 33.1% larger than the FE result on average.
Figure 3.14 Effect of FRP thickness on peak fibre direction strain

Figure 3.15 Effect of elastic modulus of FRP on peak fibre direction strain
3.6.5 Effect of friction

When friction exists between the FRP wrap and the split disks, the FRP fibre direction strains are no longer uniform either inside or outside the overlapping zone. This is demonstrated in Figure 3.16 for the reference case with friction coefficient of 0.3.

Figure 3.17 shows the effect of the friction coefficient on the maximum FRP fibre direction strain at Locations A to C. The maximum strains at all three locations decrease with an increased friction. When the friction coefficient increases from 0 to 0.5, the maximum FRP strain at Location C reduces by only 1.6% at Location C but by about 22% and 38% respectively at locations A and B. Therefore, friction appears to play a positive role in reducing the effects of strain concentrations in split-disk tests. It should be noted that these reductions are dependent on the locations of the two ends relative to the ends of the gap between the two half disks. The reduction of the peak strain is more significant if the end location is far away from the ends of the gap, since frictional force reduces the stresses in the FRP with distance.
3.7 Conclusions

This chapter has presented a detailed finite element investigation into the FRP ultimate strain observed and predicted in split-disk tests of FRP composites, and has presented one possible explanation for the lower tensile strength observed in from split-disk tests.
as compared with that from flat coupon tests. The predictions of the FE model have been shown to be in close agreement with selected test results. For split-disk test specimens failing at the gap between the two half disks, a model has been proposed to estimate the strain reduction factor. The following specific conclusions can be drawn:

(a) The FE results show that strain concentrations occur in the FRP in localized zone at three locations where geometrical discontinuities are present: (1) on the outer surface of the inner layer of FRP adjacent to the outer end of the wrap (Location A); (2) on the inner surface of the outer layer of FRP adjacent to the inner end of the wrap (Location B); and (3) on the inner surface of the inner layer near to the gap created by the relative movement of the two half disks (Location C). Local bending due to the geometric discontinuities at the ends of the FRP and bending of the FRP ring at the gap arising from the relative moment of the two half disks results in increased local strains in the FRP ring in the split-disk tests.

(b) Because these zones are very small it is not possible to measure such concentrated strains using conventional measurement techniques. This explains why the measured ultimate FRP strain in split-disk tests is typically lower than the rupture strain obtained from flat coupon tests.
(c) At the ultimate condition, the FE predicted strains away from the strain concentration and overlapping zones are close to those observed from split-disk tests based on limited test results available in the literature. The predicted maximum strains in the three key locations are significantly larger than those remote from these locations, and the maximum value of the peak strains at the three key locations is close to the rupture strains observed in flat coupon tests. This suggests that the FRP ruptures once one of the peak strains at these locations reaches the rupture strain, leading to lower apparent tensile strength than that from flat coupon tests.
CHAPTER 4
ON FACTORS AFFECTING THE ULTIMATE CONDITION OF FRP
WRAPPED CONCRETE COLUMNS

4.1 Introduction
Extensive research has been conducted on strengthening concrete columns using circumferential fibre reinforced polymer (FRP) wrapping. This technique is often favoured for its simplicity in implementation and economic benefits compared to traditional methods of strengthening (Teng et al. 2002). It has been applied in numerous projects around the world (Teng et al. 2002).

As reviewed in Chapter 2, there are several potential causes for the variability in (empirically derived) strain efficiencies suggested in the literature. However, no rational model to account for these based on sound theories and physical observations are currently available. Clearly, there is little consensus on this issue in the available literature.

This chapter examines the potential factors influencing failure of FRP wrapped circular columns corresponding to different failure modes. The contributory factors to the low apparent FRP hoop rupture strain are discussed, as are how these factors may affect the behaviour of FRP wrapped circular concrete columns.
4.2 Failure modes

Classification of the failure mode of FRP wrapped columns is important for predicting the ultimate condition of FRP wrapped columns, since there could be differences in the results of the FRP ultimate tensile strain and the ultimate axial strain of the columns due to the different failure modes.

4.2.1 FRP rupture

The typical failure mode reported from the majority of existing studies on FRP wrapped columns is rupture of the FRP wraps in tension in the hoop direction (Teng et al. 2002). It should be noted that it is usually not easy to determine the initial FRP rupture position after failure, since the rupture does not usually propagate along a vertical meridian once initiated. Figure 4.1 shows a schematic view of a two-layer FRP wrapped concrete column, in reference to which possible FRP rupture locations are discussed below.

(a) Three dimensional view
4.2.1.1 Rupture near the outer end of FRP

Due to geometric discontinuity of the FRP wraps at their outer end (Location A in Figure 4.1), when tested in compression FRP wrapped concrete columns sometimes fail due to FRP rupture there. This failure mode was first highlighted by Chen et al. (2007, 2010) in a theoretical study but significant strain concentration at the FRP outer end has later been confirmed by Smith et al. (2010) using the electronic speckle pattern interferometry (ESPI) measurement technique. Figure 4.2 shows an example of this failure mode in an FRP wrapped concrete cylinder with a single layer of CFRP and a 100mm overlap length. In an ideal uniform test specimen, the FRP rupture shall initiate at the outer surface of the FRP layer directly under the FRP outer end based on Chen et al.’s (2010) investigation, but this cannot be possibly confirmed using the currently available observation techniques.
4.2.1.2 Rupture near the inner end of FRP

As shown in Figure 4.3, the FRP wraps sometimes rupture at their inner end (Location B in Figure 4.1b). FRP rupture shall initiate at the inner surface in the layer immediately above the inner end in an ideal specimen (Chen et al. 2010). As indicated by Chen et al. (2010) through finite element analyses of FRP wrapped cylinders, the strain concentration is usually less significant at Location B than that at Location A. The probability of this failure mode is therefore smaller than that of FRP rupture failure at Location A; this is in agreement with both available experimental observations and FE analyses (e.g. Bisby et al. 2011). Figure 4.3 shows an example of FRP rupture initiating at the inner end of the FRP wrap.
4.2.1.3 Rupture outside the overlapping region

As reported in Au and Buyukozturk (2005), FRP jacket may rupture outside the overlapping region. Figure 4.4 shows such an example. Local stiffness enhancement in the lap joint area may be one of the causes for this failure (Au and Buyukozturk 2005) which results in higher stress in the FRP outside the FRP overlap region (Chen et al. 2010), but it is more likely caused by non-uniform deformation in the concrete as will be discussed later in this chapter.
4.2.2 Debonding failure at the overlap zone

Debonding failure at the overlapping joint is also observed in a limited number of tests in the literature (e.g. Bisby et al 2011); this failure mode is of only minor interest and is due to an insufficient lap length (e.g. Nanni and Bradford 1995) or other causes such as a misuse of adhesive during the wet lay-up procedure. Figure 4.5 shows debonding failure at the lap-joint in an FRP wrapped concrete cylinder for the batch of test reported in Bisby et al (2011). Due to the very small portion of specimens were failed due to FRP debonding, and generally a sufficient lap length can avoid this type of premature failure (Nanni and Bradford 1995), only the FRP rupture failure mode is further examined in this chapter.

![Figure 4.5 Debonding failure of FRP](image)

4.2.3 Other failure modes

A mixed FRP debonding and fracture failure has also been observed in the tests reported in Bisby et al. (2011). Figure 4.6 shows a typical photo of this failure mode. Due to the explosive nature of failure, it has not been possible to establish whether FRP rupture or debonding occurred first. Again, this failure mode is not often observed in practice if a
proper installation procedure is carefully followed so no further discussion is attributed to it in this chapter.

The failure generally occurs near the mid-height of the columns (Figure 4.6a), but it is also possible to occur near one of the ends of the column (Figure 4.6b) if the specimens are not aligned properly or their ends are not properly capped. This applies to all failure modes.

![Image of FRP ruptures at the middle of the column and near the end of the cylinder]

a) FRP ruptures at the middle of the column b) FRP ruptures near the end of the cylinder

**Figure 4.6 Mixed FRP debonding and fracture failure**

### 4.3 Contributory factors

Lam and Teng (2004) were among the first to conduct a methodical experimental investigation into the different contributory causes for the reduction of the apparent hoop
FRP strain in FRP wrapped circular concrete columns. They concluded that there are at least three factors, including:

a) curvature of the FRP which reduces its strain capacity;

b) the cracked concrete resulting in a non-uniform strain distribution in the FRP due to non-uniform dilation and deformation of the concrete core; and

c) FRP overlap region: the strains measured by electrical resistance strain gauges within this region are far lower than elsewhere, since up to twice the thickness of FRP exists at this location.

Several other factors may also play a role, such as the effects of the adhesive bond quality, fibre orientation and misalignment, the presence of a triaxial stress state in FRP, failure localisation and geometrical discontinuities in both the FRP and the concrete core, have all been identified and investigated by several others (e.g. Li et al. 2006, De Lorenzis and Tepfers 2003).

Extensive research on concrete filled steel tubular columns is also available in the literature. Such columns are very similar to FRP wrapped circular concrete columns in geometry, but the steel tubes usually reach their yield strength meaning that the confining pressure remains constant with increasing dilation of the concrete core up to failure. Among the differences between a steel tube and an FRP jacket, such as larger hoop and axial stiffness and lower ultimate hoop strength for normal steel tubes used in concrete filled tubular columns compared with an FRP jacket in FRP wrapped concrete
column applications, a critical one is that FRP composites used in column wrapping applications are linear elastic brittle. For a brittle FRP composite, the most highly strained fibre ruptures when its strain reaches its rupture strain. The force resisted by the ruptured fibre has to be transferred to its neighbouring fibres which may in turn rupture in an extremely rapid, progressive fashion. This invariably leads to the rupture of the whole FRP jacket and to the failure of the FRP wrapped concrete column. This means that any non-uniform deformation of the core is more important in the case of an FRP wrapped column and as a result the highest strain may control the behaviour of the column. In contrast, steel has excellent ductility and an elastic-plastic stress strain response. A local high stress in the hoop direction can lead to early yielding, but the plastic deformation of the steel allows stress re-distribution so that the presence of local high stresses is not normally detrimental to the performance of concrete filled steel tubular columns. Another important difference between steel and FRP is that the former is isotropic while the latter is anisotropic. This means that the behaviour of FRP is highly sensitive to fibre orientations.

Based on the above discussion, it is presumed that any factor which affects either the uniformity of stress distribution or the orientation of the stress in relation to the fibres can have an adverse effect on the behaviour of an FRP wrapped circular concrete column. The following factors may be considered as potentially important; some of these factors are first identified in the current study. These factors may be classified into five categories as follows:
Category 1 - Geometrical factors:

1) Geometrical discontinuities
2) FRP overlap region
3) Geometrical imperfections
4) Curvature of the FRP jacket

Category 2 - FRP material factors:

5) Fibre orientation
6) Misalignment and uneven tension of fibres
7) Damage of fibres
8) Triaxial stress state in the FRP

Category 3 - Concrete material factors:

9) Non-uniform deformation in the concrete
10) Strain localisation in the concrete

Category 4 - Adhesive material and geometry factors:

11) Mechanical properties of the adhesive
12) Geometrical details of the adhesive

13) Non-uniform bonding and partial debonding of FRP

Category 5 - Loading factors:

14) Loading eccentricity

15) Non-uniform loading

16) Frictional confinement at the supports

17) Stressing due to thermal deformation and creep

The potential effects of each of the above factors are discussed in the rest of this chapter.

4.3.1 Geometrical factors

Figure 4.1 shows a schematic view of a circular concrete column wrapped with FRP. Two layers of FRP are shown as an example, although the discussion is applicable for columns wrapped with either a single layer or multiple layers of FRP.

4.3.1.1 Geometrical discontinuities

The FRP jacket contains an overlap region, thus geometrical discontinuities exist at the inner and outer ends of the FRP (Figure 4.1). Within the overlap region the outer layer of the FRP has a larger radius than the inner layer. A transition region exists where the radius of the FRP increases from that of the inner layer to that of the outer layer (Location B in Figure 4.1b). The size of the transition region usually does not have a
significant effect on the maximum tensile strain in the FRP unless it becomes very small (Chen et al. 2010).

An early finite element study (Chen et al. 2010) showed that, due to geometrical discontinuities, stress concentrations are experienced at the ends of an FRP wrap:

a) in the inner layer under the outer end of the FRP (location A in Figure 4.1b); and

b) in the outer layer above the inner end of the FRP (location B in Figure 4.1b).

Large strains at both locations are clearly shown in Figure 4.7 for an example column with a radius of 100mm wrapped with a single layer of 1.6mm thick GFRP using a 0.1mm thick adhesive layer. The details of the model can be found in Chapter 6.

Note that it is shown in Figure 4.1b that the peak tensile strain at Location B is smaller than that at Location A, although this relationship can change when the thickness and properties of the adhesive change. FRP rupture failure at both locations A and B can thus be expected in practice. However, the failure of FRP wrapped columns is usually explosive and so it is very difficult after failure to identify the location where FRP rupture initiates. Furthermore, FRP rupture is expected to initiate from the inner surface if failure is at Location B, making it impossible to observe. Specific attention has been paid to the observation of rupture initiation of FRP hoop wraps in a number of FRP confined column experiments performed at the Hong Kong Polytechnic University (Li et
al. 2011; Hu et al. 2011) and the University of Edinburgh (Bisby et al. 2011), and it has been observed that FRP rupture initiates near one of the ends of the FRP for about 40% of specimens tested. It therefore appears likely that the modelled stress concentrations play an important role in initiating FRP rupture.

Figure 4.7 FRP circumferential strain distribution along the inner and outer surfaces of the FRP wrap
Using a simple hoop model to design the confinement does not predict the additional strains that occur due to the local discontinuities. In the example shown in Figure 4.7, the maximum tensile strain at Location A is about 30% higher than the strain outside the overlap zone (Figure 4.7b). Brittle failure of the FRP composite is expected to occur when its ultimate strain capacity is reached at Location A. Consequently, the strain remote from the overlap is about 2/3 of the ultimate tensile strain when failure occurs. This value is well within the range of experimental observations quoted in the literature.

It is noteworthy that traditional strain measurement techniques (such as electrical resistance strain gauges) are unable to detect the peak strain with confidence, as it occurs usually over a very short (sub-millimetre) length (Bisby and Take 2010; Chen et al. 2010). This explains why the experimentally observed value of hoop strain efficiency is less than 1.0 in the majority of tests.

It may be noted that an analytical solution for evaluating the stress concentrations near the ends of the FRP jacket has recently been presented by Zinno et al. (2010). Due to the limitation of analytical model, the plasticity of adhesive and the bending deformations are neglected in the solution, but these can be examined in FE analyses (Chen et al. 2010; Li et al. 2011).

4.3.1.2 FRP overlap region

The strains measured within the FRP overlap region are usually much lower than elsewhere. As noted by Lam and Teng (2004), if these strain readings are included together with those measured away from the overlap region in calculating the average
hoop strain, the calculated FRP ‘rupture’ hoop strain at failure will clearly be lower than the actual value, as shown in Figure 4.7a.

4.3.1.3 Geometrical imperfections

All test specimens are imperfect, both in terms of slight non-circularities and in terms of surface deformities. If the imperfections are large, leading to significant changes of local curvature, they may result in localized bending of the FRP and therefore large local tensile strains, as illustrated in Figure 4.8. This can reduce the apparent FRP rupture strain when it is measured away from the position where local maximum tensile strain occurs. No research is currently available examining this issue. It should be noted that if geometrical imperfections occur near either ends of FRP, it may also contribute to increased risk of FRP rupture near the ends of the wrap.

Figure 4.8 Geometrical imperfection
4.3.1.4 Curvature of the FRP jacket

The curvature of the FRP wrap has been identified as a possible contributory factor to the lower apparent rupture strength by Lam and Teng (2004). If the FRP jacket is formed by rolling from a pre-fabricated flat FRP sheet or plate, the rolling process produces a uniform bending in the jacket and this can be easily quantified because the change of the curvature is known. Assumed that in a prefabricated plate with a thickness of \( t_f \) (Figure 4.9a), the maximum bending strain in the plate caused by the change of curvature is \( t_f/(2R) \). If the jacket is formed in a wet lay-up process, it is more difficult to quantify the effect of curvature. However, if the fibres are all aligned neatly around the circumference of the column, the initial bending due to rolling is minimal because of the small thickness (diameter) of fibres (Figure 4.9b). This factor reduces the general ultimate strain of FRP, but does not cause any strain localization. Thus this factor is not reflected in the failure mode.

For an example column with a radius \( R = 150 \text{ mm} \), a prefabricated FRP plate with \( t_f=1 \text{ mm} \) can experience a bending strain \( \varepsilon = 0.33\% \); if a wet-lay up sheet is used with a typical fibre diameter in the range of 5 to 25 \( \mu\text{m} \), the bending strain will be in the range of \( \varepsilon = 0.0017\% \) to 0.0085\%. It is obvious that the wet-lay up sheets, which are much more widely used for strengthening concrete columns, have much smaller initial bending strains than those occur using prefabricated plates. Noting that the ultimate strain of the fibres is usually in the range of 1.5\% to 2.0\%, the strain due to the curvature of the columns when using wet lay-up sheets is thus insignificant.
4.3.2 FRP material factors

4.3.2.1 Fibre orientation
FRP composites applied in column wrapping are usually uni-directional so that the material is orthotropic and its properties are highly sensitive to the fibre orientation. Fibres are usually designed to be oriented in the circumferential direction and this is commonly assumed in design calculations. However, the fibres may sometimes be accidentally or intentionally aligned along a direction with a certain angle to the hoop direction, as illustrated in Figure 4.10. If the fibre orientation is not in a pure hoop direction, the hoop strain, and more importantly the hoop strength at FRP rupture, will be markedly different from those obtained from flat coupon tests in the fibre direction (Li et al. 2006).

Several studies have been conducted to experimentally investigate FRP tube encased concrete columns (e.g. Mirmiran and Shahawy 1997, Fam and Rizkalla 2001, Pessiki et
al. 2001) and FRP wrapped concrete columns (e.g. Au and Buyukozturk. 2005; Li et al. 2006) using non-hoop direction fibres. Pessiki et al. (2001) presented test results with both non-hoop direction and hoop direction fibres. The results show that the non-hoop direction fibres result in lower elastic modulus and strength of FRP composites, therefore, reduce the compressive strength of concrete-filled FRP tubes. However, the strains at rupture in the FRP with non-hoop direction fibres are larger than with hoop direction fibres. Au and Buyukozturk (2005) also investigated the effect of fibre orientations on FRP wrapped concrete columns. All the elastic modulus, ultimate tensile strain and strength of angular fibre jackets were found to be lower than that of non-angular jackets, based on the uni-axis properties from the manufacturer. Furthermore, the experiments showed that angular fibre jackets were less efficient in strength enhancement of concrete columns than the non-angular jackets. However, the former tends to yield a more ductile failure, because its distinct fibre reorientation mechanism is able to absorb more energy. Three off-axis GFRP laminates were tested using flat coupons in Li et al. (2006). The angles were 45 and 90 degrees respectively. The results showed that the elastic modulus, ultimate tensile strain and strength of angular fibre laminates were significantly lower than those of non-angular fibres. Additionally, the three properties of 90 degree angle-ply laminates were smaller than those of 45 degree laminates. Similar to previous studies, the compressive strength of concrete columns strengthened with angle-ply laminates were lower than that of hoop-direction fibre laminates.
4.3.2.2 Misalignment and uneven tension of fibres

In wet lay-up systems, if the fibres are not aligned properly so that some of the fibres are curved with different local curvatures to the column, the strength of the FRP is reduced because the fibres are not stressed uniformly even if everything else is axisymmetrical. However, this should not lead to any reduction in the strain capacity of the FRP wrap: indeed the FRP strain capacity should be increased because the forces resisted by initially more stressed fibres can be gradually transferred to the initially relaxed fibres after the rupture of the former, leading to a nonlinear behaviour of the FRP. The effect is often related to uneven tension of the FRP sheet during the wet-lay up process. This factor reduces the effectiveness of FRP confinement but not the FRP rupture strain capacity.

![Figure 4.10 Effect of fibre orientation on strain efficiency](image-url)
4.3.2.3 Damage of fibres

Any damage to the fibres during installation can obviously reduce the ultimate strength (although not necessarily the strain capacity) of the FRP. The effects of misalignment, uneven tension and damage may be minimised by proper design and careful application of the FRP wraps. It should be noted that damage, misalignment and uneven tension of fibres may also occur in an FRP coupon specimen which is used to define the coupon failure strain. However, as indicated by Xiao and Wu (2000), the more complex process of making a cylindrical shaped jacket on an FRP wrapped column (which is applied in the field) than flat coupons (which is usually conducted in a laboratory environment) leads to a lower quality of FRP composites in jackets than that in coupons.
4.3.2.4 Three dimensional stress state in the FRP

The use of a simple, uniform ring model in design calculations for FRP wrapped columns implies that the FRP ring is under uni-directional pure tension in the circumferential (hoop) direction, neglecting any stresses in the radial and axial directions. In a real column, the FRP is under a three dimensional (triaxial) stress state, as shown in Figure 4.13. It is clearly under tension in the hoop direction when the column expands laterally. The development of the lateral confinement pressure means that the FRP wrap is under an internal pressure, resulting in a radial stress with its value varying from the internal pressure on the inner surface to zero on the outer surface. The FRP is also under axial compression when the column is loaded axially in compression, assuming that it is bonded to the concrete so that axial force transfer can occur. It is reasonable in most cases to assume that the FRP deforms compatibly with the column in the longitudinal direction; even if there is no bond, the friction developed between them should normally
be sufficient to ensure this. Various shear stresses shall also exist in the FRP when non-uniform deformation due to various factors are considered.

The failure of an FRP composite in a complex triaxial stress state is different from that in a simple plane stress state, and the effects can be significant. Various failure criteria for materials which are orthotropic at a macroscopic level are available in the literature (Yu et al. 2009b), including the maximum stress criterion, the maximum strain criterion, the Tsai-Hill criterion, the Tsai-Wu criterion, etc. Two main failure modes are normally considered at a macroscopic level: (1) failure inside individual laminae (e.g. FRP rupture as described in Section 2), and (2) failure between adjacent laminae (e.g. delamination and debonding as discussed in Section 2). Fraldi et al. (2008) investigated the ultimate condition of FRP composites using the Tsai-Hill failure criterion, considering a two-dimensional stress state (ignoring the vertical stress $\sigma_z$). The results show that the ultimate condition is largely independent of the radial strength of the FRP. Lu et al. (2006) presented a model, also based on the Tsai-Hill failure criterion, considering a two-dimensional stress state with the radius stress $\sigma_r$ neglected. The results show that this model can predict the ultimate strain reasonably well for their own set of test data.

The hoop strain measured on the external surface of the specimen is lower than the strain in the innermost surface of FRP wrap, since the inner surface of FRP is affected by the internal pressure (De Lorenzis and Tepfers 2003). The difference between the inner and
outer surface of FRP could be non-negligible in concrete encased FRP tubes due to the relatively large thickness of FRP, compared to FRP wrapped specimens.

![Diagram of three dimensional stress state in the FRP on an FRP wrapped concrete column](image)

**Figure 4.13 Diagram of three dimensional stress state in the FRP on an FRP wrapped concrete column**

### 4.3.3 Concrete material factors

#### 4.3.3.1 Non-uniform deformation in the concrete

Available design and analysis models assume that the deformation of an FRP wrapped circular concrete column under axial compression is axisymmetric and uniform. However, concrete is a heterogeneous material; the stiffness of the coarse aggregate is usually much higher than that of the cement paste. Consequently, local deformation must be non-uniform even when the column is axisymmetric and the loading is axial and uniform, as illustrated in Figure 4.14. This non-uniform local deformation of concrete leads to a non-uniform strain distribution in the FRP in both the circumferential and axial directions. This may explain the phenomenon observed in practical tests that the
measured strain outside the overlap region has a large variability, even when the loading is significantly below the ultimate load (Bisby and Take 2009, Bisby et al. 2011).

![Figure 4.14 FRP strain localisation due to non-uniform deformation of concrete](image)

**Figure 4.14 FRP strain localisation due to non-uniform deformation of concrete**

### 4.3.3.2 Strain localisation in the concrete

At loads approaching failure, cracks start to appear in the FRP confined concrete, as shown in Figure 4.15. Strain localisation occurs when concrete cracks, because in a continuum sense the strain at the crack location must be infinite in order to develop a crack of finite width. Because the FRP is usually bonded to the concrete, increased local strains are expected in the FRP, often accompanied by local softening and debonding at the FRP to concrete interface. These strains are needed to accommodate the development of the concrete cracks. Being a heterogeneous material with random features, concrete cracks can occur at different places, both around the circumference (Figure 4.15a) and along the height (Figure 4.15b), leading to a non-uniform strain distribution in the FRP in both directions. Recent experimental as well as numerical
studies have confirmed the existence of such non-uniform strain distributions (Bisby and Take 2009; Bisby et al. 2011; Haskett et al. 2010; Haskett et al. 2011).

(a) Cross view of cracked concrete

(b) Concrete cracking due to the formation of shear failure planes (Haskett et al. 2011)

Figure 4.15 FRP strain localisation due to concrete cracking
4.3.4 Adhesive material and geometry factors

4.3.4.1 Mechanical properties of the adhesive

Because the FRP jacket is usually bonded to the concrete column with an adhesive, the properties of the adhesive may affect the rupture strain of the FRP. There are two main issues to investigate here: (1) the bond between the FRP wrap and the concrete column and (2) the bond between the FRP layers within the overlap region. Harries and Carey (2003) conducted a study on circular concrete columns with bonded and unbonded FRP jackets. It was concluded that the initial gap between the concrete column and the FRP jacket in the unbonded specimens does not affect the FRP rupture strain although it reduced the maximum attainable concrete strength (thus the confinement effectiveness).

Figure 4.16a shows an FRP wrap under an internal pressure. The stress-strain curve of adhesive may be significantly different with each other (Figure 4.16b). Figure 4.16c shows the FE predicted load-FRP maximum strain curve at Location A for the loading case shown in Figure 4.16a, with the adhesive being either linear elastic or linear elastic perfectly plastic. Clearly the properties of the adhesive can have a significant effect on the local strain distributions near the outer end of the FRP. Further details of the FE model can be found in Chapter 6.
(a) Geometry of the FRP in an FRP wrapped column

(b) Stress-strain relationship of different adhesives

(c) Load-FRP maximum strain curve at Location A with different adhesive properties

Figure 4.16 Effect of adhesive properties
4.3.4.2 Geometrical details of the adhesive

Two geometrical factors are potentially important here: geometrical details of adhesive at the outer end of FRP and the thickness of the adhesive layer. Figure 4.17 shows several different adhesive geometries at the outer end of the FRP. These differences may significantly influence the strain concentration at the outer end of FRP, and thus the local strain concentration on the FRP layer under it. Several studies have been conducted on the effect of adhesive geometry on the bond stress in FRP strengthened metallic beams (e.g. Deng et al. 2004; Deng 2005; Stratford and Chen 2005), but no study has been reported on FRP wrapped columns to the best knowledge of the author.
In a wet lay-up process of forming an FRP jacket from a dry fibre sheet, the impregnating resin (i.e. the adhesive) saturates the fibres through the sheet to form an FRP layer. In this case it is rather difficult to determine the actual thickness of adhesive between the FRP layers. The effect of adhesive is further discussed in this thesis.

### 4.3.4.3 Non-uniform bonding and partial debonding of the FRP

Büyüköztürk and Yu (2009) investigated the effect of artificial debonding of FRP from the concrete, and found that the effect can be significant. If the FRP is partially debonded, as shown schematically in Fig. 18, local bending effect at the fronts of the debonding crack can be experienced in the FRP. If FRP rupture occurs at these locations, the apparent FRP strain measured elsewhere at failure can be lower. Partial bonding or non-uniform bonding of the FRP wrap to the column will have a similar effect.

**Figure 4.18 Schematic diagram of non-uniform bonding and/or partial debonding of the FRP**


4.3.5 Loading factors

4.3.5.1 Loading eccentricity

If a notionally axisymmetric specimen is non-axisymmetrically loaded, which is essentially impossible to avoid in practice, the column specimen is subjected to an eccentric compressive load instead of a perfectly axial compressive load as is assumed in simple design calculations (and even in most advanced numerical modelling). In practice the column is also non-axisymmetric because of the existence of the overlap region of FRP even if the concrete column can be considered axisymmetric. The loading eccentricity (Figure 4.19a) leads to a non-uniform strain distribution in both the concrete and the FRP around the circumference as shown in Figure 4.19b (Ranger and Bisby 2007; Fitzwilliam and Bisby 2010). The FRP confinement to the concrete is consequently reduced. This effect can also lead to a lower apparent FRP hoop strain at failure when it is averaged from point strain observations made with bonded foil gauges.

(a) Schematic diagram of eccentric loading
(b) FRP hoop strain distribution in the FRP for the case of an eccentrically loaded column (after Fitzwilliam and Bisby 2010)

Figure 4.19 FRP wrapped column subject to eccentric axial compression loading

4.3.5.2 Non-uniform loading

Any non-axisymmetric and non-uniform contact at the supports would lead to non-uniform strain distribution in the FRP remote from the supports, similar to the effect of loading eccentricity. Furthermore, premature local failure can occur near the supports due to stress concentration, leading to lower hoop strain at the ultimate state.

Figure 4.20 Schematic diagram of non-uniform support in FRP wrapped column
4.3.5.3 Frictional confinement at the supports

For short column specimens, the restraint at their two ends due to friction between the column ends and the loading platens can lead to a highly non-uniform strain distribution along the height, as shown in Figure 4.21. This may have two effects. First, the end restraint may enhance the loading capacity compared with a longer specimen (even before the effect of slenderness becomes important). The variation of hoop strains over the height has been confirmed by Bisby and Take (2009), using an optical strain measurement technique based on digital image correlation. Second, as experimentally observed by Bisby and Stratford (2010), the restraints result in smaller strains in most parts of the column, leading to smaller strain observations if they are not directly measured at the height with maximum deformation, which may be at the mid-height but can vary and can be very unpredictable. The maximum deformation may also occur at different locations around the circumference. These effects also exist in long columns, however the affected zones are small relative to the length of the column.

![Figure 4.21 FRP hoop strain variability (schematized) over the height in FRP wrapped column](image-url)
4.3.5.4 Stressing due to thermal deformation and creep

Stressing (or relaxation) of the FRP jacket may arise due to factors such as creep of the concrete under axial loading and differential thermal deformation between the concrete and the FRP. If these are tensile but not measured in a test, the apparent strain is reduced by the same amount. As illustrated in Figure 4.22, an increase of temperature $\Delta T$ produces a stress in the FRP $\varepsilon_{fT}$ due to the different thermal properties of concrete and FRP

$$\varepsilon_{fT} = (\alpha_c - \alpha_f)\Delta T$$

(4.1)

in which $\alpha_c$ and $\alpha_f$ are respectively the coefficient of thermal expansion for concrete and FRP. Note that this thermal effect is likely to be greater for CFRP than for GFRP since carbon has a very low (or slightly negative) coefficient of thermal expansion, whereas glass has a coefficient of thermal expansion in the range of concrete.

Figure 4.22 Unintentional stressing in FRP due to differential thermal expansion between the FRP and the concrete
4.4 Future research

Although a number of studies have been conducted on the effects of some of the factors identified above, they are far from sufficient to quantify their effects. The effects of some of the factors are also extremely complex and many challenges remain to quantify them. Furthermore, many of these factors are likely to exist concurrently, making isolating and understanding them a very difficult task indeed. The interaction between the various factors can only make the matter more complex. A significant amount of further research is required in order to better understand the behaviour of FRP wrapped concrete columns and to quantify the apparent FRP rupture strain.

4.5 Conclusions

FRP wrapping has been widely adopted as an effective technique for enhancing both the strength and ductility of circular columns worldwide. However, the measured apparent (average) circumferential (hoop) strain in FRP at failure is usually much lower than the rupture strain obtained from flat coupon tensile tests. Although several factors contributing to this condition have been proposed in the literature, the examinations have been fragmented and no rational model exists to quantify their effects. This chapter represents an attempt to systematically examine the factors affecting this ultimate condition based on an extensive literature survey.

The chapter first classified the failure modes of FRP wrapped circular concrete columns under uniaxial compression, because factors affecting the ultimate condition are failure
mode dependent. There are mainly two types of failure modes: FRP rupture and debonding failure at the overlap zone. The former is divided into three sub-types: rupture near the outer end of FRP, rupture near the inner end of FRP and rupture outside the overlap region. Mixed debonding and rupture failure has also been observed, but it is difficult in test to determine which of these occurs first because of the explosive nature of failure. Most failures start near the mid height for short test columns, but failure near the ends is possible and has been observed. Only factors affecting FRP rupture failure are examined because FRP debonding failure can be avoided by 1) using a sufficient overlap region size, 2) appropriate adhesive and 3) carefully following an appropriate installation procedure.

For FRP rupture failure, this chapter has highlighted the importance of recognising the fact that most FRP composites used in construction are linear elastic brittle so rupture will occur when the most highly stressed fibre reaches its rupture strain because the materials do not allow any stress re-distribution due to a lack of plasticity. The force resisted by the ruptured fibre has to be transferred to its neighbouring fibres which may in turn rupture, leading to the rupture of the FRP jacket and failure of the concrete column. Therefore, it is crucially important to identify the highest tensile strain in the FRP due to either local stress concentrations or more general non-uniform strain distribution. Because failure can occur when the most highly stressed fibre reaches its rupture strain, the average hoop strain in an FRP wrapped circular column is clearly
lower. This explains the low apparent hoop strain observed in tests using isolated bonded foil strain gauges.

Seventeen contributory factors possibly affecting the apparent FRP rupture strain in FRP wrapped columns, ranging from geometrical discontinuity, triaxial stress states, geometrical imperfections and non-uniform supports in test setup, have been identified. Many of these are first identified in this study. These factors have been divided into five categories and their potential effects on the ultimate condition are discussed. The effects of many of these factors have not been quantified to date, such as the geometrical imperfections, fibre orientations, misalignment and uneven tension of fibres, 3D stress state in the FRP. Furthermore, the interaction between them makes the issue even more challenging. It is therefore concluded that a significant amount of further research is required in order to better understand the behaviour of FRP wrapped concrete columns and to quantify the apparent FRP rupture strain.
5.1 Introduction

As mentioned in Chapter 4, several causes, proposed by a number of studies, possibly contribute to the lower apparent ultimate circumferential tensile strain than the rupture strain obtained from straight coupon tensile test. Among these factors, the triaxial stress state of FRP may be a very important one. The use of the simple ring model in design calculations implies that the FRP ring is under uni-directional pure tension in the circumferential direction, neglecting any stress in the radial and axial directions. In a real column, the FRP is under a triaxial stress state. It is clearly under tension in the circumferential direction when the column expands laterally. The development of the lateral confinement pressure means that the FRP wrap is under an internal pressure, resulting in a radial stress with its value varying from the internal pressure on the inner surface to zero on the outer surface. The FRP is also under axial compression when the column is loaded axially in compression. It should be reasonable to assume that the FRP deforms compatibly with the concrete column in the longitudinal direction: even if there is no bond between them the friction developed between them should normally be sufficient to ensure this. The failure of an FRP composite in a complex triaxial stress state is different from that in a simple plane stress state (Yu et al. 2009b). However, only a very few studies are available in literature on the effect of triaxial stress state on FRP strain efficiency (e.g. Fraldi et al. 2008, Lignola et al. 2008b; Lignola et al. 2009).
This chapter investigates the effect of triaxial stress state on the FRP strain efficiency on FRP-confined concrete columns. FRP composite is considered as a transverse isotropic material. A procedure to determine the strain reduction factor is developed and various failure criteria are considered. Finally, a parametric study is conducted to investigate how the parameters affect the strain reduction factor.

5.2 Stresses in FRP

Common FRP composites used in structural strengthening are orthotropic. For simplicity, the FRP composites are treated as transverse isotropic material in this study. Their mechanical properties depend on the fibre orientation and distribution and the relative proportions of fibre and matrix. Their macro properties may be estimated from the fibre and adhesive properties using various methods (e.g. Vinson and Sierakowski 2002). In this study, the matrix is assumed to have the same properties as the adhesive. Thus, the properties of the FRP composites can be deduced from the fibre and adhesive properties and fibre volume ratio based on micromechanics approaches reviewed in Chapter 2.

5.2.1 3-D stress state

In a real column, the FRP is under a triaxial stress state, as shown in Figure 4.13. There are five elastic constants in transverse isotropic constitutive equations, including the Young's modulus and Poisson’s ratio in the \( r-z \) symmetry plane, \( E_p \) and \( \nu_p \), the Young's modulus \( E_\theta \), Poisson’s ratio in the \( p-\theta \) plane, \( \nu_p\theta \), and the shear modulus in \( p-\theta \) plane \( G_{p\theta} \) (Vinson and Sierakowski 2002). Here \( p \) represents both the \( r \) and \( z \) directions, which are assumed to have the same properties in this transverse isotropic material.
The stress-strain relationship takes the form as follow (Jones 1999):

\[
\begin{bmatrix}
\sigma_{fr} \\
\sigma_{fz} \\
\sigma_{f\theta} \\
\tau_{f\theta} \\
\tau_{f\theta} \\
\tau_{frz}
\end{bmatrix} =
\begin{bmatrix}
1 - \nu_{p\theta} \nu_{\phi p} & \nu_{p} + \nu_{\phi p} \nu_{\phi p} & \nu_{\phi p} + \nu_{p} \nu_{\phi p} & 0 & 0 & 0 \\
\frac{E_{p}}{E_{\theta}} \Delta & \frac{E_{p}}{E_{\phi}} \Delta & \frac{E_{p}}{E_{\phi}} \Delta & 0 & 0 & 0 \\
\nu_{p} + \nu_{\phi p} \nu_{\phi p} & 1 - \nu_{\phi p} \nu_{\phi p} & \nu_{\phi p} + \nu_{p} \nu_{\phi p} & 0 & 0 & 0 \\
\nu_{p\theta} (1 + \nu_{p}) & \frac{E_{p}}{E_{\theta}} \Delta & \frac{E_{p}}{E_{\phi}} \Delta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{E_{p}}{1 + \nu_{p}}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{p} \\
\varepsilon_{fz} \\
\varepsilon_{f\theta} \\
\varepsilon_{f\theta} \\
\varepsilon_{f\theta} \\
\varepsilon_{frz}
\end{bmatrix}
\]

(5.1)

Where

\[
\Delta = \frac{(1 + \nu_{p})(1 - \nu_{p} - 2\nu_{p\theta} \nu_{\phi p})}{E_{p}^{2} E_{\theta}}
\]

(5.2)

in which, \(\sigma_{fr}, \sigma_{fz}\) and \(\sigma_{f\theta}\) are the normal stresses of FRP in \(r, z\) and \(\theta\) directions respectively; \(\tau_{f\theta}, \tau_{f\theta}\) and \(\tau_{frz}\) are the shear stresses of FRP in \(z-\theta, \theta-r\) and \(r-z\) planes respectively; \(\varepsilon_{fr}, \varepsilon_{fz}\) and \(\varepsilon_{f\theta}\) are the normal strains of FRP in \(r, z\) and \(\theta\) directions respectively; and \(\varepsilon_{f\theta}, \varepsilon_{f\theta}\) and \(\varepsilon_{frz}\) are the shear strains of FRP in \(z-\theta, \theta-r\) and \(r-z\) planes respectively.

Based on the compliance matrix (Jones, 1999),

\[
\frac{\nu_{p\theta}}{E_{p}} = \frac{\nu_{\phi p}}{E_{\phi}}
\]

(5.3)

the following relationships hold in the matrix (Jones, 1999)
Assuming that the FRP confined column is axis-symmetrical, all the shear strains can be considered to be zero. Therefore, all the shear stresses are also zero and the normal stress components can be obtained from Eq 5.1 as:

\[
\begin{align*}
\sigma_{fr} &= \frac{1 - V_{p\theta} V_{\phi}}{E_p E_\theta \Delta} \varepsilon_{fr} + \frac{V_p + V_{\phi} V_{\theta}}{E_p E_\theta \Delta} \varepsilon_{fz} + \frac{V_{\phi} + V_p V_{\theta}}{E_p E_\theta \Delta} \varepsilon_{f\theta} \\
\sigma_{fz} &= \frac{V_p + V_{\phi} V_{\theta}}{E_\theta E_p \Delta} \varepsilon_{fr} + \frac{1 - V_{\phi} V_{p\theta}}{E_\theta E_p \Delta} \varepsilon_{fz} + \frac{V_{\phi} + V_{\theta} V_p}{E_\theta E_p \Delta} \varepsilon_{f\theta} \\
\sigma_{f\theta} &= \frac{V_{p\theta} + V_{\phi} V_{p\theta}}{E_p \Delta} \varepsilon_{fr} + \frac{V_{p\theta} (1 + V_p)}{E_p \Delta} \varepsilon_{fz} + \frac{1 - V_p^2}{E_p \Delta} \varepsilon_{f\theta}
\end{align*}
\]

There has been extensive research on FRP confined concrete columns. Various models for FRP confined concrete have been developed, resulting in accurate predictions of the axial stress versus axial strain and hoop strain behaviour of FRP confined concrete columns. In this study, the analysis-oriented stress-strain model developed by Jiang and Teng (2007) as reviewed in Chapter 2 is adopted for predicting the effect of triaxial stress state on FRP failure in FRP confined circular concrete columns.

The confining pressure on concrete core supplied by the FRP jacket is given by
\[ \sigma_f = \frac{-\sigma_{\theta}t}{R} \quad (5.6) \]

Through the thickness of FRP composites, the radial stress \( \sigma_{\theta} \) varies between the confining pressure \( \sigma_t \) at the inner surface and zero at the outer surface, so the largest radial stress (which is at the inner surface) is

\[ \sigma_{\theta i} = \frac{-\sigma_{\theta}t}{R} \quad (5.7) \]

Jiang and Teng’s (2007) lateral strain-axial strain relationship is given by Eq. 2.27. Assuming deformation compatibility between the FRP and concrete, the axial and lateral strains of FRP, \( \varepsilon_{fz} \) and \( \varepsilon_{f\theta} \), are equal to the axial and lateral strains of concrete, \( \varepsilon_{cz} \) and \( \varepsilon_{c\theta} \), respectively. Therefore, for a given FRP circumferential strain \( \varepsilon_{f\theta} \), the axial strain of FRP composites \( \varepsilon_{fz} \) can be found using Eq. 2.27.

By determining \( \sigma_{fz} \) and \( \sigma_{\theta} \) by substituting \( \varepsilon_{\theta} \) from Eq. 2.27 into Eq. 5.5, the radial strain at inner surface of FRP \( \varepsilon_{f\theta} \) can be found by substituting Eq. 5.7 to Eq. 5.5a, c:

\[
\left( \frac{t}{R} \times \frac{V_{p\theta} + V_{p}V_{p\theta}}{E_{p}^{2}A} + \frac{1 - V_{p\theta}V_{p}}{E_{p}E_{\theta}A} \right) \varepsilon_{fz} + \left( \frac{t}{R} \times \frac{V_{p\theta} + V_{p\theta}V_{p} + V_{p} + V_{p\theta}V_{p\theta}}{E_{p}^{2}A} \right) \varepsilon_{f\theta} \\
+ \left( \frac{t}{R} \times \frac{1 - V_{p}^{2}}{E_{p}^{2}A} + \frac{V_{p\theta} + V_{p}V_{p\theta}}{E_{p}E_{\theta}A} \right) \varepsilon_{f\theta} = 0
\quad (5.8)
5.2.2 Simplified 2-D stress states

In the literature, a two dimensional stress state is commonly assumed by neglecting the stress in either \( r \) or \( z \) direction (e.g. Fraldi et al. 2008), giving the following two 2D stress states.

**2-D stress state in \( z-\theta \) plane**

If the radius stress \( \sigma_r \) is assumed to be 0, which is the stress state at the outer surface of the FRP wrap, the stress-strain relationship takes the following form:

\[
\begin{bmatrix}
\sigma_z \\
\sigma_{\theta} \\
\tau_{z\theta}
\end{bmatrix} =
\begin{bmatrix}
E_p & \nu_{\phi}E_p & 0 \\
1-\nu_{\phi}\nu_{\theta} & 1-\nu_{\phi}\nu_{\theta} & 0 \\
\nu_{\theta}E_{\theta} & \nu_{\theta}E_{\theta} & 0 \\
1-\nu_{\phi}\nu_{\theta} & 1-\nu_{\phi}\nu_{\theta} & 0 \\
0 & 0 & 2G_{\phi\theta}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_z \\
\varepsilon_{\theta} \\
\varepsilon_{z\theta}
\end{bmatrix}
\] (5.9)

As mentioned previously, the shear strain is considered to be zero due to axisymmetry. Thus the stress components can be expressed as:

\[
\begin{cases}
\sigma_z = \frac{E_p}{1-\nu_{\phi}\nu_{\theta}} \varepsilon_z + \frac{\nu_{\theta}E_p}{1-\nu_{\phi}\nu_{\phi}} \varepsilon_{\theta} \\
\sigma_{\theta} = \frac{\nu_{\theta}E_{\theta}}{1-\nu_{\phi}\nu_{\theta}} \varepsilon_z + \frac{E_{\theta}}{1-\nu_{\phi}\nu_{\phi}} \varepsilon_{\theta}
\end{cases}
\] (5.10)

For a given value, \( \sigma_z \) and \( \sigma_{\theta} \) can be obtained by substituting Eq. 2.27 into Eq. 5.10.

**2-D stress state in \( r-\theta \) plane**
If the axial normal stress is ignored, i.e. $\sigma_z = 0$, and assuming that $\sigma_{fr}$ can be calculated using Eq. 5.7, the compliance matrix takes the form as following,

$$
\begin{bmatrix}
\varepsilon_{r}
\varepsilon_{\theta}
\varepsilon_{r\theta}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_p} & -\frac{v_{p\theta}}{E_p} & 0 \\
\frac{v_{p\theta}}{E_p} & \frac{1}{E_\theta} & 0 \\
0 & 0 & \frac{1}{2G_{p\theta}}
\end{bmatrix}
\begin{bmatrix}
\sigma_{fr}
\sigma_{f\theta}
\tau_{r\theta}
\end{bmatrix}
$$

(5.11)

The circumferential stress $\sigma_{f\theta}$ can be obtained by substituting Eq. 5.7 into Eq. 5.11:

$$
\sigma_{f\theta} = \frac{\varepsilon_{f\theta}}{1/E_\theta + v_{p\theta} t / E_p / R}
$$

(5.12)

### 5.3 Failure criteria of FRP

The failure criteria of composite materials for a single lamina have been studied by many researchers (e.g. Daniel and Ishai 2006; Yu et al. 2009b; Jones 1999). In this study, four commonly used failure criteria are used to examine the ultimate condition of FRP. They are the maximum stress theory, the maximum strain theory, the Tsai and Wu theory and the Tsai and Hill theory.

The strain efficiency considered the three stress state is defined as follows:

$$
\kappa = \frac{\varepsilon_{fr\theta}}{f_\theta / E_\theta}
$$

(5.13)
where $\varepsilon_{\theta\theta}$ is the ultimate tensile strain of FRP considering the effect of three dimensional stress state, $f_{\theta\theta}$, $E_{\theta\theta}$ are the circumferential strength and elastic modulus of the FRP.

### 5.3.1 Maximum stress theory

According to maximum stress theory, the stresses in the principal material directions must not exceed the corresponding strengths, otherwise fracture is to occur (Yu et al. 2009b). Assuming that the compressive strength equals to the tensile strength, this criterion can be described by the following equations:

\[
\sigma_{fr} = \begin{cases} 
+f_r & \text{for } \sigma_{fr} > 0 \\
-f_r & \text{for } \sigma_{fr} < 0
\end{cases}
\]  

(5.14)

\[
\sigma_{f\theta} = \begin{cases} 
+f_{\theta} & \text{for } \sigma_{f\theta} > 0 \\
-f_{\theta} & \text{for } \sigma_{f\theta} < 0
\end{cases}
\]  

(5.15)

\[
\sigma_{fz} = \begin{cases} 
+f_z & \text{for } \sigma_{fz} > 0 \\
-f_z & \text{for } \sigma_{fz} < 0
\end{cases}
\]  

(5.16)

\[
|\tau_{fr\theta}| = f_{r\theta}
\]  

(5.17)

\[
|\tau_{f\theta\theta}| = f_{\theta\theta}
\]  

(5.18)

\[
|\tau_{frz}| = f_{rz}
\]  

(5.19)

where $f_r$, $f_{\theta\theta}$ and $f_z$ are the radial, circumferential and axial strength of the FRP, respectively; $f_{r\theta}$, $f_{rz}$ and $f_{\theta\theta}$ are the shear strengths in z-\(\theta\), \(\theta-r\) and \(r-z\) planes respectively.
5.3.2 Maximum strain theory

According to the maximum strain theory, the strains in the principal material directions must not exceed the corresponding ultimate strain, otherwise fracture is said to have occurred. Assuming that the compressive ultimate strain is equal to the tensile ultimate strain, this criterion can be described by the following equations:

\[
\varepsilon_{fr} = \begin{cases} 
\varepsilon_r^u & \text{for } \varepsilon_{fr} > 0 \\
-\varepsilon_r^u & \text{for } \varepsilon_{fr} < 0
\end{cases}
\] (5.20)

\[
\varepsilon_{f\theta} = \begin{cases} 
\varepsilon_{\theta}^u & \text{for } \varepsilon_{f\theta} > 0 \\
-\varepsilon_{\theta}^u & \text{for } \varepsilon_{f\theta} < 0
\end{cases}
\] (5.21)

\[
\varepsilon_{fz} = \begin{cases} 
\varepsilon_z^u & \text{for } \varepsilon_{fz} > 0 \\
-\varepsilon_z^u & \text{for } \varepsilon_{fz} < 0
\end{cases}
\] (5.22)

\[
|\varepsilon_{fr\theta}| = \varepsilon_{r\theta}^u
\] (5.23)

\[
|\varepsilon_{f\theta\theta}| = \varepsilon_{\theta\theta}^u
\] (5.24)

\[
|\varepsilon_{fz}| = \varepsilon_{rz}^u
\] (5.25)

where \(\varepsilon_r^u, \varepsilon_{\theta}^u, \text{ and } \varepsilon_z^u\) are the radial, circumferential and axial ultimate strain of the FRP respectively; \(\varepsilon_{r\theta}^u, \varepsilon_{\theta\theta}^u, \text{ and } \varepsilon_{rz}^u\) are the ultimate shear strain of the FRP in \(z-\theta, \theta-r\) and \(r-z\) planes, respectively.

For a transverse isotropic material, Eqs 5.20 to 5.25 can be rewritten as:
\[
\sigma_{fr} - \nu_p \sigma_{\theta} - \nu_p \sigma_{f\theta} = \begin{cases} f_r & \text{for } \varepsilon_{fr} > 0 \\ -f_r & \text{for } \varepsilon_{fr} < 0 \end{cases}
\]
\begin{equation}
(5.26)
\end{equation}

\[
-\nu_p \sigma_{fr} - \nu_p \sigma_{f\theta} + \sigma_{f\theta} = \begin{cases} f_\theta & \text{for } \varepsilon_{f\theta} > 0 \\ -f_\theta & \text{for } \varepsilon_{f\theta} < 0 \end{cases}
\]
\begin{equation}
(5.27)
\end{equation}

\[
-\nu_p \sigma_{fr} + \sigma_{\theta} - \nu_p \sigma_{f\theta} = \begin{cases} f_z & \text{for } \varepsilon_z > 0 \\ -f_z & \text{for } \varepsilon_z < 0 \end{cases}
\]
\begin{equation}
(5.28)
\end{equation}

\[
|\tau_{ij}| = f_j \quad (i, j = r, \theta; \ i \neq j)
\]
\begin{equation}
(5.29)
\end{equation}

### 5.3.3 Tsai-Hill theory

The Tsai-Hill failure criterion (Tsai 1968) takes the following form in triaxial problems, assuming that the tensile and compressive strengths are the same:

\[
\left( \frac{\sigma_{fr}}{f_r} \right)^2 + \left( \frac{\sigma_{f\theta}}{f_\theta} \right)^2 + \left( \frac{\sigma_{\theta}}{f_z} \right)^2 + \left( \frac{\tau_{fr\theta}}{f_{r\theta}} \right)^2 + \left( \frac{\tau_{rz}}{f_{rz}} \right)^2 + \left( \frac{\tau_{\theta z}}{f_{\theta z}} \right)^2 - \sigma_{fr} \sigma_{f\theta} \left( \frac{1}{f_r^2} + \frac{1}{f_\theta^2} - \frac{1}{f_z^2} \right)
\]
\begin{equation}
(5.30)
\end{equation}

If only \( \sigma_{fr} \) and \( \sigma_{f\theta} \) are considered (i.e. \( \sigma_{\theta} = 0 \)), the Tsai-Hill failure criteria can be simplified as

\[
\left( \frac{\sigma_{fr}}{f_r} \right)^2 + \left( \frac{\sigma_{f\theta}}{f_\theta} \right)^2 + \left( \frac{\tau_{fr\theta}}{f_{r\theta}} \right)^2 - \frac{\sigma_{fr} \cdot \sigma_{f\theta}}{f_{\theta}^2} \leq 1
\]
\begin{equation}
(5.31)
\end{equation}

Substitute Eqs. (5.7), (5.12) and Eq. (5.31) into Eq. (5.13), the FRP strain efficiency \( \kappa \) can be found using following equation:
If only $\sigma_{fz}$ and $\sigma_{f\theta}$ are considered (i.e. $\sigma_{fr} = 0$), the Tsai-Hill failure criteria can be simplified as:

$$\left(\frac{\sigma_{fz}}{f_z}\right)^2 + \left(\frac{\sigma_{f\theta}}{f_{\theta}}\right)^2 + \left(\frac{\tau_{f\theta}}{f_{z\theta}}\right)^2 - \frac{\sigma_{fz} \cdot \sigma_{f\theta}}{f_{\theta}^2} \leq 1$$

(5.33)

The strain efficiency of FRP can be found using Eqs. (2.27), (5.10) and (5.33).

5.3.4 Tsai and Wu's theory

Based on various failure criteria, including the Tsai and Hill theory, Tsai and Wu developed a modified strength criterion, which was verified using experimental data from tubular specimens (Tsai and Wu 1971). Composite tubular specimens are believed to have several advantages over flat coupon specimens in determining the properties of composite materials. One of which is a tube can provide data under combined loading conditions. Because the angle-ply laminates tube is not subjected to the so called free
edge effects which cannot be avoided in an off-axis test of a flat coupon specimen (Tsai 1977). This failure criteria was found to be able to closely predict the failure of FRP composites by Al-Khalil et al. (1995), in which FRP tubular specimens were used. It was also recommended as one of the best available criteria for prediction of FRP lamina by Hinton et al. (2002) and Soden et al. (2004).

The Tsai-Wu failure criterion (Tsai and Wu 1971) for transversely isotropic composites takes the following form in 3-dimensional problem, assuming that tensile and compressive strengths are the same:

\[
\left( \frac{\sigma_{r}}{f_{r}} \right)^{2} + \left( \frac{\sigma_{\theta}}{f_{\theta}} \right)^{2} + \left( \frac{\sigma_{z}}{f_{z}} \right)^{2} + \left( \frac{\tau_{r\theta}}{f_{r\theta}} \right)^{2} + \left( \frac{\tau_{rz}}{f_{rz}} \right)^{2} + \left( \frac{\tau_{f\theta}}{f_{f\theta}} \right)^{2} + 2F_{r\theta}\sigma_{r}\sigma_{\theta} + 2F_{rz}\sigma_{r}\sigma_{z} + 2F_{f\theta}\sigma_{f\theta}\sigma_{fz} \leq 1
\]

where \( F_{r\theta}, F_{rz} \) and \( F_{f\theta} \) are parameters that can be determined using biaxial tests. It is suggested by Daniel and Ishai (2006), and applied in Fraldi et al. (2008), using the following equation:

\[
\begin{align*}
F_{r\theta} &= -\frac{1}{2} \sqrt{\frac{1}{f_{r}^{2} f_{\theta}^{2}}} \\
F_{rz} &= -\frac{1}{2} \sqrt{\frac{1}{f_{r}^{2} f_{z}^{2}}} \\
F_{f\theta} &= -\frac{1}{2} \sqrt{\frac{1}{f_{\theta}^{2} f_{z}^{2}}}
\end{align*}
\]
Since the interaction between the stress components is not considered in the maximum stress criterion, and considered only through the Poisson’s effect in the maximum strain criterion, the Tsai-Wu theory has obvious advantages compared to either the maximum stress criterion or the maximum strain criterion because of the consideration of the interaction between strengths in different directions. Furthermore, this theory is also believed to have advantages compared to the Tsai-Hill theory because of the additional terms in the equation, and the invariance of under rotation or redefinition of coordinates (Jones 1999).

If only $\sigma_{fr}$ and $\sigma_{f\theta}$ are considered (i.e. $\sigma_{f\varphi} = 0$), the Tsai-Wu failure criteria reduces to

$$
\left(\frac{\sigma_{fr}}{f_r}\right)^2 + \left(\frac{\sigma_{f\theta}}{f_\theta}\right)^2 + \left(\frac{\tau_{fr\theta}}{f_{r\theta}}\right)^2 - \frac{\sigma_{fr} \cdot \sigma_{f\theta}}{\sqrt{f_r^2 \cdot f_\theta^2}} \leq 1
$$

(5.36)

Substituting Eqs (2.27), (5.12) and (5.36) into Eq. (5.13) gives

$$
\kappa = \frac{R f_r}{\sqrt{t^2 f_\theta^2 + R^2 f_r^2 + t \cdot R \cdot f_r \cdot f_\theta}} \cdot \left(1 + \frac{E_\theta \cdot v_{p\theta} \cdot t}{E_p R}\right)
$$

(5.37)

If only $\sigma_{f\varphi}$ and $\sigma_{f\theta}$ are considered (i.e. $\sigma_{fr} = 0$), the Tsai-Wu failure criteria is reduced to

$$
\left(\frac{\sigma_{f\varphi}}{f_z}\right)^2 + \left(\frac{\sigma_{f\theta}}{f_\theta}\right)^2 + \left(\frac{\tau_{f\varphi\theta}}{f_{\varphi\theta}}\right)^2 - \frac{\sigma_{f\varphi} \cdot \sigma_{f\theta}}{\sqrt{f_z^2 \cdot f_\theta^2}} \leq 1
$$

(5.38)

The strain efficiency of FRP can be found from Eqs. (2.27), (5.10) and (5.38).
CHAPTER 5

5.4 Comparison with results from existing analysis and test

5.4.1 Comparison with Fraldi et al’s analysis

Fraldi et al. (2008) adopted the Tsai-Hill failure criterion by considering $\sigma_r$ and $\sigma_\theta$ in an FRP confined column to validate the criterion. The concrete column had a diameter of 152mm and unconfined concrete strength of 43.8 MPa. The FRP composites had: $E_r = E_z = 3$ GPa, $E_\theta = 105$ GPa, material strength are: $f_\theta = 1577$ MPa. They adopted four values for the transverse strengths: $f_z = f_r = 5, 13, 40$ and 80 MPa. The results from tests Fraldi et al.’s analysis, and the criteria described above are compared in Table 5.1.

Table 5.1 Comparison of failure criteria with Fraldi et al. (2008)

<table>
<thead>
<tr>
<th>Failure criteria</th>
<th>$f_z = f_r = 5$ MPa</th>
<th>$f_z = f_r = 13$ MPa</th>
<th>$f_z = f_r = 40$ MPa</th>
<th>$f_z = f_r = 80$ MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental result</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Results by Fraldi et al.</td>
<td>0.54</td>
<td>0.85</td>
<td>0.97</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_r = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tsai-Hill</td>
<td>0.54</td>
<td>0.85</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>Tsai-Wu</td>
<td>0.44</td>
<td>0.71</td>
<td>0.90</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma_r = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tsai-Hill</td>
<td>0.03</td>
<td>0.31</td>
<td>0.75</td>
<td>0.90</td>
</tr>
<tr>
<td>Tsai-Wu</td>
<td>0.03</td>
<td>0.26</td>
<td>0.62</td>
<td>0.79</td>
</tr>
<tr>
<td>3D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tsai-Hill</td>
<td>0.03</td>
<td>0.38</td>
<td>0.80</td>
<td>0.93</td>
</tr>
<tr>
<td>Tsai-Wu</td>
<td>0.03</td>
<td>0.25</td>
<td>0.60</td>
<td>0.76</td>
</tr>
<tr>
<td>Maximum stress</td>
<td>0.03</td>
<td>0.31</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Maximum strain</td>
<td>0.03</td>
<td>0.21</td>
<td>0.81</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In Table 5.1, the predictions from this analysis, using the Tsai-Hill criterion considering two dimensional stresses in radial and hoop directions, match those of Fraldi et al. (2008) using the same criterion, verifying each other, the slight difference when $f_z = 40$MPa may be caused by the round error. The table also shows that the one using the triaxial stress state, with the Tsai and Wu criterion provides most conservative FRP strain efficiency results in the case studied here. The result also shows that significant discrepancies exist
between predictions using different criteria, and also from different stress state. The simplified 2D stress state considering $\sigma_{rz}$ and $\sigma_{r\theta}$ has a much closer value to the 3D stress state than the one considering $\sigma_{r}$ and $\sigma_{\theta}$. It is also evident that the transverse strength of FRP can considerably affect the strain efficiency when 3D stress state is considered. This issue is examined further in this chapter.

5.4.2 Comparison with test results

5.4.2.1 Experiments

The predictions from the different failure criteria are also compared with the test results of six CFRP wrapped concrete columns conducted by Bisby et al. (2011). All the specimens were un-reinforced concrete cylinders of 100mm in diameter and 200mm in height. They were cast from a single batch of C25/30 ready-mix concrete with a maximum aggregate size of 10mm. The cylinder strength of the plain concrete was 31 MPa in average of two specimens in the same size as the FRP wrapped specimens. The columns were wrapped in the hoop direction with a single layer of SikaWrap Hex 230C unidirectional carbon fibre/epoxy FRP strengthening system. The manufacturer specified properties for the fibres had an ultimate tensile strength of 4100 MPa at a tensile strain of 1.7% with a nominal thickness of 0.12mm. The tensile modulus of elasticity was 231000 MPa. The resin was Sikadur 330 as suggested by the supplier, which had a tensile modulus of elasticity was 4.5 GPa and ultimate strain of 0.9%, giving a tensile strength of 40.5 MPa. The FRP was applied using a wet lay-up procedure as recommended by the manufacturer. The test results and predictions using Jiang and Teng (2007)’s model are shown in Table 5.2. The axial stress-strain curves and axial
strain-hoop strain curves are shown in Figure 5.1. It is shown that both Lam and Teng (2002)’s strength model and Jiang and Teng (2007)’s model conservatively predict the confined strength of concrete. Furthermore, the predicted ultimate axial strain using Jiang and Teng’s (2007) model is larger than the test results.

Table 5.2 Test results of Bisby et al. (2011)

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Confined strength $f'_{cc}$ (MPa)</th>
<th>Ultimate axial strain $\varepsilon_u$ (%)</th>
<th>Ultimate hoop strain $\varepsilon_{h,rup}$ (%)</th>
<th>Strain efficiency ($\varepsilon_{h,rup}/1.7%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen-1</td>
<td>55</td>
<td>0.97</td>
<td>1.08</td>
<td>0.64</td>
</tr>
<tr>
<td>Specimen-2</td>
<td>59</td>
<td>1.24</td>
<td>1.19</td>
<td>0.70</td>
</tr>
<tr>
<td>Specimen-3</td>
<td>57</td>
<td>1.24</td>
<td>1.23</td>
<td>0.72</td>
</tr>
<tr>
<td>Specimen-4</td>
<td>63</td>
<td>1.38</td>
<td>1.27</td>
<td>0.75</td>
</tr>
<tr>
<td>Specimen-5</td>
<td>61</td>
<td>1.33</td>
<td>1.11</td>
<td>0.65</td>
</tr>
<tr>
<td>Specimen-6</td>
<td>53</td>
<td>1.09</td>
<td>1.03</td>
<td>0.61</td>
</tr>
<tr>
<td>Average</td>
<td>58</td>
<td>1.20</td>
<td>1.15</td>
<td>0.68</td>
</tr>
<tr>
<td>Lam &amp; Teng</td>
<td>52.78</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Jiang &amp; Teng</td>
<td>52.14</td>
<td>1.58</td>
<td>1.15*</td>
<td>-</td>
</tr>
</tbody>
</table>

* Jiang and Teng’s prediction is based on the average $\varepsilon_{h,rup}$ from test.
Figure 5.1 Test results of FRP-wrapped concrete columns
5.4.2.2 Mechanical properties of CFRP composites

As stated in Chapter 2, the mechanical properties of FRP composites may be deduced based on the micromechanics of composites. Three methods are adopted to calculate the mechanical properties of the CFRP composite here. The nominal thickness of the CFRP was 0.12 mm. The actual thickness of the FRP composite and adhesive layer was measured using a microscope and their average values were 0.47 mm and 0.14 mm, respectively. The details of the measurement procedure can be found in Chapter 7. The volume ratio of fibres in the CFRP composites can be deduced from dividing the actual thickness of FRP into the nominal thickness of fibres, giving a value of 0.26.

Table 5.3 FRP Mechanical properties calculated using different methods

<table>
<thead>
<tr>
<th>Methods</th>
<th>Mechanical properties</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{\vartheta}$(GPa)</td>
<td>$E_{\varphi}$(GPa)</td>
<td>$G_{\vartheta\varphi}$(GPa)</td>
<td>$G_{\varphi z}$(GPa)</td>
<td>$\nu_{\varphi\vartheta}$</td>
<td>$\nu_{\varphi z}$</td>
</tr>
<tr>
<td>Isotropic fibres</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mechanics of materials</td>
<td>63.39</td>
<td>6.04</td>
<td>2.24</td>
<td>2.30</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>Halpin-Tsai</td>
<td>63.39</td>
<td>8.89</td>
<td>2.79</td>
<td>3.39</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>Hahn</td>
<td>63.39</td>
<td>6.75</td>
<td>2.78</td>
<td>2.59</td>
<td>0.31</td>
<td>0.30</td>
</tr>
<tr>
<td>$E_{\varphi} = 5% \times E_{\vartheta}$</td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Mechanics of materials</td>
<td>63.39</td>
<td>5.34</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Halpin-Tsai</td>
<td>63.39</td>
<td>5.81</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Hahn</td>
<td>63.39</td>
<td>5.10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

The properties of the FRP derived using the three methods reviewed in Chapter 2 are shown in Table 5.3. The $E_2$ and $E_3$ of carbon fibres are assumed to be 5% of $E_{\vartheta}$ in Table
5.3 when they are considered to be orthotropy, which is more realistic as stated in Chapter 2 that the carbon fibres are more likely to be an orthotropic material. It can be found from Table 5.3 that the different methods predict very similar mechanical properties of the FRP. Furthermore, the consideration of orthotropy of carbon fibres also only slightly reduces $E_p$. Since the orthotropic properties of fibres are not available, fibres are assumed isotropic in the rest of this study unless otherwise stated. The most recent micromechanics method of Hahn is applied for prediction of mechanical properties of FRP composites in this chapter.

5.4.2.3 Comparison of different failure criteria

Table 5.4 shows that the predicted strain efficiency based on the different failure criteria. Three cases with different transverse strengths are considered. The first one assumed that the $f_r$ and $f_z$ of FRP composites are the same as the adhesive’s compressive strength, 80 MPa, implying that the fibres are the same stress as the adhesive until the adhesive fails due to compression. The second one assumes the adhesive does not carry any load, and the fibres as isotropic materials. Therefore, the transverse strength of FRP can be found as: $f_\theta/t_f = 1002.64$ MPa. The third case is the transverse strength of FRP composites as the tensile strength of adhesive, 40.5 MPa. The results show that the strain efficiency highly depends on the transverse strength of FRP composites.
### Table 5.4 Strain efficiency predicted from different failure criteria

<table>
<thead>
<tr>
<th>Failure criteria</th>
<th>Strain efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( f_p = 80 \text{ MPa} )</td>
</tr>
<tr>
<td>Tsai-Hill ((\sigma_r \text{ and } \sigma_\theta))</td>
<td>0.99</td>
</tr>
<tr>
<td>Tsai-Hill ((\sigma_r \text{ and } \sigma_\theta))</td>
<td>0.56</td>
</tr>
<tr>
<td>Tsai-Hill ((3D))</td>
<td>0.58</td>
</tr>
<tr>
<td>Tsai-Wu ((\sigma_r \text{ and } \sigma_\theta))</td>
<td>0.94</td>
</tr>
<tr>
<td>Tsai-Wu ((\sigma_z \text{ and } \sigma_\theta))</td>
<td>0.47</td>
</tr>
<tr>
<td>Tsai-Wu ((3D))</td>
<td>0.47</td>
</tr>
<tr>
<td>Maximum stress</td>
<td>0.64</td>
</tr>
<tr>
<td>Maximum strain</td>
<td>0.53</td>
</tr>
<tr>
<td>Test</td>
<td></td>
</tr>
</tbody>
</table>

#### 5.5 Parametric study

The parametric study is conducted based on the reference example reported in Xiao and Wu (2000) in Section 5.4.1, used by Fraldi et al. (2008). Only one parameter is changed at a time unless otherwise stated.

#### 5.5.1 Strength in \( r-z \) plane

Figure 5.2 shows the effect of transverse FRP composite strength to the FRP strain efficiency. It is evident that the transverse strength of FRP composites has significant effect on the FRP strain efficiency, and a higher transverse strength leads to higher strain efficiency. Unfortunately, this parameter is not available in literature. Generally speaking, the adhesive strength is in the range of 30 MPa to 70 MPa. It is reasonable to
assume that the transverse strength of FRP composites is the same as the adhesive strength, since the transverse failure is usually dominant by the failure of adhesive and delaminating of fibres. The strength of 40 MPa is used as the reference case in the rest of the study.

![Diagram showing FRP strain efficiency and strength in the r-z plane](image)

**Figure 5.2 Effect of out-plane strength $f_p$ on FRP strain efficiency**

### 5.5.2 Concrete strength

Figure 5.3 shows the effect of unconfined concrete compressive strength on strain efficiency of FRP. It is clear that the FRP strain efficiency increases with the concrete strength, based on the prediction of Tsai-Wu and Tsai-Hill criteria. The predictions of Maximum strain and Maximum stress criteria do not depend on the concrete strength, in
the reference case considered here. However, the experiment results show the contrary trend. This may be attributed to other contributing factors for the strain efficiency of FRP, as discussed in Chapter 4. The test results are from Xiao and Wu (2000).

![Figure 5.3 Effect of concrete strength on FRP strain efficiency](image)

**5.5.3 The number of FRP layers**

The effect of number of FRP layers is shown in Figure 5.4. The predicted strain efficiency decreases with an increase of the number of FRP layers for the failure criteria. Again, the test results shown are from Xiao and Wu (2000). Tsai-Wu criterion considering the triaxial stress has the most conservative strain efficiency predictions. Meanwhile, because of the advantages of the Tsai-Wu criterion compared to the other criteria mentioned in Section 5.3.4, this criterion is believed to be more applicable to
failure prediction of composite materials. Therefore, this failure criterion is suggested to be used in the triaxial stress analysis. The inner surface of the FRP composites (i.e. $\sigma_r = \sigma_l$) is found to be more critical than the outer surface (i.e. $\sigma_r = 0$), if Tsai-Wu criterion is considered.

![Figure 5.4 Effect of number of FRP layers on strain efficiency](image)

### 5.6 Conclusions

This study presents an analytical solution for FRP strain efficiency in FRP confined circular concrete columns, considering the triaxial stress state of FRP. The following conclusions can be drawn from the results and discussions presented in the chapter:
a) The triaxial stress state does affect the ultimate condition of FRP composites in FRP confined concrete columns; the effect can be significant in certain aspects. Therefore, it is important to consider the triaxial stress state in the prediction of FRP strain efficiency. Significant discrepancies exist between predictions using different criteria, when all three stress components are considered.

b) The prediction of various failure criteria for triaxial stress state significantly depends on the material properties of FRP confined concrete columns, such as the transverse strength of FRP, unconfined concrete strength, number of FRP layers. A higher vertical strength of FRP, higher unconfined concrete strength and fewer FRP layers lead to a higher FRP strain efficiency.

c) The Tsai-Wu criterion considering all dimensional stress has the most conservative strain efficiency prediction, meanwhile, it has several advantages compared to the other failure criteria. Therefore, this failure criterion is suggested to be used in the triaxial stress analysis. The results considering the stress in vertical and hoop directions are much closer to the three-dimensional results, comparing with the one considering the stress in radial and hoop directions.
CHAPTER 6

STRAIN EFFICIENCY OF FRP JACKETS IN FRP-CONFINED
CONCRETE-FILLED CIRCULAR STEEL TUBES

6.1 Introduction

There has been extensive research into the use of FRP jackets (or wraps) to strengthen reinforced concrete columns (e.g. Teng et al. 2002; Lam and Teng 2004; Jiang and Teng 2007). More recently, researchers (Xiao 2004; Xiao et al. 2005; Teng and Hu 2006) have explored the benefits of using FRP jackets to provide additional confinement to concrete-filled steel tubes.

This chapter investigates the effects of geometrical discontinuities at the ends of an FRP jacket on its strain efficiency when used to confine a concrete-filled circular steel tube through a detailed FE study. In such columns, as the FRP jacket is separated by a steel tube from the concrete core, non-uniform deformation of the cracked concrete core, which is believed to be an important factor responsible for the observed rupture strain reduction in an FRP jacket confining a concrete column, is much less important. The predictions are shown to be in reasonable agreement with available test data, confirming the important role played by these discontinuities in controlling the rupture failure of such FRP jackets.
6.2 Experiments

A brief summary of a series of experiments conducted at The Hong Kong Polytechnic University (Hu et al. 2011), with particular attention to the ultimate failure mode and the observed hoop rupture strain of the FRP jacket is given here to facilitate the subsequent comparisons and discussions. A full report of this experimental study can be found elsewhere (Hu et al. 2011). Nine FRP-confined concrete-filled steel tubes were recently tested under concentric axial compression in which the hoop strains in the FRP jackets were carefully measured. The steel tubes all had a length of 400mm and an FRP hoop overlapping zone of 200mm (the zone where the beginning part and the ending part of a continuous fibre sheet used to form the FRP jacket overlaps). The concrete cores had a diameter of 200mm. The elastic modulus and tensile strength of the GFRP from flat coupon tests, calculated on the basis of a nominal fibre sheet thickness of 0.17mm, are 80.1GPa and 1,826MPa, respectively, giving an ultimate tensile strain of 0.0228 (Teng and Hu 2006). The ultimate tensile strain of FRP provided by the manufacturer is 0.028. Details of the specimens are given in Table 6.1.

Six lateral strain gauges were installed at the mid-height of each column specimen to measure the hoop strains of the FRP jacket. One strain gauge was installed within the overlapping zone, while the other five were located outside the overlapping zone. The strain efficiency, \( \kappa \), for a given test is taken as the average of the five strain gauge readings outside the overlapping zone at the ultimate state divided by the average FRP ultimate tensile strain obtained from coupon tests or from the manufacturer.
Table 6.1 Details of test specimens and selected test results

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Nominal Thickness of steel tube (mm)</th>
<th>Steel yield stress (MPa)</th>
<th>Concrete cylinder strength (MPa)</th>
<th>No. of FRP layers</th>
<th>( \kappa ) (manufacturer)</th>
<th>( \kappa^* ) (coupon test)</th>
<th>FRP rupture location</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1-102</td>
<td>1</td>
<td>0.639</td>
<td>0.784</td>
<td>0.784</td>
<td>Elsewhere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2-102</td>
<td>2</td>
<td>0.639</td>
<td>0.802</td>
<td>0.802</td>
<td>Elsewhere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F3-102</td>
<td>3</td>
<td>0.672</td>
<td>0.784</td>
<td>0.784</td>
<td>Elsewhere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2-135</td>
<td>2</td>
<td>226</td>
<td>41.6</td>
<td>0.575</td>
<td>Outer end</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F3-135</td>
<td>2</td>
<td>226</td>
<td>42.1</td>
<td>0.598</td>
<td>Elsewhere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F4-135</td>
<td>4</td>
<td>242</td>
<td>42.1</td>
<td>0.638</td>
<td>Outer end</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2-202</td>
<td>2</td>
<td>0.772</td>
<td>0.948</td>
<td>0.948</td>
<td>Elsewhere</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F3-202</td>
<td>3</td>
<td>0.683</td>
<td>0.839</td>
<td>0.839</td>
<td>Outer end</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F4-202</td>
<td>4</td>
<td>0.685</td>
<td>0.841</td>
<td>0.841</td>
<td>Outer end</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All of the specimens failed by the rupture of the FRP jacket in hoop tension within the mid-height region (Figure 6.1). Once rupture of the FRP jacket occurred, the confinement effect of the FRP in the ruptured zone vanished, leading to local buckling of the steel tube in the vicinity of the location where the FRP had ruptured. The load carried by the tube was reduced immediately at FRP rupture. Before this final failure, localized FRP rupture occurred near one end in some of the specimens due to the localized outward buckling deformation of the steel tube, but this local FRP rupture only had a small effect on the load-carrying capacity of the specimen. More details can be found in Hu et al. (2011).

Among the nine specimens, the FRP jacket ruptured near the outer end of the overlapping zone in four cases. Figure 6.1 shows the locations of FRP rupture in these four specimens. Therefore, geometric discontinuities at the ends of an FRP jacket may play a significant role in influencing the hoop rupture strain of FRP jackets in FRP-
confined concrete-filled steel tubes, similar to the phenomenon observed in FRP-confined concrete columns (Chen et al. 2010). It should be noted that once FRP rupture begins, it can propagate to different locations. The other five specimens experienced FRP rupture at locations away from the ends of the overlapping zone.

Figure 6.1 FRP rupture near the outer end of the FRP overlapping zone
6.3 Geometry and materials

6.3.1 Geometry

A circular column confined with an FRP jacket formed from a single continuous fibre sheet and comprising $N$ layers (laps of the fibre sheet) is considered. Figure 6.2 shows schematically an example column with two layers of FRP. A polar coordinate system is used to describe positions (Figure 6.2), with the circumferential angular coordinate denoted by $\theta$. The wrapping process starts at $\theta = 0^\circ$ (the inner end) and finishes at $\theta = 360N + \alpha$ (the outer end), giving an overlapping zone of angular length $\alpha$.

The change in radius necessary for an outer layer of FRP to overlap the innermost layer is assumed to occur within a transition zone of angular length $\beta$. The shape of this transition is assumed to be sinusoidal, as in Chen et al. (2010).

![Diagram](image_url)

**Figure 6.2 Idealized cross-section of FRP-confined concrete-filled steel tube with a two-layer FRP jacket**
Reference values of radius $R = 200\text{mm}$ (as of the test specimen), FRP layer thickness $t_f = 1.6\text{mm}$ (based on the thickness of test coupons), adhesive layer thickness $t_a = 0.1\text{mm}$, and angular overlap length $\alpha = 112.3^\circ$ (as in test specimens) are used herein. The transition angle $\beta$ is assumed to be $30^\circ$. Chen et al. (2010) have shown that the value of $\beta$ does not affect predictions unless it becomes unrealistically small. For such a system under radial expansion, stress concentration is expected to occur on the outer surface of each layer ($n = 1$ to $N$) of FRP at $\theta = 360 \times (N - 1) + \alpha$, adjacent to the outer end of the FRP jacket with that at the outmost FRP layer (Location A in Fig. 2, $\theta = 472.3^\circ$) being the most severe, and on the inner surface of each layer of FRP at $\theta = n \times 360^\circ$, adjacent to the inner end of the FRP jacket with that at the innermost FRP layer (Location B in Figure 6.2, $\theta = 360^\circ$) being the most severe.

6.3.2 Adhesive properties

Two adhesive constitutive models, representing the two possible extremes, were adopted in this study: a linear elastic model and an elastic-perfectly plastic model. Tensile tests showed that the adhesive used in the present tests exhibited some plasticity before tensile failure and its elastic modulus and tensile strength were 4.82GPa and 31.3 MPa respectively (Hu 2011). The Poisson’s ratio was assumed as 0.35.

The actual tensile stress-strain behaviour of adhesives can be much more complex. Some adhesives are almost linear elastic-brittle in uniaxial tension, but others exhibit considerable plasticity. Accurate modelling of a linear elastic-brittle material is difficult
without special treatment due to mesh sensitivity. The above two idealised models were used in the present numerical study to investigate the effect of adhesive behaviour on strain efficiency for the present problem. The actual behaviour of adhesives in a complex stress state, as well as its appropriate modelling, requires further research.

6.3.3 FRP properties

The test concrete filled steel tubes were jacketed with FRP using a wet lay-up process, in which a dry fibre sheet was impregnated with epoxy resin and wrapped around the steel tube. It has been shown in Chen et al. (2010) and Chapter 3 that the use of the actual thickness of the FRP (instead of the nominal thickness) is more appropriate in an FE model because the fibres are likely to be distributed through the actual thickness. The actual thickness of the FRP per layer, which was approximately 1.6mm based on measurements of flat coupon test specimens of the same FRP material, was adopted in the FE model. Since the actual thickness of FRP layer represents an average value with the thickness varying in practice, the effect of varying the assumed FRP layer thickness is further examined later.

FRP composites are orthotropic materials; their mechanical properties depend on the fibre orientations and distribution and the relative proportions of fibre and matrix. The macro-properties of an FRP can be estimated from the fibre architecture and fibre volume fraction using the rule of mixtures, as discussed in Chapter 2. The adhesive (matrix) properties used in the present analysis are stated in Section 6.3.2. The elastic modulus of the fibres was found to be 39.6GPa based on the GFRP coupon test results.
(see Section 6.3.2) and the fibre volume ratio of the coupons was taken equal to the nominal thickness (0.17 mm) divided by the actual thickness of the FRP (1.6 mm). A 0.1 mm thick pure adhesive layer was assumed between adjacent layers of the FRP.

6.4 Finite element modelling

The multi-purpose finite element analysis package Ansys (2007) was used to conduct all of the analyses presented in this chapter. Only the FRP jacket, including the adhesive between the FRP layers, was modelled as in Chen et al. (2010). The specimens were modelled as a plane strain problem using eight-node quadrilateral elements. FRP rupture is assumed to occur when the predicted maximum FRP hoop strain reaches the ultimate tensile strain of FRP determined from flat coupon tests. A predicted strain efficiency factor is then obtained by dividing the predicted FRP hoop strain outside the overlapping zone by the ultimate tensile strain of FRP from flat coupon tests. Test specimen F1-102 (refer to Table 6.1) was selected as the reference specimen.

6.4.1 Loading and boundary conditions

Only the FRP and the bonding adhesive were modelled, with their interaction with the column simulated as either an internal radial pressure or a prescribed radial displacement at the adhesive-steel interface. The same four loading and boundary conditions as in Chen et al. (2010) were examined during the construction of the finite element model: the loading is applied either as a uniform displacement or a uniform internal pressure while the circumferential displacement is either restrained or free to move, giving four combinations of the loading and boundary conditions as LB1 to LB4 in Table 6.2.
However, only LB1 and LB4 are discussed in this chapter to examine strain concentration at both ends of the FRP jacket because LB2 and LB3 are unrealistic: LB2 assumes that the FRP jacket is free to move around the steel tube (i.e. all bond is lost and there is no friction) whilst LB3 results in a non-circular deformed shape of the FRP jacket. A more detailed discussion can be found in Chen et al. (2010).

Table 6.2 Loading schemes and boundary conditions

<table>
<thead>
<tr>
<th>Loading scheme</th>
<th>Short title</th>
<th>Circumferential DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform displacement</td>
<td>LB1</td>
<td>Fixed</td>
</tr>
<tr>
<td></td>
<td>LB2</td>
<td>Free</td>
</tr>
<tr>
<td>Internal pressure</td>
<td>LB3</td>
<td>Free</td>
</tr>
<tr>
<td></td>
<td>LB4</td>
<td>Fixed</td>
</tr>
</tbody>
</table>

6.4.2 Mesh convergence

The existence of stress concentration at locations A and B in Figure 6.2 presented a challenge to the choice of a suitable finite element mesh. A mesh convergence study was thus conducted for the reference specimen to determine a suitable mesh. Four meshes with a minimum element circumferential width of 0.16, 0.32, 0.64 and 1.28 mm were investigated. The corresponding number of elements through the thickness of an FRP layer was, respectively, 7, 5, 3 and 1 for the four meshes. The adhesive layer was divided into two elements in the radial direction. The element size in the circumferential direction was the smallest at the two ends of the FRP jacket and increased gradually.
away from them using a bias factor of 1.03. Figure 6.3 shows details of the finite element mesh using minimum element circumferential width of 0.32 mm near Locations A and B. The blue and red elements represent the FRP and adhesive layers, respectively. Figure 6.4 shows the effect of element size on the FRP strain efficiency for different loading conditions and the two adhesive models. The FRP ultimate tensile strain of 2.8% provided by the manufacturer was used in calculating the strain efficiency factor. The predicted strain efficiency factor based on Location A is shown for all cases. Strain concentration at Location B was much less severe than at Location A for all cases except for the loading condition of LB1 in combination with an elastic-perfectly plastic adhesive, which is the reason why only one curve is shown in Figure 6.4 for Location B. Figure 6.4 clearly shows that a coarse mesh underestimates the peak strain and the relationship between the predicted strain efficiency with element size is almost linear for all cases. The predicted strain efficiency thus converges to a value which can be found by extrapolating its respective curve with different element size in Figure 6.4 to intersect the vertical axis. The mesh with a minimum circumferential element size of 0.32mm and 5 elements through the thickness of an FRP layer was deemed sufficiently accurate without further correction and used in all subsequent analyses.
The linear elastic adhesive model led to much higher strain concentration and resulted in lower strain efficiency factors, as shown in Figure 6.4. This model is unrealistic because it implies that the adhesive has an infinite strength. It is also shown later that the predictions using the elastic-perfectly plastic adhesive model are in better agreement with the test results. Therefore, most of the discussions hereafter are based on results obtained using the elastic- perfectly plastic adhesive model.
6.4.3 FRP strain distributions

Figure 6.5 shows the predicted distribution of FRP hoop strain on the inner and outer surfaces of the FRP jacket when the maximum FRP hoop strain reaches its rupture strain from flat coupon tests (provided by the manufacturer) for loading schemes LB1 and LB4 in combination with an elastic-perfectly plastic adhesive. The predicted strain distributions for specimens with 1 to 4 layers of FRP (Figure 6.5a-d) as well as strain measurements from relevant test specimens are shown. These results clearly show that severe strain concentration occurs at two locations in the FRP jacket: (a) on the outer surface of the layer of the FRP adjacent to the outer end of the jacket (Location A), and (b) on the inner surface of the layer of FRP adjacent to the inner end of the jacket (Location B) (Figure 6.2). Strain concentration near both the inner and outer ends of the FRP jacket also occurs in layers other than the innermost and outermost layers, but the...
degree of strain concentration is much smaller than that at locations A and B. Figure 6.5 also shows that the strain concentration is substantially larger at Location A than at Location B for LB4. This is in agreement with the experimental observation that the former is a more likely location for rupture failure than the latter. However, as indicated in Figure 6.5a, severe strain concentration is predicted at Location B for LB1 using the elastic-perfectly plastic adhesive model, although the degree of concentration reduces as the number of FRP layers increases (Figure 6.5b-d). By contrast, no significant strain concentration is observed at Location B for a linear elastic adhesive under either loading condition, as was also shown by Chen et al. (2010).

Figure 6.5 shows that the experimental strains are scattered around the predicted distribution, with reasonable overall agreement. The experimental strains are in better agreement with predictions for LB4 than LB1. The non-uniformity of the experimental hoop strain distribution is probably due to the non-uniform deformation of the concrete, which is also in agreement with continuous optical strain measurements of FRP-confined concrete (Bisby and Take 2009). The modelling of such non-uniformity is beyond the scope of the idealised FE model in the present study.

Figure 6.6 shows the detailed strain distributions predicted with the elastic-perfectly plastic adhesive model for the vicinities of locations A and B where strain concentration occurs.
a) Specimens with one layer of FRP

b) Specimens with two layers of FRP
c) Specimens with three layers of FRP

d) Specimens with four layers of FRP

Figure 6.5 Distributions of hoop strains from an elastic-perfectly plastic adhesive model
Figure 6.6 Distributions of FRP hoop strains near locations A and B
6.5 Comparison between test results and FE predictions

Rupture of the FRP jacket is assumed to occur when the predicted maximum FRP hoop strain at Location A reaches the ultimate tensile strain of the FRP from flat coupon tests. The predicted strain efficiency factor is defined as the ratio between the predicted FRP hoop strain outside the overlapping zone and this peak strain. The test strain efficiency factor is defined as the ratio between the average FRP hoop strain outside the overlapping zone and the FRP ultimate tensile strain. Two ultimate tensile strain values are explored herein for the FRP: 2.28% and 2.8%. The former was from the authors’ own flat coupon tests, while the latter was provided by the FRP manufacturer. These two ultimate tensile strains lead to two sets of strain efficiency factors and eight sets of predicted values covering two ultimate tensile strains, two adhesive constitutive models and two loading conditions. Figure 6.7 and Figure 6.8 compare the predicted strain efficiency factors with the test FRP strain efficiency factors for all nine specimens for loading conditions LB1 and LB4, respectively. The same data are also listed in Table 6.3.

These results show that the strain efficiency factors predicted using the elastic-perfectly plastic adhesive model are in much closer agreement with the test results than those predicted using the linear elastic adhesive model. Most of the test results lie between the two sets of predicted values obtained using the two FRP ultimate tensile strains and the elastic-perfectly plastic adhesive model. This observation suggests that the actual ultimate tensile strain of the FRP in FRP-confined columns may lie between these two values.
The results shown in Figure 6.7 and Figure 6.8 as well as Table 6.3 also show that there is no obvious difference between the FE results for the two different loading conditions based on the elastic-perfectly plastic adhesive model, although predictions for LB4 are in slightly better agreement with test results in terms of average value and scatter as well as overall strain distribution.

Table 6.3 shows that the ratios of FE prediction to test result all have low variance, with the coefficients of variation (CoV) being less than 10% in all cases. The results also show that the strain efficiency factors for LB1 are on average 4% larger than the results for LB4. Therefore, loading condition LB4 is more conservative for the cases studied here. The average test strain efficiency factor is 0.807 based on the flat coupon test ultimate strain and 0.657 based on the manufacturer’s data. These two values are both in the range of strain efficiency factors (0.58–0.91) for FRP-confined concrete columns previously observed by Lam and Teng (2004).
Figure 6.7 Predicted versus test strain efficiency factors under uniform radial displacement (loading condition LB1)

Figure 6.8 Predicted versus test strain efficiency factors under internal pressure (loading condition LB4)
### Table 6.3 Strain efficiency factors for different loading conditions and adhesive models

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Efficiency factor</th>
<th>Prediction/test: LB1</th>
<th>Prediction/test: LB4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elastic-perfectly plastic</td>
<td>Elastic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Coupon</td>
<td>Supplier</td>
<td>Coupon</td>
</tr>
<tr>
<td>F1-102</td>
<td>0.784</td>
<td>0.639</td>
<td>0.904</td>
</tr>
<tr>
<td>F2-102</td>
<td>0.802</td>
<td>0.653</td>
<td>0.824</td>
</tr>
<tr>
<td>F3-102</td>
<td>0.825</td>
<td>0.672</td>
<td>0.778</td>
</tr>
<tr>
<td>F2-135</td>
<td>0.706</td>
<td>0.575</td>
<td>0.935</td>
</tr>
<tr>
<td>F3-135</td>
<td>0.735</td>
<td>0.598</td>
<td>0.874</td>
</tr>
<tr>
<td>F4-135</td>
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<td>0.638</td>
<td>0.811</td>
</tr>
<tr>
<td>F2-202</td>
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<tr>
<td>F3-202</td>
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</tr>
<tr>
<td>F4-202</td>
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<td>0.755</td>
</tr>
<tr>
<td>Average</td>
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<td>0.657</td>
<td>0.816</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>8.6%</td>
<td>8.6%</td>
<td>9.4%</td>
</tr>
</tbody>
</table>

### 6.6 Parametric study

A parametric study was undertaken to investigate the effects of the following parameters on the predicted strain efficiency of FRP jackets:

- FRP thickness;
- FRP elastic modulus;
- FRP orthotropy;
- adhesive yield stress;
- adhesive thickness; and
- column size.
A uniform internal pressure with circumferential displacements prevented (i.e. LB4) was chosen as the loading and boundary condition in the parametric study because, as indicated earlier, the results for the LB4 case are more conservative and are in slightly better agreement with the test results than for LB1 when the elastic-perfectly plastic adhesive model is used. Specimen F1-102 with a single-layer FRP jacket was again taken as the reference specimen in the parametric study. In performing the study, one of the parameters was varied each time with all other parameters as defined in Section 3.1, unless otherwise stated. The focus of the parametric study was the prediction of the peak FRP strain in the direction of the fibres, which occurred at Location A in all cases unless otherwise indicated.

6.6.1 FRP thickness
Since it is difficult to control the actual thickness of FRP in a wet lay-up application, Figure 6.9 examines the effect of varying the FRP layer thickness on the predicted FRP strain efficiency. The total thickness of the FRP jacket (i.e. in a general case, the total thickness of all FRP layers plus the adhesive between FRP layers) was kept constant (=1.7mm). The thickness of the FRP layer was varied from 0.1mm to 1.6mm, with the reference nominal layer thickness of 0.17mm kept constant. This means that the corresponding thickness of the adhesive layer varied from 1.6mm to 0.1mm, and the overall membrane stiffness of the FRP plus the adhesive remained constant. Note that the variation of the stiffness ratio $E_r/E_\theta$ with the FRP thickness was considered by keeping $E_r$ constant as in the reference case. Figure 6.9 shows clearly that the strain efficiency of the FRP jacket reduces as the FRP layer thickness increases, indicating that
there is a reduced ‘kink’ effect in the thinner FRP. This suggests that the strain efficiency for a prefabricated FRP shell strengthening would be higher than that of a wet lay-up FRP sheet strengthening because the former is usually thinner than the latter for the same overall circumferential membrane stiffness. However, further experimental research is required to confirm this observation.

![Figure 6.9 Effect of FRP thickness on FRP strain efficiency](image)

**6.6.2 FRP membrane stiffness**

Figure 6.10 shows the effect of FRP membrane stiffness $E_{ft}$ on strain efficiency where the FRP nominal elastic modulus $E_f$ varies from 50GPa to 300GPa while the thickness remains constant with $t_f=1.6$mm and the stiffness ratio $E_f/E_r$ is also kept constant as in the reference case. It is evident that an increase in the FRP elastic modulus leads to an increase in the FRP strain efficiency. This observation is similar to the conclusion
reached using an FE model for loading condition LB1 and a linear elastic adhesive
(Chen et al. 2010). The results suggest that the strain efficiency is increased with a higher modulus FRP for the same actual thickness of FRP.

![Graph showing the effect of FRP membrane stiffness on FRP strain efficiency.](image)

**Figure 6.10 Effect of FRP membrane stiffness on FRP strain efficiency**

### 6.6.3 FRP orthotropy

Figure 6.11 shows the effect of FRP orthotropy on the FRP strain efficiency, where $E_r$ is the elastic modulus of FRP in the radial direction while $E_\theta$ is the elastic modulus of FRP in the circumferential direction. It is evident that the effect of FRP modulus ratio $E_r/E_\theta$ is significant when it is small. In practice, the $E_r/E_\theta$ ratio of FRP is likely in the range of 0.1 to 0.6. $E_r/E_\theta=0.53$ for the reference case. Figure 6.11 shows that the strain efficiency increases when $E_r/E_\theta$ increases but the effect is insignificant in the range of interest. The use of an orthotropic material model is more conservative than an isotropic model ($E_r/E_\theta=1$), at least for the reference case.
6.6.4 Adhesive yield stress and modulus

Figure 6.12 shows the effect of the yield stress of adhesive on the FRP strain efficiency where the adhesive yield stress varies from 20 MPa to 60 MPa. The result for a linear elastic adhesive (yield stress = infinite) is also included for comparison. Clearly a higher adhesive yield stress leads to lower strain efficiency; the linear elastic adhesive results in the lowest strain efficiency. Therefore, adhesives with greater plasticity but a lower yield stress lead to higher FRP strain efficiency.

The elastic modulus of adhesive was found in the parametric study to have almost no effect on the predicted FRP strain efficiency within the explored range of 2 to 20 GPa.
6.6.5 Adhesive thickness

In a wet lay-up process of forming an FRP jacket from a dry fibre sheet, the impregnating resin (i.e. the adhesive) saturates the fibres across the sheet thickness to form an FRP layer. Within the idealized jacket cross-section of the present study (Figure 6.13), some of the adhesive is assumed to form a pure adhesive layer between the two adjacent FRP layers. The exact thickness of such an adhesive layer is difficult to control or to measure in practice. Thus, the effect of the assumed adhesive thickness on the FRP strain efficiency is examined in Figure 6.13, where the adhesive thickness varies from 0.1mm to 1.0mm. It is evident that an increase of the adhesive thickness increases the FRP strain efficiency within the studied range, but this effect is rather small (within 5% within the range in Figure 6.13).
6.6.6 Column size

The effect of column diameter on the FRP strain efficiency was investigated by varying the column diameter from 100 to 600 mm, with the FRP and the adhesive thicknesses being kept at their reference values. The numerical results indicated that the column diameter has little effect on the FRP strain efficiency, which is similar to the observation made by Chen et al. (2010) for FRP-jacketed concrete columns.

6.7 Conclusions

An FRP jacket for confining a column is commonly formed by impregnating a dry fibre sheet with an epoxy resin and wrapping it around the column, with each lap of the fibre sheet forming an individual layer of the FRP jacket. Geometrical discontinuities exist in such an FRP jacket at the beginning and finishing ends of the wrapping process (or simply the ends of the FRP jacket). This chapter has presented a study to find out
whether these geometrical discontinuities are largely responsible for the apparent rupture strain reduction of an FRP jacket in FRP-confined concrete-filled circular steel tubes as observed in recent tests. Similar rupture strain reductions have been observed in FRP-confined concrete columns, where the non-uniform deformation of the cracked concrete core is also believed to play an important role (Lam and Teng 2004); this factor is, however, believed to be much less important for the present columns as the FRP jacket is separated from the concrete core by the steel tube. The following conclusions can be drawn from the results and discussions presented in the chapter:

a) Of the nine tests on FRP-confined concrete-filled steel tubes, four failed at the finishing end of the FRP jacket, thus demonstrating the potential importance of geometric discontinuities at the two ends of the FRP jacket on its ultimate condition.

b) The FE results have shown that strain concentration occurs in small zones near the two ends of the FRP jacket and that these highly localized strains cannot be measured using conventional techniques. Based on the FE results, this strain concentration offers a plausible explanation for the reduction in the apparent rupture strain of the FRP jacket. This conclusion is consistent with the experimental observation that four of the nine tests examined in this chapter failed at the finishing end of the FRP jacket.
c) Severe stress concentration occurs at two key locations on the FRP jacket: on the outer surface of the layer of FRP adjacent to the outer end of the jacket; and on the inner surface of the layer of FRP adjacent to the inner end of the jacket. The former location is more critical under the boundary and loading conditions examined in this chapter, as confirmed by experimental observations given in (a) above.

d) The adhesive properties can affect the strain values at the ends of the FRP jacket significantly, thus affecting significantly the strain efficiency of the FRP jacket. An adhesive with a low yield stress but substantial plasticity reduces stress concentration and increases the FRP strain efficiency. FE results obtained from the present study assuming an elastic-perfectly plastic adhesive have been found to be in reasonable agreement with the available test results.

e) The results of a parametric study have shown that the FRP strain efficiency increases with the FRP elastic modulus and the FRP isotropy but decreases with the FRP thickness and the adhesive yield stress. The effects of column diameter and adhesive thickness on the FRP strain efficiency are insignificant.
CHAPTER 7
FRP STRAIN EFFICIENCY IN FRP-WRAPPED CIRCULAR CONCRETE COLUMNS

7.1 Introduction
This chapter investigates the effects of geometrical discontinuities at the ends of an FRP jacket on its strain efficiency when used to confine a circular concrete column through a detailed FE study, using 3-dimensional FRP wrapped concrete columns with the thickness of 1mm, as shown in Figure 7.1. The red, grey and green elements represent the FRP, adhesive and concrete, respectively. An improved model based on concrete damage plasticity, developed by Yu et al. (2010), was applied for concrete. Furthermore, the particle image velocimetry (PIV) technique is used to experimentally investigate and quantify the strain concentration at the finishing end of the FRP wrap. The comparison of FE and the PIV results is conducted to verify the FE model, and the FE results are further examined in this chapter.
7. Test of FRP wrapped concrete columns

Bisby et al. (2011) conducted a series of tests on FRP wrapped concrete, where some of the concrete was exposed to high temperature prior to FRP wrapping. Among these test specimens, six FRP wrapped concrete columns which did not involve fire damage of the concrete are used here for comparison with the FE predictions. These test data have been published in Bisby et al. (2011), but most of the PIV analysis presented below is new and solely conducted by the author.

The columns were wrapped in the hoop direction with a single layer of SikaWrap Hex 230C unidirectional carbon fibre/epoxy FRP strengthening system. The manufacturer specified properties for this FRP include an ultimate tensile strength of 4100 MPa at an ultimate tensile strain of 1.7%, a nominal thickness of 0.12mm, and a tensile Young’s
CHAPTER 7

modulus of 231000 MPa. The FRP was applied using a hand lay-up procedure as recommended by the manufacturer, with an overlap length of 100 mm.

### 7.2.1 Test setup

Figure 7.2 shows the dimensions of test specimens, the FRP wrap configuration, and the locations of the two cameras, one to look at the FRP opposite the FRP overlapping zone to observe the hoop/axial deformation and the other to observe the deformation at the outer end of FRP wrap (Location A in Figure 7.2). The cameras were focused at the middle height of the cylinder. The shorter side of the photos covered about the full height of the cylinders. All the specimens were standard, un-reinforced concrete cylinders 100mm in diameter and 200mm in height. They were cast from a single batch of C25/30 ready-mix concrete with a maximum aggregate size of 10mm. The cylinder strength of the plain concrete was 31 MPa in average of two specimens, using the same size of concrete cylinders as the FRP wrapped specimens.
7.2.2 Axial stress-strain curve

Figure 7.3a-c show the axial stress-strain curves, the axial strain-hoop strain curves and the axial stress-hoop strain curves of the test specimens, respectively. The strains were obtained by using PIV analysis of the images taken from the camera focusing on the opposite side of the FRP overlap. Jiang and Teng (2007)'s model was used to compare with the stress-strain relationship of FRP wrapped concrete columns. As suggested in Lam and Teng (2004), the average FRP rupture strain measured from the ultimate stage of the FRP-wrapped concrete column tests (1.15%) was used in the Jiang and Teng’s (2007) model. The predictions were referred to as ‘Equation’ in Figure 7.3. Figure 7.3 shows that Jiang and Teng’s (2007) predictions of axial stress-strain curve, axial-hoop strain curve and axial stress-hoop strain curve are reasonably close to the test results, despite the predicted axial stress is generally lower than that from the test at a given axial or hoop strain (Figure 7.3a and c). Given the fact of large variability of test data and the fact that Jian and Teng’s (2007) model was developed based on a large test
database, the small discrepancies between the prediction and test data here are not considered a significant issue. Jiang and Teng’s (2007) model forms the basis of FE model of FRP confined concrete columns presented next in this chapter.

(a) Axial stress-strain behaviour of FRP wrapped concrete columns

(b) Axial-hoop strain curve of FRP wrapped concrete columns
7.2.3 PIV analysis

The strain field near the outer end of the FRP wrap was examined using the photos taken from the camera focused on Location A in Figure 7.4, which were further processed using PIV program GeoPIV (White et al. 2003) to obtain the strain distribution. A set of patches of pixels in the initial image are first defined, and they are tracked in subsequent images. The location and size of the patch can be chosen anywhere within the field of view of the camera, and it can be tracked in any direction within the plane of the image. The in-plane strains can be calculated based on a pair of patches by using any chosen gauge length and patch size, in any direction (Bisby et al. 2011). Generally, a smaller patch size leads to a better spatial resolution of the displacement field, which provides an improved localised deformation. However, smaller patches offer a lower measurement
precision, and more sensitive to distortion or unsteady lighting, which may lead to wild vectors (White et al. 2003).

![Diagram showing distance from the outer end of FRP wrap](image)

**Figure 7.4 Distance from the outer end of FRP wrap**

Therefore, a convergence study is necessary to be performed since the precision of strain measurement in PIV is a function of patch size and the gauge length (Lesniewska and Muir Wood 2009). A convergence study was performed on two images taken at the beginning and at the end of the loading process for Specimen-1. Meshes using five different gauge lengths and five different patch sizes were investigated, as illustrated in Figure 7.5 and Figure 7.6. Figure 7.5 shows the results using 48×48 patch size, with five different gauge lengths. Figure 7.6 shows the results using 5 mm gauge length, with five different patch sizes. Figure 7.5 shows that the strain approaches zero at near the origin from the negative side of the horizontal coordinate (i.e. on the outer layer approaching the outer end of the FRP wrap) for all gauge lengths, but the deduced strain distribution fluctuate significantly when the gauge length is very small (e.g. 1mm). A large gauge length results in smooth strain distribution but it can significantly reduce (smooth) the
interesting peak strains. As a compromise, a 5mm virtual gauge length was adopted for the rest of the analysis. Figure 7.6 shows that larger patches result in smoother strain distributions with smaller peaks. A patch size of 48×48 was adopted as a compromise in the rest of the analysis in this chapter unless otherwise stated. Note that there is no data near around a coordinate value of 0 (i.e. around the outer end of the overlap) because no virtual strain gauges were placed across this position where slip between two layers of FRP is expected to occur (so the ‘measured’ strain would not represent the real strain in the FRP). This will be further examined in this chapter.

Figure 7.5 Effect of gauge length on PIV strain distribution (patch size =48×48)
7.2.4 Measurement of FRP thickness

The specimens were wrapped using a wet lay-up process, with the FRP formed during the application process from the impregnation of fibres with resin. It has been shown in Chapter 3 that the use of the actual instead of the nominal thickness of the FRP wrap is more appropriate in an FE model because the fibres are likely distributed through the actual thickness. To measure the actual thickness of the FRP wrap, 3 samples were produced following the same wrapping process as that of wrapping the above concrete cylinders, but a 100mm diameter plastic tube was used instead. The FRP wrapped plastic tubes (Figure 7.7a) were cut into strips after curing for 7 days. The actual thickness of the FRP was measured from the photos taken using an electronic microscope (Figure 7.7b-g).
The measurement of FRP thickness in the I-I section (perpendicular to the longitudinal axis of the tube, see Figure 7.7a) was in the range of 0.307 mm to 0.384 mm (Figure 7.7b-e) with an average of 0.355mm and standard deviation of 0.027mm. It was in the range of 0.277 mm to 0.471 mm from the II-II section (a diametric section, see Figure 7.7a), with an average of 0.355 mm and a standard deviation of 0.053 mm (Figure 7.7f and g). It appeared to have a large scatter in the measurements from the II-II section.

The thickness of the adhesive layer was measured to be in the range of 0.087 mm to 0.13 mm, with an average of 0.106 mm and standard deviation of 0.016mm in section I-I (Figure 7.7b-e). It was in the range of 0.085 mm to 0.264 mm with an average of 0.176 mm and standard deviation of 0.090 mm in section II-II (Figure 7.7f and g). The average thickness of adhesive was taken as 0.141 mm, the average result of measurements in both sections. Since a thicker FRP layer was more critical as discussed in Chapter 6, the maximum thickness of 0.47 mm was used in the FE model. Thus the thickness of FRP layer and the adhesive layer in the FE model was taken to be 0.47 mm and 0.14 mm, respectively. It should be noted that in a real cylinder test, the length of the cylinder is larger than the length of tube used in the measurement, this may lead to an even larger scatter in the real cylinder tests, resulting in a larger FRP thickness in real tests.
a) FRP wrapped plastic tube

b) Section I-I: one layer FRP wrap
d) Outer end in section I-I

c) Section I-I: two layers FRP wrap
e) Inner end in section I-I
7.3 FE modelling

The multi-purpose FE analysis package ABAQUS (2008) was used to conduct all the FE analyses presented in this chapter. The specimens were modelled as a 3 dimensional problem using eight-node solid elements. FRP rupture is assumed to occur when the predicted maximum FRP hoop strain reaches the ultimate tensile strain of the FRP provided by the manufacturer. A predicted strain efficiency factor is then obtained by dividing the predicted average FRP hoop strain outside the overlapping zone by the tensile rupture strain of the FRP.

7.3.1 Geometry

The constraints in both ends of a column were assumed to have little effect on the behaviour of the FRP wrapped concrete column near the mid-height in this chapter. This effect of end constraints will be further examined in Chapter 8. Following this assumption, the FRP wrapped concrete column was simulated using a 1mm thick single
layer 3D FE model. The interface between the concrete and adhesive, and that between the adhesive and FRP were assumed to have no slip so that the elements for different materials can share the same nodes at the interfaces.

A circular concrete column confined with a single continuous FRP wrap comprising N layers (plies) was considered (Figure 7.8). A polar coordinate system was used to describe positions (Figure 7.8), with the circumferential angular coordinate denoted by $\theta$. The FRP wrap starts at $\theta = 0^\circ$ (the inner end) and finishes at $\theta = 360^\circ N + \alpha$ (the outer end), giving an overlapping zone of $\alpha$, similar to Chen et al. (2010).

The change in radius is necessary for the outer layer of FRP to overlap the inner layer occurs within a transition zone of $\beta$ (Chen et al. 2010). The shape of the transition is
assumed to be sinusoidal as in Chen et al. (2010). $\beta$ was assumed to be 30 degree in this chapter.

### 7.3.2 Concrete

The concrete model used in this chapter is an improved model based on concrete damage plasticity in ABAQUS (Yu et al. 2010). As stated in Yu et al. (2010), the improved model takes account of concrete plasticity, confinement-dependent damage and other distinct characteristics of non-uniformly confined concrete. Because of the existence of the overlap in the FRP wraps, the concrete core is under non-uniform confinement so non-uniform axial stress distribution is expected to occur in the current model. As mentioned previously, the analysis-oriented model presented by Jiang and Teng (2007) for uniformly-confined concrete was adopted herein to produce the necessary material parameters for this concrete constitutive model. Jiang and Teng’s (2007) model is able to predict the entire axial stress-axial strain curve and lateral strain-axial strain curve, and thus provides sufficient information to define the hardening/softening rule and the flow rule for concrete under uniform confinement. Additionally, in order to consider the non-uniform confinement of concrete, the effective confining pressure $\sigma_{t,\text{eff}}$ is proposed using the following equation (Yu et al. 2010):

$$\sigma_{t,\text{eff}} = \frac{2(\sigma_2 + \alpha f_{\text{co}}')(\sigma_3 + \alpha f_{\text{co}}')}{(\sigma_2 + \sigma_3 + 2af_{\text{co}}')} - af_{\text{co}}'$$

(7.1)

where $\sigma_2$ and $\sigma_3$ are the two principal lateral stresses respectively; $f_{\text{co}}'$ is the cylinder compressive strength of concrete; and $\alpha$ is a constant to be determined on test results, as stated in Yu et al. (2010), the best-fit value for $\alpha$ is 0.039.
7.3.3 Adhesive

A two-part epoxy impregnation resin Sikadur-330 was used in the experiments (Bisby et al. 2011). The elastic modulus and the strength of the adhesive from the manufacturer were 4.5GPa and 40.5MPa, respectively. The Poisson’s ratio was assumed to be 0.35 (Dean and Crocker 2001). The elastic-perfectly plastic adhesive model was found to have better predictions than the linear elastic model in Chapter 6. Thus the elastic-perfectly plastic adhesive model was used in the present FE analysis.

7.3.4 FRP

The FRP mechanical properties can be deduced from the fibre and adhesive properties and the fibre volume ratio based on the nominal and actual thickness of the FRP wrap using Hahn (1980)’s equations. For the specimens presented in Section 7.2, the elastic modulus in radial ($E_r$), hoop ($E_\theta$) and vertical ($E_z$) directions of the FRP were calculated to be 6.75GPa, 63.39GPa, 6.75GPa, respectively. The shear modulus $G_{r\theta}$, $G_{rz}$, $G_{\theta z}$ were found to be 2.78GPa, 2.59GPa and 2.78GPa respectively. The Poisson’s ratios were 0.033, 0.31, 0.31 for $\nu_{r\theta}$, $\nu_{rz}$, $\nu_{\theta z}$, respectively.

7.3.5 Mesh convergence

A mesh convergence study was conducted to determine the optimal element sizes in both concrete and FRP. Corresponding to the four minimum FRP element lengths of 0.09, 0.12, 0.18 and 0.24 mm, the number of elements used through the thickness of the FRP layer was 5, 4, 3 and 2 respectively. The number of elements used in the radial direction of concrete core was also varied, with 10, 20, 30 and 40 elements, using the
minimum FRP element width of 0.012 mm. The elements in the circumferential direction were the smallest at the ends of the FRP and increase gradually away from them using a bias factor of 1.03. The adhesive layer was divided into two elements in the radial direction. Figure 7.1 shows details of the finite element mesh using minimum element circumferential width of 0.12mm.

The effects of the FRP mesh size and the number of concrete elements in the radial direction on the predicted FRP strain efficiency are shown in Figure 7.9a and b respectively. Figure 7.9a shows that the results from element size of 0.09 mm and 0.12 mm are very close to each other. Figure 7.9b shows that the concrete radial mesh has very limited effect on the FRP strain efficiency. Therefore, the mesh with a minimum circumferential element size of 0.12 mm with 4 elements through the FRP layer thickness, and 30 concrete elements in the radial direction was deemed sufficient and used in all the subsequent analyses.
Figure 7.9 Effect of mesh on FRP strain efficiency
7.3.6 Predicted FRP strain distributions

Figure 7.10 shows that the predicted distribution of FRP hoop strain on the inner and outer surfaces of the FRP wrap when the maximum FRP hoop strain reaches its rupture strain provided by the manufacturer. This clearly shows that strain concentrations occur in two locations in the FRP wrap: (a) on the outer surface of the inner layer of the FRP adjacent to the outer end of the wrap (Location A in Figure 7.8) and (b) on the inner surface of the outer layer of FRP adjacent to the inner end of the wrap (Location B in Figure 7.8).

Figure 7.11 shows the detailed strain distributions predicted near locations A and B where the strain concentrations occur. It is evident that the strain concentration at Location A is more significant than that at Location B.
Figure 7.10 Distribution of hoop strains on the inner and outer surfaces of the FRP wrap

(a) Location A
7.4 Verification of FE model

7.4.1 Comparison of stress-strain curves

The FE predicted axial stress-strain curve, axial-hoop strain curve and axial stress-hoop strain curve are plotted in Figure 7.3a-c. In FE results, the hoop strains were obtained from the outer surface at the middle point of the FRP wrap outside the overlap zone, and the axial stresses were obtained from dividing the total reaction force by the cross-sectional area of the concrete column (the thickness of the FRP and adhesive are neglected).
The FE predicted axial stress in Figure 7.3a and c is larger than that predicted from Jiang and Teng’s (2007) model at a given same axial or hoop strain. Two reasons may be attributed to this difference: the contribution of the FRP and adhesive in resisting the axial force which is neglected here as is usually done in analysis test data, and the non-uniform stress distribution in concrete due to the existence of FRP overlap. The latter will be further investigated in this study. The ultimate state of the column in the FE analysis is assumed to be achieved when the maximum hoop strain of FRP reaches the ultimate tensile strain provided by the supplier (1.7%). Both of the ultimate compressive strain and the lateral strain from the FE predictions are higher than that from the experiments, this may be attributed to the fact that many factors, other than strain concentration at the ends of FRP wrap, affect the ultimate conditions of FRP wrapped concrete columns, as stated in Chapter 4.

### 7.4.2 Comparison of FE predictions with PIV results

Although detailed strains at each node are available from the FE analysis, they are not directly comparable with the strains from the PIV analysis because the latter represents the average strain within the gauge length. The FE predictions were thus further processed to deduce virtual strains using the same gauge length as in the PIV analysis at the ultimate state. The two sets of results are compared in Figure 7.6 and Figure 7.12. In Figure 7.12, the local nodal strain directly from the current 3D FE model is labelled ‘3D model original strain’, and the ‘3D model virtual strain’ is the calculated strain from the deformation of a 5mm gauge length. The two sets of results are almost identical, except
that original strain is available right to both side of the outer end of the FRP wrap. The virtual strain can only be available at a distance of half of the gauge length away from the outer end. Note that the results for specimen-5 are not available because the camera was not focused on Location A by mistake. The PIV and FE results both show that the hoop strain inside the overlap zone is obviously lower than that outside it (Figure 7.12) due to the difference in FRP thickness in the two regions. It is consistent that all results indicate that the strain approaches zero at the outer end of the FRP wrap on the outer surface of FRP in the overlap zone.

The FE predictions slightly overestimate the hoop strain outside the overlap zone compared with the PIV strains. This is probably due to factors other than the geometrical discontinuities that affect the strain efficiency results in the tests, as examined in Chapter 4. This results in a lower strain efficiency from the PIV strains than that from FE predictions.
Figure 7.12 Hoop strain distribution near the outer end of the FRP wrap

Figure 7.13 shows the axial stress versus deformation in a 5mm gauge centred at the outer end of the FRP from both the PIV measurement and FE prediction. It is seen that the FE prediction is in good agreement with the PIV data in average. This large deformation includes two components: that due to the hoop strain in the FRP within the gauge length, and the slip between the inner and outer layer of the FRP and the latter is usually dominant. This means that installing foil gauges across the finishing end bridging the inner and outer layers cannot measure the true strain there in the FRP, since the foil gauges can only measure the total deformation within the gauge length.
Figure 7.13 Deformation at the finishing end of FRP wrap: PIV measurement versus FE predictions

7.5 Examination of FE results

7.5.1 FE virtual strain of FRP

Figure 7.14 shows the strains calculated from the FE predictions using virtual gauge length varying from 1 mm to 20 mm, in order to simulate the actual strain gauge measurement in physical test. True strain calculated from the following equation is used in Figure 7.14:

\[ \varepsilon = \ln \left( \frac{l}{L} \right) \]  

(7.2)

where \( l \) is the final length and \( L \) is the original length of the gauge.
Foil strain gauges used on the FRP is usually 10 or 20 mm (e.g. Lam and Teng 2004; Jiang and Teng 2007 and Bisby and Take 2009). It is evident from Figure 7.14 that the strain concentration and the strain reduction near the outer end of the FRP wrap (Location A) is difficult to identify using 10 mm or 20 mm virtual strain gauge length. The peak strain would be significantly smaller than the FE prediction even if 1mm gauge is used. Furthermore, it is impossible to install a sufficient number of foil gauges in such a small zone to determine the strain distribution with sufficient details. Therefore, the strain concentration and distribution near Location A are difficult to measure using traditional foil strain gauges.

![Figure 7.14 Virtual strain distribution from of FE predictions](image)

*Figure 7.14 Virtual strain distribution from of FE predictions*
7.5.2 3D versus 2D models

In Chapter 6, the behaviour of FRP was modelled using a 2D plane strain FE ring model under either uniform internal pressure or uniform outwards displacement. Their accuracy is investigated here by comparing with the predictions of the 3D model in this Chapter. Details of the 2D models can be found in Chapter 6. All the models assume the ultimate state of the loading as the maximum FRP hoop strain over the circumference is equal to the ultimate tensile strain of FRP provided by the supplier (1.7%).

Figure 7.15 shows that considerable differences exist between the predicted FRP strains from these three models, with the predictions from the 3D model lie in between those from the two 2D models. Figure 7.16 compares the detailed strain distributions predicted by these three models at the vicinities of locations A and B where strain concentration occurs. Furthermore, the PIV hoop strain distribution near Location A at the ultimate state is also compared with the FE results from the three models in Figure 7.12, where ‘2D-dis’ represents the 2D model under uniform expansion, and ‘2D-pre’ represents the 2D model under uniform internal pressure. It can be found that the 2D uniform pressure model and the 3-D model predict similar distribution as the PIV results. The former predicts almost the same strain distribution at Location A as the latter, but smaller strains at Location B. The predicted maximum strain by the 2D uniform displacement model near Location A is smaller than that near Location B, which is contradictory to the results from the other two models.
Figure 7.15 Predicted hoop strain distributions on the inner and outer surfaces of FRP wrap from different models
Figure 7.16 FRP strain distribution near the ends of the FRP wrap

(a) Confining pressure on concrete
Figure 7.17 shows the predicted displacement and confining pressure on the concrete from the three models. Clearly the predicted displacement and pressure from the 3D model lies between those from the two 2D models. Therefore, the FRP in an FRP wrapped concrete column under uniform compression is neither under uniform internal displacement nor uniform internal pressure, but somewhere in between. Based on the numerical example analysed above, the 2D uniform pressure model (compared with the 2D uniform expansion model) is a closer representation to the 3D model in prediction of the FRP strain distribution.
7.5.3 Concrete stress distribution

Figure 7.18 shows the distribution of stresses in the concrete at the ultimate state. Figure 7.18a-d show that the stress distribution in the concrete is non-uniform. This is attributed to the presence of the overlap. The shear stress distribution in radial-vertical plane and hoop-vertical plane are not plotted because both stresses are very close to zero. The normal stress distribution from the middle point of the overlap zone (Point C) to its diametrically opposite point D, referred as Path C-D (Figure 7.18a-c), is shown in Figure 7.17e. Despite that the uniform vertical displacement is applied in this model (as in practical tests) the axial, hoop and radial stress distributions are all non-uniform due to the existence of FRP overlap. The radial, hoop and axial stresses at Point C are about 17%, 81% and 15% larger than those at Point D, respectively.
(b) Hoop stress (MPa)

(c) Axial stress (MPa)
(d) Shear stress $\tau_{r\theta}$ (MPa)

(e) Stress distribution at the diametric section C-D

Figure 7.18 Stress distribution in concrete
Figure 7.18a-d also shows that the stresses in the concrete vary significantly near both ends of the FRP wrap. Both the axial and the confining stresses at these locations are considerably lower than those in their adjacent regions. The lower confinement in turn causes a larger expansion of the concrete, which may amplify the local circumferential bending of the FRP wrap; such local bending has been identified as a major cause for the premature failure of the FRP wrap at these locations (Chen et al. 2010).

### 7.6 Conclusions

This chapter has presented a 3D FE study on the effect of geometric discontinuities at the ends of FRP wraps on FRP-confined circular concrete columns. A detailed FE analysis was presented and the predictions were compared with test results of six FRP wrapped concrete columns. The actual thickness of FRP layer and adhesive interface were measured. The PIV technique was used to analyse strain concentration at the outer end of the FRP overlapping zone. The predictions of the model have been shown to be in reasonably close agreement with the test results. This stress concentration represents one of the main causes for the lower apparent tensile strain of FRP from FRP-confined circular concrete columns than that observed in flat coupon tests. The results and discussions presented in this chapter also allow the following conclusions to be drawn:

a) The FE results have shown FRP hoop strain localizations occur in small zones near the ends of the FRP. Strain concentrations occur in two key locations on the FRP: on the outer surface of the inner layer of FRP adjacent to the outer end of the wrap; and on the inner surface of the outer layer of FRP adjacent to the
inner end of the wrap. The former location is more critical from the FE model examined in this chapter.

b) The actual thickness of FRP composites measured using a microscope in an FRP wrapped on plastic tube installed following the same process as in concrete columns using a wet-lay up process is much larger than the nominal thickness of the fibre sheet. A thin layer of adhesive does exist between two layers of the wet-lay up FRP, but the thickness of the adhesive layer is very small.

c) A detailed PIV analysis of slip and strain distribution at the outer end of the FRP wrap has been conducted on six FRP wrapped concrete columns. The PIV results are in reasonably good agreement with the FE predictions in terms of both the local slip and strain distributions, confirming the presence of the strain localisation.

d) Installing foil gauges across the finishing end bridging the inner and outer layers cannot measure the true strain in the FRP, since the foil gauges can only measure the total deformation within the gauge length, including the slip between two FRP layers. Furthermore, the virtual strain analysis from the FE results has shown that common foil strain gauge cannot be used to measure the localised strain concentration near the outer end of the FRP wrap as the area of strain localisation is much smaller than the length of these gauges.

e) The FE predictions from a 3D model lie in between those from 2D models under either uniform internal pressure or uniform displacement, with the predictions from the 2D uniform pressure model closer to the 3D model.
f) The radial, hoop and axial stress distribution in concrete is non-uniform due to the presence of the FRP overlap. The stresses in the concrete vary significantly near both circumferential ends of the FRP wrap. Both the axial and the confining stresses at these locations are considerably lower than those in their adjacent regions. The lower confinement in turn causes a larger expansion of the concrete, which may amplify the local bending of the FRP wrap; leading to premature failure of the FRP wrap at these locations.
CHAPTER 8
COMBINED EFFECT OF END CONSTRAINT AND FRP OVERLAP ON FRP WRAPPED CONCRETE COLUMNS

8.1 Introduction
As indicated in Chapter 4, many factors may lead to the lower-than expected FRP rupture strain, among all these factors, the end constraint and the FRP overlap of the columns are very important ones. However, the combined effects of these two factors are unknown. This chapter investigates the combined effect of end constraint and FRP overlap on the behaviour of FRP wrapped concrete columns using a three dimensional finite element (FE) model considering half the length of an FRP-wrapped concrete column. Comparison of different FE models is also conducted. Finally, different composite failure criteria are employed to investigate the ultimate condition of the FRP in FRP wrapped concrete columns.

8.2 Finite element modelling
Similar to Chapter 7, the multi-purpose finite element analysis package ABAQUS (2008) was used to conduct all of the analyses presented in this chapter. The specimens were modelled in three dimensions. FRP rupture was assumed to occur when the predicted maximum FRP hoop strain reached the ultimate tensile strain of the FRP provided by the manufacturer, unless otherwise stated. A predicted strain efficiency factor was then obtained by dividing the predicted average FRP hoop strain outside the overlapping zone
by the tensile rupture strain of the FRP. All the materials are modelled using three-dimensional solid element.

### 8.2.1 Geometry

With the aim of reducing the computational workload, a half height of the column was modelled. A circular concrete column confined with a single continuous FRP wrap comprising N layers/plies was considered (Figure 8.1). A polar coordinate system was used to describe positions (Figure 8.1), with the circumferential angular coordinate denoted by $\theta$. The FRP wrap starts at $\theta = 0^\circ$ (i.e. the inner end) and finishes at $\theta = 360N + \alpha$ (i.e. the outer end), giving an overlapping zone of $\alpha$, similar to the case assumed in Chapter 7.

![Figure 8.1 Assumed characteristics of model of FRP-confined concrete column](image-url)

Figure 8.1 Assumed characteristics of model of FRP-confined concrete column
8.2.2 Materials

The compressive behaviour of concrete is the same as described in Chapter 7 unless otherwise stated. The tensile model of concrete suggested in Yi and Chen (2010) is adopted in this study. The stress-crack opening displacement relationship proposed by Hordijk (1991) is adopted. This is given by:

\[
\frac{\sigma_t}{f_t} = \left[ 1 + \left( c_1 \frac{w_t}{w_{cr}} \right)^3 \right] \left( \frac{c_2 w_{cr}}{w_t} \right)^{-1} - \frac{w_t}{w_{cr}} \left( 1 + c_1^3 \right)^{(-c_2)}
\]

(8.1)

with

\[
w_{cr} = 5.14 \frac{G_F}{f_t}
\]

(8.2)

where \(w_t\) is the crack opening displacement, \(w_{cr}\) is the crack opening displacement at the complete loss of tensile stress, \(\sigma_t\) is the tensile stress normal to the crack direction, \(f_t\) is the concrete uniaxial tensile strength, and \(c_1 = 3.0\) and \(c_2 = 6.93\) are constants determined from tensile tests of concrete. \(f_t\) and the fracture energy, \(G_F\), may be estimated from the CEB-FIB (1993) model given by:

\[
f_t = 1.4 \left( \frac{f_c^{0.8} - 8}{10} \right)^2, \text{ MPa}
\]

(8.3)

\[
G_F = (0.0469d_a^2 - 0.5d_a + 26) \left( \frac{f_c}{10} \right)^{0.7}, \text{ N/m}
\]

(8.4)

where \(d_a\) is the maximum aggregate size. In the present study, \(d_a\) is assumed to be 20mm if no test data are available.
Once the stress-crack opening displacement relationship is known, the stress-strain relationship can be determined for each element based on its size through Eq. 8.1. A simple plastic degradation model proposed by Lubliner et al. (1989) is used for tensile damage. The plastic degradation occurs only in the softening range and the stiffness is proportional to the cohesion. Under uni-axial tension, the plastic degradation variable $d$ can be deduced from:

$$d = 1 - \frac{\sigma_t}{f_t}$$

(8.5)

The material properties of the FRP and adhesive used in this study are assumed to be the same as those used in Chapter 7.

### 8.2.3 Boundary conditions

The uniform vertical displacement was applied at the bottom-end of the model, the lateral displacements at the bottom-end and the vertical displacements at the mid-end were set to zero, unless otherwise stated. These boundary conditions were selected to represent the conditions of very high friction being applied between the ends of the FRP wrapped concrete column and the loading platens. The effect of the friction is examined further later in this chapter using an axis-symmetric model.

### 8.2.4 Mesh convergence

The effect of element size is very important when the materials being modelled with FE analysis have strain softening in the constitutive laws, i.e. with decreasing stress under
increasing strain. This is due to the fact that the FE result is highly sensitive to the element size when a strain softening is experienced in the analysis (Bazant 1976; Grassl and Jirásek 2006). Generally, FRP confined concrete has a hardening behaviour in the plastic stage, except if the amount of FRP is relatively low (Teng and Lam 2004). However, strain softening is quite pronounced in plain (unconfined) concrete under both compressive and tensile loading.

A mesh convergence study was conducted to determine the optimal element size in both the concrete and the FRP. The minimum circumferential element width was 0.12, 0.16, 0.24, and 0.47 mm, the number of elements used across the FRP layer were, 4, 3, 2 and 1, respectively. Furthermore, the number of elements used in the radial direction in concrete core was also investigated, with 5, 4, 3 and 2 elements being used. The number of layers along the height of the model were, 10, 8, 6 and 4, respectively. The illustrated FE model using minimum circumferential element width of 0.12mm is shown in Figure 8.2. The red, grey and green elements represents FRP, adhesive and concrete, respectively.

The adhesive layer was divided into two elements in the radial direction. The elements in the circumferential direction were smallest at the ends of the FRP and increase gradually away from the ends using a bias factor of 1.03. The effects of different meshes on the axial stress-strain curve and axial strain-hoop strain curve are shown in Figures
8.2a and b, respectively. The ‘10 layers’ to ‘4 layers’ represent the different models with 10 to 4 layers of elements in the vertical direction.

Figure 8.2 3-dimensional model of half of a FRP wrapped concrete column
As shown in Figure 8.3, The results from the mesh with a minimum circumferential element size of 0.12 mm and 4 elements through the FRP layer thickness, with an
element number of 5 in radial direction of concrete core, and 10 layers along the height has very little difference with the results from 8 layers mesh. Therefore, this mesh is believed to be deemed sufficiently accurate and used in all subsequent analyses.

8.3 Axisymmetric model

8.3.1 Effect of friction

In order to reduce the computational effort, an axisymmetric model was used to investigate the effect of friction, by ignoring the effect of overlapping zone of FRP. As discussed in Chapter 7, this effect causes the non-uniform horizontal stress distribution in the concrete. However, the interest in this section is the effect of friction on the vertical strain distribution or behaviour of the FRP confined columns, where the horizontal stress distribution should have very limited influence. The lower half of the column was simulated; the loading platen was modelled as a rigid body, the movement of which was governed by a reference node. The FRP wrapped concrete cylinders presented in Chapter 7 was modelled in this section, with a radius of 50 mm and a height of 200 mm. The failure criterion of the model was assumed to be by reaching the maximum FRP hoop strain, which is the ultimate tensile hoop strain, or 1.7%. The geometry, boundary conditions, and mesh are shown in Figure 8.4.
Figure 8.4 Axis-symmetry model of FRP wrapped concrete columns

The axial stress-strain curves for different amounts of assumed friction between the concrete cylinder and the loading platens are shown in Figure 8.5. These show that friction has little effect on the shape of axial stress-strain curves. The stress from FE results is deduced from total vertical reaction force applied on the cylinders divided by the area of the concrete, and the axial strain is taken as the true strain of the column over a 100 mm height at its middle; this is the typical way to measure the axial strain in experiments where conventional displacement instrumentation is used (e.g. Xiao et al. 2010). However, the ultimate condition of the column is predicted to be significantly influenced by the end constraint, particularly when the friction ratio is lower than 0.3. This phenomenon is further supported by the data shown in Figure 8.6 and Figure 8.7. Figure 8.6 shows the column strength and friction relationship, where it is clear that the axial strength changes significantly when friction is assumed in the range of 0 to 0.4.
Figure 8.7 shows the FRP hoop strain distribution along the height. These data agree well with overall experimental trends observed by Bisby and Take (2009) and Bisby and Stratford (2010). The FE results only present the 0 to 100 mm results, the results of 100 to 200 mm were generated using symmetry. Since the friction between the ends of the column and the loading platens provides constraint to the column, the strain and stress distribution are non-uniform in the axial direction (Ottosen 1984; Xiao et al. 2010). The FRP hoop strain near both ends is smaller than at mid-height. It is obvious that the curves change only very slightly when the friction ratio is larger than 0.3. Based on BS EN 12812: 2004, the minimum friction ratio between steel and concrete is 0.3, the maximum value of this is 0.4. The result using friction ratio of 0.3-0.4 should be close to the full constraint case. Therefore, full constraint at the ends of columns is considered in the following three-dimensional models.
Figure 8.5 Predicted axial stress strain curve of axis-symmetry model

Figure 8.6 Predicted axial strength-friction curve
Figure 8.7 Predicted effect of friction on the hoop strain distribution in the vertical direction

8.3.2 Effect of concrete constitutive model

Two concrete constitutive models are used in this section, one uses the concrete damaged plasticity model described previously, and the other uses a Mohr-Coulomb model. Tabbara and Karam (2008) used an elastic-perfectly plastic Mohr-Coulomb model to model concrete. The friction angle $\phi$ and dilation angle $\psi$ proposed in their paper have been used in a model. These are 35° and 12°, respectively. Therefore, the cohesion $c$ can be calculated using following equations (Chen and Han 1995),

$$
c = f'_c \frac{1 - \sin \phi}{2 \cos \phi}
$$

(8.6)

Young’s modulus of concrete, $E_c$, is assumed as:
End constraints are applied in the models. The stress-strain curves from analysis using each of the concrete models are compared in Figure 8.8. The failure criterion for both of the models was that the maximum FRP hoop strain reaches the FRP rupture strain. Figure 8.9 shows the maximum principal strain contour in the concrete at failure, the whole height of the concrete column across its diameter has been plotted based on the original axisymmetric model. It is evident that the shear band is more obvious when using a Mohr-Coulomb concrete model in the present case; a similar phenomenon of the formation of shear bands in concrete columns under compression has also been studied by Haskett et al. (2010) and Haskett et al. (2011). The model using a concrete damage model has less a obvious shear band when the FRP ruptures, but a triangle of concrete at the bottom appears to have little damage or strain due to the end restraint, similar to the model using a Mohr-Coulomb model for the concrete. The concrete damage plasticity model is used in the rest of the study, since the concrete damage plasticity model used in this study was developed based on analysis oriented model from Jiang and Teng (2007) which has close predictions for a number of test results in the literature. Besides this, the ratio of concrete tensile strength and compressive strength $n$ is fixed in a Mohr-Coulomb model for a given friction angle, as presented in Eq. 8.8, and the ratio is generally much higher than for concrete used in practice. The calculated ratio is 0.27 for the case studied here. This ratio is commonly assumed to be 0.1 to 0.15 in design.

$$n = \frac{1 - \sin \phi}{1 + \sin \phi}$$  
(8.8)
Figure 8.8 Axial stress-strain curves using two concrete models

Figure 8.9 Predicted maximum principal strain contour of concrete
8.4 Verification of 3D model

The experiments presented in Chapter 7 were used to compare against the 3D FE results in this study. The axial stress-strain curves and axial strain-hoop strain curves for both FRP-wrapped concrete columns are shown in Figures 9a and b, respectively. As suggested by Jiang and Teng (2007), the actual FRP rupture strain measured from the column tests is applied in the equation result. The average FRP rupture strain is taken as 0.0115.

Since the ends of the columns are frictionally constrained, the strain and stress distributions are non-uniform in the axial direction. Furthermore, because of the existence of the overlapping zone in the FRP, the stress and strain distributions are also non-uniform in the lateral direction. Similar to the Axisymmetric model, the stress from FE results is deduced from the vertical total load applied on the columns divided by the cross sectional area of the concrete, and the axial strain is the true strain of the 100 mm height in the middle of the column. It is obvious in Figure 8.10a that the stress-strain curve of FE model is higher than that predicted by Jiang and Teng’s model. This can be attributed to the existence of FRP overlapping zone and the FRP vertical stiffness, both of which are ignored in Jiang and Teng’s model but included in the FE analyses.

Figure 8.10b shows the comparison of axial strain-hoop strain curves. The hoop strain predictions of the FE analyses are taken as the hoop strain opposite the overlap, which is the same location as the measured strain in the experiments. It can be found that the FE
result is closer to the experiments than the prediction from Jiang and Teng’s model. The considerable difference between the FE model and the equation can not only be attributed to the existence of the overlap, but also largely to depend on the constraints at the ends of columns. The experimental evidence of the effect of constraints at the ends can be found in Bisby and Take (2009).

(a) Comparison of axial stress-strain curves from FE and tests (Bisby et al. 2011)
As shown in Figure 8.10a the predicted ultimate axial strain and hoop strain of columns are 1.49% and 1.44%, respectively. The average ultimate axial strain and hoop strain from the experiments are 1.21% and 1.19%. The prediction of ultimate axial strain from Jiang and Teng’s model is 1.57%, based on the FRP rupture strain from the tests. The FE model therefore overestimates both the ultimate hoop and axial strains of the FRP-wrapped concrete. More advanced FRP composite failure criteria are therefore examined further in this chapter.

### 8.5 Examination of 3D FE results

#### 8.5.1 FRP Hoop strain

Figure 8.11 shows the predicted distribution of FRP hoop strain on the inner and outer surfaces of the FRP wrap when the maximum FRP hoop strain reaches its rupture strain;
provided by the manufacturer. This clearly shows that strain concentrations occur in two locations in the FRP wrap: (a) on the outer surface of the inner layer of the FRP adjacent to the outer end of the wrap (Location A) and (b) on the inner surface of the outer layer of FRP adjacent to the inner end of the wrap (Location B) (Figure 8.1). It is also evident that the maximum hoop strain occurs at Location A in Figure 8.1, similar to the result of the one-layer FRP-wrapped concrete column model presented in Chapter 7.

Figure 8.11 Predicted inner and outer surfaces of FRP hoop strain distributions at mid-height

Figure 8.12 shows the hoop strain distribution on the outer surface of the FRP at different heights, from 0 to 100 mm, with a spacing of 20 mm. A height of 0 mm represents the bottom of the column, and a height of 100 represents the column mid-height. It is evident that the hoop strain at the mid-height along the FRP circumstance is
the largest over the height of the column. Figure 8.13 shows the hoop strains at different circumferential locations over the height of the column. Similar to Figure 8.6, the FE results only show the 0 to 100 mm results, the results of 100 to 200 mm are generated due to symmetry. Figure 8.13 shows the hoop strain distribution along height of the column at Location A, the middle of the overlap ($\theta = 237.3^\circ$ as shown in Figure 8.1b) and at the middle of the outside overlap ($\theta = 417.3^\circ$ as shown in Figure 8.1b). Figure 8.13 also clearly shows that the maximum hoop strain occurs at the mid-height of the columns. The peak strain is a maximum strain near Location A in Figure 8.1.

Figure 8.12 Predicted hoop strain distribution at the outer surface of the FRP at different heights
8.5.2 Concrete stress and strain distributions

In an FRP-wrapped concrete column the vertical and horizontal stress or strain distributions in the concrete are both non-uniform. The first is due to the friction between the both ends of columns and test platens, which provides an additional friction confinement to both ends of the concrete and results in non-uniform stress distributions in the vertical direction of the columns, as discussed previously, in particular near the cylinder’s ends (Ottosen 1984; Xiao et al. 2010). The second is due to the existence of the FRP overlap as discussed above. The horizontal non-uniform stress distribution of the concrete was investigated in Chapter 7. The non-uniform vertical strain distribution of the concrete is investigated in the following section.
The I-I section (shown in Figure 8.1) of stress and strain distributions in the concrete when the FRP hoop strain reaches the ultimate tensile rupture strain are shown in Figure 8.14 and Figure 8.15. The I-I section is the plane from the middle of the concrete edge inside overlapping zone to the middle of the concrete edge outside overlapping zone along the height of the column. The dashed line represents the axial axis of the column; the overlap zone is therefore to the right of the axis. It is evident that due to the end constraints on the end of the column, a wedge-shape of concrete at the bottom remains at a small strain, similar as for the axisymmetric model presented previously. As shown in Figures 8.13a and b, the hoop and radial strain of concrete outside overlapping zone is higher than that inside overlapping zone, due to higher confinement FRP overlap provides. Figure 8.14e shows significant radial-axial shear strain is experienced near the radial edge of the bottom, due to the end constraint. Only shear strain in the radial and vertical planes is shown in Figure 8.14, since the other two shear strain distributions are almost zero and nearly uniform.

(a) Maximum principal strain  
(b) Minimum principal strain
Figure 8.14 Strain distribution of concrete

(a) Radial stress               (b) Hoop stress
(c) Radial strain               (d) Hoop strain
(d) Vertical strain             (e) Radial-axial shear strain
Figure 8.15 Predicted stress distributions in the concrete

8.5.3 Comparison of different FE models

(a) Comparison of predicted axial stress-strain curves from FE models
Figures 8.15a and b compare the axial stress-strain curves and axial strain-hoop strain curves from the various FE models, respectively. The ‘Half-length model’ and ‘Axisymmetric model’ are the models presented in this chapter, and the ‘One layer model’ was presented in Chapter 7.

It is evident from Figure 8.16a that the shape of axial stress-strain curves is very similar in the half length model and the one layer model. However, the half length model has a lower ultimate axial strain, only 84% of that predicted by the one layer model; likely because of the effect of the frictional end constraints. The axial stress of these two
models is larger than that of the axisymmetric models corresponding to the same axial strain; likely due to the existence of the FRP overlap.

Figure 8.16b shows the shape of axial stress-strain curves is similar in the half length model and the axisymmetric model. The half length model has a lower ultimate axial strain due to the existence of FRP overlap. The absolute value of hoop strain from these two models is larger than that from the one layer models corresponding to the same axial strain, likely due to the effect of the end constraint. It should be noted that despite the differences between the axial strain-hoop strain curves, the FRP strain efficiency (i.e. ultimate hoop strain divided by 1.7%) obtained from the half-height model and the one layer model are very close to each other, 82.5% and 84.7%, respectively. This is because the end constraint has little effect on the strain concentration due to the FRP overlap in these two models. Therefore, it is reasonable and computationally effective to use the one layer model to investigate the strain concentrations associated with the FRP overlap. However, the one layer model is not recommended to study the overall behaviour of FRP wrapped columns due to its overestimation of ultimate axial strains.

**8.6 Strain efficiency prediction using different failure criteria**

In the previous studies the failure criterion for the FE model was that the FRP maximum hoop strain reached the ultimate tensile strain of the FRP. As examined in Chapter 5, many failure criteria have been developed for FRP composites, and some of the failure criteria have close predictions for a single lamina of FRP. The stress and strain results
from the FE model at Location A are used to predict the ultimate condition of FRP-wrapped concrete columns in this section. The strain efficiency results are shown in Table 8.1.

In Chapter 5, the compressive and tensile strengths of the adhesive were assumed to be the same due to the lack of available strength properties. In this section, the tensile and compressive strengths of the adhesive are reported from the adhesive supplier as 40.5 and 80 MPa, respectively. Therefore, the Tsai-Wu failure criterion considering the difference of compressive and tensile strengths is used. This criterion is expressed as:

\[
\sigma_r \left( \frac{1}{f_{rt}} - \frac{1}{f_{rc}} \right) + \sigma_t \left( \frac{1}{f_{tt}} - \frac{1}{f_{tc}} \right) + \sigma_z \left( \frac{1}{f_{zt}} - \frac{1}{f_{zc}} \right) + \frac{\sigma_r^2}{f_{rt} f_{rc}} + \frac{\sigma_t^2}{f_{tt} f_{tc}} + \frac{\sigma_z^2}{f_{zt} f_{zc}} + \frac{\tau_{rs}^2}{f_{rs}^2} + \frac{\tau_{ts}^2}{f_{ts}^2} + \frac{\tau_{zs}^2}{f_{zs}^2} \leq 1
\]

(8.8)

The predictions using Tsai-Hill and Tsai-Wu criteria are listed in Table 8.1. \(f_{rt}\) and \(f_{pc}\) are the transverse tensile strength and the compressive strength of FRP composites, respectively, and are considered as the same as the adhesive tensile strength and compressive strength. The shear strength of the FRP composite is usually difficult to quantify. Thus, it is assumed to be \(1/\sqrt{3}\) of the adhesive tensile strength, following the von Mises yield criterion, since the shear failure may occur within matrix. Unfortunately, the compressive strength of FRP composites in fibre direction is not available (and not
well known). Therefore, compressive strength is assumed to be the same as the tensile strength of the FRP composites in the fibre direction. The details of other properties are listed in Chapter 5.

### Table 8.1 Comparison of failure criteria

<table>
<thead>
<tr>
<th>Failure criteria</th>
<th>Strain efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{pt} = f_{pc}$ = 40.5 MPa</td>
</tr>
<tr>
<td>Tsai-Hill ($\sigma_r$ and $\sigma_\theta$)</td>
<td>0.67</td>
</tr>
<tr>
<td>Tsai-Hill ($\sigma_z$ and $\sigma_\theta$)</td>
<td>0.38</td>
</tr>
<tr>
<td>Tsai-Hill (3D)</td>
<td>0.26</td>
</tr>
<tr>
<td>Tsai-Wu ($\sigma_r$ and $\sigma_\theta$)</td>
<td>0.76</td>
</tr>
<tr>
<td>Tsai-Wu ($\sigma_z$ and $\sigma_\theta$)</td>
<td>0.35</td>
</tr>
<tr>
<td>Tsai-Wu (3D)</td>
<td>0.31</td>
</tr>
<tr>
<td>Maximum stress</td>
<td>0.55</td>
</tr>
<tr>
<td>Maximum strain</td>
<td>0.49</td>
</tr>
<tr>
<td>Experimental result</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.1 shows that the predictions using the Tsai and Wu criterion considering that all the stress components are the lowest ones among all the predictions. It is evident that the predictions using different failure criteria have significant differences with each other. The prediction from the maximum stress and maximum strain failure criteria considering both of the tensile and compressive strengths is much higher than the experimental result. This is because these two failure criteria do not consider any interaction between failure
strength and strain in the axial, transverse, or shear directions. Thus, these two failure criteria are not recommended for use in the prediction of FRP rupture.

The predictions considering all the stress components (i.e. Tsai-Hill 3D and Tsai-Wu 3D) are lower than the ones considering only two dimensional stress components. The strength for Tsai-Hill criterion is equal to tensile strength when the FRP is under tension, and equal to compressive strength when the FRP is under compression. The 3-D Tsai-Hill criterion considering both the tensile and compressive strength is not able to predict the failure even if the FRP reaches its ultimate tensile strain. The 3-D Tsai-Wu criterion considering the tensile and compressive strength is able to provide a conservative prediction of the strain efficiency. Therefore, despite the fact that the prediction of the Tsai-Wu criterion is lower than the experimental result, this criterion is considered to be the most appropriate among the four failure criteria examined above.

8.7 Conclusions

This chapter has presented results of three dimensional FE modelling of FRP-wrapped concrete columns. The predictions of the model are shown to be in reasonable agreement with test results. The effect of the geometrical discontinuity of the FRP overlap and frictional confinement between the ends of column and the loading platens were examined. Furthermore, different composite failure criteria were employed to investigate the ultimate condition of the FRP wrap. The results and discussions presented in this chapter allow the following conclusions to be drawn:
a) The friction between the both ends of columns and the loading platens can significantly affect the FRP hoop strain development, and thus influence the ultimate condition of the column. On the other hand, friction has little effect on the axial stress-strain behaviour. Friction larger than 0.3 in the case studied here is predicted to have little effect on the behaviour of columns.

b) The FE results showed that the maximum FRP hoop strain occurs at the mid-height of the column, near the finishing end of the FRP wrap. The ultimate axial strain predicted by the FE model is close to the experimental results, provided that the failure criterion of maximum FRP hoop strain reaching the ultimate tensile strain is considered; however the prediction of strain efficiency is higher than the experimental results. The end constraint has little effect on the strain concentration caused by the geometrical discontinuities of the FRP overlap, but the ultimate axial strain is significantly overestimated if the end constraint is not considered.

c) Despite the prediction of a Tsai-Wu failure criterion being lower than the experimental results, this criterion is considered to be the most appropriate one among the four failure criteria examined. None of the four failure criteria examined above, considering all the stress or strain components, gives a close prediction with all the experimental results, and further investigation of failure criteria is required for a better numerical estimation of FRP hoop strain efficiency.
CHAPTER 9
CONCLUSIONS AND FUTURE RESEARCH

9.1 Introduction
The main objective of this study was to investigate the phenomenon of lower apparent FRP rupture hoop strain measured in FRP confined concrete columns than the ultimate rupture strain obtained from the flat coupon test, and the fundamental mechanism behind it. This phenomenon is widely observed in almost all kinds of FRP confined circular columns, such as reinforced concrete (RC) columns, concrete-filled steel tubes and FRP-concrete-steel hybrid double-skin tubular columns. The low apparent FRP rupture strain has been one of the major concerns for the design of FRP-confined columns, despite the fact that a number of FRP-confined columns models have been developed. To achieve this objective, an extensive literature review was conducted, FE models and analytical solutions were developed, validated, and used to conduct parametric studies and examine the fundamental mechanics; a series of FRP-wrapped concrete cylinders were also tested with careful examination on FRP local rupture strains.

An FE study of FRP rupture strain in split-disk test was first carried out, since the experimental results in the literature show that the FRP rupture strain from split-disk test is considerable lower than that from flat coupon tests, and closer to that from FRP-wrapped concrete columns than that from flat coupon tests. The failure mode of FRP
wrapped circular columns were classified, with the potential contributory factors to the low apparent FRP rupture strain identified. How these factors may affect the behaviour of FRP wrapped circular concrete columns was also discussed. One of the contributory factors, the triaxial stress state of FRP was then examined in detail using analytical solutions.

Another important contributory factor, geometrical discontinuities at the ends of an FRP jacket, has also been investigated. FRP-wrapped concrete-filled steel tubes and FRP-wrapped concrete columns were examined. Both two and three dimensional FE models have been developed. The combined effect of end constraint and FRP overlap was also examined. Furthermore, different composite failure criteria were employed to investigate the ultimate condition of FRP.

### 9.2 Overall conclusions

This study has for the first time revealed the fundamental mechanics in split-disk test of composites and provided a thorough explanation why the rupture strain (stress) obtained in this test is significantly lower than that obtained from the flat coupon test. Large strain localisation exist in the FRP ring in the split-disk test due to the presence of geometric discontinuities at the ends of the FRP and circumferential bending of the FRP ring at the gap arisen from the change of curvature there caused by the relative moment of the two half disks. Being a linear elastic brittle material, the FRP ruptures once the strain at one
of these locations reaches the FRP rupture strain, leading to a lower apparent tensile strength than that obtained from flat coupon tests.

This study has also classified the failure modes of FRP wrapped concrete columns under uniaxial compression, and identified for the first time a list of contributory factors affecting the lower apparent FRP rupture strain in FRP wrapped columns. One of the important factors, the triaxial stress state in the FRP wrap, has been shown to have a potentially significant effect on the failure of the FRP wrap but considerable discrepancies exist between predictions using different failure criteria and further research has been identified in this area.

Both test data and FE analysis conducted in this study have shown that severe FRP hoop strain concentrations occur in very small zones near the ends of the FRP wrap in FRP wrapped concrete-filled circular steel tubes and FRP wrapped concrete columns, which can lead to premature FRP rupture in an FRP wrapped column, thus significantly reduce the apparent hoop strain of FRP. It is also for the first time identified in this study that FRP wraps with an overlapping zone lead to non-axisymmetric confinement to the concrete and hence a non-uniform stress distribution in the concrete, even when the concrete is under uniform axial shortening.
The friction between both ends of a column and the loading platens provides constraints to ends of the column, but this constraint has little effect on the strain concentration caused by the geometrical discontinuities of the FRP overlap. However, the ultimate axial strain of the FRP wrapped columns can be significantly overestimated if the end constraints are not considered.

9.3 Detailed conclusions

9.3.1 FRP rupture strains in split-disk tests

Chapter 3 presented an FE analysis of strain distributions in the split-disk test of FRP wraps, with the aim of helping to explain the observed reductions of ultimate FRP rupture strains from the split-disk tests as compared with those observed from flat coupon tests. FE modelling issues were first examined, with particular attention paid to the modelling of orthotropic FRP and the adhesive. Comparisons between the predictions of the model and test results were conducted to demonstrate the validity of the FE model. The effects of key test parameters were also examined. The following conclusions can be drawn from this study:

a) The FE predictions show that strain concentrations occur in the FRP in localized zones at three locations where geometrical discontinuities are present: (1) on the outer surface of the inner layer of FRP adjacent to the outer end of the wrap; (2) on the inner surface of the outer layer of FRP adjacent to the inner end of the wrap; and (3) on the inner surface of the inner layer near to the gap created by the relative
movement of the two half disks. Local bending due to the geometric discontinuities at the ends of the FRP and circumferential bending of the FRP ring at the gap arising from the relative moment between the two half disks results in increased local strains in the FRP ring in the split-disk tests.

b) Because these zones are very small it is not possible to measure such concentrated strains using conventional measurement techniques. This explains why the measured ultimate FRP strain in split-disk tests is typically lower than the rupture strain obtained from flat coupon tests.

c) At the ultimate condition, the FE predicted strains away from the strain concentration and overlapping zones are close to those observed in split-disk tests based on limited test results available in the literature. The predicted maximum strains in the three key locations are significantly larger than those remote from these locations, and the maximum value of the peak strains at the three key locations is close to the rupture strains observed in flat coupon tests. This suggests that the FRP ruptures once one of the peak strains at these locations reaches the rupture strain, leading to lower apparent tensile strength than that in flat coupon tests.

9.3.2 Contribution factors for premature FRP rupture

The failure modes examined in the literature and in tests have been discussed in Chapter 4 to discover the causes of the low apparent hoop strain. A list of 17 contributory factors affecting the apparent FRP rupture strain in FRP wrapped columns, ranging from
geometrical discontinuity, 3D stress state to geometrical imperfections and non-uniform support in test setup, has been identified through this exercise. The effects of many of these factors have not been quantified to date and the interaction between them makes the issue even more complex. Therefore, a substantial amount of future research is required to better understand the behaviour of FRP confined columns and to quantify the apparent FRP rupture strain for design.

9.3.3 Effect of triaxial stress state in FRP

Chapter 5 developed an analytical procedure to determine the strain reduction factor of FRP by considering various failure criteria. A parametric study was also conducted. The following conclusions can be drawn based on the analysis:

a) The triaxial stress state in FRP does affect the ultimate condition of FRP composites in FRP confined concrete columns and the effect can be very significant.

b) The prediction of various failure criteria for triaxial stress state significantly depends on the parameters of material properties and FRP-wrapped columns, such as transverse strength of FRP, the unconfined concrete strength, and the number of FRP layers. A higher vertical strength of FRP, higher unconfined concrete strength and less number of FRP layers lead to a higher FRP strain efficiency, if only the effect of three-dimensional stress is considered. However, some of these predictions are not in agreement with test results, indicating that either other factors are playing more significant roles, or the failure criteria are not suitable for the specific case, or both. Further research is required to clarify this.
c) The Tsai-Wu criterion considering all stress components has the most conservative strain efficiency prediction, and is believed to be more applicable to failure prediction of composite materials. The results considering the vertical and hoop directions (neglecting the radial stress) is much closer to the three-dimensional results, comparing with the one considering the radial and hoop stresses (neglecting the vertical stress).

9.3.4 Effect of geometrical discontinuities on FRP-wrapped concrete-filled steel tubes

Chapter 6 presented a two dimensional model for the FRP-filled steel tubes. In such a column, the FRP jacket is separated by a steel tube from the concrete core, so that the non-uniform deformation of the cracked concrete core, which is believed to be an important factor responsible for the observed reduction of rupture strain in FRP confining concrete columns, may play a less important role. The following conclusions can be drawn based on this work:

a) Of the nine test specimens of FRP-confined concrete-filled steel tubes, four failed at the finishing end of the FRP jacket, demonstrating the potential importance of geometric discontinuities at the two ends of the FRP jacket on its ultimate condition.

b) The FE predictions show that severe stress concentration occurs at two key locations on the FRP jacket: on the outer surface of the layer of FRP adjacent to the outer end of the jacket; and on the inner surface of the layer of FRP adjacent to the inner end of the jacket.
c) These highly localized strain concentrations occur in very small zones near the two ends of the FRP jacket and cannot be measured using conventional techniques. This offers a plausible explanation for the reduction in the apparent rupture strain of the FRP jacket. This is also consistent with the test observation that four of the nine specimens examined failed at the finishing end of the FRP jacket.

d) The properties of the adhesive can affect the strain values at the ends of the FRP jacket significantly, thus affecting significantly the strain efficiency of the FRP. An adhesive with a low yield stress but substantial plasticity reduces stress concentration and increases the FRP strain efficiency. FE results assuming an elastic-perfectly plastic behaviour for the adhesive are in reasonable good agreement with available test data.

e) A parametric study has shown that the FRP strain efficiency increases with the FRP elastic modulus and the FRP isotropy but decreases with the FRP thickness and the adhesive yield stress. The effects of column diameter and adhesive thickness on the FRP strain efficiency are insignificant.

9.3.5 Effect of geometrical discontinuities on FRP-wrapped concrete columns

Chapter 7 presented three dimensional FE models for investigating the effect of geometrical discontinuities at the ends of an FRP jacket on FRP confined circular concrete columns. An improved concrete model developed by Yu et al. (2010) based on concrete damage plasticity was used. Furthermore, the PIV technique was used to
experimentally investigate and quantify the strain concentration at the finishing end of FRP wrap. Finally, a comparison of FE predictions and PIV measurements was conducted to verify the FE model. The following conclusions can be drawn based on this work:

a) The FE results have shown FRP hoop strain localizations occur in small zones near the ends of the FRP similar to the case of FRP confined concrete filled steel tubes. There is relatively lower confinement to the concrete adjacent to the two locations where strain concentrations occur in FRP wrap, this may also lead to a premature failure of FRP due to the fact that the concrete is relatively weak under low confinement.

b) A detailed PIV analysis of slip and strain distribution at the outer end of the FRP wrap has been conducted on six FRP wrapped concrete columns. The PIV results are in reasonably good agreement with the FE predictions in terms of both the local slip and strain distributions, confirming the presence of the strain localisation.

c) FRP wraps with an overlapping zone lead to non-axisymmetric confinement to the concrete and hence a non-uniform stress distribution in the concrete, even when the concrete is under uniform axial shortening.

### 9.3.6 Combined effect of end constraints and FRP overlap

Chapter 8 investigated the combined effect of end constraint and FRP overlap on the behaviour of FRP wrapped concrete columns using a three dimensional FE model considering one half of the length of an FRP-wrapped concrete column. Comparison of
different FE models was also conducted. Different composite failure criteria were employed to investigate the ultimate condition of the FRP in FRP wrapped concrete columns. The following conclusions can be drawn:

a) The friction between the ends of the column and the loading platens can significantly affect the FRP hoop strain development, but has little effect on the axial stress-strain behaviour. A friction coefficient larger than 0.3 in the case studied was predicted to have little effect on the behaviour of columns.

b) The FE results showed that the maximum FRP hoop strain occurs at the mid-height of the column, near the finishing end of the FRP wrap. The ultimate axial strain predicted by the FE model is close to the test results. However the prediction of strain efficiency is higher than the experimental results. The end constraint has little effect on the strain concentration caused by the geometrical discontinuities of the FRP overlap, but the ultimate axial strain is significantly overestimated if the end constraint is not considered.

c) Despite the prediction of a Tsai-Wu failure criterion being lower than the experimental results, this criterion is considered to be the most appropriate one among all the four failure criteria examined. None of the four failure criteria examined, considering all the stress and strain components, gives a close prediction with all the experimental results. Further investigation of failure criteria is required for a better estimation of the FRP hoop strain efficiency.
9.4 Recommendation for future research

The research work that has been done in this study covers an in-depth investigation into the lower-than-apparent FRP rupture strain in FRP-wrapped concrete columns. This research has led to a good understanding of this phenomenon and the fundamental mechanisms. However, this research cannot be exhaustive due to time limitations. The effects of many of the identified contributory factors have not been investigated. Further research is required to investigate their effects and develop better understanding so that more rational and economical design methods can be developed. Example areas of further investigation are listed as follows.

a) Only circular columns have been studied in this thesis. Examinations of strain efficiency in rectangular and elliptical columns are required.

b) The effect of fibre orientations on FRP strain efficiency in FRP-wrapped columns, including multi-layer FRP composites with different fibre orientations in each layer.

c) Effect of slenderness on FRP strain efficiency in long columns.

d) Some of the contributory factors have not been investigated in this study and in literatures, to the best of the author’s knowledge, such as unintentional stressing and non-uniform supports, etc. Further study is required to understand the effects of these factors.

e) The effect of cyclic loading on the strain efficiency of FRP-confined columns.

f) The strain efficiency of FRP-confined columns under loading other than uni-axial compression, such as flexural loading and eccentric loading.
g) Development of design guidelines for the FRP strain efficiency that can be easily used by engineers for the design of FRP-confined columns.
REFERENCES


ACI. (2008). “Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures, ACI 440.2R-08.” American Concrete Institute, Detroit.


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