AN ANALYTICAL STUDY OF PRICE LEADERSHIP

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Ph.D. University of Edinburgh 1975
I declare that this Thesis has been composed by myself, and that the work incorporated in it is my own, with the exception of the Appendix to Chapter 2 where due credit is given.
ACKNOWLEDGEMENTS

In the writing of this thesis, my major debt has been to my two supervisors, Professor J.N.Wolfe and Mr. W.D.C.Wright. They have given me the benefit of their advice on many matters ranging from the general to the highly specific. I am most grateful to Mr. G.E. Meeks, Research Associate, Department of Accounting and Business Method, for translating an important article by Forchheimer. I have also benefited from discussing this article with Clemens Hackmann, an undergraduate from the University of Berlin, who is currently visiting Edinburgh. I should also record my debt to Mr. Brian Hilton, who was kind enough to undertake the arduous task of checking the mathematics.

Chapter 5 was previously read as a paper before the Staff Seminar, University of Durham in November 1973. Chapter 7, and parts of Chapter 2 have been read to various audiences in the University of Edinburgh, including the staff ABSE workshop and the Economics Society. I am grateful for the comments of various members of these seminars.

This is a slightly modified version of a thesis examined in May 1975. The present version has been revised in terms of Regulation 2.4.20. I am grateful to the examining board for their comments and in particular for the very detailed and constructive comments of /
of Professor C.K. Rowley, University of Newcastle-Upon-Tyne who was acting as external examiner. Any remaining deficiencies in this thesis remain my own responsibility.

Finally, my thanks go to Mrs. C. Barton for the expert typing of various drafts of this thesis, and also the final and revised versions.

Edinburgh, August 1975. 

G.C.R.
The purpose of this Thesis is to provide an analysis of the economic model of price leadership. Price leadership is said to prevail in a market when the pricing policy of a particular firm is followed exactly, or in close parallel, by other firms in the market. An account of the theoretical work on price leadership which has been accomplished up to the present time is given. A translation of the important early work by Forchheimer is bound in with the thesis. An algebraic and geometrical analysis of price leadership under various assumptions is undertaken. The method of comparative statics is used to analyse the effect on market equilibrium of marginal shifts in taxes, subsidies, royalties etc. Finally, a particular type of price leadership is incorporated into a framework of reconstruction which has been suggested by the industrial economist P.W.S. Andrews.
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Chapter 1

Introduction

1-1. Background

Consider a market for a homogeneous or mildly differentiated good. This good is produced by a number of firms, of which all or some, are in significant interaction. By this, we mean that quantity and price variations of some or all firms have significant effects upon the profits of some or all firms in the market. This definition embraces three possible structures: pure oligopoly; oligopoly of a core of firms with a competitive fringe; and partial monopoly, in which one firm is the price or quantity setter and all the other firms behave competitively. The interdependence of firms within the market distinguishes this sort of behaviour from that of perfect, or monopolistic, competition. The good is sold in the market either at a uniform price, or else at a virtually uniform price, some prices being "shaded" below others. Suppose one firm consistently initiates price changes, either in response to the free play of economic forces (e.g. price variation in the factor market), or in response to the modification of its circumstances by some external authority (e.g. by variation in a tax rate). If other firms in the market follow these price changes exactly, or in close parallel, then the initiator of such price changes is known as a price leader, and the market structure is said to be one of price leadership - followership.

1/ See Henderson et al. (1971, pp. 222-223). We shall be largely concerned with the third type.
followership, or more simply, price leadership.

Expressed in its simplest terms, this is the sort of behaviour which our thesis intends to examine. We approach the task from an analytical (as distinct from empirical) viewpoint. The original stimulus to undertake this study was the article by Wolfe (1954). There, a model is developed in which the equilibrium market price is the lowest price which any firm is willing to set. It is argued that the firm willing to set this price will become the price leader, and that if all firms have constant average and marginal costs, the price leader will in all probability be the lowest cost firm.

We will show that analytical treatments of price leadership go back nearly fifty years before this article. Furthermore, we shall diverge from the main line of argument at several points, to examine topics of special interest, such as survivorship, and the problem of discontinuity in market supply. Nevertheless, the influence of this article will be evident at various points in the thesis, most especially so in the final analytical chapter (Chapter 7) which is founded on the work of a common influence, P.W.S. Andrews.

1-2. Purpose

The purpose of this thesis is to provide an analytical study of price leadership. The study develops in three principal directions. Firstly, it provides a survey and critique of existing theoretical work directly
on, or closely related to, price leadership. Secondly, it undertakes extensions of standard theoretical work on price leadership. Thirdly, it suggests and develops a possible alternative to the existing models of price leadership.

In two senses, price leadership provides a possible approach to "the oligopoly problem". In a theoretical sense, it provides a plausible mode of behaviour which is analytically tractable. In a real-world sense, as a recent survey by Hawkins (1973, p.36) suggests, price leadership may arise "simply because this provides a practical means of solving the complexities which have been shown to be involved in the oligopoly problem". Empirical studies have indicated that price leadership is common in market economies. At the same time, it is also true that relatively little theoretical work has been done on price leadership, compared to, for example, the extensive work on the kinked demand curve and sales maximizing models of Hall et al. (1939), Sweezy (1939) and Baumol (1958). Reflecting this state of the literature there is as yet no generally accepted way of categorising price leadership. For these reasons, the writing of an analytical study on price leadership seems an objective worth pursuing.

1/ That is, the problem of determinacy in a market situation in which all firms are in significant interaction.

2/ For example: Kaplan et al. (1958), Adams ed. (1961) for the U.S.A.; Fog (1960) for Denmark; and Saxton (1942), Hall et al. (1939), Andrews (1949), Barback (1964), Heath (1961), Swann et al. (1974), Shaw (1974), Maunder (1972) and Hague (1971) for the U.K.
1-3. **Scope**

A necessary step in defining an area of study is the negative process of delimiting the boundaries of investigation. Technically speaking, we have limited ourselves to the use of calculus, algebra and geometry at a level which is fairly common in the microeconomics field 1/. At the economic level, we have, in the first instance (Chapters 3 to 6) worked within the framework of marginalism. Thus, using conventionally defined cost and revenue functions, we construct profit functions. Firms are assumed to maximize such profit functions, and necessary conditions for doing so may be formulated in marginalist terms. Thus we exclude satisficing behaviour in which an individual (or firm) engages in search until some aspiration level (like minimum acceptable profit) is achieved 2/. We also exclude the use of a more general maximand than a profit function such as a managerial utility function 3/. At a later stage however (Chapter 7), we drop the marginalist approach, and substitute for it behaviour which is more akin to satisficing. So far we have considered restrictions in techniques. Now let us turn to restrictions in subject matter.

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1/ This level is approximately that of the more technical papers reprinted in recent books of readings by Rowley ed. (1972) and Archibald ed. (1971).

2/ See, for example, Simon (1959).

3/ See, for example, Williamson (1964), who uses a utility function with staff, emoluments, and discretionary profits as arguments.
One issue which will not be considered is the efficiency aspect of price leadership. This would require the definition of a suitable "competitive norm". At the present moment, the issue is unresolved. Stigler (1947) and Markham (1951), for example, have emphasised the competitive character of one particular type of price leadership, whereas Swann *et al.* (1974) regard price leadership as being *prima facie* uncompetitive.

Finally, as we have indicated earlier, another limitation on the scope of this thesis is that we confine ourselves to an analytical, rather than empirical treatment of price leadership. This is not to deny the importance of confronting the conclusions reached by theorising with data. This must be one of the major objectives of all theorising. However, as analytical treatments of price leadership appear to be rather under-developed at the present moment, and a substantial amount of descriptive material on price leadership is already available, it is in the direction of theory that the weight of effort appears to be immediately required.

1-4. Contents

This section provides a brief guide to the contents of the rest of the thesis. Chapter 2 is primarily a survey of price leadership, but also considers models of related interest, such as the Stackelberg duopoly model.

As Rowley ed. (1972, pp. X-XV) has pointed out, the selection of such a "norm" implies the selection of a particular social welfare function.
The early contribution by Forchheimer (1908) is often referred to, but very rarely discussed. Surprisingly, no English translation of the article has yet appeared. For this reason, we have included a translation of this work (Appendix to Chapter 2) and a critical commentary on that material and the closely related work of Zeuthen (1930). Another rather inaccessible reference, the thesis of Nichol (1930) on the theory of price leadership, is examined in Section (2-4).

Any research must, perforce, depend to a greater or lesser extent on the work of established authorities. This thesis is no exception to that rule, and the heavy debt we owe to the writings of the leading price theorists will become very evident in the chapters that follow. Nevertheless, so far as we are aware, the results contained in Chapters 3 to 7 have not yet been stated in the literature. Chapter 3 considers the behaviour of the price leadership model under the assumption that the total cost function is linear. This assumption is not unrealistic, empirically speaking, but produces some paradoxical results. In particular, we show that only one firm can be the survivor in the long run, and show how that firm may be identified. It appears that this result has not yet been stated in the literature on price leadership. Chapter 4 is strictly analytical and investigates the properties of what we call "the linear case" - this being the case in which average and marginal costs are straight lines through the origin. The method of /
of comparative statics, which has been employed successfully by theorists such as Williamson (1964), is used to investigate the effect of parameter shifts on equilibrium prices and outputs. Although Hadar (1971) has provided a simple mathematical treatment of price leadership, he did not investigate the comparative statics properties of his model. To our knowledge this is the first time that the comparative statics properties of a price leadership model have been investigated. In Chapter 5, we employ weaker assumptions, and examine the behaviour of the model under increasing costs. The effect of assuming that the leader has lower costs than any follower is examined, and an Appendix takes a detailed look at the nature of the model's solution. This section proceeds beyond the treatment of Hadar (1971) in its level of rigour and also in its attempt to examine the relationship between profits costs of the leader and followers. Both the comparative statics results and the Appendix on the model's solution contain material which, to our knowledge, has not yet been stated elsewhere.

Chapter 6 looks at the price leadership model in a fairly general context, and gives some attention to the pathological cases which are usually not discussed in models of this kind. In particular, the problem of discontinuity in market supply, and by implication in the leader's demand schedule, is examined. The textbook of Boulding (1966) is notable for its treatment of discontinuities in various contexts, but this appears to be /
be the first time that these problems have been examined specifically in a price leadership model. Finally Chapter 7 departs from the strictly marginalist approach, and looks at the behaviour of a particular type of price leadership market over time. Some of the assumptions adopted (for example that the leader is the least cost firm, and that unit direct costs are constant up to capacity) bear an obvious relationship to the treatment of Wolfe (1954), although probably the major influences are Saxton (1942) and Andrews (1964). In this Chapter, the precursors of our work are obvious, but the particular development appears to be novel, as do the conclusions reached. A fairly general view of the cost-plus school is that their theories are unrigorous and loosely formulated. One of the purposes of Chapter 7 is to show how some elements of the cost-plus theory can be incorporated in a rigorous deductive framework. Sylos-Labini (1969) came some way in this direction, but his results were largely developed in terms of numerical examples, in contrast to the algebra we use. Our treatment also differs from his in that we permit technical progress, rather than assuming a fixed spectrum of techniques in an industry.

In conclusion, our principal purpose in this thesis is to survey, codify, and, where possible, extend, the existing theoretical work on price leadership.

1-5. Methodological Considerations

We shall end this chapter by taking up some methodological points. Although this thesis is exclusively concerned /
concerned with deductive analysis, there are few economists who would claim that such an activity may be regarded as an end in itself. There is a fairly general agreement that one of the primary purposes of a theory is to enable one to predict the behaviour of a particular class of phenomena under specified circumstances. There is far less agreement on other necessary attributes of a theory. One may ask the following questions, for example. How realistic should the assumptions of a theory be? To what extent should a theory provide an explanation of events? May one judge a theory on such aesthetic grounds as simplicity, economy or elegance? The purpose of this section is to broach such questions, in the hope that we will thus provide a suitable methodological backdrop to the succeeding chapters of this thesis.

We start by discussing the anatomy of a theory. This leads us to consider whether one can evaluate a theory by reference to its assumptions: an issue which has been central to the debate over Friedman's essay "The Methodology of Positive Economics". We continue by examining such matters as as explanatory power and elegance. We conclude by relating these discussions to our own methodological position.

Our first concern is to identify the principal elements of a theory. Following Nagel (1963)  1/ we may distinguish three classes of statements which constitute a theory:

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1/ Page references are to the reprint of Breit et al. (1971)
theory:

(a) **Fundamental statements**, variously known as "assumptions", "basic hypotheses" etc.

(b) **Theorems**, that is, statements arrived at by a process of logical deduction from assumptions.

(c) **Theoretical terms** whose existence are postulated solely for the purposes of the theory, and may be in either or both of the statements described under (a) and (b).

In the theory of perfect competition we assume that firms produce a homogeneous commodity which is infinitely divisible. This is a **fundamental statement** and involves two **theoretical terms** "homogeneous commodity" and "infinitely divisible". A **theorem** we may derive from the assumptions is that the effect of imposing a specific sales tax on all firms is to increase market price and diminish aggregate production. The concatenation of statements under (a) and (c) is often described as a **model**.

A fairly conventional concept of science is implied by the following words of Harre (1960, p.107): "A science enables us to satisfy three main needs, the condensation of our knowledge, the prediction of the future course of nature, and the explanation of natural phenomena". We think it unlikely that Friedman would dissent entirely from such a view of economics as a science, but probable that he would give greater emphasis to the role of prediction.

In evaluating the worth of a theory, Friedman (1953, p.8) writes:

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1/ Cf. Friedman (1953, p.7)
writes: "Viewed as a body of substantive hypotheses, theory is to be judged by its predictive power for the class of phenomena which it is intended to 'explain'. Furthermore, Friedman would deny that one can judge a theory by reference to the realism of its postulates. Thus he writes \[1/\] "the belief that a theory can be tested by the realism of its assumptions independently of the accuracy of its predictions is widespread and the source of much of the perennial criticism of economic theory as unrealistic. Such criticism is largely irrelevant, and, in consequence, most attempts to reform economic theory that it has stimulated have been unsuccessful". His position on the realism of assumptions has been subject to criticism by writers such as Nagel (1963) and Samuelson (1963). It is to this issue that we now turn. Following Nagel (1963) we may distinguish three senses in which an assumption may be unrealistic. Firstly, an assumption may be unrealistic in the sense that it does not give an exhaustive description of some real world object or event. This is the most trivial sense of the term "unrealistic assumption" because no statement of finite length could completely specify all characteristics of a concrete object or situation. Secondly, an assumption may be unrealistic in the sense that it is categorically false or highly improbable. Thirdly, statements containing "theoretical terms" (in the special sense mentioned above) may be judged to be unrealistic.

Assumptions /

1/ Friedman (1953, p.41)
Assumptions of this third type must inevitably be unrealistic in the sense that they do not and were not intended to apply to something actual. Let us give some examples from economics. The unreality of assumptions in the first sense has been well explained by Friedman (1953, p.32) in his discussion of the problem of formulating a completely realistic theory of the wheat market. We would not only have to specify supply and demand conditions, but also "the kind of coins or credit instruments used to make exchanges; the personal characteristics of wheat traders such as the colour of each wheat trader's hair and eyes, his antecedents and education, the number of members of his family ...." and so on. His view on this aspect is, as Nagel (1963, p.51) concedes, "fully conclusive". However, the first sense is the most trivial sense in which assumptions can be regarded as unrealistic and has never been a bone of contention amongst economists. Far more contentious is the notion of unrealism in the second sense. When Frank Knight (1921, p.76) is constructing his theory of choice and exchange, his first assumption is the following:

The members of the society are supposed to be normal human beings in essential respects as to inherited and acquired dispositions, differing among themselves in the ways and to the degrees familiar in a modern Western nation - a "random sample" of the population of the industrial nations of today.

It is safe to assume that most economists would agree that this assumption is not unrealistic in the second sense /
sense of the word. When Knight (1921, p.77) assumes that "there must be 'perfect mobility' in all economic adjustments", some economists might argue that this assumption is unrealistic: but they would be in error to conclude that it is unrealistic in the second sense. The expression "perfect mobility" is a theoretical term (in the sense defined above) as are "vacuum" in Galileo's law of gravitation or "gene" in the biological theory of inheritance. This sentence of Knight has no reference to actuality, and is therefore, if we must use the word "unrealistic". A theory which uses theoretical terms in its formulation may be described as the "pure case". Such pure cases can render useful information because it is very often possible to identify those factors which are not considered in the pure case and which can account for any discrepancy between what a law asserts will happen and what actually happens. Nagel (1963, p.52) develops this argument in greater detail and reaches the following conclusion: "In short, unrealistic theoretical statements (in the third sense of the word) serve as a powerful means for analyzing, representing, and codifying relations of dependence between actual phenomena".

The discussion of Nagel (1963) may be put in more rigorous forms by employing the notation of Samuelson (1963).
1963. Let B define a theory, C the consequences of B, and A the assumptions that are antecedent to B. Samuelson uses this notation as the basis for showing that what he calls the "F - twist" is fallacious. The F - twist according to Samuelson (1963, p. 1775) says that "empirical realism, at least up to some 'tolerable degree of approximation' of C is important. If C is empirically valid (realistic), then B is important even if A - and for that matter B itself - is not empirically valid". The first point to be made is that

\[ A \equiv B \equiv C \]

The realism of A cannot differ from the realism of B and C, because B and C are logically equivalent to A, as they were arrived at by a sequence of tautologies. Consider a proper subset of C, say C\(^{-}\), and a superset of A, say A\(^{+}\) (this latter being a set which contains A). Expressed in symbols,

\[ A^{+} \supset A \equiv B \equiv C \supset C^{+} \]

If C has satisfactory empirical validity, then the theory B may also be judged satisfactory, as also are the assumptions A on which it is based. This implies nothing satisfactory about A\(^{+}\), unless we can show that \( A^{+} \equiv B^{+} \equiv C^{+} \) has empirical validity. Now if C is empirically valid this does not imply that the inaccuracy or otherwise of \((A^{+} - A)\) can be ignored. Equally, the empirical validity of /
of $C^*$ does not imply that $B$ is valid. Samuelson illustrates the F-twist fallacy with an example from consumer theory. Let $B$ be maximizing an ordinal utility function subject to a budget constraint, income and prices given. Let $C$ be the strong and weak axioms of revealed preference. Then $B$ implies and is implied by $C$; and if $B$ is realistic so is $C$, and vice versa. Suppose the weak axiom (call it $C^*$) is valid and the strong axiom is invalid. Then proponents of the F-twist would argue that because $C^*$ is empirically realistic, the unreality of $B$ is irrelevant. But the validity of $C^*$ (which is not the full implication of $B$) cannot ameliorate the falseness of $(C - C^*)$. Samuelson's suggested solution is to remove $(B - B^*)$ and put forward the modified theory, $B^*$. It is clear then that Friedman's argument on the realism of assumptions may be regarded as incomplete, in that it does not distinguish between the various senses in which lack of realism may occur. Nevertheless, suitably modified along the lines suggested by Nagel (1953, p.54) we are in agreement that "the main thesis he is ostensibly defending is nonetheless sound".

Having agreed that all acceptable theories must yield valid predictions, and that at least in some senses a theory cannot be discarded because of the unreality of its assumptions, we are still at liberty to invoke additional criteria for an acceptable theory. Probably the

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1/ See Henderson et al. (1971, p.40)
the most important additional criterion is that a theory should provide a plausible explanation of the events under consideration. It has been pointed out that prediction and explanation are intimately linked, in that by predicting events one has to an extent 'explained' them. However, explanation means more than prediction alone, for a theory will not be easily accepted if it does not provide a plausible account of a sequence of events. 

The origin of scientific curiosity is a desire to understand the world, and the less a theory does to gratify this desire, the less likely it is to be accepted. In economics it is common to invoke other criteria for a satisfactory theory. For example, it is usually deemed necessary to show that a model is determinate. As variables are by their nature determined in the real world so too a plausible model must show how in principle these variables are determined. Finally, it is not uncommon for quasi-aesthetic criteria for judging a theory to be invoked. One of these is the criterion of elegance. A theory may be said to lack elegance if it proceeds by a clumsy series of steps or if it can only explain events by reference to many special cases. Closely related is the criterion of simplicity. A principle which is often invoked is that of Occam's Razor. If two rival theories are equally satisfactory from a predictive point of view, then under this principle we would accept the theory which is /

1/ See Harre (1960, p.148)

2/ See Harre (1960, pp. 174-176)
is based on fewer assumptions. Such criteria as elegance and simplicity only have meaning if they are used to contrast rival theories which are more or less simple and more or less elegant. As philosophers of science, such as Harre (1960) have shown, this by no means exhausts the possible criteria for judging a theory, though in the pages that follow we have not required of a theory than that it satisfies additional criteria.

Our general position may be summarised thus. We entirely concur with Friedman's view that adequate predictive power is an essential attribute of a theory. In this sense, the results stated in the following pages are only part of the scientific process. Only once the predictions implied by the theorizing have been subject to empirical testing will the scientific process have been completed. However, we are in agreement with Harre (1960, p. 148) that: "The power of prediction, what might be called the logical power of the theory, is a necessary but not a sufficient condition for a favourable assessment."
Chapter 2
Theoretical Studies

2-1. Introduction

Because the approach in this thesis is analytical, it is necessary to explore previous theoretical work on price leadership. In the first place, this prevents repetition of established results; and in the second place it suggests the areas in which elucidation and extension are required.

We start by looking at the various categories of price leadership which have been discussed in the literature. In common with much of the literature, this discussion of categories is not particularly rigorous. However, at a minimum, it provides a descriptive (rather than analytical) frame of reference, and, it is to be hoped, a means of comparing different terminology in this field. The survey of theoretical work, in its pure sense, starts with the section on Forchheimer and Zeuthen. We provide an algebraic re-statement of the numerical examples and geometry of these writers. This is followed by a discussion of the work by Nichol (1930) - an author who is often referenced in this field, but very rarely discussed. The next section is concerned with the textbook treatments provided by Boulding (1948) and Hadar (1971). An attempt has been made to discuss these writers using a unified notation, and this has necessitated transforming the geometrical analysis of Boulding /
Boulding into algebra. This section concludes the material which is exclusively on static models. The next section is transitional, discussing the quasi-dynamic argument of Worcester (1957) on dominant firms, and the fully-fledged dynamic models of Gaskins (1971) and others. The subsequent section goes one step further from partial equilibrium dynamics to the general equilibrium dynamics of Hadar (1971). This leads into a discussion of the related work of Arrow et al. (1971). Finally, we discuss the general framework proposed by Frisch (1933) for analysing conjectural behaviour. We relate this to the duopoly models of Cournot and Stackelberg, and also to the more recent work of Cyert et al. (1971). Price leadership is interpreted in terms of conjectural behaviour; and its relationship to the reversed kinked demand curve of Sweezy (1939) is examined.

Let us summarize. We first discuss general categories of price leadership. We then examine various models of price leadership, proceeding in an order which is, roughly speaking, one of increasing complexity. Thus we start with the numerical schemas of Forchheimer and conclude with Frisch's general theory of conjectural behaviour.

2-2. Categories of Price Leadership

The purpose of this section is taxonomic rather than analytic. We propose to examine the various categories of price leadership which have been suggested by writers such /
such as Stigler (1947), Markham (1951), Oxenfeldt (1952), Lanzillotti (1957) and Bain (1960).

The most noted classification of price leadership is due to Stigler (1947) who distinguishes dominant firm price leadership from barometric price leadership, tacitly implying that these are mutually exclusive (and exhaustive) categories of price leadership. By dominant firm price leadership, Stigler means the sort of partial monopoly situation which has been discussed by Forchheimer (1908) and Zeuthen (1930). Stigler (1947, p.148) defines a barometric price leader as a firm which "commands adherence of rivals to his price only because, and to the extent, that his price reflects market conditions with tolerable promptness".

Markham (1951) accepts the categories of dominant firm and barometric price leadership, but adds to this latter category what is really an extreme form of barometric price leadership, which he describes as "price leadership in lieu of overt collusion". He recognises three theoretical models of price leadership. The first is partial monopoly. The other two are models suggested by Boulding (1948), which will be discussed in detail later in this chapter. Putting the matter briefly for the moment, we would claim that Boulding's "models" are more correctly described as "variants", one being close to the dominant firm, or partial monopoly analysis, and the other being close to the market share analysis. Markham (1951, p.180) makes two principal /
principal points. His first is that partial monopoly should be considered under the general category of monopoly pricing. His second point is that in terms of Stigler's definition, the price set by a barometric firm is merely the price which would otherwise be set by competition. To recognise that not all barometric cases are competitive, Markham introduces a non-competitive, or monopolistic, sub-case, which he calls price leadership in lieu of overt agreement. He lists the following five pre-requisites for this type of price leadership: (a) few firms, with pricing policies that interact; (b) substantial barriers to entry; (c) close substitution within the product group; (d) an elasticity of industry demand which does not greatly exceed one; and (e) broadly similar costs between firms. Oxenfeldt (1952) suggests that several of Markham's pre-requisites are not necessary. His critique is a mixture of opinion, logical criticism and empirical counter-example. As regards opinion, for example, Oxenfeldt (1952, p.383) argues that condition (c) is unnecessary because "we must not expect all prices to be identical, or to differ by unvarying amounts under price leadership." However, if we admit of looser price behaviour than this, the very detection of price parallelism would be difficult, though one could perhaps look at it in terms of a relatively high correlation of price changes. In defence of Markham, it could be argued that the notion of a product group would lose much of its meaning were not goods within this group close /
close substitutes. Markham's fourth pre-requisite, that the elasticity of industry demand should not greatly exceed one is presumably meant to imply that in this case price wars would be an unattractive proposition. Oxenfeldt appears to contest this on logical grounds, though the nature of his argument is unclear. He argues that both price cuts and price increases could be disadvantageous and advantageous respectively, if pursued by all sellers. His point is presumably that in the increasing marginal cost case maximum net revenue is generally achieved at an output less than the turning point on the total revenue curve (i.e. the unit elasticity point). In this sense Markham's argument is certainly imperfect. Markham's fifth condition, that costs be similar between firms is really based on a priori argument. If there were substantial cost advantages enjoyed by any single firm, and condition (c) of close substitutes were also satisfied, then this firm would tend to capture the market by undercutting its rivals' prices. Oxenfeldt dismisses this argument, not because it is implausible, but because one can produce counter-examples - in this case, the United States Steel Corporation in the 1930's. As a result of these criticisms, Oxenfeldt concludes that Markham has relegated the practice of price leadership to "an ineffectual or exotic market practice". Whilst it is true that several of Oxenfeldt's criticisms are of some dubiety, his basic point, namely that we must regard price leadership (of whatever form) as being perceptibly different /
different from independent price behaviour, is well taken.

Bain (1960) has pursued this point further, arguing that the misleading element in both Stigler's and Markham's analysis is the suggestion that in some sense barometric price leadership is "competitive". In the first place, they are clearly using the term in a different (but unspecified) way from the economic theorist, hence the use of the word is ambiguous. In the second place, Bain argues that what is important is the structural situation of an industry (in terms of the degree of buyer and seller concentration, height of barriers to entry etc.) rather than a phenomenon, like price leadership, which merely reflects this structural situation. Bain himself regards the basic distinction as being between dominant firm (partial - monopoly) price leadership and oligopolistic price leadership. The former he regards as being of lesser theoretical interest and practical relevance than the latter. The latter he considers to be the "common or garden" variety of price leadership. Bain (1960, p.150) defines it as "an oligopoly of several large firms (at least two) in a market, with circular interdependence inter se, these firms being surrounded or not, as the case may be, by a competitive fringe of small sellers." Later writers in the policy field, such as Polanyi et al. (1974) have chosen to adopt Stigler's presumption that barometric price leadership is essentially competitive. This view cannot be /
be entirely supported by the article of Markham on the subject which maintains that certainly some types of barometric price leadership are uncompetitive (price leadership in lieu of overt agreement) but case-by-case examination will reveal barometric situations in which price competition is intense. Indicators of this latter situation are a tendency for the leadership role to be transient for any one firm, and a tendency for market shares to fluctuate especially when prices are changing.

The Stigler and Markham categories were the basis of a "critique" written by Lanzillotti (1957). He regards as the traditional models: dominant firm price leadership; competitive price leadership; and price leadership in lieu of overt agreement. His major criticism of standard categories is that they ignore dynamic aspects of market behaviour. A secondary criticism is that traditional models refer to rather stationary markets structures ignoring markets lying between these and free competition (which he presumably regards as a market which is not stationary). He argues that price determination is best described as essentially a competitive process rather than a quasi-collusive process. It is for this reason that Lanzillotti describes his model as one of "competitive price leadership". He enumerates seven features of this competitive price leadership:

1) /
1) Few firms whose output decisions interact

2) Mild (but not necessarily uniform) product diversification

3) Mild product differentiation

4) Moderately easy entry (patent and resources control being excluded)

5) Moderately high concentration (but declining secularly)

6) A sufficient disparity in cost between large and small firms that no single set of prices for substitutes would maximize each firm's profit

7) The demand curve for each firm should be more elastic than the market demand curve; and the market demand curve should have an elasticity of one or more

The unsatisfactory feature of this "model" is that it is not so much a model as something approaching a one-to-one map: that is, it is an exhaustive description of reality which makes little attempt to abstract key features. Furthermore, Lanzillotti makes no attempt at a formal manipulation, be it literary, geometrical or algebraic, of the assumptions: indeed we cannot even be sure that the assumptions are consistent. It becomes clear that what Lanzillotti has called a model is none other than a listing of the principal characteristics of the U.S. hard-surface floor covering industry. He then indicates that the three traditional models do not fit this industry very well, and leaves the reader to arrive at the unwritten conclusion that his does. Perhaps it is true that there can be no general model of price leadership, /
leadership, but this should not lead us to the polar view that we must construct a new model for every industry.

Some of the categories of price leadership which we have discussed so far are described in a different terminology by any individual author. Table 2.1 summarises the principal variants. Stigler (1947), Markham (1951), Bain (1960) and Scherer (1971) all mean the same when they refer to dominant firm price leadership. Stigler recognises only barometric price leadership as another distinct type. Markham's contribution was to extend the range of categories to a type not recognised by Stigler. Markham accepts Stigler's identification of barometric price leadership, and gives as an example the U.S. rayon industry in the 1940's in which American Viscose was the price leader. Markham concurs with Stigler in recognising the competitive nature of such a market, but does not give it a special name. Bain was later to describe such a market as one of "competitive price leadership". Scherer (1971, pp. 170-173) describes it simply as barometric price leadership. The second row of Table 2.1 demonstrates the equivalence of these different terms. Markham goes beyond Stigler in recognising that certain markets which appear to have the price set by a barometric firm are nevertheless essentially uncompetitive. This situation is described by Markham as price leadership in lieu of overt collusion. The equivalent terms adopted by Bain and Scherer are indicated in the third row of Table 2.1. At the foot of the table we also indicate the way some authors group different types /
<table>
<thead>
<tr>
<th>Category</th>
<th>Price Leadership</th>
<th>Barometric</th>
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<tbody>
<tr>
<td>Monopolistic</td>
<td>1.</td>
<td>2.</td>
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<tr>
<td>Oligopolistic</td>
<td>2.</td>
<td>3.</td>
</tr>
<tr>
<td>Collusive</td>
<td>3.</td>
<td>3.</td>
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<td>Competitive</td>
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<tr>
<td>Dominate</td>
<td>1.</td>
<td>1.</td>
</tr>
<tr>
<td>Scherer (1971)</td>
<td>Markham (1951)</td>
<td>Bain (1960)</td>
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Table 2.1 Categories of Price Leadership

- 1. Monopolistic
- 2. Oligopolistic
- 3. Collusive
- 4. Competitive
- 5. Dominate

Legend:
- Monopolistic: Recognized but not overt agreement
- Oligopolistic: Recognized
- Collusive: Recognized in lieu of overt agreement
- Competitive: No additional category

Stigler (1947)
types of market. At the present time, Scherer's classification is becoming increasingly accepted.

Apart from these widely accepted categories of price leadership, there are others which might lay claim to recognition. Parallel pricing, or "conscious parallelism" as it is often described in the U.S.A., is the manifestation of price leadership with no easily identifiable mechanism of collusion. The doctrine of conscious parallelism has developed through the anti-trust cases coming before American courts. Phillips (1962, Ch.3) describes the growth of the doctrine in American legal case history, and defines the practice itself as "a uniformity of pricing policy in a group of firms which is not based on any written agreement, but is nevertheless so complete that conspiracy is implied." In the case U.S. v Paramount Pictures Inc. ²(1948) which also involved other major film companies such as Warner Bros., and Twentieth-Centry-Fox, the substantial parallelism in several practices, including minimum admission charges at cinemas, was held to involve a conspiracy even though no written or oral evidence implying direct conspiracy was produced. This was a particularly stringent application of the doctrine. In the U.K., the Parallel Pricing Report ³ has adopted a less polarised view, and we have yet to see its effects upon public policy. Saxton (1942) described a variety of price leadership which he dubbed "aggressive".

Aggressive price leadership appears in markets in which there /
there is continual cost-reducing technical progress. The leader retains his competitive advantage by improving his method of production faster than his rivals. Both Maunder (1971) and Lanzillotti (1957) urge that a special category of price leadership should be recognised, in which we admit the existence of the practice of granting discounts on the list price to favoured customers.

Finally, we mention a category of behaviour which will be excluded from detailed discussion. A practice which is often mentioned in the context of price leadership is that of basing-point pricing. This may be of several forms, but we shall mention only one, the Pittsburgh-plus system. Following Hotelling (1929) we shall assume, for the sake of simplicity, that freight cost increases in direct proportion with distance. If Pittsburgh is the basing-point, then the price charged at any other location will be equal to the Pittsburgh mill price plus the freight charge from Pittsburgh, irrespective of the true origin of the steel. Thus if Indianapolis is $x$ miles from Pittsburgh, the price of steel delivered there will be $p + cx$ (where $c$ is freight cost per unit per mile) even though the steel may not have actually been manufactured in Pittsburgh. Suppose that the steel actually came from Chicago, where it is produced at the same cost as in Pittsburgh, and that Indianapolis is $y$ miles from Chicago ($y < x$). Then the true cost at Indianapolis is $p + cy$, whereas the actual charge is $p + cx$. The difference, $c(x - y)$, is described /
described as "phantom freight", and represents price discrimination. Although writers such as Kaysen (1949) have pointed out that such a system is often operative in a market characterised by price leadership, we would argue that basing-point pricing *per se* should correctly be analysed under the heading of price discrimination. This is in keeping with our general approach, namely to look at the analytical consequences of price leadership, rather than to examine the institutional conditions which may foster such an arrangement.

2-3. The Contributions of Forchheimer and Zeuthen

So far as we have been able to determine, the earliest contribution to the subject is the article "Theoretisches zum unvollständigen Monopole" ("The theory of Partial Monopoly") by Karl Forchheimer (1908). In view of the importance of this article, and the fact that no translation in the English language seems to be available, we have included a fairly literal translation as an Appendix to this chapter.

Forchheimer's starting point is to criticise the misuse of the term "monopoly" in two particular respects. Firstly the term is sometimes wrongly applied to firms which do not command the entire market, but merely a substantial part of it. Secondly the term is often incorrectly applied to situations of natural scarcity, merely because price is well above production cost, rather than to cases in which there is a genuine restriction of supply /
supply. His subsequent discussion runs in terms of four models: pure monopoly with zero costs; pure monopoly with decreasing costs; partial monopoly with the output of competitors fixed and zero costs; and partial monopoly with the output of competitors increasing with the price set by the partial monopolist and zero costs. All these models are explained by numerical example, and although Forchheimer insists at several points that he is using "quite arbitrarily chosen numbers", it is clear that underlying these numbers are algebraic relationships of quite a restrictive kind. In subsequent discussion we have chosen to analyse his models by reference to our algebraic interpretation of his tables of figures.

Model I is a simple monopoly model. Forchheimer ignores total costs or assumes they are close to zero: in effect making his profit column a total revenue column. From the price and quantity figures it is clear that the underlying demand schedule is linear, being

$$q = 2100 - 200p$$

Thus the total profit function is

$$\pi(p) = pq = 2100p - 200p^2$$

which is maximized when \(p = 5.25\); and thus \(q = 1050\). These are close to Forchheimer's given figures of \(p = 5\) and \(q = 1100\). Because he only worked in discrete terms, his figures generally /
generally only give an approximation to the optimal values. It is generally Model I that is later used for making comparisons with the partial monopoly solution.

Forchheimer presents the case in which costs are decreasing in his Model II, largely to show no new principles are introduced by introducing costs explicitly. From his table of values, it is possible to infer that the demand and average cost curves are given by

\[ q = 3100 - 100p \]  
\[ p = \frac{4100 - q}{200} \]

Therefore the profit function of the monopolist is

\[ \Pi(q) = \frac{(3100 - q)q}{100} - \frac{(4100 - q)q}{200} \]

which is maximized when \( q = 1050 \), implying a price of 20.5, compared to Forchheimer's figures of \( p = 20 \) and \( q = 1100 \).

The innovation of this paper is the introduction of a group of competitors who produce a constant output of 400 units. For the monopolist of Model I, Forchheimer substitutes a cartel, in Model III, but this makes no analytical difference. Because of the output of competitors, the cartel faces a residual demand curve given by

\[ q = 1700 - 200p \]
If costs are ignored, the leader's profit function is

\[ \Pi(q) = (1700 - q)q \left( \frac{200}{200} \right) \]

which is maximized when \( q = 850 \), implying a price of 4.25 and a profit of 3712.5. Forchheimer's own figures are \( q = 900 \), \( p = 4 \) and \( \Pi = 3500 \). A comparison of Models I and III indicate that the effect of the competitors is to reduce the absolute monopoly level of price, and increase the total level of output.

The final refinement considered by Forchheimer is to allow the capacity of competitors to increase as the cartel raises its price. This is examined in Model IV. From an examination of the given figures, it may be concluded that the following relationships hold:

\[ q = 2100 - 200p \] (market demand)

\[ q = 300 + 100p \] (competitive supply)

\[ q = (2100 - 300) - 200p - 100p = 1800 - 300p \] (residual demand)

Therefore the cartel's profit function is given by

\[ \Pi(q) = \left( \frac{1800 - q}{300} \right) q \]

which is maximized when \( q = 900 \). By implication \( p = 3 \) and /
and \( \Pi = 2700 \), which by coincidence are exactly Forchheimer's figures. By comparison with Model III, we see that the effect of increased competition has been to reduce even further the price set by the cartel.

Although these results are not general, because they have only been worked out by numerical examples, they suggest what further results might be obtained by more refined and general analysis. In Chapter 4, we shall return to a model akin to Model IV of Forchheimer and develop his, and many other results, with greater rigour.

It is appropriate that we should also deal with the works of Zeuthen (1930, 1955) at this stage. This is because Zeuthen (1930, p.17) refers to Forchheimer in his treatment of partial monopoly, and his geometry can be seen as an extension of Forchheimer's numerical examples. Though it is true that writers often refer to Zeuthen as though he were the originator of the ideas of partial monopoly, the priority clearly lies with Forchheimer. The earliest publication of Zeuthen on this matter appears to have been an article in Nationaløkonomisk Tidsskrift in 1929 entitled "Mellem Monopol og Konkurrence" ("Between Monopoly and Competition") 6/. This article was later used as the basis for Chapters I and II of Zeuthen's famous "Problems of Monopoly and Economic Warfare" (1930). In turn part of this material appeared in a revised form in a text-book treatment /
treatment by Zeuthen (1955). The definition of partial monopoly by Zeuthen (1930, p.17) is of particular relevance to this thesis, and was expressed in the following words:

A partial monopoly exists when one enterprise has so much power in the market that it is to its interest to charge a price which exceeds costs, even if sales are reduced thereby, whilst at the same time there are other enterprises which at each price have certain different sales irrespective of the policy of the other firms, the monopoly included.

At a later point, Zeuthen (1930, p.22) continues: "the supply curve SS' of the competitors has been plotted from a special y-axis. The distance of the curve from this axis indicates the total supply of the competitors at the different prices (their marginal costs) .... we have constructed the demand curve D₂D₂ such as it appears to the monopolist by deducting at each price the supply of the competitors from the actual demand curve D₁D₁ which is common to all the producers." As we would put it, D₁D₁ is market demand, D₂D₂ is the residual demand schedule facing the leader, and SS' is the competitive aggregate supply curve of the followers. A device that was not then in use (at least explicitly) was the marginal revenue curve, but apart from this, Zeuthen's diagrams have a similar structure to those later produced in price theory textbooks, such as Stigler's and Boulding's. The later textbook treatment of /
of Zeuthen (1955, pp. 243-245) himself does not differ in essentials from his earlier treatment.

A point of interest in the earlier treatment is Zeuthen's analysis of "the monopolist's line of retreat from the absolute monopoly" (p. 19) as competitors make incursions into the market. This "line of retreat" (the m m curve in Zeuthen's diagram) is the locus of the profit maximizing price and quantity coordinates. As in Forchheimer's Model III, it is assumed that the output of the competitors is a constant. Zeuthen then examines what happens when different values of this constant are assigned. He explains the outcome in terms of four diagrams, but the argument can be presented more rigourously in terms of a little algebra.

Following the assumption of linearity employed in Zeuthen's diagrams, we adopt the notation below:

\[ q = a - \beta p \]  
\[ p = cq \] (marginal cost; hence \( av.\text{cost} = \frac{cq}{2} \))

\[ q = k \] (output of competitors)

We assume \( a, \beta \) and \( k \) are positive.

The partial monopolist's residual demand is

\[ q = a - \beta p - k, \text{ from which } p = \frac{a - k - q}{\beta} \]

Thus his profit function is:

\[ \left(\frac{a - k - q}{\beta}\right)q - \frac{cq^2}{2} \]

which /
which is maximized when

\[
\frac{(a - k - q)}{\beta} - q - cq = 0
\]

from which profit maximizing output is

\[
q = \frac{(a - k)}{\beta (bc + 2)}, \text{ and the price set is}
\]

\[
p = \frac{(a - k) (bc + 1)}{\beta (bc + 2)}, \text{ by substitution}
\]

in the residual demand curve. Zeuthen's "line of retreat" is found by eliminating \( k \) between these two equations giving

\[
p = q \left( \frac{bc + 1}{\beta} \right)
\]

As \( \beta \) is positive, we conclude that the slope of the "line of retreat" depends on the costs of the partial monopolist. Thus

\[
\frac{dp}{dq} > 0 \quad \text{for} \quad c > -\frac{1}{\beta}
\]

\[
\frac{dp}{dq} = 0 \quad \text{for} \quad c = -\frac{1}{\beta}
\]

\[
\frac{dp}{dq} < 0 \quad \text{for} \quad c < -\frac{1}{\beta}
\]

For all increasing cost cases and for a range of decreasing cost cases, price will fall with output as the partial monopolist/
monopolist loses out to its competitors. What Zeuthen is approaching in his diagrams is what we would now call a "comparative statics" investigation. However, he took the matter no further, and did not apply his "line of retreat" technique to the case in which the supply of competitors increases with price, or attempt variations on the technique. In Chapter 4 we will see some of the extensions which may be undertaken in a comparative statics framework.

2-4. The Contribution of Nichol

One of the early works on price leadership is due to A.J. Nichol (1930). However, although frequently cited, the work is very rarely discussed, a notable exception being the article by Kahn (1937), on duopoly. This is probably because the work is not very accessible, having been a thesis presented to the University of Columbia in 1930.

In order to remedy the lack of discussion of Nichol (1930) in the literature, we intend to devote this section to a consideration of his thesis "Partial Monopoly and Price Leadership: A Study in Economic Theory".

Nichol (1930, p.11) is concerned with the situation in which "an industry is dominated by one concern (a trust for example) or an association of concerns (such as a cartel or syndicate), though other concerns also participate in the business". This is described as a situation of partial monopoly. He later defines it (for technical reasons which will soon become apparent) as "a concern controlling /
controlling 51% or more of the market". Chapters 1 and 3 are not strictly relevant to this thesis, the former being a theoretical introduction, discussing the analyses of monopoly by people such as Cournot and Marshall, and the latter being a detailed empirical inquiry into price leadership in the U.S. petroleum industry. Here we shall be exclusively concerned with Nichol's second chapter, which is an entirely analytical investigation of the extent to which a partial monopolist can control price.

Nichol (1930, Ch.2) is less concerned with how an exact maximum profit may be calculated, than with establishing why a firm should have the power to control price in the first instance. He establishes six propositions, largely through geometrical analysis, of which the most important is his fourth proposition. Proposition IV in Nichol (1930, p.24) states that within a certain range the partial monopolist can dictate a price to the rest of the market, and such a price cannot be altered by competition. Diagram 2.1, which is based on that in Nichol (1930, p.23), can be used to demonstrate this proposition. D is the market demand curve, OF is the maximum that can be supplied by all firms but one, this one being the leader, or partial monopolist, who can supply a maximum of OL. It is assumed that OL exceeds OF, this being the "51% or more" definition mentioned earlier. The rate at which the partial monopolist is able to supply the market is assumed to be perfectly flexible, and it /
Diagram 2.1
it is assumed that this firm will set its price without regard to short run profits. If all the competitors produce their maximal outputs, then price cannot be greater than \( p_1 \). The partial monopolist cannot diminish this supply, and if he were to produce nothing at all, then price would be exactly \( p_1 \). If the competitors produced nothing at all, and the partial monopolist produced at a maximum rate of \( OL \), then the price established would be \( p_2 \). This price could not be deliberately raised by competitors, but could only be diminished. If competitors supplied less than \( OF \) in total, either through agreement or as a result of competition amongst themselves, the partial monopolist could always augment supply such that, at least between the limits of \( p_1 \) and \( p_2 \), it had total control of price. Nichol later relaxes the assumption that the partial monopolist ignores profit considerations entirely, and assumes that he at least breaks even. It can be shown that the consequence of this is that the effective range of price control by the partial monopolist is diminished. The existence of such a range of price control warrants describing the partial monopolist as a \textit{price leader}. 

This thesis does not pursue the line of enquiry started by Nichol. We are more concerned with examining the effects of structural shifts within an existing leadership-followership relationship. That is, the dominance relationship is assumed to be exogenously determined /
determined. It is interesting to note that Nichol (1930, p.48) himself, near the end of his study concluded that "Standard Oil leadership, therefore, may be considered more a psychological phenomenon than the result of current economic forces".

2-5. Textbook Treatments of Price Leadership

Many microeconomics textbooks now contain treatments of price leadership. Most of these are derivative, and few attempt to acknowledge original sources. Two notable exceptions are the textbooks by Boulding (1948) and Hadar (1971). The first contained detailed geometrical analyses which, in their time, were original, and were often referred to by writers in the industrial economics and theory of the firm fields. The second is a treatiselike account of mathematical aspects of microeconomics. It is, so far as we know, the only book which contains a formal mathematical model of price leadership, be it all in a rather undeveloped form. Because of the relative originality of these two textbooks, we have chosen to examine their contributions in detail in this section.

The first revised edition of the textbook of Kenneth Boulding (1948), contained two models of price leadership which were later widely discussed, most notably, by Markham (1951). Boulding's models sought to explain two empirical observations. Firstly, that the high capacity firms in a market (such as the United States Steel Corporation in the American steel industry) tended to be price leaders. Secondly, /
Secondly, that in some markets, such as that for the retailing of petrol, a firm with a fairly insignificant market share could yet be the price leader. Both conclusions depend critically on the assumptions made about costs.

Boulding tends to associate high capacity with low marginal costs. Letting \( f(p) \) be market demand, with \( p \) being price, he assumes that in the two firm case the market will be equally divided, so the demand curve for each firm is effectively \( \frac{1}{2} f(p) \). Provided costs are identical, there is no problem. The profit function for each firm is

\[
\frac{1}{2} p f(p) - C \left[ \frac{1}{2} f(p) \right]
\]

(2.1)

where \( C[ \ ] \) is the total cost function. If the equation of the first derivative of this expression has a solution \( \hat{p} \), then this will be the price each firm sets, and each firm will produce \( \frac{1}{2} f(p) \). When costs differ, the situation is not so clear-cut. The profit functions may now be written

\[
\frac{1}{2} p_i f(p_i) - C_i \left[ \frac{1}{2} f(p_i) \right]
\]

(2.2)

and

\[
\frac{1}{2} p_2 f(p_2) - C_2 \left[ \frac{1}{2} f(p_2) \right]
\]

(2.3)
Boulding points out that the prices which maximize these functions (2.2) and (2.3), say $p_1$ and $p_2$, will generally be different if $C_1$ and $C_2$ are different. He chose to compare what he calls a low-cost (or high-capacity firm) with a high-cost (or low-capacity firm) $^8$. He conjectures that the final outcome will be that the high-capacity firm will eventually determine the market price, because its profit maximizing price is below that of the low capacity firm $^9$. Boulding argues that provided the low capacity firm is still making a reasonable level of profit then the industry will be stable.

A refinement on this reasoning by Boulding (1948, p. 584) is worth discussing. He produces an analysis which is related to the "reversed kinked demand" curve first analysed by Sweezy (1939), and later mentioned in a textbook by Bain (1952, p.353) and in the empirical study of pricing in Danish manufacturing industries undertaken by Fog (1960, p.142). Boulding's analysis can be explained by reference to Diagram 2.2. JD, JS, and JM are the market demand curve, market share demand curve and marginal revenue (of market share demand) curve, respectively. $C_1$ and $C_2$ are the marginal cost curves of the low-cost (high capacity) and high cost (low capacity) firms, respectively. If each firm were to get an equal market share, then each would face the market share demand curve JS. The marginal revenue curve corresponding to this is JM. In this situation, Firm /
Diagram 2.1
Firm 2 would desire to charge a price $p_2$ and Firm 1, a price of $p_1$. In a market with a homogeneous good, such a situation is unstable. Boulding suggests therefore that Firm 1 will become the price setter, and Firm 2 will regard this price as given. He argues that if Firm 1 sets a price of $p_4$, then Firm 2's average revenue curve will be $p_4 YS$. This being the case, Firm 2 will be willing to produce an output equal in magnitude to $p_4 X$ and Firm 1 will therefore produce half what the market will demand at this price (namely $p_4 Y$) plus what Firm 2 is unwilling to supply at this price (namely $XY$, which is equal to $YZ$ by construction). In this way a "reversed kinked demand curve" for Firm 1 is constructed, the kink occurring at a price of $p_3$, and $Z$ being a typical point on the curve below the kink. The geometry of such a curve is easy to investigate, and a useful analysis is given by Joan Robinson (1933, p.37) in "The Economics of Imperfect Competition". There it is shown that the appropriate marginal revenue curve is made up of two segments, like $JA$ and $BC$ in Diagram 2.2. In this particular case, the outcome suggested by Boulding is that Firm 1 will set a price of $p_4$ (which will be accepted by Firm 2), and produce an output equal in magnitude to $p_4 Z$ whilst Firm 2 produces an output equal to $p_4 X$. However, there are several criticisms that can be made of this analysis.

The most important criticism focuses on Boulding's use /
use of a model with equal market shares as a yardstick for comparison or "reference model". If firms have identical costs, then the assumption of equal market shares is reasonable, but when firms have such different costs that they may be characterized as "low-cost", high-capacity" and "high-cost", low-capacity", the assumption becomes rather implausible. It also leads to some technical confusions. Why, for example, should the kink in Firm 1's demand curve occur at a price of $p_3$?

It can be shown that $p_4X$ must equal $ZW$. It follows that the construction of Firm 1's demand curve is the same as that for the residual demand curve in the dominant firm model - at least below a price of $p_3$. In fact the solution given in Diagram 2,2 is identical to the solution that would emerge in the dominant firm model. The question then arises as to what happens above a price of $p_3$, but below $p_2$. It is clear that such a price could occur under Boulding's assumption that $C_1$ is everywhere below $C_2$. Boulding is silent on this point, but the implication behind the reversed kinked demand curve is that Firm 1 will only take a half of the market for this range of prices. Unfortunately it is unclear why the reasoning that was suggested for prices below $p_3$ should be implausible for prices below $p_2$. This emphasises the arbitrary nature of the assumption that Firm 1 will take at least one half of the market, rather than any other proportion.

The /
The second price-leadership model considered by Boulding is really a market share analysis, similar to one by Hadar (1971, p.117) to be discussed shortly. It is assumed there are two firms with market shares of, say, $\lambda$ and $(1 - \lambda)$, and that costs one identical. The profit functions are therefore given by

\begin{align}
(2.4) \quad \pi_1(p_1) &= p_1 \lambda f(p_1) - C \left[ \lambda f(p_1) \right] \\
(2.5) \quad \pi_2(p_2) &= p_2 (1 - \lambda) f(p_2) - C \left[ (1 - \lambda) f(p_2) \right]
\end{align}

The profit maximizing prices for these two functions will generally only be the same if $\lambda = 1 - \lambda$, that is $\lambda = \frac{1}{2}$, meaning that the market is equally divided. Boulding is particularly interested in the case in which $\lambda$ is substantially smaller than $(1 - \lambda)$. If it is assumed that the marginal revenue functions for both firms are falling and that the (common) marginal cost curve is rising in the neighbourhood of output equilibria, then the firm with the smaller market share will desire to set the lower price. Suppose the profit-maximizing prices for (2.4) and (2.5) are $\bar{P}_1$ and $\bar{P}_2$ and that $\lambda < (1 - \lambda)$. Then Boulding concludes that $\bar{P}_1 < \bar{P}_2$. Furthermore, he argues that Firm 2 will be obliged to accept the price $\bar{P}_1$ in a homogeneous market, and must therefore be content with a profit of $\pi_2(\bar{P}_1)$, which will generally be suboptimal.

Hadar /
Hadar (1971, pp.115-120) has provided the most recent treatment of static price leadership models. He distinguishes two basic models: (a) what is usually called the dominant firm model; and (b) the market share model (for both homogeneous and differentiated products). The simplest way to distinguish between these models is to inspect the form of the leader's profit function. If \( f(p) \) is the market demand function and \( S(p) \) the supply function of a competitive fringe of firms, then the leader's demand function is given by \( f(p) - S(p) \). Then if \( \Pi(p) \) represents the leader's profit function, it may be written

\[
\Pi(p) = p[f(p) - S(p)] - c[f(p) - S(p)]
\]

where \( c(\cdot) \) is defined as the leader's total cost function. The first order condition for maximizing \( \Pi(p) \) with respect to \( p \) provides an equation in \( p \) which it is assumed has a root say \( \hat{p} \). Then the leader produces \( f(\hat{p}) - S(\hat{p}) \) and the fringe produces \( S(\hat{p}) \) in aggregate. We have considerably extended and refined this model in the ensuing chapters, and have, amongst other matters, investigated its comparative statics properties. Hadar contents himself (rightly so, in a textbook context) with giving first- and second-order conditions for maximizing (2.6), and an example in which the market demand and competitive supply functions are hyperbolae, and the leader's /
leader's total cost function is linear. Hadar's second model is a formalization of a market share model akin to that described by Boulding (1948, p.586). If \( \lambda \) is the leader's market share \((0 < \lambda < 1)\), then the leader's demand function is \( \lambda f(p) \) and the leader's profit function is

\[
(2.7) \quad \pi(p) = p \lambda f(p) - c(\lambda f(p))
\]

It is a routine matter to set up conditions for maximizing this function. Hadar (1971, p.118) notes two interesting properties of the model. Firstly, the leader's profit is an increasing function of his market share. Secondly, the market price is an increasing function of the leader's market share, under the assumption that the leaders total cost function is convex. It also is interesting to note that (2.7) may be regarded as a special case of (2.6) in which we have set \( S(p) = (1 - \lambda) f(p) \).

Hadar (1971, pp.119-120) develops a variant of this model for a market with a differentiated product. Suppose there are \( n \) firms, and the first firm is the leader. The (inverse) demand function of the \( i \)th firm is \( p_i = f_i(q_1, q_2, \ldots, q_n) \), where \( q_i \) is the output of the \( i \)th firm. Denote the \( i \)th follower's share of the leader's output by \( \lambda_i \), and then \( q_i = \lambda_i q_1 \) \((i = 2, 3, \ldots, n)\). The leader's profit function can be written:

\[
(2.8) \quad \Pi(q_1) = f_1(q_1, \lambda_2 q_1, \ldots, \lambda_n q_1)q_1 - c(q_1)
\]
First-order conditions for maximizing \( \pi(q_1) \) are assumed to provide an equation with a unique solution \( \tilde{q}_1 \), from which \( \tilde{q}_i = \lambda_i \tilde{q}_1 \) (\( i = 2, 3, \ldots, n \)): thus the outputs of the followers are determined. Finally we determine the market prices which will be established from the expressions

\[
\Pi_i = \sum_{i=1}^{n} \tilde{q}_i^2 \quad (i = 1, 2, \ldots, n)
\]

A possibility which Hadar did not consider, but which is a fairly simple extension of his analysis involves a hybrid version of the dominant firm and market share models. In this variant the market is composed of a group of dominant firms who have agreed to divide the major part of the market amongst themselves, whilst permitting the existence of a competitive fringe. An example of such a hybrid market, is provided by the American aluminium industry in the 1950's.

It suggests the following extension of Hadar's model. The residual demand schedule facing the dominant group is \( f(p) - S(p) \), where we continue using the same notation. Suppose there are \( n \) firms in the dominant group who charge a uniform price, and that the \( i \)'th firm in this group takes \( \lambda_i \) of the residual demand. As the evidence cited above suggests, it is reasonable to suppose that costs are so similar in the dominant group that they may safely be assumed identical. Then the profit function of the \( i \)'th firm in the dominant group is

\[
\pi_i(p) = p\lambda_i \left( f(p) - S(p) \right) - c \sum_{i=1}^{n} \lambda_i \left( f(p) - S(p) \right)
\]
where \( \sum_{i=1}^{n} \lambda_i = 1 \) and \( c \frac{\lambda}{2} \) is the cost function. Clearly if each member of the dominant group maximizes \( \pi_i \) independently, a common price will not emerge, contrary to our assumption. Two possible outcomes are the following. The first is that one of the group, say the one with the largest capacity and market share (greatest \( \lambda_i \)), will set the price which maximizes his own profit, and which other firms in the group will accept — behaving, in effect, like the fringe group. The second possibility is that the desire to have a uniform price will outweigh the desire to have the largest market-share, and agreement will be reached to divide the market equally. That is \( \sum_{i=1}^{n} \lambda_i = n \lambda = 1 \), and the typical profit function of a dominant firm becomes

\[
\pi (p) = \frac{p}{n} [f(p) - s(p)] - c \frac{1}{n} [f(p) - s(p)]^2
\]

Maximizing (2.11) is equivalent to maximizing (2.6), and the model reduces to the simple dominant firm model from an analytical viewpoint.

In conclusion, it is clear that the models of Boulding and Hadar are original in several aspects, and therefore deserve examination. Furthermore, it seems that most variants reduce, in the final analysis, to two basic types: the dominant firm model; and the market share model. In subsequent chapters we have chosen to regard the market share model as being distinct from the price /
price leadership model. We have also abandoned Boulding's artificial distinction between high and low capacity firms. On the one hand, a linear marginal cost curve tells us nothing about capacity, which is usually identified with the minimum point on the long run average cost curve. On the other hand, any variant of the type suggested by Boulding can be incorporated in a more general model by making suitable restrictions on the cost parameters.

2-6. The Behaviour of Dominant Firms Over Time

The purpose of this section is to examine in some detail the theory underlying explanations for the decline of dominant firms. We concentrate attention on the essentially static analysis of Worcester (1957) and the dynamic analysis of Gaskins (1971). Some possible amendments to the model of Gaskins which have been suggested by Ireland (1972) and Jacquemin et al. (1972) are also considered. The origins of the models constructed by both Worcester and Gaskins lie in the dominant firm model of price leadership.

Worcester's analysis concentrates on the special case in which the combined capacity of all the firms in the fringe is equal to the capacity of the leader at any given price. Worcester deliberately constructs an equilibrium solution in which the leader's profit is zero and yet the followers make positive profits, in order to illustrate the disadvantage the leader may bring on himself by accepting the burden of output restriction. This ignores the possibility that the raison d'être for the existence of /
of price leadership may be that the leader has substantial cost advantages over the follower. It seems that Worcester is reluctant to admit this possibility, to the extent that he even claims the leader must be less efficient in the multi-plant case. Even if one accepts his postulate that long-run marginal and average costs are identical (and constant) for the leader and follower, it is not necessarily true that it will be less administratively efficient to co-ordinate a collection of plants than it is (in aggregate) to run a group of individual plants. If there are economies in management, the reverse might well be true.

Worcester's central argument distinguishes two subcases: (a) **Subcase 1** "single-plant dominant firm"; and (b) **Subcase 2** "optimal multiplant dominant firm".

Under Subcase 1, the leader's average cost curve is tangential to the residual demand curve. Certainly the leader has no incentive to expand, but neither has he an incentive to contract, as Worcester (1957, p. 340) claims, because it is equally true that to the left of the tangency position, average cost will exceed average revenue. Worcester's argument, that plant of a smaller capacity would be profitable, is not valid in the context of the short-run argument of Subcase 1. It is also not true that it would pay the followers to expand, thus encouraging the demise of the leader, if we limit discussion to fixed plant - as we should correctly do under Subcase 1. It is true however, that entry by additional followers would encourage /
encourage the decline of the leader, as such entry would detach the leader's residual demand curve from its tangency position and render any output unprofitable. Under Subcase 2, variation of plant is permitted and the argument is couched in terms of the long-run. The leader sets a price at which long-run marginal cost equals marginal revenue. However the situation is unstable. In view of the fact that long-run marginal costs are assumed identical and the leader sets a price which is a mark-up on this cost level, the profits of the followers are limited only by the extent of the market - it will always pay for them to increase output. In this way, Worcester hopes to reinforce his argument that the leader will decline. There are two major criticisms which can be made of this argument. The first, is that Worcester's conclusion under Subcase 2 should really be that the model collapses, or that its conclusions contradict its assumptions. If it really were true that the leader set a price such that followers increased output until they saturated the market, then the leader's residual demand curve would cease to exist, and with it, the possibility that the leader could ever have set such a price in the first place. The second criticism is that unless one postulates some reason why the leader might have assumed its role (e.g. cost advantages, along the lines suggested in Chapter 6) then instability is especially likely to arise. If the technologies are identical between leader and follower, and one can identify no exogenous factor giving the /
the leader market power, then it is not surprising that its role as leader is untenable.

One way of avoiding the passivity which must be imputed to the leader in Worcester's analysis is to adopt the price shading argument advanced in Chapter 5 or the priority pattern argument of Chapter 7. The price shading argument has been used by writers such as Vickrey (1964) to explain why the followers should get first bite of the market cherry: it is simply that in order to get any share of the market at all, the followers must shade their price slightly below the leader. It implies the mildest degree of product differentiation. The priority pattern argument of Chapter 7 has its origins in the market research literature on brand loyalty. It postulates that customers will tend to buy, in the first instance, from high turnover firms which operate the latest vintages of plant. A reinforcing factor mentioned in the analysis of that chapter is that firms high in customers' purchase priorities also tend to be relatively more profitable, and hence are able to allocate greater funds to advertising than their less-favoured competitors.

More satisfactory as a dynamic analysis of the dominant firm model is the paper of Gaskins (1971). In Gaskins' model, a dominant firm is threatened by the entry of a competitive fringe. There exists a limit price, $\tilde{p}$, which will totally exclude all competitors, and the greater is the difference between the actual price set by the leader and the limit price, the greater will be the rate /
rate of entry of competitors into the market. It is assumed that the limit price exceeds the unit total cost of the leader, implying that the leader has natural cost advantages over the followers. The problem for the leader is to choose that configuration of prices over time which will maximize the present value of its profits. Put technically, the problem posed by Gaskins (1971, p.308) is to maximize

$$\int_{0}^{\infty} \left[ p(t) - c \right] \left[ f(p) - x(t) \right] e^{-rt} dt$$

subject to

$$\dot{x}(t) = k \left[ p(t) - \bar{p} \right] \quad x(0) = x_0$$

The first bracket under the integral is an expression for unit profit, where $p$ is price and $c$ is unit total cost (assumed constant). The second bracket is the familiar "residual demand curve", where $f$ is the market demand function and $x$ the supply of followers. The meaning of the first constraint has already been explained, and the second constraint is merely an initializing value for the output of the followers. The selection of a suitable function $p(t)$ to maximize the profit integral is a problem in control theory, which Gaskins solves by advanced methods.

It turns out that the optimal choice of $p(t)$ depends, naturally enough, on the relationship between $x_0$ and a critical value of followers' output: $\hat{x}$ in Gaskins' notation. If $x$ exceeds $x_0$ then the optimal strategy is /
is to let price fall, approaching the limit price over time and thus reducing the leader's market share towards a long-run equilibrium value. If $x_0$ exceeds $x$, then the optimal strategy is to price below the limit price, which (in view of the absolute cost advantage of the leader) will continuously eliminate followers from the market. If the leader's and followers' unit costs are identical, as was the case in Worcester's analysis, then the followers will approach total industry output over time. This is an intuitively satisfying result, and illustrates the limited analytical interest of this case. Another result of interest is that the optimal price path over time will always lie below the short-run profit-maximizing price. Expressed intuitively this means that it will not pay the firm to exert full market power because (in the long-run) this will induce too high a rate of entry of competitors. An interesting extension considered by Gaskins is the case in which product demand is growing exponentially. Most of the results coincide with the stable product demand case, except for one. If the leader has no cost advantage over his followers, he will certainly not price himself out of the market in the long run, and provided the market demand function is concave at the equilibrium price, then the long run market share of the leader will increase with the growth in market demand. This last result is the only one which is rather counter-intuitive, and suggests that a slight modification of assumptions may be in order. In an unpublished paper by Ireland (1972) it is pointed out that Gaskins assumes that any increase in demand is taken up /
up by the dominant firm. It is more natural to assume that the dominant firm and fringe acquire a share of the growing market which is in proportion to their existing market share. When this modification has been made, Ireland shows that again, as in the static case, the dominant firm with no cost advantage will ultimately decline. As a final refinement, Jacquemin et al. (1972) have suggested that if the dominant firm has other variables under its control in addition to price it may still maintain its market share, even if it has no cost advantages. They assume, as an example, that the rate of exit depends on the level of expenditure undertaken by the dominant firm (through merger, take-over, etc.) to sustain seller concentration. In such a case, it is shown that a dominant firm with no cost advantages still has a positive equilibrium market share.

In summary, it is clear that the analysis of the progress of dominant firms through time is a problem of great conceptual difficulty. Worcester sought to show that the decline of dominant firms was inevitable. By contrast, the latest theoretical developments suggest that provided the dominant firm has a sufficient number of variables under its control it may not decline even if it has no cost advantages.

2-7. General Equilibrium and Monopolistic Competition

Another model developed by Hadar (1971, pp. 322-327) has quite a different flavour from those described so far. Hadar is concerned with an exchange economy over time, in which /
which all traders but one are price takers and one trader is a monopolistic price setter. Thus the pricing behaviour is very similar to that in a price leadership model. The main dissimilarity is that production does not appear in a meaningful way in Hadar's model. In the t\textsuperscript{th} time period the j\textsuperscript{th} competitive trader maximizes a utility function 

\[ u^j_t = \phi^j \left( \mathbf{x}^j_t \right) \]

subject to the constraint

\[ \sum_{i=0}^{n_l} p_i^t \left( x_i^j_t - x_i^{j-1} \right) = 0 \]

where \( \mathbf{x}^j_t \) denotes the vector of goods allocated to the j\textsuperscript{th} trader in time period t, and \( p_i^t \) is the price of the i\textsuperscript{th} good in time period t, and is assumed to be chosen by the monopolist. The constraint says simply that the value of goods before trade is equal to the value of goods after trade. Solving this maximum problem yields a set of individual excess demand functions which can be summed over all the competitive traders, giving a market excess demand function of the form

\[ x_i^t - x_i^{t-1} = h \left( p_t \mathbf{x}^{\xi}_{t-1} \right) \]

where \( \mathbf{x}^{\xi}_{t-1} \) is a set of vectors denoting the distribution of competitive holdings amongst the competitive traders. In this model, the monopolist is rather like a multi-product firm which is a price leader in every commodity market. The problem facing the monopolist is to maximize his utility function by choosing a suitable set of \( p_t \)'s. The monopolist /
monopolist is constrained by the condition that he must undertake the trading offered by the competitive firms at the prices he has declared.

Hadar is able to show that every trader's utility level is a bounded infinite sequence and, in consequence, is convergent. This also implies that the $x_{ij}$ will converge. Convergence is achieved by a new price being set in each period, provided what Hadar (1971, p.324) calls a Pareto-superior distribution may be achieved. His definition of "Pareto-superior" is rather contentious. It has often been held that a divergence of price from marginal cost violates a condition of Pareto optimality. Hadar has avoided this problem because his model is one of pure exchange, with no production. Furthermore by specifying the traders' behaviour in terms of utility functions, rather than profit functions, he is able to ensure that his brand of Pareto optimality emerges. This ingenious method of sidestepping the conventional argument has also been employed by Chamberlin (1957, p.98): "The fact that equilibrium for the firm when products are heterogeneous normally takes place under conditions of falling average costs of production has generally been regarded as a departure from ideal conditions, these latter being associated with the minimum point on the curve; and various corrective measures have been proposed. However, if heterogeneity is part of the welfare ideal, there is no prima facie case for doing anything at all." There is some danger in this approach however, because it means that with a sufficiently wide definition of utility function almost /
almost any type of behaviour can be construed as optimal.

The discussion of Hadar's monopolistic competition model leads us naturally to a consideration of monopolistically competitive general equilibrium models. Negishi (1960-61) provided the earliest treatment of this type of problem, and more recently Arrow (1971), and Arrow and Hahn (1971, pp. 151-167) have carried the analysis further. The theory is in its infancy, witness the fact that a recent book by Hansen (1970) which surveys general equilibrium theory contains no mention at all of monopolistic competition. The theory is also very advanced mathematically, some aspects of it being beyond the technical competence of the writer. Fortunately, Arrow and Hahn (1971), in particular, have provided an excellent heuristic account of their detailed proof.

This type of analysis is interesting, as it attempts to formalize the work of Chamberlin (1933) on monopolistic competition with many firms - though some aspects of Chamberlin's model, such as free entry and exit, are absent from recent models.

In Negishi's model, there are two types of firm: monopolistically competitive and perfectly competitive. Unlike Hadar, Negishi introduces production in a meaningful way, and the monopolistic firms maximize profit functions rather than utility functions. Each monopolistic firm has an inverse subjective demand or supply function for its outputs and inputs. The analysis can be extended to the kinked /
kinked demand or supply curve cases. The "oligopoly problem" is assumed away in the sense that there is no interaction between firms in the monopolistic group. In Negishi's model, each monopolistic firm produces a single good and maximizes profits according to a subjective demand curve which is a function of all prices and the entire allocation of production and consumption. In such a model, Negishi is able to show by advanced methods that an equilibrium exists, and that under certain additional assumptions this equilibrium is stable.

Arrow and Hahn (1971, pp. 157-158, 165-167) give a useful explanation of the subjective demand curve technique, and a set of comments on their formal model. We base the following account on their work.

Suppose a monopolistic firm has an (inverse) subjective demand curve $p = f(q, \Theta)$, where $\Theta$ is a parameter dependent on the observed prices and allocations in the economy. We will denote the corresponding subjective marginal revenue curve by $R(q, \Theta)$ and the marginal cost curve by $M(q)$. For a given $\Theta$, the monopolistic firm's output and price are the solutions to $p = f(q, \Theta)$ and the profit maximization condition $R(q, \Theta) = M(q)$. By eliminating $\Theta$ between these two equations we get a function $q = \phi(p)$ which Arrow and Hahn (1971, p.157) term "the monopolist's supply curve". As in Negishi's model, it is not assumed that monopolistic firms have knowledge of the true demand curve, say $p = F(q)$, nor even /
even of its elasticity, but only that equilibrium prices and quantities are consistent with the true demand curve. In the fully-fledged model the expression \( p = f(q, \theta) \) is generalized to a production sector which is a continuous function of prices and allocations, such that profits are non-negative for each monopolistic firm for all prices and allocations. This is actually quite a strong assumption, because it rules out the possibility of firms entering or exiting at substantial levels of capacity. Such behaviour would imply discontinuity in the function. The problem is very similar to that tackled in the model of Newman and Wolfe (1961, p.51), where it is assumed that newly emergent firms in an industry very quickly grow from zero size in a "limbo class" through lower size classes, appearing almost instantaneously at a finite scale of operation. As we shall see in Chapter 6, discontinuities of this kind can be very hard to treat in a general way.

2-8. Theories of Conjectural Behaviour

Frisch (1933) devised a general framework for analysing monopolistic competition in terms of the "action parameters" of economic agents (or "polists" in his nomenclature). In this section we will give a brief outline of Frisch's theory, giving consideration to his theory of conjectural elasticities, amongst other matters. His work will then be related to some of the types of conjectural behaviour considered in oligopoly models.

We /
We will start by defining $z_{ij}$ as the $j$'th action parameter of the $i$'th polist ($i = 1, 2, \ldots m; \ j = 1, 2, \ldots n_i$). It is assumed that the $n_i$ action parameters open to the $i$'th polist are all independent. In the simplest price leadership model, for example, the two polists are the leader and the follower. The action parameters are; price, for the leader; and output, for the follower. The leader's output is not an action parameter, as it is not independent of price. In general, different polists will have different numbers of action parameters, totalling, say, $N$ in aggregate. Frisch assumes the profit of the $i$'th polist is a continuous function of all $N$ action parameters. He proposes classifying polists into three groups, depending on the magnitude of "conjectural coefficients" (or "conjectural elasticities"). He defines a typical such coefficient as

$$ z_{ij} = \frac{\partial z_i}{\partial z_j}$$

which expresses the believed elasticity of response of the $h$'th polist's $i$'th action parameter to a change in the $k$'th rival's $j$'th action parameter. The special symbol $\partial$ is used rather than the more familiar symbol for a partial derivative, to emphasize the point that the elasticity is defined in terms of conjectures or beliefs, rather than in terms of actual outcomes. Frisch defines autonomous adaption as /
as the case in which $Z_{i,j}^{hk} = 1$ for $h = k, i = j$, but is zero otherwise. It is obviously true that we will always have $Z_{i,i}^{hh} = 1$. Under conjectural adaptation, $Z_{i,j}^{hk}$ will normally be non-zero for distinct $h, k$ and $i, j$. In this case each polist assumes that a change in his rivals' action parameters will be a continuous function of his own action parameters. The most general case is superior adaption. In a market in which some polists act conjecturally and some autonomously, the former group will be in a dominant position provided members of it are aware of two facts: firstly, that other polists are acting autonomously; and secondly, the profit schedules which the autonomous adaptors are attempting to maximize. If these two conditions hold, then the conjectural adaptors are described as superior adaptors. Frisch suggests that this idea might be generalized to situations in which there are groups of more or less superior adaptors, or, as Frisch (1933, p.32) himself puts it, "a hierarchy of superior adaptors". The final element of the general framework is the coefficient of attraction, which is defined as

$$\omega_i^h = \frac{d\pi^h}{dZ_i^h} \frac{Z_i^h}{\pi^h}$$

Technically this is the elasticity of profits with respect to an action parameter: less formally, it is a measure of the profit inducement for the $h^{th}$ polist to increase or decrease his $i^{th}$ action parameter. Frisch shows how a field /
field of force in terms of the vector of $\omega_i$ can be defined, and how such a field may enable one to investigate whether an equilibrium will be reached in the market. In the earliest models of oligopoly, conjectural variation was generally assumed to be zero. This is true, for example of Cournot's duopoly model. If the good is homogeneous, with market price a function of total output, $p = f(q_1 + q_2)$, then the two profit functions may be written

$$\pi_1(q_1, q_2) = q_1 f(q_1 + q_2) - C_1(q_1)$$
$$\pi_2(q_1, q_2) = q_2 f(q_1 + q_2) - C_2(q_2)$$

Where $C_1$ and $C_2$ are cost functions for the first and second duopolists. The zero conjectural variation assumption implies that each duopolist attempts to maximize profit by varying his own output, under the assumption that his rival's output will remain constant.

First-order conditions for maximization of each duopolists profit function with respect to his output level are

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = 0 \quad \text{and} \quad \frac{\partial \pi_2(q_1, q_2)}{\partial q_2} = 0$$

By solving these equations for $q_1$ and $q_2$, we can obtain the /
the optimal outputs for each firm, and by substitution of these values, say \( \bar{q}_1 \) and \( \bar{q}_2 \) back into the market demand schedule, can determine the market price, namely 
\[ \bar{p} = f(\bar{q}_1 + \bar{q}_2). \]
A less direct method, but one which is of analytical interest, is to solve the first of these equations for \( q_1 \) and the second for \( q_2 \), giving
\[
q_1 = F_1(q_2) \quad \text{and} \quad q_2 = F_2(q_1)
\]

\( F_1 \) and \( F_2 \) are the so-called "reaction functions" of the first and second duopolist. \( F_1 \), for example, is an equation defining (for any output level \( q_2 \) of the second duopolist) that output which will maximize the profit of the first duopolist. Solving the reaction functions gives the equilibrium outputs already discussed.

From the viewpoint of this thesis, the leader-follower analysis of Stackelberg is of greater interest. A leader does not obey his reaction function, but assumes that his rival does. A follower obeys his reaction function, maximizing his profit, given the leader's output level. Suppose that the two profit functions and reaction functions of the duopolists are given by
\[
\pi_1 = \pi_1(q_1, q_2) \quad \pi_2 = \pi_2(q_1, q_2) \\
q_1 = F_1(q_2) \quad q_2 = F_2(q_1)
\]

If /
If the first duopolist assumes the role of leader and that his rival will be a follower, his profit function may be written

\[ \pi_1 = \pi_1 \left[ q_1, E_2(q_1) \right] \]

which is a function of \( q_1 \) alone. This may be maximized directly with respect to \( q_1 \). Each duopolist can compute the profit that would accrue to him in playing either role, and then decide which is most favourable. Let us call the duopolists Firm 1 and Firm 2. Then there are four possible role combinations: (1) Firm 1 leader, Firm 2 follower (2) Firm 2 leader, Firm 1 follower (3) Firms 1 and 2 leaders (4) Firms 1 and 2 followers. Under (1) and (2) the desired roles are consistent and the model is determinate. Under (4), the Cournot solution emerges as conjectural variation is effectively zero. Under (3) the desires of each duopolist are inconsistent and conflicting and the so-called Stackelberg disequilibrium emerges.

It should be noticed that the similarity between this model and the price leadership model is more apparent than real. In the Stackelberg model, a firm is a leader or follower depending on whether he does not, or does, follow his reaction function - and these reaction functions are set up with outputs as the action parameters. In the price leadership model the roles of leader and follower are already decided. Furthermore the action parameters are different /
different, being price in the case of the leader and output in the case of the follower.

Closer to our interests is the analysis of Cyert and de Groot (1971). The kinked demand curve model of Sweezy (1939) and Hall and Hitch (1939) is analysed in terms of the following conjectural variations

\[
\frac{dp_1}{dp_2} = 0 \quad \text{when} \quad p_2 > p_0
\]

\[
\frac{dp_2}{dp_1} = 1 \quad \text{when} \quad p_2 < p_0
\]

This is actually a special case of the kinked demand curve, in which the upper segment is infinitely elastic. The implied reaction curves are given in Diagram 2.3, where OAC is Firm 1's reaction curve and OAB is Firm 2's reaction curve. The price \( p_0 \) is the point at which the demand curve is kinked. If either Firm 1 or Firm 2 reduces price below \( p_0 \), he will be followed by his rival. If either firm raises price above \( p_0 \), he will do so alone. This suggests that the reaction curves in the price leadership case should be as in Diagram 2.4. OAC is the follower's reaction curve: whatever price the leader sets, he will follow. The leader's reaction curve is \( p_0AB \), and is horizontal because it is not contingent on the follower's price. The equilibrium /
equilibrium price is $p_0$.

An interesting modification to the basic kinked demand curve analysis has been suggested by Sweezy (1939). He suggests that a reversed kinked demand curve might be applicable to certain types of price leadership. If the leader assumes that if he raises prices he will be followed, but if he secretly undercuts price he will not necessarily be followed, his demand curve will be kinked, with the elasticity to the immediate left of the kink being less than the elasticity to the immediate right of the kink. As Stigler (1947, p. 127) has pointed out, maximum profit will no longer be achieved at the price corresponding to the kink.

In this section, we have seen that the price leadership model may be related to three conjectural approaches. First, in Frisch's terminology, we may regard the price leader as a superior adaptor over the follower. Second, the model may be described in terms of two reaction curves between price axes: one a 45° line and the other a straight line at a constant price. Third, a particular type of price leadership (in which secret under-cutting takes place) may be regarded as a special type of kinked demand curve model.

2-9. **Summary**

This Chapter has examined some theoretical work on the price leadership model over a period of approximately seventy years. The most thorough analytical works /
works on price leadership were seen to be due to Forchheimer (1908), Zeuthen (1930), Nichol (1930), Boulding (1948), Worcester (1957) and Hadar (1971), all of these being static analyses, with the exception of the study by Worcester which is quasi-dynamic. The models of Gaskins (1971) and Hadar (1969), which are essentially dynamic, were shown to be closely related to these basic works. We also gave some consideration to peripheral material on duopoly and general equilibrium models.

It should be noticed that a preoccupation of most writers has been with explaining price leadership in terms familiar to economists (i.e. using notions of costs, profits, demand etc.), and in certain circumstances with demonstrating the determinacy of a particular model. With the notable exception of the "line of retreat" argument advanced by Zeuthen (1930), few writers have taken the next step of making predictions from their models. In the static analysis of the following four Chapters we have attempted to formalize, modify and extend the basic static model, namely that of dominant firm price leadership. As with previous works we shall be concerned to explain price leadership in economic terms and to show that the model developed is determinate. In addition, we shall also show how the technique of comparative statics may be used to derive predictions from the model.
Footnotes to Chapter 2

1/ Page references are to reprints in the case of the articles by Bain (1960), Stigler (1947) and Markham (1951). These reprinted sources are, in order, Bain (1972), Archibald ed. (1971), Heflebower and Stocking ed. (1958).


4/ A model of this type of behaviour is developed in some detail in Chapter 7.

5/ Practical examples of the operation of this system are in Adams ed. (1961, p.161).


7/ I should express my gratitude to the staff of the University of Edinburgh Library for tracing Nichol's work and obtaining a microfilm of his thesis.

8/ Although he speaks as though low-cost is synonymous with high capacity, this is clearly not always so, and is a consequence of the fact that he does not permit the marginal cost curves to intersect.
This result on prices follows because Boulding assumes that $c_1(q) < c_2(q)$ for a given $q$, where $c_1$ and $c_2$ are the marginal cost curves of the high and low capacity firms, respectively. It is also assumed that the $c_i$ are increasing in the neighbourhood of profit maximizing outputs, and that the marginal revenue curve is decreasing everywhere. It follows that the profit maximizing output of the high capacity firm exceeds that of the low capacity firm, and hence that the profit maximizing price will be lower for the high capacity firm.

$P_4 Y = YW$ and $XY = YZ$, both by construction. Subtracting, we get $P_4 Y - XY = YW - YZ$, that is $P_4 X = ZW$.

Empirical details on this industry may be obtained from Adams ed. (1961, Ch.6).

Specifically, by the application of Pontryagin's maximum principle, which has recently become widely used in economic theory. In fairness to Worcester, it should be said that these control techniques were not available at his time.

The original article is Hadar (1969).

See, for example, Henderson and Quandt (1971, p. 266).

In particular to the type suggested by Lanzillotti
Lanzillotti (1957) and Maunder (1972).
Appendix to Chapter 2

The Theory of Partial Monopoly*

by

Karl Forchheimer 1/

List of Contents

The problem, p.1; empirical material, p.1; the meaning of monopolistic power, p.2; the theory of price under monopoly, p.5; its application to partial monopoly, p.7; the relationship between competition and monopoly, p.9; the real situation, p.10; the power to undersell, p.11.

A monopolist is generally understood to be a sole supplier who, as near as makes no difference, controls the total output of a good. It is obvious that, in practice such a situation (conforming in every respect) can be approached at most by the so called legal monopolies, but that in actual monopolies part of the output of a good is mostly supplied by other producers. In the theoretical treatment of monopoly too there is always a distinction between "complete" and "partial" or "absolute" and "relative" monopoly. Since in the latter case total supply is no longer controlled by a single producer, should it nevertheless be considered as a monopoly?

The inquiry of the "industrial commission" which was held in Washington from 18.6.1898 on the instructions of Congress, has brought to light a wealth of empirical material /
material on the question of partial monopoly. The representatives of big companies stated there that their combines mostly commanded in the region of 70 to 90% of the entire supply, but not more than such a percentage. So, for example, the American Sugar Refining Company for a time produced 75-80% of the U.S's refined sugar, the Standard Oil Co. controlled 80-95% of production, the American Smelting and Refining Co. 85%, the International Paper Company 70-80%, and so on. It was generally affirmed by the representatives who were examined that the combinations dictated price. The question repeatedly raised in the hearings, namely whether partial monopoly does in fact bestow this power, could nevertheless only be answered for these cases by looking at the facts.

In economic theory, the question of the frontier between partial monopoly and a non-monopoly situation, and the problem whether a partial monopoly controls price, are usually not raised. So far as I know, only Cournot has attempted an explanation (in a complicated mathematical form) in his "Recherches sur les principes mathematiques de la theorie des richesses" which is not illuminating and is rejected by Lexis in the Handworterbuche der Staatswissenschaften. Even Ely in his book "Monopolies and Trusts" (New York, 1900), who describes many problems of relevance to this area, gives no theoretical clarification on this point. And yet the model they usually use in their discussions, which is within the framework of orthodox price theory, is freely called upon to illustrate partial monopoly.
monopoly too. I think it is worthwhile, in what follows, to give a short presentation of the theory of price under monopoly, for the sake of coherence.

The significance of monopolistic power and of its consequences is obviously not confined to the sphere of price policy. By concentrating the whole of supply in one unit, the holder of this power is able to achieve certain results which do not directly imply a higher profit and are achieved through means other than a change in price. However, the monopoly's control over price is the outstanding aspect of this phenomenon and the only one which can be described generally.

When we speak of monopoly prices we use the word "monopoly" in a suggestive sense, meaning by it not only the control of supply but also the complete exploitation of this control. Monopoly price does not differ from the price under free competition in that one seller faces many buyers; the distinguishing feature of monopoly price lies rather in the fact that not the whole available supply is put on the market, or not as much is produced as could be at a profit. Instead supply is restricted in order that a higher price and thereby a higher profit can be obtained. At any given time this restriction is to the monopolist's advantage only up to a certain point. If supply is reduced beyond this point, then the higher price may no longer compensate for the lower sales. The ideal monopoly price lies at this point. Of course, in practice price only more or less approaches this point — proceeding /
proceeding from the lower limit of production costs. In this, restriction is taken to be the element of monopoly which is economically harmful. The interest of the monopolist in restricting output conflicts with the economic interests of society, namely that as much be produced as can be at a profit. This is illustrated particularly forcefully by the fact that part of the current crop in Greece is destroyed in years of good harvests, causing price to rise. In order that such a restriction of supply be achieved it is of course necessary that supply be in the hands of a single person or of a group of people with an individual determination to act to this end. As soon as goods of a particular type are to be found in the hands of several people who cannot come to terms over such a restriction - i.e. under the regime of free competition - then it must necessarily be the aim of each individual to sell as much as he can. For the reduction of his own supply would not be sufficient to raise the price significantly, so that the reduced sales volume at a higher price would prevent bigger profits being obtained. Hence in a competitive regime the total supply is increased as much as possible: as a rule, by as much as is consistent with the attainment of average profit. The monopolist alone is in a position to enjoy the fruits of the restriction he has undertaken, and thereby to attain a "monopoly profit".

Familiar though these facts are, they cannot be emphasised too often; for this simple matter of fact is frequently overlooked, and the higher price which certain goods /
goods attain by virtue of their natural scarcity is confused with the inflated prices achieved through monopoly. For example, the few extant pictures of a dead master, or certain exceptional sorts of wine which only grow in limited areas, are adduced to illustrate monopoly. The common element is the high price (well in excess of production costs) which accords a surplus; but the difference is to be found in the fact that in the one case (that of natural scarcity value) this surplus is founded on the object. In this connection it does not matter whether these objects are concentrated in the hands of one seller, or dispersed over several competitors; for the superior price will be achieved by these goods if so many are sold as really are available, or as many produced as is technically feasible. In the other case (that of monopoly), on the other hand, the superior price is founded in the power of the person, since there was no essential scarcity before one was created artificially by the monopoly.

Indeed the confusion occurs most frequently when municipal ground rent, or ground rent in general, is labelled monopoly rent. The high value and yield of municipal plots rests on their scarcity, and, under certain circumstances, on the fact that they are unique in some respect. In connection with this, however, whether they belong to one particular person or not is quite without significance. Only when a few land speculators buy up land around the town, and in the course of the town's expansion hold back part of the land in order to drive /
drive up the price, does price contain a monopoly element. Likewise, to describe as "monopoly" the advantageous situation of the owners which results from the scarcity of goods is pointless on grounds of terminology, and does not accord with the significance of the word "monopoly". Of course, where it is a matter of an artificially created scarcity - of a monopoly - then in certain cases it is appropriate and possible to combat this phenomenon; but the natural form of scarcity can never be dealt with. Indeed, even in a communist economic system scarce factors of production and final goods would have to be given a different valuation and, accordingly, a different treatment and use from less scarce ones. This had to be pointed out here in order to establish that the sole significant characteristic of monopoly is the monopolist's gaining of an advantage at the cost of the purchasing public by restriction of supply.

The model which is commonly invoked to illustrate the situation of the monopolist is roughly as follows:

<table>
<thead>
<tr>
<th>Model I</th>
<th>Price</th>
<th>Quantity Demanded at this Price</th>
<th>The Profit Produced at this Price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>100</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>300</td>
<td>2700</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>500</td>
<td>4000</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>700</td>
<td>4900</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>900</td>
<td>5400</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1100</td>
<td>5500</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1300</td>
<td>5200</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1500</td>
<td>4500</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1700</td>
<td>3400</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1900</td>
<td>1900</td>
</tr>
</tbody>
</table>
The monopolist who was in a position to produce 1900 and sell at 1 (a price which - let us assume - would bring him an acceptable profit) benefits by restricting the quantity supplied to 1100, because total profit is at a maximum at a price of 5. In a regime of competition, on the other hand, so much will be produced that the price will be depressed to that level at which is exceeds costs of production by just the standard rate of profit.

By the way it is very simple in this scheme to take falling costs into account - in roughly the following way:

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity Demanded at this Price</th>
<th>Unit Cost</th>
<th>Net Profit Unit</th>
<th>Net Profit Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>100</td>
<td>20</td>
<td>10</td>
<td>1000</td>
</tr>
<tr>
<td>28</td>
<td>300</td>
<td>19</td>
<td>9</td>
<td>2700</td>
</tr>
<tr>
<td>26</td>
<td>500</td>
<td>18</td>
<td>8</td>
<td>4000</td>
</tr>
<tr>
<td>24</td>
<td>700</td>
<td>17</td>
<td>7</td>
<td>4900</td>
</tr>
<tr>
<td>22</td>
<td>900</td>
<td>16</td>
<td>6</td>
<td>5400</td>
</tr>
<tr>
<td>20</td>
<td>1100</td>
<td>15</td>
<td>5</td>
<td>5500</td>
</tr>
<tr>
<td>18</td>
<td>1300</td>
<td>14</td>
<td>4</td>
<td>5200</td>
</tr>
<tr>
<td>16</td>
<td>1500</td>
<td>13</td>
<td>3</td>
<td>4500</td>
</tr>
<tr>
<td>14</td>
<td>1700</td>
<td>12</td>
<td>2</td>
<td>3400</td>
</tr>
<tr>
<td>12</td>
<td>1900</td>
<td>11</td>
<td>1</td>
<td>1900</td>
</tr>
</tbody>
</table>

But since, for our purpose, nothing new in principle is expressed in this, and it only complicates one's train of thought, in what follows we will go back to Model I.

There the numbers used to denote the demand relationship are quite arbitrarily chosen. In the market, demand might generally rise more steeply as price is reduced, probably /
probably rather in a geometric progression and less regularly. But demand must always increase as price falls. Starting from an exorbitant price, where both demand and profit are zero, profit increases as price falls up to a certain point, then falls again and at price 0 and a very big demand reaches zero again. It is also conceivable that profit does not move in an ascending and descending line, but rather in a zig-zag, in which case a levelling-off at the peak, or such-like, results. It will always pay most to raise price (and so reduce sales) up to a specific point; but since the numbers serve only as an illustration, and not a proof of this point, the discussion will not be affected by their arbitrariness.

Now we come to the question of partial monopoly. Is such an arrangement capable of attaining a monopoly price even when some competition persists which aims to undercut the monopolist? If the relative monopolist attains a monopoly price, is this just as high as the absolute price, and is the partial monopoly's lower profit simply a matter of lower sales? How great must competition be to eradicate partial monopoly and - in effect - for free competition to emerge?

Let us assume that, in a closed region in a single production period, 1900 units of a product were produced, and the price of 1 resulting in this period (following Model I) still secures an acceptable profit in addition to costs. If a cartel of collective producers set a price of 5 (the absolute /
absolute monopoly price under Model I), and a restriction of production to 1100 would be most favourable for them, then the cartel would strive for this price - whether through price-fixing or quota-fixing. Now let the cartel secure only some of the producers, whose share totals 1500 units, whilst the rest, who together produce 400 (rather over 20%) remain outside the combination competing both with the cartel and amongst themselves. In talking of the quantities produced, let these quantities mean at the same time the maximum that the producers in question can make. This is also important if we are considering just monopoly. The assumption that productive capacity is limited in this way conforms roughly to reality. The available means of production permit only a certain output and the setting up of new means of production (construction of factories and workshops) implies great costs and would presumably not pay since the quantity supplied yields just a normal profit, and if more were supplied they would have to put up with an unprofitable price. Besides, the effect of these new establishments would not come into question in the production period under consideration and in which we want to examine price formation. Only products which by and large meet such conditions are suitable for cartelisation.

Now it is noteworthy that in this supposition a sort of potential scarcity value certainly does not rest in the object, and without it no monopoly can persist. But it is important that monopoly profit, which exceeds normal profit, does not rest on scarcity value. For there could be sufficient /
sufficient goods produced (and there would actually be if competition prevailed) to force price down to that level which secures the lowest possible entrepreneurial profit. Only the artificial restriction of production (which goes beyond this scarcity) and the scarcity value achieved by this action brings about the higher price which yields monopoly profits. Of course, the fact that this potential scarcity value appears as a prerequisite of monopoly leads to the confusion between monopoly and the operation of objective scarcity.

So the cartel must also reckon with the 400 units that will be produced by the outsiders, and can only control the domain left over. The outsiders compete both among themselves and with the cartel, and so are willing to sell their product at any price, or at least when it exceeds 1 (prime costs plus profit). In setting price the cartel can therefore only reckon on sales corresponding to demand at a particular price, less 400. With the demand relationship of Model I, the cartel can hope for the following profits:

<table>
<thead>
<tr>
<th>Price</th>
<th>Sales at this Price that the Cartel can count on</th>
<th>Profit of the Cartel</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>(500 - 400 = )</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>(700 - 400 = )</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>(900 - 400 = )</td>
<td>500</td>
</tr>
<tr>
<td>5</td>
<td>(1100 - 400 = )</td>
<td>700</td>
</tr>
<tr>
<td>4</td>
<td>(1300 - 400 = )</td>
<td>900</td>
</tr>
<tr>
<td>3</td>
<td>(1500 - 400 = )</td>
<td>1100</td>
</tr>
<tr>
<td>2</td>
<td>(1700 - 400 = )</td>
<td>1300</td>
</tr>
<tr>
<td>1</td>
<td>(1900 - 400 = )</td>
<td>1500</td>
</tr>
</tbody>
</table>
It still pays the cartel pretty well to restrict output, though not by as much as if it had absolute monopoly, for in Model I production was restricted in total from 1900 to 1100, whilst in this case a total of 1300 is still produced. It should be noted that the outsiders cannot take action against this price. If they try to undercut it, then they only harm themselves and perhaps even give the cartel a chance to sell even more at a price of 4, despite them.

Here there emerges a price which exceeds the competitive price and brings a monopoly profit; this price is, however, lower than the absolute monopoly price - it has elements of both monopoly and competition. The competitive and monopoly price are not in general mutually exclusive, but are opposite poles between which lie many intermediate possibilities. For instance, when we consider the outsiders' share of output being successively increased to 500, 600, 700 and so on, and each time make this same calculation, we find that, if the outsiders produce 700, the cartel can only get a price of 2; and if the outsiders' share is 1500, a restriction of output is no longer worthwhile for the cartel; on the contrary the pure competition price becomes manifest.

The gradual diminution of the monopoly element should be indicated by this, but in this case only whole numbers are taken into account, and so the finer gradations are not considered. But the fact that a monopoly element is still present /
present when the cartel controls only about 1/4 of supply rests only on the arbitrarily assumed demand relationship. Hence it happens that in our model, firstly the absolute monopoly price stands very much in excess of the competitive price, being five times as great (5:1); and further that, in the example, the quantity supplied goes up only gently as price declines - more slowly, I daresay, than is generally the case with actual market relationships.

One cannot establish in general the specific percentage of supply controlled by outsiders up to which a partial monopoly will exist - that depends particularly on both the conditions indicated, which are really essential for the existence and success of a monopoly and are of course different in each real instance. The outsiders' percentage can of course be all the bigger, the greater, on the one hand, the difference between the absolute monopoly price and the competitive price (which depends on costs, or else, in the case of scarce goods, on the quantity available), and the more gently, on the other hand, demand rises as price declines, or, put differently, the more rapidly price rises as supply is reduced. If the result of a significant increase in price is a substantial change in the value of goods purchased, then for a small restriction of quantity on offer, the restriction will very soon pay; but if, on the other hand, a significant reduction in supply is required to raise price only slightly, then such action will only benefit those who can either reap the /
the whole reward of the restriction, or else a large part of it.

Abstract discussions are of course remote from a representation of price formation in real life under conditions of partial monopoly. Above all, the full picture of the demand relationships is never available as we have assumed here, and at any given time the actual price can at least only approach this ideal monopoly price after a number of adjustments. Moreover, many other things play a part in the pricing policy of the cartel (this being the most important case of relative monopoly) - and in particular a major role must be accorded to the fact that producers do not all operate with the same costs. This chiefly results in a superior income for the low cost producers, though this is not a monopoly income in the sense in which we have used this term above, but has its roots in purely objective relations. With the calculation of costs a quite new factor generally enters pricing policy, a factor which our model cannot take into account, being supposed - in the tradition of the simplifying method - only to illustrate an active tendency: the balancing of monopolistic and competitive forces.

We would like to introduce costs into the argument in just one respect, not because the line of argument suffers a major modification as a result, but because one objection is thereby taken into account which could be raised against its applicability. We assumed for our presentation that the possibilities for competition were narrowly limited, that the outsiders /
outsiders can together produce only 400. But now we come closer to reality when we assume that only this level of competition does indeed exist, and that at a price of 1, which just yields an acceptable level of profit, no more competition is to be expected; but as soon as price rises substantially above this point, then competition becomes more intense; and the higher price rises, the more competition the cartel should fear.

We adapt our model accordingly for such a case:

Model IV

<table>
<thead>
<tr>
<th>Price</th>
<th>Sales at this Price</th>
<th>Competitors Sales at this Price.</th>
<th>Sales left over for the Cartel</th>
<th>Profit of Cartel</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>100</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>300</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>500</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>700</td>
<td>900</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>900</td>
<td>900</td>
<td>300</td>
<td>1500</td>
</tr>
<tr>
<td>5</td>
<td>1100</td>
<td>800</td>
<td>600</td>
<td>2400</td>
</tr>
<tr>
<td>4</td>
<td>1300</td>
<td>700</td>
<td>600</td>
<td>2400</td>
</tr>
<tr>
<td>3</td>
<td>1500</td>
<td>600</td>
<td>900</td>
<td>2700</td>
</tr>
<tr>
<td>2/2</td>
<td>1700</td>
<td>900</td>
<td>1200</td>
<td>2400</td>
</tr>
<tr>
<td>1</td>
<td>1900</td>
<td>400</td>
<td>1500</td>
<td>1500</td>
</tr>
</tbody>
</table>

The numbers are again arbitrarily chosen. But even if one chooses different ones, an essentially similar picture will always emerge. The increased capacity of competitors at higher prices of course has an effect on the relative monopoly price; but a monopoly price can still develop if the restriction of supply still pays the cartel - and that can still be the case if the higher price attracts more fresh competitors.
competitors for the cartel - then a point can always be found up to which restriction still pays. Our other assumptions may be modified along these lines; but since a comprehensive picture of reality will not be attained by doing so, and since, on the other hand, the main features of the conflict between monopoly and competition will not be displaced, we may disregard such modifications.

As against this, the presentation needs further development in another direction. So far we have considered to what extent the monopolist can benefit from a price higher than that prevailing under free competition. Sometimes the cartel or trust pricing policy exploits a monopoly position for opposite ends, namely to dictate to the market as low a price as possible for a while, which harms the competitor, perhaps ruins him, or at least puts him in an embarrassing situation. Therefore the question which presents itself is what is the lowest price the dominant firm can force on the market. In this we must of course completely neglect the question whether the monopolist himself can financially sustain such a low price. That cannot be answered in general, but depends on the situation in each instance. Here it is only a matter of what price the market, or all the outsiders, can be forced down to. For here too the monopolist is faced with a limit which he can perhaps overstep for a while, but not for an extended period.

The absolute monopolist can only dictate a price for a long time if he can fully meet the demand corresponding to /
to that price, and he cannot dictate a lower price than this. For if he wanted to sell at such a lower price, then the difference would only benefit middlemen who would obtain the appropriate price. But there is an analogous situation in the case of partial monopoly. If the holder of the partial monopoly establishes too low a price (in the sense used above), then - as can easily be demonstrated numerically in Model I - the competitors can still maintain the corresponding higher price, since a corresponding portion of demand is reserved for them at this price.

This theoretical lower limit is of course not reached in practice, where it is only a matter of temporary undercutting. For the outsiders will not generally dare to maintain a higher price than the cartel. They cannot easily obtain a sufficiently complete view of the market to be able to know with certainty that the stocks and productive capacity of the cartel will be insufficient to meet requirements, and hence that people will eventually have to buy their products at the higher price. And frequently they are not in a position to await this time - for reasons of their real endowment of wealth.
Footnotes to Appendix (Chapter 3)

* This translation was very kindly provided for me by Geoff Meeks, Research Associate, Department of Accounting and Business Method, University of Edinburgh. It has also benefited from the advice of Clemens Hackmann, an undergraduate from the University of Berlin, currently studying economics for one year at the University of Edinburgh. The original source is K. Forchheimer, "Theoretisches zum unvollstandigen Monopole", Schmollers Jahrbuch, 1908, Vo. XXXII, No. 1, pp. 1-12.

1/ Lecture given in the economics seminar of Professor Alfred Weber, in the Winter Term 1906/7 in Prague. The criticisms of members of the seminar have influenced the paper.


3/ Cf. Ely "Monopolies and Trusts" p. 15 (remark) and p. 32.

4/ See Ely p. 97 where interesting examples are used.

5/ In this paper we are only talking about seller's monopoly; monopsony (buyer's monopoly) is outside our discussion; although the argument can be altered to accommodate it.

6/ What in this paper is called a rare good is in economic theory called "not freely increaseable"; this term is not accurate because no good is literally "freely increaseable".

7/
Footnotes (contd).

7/ Compare Fritz Pabst, "Is the ground rent in the suburbs of towns a general monopoly rent?".
(Jahrbuck für Nationalökonomie und Statistik, III, 33, Paper 1, p.4) and also Ely p.33.

8/ Compare Ely, and Zuckerkandl's article "Price" in Handwortenbucke der Staatswissenschafiten.

9/ Cf. Ely, pp. 115, 121, 123, 125, 126, 128.

Chapter 3
The Constant Marginal Cost Case

3-1. Introduction

In the following two chapters we intend to examine the behaviour of the price leaderships model under fairly specialized assumptions. As is usual in such cases, one is frequently able to derive quite exact results. The cost of generality in formulation is often the weakening of conclusions, and we will therefore leave more general analysis to later chapters, concentrating for the moment on results which are readily established. The two main cases we wish to consider are: (a) The constant marginal cost case, which will lead to a discussion of survivorship in this chapter and (b) The linear case, for which various comparative statics results will be derived in the next chapter.

3-2. The Constant Marginal Cost Case.

Diagram 3.1 recapitulates the standard analysis. DD is market demand, and S is the supply curve of the followers: analytically speaking, the horizontal sum of the followers' marginal cost curves. The leader takes up the residual demand not met by the followers at any particular price. Thus the leader's demand curve is ddD, the kink occurring at that price below which the followers will /

1/ The precise sense in which we distinguish the constant marginal cost case from the linear case will become apparent in section (4-1) of Chapter 4. It rules out the possibility of a falling AC curve which is permitted in the analysis of this chapter.
will supply nothing at all. If MC is the leader's marginal cost curve, then the price set will be \( P \), and the leader and followers will supply the amounts \( q \) and \( q_f \) respectively. Thus, typically the model is determinate and the leader and followers co-exist.

3-3. Cost and Revenue Curves.

It is usually assumed [e.g. by Ryan (1958, p.65), Henderson and Quandt (1971, p.71)] that the total cost function should have a shape which, roughly speaking, is cubic. More precisely, if the total cost function is \( f(q) \), with \( f' > 0 \), then there exists an output level at which \( f'' = 0 \), below which \( f'' < 0 \) and above which \( f'' > 0 \). However, many empirical studies, extending back to those of Joel Dean, have tended to support the idea that total costs are linear. This being the case, marginal costs are invariant with respect to output level. At first sight this may appear to be merely a pathological case; but the fact that it has some empirical support means that it is worth considering how the model behaves in this instance. Suppose the following relationships hold.

\[
\begin{align*}
(3.1) \text{ Costs: } & \quad TC_i = a_i + b_i q \\
(3.2) \text{ AC: } & \quad AC_i = \frac{a_i}{q} + b_i \\
(3.3) \text{ MC: } & \quad MC_i = b_i, \quad a_i, b_i > 0, \; i=1,2 \\
(3.4) \text{ Market Demand: } & \quad q = \alpha - \beta P, \quad \alpha, \beta > 0
\end{align*}
\]

\( TC, AC \) and \( MC \) denote total, average and marginal costs. For the sake of simplicity, the market demand curve is assumed to be linear.

\[1/\] A convenient summary of the evidence is in Johnston (1960, Ch.5)
Diagram 3.1

Diagram 3.2
As an additional simplification, we are supposing that there is only one follower. This would not change the presentation of Diagram 3.1, but merely alter its interpretation, the curve $S$ being the marginal cost curve for a single follower. The subscripts 1 and 2 refer to the leader and the follower. The average cost curve is one branch of a hyperbola which also lies in the second and third quadrants. Only the segment defined for positive $q$ is relevant.

3-4. The Follower.

Maintaining the basic behavioural postulate of the standard model - in essence that the leader is a price maker and the follower a price taker - radically alters the situation for the follower. The average and total profit functions are given by

\[
\begin{align*}
\text{(3.5)} & \quad AT\Pi_2 = P_1 - \left( \frac{a_2}{q} + b_2 \right) \\
\text{(3.6)} & \quad TT\Pi_2 = qP_1 - \left( \frac{a_2}{q} + b_2 \right)q = q(P_1 - b_2) - a_2
\end{align*}
\]

for a given $P_1$

It is to be noted that the profit function for the follower is linear, and hence has no stationary value. Although total profit is therefore theoretically unbounded, average profit is bounded by $P_1 - b_2$ in general (and by $b_2$ in Diagram 3.2 where the special case $P_1 = 2b_2$ is shown), which can be seen by letting $q \to \infty$ in equation (3.5).
In fact, maximum profit for the follower is earned when he supplies total market demand, for a price greater than \( b_2 \). The follower will produce until his average revenue curve cuts market demand at output \( q = a - \beta p_1 \). At this point, average cost is given by

\[
\frac{a_2}{\alpha - \beta p_1} + b_2
\]

and average profit, the difference between average revenue and average cost, is given by

\[
p = \frac{a_2}{\alpha - \beta p_1} - b_2
\]

Thus for a price \( p_1 \), prescribed by the leader, this maximum profit is

\[
q (p_1 - \frac{a_2}{\alpha - \beta p_1} - b_2)
\]

\[
= (\alpha - \beta p_1)(p_1 - b_2) - a_2
\]

(Area \( p_1 \ ABC = \text{area } DEFO \) in Diagram 3.2), which is achieved at output level \((a - \beta p_1)\), \( (F \text{ in Diagram 3.2}) \). In general, the follower will earn less profit than he could in the pure monopoly situation, though there will exist one price \( (p_2 \text{ in Diagram 3.2}) \) at which the follower's actual profit and potential monopoly profit are identical. Below a certain price \( (p_3 \text{ in Diagram 3.2}) \), the follower would be making a loss and would drop out of the market. This price is defined by the intersection of \( AC_2 \) and the market demand curve. That is, it is that value of \( p \) which satisfies:

\[
p = \frac{a_2}{\alpha - \beta p} + b_2
\]
or \[ \alpha p - \beta p^2 = a_2 + b_2 \lambda - b_2 \beta p \]
or \[ \beta p^2 - (\alpha + b_2 \beta) p + (a_2 + b_2 \lambda) = 0 \]

which is a quadratic in \( p \). Considering only the case in which two real roots occur, \( p', p'' \) we have

\[
p', p'' = \frac{(\alpha + b_2 \beta) \pm \sqrt{(\alpha + b_2 \beta)^2 - 4 \beta (a_2 + b_2 \lambda)}}{2 \beta}
\]

\( p'' \) is smaller than \( p' \) when we use the negative sign in front of the discriminant. In terms of the diagram, \( p_3 \leq p'' \).

In summary of this section, confronted with any price greater than or equal to \( p_3 \), the follower will attempt to meet the whole market demand; whilst for any price below \( p_3 \) the follower will drop out of the market, and by implication the leader will meet the total market demand.

3-5. The Leader

So far, the analysis has been conducted under the assumption that the leader can set a price which will be accepted by the follower, without enquiring how the leader actually goes about setting his price. Diagram 3.3 illustrates the typical case. Because, in general, \( p_3 > b_2 \), this figure does not necessarily portray the case in which the leader has lower costs than the follower. The demand curve faced by the leader is a segment of the total demand.
demand curve for the market (labelled \( AR_m \)), being
defined only for those prices at which the follower would
make a loss (i.e. below \( p_3 \) in Diagram 3.2 and 3.3). In
Diagram 3.3 we have \( b_1 < p_3 \) and no point of intersection
for the marginal cost and marginal revenue curves. The
total profit function for the leader is given by

\[
(3.7) \quad \Pi_l = pq - (a_1 + b_1 q) \\
= \left( \frac{d-q}{p} \right) q - a_1 - b_1 q \\
= \left( -\frac{1}{p} \right) q^2 + \left( \frac{d}{p^2} - b_1 \right) q - a_1.
\]

A necessary condition for a maximum is

\[
\frac{d \Pi_l}{dq} = -\frac{2}{p} q + \left( \frac{d}{p^2} - b_1 \right) = 0
\]

whence

\[
q = \frac{p}{2} \left( \frac{d}{p^2} - b_1 \right)
\]

which is positive provided

\[
\frac{d}{p^2} > b_1
\]

Also \( \frac{d^2 \Pi_l}{dq^2} = -2/p < 0 \), establishing the
concavity of \( \Pi_l \) and (with the previous condition)
establishing sufficient conditions for a unique local
maximum at \( q = \frac{p}{2} \left( \frac{d}{p^2} - b_1 \right) \). But will this maximum
be achieved? The profit function defined by equation (3.7)
would reach its maximum to the left of the \( AR_m \) segment
confronting /
confronting the leader (actually at J in Diagram 3.3) if only it were defined at such output levels. As it is, because (3.7) indicates that $\Pi$ is quadratic, and $OH > OJ$, total profit must be falling beyond $OH$. Thus the achievable maximum profit will be attained at output level $OH$. There is a slight problem of interpretation here, because strictly speaking exactly at price $p_3$ the follower will supply $OH$ and the leader will supply nothing. But for a point on $AR_m$ below $p_3$ and to the right of $OH$, no matter how slightly, the leader will be the exclusive producer. Thus the profit associated with output $OH$ can never actually be achieved: more technically, it defines a supremum rather than a maximum.

That the maximum is achieved as a corner solution highlights the leader’s dilemma when the constant marginal cost assumption is introduced into the model. If the leader regards the $AR_m$ segment in Diagram 3.3 as his average revenue (or residual demand) curve, he will maximize profits by selling $OH$ units at price $p_3$ per unit. 1/ But if he sets a price of $p_3$, the follower too will attempt to meet the total market demand at this price by putting an equivalent supply on the market, though unlike the leader, he will be making zero profit.

3-6. The Determinacy of the Model

Taken literally, the model is obviously not determinate. However, were we permitted to go outside the strict framework of the model then a solution of sorts might emerge. Consider the /

1/ This is because profit is a decreasing function of output beyond $OH$, and below $OH$ all output will go to the follower.
the following argument, for example.

If the leader and follower both put OH onto the market at a price $p_3$, there will be a substantial, in fact one hundred percent, excess supply of the good. Under Walrasian stability assumptions this will cause the price to fall. But should price fall, even to a slight extent, the follower will go out of business, and the acute condition of excess supply will be ameliorated. Thus the likely final outcome is that the leader will shade his price below $p_3$, but without moving too far to the right on his falling profit schedule; and the follower will drop out of the market.

I have said above that Diagram 3.3 does not imply that the leader has lower costs than the follower, and now this statement should be amplified. The figure certainly does indicate that at output level OH, the follower has higher average costs than the leader, though of course this could be perfectly consistent with the follower having lower marginal costs than the leader at all output levels (cf. Diagram 3.4). The firm which must be the leader and hence also the survivor, is the firm whose average cost curve cuts the market demand curve at the lower point. This proposition can be demonstrated using Diagram 3.4. If firm 1 is the leader, he will face the segment XZ of market demand, and produce an output slightly more than $q_x$ at a price slightly less than $P_x$. Firm 2, the follower, will be forced out of the market /
market. Firm 2 could not be the leader. For, supposing that he tried to be the leader, he would face the segment YZ of market demand. But even at price $p_y$, he will make a loss, hence his role as leader is untenable.

3-7. Long Run vs. Short Run

Up to now, we have made no distinction between the long run and the short run, and have merely assumed that a firm cannot operate with negative profits. However, standard analysis recognizes that a firm will be prepared to bear short run losses provided average variable costs are covered, even though average total costs may not be covered.

To incorporate this feature into our analysis, we may re-write (3.1) in a familiar way as:

\[
(3.8) \quad T C_i = F C_i + VC_i = a_i + b_i q
\]

whence

\[
ATC_i = AFC_i + AVC_i = \frac{a_i}{q} + b_i
\]

where $FC_i$ and $VC_i$ stand for fixed costs and variable costs, and $AFC_i$ and $AVC_i$ for their average equivalents. Then $AVC_i = MC_i = b_i$ and the analysis simplifies greatly.

Consider Diagram 3.2 for example. If Firm 1 is the leader, then /
then Firm 2 will be willing to supply the whole market at any price greater than or equal to $b_2$, in the short run. Suppose $p_3 = b_2$ in Diagram 3.3. Then the solution is as before, with Firm 1 shading its price below $p_3$, and forcing Firm 2 out of the market.

However, the conclusion on survivorship is altered. In Diagram 3.4, if Firm 1 tries to become the leader, it will face the residual demand curve $WZ$, because now we are admitting that if a price below $AC_2$ is charged, Firm 2 will continue producing (in the short run) as long as price exceeds $MC_2$. Thus Firm 1 cannot be the leader. Only Firm 2 can be the leader and in doing so will become the sole survivor, because it would face a residual demand curve defined by $VZ$ in Diagram 3.4. By shading price below $MC_1$, it can put Firm 1 out of business in the short run. This reversal of conclusion when the distinction between the long run and the short run is made emphasizes that the analysis does not provide a satisfactory model of survivorship.

3-8. Models of Economic Survival

The conventional Marshallian argument for monopolization of an industry (or "natural monopoly" argument) provides one means of determining which firm will survive in the long run. In this case, the firm with falling unit costs, or most rapidly falling unit costs, will become the sole survivor. On a less obvious level, the analysis of
the path to long run equilibrium in perfect competition may be regarded as an analysis of economic survival. Under one interpretation, no differences in efficiency are admitted in the survivors, and in the long-run equilibrium of the industry identical firms all produce at minimum long-run average cost, implying that the long-run supply curve of the industry is infinitely elastic. Kaldor (1934) has strongly criticised this analysis. In order that the model of perfect competition should be logically valid, there must be increasing costs for all firms, otherwise the industry will become monopolised. But if all factors are in infinitely elastic supply in the long-run (as is usually assumed in neoclassical analysis) then the logical basis of increasing costs (namely that increasing amounts of variable factors are combining with one or more fixed factors until marginal products fall), is shown away. Kaldor argues, therefore, that for the theory to be consistent, there must be a factor which is fixed from the viewpoint of the individual firm, but variable from the viewpoint of the industry. He concludes that it is the supply of what he calls "co-ordinating ability" which meets this requirement. Under another interpretation, the effect of free entry and exit is to reduce market price until one or more marginal firms just survive, producing at the minimum point on their long-run average cost curves. Firms of lesser efficiency leave the industry, and firms of greater efficiency /
efficiency make positive (or super-normal profits). Newman and Wolfe (1961) tried to maintain elements of the Marshallian analysis in their stochastic analysis of the growth and decay of firms. Consider a size distribution of firms, where the number of size classes is large. Then Newman and Wolfe introduce a "limbo class" defined for all firms with zero output. A movement from the limbo class to any other class involves a firm being "born" and a movement from any positive output class to the limbo class involves "dying". This is a satisfactory technical device for dealing with survivorship, but unfortunately it leaves open the important question of what projects a firm back into the limbo class. Worcester (1957, pp. 342-343) discusses instability of the leader-followership roles when there are increasing returns to scale. His general conclusion is that the dominant firm or leader will decline, even if it has absolute cost advantages. The argument is quasi-dynamic. If the leader has set a long-run equilibrium price, there will be no incentive for it to increase output. If the followers are able to expand plant by moving along the same long-run average cost curve as the leader (i.e. the "envelope" curve) there will always be an incentive for followers to expand; and in the long-run this will be to the disadvantage of the leader. Worcester suggests that two outcomes are possible. The first is that the leader will start to become more aggressive, and through its /
its short-run cost advantage will be able to drive the followers out of the market and achieve a "natural monopoly". A second outcome is more likely if the leader's cost advantages are not substantial. In this case, the followers will start to merge or collude before they are driven out, price leadership as such will vanish, and genuine oligopolistic inter-dependence will take its place. We have some reservations, however, about applying dynamic arguments to what is essentially a static model. Whilst it is true that we may venture opinions on what might be the outcome, these are no more than conjectures, like the argument in section (3-6), and may not be vindicated by a genuinely dynamic analysis.

Probably the most satisfactory attempt to deal with models of economic survival has been in "Strategy and Market Structure" by Shubik (1959). His general tool of analysis is game theory, and he defines economic survival in the following way:

A game of survival is a natural extension of the two-person, zero-sum game when asset structure and dynamic features are introduced. At the beginning of the game each player is in control of a given amount of capital, his assets or 'fortune'. During every time period a two-person zero-sum game is played and the appropriate payments are made until one player runs out of funds or is 'ruined'. The goal of each player is to ruin his opponent; hence his payoff consists of the valuation he places upon survival and ruin.

Shubik (1959, p.204)

It turns out that models formulated along these lines /
lines produce a rich variety of solutions depending on the assumptions made about the pay-off matrices, the firms' assets, the rate of discount and the potential gains of survivorship. Solutions range from the case in which one firm can impose a loss on the other firm at every play of the market game, to the co-operative case in which two firms agree that one should leave the market in order that they may benefit jointly from the monopoly profits of the survivor.

3-9. Conclusion

From the previous section, it is clear that the price leadership model under constant marginal cost conditions does not provide an explanation of survivorship that is a significant advance on the "natural monopoly" analysis, and it is certainly a less adequate explanation than that provided by Shubik (1959). However, to require this of the analysis in sections (3-1) to (3-7) would be to mistake our intentions. We make three observations:

(a) price leadership is common in manufacturing business;
(b) some studies indicate that total cost curves are linear; and (c) there is a standard model of price-leadership. By (a), we justify consideration of the problem in the first place. We then ask where the "received wisdom" of (b) and (c) will lead us. We conclude there is a paradox - or, as we might put it, the conclusion obtained from an analysis of the model (viz. that only one firm /
firm can survive) is inconsistent with one of the postulates of the model (viz. that there are two firms, one assuming the role of price leader and the other the role of price follower). For this reason it is clear that we should develop alternative and more satisfactory models of price leadership. As declining average costs are so destructive to the content of the model, the models we have analysed more fully in the next three chapters have either increasing average costs everywhere, or U-shaped average costs (i.e. eventually increasing average costs). In Chapter 7, however, we return again to the problem of price leadership when unit costs are falling.
Chapter 4

The Linear Case

4-1. Outputs and Price

As in the previous chapter, we will stick to the assumption that there is one leader and a single follower. Suppose that total cost curves were quadratics of the form:

\[ TC_i = \frac{b_i q_i^2}{2}, \quad b_i > 0 \quad (i=1,2) \]  

It follows that average and marginal cost curves are given by:

\[ AC_i = \frac{b_i q_i}{2} \]

\[ MC_i = b_i q_i \]

where, as before, the subscript 1 refers to the leader, and the subscript 2 to the follower.

Suppose, as in Ch. 3, that market demand is defined by the equation

\[ q = \lambda - \beta p, \quad \lambda, \beta > 0 \]

Equations (4.1) through (4.4) summarize what we have called the linear case.

The follower's total profit function is

\[ \pi_2(q_2) = pq_2 - \frac{b_2 q_2^2}{2} \]
where \( p \), the price set by the leader, is regarded as given.

First and second order conditions for maximization of this function are given by:

\[
\frac{d\pi_2}{dq_2} = p - b_2 q_2 = 0, \quad \text{whence} \quad p = b_2 q_2
\]

\[
\frac{d^2\pi_2}{dq_2^2} = -b_2 < 0
\]

establishing sufficient conditions for a maximum and that the follower's supply function is

\[
q_2 = \frac{p}{b_2}
\]

The leader's demand function is

\[
q_1 = d - \beta p - q_2 = d - \left(\beta + \frac{1}{b_2}\right)p
\]

from (4.8); and his profit function is

\[
\pi_1(q_1) = \frac{dq_1 - q_1^2}{\left(\beta + \frac{1}{b_2}\right)} - \frac{b_1 q_1^2}{2}
\]

First and second order conditions for the maximization of (4.10) are:

\[
\frac{d\pi_1}{dq_1} = \frac{d - 2q_1}{\beta + \frac{1}{b_2}} - b_1 q_1 = 0
\]

from which

\[
q_1 = \frac{d}{2 + b_1 \left(\beta + \frac{1}{b_2}\right)}
\]
\begin{align}
(4.12) \quad \frac{d^2 T_1}{dq^2} = & - \frac{z}{\lambda + \frac{1}{b_2}} - b_1 < 0, \quad \text{as} \quad \beta > b_1, b_2 > 0
\end{align}

which provide a sufficient condition for a maximum. The symbol \( \tilde{q} \) is used to indicate the solution to equation (4.11). By substitution of the solution to (4.11) into (4.9), we establish that the price set by the leader is

\begin{align}
(4.13) \quad \overline{p} = & \quad x - \frac{x}{z + b_1 (\lambda + \frac{1}{b_2})} \\
= & \quad \frac{x}{(\beta + \frac{1}{b_2})} \left[ 1 - \frac{1}{z + b_1 (\beta + \frac{1}{b_2})} \right]
\end{align}

Now \( \overline{p} \) must be positive, because the term

\begin{align}
\frac{1}{z + b_1 (\beta + \frac{1}{b_2})} < 1
\end{align}

Substitution of \( \overline{p} \) defined by (4.13) into the equation for the follower's supply, (4.8), provides an expression for the follower's output

\begin{align}
(4.14) \quad \overline{q}_2 = & \quad \overline{p} \frac{\beta}{b_2} \left[ 1 - \frac{1}{z + b_1 (\lambda + \frac{1}{b_2})} \right] > 0
\end{align}

Thus equations (4.11), (4.13) and (4.14) establish that the model /
model is determinate and provide expressions for the (necessarily positive) market price and firms' outputs.

Diagram 4.1 graphs the model for the following parameter values

\[ b_1 = 0.4, \quad b_2 = 1.0, \quad d = 20.0, \quad \beta_1 = 1.54; \]

for which the solutions are \( P = 5.27, \quad q_1 = 6.60 \) and \( q_2 = 5.27 \)

The extension of the analysis to the case of many followers is elementary in the linear case. Suppose there are \( n \) firms in all, and that Firm 1 is the leader. Then there will be \( n \) versions of equations (4.1), (4.2) and (4.3), and \((n - 1)\) versions of equation (4.5). The followers' supply function is modified to

\[
\sum_{i=2}^{n} q_i = P \sum_{i=2}^{n} \frac{1}{b_i}
\]

The price and outputs established are given by

\[
P = \frac{\lambda}{\beta + \sum_{i=2}^{n} \frac{1}{b_i}} \left[ \frac{1}{2 + b_1(\beta + \sum_{i=2}^{n} \frac{1}{b_i})} \right]
\]

\[
q_1 = \frac{\lambda}{2 + b_1(\beta + \sum_{i=2}^{n} \frac{1}{b_i})}
\]

\[
q_j = \frac{\lambda}{b_j(\beta + \sum_{i=2}^{n} \frac{1}{b_i})} \left[ \frac{1}{2 + b_j(\beta + \sum_{i=2}^{n} \frac{1}{b_i})} \right]
\]

\( (j = 2, 3, \ldots, n) \)
\[ q = 20 - 1.54p \]
\[ MC_1 = 0.4q_1 \]
\[ MC_2 = q_2 \]
\[ \bar{q}_1 = 6.6 \]
\[ q_2 = 5.3 \]

Diagram 4.1
Equations (4.16), (4.17) and (4.18) enable us to reach conclusions about the effect of an additional follower entering the industry. We may state the following three propositions:

1) **The greater the number of followers, the smaller is the leader's output.**

2) **The greater the number of followers, the lower is the market price.**

3) **From the viewpoint of any follower already in the market the greater the number of followers, the smaller is his output.**

In each case, we are assuming a ceteris paribus clause. The first proposition is clear from an inspection of equation (4.17). Let the term under the summation sign increase, and \( \bar{q} \) will decrease. At first sight, the second proposition is not at all obvious, because allowing the terms under the summation sign to increase will increase the expression in square brackets, but, decrease the term outside it; and it is not clear which influence will predominate. In fact, the proof is rather tedious, but it can be simplified by a change in notation. Suppose \((2, \ldots, n)\) followers are already in the market when price is \( \bar{p} \), and that once follower \((n + 1)\) enters the market, price becomes \( \bar{p} ' \). The term \( \sum_{i=2}^{n} \frac{1}{b_i} \) will become \( \sum_{i=2}^{(n+1)} \frac{1}{b_i} \). For the sake of convenience we shall adopt the notation

\[ A = \]
\[ A = \beta + \sum_{i=1}^{n} \frac{1}{b_i} \quad \text{and} \quad S = \frac{1}{D_{n+1}} \]

To prove the second proposition, we require to show that
\[ (\bar{P} - \bar{P}') > 0 \]

In the new notation
\[
\bar{P} - \bar{P}' = \frac{\alpha}{A} \left[ 1 - \frac{1}{2 + b, A} \right] - \frac{\alpha}{A+\delta} \left[ 1 - \frac{1}{2 + b, (A+\delta)} \right]
\]
\[
= \{ \frac{1}{A} - \frac{1}{A(2+b,A)} \} \cdot \frac{1}{(A+\delta)[2+b,(A+\delta)]}
\]
\[
= \left\{ (2+b,A)(A+\delta)[2+b,(A+\delta)] - (A+\delta)[2+b,(A+\delta)] \right\}
\]
\[
- A (2+b,A)[2+b,(A+\delta)] + A (2+b,A)^2 \times \frac{\alpha}{A(2+b,A)(A+\delta)[2+b,(A+\delta)]}
\]

The term after the multiplication sign must be positive, so it remains to show that the term contained within the curly brackets is positive.

After considerable, but routine, manipulation, it can be shown that this term reduces to
\[ \delta b, A^2 + 2 \delta + b, \delta^2 + b, \delta^2 A s^2 + 2 \delta b, A \]

which /
which must be positive, completing the proof.
The proof of the third proposition follows directly.

4-2. The Method of Comparative Statics

In the "Foundations of Economic Analysis" of Samuelson (1947, p.8), we find the method of comparative statics defined as "the investigation of changes in a system from one position of equilibrium to another without regard to the transitional process involved in the adjustment". The scope and the limitations of the method have since been discussed by, among others, Lancaster (1961, 1965, 1966), and Gorman (1964). However, our purpose here is not to follow these developments, but to explain the method briefly, as a preliminary to applying it.

The starting point of the method is the statement of a set of equilibrium conditions, which we may write as

\[
\begin{align*}
\frac{\partial f}{\partial x_1} &+ \frac{\partial f}{\partial x_2} + \cdots + \frac{\partial f}{\partial x_n} = 0 \\
\frac{\partial f}{\partial y_1} &+ \frac{\partial f}{\partial y_2} + \cdots + \frac{\partial f}{\partial y_m} = 0 \\
\cdots &
\end{align*}
\]

which is a system of equations in \( n \)-variables and \( m \)-parameters. In vector notation, the system (4.19) may be written

\[
\mathbf{f}(\mathbf{x}, \mathbf{y}) = \mathbf{0}
\]
The so-called "Jacobian" matrix of this system is defined as

\[
J(x) = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n}
\end{bmatrix}
\]

(4.21)

Assuming that this matrix is non-singular, then by the implicit function theorem the equilibrium conditions of (4.19) can be solved to give

\[
\begin{align*}
x_1 &= \psi^1(d_1, d_2, \ldots, d_m) \\
&\quad \vdots \\
x_n &= \psi^n(d_1, d_2, \ldots, d_m)
\end{align*}
\]

(4.22)

or \( x = \boldsymbol{\psi}(d) \) in vector notation.

Substitution of each of these expressions for \( x_i \) into (4.19) gives

\[
\begin{align*}
f^1(\psi^1, \psi^2, \ldots, \psi^n, d_1, d_2, \ldots, d_m) &= 0 \\
&\quad \vdots \\
f^n(\psi^1, \psi^2, \ldots, \psi^n, d_1, d_2, \ldots, d_m) &= 0
\end{align*}
\]

(4.23)

or, more concisely, \( \boldsymbol{f}(\psi, d) = 0 \)

By virtue of the fact that the \( \psi^i \) are all functions of the \( d_i \), it follows that the \( f^i \) are also functions of the \( d_i \). The variables.

---

1/ See any advanced calculus. For example, Gillespie (1951, pp. 104-105.)
variables \( x_i \) will take on particular equilibrium values, \( x_i^* \), once we have fixed the values of the \( d_i \) to be \( d_i^* \). In employing the method of comparative statics, we are posing the question "What will happen to all of the \( x_i \) when, say, \( d_j^* \) changes its value?". Differentiating (4.23) with respect to \( d_j \) we get

\[
\left[ \frac{\partial x_i^*}{\partial d_j^*} \right] \frac{\partial x_i}{\partial d_j} + \cdots + \left[ \frac{\partial x_n^*}{\partial d_j^*} \right] \frac{\partial x_n}{\partial d_j} = - \left[ \frac{\partial f_j}{\partial d_j} \right]^*
\]

(4.24) .................................

\[
\left[ \frac{\partial x_i^*}{\partial d_j^*} \right] \frac{\partial x_i}{\partial d_j} + \cdots + \left[ \frac{\partial x_n^*}{\partial d_j^*} \right] \frac{\partial x_n}{\partial d_j} = - \left[ \frac{\partial f_j}{\partial d_j} \right]^*
\]

where the asterisk superscript indicates that the corresponding partial derivatives are evaluated at the initial equilibrium point \( (x_1^*, \ldots, x_n^*; d_1^*, \ldots, d_m^*) \). Expressed in matrix form, (4.24) is

\[
J(x^*) \frac{\partial x}{\partial d_j} = - f_j^*
\]

We may regard this as a system of \( n \) equations in the \( n \) unknowns \( \frac{\partial x_1}{\partial d_j}, \ldots, \frac{\partial x_n}{\partial d_j} \). As the Jacobian has been assumed to be non-singular, this may be transformed to

(4.25)

\[
\frac{\partial x_i}{\partial d_j} = - J^{-1}(x^*) f_j^* \quad 1 \leq i \leq n
\]

The importance of (4.25) in economics depends upon the sign of the elements in the Jacobian and the vector \( f_j^* \). One usually has \textit{a priori} expectations about the signs of the terms /
terms \( \left( \frac{\partial f_j}{\partial x_j} \right)^k \) in the Jacobian, and very often elements of the \( f_j \) are either zero or of known sign. It is frequently convenient to solve the system \((4.24)\) by employing Cramer's rule, as described by Aitken (1956, p.56). Then the solution may be written

\[
\frac{\partial z_i}{\partial x_j} = \sum_{k=1}^{m} \frac{\partial f_k}{\partial x_j} \Delta k_i \quad (i=1,2,\ldots,m)
\]

where \( \Delta \) is the determinant of the Jacobian, written \( |J| \), and \( \Delta k_i \) if the cofactor of its \( k \)'th row and \( i \)'th column.

### 4-3. Comparative Statics in the Linear Case

In this section, we intend to apply the general method of comparative statics described above, to what we have called the linear case. We may regard the "equilibrium conditions" of the model as:

\[
\begin{align*}
\frac{d-2q_1}{\beta + \frac{1}{b_2}} - b_1q_1 &= 0 \\
p - b_2q_2 &= 0 \\
(q_1 + q_2) - d + \beta p &= 0
\end{align*}
\]

the first two equations being the profit maximizing conditions for the leader and the follower, and the third equation being a market clearing condition. Comparing \((4.27)\) with \((4.19)\) we see it is a system of three equations in three unknowns (viz. \( q / \))
with four parameters \( \text{viz.} \alpha, \beta, b_1, b_2 \). It is readily confirmed that the solutions to these equations are indeed given by the expressions for \( \overline{q}_1 \), \( \overline{q}_2 \) and \( \overline{p} \) in equations (4.11), (4.14) and (4.13) above.

Suppose the follower were to receive a subsidy of £s per unit of output. Then the follower's total profit function changes from (4.5) to become

\[
\Pi_2(q_2) = pq_2 + sq_2 - \frac{b_2 q_2^2}{2}
\]

and a similar development of the model confirms that the equilibrium conditions given by (4.27) now become

\[
\left( \alpha - \frac{s}{b_2} \right) - \frac{2q_1}{\left( \beta + \frac{1}{b_2} \right)} - b_1 q_1 = 0
\]

(4.28)

\[
p + s - b_2 q_2 = 0
\]

\[
(q_1 + q_2) - \alpha + \beta p = 0
\]

Differentiating these equilibrium conditions with respect to \( s \) gives:

\[
- \left[ \frac{2}{\beta + \frac{1}{b_2}} + \frac{b_1}{b_2} \right] \frac{\partial q_1}{\partial s} = \frac{1}{b_2 \left( \beta + \frac{1}{b_2} \right)}
\]
\[
\frac{\partial p}{\partial s} - b_2 \frac{\partial q_2}{\partial s} = -1
\]

\[
\frac{\partial q_1}{\partial s} + \frac{\partial q_2}{\partial s} + \beta \frac{\partial r}{\partial s} = 0
\]

which is a set of linear equations in \( \frac{\partial q_1}{\partial s}, \frac{\partial q_2}{\partial s} \) and \( \frac{\partial r}{\partial s} \). Solving by Cramer's rule (Aitken, 1956, p. 56)
gives the following expressions:

\[
\frac{\partial p}{\partial s} = \frac{\begin{bmatrix}
1 & 0 & 0 \\
-b_2 & \beta & 0 \\
0 & 0 & 1
\end{bmatrix}}{|A|} \]

\[
\frac{\partial q_1}{\partial s} = \frac{\begin{bmatrix}
0 & 0 & 0 \\
1 & -1 & -b_2 \\
0 & 0 & 1
\end{bmatrix}}{|A|}
\]

\[
\frac{\partial q_2}{\partial s} = \frac{\begin{bmatrix}
0 & -\left(\frac{2}{\beta + \frac{1}{b_2}} + b_1\right) & -\frac{1}{b_2 (\beta + \frac{1}{b_2})} \\
1 & 0 & -1 \\
0 & 1 & 0
\end{bmatrix}}{|A|}
\]
As the Jacobian determinant, $|A|$, is positive, the signs of $\partial \rho / \partial s$, $\partial q_1 / \partial s$ and $\partial q_2 / \partial s$ are given by the signs of the determinant in each numerator. Thus

\[
\begin{align*}
\frac{\partial \rho}{\partial s} &= \left[ \frac{b_2}{b_2 (\beta + \frac{1}{b_2})} - \left( \frac{2}{\beta + \frac{1}{b_2}} + b_1 \right) \right] \div |A| \\
&= - \left( \frac{1}{\beta + \frac{1}{b_2}} + b_1 \right) \div |A| < 0
\end{align*}
\]

\[
\begin{align*}
\frac{\partial q_1}{\partial s} &= - \frac{1}{b_2 (\beta + \frac{1}{b_2})} (1 + b_2 \beta) \div |A| \\
&= - |A|^{-1} < 0
\end{align*}
\]
We may summarize (4.30), (4.31) and (4.32) as follows:

A subsidy of £s per unit of the follower output results in a decrease in price, a decrease in the leader's output, and an increase in the follower's output. An obvious corollary of this is that aggregate supply is increased.

As a tax of £t per unit of the follower's output may be regarded as a negative subsidy, the following will be true: \( \frac{\partial p}{\partial t} > 0, \frac{\partial q_1}{\partial t} > 0, \frac{\partial q_2}{\partial t} < 0. \) Also of interest is the case in which the leader is subject to a tax.

Suppose the leader were subject to a tax of £t_1 per unit of output. What effect would this have on the outputs and price? The equilibrium conditions become:

\[
\begin{align*}
\frac{\partial q_1}{\partial t_1} - \beta q_1 - t_1 &= 0 \\
q_1 + q_2 &= 0
\end{align*}
\]

(4.33)

Differentiating with respect to \( t_1 \), and solving for the partial derivatives \( \frac{\partial p}{\partial t_1}, \frac{\partial q_1}{\partial t_1}, \frac{\partial q_2}{\partial t_1} \), we get
\[
\frac{\partial p}{\partial t_1} = \begin{vmatrix}
1 & -\left(\frac{2}{\beta + \frac{1}{b_2}} + b_1\right) & 0 \\
0 & 0 & -b_2 \\
0 & 1 & 1
\end{vmatrix} \div |A|
= \frac{b_2}{|A|} > 0
\]

\[
\frac{\partial q_1}{\partial t_1} = \begin{vmatrix}
1 & 0 & -b_2 \\
\beta & 0 & 1
\end{vmatrix} \div |A|
= -\left(1 + b_2 \beta \right) < 0
\]

\[
\frac{\partial q_2}{\partial t_1} = \begin{vmatrix}
0 & -\left(\frac{2}{\beta + \frac{1}{b_2}} + b_1\right) & 1 \\
1 & 0 & 0 \\
\beta & 1 & 0
\end{vmatrix} \div |A|
= |A|^{-1} > 0
\]
where \( |A| \), given by (4.29) above, is already known to be positive. Summarizing:

The imposition of a specific sales tax of \( \ell t \) per unit of the leader's output results in an increase in price, a decrease in the leader's output, and an increase in the follower's output.

An immediate consequence of this is that the follower's market share increases. Let primes distinguish the post-tax from the pre-tax outputs and prices. Then

\[
\bar{p}' > \bar{p} \quad \text{implies} \quad (\bar{q}'_1 + \bar{q}'_2) < (\bar{q}_1 + \bar{q}_2)
\]

Also

\[
\bar{q}'_2 > \bar{q}_2.
\]

Therefore

\[
\frac{\bar{q}_2}{\bar{q}_1 + \bar{q}_2} < \frac{\bar{q}'_2}{\bar{q}'_1 + \bar{q}'_2}
\]

(4.34)

where the expressions to the left and right of the inequality (4.34) represent the follower's market share before and after the imposition of the tax, respectively.

A case which is of considerable interest is that in which the leader exacts from the follower a royalty payment which is related to the volume of sales [cf. Pickering (1974, pp.86-87)]. Suppose that the leader collects a revenue of \( \ell r \) per unit of the follower's output. There will therefore be an increase of \( \ell r q_2 \) in the follower's cost, and this same amount will swell the leader's revenue. The follower's profit function becomes:
\[ \pi_2(q_2) = pq_2 - \frac{b_2 q_2^2}{2} - rq_2 \]

from which one obtains the "equilibrium condition":

\[ (4.35) \quad p - \frac{b_2 q_2}{2} - r = 0 \]

The follower's supply curve is therefore given by
\[ q_2 = \frac{(p-r)}{b_2} \]
from which we may derive the leader's (inverse) demand function:

\[ (4.36) \quad p = \frac{(\alpha + \frac{r}{b_2}) - q_1}{\left(\beta + \frac{1}{b_2}\right)} \]

The leader's profit function is given by

\[ \frac{(\alpha + \frac{r}{b_2})q_1 - q_1^2}{\left(\beta + \frac{1}{b_2}\right)} - \frac{b_1 q_1^2}{2} + rq_2 \]

where \( rq_2 \) is the revenue raised from the royalty payment of the follower. By substitution, using (4.35) and (4.36), the leader's profit function can be written

\[ (4.37) \quad \pi_1(q_1) = \frac{(\alpha + \frac{r}{b_2})q_1 - q_1^2}{\left(\beta + \frac{1}{b_2}\right)} - \frac{b_1 q_1^2}{2} + r \left[ \frac{\left(\frac{\alpha + \frac{r}{b_2}}{b_2}\right)q_1}{\left(\beta + \frac{1}{b_2}\right)} - \frac{1}{b_2} \right] \]
A necessary condition for a maximum is that
\[ \frac{d\Pi_i}{dq_1} = \left( \frac{x + \frac{1}{b_2}}{b + \frac{1}{b_2}} \right) - 2q_1 - b_1q_1 - \frac{r}{b_2(b + \frac{1}{b_2})} = 0 \]
which provides the second equilibrium condition of the model namely that
\[ q_1 \left( \frac{\beta}{\beta + \frac{1}{b_2}} \right) = 0 \]

We notice that the solution to (4.38) is identical to that of the model with no royalties or taxes. The three equilibrium conditions of the model are (4.38), (4.35) and the market clearing condition that \( (q_1 + q_2) = d + \beta p \). It is clear that the Jacobian determinant of this system has the same value as \( |A| \) evaluated in (4.39) above. Routine application of comparative statics along the lines already demonstrated reveals that \( \frac{\partial q_1}{\partial r} > 0 \), \( \frac{\partial q_1}{\partial r} = 0 \) and \( \frac{\partial q_2}{\partial r} < 0 \). That is:

A royalty payment of \( \varkappa r \) per unit of the follower's output, exacted by the leader, results in an increase in price, a decrease in the follower's output, but no change in the leader's output.

By a similar argument to that advanced for inequality (4.34) above, it is clear that a marginal increase in the royalty fee will marginally increase the leader's market share.
The result that $\frac{\partial q_1}{\partial r} = 0$ is interesting in that it indicates the two opposing effects which a change in the royalty payment would have. On the one hand it is effectively a tax on the follower, and will reduce his output for any given price. Consequently it will expand the leader's residual demand, hence increasing marginal revenue at any particular output. On the other hand, the fall in the followers output is decreasing the leader's revenue proceeds, and, if we care to think of these proceeds as offsetting costs, the leader's costs will be effectively increased. Thus revenue is increasing, but so are costs, and at least in the linear case, the optimal output of the leader remains unaltered.

Finally, following the popular presentation first used in Samuelson (1947, p. 280) we give below the comparative statics sign matrix for the whole model. The technique of derivation is exactly the same as that used throughout this section for parameters $s$, $t$, $t_1$ and $r$. A typical element in this table gives the sign of the partial derivative of a variable with respect to a parameter.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$\beta$</th>
<th>$b_1$</th>
<th>$b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$q_2$</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$p$</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
These results accord with intuition. For example, a marginal shift in $a$ is equivalent to a marginal expansion in market demand. Part of this increase will go to the leader, and part to the follower. The expansion of the leader's residual demand implies a higher market price. The $\beta$ case is almost the exact reverse. A marginal increase in $\beta$ is equivalent to a marginal contraction of the market, the slight difference being that in this case the contraction is greater the higher the price.

The $b_1$ case is quite straightforward. A marginal increase in the leader's cost will reduce his optimal output and hence increase his price. This higher price induces a greater output from the follower. Finally the $b_2$ case is most easily understood by regarding it as the converse of propositions (1), (2) and (3) in section (4-1) where results on new entry were derived. The effect of new entry is to increase the follower's total output at a given price. Conversely the effect of increasing a single follower's cost (which is what a marginal positive shift in $b_2$ implies) is to decrease the follower's output at a given price. Thus, as one would expect, the signs derived in the section on entry are reversed in the $b_2$ case.

4-4. **Summary**

In this chapter, we have applied the method of comparative statics to a linear version of the price leadership model. We have seen that the model is determinate, provides unique price and quantity solutions, and /
and is also stable in the sense that second-order conditions (or "stability conditions") for a maximum are satisfied. It has been shown that the general effect of additional entry by competitors or subsidisation of followers is to decrease the extent to which the market is monopolistic. Taxation of the leader (by specific sales tax) and the raising of royalty revenue by the leader from the follower(s) generally has the reverse effect. Finally the comparative statics sign matrix for the whole model was given and shown to accord with a priori expectations.

As a final comment, it should be said that the clear-cut nature of many of the comparative statics predictions which we have derived is due to the assumption of linearity embodied in equations (4.1) to (4.4). When the assumptions of linearity is dropped, comparative statics results are harder to obtain. However, it may be argued that there is nothing unscientific about making sufficiently strong assumptions for a model to yield unambiguous predictions. This is the line that Rowley (1972, Vol. 1, p. XIX) has taken, arguing that "in the last analysis, the validity of the model still rests upon the conformity or otherwise of those predictions with evidence drawn from the real world".
5-1. Introduction

In this chapter, we intend to consider the case in which there is strong competition for scarce factors and, in consequence, all firms operate under increasing costs. In the case examined the leader enjoys a favourable reputation for its product; and, for reasons of technical efficiency and/or superior bargaining power in the factor markets, produces at lower costs than the followers. The principal version of the model in the literature, and that which has already been examined for a number of special cases in Chapters 3 and 4, is one in which a single dominant firm sets a price which maximises (monopoly) profit. There are many followers who are prepared to accept this price. In short, we have a market with one price maker and many price takers.

Simple geometrical analyses of the model with just one follower have been presented for many years in microeconomics text books, some of the most notable treatments being in Boulding (1966) and Vickrey (1964); whilst mathematical versions of an undeveloped form have been provided by Hadar (1971). These models were discussed in Chapter 2. Diagram 5.1, which is an extended version of Diagram 3.1, illustrates the typical sort of analysis. The curves $M_f$, $A_f$, $M_l$, and $A_l$ are the marginal and /
and average cost curves of the follower and leader respectively. Because the follower is a price taker, he will supply output up to the point where a horizontal line at the going price (viz. his average revenue curve) intersects his marginal cost curve, $M_f$. The greatest amount he can supply is determined by the intersection of $M_f$ with $DD'$. The follower will then be meeting total market demand at price $O_d$. Below the point of minimum average cost (i.e. below $O_p'$ the Figure 5.1), the follower will go out of business. The segments $de$ and $fB'$ constitute the residual demand curve facing the leader. This curve is constructed by taking the horizontal difference between $DD'$ and that segment of $M_f$ which lies above the minimum point of $A_f$. The segments $dg$ and $hk$ constitute the marginal revenue function facing the leader. The solution to the model involves the market price $p$ being established, with the amounts $q_L$ and $q_f$ being produced by the leader and follower respectively. In this particular case, the leader is totally dominant. It has lower costs than the follower in every sense; produces more of the good; and earns a greater profit. In fact, this is by no means the only type of solution to the model; and if the discontinuity (ef) in the leader's average revenue curve is at all large there may be no solution whatsoever. This point is considered in greater detail in Chapter 6.

Many of the problems of developing a general solution to the price leadership model arise from discontinuities in /
in the market supply function and by implication in the leader's average revenue function, as exemplified above. These discontinuities only arise when the followers have U-shaped average cost curves (cf. Ryan, 1958, p.337). In general, for this problem to arise it is not necessary for the marginal cost curve to be U-shaped as well, although this is very often the case. For example, a quadratic total cost curve implies a U-shaped average cost curve, but a linear marginal cost curve. In the case in which the marginal and average cost curves are increasing and emanate from a common positive ordinate, the leader's average revenue will no longer be discontinuous, as in Figure 5.1, but merely kinked.

As we have seen, the price at which a discontinuity in the supply curve arises is equal to the point of minimum average cost for a particular firm. Below this price the firm will go out of business, and from producing a finite amount of the good it produces nothing, hence the discontinuity in the supply function. Of course, this problem of discontinuity is also present in the model of perfect competition. However, in this case the problem is not so severe as there is a multitude of firms in the market; and possibly many firms have different plant size but the same minimum average cost. In view of these features it is reasonable to argue that the supply curve is approximately continuous. But in the case under consideration this is no longer reasonable. There are fewer firms and the disappearance /
disappearance of any one firm may cause a substantial shortfall in the market supply. In cases such as this the model very often breaks down or else can only be patched up by ad hoc assumptions. In order to treat the model in some generality, without having to consider a host of special cases, attention has been concentrated on the increasing cost case. The exact definition adopted is expressed by (5.1) below.

5.2 The Model

Consider a market in which there are \( n \) firms: \((n-1)\) followers and one leader. The leader (arbitrarily the 1st firm) is in a hegemonic position by virtue of its low costs of production. This firm aims to maximise its monopoly profit and in doing so sets a monopoly price which is accepted by the market followers. The followers thus behave similarly to perfect competitors, the important difference in this case being that there is now an opportunity to make substantial supernormal profit, as the propensity of followers to do so is not circumscribed by a free entry condition. It is possible to refine this argument slightly, to embrace product differentiation between the leader's product on the one hand, and the product of the followers, considered as a group, on the other hand. We suppose that the leader is a well-established firm which enjoys limited monopolistic power by virtue of reputation. If the leader and followers all tried to charge the same price, then we will suppose that consumers would purchase exclusively /
exclusively from the leader, and the followers would all be forced out of the market. The followers can only gain some share of the market at all by "shading" their price below that of the leader. Thus consumers who wish to buy the prestige product from the leader must be willing to purchase at a slight premium on the followers' price. This sort of argument is supported by the study of Fog (1960, p. 136) where it is stated that "the general rule in the case of price leadership is that the followers do a certain amount of undercutting". More recently, Shaw (1974, p. 69) has observed this behaviour in the U.K. retail petrol trade where, it is said, "the cut-price firms were using the major company prices as reference points so that the cut-price firms' prices tended to move in parallel and after the initiating change of the leaders". This is also the argument advanced in the text book treatment of Vickrey (1964, pp. 309-314).

We define $T_i(q)$ as the total cost curve for the $i$'th firm, where $q$ is quantity per unit time. As usual, the marginal and average cost functions are defined by:

$$M_i(q) = T_i'(q) \quad \text{and} \quad A_i(q) = \frac{T_i(q)}{q}$$

Let us suppose that we have increasing costs at all output levels for all firms, with average cost curves possessing the following properties 1/.

(5.1) /
These restrictions on $A_i$ provide the definition of increasing costs to be used in this article. By implication, every $A_i$ is convex and monotonically increasing. Because we are assuming that $A_i(0) = 0$, we are also implicitly assuming that fixed costs are zero. We may either adopt the interpretation that fixed costs are negligible, or that we are working with long run cost curves.

It will be assumed that the 1st firm, or leader, is able to establish itself as market leader because its costs are lower than its rivals'. Having lower costs at all output levels, it could always "punish" (by price cutting) followers who got out of line. Followers would suffer, because in order to get any market share at all, they must shade their price below the leader. If a follower were slow to react to the leader's price cut, he might be forced out of the market. It is interesting to note that, working with a model akin to a dynamic version of the one developed here, with "a dominant firm able to deduce a residual demand schedule through knowledge of a well-behaved competitive /
competitive fringe", Gaskins (1971, p. 320) concludes that "optimizing dominant firms will ultimately decline if they have no substantial long-run cost advantage".

Formally we assume that the inequality

\[(5.2) \quad A_i(q) > A_1(q) \quad (i = 2, 3, \ldots, n)\]

holds for all positive \(q\). It follows that

\[T_i(q) > T_1(q)\]

We will say that this state of affairs characterises the leader as a **totally cost dominant** firm. It is easily shown that

\[M_1(q_1) = A_i(q_1) + q_1A_i'(q_1)\]

from which \(M_1(q_1) > A_i(q_1)\) and \(M_1(q_1) > 0\) using (5.1). Also \(M_1(0) = 0\). The market demand function is assumed to be single valued, bounded, monotonically decreasing, and defined for all positive \(p\):

\[(5.3) \quad q = f(p), \quad f'(p) < 0\]

In view of the cost dominance of the leader, the followers will regard any price he sets as a datum. As in the model of /
of perfect competition, the followers will have a common horizontal average revenue curve which will cut the vertical axis at an ordinate equal to the price established by the leader. Stability requires that the follower's marginal cost functions be rising, which is ensured by the assumptions of (5.1). As each $M_i$ is single valued and monotonically increasing with $q$, this will also apply to the unique inverse $M^{-1}_i$. Thus the amount supplied by the $i$'th follower for a price $p$, prescribed by the leader is

$$q_i = M^{-1}_i(p) \quad (i = 2, \ldots, n)$$

and the total supply from followers which is forthcoming at this price is

$$\left(5.4\right) \quad \sum_{i=2}^{n} q_i = \sum_{i=2}^{n} M^{-1}_i(p)$$

As (5.4) is a sum of monotonically increasing functions, it is itself increasing. The demand function faced by the market leader is

$$q_i = q - \sum_{i=2}^{n} q_i \quad \text{or}$$

$$F(p) = s(p) - \sum_{i=2}^{n} M^{-1}_i(p) \quad \left(5.5\right)$$
which is monotonically decreasing, because \( f' < 0 \) and 
\( M_i^{-1} > 0 \) for all \( i \). Therefore \( f'(p) > F'(p) \). Also
from (5.5), \( F(p) < f(p) \). As \( f'(p) < -F'(p) \), it follows
that for a given \( p \),
\[
\frac{-P}{F(p)} > \frac{-P}{f(p)} \cdot f'(p)
\]
That is, for a given \( p \), the leader's demand curve is more
elastic than the market demand curve. From (5.5) we
get the leader's average revenue function
\[
(5.6) \quad p = F^{-1}(q)
\]
as a unique inverse is defined. Clearly \( F^{-1} \) is monotonically
decreasing, by the inverse function rule.

The construction of the leader's demand curve is
related to the "contingent demand curve" technique developed
by Shubik (1949, pp. 82-91, 143-148). By the product
differentiation argument advanced at the beginning of this
section, followers are in no position to contest the price
set by the leader. They cannot set exactly the same price
but must shade it below (be it every so slightly) the
leader's price. There is a slight analytical fudge in
the sense that in equation (5.5), the price of the first
right hand term must in fact be very slightly higher than
the price of the second right hand term; but for the sake
of analysis, these prices must be assumed equal.

The /
The market leader's profit function is defined as

\[(5.7) \quad \Pi_i(q_i) = q_i F^{-1}(q_i) - T_i(q_i)\]

The first - and second - order conditions for maximization of (5.7) are:

\[(5.8) \quad MR_i(q_i) = q_i F^{-1}(q_i) + F^{-1}(q_i) = M_i(q_i)\]

\[(5.9) \quad [q_i F''^{-1}(q_i) + 2 F'^{-1}(q_i) - M'_i(q_i)] < 0\]

where \(MR_i\) indicates the leader's marginal revenue function. Supposing that a solution to (5.8) is given by \(\bar{q}_1\), the price that will be set is given by \(\tilde{p} = F^{-1}(\bar{q}_1)\). Then the optimum output of the \(i^{th}\) follower will be \(\bar{q}_i = M_i^{-1}(\tilde{p})\).

In summary, it appears to be the case \(^4\) that this sort of market structure ensures the selection of a set of equilibrium outputs, \(\bar{q}_i\) (\(i = 1, 2, \ldots, n\)), which are optimal with respect to the postulated behaviour of each corresponding firm, be it leader or follower, and which just clear the market a price \(\tilde{p}\). Thus the model is determinate.

To establish this fact is a necessary first step in model building. It is perhaps not immediately useful from /
from the viewpoint of prediction, but as real world events are by their nature determined, it would be disturbing if one devised a model which purported to explain the real world, but yet was indeterminate.

5-3. Price as the Independent Variable

An alternative development which is logically equivalent and has some intuitive appeal is to express the leader's profit as a function of price. Thus

\[(5.10) \quad \pi_i(p) = p F(p) - T[F(p)]\]

A necessary condition for a maximum is that

\[(5.11) \quad p F'(p) + F(p) = \frac{dT_i}{dF} = F'(p) = \frac{dT_i}{dq_i} \frac{dq_i}{dp} \]

that is,

\[p \frac{dp}{dq_i} \frac{dq_i}{dp} + q_i \frac{dp}{dq_i} = \frac{dT_i}{dq_i} \]

or

\[-F^{-1}(q_i) + q_i F^{-1}(q_i) = M_i(q_i)\]

which is identical to (5.8) above, confirming the equivalence of each approach. The merit of setting up the profit function as in (5.10) is that it provides a metaphor for the leader's behaviour. If \(\tilde{p}\) is the solution of (5.11), then \(\tilde{q}_i = F(\tilde{p})\) and \(\tilde{q}_i = M_i^{-1}(\tilde{p})\) for \((i = 2, \ldots, n)\).

Although /
Although this approach has some appeal on economic grounds, we have generally chosen to regard the leader's profit as a function of quantity. This is more convenient analytically, and of course makes no difference to the conclusions reached.

5-4. Comparative Statics

Corresponding to the equilibrium conditions of the linear case, given by (4.27) in the previous chapter, we have the extended equilibrium conditions

\[(5.12) \quad p - M_i(q_i) = 0 \quad (i=2,3,\ldots,n)\]

\[q_i F^{-1}(q_i) + F^{-1}(q_i) - M_i(q_i) = 0\]

\[\sum_{i=1}^{n} q_i - f(p) = 0\]

which is a system of \((n+1)\) equations in the \((n+1)\) variables \((q_1, q_2, \ldots, q_n, p)\).

The first analytical problem is to determine the sign of the Jacobian determinant for (5.12). The Jacobian is given by:
We have the following sign restrictions

\[ M_i' > 0 \quad (i = 1, 2, \ldots, n) \quad (5.14) \]

\[ F^{-1'} < 0 \quad \text{and} \quad f' < 0 \]

Unfortunately these sign restrictions in themselves do not enable us to determine the sign of the determinant of the Jacobian, \(|J|\). As a counter-example, consider \(|J|\) in the case of just one follower. It is given by

\[
\begin{vmatrix}
-M_2^1 & 0 & 1 \\
0 & z & 0 \\
1 & 1 & -f_1^1
\end{vmatrix} = Z (M_2^1 f_1 - 1)
\]
The term inside the bracket in (5.15) is certainly negative, because \( M_2^1 > 0 \) and \( f^1 < 0 \). But what about the term outside the brackets? As we indicate in the Appendix to this chapter, the restrictions on costs and demand which we have adopted do not, in themselves, allow us to sign \( Z \). However, there is another possible line of attack.

We observe that \( Z \) is nothing other than the expression (5.9) above. If we are prepared to assume that this second-order condition ("stability condition") is satisfied, we can conclude that (5.15) is positive. This suggests the possibility that we may be able to sign \( |J| \) for any order. Expansion of the relevant determinants yields the following expressions.

\[
(3 \times 3) \text{ case } \quad Z (M_2^1 f^1 - 1) > 0
\]
\[
(4 \times 4) \text{ case } \quad Z (-M_2^1 M_3^1 f^1 + M_2^1 + M_3^1) < 0
\]
\[
(5 \times 5) \text{ case } \quad Z (M_2^1 M_3^1 M_4^1 f^1 - M_2^1 M_3^1 M_4^1 - M_2^1 - M_3^1 M_4^1) > 0
\]
\[
(6 \times 6) \text{ case } \quad Z (-M_2^1 M_3^1 M_4^1 M_5^1 f^1 + M_2^1 M_3^1 M_4^1 + M_2^1 M_3^1 M_5^1 + M_2^1 M_3^1 M_4^1) < 0 \quad \text{etc.}
\]

The general expression for \( |J| \) becomes clear, and its sign is evidently given by \( (-1)^{n-1} \). That is, \( |J| \) is negative for \( / \)
for even orders and positive for odd orders.

As in Chapter 4, we may work out comparative statics properties of the model, although the expressions become harder to handle, analytically speaking. A simple point worth noting is that neither a profits tax (\( t_1 \)), nor a lump-sum tax (\( t_2 \)) will affect the second equilibrium condition of the model. The profit function of the leader in each case will be

\[
\Pi_1(q_i) = (1 - \xi_1)[q_i F^{-1}(q_i) - T_1(q_i)]
\]

\[
\Pi_1(q_i) = q_i F^{-1}(q_i) - T_1(q_i) - t_2
\]

On setting the derivative of each of these expressions equal to zero, we arrive at the second equation in (5.12).

To extend the sort of analysis provided in Chapter 4, let us consider, for example, the effect of a specific sales tax (\( t_3 \)) in the two-follower case. We get the system:

\[
\begin{bmatrix}
-M_2' & 0 & 0 & 1 \\
0 & -M_3' & 0 & 1 \\
0 & 0 & 0 & 0 \\
1 & 1 & 1 & -f'
\end{bmatrix}
\begin{bmatrix}
\partial q_2/\partial t_3 \\
\partial q_3/\partial t_3 \\
\partial q_1/\partial t_3 \\
\partial p/\partial t_3
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}
\]

We already know that \(|J| < 0\) in the (4 x 4) case. It can then be shown that
Thus, as in the linear case, a specific sales tax on the leader lowers his output, raises price, and increases the output of each follower.

5-5. Cost, Output and Profit Relationships

The profit of the \( i \)'th follower \( (\pi_i) \) is given by

\[
\pi_i = \bar{q}_i \left[ \bar{p} - A_i (\bar{q}_i) \right] (\text{i = 2, 3, ..., n})
\]

from (5.7)

\[
\pi_i = \bar{q}_i F^{-1} (\bar{q}_i) - T_i (\bar{q}_i)
\]

\[
= \bar{q}_i \left[ F^{-1} (\bar{q}_i) - T_i (\bar{q}_i) / \bar{q}_i \right]
\]

(5.19)

\[
= \bar{q}_i \left[ \bar{p} - A_i (\bar{q}_i) \right]
\]

using (5.6), and a basic definition. It is not necessarily true that the total cost dominance of the leader will ensure that in equilibrium it will also be output dominant. By output dominance we mean that \( \bar{q}_1 > \bar{q}_i \) (i = 2, 3, ..., n), which /
which implies that the leader has the greatest market share; rather than the very much stronger condition that in equilibrium he produces more than the sum of the followers' outputs. Diagram 5.2, in which all functions have been accurately drawn, provides a suitable counterexample to the conjecture that \( \tilde{q}_1 > \tilde{q}_i \) will always hold. What we **can** say is that in such instances as \( \tilde{q}_1 = \tilde{q}_i \), then it will be true that \( \pi_1 > \pi_i \) by application of inequality (5.2) to equations (5.18) and (5.19). In this sort of situation, the leader must be profit dominant because he can sell the same amount as the follower at a price above marginal costs, whereas any follower's average profit must always be exactly equal to the excess of marginal cost over average cost at any output level. It is not necessarily true that \( \tilde{q}_1 > \tilde{q}_i \) implies \( \pi_1 > \pi_i \). This is because the leader, although producing more than the follower may be doing so at substantially higher costs. Although we have assumed that the leader has lower costs than the followers at equal output levels, this does not preclude the possibility that its costs might exceed the followers' at higher output levels.

**5-6. Conclusion**

We have seen that the linear model of Chapter 5 may be generalized to the increasing cost case. The model remains determinate, although (as the Appendix shows) there may not be a unique equilibrium solution. The sign of /
of the Jacobian determinant of the equilibrium conditions was shown to be \((-1)^{n-1}\) in the \((n \times n)\) case. This result was used in a comparative statics exercise along the lines of Chapter 4. Finally, it was shown that there are no strong results on the relationship between cost dominance and output or profit dominance. This implies that the least cost producer may not be so advantaged as appearances indicate: the configuration of his rivals' costs and of market demand are equally important in determining his market share and profits.
Footnotes to Chapter 5

1/ Throughout this chapter we implicitly assume continuity and differentiability (to the required order) of all functions introduced.

2/ \[
\lim_{q \to +0} A(q) = \lim_{q \to +0} \frac{T(q)}{q} = \lim_{q \to +0} \frac{T'(q)}{1}, \text{ by l'Hopital's rule,}
\]

But \( \lim_{q \to +0} A(q) = 0 \), by assumption

\[
\lim_{q \to +0} M(q)
\]

whence also \( \lim_{q \to +0} M(q) = 0 \), or as we have expressed it, \( M(0) = 0 \).

3/ By the inverse function rule.

4/ We use caution in our expression at this point, because as the Appendix to this chapter indicates there may not be a unique solution to equation (5.8). However a solution will exist.
Appendix to Chapter 5
The Nature of the Solution

In microeconomic models of general equilibrium the theorists' major concern is to prove the existence of an equilibrium solution. In certain instances, it can be shown that a unique solution exists. In the substantially simpler model with which we are now working such rigour is out of place. In the linear case discussed in Chapter 4, the restrictions on the parameters of the model guaranteed that the model would have a unique solution. In the increasing cost case now under consideration the outcome is not so clear-cut, but we can still say a little about the nature of the solution.

Let us slightly strengthen the restrictions on the market demand function defined by (5.3). In particular, we will assume that at some positive \( \tilde{p} \) (which may be large) \( f(\tilde{p}) = 0 \); and that \( f(0) = \tilde{q} \), for some positive \( \tilde{q} \) (which may be large), this being, in effect, an assumption of satiation. This latter assumption can be relaxed at the expense of additional complications, but doing so would leave the basic argument unaffected.

We have already noted that \( M_i(0) = 0 \), whence \( M_i^{-1}(0) = 0 \); and have assumed that \( f(0) = \tilde{q} \). Therefore, from (5.5) it follows that \( F(0) = f(0) = \tilde{q} \), or \( F^{-1}(\tilde{q}) = 0 \). As all the \( M_i^{-1} \) are continuously increasing from the origin, and \( f(p) \) is decreasing and intersects both axes, there will exist a /
a \( \tilde{p} \) for which

\[
\int_{1}^{\tilde{p}} f'(p') \, dp' = \sum_{i=2}^{n} M_i^{-1}(p')
\]

implying that \( F'(\tilde{p}) = 0 \). So we have established that the leader's average revenue function intersects the price axis at \( \tilde{p} \) and the quantity axis at \( \tilde{q} \).

From (5.8), for quantity \( \tilde{q} \) we have \( MR_1(\tilde{q}) < 0 \) because \( F^{-1}(\tilde{q}) = 0 \) and \( F^{-1}(\tilde{q}) < 0 \). Also \( MR_1(0) = F^{-1}(0) = \tilde{p} > 0 \). Thus \( MR_1 \) cuts the quantity axis because it changes sign between 0 and \( \tilde{q} \). To complete our investigation, we note that \( MR_1 < F^{-1} \) as \( F^{-1} < 0 \). Thus \( MR_1 \) lies everywhere below \( F^{-1} \) and intersects the price axis at \( \tilde{p} \) and the quantity axis at least once between 0 and \( \tilde{q} \). As \( M_1 \) is increasing from zero, we can be sure that at least one root exists for equation (5.8).

It will be noted that what we have not shown is that a unique root exists. Under the assumptions of the model it is not possible to arrive at this conclusion. Were we also to show \( MR_1 \) to be monotonically decreasing then uniqueness would follow. But

\[
MR_1' = q \left( F^{-1} \right)' + \frac{1}{2} F^{-1}''
\]

We do not know the sign of \( F^{-1}'' \) in this expression and cannot safely assume that it would be negative, implying concavity of the leader's demand curve. The relationship between the concavity or convexity of the functions may be examined /
examined by inspecting the second derivative of (5.5), namely

\[ F'' = f'' - \sum_{i=2}^{n} M_i'' \]

Unfortunately, the signs of \( f'' \) and the \( M_i'' \) are unknown. The most plausible assumptions are that \( f'' > 0 \) and \( M_i'' > 0 \), but even these will not ensure that \( F'' < 0 \) [and by implication \( F^{-1}'' < 0 \)].
Chapter 6

Discontinuity Problems in the Price Leadership Model

6-1. Introduction

In Chapters 3 through 5, we provided the basis for constructing a fairly general static model of price leadership. The purpose of this chapter is to generalize in three directions the existing treatments of the price leadership model. Firstly, we give a mathematical statement of the model, generalizing existing geometrical and literary approaches, and extending the mathematical version of Hadar (1971, pp.115-117). Secondly, we admit of many followers, rather than adopting the conventional assumptions of just one follower, or an anonymous aggregate of followers regarded as a competitive "fringe". Thirdly, we attempt to take seriously the problems of discontinuity which might arise in this type of market if a follower is forced out of business: a matter which is frequently ignored in the theorizing of economists.

Although we are attempting to achieve a greater generality than previously, there is a danger that the whole exercise becomes merely taxonomic. We regard this as being unsatisfactory, but largely inevitable. As Archibald (1961) pointed out when investigating the comparative statics properties of the monopolistic competition model, "we require more facts, not for their own sake, but in order to put into the theory sufficient content for it to yield significant predictions". This statement is related to a methodological point /
point that we made earlier in Section 4-4. There we emphasized that in order to get unambiguous comparative statics results one often has to impose fairly tight restrictions on the model. In the present chapter, we have relaxed some of the restrictions used earlier, but will discover that the model therefore loses part of its potential for prediction. We shall leave until the next chapter the problem of developing a price leadership model with greater empirical content.

6.2. The Assumptions

Modifying and extending Ryan (1958, p.337) the assumptions of the model are as follows:

(1) There are \( n \) independent firms. Arbitrarily, the first is the leader, and the remaining \( n \) firms with indices 2, 3, \( \ldots \), \( n \) are the market followers. Without loss of generality, we assume that the followers are ranked according to their costs, with Firm 2 having lowest costs, Firm 3 the next lowest and so on. The integer \( n \) is to be conceived of as fairly large, for the number of firms which it defines includes merely potential followers who may not be able to stay in business when the market is in equilibrium.  

(2a) All firms produce a good that is effectively homogeneous. An alternative, and perhaps more realistic assumption is the following:

(2b) All firms produce a similar product, but there is a priority pattern in the purchases of consumers. In particular /
particular, consumers will prefer to purchase what they can from the leader in the first instance, and only after that supply is exhausted will they take up the supply of the followers. In cases such as this, the leader will be well established, and enjoys this first-purchase privilege because of the prestige value attached to its product. It is possible that the followers might have to "shade" their prices very slightly below the leader, in order to winnow away their market share. Otherwise there would be a possibility that consumers would want to purchase exclusively from the leader.

(3) The good is perishable, so that the total output of any firm in each period must be sold. Alternatively, we must admit that we ignore the possibility that firms may hold varying levels of stocks.

(4) The consumers are numerous and knowledgeable, and are liable to favour the leader's product over that of any follower, ceteris paribus, as outlined in (2b).

(5) The leader knows the pattern of demand for his product under the assumption that other firms in the market act as followers.

(6) Each firm seeks to maximise its profit per period in terms of its role, be it leader or follower.

(7) Firm 1, the leader, assumes that the (n-1) other firms, the followers (both potential and actual, or merely potential), will always charge the price he sets, or shade their's slightly below. Furthermore, the leader is correct in making this assumption.

6-3. The Model

We have listed the principal economic assumptions.
In developing the model, further assumptions of a mathematical nature will be introduced, but even these will have an economic interpretation. For example, the assumption that marginal cost functions are convex presumes that the law of non-proportional returns is operative. That is, marginal costs are initially high, when production is well below the technically efficient level, begin to fall as the engineering optimum is approached, and then rise again as plant is operated at strained capacity.

The aggregate or market demand function is defined as

\[ (6.1) \quad q = D(p) \quad D'(p) < 0 \]

where \( q \) is expected demand per unit time, and \( p \) is the price per unit of the good.

There are \( n \) total cost curves, all distinct:

\[ (6.2) \quad C_i = C_i(q) \quad (i = 1, 2, \ldots, n) \]

As usual, marginal and average cost curves are defined by:

\[ (6.3a) \quad M_i(q) = C_i'(q) \quad (6.3b) \quad A_i(q) = \frac{C_i(q)}{q} \]

where \( M_i \) and \( A_i \) are convex, the \( A_i \) all achieving a minimum for some positive \( q \). If we call these minima of the average cost curves of followers \( \min(A_i) \), then, as a convenience /
convenience, we may rank the firms such that

\[(6.4) \quad \min(A_2) < \min(A_3) < \ldots < \min(A_n)\]

this being a precise statement of part of Assumption (1) above. Corresponding to these \( \min(A_i) \), there are minimal output levels which we will call \( \min(q_i) \). The \( \min(q_i) \) will not necessarily be ranked as in (6.4), though in practice there is some likelihood that their rankings would be, broadly speaking, reversed.

The marginal cost function for the \( i \)th follower is only effectively defined at a price no less than \( \min(A_i) \). Below this price, the follower will shut down production. Above this price, the marginal curve is convex, one - one and monotonically increasing.

As before, the basic behavioural postulate (see Assumption (7) above) is that the leader is in a hegemonic position with respect to price setting, and will set that price which maximizes his (monopolistic) profit. The market followers are price takers, and behave in a similar manner to firms under perfect competition. However, in this case at least a few of the followers will probably have a chance to make substantial supernormal profits. This will arise if any follower has costs which are relatively low, because he will be accepting the monopolistic price set by the leader and thus his unit profits are likely to be high. For any price, \( p \), set by /
by the market leader, followers will set marginal cost equal to price

\[ M_i(q) = P \quad (i = 2, 3, ... , n) \]  

A sufficient condition for the solutions to (6.5) to provide profit maximizing outputs

\[ q_i = M_i^{-1}(P) \quad \min(A_i) \leq P \]

\[ q_i = 0 \quad P < \min(A_i) \]

is that \( M'_i(q) > 0 \) for all \( i \) at these output levels. The assumed restrictions on \( M_i \) ensure that this condition is satisfied and that we may define a unique inverse function in the manner of (6.6).

In Diagram 6.1 the cost curves (Eqn. 6.3) for merely three followers have been drawn to keep the geometry simple. \( D \) is the market demand curve (Eqn. 6.1).

The expression given by (6.6) in fact defines the supply function of the \( i \)'th follower. Thus the aggregate supply of the followers is given by

\[ S(p) = \sum_{i=2}^{n} M_i^{-1}(p) \]

A special case of this function, constructed from the marginal /
Diagram 6.1

Diagram 6.2
marginal cost curves of the three followers in Diagram 6.1, is given in Diagram 6.2 as the curve S. The supply curve of market followers has a saltus or "jump" at each $\min (A_j)$ price, and at an output level $\min (q_j)$, this being the minimum non-loss-incurring output. As $S(p)$ is a sum of monotonically increasing functions, it is itself monotonically increasing by segments.

Let us assume for the moment that a root of the equation $S(p) - D(p) = 0$ exists. In view of the properties of $S$ and $D$, if this root (say $p^*$) exists, it will be unique and positive. In fact $p^*$ is the greatest realizable market price under normal circumstances. The leader's demand function $F(p)$ is defined by the segments

(6.8a) \[ F(p) = D(p) - \sum_{i=2}^{j} M_i'(p) \text{ for } \min (A_j) \leq p < \min (A_{j+1}) \]

where $\min (A_2) \leq p \leq p^*$

(6.8b) \[ = 0 \text{ for } p > p^* \]

(6.8c) \[ = D(p) \text{ for } 0 \leq p < \min (A_2) \]

The cases for which $p^* \leq p$ or $p < \min (A_2)$ require special treatment, so let us concentrate on (6.8a). $F(p)$ is clearly monotonically decreasing by segments, and uniquely invertible by segments. We will define the inverse of /
of $F$ as

$$\tag{6.9} p = F'(q) = \Phi(q)$$

which is to be interpreted as the market leader's average revenue function. We notice that the slope of the leader's demand curve is always less than that of the market demand curve for a given $p$. For, from (6.8a) we have

$$\tag{6.10} F'(p) = D'(p) - \sum_{i=2}^{j} M_i''(p) \quad \text{for all } j$$

Now $D'(p) < 0$ by assumption, and $M_i'' > 0$ for the relevant range of $p$ implies that $M_i'' > 0$ for all $i$ by the inverse function rule. Thus

$$D'(p) > [D'(p) - M_2''(p)] > [D'(p) - M_2''(p) - M_3''(p)] \text{ etc.}$$

That is, $D'(p) > F'(p)$ for all segments of $F$, the inequality becoming more marked the greater the number of segments (i.e. the greater is $j$, the upper limit of the summation in (6.10) above). From (6.8a) it is clear that by a similar argument we have $F(p) < D(p)$ for a given $p$. We can now establish that the elasticity of the leader's demand curve $F(p)$, is greater than the elasticity of the market demand curve, $D(p)$, for any given $p$. From $D'(p) > F'(p)$ we have $-D'(p) < F'(p)$; and from $F(p) < D(p)$ we have

$$\tag{6.11} -\frac{D'(p)}{D(p)} \cdot \frac{p}{F(p)} < -\frac{F'(p)}{F(p)} \cdot \frac{p}{D(p)}$$
where the left hand and right hand sides of the inequality represent the elasticity of the market demand curve and the elasticity of the leader's demand curve respectively. This inequality leads us to the important conclusion that a dominant firm which permits the existence of a fringe of competitive firms, rather than driving them out and acting as a pure monopolist, will face a more elastic demand curve and hence will have reduced its own monopolistic powers. The practical effect of this is that price will always be lower than it would under pure monopoly.

From the leader's average revenue function given by (6.9), we can derive the marginal revenue function

\[ R(q_i) = q_i \, \Phi'(q_i) + \Phi(q_i) \]  

Diagram 6.3 graphs the marginal revenue function for the curves given in Diagrams 6.1 and 6.2. Geometrically, the leader's demand (average revenue) curve is obtained by taking the horizontal difference between the market demand curve (d in Diagrams 6.1 and 6.2) and the followers' aggregate supply curve (s in Diagram 6.2). As a direct consequence of the segmented nature of the leader's demand curve, the leader's marginal revenue curve is segmented in a similar manner.

Before considering any complications, let us indicate the simplest solution to the model. The leader's profit /
Diagram 6.3

Diagram 6.4
profit function is given by

\begin{equation}
(6.13) \quad \Pi_1 = pq_1 - c_1(q_1) = \Phi(q_1)q_1 - c_1(q_1)
\end{equation}

which is maximized when

\begin{equation}
(6.14) \quad \Phi'(q_1)q_1 + \Phi(q_1) = M_1(q_1)
\end{equation}

Suppose the root of (6.14) is \( \overline{q}_1 \); then the price set by the leader will be, from (6.9),

\begin{equation}
(6.15) \quad \overline{p} = \Phi(\overline{q}_1)
\end{equation}

Market followers will accept this price, and the equilibrium outputs for the followers will be

\begin{equation}
(6.16) \quad \overline{q}_i = M_i^{-1}(\overline{p}) \quad (i=2, \ldots, k) \quad k \leq n
\end{equation}

for \( \min (A_i) \leq \overline{p} \) \((i=2, \ldots, k)\), in which case \((n-k)\) potential followers will be unable to operate profitably in the equilibrium state of the market. The profit per unit for the \(i\)'th follower still in business at price \( \overline{p} \) is \( \overline{p} - A_i(q_i) \) whence the profit of the \(i\)'th follower is

\begin{equation}
(6.17) \quad \Pi_i = \overline{q}_i \left[ \overline{p} - A_i(\overline{q}_i) \right]
\end{equation}
The model is thus determinate in this, the least awkward case, and achieves the selection of a set of optimal outputs $\bar{q}_i (i = 1, 2, \ldots, k)$ at a price $\bar{p}$, with (k-1) market followers in operation.

A particular solution is given in Diagrams 6.1 and 6.3. If the leader has average and marginal cost curves $A_1, M_1$ (Diagram 6.3), it will set price at $\bar{p}$. Once set, the optimum output, $\bar{q}_1$, is defined. Market followers take the price as given, and thus their marginal and average revenue curves are coincident horizontal lines cutting the price axis at this ordinate - or, perhaps, as we have suggested already, very slightly below. In Diagram 6.1, the optimal outputs of the followers are $\bar{q}_2$ and $\bar{q}_5$. The fourth firm has been forced out of the market.

6-4. A Numerical Example

Consider the following example in which there are four potential followers; and a leader with substantial cost advantages.

Market Demand (cf. Eqn. 6.1) $D(p) = 12 - 0.5p$

**Leader's Cost Curves**

$MC_1: \quad p = 0.05(q-7)^2 + 0.1q(q-7) + 1$

$AC_1: \quad p = 0.05(q-7)^2 + 1$

**Followers' Cost Curves**

$MC_2: \quad p = (q-1)^2 + 2q(q-1) + 3$

$AC_2: \quad p = (q-1)^2 + 3$

$MC_3: \quad p = (q-2)^2 + 2q(q-2) + 4$
AC₃: \[ p = (q - 2)^2 + 4 \]
MC₄: \[ p = (q - 4)^2 + 2q(q - 4) + 6 \]
AC₄: \[ p = (q - 4)^2 + 6 \]
MC₅: \[ p = (q - 11)^2 + 2q(q - 11) + 10 \]
AC₅: \[ p = (q - 11)^2 + 10 \]

The equation of the supply curve (cf. Eqn. 6.7) in the neighbourhood of the maximum achievable market price \( p^* \) is given by:

\[ S(p) = 4.66 + 0.66 \sqrt{3p - 8} + 0.33 \sqrt{3p - 2} \]

The value of \( p^* \) is found to be approximately 7. But the average cost curve of Firm 5 reaches a minimum at 10, and therefore this firm could not operate in the market, irrespective of the leader’s pricing policy.

The leader’s demand curve (cf. Eqn. 6.8) is given by:

\[ F(p) = 11.66 - 0.33 \sqrt{3p - 8} \quad \text{for} \ 3 \leq p < 4 \]
\[ = 10.0 - 0.66 \sqrt{3p - 8} - 0.5p \quad \text{for} \ 4 \leq p < 6 \]
\[ = 7.33 - 0.66 \sqrt{3p - 8} - 0.33 \sqrt{3p - 2} - 0.5p \quad \text{for} \ 6 \leq p < 7 \]
\[ = 0 \quad \text{for} \ p > 7 \]
\[ = 12 - 0.5p \quad \text{for} \ p \leq 3 \]

In /
In this case, there is no admissible root to Eqn. (6.14), and the profit maximizing output for the leader has to be found by inspection of the total profit function. By exhaustion, approximate figures were obtained. Maximum profit for the leader is 24.8, which is achieved at an output level of 8.7 and a price of 4. At this price, Firm 3 is just forced out of the market, and only one follower, Firm 2, remains, earning a profit of 5.3 and producing an output of 1.3. The cost advantages of the leader are so great that only a single follower can survive in the equilibrium state of the model, producing a small proportion of aggregate output.

6.5. The Discontinuous Demand Curve Problem

In Section (6-3) above, a straightforward solution was presented for the sake of clarity. However, as the numerical example of section (6-4) indicates, this tidy solution may not always emerge. In the next two sections we will consider the two principal departures from this simple solution, which we will classify as the discontinuous demand curve and discontinuous supply curve problems; and we will discover that assumptions going beyond those included in (6-2) are required.

Closely related to the discontinuous demand curve is the "plateau demand curve" due to Nutter (1955), who built on the earlier work of Nichol (1934). Nichol argued that the competitive solution for a firm might emerge, even under monopolistic conditions, if the firm had an infinitely elastic /
elastic segment on its demand curve. This "plateau" on the demand curve might arise at a certain price if consumers were indifferent from whom they made their purchases at this price and were motivated to buy from Firm A rather than Firm B by incidental circumstances. Nutter describes a group of consumers who behave in this way as "marginal buyers", and points out that their existence contradicts the Principle of Diminishing Marginal Rate of Substitution.

The question to be answered here is whether the plateau demand curve theory has any relevance to the price leadership model under consideration. In particular, will it indicate how to cope with the situation depicted in Diagram 6.47?

In the diagram, the discontinuity in the leader's average revenue curve $\bar{f}$ is due to the disappearance of the sole follower at prices not greater than $p'$. The leader is evidently at liberty to produce between output levels $q^3$ and $q^2$, but his choice of some such output, say $q^1$ as in the plateau curve analysis, introduces problems into the analysis. Prima facie there are two cases to be considered: (a) the follower stays in the market; (b) the follower drops out of the market. If (a) holds, the follower stays in the market at price $p'$, and it must produce an amount equal to $AB$ (i.e. $q^2 - q^3$), as this is its minimum scale of operation. If (b), the leader must produce the amount $q^2$ at price $p'$. If Assumption (2b) of section (6.2) above is admitted, then this latter case is /
is more likely to hold. The leader sets price $p'$, and the follower, to gain its market share ought to shade its price below $p'$ - but this the follower cannot do profitably. Typically one would expect other followers to be present in the market, but in this case we are presented with the interesting possibility that $p'$ is a total entry-preventing price. It would not be wise for the leader to regard his (ex ante) demand curve as now being the aggregate market demand curve, for doing so would entail setting a higher monopoly price which would merely attract back a fringe of followers. In many institutional settings, the leader would not even desire to become totally dominant because possessing too great a market share might make it subject to investigation by anti-trust authorities.

A possibility we have still to consider is that the plateau demand curve solution per se might apply. In the case considered by Nutter (1955) there is a horizontal segment on the demand curve, and along this segment the average and marginal revenue functions are coincident. If the marginal cost curve cuts this horizontal segment from below, the point of intersection defines the profit maximizing output. Thus Diagram 6.4 bears some resemblance to Nutter's diagram.

The principal difference between the two diagrams is that in Diagram 6.4, the average revenue function does not seem to exist in the range $AB$: it being defined for outputs up to $q^3$ and beyond $q^2$, but vanishing between outputs /
outputs $q^3$ and $q^2$. Thus the debate really hinges on the consequences of the leader producing at outputs between $q^2$ and $q^3$. The plateau demand curve solution would require, for example, that the leader produce at $q^1$, charging a price $p'$. The market is demanding an output of $q^2$ at this price and hence there is an excess demand of $(q^2 - q^1)$. Were supply to be restricted to $q^1$, market price would be forced up to $p''$. In practice, a leader might do one of two things. Firstly, he might maintain the price $p'$ but lengthen his order books, using this expedient, in effect, as a rationing device. Secondly, he might choose to "make hay", at least in the short term, and hastily snatch a bit of excessive profits. On the longer term, neither of these strategies is likely to prevail. The high price $p''$ might attract a new follower or followers able to mop up the excess demand and still not run at a loss. Or, more probably, the leader will fear the loss of good will which might result from a prolonged high price policy and gradually reduce its price from $p''$ to $p'$ and increase its output from $q^1$ to $q^2$ by moving along the broken line continuation of $\varnothing$ to point $B$.

Thus it seems there is no presumption that the plateau demand curve solution will emerge. Therefore although this concept has provided a useful peg for discussion, it does not provide a completely satisfactory answer to the analytical problems with which we are concerned here.

6-6. The Discontinuous Supply Curve Problem /
6-6. The Discontinuous Supply Curve Problem

The discussion of the previous section provides a useful background for tackling the discontinuous supply curve problem. This problem arises when a root for the equation $D(p) - S(p) = 0$, which we have called $p^*$ in section 6.3, does not exist. Graphically, we have the sort of situation depicted in Diagram 6.5.

Difficulties arise at prices above $\min (A_4)$. Exactly at $\min (A_4)$ the third follower, Firm 4, will be forced out of the market and hence the output $(q^3 - q^1)$ could be produced by Firm 1, the leader. At a price $p^* > \min (A_4)$, the supply from the fringe of followers will remain at the level $q^1$ and hence the difference $(q^2 - q^1)$ will be made up by output from the leader. At $p^*$ itself the market is cleared by the output $q^1$ of the followers alone; and this price is once again the maximum feasible market price, though it is achieved in a slightly different manner to the $p^*$ of Section 3 above. Diagram 6.6 is based on the curves given in Diagram 6.5.

The segment of the average revenue curve $\overline{\gamma}$ which lies between $\min (A_4)$ and $p^*$ is merely the segment of $D(p)$ in Diagram 6.5 lying in the same price range, but shifted laterally to the left until it meets the price axis. The upper two segments of $\overline{\gamma}$ resemble the kinked demand curve analysed by Stigler, Sweezy, Hall and Hitch. Thus the discontinuous supply curve in itself introduces no additional /
additional problems into the analysis.

We introduce the possibility that a follower might not be able to operate in the market, even though the prevailing price is well above its minimum average cost. This is illustrated by the price $p_1$ in Diagram 6.6. The possibility arises because were Firm 4 to try and return to the market in its role as follower, it would merely frustrate its own intentions. It must perforce operate at its minimum feasible scale, but in doing so would merely create a situation of excess supply once more, drive down the market price, and eventually force itself out of the market once again.

6-7. Conclusion

Economists such as Akerman (1958) and Boulding (1966) have emphasised that the concept of discontinuity should not be neglected in economic analysis. In this chapter, we have attempted to tackle seriously the problem of discontinuity which arises when a firm closes down.

In the first place, we have given a mathematical generalization of the price leadership model of oligopoly to the case of one leader and many followers. In the second place, we have concentrated attention on the problem of discontinuity in the market supply function and in the average and marginal revenue functions of the leader. It has been shown that the solution may be less tidy than in versions of the model which ignore such problems.
Footnotes to Chapter 6

1/ This device is similar to that adopted by Newman and Wolfe (1961, p. 53), where it is assumed, firstly, that "the total number of firms is arbitrarily fixed at some number $K$ .... although the number of those actually in production will vary"; and, secondly, that $K$ is large.

2/ We add this proviso, because in some models (e.g. Cournot's duopoly model), a firm consistently makes a wrong assumption about his rival.

3/ The notation of Chapter 5 has been very slightly altered in this chapter. The main point of difference is that $T_1$ of Chapter 5 and $C_1$ of the present chapter have different shapes.

4/ We will assume that a firm goes out of business if it sets a price below average cost. The natural interpretation of $A_1$, therefore, is that of average variable cost. This introduces a slight problem of definition in the leader's profit function, given later by equation (6.13), where $C_1$ is treated as total cost, rather than total variable cost, though of course this problem could easily be overcome by introducing yet more notation. The simplest, consistent alternative is to assume that $C_1$ and $A_1$ refer to long-run costs.

5/ It is well known that when marginal cost is equal to marginal revenue, Lerner's "degree of monopoly" /
monopoly is equal to the reciprocal of the elasticity of demand. Hence the greater this elasticity, the lesser is the degree of monopoly. The most usual reasons why a firm might permit the existence of a fringe of competitors is an anti-trust law making a firm liable to investigation if it exceeds a certain size. One measure of size might be market share, for example.

In terms of nomenclative introduced by Sylos-Labini, can be regarded as the entry-preventing price from the viewpoint of the (n-k) firms forced out of the market.

Bain (1968, p.255) lists three principal barriers to entry: absolute cost advantages; product differentiation advantages; and scale economies advantages. Perhaps it is just tenable that the second barrier always holds to a minor degree in view of the postulated priority pattern of consumers' purchases; but in this particular case the first barrier is most significant from the viewpoint of Firm 4.

It should be noted that the diagrams referred to in this section (and indeed throughout the chapter) are sketches rather than mathematically accurate drawings.
Chapter 7
Towards a Reconstructed Theory of the Firm

7-1. Introduction

In Chapter 6 we saw that a generalized version of a standard price leadership model provided an untidy set of solutions. Many of the solutions investigated had an ad hoc character (involving boundary solutions, no real solutions etc.) Because this causes one some methodological discomfort, we have chosen to take a different direction in this chapter. It should be understood that the model developed in Chapters 3 - 6 may have predictive usefulness in specific contexts. It is to be hoped that eventually empirical work will emerge using this theoretical work.

In this chapter we have attempted to follow the scheme proposed by P.W.S. Andrews for developing a reconstructed theory of the firm. In doing so we have abandoned many of the standard accoutrements of economic theorists, including marginalism, Walrasian notions of equilibrium, and static analysis. The model constructed is one of an oligopolistic industry in which technical progress is led by a firm which is said to be following an "aggressive pricing policy" - a term elaborated in section (7-4). The model is presented in three versions, proceeding from the simple to the more complex. The final version involves considering interactions with other industries, providing, we hope, a possible approach to "general disequilibrium" analysis in the context of the /
the theory of the firm.

7-2. Elements of a Reconstruction of the Theory of the Firm

A major purpose of "On Competition in Economic Theory" by P.W.S. Andrews was to issue a challenge to students of the theory of the firm. He proposed a reconstruction of this theory which would differ radically from existing doctrines. We may distinguish three elements in this proposed reconstruction: the critical, involving a rejection of many of the economic "axioms" cherished by theorists, including the profit maximization axiom; the constructive, introducing both alternative and additional assumptions derived mainly from experience rather than introspection; and, finally, the synthetic, involving the integration of any reconstructed theory of the firm into a theory of industries, eventually drawing together these individual theories into a theory of activity in the economy as a whole.

The critical element in the reconstruction should, above all, involve the rejection of marginalist theory. Andrews tended to identify "marginalist" and "equilibrium" theories mainly because the great majority of equilibrium theories are constructed on a marginalist basis. His attack was not really focussed on the attempts of economists such as Adelman (1958), Newman and Wolfe (1961) and Simon and Bonini (1958) to develop theories of dynamic statistical equilibrium at the industry level. His real objection was to equilibrium theory in the Walrasian vein. Thus we may say that his critical stance was anti-marginalist.
anti-marginalist and anti-Walrasian.

If the first step towards a reconstructed theory of the firm is to reject a number of basic tenets of traditional theory, such as marginalism and static micro-equilibrium, the second step must be to put forward alternative ground rules within which analysis can proceed. The approach to this problem which Andrews proposed was the adoption of more realistic assumptions. These assumptions should be founded on empirical evidence - on actual experience in firms - rather than on a priori reasoning. Not only should we substitute more realistic for less realistic assumptions, but we should introduce additional assumptions to broaden our scope of analysis to the major practical problems confronting firms. These new assumptions should provide a basis for tackling, among other things, problems of technical progress, growth and uncertainty. As we saw in Section (1-5), this view on the importance of making realistic assumptions has been subjected to detailed criticism. The counter arguments hinge on the view that a theory should be judged on the accuracy of its predictions, rather than on the apparent realism, or otherwise of its postulates. We have indicated in Chapter 1 that there are several senses in which an assumption may be said to be unrealistic, some of which are innocuous but one of which is of some consequence. In particular, if an assumption is unrealistic in the second sense in which the term was to be used, the consequences are not trivial. It is the view of Nagel (1963, p.51) that
"if by an assumption we understand one of the theory's fundamental statements .... a theory with an unrealistic assumption ..... is patently unsatisfactory; for such a theory entails consequences that are incompatible with observed facts, so that on pain of rejecting elementary logical canons the theory must also be rejected". Provided we regard the concept of a "rational profit maximizer" as a theoretical term, rather than a descriptively accurate assumption, then we cannot attack the neoclassical theory of the firm for using such expressions. Nevertheless we are still at liberty to formulate fundamental statements (i.e. assumptions) which do not involve theoretical terms in a manner which we believe to be empirically valid. It is interesting to note that Friedman himself is not unsympathetic to this view. Friedman (1953, p.31) admits that the work of Andrews, Hall and Hitch may be "extremely valuable .... in constructing new hypotheses or revising old ones". Naturally no new theory can be excused from the ultimate test, namely that it should enable one to make valid predictions, no matter how attractive are its other attributes.

The synthetic element in Andrews' proposed reconstruction is at the same time both the most important and the most difficult to develop. Most industrial analysis is partial equilibrium in form, largely as a simple consequence of the fact that any industry is but a part of the whole economy. However, that an industry is /
is regarded as a legitimate object of study in its own right, should not blind us to what is happening outside its boundaries. Because of the manifold linkages between industries and the growth of conglomerates which cut across industrial divisions, nowhere is it more true than of the industry that, in John Donne's words: "No man is an Island entire of it self". More prosaically, the words of Andrews (1964, p.91) himself were: "We should thus hope for a framework for further work on individual industries of a truly analytical kind, in which relevant factors in the internal structure of the firm would be linked with the analysis of the external circumstances in the industry, where again, a structured theory would tie on to the analysis of the economy as a whole". So clearly the synthetic element is important. But how is it to be achieved? The welding together of theories of individual industries into an all-embracing industrial theory is of course a monumentally difficult, and unsolved, problem. The purpose of this chapter is to take one small step in that direction.

7-3. A New Model

In the following pages, we propose to present a new model of an industry embodying many of the features suggested by cost-plus theorists. Initially, this involves setting out some definitions in considerable detail, under the general headings of aggressive price leadership, cost-plus pricing, and technical progress. An /
An analysis of the model follows, progressing through three versions which attempt to become increasingly realistic.

Unfortunately, we have not managed to relax all the unrealistic assumptions conventionally adopted in the theory of the firm, and we assume that all firms have a single plant (though perhaps with a very high capacity) which produces one type of good. Our analysis is dynamic and is conducted in terms of time periods which should be regarded as "seasons of production". Within each season of production, price does not vary, nor is any firm free to alter its plant. Here we were guided by the words of Saxton (1942, p.127): "In fact the essence of price fixing is that once price is decided upon it shall not be subject to alteration for some given length of time". More recent evidence, summarised by Hague (1971, pp.257-258) suggests that this is still general practice. In his sample, Hague found that price changes were infrequent, and tended to occur at periodic (often annual) price reviews.

We assume that all firms have identical production seasons, and make price revisions at the beginning of each new season, having decided what plant they will use. Again we were guided by Saxton (1942, p.133): "If he is following an aggressive price policy we find that changes in price may take place at the beginning of each production period to reflect changes in cost".
It may be convenient, but it is not mandatory, to think of each production season as following strict calendar time (e.g. Jan 1968 - Jan 1969 - Jan 1970. etc.) An alternative interpretation, though perhaps a rather artificial one, is that the production season, is defined by the period which elapses before the leader adopts new plant. This may not occur at regular calendar intervals, but we may still label such consecutive seasons by the indices \( t, t + 1, t + 2, \ldots \) and so on. As price only changes when the leader's costs change and followers always accept the leader's price, the production season of every follower must be identical to that of the leader. However, there is no doubt that extreme irregularity, in calendar time terms, of the production periods could detract considerably from the accuracy of the analysis.

7-4. Aggressive Price Leadership

In this section we wish to combine the notion of an aggressive pricing policy with that of price leadership. The concept of an aggressive pricing policy is due to J.M. Cassels (1936, p.435) who defined it in the following words: "What is meant by the individual being aggressive in their price policies is that there is nothing in their psychological make-ups, or in their institutional arrangements which will prevent them cutting price when the competitive situation calls for price cutting". Early British studies by Andrews (1919), Hall and Hitch (1939) and Saxton (1942) indicated that price leadership was common in oligopolistic industries; and both the importance and prevalence of this type /
type of market structure have been confirmed by more recent studies such as those of Heath (1961), Maunder (1972) and Shaw (1974). Price leadership may exist in oligopolies with either a homogeneous or a differentiated product. In the case of a homogeneous product all firms usually charge an identical price; whereas in the case of a differentiated product, the price of any follower tends to differ from the leader's price by a constant proportion. In the latter case, the price of a follower will often slightly undercut that of the leader, this being the only way in which a follower can overcome the marketing advantages of the leader and winnow away a share of the market. At a later stage, we shall introduce a combination of these two situations, in which a uniform price prevails, but the mildest degree of differentiation exists. This establishes what we shall call a "priority pattern" of customers. Finally, we shall be concerned exclusively with price leadership of the dominant firm type.

Clive Saxton, one of the first contributors to the cost-plus versus marginalist pricing debate, devoted a considerable part of his classical study "The Economics of Price Determination" to the phenomenon of price leadership. In particular, he gave a fairly detailed treatment of what we will call here aggressive price leadership: this being, as the term suggests, the combination of an aggressive price policy with the role of price leadership. This sort of oligopolistic behaviour /
behaviour has been described by Saxton (1942, p.130) in the following terms: "In the newer industries some price leaders have adopted and followed what has become known as an aggressive price policy ... An aggressive price policy may be described as a systematically designed scheme of reducing the price of a product by so changing technical conditions and costs that it is profitable to force on output rapidly beyond certain dead points, so that a new and more favourable productive position is reached". The prime purpose of this chapter is to develop a model of aggressive price leadership in a systematic fashion, following the scheme proposed by Andrews.

This section should be regarded as performing a classificatory function. An aggressive price policy might be followed in a variety of industrial structures and indeed Cassels had a competitive, rather than oligopolistic market foremost in his mind when he coined this expression. For the purpose at hand, we have chosen to restrict the term to oligopolies. Within the class of aggressively pricing oligopolists we have isolated dominant firms, and within that sub-class itself we have located the object of our study: the dominant oligopolist following an aggressive price leadership policy.

7-5. Cost-Plus Pricing

The first assumption of our theory is that prices are determined on a "cost-plus" basis. By a cost-plus pricing policy we mean that all firms will compute their selling /
selling price by adding a constant profit margin to their average costs under "partial adaption" 3/. So many studies indicate that some sort of cost-plus pricing policy is followed by the majority of firms, that we consider this to be a vital element in any realistic theory of the firm. Also in line with empirical evidence, we will assume that average costs (and, by implication, marginal costs) are constant, under partial adaption.

There are several versions of this cost-plus pricing technique, variously described as "mark-up", "full-cost" and "target" pricing. Probably the most familiar statement of the technique is that given by Hall and Hitch: "prime (or 'direct') cost per unit is taken as the base, and a percentage addition is made to cover overheads (or 'oncost', or 'indirect' cost), and a further conventional addition (frequently 10 per cent) is made for profit". A contemporary commentator on the Hall and Hitch study, Silberston (1970, p.51), has observed that "if an industry is competitive in some sense firms will not each be able to base their price on their own full costs, regardless of the costs of others. The possibility of price leadership must then be considered, but this subject was not dealt with satisfactorily by Hall and Hitch". This view harks back to an earlier review article by Kahn (1952) of "Oxford Studies in the Price Mechanism". Hall /
Hall and Hitch hold that a firm bases its price on its own full costs and maintain a uniform mark-up. They observed that frequently a dominant firm would set its price according to the full cost principle, and that other weaker firms adopted this price. What Kahn disputes is that the view of Hall and Hitch that price is a mark-up on the full costs of the representative firm is consistent with a conventional mark-up. If costs vary between firms, then there cannot be a conventional mark-up on costs, because price is uniform.

Where we use the full cost principle in our model, we take Kahn's criticism as valid and admit that if followers have higher costs than the leader, then they must have smaller mark-ups. We will adopt the simplest possible version of the theory. Consider a particular firm (say, the i'th) which is producing in an industry at a particular moment in time (say, period t). Then

\[ \text{price} = \text{unit direct cost} + \text{unit gross profit} \]

or, in symbols:

\[ p(t) = c_i(t) + \pi_{i}(t) \]

\[ = \left[ 1 + k_i(t) \right] c_i(t) \quad \text{when } k_i(t) > 0 \]

(7.1)

where \( p(t) \) is the price set by the leader, \( \pi_{i}(t) \) is the gross profit margin, \( c_i(t) \) is unit direct cost (average variable, or prime, cost) and \( k_i(t) \) is the percentage gross profit mark-up, which is always positive and
and should generally be thought of as substantially less than one. All variables are dated, as we wish to undertake a dynamic analysis of price leadership. We have chosen to ignore the distinction between the indirect cost and profit mark-ups, and have lumped the two together into a gross profit mark-up. In our analysis, gross profit will be used to meet: indirect cost; any one, or all, of research and development expenditure, inventors' royalty fees, firms' production licence fees; with the residual being regarded as pure profit. If we gave each item an individual mark-up this would probably result in a loss, rather than a gain, in realism, and furthermore would render the analysis unnecessarily intricate.

It has long disturbed economists that a mark-up rule is inconsistent with the assumption of profit maximization which is conventionally adopted in much of the theory of the firm literature. It is easily shown [e.g. Scherer (1971, p.176)] that for the profit maximizing firm the ratio of the net profit margin to price is equal to the reciprocal of the elasticity of the demand curve. A mark-up which does not vary with shifts in demand cannot therefore permit profit maximization. Three comments on this criticism may be made. Firstly, long-run and short-run profit maximization should be distinguished. A firm might prefer an apparently non-optimal short-run pricing rule which does not require too much knowledge of market conditions (e.g. elasticity of demand), nor too frequent price revisions, such revisions /
revisions being costly and inconvenient to customers and salesmen alike. In the long-run we postulate (cf. Footnote 6) that firms seek increasing profits over time, even though they adopt a mark-up rule for any given plant. Secondly, it might not be unreasonable to assume, as Langholm (1969, p.20) suggests, that the demand curve is iso-elastic. As he points out, over the relevant range of price variation, this is not a particularly strong assumption. Thirdly, the role of information and uncertainty should not be ignored. Information on costs is more easily obtained than is information on demand conditions. As Cyert and March (1963, p.120) have emphasized, rules-of-thumb may often be adopted as a means of dealing with uncertainty. In a world with no uncertainty about the future, and full information in the present, such rules may non-optimal. But the practical requirements of making day-to-day decisions in an uncertain environment tend to foster such rules. For these reasons, the incorporation of a mark-up assumption in our model appears justifiable, if not entirely free from criticism. Another criticism of this approach is that the size of the mark-up will depend on the output of the firm. There are disagreements as to what this output should be. Possibilities are that it should be a certain percentage of engineering capacity that it should be expected average output and so on. We have listed the outlays which gross profit must meet, and once this sum is computed and the output level has been determined /
determined, then the size of the mark-up on prime costs can be determined. The first point to be made follows directly from Kahn's criticism of the mark-up approach, and is that from the viewpoint of a follower, price (and hence the mark-up) is given. In Section (7-6) we shall assume that the unit direct costs of the followers always exceed that of the leader. Hence it will always pay the followers to produce at capacity, given the ruling price. The second point is that what we define as "capacity" in Section (7-6) may be any one of the concepts suggested. Such a question is not perturbing from an analytical point of view. What is important is the relationship of the capacity of the leader to the capacity of each follower. It is to these matters that we now turn.

7-6. Technical Progress

Aggressive price leadership is a common feature of oligopolies in which technical progress is pervasive, if not continuous. Technical progress is customarily regarded as being either embodied or disembodied, the former being caused by improvements in kind of the capital employed in the production of a good or a range of goods, and the latter being the result of a rearrangement of the factors of production employed in an existing process in a fashion which reduces costs. From the viewpoint of this chapter, both types of technical progress might be important in achieving persistent cost reduction for the leader, but the relative importance of each is of little significance.
A more important distinction for our present purpose is between what Nordhaus (1969, p.5) calls *product inventions* and *process inventions*. A product invention involves the discovery of a new type of good altogether, whereas a process invention is a technique for producing an existing type of good at a lower unit cost than hitherto achieved. As we are concentrating our attention on the development through time of a particular industry producing a predefined product, this chapter will discuss only process inventions. In doing so, we do not limit ourselves as much as one might judge at first sight. If a product invention, X, is in fact a new type of capital good which is designed to produce a good, Y, for which an established market already exists, then X should be regarded as a process invention with respect to Y, even though it is a product invention itself. In further discussion we shall also make the traditional distinction between invention, which is the discovery of a new method of production, and innovation, which is the industrial application of such a new method.

As Saxton's words indicate, the aggressive price leader retains the initiative by achieving a higher rate of innovation than the followers. We will assume that the innovating leader always possesses the latest vintage /
vintage of capital equipment and the followers will be consigned to using plant which is at least one vintage behind the leader. Technical progress takes the form of cost reduction and capacity extention. If we adopt the convention of earlier chapters of calling the leader Firm 1, then the nature of the leader's technical progress may be described by the inequalities:

\[(7.2) \quad c_i(t) < c_i(t-\tau) \quad \text{and} \quad a_i(t) > a_i(t-\tau)\]

\[\tau = 1, 2, 3, \ldots.\]

where \(a_i\) refers, in general, to the capacity output of the \(i^{th}\) firm. This is similar to the assumption about technology made in the important work of Sylos-Labini (1969, p.38), but differs from that analysis in that we do not restrict ourselves to a fixed number of technologies in the industry, but allow for the development of new technologies.

A follower can obtain plant in three different ways: by purchasing secondhand plant either from a leader, or from/
from another follower selling off a more recent vintage of plant than he possesses; by undertaking himself the research necessary to develop the lower cost technique which the leader has already developed but kept secret; or by paying a licence fee to the leader in order to benefit from accomplished, but "classified", unreported research.

Whichever method is used to obtain plant, we will assume that it is obtained by the payment of a lump-sum, and that therefore marginal cost under partial adoption is unaffected. We may express the fact that followers always operate later vintages than the leader by the simple equations

\[ c_i(t) = c_i(t - \tau_i) \]
\[ a_i(t) = a_i(t - \tau_i) \]

(7.3)

for some positive integral \( \tau_i \). It is natural that if \( \tau_i > \tau_j \), we should say that "the \( i \)'th firm has an older vintage of plant than the \( j \)'th firm", meaning by this that the \( i \)'th firm has higher unit cost and smaller capacity than the \( j \)'th firm. An elementary consequence of (7.2) and (7.3) is that

\[ c_i(t) > c_j(t) \]
\[ a_i(t) < a_j(t) \]

(7.4)

\( (i = 2, 3, 4, \ldots) \)

We /
We shall say that this state of affairs characterises the leader as being both cost dominant and output dominant, harking back to the terminology of Chapter 5.

To explain the conditions under which the leader will innovate, we will introduce the notion of a valuable invention. By this we mean a process which is potentially more profitable than the newest vintage of plant actually operating. We assume that a leader will innovate (i.e. adopt an invention) only if the invention will increase its gross profit. This is an important point, because it means that the discovery of a cost-reducing invention is not sufficient in itself for that invention to be adopted - one must also consider the scale at which the new plant would operate. The neglect of this point is a major weakness of the discussion of cost-reducing innovation by Amdw (1962), Demsetz (1969) and Kamien et al. (1972) who all ignore the capacity problem, and tacitly assume that new plant can be replicated at will.

Assuming that the leader is always working at capacity, the gross profit before the introduction of a valuable invention is \( \Pi_1(t) \alpha_1(t) \), and after is \( \Pi_1(t+1) \alpha_1(t+1) \). We require that:

\[
\Pi_1(t) \alpha_1(t) < \Pi_1(t+1) \alpha_1(t+1)
\]

or, in simpler notation,

\[
(7.5) \quad \Pi_1(t) < \Pi_1(t+1)
\]
The rationale behind this condition is threefold. Firstly, if a firm is to have any incentive to change at all, there must be some compensation for the inconvenience of switching to new plant. Secondly, the process of innovation is subject to uncertainty, in the sense made familiar by Frank Knight, and compensation for this bearing of uncertainty is embodied in the condition under (7.5). Thirdly, because plant is expanding in capacity and presumably increasing in complexity over time, it is increasingly more expensive to purchase or build. If the leader is to maintain as smooth a rate of technical progress as is possible in an uncertain world, then the more modern the plant, the greater the amount he must be able to put aside per unit time towards the purchase (by a lump-sum) of new plant. Now this contribution towards what we might care to think of as an ever-enlarging sinking fund, must be drawn from gross profit: providing another justification for (7.5).

Finally, the possibility of cost-reducing technical progress is not unlimited. There may be diminishing returns to research and development; and in order for a tradeable object to exist at all, it must be made up of material of non-zero cost. By contrast, there is no foreseeable limit to the growth in capacity of new plants. It seems probable that over time the major factor in maintaining the inequality (7.5) will be growth in capacity. We assume,
assume, therefore, that unit cost, \( c(t) \) is bounded from below and away from zero, whereas capacity, \( a(t) \), is not bounded from above. An interesting consequence of this, when considered in conjunction with (7.3), is that the leader's unit cost curve under "total adaption" is L-shaped, in line with empirical evidence on long-run average costs. Strictly speaking, there is no continuous unit cost curve under total adaption, but rather a series of points at capacity through which a flattened L-shaped curve could be drawn. A similar relationship holds for those followers working at full capacity.

Let us summarize. The leader is the innovator, and is both cost dominant and output dominant as a consequence. Only valuable inventions, capable of providing higher profit than existing plant, will be adopted by the leader.

7-7. Trends in Profits and Concentration

In the ensuing pages, we have found it convenient to analyse the behaviour of profits and concentration over time under three headings:

1) Constant Price and Stable Demand
2) Constant Mark-Up and Stable Demand
3) Constant Mark-Up and Expanding Demand

The conclusions will differ, depending on what we assume about prices and demand. To take price first, evidence suggests that the leader would tend to keep his mark-up /
mark-up constant, and hence would allow price to fall with cost. This conclusion has been admirably expressed by Andrews (1949, p.272): "as it grows, a business will let any actual fall in costs influence its price through the operation of the normal pricing policy ....an expanding business will tend to fix its costing margin on the basis of average over-heads on the last account plus an allowance for normal profit. It will, therefore, tend to reduce its price. Granted a rapidly expanding market, it may even look ahead a little and take account of the lower costs which it expects at the larger outputs". Initially we avoid this complication, but under the second and third headings, we incorporate this realistic assumption in our analysis.

Concerning demand, the main point to put forward is that it may take the form of either derived demand or consumer demand. For this reason, we shall always talk of "customers" for the product of the industry, meaning either firms or consumers. This point has been emphasised by Andrews in Wilson et al. (1951, p.156). Whether or not one assumes demand is stable depends on the perspective one takes. In a partial equilibrium analysis, the assumption of a stable demand schedule is acceptable. For reasons which will become apparent, the only important alternative assumption in a general equilibrium analysis of industries is that demand is expanding over time.
The case in which both price and demand are constant is the least realistic, but the simplest to study. We use it as our starting point merely to introduce the model in action. Modifying this simple case by allowing price to fall but maintaining demand constant, brings us into the familiar realm of partial equilibrium analysis, where we ignore effects beyond the industry. Finally, the third and most realistic case analyses the interplay between industries under the assumption of general increasing returns.

7-8. The Constant Price and Stable Demand Case

Suppose that both market demand and the price set by the leader were stable over time. This implies that the leader has an increasing mark-up over time, and effectively defines the oldest vintage of plant which can operate profitably. We assume that if the price set by the leader is below any follower's unit direct cost, then that follower will go out of business. Any such price for which \( p < c_j(t) \) holds has been called an elimination price by Sylos-Labini (1969, p.40). He points out that even prices above \( c_j(t) \) may be entry-preventing in the long run if they permit only a low level of profit. This is true, but it introduces an unnecessary complication which even Sylos-Labini elected to ignore in much of his analysis. The concept of an elimination price is less important in this section, where price is constant, than in the next section, where price /
price is falling.

No follower will go out of business if it is selling at above unit direct cost and operating at capacity. However, as technical progress proceeds, the total capacity of industry becomes greater, even though customers' demand is unaltered at the constant market price. This implies that over time an increasing number of firms will be forced to work below capacity. 10/

At this point we will simplify the analysis by assuming a mild degree of product differentiation exists; sufficient, at least, to establish what we shall call a priority pattern of customers. 11/ Customers follow a priority pattern in that, ceteris paribus, they will purchase the good produced by the most recent vintage of plant. As a uniform price prevails other things are indeed equal, and we may think of customers having the mildest preference for the product of a recent rather than a later vintage. But if the former is not available, they will be content to purchase the latter, rather than having to queue, register on a waiting list, or accept a delay in delivery. There may be several reasons for this priority pattern. For example, the possession of a good from the new vintages may confer a certain prestige upon the customer, even though the product is homogeneous in a physical sense. It implies that customers tend to buy from firms with the greatest turnover /
turnover, which is rather plausible. We shall shortly discover that the newer the vintage of plant, the greater the profit. Hence firms with newer vintages are capable of undertaking greater advertising expenditure than firms with older vintages, and in this way too, a priority pattern could be established.

Thus the leader will always operate at capacity because he has a "first purchase" advantage; whereas firms with older vintages already working with relatively low profit margins may have their miseries compounded by having to operate below capacity. In terms of the strict logic of the model, only firms with the oldest vintage of plant which can be profitably operated at the prevailing price will have to work below capacity, but in practice it is unlikely that customers will follow a priority pattern with complete consistency, and there will be merely a tendency for excess capacity to be greater for firms with older vintages of plant.

Because price is constant, and unit cost is falling over time, unit profit for each firm, and the leader in particular, is increasing over time. This fact, when considered in conjunction with the extension of capacity caused by technical progress, means that the gross profit of the leader is increasing over time \(^{12/}\). Thus the condition for an invention to be valuable (cf. (7.5) above), is automatically satisfied. At least some of the followers will work at capacity. Because followers employ plant which is /
is at least one vintage behind the leader, they will all have lower unit profits and smaller capacity outputs - indeed some may even operate below capacity. It follows that the leader's gross profit exceeds that of any follower. In other words, it is automatically true that the leader is cost, output and profit dominant: this firm is in a position of total hegemony.

The leader's position with respect to market share will also be dominant in the long run. As technical progress proceeds and firms expand capacity the firms with the oldest vintages of plant will find themselves losing their market shares. They may try to counteract this tendency by buying or developing more modern plant, but in fact this will only exacerbate the situation. There is a fixed quantum of demand at the prevailing price but an increasing aggregate capacity caused by technical progress. Only the leader cannot lose. Eventually he will become the sole producer after a period of increasing, and probably accelerating, concentration.

7-9. The Constant Mark-Up and Stable Demand Case

As a previously cited quotation by Andrews suggests, it is more realistic to assume that the percentage profit mark-up remains constant over time, rather than price itself. This is in agreement with Saxton (1942, p.134) who argues: "If the producer is following an aggressive price policy as many of the newer industries do, the price may be reduced every season of production for a series /
series of years to pass on the economies of large scale operation in reduced prices and to widen the market. If the leader maintains a constant mark-up and demand is stable over time, there are two immediate consequences. First, we observe that if unit direct cost is falling and the mark-up is constant, then price per unit and unit gross profit will fall. Second, only the leader is in a position to maintain a constant mark-up because it is not to be expected in general that the rate of diffusion of cost-reducing innovations to followers will exactly offset the rate of decrease in the price set by the leader. In fact, because the unit direct costs of the followers are always above the leader's, the leader must have the greatest mark-up and the greatest gross profit. Unlike in the previous case, it is no longer true that any cost-reducing invention is valuable. We now require that the cost-reducing effect must be outstripped by the capacity extension effect. This means that the rate of innovation will be slower than in the "constant price, stable demand" case. We will assume that although inequality (7.5) does not automatically hold, the leader will be sufficiently prudent to ensure that any innovation it makes will meet this criterion. If this is true, then by definition, the leader's gross profit is increasing over time. In passing, it is of interest to note that firms may be subject to a falling rate of profit even if they observe this.
this valuable invention criterion. Insofar as capacity is a measure of capital employed, the value of capital is \((\text{cost per unit of capacity}) \times (\text{capacity})\). Then the rate of profit is:

\[
\frac{(\text{unit gross profit}) \times (\text{capacity})}{(\text{cost per unit of capacity}) \times (\text{capacity})}
\]

or, more simply, the ratio of unit gross profit to cost per unit of capacity.

We have seen that unit gross profit is falling over time (Footnote 14), so the behaviour of the rate of profit depends on cost per unit of capacity \(\xi(t)\). We have made no particular assumption about this, and the evidence is not clear-cut. On the one hand we have the engineers "0.6 rule", described by writers like Moore (1959) which says that the increase in capital cost of a plant is given by the increase in capacity raised to the 0.6 power. In other words, cost per unit of capacity falls as capacity increases. However this rule is based on production processes in which the amount of material required to enclose a given volume (e.g. in boilers, gas tanks, compressors) is the basic cost to capacity ratio. As Moore (1959, p.127) puts it: "area varies as the volume to the 2/3 power, or in other language, cost varies as capacity to the 2/3 power". On the other hand, Moore (1959, p.128) admits that "complicated industrial machinery does not necessarily exhibit /
exhibit the same relationship between area (cost) and volume (capacity) as do simple structures like tanks and columns. If a process invention involved a fair degree of automation and/or computerization of the production process, then it is probable that the "0.6 rule" would be violated, and cost per unit of capacity might well increase. In such cases, the rate of profit must certainly fall. The rate of profit will only rise if something akin to the "0.6 rule" is working sufficiently strongly to offset falling unit gross profits. Recently, economists of the New Left have made great play out of the falling rate of profit in the company sector of U.K. manufacturing industry. For example, Glyn and Sutcliffe (1971, p.3) have concluded that: "for British capitalism it looks as if this time the wolf is really at the door". Our analysis shows that a falling rate of profit may not be symptomatic of any crisis at all, but may be merely a mechanical consequence of pursuing a constant mark-up policy in an environment of persistent innovation. One could argue that this is not undesirable. Firms increase their gross profits as they innovate, and consumers enjoy the benefits of this innovation through the falling price of the good produced by the industry.

Another point of considerable interest in this "constant mark-up, stable demand" case is that followers are always at a competitive disadvantage in the innovatory process. The argument is as follows. Each follower /
follower is using plant of a type which the leader used several periods previously. But in those past periods, price was higher. Hence any follower is operating plant which had earlier yielded a higher gross profit to the leader. If a firm's capability for introducing new plant is an increasing function of its gross profit, then a follower's position is clearly weaker than the leader's. For, if the level of gross profit which the leader has earned in each period is always necessary to achieve the next innovation, then followers may simply be debarred from undertaking their own research and development. However, it is unlikely that this in itself will prevent them from adopting new plant. Second-hand plant should be available, and even new plant of an older vintage will be less desirable than when first introduced and hence will command a lower price.

Unlike the situation discussed under the "constant price, stable demand" heading, firms will be continually exiting from the industry as the leader's price falls below the unit direct costs of followers with the oldest vintages of plant. Previously, firms only shut down when they were forced to work well below capacity. This effect will still exist, but will now be supplemented by one of greater importance, namely, a persistently declining "elimination price" whittling away the profits of marginal firms. As before, this will mean that there is a tendency to increasing concentration over time, and the /
the priority pattern of customers implies that eventually the leader will become the sole producer. However, in contrast to the previous case, price will be lower and output greater, in this, the more realistic case, as a consequence of the leader's falling profit margin.

Several of the characteristics of the "constant price, stable demand" case carry over to the "constant mark-up, stable demand" case: cost, output and profit dominance of the leader; increasing concentration over time, terminating with monopolisation by the leader; and the continuous exiting of firms as technical progress proceeds. There are some important differences however, and two in particular are noteworthy. Firstly, profit margins and price are falling over time, and as a result there may be a tendency for the leader to experience a falling rate of profit. Secondly, the leader, by adopting a constant mark-up policy, always has a competitive advantage over the followers in the innovatory process. Indeed, we may regard this as the rationale for adopting such a policy.

7-10. The Constant Mark-Up and Expanding Demand Case

So far the analysis has proceeded in dynamic, but nevertheless, partial equilibrium terms. We have yet to broach the problem of developing the synthetic element in Andrews' proposed reconstruction, namely the integration of our particular theory of the industry into a theory of all industries.
We take a recent article by Kaldor (1972) as our starting point. Building on a famous paper by Allyn Young (1928), Kaldor argues that theorists should re-instate increasing returns as an economic "axiom", because it is more realistic than the assumption of constant returns which is at the heart of general equilibrium analysis in the Walrasian tradition. As we shall see, if we follow Kaldor and Young, the notion of equilibrium in an industry becomes meaningless. If increasing returns prevail in all industries, then in the words of Young (1928, p.528): "no analysis of the forces making for economic equilibrium, forces which you might say are tangential at any moment of time, will serve to illumine this field, for movements away from equilibrium, departures from previous trends, are characteristic of it". In the model of aggressive price leadership which we have been analysing, unit costs under total adaption have been falling over time, producing, in effect, increasing returns in the long run. Whilst not assuming that other industries also have a structure of aggressive price leadership, we do assume, at least, that all industries are subject to increasing returns for some reason or other. With the addition of this fairly realistic assumption, we are able to lift the partial equilibrium argument of section 7-9 and put it into what might be described as a "general disequilibrium" setting.
Young's basic argument is a variant of Say's Law. Before considering distracting complications, let us discuss it at a simple level. Imagine a barter economy, in which a firm in an industry has adopted a labour saving innovation, as a result of which greater output can now be produced with the same number of men. This is the simplest example of a process invention resulting from disembodied technical progress. Because there is a greater supply of this good, say A, an increased demand has been generated, in effect, for the goods, B, C, D, E etc (produced in other industries) for which A can be exchanged. Supposing for the sake of argument that the producers of goods B, C, and D try to take advantage of this increased demand by enlarging their supply in a labour saving fashion, they too will generate increased demand: mutually, for each others goods; and globally, for all other goods, A, E, F, G etc. for which B, C, and D are exchangeable. Provided increasing returns prevail generally, this process will continue, and as Young (1928, p.533) put it: "change becomes progressive and propagates itself in a cumulative way".

Now to some complications. A condition which must be satisfied in order that the process may continue, is that an increase in the supply of any good should generate an increase in the demand for all other goods. Young thought this condition would be satisfied if the demand for each good was relatively elastic, the reasoning /
reasoning being that if a given percentage fall in supply price engendered a greater percentage increase in sales, the increased revenue, when distributed to factors, would generate increased demand for other goods. The fallacy in this argument, which Kaldor pointed out, is that the increased revenue is only gained by diverting expenditure from other goods, that is, by decreasing demand for them. Kaldor offers an alternative, more complex, argument which is Keynesian in spirit. Briefly, it says that in disequilibrium situations (which he considers characteristic of decentralized economies) excess supply and excess demand are absorbed by producers and merchants. In rather imperfectly competitive markets, an increased demand causes producers to deplete stocks, and they then undertake induced investment in the form of increased value of goods in process, and, if the excess demand continues, in the purchase of new plant to expand capacity. In relatively competitive markets, excess supply is absorbed by merchants, and provided they have sufficient confidence in future prices to increase not just the volume, but the value of their stocks, they too will have undertaken induced investment. The addition to income of these induced investments generates an increase in effective demand which is the motor driving this process of cumulative expansion described by Young.

In section (7-9) the dynamic process of innovation finally stopped when the leader was supplying the whole market /
market at a price he had set. At no stage did we consider how firms in the industry were interacting with firms in other industries. As firms in our price leadership model are subject to increasing returns, and we are following Kaldor in assuming increasing returns in other industries, the expansionary process described by Young will be in operation.

In this general disequilibrium framework, some of the conclusions of the "constant mark-up, stable demand" case remain valid. The leader will still be totally dominant with respect to cost, output and gross profit. Price and the profit margin will fall over time. The leader will be subject to a falling rate of profit. Followers will be at a competitive disadvantage in the innovatory process and will shut down if the leader's price falls below their unit direct costs. The major difference is that, prima facie, concentration does not necessarily increase over time. Previously, capacity output was unbounded but demand was stable, hence the capacity of the leader would eventually satisfy total demand at some price. By contrast, demand is now continually expanding, and even though firms with recent vintages of plant must have relatively high market shares, compared to other firms, these market shares could be small compared to the aggregate output of the industry.

One qualification should be made to this argument. We have not yet considered what happens when the lower bound /
bound on cost is approached. In order for an invention to be valuable when cost is very close to this bound, new plant must have enormously enlarged capacity. In this situation, which no advanced economy seems to have approached as yet, it may be that the tendency to concentration will re-assert itself as technical leaders, being unable to achieve appreciable cost reduction, concentrate instead on substantial capacity extension.

7-11. Conclusion

Probably the most important conclusions of this chapter are contained under the "constant mark-up, stable demand" and "constant mark-up expanding demand" headings, and results under the former are only important insofar as they carry through to the latter. The principal results were:

1) The leader may be subject to a falling rate of profit, even if gross profits are expanding over time.

2) Followers are at a competitive disadvantage compared to the leader in the innovatory process.

3) There is no necessary tendency to concentration over time, unless the lower bound on cost is being approached.

4) There is no necessary tendency of any firm to an equilibrium position.

5) /
5) There is in principle no limit to the growth of aggregate output of the industry.

Insofar as we followed Andrew's scheme to achieve these results, we consider they vindicate his belief that "the ending of marginalist static equilibrium at the level of the firm as the seat of a sovereign general theory would free us to use our economists' tools on change and chance in the real world in a way that our supposed 'legacy' has denied us".
Footnotes to Chapter 7

1/ See Andrews (1964, pp. 90-93).

2/ The most celebrated critic is of course Friedman, (1953, pp. 1-43).

3/ A term due to Wiles (1961, p.8). Behaviour under partial adaption is roughly equivalent to behaviour "in the short run".


5/ So far as we have been able to determine, this type of technical progress was first described by von Mangoldt (1863). See the translation by R. Henderson (1962, p.36) where we find: "Economic progress and the advance of civilization tend to cheapen the supply of good through better production and to extend it through increased knowledge and mastery over nature".

6/ This assumption may be subjected to some criticism, as it ignores the rate of return on what we call a "valuable invention". As we shall see, this may imply a falling rate of profit over time. The assumption is justified by its generic similarity to growth maximizing and profit satisficing behaviour. A quotation from Baumol (1962, p.1085) makes /


makes this clear: "From the point of view of a long-run growth (or sales) maximizer, profit no longer acts as a constraint. Rather, it is an instrumental variable - a means whereby management works towards its goals. Specifically, profits are a means for obtaining capital to finance expansion plans". Because we have assumed that gross profit in period \((t + 1)\) merely exceeds gross profit in period \(t\), ignoring magnitudes, we are also implying a kind of satisficing behaviour.

7/ We mention this because it would be quite possible for the leader to accumulate sufficient profits to undertake a valuable invention of any cost, no matter how high, given sufficient time. We made this point in section (7-4), saying "extreme irregularity, in calendar time terms, of the production periods could detract considerably from the accuracy of the analysis".

8/ A term due to Wiles (1961, p.8). Behaviour under total adaption is roughly equivalent to behaviour "in the long run".

9/ Now \(p(t) = p\), a constant for all \(t\). From (7.1) \(p = [1 + k_1(t)] c_1(t)\). From (7-2), \(c_1(t) < c_1(t - \tau)\), whence \(k_1(t - \tau) < k_1(t)\).

10/ It has been pointed out that our analysis of vintages of plants bears some relationship to the work of Salter /
Salter (1966, pp. 58-60). However there are important differences. In Salter's analysis technical change is such that the best-practice technique in any time period has a higher labour productivity than that of its predecessors. Market price is in fact determined by the marginal technique - that is by the technique which is only just profitable. This is basically a competitive type of analysis. It is also assumed by Salter (1966, p. 51) that all plants work at full (or "normal") capacity. In our analysis, prices are not competitively determined in the narrow sense of the word. Price is set by the aggressive leader and in this sense is an administered price rather than one determined by the forces of supply and demand. It is also not the case that firms must operate at full capacity in our model, there being a tendency for firms with older vintages of plant to operate below capacity.

The original stimulus to adopt this assumption was a study by Tucker (1964) on the development of brand loyalty. The first two conclusions of his study were (p. 35): "(1) Some consumers will become brand loyal even when there is no discriminable difference between brands other than the brand itself. (2) The brand loyalty established under such conditions is not trivial, although it may be /
be based on what are apparently trivial and superficial differences". On the other hand, some consumers were conspicuously non-loyal, and exploratory in their purchases. We also wish to extend these ideas to purchasers of intermediate goods. Provided the exploratory behaviour of non-loyal customers is fairly evenly spread over purchases from all vintages, and the brand-loyal customers tend to buy goods produced by the most recent vintages, the priority pattern will tend to be established.

From (7.1), $p - c_1(t) = \Pi_i(t)$, and from (7.2) $c_1(t) < c_1(t - \gamma)$, whence $\Pi_i(t) > \Pi_i(t - \gamma)$.

From (7.2) $a_1(t) > a_1(t - \gamma)$ whence $\Pi_i(t) > \Pi_i(t - \gamma)$. Thus condition (7.5) is satisfied.

From (7.1), $p = c_1(t) + \Pi_i(t) = c_1(t) + \Pi_i(t)$. But from (7.4), $c_1(t) > c_1(t)$ whence $\Pi_i(t) > \Pi_i(t)$.

From (7.4), $a_1(t) > a_2(t)$, whence $\Pi_i(t) < \Pi_i(t)$.

Leader's mark-up $k_1(t) = k$, a constant for all $t$ under the assumption of section (7-9). From (7.1) $p(t) = (1 + k)c_1(t)$. From (7.2), $c_1(t) < c_1(t - \gamma)$ whence $p(t) < p(t - \gamma)$ i.e. price is falling over time. Similarly, as $\Pi_i(t) = kc_1(t)$, we have $\Pi_i(t) < \Pi_i(t - \gamma)$, i.e. unit gross profit is falling over time.
For the leader: \( p(t) = (1 + k)c_1(t) \), and the same price \( p(t) = [1 + k_1(t)]c_1(t) \) for the \( i \)'th follower. Subtracting \( (1 + k)c_1(t) = [1 + k_1(t)] \).

\( c_1(t) \) But from \((7.4)\), \( c_1(t) = c_1(t) \), whence \( k_1(t) < k \). By a similar argument to Footnote 13, it follows that
\[
\pi_i(t) < \pi_1(t) \quad \text{and} \quad \overline{\pi}_i(t) < \overline{\pi}_1(t) \).
\]

As \( a_1(t) > a_1(t - \gamma) \), but \( \pi_1(t) < \pi_1(t - \gamma) \).
Therefore the relative size of \( \overline{\pi}_1(t) \) and \( \overline{\pi}_1(t - \gamma) \) is unknown.

In the notation of Moore (1959), \( C_2 = C_1 \left( \frac{X_2}{X_1} \right)^{0.6} \), where \( C_1 \) and \( C_2 \) are the costs of two pieces of machinery with capacities of \( X_1 \) and \( X_2 \). This implies that
\[
\frac{C_2}{X_2^{0.6}} = \frac{C_1}{X_1^{0.6}} \quad \text{i.e.} \quad \frac{C_2}{X_2^{0.6}} \div \frac{C_1}{X_1^{0.6}} = 1
\]

Now \( \frac{C_2}{X_2} \div \frac{C_1}{X_1} = \left( \frac{C_2}{X_2^{0.6}} \div \frac{C_1}{X_1^{0.6}} \right) \left( \frac{X_1}{X_2} \right)^{0.4} \) The first term in brackets is unity and the second term is less than unity for \( X_2 > X_1 \). Therefore
\[
\frac{C_2}{X_2} < \frac{C_1}{X_1} \quad \text{for} \quad X_2 > X_1 \].
That is, cost per unit of capacity declines as capacity increases.

This relationship was pointed out to me by Mr. I.M.S. White, Department of Statistics, University of
of Edinburgh.

18/ The costs of capital before and after innovation are $\kappa(t)\alpha_1(t)$ and $\kappa(t+1)\alpha_1(t+1)$. The general expression for the rate of profit is:

$$\rho(t) = \frac{\pi_1(t) \alpha_1(t)}{\kappa(t) \alpha_1(t)}$$

or $\pi_1(t) / \kappa(t)$. If $\kappa(t+1) > \kappa(t)$ and from Footnote 14 $\pi_1(t+1) < \pi_1(t)$, then $\rho(t+1) < \rho(t)$.

19/ Their measure, however, is different though they are apt to argue as though their conclusions generalize to all measures of the rate of profit. Their measure of the pre-tax rate of profit was the ratio of profits earned by shareholders before tax to the book value of shareholders' assets.

20/ It has been pointed out by several writers, most notably by Pearce and Gabor (1952) that profit maximizing behaviour will not in general maximize the rate of return on capital. In the case of the marginal analysis in competitive markets, firms will tend to increase output beyond the point at which the rate of return would be maximized. To an extent, therefore, it is not surprising that in the present analysis, where the /
the firm seeks merely to **improve** profits in successive periods, the rate of profit might actually fall.

21/ For the leader \( x_i \) periods ago, the basic relationships are

\[
p(t - x_i) = c_i(t - x_i) + r c_i(t - x_i)
\]

\[
= c_i(t - x_i) + \Pi_i(t - x_i)
\]

and

\[
\Pi_i(t - x_i) = a_i(t - x_i)\Pi_i(t - x_i)
\]

i'th follower, currently working with the same type of plant.

\[
P(t) = c_i(t) + \Pi_i(t)
\]

\[
= [1 + k_i(t)] c_i(t - x_i) \text{ from (7.3)}
\]

But \( k_i(t) < k \), from Footnote 15, from which

\[ k_i(t) c_i(t - x_i) < k c_i(t - x_i) \text{ or } \Pi_i(t) < \Pi_i(t - x_i) \]

As \( a_i(t) = a_i(t - x_i) \text{ from (7.3)}, \) it follows that

\[ \Pi_i(t) < \Pi_i(t - x_i) \]

22/ See Andrews (1964, p.92)
Chapter 8

Summary and Conclusions

In Chapter 1, our declared intention was to codify and extend the existing theoretical work on price leadership. The purpose of this, the final chapter of the thesis, is to examine the extent to which this intention has been fulfilled, and to make tentative suggestions on what direction further work in this field might take.

Chapter 2 provided survey material on the theoretical literature. The survey examined models of price leadership from the earliest contributions by Forchheimer (1906) and Nichol (1930) to the recent work of theorists such as Gaskins (1971) and Hadar (1971).

It emerged that, although formal statements of price leadership models have existed for some time, these models have not yet been developed to any great extent. For example, their comparative statics properties have not been examined. In discussing the writings of various theorists, an attempt was made to use a uniform notation. In certain cases [e.g. Forchheimer (1908), Zeuthen (1930), Boulding (1948)] this involved re-interpreting numerical examples or geometry in terms of algebra and calculus.

Chapters 3 to 6 examined price leadership analytically, from a static, marginalist viewpoint. As regards the nature of the solution three basic possibilities emerged.
The first (as in the linear case) was that a solution existed and was unique. The second (as in the increasing cost case) was that a solution existed, but was not unique. The third, was that no solution existed. This last possibility manifested itself in several ways. For example, in the constant marginal cost case, the model broke down entirely, producing various paradoxes. The leader and follower could not co-exist. A firm which could have been the long-run sole survivor if both firms followed long-run strategies might nevertheless be vanquished if both firms followed short-run strategies. The possibility of no solution arose again in the fairly general model of Chapter 6. There, the difficulty was that the appreciable scale of firms behaving as followers might induce a discontinuity in the market supply with the consequence that the usual marginal conditions for profit maximization could not obtain.

In the linear case, examined in Chapter 4, comparative statics results were derived. These were summarised in section (4-3), and will only be briefly recapitulated. It was shown that the effect of entry is to reduce the leader's output, lower the market price, and reduce the outputs of existing followers. A subsidy on the follower decreases price, decreases the leader's output and increases the follower's output. A sales tax on the leader increases price, decreases the leader's output and increases the follower's output. A royalty payment from /
from the follower to the leader increases price, decreases the follower's output and leaves the leader's output unchanged. The comparative statics sign matrix for the whole model was given. It indicated, for example, that an overall expansion in demand increases price, the follower's output and the leader's output. In Chapter 5 we derived the Jacobian determinant for the increasing cost case. A priori restrictions were used to sign this determinant, and the comparative statics properties of a specific sales tax on the leader were derived and shown to agree with those of Chapter 4. An interesting negative conclusion emerged in section (5-5). It seems that it is not necessarily true that a leader with lowest costs and the greatest market share must have the greater profit. This has some bearing on the conclusion reached by Wolfe (1954, p. 182) to the effect that the leader will be the least cost firm. If costs are increasing, this may be an unstable situation because the leader might earn less profit than it could in a more passive role. In Wolfe's case, there is no problem, as constant average and marginal costs are assumed. We followed this assumption in Chapter 7.

In the final analytical chapter (Chapter 7) we followed a scheme proposed by Andrews (1964) for developing an alternative theory of the firm. Our purpose was to explore just one area in which Andrews' methodology might be applied. In this case we used it to /
to develop a model of aggressive price leadership. This model had three basic elements. Firstly, the analysis was conducted in terms of a sequence of time periods, each of which could be identified with a production period. Secondly, pricing was assumed to be based on a cost-plus formula. Thirdly, technical progress was led by an aggressive price leader. We were able to show that such a model had no necessary tendency to equilibrium, though this did not imply its behaviour could not be analysed. Provided other industries experienced increasing returns, the market continuously expanded, with no necessary tendency to increasing concentration. In this process the price leader always had a competitive advantage over the followers, though it might have been subject to a falling rate of profit.

It is probably the material of Chapter 7 which is capable of greatest extension. The model has a number of weaknesses. Firstly, there is no explanation of the magnitude of the mark-up. This is a problem which has vexed the cost-plus school for many years. Very recently, Bichner (1973, 1974, 1975) has claimed to have a solution. The claim of Bichner (1973, p. 1189) is that "the size of the 'plus' factor depends on the demand for and supply of additional investment funds by the firm or group of firms with the price-setting power within the industry". However, the analysis has evoked extremely strong criticisms from Hazledine (1974), Robinson (1974) and Rubin et al. (1975). It seems, for example, that the analysis does not /
not allow for mark-up reductions, but only mark-up increases. A crucial assumption of Bichner's model is that demand for the price setting firm or group is inelastic in the short run, but elastic in the long run. This too has been subject to criticism, on empirical grounds. It seems probable that the dispute over Bichner's analysis will continue for some time, but there is some hope that what will eventually emerge will be a model which no longer requires the mark-up to be determined by exogenous factors. A second criticism of the model of Chapter 7 is its relatively crude dynamics, and neglect of uncertainty (or lack of information).

Hadar and Hillinger (1969) have developed a model which suggests what might be done to remedy this deficiency. Their model reduces to a single-equation adjustment process for each firm. It is assumed that firms follow a cost-plus formula and adjust the level of mark-up over time. It is shown that the price adjustment process is stable, and that the equilibrium price vector is unique, even though firms are assumed to have no more information about their demand schedules than that they are negatively sloped. Another study which is suggestive of what course developments might take is that of Cyert et al. (1967). They treat uncertainty in probabilistic terms, rather than in terms of lack of information. On the basis of two simple behavioural decision rules on price adjustment, it is shown how an optimum price will be achieved over time.

Clearly /
Clearly the incorporation of such devices into a more general analysis of the firm's behaviour over time is no elementary task: the mathematical refinements required to analyse relatively simple adjustment processes suggest this. However, progress will probably proceed in a piecemeal fashion, as theorists slowly discover how to solve the analytical problems of incorporating these types of behaviour into more general models.

Eventually the problem of empirical application of theory must be broached. The comparative statics results provide qualitative predictions which may in principle be tested. However, there are practical problems in performing such tests at the present moment. Firstly, as Archibald (1961) has pointed out in a similar context, with the techniques of observation available to us at present, the theory may not be testable. Most data are of the case-study type, which do not readily lend themselves to the testing of propositions like "a subsidy on the followers will increase their output". Secondly, despite the vigorous activity of econometricians since the Second World War, we are still not in a situation in which we have complete confidence in the statistical tools for testing predictions. Such procedures themselves are based on assumptions which are often only very crudely approximated to in the data. However, progress continues to be made, and it may be that the problems of the econometrician are closely linked with the inadequacy of data. There is some hope that this latter deficiency will /
will slowly be remedied, as government bodies and research institutes increasingly involve themselves in the collection of micro-data from business enterprises.
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