Three-stage Turbine Pump.

Figs. 6 and 7.

W. Blackadder, D.Sc. 1923.
EXPERIMENTAL INVESTIGATION OF THE LOSSES IN THE COMPONENT PARTS of a THREE-STAGE TURBINE PUMP

with analysis of the results and discussion of their relation to the ordinary theory.
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EXPERIMENTAL INVESTIGATION OF THE LOSSES IN THE COMPONENT PARTS of a THREE-_STAGE TURBINE PUMP. with analysis of the results and discussion of their relation to the ordinary theory.

1 Introduction.

Consultation of publications on the testing of centrifugal pumps seems to show that while much experimental work has been carried out, yet the work has been confined mainly to overall tests.

Overall tests of a pump may show slight changes in efficiency with alterations in the proportions of one component part of the pump, but it is not possible to make definite conclusions as to whether this change is entirely due to the alteration in that one part of the pump. For new conditions of flow introduced by the alteration in proportion may affect the efficiency of and the losses in other component parts. A change in curvature of impeller blades or in the rate of change of cross sectional area of the passages between the impeller vanes — without alteration of the angles or radial areas at entry or exit — may affect the actual angle of discharge from the impeller tip and hence also the losses in the guide passages; and thus the change in overall efficiency may not be due entirely to the alteration in the impeller.

The total losses in the pump at any delivery may be known but their allocation among impeller, guide passages etc., is very much a matter of conjecture from analogy with results from flow through stationary passages; and in the case of shock losses, deductions from such analogy are very uncertain.

Experiments were carried out by Dr. Stanton (†) on the efficiency of a vortex chamber with and without guide vanes and with various/

various types of impellers; and this seems to be the only published record of experiments carried out to determine separately the efficiency of a certain component part of a centrifugal pump.

The experiments described in the present paper were undertaken primarily to determine the separate losses and efficiencies of the impellers, guide passages, volute, and the various connecting passages of a turbine pump, and how these losses and efficiencies varied with the discharge and the impeller speed. But during the analysis of the results it was found necessary to consider other questions. For the ordinary theory of the centrifugal pump, while indicating the general laws connecting discharge, head, speed and power and explaining the effect of various factors in design, is admittedly not quite in accordance with actual test results as regards the relative proportions of these items.

The additional matters considered are, the amount of leakage, the pressure in the casing at the side of the impeller, the manner of flow through the impeller, and the relation of the ordinary theory to the experimental results. In addition, the pump tested presents a peculiarity in the abrupt rise (with increasing discharge) of the head quantity curve at constant speed. (See Fig. 9). Additional tests were therefore made at the discharge at which the abrupt rise takes place (termed the "critical" discharge hereafter); the results have been analysed and a possible explanation of this detrimental feature put forward.

§2. General description of testing plant.

The tests were carried out in the Hydraulics Laboratory of the Royal Technical College, Glasgow, on a three-stage turbine pump, driven by a variable speed electric motor. Water was taken from a well below the pump, and after passing through the pump was delivered into an overhead channel through a 6 inch delivery pipe on/
on which was placed a Venturi Meter. From the channel the water passed over a rectangular or a V-notch weir (or over both); the discharge could thus be measured at both the Venturi meter and at the weirs. The pump was driven at constant speed, the revolutions being determined by a tachometer driven from the pump shaft; but during the tests the pump itself became a check on the tachometer and on any variation of its own speed as is described later. Changes of head were obtained by operation of a valve on the delivery pipe. Pressures were determined at the inlet to and at the outlet of each impeller, in the casing at the side of each impeller, at the outlet from the guide passages, at the collecting chamber (or volute), and in the discharge pipe at its connection with the outlet branch from the pump. All pressures were read direct on mercury columns except that at the inlet to the first impeller which was read on a vacuum gauge.

§3. Details of apparatus and calibration.

Since it was expected that many of the losses to be determined would be of small magnitude, it was necessary to determine the values of the observed quantities as accurately as possible.

Motor. Power was obtained from a variable speed shunt wound D.C. Motor with interpoles, run at constant voltage. The power delivered to the pump shaft was obtained by careful determination of the armature, iron, eddy, wind &c., losses. The values of these losses from two tests plotted on a speed basis are shown on Fig. (1) in order to consider later the effect of possible error in motor output on the estimation of the pump losses. During the pump tests the speed was kept as constant as possible, and for this purpose additional finely adjustable resistance was placed in the field circuit, making the speed adjustable to about 1 in 1000 (See §4). Precision volt meters and ammeters were used in all readings, the volt meter being connected across the brushes, and the ammeter placed in the armature circuit only. The field current was/
was measured separately and at constant speed was independent of the load. Amps were read on a scale such that readings were possible to \(\frac{1}{10}\)th. of an amp. Volts could be read with an error of not more than 1 in 500. The possible error due to oscillations of the pointers is considered in \(\S 4\).

**Tachometer.** The tachometer used was of the centrifugal type, with 9" reading dial, and belt driven from the pump shaft. During calibration and all tests the belt was kept in the same moist condition by a fine spray. At the 1100 revs/min mark on the dial the indicated revolutions could be read to within 1\(\frac{1}{2}\) revs/min. and at the 500 revs/min mark to within 1 rev/min. The tachometer was carefully calibrated by a counter driven direct from the pump shaft. The calibration curve is shown on Fig. (2); these are plotted from several trials which among themselves differed by under \(\frac{1}{2}\) per cent; and, the pump speed at the time of calibration could be relied on to within \(\frac{1}{2}\) per cent. The questions of any subsequent change in the ratio of tachometer to pump speed (through variation of slip of belt &c.) is considered in \(\S 4\).

**Weirs and Venturi meter.** These were carefully calibrated and the calibration curves were adopted to determine the discharge. They were calibrated by means of four fixed tanks each of 70 cubic feet capacity, the quantity at any depth in the tank being known by previous calibration of the tanks. Calibration curves (on a reduced scale) of the Weirs and meter are shown Fig. (3). The possible error on the estimated discharge is best seen by comparison of the discharges as determined by the two methods. (See Tables \(I_a, I_b\) and \(I_c\)).

**Pressure gauges.** All pressure heads were read directly on two mercury columns except that at inlet to the first impeller. A whole series of tests had been carried out previously to those described here by reading impeller outlets and discharge pressures directly/
directly on two mercury columns and the other pressures (inlet to impellers, guide passages, and volute) on differential mercury gauges. These indicated at one and the same time practically all the pressures to be read, thus avoiding delay and errors due to slight variations in the motor and pump speed - the additional finely adjustable field resistance referred to in this paragraph under "Motor" not having then been inserted. But changes of speed or other sources of error did occur; and on analysis of the test readings absolutely discordant results were obtained especially where small differences were involved. The differential gauges were therefore discarded and, by suitable coupling up with strong rubber and canvas hose, all pressures read previously on these gauges were transferred to the two mercury columns and read directly; and arrangements were made to keep the speed of the pump almost absolutely constant by the additional field resistance, and by methods described below (§ 4). The mercury columns gave very accurate readings of the pressure-heads; the mercury remained practically steady with occasional "lurches" up or down about 1 inch, except at discharges below the "critical" discharge, it being then in a constant state of "tremor". Above this discharge the pressure head indications could be read to within 3 inches of water and in many cases to less.

The vacuum gauge on the first impeller inlet was calibrated with a mercury column; its possible error should not be more than that of the mercury columns on account of the size of dial-scale; but any excess of error over that of the mercury columns would only be taken at one third value on account of the necessity of averaging energy gains on the three impellers as described later in the analysis of the results. (§ 9, p/6)

Pump and connections for pressure head. The pump is shown in longitudinal and cross section in Figs. 4 and 5 respectively; a section (as a two-stage pump) in fig. 6; a section of impeller and/
and guide passages in a plane perpendicular to pump shaft in fig. 7 and an exact copy from an impression of the guide passage vanes on cardboard in fig. 8. The curve of the impeller vanes was obtained by carefully fitting and trimming a piece of cardboard between two vanes; this could be fitted very closely except towards the inner periphery of the impeller.

Referring to fig. 4; AA, shows the passage transmitting the pressure at A (for impeller inlet) to A, the rib in the casing being slightly increased in thickness there. The passage is shown for one impeller only but a similar arrangement was put (at different angles) for each impeller and similarly for all other places at which pressures were to be read. The positions of these are shown on fig. 7 by corresponding letters. For the first impeller the vacuum gauge was placed at SS, BB, shows similarly the type of passage for pressure at impeller outlets; CC, that at the casing top for the pressures at outlets from the guide passages; EE, that for the collecting chamber (four connections made and all joined to one pipe finally); FF, is that for the discharge pipe (two connections).

As stated before, with a first series of tests the differential gauges were unsatisfactory; further, the pressure readings from the impeller outlets (B fig. 4 or 7) which were read direct on mercury columns and led to impossible results on analysis; the holes were therefore plugged and re-bored smaller \( \frac{3}{16} \) inch diameter, and finished more smoothly and placed exactly midway between the tips of the guide vanes. In addition it seemed that probably some conversion from velocity to pressure energy had taken place when the water had reached these holes; so it was decided to read the pressure in the casing at the sides of the impellers at various radii. For the second and third impellers the sliding tube GG, Fig. 4 was therefore added; it passed through a gland at G, permitting G (detail on same figure) to be placed at any desired radius. For the/
the first impeller shows the arrangement; pressure could be read at the impeller outer radius only. Further difficulty had also been encountered in the first series of tests; a rise of pressure was read from the outlet of the preceding guide passages to the inlet of the third impeller; it was found to be due to a small hole in the casting leading pressure from a compartment of the pump under higher pressure; but when fitting the tubes and two copper pipes and brass ferrules fig. (4) were added, allowing the pressure for inlets to the second and third impellers each to be read at an additional point, so enabling a check to be obtained and giving an idea of the accuracy of the arrangements. Pressures were transmitted to the mercury columns by \( \frac{3}{4} \) inch bore copper piping in the original fittings, and by rubber and canvas hose pipe in the additions. From the measurement of the pump the following angles and areas were estimated and adopted in the analysis:

**Impeller.**

- Angle at outer periphery with tangent \( \beta = 47^\circ \) (see fig. 2)
- Angle at inner tangent \( \gamma = 14.3^\circ \) (do.)
- Radial area at outlet \( A_r = 0.122 \) sq.ft.nett.
- Radial area at inlet \( A_i = 0.1050 \) sq.ft.
- Area at \( A \) fig. 4 (for impeller inlet) \( A = 0.1274 \) sq.ft.

**Guide passages.**

- Angle at entrance with tangent \( \theta_o = 25^\circ \) (see fig. 3)
- Area at outlet at position of pressure-hole \( C \) fig. 4. \( A_c = 0.271 \) sq.ft.nett.

**Discharge Pipe.**

- Area at \( F \) (fig.4) \( A_d = 0.196 \) sq.ft.

At the position of the vacuum gauge fig. 4, allowing for the boss supporting the pump shaft the area does not differ materially from that at the position of the pressure holes at \( A \) for the second and third impellers.

**§ 4. Procedure on tests.**

The/
The tachometer was set to the speed decided on and after allowing flow through the pump the delivery valve was shut down and the pressure head at the discharge pipe was read on No. 1 mercury column and that of the second impeller outlet on No. 2 Column and both readings were booked. The discharge valve was then opened to give the largest discharge at the speed - the speed being re-adjusted as closely as possible by the fine resistance and the tachometer to limits stated in §3. Readings of discharge pipe and second impeller outlet were again taken and booked as before. The pressures at the other points in the pump at the discharge were then read, some on No. 1 and the others on No. 2 column. If read on No. 2 then No. 1 was kept open to discharge pipe pressure as a reference pressure, while the readings were being taken. Thus any small change in pump speed was detected by the reference pressure. If that took place the tachometer, which was watched continually, was found to corroborate it; and the speed was brought back by tachometer and reference pressure. The reference column was found to agree with indications of the tachometer but was more sensitive. No. 2 column was similarly used as reference column while readings were being taken in Column 1. For each discharge the procedure was similar; finally, at zero discharge, pressures at the various points in the pump were similarly determined and the readings for discharge and second impeller outlet were compared with those initially read at zero discharge. The difference in the height of the mercury column on recording the discharge pipe pressure varied from $\frac{1}{4}$ to $\frac{1}{2}$ inch; a difference of $\frac{1}{2}$ inch at 900 revs/min would mean a change in pump speed of about 3 revs per min which is therefore the maximum probable variation of speed on setting the pump by the tachometer at each different discharge; and on the average it would be less. At each speed the same procedure was adopted but, before commencing, the pressure head at the discharge pipe was read for zero discharge for all speeds at which trials had previously been made, and compared with the readings then taken.
taken; the differences were of the same order as mentioned before - to inch of mercury. Discharges were read on the gauges of the weirs and Venturi meter for each discharge before pressure readings were taken, and after they had all been taken; they were read oftener at first but with the constancy of speed this was found to be unnecessary. Power readings were read several times at each discharge and the average taken; the ammeter pointer oscillated with occasional surges; estimation by different observers gave variations usually under amp and less than this at low powers.

The trouble taken to keep the speed as constant as possible was partly to avoid the large amount of labour of interpolation with pressures recorded at so many points in the pump; but especially because it was desired to discover if possible a cause for the abrupt rise in the head discharge curve and to trace as closely as possible the changes of pressure in the guide passages as the discharge increased; from previous trials it had been found that interpolation led to irregularities and smoothed over the abruptness of the sudden rise in pressure at the "critical" discharge.

5. Observed Pressures &c.

The observed results of the tests are shown in Tables Ia, Ib and Ic. For each impeller is the pressure head at the impeller inlets (A, fig.4). These were read at two points for impellers Nos. 2 and 3. is used for the pressure head (suction) at the inlet to the first impeller. is that at impeller outlets (B, Fig.4); that in the casing at the side of the impeller (G or D, fig.4); that/

† Pressure heads are reduced to the level of the pump-shaft.
that at the outlet from the guide passages (C, fig. 4); \( \frac{\rho g}{\omega} \) that in the volute or collecting chamber (E, fig. 4) and \( \frac{\rho g}{\omega} \) that in the discharge pipe (F, Fig. 4). The discharge as measured by weirs and Venturi meter are also tabulated; where enclosed in brackets the quantity was so small that it could not be measured accurately and is approximate only.

From Tables Ia, Ib, and Ic along with Fig. 1, Table II was obtained.


Taking \( Q \) as the discharge and \( q \) as the leakage (so that \( q = Q \)), is the quantity passing through the impeller in cubic feet per second and \( K \) the horse power imparted to the impeller (i.e. the shaft horse power less bearing and disc friction loss so that \( K \) is the power really imparted to the impeller for pumping) then:

\[
\frac{\omega(q+q)}{550} = K
\]

\( H_0 \) being the head corresponding to the impeller power input.

The power imparted to the water as it passes through each impeller is

\[
\frac{\omega(q+q)}{550} \left\{ \frac{p_x}{\omega} + \frac{v_x^2 + v_y^2}{2g} \right\} = K
\]

where \( \frac{p_x}{\omega} \) is the pressure head, and \( v_x \) the absolute velocity at exit from the impeller and \( \frac{p_x}{\omega} \) and \( v \) the corresponding items at inlet to the impeller (see fig. 2)

Hence

\[
H_o = \left( \frac{p_x}{\omega} + \frac{v_x^2 + v_y^2}{2g} + l_1 \right) + \left( \frac{p_x}{\omega} + \frac{v_x^2 + v_y^2}{2g} + l_2 \right) + \left( \frac{p_x}{\omega} + \frac{v_x^2 + v_y^2}{2g} + l_3 \right)
\]

\( l_1 \) is the loss in each impeller and the subscripts refer to the first, second and third impellers.

The power given to each impeller cannot be measured separately.

Hence \( H_o \), the gross head per impeller is \( H_o/3 \) and

\[
H_o = \frac{p_x}{\omega} + \frac{v_x^2 + v_y^2}{2g} + l = H_1 + l_1
\]

where:

\( \frac{p_x}{\omega} \) is the true pressure head at impeller outlet; while \( \frac{p_x}{\omega} \) and \( \frac{p_x}{\omega} \) (figs) are observed pressure heads near the impeller outlet.
where $\theta$ is the total head imparted to the water as it passes through each impeller and $\zeta$, the loss in the impeller.

Also if $h'_0$ and $u'_g$ are the pressure head and velocity at the outlet from the guide passages,

$$h'_0 + u'_g^2 + 2g = h'_0 + {u'_g^2 \over 2g} + \zeta,$$

being the loss in the guide passages.

Hence from (4) $h'_0 = h'_0 + u'_g^2 + 2g + \zeta = \zeta + Z = \zeta + Z + Z_g$.

or $h'_0 = h'_0 + \zeta + Z_g$.

Where $h'_0$ is the net head gained by the water as it passes from any impeller-inlet to the corresponding guide passage outlet.

For flow from the first guide passages outlet to the inlet of the second impeller

$$\left( h'_0 + u'_g^2 \right) = \left( h'_0 + u'_g^2 \right) + \zeta,$$

$Z_a$ being the loss incurred in the connecting passage, and subscripts as before.

Hence from (5) $h'_0 = \left( h'_0 + u'_g^2 \right) - \left( h'_0 + u'_g^2 \right) + \zeta + Z_g + Z_a$

For flow from the inlet to the first impeller to that of the second impeller. Similarly from the second impeller inlet to the third impeller inlet.

$$h'_0 = \left( h'_0 + u'_g^2 \right) - \left( h'_0 + u'_g^2 \right) + \zeta + Z_g + Z_a$$

and from the third impeller inlet to the discharge-pipe gauge position

$$h'_0 = \left( h'_0 + u'_g^2 \right) - \left( h'_0 + u'_g^2 \right) + \zeta + Z_g + Z_v + Z_d$$

$Z_v$ being as defined in $\xi(5)$, $v'_g$ the corresponding velocity, $Z_v$ the loss in the volute, and $Z_d$ that from volute to the discharge pipe. These last two equations and equations (5) and (6) follow at/
at once from (4) since all power is imparted only during passage through the impellers.

Adding the last three equations:

\[ 3h_o = H_o = \left( \frac{\rho g}{\cos \beta} \cdot \frac{v_1^2}{2g} - \left( \frac{\rho g}{\cos \beta} \cdot \frac{v_2^2}{2g} \right) + s_2 + \frac{v_2}{2} z + \frac{v_2}{2} l_d \right) \]

\[ = H + \mathcal{E}(L) \]  

H is the actual head (pressure and kinetic) imparted to the water as it passes through the pump and \( \mathcal{E}(L) \) is the sum of all the separate losses.

The hydraulic efficiency of the pump is \( \eta_h = \frac{H}{H_o} \); that of the impellers \( \frac{h}{h_o} \), and that of the impellers and guide passages combined is \( \frac{h_o}{h_o} (= \eta_u) \). The "conversion" efficiency of the guide passages is the ratio of the actual rise of pressure head in the guide passages to the possible rise, and is

\[ \eta_i = \frac{p_y - p_1}{\rho g} \frac{v_1^2 - v_2^2}{2g} \]

The gross efficiency \( \eta_g \) is as usual the ratio of the water horse power to the impeller shaft horse-power.

In the ordinary theory of the centrifugal pump

\[ \frac{3\mu x_i}{g} = \frac{3\mu_i (a - y cot \beta)}{g} = 3 \left( \frac{p_j}{\rho g} + \frac{v_1^2 - v_2^2}{2g} + \frac{z_f}{2} \right) \]  

where \( x, y \) being the tangential and radial components of \( v \), and \( \beta \) the impeller vane angle at exit (see Fig. 2). It is known that in all pumps this equation is not quite in accordance with experiments; \( \frac{3\mu x_i}{g} \) is greater than \( H_o \).

If \( H_o \) is accurately determined from the power input and \( \frac{p_j}{\rho g}, \frac{p_j}{\rho g}, v_1, \text{ and } v \) be known the losses and various efficiencies may be determined and, by comparison of \( \frac{3\mu x_i}{g} \) and \( H_o \), the Theory may be compared with actual results.
For the power distribution, the power imparted to the water at discharge (water horse power) is \( \frac{\omega}{550} \) ; that lost in the impellers, guide passages, connecting passages (from guides to next impeller), volute and discharge is given by replacing \( H \) in the above expression by \( 37, 32g, 2L_a, L_v \) and \( L_d \) respectively; that lost in leakage is \( \frac{\omega}{550} \frac{3h_o}{3} = \frac{\omega}{550} \frac{Q}{H_o} \);

and with equation (7) or (8) and (1) these sum up to \( K \).

To determine the above mentioned losses at discharges below the critical a value \( H_o' \) is used in place of \( H_o \) as determined from (1) as is described in \( \xi \xi \), 13 and 20.

\( \xi 7 \). POWER APPLIED TO THE IMPELLERS.

The nett power applied to the impellers (denoted by \( K \)) is the shaft horse power less losses due to disc friction and bearings. Trials were made to estimate these losses by speeding up the pump and then noting the rate of retardation, but this rate was entirely irregular, results with the same initial speed varying nearly 100 per cent.

Further even though these results had been regular, yet from subsequent analysis it seems they would not represent the disc friction loss only, as during retardation a small quantity of leakage water would be passing through the impellers and in addition to disc friction loss there would be the special loss at small discharges dealt with later in \( \xi \xi \) 13 and 20.

The disc friction loss was estimated from the best available experiments on such losses. In place of using the formula there deduced, the loss has been estimated from that found in the paper referred to for a disc of very nearly the same diameter as that of the pump impellers. Experiments with a 12 inch polished brass disc in a painted casing gave the H.P. absorb:

\[ \text{ed at 1500 and 2000 revs/min. as 0.928 and 2.07 respectively; and} \]

from the experiments in general that the power loss varied as
\[
\left(\frac{\text{Revs}}{\text{min}}\right)^n \times (\text{Diam})^{n+3}
\]
where \(n\) varies from 1.8 to 2.0.
The pump impeller is 11 inches diameter and at a speed of 885
revs/min. the corresponding loss is \((with \ n=2)\ 0.125\ \text{H.P.}
\]
estimated from the 1500 revs/min. value above and 0.117 H.P.
if estimated from the 2000 revs/min. value. Taking the loss at
0.13 H.P., for three impellers this gives 0.39 H.P. loss in disc
friction.

The bearings loss was determined by finding the actual
torque required to turn the pump slowly in its bearings with the
pump free of water; the torque was measured by wrapping a cord
round a pulley, on the pump shaft. The value found was 3 foot
lbs. practically, corresponding to a power loss of 0.5 H.P. at
885 revs./min. This will be slightly increased due to end
thrust during actual pumping. As seen from figs. 4 and 6, to
diminish end thrust water is admitted from the suction inlet to
the far side of the impeller by 4 holes in the impeller at the
entrance. Allowing 0.11 H.P. additional for this and for other
errors in estimation the allowances made are shown in Table II,
the values at other speeds being similarly determined. An
attempt was made to determine the bearings and gland losses by
noting the rate of retardation of the rotating shaft and impellers
after emptying the pump of all water; as was to be expected the
gland packings heated up and the friction greatly increased.
Further at any except slow speeds extra power must be used on
account of wind resistance. If \(n = 1.8\) the value for disc
friction loss should be slightly higher, but the effect of error
in this estimation will be considered later along with errors in
other power estimations \((\approx 17)\).

The correct estimation of this loss affects the ratio
between \(H_0\ and \ \frac{3 \text{H.P.}}{Q}\) and so also, any comparison between
the ordinary theory and the actual results of tests. In some
cases this comparison has been made by taking the power imparted
to the impeller at any discharge as the shaft horse power at that
discharge less the shaft horse power at zero discharge.\textsuperscript{x} But an additional source of loss comes into play at small discharges; this is generally not mentioned in the theory of centrifugal pumps. If the impeller input be determined by deducting the power at zero discharge in this series of tests, it leads to a hydraulic efficiency of over 100 per cent for the impeller; 99\% for impeller and guides and 95\% for the pump at 885 revs/min. and 1.09 cub/ft./sec. discharge and leakage allowance; while if no allowance be made for leakage these efficiencies will be reduced by from 5 to 6\%.

In Table II the shaft horse power allowing for electrical losses (fig. 1) and the impeller horse power after allowance for disc friction and bearing losses estimated as above are shown:

\textbf{8 Overall test characteristics.}

From the values in Tables I & II, Table III is prepared; \(H_0\) and \(H\) are determined from equations (1), (7) and (8). The leakage allowance made is shown on table IV and is discussed later (\S\ 12). In this Table \(\frac{3u}{\gamma}x_i\) is determined from \(\frac{3u}{\gamma}x_i(\frac{4}{\gamma} - \frac{y_1 - \alpha_1}{\gamma})\) (equation 9) \(y_1\) being \(\frac{Q}{\alpha_1}\) where \(\alpha_1\) is the radial area at outlet of the impeller as stated \(\S\ 3\). Below the critical discharge the values of \(H_0\) are shown in brackets as explained later on dealing with impeller and guide passages losses (\S\ 13, 20) Values of \(H, H_0, \frac{3u}{\gamma}x_i, E_H\) and \(E_a\) are shown on figs. 11A, 11B & 11C for 695, 885 and 1079 revs. per min. on \(Q\) as basew; the shaft and water horse power are shown for 885 revs per min. on fig. 10; and \(H_0\) alone, estimated with and also without leakage allowance, (13. \(\alpha = 0\) in equation \(01\)) in fig. 30. The curves of \(H E_a\) and water horse power present the usual features except below the critical delivery where they differ radically from the usual type of such curves. On fig. 10 the change in the general trend of the impeller horse power curve \(ab\) at the critical discharge is very evident; and at the same discharge the rapid increase of

\textsuperscript{x}Bulletin, University of Wisconsin, 1907 No. 173. C.B. Stewart.
Pressure rises and falls through the pump.

On figs. 12A, 12B & 12C are shown the rises of pressure head from the impeller inlets to the outlets from the corresponding guide passages for the three chosen speeds, i.e. from A to C fig. (4), or \( \frac{H_y - H_x}{\omega} \) Tables I or IV. At the inlets the pressure heads were measured for each impeller at two points as stated before and at the guide passage outlets the speed is low and the pressure readings should therefore be reliable. It is noticeable that even with three impellers and three sets of guide passages intended to be similar there is an appreciable difference between the pressure rises even at about normal delivery. The scale of rise of pressure is however fairly open and there the greatest difference from the mean varies from \( \frac{3}{8} \) to \( \frac{6}{8} \) in the cases of the three chosen speeds, a variation which seems quite reasonable owing to slight differences in angles of the impeller or guide vanes or in their curvature.

These variations show, however, the advantage of testing a multi-stage pump; an average value of the losses and efficiency of a certain intended type of construction is obtained and the effects of minor irregularities in any one stage are minimised. As stated before the power given to each impeller could not be measured separately and the pressure rises show that there would be no advantage in doing so if it were possible; so also the taking of \( h_a = \frac{H_y}{3} (e, e) \) in the analysis leads to a true estimate of the losses and efficiencies of impellers, guide passages etc., constructed as nearly as possible to certain outlines and proportions.

On the same sheets are shown, (a) the falls of pressure head in the connecting passages from guide passages outlet to the next impeller inlet, that is \( (\omega)_{a_2} (\omega)_{a_3} \) and \( (\omega)_{a_2} (\omega)_{a_3} \) the subscripts referring to the impeller number and \( \frac{1}{\omega} \) and \( \frac{1}{\omega} \) being taken from Tables I or IV; (b) the falls from the outlet from
the third or last set of guide passages to the discharge pipe at the position of the pressure gauge i.e. \( \left( \frac{p G}{\omega} \right)_3 - \left( \frac{p G}{\omega} \right) \)

Tables I or IV.

If the loss varies as the square of the discharge, i.e. \( Q^2 \) the pressure fall will vary as \( Q^2 \) also; the parabola \( 1.91 Q^2 \) is drawn among the experimental results from guide passages outlets to inlets for each speed; it will be seen that it represents very closely the law of pressure fall. For the falls to the discharge pipe, the parabola \( 1.40 Q^2 \) is drawn among the experimental points. This also represents very closely the law of pressure fall. Below the "critical" discharge the pressure falls were more irregular and, while smaller, could not be read with the same accuracy; in fact their values were sometimes less than the possible error of reading and so the values from the Tables I & IV are not in general plotted in these figures. Thus practically the losses in these parts of the pump vary as \( Q^2 \) and as would be anticipated, are independent of the pump speed.

Figs. 13A, 13B and 13C show on the lower portion of the figures, the rises from the impeller inlets to the impeller outlets (B Fig. 4) i.e. \( \frac{p_0 - p}{\omega} \) Table I; and, on the upper portion, the rises from the impeller inlets to the casing at the sides of the impellers (G or D fig. 4) i.e. \( \frac{p_0 - p}{\omega} \) Table I. Dealing first with \( \frac{p_0 - p}{\omega} \), it is seen that though among the impellers differences occur yet at the various speeds the relative position of the pressure rise curves and the proportionate differences at corresponding quantities are very similar.

Next considering the pressure rises to the casing at the sides of the impeller, i.e. \( \frac{p_0 - p}{\omega} \) as determined by the sliding tube (GG, fig. 4) and plotted in the upper portion of figs. 13A, B & C, variations among the different impellers are again seen to occur though the differences are proportionately not quite so great as with \( \frac{p_0 - p}{\omega} \).

For impellers Nos. 1 and 2 the pressure rises follow
the type of the \( \frac{P_2 - P_1}{\omega} \) curves i.e., less fall in No. 2 as discharge increases than in No. 1, but with each speed the pressure rise remains more constant in value over the whole range of discharge than in the case of \( \frac{P_2 - P_1}{\omega} \). The difference between the curves for these impellers, Nos. 1 & 2, at large discharges might be considered to be due to variable pressure at different points at the impeller outer periphery and to irregularities in recording of pressure with a high value of the absolute velocity of discharge from the impeller. But at the outlet from the guide passages the speed of the water is not high and the inlet pressure head \( \frac{P_1}{\omega} \) is the common base from which all these pressure rises are measured. It will be seen from figs. 12 A, B & C that \( \frac{P_2 - P_1}{\omega} \) for No. 2 impeller is relatively higher than for No. 1 impeller quite as much as in the cases of \( \frac{P_2 - P_1}{\omega} \) and \( \frac{P_3 - P_2}{\omega} \). No doubt a certain amount of pressure variation will occur at the impeller tip and in future experimental work it would be preferable to obtain pressures corresponding to \( \frac{P_1}{\omega} \) at various points round the circumference; but the differences between these two impellers seem to be due to inherent small variations in the details of the impeller angles or proportions, or in the guide vane angles at the tips.

But with No. 3 impeller the constancy of the value of \( \frac{P_2 - P_1}{\omega} \) is very marked; it would be expected that the curve of \( \frac{P_2 - P_1}{\omega} \) would follow the corresponding curve for No. 1 impeller as in the case of the \( \frac{P_1}{\omega} \) curve. Tests were repeated but the same results were obtained. The probability of the pressure being affected by communication with another compartment of the pump as occurred with a pressure at the impeller inlet in the previous set of tests was considered. But from Table I it will be seen that at \( Q = 1.61 \) and \( Q = 1.56 \) the value of \( \frac{P_1}{\omega} \) is higher than that at any other point in the pump. Referring to figs. 12a, 12b and 12c for \( \frac{P_2 - P_1}{\omega} \) with this impeller, it is seen to lie as a whole between the values of \( \frac{P_2 - P_1}{\omega} \) for
impellers Nos. 1 and 2; and the mean of $\frac{\beta_3 - \beta_1}{\omega}$ and $\frac{\beta_3 - \beta_2}{\omega}$ for impeller No. 3 at large discharges lies between the corresponding means for impellers Nos. 1 and 2; so that some local circumstance seems to have increased $\frac{\beta_3}{\omega}$ and reduced $\frac{\beta_1}{\omega}$ in the case of No. 3 impeller.

Fig. 14 shows, for 885 revs/min. $\frac{\beta_3 - \beta_1}{\omega}$ and $\frac{\beta_3 - \beta_2}{\omega}$ for each impeller plotted together. Similar figures are obtained for the other speeds if plotted from Tables Ia, b & c. The values of $\frac{\beta_3 - \beta_1}{\omega}$ and $\frac{\beta_3 - \beta_2}{\omega}$ are in all cases more constant with variation of discharge than these of $\frac{\beta_0 - \beta_1}{\omega}$. Comparing Nos. 1 & 2 impellers, as $\frac{\beta_0 - \beta_1}{\omega}$ falls more, so also does $\frac{\beta_3 - \beta_1}{\omega}$ but the irregularity of No. 3 impeller is very evident.

The actual value to be taken for $\frac{\beta_3}{\omega}$ (the pressure head at the impeller outlet for analysis purposes) will be between $\frac{\beta_0}{\omega}$ and $\frac{\beta_2}{\omega}$, and is considered later ($\xi$, 12).

\xi 10. Pressure variation in the casing at the side of the impeller.

The possibility of rotation of the water in the casing chamber adjacent to the impellers with the formation of a forced vortex there and its effect on leakage has been discussed; and the problem of end thrust has been investigated on the basis of an existing forced vortex. The sliding tube was fitted to determine any pressure variation with the position of G (Fig. 4). At each discharge the pressure was found to be practically constant and absolutely independent of the radius at which G was placed; slight differences of pressure head of about three inches (of water) were noticed irregularly at different radii but these differences were within the limit of possible error. In the paper previously referred to \xii 12 inch diameter discs were rotated in a rough casing, 13 inches diameter at 1200 revs/min. With

Loewenstein and Crissey "Centrifugal Pumps".


2\(\frac{1}{2}\) inches side clearance the pressure head at the circumference of the casing was practically 10 ft. of water, that at the centre being atmospheric. Assuming a parabolic law of radial distribution of pressure this would mean a rise of pressure head of about 7\(\frac{1}{2}\) feet from a point midway radially on the disc to a point on outer circumference.

The tube GG, was placed as close to the impeller as possible; the point D at which the pressure head \(P_c\) for No. 1 impeller was read is close to the impeller side; and comparing the curves of \(P_c/P_0\) with those of \(P_c/P_0\) for impellers Nos. 1 and 2 (fig. 14) the distance of the tube from the impeller does not seem to be of importance. If a forced vortex of any practical amount were in existence it is possible that a tube of so large dimensions in comparison to the size of chamber might give incorrect readings of pressure variation with the radius, but it is hardly probable that it would record no variation.

Values of \(P_c/P_0\) so constant in comparison with those of \(P_0/P_0\) are not what would be anticipated; and the question of the pressure at the side of the impeller and its relation to that just at entrance to the guide passages and to leakage is a subject for further experimental investigation. The values obtained here are considered further in \(\xi\) 12.

\(\xi\) 11. Comparison of experimental results with the ordinary Theory.

In the ordinary theory of the centrifugal pump

\[
\frac{3\alpha x_l}{g} = \frac{3\alpha_c (u - \gamma \alpha x_l \beta)}{g} = 3\left(\frac{P_c}{\omega} + \frac{\beta^2 v^2}{2g} + \frac{L_f^2}{g} + \frac{3h_L^2}{2g}\right) \tag{10}
\]

taking as before equal values of energy to be transmitted to the water by each impeller. Values of \(\frac{3\alpha x_l}{g}\) from (10) and of \(H_q\) from (1) are shown on Table III calculated for each speed, and with leakage allowance as in Table IV. These are also shown plotted on figs. 11A, 11B and 11C. Dealing with 885 rev. per
min. Fig. 15 shows these items in full lines marked C and \( H_0 \) respectively. Dealing next with energy losses; at \( Q = 1.03 \) and 885 revs/min. and with leakage = 0.06

\[
y_1 = \frac{1.09}{1.122} = 0.971 \text{ ft},\quad \text{sec}^2 = 42.5 \text{ ft},\quad \text{sec}^2,
\]
whence with \( \beta = 47\frac{\pi}{180}, \quad x_1 = 33.61 \text{ sec.} \)

\[
3\sqrt{\frac{y_2}{3}} = 133.4 \text{ ft}.
\]

For the energy losses, \( y_1 = \frac{\sqrt{x_1^2 + y_1^2}}{\sqrt{x_1^2 + y_2^2}} = 34.95 \text{ ft} \)
and (Table III) \( v = 8.10 \text{ ft/sec} \).

For the value of \( \beta_0 \) in (10), it may be considered for the present to be either \( \beta_2 \) or \( \beta_3 \).

If taken as \( \beta_0 \) then from Table I \( \) the average value of

\[
\beta - \beta_0 = 26.37 \text{ ft}; \quad \text{and from these values} \]

\[
\frac{\beta - \beta_0}{\beta_0} + \frac{\beta - \beta_0}{\beta_0} = h_1 = 26.37 + 18.08 = 44.45 \text{ ft}.
\]

Also from (4) \( h_0 = h + \frac{\beta_0}{2} \); and \( h_0 = \frac{h_0}{2} = 40.43 \text{ ft} \).

Thus leading to \( z = -4.02 \text{ ft} \).

If \( \beta_0 \) be taken as equal to \( \beta_0 \) then the average value of

\[
\beta - \beta_0 \text{ is } 22.80 \text{ ft}, \quad \text{leading to } z = -0.45 \text{ ft}.
\]

In similar manner, for various discharges, values of \( z \) have been calculated and are shown in Fig. 15 by the full line graphs marked A when \( \beta_0 \) is taken equal to \( \beta_0 \), and marked B with \( \beta_0 = \beta_0 \).

It has been found from trials with many pumps that, if the usual theory be adopted with a mean radius of impeller outlet to such a point as \( P \) in Fig. 7, values of \( \frac{3}{2} = \) so estimated show better agreement with \( H_0 \) as estimated from equation (1) i.e. from the actual power input. Taking the radius to \( R \) as 5.05 inches and \( u_r \), correspondingly reduced, the values of \( z \), calculated as above for \( \beta_0 = \beta_0 \), and for \( \beta_0 = \beta_0 \), are shown in Fig. 15 and marked A and B.

Also values of \( \frac{3}{2} \), calculated from the reduced values of \( u_r \) and \( x_0 \), are shown by the line marked C.

Thus the ordinary theory, with the outlet radius taken to be the actual impeller radius is at variance with observed results even with the lowest estimate for \( \beta_0 \); viz. \( \beta_0 \); for both \( \frac{3}{2} \) and \( 3h \) are greater than \( H_0 \); so both from the momentum principle and from the energy principle the actual

\[\text{See foot note to Equation (2)} \equiv 6.\]

\[\text{p. 10.}\]
power input is less than that required by the ordinary theory. Using the mean radius to $P$ then from curve $A$, (with $L_{0}^{2} = \frac{1}{5} L_{0}$)

$\zeta$ is negative, i.e. $5 \theta_{t}$ is greater than $H_{0}$, while the

graph of $\frac{3\zeta L_{0} x_{t}}{\theta_{t}}$ calculated with the mean and reduced radius, and marked $\zeta_{t}$ fig. 15 shows that $\frac{3\zeta L_{0} x_{t}}{\theta_{t}}$ so calculated is less than $H_{0}$; the theory is again at variance with observed results; and further it is at variance in opposite directions according as comparison is made from momentum or from energy considerations.

With the mean radius, and $\frac{L_{0}^{2}}{\theta_{t}} = \frac{L_{0}}{\theta_{t}}$, $\zeta_{t}$ is positive as shown by curve $B_{t}$, fig. 15; that is $3 \theta_{t}$ is less than $H_{0}$. $\frac{3\zeta L_{0} x_{t}}{\theta_{t}}$ calculated with the mean reduced radius remains as before and is represented by $C_{1}$, and so $\frac{3\zeta L_{0} x_{t}}{\theta_{t}}$ is also less than $H_{0}$. Curve $B_{1}$ cannot, however, reasonably be accepted as the law of variation of impeller loss with the discharge. The total losses, $H_{0} - H$, equation (5), and figs. 11A, B & C show losses decreasing up to a certain point and then increasing again, clearly illustrating the existence of shock loss; and it is improbable that the impeller should differ, in the nature of its losses from other analogous parts of the pump; also in analysis for determination of the losses the two equations

$$3 \zeta L_{0} x_{t} / \theta_{t} = H_{0} = 5 \theta_{t}$$

and

$$3 \left( \int \frac{\theta_{t}^{2}}{\theta_{t}} + \frac{\zeta_{t}^{2}}{\theta_{t}} \right) = 3 \left( \theta_{t} + \zeta_{t} \right) = H_{0} = 5 \theta_{t}$$

should both be satisfied.

### §12. Consideration of values of $\zeta_{t}$, $\theta_{t}$, and leakage.

If the impeller input power $K$ (§ 7) be accurately determined then the true average value of $x_{t}$ (denoted hereafter by $x'_{t}$) may be determined from equation (11) independent of losses as the equation is based on the rate of change of angular momentum. (a) If the full radial area at outlet from the impeller be presumed to be utilised the average value of $y_{t}$ is $\frac{\zeta_{t} x_{t}}{\theta_{t}}$ and that of $\zeta_{t}$ is $\sqrt{\frac{x_{t}^{2} + \zeta_{t}^{2}}{2}}$, $x_{t}'$ be taken to represent the tangential component of $\zeta_{t}$ as determined from (11). The average value of the angle of discharge with a tangent to the impeller at
outlet, if denoted similarly by $\beta'$ (in place of $\beta$ the actual vane angle, fig. 2) will be given by

$$
\tan \beta' = \frac{\tan \beta}{\cos \alpha - \cos \beta'},
$$

(3)

(b) The water may be presumed to leave the impeller practically at the actual vane angle $\beta$, the full radial area at outlet not being utilised for flow, and the remainder to be occupied by "dead" water and eddies. The possibility of this occurring under certain conditions has been shown experimentally, though the conditions and the proportions of the experimental apparatus were very different from those occurring during delivery of the usual type of centrifugal pump.

Considering next the values of $\frac{\beta_0}{\alpha}$ and $\frac{\beta_c}{\alpha}$ Figs. 13A, B & C and Fig. 14; at zero discharge $\frac{\beta_0}{\alpha}$ is in all cases greater than $\frac{\beta_c}{\alpha}$ and a further small rise takes place from $\frac{\beta_0}{\alpha}$ to $\frac{\beta_c}{\alpha}$ (Tables I); at zero discharge the mercury was very steady in the columns. Immediately discharge commences $\frac{\beta_0}{\alpha}$ rises more than $\frac{\beta_c}{\alpha}$; the difference remains fairly constant up to the critical discharge at which point the rise in $\frac{\beta_0}{\alpha}$ is much more marked than in $\frac{\beta_c}{\alpha}$. After discharge commences it seems them probable that the pressure head $\frac{\beta_0}{\alpha}$ may be greater than the value at the actual tip of the impeller, on account of some conversion from kinetic to pressure energy; even at zero discharge of the pump this is probable as there is also a small rise from $\frac{\beta_0}{\alpha}$ to $\frac{\beta_c}{\alpha}$. At zero discharge $\frac{\beta_c}{\alpha}$ may also be less than the pressure head at impeller tips - the difference being due to leakage at the impeller tips to the casing chambers at the sides of the impellers. The leakage from the casing to the suction side of the impeller will tend to vary as $\sqrt{\frac{\beta_0 - \beta_c}{\alpha}}$, which increases with the speed. If the leakage depends also on $\sqrt{\frac{\beta_0 - \beta_c}{\alpha}}$ then for various speeds if $\frac{\beta_0 - \beta_c}{\alpha}$ be plotted against $\frac{\beta_0 - \beta_c}{\alpha}$, the points should lie on a straight line. For zero discharge values of $\frac{\beta_0}{\alpha}$, $\frac{\beta_c}{\alpha}$ and $\frac{\beta_c}{\alpha}$ were obtained for 502, 595, 695, 788, 885, 983, 1079 & 1200 revs/min.

Fig. 16 represents the results plotted as above, - revs/min marked at each point. At 885 revs. $\frac{R_o - R_c}{c}$ is on the average about 2$\frac{1}{2}$ feet, and taking this as the head causing leakage through the space between impeller tips and casing which from measurement was found to average about 1/50 inch, the leakage would be 0.70 cub. ft. per sec. with a coefficient of discharge of 0.6. This is slightly more than 5 to 6 per cent of the normal delivery which is generally stated to be the average leakage of centrifugal pumps; though no details of experimental work on the subject seem to have been published.

The pressure at impeller outlet for analysis, $\frac{p_y}{c}$, is then probably less than $\frac{p_o}{c}$ on account of some conversion from kinetic to pressure head, is greater than $\frac{p_c}{c}$ on account of leakage at the impeller tips. From the value of $\frac{R_o - R_c}{c}$ at zero discharge and speed mentioned above $\frac{p_y}{c}$ is probably greater than $\frac{p_c}{c}$ by about 2 to 2$\frac{1}{2}$ feet. For small deliveries and up to about normal delivery this is very nearly equivalent to taking $\frac{p_y}{c} = 1/2(\frac{R_o + R_c}{c})$

For large deliveries little conversion of energy from $\frac{p_o}{c}$ to $\frac{p_y}{c}$ takes place (see Table I) and so probably also from impeller tip to pressure hole for $\frac{p_o}{c}$, and then the value of $\frac{p_o}{c}$ will approach $\frac{p_c}{c}$. This means (see fig. 14) that it approaches also $\frac{p_c}{c}$ (omitting the already mentioned exceptional case of impeller No. 3) and that leakage tends to a zero value. In the case of impeller No. 3 as mentioned in $\xi, \eta, \eta$, the mean of $\frac{R_o - R_c}{c}$ and $\frac{p_c - p_o}{c}$ is quite regular in comparison with impellers Nos. 1 and 2; and hence if $\frac{p_o}{c}$ be taken equal to $\frac{1}{2}(\frac{R_o + R_c}{c})$, the effect of the irregularities in $\frac{p_o}{c}$ and $\frac{p_c - p_o}{c}$, which seem to occur with this impeller, tends to be neutralised.

Analysis has therefore been carried out with $\frac{p_o}{c} = \frac{1}{2}(\frac{R_o + R_c}{c})$; it should be very nearly the real value at the normal delivery where the losses in the impellers will be near their minimum value and where the efficiency of the guide passages is close to its maximum value. The nature of flow through the impellers has
been taken in the analysis to be according to (a) at the commencement of this paragraph; and the effect of a change of estimate of \( \frac{A_p}{K} \) and of flow through the impeller according to (b) are considered in \( \xi 13 \).

The leakage allowed at 885 revs. is 0.6 cubic ft/sec. and at 695 and 1079 revs. as in Table IV. The values so chosen are practically proportional to \( \sqrt{\frac{A_p}{K}} \) or \( \sqrt{\frac{A_p}{F}} \) at zero discharge for the various speeds and are also about 6 per cent of the normal delivery. As this, however, is a quantity not quite constant with the discharge and of uncertain value the analysis has been carried out with the above allowances and also with no allowance at all for leakage.

\[ \xi 13 \] Losses in the Impellers.

Table IV gives the values of the various pressure heads made up from Table I and with \( \frac{A_p}{K} \) as decided on from \( \xi 12 \), and also the delivery and leakage allowance made.

Taking at 885 revs., \( Q = 1.0 \) the average value of \( \frac{A_p}{K} \) for the three impellers is 24.58 ft. From equation (1) with

\( K = 15.02 \) H.P., \( h_o = 121.3 \) ft \( \times h_o = 40.43 \). For \( x'_r = \frac{\tau_o x'_r}{3} \) (3), \( x'_r \) being the average tangential component of \( v_o \) as stated in \( \xi 12 \) and thence \( x'_r = 30.44 \) ft./sec.; \( y_r = \frac{Q + \tau_o}{4} = 9.71 \) ft/sec, and thence \( v_r = 31.95 \) ft./sec. \( v = \frac{Q}{2}(x'_r) = 8.10 \) ft./sec. Hence (equation 4)

\[ h_r = \left( \frac{A_p}{K} \right) \left( \frac{Q + \tau_o}{x'_r} \right) = (24.58) + (15.94 - 1.03) = 39.49 \) ft.

and \( h_o = h_r + 2, \) giving \( z = 0.94 \) ft.

For the average angle of discharge from the impeller tip (relative to the impeller tangent at the tip) using (13) \( \xi 12 \), \( \beta' = 39^\circ \) and for the angle of the average absolute velocity \( v_o \) with a tangent at the impeller tip \( (\theta, fig.2) \)

\[ \tan \theta = \frac{y_r}{x'_r}, \] whence \( \theta = 17^\circ.40^\circ \).

In similar manner for the three speeds and for discharges above the "critical" these quantities and angles have been estimated and are shown on figs. 17 and 19 while fig. 18 shows \( \xi z \) for 885 revs. only. Similarly with no leakage allowance (involving a higher value of \( h_o \) since \( v_0 > \xi \) in (1)), the
corresponding quantities are plotted in figs. 17 A and 19 A (same sheet as fig. 19)

Dealing next with losses at discharges below the critical, reference to figs. 11A, 11B and 11C show an immediate change in the general course of the graph of $H_o$ calculated from (1), $H_o$ estimated from (1) must rise to a high value as Q diminishes (to infinity if the leakage at zero discharge be neglected) - since the shaft horse power at zero discharge is greater than that required for disc friction and mechanical losses; and the hydraulic efficiency will be practically zero at zero discharge. Reference to fig. 10 shows also a change in the general course of the shaft horse power graph at the critical discharge and so also in the impeller horse power curve at $\theta$, the values of the latter power being merely the shaft horse power less the constant disc friction and bearings losses. A source of loss of a different nature seems to have come into action. This additional loss has been termed the "impeller tips" or "the churning" loss. Of the total loss in the impeller below the critical discharge, part must be of the same nature as with discharges above the critical, the remainder being the "churning" loss.

Above the critical discharge $H_o$ follows practically a straight law, see figs. 11A, 11B, 11C and also Fig. 30. Continuing the straight lines $c\theta$ in these figures to 'a' the ordinates to $b\alpha$ represent the values of $H_o$ if causes of loss and conditions of flow in the impellers were the same as with discharges greater than the critical, i.e. if the cause of churning loss were absent. The lines $c\theta\alpha$ cut the ordinate at zero discharge at heads practically varying as $\frac{\mu^2}{\nu^3}$, and the variation of $H_o$ with Q is similar to that of $\frac{\mu^2}{\nu^3}$; theory and actual results may probably be related by some experimental coefficients as in all hydraulic formulae. The line $b\theta$ is used here as a basis for differentiating between the ordinary impeller loss and the additional "churning" loss. In reckoning the hydraulic efficiency it is taken, below the critical discharge, as $\frac{H}{H_o}$
where $H'_{0}$ is the ordinate to $Z_0$ in these figures (shown in brackets in Table III); and if $H_0'$ represents the "churning" loss then $H_0' + H_0 = H_0$. The question of the two types of loss in the impellers at small discharges is considered further in $\xi_{20}$.

With $H_0$ from these figures the values of $Z_0$ below the critical discharge are plotted in fig. 18 and are shown by the broken line $pqrs$. This broken line for $Z_0$, in place of a continuous curve, is due to the peculiarity of the pump at the critical discharge. Not all pumps show this feature (though all do show the additional churning loss); if this feature were absent the graphs of $H, H_0', H_0'', \ldots$ etc., in figs. 11, 12 and 13 (A, B & C) would be continuous curves below the critical discharge in place of broken lines. Estimating $H_0$ from such continuous curves and adopting the value so found the calculated values of $Z_0$ below critical discharge would lie on the curve $ps$. The broken line is replaced by this curve in this analysis merely to determine the equation to the impeller loss curve, the equation holding only above critical discharge; though probably it would hold over the whole range if the cause of the abrupt change of head at critical discharge were removed ($\xi_{19}$).

With this substitution and with $H_0'$ in place of $H_0$ below the critical discharge, the values of $Z_0$ for the three speeds are shown in fig. 19. Considering their form only the effect of shock loss is clearly seen. The minimum values occur practically at $Q = .61, .74 & .92$ — values practically proportional to the speeds. The curves cross each other as they should from consideration of the magnitude of the sudden change of velocity at the entrance to the impeller with the same discharge, but with different impeller speeds. The values of $Z_0$ at the minimums should increase progressively with the speed; that at 885 revs. is rather high, but considering the small values of the losses, the error is not large; in fact as all
values of \( L \) are estimated from actual individual readings during tests, and not from average results from graphs (except for \( H_0 \) as explained) the uniformity of the curves lying among the calculated values is much better than would be anticipated.

With regard to the actual value of \( L \), during tests water was run through the pump from an elevated tank, the impeller being stationary. The drop of pressure from impeller inlet (at A fig. 4) to the casing at the side of the impeller tips was read for Nos. 2 and 3 impellers. The drops were as below (\( Q = 0.75 \) was the maximum flow obtainable)

\[
\begin{array}{cccccc}
Q & 0.3 & 0.46 & 0.51 & 0.62 & 0.69 & 0.75 \\
\text{Fall through} & \\
\text{im} & 0.4 & 0.5 & 0.8 & 0.9 & 1.2 & 1.3 \\
\text{No.2} & \\
\text{" No.3} & 0.3 & 0.6 & 0.7 & 0.6 & 0.6 & 1.0 \\
\text{ft.} & \\
\end{array}
\]

The pressure falls to the impeller outlets B (fig. 4) were absolutely irregular as would be expected from the angle of discharge of the water from a stationary impeller. For flow through the stationary impeller as an ordinary pipe or channel

\[
\frac{P_0}{2g} + \frac{V^2}{2g} = \frac{P_0}{2g} + \frac{V^2}{2g} + L,
\]

\( L \) being the loss and \( V \) the velocity at discharge. Taking \( V = \frac{Q}{\sin \alpha} \) and the fall of pressure head with \( Q = 0.75 \) to be 1.3 ft. then \( L = 0.55 \) ft. and with \( Q = 0.46 \) and fall as .60 ft. \( L = 0.26 \) ft. These are respectively 0.42 \( \frac{V^2}{2g} \) and 0.47 \( \frac{V^2}{2g} \). The loss in the wheel passages of a turbine is frequently stated to be from 0.1 to 0.2 \( \frac{V^2}{2g} \), \( V \) being the relative velocity on leaving these passages. The above values do not seem too high when account is taken of the loss at entry to the impeller on account of the sudden change of direction with a stationary impeller.

Above fig. 19, on the same sheet, is fig. 19 A showing the losses in the impeller (while pumping) with no leakage allowance included, all calculated in the manner described when such allowance is made. They present the same characteristics as those with leakage allowance but the losses are larger. If the losses
obtained with flow through the stationary impeller be compared with figs. 19 & 19A it will be seen that they tend to be above those of fig. 19 and below those of fig. 19A. The losses with stationary impeller being merely an approximation to possible losses can hardly be used as a criterion of the leakage estimate, but they show that the results obtained by this analysis and estimated from power input are a close approximation to the real values; the impeller losses at their minimum values are so small that it is only possible to place them within certain limits, but their law of variation with change in discharge and speed is clearly shown in these figures.

From equations (4) and (5), \( h_0 = h + l_i = h_9 + 2Z_2 \).

\( h_9 \) depends on the value of \( k \); \( h_9 \) depends on \( \beta_9, v_9, \beta_2 \) and \( v_2 \), thus both are independent of \( \beta_6 \) and \( v_6 \), which are debatable quantities. And as \( l_1 + Z_2 = h_0 - h_9 \) any increase of \( l_1 \) involves a corresponding decrease in \( Z_2 \). Of the total loss \( l_1 + Z_2 \), it is only a question of how much is \( l_1 \) and how much is \( Z_2 \) that is affected by the values taken for \( \beta_6 = v_6 \). This will affect any estimate of guide passage efficiency; and so the curves of fig. 19 are analysed further even though probably rather low, and the effect of an increase in \( Z_2 \) is considered later when dealing with the guide passages efficiency (§17).

§14. Losses in Guide Passages

Values of \( \frac{Z_2}{h_9} \) are shown in Table IV (or Tables I).

Allowing leakage and taking as before 885 revs. \& \( Q = 1.03 \),

\( \frac{Z_2}{h_9} = \frac{Q}{Q_9} = 3.8 \text{ ft.} / \text{sec.} \). The possible rise of pressure head in the guides is \( \frac{Z_2}{h_9} = 15.72 \text{ ft.} \). From Table IV the average rise for the three impellers is 9.65 ft. and thus the loss \( Z_2 = 9.87 \text{ ft.} \). The "conversion" efficiency \( \varepsilon = \frac{Z_2}{h_9} = \frac{9.87}{15.72} = .627 \); while the hydraulic efficiency of the impellers and guides together is \( \eta = \frac{h_9}{h_9} = \frac{(\beta_3 + v_3)^2 + (\beta_2 + v_2)^2}{h_9} \). For the three impellers with the values of \( \beta_3 \) and \( v_3 \) from Table IV, and with
\[ v_2 = 3.8 \text{ ft./sec.}, \quad v = 8.10 \text{ ft./sec.} \quad \text{and} \quad \gamma_0 = 40.43 \text{ ft. as in} \ \text{(13)} \]
\[ \epsilon_2 = \frac{33.62}{40.43} = 0.822. \] i.e. slightly higher than that of the whole pump (Table III) on account of further losses in the connecting passages etc., Values of \( Z_1, Z_2, Z_2' \), and \( \epsilon_2 \), so calculated for various discharges and for 885 revs. only are shown on fig. 18; below the critical discharge \( \gamma_0' \) is again used in place of \( \gamma_0 \); and the curve is shown replacing the broken line as with the impeller loss.

For the three speeds, losses so calculated are shown by the full lines in fig. 22, and the values of \( \epsilon_2 \) and \( \epsilon_1 \) on fig. 23; with no leakage allowance, fig. 24 shows the values of \( Z_1 \) plotted and fig. 25 those of \( \epsilon_1 \) and \( \epsilon_2 \). The effect of shock loss and all the characteristics of the position of the minimum values etc., as depending on speed referred to in the impeller loss curves are clearly shown in figs. 22 and 24; but the relative position of the curves is better as the losses are larger and possible errors in observed results are not proportionately so large.

\( \S \) 15. Losses in Connecting passages and Volute.

The falls of pressure in the connecting passages from guide passages outlets to the next impeller inlet have been referred to in \( \S \) 9 and are plotted in figs. 12A, 12B & 12C. The falls are seen to be practically represented by \( 1.91Q^2 \) and so are independent of the pump speed. As in \( \S \) 6,
\[ (\frac{P_0}{\rho} + \frac{v_2^2}{2}) \gamma_2 + \gamma_1 \epsilon_2 = (\frac{P_0}{\rho} + \frac{v_1^2}{2}). \]
also \( v = \frac{Q}{d} \) and \( v_2 = \frac{Q}{c} \) (leakage water not passing the points where \( v \) and \( v_2 \) are measured). With values of \( \epsilon_1' \) and \( \epsilon_2' \) as in \( \S \) 3, \( v = 7.85Q \) and \( v_2 = 3.69Q \); taking also \( \frac{P_0}{\rho} - \frac{1}{2} = 1.91Q^2 \) and substituting in the above equation, \( \gamma_2 = 1.16Q^2 \) with the same difference from actual test values as \( 1.91Q^2 \) has from the plotted results in figs. 12A, B & C i.e. within the limit of error of readings pressure heads, or about 3 inches of water.

For the falls from the outlet of the third set of guide
passages to the discharge pipe plotted on the same sheets they are a little more irregular but still are represented in general trend by the parabola $1.40 Q^2$. By usual principles

$$\frac{p_d}{Q} + \frac{u_d^2}{2} + 2u + 2d_0 = \left( \frac{p_g}{Q} + \frac{u_g^2}{2} \right)$$

With $u_g = \frac{Q}{a_d}$ and $u_d = \frac{Q}{a_d}$, $u_g = 3.69 Q$ and $u_d = 5.1 Q$ and substituting in the above equation (with $\frac{p_g - p_d}{Q} = 1.40 Q^2$)

$$2u + 2d = 1.20 Q^2$$

Thus the losses in these connecting passages and in the collecting chamber and volute follow practically the laws of flow in pipes or channels.


The losses in these parts plotted on figs. 19, 19A, 22 and 24 show clearly the effect of shock loss; the outline of the curves suggest that the losses might be expressed by an equation of the type $l = a u^2 - b u + c Q^2$. In all cases the minimum values occur at values of $Q$ practically proportional to the speed. For the minimum value of $l$ in the above equation $Q = \frac{b}{2c} u$, and denoting this value of $Q$ by $Q_m$ then

$$Q_m = \frac{b}{2c} u, \quad \text{where} \quad \frac{b}{2c} = k$$

Thus probably

$$l = a u^2 - 2 k u + c Q^2$$

For the guide passages (with leakage allowance) when $Q = 0$, $a = \frac{b}{c}$ and for the 3 speeds it is found that $a = 0.0104, 0.0100$ and $0.0106$; the minimum losses occur at $Q = 0.78, 0.97$, and $1.22$ giving as values of $k$, $0.0236$, $0.0226$ and $0.0234$ respectively. The curves, therefore, seem to follow the law of equation (14); but to agree best with the losses as estimated in the analysis and shown by the full line curves - especially beyond the critical discharge, the most essential portion for consideration - 'a' has to be slightly
increased. Adopting $a = .01125$, $k = .0233$ & $c = 14.6$

$$Z_g = .01125 \, u^2 - 0.68 \, u \times Q + 14.6 \, Q^2 - - - - (15)$$

Values from this equation are shown dotted on fig. 22 and are seen to follow closely the actual curves above the critical discharge, and for the values of $Q_m$, since $k = .0233$, $Q_m$ were practically the same points as those of the curves of the losses calculated from the test results. In the same manner for the impellers the equation

$$Z_g = .00157 \, u^2 - .1472 \, u \times Q + 4.05 \, Q^2 - - - - (16)$$

is plotted on fig. 20 in dotted lines, the curve through the actual calculated losses being shown by the adjacent full lines; in (16) $k = .0182$ and $c = 4.05$ and so the minimum values of from (16) occur at $Q_m = .61$, .77, and .94 thus agreeing practically with those of full line curves. Since the losses $Z_g$ can be so closely represented by an equation of the type of (14), there is seen clearly the effect of shock loss superimposed on a loss varying as the square of the discharge. For, drawing a tangent parabola $y = BQ^2$ from the origin to the loss curves in figs. 20 and 22, and taking $Q_o$ as the discharge at the tangent point and equating the slopes there,

$$2cQ_o - 2kcu_o = 2BQ_o - - - - - - - - - - - - - - - - (17)$$

and equating the ordinates there

$$au^2 - 2kcu_oQ_o + cQ_o^2 = BQ_o^2 - - - - - - - - - - (18)$$

and from these two equations $Q_o = \frac{au^2}{kc}$

$$- - - - - - - - - - - - - - - - (19)$$

and so, $Q_o$ varies as the speed.

Substituting for $Q_o$ in $Z = au^2 - 2kcu_oQ_o + cQ_o^2$ and denoting the value of $Z$ corresponding to $Q_o$ by $Z_o$

$$Z_o = au^2\left(\frac{a}{k^2c} - 1\right) - - - - - - - - - - - - - - - - (20)$$

and hence for $B$,

$$au^2\left(\frac{a}{k^2c} - 1\right) = B\left(\frac{au}{kc}\right)^2$$

giving

$$B = c\left(1 - \frac{k^2}{ae}\right)$$

The total loss $Z$ may, therefore, be looked on as (1)

† Fig. 20 shows the $Z$ curves of fig. 19, plotted separately.
a loss \( y \), varying as \( Q^2 \), where \( y = \beta Q^2 = c \left( \frac{\delta V}{\alpha} \right) Q^2 \) and (2) a loss \( L_s = z - y \) depending on shock and of value
\[
L_s = \alpha \omega^2 \alpha + c Q^2 - c \left( \frac{\delta V}{\alpha} \right) Q^2 \left( \frac{\omega^2}{\alpha} \right) = (\alpha \omega - \kappa c Q)^2
\]
For the guide passages from equation (15) the tangent parabola is \( y = 4.34 Q^2 \) and is shown on fig. 22; for the impeller losses on fig. 19, the three curves are shown separately on fig. 20; the tangent parabola from equation (16) is \( y = 0.596 Q^2 \), and is shown on the latter figure.

From fig. 20 it is seen that at 885 revs/min. the values of \( L_s \), determined from Equation (16) are slightly lower than those obtained by analysis of the observed results; while at 1079 revs/min, and at the discharge for minimum loss in the impellers (16) gives larger values than those determined from the analysis. The differences are small — only about 0.2 ft. and within the possible error of observations.

On fig. 21A (above fig. 21) are shown curves of the type of equation (14) with constants such that the minimum loss at 885 revs/min. is raised by about 0.2 ft. above the values obtained from analysis of the observed results. The tangent parabola \( y = 0.93 Q^2 \) is drawn on the same figure. This loss \( y = 0.93 Q^2 \) agrees closely with values found for the losses when water was passed through the stationary impeller (\( \xi \), 13) viz. 0.55 ft. with \( Q = 0.75 \) cub. ft/sec. and 0.26 ft. with \( Q = 0.46 \) cub. ft/sec.

The loss \( y = \beta Q^2 \) (i.e. the loss in the impellers exclusive of shock loss) may be expressed as \( \frac{F V}{2g} \) where \( V \) is the velocity of discharge from the impeller at the outlet. Taking \( V = \frac{y}{s m \beta} \) the impeller loss exclusive of shock loss, is found to be 0.26 \( \frac{V}{2g} \) with \( B = 0.596 \), and 0.43 \( \frac{V}{2g} \) with \( B = 0.93 \).
so that while the plotted curves of loss in the impeller clearly show the shock loss by the relative position of the curves for different speeds at any one discharge yet the actual value of the total loss is so small (except at large discharges) that it is only possible to place this \( y \)-loss within certain limits.

Comparing the results for no shock loss from equations (15) and (16) with the actual vane angles; for no shock loss at guide vane tips at the entry \( Q_0 = \frac{c w}{\kappa c} \) from (19), and with \( \alpha, \kappa, c \) as in equation (15) \( Q_0 \) is 1.10, 1.41 and 1.71 for 695, 885 and 1079 revs/min. respectively. Referring to fig. 17 these are marked on the graph of \( \theta \); the angle of the absolute velocity of discharge (\( \theta \) fig. 2) is \( 25^\circ, 25^\circ \) and \( 24^\circ \) for the three speeds. The angle of the guide vanes at the tip (\( \theta_0 \)) is shown on fig. 8; the guide vanes are there shown to be practically involutes and the angle \( \theta_0 \) to be about \( 25^\circ \). Thus the discharge for no shock determined from the losses agrees well with the actual guide vane angle; if no leakage allowance be made, the tangent parabola to the loss curves deduced from tests is shown on fig. 24. The points of tangency and so of no shock loss, are practically at \( \Theta = 1.06, 1.36 \) and 1.70 cub. ft/sec. Referring to fig. 17A the corresponding values of \( \theta \) are \( 22^\circ, 22^\circ \) and \( 23^\circ \). The angles are not quite so well in accordance with \( \theta_0 \) as in the case allowing for leakage; and on account of slight taper of the vanes at the tips, while the general angle is \( 25^\circ \), yet the angle which can truly be called the vane tip angle is not quite definite.

No argument as to the nature of flow through the impeller could be founded on this agreement of no shock discharge and the value of \( \theta_0 \); for even with the ordinary theory that \( x = \alpha - \frac{\gamma \beta}{\kappa} \) and \( \beta = 47^\circ \) the values of \( \theta \) do not vary much from those obtained by using \( x' \), as determined from \( H_0 \), in place of \( x \); but the agreement when the value of \( Q_0 \) is determined from curves of loss in the guide passages seem to show that the losses may be divided into two types one caused by shock, and one depending
on \( Q^2 \) as with ordinary hydraulic channels.

Dealing next with the impellers for no shock \( Q_o = \frac{a v}{k c} \)
and with the values of \( a, k, \) and \( c \) in (16), \( Q_o = 0.71, 0.906 \) and 1.10 (fig. 20) for the three speeds

At entry the radius is 4.7 inches giving \( \omega = \frac{g l}{H}, \)
\( \nu = \frac{Q_o + \frac{g l}{H}}{105} (c, b) \) and \( \tan \varphi = \frac{v}{u}. \) For the 3 speeds it is thus found that \( \varphi' = 26^\circ, 26^\circ, \) and \( 27^\circ \) respectively and \( (180^\circ - \varphi') = 153^\circ, 153^\circ, \) and \( 153^\circ; \) while the actual vane angle is \( 149^\circ \) by measurement from the impeller. These values are in as close agreement as might be expected on determining them from the impeller loss curve as the losses are so small; to enter the impeller at \( 149^\circ \) (relative) the water would require to have a small tangential component in the direction of the impeller rotation; in fact an increase in the losses of the impeller near the minimum values as in fig. 21A would so alter the value of \( Q_o \) for no shock that the angle \( (180^\circ - \varphi') \) calculated from \( \nu \) is practically \( 149^\circ \).

Any increase of \( I \), involves a decrease of \( L_g (413) \), but \( L_g \) being large in comparison with \( I \), such alterations in \( I \) will not materially affect the value \( Q_o \) of discharge for no shock loss at the entry to the guide passages; as stated before even the assumption of no leakage only reduces \( Q_o \) from 1.41 to 1.38 at 885 revs.

\( \xi(6a) \) In the absence of experimental evidence the value of the shock loss is usually taken to be proportional to the square of the sudden change in velocity on meeting the impeller or guide vane.

The losses so estimated are given by -
for the impeller entrance - \( L_s = \left( \frac{v - v \cot \varphi}{2g} \right) \) and for the guide vane tips - \( L_g = \left[ \frac{v - v \cot (\beta + \cot \theta)}{2g} \right] \)
Since \( \varphi, \beta, \) and \( \theta \) are constant these are of the same form as equation (21). With the values of \( a, k, \) and \( c \) of equations (15) and (16) equation (21) becomes -
for the impeller entrance - \( L_s = \left( 0.75 w - 0.74 v \cot (\varphi) \right) \)
\( \delta \) \( \nu, \) being the value of \( \nu, \) and \( \varphi' \) that of \( \varphi, \) corresponding to \( Q_o. \) See Fig. 2.
and for the guide vane tips; \( \beta_s = \frac{0.94 \omega - 0.94 y_s (c_t \beta \cos \theta_s)}{2g} \) 

The corresponding equations \( \alpha \) and \( \alpha' \), \( \beta \) and \( \beta' \) have different values of \( \nu \) and \( y_s \) for no shock loss. For the impeller \( \beta_s \) from \( (a) \) is zero if \( Q = 1.14 \) and from \( (\alpha') \) if \( Q = 0.906 \) cub. ft./sec; while for the guide vane tips \( \beta_s \) from \( (b) \) is zero if \( Q = 1.56 \) and from \( (\beta') \) if \( Q = 1.41 \) cub. ft/sec. The graphs of the corresponding curves must cross and so the form of these equations is not suitable for purposes of comparison.

Comparing, however, the values of \( \beta_s \) from corresponding equations \( (\alpha = \alpha', \beta = \beta') \) when the value of \( \nu \) or of \( y_s \) in each case differs by the same amount, from that at which shock loss is zero, let \( y_s \) be the value of \( y_s \) in \( (\beta) \) when \( \beta_s = \alpha \) so that \( y_s = \frac{\omega_s}{c_t \beta + c_t \theta_s} \n \)

At a discharge \( y' \) greater or less than \( y_s \) (so that \( y = y_s + y' \)) then from \( (\beta) \)

\[
\beta_s = \left[ \alpha - (y_s + y') \left( c_t \beta + c_t \theta_s \right) \right] / 2g \]

and thus if the discharge differs by a certain amount from that at which no shock loss takes place then the shock loss is proportional to the square of that amount and is the same for all impeller speeds - a result which follows also from the usual figure from which the value of \( \beta_s \) in \( (\beta) \) is obtained.

In the same manner from equation \( (\beta') \)

\[
\beta_s = \left[ 0.94 (y_s - y_s) (c_t \beta + c_t \theta_s) \right] / 2g \]

and corresponding results will hold with \( (\alpha) \) and \( (\alpha') \); so, for equal variations of \( Q \) from that at which no shock loss takes place, the shock loss as determined from the analysis of this series of tests is \((0.94)^2\) (or 0.88) of that estimated on the usual assumptions and as expressed in \( (\alpha) \) and \( (\beta) \); and thus as \( Q \) departs from the value for which no shock loss takes place the variation in shock loss may be taken to be represented approximately by the usual equations \( (\alpha) \) and \( (\beta) \).
in the case of impact with a solid vane; in the case of the pump it is 0.88 of that represented by equations (a) or (b).

But for discharges measured from zero the difference of \( L_5 \) as estimated from (b) or (b') is more marked. For (b) gives a higher value for the no shock discharge than (b') gives; and the parabola represented by (b) is displaced to the right (i.e. in the direction of Q increasing) relative to that represented by (b'). At \( Q = 0.70 \) and 885 revs/min. the value of \( L_5 \) determined from (b) is 8.58 ft; while the value of \( L_5 + L_3 \) (the total loss in impellers and guide vane passages) as experimentally determined is only 7.56 ft; and the value of \( L_5 + L_3 \) is independent of the disagreeable values of \( \beta \) and \( v_i \).

Equation (b') may be written: \[ L_5 = 0.72 \left( \alpha - 1.11 \gamma \frac{\cos \beta + \cos \theta}{\gamma} \right)^2 \]

Comparing this with (b); in the ordinary expression for the shock loss, \( \left( \frac{\alpha - v_i (\cos \beta + \cos \theta)}{\gamma} \right) \) is the sudden change of velocity as obtained from the usual triangle of velocities for the impeller outlet. The actual tangential component of \( v_i \) being less than that of the usual theory (since \( \gamma \) is less than \( \frac{3 \alpha_{ij}}{2} \)), the angle \( \beta \) is increased; so the sudden change of velocity is reduced if \( \alpha \) is less than \( \theta \), and is increased if \( \alpha \) is greater than \( \theta \). Hence the coefficient \( \gamma \) should be greater than unity as it is in the above equation; and the coefficient \( \alpha \) being less than unity, the loss is less than that due to the sudden change of velocity.

For the impeller, (a') becomes: \[ L_5 = 0.562 \left( \alpha - 1.25 \gamma \cos \phi \right)^2 \]

and illustrates the same points; but the values of the numerical factors must be approximate only just as with the losses in the impeller due to other causes than shock which, as stated previously in this paragraph could only be estimated within certain limits \( v_i = 0.26 \) to \( 0.46 \sqrt{\frac{a_{ij}}{2a_i}} \).

\[ \Sigma 17. \] "Conversion" efficiency of the guide passages.

This is shown on fig. 23 with leakage allowance and on fig. 25.
with no leakage allowance. The main loss of the pump is in the guide passages. The possible efficiency of conversion is estimated at 70 to 75 per cent; the maximum value found by this analysis is approximately 66 per cent. If the efficiency be reckoned by the ordinary theory, then

\[ \varepsilon = \eta - \gamma \cos \beta \]

At \( Q = 1.03 \) & 885 revs/min. and with \( \gamma = 9.71 \) ft./sec. as before and \( \beta = 47^{1/2} \), \( \varepsilon = 33.61 \times \frac{\gamma^2}{2g} = 19.61 \) ft.

The possible rise of pressure head is then (with \( \nu = 3.8 \) ft/sec as \( \varepsilon \), 14), \( \frac{\nu^2}{2g} = 19.61 - 0.22 = 19.39 \) ft. The actual rise is 9.85 ft and so \( \varepsilon = 9.85 \times 0.508 \) only, as against 0.627 in \( \varepsilon \) 14.

Referring to numerical values in \( \varepsilon \) 14 if the conversion efficiency were 75 per cent the actual rise of pressure head would have to be \( 0.75 \times 15.72 = 11.80 \) feet and, since \( \frac{\beta}{\nu} \) is a definite reliable pressure head, to obtain this value of \( \frac{\beta}{\nu} \) would have to be reduced by 11.80 - 9.85 = 1.95 ft. This would reduce \( \gamma \) to 5.87 - 1.95 = 3.92 ft, and correspondingly \( \zeta \) (\( \varepsilon \) 13) would be increased to 0.94 + 1.95 = 2.89 ft. This value of \( \zeta \) seem excessive in proportion to \( \gamma \); and to estimate \( \frac{\beta}{\nu} \) as 1.95 ft. lower than that adopted in this analysis, i.e. \( \approx 22.63 \) feet, brings it down almost to \( \frac{\beta}{\nu} \) (fig. 14) which seems lower than can be reasonably taken as an estimate of the pressure head at the impeller outlet.

An increase in the estimated values of \( \gamma \), as in fig. 21A will increase the estimated conversion efficiency to a slight extent. At \( Q = 1.03 \), \( \gamma \) is about 0.4 ft. greater than in fig. 21; this involves a decrease of the same amount in the estimated value of \( \frac{\beta}{\nu} \) and a corresponding increase in \( \frac{\beta}{\nu} \) (fig. 14). Therefore (See \( \varepsilon \) 14) \( \zeta = \frac{10.25}{15.72} = 0.652. \) But while the efficiency estimate may vary slightly it is seen from a comparison of figs. 22 & 23 that the value of the discharge for no shock loss is much larger than that at minimum loss in the guide passages or at maximum conversion efficiency; and that a lower value of the guide vane angle (\( \theta \)) would give higher efficiency of conversion and also of
the pump. For with \( \theta \) less, \( Q \) would also be less and at maximum conversion efficiency the shock loss and the \( y \)-loss (6.16) would each be less in value.

From the guide vane loss curve and equation (21) an estimate may be made as follows. Since all energy is given only in passing through the impeller, \( H_0 \) is a quantity determined by the impeller only; and therefore \( H_0, \theta', \) and \( \theta \) are independent of guide vane angles and all other passages (though these must influence \( H \) the final head obtained from \( H_0 \)); and so at any chosen discharge \( \theta' \) and \( \theta \) are independent of the guide vane angles.

Referring to fig. 17 it will be seen that above the critical discharge the graphs of \( \theta \) on \( \theta \) are very nearly straight lines passing through the origin.

Thus for discharges \( Q \) and the corresponding angles \( \theta \),

\[
\theta' = \frac{\theta}{\theta'}
\]

and when \( \theta' = \theta \) and \( \theta = \theta_0 \) in the pump

and thus

\[
\frac{Q}{H_0} = \frac{\theta}{\theta_0}; \text{or } Q = \frac{H_0 \theta}{\theta_0}.
\]

Also \( Q = \frac{\alpha_\theta^2}{\alpha_\theta^2} \) from (19) and substituting in equation (21), on reduction

\[
I_s = \alpha_\theta^2 \left( 1 - \frac{Q}{H_0} \right)^2.
\]

This gives the variation of \( I_s \) with differences between \( \theta \) and \( \theta_0 \).

With a change in guide vane angle 'a' may vary slightly. For at zero discharge the loss in the guide passages is from (21) or (22), \( I_s = \alpha_\theta^2 \) and this must contribute to the final value of \( H \) at zero discharge. With different pumps \( H \) at zero discharge varies with widely different types of impellers, guide vanes etc., from about \( 0.9 \frac{\mu_\theta^2}{\alpha_\theta^2} \) to \( 1.1 \frac{\mu_\theta^2}{\alpha_\theta^2} \). For a small change in the guide vane angle only 'a' therefore does not probably vary much, Though an allowance may be made in the following estimate for such a small change.

Taking a guide vane angle of \( 19\frac{1}{2}^\circ \), then from fig. 17 this corresponds to the angle of \( \theta \) at \( Q = 1.10 \) for 385 rews/min.

Taking 'a' as before, \( \theta_0 \) at \( 19\frac{1}{2}^\circ \) and the value of \( \theta \) for various discharges from figs 17, \( I_s \) may be obtained from (22). Or, using (19) with \( \theta_0 = 1.10, \kappa c = .434, \) 'a' being as before;
also as the 

\[ y = 4.34Q^2 \]

\[ c = 21.07, \text{ since } c(1 - \frac{d^2}{a^2}) = 4.34 \]

The equation for \( L_g \) then becomes on reduction

\[ = 0.01125 \eta^2 - 0.87 \quad \ldots \quad 21.07 Q^2 \quad \ldots \quad (23) \]

in place of (15)

From either of these methods the values of \( L_g \) are plotted in dotted lines in fig. 26 along with the values of \( L_g \) previously deduced by analysis from the tests; and it is seen that even if \( 'a' \) in the above calculation were increased by 5 per cent yet the values of \( L_g \) are still much lower than previously. The other losses in the pump being the same, and allowing for the decrease in \( L_g \), the new value of \( H \) is shown in dotted lines by adding to the experimental value of \( H \), thrice the value of the reduction in the guide passages loss; similarly the increased values of \( E_{\nu} \) and \( E_{\theta} \) are shown; these are estimated from \( H_0 \) and the shaft horse power which remain independent of changes in parts other than the impeller.

At \( Q = .78 \) which is very nearly the discharge for maximum hydraulic efficiency, the previous value of the conversion efficiency was 0.602 as estimated from a possible rise of \( \eta^{2} - \eta^{1} \)

\[ = 16.47 \text{ ft. and an actual rise of } 9.90 \text{ ft.} \]

The previous value of \( L_g \) viz. 6.57 ft. is reduced (from fig. 26) to 4.5 ft. increasing the probable rise of pressure head to 11.97 ft and the conversion efficiency to almost 73 per cent; A slight change in \( B \) in the loss \( y = \beta Q^2 \) due to alteration of the angle \( \theta \) may affect the total loss \( L_g \); but the analysis seems to show that the guide vanes loss in this pump can be appreciably reduced and that the general estimation of possible conversion efficiency of guide passages of 70 to 75 per cent is quite attainable.

Method of flow through the impeller according to \( (\beta) \cdot 12 \) leads to an increased value of \( \eta \) and, therefore, of possible rise of pressure head in the guide passages; and thus to a reduced estimate of conversion efficiency; the efficiencies are considered in \( \xi \).
Fig. 25 shows that if no leakage allowance be made the estimated conversion efficiencies are appreciably lower.

As stated in §4, readings of the ammeter might vary to about 1/4 amp. which would lead to an error of 1/5 horse power in K. From comparisons of the discharge as determined by the two modes of measurement differences of from about 1 per cent to zero occur. Irregularity in the power or discharge readings to these amounts would lead to greater irregularity of the plotted values of \( H_0 \) than are shown on fig. 30. The main source of error in the input head \( H_0 \) will be due to estimation of disc friction and bearing loss and of leakage.

With \( Q = 1.03 \text{ cub. ft/sec.} \) and 885 revs/min. allowing an error of 1 per cent in power estimation and of 25 per cent in the allowance of 1 H.P. for disc friction etc., the value of \( K \) is increased from 15.02 to practically 15.40 H.P. This reduces \( \varepsilon_1 \) from 0.627 to 0.600 and \( \varepsilon_2 \) from 0.833 to 0.813. If the value of \( K \) be correspondingly reduced the values of \( \varepsilon_1 \) and \( \varepsilon_2 \) become 0.657 and 0.852 respectively. Variation of power on the calculated value of \( H_0 \) is similar in effect to variation of \( Q \) and, as the leakage allowance is 0.6 cub. ft/sec. at this discharge, the values of \( \varepsilon_1 \) for various discharges with the above increase in power (2 per cent) lie between the values previously estimated with leakage and without leakage as shown on figs. 23 and 25 but nearer the value as estimated with leakage. But from consideration of the values of the impeller losses obtained in this analysis, \( K \) is probably not less than 15.02 and thus the conversion efficiency is not greater than that shown on fig. 22.

As mentioned in §12 at zero discharge the value of \( \frac{\beta_{30}}{\alpha_0} \) is probably from 2 to 2½ ft above that of \( \frac{\beta_{30}}{\alpha_0} \). If at this discharge and speed \( (Q = 1.03 \text{ revs 885 per min}) \frac{\beta_{30}}{\alpha_0} \) be taken at \( \frac{\beta_{30}}{\alpha_0} + 2' \) the estimated conversion efficiency is reduced from 0.627 to 0.612.

\( \S 18. \) Losses estimation by method of flow \((\beta)\) \( \S 12. \)

In the analysis it has been considered that flow takes place
over the whole outlet area at the tip of the impeller, that is

\[ y = \frac{Q + Q_0}{Q_0} \]

As mentioned under (3) 12 p.23 it has been considered possible that flow may take place over a portion only of the impeller outlet periphery between adjacent vanes, the remaining portion between the vanes being occupied by "dead" water eddies etc. This has been looked on as a probable explanation of the fact that the manometric head \( \frac{Q}{Q_0} \) is always found to be greater than the head corresponding to the measured power input; for, still presuming that over such portions of the impeller rim the discharge from the impeller follows the impeller angle \( \beta ( = 47\frac{1}{2}^\circ) \) it will lead to a reduced value of the tangential component of the absolute velocity of discharge.

Referring to Fig. 29A (same sheet as fig. 29) \( \sigma \beta \) represents determined from the exit velocity parallelogram by the ordinary theory. Since discharge over part only of the impeller circumference increases \( v \) (the relative velocity at exit) the absolute velocity of discharge becomes \( \sigma \beta \) with a lessened tangential component \( x' \). The experimental evidence on which the possibility of this type of delivery taking place has also been mentioned in 12.

From fig. 29A, if \( x' \) be the tangential component of \( \sigma \beta \) being determined from \( H_0 \) as in the previous analysis, then the new and increased value of \( y_0 \) corresponding to \( \sigma \beta \) and denoted by \( y' \) is given by

\[ y' = (u - x') \tan \beta \quad (24) \]

Values of \( y' \) so calculated will be found at the speeds in this test to agree more closely with \( y = \frac{Q + Q_0}{Q_0} \) as the discharge is increased, as would be expected. Values of the ratios of the value of \( y_0 = \frac{Q + Q_0}{Q_0} \) and of \( y' \) obtained from (24) are shown on fig. 29B and it will be seen that at a given discharge the proportion of radial area utilised becomes less as the speed increases.
Referring to fig. 17 the values of \( \beta' \), the reduced angle of discharge relative to the impeller as estimated by the previous analysis in which the full radial outflow area is presumed to be utilised (see \( \& 13 \)), are shown thereon for the three speeds. It will be seen that at large discharges the water tends to move as a body through the passage between the vanes and \( \beta' \) approaches \( \beta \) in value; that as discharge is decreased \( \beta' \) decreases and the average angle of discharge from the impeller becomes much less than \( \beta \), "slip" taking place increasingly as discharge decreases; and that for a given discharge, the "slip" is greater at higher speeds. Fig. 17A shows the same variation of \( \beta' \) when no leakage allowance is made except that the variation with speed at a definite discharge is irregular (see 20, p. 57). For the two methods of flow, the variation of the proportion of radial area utilised in the one case, and of \( \beta' \) in the other, are both reasonable; and either method of flow is equally possible.

But while the discharge over part of the radial area at the impeller outlet may, by reducing the rate of change of angular momentum, explain the fact that the actual power required is less than that estimated from the ordinary theory, yet the increased value of \( \nu \) leads to difficulty when the pressures at the impeller outlets are measured.

The values of \( \zeta \) and \( \theta \) with the increased values of \( \nu \) viz.

\[
\text{from (24) are shown on Fig. 27. The curves for these quantities as previously calculated on the first hypothesis are marked "A (or B) with full radial area"; on this second hypothesis of flow they are marked "A, (or B) with } \beta' = 47\frac{1}{2}^\circ\text{". It will be seen that } \zeta \text{ is lower on the second hypothesis; and since } \zeta + 2q \text{ (the total loss from impeller inlet to the guide passages outlet) is constant, being independent of any questions of the nature of flow through the impeller, (\& 13 p. 29) the value of } 2q \text{ must be increased. Also the increased value of } \nu \text{ involves a greater possible rise in the guide passages and a reduced conversion efficiency. In fig.}
\]
27, \( \dot{i} \) is negative when leakage allowance is made; to give reasonable values of \( \dot{i} \), the pressure head \( \frac{p_i}{\omega} \) would have to be dropped about 1.5 to 1.75 feet at 885 revs/min. At \( Q = 0.78\) and 885 revs/min this reduces \( \frac{p_i}{\omega} \) from 25.78 to about 24.20 ft. At this value of \( Q \) it is seen from figs. 13B and 14 that this brings \( \frac{p_i}{\omega} \) to a value very close to that of \( \frac{p_i}{\omega} \); at higher discharges \( \frac{p_i}{\omega} \) could not not be reduced by that amount without in general being lower than both \( \frac{p_i}{\omega} \) and \( \frac{p_i}{\omega} \).

Considering next the conversion efficiency, on fig. 28 the curves marked \( A \) are the values of \( \dot{g} \) and \( e \), for 885 revs/min, as plotted previously on figs. 22 and 23. With \( \dot{i} \), negative as shown on fig. 27 with this second hypothesis as regards flow through the impeller, values of \( \dot{g} \) or \( e \) cannot be obtained without assuming a lower value of \( \frac{p_i}{\omega} \). It has, therefore, been reduced 1.5 ft. merely to estimate (at about normal delivery) the conversion efficiency on the assumption that this reduced pressure head is a possible value of \( \frac{p_i}{\omega} \). The curves of \( \dot{g} \) and \( e \), on this second hypothesis are marked \( A_1 \). Comparing curves \( A \) and \( A_1 \), it is seen the efficiencies are higher in \( A_1 \); at \( Q = 1.03 \) it is increased from .627 to .669. But if in the previous analysis the value of \( \frac{p_i}{\omega} \) be reduced in the same manner the efficiency rises from .627 to .723; and thus the comparative:ly low conversion efficiencies obtained on the first hypothesis are not increased by adoption of the second hypothesis.

From the curve of \( \dot{i} \) marked \( A \), fig. 27 (with leakage allowance) the discharge at which \( \dot{i} \) is equal to the guide vane angle, viz. 25°, is \( Q = 1.14 \) cub. ft/sec. when the speed is 885 revs/min. If tangent parabolas be tried on fig. 28 to the curve of \( A_1 \) for the guide passages loss, the most suitable tangent parabola does not touch the curve at that discharge; though this is a minor point as irregularities in the \( \dot{g} \) curve would influence the tangent point. But with this second hypothesis the values of \( \dot{i} \), at the critical discharge at each speed lead to difficulty in accounting for the break in the quantity — head curve at that
If no leakage allowance be made the values of \( I \) and \( \theta \) on the second hypothesis are shown marked \( B \), on fig. 27; those of \( I_2 \) and the conversion efficiency are shown on fig. 28 marked \( B \). Results as previously found by the first hypothesis are similarly marked \( B \). In the case also \( I \) and the conversion efficiency are reduced below values by the first hypothesis; while \( I_2 \) is increased. The value of \( I \), though reduced does not become negative, and results as a whole are more reasonable than in the case where leakage allowance is made. No leakage is, however, a condition which is most improbable; and flow through the whole radial outlet of the impeller with "slip" varying with the discharge and the speed seems from the results of this series of tests to be the more probable cause of the actual power input being less than that required by the ordinary theory at all except small discharges.

\section*{\textsection19. Critical discharge.}

This is seen on fig. 9 to occur at all speeds. Just as discharge commences a small rise in head takes place, and, thereafter, the head remains practically constant till the critical discharge is reached at which discharge it suddenly rises. There is apparently very little reference to this feature in the published literature of centrifugal pumps. M. Rateau mentions it\(^x\) as occurring in turbine pumps and refers to it as a serious disadvantage; the question of its cause is not taken up in the paper referred to. The only other reference found is an abstract\(^{x1}\). There it is stated that at a certain discharge there was a sudden drop in the head, the motion became turbulent and a periodic noise was heard in the pump; and that for various speeds the boundary line between normal and turbulent flow was a parabola passing through the origin and pointing upwards - discharges and heads being abscissae and ordinates respectively.

\begin{flushleft}
\textbf{Head-quantity characteristic curves made up from tests in \\
\textsection"Pompes Centrifuges". Memoires de la Société des Ingenieurs Civils. 1910 Pt.I. \\
\end{flushleft}
the works of pump manufacturers show in some cases the curve concave upwards as discharge commences and thereafter rising steeply and convex upwards (as shown dotted on fig. 9 on the 983 revs. H-Q curve) This occurs only in the case of turbine pumps, but usually the H-Q curve rises continuously to a maximum without any point of inflexion. It is possible that, in the cases where the H-Q curve was initially concave upwards, the feature at the critical discharge so markedly shown by this pump may have occurred in a minor degree and have been unnoticed on account of pressure heads being read on an ordinary pressure gauge and of more attention having been paid to the nature of the H-Q curve at about normal delivery.

In this pump at normal delivery the mercury in the pressure reading columns was steady with occasional surges as stated in §3. As discharge was reduced the head rose to a maximum; on further reduction of the discharge, the head fell slightly till the sudden decrease took place. The manner of motion of the mercury in the columns then changed; it was in a state of rapid pulsation or tremor, the movement being about \( \frac{3}{8} \) to \( \frac{1}{2} \) inch amplitude. The head remained very nearly constant as did also the rate of pulsation, even though the discharge was reduced to a quantity that could not be measured with accuracy, as shown in brackets in Tables I a b & c. On finally closing the discharge valve the pulsation entirely ceased and the mercury was almost absolutely steady, while the head and shaft-horse-power fell appreciably as shown on figs. 9 and 10 and in Tables II and III. The tremor of the mercury in the columns was most marked when the columns were connected to read \( \frac{F_2}{\alpha^2} \) or \( \frac{F_3}{\alpha^2} \) (B or C fig. 4); it was much decreased when they were connected to read \( \frac{F_2}{\alpha^2} \) or \( \frac{F_3}{\alpha^2} \); and with volute or discharge pressure head readings, it was just perceptible; thus pointing to some occurrence at the impeller outlets. At the critical discharge though the general course of the shaft horse power changes yet no "kick" of the ammeter pointer was observed; at that discharge there is then no abrupt
rise or fall in the power absorbed by the impeller and the
cause seems, therefore, not to be due to a sudden alteration
of conditions within the impeller itself.

On opening the discharge valve the same sequence of
events took place in reverse order; the critical discharge
and head at which it occurred were practically the same
whether obtained with increasing or decreasing discharge.
Repeated tests gave critical discharges varying about 5 per
cent but the head at which these occurred was practically
constant as the H-a curve is very flat at that point.

Referring to figs. 13A, 13B, 13C and 14 it will be
seen that after a small rise just after discharge commences the
values of \( \frac{\beta - \beta}{\omega} \) fall slightly up to a discharge just below
the critical. The values of \( \frac{\beta - \beta}{\omega} \) fall rather more, but
the value of \( \frac{\beta - \beta}{\omega} \) over this range is fairly constant. At
the critical discharge, however, the rise in \( \frac{\beta - \beta}{\omega} \) is distinctly
greater than that in \( \frac{\beta - \beta}{\omega} \).

Figs. 12A, B, & C show a continual, though small,
rise in \( \frac{\beta - \beta}{\omega} \) up to a discharge just below the critical,
but at the critical discharge a rapid rise takes place in \( \frac{\beta - \beta}{\omega} \).

Fig. 14a shows \( \frac{\beta - \beta}{\omega} \) plotted on Q as base (for
885 & 1079 revs raised 5 to 10 ft. respectively to separate
the graphs.) It is thus seen that even below the critical
discharge a continual though imperfect rise is taking place
in the guide passages as a whole; but that just as the
critical value is reached a sudden change takes place and
conversion of energy from kinetic to pressure type becomes much
more efficient.

Below critical discharge, however, the conversion is
so inefficient that the imperfect rise \( \frac{\beta - \beta}{\omega} \) is lost in the
connecting passages and volute; for the total head H over
that range remains more constant (fig. 9) except for the initial
rise.
After the initial rise and at discharges below the critical, the absence of any rise in $\frac{P_0 - P}{P}$ along with the imperfect rise of $\frac{P_0 - P}{\omega}$ seems to indicate some condition at the entrance to the guides as the cause of the critical discharge, and the greater increase in $\frac{P_0 - P}{\omega}$ than in $\frac{P_0 - P}{\omega}$ at this discharge to indicate that this condition no longer exists and that some conversion of energy has taken place even at the point B (fig. 4); the condition affecting $\frac{P_0 - P}{\omega}$ being removed more efficient conversion from B to C (fig. 4) as shown in fig. 14A would follow naturally.

Figs. 12A and 12B show an initial rise in $\frac{P_0 - P}{\omega}$ and figs. 13A and 13B show a corresponding rise just as flow commences bodily through the pump; thus conversion of energy takes place initially, but any further conversion as discharge increases is much impaired till the critical discharge is reached. The drop in the absolute values of $\frac{P_0}{\omega}$ when the very small delivery was cut off by complete closing of the valve was very evident in the case of the higher stages of the pump (See Tables I).

Details of the critical discharges and corresponding heads for various speeds are shown in Table V. At speeds other than the three analysed here, observations and reductions were made in a similar manner.

By the ordinary theory of the centrifugal pump, the discharge and the square root of the head vary as the speed for similar conditions.

Referring to fig. 29, values of the critical discharge $Q_c$ (col.11 Table V) plotted on revs/min as base are shown by small circles; and if taken from column (13) i.e. with leakage allowance included the values are shown by small crosses. The $\frac{P_0}{\omega}$ for 1079 revs, $\frac{P_0}{\omega}$, $\frac{P_0}{\omega}$, and $\frac{P_0}{\omega}$ were not read at a small discharge of $\frac{P_0}{\omega}$ but as in the cases of 695 and 885 revs, $\frac{P_0}{\omega}$ and $\frac{P_0}{\omega}$ only were read. An initial rise of $H$ at all three speeds is seen on figs 11a, 11b, 11c. Thus at 1079 revs the $\frac{P_0}{\omega}$ and $\frac{P_0}{\omega}$ curves would show an initial rise as occurs at 695, 885 revs. This is shown dotted on figs 12c and 13c.
points lie very nearly on the lines $o\theta$ and $oa$ passing through the origin. The value of the critical discharge is not a perfectly definite quantity, as would be expected at a change from normal to turbulent flow, and thus irregularities occur among the plotted points. The head varies very slowly near the critical discharge and values of $\sqrt{H}$ (Col. 8) plotted on revs/min. lie almost exactly on the straight line $oc$.

In the method of analysis adopted here in evaluating losses

$$\tan \theta = \frac{y}{z} \quad \text{where} \quad \frac{y}{z} = \frac{Q+Q_1}{a} \quad \text{and} \quad \frac{\mu z}{9} = \mu_0$$

and thus $\tan \theta$ varies as $\frac{Q+Q_1}{\mu_0 a}$ or as $\frac{Q+Q_1}{\mu_0 \text{revs/min}}$.

Values of $Q_c$ and also of $Q_c+Q_1$ plotted on $\mu_0 / \text{revs/min.}$ from Table V col. 10 are shown on fig. 29; in the first case by circles and in the second by crosses. The points again lie practically on the straight lines $oc$ and $od$ with the exception of that for 788 revs/min, which seems due to some error in observation.

So from the ordinary theory and from the method of analysis adopted here the critical discharge takes place at a certain value of $\theta$; that is, with a certain direction of the absolute velocity of discharge from the impeller.

Referring to fig. 17 this angle is seen to be about 10° and from fig. 17A to be about 8° if no leakage allowance is made. Fig. 17 also shows that when discharge is presumed to take place over the whole radial area at the outlet of the impeller, the amount of 'slip' is practically the same at each speed as the values of $\beta'$ are very nearly equal at each speed at the critical discharge.

The rate of pulsation of the mercury in the pressure reading columns at the critical discharge was as in Table VI; it is seen approximately proportional to the speed of the pump; as discharge was further reduced the rate of pulsation diminished by about 10 per cent.
From fig. 29 the critical discharge at various speeds is approximately proportional to the speed and also to \( \frac{H}{u} \), and as \( \frac{3u_x}{2} - H_0 \) and \( \gamma_1 = \frac{\rho - p}{\alpha_a} \) it follows that the rate of pulsation just after the drop has taken place in the \( H - Q \) curve is practically proportional to the pump speed, to the critical discharge for that speed and to the absolute velocity of the water on leaving the impeller at the critical discharge.

Drawing a line \( a\delta \) at the guide vane tip on fig. 8 at the average value of \( \theta \) at critical discharge from figs. 17 and 17a, i.e. 8° to 10° the conditions of discharge shown, with a large guide vane angle, a space for "dead" water and eddies, a narrow throat at entry to the guide vane passage proper, and close proximity of the guide vane tips to the outer periphery of the impeller, are such as could lead to the change from normal to turbulent conditions at the entry to the guide passages and so to inefficient conversion from kinetic to pressure head energy, though probably such results might not be anticipated.

The possibility of the critical discharge being due to the impeller seems less probable as this is a feature observed only in turbine pumps (so far as records and investigation show); and the pulsations of the mercury at the impeller inlet (and also at the guide passages outlet and volute) were much less in magnitude than they were at the impeller outlet.

If flow were presumed to take place according to the second hypothesis \( \beta \) \( \approx \) 12 the pulsation might be attributed to the "dead" water spaces at the impeller outer periphery. From fig. 27 the value of \( \theta \) is seen to be about 17° at the critical discharge, and if this angle be plotted on fig. 8 there is no apparent reason why a sudden change in the action of the guide vanes and passages should take place with that angle of discharge; the abrupt change at the critical discharge thus appears to indicate that flow through the impeller takes place more after the type of hypothesis \( \alpha \) than after that of hypothesis \( \beta \) \( \approx \) 12.
§ 20. Power at small discharges and comparison of theory and test results.

On fig. 30 it is seen that as the discharge becomes large the values of $H_0$ plotted on $\varphi$ lie very closely on a straight line. At a certain discharge (which in this pump happens to be the critical discharge) $H_0$ begins to rise abruptly and rapidly and at zero discharge would be infinite if the small amount of leakage, which passes through the impeller then, were neglected. At small discharges, if $x'$ be determined from

$$\frac{3\mu x'}{\varphi} = H_0$$

then $x'$ is found to be greater than $u$. This result and the form of the $H_0$ graph indicate that some other source of loss has come into action.

From the ordinary theory

$$\frac{3\mu x'}{\varphi} = u_e (\alpha - \beta \phi)$$

where

$$H_m$$

is the "manometric" head. Denoting the input power to the impeller (corresponding to $K$) by $K_m$, then

$$3\mu \frac{K_m}{\varphi} = \frac{3\mu x'}{\varphi} (\alpha - \beta \phi); \text{ or } K_m = \frac{3\mu x'}{\varphi} (\alpha - \beta \phi)$$

where $A$ & $B$ are constants.

In most pumps it is found that the actual shaft horse power is given closely by an equation of the form.

$$S = \alpha^3 \omega^3 + \beta \omega^2 \varphi + \gamma \omega^2$$

(25)

$d^2$ includes the disc friction and bearings loss which will vary approximately as $\omega^3$ but is greater than these losses for, as stated in § 7, if the zero shaft horse power be deducted to give $K$ then impossible hydraulic efficiencies are obtained in this pump, and in general.

Deducting the estimated loss in disc friction and bearings from $S'$, then in most pumps, $K$ will be given by an equation of the form

$$K = \alpha \omega^3 + \beta \omega^2 \varphi + \gamma \omega^2$$

(26)

With suitable values of the constants, values of $K$ from (26) usually agree closely with experimental values at all except small discharges; at zero and small discharges the actual value of $K$ is found to be greater than $a\omega^3$ with the value of 'a' which suits best over the main range of discharge.\(^x\)

Expressing $K$ in terms of $Q$ in (26) in place of $Q+Q_t$, the actual quantity of water passing through the impeller, might partly, but could not wholly, account for this; and thus there is some special loss at zero discharge, which tends to decrease as discharge increases. This loss is referred to usually as the impeller tip loss or "churning" loss.

Since $H_o$ varies as $\frac{K}{Q}$ then from (26) $H_o$ may be expressed very closely by

$$H_o = c_1 Q^3 + c_2 Q^2 + c_3 Q$$

where $c_1, c_2, c_3$ are new constants; and so $H_o$ could be represented by the line $y = c_1 Q^3 + c_2 Q^2 + c_3 Q$ to which are added the ordinates $\frac{c_1 Q^3}{Q}$ which represent the addition on account of the "tip" or "churning" loss; correctly 'a' should vary having a maximum value at zero discharge and decreasing as discharge increases.

With the value of $a$, as determined from 'a' of (26) for the usual types of impeller, the graph of $H_o$ becomes very nearly straight after moderate values of $Q$; for this pump the straight line portion is very evident. Equation (26) is an equation built up to represent the actual impeller power over the whole range of $Q$ both when $Q$ is small and the churning loss is appreciable and when $Q$ is large and the churning loss has disappeared or is negligible - though at zero discharge it tends usually to give too small a value, as stated before.

The actual plotted values of $H_o$ (which lie approximately on a straight line for large discharges) are the basis of any equation for $H_o$ or $K$; and, therefore, the power input $K$ and also $H_o$ might be expressed by two equations; over a certain discharge, values of $H_o$ lie practically on a straight line and can be represented by an equation of the type

$$H_o = c_2 Q^2 + c_3 Q$$

which expresses the head required for normal losses and that in

\(^X\) Daugherty "Centrifugal Pumps".
the water at discharge; as discharge decreases another source of loss comes in; and \( H_0 \) has to be increased over that given by (28) by an amount which increases as discharge decreases and which at a given discharge is greater when the speed is greater. Thus at small discharges the loss is considered to be of two types (1) a loss of the same nature as at large discharges when \( H_0 \) follows the law of (28); (2) a loss due to eddies etc., at the impeller tip which increases towards zero discharge and is non-existent or practically so at large discharges. This mode of differentiation is adopted in \( \xi_1 \) & 13 as basis; in fig. 30 \( c'b' \) is produced to \( a \), and \( H_0' \) is used in place of \( H_0 \) as shown by \( \overline{ba} \). The additional "churning" loss is \( H_0 - H_0' \).

In some cases with special types of impeller the above conditions do not hold; the shaft horse power may be almost constant with discharge owing to very high "tip" losses at zero discharge, which also continue to be of appreciable importance even at high discharges. The \( H_0 \) graph will not then show a practically straight portion; but neither could the power input then be expressed by the usual type of equations (25) or (26), which hold for most normal pumps.

Fig. 30 shows \( H_0 \) for this pump. For discharges above the critical, the values of \( H_0 \) lie round the straight lines \( c'b' \). There are no signs of the plotted points showing a tendency to lie on a flat curve, concave upwards; the churning loss, so evident below the critical discharge, seems to have disappeared or to be negligible.

Referring to fig. 10, drawn for 885 revs/min (the curves for the other speeds being similar) the power corresponding to the various losses, (estimated by the equations at the end of \( \xi_6 \)) is there shown. \( k \) is represented by the curve \( abc' \). The vane tip or churning loss is determined from \( \frac{\alpha'g}{350}(H_0 - H_0') \). From \( e \) to \( f \) the course of the shaft horse power shows a distinct "hump" which is usually absent in such curves. Ordinarily the curve between points corresponding to \( e \) and \( f \) is nearly straight in place of (28).
or shows a tendency to be slightly curved with concavity upwards and to the left.

The turbulent conditions and pulsation of the water as observed on the mercury pressure columns to exist just outside the impeller at discharges below the critical must cause the losses and the power consumed to be greater than when these conditions are absent. As stated in §19 the pulsation ceased at zero discharge. The drop in the power curves in fig. 10 as discharge reaches zero is very marked; and so while the addition:...al impeller tip loss at small discharges occurs with all pumps yet the turbulent conditions which commence at the critical discharge in this pump seem to initiate this type of loss and to increase its magnitude.

After a certain discharge $H_o$ is expressed by an equation of the type of (23); the manometric head is expressed by an equation of the same type since

$$H_m = \frac{3\alpha u (\alpha - \gamma \cos \beta)}{\alpha} = \frac{3\alpha u}{\alpha} \left( \alpha - \frac{\gamma \cos \beta}{\alpha} \right)$$

in which $\gamma = \frac{\cos \beta}{\alpha}$; ($\alpha$ as defined $\approx 3.16$)

and this equation holds over the whole range of discharge. This suggests some relation between the actual head and the theoretical or manometric head for all speeds provided the discharge is such that the churning loss is negligible or inappreciable; that is for discharges where the plotted values of $H_o$ lie practically on a straight line; and if the churning loss be deducted by producing $c\delta$ to $\alpha$ (fig. 30) then the relationship will hold for all discharges.

At normal delivery the ratio of $H_o$ to $\frac{3\alpha u}{\alpha}$ is known to have varying values depending on the number of vanes and tip angle, but it is not constant over the whole range of discharge. In this pump it varies from 0.86 to 0.93 for the three speeds (See figs. 11A, B & C).

Determining equations for $H_o$ from fig. 30 for each speed and of the form

$$\frac{3\alpha u}{\alpha} \left( \alpha - \frac{\gamma \cos \beta}{\alpha} \right) = H_o$$

in place of (28) then,
then, with leakage allowance (lines \( c \delta a \) fig. 30) the equation becomes
\[
\frac{564 x 3.6}{J} (\Delta \gamma - 7.2 Q) = H_o \tag{31}
\]

Values of the \( H_o \) from this equation for comparison with fig. 30 are shown in Table VII; if these are plotted on fig. 30 the straight lines so obtained lie as satisfactorily among the test values of \( H_o \) as do the originally drawn lines \( c \delta a \). (For 695 revs/min. at which errors in the estimation of leakage and disc friction losses would be of greater effect, the line represented by (31) is slightly above the mean line through the plotted points.)

Equation (31) is an impeller equation only; to determine the head generated by the pump
\[
\frac{564 x 3.6}{J} (\Delta \gamma - 7.2 Q^2) = H_o = H + \xi(Q) \tag{31a}
\]

Expressing the losses \( \xi(Q) \) for the impeller by equation (16); for the guide passages by equation (15); the losses in the two connecting passages by \( 2 \times 1.16 Q^2 \) and in the volute and discharge pipe by \( 1.20 Q^2 \) (\( \xi \leq 15 \)), there is obtained on reduction,
\[
H = 0.425 \Delta \omega^2 + 1.94 \Delta \omega Q - 89.47 Q^2 \tag{32}
\]
as the equation for the pump.

Values of \( H \) from (32) for speeds from 502 to 1079 revs/min. are shown by circles on fig. 9 for discharges above the critical. At zero discharge the values of \( H \) from equation (32) are rather lower than the actual test results; for equations (15) and (16) which express the losses in the guide passages and impellers closely at moderate discharges give rather high values to these at zero discharge (Figs. 20 and 22).

The results from equation (32) plotted on fig. 9 for pump speeds of 502, 595, 788 and 983 revs/min. lie as closely on the \( H \sim Q \) experimental curves as in the cases of the three speeds analysed in detail; and thus it may be presumed that for all speeds and discharges, the relation between \( H_o \) and \( Q \)
is given by equation (31); and that the total losses are given by the equations here found for the losses in the various component parts of the pump.

Equations (30 and 31) depend on the impeller only and so should preferably be expressed in terms of \( \alpha + \alpha_k \), \( \alpha_k \), being the leakage, which varies with the speed. From fig. 30 and with leakages as in Table IV, it is found that \( H_0 \), with leakage, allows: once, may be expressed by

\[
\frac{871 \times 3\alpha}{\alpha_k} \left[ \alpha_k - \gamma \cdot H (\alpha + \alpha_k) \right] = H_0 
\]

values from which are shown in Table VIII for comparison with the experimental values of \( H_0 \) plotted on fig. 30. But since the pump losses are expressed in terms of \( Q \), and pump characteristics are plotted on \( Q \) as base, it is more convenient to use (31) in place of (33) — \( K \) however being determined in either case by \( \frac{\alpha (\alpha + \alpha_k)}{550} \).

The ordinary theory is expressed by

\[
\frac{3\alpha}{\alpha_k} \left( \alpha_k - \gamma \cdot \operatorname{col}(\beta) \right) = H_0 + H + \varepsilon (L) 
\]

and taking \( \gamma = \frac{Q + \alpha_k}{\alpha_k} \) and \( \beta = 47.5^\circ \) this becomes

\[
\frac{3\alpha}{\alpha_k} \left[ \alpha_k - 0.16 (\alpha_k + \alpha_k) \right] = H_0 + H + \varepsilon (L) 
\]

By experiment on this pump from (33) above,

\[
\frac{871 \times 3\alpha}{\alpha_k} \left[ \alpha_k - \gamma \cdot H (\alpha + \alpha_k) \right] = H_0 + H + \varepsilon (L) 
\]

holds for all speeds. This may be put in the form

\[
\alpha \cdot \frac{3\alpha}{\alpha_k} \left( \alpha_k - \gamma \cdot \operatorname{col}(\beta) \right) = H + \varepsilon (L) 
\]

where \( \alpha \) and \( \gamma \) will depend on the type of the impeller and the coefficients in \( \varepsilon (L) \) will depend on the type of the impeller and of the other passages in the pump; Thus for the impeller only or for the pump as a whole, theory and experiment are no more divergent than in other types of hydraulic problems; they are connected by coefficients which may differ with changes in the proportions of the component parts of the pump, but which for each pump remain practically constant with variation of speed and discharge.
If $H_0$ be determined from $Q$ only neglecting leakage (as shown by lines $C'D'$ fig. 30) so that the power input $K$ is given by $\frac{\omega^2 H}{550}$, similar results are obtained. $H_0$ may in this case be expressed approximately by

$$-945 \times 3\sec^2 \frac{u_r}{g} - 8.65 \frac{v_r}{g} = H_0 \quad \text{(37)}$$

Values of $H_0$ from this equation are shown in Table IX for comparison with the actual values of $H_0$ as shown on fig. 30.

In (36) $\alpha = (u_r - \gamma y_c \cot \beta)$ is the tangential component of the water on leaving the impeller. Hence by comparison with the ordinary theory, the virtual vane angle $\beta'$ is given by

$$u_r - \gamma y_c \cot \beta' = \alpha (u_r - \gamma y_c \cot \beta)$$

from which

$$\cot \beta' = \alpha \gamma \cot \beta + \frac{u_r}{y_c} (1 - \alpha) \quad \text{(38)}$$

With the values of $\alpha$ and $\gamma$ in (35a), $\beta'$ can only be equal to $\beta$ at values of $y_e$, or of $\omega - \alpha$, outside the range of the pump at each speed, e.g. at 885 revs/min. when $\omega - \alpha$, is 2.74 cub. ft/sec. which from fig. (9) is seen to be outside the range of possible discharges at this speed. Also since $(1 - \alpha)$ is positive, $\beta'$ diminishes with the discharge at constant speed and with increase of speed at constant discharge.

So if values of $H_0'$ plotted on $Q$ lie practically on straight lines, all expressible by an equation of the form of (30), and lying below the straight lines of the $H_n$ values similarly plotted; and if the full radial area at the impeller tip be presumed to be utilised, then $\beta'$ must vary with $Q$ and $u_r$, in the manner shown on fig. 17; and this will hold even if the gradient of the $H_0$ line is greater than that at the corresponding $H_n$ line.

The values of $\beta'$ on figs. 17 and 17a are calculated from the actual values of $H_0$ as determined from the test values of $K$ (Table II). In the case of Fig. 17a the graphs of $\beta'$ cross and do not show the regularity of these in fig. 17.

This may be partly due to the extreme assumption of no leakage; but if $\beta'$ be estimated from the slightly different values of $H_0$ determined from (37) the graphs of $\beta'$ lie relatively as in fig. 17.
In the analysis, the normal losses at small discharges were estimated from a power input corresponding to ordinates to $\delta a$ in figs. 11A, B, C or fig. 30. This was adopted merely as a reasonable basis for the separation of the total loss into two types - the normal and the special impeller tip loss; and if in (31) and (31a) both $H_0$ and $\delta a$ are estimated in this manner then (32) will hold over the whole range of discharge.

But if $H_0$ did follow at small discharges the straight line law which it does practically at higher discharges, then, from (38), $\beta'$ would gradually decrease, as shown by dotted lines on fig. 17. Such conditions are impossible of realisation; at some discharge, eddies and reverse currents must come to be of importance and $H_0$ must rise. But at large discharges, though $\beta'$ is less than $\beta$, the larger volume of water passing through the impeller prevents the formation of these eddies and reverse currents to any appreciable extent and $H_0$ follows practically a straight line law; and then theory and actual results may be connected by coefficients as in (34) and (36).
Tables
Table Ia

Revs/min. Tachometer 705. Corrected 695. All quantities in ft or cals/hr/sec, unless stated.

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**Note:** Table 1.2: This table contains corrected data. All entries in 1.2 are marked with an asterisk (*) unless otherwise noted.
Table Ic.

RPM/min. Tachometer 1/100. Corrected 1/79. All quantities in ft. or cubic feet unless stated.

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Note: The table contains RPM values for different equipment numbers and impeller positions, indicating the performance of the tachometer under various conditions.
### Table II.

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**Table continued...**

| 885      | 1        | 3510 | 502   | 17600| 390  | 1360        | 15850                 | 901        | 21.80          | 100                             | 20.60            | 1.61                      |
|          | 2        | 3475 | 500   | 17370| 382  | 1352        | 15624                 | 901        | 21.50          | 90                              | 19.00            | 1.52                      |
|          | 3        | 3350 | 502   | 16170| 345  | 1124        | 14665                 | 899        | 19.90          | 80                              | 16.90            | 1.45                      |
|          | 4        | 3135 | 502   | 15740| 312  | 978         | 14064                 | 894        | 18.85          | 70                              | 15.85            | 1.31                      |
|          | 5        | 2675 | 506   | 13540| 227  | 968         | 11953                 | 893        | 16.62          | 60                              | 13.85            | 1.03                      |
|          | 6        | 22.25 | 506   | 13400| 155  | 9823        | 9823                 | 865        | 13.17          | 50                              | 12.17            | 0.70                      |
|          | 7        | 2100 | 500   | 10500| 140  | 9079        | 9079                 | 857        | 12.04          | 40                              | 11.08            | 0.70                      |
|          | 8        | 1810 | 504   | 9120 | 104  | 9656        | 9656                 | 840        | 10.24          | 30                              | 9.28             | 0.585                     |
|          | 9        | 1075 | 506   | 8460 | 87   | 7631        | 7631                 | 830        | 9.42           | 20                              | 8.42             | 0.470                     |
|          | 10       | 1240 | 506   | 6370 | 49   | 4901        | 4901                 | 777        | 6.57           | 10                              | 5.57             | 0.148                     |
|          | 11       | 920  | 504   | 4830 | 28   | 3461        | 3461                 | 712        | 4.61           | 60                              | 3.61             | (0.04)                    |
|          | 12       | 770  | 506   | 3915 | 19   | 2536        | 2536                 | 644        | 3.40           | 40                              | ...              | ...                       |

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Table III (cont.)

Footnote:

* WP Gross: 18% HWP, 30% HWP, 50% HWP, 75% HWP
* WP Simpson: 18% HWP, 30% HWP, 50% HWP, 75% HWP
* WP Ex: 18% HWP, 30% HWP, 50% HWP, 75% HWP
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<th>Impeller No. 2</th>
<th>Impeller No. 3</th>
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<th>Ad</th>
<th>Q</th>
<th>Total Q</th>
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**Table IV.**
| Rev\min | Exp No | Impeller No 1 | | | Impeller No 2 | | | Impeller No 3 | \( Q \) | \( \Delta Q \) | Total \( Q \+\Delta Q \) |
|--------|-------|---------------|---|---|---|---|---|---|---|---|
| 1077   | 1     | 1.50          | 18.75 | 26.20 | 20.60 | 54.80 | 64.30 | 59.70 | 91.25 | 98.40 | 96.20 | 95.00 | 1.70  | 96.90 | 1.35  |
| 2      | 2     | 10.50         | 22.00 | 32.30 | 22.70 | 62.65 | 73.50 | 70.60 | 103.15 | 113.60 | 115.30 | 116.00 | 0.70  | 14.55 |
| 3      | 3     | 9.00          | 23.00 | 33.85 | 24.40 | 72.10 | 83.30 | 82.00 | 117.85 | 131.60 | 131.90 | 132.50 | 1.80  | 1.45  |
| 4      | 4     | 7.70          | 29.90 | 42.70 | 30.70 | 83.25 | 96.50 | 91.60 | 120.25 | 144.00 | 143.50 | 144.00 | 1.50  | 1.25  |
| 5      | 5     | 6.40          | 31.60 | 45.50 | 44.40 | 93.35 | 106.50 | 76.60 | 135.50 | 152.00 | 152.00 | 152.00 | 2.00  | 2.00  |
| 6      | 6     | 5.50          | 32.95 | 46.00 | 56.00 | 103.95 | 110.00 | 70.70 | 135.00 | 150.00 | 150.00 | 150.00 | 3.00  | 3.00  |
| 7      | 7     | 5.40          | 33.45 | 46.70 | 56.10 | 103.50 | 110.00 | 70.70 | 135.00 | 150.00 | 150.00 | 150.00 | 3.00  | 3.00  |
| 8      | 8     | 5.40          | 33.45 | 46.70 | 56.10 | 103.50 | 110.00 | 70.70 | 135.00 | 150.00 | 150.00 | 150.00 | 3.00  | 3.00  |
| 9      | 9     | 4.80          | 33.75 | 46.80 | 59.50 | 110.85 | 131.00 | 82.70 | 128.35 | 145.70 | 145.70 | 145.70 | 4.00  | 4.00  |
| 10     | 10    | 4.60          | 34.80 | 46.80 | 62.00 | 113.85 | 131.50 | 83.50 | 128.75 | 146.70 | 147.00 | 147.00 | 4.30  | 4.30  |
| 11     | 11    | 4.50          | 35.30 | 46.80 | 64.50 | 115.30 | 132.00 | 84.80 | 129.75 | 148.50 | 148.50 | 148.50 | 4.50  | 4.50  |
### Table V & 19.
Critical Discharge, Qc.

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<th>$\frac{p}{w}$ ft</th>
<th>$\sqrt{H}$ ft</th>
<th>$\frac{H}{H_0}$ ft</th>
<th>$t_{revs}$ min</th>
<th>$Q_c$ cu. ft/sec</th>
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### Table VI & 19.

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Values of $H_0$ from Eqn. (31)

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### Table VIII

Values of $H_t$ in feet from (33) $\xi, 20$

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### Table IX

Values of $H_t$ in feet from (37) $\xi, 20$

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</tbody>
</table>
Figs. 4, 5, and 8.
DELIVERY of CROSS-CONNECTED PRESSURE MAINS
of different materials or internal surface conditions; with deduction of formulae for practical use.

1. The practical advantages of the cross-connection of pressure mains between reservoirs have been referred to in two recent papers in the Proceedings of the Institution of Civil Engineers. In the second of these papers the additional advantages of cross connection in the case where gradual increase of supply is required are also dealt with; and from the discussion on the paper it seems that while cross connection of pressure mains between reservoirs is not an entirely novel procedure, yet the advantages to be derived from such cross connection are not so well known to Engineers as they should be.

The principle of the calculation of the delivery of a number of cross connected mains where portions of the mains between cross connections are put out of action may be found in any standard treatise on Hydraulics; but the method of calculation is left as a rule in general form; and further, the pipes are usually presumed to be of similar nature of internal surface and so to have the same frictional resistance coefficients.

Even when the internal surfaces of the several cross-connected mains is such that, for practical purposes, the same frictional resistance coefficients may be assumed for all the pipes, the estimation of the delivery from general principles involves a type of calculation not common in practice; and when the internal surfaces are markedly different in nature the

x (a) "The Derwent Valley Waterworks" by Edward Sandeman, M.Sc., M.Inst.C.E. Vol. 206, p. 175.
calculations required are long and laborious, and while the problem may be simplified by assuming the frictional coefficients of all pipes to be the same, yet the error involved in so doing is not estimated and may be of relatively large magnitude.†

The cross connection of pressure mains of materials with very different frictional coefficients will probably occur more frequently in the future. On account of certain advantages, reinforced concrete pipes are now being more extensively used than formerly; and such pipes may be cross connected to cast iron pipes which have been in use for a considerable period. In the works described in the second of the papers referred to above, it is proposed to increase the supply by cross connecting a reinforced concrete pipe (laid over part of the length between the reservoirs) to two existing cast iron mains.

In practice a simple and practically accurate formula is desirable and this formula must be applicable to cross connected pipes entirely different as regards nature of internal surface and frictional resistance coefficients.

The equation \( h = \frac{\mu v^2}{2gD} \), in the usual meaning of the letters, expresses in the best form the relation between head, length, diameter and speed of flow in a simple pipe. Many engineers however use the equation \( v = c\sqrt{mL} \), \( c \) being determined from Kutter's or some other similar formula; and from long experience and the results of many tests this equation has become so established in engineering practice that any practical formula for the delivery of cross-connected mains must be stated in such terms that it may be used in conjunction with either of the above two formulae.

A simple and practically accurate formula will be derived from the first of the above equations; but it will be expressed finally in terms of the discharges of the separate pipes as single pipes between the reservoirs; and these discharges may be estimated by means of either of the two formulae stated above or by any other formula considered reliable and

†See Appendix. Cases 7 and 8. p.30
preferable as the result of personal experience.

2. **Two Pipes cross-connected.**

Fig. (1) represents a pipe of length \( L \) and diameter \( d \) connecting two reservoirs \( A \) and \( B \). For a portion \( kl \) an: other pipe of diameter \( D \) is laid alongside and joined at its ends to the pipe of diameter \( d \). Alternatively the \( D \)-pipe may be supposed laid the full length between reservoirs and cross-connected to the \( d \)-pipe; but by closing of suitable valves only the portion shown contributes to the delivery. The Hydraulic Gradient is shown on the figure. In addition to the heads and velocities shown in Fig. (1) let

\[
\begin{align*}
\nu_o &= \text{speed of flow in the } d \text{-pipe as a single pipe between Reservoirs, i.e., with Hydraulic Gradient } \gamma \\
Q_o &= \text{discharge of } d \text{-pipe under the same conditions,} \\
\mu, n, r, m &= \text{friction constants for the } d \text{-pipe,} \\
\bar{V}, Q', \mu', n, r, m &= \text{the above items for the } D \text{-pipe.} \\
Q &= Q_o + Q' = \text{the combined discharge of the two pipes as separate pipes laid between the Reservoirs.} \\
Q &= \text{discharge under conditions in Fig. (1)}
\end{align*}
\]

In practical cases the ratio of the length to diameter of a pipe is large and all losses except those due to friction may be neglected.

Hence from Fig. (1) \( h = h_1 + h_2 + h_3 \)

\[
\begin{align*}
h &= \frac{\mu v_o^n (L-k)}{d^m} + \frac{\mu v_o^n k \ell}{d^m} \\
\text{or } h &= \frac{\mu v_o^n L}{d^m} + \frac{\mu v_o^n k \ell}{d^m} \tag{1}
\end{align*}
\]

\[
\begin{align*}
\text{also } \frac{\mu v_o^n k \ell}{d^m} &= \frac{\mu' \bar{V}'^n k \ell}{D^m} \tag{2}
\end{align*}
\]

\[
\begin{align*}
vd^2 &= v_i d^2 + \bar{V} D^2 \tag{3}
\end{align*}
\]

From (3) \( v = v_i \left\{ 1 + \frac{\bar{V} D^2}{v_i c_x} \right\} = v_i \left\{ 1 + \frac{u}{c_x} \right\} \)
where \( y \) and \( x \) are the deliveries of the \( D \) and \( d \) pipes respectively at the Hydraulic Gradient \( \frac{h}{k} \) as in Fig. (1).

If now the two pipes are of similar surface i.e., \( n = n \), etc., equation (2) shows that \( \frac{y}{y} \) is constant and so the ratio of the deliveries of the two pipes is independent of the Hydraulic Gradient.

Hence \( \frac{y}{x} = \frac{Q'}{Q_o} \) : and so \( v = \frac{y}{y} \frac{Q'}{Q_o} = \frac{v}{Q_o} \) \( Q' \), \( Q_o \) \( \cdots \) \( (4) \)

Substituting this in (1) and noting that \( h = \frac{\mu v_o}{2} \) gives \( n \) and also that

\[
\frac{v}{v_o} = \frac{Q}{Q_o} \quad \text{gives on reduction,} \quad k = \left( \frac{1 - \left( \frac{Q_o}{Q} \right)^n}{1 - \left( \frac{Q_o}{Q} \right)} \right)^n \quad \text{----- (5)}
\]

Equation (5) gives the proportion of the length over which the \( D \)-pipe must be laid to increase the discharge from \( Q_o \) to \( Q \), or if the \( D \)-pipe is laid over the whole length \( l \), it gives the delivery \( Q \) when only a portion \( kl \) is contributing to the delivery, the portion \( l(1-k) \) being shut off.

In actual cases \( n \) varies from about 1.75 for smooth pipes to about 2 for rough pipes. If, for values of \( \frac{Q}{Q_o} \) up to a practical maximum of about 3, values of \( k \) from (5) be plotted on \( \frac{Q}{Q_o} \) as base with \( n = 1.75 \) and with \( n = 2 \) it will be found that the corresponding curves lie very close together. See Fig. (2). Even with \( \frac{Q}{Q_o} = 3 \), the greatest difference in \( k \) is only about 2 per cent, and this value of \( \frac{Q}{Q_o} \) is rather higher than would generally be adopted in practice; and a value of \( n \) as low as 1.75 is only obtained in the smoothest of pipes, and is usually exceeded after they have been in use for some time. Further, no matter how carefully \( n \) is determined its value changes as the pipe surface deteriorates with age and also varies along the length of the pipe, and minor losses at valves, cross-connections, etc. are neglected in the above investigation. Thus the equation

\[
k = \frac{1 - \left( \frac{Q_o}{Q} \right)^n}{1 - \left( \frac{Q_o}{Q} \right)^n} \quad \text{----- (6)}
\]
may be used in place of (5) for all types of pipes of similar surface and the results are just as likely to be in accordance with actual discharges as are results calculated by the use of (5) with the best determined values of \( n \). Also any error so made will be on the safe side as \( n \) usually does not exceed \( Z \).

But, of course, \( Q_o \), \( Q' \) and \( Q \), must be determined with the best available value of \( \beta \), \( n \), and \( m \) if the equation
\[
\frac{\alpha}{\alpha m} = \frac{\alpha}{\alpha m} (\text{the pipes being similar and } n = n, \text{ etc.)}
\]
is used; and also the equation
\[
\nu = \sqrt{\frac{\frac{\alpha}{\alpha m}}{\alpha m}}
\]
is used with the best estimated values of \( c \) in order to determine these quantities. \( Q_o \) may be known by actual measurement; and in the case when the \( D \)-pipe extends over the full length (but a portion \( \ell(\ell - k) \) is shut off), \( Q' \) can be measured also.

Graphical Method to determine \( k \) and the Hydraulic Gradient.

Since
\[
\nu = \nu \frac{Q_o}{Q_o}, \quad \text{and} \quad \nu = \nu \frac{Q}{Q_o}
\]
a very simple graphical construction, determines \( k \) and the corresponding hydraulic gradient for any value of \( \frac{Q}{Q_o} \).

Equation (1) shows that the duplicate portion of the pipe may be laid at any part of the length; suppose it laid at the lower end of the pipe line.

Taking \( n = 2 \), the slope of the Hydraulic Gradient line varies as (velocity). Set off Fig. (3) \( \ell \nu = h \frac{\nu^2}{\nu^2} = h \left( \frac{Q_o}{Q_o} \right)^2 \) and join \( ac \). Set off \( \ell g = h \frac{\nu^2}{\nu^2} = h \left( \frac{Q_o}{Q_o} \right)^2 \) and join \( cg \). Then \( abc \) is the Hydraulic Gradient and the portion of the length over which the \( D \)-pipe is laid (or is in action if fully laid) is that corresponding to \( bc \); or \( h = \frac{bc}{Q_o} \). If \( n \) is not equal to 2 then \( h \left( \frac{Q_o}{Q_o} \right)^n \) and \( h \left( \frac{Q_o}{Q_o} \right)^n \) should be set off instead of the quantities given above. A slight error will be introduced by taking \( n = 2 \) for simplicity when this is not the true value; this slight error corresponds to taking (6) in place of (5).

It is seen from the figure that as the straight line joining \( a \) and \( c \) is the Hydraulic Gradient for a simple pipe between the reservoirs, the placing of the portion of the \( D \)-pipe in action at the end next the lower reservoir depresses the

---

\( ^{1} \) See Appendix. Case 1, p. 29.
Hydraulic Gradient very largely. Completing the upper part of the parallelogram by \( a'c \), then \( a'b' \) is the hydraulic gradient if the portion corresponding to \( ab' \) is in duplicate; and now the Hydraulic Gradient is raised in position. If next the actual pipe line be plotted, Fig. (3) may be used to fix suitable positions for the duplicated portion of the pipe while keeping it always below the Hydraulic Gradient.

Thus if \( \delta_i \delta_3 \) be drawn parallel to \( ab' \), \( a'\delta_2 \delta_5 \) is the Hydraulic Gradient with the portion corresponding to \( ab' \) duplicated; or drawing \( \delta_4 \delta_5 \) parallel to \( ab \) and \( \delta_4 \delta_6 \) parallel to \( ab' \) then \( a'\delta_2 \delta_5 \delta_6 \) is the Hydraulic Gradient with the separate portions of the pipe corresponding to \( \delta_2 \delta_5 \) and \( \delta_5 \delta_6 \) duplicated as shown on the figure. In all these cases the portions duplicated are clearly equal to \( \delta_k \) in length.

If the pipes are not of similar surface i.e. \( h \) is not equal to \( n \delta_0 \), then as before \( v = \gamma/\gamma_k \) but \( \gamma/\gamma_k \) changes with \( \gamma_k \), as \( k \) and \( h_2 \) change. If \( \gamma/\gamma_k \) be taken equal to \( a'\delta_0 \) as before, this is equivalent to substituting for the actual \( D \)-pipe one of surface similar to that of the \( \delta'\)-pipe and with diameter such as to give the same discharge \( Q' \) with Hydraulic Gradient \( \gamma_k \) as the actual \( D \)-pipe gives. The possible error may be shown graphically.

Let \( k \) be determined from (6) (i.e. with this substitution) for a given value of \( a'\delta_0 \) and take the duplicated portion to be at the lower end; and let \( abc \) Fig. (4) be the hydraulic gradient. Suppose now that the original \( D \)-pipe is replaced and in action over the length corresponding to \( \delta c \) Fig. (4); also that it is smoother than the \( \delta \)-pipe.

The actual \( D \)-pipe and its substituted pipe gave the same delivery at Hydraulic Gradient \( \gamma_k \); but as over \( \delta c \) this gradient is reduced the actual pipe will, over \( \delta c \), give less delivery than the substituted pipe. Thus the gradient must really be greater than that of \( \delta c \), so \( b \) moves to \( \beta \), and \( a'\delta_2 \delta_5 \delta_6 \) is the Hydraulic Gradient. The length in duplicate has to be increased from \( c\delta \) to \( c\delta_0 \).
If the D-pipe is rougher than the d-pipe, the Hydraulic Gradient is \( \alpha_2 c \) and the length in duplicate has to be slightly decreased.

The error in \( k \) by assuming \( \frac{y}{x} = \frac{Q'}{Q_o} \) is probably not large. Determining the correct value of \( k \)

\[
\frac{y}{x} = \frac{V D^2}{V d^2} \quad \text{and from (2) on reduction}
\]

\[
v = v_1 \left( 1 + \frac{m_1}{m_1 + m_2} \right) \left( \frac{V d^2 + \alpha_2}{V d^2} \right)
\]

and denoting the term in the bracket by \( \mathcal{A} \), then \( v_1 = \mathcal{A} v \), but \( \mathcal{A} \) is not constant as it varies with \( v_1 \) and so with \( \frac{Q'}{Q_o} \); and only has the value \( \frac{Q_0}{Q_o} \) when \( v_1 = v_o \), that is to say when \( k \) is equal to unity and the Hydraulic Gradient is \( \frac{1}{2} \).

If \( \mathcal{A} \) be determined for any value of \( \frac{Q'}{Q_o} \) then corresponding to (5) there is obtained in the same manner as before

\[
-k = \frac{1 - \left( \frac{Q_0}{Q_o} \right)^n}{1 - \mathcal{A}^n}
\]

Without error of practical importance \( n \) may be taken as \( 2 \), as mentioned before and

\[
k' = \frac{1 - \left( \frac{Q_0}{Q_o} \right)^2}{1 - \mathcal{A}^2}
\]

\( k' \) being the practically correct value of the proportion of pipe in duplicate in the case of pipes of dissimilar surface.

In practical cases the variation of \( \mathcal{A} \) is limited. The greatest value of \( v_1 \) is \( v_o \), which has a practical maximum. When this maximum is fixed the minimum value of \( v_1 \) is given by (7) when \( v \) has its lower limit, viz. \( v_o \), and \( k \) is just approaching zero.

Further taking \( \mathcal{A}^2 = \left( \beta \frac{m_1}{m_2} \right)^2 \), \( \beta \) varying with \( v_1 \), then

\[
\frac{k' (\text{from } \mathcal{A})}{k (\text{from } \alpha)} = \frac{1 - \left( \frac{Q_o}{Q} \right)^2}{1 - \beta \left( \frac{Q_o}{Q} \right)^2} \quad \text{so that not only the variation in } \mathcal{A} (= \frac{y}{x}) \text{ but its value compared with } \left( \frac{Q_o}{Q} \right) \text{ affects the error in using (6) in place of (6'); and also in any one case only a proportion of the range of } v_1 \text{ from its maximum to zero is required, and this proportion varies with } \frac{Q}{Q_o}.\]
In these circumstances in place of investigating this error generally for all possible value of $v_o, c_l, D, n, n_1, \ldots$, etc. extreme cases will be investigated first and the error due to using (6) in place of (6') evaluated.

Case (a) Taking $c_l = D = 1 ft.; v_o = 3 ft/sec, \mu = 2, \mu_1 = \pi = 175$

$$h = 1.25, \quad m = 1.10 \quad \text{i.e. } D \text{-pipe is smoother than the } d \text{-pipe,}$$

(7) becomes $u = v_o + 1.486 \frac{D}{v_o}$, i.e. $D$-pipe is smoother than the $d$-pipe.

The maximum value of $u$ occurs when $\gamma = \gamma_0 = \pi \frac{D}{v_o}$. and reaches the rather high value of $3.22 \frac{D}{v_o}$, and thus $\gamma_0 = \gamma_0 = 2.74$.

Values of $u$ corresponding to those of $v_o$, varying from 0 to 3 calculated from (7) are plotted in Fig. (5); from this graph values of $u$, corresponding to values of $v$ in column 3 of Table I below, are obtained and these are shown in column 4.

These values of $u$ are chosen for values of $\frac{\gamma_0}{\gamma_o}$ from 1 to 2.74. The variation of $\gamma$ is seen in column 5; $k$ and $k'$ are shown in columns (7) and (8). Thus for values of $\frac{\gamma_0}{\gamma_o}$ approaching unity the difference between $k$ and $k'$ is about one per cent if (6) be adopted in place of (6'), and this error is reduced as $\frac{\gamma_0}{\gamma_o}$ increases and approaches $\frac{\gamma_0}{\gamma_o}$. This amount of error occurs in the cases where the two pipes are extremely different as regards frictional resistance.

Values of $k$ and $k'$ plotted on $u$; as base (for convenience of arrangement) are also shown in Fig. (5).

**Table I.**

<table>
<thead>
<tr>
<th>Case (a)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CI = D = 1 ft.$</td>
<td>1.00</td>
<td>3.00</td>
<td>1.19</td>
<td>1.97</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$v_0 = 3 ft/sec$</td>
<td>1.25</td>
<td>3.35</td>
<td>1.46</td>
<td>1.39</td>
<td>3.60</td>
<td>1.45</td>
<td>1.15</td>
<td>1.15</td>
</tr>
<tr>
<td>$q_o = 2.74$</td>
<td>1.50</td>
<td>4.50</td>
<td>1.98</td>
<td>1.85</td>
<td>5.55</td>
<td>0.65</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>$\mu = 2.0$</td>
<td>1.75</td>
<td>5.25</td>
<td>2.99</td>
<td>2.79</td>
<td>7.73</td>
<td>0.77</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>$n = 2, m = 2.58$</td>
<td>2.25</td>
<td>6.25</td>
<td>3.25</td>
<td>3.17</td>
<td>8.75</td>
<td>0.87</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>$M_1 = 1.8, m_1 = 1.4$</td>
<td>2.25</td>
<td>6.75</td>
<td>3.25</td>
<td>3.17</td>
<td>8.75</td>
<td>0.87</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>$n_1 = 1.75, m_1 = 1.94$</td>
<td>2.50</td>
<td>7.50</td>
<td>3.75</td>
<td>3.65</td>
<td>9.40</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>$v_o = 3 ft/sec$</td>
<td>2.00</td>
<td>6.00</td>
<td>2.25</td>
<td>2.16</td>
<td>8.15</td>
<td>0.81</td>
<td>0.81</td>
<td>0.81</td>
</tr>
<tr>
<td>$\gamma_0 = 2.74$</td>
<td>6.22</td>
<td>3.00</td>
<td>3.65</td>
<td>0.26</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Other cases (b) and (c) worked out in a similar manner are shown in Fig. (5); corresponding cases marked a', b', and c' in which the D-pipe is rougher than the a-pipe are similar and shown graphically on Fig. 5'. Values of $\frac{D}{\ell}$ and $\mu, \eta, m,$ etc. are shown on the figures. The actual diameters shown for the a-pipe are chosen to increase or decrease the variable factor inside the bracket in (7) along with $(\mu_D)\eta_D$.

Considering the difference between $k$ and $k'$ in a more general manner, two pipes, diameters a and D, of the same nature of surface and giving a certain value of $\frac{Q'}{Q_0}$ may be chosen as basis. For any value of $\frac{Q}{Q_0}$, (6) gives the corresponding value of $k$. The nature of surface of the D-pipe may then be assumed to change subject to the condition that $Q'$ remains the same, so that the diameter of the pipe changes to $D'$.

To determine $D'$ in terms of $D \times a$, let $\overline{v}_0$ and $\overline{v}_0'$ be the velocities in these pipes at Hydraulic Gradient $\frac{h}{L}$ (at which gradient they give the same delivery)

Then

$$\left(\frac{\overline{v}_0}{D'}\right)^2 = \frac{\overline{v}_0 D}{D'}$$

and

$$\frac{\mu \overline{v}_0}{D} = \frac{\mu_D (\overline{v}_0')^2}{D'}$$

and hence

$$\left(\frac{\mu}{\mu_D}\right)\left(\frac{D'}{D}\right)^{2+\frac{m}{\eta}} \frac{\overline{v}_0}{\overline{v}_0'} = \frac{\overline{v}_0 D^2}{D'}.$$

But since the a and D pipes have the same frictional coefficients

$$\frac{\mu \overline{v}_0}{D} = \frac{\mu_D \overline{v}_0'}{D'},$$

and substituting in the equation above and reducing

$$\left(\frac{D'}{D}\right)^{2+\frac{m}{\eta}} = \left(\frac{\mu}{\mu_D}\right)^{2+\frac{m}{\eta}} \frac{\overline{v}_0}{\overline{v}_0'}.$$

In (7) $D$ is now represented by $D'$, and hence writing $D'$ for $D$ in that equation

$$\overline{v}' = \overline{v}_1 \left\{ 1 + \left(\frac{\mu}{\mu_D}\right)^{2+\frac{m}{\eta}} \frac{(D')^{2+\frac{m}{\eta}}}{(D)^{2+\frac{m}{\eta}}} \right\} \overline{v}_0.$$
Substituting for \( D' \) from the equation just above, then on re-duction,

\[
v = v_0 + v_1 \frac{D^{2r - \frac{n}{2}}}{\alpha^{2r - \frac{n}{2}}} \quad \text{(7')}
\]

(7') gives the relation between \( v_1, v \) for pipes with different values of the frictional constants but retains (in the equation) the diameter \( D \) of the pipe which, with frictional constants of the same value as those of the \( d \)-pipe, will give the same value of \( Q' \) or \( \alpha'/\alpha_0 \)

Various ratios of \( D \) to \( d \) may be chosen to cover all practical ratios of \( \frac{\alpha'}{\alpha_0} \) — say from 1.20 to 3.0. Higher values of \( \frac{\alpha'}{\alpha_0} \) would lead to extreme velocities in the single portion of pipe line when \( k \) is nearly unity; and by means of (7') the ratio of \( \frac{v}{v_1} \) and \( k' \) may be determined; and then by (6'), \( k' \) may be estimated and compared with \( k \) from (6).

The procedure for the six cases given above is virtually the same as if this latter method were adopted; and from the data on Figs. (5) and (5') it is seen that they cover the practical ratios of \( \frac{\alpha'}{\alpha_0} \). In these figures the value of \( D \) stated is that to give the value of \( \frac{\alpha'}{\alpha_0} \) with the corresponding values of \( \mu, \eta, \frac{m}{\nu}, \) as the results were worked out from equation (7).

In some of these cases the higher values of \( v \) are greater than would be practically adopted, but extreme values of the ratios \( \frac{D'}{D}, \frac{n}{2}, \) etc., have been taken. Results worked out for \( v_0 = 2.5 \text{ in sec} \) and \( \frac{\mu}{\nu} = 1 \text{ in sec} \) show differences of the same degree between \( k \) and \( k' \).

The values of \( k \) and \( k' \) shown in Figs. (5) and (5') lie so close together that, from the practical point of view, the simple equation (6) may be used in the case of dissimilar pipes just as it may with similar pipes.\(^{+}\) Any error introduced by using (6) in place of (6') is not of great magnitude in

\[^{+}\text{See Appendix, Cases 1, 2, and 3, p. 20.}\]
comparison with errors introduced due to variability of the friction factors \( n, m, \mu \); and results found by (6) are just as likely to be in agreement with actual measurements as results obtained from the more laborious calculations involved in the use of (6').

As with similar pipes, if \( \alpha \) and \( \alpha' \) cannot be actually measured they may be estimated by any formula, but this should be done as carefully as possible and with the best available values of the friction constants.

3. **Three pipes cross-connected.**

The two \( d \)- and \( D \)-pipes previously referred to may be laid the whole length between the reservoirs and cross-connected. To increase supply a third pipe of diameter \( D_2 \) may be laid along:

- side and cross-connected to the adjacent \( d \) or \( D \)-pipe at the same point as the \( d \) and \( D \)-pipes are themselves cross-connected;
- the \( D_2 \)-pipe may be presumed also to be laid the full length and only a portion \( k \ell \) in length between cross connections to contribute to delivery by the closing of suitable valves. See Fig. (6).

The head being \( h \), the Hydraulic Gradient will be as in Fig. (1), and hence dealing with the \( d \)-pipe equation (1) holds as before, that is

\[
h = \frac{\mu \ell}{d^m} (1 - k) + \frac{\mu \ell}{d^m} k \ell
\]

In addition to the terms stated before

Let

\[
\phi_o = \text{discharge of } d \text{-pipe alone as single pipe between the reservoirs.}
\]

\[
\phi_o = \text{discharge of } D \text{-pipe under above condition.}
\]

\[
\mu_{x_1}, n_{x_1}, m_{x_1} = \text{friction constants for the } D_2 \text{-pipe}
\]

\[
\phi \times \phi = \text{quantity flowing in the } d \text{ and } D \text{-pipes respectively in the lengths where the } D_2 \text{-pipe is not in action, i.e., where the velocities are } v - v \text{ in Fig. (6).}
\]

Also as there are now two full length pipes \( \phi_o, \phi, \phi', \phi \), and while representing corresponding quantities will be redefined.
\[ Q_0 = \text{combined discharge of the two full length (d and D) pipes as single unconnected pipes between the reservoirs; or } Q_0 = \frac{Q_o + Q'_o}{2} \]

\[ Q' = \text{discharge of the } D\text{-pipe under the above condition.} \]

\[ Q'_e = Q_o + Q'_o = \text{combined discharge of the three pipes under above conditions.} \]

\[ Q = \text{discharge under condition in Fig. (6) or } Q = \frac{Q_o}{2} + Q'_o \]

Then corresponding to (2)

\[ \frac{\mu_v \eta^2 k^2}{d_m} = \frac{\mu_v \eta^2 k^2}{D_m} = \frac{\mu_v \eta^2 k^2}{d_m} \quad (\text{see Fig. (6), for } \overline{D}_f) \]

and

\[ \frac{v_d^2 + v_D^2}{\eta d^2 + \eta D^2} = \frac{v_d^2 + v_D^2}{\eta d^2 + \eta D^2} \]

or

\[ v_d = \left( \frac{v_d^2 + v_D^2}{\eta d^2 + \eta D^2} \right) \]

From this,

\[ v \left( \frac{Q'}{Q} \right) = v_d \left( \frac{Q_o + Q'_o}{Q_o} \right) \]

where \( x, y, \) and \( z \) denote the quantities flowing in the \( d, D \) and \( D_2 \) pipes respectively in the portion \( k \) in Fig. (6).

If now the three pipes are of similar surface i.e.

\[ n = n_2 = n_2 \text{ etc. } \]

(8) shows that the ratios \( \frac{Q}{Q_o} \) and \( \frac{Q'_o}{Q} \) are independent of the velocities since \( \frac{v_d}{v} \) and \( \frac{v}{v_d} \) are constant, so

\[ \frac{Q}{Q_o} = \frac{Q_o}{Q_o} \]

and

\[ \frac{Q'_o}{Q} = \frac{Q'_o}{Q'_o} \]

; substituting in (10) and noting that \( Q_o + Q'_o + Q'_e = Q_o \)

; then on reduction

\[ v = v_d \frac{Q_o}{Q_o} \]

which is similar to equation (4); and, inserting this relation:

\[ : \text{ship in (1), equation (5) is again obtained. As before } n \text{ may be taken at 2 in all cases in (5) and thus equation (6) i.e.} \]

\[ \eta = \frac{1 - \left( \frac{Q_o}{Q} \right)^2}{1 - \left( \frac{Q'_o}{Q'_e} \right)^2} \]

may be taken in the case of 3 pipes just as it may in that of two pipes. \( Q_o \) is the analogous quantity in the two cases, viz.

the discharge of the whole length pipes as simple pipes between the reservoirs, and \( Q'_o \) that of the whole three pipes under these conditions; and both \( Q_o \) and \( Q'_o \) must be estimated with correct
values of $n, n_1, n_2$ etc. or with correct values of $c$ in
\[ v = c \sqrt{m} \] if that equation be used.

This result might have been anticipated; the two $\alpha$ and $D$-pipes might clearly be replaced by one giving the dis:

:charge \( q_0 q_0 = Q_0 \). Secondly, if the two full length
\( (\alpha + D) \) pipes are of similar surface but the $D$-not of the
same frictional resistance, then as before

\[ \frac{\gamma}{x} = \frac{L}{v} = \frac{Q_0}{Q_0}, \text{ and (10) becomes} \]

\[ v = v_i \left( 1 + \frac{3/\alpha}{1 + \frac{Q_0}{Q_0}} \right) \]

Here \( 3/\alpha \) only is variable; when \( k \) is just approaching unity,

\[ \frac{3/\alpha}{1 + \frac{Q_0}{Q_0}} \]: and then substituting this in (11)

\[ v = v_i \frac{Q_1}{Q_0} \]

Thus only when the Hydraulic Gradient is \( h/2 \) (and \( v_i - v_0 \))
does \( v_i/Q_1 = Q_0/Q_0 \).

Equation (11) is exactly similar in form to the equa:

:tion \( v = v_i (1 + \frac{3/\alpha}{1 + \frac{Q_0}{Q_0}}) \) for the case of two dis-similar pipes

cross-connected; taking \( v = v_i \frac{Q_0}{Q_0} \) is equivalent to replacing

the $D$-pipe by one giving the same delivery at Hydraulic Gradient

\( h/2 \) but of surface similar to that of the $\alpha$ and $D$ pipes;

and the error in making this substitution may be graphically

realised by consideration of the slight alterations in Hydraulic

Gradient as explained in reference to Fig. (4); and is dealt

with further in the case of three dis-similar pipes later, on p. 14.

Errors in taking \( v = v_i \frac{Q_0}{Q_0} \) will be small just as in

the two pipe case; the variable \( 3/\alpha \) of the two pipe case is

replaced by the variable \( 3/\alpha \) divided by the constant \( 1 + \frac{Q_0}{Q_0} \).

Examples are given later. (p.17 and 18 and Table III.)

So (6) may be used for relation between $k$ and \( \frac{Q_0}{Q_0} \)
in this case also with no error of practical amount.

Lastly when all three pipes are of different surface,
In (10) \( \frac{d\theta}{dx}, \frac{3}{4x} \) and \( \frac{3}{x} \) are all variable. When \( k \) is just approaching unity \( \frac{d\theta}{dx} = \frac{d}{dx} \) and \( \frac{3}{x} \) but \( \frac{3}{x} \) does not then equal \( \frac{d}{dx} \), so that even when \( k \) is approaching unity \( \frac{\theta}{x} \) does not become equal to \( \frac{d}{dx} \) as in the previous cases, though it may not in practical cases differ much from this value.

Taking \( \frac{\theta}{x} = \frac{d}{dx} \) leads to equation (6) again, and is equivalent to considering all the pipes to be of the same nature of surface as the \( d \)-pipe, that is to substituting for the \( D \) and \( D_2 \)-pipes, pipes of the same nature of surface as that of the \( d \)-pipe and with changed diameters so that at the Hydraulic Gradient \( \frac{h}{l} \) they give the same discharges as the actual pipes, namely \( \theta_0 \) and \( \theta' \).

The error incurred in doing so may be illustrated graphically as was done in the case of the two pipes.

The actual pipes all differ in the nature of surface. Take the \( d \)-pipe "rough" i.e. \( n = 2, m = 0.15 \) and \( \mu \) at any suitable high value. Substituting, for the \( D \) and \( D_2 \)-pipes, rough pipes giving \( \theta c \) at Hydraulic Gradient \( \frac{h}{l} \) and determining \( k \) for a chosen value of \( \frac{\theta}{\theta_0} \) from (6), let \( \theta c \) (Fig.7) be the Hydraulic Gradient, \( \theta c \) being the proportion in "triplicate", that is over which the \( D_2 \)-pipe is laid, or in action and contributing to delivery.

If now the actual \( D \) and \( D_2 \)-pipes are "rough" or "smooth" - smooth when \( n = 1.95 \) and \( m = 1.10 \) and \( \mu \) is one half its value when rough - the effect may be shown as follows.

(a) \( \theta, D, \) and \( D_2 \)-pipes rough, rough, smooth respectively. Considerations as with the two pipes show that the Hydraulic Gradient changes to \( \theta_2 \) Fig. 7.

(b) \( \theta, D \) and \( D_2 \)-pipes rough, smooth, smooth respectively. Hydraulic Gradient is \( \theta_2 \) Fig. 7.

(c) \( \theta, D \) and \( D_2 \)-pipes rough, smooth, rough respectively. Hydraulic Gradient is \( \theta_2 \) Fig. 7.

The \( d \)-pipe may be smooth and the \( D \) and \( D_2 \)-pipes first considered all to be smooth also, and a similar diagram may be drawn, but
the corresponding altered hydraulic gradients will lie below \( \frac{d \varphi}{dc} \).

Intermediate values of roughness and smoothness may be considered and the results will lie within the limits for extreme variations of the nature of the pipe surfaces.

The exact proportion over which the \( D_2 \)-pipe must be laid for given values of \( \frac{d \varphi}{d \rho} \) and \( \frac{d h}{d \rho} \) may be calculated thus:

From (6) and (9), and also since \( \nu = F \) are the velocities in the \( \alpha \) and \( D \)-pipes under the same Hydraulic Gradient so that

\[
\frac{\mu_1 \nu_1^n}{D_2^m} = \frac{\mu_\alpha \nu_\alpha^n}{D_\alpha^m}
\]

there is obtained

\[
u + \nu_2 F \left( \frac{\mu_\alpha}{\mu_1} \right) \frac{D_2}{D_\alpha} \frac{Z_\alpha}{Z_2} = \nu_1 + \nu_2 F \left( \frac{\mu_\alpha}{\mu_1} \right) \frac{D_2}{D_\alpha} \frac{Z_\alpha}{Z_2} + \nu_\alpha \left( \frac{\mu_\alpha}{\mu_1} \right) \frac{D_2}{D_\alpha} \frac{Z_\alpha}{Z_2} \quad \cdots (12)
\]

so that \( \frac{\nu}{\nu_1} \) is not constant unless all pipes are similar.

Putting \( \nu_1 = \beta \nu \) where \( \beta \) varies with \( \nu \), and noting that

\[
h = \frac{\mu \nu_1}{\beta^{\frac{1}{2}}} \frac{\mu_1 \nu_\alpha}{\mu_\alpha}
\]

and that \( \frac{\varphi}{\varphi_0} = \frac{\nu}{\nu_0} \) then from (1), corresponding to (9), there is obtained

\[
k' = \left( 1 - \frac{\nu}{\nu_0} \right)^n \quad \cdots (13)
\]

When \( k' \) is just approaching the value unity, \( \nu \) becomes equal to \( \nu_0 \). If \( \varphi_0 \) is the discharge through the \( \alpha \)-pipe at that instant (in the portions in Fig. 6 where the velocity is marked \( \nu \) ) then

\[
\beta = \frac{\nu_0}{\nu} = \frac{\varphi_0}{\varphi}
\]

Also comparing velocities under corresponding Hydraulic Gradients in the \( \alpha \) and \( D \)-pipes

\[
\left( \frac{\varphi_0}{\varphi} \right)^n = \left( \frac{\nu_0}{\nu} \right)^n \quad \text{and so} \quad \left( \frac{\varphi_0 \alpha}{\varphi_\alpha} \right)^n = \left( \frac{\nu_0 \alpha}{\nu_\alpha} \right)^n
\]

or \( \left( \frac{\varphi_0}{\varphi} \right)^n = \left( \frac{\nu_0}{\nu} \right)^n \) and thus

\[
\left( \frac{\varphi_0}{\varphi_0} \right)^n = \left( \frac{\nu_0}{\nu_0} \right)^n \quad \text{when} \quad \varphi = \varphi_0
\]

\( \varphi_0 \) being for the \( D \)-pipe what, \( \varphi_\alpha \) is for the \( \alpha \)-pipe (as defined above)
So that (13) may be put approximately

\[
k' = \frac{1 - \left(\frac{\alpha}{\theta}\right)^n}{1 - \left(\frac{\alpha}{\theta}\right)^m} = \frac{1 - \left(\frac{\alpha}{\theta}\right)^m}{1 - \left(\frac{\alpha}{\theta}\right)^n}. \tag{14}
\]

This might also be deduced at once by writing an equation similar to (1) for the \( D \)-pipe as obviously either of these pipes might be taken as basis in equation (1). As before, in all cases of use of equation (13) or (14) \( n \) or \( m \), may be taken at 2 without practical error as explained previously. But it would clearly be better to use either equations (12) and (13) or the corresponding equations for the \( D \)-pipe according as \( h \) or \( n \), is nearer 2 in actual value.

As an example of the difference between results given by (6), or by (13) or (14) with \( n = n_1 = 2 \), and of the method of calculation, take \( D = D = \frac{2}{3} D_2 = 1.4 \) so that the diameter of the part length pipe is 1\( \frac{2}{3} \) times that of the full length pipes; and let the pipes be rough, smooth, rough, (as defined before) respectively. Also take \( v_c = 3 \) feet per second.

With \( n = n_1 = 2; m = m_2 = 1.25; \mu = \mu_2 = 2 \mu_1; \eta = 175, \mu = 1.10 \)
equation (12) becomes

\[
u + 1.486 v^{\frac{6}{7}} = 3.7 v_0 + 1.486 v^{\frac{6}{7}}. \tag{15}
\]

Plotting both sides of this equation on \( v \) and on \( v_c \) as base up to \( v = 3 \frac{H}{g} \) as a maximum, corresponding values of \( v \) and \( v_c \) may be found; and hence columns (4) and (5) in Table II obtained for selected values of \( v \) in column (2)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>( \eta )</th>
<th>( \mu )</th>
<th>( \eta )</th>
<th>( \mu )</th>
<th>( v_c )</th>
<th>( v_c )</th>
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<tr>
<td>1</td>
<td>1.75</td>
<td>3</td>
<td>20</td>
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<td>100</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
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<td>4</td>
<td>25</td>
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<td>3</td>
</tr>
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<td>6</td>
<td>30</td>
<td>20</td>
<td>300</td>
<td>600</td>
<td>2.00</td>
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<td>5</td>
</tr>
<tr>
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<td>2.5</td>
<td>7</td>
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<td>25</td>
<td>200</td>
<td>500</td>
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<td>7</td>
</tr>
<tr>
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<td>3.0</td>
<td>8</td>
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<td>200</td>
<td>6.00</td>
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<td>5.0</td>
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<td>60</td>
<td>50</td>
<td>5</td>
<td>50</td>
<td>8.00</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
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<td>5.5</td>
<td>13</td>
<td>65</td>
<td>55</td>
<td>2.5</td>
<td>25</td>
<td>9.00</td>
<td>11</td>
<td>19</td>
</tr>
</tbody>
</table>

**TABLE II.**
Then \( \left( \frac{C}{C_0} \right)^n \) gives values in column (6).

Also \( C_0 = \frac{C}{C'_{10}} \); and \( C \) being determined from \( C_0 \) by usual principles and \( C \) taken at unity, columns 7, 8, 9, and 10 are obtained. Columns 11 and 12 give corresponding values of \( k' \) and \( k(\alpha_n') \), and, from the slight difference in the two values, and if \( k' \) and \( k \) be plotted on \( \alpha_n \) as base as in Fig. (2), it is seen that equation (6) may be used for all practical purposes and that the laborious work involved in the use of (13) is unnecessary.

To determine the difference between \( k \) and \( k' \) generally as obtained from (6) and (13) within practical limits of velocities and pipe diameters the procedure outlined in the case of two pipes may be adopted. A set of three pipes all of the same nature of surface and giving a certain value of \( \frac{\partial \nu}{\partial \alpha_o} \) may be chosen. Then the nature of the surface and the diameters of the pipes may be assumed to change subject to the condition that \( \alpha'_n \) and \( \alpha_n \) remain the same so that \( \frac{\partial \nu}{\partial \alpha_o} \) remains constant. This may be done for a few such sets for \( \frac{\partial \nu}{\partial \alpha_o} \) varying from about 1.5 to 3.0 — about the practical limits.

Let \( \alpha' \), \( D' \), and \( D_2' \) be the diameters of the set of three pipes all of the same nature of surface and \( D' \) and \( D_2' \) be the new diameters of the \( D \) and \( D_2 \)-pipes when the nature of the surfaces is changed; also \( C' \) and \( C' \) the velocities at Hydraulic Gradient \( \frac{h}{l} \) in the \( D \)-pipe and in the pipe of different nature of surface and of diameter \( D' \) which replaces it.

Then

\[
\frac{C'_0 (D')^2}{\alpha'} = \frac{C}{C'} \frac{D'^2}{D} \quad \text{and} \quad \frac{C' \nu'^{n'}}{D'^{m'}} = \frac{C \nu^{n}}{D^{m}}
\]

and on reduction from these equations as previously (6.7)

\[
(D')^{2 + \frac{\nu}{\alpha'}} = \left( \frac{C'}{C} \right)^{\frac{n}{m}} \frac{D'^{2 + \frac{\nu}{\alpha'}}}{\alpha' \nu'^{n'}}
\]

and a corresponding equation is obtained for the \( D \)-pipe by substituting \( \frac{\alpha'}{\alpha} \) for \( D' \) and \( \frac{D}{D'} \) for \( \frac{\nu'}{\nu} \). Since \( D \) and \( D_2 \) in (12) are now represented by \( D' \) and \( D_2' \), re-write it with this alteration; and then substitute for \( (D')^{2 + \frac{\nu}{\alpha'}} \) the
value as given by (16); and similarly for \((\varphi_3')^{2+\varphi_3}\) from the
equation corresponding to (16) and

\[
v + v' \frac{\varphi_3}{D^{2+\varphi_3}} = v_0 + v'_0 \frac{\varphi_3}{D^{2+\varphi_3}} - \frac{\varphi_3}{D_0^{2+\varphi_3}} - \frac{\varphi_3}{D_0^{2+\varphi_3}}\quad (17)
\]
gives the relationship between \(v\) and \(v'\) for different values
of the frictional constants while retaining (in the equation) the
diameters \(d\), \(D\) and \(D_2\) of the original pipes chosen, though these
actually alter as the frictional constants and nature of the pipe
surfaces is changed.

For any values of \(d\), \(D\) and \(D_2\) this equation may be used
as (12) was in the last example and values corresponding to those
in Table II obtained. Carrying out this for values of \(\varphi_{20}\) at
1.50, 2.07 and 2.46, results are given in Table III. Therein
the letters such as R S S mean that the \(d\), \(D\) and \(D_2\)-pipes have been
changed from all rough, to rough, smooth and smooth (as defined
before) respectively. As both \(k\) and \(k'\) plotted on \(\varphi_{20}/\varphi_0\) give
curves which meet when \(\varphi_{20}/\varphi_0 = 1\) and also at \(\varphi_{20}/\varphi_0 = \varphi_{20}/\varphi_0\) a comparision
for \(\varphi_{20}/\varphi_0\) about midway between 1 and \(\varphi_{20}/\varphi_0\) gives a close
approximation to the maximum difference between \(k\) and \(k'\).

<table>
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<th>(d) : (D = \frac{3}{2}D_2)</th>
<th>(\varphi_{20}/\varphi_0 = 2.46)</th>
<th>(\varphi_{20}/\varphi_0 = 1.50)</th>
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<td>RRR 1.50</td>
<td>RRR 1.50</td>
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<td>RRR 1.46</td>
<td>RRR 1.46</td>
</tr>
<tr>
<td>(\varphi_{20}/\varphi_0 = 1.37)</td>
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<td>RRR 1.37</td>
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</table>

<table>
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<th>(d) : (D = \frac{2}{3}D_2)</th>
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<table>
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<td>RRR 1.23</td>
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</table>

| \(\varphi_{20}/\varphi_0 = 1.25\) | RRR 1.25 | RRR 1.25 | RRR 1.25 |

| \(\varphi_{20}/\varphi_0 = 1.23\) | RRR 1.23 | RRR 1.23 | RRR 1.23 |

| \(\varphi_{20}/\varphi_0 = 1.22\) | RRR 1.22 | RRR 1.22 | RRR 1.22 |

| \(\varphi_{20}/\varphi_0 = 1.21\) | RRR 1.21 | RRR 1.21 | RRR 1.21 |

| \(\varphi_{20}/\varphi_0 = 1.20\) | RRR 1.20 | RRR 1.20 | RRR 1.20 |

| \(\varphi_{20}/\varphi_0 = 1.19\) | RRR 1.19 | RRR 1.19 | RRR 1.19 |

| \(\varphi_{20}/\varphi_0 = 1.18\) | RRR 1.18 | RRR 1.18 | RRR 1.18 |

| \(\varphi_{20}/\varphi_0 = 1.17\) | RRR 1.17 | RRR 1.17 | RRR 1.17 |
It is seen that the differences are small; equation (6) is quite satisfactory from the practical point of view.

In the cases taken in Table III $\alpha$ is taken equal to $D$. Without altering $\alpha_o$ and $\frac{\alpha}{\alpha_o}$ the diameter of the two original similar $\alpha$ and $D$ pipes might be altered; e.g. $\alpha$ reduced and $D$ increased or vice versa; and this would involve an alteration in the ratio $\frac{D^2}{\alpha}$. But if this be done and results be worked out the difference between $k$ and $k'$ cannot be of practical importance. For if either $\alpha$ or $D$ be assumed to be gradually reduced, finally reaching zero, the difference between $k$ and $k'$ will change gradually also, finally reaching the value for two dissimilar pipes with the same value of $\frac{\alpha}{\alpha_o}$. It has already been shown that the difference between $k$ and $k'$ is not of practical importance in that case. If the three original similar surfaced pipes be presumed all smooth differences of similar magnitude are obtained but the points $\beta$, $\beta_2$ and $\beta_3$, Fig. (7) will lie below $\alpha \beta c$.

Thus equation (6) is applicable also to the case of three pipes all of different surface without practical error; and so also the graphical construction of Fig. 7 for the Hydraulic Gradient.

While any number of pipes may be cross-connected yet for practical reasons the system would probably be worked in sets with not more than three cross-connected pipes in each set; but in the case of more than three pipes, equations corresponding to (1) (8) (9) and (10) may be written. If the pipes are all the same as regards nature of internal surface, then $\theta = \frac{\alpha}{\alpha_o}$ and thus (6) holds as before.

In the case when the pipes are not of the same nature of surface, considerations of the Hydraulic Gradient changes similar to those in reference to Fig. (7) show that small errors will occur if (6) is adopted to determine $k$ just as with the case of three pipes, but that these will not be of practical importance.

\* See Appendix. Cases 4, 5, and 6. p. 29.

The investigations so far have been made on the supposition that all pipes except one are full length pipes between the reservoirs; in the case of two or more pipes laid the full length between the reservoirs and cross-connected, portions of each of the pipes may be put out of action and it may be required to estimate the delivery under such conditions.

Fig. 8 (1) shows, in Plan, 3 pipes cross-connected. The portion $AA'$ of the $D_1$-pipe may be cut out; next the portion $BB'$ of the $D$-pipe also, and finally, along with these two portions, the length $CC'$ of the $D'$-pipe may be shut off. General methods might be adopted to obtain a solution of the problem but these become cumbersome and (6) may again be used subject to a correction. It will be seen from the previous work that it is immaterial whether the portions cut out are adjacent as in the Figure or not; these may be placed anywhere provided that not more than one pipe is shut off between the same cross-connections, and, from the practical point of view, that the position of the hydraulic gradient is satisfactory with reference to the pipeline position.

At first it will be presumed that all the pipes are of similar nature of surface and that \( n = 2 \). When the portion $AA'$ of the $D_2$-pipe is cut out, (6) gives the new delivery, \( k \) being of value \( \frac{1 - \frac{q^2}{L}} \). Denoting this delivery — with the length $AA$ cut out — by \( Q_a \) then $\delta c$ Fig. 8 (2) is the hydraulic gradient and \( \delta b = h \left( \frac{Q_a}{\alpha} \right)^2 \) as explained in reference to the construction of Fig. (3); and \( \delta a = k h \left( \frac{Q_a}{\alpha} \right)^2 \).

Next when in addition the portion $BB'$ of the $D$-pipe is cut out, the portion of the system from $B$ to the lower reservoir may be looked upon as a system of three pipes delivering at hydraulic gradient $\frac{3}{h}$ and that it is required to find the new delivery when the portion $BB'$ of the $D$-pipe has been cut out. This may be done by another application of (6); but so far the
result will be approximate only; the assumption is made that \( a^2 = \bar{a} \) remains constant.

In using (6) \( k \) is now \( 1 - \frac{b}{2-a} \); \( a_e \) represents the delivery of the \( a, a_e \) pipes at hydraulic gradient \( \frac{3}{2-\alpha} \) i.e.

\[
q_a \left[ \frac{q_a}{a} + \frac{q_e}{a_e} \right]
\]

and \( q_e \) (in the denominator of (6)) now represents the delivery of the three pipes at hydraulic gradient \( \frac{3}{2-\alpha} \) viz., \( q_a \).

Denote this first approximation by \( \tilde{q}_a \). This under:

estimates the delivery; the pressure head at \( \beta \) or \( \beta' \) must rise as \( q_a \) is greater than \( \tilde{q}_a \). If it rises by the amount \( \bar{bc} \) then \( \alpha \) Fig. 8 (2) is the hydraulic gradient and it is required to find the true delivery - lying between \( \tilde{q}_a \) and \( q_a \). Note that in the Fig. \( \alpha^2 \) must cross \( \bar{bc} \), \( \bar{fc} \) being at a flatter gradient than \( \bar{bc} \), but it is shown above for clearness in the figure. The delivery of a system will vary as the square root of the head if \( h=2 \), hence if \( \tilde{q}_b \) is the actual delivery with \( \alpha, \alpha' \) and \( \beta, \beta' \) cut out, and referring to Fig. 8 (2),

\[
\frac{\tilde{q}_b}{\tilde{q}_a} = \sqrt{\frac{h-2}{h-\bar{a}}}
\]

and

\[
\frac{q_b}{\tilde{q}_a} = \sqrt{\frac{y}{\bar{a}}}
\]  \[\text{(18)}\]

from which

\[
h = \frac{\tilde{q}_b^2}{q_a^2} \bar{a} + \frac{q_b}{q_a} (h-\bar{a}) \]

\[\text{(19)}\]

Also from the figure

\[
j = \frac{k}{\tilde{q}_a} \frac{(q_a)^2}{q_a} \]

\[\text{(20)}\]

and from (19) and (20)

\[
\frac{\tilde{q}_b^2}{\tilde{q}_a^2} = \frac{\bar{a} \cdot \tilde{q}_a^2}{k \cdot q_a (q_a^2 - \tilde{q}_a^2) + \tilde{q}_b^2 \cdot \tilde{q}_a^2}
\]  \[\text{(21)}\]

For the hydraulic gradient \( j \) is obtained from (20) and \( y \) from (18). \( f_j \) is determined from \( f_j = k h \frac{\tilde{q}_b^2}{q_a} \) as explained below in dealing with cut out of length \( \alpha \); and so the slope of the hydraulic gradient is known at all positions of the pipe where cut-outs are made.

In numerical cases it may be in preference to using (21), \( j \) may be determined from (20) and \( \tilde{q}_b \) is then obtained at
denoting \( f_g \) by \( j \), and \( f'_g \) by \( j' \), then

\[
\frac{Q_c}{Q_3} = \sqrt{\frac{j'}{j}}
\]

corresponds to (18) and determines \( j' \), as \( j' \) or

\( f_g \) is equal to \( \frac{k_k}{k} \frac{Q_g}{Q_i^2} \) as shown above; \( mm = \frac{k_k}{k} \frac{Q_g}{Q_i^2} (1 - \frac{a_{ik}}{2}) \)

by analogy with \( f_g \); and finally, as the slope of \( ad' \) depends on \( Q_c \)

\[
\frac{h - ea'}{h (\frac{Q_c}{Q_o})^2} = \frac{2(r - k)}{\frac{1}{2}} = 1 - k
\]
determines the value of \( ad' \).

If the pipes of the system are not all of similar nature of surface it has been seen that no error of practical importance is incurred in the repeated use of (6) provided the deliveries of the two full length pipes in each case are estimated correctly. In the case of cutting out \( AA' \), \( Q_o \) and \( Q_o \) are known, but in the case of \( BB' \) it will be necessary to determine the deliveries of the \( D_1 \) and \( D_2 \) pipes under gradient \( \frac{3}{2} h \); and of the \( D_1 \) and \( D_2 \) pipes under gradient \( \frac{2}{h} k \) in the case of cutting out the length \( CC' \) out of the \( C \)-pipe. This may be done by methods previously explained in the case of three pipes of different nature of surface. The sum of these deliveries for each case is the value to be adopted for \( Q_o \) in the use of (6). But since in (6) \( Q_o \) occurs both in numerator and denominator the difference in its value from that obtained if the ratios of the deliveries of the pipes are presumed to be the same as that at gradient \( \frac{1}{h} \) will not cause errors of practical importance in the estimation of \( \alpha_1 \) and \( \alpha_3 \). Errors of small amount are introduced in \( \alpha_1 \) and \( \alpha_3 \) by the use of (18) if \( n_1 \), \( n_r \), and \( n_2 \) are not equal. For all practical purposes the method outlined for the case of pipes of similar nature of surface may be adopted provided the deliveries \( Q_o \), \( Q_o \) and \( Q' \) at gradient \( \frac{1}{h} \) are estimated with the best possible values of the friction constants in whatever equation is used to do so.

5. Cross-connection of branched mains. In many cases subsidiary branch mains may be taken off from the principal main at
various points on its length between the reservoirs. When increased deliveries in the various portions of the principal main are required, the portion of the length over which a second main must be laid in order to give these new increased deliveries may be determined very simply by the application of the formulae here derived; and by suitable location of this length of second main the hydraulic gradient may be made to conform to certain requirements of increased pressure at selected points.

Thus in Fig. (9) \( ABCDE \) is a principal main between reservoirs at \( A \) and \( D \), while branches or subsidiary mains are taken off at \( BC, B \) and \( E \). In the figure let \( abcdef \) be the hydraulic gradient with the existing deliveries; from this the values of \( C \) in \( v = C \sqrt{m^2} \) may be determined for the various portions of the existing pipe. Also let it be required with the new deliveries to increase the pressure head at \( D \) from \( Da \) to \( Dd \), so that the new hydraulic gradient must pass through \( d \).

The values of \( C \) being known for the existing main let \( h_1 \) and \( h_2 \) be the estimated heads lost respectively in the portions \( DE \) and \( EF \) if there the existing pipe alone gave the increased deliveries required. Then if \( h_1 + h_2 \) is less than the fall from \( d \) to \( f \), no second main is required; if it is greater, then a second main is required for a certain portion of \( DF \) depending on the diameter of this second main, on the nature of its surface and on its frictional coefficients.

In this case suppose next that over the length \( EF \), no second main is laid, and that to give the new increased delivery in \( EF \) the hydraulic gradient must rise at \( E \) to the point \( g \). With the fall \( d \varphi \), over the length \( DE \) estimate the delivery of the existing main and of the proposed new main. Then to determine the portion of the length \( DE \) over which the second main must be laid equation (6) may be used; \( \varphi_0 \) is the delivery of the existing main over \( DE \) with fall \( d \varphi \), and \( \varphi_1 \), that of the existing and the proposed new main combined with the same fall \( d \varphi \), while \( \varphi \) is the new discharge required in the portion \( DE \).
If, however, the delivery of the two mains laid the full length from $D$ to $E$ with the fall $\alpha_2$, is less than the new delivery required in the length $DE$, the hydraulic gradient must be lowered below $\varepsilon$, at the point $E$ on the pipe, and the second main must continue beyond $E$. Exactly analogous procedure determines how far it must continue. With the second main laid from $D$ to $E$ determine the loss of head over the length $DE$ when these mains together give the new delivery required in this length; and with this loss of head, let the hydraulic gradient at $E$ be at elevation $\varepsilon_2$, (not shown on figure but below $\varepsilon_1$).

Then estimating the deliveries of the existing and the new main with the fall $\varepsilon_2$, over the length $EF$, the proportion of $EF$ over which the second main must be laid may be determined from equation (6) in manner similar to that described in the case of $DE$. Of course, preliminary calculation would have fixed the diameter of the second main so that with the fall $\alpha_2$, the second main along with the existing one would at least give the new deliveries required.

The length or lengths of second main between $A$ and $D$ may similarly be determined; and if it is laid from $A$ to a point between $B$ and $C$; and from $D$ to a point between $D$ and $E$ then the type of hydraulic gradient is $a b c d e f$; which may be determined by estimating the losses of head in the various portions. In all cases a cross-connection must be made between the mains at the points where branch or subsidiary mains are taken off.

By the use of equation (6) the calculations are equally simple in principle whether the existing and new mains are similar in nature of surface or extremely different; and the method is equally applicable in the case of more than one original main provided the original mains are cross-connected at the points where the subsidiary mains are taken off. But since the values of $\alpha_0$ and $\alpha_1$ in the repeated use of (6) have to be estimated for various hydraulic gradients, more reliance can be placed on
the calculated results if recent determinations of the values of
$C$ in $u = C D m$ are made on the existing main at various
hydraulic gradients, and speeds of flow in the pipe.

6 Corrections for Minor Losses.

Minor losses such as those at the cross connections
valves etc. have been omitted in this investigation. With the
usual ratio of pipe length to diameter the error introduced by
the omission of these losses is negligible in comparison with
other errors due to estimation of the friction coefficients for
the pipes. If, however, the $D$-pipe consists of several separate
short segments, the total losses at the various cross connections
might become appreciable.

These losses might be allowed for in the initial equa-
tions; but if this be done, the estimation of the delivery
involves much labour of the type required in the use of equations
(7) and (12).

But as these losses should always be small and cannot
be exactly determined, probably any estimation of their effect
would be made merely to justify the assumption that they are
negligible.

An estimation of the correction required for these
losses may be made by using a slightly increased value of $\mu',$
say $\mu'_1$ as follows:

Let the loss at the cross connections be expressed by
$\alpha u^2$, where $\alpha$ is some numerical factor; and (2) may be then
written

$$\frac{\mu' u'' \frac{k^2}{m}}{D^m} = \frac{\mu u'' \frac{k^2}{m}}{D^m} + \alpha \frac{u^2}{D^m} = \frac{\mu'_1 u'' \frac{k^2}{m}}{D^m}$$

Taking $n = 2$ and $m = 1$ for this estimation

$$\mu'_1 = \mu + \frac{\alpha D}{k^2}$$

Thus $(\mu'_1 - \mu)$ depends on the ratio of the pipe diameter to the
actual length of the duplicated portion. $\alpha$ must be estimated
from the losses known for analogous types of construction or
may be measured if to do so is possible. Thus it will consist
of loss at entry to the cross connection, extra friction loss
due to higher speed in the cross connection, if it is of smaller
diameter than the D - pipe, loss at valve in the cross connection,
and loss at the junction with the D-pipe due to the change of
direction and reduction of velocity.

$\Phi^2$ may be determined approximately by using (6) and then $\mu'$
is obtained. Then re-estimating $k$ by means of (6) with the new
value of $Q_1$ corresponding to $\mu'$, i.e. $Q = Q_0 + Q_1(\mu')^2$
(if $n_1$ be taken as 2) the new value of $k$ will show if these
losses have any appreciable effect.

7. GENERAL SUMMARY.

For practical purposes the delivery in the case of a set of
cross-connected pipes may be determined by the use of the equation

$$K = \frac{1 - \left(\frac{V}{V_0}\right)^{p.4}}{1 - \left(\frac{V}{V_0}\right)^{p.5}}$$

the equation is applicable to any
number of pipes which may be of various diameters and all differing
as regards nature of internal surface. The additional equations

$$Q_i = V \frac{Q_0}{\theta_i} \quad (p. 4) \quad or \quad Q_i = V \frac{Q_0}{\theta_i}$$

$p. 5$ and

$v = \frac{V}{\theta_0}$

enable the hydraulic gradient to be determined by graphical con:
struction (p. 5) or by calculation (p. 28 ). By adopting the
deliveries of the various pipes at the general gradient of the
pipe line as basis, the problem of the delivery of a set of cross
connected pipes is reduced to the simplicity of that of a single
pipe.

In the case where the pipes are of different nature of
internal surface the use of equation (6) is equivalent to considering
the actual pipes all to be of similar nature of internal surface
but with diameters changed so that the delivery at the general
gradient of the pipe lines is unaltered. A small error is thereby
introduced, the proportionate magnitude of which is indicated in
Tables I, II and III and on figs. 5 and 5 and in the appendix (p. 28.).
The methods of calculation established here are equally applicable
to branched pipes, or between any two points in the pipe line system
at which the pressure heads are fixed; and also, with an additional
correction equation, in the general case where different portions
of the various pipes are not contributing to the delivery. 4 & 5.

(8) APPENDIX.

For all cases \( h = 30 \) feet, \( l = 24,000 \) feet, so that the general gradient of the pipe line is \( \frac{h}{l} = \frac{30}{24,000} \). All discharges are stated in cub. ft. per second.

**Case (1)** Two pipes \( d = 18" \), \( D = 20" \), both pipes clean metal; \( c (\text{in } \text{cu. ft. per sec.}) \) approximately 120. At \( \frac{h}{l} \) gradient delivery of \( d\)-pipe, \( 4.57 = Q_o \); of D-pipe, \( 6.05 = Q' = Q_o + Q' = 10.72. \frac{Q}{Q_o} = 2.295 \)

For a delivery \( Q = 7.0 \), the proportion of length over which the D-pipe must be laid is from (6)

\[
k = \frac{1 - \left(\frac{4.67}{4.7}\right)^2}{1 - \left(\frac{6.05}{10.72}\right)^2} = 0.686.
\]

Hence length of pipe in duplicate 16490 ft. = \( kl \)

" single 18" pipe 7520 ft. = \( l(i-k) \)

Over length \( kl \), hydraulic gradient, \( i = \frac{h}{l} \), \( i = \frac{30}{24,000} \), \( i = \frac{30}{24,000} \), \( i = \frac{1.25}{80} \).

" \( l(i-k) \), " " \( l = \frac{h}{l} \). \( i = \frac{30}{24,000} \), \( i = \frac{30}{24,000} \), \( i = \frac{1.25}{80} \).

For a smooth pipe of the clean metal, \( n = 1.75 \) approximately:

hence with \( \frac{Q}{Q_o} = 2.295 \) the error in taking \( n = 2 \) in the above use of (6) is under 1 per cent (see fig. 2); thus the error in \( kl \) due to using (6) in place of (5) p. (4), is only 80 yards (excess) in 3.12 miles (16480 ft.)

**Case (2)** Two pipes. \( d = 22" \), rough encrusted metal, \( c = 73; D = 20" \), smooth concrete, \( c = 120. \) At \( \frac{h}{l} \), delivery of d-pipe is \( 4.67 = Q_o \); of D-pipe, \( 6.05 = Q' = Q_o + Q' = 10.72. \frac{Q}{Q_o} = 2.295 \).

To find the proportion of the length over which the smooth D-pipe must be laid to give a delivery \( Q = 7 \). \( Q_o, Q', \text{and } Q \), are as in case (1). Hence using (6) the length in duplicate and the hydraulic gradients are as in case (1).

For a rough pipe with \( c = 73; n \) is approximately 2 and thus no appreciable error is introduced by taking \( n = 2 \), as in (6) or (6)'- (p. 7) From table I or fig. 5, the error in using (6) in place of (6)' is about 1 per cent. Values of \( k \) from (6)' are larger...
than from (6); hence the above value of $k^2$ is about 30 yards too small in a length of 3.12 miles (16480 ft.)

Case 3. Two pipes. With the two pipes of Case 2, both laid the 24000 feet, but only 16480 ft. of the 22" rough pipe between cross connections contributing to the delivery;

$$\frac{q_0}{q'} = 6.05, \quad q' = 4.67, \quad \frac{q_0}{q_1} = 10.72. \quad \frac{q_0}{q_1} = 1.77 \quad k = 0.686.$$  

To determine the delivery. From (6),

$$0.686 = \frac{1 - \frac{q_0}{q_e}}{1 - \frac{q_0}{q_1}}$$

from which $q = 8.29$

For the hydraulic gradient (1)

In the duplicated length $k^2$, $i = \frac{l}{600} \frac{q_e^2}{q_0^2} = \frac{1}{1840}$

" = single length $l/(r-k)$, $i = \frac{l}{600} \frac{q_e^2}{q_0^2} = \frac{1}{428}$

From fig. 5', the value of $\frac{q_0}{q_0}$ from (6) in place of (6') when $\frac{q_0}{q_0} = 1.77$ and $k = 0.686$ is approximately 0.4 per cent too small. The full length pipe in this case is smooth, so that $n$ is about 1.75; from fig. (2) at $k = 0.686$ and $\frac{q_0}{q_e} = 2$, if $n = 1.75$ in place of 2, $\frac{q}{q_0}$ will be increased by about 1.00 per cent. Hence estimation by (5') (p. 7) in place of (6) would give $Q = 8.39$, compared with 8.29 as above.

Case 4. Three pipes, $d = 12"$, $D = 15"$, $D_2 = 20"$, all pipes clean metal, $C$ approximately 120. At $\frac{l}{600}$ gradient, delivery of the $d$-pipe 1.67 = $q_0$, of the $D$ pipe 3.00 = $q_0$, of the $D_2$ pipe, 6.05 = $q'$. To determine the proportion of the length over which the $D_2$ pipe must be laid to give a delivery $Q = 7$.

Here, $q_0 = q_0 + q_0 = 4.67$; $q_0 + q_0 + q' = 10.72$; $\frac{q_0}{q_0} = 2.295$.

These values are as in case (1). So results from (6) are as in that case; and $k^2 = 16480$ ft; hydraulic gradient over the length where the $d$ and $D$ pipes only are laid is $\frac{1}{1840}$, and where the three pipes are laid is $\frac{1}{1840}$. The length in triplicate is as before 80 yards in excess of that determined from (5).

Case 5. Three pipes $d = 14"$, rough encrusted metal, $c = 72$; $D = 18"$ encrusted metal $c = 85$, $D_2 = 20"$, smooth concrete, $c = 120$
At \( \frac{Q}{300} \), delivery of the D-pipe is \( 1.40 = \beta_0 \); of the D-pipe \( 3.27 = \beta_0 \); of the \( D_2 \)-pipe \( 6.05 = \beta' \). So \( \beta_0 = \beta_0 + \beta' = 4.67 \), \( \beta_0 + \beta' = 10.72 \), \( \frac{\beta_0}{\beta_0} = 2.295 \). To determine the length over which the \( D_2 \)-pipe must be laid to give a delivery \( \beta = 7 \).

The values are as in case (1), and thus from (6) the values of \( k \) and of the hydraulic gradients are as in that case. The value of \( \frac{\beta_0}{\beta_0} \) here is 2.295, and the pipes are \( RRS \) (Table III) and from that table \( k \), determined from (6), is approximately 1 per cent less than from (13) with \( n = 2 \), and thus the error is also as in case 2.

Case 6. Three pipes. With the three pipes of case (5) all laid the full length, but only 16480 feet of the 14" rough encrusted pipe between cross connections contributing to the delivery, then (see Case 5) \( \beta_0 = 3.27 + 6.05 = 9.32 \), \( \beta' = 1.40 \), \( \beta_1 = 10.72 \), \( \beta_0 = 1.15 \) and \( k = 0.686 \); to determine the delivery \( \beta \). From (6)

\[
\frac{0.686}{1 - \left( \frac{\beta_0}{\beta} \right)^2} = 1 - \left( \frac{0.32}{10.72} \right)^2
\]

and thus \( \beta = 10.18 \)

For the hydraulic gradient (\( i' \))

In the triplicated length \( kl \), \( i' = \frac{1}{800} \frac{\beta_1}{\beta} \approx \frac{1}{865} \)

In this case the pipes are \( RSR \) (Table III) and from that table with the small value of \( \frac{\beta_0}{\beta_0} \) of this case (1.15) there is little error in \( \frac{\beta_0}{\beta_0} \) by using (6) in place of (13).

Cases 7 & 8. Errors due to assuming all pipes to be of similar nature of surface (referred to p. 2)

Case 7. Two pipes as in case (2); and as in that case to find the length of D-pipe to give a delivery \( \beta' = 7 \).

(a) Take both pipes rough (with \( c = 73 \)). Delivery of d-pipe \( \beta_0 = 4.67 \) as before; of D-pipe \( \beta' = 3.64 \) (v. 6.05); and \( \beta_1 = 8.31 \) Using (6). \( k = \frac{1 - (\frac{\beta_0}{\beta})^2}{1 - (\frac{\beta_0}{\beta})^2} \approx 0.814 \)

Length in duplicate \( k = 19500 \) ft. (v. 16480 ft, case(2) )
giving an error of $18\frac{1}{2}$ per cent.

(b) Take both pipes "smooth" (with $c = 120$) Delivery of d-pipe $Q'_o = 7.60$.

Thus $Q'_o$ is greater than $Q$, and length in duplicate is nil. (v. 16480 feet, case (2))

Case 8. Taking the same two pipes as in case (2). To determine the length over which the d-pipe must be laid to give a delivery $Q = 7.0$.

By methods as in case (2) the required length is 8900 ft. with an error of about $1\%$ by the use of (6) in place of (5').

(a) If both pipes be assumed to be "smooth" (with $c = 120$) the delivery of the D-pipe $Q'_o = 6.05$; of the d-pipe (with $c = 120$) is $Q' = 7.60$, and $Q = Q'_o + Q' = 13.65$.

From (6) $k = .314$, and the length in duplicate, $kl = 7540$ ft. (v. 8900 above) an error of $15\frac{1}{2}$ per cent.

(b) If both pipes be taken "rough" (with $c = 73$), delivery of D-pipe $Q'_o = 3.64$; of d-pipe $Q' = 4.67$ and $Q = Q'_o + Q' = 8.31$.

From (6) $k = .903$, and the length in duplicate, $kl = 21600$ ft. (v. 8900 above), an error of $143$ per cent.
Experimental Investigation of losses in a Turbine Pump.

Figures.

Motor Losses Windage, Iron etc.

\[ C_i = \text{Armature Current} \]
\[ V = \text{Voltage across brushes.} \]
\[ R_a = \text{Armature Resistance. } 0.317 \text{ Ohm.} \]

Revs. (Tachometer)

Fig. 1.
Fig. 2
Tachometer Calibration
Fig. 3. Calibration Curves.

Readings in feet of mercury.
Figs 4, 5, and 8 with text of paper

Full size Figs 6 and 7 under separate cover.
Fig 11c
Revs 1079

Guide Passages Loss
Connecting Passages Loss

H

Impellers Loss

Impellers Loss

Impeller No. 3

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 Q. cu.d/st/sec.
Rises. Impeller inlets & Guide passages outlets.

Table IV.

Fig 12a
Revs. 695.
Fig 126
Revs. 885

Rises: Impeller Inlets to Guide Passages Outlets

No 2 Impeller
No 3 Impeller
No 1 Impeller

Falls: last guide passages outlets to discharge pipe.
Dotted curve; Fall = 1.40 Q°²

Falls: guide passages outlets to inlet of next impeller
Dotted curve; Fall = 1.91 Q°²

Q cu.ft/sec

Rises Inlet outlets to Guide Passages outlets
Fig 12c
Revs. 1079.

Rises. Impeller Inlets to Guide passages outlets
$\frac{p_2 - p_1}{\omega}$ Table IV

Falls; last guide passages Outlets to discharge pipe
Dotted curve; Fall = 1.40 $Q^2$

Falls; Guide passages outlets to inlet of next impeller
Dotted curve; Fall = 1.91 $Q^2$
Fig 13A
Revs 695
Fig. 13c
Revs. 1079.

Rises in Impellers $\beta_2 - \beta$ Table I.

Rises, Impeller Inlets to Casing (at side of Impellers) $\beta_2 - \beta$ Table I.

No. 3 Impeller
No. 2 Impeller
No. 1 Impeller
Fig 15

885 Revs.
Fig. 16.
Values of $\beta^\prime$ (with leakage allowance)

Values of $\theta^\prime$ (with leakage allowance)

Fig. 17
With leakage allowance

Fig. 18
Revs. 885

For 
\[ z = \text{loss in guide passages} \]
\[ y = \text{loss in impeller} \]
\[ z + y = \text{sum of losses in impeller and guide passages} \]
\[ z = \text{loss in guide passages} \]
\[ x_1 = \text{conversion efficiency of guide passages} \]
\[ x_2 = \text{conversion efficiency of impeller} \]

Q. cu ft/sec

-0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-7.5
-8
-8.5
-9
-9.5
-10
-10.5
-11
-11.5
-12
-12.5
-13
-13.5
-14
-14.5
-15
-15.5
-16

0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
11
12
13
14
15
16

-0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5.5
-6
-6.5
-7
-7.5
-8
-8.5
-9
-9.5
-10
-10.5
-11
-11.5
-12
-12.5
-13
-13.5
-14
-14.5
-15
-15.5
-16

---0%
Impeller losses

With no leakage allowance

Fig. 19a

For \( L_2 = \text{loss in impeller (no leakage)} \)

Revs 1079
Revs 695
Revs 885

With leakage allowance

Fig. 19

For \( L_2 = \text{loss in impeller (with leakage)} \)

Revs 1079
Revs 695
Revs 885

Q. cu.ft/sec
Impeller losses, Experimental and Equation Values.

(with leakage allowance)

Revs. 1079

Revs. 885

Revs. 695

Q cu.ft/sec.
Impeller losses.

\( z_1 = \text{loss in impeller} \)

\( z \), as given by dotted curves of Fig 20 superimposed.

Fig 20

\( Q \) cub. ft/sec.

\( z_2 = \text{loss in impeller} \)
Guide Passages losses. (with leakage allowance)

Free curve through experimental values, thus
Values from equation (15) & (16), thus

\[ y = 4.34 Q^2 \]
Hydraulic Efficiency of Impeller and Guide Passages (with leakage allowance)

Conversion Efficiency of Guide Passages (with leakage allowance)

$Q = \text{conversion efficiency \ of \ guide \ passages}$

Fig 23.
Guide Passages losses
(No leakage allowance)

Fig 24
Hydraulic Efficiency of Impellers and Guide Passages (no leakage allowance)

Fig 25
Guide Passages losses

"Conversion" Efficiencies

A: Full radial area and leakage.
A', with $\beta=47.5^\circ$ do. do.

B: Full radial area and no leakage.
B', with $\beta=47.5^\circ$ do. do.

885 revs.

Fig. 28.
Fig 29

\[ Q_c = \text{Critical Discharge} \]

**Fig 29a.**
Proportion of Radial Outlet area utilised = \( \frac{3}{5} \) to 18

Fig. 29 B.
Cross-Connected Pressure Mains

Figures.

W. Blackadder, D.Sc., 1928.
In full lines \[ k = \frac{1 - \left( \frac{q}{q_0} \right)^2}{r - \left( \frac{q}{q_0} \right)^2} \]

In dotted lines \[ k = \frac{1 - \left( \frac{q}{q_0} \right)^{1.75}}{r - \left( \frac{q}{q_0} \right)^{0.75}} \]
Experimental Investigation of Losses in a Turbine pump.

Figures.