THE SCATTERING AND ABSORPTION
OF HETEROGENEOUS X-RADIATION

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INTRODUCTION.

When a substance is traversed by a homogeneous X-radiation it is observed that the substance itself becomes a source of secondary radiations of two distinct types, one being a corpuscular radiation and the other an electromagnetic radiation similar in character to the original primary. If the primary radiation is not homogeneous similar secondary radiations are still observed, but in this case the effects are complicated by the fact that more than one wavelength is present in the primary beam. The secondary X-radiation is not in general of the same quality as the primary exciting it, but can be divided into two distinct parts. The first of these is a radiation characteristic of the element traversed by the primary beam, and, except for extremely slight variations observed with different chemical combinations of the element, dependent on no other factor. This is the 'characteristic' or 'fluorescent' radiation of the element the conditions of excitation of which by a given primary radiation are well known.

In the region of medium X-ray wavelengths, the other part of the secondary radiation never differs greatly in quality from the primary radiation exciting it, but depends, in varying degrees, on the scattering substance itself, on the wavelengths present in the primary beam, and on the direction of propagation of the secondary beam relative to the primary. It is with this radiation that we propose to deal, and which we may refer to as the X-radiation 'scattered' by the substance.

The scattered radiation can itself be analysed into two parts, one having a wavelength and penetrating power identical with those of
the original primary radiation, the other having a slightly longer wavelength and a less penetrating power. This latter is the so-called 'modified' scattered radiation of Compton; it is doubtful whether the modified radiation should be regarded as a truly 'scattered' radiation or not, since the process of its production appears distinct from that of the 'unmodified' radiation, however it has become customary to include both the modified and the unmodified radiations under the heading of scattered radiation.

The scattering of X-rays receives a simple, and for many purposes satisfactory, explanation from the classical electromagnetic theory, according to which the X-rays are to be regarded as electromagnetic waves propagated with the velocity of light. On passing through matter, X-rays should set into forced oscillation, therefore, the electrons over which they move, and these electrons by virtue of their accelerations must themselves radiate energy as an electromagnetic radiation having precisely the same form and quality as the original primary X-rays. This is the classical picture of the process of scattering of X-rays, the scattering units being the electrons. Using this idea, J. J. Thomson\(^2\) obtained the intensity scattering function for a single free electron in the form

\[
I_\phi = I_0 \left( \frac{e^2}{2 m^2 r^2 c^4} \right) (1 + \cos^2 \phi) \tag{1}
\]

in which \(I_\phi\) gives the intensity of the scattered radiation proceeding in a direction making an angle \(\phi\) with the direction of the primary radiation, at a distance \('r'\) from the scattering electron. The mass of the electron is \('m'\), and its charge \('e'\) in e.s.u. The primary radiation is assumed to be unpolarised, to have an intensity \(I_0\), and to be propagated with a velocity \('c'\). If there are \('n'\) electrons independently effective in scattering, then the total intensity of the scattered radiation is simply \('n'\) times that from a single electron,
i.e., $nI\phi$. This is valid only if the wavelengths in the primary radiation are small compared with the distances between the electrons effective in scattering. If the wavelengths are large compared with the inter-electronic distances, the phase differences between the waves scattered by the individual electrons become small, when superposition of these waves occurs. Ultimately, when the wavelengths are very great compared with the distances between the electrons, these electrons scatter as a group, and the total intensity scattered by 'n' electrons becomes in this case proportional to $n^2$ instead of proportional to 'n'.

Most of the early experiments on X-ray scattering yielded results which confirmed the principles underlying the simple classical theory. For instance, according to the theory, the radiation scattered in a direction at 90° to the direction of the primary beam should be completely polarised; further, the primary beam itself should show a partial polarisation in a particular direction. Both of these predictions have been verified experimentally. The observed angular distribution of intensity also was, for soft radiations, in close agreement with equation (1), except for small values of $\phi$ where $I\phi$ became greater than the formula predicts; even this deviation could be explained qualitatively by considering the superposition of the secondary waves scattered from the various electrons, at small angles the phase differences between these waves would be small thus giving rise to an 'excess scattering' for small values of $\phi$. Again, the first experiments to give a correct estimate of the number of extra-nuclear electrons in light atoms were based on Thomson's formula. It is important, however, to observe that in these early experiments the substances used to scatter the radiation consisted of light elements only, while the primary beam itself consisted of
radiations of medium or long wavelengths — longer than about 0.3 A. With shorter wavelengths deviations from the simple scattering formula were observed which later were to prove of great importance.

The most significant feature of Thomson's formula is that it is obtained without assuming any particular form for the primary electromagnetic radiation, which may therefore consist of a regular wave-train or an irregular pulse. Thus the expression for \( I \) is independent of the wavelength and of the degree of homogeneity of the primary radiation. Returning to equation (1) and considering a particular angle of scattering, say 90°, this means that

\[
\frac{I_{90}}{I_0} = \text{constant} \tag{2}
\]

for any particular scattering substance provided the wavelengths present in the primary beam are short compared with the distances between the scattering electrons. A second important consequence of this process of scattering is that the primary and scattered radiation must be of precisely the same quality, where by quality is meant any property of the radiations (such as ionising power, or absorbability in any given element) depending on the distribution of energy among the different wavelengths.

Both of these conclusions can be compared directly with experiment; the former, as expressed by equation (2), is found to be approximately true so long as the primary radiation used consists of medium wavelengths and is scattered from substances containing only elements of low atomic number. For short wavelengths, less than about 0.1 A, the experiments of Read and Lauritsen\(^7\) with hard X-rays and those of Chao\(^8\) with \(\gamma\)-rays have shown that the intensity of scattering becomes less than would be expected on the electromagnetic theory, while if heavy elements are used as scatterers the superposition of the waves scattered by different electrons becomes
appreciable and 'group scattering' occurs. This latter receives a satisfactory explanation on the classical theory as has already been indicated, but the decrease of scattered intensity with decreasing wavelength is not explained by any application of classical ideas.

The quality of the scattered radiation may be compared with that of the primary by a variety of methods, the simplest probably being a comparison of the relative absorbabilities of the two beams in some given element. The earliest experiments of this kind showed that the penetrating power of the scattered radiation was in general less than that of the primary; i.e., the qualities of the two beams were not precisely the same. This result, though at variance with the classical theory of scattering, is confirmed by almost all the observations designed to test the point. There are however some experiments, using ionisation-absorption methods of measurement, and generally heterogeneous primary radiations, which yield results that may be interpreted as showing no difference in quality between the primary and scattered radiations; these will be considered in detail later.

In 1922 the first spectroscopic investigations of the secondary X-rays from light elements were made by A. H. Compton. According to the classical theory of scattering by electrons the scattered rays must be of the same frequency as the forced oscillations of the electrons which emit them, and hence will have the same frequency as the primary waves which set the electrons in motion. Compton however, using a homogeneous primary radiation, observed that the rays scattered in any given direction were not homogeneous but consisted in general of two radiations, one having the same wavelength as the primary and the other being slightly displaced towards the longer wavelengths by an amount depending on the angle of scattering. This observation at once explains the greater absorbability of the scattered radiation
which is usually found.

Assuming a scattering process entirely different from that of the electromagnetic theory, Compton was able satisfactorily to account for the observed change of wavelength of the scattered radiation. Compton regarded the radiation from the quantum point of view and envisaged the scattering process as a collision between two elastic bodies, the electron and the photon, conservation laws being obeyed. With these assumptions the well known formula for wavelength change can be simply deduced,

$$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \phi)$$  \hspace{1cm} (3)

or in terms of frequency,

$$\nu/\nu' = 1 + \frac{\hbar}{mc^2} (1 - \cos \phi)$$  \hspace{1cm} (4)

In these expressions, $\lambda$ is the wavelength and $\nu$ the frequency of the primary radiation, scattered at an angle $\phi$, the wavelength and frequency of this scattered radiation being $\lambda'$ and $\nu'$ respectively. $\hbar$ is Planck's constant and the other symbols have their usual significance. E. Schrödinger has shown how the results expressed by equations (3) and (4) may be deduced by a simple application of de Broglie's wave-mechanics.

It can easily be seen that the scattering process assumed by Compton must give an intensity scattering function different from equation (1). Thus, in the collision between the photon and the scattering electron some of the initial energy $h\nu$ of the photon is transferred to the electron, while the photon itself proceeds in the direction $\phi$ with an energy $h\nu'$, $\nu'$ being given by equation (4). Now the total intensity of the radiation scattered in any given direction is merely a measure of the energy of the total number of photons proceeding in that direction, and therefore depends on $\nu'$ as well as on the number. Hence the intensity distribution varies with the
angle $\phi$ in a way different from that expressed by equation (1); and, further, the intensity of the radiation scattered in any given direction will on this theory depend on the wavelength of the primary radiation. Actually it can be shown that, due to the motion of the electron during the collision process, the ratio of the scattering by the recoiling electron to that by an electron at rest is given by

$$\frac{I'}{I} = \left(\frac{\nu'}{\nu}\right)^3$$

where $I$ represents Thomson's scattering function expressed by equation (1). Hence in place of (1) we have as the scattering function for a single free electron

$$I'_{\phi} = I_0 \frac{a}{2\pi^2 r^2 c^4} \frac{(1 + \cos^2 \phi)}{\left[1 + \alpha(1 - \cos \phi)\right]^3}$$

in which $\alpha$ is written for $h \nu / m e^2$ and the other symbols have the same significance as before.

This formula (6) was first suggested by Breit, and later derived by Dirac using the quantum dynamics of Heisenberg and Born. Other formulae of similar type have been devised, for instance by Klein and Nishina and by Compton; but as the deviations between these formulae and the Breit-Dirac expression are only of the order of $\alpha^2$ the differences to be expected in the region of medium X-ray wavelengths are negligible. The quantum formula (6) however, differs from the classical Thomson formula by terms of the first order in $\alpha$, and therefore the predicted deviations from (1) are appreciable even in the normal X-ray region.

Many of the experiments performed on the scattering of X-radiation have been designed to test the various quantum formulae mentioned above. Direct tests of the scattering as a function of angle meet with considerable difficulties, principally because the rays scattered at different angles have, according to the quantum
theory, different wavelengths, and hence different ionising efficiencies. The corrections necessary on this account are of the same order of magnitude as the differences to be expected between the predictions of the classical and quantum theories, hence the observed results are somewhat difficult to interpret. Nevertheless, all the more recent experiments of this type (for instance those carried out by Friedrich and Goldhaber, Chylinski, and others) show good agreement with the quantum theories and wide divergence from the classical theory of Thomson. If we consider the radiation scattered in one particular direction only, say at 90° to the direction of the primary radiation, the corrections are less uncertain than in the previous case and the results more easy of interpretation. In the present work this procedure is adopted, the relation between the scattered and primary radiations being observed for various primary radiations, and the influence of various factors on the scattering curves so obtained being investigated. From the results obtained it will be clear that there exist factors which can alter the form of the scattering curves and yet which are not contained in any of the formulae previously considered. The theory underlying these formulae cannot therefore be regarded as adequate to account for the observed experimental results on scattering; in fact, the results appear irreconcilable with any accepted theory. The same conclusion follows from a consideration of the experimental results on a comparison of the relative absorbabilities of the scattered and primary radiations; the salient features of such comparisons which have been made in this laboratory may be briefly enumerated:—

(1) Many observations made of the radiation scattered from thin plates of substances containing light elements have indicated that the penetrating power of this radiation often appears precisely the
same as that of the primary radiation exciting it, at any rate within a small experimental error. This is a result which seems quite inconsistent with the quantum theory of Compton.

(2) When differences in the penetrating powers are evident they take the form of very abrupt increases in the absorbability of the scattered radiation, the abrupt increases occurring at fairly well-defined values of penetrating power of the primary radiation (that is, 'penetrating power' as understood in this work and as defined on page 30).

(3) Between these abrupt changes the difference in the penetrating powers of the scattered and primary radiations in general remains constant.

Results such as the above have been evident in many of the experiments involving the progressive absorption of the scattered and primary radiations, those for instance in which the two beams of radiation have been intercepted by successive equal thicknesses of aluminium$^{(7)}$, or of silver$^{(8)}$; (similar absorption discontinuities have been observed even when a radiation consisting largely of the characteristic rays from a heavy element has been absorbed by increasing thicknesses of aluminium$^{(9)}$).

In these progressive absorption experiments, it will be observed, the thickness of absorbing material is varied while the radiation incident on the scattering plate is kept as nearly as possible constant. Precisely similar conclusions regarding the difference in penetrating powers of the primary and scattered radiations can be drawn from experiments in which the absorbing material in the paths of the primary and scattered beams is kept constant while the primary radiation (and consequently the scattered radiation from a given scatterer) is varied. We may summarise the important features of
observations of this type as follows: —

(1) Experiments described in the present work, along with many others, have shown that under appropriate conditions the ratio of the ionisations produced by the primary and scattered radiations is independent of the penetrating power of the primary radiation over a large range of penetrating power. This fact is itself of considerable interest, but the point of present importance is that given in (2).

(2) If the primary and scattered beams are intercepted by equal thicknesses of some absorbing material the ionisation ratio above defined is still independent of the penetrating power of the primary beam. The 'intercepted' and the 'unintercepted' ratios may, or may not coincide, the constant difference — if any — between them giving a measure of the (constant) difference in penetrating powers of the scattered and primary radiations. Such results have been obtained when intercepting the radiations with aluminium, copper, silver, and gold.

(3) As in the case of progressive absorption experiments previously quoted, differences in the penetrating powers of the scattered and primary radiations have been observed to occur in the form of abrupt increases in the absorbability of the scattered radiation; these increases again occurring at definite values of the penetrating power of the primary beam.

The type of result indicated above has, in general, been obtained when the radiation was scattered from a material in the form of a thin plate; with thick scatterers the occurrence of absorption discontinuities, or of constant ionisation ratios, has been much less frequent; the experiments have suggested, in fact, that a thin scatterer is a necessary condition in order that these may occur.
The systematic series of scattering experiments carried out in the present work show, however, that a region of constant ionisation ratio would probably be obtained for all thicknesses of scatterer — or at least for a wide range of thicknesses — with appropriate experimental conditions, provided only that the voltage operating the X-ray tube was sufficiently high.

The most remarkable feature of all the experiments to which reference has been made is that the occurrence of these absorption discontinuities appears to be dependent on some unrecognised factor, for they may occur or not occur under what are apparently identical experimental conditions; this fact has been noted on many occasions, and is a characteristic feature of the J-phenomenon, but experiments have not yet allowed of any definite conclusions being drawn as to the cause of the appearance or non-appearance of the J-discontinuities.

The experiments described in the present work form an attempt to determine the influence of various experimental conditions on the scattering, and on the absorption, of heterogeneous radiations, to reduce the experimental error to a minimum, and to make a more systematic study of the influence of the significant factors than has hitherto been possible. One important fact emerging from the results is the profound influence, on both the scattering and the absorption measurements, of the lateral dimensions of the apertures limiting the primary beam of radiation entering the ionisation chamber. This work contains an account of the first systematic experiments on this important feature, and the results indicate that a systematic quantitative variation in the scattering curves is produced merely by a progressive alteration in the size of the aperture limiting the primary beam of radiation, provided this aperture is less than a certain relatively small size. With such apertures the significant feature
of the scattering curves is the existence of a region (variable in extent) over which the previously mentioned ionisation ratio remains independent of the penetrating power of the primary beam; variation of the aperture causes a variation of the position of this region. With all larger primary apertures the scattering curves remain essentially similar to one another, and show no region of constant ionisation ratio. These results are quite unexpected, yet a critical survey of the measurements leaves no doubt as to their reality.

In contrast to the recorded results of most observers in this laboratory, the results of the present work have been remarkably constant; similar experimental conditions have, invariably, given similar results, irrespective of the intervals of time elapsing between the experiments. This fact, of course, can not in any way detract from the experiments which show alternate results under apparently similar experimental conditions; however, it seems worth noting since the experimental conditions in the work to be described are probably more rigidly defined than in any previous experiments, principally on account of the use of a constant potential direct current apparatus to operate the tubes in many of the experiments. This, together with the careful control of a sensitive electrometer to measure the intensity of the scattered radiations, results in a relatively high order of accuracy in the measurements. (The question of experimental error is more fully discussed later).

Throughout the work no absorption discontinuities of the type described above have been observed, but over limited ranges apparently equal absorbabilities have been found for the scattered and primary radiations under appropriate experimental conditions. This result has been obtained both by intercepting the primary and scattered radiations by successively increasing equal thicknesses of absorbing
material (progressive absorption), and by intercepting a varied primary and scattered radiation by constant equal thicknesses of absorbing material. An important fact established by these observations is the correlation of the two types of experiments, the progressive absorption experiments only showing equal absorbabilities of the scattered and primary beams so long as the relative ionisations produced by them remain independent of the penetrating power of the primary beam.
APPARATUS AND EXPERIMENTAL PROCEDURE.

Earlier experiments appear to have shown that the experimental arrangements and the procedure adopted in making the observations may have considerable influence on the type of result obtained, consequently the apparatus and the methods of measurement will be described in some detail. The general arrangements however may first be briefly described as follows: heterogeneous beams of radiation from various hot-cathode tubes, operated from different types of high potential sources, are scattered at an angle of approximately 90° from plates of paraffin wax, aluminium, or filter paper, the intensities of the scattered and the transmitted primary radiations being measured by ionisation methods. Figure 1 shows diagrammatically the arrangement of the scatterer in relation to the primary and scattered radiations.

The scattering plate is arranged so that the normal to its surface makes an angle of 45° with the directions of the scattered and the transmitted primary radiations, this position gives equal path lengths through the scatterer of the primary and scattered radiations after scattering has occurred at any given depth in the plate. Hence if
the scattered and primary radiations differ little in quality the absorptions undergone by each after the scattering process has occurred are very nearly equal.

The actual arrangement of the apparatus is shown in detail in figure 2. The X-ray tube is enclosed and screened with lead except for an aperture 4.4 cms. in diameter which allows the beam of primary radiation to fall on the scatterer; this primary radiation can be cut off when necessary by the lead shutter moving in front of the aperture. The angular spread of the primary beam incident on the scattering plate can be varied by means of the adjustable aperture at 'i' (this aperture will be referred to as the 'incident aperture'); with the aperture at 'i' fully opened the primary beam consists of a cone of radiation having a total spread of about $70^\circ$. The primary beam transmitted through the scatterer is further limited by an adjustable aperture or apertures at 'p', this primary aperture being small in order to reduce the ionisation caused by the primary beam sufficiently for accurate measurement. Secondary apertures at 's' and 's\text{\textsubscript{2}}' limit the beam scattered at approximately $90^\circ$ to the direction of the primary beam; but since the scattered intensity is relatively small, the secondary apertures have to be large compared with the primary aperture, consequently the direction of the scattered beam in relation to the primary is not perfectly definite but may vary through a small angle. Actually the maximum deviation from a scattering angle of $90^\circ$ which the extreme rays entering the secondary ionisation chamber may undergo is approximately $10^\circ$, though generally, of course, the deviation is somewhat less.

The primary and scattered radiations so obtained then enter ionisation chambers suitably placed to receive them, and the ionisations produced are measured by suitable instruments connected
PLAN of APPARATUS

Fig. 2.

SCALE: 1/5 FULL SIZE
to the ionisation chamber electrodes. These measurements have been
effected by different arrangements of ionisation chambers which will
therefore be described separately; the distances of the primary and
secondary ionisation chamber electrodes from the scatterer have,
however, been kept as nearly as possible equal in the various cases.
The whole of the shielding of the apparatus is of lead 3 mms. in
thickness, and appears from tests to be sufficient to eliminate
practically all effects due to stray radiations. (Provision is made
in the apparatus for scattering at angles of 60° and 120° but these
angles have not been used in the present experiments).

In order to measure the intensity of a given X-radiation
ionisation methods are by far the most convenient, though the results
so obtained are liable to misinterpretation; uncertainties arise
because when X-rays of different wavelengths enter the ionisation
chamber the saturation currents measured are not in general
proportional to the intensities of the radiations. This point must
be borne in mind when considering the results of experiments; it
will be discussed in more detail later. In the present work two
distinct ionisation methods have been used to measure the intensities
of the radiations: in the one, different gases have been ionised and
the ionic saturation currents produced have been measured by suitable
electrometers or electroscopes; in the other, the electronic
emission from gold has been measured by the ionisation it produces in
hydrogen.

The first type of ionisation chamber used is shown in figure 3.
It consists of a brass cylinder of internal diameter approximately
14.5 cms. and internal depth 10 cms. The radiation is allowed to
enter one end of the chamber through an aluminium window 0.01 cm.
thick, the clear aperture of this window being 4.4 cms. To reduce ionisation due to corpuscular radiation from the window and the back wall, the chamber is completely lined with aluminium 0.01 cm. thick and filter paper of surface density 0.064 gms. per sq. cm. Inlet and outlet pipes are provided in order to introduce the gas to be ionised. The electrode system will be clear from the diagram; the collecting electrode itself is in the form of a circle of aluminium wire of 10.5 cms. diameter with cotton gauze stretched across it, the whole electrode then being painted with Indian ink. Connection to the measuring electroscope is made by a rod passing down the centre of the ebonite insulating plug. In use the ionisation chamber is held on insulating supports and raised to a suitable potential.

Figure 4 shows the second type of ionisation chamber, used principally in the experiments involving electronic emission from gold. The gold is in the form of sheets held near the front and back walls of the chamber at a distance apart of 4.4 ± 0.2 cms.; the other internal dimensions of the chamber being 11 cms. each way. The radiation is again admitted through an aluminium window 0.01 cm. thick and of 4.4 cms. clear aperture, while the whole ionisation
chamber is lined with aluminium and filter paper as before. The electrode system is identical with that of the first ionisation chamber except that the diameter of the collecting electrode is reduced to 8 cms. Other details are sufficiently clear from the diagram.

In order that the measurements of ionisation currents may have any value for the purposes of these experiments there are several precautions that must be observed. In the first place it is necessary that the ionisation chambers used to receive the primary and scattered radiations are, as nearly as possible, identical in all their dimensions. Secondly, the radiation must not be able to eject electrons from any part of the chamber other than the gold sheets or the gas as the case may be; the radiation, therefore, should not be allowed to strike the metal parts of the collecting electrode. Finally, if the intensity measurements are by the ionisation of a gas, then the gas must be identical in both primary and secondary ionisation chambers otherwise the absorptions in the two chambers will be different. Any discontinuous effect in the gas, say by the excitation of fluorescence radiation within the region investigated, must
also be considered.

Measurement of the ionic saturation current produced by the primary radiation is carried out by connecting the primary ionisation chamber electrode to an ordinary cubical air electroscope, the complete electrode system being charged originally to a potential of 240 volts positive. The primary chamber itself is maintained at a potential of about 180 volts negative, hence during ionisation of the gas negative ions are driven to the insulated collecting electrode and its original positive charge therefore reduced. The resulting decrease in deflection of the gold leaf of the electroscope gives a measure of the charge collected by the electrode, and therefore of the ionisation produced by the given primary radiation. Since the intensity of the scattered or secondary radiation is very much less than that of the primary it is preferable to use an instrument with a sensitivity higher than that of an ordinary cubical electroscope. In the present experiments an electrometer of the type described by H.A.Bumstead\(^{(2)}\) is used; it is shown diagramatically in figure 5.

![Diagram of electrometer](image)

The two tilted plates are maintained at equal and opposite potentials, while the gold leaf, initially at earth potential, hangs symmetrically between them. If a charge is communicated from the electrode to the
gold leaf, the leaf is deflected from its symmetrical position, this deflection is used to measure the ionisation produced by the radiation. The sensitivity of the electrometer may be altered, either by varying the potentials on the plates, or by adjusting the positions of the plates in relation to the gold leaf. During these experiments the plate potentials used varied between 180 volts and 220 volts, depending on the sensitivity required, while the secondary ionisation chamber was maintained at potentials between 360 and 480 volts negative.

It is, of course, necessary that the ionisation currents measured are in all cases the saturation values; this may be ensured either by making a large change in the intensity of the radiation measured and observing any change in the relative values of the primary and secondary ionisations, or by a direct measurement of the ionisation currents for different values of potentials between the case of the ionisation chamber and the collecting electrode. This latter procedure gives the well known curves in which the ionisation current rises to a maximum as the potential difference between collecting electrode and ionisation chamber is increased and thereafter remains constant. In these experiments both methods have been used, and in the case of the most intense ionisation measured the values of the potential differences quoted above are more than double those required to give saturation.

The method of observation in the present work has been as follows: a certain total primary ionisation was allowed to take place in the primary ionisation chamber, this ionisation being measured by the deflection of the leaf of the electroscope over a definite range on the scale of the reading microscope. The edge of the leaf was
always brought into coincidence with a scale division at the commencement of an observation and then allowed to move into coincidence with another scale division, generally between ten and twenty divisions away from the first. (Although the accuracy of reading is much less near a scale division than midway between two scale divisions coincidence between the leaf edge and a scale division can be judged very precisely). The range over which the leaf travelled was kept fixed during any one experiment, and thus errors due to calibration of the scale or to estimation of fractions of a scale division were avoided. In the case of the secondary electrometer fractions of a scale division had of necessity to be estimated; the calibration was, however, nearly linear over the range used. The procedure was to observe the number of scale divisions moved over by the leaf of the secondary electrometer while the leaf in the primary electroscope moved over a definite fixed range. In effect, therefore, the primary electroscope was used as a standard while the secondary ionisation was measured when different primary radiations were incident on the scatterer. (On a few occasions the procedure has been reversed, the secondary electrometer being used as standard and the primary ionisation observed, but this process was found to be less accurate than the former and was only used to show that the form of the scattering curves was not altered in so doing). The quantity directly measured in the above procedure is the ratio of ionisations produced by the scattered and primary radiations.

In many of the observations the potential taken up by the secondary collecting electrode has been directly measured by comparison of the observed deflection with that produced by a known voltage applied to the electrode from a standard cell. This method has the advantage of eliminating errors due to changes in the
sensitivity of the secondary electrometer, which, especially at higher sensitivities, sometimes amount to more than the estimated possible error of a single observation. The normal electrical leak of the system was in practically all cases entirely negligible, on the average not affecting the observations by more than one part in about 300, and therefore considerably less than the possible error of any one observation; the probable errors in the measurements will be discussed in more detail later. In the few cases in which it has been considered necessary corrections for normal electrical leak have been made.

As sources of X-radiation five different tubes have been used, all being of the hot-cathode type, and in operation these have been placed with the cathode stream parallel to the direction of the scattered radiation studied. A brief description of the tubes will suffice:

(i) Müller media-focus tube No. 311704. The radiation is allowed to emerge from this tube through a small window of ordinary glass attached to a metal centre-piece. The anode is of tungsten, set at 45° to the direction of the cathode stream and cooled by boiling water circulating behind it. Maximum operating voltage 110 kV.

(ii) Philips Structure-Research tube No. 354624. The radiation from this tube emerges through a Lindemann glass window attached to a metal centre-piece. The anode is of molybdenum and is set normal to the direction of the cathode stream, so that in use the tube has to be inclined in such a way that the radiation leaves it in a direction making an angle of approximately 78° with the direction of the cathode
stream. Cooling of the anode takes place by means of cold water passing continuously behind it from a suitable circulator. Maximum operating voltage 70 kV.

(iii) Philips Metalix tube No. 362619. This tube also has a Lindemann glass window; the anode is of tungsten, cooled by boiling water, and set at 45° to the direction of the cathode stream. Maximum operating voltage 120 kV.

(iv) Coolidge tube No. XP 3. The glass wall of this tube is ground down near the anode to reduce the absorption of the radiation in passing through it; the anode is of tungsten and is air-cooled.

(v) Siemens Multex tube No. 234303. The anode is of tungsten cooled by boiling water and set at 45° to the direction of the cathode stream. The glass window of the tube is approximately equivalent in its absorbing power to 0.7 mm. of aluminium, so that the radiation leaving the tube is highly filtered.

Three different sources of high potential have been used to operate the tubes.

In the first (figure 6) a rotary converter (2 kw., 50 cycles) is driven from the 230 volts d.c. mains. From this, alternating current at 160 volts is passed through an auto-transformer tapped suitably to operate the high-tension transformer, and further regulated by the variable series resistance R₁. The secondary of the high-tension transformer is earthed at the middle point and the high-tension current passed through a milli-ammeter, so that the current through the tube is known. The filament heating circuit is clear from the figure; the variable resistance R₂ being used to adjust
the current through the tube; the filament current is derived from

\[ \text{230 v. D.C.} \]

\[ \text{To Rotary Converter} \]

\[ \text{A.C.} \]

\[ \text{Auto-Transformer} \]

\[ \text{Filament Heating Circuit} \]

\[ \text{Filament} \]

\[ \text{To X-ray Tube} \]

\[ \text{Filament} \]

**FIG. 6**

a 12 volt battery of accumulators. The maximum voltage obtainable from this transformer is slightly above 70 kV.

The second high potential source used was another transformer operated from the a.c. mains, (figure 7). The voltage to the primary winding of the transformer is regulated by the auto-transformer and variable series resistance \( R_1 \), and the maximum voltage delivered by the transformer is 90 kV. The filament heating circuit is identical with that of figure 6, and other details are similar.

(It will be noticed in the case of figures 6 and 7 that the
unrectified secondary current is supplied direct to the X-ray tube, and the tube therefore acts as a valve, rectifying its own current. This has the effect that when secondary voltages are measured by placing a spark-gap in parallel with the tube the inverse values are observed, and not the values existing during the actual production of X-radiation. However, as only peak values of voltage are observed, the difference between the inverse and the operating values is no greater than the uncertainty attached to the measurement of peak voltages.

The operating voltage for the tube was derived in the third case from a constant potential direct current apparatus, the circuit of which is shown in figure 8. In this the two valves $V_1$ and $V_2$ allow the current to pass to and from the tube respectively in one direction only; this pulsating current is smoothed by the two condensers $C_1$ and $C_2$, so that an approximately constant potential is applied to the tube, having a value twice that of the peak voltage of the transformer. The current passing through the tube is measured by the milli-ammeter placed on the negative side. Primary current is supplied to the
high-tension transformer from the a.c. mains, the primary voltage being regulated by a variable series resistance as shown. The valve filaments are heated by step-down transformers also operated from the a.c. mains, the filament of the X-ray tube however is heated by current from accumulators as in the previous apparatus. High resistances each of 50,000 ohms are inserted in the positions shown. The apparatus can deliver direct current to the tube at a maximum potential of 100,000 volts, the voltage being measured by a 10 cm sphere spark-gap permanently connected in parallel with the tube.
The general Relation between the scattered and primary Radiations:

The general type of result obtained in these investigations when a heterogeneous beam of X-radiation is scattered at 90° from a thin plate of a light substance, the experimental conditions being as previously described, may be illustrated by figure 9. In this the ionisations produced by the scattered and primary radiations are measured and their ratio plotted as a function of the quality of the primary radiation giving rise to the scattered beam. (For the moment the 'quality' of the primary may be thought of as its penetrating power, or absorbability in some given element—the penetrating power of the radiation increasing, of course, as the average wavelength is decreased). Except in the region of short wavelengths, this curve gives the relative intensities of the radiation scattered at 90° and the primary radiation, assuming ionising power to give a true measure of intensity. The general, unfiltered radiation from the tube is used, the tube being placed with the cathode stream parallel to the direction of the scattered radiation investigated, and the absorbability of the primary radiation is altered by varying the potential operating the tube—not by filtering the radiation incident on the
scatterer.

The main features of the scattering curve of figure 9 are as follows:—

(i) Over a considerable range of absorbability the ionisations produced by the scattered and primary radiations maintain a constant relationship to each other. Over this range the slope of the experimental curve is, within a small experimental error, zero.

(ii) At a certain region D there is a rapid variation in the slope of the curve, the secondary ionisation (i.e. ionisation due to the scattered radiation) begins to decrease in relation to the primary as the absorbability of the primary radiation is increased. The position of the region D is, for any given set of experimental conditions, fairly well defined.

(iii) Beyond the region D, on the side of increasing absorbability of the primary radiation, the ionisation produced by the scattered rays shows a continuous decrease compared with that produced by the primary, the apparent rate of decrease depending, of course, on the definition of absorbability.

The conditions essential in order to obtain scattering curves of the form shown in figure 9, having the features cited above, are that the plate of scattering material is relatively thin, and that the apertures at 'p' (figure 2) limiting the cross-section of the beam entering the primary ionisation chamber are small; so far as the present experiments are concerned, these may be called the necessary and sufficient conditions. As a typical experimental result, figure 10 shows the scattering curve obtained when the unfiltered general radiation is scattered from a slab of paraffin wax of thickness 3 mms. and surface density 0.276 gms. per sq. cm. The apertures
at 'p' consisted of four drilled holes, each 0.6 mm. in diameter on the corners of a square of 6 mm. side. The tube used was No. 362619 operated from the constant potential direct current apparatus, secondary voltages being measured by the 10 cm. sphere spark-gap in parallel with the tube (these measured values are probably accurate to within 3 kV, the constancy of the potential from the apparatus being specified as 5%). Secondary current was constant to about 5%.

Other experimental details are as indicated in figure 10.

The method of indicating the absorbability of the primary radiation requires explanation, since the absorbability of a heterogeneous radiation is not defined by a single quantity, at least according to present ideas. But if both the secondary voltage producing the radiation and the thickness of some pure element absorbing say 50% of that radiation are known, then the quality of the radiation may be taken as fairly completely specified. It has been customary in this laboratory to define an 'average mass-absorption coefficient' by obtaining the thickness of aluminium required to reduce by 50% the ionisation produced in a short ionisation chamber by the primary beam after transmission through the scatterer, then, treating the absorption as exponential we have

$$P = P_0 e^{-\mu x}$$

where \(x\) is the thickness of aluminium required to make \(P = \frac{1}{2}P_0\) and \(\mu\) is the 'average absorption-coefficient'. From this it follows that

$$\mu/\rho = \log_e 2/\rho x$$

\(\rho\) being the density of aluminium. This is to be taken as the definition of the 'average mass-absorption coefficient' \(\mu/\rho\) in aluminium of the transmitted primary radiation, which, together with the applied secondary voltage, completely defines the quality of the radiation.

In figure 10 the relative ionisation, \(S/P\), is shown as a
function of secondary voltage and of average mass-absorption coefficient. The general form of this curve is as previously stated: from an applied secondary voltage of 96 kV to one of approximately 55 kV,
corresponding to average mass-absorption coefficients of 6.4 and 9.8 respectively, the relative ionisations produced by the scattered and primary radiations remain constant. As the voltage is reduced below
55 kV a fairly rapid decrease of the secondary ionisation relative to the primary sets in, the rate of this decrease then remaining nearly uniform. Plotted as a function of average mass-absorption coefficient the ratio $S/P$ shows similar variations. The abruptness of the change in slope of the scattering curve in the region of 55 kV may perhaps be best illustrated by considering the rate of change of the ionisation ratio with the absorbability of the primary beam. This is done in figure 11, where the change of the ratio $S/P$ for 1 kV change in the secondary voltage is plotted as a function of the secondary voltage; the derivative of $S/P$ is obtained by measurement from large-scale graphs.

![Graph showing rate of change of ionisation ratio](image)

Quantitative Variations of the scattering Curves:

The features shown by the curves of figures 10 and 11 may be varied, quantitatively, merely by changing the thickness of the scattering plate, all other experimental conditions remaining constant. (This variation is distinct from qualitative changes which may be produced by other means to be considered later). From several series of experiments with various thicknesses of scattering materials two general conclusions can be drawn: as the thickness of the scattering...
plate is increased

(i) the general features of the scattering curve remain unchanged;
(ii) the point D moves towards higher secondary voltages, i.e.,
towards higher values of penetrating power of the primary
transmitted radiation.

This means that a thick scatterer exhibits a region of constant
ionisation ratio S/P, but over a less extensive range than a relatively
thin scatterer.

Figures 12 show the scattering curves obtained using paraffin wax in the form of plates of various thicknesses as scatterer. The ionisation ratio S/P is again plotted as a function of applied secondary voltage (in this case peak voltage of the transformer) and of average mass-absorption coefficient. The possible error in the determination of the position of the point D in this series of experiments is rather large, since, in some cases, it was not possible to vary the secondary voltage appropriately to obtain observations near D. Nevertheless a systematic decrease in the range of constant ionisation ratio S/P is obvious, and is confirmed by the considerably more accurate results shown in figures 14.

The two series of curves of figures 12a and 12b show clearly the displacement of the point D towards higher penetrating powers as the thickness of the scatterer is increased. The fact that over a range of thicknesses of scatterer from 2 mm. to 11 mm. the general form of the scattering curve remains unaltered also suggests that with a sufficiently high secondary voltage still thicker scatterers would show a region in which the ionisation ratio S/P would be constant. These results are summarised in Table 1 and figures 12c and 12d.

In this series of experiments the tube used was the Müller No. 311704 with the cathode stream horizontal, operated from the
The intensities of the primary and scattered radiations were measured by the ionisations produced in SO₂ at atmospheric pressure, the two ionisation chambers (figure 3) being in gas connection to maintain the gas in both chambers as nearly identical as possible. The aperture
at 'p', limiting the primary beam, was in all cases a series of small holes (16 in number) varying in size from 0.2 to 0.7 mm. diameter approximately, arranged over a square of 6 mm. side; (for reasons which will appear later, it is important that the apertures at 'p' are unaltered during such observations as the above). The sizes of
the apertures limiting the scattered radiation are given in Table 1.

**TABLE 1.**

<table>
<thead>
<tr>
<th>SCATTERER</th>
<th>THICKNESS</th>
<th>SURFACE DENSITY</th>
<th>POS. OF D</th>
<th>SEC. PEAK VOLTAGE</th>
<th>p(μ/π)/m</th>
<th>SECONDARY CURRENT mA</th>
<th>S1 DIAM.</th>
<th>S2 DIAM.</th>
<th>i DIAM.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paraffin Wax Scatterers.</td>
<td>2.0 mm.</td>
<td>0.18 cm²</td>
<td>4.50 kV</td>
<td>7.3</td>
<td>0.33 e 0.45</td>
<td>4.4 cm.</td>
<td>4.4 cm.</td>
<td>4.4 cm.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>0.28</td>
<td>4.65</td>
<td>5.0</td>
<td>0.25</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.7</td>
<td>0.43</td>
<td>4.80</td>
<td>4.3</td>
<td>0.30</td>
<td>4.4</td>
<td>4.4</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.5</td>
<td>0.69</td>
<td>5.45</td>
<td>3.5</td>
<td>0.33</td>
<td>2.3</td>
<td>4.4</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11.0</td>
<td>1.01</td>
<td>6.60</td>
<td>2.7</td>
<td>0.45 e 1.40</td>
<td>2.2</td>
<td>4.4</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>18.5</td>
<td>1.70</td>
<td>-</td>
<td>-</td>
<td>0.50</td>
<td>2.1</td>
<td>4.4</td>
<td>4.4</td>
<td></td>
</tr>
</tbody>
</table>

For secondary voltages calibration curves were obtained for various secondary currents by measuring with a 10 cm sphere spark-gap in parallel with the tube the secondary peak voltages corresponding to different primary voltages; the resulting curves were practically linear. Such a procedure is not very accurate, but the tabulated values of secondary voltage should not be in error by more than about 5 or 6 kV; the estimated probable errors in the peak voltages of the
points D on account of this measurement alone are shown by the vertical lines through the experimental points. The small values of secondary currents used render the tabulated values of these liable to a fairly large percentage error.

Similar results are obtained when filter paper is used as a scatterer; proceeding in the direction of increasing secondary voltage the secondary ionisation increases relative to the primary until a certain value of secondary voltage is reached, this is the point designated D in the curves. For further increases in the value of the secondary voltage the ionisation ratio S/P remains unchanged. Table 2 and figures 13 summarise a series of results obtained with filter paper scatterers.

**TABLE 2.**

**Filter Paper Scatterers.**

<table>
<thead>
<tr>
<th>SCATTERER</th>
<th>POSITION of D</th>
<th>SECONDARY CURRENT mA.</th>
<th>APERTURES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P(µ)/c_AL</td>
<td>s₁</td>
</tr>
<tr>
<td>3 sheets</td>
<td>0.02 cms/ct.</td>
<td>45.0 kV</td>
<td>5.28</td>
</tr>
<tr>
<td>5</td>
<td>0.03</td>
<td>45.5</td>
<td>5.26</td>
</tr>
<tr>
<td>10</td>
<td>0.06</td>
<td>53</td>
<td>4.70</td>
</tr>
<tr>
<td>15</td>
<td>0.10</td>
<td>66</td>
<td>3.30</td>
</tr>
<tr>
<td>20</td>
<td>0.13</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
In this series of experiments the primary aperture at 'p' consisted of a horizontal lead slit 0.095 mm. by 7 mm. in all cases; the jaws of the slit were square-cut, 3 mm. in thickness, and worked by a micrometer screw. The slit width was measured by a travelling microscope. All other experimental conditions and methods of measurement were identical with those of the previous series, and the probable errors of all quantities, except secondary currents, were about the same; owing to the increased values of the secondary currents used the percentage error of the tabulated values are considerably less than in the previous series.

Variations of the scattering curves, brought about solely by changes in the thickness of the scatterer, were observed by Sen Gupta in the case of scatterers of filter paper. Using one sheet of filter paper as scatterer he found the ratio of secondary to primary ionisations to remain constant over the whole range of absorbabilities investigated, viz. from \( \rho(\mu/\rho)_{Al} 1.5 \) to 6.5; with a scatterer consisting of 16 sheets the secondary ionisation decreased continuously relative to the primary over the same range. Scatterers of intermediate thicknesses gave curves showing a region of constant ionisation ratio until certain values of absorbability were reached. Further increase in the values of the absorbability resulted in a decrease of the secondary ionisations relative to the primary. Sen Gupta's curves agree in all essential features with the results of the present experiments using filter paper scatterers; in particular, the rapid displacement of the point D as the thickness of the scatterer is changed is evident in both cases. No measurements of secondary voltages, however, are given in the work of Sen Gupta, nor is any indication given of the primary apertures used — this latter is a
factor which is now known substantially to influence the result obtained. The same type of variation with thickness of scatterer was also observed in the case of filter paper by Kay\(^{(a)}\), with, however, one significant qualitative difference; Kay observed a region of constant ionisation ratio \(S/P\) even with scatterers as thick as 100 sheets, the point D however was displaced as the thickness of scatterer was changed in the same manner as found by Sen Gupta. A probable cause of this difference will be apparent later when the effect of the primary aperture on the scattering curve is discussed. A limited number of observations by Kay using paraffin wax scatterers indicated changes identical with those considered above as the thickness of the scattering plate was varied.

A considerably more accurate series of results with paraffin wax scatterers of various thicknesses, in which the ionisations in both air and \(SO_2\) at atmospheric pressure were used to measure intensities, is shown in figure 14. In this series the tube employed was the Müller No. 311704 operated from an a.c. mains transformer, using the circuit shown in figure 7, the cathode stream of the tube being horizontal. Secondary peak voltages were measured by a 10 cm. sphere spark-gap in parallel with the tube, every value of peak voltage employed being measured at the time of observation. Any peak voltage could be repeated with an accuracy of 1 or 2 kV and the tabulated values are probably correct within 3 kV throughout the range used — this is, of course, the inverse value. Secondary currents are accurate in every case to about 8%. The apertures at 'p' consisted of four drilled holes each 0.6 mm. in diameter spaced on a square of side 6 mm.; the other apertures being as indicated in Table 3.

Change of thickness of scatterer in this series of experiments produces variations in the scattering curves of precisely the same
nature as those evident in figures 12 and 13, the rate of displacement of the point D being, however, different from that of figure 12. A
feature of importance which emerges from this series of experiments is that the form of the scattering curve, plotted as a function of secondary voltage, is independent of the ionised gas; within a small experimental error, the secondary voltage of the point D is the same whether the intensities are measured by ionisation of air or of SO₂, and, further, the slope of the curve to the right of the point D is practically identical in both cases. If, on the other hand, the ionisation ratio S/P is plotted as a function of average mass-absorption coefficient differences between the curves obtained using air as the ionised gas and SO₂ as the ionised gas become apparent. The difference takes the form of a general displacement of the curve towards the direction of greater absorbability when air is used as the gas ionised as compared with when SO₂ is used; the displacement decreases as the thickness of the scatterer is increased. It is unnecessary to show in detail the scattering curves as a function of average mass-absorption coefficient; Table 3, however, gives the mass-absorption coefficients of the points D for the various thicknesses of scatterer.

TABLE 3.

<table>
<thead>
<tr>
<th>SCATTERER</th>
<th>POSITION of D</th>
<th>SECONDARY CURRENT mA</th>
<th>APERTURES</th>
<th>GAS IONISED</th>
</tr>
</thead>
<tbody>
<tr>
<td>THICKNESS</td>
<td>SURFACE DENSITY</td>
<td>REC. FORM VOLTAGE</td>
<td>t(M/V) MÅ</td>
<td>S₁</td>
</tr>
<tr>
<td>2.0 MMS.</td>
<td>0.18 gms/cm²</td>
<td>45 KV</td>
<td>5.80</td>
<td>3.0</td>
</tr>
<tr>
<td>3.0</td>
<td>.28</td>
<td>48</td>
<td>5.20</td>
<td>1.5</td>
</tr>
<tr>
<td>4.7</td>
<td>.43</td>
<td>53</td>
<td>4.25</td>
<td>1.0</td>
</tr>
<tr>
<td>7.5</td>
<td>.69</td>
<td>79</td>
<td>2.80</td>
<td>1.0</td>
</tr>
<tr>
<td>2.0 MMS.</td>
<td>0.18 gms/cm²</td>
<td>46 KV</td>
<td>4.90</td>
<td>1.0</td>
</tr>
<tr>
<td>3.0</td>
<td>.28</td>
<td>NOT FOUND</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.7</td>
<td>.43</td>
<td>54</td>
<td>3.75</td>
<td>1.0</td>
</tr>
<tr>
<td>7.5</td>
<td>.69</td>
<td>79</td>
<td>2.70</td>
<td>1.0</td>
</tr>
</tbody>
</table>
A consideration of the experimental results described so far, reveals two features in particular which call for comment. In the first place, the region of constant ionisation ratio $S/P$ has a definitely limited extent in every case investigated; in fact the curves suggest that as the scattering substance is made indefinitely thin the point $D$ moves towards a limiting position. The second feature referred to above will be evident from a comparison of figures 12c and 14b, showing results with paraffin wax scatterers. These two curves show the rate of displacement of the point $D$ as the thickness of the scatterer is varied; it will be evident that this rate of displacement is not the same in the two cases, and the difference must result from a difference in the experimental conditions in the two series of experiments. The factors which might conceivably affect the scattering curve are:

(i) the distribution of energy among the different wavelengths in
the primary radiation.

(ii) the cross-sectional area of the beams of radiation entering the ionisation chambers.

(iii) the gas ionised by the radiations.

(In addition, Key's experiments\textsuperscript{20} have shown that the scatterer itself has some influence on the scattered radiation, two apparently identical scatterers giving totally different results; however, no such effect has been observed in the present work). Experimental evidence will be presented that, so long as we consider the ionisation ratio $S/P$ as a function of secondary voltage only, the factors (i) and (iii) above have no observable effect on the scattering curves; since the energy-wavelength distribution of the primary radiation depends on the X-ray tube itself, on the secondary current passing through it, and on the high potential source operating it, this statement means that neither the tube, the secondary current, nor the high potential source has any appreciable influence on the results obtained. Some evidence has already been given that the curves are independent of the ionised gas (figure 14a). The only remaining factor, namely the cross-sectional area of the beams of radiation, must therefore account for changes in the scattering curves which cannot be attributed simply to changes in the thickness of the scatterer. Experiment shows that the apertures at 'i', 's.', and 's_{z}', defining respectively the incident and the scattered beams of radiation have little observable effect, and certainly no fundamental influence, upon the shape of the scattering curve. This is not the case, however, with the primary aperture at 'p'; it appears as an unexpected, yet undoubted result, that the dimensions of the aperture limiting the beam of radiation entering the primary ionisation chamber can alter completely the general features of the scattering curve obtained in these experiments.
Regarding the scattering as a function of average mass-absorption coefficient, however, the factors (i) and (iii) can not be dismissed without consideration. The value of $P(\mu/\rho)_{Al}$ must obviously depend in part on the tube itself, since the absorption of the radiation leaving the anode varies according to the material of the window of the tube. In addition, the secondary current passing through the tube influences the measured value of absorption coefficient, especially if the secondary current passing is considerable, while it has already been noted that the observed absorbability of the radiation depends to some extent on the gas ionised by it. For these, and other reasons, it would appear more desirable always to express the ionisation ratio $S/P$ as a function of secondary voltage, rather than as a function of average mass-absorption coefficient.

It has already been noted that Kay, using filter paper scatterers of considerable thickness, observed regions of constant ionisation ratio $S/P$, whereas in the present experiments, and in those of Sen Gupta, the region of constant ionisation ratio had disappeared even with relatively thin scatterers. The fact that different rates of displacement of the point D are observed for different primary apertures suggests a reason for this discrepancy; it is probable that the aperture used in these experiments was of such a size as to give a much more rapid rate of displacement than the aperture used in Kay's experiments — this latter was a single circular hole, the size, however, not being indicated. The effect of the primary aperture was apparently unknown to Sen Gupta who, therefore, would probably vary this aperture indiscriminately to obtain a suitable ratio of $S/P$. The irregular rate of displacement of the point D evident in his experiments probably arises from this cause.
Qualitative Variations of the scattering Curves:

In the present experiments the pronounced influence of the primary aperture was observed when using two different apertures alternately, every other experimental condition remaining unaltered. Figure 15 shows the two scattering curves obtained in the case of paraffin wax 4.7 mms. in thickness. The radiation used was from the Müller tube No. 311704 operated on the circuit shown in figure 6, and passing a constant secondary current of 0.20 mA throughout the whole series of observations. Radiation intensities were measured by the ionisations produced in SO₂ at atmospheric pressure contained in the ionisation chambers shown in figure 3. Incident and secondary apertures were all of diameter 4.4 cms. and remained unchanged in both cases. The obvious difference between the curves (a) and (b) of figure 15 can be attributed only to the change in the primary aperture.
In (a) the aperture used consisted of a single hole of 2 mm. diameter; in (b) the aperture was that used in the series of experiments summarised in figures 12, consisting of 16 holes of varying diameters from 0.2 to 0.7 mm. The two curves are entirely distinctive, (a) showing an almost uniform decrease of the secondary ionisation relative to the primary as the operating voltage across the tube is reduced, while (b) shows all the characteristics of the scattering curves previously described. This remarkable difference is quite unmistakable and has been observed many times. It cannot be regarded as due to any observational error, for, apart from the fact that the difference is greater than any reasonable estimation of possible error would allow, the method of observation in the present case eliminates any variations due to instrumental peculiarities, — at least so far as such variations can be eliminated. The experimental points of figure 15 are obtained thus: four to six readings of deflection of the secondary electrometer using aperture (b) are observed for a given deflection of the primary electroscope; three or four similar readings are observed for aperture (a); these are followed by a further one or two readings with (b). The points as plotted are the averages of these readings. It must be evident, therefore, that any variation of the measuring instruments would be extremely unlikely to give rise to the observed curves, even in a single case; when the difference in the curves is obtained as frequently as the experimental conditions are made appropriate, such a possibility of error may be dismissed. This point has been emphasised here because many experimenters, in critical surveys of work performed in this laboratory, have questioned the accuracy claimed for their results by various observers. It is usually stated — admittedly without much evidence in support — that the accuracy of observation is from 1% to 2% of the measured deflec-
tions of the secondary electrometer for a deflection of 12 scale divisions of the primary electroscope at various secondary voltages with the apertures (a) and (b), the observations being in the order given.

It will be observed that the deviations of any single reading from the mean value vary from zero to a maximum of slightly over 1%, while the average root-mean-square deviation is easily found to be $0.04$, in the case of both sets of observations (a) and (b). In the case of the present observations the probable error of any one reading is therefore $0.03$. It must be clearly understood that this estimate of probable error applies only to this particular experiment, and, further, that it takes no account of errors arising from the measurement of secondary voltages (which, incidentally, are rather large in this experiment). To obtain any reliable idea of the probable errors of observation in an experiment, the above analysis must always be carried out for that particular experiment. The deviations shown in Table 4 may, however, be taken as typical of the observations on scattering described in this work.

From the above considerations it must be evident that the changes observed in the form of the scattering curves when variations are made in the primary apertures are not the result of observational uncertainties, but denote some significant variation in the intensity relation between the primary and scattered beams of radiation. A further example of the complete qualitative change in the nature of the scattering curve when the only change in the experimental...
<table>
<thead>
<tr>
<th>Aperture Voltage</th>
<th>S</th>
<th>ε</th>
<th>S - S</th>
<th>Aperture Voltage</th>
<th>S</th>
<th>ε</th>
<th>S - S</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b) 65 kV</td>
<td>11.95</td>
<td>11.94</td>
<td>-0.01</td>
<td>(b) 49 kV</td>
<td>11.92</td>
<td>11.95</td>
<td>+0.03</td>
</tr>
<tr>
<td>&quot;</td>
<td>11.95</td>
<td>&quot;</td>
<td>-0.01</td>
<td>&quot;</td>
<td>11.98</td>
<td>&quot;</td>
<td>-0.03</td>
</tr>
<tr>
<td>&quot;</td>
<td>11.98</td>
<td>&quot;</td>
<td>-0.04</td>
<td>&quot;</td>
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<tr>
<td>&quot;</td>
<td>11.9</td>
<td>&quot;</td>
<td>+0.04</td>
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</tr>
<tr>
<td>&quot;</td>
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<td>&quot;</td>
<td>+0.09</td>
<td>(a)</td>
<td>12.45</td>
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</tr>
<tr>
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<tr>
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<td>&quot;</td>
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</tr>
<tr>
<td>&quot;</td>
<td>12.8</td>
<td>&quot;</td>
<td>0</td>
<td>&quot;</td>
<td>11.95</td>
<td>11.95</td>
<td>0</td>
</tr>
<tr>
<td>(b) 72 kV</td>
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conditions is a variation of the primary aperture is illustrated in figure 16. This shows the scattering from a plate of paraffin wax 3 mms. in thickness, the intensities of the scattered and transmitted radiations being measured by the ionisations produced in air at atmospheric pressure. The tube used was the Philips No. 362619 operated from the constant potential direct current apparatus shown in figure 8, passing a constant secondary current of 1.0 mA. The aperture at '1'

(see figure 2) was of 4.4 cms. diameter, while 'a₁' and 'a₂' were 2.8 cms. and 4.4 cms. in diameter respectively. Curve (a) refers to a primary aperture consisting of four drilled holes, each 0.6 mm. in diameter, on the corners of a square of side 6 mms. Curve (b) is for a primary aperture consisting of a single hole 1 mm. in diameter. (The ratios S/F are reduced to the same scale).

Figures 15 and 16 illustrate only two of a large number of experiments in which this effect has been observed, both exhibiting a
feature, however, which has proved perfectly general throughout the present work. It will be observed that the region of constant ionisation ratio $S/P$ appears, in both cases, in the curve relating to the smaller primary aperture. All the experiments performed in connection with this question of primary apertures have indicated that the relative ionisations produced by the secondary and primary radiations vary continuously with voltage unless the aperture is small, where by a 'small' aperture is meant one with a diameter of less than 1 mm. To test the generality of this conclusion a series of observations was made using a slit of variable width to limit the primary beam entering the ionisation chamber. The slit was of lead 3 mm's. in thickness, the jaws being square-cut and worked by a micrometer screw; a calibration of slit width against micrometer screw reading was made using a travelling microscope, and this calibration checked at various points by standard gauges. A fairly accurate knowledge of slit width was thus obtained. In use the slit was placed with its length horizontal. Figure 17 illustrates the results obtained using the Philips tube No. 362619 on the constant potential direct current apparatus; radiation intensities were measured by the ionisation produced in air at atmospheric pressure. The apertures at 's$_2$' and 'i' were each of diameter 4.4 cms., the primary and 's,' apertures being as indicated. From figure 17 it is obvious that a rapid displacement of the point D towards the direction of lower voltages occurs as the width of the primary aperture is decreased. In addition, the smaller apertures produce a complete modification of the curve on the side of higher secondary voltages, a gradual diminution in the secondary ionisation relative to the primary occurring as the secondary voltage is increased beyond certain values. In this series of curves the measured voltages are considered accurate to within 3 kV,
while the average probable error in the determination of ionisation

![Graph showing scattering from 3mm Paraffin Wax at 45°](image)

ratios is not greater than \( \frac{1}{2} \% \).

Figure 18 shows three further curves using the unfiltered radiation from the Müller tube No. 311704 operated from the constant potential direct current apparatus. Ionisations are again measured in air at atmospheric pressure. The primary apertures in this series consist of circular holes of the diameters indicated on the figure. While the scattering curves indicate a displacement of the point D of a magnitude not outside the experimental error in determining the
voltages, they do nevertheless suggest a displacement with decreasing size of aperture in the same direction as that demonstrated unmistakably in the series of experiments summarised in figure 17. The curve (figure 19) showing the position of D as a function of
width, or diameter, of the primary aperture indicates why this latter observed variation is small. It will be evident that the displacement of the point D is not very rapid until the width, or diameter, of the primary aperture is reduced below about 0.3 mm. For sizes less than this figure it would appear that the curve tends towards the origin. This means therefore, that plotted as a function of average mass-absorption coefficient, the point D would move indefinitely towards the right as the width of the primary aperture was indefinitely decreased. It is obvious, however, that towards the region of smaller mass-absorption coefficients the curve would show a considerable modification as compared with the typical form of figure 10.

The reason for this gradual decrease in the relative value of the secondary ionisation is not clear. It is true that the quantum theory predicts a decrease in the intensity of the scattered radiation as the frequency of the incident radiation is increased — in accordance with equation (5) — but this obviously cannot be the explanation in the present case since the decrease is the result of a change in the primary aperture only; in fact, in the range investigated, it does not appear at all with some apertures. The only evident effect of a very small aperture is a possible filtering of the heterogeneous beam at its edges, — an effect which would tend to give rise to the observed increase of the primary ionisation relative to the secondary, as can be seen from the following consideration. The primary beam entering the ionisation chamber may be supposed to consist of two parts, one part passing through the clear aperture and the other part being filtered through the edges of the aperture. This latter part must increase relatively to the former as the penetrating power of the radiation is increased and contribute to the ionisation produced by the primary beam. A decrease in the ionisation ratio S/P would
therefore result.

The Effect on the scattering Curves of Variations in the Experimental Conditions:—

The results so far considered demonstrate clearly that the nature of the scattering curve observed in these experiments is to a large extent, if not completely, determined by two factors, viz. the thickness of the plate of scattering material, and the dimensions of the aperture limiting the beam of radiation entering the primary ionisation chamber. Before proceeding to a discussion of the experiments it is necessary to ascertain the extent of the influence of other factors on the nature of the observed scattering. This is done in a series of observations which will be described; it may be stated at once however, that so far as the present experiments are concerned no variation in the form of the scattering curves has ever been observed to result from a variation in experimental conditions other than the two above mentioned.

Variation of the X-ray Tubes and the Sources of high Potential:—

The principal factors which may at first sight appear likely to show some evident influence on the scattering function have already been enumerated on pages 42 and 43; there are others which, with one exception, are of minor importance.

When heterogeneous radiation is used the energy-wavelength distribution depends, obviously, on the X-ray tube itself, the material of the window through which the radiation leaves the tube affecting the amount of filtering undergone by it. For instance, the tubes fitted with a Lindemann glass window transmit a much greater proportion of radiation of long wavelength than do the tubes in which the window is of a material of greater absorbing power. The result
is that the constitution of the beam of radiation incident on the scatterer, and consequently also that of the scattered radiation, depends to a large extent on the X-ray tube. In the present work the perfectly definite result has been obtained that the scattering curve, expressed as a function of secondary voltage, depends in no observable way on the tube giving the radiation incident on the scatterer. On the other hand, it is obvious that the average mass-absorption coefficient of the incident radiation must, for any given voltage across the tube, depend on the amount of filtering to which it is subjected on emerging through the window. Hence, plotted as a function of average mass-absorption coefficient, the scattering curves for the radiations from two different tubes may be situated in altogether different regions of absorbability. Figure 20 illustrates scattering curves obtained from a plate of paraffin wax of 3 mm thickness, the tubes used being the three previously described, viz., Müller No. 311704, Philips No. 362619, and Siemens No. 234303, operated on the constant
potential direct current apparatus. Except for the change of tube, all the other experimental conditions are identical in the two cases. Ionisation in air was used to measure radiation intensities, and the primary aperture was one used previously, four holes each 0.6 mm. in diameter. The aperture at 's' was 2.8 cms. in diameter, those at 's₁' and 'i' each being of 4.4 cms. diameter. It will be seen that the positions of the point D are, within experimental error, nearly identical, while the form of the curves also remains unaltered. This particular comparison is merely one example taken from a number of such experiments, which though involving different apertures and scatterers, all substantiate the fact that the X-ray tube has no appreciable effect on the curve so long as the scattering is considered as a function of secondary voltage only. As regards the marked difference when absorption coefficients are considered, it may be interesting to note that the value of \( p(np)_{Al} \) of the point D is, in the case of tube No. 311704 about 4.4, while for the tube No. 362619 it is 9.8, and for the tube No. 234303 it is 2.38.

Examination of the curves obtained using different sources of high potential to operate the tube, other experimental conditions remaining unchanged, reveals an interesting fact. It appears that the secondary peak voltage observed for the point D when the tube is operated from a high tension transformer has the same value as the constant voltage observed for that point when the tube is operated on c.p.d.c. In figure 21 are shown the curves obtained when the tube No. 311704 was operated on c.p.d.c. and on two different high tension transformers, one having a step-up ratio twice that of the other, both being operated from the a.c. mains. The curve marked a.c. transformer II is considerably less accurate than the average curve illustrating this work, and should not be attributed much weight; nevertheless
the three curves show clearly that there is no appreciable difference between the peak value of secondary voltage and the constant value of voltage of the region D. This, of course, means that the position

![Diagram](image)

of the point D is dependent on the maximum frequency of the radiation rather than on the distribution of energy among the different wavelengths; the fact that there is no observable dependence on the gas ionised also suggests the same conclusion.

A third feature which merits consideration at this point is the effect of filtration of the primary beam incident on the scatterer, say by interposing aluminium in the path of the radiation before it falls on the scatterer. The effect of so doing is, of course, to increase the average penetrating power of the beam of radiation as a whole, and it might be expected that some effect on the scattering curve would be apparent. Considered, as before, as a function of
secondary voltage, it is observed that the ionisation ratio \( S/P \) shows almost identical variations whatever the thickness of filter placed before the incident beam. Figures 22a and 22b illustrate this fact;

![Graph showing scattering from 3mm paraffin wax at 45°](image)

it will be evident that, within the limits of error, the secondary voltage of the point D remains unchanged for different thicknesses of aluminium filtering the incident beam. Figure 22a gives the results of an experiment designed specifically to test this feature, the radiation from tube No. 362619 operated on c.p.d.c. being scattered from 3 mm of paraffin wax, radiation intensities being measured by the ionisation produced in air at atmospheric pressure. Figure 22b gives a very satisfactory confirmation of this observation in that it represents results obtained at widely different times, though under the same experimental conditions, the primary beam having been filtered for other purposes. Despite this the curves show clearly that the filtering aluminium has no effect, outside the possible error of measurement, on the position of D. In figure 22b the radiation from the tube No. 354624 operated on the circuit of figure 6 was
scattered by 4.7 mm. of paraffin wax, intensities being measured by the ionisations produced in SO₂ at atmospheric pressure. In the two lower curves of figure 22b the voltages taken up by the secondary electrode system were measured directly by comparisons with the deflections produced by a standard cell. These voltages varied between 1.68 and 1.90 volts approximately, the values all being reduced to the same scale.

From the results expressed in figures 22 it will immediately be clear why, when plotted as a function of secondary voltage, the scattering curves are practically independent of the tube used to produce the radiation. This follows as a consequence of the fact that the only essential difference between the X-ray tubes is a difference in the filtering effect of their windows.

The magnitude of the secondary current passing through the tube is a factor on which the energy-wavelength distribution of the radiation depends, and may on that account be thought to have some influence
on the type of result obtained, but throughout the present work extreme changes made in the value of the secondary current have produced no observable changes, either in scattering or absorption experiments. In this work it has been a common procedure to make observations over the same regions of a scattering curve with different values of secondary currents. Provided that the saturation value of the ionisation current is being measured, this practice has never resulted in any appreciable change in the ionisation ratio, even although the secondary current has been varied by a factor of three or four. Several experiments have been made at different times designed specifically to observe any variation in the position of the point D as the secondary current is varied. One such experiment is illustrated in figure 23, the value of the secondary current in one case being

![Graph showing scattering from 3 mms. paraffin wax at 45°](image)

five times that in the other; all experimental conditions apart from this being the same. In this experiment the radiation from tube No. 311704 operated on the circuit of figure 7 was scattered from 3 mms. of paraffin wax, radiation intensities being measured by the ionisation produced in air at atmospheric pressure. A point worth noting
is the large difference observed in this experiment between the two measured values of average mass-absorption coefficients for any given secondary peak voltage. For instance, the value of $P(\mu/\rho)_{Al}$ at 50 kV is 4.32 with a secondary current of 1.5 mA, and 6.40 with 7.5 mA. Thus, plotted as a function of average mass-absorption coefficient, the scattering curves in these two cases would not be the same. It is of course realised that for the different values of secondary current the effective secondary voltage may vary while the observed inverse value is unaltered; this ambiguity does not apply, however, when the tube is operated with direct current, using which yields results of exactly similar type. Perhaps the most conclusive evidence in the present work that secondary current does not affect the observed results is to be found in the absorption experiments which will be considered later; in these a variation in secondary current from 0.2 mA to 2.0 mA has made no appreciable variation in the form of the curve obtained.

Taken in conjunction, the four features just established for the scattering curves observed in these experiments, viz. their independence of the X-ray tube, of the method used to excite it, of the secondary current passing through it, and of the amount of filtering undergone by the beam before falling on the scatterer, allow of an important general conclusion. It may be stated that the ratio of the ionisations produced by the scattered and primary radiations observed in these experiments, in which the penetrating power of the primary radiation is altered only by altering the voltage operating the tube, does not appreciably depend on the energy-wavelength distribution of the radiation scattered. In fact the results suggest that the limiting frequency of the radiation scattered is the important factor in experiments of this type.
Variation of the cross-sectional Area of the scattered Beam of Radiation:

The cross-sectional area of the primary beam of radiation has been shown to be a determining factor of the nature of the scattering curve, but no experiment has ever demonstrated any well-marked effect that can be attributed to a variation in the cross-sectional area of the scattered beam. In the present work systematic experiments have been carried out with apertures of various sizes limiting the incident and scattered beams of radiation, so that both wide and narrow angle cones of radiation have entered the secondary ionisation chamber. This has necessitated an alteration in the sensitivity of the secondary electrometer in order to maintain the two ionisations comparable, but all other experimental conditions have been unchanged. Without quoting the results in detail it is sufficient to say that no combination of incident and secondary apertures has given a scattering curve differing from the average by more than the amount of the experimental error. In this investigation the apertures at 's,' have been varied from 4·4 cms. in diameter, through smaller sizes, to a group of some 400 holes each 1 mm. in diameter spread over a circle of 4·4 cms. diameter; the apertures at 'i' have been varied from 4·4 cms. to 1·1 cms. in diameter. The fact that apertures of these sizes have no effect on the scattering does not, however, preclude the possibility that scattered beams having a cross-sectional area similar to that of the primary may exhibit features different from those which appear normal in the present experiments.

Variation of the Method of Measurement of Ionisation:

The intensity of a given beam of X-radiation is very frequently measured by the ionisation it produces in a gas under specified
conditions; it is the method of measurement invariably adopted in experiments such as those described in this work, and therefore it is a matter of some importance to determine the extent to which these results depend on the particular ionisation measurements involved. It has already been shown that, if the observed ionisation ratio S/P is plotted against secondary voltage, identical scattering curves result, whether the ionisations are measured in air or in SO₂ (see page 40). This result has been obtained under a variety of conditions and seems fairly general. A much more stringent test of this conclusion is, however, illustrated in figure 24, which shows the

Scattering from 3 mm. Paraffin Wax at 45°

Secondary Voltage (c.p.d.c.)

FIG. 24

scattering curves obtained when the radiation from the Philips tube No. 362619 operated on c.p.d.c. is scattered from 3 mm. of paraffin
wax, the radiation intensities being measured in the following manners:

In curve (a) by the ionisations produced in air at atmospheric pressure and contained in the ionisation chambers shown in figure 3.

In curve (b) by the ionisations produced in hydrogen at atmospheric pressure contained in the ionisation chambers shown in figure 4. The hydrogen is ionised by the electrons emitted from gold sheets on the inner front and back walls of the chambers; in these experiments the gold sheets being of thicknesses 0.000302 cms. and 0.000760 cms. respectively. (Hydrogen is ionised only to a negligible extent, compared with any other gas, by the action of the X-radiation itself).

In curve (c) by the ionisation produced in air at atmospheric pressure and contained in the ionisation chambers shown in figure 4.

In curve (d) by the ionisation produced in air in ordinary cubical gold-leaf electrosopes, the usual precautions being taken to prevent the emission of electrons by the metal parts of the electrosopes.

It will be apparent that the curves are essentially similar. (Curve (d), it should be mentioned, has a somewhat lower accuracy than is usual in these experiments). This similarity is very surprising when one considers the great difference in the mode of ionisation in, say, (a) and (b) above; in the one case a fixed mass of gas is being ionised, while in the other the ionisation is caused by electrons which emerge from a depth of metal increasing as the frequency of the radiation increases. If, however, the hydrogen is replaced by air the nature of the curve is altogether changed; there occurs a gradual increase of the secondary ionisation relative to the primary followed later by a decrease, as the secondary voltage across the tube is
reduced. Figure 25 illustrates the type of result obtained in this case. A curve of this general form would result from an application of Compton's theory, in which the scattered radiation, being of slightly longer wavelength than the primary, would not excite the L characteristic radiation of gold until after the primary had done so. Until the scattered radiation was of sufficiently short wavelength to excite the L characteristic radiation it would be less absorbed by the gold sheets than the primary and produce a relatively greater ionisation in the air, resulting in a gradual increase in the ratio S/P as the voltage across the tube is increased. After the scattered radiation is itself able to excite the characteristic this selective effect ceases and the ratio S/P begins to decrease with increasing voltage. This effect is considerably complicated in the present case, however, by the fact that the L electrons from the gold themselves ionise the air in the ionisation chambers, thus tending to neutralise the effect due to the Compton change of wavelength. Similar variations in the scattering curve have been observed when the scattered and primary radiations are both made to pass through equal
thicknesses of silver placed outside the ionisation chambers. The curves are more easily interpreted in this case and will be considered more completely when dealing with absorption. The observations illustrated in figure 25 are not of themselves of much importance but have some interest from the fact that they show a complete disappearance of a constant ionisation ratio \( S/P \) when the ionisation ceases to be due to a simple absorption of the radiation.
DISCUSSION.

Comparison of these results - and, in fact, of practically all the results obtained in this laboratory - with the various theoretical predictions is rendered difficult owing to the fact that the radiation employed is almost invariably heterogeneous. When such is the case a 'mean' or 'effective' wavelength is assumed for the radiation, corresponding to its mean mass-absorption coefficient, and comparisons made accordingly. Such a procedure can not, for obvious reasons, afford a true test of theory since any true comparison of theory and experiment must take into account the heterogeneity of the radiation. If, however, a quantitative relationship between the observed ionisations produced by the primary and scattered radiations is obtainable which does not contain the wavelength explicitly then comparison with theory becomes possible. Such a relationship follows at once from the classical theory. In this the scattering coefficient is the same for all wavelengths, provided the electrons can be regarded as scattering independently, so that at each wavelength of the primary beam the fraction of the energy scattered, at that wavelength, is the same. Hence for all energy-wavelength distributions of the primary beam there results a scattered beam having a similar energy-wavelength distribution and having a total energy which is a constant fraction of that of the primary. The ionisation ratio $S/P$, as defined in the experiments described, should therefore remain constant for all primary radiations, i.e. for all values of voltage across the X-ray tube, provided the primary beam is unpolarised. Experiment has shown, however, that the primary radiation is partially polarised; if we regard this radiation as made up of two parts, one unpolarised, and one plane polarised with the electric vector parallel to the cathode
stream, this latter part is, in these experiments, not effective in producing ionisation in the secondary ionisation chamber; and since the amount of polarisation varies appreciably with the voltage across the tube a correction is necessary on this account. It is a simple matter to calculate the curve to be expected according to the classical theory provided the amount of the polarisation of the primary beam is known. In the present experiments no measurement of polarisation has been made; since, however, various observers have found amounts of polarisation of the primary beam which vary very little for different tubes and scatterers, it appears allowable to take a given set of observations and calculate the curve which would result by applying a polarisation correction factor. Table 5 gives the ratio of the polarised to the unpolarised part of the primary radiation from a Müller tube at various peak voltages.

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<th>Ip/Iu</th>
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<tr>
<td>40</td>
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</table>

We can obtain the polarisation correction factor as follows. Suppose I is the intensity of the partially polarised incident beam, and assume this beam to be made up of an unpolarised part of intensity I_u and a plane polarised part of intensity I_p. Further suppose s_H is the scattering coefficient per unit solid angle in the horizontal direction, and s_V that in the vertical direction, then, since the electric vector of the polarised component of the primary beam is in

*From results kindly supplied by H.K. Pol.*
a horizontal plane, we have, considering scattering at 90°

\[ s_H I = s I_u/2 \]

\[ s_V I = s I_u/2 + s I_p \]

\( s \) being the scattering coefficient per unit solid angle of an unpolarised incident radiation of intensity \( I \). The ratio \( s_V/s_H \) can be directly measured by observing the ratio of the ionisations produced when the cathode stream of the tube is vertical and horizontal respectively, and since \( s_V/s_H = 1 + 2I_p/I_u \) the amount of polarisation in the primary beam is at once obtainable. Noting that the time average of the intensity of the unpolarised part of the primary radiation which is scattered is \( sI_u/2 \), a further simple consideration shows that the ionisation ratio \( (S/P)_c \) which would be observed if the incident radiation \( I \) were unpolarised is given in terms of the ratio \( S/P \) actually observed by the relation

\[ (S/P)_c = (S/P) (1 + I_p/I_u). \]

According to the simple classical theory the scattering curve plotted
as a function of, say, secondary voltage would be a horizontal line, extending over all voltages, as shown by \((S/P)\) in figure 26. Applying the polarisation correction factor to this theoretical curve we obtain the result shown by the curve marked \((S/P)\) in figure 26, which may be compared with the results actually observed in these experiments.

Considering the curve (a) of figure 15 and (b) of figure 16 it is evident that a marked similarity exists between the result to be expected on the classical theory and the result actually observed in these cases. In fact, if allowance is made for the slight uncertainty in the polarisation correction, the agreement may be said to be exact. This applies not only to the two cases cited above but also to large numbers of results which have been obtained using either thick scatterers, or relatively large primary apertures, or both simultaneously.

The predictions of the quantum theory may be obtained in a fairly simple manner. Considering the radiation scattered at 90° equation (6) may be written as

\[
I' = I_0 \frac{\varepsilon^4 r^2 e^2}{2m^2 r^2 e^4} \frac{1}{(1 + \alpha)^2}
\]

This equation gives the intensity \(I'\) of the radiation scattered at 90° to the primary radiation of intensity \(I_0\) and frequency \(c = \frac{\alpha m c^2}{\hbar}\).

Equation (7) may be written as

\[
\frac{I'}{I_0} = \frac{C}{(1 + \alpha)^2}
\]

Now the ionisation produced in a gas by a given radiation is very nearly proportional to its absorption coefficient in that gas, so that if \(i'\) and \(i_0\) are the ionisations produced by the scattered and primary radiations respectively we have that

\[
\frac{i'}{i_0} = \frac{\mu'}{(1 + \alpha)^2} \frac{\mu'}{\mu}
\]

where \(\mu'\) and \(\mu\) are the absorption coefficients of the scattered and
primary radiations respectively - \( \mu' \) not being equal to \( \mu \) according to the quantum theory. The absorption coefficient of a homogeneous X-radiation is approximately proportional to the cube of its wavelength, so that (8) may be rewritten in the form

\[
\frac{I'}{I_0} = \frac{C}{(1 + \alpha)^3} \left( \frac{\nu}{\nu'} \right)^3
\]

(9)
in which \( \nu' \) is the frequency of the radiation scattered at \( 90^\circ \), related to that of the primary by the Compton equation

\[
\frac{\nu}{\nu'} = 1 + \alpha
\]

(10)
Equation (9) gives, then, the ratio of the ionisation produced by the scattered radiation of frequency \( \nu' \) to that produced by the primary of frequency \( \nu \). Applying this to all the constituents of the heterogeneous primary beam we have

\[
\sum \frac{I'}{I_0} = C \sum \frac{1}{(1 + \alpha)^3} \left( \frac{\nu}{\nu'} \right)^3
\]

But the left hand side of this equation is simply the ionisation ratio \( S/P \) observed in the experiments described, so that

\[
\frac{S}{P} = C \sum \frac{1}{(1 + \alpha)^3} \left( \frac{\nu}{\nu'} \right)^3
\]

whence, by equation (10),

\[
\frac{S}{P} = \text{constant}
\]

(11)
Thus this simple quantum theory, based on the scattering function given by equation (6), predicts a scattering curve which is very nearly identical with that of the classical theory. For quantum scattering functions other than (6) it is not possible to obtain the expression for \( S/P \) for a heterogeneous radiation in simple terms. Since, however, other scattering functions (that of Klein and Nishina for instance) differ from (6) only by quantities of the second order in \( \alpha \), the variations to be expected between the various quantum formulae are small in the region of medium wavelengths.
It should be observed, also, that in obtaining equation (11) the radiation has been assumed to be completely modified on scattering; this assumption is not strictly true, but with scatterers of the low atomic number used in these experiments it becomes sufficiently nearly so. Further, as the frequency of the radiation increases the proportion of modified to unmodified scattered radiation increases slightly. A quantitative allowance for the effect of these two factors is, with a heterogeneous radiation, scarcely possible; they must, however, be recognised as present and modifying to a small extent the form of equation (11). To a first approximation, therefore, figure 26 may be taken also to represent the curve to be expected according to Compton's theory.

If now we consider the typical curve obtained when using thin plates of scattering material and small primary apertures (see, for instance, figure 10) it is evident that neither theory appears to be in agreement with observation. In the first place, making due allowance for the varying degree of polarisation of the primary radiation, no region of constant ionisation ratio $S/P$ would be expected, nor would any sudden relative decrease in the rate of ionisation by the scattered beam take place at lower voltages, both of which features are actually observed. Again, the marked difference which appears when different apertures are used to limit the primary beam of radiation is not predicted by either theory. (It is perhaps worth noting at this point that the constant ionisation ratio $S/P$, observed in these experiments, does not, as has often been stated, signify agreement with the classical theory of scattering except over very short ranges in the region of greater penetrating power of the primary radiation, otherwise, as is obvious from figure 26, the polarisation correction becomes considerably greater than the possible experimental
error. With the exception of Dunbar(2c), other observers appear to have neglected this point, because almost invariably the X-ray tube has been used with the cathode stream parallel to the direction of the scattered radiation studied).

The cause of the rapid change in the relative rates of ionisation produced by the scattered and primary radiations — the point designated D in the experiments — is obscure. How far it is justifiable to regard the slope of the scattering curve as discontinuous at this point depends, of course, on the possible error of observation allowed. If a deviation of 1% of the observed value of the ratio S/P is allowed, then it is possible in perhaps one half the number of examples quoted to draw a curve having a continuous slope through the observational points, retaining these within 1% of this curve. In many cases, however, such a procedure is not possible, and we must then regard a real discontinuity in slope as existing. This reality is more convincingly established when it is remembered that the position of the discontinuity can be systematically varied by a systematic change in the dimensions of the primary aperture. Four other experimental variations which these experiments have shown to be without any appreciable effect on the scattering curve are:

(a) the X-ray tube,

(b) the method of excitation of the tube, i.e., whether the current is unrectified alternating current or constant potential direct current,

(c) the amount of filtering aluminium placed in the path of the incident beam before falling on the scatterer,

(d) the secondary current passing through the X-ray tube.

The only factor in the primary radiation not affected by any one of these conditions is the limiting frequency excited, which might
therefore be considered as responsible in some way for the rapid change in the relative rates of ionisation. But again, if this is so, it is extremely difficult to see why the thickness of the scatterer or the size of the primary aperture should exert so marked an influence on the position of the point D.

In this connection some calculations carried out by S.R. Khastgir are of considerable interest. He calculates the scattering curves to be expected from an application of the quantum theory allowing for the fact that the primary radiation is heterogeneous, using C. T. Ulrey's empirically determined energy-wavelength distribution curves as representing the primary radiation falling on the scatterer. The area included under these curves gives a measure of the ionisation produced by the radiations they represent. Then, employing an equation of the form (9) above, Khastgir calculates the energy-wavelength distribution of the scattered beam for various voltages, the areas under these curves giving the secondary ionisations in the various cases. Thus $S/P$ can be obtained. Table 6 and figure 27 illustrate Khastgir's calculations.

$$\begin{array}{|c|c|c|}
\hline
\text{Voltage} & S/P & S'/P' \\
\hline
50 \text{ kV} & 1.0 & 0.93 \\
40 & 1.0 & 0.94 \\
35 & 1.0 & 0.93 \\
30 & 1.0 & 0.95 \\
25 & 1.01 & 0.90 \\
20 & 1.01 & 0.92 \\
\hline
\end{array}$$

$S/P$ represents the ratio of ionisations when both beams are unintercepted, $S'/P'$ when both are intercepted by $0.15 \text{ cm. of Al}$. The curve marked $S/P$ in figure 27 is in almost exact agreement with equation (11). In any actual experiment, however, the two beams
of radiation must of necessity be intercepted by some material before reaching the gas in the ionisation chamber — by the material of the ionisation chamber window and, on the average, by half the thickness of the scattering material in addition to the intervening air — so that the curve $S'/P'$ must more nearly represent the true experimental conditions. The remarkable similarity between this curve and those actually observed in the experiments described will be at once evident, suggesting an agreement with the quantum theory. But this makes the origin of the slope-discontinuity at $D$ even more obscure. It is, in fact, incomprehensible how the point $D$ — which, if Khastgir's calculations are correct, quite obviously exists — can arise from quantum calculations involving only functions which are themselves continuous.

The occurrence of a region of constant ionisation ratio $S/P$ in these experiments is a result of such extreme simplicity that some explanation, equally simple, should be evident. As shown by equation (9) the quantum theory would explain the constant ratio for an unpolarised radiation as due simply to a compensation between the falling off in the intensity of the scattered radiation and the relat-
ive increase in its ionising efficiency (owing to its greater absorbability as compared with the primary) as the frequency is increased. Such a compensation would involve the exact balance between two factors which vary, over the range used in the present experiments, by as much as 30%. For a balance of this kind to occur in every case in which a constant ionisation ratio has been observed seems scarcely probable; not only so, but such a balance would tend to be upset by other factors which must come into consideration, as, for instance, the slight variation in the ratio of modified to unmodified scattered radiation, the occurrence of double-scattering, the absorption of the radiation in the scatterer itself, as well as a variation in the amount of polarisation in the primary beam. This latter has already been considered, and shown to give rise to a difference of some 9% in the ionisation ratio $S/P$ over the range of voltages used. According to the figures given by Backhurst\(^{(29)}\), the ratio of modified to unmodified scattered radiation does not vary by more than about 5% with the scattering materials used in the present experiments; the intensity of twice-scattered radiation may, however, reach a considerable fraction of that of the radiation undergoing only single-scattering. In the case of a scatterer of paraffin wax 0.5 cm. in thickness penetrated by a beam of medium hardness — say having a $p(\mu/\rho)_{Al}$ value of 5 — approximately 8% of the total scattered intensity consists of twice-scattered radiation.

That an exact compensation between all of these variables actually occurs seems extremely unlikely. It seems more satisfactory to explain the constant ratio of the ionisations by supposing that the primary and scattered beams of radiation have equal 'ionising coefficients', irrespective of whether their actual constitutions are the same or not, and that the intensity of the scattered radiation relative
to that of the primary is independent of the wavelength, at least over the range of constant ionisation ratio found in these experiments. A considerable amount of evidence in support of this conclusion is found in the experiments of Richtmyer and of Hewlett, both of whom find the scattering coefficients in the case of light elements (aluminium and carbon) to be practically independent of the wavelength over the range 0.1 Å to 0.4 Å approximately. More discussion can be devoted to this point after the absorption experiments have been considered.
The laws governing the absorption of X-radiation of a single wavelength are comparatively simple, equal fractions of the radiation intensity being cut off by equal thicknesses of any given absorbing element. This at once gives rise to an exponential law of absorption, so that the intensity, \( I \), of a beam of homogeneous radiation on passing through an absorbing layer of thickness \( x \) is given in terms of the initial intensity \( I_0 \) by the equation
\[
I = I_0 e^{-\mu x}
\]
where \( \mu \) is the linear-absorption coefficient of the radiation in the particular absorber. For a single wavelength, \( \mu \) is a characteristic constant and may be taken as completely specifying the quality of the radiation of that wavelength. For any given absorbing element the linear-absorption coefficient at any wavelength is related to that wavelength by the approximate empirical equation
\[
\mu = \sigma + \alpha \lambda^3
\]
It is in general more convenient to use in place of a linear-absorption coefficient the mass-absorption coefficient \( \mu/\rho \), \( \rho \) being the density of the absorbing element. Equation (13) then becomes
\[
\frac{\mu}{\rho} = \frac{\sigma}{\rho} + \alpha \lambda^3
\]
The term \( \sigma/\rho \) represents, as is well known, that part of the absorption which is due to scattering, and for absorbing materials consisting of light elements \( \sigma/\rho \) has a practically constant value. The experiments of Richtmyer and of Hewlett previously referred to have shown that the mass-scattering coefficient has a value varying between 0.15 and 0.16 over a wavelength range of 0.1 to 0.7 Å in the case of aluminium; for carbon the value is about 0.175 over a similar range, decreasing
slightly for longer wavelengths. For radiations of medium wavelengths, this value is small compared with that of $\mu/\rho$, and from this fact is obtained the relation used to derive equation (9), viz., that $\mu$ is approximately proportional to $\lambda^2$.

With a heterogeneous radiation these simple absorption-wavelength relations no longer apply, except presumably to each homogeneous constituent. It is possible, however, to consider a 'mean' or 'effective' mass-absorption coefficient of the whole heterogeneous radiation as has been previously explained (see page 30). In the experiments to be described an empirical equation is developed, involving the 'mean mass-absorption coefficient' as already defined, which may be regarded as taking the place of equation (14).

Results which are, from a theoretical point of view, of more significance are obtained from a study of the relative absorptions of a primary and scattered radiation. Figure 28 illustrates diagrammatically the simple method which may be used in such observations. A and A' represent equal thicknesses of any given absorbing material placed in the paths of the transmitted primary and the scattered
radiations respectively; the intensities of these radiations are correspondingly reduced by an amount depending on the thickness of the absorber. If the ratio of the intensities of the scattered and primary radiations is observed as a function of the thickness of the absorbing element, then the absorption of the scattered radiation relative to the primary in that element is at once obtained. The intensities may be measured, as before, by the ionisations produced in a gas under stated conditions. It is fairly evident that according to the classical theory the ionisation ratio should remain unchanged as the thickness of the absorbing material is varied, because the scattered and primary radiations are of exactly the same quality and must undergo equal diminutions in intensity on passing through the same absorbing medium.

The quantum theory on the other hand leads to a different result on account of the change in wavelength of the radiation which is assumed to occur in the process of scattering. According to equation (3) the wavelength of a radiation scattered at 90° to the primary is longer by 0.0242 A than the wavelength of the primary radiation. This it will be observed is independent of the wavelength of the primary radiation, so that the relative difference between the two radiations is greater for shorter wavelengths. Hence with heterogeneous beams the scattered radiation must be, on the whole, less penetrating than the primary, provided the scattering coefficient is the same for all wavelengths, since it has on the average a longer wavelength. Increasing thicknesses of the absorbing material would therefore reduce the intensity of the scattered radiation relative to that of the primary. It is possible that a decrease in the scattering coefficient with increase in wavelength might result in the scattered beam having an energy-wavelength distribution such that it
appears of the same, or even of less, absorbability than the primary. This possible 'compensation effect' might cause the scattered and primary beams to appear equally absorbable over limited ranges of absorbability, a fact actually observed in the experiments to be described. The possible validity of this explanation will be discussed after a consideration of the experimental results.

Though a theoretical calculation is scarcely possible for heterogeneous radiations the case of a single wavelength is simple, provided the differential absorption between the primary and scattered beams in the scatterer is neglected. For absorbing materials at A, A' of thicknesses x we have

\[ I_s = sI_0 e^{-\mu'x} \]
\[ I_p = I_0 e^{-\mu x} \]

in which \( s \) is the scattering per unit solid angle, for the given experimental conditions, of the initial beam of intensity \( I_0 \). \( I_s \) and \( I_p \) are respectively the intensities of the scattered and primary radiations after passing through the absorbers, their linear-absorption coefficients in the absorbers being \( \mu' \) and \( \mu \). From these two equations we obtain

\[ \frac{I_s}{I_p} = s e^{-(\mu' - \mu)x} \]

From this it follows that

\[ S/P = \left( \frac{\mu'}{\mu} \right) s e^{-(\mu' - \mu)x} \] (15)

in which the ordinate \( S/P \) decreases exponentially as \( x \) increases.

In general then, one would expect the simple experiment of figure 28 to throw considerable light on the process of X-ray scattering and absorption, and to be capable of distinguishing clearly between classical and quantum theories. It appears in addition to be the type of experiment most suited to exhibit the \( J \)-absorption discontinuities which occur in the radiation scattered from light
elements. Dunbar(5), however, has attempted to show that if heterogeneous radiations are used in the experiment then the results may be inconclusive as regards differentiation between classical and quantum theories on account of the possible compensation effects which have been mentioned above—just as in the case of scattering when medium wavelengths are considered.
Empirical Absorption Formulae:

If the 'average mass-absorption coefficient' of the general unfiltered radiation from an X-ray tube is plotted against the secondary voltage (either a.c. peak, or c.p.d.c.) exciting that radiation the curve obtained is that shown in figure 29a. This represents a typical set of results obtained with the tube No. 311704 operated on constant potential direct current (figure 8) with a constant secondary current of 2.0 mA. The values of mass-absorption coefficients have been measured in aluminium as previously described, by observing the thickness of aluminium required to reduce the intensity of the primary beam transmitted through the scatterer by approximately 50%, intensities being measured by the ionisations.
produced in air at atmospheric pressure. In this set of observations the intensity diminution has been kept in all cases within 2% of the 50. It may easily be verified that these observations can be represented with a high degree of accuracy by the equation

\[(V + 9)(\mu/\rho + 0.39) = \text{constant}\]

where \(V\) is the secondary voltage in kV. This is made more clear by figure 29b in which \(\log(V + 9)\) is plotted against \(\log(\mu/\rho + 0.39)\), the slope of the resulting straight line being, very closely, -1.

That this simple result is not peculiar to the Müller tube is made evident by figures 30, in which an exactly similar kind of relation is found to hold for the unfiltered radiation from the Philips tube No. 362619 operated on c.p.d.c. This tube has a Lindemann glass window, so that the composition of the radiation is very different from the previous case, a fact evident from the much
higher values observed for mass-absorption coefficients. The

![Graph image]

observations are in this case well represented by the equation

\[(V + 25)(\mu/\rho - 0.66) = \text{constant}\]

as is shown again by the logarithmic plot of \((V + 25)\) against \((\mu/\rho - 0.66)\) in figure 30b.

It may be said then that the average mass-absorption coefficient
of the unfiltered radiation from the tube is related to the secondary
voltage by an equation of the general form
This conclusion is based not only on the two sets of results quoted but it has been found that an equation of the form (16) accurately represents the observations in every case to which it has been applied; this is true whether the exciting voltage has been alternating or constant. The generality of the equation may perhaps be best illustrated by figure 31, which includes observations for scattering materials of paraffin wax and filter paper of various thicknesses. It will be evident that, by a suitable choice of constants, the observations can be made to fit fairly accurately to a straight line of slope -1. The results are still more closely represented by (16) if a slight lateral shift of the line is allowed in the various cases. A further important point is that the equation does not appear to be valid only over limited ranges; it has been found still applicable throughout the most extensive range of absorbability of the unfiltered radiation studied in these experiments.
from $p(\nu/\rho)_{Al}$ values of 5.0 to 34.5, obtained using the tube No. 354624. It is not felt, however, that this equation (16) has any important significance, but that it represents merely a convenient method of obtaining average mass-absorption coefficients at any voltage when these are known accurately for three voltages. The significance of the constants $\alpha$ and $\beta$ is not clear, though they obviously depend on the thickness of the scatterer and to some extent on whether the secondary voltage is constant or alternating. The value of $\beta$ in the observations of figure 31 is negative and appears to vary systematically with the thickness of the scatterer, while $\alpha$ does not exceed $-4$ kV and generally is, within the limits of accuracy of measurement, zero. With c.p.d.c. however, the value of $\alpha$ appears to be much larger, while $\beta$ may become positive, so that no definite conclusion can be drawn with regard to these constants. As has already been mentioned, the measured value of average mass-absorption coefficient depends on the gas ionised, so also therefore must the value of $\beta$. 
Experiments on selective Absorption:

It is well known that, in order to excite a characteristic radiation of any element by a given primary radiation, the primary must contain a wavelength at least as short as that of the corresponding absorption edge of the element. This fact at once suggests a simple way of exhibiting the Compton change of wavelength on scattering, because if such change does occur it is possible for the primary radiation (of figure 28) to excite a characteristic radiation of absorber A while the modified secondary is still unable to excite the similar characteristic radiation in the absorber \( A' \) \( = \) \( A \) and \( A' \) being supposed identical. The results of such an experiment are shown in figure 32. Curve (a) is the usual type of scattering curve observed in these experiments; (b) represents observations in which equal absorbers of silver 0.0044 cm. were placed at \( A \) and \( A' \) (see figure 28). The tube No. 354624 was used on the circuit of figure 6, radiation intensities being measured by the ionisation produced in \( \text{SO}_2 \) at...
atmospheric pressure; the beam of radiation from the tube was filtered by 0.164 cm. of aluminium before falling on the scatterer. The primary aperture was the series of irregular holes (16 in number) previously referred to; ordinates represent the voltage taken up by the secondary electrode for a given deflection of the primary electroscope.

Curve (b) shows almost exactly the type of variation to be expected in $V_s$ if the Compton increase of wavelength actually occurs on scattering. Proceeding from low to high voltages, when a voltage exceeding about 26 kV is applied to the tube the primary beam of radiation is able to excite the K characteristic radiations of silver in the silver absorbing sheet A, since radiations of just sufficiently short wavelengths are present in the primary beam at that voltage. This involves an increased absorption of the primary beam (the K absorption in the silver) but not of the scattered, because the scattered beam does not yet contain wavelengths sufficiently short to excite the K radiations of silver, hence the ratio $S/P$ must begin to rise; the curve (b) shows this to occur, and at about the correct secondary voltage. With a slight increase in voltage the scattered radiation will become able to excite the characteristic radiations in A', but now only to a small extent compared with the primary, so that the primary radiation still suffers a greater absorption than the secondary and consequently $S/P$ continues to rise. This continues until the secondary voltage is considerably above the limiting value and the two beams excite the characteristic radiation to practically the same extent; after this the secondary beam becomes more absorbed than the primary and $S/P$ falls. (In the figure variations in $V_s$ are shown, but these are precisely the same as in the corresponding ratio $S/P$).
All the experiments performed in this work with absorbers of silver have shown a similar result to that indicated above, and can therefore be interpreted as confirming the existence of the Compton effect in scattering. The same result (though with complications due to the emission of L electrons) has been shown to occur when gold sheets are placed inside the ionisation chambers (see page 35 and figure 25). In the case of gold the whole L series is excited by a voltage of about 14 kV, (there are, of course, three separate excitation values, but these are close together), and it will be observed that between 20 kV and 50 kV the primary beam is being absorbed to a greater extent than the secondary, after which the reverse takes place just as in the case of silver.

These experiments showing selective absorption must be regarded, then, as fully in accord with the Compton increase of wavelength on scattering — the so-called 'modification'. That such modification does not invariably occur, however, is indicated by many experiments performed in this laboratory — for instance those described by C.G. Barkla and S.R. Khastgir\(^{(2)}\), in which the ionisation ratios \(S/F\) remained constant and equal over a considerable range of frequencies of the primary beam, both when the two beams were unintercepted and when they were intercepted by 0.006 cm. of silver. Yet these same experiments, under apparently identical conditions, yielded results of precisely the same nature as those illustrated by figure 32, leading the authors to conclude that the modification depends on a relationship between the radiation tested and the testing substance.

Aluminium has no characteristic absorption edge within the range of voltage used in these experiments, so that if the primary and scattered beams are intercepted with this element results of a simpler nature might be expected. In the work referred to above, Barkla
and Khastgir found that the ionisation ratio $S'/P'$ when the two beams were intercepted by 0.16 cm. Al was markedly lower than the corresponding ratio $S/P$ for the unintercepted beams. This result indicated that the scattered beam was more absorbable than the primary, or, in other words, the scattered radiation was modified. The modification, however, followed a law not to be expected from the quantum theory in that (except for high frequencies) the two ratios $S/P$ and $S'/P'$ were represented by two parallel and horizontal lines, a fact which indicates that there is a constant difference between the average mass-absorption coefficients of the primary and scattered radiations. In the present experiments the general type of result observed has been that shown in figure 33; (the absorbing aluminium cuts off an amount of the primary radiation varying from 17% to 34%). In this particular experiment the radiation from the tube No. 362619 operated on c.p.d.c. was scattered from 3 mms. of paraffin wax, the radiation intensities being measured by ionisation produced in hydrogen by the electrons emitted from gold. (It was thought that a difference might be evident if the absorbing aluminium was placed inside the ionisation
chambers, but inserted so that the radiation passed through it before penetrating the gold sheets. No essential difference was observed, the curve being almost identical with that for the intercepted beams in figure 33). When the absorbing sheets of aluminium were considerably thinner, however, the type of result obtained was quite different. Figure 34 shows the observations obtained when intercepting both beams of radiation by 0.01 cm. of aluminium, absorbing approximately the amounts shown in the figure. It will be evident that, within the limits of experimental error, the intercepted and unintercepted ionisation ratios are equal, and the result can be interpreted as showing no modification of the scattered radiation. The probable cause of this equality will become apparent when the results on relative absorptions of the primary and scattered radiations have been considered.

Relative Absorptions of primary and scattered Radiations:

The relative rates of absorption of the primary and scattered radiations may be very easily studied with the experimental arrangement
of figure 28, varying the thickness of the absorbers $A$ and $A'$ and observing the changes in the relative intensities of the primary and scattered beams. The theory of the experiment is extremely simple for homogeneous radiations; if $I_s$ is the intensity of the scattered beam after passing through a thickness $x$ of absorbing material we have that

$$I_s = (I_s)_0 e^{-\mu' x}$$  \hspace{1cm} (17)$$

where $(I_s)_0$ is the original intensity of the scattered beam and $\mu'$ is its linear-absorption coefficient in the material of the absorber.

Using a similar notation we have for the primary beam

$$I_p = (I_p)_0 e^{-\mu x}$$  \hspace{1cm} (18)$$

so that

$$I_s/I_p = (I_s/I_p)_0 e^{-(\mu' - \mu) x}$$

or

$$S/P = (\mu'/\mu)(I_s/I_p)_0 e^{-(\mu' - \mu) x}$$  \hspace{1cm} (19)$$

According to quantum theories $\mu'$ is not equal to $\mu$ so that (19) represents an exponential curve whose ordinate $S/P$ decreases as $x$, the thickness of the absorbing material, increases. A constant value for $S/P$ with increasing $x$ might be interpreted as showing that $\mu'$ and $\mu$ are equal, or in other words that the radiation is not modified on scattering. When the beam is heterogeneous various complicated balancing effects may occur, and the curve obtained is in general no longer exponential. This is illustrated by figure 35 which shows the typical relative absorption curve obtained using either a thick scatterer or a relatively large primary aperture; it may easily be verified that the curve is not exponential. In this example the radiation from the tube No. 354624, operated on the circuit of figure 6, was scattered from 18.5 mm. of paraffin wax, the two beams being absorbed in aluminium. The primary beam was filtered by 0.058 cm. of
aluminium before falling on the scatterer and was limited by an aperture consisting of a circular hole 2 mms. in diameter, the gas ionised being SO$_2$ at atmospheric pressure.

This curve may be taken as typical of a large number of such experiments in which a thick scatterer, or a large primary aperture, or both simultaneously, were used, and may be interpreted as showing a modification of the scattered beam of radiation. If the primary and scattered radiations are absorbed with silver a similar variation in the ionisation ratio $S/P$ is found; figure 36 illustrates the results obtained when filtering the radiation transmitted through, and scattered from, 18.5 mms. of paraffin wax, using as primary apertures a single hole 2 mms. in diameter and the series of irregular holes previously referred to. Other experimental conditions were similar to those of figure 35 except that the primary radiation was unfiltered before falling on the scatterer. Absorbing the primary and scattered beams with filter paper leads to a similar result, as shown by
figure 37; in this experiment all conditions were identical with

... those of figure 35, and it will be seen that the slopes for

... equivalent absorptions of the primary beam are practically equal.
As has been pointed out, equation (19) refers to a homogeneous primary beam; it is, of course, not possible to obtain radiation of a single wavelength direct from the X-ray tube, but approximations to this may be obtained by the use of suitable filters in the primary beam. Figure 38a illustrates the results obtained by scattering the radiation from the tube No. 354624 from 18.5 mm. of paraffin wax after it had been passed through 0.011 cm. of molybdenum. This tube has a molybdenum anode, so that by passing the primary radiation through molybdenum the wavelengths shorter than the K characteristic wavelengths of molybdenum will be strongly absorbed, while the characteristic radiations themselves will be relatively little absorbed. We should thus expect a primary radiation approximating to a homogeneous beam and a relative absorption curve which approximates to an exponential. How far this is realised will be clear from figure 38b, representing the variation of log5/P with x; the result curve is very nearly linear. Taking the mean wavelength of the radiation as 0.62 A, corresponding to the measured value of mass absorption coefficient 3.66 we easily obtain for \( \mu' - \mu \) the value
1.426. Assuming the well-known relation between wavelength and mass-absorption coefficient we obtain the approximate relation

\[(\mu' - \mu)/\mu = 3(\lambda' - \lambda)/\lambda\]

whence for \(\lambda' - \lambda\) we obtain the value 0.029 A, which is in fair agreement with the value 0.024 A to be expected on Compton's theory; the discrepancy might easily be accounted for by double-scattering in the plate of paraffin wax.

The results described so far may be taken as agreeing qualitatively, and where the conditions allow of comparison, quantitatively, with the predictions of the quantum theory. This is no longer the case when thin plates of scattering material or primary apertures of small dimensions are used. The typical result obtained under such conditions is illustrated in figures 39a and 39b, the absorbing materials being filter paper and aluminium respectively. The characteristic and important feature of these curves is the region of constant ionisation ratio \(S/F\), which persists until the primary radiation has undergone an absorption of approximately 20%.
Considering equation (19) this constant ratio of the ionisations is

![Graph 3.9a](image)

![Graph 3.9b](image)

most easily interpreted as resulting from an equality of $\mu$ and $\mu'$ over the range in question; in other words, the radiation as a whole is

unmodified in the process of scattering. However, it must be borne in mind that all we actually observe in the experiments is that the
ionisation produced by the scattered beam is a constant fraction of that produced by the primary beam, over a relatively small range of thickness of the absorbing material. There can be no question as to the accuracy of this result; although the range over which the ratio \( S/P \) remains constant is small it is very definitely evident, and these experiments in which the features shown by figures 39 are exhibited are among the most accurate that have been performed in this work. It is unnecessary to give in detail an example of individual readings, but a consideration of the mean deviations shows that the average accuracy of the observations is at least as good as that of Backhurst\(^{(25)}\) and considerably better than that of Gaertner\(^{(26)}\). Both of these experimenters, incidentally, remark on the possible accuracy of the work performed in this laboratory in connection with the J-phenomenon.

It was observed in these experiments on relative absorption, that a range of constant ionisation ratio \( S/P \) was no longer obtained if the voltage operating the X-ray tube was lower than that corresponding to the point D in the scattering curves previously discussed, the conditions in the relative absorption experiments being such as would give a scattering curve of the form shown in figure 10. This fact suggests a connection between the two sets of experiments and systematic observations were carried out to determine this connection. In figure 40 are shown the relative absorption curves obtained with a paraffin wax scatterer of 3 mm thickness and a primary aperture consisting of two holes 0.6 mm. in diameter; the tube used was the Philips No. 362619 operated on c.p.d.c., the gas ionised being air at atmospheric pressure. With these experimental conditions the scattering curve obtained shows a region of constant ionisation ratio \( S/P \) from 95 kV to 50 kV secondary voltage, at voltages below
50 kV the ratio steadily diminishes. From figure 40 it will now be evident that the relative absorptions of the two beams appear equal only so long as the voltage operating the X-ray tube is maintained above that corresponding to the point D of the scattering curve. To make this conclusion more general, exactly similar relative absorption experiments were carried out using filter paper as the absorbing material (1 sheet = 0.0064 gms./cm.). The general result is the same as before; at voltages above that of the point D the primary and
scattered beams appear equally absorbable over a short range, while at lower voltages no such equality appears. This result is not confined merely to the two series of observations just described, but has been observed on many occasions during the present work, always occurring with experimental conditions appropriate to the appearance of a constant ionisation ratio S/P in the scattering curve. This apparent equality of absorptions of the two beams of radiation appears then, to have some intimate connection with the constancy of the ionisation ratio S/P observed in scattering, and, so far as these experiments show, the one result is a necessary accompaniment of the other. A
satisfactory confirmation of this conclusion is obtained from a further consideration of the result illustrated by figures 22a and 22b, which shows that filtering the radiation incident on the scatterer is without any appreciable effect on the scattering curve provided this is regarded as a function of secondary voltage. In view of this it might be expected that the relative absorption curve obtained with a filtered incident radiation would show the same characteristics as that obtained with an unfiltered radiation. That such is actually the case is clearly shown by the experiment illustrated in figure 42.

The absorbability of the two radiations again appears equal until the absorption of the primary beam approaches 20%, after which the scattered beam appears more absorbable than the primary.

This simultaneous occurrence of the constant ionisation ratios in the scattering and absorption experiments appears to be of fundamental importance. It is more clearly brought out in figure 43 by plotting the absorption curves observed at the various points on the scattering curve. It will be quite evident from this that only so long as the scattered and primary radiations show equal absorbabilities
does the ionisation ratio $S/P$ of the scattering curve remain constant. When a difference is observed between the two absorbabilities the ratio $S/P$ in the scattering curve is no longer constant.

\[\text{FIG. 43}\]

The $J$-Phenomenon:

Experiments such as those described under the heading of relative absorptions of primary and scattered radiations have in general been regarded as the most suitable for exhibiting the $J$-absorption discontinuities. These take the form of abrupt, discontinuous variations, on the average about 7%, in the value of $S/P$ at definite thicknesses of the absorbing materials, very often the ratio remaining constant between these discontinuities. One important feature of the discontinuities is that they appear to occur when approximately
definite values of average mass-absorption coefficients of the primary beam are reached, and may therefore be said to depend on some quality of the radiation as a whole rather than on the separate constituents of the radiation. A second feature of the discontinuities is that they are by no means easily reproducible; at different times various factors have been regarded as the governing conditions for the appearance of the discontinuities (as, for instance, the method of excitation of the X-ray tube, and the state of the scattering material or the absorbing sheets\(^{(a2)}\)), but no single experimental condition has actually been discovered which can really be regarded as the controlling feature for the occurrence of these J-discontinuities. The discontinuities may appear or not appear under what are apparently identical experimental conditions, and this fact adds considerably to the difficulty of their systematic investigation. It will be at once clear that none of the experiments described above on the relative absorptions of the primary and scattered radiations shows any evidence of discontinuities of the kind which occur in connection with the J-absorption. The same is true of every experiment which has been made in this connection during the present work. One further experiment may be quoted in which the primary and scattered radiations were absorbed separately; if any discontinuity were present this method of observation would show which beam of radiation was responsible for the discontinuity. Figure 44 shows the ionisations produced by the primary and the secondary radiations in S\(_2\) at atmospheric pressure when each beam is absorbed separately by increasing thicknesses of aluminium. The resulting relative absorption curve is also shown, and although this method of measurement of S/P necessarily yields results less accurate than those of the experiments previously described it is obvious that no discontinuities of an amount approaching 7\%
are present. In several other experiments of this kind results of an exactly similar nature have been obtained. It would appear, therefore, that in none of the experiments carried out during this
work have the experimental conditions been appropriate to the occurrence of J-discontinuities of this kind.

Scattering from Aluminium:

The results obtained using aluminium as a scatterer are interesting in that they show the transition between the scattering curves obtained from light and from heavy substances. They do, in addition, show that even a thin scatterer is not the only necessary condition in order to obtain a constant ionisation ratio in the scattering curves, but that the scatterer must also consist of a light substance.

The results dealing with scattering and relative absorptions have shown that so long as the absorption undergone by the primary radiation remains less than approximately 20% of its original intensity a constant ionisation ratio $S/P$ is obtained in scattering curves and apparently equal absorbabilities in relative absorption curves. In addition to the decrease of intensity due to the absorbing sheets themselves there is, of course, a certain amount of absorption in the material of the scatterer, in the window of the ionisation chamber, and in the ionised gas. It has also been shown that with a sufficiently thick scatterer the constant ratio $S/P$ and the apparently equal absorbabilities both disappear. The inference is that the total absorption undergone by the primary radiation is considerably greater than the maximum value associated with the appearance of these two results, due, presumably, to the increased absorption by the scatterer.

In view of this, also, the results obtained by scattering heterogeneous radiation from aluminium are of some interest, and, although belonging more properly to the section dealing with scattering, will be considered here on account of their apparent connection with absorption. Figure 45 shows the results obtained on scattering the unfiltered radiation
from the Siemens tube No. 234303 operated on c.p.d.c. from various thicknesses of aluminium. In this series of experiments the radiation intensities were measured by the ionisations produced in air at atmospheric pressure, and the primary beam was limited by an aperture consisting of 4 holes each 0.6 mm. in diameter. There is obviously little similarity between these curves and those which have been described as typical of thin plates of scattering material. The two lower curves are, in fact, very like those obtained using thick scatterers or relatively large primary apertures. Actually, the only feature bearing any resemblance to the results with thin scatterers is the systematic displacement of the maximum, evident in these curves, towards lower voltages as the thickness of the plate of scattering
material is reduced. A comparison of the rates of ionisation at different secondary voltages shows that 3 mms. of paraffin wax is, on the average, equivalent in its absorbing power to 0.16 mm. of aluminium, so that the thinnest scattering plate used in this series of experiments has an equivalent thickness of approximately 6 mms. of paraffin wax. With the primary aperture used in these experiments a paraffin wax scatterer of this thickness would show a short range of constant ionisation ratio in the scattering curve (see page 40) so that the increase in absorption, due to the increase in equivalent thickness, while appearing to be the major factor contributing to the difference in the form of the scattering curves, is not the only one. The necessity for the absorptions of the primary and scattered beams to be small in order to obtain these constant ionisation ratios will have been further evident from figures 33 and 34.

The Effect of filtering the Radiation before Scattering:

If an absorbing material is placed in the path of the primary radiation before it falls on the scatterer the average penetrating power of the beam is increased, due to the relatively greater absorption of the longer wavelength constituents of the beam. The energy-wavelength distribution is therefore completely altered, and some effect on the scattering curves might be expected. It has already been shown that a constant amount of filtering aluminium placed in the beam before falling on the scatterer is without any appreciable effect on the form of the scattering curve obtained (see pages 58 and 59) provided the ionisation ratio S/P is plotted against secondary voltage and not against average mass-absorption coefficient. If, however, the secondary voltage is kept constant and the absorbability of the primary radiation varied by altering the thick-
ness of the filtering aluminium only, the results obtained become much more variable and difficult of interpretation than those hitherto described. The general type of result found may be illustrated by figure 46; there are, however, differences in detail caused by different experimental conditions which will be considered later.

From figure 46 it is clear that with a constant amount of filtering aluminium the ratio falls in the normal way for thick scatterers as the secondary voltage is reduced. If, however, the voltage is kept constant and the thickness of filtering aluminium increased the ratio again falls. This fall in the ratio $S/P$ with increasing thickness of aluminium filter is the general characteristic of the scattering curves when the primary beam has already been well filtered (either by passing through aluminium or through a thick-walled tube). One further example of this type may be quoted, for the tube No. 311704 operated on c.p.d.c., the radiation being scattered from 3 mms. of paraffin wax and the intensities being measured by the ionisations
produced in air at atmospheric pressure. In this experiment the primary aperture consisted of 14 holes each approximately 0.45 mm in diameter, and the scattering curve showed a constant ionisation ratio $S/P$ over a range of secondary voltage from about 50 kV to 95 kV. Figure 47 shows that with this tube the ratio is lowered with even a small amount of aluminium in the primary radiation. A similar variation occurs when the radiation incident on the scatterer is filtered with tin, as is shown in figure 48. If the radiation incident on the scatterer has a considerable proportion of its intensity in the region of very long wavelengths then the ratio $S/P$ no longer falls with very thin sheets of filtering aluminium but remains practically constant until a considerable absorption of the incident radiation has taken place. This experimental condition is realised with the Philips tube No. 362619 which has a Lindemann glass window and allows the passage of long wavelengths without much absorption. Figure 49
illustrates the type of result obtained with this tube operated on c.p.d.c., the radiation being scattered from 3 mms. of paraffin wax

![Scattering from 4.7 mms. Paraffin Wax at 45°](image1)

and the primary beam being limited by 4 holes each 0.6 mm. in diameter. Radiation intensities are measured by the ionisations produced in air at atmospheric pressure. From this experiment it will be clear that
no appreciable reduction in the ratio $S/P$ occurs until more than
0.09 cm. of aluminium has been placed in the incident beam, absorbing
approximately 64% of the unfiltered radiation from the tube.

From the results of these experiments one might conclude that
the ionisation ratio $S/P$ tends to remain constant so long as there is
an appreciable proportion of the intensity in the region of long
wavelengths, when this is filtered out — either by being passed
through aluminium or in passing through the window of the tube — the
ratio begins to decrease. However, this conclusion can not be
regarded as completely general, for a change in the primary aperture
again produces changes in the form of the scattering curve in the
region in which the value of $F(\mu/\rho)_{Al}$ is varied by filtering the
incident radiation. Further, in these experiments no definitely
systematic variation of the curve in this region has been found to
result from a systematic variation of the primary aperture, except
perhaps a slight accentuation of the fall in $S/P$ with smaller apertures.
The only justifiable conclusion that can be drawn from these results
is that the variation of $S/P$ when the radiation incident on the scat-
terer is filtered, the voltage remaining constant, depends on the
energy-wavelength distribution of the unfiltered radiation; this
conclusion should be compared with that expressed on page 61
relating to a variation of the secondary voltage alone.
DISCUSSION.

Before proceeding to a detailed discussion of the experiments, it is appropriate to give a summary of the three most important results established by this work.

(1) It has been shown that under suitable experimental conditions results are obtained which may be interpreted as showing the scattered and primary radiations to have precisely equal absorbabilities over a certain range of absorption of the primary radiation; this range amounting, at most, to about 20%; and decreasing systematically as the voltage operating the X-ray tube is reduced. For absorptions of an amount greater than this value the absorbability of the scattered radiation appears greater than that of the primary.

The experimental conditions which these observations have shown to be essential in order to obtain equal absorbabilities are two in number: the scattering material must be in the form of a thin plate of a light substance, and the cross-sectional area of the beam of radiation entering the primary ionisation chamber must be small. If either of these conditions is not fulfilled, the absorbability of the scattered radiation always appears greater than that of the primary.

(2) Under suitable experimental conditions, the ratio of the ionisations produced by the scattered and primary beams of radiation remains constant and independent of the voltage operating the X-ray tube, over a certain range of voltage. In the direction of increasing voltage this ionisation ratio remains constant up to the highest voltage it has been possible to use in this work, viz. up to 100 kV.

Again the essential experimental conditions for the occurrence
of this constant ionisation ratio are that the scattering material is in the form of a thin plate of a substance consisting only of light elements, and that the cross-sectional area of the primary beam is small. With thicker plates of scattering materials a region of constant ionisation ratio is still observed but over a less extensive range; this means that decreasing the thickness of the scatterer does not change the slope of the scattering curve but merely extends the range of constant ionisation ratio.

(3) The most important fact established by the present work is the connection between the two types of experiment, (1) and (2) above, illustrated by figure 43. From this figure it is clear that only so long as the two beams of radiation appear equally absorbable does the ionisation ratio $S/P$ remain constant in the scattering curve. When the two absorbabilities differ in the manner assumed by Compton's theory, the ratio $S/P$ is no longer constant. The conclusion suggested by this result is that so long as the primary and scattered radiations appear of equal absorbability, the scattering coefficient is independent of the wavelength of the radiation scattered.

From the point of view of their theoretical significance the most important results obtained on the absorption of heterogeneous radiation are those dealing with the relative absorptions of the scattered and primary radiations. It has been shown that these results are, in certain cases, in agreement with what might be expected if the scattered radiation were more absorbable than the primary, a fact which according to the quantum theory is actually the case. As is well known, the quantum theory attributes this difference in the two absorbabilities to an increase in the wavelength of the scattered
radiation, and these experiments have shown that when the radiation is approximately homogeneous the results obtained do in fact find a satisfactory explanation in terms of this idea of increased wavelength the increase required to explain the results being very nearly that predicted by the theory (see pages 96 and 97). Those results in which the relative ionisations produced by the scattered and primary radiations remain constant over a certain range of absorption (see pages 98 to 102) are, however, not to be expected on the quantum theory of scattering. The simplest interpretation is that the scattered and primary radiations are identical in penetrating power, over this range of absorption. It is conceivable, however, as has already been pointed out on page 81, that with a heterogeneous beam compensation effects may occur which alter completely the form of the curves to be expected, so that while each homogeneous component of the radiation is scattered in accordance with the quantum theory, the absorption of the beam as a whole might appear in conformity with the predictions of classical theory. Dunbar has endeavoured to show that such a process may actually occur, and his calculated results are worthy of consideration, if only to point out that they are not such as would in practice generally occur. The theory of Dunbar's calculations is fairly simple. Equation (19) may be written in the form

$$\frac{i_s}{i_p} = \frac{S_0}{P_0} \frac{e^{-(\mu_\lambda + \delta \mu_\lambda) x}}{e^{-\mu_\lambda x}}$$

where $i_s/i_p$ gives the ratio of the ionisations produced by the scattered and primary radiations after traversing a thickness $x$ of absorbing material. $S_0/P_0$ is the corresponding ratio for the unintercepted radiations. The wavelength of the primary radiation is $\lambda$ and its linear-absorption coefficient $\mu_\lambda$; due to the assumed increase in
wavelength on scattering the linear-absorption coefficient of the scattered radiation is increased by an amount \( \delta \mu_n \). For a series of different wavelengths the above equation may be written as

\[
\sum \frac{i_\lambda}{i_p} = S = \text{constant} \times e^{-(\mu_n + \delta \mu_n)x}
\]

where \( i_\lambda \) is the intensity of the wavelength \( \lambda \). Equation (20) is the basis of Dunbar's calculations; by assuming appropriate values for \( i_\lambda \) at various values of \( \lambda \) and using the quantum theory to obtain \( \delta \mu_n \) at these wavelengths it is possible to evaluate \( S/P \) for different thicknesses \( x \) of absorbing material. Table 7 and figure 50 are reproduced from Dunbar's results.

**TABLE 7.**

Wavelength Components and their relative Intensities.

<table>
<thead>
<tr>
<th>No. of Curve</th>
<th>0.20 A %</th>
<th>0.35 A %</th>
<th>0.50 A %</th>
<th>0.70 A %</th>
<th>0.85 A %</th>
<th>1.0 A %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>67.1</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>39.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>60.3</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>46.5</td>
<td>53.5</td>
<td>77.75</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>22.2</td>
<td>-</td>
<td>59.8</td>
<td>-</td>
<td>-</td>
<td>40.2</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>25</td>
<td>-</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

The composition of all of these beams of radiation is such as would give an average mass-absorption coefficient in aluminium of 3.2. (The curve marked H represents the absorption curve for a homogeneous radiation of wavelength 0.6 A). A consideration of Table 7 will show that the only energy-wavelength distributions which can be regarded as approximating in any degree to that of the primary radiation from an X-ray tube are those corresponding to curves 2, 3, and 5; these curves are actually very similar to those observed in most of the present experiments with thick scatterers or large primary apertures. The distributions relating to curves 1 and 6 are not normally obtained
in practice (unless a special filtering of the primary radiation is made), yet these are the only curves in which the ionisation ratio remains practically constant although a quantum scattering process is assumed to occur. It appears irrelevant, therefore, to introduce the consideration of distributions and curves such as 1 and 6.

Another feature of these calculated curves worth noting is that in all cases the decrease in the ratio $S/P$ is more rapid at the commencement of the absorption; in the experiments with thin scatterers and small primary apertures (described on pages 98 to 102) the initial slope of the curve has, over certain ranges of secondary voltage, been negligible. It appears therefore that compensation effects in the absorbing materials would not give rise to the curves obtained in the present experiments.

There still remains the possibility of a compensation effect in the material of the scatterer such as would make the scattered beam appear of the same penetrating power as the primary; this would
necessitate that the increase in average mass-absorption coefficient of the scattered beam due to the increase in wavelength of each of its components be almost balanced by the decrease in average mass-absorption coefficient due to absorption in its passage through the scattering material. Such a balance might occur under very special conditions, but would obviously be dependent for its occurrence on the energy-wavelength distribution of the radiation incident on the scatterer. Now this apparent equality of absorptions has been observed with various X-ray tubes, and with filtered and unfiltered radiations, so that altogether primary radiations of very different constitutions have been employed.

A very strong argument against the occurrence of any compensation effect is evident from a consideration of the slopes which might be expected in the case of the absorption curves. If various factors which tend to give slopes in opposite directions are combined in different degrees one would expect absorption curves to be obtained having, within limits, slopes of all magnitudes, i.e., slopes varying from zero through all values, both positive and negative, within these limits. This is never observed. All the absorption curves obtained in these experiments either have a slope which is, within very narrow limits, zero over an appreciable range, or have a negative slope of quite an appreciable magnitude—of the order of 3% or 4% per 0.01 cm. of absorbing aluminium over the initial parts of the curves. The curves in no case have a positive gradient. Throughout the present work no absorption curves have been obtained having slopes lying within these two limits, but only with a combination of the two as in figures 39a and 39b.

A similar argument can be advanced in favour of the reality of the constant ionisation ratio S/P observed in the scattering experi-
ments. In these, also, the gradient has an appreciable, finite, and negative value to the right of the point D, while over the remaining region the gradient is again zero within very narrow limits. This point is brought out clearly in figure 11.

In view of these considerations it seems extremely improbable that a balance of the nature considered above could have occurred in all cases. It is more satisfactory to regard the observed equality of the relative ionisations produced by the secondary and primary radiations as some fundamental property of these radiations, apart from any considerations of change of wavelength, double-scattering, polarisation, or similar features. The intimate relationship which has been shown to exist between these experiments and those on scattering suggests a similar conclusion, and one is therefore led to suppose that present theories, while affording a fairly accurate description of the results obtained with homogeneous radiations, are not adequate to account for the behaviour of heterogeneous beams of radiation.
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