Progressive pressure measurements beneath a granular pile with and without base deflection

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ABSTRACT: This paper describes an experimental investigation to measure the pressure distribution underneath a conical pile of granular material as the pile grows. Several factors have been suggested to explain the pressure dip observed under the apex of a pile but the relative significance of these factors is far from clear. This study was conducted using quite round rough particles in a relatively large scale experiment to avoid the possibility that particle scale effects would mask the macroscopic pressure distribution. A reproducible pressure profile with a significant central dip was observed in each test, which confirms that at macroscopic scale, the pressure dip is a stable and robust phenomenon when the pile is formed from a localised jet. The normalised data provide quantitative information on the vertical force redistribution away from the central zone. The results also show that base deflection is not a prerequisite for the pressure dip, but that it enhances both the magnitude and the width of the dip.

1 INTRODUCTION

Granular materials are in abundance in nature and are also estimated to constitute over 75% of all raw material feedstock to industry (Nedderman 1992). They have been extensively studied by both the scientific and engineering communities, and yet they sometimes display behaviour that is counter-intuitive and a full understanding remains elusive. One classic granular mechanics problem is that of a humble ‘sandpile’ in which a significant dip in the vertical pressure on the base is observed underneath the apex of a poured pile, at the location where a simple interpretation might expect the maximum pressure. This ‘sandpile’ phenomenon is relevant to the bulk handling of industrial solids because many different bulk solids are commonly stored in open stockpiles, particularly in the mining industry (Fig. 1). The design of a gravity reclaim system for a stockpile requires knowledge of the base pressure distribution underneath the stockpile. The same phenomenon must also occur in silos that are filled from a ‘point source’ which might be expected to result in increasing the silo wall pressures near the highest wall contact. But this phenomenon is not recognised at all in the silos experimental literature.

The sandpile problem has been the subject of many analytical, numerical and experimental studies and good reviews of the problem can be found elsewhere (e.g. Atman et al. 2005, Cates et al. 1998, Savage 1997). There is little consensus on the fundamental physics and mechanics assumptions made between the many mathematical models of this apparently simple system, and quite contradictory results are often claimed. Several factors have been suggested to explain the pressure dip observed under the apex of a pile. These include the presence of a base deflection (Savage 1997), the pile construction history (Geng et al. 2001, Vanel et al. 1999), formation of a granular skeleton (Savage 1997), particle shape (Zuriguel et al. 2007) and “reduced local density due to increased filling rate” (Smid & Novosad 1981). However neither the relative importance nor the interplay between these factors is at all clear and a comprehensive understanding of this phenomenon remains elusive. This paper describes experiments used to investigate the base pressure profile under a granular pile of approximately spherical particles, with and without base deflection.
A variety of measurement techniques have been used to measure the pressure distribution on the base of a granular pile, including pressure cells (McBride 2006, Smid & Novosad 1981), registering the load on articulated base strips (Lee & Herington 1971), strain gauges mounted on base plates (Jotaki & Moriyama 1979), elasto-optical method (Brockbank et al. 1997), and photoelastic methods using the gradient of the light intensity (Geng et al. 2001, Zuriguel et al. 2007). The majority of these experiments were relatively small scale or suffered from significant fluctuations in the deduced pressures, sometimes of an order comparable with the magnitude of the dip being measured (e.g. Lee & Herington 1971), or required the averaging of many repeated experiments (e.g. Geng et al. 2001, Zuriguel et al. 2007) before the pressure dip could be seen as a mean phenomenon. These results have led some to believe that the pressure dip beneath a conical pile is not a securely reproducible phenomenon and its formation can be sensitive to numerous factors. In this study, relatively large scale granular pile experiments were conducted in which the base pressure can be measured relatively accurately. The measurements show a high degree of repeatability with relatively small scatter. The experiments also captured the progressive development of the base pressure form as the pile grew.

The overall experimental plan involves a series of relatively large scale granular pile tests to investigate several factors affecting the base pressure profile. These factors are the base deflection, construction history (pouring jet dimension and drop height), particle shape and size variation. All these factors have been speculated by others to be probable causes of the pressure dip, as outlined above. In this paper, the granular pile experiments using a concentrated pouring jet with and without base deflection are reported and discussed.

2 EXPERIMENT SETUP AND TEST MATERIAL

In the tests reported here, the granular particles were small and approximately spherical iron ore pellets which had a very rough surface (Fig. 2). These pellets were relatively uniform in size with a mean diameter of $d_p=3.0 \text{ mm}$ and a size range of $2.5<d_p<3.8 \text{ mm}$ for 10% to 90% passing in particle size analysis by dry sieving. These particles are interesting because they are approximately spherical but sufficiently non-spherical to destroy the degenerate symmetry observed in spherical assemblies. This choice allows a comparison to be made with a recent study involving elongated particles, which are thought to significantly enhance the pressure dip when compared with circular particles (Zuriguel et al. 2007). The pellets also have the added advantages that they have: i) a high density, allowing a greater sensitivity in pressure measurement; and ii) a relatively uniform bulk density that is insensitive to packing (the loosest and densest bulk densities achieved in control tests being 2260 and 2370 kg/m$^3$) thus minimising the effect of bulk density variation during pile formation. Density variation should therefore not be a key part of any explanation for the phenomena observed here. Using a direct shear tester, the internal angle of friction for the pellets was measured to be $34^\circ$.

Free-field pressure cells have been widely used to observe pressures in granular media (Askegaard 1978, 1981, 1986, Munch-Andersen 1982). The cells were designed and manufactured by Askegaard (1989) using well-established procedures that are described in detail elsewhere. Figure 3 shows the Askegaard pressure cell used in this study. It has a diameter of 75mm which is 25 times larger than the mean particle size, giving more than 400 contacts on each cell face. This makes the measurement effectively independent of the force chain structure in the solid. The cell face is very stiff (face flexibility $10^{-2} \mu\text{m/kPa}$). Each cell was calibrated with the cell embedded in a stiff granular solid in a specially designed calibration chamber.
In the experiments, the pressure cells were first placed carefully at fixed positions along a radial line on the flat wooden base plate. A layer of pellets approximately 25mm thick was then spread evenly so that the pressure cells were firmly embedded, with a thin layer covering over each cell face. The top surface of this layer was taken as the nominal base and the pressure cell readings were taken as zero at this point. The granular pile was then constructed using a concentrated pouring jet with a radius $R_j=16\text{mm}$ ($R_j/d_p=5.3$) located centrally at $674\text{mm}$ above the base. The pressures resulting from this process were recorded during the pile formation. A schematic diagram of the granular pile test showing the positions of the surface profile measurement is given in Figure 4. The symbols used in this paper for the various parameters describing the pile geometry and the base pressure profile are shown in Figure 5.

3 RESULTS AND DISCUSSION

3.1 Rigid base

Five repeat tests were conducted with a stiff base producing piles with a mean radius of $R_p=554\text{mm}$ and a coefficient of variation (CoV) of 2.6%. These five tests can thus be taken as repetitions of the same configuration. The surface profile may be characterised by a parabolic crown with a linear conical slope (Fig. 5). Figure 6 shows the final surface profiles for the five tests. The angle of repose had a mean value of $\beta=29.0^\circ$ with a CoV=2.3%, determined from the middle of the conical slope and away from both the apex and the tail of the slope. Figure 7 shows the vertical pressures measured at the end of pile formation on the base at seven radial positions, plotted against the normalized radius (relative to the mean base radius $R_p$ deduced from diametral measurements). The measured vertical coordinate $z$ of the surface of the pile at six radial positions is also shown for reference. The results show a robust and reproducible pressure profile with a significant dip under the apex, rising steadily from a minimum pressure at the centre to a peak at the radius of $r \approx 0.3R_p$, before falling off towards the edge of the pile. The pressure profile is very similar in form to the smaller conical pile results of Vanel et al. (1999) and the much larger gravel pile of McBride (2006). The results support the commonly stated proposition that an arching effect of some kind results from the formation process and causes a significant part of the weight of solid in the central zone to be supported by an annular zone at larger radii.

The pressure dip under investigation is best explored by comparing the measured base pressure with the hydrostatic value $\gamma z$ associated with the local vertical distance from the nominal base to the pile surface $z$. The vertical coordinate of the sur-
face profile $z$ has also been normalised using the mean base radius of the pile $R_p$. The mean and the standard deviation of the normalised pressure and normalised surface profiles for the same five tests are shown in Figure 8. The central pressure minimum is seen to fall below 50% of the hydrostatic value and remains below the hydrostatic value (unity) throughout the central zone up to a radius of $r \approx 0.6 R_p$. For vertical equilibrium, the vertical pressure must exceed the hydrostatic values in the outer zone to account for the much reduced pressures in the central zone: this is evident from this normalised plot. The error bars plotted in Figure 8 represent ± one standard deviation from the mean: the larger error bar for the last pressure measurement near the pile edge is caused by the very small magnitudes of pressure measured there. Elsewhere, the standard deviations of the measurements are all quite small, indicating the good repeatability of the pressure measurements.

The results of these tests thus confirm that the base pressure under a conical pile with a central local minimum is a robust event that occurs naturally when the pile has been constructed using a concentrated pouring jet. The proposition that size segregation during pouring is a main cause for the pressure dip is not supported here since the size variation in these pellets is too small to permit significant size segregation. The proposition that elongated particles, which can form a preferential anisotropic packing structure upon pouring, may be essential for a significant pressure dip is also not supported by this study. However it is still likely that elongated particles with strong orientation effects exaggerate the dip effect (Zuriguel et al. 2007).

A key aspect of the pressure distribution beneath the pile is its progressive development. The evolution of the base pressure profile recorded during the construction of one pile is shown in Figure 9. There is little evidence of a significant dip when the pile is small. This may indicate that the relationship between the width of the jet and the pile diameter may play a significant role in affecting the depth of the pressure dip. It may be noted that Vanel et al. (1999) showed that no pressure dip occurred when the pouring jet was as wide as the base radius. As the pile grew bigger, the pressures in the outer zone continued to increase slightly faster than the reference hydrostatic pressure value. The pressure in the central zone also increased, but at a much slower pace, so that the pressure dip became steadily more pronounced. This supports the proposition that, for any macroscopic granular pile where the pile dimension is much larger than the dimension of the concentrated pouring jet, a robust pressure profile with a central dip is a natural formation which occurs reproducibly. By contrast, some published results (e.g. Brockbank et al. 1997, Geng et al. 2001, Zuriguel et al. 2007) show considerable fluctuations and a much less well defined pressure dip even after a considerable amount of averaging over many repeated experiments. It is thus probable that these fluctuations are caused by the relatively small ratio of pile size to particle size in those experiments.
For this set of experiments, the central pressure dip began to emerge when the pile outer edge reached $r/R_p \approx 0.3$ which corresponds to a base dimension of about $55d_p$ where $d_p$ is the mean particle size. This indicates that with a pouring jet of width $5.3d_p$ used here, the pile had to grow to a base radius of about 10 times the pouring jet dimension before a dip could be clearly seen. Further explorations are planned to investigate the importance of the two length ratios: the pouring jet width $R_j$ and the pile radius $R_p$ relative to the mean particle size $d_p$ in affecting the formation process that leads to a pressure dip.

### 3.2 Flexible base

In the tests to explore base deflection, a 1220mm square thin plate was supported on its four corners to permit the base to deflect into a dome shape. At the end of pile formation, the vertical deflection of the plate was 30.0mm at the centre and 17.3mm at the edge of the pile. Thus the relative deflection of the base beneath the pile was approximately $4.2d_p$. The pressure profiles found for the rigid base and in two tests using this flexible base are compared in Figure 10. It shows that the base deflection reduced the pressure in the central zone further and caused the width of the pressure dip to increase. This base deflection therefore increases the arching effect in the granular solid, causing even more of the weight to be carried on the annular outer zone.

![Figure 10. Influence of base deflection on pressure profile](image)

### 4 CONCLUDING REMARKS

These experiments explored the base pressure profile under a conical granular pile that has been centrally poured. They showed that this base pressure distribution, at the macroscopic scale, has a central dip beneath the apex of the pile that is a repeatable and robust phenomenon. The results also show that deflection of the base is not a prerequisite for this dip in the pressure profile, such a flexible base does enhance both the magnitude and the width of the dip. It has also been shown that elongated particles and particle size segregation are not prerequisites for the pressure dip formation. However the construction history, in the form of a concentrated pouring jet leading to avalanching down the conical slope of the pile during formation, does appear to be an important factor. These aspects are being further investigated using both this experimental setup and discrete particle computational studies.

### 5 ACKNOWLEDGEMENTS

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Finite Element Prediction of Progressively Formed Conical Stockpiles

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Abstract: Conical piles of granular solids can be found in many industrial sites. These piles are usually progressively formed by depositing from above. A classic question concerning such simple piles is the observation that the pressure distribution beneath the pile shows a marked local minimum beneath the apex which is counter-intuitive as this should be the location expected to have the maximum pressure. Numerous experimental, analytical and computational studies have been conducted to investigate this classical problem over the last few decades, but a comprehensive understanding of the problem remains elusive. A number of recent finite element simulations of the pile have considered the effects of construction history, plasticity and stress-dependence of modulus of the granular solids. Whilst a pressure dip beneath the apex has been predicted, significant uncertainties remain about the effects of these factors on the pressure dip and their interaction.

This paper presents the finite element modelling of a conical stockpile using ABAQUS. The effect of construction history was realized by simulating the progressive formation of the conical pile. This was achieved by discretising the final geometry of the stockpile into multiple conical layers and then activating each layer sequentially. The effects of the elastic and plastic parameters were explored. The results show that a pressure dip may or may not be predicted depending on the constitutive model and the values for the model parameters. The study also shows that modelling the conical pile in one single step does not produce the pressure dip. It further shows that the central pressure dip is predicted using a relatively small number of layers and the magnitude of the dip is not sensitive to increasing number of layers, which is in contrast with one previous study.

Keywords: Sandpile, Stockpile, Stress distribution, Pressure dip, Pressure dependent modulus, Progressive layering, Incremental construction.

1. Introduction

The behaviour of granular solids has attracted much attention of researchers from many communities such as Applied Mechanics, Geotechnical Engineering, Chemical Engineering, Materials Handling, Agricultural Engineering and Geophysics. The storage and handling of granular materials is essential to many industries (Nedderman, 1992). Where the material is held in very large quantities, it is often stored in a stockpile, formed by dumping the solid (e.g. coal and
mineral ore) to form a pile whose overall shape is typically conical, but may be prismatic, depending on the method of placement (Figure 1). The solid is often recovered from the stockpile using a conveyor beneath its centre. In such a case, the structure containing the conveyor needs to withstand the pressures exerted by the stockpile. Consequently one key aspect of stockpile design is to determine the pressure pattern beneath it. The experimental finding that there is a significant local reduction in pressure (Figure 2) beneath the apex of the pile below the values one might expect can have a strong impact on the design requirements. This reduction in pressure which is commonly known as the “sandpile” problem has mostly been studied in the past as an interesting scientific anomaly, but the stockpiles make it of considerable economic importance.

Figure 1. A typical industrial stockpile

Despite extensive studies by both the physics and engineering communities over several decades, a comprehensive understanding of the counter-intuitive phenomenon of the pressure dip remains elusive. Good reviews of previous analytical, numerical and experimental studies of the problem

Figure 2. Vertical base pressure underneath a granular pile

a) Smid and Novasad (1981)  
b) Ooi et al. (2008)
can be found in Savage (1997, 1998) and Cates et al. (1998). Among these studies, very few adopted the finite element method (FEM), even though it is very powerful and flexible in dealing with complex loading and boundary conditions. This paper investigates the feasibility of adopting the FEM to model the sandpile problem and explores how the observation from such modelling may provide further insight into the physical mechanism of the problem from a continuum point of view.

It has been observed experimentally that the pile construction history is an important factor in the occurrence of central pressure dip beneath the stockpile (Vanel et al., 1999; Geng et al., 2001). For a sandpile formed by distributed deposition (“raining procedure”), no dip was found while for a concentrated deposition (also termed “localized source” or “funnel procedure”), a pronounced dip was observed. It has also been observed that base deflection (e.g. Trollope, 1957; Lee and Herington, 1971) and spatial variation of material stiffness (as introduced in Savage, 1997) can have a significant influence on the extent of the pressure dip. The present study is concerned with the general case that a conical pile is formed on a rigid flat and rough base by concentrated deposition. Finite element simulations of such a general case have been conducted by several researchers. These studies are summarised as below.

Savage (1998) reported results from elastic and elastic-rigid plastic finite element computations for wedges and cones using three different finite element analysis (FEA) packages including ABAQUS. They adopted an elastic-rigid plastic Mohr-Coulomb model and modelled the pile using the 8-node quadrilateral element. Many calculations were undertaken to investigate the effects of the internal friction angle \( \phi \), dilation angle \( \psi \), Poisson’s ratio \( \nu \), cohesion \( c \) and elastic modulus \( E \) of the granular solid. It was found that the results were relatively insensitive to all of these parameters except \( \phi \). The predicted vertical base pressure distribution showed no central dip and was almost indistinguishable from the active limit state solution. Simulations using purely elastic, Drucker-Prager and Drucker-Prager/Cap constitutive models produced similar predictions.

Anand and Gu (2000) conducted elastic-plastic calculations of a static conical granular pile with an angle of repose of \( \beta = 31.5^\circ \) using the “double-shearing” constitutive model through a user-material subroutine in ABAQUS/EXPLICIT. Two sets of parameters were investigated: one with a constant internal friction angle of 31.5\(^{\circ}\), the other with a mobilized internal friction angle which evolved from 0\(^{\circ}\) to 30.0\(^{\circ}\) with strain hardening. The former produced a state with no plastic deformation and the vertical stress distribution showed a peak under the apex. The latter generated a fully plastic state and the vertical stress distribution showed a pronounced dip under the apex of the conical pile. They concluded that the cause of the dip was the nonhomogeneous plastic strain occurred during the formation of a sand pile, which resulted in a nonuniform internal friction coefficient. The largest plastic shear strain concentrated in a wide inclined band which lies slightly below the pile surface.

Al Hattamleh et al. (2005) also argued that strain localization is the main cause of the pressure dip. In their model, the construction of the granular heap was simulated by incrementally layering in five stages. They adopted a double-slip formulation of “double-shear-type” constitutive model which is similar to that of Anand and Gu (2000) but permits the user to assign the orientations of initial slip lines. Very pronounced stress dips were predicted in all cases except the case of homogeneous state where the initial slip orientations equal \( \pm \pi/4 \pm \phi/2 \). They predicted localized vertical plastic strain around the apex with the rest of the pile in an elastic state.
Tejchman and Wu (2008) analysed the pressure distribution under both prismatic and conical sandpiles using a micro-hypoplasticity model which considers the effect of the direction of deformation rate. The construction of the pile was simulated in ten stages with two different methods: horizontal layers and inclined layers, namely the raining procedure and funnel procedure. The results were in qualitative agreement with the experimental results reported by Vanel et al. (Vanel et al., 1999) and the numerical results reported by Al Hattamleh et al. (2005).

Jeong (2005) conducted an extensive FE study on sandpiles using several loading and boundary conditions. In contrast to the other recent studies that adopted complicated constitutive relations, a simple elastic-rigid plastic Mohr-Coulomb model was deployed. A significant effort was invested in developing the “incremental construction” scheme in which the pile was constructed in many stages, which has been used by the silo research community (e.g. Yu, 2004) and Geotechnical Engineering (e.g. Clough and Woodward, 1967; Kerry Rowe and Skinner, 2001). This procedure was shown to be capable of producing a central pressure dip underneath a conical pile. Jeong (2005) also observed that the results are very sensitive to the number of construction layers adopted. A larger number of construction layers predict a bigger pressure dip. The plastic zone in the final stage was predicted to occupy the majority of the pile except the part near the base and the tail ends of the conical surface.

Although some FE simulations of stockpile reported above have successfully predicted a pressure dip, they differ considerably in both the analysis procedure and the final distribution of plastic strain. Some adopted very complicated constitutive models (Anand and Gu, 2000; Al Hattamleh et al., 2005; Tejchman and Wu, 2008), while others used more general elastic-rigid plastic model (Jeong, 2005). Strain localization was argued in two of these studies to be the origin of stress dip (Anand and Gu, 2000; Al Hattamleh et al., 2005), while incremental construction was shown to be a key issue in another two studies (Jeong, 2005; Tejchman and Wu, 2008). There might be some links between these two mechanisms, but they are still unclear.

This study attempts to evaluate the capability of simple constitutive models in predicting the pressure dip in conical stockpiles and to investigate in detail the effect of incremental construction scheme on the pressure profile. The evolution of stress distribution during progressive formation of the stockpile is also shown. The results provide further insight on the potential mechanisms responsible for the pressure dip using such modelling scheme.

2. Reference test data

The stockpile tests conducted by Ooi et al. (2008) with mini iron pellets centrally poured on a rigid base were used as reference data in this study. The pellets are approximately spherical and have a relatively uniform bulk density that is relatively insensitive to packing: the loosest and densest bulk densities achieved in control tests being 2260 and 2370 kg/m$^3$. The internal angle of friction for the pellets was measured to be 34º using a direct shear tester. Five repeat tests were conducted producing a mean pile radius at the base of $R_p = 554mm$ and an average angle of repose of $\beta = 29.0^\circ$. Free-field pressure cells were used to record the normal base pressures underneath the pile. Figure 2b shows the normal base pressure distribution at the final stage averaged from 5 repeat tests, where a pressure dip in the centre is clearly evident. The average height above the base at different radial positions is also shown.
3. Finite element modelling

3.1 Constitutive models

In this study, relatively simple elastic and elastic-rigid plastic constitutive relations are used in modelling the sandpile. These include linear elasticity (LE), pressure-dependent elasticity (PDE), linear elasticity with Mohr-Coulomb rigid-plasticity (LEMC) and pressure-dependent elasticity with Mohr-Coulomb rigid-plasticity (PDEMC). By adopting simple elastic-plastic models, the role of elastic and plastic parameters in producing the numerical solution can be explored more clearly.

The PDE model used is a Janbu-type relation (Janbu, 1963; Chen and Mizuno, 1990) which is expressed as:

$$\frac{E_t}{P_a} = K \left( \frac{P}{P_a} \right)^m$$  \hspace{1cm} (1)

where $E_t$ is the tangent modulus of elasticity of the granular solids, $P_a$ is the atmospheric pressure (101.3 kPa), $p = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ is the mean pressure, and $K$ and $m$ are experimentally determined parameters. Because such a PDE relation is not readily available in ABAQUS, it was implemented as a solution-dependent modulus based on the LE model through the user-subroutine USDFLD. Note that the LE model in ABAQUS only accepts the secant modulus. Consequently, Equation 1 is transformed to the following form in terms of the secant modulus and implemented in ABAQUS:

$$\frac{E_s}{P_a} = K_s \left( \frac{P}{P_a} \right)^m$$  \hspace{1cm} (2)

where:

$$K_s = K(1 - m)$$  \hspace{1cm} (3)

Because the routine USDFLD provides access to material point quantities only at the start of each numerical time increment, the solution clearly depends on the time increment size or the number of time increment because the material properties remain constant during each increment. Numerical calibration tests were conducted to ensure that this PDE relation was correctly implemented. A frictionless uniaxial compression test with a radius of $r=1.0m$ and a height of $z=1.0m$ was modelled (Fig. 3a). The parameters were chosen as $K_s = 100$ and $m = 0.4$. To avoid numerical difficulties caused by zero elastic modulus at the beginning of the compression, a small initial elastic modulus of $E_0 = 0.5 MPa$ was adopted. In this uniaxial compression test, the relation of vertical stress $\sigma_z$ and vertical strain $\varepsilon_z$ for an elastic material can be derived as:
where $\nu$ is the Poisson’s ratio of the solid. The comparison between the input PDE relation and the computation outputs using different number of time (loading) increments $N_t$ is shown in Figure 3b. It is shown that the output curve from the explicit solution approaches the input curve quickly when number of increments increases. Similar convergence tests were also performed for the pile simulations to ensure accurate implementation of the PDE nonlinear elastic treatment.

\[
\frac{\sigma_z}{\varepsilon_z} = K_s \left( \frac{1 + \nu}{3(1 - \nu)} \right)^m \left( \frac{1 - 2\nu^2}{1 - \nu} \right)
\]  

Figure 3. Verification of Janbu elasticity user-subroutine

3.2 Problem configuration

Assuming axisymmetry, the conical sandpile was simulated as a triangle in two dimensions. The final sandpile geometry was further discretised into triangular elements using the quadratic 6-node triangular axisymmetric element CAX6. The only load considered is the self-weight of the solids. The bottom boundary of the pile was fixed in both vertical and horizontal directions, representing a rigid and completely rough base. The simulation process was treated as a static problem so the effect of inertia was neglected.
3.3 Incremental construction scheme

The effect of construction history due to the progressive loading of the conical pile was explored by modelling the progressive formation (or incremental construction) of the conical pile. This was achieved by discretising the final geometry of the pile into many conical layers and then activating each layer sequentially, starting from the bottommost layer. This incremental construction process was implemented in ABAQUS by using the element removal and reactivation technique through the Model Change keyword. A sketch of the incremental construction with FE mesh arrangements is shown in Figure 4 where the final geometry of pile is divided into several construction layers (denoted by alternative dark and light grey layers). Each construction layer may contain one (Figure 4a) or more layers (Figure 4b) of elements. As a result, the total number of layers of element $N_{el}$ is a multiple of number of construction layers $N_{IC}$. In the present study, the sensitivity of numerical results to both these values ranging from 1 to 60 has been explored.

![Figure 4. Sketch of incremental construction for sandpile](image)

3.4 Input parameters

Unless stated otherwise, the input parameters adopted in all the simulations are listed in Table 1 based on the experimental data described above. The density was chosen as the minimum value from the control tests. This is a reasonable assumption because the particles in a stockpile undergo avalanching during the formation process leading to a relatively loose packing. As suggested by Jeong (2005), the particles are in a state of constant volume condition during avalanches, which corresponds to a Poisson’s ratio of around 0.5. As a result, a large Poisson’s ratio of 0.45 was chosen. The pellets tested were dry and non-cohesive, but a small value of cohesion of 1 $P_a$ is assumed to avoid numerical difficulties.

<table>
<thead>
<tr>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Angle of repose $\beta$</th>
<th>Angle of internal friction $\phi$</th>
<th>Angle of dilation $\psi$</th>
<th>Cohesion $c$</th>
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<td>34°</td>
<td>20°</td>
<td>1 (Pa)</td>
</tr>
<tr>
<td>Elastic modulus $E$</td>
<td>Poisson’s ratio $\nu$</td>
<td>Initial elastic modulus for Janbu relation $\kappa_\nu$</td>
<td>$m$</td>
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<td>2.0 (MPa)</td>
<td>0.45</td>
<td>0.5 (MPa)</td>
<td>100</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 1. Input parameters for sandpile simulations
4. Key results

Two groups of simulations were conducted: one without incremental construction and the other adopting incremental construction. Figures 5a and b show respectively the predicted vertical base pressure distributions underneath the pile for the two groups. The pressure has been normalised by the hydrostatic pressure under the apex at the base $p = \gamma H$ where $H$ is the height of the pile. When the whole sandpile was analysed as a single layer (Fig 5a), none of the four constitutive models produced a central dip. This result concurs with the conclusion by Savage (1998). The base pressure is lower than the hydrostatic value towards the centre (at radius $r=0$) and slightly higher than the hydrostatic value elsewhere, satisfying the global equilibrium in the vertical direction. It is interesting to note that the linear elastic LE model and the non-linear elastic PDE model produced very similar results whilst the two elastic-plastic models LEMC and PDEMC also produced very similar results, but the introduction of plasticity has further increased the shedding of the load away from the centre.

The inclusion of incremental construction produced different effects in each constitutive model, as shown in Figure 5b. The linear elastic-plastic LEMC model produced a shallow dip in pressure whilst the non-linear elastic-plastic PDEMC model produced the most pronounced dip. Both elastic models (LE and PDE) did not predict any dip in pressure. The simulation using incremental construction appears to give rise to a larger shedding of the load away from the centre, thus giving rise to the manifestation of a pressure dip. The results also support the proposition that material...
plasticity is a requirement for predicting the sandpile phenomenon and the progressive loading history during sandpile formation also plays a vital role.

It is noted that the computed base pressure at the centre of the pile on the axis of symmetry shows a larger value that disrupts the smoothness of each curve, except the linear elastic case. Further calculations were carried out exploring different number of construction layers and element layers to investigate the origin of this problem. It was observed that this larger pressure at the central node is always confined to only within the first column of elements next to the central axis. The source of the problem may thus be associated with the computation of the nodal stress in the elements containing nodes where zero radial displacement is imposed and stepwise incremental construction scheme is used. This issue is further illustrated in Figure 6a which shows five calculations with increasing number of construction layers. The nodal pressure at the centre was always predicted to be larger no matter how many incremental layers were used. This issue is under further investigation. Figure 6b shows the same results as Fig. 6a with the central point of data omitted. It is evident that this very local occurrence does not influence the overall prediction, so in the rest of this paper, the central point will not be plotted. It should be noted that no previous FEA studies have reported this phenomenon.

Figure 6b reveals that the shape of predicted pressure distribution and size of central pressure dip are not sensitive to the number of construction layers in the range tested in this study ($N_{IC} = 5$ to 60). This is contradictory to the observation of Jeong (2005) where the size of the dip increases with an increasing number of construction layers. In another independent study of progressive filling silo wall pressure using a similar incremental scheme, Yu (2004) concluded that the
calculated peak wall pressure converged quickly and was not sensitive to the number of filling layers used, which is in agreement with the observation here.

Since the nonlinear elastic-plastic PDEMC model produced the best prediction, the rest of this paper will focus on the results using the PDEMC constitutive model with incremental construction. Figures 7-9 show the FEA results using 40 layers of elements and 40 construction layers ($N_{el} = N_{ic} = 40$). Figure 7a shows the vertical stress along horizontal paths at different heights in the pile at the final stage of construction. The FEA predicted that the central dip in vertical stress also exists within the pile but it reduces quickly from the base upwards. This conclusion is consistent to that drawn by Anand and Gu (2000). Figure 7b shows the evolution of the normal base pressure during the incremental simulation process. It is clear that the pressure dip is experienced throughout the pile formation process from the very beginning.

![Figure 7. Vertical stress along horizontal paths using PDEMC elastic-plastic model](image)

The contours of the vertical stress $\sigma_v$ and the mean pressure $p = -(\sigma_1 + \sigma_2 + \sigma_3)/3$ for the whole pile are shown in Figures 8a and b respectively. Because the tangent elastic modulus is dependent on the mean pressure according to Eq. 1, the variation of the elastic stiffness in this model is directly related to the mean pressure. The stiffness is predicted to be increasing with depth as one would expect. More importantly, the largest stiffness at each level is some radial distance away from the centre, giving rise to a softer central core surrounded by stiffer surrounding regions. The analysis has thus identified an arching mechanism arising from the stress dependency of the bulk stiffness in which the vertical load is attracted to the stiffer zone, away from the softer zone.
a) Vertical stress  

b) Mean pressure

Figure 8. Stress distribution in pile (Pa)

Figure 9. Comparison between FEM predictions and test results

The predicted normal base pressure distribution is compared with the experimental result in Figure 9. The FEA predicted a smaller dip than observed in the experiments. One of the possible causes for his discrepancy is that the test piles were slightly rounded at the top due to the impact of the pouring pellets whilst a perfect conical pile was assumed in the numerical simulations, resulting in a smaller apex height in the actual pile than in the numerical simulation. It is also possible that a better match can be produced by varying some of the input parameters including the dilation angle, the Poisson’s ratio and the Janbu elastic parameters. Further parametric investigation is being undertaken and will be reported elsewhere.

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5. Conclusion

A finite element analysis of a conical sandpile has been presented and compared with experimental observations. The key aspects of modelling a sandpile using the finite element method have been discussed and the outcomes for several elastic and elastic-plastic models, with and without incremental layer construction scheme, have been presented. The chief conclusions of the study are:

1. Incorporating plasticity and simulating the progressive construction of the sandpile are both necessary for the finite element method to predict the classic sandpile pressure distribution where a significant dip exists beneath the apex of the pile.

2. Whilst the FE calculations have produced a reasonable prediction of the pressure distribution, they under predict the size of the central dip. A closer match may be achieved by modelling the actual shape of the test pile and adjusting the input parameters.

3. The size of the pressure dip and the overall pressure profile are found to be insensitive to the number of construction layers used. This contrasts with the observation of Jeong (2004) where the dip was reported to become larger as the number of layers increases.

4. The largest stiffness was predicted to be some radial distance away from the centre, giving rise to a softer central core surrounded by stiffer surrounding regions. The analysis has thus identified an arching mechanism arising from the stress dependency of the material stiffness in which the vertical load is attracted to the stiffer zones, away from the softer central zone.

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7. References


A numerical and experimental study of the base pressure distribution beneath a stockpile

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Abstract

Stockpiles are common for storage of bulk solids in many industrial sectors. One interesting phenomenon is that there is a significant dip of the base pressure beneath the apex of the pile which may have significant implications in the design of stockpile facilities and related support structures. This paper presents a numerical and experimental study of this phenomenon. Experiments have been conducted to measure the base pressure distribution under a stockpile formed with iron ore pellets and significant central stress minimum was revealed. Continuum analysis using the finite element method (FEM) was conducted to simulate the stress distribution in the test pile. It showed the critical importance of progressive pile development and nonlinear constitutive models. To investigate the underlying mechanisms further, simulations using the discrete element method (DEM) were conducted, which related well to the FEM predictions and revealed key aspects of the inter-particle force patterns. Both the FEM and DEM predictions made good agreement with test results, revealing new key features of the mechanics of such piles.

Keywords: Granular pile, pressure dip, contact orientation, discrete element modelling, finite element modelling, experiment
Introduction

Storage and handling of granular materials is essential to many industries [1]. Many different bulk solids are commonly stored in open stockpiles in very large quantities, particularly in mining industry. The shape of stockpiles is typically conical (Fig. 1), but can be prismatic, depending on the method of placement. The design of a gravity reclaim system underneath a stockpile requires knowledge of the base pressure distribution. The counter-intuitive finding that there is a significant central pressure minimum beneath the apex of the pile can have significant implications for the design of the support structure of the stockpile and the reclaim facilities. This “pressure dip” phenomenon has been extensively studied by both the physics and the engineering communities in the last few decades. However, there is still a lack of consensus on the fundamental mechanics. Many mathematical models of this seemingly simple system have been developed based on different assumptions, but they often lead to contradictory results. A comprehensive understanding of the phenomenon remains elusive. Good reviews of existing analytical, numerical and experimental studies of this problem can be found in Savage [2, 3], Cates et al. [4] and Atman et al. [5].

This paper describes the key findings from an extensive study of this phenomenon, including stockpile experiments with mini iron ore pellets, continuum analysis using the finite element method (FEM) aided by a progressive layering scheme, and a particle scale investigation employing the discrete element method (DEM). The FEM analysis studied the influence of constitutive relations adopted in the computations and explored whether a continuum treatment is capable of reproducing the sandpile phenomenon. The main part of this paper employs the DEM method to investigate the particle scale behaviour arising from the sandpile formation and attempts to establish whether there is an agreement between the particle-scale information arising from DEM with the continuum prediction arising from FEM. Based on this study, we propose that mobilisation of base friction during the formation of the pile is a key factor in relation to the internal arching mechanism which leads to the pressure dip phenomenon.
Conical pile experiments

The experimental study involved a series of granular pile tests to investigate several factors that researchers have speculated would affect the base pressure distribution. These factors include base deflection, construction method (pouring jet dimension, pouring flow rate and dropping height), and particle shape and size variation. The tests using a concentrated pouring jet onto a rigid base are described below to show the nature of the sandpile phenomenon and to compare with the numerical results. The full results of the stockpile tests can be found in [6].

The conical pile tests involved nearly spherical mini iron ore pellets which had a very rough surface. The pellets were relatively uniform in size, with a mean diameter of $d_p=3.0\text{mm}$ and a size range of $2.5<d_p<3.8\text{mm}$ for 10% to 90% passing in particle size analysis by dry sieving. Their bulk density measured in control tests was in the range of 2260 and 2370 $\text{kg/m}^3$, giving a relatively uniform state that is insensitive to packing. The internal friction angle ($\phi$) measured in a Jenike shear test was $34^\circ$.

Relatively large laboratory granular pile experiments were conducted in which the base pressure was measured using Askegaard free-field pressure cells which have been widely used to observe pressures in granular media [e.g. 7, 8]. The pressure cells were carefully placed at fixed positions along a radial line on a flat wooden base ($t=20\text{mm}$) which was in turn laid on a rigid concrete floor. The stockpile was constructed by pouring particles from a narrow jet with a radius $R_j=16\text{mm}$ located centrally at 674mm above the base (Fig. 2a).

The progressive development of the base pressure during the pile formation process was measured. Figure 2b shows the measured final pile surface profile.
(right vertical axis) and the mean base pressure distribution (left vertical axis) for five repeat tests. The measured surface profile had a mean angle of repose of $\beta = 29^\circ$ with a coefficient of variation $\text{CoV}=2.3\%$ and a mean pile radius of $R_p=554\, \text{mm}$ with $\text{CoV}=2.6\%$. A clear and significant pressure dip was evident in every individual test and the measurements show a high degree of repeatability with small scatter. The scatter is indicated in Figure 1 as $\pm$ one standard deviation. The measured base pressure profile with a central dip and a maximum pressure at a radius of $r \approx 0.3R_p$ is very similar in form to that reported by Vanel et al. [9] for a smaller sand pile and that by McBride [10] for a much larger gravel pile.

Fig. 2  Vertical base pressure in a conical mini-iron ore pellets pile. a) Description of stockpile surface and base pressure profile; b) vertical base pressure from 5 tests

The evolution of the base pressure profile recorded during the formation of one pile is shown in Figure 3. Each measurement represents the average pressure on the pressure cell (cell diameter = $75\, \text{mm} = 25d_p = 0.14R_p$). The measured pressure is consequently slightly higher than the actual value when the pressure cell is located at a dip but lower when it is located at a peak. A clear pressure dip started to emerge from an early stage and appeared to increase with the progressive build-up of the pile. The pressure at the centre relative to the peak pressure at each instant gradually reduced and the position of the peak pressure gradually moved outwards. The first two sets of measurements cannot be used to infer the pressure pattern, as the spacing of the cells is too wide relative to the small size of the pile at that stage.
The results support the widely stated proposition that an arching effect arising from the formation process causes a significant part of the weight of the solid in the central zone to be transferred gradually to the outer zones. The pressure dip phenomenon has been shown to be a repeatable and robust phenomenon when the stockpile is constructed on a rigid flat base by concentrated jet deposition.

**Finite element (FE) prediction**

Even though the pressure dip phenomenon has generally been attributed to an arching effect, it is yet to be clearly understood how the flow and deformation of the solid result in such an arching mechanism. Very few studies have numerically modelled stockpiles using continuum methods such as the FEM. This is because classical FEM analysis applied to a complete pile using elasto-plastic constitutive models cannot easily predict the central pressure dip, as first reported by Savage [2, 3]. Although some more recent FE simulations that implemented more realistic construction histories have successfully predicted a pressure dip [5, 11-15], they differ considerably in the construction calculation procedure, constitutive models and the predicted distribution of plastic strain, as reviewed by Ai [16].

The conical pile test described above was modelled here using the FEM. The pellets were modelled using two material constitutive models: linear elastic with Mohr-Coulomb perfectly-plastic (LEMC) model and stress-dependent elastic with
Mohr-Coulomb perfectly-plastic (PDEMC) model. The base was assumed to be ideally rough (all base nodes were restrained vertically and horizontally). The effect of construction history was explored by modelling the progressive formation of the pile [16, 17]. This was achieved by discretising the final geometry of the pile into many layers and then activating each layer sequentially in the FE analysis, starting from the bottom layer. Figure 4 shows a sketch of the incremental construction, where different construction layers are denoted by alternate dark and light grey layers. It is important to note, however, this incremental numerical process ignores the dynamic process of particle impacting, flowing and avalanching during the deposition process. It allows us to explore whether this dynamic effect is important in the dip formation.

The measured geometry and internal friction angle of the test pile ($\beta=29^\circ$, $R_p=554\text{mm}$, $\phi=34^\circ$) were adopted in the FE analysis. The density of the pellets was chosen as the minimum value (2260$\text{kg/m}^3$) from the control tests. This is a reasonable assumption because the particles in a stockpile undergo avalanching during the formation process, leading to a relatively loose packing, and the stresses in the stockpile are small compared with those in the control tests. The pellets were also assumed to have a Poisson’s ratio $\nu=0.3$ and dilation angle $\varphi=20^\circ$ as no test data were available. The tested pellets were dry and non-cohesive, but a small value of cohesion $c=1.0\text{Pa}$ was adopted to avoid numerical difficulties at very low stresses. For the PDEMC material model, the stress dependent secant modulus was evaluated based on simple deduction using confined compression tests of pellets and Australian and Eurocode Standards [18, 19]:

$$E_s = Kp + E_0$$ (1)

where $E_s$ is the secant modulus; $p$ is the mean stress; and $E_0$ is the initial modulus. The values of $K$ and $E_0$ were estimated to be 100 and 0.02 $\text{MPa}$ respectively. For the LEMC material model, the elastic modulus of the pellets was chosen as 2.0 $\text{MPa}$, which was estimated from the confined compression tests and the average mean stress in the final stockpile. The influence of these elastic and plastic parameters has been explored in a parametric investigation [16].
Figure 5 compares the predicted base pressure distribution from the two material models with the experimental data. A very shallow dip was predicted by the linear elastic-plastic LEMC model but a much deeper dip closer to the experimental observations was predicted by the stress dependent elastic-plastic PDEMC model. The significant effect of the stress dependent elastic properties on the size of the dip suggests that stress dependent elasticity plays a crucial role in the pressure dip phenomenon.

![Graph showing predicted base pressure distribution](image)

**Fig. 5** Vertical base pressure: FEM predictions versus experimental observations ($N_{cl}=40$)

The natural question that follows is to ask how the stress dependent elastic stiffness assumption gives rise to a much improved prediction in this FEM computation. Figure 6 shows the predicted plastic zones in the pile for the two models, first at an intermediate stage and then at completion of the construction process. Both constitutive models predicted that the solid undergoes plastic yielding at the intermediate stage, followed by unloading to an elastic state as further solid was laid above. The stress dependent PDEMC model predicted plastic zones along the instantaneous top sloping surface which moved progressively with subsequent layers, akin to a layer of solid under shear failure associated with avalanching. The resulted state consists of an elastic inner core surrounded by superficial plastic layers. It may be noted that such a pattern is qualitatively similar to those found in earlier solutions of admissible stress field that contain a pressure dip [20-23]. On the other hand, the LEMC model predicted a central conical plastic zone at the apex of the cone away from the bottom
boundary. The PDEMC model also predicted larger elastic zones in the final state than the LEMC model, which may be partially linked to the larger pressure dip predicted. Further investigation of the effect of plastic yield progression may provide further insight into the mechanics.

**Fig. 6** Plastic zone at middle stage (upper) and final stage (lower), note the darker zones represent solids under active yielding. a) LEMC; b) PDEMC

Michalowski and Park [23] suggested that base frictional shears can play an important role in the internal arching behaviour of the pile. Figure 7 shows the base shear predicted by the two models. The form of the base shear distribution for both models is as expected: increasing from zero at the axis of symmetry ($r/R_p=0$) to a peak value and then decreasing to zero at the outer edge of the pile ($r/R_p=1$). However it is significant that the PDEMC model predicted a much greater rate of increase of shear from the centre (at $r/R_p=0$), reaching the peak at a smaller radius than the LEMC model. This observation supports the proposition that a greater rate of base shear mobilisation beneath the apex in the PDEMC model amplifies the arching effect and moves the vertical forces away from the centre, resulting in a greater pressure dip.
The FEM results presented above suggest that: a) an elastic-plastic model aided by the progressively layering modelling scheme can predict a plausible base pressure distribution with a central pressure dip; b) a constitutive model with pressure dependent modulus produces a significantly enhanced dip, bringing it closer to the experimental observations; c) there is a correlation between the development of the base shear, plastic zones in the pile and the pressure dip formation. The FEM calculations have provided a useful insight into the relationship between constitutive model assumptions and numerical predictions that adopt incremental construction. The next investigation used particle scale modeling to seek further evidence for these observations based on continuum predictions.

**Discrete element (DE) prediction**

The DEM [24, 25] has become a popular tool for simulating particulate systems because it is capable of particle scale studies in which the packing structure of the assembly is observable and can be traced throughout the time history. Extensive DEM simulations have been conducted in this study to investigate the pressure dip phenomenon. Due to the high computational cost associated with DEM, the test of a small scale two dimensional pile with 3,000 binary sized photoelastic circular disks conducted by Zuriguel *et al.* [26] was chosen for this modeling, instead of the 3D conical pile tested by the authors. The calculations were performed using
PFC2D [27]. A Hertz-Mindlin no-slip contact model with damping and a frictional slider in the tangential direction [28] was adopted. A rolling resistance model was employed through a User Written C++ [27] to achieve a stable pile matching the experimentally observed angle of repose [16]. Figure 8 shows the setup of the DEM model. Particles were randomly generated in a shallow pipe located at a fixed height of $H_{dep}=570mm$ above a rigid horizontal base. A pile was formed by allowing the generated particles to fall and accumulate on the base. DEM computations from two particle generation schemes are compared here: concentrated deposition using a jet radius $R_j=16mm$ and distributed deposition with particles randomly falling over the full width of the pile. These two particle deposition modes correspond respectively to the localised-source procedure and raining procedure adopted in sandpile experiments by Vanel et al. [9] and Geng et al. [29]. A full description of the model and parameters used in the calculations can be found in Ai [16].

**Base pressure distribution**

Figure 9 shows the progressive development of the normal base pressure that was averaged from 100 random DEM piles formed with a concentrated jet. When small scale piles with limited number of particles are modeled using DEM [e.g., 30, 31], many discrete runs are indispensible to overcome the large random fluctuations in evaluated pressure profiles [16]. To transform the discrete particle contact forces into local base pressures, a running average scheme was used to determine the global profile. The averaging length $L_{av}$ for each stage was set at 1/8 of the pile radius. Figure 9 reveals a clear pressure dip that develops at an early stage, consistent to the experimental observation in the conical stockpile tests described above. The peak of the base pressure distribution at 1s is slightly higher than that at 2s because the influence of the impact is more significant when the pile is very small.
**Fig. 9** Evolution of normal base pressure averaged from 100 DE piles [running average length $L_{av} = 1/8R_p(t)$]

**Fig. 10** Base pressures from different construction histories

Figure 10 shows the base pressures distributions at pile completion and compares the outcomes of concentrated deposition (localised-source procedure) and distributed deposition (raining procedure). The horizontal coordinate was normalised by the pile radius $R_p$, and the base pressures by the hydrostatic vertical pressure at the centre: the integrated force due to vertical pressure is then unity in both cases.
The effect of the deposition method on the base pressure distribution in sandpile experiments has been reported before [e.g. 9, 29]. Distributed deposition produced no pressure dip but concentrated deposition did. The DEM results in Fig. 10 show a convincing match to this finding. Further, the base shear profiles (Fig. 10) show that concentrated deposition produces a much steeper slope near the centre, indicating a higher rate of base shear development in the core of the pile. This feature was already identified in the continuum FEM prediction. We therefore propose that the faster development of base shear in the core region is the mechanism that causes arching, resulting from the avalanches occurring under concentrated deposition, by contrast with the in-situ placement of particles in horizontal layers from distributed deposition.

**Probability distribution of contact and contact force orientations**

Since forces in a sandpile are carried by particles through contacts, the macroscopic features discussed above (layer pattern and base pressures), should be reflected at the particle scale. This pair of DEM predictions, using two different particle deposition modes, and giving base pressure predictions with and without the pressure dip, provides an excellent opportunity to probe the underlying particle scale mechanism of the pressure dip phenomenon.

The DEM predicted granular pile shape and the force chain network for the two construction histories is shown in Figure 11. The thickness of the force chain markers is proportional to the contact force. Distributed deposition clearly produces a steeper pile. The force chains in the central zone near the base also appear to be closer to vertical after distributed deposition, reducing the base shear mobilization. This corresponds to the base shear pattern found in the FE calculations.

![Fig. 11 Force chain network. (a) Concentrated deposition; (b) distributed deposition](image)
It has been suggested that contact network anisotropy may be closely related to the pressure dip under a pile [e.g., 32, 33]. Since each particle contact is frictional in nature, the contact orientation differs from the associated contact force orientation, depending on the magnitude of the tangential contact force. The difference between these two angles indicates the mobilised friction angle at each contact. Here, the orientation of each contact ($\theta_c$) and contact force ($\theta_{cf}$) is defined as shown in Figure 12. The treatment of Geng et al. [29] was used to evaluate the probability distribution of the orientation of contacts $P(\theta_c)$ and contact forces $P(\theta_{cf})$ independently for the left and right halves of the pile. The probability distributions $P(\theta_c)$ and $P(\theta_{cf})$ for different zones in the right half are shown in Figures 13 and 14 respectively.

The contact orientation in the core region is significantly affected by the construction history (Figure 13a). For distributed deposition, the distribution is almost symmetrical about the vertical (gravitational) direction and contains high concentrations of contacts at $0^\circ$, $60^\circ$, $120^\circ$, $180^\circ$, $240^\circ$ and $300^\circ$ due to a significant development of crystalline packing. For concentrated deposition, the contact orientation in the core region does not contain these sharp spikes and is more random in nature with preferred orientations in the $105^\circ$–$150^\circ$ region and the corresponding $285^\circ$–$330^\circ$ since each contact is always in pair. In the surface region (Figure 13b), the contact orientation distribution is not affected by the construction history and is very similar to that observed in the core region for concentrated deposition.

The contact force orientation distributions from the two construction histories are compared in Figure 14. Again, the two cases are little different in the surface
region, but differ significantly in the core region. After distributed deposition, the sharp concentration of the contact orientations in the six directions is not present in the contact force orientation. Inter-particle friction is mobilised to produce contact force orientations principally aligned vertically despite the six preferred directions in the contact orientations. The results show that the deposition history has a strong influence on the contact and contact force orientations in the core zone, which are highly correlated with the base shear development and pressure dip phenomena.

Fig. 13  Distribution of contact orientations for concentrated and distributed deposition. a) core region; b) surface region
Conclusions

This paper has presented a summary of key findings from an extensive study of the pressure dip phenomenon in a granular stockpile. Conical pile experiments using mini iron ore pellets showed a robust and repeatable base pressure distribution with a significant central pressure dip. Both continuum analysis using the finite element method (FEM) and particle scale discrete analysis using the discrete element method (DEM) have been conducted and compared with

Fig. 14  Distribution of contact force orientations for concentrated and distributed deposition. a) core region; b) surface region
experimental observations.

The FEM investigation has shown that an elastic-plastic model aided by a progressively layering pile construction modelling scheme can reproduce the pressure dip. A material model that includes the pressure dependence of the elastic modulus has been shown to significantly enhance the magnitude of the dip, bringing it much closer to experimental observations. The DEM study has revealed that the deposition history has a strong effect on the formation of the pressure dip and the contact and contact force orientations in the core zone. Distributed deposition produces almost symmetrical contact orientation distributions about the vertical and no pressure dip is found, whilst concentrated deposition results in inclined contact orientation distributions and a pressure dip.

Both the FEM and DEM investigations have revealed that there is a close relationship between the development of base frictional shears and the pressure dip. A larger pressure dip is associated with a rapid base shear development near the pile centre. This is further linked to the particle scale behaviour of the pile in the central zone, shown by the contact orientation and contact force orientation distributions.

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