Chapter 1

INTRODUCTION

1.0 INTRODUCTION

Generally, in a masonry building there are two types of panels that resist the lateral load due to wind or explosion. They can be classified as a panel with or without pre-compression. The panels with precompression are those which are subjected to both lateral and in-plane compressive loading. Such panels can be identified as infilled panels in a framed structure and walls two or more storeys below roof level in a loadbearing brickwork structure. The method of design for this type of panel has already been established and the ultimate strength can be predicted by idealising them as a three-pinned arch at the time of failure. The second type of panel classified as cladding panels carry very little or no precompressive in-plane loads. They can be found on the top of load-bearing multi-storey buildings, in framed structures or low rise buildings. The main function of the masonry cladding panel is to resist lateral loading and to keep the building weather proof. The panels have to be sufficiently strong and stiff enough to transfer the lateral forces through floor slabs to shear walls or to the main frame in a framed building. Thus, they have to be designed to resist the bending moment developed due to wind pressure or explosion. It poses no problem when the brickwork cladding panels are supported on top and bottom or on the two vertical edges. The failure load is thus purely dependent on the moment of resistance of the interface bond strength between the brick and mortar or the bond strength of perpendicular joints and the tensile brick strength. The problem becomes more complicated when brickwork cladding panels are supported on three or four sides, and are therefore subject to bi-axial bending. The complexity
increases as masonry exhibits both strength and stiffness orthotropies. The most common analytical methods used are the conventional finite element method based on elastic plate bending theory or the yield line method based on plastic theory.

Elastic plate bending theory may be the most appropriate analytical method to predict the cracking of brickwork cladding panel since brickwork behaves as an elastic brittle material. However, this method does not explain the post-cracking behaviour nor the considerable reserve of strength after initial cracking of the wall panels. In most cases, this method seems to underestimate the failure load.

The British Code of Practice gives the coefficients for the design of such panels. The code does not mention the analytical basis of these coefficients but they are similar to the yield line coefficients. The main reason for using the yield line theory could be the resemblance of the crack patterns in brickwork panels at failure. The pattern seems similar to the yield line in laterally loaded concrete slabs. The assumptions made in yield line theory cannot be satisfied by the brittle nature of brickwork or masonry. Brickwork cannot resist moment after cracking. In addition, ignoring the stiffness orthotropy violates the equilibrium condition of panels subjected to lateral loading. Therefore, the application of yield line theory to masonry panels subjected to lateral loading is dubious.

As explained earlier, neither elastic nor plastic methods are capable of predicting the strength of brickwork cladding panels subjected to lateral loading. One of the main reasons is that the material behaviour and failure criterion in bi-axial bending have not been defined for brickwork. Therefore, an extensive experimental research programme was carried out to study the pre- and post-cracking behaviour of brickwork and to establish the failure criterion in bi-axial bending. The bi-axial
failure criterion was incorporated into a conventional plate bending finite element program to predict the failure pressure of the brickwork cladding panel subjected to lateral loading. To verify this numerical model, some experimental investigations of panels subjected to lateral pressure were also carried out.

The outline of the thesis is briefly described here. A literature review is presented in Chapter 2 to summarise those important findings related only to brickwork panels subjected to lateral loading. The experimental work is presented in Chapters 3, 4 and 5. Chapter 3 presents the tests on small wallettes to define the strength and the modulus of elasticity in two orthogonal directions which are required for the theoretical analysis. A novel cross-beam test is described in Chapter 4. The aim of this test was to study the distribution of load before and after cracking, and also to establish the failure criterion for brickwork in bi-axial bending. A finite element program including the failure criterion was developed. Two parametric studies are also included. The first investigates the effect of modulus of elasticity on cracking and failure loads by the finite element method using the failure criterion. The second investigates the effect of changing hypothetically the flexural strength both in the weaker and the stronger directions on the failure load obtained by the yield line method. Some tests on walls with different aspect ratios under idealised boundary conditions are described in Chapter 5. Some parametric studies and a comparison of the test results of other researchers with the proposed theory are also presented in Chapter 5.

Tables, drawings and photographs are placed at the appropriate location for easier reference to the reader.
Chapter 2

LITERATURE REVIEW

2.1 GENERAL

Brickwork or stonework has been used in construction for a long time. Many important ancient historical buildings, especially during the Roman's time\(^{(1,2)}\) can be found which have been built in masonry. Up until the 19th century, the 'rule of thumb' was the only design method used and the structural stability was achieved by using massive thickness of walls. Such construction concepts continued for centuries. The problem of thin panel design did not arise at all. Only during the late 20th century, did the rapid progress in construction research and the advent of new construction technology lead to the use of steel and concrete framed structures. This has dominated the whole construction industry. Brickwork was mainly used as cladding panels. The change of construction method from traditional load bearing to non-load bearing wall poses a very important question. How strong is non-load bearing brickwork panel in lateral loading? These types of wall can be found in low-rise buildings or in the upper floors of multi-storey buildings. Usually wind pressure is the critical lateral loading on these walls. The investigations have mainly been carried out to understand the behaviour and strength of such panels subjected to lateral pressure.
2.2 SURVEY OF LITERATURE

It is well documented that brickwork is very weak in tension. A huge amount of work (3 to 16) has been done to identify the factors which affect the flexural strength of brickwork, such as:

- suction rate of brick (moisture content of brick before laying).
- surface roughness of brick.
- water retention in the mortar.
- thickness of the bed-joint.
- workmanship.
- curing condition.

These investigations also suggest that the brickwork exhibits both stiffness and strength orthotropies. However, many researchers ignored the stiffness orthotropy which is an important factor in the load distribution under lateral loading. The factors affecting the flexural strength mentioned earlier have been investigated and well documented. Therefore in this chapter only those works related to the brickwork cladding panels subjected to lateral loading will be presented in a chronological order.

The first British tests on lateral loaded brickwork panels with four sides simply supported with no in-plane restraint were reported by Davey and Thomas (17) in 1950. The failure cracks patterns were similar to the yield lines. They also observed that the ultimate failure load increased if the wall was built within the steel frame. However, the problems of rotational restraint at the supports and the
dead weight were ignored in the analysis. The results might have led to the
formulation of the working stress for the code of practice CP111\(^{(18)}\).

In 1971, a paper summarising the results of work done at the British Ceramic
Research Association\(^{(19)}\) was presented at the 4th Symposium on Load Bearing
Brickwork. More than 100 walls were tested under lateral loading but the study
mainly dealt with the presence of pre-compression. The analysis of those walls
with precompression has been established and their strength can easily be assessed
by using a three-pinned arch theory\(^{(20)}\).

In 1972, Baker\(^{(21)}\) used one-third scale model brickwork to carry out some wall
panels subjected to lateral loading. He found that the elastic methods\(^{(22)}\) predict
the cracking load reasonably well for some panels. On the other hand, yield line
analysis\(^{(23)}\) always over-estimated the failure loads. He then used the strip
method\(^{(24)}\) and concluded that this theory corresponded best with his experimental
results. The strip method is an empirical approach. It is based on the strength of
independent strips either spanning vertically or horizontally between the panel
supports. No interaction is considered between the strips. Also such a method in
most cases does not satisfy the conditions of deflection compatibility.

A paper was published by Hendry\(^{(25)}\) in 1973. He stressed that yield line analysis
has no rational justification for its use on brittle materials like brickwork. He
pointed out that the main problem in using the elastic theory was that the stiffness
of the wall is not linear and does not remain constant after cracking. He also
mentioned that the failure criteria of brickwork in bi-axial bending is still
unknown. He suggested the use of moment coefficients based on a horizontal strip
to predict the failure load of his test panels.
Hellers and Sahlin(26) (1972) and Lindsay(27) (1973) discussed the methods of analysis of brickwork panels subjected to lateral loading. The former concluded that elastic theory predicted the cracking load and that ultimate load could be calculated by yield line theory(23). Lindsay recommended the use of elastic analysis for the design of brickwork panels using permissible stresses which were twice as high as those given in the Australian code(28). Again, they did not justify why the yield line analysis should be applied to brickwork panels and provided no rational justification for using the flexural strengths twice as high as those given in the Code(28).

At the Fifth Australian Ceramic Conference, Base and Baker(29) presented a paper in which they concluded that the empirical strip theory gave good agreement with the experimental results of panels supported on four sides, but underestimated the strength of panels supported on three sides.

At the Third International Brick-Masonry Conference in Essen in 1973, two papers were presented by West, Hodgkinson and Webb(30), and Haseltine and Hodgkinson(31). One of the papers described the lateral load test results for panels having different degrees of edge fixity. The walls were built with different types of brick and mortar strength, and some walls with openings were also included. They concluded that the type and amount of edge restraint are the important factors in determining the failure load. In the second paper, they examined two possible design methods. They found that yield line theory might be satisfactory for design. However, they also understood that the yield line method had no rational justification when applied to a brittle material like brickwork. Elastic theory would be easier for the designer to use, provided some appropriate safety factors were available.
A paper was presented by Cajdert and Losberg (32) on lateral loaded panels. They pointed out that the elastic and yield line methods agreed quite well with the experimental cracking and ultimate failure loads respectively, provided the mean values of the flexural strength in the horizontal, diagonal and vertical directions were used.

In 1974, Baker (33,34) presented two more papers on lateral load tests on walls supported on four sides. His first paper confirmed his earlier findings of model tests in which the cracking load was predicted by elastic method and the ultimate load by strip theory. In his second paper, he used computer simulation to demonstrate that the flexural strength of brickwork is solely dependent on the number of joints in the span, and the shape of the applied bending moment diagram. He also used this method to compare his previous work (35) and good agreement was found between the theory and experimental results.

A paper was presented by Hodgkinson, West and Haseltine (36) at the Fourth International Brick Masonry Conference in 1976. The paper summarised the results of walls test built within rigid supports. They found that the cracking load was about twice the strength of a simply supported wall and that the ultimate strength was even higher. This was due to the arching effect, as the walls were built between rigid supports. No theoretical approach was given in the paper.

In the same conference, Hendry and Kheir (37) presented a paper on lateral loading tests on one-sixth scale model brickwalls with various aspect ratios and support conditions. Elastic, strip method and yield line theory were used to analyse the experimental failure pressures and it was found that the yield line method gave the
best prediction. No rational justification for using such a method was discussed in the paper.

A paper was presented by Baker\(^{(38,39,40)}\) at the Load-Bearing Brickwork Conference in London (1977). He presented a series of overall subjective views based on his visits to the various research centres. He pointed out that the wall test results were greatly affected by the method of loading, self weight, arching, restraint at the supports, translational yielding at supports and methods of measuring flexural strength from small specimens.

The tests\(^{(41 \text{ to } 44)}\) carried out on laterally loaded panels at the British Ceramic Research Association were presented at the London conference in 1977, which formed the basis of the amendments to the British masonry code\(^{(45)}\) for the design of such panels. The test results were compared with the elastic and yield line theories. As a result, a table of moment coefficients based on the yield line theory for the design of unreinforced masonry walls was produced. The method was also recommended for irregularly shaped walls and for walls with openings. Again they did not explain how the assumptions made in yield line theory could be applied to a brittle material like brickwork.

A fracture line analysis was proposed by Sinha\(^{(46)}\) in 1978. This analysis was actually based on conventional yield line analysis. The only difference was that this method considered the load distributed according to the flexural stiffness in two orthogonal directions. Therefore, the ultimate moment of resistance in horizontal bending was reduced by dividing by the ratio of \(E_x/E_y\), i.e. modulus of elasticity in the horizontal direction to the modulus of elasticity in the vertical direction. Good agreement was found between the theory and experimental results.
At the Fifth International Brick Masonry Conference in 1979, a paper was presented by Baker(47). He suggested an elliptical failure criterion for brickwork panels subjected to bi-axial bending. He further suggested that with the presence of vertical compressive force both the bending strengths parallel and normal to the bed-joint were increased but the shape of the elliptical failure criterion remained the same. However, such elliptical failure criterion was obtained by carrying out test on a single joint. The main problem in using this method is that the orthogonal moment distributions cannot be recorded after cracking because when any one side of the single joint fails, the whole specimen will also fail.

Lawerence(48) presented some results on full-scale lateral load tests. He stressed that because of the importance of the stiffness of the test frame, any deformation of the test frame could alter the load distribution in two orthogonal directions. He also applied strip method and yield line theory to predict the failure pressure. He found that yield line analysis always overestimated the failure load. The strip method generally gave better predictions for four sides supported but it overestimated the failure load for high aspect ratios (length/height). For the three sides supported panel with top edge free the strip theory gave conservative results, as it ignored two way action.

Sinha, Loftus and Temple(49) investigated the behaviour of lateral loaded panels. They also concluded that elastic theory underestimated and yield line overestimated the failure pressure. The fracture line theory agreed very well with the experimental results for panels with two sides simply supported and two sides continuous.
Sinha(50) carried out some experimental work on third scale brickwork triangular and octagonal, and also rectangular panels with openings to confirm his earlier findings(46). The results obtained from the fracture line theory, taking into account the strength and stiffness orthotropies, agreed very well with the experimental results. He concluded that this method could be used with some confidence for the design of laterally loaded brickwork panels.

In 1980, Cajdert(51) from Sweden carried out some wall tests. It was generally found that the elastic theory was conservative in predicting cracking loads and the yield line theory was always conservative in predicting ultimate loads. He attributed this to secondary effects, such as arching, support restraint, self weight and crack patterns deviating from theoretical paths. It seems he is the only one who found the yield line analysis conservative!

Seward(52) in 1982 used the elastic principal moment and a method tracing the point of failure which was very similar to Baker’s(53). The only difference was that Seward did not consider the random variation in the flexural tensile strength of the brickwork. He claimed that his method had more rationale than the yield line theory. He also pointed out the moment coefficients given in British Code BS5628(45) might have overestimated the failure load in some cases.

Lawrence(54) submitted his Ph.D. thesis in 1983. He carried out 32 full-scale lateral loaded wall tests. His results were compared with the elastic plate theory, yield line theory, strip method and Monte Carlo simulation. He found that the elastic approach was good for predicting the cracking load. Failure pressure was always overestimated by the yield line analysis, underestimated by Monte Carlo simulation and reasonable by the strip method. He reckoned that there is no
theoretical basis for the strip method. However, he suggested that the strip method can be used as a design tool for unreinforced brickwork panels with some confidence.

In 1984, Brincker(55) tried to relate the behaviour of horizontal and oblique yield lines on small specimens of lateral loaded panels. He used eccentric compression tests on small piers to investigate the strengths of a horizontal and an oblique yield line. He proposed that the stress-strain relationship could be divided into three phases: a linear elastic phase, a plastic phase with strain-hardening, and finally a fracture phase. However, this was just a compression test which did not truly reflect the actual behaviour of a flexural test. His reasons for supporting the use of yield line based on his findings as a design method for laterally loaded brickwork panels are very doubtful.

In 1985, Lovegrove(56) investigated the effect of length, height and thickness of single leaf masonry walls on the ultimate lateral load. He found that the total load carried at failure was proportional to some power (approximately 0.3 to 0.4) of the aspect ratio (height/length). It also appeared that there was a 'thickness effect' in masonry which caused the wall-strengths to be approximately proportional to thickness to the power 1.4 rather than to the power 2. Hence, he concluded that if the design method was based on the normal way of obtaining the section modulus the failure load could be overestimated.

A non-linear macroscopic finite element model for a block was developed by Essawy, Drysdale and Mirza(57) in 1985. This program included both transverse shear effects and non linearity due to cracking. A multi-layered model was used in this program. This model can handle general in-plane and out-of-plane loading
conditions simultaneously. The main feature of this program was its ability to trace the progress of cracking. The stiffness of each cracked element was modified only in that particular cracked layer. So far this program was only been applied to hollow block walls. Further comparisons with test data on brickwork panels are needed in order to prove its usefulness.

A non-linear finite element program for the design of reinforced and unreinforced brickwork was developed by May and Tellett(58) in 1986. In this program, unreinforced brickwork has been modelled as an isotropic material in the elastic range. Poisson's ratio was taken as 0.2, but assumed to be zero after cracking. They claimed that a good agreement was obtained between the theoretical and experimental work. However, only one type of boundary condition (two sides simply supported with bottom fixed and free on at top edge) of unreinforced brickwork panel was analysed. Different types of boundary conditions should be analysed in order to verify the program. It is also not justifiable to assume brickwork is an isotropic material as it is a composite material (i.e. consist of bricks and mortar).

In 1986, Sinha and Mallick(59) carried out tests on laterally loaded brickwork panels with fixed supports. A uniformly distributed load was simulated through 16 points loads. They found that the ultimate failure load was about 2.5 times the cracking load. They suggested that the cracking load should be treated as the failure load for safety reasons. However, for limit state design, there are some safety factors already included in the material properties and also in the design load. Hence, using the cracking load as failure load imposed a very high factor of safety which leads to uneconomic design.
In 1986, Ma and May(60) presented a failure criterion for brick masonry biaxial stress which covered the conditions of tension-tension, tension-compression and compression-compression. They commented that Baker's tests(47) only cover the orientation of 0° & 90° to the bed joint. They attempted to extend Baker's test results to establish the complete bi-axial stress failure criterion by assuming a linear relationship between the change of stresses and the change of bed joint orientation. Later on, they applied the failure criterion(62) to a non-linear finite element program to analyse some laterally loaded brickwork panels tested at the British Ceramic Research Association(44). Generally good agreement was found between the analytical and the experimental works. However, their results were not convincing since they only considered one type of boundary condition which was three sides supported and top side free. In order to verify their results, the work should be extended to different types of aspect ratio and boundary conditions.

In 1987, Essawy and Drysdale(63) used the finite element model as a basis to evaluate the accuracy and weaknesses predicted by various methods, such as the yield line, British Code coefficients and strip method. They concluded that all these methods were applicable only to some cases but not for the whole range of aspect ratios and boundary conditions. All these methods required justification.

In 1988, several papers were presented at the 8th International Brick/Block Masonry Conference. Gairns and Scrivener(64) carried out lateral load tests on five hollow blocks and two clay brick panels. These panels were analysed by elastic plate theory, yield line theory and strip methods. They found that these two materials behave differently in panel flexural behaviour. For solid brick, the ultimate failure load was under-estimated by the strip method but over-estimated by
the yield line analysis. In the case of hollow blocks, all the above analyses underestimated the failure load.

Lawrence, and Cao(65) presented a paper in the 8th International Brick/Block Masonry Conference, at Dublin, in 1988. They analysed the cracking pressure of 32 full scale tests. Monte Carlo simulation based on isotropic elastic plate analysis gave good agreement for the initial cracking. Random variation in flexural tensile strength was considered.

In 1991, Goldong(66) presented an integrated and practical approach to the design of laterally loaded masonry panels. Unreinforced, reinforced or prestressed panels were all covered together with different boundary conditions. He suggested that yield line analysis was a unified solution. He also stressed that yield line design could be carried out solely in term of the primary variables of lateral pressure, moment, and panel dimensions. His suggestion is very doubtful as brickwork is a brittle material which cannot behave as a rigid-plastic material on which the yield line method is based.

An investigation of laterally loaded masonry panels using non-linear finite element analysis was developed by Chong, May, Southcombe, and Ma(67) in 1991. The failure criterion was incorporated into the program as proposed by Ma and May(60). Good agreement was found between the analytical and experimental results. However, only panels supported on three sides with the top side free were considered. They should extend their investigations to other boundary conditions. They also commented that the yield line method tends to overestimate the failure strength especially for high aspect ratios(H/L).
In 1993, Duarte\(^{(68)}\) presented his PhD thesis. He carried out tests on some specially made cross-beams which were proposed by Sinha, to establish the bi-axial cracking criterion for brickwork panels. The test arrangement allowed the moments in two orthogonal directions (i.e. parallel and normal to the bed-joint) to be measured. The results show that there were some load shedding from the weaker (normal to bed-joint) to the stronger direction (parallel to bed-joint) after cracking. He also pointed out that due to moment interaction the flexural tensile strength perpendicular to the bed joint (i.e. y-direction) can be enhanced beyond its ultimate value obtained from the uni-axial beam test (also called wallets test). However, only three different aspect ratios were tested which is insufficient data to establish the failure criterion. Extensive theoretical work on the yield line solution for panels with different boundary conditions containing window openings was also presented. He attempted to analyse such wall panels with openings using the yield line method. However, he did not explain the basic assumptions of how the yield line theory can apply to a brittle material like brickwork.

In 1994, a paper on numerical simulation of cracking and collapse of masonry panels subject to lateral loading was presented by Pande, Middleton, Lee and Kralj\(^{(69)}\). They used the homogenisation technique (also called equivalent elastic) based on elasto-plastic theory to establish the equivalent orthotropic uncracked and cracked material properties. Detail of this technique was published elsewhere\(^{(70)}\). A three-dimensional finite element model based on this technique had been developed. Good agreement was obtained between the theoretical model and experimental results. However, only one test wall result was compared. This homogenisation technique was discussed in the Liang, J. X\(^{(71)}\) PhD thesis. He explained the disadvantages of using this method. Firstly, the head joint was replaced by continuous brick which may lead to a wrong prediction of failure load.
The influence of the head joint had been reported by Sinha(46) previously. He tested wallettes with and without completely filled head joints. He concluded that there was a 44% drop of strength if the head joint was not considered. Secondly, the complexity in formulation and numerical implementation poses a limitation in the use of the model in practice.

2.3 SCOPE OF THE PRESENT RESEARCH

After critically examining the work done by previous researchers and the available design methods adopted, it seems that there is no definitive mathematical solution available for the analysis of brickwork panels subjected to lateral loading at this moment. Both the strip and yield line methods lack rational justification when applied to a brittle material like brickwork. The only reasonable approach is elastic plate theory which seems incapable of explaining the fundamental behaviour of brickwork panels subjected to lateral loading. Hence there is a great need for additional research in order to understand the fundamental behaviour of panels subjected to lateral loading. However, a rational approach can only be developed provided the failure criterion and the material behaviour of masonry in bi-axial bending are fully known. There is a compelling need to develop the biaxial bending (i.e tension-tension) material failure model. The present work attempts to fill this gap by the following:

1. A novel cross-beam test was developed and used to establish the failure criterion of brickwork panels subjected to bi-axial bending.
2. A number of half-scale brickwalls with different aspect ratios and boundary conditions subject to lateral loading will be tested. All the walls will be tested under ideal boundary conditions to develop and check with the numerical model.

3. A finite element program using the failure criterion will be developed. The programme will use the stiffness orthotropies and analyse the behaviour in pre- and post-cracking of the walls.

In addition to all of this, some small scale wallets tests will be carried out to determine the material properties of brickwork such as flexural strength parallel and normal to the bed-joint, Poisson's ratios and moduli of elasticity in both orthogonal directions.
Chapter 3

MATERIAL PROPERTIES

3.1 INTRODUCTION

Brickwork is a composite construction of bricks and mortar. The individual and the combined properties greatly influence the load carrying capacity of the brickwork. The flexural strength of the brickwork under lateral loading is dependent on the bond strength between the interface of brick and mortar. However, there are many factors affecting the flexural bond strength (3 to 16). In order to keep the variable factors affecting the flexural strength as minimal as possible, only one type of mortar and brick were used in this project; also all specimens were built by the same bricklayer, so that the same level of workmanship could be maintained throughout the whole project.

This chapter presents the results of tests to determine the basic material properties of bricks, mortar and brickwork. The main properties for the brickwork were the moduli of elasticity and flexural bond strengths in both vertical (normal to bed-joint) and horizontal directions (parallel to bed-joint). Such properties will be used for theoretical analysis at a later stage.

3.2 PROPERTIES OF BRICKS AND MORTAR

3.2.1 Properties of brick

Half-scale bricks were used in the construction of all test specimens. These bricks were tested according to BS3921 (45) to obtain the dimensions, tensile and
compressive strength, initial rate of suction and the water absorption. The average results of each test are given in Tables 3.1 and 3.2.

**Table 3.1 Co-ordinating Size Of The Bricks**

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>Height (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>53</td>
<td>36</td>
</tr>
</tbody>
</table>

Research work done at the Structural Clay Products Research Foundation (SCPRF)(72) suggested that the compressive strength of bricks does not directly influence the flexural strength of brickwork. This test is not relevant for this research, but as most national codes give the compressive strength as a measure of the quality of the brick, the test was performed. On the other hand, the tensile strength of bricks is more important than the compressive strength especially when the panel is subjected to lateral loading, hence the test was done; the results are given in Table 3.2.

The present code(45) only gives the relationship between the characteristic flexural strength and the water absorption of bricks. The water absorption of a brick is the amount of water, as a proportion of the dry weight of the brick, absorbed under prescribed conditions. The most reliable method of testing is to leave the brick in boiling water for 5 hours. The relationship between the water absorption and flexural strength had been investigated by some researchers(43,53). Some of their results are reproduced in Figures 3.1a and 3.1b. It can be seen that the experimental points for both cases (i.e normal and parallel to bed joint) are very scattered. It seems to the author that there is poor correlation between the flexural strength and water absorption. However, this has been incorporated in the British Code BS5628(45).
Figure 3.1a Flexural strength/water absorption (normal to bed-joint), 1:4:3

Figure 3.1b Flexural strength/water absorption (parallel to bed-joint), 1:4:3
Earlier the American tests\(^{(72)}\) claimed that the flexural strength of brickwork is more dependent on the initial rate of suction rather than water absorption, which seems contrary to the conclusion drawn by the British Researchers. The initial rate of suction of a brick is the average rate, measured over 1 min, starting from the first contact, at which the bed face of the brick absorbs water at ambient temperature. For completeness, both tests were carried out and the detail of their results is given in Table 3.2.

**Table 3.2 The Average Properties Of Bricks**

<table>
<thead>
<tr>
<th></th>
<th>Tensile strength (N/mm(^2))</th>
<th>Compressive strength (N/mm(^2))</th>
<th>Initial rate of suction (gms/mm(^2)/min)</th>
<th>Water absorption (% by weight at 5hr. boiling)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>4.46</td>
<td>36.0</td>
<td>1.91</td>
<td>14.74</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.46</td>
<td>3.9</td>
<td>0.33</td>
<td>0.54</td>
</tr>
</tbody>
</table>

### 3.2.2 Properties of mortar

The mortar used in all test specimens was 1:3 (rapid-hardening cement:sand) by volume. The water:cement (w/c) ratio was adjusted by the bricklayer in order to achieve a workable mix. All the specimens were built by the same bricklayer throughout the whole experimental programme.

Three 100mm cubes of mortar were prepared for each wall test. The average compressive strengths was 18.5 N/mm\(^2\) and the results for each wall are given in Table 3.3.
<table>
<thead>
<tr>
<th>Wall no.</th>
<th>Mortar cube no.</th>
<th>Compressive strength N/mm²</th>
<th>Average result N/mm²</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>16.5</td>
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Table 3.3 Average Compressive Strength of Mortar Cubes
3.3 DETERMINATION OF FLEXURAL STRENGTH AND MODULUS OF ELASTICITY

3.3.1 General

To predict the strength of a laterally loaded panel, it is essential to have an adequate knowledge of the strength and stiffness properties of brickwork in bending. Since masonry is a composite material, it exhibits both stiffness and strength orthotropies in the vertical (y) and the horizontal (x) directions. Therefore, the major axes of interest are those parallel to (i.e. x-direction) and perpendicular to (i.e. y-direction) the bed joint. The ratio of $E_x/E_y$ is required to enable the distribution of load to be determined. However, it does not require the absolute individual value of the modulus of elasticity. The reason will be discussed in Section 5.3.4, in Chapter 5. The flexural strengths are also required in order to predict the failure load. Both the strength and stiffness properties can be determined by performing flexural tests on the wallettes.

There are two methods of obtaining the modulus of elasticity. One is the flexural test by measuring the load-deflection or stress-strain relationship and the second is the compression test. The advantage of the compression test is that the values of Poisson's ratio can be easily obtained. The values of modulus of elasticity obtained by the compression test are very similar to those obtained from the flexural tests, as confirmed by Sinha(46) and Durate(68). Hence, only the flexural tests were carried out in this project.
3.3.2 Modulus of elasticity obtained from the flexural tests

Two types of wallettes, denoted as x-beam and y-beam, were either built separately or extracted from the uncracked brickwork from the tested walls. The dimension of the wallettes in the x-direction were four bricks long in the main direction and four bricks wide, whereas the wallettes in the y-direction were eight or nine brick-courses high and two bricks wide. Figure 3.2 shows the configuration of the wallettes in the x and y-directions.

Figure 3.3 shows how these wallettes were tested as simply supported beams subjected to four line loads. The distance between the two line loads was approximately one-third of the length. A layer of "dental plaster" was used over the steel plate as shown in Figure 3.3, to avoid any irregularities in support height.

![Figure 3.2 Configuration of the wallettes in x and y directions](image)
An 'Instron' testing machine was used to apply constant increments of load to each specimen. Three mechanical dial gauges, with a resolution of 0.002mm, were used to measure the centre deflection and also the deflections close to the two line loading. Dial gauges were fixed to a frame which was resting directly over the supports. This arrangement was to eliminate any support settlement. In addition, six electrical strain gauges were placed within the pure bending zone as shown in Figure 3.3. Half of these were placed on the tension and half were placed on the compression side of each beam. Both types of wallette were tested as horizontally spanning rather than in the vertical attitude recommended by the BS5628(45), thus eliminating the rotational restraint due to dead weight, if any.

The moduli of elasticity in the x and y directions were calculated using either the load-deflection or stress-strain relationships from the expressions shown in equations (3.2) and (3.4).
a) Load-deflection method

The maximum central deflection of a beam under 4 line loading (Figure 3.4) is given by:

$$\delta_{\text{cent}} = \frac{23PL^3}{648EI}$$  \hspace{2cm} \text{(3.1)}

By rearranging the equation 3.1, the value of $E$ can be obtained as

$$E = \frac{23PL^3}{648\delta_{\text{cent}}}$$  \hspace{2cm} \text{(3.2)}
b) Stress-strain method

The maximum bending moment for such load arrangement is given by:

\[ M = \frac{PL}{3} \]

and the stress is,

\[ \sigma = \frac{My}{I} = \frac{PL}{3Z}, \quad \text{where} \quad Z = \frac{I}{y} \]

From Hooke's law theory, \( E = \frac{Stress}{Strain} \)

Hence,

\[ E = \frac{PL}{3Ze} \]

Where
\( \delta_{\text{cent}} \) is the maximum central deflection,
\( \varepsilon \) is the strain,
\( E \) is the modulus of elasticity,
\( \sigma \) is stress in the beam,
\( I \) is the second moment of area,
\( L \) is the total span,
\( M \) is bending moment,
\( P \) is the point load,
\( y \) is the maximum distance from the neutral axis and
\( Z \) is the section modulus.
3.3.3. Measurement of moment curvature

The current British Code BS 5628(45) gives coefficients for the design of panels, which are derived from the yield line theory developed for under-reinforced concrete slabs. A brittle material cannot behave in the rigid-plastic manner on which the yield line theory is based (Figure 3.5). Brickwork being brittle may not be capable of resisting moment after cracking. In order to verify whether brickwork behaves as a rigid-plastic material, the moment curvature relationship was established for both orthogonal directions. Figures 3.6 and 3.7 show how the curvatures were calculated by measuring the strains and deflections from a beam subjected to pure bending.

![Moment-curvature relationship for idealised rigid-plastic material](image)

Figure 3.5 Moment-curvature relationship for idealised rigid-plastic material

a) Curvature obtained from the measurement of strains:

When a beam is subjected to lateral loading as shown in Figure 3.4, the top surface will be under compression and bottom surface of the beam will be under tension. The relationship of the strain in compression and tension is shown in Figure 3.6. The
curvature within the pure bending zone can be obtained by dividing the total strains (i.e compressive strain and tensile strain) by the overall depth of that section. The advantage of using the total strains is that the curvature will not be affected even if the neutral axis is not at the centre of the section.

![Diagram of Compressive and Tensile Strains](image)

Figure 3.6 Showing the curvature

The curvature of the beam is given by:

\[ \varphi = \frac{\varepsilon_t + \varepsilon_c}{d} \]  \hspace{1cm} (3.5)

b) Curvature obtained from the measurement of deflection:

When the beam is under transverse loading as shown in Figure 3.4, the central part of the beam between the two line loads will be subjected to pure bending and will bend into a circular arc as shown in Figure 3.7.
Using the Pythagorean theorem, the radius of the circular arc is given by:

\[ R^2 = (R - \delta)^2 + \left( \frac{L_{\text{ref}}}{2} \right)^2 \] .................................(3.6)

or,

\[ R^2 = R^2 - 2R\delta + \delta^2 + \frac{L_{\text{ref}}^2}{4} \] .................................(3.7)

In equation (3.7) the value of \( \delta^2 \) is too small, hence can be omitted. After rearranging,

\[ \varphi = \frac{1}{R} = \frac{8\delta}{L_{\text{ref}}^2} \] .................................(3.8)

where

\( \varepsilon_t \) and \( \varepsilon_c \) are the tensile and compressive strains respectively,

\( \varphi \) is the curvature,

\( d \) is the thickness of the wallettes,

\( \delta \) is the centre deflection between the two reference points,

\( L_{\text{ref}} \) is the span between the two reference points and

\( R \) is the radius of the circular arc.
3.4 DISCUSSION OF EXPERIMENTAL RESULTS

3.4.1 Modulus of elasticity

The moduli of elasticity in two orthogonal directions could be calculated by using the two methods described in Section 3.3.2. The average results are given in Table 3.4. The difference between these two methods is very small as can be seen in Table 3.4. The difference may be due to the fact that the electrical strain gauges are more sensitive than the dial gauges. Hence, the modulus of elasticity obtained by the electrical strain gauges was used in the analysis.

<table>
<thead>
<tr>
<th>Wallettes no.</th>
<th>Modulus of elasticity obtained from deflection (N/mm²)</th>
<th>Modulus of elasticity obtained from the strain (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In x-direction</td>
<td>In y-direction</td>
</tr>
<tr>
<td>1</td>
<td>16288</td>
<td>9545</td>
</tr>
<tr>
<td>2</td>
<td>15862</td>
<td>9853</td>
</tr>
<tr>
<td>3</td>
<td>14822</td>
<td>9715</td>
</tr>
<tr>
<td>4</td>
<td>13108</td>
<td>11251</td>
</tr>
<tr>
<td>Average</td>
<td>15020</td>
<td>10051</td>
</tr>
</tbody>
</table>

3.4.2 Load-deflection and stress-strain relationships

Typical load-deflection and stress-strain relationships for both types of beam are shown in Figures 3.8 to 3.11. It can be seen that the relationship in both cases in the y-beam is linear up to the failure. Once the ultimate tensile strength is reached, the load drops immediately to zero.
The stress-strain and load-deflection relationships in the x-beam linear up to 60-80% of the failure load and later become non-linear till failure. In this case the failure load also drops to zero when the ultimate tensile strength is reached. At failure, the applied load normally "dropped" very quickly. The drop was so fast that even with a very advanced and fast reading data logger capable of recording 9 readings per second, it was not possible to capture the "dropping" of the applied load (Figures 3.10 and 3.11). This clearly shows that the brickwork is a brittle material.

![Figure 3.8 Typical load-deflection relationship in x-direction](image)

![Figure 3.9 Typical load-deflection relationship in y-direction](image)
Figure 3.10 Typical stress-strain relationship in x-direction

Figure 3.11 Typical stress-strain relationship in y-direction
3.4.3 Moment-curvature

The moment-curvature relationships obtained by both methods, equations (3.5) & (3.8), are similar for both the x and y-beams. The moment curvature relationship in the y-beam is linear up to the failure (Figure 3.12). Once the ultimate moment is reached, the load "drops" immediately to zero. The failure was of a brittle nature and normally cracks developed along the bed-joint as shown in Figure 3.15a.

![Figure 3.12 Typical moment-curvature in y-direction](image)

The moment-curvature relationship in the x-beam is linear up to 60-80% of the failure load and later becomes slightly non-linear till failure. Two types of moment-curvature relationships were obtained in the tests. The moment-curvature relationships in Figures 3.13 and 3.14 correspond to the first and second types of failure as shown in Figure 3.15b & 3.15c. The first type is when the moment immediately "dropped" to zero as soon as the tensile strength reached the ultimate value, which is expected of a brittle material. The second type is when the moment
dropped to about 30% of the ultimate moment just after cracking and the curvature continues to increase which is expected of an elasto-plastic material; in this case some plastic deformation can be seen.

Figure 3.13 Typical moment-curvature in x-direction

Figure 3.14 Typical moment-curvature in x-direction
All these results indicate that the brickwork behaves more as a brittle material and once cracked it cannot support the maximum moment any longer in both the x and y directions. Such behaviour is contradictory to the assumptions made in the classical yield line theory which has been used in the current British Code BS5628(45).

3.4.4 Flexural strength in two orthotropic directions

Having established the elastic constants, the other important properties needed are the flexural strengths in two orthogonal directions (i.e. x and y directions). The beams were tested to failure to obtain the modulus of rupture which is calculated by dividing the failure moment by its section modulus. The modulus of rupture in both directions is required to predict the cracking and failure loads at a later stage.

As explained earlier the wallettes were tested horizontally. Hence, the dead weight of the specimen was taken into account in calculating the flexural tensile strength. The average results of flexural strength in both directions are given in Tables 3.5 to 3.7.

3.4.4.1 Discussion of the experimental flexural strength results

All y-beams failed at the interface of the mortar joint which was either under the line loading or at the centre of the wallettes. It is understood that for structural brickwork the weaker part is the interface of the brick/mortar joint. In the case of the x-beam, two different modes of failure were observed. In the first type, cracks passed through the brick units and head joint. In the second case, failure occurred through the brick units, head and bed joints forming a zigzag pattern. Approximately 70% of the failure belonged to the first type and 30% to the second type. All the failure patterns are shown in Figures 3.15a, b and c.
It had been reported\(^{(73)}\) that the flexural strength of wallettes built either along side of each wall or extracted from the uncracked section of the tested walls are almost the same, hence in this project some wallettes built along side of each wall were tested to confirm the previous finding. The comparative results are given in Table 3.5. It can be seen that there is no appreciable difference between the strength of wallettes built either along the wall or extracted from the undamaged portion of the test walls. The flexural strengths in both the x and y directions were obtained only from the wallettes extracted from the undamaged portion of the tested walls. This did save material and the labour cost. The results of the flexural strengths in both the y and x directions together with the average, standard deviation and coefficient of variation are given in Tables 3.6 and 3.7.
Table 3.5 Comparison Of The Flexural Strength Of The Wallettes Built Along Side Of The Wall and Wallettes Extracted From The Untracked Section Of The Tested Wall

<table>
<thead>
<tr>
<th>Wallettes no.</th>
<th>Wallettes built along side of the wall</th>
<th>Wallettes extracted from the uncracked section of tested wall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_X (N/mm^2)$</td>
<td>$F_Y (N/mm^2)$</td>
</tr>
<tr>
<td>1</td>
<td>2.98</td>
<td>0.79</td>
</tr>
<tr>
<td>2</td>
<td>2.65</td>
<td>0.61</td>
</tr>
<tr>
<td>3</td>
<td>2.72</td>
<td>0.72</td>
</tr>
<tr>
<td>4</td>
<td>3.12</td>
<td>0.85</td>
</tr>
<tr>
<td>5</td>
<td>2.63</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>3.15</td>
<td>0.68</td>
</tr>
<tr>
<td>7</td>
<td>2.77</td>
<td>0.58</td>
</tr>
<tr>
<td>8</td>
<td>2.97</td>
<td>0.63</td>
</tr>
<tr>
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<tr>
<td>10</td>
<td>2.99</td>
<td>0.80</td>
</tr>
<tr>
<td>11</td>
<td>2.67</td>
<td>0.74</td>
</tr>
<tr>
<td>Average</td>
<td>2.91</td>
<td>0.74</td>
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Table 3.6 Flexural Tensile Strength Normal To The Bed-joint From The Wallettes Extracted From The Undamaged Portion Of The Test Walls

<table>
<thead>
<tr>
<th>No.</th>
<th>Normal to bed-joint $F_y$(N/mm²)</th>
<th>No.</th>
<th>Normal to bed-joint $F_y$(N/mm²)</th>
<th>No.</th>
<th>Normal to bed-joint $F_y$(N/mm²)</th>
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<td>71</td>
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<td>81</td>
<td>0.87</td>
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<tr>
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<td>1.05</td>
<td>97</td>
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<td>98</td>
<td>1.27</td>
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<tr>
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<td>0.65</td>
<td>66</td>
<td>0.96</td>
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</tr>
</tbody>
</table>

Average = 0.87 N/mm²  
S.D. = 0.17 N/mm²  
C.V = 0.19 %
Table 3.7 Flexural Tensile Strength Parallel To The Bed-joint From The Wallettes Extracted From The Undamaged Portion Of The Test Walls

<table>
<thead>
<tr>
<th>No.</th>
<th>Parallel to bed-joint $F_A$ (N/mm²)</th>
<th>No.</th>
<th>Parallel to bed-Joint $F_A$ (N/mm²)</th>
<th>No.</th>
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<td>33</td>
<td>3.05</td>
<td>66</td>
<td>3.86</td>
<td></td>
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</table>

Average = 3.22 N/mm²
S. D. = 0.39 N/mm²
C.V = 0.12 %
3.5 CONCLUSIONS

On the basis of these tests it can be concluded that:

1. The brickwork possesses both strength and stiffness orthotropies.

2. The flexural tensile strengths normal and perpendicular to the bed-joint obtained from the wallettes built independently along the test wall or extracted from the undamaged portion of the tested walls are similar.

3. The load-deflection, stress-strain and moment-curvature relationships show the brittle nature of brickwork. Once cracked, the cracked section cannot support the ultimate moment any longer.
Chapter 4

Cross-Beam Test

4.1 INTRODUCTION

In chapter 3, the experimental work carried out mainly to determine the uniaxial flexural strength of brickwork beams or slabs was described. Such tests on statically determinate structures are easy to perform and simple to analyse. In practical terms they are only representative of cladding panels supported either on the top & bottom or on the two vertical edges (i.e uniaxial bending). When the panel is subjected to bi-axial bending the situation is considerably more complex and the tests described in Chapter 3 do not represent the behaviour under bi-axial bending. So far, no "proper" method has been developed to apply both vertical and horizontal moments simultaneously to define the behaviour of brickwork in bi-axial bending. Baker (47) attempted to use a "single joint" (Figure 4.1) to simulate the bi-axial behaviour of a wall panel. On the basis of his test, he proposed an elliptical failure criterion for combined vertical and horizontal moments. This can be represented by the equation:

\[ \left( \frac{F_y}{F_{uy}} \right)^2 + \left( \frac{F_x}{F_{ux}} \right)^2 = 1 \] ..........................(4.1)

where

F_y and F_x are the maximum flexural stresses in the y and x directions.

F_{uy} and F_{ux} are the ultimate flexural strengths in the weaker (y) and stronger (x) directions.
This method ignored the stiffness orthotropy and thus did not improve the understanding of the distribution of loads in two orthogonal directions. The loading arrangement was such that once any one side of the specimen cracked the whole specimen failed immediately.

![Figure 4.1 Single joint specimen](image)

The behaviour and strength of brickwork is complex when subjected to bi-axial bending. The behaviour depends on the physical and mechanical properties of bricks and mortar, as well as the nature of the loading. A brickwork panel simply supported on four or three sides when subjected to lateral loading will bend in two directions, as shown in Figure 4.2. The panel will be subjected to bi-axial bending. The tensile stresses developed due to the bi-axial bending will control the mode of failure of the panel.

![Figure 4.2 Brickwork panel bending in two orthogonal directions](image)
The failure criterion under such a stress condition (tension-tension) is not yet clearly defined. For this reason, no single acceptable rational method is available which can predict the cracking and failure load of a brickwork panel subjected to lateral loading with confidence. Of course, some conventional failure theories such as Tresca, Von Mises and Rankine are available, but they are mainly applicable to ductile or to some extent brittle isotropic material. These theories may not be applicable directly to masonry in bending because brickwork panels exhibit both strength and stiffness orthotropies. A rational approach can only evolve, if the failure criterion in bi-axial bending is known. Any attempt to define the failure criterion must be appropriate to the anisotropic behaviour of the material. In this chapter, a novel test method to establish the failure criterion in bi-axial bending is described. This novel test method and specimen was devised by Sinha\textsuperscript{(68,74)}.

4.2 EXPERIMENTAL INVESTIGATION

4.2.1 Construction of the cross-beam

The bricks used for the cross-beams were those used for the wallette tests described in Chapter 3. All the cross-beams were single leaf construction. The central portion of the cross-beams was built separately from the arms. The central part of the cross (Figure 4.3) which is representative of a panel, was built with half-scale bricks in 1:3 (rapid hardening cement: sand) mortar.

![Half-scale Bricks in 1:3 mortar](image)

Figure 4.3 Centre portion of the cross-beam
The four arms attached to the central portion were comb-like structures built with similar bricks in epoxy/sand mortar. This was done to prevent premature failure within the arm either in bending or shear. In addition, the comb-like structure allowed unhindered crack propagation within the central portion. The four individual arms were glued to the central portion of the brickwork after it had set properly.

Figure 4.4 shows a typical cross ready for gluing the arms. The advantage of using such a construction method was that the individual arms can be re-used again after each test. It also proved quicker and easier for the bricklayer to build only the central part of the cross-beam.
4.2.2 Experimental details

After construction the cross-beams were tested as simply supported beams subjected to a point load as shown in Figure 4.5. The aspect ratios, normal to the bed-joint in the direction of y and parallel to bed-joint in the direction of x, varied from 0.5 to 2.0. Three specimens were tested for each aspect ratio, giving a total of 33 specimens.

![Figure 4.5 Configuration of cross-beam](image)

Figure 4.5 Configuration of cross-beam
The central load was applied through a 40mm diameter steel disc, which was bedded on the centre of the cross with "dental plaster" to ensure an even spread of the load. The load was applied by a hydraulic jack, which allowed the load to be applied in small increments up to cracking and failure of the cross beam. The applied load including reactions was measured by the load-cells. Three types of load cell were used for this investigation. A three-tonne load cell measured the central applied load. Two half-tonne load cells were placed under the supports in the direction receiving smaller reactions (i.e normal to bed-joint). Two one-tonne load cells were positioned under the supports of the cross-beam in the stronger direction. The four load cells measuring the reactions rested on stands; the height of which could be easily altered by screw action. This arrangement ensured the levelling of the specimens. All the load cells were connected to a data-logger. The applied load and the reactions were measured to check any discrepancies between the results. No difference was recorded between the applied load and measured reactions.

The advantage of this arrangement is that not only the cracking load can be pinpointed but also the redistribution of loading between the two directions after one has failed. The orthogonal moments at any stage of loading can be obtained accurately from the measured reactions multiplied by the lever arm. The test arrangement for the cross-beam is shown in Figure 4.6.
Figure 4.6 Test arrangement for the cross-beam
4.3 THEORETICAL ANALYSIS

It has already been established in Chapter 3, that the moment-curvature relationship of brickwork in two orthogonal directions is linear in tension. Only, when the failure was step-wise fashion can some elasto-plastic behaviour be detected. Most of the wallets failed in a brittle manner and the load dropped immediately to zero except in those cases where failure was step-wise. In a step-wise failure, the moment also dropped after cracking to 30% or less compared to the ultimate moment. Hence, this has been ignored in the analysis and the brickwork is assumed to behave in a purely elastic manner up to failure. Two different elastic methods were used to predict the cracking and the failure loads of the cross beams. The methods used are the Rankine maximum stress theory (75) and an in-house finite element plate bending programme (76).

The fracture line, the yield line and the finite element methods together with the failure criterion derived for bi-axial bending were used also to predict the failure loads of the crosses.

4.3.1 Rankine maximum stress theory

The Rankine maximum stress theory was used to obtain the failure load. It was assumed that failure in the central part of the cross will take place if the moment in any one direction reaches its ultimate moment of resistance. Consider the simply supported cross beam subjected to an applied point load P at the centre. Neglecting the effect of Poisson's ratio, the point load is shared by the two orthogonal strips (i.e. x and y directions) as shown in Figure 4.7.
If the effects of torsion and Poisson's ratio are neglected, the point load will be shared as:

\[ P_x + P_y = P \]  \hspace{1cm} (4.2)

where \( P_x \) and \( P_y \) are the loads carried by x and y directions as shown in Figure 4.7

From compatibility of mid span deflection, we can write

\[ \frac{P_x (L_x)^3}{48E_x I_x} = \frac{P_y (L_y)^3}{48E_y I_y} \]  \hspace{1cm} (4.3)

or

\[ P_x = P_y \left( \frac{L_y}{L_x} \right)^3 \left( \frac{E_x}{E_y} \right) \], since \( I_x = I_y \) \hspace{1cm} (4.4)

The moduli of elasticity of the arm and the central portion of the cross is slightly different but they are assumed constant for simplification. If the exact values are
used, the change in result will be of the order of only 2.5%, which is very small and has therefore been neglected.

Substituting, the value of $P_x$ from equation (4.4) into (4.2), one can get the value of $P_y$ in term of applied load $P$. Hence, the applied moments in both directions can be obtained from:

$$M_y = \frac{P_y L_y}{4} = \frac{P L_y}{4} \left( \frac{1}{1 + \left( \frac{L_y}{L_x} \right)^3 \frac{E_x}{E_y}} \right)$$

(4.5)

similarly,

$$M_x = \frac{P_x L_x}{4} = \frac{P L_x}{4} \left( \frac{1}{1 + \left( \frac{L_x}{L_y} \right)^3 \frac{E_y}{E_x}} \right)$$

(4.6)

According to Rankine's theory, failure will take place in the direction which reaches its ultimate moment of resistance first, i.e

$$M_y \geq M_y^{\text{u}}$$

(4.7)

or

$$M_x \geq M_x^{\text{u}}$$

(4.8)

The theoretical cracking loads $P_y$ and $P_x$ were calculated from equations (4.5) and (4.6) and equated to the ultimate moment given in equations (4.7) and (4.8) respectively. The minimum of the two values define the cracking load. The ultimate moments in two orthotropic directions were obtained from the flexural tests described in chapter three. In many cases during the tests, it was observed that once the $y$-direction cracked, it could not take any more load and hence any subsequent
resistance to the load offered by the cross beam was solely due to its strength in the x-direction. Based on these observations, the theoretical analysis at failure can be simplified by assuming the cross-beam acts as a beam subjected to uni-axial bending in the x-direction. Therefore, the failure of the cross beam can only occur when the moment in this direction reaches its ultimate moment of resistance. The theoretical results are compared with the experimental values for the cross-beam in Tables 4.2 and 4.3.

4.3.2 Finite element method

When a brickwork panel is subjected to lateral loading, the panel behaves almost in a linear elastic manner up to cracking. After cracking, large deformations take place and the panel behaves in a non-linear manner. This can be seen in some typical load-deflection relationships of wall test results in Chapter 5. This non-linearity is not due to material properties, but is due to cracking in weaker direction. The post-cracking behaviour is not considered in the conventional finite element program based on plate bending theory. Conventional finite element programs can only be applied to a brickwork panel up to cracking but not up to the failure load. Hence, an in-house finite element program(76) developed at the University of Edinburgh was used and modified by incorporating a suitable crack modelling technique to predict the pre- and post- cracking load distribution in two orthogonal directions.

Currently, three different crack modelling techniques(77) are used:

- Smeard crack modelling.
- Discrete crack modelling and
- Fracture mechanics modelling.
The model to be selected depends on the purpose of the analysis. If overall structural behaviour is desired, smeared crack modelling is probably the best choice. If detailed local behaviour is of interest, adaptations of the discrete cracking modelling are useful. For a very special problem like stress concentrations at crack tips, crack width, bond and dowel effects in reinforced concrete, fracture mechanics modelling is more accurate and is the most appropriate tool to be used.

In this investigation, the overall behaviour of brickwork panels subjected to lateral loading is being studied, hence smeared crack modelling appeared to be the most suitable technique to be incorporated into the FEM program for predicting the load distribution in two orthogonal directions after cracking.

### 4.3.2.1 Smeared crack modelling

In the smeared crack model, the element of brickwork is represented by an elastic orthotropic material with reduced modulus normal to the cracking plane. The crack appears when the flexural tensile strength normal to the crack is equal to the ultimate flexural tensile strength.

In this approach, the cracked section is assumed to remain as a continuum, that is, the cracks are "smeared out" in a continuous fashion. The main difference between smeared crack and discrete crack modelling is that the smeared crack model represents a crack as a stress discontinuity rather than as a discrete separation within the material. The discrete crack model requires redefinition of the finite element topology each time a crack propagates or new crack forms.
4.3.2.2 Checking the validity of the finite element program

As the in house package written by others was used, it was felt essential to check its validity. The best way was to compare the results obtained by this program with the analytical solutions given by Timoshenko\(^{(22)}\) for plate bending. The solution of a number of elastic isotropic square plates with different boundary conditions were used to check its validity. The results are given in Table 4.1. It can be seen that there is a good agreement between the theoretical results of Timoshenko and the finite element program.

**Table 4.1 Comparison Between The Results Obtained By Finite Element And Timoshenko's Plate Theory (aspect ratio 1:1)**

<table>
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<tr>
<th>Boundary conditions</th>
<th>FEM program (76)</th>
<th>Timoshenko's results (22)</th>
</tr>
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<tr>
<td></td>
<td>Bending moment coefficients</td>
<td>Maximum deflection coefficients</td>
</tr>
<tr>
<td></td>
<td>(\alpha_x) (\alpha_y) (\delta_{\text{def}})</td>
<td>(\alpha_x) (\alpha_y) (\delta_{\text{def}})</td>
</tr>
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<td>Simply Supported on Four Sides</td>
<td><strong>0.0229</strong> *0.0229 0.0511</td>
<td><strong>0.0231</strong> *0.0231 0.0513</td>
</tr>
<tr>
<td>Fixed on Four Sides</td>
<td><strong>0.0246</strong> *0.0334 0.0692</td>
<td><strong>0.0244</strong> *0.0332 0.0697</td>
</tr>
<tr>
<td>Two sides fixed &amp; two sides simply supported</td>
<td><strong>0.053</strong> *0.058 0.00156</td>
<td><strong>0.055</strong> *0.059 0.00156</td>
</tr>
</tbody>
</table>

* at the centre of panel
** at the fixed support
4.3.2.3 Modification of the cracked elements

Having checked the validity of the in-house program, the crosses were analysed using this program. Masonry was considered as a homogeneous orthotropic material. The rigidity matrix was derived by using the properties obtained from the experiments described in Chapter 3.

Before cracking, the rigidity matrix (D) is given as:

\[
D = \frac{t^3}{12} \begin{bmatrix}
E_x & \nu_{xy}E_y & 0 & 0 & 0 \\
(1-\nu_{xy}^2) & (1-\nu_{xy}^2) & 0 & 0 & 0 \\
\nu_{xy}E_y & \frac{E_y}{(1-\nu_{xy}^2)} & 0 & 0 & 0 \\
(1-\nu_{xy}^2) & (1-\nu_{xy}^2) & 0 & 0 & 0 \\
0 & 0 & G_{xy} & 0 & 0 \\
0 & 0 & 0 & G_{xz} & 0 \\
0 & 0 & 0 & 0 & G_{yz}
\end{bmatrix}
\]

When the maximum moment output from the FEM program exceeded the ultimate moment of resistance in the y-direction (i.e. flexural strength normal to bed-joint) of any element, the rigidity of that element (or cracked element) is modified. The rigidity crack matrix (\(D_{cr}\)) is given by:

\[
D_{cr} = \frac{t^3}{12} \begin{bmatrix}
E_x & \nu_{xy}E_y & 0 & 0 & 0 \\
\phi \frac{E_x}{(1-\nu_{xy}^2)} & \phi \frac{E_y}{(1-\nu_{xy}^2)} & 0 & 0 & 0 \\
0 & 0 & \beta G_{xy} & 0 & 0 \\
0 & 0 & 0 & G_{xz} & 0 \\
0 & 0 & 0 & 0 & \beta G_{yz}
\end{bmatrix}
\]
After the rigidity of the cracked elements was modified, the program was run again to check if any other element exceeded the ultimate moment of resistance. During the modification stages, the applied load still remains constant. The procedure is repeated until no further cracked element/elements were found. The applied load is then increased in small increments and the process repeated. The program will only stop when the maximum moment exceeds the ultimate moment of resistance in the x-direction (i.e. parallel to bed-joint).

In equation 4.10, two reduction factors $\phi$ and $\beta$ were used in the rigidity crack matrix ($D_{cr}$) to model the cracked section. It is assumed that a cracked section is not able to carry any moment normal to the crack. Hence, the value of $\alpha$ can be considered as very small ($\phi \approx \text{zero}$). The factor $\alpha$ is not set to zero in order to avoid the possibility of a singular stiffness matrix. In line with the above assumption, the cracked section can be treated as a pinned-support which means it cannot carry any moment but possibly carry some shear force. The experimental results show that some residual load was carried in the weaker direction after cracking which was on average 9% of the total load (Table 4.3). This may be due to the effect of friction between the interface of brick and mortar or some frictional restraint at the support. After cracking, a reduced shear modulus is assumed and the value of $\beta$ is taken\cite{54,77,78,80} between 0 to 1. In this case, the value of $\beta$ is taken as 0.09 (i.e. 9% of the total load) for the analysis.

In any finite element analysis\cite{81}, the theoretical results will be affected by the number of the elements used in the analysis. In this investigation, the centre part of the cross-beam was divided into two different sets of elements (11x11 and 13x13) to check the results. The number of elements used in both cases were so small that the results obtained from these sets of elements differed by less than 1%. Since the
difference in the results was so small, the 11x11 elements were used for the rest of the finite element analysis. The preparation of the elements' mesh (11 x 11) for the finite element analysis is given in Appendix B.

For the applied loading, a constant value of 0.0002 N/mm² was used for each load increment. The load increment has to be kept reasonably small so that any change of stress in individual elements can be captured. The flow chart of the finite element program, the input data and the listing of the modified finite element program are presented in the Appendix C, D, and E respectively.

4.3.3 Fracture line analysis

Fracture line analysis was proposed by Sinha(46) in 1978. This analysis has been used for lateral loaded brickwork panels, hence the method was used for all the crosses.

4.3.3.1 Assumptions

It is assumed that the material behaves as linear elastic. Deformations take place along the fracture lines only, and the individual parts of the cross rotate as rigid bodies. The load distributes according to the stiffness in the respective direction. The fracture lines develop when the flexural strength reaches its relevant strength simultaneously in both directions.

To deal with the stiffness orthotropy, the actual cross has been converted to an affine slab having the same modulus of elasticity in both directions.
4.3.3.2 Conversion to an affine slab

Figure 4.8 shows a cross-beam simply supported on four sides and subjected to a central point load. The load is shared between strips of unit width running in the two orthogonal directions as given in equation 4.11.

![Figure 4.8 Showing a simply supported cross-beam subjected to a central point load](image)

\[ P_x + P_y - P \] \hspace{1cm} (4.11)

From the compatibility of mid span deflection

\[ \frac{P_y L_y^3}{48E_y I_y} = \frac{P_x L_x^3}{48E_x I_x} \] \hspace{1cm} (4.12)

or

\[ P_y - P_x \left( \frac{E_y}{E_x} \right) \left( \frac{L_x}{L_y} \right)^3, \quad \text{since} \ I_x - I_y \] \hspace{1cm} (4.13)

Substitute equation (4.13) into equation (4.11),
Re-arrange equation 4.14,

\[ P_s \left[ 1 - \left( \frac{L_x}{L_y} \right)^3 \frac{E_x}{E_y} \right] = P \] ..........................(4.14)

Any affine isotropic cross must have the same load distribution as before (equations 4.14 & 4.15), hence one of the spans has to be modified. From compatibility, equation 4.15 can be expressed as:

\[
\begin{array}{c}
\text{Orthotropic} \\
P
\end{array}
\begin{array}{c}
\left[ 1 - \left( \frac{L_x}{L_y} \right)^3 \frac{E_x}{E_y} \right]
\end{array}
\begin{array}{c}
\text{isotropic} \\
P
\end{array}
\begin{array}{c}
\left[ 1 - \left( \frac{L_x}{L_m} \right)^3 \frac{E_x}{E_z} \right]
\end{array}
\] ..........................(4.16)

where \( L_m \) is the modified length,

after re-arranging the equation (4.16), one gets

\[ L_m = L_y \sqrt[3]{\frac{E_x}{E_y}} \] ..........................(4.17)

In this case, the span in the y-direction was modified while the span in the x-direction was kept constant. Instead of modifying the span in the y-direction, the same result can be obtained by modification of the span in the x-direction.
4.3.3.3 Derivations of fracture line analysis

The idealised fracture lines for the affine cross with four sides simply supported is given in Figure 4.9. A virtual deflection of unity is assumed at the centre(O) at failure.

Therefore, the external work done = \( \frac{1}{2} P \).

![Figure 4.9 Crack pattern for fracture line analysis of cross-beam](image)

The external work done must be absorbed by the internal work done on the fracture lines.
Internal work done on one of the fracture lines (PO) = \( \frac{1}{2} \left[ m b \theta_x + \mu m a \theta_y \right] \) .... (4.19)

where the lengths "a" and "b" are the projected lengths of the fracture lines onto two orthogonal directions and \( \theta_x \) and \( \theta_y \) are referred to normal rotations of the two orthogonal projected lengths of the fracture lines, which are \( \frac{1}{L_y/2} \) and \( \frac{1}{L_x/2} \) respectively.

Total internal work done on all four fracture lines = \( \frac{1}{2} \left[ \frac{4mb}{L_y} + \frac{4\mu ma}{L_x} \right] \) .... (4.20)

\[ = 4m \frac{1}{2} \left[ \frac{b}{L_y} + \frac{\mu a}{L_x} \right] \] .... (4.21)

Therefore,

External work done must equal internal work done. Hence,

\[ P = 4m \left[ \frac{b}{L_y} + \frac{\mu a}{L_x} \right] \] .... (4.22)

The results obtained from this equation 4.22 are given in Table 4.3. Equation 4.22 becomes the same as the yield line equation, if stiffness orthotropy is neglected, i.e. the deflection compatibility is not met.

4.4 TEST RESULTS OF CROSS-BEAM

4.4.1 Load distribution

Initially, the applied load was shared linearly according to the stiffness in the respective directions. The load distributions for the cross-beams of various aspect ratios are shown in Figures 4.10 to 4.20. Once the strength in any one direction reached its ultimate value, the specimen cracked and the value of the reaction...
dropped and the load was shed to the stronger direction. Three types of failure were detected in the experiments. In the first type of failure, both directions failed simultaneously without cracking (Figure 4.20). The typical diagonal crack pattern for this type of failure can be seen in Figures 4.29 to 4.31. The load distributions for the second type of failure are shown in Figures 4.18 and 4.19. After cracking in the weaker (y) direction, a further slight increase of applied load would result in immediate failure of the crosses, as the shed load was sufficient to cause failure in the stronger (x) direction. Figures 4.10 to 4.17 show the load distribution for the third type of failure. In this type, the cross-beam first cracked in the weaker y-direction (i.e. normal to the bed joint) and the load was shed to the stronger (x-direction) direction immediately after cracking but the shed load is insufficient to cause failure. The applied load can be increased till the ultimate strength in the stronger direction is reached. From the results, it is very clear that the cracked section cannot support any moment due to the brittle nature of the material. Some residual load in the y-direction was measured, but this is due to the dead weight and probably some frictional restraint at the support. The cracking patterns for the second and third type of failures are shown in Figures 4.21 to 4.28. No diagonal cracking was noticed.

Figure 4.10 Distribution of the central applied load and reactions in x and y directions (L_x = 300mm & L_y = 585mm)
Figure 4.11 Distribution of the central applied load and reactions in x and y directions ($L_x = 445\text{ mm}$ and $L_y = 585\text{ mm}$)

Figure 4.12 Distribution of the central applied load and reactions in x and y directions ($L_x = 585\text{ mm}$ and $L_y = 585\text{ mm}$)
Figure 4.13 Distribution of the central applied load and reactions in x and y directions ($L_x=690\text{mm}$ and $L_y=585\text{mm}$)

Figure 4.14 Distribution of the central applied load and reactions in x and y directions ($L_x=860\text{mm}$ and $L_y=585\text{mm}$)
Figure 4.15 Distribution of the central applied load and reactions in x and y directions ($L_x=1140\text{mm}$ and $L_y=585\text{mm}$)

Figure 4.16 Distribution of the central applied load and reactions in x and y directions ($L_x=585\text{mm}$ and $L_y=300\text{mm}$)
Figure 4.17 Distribution of the central applied load and reactions in x and y directions ($L_x = 585\text{mm}$ and $L_y = 445\text{mm}$)

Figure 4.18 Distribution of the central applied load and reactions in x and y directions ($L_x = 585\text{mm}$ and $L_y = 690\text{mm}$)
Figure 4.19 Distribution of the central applied load and reactions in x and y directions ($L_x=585\,\text{mm}$ and $L_y=860\,\text{mm}$)

Figure 4.20 Distribution of the central applied load and reactions in x and y directions ($L_x=585\,\text{mm}$ and $L_y=1140\,\text{mm}$)
Figure 4.21 Typical crack pattern of cross-beam (cb 0.5:1) ($L_x=300\text{mm}$ and $L_y=585\text{mm}$)

Figure 4.22 Typical crack pattern of cross-beam (cb 0.75:1) ($L_x=445\text{mm}$ and $L_y=585\text{mm}$)
Figure 4.23 Typical crack pattern of cross-beam (cb 1:1) \( L_x = 585 \text{mm} \) and \( L_y = 585 \text{mm} \)

Figure 4.24 Typical crack pattern of cross-beam (cb 1.2:1) \( L_x = 690 \text{mm} \) and \( L_y = 585 \text{mm} \)

Figure 4.25 Typical crack pattern of cross-beam (cb 1.5:1) \( L_x = 860 \text{mm} \) and \( L_y = 585 \text{mm} \)
Figure 4.26 Typical crack pattern of cross-beam (cb 2:1) \((L_x=1140\text{mm} \text{ and } L_y=585\text{mm})\)

Figure 4.27 Typical crack pattern of cross-beam (cb 1:0.5) \((L_x=585\text{mm} \text{ and } L_y=300\text{mm})\)

Figure 4.28 Typical crack pattern of cross-beam (cb 1:0.75) \((L_x=585\text{mm} \text{ and } L_y=445\text{mm})\)
Figure 4.29 Typical crack pattern of cross-beam (cb1:1.2) ($L_x=585\text{mm}$ and $L_y=690\text{mm}$)

Figure 4.30 Typical crack pattern of cross-beam (cb1:1.5) ($L_x=585\text{mm}$ and $L_y=860\text{mm}$)

Figure 4.31 Typical crack pattern of cross-beam (cb1:2) ($L_x=585\text{mm}$ and $L_y=1140\text{mm}$)
4.5 DISCUSSION OF THE TEST RESULTS

4.5.1 Comparison of the experimental and theoretical results

Tables 4.2 and 4.3 show the theoretical and experimental results for the crosses. It can be seen that the finite element method underestimates the cracking and failure load from 1 to 26% and 3 to 18% respectively. The Rankine's theory underestimates the cracking load by 8 to 24% and the ultimate load by 3 to 19%. The results obtained by the finite element method including the smeared crack technique provides some improvement over the Rankine theory but not very significantly. Both analyses did not agree very well with the experimental results due to the fact that the bi-axial bending failure criterion was not considered in the analysis. The theoretical results may improve if the failure criterion in bi-axial bending is included in the finite element program. This will be discussed later in Section 4.7.

In many cases the cross-beams failed in the weaker direction first and the load was then carried only by the stronger direction, hence it was felt appropriate to check the results with the wallettes tested under a three point loading system. The results of three point loading wallette tests are given in Appendix A. The average values of the ultimate moments normal and parallel to the bed-joint were 1.4% and 1.8% higher compared to the wallettes tested with four-point loading. The theoretical results (Rankine and finite element methods) based on the flexural strength obtained from the three-point loading system are also given in Tables 4.2 and 4.3. The theoretical failure load is about 1 to 2% higher than the results obtained using the wallettes results from a four-point loading system. This difference is insignificant, hence the ultimate moments obtained from wallettes with four-point loading were used to comply with the British Code (45).
Table 4.2 Comparison of Experimental and Theoretical Cracking Load Of The Cross-Beam

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<tr>
<th>Lx (mm)</th>
<th>Ly (mm)</th>
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<th>Experimental</th>
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<th>Finite element analysis</th>
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^a^ flexural strength obtained from a four-point loading system

^b^ flexural strength obtained from a three-point loading system
Table 4.3 Comparison of Experimental And Theoretical Failure Load Of The Cross-Beam

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<thead>
<tr>
<th>Lx (mm)</th>
<th>Ly (mm)</th>
<th>Applied Load</th>
<th>Experimental results</th>
<th>Rankine's theory</th>
<th>Finite element analysis</th>
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</table>

a flexural strength obtained from a four-point loading system
b flexural strength obtained from a three-point loading system

4.6 THE FAILURE CRITERION IN BI-AXIAL BENDING

The results of the test have been plotted in non-dimensional form as shown in Figure 4.32. The average values of uniaxial strength in two orthogonal directions were taken from the wallettes' tests described in Chapter 3. The best envelope which fits all the points can be represented by:
\[
\left( \frac{M_x}{M_{wx}} \right)^2 + \left( \frac{M_y}{M_{wy}} \right)^2 - 0.75 \times \frac{M_x}{M_{wx}} \times \left( \frac{M_y}{M_{wy}} \right)^2 - 0.25 \times \frac{M_x}{M_{wx}} \times \frac{M_y}{M_{wy}} = 1.0......(4.23)
\]

Cracking will precede the failure, if any point on the failure envelope lies to the left of the intersection point (1,1). The final failure will result due to the load shedding from the weaker to the stronger direction. When the moment reaches the value of the moment of resistance in the stronger direction, failure takes place. Below the intersection point (1,1), failure will happen together or due to failure of the stronger direction, thus the weaker direction may or may not reach its ultimate strength.
To evaluate the results against the Rankine maximum stress theory and Baker's failure criterion, both failure envelopes were also plotted on Figure 4.32. The poor agreement between the experimental results and existing failure theories suggests very strongly that both theories are not applicable to masonry in bi-axial bending. Baker's failure criterion does not predict cracking nor the ultimate failure of the cross-beams.

4.7 COMPARING THE EXPERIMENTAL RESULTS WITH THE FINITE ELEMENT PROGRAM USING THE FAILURE CRITERION

Having established the failure criterion from the test results, the next step was to incorporate this criterion into the finite element program to predict the cracking load and the failure load. Tables 4.4 and 4.5 show the results of the analysis. The average values of the moduli of elasticity and the flexural strength were used for the theoretical prediction. Three typical load-distributions predicted by the finite element analysis and the experimental results for cross-beams are shown in Figures 4.33 to 4.35. It can be seen that the results obtained by considering the proposed failure criterion gave a better prediction of the trend in both the pre- and post-cracking phases of the experimental results (Tables 4.4 and 4.5, Figures 4.33 to 4.35).

The theoretical results obtained by using the failure criterion and average ratio of $E_x/E_y$ improves the predicted cracking and failure loads (Tables 4.4 and 4.5). The theoretical results lie between 1 to 13% and 1 to 11% of the experimental results for cracking and failure load respectively. These are significantly improved compared to the Rankine method or FEM results described in Section 4.51 and Tables 4.2 & 4.3.
Table 4.4 Comparison Of Experimental And Theoretical Cracking Load Of The Cross-Beam

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<tr>
<th>Lx (mm)</th>
<th>Ly (mm)</th>
<th>P_X Normal</th>
<th>P_Y Normal</th>
<th>Apply Load</th>
<th>P_X F. E. M</th>
<th>P_Y F. E. M</th>
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<th>FEM/Exp.</th>
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aThree specimens
bTwo specimens

78
### Table 4.5 Comparison Of Experimental And Theoretical Failure Load Of The Cross-Beam

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<tr>
<th>Lx (mm)</th>
<th>Ly (mm)</th>
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$^a$Three specimens

$^b$Two specimens

![Figure 4.33 Typical load distribution in cross-beam (failure type 1)](image-url)
Figure 4.34 Typical load distribution in cross-beam (failure type 2)

Figure 4.35 Typical load distribution in cross-beam (failure type 3)
The numerical modelling was further refined to detect any changes in the cracking or failure load. Instead of using the average modulus of elasticity in two orthogonal directions, the actual ratio of $E_X/E_Y$ from each individual cross was used in the finite element program. The initial measured load distribution was used to obtain the ratio of $E_X/E_Y$. The results of this analysis are given in Table 4.6.

Table 4.6 shows that the theoretical results obtained from the ratio of $E_X/E_Y$ obtained as explained above gave better prediction than using the average ratio of $E_X/E_Y$ from the wallette test. This refined modelling technique improved the prediction of cracking and failure loads by 5% compared to using the average values of modulus of elasticity in two orthogonal directions.
### Table 4.6 Comparison Of Experimental And Theoretical Cracking And Failure Load Of The Cross-Beam

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<th>Cross beams no.</th>
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4.8 COMPARISON OF THE EXPERIMENTAL FAILURE LOAD WITH OTHER THEORETICAL METHODS

4.8.1 Yield line, Fracture line and Finite element with proposed criterion

Table 4.7 compares the experimental results of all the crosses with the yield line method(23), the fracture line theory(46) and the finite element method using the failure criterion defined by equation (4.23). The yield line method consistently over-estimates the failure loads in all cases. In the majority of cases, the fracture line method gives a slightly better prediction of the failure loads compared to the yield line method. However, both methods give good agreement with the experimental results of the test panels where both directions failed simultaneously. The yield line and the fracture line methods can safely be applied to these cases only. These methods are not capable of predicting the failure load correctly, if cracking in any direction precedes the failure. After the section cracks, it does not support any moment in that direction, which is contrary to the assumptions made in the yield line or fracture line methods. The finite element method, using the failure criterion proposed in chapter 4, slightly underestimates the failure load (Table 4.7) and thus can safely be used for predicting the strength of panels.
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* both directions failed simultaneously.
4.9. PARAMETRIC STUDY

Some parametric studies have been carried out to determine:

i) The effect of the modulus of elasticity on cracking and failure loads by the finite element analysis using the failure criterion developed in this project.

ii) The effect on the failure load obtained by the yield line method by changing hypothetically the flexural strength both in the weaker and stronger direction.

4.9.1 The effect of changing modulus of elasticity on the cracking and failure of a cross-beam.

The cross-beam test results have clearly demonstrated that the load distribution depends on the ratios of $E_x/E_y$ in the x and y directions. It would be very interesting to see how the cracking and failure loads of the cross-beam are affected by using a range of ratios of $E_x/E_y$. Ratios of moduli of elasticity ranging from 0.5 to 2.0 were used. The main reason for using these is to cover the whole range of masonry materials. In this study, only the ratio of modulus of elasticity is changed, while other parameters were kept constant. The finite element method in corporating the bi-axial failure criterion was used in this parametric study. The analysis results are discussed in the following sections:

4.9.1.1 Cracking

Figure 4.36 shows the variation in the cracking loads of crosses of different aspect ratios. It can be seen that the cracking load decreases with decreasing ratio of
$E_x/E_y$ and increasing aspect ratio. With decreasing ratio of $E_x/E_y$, more load is carried in the weaker (y) direction which causes earlier cracking.

In contrast, for the crosses in which both the stiffness and strength are considered as isotropic (i.e. darker line) the cracking load is much lower for aspect ratios less than 1 and stays constant for aspect ratios higher than 1.0.

![Graph showing relationship between cracking load and aspect ratio](image)

Figure 4.36 Relationship between cracking load against aspect ratio of the cross

### 4.9.1.2 Failure

After cracking, the applied load in the y-direction (i.e normal to bed-joint) was shed immediately to the x-direction (i.e. parallel to bed-joint). Once it cracks, a cracked section will no longer take further load; most of the applied load will be carried in the x-direction. Thus, the failure loads are not affected significantly by different ratios of the modulus of elasticity, because the modulus of elasticity in the weaker direction has no effect on failure (Figure 4.37). The failure load decreases with the increase in the aspect ratio. The increase is not significant between aspect...
ratios 1 to 2. The aspect ratio has no effect on the failure load of the isotropic (stiffness and strength) crosses.

![Graph showing relationship between failure load and aspect ratio](image)

Figure 4.37 Relationship between failure load against aspect ratio of the cross

4.9.2 The effect of changing the flexural strength on the failure load of cross-beam

In the cross beam tests, it was found that the load is shed from one direction to the other direction, hence it was thought to see the effect of reducing the actual strength in one direction by a certain percentage while keeping the strength in other direction constant.

4.9.2.1 Percentage reduction in flexural strength in y-direction

From Figure 4.38, it can be seen that the theoretical failure loads did not decrease significantly when the flexural strength in the y-direction decreased to a very low value. This is true for all aspect ratios. The failure load only dropped by 6% while
the flexural strength in y-direction dropped to 20% of the ultimate flexural strength.

![Graph showing relationship between failure load and % drop in flexural strength in weaker y-direction](image)

Figure 4.38 Relationship between the failure load and % drop in flexural strength in weaker y-direction

4.9.2.2 Percentage reduction in flexural strength in x-direction

Figure 4.39 shows the effect of reducing the strength in the stronger direction while the flexural strength in the weaker (y) direction is kept constant. It is clear from Figure 4.39 that the ultimate load is significantly affected by the percentage reduction in the flexural strength of the stronger direction. The relationship between failure load and the percentage reduction is linear for all aspect ratios. From these it can be concluded that the failure load is mainly dependent on the strength in the stronger direction.
Figure 4.39 Relationship between the failure load and the percentage reduction in flexural strength in weaker x-direction

4.10 CONCLUSIONS

1. The applied lateral load distributes according to the stiffness orthotropy of the brickwork. Hence, the ratio of the modulus of elasticity in two orthogonal directions exerts great influence on the behaviour of the panel.

2. Once the weaker direction cracks, the cracked section does not support any load. After cracking, most of the applied load is shed to the stronger direction till final failure.

3. In bi-axial bending, the strength in the weaker direction is enhanced beyond the uniaxial ultimate moment of resistance.
4. Both the Rankine maximum stress theory and Baker's failure criterion are not applicable to a brickwork panel subjected to bi-axial bending.

5. The yield line method consistently overestimates the failure load. The fracture line appears to be slightly better in predicting the failure load compared to the yield line. Both methods predicted very closely the failure load of the cross where both directions failed simultaneously. These methods are not capable of predicting satisfactorily the failure load for all the crosses.

6. The cracking and failure loads of the masonry panel can be predicted reasonably well by the finite element program using the bi-axial failure criterion developed in this work.

7. Smeared crack modelling proved to be a useful tool in modelling the post-cracking behaviour of brickwork panels.
Chapter 5

Wall Tests

5.1 INTRODUCTION

Brickwork cladding panels bend out of plane when subjected to lateral loading due to wind pressure or explosion. Lateral loading exerts a uniform load over the wall panel. The wall must, therefore, be strong and stiff enough to withstand such forces. The current recommended moment coefficients for the design of brickwork panel in the British Code BS5628(45) are derived from the yield line theory developed for under-reinforced concrete slabs. These coefficients are not strictly applicable to brickwork panels. Firstly, brickwork is brittle in nature, it does not behave in a fully rigid-plastic manner and thus it is not capable of resisting moment after cracking. Once cracking occurs brickwork loses all its strength in the direction perpendicular to the crack. Secondly, the code(45) only considers the orthogonal strength ratio but ignores the stiffness orthotropy. This is not justified, since it has been shown in Chapter 4 that the stiffness orthotropy has a significant influence on the load distribution. Because of these anomalies, the failure pressures calculated by the yield line theory are consistently over-estimated in most cases(37,40,49). However, in some cases(42,43,44,61) the yield line theory has been shown to predict correctly the strength of laterally loaded panels without openings. This may be just a coincidence, as many factors such as rotational restraint at the supports and the effect of self-weight have been ignored in the analysis.
At present, no analytical method is available to predict the ultimate load of brickwork panels supported on three or four sides and subjected to lateral loading. A rational approach can only evolve provided the material behaviour and failure criterion in bi-axial bending are established. This criterion has already been established in Chapter 4 and it was incorporated into a conventional plate bending finite element program which takes into account both the strength and stiffness orthotropies together with the load-shedding behaviour observed in the cross-beam tests to predict the experimental results. Good agreement was obtained between the modified finite element analysis and the cross-beam test results. However, the validity and applicability of the numerical model needs to be checked with reference to representative wall tests.

Therefore, walls with different aspect ratios were tested under ideal boundary conditions to verify this model. Reactions and strains in two orthogonal directions were measured in both the pre- and post-cracking phases. The numerical model was used to compare the available results\(^{19,35,37,41,49,54,68,82,84}\) for panels with and without openings and having different aspect ratios and boundary conditions.

### 5.2 Experimental Investigation

#### 5.2.1 Test arrangements

All walls were tested in a specially designed frame (Figure 5.1). Both the reacting and supporting frames were bolted to the strong floor. In order to reduce or eliminate the frictional restraint due to the dead weight all walls were built on the
top of a steel plate supported on stainless steel rollers. The supporting frame for the wall was designed in such a way that it enabled the reactions to be measured on the four sides. In order to prevent any rotational restraint at the supports, some frictionless materials (PTFE) were used and placed along each support to prevent any bond friction between the wall and the supports as shown in Figure 5.2. The entire test program took nearly two years to complete. A total number of 15 walls was built and tested during this investigation. The variables considered were:

i) Aspect ratios (Length/Height): 0.67, 1.0 and 1.5
   The height of the wall was generally maintained between 1140mm to 1200mm. However, owing to the restriction on the length of the testing frame, some walls (walls 14 and 15) were built to a height of 755 mm, enabling the aspect ratio of 1.5 to be achieved.

ii) Boundary conditions:
   a) Four sides simply supported.
   b) Three sides simply supported with vertical edge free,
   c) Three sides simply supported with top edge free.

The lateral pressure was applied by an air-bag sandwiched between the wall and reacting frame. This was chosen as the best means of achieving a uniform distribution of pressure. The pressure was measured by a water manometer. Each airbag was tailor-made to suit the size of the test wall. A 15 mm thick foam sheet was inserted between the air-bag and the wall to prevent damage to the airbag by broken bricks or mortar as the wall failed. Initially, wallettes were built along side each corresponding wall. However, this practice of building wallettes was stopped at a later stage as the flexural strength of the wallettes built along side the test wall
and those extracted from the undamaged portion of the test walls were very consistent (Table 3.5).

5.2.2 Experimental Details

5.2.2.1 Construction of the test walls

All test walls were of single leaf construction. The bricks used for the construction of test walls were similar to those described in Chapter 3. 1:3 (rapid hardening Portland cement: sand) mortar was used for the construction of the walls. In order to maintain consistent workmanship, all walls were built by the same bricklayer. After construction, the walls were covered by plastic sheets and cured under ambient conditions until testing. Three mortar cubes were made on the same day as each test wall. They were also covered by plastic sheets and immersed in water the day after building the test wall. They were crushed on the same day as the testing of the wall. The compressive strengths of the mortar cubes are given in Chapter 3, Table 3.3.

5.2.2.2 Instrumentation

a) Measurement of the Reactions:

Three different capacities of load cells were used in this investigation. Four 3-tonne load cells (two on each side of the support) were positioned at the supports in the x-direction (i.e. parallel to bed-joint) which was the stronger direction. The other two half-tonne (at the top support) and two 1-tonne load cells (at the bottom support) were placed at the supports in the y-direction (i.e. normal to bed-joint) which were expected to receive much smaller reactions, thus a total of 8 load cells
was used. All load cells were calibrated prior to their use. The reason for measuring the reaction in two orthogonal x and y reactions was to detect the occurrence of cracks and transfer of load from one direction to the other direction in the test wall as indicated by a change in the load-cell readings.

b) Strain Measurement:

Electrical strain gauge rosettes were used to measure strains in two orthogonal directions of wall panels subjected to lateral loading. The locations of the electrical strain gauges on the walls with different support conditions are shown in Section 5.4.1.2. The electrical strain gauges (10 mm) were used to pick up any changes in strain in two orthogonal directions due to the cracking. An "Orion" data logger was used to record the instantaneous increase or decrease of loads and strains in two orthogonal directions at the onset of cracking.

c) Deflection Measurement:

Dial gauges were used to measure the deflection of the wall. Also, dial gauges were placed at the supporting frame to check any settlement. Deflections were measured along the vertical and horizontal profiles through to the mid-height and mid-length of the test walls. The location of the dial gauges for the test walls is shown in Section 5.3.3, Figure 5.19. In order to avoid any damage to the dial gauges and also for safety reasons in the event of a sudden collapse of the wall, all dial gauges were removed from the test wall when the applied pressure reached approximately about 80% to 90% of the theoretical failure pressure.

An overall view of the test set-up for the wall is shown in Figure 5.3.
Figure 5.1 Side view of the wall test set-up
Figure 5.2 Preparation of the wall testing frame

Figure 5.3 Overall view of the set-up for the testing of the wall
5.3 EXPERIMENTAL RESULTS

5.3.1 Wall test results

The results of the wallettes and wall tests are given in Table 5.1. The flexural strengths in both the x and y directions (i.e. $F_x$ and $F_y$) were obtained from the wallettes extracted from the undamaged part of the tested walls.

It can be seen that the failure loads are lower for walls with higher aspect ratios and similar boundary conditions. In the case of walls 14 and 15 with aspect ratio of 1.5, the failure loads were much higher than walls 2 and 3 with aspect ratio, 1.0. The reason was that the height of the walls 14 and 15 was only 755 mm compared to 1140 mm or 1150 mm for others.

The failure pressure of wall 1 was much lower than walls 2 and 3 with similar boundary conditions. This was due to the fact that an older batch of bricks giving lower tensile strengths in two orthogonal directions was used for the construction of wall 1. The remainder of the test walls were built using the new batch of bricks. The average tensile strengths of the old and the new bricks were 2.95 N/mm$^2$ and 4.45 N/mm$^2$ respectively. The flexural tensile strength parallel to the bed-joint ($F_x$) is to some extent dependent on the strength of brick; whereas the flexural strength normal to the bed-joint ($F_y$) is dependent only on inter-face bond between the bricks and mortar which is not affected by the tensile strength of the bricks.
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<td>Wall 4</td>
<td>3 sides simply</td>
<td>1140x1140</td>
<td>1.0</td>
<td>2.9</td>
<td>0.75</td>
</tr>
<tr>
<td>Wall 5</td>
<td>supported and</td>
<td>1140x1140</td>
<td>1.0</td>
<td>2.9</td>
<td>0.75</td>
</tr>
<tr>
<td>Wall 9</td>
<td>top side free</td>
<td>795x1190</td>
<td>0.67</td>
<td>3.5</td>
<td>0.98</td>
</tr>
<tr>
<td>Wall 13</td>
<td></td>
<td>795x1190</td>
<td>0.67</td>
<td>3.85</td>
<td>1.1</td>
</tr>
<tr>
<td>Wall 6</td>
<td>3 sides simply</td>
<td>1200x1200</td>
<td>1.0</td>
<td>3.0</td>
<td>0.74</td>
</tr>
<tr>
<td>Wall 7</td>
<td>supported and</td>
<td>1200x1200</td>
<td>1.0</td>
<td>2.95</td>
<td>0.71</td>
</tr>
<tr>
<td>Wall 10</td>
<td>one side free</td>
<td>795x1190</td>
<td>0.67</td>
<td>3.65</td>
<td>1.1</td>
</tr>
<tr>
<td>Wall 11</td>
<td></td>
<td>795x1190</td>
<td>0.67</td>
<td>3.5</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 5.1 Summary of The Test Results of Walls

Panel size (mm) Length x Height

Flexural strength (N/mm²) Fx Fy

Experimental result (kN/m²) Cracking Pressure Failure Pressure
5.3.2 Crack patterns

When the brickwork panels failed, a certain crack pattern formed. The crack patterns were dependent on the boundary conditions and aspect ratios. The crack patterns of all walls are shown in Figures 5.4 to 5.18. In order to show the crack patterns more clearly, dark lines were used to highlight the cracked lines of each individual wall test. In some walls, the crack patterns are similar to those obtained in the yield line analysis.

5.3.2.1 Wall panel simply supported on four sides

For this type of wall panel, two types of crack patterns were observed during the tests. The first type was found in walls 1 and 3 (Figures 5.4 and 5.6), where cracks occurred in the centre of the wall (mid-height along a horizontal bed-joint) at the maximum bending moment. The second type of crack patterns can be seen in walls 2, 8, 12, 14 and 15 corresponding to Figures 5.5, 5.7, 5.8, 5.9, 5.10, where cracks occurred nearer to one-third of the wall height rather than at the maximum bending moments. It may be possible for cracks to develop at a section where the flexural strength is lowest. As shown in the Chapter 3, the flexural strength in the weaker y-direction (normal to bed-joint) is very variable with a coefficient of variation of 0.19. Cracking will occur when the stress due to the applied bending moment exceeds the moment of resistance of the bed-joint (or bond strength). After the initial crack, the final cracks extended to the four corners of the supports.
i) Wall panel of aspect ratio 1.0:1.0

Figure 5.4 Crack pattern of Wall 1

Figure 5.5 Crack pattern of Wall 2

Figure 5.6 Crack pattern of Wall 3
ii) Wall panel of aspect ratio 0.67:1.0

Figure 5.7 Crack pattern of Wall 8

Figure 5.8 Crack pattern of Wall 12

iii) Wall panel of aspect ratio 1.5:1.0

Figure 5.9 Crack pattern of Wall 14

Figure 5.10 Crack pattern of Wall 15
5.3.2.2 Wall panel simply supported on three sides and free at the top edge

The crack patterns for walls with different aspect ratios are shown in Figures 5.11 to 5.14. Cracking normally started at the centre of the free edge at the maximum bending moment. The crack quickly extended vertically downward and progressed towards the bottom corners of the test wall. For this type of wall, the cracking and the failure usually happened simultaneously which is confirmed by the measured distribution of loads and strains (sections 5.4.1.1 & 5.4.1.2).

i) Wall panel of aspect ratio 1.0:1.0

![Figure 5.11 Crack pattern of Wall 4](image1)

![Figure 5.12 Crack pattern of Wall 5](image2)
ii) Wall panel of aspect ratio 0.67:1.0

The crack patterns for this type of wall with different aspect ratios are shown in Figures 5.15 to 5.18. Two types of crack patterns were observed in the experiment. In the first case, the crack began at a distance one third away from the top and bottom of the free edge as in wall 6 (Figure 5.15). In the second case, as shown in wall 7 (Figure 5.16), the first crack appeared at the middle of the free edge. A crack pattern similar to wall 6 was also found in walls 10 and 11 of aspect ratio 0.67:1 (Figures 5.17 & 5.18). After the initial crack, the cracks propagated horizontally towards the vertical support and extended to the top and bottom corner of the wall.

5.3.2.3 Wall panel three sides simply supported with vertical edge free

The crack patterns for this type of wall with different aspect ratios are shown in Figures 5.15 to 5.18. Two types of crack patterns were observed in the experiment. In the first case, the crack began at a distance one third away from the top and bottom of the free edge as in wall 6 (Figure 5.15). In the second case, as shown in wall 7 (Figure 5.16), the first crack appeared at the middle of the free edge. A crack pattern similar to wall 6 was also found in walls 10 and 11 of aspect ratio 0.67:1 (Figures 5.17 & 5.18). After the initial crack, the cracks propagated horizontally towards the vertical support and extended to the top and bottom corner of the wall.
i) Wall panel of aspect ratio 1.0:1.0

Figure 5.15 Crack pattern of Wall 6

Figure 5.16 Crack pattern of Wall 7

Figure 5.17 Crack pattern of Wall 10

Figure 5.18 Crack pattern of Wall 11

ii) Wall panel of aspect ratio 0.67:1.0
5.3.3 Deflection

Figure 5.19 shows the location of dial gauges to measure the vertical and horizontal deflection of the walls.

Figure 5.19 Positions of dial gauges to measure the deflection of walls
Some typical theoretical and experimental load-deflection relationships and the central deflection of the walls in horizontal and vertical directions are shown in Figures 5.20 to 5.40. The theoretical deflections were calculated from the modified finite element program.

It can be seen from Figures 5.20, 5.23, 5.26, 5.29, 5.32, 5.35 and 5.38 that the load-deflection relationship is linear up to the cracking pressure. In most cases, after cracking, the load-deflection relationship becomes non-linear. This non-linearity is due to cracking of the wall in the weaker direction and not due to the material properties.

At failure, some of the test walls burst without prior warning (Figure 5.7). This sudden collapse of the wall can cause damage to the deflection gauges. Therefore, for the safety reason the deflection was measured only up to 80% to 90% of the theoretical failure pressure. In addition, it was not feasible to read the dial gauges near failure since the needle started to move very fast.
5.3.3.1 Wall panel simply supported on four sides
i) Wall panel of aspect ratio 1:1

Figure 5.20 Typical load-deflection relationship of Wall 1

Figure 5.21 Deflection at the centre along the height of wall 1 (y-direction)

Figure 5.22 Deflection at the centre along the length of wall 1 (x-direction)
ii) Wall panel of aspect ratio 0.67:1

Figure 5.23 Typical load-deflection relationship of Wall 8

Figure 5.24 Deflection at the centre along the height of wall 8

Figure 5.25 Deflection at the centre along the length of wall 8
iii) Wall panel of aspect ratio 1.5:1

![Figure 5.26 Typical load-deflection relationship of Wall 14](image1)

![Figure 5.27 Deflection at centre along the height of wall 14](image2)

![Figure 5.28 Deflection at centre along the length of wall 14](image3)
5.3.3.2 Wall panel three sides simply supported with top edge free

i) Wall panel of aspect ratio 1:1

Figure 5.29 Typical load-deflection relationship of Wall 4

Figure 5.30 Deflection at centre along the height of wall 4

Figure 5.31 Deflection at centre along the length of wall 4
i) Wall panel of aspect ratio (0.67:1)

Figure 5.32 Typical load deflection relationship of Wall 9

Figure 5.33 Deflection at centre along the height of wall 9

Figure 5.34 Deflection at centre along the length of wall 9
5.3.3.3 Wall panel three sides simply supported with vertical edge free

i) Wall panel of aspect ratio 1:1

![Graph showing load-deflection relationship](image1)

**Figure 5.35 Typical load-deflection relationship of wall 6**

![Graph showing deflection along height](image2)

**Figure 5.36 Deflection at centre along the height of wall 6**

![Graph showing deflection along length](image3)

**Figure 5.37 Deflection at centre along the length of wall 6**
i) Wall panel of aspect ratio 0.67:1

Figure 5.38 Typical load-deflection relationship of Wall 10

Figure 5.39 Deflection at centre along the height of wall 10

Figure 5.40 Deflection at centre along the length of wall 10
5.3.4 Modelling the theoretical deflections

Figures 5.20 to 5.40 show that there are discrepancies between the measured and theoretical results. The reason for these discrepancies could be the use of the average modulus of elasticity obtained from the wallette tests in the finite element method. This may be different from the absolute values for the two directions of the individual wall. A numerical modelling technique was used to demonstrate the effect on the deflection of choosing arbitrary values of modulus of elasticity in two directions while keeping the ratios of $E_x/E_y$ constant. A four sides simply supported square panel (600 x 600 mm) was used in this study. The results of this analysis are given in Table 5.2.

Table 5.2 clearly shows that the central deflections are greatly affected by the chosen arbitrary values of the modulii of elasticity even though the various ratios of $E_x/E_y$ are kept constant. Therefore, the theoretical deflection could differ from the experimental test results because the absolute values of moduli of elasticity in two directions for each panel may differ. On the other hand, the bending moments are not affected by the individual modulus of elasticity which is a very important point to note when designing laterally loaded brickwork panels.

The above numerical modelling has shown that deflection is very much dependent on the values of the moduli of elasticity in the two directions. Hence, if one can adjust the actual individual value of modulus of elasticity while keeping the ratio of $E_x/E_y$ constant to maintain the correct load distribution. The theoretical deflection could be improved further as shown by Figures 5.20 to 5.22. The theoretical deflection for wall 1 has greatly improved in both the vertical and horizontal directions due to this adjustment in the values of the moduli of elasticity.
Table 5.2 Effect of The Modulii of Elasticity On The Deflection and Bending Moment For Simply Supported Square Panel (600 x 600mm)

<table>
<thead>
<tr>
<th>Arbitrary values of modulus of elasticity</th>
<th>Ratio of $E_x/E_y$</th>
<th>Maximum bending moment (Nmm) in x and y directions (at pressure of 5.7 kN/m²)</th>
<th>Central deflection (mm) (at pressure of 5.7 kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$</td>
<td>$E_y$</td>
<td>$E_x/E_y$</td>
<td>$M_x$</td>
</tr>
<tr>
<td>6552</td>
<td>7200</td>
<td>0.91</td>
<td>81.75</td>
</tr>
<tr>
<td>8372</td>
<td>9200</td>
<td>0.91</td>
<td>81.75</td>
</tr>
<tr>
<td>16530</td>
<td>18130</td>
<td>0.91</td>
<td>81.75</td>
</tr>
<tr>
<td>8800</td>
<td>6200</td>
<td>1.42</td>
<td>108.5</td>
</tr>
<tr>
<td>10224</td>
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<td>13064</td>
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<tr>
<td>15300</td>
<td>8650</td>
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<td>123.2</td>
</tr>
<tr>
<td>16192</td>
<td>9200</td>
<td>1.76</td>
<td>123.2</td>
</tr>
</tbody>
</table>

116
5.4 DISCUSSION OF THE TEST RESULTS

5.4.1 Load and strain distribution

The behaviour of a brickwork panel when subjected to lateral loading cannot be fully understood by merely looking at the final crack patterns or the deflection results of tested walls. A crack pattern is only the end result of the final failure of the wall. It does not indicate how walls of different strengths and ratios of $E_x/E_y$ behaved before and after cracking. In order to study the behaviour of the brickwork panel more clearly, two other types of measurements were recorded during the wall tests, namely the measurement of load and strain distribution in two orthogonal directions.

5.4.1.1 Load Distribution

The distribution of the experimental and theoretical reactions in two directions for the test walls due to applying the load are shown in Figures 5.41 to 5.55.

The theoretical reactions were obtained from the modified finite element method. Initially, the applied load was shared according to the stiffness in both the vertical (i.e normal to the bed-joint) and horizontal (i.e parallel to the bed-joint) directions before cracking. Figures 5.47, 5.48 and 5.49 (corresponding to walls 1, 2 and 3) show that the applied load was not distributed equally in two directions (i.e vertical and horizontal) for square panels even before cracking. The distribution of the applied load in two directions would be the same for square panels made of an isotropic material, but the results conclusively prove that brickwork panels exhibit...
stiffness orthotropy and the load distributes according to the stiffness in two directions.

In Chapter 3 it was shown that the flexural strength in the weaker y-direction is about 2.5 to 4.0 times lower than the flexural strength in the stronger x-direction. Thus, the weaker y-direction (normal to bed joint) would normally crack first; the load in this direction decreased immediately. The load was then shed to the stronger x-direction (i.e. parallel to bed-joint). This load shedding behaviour from the weaker to the stronger direction clearly indicated that brickwork exhibits strength orthotropy. The bending moment coefficients given in the British Code BS5628(45) are derived from the yield line theory which considers only the strength orthotropy and neglects the stiffness orthotropy. This is not justified. Both the strength and stiffness orthotropies should be considered in any theoretical analysis.

Two different types of failure were observed for the walls tested. Firstly, cracking preceded the failure in walls simply supported on four sides or on three sides with vertical edge free and further load was carried after cracking. Secondly, cracking and failure occurred simultaneously in wall panels simply supported on three sides with the top edge free (walls 4, 5, 9 and 13) and simply supported on four sides with an aspect ratio of 0.67 (walls 8 and 12).

5.4.1.1.1 Wall panels where both cracking and failure occurred simultaneously

For wall panels simply supported on three sides with the top edge free or wall panels simply supported on four sides with the smallest aspect ratio ($L_x:L_y = 0.67:1.0$), both cracking and failure usually occurred simultaneously. The
distributions of the applied load in both the x and y directions for this type of failure are shown in Figures 5.41 to 5.46. As a result of the support conditions and the aspect ratio, a larger load was carried in the x-direction. Hence, the wall panels tended to fail in the x-direction before the y-direction reached its capacity. These walls normally failed suddenly without warning. Cracking could not be detected before failure.

Figure 5.41 Distribution of the applied load in both x and y directions (wall 4)

Figure 5.42 Distribution of the applied load in both x and y directions (wall 5)
Figure 5.43 Distribution of the applied load in both x and y directions (wall 8)

Figure 5.44 Distribution of the applied load in both x and y directions (wall 12)
Figure 5.45 Distribution of the applied load in both x and y directions (wall 9)

Figure 5.46 Distribution of the applied load in both x and y directions (wall 13)
5.4.1.1.2 Wall panels where cracking preceded the failure

For this group, the distributions of the applied load in both the x and y directions are shown in Figures 5.47 to 5.55.

In wall 1, after cracking, most of the load in the weaker y-direction was shed to the stronger x-direction. The amount of load shed was sufficient to cause failure in the x-direction. Hence, a slight increase of applied load resulted in the immediate collapse of the wall. With this type of panel, the cracking load was very close to the failure load, as shown in Figure 5.47.

However, in many cases, the amount of load shed after cracking was insufficient to cause immediate failure of the wall. More load was required to cause failure of the test wall. The distributions of the applied load in both x and y directions in these cases are shown in Figures 5.48 to 5.55. The wall only failed when the ultimate strength in the stronger x-direction was reached. The load carried in the y-direction did not drop to zero because the adjacent uncracked sections were still able to carry the applied load.
Figure 5.47 Distribution of the applied load in both x and y directions (wall 1)

Figure 5.48 Distribution of the applied load in both x and y directions (wall 2)
Figure 5.49 Distribution of the applied load in both x and y directions (wall 3)

Figure 5.50 Distribution of the applied load in both x and y directions (wall 6)
Figure 5.51 Distribution of the applied load in both x and y directions (wall 7)

Figure 5.52 Distribution of the applied load in both x and y directions (wall 10)
Figure 5.53 Distribution of the applied load in both x and y directions (wall 11)

Figure 5.54 Distribution of the applied load in both x and y directions (wall 14)
Strain distribution

Another qualitative way to study the behaviour of wall panel subjected to lateral loading was to measure the strains in two orthogonal directions (normal and parallel to the bed-joint). Both the experimental and theoretical results are shown in Figures 5.56 to 5.58. The theoretical strains in two directions can be obtained from the output of the normal stresses ($\sigma_x$ and $\sigma_y$) in the modified finite element program. Before cracking, strains in the x and y directions were calculated from:

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \nu \frac{\sigma_y}{E_y}$$  \hspace{1cm} (5.1)

$$\varepsilon_y = \frac{\sigma_y}{E_y} - \nu \frac{\sigma_x}{E_x}$$  \hspace{1cm} (5.2)
It can be seen that the theoretically predicted strain are very similar to the experimental results. The slight difference could be due to the use of average values of moduli of elasticity and Poisson's ratios.

Some electrical strain gauges were placed at those locations where the maximum bending moment in that relevant direction was expected to occur. As mentioned before the purpose of measuring the strains was to observe the change of strains in two directions after cracking and to detect the effect of load shedding. Electrical strain gauges are expensive and can only be used once. Hence, only a few typical square panels with different boundary conditions were tested with strain gauges and their results presented here.

5.4.1.2.1 Wall panel simply supported on four sides

Figure 5.56 shows the strains in the x and y directions at the centre of wall. For this type of panel the strain in the y-direction is higher than the strain in the x-direction because of the low value of the modulus of elasticity in the y-direction. After cracking, the strain in the weaker (y-direction) direction dropped and the strain in the stronger x-direction increased until the x-direction reached its ultimate flexural strength. Thus, the strain measurements confirm the load shedding behaviour after cracking.
5.4.1.2.2 Three sides simply supported with top edge free

In this type of panel, the maximum bending moment (or the maximum strain) is expected to occur at the middle of the free edge. Figure 5.57 shows the strain at point 'a' where the bending moment is maximum. Once the tensile stress reached its maximum value the panel cracked and the strain in the x-direction at 'a' dropped with simultaneous increase in the strain at point 'b'. Due to the support condition a larger load was carried in the x-direction, hence the strain in the y-direction (at point c) was smaller. It can be seen that a slight increase of load caused the wall to fail immediately as the flexural stress at point 'b' in the x-direction reached its ultimate capacity. This also explains why the cracking and the failure load are of similar values.
Figure 5.57 Strain distribution at various locations in the test wall

5.4.1.2.3 Three sides simply supported and vertical edge free

For this type of wall, most of the load was carried in the y-direction. The bending moment (or maximum strain) is a maximum at the middle of the free edge which is shown at point 'a' in Figure 5.58. Initially, the crack appears at point 'a' which causes the strain to decrease at this location with simultaneous increase of strain at location 'b'. The stress at point 'b' quickly reaches its maximum causing the crack to extend up to that point and the strain then decreases. The strain at location 'c' in the x-direction increases immediately (dark line), thus indicating that load has been shed to the stronger x-direction.
Both the strain and reaction measurements during the wall test indicated strongly the load shedding behaviour of brickwork panels subjected to lateral loading. Therefore, it was decided not to use the electrical strains gauges on other walls to reduce the cost.

5.4.2 Support reactions

In addition, apart from looking at the distributed load in the two orthogonal directions, it may be worthwhile expressing the orthogonal support reactions in terms of the percentages of the total cracking and failure loads. These are given in tabular form in Table 5.3. The table also shows the theoretical values obtained from the modified finite element analysis.

In general, the percentage of the support reactions carried in the y-direction at cracking is higher than at the time of failure, which means that the load is shed to
the stronger x-direction after cracking. The cracking always occurred first in the weaker y-direction. The percentage drop in load at failure is not very high as the adjoining sections of the panels have not attained their strength in the y-direction but the shed load was sufficient to cause failure in the stronger x-direction leading to the ultimate collapse of the wall.

In the case of a wall panel simply supported on three sides with the top edge free, also for panels simply supported on four sides with an aspect ratio of 0.67, there is no percentage drop in load from cracking to ultimate. In this type of wall, failure is precipitated due to the flexural tension reaching its ultimate value in the stronger x-direction. Hence, no redistribution or load shedding takes place from the stronger to the weaker direction.

The measured support reaction expressed as a percentage, in two orthogonal directions for wall 1 before cracking is different compared to walls 2 and 3. The difference could be due to the fact that the ratio of $E_x/E_y$ for wall 1 is different from the other two walls because a different batch of bricks was used.

Generally good agreement was obtained between the modified finite element program and the experimental results. The differences lie between 1 to 10% and 1 to 6% for the cracking and failure loads respectively. Such small differences can be attributed to many uncertain factors such as the workmanship, the modulus of elasticity and the variation of flexural strength found in the wallette tests. In addition, the theoretical analysis is based on the average ratio of $E_x/E_y$ which might affect the theoretical prediction as has been discussed in Chapter 4, Section 4.4.1.
Table 5.3. Comparison of Experimental & Theoretical Support Reactions

<table>
<thead>
<tr>
<th>Wall no.</th>
<th>Support conditions</th>
<th>Aspect ratio</th>
<th>Percentage of average support reactions in term of total load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>L_x:L_y (%)</td>
<td>Experimental results</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X (%)</td>
<td>Y (%)</td>
</tr>
<tr>
<td>At cracking</td>
<td>At failure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W1</td>
<td>4 sides simply supported</td>
<td>1.0</td>
<td>70</td>
</tr>
<tr>
<td>W2</td>
<td>4 sides simply supported</td>
<td>1.0</td>
<td>63</td>
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<td>W3</td>
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<td>1.0</td>
<td>59</td>
</tr>
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</tr>
<tr>
<td>W12</td>
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<td>69</td>
</tr>
<tr>
<td>W14</td>
<td>4 sides simply supported</td>
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<td>39</td>
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</tr>
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</tr>
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<td>W11</td>
<td>one side free simply supported</td>
<td>0.67</td>
<td>24</td>
</tr>
</tbody>
</table>

133
5.5 Numerical Modelling Of The Support Reactions

5.5.1 Modelling the support reactions

It was mentioned in Section 5.2.2.2 that the individual support reaction was measured by two load cells. Thus, the support reaction obtained from the wall test was the average of the horizontal and vertical support reactions. However, the support reactions obtained from the modified finite element program are parabolic in pattern as shown in Figures 5.59 and 5.60. In practice, it is very difficult to measure the reaction at every single point along the support as assumed in the finite element program. In the theoretical modelling the theoretical support condition can be treated as several point supports. Hence, it will be very interesting to use the numerical model to look at how the support reactions in two directions varied along the height (x-direction) and the length (y-direction) of the wall with different boundary conditions at cracking and failure. Some typical results are shown in Figures 5.59 to 5.64.

5.5.1.1 Wall panel simply supported on four sides

Typical support reactions of a wall panel simply supported on four sides at cracking and failure are shown in Figures 5.59 and 5.60. It can be seen that after cracking more load was carried in the x-direction (at the middle section of the support). On the other hand, the load dropped in the y-direction. The reason was that the applied load was shed from the weaker y-direction to the stronger x-direction after cracking.
Figure 5.59 Load distribution along the height of the wall

Figure 5.60 Load distribution along the length of the wall
5.5.1.2 Wall panel three sides simply supported with vertical edge free

Figures 5.61 and 5.62 show typical load distributions in two directions for a wall panel simply supported on three sides with the vertical edge free. At failure, the load carried in the x-direction does not increase very significantly across the full length of the support. However, a sharp increase of load occurs at approximately 200mm from the centre horizontal line of the wall. It seems that the first crack propagated horizontally to a certain distance (i.e. near to the centre of the wall) from the free edge resulting in a rapid extension of the crack to the two corners of the wall and caused the collapse.

Figure 5.61 Load distribution along the height of the wall
5.5.1.3 Wall panel three sides simply supported with top edge free

The reactions along the supports in the x and y-directions for a panel simply supported on three sides and free on the top are shown in Figures 5.63 and 5.64. The maximum value (Figure 5.63) of the support reaction is fairly constant from the free edge to the centre of the panel. Thus, the cracking which extends from the centre of the free edge to the middle of the panel precipitates the sudden failure. For this type of support condition cracking and failure usually happened simultaneously. No load shedding was detected.
Figure 5.63 Load distribution along the height of the wall (N)

Figure 5.64 Load distribution along the length of the wall
5.6 THEORETICAL METHODS OF ANALYSIS

5.6.1 Introduction

Three methods have been used to analyse two-way spanning brickwork panels subjected to lateral loading:

(i) Modified finite element analysis based on elastic theory
(ii) Yield line analysis (or British Code BS5628(45))
(iii) Fracture line analysis

A detailed comparison between the theoretical analysis and the test results is given in Table 5.4.

5.6.2 Modified finite element method

The conventional finite element method is one of the most powerful engineering tools in solving complex structures. It has been shown that the finite element method which included the bi-axial bending failure criterion together with the load shedding behaviour agreed very well with the cross-beam test results. Hence, the finite element method which is based on the elastic theory seems to be an obvious choice for an elastic brittle material such as a brickwork panel.

In analysing the flexural behaviour of a brickwork panel, the panel was divided into a small number of elements interconnected at a finite number of nodes. Both the brick and mortar were modelled as homogeneous materials to reduce the number of elements required without any loss of accuracy.
The assembly of the elements with consideration of external loads and boundary conditions results in a system of equations describing the equilibrium of the structure, which has to be solved to obtain the nodal displacements of the structure. From these displacements it is possible to obtain strains and stresses at the integration points. Finally, the bending moments can be obtained from the stresses.

In solving any finite element problem, the accuracy of the theoretical prediction is very much dependent on the grid size of each element and the load increment. Of course using the smaller element and small load increment, more accurate results can be obtained, but it means that more memory and longer computer running time is required. To compromise between the accuracy and efficiency of the theoretical prediction, one way to reduce the element grid size was to use either half or quarter panel of the wall if the support conditions were the same on one or both sides of the wall.

The other way to minimise the amount of memory and the running time of the computer was to avoid excessive numbers of iteration due to small load increments between the cracking and the failure pressures. Wall 14, which had the largest difference between the cracking and failure pressure equal to 0.0107 N/mm² was chosen. Thus, a constant value of 0.0002 N/mm² was used for the theoretical load increment. On the other hand, the load increment had to be kept reasonably small so that any change of stress in each individual element could be captured.

The element stiffness matrix before and after cracking for the brickwork panel is given in Chapter 4 and Section 4.3.2.3.
5.6.3 Yield line analysis

Yield line theory has been proposed by some researchers(42,43,44,61) for the analysis of masonry and seems to be the basis for the design of laterally loaded brickwork panels in the British code of practice, BS5628(45). There is reason to doubt this approach because brickwork is inherently brittle. It cannot carry any moment after cracking whereas the yield line method assumes ductile behaviour of the material. However, the yield line method is simpler to use. It is applicable to slabs of any shape, size, loading and boundary conditions. Hence, it was also used to compare the results of the wall tests.

Yield line theory was originally developed for the under-reinforced concrete slab. It is assumed the elastic deformations are negligible in comparison with plastic deformations, and that the slab elements between the yield lines remain rigid plane sections. Yield lines are the intersections between the plane elements and are also straight lines. Additionally, yield lines must pass through the intersection point of their axes of rotation. Failure of the slab is assumed when all the steel crossing the yield lines reaches its yield stress. The assumed collapse mechanism is defined by a pattern of yield lines. Once the yield line pattern is assumed, the ultimate resistance moment along the yield lines can be calculated based on the flexural strength of the brickwork panel rather than the yield strength of the steel. The failure load can be found by equating with these moments. This is now being applied to masonry panels. Since the bending moment coefficients given in the British Code BS5628(45) are derived from the yield line theory, it will be more straightforward to use the bending moment coefficients given in the code for comparison with the wall test results.
5.6.4 Fracture Line method

The fracture line method is very similar to the yield line method except that stiffness orthotropy is considered in the analysis. Detail of the fracture line analysis is given in Chapter 4, Section 4.3.3.3.

5.7 COMPARISON OF THE EXPERIMENTAL FAILURE PRESSURE WITH THE THEORETICAL PREDICTION

5.7.1 Analysis by British Code BS5628

Table 5.4 shows that in most cases the British Code BS5628(45) overestimates the experimental failure pressures by between 8 and 30%. This difference is understandable as the code is based on the yield line method. It is difficult to imagine how a brittle material like brickwork can resist any moment after cracking. The stiffness orthotropy of the brickwork panel was not considered in the analysis. In the actual wall tests, there is always a preferential location of cracks along the mortar joints. In addition, for panels with similar aspect ratios and boundary conditions, the failure crack pattern can be completely different to one another. This can be seen in Section 5.3.2.

Some walls, (walls 4, 5, 8, 9,12 and 13) failed suddenly with no prior cracking. It was thought that for this type of failure all the assumed yield lines attained the maximum moment simultaneously and thus the results obtained from the yield line method would agree reasonably well. However, the experimental results did not agree very well with the theoretical prediction. This can be seen (Table 5.4) from the experimental results of walls 4, 5, 8, 9, 12 and 13 where the failure pressures
were 11% to 23% lower than those calculated using the British Code\(^{(45)}\). Therefore, the application of the yield line theory to the design of brickwork panels subjected to lateral loading remains questionable.

5.7.2 Analysis by Fracture Line method

Fracture line analysis using both strength and stiffness orthotropy was used and the results are given in Table 5.4. It can be seen that the results obtained from the fracture line method and those obtained from the experiments agree to within 3 to 20% which is a slight improvement compared to the yield line method. The difference is understandable as this method was based on the yield line method.

5.7.3 Analysis by modified finite element method

The finite element method together with the bi-axial bending failure criterion and smeared cracked modelling was used to model the load-shedding behaviour observed in the cross-beam tests. This numerical model was used to predict the theoretical failure loads of walls. As can be seen from the Table 5.4 good agreement was found between the experimental and theoretical results. It can be seen that for walls tested under an ideal boundary condition the overall difference between the experimental and theoretical values lie between 1% to 6%. The difference is small and the reason for this has been discussed in Section 5.4.2. The effect of varying the ratio of \(E_x/E_y\) on the failure loads by the finite element analysis using the failure criterion will be discussed later in the parametric study.
Table 5.4 Comparison of the Theoretical and Experimental failure pressure of the walls

<table>
<thead>
<tr>
<th>Wall no.</th>
<th>Support conditions</th>
<th>Length x height</th>
<th>Experimental failure pressure (kN/m²)</th>
<th>British Code BS 5628 Code Exp.</th>
<th>Fracture Line (FL) Exp.</th>
<th>FL Exp.</th>
<th>FEM Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall 1</td>
<td></td>
<td>1140 x 1140</td>
<td>7.46</td>
<td>9.69</td>
<td>1.30</td>
<td>8.97</td>
<td>1.20</td>
</tr>
<tr>
<td>Wall 2</td>
<td></td>
<td>1150 x 1150</td>
<td>12.20</td>
<td>14.11</td>
<td>1.16</td>
<td>13.50</td>
<td>1.12</td>
</tr>
<tr>
<td>Wall 3</td>
<td>4 sides simply supported</td>
<td>1150 x 1150</td>
<td>12.55</td>
<td>14.11</td>
<td>1.12</td>
<td>13.85</td>
<td>1.11</td>
</tr>
<tr>
<td>Wall 8</td>
<td></td>
<td>790 x 1190</td>
<td>25.0*</td>
<td>29.2</td>
<td>1.17</td>
<td>27.50</td>
<td>1.10</td>
</tr>
<tr>
<td>Wall 12</td>
<td></td>
<td>790 x 1190</td>
<td>31.8*</td>
<td>34.1</td>
<td>1.08</td>
<td>32.10</td>
<td>1.02</td>
</tr>
<tr>
<td>Wall 14</td>
<td></td>
<td>1130 x 755</td>
<td>20.6</td>
<td>23.13</td>
<td>1.12</td>
<td>22.80</td>
<td>1.11</td>
</tr>
<tr>
<td>Wall 15</td>
<td></td>
<td>1130 x 755</td>
<td>18.9</td>
<td>22.33</td>
<td>1.19</td>
<td>22.10</td>
<td>1.16</td>
</tr>
<tr>
<td>Wall 4</td>
<td>3 sides simply supported and top side free</td>
<td>1140 x 1140</td>
<td>8.54*</td>
<td>10.6</td>
<td>1.23</td>
<td>10.10</td>
<td>1.18</td>
</tr>
<tr>
<td>Wall 5</td>
<td></td>
<td>1140 x 1140</td>
<td>8.55*</td>
<td>10.6</td>
<td>1.23</td>
<td>10.10</td>
<td>1.18</td>
</tr>
<tr>
<td>Wall 9</td>
<td></td>
<td>790 x 1190</td>
<td>23.5*</td>
<td>27.9</td>
<td>1.19</td>
<td>26.90</td>
<td>1.15</td>
</tr>
<tr>
<td>Wall 13</td>
<td></td>
<td>790 x 1190</td>
<td>27.8*</td>
<td>30.8</td>
<td>1.11</td>
<td>29.60</td>
<td>1.06</td>
</tr>
<tr>
<td>Wall 6</td>
<td>3 sides simply supported and one side free</td>
<td>1200 x 1200</td>
<td>5.20</td>
<td>5.8</td>
<td>1.12</td>
<td>4.80</td>
<td>0.93</td>
</tr>
<tr>
<td>Wall 7</td>
<td></td>
<td>1200 x 1200</td>
<td>4.51</td>
<td>5.4</td>
<td>1.20</td>
<td>4.70</td>
<td>1.04</td>
</tr>
<tr>
<td>Wall 10</td>
<td></td>
<td>790 x 1190</td>
<td>12.2</td>
<td>13.8</td>
<td>1.14</td>
<td>12.55</td>
<td>1.03</td>
</tr>
<tr>
<td>Wall 11</td>
<td></td>
<td>790 x 1190</td>
<td>11.9</td>
<td>12.8</td>
<td>1.10</td>
<td>11.10</td>
<td>0.93</td>
</tr>
</tbody>
</table>

* wall failed suddenly without prior cracking
5.7.4 Relationship between the theoretical bending moments contour and the actual crack patterns

From the experiments it was clear that the load was shed from the weaker to the stronger direction after cracking. In other words, any redistribution of load in the two orthogonal directions indicated the presence of cracks. Therefore, if one can plot the theoretical contour lines of bending moment before and after cracking, and also at failure, the theoretical crack pattern can be traced by looking at those changes of the bending moment contour lines due to cracking. The theoretical bending moments in the x and y directions of the brickwork panel can be determined from the output stresses from the modified finite element program. Because the mortar joints were much weaker than the brick unit itself, the actual crack lines tend to propagate through the mortar joints (i.e. bed or head joints) rather than the brick itself. Hence, it is difficult to have similar theoretical and experimental crack pattern. Some typical bending moment contour lines for square panels with different boundary conditions are presented in Figures 5.65 to 5.68.

Figures 5.65 and 5.66 show some typical bending moment contour lines for the x and y directions for square panels simply supported on four sides. Since it is a square panel, the bending moment contour for quarter of the panel was plotted. It can be seen that before cracking all the bending moment contour lines in the x and y directions are smooth curves (see Figures 5.65a and 5.66a). However, after cracking (Figures 5.65b and 5.66b) and at failure (Figures 5.65c and 5.66c), the contour line patterns changed. These changes were due to cracks which occurred in the weaker y-direction. Load was shed from the weaker y-direction to the stronger x-direction. One interesting point to note is that the changes were similar to the crack patterns of wall 1 (Figure 5.4) as observed in the wall tests.
Figure 5.67 shows that the bending moment contour lines in the y-direction of a wall panel simply supported on three sides with one vertical edge free. Since the top and bottom of the wall panel are symmetrical, only half of the wall panel was considered. In this case, the maximum bending moment in the y-direction was at the top right corner of the free edge (Figure 5.67a). After cracking, the maximum bending moment moved towards to the left side of the wall panel (Figure 5.67b). This move indicated that the propagation of the crack was from the free edge to the inner side of the wall. At failure, the bending moments in the y-direction along the top and left lower parts of the wall panel had much lower values than at others parts of the wall (Figure 5.67c). Those areas which have lower bending moment values indicated the presence of cracks. The bending moment contour lines pattern was very similar to the actual crack pattern of wall 7 (Figure 5.16).

Figure 5.68 shows the bending moment contour lines in both the x and y directions of a wall panel simply supported on three sides with the top edge free. Only half of the wall panel bending moment contour is plotted due to symmetry. As mentioned earlier for walls with such boundary conditions, cracking and failure occur almost simultaneously, hence, only the bending moment at failure is presented. It can be seen that the maximum bending moment in the x-direction is at the top corner of the free edge (Figure 5.68a), whereas the maximum bending moment in the y-direction is close to the mid-height of the wall panel (Figure 5.68b). For such support conditions, the crack propagation begins from the top free edge and extends vertically downward to the two bottom corners of the wall. The crack pattern is not shown as the program terminates when the bending moment in the x-direction exceeds the ultimate moment of resistance.
a) Before cracking

b) After cracking
c) At failure

Figure 5.65 Bending moment (Nmm) contour in the x-direction for a brickwork panel simply supported on four sides (only quarter of the panel shown)
Figure 5.66 Bending moment (Nmm) contour in the y-direction for a brickwork panel simply supported on four sides (only quarter of the panel shown)
Figure 5.67 Bending moment(Nmm) contour lines in the y-direction of brickwork panel simply supported on three sides with one vertical edge free (only a half of the panel shown)
a) Bending moment in y-direction

b) Bending moment in x-direction

Figure 5.68 Bending moment (Nmm) contour at failure in both x and y directions of brickwork panel simply supported on four sides with the top edge free (only a half of the panel shown)
5.8 COMPARISON BETWEEN THE FAILURE PRESSURES OBTAINED FROM THE MODIFIED FINITE ELEMENT METHOD AND SOME PUBLISHED TEST RESULTS

5.8.1 Basis of choosing published wall test results

In previous sections, it has been shown that there is a good agreement between the failure pressures obtained by the modified finite element method and the experiments carried out for this thesis. It would be helpful if the modified finite element method was able to predict the wall test results of other researchers who have done work on panels with and without openings and with different aspect ratios and boundary conditions, so that the usefulness of the program could be verified. A large number of experimental results are available. However, most of these results cannot be used for comparison due to the lack of parameters, such as the moduli of elasticity - which are very important for the load distribution in two orthogonal directions as discussed in Chapter 4; and the corresponding flexural strength of wallettes for particular walls. Also, the boundary conditions were not defined properly, for example wall ties were used to tie the wall to the supporting frame. All these precluded valid comparison. Therefore, there were only few published test results that were directly useful for comparative analysis.

The most appropriate published results which contained all the relevant information necessary for comparative studies belonged to Lawrence(54), Baker(35,82), Sinha(49), Kheir(37,84) and BCRA(19,41) for single leaf solid walls. For wall with openings the results of Duarte(68) were used.
5.8.2 Comparison of the failure pressure of walls obtained by the modified FEM and results of Lawrence\(^{(54)}\)

Lawrence\(^{(54)}\) carried out some full-scale wall tests with five different types of boundary conditions. The length of wall panel varied from 2.5 to 6.0 m and the height of the wall was from 2.5 to 3.0 m. The aspect ratio ranged between 1.0 to 2.4. The individual orthogonal strength and the ratios of \(E_x/E_y\) were both given for each wall test. The theoretical failure pressure for each wall was obtained from its relevant properties. A detailed comparison between the failure pressures obtained by the theoretical analysis and experimental result is given in Table 5.5. Generally, good agreement was obtained between the theoretical and experimental results. The differences lie between 3 to 13\%. For walls T9 and T18, the difference between the experimental and theoretical pressures was higher because of the high coefficient of variation (CV) in the flexural strength. In addition, the walls were not tested in an ideal condition, hence the rotational restraint at the support and the frictional restraint at the base of the wall due to the self-weight could not be avoided. These two factors are not considered in the modified finite element program which may certainly affect the theoretical prediction and further reduce the difference.
### Table 5.5 Comparison of failure pressure between the modified FEM and wall test results carried out by Lawrence (54)

<table>
<thead>
<tr>
<th>Aspect ratios (length/height) (M)</th>
<th>Wall no.</th>
<th>Support conditions</th>
<th>Flexural strength (N/mm²)</th>
<th>Experimental failure pressure (kN/m²)</th>
<th>FEM result (kN/m²)</th>
<th>FEM Exper.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 (2.5/2.5) T13</td>
<td>Four sides</td>
<td>2.08 0.86</td>
<td>9.1</td>
<td>10.2</td>
<td>1.12</td>
<td></td>
</tr>
<tr>
<td>1.5 (3.75/2.5) T20</td>
<td>Four sides</td>
<td>1.9 0.79</td>
<td>5.2</td>
<td>4.6</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>2.0 (5/2.5) T23</td>
<td>Fixed</td>
<td>2.22 1.21</td>
<td>5.5</td>
<td>5.2</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>2.0 (6/3) T6</td>
<td></td>
<td>1.9 1.54</td>
<td>4.4</td>
<td>4.6</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td>1.0 (2.5/2.5) T12</td>
<td>Four sides</td>
<td>1.93 0.81</td>
<td>8.6</td>
<td>9.1</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>1.5 (3.75/2.5) T18</td>
<td>Four sides</td>
<td>2.06## 0.81</td>
<td>4.9</td>
<td>6.6</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>2.0 (5/2.5) T22</td>
<td>simply</td>
<td>1.98 0.96</td>
<td>4.7</td>
<td>4.2</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>2.0 (6/3) T32</td>
<td>supported</td>
<td>2.33 1.79</td>
<td>3.5</td>
<td>3.8</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>1.0 (2.5/2.5) T15</td>
<td>Three sides</td>
<td>2.09 1.03</td>
<td>7.8</td>
<td>7.6</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>1.5 (3.75/2.5) T17</td>
<td>simply</td>
<td>1.93 0.87</td>
<td>3.4</td>
<td>3.2</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>2.0 (5/2.5) T26</td>
<td>supported with</td>
<td>1.99 1.0</td>
<td>2.7</td>
<td>2.4</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>2.0 (6/3) T36</td>
<td>vertical</td>
<td>1.36 1.23</td>
<td>1.9</td>
<td>1.8</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>2.4 (6/2.5) T28</td>
<td>edge free</td>
<td>2.06 1.5</td>
<td>2.3</td>
<td>2.4</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td>1.0 (2.5/2.5) T14</td>
<td>Two sides</td>
<td>2.06 1.07</td>
<td>11.3</td>
<td>10.8</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>1.5 (3.75/2.5) T19</td>
<td>fixed and</td>
<td>1.76 0.76</td>
<td>4.8</td>
<td>4.4</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>2.0 (5/2.5) T24</td>
<td>top &amp; bottom</td>
<td>2.56 1.31</td>
<td>5.0</td>
<td>5.4</td>
<td>1.08</td>
<td></td>
</tr>
<tr>
<td>2.0 (6/3) T9</td>
<td>simply supported</td>
<td>2.32 1.06#</td>
<td>2.55</td>
<td>3.0</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td>1.5 (3.75/2.5) T21</td>
<td>Two sides</td>
<td>1.87 0.86</td>
<td>3.9</td>
<td>3.8</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>2.0 (5/2.5) T25</td>
<td>fixed and</td>
<td>2.09 1.15</td>
<td>2.6</td>
<td>2.95</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>2.0 (6/3) T35</td>
<td>bottom</td>
<td>1.79 1.36</td>
<td>1.7</td>
<td>1.8</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>2.4 (6/2.5) T29</td>
<td>simply supported</td>
<td>2.12 1.54</td>
<td>2.4</td>
<td>2.6</td>
<td>1.09</td>
<td></td>
</tr>
</tbody>
</table>

Average 1.02

#  C.V. of the flexural strength in y-direction is high (> 0.27)

##  C.V. of the flexural strength in x-direction is high (>0.38)
5.8.3 Comparison of the failure pressure of walls obtained by the modified FEM and results of Baker(35,82)

Baker(35,82) carried out some one-third scale wall tests with three different types of boundary conditions. The height of the wall was maintained at 690 mm and the lengths of wall panel were 690, 1035 and 1380 mm to achieve aspect ratios of 1.0, 1.5 and 2.0. Since shorter beams were used to obtain the flexural strengths in two directions, Baker suggested that the given flexural strengths in both directions should be reduced by 75% (in y-direction) and 80% (in x-direction) respectively. Only an average modulus of elasticity was given. The modulus of elasticity in the y-direction ($E_Y = 21425$ N/mm$^2$) was slightly higher than the modulus of elasticity in the x-direction ($E_X = 21287$ N/mm$^2$). The theoretical failure pressure for each wall was obtained from its relevant properties. A detailed comparison between the failure pressures obtained by the theoretical analysis and the experimental result is given in Table 5.6. Reasonably good agreement was obtained between the theoretical and experimental results. The differences lie between 3 and 18%. Part of the reason could be due to the fact that the walls were tested horizontally, and the pressure was applied upward.
Table 5.6 Comparison of failure pressure between the modified FEM and wall test results carried out by Baker(35,82)

<table>
<thead>
<tr>
<th>Aspect ratios (length/height)</th>
<th>Support conditions</th>
<th>Flexural strength (N/mm²)</th>
<th>Experimental failure pressure (kN/m²)</th>
<th>FEM result (kN/m²)</th>
<th>FEM/Exper.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 686x686</td>
<td>Four sides</td>
<td>Fₓ 1.91 Fᵧ 0.71</td>
<td>12.33</td>
<td>14.5</td>
<td>1.17</td>
</tr>
<tr>
<td>1.5 1029x686</td>
<td>simply supported</td>
<td>Fₓ 2.3 Fᵧ 0.81</td>
<td>5.9</td>
<td>6.9</td>
<td>1.17</td>
</tr>
<tr>
<td>2.0 1372x686</td>
<td>supported</td>
<td>Fₓ 2.55 Fᵧ 0.6</td>
<td>5.13</td>
<td>4.4</td>
<td>0.86</td>
</tr>
<tr>
<td>1.0 686x686</td>
<td>Three sides</td>
<td>Fₓ 2.13 Fᵧ 0.77</td>
<td>7.7</td>
<td>8.8</td>
<td>1.14</td>
</tr>
<tr>
<td>1.5 1029x686</td>
<td>simply supported</td>
<td>Fₓ 1.74 Fᵧ 0.44</td>
<td>3.21</td>
<td>3.4</td>
<td>1.06</td>
</tr>
<tr>
<td>2.0 1372x686</td>
<td>with top edge free</td>
<td>Fₓ 1.78 Fᵧ 0.71</td>
<td>2.74</td>
<td>2.4</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td></td>
<td></td>
<td>1.05</td>
</tr>
</tbody>
</table>

5.8.4 Comparison of the wall test results of Sinha(49) with the modified FEM

Some tests on one-third scale model brickwork panels were carried out by Sinha(49). The parameters of the wall tests included panels with different aspect ratios (ranging from 0.5 to 2.0) and boundary conditions. Both the ratio of \(E_x/E_y\) and the flexural strengths in two orthogonal directions for individual brickwork panels were given. A detailed comparison between the failure pressures obtained by the theoretical analysis and by experimental is given in Table 5.7. The differences lie between 6 and 13%.

155
Table 5.7 Comparison of failure pressure between the modified FEM and wall test results carried out by Sinha\textsuperscript{(49)}

<table>
<thead>
<tr>
<th>Support conditions</th>
<th>Aspect ratios</th>
<th>Flexural strength (N/mm$^2$)</th>
<th>Experimental failure pressure (kN/m$^2$)</th>
<th>FEM result (kN/m$^2$)</th>
<th>FEM/Exper.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L/H</td>
<td>$F_X$ $F_Y$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One side fixed and top &amp; bottom</td>
<td>0.5</td>
<td>1.14 0.413</td>
<td>8.8*</td>
<td>9.4</td>
<td>1.06</td>
</tr>
<tr>
<td>Simply</td>
<td>1.06</td>
<td>1.36 0.43</td>
<td>4.22*</td>
<td>3.67</td>
<td>0.87</td>
</tr>
<tr>
<td>Supported</td>
<td>1.5</td>
<td>1.02 0.39</td>
<td>2.06</td>
<td>2.2</td>
<td>1.06</td>
</tr>
<tr>
<td>Two sides simply supported and Two sides Fixed</td>
<td>2.14</td>
<td>0.95 0.26</td>
<td>1.37</td>
<td>1.2</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.62 0.51</td>
<td>10.7</td>
<td>12.0</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>1.24 0.53</td>
<td>5.35*</td>
<td>5.8</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1.96 0.8</td>
<td>5.95*</td>
<td>6.4</td>
<td>1.08</td>
</tr>
<tr>
<td>Average</td>
<td>1.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* average of two wall tests

5.8.5 Comparison between the failure pressure of walls tested by Duarte\textsuperscript{(68)} and the modified FEM

Duarte\textsuperscript{(68)} carried out tests on walls simply supported on four sides and on three sides. The aspect ratios of the walls varied from 1.0 to 1.5 (L/H). All the test walls were built from half-scale model bricks. A central window opening of 400 x 400mm was provided in each wall. This opening was covered by a piece of plywood sheet representing a closed window. A closed window was used to simulate the most critical situation when the wall would be expected to receive the maximum lateral loading. The applied lateral loading from the plywood board was spread into four corners of the opening instead of line load. A detailed comparison between the failure pressures obtained by the theoretical analysis and by experimental is given in Table 5.8. The differences lie between 8 and 14%. Again
this difference could be due to the fact that the average ratio of $E_x/E_y$ was used in the theoretical analysis. In addition, pressure from the closed window was transferred as point loads which might cause high stress concentration at the four corners of the opening. This high stress concentration may cause the wall to crack or fail much earlier if any slight uneven distribution of applied load occurred at any corner of the opening.

Table 5.8 Comparison of wall failure pressure between the modified FEM and wall test results carried out by Duarte

<table>
<thead>
<tr>
<th>Wall no.</th>
<th>Support conditions</th>
<th>Panel size (m)</th>
<th>Flexural strength (N/mm$^2$)</th>
<th>Experimental failure pressure (kN/m$^2$)</th>
<th>FEM result (kN/m$^2$)</th>
<th>FEM/Exper.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>L x H</td>
<td>$F_x$ $F_y$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wall 1 &amp; 2</td>
<td>4 sides</td>
<td>1.2 x 1.2</td>
<td>2.08 0.82</td>
<td>9.1*</td>
<td>10.2</td>
<td>1.12</td>
</tr>
<tr>
<td>Wall 7 &amp; 8</td>
<td>simply supported</td>
<td>1.8 x 1.2</td>
<td>1.88 0.67</td>
<td>5.9*</td>
<td>6.5</td>
<td>1.10</td>
</tr>
<tr>
<td>Wall 3 &amp; 4</td>
<td>3 sides simply supported</td>
<td>1.2 x 1.2</td>
<td>2.40 0.83</td>
<td>7.4*</td>
<td>6.8</td>
<td>0.92</td>
</tr>
<tr>
<td>Wall 9 &amp; 10</td>
<td>and top side free</td>
<td>1.8 x 1.2</td>
<td>2.26 1.05</td>
<td>3.5*</td>
<td>3.7</td>
<td>1.06</td>
</tr>
<tr>
<td>Wall 5 &amp; 6</td>
<td>3 sides simply supported</td>
<td>1.2 x 1.2</td>
<td>2.08 0.88</td>
<td>7.1*</td>
<td>6.2</td>
<td>0.86</td>
</tr>
<tr>
<td>Wall 11 &amp; 12</td>
<td>and one side free</td>
<td>1.8 x 1.2</td>
<td>1.83 0.68</td>
<td>2.6*</td>
<td>2.8</td>
<td>1.08</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.02</td>
</tr>
</tbody>
</table>

* average of two wall tests

5.8.6 Comparison of the failure pressure of walls obtained by the modified FEM and results of Kheir

Kheir carried out some lateral loading tests on walls built of 1/6 scale brick. The walls were simply supported on either three or four sides. The thickness of the wall was 19mm and the length to height ratios were between 0.5 to 2.0. The
orthogonal strength for each wall was given but the modulus of elasticity was given as an average value. The ratio of $E_x/E_y$ was equal to 1 for Kheir’s panels. A detailed comparison between the failure pressures obtained by the theoretical analysis and by experimental is given in Table 5.9. Reasonably good agreement was obtained between the theoretical and experimental results. The theoretical prediction underestimates the experimental results by 17%. This difference is acceptable as for this type of small scale brickwork, the thickness of the bed-joint may effect the wall strength more significantly. A mercury manometer was used to measure the pressure. This may not be as sensitive as a water manometer.

Table 5.9 Comparison of wall failure pressure between the modified FEM and wall test results carried out by Kheir\(^{(37,84)}\)

<table>
<thead>
<tr>
<th>Wall no.</th>
<th>Support conditions</th>
<th>Panel size (m) $L \times H$</th>
<th>Flexural strength (N/mm(^2)) $F_x$ $F_y$</th>
<th>Experimental failure pressure (kN/m(^2))</th>
<th>FEM result (kN/m(^2))</th>
<th>FEM/Exper.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3 sides simply</td>
<td>190x380</td>
<td>1.3 0.32</td>
<td>8.4</td>
<td>7.2</td>
<td>0.86</td>
</tr>
<tr>
<td>2</td>
<td>supported and</td>
<td>190x380</td>
<td>1.62 0.64</td>
<td>10.0</td>
<td>8.4</td>
<td>0.84</td>
</tr>
<tr>
<td>3</td>
<td>vertical edge free</td>
<td>380x380</td>
<td>1.43 0.52</td>
<td>4.7</td>
<td>4.0</td>
<td>0.85</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>380x380</td>
<td>1.49 0.46</td>
<td>4.6</td>
<td>3.8</td>
<td>0.83</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>760x380</td>
<td>1.3 0.4</td>
<td>2.3</td>
<td>1.9</td>
<td>0.83</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>760x380</td>
<td>1.56 0.5</td>
<td>2.9</td>
<td>2.4</td>
<td>0.83</td>
</tr>
<tr>
<td>7</td>
<td>4 sides</td>
<td>400x200</td>
<td>1.46 0.45</td>
<td>18.0</td>
<td>16.6</td>
<td>0.92</td>
</tr>
<tr>
<td>8</td>
<td>simply supported</td>
<td>400x400</td>
<td>1.35 0.47</td>
<td>8.4</td>
<td>7.0</td>
<td>0.83</td>
</tr>
<tr>
<td>9</td>
<td>vertical edge free</td>
<td>400x800</td>
<td>1.53 0.48</td>
<td>7.0</td>
<td>6.0</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Average 0.85
5.8.7 Comparison of the failure pressure of walls obtained by the modified FEM and results of BCRA(19,41)

The British Ceramic Research Association (BCRA)(19,41) carried out a large number of full-scale tests under lateral loading. The test walls were single leaf construction of different heights, lengths and thickness. They were supported on three sides, the top edge being free; the vertical edges were tied back to a steel frame at every fourth course; the bottom edge was restrained by an angle iron. A bitumen dpc was placed at the top of the first course. These tests simulated the loading and construction of brickwork panels on a steel frame building. As the boundary conditions of the test walls and the ratio of $E_x/E_y$ were not well defined, it was assumed that the panel was simply supported and that the ratio $E_x/E_y$ was 1.0 for the theoretical analysis. A detailed comparison between the failure pressures obtained by the theoretical analysis and by experiment is given in Table 5.10. From the Table 5.10 it can be seen that the theoretical results were 12 to 37% lower than the experimental results. The walls were tied to the steel frame by ties which may provide some degree of fixity in the actual test. This may result in higher failure loads than those obtained by the FEM model.
Table 5.10 Comparison of wall failure pressure between the modified FEM and Wall test results carried out by BCRA(19,41)

<table>
<thead>
<tr>
<th>Wall no.</th>
<th>Thickness (mm)</th>
<th>Panel size (m)</th>
<th>Flexural strength (N/mm²)</th>
<th>Experimental failure pressure (kN/m²)</th>
<th>FEM result (kN/m²)</th>
<th>FEM/Exper.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>L</td>
<td>H</td>
<td>FX</td>
<td>FY</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>102</td>
<td>5.5</td>
<td>2.6</td>
<td>1.19</td>
<td>0.32</td>
<td>3.3</td>
</tr>
<tr>
<td>2</td>
<td>&quot;</td>
<td>5.5</td>
<td>3.6</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.7</td>
</tr>
<tr>
<td>3</td>
<td>&quot;</td>
<td>5.5</td>
<td>1.3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2.8</td>
</tr>
<tr>
<td>4</td>
<td>&quot;</td>
<td>5.5</td>
<td>5.2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1.4</td>
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<td>&quot;</td>
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<td>2.7</td>
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<td>4.4</td>
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<tr>
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<td>3.6</td>
<td>&quot;</td>
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<tr>
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<td>5.2</td>
<td>1.19</td>
<td>0.32</td>
<td>3.4</td>
</tr>
<tr>
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<td>215</td>
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<td>2.6</td>
<td>1.15</td>
<td>0.53</td>
<td>6.6</td>
</tr>
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<td>&quot;</td>
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<td>4.8</td>
</tr>
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<td>19</td>
<td>&quot;</td>
<td>2.7</td>
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<td>3.6</td>
<td>&quot;</td>
<td>&quot;</td>
<td>12.8</td>
</tr>
<tr>
<td>21</td>
<td>215</td>
<td>2.7</td>
<td>4.5</td>
<td>1.15</td>
<td>0.53</td>
<td>11.8</td>
</tr>
<tr>
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<td>327</td>
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<td>2.6</td>
<td>0.71</td>
<td>0.39</td>
<td>12.5</td>
</tr>
<tr>
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<td>3.6</td>
<td>&quot;</td>
<td>&quot;</td>
<td>8.0</td>
</tr>
<tr>
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</tr>
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<td>2.6</td>
<td>&quot;</td>
<td>&quot;</td>
<td>14.5</td>
</tr>
<tr>
<td>26</td>
<td>327</td>
<td>2.7</td>
<td>2.6</td>
<td>0.71</td>
<td>0.39</td>
<td>21.5</td>
</tr>
</tbody>
</table>

Average 0.78
5.8.8 Graphical representation of the theoretical and experimental results

The relationship between the theoretical and experimental failure pressures of walls of the author and others researchers (19, 35, 37, 41, 49, 54, 68, 82, 84) is shown in Figure 5.69. In an ideal situation, all the test results should lie on the line of equality (shows as the darker line). In this investigation, it can be seen that the results analysed by the modified finite element method were very close to the line of equality. This suggests that this numerical model could be the best analytical tool to predict the failure pressure of the unreinforced brickwork panel subjected to lateral loading. It is, therefore, suggested that the design for laterally loaded brickwork panels should be done by this method rather than by the coefficients given in the British Code of Practices BS 5628(45).

![Graphical representation of the theoretical and experimental results](image)

Figure 5.69 Comparison between the experimental and theoretical failure pressures for brickwalls
5.9 PARAMETRIC STUDY

The main purpose of this parametric study was to examine the effect of the following variables:

1. Boundary conditions:
   a. Four sides simply supported,
   b. Three sides simply supported with vertical edge free,
   c. Three sides simply supported with top edge free.

2. Stiffness orthotropy: varied from 0.5, 1.0 (for isotropic case), 1.5 and 2.0.

3. The range of aspect ratio (0.5, 1.0, 1.5 and 2.0).

on the failure pressure of walls.

An orthogonal strength ratio ($\frac{F_x}{F_y}$) of 3 was used for the analysis. The orthogonal strength ratio was defined as the flexural strength parallel to the bed-joint ($F_x$) over the flexural strength normal to the bed-joint ($F_y$). In this case, the value of $F_x$ was considered as 3.0 and the value of $F_y$ as 1.0.

Based on the above parameters, 48 combinations of different parameters were analysed using the modified finite element method. All the results were grouped according to their respective boundary conditions. The failure pressures for the panels with various ratios of $E_x/E_y$ are shown in Figures 5.70 to 5.72. In order to show the contrast between results obtained for the isotropic panel (i.e $E_x/E_y = 1.0$) and orthotropic panel, a darker line was used to highlight results belonging to the isotropic panels.
5.9.1 Panel simply supported on four sides

Figure 5.70 shows the relationship between the failure load and aspect ratios \( \left( \frac{L_x}{L_y} \right) \) for different ratios of \( \frac{E_X}{E_Y} \). The failure loads were significantly affected by the ratio of \( \frac{E_X}{E_Y} \) especially when the aspect ratio is between 1.0 and 1.5. The effect was insignificant when the aspect ratio was between 0.5 or 2.0. The panel behaved as a one-way slab for these two aspect ratios. On the whole the higher the aspect ratio the lower is the failure load.

![Graph showing the relationship between the failure load and the aspect ratios for panels with four sides simply supported.](image)

Figure 5.70 Relationship between the failure load and the aspect ratios for panels with four sides simply supported

5.9.2 Panel simply supported on three sides with top edge free

Figure 5.71 shows that for this type of support condition, the failure load was not significantly affected by the ratio of \( \frac{E_X}{E_Y} \). In this type of panel more load is carried in the x-direction, hence once the load in the x-direction reached its
ultimate moment of resistance, the panel failed immediately. Again the failure load is lower for higher aspect ratio.

Figure 5.71 Relationship between the failure load and the aspect ratios for panel three sides simply supported with top edge free

5.9.3 Panel simply supported on three sides with vertical edge free

Figure 5.72 shows that for this type of panel, the ratio of $E_x/E_y$ exerts great influence on the failure load when the aspect ratio is between 0.5 to 1.5. Beyond this range, the ratio of $E_x/E_y$ has no influence on the load carrying capacity of the panel.
Figure 5.72 Relationship between the failure load and the aspect ratios for panel three sides simply supported with vertical edge free

5.9.4 Summary of the parametric study

From the parametric study it can be concluded that the failure load very much depends on the boundary conditions, the aspect ratio and also the ratio of $E_x/E_y$. The difference between the failure loads for isotropic and anisotropic panels simply supported on three sides with the top edge free and also for panels with an aspect ratio $(L_x/L_y)$ larger than 1.5 is insignificant. In most cases, isotropic wall panels resisted higher failure loads due to the strength orthotropy. Thus, masonry cannot be regarded as an isotropic elastic material. It would be wrong and unsafe to neglect the stiffness orthotropy of the brickwork.
5.10 THE EFFECT OF WALL THICKNESS ON THE FAILURE LOAD

Lovegrove presented the BCRA\(^{(19,41)}\) test results by plotting the aspect ratio against the failure load in the log-log scale as shown in Figure 5.73 for different wall thicknesses. The best fit line has been shown by the lighter lines. This is slightly different from those given by Lovegrove for the best fit parallel lines shown by thicker lines for the various thicknesses of the test walls.

From the plot, Lovegrove\(^{(56)}\) concluded erroneously that the wall strength is proportional to wall thickness to the power 1.4 rather than thickness squared. According to his analysis the wall strength is governed by equation 5.3; i.e.
\[ W(\alpha) = \alpha^k C t_i^{n_i} \] (5.3)

or

\[ \log W(\alpha) = k \log \alpha + \log C + n_i \log t_i \] (5.4)

where:
- \( W \) = total applied load
- \( k \) = gradient of the best fit parallel lines
- \( \alpha \) = aspect ratio
- \( n_i \) & \( C \) = constant values
- \( t_i \) = thickness of wall

Equation (5.4) can be rewritten as equations (5.5) to (5.7) for various thicknesses of walls:

102 mm: \[ \log W(\alpha) = 0.38 \log \alpha + 1.6 + n_1 \log \left( \frac{t_1}{t_i} \right) \] (5.5)

215 mm: \[ \log W(\alpha) = 0.38 \log \alpha + 2.04 + n_2 \log \left( \frac{t_2}{t_i} \right) \] (5.6)

327 mm: \[ \log W(\alpha) = 0.38 \log \alpha + 2.29 + n_3 \log \left( \frac{t_3}{t_i} \right) \] (5.7)

These equations will give three lines parallel to each other, if the best fit thin line is ignored. Considering the equation for the line representing the 102 mm \( (t_1) \) thick wall as a datum from which walls of other thicknesses can be compared, one can see that the distances between the lines representing 102 mm with 215 and 327 mm thick walls are 0.44 and 0.69 respectively. This implies that the values \( n_2 \) and \( n_3 \) equal 1.36 which represents the power of the thickness of the wall. In other words, the ultimate moment of resistance does not increase as rapidly as the expected square law based on section properties. Lovegrove termed this as 'thickness effect'. In the following section, it will be shown that there is no 'thickness effect' and the strength is directly proportional to the thickness squared.
In fact equation (5.3), given by Lovegrove, is only applicable to one-way spanning slabs. He did not consider the load distribution in two directions (i.e. x and y directions). This can be demonstrated by considering a rectangular wall panel simply supported on three or four sides and subjected to uniformly distributed pressure \( w \) as shown in Figure 5.74.

![Diagram of panels simply supported on three or four sides](image)

**Figure 5.74** Panels simply supported on three or four sides

Considering a wall panel simply supported on three or four sides, the applied pressure is shared by the two orthogonal strips (i.e. x and y directions).

\[
wx + wy = w \quad \text{.................................................................(5.8)}
\]

From the compatibility of deflection, we can write

\[
K_1 \frac{wx (L_x)^4}{E_s I_x} = K_2 \frac{wy (L_y)^4}{E_y I_y} \quad \text{..................................................(5.9)}
\]

where \( K_1 \) & \( K_2 \) are deflection coefficients and equal to 1.0 for panel with four sides simply supported.
Re-arranging the equation (5.9), and assuming $E_X$ is equal to $E_Y$ gives

$$w_y = \lambda w_x \left( \frac{L_x}{L_y} \right)^4$$

since $I_X = I_Y$ and where $\lambda = \frac{K_1}{K_2}$ ............................................(5.10)

Substituting $w_y$ from equation (5.10) into (5.8), gives the value of $w_x$ in terms of the applied pressure $w$,

$$w_x = \frac{w}{\left( 1 + \lambda \left( \frac{L_x}{L_y} \right)^4 \right)}$$ ............................................(5.11)

If we consider that failure is governed by the ultimate flexural strength in the x-direction, the applied moment in the x-direction is given by:

$$M_x = \frac{w_x L_x^2}{8} - \frac{w L_x^2}{8} \left[ \frac{1}{1 + \lambda \left( \frac{L_x}{L_y} \right)^4} \right]$$ ............................................(5.12)

or

$$\frac{F_x t^2}{6} = \frac{w L_x^2}{8} \left[ \frac{1}{1 + \lambda \left( \frac{L_x}{L_y} \right)^4} \right], \quad \text{where} \quad M_x = \frac{F_x t^2}{6}$$ ............................................(5.13)

Re-arranging equation (5.13):

$$w = \frac{8 F_x t^2}{6 L_x^2} \left[ 1 + \lambda \left( \frac{1}{\alpha} \right)^4 \right], \quad \text{where} \quad \alpha = \frac{L_y}{L_x}$$ ............................................(5.14)

or

$$w \cdot L_x \cdot L_y = \frac{8 F_x t^2}{6 L_x^2} \left[ 1 + \lambda \left( \frac{1}{\alpha} \right)^4 \right] \cdot L_x \cdot L_y$$ ............................................(5.15)

or
Total load, $W = \frac{8F_xt^2}{6L_x^2} \left[ \alpha + \left[ \frac{\lambda}{\alpha} \right. \right] \left\frac{1}{\alpha^3} \right\right] \tag{5.16}$

Re-arranging the equation (5.16) gives the general term:

$W(\eta) = ct^n\eta^k \tag{5.17}$

where $c = \frac{8F_x}{6}$, $W = \text{Total load}$ and $\eta(\alpha) = \left[ \alpha + \frac{\lambda}{\alpha^3} \right]$

The main difference between equations (5.17) and (5.3) is the term $\eta$ on the right hand side. The failure load is not only dependent on the flexural strength in $x$-direction but the term $\eta$ which is a function of the aspect ratio ($\alpha$) and the distribution factor ($\lambda$). The value of $\lambda$ lies between 0 to 1.0 for an isotropic slab. In the case of a wall panel simply supported on four sides, the deflection coefficients $K_1$ and $K_2$ are the same thus $\lambda$ is equal to one. For panels simply supported on two sides, the $\lambda$ value is equal to zero which is exactly same as Lovegrove's equation. The value of $\lambda$ will lie between the two extremes for a panel simply supported on three sides.

In order to substantiate the above argument, both the test results of BCRA and the theoretical results obtained from the modified finite element program for their walls are presented in Figures 5.75 and 5.76. Since the flexural strength $F_x$ for the 327mm ($t_3$) thick wall is only 0.71N/mm$^2$ compared to 1.19N/mm$^2$ for the 102mm ($t_1$) thick wall, the failure load for 327mm wall has to be increased by a factor of $1.19/0.71$ for the purpose of comparison. It can be seen from Figure 5.75 that the distances between the line representing $t_1$ and the best fit parallel lines for $t_2$ and $t_3$ are 0.60 and 0.99 respectively for the BCRA test results. The values of $n_i$ for $t_2$ and $t_3$ have improved from 1.36 to 1.85 and 1.96 respectively which is
approximately equal to 2.0. In the case of the theoretical results, the distances between the line representing $t_1$ and the best fit parallel lines for $t_2$ and $t_3$ are 0.65 and 1.02 respectively, which means that the values of $n_i$ for $t_2$ and $t_3$ are 2.0 for the two different thicknesses of walls.

$$\log_{10}(W) = 0.31 \log_{10}(\lambda) + 2.41$$

$$\log_{10}(W) = 0.51 \log_{10}(\lambda) + 2.02$$

$$\log_{10}(W) = 0.51 \log_{10}(\lambda) + 1.42$$

where $\eta = (\alpha + \lambda(1/\alpha^2))$

Figure 5.75 Relationship between the failure load and the aspect ratio for different wall thicknesses

$$\log_{10}(W) = 0.525 \log_{10}(\eta) + 2.28$$

$$\log_{10}(W) = 0.525 \log_{10}(\eta) + 1.92$$

$$\log_{10}(W) = 0.525 \log_{10}(\eta) + 1.27$$

where $\eta = (\alpha + \lambda(1/\alpha^2))$

Figure 5.76 Relationship between the failure load and the aspect ratio for different wall thicknesses
The previous study certainly proves that Lovegrove reached a wrong and misleading conclusion by ignoring the effect of the distribution factor and the different flexural strengths in the x-direction for comparison of walls of different thicknesses (see equation 5.14).

To reinforce and prove conclusively that masonry does not exhibit the 'thickness effect', further numerical analyses of wall panels simply supported on three or four sides were studied. The thicknesses of walls considered were 102mm, 215mm and 327mm, i.e very similar to the BCRA tests. The aspect ratios used varied from 0.5 to 2.0.

The wall panels were simply supported on three or four sides

i) Isotropic panel \((E_x/E_y = 1)\) with strength orthotropy \((F_x = 1.19 \text{ N/mm}^2, F_y = 0.32 \text{ N/mm}^2)\)

ii) Orthotropic panel \((E_x/E_y = 1.5)\) panel with strength orthotropy \((F_x = 1.19 \text{ N/mm}^2, F_y = 0.32 \text{ N/mm}^2)\)

Since the investigation is to study the effect of wall thickness on the failure load, it was felt that it would be more useful and informative to present the theoretical results by plotting the failure load against the wall thickness in log-log scale with the aspect ratio kept constant. Equation 5.17 was used. The main advantage of plotting in this manner is that the gradients of the best fit parallel lines give directly the power of the wall thickness i.e the \(n_i\) values. The results of the analysis for the four cases are shown in Figures 5.77 to 5.80.

Figures 5.77 to 5.80 show that the relationships between \(\log_{10}(W)\) and \(\log_{10}(t)\) is linear and are parallel to one another. The gradients for all the lines were very
close to 2.0. These indicate conclusively that the failure pressure is directly proportional to the thickness squared.

Figure 5.77 Relationship between the wall thicknesses and the failure load for various aspect ratios (isotropic panel with three sides simply supported)

Figure 5.78 Relationship between the wall thicknesses and the failure load for various aspect ratios (orthotropic panel with three sides simply supported)
Figure 5.79 Relationship between the wall thicknesses and the failure load for various aspect ratios (isotropic panel with four sides simply supported)

Figure 5.80 Relationship between the wall thicknesses and the failure load for various aspect ratios (orthotropic panel with four sides simply supported)
5.11 CONCLUSIONS

The experimental results clearly show that brickwork cannot be idealised as a fully rigid-plastic material. On the basis of the tests, the following conclusions can be drawn:

1. Brickwork panels possess both strength and stiffness orthotropies. It is incorrect to assume a brickwork panel as an isotropic rigid-plastic material. The load distribution and measured strains in two orthogonal directions clearly indicate that a cracked section cannot support any further moment.

2. The load-deflection relationships of the wall panel is linear up to cracking. After cracking the behaviour becomes non-linear. This non-linear behaviour is not due to material properties but is due to cracking in the weaker y-direction.

3. After cracking, the load is shed from the weaker y-direction to the stronger x-direction until failure of the wall. Failure occurred only when the strength in stronger direction reached its ultimate tensile strength.

4. The British Code BS5628 invariably overestimated the failure pressures of the test walls.

5. The modified finite element program incorporating bi-axial failure criterion predicts the failure pressures reasonably well for the previous wall test results including the work described in this thesis. Hence, this modified finite element program can be used for the design of laterally loaded brickwork panels with confidence.
6. The difference between the failure loads for isotropic and anisotropic panel simply supported on three sides and free on the top and others types of boundary conditions where the aspect ratio ($L_x/L_y$) is larger than 1.5 is insignificant. In most cases, isotropic panels ($E_x/E_y=1$) carry higher failure loads.

7. Masonry walls do not exhibit a 'thickness effect' i.e the ultimate moment of resistance is proportional to the square of the thickness - a well established theoretical fact, rather than to the power of 1.4 as postulated by Lovegrove.
Chapter 6

Conclusions

6.1 SUMMARY AND CONCLUSIONS

The work described in this thesis clearly shows that brickwork cannot be idealised as a fully rigid-plastic material. After cracking, a cracked section is unable to resist any load. Most of the applied load is shed from the weaker y-direction to the stronger x-direction. A bi-axial bending failure criterion was established through the novel cross-beam test. A modified finite element program which takes into account the load-shedding behaviour and the bi-axial bending failure criterion has been developed. This numerical model is able to predict the experimental results of lateral loaded brickwork panels reasonably well. On the basis of the present work, the following detailed conclusions can be drawn:

1. The brickwork wall possesses both strength and stiffness orthotropies. It is not justified to consider brickwork panels as isotropic rigid-plastic material. The load-deflection, stress-strain and the moment-curvature relationships in the wallette tests clearly indicated that a cracked section cannot resist any further moment.

2. The applied lateral load distributes according to the stiffness orthotropy of the brickwork. Hence, the ratio of the modulus of elasticity in two orthogonal directions exerts great influence on the behaviour of the panel.
3. The distributions of load in both the cross-beam and wall tests show very clearly that the applied load is shed from the weaker to the stronger direction after cracking.

4. No significant difference was found between the flexural tensile strengths normal and perpendicular to the bed-joint obtained either from the wallets built independently along-side the test wall or extracted from the undamaged portion of the tested walls, hence it can be concluded that the wallets can reflect the uni-axial strength of the material in the wall. However, in bi-axial bending, the strength in the weaker direction is enhanced beyond the uniaxial strength.

5. The flexural tensile strength perpendicular to the bed-joint is 2.5 to 4.0 times higher than the flexural tensile strength normal to the bed-joint.

6. The ratio of the moduli of elasticity rather than the absolute individual value of modulus of elasticity exerts a significant influence on the bending moments in the bi-axial bending of the brickwork panels.

7. The deflection of a brickwork panel is very much dependent on the individual value of modulus of elasticity rather than the ratio of the modulus of elasticity in x and y directions. The load-deflection relationship of walls is non-linear after cracking. This non-linearity is not due to material properties but is due to the cracking in weaker y-direction (i.e. normal to the bed-joint).

8. Both the Rankine maximum stress theory and Baker's failure criterion are not applicable to brickwork panels subjected to bi-axial bending.
9. The yield line and fracture line both predict very closely the failure load of the crosses where failure occurred in both directions simultaneously, but are not capable of predicting the failure load of other cases, hence they should not be used for design.

10. The difference between the failure loads for isotropic or anisotropic panels simply supported on three sides and free on the top edge and others types of boundary conditions where the aspect ratio \( \frac{L_x}{L_y} \) is larger than 1.5 is insignificant. In most cases, isotropic panels \( \frac{E_x}{E_y}=1 \) give higher failure loads.

11. Masonry walls do not exhibit a 'thickness effect' i.e. the ultimate moment of resistance is directly proportional to the thickness squared.

12. The modified finite element program with smeared crack modelling incorporating the bi-axial bending failure criterion predicted reasonably well the experimental results of the cross-beams and all published test results on walls including the work described in this thesis. It is recommended that this method may be used universally for the design of laterally loaded panels.

6.2 SUGGESTION FOR FURTHER RESEARCH

The numerical model proposed in this thesis proved to be a very successful tool in predicting the failure pressure of laterally loaded brickwork panels with different boundary conditions and aspect ratios. However, some practical problems such as
partial fixity in actual construction is yet to be thoroughly considered. Some suggestions for further works are outlined below:

1. To study the influence of the bi-axial failure criterion with the presence of precompressive forces using a similar cross-beam test.

2. It will be particularly interesting to examine the bi-axial bending failure criterion for isotropic panels.
REFERENCES


181


Appendix A

Table A.1 Flexural Strength Obtained From A Three-Point Loading System

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<th>Wallettes</th>
<th>Wallettes built along side of the cross-beams</th>
</tr>
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<td></td>
<td>F&lt;sub&gt;x&lt;/sub&gt; (N/mm²)</td>
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<tr>
<td>1</td>
<td>3.2</td>
<td>0.96</td>
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<tr>
<td>2</td>
<td>3.87</td>
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<td>6</td>
<td>3.2</td>
<td>0.97</td>
</tr>
<tr>
<td>7</td>
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<td>1.05</td>
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<tr>
<td></td>
<td>Average</td>
<td>3.44</td>
</tr>
<tr>
<td></td>
<td>Ultimate Moment (Nmm/mm)</td>
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<td></td>
<td>Coefficient of Variation For Moment</td>
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Appendix B

B.1 MESH USED IN THE FINITE ELEMENT ANALYSIS

When running any FEM analysis, the element mesh have to be arranged first. Since the centre portion of cross-beam was the most important one, it was divided into smaller elements 121 (11 x 11) elements. The arms are less important, it was divided into 16 elements only, giving the total of 64 elements for the four arms. Hence a total number of 185 elements was used for each cross-beam. The element arrangement for the arms and the central portion of the cross beam are shown in Figure B.1.

![Element Mesh Arrangement For The Finite Element Analysis](image)

Central portion have 121 elements (11 x 11 elements)

Figure B.1 Element Mesh Arrangement For The Finite Element Analysis
Appendix C

C.1 THE FUNCTION OF EACH MAIN SUBROUTINE

The name of the program is called wallrun. The program is subdivided into seven main subroutines. There are INPUT, MODIFY, LOADPB, STIFPB, FRONT, STREPB and COMPARE. Within each subroutine, some of them may divide further into sub-subroutines. Most of the subroutines and sub-subroutines are standard programs which can be found in most general finite element text books\(^{(1)}\). Hence, only the main subroutines are briefly discussed here.

**Briefs notes of the function of each main subroutine**

**INPUT:** geometry of the structure, support conditions, material properties and applied load.

**MODIFY:** the modulus of elasticity of any cracked elements were reduced to zero by using the smeared crack modelling technique.

**LOADPB:** to evaluate the consistent element forces for each element due to the applied uniformly distributed load.

**STIFPB:** to formulate the stiffness matrix for each element and store them prior to their use in the assembly and equation solving routines.

**FRONT:** to assemble the contributions from each element to form the global stiffness matrix and global load vector and to solve the resulting set of simultaneous equations by Gaussian direct elimination.

**STREPB:** to compute the bending moments and others stresses in both the x and y directions.

**COMPARE:** to check whether the output of the bending moments in two orthogonal (x and y ) directions will violate the bi-axial bending failure criterion.
C. 2 OVERALL VIEW OF THE FLOW CHART

CONVENTIONAL
FINITE ELEMENT PROGRAM

MAIN PROGRAM

→ INPUT DATA

→ LOADPB(A)

→ STIFPB

(no cracked element go to MODPB-1
if yes, then go to MODPB-2)

→ FRONT

→ STREPB

COMPARc
Conditions of the output results (Mx and My) violating the failure criterion:
1. If the results are less than Muy or Mux, then go to subroutine LOADPB(A) and only increase the applied load.
2. If the results are more than Muy then go to the subroutine MODIFY(B), the applied load remain the same, only the E-value of that cracked element changed.
3. If the results are more that Mux, then the program will STOP

ADDITIONAL FEATURES

SUBROUTINE PROGRAM

→ NODXY

→ GAUSSQ

→ SRF3

→ JACOB2

→ MODPB-1

→ SRF3

→ JACOB2

→ RMATEPB

→ DBE

MODIFIED PROGRAM

→ From Compare

→ MODIFY(B)

→ LOADPB-2

(go to STIFPB)

→ MODPB-2
Appendix D

D.1  FINITE ELEMENT ANALYSIS OF ORTHOTROPIC BRICKWORK PANEL

D.1.1 Input of Data

The cross-beams and the wall panels were analysed by using a modified finite element programme based on plate bending theory. For any analysis the input data required are as follows:

- dimensions of the panel (mm)
- thickness of the panel (mm)
- modulus of elasticity in vertical and horizontal directions (N/mm²)
- Poisson's ratios in the vertical and horizontal directions (νₓᵧ and νᵧₓ)
- shear modulus (N/mm²)
- uniformly distributed load (N/mm²)
- flexural strength in vertical and horizontal directions (N/mm²)

Since the output of the program is too large it is impossible to show everything, so only the input source file is presented.
### D 1.2 Source file (data.sor)

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Appendix E

LISTING OF THE
FINITE ELEMENT PROGRAM
PROGRAM WALLRUN

C ISOPARAMETRIC FINITE ELEMENT PROGRAM
C FOR PLATE BENDING
C THIS VERSION WALLRUN INCLUDES
C DISTRIBUTED PRESSURES BY MATERIAL TYPE
C LINEAR, QUADRATIC AND CUBIC SERENDIPITY ELEMENTS INCLUDED
C
C JMR ORIGINAL DATES FROM: 24 NOV 1980
C MOST RECENT MODIFICATION: By C.L. Ng 20 FEB 1996
C
C CURRENTLY DIMENSIONED TO 269 ELEMENTS, 997 NODES
C 297 RESTRAINED NODES, FRONTWIDTH OF 150
C WHEN CHANGING FRONTWIDTH, BE SURE TO CHANGE RECORD LENGTH IN OPEN
C
C
C$INSERT SYSCOM>A$KEYS
C$INSERT SYSCOM>KEYS.F
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)

C INTEGER DATNAM(4), ANSNAM(4), GRANAM(4)

C COMMON / CONTRO / NPOIN, NELEM, NNODE, NDOFN, NDIME,
+ NSTRE, NTYPE, NGAUS, NPROP, NMATS, NVFIX, NEVAB,
+ ICASE, NCASE, ITEMP, IPROB, NPROB, LUTERM
C COMMON / LGDATA / COORD(997,2), ASDIS(2991),
C COMMON / LGDATA / COORD(997,2), ASDIS(3991),
+ ELOAD(201,36), LNODS(201,12), MATNO(269),
+ PROPS(31,10),
+ PRESC(297,3), NOFIX(297), IFPRE(297,3)
C
C COMMON / WORK / ELCOD(2,12), SHAPE(12),
+ DERIV(2,12), dmatx(5,5),
+ CARTD(2,12), BMAT(5,36), BMATX(5,36),
+ smatx(201,5,36,9), POSGP(3),
+ WEIGP(3), gpcod(201,2,9), NEROR(24)
C
C 196
COMMON / new / md, mxult, myult, diff, zzz, iudl, udld, ich, fx, fy, thick

COMMON / newdim / xstrs(201,12,2), ystrs(201,12,2),
+ chudl(30,390), CHEM(30,390), maxmy(53), disp(3,3), react(3,3),
+ ex(269), pox(269)

COMMON / NEWELE / IELEM

DATA YES / 'Y' /
DATA YESL / 'y' /

LUTERM = 0

C*****************************************************************
C fcrack=0
C no = 0
C the factor which is being used to increase loading
C diff=1
C*****************************************************************

WRITE (LUTERM, 1005)
1005 FORMAT (// '**************************************************************************/
+ ' PROGRAM WALLRUN '/
+ ' FINITE ELEMENT ANALYSIS OF MINDLIN PLATES',
+ '/ ' USING LINEAR, QUADRATIC OR CUBIC',
+ ' ISOPARAMETRIC ELEMENTS' '/
+ '**************************************************************************/
+ ' AUTHOR: C. L. NG, UNIVERSITY OF EDINBURGH'/
+ ' AFTER AN ORIGINAL BY HINTON AND OWEN'/
+ ' AND J. M. ROTTER'/
+ ' MOST RECENT MODIFICATION :By C.L. Ng 20 FEB 1996'/
+ ' (C) Copyright C.L. NG 1996: All rights reserved'/
+ '**************************************************************************/

WRITE (LUTERM, 1001)
1001 FORMAT (' GIVE THE ROOT NAME FOR YOUR SET OF FILES' /)
READ (LUTERM, 23) ROOTN
23 FORMAT ( A )

WRITE (LUTERM, 1002)
1002 FORMAT (// ' HAVE YOU ALREADY RUN THIS PROBLEM BEFORE,'/
+ ' SO THAT AN OUTPUT FILE ALREADY EXISTS?' // )
READ (LUTERM, 23) ANSFIL

FNAM5 = '
FNAM6 = '
C FIND HOW LONG THE OFFERED ROOT NAME IS AND ADD TAIL
C
LEN = 8
110 ANS = ROOTN(LEN:LEN)
   IF ( ANS .NE. '' ) GO TO 120
   LEN = LEN - 1
   GO TO 110
120 L1 = 1
   L2 = 8
   FNAMS(L1:L2) = ROOTN
   FNAMG(L1:L2) = ROOTN
   LP1 = LEN + 1
   LP4 = LEN + 4
   FNAMS(LP1:LP4) = '.DAT'
   FNAMG(LP1:LP4) = '.OUT'
C
   IF ( ANSFI L .EQ. YES .OR. ANSFIL .EQ. YESL ) GO TO 125
   OPEN (6,FILE=FNAM6,STATUS='unknown')
   GO TO 128

125 OPEN (6,FILE=FNAM6,STATUS='OLD')
C
128 OPEN (5,FILE=FNAM5,STATUS='OLD')
c*   OPEN (7,FILE='TAPE7',STATUS='unknown',
   + ACCESS='SEQUENTIAL',FORM='UNFORMATTED')
   OPEN (8,FILE='TAPE8',STATUS='unknown',
   + ACCESS='DIRECT',FORM='UNFORMATTED',RECL=1216)
C (150 + 1)*8 + (1+1)*4 = 1216
c*   OPEN (9,FILE='TAPE9',STATUS='unknown',
   + ACCESS='SEQUENTIAL',FORM='UNFORMATTED')
   OPEN (11,FILE='TAPE11',STATUS='unknown',
   + ACCESS='DIRECT',FORM='UNFORMATTED',RECL=8)
C 1*8 = 8
C
C   READ (5,900) NPROB
900 FORMAT (15)
C   DO 20 IPROB = 1,nprob

   REWIND 8
   REWIND 11
C
   READ(5,910) (TITLE(IT),IT=1,72)
910 FORMAT (72A1)
   IF ( NPROB .EQ. 1 ) GO TO 5
   WRITE (6,915) IPROB,(TITLE(IT),IT=1,72)
   WRITE (LUTERM,915) IPROB,TITLE
915 FORMAT ( 1X,'PROBLEM NO.',I3,10X,72A1 )
    GO TO 7
5 WRITE (6,910) (TITLE(IT),IT=1,72)
    WRITE (LUTERM,910) (TITLE(IT),IT=1,72)
7 CONTINUE
C
    WRITE (6,1005)
    WRITE (6,1010) FNAM5,FNAM6
    WRITE (LUTERM,1010) FNAM5,FNAM6
1010 FORMAT ( ' THE DATA FILE WAS ',A /
             ' THE OUTPUT FILE WAS ',A )
C
C CALL THE INPUT S/R
C
    CALL INPUT (no,ey,poy,shmod)
    WRITE (LUTERM,1111)
1111 FORMAT ( ' FLEXURAL STRENGTH IN X AND Y DIRECTIONS FX,FY '
             /
             ' )
    READ (LUTERM,*) FX,FY
    MXULT=FX * THICK*THICK/6
    MYULT=FY * THICK*THICK/6
C**************************************************************
    write(luterm,1112)no,zzz,thick,mxult,myult
1112 format('***ll12',2i6,3fl4.6)
C*****************************************************************
C DO 10 ICASE = 1,NCASE
 ICASE = 1
250 no=no+l
    write(6,17)no,zzz,md,ich
17 format('l7* * no,zzz,md,ich',3i7,i15)
251 IF (zzz.GT.0) CALL MODIFY(no,ey,poy,shmod)
C*****************************************************************
C READ LOAD DATA AND DEVELOP NODAL LOADS
C
    CALL LOADPB (no)
 C IF ( ICASE .GT. 1 ) GO TO 40
C
C DEVELOP THE ELEMENT STIFFNESS FILE
C
    CALL STIFPB (no,ey,poy,shmod)
C
C MERGE AND SOLVE THE RESULTING EQUATIONS

C

199
C USING THE FRONTAL SOLVER
C
40 CALL FRONT (no)
C
C EVALUATE STRESSES IN ALL ELEMENTS
C
CALL STREPB(no)
C*******************************************************************
*  
C got0 20
C
CALL COMPARE (no,EY,POY,SHMOD)
C
write(6,14)no,zzz,ich
C 14 format('14**no, zzz,ich',2i7,i15)
IF (ZZZ .lt. 3) goto 250
C THE LOADING, GOTO THE FRONT OF THE WALLRUN PROGRAM
C
20 WRITE (LUTERM, 10 10) FNAMS,FNAM6
C
CLOSE (S,STATUS='KEEP')
CLOSE (6,STATUS='KEEP')
CLOSE (8,STATUS='DELETE')
CLOSE (11 ,STATUS='DELETE')
C
STOP
END
C
C **************************************************
C S/R SIR SIR
C **************************************************
C
SUBROUTINE INPUT (no,ey,poy,shmod)
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
C
dimension ey(269),poy(269),shmod(269)
C
COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM
C
COMMON / LGDATA / COORD(997,2),ASDIS(399 l),
+ ELOAD(201,36),LNODS(201,12),MATNO(269),
+ PROPS(31,10),
+ PRESC(297,3), NOFIX(297), IFPRE(297,3)

COMMON / WORK / ELCOD(2,12), SHAPE(12),
+ DERIV(2,12), DMATX(5,5),
+ CARTD(2,12), DBMAT(5,36), BMATX(5,36),
+ smatx(201,5,36,9), POSGP(3),
+ WEIGP(3), gpcod(201,2,9), NEROR(24)

COMMON / new / md, mxult, myult, diff, zzz, iudl, udlod, ich, fx, fy, thick

COMMON / newdim / xstrs(201,12,2), ystrs(201,12,2),
+ chudl(30,390), CHEM(30,390), maxmy(53), disp(3,3), react(3,3),
+ ex(269), pox(269)

COMMON / NEWELE / IELEM

MAXNOD = 997
MAXEL = 269
MAXFIX = 297
MAXMAT = 31

MAX FRONTWIDTH CURRENTLY SET AT 150

WRITE (LUTERM, 1000)
1000 FORMAT ( ' ENTERING S/R INPUT ' )

C READ THE MASTER DATA CARD

READ (5,900) NPOIN, NELEM, NVFIX, NCASE, NTYPE,
+ NNODE, NDOFN, NMATS, NPROP, NGAUS, NDIME, NSTRE
900 FORMAT (14I5)

NEVAB = NDOFN * NNODE
WRITE (6,905) NPOIN, NELEM, NVFIX, NCASE, NTYPE,
+ NNODE, NDOFN, NMATS, NPROP, NGAUS, NDIME,
+ NSTRE, NEVAB
905 FORMAT ( // ' TOTAL NO. OF NODAL POINTS =' , I4 /
+ ' TOTAL NO. OF ELEMENTS =' , I4 /
+ ' NO. OF RESTRAINED NODES =' , I4 /
+ ' NO. OF LOAD CASES =' , I4 /
+ ' ELEMENT TYPE =' , I4 /
+ ' NO. OF NODES PER ELEMENT =' , I4 /
+ ' DEGS OF FREEDOM PER NODE =' , I4 /
+ ' NO. OF DIFFERENT MATERIALS =' , I4 /
+ ' NO. OF PROPERTIES PER MATL =' , I4 /
+ ' ORDER OF GAUSSIAN INTEGN =' , I4 /
+ 'NO. OF COORD DIMENSIONS  =',I4 /
+ 'NO. OF STRESS RESULTANTS  =',I4 /
+ 'NO. OF IND VARS PER ELEM  =',I4 /
C
IF ( NPOIN .LE. MAXNOD .AND. NELEM .LE. MAXEL .AND.
+ NVFIX .LE. MAXFIX .AND. NMATS .LE. MAXMAT .AND.
+ NDOFN .EQ. 3 ) THEN
  GO TO 130
ELSE
  IF ( NPOIN .GT. MAXNOD ) WRITE (LUTERM,l001) MAXNOD
1001 FORMAT (//' TOO MANY NODES : MAX = ',I5)
  IF ( NELEM .GT. MAXEL ) WRITE (LUTERM,1002) MAXEL
1002 FORMAT ( N ' TOO MANY ELEMENTS : MAX = ',I5)
  IF ( NVFIX .GT. MAXFIX ) WRITE (LUTERM,1003) MAXFIX
1003 FORMAT (// ' TOO MANY RESTRAINTS : MAX = ',I5)
  IF ( NMATS .GT. MAXMAT ) WRITE (LUTERM,1004) MAXMAT
1004 FORMAT (// ' TOO MANY MATERIALS : MAX = ',I5)
  IF ( NDOFN .NE. 3 ) WRITE (LUTERM,1005) NDOFN
1005 FORMAT (// ' YOU ARE PROBABLY RUNNING THE WRONG
    PROGRAM. ',
      + '/ YOU HAVE',I3,' DEGREES OF FREEDOM AT A NODE/
      + ' THERE SHOULD BE 3 '/'
  STOP
ENDIF
C
C ZERO ALL NODAL COORDINATES BEFORE READING SOME OF THEM
C
130 DO 140 IPOIN = 1 ,NPOIN
  DO 140 IDIME = 1 ,NDIME
  COORD(IPOIN,IDIME) = 0.0
140 CONTINUE
C
C READ SELECTED NODAL COORDINATES
C FINISHING WITH THE LAST NUMBERED NODE
C
150 READ (5,930) IPOIN, (COORD(IPOIN,IDIME),IDIME=1,NDIME)
930 FORMAT ( 15,2D15.7)
  IF ( IPOIN.NE. NPOIN ) GO TO 150
C
C READ THE ELEMENT NODAL CONNECTIONS
C THE PROPERTY NUMBERS
C
WRITE (6,910)
910 FORMAT (//' ELEMENT ',14X,'NODE NUMBERS',14X,'PROPERTY')
  DO 160 IELEM = 1,NELEM
  READ (5,900) NUMEL,
    + (LNODS(NUMEL,INODE),INODE=l ,NNODE),MATNO(NUMEL)
160 CONTINUE

WRITE (6,915) NUMEL,
+ (LNODS(NUMEL,INODE),INODE=1,NNODE),MATNO(NUMEL)
160 CONTINUE
915 FORMAT (1X,I4,13I5 )
C
C INTERPOLATE COORDINATES OF MID-SIDE NODES
C
IF ( NNODE .NE. 4 ) CALL NODEXY(no)
WRITE (6,920)
920 FORMAT ( // ' NODAL POINT COORDINATES ' )
WRITE (6,925)
925 FORMAT ( ' NODE',7X,'X',9X,'Y' )
DO 180 IPOIN = 1,NPOIN
180 WRITE (6,935) IPOIN,
+ (COORD(IPOIN,IDIME),IDIME=1,NDIME)
935 FORMAT ( 1X,I5,1P2D15.7 )
C
C READ THE FIXED VALUES
C
WRITE (6,940)
940 FORMAT ( // ' RESTRAINED NODES ' )
WRITE (6,945)
945 FORMAT ( ' NODE CODE FIXED VALUES' )
C
DO 190 IVFIX = 1,NVFIX
READ (5,955) NOFIX(IVFIX),
+ (IFPRE(IVFIX,IDOFN),IDOFN=1,NDOFN),
+ (PRESC(IVFIX,IDOFN),IDOFN=1,NDOFN)
955 FORMAT ( 1X,I4,2X,311,3E15.7 )
WRITE (6,956) NOFIX(IVFIX),
+ (IFPRE(IVFIX,IDOFN),IDOFN=1,NDOFN),
+ (PRESC(IVFIX,IDOFN),IDOFN=1,NDOFN)
956 FORMAT ( 1X,I4,2X,311,1P3D15.7 )
190 CONTINUE
C
C READ THE AVAILABLE SELECTION OF ELEMENT PROPERTIES
C
WRITE (6,960)
960 FORMAT ( // ' MATERIAL PROPERTIES ' )
WRITE (6,965)
965 FORMAT( ' NUMBER THICKNESS YOUNG MOD XX YOUNG
MOD YY',
+ ' POISSON XY POISSON YX SHEAR MOD' )
DO 310 IMATS = 1,NMATS
READ (5,970) NUMAT,
+ (PROPS(NUMAT,IPROP),IPROP=1,NPROP)
970 FORMAT (15,5D15.7 / 5X,5D15.7 )
WRITE (6,975) NUMAT,
+ (PROPS(NUMAT,IPROP),IPROP=1,NPROP)
975 FORMAT (13,2X,1P7D15.7 / 5X,7D15.7)
310 CONTINUE

C*******************************************************************
DO 305 IELEM =1,NELEM
THICK = PROPS(1,1)
EX(ielem) = PROPS(1,2)
EY(IELEM) = PROPS(1,3)
POX(IELEM) = PROPS(1,4)
POY(IELEM) = PROPS(1,5)
SHMOD(IELEM) = PROPS(1,6)

305 CONTINUE
C WRITE (LUTERM,975)
NUMAT,EX(11),EY(11),POX(11),POY(11),SHMOD(11)
C
C SET UP GAUSSIAN INTEGRATION CONSTANTS
C CALL GAUSSQ (no)
RETURN
END

C************************************************************************
C* SIR SIR S/R *
C************************************************************************
subroutine modify (no,Ey,poy,shmod)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
DIMENSION ey(269),poy(269),shmod(269)
COMMON I CONTRO I NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM
C
COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATN0(269),
+ PROPS(31,10),
+ PRESC(297,3),NOFIX(297),IFPRE(297,3)
C
COMMON / WORK / ELCOD(2,12),SHAPE(12),
+ DERIV(2,12),DMATX(5,5),
+ CARTD(2,12),DBMAT(5,36),BMATX(5,36),
C
COMMON / new / md,mxult, myult,diff,zzz,iudl,udlod, ich,fx,fy,thick
C
COMMON / newdim / xstrs(201,12,2),ustrs(201,12,2),
+ chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3),
+ ex(269),pox(269)
C COMMON / NEWELE / IELEM
C READ THE AVAILABLE SELECTION OF ELEMENT PROPERTIES
C
C*******************************************************************
* write(6,701)no,zzz,md,ich
701  format(*701**no,zzz,md,ich',4i15)
   if (md .gt. 0) THEN
      DO 310 MOD=1,MD
      call modd (no,mod,youngy)
      DO 310 CH=1,ICH

      Ey(chem(mod,ch))=0.0

310  CONTINUE
END
C **************************************************
C SIR SIR S/R
C **************************************************
C
SUBROUTINE NODEXY(no)
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
C
COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM
C
COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATNO(269),
C COMMON / WORK / ELCOD(2,12),SHAPE(12),
    + DE:RI V(2,12),DMATX(5,5),
    + CARTD(2,12),DBMAT(5,36),BMATX(5,36),
    + smatx(201,5,36,9),POSGP(3),
    + WEIGP(3),gpcod(201,2,9),NEROR(24)
C
C COMMON / new / md, mxult, myult, diff, xxz, iudl, udlod, ich, fx, fy, thick
C
C COMMON / newdim / xstrs(201,12,2), ystrs(201,12,2),
   + chudl(30,390), CHEM(30,390), maxmy(53), disp(3,3), react(3,3),
   + ex(269), pox(269)
C
C COMMON / NEW ELE / IELEM
C
WRITE (LUTERM, 1000)
1000 FORMAT (' ENTERING S/R NODEXY ')
C
C LOOP OVER ELEMENTS
C
DO 30 IELEM = 1 ,NELEM
C
C LOOP OVER EACH EDGE OF THE ELEMENT
C
ISIDE = NNODE / 4
DO 20 INODE = 1, NNODE, ISIDE
C
C COMPUTE THE NODE NUMBER OF THE FIRST NODE
C
NODST = LNODS(IELEM, INODE)
ILAST = INODE + ISIDE
IF ( ILAST .GT. NNODE ) ILAST = 1
C
C FIND THE LAST NODE ON THIS SIDE
C
NODFN = LNODS(IELEM, ILAST)
C
C DETERMINE THE NODE NUMBERS OF THE MID-SIDE NODES
C
NMID = ISIDE - 1
FAC0 = 1.0D0 / FLOAT(NMID + 1)
DO 20 IMID = 1, NMID
   MIDPT = INODE + IMID
206
NODMD = LNODS(IELEM,MIDPT)
TOTAL = DABS(COORD(NODMD,1)) + DABS(COORD(NODMD,2))

C IF THE COORDINATES OF THE INTERMEDIATE NODE
C ARE BOTH ZERO, INTERPOLATE USING A STRAIGHT LINE
C
IF ( TOTAL .GT. 1.0D-20 ) GO TO 20
FAC = FLOAT(IMID) * FAC0
FACCOM = 1.0D0 - FAC
KOUNT = 1
10 COORD(NODMD,KOUNT) = FACCOM * COORD(NODST,KOUNT) +
+ FAC * COORD(NODFN,KOUNT)
KOUNT = KOUNT + 1
IF ( KOUNT .EQ. 2 ) GO TO 10
20 CONTINUE
30 CONTINUE
RETURN
END

C **************************************************
C S/R S/R S/R
C **************************************************
C SUBROUTINE GAUSSQ (no)
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)

COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM

COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATN0(269),
+ PROPS(31,10),
+ PRESC(297,3),NOFIX(297),IFPRE(297,3)

COMMON / WORK / ELCOD(2,12),SHAPE(12),
+ DERIV(2,12),DMATX(5,5),
+ CARTD(2,12),DBMAT(5,36),BMATX(5,36),
+ smatx(201,5,36,9),POSGP(3),
+ WEIGP(3),gpcod(201,2,9),NEROR(24)

COMMON / new / md,mxult,myult,diff,zzz,iudl,udlod,ich,fx,fy,thick

COMMON / newdim / xstrs(201,12,2),ystrs(201,12,2),
C COMMON /NEWELE/IELEM

WRITE (LUTERM,1000)
1000 FORMAT (' ENTERING S/R GAUSSQ ')
C
NGP = NGAUS + 1
NGS = (NGAUS+1)/2 + 1
C
GO TO (10.20,30,40,50),NGAUS
C
10 POSGP(1) = 0.000000000000000D0
WEIGP(1) = 2.000000000000000D0
GO TO 90
C
20 POSGP(1) = -0.577350269189626D0
WEIGP(1) = 1.0D0
GO TO 80
C
30 POSGP(1) = -0.774596669241483D0
WEIGP(1) = 0.555555555555556D0
POSGP(2) = 0.000000000000000D0
WEIGP(2) = 0.888888888888889D0
GO TO 80
C
40 POSGP(1) = -0.861136311594053D0
WEIGP(1) = 0.347854845137454D0
POSGP(2) = -0.339981043584856D0
WEIGP(2) = 0.652145154862546D0
GO TO 80
C
50 POSGP(1) = -0.906179845938664D0
WEIGP(1) = 0.236926885056189D0
POSGP(2) = -0.538469310105683D0
WEIGP(2) = 0.478628670499366D0
POSGP(3) = 0.000000000000000D0
WEIGP(3) = 0.568888888888889D0
C
80 DO 85 IG = NGS,NGAUS
POSGP(IG) = -POSGP(NGP-IG)
WEIGP(IG) = WEIGP(NGP-IG)
85 CONTINUE
90 CONTINUE
2000 RETURN
SUBROUTINE STIFPB (no,ey,poy,shmod)

CALCULATES ELEMENT STIFFNESS MATRIX
FOR PLATE BENDING ELEMENT

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
dimension ey(269),poy(269),shmod(269)

COMMON / ELSTIF / estif(201,36,36),LOCEL(36),NDEST(36)

COMMON / CONTRO / estif(201,36,36),LOCEL(36),NDEST(36),
+ NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,PROP,NMATS,NVFIX,NEVAB,
+ NCACT,NCASE,ITEMP,IPROB,NPROB,LUTERM

COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELAD(201,36),LNODS(201,12),MATNO(269),
+ PROPS(31,10),
+ PRESC(297,3),NOFIX(297),IFPRE(297,3)

COMMON / WORK / ELCOD(2,12),SHAPE(12),
+ DERIV(2,12),DMATX(5,5),
+ CARTD(2,12),DBMAT(5,36),BMATX(5,36),
+ smatx(201,5,36,9),POSGP(3),
+ WEIGP(3),gpcod(201,2,9),NEROR(24)

COMMON / new / md,mxult,myult,diff,zzz,iudl,udlod,ich,fx,fy,thick

COMMON / newdim / xstrs(201,12,2),ystrs(201,12,2),
+ chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3),
+ ex(269),pox(269)

COMMON / NEWELE / IELEM

WRITE (LUTERM,1000)
1000 FORMAT (' ENTERING S/R STIFPB ')

LOOP OVER EACH ELEMENT
DO 70 IELEM = 1,NELEM
LPROP = MATNO(IELEM)
70 CONTINUE
C
IF (10*((IELEM-1)/10) .EQ. IELEM-1) WRITE (LUTERM,1001) IELEM
1001 FORMAT (' FORMING STIFFNESS MATRIX FOR ELEMENT NO ',15)
C
EVALUATE THE COORDINATES OF THE ELEMENT NODAL POINTS
C
DO 10 INODE = 1,NNODE
  LNODE = LNODS(IELEM,INODE)
  DO 10 IDIME = 1,NDIME
    ELCOD(IDIME,INODE) = COORD(LNODE,IDIME)
10 CONTINUE
C
INITIALISE THE ELEMENT STIFFNESS MATRIX
C
DO 20 IEVAB = 1,NEVAB
  DO 20 JEVAB = 1,NEVAB
    ESTIF(ielem,IEVAB,JEVAB) = 0.0
20 CONTINUE
C
C
C
C
C
C
C
C
C
C
C
C
C
CALCULATE MATRIX OF ELASTIC RIGIDITIES
CALL MODPB(LPROP,IELEM,no,ey,poy,shmod)
KGASP = 0
ENTER LOOPS FOR NUMERICAL INTEGRATION
DO 50 IGAUS = 1,NGAUS
  EXISP = POSGP(IGAUS)
  DO 50 JGAUS = 1,NGAUS
    ETASP = POSGP(JGAUS)
    KGASP = KGASP + 1
50 CONTINUE
EVALUATE THE SHAPE FUNCTIONS
THE ELEMENT AREA
IF ( NNODE .EQ. 4 ) CALL SFR1(EXISP,ETASP,no)
IF ( NNODE .EQ. 8 ) CALL SFR2(EXISP,ETASP,no)
IF ( NNODE .EQ. 12 ) CALL SFR3(EXISP,ETASP,no)
CALL JACOB2(IELEM,DJACB,KGASP,no)
DAREA = DJACB * WEIGP(IGAUS) * WEIGP(JGAUS)
C
EVALUATE THE B AND DB MATRICES
CALL BMATPB (no)
CALL DBE (ielem,no)

C CALCULATE THE ELEMENT STIFFNESSES
C
DO 30 IEVAB = 1,NEVAB
DO 30 JEVAB = IEVAB,NEVAB
DO 30 ISTRE = 1,NSTRE
ESTIF(ielem,IEVAB,JEVAB) = ESTIF(ielem,IEVAB,JEVAB) +
+ BMATX(ISTRE,IEVAB) * DBMAT(ISTRE,JEVAB) * DAREA
C*****************************************************************
30 CONTINUE
C
C STORE THE COMPONENTS OF THE DB MATRIX FOR THE ELEMENT
C
DO 40 ISTRE = 1,NSTRE
DO 40 IEVAB = 1,NEVAB
SMATX(ielem,ISTRE,IEVAB,KGASP) = DBMAT(ISTRE,IEVAB)
C*******************************************************************
* 40 CONTINUE
50 CONTINUE
C
C CONSTRUCT THE LOWER TRIANGLE OF THE STIFFNESS MATRIX
C
DO 60 IEVAB = 1,NEVAB
DO 60 JEVAB = 1,NEVAB
ESTIF(ielem,JEVAB,IEVAB) = ESTIF(ielem,IEVAB,JEVAB)
60 CONTINUE
C
C STORE THE STIFFNESS MATRIX, THE STRESS MATRIX,
C AND THE SAMPLING POINT COORDINATES ON DISC FILE
C
70 CONTINUE
2000 RETURN
END

C ***********************************************************************
C S/R  S/R  S/R
C ***********************************************************************
C
C subroutine MODPB(1,PROP,IFI,F,M,no,ey,poy,shmod)
C
C CALCULATES D MATRIX OF ELASTIC RIGIDITIES
C FOR THE PLATE BENDING ELEMENT
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
dimension ey(269),poy(269),shmod(269)

C
COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,NTEMP,NIPROB,LPROB,LUTERK

C
COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATNO(269),
+ PROPS(31,10),
+PRESC(297,3),NOFIX(297),IPRE(297,3)

C 0
COMMON / WORK / ELCOD(2,12),SHAPE(12),
+DERIV(2,12),DMATX(5,5),
+CARTD(2,12),DBMAT(5,36),BMATX(5,36),
+smatx(201,5,36,9),POSGP(3),
+WEIGP(3),gpcod(201,2,9),NEROR(24)

C

C
COMMON / new / md,mxult,myult,diff,zzz,iudl,udlod,ich,fx,fy,thick

C
COMMON / newdim /xsts(201,12,2),ysts(201,12,2),
+chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3),
+ ex(269),pox(269)

C

C
COMMON / NEWELE / IELEM

C
*----------------------------------------------------------------------
DO 10 ISTRE = 1,NSTRE
DO 10 JSTRE = 1,NSTRE
DMATX(ISTRE,JSTRE) = 0.0
10 CONTINUE

C
THICK = PROPS(1,1)

C
*----------------------------------------------------------------------
IF ( EY(IELEM) .EQ. 0) THEN
POX(IELEM)=PROPS(1,4)
ex(ielem)=props(1,2)

TEMP = THICK**3 / (12.0 * (1.0 - POX(IELEM)) )
DMATX(1,1) = TEMP * EX(IELEM)
POX(IELEM)*P OY(IELEM)) * SHMOD(IELEM) * TEMP
DMATX(1,2) = 0.0
DMATX(2,2) = 0.0
DMATX(2,1) = 0.0

DMATX(3,3) = 0.09*Thick**3*ex(IELEM)/(12*2*(1+pox(ielem)))

C
YOUNG = DSQRT( EX(IELEM) )
POISS = DSQRT( POX(IELEM) )
TEMP = YOUNG * THICK / ( 2.40 * (1.0 + POISS) )
DMATX(4,4) = Ex(ielem)*thick/(2.4*(1+pox(ielem)))

DMATX(5,5) = 0.09*Ex(ielem)*thick/(2.4*(1+pox(ielem)))
GOTO 30
ENDIF

C******************************************************************
TEMP = THICK**3 / (12.0 * (1.0 - POX(IELEM)*POY(IELEM)) )
DMATX(1,1) = TEMP * EX(IELEM)
DMATX(1,2) = POY(IELEM) * DMATX(1,1)
DMATX(2,2) = TEMP * EY(IELEM)
DMATX(2,1) = POX(IELEM) * DMATX(2,2)
DMATX(3,3) = (1.0 - POX(IELEM)*POY(IELEM)) * SHMOD(IELEM)
C
YOUNG = DSQRT( EX(IELEM) * EY(IELEM) )
POISS = DSQRT( POX(IELEM) * POY(IELEM) )
TEMP = YOUNG * THICK / ( 2.40 * (1.0 + POISS) )
DMATX(4,4) = TEMP
DMATX(5,5) = TEMP
C******************************************************************
30 WRITE (6,976) IELEM,EX(IELEM),POX(IELEM),EY(IELEM),POY(IELEM)
976 FORMAT ('976**ielem,ex,pox,ey,poy', 15,4fl2.5)
RETURN
END

C **************************************************
C SIR SIR SIR
C **************************************************
C SUBROUTINE SFR1(S,T,no)
C
C CALCULATES SHAPE FUNCTIONS AND THEIR DERIVATIVES
C FOR TWO DIMENSIONAL LINEAR ELEMENTS
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
C
S/R S/R S/R
C **********************************************
C S/R S/R S/R
C **********************************************
C SUBROUTINE SFR1(S,T,no)
C
C CALCULATES SHAPE FUNCTIONS AND THEIR DERIVATIVES
C FOR TWO DIMENSIONAL LINEAR ELEMENTS
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
C
COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME, 
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB, 
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM 
C 
COMMON / LGDATA / COORD(997,2),ASDIS(3991), 
+ ELOAD(201,36),LNODS(201,12),MATNO(269), 
+ PROPS(31,10), 
+ PRESC(297,3),NOFIX(297),IFPRE(297,3) 
C 
COMMON / WORK / ELCOD(2,12),SHAPE(12), 
+ DERIV(2,12),DMATX(5,5), 
+ CARTD(2,12),DBMAT(5,36),BMATX(5,36), 
+ smatx(201,5,36,9),POSGP(3), 
+ WEIGP(3),gpcod(201,2,9),NEROR(24) 
C 
COMMON / new / md,mxult, myult,diff,zzz,iudl,udlod, ich,fx,fy,thick 
C 
COMMON / newdim /xstrs(201,12,2),ystrs(201,12,2), 
+chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3), 
+ ex(269),pox(269) 
C 
COMMON / NEWELE / IELEM 
C 
S1S = 1.0D0 + S 
T1S = 1.0D0 + T 
S1D = 1.0D0 - S 
T1D = 1.0D0 - T 
C 
C SHAPE FUNCTIONS 
C 
SHAPE(1) = S1D * T1D / 4.0 
SHAPE(2) = S1S * T1D / 4.0 
SHAPE(3) = S1S * T1S / 4.0 
SHAPE(4) = S1D * T1S / 4.0 
C 
C SHAPE FUNCTION DERIVATIVES 
C 
DERIV(1,1) = -T1D / 4.0 
DERIV(1,2) = T1D / 4.0 
DERIV(1,3) = T1S / 4.0 
DERIV(1,4) = -T1S / 4.0 
C 
DERIV(2,1) = -S1D / 4.0 
DERIV(2,2) = -S1S / 4.0 
DERIV(2,3) = S1S / 4.0 
DERIV(2,4) = S1D / 4.0
SUBROUTINE SFR2(S,T,no)

CALCULATES SHAPE FUNCTIONS AND THEIR DERIVATIVES FOR TWO DIMENSIONAL QUADRATIC ELEMENTS

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)

COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM

COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATNO(269),
+ PROPS(31,10),
+ PRESC(297,3),NOFIX(297),IFPRE(297,3)

COMMON / WORK / ELCOD(2,12),SHAPE(12),
+ DERIV(2,12),DMATX(5,5),
+ CARTD(2,12),DBMATX(5,36),BMATX(5,36),
+ smatx(201,5,36,9),POSGP(3),
+ WEIGP(3),gpcod(201,2,9),NEROR(24)

COMMON / new / md,mxult, myult,diff,zzz,iudl,udlod, ich,fx,fy,thick

COMMON / newdim / xstrs(201,12,2),ystrs(201,12,2),
+chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3),
+ ex(269),pox(269)

COMMON / NEWELE / IELEM

S2 = S * 2.0
T2 = T * 2.0
SS = S * S
TT = T * T
ST = S * T
SST = S * S * T
S'T = S * T * T
ST2 = S * T * 2.0

SHAPE FUNCTIONS

SHAPE(1) = (-1.0 + ST + SS + TT - SST - STT) / 4.0
SHAPE(2) = (1.0 - T - SS + SST) / 2.0
SHAPE(3) = (-1.0 - ST + SS + TT - SST + STT) / 4.0
SHAPE(4) = (1.0 + S - TT - STT) / 2.0
SHAPE(5) = (-1.0 + ST + SS + TT + SST + STT) / 4.0
SHAPE(6) = (1.0 + T - SS - SST) / 2.0
SHAPE(7) = (-1.0 - ST + SS + TT + SST - STT) / 4.0
SHAPE(8) = (1.0 - S - TT + STT) / 2.0

SHAPE FUNCTION DERIVATIVES

DERIV(1,1) = (T + S2 - ST2 - TT) / 4.0
DERIV(1,2) = (-S + ST)
DERIV(1,3) = (-T + S2 - ST2 + TT) / 4.0
DERIV(1,4) = (1.0 - TT) / 2.0
DERIV(1,5) = (T + S2 + ST2 + TT) / 4.0
DERIV(1,6) = (-S - ST)
DERIV(1,7) = (-T + S2 + ST2 - TT) / 4.0
DERIV(1,8) = (-1.0 + TT) / 2.0
DERIV(2,1) = (S + T2 - SS - ST2) / 4.0
DERIV(2,2) = (-1.0 + SS) / 2.0
DERIV(2,3) = (-S + T2 - SS + ST2) / 4.0
DERIV(2,4) = (-T - ST)
DERIV(2,5) = (S + T2 + SS + ST2) / 4.0
DERIV(2,6) = (1.0 - SS) / 2.0
DERIV(2,7) = (-S + T2 + SS - ST2) / 4.0
DERIV(2,8) = (-T + ST)

RETURN
END

******************************************************************************************
******************************************************************************************

SUBROUTINE SFR3(S,T,no)

CALCULATES SHAPE FUNCTIONS AND THEIR DERIVATIVES FOR TWO DIMENSIONAL CUBIC ELEMENTS

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)

******************************************************************************************
******************************************************************************************
COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME, 
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB, 
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM
C
COMMON / LGDATA / COORD(997,2),ASDIS(3991), 
+ ELOAD(201,36),LNODS(201,12),MATNO(269), 
+ PROPS(31,10), 
+ PRES(297,3),NOFIX(297),IPFRE(297,3)
C
COMMON / WORK / ELCOD(2,12),SHAPE(12), 
+ DERIV(2,12),DMATX(5,5), 
+ CARTD(2,12),DBMAT(5,36),BMATX(5,36), 
+ smatx(201,5,36,9),POSGP(3), 
+ WEIGP(3),gpecod(201,2,9),NEROR(24)
C
COMMON / new / md,mxult,myult,diff,zzz,iudl,udlod,ich,fx,fy,thick
C
COMMON / newdim /xstrs(201,12,2),ystrs(201,12,2), 
+chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3), 
+ex(269),pox(269)
C
COMMON / NEWELE / IELEM
C
S18 = 18.0D0 * S
T18 = 18.0D0 * T
S1S = 1.0D0 + S
T1S = 1.0D0 + T
S1D = 1.0D0 - S
T1D = 1.0D0 - T
SS1D = S1D * S1S
TT1D = T1D * T1S
S3S = 1.0D0 + 3.0D0*S
T3S = 1.0D0 + 3.0D0*T
S3D = 1.0D0 - 3.0D0*S
T3D = 1.0D0 - 3.0D0*T
Q = 9.0D0 * ( S*S + T*T ) - 10.0D0
SQP = ( 9.0D0*S - 2.0D0 ) * S - 3.0D0
TQP = ( 9.0D0*T - 2.0D0 ) * T - 3.0D0
SQN = ( -9.0D0*S - 2.0D0 ) * S + 3.0D0
TQN = ( -9.0D0*T - 2.0D0 ) * T + 3.0D0
F = 1.0D0 / 32.0D0
F9 = 9.0D0 * F
C
C SHAPE FUNCTIONS
C
SHAPE(1) = F * S1D * T1D * Q

217
SHAPE(2) = $F9 \cdot S3D \cdot T1D \cdot SS1D$
SHAPE(3) = $F9 \cdot S3S \cdot T1D \cdot SS1D$
SHAPE(4) = $F \cdot S1S \cdot T1D \cdot Q$
SHAPE(5) = $F9 \cdot S1S \cdot T3D \cdot TT1D$
SHAPE(6) = $F9 \cdot S1S \cdot T3S \cdot TT1D$
SHAPE(7) = $F \cdot S1S \cdot T1S \cdot Q$
SHAPE(8) = $F9 \cdot S3S \cdot T1S \cdot SS1D$
SHAPE(9) = $F9 \cdot S3D \cdot T1S \cdot SS1D$
SHAPE(10) = $F \cdot S1D \cdot T1S \cdot Q$
SHAPE(11) = $F9 \cdot S1D \cdot T3S \cdot TT1D$
SHAPE(12) = $F9 \cdot S1D \cdot T3D \cdot TT1D$

C
SHAPE FUNCTION DERIVATIVES
C

DERIV(1,1) = $F \cdot T1D \cdot (S18 \cdot S1D - Q)$
DERIV(1,2) = $F9 \cdot T1D \cdot SQP$
DERIV(1,3) = $F9 \cdot T1D \cdot SQN$
DERIV(1,4) = $F \cdot T1D \cdot (S18 \cdot S1S + Q)$
DERIV(1,5) = $F9 \cdot T3D \cdot TT1D$
DERIV(1,6) = $F9 \cdot T3S \cdot TT1D$
DERIV(1,7) = $F \cdot T1S \cdot (S18 \cdot S1S + Q)$
DERIV(1,8) = $F9 \cdot T1S \cdot SQQ$
DERIV(1,9) = $F9 \cdot T1S \cdot SQN$
DERIV(1,10) = $F \cdot T1S \cdot (S18 \cdot S1D - Q)$
DERIV(1,11) = $-F9 \cdot T3S \cdot TT1D$
DERIV(1,12) = $-F9 \cdot T3D \cdot TT1D$

C

DERIV(2,1) = $F \cdot S1D \cdot (T18 \cdot T1D - Q)$
DERIV(2,2) = $-F9 \cdot S3D \cdot SS1D$
DERIV(2,3) = $-F9 \cdot S3S \cdot SS1D$
DERIV(2,4) = $F \cdot S1S \cdot (T18 \cdot T1D - Q)$
DERIV(2,5) = $F9 \cdot S1S \cdot TQP$
DERIV(2,6) = $F9 \cdot S1S \cdot TQN$
DERIV(2,7) = $F \cdot S1S \cdot (T18 \cdot T1S + Q)$
DERIV(2,8) = $F9 \cdot S3S \cdot SS1D$
DERIV(2,9) = $F9 \cdot S3D \cdot SS1D$
DERIV(2,10) = $F \cdot S1D \cdot (T18 \cdot T1S + Q)$
DERIV(2,11) = $F9 \cdot S1D \cdot TQN$
DERIV(2,12) = $F9 \cdot S1D \cdot TQP$

C

2000 RETURN
END

C *************************
C S/R S/R S/R
C *************************
SUBROUTINE JACOB2(IELEM,DJACB,KGASP,no)

CALCULATES COORDINATES OF GAUSS POINTS
JACOBIAN MATRIX AND ITS DETERMINANT
INVERSE OF THE JACOBIAN
FOR TWO DIMENSIONAL ELEMENTS

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)

DIMENSION XJACM(2,2),XJACI(2,2)

COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAR,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM

COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATNO(269),
+ PROPS(31,10),
+ PRESC(297,3),NOFIX(297),IFPRE(297,3)

COMMON / WORK / ELCOD(2,12),SHAPE(12),
+ DERIV(2,12),DMATX(5,5),
+ CARTD(2,12),DBMAT(5,36),BMATX(5,36),
+ smatx(201,5,36,9),POSGP(3),
+ WEIGP(3),gpcod(201,2,9),NEROR(24)

COMMON / new / md,mxult, myult,diff,zzz,iudl,udlod, ich,fx,fy,thick

COMMON / newdim /xstrs(201,12,2),ystrs(201,12,2),
+chudl(30,390),CHIE(30,390),maxmy(53),disp(3,3),rcact(3,3),
+ ex(269),pox(269)

COMMON / NEWELE / IELEM

WRITE (1,1000)
C1000 FORMAT (' ENTERING S/R JACOB2 ')

CALCULATE COORDINATES OF SAMPLING POINT

DO 10 IDIME = 1,NDIME
  GPCOD(ielem,IDIME,KGASP) = 0.0
DO 10 INODE = 1,NNODE
  GPCOD(ielem,IDIME,KGASP) = GPCOD(ielem,IDIME,KGASP) +
  ELCOD(IDIME,INODE) * SHAPE(INODE)
10 CONTINUE
C SET UP JACOBIAN MATRIX
C
DO 20 IDIME = 1,NDIME
DO 20 JDIME = 1,NDIME
XJACM(IDIME,JDIME) = 0.0
DO 20 INODE = 1,NNODE
XJACM(IDIME,JDIME) = XJACM(IDIME,JDIME) +
+ DERIV(IDIME,INODE) * ELCOD(JDIME,INODE)
20 CONTINUE
C
C CALCULATE DETERMINANT AND INVERSE OF JACOBIAN
C
DJACB = XJACM(1,1)*XJACM(2,2) - XJACM(1,2)*XJACM(2,1)
C
IF( DJACB .GT. 0.0 ) GO TO 30
WRITE (6,900) IELEM
900 FORMAT ( // 2X,'PROGRAM HALT IN JACOB2' /
+ 2X,'ZERO OR NEGATIVE AREA IN ELEMENT NUMBER',15 /)
STOP
C
30 XJACI(1,1) = XJACM(2,2) / DJACB
XJACI(2,2) = XJACM(1,1) / DJACB
XJACI(1,2) = -XJACM(1,2) / DJACB
XJACI(2,1) = -XJACM(2,1) / DJACB
C
C CALCULATE CARTESIAN DERIVATIVES
C
DO 40 IDIME = 1,NDIME
DO 40 INODE = 1,NNODE
CARTD(IDIME,INODE) = 0.0
DO 40 JDIME = 1,NDIME
CARTD(IDIME,INODE) = CARTD(IDIME,INODE) +
+ XJACI(IDIME,JDIME) * DERIV(JDIME,INODE)
40 CONTINUE
C
2000 RETURN
END
C **************************************************
C SIR SIR SIR
C **************************************************
C
SUBROUTINE BMATPB (no)
C
C CALCULATES STRAIN MATRIX B
FOR THE PLATE BENDING ELEMENT

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)

COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM

COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATNO(269),
+ PROPS(31,10),
+ PRESC(297,3),NOFIX(297),IFPRE(297,3)

COMMON / WORK / ELCOD(2,12),SHAPE(12),
+ DERIV(2,12),DMATX(5,5),
+ CARTD(2,12),DBMAT(5,36),BMATX(5,36),
+ smatx(201,5,36,9),POSGP(3),
+ WEIGP(3),gpcod(201,2,9),NEROR(24)

COMMON / new / md,mxult, myult,diff,zzz,iudl,udlod, ich,fx,fy,thick

COMMON / newdim / xstrs(201,12,2),ystrs(201,12,2),
+ chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3),
+ ex(269),pox(269)

COMMON / NEWELE / IELEM

WRITE (LUTERM,1000)
C1000 FORMAT ( ' ENTERING S/R BMATPB ' )

DO 10 ISTRE = 1,NSTRE
DO 10 IEVAB = 1,NEVAB
BMATX(ISTRE,IEVAB) = 0.0
10 CONTINUE

JGASH = 0
DO 20 INODE = 1,NNODE
IGASH = JGASH + 1
BMATX(4,IGASH) = CARTD(1,INODE)
BMATX(5,IGASH) = CARTD(2,INODE)
20 CONTINUE

IGASH = IGASH + 1
JGASH = IGASH + 1
BMATX(1,IGASH) = -CARTD(1,INODE)
BMATX(3,IGASH) = -CARTD(2,INODE)
BMATX(4,IGASH) = SHAPE(INODE)
BMATX(2,JGASH) = -CARTD(2,INODE)
BMATX(3,JGASH) = CARTD(1,INODE)
BMATX(5,JGASH) = SHAPE(INODE)

20 CONTINUE
RETURN
END

C **************************************************************
C S/R S/R S/R
C **************************************************************

SUBROUTINE DBE (ielem,no)
C
CALCULATES D X B
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
C
COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM
C
COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATNO(269),
+ PROPS(31,10),
+ PRES(297,3),NOFIX(297),IFPRE(297,3)
C
COMMON / WORK / ELCOD(2,12),SHAPE(12),
+ DERIV(2,12),DMATX(5,5),
+ CARTD(2,12),DBMAT(5,36),BMATX(5,36),
+ smatx(201,5,36,9),POSGP(3),
+ WEIGP(3),gpcod(201,2,9),NEROR(24)
C
C
COMMON / new / md,mxult,myult,iff;zzz,iudl,udlod,ich,fx, fy,thick
C
COMMON / newdim /xstrs(201,12,2),ystrs(201,12,2),
+chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3),
+ex(269),pox(269)
C
COMMON / NEWELE / IELEM
C
C WRITE (LUTERM,1000)
C1000 FORMAT (' ENTERING S/R DBE ')

222
DO 10 ISTRE = 1,NSTRE
DO 10 IEVAB = 1,NEVAB
DBMAT(ISTRE,IEVAB) = 0.0
DO 10 JSTRE = 1,NSTRE
DBMAT(ISTRE,IEVAB) = DBMAT(ISTRE,IEVAB) +
                     DMATX(ISTRE,JSTRE) * BMATX(JSTRE,IEVAB)

10 CONTINUE
2000 RETURN
END

C **************************************************
C SIR SIR SIR
C **************************************************

SUBROUTINE LOADPB (no)
C
C CALCULATE NODAL FORCES FOR THE PLATE ELEMENT
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)

DIMENSION POINT(4),PRESS(4,4),PGASH(4),DGASH(4),
          NOPRS(4)
CHARACTER*1 TITLE(72)

COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
          NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
          ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM

COMMON / LGDATA / COORD(997,2),ASDIS(3991),
          ELOAD(201,36),LNODS(201,12),MATNO(269),
          PROPS(31,10),
          PRESC(297,3),NOFIX(297),IFPRE(297,3)

COMMON / WORK / ELCOD(2,12),SHAPE(12),
          DERIV(2,12),DMATX(5,5),
          CARTD(2,12),DBMAT(5,36),BMATX(5,36),
          smatx(201,5,36,9),POSGP(3),
          WEIGP(3),gpcod(201,2,9),NEROR(24)

COMMON / new / md,mxult,myult,diff,zzz,iudl,udlod,ich,fx,fy,thick

COMMON / newdim /xstrs(201,12,2),ystrs(201,12,2),
          +chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3),
C COMMON / NEWELE / IELEM
C
WRITE (LUTERM,1000)
1000 FORMAT ( ' ENTERING S/R LOADPB ' )
C
DO 10 IELEM = 1,NELEM
DO 10 IEVAB = 1,NEVAB
ELOAD(IELEM,IEVAB) = 0.0
10 CONTINUE
C***********************************************************
IF (NO .gt. 0) GOTO 255
C
READ(5,900) (TITLE(IT),IT=l,72)
900 FORMAT ( 72Al)
WRITE (6,905) ICASE,(TITLE(IT),IT=1,72)
WRITE (LUTERM,905) ICASE,(TITLE(IT),IT= 1,72)
905 FORMAT ( I SX,'LOAD CASE NO.',15 I 5X,72Al)
C
READ NO. OF LOADED NODES AND POINT LOAD DATA
C
C***********************************************************
* 
IF (NO .gt. 0) GOTO 255
READ(5,910) NPLOD
910 FORMAT (215)
WRITE (6,915) NPLOD
WRITE (LUTERM,915) NPLOD
915 FORMAT ( 5X,'NO. OF NODAL POINT LOADS =',I5 )
255 write(6,21)no,zzz,point( 1)
21 format('********testing********',2i6,i8)
C
IF ( NPLOD .EQ. 0 ) GO TO 65
C
DO 60 IPLOD = 1,NPLOD
20 READ (5,920) LODPT,(POINT(IDOFN),IDOFN=1,NDOFN)
C***********************************************************
if ( zzz .LT. 2 ) then
write (LUTERM,19)no,zzz,ielem,point( 1)
19 format(' checking the no.=',3i6,2fl2.4)
point( l)=point( l)-25
WRITE(6,926) NO,ZZZ,POINT( 1)
926 FORMAT ('checking the point load incrememt =',216$9.4 )
C
IF ( NPLOD .EQ. 0 ) GO TO 65
C
DO 60 IPLOD = 1,NPLOD
20 READ (5,920) LODPT,(POINT(IDOFN),IDOFN=1,NDOFN)
endif

920 FORMAT ( 15,3D15.7 )
WRITE(6,925) LODPT,(POINT(IDOFN),IDOFN=l ,NDOFN)
WRITE(LUTERM,925) LODPT,(POINT(IDOFN),IDOFN=l ,NDOFN)
925 FORMAT ( 15,1P3D15.7 )

C ASSOCIATE THE NODAL POINT LOADS WITH AN ELEMENT
C
DO 30 IELEM = 1,NELEM
DO 30 INODE = 1,NNODE
NLOCA = LNODS(IELEM,INODE)
IF ( LODPT .EQ. NLOCA ) GO TO 40
30 CONTINUE

C 40 DO 50 IDOFN = 1,NDOFN
NGASH = (INODE - 1) * NDOFN + IDOFN
ELOAD(IELEM,NGASH) = POINT(IDOFN)
50 CONTINUE
60 CONTINUE

65 continue

C*******************************************************************
*
IF (NO .gt. 0) GOT0 256
READ(5,910) NEDGE
256 IF ( NEDGE .EQ. 0 ) GO TO 300
C
C DISTRIBUTED EDGE LOADS
C
WRITE (LUTERM,975) NEDGE
WRITE (6,975) NEDGE
975 FORMAT ( //5X,'NO OF LOADED EDGES =' ,I4 )
WRITE (6,940)
940 FORMAT ( /5X,'LIST OF LOADED EDGES AND APPLIED LOADS ' )
NODEG = 1 + NNODE / 4
C
C LOOP OVER EACH LOADED EDGE
C
DO 160 IEDGE = 1,NEDGE

C READ DATA LOCATING THE LOADED EDGE
C
READ(5,945) NEASS,(NOPRS(IODEG),IODEG=1,NODEG)
C READ DATA DEFINING THE EDGE LOADING
C
DO 162 IDOFN = 1,NDOFN
READ(5,955) (PRESS(IODEG,IDOFN),IODEG=1,NODEG)
WRITE(6,955) (PRESS(IODEG,IDOFN),IODEG=1,NODEG)
955 FORMAT ( 1P4D15.7 )
162 CONTINUE
ETASP = -1.0D0
C
C CALCULATE THE COORDINATES OF THE NODES OF THE ELEMENT EDGE
C
DO 100 IODEG = 1,NODEG
LNODE = NOPRS(IODEG)
DO 100 IDIME = 1,NDIME
100 ELCOD(IDIME,IODEG) = COORD(LNODE,IDIME)
C
C ENTER LOOP FOR LINEAR NUMERICAL INTEGRATION
C
DO 150 IGAUS = 1,NGAUS
EXISP = POSGP(IGAUS)
C
C EVALUATE THE SHAPE FUNCTIONS AT THE SAMPLING POINTS
C
IF ( NNODE .EQ. 4 ) CALL SFR1(EXISP,ETASP,no)
IF ( NNODE .EQ. 8 ) CALL SFR2(EXISP,ETASP,no)
IF ( NNODE .EQ. 12 ) CALL SFR3(EXISP,ETASP,no)
C
C CALCULATE COMPONENTS OF THE EQUIVALENT NODAL LOADS
C
DO 105 IDOFN = 1,NDOFN
PGASH(IDOFN) = 0.0
DO 105 IODEG = 1,NODEG
105 PGASH(IDOFN) = PGASH(IDOFN) +
PRESS(IODEG,IDOFN) * SHAPE(IODEG)
C
DO 110 IDIME = 1,NDIME
DGASH(IDIME) = 0.0
DO 110 IODEG = 1,NODEG
110 DGASH(IDIME) = DGASH(IDIME) +
+ ELCOD(IDIME,IODEG) * DERIV(1,IODEG)
DVOLU = WEIGP(IGAUS)
RMXCOM = DGASH(1) * PGASH(3) - DGASH(2) * PGASH(2)
RMYCOM = DGASH(1) * PGASH(2) + DGASH(2) * PGASH(3)
PZCOM = PGASH(1) * DSQRT( DGASH(1)**2 + DGASH(2)**2 )

C ASSOCIATE THE EQUIVALENT NODAL EDGE LOADS WITH AN ELEMENT

DO 120 INODE = 1,NNODE
    NLOCA = LNODS(NEASS,INODE)
    IF ( NLOCA .EQ. NOPRS(1) ) GO TO 130
120 CONTINUE

130 JNODE = INODE + NODEG - 1
    KOUNT = 0
    DO 140 KNODE = INODE,JNODE
        KOUNT = KOUNT + 1
        LGASH = ( KNODE - 1 ) * NDOFN + 1
        NGASH = ( KNODE - 1 ) * NDOFN + 2
        MGASH = ( KNODE - 1 ) * NDOFN + 3
        IF ( KNODE .GT. NNODE ) LGASH = 1
        IF ( KNODE .GT. NNODE ) NGASH = 2
        IF ( KNODE .GT. NNODE ) MGASH = 3
        ELOAD(NEASS,LGASH) = ELOAD(NEASS,LGASH) + SHAPE(KOUNT) * PZCOM * DVOLU
        ELOAD(NEASS,NGASH) = ELOAD(NEASS,NGASH) + SHAPE(KOUNT) * RMXCOM * DVOLU
        ELOAD(NEASS,MGASH) = ELOAD(NEASS,MGASH) + SHAPE(KOUNT) * RMYCOM * DVOLU
140 CONTINUE
150 CONTINUE
160 CONTINUE

C UNIFORMLY DISTRIBUTED PRESSURES BY MATERIAL TYPE
if (no.eq.0) then
    READ(5,910) NUDL

    WRITE (6,947) NUDL
    WRITE (LUTERM,947) NUDL
947 FORMAT ( 5X,'NO. OF MATERIAL TYPES CARRYING UNIFORMLY',
       + ' DISTRIBUTED LOADING =',I5 I
       + ' MATL U.D.LOAD ' )
952 FORMAT (I5,D15.9)
954 FORMAT (15,D15.9)
C WRITE (LUTERM,954) MAT,UDLOD
C WRITE (6,954) MAT,UDLOD
C open area, line area, open size area of vertical(vdist) and horizontal distance(hdist)
   openarea=400*400
   linearea=2*40*((400+80)+400)
   openfact=openarea/linearea
   ulineld=(1+openfact)*udlod
   write(6,956)openarea,linearea,openfact
956 format(3f10.3)
   ulinld=udlod
ENDIF
C
   if (zzz .lt. 2) then
   C increase load, but no modification of element
   c   UDLOD=UDLOD*DIFF

   ulineld=ULINELD-(openfact*0.0002)
   udlod=udlod+0.0002
C*******************************************************************
* 927 az=l
1013 write(luterm,1014)no,zzz,frack,udlod
1014 format('1014 no,zzz,frack,applied pressure=',3i4,fl6.9)
   write(6,1015)no,zzz,udlod,mxult,myult
1015 format('1015 no,zzz,udlod,mxult,myult',2i4,f16.9,2f12.6)
endif
C
C LOOP OVER EACH ELEMENT FOR UNIFORMLY DISTRIBUTED
LOADING
C
   DO 380 IELEM = 1,NELEM
C*******************************************************************
*   IF ( MATNO(IELEM) .NE. MAT ) GO TO 380
*   EVALUATE THE COORDINATES OF THE ELEMENT NODAL POINTS
C
   DO 340 INODE = 1,NNODE
   LNODE = LNODS(IELEM,INODE)
   DO 340 IDIME = 1,NDIME
      ELCOD(IDIME,INODE) = COORD(LNODE,IDIME)
340 CONTINUE
C
   THESE STATEMENTS REMOVED FROM ORIGINAL TO ALLOW
   NODAL POINT LOADS AND UD LOADS TO BE USED
   SIMULTANEOUSLY
C
KGASP = 0
C
C ENTER LOOPS FOR NUMERICAL INTEGRATION
C
DO 370 IGAUS = 1,NGAUS
EXISP = POSGP(IGAUS)
DO 370 JGAUS = 1,NGAUS
ETASP = POSGP(JGAUS)
C
C EVALUATE THE SHAPE FUNCTIONS AT THE SAMPLING POINTS
C AND THE ELEMENT AREA
C
KGASP = KGASP + 1
C
IF (NNODE .EQ. 4 ) CALL SFR1(EXISP,ETASP,no)
IF (NNODE .EQ. 8 ) CALL SFR2(EXISP,ETASP,no)
IF (NNODE .EQ. 12 ) CALL SFR3(EXISP,ETASP,no)
call JACOB2(IELEM,DJACB,KGASP,no)
DAREA = DJACB * WEIGP(IGAUS)* WEIGP(JGAUS)
C
C CALCULATE LOADS AND ASSOCIATE WITH ELEMENT NODAL POINTS
DO 360 INODE = 1,NNODE
NPOSN = (INODE - 1) * NDOFN + 1
ELOAD(IELEM,NPOSN) = ELOAD(IELEM,NPOSN) +
+ SHAPE(INODE) * UDLOD * DAREA
360 CONTINUE
370 CONTINUE
380 CONTINUE
C
C PRESSURE LOADS DEFINED AT NODAL POINTS
C*******************************************************************
* IF (NO .gt. 0) goto 257
READ(5,910) NPRNOD
257 IF (NPRNOD .EQ. 0 ) GO TO 490
C*******************************************************************
*
WRITE (6,960) NPRNOD
WRITE (LUTERM,960) NPRNOD
960 FORMAT (5X,'NO. OF NODES WITH NON-ZERO ',I5 /
+ ' PRESSURE LOADING =',I5 /

229
READ THE NODE NUMBERS AND PRESSURES

DO 485 IPRNOD = 1,NPRNOD
READ (5,952) NODPR,PRLOD
WRITE (LUTERM,954) NODPR,PRLOD
WRITE (6,954) NODPR,PRLOD

DO 480 IELEM = 1,NELEM

DO 410 JNODE = 1,NNODE
    LJNODE = LNODS(IELEM,JNODE)
    IF ( NODPR .EQ. LJNODE ) GO TO 420
GO TO 480

DO 440 INODE = 1,NNODE
    LNODE = LNODS(IELEM,INODE)
    DO 440 IDIME = 1,NDIME
        ELCOD(IDIME,INODE) = COORD(LNODE,IDIME)
    CONTINUE

KGASP = 0

DO 470 IGAUS = 1,NGAUS
    EXISP = POSGP(IGAUS)
    DO 470 JGAUS = 1,NGAUS
        ETASP = POSGP(JGAUS)
        KGASP = KGASP + 1
        IF ( NNODE .EQ. 4 ) CALL SFR1(EXISP,ETASP,no)
        IF ( NNODE .EQ. 8 ) CALL SFR2(EXISP,ETASP,no)

CONTINUE
IF ( NNODE .EQ. 12 ) CALL SFR3(EXISP,ETASP,no)
CALL JACOB2(IELEM,DJACB,KGASP,no)
DAREA = DJACB * WEIGP(IGAUS)* WEIGP(JGAUS)

C
C  CALCULATE LOADS AND ASSOCIATE WITH ELEMENT NODAL
POINTS
C
DO 460 INODE = 1,NNODE
NPOSN = (INODE - 1) * NDOFN + 1
ELOAD(IELEM,NPOSN) = ELOAD(IELEM,NPOSN) +
+ SHAPE(INODE) * SHAPE(JNN) * PRLOD * DAREA
460 CONTINUE
470 CONTINUE
480 CONTINUE
485 CONTINUE

C
490 CONTINUE
C
RETURN
END

C **************************************************
C S/R S/R SIR
C **************************************************
C
SUBROUTINE FRONT (no)
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
C
COMMON / ELSTIF / estif(201,36,36),LOCEL(36),NDEST(36)
COMMON / TT1 / EQUAT(150),GLOAD(150),NACVA(150),VECRV(150)
COMMON / TT2 / GSTIF(11325)
COMMON / TT3 / FIXED(3991),IFFIX(3991)
C
INTEGER*4 IPO8S,IPO8D,IPO11S,IPO11D,IPOS
C
COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM
C
COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATNO(269),
+ PROPS(31,10),
+ PRESC(297,3),NOFIX(297),IFPRE(297,3)
C
231
COMMON / WORK / ELCOD(2,12),SHAPE(12),
+ DERIV(2,12),DMATX(5,5),
+ CARTD(2,12),DBMAT(5,36),BMATX(5,36),
+ smatx(201,5,36,9),POSGP(3),
+ WEIGP(3),gpcod(201,2,9),NEROR(24)
C
COMMON / new / md,mxult, myult,diff,zzz,iudl,udlod, ich,fx,fy,thick
C
COMMON / newdim /xstrs(201,12,2),ystrs(201,12,2),
+chudl(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3),
+ ex(269),pox(269)
C
COMMON / NEWELE / IELEM
C
NFUNC(I,J) = ( J*J - J ) / 2 + 1
C
WRITE (LUTERM,1000)
1000 FORMAT ( ' ENTERING S/R FRONT ' )
C
MFRON = 150
MSTIF = NFUNC(MFRON,MFRON)
C
INTRERPRET FIXITY DATA IN VECTOR FORM
C
NTOTV = NPOIN * NDOFN
C*****************************************************************************
C
WRITE (6,7)NTOTV,NPOIN,NDOFN,NVFIX
C 7 FORMAT('**7**',418)
DO 100 ITOTV = 1,NTOTV
IFFIX(ITOTV) = 0
100 FIXED(ITOTV) = 0.0
C
DO 110 IVFIX = 1,NVFIX
NLOCA = ( NOFIX(IVFIX) - 1 ) * NDOFN
DO 110 IDOFN = 1,NDOFN
NGASH = NLOCA + IDOFN
IFFIX(NGASH) = IFPRE(IVFIX,IDOFN)
110 FIXED(NGASH) = PRESC(IVFIX,IDOFN)
C*****************************************************************************
C
CHANGE THE SIGN OF THE LAST APPEARANCE OF EACH NODE
C
DO 140 IPOIN = 1,NPOIN

232
KLAST = 0
DO 130 IELEM = 1,NELEM
DO 120 INODE = 1,NNODE
IF ( LNODS(IELEM,INODE) .NE. IPOIN ) GO TO 120
KLAST = IELEM
NLAST = INODE
120 CONTINUE
130 CONTINUE
IF ( KLAST .NE. 0 ) LNODS(KLAST,NLAST) = -IPOIN
140 CONTINUE
C
C START BY INITIALISING EVERYTHING THAT MATTERS TO ZERO
C
DO 150 ISTIF = 1,MSTIF
150 GSTIF(ISTIF) = 0.0
DO 160 IFRON = 1,MFRON
GLOAD(IFRON) = 0.0
EQUAT(IFRON) = 0.0
VECRV(IFRON) = 0.0
NACVA(IFRON) = 0
160 CONTINUE
C
PIVMAX = -1.0E30
PIVMIN = 1.0E30
C
C PREPARE FOR DISC READING AND WRITING OPERATIONS
C******************************************************************
REWIND 8
REWIND 11
C
C ENTER MAIN ELEMENT ASSEMBLY : REDUCTION LOOP
C
KWR8 = 0
KWR11 = 0
C
NFRON = 0
MAXFRO = 0
KELVA = 0
C******************************************************************
DO 380 IELEM = 1,NELEM
KEVAB = 0
C
IF (20*((IELEM-1)/20) .EQ. IELEM-1) WRITE (LUTERM, 10) IELEM
10 FORMAT ( ' SOLUTION FRONT IS ENTERING ELEMENT NO ',I4 )
C******************************************************************
C
READ(7) ESTIF

C*******************************************************************
C WRITE (6,10)IElem,NEVAB,estif
C 10 FORMAT ('10 ielem,nevab,estif***',I5,3x,i5,3x,lple14.6)
c 10 FORMAT (316)
   DO 170 INODE = 1,NNODE
   DO 170 IDOFN = 1,NDOFN
   NPOSI = (INODE - 1) * NDOFN + IDOFN
   LOCNO = 1.NODS(IELEM,INODE)
   IF ( LOCNO .GT. 0 ) LOCEL(NPOSI) = (LOCNO - 1)*NDOFN + IDOFN
   IF ( LOCNO .LT. 0 ) LOCEL(NPOSI) = (LOCNO + 1)*NDOFN - IDOFN
C*******************************************************************
*
170 CONTINUE
C C START BY LOOKING FOR EXISTING DESTINATIONS
C
   DO 210 IEVAB = 1,NEVAB
      NIKNO = IABS( LOCEL(IEVAB) )
C******************************************************************
c 14 FORMAT (316)
   KEXIS = 0
   DO 180 IFRON = 1,NFRON
C*******************************************************************
   IF ( NIKNO .NE. NACVA(IFRON) ) GO TO 180
   KEVAB = KEVAB + 1
   KEXIS = 1
   NDEST(KEVAB) = IFRON
180 CONTINUE
   IF (KEXIS .NE. 0 ) GO TO 210
C C NOW SEEK NEW EMPTY PLACES FOR THE DESTINATION VECTOR
C
   DO 190 IFRON = 1,MFRON
      IF ( NACVA(IFRON) .NE. 0 ) GO TO 190
      NACVA(IFRON) = NIKNO
      KEVAB = KEVAB + 1
      NDEST(KEVAB) = IFRON
      GO TO 200
190 CONTINUE
C
   WRITE (6,950)
   WRITE (LUTERM,950)
C
C THE NEW PLACES MAY DEMAND AN INCREASE IN CURRENT
FRON WIDTH
C
200 IF ( NDEST(KEVAB) .GT. NFRON ) NFRON = NDEST(KEVAB)
   IF ( NFRON .GT. MAXFRO ) MAXFRO = NFRON
210 CONTINUE
C
C ASSEMBLE ELEMENT LOADS
C
DO 240 IEVAB = 1,NEVAB
   IDEST = NDEST(IEVAB)
   GLOAD(IDEST) = GLOAD(IDEST) + ELOAD(IELEM,IEVAB)

C*******************************************************************
C ASSEMBLE THE ELEMENT STIFFNESSES
C - BUT NOT IN RESOLUTION
C
IF ( ICASE .GT. 1 ) GO TO 230
DO 220 JEVAB = 1,IEVAB
   JDEST = NDEST(JEVAB)
   NGASH = NFUNC(IDEST,JDEST)
   NGISH = NFUNC(JDEST,IDEST)
   IF ( JDEST .GE. IDEST ) GSTIF(NGASH) =
      GSTIF(NGASH) + ESTIF(IELEM,IEVAB,JEVAB)
   IF ( JDEST .LT. IDEST ) GSTIF(NGISH) =
      GSTIF(NGISH) + ESTIF(IELEM,IEVAB,JEVAB)
220 CONTINUE
230 CONTINUE
240 CONTINUE
C
C REEXAMINE EACH ELEMENT NODE
C TO FIND THOSE WHICH CAN BE ELIMINATED
C
DO 370 IEVAB = 1,NEVAB
   NIKNO = -LOCEL(IEVAB)

C*******************************************************************

* IF ( NIKNO .LE. 0 ) GO TO 370
C
C FIND POSITIONS OF VARIABLES READY FOR ELIMINATION
C
DO 350 IFRON = 1,NFRON
IF ( NACVA(IFRON) .NE. NIKNO ) GO TO 350
C
C EXTRACT THE COEFFICIENTS OF THE NEW EQUATION
C FOR ELIMINATION
C
IF ( ICASE .GT. 1 ) GO TO 260
DO 250 JFRON = 1,MFRON
IF ( IFRON .LT. JFRON ) NLOCA = NFUNC(IFRON,JFRON)
IF ( IFRON .GE. JFRON ) NLOCA = NFUNC(JFRON,IFRON)
EQUAT(JFRON) = GSTIF(NLOCA)
C*******************************************************************
250 GSTIF(NLOCA) = 0.0
260 CONTINUE
C
C EXTRACT THE CORRESPONDING RIGHT HAND SIDES
C
EQRHS = GLOAD(IFRON)
GLOAD(IFRON) = 0.0
KELVA = KELVA + 1
C
C WRITE EQUATIONS TO DISC OR TAPE
C
IF ( ICASE .GT. 1 ) GO TO 270
C
KWR8 = KWR8 + 1
C
269 WRITE (8,REC=KWR8) EQUAT,EQRHS,IFRON,NIKNO
GO TO 280
C
270 KWR11 = KWR11 + 1
C
275 WRITE (11,REC=KWR11) EQRHS
READ(8) EQUAT,DUMMY,IDUMM,NIKNO
C
280 CONTINUE
C
C DEAL WITH PIVOT
C
PIVOT = EQUAT(IFRON)
EQUAT(IFRON) = 0.0
C
C ENQUIRE WHETHER PRESENT VARIABLE IS FREE OR PRESCRIBED
C*******************************************************************
IF ( IFIX(NIKNO) .EQ. 0 ) GO TO 300

236
C DEAL WITH A PRESCRIBED DEFLECTION
C
DO 290 JFRON = 1, NFRON
290 GLOAD(JFRON) = GLOAD(JFRON) - FIXED(NIKNO) * EQUAT(JFRON)
C*******************************************************************************
GO TO 340
C
C ELIMINATE A FREE VARIABLE
C DEAL WITH THE RIGHT HAND SIDE FIRST
C
300 IF ( DABS(PIVOT) .LT. 1.0D-80 ) GO TO 580
C 300 dummy=1
DO 330 JFRON = 1, NFRON
C***************************************************************************
GLOAD(JFRON) = GLOAD(JFRON) - EQUAT(JFRON) * EQRHS / PIVOT
C***************************************************************************
C DEAL WITH THE COEFFICIENTS IN CORE
C
IF ( ICASE .GT. 1 ) GO TO 320
IF ( EQUAT(JFRON) .EQ. 0.0 ) GO TO 330
NLOCA = NFUNC(0, JFRON)
DO 310 LFRON = 1, JFRON
NGASH = LFRON + NLOCA
C***************************************************************************
310 GSTIF(NGASH) = GSTIF(NGASH) - EQUAT(JFRON) * EQUAT(LFRON) / PIVOT
320 CONTINUE
330 CONTINUE
340 EQUAT(JFRON) = PIVOT
C
C RECORD THE NEW VACANT SPACE
C AND REDUCE FRONTWIDTH IF POSSIBLE
C
NACVA(JFRON) = 0
GO TO 360
C
C COMPLETE THE ELEMENT LOOP IN THE FORWARD ELIMINATION
C
350 CONTINUE
360 IF ( NACVA(NFRON) .NE. 0 ) GO TO 370
NFRON = NFRON - 1
IF ( NFRON .GT. 0 ) GO TO 360
370 CONTINUE
380 CONTINUE
C
C ENTER BACKSUBSTITUTION PHASE
C
C LOOP BACKWARDS THROUGH VARIABLES
C
WRITE (LUTERM,1016)
1016 FORMAT ( //" BACKSUBSTITUTE "/ )
C
IF ( ICASE .EQ. 1) MKWR8 = KWR8
KWR8 = MKWR8
WRITE (LUTERM,1017) KWR8,KWRll
1017 FORMAT (' MAX RECORDS STORED ARE KWR8 =',I4,' KWR11 =',I4 )
C
DO 410 IELVA = 1 ,KELVA
C
C READ A NEW EQUATION
C
C*****************************************************************
READ(8,REC=KWR8) EQUAT,EQRHS,IFRON,NIKNO
C
IF ( ICASE .EQ. 1) GO TO 390
C
C******************************************************************
READ (11 ,REC=KWR11) EQRHS
KWRll = KWRll - 1
C
390 CONTINUE
C
C PREPARE TO BACKSUBSTITUTE FROM THE CURRENT EQUATION
C
PIVOT = EQUAT(IFRON)
C
IF ( PIVOT .LT. 0.0 ) WRITE (LUTERM,1018)
IF ( PIVOT .LT. 0.0 ) WRITE (6,1018)
1018 FORMAT ( ' ***** NEGATIVE PIVOT **** ',
+ ' EXAMINE YOUR DATA CAREFULLY ' )
KWR8 = KWR8 - 1
IF ( PIVMAX .LT. DABS(PIVOT) ) PIVMAX = DABS(PIVOT)
IF ( PIVMIN .GT. DABS(PIVOT) ) PIVMIN = DABS(PIVOT)
C
IF ( IFFIX(NIKNO) .EQ. 1) VECRV(IFRON) = FIXED(NIKNO)
IF ( IFFIX(NIKNO) .EQ. 0 ) EQUAT(IFRON) = 0.0
C
C BACKSUBSTITUTE IN THE CURRENT EQUATION
DO 400 JFRON = 1,MFRON
400 EQRHS = EQRHS - VECRV(JFRON) * EQUAT(JFRON)

C PUT THE FINAL VALUES WHERE THEY BELONG
C
C******************************************************************************
IF ( IFFIX(NIKNO) .EQ. 0 ) VECRV(IFRON) = EQRHS / PIVOT
409 IF ( IFFIX(NIKNO) .EQ. 1 ) FIXED(NIKNO) = -EQRHS
    ASDIS(NIKNO) = VECRV(IFRON)
CONTINUE
C
WRITE (6,895) MAXFRO,PIVMAX,PIVMIN
WRITE (LUTERM,895) MAXFRO,PIVMAX,PIVMIN
895 FORMAT (/// ' MAXIMUM FRONT WIDTH USED =',I4 //
          + ' MAXIMUM PIVOT ENCOUNTERED =',1PE10.3 /
          + ' MINIMUM PIVOT ENCOUNTERED =',1PE10.3 //)
C
WRITE (6,900)
900 FORMAT (/// 5X,'DISPLACEMENTS' )
C
430 WRITE (6,915)
915 FORMAT ( /3X,'NODE',5X,'LATERAL ',7X,'THETA X',7X,'THETA Y' /
          + 4X,'NO.',4X,'DEFLECTION',6X,'ROTATION',6X,'ROTATION' /)
C
DO 450 IPOIN = 1,NPOIN
    NGASH = IPOIN * NDOFN
    NGISH = NGASH - NDOFN + 1
C******************************************************************************
    WRITE (6,920) IPOIN,(ASDIS(IGASH),IGASH=NGISH,NGASH)
920  FORMAT ( 15,3X,lP3E14.6 )
450 CONTINUE
C
WRITE (6,925)
925 FORMAT (/// 5X,'REACTIONS: ',
          + 'FORCES AND MOMENTS AT RESTRAINED NODES' /)
470 WRITE (6,940)
940 FORMAT ( 3X,'NODE',2X,'VERTICAL FORCE',3X,'MOMENT MX',
          + 5X,'MOMENT MY' )
C
DO 510 IPOIN = 1,NPOIN
    NLOCA = (IPOIN - 1) * NDOFN
    DO 490 IDOFN = 1,NDOFN
        NGUSH = NLOCA + IDOFN
        IF ( IFFIX(NGUSH) .GT. 0 ) GO TO 500
        NGUSH = NLOCA + IDOFN
    CONTINUE
DO 510
510 CONTINUE
490  CONTINUE
   GO TO 510
C
500  NGASH = NLOCA + NDOFN
    NGISH = NLOCA + 1
C**********************************************************************************************
C*** KEEP ALL THE REACTION AND MOMENT IN BIG ARRAY
C**********************************************************************************************

   WRITE (6,945) IPOIN,(FIXED(IGASH),IGASH=NGISH,NGASH)
510 CONTINUE
945 FORMAT ( 15,3X,I3P3E14.6 )
C
C POST FRONT
C RESET ALL ELEMENT CONNECTION NUMBERS TO POSITIVE VALUES FOR
C SUBSEQUENT USE IN STRESS CALCULATION
C
   DO 520 IELEM = 1,NELEM
      DO 520 INODE = 1,NNODE
   520 LNODS(IELEM,INODE) = IABS( LNODS(IELEM,INODE) )
C
   RETURN
C
C SINGULAR MATRIX
C
580 WRITE (6,590) PIVOT,IELEM,IEVAB,IFRON
   WRITE (LUTERM,590) PIVOT,IELEM,IEVAB,IFRON
590 FORMAT (// ' $$*************$$*$*******1//' + ' ZERO PIVOT ENCOUNTERED IN EQUATION SYSTEM'/' + ' Calculated pivot = ',1PD16.9 // + ' YOUR EQUATIONS ARE SINGULAR'/' + ' Check your data for inadequate restraints'/' + ' zero elastic properties'/' + ' zero sized elements'/' + ' problem too large'//' + ' ********************************1/' + ' Program stopped at element no.',I4,' variable no.',I3,' ifron =',I3) STOP
C
END
C
C **************************************************
C SIR S/R SIR
240
**SUBROUTINE STREPB(no)**

**CALCULATES STRESS RESULTANTS AT GAUSS POINTS**

**FOR THE PLATE BENDING ELEMENT**

**IMPLICIT DOUBLE PRECISION (A-H,O-Z)**

**IMPLICIT INTEGER*4 (I-N)**

**DIMENSION ELDIS(3,12),STRSG(5),STRSP(3)**

**COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,**

**+ NSTRE,NTYPE,NGAUS,NPROP,NEZ,NEVAB,**

**+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM**

**COMMON / LGDATA / COORD(997,2),ASDIS(3991),**

**+ ELOAD(201,36),LNODS(201,12),MATNO(269),**

**+ PROPS(31,10),**

**+ PRES(297,3),NOFIX(297),IFPRE(297,3)**

**COMMON / WORK / ELCOD(2,12),SHAPE(12),**

**+ DERIV(2,12),DMATX(5,5),**

**+ CARTD(2,12),DBMAT(5,36),BMATX(5,36),**

**+ smatx(201,5,36,9),POSGP(3),**

**+ WEIGP(3),gpcod(201,2,9),NEROR(24)**

**COMMON / new / md,mxult,myult,diff,zzz,iudl,udlod, ich,fx, fy, thick**

**COMMON / newdim / xstrs(201,12,2),ystrs(201,12,2),**

**+ chudl(30,390),CIIEM(30,390),maxmy(53),disp(3,3),rcact(3,3),**

**+ ex(269),pox(269)**

**COMMON / NEWELE / IELEM**

**WRITE (LUTERM,1000)**

**1000 FORMAT ( ' ENTERING S/R STREPB ' )**

**WRITE(6,900)**

**LOOP OVER EACH ELEMENT**

**DO 70 IELEM = 1,NELEM**

**IF ( 1+(IELEM/20)*20 .EQ. IELEM ) WRITE(6,905)**

**READ THE STRESS MATRIX AND SAMPLING POINT COORDINATE**
C*******************************************************************
C READ(9) SMATX,GPCOD
WRITE (6,9 10) IELEM
C
C IDENTIFY THE DISPLACEMENTS OF THE ELEMENT NODAL POINTS
C
DO 10 INODE = 1,NNODE
  LNODE = LNODS(IELEM,INODE)
  NPOSN = (LNODE - 1) * NDOFN
  DO 10 IDOFN = 1,NDOFN
    NPOSN = NPOSN + 1
    ELDIS(IDOFN,INODE) = ASDIS(NPOSN)
10  CONTINUE
C
KGASP = 0

C ENTER LOOPS OVER EACH SAMPLING POINT
C
DO 60 IGAUS = l,NGAUS
  DO 60 JGAUS = l,NGAUS
    KGASP = KGASP + 1
    DO 20 ISTRE = l,NSTRE
      STRSG(ISTRE) = 0.0
      KGASH = 0

C COMPUTE THE STRESS RESULTANTS
C
DO 20 INODE = 1,NNODE
  DO 20 IDOFN = l,NDOFN
    KGASH = KGASH + 1
    STRSG(ISTRE) = STRSG(ISTRE) +
    + SMATX(ielem,ISTRE,KGASH,KGASP) * ELDIS(IDOFN,INODE)
20  CONTINUE
C
C COMPUTE THE PRINCIPAL STRESSES
C
40 XGASH = ( STRSG(1) + STRSG(2) ) * 0.5
XGISII = ( STRSG(1) - STRSG(2) ) * 0.5
XGESH = STRSG(3)
XGOSH = DSQRT( XGISII**2 + XGESH**2 )
STRSP(1) = XGASH + XGOSH
STRSP(2) = XGASH - XGOSH
IF ( XGISII .EQ. 0.0 ) XGISII = 1.0D-20
STRSP(3) = DATAN2( XGISH , XGISII ) * 28.647889757D0
C OUTPUT THE STRESS RESULTANTS
C
if (zzz .lt. 2) then
  WRITE (6,915) KGASP,(GPCOD(ielem,IDIME,KGASP),IDIME=1,NDIME),
  + (STRSG(ISTRE),ISTRE=1,NSTRE),
  + (STRSP(ISTRE1),ISTRE1=1,3)
endif
if (zzz .eq. 3) then
  WRITE (6,915) KGASP,(GPCOD(ielem,IDIME,KGASP),IDIME=1,NDIME),
  + (STRSG(ISTRE),ISTRE=1,NSTRE),
  + (STRSP(ISTRE1),ISTRE1=1,3)
endif
C******************************************************************
C*** ASSEMBLY ALL THE MX AND MY INTO A BIG ARRAY
59 XSTRS(IELEM,KGasp,1)=STRSG(1)
    YSTRS(IELEM,KGasp,2)=STRSG(2)
    IF (MAXMY(NO) .LT. STRSG(2)) MAXMY(NO) = STRSG(2)
60 CONTINUE
70 CONTINUE
900 FORMAT (/// 10X,'STRESS RESULTANTS: MOMENTS AND SHEARS',
      + ' PER UNIT WIDTH OF PLATE' /)
905 FORMAT ( /'GAUSS',2X,'COORDINATES',7X,
      + 'BENDING',4X,'BENDING',3X,'TWISTING',3X,
      + 'VERTICAL',3X,'VERTICAL',,
      + '4X','MOST +VE','3X','MOST -VE','2X','ORIENTATION'/
      + 'POINT',4X,'X',7X,'Y',7X,
      + 'MOMENT',5X,'MOMENT',5X,'MOMENT',2X,
      + 'SHEAR FORCE',1X,'SHEAR FORCE',
      + '1X','PRINC MOM',2X,'PRINC MOM',1X,'ANTICLOCK ANGLE'/
      + 'NO',',2X','COORD',',2X','COORD',',5X,
      + 'MXX',8X,'MYY',8X,'MXY',8X,
      + 'VXZ',8X,'VYZ',,
      + '9X','MUU',7X,'MVV',5X,'OF U FROM X' )
910 FORMAT (/ 5X,'ELEMENT NO',15 )
915 FORMAT ( I3,2F10.4,1P8D11.3 )

return
end
C*******************************************************************
C COMPARE MY AND MX WITH MYult AND MXult respectively
SUBROUTINE COMPARE (no,EY,POY,SHMOD)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
IMPLICIT INTEGER*4 (I-N)
dimension ey(269),poy(269),shmod(269)

C DIMENSION ELDIS(3,12),STRSG(5),STRSP(3)
C COMMON / CONTRO / NPOIN,NELEM,NNODE,NDOFN,NDIME,
+ NSTRE,NTYPE,NGAUS,NPROP,NMATS,NVFIX,NEVAB,
+ ICASE,NCASE,ITEMP,IPROB,NPROB,LUTERM
C COMMON / LGDATA / COORD(997,2),ASDIS(3991),
+ ELOAD(201,36),LNODS(201,12),MATNO(269),
+ PROPS(31,10),
+ PRESC(297,3),NOFIX(297),IFPRE(297,3)
C COMMON / WORK / ELCOD(2,12),SHAPE(12),
+ DERIV(2,12),DMATX(5,5),
+ CARTD(2,12),DBMAT(5,36),BMA TX(5,5),
+ smatx(201,5,36,9),POSGP(3),
+ WEIGP(3),gpcod(201,2,9),NEROR(24)
C COMMON / new / md,mxult, myult,diff,zzz,iudl,udlod,  ich,fx, fy,thick
C COMMON / newdim /xstrs(201,12,2),ystrs(201,12,2),
+chud1(30,390),CHEM(30,390),maxmy(53),disp(3,3),react(3,3),
+ ex(269),pox(269)
C COMMON / NEWELE / IELEM

mod =0
iudl=0
zz=0
zzz =0
ich=0

C**************************************************************
write(6,9991)udlod
write(LUTERM,9991)udlod
9991 format('applied pressure (udlod) =',f16.9)
DO 10 IELEM =1,NELEM
DO 10 KG =1,9
C***********************************************************
xx=dabs(xSTRS(IELEM,KG,1))/mxult
yy=dabs(YSTRS(IELEM,KG,2))/myult
if (xSTRS(IELEM,KG,1) .lt. 0 ) then
ctime=-(xx*xx)+(yy*yy)-(0.25*xx*yy)-(0.75*xx*yy*yy))
goto 801
endif

244
\[
\text{crite} = (xx*xx) + (yy*yy) - (0.25*xx*yy) - (0.75*xx*yy*yy)
\]

801 IF (crite .GT. 1) THEN
    IF (yy .LT. 1) THEN
        IF (xSTRS(IELEM,KG,1).GT.mxult) THEN
            ICH = ICH + 1
            IUDL = IUDL + 1
            md = md + 1
            CHUDL(md,IUDL) = UDL
            CHEM(md,ICH) = IELEM
            ZZZ = 3
            fcrack = 2
        endif
    endif
ENDIF
endif

CLOSE (5,STATUS='KEEP')
CLOSE (6,STATUS='KEEP')
CLOSE (8,STATUS='DELETE')
CLOSE (11,STATUS='DELETE')

stop
endif
ENDIF
endif

C RECORD THE FIRST CRACKING LOAD, (ONCE ONLY) PREPARE FOR THE LAST PRINT
C #COMPARE THE MOMENT AND THE ULTIMATE MOMENT
C CHECK THE MOMENT AND THE ULTIMATE MOMENT
IF (ZZ .EQ. 0) THEN
    IF (MOD .EQ. 0) THEN
        IF (crite .GT. 1) THEN
            MD = MD + 1
            MOD = 1
            ICH = ICH + 1
            IUDL = IUDL + 1
            CHUDL(md,iudl) = UDL
            CHEM(MD,ICH) = IELEM
            zzz = 2
            fcrack = 1
        endif
    endif
    goto 108
celse IF (crite .GT. 1) THEN
        ICH = ICH + 1
        IUDL = IUDL + 1
        ZZ = 1
C FIRST PRINT NO(FPN)=NO

245
CHUDL(md, iudl) = UDL
CHEM(MD, ICH) = IELEM
ZZZ = 2
fcrack = 0
endif
GOTO 108
else IF (crit .GT. 1 ) THEN
ICH = ICH + 1
IUDL = IUDL + 1
CHUDL(md, IUDL) = UDL
CHEM(MD, ICH) = IELEM
ZZZ = 2
fcrack = 0
endif

C*******************************************************************
108 i = 1
write(6, 1014) no, ielem, zzz, ich, iudl, md, mod, zz
1014 format(* * * * * 10 14 no, ielem, zzz, ich, iudl, md, mod, zz', 8i6)

10 CONTINUE

900 FORMAT (//// 10X, 'STRESS RESULTANTS: MOMENTS AND SHEARS',
+ ' PER UNIT WIDTH OF PLATE' /)
905 FORMAT (/'GAUSS', 2X, 'COORDINATES', 7X,
+ 'BENDING', 4X, 'BENDING', 3X, 'TWISTING', 3X,
+ 'VERTICAL', 3X, 'VERTICAL',
+ 4X, 'MOST +VE', 3X, 'MOST -VE', 2X, 'ORIENTATION'/
+ 'POINT', 4X, 'X', 7X, 'Y', 7X,
+ 'MOMENT', 5X, 'MOMENT', 5X, 'MOMENT', 2X,
+ 'SHEAR FORCE', 1X, 'SHEAR FORCE',
+ 1X, 'PRINC MOM', 2X, 'PRINC MOM', 1X, 'ANTICLOCK ANGLE'/
+ 'NO.', 2X, 'COORD.', 2X, 'COORD.', 5X,
+ 'MXX', 8X, 'MYY', 8X, 'MXY', 8X,
+ 'VXZ', 8X, 'VYZ',
+ 9X, 'MUU', 7X, 'MVV', 5X, 'OF U FROM X' )
910 FORMAT (/ 5X, 'ELEMENT NO', I5 )
915 FORMAT (I3, 2F10.4, 1P8D11.3)
RETURN
END

C HERE ENDS PROGRAM WALLRUN

246