

Gradable Adjectives and the Semantics of Locatives

Johannes C. Flieger

Doctor of Philosophy
Institute for Communicating and Collaborative Systems
School of Informatics
University of Edinburgh
2009

Abstract

This dissertation develops a semantic model of gradable adjectives such as ‘tall’, ‘good’, ‘big’, ‘heavy’, etc., within a formal semantic theory of locatives we call *Locative Structure Semantics* (LSS).

Our central hypothesis is that gradable adjectives are, semantically, a species of locative expression. The view of gradable adjectives as locatives is inspired by the vector-based semantic models of Vector Space Semantics (VSS), as well as the notion of perspective or point of view, as found in Leonard Talmy’s research on spatial expressions (Talmy [153]) and the tradition of Situation Semantics (cf. Barwise and Perry [9, p. 39]). Following Barwise and Seligman [11], we construe the contextual variability that characterises gradable adjectives in terms of shifts in cognitive perspective.

We argue that perspectives are a formal part of a semantic representational structure that is shared by expressions from several different domains, which we refer to as a *locative structure* (L-structure). The notion of an L-structure is influenced by Reichenbach’s notion of tense, and can be thought of as a generalisation of the Reichenbachian notion of tense to the realm of concepts. Reichenbach [134] proposed that each temporal expression is associated with three time points: a speech point, *S*, an event point, *E*, and reference point, *R*, where *E* refers to the time point corresponding to the event described by the tensed clause, *S* is (usually) taken to be the speaker’s time of utterance, and *R* is a temporal reference point relevant to the utterance. In LSS we extend this tripartite scheme to locative expressions in general, to which we assign a ternary structure comprising a Perspective, a Figure, and a Ground, represented symbolically as *P*, *F*, and *G*, and which are generalisations of the Reichenbachian *S*, *E*, and *R*, respectively. We show that a formal semantics based on L-structures enables us to capture important cross-categorical similarities between gradable adjectives, tenses, and spatial prepositions.

Acknowledgements

My thanks go, in the first place, to my supervisor, Ewan Klein, for his patient guidance and insightful advice during the writing of this dissertation. I extend my thanks also to my second supervisor, Jon Oberlander, and the other members of my thesis committee, Alex Lascarides and Mark Steedman, who provided both encouragement and helpful criticism of my ideas during their development.

I wish to thank the staff and students of the School of Informatics at the University of Edinburgh, and in particular the members of the Institute for Communicating and Collaborative Systems (ICCS). In no particular order, I thank Bonnie Webber, Miles Osborne, Claire Grover, Colin Stirling, Chris Callison-Burch, Jochen Leidner, Andrew Smith, Amittai Axelrod, Sebastian Riedel, James Clarke, Ben Hachey, Bea Alex, Ruken Cakici, Abhishek Arun, Ingo and Carla Battenfeld, and all those who have helped make my time in Edinburgh an interesting and pleasant one. I also gratefully acknowledge the financial assistance provided by the University of Edinburgh and the EPSRC, which has made this work possible.

Among former teachers, several stand out not only for their ability as scholars and researchers, but also in virtue of their professional and personal integrity. I wish to thank Robyn Carston (University College London) for her long-standing encouragement and friendship, as well as Marek Sergot (Imperial College London), Francesca Toni (Imperial College London), Jeroen Groenendijk (ILLC, Amsterdam), Martin Stokhof (ILLC, Amsterdam), David Bennett (SOAS, London), Paul Thompson (SOAS, London), Alexander Piatigorski (SOAS, London), and Tadeusz Skorupski (SOAS, London), for their guidance and help during my studies.

Thanks are also due to my past colleagues at LShift Ltd., one of the most talented and inspired group of programmers to be found anywhere, and especially Matthias Radestock, Andy Wilson, Michael Bridgen, and Tony Garnock-Jones.

Friends and family have been a constant source of advice, encouragement and support. I owe special thanks to my cousin Erik-Jan Flieger, and my good friends Rafael de la Fuente, Georg Roth, Dimitris Dimitriadis, Nikolaos Papadakis, Julia Bindman, and Julia Vogl.

Thanks are also due to Guro Shamim Haque and the Kalis Ilustrisimo family, as well as Maurição Motta Gomes, Roger Gracie, and the members of the Roger Gracie Academy, for teaching me that real gold does not fear the fire. Practice truth, and fear nothing.

Above all, I wish to thank my parents, Johannes Franciscus Josephus Flieger and Cynthia Ma Flieger, and my beloved wife, Cindy Liu Kun. Their love is ultimately what has made everything possible, and it is to them that I dedicate this dissertation.

Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified.

(Johannes C. Flieger)

To my beloved parents, Jan and Cynthia, and my dearest wife Liu Kun.

Table of Contents

1	Towards a Locative Theory of Gradable Adjectives	1
1.1	Introduction	1
1.2	The Formal Semantics of Gradable Adjectives	2
1.2.1	Adjectives and Gradability	2
1.2.2	Predicative and Attributive Adjectives	5
1.2.3	Degree Adjectives as Semantic Attributives	7
1.2.4	Degree Adjectives as Semantic Predicatives	12
1.2.5	Scalar Theories	17
1.2.6	From Degrees to Vectors	20
1.3	Vector Space Semantics	21
1.3.1	Measure Phrases in VSS	24
1.3.2	Gradable Adjectives in VSS	25
1.3.3	Adjectives and Measure Phrase Modifiers	29
1.3.4	The Modification Condition	31
1.3.5	A Critique of VSS	35
1.3.5.1	The Divergent Type Problem	36
1.3.5.2	The Coordination Problem	36
1.4	Aims and Scope	38
1.5	Chapter Overview	40
2	A New Perspective on the Semantics of Locatives	41
2.1	Introduction	41
2.2	Perspectives	42
2.2.1	Figure and Ground	42
2.2.2	Perspective and the Figure-Ground Relation	44
2.3	Tense	47
2.3.1	From Tenses to Locative Structures	49

2.4	Adjectives as Locatives	51
2.4.1	The Deictic Centre	52
2.4.2	Conceptual Perspectives	53
2.4.3	Some Structural Analogies between Tenses, Adjectives and Pronouns	55
2.5	Extended Locations	58
2.6	Conclusion	61
3	Locative Structures and the Semantics of Gradable Adjectives	62
3.1	Introduction	62
3.2	Vectors, Dimensions, and Locative Structures	63
3.2.1	Polarity	64
3.2.2	Magnitude	66
3.3	A Locative Structure Semantics for Gradable Adjectives	67
3.3.1	A Simple Fragment	69
3.3.2	The <i>Than</i> -Phrase as Modifier	76
3.4	Measures and Measure Phrases	79
3.4.1	The Structure of Measure Phrases	81
3.4.2	Measure Phrases as Adjectival Modifiers	82
3.4.3	Sortal Restrictions on Measure Phrases	85
3.5	Constraints and Compositionality	87
3.6	Adjectival Anaphora	93
3.6.1	Dynamic Semantics	93
3.6.2	Adjectival Anaphora from a Dynamic Perspective	95
3.6.3	Vagueness in Dynamic Semantics	98
3.6.3.1	Metalinguistic Sharpening	99
3.7	Conclusion	101
4	A Dynamic Semantics for Locatives	103
4.1	Introduction	103
4.2	The Problem with Intersective Constructions	104
4.2.1	Coordination in VSS	105
4.3	Existential Closure	107
4.4	Dynamic Locative Structure Semantics (DLSS)	110
4.4.1	Dynamic Intensional Logic	110
4.4.2	Model Theory	113

4.4.3	Context Change Potential (CCP)	116
4.5	Existential Disclosure	119
4.5.1	The <i>Than</i> -Phrase as Modifier	124
4.6	Coordination and Existential Disclosure	127
4.6.1	Existential Disclosure Revisited	130
4.6.2	Modifiers as Arguments	136
4.7	Linking L-Structures and Sentences	137
4.8	Conclusion	140
5	Loose Ends and Conclusions	142
5.1	Introduction	142
5.2	Vagueness	142
5.2.1	Vagueness and Locatives	142
5.2.2	Vagueness versus Gradability	143
5.2.3	Higher-Order Vagueness and Sharpness in Semantics	144
5.2.4	The Sorites Paradox	145
5.2.5	Theories of Vagueness	146
5.2.5.1	Supervaluationism	147
5.2.5.2	Many-Valued Theories	148
5.2.6	Epistemicism	150
5.3	Scales and Vectors	151
5.3.1	Vector vs. Numbers	151
5.3.1.1	Dimensions and Scale Structure	151
5.3.2	The Informational Properties of Scales	152
5.4	Adverbs and Gradability	154
5.4.1	From Adjectives to Adverbs	154
5.4.2	Extending LSS to Adverbs	155
5.5	Conclusion	158
	Bibliography	162

Chapter 1

Towards a Locative Theory of Gradable Adjectives

1.1 Introduction

This dissertation develops a semantic model of gradable adjectives such as ‘tall’, ‘good’, ‘big’, ‘heavy’, etc., within a formal semantic theory of locatives we call *Locative Structure Semantics* (LSS).

Our central hypothesis is that gradable adjectives are, semantically, a species of locative expression. The view of gradable adjectives as locatives is found in the vector-based accounts of Faller [46] and Winter [176], who employ a formal vector-based account within the framework of Vector Space Semantics (Zwarts [180], Zwarts and Winter [182]).

At the heart of our proposed account of gradable adjectives is the notion of *perspective* or *point of view*. The concept of perspective is given particular importance in Leonard Talmy’s research on spatial expressions (Talmy [153]), and is also a central principle of Situation Semantics (e.g., Barwise and Perry [9, p. 39]). In particular, we follow Barwise and Seligman [11] in regarding the contextual variability that characterises gradable adjectives as a matter of perspective, and present a formal account in which attributing this variability to a hidden perspectival parameter in the semantic representation of the adjective.

Furthermore, we claim that there is an abstract level of semantic representation that is shared by gradable adjectives, spatial and temporal prepositions, and tense, which we refer to as a *locative structure* (L-structure). The notion of a locative structure is influenced by Reichenbach’s notion of tense, and can be thought of as a generalisation of the Reichenbachian notion of tense to the realm of concepts. Reichenbach [134] proposed that each temporal expression is associated with three time points: a *speech point*, *S*, an *event point*, *E*, and a *reference point*, *R*, where

E refers to the time point corresponding to the event described by the tensed clause, *S* is (usually) taken to be the speaker's time of utterance, and *R* is a temporal reference point relevant to the utterance. LSS extends this tripartite scheme to spatial expressions gradable adjectives; like Reichenbachian tenses, locative structures have a ternary structure, comprising a *Perspective*, a *Figure*, and a *Ground* (cf. Talmy [153]), represented symbolically as *P*, *F*, and *G*, respectively, which are generalisations of the Reichenbachian *S*, *E*, and *R* points from the temporal to the conceptual domain.

We begin this chapter with a discussion of the semantic properties of gradable adjectives, followed by a survey of several different treatments they have been accorded in the formal semantics literature (section 1.2). Of particular relevance to the present work is the theory of *Vector Space Semantics* (VSS), as developed by Zwarts [180], Zwarts and Winter [182], which we discuss in section 1.3. The VSS accounts of gradable adjectives (Faller [46], Winter [175, 176]) extend mechanisms originally developed to model the semantics of spatial prepositions, effectively treating adjectives as locatives. In section 1.4 we present the principal claims that comprise our thesis, and conclude with an overview of the following chapters, in section 1.5.

1.2 The Formal Semantics of Gradable Adjectives

1.2.1 Adjectives and Gradability

Certain properties have the characteristic that entities do not simply possess them, but possess them to a certain *degree*. Adjectives whose denotations are ordered according to such properties are accordingly called 'gradable' or 'degree' adjectives. For example, the adjectives 'tall' and 'short' have denotations that are ordered according to the same gradable property, viz. HEIGHT. However, although both adjectives refer to the same underlying property, they do so in contrasting ways; informally, 'tall' indicates a height greater than some threshold, while 'short' indicates a height that is lower.¹ This threshold tends to vary according to the context, and may be influenced by a broad range of factors, including the kind of entity the adjective is being used to describe; for example, it might be appropriate to describe a five-year-old child as 'tall', yet a grown man of the same height might be considered 'short'.

Klein [95, p. 6] takes an adjective *A* to belong to the class of degree adjectives if and only if

1. *A* can occur in predicative position, in copular constructions with verbs such as *be*, *seem*, *become*, et cetera;

¹It is not necessary to think of the threshold as a *point*; it can be construed as an *interval* instead, in which case it would be possible for something to be neither 'short' nor 'tall'.

2. A can be preceded by degree modifiers such as *very*, *quite*, *extremely*, et cetera.

Another traditional hallmark of degree adjectives is usage in comparative constructions, where depending on properties of the associated ordering, the potential for comparison may be total or partial:

- (1) a. Bill is very rich.
- b. Her watch was more expensive/cheaper than my car.
- c. Bob is richer/poorer than Bill.
- d. The bedroom was hotter/colder than the kitchen.
- e. My car is faster/slower than your motorcycle.

The criteria for applying the standard and comparative forms of a given adjective do not always coincide; it is often possible to describe a thing using the comparative form of an adjective even where one would use an antonym or a negation of the standard form.

- (2) a. Bob is not tall, but he is taller than Bill.
- b. The film was not short, but it was shorter than the concert.
- c. John is old, but he's younger than Alice.

By the criteria given so far, the set of gradable adjectives includes items such as *big*, *small*, *tall*, *short*, *old*, *young*, *high*, *low*, etc., as well as 'evaluative' adjectives such as *superb*, *excellent*, *skilful*, *beautiful*, *good*, *fair*, *poor*, *bad* and *awful*.

Another mark of gradability is *measurability*, though this is a sufficient rather than a necessary criterion for gradability. Measurement can be viewed as a form of comparison in which an interval or extent (a length, duration, or the degree to which an entity possesses a given property) can be numerically expressed as a ratio of some known interval, known as a *measure* or *unit of measurement*. Some measures are fixed by formal social or scientific convention (e.g., centimetre, fathom, kilogramme, ampere), but everyday measure terms vary considerably in their degree of exactness (e.g., *handful*, *day's walk*), and can sometimes be improvised in an ad hoc manner.

In applying a measure, it is necessary to identify the bounds of the interval being measured. In English, when the measure phrase modifies a comparative form, as in (3), what is measured is the *difference* in degree between the things being compared:

- (3) a. Bob is 60 kilogrammes heavier/lighter than Alice.
- b. Her watch was 400 dollars more expensive/cheaper than my car.
- c. Bob is 2 million dollars richer/poorer than Bill.

- d. The bedroom was 30 degrees hotter/colder than the kitchen.
- e. My car is 10 metres-per-second faster/slower than your motorcycle.

By contrast, when a measure phrase modifies the positive (non-comparative) form of a gradable adjective, as in (4), the measure term invariably qualifies the *total* or *absolute* extent of possession of the property in question; if we think of the property as represented by a line, then measurement is taken from the zero point or origin.

- (4) a. Bob is 2 metres tall.
- b. The hotel is 4 storeys high.
- c. John is 30 years old.
- d. The film was 2 hours long.
- e. Her watch is 5 minutes slow.

While we may take the presence of a measure phrase modifier as a sure indicator of gradability in an adjective, the converse is not the case. First, there are adjectives for which measure terms are simply not readily available (e.g., ‘beautiful’); moreover, there are also many gradable adjectives which readily take a measure phrase modifier in their comparative form, yet seem unable to do so in their absolute form, as we can see if we compare the sentences in (3) with those in (5).

- (5) a. * Bob is 60 kilogrammes heavy/light.
- b. * My watch was 400 dollars expensive/cheap.
- c. * Bob is 2 million dollars rich/poor.
- d. * The bedroom was 30 degrees hot/cold.
- e. * My car is 10 metres-per-second fast/slow.

Even in those cases where measure phrase modification is felicitous, however, the absolute form is constrained in other ways. Given an antonymic pair of gradable adjectives in positive form, as in the sentences in (6), we often find that measure phrase modification is acceptable with one member of the pair, but not the other—by contrast, with the comparative form, measure phrase modifiers are typically acceptable with both members of the pair, as in (3) above.

- (6) a. Bob is 2 metres tall/*short.
- b. The table is 1 metre wide/*narrow.
- c. The hotel is 4 storeys high/*low.
- d. John is 30 years old/*young.

However, there are some pairs, such as those in (7), where this asymmetry is not present.

- (7) a. My watch is 2 minutes fast/slow.
- b. Bob was 2 hours late/early.

One feature of adjectival measure phrase modification is that the meaning of a gradable adjective in the presence of a measure phrase appears to be different from what it means on its own. The adjective appears to lose the ‘evaluative’ aspect of its meaning in the presence of a measure phrase modifier: objects of *any* height can be described using the construction ‘x metres tall’, not just the ones judged to be ‘tall’ *simpliciter*. For example, a child who is described as ‘five years old’ would not be described as ‘old’ in absolute terms, nor would a man who is ‘four-foot tall’ be described as ‘tall’.

Measure phrases do not only appear with gradable adjectives, of course, but also with spatial and temporal locative expressions in general, as in (8) and (9) below.

- (8) a. five metres in front of/behind/outside/*near/*on the house
- b. five metres tall/*short/wide/*narrow/deep/*shallow
- (9) a. two years before/after the storm
- b. two years old/*young

The possibility of measure phrase modification with gradable adjectives has led to attempts to develop a cross-categorial account of measure phrase modifiers, notably by Martina Faller [46] and Yoad Winter [175, 176] in the framework of Vector Space Semantics (VSS).

1.2.2 Predicative and Attributive Adjectives

In English, adjectives occur in two canonical positions within the clause, in what are often called ‘attributive’ and ‘predicative’ position. If *A* is an adjective and *N* a common noun, then we shall say that *A* is in *attributive position* when it occurs as a nominal modifier in a construction of the form *A + N*, as in the phrases ‘blue sky’, ‘big flea’ and ‘good plumber’, and in *predicative position* when it appears without an accompanying noun in the VP, after a copular *be* or a verb such as *become*, as in the corresponding phrases ‘The sky is blue’, ‘The flea is big’, and ‘The plumber is good’.

It has become commonplace to employ the terms ‘attributive’ and ‘predicative’ to describe the *semantic* properties of adjectives. Those adjectives whose occurrences, whether in attributive and predicative position, can be given a standalone or intersective interpretation are classified as *semantic predicatives*, and those which cannot are classed as *semantic attributives*.

Much of the debate over the semantics of degree adjectives has centred around whether they should be treated as predicatives or attributives.

The traditional logical analysis of adjectives treats them as predicates, which works well for some cases; for example, if we know that some entity x is a three-legged dog, then we know that x is a dog, and also that x is three-legged. Moreover, a three-legged dog doesn't cease to be three-legged when we consider the supertypes of dog: a three-legged dog is also a three-legged mammal, and a three-legged animal.

(10) x is a three-legged dog.

(11) a. $\Rightarrow x$ is a dog

b. $\Rightarrow x$ is three-legged

The meaning of ' x is a three-legged dog' can be given an analysis in terms of conjunction or (extensionally) set intersection: if A is an adjective and N a noun, with denotations A' and N' respectively, then $\llbracket A + N \rrbracket = A' \cap N'$. It is possible to think of 'three-leggedness' as a standalone property, independent of whatever noun it modifies.

While many adjectives can occur in both attributive and predicative positions, some adjectives cannot appear as standalone predicates, and are only found in attributive position; for example, the attributive phrases 'former president', 'alleged fraudster', 'pretend policeman', etc., do not appear to have acceptable predicative versions, cf. *'The president is former', *'The fraudster is alleged', *'The policeman is pretend'. Accordingly, these are sometimes called 'attributive' or 'relational' adjectives; this group includes, for example, the adjectives 'former', 'future', 'alleged', 'supposed', 'purported', 'pretend', and 'imitation', among others.

Parsons [123, p. 323] refers to these adjectives as 'non-standard modifiers';² semantically, they cannot be modelled using a straightforward intersective analysis. If we know that person x is an 'alleged' fraudster, then we cannot conclude that x is a fraudster, while it makes no sense to say that ' x is alleged'; moreover, if x is a 'pretend' policeman, then we must conclude that he or she is in fact *not* a policeman, while a 'former' policeman is someone who is not a policeman at the time of utterance, but has been at some point in the past.

(12) x is an alleged fraudster.

(13) a. $\nRightarrow x$ is a fraudster

b. $\nRightarrow *x$ is alleged

²The class of 'non-standard modifiers' also includes certain adverbs, such as 'allegedly' and 'supposedly'; see Parsons [123, p. 323] for discussion.

In most of these cases, the adjective does not appear to denote a standalone property: adjectives such as ‘former’ or ‘alleged’, for example, do not make sense in isolation, but can only be interpreted in construction with a common noun N , as in ‘former N ’ or ‘alleged N ’. In the logical semantics, these adjectives are functors that must first apply to the semantic value of N , only then yielding a one-place predicate which can be combined with a subject. In other words, if A is an adjective and N a noun, with interpretations A' and N' respectively, then the interpretation of the phrase $A + N$ is given by $\llbracket A + N \rrbracket = A'(N')$.

Some theorists have proposed that certain other types of adjective, notably degree and evaluative adjectives, should be given the same semantic treatment as relational adjectives even when occurring by themselves in predicative position without an accompanying noun. As Geach [55, p. 33] puts it:

‘Big’ and ‘Small’ are attributive; ‘x is a big flea’ does not split up into ‘x is a flea’ and ‘x is big’, nor ‘x is a small elephant’ into ‘x is an elephant’ and ‘x is small’; for if these analyses were legitimate, a simple argument would show that a big flea is a big animal and a small elephant a small animal.

Geach’s claim is that ‘big’ and ‘small’, as well as other gradable adjectives, are *semantic attributives*, a view that is also found in Montague’s proposed semantics for English in [117]. We describe this position in the next section, and the resurgence of predicative analysis in the work of Kamp and others in Section 1.2.4 below.

1.2.3 Degree Adjectives as Semantic Attributives

Montague’s [117] describes gradable and evaluative adjectives as ‘subsective’ (other terms are ‘affirmative’ in Kamp [84, p. 125]) and ‘restrictive’ in Keenan and Faltz [91, p. 68]), which means simply that the set denoted by the result of applying such an adjective to a noun is always a subset of the denotation of the noun by itself. In other words, given adjective A and noun N , with denotations A' and N' respectively, we have $A'(N') \subseteq N'$, as in (1.2.3) below.

$$(14) \quad \textit{big}'(\textit{flea}') \subseteq \textit{flea}'$$

$$(15) \quad \textit{good}'(\textit{hitman}') \subseteq \textit{hitman}'$$

Clearly, the class of subsective adjectives includes the class of intersective adjectives, but not vice versa: in Geach’s example, if we know that an entity is a big flea, for instance, we cannot say that the entity is big, since a big flea can be considered a small animal, but at least we know that the entity is a flea. Similarly, if we know that someone is a good hitman, then although we cannot conclude that he or she is good simpliciter—indeed, one would usually consider a good hitman to be a rather *bad* person—we do know that the person is a hitman.

- (16) x is a big flea.
- (17) a. $\Rightarrow x$ is a flea
 b. $\not\Rightarrow x$ is big
- (18) x is a good hitman.
- (19) a. $\Rightarrow x$ is a hitman
 b. $\not\Rightarrow x$ is good

In the case of the relational or attributive approach, instead of saying simply that some entity is ‘big’ or ‘tall’, one says that that entity is ‘a big N ’ or ‘a tall N ’, where N specifies the comparison class. In similar vein, instead of saying that some entity is ‘good’, one says that it is ‘a good N ’ where N indicates the relevant role or respect in terms of which goodness is judged. In logical syntax, the adjective is applied first to this N , yielding a one-place predicate which only then can be predicated of the subject like the one-place predicate *three – legged*’.

On this view, ‘big’ is always interpreted relative to some comparison class, and it makes no sense to call something big or small except relative to such a class: a flea can be ‘big for a flea’, while at the same time ‘small for an animal’. In the case of evaluatives like ‘good’ and ‘bad’, interpretation is relative to some relevant criterion, such as the *function* performed by the entity being evaluated: thus, a man can be at once ‘good as a husband’ but ‘bad as a father’, without contradiction.

However, Montague [117] rejects the idea of defining multiple classes of adjectives in the grammar itself, opting instead for a uniform treatment in which *all* adjectives are relational (two-place) predicates, denoting functions from intensions of properties to properties. Moreover, all adjectives, in common with other functors, operate on the *intensions* of their arguments (this is motivated by the fact that many adjectives, especially the relational ones, are indeed intensional in character).

While Montague’s approach avoids generating invalid entailments, this is only because, in its raw form, it does not give rise to any entailments at all; in order to model the properties of intersective and subsective adjectives (e.g., the knowledge that a three-legged dog is both three-legged and a dog), Montague resorts to meaning postulates such as those in Definition 1.

Definition 1 (Meaning postulates (Montague [117]))

1. $[every \delta\zeta \text{ is a } \zeta]$

where δ is an intersective or subsective adjective and ζ is a common noun.

2. [*every* $\delta\zeta$ *is* δ]

where δ is an intersective adjective and ζ is a common noun.

For example, ‘three-legged’ is an intersective adjective and ‘big’ is a subjective adjective. The first meaning postulate in Definition 1 then allows one to infer that a three-legged dog is a dog and a big flea is a flea; similarly, the second postulate allows us to infer that a three-legged dog is three-legged, but not that a big flea is big. These rules allow Montague to assign all adjectives the same syntactic category and logical translation, while still obtaining the right entailments. Of course, while Montague manages to avoid introducing adjectival subclasses in the grammar proper, ultimately he does have to make reference to such subclasses.

Parsons [123] and Wheeler [170] also advocate a non-predicative analysis of degree adjectives. Parsons analyses attributives like ‘fake’ and ‘alleged’ as semantic operators, so examples like ‘ x is a fake gun’ become $F(Gx)$ (Parsons [123, p. 326]). Wheeler [170] translates a gradable adjective like ‘tall’ as a two-place relation holding between an entity and a set of entities, where the latter acts as a comparison class by which tallness is to be judged; a phrase such as ‘John is a tall man’ receives a two-part translation, as shown in (20):

(20) John is a tall man.

$$tall(john, \lambda x.man(x)) \wedge john \in \lambda x.man(x)$$

In the first part of the translation given in (20), the adjective ‘tall’ is translated as a two-place predicate holding between an entity and a comparison class, and is properly read as “John is tall relative to the class of men”; the second part states that John is a man, and supports the inference from ‘John is a tall man’ to ‘John is a man’. In contrast to the Montague-Parsons translation of ‘tall man’, Wheeler’s translation supports this inference directly. The analysis of the adjective as a two-place relation allows for the variability of ‘tall’ to be modelled in terms of the variability of the comparison class: for example, ‘John is a tall jockey’ would receive the interpretation in (21), which indicates that John’s height is now to be considered relative to the set of jockeys.

(21) John is a tall jockey.

$$tall(john, \lambda x.jockey(x)) \wedge john \in \lambda x.jockey(x)$$

Thus, as the comparison class changes from men to jockeys to basketball players, or whatever, the tallness of John may also vary significantly.

At first sight, the attributive approach appears to be a simple and adequate solution for capturing the variability of gradable adjectives: any syntactically attributive construction $A + N$,

where *A* is a degree adjective and *N* is a noun, will be translated with *N* representing the relevant comparison class. However, several researchers, including McConnell-Ginet [113], Siegel [144], [146], and Beesley [12, p. 200–202], have argued that a syntax-based attributive analysis along the lines of (20) is inadequate, and that the relativity of degree adjectives can be better handled by appealing to context.

One problem with this approach is that, while it is natural in simple cases to construe the modified noun as indicating the comparison class, it is not always easy or even possible to derive the relevant comparison class straightforwardly from the syntax, in a mechanical fashion, once the noun phrases get more complex. In the case of noun phrase constructions involving multiple degree adjectives, like the ones in (22) below, we find that a syntax-based approach such as Wheeler's generates multiple candidate comparison classes, the number of which rises quickly with the number of modifiers.

- (22) a. Jones is a tall old man.
b. Jones is a tall old fat man.
c. Jones is a tall old fat ugly man.
d. Jones is a tall old fat ugly evil man.

According to Wheeler [170, p. 314], a sentence such as (22a) has two readings, one of which describes Jones as tall relative to the class of old men, while the other describes him as both tall and old relative to the class of men in general; in turn, a sentence like (22b) has a reading where Jones' tallness is relative to the class of old fat men, plus (at least) two other readings, and sentences (22c) and (22d) have more still. Thus, Wheeler's syntax-based approach leads to considerable semantic ambiguity in these cases: rather than a single comparison class, we have a set of candidate comparison classes, from which the hearer will still need to select the correct or intended one, at least partly on the basis of contextual information. Moreover, it also appears that the comparison class does not have to correspond to a continuous subconstituent of the noun phrase. Siegel, who has discussed the problems of interpretation of multiple degree adjectives at some length in [144] and [146], observes that (23) has a reading where tallness is understood relative to the class of little basketball players, leaving out red-headedness altogether, which she takes as an indication that context rather than syntax has the final say in determining the comparison class.

- (23) Billy is a tall little red-headed basketball player.

Indeed, Kenneth Beesley [12, p. 202] has argued that the comparison class, even in the simplest *A + N* constructions, may not even be the class denoted by the noun at all. He asks us to consider the conversation in (24):

(24) Q: Which of the men over there is Quang?

A: Quang is the short Vietnamese.

As Beesley points out, there is a perfectly natural reading of ‘short Vietnamese’ where *men* rather than *Vietnamese* serves as the comparison class for ‘tall’; that is, ‘short Vietnamese’ can be paraphrased as “short compared to men in general, and Vietnamese”, and it is moreover perfectly consistent to claim that Quang is a short Vietnamese but that he is nevertheless not short *for* a Vietnamese. But if a syntactically attributive $A + N$ construction may in fact involve a comparison class distinct from N , then this undermines any attempt to develop a syntax-based attributive account of gradable adjectives.

Another central problem for any syntactic attributive theory is how to account for gradable adjectives appearing in predicate position. If a sentence such as ‘Bob is tall’ is to be analysed as an attributive, then the noun denoting the comparison class must be present at some point in the underlying syntactic representation. As Beesley [12] notes, this can be done in basically two ways: the first is to derive a predicative sentence such as ‘Bob is tall’, for example, from the sentence ‘Bob is a tall jockey’ by a syntactic mechanism of noun phrase reduction, but which leaves the predicate corresponding to the noun, in this case *jockey*, in the semantic representation; the second is to take ‘tall’ as applying to a phonologically empty dummy noun, which is then assigned a nominal predicate in the semantics. On the first approach, we end up with an indefinite number of syntactic representations for the same surface string (one for each noun that could be modified by the adjective), while on the second the indeterminacy is transferred to the semantic representation. Either way, this approach leads to what appears to some as an unacceptable indeterminacy at the heart of the grammar itself, where we cannot even say exactly how many syntactic (semantic) representations a simple sentence might have, nor determine which syntactic or semantic structure is the correct one without appealing to context.

An alternative approach is to derive the comparison class for predicative constructions from lexically coded semantic features. One proposal, due to Katz [86], is that concepts are ordered in a semantic hierarchy, and the choice of comparison class for an entity is the ‘lowest’ category that includes it (see Katz [86, p. 186]). Thus we have, for example, skyscraper \in buildings, man \in humans, flea \in insects, United States \in countries, etc. However, Katz restricts the number of possible interpretations by maintaining that the comparison class can *only* be the immediately subsuming category—going so far as to claim that we cannot interpret ‘The skyscraper is big’ as *The skyscraper is big for a physical object*, or ‘The flea is big’, as *The flea is big for an animal*.

One obvious problem with Katz's proposal is that the restriction he imposes is far too stringent: even if it might be plausibly argued that the *default* comparison class is given by the immediately subsuming category, there are contexts where it is quite reasonable to select a different category as the basis for comparison. For example, in the context of a science fiction novel featuring giant mutated fleas, *The flea is big for an animal* might be a reasonable, or even preferred, interpretation of the sentence 'The flea is big' (cf. Beesley [12, p. 203]), and there are buildings large enough to warrant interpreting a sentence such as 'The skyscraper is big' as *The skyscraper is big for a physical object*. Moreover, the lexical entry for certain words, such as proper names, may provide little or no indication of which category they should be subsumed under (see Bartsch [7, p. 165] and Sampson [140, p. 257]). Further analysis of the inadequacy of trying to assign comparison classes mechanically on the basis of lexical codings can be found in the work of McConnell-Ginet [113, p. 89], Siegel [144, p. 129], [146, p. 243], and Bierwisch [13, p. 165 ff.]. What emerges from this literature is that, while the lexicon may provide useful clues for the interpretation process, context is the dominant factor in determining the appropriate comparison class.

1.2.4 Degree Adjectives as Semantic Predicatives

A number of researchers in the general tradition of formal semantics and Montague Grammar, including Dowty [39], Kamp [84], Bartsch [8], and Klein [95, 96, 97], have disagreed with the Montague-Parsons analysis of all adjectives as attributive. Kamp [84] defends the traditional translation of degree adjectives as one-place predicates, though he expresses doubts over extending one-place predicate status to relational adjectives like 'alleged'. Siegel [145], [146] distinguishes between degree adjectives and evaluatives, analysing the former as one-place predicates (together with intersective adjectives), and treating the latter as semantic attributives. Beesley [12], however, has argued that even evaluatives should be analysed as one-place predicates.

A number of arguments have been advanced to show that degree adjectives behave syntactically like predicative adjectives, which lends support to a common translation. For example, unlike most relational adjectives, degree adjectives are typically able to appear grammatically in predicate position.

- (25) a. Alice is alive/two-legged/pregnant. (predicative)
- b. Alice is tall/rich/slim. (degree)
- c. * Alice is supposed/alleged/former. (relational)

Siegel [145] notes that only one-place predicates can function as non-restrictive modifiers of noun phrases, and observes that in this regard degree adjectives pattern with predicatives rather than attributives:

- (26) a. I saw pregnant/divorced Alice. (predicative)
b. We spoke with big/rich Bertha. (degree)
c. * Mere/alleged Bob came to see us. (relational)

There also appears to be some correlation between predicative status and having a meaningful nominalisation or nominal form; the key seems to be that we can conceive of *redness*, *three-leggedness*, or *tallness* as independent qualities, but not of **mereness* or **allegedness*. Siegel also argues that in the case of verbs and verb constructions with ‘making *A*’ or ‘becoming *A*’ interpretations, the adjective *A* typically corresponds to a predicative rather than a relational adjective: we can make something red (‘redden’), or make something big (‘enlarge’), or make someone pregnant (‘impregnate’), but there are no verbal constructions corresponding to **‘make former’* or **‘make alleged’* (the verb ‘allege’, though related in meaning, is not interpretable as **to make alleged*).³

Of course, the main problem in treating degree adjectives as one-place predicates is the apparent failure of an intersective analysis, which is what prompted their classification as semantic attributives in the first place. But advocates of a predicative analysis argue that this problem can be resolved through a proper understanding of the role of context. Proponents of an attributive analysis generally assume that the comparison class for a sentence of the form ‘*x* is a *A N*’ must necessarily be the class denoted by *N*, so that for ‘Dumbo is a small elephant’, for example, the comparison class must necessarily be the set of elephants. But as Beesley and others have argued, this assumption is simply false: even for simple attributive constructions like ‘short Vietnamese’ and ‘big flea’, context may introduce comparison classes altogether distinct from *N*. Though there may often be strong pragmatic reasons to evaluate shortness relative to Vietnamese in a construction like ‘short Vietnamese’, we should not allow ourselves to be misled into thinking that the usual or default readings for such constructions are the *only* readings available. Rather, it is context that ultimately determines the comparison class by which any sentence of the form ‘*x* is an *A N*’ or ‘*x* is *A*’ will be evaluated.

One criterion commonly proposed for distinguishing between gradable and non-gradable adjectives is that the former allow modification by degree modifiers such as ‘very’, ‘extremely’,

³Here the argument is less convincing, however, given apparent counterexamples such as ‘fire’, ‘dismiss’, ‘impeach’, etc. (Ewan Klein, personal communication); one could still argue that these are rare or exceptional, but this would require a more detailed empirical investigation.

‘remarkably’, ‘quite’, and the like (e.g., see Klein [95, p. 6]). Many intersective adjectives that are not usually classified as gradable also allow modification by modifiers such as ‘very’, though they usually cannot do so without taking on a secondary (and sometimes figurative) meaning (see Vendler [166, p. 109], Bolinger [15, p. 15], and Keenan and Faltz [91, p. 164]). For example, a woman is either pregnant or not pregnant, but ‘very pregnant’ might refer to a range of properties associated with being pregnant, such as a woman’s size, emotional state, etc.; similarly, being alive is an all-or-nothing affair, but ‘very alive’ might be used to describe qualities that typically indicate life: high levels of activity, healthy appearance, a quick mind, and so on. By contrast, relational adjectives usually cannot take degree modifiers at all, which lends further supports treating degree adjectives as predicative rather than relational.

- (27) a. A very pregnant woman walked into the room.
b. We saw a remarkably tall man in the park.
c. * The plumber was a quite former academic.
d. * The extremely alleged murderer was arrested.

Given that an appeal to context appears to be both necessary and natural in selecting a comparison class, we can capture this in the semantics in at least two ways: the first is to translate the adjective as a logical relation, which takes a comparison class as an explicit argument; the second is to translate the adjective as a one-place predicate, but one whose semantic value is contextually variable. In the former case, the interpretation of the adjective is kept constant, and it is the value of the additional parameter that varies; in the latter, the interpretation of the adjective itself is made context-dependent.

One way to make the interpretation of the adjective variable is by employing contextual variables, sometimes called ‘delineations’ (e.g., McConnell-Ginet [113, p. 115]), to fix the extension of degree adjectives. The context picks out a relevant delineation, relative to which predicates are assigned an interpretation. Some version of this idea can be found in the work of McConnell-Ginet [113], Klein [95], Bartsch [7], and Keenan and Faltz [91], among others. These theories all translate degree adjectives as one-place predicates, while relativising their interpretation to contextually specified comparison classes. We shall refer to these as ‘delineation theories’.

Semantically, the difference between gradable and non-gradable predicatives lies in the way an adjective divides up the universe of a model: a predicate like ‘dead’ sharply divides all entities into the dead and the not-dead, with no remainder. The boundaries of tall, on the other hand, may be somewhat unclear, and between the group of tall entities and the group of short entities there may be a group of entities which are neither short nor tall.

While contextual variability is taken care of by relativising interpretation to contexts, vagueness is commonly handled in terms of semantic partiality. According to Klein [95], who extends the theory of adjectives put forward in Kamp [84], gradable and non-gradable adjectives are distinguished by the fact that the latter always denote *complete* functions from individuals to truth values, while the former may denote *partial* functions from individuals to truth values. Thus, for a logical language \mathcal{L} , the interpretation function F is subject to the following constraint (see Klein [95, p. 10]):

$$\text{Whenever } \zeta \in \text{Adj} \text{ and } c \in C, F_{\zeta}(c) \in \{0, 1\}^{(U)}.$$

Here, Adj denotes the subset of one-place predicates that correspond to adjectives in English, C is the set of contexts, and $\{0, 1\}^{(U)}$ is the set of *partial* functions from the universe of discourse, U , into the set of truth values. The constraint ensures that the interpretation function, F , assigns denotations to non-logical constants relative to contexts of use.

Given α an arbitrary expression of \mathcal{L} , and \mathcal{U} a partial context dependent interpretation, the extension of α is relative to \mathcal{U} , a context c and assignment a , and written $\llbracket \alpha \rrbracket_{\mathcal{U}, c, a}$ (cf. Klein [95, p. 11]). The adjective ‘tall’ in a sentence like ‘Sean is tall’ is translated as a one-place predicate, which is then assigned the interpretation in (28b) below.

- (28) a. Sean is tall.
 b. $\llbracket \text{tall}(\text{Sean}) \rrbracket_{\mathcal{U}, c, a}$

Klein associates each predicate ζ with a pair of functions, pos and neg , from contexts to sets of entities, namely, the set of entities x for which $A(x)$ is definitely true (what Klein calls the ‘positive extension’, following Kamp [84]), and the set for which $A(x)$ is definitely false (the ‘negative extension’).

For any context $c \in C$:

1. $\text{pos}_{\zeta}(c) = \{u \in U : F_{\zeta}(c)(u) = 1\}$
2. $\text{neg}_{\zeta}(c) = \{u \in U : F_{\zeta}(c)(u) = 0\}$

A comparison class is the subset of the domain of a gradable adjective that is relevant in a given context of use. Since different subsets of the domain may be relevant in different contexts, when we evaluate a sentence of the form ‘ x is ζ ’ in a context c , we first apply the interpretation function corresponding to ζ to the c -relevant subset of \mathbb{D}_{ζ} , and then check whether the denotation of x is contained in the positive extension.

Since the semantics is partial, some entities may not be assigned to either set; those entities in the domain which are not assigned to either the positive or negative extension of ζ are said

to belong to the ‘extension gap’ of ζ (see Klein [95, p. 11]). The existence of borderline cases is the hallmark of vagueness: in any given context of use, there will be people we consider to be definitely tall, others who are definitely not tall, and yet others who are borderline (that is, neither definitely tall nor definitely not tall). Although a vague predicate can be used without having to draw a clear boundary between the positive and negative extensions, this boundary can often be made more precise when the circumstances require; however, while the boundary may vary from context to context, the assignment of entities to partitions is typically constrained by the underlying ordering of the domain, which is assumed to remain constant across contexts; Klein [96, p. 126] stipulates that the ordering on a comparison class must preserve the initial ordering on the domain of the adjective.

Suppose that Sean is on the borderline between tall and not tall in a context c ; then (28a) will be neither definitely true nor definitely false. However, the statement might be “sort of true” or “true-ish”. One of Kamp’s aims in [84] is to capture the degree of truth of a vague sentence in terms of the set of complete (classical) valuations of ‘tall’ which close up the extension gap in a consistent manner (see also Lewis [111]). One way to do this is to introduce a two-place function \mathcal{S} , which assigns to any $c \in C$ and $\zeta \in Adj$ a set of new contexts $\mathcal{S}(c, \zeta)$. $\mathcal{S}(c, \zeta)$ is intended to represent the set of contexts in which ζ has been made fully precise. Relative to any $c^+ \in \mathcal{S}(c, \zeta)$, F then assigns to ζ a classical extension; i.e. $F(c^+, \zeta)$ is a total function. Then, we can capture the degree of truth of a sentence α by examining the proportion of full specifications in which α is true.

The analysis of gradable adjectives in terms of full specifications can also be used to model the semantics of comparatives. Both Lewis [111] and Kamp [84] attempt to explain the connection between a positive form, such as ‘tall’, and its associated comparative, ‘taller than’, by taking into consideration the total set of full specifications, and propose that a comparative such as ‘ a is taller than b ’ is true iff the set of specifications on which ‘ b is tall’ is true is a proper subset of those on which ‘ a is tall’ is true (see Kamp [84, pp. 138–140] and Lewis [111, p. 65]). However, both Lewis [111] and Kamp [84] have difficulty with cases where the compared elements both definitely have some property; for example, if Bob and Bill both count as definitely tall (say 1.98 and 2.1 metres respectively), then ‘Bob is tall’ and ‘Bill is tall’ will be true in exactly the same set of admissible specifications (that is, all of them), and it follows that the set of specifications in which ‘Bob is tall’ is true will not be a proper subset of those in which ‘Bill is tall’ is true. Another example, due to Klein [95, p. 12], is given below in (29).

- (29) a. ‘an’ is a longer word than ‘a’.
 b. ‘an’ is a long word.

c. ‘a’ is a long word.

As Klein points out, on Kamps approach, (29a) will only be true in a context c if there is some context c' in which the standards for what counts as a long word have been shifted in such a manner that (29b) comes out true in c' but (29c) comes out false. But the mechanisms Kamp employs for making vague predicates more precise, being monotonic in character, cannot produce such a context c' .

Klein [95] proposes a solution to this problem using comparison classes. As Klein [95, p. 126] observes, what is required within Kamp’s overall framework is a context in which ‘an’ can be considered an orthographically long word; such a context would be one in which we could find a way to exclude certain elements from consideration, for example, all words longer than two letters. By employing comparison classes, we can produce such a context: namely, the context in which the comparison class for long is a set of word forms consisting of at most two letters, such as $X = \{‘an’, ‘we’, ‘on’, ‘to’, \dots, ‘I’, ‘a’\}$. Once we are able to restrict the interpretation of the adjective to a subset of the domain (excluding the larger set of all words from consideration), we can apply the predicate ‘long’ to the remaining, smaller subset.

1.2.5 Scalar Theories

All the theories we have discussed so far treat gradable and non-gradable adjectives as belonging to the same semantic type. There are approaches, however, which analyse gradable predicates in terms of abstract entities such as dimensions, scales, degrees, intervals, or vectors. We shall refer to theories which employ such abstract measures as ‘scalar’ theories, and to those which do not as ‘non-scalar’ theories.

The basic notion underlying scale-based analyses of gradable adjectives is that an individual can possess or exhibit a property to a certain *degree* or *extent*, where a degree is interpreted as a position along, or section of, a (totally or partially) ordered scale associated with a particular property (e.g., HEIGHT in the case of ‘tall’ and ‘short’), and are often measurable in terms of some unit associated with that dimension, such as metres or feet for height, kilos and pounds for weight, and so on. Gradable adjectives are analysed as relations between individuals and abstract representations of measurement, such as degrees or intervals. Comparisons between individuals relative to a dimension are then evaluated in terms of the relative ordering of their associated degrees on a scale associated with that dimension, while absolutes can be analysed as *covert* comparisons between the degree associated with an individual and some contextually determined standard.

Most (if not all) non-scalar theories assign the same semantic type to both gradable and

non-gradable adjectives; on a predicative analysis, they are usually interpreted extensionally as sets of entities (or characteristic functions thereof), and share the same logical type, $(e \rightarrow t)$.⁴ The principal difference between gradable and other predicative expressions is that gradable adjectives have domains that are ordered (totally or partially) with respect to some property or set of properties that permits gradation, such as such as HEIGHT, WEIGHT, AGE, and so on. Scalar theories, on the other hand, typically introduce some abstract representations of measurement into the semantic ontology, and analyse the semantics of gradable adjectives in terms of relations between entities and those representations; examples include Cresswell [29], Hellan [67], Hoeksema [73], von Stechow [169], Bierwisch [14], Moltmann [115], Kennedy [92], as well as the vector-based analyses of Faller [46] and Winter [175], [176].

From a methodological standpoint, there is a bias in favour of a non-scalar approach: in accordance with the principle of ontological parsimony, or *Occam's Razor*, when given the choice between two explanatorily adequate theories, we should prefer the one which makes the fewest ontological assumptions. In other words, we should only allow additional types into the ontology as a last resort; *all else being equal*, parsimony dictates that a non-scalar approach is preferable on methodological grounds. Of course, proponents of scalar theories argue that such entities *are* in fact necessary to capture the full range of behaviour that gradable adjectives exhibit, and justify the introduction of scales and degrees on the grounds that this enables them to explain certain phenomena that are problematic for non-scalar approaches.

One such phenomenon is *incommensurability*, which has been discussed by Klein [97] and Kennedy [92, ch. 1], among others. In (30) below, we see that comparisons are not possible between adjectives associated with different dimensions; while it is simple and intuitive to account for this by stipulating that semantically well-formed comparatives must have the same dimensional parameter, it is hard to see how this could be explained without introducing dimensions into the semantic ontology (cf. Kennedy [92, p. 17]).

- (30) a. Bob's boat is wider than Bill's car is long.
 b. * The film was heavier than Bob's caravan.
 c. * Bob is shorter than Bill is intelligent.
 d. * The speech was longer than the

Another phenomenon, which Kennedy [92, p. 19] refers to as *cross-polar anomaly* concerns comparisons between adjectives of opposite polarity: comparatives that involve positive-positive

⁴It would be possible for a non-scalar, delineation-based theory to interpret gradable adjectives as relationships between individuals and delineations (Ewan Klein, p.c.); however, to the best of our knowledge, none of the non-scalar theories do so—for good reasons, as we shall see.

or negative-negative pairs of adjectives, as in (31a) and (31b) below, are generally acceptable, but those constructed out of positive-negative pairs, such as (31c) and (31d), are typically anomalous. In [92, ch. 3], Kennedy shows that it is possible to provide an account for this in terms of the scale structure associated with the adjectives.

- (31) a. The desk is longer than the table is wide.
 b. The Christmas tree is shorter than the ceiling is low, so it'll fit in the room.
 c. * Bob is poorer than Bill is rich.
 d. * Bob is fatter than Bill is slim.

A third phenomenon which scalar approaches are arguably better equipped to deal with is the distribution of measure phrases, and specifically the distribution of measure phrases with antonymic pairs of adjectives. In order to accommodate measure phrases, some theorists employ the notion of a *difference degree*, which, as the name suggests, is the difference on a given scale between one degree and another (see Hellan [67], von Stechow [169], and Bierwisch [14]). Bierwisch [14], for example, proposes that difference degrees have a *direction* as well as a magnitude, and that comparison takes place on a directed scale $(D, 0)$ with a zero point, and the degrees d_i corresponding to the entities being compared are sections of D beginning at 0. The difference δ between two degrees d_1 and d_2 is then a section of D that starts at one degree and ends at the other, and depending on its starting point may have the same direction as the scale $(D, 0)$, or the opposite one (see Bierwisch [14, pp. 111–114]). Bierwisch exploits this contrast in directionality in order to provide an account of the contrary nature of antonymic pairs of adjectives such as *tall/short*, *heavy/light*, etc., and associated entailments such as *Bob is taller than Alice* \Leftrightarrow *Alice is shorter than Bob*.

It is possible to construct the notion of a degree within a non-scalar approach. For example, Klein [95, p. 3] endorses Cresswell [29, p. 281] formal definition of the notion of ‘degree’ in terms of equivalence classes of objects: if d is the degree to which Bill is tall, then d is the equivalence class consisting of all things which are neither less tall nor more tall than Bill. For any given adjective, such as ‘tall’, it is possible to define an equivalence relation, \approx_{tall} , as in Definition 2 below:

Definition 2 $u \approx_{tall} u'$ iff for all v :

1. u is taller than v iff u' is taller than v ,
2. v is taller than u iff v is taller than u' .

Bills degree of tallness is thus $\{u: u \approx_{tall} Bill\}$.

Klein [95, pp. 27–30] goes on to discuss constructions involving adjectives and measure phrases, including examples such as (32).

(32) Mona is six foot tall.

On a reading where ‘*x* foot tall’ is interpreted as ‘exactly *x* foot tall’ (cf. Klein [95, pp. 28–29]), the predicate ‘six foot tall’ is true of *u* iff *u* bears the relation \approx_{tall} to some object which measures six feet, i.e. some object in the set of objects that are six foot tall, written $6\bar{f}$. As Klein [95, p. 30] observes, if we regard $6\bar{f}$ as a *degree* of tallness, then his treatment of degrees turns out to be very similar to that proposed by Cresswell [29, p. 281]—the main difference being that, unlike Cresswell, Klein’s degrees are derived rather than primitive elements in the semantic ontology.

However, as pointed out by Kennedy [92, p. 40], this analysis fails to make a distinction between positive and negative adjectives, and thus does not provide an explanation of the fact that negative adjectives such as ‘short’ do not allow measure phrases:

(33) *Mona is six foot short.

The problem is that, since ‘tall’ and ‘short’ are extensionally equivalent, the relation \approx_{short} is equally well-defined according to Definition 2, and (33) should be acceptable.

1.2.6 From Degrees to Vectors

The notion of a directed difference degree put forward by Bierwisch [14] is very similar to the intuitive (or “naive”) notion of a vector; in both cases, we are dealing with an abstract object with a *magnitude* and a *direction*. Vector-based accounts of gradable adjectives have been proposed by Faller [46] and Winter [175, 176], who both attempt to extend the semantic analysis of locative PPs originally developed in Zwarts [180] and Zwarts and Winter [182] to the case of gradable adjectives.

Purely in terms of the study of gradable adjectives, little is gained merely by employing vectors *per se*, instead of difference degrees *à la* Bierwisch [14], or the intervals of Kennedy [92]. The principal benefit of adopting a specifically *vector*-based account of gradable adjectives, rather than one based on degrees or intervals, is that it provides a format in which to analyse cross-categorial aspects of semantic phenomena, one example of which being measure phrase modification. As noted earlier, measure phrase modifiers occur not only with adjectives, but also with expressions of other categories, such as the spatial and temporal PPs in (34) below.

- (34) a. The tree is 60 metres in front of the palace.
b. The bird is 20 feet above the house.

- c. Your suitcase is 40 kilogrammes above the limit.
- d. The gala dinner took place 30 minutes after the ceremony.
- e. The fireworks will begin 20 minutes before the presentation.

In (34), the measurement in each case applies to the relations expressed by the locative PPs; specifically, what is measured is the *relative position* (in space or time) of two objects or events. This bears some resemblance to the way measurement functions with comparative adjectives, where what is measured is the *relative degree* of some property, and suggests the possibility of a unified theory that accounts for the range of cross-categorial modification phenomena. In the next section, we discuss the attempts by Faller [46] and Winter [175, 176] to extend the VSS account of measure phrase modification to adjectival degree constructions.

1.3 Vector Space Semantics

Vector Space Semantics (VSS) is a framework for model-theoretic semantics that includes vectors in its basic ontology, and was originally developed by Zwarts [180] and Zwarts and Winter [182] in order to account for the semantics of spatial expressions such as locative and directional prepositions.

A locative preposition combines syntactically with an NP that refers to a spatially located object, such as ‘the house’, ‘the cloud’, etc., in (35) below. Locative prepositions are usually relational in nature, and describe the position of an object relative to that of another; for example, in sentence (35a), the bird is assigned a location relative to the house, while in (35b) it is assigned a position relative to the cloud. VSS refers to the object whose location is being determined to as the *located object* (LO), and the object functioning as a reference point as the *reference object* (RO).

- (35)
- a. The bird is above the house.
 - b. The bird is below the cloud.
 - c. The bird is above the house and below the cloud.

VSS interprets locative prepositions as maps from the reference object (denoted by the preposition’s NP complement) to a region in space which is related to the reference object, where the exact nature of the relation depends on the preposition involved.⁵ These regions are formally modelled as sets of vectors emanating from the reference object, so a preposition is a function from individuals to sets of vectors, and a locative PP is simply a set of vectors. For example,

⁵See Jackendoff [77] and Landau and Jackendoff [104] for an earlier proposal of this idea.

in the case the sentences in (35) above, the PP ‘above the house’ denotes a set of vectors emanating from the house in an upwards direction, and ‘below the cloud’ denotes the set of vectors emanating from the cloud in a downwards direction. The proposition expressed by a sentence such as (35a), ‘The bird is above the house’, for example, is then true iff there exists a vector v in the set of vectors emanating upwards from the house which has the location of the bird as its endpoint.

The located object can be described in relation to more than one reference point, as in example (35c). Intuitively, the sentence (35c) will be true iff there exists a vector emanating upward from the house as well as another vector emanating downward from the cloud, both of which end at the location of the bird. Surprisingly, however, it turns out that the formal modelling of coordinate structures is highly problematic for VSS; we shall go into the reasons for this in Section 1.3.5.2 below, after we have examined the theory in greater detail.

VSS employs a standard definition of vector spaces, as in Definition 3 below.

Definition 3 (Vector space) *Let \mathbb{F} be a number field. Then a vector space over \mathbb{F} is a structure $\langle \mathcal{V}, \vec{0}, +, \cdot \rangle$ comprising the following elements:*

- a set \mathcal{V} ;
- a distinguished zero element $\vec{0} \in \mathcal{V}$;
- a vector addition operation, $(+): \mathcal{V} \times \mathcal{V} \rightarrow \mathcal{V}$;
- a scalar multiplication operation, $(\cdot): \mathbb{F} \times \mathcal{V} \rightarrow \mathcal{V}$.

for which the following properties hold:

1. $\forall \vec{u}, \vec{v} \in \mathcal{V}: \vec{u} + \vec{v} = \vec{v} + \vec{u}$;
2. $\forall \vec{u}, \vec{v}, \vec{w} \in \mathcal{V}: (\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$;
3. $\forall \vec{v} \in \mathcal{V}: \vec{v} + \vec{0} = \vec{v}$;
4. $\forall \vec{v} \in \mathcal{V}: \exists (-\vec{v}) \in \mathcal{V}: \vec{v} + (-\vec{v}) = \vec{0}$;
5. $\forall k \in \mathbb{F}: \forall \vec{u}, \vec{v} \in \mathcal{V}: k \cdot (\vec{u} + \vec{v}) = k \cdot \vec{u} + k \cdot \vec{v}$;
6. $\forall k, m \in \mathbb{F}: \forall \vec{v} \in \mathcal{V}: (k + m) \cdot \vec{v} = k \cdot \vec{v} + m \cdot \vec{v}$;
7. $\forall k, m \in \mathbb{F}: \forall \vec{v} \in \mathcal{V}: (km) \cdot \vec{v} = k \cdot (m \cdot \vec{v})$;
8. $\forall \vec{v} \in \mathcal{V}: 1 \cdot \vec{v} = \vec{v}$.

This definition is more general than required, as it is defined over arbitrary fields. VSS limits itself to finite n -dimensional vector spaces over the real numbers \mathbb{R} . VSS also assumes the existence of a *norm function* $|\cdot|: V \mapsto \mathbb{R}^+$, satisfying the following conditions:

Definition 4 (Norm) A norm on a real vector space \mathcal{V} is a map $|\cdot|: \mathcal{V} \mapsto \mathbb{R}^+$ such that the following hold for all vectors $\vec{u}, \vec{v} \in \mathcal{V}$ and scalars $k \in \mathbb{F}$:

1. $|\vec{v}| \geq 0$, and $|\vec{v}| = 0 \Leftrightarrow \vec{v} = \vec{0}$;
2. $|k \cdot \vec{v}| = |k| |\vec{v}|$;
3. $|\vec{u} + \vec{v}| \leq |\vec{u}| + |\vec{v}|$

VSS distinguishes between a set of normal, or ‘basic’, vectors V , and a domain of *located vectors* (essentially directed line segments), where a located vector consists in an ordered pair of basic vectors, $\mathbf{w} = \langle p, v \rangle \in V \times V$. Although both the first and second members of each pair $\langle p, v \rangle$ are vectors in V , they are interpreted differently: the first component indicates a *location*, while the second component indicates a *displacement*. Each point p defines a *located vector space*, V_p , centred at p .

The *starting point* (spo) and *end point* (epo) of a located vector are locations in the vector space, defined as follows (we use the symbol ‘ \triangleq ’ for definitional equations).

Definition 5

1. $\text{spo} \langle p, \vec{v} \rangle \triangleq p$,
2. $\text{epo} \langle p, \vec{v} \rangle \triangleq p + \vec{v}$.

According to Zwarts and Winter [182], all locative prepositions denote functions from sets of (simple) vectors to sets of located vectors. The location of a reference object is represented as a set $A \subseteq \mathcal{V}$, the locative expression denotes a function $\text{loc}: \wp(\mathcal{V}) \mapsto \wp(\mathcal{V} \times \mathcal{V})$ from sets of vectors to sets of located vectors. For example, the preposition ‘behind’ denotes a function from the set of vectors α whose endpoints indicate the position of the reference object (denoted by the prepositional complement) to the set of vectors $\llbracket \text{behind NP} \rrbracket$ that start on the reference object and end at a point behind it; the phrase *behind the table*, therefore, denotes the set of located vectors that start on the table and end at a point behind the table.⁶

⁶Properly speaking, in VSS the set $\llbracket P_{loc} \text{ NP} \rrbracket$ comprises those located vectors that are *internally* or *externally closest* to $\llbracket \text{NP} \rrbracket$; informally, a located vector v is internally (externally) closest to a set A when v starts at a boundary point of A , has its endpoint inside (outside) A , and there is no shorter connection between the boundary of A and the endpoint of v ; for a formal definition of the notion, see Zwarts and Winter [182].

In terms, sentence (35c) is analysed as making an existence claim regarding two located vectors, call them $\langle w_1, v_1 \rangle, \langle w_2, v_2 \rangle$, where the pair of vectors $w_1, v_1 \in \mathcal{V}$ in the first located vector $\langle w_1, v_1 \rangle$ correspond to the location of the house and the location of the bird in relation to the house, and the vectors w_2, v_2 in the second located vector $\langle w_2, v_2 \rangle$ describe the location of the cloud and location of the bird relative to the cloud.

- (36) a. $\llbracket \text{above the house} \rrbracket =$
 $\lambda \mathbf{v}. \mathbf{v} \text{ points up} \wedge \mathbf{v} \text{ starts at (the location of) the house}$
 b. $\llbracket \text{below the cloud} \rrbracket =$
 $\lambda \mathbf{v}. \mathbf{v} \text{ points down} \wedge \mathbf{v} \text{ starts at (the location of) the cloud}$
- (37) a. $\llbracket \text{the bird is above the house} \rrbracket =$
 $\exists \mathbf{v}. \mathbf{v} \text{ points up} \wedge \mathbf{v} \text{ starts at the house} \wedge \mathbf{v} \text{ ends at the bird}$
 b. $\llbracket \text{the bird is below the cloud} \rrbracket =$
 $\exists \mathbf{v}. \mathbf{v} \text{ points down} \wedge \mathbf{v} \text{ starts at the cloud} \wedge \mathbf{v} \text{ ends at the bird}$

However, the interpretation of spatial PPs as properties of vectors introduces certain complications into the semantic interpretation process: in order to apply the vector predicate in (36a) to the non-vector individual denoted by *the bird* to obtain (37a), we need to perform some sort of type conversion operation to overcome the type mismatch between subject and predicate. The mechanism introduced by Zwarts and Winter [182] makes use of a pair of complementary ‘location’ and ‘anti-location’ functions, written loc and loc^- , which map non-vector entities into vector entities and (sets of) vector entities into (sets of) non-vector entities, respectively. The type mismatch between the subject and the predicate triggers the application of loc^- , which transforms the predicate from a property of vectors into a property non-vector individuals, namely, those objects that are located at the endpoint of *some* vector in the original set—see Zwarts and Winter [182] for further details.

$$\begin{aligned} \llbracket \text{the bird is above the house} \rrbracket &= \\ &= \text{loc}^- (\llbracket \text{above the house} \rrbracket) (\llbracket \text{the bird} \rrbracket) \\ &= (\lambda x. \exists \mathbf{v}. \mathbf{v} \text{ points up} \wedge \mathbf{v} \text{ starts at the house} \wedge \mathbf{v} \text{ ends at } x) (\llbracket \text{the bird} \rrbracket) \\ &= \exists \mathbf{v}. \mathbf{v} \text{ points up} \wedge \mathbf{v} \text{ starts at the house} \wedge \mathbf{v} \text{ ends at the bird} \end{aligned}$$

1.3.1 Measure Phrases in VSS

Measure phrases may occur in spatial and temporal PPs with the structure shown in Figure 1.1.

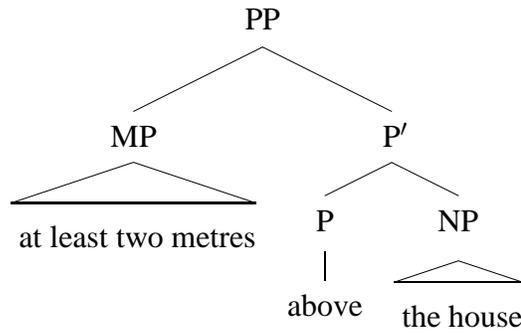


Figure 1.1: PP with MP modifier

In VSS, measure phrases denote properties of vectors; e.g., an MP such as *two metres* has as its extension the set of vectors that is 2 metres in length:

$$\llbracket 2 \text{ metres} \rrbracket = \{\mathbf{w} : |\mathbf{w}| = |2 \text{ metres}|\} \quad (1.1)$$

Since locative phrases are also interpreted as sets of vectors, VSS provides a straightforward compositional analysis of measure phrase modification in terms of set intersection. Thus, the meaning of the phrase ‘2 metres above the house’ is given by:

$$\begin{aligned} & \llbracket 2 \text{ metres above the house} \rrbracket \\ &= \llbracket 2 \text{ metres} \rrbracket \cap \llbracket \text{above the house} \rrbracket \\ &= \{\mathbf{w} : |\mathbf{w}| = |2 \text{ metres}|\} \cap \{\mathbf{u} : \mathbf{u} \text{ points up} \wedge \mathbf{u} \text{ starts at the house}\} \\ &= \{\mathbf{u} : \mathbf{u} \text{ points up} \wedge \mathbf{u} \text{ starts at the house} \wedge |\mathbf{u}| = |2 \text{ metres}|\} \end{aligned}$$

We find exactly the same modifier construction in the case of temporal expressions:

$$\begin{aligned} & \llbracket 2 \text{ hours after the concert} \rrbracket \\ &= \llbracket 2 \text{ hours} \rrbracket \cap \llbracket \text{after the concert} \rrbracket \\ &= \{\mathbf{w} : |\mathbf{w}| = |2 \text{ hours}|\} \cap \{\mathbf{u} : \mathbf{u} \text{ points to the future} \wedge \mathbf{u} \text{ starts at the concert}\} \\ &= \{\mathbf{u} : \mathbf{u} \text{ points to the future} \wedge \mathbf{u} \text{ starts at the concert} \wedge |\mathbf{u}| = |2 \text{ hours}|\} \end{aligned}$$

1.3.2 Gradable Adjectives in VSS

The motivation for Martina Faller’s [46] extension of the VSS framework to degree adjectives and comparatives is the possibility of modelling the cross-categorial semantics of measure phrase modifiers. Faller [46] develops an interpretation process for adjectives modelled

on the one for locative expressions: first, a given denotation of type e (corresponding to the reference object) is mapped to a vector that indicates its position on some dimensional scale (e.g., HEIGHT); then, semantic operations involving the adjectival expression yield as result a set of located vectors, which is mapped back into a set of entities of type e for type-commensurability with the subject of the predication. In order to carry out the necessary type shifts, Faller [46] introduces a pair of ‘dimension’ and ‘anti-dimension’ functions $\text{dim}_S: e \rightarrow \wp(\mathcal{V})$ and $\text{dim}^-_S: \wp(v \times v) \rightarrow \wp(e)$, which are explicitly modelled on the equivalent functions for locative expressions, namely, the “location” function, $\text{loc}: e \rightarrow v$, which maps entities to a position in space, and the “anti-location” function, $\text{loc}^-: \wp(v \times v) \rightarrow \wp(e)$, which maps sets of vectors to sets of entities, viz., those located at the endpoints of those vectors (cf. Zwarts and Winter [182]).

Faller [46] takes dimensional adjectives to be sets of located vectors in some dimension d , where antonymic pairs of adjectives correspond to vector sets with opposed polarities; she follows Kennedy [92] in deriving both comparative and absolute forms from a common adjectival root; for example, the adjectival roots *tall-* and *short-* are interpreted as sets of dimensional vectors:

1. $\text{tall} \Rightarrow \lambda \mathbf{u}.\text{HEIGHT}(\mathbf{u}) \wedge \text{pos}(\mathbf{u})$
2. $\text{short} \Rightarrow \lambda \mathbf{u}.\text{HEIGHT}(\mathbf{u}) \wedge \text{neg}(\mathbf{u})$

Here S is a position vector which corresponds to some contextually-given standard for tallness or shortness (for present purposes, we make the simplifying assumption that the standards for ‘tall’ and ‘short’ are the same, even though in reality they might not be).

Following Kennedy [92], Faller adopts an analysis in which the adjectival roots combine with a comparative marker, such as the suffix *-er* and the adverbs *more* and *less*, or a phonetically null absolute marker, \emptyset^{ABS} (Definition 6).

Definition 6 (comparative and absolute markers)

1. $\text{more} / \text{-er} \Rightarrow \lambda W.\lambda V.\lambda \mathbf{v}.W(\mathbf{v}) \wedge (\exists w.V(w) \wedge w = \text{spo}(\mathbf{v}))$
2. $\text{less} \Rightarrow \lambda W.\lambda V.\lambda \mathbf{v}.W(\tilde{\mathbf{v}}) \wedge (\exists w.V(w) \wedge w = \text{spo}(\mathbf{v}))$
3. $\emptyset^{\text{ABS}} \Rightarrow \lambda W.\lambda v.\exists \mathbf{w}.W(\mathbf{w}) \wedge v = \text{epo}(\mathbf{w}) \wedge S = \text{spo}(\mathbf{w}) \wedge |\mathbf{w}| > 0$

where $\tilde{\mathbf{v}}$ is defined as follows: if $\mathbf{v} = \langle s, u \rangle$, then $\tilde{\mathbf{v}} = \langle s, -u \rangle$. Thus the effect of *less* is to reverse the basic polarity of the vector associated with the adjective (cf. Faller [46, p. 158]).

Let us consider first the comparative form *taller*.⁷ By Definition 6, we obtain the interpretation in (1.4) below:

$$\begin{aligned}
[\text{Deg}' \text{ taller}] &\Rightarrow (\lambda W. \lambda V. \lambda \mathbf{v}. W(\mathbf{v}) \wedge (\exists w. V(w) \wedge w = \text{spo}(\mathbf{v}))) (\lambda \mathbf{u}. \text{HEIGHT}(\mathbf{u}) \wedge \text{pos}(\mathbf{u})) \\
&= \lambda V. \lambda \mathbf{v}. (\lambda \mathbf{u}. \text{HEIGHT}(\mathbf{u}) \wedge \text{pos}(\mathbf{u}))(\mathbf{v}) \wedge (\exists w. V(w) \wedge w = \text{spo}(\mathbf{v})) \\
&= \lambda V. \lambda \mathbf{v}. \text{HEIGHT}(\mathbf{v}) \wedge \text{pos}(\mathbf{v}) \wedge (\exists w. V(w) \wedge w = \text{spo}(\mathbf{v}))
\end{aligned} \tag{1.2}$$

Both comparative markers license a *than*-phrase, where the complement of *than* may be nominal, as in *taller than John*, or clausal, as in *taller than John is wide*; we shall follow Faller [46] and Winter [175, 176] in limiting our discussion to the case where the complement is a nominal expression. For this case, Faller identifies *than* with the function *dim*, so the phrase *than John*, for example, returns the set of dimensional vectors corresponding to the denotation of John.

Definition 7 (than)

$$\text{than} \Rightarrow \lambda y. \lambda u. u \in \text{dim}(y)$$

For example, given that ‘John’ $\Rightarrow j$, we have

$$\begin{aligned}
[\text{PP than John}] &\Rightarrow (\lambda y. \lambda u. u \in \text{dim}(y))(j) \\
&= \lambda u. u \in \text{dim}(j)
\end{aligned} \tag{1.3}$$

Then we can combine the interpretation of the comparative adjective in (1.4) with that of the *than*-phrase to obtain the following translation of *taller than John*.

$$\begin{aligned}
&[\text{DegP} [\text{Deg}' \text{ taller}] [\text{PP than John}]] \\
&\Rightarrow (\lambda V. \lambda \mathbf{v}. \text{HEIGHT}(\mathbf{v}) \wedge \text{pos}(\mathbf{v}) \wedge (\exists w. V(w) \wedge w = \text{spo}(\mathbf{v}))) (\lambda u. u \in \text{dim}(j)) \\
&= \lambda \mathbf{v}. \text{HEIGHT}(\mathbf{v}) \wedge \text{pos}(\mathbf{v}) \wedge (\exists w. (\lambda u. u \in \text{dim}(j))(w) \wedge w = \text{spo}(\mathbf{v})) \\
&= \lambda \mathbf{v}. \text{HEIGHT}(\mathbf{v}) \wedge \text{pos}(\mathbf{v}) \wedge (\exists w. w \in \text{dim}(j) \wedge w = \text{spo}(\mathbf{v}))
\end{aligned}$$

Since in Faller’s theory there is at most *one* dimensional vector in $\text{dim}(j)$ from each dimension, it follows that there can be at most one vector $w \in \text{dim}(j)$ such that $\text{HEIGHT}(w)$ and $w = \text{spo}(\mathbf{v})$, namely, the vector corresponding to John’s height, which Faller [46, p. 158] writes as $\text{dim}_{\langle H \rangle j}$. We can simplify the above expression as (1.4):

$$[\text{DegP} [\text{Deg}' \text{ taller}] [\text{PP than John}]] \Rightarrow \lambda \mathbf{v}. \text{HEIGHT}(\mathbf{v}) \wedge \text{pos}(\mathbf{v}) \wedge \text{dim}_{\langle H \rangle j} = \text{spo}(\mathbf{v}) \tag{1.4}$$

However, since the semantic interpretation of the predicate ‘is taller than John’ in (1.4) is a property of (located) vectors rather than individuals, we cannot combine this directly with an

⁷We follow Faller and Kennedy in assuming that *more* is semantically equivalent to *-er*.

individual-denoting subject, say ‘Bob’. In order to assign an interpretation to a sentence such as ‘Bob is taller than John’, we require some way to overcome the type mismatch between the subject and the predicate. For this, Faller [46] introduces a pair of functions dim and dim^- , explicitly modelled on the loc and loc^- functions in Zwarts and Winter [182]; like loc^- , the dim^- function transforms the predicate from (the characteristic function of) a set of vectors into (the characteristic function) of a set of non-vector individuals (cf. Faller [46, pp. 158–159]): namely, the set of all things that are located at the endpoint of some vector in the original set; note also that dim^- also implicitly performs existential closure at the clausal level (cf. Definition 8 below).

Definition 8 (dim^-)

$$\text{dim}^- \triangleq \lambda W. \lambda x. \exists \mathbf{v}. \exists u. W(\mathbf{v}) \wedge u \in \text{dim}(x) \wedge u = \text{epo}(\mathbf{v}) \wedge |\mathbf{v}| > 0$$

As in the case of loc^- , dim^- is not associated with a particular lexical item; rather, it is an operation that is triggered by the type mismatch between subject and predicate. Applying dim^- to the interpretation of ‘tall’, we obtain an expression of type $(e \rightarrow t)$, which is then able to combine semantically with an expression of type e :

$$\begin{aligned} & \text{dim}^-([\text{taller than John}]) \\ \Leftrightarrow & (\lambda W. \lambda x. \exists \mathbf{v}. \exists u. W(\mathbf{v}) \wedge u \in \text{dim}(x) \wedge \text{epo}(\mathbf{v}) = u \wedge |\mathbf{v}| > 0) \\ & (\lambda \mathbf{w}. \text{HEIGHT}(\mathbf{w}) \wedge \text{pos}(\mathbf{w}) \wedge \text{dim}_{\langle H, j \rangle} = \text{spo}(\mathbf{w})) \\ = & \lambda x. \exists \mathbf{v}. \exists u. (\lambda \mathbf{w}. \text{HEIGHT}(\mathbf{w}) \wedge \text{pos}(\mathbf{w}) \wedge \text{dim}_{\langle H, j \rangle} = \text{spo}(\mathbf{w}))(\mathbf{v}) \wedge u \in \text{dim}(x) \\ & \wedge u = \text{epo}(\mathbf{v}) \wedge |\mathbf{v}| > 0 \\ = & \lambda x. \exists \mathbf{v}. \exists u. \text{HEIGHT}(\mathbf{v}) \wedge \text{pos}(\mathbf{v}) \wedge \text{dim}_{\langle H, j \rangle} = \text{spo}(\mathbf{v}) \wedge u \in \text{dim}(x) \\ & \wedge u = \text{epo}(\mathbf{v}) \wedge |\mathbf{v}| > 0 \end{aligned}$$

Taking ‘Bob’ $\Leftrightarrow b$, b of type e , we obtain the following translation for ‘Bob is taller than John’:

$$\begin{aligned} & [\text{Bob} [\text{is} [\text{taller than John}]]] \\ \Leftrightarrow & (\lambda x. \exists \mathbf{v}. \exists u. \text{HEIGHT}(\mathbf{v}) \wedge \text{pos}(\mathbf{v}) \wedge \text{dim}_{\langle H, j \rangle} = \text{spo}(\mathbf{v}) \wedge u \in \text{dim}(x) \\ & \wedge u = \text{epo}(\mathbf{v}) \wedge |\mathbf{v}| > 0)(b) \tag{1.5} \\ = & \exists \mathbf{v}. \exists u. \text{HEIGHT}(\mathbf{v}) \wedge \text{pos}(\mathbf{v}) \wedge \text{dim}_{\langle H, j \rangle} = \text{spo}(\mathbf{v}) \wedge u \in \text{dim}(b) \\ & \wedge u = \text{epo}(\mathbf{v}) \wedge |\mathbf{v}| > 0 \end{aligned}$$

The translation states that there is a positive height vector \mathbf{v} which has its starting point at the vector $\text{dim}_{\langle H, j \rangle}$, corresponding to John’s height, and its end point at another vector u belonging

to the set of vectors associated with Bob, $\text{dim}(b)$, which must therefore be the (unique) height vector corresponding to Bob's height. If we represent John's height as j_H and Bob's height as b_H , and allow for addition of vectors, then we can simplify the translation in (1.5) to the following (cf. Faller [46, p. 159]):

$$[\text{Bob [is [taller than John]]}] \Leftrightarrow \exists \mathbf{v}. b_H = j_H + \mathbf{v} \wedge \text{pos}(\mathbf{v}) \wedge |\mathbf{v}| > 0 \quad (1.6)$$

The interpretation of a standard (non-comparative) form of the adjective proceeds in similar fashion, but here the adjectival root combines with the phonologically empty absolute morpheme \emptyset^{ABS} . Definition 6 gives us the following interpretation to the adjective *tall*:

$$\begin{aligned} & [\emptyset^{\text{ABS}} \text{ tall}] \\ & \Leftrightarrow (\lambda W. \lambda v. \exists \mathbf{w}. W(\mathbf{w}) \wedge v = \text{epo}(\mathbf{w}) \wedge S = \text{spo}(\mathbf{w}) \wedge |\mathbf{w}| > 0) \\ & \quad (\lambda \mathbf{u}. \text{HEIGHT}(\mathbf{u}) \wedge \text{pos}(\mathbf{u})) \\ & = \lambda v. \exists \mathbf{w}. (\lambda \mathbf{u}. \text{HEIGHT}(\mathbf{u}) \wedge \text{pos}(\mathbf{u}))(\mathbf{w}) \wedge v = \text{epo}(\mathbf{w}) \\ & \quad \wedge S = \text{spo}(\mathbf{w}) \wedge |\mathbf{w}| > 0 \\ & = \lambda v. \exists \mathbf{w}. \text{HEIGHT}(\mathbf{w}) \wedge \text{pos}(\mathbf{w}) \wedge v = \text{epo}(\mathbf{w}) \wedge S = \text{spo}(\mathbf{w}) \wedge |\mathbf{w}| > 0 \end{aligned} \quad (1.7)$$

1.3.3 Adjectives and Measure Phrase Modifiers

There is a considerable literature on the syntax of comparatives and gradable adjectives; within a broadly generative framework, we find, among others, Lees [109], Smith [147], Pilch [130], Huddleston [75], Hale [63], and Hendrick [68], as well as Joan Bresnan's [17, 18] landmark studies of the comparative clause in English. Much of this work, especially Bresnan's, influenced subsequent developments in X' theory, from Jackendoff [76] to Abney [1], whose work on phrasal projection forms the basis of the syntactic framework adopted by Kennedy [92], where measure phrases occur as specifiers of the functional head of a degree phrase (DegP). The head position of the degree phrase is occupied by a comparative marker ('less', 'more') or a (phonologically empty) 'absolute' degree morpheme which takes a gradable adjective as its complement, as in Figures 1.2 and 1.3.

Faller [46] claims that the same simple mechanism that VSS uses for measure phrase modification in the case of spatial PPs can be applied to the measure phrase modification of gradable adjectives. Thus, we take the phrase *two metres* to have the translation given in (1.8) below,

$$[2 \text{ metres}] \Leftrightarrow \lambda W. \lambda v. W(v) \wedge |v| = 2m \quad (1.8)$$

Given an interpretation in which *tall* denotes a set of vectors, as in (1.7) above, we can combine this with the above semantics for the measure phrase to obtain the interpretation for the phrase

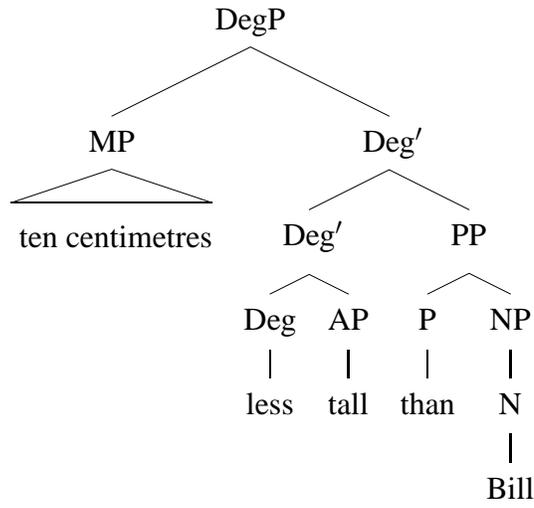


Figure 1.2: comparative degree phrase with MP modifier

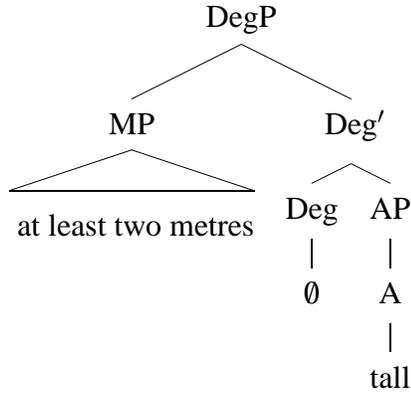


Figure 1.3: absolute degree phrase with MP modifier

2 metres tall in (1.9).

$$\begin{aligned}
 & [[2 \text{ metres}] [\emptyset^{\text{ABS}} \text{ tall}]] \\
 \Rightarrow & (\lambda W. \lambda v. W(v) \wedge |v| = 2m) \\
 & (\lambda v. \exists w. \text{HEIGHT}(w) \wedge \text{pos}(w) \wedge v = \text{epo}(w) \wedge S = \text{spo}(w) \wedge |w| > 0) \\
 = & \lambda v. (\lambda v. \exists w. \text{HEIGHT}(w) \wedge \text{pos}(w) \wedge v = \text{epo}(w) \wedge S = \text{spo}(w) \wedge |w| > 0)(v) \wedge |v| = 2m \\
 = & \lambda v. \exists w. \text{HEIGHT}(w) \wedge \text{pos}(w) \wedge v = \text{epo}(w) \wedge S = \text{spo}(w) \\
 & \wedge |w| > 0 \wedge |v| = 2m
 \end{aligned} \tag{1.9}$$

As Faller herself notes, the semantics of \emptyset^{ABS} , as defined in Definition 6 above, require the

d-vector to be longer (or shorter) than a contextually-given standard S_{HEIGHT} introduced by the (null) absolute morpheme; but, in the presence of a measure phrase, the contextual standard is no longer relevant—a child that is one metre tall is not necessarily tall relative to a contextually-salient standard of height. Moreover, the measurement in absolute adjectives is always taken from zero.

One approach Faller considers, and which is in fact the one adopted by Kennedy [92], is the introduction of a *second* interpretation for the null absolute morpheme which does not make reference to any standard of evaluation; in other words, in addition to the interpretation of the null morpheme in (1.10), we would have the additional interpretation in (1.11).

$$\emptyset^{\text{ABS}} \Rightarrow \lambda W. \lambda v. W(v) \quad (1.10)$$

or, alternatively,

$$\emptyset^{\text{ABS}} \Rightarrow \lambda W. \lambda v. \exists w. W(w) \wedge v = \text{epo}(w) \wedge S = \text{spo}(w) \wedge S = 0 \wedge |w| > 0 \quad (1.11)$$

The use of an ambiguous null morpheme is ultimately rejected by Faller [46, p. 161], who points out that since there is nothing to prevent either null morpheme from being applied, this predicts that a sentence such as *Bob is 2 m. tall* is ambiguous between a reading in which the contextual standard is present and a reading in which it is absent, an ambiguity that does not appear to exist.

Ultimately, Faller simply assumes that S is set to zero in measure phrase constructions containing absolutes, but, having eschewed the option of an ambiguous absolute morpheme, does not provide an explicit compositional mechanism to accomplish this.

1.3.4 The Modification Condition

As mentioned earlier in this section, VSS analyses locative prepositions as functions from entities to sets of vectors representing spatial regions. Zwarts [180] examines the characteristics of these vector sets, in particular their *closure* and *continuity* properties.

Two properties of particular importance for our discussion are those of *closure under shortening*, and *closure under lengthening*. Closure under shortening means that, if v is a vector in the region V denoted by a prepositional phrase P , then $k \cdot v$ also belongs to V , for all $0 < k < 1$, while closure under lengthening means that, if v is a vector in the region V denoted by a prepositional phrase P , then $k \cdot v$ also belongs to V , for all $k > 1$. A vector set is said to be *upward monotone* (VMON \uparrow) iff it is closed under lengthening of its members, and *downward monotone* (VMON \downarrow) iff it is closed under shortening. Zwarts argues that the class of ‘simple’ (i.e., unmodified) prepositions satisfy the following universal property:

Claim 1 *All simple locative prepositions are downward monotone (VMON ↓).*

By contrast, only a subset of simple prepositions are *upward* monotone; the class of upward monotone prepositions includes ‘behind’, ‘in front of’, ‘over’, ‘under’, ‘above’, ‘below’, ‘outside’, for example, but not ‘beside’, ‘next to’, ‘inside’, ‘near’, ‘in’, ‘on’, ‘at’, or ‘inside’. Zwarts [180] noted that upward monotonicity appears to be a distinguishing feature of prepositions that can be modified by a measure phrase, as in (38) below.

- (38) a. The bird is 10 m. in front of the house.
b. * The bird is 10 m. near the house.

The preposition ‘in front of’ is upward monotone because, if some object x is in front of the house and moves further away from it, it still remains in front of the house.⁸ By contrast, if x is ‘near’ the house and moves further away from it, at some point it will no longer be near the house, so the preposition ‘near’ is not upward monotone.

The *Modification Condition* (MC), proposed by Zwarts [180] and subsequently refined by Zwarts and Winter [182], is intended to capture the contrasting behaviour of certain spatial prepositions in the presence of measure phrase modifiers, as exemplified in (38a) and (38b) above. The MC is couched in terms of the monotonicity properties of the vector sets denoted by the prepositional phrases; the particular formulation in definition 9 below is from Winter [176, p. 252], and corresponds essentially to the version in Zwarts and Winter [182].

Definition 9 (Modification Condition (MC)) *An expression that is associated with a set of vectors W can be modified by an MP only if W is non-empty, upward and downward monotone and does not contain zero vectors.*

However, as we noted earlier, there is a similar contrast in acceptability in the use of the MP modifiers with gradable adjectives, as in (39) below (cf. example (6) in section 1.2.1 above).

- (39) a. The table is 1 metre wide/long/high.
b. * The table is 1 metre narrow/short/low.

Given the parallel between the examples in (38) and (39), it is natural to consider the possibility of a common explanation for the behaviour of MP modifiers in both cases. In [176], Winter attempts to account for the behaviour of MP modifiers with gradable adjectives in terms of the MC.

⁸It is assumed that the vectors in the set denoted by ‘in front of the house’ have their starting points at the outside boundary of the house, and point away from it.

The formal details of Winter’s account differ from Faller’s in certain respects; for example, where Faller uses a single scale structure for antonymic adjective pairs such as ‘tall’/‘short’, ‘fat’/‘slim’, etc., Winter follows Kennedy [93] in employing a pair of distinct scales with opposite orientations. An adjective A in the positive form is associated with a set of located vectors of the form $\langle z_S, w \rangle$, where z_S is a vector representing the *zero value* for the adjective A , or, as Winter [176, p. 243] puts it, “the minimal amount of A -ness relative to the scale S ”; the vector w , for its part, represents the degree of A -ness of an entity in A ’s denotation, where this must be greater than some standard d_S .

Winter [176, p. 245] then assigns an interpretation to adjectives in terms of sets of located vectors as follows: the positive form of an adjective is associated with a set of located vectors of the form $\langle z_S, t \cdot u_S \rangle$, where z_S is the zero vector, and $t \cdot u_S$ is some vector greater in magnitude than the standard vector d_S (the product of a scalar t and the unit vector u_S of the scale S).

In the case of ‘tall’ and ‘short’, for example, we end up with the interpretations in (40) below, where t_0 is a scalar derived from the contextual standard of evaluation, and u_H is a unit height vector:

- (40) a. $tall' = \{ \langle 0, t \cdot u_H \rangle : t > t_0 \}$
 b. $short' = \{ \langle 0, t \cdot u_H \rangle : 0 < t < t_0 \}$

Winter’s [176, p. 245] principal reason for treating the positive form of degree adjectives as located vectors of the form $\langle 0, w \rangle$ is the general semantics of MP modification: unlike comparatives such as ‘2m. longer than the garage’, where the length is taken relative to the length of the garage, when something is ‘2 m. long’, this is invariably taken to mean *two metres longer than zero*.

We can paraphrase the interpretations Winter assigns to the basic adjective forms very roughly as follows:

- (41) a. $\llbracket tall \rrbracket = \lambda v. v$ is on the height scale $H \wedge v$ starts at $0 \wedge v$ is greater than d_H
 b. $\llbracket short \rrbracket = \lambda v. v$ is on the height scale $H \wedge v$ starts at $0 \wedge v$ is less than d_H

The interpretation he provides for *five feet tall* is then:

$$\llbracket \text{five feet tall} \rrbracket = \lambda v. v \text{ is on the height scale } H \wedge v \text{ starts at } 0 \wedge v \text{ is greater than } d_H \quad (1.12)$$

$\wedge v$ has a magnitude of 5 feet

However, as Winter [176, p. 250] himself observes, the translation in (40), which we paraphrased informally in (1.12), is not in itself an adequate representation of the meaning of the phrase: the translation still requires the vectors in the interpretation of ‘five feet tall’ to be

greater than the contextual standard d_H . This means that, given a standard d_H for tallness somewhere around 6 feet (say), the sentence ‘Bob is 5 feet tall’ would be false even if Bob is indeed 5 feet in height, simply because Bob does not count as tall according to the contextual standard. The standard must be set to zero, as pointed out by Faller, or done away with altogether, in order to obtain the correct interpretation (cf. Section 1.3.3 above).

Winter claims that it is the Modification Condition which produces the correct interpretation; in particular, the standard d_H is set to zero in contexts of MP modification in order to satisfy the *downward* monotonicity constraint of the MC. The set of vectors corresponding to *tall'* is upward monotone for any value of d_H , but it is only downward monotone if $d_H = 0_H$; in this case, the denotation of *tall'* comprises the entire height scale and satisfies both the upward and downward monotonicity constraints, and thus the MC, as shown graphically in Figure 1.4.

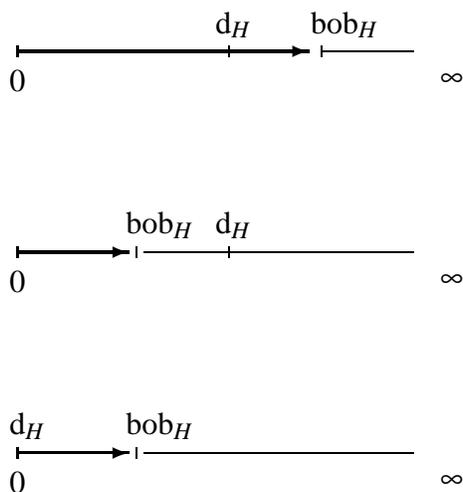


Figure 1.4: Downward monotonicity and the scale for ‘tall’

By contrast, the vector set denoted by *short'* is downward monotone for any value of d_H , but upward monotone only if $d_H = 0_H$, in which case the set is empty (because there are no vectors shorter than 0_H). It is this contrast that explains why unbounded adjectives, such as ‘tall’, can undergo modification by measure phrases, while bounded adjectives like ‘short’ do not.

1.3.5 A Critique of VSS

The inclusion of vectors in the semantic ontology makes VSS a powerful framework within which to express geometric constructs. This is immediately useful in capturing the semantics of spatial expressions, where geometric notions are directly relevant, but it is also applicable to other domains which share aspects of meaning with the spatial domain at an abstract level. One such domain is time, which in our modern conception is isomorphic to the real number line; this shared structure is manifested in the use of spatial expressions in the temporal domain, as when we speak of the ‘near’ and ‘distant’ future, or a ‘long’ or ‘short’ duration. Several researchers regard the spatial dimension of length as more fundamental than the time dimension, and regard the spatial domain as the structuring domain for many other domains; Lakoff and Johnson [102] have argued that the concept of time is *metaphorically* structured by the concept of space. Herskovits [70] presents a detailed study of spatial prepositions, and discusses several instances where spatial structure is transferred to other domains: a sentence like ‘We met Alice *at* six o’clock’, for example, shares the same abstract structure as ‘We met Alice *at* the races’, or ‘Bob is *at* the station’ (cf. Herskovits [70, p. 51], Gärdenfors [53, ch. 5]).

A vector-based semantics, being especially suited to interface with geometric models of conceptual structure, is thus well-suited to modelling these and other abstract structural parallels among conceptual domains (see Gärdenfors [53, ch. 5]). However, there are also certain areas where VSS is incomplete or inadequate.

As Herskovits has pointed out,⁹ one problem for VSS, at least in its original formulation, is that the meaning of a prepositional phrase is not fully reducible to geometric properties. In many cases, inclusion in the relevant region is often a necessary but not *sufficient* condition for the use of the preposition: e.g., ‘upon’, ‘against’ demand spatial contiguity, while ‘on’ requires both contiguity and support (see Herskovits [70], Gärdenfors [53, ch. 5]). More recent work within the framework (e.g., Zwarts [181]) has begun to address this issue, using Talmy’s [155] notion of force dynamics.

Another issue is that VSS analyses spatial relations as *two-place* relations between a located object and reference object. However, in Chapter 2, Section 2.2 below we shall see that a two-place relation is insufficient to capture the properties of spatial prepositions with a directional character, where we also need to take into account the *point of view* from which a spatial scene is described.

There are two further problems with VSS which are especially pertinent to the semantics of gradable adjectives, which we shall refer to as the ‘divergent type problem’ and the ‘coordina-

⁹Personal communication, cited by Gärdenfors [53, pp. 172–173]; see also Zlatev [178].

tion problem’.

1.3.5.1 The Divergent Type Problem

The defining feature of scalar theories is the presence of abstract representations of measure, such as degrees, intervals, or vectors, as ‘first-class’ members of the semantic ontology. Such theories typically assign gradable and non-gradable adjectives logical representations of different semantic types: on a predicative analysis, non-gradable adjectives are usually interpreted as properties of individuals (i.e., as sets of entities), while gradable adjectives are interpreted as relations between individuals and degrees, or, as in VSS, as properties of degrees or vectors. The distributional characteristics of gradable adjectives, including in particular the fact that gradable adjectives, unlike non-gradable ones, can appear with degree modifiers, are then explained in terms of this difference.

In scalar theories, gradable adjectives are analysed as relations involving abstract measures, and in some theories are assigned a logical type that is distinct from non-gradable adjectives. We see this in VSS, for example, where non-gradable adjectives are given a traditional analysis as properties of individuals, but gradable adjectives are instead analysed as properties of *vectors* (see Faller [46] and Winter [175, 176]).

While adjectives form a very diverse class, it is nevertheless the case that gradable adjectives, like other predicative adjectives, are intuitively construed as properties of individuals, not properties of vectors. This fundamental fact is obscured by the assignment of completely distinct types to gradable and non-gradable adjectives.

Thus one criticism that can be levelled at VSS, along with other scalar theories, is that it fails to capture the underlying unity of the class of adjectives. Adjectives in English exhibit a diverse range of semantic behaviour; the challenge for a semantic theory is to model this diversity, while still recognising the underlying unity of the class within the grammar. This leads to a situation where, in order to cope with even the simplest predicative and attributive constructions, VSS needs to supplement the semantic composition rules with powerful type coercion mechanisms to ensure basic type compatibility.

1.3.5.2 The Coordination Problem

The introduction of additional entity types into the ontology has other disadvantages; for example, the increase in the formal complexity of the semantic theory, in the form of additional rules for type conversion and semantic interpretation. Worse than this, however, is the fact that the introduction of vectors (or degrees) appears to give rise to paradoxical interpretations for even

quite simple constructions that are unproblematic for non-scalar approaches. One example of this is the problem VSS has with intersective constructions, such as coordinate structures and determiner phrases with restrictive modifiers, as in (42) and (43) below.

- (42) a. Bill is tall and slim.
b. Bill is taller and slimmer than John.
c. Bill is taller and slimmer than some student.
d. John is shorter and heavier than Bill.
e. Bill is taller than Jack and slimmer than Jill.
- (43) a. The tall ugly bloke is my brother.
b. My sister is a beautiful slim girl.
c. I would like a big red juicy apple for dessert.

The problem is that VSS is unable to assign interpretations to intersective constructions involving multiple gradable adjectives, or indeed spatial and temporal PPs. There exists a venerable tradition of analysing both conjunction and restrictive modification in terms of set-theoretic intersection.¹⁰ However, as Zwarts and Winter [182] point out, a Boolean approach to conjunction appears to be incompatible with the VSS account of spatial prepositions, because one cannot interpret the conjoined VSS denotations (which are sets of vectors) in terms of set-theoretic intersection. For example, VSS assigns completely disjoint denotations to the PPs *above the house* and *below the cloud* in sentence (44a) below (since the vectors have a different starting point in each set), so their intersection yields the empty set; but (44a) is not in any way contradictory or impossible to interpret, and has an interpretation roughly paraphrased by sentence (44b).

- (44) a. The bird is ten meters [above the house and below the cloud].
b. The bird is ten meters above the house and the bird is ten meters below the cloud.

As Faller [46] recognises, this problem with coordination carries over to the VSS analysis of degree adjectives, which assigns completely disjoint denotations to sentences such as those in (42) and (43) above; as in the case of (44), the conjoined elements are again not contradictory, nor otherwise semantically incompatible.

¹⁰The cross-categorial treatment of coordination in formal semantics has been developed over the years in the work of von Stechow [168], Keenan and Faltz [91], Gazdar [54], Partee and Rooth [129], and others.

1.4 Aims and Scope

The case for treating gradable adjectives along the lines of spatial and temporal expressions is based on a number of structural and conceptual analogies among these domains. These are explored in greater detail in Chapter 2.

In Chapter 2, we shall see that there are certain aspects of the meaning of spatial and temporal expressions which show that locative relations cannot be adequately analysed in terms of a binary relation between a located object and a reference object alone. This can be seen, for example, in the apparent inconsistencies that can arise in describing a given Figure-Ground pair; for example, Bob may say that the church is on the left side of the road, while Bill may equally insist that it is on the right. Of course, it is quite possible for both Bob and Bill to be right, since they might be looking at the church from different points of view. As Barwise and Etchemendy [10] observe, whenever one encounters an apparent incoherence in the world, it is often a symptom of some implicit parameter whose value is changing: in this case, the inconsistency can be resolved by recognising that one and the same arrangement of objects may be viewed from different points-of-view, or *perspectives*.

Chapter 3 presents a formal semantics for gradable adjectives within a general theory of locative semantics, which we refer to as *Locative Structure Semantics* (LSS). Our main hypothesis is that there is a common locative structure underlying the semantics of gradable adjectives, spatial expressions, and tense. We claim that gradable adjectives, along with spatial and temporal expressions, have a common underlying semantics couched in terms of ternary relational structures called *locative structures* (L-structures), and refer to the semantic framework based on locative structures as *Locative Structure Semantics* (LSS). A central notion in our theory of locative structures will be the notion of *perspective*, as found in the work of Barwise and Perry [9], Seligman [141, 142], Barwise and Seligman [11], and Talmy [157]; in particular, we will regard the norm or standard corresponding to a gradable adjective as a *conceptual perspective*, whose variability is a form of *deictic shift*.

In the locative theory we propose, the standard of evaluation is tied to the notion of a *perspective* or *point of view*. The standard refers to a location in conceptual space, a *conceptual* deictic akin to the locative indexicals ‘here’ and ‘now’; the variability of the standard of evaluation can then be viewed as a form of *deictic shift*, akin to the variability exhibited by ‘here’ and ‘now’. Moreover, the vagueness that characterises both gradable adjectives and spatio-temporal deictic expressions like ‘here’ and ‘now’ can also be seen as deriving from their shared locative nature.

The notion of perspective is also highly relevant to the phenomenon of tense, where it is

usually associated with the *moment of speech*. Here, too, there are good grounds for assuming that tenses have an underlying ternary structure. While all traditional theories of tense recognise a binary relation between the described event and a temporal deictic centre relative to which the description is made (usually, the moment of speech), Hans Reichenbach [134] famously proposed that each temporal expression is associated with three time points: a *speech point*, S , an *event point*, E , and a *reference point*, R , thought of as a situation or context that is relevant to the utterance. Reichenbach's tripartite scheme can capture subtle differences between tenses that are difficult if not impossible to express in a framework that only admits of binary relations between points E (*then*) and S (*now*).

We will argue that locative relations in general comprise three essential elements: an object, event or location being described, which we write as F and call the *Figure* (or *located entity*); an object, event or location in relation to which the target is described, which we write as G and call the *Ground* (or *reference entity*); and, a location or event corresponding to the perspective, or point of view, from which the description is made, which we write as P and call the *Perspective point*.

Our claim is that L-structures enable us to capture important parallels between the semantics of adjectives and spatial and temporal locatives. L-structures can be thought of as a generalisation of Reichenbachian tenses, abstracted from a specifically temporal setting and applied to the interpretation of locatives in general, including spatial and adjectival expressions; in much the same way that Reichenbachian theories of tense associate all tensed clauses with an event point, E , a reference point, R , and a speech point, S , we will associate locative expressions with an L-structure comprising a figure, F , a ground, G , and a perspective, P . In fact, in our view, Reichenbach's S , E , R points are simply specialised temporal instances of more abstract *locative* categories of *Perspective*, *Figure*, and *Ground*; in particular, the event point (or located event) E is the temporal Figure, the reference point R the temporal Ground, and the speech point S the temporal Perspective Point.

Another issue we shall touch on is vagueness, which we view as conceptually distinct from gradability. We provide an overview of vagueness in the context of in Section 5.2. Although we will argue in favour for a form of epistemicism, our main concern is not to provide a definitive account of vagueness per se, but rather to clarify its relationship to locative expressions in general, and gradable expressions in particular.

1.5 Chapter Overview

In the following chapters, we present a formal semantics for gradable adjectives couched in terms of a general theory of locative semantics we refer to as *Locative Structure Semantics* (LSS). The chapters are organised as follows:

Chapter 2: A New Perspective on the Semantics of Locatives We introduce the notion of *perspective* and its role in semantic accounts of spatial and tense phenomena. We explain the influence of the perspective on the interpretation of figure-ground relations in the spatial and temporal domains, and argue for a uniform treatment in terms of ternary semantic structures we refer to as *locative structures* (L-structures). We motivate the view of gradable adjectives as locatives, and discuss the relevance of the notion of perspective to the standard of evaluation. We discuss how L-structures are contextually anchored, and propose an account of gradability in terms of *deictic shift*.

Chapter 3: Locative Structures and the Semantics of Gradable Adjectives We define a formal *locative structure semantics* (LSS) for gradable adjectives. We examine the issue of measure phrase modification, and propose an *adjectival modification condition* (AMC) to capture the semantics of adjectival constructions involving measure phrases.

Chapter 4: A Dynamic Semantics for Locatives This chapter develops a dynamic model of LSS. We will argue that a dynamic semantics not only provides a means to capture anaphoric dependencies involving L-structures, but also enables us to resolve some of the problems facing vector-based theories (including both VSS and LSS), notably the *Divergent Type Problem* and the *Coordination Problem*, using a modified version of Paul Dekker's notion of *existential disclosure* (Dekker [35], [36]).

Chapter 5: Conclusion We discuss some of the issues raised by the theory developed in the previous chapters, and some directions for further research.

Chapter 2

A New Perspective on the Semantics of Locatives

2.1 Introduction

The locative theory of adjectives is based on the hypothesis that an abstract locative structure is shared by expressions from several different categories, including spatial and temporal PPs, tense constructions, and gradable adjectives.

There are two basic elements at the heart of most spatial relations: one is typically some object which is the focus of interest, while the other acts a landmark or reference point relative to which the position of the first is specified. In the view of some theorists, however, there is at least one other additional element of semantic relevance, the *perspective* or *point of view* from which the spatial scene is conceptualised or described. Following Talmy [153], we refer to these elements as the *Figure*, *Ground* and *Perspective*, and represent them symbolically as *F*, *G*, and *P*, respectively.

We shall see that the same notions can be applied to the semantics of tense. Hans Reichenbach [134] proposed that each temporal expression is associated with three time points: a *speech point*, *S*, an *event point*, *E*, and a *reference point*, *R*, where *E* refers to the time point corresponding to the event described by the tensed clause, *S* is (usually) taken to be the speaker's time of utterance, and *R* is a temporal reference point relevant to the utterance. Reichenbach argued that this tripartite scheme can capture subtle differences between tenses that are difficult if not impossible to express in a framework that only admits of binary relations. We will propose a correspondence between Reichenbach's *S*, *E*, *R* points and the categories of Perspective, Figure, and Ground; in particular, we can look upon the event point (or located event) *E* as

a temporal Figure, the reference point *R* as a temporal Ground, and the speech point *S* as a temporal Perspective point.

On the basis of the cross-categorial applicability of the notions of Figure, Ground and Perspective, we will propose a generalisation of the Reichenbachian ternary notion of tense to the domain of locative concepts. We will conjecture that there exists an abstract ternary conceptual structure we call a *locative structure*, or *L-structure*, underpinning the semantics of a broad class of ‘locative’ expressions, including both tense and spatial and temporal PPs.

We will furthermore claim that the notions of Figure, Ground and Perspective also underpin the semantic analysis of gradable adjectives. The central importance of the notion of *point of view* or *perspective* in semantic theory has been a tenet of the theory of Situation Semantics since its inception (see Barwise and Perry [9, p. 39]), and the view that the notion of perspective is applicable to the semantics of gradable adjectives is found in the work of Barwise and Seligman [11]. We will adopt the hypothesis that there is a perspectival parameter, *P*, in the semantic representation of gradable adjectives, which is responsible for the context-variable standard that characterises gradable predicates such as ‘tall’, ‘heavy’, etc., effectively treating adjectival gradability as a form of pronominal value assignment. This parametric analysis of gradability is supported by a number of structural parallels between gradable adjectives, pronouns and tenses, of the sort originally observed by Partee [127] between tenses and pronouns.

2.2 Perspectives

2.2.1 Figure and Ground

Many spatial expressions can be modelled as a relation between two elements, one of which is the theme of interest, while the other functions as a reference point or landmark relative to which the position of the first is specified. Leonard Talmy [153], [154], [156] employs the terms ‘Figure’ and ‘Ground’ for these two fundamental cognitive roles,¹ while other roughly equivalent terms found in the literature include ‘located object’ and ‘reference object’ (Zwarts and Winter [182]), ‘trajector’ and ‘landmark’ (Langacker [106]), or ‘target’ (‘cible’) and ‘landmark’ (‘site’) (Vandeloise [165, p. 21 ff.]).

Talmy characterises the Figure as a moving (or conceptually movable) entity, whose location, path, or orientation is conceived of as variable, while the Ground is a reference entity which serves as an aid in specifying the figure’s location or path (see Talmy [156, p. 312]).

¹Talmy borrows these terms from Gestalt psychology, and writes them with capitals to distinguish their specifically linguistic usage from their original, psychological usage—see Talmy [156, pp. 312–313].

Thus, the Figure is typically associated with features such as relative mobility, small size, and perceptual inaccessibility as compared with the Ground, which tends to be comparatively immobile, large in size, and of known or easily knowable location. The Figure-Ground relation is fundamentally asymmetric, and is moreover associated with a contrast between new and given information: the position of the Figure (located object) is new information, while that of the Ground (reference object) is given or old information. The asymmetric nature of the relation means that we find many examples where interchanging the roles of Figure and Ground is not acceptable, even though the purely physical relationship between the objects is unchanged (e.g., compare *the cat is near the house* with *?the house is near the cat*). In cases where the Ground is not explicitly specified, it is usually identified with a contextually salient location, such as the speaker or hearer's location; thus, an utterance of *the station is very far away* might naturally be taken to mean *very far away from here*.

For example, in a physical situation perceived as involving an object *A* located or moving relative to another object *B*, *A* takes on the role of the figure while *B* becomes the ground; thus, “Bob” is the Figure and “the house” is the Ground in (45a) below, while in (45b), the Figure is “the bird” and the Ground is “the cloud”.

- (45) a. Bob (F) sat in front of the house (G).
 b. The bird (F) flew through the cloud (G).

Talmy [156, p. 334] observes that the linguistic expression of Figure-Ground relations appears to be governed by a ‘precedence principle’: there is a pronounced tendency for the Figure to enjoy syntactic precedence over the Ground. Thus, in complex sentences, the Figure tends to be specified in the main clause and the Ground in a subordinate clause, while within a single clause the precedence takes the form of a case hierarchy: in non-agentive clauses, the Figure may be expressed as subject and the Ground as (oblique) object, while in agentive clauses, with an Agent as subject, the Figure appears as direct object and the Ground as oblique object. Talmy [156, p. 334 ff.] provides reasons for taking sentences that violate of the precedence principle as non-basic or derived.

As Talmy and others note, Figure-Ground relations are evident in other domains, notably in the temporal use of prepositions, where the Figure is an event whose location in time is the relevant issue, and the Ground is a reference event with reference to which the Figure's temporal location is characterised (cf. Talmy [156, p. 320 ff.]).

- (46) a. The reception (F) will come after the wedding (G).
 b. The explosion (F) occurred at noon (G).
 c. Bob will arrive (F) before John signs the contract (G).

- d. Alice slept (F) through the raising of the flag (G).

In (46a) and (46b), the Figure is indicated by the grammatical subject and the Ground as prepositional object, just as in example (2.2.1) above; in (46c) and (46d), both the Figure and Ground correspond to clausal constituents.

2.2.2 Perspective and the Figure-Ground Relation

While a Figure-Ground relation is apparent, in some form, in many spatial and temporal expressions, a binary relation is not in general sufficient to model the meaning of spatial expressions. This can be seen in the apparent inconsistencies that can arise in describing even quite simple Figure-Ground relations. For example, Bob says that the church is on the left side of the road, but Bill insists that it is on the right; it is of course quite possible for both Bob and Bill to be right, since they might be looking at the church from different points of view. As Barwise and Etchemendy [10] observe, whenever one encounters an apparent incoherence in the world, it is often a symptom of some implicit parameter whose value is changing: once one recognises that one and the same arrangement of objects may be viewed from different *perspectives*, the apparent inconsistency melts away.

One and the same situation or event can be viewed, or, in a broader sense, *experienced*, from different points of view. Bob, looking down the road at the church, sees an image that is dependent on his location and the direction of his gaze: his view of the church is partial, since he cannot see the far side; objects of the same size may appear to be larger or smaller, depending on how far away they are; the sides of the road, although parallel to each other, seem to converge; and so on. There is a correspondence between how the entities and relations that constitute a situation are perceived, and the particular location from which that situation is observed (as well as the perceptual and inferential abilities of the observer, of course); we refer to the observer's point of view as the *Perspective point*. A Perspective, being relative to the location of the observer, has a subjective aspect: Bob's perception and experience of a situation is specific to his location: observers at other locations will usually perceive the same situation or event in a different way. However, Bob's perspective on the situation is not completely subjective, since if another similarly-endowed observer were to occupy exactly the same position as Bob, he or she would be able to perceive the situation in the same or similar way. Thus perspectives ultimately have an objective basis, and are, in principle at least, intersubjectively shareable (cf. Seligman [141, p. 151]).

The phenomenon of perspectival relativity appears to be so ubiquitous that some researchers, notably Barwise and Perry [9], Seligman [141, 143, 142], and Talmy [154, 157, 158], have pro-

posed that the notion of perspective be recognised as a fundamental part of semantic theory as a whole.

There are many words, such as ‘come’, ‘go’, ‘left’, ‘right’, and so on, whose meaning appears to make reference to the point of view or perspective of an actual or hypothetical observer. Fillmore [49] has shown that references to points of view is ubiquitous in discourse, while Talmy [154, 157, 158], Vandeloise [165], and others, have investigated in considerable detail the role of perspective in the analysis of spatial prepositions: for example, the sentence ‘The chicken is in front of the chair’, can be truthfully used to describe (at least) two quite distinct configurations, illustrated in Figures 2.1 and 2.2.



Figure 2.1: *The chicken is in front of (behind) the chair*



Figure 2.2: *The chicken is in front of (behind) the chair*

Conversely, both spatial configurations can also be truthfully described by the sentence ‘The chicken is behind the chair’. In all cases, the chicken is the Figure and the chair is the Ground. However, in the first case the Figure is between the speaker and the Ground (the ‘front’ of the Ground is taken to be the side facing the speaker), while in second the Figure is on the far side of the chair (the ‘front’ of the Ground is the side facing *away* from the speaker). This difference can be attributed to a change in the point of view adopted by the speaker: in one case, the point of view (Perspective point) is the speaker’s own physical location, whereas in the other the point of view corresponds to the Ground. The point of view need not correspond to the speaker’s location or that of the reference object; the speaker might adopt the point of

view of the hearer or addressee of the utterance. For example, in a situation where Alice is helping Bob to find something, the pragmatically preferred option might well be for Alice to describe the spatial environment from Bob's point of view, rather than her own.

Vandeloise [165, ch. 2] describes this in terms of a contrast between *contextual* orientation and *intrinsic* orientation. People recognise certain objects, such as chairs and houses, as having canonical front and back sides, regardless of the position of the objects around them; this is their *intrinsic* orientation. However, orientation is also sometimes contextually determined, as when, for example, the speaker regards the 'front' of an object as that side which she is facing or interacting with. The contrast between contextual and intrinsic orientation can be explicated in terms of a difference in point of view: in cases of intrinsic orientation, the point of view coincides with the object itself, whereas in the case of contextual orientation the point of view is that of some other object.

Depending on the point of view, a Figure object (such as the cat in the above example) may concurrently bear several distinct spatial relations to one and the same Ground object (the chair). Indeed, there will generally be an indeterminate number of possible points of view in a given situation, some of which may be 'embedded' within others. This is illustrated particularly well in Talmy's [157, p. 226] example of the church, illustrated in Figure 2.3 below (as in Talmy's original diagram, the circles represent people, and their "noses" indicate the direction in which they are facing): in this scenario, a speaker ('S') and a hearer ('H') are in the back of a church (near the entrance), where a queue of people runs from left to right (facing the right wall); John ('J') is standing in the queue, but has turned completely around so that he is facing the person behind him; on either side of John, perpendicular to the queue, are two people facing the altar (one slightly closer to the entrance than John, the other closer to the altar). Given this scenario, Talmy [157, pp. 226–227] points out that the answer to the question *who is in front of John?* will vary according to what we take as our reference frame, and in fact may be any of the four persons labelled by numbers in the diagram: person 1 is in front of John with respect to John's own intrinsic orientation; person 2 is in front of John with respect to the contextual orientation provided by the queue; person 3 is in front of John with respect to the contextual orientation provided by the church's interior; and person 4 is in front of John with respect to the vantage point of the speaker and hearer (cf. Figure 2.1 above). If we extend Talmy's scenario to consider the spatial surroundings of the church itself, we find that the church may be assigned an *external* orientation distinct from its *internal* one: whereas the internal orientation of the church places the 'front' of the church at the altar, the external orientation typically locates the 'front' of the church at the entrance of the building.

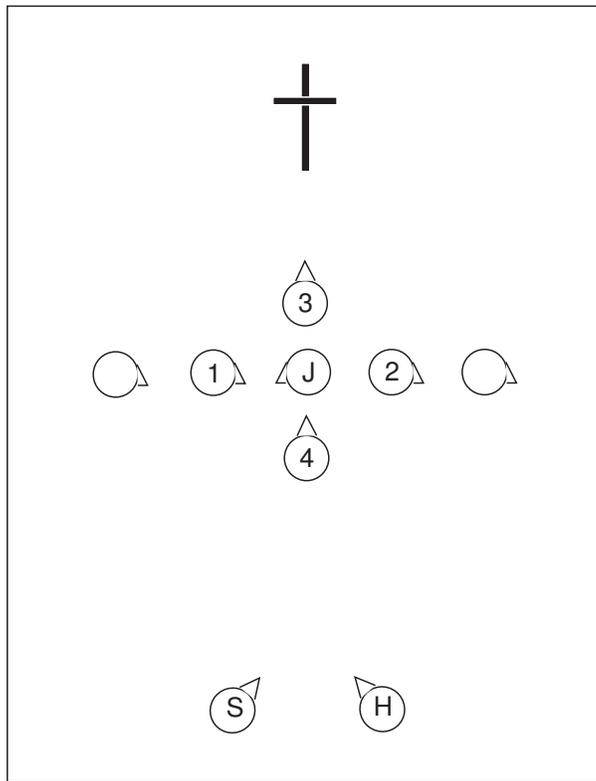


Figure 2.3: Multiple orientation

2.3 Tense

Tense has been traditionally regarded as the grammaticalised expression of location in time (Comrie [26, p. 9]). If we adopt a simple representation of time as a straight line, with the past represented to the left and the future to the right (as per the usual convention), and indicate the present moment by 0, we arrive at the conventional timeline shown in Figure 2.4 below (cf. Comrie [26, p. 2]):

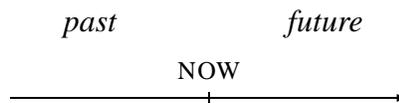


Figure 2.4: Timeline

Traditional theories of tense recognise two important temporal locations: an event which is the focus of interest, and a point relative to which the event is located (usually, but not always, the moment of speech). If we designate the former by *E* and the latter by *S*, then the relative precedence of the points on the timeline gives rise to three basic temporal categories, shown in Figure 2.5 below, according to whether the event is located before, at, or after the deictic centre. A binary precedence relation on a linear timeline gives rise to the three basic

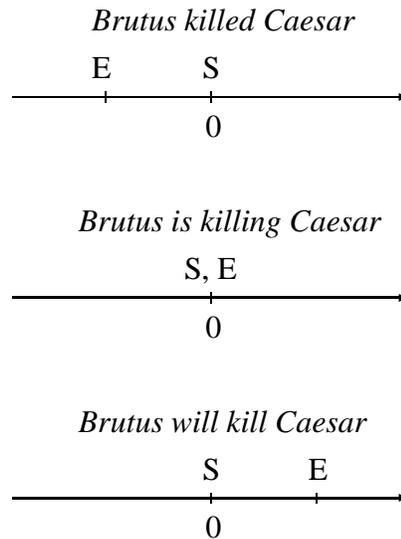


Figure 2.5: Simple Tenses

configurations in Figure 2.5, which is clearly not enough to capture the range of tenses to be found in English, let alone other languages. Hans Reichenbach [134] famously proposed that each temporal expression is associated with three time points: a *speech point*, *S*, an *event point*, *E*, and a *reference point*, *R*. As above, *E* refers to the time point corresponding to the event described by the tensed clause, and *S* is (usually) taken to be the speaker's time of utterance. Reichenbach's innovation was to introduce the reference point *R*, to be thought of as a situation or context that is relevant to the utterance. Reichenbach's tripartite scheme can capture subtle differences between tenses that are difficult if not impossible to express in a framework that only admits of binary relations between points *E* (*then*) and *S* (*now*), such as the difference between the simple past and present perfect: the former tense makes a statement about a past time, while the latter makes a statement about the present (see Figure 2.6 below).

Another characteristic of Reichenbach's analysis of tense is that the reference point *R* is present even in the simple tenses, where it is apparently superfluous (see Figure 2.7 below).

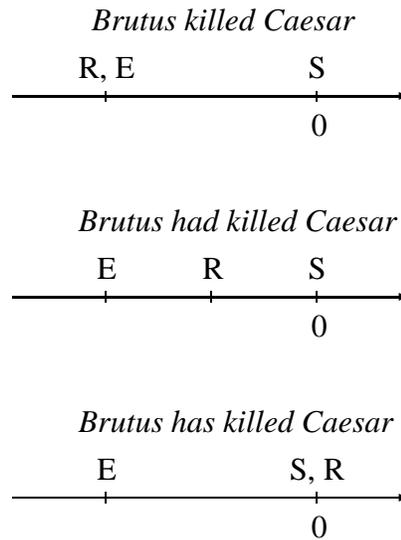


Figure 2.6: English Past Tenses: Perfect, Pluperfect, and Present Perfect

While there is much more to tense phenomena than a purely structural account can capture,² Reichenbach’s proposal has been widely influential; in particular, the idea that tense relations have a *ternary* structure, with a reference point *R* distinct from the speech point *S*, has been incorporated into several contemporary theories of tense (e.g., see Comrie [26], Hornstein [74], and Steedman [151]).

2.3.1 From Tenses to Locative Structures

As in the case of space (Section 2.2.2 above), the conceptualisation of time embodied in the English tense system is inherently perspectival in nature. The point of speech, *S*, marks the fundamental boundary between past, present and future; it is the component of tense that relates the timeline to the speech situation and its participants (above all, the speaker), and thus corresponds to the *temporal perspective point* in terms of which events are conceptualised. Reichenbach’s event time *E* and reference point *R*, for their part, stand in a Figure-Ground

²There are aspects of tense that are arguably better modelled using tense operators, as in Priorian tense logic, and there also appear to be tense phenomena that cannot be captured in terms of temporal relations alone; in particular, there is experimental evidence that suggests a strong connection between tense and goal-oriented reasoning (see Trabasso and Stein [159]), a notion that has recently been advanced within computational semantics by van Lambalgen and Hamm [164].

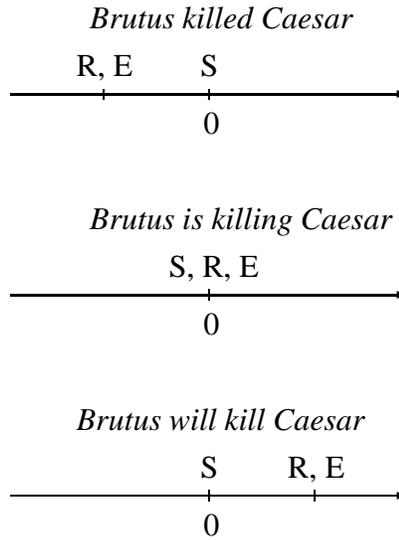


Figure 2.7: Simple Tenses Redux

relation: *E* indicates the temporal point of interest, and displays the characteristics of Talmy’s Figure, while *R* functions as a temporal landmark or Ground.

Thus it appears plausible to regard Reichenbach’s *S*, *E*, *R* points as specialised temporal instances of the categories of Perspective, Figure, and Ground; in particular, we can look upon the event point (or located event) *E* as a temporal Figure, the reference point *R* as a temporal Ground, and the speech point *S* as a temporal Perspective Point. Indeed, given the importance of these three elements in the semantics of both spatial and temporal expressions, we can regard a Reichenbachian tense *S*, *E*, *R* triple as a specific instance of a more general ternary structure, shared by tense expressions as well as spatial and temporal PPs, which we call a *locative structure* (L-structure).

One way to represent an L-structure is in *record format*, as in (2.1) below, where we refer to the Perspective, Figure and Ground components *P*, *F* and *G* in lowercase using ‘dot’ notation (so that **u.g** denotes the Ground of the L-structure **u**, for example).

$$\mathbf{u} = \begin{bmatrix} p = v_1 \\ f = v_2 \\ g = v_3 \end{bmatrix} \quad (2.1)$$

A Reichenbachian tense, in our framework, is simply a temporal L-structure, in which *P* (or

u.p) is Reichenbach’s speech point, *S*, *F* (or **u.f**) is Reichenbach’s event point, *E*, and *G* (or **u.g**) is Reichenbach’s reference point, *R*. Just as Reichenbachian theories associate a tensed clause with a ternary tense structure comprising *S*, *E*, and *R* points, we claim that spatial PPs are associated with a spatial locative structure, where the *P*, *F* and *G* parameters are instantiated as spatial locations, and that gradable adjectives are associated with *conceptual* L-structures comprising *P*, *F*, and *G* points. Moreover, we claim that these three components are present in *all* cases, just as all three time points are always present in a Reichenbachian tense, even where this may not be immediately obvious (as in the case of the ‘basic’ tenses). Similarly,

2.4 Adjectives as Locatives

Certain adjectival constructions appear to exhibit the same Figure-Ground relation as spatial and temporal locative constructions; this is particularly evident in comparative constructions, such as those in (47) below. The focus of interest in the adjectival case is the degree of possession of a given property by a given entity (the Figure), where this is often specified with respect to another entity which manifests this property to a known extent (the Ground).

- (47) a. Bob (F) is older/younger than Alice (G).
 b. Bob (F) is older/younger than Alice is (G).
 c. Bob (F) is older/younger than I thought he was (G).
- (48) a. Bob (A) likes Alice (F) more than Eve (G).
 b. Bob (A) likes Alice (F) more than I like Eve (G).
 c. Bob (A) likes Alice (F) more than Eve does (G).
 d. Bob (A) likes Alice (F) more than she likes him (G).

The Figure is grammatical subject in the sentences in (47), while in (48) the Agent, “Bob”, is subject and the Figure is direct object; the Ground is expressed as the complement of *than* throughout these examples, where it can take the form of a noun phrase, as in (47a), (48a), or a clause, as in (47b), (47c), and (48b), (48c), (48d).

Typically, we find that the expression of the Figure-Ground relationship in comparative constructions also conforms to Talmy’s precedence principle: as in the case of spatial and temporal locatives, thus, the Figure typically enjoys syntactic precedence over the Ground, which usually appears lower than the Figure in the case hierarchy, often as the nominal or clausal complement of *than*, as in the sentences in (47) and (48) above.

2.4.1 The Deictic Centre

Deixis or indexicality is a universal feature of language that grounds the use of language with respect to a particular time, place, and person(s); it is the function that connects all language to situations, and thus often involves extralinguistic, occasion-specific and subjective considerations.

Most contemporary theories of tense agree that tense expressions are indexical, because they may refer to different times, or time intervals, according to the properties of the context, such as the time of utterance (see Reichenbach [134], Partee [127], Salmon [139], Enc [41], Ogihara [121], and King [94]). In Reichenbach's [134] theory of tense, *S* is a deictic element that is typically anchored to the moment of speech; as several linguists and philosophers have noted, it is similar in this regard to pronouns such as 'I' and 'here', which are also interpreted deictically.

The psychologist Karl Bühler [20] introduced the notion of a 'deictic field' (*Zeigfeld*), whose origin point, or *Origo*, is associated with the notions of HERE, NOW, and I. The most straightforward instance of a deictic system is one where the speech situation is taken as deictic centre: for the category of person, this defines the first person as the speaker and second person as the hearer or hearers, with everything else being third person; for the category of place, the location of the speech situation is defined as *here*, everywhere else as *there*; for the category of time, the temporal location of the speech situation is defined as *now*, every other time is *then* (see Comrie [26, pp. 14–18]). We refer to this as the *egocentric perspective*.

There are cases, though, where the deictic centre is not identical with the speech situation. We saw in Section 2.2.2 above that it is possible for the speaker to adopt a vantage point that is not her own. As in the case of spatial relations, where the point of view does not have to correspond to the speaker's location, the pronoun *I* does not always indicate the speaker, nor the temporal point of view always coincide with the time of utterance; we find this, for example, in the "narrative" tenses of English, as in (49) below.

- (49) It is the morning of 16th of June, 1815, outside Waterloo, and the moment of battle has now arrived. Having noticed the exposed Prussian front, now clearly visible in the morning sun, the general orders a furious artillery barrage as his men surge forward towards the enemy positions.

In its most basic form, deixis sets the position of the perspective point at the speaker's current location and time. In this case, however, the *here* is somewhere outside Waterloo, and the *now* of the narration is itself in the past: the (syntactic) present and perfect are interpreted relative to a past point *t*, distinct from the current speech point 0. A time is specified in the first clause

of (49) and referred to in subsequent clauses, suggesting an anaphoric dependency between the clauses.

This example demonstrates how the hearer can project herself to a deictic centre distinct from the actual here and now. Just as a visual scene can often be described from more than one viewpoint, speakers can adopt temporal viewpoints other than the actual moment of speech. There are multiple possible temporal perspectives, just as there are many possible spatial perspectives; the *here* and *now* of the speaker are just two contextually prominent ones.

Indeed, through the use of the appropriate linguistic expressions, the hearer may be induced to project her viewpoint to a number of virtual locations, or to adopt the viewpoint of a particular individual, by employing such devices as descriptions of subjective experience, perception or thought (cf. Zubin and Hewitt [179]).

2.4.2 Conceptual Perspectives

As with tenses and pronouns, there is also widespread agreement that gradable adjectives are contextually sensitive, based on the variability in the truth value of sentences in which such adjectives occur; for example, the truth of a sentence such as ‘Bill is rich’ appears to vary from context to context, and it seems plausible to attribute the responsibility for this contextual variation to the semantic properties of ‘rich’ (cf. Richard [135], McFarlane [114]).³ Gradable adjectives such as ‘tall’, ‘rich’, ‘good’, etc., appear to have *variable standards of application*. Some researchers (Partee [128], Condoravdi and Gawron [27]) have argued for a hidden parameter analysis for certain adjectives, such as ‘local’. A typical speaker who utters ‘A local pub is selling cheap beer’ is likely to mean (roughly) that there is a bar *near him* that is selling cheap beer. But a speaker who utters ‘Every football fan watched the Cup Final at a local pub’ probably does not mean that every football fan watched the Cup Final at a pub near him (that is,

³There is also the view that adjectival meaning is not context sensitive (appearances to the contrary notwithstanding). Cappelen and Lepore [22], for example, have claimed that comparative adjectives cannot be context-sensitive, basing their argument partly on the behaviour of adjectives in assertion reports. If Alice says ‘Bill is rich’, then Bob can truly say ‘Alice said that Bill is rich’; but if ‘rich’ were context-sensitive, then ‘rich’ could have a different content in Alice’s context than it does in Bob’s: the content of ‘rich’ in Alice’s context might be *rich for a lecturer*, while the content of ‘rich’ in Bob’s context might be, say, *rich for an investment banker*; but then Bob’s report would be incorrect, so rich cannot be context-sensitive. In Cappelen and Lepore’s view, this supports the conclusion that the content of ‘rich’ is a single, uniform property across *all* contexts. The contextualist, by contrast, holds that the truth of *Bill is rich* depends on some contextual factor, perhaps a standard of ‘richness’ which may vary according to a variety of factors, such as social class, culture, age group, and profession. See Atlas [2], Korta and Perry [98], Richard [135], McFarlane [114]), and several other articles in [133] for a critique of Cappelen and Lepore’s claims and a defense of the contextualist position.

the speaker); rather, he is likely to mean that each football fan x watched the Cup Final at some pub that is local to x . This variation can be explained by supposing that ‘local’ has a hidden variable associated with it (y is local to x), whose value is supplied by context in the first case, but which gets bound by the quantifier ‘every football fan’ in the second.

Like Barwise and Seligman [11, Lecture 18], we interpret the contextually variable (and often vague) *norms* or *thresholds* associated with gradable adjectives such as ‘tall’, ‘rich’, ‘good’, etc., in terms of shifting perspectives or points of view. In our approach, the norm corresponds to a hidden perspectival parameter in the semantic representation of the adjective, which is assigned a value in a context-sensitive manner.

The locative analysis of adjectives generalises Bühler’s *Zeigfeld* to the conceptual realm, and treats properties as dimensions in a conceptual space. The notion of a ‘perspective’ is applicable to conceptual structure in general because there is a level of mental structure that is, in fact, geometric in nature; this claim is at the heart of the theory of *conceptual spaces*, as developed by Peter Gärdenfors [53]. Gärdenfors’ theory is based on a reification of geometrical models as a level of mental structure: concepts are represented as n -dimensional regions in a *conceptual space*, which is defined as a set of quality dimensions with a certain geometric structure; the notion of conceptual similarity is defined in terms of the notion of metric distance in the space.⁴

As in the case of tense, where the temporal Perspective point (Reichenbach’s “moment of speech”, S) divides the time line into past and future, a conceptual perspective point divides a given conceptual dimension into non-overlapping segments. If we represent HEIGHT in terms of a unidimensional scale, bounded by zero at one extreme and unbounded at the other, then we may represent a given height h by an interval bounded by zero and a particular location a on this scale; similarly, we may represent the conceptual perspective point from which we evaluate or describe this interval by another location x on this scale; we claim that the nature of the evaluation is sensitive to the geometric relation between x and a , so that we evaluate h as ‘tall’ if a is further from zero than the perspective point x , while, conversely, we regard the same height h as ‘short’ if a is closer to zero than x . A given height, say 6 feet, may be described as ‘tall’ or ‘short’ according to the speaker and circumstance; Bill might be ‘short’ according to Bob but ‘tall’ according to Eve; but Bob might reckon that Bill is ‘tall’ for a thirteen-year-old, and Eve might consider him ‘short’ for a basketball player, and so on. Thus, the same *objective* property (in this case, a given height) is evaluated differently (as ‘tall’ or ‘short’) according

⁴In fact, Gärdenfors’ theory does not require all dimensions to have a metric: a dimension can consist of a simple ordering, or even a bare graph with no associated notion of order or distance whatsoever; cf., Gärdenfors [53, Section 1.8].

to the possibly *subjective* perspective adopted by the evaluator. However, while an *egocentric perspective* may be the most natural and basic one for someone to adopt, interpersonal interaction and especially communication depends on the ability to envisage points of view other than one's own.

2.4.3 Some Structural Analogies between Tenses, Adjectives and Pronouns

In [127], Barbara Partee suggests that tenses in natural languages might not be operators, but pronouns. Like pronouns, they have indexical, anaphoric, and bound variable uses. For example, there are cases where tense is not interpreted deictically at all, a phenomenon traditionally called sequence of tense, as in (50):

- (50) a. It was Alice's birthday last Saturday and Bob bought her a present.
b. Bill decided last Friday that as soon as he finished his dissertation he would tell his wife that they were moving to Italy.
c. Bob said he would look for a pharmacy that was still open.

The underlined tenses in (50) are not necessarily interpreted as past tenses; all three sentences have readings where the tenses seem to merely agree with a previous governing past tense, without making any semantic contribution of their own (cf. Partee [127, p. 605]). In (50a), a time is specified in the first clause and then referred to by the tense of the subsequent clause. Partee notes that this is akin to the phenomenon of anaphoric dependency in pronouns, such as the relation between the underlined pronouns and their antecedents in sentence (51) below:

- (51) a. Alice bought **a cake** yesterday, and Bob ate it this morning.
b. **Alice** loves her child.
c. **Bob** bites his nails when he is nervous.

Sometimes a distinction is made among those indexicals whose contextual assignment appears automatic and rule-governed, and those whose assignment appears to have a pragmatic basis: for example, the reference of 'I' is usually determined automatically on the basis of a linguistic rule, without taking into account such factors as the beliefs and intentions of the speaker, while a possessive phrase such as 'Bill's car' may mean something vague like *the car that bears relation P to Bill*; the free variable *P* must be assigned a value in context, but here the assignment is not fully determined by the meaning of the possessive, and is at least partly dependent on the speaker's intention.

In actuality, the matter is far from clear-cut; although the pronoun ‘I’ may appear at first blush to *only* have a deictic use, this is not the case, as we can see from example (52), which Kratzer [99] attributes to Irene Heim.

(52) Only I got a question that I understood.

(52) has (at least) two readings, depending on the interpretation of the second (underlined) occurrence of ‘I’. On one reading (the strict reading), ‘I’ has its usual indexical interpretation: that is, the sentence describes a situation where nobody else got a question that ‘I’ (the speaker) understood. On the second reading (the loose reading), the sentence says that, apart from me, there was no person x such that x got a question x understood; the person and number features of the second ‘I’ are not interpreted, and the pronoun has a bound variable interpretation. Thus, we have cases of *underdetermination*, where the pronominal features are insufficient in themselves to fully determine the referent, and we also have cases where certain pronominal features are ignored. Kratzer [99] argues that this loss of interpretable features is another property that is shared by pronouns and tenses.

We regard deixis and referential anaphora as special cases of the same phenomenon: in both cases, the pronoun refers to an entity intended by the speaker that is present in the context of interpretation. On this view, deictic pronouns and cases of referential anaphora are interpreted using the same general strategy: in disambiguating the pronoun’s reference, the hearer assigns to it the most salient individual that allows them to make sense of the utterance—subject to pragmatic constraints of relevance, informativeness, etc. (see Sperber and Wilson [150], Heim and Kratzer [66, Chapter 9]).

We can observe cases of deictic and anaphoric dependency in adjectival constructions, involving both the standard and comparative forms. For example, the adjective in (53a) below, considered as an isolated utterance, can be construed deictically as referring to the height of some contextually salient person or thing, such as Bob’s own height at some point in the past; in (53b), however, the positive form ‘tall’ is contextually anchored to some appropriate norm or standard of height, s_H , which Bill’s height is claimed to exceed.

- (53) a. Bob is taller.
b. Bill is tall.

The kind of contextual dependency involved in each case is quite distinct. This is most clearly seen in examples involving anaphoric dependency, such as (54) below.

- (54) a. Bill is tall. Bob is also tall.
b. Bill is tall. Bob is also tall (for his age).

- c. Bill is tall. Bob is taller.
- d. Bill is short. Bob is taller.
- e. Bill is strong, but Bob is taller.

In (54a), the first clause ‘Bill is tall’, interpreted relative to some contextual standard, call it s_H , makes up part of the interpretive context for the second occurrence of ‘tall’. One natural interpretation is for this to be interpreted relative to the same standard s_H , as may further uses of the same adjective.

Where the standard is not itself precise, these further references to the same s_H may have the effect of providing further information about s_H . Several researchers have noted that the use of a vague predicate may itself have an effect on the discourse context (see Kamp [84], Klein [95], Pinkal [132], Parikh [122], Eikmeyer and Reiser [40], Ballweg [3], Kyburg and Morreau [100], Barker [5]), and in particular contribute to its ‘precization’ (Pinkal [131]) or ‘sharpening’ (Williamson [172]). In (55a) below, the first clause establishes a standard of height, which is referred back to in the subsequent clauses which assert that Bob, Bill, and Alice are tall. In this case, the use of the adjective is *descriptive*: the attribution of tallness tells us something about Bob, Bill, and Alice—namely, that they are all over 6 ft. in height. However, where the standard is unclear, but the heights of various exemplars are known, such attributions may have a *metalinguistic* use, in which they are used to convey information about the standard of height itself, as in (55a) and (55b) below (cf. Barker [5, p. 2]).

- (55) a. According to regulations, anybody over 6 ft. is tall: so, Bob is tall, Bill is tall, and Alice is not tall.
- b. Well, according to Eve Bob is tall, Bill is tall, but Alice is not.

In general, however, it cannot be assumed that in a discourse there can only be a single standard for tokens of the same adjective to refer to. While it is possible that a single standard may underpin adjectival use over an extended segment of discourse, new standards of evaluation may be introduced, and there may even be several standards in play at a given point. In example (54b), the hedge ‘for his age’ suggests that the standard of tallness for Bob is different to that for Bill (Bob might be Bill’s five-year-old son, for instance). The standard may vary according to the characteristics of the object described (e.g., ‘big’ as applied to elephants and fleas, or ‘tall’ in the case of adults and children, etc.), or to the point of view of an observer (e.g., Eve may have a different standard for what counts as ‘rich’ than Bill), or any practical tasks or goals relevant to the conversation (e.g., ‘clean’ has different standards of application in a kitchen and a surgical theatre).

In (54c), the first clause ‘Bill is tall’, interpreted relative to some standard s_H , again provides an antecedent for the comparative ‘taller’ in the second clause. In this case, however, the comparative form does not make reference to the standard of tallness, s_H , but rather to Bill’s height; the claim made by the sentence is that Bob is taller *than Bill*. Of course, in (54c), if Bob’s height exceeds Bill’s, it follows by transitivity that it also exceeds s_H , and thus that Bob also counts as ‘tall’.⁵ As (54d) and (54e) demonstrate, it is ‘Bill’ that is taken as the antecedent of the comparative, rather than the adjective, which need not even involve the same dimensional scale.

If we compare these cases, we find that the anaphoric dependency involving the comparative in (54c) has a strikingly different quality from that displayed by the standard form of the adjective in (54a). Our view is that the different forms of context-dependence exhibited by the comparative and standard forms of the adjective involve different parameters of the L-structure: the anaphoric dependency in (54c) involves the Ground parameter of the L-structure associated with the comparative adjective ‘taller’, while in (54a) it is the Perspectival parameter that is assigned a value in context. We shall present this in formal terms in Chapter 3, and give a treatment in terms of dynamic semantics in Chapter 4.

2.5 Extended Locations

Thus far we have been speaking of locations as if they were entirely punctual in nature, but in fact locations typically have an extension in time or space. For many purposes, it is perfectly valid to abstract away from this: for instance, if we are concerned with determining the configurational relationship among several locations, as when drawing a map to show a friend the route from the station to our house, there may well be harm choose to represent buildings and landmarks as points rather than extended regions. However, there are circumstances where the extension of an object or event is relevant; In the case of spatial relations, for instance, certain prepositions make reference to the interior of objects. Also, measurement of distance is often made relative to the exterior boundary of a reference object; for example, if we say “Bob is 5 metres in front of (behind) the house”, then Bob’s position is typically measured from the boundary of the house, not from some point within the building. In general, the fact that the reference object has a spatial extension means that there is a region that is neither ‘behind’ nor ‘in front of’ the reference object; thus, if Bob is inside the house, he is neither in front of it nor behind it.

⁵Assuming that the relevant standard of height is applicable to Bob.

Temporal extension is relevant to the aspectual system of some languages. For example, the *imparfait* and *passé défini* in French differ in that the *imparfait* is an extended tense, whereas the *passé défini* is not—cf. Figure 2.8 below, from Reichenbach [134]. This corresponds roughly to the contrast between “I was seeing John” and “I saw John” in English, where the present participle conveys that the event covers a certain amount of time.

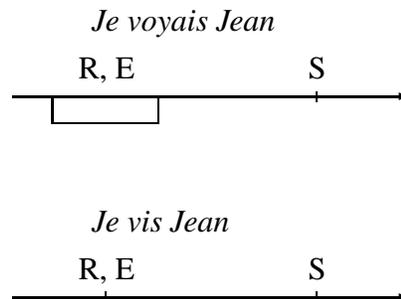


Figure 2.8: French *imparfait* and *passé défini*

The boundaries of a location are also typically both contextually variable and vague.⁶ For example, the spatial locatives ‘here’ and ‘there’ contrast a region that (typically) includes the speaker’s location, and another region that does not; a similar contrast holds between the temporal locatives ‘now’ and ‘then’. The boundaries of the denoted spatial and temporal locations may vary according to the context: thus, ‘here’ in one conversation may refer to a region within arm’s reach of where I am standing (“Come over here and take this!” said to a child), while in another it may refer to the city where I live, or a continent, even the entire planet (“here in the United Kingdom”, “over there in the US”, “here on planet earth”); ‘now’ may refer to a brief interval of a few minutes, or a period lasting years (“we are now living through a period of rapid climate change”)—cf. Figure 2.9 below. In addition to this contextual variation, locations are also typically vague: that is, the boundary of the region will still be ‘fuzzy’ even after contextual variability has been taken into account (see Chapter 5, Section 5.2).

We find the same phenomenon in the case of gradable adjectives, where we often find a single dimension associated with an antonymic pair (e.g., ‘tall’ / ‘short’, ‘wide’ / ‘narrow’, ‘old’ / ‘young’, etc.). These pairs are usually semantically contrary rather than complementary; that is, one is not short simply in virtue of being not-tall, because being not-tall includes the possibility

⁶This is often cited as a canonical instance of ‘ontic’ vagueness, or vagueness in the world. See Chapter 5, Section 5.2 for further discussion and citations.

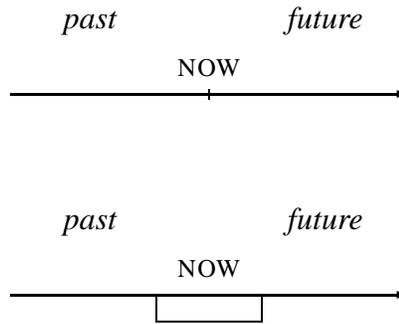


Figure 2.9: Temporal extension of 'now'

of being neither short nor tall. One way to model this is by assigning distinct thresholds to 'tall' and 'short'; however, if we separate the thresholds in this way, we need to provide a system of what the Kyburg and Morreau [100] refer to as 'penumbral connections' which will ensure, among other things, that the threshold for tall must be greater or equal to the threshold for short in the same context of evaluation.

An alternative approach, we assume a single threshold between the members of an antonymic pair, thought of as a *conceptual location* corresponding to an extended continuous region on the height scale rather than a single, extensionless point, as illustrated in Figure 2.10 below.

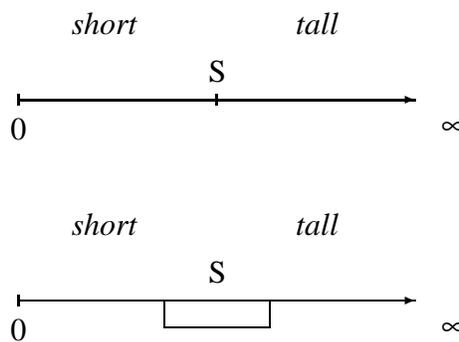


Figure 2.10: The extended height threshold for 'tall' / 'short'

On this account, tallness and shortness are determined relative to the upper and lower boundaries of the threshold. Note that, by treating the threshold as a single entity with upper and

lower boundaries, we automatically ensure that the appropriate penumbral connections between instances of ‘tall’ and ‘short’ are maintained, without resorting to an explicit system of penumbral constraints. Moreover, in line with the properties of spatial and temporal locations, the boundaries for conceptual locations are typically vague, even after contextual variability has been taken into account.

We shall revisit the topic of vagueness and locatives in Section 5.2 of Chapter 5. However, since we shall be primarily concerned with relations between locations in the following chapters, we shall generally employ a simple representation of locations as points.

2.6 Conclusion

This chapter has introduced the foundational ideas underpinning the locative view of adjectival semantics. At the heart of our approach is the notion of *point of view* or *perspective*. Gradable properties are associated with dimensions in a *conceptual space*, wherein specific locations can function as *conceptual* perspective points and reference points. As in the case of tense, where the temporal perspective point *S* acts as a threshold between past and future, a perspective point *P* along a conceptual dimension acts as a standard or norm separating entities which have a property from those which do not.

We have introduced the notion of an L-structure, a generalisation of a Reichenbachian ternary tense structure to the realm of concepts, and have provided some motivation for the view that the notions of *Perspective*, *Figure*, and *Ground* can be extended from the spatio-temporal domain to the semantics of gradable adjectives; we have claimed that, just as tensed clauses are associated with ternary tense structures, gradable adjectives are associated with ternary *conceptual* structures.

In a similar way to how a Reichenbachian theory models certain features of tense in terms of configurations of *S*, *E*, and *R* points on a timeline, we propose to account for certain semantic properties of adjectives in terms of configurations of *P*, *F*, and *G* points on a dimensional scale. This is the task we undertake in the next chapter, where we develop a formal semantics for gradable adjectives along these lines.

Chapter 3

Locative Structures and the Semantics of Gradable Adjectives

3.1 Introduction

Chapter 2 introduced the hypothesis that gradable adjectives are semantically a species of locative expression, sharing certain aspects of their semantic structure with certain spatial and tense expressions. In particular, we proposed the idea that there is an abstract semantic representation, akin to a Reichenbachian tense, called a *locative structure* (L-structure), that is shared by gradable adjectives, spatial and temporal prepositions, and tense, and that this L-structure has a ternary structure comprising elements we represented symbolically as *F*, *G*, and *P*, standing respectively for *Figure*, *Ground*, and *Perspective*.

In this chapter we consider several of the topics introduced in Chapter 2 from a more formal standpoint. Using the notion of L-structure as our starting point, we develop a formal *locative structure semantics* (LSS) for gradable adjectives in Sections 3.2 and 3.3. In Section 3.4, we consider measures and measure phrases, and propose an *adjectival measurability condition* (AMC) to account for the phenomena related to measure phrase modification, including sortal restrictions (Section 3.4.3). We then consider the issue of how to incorporate semantic constraints such as the AMC into the compositional semantic definition of the language in Section 3.5, where we propose a *construction-based* approach to semantic constraints. In Section 3.6, we re-examine the structural parallels between adjectives, tense and pronominals that we discussed in Section 2.4.3 of Chapter 2, using the concepts of dynamic semantics.

3.2 Vectors, Dimensions, and Locative Structures

In Chapter 2, we proposed that the semantics of gradable adjectives, along with spatial and temporal expressions, involves ternary structures called *L-Structures* comprising a Figure f , a Ground g , and a Perspective point p , as set out in record format in (3.1).

$$\mathbf{u} = \begin{bmatrix} f & = & \vec{v}_1 \\ g & = & \vec{v}_2 \\ p & = & \vec{v}_3 \end{bmatrix} \quad (3.1)$$

In line with VSS and certain other trends in Cognitive Science (notably Gärdenfors [53]), we shall think of properties in geometric terms as qualitative dimensions in a multidimensional vector space, and assume a set of *individuals* \mathcal{U} , a set of *dimensional vectors* \mathcal{V} , and a set of *dimensions* \mathcal{D} , which is a partition of \mathcal{V} . We also assume a *constant interpretation function* \mathfrak{I} , whose role is to assign semantic values to the basic lexical and logical constants of the language; e.g., $\mathfrak{I}(\text{'Bob'}) = b$ ($b \in \mathcal{U}$).

In addition, we will have a *projection function*, ∂ , which takes as argument an individual $a \in \mathcal{U}$, and yields a function $\partial(a)$ from property dimensions $d \in \mathcal{D}$ to position vectors in d ; applying function $\partial(a)$ to a particular dimension d' returns a particular position vector in d' , which we call the *projection of a onto (the dimension) d'* .¹ For example, if $\mathcal{D} = \{\text{HEIGHT, WEIGHT, AGE, ...}\}$, and $\mathfrak{I}(\text{Bob}) = b$ ($b \in \mathcal{U}$), then

$$\partial(b) = \begin{bmatrix} \text{HEIGHT} & \mapsto & b_H \\ \text{WEIGHT} & \mapsto & b_W \\ \text{AGE} & \mapsto & b_A \\ & & \vdots \end{bmatrix}$$

Thus, given $\mathfrak{I}(\text{Bob}) = b$ ($b \in \mathcal{U}$) and a dimension $\text{HEIGHT} \in \mathcal{D}$, then $\partial(b)(\text{HEIGHT}) = b_H$ is a position vector belonging to the dimension HEIGHT which represents Bob's height. Since $\partial(b)$ yields a position vector for every dimension $d \in \mathcal{D}$,² it follows that ∂ assigns to each individual $b \in \mathcal{U}$ a location in the multidimensional vector space \mathcal{V} , corresponding to the *sum* of all its projections.

Since each dimensional vector in \mathcal{V} belongs to a unique $d \in \mathcal{D}$, we can define a function ρ that, given a vector, returns its *dimensional type*, as in Definition 10.

¹This can be straightforwardly generalised to an intensional context, if so desired, by relativising the projection function to a time and/or possible world.

²Note that sortal restrictions may require abandoning the assumption that ∂ is a total function; see Section 3.4.3 below.

Definition 10 (ρ) For all dimensions $d \in \mathcal{D}$ and vectors $v \in d$,

$$\rho(v) \triangleq d$$

We define the set TYPE of LSS types as follows:

Definition 11 (LSS types) TYPE is the smallest set such that

1. $e, v, t \in \text{TYPE}$.
2. if $a, b \in \text{TYPE}$, then $(a \rightarrow b) \in \text{TYPE}$.
3. if $a, b \in \text{TYPE}$, then $(a \bullet b) \in \text{TYPE}$.

where

- e is the type of individuals;
- v is the type of (basic) vectors;
- t is the type of truth values;
- $(a \rightarrow b)$ is the type of functions from type a into type b .
- $(a \bullet b)$ is the type of products of type a and type b .

The values v_1, v_2, v_3 in (3.1) are vectors in a multidimensional *conceptual* space, whose dimensions represent properties. Each of these elements is a basic vector, so L-Structures are triples of basic vectors, and therefore of type $(v \bullet v \bullet v)$, which we abbreviate to v^3 . Where convenient, we shall represent L-Structures in record format, as in (3.1) below, and specify their components using ‘dot’ notation (so that $\mathbf{u.g}$ denotes the Ground or reference point of the L-Structure \mathbf{u} , for example).

3.2.1 Polarity

In Chapter 2, Section 2.2, we discussed some examples of spatial and temporal domains, and proposed that expressions with a directional character³ cannot be modelled in terms of a binary relation between Figure (located object) and Ground (reference object), but require consideration of the *point of view* or *perspective* from which the scene is described.

³E.g., ‘in front of’, ‘behind’, ‘left of’, ‘right of’, etc., as opposed to ‘next to’, ‘near’, ‘around’, and so on.

Consider again the situation represented in Figure 2.1 of Chapter 2, repeated here for convenience as Figure 3.1. As we observed in Chapter 2, the same scene can be described using either member of the contrary pair of prepositions ‘in front of’ or ‘behind’, according to whether we adopt the point of view of the observer, or the point of view associated with the canonical use of the chair.



Figure 3.1: *The chicken is in front of / behind the chair*

The primacy of the Perspective point in fixing orientation is particularly evident in the case of tense, where it is the relative position of the event time (the Figure) and the moment of speech (the Perspective point) on the timeline that determines the fundamental classification of the tense as Past, Present, or Future (cf. Chapter 2, Figure 2.5). The relative position of Figure and Ground (Reichenbach’s reference time R) constitutes a *secondary* orientational principle, which modulates the initial classification; for example, within the class of Past tenses, the relative position of Figure-Ground distinguishes between the Perfect, Pluperfect, and Present Perfect (cf. Chapter 2, Figure 2.6).

Along a single dimension, the orientational possibilities reduce to two opposed *polarities*, which we call ‘positive’ and ‘negative’, plus the absence of polarity. Polarity is simply *orientation along a single dimension*; for example, the vectors u and $(-u)$ have opposite polarity, while $2 \cdot u$ has the same polarity as u , but twice the magnitude. In the case of tense, polar opposition takes the form of the contrast between past and future, while in the case of gradable adjectives it is evident in antonymic pairs associated with the same dimension, e.g., ‘tall’ / ‘short’ for HEIGHT, ‘rich’ / ‘poor’ for WEALTH, etc.

As far as gradable adjectives are concerned, we find that it is sufficient to consider the primary orientation determined by the relative position of the Figure and Perspective points, as set out in Definition 12.

Definition 12 (Polarity) *Given an ordering $<$ on a dimension d , an L -structure \mathbf{u} is said to be*

1. *positive* if $\mathbf{u.f} > \mathbf{u.p}$,

2. *negative* if $u.f < u.p$, and

3. *neutral* if $u.f = u.p$.

The members of an antonymic pair, such as ‘tall’ and ‘short’, are associated with positive and negative polarities in accordance with the ordering on the dimension.

Definition 12 has the effect of partitioning a dimension into three parts, relative to the position of the Perspective. The Perspective thus plays the role of a boundary or threshold between segments of the dimensional scale. For the most part, we shall ignore the extension of the Perspective itself in the following, and treat it as a point on the scale, even though strictly speaking the Perspective should be treated as a *region* (i.e., a set of points) rather than a single point. This will make little difference to the issues we consider.

3.2.2 Magnitude

In contrast to polarity, the magnitude of a spatial relation is often largely independent of Perspective. In the situation depicted in Figure 3.2, for example, the distance between the chicken and the chair remains constant at one metre, regardless of whether we describe the chicken as being ‘in front of the chair’ or ‘behind the chair’.

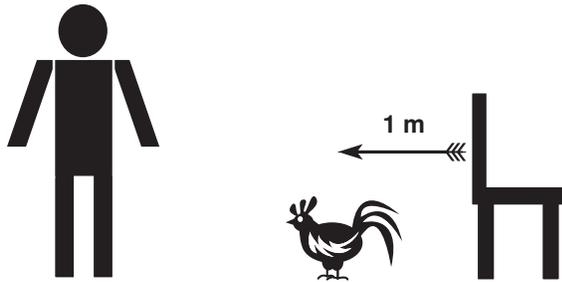


Figure 3.2: Separation between Figure and Ground

Here the magnitude of the interval between the Figure and the Ground is invariant across the change in viewpoint. The same phenomenon is apparent in (56), where the magnitude of the relation between church and road remains constant, even though the orientation of the relation changes with the shift in perspective. This perspective-invariance is also apparent in the case of certain temporal example in (57), where the interval between Figure and Ground is the same regardless of the shift in temporal perspective point.

- (56) a. The church (F) is 10 metres to the left of the road (G).
b. The church (F) is 10 metres to the right of the road (G).

- (57) a. The meeting (F) took place two hours before the inauguration (G).
 b. The meeting (F) will take place two hours before the inauguration (G).

In light of this, we define the magnitude of an L-structure in terms of the interval between the Figure and Ground.⁴ The expression $\mathbf{u}.\vec{xy}$ in Definition 13 denotes the vector $\mathbf{u}.y - \mathbf{u}.x$.

Definition 13 (Magnitude) *If \mathbf{u} is an L-structure, then its magnitude is defined as*

$$|\mathbf{u}| \triangleq |\mathbf{u}.\vec{gf}|$$

In other words, we take the magnitude of the L-structure to be equivalent to the magnitude of the interval between the Figure and the Ground.

In sum, we have defined the polarity and magnitude of an L-structure in terms of two distinct relations among its elements: the Figure-Perspective relation in the case of polarity, and the Figure-Ground relation in the case of magnitude. As we shall see in the next section, the interesting feature of this arrangement, as far as the analysis of gradable adjectives is concerned, is that it allows for cases where the standard of evaluation (the Perspective) may not coincide with the reference point for measurement (the Ground).

3.3 A Locative Structure Semantics for Gradable Adjectives

Along the lines of the Reichenbachian representation of tenses as configurations of S , E and R points on a timeline (cf. Chapter 2, Section 2.3), we shall represent L-structures as configurations of P , F , and G points along a linear property dimension, as shown in Figure 3.3 and Figure 3.4. Certain aspects of the representation differ from that of tense, reflecting certain important differences between property dimensions and the timeline: first, whereas the timeline is not bounded a priori, a property dimension such as height, weight, etc., may be bounded at one end (or conceivably both); second, while the timeline has no natural landmark intrinsically associated, for many if not most properties we can identify a natural ‘zero’ point.

As we discussed in Section 2.4 of Chapter 2, we shall represent the standard of evaluation for a gradable adjective as a Perspectival parameter in an L-structure. In the case of the standard (non-comparative) form of the adjective, the Perspectival component is free to be assigned a value implicitly from context, as in (58a) below, or a ‘for’-phrase may be used to

⁴It is important to stress, as in the case of polarity above, that this is a simplification. For example, the point of view is central to determining the *perceived* distances in a spatial scene, a fact exploited in systematic ways by painters since the Renaissance.

specify a relevant comparison class, as in (58c); however, as we can see in (58b), the Perspective cannot be designated using a ‘than’-phrase.

- (58) a. Bob is tall.
b. * Bob is tall than Bill.
c. Bob is tall for a five-year-old.

The variable nature of the Perspective means that, even if the Figure remains constant across contexts, the ordering relation between the Figure and the Perspective point may vary, causing a shift in polarity; thus the same object may be described by a negative form in one context (e.g., ‘short’), and a positive form in another (e.g., ‘tall’).

Each L-structure describes a particular region or interval, which is bounded by the locations of the Ground and the Figure; for example, in the scene depicted in Figure 3.2, this region lies between the chicken and the chair. One way to determine the boundaries of the relevant region is to take into account the semantic contribution of modifiers such as measure phrases, which describe properties of the region in question; for example, in a sentence such as ‘The chicken is one metre in front of the chair’, uttered as a description of the scene in Figure 3.2, the measure phrase modifier ‘one metre’ describes the region between the chicken and the chair.

In the case of the standard, non-comparative form of a gradable adjective, the relevant interval is always the *total* or *absolute* extent of possession of the property in question; for example, in the sentence ‘Bob is two metres tall’, the relevant interval described by the measure phrase spans Bob’s total height, extending from zero (the Ground point) to the position vector b_H representing Bob’s height (the Figure). This is represented diagrammatically in Figure 3.3, where the Ground point is fixed at zero in the configurations corresponding to the adjectives ‘tall’ and ‘short’.

One of the obvious differences between the standard and the comparative form is that, in the comparative case, the Ground is an independent variable that can be specified either implicitly, as in (59a), or by means of a ‘than’-phrase, as in (59b). A further difference is that it is not possible to freely specify the Perspective in the comparative case, as exemplified by the contrast between (58c) on the one hand and (59c) on the other. Specifically, what we find is that the Perspective is *bound* to the Ground point, in the sense that the same point that is used as a reference point for comparison is also necessarily the determinant of the polarity of the adjective; that is, given a vector b_H corresponding to Bill’s height, then b'_H is not only the Ground of the comparison in (59b) below, but also the point relative to which polarity is determined. The configurations for positive and negative comparative forms are illustrated in Figure 3.4.

Definition 14 (Box notation) For $\spadesuit \in \{\lambda, \forall, \exists\}$:

$$\begin{array}{l} \spadesuit x_1 \\ \dots \\ \spadesuit x_m \end{array} \boxed{\begin{array}{c} \phi_1 \\ \phi_2 \\ \dots \\ \phi_n \end{array}} \triangleq \spadesuit x_1 \dots \spadesuit x_m. \phi_1 \wedge \phi_2 \wedge \dots \wedge \phi_n$$

Following Kennedy [92] and Faller [46], we shall employ the syntactic analysis described in Section 1.3.2 of Chapter 1, in which a DegP is headed by a (possibly null) degree term with an adjective root as its complement (cf. Figures 1.2 and 1.3 in Section 1.3.3 of Chapter 1), and the basic semantic definitions follow the same pattern as those described in Chapter 1, Section 1.3.2.

Definition 15 (Adjective roots)

$$1. \textit{ tall} \Rightarrow \lambda \mathbf{u} \boxed{\begin{array}{c} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \end{array}}$$

$$2. \textit{ short} \Rightarrow \lambda \mathbf{u} \boxed{\begin{array}{c} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p > \mathbf{u}.f \end{array}}$$

Definition 16 (Degree terms)

$$1. \textit{ more / -er} \Rightarrow \begin{array}{l} \lambda W \\ \lambda V \\ \lambda \mathbf{w} \end{array} \boxed{\begin{array}{c} W(\mathbf{w}) \\ V(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array}}$$

$$2. \textit{ less} \Rightarrow \begin{array}{l} \lambda W \\ \lambda V \\ \lambda \mathbf{w} \end{array} \boxed{\begin{array}{c} W(\tilde{\mathbf{w}}) \\ V(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array}}$$

$$3. \emptyset^{\text{ABS}} \Rightarrow \begin{array}{l} \lambda W \\ \lambda \mathbf{w} \end{array} \boxed{\begin{array}{c} W(\mathbf{w}) \\ \mathbf{w}.g = 0 \end{array}}$$

Definition 17 ('Than')

$$\textit{ than} \Rightarrow \begin{array}{l} \lambda y \\ \lambda \mathbf{u} \end{array} \boxed{\mathbf{u}.g = \partial(y)(\rho(\mathbf{u}))}$$

In the standard (non-comparative) case, the phrase [\emptyset^{ABS} tall] is assigned the interpretation in (3.2) below.

$$\begin{aligned}
[\emptyset^{\text{ABS}} \text{ tall}] &\Rightarrow \lambda W \begin{array}{|c|} \hline W(\mathbf{w}) \\ \hline \mathbf{w.g} = 0 \\ \hline \end{array} \lambda \mathbf{u} \begin{array}{|c|} \hline \text{HEIGHT}(\mathbf{u}) \\ \hline \mathbf{u.p} < \mathbf{u.f} \\ \hline \end{array} \\
&= \lambda \mathbf{w} \begin{array}{|c|} \hline \lambda \mathbf{u} \begin{array}{|c|} \hline \text{HEIGHT}(\mathbf{u}) \\ \hline \mathbf{u.p} < \mathbf{u.f} \\ \hline \end{array} (\mathbf{w}) \\ \hline \mathbf{w.g} = 0 \\ \hline \end{array} \\
&= \lambda \mathbf{w} \begin{array}{|c|} \hline \begin{array}{|c|} \hline \text{HEIGHT}(\mathbf{w}) \\ \hline \mathbf{w.p} < \mathbf{w.f} \\ \hline \end{array} \\ \hline \mathbf{w.g} = 0 \\ \hline \end{array} \\
&= \lambda \mathbf{w} \begin{array}{|c|} \hline \text{HEIGHT}(\mathbf{w}) \\ \hline \mathbf{w.p} < \mathbf{w.f} \\ \hline \mathbf{w.g} = 0 \\ \hline \end{array}
\end{aligned} \tag{3.2}$$

Note that, while the Ground component of the L-structure is set to zero in (3.2), the value of the component $\mathbf{w.p}$ is not specified directly, and may take a range of contextual values; however, the set of explicit constraints, together with the basic geometry of the HEIGHT dimension (which is bounded below by zero), conspire to limit the range of values that the parameter can take.

In the comparative case, the complement of ‘than’ in the PP may be nominal (as in ‘ x is taller than John’) or clausal (as in ‘ x is taller than John is wide’). For the sake of simplicity, the interpretation given in Definition 17 considers only the case where the complement of ‘than’ is nominal (as indeed Faller and Winter do).⁵ This definition states that ‘than’ takes an argument of type e and produces a ‘than’-phrase of type $(v^3 \rightarrow t)$. Given ‘John’ $\Rightarrow j$, we then have

$$\begin{aligned}
[\text{than John}] &\Rightarrow \lambda y \begin{array}{|c|} \hline \mathbf{u.g} = \partial(y)(\rho(\mathbf{u})) \\ \hline \end{array} (j) \\
&= \lambda \mathbf{u} \begin{array}{|c|} \hline \mathbf{u.g} = \partial(j)(\rho(\mathbf{u})) \\ \hline \end{array}
\end{aligned} \tag{3.3}$$

⁵One strategy for providing a general account of the complements of ‘than’ would be to treat all cases as involving a covert clausal complement, so that, for example, ‘Bob is taller than John’ is analysed as ‘Bob is taller than John is tall’.

Then by Definition 16 above, we have the following translation of ‘taller’:

$$\begin{aligned}
 [\text{taller}] &\Rightarrow \lambda W \begin{array}{|l} W(\mathbf{w}) \\ V(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \lambda \mathbf{u} \begin{array}{|l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \end{array} \\
 &= \lambda V \begin{array}{|l} \lambda \mathbf{u} \begin{array}{|l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \end{array} (\mathbf{w}) \\ V(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \\
 &= \lambda V \begin{array}{|l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ V(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array}
 \end{aligned} \tag{3.4}$$

Combining this with the translation of ‘than John’ gives us:

$$\begin{aligned}
 [\text{taller than John}] &\Rightarrow \lambda V \begin{array}{|l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ V(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \lambda \mathbf{u} \begin{array}{|l} \mathbf{u}.g = \partial(j)(\rho(\mathbf{u})) \end{array} \\
 &= \lambda \mathbf{w} \begin{array}{|l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \lambda \mathbf{u} \begin{array}{|l} \mathbf{u}.g = \partial(j)(\rho(\mathbf{u})) \end{array} (\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \\
 &= \lambda \mathbf{w} \begin{array}{|l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = \partial(j)(\rho(\mathbf{w})) \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \\
 &= \lambda \mathbf{w} \begin{array}{|l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = \partial(j)(\rho(\mathbf{w})) \\ \mathbf{w}.p = \mathbf{w}.g \end{array}
 \end{aligned}$$

Whereas the translation for ‘tall’ in (3.2) allows for the Perspective point to be contextually specified (e.g., ‘tall for a five-year old’), this is not possible in the comparative case (e.g., *‘Bob was taller than Bill for a five-year old’), since the translations of the comparative markers ‘more’ and ‘less’ given in Definition 16 incorporate the constraint ($\mathbf{w}.p = \mathbf{w}.g$). On the other

hand, the comparative form of the adjective in English allows for the explicit specification of the Ground point g (by means of a ‘than’-phrase), whereas the base or standard form of the adjective does not, since the Ground point $w.g$ is fixed at zero. This fits in with our earlier observation that magnitude in the case of the non-comparative form of the adjective is always determined relative to the *zero point* of the scale: e.g., in the sentence ‘Bob is two metres tall’, the interval measured spans Bob’s *total* height, i.e., from zero to two metres, whereas in ‘Bob is two centimetres taller than Bill’, the measured interval is the difference between Bob’s height and the reference point corresponding to Bill’s height.

In order for the adjective to combine predicatively with a subject, however, we need to convert the expression into one of the appropriate type. Definition 18 below presents a modified form of the dim^- function proposed by Faller [46, pp. 158–159], adapted to L-structures (cf. Definition 8 in Chapter 1, Section 1.3.2).⁶

Definition 18 (∂^-)

$$\partial^- \triangleq \lambda W \lambda x \exists \mathbf{u} \begin{array}{|l} W(\mathbf{u}) \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array}$$

Given the translation ‘Bob’ $\mapsto b$, we can now combine the type-converted adjectival predicates ‘is tall’ or ‘is taller than John’ with the subject ‘Bob’, as shown in (3.5) and (3.9) below (we assume the copula is semantically empty):

$$[\text{Bob} [\text{is } \emptyset^{\text{ABS}} \text{ tall}]] \mapsto \partial^- \left(\lambda \mathbf{w} \begin{array}{|l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \end{array} \right) (b) \quad (3.5)$$

⁶We use the symbol ‘ ∂^- ’ instead of ‘ dim^- ’, to avoid confusion with Faller’s definition.

The application of ∂^- to the semantic translation of ‘tall’ gives us the following expression, of type $(e \rightarrow t)$:

$$\begin{aligned}
[\text{is } \emptyset^{\text{ABS}} \text{ tall}] &\Rightarrow \partial^- \left(\lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \end{array} \right) \\
&= \lambda W \begin{array}{l} W(\mathbf{u}) \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} \lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \end{array} \\
&= \lambda x \begin{array}{l} \lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \end{array} (\mathbf{u}) \\ \exists \mathbf{u} \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} \\
&= \lambda x \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} \\
&= \lambda x \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array}
\end{aligned} \tag{3.6}$$

Substituting this expression into (3.7), we obtain the following translation, of type t :

$$\begin{aligned}
[\text{Bob } [\text{is } \emptyset^{\text{ABS}} \text{ tall}]] &\Rightarrow \partial^- \left(\lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \end{array} \right) (b) \\
&= \lambda x \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} (b) \\
&= \exists \mathbf{u} \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ \mathbf{u}.f = \partial(b)(\rho(\mathbf{u})) \end{array}
\end{aligned} \tag{3.7}$$

The expression $\partial(b)(\rho(\mathbf{u}))$ denotes the projection of Bob onto the dimension to which \mathbf{u} belongs, namely, the HEIGHT dimension; thus, we have $\partial(b)(\rho(\mathbf{u})) = \partial(b)(\text{HEIGHT}) = b_H$, where b_H is simply the vector corresponding to Bob’s height (cf. Section 3.2 above). Hence, we can

simplify the expression further as follows:

$$[\text{Bob [is } \emptyset^{\text{ABS}} \text{ tall}]] \Rightarrow \exists \mathbf{u} \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ \mathbf{u}.f = b_H \end{array} \quad (3.8)$$

Informally, this states that there exists an L-structure \mathbf{u} associated with the HEIGHT dimension (i.e., all the vector components of the L-structure belong to the set of vectors HEIGHT) such that Bob's height b_H is the Figure of \mathbf{u} , and Bob's height is greater than the criterion of tallness $\mathbf{u}.p$ in the ordering corresponding to the dimension; note that the translation does not provide any information regarding the exact location of the Perspective point on the HEIGHT dimension, only the relational constraint that it is less than Bob's height, so the expression is compatible with a range of values for $\mathbf{u}.p$ ⁷.

In exactly the same fashion, we obtain the following translation for 'Bob is taller than John':

$$\begin{aligned} [\text{Bob [is [taller than John}]]] &\Rightarrow \partial^- \left(\lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = \partial(j)(\rho(\mathbf{w})) \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \right) (b) \\ &= \dots \\ &= \lambda x \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = \partial(j)(\rho(\mathbf{u})) \\ \mathbf{u}.p = \mathbf{u}.g \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} (b) \quad (3.9) \\ &= \exists \mathbf{u} \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = \partial(j)(\rho(\mathbf{u})) \\ \mathbf{u}.p = \mathbf{u}.g \\ \mathbf{u}.f = \partial(b)(\rho(\mathbf{u})) \end{array} \end{aligned}$$

Again, we have $\partial(b)(\rho(\mathbf{u})) = \partial(b)(\text{HEIGHT}) = b_H$, and $\partial(j)(\rho(\mathbf{u})) = \partial(b)(\text{HEIGHT}) = j_H$ where b_H and j_H correspond to the heights of Bob and John, respectively, so we can simplify

⁷Of course, given the that the dimension has a lower bound at zero, we can also infer that the Perspective point is greater than or equal to zero.

the translation to the following:

$$[\text{Bob [is [taller than John]]}] \Rightarrow \exists \mathbf{u} \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u.p} < \mathbf{u.f} \\ \mathbf{u.g} = j_H \\ \mathbf{u.p} = \mathbf{u.g} \\ \mathbf{u.f} = b_H \end{array} \quad (3.10)$$

Here the conjuncts ($\mathbf{u.p} = \mathbf{w.g}$) and ($\mathbf{u.p} < \mathbf{u.f}$) together entail that ($\mathbf{u.g} < \mathbf{u.f}$), that is, Bob's height b_H is greater than John's height j_H .

3.3.2 The *Than*-Phrase as Modifier

We often find cases where the 'than'-phrase is omitted, and the reference point for the comparison is understood from the context.

- (60) a. Bob is taller now.
b. John is stronger, but Bill is younger.

One way to handle this is to treat the 'than'-phrase as a *modifier* of the comparative, instead of an argument; for example, we can assign 'than' the interpretation in Definition 19.

Definition 19 ('Than' as modifier)

$$\text{than} \Rightarrow \begin{array}{l} \lambda y \\ \lambda V \\ \lambda \mathbf{u} \end{array} \begin{array}{l} V(\mathbf{u}) \\ \mathbf{u.g} = \partial(y)(\rho(\mathbf{u})) \end{array}$$

Here 'than' is a mapping from sets of dimensional vectors to sets of dimensional vectors. Since the comparative forms no longer take a 'than'-phrase complement, we can simplify the definitions of the degree terms as follows (cf. Definition 16 above):⁸

Definition 20 (Simplified degree terms)

1. *more* / *-er* $\Rightarrow \begin{array}{l} \lambda W \\ \lambda \mathbf{w} \end{array} \begin{array}{l} W(\mathbf{w}) \\ \mathbf{w.p} = \mathbf{w.g} \end{array}$
2. *less* $\Rightarrow \begin{array}{l} \lambda W \\ \lambda \mathbf{w} \end{array} \begin{array}{l} W(\tilde{\mathbf{w}}) \\ \mathbf{w.p} = \mathbf{w.g} \end{array}$
3. \emptyset^{ABS} $\Rightarrow \begin{array}{l} \lambda W \\ \lambda \mathbf{w} \end{array} \begin{array}{l} W(\mathbf{w}) \\ \mathbf{w.g} = 0 \end{array}$

⁸Note that the definition of \emptyset^{ABS} remains unchanged.

Using the same definition of the root form (Definition 15), we now have the following interpretation for ‘taller’:

$$\begin{aligned}
 [\text{taller}] &\Rightarrow \lambda W \begin{array}{|l} W(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \lambda \mathbf{u} \begin{array}{|l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \end{array} \\
 &= \lambda \mathbf{w} \begin{array}{|l} \lambda \mathbf{u} \begin{array}{|l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \end{array} (\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \\
 &= \lambda \mathbf{w} \begin{array}{|l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.p = \mathbf{w}.g \end{array}
 \end{aligned} \tag{3.11}$$

Given this interpretation, we can combine ‘taller’ with a ‘than’-phrase such as ‘than John’, as in (3.12) below:

$$\begin{aligned}
 [\text{taller than John}] &\Rightarrow \lambda V \begin{array}{|l} V(\mathbf{u}) \\ \mathbf{u}.g = \partial(j)(\rho(\mathbf{u})) \end{array} \lambda \mathbf{w} \begin{array}{|l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \\
 &= \lambda \mathbf{u} \begin{array}{|l} \lambda \mathbf{w} \begin{array}{|l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.p = \mathbf{w}.g \end{array} (\mathbf{u}) \\ \mathbf{u}.g = \partial(j)(\rho(\mathbf{u})) \end{array} \\
 &= \lambda \mathbf{u} \begin{array}{|l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.p = \mathbf{u}.g \\ \mathbf{u}.g = \partial(j)(\rho(\mathbf{u})) \end{array} \\
 &= \lambda \mathbf{u} \begin{array}{|l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.p = \mathbf{u}.g \\ \mathbf{u}.g = \partial(j)(\rho(\mathbf{u})) \end{array}
 \end{aligned} \tag{3.12}$$

The translation for ‘taller’ in (3.11) can also combine semantically with a subject NP, such as ‘Bob’, by means of the ∂^- function (cf. Definition 18). First, the application of ∂^- to the bare comparative ‘taller’ gives us an expression of type $(e \rightarrow t)$:

$$\begin{aligned}
[\text{taller}]^- &\mapsto \partial^- \left(\lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \right) \\
&= \lambda W \begin{array}{l} W(\mathbf{u}) \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} \left(\lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \right) \\
&= \lambda x \begin{array}{l} \lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.p = \mathbf{w}.g \end{array} (\mathbf{u}) \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} \\
&= \lambda x \begin{array}{l} \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.p = \mathbf{w}.g \end{array} \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} \\
&= \lambda x \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.p = \mathbf{w}.g \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array}
\end{aligned} \tag{3.13}$$

Combining (3.13) with the translation ‘Bob’ $\mapsto b$, we get the following translation for ‘Bob is taller’ (as before, we assume the copula is semantically empty):

$$\begin{aligned}
[\text{Bob [is taller]}] &\mapsto \partial^- \left(\lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.p = \mathbf{w}.g \end{array} \right) (b) \\
&= \lambda x \begin{array}{l} \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.p = \mathbf{w}.g \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} \\ \mathbf{u}.f = \partial(x)(\rho(\mathbf{u})) \end{array} (b) \\
&= \lambda x \begin{array}{l} \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.p = \mathbf{w}.g \\ \mathbf{u}.f = \partial(b)(\rho(\mathbf{u})) \end{array} \\ \mathbf{u}.f = \partial(b)(\rho(\mathbf{u})) \end{array}
\end{aligned} \tag{3.14}$$

The expression $\partial(b)(\rho(\mathbf{u}))$ denotes the projection of Bob onto the dimension to which \mathbf{u} belongs, namely, the HEIGHT dimension; thus, we have $\partial(b)(\rho(\mathbf{u})) = \partial(b)(\text{HEIGHT}) = b_H$, where

b_H is simply the vector corresponding to Bob's height. The above expression then simplifies further to

$$[\text{Bob [is taller]}] \Leftrightarrow \exists \mathbf{u} \begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.p = \mathbf{w}.g \\ \mathbf{u}.f = b_H \end{array} \quad (3.15)$$

Informally, this states that there exists an L-structure associated with the HEIGHT dimension (i.e., all the vector components of the L-structure belong to the set of vectors HEIGHT) such that the L-structure is *positive* (the Figure is above the Perspective point in the ordering corresponding to the dimension), the Perspective point coincides with the Ground point ($\mathbf{u}.p = \mathbf{w}.g$), and the Figure corresponds to Bob's height, b_H . Since the Perspective point and the Ground point are the same, it follows that $\mathbf{u}.g < \mathbf{u}.f$, so Bob's height b_H is greater than the height of whatever it is being compared to. Since the Ground point is unspecified, all this says is that Bob is taller than *something*.⁹

We have discussed the basic semantics for both standard and comparative adjectival constructions. In the next section, we turn to the interpretation of measure phrases.

3.4 Measures and Measure Phrases

Measurement can be viewed as a form of comparison in which the degree of possession of a property can be numerically expressed as a ratio of some identifiable extent, known as a *measure* or *unit of measurement*. We will treat measures as abstract entities, and refer to the terms used to denote them as *measure terms*. In English, when a measure phrase modifies the standard, non-comparative form of a gradable adjective, as in (61), what is measured is the total or absolute extent of possession of the property it denotes.

- (61) a. Bob is 2 metres tall.
 b. The hotel is 4 storeys high.
 c. John is 30 years old.
 d. The film was 2 hours long.
 e. Her watch is 5 minutes slow.

By contrast, when the measure phrase modifies a comparative form, as in (62), what is measured is the *difference* in degree between the things being compared.

- (62) a. Bob is 60 kilogrammes heavier/lighter than Alice.

⁹Note that the the existence of an L-structure \mathbf{u} entails the existence of its component elements.

- b. Her watch was 400 dollars more expensive/cheaper than my car.
- c. Bob is 2 million dollars richer/poorer than Bill.
- d. The bedroom was 30 degrees hotter/colder than the kitchen.
- e. My car is 10 metres-per-second faster/slower than your motorcycle.

While the admissibility of a measure phrase modifier is a sure indicator of gradability in an adjective, the converse is not the case. First of all, there are adjectives for which measure terms are simply not readily available (e.g., ‘beautiful’); however, there are also many gradable adjectives which readily take a measure phrase modifier in their comparative form, yet seem unable to do so in their absolute form, as we can see if we compare the sentences in (62) with those in (63).

- (63)
- a. * Bob is 60 kilogrammes heavy/light.
 - b. * My watch was 400 dollars expensive/cheap.
 - c. * Bob is 2 million dollars rich/poor.
 - d. * The bedroom was 30 degrees hot/cold.
 - e. * My car is 10 metres-per-second fast/slow.

Even in those cases where measure phrase modification is felicitous, the absolute form is constrained in other ways. Given an antonymic pair of gradable adjectives in absolute form, as in the sentences in (64), we often find that measure phrase modification is acceptable with one member of the pair, but not the other—by contrast, with the comparative form, measure phrase modifiers are typically acceptable with both members of the pair, cf. (62).

- (64)
- a. Bob is 2 metres tall/*short.
 - b. The table is 1 metre wide/*narrow.
 - c. The hotel is 4 storeys high/*low.
 - d. John is 30 years old/*young.

However, there are some pairs, such as those in (65), where this asymmetry is not present.

- (65)
- a. My watch is 2 minutes fast/slow.
 - b. Bob was 2 hours late/early.

One of the challenges facing a theory of adjectival semantics is to provide an account of these facts.

3.4.1 The Structure of Measure Phrases

We follow Landman [105] in taking a numerical measure phrase to be a species of determiner phrase whose *semantics* comprises three elements (which may not all be phonologically present) shown in Figure 3.5 below: a numerical relation Rel (e.g., ‘exactly’, ‘at least’, ‘at most’, ...), a number term Num (e.g., ‘one’, ‘two’, ‘four-and-a-half’, ...), and a measure term M (e.g., ‘metre’, ‘inch’, ‘litre’, ‘gramme’, ‘ounce’, ‘kilogramme’, ‘storey’, ...). As shown in the diagram, we will treat the relation and number as a quantificational phrase modifying a measure term in head position, although there exist other plausible syntactic analyses (we refer the interested reader to [105, Chapter 1] for discussion).

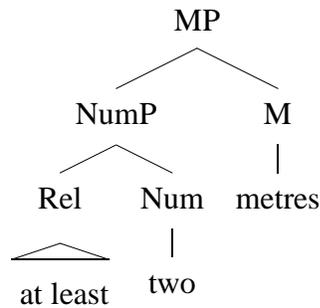


Figure 3.5: Measure phrase

We translate numerical expressions using numerals, which in turn denote numbers (e.g., ‘two’ $\mapsto 2$), and we interpret measure terms as constants that denote an abstract entity, a *measure*. In the case of measure phrase modifiers, we analyse numerical relations as shown in Definition 21 (this particular formulation assumes an order of combination in which the numerical relation combines with the number and then with the measure; clearly, it would be straightforward to change the definition to suit a different order of combination).

Definition 21 (Numerical relations)

1. ‘at least’ \Rightarrow

$$\lambda y \lambda x \lambda W \lambda \mathbf{u} \boxed{\begin{array}{l} W(\mathbf{u}) \\ |\mathbf{u}| \geq y \cdot |\partial(x)(\rho(\mathbf{u}))| \end{array}}$$
2. ‘at most’ \Rightarrow

$$\lambda y \lambda x \lambda W \lambda \mathbf{u} \boxed{\begin{array}{l} W(\mathbf{u}) \\ |\mathbf{u}| \leq y \cdot |\partial(x)(\rho(\mathbf{u}))| \end{array}}$$

$$3. \text{ 'exactly' } \Rightarrow \begin{array}{l} \lambda y \\ \lambda x \\ \lambda W \\ \lambda \mathbf{u} \end{array} \boxed{\begin{array}{l} W(\mathbf{u}) \\ |\mathbf{u}| = y \cdot |\partial(x)(\rho(\mathbf{u}))| \end{array}}$$

$$4. \emptyset \Rightarrow \begin{array}{l} \lambda y \\ \lambda x \\ \lambda W \\ \lambda \mathbf{u} \end{array} \boxed{\begin{array}{l} W(\mathbf{u}) \\ |\mathbf{u}| = y \cdot |\partial(x)(\rho(\mathbf{u}))| \end{array}}$$

The above definitions assign the following interpretation to the measure phrase *at least two metres* as follows:

$$\begin{aligned} [[\text{at least two}] \text{ metres}] &\Rightarrow \begin{array}{l} \lambda y \\ \lambda x \\ \lambda W \\ \lambda \mathbf{u} \end{array} \boxed{\begin{array}{l} W(\mathbf{u}) \\ |\mathbf{u}| \geq y \cdot |\partial(x)(\rho(\mathbf{u}))| \end{array}}^{(2)(m)} \\ &= \begin{array}{l} \lambda x \\ \lambda W \\ \lambda \mathbf{u} \end{array} \boxed{\begin{array}{l} W(\mathbf{u}) \\ |\mathbf{u}| \geq 2 \cdot |\partial(x)(\rho(\mathbf{u}))| \end{array}}^{(m)} \\ &= \begin{array}{l} \lambda W \\ \lambda \mathbf{u} \end{array} \boxed{\begin{array}{l} W(\mathbf{u}) \\ |\mathbf{u}| \geq 2 \cdot |\partial(m)(\rho(\mathbf{u}))| \end{array}} \end{aligned}$$

3.4.2 Measure Phrases as Adjectival Modifiers

We mentioned earlier the fact, pointed out by Faller [46] and Winter [176], among others, that in cases where a measure phrase modifies a standard (non-comparative) adjectival form, the interval measured is always between the Figure and the ‘zero’ point on the relevant scale. Moreover, in English the standard form of the adjective loses its evaluative quality in the presence of a measure phrase modifier, a phenomenon that Winter [176] refers to as “neutralisation”: an entity of *any* height can be described as ‘*x units tall*’, even if it would be described as *short* in the absence of the measure phrase. Further evidence that the adjective is no longer evaluative in character is the fact that it is no longer possible to contextually specify the standard of evaluation in the presence of a measure phrase, as in example (66) below.

(66) * Alice is 5 ft. tall for a five year-old.

One way to explain this loss of evaluative character is to view the introduction of the measure phrase as ‘binding’ the Perspective parameter in the L-structure in some way, so that it is no longer free to take a value from the context. Thus Definition 22 below states an *adjectival measurability condition* (AMC) on gradable adjectives, which requires that the Perspective parameter be bound to the value of the Ground component of the L-Structure in the presence of a measure phrase.

Definition 22 (Adjectival Measurability Condition)

An adjectival L-Structure \mathbf{u} is measurable only if $\mathbf{u.p} = \mathbf{u.g}$.

Note that, by definition, the interpretation of the comparative already satisfies the AMC (cf. Figure 3.4 and Definition 16 in Section 3.3 above). In the non-comparative case, however, where the Ground parameter is the zero point of the dimension (cf. Figure 3.3 and the definition of \emptyset^{ABS} in Definition 16), the combination of the constraints $\mathbf{u.g} = 0$ and $\mathbf{u.p} = \mathbf{u.g}$ entails $\mathbf{u.p} = 0$; that is, the standard of ‘tallness’ is set to zero, whereupon it follows that objects of *any* height can be described as ‘tall’. The three acceptable configurations licensed by the AMC are shown in Figure 3.6 below; note that there is no configuration corresponding to the adjective ‘short’.

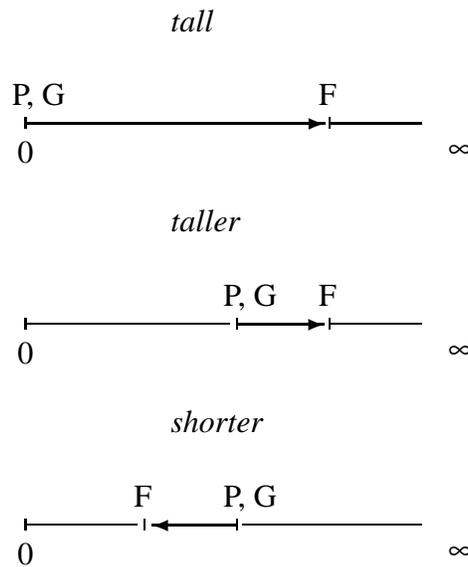


Figure 3.6: Configurations satisfying the measurability condition

There are several possible ways of integrating this measurability condition into a compositional semantic theory. Since the condition only applies in the presence of MP modification,

a natural and technically straightforward choice is to incorporate the measurability condition into the semantics of the measure phrase, as in Definition 23 below (although see Section 3.5 below for an alternative).

Definition 23

$$2 \text{ metres} \Rightarrow \begin{array}{l} \lambda W \\ \lambda \mathbf{u} \end{array} \boxed{\begin{array}{l} W(\mathbf{u}) \\ |\mathbf{u}| = 2m \\ \mathbf{u}.p = \mathbf{u}.g \end{array}}$$

This in turn gives us the following semantics for the phrase *2 metres tall*:

$$[[2 \text{ metres}] [\emptyset^{\text{ABS}} \text{ tall}]] \Rightarrow \lambda \mathbf{u} \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ |\mathbf{u}| = 2m \\ \mathbf{u}.p = \mathbf{u}.g \end{array}} \quad (3.16)$$

In similar fashion, the phrase **2 metres short* receives the interpretation shown in (3.17) below.

$$[[2 \text{ metres}] [\emptyset^{\text{ABS}} \text{ short}]] \Rightarrow \lambda \mathbf{u} \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.p > \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ |\mathbf{u}| = 2m \\ \mathbf{u}.p = \mathbf{u}.g \end{array}} \quad (3.17)$$

The constraints in (3.17) simultaneously require that $\mathbf{u}.f < 0$ and $|\mathbf{u}| = 2m$; however, because the height scale has a lower bound at zero, $\mathbf{u}.f$ cannot take values less than zero and so these constraints cannot be jointly satisfied. The upshot is that the negative form ‘short’ cannot be combined with a measure phrase modifier. This is consistent with the evidence from English, where the negative absolute form is often unacceptable in the presence of a measure phrase, while the positive absolute and both positive and negative comparative forms are permitted—cf. the examples in (67) below.

- (67) a. Bob is 2 m. tall.
 b. * Bob is 2 m. short.
 c. Bob is 4 cm. taller than Bill.
 d. Bill is 4 cm. shorter than Bob.

It is important to note, however, that it is not always the case that at most one of a pair of complementary adjectives in absolute form can be used with a measure phrase; for example,

in the complementary pairs ‘late/early’ and ‘fast/slow’, both members admit measure phrase modification—cf. the sentences in (68) and (69) below.

- (68) a. Bob was 2 hours late.
b. Bob was 2 hours early.
c. Bob was 2 hours later than Bill.
d. Bill was 2 hours earlier than Bob.
- (69) a. My watch is 2 min. fast.
b. Bob’s watch is 10 min. slow.
c. My watch is 12 min. faster than Bob’s.
d. Bob’s watch is 12 min. slower than mine.

The crucial difference in this case is that in both pairs the temporal dimension in question is unbounded in *both* directions, whereas the dimension of ‘height’ is unbounded in one direction alone, viz., the ‘positive’ one. This is consistent with the semantics we have presented, since when the dimension is unbounded in the negative direction, the condition $u.f < 0$ can be satisfied.

In short, the AMC appears to account for the phenomena involving measure phrases mentioned at the beginning of this section, and it can be incorporated directly into the compositional semantic machinery, for instance in the semantic definition of the measure phrase itself.

3.4.3 Sortal Restrictions on Measure Phrases

We can use the projection function ∂ to apply to measures, just as to other kinds of entity; however, we will associate measures with a very restricted set of dimensions (usually one): thus *year* and *minute* are associated with TIME, *kilogramme* with WEIGHT, and so on. This is motivated by the sortal restrictions governing the possible combinations of measure phrases and dimensional adjectives, as in (70), (71), (72), and (73) below.

- (70) a. Bob is 2 m. tall.
b. * Bob is 2 m. old.
- (71) a. Bob is 3 kg. heavier than John.
b. * Bob is 3 kg. taller than John.

There are also sortal restrictions on subject-predicate combinations:

- (72) a. The film was 2 hours long.

- b. * The chair was 2 hours long.
- (73)
- a. The lake was 2 fathoms deep.
 - b. * The theorem was 2 fathoms deep.

Measures may be associated with a single dimension, or more than one; for example, there are measures of extension, such as ‘metre’, ‘foot’, and so forth, which are compatible with several distinct dimensions, such as HEIGHT, LENGTH, DEPTH, etc., but not with AGE or WEIGHT.

- (74)
- a. Bob is 6 ft. tall.
 - b. The swimming pool 3 m. deep.
 - c. The rampart is 50 m. high.

One way to can capture sortal restrictions on measure phrases, as well as sortal restrictions on entities in general (cf. Lappin [107]), is by means of *partial* functions. This approach treats the ill-formed ‘(b)’ sentences in examples (70), (71), (72), and (73) as semantically undefined.

We can model the sortal restrictions between subject and predicate by means of a partial interpretation function: we assume a domain of individual entities \mathcal{U} , a denotation function \mathfrak{S} , a set of dimensions \mathcal{D} , a *sortal function* $\sigma: \mathcal{U} \mapsto 2^{\mathcal{D}}$, and a projection function $\partial: \mathcal{U} \mapsto \mathcal{D} \mapsto \mathcal{V}$ that assigns to each element $a \in \mathcal{U}$ a (point) vector from each dimension d in $\sigma(a)$. In effect, $\sigma(a)$ assigns a *sort* to the entity a , and $\partial(a)(d)$ becomes a *partial* function that assigns a location in dimension d to entity a (possibly zero) just for those $d \in \sigma(e)$. The *meaning* $\llbracket t \rrbracket$ of a term t is then a location in a subspace of dimensionality $n = |\sigma(t)|$; for example, if we suppose that the denotation function \mathfrak{S} assigns the individual b to the proper name ‘Bob’, $\mathfrak{S}(\text{Bob}) = bob$, then its *meaning* is given by

$$\llbracket \text{Bob} \rrbracket = \{\partial(\mathfrak{S}(\text{Bob}))(d) : d \in \sigma(\mathfrak{S}(\text{Bob}))\} = \{\partial(b)(d) : d \in \sigma(bob)\}$$

Each vector belongs to a dimension, given by $\rho(v)$. Since the predicate, according to VSS, is a set of co-dimensional vectors, we simply require that the dimension associated with the predicate belong to the sort of the subject: $\rho(v) \in \sigma(b)$

The same basic mechanism applies to the relation between measure phrase and adjective within the predicate. As with proper nouns, we can let a measure term such as ‘metre’ have a denotation $\mathfrak{S}(\text{metre}) = m$ (an abstract entity),¹⁰ and thus an associated sort

$$\sigma(m) = \{\text{HEIGHT, LENGTH, WIDTH, DEPTH, } \dots \}.$$

¹⁰An alternative would be to treat ‘metre’ as a common noun, by letting its denotation consist of all entities that are one metre long.

In order to ensure that the dimension associated with the adjective (in this case, HEIGHT) is compatible with the sort of the measure phrase in a phrase such as *2 metres tall*, it is sufficient to require that the dimension of the measured vector belongs to the sort of the measure term: $\rho(v) \in \sigma(m)$.

Thus, by treating measure terms as nouns, we are able to employ the same mechanism for ensuring sortal compatibility between measure phrases and adjectives as we use for clausal subjects and predicates; indeed, the choice to treat measure terms just like other nouns means that we can also assign them a meaning, namely, the set of projections of the entity it denotes onto its sortally compatible dimensions:

$$\llbracket \text{metre} \rrbracket = \{m_{\text{HEIGHT}}, m_{\text{LENGTH}}, m_{\text{WIDTH}}, m_{\text{DEPTH}}, \dots\}$$

This allows us to handle sentences in which a measure term is itself in subject position, without modifying the semantics; e.g., *One metre is just over three feet*.

Since we are modelling sorts as sets of dimensions, the subset relation effectively gives rise to a *sortal hierarchy*, in which sorts may have *supersorts* and *subsorts*. For example, the spatial measure terms ‘metre’, ‘foot’, ‘yard’, etc., are associated with a *supersort*

$$\{\text{HEIGHT}, \text{WIDTH}, \text{DEPTH}, \dots\},$$

corresponding to the full set of physically measurable spatial dimensions, while certain others may be more restricted in application; this appears to be the case for measures such as ‘storey’ and ‘fathom’, which are usually employed as measures of vertical extension, and which therefore correspond to the subsort $\{\text{HEIGHT}, \text{DEPTH}\}$.¹¹

3.5 Constraints and Compositionality

In Section 3.4.2 above, we integrated the AMC into the compositional semantic mechanism by making it part of the representation of the measure phrase. It is important to note that this move has certain cross-categorial ramifications, arising from the fact that measure phrases do not only appear with adjectives, but with spatial and temporal expressions as well. So, by incorporating the AMC into the measure phrase itself, we are implicitly claiming that the AMC is valid for other locative expressions, such as spatial and temporal PPs. However, the AMC is far too restrictive to apply to L-structures in general—at least in its current form. In the spatial case, for example, measurability clearly does not require the Perspective and Ground points to

¹¹This is true of current usage, although ‘fathom’ was originally used as a general measure of spatial extension.

coincide. This is evident in the example ‘The chicken is one metre in front of/behind the chair’, given in Figure 3.2 of Section 2.2.2 above, where the location of the Perspective point clearly does not affect the acceptability of measure phrase modification: that is, we can express the fact that the chicken is one metre away from the chair, regardless of whether we take the chicken to be ‘in front of’ the chair or ‘behind’ the chair.

It is important to note that this counterexample does not by itself rule out the correctness of the AMC in the specific case of adjectival constructions, even though it does establish that the AMC is not an accurate characterisation of the cross-categorial constraints on measurability. While it might of course be the case that the AMC is simply incorrect, or correct only in virtue of some accidental feature, the possibility remains that the AMC is an accurate, if partial, characterisation of measurability.

This, in itself, should not be controversial: it is reasonable to expect that L-structures, while sharing the same basic structure and properties, may be subject to constraints that are domain-specific and not universal. We expect there to be different constraints on spatial and tense L-structures, for instance, arising in part from the different underlying geometries of their domains; for a start, tense is typically associated with a one-dimensional timeline, unbounded in both directions (for all linguistic purposes), whereas the semantics of spatial expressions displays a much richer, three-dimensional structure. Certainly, the thesis that there are underlying structural commonalities at the semantic level among spatial, temporal, and adjectival expressions does not commit us to the (*much* stronger) claim that these domains do not have domain-specific constraints; we should not expect, for instance, to find an exact spatial analogue of the Future Perfect tense.

Given that we should expect there to be domain and even category-specific constraints on grammaticality, we need some way of incorporating such constraints into the grammar while at the same time limiting the extent of their application. One common way of coping with such cases is by introducing an ambiguity of some sort; for instance, we could claim that measure phrases are ambiguous among MPs that incorporate the AMC and others which do not; however, in the absence of any other independent evidence of ambiguity, this ‘solution’ is completely ad hoc, and hardly deserving of the name. A more satisfactory approach, which we will pursue in this section, is to find a way to state local, domain-specific constraints on L-Structures in a way that integrates smoothly with the overall compositional machinery.

The Fregean principle of compositionality holds that the meaning of a complex expression is a function of the meanings of its (immediate) constituents and their mode of combination. In

the mainstream of formal semantics,¹² this is usually interpreted as the requirement that there be a structure-preserving mapping, or homeomorphism, from the syntax to the semantics; that is, if \textcircled{S} is a syntactic combination rule, A_1, \dots, A_n the syntactic constituents to which the rule applies, and $\llbracket \cdot \rrbracket$ is some mapping from syntax to semantics (whether direct or via translation), then compositionality requires that

$$\llbracket \textcircled{S}(A_1, \dots, A_n) \rrbracket = \llbracket \textcircled{S} \rrbracket (\llbracket A_1 \rrbracket, \dots, \llbracket A_n \rrbracket) \quad (3.18)$$

So far, we have assumed a type-theoretic semantics in which the principal semantic combination rule is function application, $@$ in Definition 24 below; one of the constituents (typically the head of the phrase) is a functor that takes the other constituents as its arguments. In the case of degree phrases, the interpretations of the Deg' and the MP are of type $(\nu^3 \rightarrow t)$ and $((\nu^3 \rightarrow t) \rightarrow (\nu^3 \rightarrow t))$, respectively.

Definition 24 (@)

$$@ \triangleq \lambda\beta.\lambda\alpha_1 \dots \lambda\alpha_n.\beta(\alpha_1) \dots (\alpha_n),$$

where each $\alpha_i : a_i$ and $\beta : (a_1 \rightarrow (a_2 \rightarrow \dots (a_n \rightarrow b) \dots))$, for $a_i, b \in \text{TYPE}$.

However, it is only on a particularly restrictive view of semantic composition, in which the semantic combination rules are “bleached” of all construction-specific content, that we are forced to incorporate the AMC into the semantics of the measure phrase; there is nothing in the notion of compositionality, as described above, that prevents us from associating meanings with the syntactic constructions themselves. If we instead view syntactic constructions themselves as bearers of content, then we can associate the measurability condition not with the measure phrase but with the syntactic structure of the degree-modified adjective phrase itself (for the purposes of this discussion, we will follow Faller [46] and Kennedy [92] in assuming the structure shown in Figure 3.7).

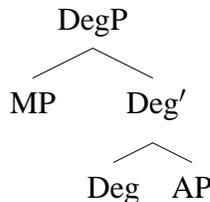


Figure 3.7: Degree phrase

¹²We take this to be the tradition of which Montague [117], [118] and [116] are representative.

The view that syntactic constructions themselves have content is associated with traditional, construction-oriented conceptions of grammar, as well as contemporary theories such as Construction Grammar.¹³ In a construction-based framework, we can assign a distinct semantic combination rule, $@_C$, to each syntactic construction C . There are several ways in which this can be done, but for the moment we will assume that Deg and MP both denote simple properties of L-structures, and thus have translations of type $(v^3 \rightarrow t)$, and assign the translation specified in Definition 25 below to the degree phrase construction $[\text{DegP MP Deg}']$.

The simplification of the type of the MP to $(v^3 \rightarrow t)$ is possible because the mechanism for semantic combination mechanism no longer relies on the MP to serve as a functor which takes Deg' as its argument; it would of course be straightforward to preserve the assumption that MP is of type $((v^3 \rightarrow t) \rightarrow (v^3 \rightarrow t))$, by defining

$$[\text{DegP MP Deg}'] \Rightarrow \lambda M. \lambda A. \lambda \mathbf{w} \begin{array}{l} M(A(\mathbf{w})) \\ \mathbf{w}.p = \mathbf{w}.g \end{array}$$

This version of the rule encapsulates the original mechanism of functional application, where the modifying expression is the functor and the modified constituent is the argument, while adding the constraint $\mathbf{w}.p = \mathbf{w}.g$.

There are several ways in which one can organise a construction-based system, including the assignment of semantic combination rules to syntactic schemata. For example, one might isolate the mechanism for semantic combination from the construction-specific content (in the form of constraints), and introduce a variant of functional application, $@_2$, that takes as its arguments not only the semantic interpretations of the immediate constituents of a construction, but also a construction-specific constraint, expressed as a λ term.

$$@_2 \triangleq \lambda \gamma. \lambda \beta. \lambda \alpha. \gamma(\beta(\alpha)),$$

where $\alpha : a$, $\beta : (a \rightarrow b)$, and $\gamma : (b \rightarrow c)$, for $a, b, c \in \text{TYPE}$.

The function $@_2$ is thus *a rule that generates construction-specific semantic combination rules*; that is, it takes as its argument the semantic content associated with a given syntactic construction, C , to give a semantic composition rule, $@_C$, corresponding to that construction. We can therefore redefine $@_{\text{DegP}}$ using $@_2$ as follows:

$$[\text{DegP MP Deg}'] \Rightarrow @_2(\lambda W. \lambda \mathbf{w} \begin{array}{l} W(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array})$$

¹³This view is most pronounced within Construction Grammar, as found in the work of Fillmore et al. [48], Fillmore [50], Fillmore and Kay [47], Goldberg [57], [58], Goldberg and Jackendoff [59], Lakoff [101], [19] [19], and Lambrecht [103], among others; a construction-oriented approach is also evident in some of Jackendoff's recent research (Jackendoff [78], [79], [80]).

By λ -conversion, RHS is equivalent to

$$\lambda\beta.\lambda\alpha.(\lambda W.\lambda\mathbf{w} \begin{array}{l} W(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array})(\beta(\alpha)).$$

Definition 25

$$[{}_{DegP} MP Deg'] \Rightarrow \lambda M.\lambda A.\lambda\mathbf{w} \begin{array}{l} M(\mathbf{w}) \\ A(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array}$$

Thus the meaning of the phrase is given by (3.19), which is consistent with the scheme in (3.18).

$$\llbracket [{}_{DegP} [MP \alpha] [{}_{Deg'} \beta]] \rrbracket = \llbracket [{}_{DegP} MP Deg'] \rrbracket \llbracket [MP \alpha] \rrbracket \llbracket [{}_{Deg'} \beta] \rrbracket \quad (3.19)$$

We then have the following translation of the degree phrase *two metres tall*, where $@_{DegP_1}$ abbreviates the translation of $[{}_{DegP} MP Deg']$ given in Definition 25.

$$\begin{aligned}
& [\text{DegP} [\text{MP } 2 \text{ metres}] [\text{Deg}' \theta^{\text{ABS}} \text{ tall}]] \\
& \Rightarrow @_{\text{DegP}_I} (\lambda \mathbf{u} \boxed{|\mathbf{u}| = 2m}) (\lambda \mathbf{v} \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{v}) \\ \mathbf{v}.p < \mathbf{v}.f \\ \mathbf{v}.g = 0 \end{array}}) \\
& = (\lambda M. \lambda A. \lambda \mathbf{w} \boxed{\begin{array}{l} M(\mathbf{w}) \\ A(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array}}) (\lambda \mathbf{u} \boxed{|\mathbf{u}| = 2m}) (\lambda \mathbf{v} \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{v}) \\ \mathbf{v}.p < \mathbf{v}.f \\ \mathbf{v}.g = 0 \end{array}}) \\
& = (\lambda A. \lambda \mathbf{w} \boxed{\begin{array}{l} (\lambda \mathbf{u} \boxed{|\mathbf{u}| = 2m}) (\mathbf{w}) \\ A(\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array}}) (\lambda \mathbf{v} \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{v}) \\ \mathbf{v}.p < \mathbf{v}.f \\ \mathbf{v}.g = 0 \end{array}}) \\
& = \lambda \mathbf{w} \boxed{\begin{array}{l} (\lambda \mathbf{u} \boxed{|\mathbf{u}| = 2m}) (\mathbf{w}) \\ (\lambda \mathbf{v} \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{v}) \\ \mathbf{v}.p < \mathbf{v}.f \\ \mathbf{v}.g = 0 \end{array}}) (\mathbf{w}) \\ \mathbf{w}.p = \mathbf{w}.g \end{array}} \tag{3.20} \\
& = \lambda \mathbf{w} \boxed{\begin{array}{l} \boxed{|\mathbf{w}| = 2m} \\ \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \end{array}} \\ \mathbf{w}.p = \mathbf{w}.g \end{array}} \\
& = \lambda \mathbf{w} \boxed{\begin{array}{l} |\mathbf{w}| = 2m \\ \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.p = \mathbf{w}.g \end{array}}
\end{aligned}$$

Note that we obtain the same result (modulo ordering of the constraints) as (3.16) in Section 3.4.2 above. Moreover, since the measurability condition is no longer part of the lexical semantics of the measure phrase, the translation of the measure phrase *2 metres* can be simplified, and is now of type $(v^3 \rightarrow t)$.

Some support for adopting a construction-oriented approach can be found in certain other phenomena discussed earlier in this chapter. In the introduction, we remarked that there are many gradable adjectives which readily take a measure phrase modifier in their comparative

form, yet seem unable to do so in their absolute form; cf. the sentences in (63), reproduced here for convenience as (75).

- (75) a. * Bob is 60 kilogrammes heavy/light.
b. * My watch was 400 dollars expensive/cheap.
c. * Bob is 2 million dollars rich/poor.
d. * The bedroom was 30 degrees hot/cold.
e. * My car is 10 metres-per-second fast/slow.

In fact, the list of adjectives that *do* take a measure phrase modifier with their absolute form in the structure $[_{\text{DegP}} \text{MP} [_{\text{Deg}'} \emptyset \text{AP}]]$ is severely restricted, and appears to be limited to a handful of adjectives denoting basic spatial and temporal properties, such as LENGTH (of spatial extension or temporal duration), HEIGHT, WIDTH, DEPTH, AGE, and so on. The apparent absence of any systematic feature, syntactic or semantic, that distinguishes these adjectives from the others supports the idea that the structure $[\text{MP} [\emptyset \text{Adj}]]$ perhaps ought to be analysed as a *constructional idiom*, one which specifically licenses only a closed list of gradable adjectives.

3.6 Adjectival Anaphora

In Chapter 2, we discussed cases of deictic and anaphoric dependency in adjectival constructions, involving both the standard and comparative forms of the adjective (see Chapter 2, Section 2.4.3). We will now re-examine this topic from a slightly more formal point of view, using a dynamic model of anaphoric dependency.

3.6.1 Dynamic Semantics

Dynamic semantics is one of the principal contemporary approaches to the study of anaphora within formal semantics. In dynamic semantics, the characteristic phenomena of discourse anaphora arise from the semantics of indefinite expressions, which differ from other quantified expressions in that they introduce discourse referents that can be picked up in subsequent discourse; this idea, which can be found in the work of Lewis [110] and Karttunen [85], lies at the heart of Heim's [65] theory of File Change Semantics (FCS) and Kamp's [82] Discourse Representation Theory (DRT), both developed in the early 1980s as formal theories of discourse anaphora (see also Kamp and Reyle [83], Groenendijk and Stokhof [61], [60], and van Eijk and Kamp [163], among others, for some later developments). The central insight of dynamic

semantics is that the meaning of a discourse can be seen as a series of transitions between contexts. Each sentence in a discourse transforms the context, e.g., by introducing new discourse referents or supplying new information about already existing ones, and dynamic semantics claims that it is precisely this potential for transforming the context, which Heim calls the *context change potential* (CCP), that should be taken as the basis of linguistic meaning.¹⁴

Discourse Representation Theory (DRT) (Kamp [82], Kamp and Reyle [83]) is a paradigmatic example of a dynamic semantic theory. Like other theories of discourse interpretation, DRT comprises two components: a language with a model-theoretic semantics in which the logical forms of discourse are represented, and an account of how these logical forms are modified or extended as discourse proceeds. In DRT, the discourse is represented as a discourse representation structure (DRS); a DRS is a pair of sets $\langle U, C \rangle$, where U is a set of discourse referents and C is a set of discourse conditions; these conditions may be properties and relations involving discourse referents, or other DRSs.

One basic way in which DRT departs from traditional, ‘static’ semantic theories (such as those of Montague [117], [116] and Davidson [31], [32]) is that it claims that indefinite noun phrases such as a man or a donkey are essentially predicates with free variables, $man(x)$, $donkey(y)$ rather than existential quantifiers. In effect, an indefinite introduces a new variable, or discourse marker, and a pronoun anaphoric on an indefinite is interpreted as the same variable as was introduced by its indefinite antecedent. For example, let us consider a simple discourse consisting of the two sentences in (76):

(76) John sees a woman. She is singing.

In DRT, the first sentence can be represented using the following *Discourse Representation Structure* (DRS):

(77)	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-bottom: 1px solid black; padding: 2px;">$x\ y$</td> </tr> <tr> <td style="padding: 2px;">$x = john$ $woman(y)$ $sees(x, y)$</td> </tr> </table>	$x\ y$	$x = john$ $woman(y)$ $sees(x, y)$
$x\ y$			
$x = john$ $woman(y)$ $sees(x, y)$			

¹⁴While dynamic semantics arguably provides, at present, the most detailed and extensive model-theoretic framework for analysing intersentential anaphoric dependencies, it is not the only approach; the *descriptive pronoun* or *E-type* theory, for instance, attributes the special properties of discourse anaphora to a special interpretive strategy governing the anaphoric pronominal expression, rather than its indefinite antecedent; on this view, anaphoric pronouns basically act as proxies for definite descriptions (Geach [56], Evans [42], [44], [45], Parsons [124], Cooper [28], Davies [34]), and pronouns used in this fashion are sometimes referred to as “pronouns of laziness” (Geach [56]) or “E-type pronouns” (Evans [44], [45]).

On an anaphoric reading of the pronoun, the interpretation of the complete example is then represented as in (78):

$$(78) \begin{array}{|l} \hline x \ y \\ \hline x = john \\ woman(y) \\ see(x,y) \\ sing(y) \\ \hline \end{array}$$

Since DRT builds in to the assignment of truth conditions *default existential quantification* over free variables, the DRS in (78) is true if there is something that is John and something John sees that is a woman who is singing. Crucially, the fact that indefinites appear to have the force of existential quantifiers is not because they are existential quantifiers, but arises from the default existential quantification over free variables.

3.6.2 Adjectival Anaphora from a Dynamic Perspective

Armed with the basic notions of dynamic semantics, we can now revisit some of the examples we considered at the end of Chapter 2, repeated here for convenience as (79) and (80).¹⁵

- (79) a. Bob is taller.
b. Bill is tall.
- (80) a. Bill is tall. Bob is also tall.
b. Bill is tall. Bob is also tall (for his age).
c. Bill is tall. Bob is taller.
d. Bill is short. Bob is taller.
e. Bill is strong, but Bob is taller.

Recall that in Section 3.3.2 above, we assigned (79a) the following interpretation:

$$[\text{Bob [is taller]}] \Rightarrow \exists \mathbf{u} \begin{array}{|l} \hline \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u}.f > \mathbf{u}.p \\ \mathbf{u}.p = \mathbf{w}.g \\ \mathbf{u}.f = bob_H \\ \hline \end{array} \quad (3.21)$$

As mentioned in Chapter 2, an isolated utterance of sentence (79a) can be construed deictically as referring to the height of some contextually salient person or thing, such as Bob's own height

¹⁵These corresponds to examples (53a) and (54), respectively, in Section 2.4.3 of Chapter 2.

at some point in the past; in (79b), however, the positive form ‘tall’ is contextually anchored to some appropriate norm or standard of height, s_H , which Bill’s height is claimed to exceed. The kind of contextual dependency involved in each case is quite distinct. This is most clearly seen in examples involving anaphoric dependency, such as (80).

In (80a), the first clause ‘Bill is tall’, interpreted relative to some contextual standard, call it s_H , makes up part of the interpretive context for the second occurrence of ‘tall’. One natural interpretation is for this to be interpreted relative to the same standard s_H , as may further uses of the same adjective.

A standard DRS for the sentence ‘Bill is tall’ would be (81) below.

(81) a. Bill is tall.

	x
b.	$x = bill$ $tall(x)$

In LSS, each adjective token introduces an L-structure into the discourse; thus in (82) below, ‘tall’ introduces the L-structure \mathbf{u} into the DRS universe. In the zero context, there is no antecedent available for an anaphoric interpretation, so the Perspectival parameter is assigned the value s_I deictically (for clarity, we omit reference to the Figure and Ground parameters, which we assume to be assigned in accordance with the compositional machinery set out in the previous chapter).

(82) a. Bill is tall₁.

	x \mathbf{u}
b.	$x = bill$ $\mathbf{u}.f = bill_H$ $\mathbf{u}.f > \mathbf{u}.p$ $\mathbf{u}.g = 0$ $\mathbf{u}.p = S_I$

In the two-clause discourse fragment in (80a), the addition of the sentence ‘Bob is also tall’ has the DRS in (83) as one of its possible representations, where the perspectival parameter of the L-structure \mathbf{w} , introduced by the second occurrence of ‘tall’, is anaphorically identified with the value of the perspectival parameter of the first occurrence.

(83) a. Bill is tall₁. Bob is also tall₁.

	x	y	\mathbf{u}	\mathbf{w}
	$x = bill$			
	$y = bob$			
	$\mathbf{u}.f = bill_H$			
	$\mathbf{u}.f > \mathbf{u}.p$			
b.	$\mathbf{u}.g = 0$			
	$\mathbf{u}.p = S_1$			
	$\mathbf{w}.f = bob_H$			
	$\mathbf{w}.f > \mathbf{w}.p$			
	$\mathbf{w}.g = 0$			
	$\mathbf{w}.p = \mathbf{u}.p$			

As with pronouns, it would be possible for the second token of ‘tall’ to refer deictically to some *different* standard, call it s_2 ; we see this more clearly in (80b), where the hedge ‘for his age’ prompts the selection of a different standard, as in (84) below.

- (84) a. Bill is tall₁. Bob is also tall₂ (for his age).

	x	y	\mathbf{u}	\mathbf{w}
	$x = bill$			
	$y = bob$			
	$\mathbf{u}.f = bill_H$			
	$\mathbf{u}.f > \mathbf{u}.p$			
b.	$\mathbf{u}.g = 0$			
	$\mathbf{u}.p = S_1$			
	$\mathbf{w}.f = bob_H$			
	$\mathbf{w}.f > \mathbf{w}.p$			
	$\mathbf{w}.g = 0$			
	$\mathbf{w}.p = S_2$			

In the case of the comparative, it is the Ground parameter that is free for specification, and the relevant antecedent in this case is not a prior Perspective point, but the height of some contextually accessible entity, such as Bill, represented as h_{bill} in (85).

- (85) a. Bill is tall. Bob is taller.

	x	y	\mathbf{u}	\mathbf{w}
	$x = bill$			
	$y = bob$			
	$\mathbf{u}.f = bill_H$			
	$\mathbf{u}.f > \mathbf{u}.p$			
b.	$\mathbf{u}.g = 0$			
	$\mathbf{u}.p = S_I$			
	$\mathbf{w}.f = bob_H$			
	$\mathbf{w}.f > \mathbf{w}.p$			
	$\mathbf{w}.p = \mathbf{w}.g$			
	$\mathbf{w}.g = bill_H$			

3.6.3 Vagueness in Dynamic Semantics

One feature of dynamic semantics which is relevant to the interpretation of gradable and vague expressions is its ability to model informational updates over the course of a discourse. Kyburg and Morreau [100] have claimed that vague utterances involving gradable adjectives involve negotiation about how to use vague terms, and argue that we require the following in order to handle such cases:

1. a semantic account of vagueness (Kyburg and Morreau favour a version of supervaluationism, together with a system of ‘penumbral connections’);
2. a theory of conveyed meanings (such as implicatures and presuppositions);
3. a theory of context update; and
4. a theory of belief accommodation

Part of what is accommodated is the extension of the vague predicate. Kyburg and Morreau define the contextual update process as follows:

Definition 26 (Context Update) *If c is a context and s is a sentence uttered in c , then*

$$cg(c + s) = cg(c) + cm(s)(c)$$

where $cg(\cdot)$ indicates the common ground and $cm(\cdot)$ denotes the conveyed meanings.

The basic definition of context update takes care of cases where the information in the common ground grows monotonically. However, it cannot cope with cases where the use of a vague predicate introduces an inconsistency. This is where the theory of belief accommodation comes

in, and proceeds roughly as follows: we take the conveyed meanings of an utterance; then we add the contribution of the non-vague sentences of the common ground; finally, we add as many of the remaining vague sentences as we can, without introducing inconsistency.

3.6.3.1 Metalinguistic Sharpening

In addition to the standard *descriptive* uses, as in (86) below, gradable adjectives can also be employed *metalinguistically* to provide information regarding their own applicability. As several commentators have pointed out (e.g., Kamp [84, p. 149], Klein [95, p. 14], Pinkal [132, p. 223], among others), the very act of asserting a sentence containing a vague predicate may cause the context to change in certain ways, for instance, by contributing to the ‘sharpening’ of its intended denotation, as in (87) below.

(86) **A:** “Who is tall in this neighbourhood?”

B: “Well, Jürgen is tall, Englebert is tall, and Bob is tall, but Bill is not.”

(87) **A:** “What counts as tall around here?”

B: “Well, Jürgen is tall, Englebert is tall, and Bob is tall, but Bill is not.”

This phenomenon is addressed within a dynamic framework by Barker [5], who uses it to model cases involving the metalinguistic sharpening of vague predicates. Barker makes use of the notion of a *delineation*, as defined by David Lewis [111]. Intensions in Lewis’s system of general semantics are functions from indices to extensions, where indices are tuples of coordinates for aspects of the context which affect the evaluation of the truth-value of a sentence, and may also enter into the determination of the extension of a term, such as time, possible world, place, etc. Lewis suggests adding a *delineation coordinate* to the set of indices, which specifies how vague or indeterminate terms are to be made precise.¹⁶

Barker [5] proposes a function $d(\cdot)$ that maps possible worlds into delineations for the vague predicates in that world, where the latter are functions from gradable adjectives to degrees. For example, $d(c)$ is the set of delineations associated with world c , and $d(c)(\llbracket tall \rrbracket)$ denotes the absolute standard of evaluation for ‘tall’ in world c .

As in other dynamic theories, Barker takes propositional expressions to denote context update functions (i.e., functions that take a context as an argument and return an updated context

¹⁶In Lewis’s original proposal (see [111, pp. 64–65]), these delineations take the form of boundary-specifying real numbers, one for each vague term in the language; e.g., in a language containing the vague terms ‘tall’ and ‘hot’, the delineation coordinate would include a boundary number specifying the height threshold for tallness and the temperature threshold for hotness. For a discussion of the use of real numbers and scales, see Section ?? below.

as a value). If ϕ is a natural language expression and C and C' are contexts (modelled as sets of possible worlds), then $\llbracket\phi\rrbracket$ is an *update function* between contexts, such that $\llbracket\phi\rrbracket(C, C')$ holds only if C' is a legitimate way of updating C with the content of ϕ . The only update functions Barker considers are filters, so that $C' \subseteq C$ for all C, C' such that $\llbracket\phi\rrbracket(C, C')$. Barker sometimes writes $C \subseteq \llbracket\phi\rrbracket$ in lieu of $\llbracket\phi\rrbracket(C) = C$, and similarly for $c \in C$, he writes $c \in \llbracket\phi\rrbracket$ instead of $\llbracket\phi\rrbracket(\{c\}) = \{c\}$. Thus, if RAIN is the set of worlds in which it is raining, then $\llbracket\text{It is raining}\rrbracket = \lambda C.\{c \in C : c \in \text{RAIN}\}$ (Barker [5, p. 6]).

Letting $\text{TALL}(d, x)$ denote the set of worlds in which x is tall to (at least) degree d , then the expression $\text{TALL}(d(c)(\llbracket\text{tall}\rrbracket), x)$ is the set of possible worlds in which the degree of tallness of x is greater or equal to the standard of tallness in c . The semantic interpretation of ‘tall’ is then given as follows [5, p. 7]:

$$\llbracket\text{tall}\rrbracket = \lambda x \lambda C.\{c \in C : c \in \text{TALL}(d(c)(\llbracket\text{tall}\rrbracket), x)\} \quad (3.22)$$

Simple copular constructions involving adjectives then receive an interpretation as follows (cf. Barker [5, p. 8]):

- (88) a. Bob is tall.
 b. $\llbracket\text{Bob is tall}\rrbracket = \lambda C.\{c \in C : c \in \text{TALL}(d(c)(\llbracket\text{tall}\rrbracket), \text{bob})\}$

The update function in (88) filters out possible worlds based on two criteria: the standard of tallness in that world and Bob’s height in that world. However, where the standard of tallness is not known, as in (87) above, successive predicative uses of ‘tall’ above will have the effect of filtering out those possible worlds in which the standard for tallness is greater than the degree of tallness possessed by Jürgen, Englebert, and Bob, as well as those worlds where it is lower than the degree to which Bill is tall.

The dynamic treatment of adjectives we presented in Section 3.6 above, although quite different in certain respects, is also able to capture the difference between descriptive and metalinguistic usage. The standard of evaluation is an entity in the discourse context, and the anchoring of the Perspectival parameter of the adjective functions in a manner akin to pronominal reference and anaphora. Nevertheless, in spite of this difference with Barker’s approach, the basic dynamic nature of the framework enables us to capture the effect of metalinguistic uses of adjectives. In example (83), we saw that successive predications adds more information regarding the properties of the standard S_I ; in cases where the extent of S_I on the height scale were known, say $S_I = 1.8\text{metres}$, then subsequent reference to this standard would be interpreted descriptively; on the other hand, in cases where we know nothing about S_I , then instances of ‘tall’ that make reference to S_I may function metalinguistically to provide further information about its location.

3.7 Conclusion

This chapter has presented a formal semantics for gradable adjectives centred around the notion of an L-structure, which we introduced in Chapter 2. We defined an L-structure formally as a ternary structure whose components are vectors, which in the case of adjectival L-structures belong to dimensions in a conceptual space (cf. Section 3.2). We also defined notions of orientation and magnitude for L-structures in terms of the relations between their components (Sections 3.2.1 and 3.2.2), and presented a small formal fragment for adjectival constructions (Section 3.3).

We distinguished the semantic properties of gradable adjectives in terms of configurations of *P*, *F*, and *G* points on a dimensional scale, in a manner reminiscent of the Reichenbach treatment of tenses in terms of configurations of *S*, *E*, and *R* points on the timeline, and proposed an *adjectival measurability condition* (AMC) in order to account for the phenomena associated with MP modification (Section 3.4.2). We showed how to incorporate the AMC into the semantic composition mechanism, as part of the lexical semantic definition of the measure phrase.

The AMC has certain properties that make it appealing: (i) it is extremely simple; (ii) it applies uniformly to both comparatives and non-comparatives; (iii) it can be incorporated straightforwardly into the compositional semantic definition of the language.

Unfortunately, it turns out that the AMC, as stated in Definition 22, appears to be far too restrictive to provide an accurate cross-categorial characterisation of measure phrase behaviour. However, we noted that this does not automatically entail that the AMC is not applicable to the adjectival domain, and we suggested a way to express the AMC as a *local* rather than a global constraint, in the context of a construction-oriented semantic framework (Section 3.5).

In Section 3.6 we revisited the idea, inspired by Partee [127], that adjectives are structurally analogous to tenses and pronouns (cf. Chapter 2, Section 2.4.3), and presented a brief treatment of intersentential adjectival dependencies in a dynamic semantic framework.

The theory of Locative Structure Semantics (LSS) presented in this chapter can be regarded as both an extension of and a deviation from Vector Space Semantics (VSS). LSS shares the same basic semantic ontology, and the same preoccupation with structural parallels across domains; from a purely technical standpoint, the principal difference so far hinges on the use of ternary L-structures instead of located vectors. Conceptually, LSS deviates from VSS principally in the central role it assigns to the notion of *perspective*, as found in the spatial, temporal, and conceptual domains (cf. Chapter 2, Section 2.2), and the incorporation of notions from Reichenbach's theory of tense (cf. Chapter 2, Section 2.3).

The technical similarity between LSS and VSS means that, unfortunately, LSS inherits two of the problems with VSS described in Section 1.3.5 of Chapter 1: (i) the type incompatibility between gradable and non-gradable adjectives (and other predicates), which we called the ‘Divergent Type Problem’; and, (ii) the problem posed by intersective constructions, which Winter and Faller refer to as the ‘Coordination Problem’. Chapter 4 presents a solution to both problems in terms of a dynamic model of semantic interpretation; we shall see that, in the process, the divergence between LSS and VSS becomes more pronounced.

Chapter 4

A Dynamic Semantics for Locatives

4.1 Introduction

We introduced dynamic semantics briefly at the end of Chapter 3 (Section 3.6), where we explored its relevance to modelling anaphoric dependencies involving gradable adjectives. In this chapter, we will employ a dynamic approach to address a quite distinct issue, which we consider to be of equal or even greater importance; in particular, we will show how a dynamic model enables us to tackle the two problems faced by vector-based theories such as LSS and VSS which we discussed in Section 1.3.5 of Chapter 1, namely, the ‘Divergent Type Problem’ and the ‘Coordination Problem’.

We shall develop a dynamic version of LSS based on the theory of dynamic semantics known as ‘Dynamic Binding’ (DB), developed by Gennaro Chierchia [25], which is a variant of Groenendijk and Stokhof’s [60] Dynamic Montague Grammar (DMG).

There are two main reasons for choosing DB or DMG over other dynamic theories, such as DRT. The first is that the divergent type problem and the coordination problem are *intrasentential* rather than *intersentential* phenomena, and, although DRT is well-suited to modelling dependencies between sentences, it is less suited to modelling dynamic phenomena at a sentence-internal level (cf. Section 4.4 below); by contrast, the semantic composition mechanism used in Dynamic Binding/Dynamic Montague Grammar is the same as in traditional Montague-style formal semantics. The second reason is that our proposals in this chapter rely heavily on the *Existential Disclosure* operation (ED), originally introduced into DMG by Paul Dekker (cf. Dekker [35], [36]), and which has no equivalent in DRT (cf. Section 4.5 below).

The fundamental feature of dynamic semantics is that existentially quantified objects are not completely closed off to further specification; as Dekker originally noted in the context of Dynamic Montague Grammar, this means that implicit arguments can be ‘hidden’ by existen-

tially quantifying over them in the lexicon, while remaining accessible to further specification by modifier phrases. This has two consequences for an L-structure-based (or vector-based) theory of gradable adjectives. First, by existentially quantifying over the L-Structure parameter, it becomes possible to interpret the gradable adjective as a (dynamic) property of individuals, rather than a property of L-structures or vectors, as in VSS and the version of LSS presented in Chapter 3, which means that gradable adjectives can be assigned exactly the same type as non-gradable adjectives and other predicates. Second, we will see that by using a modified form of existential disclosure, it becomes possible to develop a dynamic form of generalised coordination for L-structures, thereby resolving the Coordination Problem.

4.2 The Problem with Intersective Constructions

There are many constructions that may involve more than one adjective; for example, adjectives may appear conjoined in a coordinate structure or as restrictive modifiers within a determiner phrase, as in (89) and (90) below, and we require our semantics to assign an interpretation to these cases (and others).

- (89) a. Bill is tall and slim.
b. Bill is taller and slimmer than John.
c. Bill is taller and slimmer than some student.
d. John is shorter and heavier than Bill.
e. Bill is taller than Jack and slimmer than Jill.
- (90) a. The tall ugly bloke is my brother.
b. My sister is a beautiful slim girl.
c. I would like a big red juicy apple for dessert.

There exists a venerable tradition of analysing both conjunction and restrictive modification in terms of set-theoretic intersection;¹ however, this Boolean approach to conjunction is incompatible with VSS as standardly formulated (cf. Zwarts and Winter [182]). The standard VSS semantics for adjectives assigns completely disjoint denotations to adjectives from different dimensions, so their intersection produces the empty set; this implies that the adjectival constructions in (89) and (90) above are contradictory, which is evidently not the case. Hence, there appears to be a conflict between the standard account of the semantics of coordination

¹The cross-categorial treatment of coordination in formal semantics has been developed over the years in the work of von Stechow [168], Keenan and Faltz [91], Gazdar [54], Partee and Rooth [129], and others.

(and other ‘intersective’ structures) and a vector-based semantics, which is referred to as the “coordination problem” in the VSS literature (e.g., Faller [46]).

4.2.1 Coordination in VSS

One of the basic accepted claims regarding coordination in natural language is that if two natural language expressions α_1, α_2 belong to the same syntactic category X , then they can be coordinated, and their coordination will also belong to X . This is usually expressed, in the case of English, by a rule schema such as the following (cf., Ross [136], Dik [37], Gazdar [54]).

Definition 27

$$X \rightarrow X_1 \cdots \mu X_n$$

where $2 \leq n$, $\mu \in \{and, or\}$, and X is any syntactic category.

A number of researchers have observed that the semantics for such coordinate structures is generally Boolean in nature, with *and* interpreted as set-theoretic intersection and *or* as union (see von Stechow [168], Keenan and Faltz [91], Gazdar [54], and Partee and Rooth [129], among others).

Boolean semantics generally requires that the coordinated elements belong to the same type. In a Montague-style theory, this is usually enforced by the application of type-shifting operations which raise (or possibly lower) the type of one or both conjuncts until they both belong to the same conjoinable Boolean type.

Definition 28 (Boolean conjunction)

1. S_1 and $S_2 \mapsto S_1' \wedge S_2'$
2. NP_1 and $NP_2 \mapsto \lambda X.NP_1'(X) \wedge NP_2'(X)$
3. A_1 and $A_2 \mapsto \lambda x.A_1'(x) \wedge A_2'(x)$
4. VP_1 and $VP_2 \mapsto \lambda x.VP_1'(x) \wedge VP_2'(x)$
5. V_1 and $V_2 \mapsto \lambda x_1 \dots \lambda x_k.V_1'(x_1) \dots (x_k) \wedge V_2'(x_1) \dots (x_k)$

As we noted earlier in Section 1.3.5.2 of Chapter 1, Zwarts and Winter [182] acknowledge that the standard Boolean account of conjunction appears to be incompatible with the VSS account of spatial prepositions, because one cannot interpret the conjoined VSS denotations (which are sets of vectors) in terms of set-theoretic intersection. For example, VSS assigns completely disjoint denotations to the PPs *above the house* and *below the cloud* in sentence (91a) below

(since the vectors have a different starting point in each set), so their intersection yields the empty set; but (91a) is not in any way contradictory or impossible to interpret, and has an interpretation roughly paraphrased by sentence (91b).

- (91) a. The bird is ten meters [above the house and below the cloud].
b. The bird is ten meters above the house and the bird is ten meters below the cloud.

As Faller [46] points out, this problem with coordination carries over directly to the VSS analysis of degree adjectives, which assigns completely disjoint denotations to sentences such as those in (89) and (90) above; as in the case of (91), the conjoined elements are again not contradictory, nor otherwise semantically incompatible.

Note that a strategy of paraphrasing a sentence such as (92a) below as (92b) cannot work in the general case, as the two forms are not always equivalent; this is exemplified by sentences (93a) and (93b) (cf. Faller [46, p. 167, footnote 16]).

- (92) a. Bob is taller and slimmer than Alice.
b. Bob is taller than someone and Bob is slimmer than someone.
(93) a. Bob is taller and slimmer than someone.
b. Bob is taller than someone and Bob is slimmer than someone.

Zwarts and Winter [182] perceive certain structural and semantic parallels between conjunction in VSS and certain cases of ‘wide-scope’ conjunction in NPs, illustrated by (94) below:

- (94) a. Most [cats and dogs] swim.
b. Most cats swim and most dogs swim.

Examples such as (94) are well-known in the literature (cf. Partee and Rooth [129], Keenan [90], Dowty [38]). As in sentence (91) above, instances of wide-scope conjunction are characterised by the fact that the compositional intersection of the ‘standard’ denotations of the N’ predicates (e.g., *cats* and *dogs*) yield a set that is empty or corresponds to a peculiar and unintended interpretation (such as the set of all cat-dog hybrids), yet have a standard interpretation that is neither contradictory nor bizarre, and indeed roughly equivalent to the sentence-level conjunction in (96b).

Zwarts and Winter [182] conjecture that the same mechanism that is responsible for wide-scope coordination in NPs might also account for the interpretation of conjoined PPs in VSS, but they do not develop this idea any further (cf. Zwarts and Winter [182, section 4.2]), nor do they provide a strategy for how this might be formally implemented.²

²As far as we can ascertain, the coordination problem in VSS has not been resolved to date (Yoad Winter, personal communication).

4.3 Existential Closure

The LSS semantics for gradable adjectives in Section 3.3.1 of Chapter 3 assigned the translation in (4.1) below to the standard form of the adjective ‘tall’.

$$[\emptyset^{\text{ABS}} \text{ tall}] \Rightarrow \lambda \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \end{array} \quad (4.1)$$

This expression is of type $(v^3 \rightarrow t)$. Consider now the alternative translation in (4.2) below:

$$[\emptyset^{\text{ABS}} \text{ tall}] \Rightarrow \begin{array}{l} \lambda x \\ \exists \mathbf{w} \end{array} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.f = \partial(x)(\text{HEIGHT}) \end{array} \quad (4.2)$$

(4.2) differs from (4.1) in three simple respects:

1. the projection function ∂ is part of the hidden or “subatomic” semantics of the adjective;³
2. the L-structure parameter is not abstracted over, but *existentially quantified*;
3. the type of the logical expression is $(e \rightarrow t)$ (a property of individuals), not $(v^3 \rightarrow t)$ (a property of L-structures).

Note in particular that the semantic type $(e \rightarrow t)$ in (4.2) is the type traditionally assigned to adjectives and common nouns in a Montague-style grammar. Thus, there is no longer a type mismatch between the subject and predicate interpretations, and the translation of the adjective can combine directly with an argument of type e to yield an expression of type t , without the need to resort to type-conversion operations like dim^- in VSS (Definition 8, Chapter 1) or its LSS correlate ∂^- (Definition 18, Chapter 3).⁴

On the assumption that the relevant use of the copula ‘be’ is semantically transparent,⁵ then

³In the sense of Parsons [126].

⁴Recall that these type conversion operations are triggered by the type mismatch between the predicate and its argument, as explained in Chapter 1, Section 1.3.

⁵E.g., ‘be’ $\Rightarrow \lambda X.X$, and ‘Bob’ $\Rightarrow b$.

we have the translation for ‘Bob is tall’ shown in (4.3).

$$\begin{aligned}
 [\text{Bob [is tall]}] &\Rightarrow \lambda x \exists \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.f = \partial(x)(\text{HEIGHT}) \end{array} (b) \\
 &= \exists \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.f = \partial(b)(\text{HEIGHT}) \end{array}
 \end{aligned} \tag{4.3}$$

Note that, unlike (3.7) in Chapter 3, there is no longer any appeal to the type-conversion operation ∂^- in the course of the derivation.

In line with (4.2) above, we get the translation for ‘heavy’ shown in (4.4):

$$[\emptyset^{\text{ABS}} \text{ heavy}] \Rightarrow \lambda x \exists \mathbf{u} \begin{array}{l} \text{WEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ \mathbf{u}.f = \partial(x)(\text{WEIGHT}) \end{array} \tag{4.4}$$

Given (4.2) and (4.4), we can employ the definition of Boolean conjunction in Definition 28 to assign an interpretation to the conjunction of ‘tall’ and ‘heavy’, as in (4.5):

$$\begin{aligned}
 [[\emptyset^{\text{ABS}} \text{ tall}] \text{ and } [\emptyset^{\text{ABS}} \text{ heavy}]] &\Rightarrow \lambda y \begin{array}{l} \lambda x \exists \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.f = \partial(x)(\text{HEIGHT}) \end{array} (y) \\ \lambda x \exists \mathbf{u} \begin{array}{l} \text{WEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ \mathbf{u}.f = \partial(x)(\text{WEIGHT}) \end{array} (y) \end{array} \\
 &= \lambda y \begin{array}{l} \exists \mathbf{w} \begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.f = \partial(y)(\text{HEIGHT}) \end{array} \\ \exists \mathbf{u} \begin{array}{l} \text{WEIGHT}(\mathbf{u}) \\ \mathbf{u}.p < \mathbf{u}.f \\ \mathbf{u}.g = 0 \\ \mathbf{u}.f = \partial(y)(\text{WEIGHT}) \end{array} \end{array}
 \end{aligned} \tag{4.5}$$

At this point, it would appear that the modified interpretation of ‘tall’ in (4.2) does not suffer from the two problems with (4.1) we originally discussed in Section 1.3.5 of Chapter 1 in connection with VSS, namely, the divergent type problem and the coordination problem. However, there is a crucial problem with the new approach, as presented thus far: the existential closure in (4.2) means that the adjective is not accessible to any modifiers involving the L-structure parameter. In particular, the parameter is no longer accessible to measure phrase modifiers.

One solution to this problem is to turn all modifiers into logical arguments of the adjectival expression. Indeed, in some treatments of gradable adjectives, such as Heim [64], measure phrases are treated as formal arguments of the adjective, not modifiers. While this does indeed work for individual cases of measure phrase modification, it is very hard to generalise this strategy to *all* modifiers, including adverbial and prepositional phrases, where it is often difficult or impossible to list in advance the number of possible modifiers an expression may take. It is concerns such as these regarding the possibility of treating all modifiers as arguments that lead Winter [176] to reject this as a comprehensive solution to the coordination problem.

In Section 4.6.1 below, we will present a ‘hybrid’ theory of modification in which all modifiers behave as arguments of the expressions they modify. In informal terms, the basic idea is to define a mechanism whereby the modified expression can *systematically* generate as few or as many argument places as required to accommodate its modifiers. In the case of measure phrases, this approach retains the analysis of MPs as modificational expressions, while treating them as arguments of the adjective.

This is where a dynamic semantics can help. In a dynamic semantics, indefinite objects, that is, objects which are existentially quantified over, are available for further specification. This means that, within a dynamic theory, implicit arguments can be assumed to be existentially quantified over in the lexical translation itself; since existential quantification is dynamic, such arguments are still accessible to further specification by modifier phrases. As Dekker [35, p. 585] points out, the advantage of using a dynamic framework is that we are able to deal with implicit arguments at the level of lexical specification; the alternative, in a static framework, would be to complicate the syntax-semantics mapping with optional and potentially ad hoc closure operations. Furthermore, since the implicit arguments are ‘hidden’ by the quantifier, it enables us to give a *uniform* treatment of expressions within the same syntactic category, regardless of what implicit arguments they might have: for example, relational and non-relational nouns, eventive and non-eventive verbs, and, as we shall see, gradable and non-gradable adjectives.

In the following sections, we define a dynamic semantics for L-Structures that is compatible with a generalised Boolean notion of conjunction, yet preserves the modificational character of

measure phrases.

4.4 Dynamic Locative Structure Semantics (DLSS)

We introduced dynamic semantics briefly in Section 3.6 of Chapter 3, where we explained its relevance to modelling anaphoric dependencies involving gradable adjectives. In the following sections we shall employ a dynamic version of LSS based on the theory of dynamic semantics known as ‘Dynamic Binding’ (DB), developed by Gennaro Chierchia [25], and which is itself a variant of Groenendijk and Stokhof’s [60] Dynamic Montague Grammar (DMG).

There are two main reasons for employing DB/DMG instead of DRT. The first is that the semantic composition process used in Dynamic Binding is more in keeping with traditional Montague-style formal semantics, including the version we have been employing in the previous chapters: “classical” DRT (e.g., Kamp [82], Kamp and Reyle [83]) does not employ a lexically-based bottom-up compositional mechanism (cf. Groenendijk and Stokhof [61]), and, although there are more recent versions of DRT that do employ a bottom-up compositional mechanism, such as, e.g., the λ -DRT of Pinkal and others (Latecki and Pinkal [108], Bos et al. [16]) and Muskens’ [120] Compositional DRT, these are still less developed and/or widespread than DMG. The second reason is that our proposals in this chapter rely on the *Existential Disclosure* operation in DB/DMG (cf. Section 4.5 below), which is not available in DRT. This is because discourse markers in DRT are *free variables* which are interpreted existentially at the level of the discourse; the fact that discourse markers in DRT have existential force comes about, not because they are explicitly bound by existential quantifiers, but because there is a *global* assumption of default existential quantification over free variables (cf. Chapter 3, Section 3.6.1). In DB/DMG, by contrast, discourse markers are *explicitly* bound by existential quantifiers; however, the dynamic interpretation of the quantifiers and connectives allows existential quantifiers to bind variables outside their syntactic scope, and, under certain conditions, also allows modifiers to make reference to existentially quantified variables that would be inaccessible in a ‘static’ semantics.

4.4.1 Dynamic Intensional Logic

As before, we translate natural language expressions into a formal language with λ -abstraction and application. The language of DLSS is basically that of a dynamic version of Montague’s Intensional Logic (DIL), with an expanded set of types that includes vectors and product types. We will briefly outline those essential features of Intensional Logic and its dynamic variant

that are required to understand the rest of the chapter, referring the reader to the literature for a detailed exposition of the theory.⁶

The set of *extensional* types in Montague’s IL comprises the basic types e (for ‘entity’) and t (for ‘truth value’), and the complex type $(a \rightarrow b)$, where a and b are themselves types. For each extensional type a , there corresponds an *intensional type* of the form $(s \rightarrow a)$;⁷ this has an *intension* as its semantic value, which is modelled as a function from a set of indices or parameters⁸ into an extensional object of type a . We will employ an expanded set of types, in which we have an additional basic type v (for ‘vector’), and an additional product type of the form $(a \bullet b)$. The result is the set TYPE set out in Definition 29 below, an intensional version of the type system in Winter [176]:

Definition 29 (TYPE) *TYPE is the smallest set such that*

1. $e, v, t \in \text{TYPE}$.
2. if $a, b \in \text{TYPE}$, then $(a \rightarrow b) \in \text{TYPE}$.
3. if $a, b \in \text{TYPE}$, then $(a \bullet b) \in \text{TYPE}$.
4. if $a \in \text{TYPE}$, then $(s \rightarrow a) \in \text{TYPE}$.

We will adopt the standard policy of simplifying expressions where possible by omitting parentheses, subject to the convention that the functional type operator, \rightarrow , is right associative and has a lower precedence than \bullet , and by employing abbreviations where this does not give rise to confusion: as before, L-Structures have type $(v \bullet v \bullet v)$, abbreviated as v^3 .

Definition 30 (SYMB) *The symbolic repertoire SYMB includes that of standard intensional logic:*

1. for every type a , an infinite set VAR_a of variables of type a .
2. for every type a , a (possibly empty) set CON_a of constants of type a .

⁶The collection of papers in Montague [119] is the ‘classical’ reference for Montague’s Intensional Logic and its application to grammar, while extensive and authoritative introductions can also be found in Dowty et al. [39] and Gamut [52, Chs. 5 & 6], inter alia. Groenendijk and Stokhof’s [60] Dynamic Montague Grammar is based on the Dynamic Intensional Logic developed by Janssen [81], which employs the notion of a ‘state switcher’ to achieve dynamic effects; unlike DMG, Chierchia’s [25] Dynamic Binding does not employ state switchers.

⁷Note that here we introduce the type s syncategorematically, as is done in most treatments of IL.

⁸Examples of intensional parameters include possible worlds, points in time, spatial locations, or computational states.

3. *the connectives:* $\neg, \wedge, \vee, \rightarrow$.

4. *the quantifiers:* \forall, \exists .

5. *the abstraction symbol:* λ .

6. *the identity symbol:* $=$.

7. *the brackets:* '(' and ')

In addition, SYMB also contains symbols for discourse markers as well as the dynamic quantifiers and connectives:

8. *for every type a , an infinite set DM_a of dynamic markers of type a .*

9. *the dynamic connectives:* $\sim, ;, \Upsilon, \rightsquigarrow$.

10. *the dynamic quantifiers:* $\mathfrak{A}, \mathfrak{E}$.

11. *the dynamic identity symbol:* \cong .

Definition 31 specifies the set of meaningful expressions of each type, where the total set of meaningful expressions is given by $EXP = \bigcup EXP_a$.

Definition 31 (EXP_a) *For each type $a \in \text{TYPE}$, the set EXP_a of expressions of type a is the smallest set such that:*

1. $DM_a \subseteq EXP_a$.

2. $CON_a \cup VAR_a \subseteq EXP_a$.

3. *if $\alpha \in VAR_a^+$ and $\beta \in EXP_b$ then $\lambda\alpha.\beta \in EXP_{(a \rightarrow b)}$.*

4. *if $\alpha \in EXP_{(a \rightarrow b)}$ and $\beta \in VAR_a^+$ then $(\alpha(\beta)) \in EXP_b$.*

5. *if $\phi, \psi \in EXP_t$ then $(\neg\phi), (\phi \wedge \psi), (\phi \vee \psi), (\phi \rightarrow \psi) \in EXP_t$.*

6. *if $\alpha, \beta \in EXP_a$ then $(\alpha = \beta) \in EXP_a$.*

7. *if $\alpha \in VAR_a^+$ and $\phi \in EXP_t$ then $(\forall\alpha.\phi), (\exists\alpha.\phi) \in EXP_t$.*

8. *if $\alpha \in EXP_a$, then $\wedge\alpha \in EXP_{(s \rightarrow a)}$.*

9. *if $\alpha \in EXP_{(s \rightarrow a)}$, then $\vee\alpha \in EXP_a$.*

where $\text{VAR}_a^+ = \text{VAR}_a \cup \text{DM}_a$.

Clauses 2–7 in Definition 31 are familiar from extensional systems of logic: clause 2 states that there is a set of logical variables and constants for each type; clauses 3 and 4 define λ -abstraction and λ -application function in the standard fashion; clause 5 states that formulae are closed under the standard set of logical connectives; clause 6 states that there exists an equality statement for each type; and, clause 7 states that quantifiers can bind variables of any type.

Clauses 8 and 9 introduce the “cap” operator, \wedge , and the “cup” operator \vee . These operators express *abstraction* over and *application* to indices, respectively,⁹ and thus enable us to switch between intensions and extensions, in a manner described in Section 4.4.3 below.

Clause 1 introduces a set of *discourse markers* (DM) for each type, distinct from the set of logical variables in VAR_a . Discourse markers can be abstracted over using λ and bound by the same quantifiers in the same way as ordinary variables; however, with regard to scope, discourse markers behave in a dynamic manner which is quite unlike the behaviour of variables in classical logic: if x_i is a discourse marker, then a quantifier expression of the form Qx_i will be able to bind DMs beyond its syntactic scope, in ways that we will define below.

4.4.2 Model Theory

Our version of DIL is a two-sorted theory in which we have distinct terms for vector and non-vector entities. As in Section 3.2 of Chapter 3, we assume a set of *individuals* \mathcal{U} , a set of *dimensional vectors* \mathcal{V} , and a set of *dimensions* \mathcal{D} , which is a partition of \mathcal{V} . We take a model $M = \langle \mathcal{W}, \mathfrak{I} \rangle$ to be a structure containing the set $\mathcal{W} = \mathcal{U} \cup \mathcal{V}$ ($\mathcal{U} \cap \mathcal{V} = \emptyset$) of individual and vector entities, and a *constant interpretation function* \mathfrak{I} such that, for any $a \in \text{TYPE}$ and $\alpha \in \text{CON}_a$: $\mathfrak{I}(\alpha) \in \text{DOM}_a$. For each type a , the set DOM_a of denotations of type a is given by Definition 32.

Definition 32 (Domains of interpretation) *For each type $a \in \text{TYPE}$, the set DOM_a of denotations of type a (relative to \mathcal{U}, \mathcal{V}) is defined as follows:*

1. $\text{DOM}_e = \mathcal{U}$
2. $\text{DOM}_v = \mathcal{V}$
3. $\text{DOM}_t = \{0, 1\}$
4. $\text{DOM}_{(a \rightarrow b)} = \text{DOM}_b^{\text{DOM}_a} = \{h \mid h: \text{DOM}_a \rightarrow \text{DOM}_b\}$
5. $\text{DOM}_{(a \bullet b)} = \text{DOM}_a \times \text{DOM}_b$

⁹Note the parallel with the clauses for λ abstraction and application.

$$6. \text{DOM}_{(s \rightarrow a)} = \text{DOM}_a^{\mathcal{S}} = \{h \mid h: \mathcal{S} \rightarrow \text{DOM}_a\}$$

The set \mathcal{S} is a set of indices relative to which interpretation depends; these normally include possible worlds and times, as in Montague's IL, but we shall take *discourse states* as our indices, as explained in Section 4.4.3 below.

We define the set of all logical variables as $\text{VAR} = \bigcup \text{VAR}_a$ and the set of all discourse markers as $\text{DM} = \bigcup \text{DM}_a$. Semantically, we employ two distinct assignment functions, one for the logical variables in VAR , written γ , and another for the discourse markers in DM , written σ .

Definition 33 (Assignment functions) *We have two assignment functions, one for logical variables (VAR) and one for dynamic markers (DM).*

- *A variable assignment for a variable $\alpha \in \text{VAR}_a$ is a function $\gamma: \text{VAR}_a \rightarrow \text{DOM}_a$, and we write $\gamma[\alpha/d]$ for the assignment identical to γ for all variables in VAR_a , except possibly for α , where $\gamma[\alpha/d](\alpha) = d$.*
- *A DM-assignment σ for a dynamic marker $\alpha \in \text{DM}_a$ is a function $\sigma: \text{DM}_a \rightarrow \text{DOM}_a$, and we write $\sigma[\alpha/d]$ for the DM-assignment identical to σ for all dynamic markers in DM_a , except possibly for α , where $\sigma[\alpha/d](\alpha) = d$.*

We now define the semantic interpretation for the set EXP of expressions of the language. In Definition 34 below, the interpretation function $\llbracket \cdot \rrbracket_{M, \gamma, \sigma}$, for a model $M = \langle \mathcal{W}, \mathcal{I} \rangle$, associates with every expression α of type a , written $\alpha : a$, a member $\llbracket \alpha \rrbracket_{M, \gamma, \sigma}$ of DOM_α , relative to a γ and a σ .

Definition 34 (Interpretation function) *Given a model $M = \langle \mathcal{W}, \mathcal{I} \rangle$, we define $\llbracket \alpha \rrbracket_{M, \gamma, \sigma}$, the interpretation of α in M relative to γ and σ , as follows:*

1. dynamic markers:

$$\text{if } \alpha \in \text{DM}_a \text{ then } \llbracket \alpha \rrbracket_{M, \gamma, \sigma} = \sigma(\alpha).$$

2. constants:

$$\text{if } \alpha \in \text{CON}_a \text{ then } \llbracket \alpha \rrbracket_{M, \gamma, \sigma} = \mathcal{I}(\alpha).$$

3. variables:

$$\text{if } \alpha \in \text{VAR}_a \text{ then } \llbracket \alpha \rrbracket_{M, \gamma, \sigma} = \gamma(\alpha).$$

4. functional abstraction and application:

$$4.1. \llbracket \lambda \alpha_a. \beta \rrbracket_{M, \gamma, \sigma} = \lambda d \in \text{DOM}_a. \llbracket \beta \rrbracket_{M, \sigma, \gamma[\alpha_a/d]}$$

$$4.2. \llbracket (\alpha(\beta)) \rrbracket_{M,\gamma,\sigma} = \llbracket \alpha \rrbracket_{M,\gamma,\sigma} (\llbracket \beta \rrbracket_{M,\gamma,\sigma})$$

5. **connectives:**

$$5.1. \llbracket \neg\phi \rrbracket_{M,\gamma,\sigma} = 1 \text{ iff } \llbracket \phi \rrbracket_{M,\gamma,\sigma} = 0.$$

$$5.2. \llbracket (\phi \wedge \psi) \rrbracket_{M,\gamma,\sigma} = 1 \text{ iff } \llbracket \phi \rrbracket_{M,\gamma,\sigma} = 1 \text{ and } \llbracket \psi \rrbracket_{M,\gamma,\sigma} = 1.$$

$$5.3. \llbracket (\phi \vee \psi) \rrbracket_{M,\gamma,\sigma} = 1 \text{ iff } \llbracket \phi \rrbracket_{M,\gamma,\sigma} = 1 \text{ or } \llbracket \psi \rrbracket_{M,\gamma,\sigma} = 1.$$

$$5.4. \llbracket (\phi \rightarrow \psi) \rrbracket_{M,\gamma,\sigma} = 0 \text{ iff } \llbracket \phi \rrbracket_{M,\gamma,\sigma} = 1 \text{ and } \llbracket \psi \rrbracket_{M,\gamma,\sigma} = 0.$$

6. **identity:**

$$\llbracket \alpha = \beta \rrbracket_{M,\gamma,\sigma} = 1 \text{ iff } \llbracket \alpha \rrbracket_{M,\gamma,\sigma} = \llbracket \beta \rrbracket_{M,\gamma,\sigma}.$$

7. **universal quantifier:**

$$7.1. \text{ if } \alpha \in \text{DM}_a, \text{ then } \llbracket \forall\alpha.\phi \rrbracket_{M,\gamma,\sigma} = 1 \text{ iff for every } d \in \text{DOM}_\alpha : \llbracket \phi \rrbracket_{M,\gamma,\sigma[\alpha/d]} = 1.$$

$$7.2. \text{ if } \alpha \in \text{VAR}_a, \text{ then } \llbracket \forall\alpha.\phi \rrbracket_{M,\gamma,\sigma} = 1 \text{ iff for every } d \in \text{DOM}_\alpha : \llbracket \phi \rrbracket_{M,\gamma[\alpha/d],\sigma} = 1.$$

8. **existential quantifier:**

$$8.1. \text{ if } \alpha \in \text{DM}_a, \text{ then } \llbracket \exists\alpha.\phi \rrbracket_{M,\gamma,\sigma} = 1 \text{ iff for some } d \in \text{DOM}_\alpha : \llbracket \phi \rrbracket_{M,\gamma,\sigma[\alpha/d]} = 1.$$

$$8.2. \text{ if } \alpha \in \text{VAR}_a, \text{ then } \llbracket \exists\alpha.\phi \rrbracket_{M,\gamma,\sigma} = 1 \text{ iff for some } d \in \text{DOM}_\alpha : \llbracket \phi \rrbracket_{M,\gamma[\alpha/d],\sigma} = 1.$$

9. **cap:**

if $\alpha \in \text{EXP}_a$ then $\llbracket \wedge\alpha \rrbracket_{M,\gamma,\sigma}$ is that function $h \in \text{DOM}_a^{\mathcal{S}}$ such that for all $\sigma' \in \mathcal{S} : h(\sigma') = \llbracket \alpha \rrbracket_{M,\gamma,\sigma'}$.

10. **cup:**

if $\alpha \in \text{EXP}_{(s \rightarrow a)}$ then $\llbracket \vee\alpha \rrbracket_{M,\gamma,\sigma} = \llbracket \alpha \rrbracket_{M,\gamma,\sigma}(\sigma)$.

Dynamic Binding (and DMG) employs *discourse assignments* as intensional indices, instead of worlds and times as in Montague's IL; that is, assignments σ to discourse markers take the place of possible worlds in the semantics. Of course, such a minimal notion of intension is clearly only adequate for modelling extensional meanings (cf. Groenendijk and Stokhof [60], Chierchia [25, p. 80]): despite the intensional form of the theory, it is really extensional in spirit. However, although it is possible to extend the theory to incorporate modalities proper (c.f., Chierchia [25, Ch. 4]), the extensional theory is nevertheless quite sufficient for our purposes.

We will later make frequent use of the following useful fact, which is a straightforward corollary of the definitions above.

Fact 1 (Cup-Cap) $\llbracket \wedge\alpha \rrbracket_{M,\gamma,\sigma} = \llbracket \alpha \rrbracket_{M,\gamma,\sigma}$

4.4.3 Context Change Potential (CCP)

As we mentioned earlier in Section 3.6.1, at the heart of the dynamic approach to natural language semantics is the view of meaning in terms of *context change potential* (CCP), the transformative effect of a sentence on the discourse context, rather than truth conditions alone.

Consider a discourse D viewed, somewhat simplistically, as a finite series of sentences S_1, S_2, \dots, S_n . A sentence token S_i in the sequence is interpreted in the informational context C_{i-1} resulting from the contribution of previous sentences S_1, S_2, \dots, S_{i-1} ; S_i in turn transforms this context by the introduction of additional information, producing a new informational state C_i , which then acts as the informational context for the next sentence token S_{i+1} , and so on.

Semantically, a discourse can be seen as a series of transitions among informational states. The dynamic meaning of sentence (and linguistic expressions in general) is the way it constrains or determines this transition from state to state. There are several ways to model how linguistic meaning effects these transitions. One approach is to model the meaning of a sentence (or sub-sentential constituent) as a function from input contexts to output contexts, as in Heim's [65] File Change Semantics (FCS) or Groenendijk and Stokhof's [61] Dynamic Predicate Logic (DPL). Chierchia's theory of Dynamic Binding, however, models discourse transitions in terms of *continuations*, by building into the semantic representation of each sentence a placeholder for subsequent discourse (cf. Chierchia [25, pp. 81–84]).

Suppose that the truth-conditional content of a given sentence S is represented by the IL translation S' , and consider the conjunction $(S' \wedge p)$, wherein p is a free propositional variable of type t . As Chierchia explains, the variable p can function as a placeholder for what follows S in the discourse; that is, as a variable over the possible continuations of S , or the ways in which the discourse may continue after the utterance of S . If we take a conjunction of the form $(\Phi \wedge p)$ as a representation of the dynamic meaning of each sentence in the discourse $D = S_1, S_2, \dots, S_n$, we obtain a sequence of translations $(S'_i \wedge p_i)$, for $1 \leq i \leq n$. Given this choice of representation, we can integrate the meaning of each S_{i+1} into the the preceding discourse, by substituting its translation $(S'_{i+1} \wedge p_{i+1})$ into the slot p_i provided by the previous sentence S_i ; that is, given the concatenation $(S'_i \wedge p_i) + (S'_{i+1} \wedge p_{i+1})$, we substitute $(S'_{i+1} \wedge p_{i+1})$ for p_i in $(S'_i \wedge p_i)$ to obtain $(S'_i \wedge S'_{i+1} \wedge p_{i+1})$. Note that this expression has the same form $(\Phi \wedge p)$ as the original concatenated elements; in particular, it also contains a free variable p_{i+1} as a placeholder for the next sentence in the discourse; proceeding in this fashion, the individual sentential meanings can be chained together to construct a semantic representation of the discourse as a whole.

In the context of the Dynamic Intensional Logic defined in the previous sections, we can ensure that the appropriate substitution is made by abstracting over the variable p in each case;

moreover, since we are dealing with an intensional system, we will apply the cup operator \vee to the substituted element, in order to make it of the appropriate type. This gives us the expression $(\lambda p.\Phi \wedge \vee p)$ as Chierchia's proposed representation of the CCP of a sentence S , where Φ is the representation of the truth conditions of S in IL (cf. Chierchia [25, p. 84]). The type of this expression is $((s \rightarrow t) \rightarrow t)$, which we abbreviate as τ .¹⁰

Dynamic Binding employs a 'lift' operator, \uparrow , to map the truth-conditional value of a formula into the corresponding CCP, as in Definition 35.

Definition 35 (Lift \uparrow)

$$\begin{aligned}\uparrow &: t \rightarrow \tau \\ \uparrow \phi &\triangleq \lambda p.\phi \wedge \vee p\end{aligned}$$

The truth conditional meaning of a CCP is recovered by means of a 'closure' operator, which applies the CCP to a tautology (Definition 36). The tautology \top fills the continuation parameter in the CCP, giving us a standard formula of type t containing the tautology as a conjunct; since conjunction of a formula Φ with a tautology does not affect the truth value, the truth conditions of $(\Phi \wedge \top)$ will be equivalent to Φ alone.

Definition 36 (Closure \Downarrow) *Let \top denote a tautologous proposition. Then we define*

$$\begin{aligned}\Downarrow &: \tau \rightarrow t \\ \Downarrow \phi &\triangleq \phi(\wedge \top)\end{aligned}$$

Compound sentences and discourses are interpreted by means of a set of logical operators and quantifiers defined on CCPs, as given in Definition 37 (cf. Chierchia [25, p. 87], Groenendijk and Stokhof [60]).

Definition 37 (The Logic of CCP)

1. $(\alpha \cong \beta) \triangleq \uparrow \Downarrow \alpha = \Downarrow \beta$
2. $\sim \phi \triangleq \uparrow \neg \Downarrow \phi$
3. $(\phi ; \psi) \triangleq \lambda p.\phi(\wedge \psi(p))$
4. $(\phi \Upsilon \psi) \triangleq \sim(\sim \phi ; \sim \psi)$
5. $(\phi \rightsquigarrow \psi) \triangleq \sim \phi \Upsilon (\phi ; \psi)$

¹⁰In fact, for reasons explained in [60] and [25, p. 247, note 15], strictly speaking the CCP of a sentence should be an expression of the type $(s \rightarrow ((s \rightarrow t) \rightarrow t))$; for present purposes, however, the simpler formulation employed by Chierchia [25] is more than adequate, and simplifies our exposition considerably.

$$6. \mathfrak{E}x.\phi \triangleq \lambda p.\exists x.\phi(p)$$

$$7. \mathfrak{A}x.\phi \triangleq \sim \mathfrak{E}x.\sim \phi$$

In virtue of the dynamic interpretation of \exists ; and \mathfrak{E} , the existential quantifier is able to bind variable occurrences that lie outside its syntactic scope, so that even when a discourse marker d is *free* in ψ , $(\mathfrak{E}d.\phi) ; \psi$ is equivalent to $(\mathfrak{E}d.\phi ; \psi)$:

Fact 2 (Existential Associativity)

$$(\mathfrak{E}d.\phi) ; \psi \equiv (\mathfrak{E}d.\phi ; \psi)$$

This property is a straightforward corollary of Definition 37 (in particular, clauses 3 and 6), and is the cornerstone of the account of anaphora in DB and DMG.

The compositional interpretation procedure of Dynamic Binding is similar to Montague Grammar: meanings are assigned to sub-sentential constituents in accordance with the principle of compositionality, by examining the contribution these elements make to the meaning of the sentence as a whole. In traditional Montague grammar, where meaning is based on truth conditions, sentences are of type t , and the semantic values of sub-sentential constituents are assigned on the basis of their contribution to the truth conditions of the whole: thus, a sentence such as ‘The bird sings’ is assigned the type t , the common noun ‘bird’ and the predicate ‘sings’ are given the type $(e \rightarrow t)$, and the determiner ‘the’ is assigned the type $((e \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t))$; given this type assignment, the determiner first combines with the common noun to give $the'(bird')$, of type $((e \rightarrow t) \rightarrow t)$, which then applies to the predicate $sings'$, yielding the interpretation $the'(bird')(sings')$, of type t .¹¹ In the case of Dynamic Binding, the meaning of sentences is given in terms of CCPs instead of truth conditions, and sentences have type τ ; in accordance with the same principle, we simply assign semantic interpretations to the subsentential constituents in terms of their contribution to the context change potential of the whole. Thus, predicates such as ‘bird’ and ‘sings’ will be of type $(e \rightarrow \tau)$, while the determiner will have type $((e \rightarrow \tau) \rightarrow ((e \rightarrow \tau) \rightarrow \tau))$, and so on. Chierchia [25, p. 94] refers to this systematic replacement of type t by type τ as the “Basic Principle of Dynamic Interpretation”. In this way, the lift operator can be extended from formulae to *all* expressions α , which are assigned a dynamic interpretation $\uparrow \alpha$ in accordance by means of the systematic replacement of t with τ in their associated type.

¹¹Naturally, this is only one out of several possible ways in which types might be assigned; what is important is that these will all be guided by the same basic methodological principle.

4.5 Existential Disclosure

One consequence of the equivalence $(\mathcal{E}d.\phi) ; \psi \equiv (\mathcal{E}d.\phi ; \psi)$ is that indefinite objects introduced by discourse markers are semantically available for further specification. Paul Dekker [35], [36] has shown how this feature can be exploited to give an account of adverbial modification for implicit arguments of verbs and relational nouns. In Fragment 1 below, which is based on Dekker’s treatment of non-temporal adverbial phrases in [35, pp. 576–579], we exploit the dynamic nature of existential binding by including an unbound discourse marker as part of the definition of modifying expressions, including the degree terms, the numerical relations, and ‘than’. The definitions are couched in the ‘box’ notation introduced in Definition 14 of Chapter 3, adapted to the language of Dynamic Binding.

Fragment 1

1. Adjectival roots

$$1.1. \textit{ tall} \Rightarrow \lambda \mathbf{u} \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u.p} < \mathbf{u.f} \end{array}}$$

$$1.2. \textit{ short} \Rightarrow \lambda \mathbf{u} \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{u}) \\ \mathbf{u.p} > \mathbf{u.f} \end{array}}$$

$$1.3. \textit{ wide} \Rightarrow \lambda \mathbf{u} \uparrow \uparrow \boxed{\begin{array}{l} \text{WIDTH}(\mathbf{u}) \\ \mathbf{u.p} < \mathbf{u.f} \end{array}}$$

$$1.4. \textit{ narrow} \Rightarrow \lambda \mathbf{u} \uparrow \uparrow \boxed{\begin{array}{l} \text{WIDTH}(\mathbf{u}) \\ \mathbf{u.p} > \mathbf{u.f} \end{array}}$$

2. Degree terms

$$2.1. \textit{ more}_i / \textit{-er}_i \Rightarrow \lambda \mathbf{W} \uparrow \uparrow \boxed{\begin{array}{l} \mathbf{w}_i.p = \mathbf{w}_i.g \\ \mathbf{w}_i.f = \partial(x)(\rho(\mathbf{w}_i)) \end{array}} ; \mathbf{W}(\mathbf{w}_i)$$

$$2.2. \textit{ less}_i \Rightarrow \lambda \mathbf{W} \uparrow \uparrow \boxed{\begin{array}{l} \mathbf{w}_i.p = \mathbf{w}_i.g \\ \mathbf{w}_i.f = \partial(x)(\rho(\mathbf{w}_i)) \end{array}} ; \mathbf{W}(\tilde{\mathbf{w}}_i)$$

$$2.3. \emptyset^{\text{ABS}}_i \Rightarrow \lambda \mathbf{W} \uparrow \uparrow \boxed{\begin{array}{l} \mathbf{w}_i.g = 0 \\ \mathbf{w}_i.f = \partial(x)(\rho(\mathbf{w}_i)) \end{array}} ; \mathbf{W}(\mathbf{w}_i)$$

3. Numerical relations

$$3.1. \text{ at least}_i \Rightarrow \begin{array}{c} \lambda y \\ \lambda z \\ \lambda P \\ \lambda x \end{array} \boxed{P(x); \Uparrow \boxed{\begin{array}{l} |\mathbf{w}_i| \geq y \cdot |\partial(z)(\rho(\mathbf{w}_i))| \\ \mathbf{w}_i \cdot \mathbf{p} = \mathbf{w}_i \cdot \mathbf{g} \end{array}}}$$

$$3.2. \text{ at most}_i \Rightarrow \begin{array}{c} \lambda y \\ \lambda z \\ \lambda P \\ \lambda x \end{array} \boxed{P(x); \Uparrow \boxed{\begin{array}{l} |\mathbf{w}_i| \leq y \cdot |\partial(z)(\rho(\mathbf{w}_i))| \\ \mathbf{w}_i \cdot \mathbf{p} = \mathbf{w}_i \cdot \mathbf{g} \end{array}}}$$

$$3.3. \text{ exactly}_i \Rightarrow \begin{array}{c} \lambda y \\ \lambda z \\ \lambda P \\ \lambda x \end{array} \boxed{P(x); \Uparrow \boxed{\begin{array}{l} |\mathbf{w}_i| = y \cdot |\partial(z)(\rho(\mathbf{w}_i))| \\ \mathbf{w}_i \cdot \mathbf{p} = \mathbf{w}_i \cdot \mathbf{g} \end{array}}}$$

$$3.4. \emptyset_i \Rightarrow \begin{array}{c} \lambda y \\ \lambda z \\ \lambda P \\ \lambda x \end{array} \boxed{P(x); \Uparrow \boxed{\begin{array}{l} |\mathbf{w}_i| = y \cdot |\partial(z)(\rho(\mathbf{w}_i))| \\ \mathbf{w}_i \cdot \mathbf{p} = \mathbf{w}_i \cdot \mathbf{g} \end{array}}}$$

$$4. \text{ than}_i \Rightarrow \begin{array}{c} \lambda y \\ \lambda P \\ \lambda x \end{array} \boxed{P(x); \Uparrow \boxed{\mathbf{w}_i \cdot \mathbf{g} = \partial(y)(\rho(\mathbf{w}_i))}}$$

These definitions are best understood by examining some examples of their application, rather than in the abstract. We begin by considering the dynamic interpretation of ‘tall’ according to Fragment 1 (cf. the translation given in example 4.2 above):

$$[\emptyset^{\text{ABS}} \text{ tall}]_i \Rightarrow \begin{array}{c} \lambda x \\ \mathfrak{E}\mathbf{w}_i \end{array} \Uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_i) \\ \mathbf{w}_i \cdot \mathbf{p} < \mathbf{w}_i \cdot \mathbf{f} \\ \mathbf{w}_i \cdot \mathbf{g} = 0 \\ \mathbf{w}_i \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array}} \quad (4.6)$$

The interpretation of the numerical relations in Fragment 1 takes advantage of the possibilities offered by dynamic existential binding; the measure phrase *at least two metres* now receives the following interpretation:

$$\begin{aligned}
[\text{at least 2 metres}]_i &\stackrel{\lambda y}{\Rightarrow} \lambda z \lambda P \lambda x \left[P(x) ; \uparrow \uparrow \left[\begin{array}{l} |\mathbf{w}_i| \geq y \cdot |\partial(z)(\rho(\mathbf{w}_i))| \\ \mathbf{w}_i \cdot \mathbf{p} = \mathbf{w}_i \cdot \mathbf{g} \end{array} \right] \right] (2)(m) \\
&= \lambda P \lambda x \left[P(x) ; \uparrow \uparrow \left[\begin{array}{l} |\mathbf{w}_i| \geq 2 \cdot |\partial(z)(\rho(\mathbf{w}_i))| \\ \mathbf{w}_i \cdot \mathbf{p} = \mathbf{w}_i \cdot \mathbf{g} \end{array} \right] \right] (m) \\
&= \lambda P \lambda x \left[P(x) ; \uparrow \uparrow \left[\begin{array}{l} |\mathbf{w}_i| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_i))| \\ \mathbf{w}_i \cdot \mathbf{p} = \mathbf{w}_i \cdot \mathbf{g} \end{array} \right] \right]
\end{aligned} \tag{4.7}$$

Here P and x are of type $(e \rightarrow \tau)$ and e respectively, making the type of the measure phrase $((e \rightarrow \tau) \rightarrow (e \rightarrow \tau))$. This expression combines with the interpretation of *tall* obtained in (4.6) to give us the interpretation of *at least two metres tall* in (4.8) below:

[[at least two metres]₁ tall₁]

$$\begin{aligned}
&\Rightarrow \lambda P \quad \lambda x \quad P(x) ; \Uparrow \left[\begin{array}{l} |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right] \quad \lambda x \quad \mathfrak{E}\mathbf{w}_I \quad \Uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array} \right] \\
&= \lambda x \quad \lambda x \quad \mathfrak{E}\mathbf{w}_I \quad \Uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array} \right] \quad (x) ; \Uparrow \left[\begin{array}{l} |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right] \\
&= \lambda x \quad \mathfrak{E}\mathbf{w}_I \quad \Uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array} \right] ; \Uparrow \left[\begin{array}{l} |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right] \\
&= \lambda x \quad \mathfrak{E}\mathbf{w}_I \quad \Uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array} \right] ; \Uparrow \left[\begin{array}{l} |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right] \\
&= \lambda x \quad \mathfrak{E}\mathbf{w}_I \quad \Uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right] \\
&= \lambda x \quad \mathfrak{E}\mathbf{w}_I \quad \Uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right]
\end{aligned} \tag{4.8}$$

Then, applying this to the translation ‘Bob’ $\Rightarrow b$, we obtain:

[Bob [is [at least two metres]₁ tall₁]]

$$\begin{aligned}
 & \Rightarrow \lambda x \mathfrak{E}_{\mathbf{w}_I} \Uparrow \left(\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right) (b) \\
 & = \mathfrak{E}_{\mathbf{w}_I} \Uparrow \left(\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(b)(\text{HEIGHT}) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right)
 \end{aligned} \tag{4.9}$$

The *static* (i.e., truth conditional) content of this expression can be obtained using the closure operator \Downarrow (Definition 36), in combination with the reduction rules for DMG:

$$\begin{aligned}
& \Downarrow \mathfrak{E} \mathbf{w}_I \Uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I.p < \mathbf{w}_I.f \\ \mathbf{w}_I.g = 0 \\ \mathbf{w}_I.f = \partial(b)(\text{HEIGHT}) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I.p = \mathbf{w}_I.g \end{array}} \\
& = \mathfrak{E} \mathbf{w}_I \Uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I.p < \mathbf{w}_I.f \\ \mathbf{w}_I.g = 0 \\ \mathbf{w}_I.f = \partial(b)(\text{HEIGHT}) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I.p = \mathbf{w}_I.g \end{array}} \quad (\wedge \top) \\
& = \lambda p \exists \mathbf{w}_I \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I.p < \mathbf{w}_I.f \\ \mathbf{w}_I.g = 0 \\ \mathbf{w}_I.f = \partial(b)(\text{HEIGHT}) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I.p = \mathbf{w}_I.g \\ \vee p \end{array}} \quad (\wedge \top) \quad (4.10) \\
& = \exists \mathbf{w}_I \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I.p < \mathbf{w}_I.f \\ \mathbf{w}_I.g = 0 \\ \mathbf{w}_I.f = \partial(b)(\text{HEIGHT}) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I.p = \mathbf{w}_I.g \\ \vee \wedge \top \end{array}} \\
& = \exists \mathbf{w}_I \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I.p < \mathbf{w}_I.f \\ \mathbf{w}_I.g = 0 \\ \mathbf{w}_I.f = \partial(b)(\text{HEIGHT}) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I.p = \mathbf{w}_I.g \end{array}}
\end{aligned}$$

4.5.1 The *Than*-Phrase as Modifier

We can also treat the *than*-phrase complement as an optional modifier of the comparative (cf. Chapter 3, Section 3.3.2): in a dynamic setting, ‘than’ takes an argument of type e and produces a *than*-phrase of type $((e \rightarrow \tau) \rightarrow (e \rightarrow \tau))$; e.g., given that ‘John’ $\models j$, we have

$$\begin{aligned}
[\text{than John}]_i &\Rightarrow \lambda y \lambda P \lambda x \left[P(x) ; \uparrow \left[\mathbf{w}_i \cdot \mathbf{g} = \partial(y)(\rho(\mathbf{w}_i)) \right] \right] (j) \\
&= \lambda P \lambda x \left[P(x) ; \uparrow \left[\mathbf{w}_i \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_i)) \right] \right]
\end{aligned} \tag{4.11}$$

Since the *than*-phrase is now of type $((e \rightarrow \tau) \rightarrow (e \rightarrow \tau))$, this means that we can simplify the translations for the comparative markers ‘more/-er’ and ‘less’ as shown in Fragment 1. Consequently, the comparative form ‘taller’ receives the following translation, of type $(e \rightarrow \tau)$:

$$[\text{taller}]_i \Rightarrow \lambda x \mathfrak{E} \mathbf{w}_i \uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_i) \\ \mathbf{w}_i \cdot \mathbf{p} < \mathbf{w}_i \cdot \mathbf{f} \\ \mathbf{w}_i \cdot \mathbf{p} = \mathbf{w}_i \cdot \mathbf{g} \\ \mathbf{w}_i \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array} \right] \tag{4.12}$$

We then combine (4.11) and (4.12) to obtain

$$\begin{aligned}
&[[\text{taller}]_1 [\text{than John}]_1] \\
&\Rightarrow \lambda P \lambda x \left[P(x) ; \uparrow \left[\mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \right] \right] \lambda x \mathfrak{E} \mathbf{w}_I \uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array} \right] \\
&= \lambda x \left[\lambda x \mathfrak{E} \mathbf{w}_I \uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array} \right] (x) ; \uparrow \left[\mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \right] \right] \\
&= \lambda x \left[\mathfrak{E} \mathbf{w}_I \uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array} \right] ; \uparrow \left[\mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \right] \right]
\end{aligned}$$

Since $(\mathfrak{E}x.\phi) ; \psi \equiv (\mathfrak{E}x.\phi ; \psi)$, this is equivalent to:

$$\lambda x \left[\mathfrak{E} \mathbf{w}_I \uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array} \right] ; \uparrow \left[\mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \right] \right]$$

Further reduction then gives us the following translation of ‘taller than John’, of type $(e \rightarrow \tau)$:

$$\lambda x \uparrow \uparrow \mathfrak{E} \mathbf{w}_I \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \\ \mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \end{array} \right] \quad (4.13)$$

Note that the discourse marker \mathbf{w}_I is still accessible for further modification, for example by a measure phrase such as ‘at least two metres’:

$$\begin{aligned} & [[\text{at least two metres}]_1 [\text{taller than John}]_1] \\ \Rightarrow & \lambda P \uparrow \uparrow \lambda x \left[P(x); \uparrow \uparrow \left[\begin{array}{l} |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right] \right] \lambda x \uparrow \uparrow \mathfrak{E} \mathbf{w}_I \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \\ \mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \end{array} \right] \\ = & \lambda x \uparrow \uparrow \mathfrak{E} \mathbf{w}_I \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \\ \mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \end{array} \right] (x); \uparrow \uparrow \left[\begin{array}{l} |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array} \right] \end{aligned}$$

a derivation similar to (4.8) above then gives us (4.14) as the translation of ‘at least two metres taller than John’:

$$\lambda x \uparrow \uparrow \mathfrak{E} \mathbf{w}_I \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \\ \mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \end{array} \right] \quad (4.14)$$

The resulting expression is of type $(e \rightarrow \tau)$, and thus combines with a subject of type e to give a proposition of type τ .¹²

¹²For convenience, we employ the basic type rather than the principal ultrafilter, as is standard in Montague Grammar and Generalised Quantifier Theory.

[Bob [is [at least two metres taller than John]₁]]

$$\begin{aligned}
& \Rightarrow \lambda x \uparrow\uparrow \mathfrak{E} \mathbf{w}_I \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \\ \mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \end{array} \right] (b) \\
& = \mathfrak{E} \mathbf{w}_I \uparrow\uparrow \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(b)(\text{HEIGHT}) \\ \mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \end{array} \right]
\end{aligned} \tag{4.15}$$

As before, we can extract the static truth conditions using the closure operator, \Downarrow , to obtain (4.16).

$$\Downarrow \exists \mathbf{w}_I \left[\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(b)(\text{HEIGHT}) \\ \mathbf{w}_I \cdot \mathbf{g} = \partial(j)(\rho(\mathbf{w}_I)) \\ |\mathbf{w}_I| \geq 2 \cdot |\partial(m)(\rho(\mathbf{w}_I))| \end{array} \right] \tag{4.16}$$

4.6 Coordination and Existential Disclosure

We can extend generalised conjunction to a dynamic setting by systematically replacing type t with type τ :

Definition 38 (Dynamic Generalised Conjunction) *Given expressions A_1, A_2 of type $(\zeta_1 \rightarrow (\zeta_2 \rightarrow \dots (\zeta_n \rightarrow \tau)))$, their conjunction is given by:*

$$A_1 \text{ and } A_2 \Rightarrow \lambda \beta_1 \dots \lambda \beta_n . A_1'(\beta_1) \dots (\beta_n) ; A_2'(\beta_1) \dots (\beta_n)$$

where each variable β_i is of type ζ_i , for $i = 1, \dots, n$.

Let us note that this immediately gives us the intuitively correct interpretation in the case of unmodified adjectives (cf. Section 4.3 above):

$$\begin{aligned}
 & [[\emptyset^{\text{ABS}} \text{ tall}]_1 \text{ and } [\emptyset^{\text{ABS}} \text{ heavy}]_2] \\
 \Rightarrow & \lambda z \left(\begin{array}{c} \lambda x \uparrow\uparrow \\ \mathfrak{E}w_1 \uparrow\uparrow \end{array} \left[\begin{array}{l} \text{HEIGHT}(w_1) \\ w_1.p < w_1.f \\ w_1.g = 0 \\ w_1.f = \partial(x)(\text{HEIGHT}) \end{array} \right] (z); \begin{array}{c} \lambda y \uparrow\uparrow \\ \mathfrak{E}w_2 \uparrow\uparrow \end{array} \left[\begin{array}{l} \text{WEIGHT}(w_2) \\ w_2.p < w_2.f \\ w_2.g = 0 \\ w_2.f = \partial(y)(\text{WEIGHT}) \end{array} \right] (z) \right) \\
 = & \lambda z \left(\begin{array}{c} \mathfrak{E}w_1 \uparrow\uparrow \\ \mathfrak{E}w_2 \uparrow\uparrow \end{array} \left[\begin{array}{l} \text{HEIGHT}(w_1) \\ w_1.p < w_1.f \\ w_1.g = 0 \\ w_1.f = \partial(z)(\text{HEIGHT}) \end{array} \right]; \left[\begin{array}{l} \text{WEIGHT}(w_2) \\ w_2.p < w_2.f \\ w_2.g = 0 \\ w_2.f = \partial(z)(\text{WEIGHT}) \end{array} \right] \right)
 \end{aligned}$$

The resulting phrase is of the same type as the conjoined constituents, viz., $(e \rightarrow \tau)$. Given the translation ‘Bob’ $\Rightarrow b$, and assuming the semantic transparency of ‘be’, we obtain the following interpretation for the clause *Bob is tall and heavy*:

$$\begin{aligned}
 & [\text{Bob} [\text{is} [[\emptyset^{\text{ABS}} \text{ tall}]_1 \text{ and } [\emptyset^{\text{ABS}} \text{ heavy}]_2]]] \\
 \Rightarrow & \lambda z \left(\begin{array}{c} \mathfrak{E}w_1 \uparrow\uparrow \\ \mathfrak{E}w_2 \uparrow\uparrow \end{array} \left[\begin{array}{l} \text{HEIGHT}(w_1) \\ w_1.p < w_1.f \\ w_1.g = 0 \\ w_1.f = \partial(z)(\text{HEIGHT}) \end{array} \right]; \left[\begin{array}{l} \text{WEIGHT}(w_2) \\ w_2.p < w_2.f \\ w_2.g = 0 \\ w_2.f = \partial(z)(\text{WEIGHT}) \end{array} \right] \right) (b) \\
 = & \left(\begin{array}{c} \mathfrak{E}w_1 \uparrow\uparrow \\ \mathfrak{E}w_2 \uparrow\uparrow \end{array} \left[\begin{array}{l} \text{HEIGHT}(w_1) \\ w_1.p < w_1.f \\ w_1.g = 0 \\ w_1.f = \partial(b)(\text{HEIGHT}) \end{array} \right]; \left[\begin{array}{l} \text{WEIGHT}(w_2) \\ w_2.p < w_2.f \\ w_2.g = 0 \\ w_2.f = \partial(b)(\text{WEIGHT}) \end{array} \right] \right)
 \end{aligned}$$

As before, we extract the truth-conditions of the dynamic expression by means of the closure operation \Downarrow and the reduction rules in Definition 37 to obtain the static expression in (4.17) below, which is an intuitively plausible representation of the truth conditions of the sentence *Bob is tall and heavy*.

$$\begin{array}{l}
\exists \mathbf{w}_1 \\
\exists \mathbf{w}_2
\end{array}
\left[\begin{array}{l}
\text{HEIGHT}(\mathbf{w}_1) \\
\mathbf{w}_1.p < \mathbf{w}_1.f \\
\mathbf{w}_1.g = 0 \\
\mathbf{w}_1.f = \partial(b)(\text{HEIGHT}) \\
\text{WEIGHT}(\mathbf{w}_2) \\
\mathbf{w}_2.p < \mathbf{w}_2.f \\
\mathbf{w}_2.g = 0 \\
\mathbf{w}_2.f = \partial(b)(\text{WEIGHT})
\end{array} \right] \quad (4.17)$$

We also obtain the correct interpretation in cases where individual conjuncts undergo modification by a measure phrase or *than*-phrase; for example, the phrase *ten centimetres taller than Eve and 5 centimetres shorter than John* receives the following dynamic interpretation:

$[[10 \text{ cm. taller than Eve}]_1 \text{ and } [5 \text{ cm. shorter than John}]_2]$

$$\Rightarrow \lambda z \quad \mathfrak{E}\mathbf{w}_1 \uparrow \uparrow \left[\begin{array}{l}
\text{HEIGHT}(\mathbf{w}_1) \\
\mathbf{w}_1.p < \mathbf{w}_1.f \\
\mathbf{w}_1.p = \mathbf{w}_1.g \\
\mathbf{w}_1.f = \partial(z)(\text{HEIGHT}) \\
\mathbf{w}_1.g = \partial(e)(\rho(\mathbf{w}_1)) \\
|\mathbf{w}_1| = 10 \cdot |\partial(\text{cm})(\rho(\mathbf{w}_1))|
\end{array} \right]; \quad \mathfrak{E}\mathbf{w}_2 \uparrow \uparrow \left[\begin{array}{l}
\text{HEIGHT}(\mathbf{w}_2) \\
\mathbf{w}_2.p > \mathbf{w}_2.f \\
\mathbf{w}_2.p = \mathbf{w}_2.g \\
\mathbf{w}_2.f = \partial(z)(\text{HEIGHT}) \\
\mathbf{w}_2.g = \partial(j)(\rho(\mathbf{w}_2)) \\
|\mathbf{w}_2| = 5 \cdot |\partial(\text{cm})(\rho(\mathbf{w}_2))|
\end{array} \right]$$

However, examples where the modifier applies to the conjunction *as a whole* are still problematic. We have in mind sentences such as the following:

- (95) a. The painting is [20 centimetres]_? [tall₁ and wide₂]
b. Bob is [taller₁ and heavier₂] [than John]_?

We can see the problem easily if we consider that the translations assigned to measure phrases and *than*-phrases by Fragment 1 contain a *single* indexed discourse marker, while a coordinate construction contains at least two (and possibly more) discourse markers. If the conjuncts have different indices, then the modifier can at most modify one of them (viz., the one bearing the same index); if we were to assign *all* the conjuncts the same index, then the only the *last* conjunct will be modified, in accordance with the dynamic behaviour of the existential quantifier. In any case, the indexing scheme used in Fragment 1 is inadequate, and provides at best a partial solution to the problem posed by intersective constructions. While it might be possible to extend or modify the indexing mechanism to handle the case illustrated in (95), we

will develop an account that relies on a different semantic mechanism for combining modifiers and modified expressions.

4.6.1 Existential Disclosure Revisited

Zwarts and Winter [182] have suggested a parallel between the interpretation of conjoined PPs under modification and the phenomenon of wide-scope coordination in NPs. One strategy for dealing with wide-scope coordination is makes use of higher types; thus, Partee and Rooth [129] note that one can obtain the correct reading for sentences such as (94), repeated here for convenience as (96), by assigning interpretations of higher type to the common nouns, as in Definition 39 below¹³.

- (96) a. Most [cats and dogs] swim.
 b. Most cats swim and most dogs swim.

Definition 39 (Flip-Flop) *Let the phrase A have type α , and let β be any type. Then A has a translation A'' of type $((\alpha \rightarrow \beta) \rightarrow \beta)$ in addition to its translation A' of type α :*

$$A'' \triangleq \lambda F.F(A'),$$

where F is a variable of type $(\alpha \rightarrow \beta)$.

This corresponds to the application of the type-raising combinator C_* of Curry and Feys [30]:

Definition 40 (C_*)

$$C_*x \triangleq \lambda p.px$$

The effect of the higher type is to reverse the function-argument application order in the construction [Det CN], which has the effect of distributing the quantifier *most* across the conjoined elements. Although the free application of type raising expressed in the ‘Flip-Flop’ rule is ultimately at odds with Partee and Rooth’s “lazy” approach to type-shifting (according to which types combine at the lowest possible level, and only shift if required to do so by the mechanism of semantic composition), there are more “liberal” systems with flexible type raising which allow such higher types to be freely derived.¹⁴

¹³The formulation is due to Partee and Rooth [129], who credit it to Robin Cooper.

¹⁴Cf. Dowty [38], Hendriks [69]. The terms “lazy” and “liberal” are used by Winter [174, Chapter 4] to characterise the two fundamental attitudes to type shifting: the “lazy” approach, exemplified by Partee and Rooth [129], maintains that type shifting is an interpretive process of last resort, to be done only if necessary, while

Instead of making implicit use of disclosure, as in the definitions in Fragment 1, we will employ an explicit operation that both discloses and abstracts over the existentially bound discourse marker; such a disclosure operation was originally introduced by Paul Dekker in [35], [36], and Definition 41 presents the (simplified) formulation due to Chierchia [25, p. 104]).

Definition 41 (Existential Disclosure) *We define the existential disclosure of a discourse marker d of type δ in a formula ϕ , $\Lambda d.\phi$, as:*

$$\Lambda d.\phi \triangleq \lambda x.\phi ; \uparrow \{x = d\}$$

where x is a variable of type δ .

As can be seen from this definition, the operation abstracts over a variable x , which is also identified with the discourse marker d ; if ϕ is of type η , the effect of disclosure is to create and expression of type $(\delta \rightarrow \eta)$.

We can regard the Dekker-Chierchia formulation of existential disclosure in Definition 41 as a specific instance of a higher-order disclosure operation, which we refer to as ED in Definition 42 below:

Definition 42 (ED) *We define the higher-order existential disclosure of dynamic marker d of type δ in a formula ϕ , $\text{ED}_d\{\phi\}$, as:*

$$\text{ED}_d\{\phi\} \triangleq \lambda P.\phi ; P(d)$$

where P is a variable of type $(\delta \rightarrow \tau)$.

The Dekker-Chierchia formulation can be recovered in terms of ED as follows:

$$\Lambda d.\phi \triangleq \lambda x.\text{ED}_d\{\phi\}(\lambda y.\uparrow \{x = y\}) \tag{4.18}$$

It is straightforward to check that this is equivalent to Definition 41.

In order to be able to apply this operation to expressions containing lambda abstractions, we need to extend the definition slightly:

the “liberal” approach, of which Hendriks [69] is a good example, allows type-shifting to take place whenever possible; for his part, Winter [174, Chapter 4] considers both positions to be incorrect, and proposes an approach in which category shifts are triggered by syntactic mechanisms rather than semantic ones. Interestingly, such higher types are available without explicit type-shifting in “continued” grammars, such as those investigated in Barker [4, 6].

Definition 43 (ED with abstraction) Let ϕ have the form $\lambda x_1 \dots \lambda x_k. \psi$. Then we define the higher-order existential disclosure (with abstraction) of dynamic marker d of type δ in ϕ , $\hat{\text{ED}}_d\{\phi\}$, as:

$$\hat{\text{ED}}_d\{\phi\} \triangleq \lambda P. \lambda y_1 \dots \lambda y_k. \phi(y_1) \dots (y_k) ; P(d)$$

where P is a variable of type $(\delta \rightarrow \tau)$.

We will treat $\hat{\text{ED}}$ as a freely applicable type-conversion rule that generates additional translations for expressions containing hidden parameters, as set out in Definition below.

Definition 44 (Implicit Argument Disclosure) Let the phrase A have type α , and let d be a discourse marker of type δ occurring in A . Then A has a translation A'' of type $((\delta \rightarrow \tau) \rightarrow \alpha)$ in addition to its translation A' of type α :

$$A'' \triangleq \hat{\text{ED}}_d\{A'\}.$$

The selection of the appropriate translation during semantic composition will be determined by the type compatibility requirements of the elements to be combined.

Fragment 2

1. Adjectival roots

$$1.1. \text{ tall} \Rightarrow \lambda \mathbf{u} \uparrow \uparrow \begin{array}{|c|} \hline \text{HEIGHT}(\mathbf{u}) \\ \hline \mathbf{u.p} < \mathbf{u.f} \\ \hline \end{array}$$

$$1.2. \text{ short} \Rightarrow \lambda \mathbf{u} \uparrow \uparrow \begin{array}{|c|} \hline \text{HEIGHT}(\mathbf{u}) \\ \hline \mathbf{u.p} > \mathbf{u.f} \\ \hline \end{array}$$

$$1.3. \text{ wide} \Rightarrow \lambda \mathbf{u} \uparrow \uparrow \begin{array}{|c|} \hline \text{WIDTH}(\mathbf{u}) \\ \hline \mathbf{u.p} < \mathbf{u.f} \\ \hline \end{array}$$

$$1.4. \text{ narrow} \Rightarrow \lambda \mathbf{u} \uparrow \uparrow \begin{array}{|c|} \hline \text{WIDTH}(\mathbf{u}) \\ \hline \mathbf{u.p} > \mathbf{u.f} \\ \hline \end{array}$$

2. Degree terms

$$2.1. \text{ more / -er} \Rightarrow \begin{array}{l} \lambda W \\ \lambda x \\ \mathfrak{E} \mathbf{w}_i \end{array} \uparrow \uparrow \begin{array}{|c|} \hline \mathbf{w}_i.p = \mathbf{w}_i.g \\ \mathbf{w}_i.f = \partial(x)(\rho(\mathbf{w}_i)) \\ \hline \end{array} ; W(\mathbf{w}_i)$$

$$2.2. \text{ less} \Rightarrow \begin{array}{l} \lambda W \\ \lambda x \\ \mathfrak{E} \mathbf{w}_i \end{array} \uparrow \uparrow \begin{array}{|c|} \hline \mathbf{w}_i.p = \mathbf{w}_i.g \\ \mathbf{w}_i.f = \partial(x)(\rho(\mathbf{w}_i)) \\ \hline \end{array} ; W(\tilde{\mathbf{w}}_i)$$

$$2.3. \emptyset^{\text{ABS}} \Rightarrow \begin{array}{l} \lambda W \\ \lambda x \\ \mathfrak{E} \mathbf{w}_i \end{array} \uparrow \uparrow \boxed{\begin{array}{l} \mathbf{w}_i \cdot \mathbf{g} = 0 \\ \mathbf{w}_i \cdot \mathbf{f} = \partial(x)(\rho(\mathbf{w}_i)) \end{array}} ; W(\mathbf{w}_i)$$

3. Numerical relations

$$3.1. \text{at least} \Rightarrow \begin{array}{l} \lambda y \\ \lambda z \\ \lambda \mathbf{u} \end{array} \uparrow \uparrow \boxed{\begin{array}{l} |\mathbf{u}| \geq y \cdot |\partial(z)(\rho(\mathbf{u}))| \\ \mathbf{u} \cdot \mathbf{p} = \mathbf{u} \cdot \mathbf{g} \end{array}}$$

$$3.2. \text{at most} \Rightarrow \begin{array}{l} \lambda y \\ \lambda z \\ \lambda \mathbf{u} \end{array} \uparrow \uparrow \boxed{\begin{array}{l} |\mathbf{u}| \leq y \cdot |\partial(z)(\rho(\mathbf{u}))| \\ \mathbf{u} \cdot \mathbf{p} = \mathbf{u} \cdot \mathbf{g} \end{array}}$$

$$3.3. \text{exactly} \Rightarrow \begin{array}{l} \lambda y \\ \lambda z \\ \lambda \mathbf{u} \end{array} \uparrow \uparrow \boxed{\begin{array}{l} |\mathbf{u}| = y \cdot |\partial(z)(\rho(\mathbf{u}))| \\ \mathbf{u} \cdot \mathbf{p} = \mathbf{u} \cdot \mathbf{g} \end{array}}$$

$$3.4. \emptyset \Rightarrow \begin{array}{l} \lambda y \\ \lambda z \\ \lambda \mathbf{u} \end{array} \uparrow \uparrow \boxed{\begin{array}{l} |\mathbf{u}| = y \cdot |\partial(z)(\rho(\mathbf{u}))| \\ \mathbf{u} \cdot \mathbf{p} = \mathbf{u} \cdot \mathbf{g} \end{array}}$$

$$4. \text{than} \Rightarrow \begin{array}{l} \lambda y \\ \lambda \mathbf{u} \end{array} \uparrow \uparrow \boxed{\mathbf{u} \cdot \mathbf{g} = \partial(y)(\rho(\mathbf{u}))}$$

Fragment 2 gives us the same basic translation for ‘tall’ as Fragment 1, repeated here for convenience as (4.19):

$$[\emptyset^{\text{ABS}} \text{tall}]_i \Rightarrow \begin{array}{l} \lambda x \\ \mathfrak{E} \mathbf{w}_i \end{array} \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_i) \\ \mathbf{w}_i \cdot \mathbf{p} < \mathbf{w}_i \cdot \mathbf{f} \\ \mathbf{w}_i \cdot \mathbf{g} = 0 \\ \mathbf{w}_i \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array}} \quad (4.19)$$

However, in virtue of Definition 44, we now have the following additional *existentially disclosed* translation of ‘tall’, of type $((v^3 \rightarrow \tau) \rightarrow (e \rightarrow \tau))$, obtained from the basic translation (of type $(e \rightarrow \tau)$) by means of $\hat{\text{E}}\hat{\text{D}}$:

$$[\emptyset^{\text{ABS}} \text{tall}] \Rightarrow \begin{array}{l} \lambda R \\ \lambda x \end{array} \mathfrak{E} \mathbf{w}_i \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_i) \\ \mathbf{w}_i \cdot \mathbf{p} < \mathbf{w}_i \cdot \mathbf{f} \\ \mathbf{w}_i \cdot \mathbf{g} = 0 \\ \mathbf{w}_i \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array}} ; R(\mathbf{w}_i) \quad (4.20)$$

This expression takes a modifier R of type $(v^3 \rightarrow \tau)$ as an argument. Fragment 2 assigns the type $(v^3 \rightarrow \tau)$ to both measure phrases and ‘than’-phrases; for example, the measure phrase ‘53 centimetres’ has the interpretation:

$$[53 \text{ centimetres}] \Rightarrow \lambda \mathbf{u} \uparrow \uparrow \boxed{\begin{array}{l} |\mathbf{u}| = 53 \cdot |\partial(\text{cm})(\rho(\mathbf{u}))| \\ \mathbf{u} \cdot \mathbf{p} = \mathbf{u} \cdot \mathbf{g} \end{array}} \quad (4.21)$$

Combining the MP ‘53 centimetres’ with the disclosed interpretation of ‘tall’ gives us:

$$\begin{aligned} & [[53 \text{ centimetres}] [\emptyset^{\text{ABS}} \text{ tall}]] \\ \Rightarrow & \lambda R \quad \lambda x \quad \mathfrak{E} \mathbf{w}_I \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array}} ; R(\mathbf{w}_I) \quad \lambda \mathbf{u} \uparrow \uparrow \boxed{\begin{array}{l} |\mathbf{u}| = 53 \cdot |\partial(\text{cm})(\rho(\mathbf{u}))| \\ \mathbf{u} \cdot \mathbf{p} = \mathbf{u} \cdot \mathbf{g} \end{array}} \\ = & \lambda x \quad \mathfrak{E} \mathbf{w}_I \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array}} ; \lambda \mathbf{u} \uparrow \uparrow \boxed{\begin{array}{l} |\mathbf{u}| = 53 \cdot |\partial(\text{cm})(\rho(\mathbf{u}))| \\ \mathbf{u} \cdot \mathbf{p} = \mathbf{u} \cdot \mathbf{g} \end{array}} (\mathbf{w}_I) \\ = & \lambda x \quad \mathfrak{E} \mathbf{w}_I \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \end{array}} ; \uparrow \uparrow \boxed{\begin{array}{l} |\mathbf{w}_I| = 53 \cdot |\partial(\text{cm})(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array}} \\ = & \lambda x \quad \mathfrak{E} \mathbf{w}_I \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}_I) \\ \mathbf{w}_I \cdot \mathbf{p} < \mathbf{w}_I \cdot \mathbf{f} \\ \mathbf{w}_I \cdot \mathbf{g} = 0 \\ \mathbf{w}_I \cdot \mathbf{f} = \partial(x)(\text{HEIGHT}) \\ |\mathbf{w}_I| = 53 \cdot |\partial(\text{cm})(\rho(\mathbf{w}_I))| \\ \mathbf{w}_I \cdot \mathbf{p} = \mathbf{w}_I \cdot \mathbf{g} \end{array}} \end{aligned} \quad (4.22)$$

We can now consider the case where the MP modifies conjoined adjectives. We can combine the existentially disclosed translations of ‘tall’ and ‘wide’, of type $((v^3 \rightarrow \tau) \rightarrow (e \rightarrow \tau))$, in accordance with Definition 38 of (dynamic) generalised conjunction, to obtain the following interpretation of the phrase ‘tall and wide’, of type $((v^3 \rightarrow \tau) \rightarrow (e \rightarrow \tau))$:

$[[\emptyset^{\text{ABS}} \text{ tall}] \text{ and } [\emptyset^{\text{ABS}} \text{ wide}]] \Rightarrow$

$$\begin{array}{l}
 \lambda R \\
 \lambda x
 \end{array}
 \left[\left[\begin{array}{c}
 \mathfrak{E}w_1 \uparrow\uparrow \\
 \begin{array}{l}
 \text{HEIGHT}(w_1) \\
 w_1.p < w_1.f \\
 w_1.g = 0 \\
 w_1.f = \partial(x)(\text{HEIGHT})
 \end{array}
 ; R(w_1)
 \end{array} \right] ; \left[\begin{array}{c}
 \mathfrak{E}w_2 \uparrow\uparrow \\
 \begin{array}{l}
 \text{WIDTH}(w_2) \\
 w_2.p < w_2.f \\
 w_2.g = 0 \\
 w_2.f = \partial(x)(\text{WIDTH})
 \end{array}
 ; R(w_2)
 \end{array} \right] \right]
 \quad (4.23)$$

This expression takes a modifier R of type $(v^3 \rightarrow \tau)$ as argument and distributes it across both conjuncts. Combining this with the measure phrase gives us:

$[[[53 \text{ cm.}] [[\emptyset^{\text{ABS}} \text{ tall}] \text{ and } [\emptyset^{\text{ABS}} \text{ wide}]]]] \Rightarrow$

$$\lambda x
 \left[\left[\begin{array}{c}
 \mathfrak{E}w_1 \uparrow\uparrow \\
 \begin{array}{l}
 \text{HEIGHT}(w_1) \\
 w_1.p < w_1.f \\
 w_1.g = 0 \\
 w_1.f = \partial(x)(\text{HEIGHT}) \\
 |w_1| = 53 \cdot |\partial(\text{cm})(\rho(w_1))| \\
 w_1.p = w_1.g
 \end{array}
 \end{array} \right] ; \left[\begin{array}{c}
 \mathfrak{E}w_2 \uparrow\uparrow \\
 \begin{array}{l}
 \text{WIDTH}(w_2) \\
 w_2.p < w_2.f \\
 w_2.g = 0 \\
 w_2.f = \partial(x)(\text{WIDTH}) \\
 |w_2| = 53 \cdot |\partial(\text{cm})(\rho(w_2))| \\
 w_2.p = w_2.g
 \end{array}
 \end{array} \right] \right]
 \quad (4.24)$$

The result is an expression of type $(e \rightarrow \tau)$, which combines with an expression of type e to yield a context change potential. For example, letting r denote Kasimir Malevich's painting *Red Square*,¹⁵ we can describe its proportions thus:

¹⁵Kasimir Malevich (1915), *Red Square: Painterly Realism of a Peasant Woman in Two Dimensions*, oil on canvas; State Russian Museum, St. Petersburg.

[the Red Square [is [53 cm.] [[\emptyset^{ABS} tall] and [\emptyset^{ABS} wide]]]]

$$\begin{aligned}
&\Rightarrow \lambda x \quad \mathfrak{E}w_1 \uparrow\uparrow \left[\begin{array}{l} \text{HEIGHT}(w_1) \\ w_1.p < w_1.f \\ w_1.g = 0 \\ w_1.f = \partial(x)(\text{HEIGHT}) \\ |w_1| = 53 \cdot |\partial(\text{cm})(\rho(w_1))| \\ w_1.p = w_1.g \end{array} \right]; \mathfrak{E}w_2 \uparrow\uparrow \left[\begin{array}{l} \text{WIDTH}(w_2) \\ w_2.p < w_2.f \\ w_2.g = 0 \\ w_2.f = \partial(x)(\text{WIDTH}) \\ |w_2| = 53 \cdot |\partial(\text{cm})(\rho(w_2))| \\ w_2.p = w_2.g \end{array} \right] \quad (r) \\
&= \mathfrak{E}w_1 \uparrow\uparrow \left[\begin{array}{l} \text{HEIGHT}(w_1) \\ w_1.p < w_1.f \\ w_1.g = 0 \\ w_1.f = \partial(r)(\text{HEIGHT}) \\ |w_1| = 53 \cdot |\partial(\text{cm})(\rho(w_1))| \\ w_1.p = w_1.g \end{array} \right]; \mathfrak{E}w_2 \uparrow\uparrow \left[\begin{array}{l} \text{WIDTH}(w_2) \\ w_2.p < w_2.f \\ w_2.g = 0 \\ w_2.f = \partial(r)(\text{WIDTH}) \\ |w_2| = 53 \cdot |\partial(\text{cm})(\rho(w_2))| \\ w_2.p = w_2.g \end{array} \right] \quad (4.25)
\end{aligned}$$

As before, we unveil the truth-conditional content of this expression by applying the closure operation \downarrow to it, and reducing the expression in accordance with Definition 37. It is straightforward to check that this gives us the static expression in (4.26) below, which is an intuitively plausible representation of the truth conditions of the sentence *the Red Square is 53 cm. tall and wide*.

$$\begin{aligned}
&\exists w_1 \left[\begin{array}{l} \text{HEIGHT}(w_1) \\ w_1.p < w_1.f \\ w_1.g = 0 \\ w_1.f = \partial(r)(\text{HEIGHT}) \\ |w_1| = 53 \cdot |\partial(\text{cm})(\rho(w_1))| \\ w_1.p = w_1.g \end{array} \right] \\
&\exists w_2 \left[\begin{array}{l} \text{WIDTH}(w_2) \\ w_2.p < w_2.f \\ w_2.g = 0 \\ w_2.f = \partial(r)(\text{WIDTH}) \\ |w_2| = 53 \cdot |\partial(\text{cm})(\rho(w_2))| \\ w_2.p = w_2.g \end{array} \right] \quad (4.26)
\end{aligned}$$

4.6.2 Modifiers as Arguments

There are certain points worth noting in connection with the operation of Implicit Argument Disclosure (IAD), as set out in Definition 44 above: (i) IAD applies to expressions of any

category: it allows for *any* expression with a hidden parameter, x , to take modifiers of x as arguments; (ii) it does not require us to spell out in advance exactly what those modifiers will be (say, in the lexical definition), or the order of their occurrence; (iii) the disclosure operation is *iterable*, thus ensuring that there will be as many or as few modifier argument places as required; (iv) IAD is not restricted to any particular type of hidden parameter (these can be L-structures, vectors, degrees, events, etc.).

Thus in Fragment 2, MPs and *than*-phrases are interpreted as properties of L-structures, and assigned logical translations of type $(\nu^3 \rightarrow \tau)$, in keeping with the analysis of these expressions as modifying expressions. Yet IAD allows us to treat these modifiers as formal *arguments* of the (existentially disclosed) modified expression. Hence the approach to modification presented in this section blurs the line between modifiers and arguments.

Metaphorically speaking, we can describe this process as one in which an expression “grows” additional argument places to absorb modifiers of the appropriate type.

4.7 Linking L-Structures and Sentences

We have not yet considered the way in which L-Structures link up with sentences in cases where the predication applies to more than one subject, such as, for example, cases where the subject NP contains a quantifier or a conjunction, as in (97) below.

- (97) a. All footballers are rich.
b. Bob and Eve are tall.
c. All Bob’s relatives are tall.

These cases are important because there are cases where more than one standard of evaluation may enter into the interpretation of a sentence. In example (97a), one might argue that a single standard for wealth applies to the entire class of footballers, but (97c) and (97b) are more problematic; these examples both have readings where different standards of evaluation may apply to the subject of the predication. For example, there is a reading of (97b) on which different standards of tallness are associated with the different genders of Bob and Eve, making Bob tall for a man and Eve tall for a woman; in the case of (97c), there are potentially variations of not only gender but also age to consider.

Clearly, although (97c) and (97b) only contain a single token of the adjective ‘tall’, there must be more than one perspectival parameter available in order to accommodate the (potentially) multiple standards of tallness that might be relevant to the interpretation of the sentence.

We need a distinct L-Structure for each individual *act of predication*: in the case of the universal quantifier, there is an act of predication for each element in the quantifier’s domain, while in the case of the conjunction there is an act of predication for each conjoined element.

All the elements we require to handle these cases are already available in Dynamic Montague Grammar.¹⁶ First, we interpret ‘Bob’ and ‘Eve’ as higher-order functors $\lambda F.F(bob)$ and $\lambda F.F(eve)$, of type $((e \rightarrow \tau) \rightarrow \tau)$.¹⁷ In accordance with the rule for generalised conjunction, we then obtain the following translation for ‘Bob and Eve’:

$$[\text{Bob and Eve}] \Rightarrow \lambda P \boxed{P(bob) ; P(eve)} \quad (4.27)$$

Assuming the copula to be semantically transparent, we have the following translation for ‘are tall’ (see Section 4.5 above):

$$[\emptyset^{\text{ABS}} \text{ tall}] \Rightarrow \begin{matrix} \lambda x \\ \mathfrak{E} \mathbf{w} \end{matrix} \uparrow\uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.f = \partial(x)(\text{HEIGHT}) \end{array}} \quad (4.28)$$

¹⁶The DRT-based formulation we employed in Section 3.6.2 of Chapter 3 would need to be enriched in order to cope with quantification and conjunction, e.g., along the lines of Kamp and Reyle [83, Chapter 2]; the linking of L-Structures to sentences could then be accomplished by introducing the L-Structure using existential quantification, with anaphoric links in the body of the formula, in line with Kamp and Reyles semantics for pronouns (Alex Lascarides, personal communication).

¹⁷According to the canonical version of Dynamic Montague Grammar, due to Groenendijk and Stokhof [60], the higher-order translation of ‘Bob’ should be $\lambda F.F(\uparrow bob)$; however, for consistency with the foregoing discussion, we shall retain the simpler types we have been employing thus far, in line with Chierchia [25].

Applying the functor (in this case, the conjoined subject) to the translation of the predicate gives us the result in (4.29) below:

$$\begin{aligned}
 & [[\text{Bob and Eve}] [\text{are } \emptyset^{\text{ABS}} \text{ tall}]] \\
 \Rightarrow & \lambda P \boxed{P(\text{bob}) ; P(\text{eve})} \underset{\mathfrak{E}\mathbf{w}}{\lambda x} \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w.p} < \mathbf{w.f} \\ \mathbf{w.g} = 0 \\ \mathbf{w.f} = \partial(x)(\text{HEIGHT}) \end{array}} \\
 \Rightarrow & \boxed{\begin{array}{l} \underset{\mathfrak{E}\mathbf{w}}{\lambda x} \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w.p} < \mathbf{w.f} \\ \mathbf{w.g} = 0 \\ \mathbf{w.f} = \partial(x)(\text{HEIGHT}) \end{array}} \\ (bob) ; \underset{\mathfrak{E}\mathbf{w}}{\lambda x} \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w.p} < \mathbf{w.f} \\ \mathbf{w.g} = 0 \\ \mathbf{w.f} = \partial(x)(\text{HEIGHT}) \end{array}} \\ (eve) \end{array}} \quad (4.29) \\
 \Rightarrow & \boxed{\begin{array}{l} \underset{\mathfrak{E}\mathbf{w}}{\lambda x} \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w.p} < \mathbf{w.f} \\ \mathbf{w.g} = 0 \\ \mathbf{w.f} = \partial(bob)(\text{HEIGHT}) \end{array}} ; \underset{\mathfrak{E}\mathbf{w}}{\lambda x} \uparrow \uparrow \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w.p} < \mathbf{w.f} \\ \mathbf{w.g} = 0 \\ \mathbf{w.f} = \partial(eve)(\text{HEIGHT}) \end{array}} \end{array}}
 \end{aligned}$$

As we can see, the predicate is distributed across the conjunction, and so there exists a distinct L-Structure for each conjunct, each with its own perspectival parameter which can be anchored to a (potentially) different standard of evaluation. The closure operation, \Downarrow , then straightforwardly gives us the following truth conditions:

$$\begin{aligned}
 & [[\text{Bob and Eve}] [\text{are } \emptyset^{\text{ABS}} \text{ tall}]] \Rightarrow \\
 & \boxed{\begin{array}{l} \exists \mathbf{w} \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w.p} < \mathbf{w.f} \\ \mathbf{w.g} = 0 \\ \mathbf{w.f} = \partial(bob)(\text{HEIGHT}) \end{array}} \\ \exists \mathbf{w} \boxed{\begin{array}{l} \text{HEIGHT}(\mathbf{w}) \\ \mathbf{w.p} < \mathbf{w.f} \\ \mathbf{w.g} = 0 \\ \mathbf{w.f} = \partial(eve)(\text{HEIGHT}) \end{array}} \end{array}} \quad (4.30)
 \end{aligned}$$

We now consider the case of (97c), where a gradable predicate is attributed to a class of individuals via universal quantification. We assign the phrase ‘all Bob’s relatives’ the following translation (glossing over the compositional details of the possessive construction):

$$\begin{aligned}
 & [\text{all Bob's relatives}] \Rightarrow \underset{\mathfrak{Q}y}{\lambda F} \uparrow \uparrow \boxed{\text{RELATIVE-OF-BOB}(y)} ; F(y) \quad (4.31)
 \end{aligned}$$

Applying the translation of the quantifier to that of the predicate, we obtain the following:

$$\begin{aligned}
 & [[\text{all Bob's relatives}] [\text{are } \emptyset^{\text{ABS}} \text{ tall}]] \\
 \Rightarrow & \lambda F \left[\left[\left[\text{RELATIVE-OF-BOB}(y) \right] ; F(y) \right] \right] \lambda x \left[\left[\left[\text{HEIGHT}(\mathbf{w}) \right. \right. \right. \\
 & \left. \left. \left. \begin{array}{l} \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.f = \partial(x)(\text{HEIGHT}) \end{array} \right. \right. \right. \\
 \Rightarrow & \mathcal{Q}y \left[\left[\left[\left[\text{RELATIVE-OF-BOB}(y) \right] ; \lambda x \left[\left[\left[\text{HEIGHT}(\mathbf{w}) \right. \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \begin{array}{l} \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.f = \partial(x)(\text{HEIGHT}) \end{array} \right. \right. \right. \right. \left. \right. \right. (y) \quad (4.32) \\
 \Rightarrow & \mathcal{Q}y \left[\left[\left[\left[\text{RELATIVE-OF-BOB}(y) \right] ; \mathcal{E}\mathbf{w} \left[\left[\left[\text{HEIGHT}(\mathbf{w}) \right. \right. \right. \right. \right. \right. \\
 & \left. \left. \left. \begin{array}{l} \mathbf{w}.p < \mathbf{w}.f \\ \mathbf{w}.g = 0 \\ \mathbf{w}.f = \partial(y)(\text{HEIGHT}) \end{array} \right. \right. \right. \right. \left. \right. \right.
 \end{aligned}$$

This case involves both existential quantification over L-structures and universal quantification over individuals. Note that the order of the quantifiers is crucial in determining the correct interpretation: since the universal quantifier here takes wide scope over the existential, we get an interpretation in which there exists an L-Structure for each element of the domain, which may be anchored to a different standard in each case, as required.

4.8 Conclusion

One motivation for employing a dynamic semantics is to capture anaphoric dependencies involving L-structures, such as those mentioned in Section 2.4 at the end of Chapter 2, where we suggested that gradable adjectives exhibit similar types of deictic and anaphoric context dependency as pronominal expressions, and again in Section 3.6.2 of Chapter 3, where we showed how this could be modelled in the context of a dynamic theory of semantic meaning.

In this chapter, we addressed two important problems faced by LSS and similar theories, such as VSS, namely, the *Coordination Problem* and the *Divergent Type Problem* (cf. Chapter 1, Section 1.3.5). We showed that both problems disappear once the L-structure parameter is existentially closed; however, this comes at the cost of making the parameter unavailable for further modification. However, this restriction is not present in a dynamic theory such as Dynamic Binding (Chierchia [25]) or Dynamic Montague Grammar (Groenendijk and Stokhof [60]), where the objects introduced into the discourse context by existentially quantified parameters remain available for optional further specification.

The fundamental feature of dynamic semantics is that existentially quantified objects are not completely closed off to further specification. As Paul Dekker originally noted (Dekker [35], [36]), this means that implicit arguments can be ‘hidden’ by existentially quantifying over them in the lexicon, while remaining accessible to further specification by modifier phrases. This has two consequences. First, by existentially quantifying over the L-structure parameter, and thus hiding it, a gradable adjective can be interpreted as a (dynamic) property of individuals (i.e., a function from individuals to context change potentials), in exactly the same way as non-gradable adjectives and other predicates. The dynamic interpretation of the quantifiers and connectives in DB/DMG allows existential quantifiers to bind variables outside their syntactic scope, and, under certain circumstances, it also allows modifying expressions to make reference to existentially quantified variables that would be inaccessible in a ‘static’ semantics. Using Dekker’s notion of *existential disclosure*, we showed how a modifying expression could still access an existentially quantified L-structure parameter as if it were a free variable.

In Section 4.6, we discussed the extension of generalised conjunction to a dynamic setting, and noted that hiding the L-structure parameter, by itself, provides at best a *partial* resolution to the Coordination Problem. The recalcitrant cases involved constructions in which a single modifier applies to the conjoined adjectives, as in Example 95, where the MP ‘20 centimetres’ in the sentence ‘The painting is [20 centimetres [tall and wide]]’ modifies the conjunction ‘tall and wide’. This led us to adopt a more radical approach, based on a mechanism we called *Implicit Argument Disclosure* (IAD), on which certain modifiers are treated as arguments of the expressions they modify (cf. Definition 44). In Section 4.6.2, we observed that the IAD provides a systematic way of treating modifiers of hidden parameters as arguments, and noted that this undermines the distinction between arguments and modifiers.

In sum, we have shown that it is possible to construct a scalar theory of gradable adjectives in which gradable share the same (dynamic) type as non-gradable adjectives and other predicates. Moreover, we have shown that, using the same formal devices, it is possible to develop an L-structure based semantics for gradable adjectives that is compatible with a dynamic form of generalised Boolean coordination, thereby resolving the Coordination Problem inherited from VSS.

Chapter 5

Loose Ends and Conclusions

5.1 Introduction

We begin this chapter by addressing some foundational assumptions of the representational framework developed over the last few chapters. First, Section 5.2 examines the topic of vagueness, which is a characteristic of most if not all gradable adjectives. Section 5.3 then considers the informational structure of the dimensions associated with gradable adjectives, and in particular the assumption that adjectival dimensions should be based on ordered linear scales.

In Section 5.4, we move on to consider an extension of our theory to the case of adverbial modifiers. Although this still counts as work in progress, we show how several aspects of our theory can be straightforwardly adapted to an event-based framework, along the lines of Section 3.2.

Finally, section 5.5 concludes with a brief recapitulation of the main points of our thesis, and a brief statement of its contribution and possible further development.

5.2 Vagueness

5.2.1 Vagueness and Locatives

The general hypothesis that gradable adjectives are semantically akin to locatives encourages us to treat the vagueness of adjectives as a subtype of the vagueness of locatives. In itself, this view is agnostic among specific theories of vagueness; it merely requires us to handle adjectives and locatives in the same way—whatever that may be. Note that although this view may be noncommittal regarding which theory of vagueness is correct, it is still open to empirical refutation: for example, if adjectives turned out to display a very different kind of vague behaviour

from locatives, then this would count against treating the former as a species of the latter.¹

Prima facie, there appear to be some telling similarities between adjectives and locatives in this regard. As we pointed out in Section 2.5, spatial expressions, tenses, and gradable adjectives all appear to exhibit both indexicality and vagueness. For example, the spatial locatives ‘here’ and ‘there’, and the temporal locatives ‘now’ and ‘then’, are vague, in addition to being indexical: roughly, ‘here’ and ‘there’ refer to a contrast between a region that includes the speaker’s location, and another region that does not. Similarly for the temporal locatives ‘now’ and ‘then’. The extent of this region is highly variable, and is sensitive to context. But in addition to being contextually variable, the region is usually vague; that is, even when the contextual factors are fixed, the boundary of the region will still be ‘fuzzy’.

5.2.2 Vagueness versus Gradability

Gradable adjectives are a typically vague, and vagueness is an important topic in the semantics of adjectives; for example, Kamp develops a contextualised version of supervaluationism in [84], which has been influential in later work by Klein [95][96][97] and others.

Nevertheless, even though most (if not all) gradable predicates found “in the wild” are also vague, gradability and vagueness remain *conceptually* distinct notions: whereas gradability is characterised by the possession of a property to varying degrees, vagueness is (traditionally) characterised by the possession of borderline cases (see e.g. Keefe and Smith [89, ch. 1]), and the lack of well-defined extensions (cf. Sainsbury [138, pp. 251–253]). It is perfectly possible to conceive of a language which is vague but not gradable, and vice versa.

In addition to being gradable, the adjective ‘tall’ is vague because there are borderline cases which are neither clearly tall nor clearly non-tall, and for which no further empirical or analytical investigation will be able to establish whether they are tall or not. Of course, different standards of evaluation may apply to ‘tall’, according to contextual factors such as speaker intention and the kind of thing being described: e.g., Bob may consider a Rolex watch expensive, while to Bill it might seem affordable or even cheap; an adult male of height 180 cm. may be tall for a Vietnamese, for example, but may not be tall for a Dutchman; and so on. Crucially, however, this relativisation to context does nothing to eliminate the existence of borderline cases per se: there will still be Vietnamese men, shorter than 180 cm., who are borderline tall for a Vietnamese, and Dutchmen, taller than 180 cm., who are borderline tall for a Dutchman. No matter how detailed the specification, e.g., ‘expensive for a 18-carat vintage

¹Of course, we must also consider that examples of vagueness can be found across most if not all grammatical categories—e.g, see Keefe [87, ch. 1].

Swiss gold watch’, ‘tall for a fourteenth-century German adult female peasant’, etc., the use of a vague term will still generally present borderline cases. This ineliminability of borderline cases that is one of the hallmarks of true vagueness.

It is important to be clear about this distinction between gradability and vagueness, because there exist both degree-based theories of gradability and degree-based theories of vagueness, namely those which employ an infinite-valued, or ‘fuzzy’, logic—see Section 5.2.5.2 below; however, while the model of gradable adjectives we have presented is basically a type of degree-based theory, we do not advocate a degree-based treatment of vagueness. Instead, we will adopt a thoroughly epistemic view of vagueness phenomena.

5.2.3 Higher-Order Vagueness and Sharpness in Semantics

An striking feature of vagueness is that it is *recursive*, a phenomenon commonly referred to as ‘higher-order’ vagueness. Borderline cases typically have borderline cases, where it is unclear whether something counts as a borderline case (the existence of borderline borderline cases has been recognised by commentators at least since Russell [137]; e.g., see Keefe and Smith [89, pp. 14–17] for discussion).

Consider a hypothetical predicate F that has a sharply-delineated set of clear positive cases, a sharply-delineated set of clear negative cases, and a sharply-delineated set of borderline cases. Although F does have borderline cases, in the sense of there being instances which are neither clearly F nor clearly not- F , the predicate is not vague; in the same way as ‘tall’, ‘big’, ‘rich’, etc.; English predicates do not exhibit such a sharp division between positive, negative, and intermediate cases.

The fact that vagueness is itself vague has important consequences for our semantics, because the recursive nature of vagueness makes it awkward to speak of the ‘class’ or ‘set’ of borderline cases, for the simple reason that classes are *exact* objects. In classical logical semantics, a predicate has a meaning which determines its extension, the class of all things of which it is true. Classes (and therefore sets) are objects with sharp boundaries: for any class and any entity, either that entity belongs to the class, or it does not. A semantics based on exact objects, such as classical set theory, thus inescapably entails sharp boundaries (cf. Sainsbury [138, pp. 252–253]). So long as our semantics is couched in an exact metalanguage, then predicates will always be assigned exact extensions, and sentences will always be assigned exact truth values. This applies to all theories which attempt to model vagueness using exact methods, including supervaluationism and many-valued logics (cf. Section 5.2.5 below).

5.2.4 The Sorites Paradox

In addition to the existence of borderline cases and the lack of well-defined extensions, another important characteristic of vague predicates is that they give rise to sorites paradoxes (cf. Keefe and Smith [89, ch. 1]) such as the following:

(98) **Basis premise:** a collection of one billion grains of sand is a heap.

Induction hypothesis: if a collection of n grains of sand is a heap, then a collection of $n - 1$ grains of sand is a heap.

Conclusion: a collection of one grain of sand is a heap.

Starting with an uncontroversial premise, we arrive at the conclusion that even a single grain is a heap; conversely, we can take the negation of the conclusion as our starting point, as in (99), and show that no collection is a heap, no matter how many grains of sand it contains.

(99) **Basis premise:** a collection of one grain of sand is not a heap.

Induction hypothesis: if a collection of n grains of sand is not a heap, then a collection of $n + 1$ grains of sand is not a heap.

Conclusion: a collection of a billion grains of sand is not a heap.

Depending on our starting point, we end up with the result that either every collection of grains of sand is a heap, or none is. Clearly, something is amiss.

The classic sorites argument has the form of a mathematical induction, in which the first premise is obviously true, and the conclusion is obviously false. The induction hypothesis in both examples reflects a intuition that the addition or subtraction of a single grain of sand does not make a difference to whether a collection is a heap. It is important to note, however, that the intuitive reasonableness of the induction hypothesis is highly dependent on the size of the interval employed: the interval chosen must be small enough to be considered insignificant. The trick to setting up the paradox is to make the steps so small that one is not able to distinguish the last P because it is too close to the first non- P . For example, while an interval of 1 mm. is not enough to make a difference between being tall or not, as in example (100) below, an interval of 1 metre is too large for the appearance of paradox to arise, as in (101).

(100) **Basis premise:** a person of height 1 metre is not tall.

Induction hypothesis: if a person of height n metres is not tall, then a person of height $n + 0.001$ metres is not tall.

Conclusion: a person of height 2 metres is not tall.

(101) **Basis premise:** a person of height 1 metre is not tall.

Induction hypothesis: if a person of height n metres is not tall, then a person of height $n + 1$ metres is not tall.

Conclusion: a person of height 2 metres is not tall.

However, despite the difference in assertibility between the apparently reasonable induction hypothesis in (100) and the seemingly unreasonable induction hypothesis in (101), the former in fact logically entails the latter. Moreover, the point at which an interval is small enough to count as insignificant is itself indeterminate, and may vary according to the kind of entity under consideration. Thus, while the interval of 1 metre used in the induction hypothesis in (101) is clearly false with respect to people, it appears far more reasonable when ‘tall’ is applied to buildings, as in (102) below; an interval of 1 metre, though large relative to the height of a human being, is far less significant in the case of large modern buildings.

(102) **Basis premise:** a tower of height 1 metre is not tall.

Induction hypothesis: if a tower of height n metres is not tall, then a tower of height $n + 1$ metres is not tall.

Conclusion: a tower of height 1000 metres is not tall.

From the standpoint of classical logic, the falsity of the conclusion requires us to reject one of the two premises. The ‘paradox’ has a clear and simple solution: we simply reject the induction hypothesis. However, by rejecting the induction hypothesis we are thereby compelled to accept its negation, and therefore to acknowledge that there exist sharp boundaries for vague concepts: for example, there is some n such that n grains of sand form a heap, but $n - 1$ grains of sand do not; and there is some m such that a human being is tall at m centimetres, but no longer tall at $m - 1$ centimetres.

5.2.5 Theories of Vagueness

If we accept the classical solution, then many of our concepts have boundaries that we do not know, and perhaps *cannot* know. This view is known as *epistemicism*; representatives of contemporary epistemicism include Cargile [23], Campbell [21], Williamson [171][172], and Sorensen [148][149], among others (although the view itself arguably goes back as far as the Stoic logician Chrysippus; see Williamson [172, ch. 1] for discussion). On an epistemic view, vagueness is a matter of ignorance: for every gradable predicate F , there exists a precise threshold between being F and being not- F , even if we are ignorant of where it lies.

The reluctance to accept the classical solution to the sorites paradox has given rise to several alternative accounts of vagueness. These typically deviate from classical logic in some way, either in terms of their semantics or in terms of the theorems and rules of inference of the logic itself—and sometimes both (see Keefe and Smith [89, ch. 1] for a survey of the dominant contemporary approaches).

We can divide theories of vagueness into two broad groups, according to the methods they employ. The first group of theories views vagueness as in some sense irreducible; since almost all concepts lack boundaries, they regard classical logic and set theory as simply of very little use as far as natural languages are concerned. Michael Tye [160] and Mark Sainsbury [138], for example, have argued for an alternative semantics that eschews classical set theory completely in favour of one based on truly vague entities. Another possibility is to the use of a semantic metalanguage that is itself fundamentally vague, such as English or another natural language. ‘Ontic’ approaches regard vagueness primarily as a matter of the world, rather than one of logic or language; for example, many commonplace objects, such as clouds, lakes and mountains, appear to have fuzzy spatial boundaries.²

The second group, despite rejecting the standard logical account, nevertheless attempts to explain vagueness using exact methods based on logic and mathematics. Prominent among these are supervaluationism and many-valued logic; while both of these approaches depart from classical semantics and logic in some way, they attempt to preserve as much of classical logic as possible.

5.2.5.1 Supervaluationism

Consider a proposition p containing a vague predicate F . According to the supervaluationist, borderline cases give rise to truth-value gaps, where p is neither true nor false. However, borderline cases are also potentially instances of F , as well as not- F , and there are situations under which borderline cases could become clear cases.

Supervaluationism models the potential meaning of vague expressions using the notion of a *precisification*: while holding the clear cases of F constant, the borderline cases are varied. The set of all precisifications can be thought of as capturing the range of potential meanings of the vague predicate F . For each precisification of F , we can assign p a standard (total rather than partial) classical valuation, and doing this for every precisifications produces a set of

²Parsons [125], Tye [160], and Zemach [177] argue in support of the existence of vague objects, while Evans [43] presents an influential contrary view. One potential problem with recognising vague objects is that they can easily give rise to sorites paradoxes of their own—cf. Unger [161][162] and Geach [56].

classical valuations. A ‘supervaluation’ is then simply a function of the proposition’s classical valuations.

A supervaluationist takes the truth of a proposition to be its ‘supervaluation’: the proposition is True (or ‘super-true’) if it is classically true under every valuation, and False (or ‘super-false’) if it is classically false under every valuation. The remaining cases are those in which the proposition lacks a defined super-truth value, because it comes out true under some precisifications but not others; these correspond to the borderline cases, and these propositions are neither True nor False.

In virtue of the analysis of truth as super-truth, a compound statement may have a truth value even in cases where its component statements do not (cf., Lewis [111], Fine [51], Kamp [84], Keefe [87]). Thus, a disjunction of the form $p \cup \neg p$, for example, ‘Either Bill is tall or it is not the case that Bill is tall’, is True because it is super-true; i.e., it comes out true under all ways of making ‘tall’ precise.

In this way, supervaluationism promises to do justice to our intuitions regarding statements containing vague predicates, while preserving the theorems of classical logic (see Keefe [87, p. 17]). However, while supervaluationism may converge with classical logic with regard to theoremhood, it certainly does not preserve classical rules of inference, as it rejects contraposition, conditional proof, and proof by contradiction (e.g., see Williamson [172, pp. 151–152] and Machina [112]).

Ruling out the empty extension for ‘tall’, then the statement “There is a person who is tall when n centimetres tall, but not when $n - 1$ centimetres tall” is true under all admissible precisifications of ‘tall’, and therefore True, according to the supervaluationist.

5.2.5.2 Many-Valued Theories

Many-valued theories admit a range of truth values in addition to ‘True’ and ‘False’; the number of values can be finite (typically three or four), or infinite, in which case it is common to use the real numbers between 0 (False) and 1 (True). In contrast to supervaluationism, which takes borderline statements to be truth-valueless, many-valued approaches assign borderline statements some intermediate truth-value between truth and falsehood.

A central duty of many-valued and ‘fuzzy’ logics, as in any logical system, is to provide a means of calculating the truth values of complex propositions from their component parts. The rules in such theories must cover cases where the constituents may have intermediate truth-values. One desideratum that many-valued logicians accept in formulating these rules is that they should yield all the standard theorems of classical logic when the truth values have their

limiting values of 0 and 1; in other words, they see classical logic as a limiting case of many-valued logic. For example, it is common to adopt rules for conjunction and disjunction which assign to a conjunction the truth-value of the conjunct with the lowest truth-value, and assigns to a disjunction the truth-value of the disjunct with the highest truth-value.

However, although some theorems are preserved in the presence of intermediate values, many of the theorems of classical logic break down; for example, supposing that the sentence ‘Bob is tall’ has a truth value of 0.5, then even a simple contradiction such as “Bob is tall and it is not the case that Bob is tall” would be assigned a truth-value of 0.5. Similarly for the simple tautology “Bob is tall or it is not the case that Bob is tall” (see Haack [62, ch. 11]).

The many-valued rule for disjunction also causes problems in dealing with the phenomenon of hedging. If Alice regards Bob as a borderline case of tall man, then she cannot simply assert that he is tall, nor can she simply assert that he is of average height; but she could in full confidence make a hedged claim such as “Bob is tall or of average height”. Crucially, however, according to the standard definition of disjunction in a many-valued logic, the additional disjuncts in a hedged claim cannot increase the degree of truth, unless one of them is more plausible than the others. Thus, a many-valued approach is unable to explain the increased assertibility of hedged claims in terms of an increase in their degree of truth.

Epistemicist and supervaluationist approaches enjoy a clear advantage in this regard. For the epistemicist, the greater assertibility of hedged statements is due to their greater probability of truth, as opposed to a greater degree of truth; since additional disjuncts can raise probability indefinitely (up to full certainty), epistemicism correctly predicts that additional hedges increase assertibility. On a supervaluationist account, additional disjuncts lead to truth in a greater number of precisifications, so the supervaluationist can explain the effect of hedging on assertibility by quantifying over precisifications.

We observed earlier (Section 5.2.2 above) that gradability and vagueness are conceptually distinct notions. It is particularly important to bear this distinction in mind when considering degree-based theories of gradability, because one might be tempted to think that the truth of a predication attributing a property to an entity may vary with the degree to which the entity possesses the property in question. That is, one might think that, because a is more F than b , it follows that the statement ‘ A is F ’ is more true than ‘ B is F ’; for example, given that Bob is taller than Bill, one might reflect this in a many-valued logic by assigning the statement ‘Bob is tall’ a higher degree of truth than to ‘Bill is tall’. However, as Klein [95] and others have observed, we should resist making this connection between gradability and degrees of truth: for suppose that Bob is 2 metres tall and Bill is 1.9 metres tall; then, although Bob is taller than Bill, they are both unquestionably tall by normal standards. The statements ‘Bob is

tall' and 'Bill is tall' both count as fully true.

5.2.6 Epistemicism

Since the semantic framework we employ is based on the use of exact methods, our model falls squarely in the camp of the 'exact' theories. We do not think that vagueness justifies the abandonment of standard set theory and other exact methods in natural language semantics, nor are we convinced that vagueness requires us to deviate from classical logic and semantics, pace the proponents of many-valued logic and supervaluationism.

It is hard to find a simpler and more straightforward argument than the classical solution to the sorites paradox; so much so, that it is reasonable to regard it as an *existence proof* for sharp boundaries. There is nothing fundamentally 'paradoxical' about the so-called sorites paradox, beyond the counterintuitive nature of its solution; the rejection of the classical solution is not on the grounds of incorrectness, but of *incredulity*.

The main source of incredulity is the claim that vague predicates have sharp boundaries, even if we do not know where they are. This acceptance of this counterintuitive claim is not limited to epistemicism, however: it is a characteristic of *all* theories that employ an exact semantics, and is therefore part and parcel of supervaluationism and many-valued approaches as well. For example, the statement "A man of height n centimetres is tall but a man of height $n - 1$ centimetres is not" is true under all admissible precisifications of 'tall', and therefore True for the supervaluationist. Indeed, the introduction of truth-value gaps or multiple truth-values arguably makes the problem of sharp boundaries worse: for example, instead of a single sharp boundary (between the cases that satisfy the vague predicate and those which do not), supervaluationism introduces *two* sharp boundaries (one between the clearly true cases and the borderline cases, another between the borderline cases and the clearly false cases), while a many-valued logic may create any number of sharp boundaries greater than two. In the case of a fuzzy logic, the number of sharp boundaries is infinite; any real number on the interval from 0 to 1, say 0.37337, creates a sharp division between those propositions with a lower truth value and those with an equal or higher truth-value.

The fundamental choice, therefore, is whether to retain a semantics based on the exact notions of logic and set theory in the face of our metalinguistic intuitions. Among exact theories, epistemicism has the benefit of simplicity, clarity, and preserving classical logic in its entirety (cf. Williamson [173, p. 279]). It has the virtue of handling the phenomenon of higher-order vagueness in exactly the same way as it handles first-order vagueness, as a matter of semantic ignorance (cf. Section 5.2.3 above): the borderline instances of a predicate F arise from our ig-

norance of the exact boundary of F , and the borderline status of the borderline cases themselves can be attributed to uncertainty about our own ignorance.

5.3 Scales and Vectors

5.3.1 Vector vs. Numbers

In order to be able to investigate possible parallels between spatial, temporal, and adjectival expressions, we need a common formal framework in which to represent the properties of these different domains. There is a substantial and growing body of research into spatial models of cognition and language, and vectors are particularly suited to modelling locations and geometrical relations, especially *distance* and *relative location*, along a single or multiple dimensions.

In the case of adjectives, vectors and directed line segments (*anchored* vectors) can be used to represent specific locations in a conceptual space comprised of a number of property dimensions (HEIGHT, WEIGHT, etc.), as well as relations among these locations. The two fundamental properties of vectors which are relevant to modelling the behaviour of gradable adjectives are those of *magnitude* and *orientation*: we use magnitude to represent the degree of possession of a gradable property, and orientation to model the contrast or opposition between antonymic pairs along the same property dimension (e.g., ‘tall’/‘short’ in the case of HEIGHT).

Why use a vector to represent extent along a dimension, and not simply a (real) number? One important reason is that measurement is typically *relational*; it is expressed as the ratio of some unit of measurement. So Bob’s height may be expressed as 6 feet, or 72 inches, or 1.8288 metres, or 1828.8 millimetres, and so on (alternative units include furlongs, fathoms, miles, nautical miles, light years, etc.). There is no unique number n that corresponds to Bob’s height; indeed, the number will vary according to the unit of measurement employed, and we could associate Bob’s height with *any* number, given a suitable choice of measure. We could, of course, build a unit of measurement into the scale itself, in which case we would be able to represent height by a bare numerical value, and specify some n as *the* representation of Bob’s height. However, we do not think that it is appropriate to incorporate a unit of measurement into semantic scales, unless we want to claim that speaking English requires us to think in, say, feet rather than metres.

5.3.1.1 Dimensions and Scale Structure

In our semantics, we have assumed that each adjective is assigned its own dimension, whose scale structure is based on the real number line. Thus, gradable adjectives, as ‘nice’ and ‘big’,

are associated with their own scale, just as ‘height’ is. This assumption is uncontroversial in the case of adjectives such as, e.g., ‘tall’, ‘deep’, ‘heavy’, ‘big’, or ‘rich’, which are associated with a clearly identifiable, measurable dimensions; however, it may appear questionable to do so for all predicates.

There are many predicates that do not appear to be associated with a single, measurable property, such as ‘nice’, ‘beautiful’, ‘clever’, ‘good’, and so on. In many of these cases, it may be difficult to state a total ordering between elements, and it is often proposed that scales should be *partially* rather than totally ordered by the relation \leq , while preserving monotonicity with respect to this ordering: that is, if an individual is nice (or clever, or beautiful) to some extent d , then that individual is nice (or clever, or beautiful) to any degree d' where $d' \leq d$. This ensures, for example, that if Bob is tall and Bill is not tall (relative to the same standard of tallness), then Bill must be taller than Bob (see Klein [97, p. 684]).

In the case of adjectives such as ‘nice’ and ‘big’, there appears to be a connection with more than one property dimension; thus ‘big’ appears to be correlated with greater height and volume, and ‘nice’ suggests an indeterminate range of pleasing attributes; in a person, these might include politeness, generosity, consideration for others in the case a person, and the like. These and similar predicates are thus sometimes described as *multidimensional* predicates (e.g., Keefe and Smith [88, p. 5]). One might be tempted to pursue an analyse the meaning of ‘big’ or ‘nice’ as (weighted) functions of multiple dimensions, but this is not a precondition for studying many of the semantic properties of ‘nice’, including the informational structure of its associated scale.

5.3.2 The Informational Properties of Scales

It is common in the behavioural sciences to categorise scales in terms of their *informativeness*. The following four major types of scale are usually distinguished, in order of increasing informativeness: the categorical, ordinal, interval, and ratio scales.³ Each scale higher in the informational hierarchy subsumes the properties of those below it. Differences in ordinal data allow us to infer which of two statements has greater intensity, but nothing more—in particular, a difference in ordinal scale values does not carry any information about the actual *size* of the interval; thus, an ordinal scale might tell us that the phrase *extremely tall* has a higher degree than *very tall*, and that the latter in turn expresses a higher degree than *quite tall*, for example, but we would not be able to say anything about the relative size of the difference. To capture this kind of information, we need an interval scale, which allows us to determine which of two

³For a detailed explanation of the different measurement scales, see Stevens [152].

differences in sentiment intensity is the larger one. If we fix an appropriate zero point on the measurement scale, then we obtain a ratio scale, which allows us to determine the *ratio* between degrees; that a given degree is twice as large as another, for example.

The reason for assigning a dimension to each predicate is to model aspects of its semantic behaviour, such as contrasts between antonyms (e.g., ‘big’ vs. ‘small’), and constructions which express ordering, measurement, or judgements of differences or ratios. The structure of the scale determines the type of information that can be expressed. The issue we face is determining what level of informativeness is appropriate for the scales associated with gradable adjectives. For example, constructions which express *differences* and *ratios* will require interval and ratio scales, since these are the only ones with the appropriate structure to support such judgements. All else being equal, the richer the phenomena, the higher the level of informativeness of the scale associated with the adjective.

Let us consider the case of the adjective ‘nice’. There is no ready unit of measurement for niceness, and we may often be at a loss to assign a total ordering for niceness. However, there are cases where an ordering can be assigned, and where it is also possible to express judgements regarding the *size* of the interval as well, as in (104) below.

- (103) a. Bob is taller than Bill.
b. Bob is a bit taller than Bill.
c. Bob is a lot taller than Bill.
d. Bob is taller than Bill by far.
- (104) a. Bob is nicer than Bill.
b. Bob is a bit nicer than Bill.
c. Bob is a lot nicer than Bill.
d. Bob is nicer than Bill by far.

In informational terms, the judgements in (104), like (103), are richer than those associated with an ordinal scale, and correspond to an interval scale.

Furthermore, it is possible to express ratio judgements in connection with gradable adjectives such as ‘nice’, ‘smart’, ‘beautiful’, ‘big’, etc., as in (105) below:

- (105) a. Bob isn’t even half as smart as Bill.
b. Bill is twice as smart as Bob.
c. Alice is ten times more beautiful than Eve.
d. Lucy is three times nicer than her brother.

Given that the sentences in (105) are acceptable, then we need to assign them a semantic interpretation. These statements express a ratio comparison, which in informational terms requires not just an interval scale, but a ratio scale.

One might object that the ratio comparisons in (105) are not usually intended to be taken literally but figuratively; that is, they are instances of hyperbole, akin to “France is hexagonal”, and the numerical ratio is employed pragmatically to convey a sense of the extent of the difference between the things being compared. But even if this were the case, that would not absolve us from giving these sentences a literal interpretation—even if what they communicate turned out to be quite different from what they say. “France is hexagonal” has a clear literal meaning, even if it is (strictly speaking) false. Taken at face value, these sentences express ratio judgements, and one way to model the semantics thereof is in terms of a ratio scale.

Of course, there may still be alternatives to a real number scale; it might be possible to use a scale based on the rational numbers rather than the reals to handle the cases in (105), for example. Ultimately, the choice between the rationals and the reals will depend on issues such as whether adjectival dimensions are best thought of as discrete or continuous.

We will not pursue the matter any further here. We hope to have shown that the use of a real number scale is not an unreasonable choice for the representation of adjectival dimensions.⁴

5.4 Adverbs and Gradability

5.4.1 From Adjectives to Adverbs

Our proposal has focused on gradable adjectives; however, there is scope to extend the analysis to other types of expression. One possibility is to handle the phenomena of adverbial gradability in the same way as adjectival gradability.

There are strong connections between adjectives and adverbs, for example. Many adjectives have related adverbial forms, for example, ‘quick’ / ‘quickly’, ‘slow’ / ‘slowly’, ‘former’ / ‘formerly’, etc. Also, many adjectives can apply to both individuals and events, especially when they occur as adjunct predicates in the VP, as in (106) below.

- (106) a. Englebert arrived *late*.
b. Bob surrendered *quick*.

⁴Should it be necessary to adopt a more general alternative to the reals, we could always modify our model by employing *modules* instead of vector spaces, where a module is a generalisation of a vector space which is defined over rings instead of fields.

- c. Alice drove away *fast*.

Many adjectives have adverbial readings, including ‘former’, ‘sloppy’, and ‘beautiful’—indeed, ‘former’ only has an adverbial reading, as can be seen from (107) below.

- (107) a. Englebert is a *former* spy.
b. * Englebert is *former*, and a spy.
c. Englebert was *formerly* a spy.
- (108) a. Alice is a *sloppy* writer.
b. Alice is *sloppy*, and a writer.
c. Alice writes *sloppily*.
- (109) a. Alice is a *beautiful* dancer.
b. Alice is *beautiful*, and a dancer.
c. Alice dances *beautifully*.

Crucially, many adverbs appear to exhibit some of the same gradability phenomena that adjectives display. We find antonymic pairs associated with the same property, e.g., ‘quickly’ / ‘slowly’, ‘carefully’ / ‘carelessly’, and so on. Adverbs can occur in a similar range of comparative constructions, as shown in (110), (111) and (112).

- (110) a. Englebert is *quicker* than Bob.
b. Englebert ran *more quickly* than Bob.
c. Bob ran *less quickly* than Englebert.
- (111) a. Alice is *sloppier* than Eve.
b. Alice eats *more sloppily* than Eve.
c. Eve eats *less sloppily* than Alice.
- (112) a. Bob is *more fluent* than Bill.
b. Bob speaks *more fluently* than Bill.
c. Bill speaks *less fluently* than Bob.

5.4.2 Extending LSS to Adverbs

Contemporary theories of event semantics, notably those influenced by Donald Davidson [33] such as Castañeda [24], Parsons [126], Vlach [167] and Higginbotham [71][72], among others, analyse adverbial modification in terms of an underlying predication on events.

Recall that, in Section 3.2 of Chapter 3, we defined a projection function, ∂ , which takes as argument an individual a , and yields a function $\partial(a)$ from property dimensions d to position vectors in d . The subsequent application of the function $\partial(a)$ to a particular dimension d' then returns a particular position vector in d' , the projection of a onto d' .

We can extend the projection function, ∂ , straightforwardly to take events as arguments, in addition to individuals. We assume a set \mathcal{E} of events, and extend the domain of ∂ to take events as arguments as well as individuals: thus, given event $e \in \mathcal{E}$, $\partial(e)$ denotes a function from property dimensions $d \in \mathcal{D}$ to position vectors in d , and the expression $\partial(e)(d')$ therefore denotes a particular position vector in a dimension d' , which we call the *projection of e onto d'* .

We can employ the dynamic machinery of Chapter 4 to construct a thematic-role based version of event semantics, along the lines of Parsons [126], in which the sentence “Brutus killed Caesar” is assigned the semantic interpretation in (5.1) below (ignoring tense):

$$\text{Brutus killed Caesar} \Rightarrow \mathfrak{C}e_I \uparrow\uparrow \boxed{\begin{array}{l} \text{KILLING}(\mathbf{e}_I) \\ \mathbf{e}_I.\text{agent} = b \\ \mathbf{e}_I.\text{theme} = c \end{array}} \quad (5.1)$$

We will first run through a simple example of adverbial modification treating adverbs as properties of events. The example in (5.2) below shows a simple event-based interpretation for ‘quickly’:

$$\text{quickly} \Rightarrow \lambda e \uparrow\uparrow \boxed{\text{QUICK}(\mathbf{e})} \quad (5.2)$$

Since the event parameter in (5.1) is bound by the dynamic existential quantifier, the IAD disclosure operation (Chapter 4, Definition 44) provides us with the following additional translation for the sentence:

$$\text{Brutus killed Caesar} \Rightarrow \lambda E \boxed{\mathfrak{C}e_I \uparrow\uparrow \boxed{\begin{array}{l} \text{KILLING}(\mathbf{e}_I) \\ \mathbf{e}_I.\text{agent} = b \\ \mathbf{e}_I.\text{theme} = c \end{array}} ; E(\mathbf{e}_I)} \quad (5.3)$$

Combining the translation of ‘quickly’ with that for “Brutus killed Caesar” gives us the follow-

ing interpretation for *Brutus killed Caesar quickly*.

$$\begin{aligned}
\text{Brutus killed Caesar quickly} &\Rightarrow \lambda E \quad \boxed{\mathfrak{C}e_I \uparrow\uparrow \begin{array}{l} \text{KILLING}(e_I) \\ e_I.\text{agent} = b \\ e_I.\text{theme} = c \end{array}; E(e_I)} \quad \lambda e \uparrow\uparrow \boxed{\text{QUICK}(e)} \\
&= \mathfrak{C}e_I \uparrow\uparrow \boxed{\begin{array}{l} \text{KILLING}(e_I) \\ e_I.\text{agent} = b \\ e_I.\text{theme} = c \end{array}}; \lambda e \uparrow\uparrow \boxed{\text{QUICK}(e)}(e_I) \\
&= \mathfrak{C}e_I \uparrow\uparrow \boxed{\begin{array}{l} \text{KILLING}(e_I) \\ e_I.\text{agent} = b \\ e_I.\text{theme} = c \end{array}}; \uparrow\uparrow \boxed{\text{QUICK}(e_I)} \\
&= \mathfrak{C}e_I \uparrow\uparrow \boxed{\begin{array}{l} \text{KILLING}(e_I) \\ e_I.\text{agent} = b \\ e_I.\text{theme} = c \\ \text{QUICK}(e_I) \end{array}}
\end{aligned} \tag{5.4}$$

Finally, by the application of the closure operator \Downarrow , we obtain (5.5) as the representation of the truth conditions of (5.4).

$$\exists e_I \quad \boxed{\begin{array}{l} \text{KILLING}(e_I) \\ e_I.\text{agent} = b \\ e_I.\text{theme} = c \\ \text{QUICK}(e_I) \end{array}} \tag{5.5}$$

In order to incorporate gradability into the analysis, we simply replace the definition of ‘quickly’ in (5.2) by the following:

Definition 45 (quickly)

$$\text{quickly} \Rightarrow \lambda e \quad \boxed{\mathfrak{C}w \uparrow\uparrow \begin{array}{l} \text{SPEED}(w) \\ w.p < w.f \\ w.f = \partial(e)(\text{SPEED}) \\ w.g = 0 \end{array}}$$

The same derivation process as in (5.4) then gives us the following dynamic interpretation for “Brutus killed Caesar quickly”:

$$\text{Brutus killed Caesar quickly} \Rightarrow \mathfrak{C}e_I \uparrow\uparrow \boxed{\begin{array}{l} \text{KILLING}(e_I) \\ e_I.\text{agent} = b \\ e_I.\text{theme} = c \\ \text{SPEED}(w) \\ w.p < w.f \\ w.f = \partial(e_I)(\text{SPEED}) \\ w.g = 0 \end{array}} \tag{5.6}$$

Finally, we obtain the truth conditions for the sentence by means of the closure operator \Downarrow :

$$\begin{array}{l} \exists \mathbf{e}_I \\ \exists \mathbf{w} \end{array} \boxed{\begin{array}{l} \text{KILLING}(\mathbf{e}_I) \\ \mathbf{e}_I.\text{agent} = b \\ \mathbf{e}_I.\text{theme} = c \\ \text{SPEED}(\mathbf{w}) \\ \mathbf{w}.\text{p} < \mathbf{w}.\text{f} \\ \mathbf{w}.\text{f} = \partial(\mathbf{e}_I)(\text{SPEED}) \\ \mathbf{w}.\text{g} = 0 \end{array}} \quad (5.7)$$

Although we are still exploring the extension of LSS to the analysis of adverbial gradability, examples such as this suggest that many aspects of our approach can be applied straightforwardly to gradable adverbs.

5.5 Conclusion

We regard the present dissertation as a contribution to a broader research programme concerned with the investigation of the role that spatial concepts play in structuring multiple semantic domains. Although there are well-known parallels between the semantics of spatial and temporal expressions, and theories such as VSS have investigated the connection between spatial prepositions and gradable adjectives, we do not know of other work that has extended the parallel to include tenses, let alone taken the ternary structure of Reichenbachian tenses as a model of locative concepts.

The guiding hypothesis of this dissertation is that gradable adjectives share a common underlying semantic structure with spatial and temporal locative expressions. Accordingly, we have developed a formal semantics for gradable adjectives within a broader semantic framework for locative structures we have called *Locative Structure Semantics* (LSS), which employs an abstract, cross-categorial semantic structure which we refer to as a *locative structure* (L-structure).

Locative Structure Semantics (LSS) is a type of scalar, or degree-based, theory of gradable adjectives (Section 1.2.5); in many respects, it can be regarded as an extension of Vector Space Semantics (VSS) (Section 1.3), with which it shares a basic semantic ontology and the same preoccupation with structural parallels across semantic domains. Technically, LSS departs from VSS in the use of ternary L-structures instead of located vectors, a feature inspired by the Reichenbachian treatment of tense (Section 2.3). LSS also departs from VSS in according a central role to the notion of a *perspective*, as found in the spatial, temporal, and conceptual domains (cf. Chapter 2, Section 2.2), and the incorporation of notions from Reichenbach's

theory of tense. A further difference is that LSS, in its dynamic version, attributes the same (dynamic) type to both gradable and non-gradable adjectives (cf. Chapter 1, Section 1.2.5).

Like VSS, LSS employs vectors in its semantic ontology. The use of vectors and vector spaces is motivated by the need for an abstract structure in which to represent the properties of locatives from different domains. There is a substantial and still growing body of research into spatial models of cognition and language; vectors are eminently suitable to the representation of configurational relations, especially *distance* and *relative location*, along a single or multiple dimensions. This is pertinent to the semantics of gradable adjectives, where we use vectors to represent locations in a conceptual space comprised of a number of property dimensions (HEIGHT, WEIGHT, etc.), as well as the relative position of locations along those dimensions. Two fundamental vectorial properties that are especially pertinent to the semantics of all locatives are *magnitude* and *orientation*; in the case of gradable adjectives, magnitude is used to model the degree of possession of a gradable property, while orientation is used to model the contrast or opposition between antonymic pairs along the same property dimension (e.g., ‘tall’/‘short’ in the case of HEIGHT). In this regard, the ‘directed degrees’ proposed by Bierwisch [14] can be seen as vectorial in nature.

The notion of an L-structure is a generalisation of a Reichenbachian tense to the level of concepts in general: where Reichenbachian tenses are associated with a *speech point*, *S*, an *event point*, *E*, and a *reference point*, *R*, locative structures are associated with a *Perspective*, *P*, a *Figure*, *F*, and a *Ground*, *G*. Indeed, Reichenbach’s *S*, *E*, *R* points may be regarded as temporal instantiations of the more general categories *P*, *F*, and *G*; in particular, the speech point *S* is a temporal Perspective point, the event point *E* is a temporal Figure, and the reference point *R* is a temporal Ground.

While the terminology of Perspective, Figure, and Ground is drawn from Talmy’s work on the semantics of spatial and locative expressions (Talmy [153]), the importance of the notion of *perspective* or *point of view* in semantic theory has been recognised in other semantic theories, and constitutes a central tenet of the theory of Situation Semantics (see Barwise and Perry [9, p. 39]). In Chapter 2, we presented the Perspectival parameter, *P*, as the primary locus of contextual variability in the semantics of gradable predicates, a notion which we borrowed from Barwise and Seligman [11]. We have observed several structural parallels between gradable adjectives, pronouns and tenses, of the sort originally observed by Partee [127] between tenses and pronouns, and argued that adjectival gradability should be treated as a form of pronominal contextual assignment, in which the *P* (and sometimes *G*) components of an L-structure take their value by deictic or anaphoric means. Again, the treatment of adjectival anaphora as akin to tense and pronominal anaphora is original, to the best of our knowledge.

In Chapter 3, we presented a formal semantics for gradable adjectives centred around the notion of an L-structure which we introduced in Chapter 2. We defined an L-structure formally as a ternary structure whose components are vectors, which in the case of adjectival L-structures belong to dimensions in a conceptual space (cf. Section 3.2). We also defined notions of orientation and magnitude for L-structures in terms of the relations between their components (Sections 3.2.1 and 3.2.2), and presented a small formal fragment for adjectival constructions (Section 3.3). We distinguished the semantic properties of gradable adjectives in terms of configurations of P , F , and G points on a dimensional scale, in a manner reminiscent of the Reichenbach treatment of tenses in terms of configurations of S , E , and R points on the timeline, and proposed an *adjectival measurability condition* (AMC) in order to account for the phenomena associated with MP modification (Section 3.4.2). We showed how to incorporate the AMC into the semantic composition mechanism, as part of the lexical semantic definition of the measure phrase. In Section 3.6 we provided a brief treatment of intersentential adjectival dependencies using notions of dynamic semantics.

Certain aspects of the *adjectival measurability condition* (AMC), introduced in Chapter 3, stand in need of further investigation. Although the AMC, as stated in Definition 22, has certain properties that make it appealing, including the fact that (i) it is extremely simple, (ii) it applies uniformly to both comparatives and non-comparatives, and (iii) it can be incorporated straightforwardly into the compositional semantic definition of the language, it is equally true that the AMC is too restrictive when considered as a cross-categorical characterisation of measure phrase behaviour. At best, the AMC might be adequate as a category-specific constraint on adjectives, but not as a global constraint on L-structures in general; while the semantic compositional machinery can be modified to accommodate this fact (cf. Section 3.5), a global constraint on measurability, applicable to all L-structures, would be preferable. One candidate for such a constraint is the modification condition (MC) of VSS discussed in Section 1.3.4 of Chapter 1. While it would be relatively straightforward to import the MC into LSS, given that the latter theory is basically an extension of the former, we are also currently investigating the possibility that the AMC might be a special case of the MC, the result of the interaction of the MC with the particular geometric properties of the adjectival domain. We are also investigating ways of making the AMC less restrictive, in order to allow for the greater range of possible P , F , and G configurations in the spatial and temporal domains.

We have resolved certain technical issues that have proved highly problematic for VSS and potentially other degree-based theories, notably the problems we described in Section 1.3.5 of Chapter 1, which we referred to as the ‘Divergent Type Problem’ and the ‘Coordination Problem’: the first concerns the type incompatibility between gradable adjectives on the one

hand, and non-gradable adjectives and all other predicates on the other, while the second is the problem intersective constructions present to a vector-based theory. In Chapter 4, we exploited Paul Dekker's [35, 36] insight that in a dynamic model of meaning, existentially quantified objects are not completely closed off to further specification, and so implicit arguments can be 'hidden' by existentially quantifying over them in the lexicon, while remaining accessible to further specification by modifier phrases. Accordingly, we defined a dynamic version of LSS using the theory of Dynamic Binding (Chierchia [25]), a variant of Groenendijk and Stokhof's Dynamic Montague Grammar, and made use of Dekker's notion of *existential disclosure* to address the existentially quantified L-structure parameter as if it were a free variable. The result is a theory in which gradable adjectives can be interpreted as (dynamic) properties of individuals (i.e., functions from individuals to context change potentials), and therefore share the same (dynamic) type as gradable and non-gradable adjectives.

In Section 4.6, we discussed the extension of generalised conjunction to a dynamic setting, and noted that hiding the L-structure parameter, by itself, provides at most a *partial* resolution to the Coordination Problem. The recalcitrant cases involved constructions in which a single modifier applies to the conjoined adjectives, as in Example 95, where the MP '20 centimetres' in the sentence 'The painting is [20 centimetres [tall and wide]]' modifies the conjunction 'tall and wide'. This led us to adopt a more radical approach, based on a mechanism we called *Implicit Argument Disclosure* (IAD), on which certain modifiers are treated as arguments of the expressions they modify (cf. Definition 44). In Section 4.6.2, we observed that the IAD provides a systematic way of treating modifiers of hidden parameters as arguments, and noted that this undermines the distinction between arguments and modifiers.

Finally, we should point out that several of the proposals we have made in this dissertation do not depend on the use of L-structures. The proposals regarding construction-specific constraints in Section 3.5, for example, are quite general in nature. Many of the ideas put forward in Chapter 4 do not require the acceptance or use of L-structures, and can be quite straightforwardly incorporated into VSS or indeed any scalar or degree-based account. Thus, it is our hope that even those who might disagree with some aspects of our thesis may find some of value herein.

Bibliography

- [1] Steven Abney. *The English Noun Phrase in Its Sentential Aspect*. PhD thesis, MIT, Cambridge, MA, 1987.
- [2] Jay David Atlas. Meanings, propositions, context, and semantical underdeterminacy. In Gerhard Preyer and Georg Peter, editors, *Context-Sensitivity and Semantic Minimalism: New Essays on Semantics and Pragmatics*, pages 217–239. Oxford University Press, Oxford, 2007.
- [3] Joachim Ballweg. Vagueness or context-dependence? Supervaluation revisited in a semantics based on scales. In Thomas T. Ballmer and Manfred Pinkal, editors, *Approaching Vagueness*, pages 59–78. North-Holland, Amsterdam, 1983.
- [4] C. Barker. Continuations and the nature of quantification. *Natural Language Semantics*, 10:211–242, 2002.
- [5] C. Barker. The dynamics of vagueness. *Linguistics and Philosophy*, 25(1):1–36, 2002.
- [6] Chris Barker. Continuations in natural language [extended abstract]. In *Fourth ACM-SIGPLAN Continuation Workshop*, Venice, Italy, 2004. ACM.
- [7] R. Bartsch. Relative adjectives and comparison in a Montague grammar. In R. Bartsch and T. Vennemann, editors, *Semantic Structures: A Study in the Relation Between Semantics and Syntax*, pages 157–186. Athenaeum Verlag, Frankfurt am Main, 1972.
- [8] R. Bartsch. The semantics and syntax of number and numbers. In J. P. Kimball, editor, *Syntax and Semantics*, volume vol. 2, pages 51–93. Seminar Press, New York and London, 1973.
- [9] J. Barwise and J. Perry. *Situations and Attitudes*. The MIT Press, Cambridge, Mass., 1983.

- [10] Jon Barwise and John Etchemendy. *The Liar: An Essay on Truth and Circularity*. Oxford University Press, Oxford, 1987.
- [11] Jon Barwise and Jerry Seligman. *Information Flow: The Logic of Distributed Systems*. Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, Cambridge, 1997.
- [12] Kenneth Reid Beesley. Evaluative adjectives as one-place predicates in montague grammar. *Journal of Semantics*, 1(3–4):195–249, 1982.
- [13] M. Bierwisch. On certain problems of semantic representations. *Foundations of Language*, 5:153–184, 1969.
- [14] M. Bierwisch. The semantics of gradation. In M. Bierwisch and E. Lang, editors, *Dimensional Adjectives*, pages 71–262. Springer Verlag, 1984.
- [15] Dwight Bolinger, editor. *Degree Words*. Mouton, The Hague, Netherlands, 1972.
- [16] J. Bos, E. Mastenbroek, S. McGlashan, S. Millies, and M. Pinkal. A compositional DRS-based formalism for NLP applications. In H. Bunt, R. Muskens, and G. Rentier, editors, *Proceedings of the International Workshop on Computational Semantics, ITK*, pages 21–31, Tilburg, The Netherlands, 1994.
- [17] Joan Bresnan. Syntax of the comparative clause in English. *Linguistic Inquiry*, 4(3):275–343, 1973.
- [18] Joan Bresnan. Comparative deletion and constraints on transformations. *Linguistic Analysis*, 1:25–74, 1975.
- [19] Claudia M. Brugman. *The Syntax and Semantics of 'have' and its Complements*. PhD thesis, University of California, Berkeley, 1988.
- [20] Karl Bühler. The deictic field of language and deictic words. In R. J. Jarvella and W. Klein, editors, *Speech, Place and Action: Studies in Deixis and Related Topics*, Abridged translation of K. Bühler (1934): Sprachtheorie, part 2, chs. 7 and 8, pages 9–30. John Wiley & Sons Ltd., New York, 1982.
- [21] Richmond Campbell. The sorites paradox. *Philosophical Studies*, 26:175–191, 1974.
- [22] Herman Cappelen and Ernie Lepore. *Insensitive Semantics*. Blackwell, Oxford, 2005.

- [23] James Cargile. The sorites paradox. *British Journal for the Philosophy of Science*, 20(3):550–63, 1969.
- [24] Hector-Neri Castañeda. Comments. In Nicholas Rescher, editor, *The Logic of Decision and Action*, pages 104–112. University of Pittsburgh Press, Pittsburgh, PA, 1967.
- [25] G. Chierchia. *Dynamics of Meaning: Anaphora, Presupposition, and the Theory of Grammar*. The University of Chicago Press, Chicago and London, 1995.
- [26] Bernard Comrie. *Tense*. Cambridge University Press, Cambridge, England, 1985.
- [27] Cleo Condoravdi and Jean Mark Gawron. The context dependency of implicit arguments. In Makoto Kanazawa, Christopher Pinon, and Henriette de Swart, editors, *Quantifiers, Deduction, and Context*, pages 1–32. CSLI, Stanford, CA, 1996.
- [28] Robin Cooper. The interpretation of pronouns. In F. Heny and H. Schnelle, editors, *Selections from the Third Groningen Roundtable*, Syntax and Semantics 10, pages 61–92. Academic Press, New York, 1979.
- [29] M.J. Cresswell. The semantics of degree. In Barbara H. Partee, editor, *Montague Grammar*, pages 261–292. Academic Press, New York, NY, 1976.
- [30] Haskell B. Curry, Robert Feys, and William Craig. *Combinatory Logic, Vol. 1*. North-Holland, Amsterdam, 1958.
- [31] D. Davidson. *Essays on Actions and Events*. Clarendon Press, Oxford, 1980.
- [32] D. Davidson. *Inquiries into Truth and Interpretation*. Clarendon Press, Oxford, 1984.
- [33] Donald Davidson. The logical form of action sentences. In Nicholas Rescher, editor, *The Logic of Decision and Action*. University of Pittsburgh Press, Pittsburgh, PA, 1967.
- [34] M. Davies. *Meaning, Quantification, Necessity*. Routledge and Kegan Paul, London, 1981.
- [35] Paul Dekker. Existential disclosure. *Linguistics and Philosophy*, 16(6):561–587, December 1993.
- [36] Paul Dekker. *Transsentential Meditations*. PhD thesis, University of Amsterdam, Amsterdam, The Netherlands, 1993.

- [37] S. C. Dik. *Coordination: Its Implication for the Theory of General Linguistics*. North-Holland, Amsterdam, 1968.
- [38] David Dowty. Type-raising, functional composition, and non-constituent coordination. In R. T. Oehrle, E. Bach, and D. Wheeler, editors, *Categorial Grammars and Natural Language Structures*, pages 153–197. Reidel, Dordrecht, 1988.
- [39] David R. Dowty. Montague grammar and the lexical decomposition of causative verbs. In Barbara H. Partee, editor, *Montague Grammar*, pages 201–246. Academic Press, New York, NY, 1976.
- [40] Hans-Jürgen Eikmeyer and Hannes Reiser. A formal theory of context dependence and context change. In Thomas T. Ballmer and Manfred Pinkal, editors, *Approaching Vagueness*, pages 131–188. North-Holland, Amsterdam, 1983.
- [41] Muvet Enc. Anchoring conditions for tense. *Linguistic Inquiry*, 18:633–657, 1987.
- [42] Gareth Evans. Pronouns, quantifiers, and relative clauses (i). *Canadian Journal of Philosophy*, 7:467–536, 1977.
- [43] Gareth Evans. Can there be vague objects? *Analysis*, 38:208, 1978.
- [44] Gareth Evans. Pronouns. *Linguistic Inquiry*, 11:337–362, 1980.
- [45] Gareth Evans. *The Varieties of Reference*. Clarendon Press, Oxford, 1982.
- [46] Martina Faller. Dimensional adjectives and measure phrases in vector space semantics. In Martina Faller, Stefan Kaufmann, and Marc Pauly, editors, *Formalizing the Dynamics of Information*. CSLI, Stanford, CA, 2000.
- [47] C. Fillmore and P. Kay. *Construction Grammar Coursebook*. Copy Central, University of California, Berkeley, 1993.
- [48] C. Fillmore, P. Kay, and C. O’Connor. Regularity and idiomaticity in grammatical constructions: The case of *let alone*. *Language*, 64:501–538, 1988.
- [49] Charles Fillmore. Pragmatics and the description of discourse. In P. Cole, editor, *Radical Pragmatics*. Academic Press, New York, NY, 1981.
- [50] Charles Fillmore. The mechanisms of “Construction Grammar”. *BLS*, 14:35–55, 1988.
- [51] Kit Fine. Vagueness, truth and logic. *Synthese*, 30:265–300, 1975.

- [52] L. T. F. Gamut. *Logic, Language, and Meaning*, volume 2. University of Chicago Press, Chicago, IL, 1991.
- [53] P. Gärdenfors. *Conceptual Spaces: The Geometry of Thought*. The MIT Press, Cambridge, MA, 2000.
- [54] Gerald Gazdar. A cross-categorial semantics for coordination. *Linguistics and Philosophy*, 3:407–409, 1980.
- [55] Peter Geach. Good and evil. *Analysis*, 17:33–42, 1956.
- [56] Peter Geach. *Reference and Generality*. Cornell University Press, Ithaca, NY, 1962.
- [57] Adele Goldberg. *Constructions: A Construction Grammar Approach to Argument Structure*. University of Chicago Press, Chicago, 1995.
- [58] Adele Goldberg. *Constructions at Work: The Nature of Generalization in Language*. Oxford University Press, 2005.
- [59] Adele Goldberg and Ray Jackendoff. The English resultative as a family of constructions. *Language*, 80:532–68, 2004.
- [60] Jeroen Groenendijk and Martin Stokhof. Dynamic montague grammar. In L. K’alm’an and L. Pol’os, editors, *Papers from the Second Symposium on Logic and Language*, pages 3–48, Budapest, 1990. Akad’emiai Kiad’o.
- [61] Jeroen Groenendijk and Martin Stokhof. Dynamic predicate logic. *Linguistics and Philosophy*, 14:39–100, 1991.
- [62] S. Haack. *Philosophy of Logics*. Cambridge University Press, Cambridge, 1978.
- [63] A. Hale. Conditions on English comparative clause pairings. In R. Jacobs and P. Rosenbaum, editors, *Readings in English Transformational Grammar*, pages 30–55. Georgetown University Press, Washington, DC, 1970.
- [64] I. Heim. Degree operators and scope. In C. Fery and W. Sternefeld, editors, *A Festschrift for Arnim von Stechow*, Berlin, 2001. Akademie Verlag.
- [65] Irene Heim. *The Semantics of Definite and Indefinite Noun Phrases*. PhD thesis, University of Massachusetts, Amherst, 1982.

- [66] Irene Heim and Angelika Kratzer. *Semantics in Generative Grammar*. Blackwell, Oxford, 1998.
- [67] L. Hellan. *Towards an Integrated Analysis of Comparatives*. Gunter Narr Verlag, 1981.
- [68] R. Hendrick. The phrase structure of adjectives and comparatives. *Linguistic Analysis*, 4(3):255–299, 1978.
- [69] H. Hendriks. *Studied Flexibility: categories and types in syntax and semantics*. PhD thesis, University of Amsterdam, Amsterdam, 1993.
- [70] A. Herskovits. *Language and Spatial Cognition: An Interdisciplinary Study of the Prepositions in English*. Cambridge University Press, Cambridge, 1986.
- [71] James Higginbotham. On semantics. *Linguistic Inquiry*, 16(4):547–593, 1985.
- [72] James Higginbotham. *Sense and Syntax*. Clarendon Press, Oxford, England, 1995.
- [73] J. Hoeksema. Negative polarity and the comparative. *Natural Language and Linguistic Theory*, 1:403–434, 1983.
- [74] N. Hornstein. *As Time Goes By: Tense and Universal Grammar*. MIT Press, Cambridge, MA, 1990.
- [75] R. Huddleston. More on the English comparative. *Journal of Linguistics*, 3:91–102, 1967.
- [76] Ray Jackendoff. *X-bar Syntax: A Study of Phrase Structure*. MIT Press, Cambridge, MA, 1977.
- [77] Ray Jackendoff. *Semantics and Cognition*. The MIT Press, Cambridge, MA, 1983.
- [78] Ray Jackendoff. *The Architecture of the Language Faculty*. The MIT Press, Cambridge, Mass., 1997.
- [79] Ray Jackendoff. Twistin’ the night away. *Language*, 73:534–559, 1997.
- [80] Ray Jackendoff. *Foundations of Language: Brain, Meaning, Grammar, Evolution*. Oxford University Press, Oxford and New York, 2002.
- [81] T. M. V. Janssen. *Foundations and Applications of Montague Grammar*. CWI, Amsterdam, The Netherlands, 1986.

- [82] H. Kamp. A theory of truth and semantic representation. In J. Groenendijk, T. M. V. Janssen, and M. Stokhof, editors, *Formal Methods in the Study of Language: Proceedings of the Third Amsterdam Colloquium*, pages 277–322. University of Amsterdam, Mathematical Centre Tracts 135, 1981.
- [83] Hans Kamp and Uwe Reyle. *From Discourse to Logic*. Kluwer, Dordrecht, 1993.
- [84] J. A. W. [Hans] Kamp. Two theories about adjectives. In E. Keenan, editor, *Formal Semantics of Natural Language*. Cambridge University Press, Cambridge, England, 1975.
- [85] L. Karttunen. Discourse referents. In James D. McCawley, editor, *Notes from the Linguistic Underground*, Syntax and Semantics 7, pages 363–385. Academic Press, New York, 1976.
- [86] J. J. Katz. Recent issues in semantic theory. *Foundations of Language*, 3:124–194, 1967.
- [87] Rosanna Keefe. *Theories of Vagueness*. Cambridge University Press, Cambridge, England, 2000.
- [88] Rosanna Keefe and Peter Smith. Introduction: Theories of vagueness. In Rosanna Keefe and Peter Smith, editors, *Vagueness: A Reader*, chapter 1, pages 1–57. MIT Press, 1997.
- [89] Rosanna Keefe and Peter Smith. *Vagueness: A Reader*. MIT Press, Cambridge, MA and London, England, 1997.
- [90] E. Keenan. A Boolean approach to semantics. In J. A. G. Groenendijk, T. M. V. Janssen, and M. Stokhof, editors, *Truth, Interpretation and Information: Selected Papers from the Third Amsterdam Colloquium*. Foris, Dordrecht, 1984.
- [91] E. Keenan and L. Faltz. Logical types for natural language. In *UCLA Working Papers in Linguistics*, volume 3, 1978.
- [92] C. Kennedy. *Projecting the Adjective: The syntax and semantics of gradability and comparison*. Garland Press, New York, NY, 1999.
- [93] C. Kennedy. Polar opposition and the ontology of ‘degrees’. *Linguistics and Philosophy*, 24:33–70, 2000.
- [94] Jeffrey King. Tense, modality, and semantic values. *Philosophical Perspectives*, 17:195–245, 2003.

- [95] Ewan Klein. A semantics for positive and comparative adjectives. *Linguistics and Philosophy*, 4:1–45, 1980.
- [96] Ewan Klein. The interpretation of adjectival comparatives. *Journal of Linguistics*, 18:113–136, 1982.
- [97] Ewan Klein. Comparatives. In A. von Stechow and D. Wunderlich, editors, *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung*. Walter de Gruyter, 1991.
- [98] Kepa Korta and John Perry. Radical minimalism, moderate contextualism. In Gerhard Preyer and Georg Peter, editors, *Context-Sensitivity and Semantic Minimalism: New Essays on Semantics and Pragmatics*, pages 94–111. Oxford University Press, Oxford, 2007.
- [99] Angelika Kratzer. More structural analogies between pronouns and tenses. In *SALT VIII*. CLC Publications, May 1998.
- [100] Alice Kyburg and Michael Morreau. Fitting words: Vague language in context. *Linguistics and Philosophy*, 23(6):577–597, 2000.
- [101] G. Lakoff. *Women, Fire, and Dangerous Things: What Categories Reveal About the Mind*. The University of Chicago Press, Chicago and London, 1987.
- [102] G. Lakoff and M. Johnson. *Metaphors We Live By*. The University of Chicago Press, Chicago and London, 1980.
- [103] Knud Lambrecht. *Information Structure and Sentence Form: A Theory of Topic, Focus, and the Mental Representation of Discourse Referents*. Cambridge Studies in Linguistics. Cambridge University Press, Cambridge, England, 1994.
- [104] Barbara Landau and Ray Jackendoff. ‘what’ and ‘where’ in spatial language and spatial cognition. *Behavioural and Brain Sciences*, 16:217–238, 1993.
- [105] F. Landman. *Indefinites and the Type of Sets*. Blackwell, Malden, MA, 2004.
- [106] R. W. Langacker. *Foundations of Cognitive Grammar*, volume 1. Stanford University Press, Stanford, Cal., 1987.
- [107] S. Lappin. *Sorts, Ontology and Metaphor: The Semantics of Sortal Structure*. Walter de Gruyter, Berlin, 1981.

- [108] L. Latecki and M. Pinkal. Syntactic and semantic conditions for quantifier scope. In *Graduirung und Komparation*, Arbeitspapier Nr. 13. Universität des Saarlandes, 1990.
- [109] R. Lees. Grammatical analysis of the English comparative construction. *Word*, 17(2):171–185, 1961.
- [110] D. Lewis. Adverbs of quantification. In E. Keenan, editor, *Formal Semantics of Natural Language*, pages 3–15. Cambridge University Press, Cambridge, 1975.
- [111] David K. Lewis. General semantics. *Synthese*, 22:18–67, 1970.
- [112] Kenton F. Machina. Truth, belief and vagueness. *Journal of Philosophical Logic*, 5:47–78, 1976.
- [113] S. McConnell-Ginet. *Comparative Constructions in English: A Syntactic and Semantic Analysis*. PhD thesis, University of Rochester, 1973.
- [114] John McFarlane. Relativism and disagreement. *Philosophical Studies*, 132:17–31, 2007.
- [115] F. Moltmann. *Coordination and Comparatives*. PhD thesis, MIT, 1992.
- [116] R. Montague. The proper treatment of quantification in ordinary English. In J. Hintikka, J. Moravcsik, and P. Suppes, editors, *Approaches to Natural Language*. Reidel, Dordrecht, 1973.
- [117] Richard Montague. English as a formal language. In B. Visentini et al., editors, *Linguaggi nella società e nella tecnica*. Edizioni di Comunità, 1970.
- [118] Richard Montague. Universal grammar. *Theoria*, 36, 1970.
- [119] Richard Montague. *Formal Philosophy: Selected Papers of Richard Montague*. Yale University Press, New Haven, Conn., 1974.
- [120] Reinhard Muskens. Combining montague semantics and discourse representation. *Linguistics and Philosophy*, 19:143–186, 1996.
- [121] Toshiyuki Ogihara. *Tense, Attitudes, and Scope*. Kluwer, Dordrecht, 1996.
- [122] Rohit Parikh. Vagueness and utility: The semantics of common nouns. *Linguistics and Philosophy*, 17:521–535, 1994.
- [123] Terence Parsons. Some problems concerning the logic of grammatical modifiers. *Synthese*, 21(3–4):320–324, October 1970.

- [124] Terence Parsons. Pronouns as paraphrases. Unpublished Ms., University of Massachusetts, Amherst, 1978.
- [125] Terence Parsons. Entities without identity. In J. E. Tomberlin, editor, *Philosophical Perspectives, 1: Metaphysics*, pages 1–19. Ridgeview, Atascadero, CA, 1987.
- [126] Terence Parsons. *Events in the Semantics of English: A Study in Subatomic Semantics*. Current Studies in Linguistics. The MIT Press, Cambridge, Mass., 1990.
- [127] Barbara H. Partee. Some structural analogies between tenses and pronouns in english. *The Journal of Philosophy*, 70(18):601–609, 1973.
- [128] Barbara H. Partee. Binding implicit variables in quantified contexts. *Papers of the Chicago Linguistic Society*, 25:342–365, 1989.
- [129] Barbara H. Partee and Mats Rooth. Generalized conjunction and type ambiguity. In R. Bauerle, C. Schwarze, and A. von Stechow, editors, *Meaning, Use and Interpretation of Language*, pages 361–383. Mouton de Gruyter, Berlin, 1983.
- [130] H. Pilch. Comparative constructions in English. *Language*, 41(1):37–58, 1965.
- [131] Manfred Pinkal. Towards a semantics of precization. In Thomas T. Ballmer and Manfred Pinkal, editors, *Approaching Vagueness*, pages 13–57. North-Holland, Amsterdam, 1983.
- [132] Manfred Pinkal. Imprecise concepts and quantification. In R. Bartsch, J. van Benthem, and P. van Emde Boas, editors, *Semantics and Contextual Expression*, pages 221–265. Foris, Dordrecht, 1989.
- [133] Gerhard Preyer and Georg Peter, editors. *Context-Sensitivity and Semantic Minimalism: New Essays on Semantics and Pragmatics*. Oxford University Press, Oxford, 2007.
- [134] Hans Reichenbach. *Elements of Symbolic Logic*. Macmillan, London, 1947.
- [135] Mark Richard. Contextualism and relativism. *Philosophical Studies*, 119:215–242, 2004.
- [136] J. R. Ross. *Constraints on Variables in Syntax*. PhD thesis, MIT, Cambridge, MA, 1967.
- [137] Bertrand Russell. Vagueness. *Australasian Journal of Philosophy and Psychology*, 1:84–92, 1923.

- [138] R. M. Sainsbury. Concepts without boundaries. In Rosanna Keefe and Peter Smith, editors, *Vagueness: A Reader*, pages 251–264. The MIT Press, Cambridge, MA and London, England, 1997.
- [139] Nathan Salmon. Tense and singular propositions. In J. Almog, J. Perry, and H. Wettstein, editors, *Themes from Kaplan*, pages 331–392. Oxford University Press, Oxford, 1989.
- [140] G. Sampson. Good. *Linguistic Inquiry*, 1:257–260, 1970.
- [141] Jerry Seligman. Perspectives in situation theory. In R. Cooper, K. Mukai, and J. Perry, editors, *Situation Theory and its Applications*, volume 1 of *CSLI Lecture Notes*, pages 147–191. CSLI Publications, Stanford, CA, 1990.
- [142] Jerry Seligman. *Perspectives: A Relativistic Approach to the Theory of Information*. PhD thesis, University of Edinburgh, Edinburgh, 1991.
- [143] Jerry Seligman. Physical situations and information flow. In J. Barwise et al., editors, *Situation Theory and its Applications*, volume 2 of *CSLI Lecture Notes*, pages 257–292. CSLI Publications, 1991.
- [144] M. E. A. Siegel. *Capturing the Adjective*. PhD thesis, University of Massachusetts (Amherst), 1976.
- [145] M. E. A. Siegel. Capturing the russian adjective. In Barbara H. Partee, editor, *Montague Grammar*, pages 293–309. Academic Press, New York, 1976.
- [146] M. E. A. Siegel. Measure adjectives in montague grammar. In S. Davis and M. Mithun, editors, *Linguistics, Philosophy, and Montague Grammar*, pages 223–262. University of Texas Press, Austin and London, 1979.
- [147] C. Smith. A class of complex modifiers in English. *Language*, 37:342–365, 1961.
- [148] Roy Sorensen. *Blindspots*. Clarendon Press, Oxford, 1988.
- [149] Roy Sorensen. *Vagueness and Contradiction*. Oxford University Press, Oxford, England, 2001.
- [150] Dan Sperber and Deirdre Wilson. *Relevance: Communication and Cognition*. Blackwell, Oxford, 2nd edition, 1995. First edition published in 1986.

- [151] Mark J. Steedman. Temporality. In Johan van Benthem and Alice ter Meulen, editors, *Handbook of Logic and Language*, chapter 16, pages 895–938. Elsevier North Holland, Amsterdam, 1997.
- [152] S. S. Stevens. On the theory of scales of measurement. *Science*, 103:667–688, 1946.
- [153] L. Talmy. Figure and ground in complex sentences. In J. Greenberg et al., editors, *Universals of Human Language*, volume Four. Stanford University Press, Stanford, Cal., 1978.
- [154] L. Talmy. How language structures space. In H. L. Pick and L. P. Acredolo, editors, *Spatial Orientation*. Plenum Press, New York, 1983.
- [155] Leonard Talmy. Force dynamics in language and cognition. *Cognitive Science*, 12:49–100, 1988.
- [156] Leonard Talmy. Figure and ground in language. In *Toward a Cognitive Semantics, Volume 1: Concept Structuring Systems*, chapter 5. The MIT Press, Cambridge, MA, 2000.
- [157] Leonard Talmy. How language structures space. In *Toward a Cognitive Semantics, Volume 1: Concept Structuring Systems*, chapter 3. The MIT Press, Cambridge, MA, 2000.
- [158] Leonard Talmy. The relation of grammar to cognition. In *Toward a Cognitive Semantics, Volume 1: Concept Structuring Systems*, chapter 1. The MIT Press, Cambridge, MA, 2000.
- [159] T. Trabasso and N. L. Stein. Using goal-plan knowledge to merge the past with the present and the future in narrating events online. In M. H. Haith, J. B. Benson, R. J. Roberts, and B. F. Pennington, editors, *The Development of Future-Oriented Processes*, pages 323–352. University of Chicago Press, Chicago, IL, 1994.
- [160] M. Tye. Vague objects. *Mind*, 99:535–557, 1990.
- [161] P. Unger. There are no ordinary things. *Synthese*, 41:117–154, 1979.
- [162] P. Unger. The problem of the many. In P. A. French, T. E. Uehling, and H. K. Wettstein, editors, *Midwest Studies in Philosophy* 5, pages 411–469. University of Minnesota Press, Minneapolis, MN, 1980.

- [163] Jan van Eijck and Hans Kamp. Representing discourse in context. In Johan van Benthem and Alice ter Meulen, editors, *Handbook of Logic and Language*, pages 179–237. North-Holland, Amsterdam, 1997.
- [164] Michiel van Lambalgen and Fritz Hamm. *The Proper Treatment of Events*. Explorations in Semantics. Blackwell, Oxford, England, 2005.
- [165] C. Vandeloise. *Spatial Prepositions: A Case Study from French*. The University of Chicago Press, Chicago and London, 1991. Translated by A. R. K. Bosch.
- [166] Zeno Vendler. *Adjectives and Nominalizations*. Mouton, The Hague, 1968.
- [167] Frank Vlach. Temporal adverbials, tenses and the perfect. *Linguistics and Philosophy*, 16:231–283, 1993.
- [168] A. von Stechow. ε - λ kontextfrei Sprachen: Ein Beitrag zu einer natürlichen formalen Semantik. *Linguistische Berichte*, 34:1–33, 1974.
- [169] A. von Stechow. Comparing semantic theories of comparison. *Journal of Semantics: Special Issue on the Comparative Construction*, 3(1–2), 1984.
- [170] S.-C. Wheeler. Attributives and their modifiers. *Noûs*, 6(4):310–334, 1972.
- [171] Timothy Williamson. Vagueness and ignorance. In *Proceedings of the Aristotelian Society*, volume Supp. Vol. 66, pages 145–162. The Aristotelian Society, 1992.
- [172] Timothy Williamson. *Vagueness*. Routledge, London, 1994.
- [173] Timothy Williamson. Vagueness and ignorance. In Rosanna Keefe and Peter Smith, editors, *Vagueness: A Reader*, pages 265–280. The MIT Press, Cambridge, MA, 1997. Originally published in 1992.
- [174] Yoad Winter. *Flexibility Principles in Boolean Semantics*. MIT Press, Cambridge, MA, 2001.
- [175] Yoad Winter. Measure phrase modification in vector space semantics. In Karine Megerdooimian and Leora Anne Bar-el, editors, *Proceedings of the 20th West Coast Conference on Formal Linguistics*, pages 607–620, Somerville, MA, 2001. Cascadilla Press.
- [176] Yoad Winter. Cross-categorial restrictions on measure phrase modification. *Linguistics and Philosophy*, 28:233–267, 2005.

- [177] E. M. Zemach. Vague objects. *Noûs*, 25:323–340, 1991.
- [178] J. Zlatev. *Situated Embodiment: Studies in the Emergence of Spatial Meaning*. Gotab, Stockholm, 1997.
- [179] David Zubin and Lynne E. Hewitt. The deictic center: A theory of deixis in narrative. In Judith Duchan, Gail A. Bruder, and Lynne E. Hewitt, editors, *Deixis in Narrative: A Cognitive Science Perspective*. Lawrence Erlbaum, Hillsdale, NJ, 1995.
- [180] Joost Zwarts. Vectors as relative positions: A compositional semantics of modified PPs. *Journal of Semantics*, 14:57–87, 1997.
- [181] Joost Zwarts. Vectors across spatial domains: From place to size, orientation, shape, and parts. In Emile van der Zee and Jon Slack, editors, *Representing Direction in Language and Space*, chapter 3, pages 39–68. Oxford University Press, Oxford, 2003.
- [182] Joost Zwarts and Yoad Winter. Vector space semantics: A model-theoretic analysis of locative prepositions. *Journal of Logic, Language and Information*, 9:169–211, 2000.