Electrical Power Conversion
In Arrays of Linear Direct Drive
Wave Energy Converters (WECs)

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Abstract

Wave energy has drawn great interest as a renewable energy technology since it is not been fully explored and is still in its infancy. Over the last few decades several very different types of wave devices have been invented to capture this energy for the generation of clean electricity. Among the wave device technologies, direct drive wave energy converters (WEC), which use linear generators in their power take off (PTO) system, offer a simpler approach for power conversion. As the electricity is produced directly from the reciprocating motion of its translator, this type of power take off can be economically attractive compared to the conventional rotary generator which uses a hydraulic mechanical interface that normally has a high maintenance requirement. Most of the studies that have been carried out by wave energy researchers has focused on designing well-proven and reliable wave devices that can operate in the harsh environment of the open sea. Thus any devices that have low maintenance requirements and long life expectancy would be attractive to investors. The desirable features of a linear generator make it a suitable candidate for use with the direct drive WEC.

Direct drive linear generators in wave energy converters provide a simple energy extraction concept by reducing the mechanical intervention in the drive train. Instead of generating constant output power, however, the induced voltage and current generated varies in amplitude and frequency. Thus in order to generate more reliable and stable output power, it is highly recommended that they are implemented in arrays. The hypothesis made in this thesis is that the power quality generated by the linear generator can be improved by connecting several of them in an array, with either a DC link or AC link interconnection.

This thesis is concerned about the control strategy of collecting the power from arrays of direct drive WECs using linear generators. Although the study is made for the array, control is applied to each WEC to ensure the optimum power is extracted from the waves. A key requirement to ensure that the power from the WECs is effectively collected is by maintaining the DC link voltage at a constant level. This can be done by controlling the power converter at the grid side. Four different control strategies are proposed for the WEC side control. The results presented validate the performance of the control schemes. The study concludes with a proposal for the configuration and the integration of control system for a multi-array wave park.
Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text. This work has not been submitted for any other degree or professional qualification.

A.Z. Annuar
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Abbreviations

AC  Alternating Current
ACTM Air-Cored Tubular Machine
AWS Archimedes Wave Swing
AOVC Adaptive Optimal Voltage Control
CB  Circuit Breaker
CSC Current Source Converter
DC  Direct Current
Dis  Disconnector/Isolator
DRLV Deviation Reference Level Voltage
DOF Degree of Freedom
EMF Electromagnetic Field
GCRCT Group Cluster Radial Common Transmission
GCRST Group Cluster Radial Separate Transmission
GC3TCT Group Cluster Three-Winding Transformer Common Transmission
GC3TST Group Cluster Three-Winding Transformer Separate Transmission
HVAC High Voltage Alternating Current
HVDC High Voltage Direct Current
IGBT Insulated Gate Bipolar Transistor
LG  Linear Generator
KVL  Kirchhoff Voltage Law
OWC  Oscillating Water Column
PCP  Power Collecting Point
PI  Proportional and Integrator Controller Gains
PLL  Phase Locked Loop
PTO  Power Take Off
PWM  Pulse Width Modulation
SARCT Single Array Radial Common Transmission
SDC Single Direct Current Transmission line
WEC  Wave Energy Converter
VSC  Voltage Source Converter
Symbols

\( a \) Acceleration \((m/s^2)\)

\( A_{ij} (\infty), A_{33} (\infty) \) Added mass (kg)

\( b \) Mechanical damping (Ns/m)

\( B_{PTO} \) Damping force coefficient attributed to the power take-off system (Ns/m)

\( B \) Magnetic Field (T)

\( B_{ij}, B_{33} \) Added damping coefficient (Ns/m)

\( \hat{B}_g \) Air gap flux density (T)

\( C_{DC} \) DC link capacitor (F)

\( c_g \) Wave group speed (m/s)

\( K_{\phi}, K_{33} \) Hydrostatic spring stiffness coefficient (N/m)

\( ch \) Coil height (m)

\( cw \) Coil width (m)

\( \delta \) Phase angle with respect to the mains

\( e(t) \) EMF source (V)

\( E \) Energy (J)

\( F \) Force on a current-carrying conductor (N)

\( F_b \) Force attributed to the mechanical damping (N)

\( F_{\text{coil}} \) Force per coil (N)

\( F_k \) Force attributed to the spring stiffness (N)

\( F_e (t) \) Total Force source (N)

\( F_m \) Force acting on a body, \( m \) (N)

\( F_{PTO} \) Power take-off force (N)

\( \hat{F}_{r,i} \) Total hydrodynamic force acting on oscillating body (N)

\( \hat{F}_{e,i} \) Excitation force due to heaving motion (N)

\( \hat{F}_{r,i} \) Radiation force due to heaving motion (N)

\( F_{gen} \) Generator force (N)

\( F_g \) Peak generator force constant
Excitation force in frequency domain (N)

Excitation force in time domain (N)

Excitation force amplitude (non-dimensional)

Excitation force degrees (non-dimensional)

Fundamental frequency (Hz)

Controller gain

Air gap (m)

Acceleration due to gravity (m/s^2)

Wave height (m)

Water depth (m)

Current (A)

Phase current (A)

Current in phase A, phase B and phase C

DC fault current -positive terminal (A)

DC fault current -negative terminal (A)

Phase source current (A)

Phase current for array 1 (A)

DC link current for array 1 (A)

DC link current for array 2 (A)

Load current (A)

Inductor current (A)

Amplitude of current vector (A)

Reference phase current amplitude (A)

Reference phase current amplitude of WEC (A)

Phase current of WEC (A)

Inverter phase current, d-axis (A)(* reference value)

Inverter phase current, q-axis (A)(* reference value)

Displacement-proportional current vector (A)

Velocity-proportional current vector (A)

IGBT currents (positive/negative) terminal

Anti-parallel diode currents (positive/negative) terminal

DC link current (A)
\( i_c \) Capacitor current (A)

\( I_D, I_{DT} \) Collective DC current (A)

\( J \) Current density (A/m\(^2\))

\( K_{PTO} \) Spring stiffness force coefficient attributed to the power take-off system (N/m)

\( K_F \) Constant coefficient of generator

\( k_{ij}(t) \) Memory kernel term

\( k \) Spring stiffness (N/m)

\( k \) Wave number

\( L/L_m \) Inductance (H)

\( l \) Length of the conductor (m)

\( l_g \) Length of equivalent air gap (m)

\( m \) Mass (kg)

\( m \) Modulation index

\( M_L \) Mass attributed to the power take-off system (kg)

\( m_{g}, m_{33} \) Mass of the buoy (kg)

\( \phi \) Phase Angle (rad)

\( \phi_g \) Phase Angle of current vector (rad)

\( \hat{\phi} \) Peak magnetic flux linkage coil (Wb-turns)

\( P \) Power (W)

\( \rho \) Water density (kg/m\(^3\))

\( P_w \) Wave power (W)

\( Q \) Reactive Power (Var)

\( R_m \) Radius of magnets (m)

\( R_i \) Inside radius of coil (m)

\( R_o \) Outside radius of coil (m)

\( R_{ij} \) Radiation impedance due to causality

\( R/R_{on} \) Resistance (ohm)

\( S_{abc}, d_{abc} \) Switching states of PWM signal

\( S_o \) Fundamental sinusoid for irregular wave

\( T \) Wave period (s)

\( T_k \) Fundamental irregular wave period (s)

\( T_p \) Positive terminal IGBT switch
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CHAPTER 1

Introduction

1.1 Wave Energy

Wave energy is a renewable energy source which appears to have potential to offset fossil fuel for the production of electricity in the future. This energy source, still in its infancy, is originally related to natural movements and can be considered as a concentrated form of solar energy. The formation of wave energy occurs when the winds, induced from the differential heating of the earth, transfer some of their energy as they pass over the open water surface. As a result significant wave motion is produced and is only useful when converted to other types of energy, such as electricity. Rivers and lakes are well known sources of hydropower where the energy can be stored as potential energy behind dams. The motion of sea waves can also become important by harnessing the rotary movement in the open ocean to generate clean electrical energy. There is a need to explore the full potential of wave energy, and wave energy conversion research is ongoing around the world. The UK shoreline alone is reported to have an estimate capacity resource of 2TW/year [1].

In the world today electricity has become an essential service to us, which we use daily. Electricity generation from renewable energy sources is a major subject for discussion among the policy makers around the world. Since the Kyoto Protocol many countries are trying to produce electricity from renewable energy sources. UK and other European countries are leading the fight against climate change all over the world by embarking on alternative energy projects to produce electricity to create a more sustainable world environment [2]. To explore the full potential of wave energy, serious studies took place in the 1970s. Since then there have been many industrial and academic researchers concentrating on developing particular devices to capture this energy effectively, and investigating how the energy converters can survive in the harsh condition of the open sea,
but there has been less work on the electrical power take-off (PTO) and connection to the grid. Compared to other renewable energy sources such as wind and solar, wave energy is absent from the commercial world market and is a small fraction together with other ocean renewable technologies such as tidal and ocean thermal energy. The uncertainty of the wave conditions and its variation in the mild to horrific weather in the open ocean make wave energy an unpredictable resource, and it is difficult to attract investors to fund any large wave energy project. Reliability and maintenance costs are also challenging wave energy research groups around the world to design a simple but robust wave energy converter (WEC) to minimise the risk of device failure. Many issues still need to be addressed before wave energy can compete with other renewable sources and exploit the large resources available, especially on the western coasts of the UK. Figure 1.1 shows annual mean wave power around the UK coast produced by Department for Business, Enterprise & Regulatory Reform.

**Figure 1.1**: Annual mean wave power around UK coast [8]

Wave power estimation in this resource data (Figure 1.1) was based on equation (1.1) described previously by Tucker and Pit [3], which calculates wave power (W) per metre wave crest, where \( \rho \) is water density (kg/m\(^3\)), \( g \) is the gravity constant (m/s\(^2\)), \( H \) is wave
height (m) and \( c_g \) is wave group speed (m/s), and it defines the wave energy that propagates through the sea surface.

\[
P_w = 0.0623 \rho g H_w^2 c_g
\]  
(1.1)

where \( c_g \) is defined as

\[
c_g = \frac{gT}{4\pi} \left( 1 + \frac{2kh}{\sinh 2kh} \right)
\]  
(1.2)

where \( T \) is wave period (s), \( k \) is wave number and \( h \) is water depth (m).

Although research on producing electricity from wave energy was first developed and tested more than 30 years ago, only a few commercial devices are currently installed worldwide [4]–[8]. The reason is due to the high cost of developing the technology and the challenges of capturing the energy in real seas. As reported by Thorpe [9] in 2010, only the WEC built by Pelamis Ltd can be considered to have reached the commercialized stage: most of the WECs were either still in theoretical concept or are undergoing small scale tests. Table 1.1 displays some of the wave energy devices currently being developed and considered to have reached stage 4 (demonstration prototype) and stage 5 (commercial deployment).

**Table 1.1: Status development of several wave energy converter developers [9]**

<table>
<thead>
<tr>
<th>Company</th>
<th>Technology</th>
<th>Types</th>
<th>Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aquamarine Power</td>
<td>Oyster</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Finevera Renewables</td>
<td>Aqua Buoy</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>AWS Ocean Energy</td>
<td>Archimedes Wave Swing</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Carnegie Wave Energy</td>
<td>CETO</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Hydam Technology</td>
<td>McCabe Wave Pump</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Instituto Superior Tecnico</td>
<td>Pico OWC</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Muroran Institute of Technology</td>
<td>Pendular</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Ocean Power Technologies</td>
<td>PowerBuoy</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Pelamis Wave Power</td>
<td>Pelamis</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Seabased AB</td>
<td>Linear generator</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Wave Plane Production</td>
<td>Wave Plane</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Wavegen</td>
<td>Limpet</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Wave Dragon</td>
<td>Wave Dragon</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Key:
Types: 1- Oscillating Water Column (OWC); 2- Point Absorber; 3- Surge/Flap; 4- Attenuator/ Contour; 5- Overtopping
To realize the potential of wave energy, the research to collect this energy needs to be accelerated. The time has come for wave energy to stand with other sources as an economic energy source by producing power “bundles” from an array of wave energy converters, rather than collecting the power from a single wave energy converter. The research literature on capturing wave energy is growing very rapidly. As optimizing the efficiency of single wave energy converters continues to develop, research into interconnected arrays of wave energy converters is being investigated, considering small, medium and large scale offshore (> 50 km from the shore) deployment sites. Since the first offshore device developed by Pelamis was installed in real seas, many others WECs demonstration have followed such as PowerBuoy (OPT), Multi cell platform (Hydroflot) and linear generators based systems (Seabased and Trident). In the future, offshore marine energy converters could have in excess of 100 devices interconnected in arrays to generate electricity and deliver it to the onshore network. Until now there is no standard method as to how this power can be captured and converted before sending it to the grid.
1.2 Introduction to WECs in arrays

Collection of electricity from a wave energy farm is a challenging task for WEC developers. In order to design a robust device, extensive testing in real sea conditions is required. Simulation tools and production of a small scale prototype model of the WEC are common techniques used by developers to test the survivability of the devices in extreme wave conditions. Figure 1.2 shows example illustrations of future wave power plants from different WEC developer concepts interconnected in arrays. PowerBuoy, which was among the first point absorber wave energy devices installed in real seas, recently proposed up to 200 wave devices to convert the mechanical movement due to the interaction with the sea surface, using a sophisticated direct-drive system to drive an electrical generator. The device was rated at 500kW and is proposed to be installed in North America at Coos Bay, Oregon.

Figure 1.2: Illustration of a future WECs in farm. (Top)Left: Trident Energy, Right: AWS.(Centre)Left: PowerBuoy, Right: Pelamis Ltd (Bottom) Seabased.
A hydraulic system is a likely candidate to extract the energy from the waves and is suggested to be used for offshore applications until direct drive linear generator systems have been successfully developed [10-13]. Violent weather conditions producing large waves will reduce the survivability of the WEC especially when it has high mechanical interactions involved during conversion. Consequently the usage of a linear generator as the power take-off (PTO) has recently attracted attention of wave researchers around the world. Operation using direct linear motion is believed to increase the survivability of the devices since it consists of only a few mechanical parts and is seen as being very robust with less maintenance needed. There are many ways of converting wave energy into electricity [14]. This thesis focuses on a WEC in an array based on a direct drive linear generator. A review of existing concepts using linear generators is now presented.

The AWS (Archimedes Wave Swing), developed by a Dutch company, is a fully submerged device. It is a cylindrical air-filled chamber where the vertical movement of the lid of the chamber is influenced by the alternating crest-trough wave movement. The wave device is shown in Figure 1.2, and one single device rated at 2MW was successfully submerged in May 2004 and in Sept 2004 was reported to deliver power to the grid [15].

Seabased AB [6] in partnership with Uppsala University developed a WEC with a linear permanent magnet generator [13]. This type of WEC is expected to be placed on the seabed, and a back emf is induced due to the reciprocation motion of the translator which is directly coupled to the buoy by a rope. The only device that will interact with the wave is the cylindrical buoy, which drives the three phase permanent magnet generator to produce the induced voltages. This device is also illustrated in Figure 1.2. Seabased have connected all their WECs in parallel instead of in series connection, and collect the power from each WEC in the DC side before it is filtered and inverted it into a suitable grid voltage [16].

In contrast with the fully submerged devices such as the AWS and Seabased WECs, Trident Energy group uses a direct drive system which works above the sea surface and is placed on a sea structure platform to support the generators and the floats. The float is connected to the translator end and moves vertically due to movement of the waves, resulting in the generation of electricity. The generators are connected in arrays, and a fully functional test rig was expected to be deployed in the North Sea, prior to the development of a 1MW system. However during the deployment stage, the test rig sank due to the problem of the rig's buoyancy [17].
In the literature the damping factor is an essential parameter affecting power capture and plays an important role in abstracting wave power in the oscillating devices [18-19]. Having a very high damping factor can increase the power absorption but can affect the speed of the WEC's translator, which would interrupt the continuity of the power delivered as well as increasing power losses. Conversely at low damping factor, the power losses can be reduced, but it will affect the survivability of the devices due to increasing of the speed of the translator, especially during adverse weather. Therefore the damping of the buoy needs to be controlled as well as the stiffness factor, before the electrical output power can be converted by the power electronics converter to produce a constant voltage and frequency. This thesis is concerned with the control scheme of an array of direct drive WECs based on the linear generator concept. The process of converting the wave energy to electrical power by the WEC involves the control of the motion of the WEC buoy and the two electrical power conversions (the first conversion converts the varying generated output power to DC power and in the second conversion converts the DC power to be suitable for feeding into the AC power grid).

An array of WECs based on direct drive linear generators can be connected either in series or in parallel through their power electronic converter. In series connection, the negative terminal of the first WEC's rectifier (converting AC to DC), is connected to the positive terminal of the second WEC's rectifier in the array, and the negative terminal of the second WEC's rectifier is connected to the positive terminal of the third WEC's rectifier. In parallel connection, all WEC rectifiers in the array have all their positive terminals connected together and all their negative terminals connected together. All of these connections are on the DC side [33].
1.3 Background of the Project

The following is a brief description of the proposed work which will be covered in this research project.

Figure 1.3 shows the overall system structure from the interconnection of WECs within an array, the method of power transfer and the grid connection issues.

A. The interconnection of individual WEC devices offshore
B. The transfer of the power to shore (shore-line power transfer) and
C. On-shore connection issues.

![Diagram of WEC interconnection, shore line power transfer, and on-shore system]

- WEC Generating unit: WEC's PTO, converter, transformer, Circuit Breaker, Isolator. Generator unit capacity - 20kW

**Figure 1.3: Overall WEC System Scheme**

1.3.1 WEC interconnection

The power generated from each WEC in a farm varies due to the different incident wave patterns experienced by them. There are various possible arrangements for interconnecting WECs in an array (wave energy farms). Each method should be developed and tested by the simulation tools under different operating conditions. In order to model one single direct-drive PTO system, which will include the buoy, translator and generator, the power produced by each device should be fully understood before studying WEC electrical behaviour in a farm. The induced voltage and current in each linear generator varies in amplitude and frequency which makes them very suitable to be connected in parallel with
a single common link bus. The literature [19] shows that interconnecting a WEC farm in series could produce problems in keeping the output power stable since most variations occur in the current. It is evident that in parallel interconnection, stable and smooth output power can be achieved and be improved by increasing the number of WECs interconnected in the farm [20].

There are many more research studies on offshore wind farms compared to wave farms. Unlike the problem of producing stable output power faced by wave power linear generators, wind turbines can be connected in a single line string by having switchgear and rotational speed control to synchronize their output power, which is not possible to implement in direct drive linear wave generators. However, the ring and radial wind farm concept can be adopted in offshore wave farms to increase reliability and security during fault conditions [21]. Instead of having one single collection bus, it should be investigated whether having a second collection bus in the event of faults in the primary link is a viable alternative.

The rated output power for each primary and auxiliary collection bus for any proposed configuration needs to be determined together with the required cable rating. For example, a system of 6 generators each rated at 100kW, their cable has to be able to handle the full power of at least 600kW. Some of the concepts of wind offshore power conversion can be implemented in a wave energy farm, since after the interconnection issues of WECs are overcome, the collection and the transmission will be similar to the technologies currently proposed for offshore wind. However, there will be different requirements on the cabling technology, as they will be less accessible than the near shore devices, as demonstrated by the wave group from Seabased in Sweden [6].

The study of multi-arrays in larger sizes of wave farm is very important, as these can transform the prospect of wave energy as a viable option among other commercial renewable energy resources, thus these will be considered in this study.

1.3.2 Shore line power transfer

For the next few wave projects (2 - 3 years) a small number of WEC’s (<5) rated 20-1000kW are likely to be installed near shore and the power transfer to the shoreline will probably be rated at AC 11-33kV. An AC transmission system is preferable if the distance of the farm from the shoreline is less than 50 km, since this transmission technology has
been operated with high reliability and is fully proven to deliver the power within that range.

1.4.1 Aims and Objectives Of The Project

In the longer term (5 – 10 years) power bundles of 200 MW or more are expected to be deployed. These could include the collection of power from wave farms in the near shore region, with the additional collection of power from further offshore where their location and distance will determine the type of transmission. For a far offshore wave energy farm (>50 km), HVDC transmission is likely to become the only option since it has been proved that this will reduce the power losses during transmission [22]. Transmitting AC power through a very high capacitive cable lying in deep sea water results in large charging currents which are eliminated by using DC transmission.

In this study, AC transmission is chosen since in the foreseeable future, all the WEC devices are employed within 50 km. Moreover, for low-medium voltage system such as in the array system studied, AC transmission is believed to offer better efficiency thus providing a more economical solution.

1.3.3 On-shore system connection Issues

The existing system on-shore can be used to receive the power from an array. However the power collection must be converted to a level which is suitable for the grid. The location, size and the requirements of the on-shore station should be clearly defined in order to manage different power handling options.

Large arrays connecting to a rural distribution network will cause a reverse power flow when the peak power generated exceeds load demand, and forward power flow when demand exceeds generations. Where the network impedance is appreciable this will lead to a time-varying voltage rise, or voltage drop that reduces power supply quality. Other studies at the University of Edinburgh, [22-23] have identified means to mitigate the voltage impacts of distributed generators on the rural network.

The work reported in this thesis is concerned with the individual and group control of linear generators in array, connecting to the network, via AC-DC-AC power conversion system, at points of much lower impedance with higher fault levels and consequently lower impact on supply quality.
1.4 Aims and Objectives Of The Project

The major objective is to develop for an array of WECs based on direct drive linear generators feasible configurations and control schemes that can produce optimum power extraction from the incident waves. The study is made in order to gain a better understanding of the role and potential benefits of electrical power conversion systems in the delivery of electrical power from an array of wave energy converters with linear generators.

This objective will be achieved through the following:

1. The development of system models, which will include: devices; electrical power take off (PTO) and associated power converters; modes of power transfer to shore; possible array topologies ; and converters used in grid connection.

2. The application of a control strategy that can optimise the wave power capture, both individually and collectively, and convert the generated power to the form that is suitable for connection to the onshore electricity network.

3. Assessment of the feasibility of the control schemes in the array subjected to the type of the incident waves received by the WECs.

4. An investigation of the ability of the electrical system to provide a reliable network for the arrays, and the associated control strategies to make this happen.
1.5 Contribution to Knowledge

Direct drive generators provide a simple power extraction concept for wave energy by reducing the mechanical intervention in the system. In turn it implies that the power generated by a single device has varying frequency and amplitude which complicates the conversion electrically instead of mechanically. The induced voltage and current has different frequency and amplitude which needs to be addressed before it can be delivered to the grid, which requires fixed amplitude and frequency. Due to the movement of the waves, the generated power pulsates in a linear generator instead of a continuous level of power being produced as in conventional rotational generators.

Interconnecting WECs in arrays adds more complications to the power conversion process and hence is a challenging task. Although an array can reduce the pulsating shape of power produced compared to a single generator, interconnecting them in arrays or in a cluster of arrays requires a proper and an efficient control strategy for ensuring that smooth output power is delivered. This study investigates the technical aspects to the control of an array of point absorber WECs based on linear generators. Such a control scheme focuses on two practical issues and is applied to each WEC in the array. First, how the control can enhance the performance of the linear generator by generating higher output power. Second, despite the varying motion of the incoming wave, the control of the electrical power converter makes the array capable of delivering constant voltage and frequency which is vital for the integration with the power grid. Based on these two control strategies, the control signals are computed and applied to both power converters in the array (WEC side and grid side). The good agreements between the control schemes in the array can be seen when the voltage level in the DC link (where all the WECs in the array are connected) is controlled at a constant value, which indicates the balance of power exchange between the WECs and the grid sides. The study also embarks on the control strategy for a larger size of wave farm by proposing the efficient control means that not only can manage the power from the individual array, but has an ability to initiate the communication so that the power can be shared between the arrays to meet the array control objective of optimising power to the shore.

Overall, the work undertaken in this study demonstrates the potential of the deployment of linear direct drive WECs in an array.
1.6 Chapters Outline

Chapter 2 gives an overview of the components involved in the system studied and examines the variety of the available system options that could be implemented.

Chapter 3 describes the theoretical aspects of the components in the array including the mathematical derivation that is used for modelling the array power system components in this study. This chapter also discusses the fundamental concept and operation of the array’s components.

Chapter 4 analyses and demonstrates the capabilities of the control schemes proposed for the WECs when incident with the regular waves. The performance is assessed from the case studies presented. The study for regular waves was divided into two control strategies: First, the control is made based on the sub-optimal strategy which only considers the damping coefficient in the control scheme. The second strategy allows optimal (maximum) power to be extracted from the array by the inclusion of the stiffness coefficient in the control system.

Chapter 5 assesses the performance of the proposed control schemes in the array when incident with irregular waves. The fundamental frequency of the respective wave train is chosen in the equation of motion for the derivation of the control schemes. The ability of the control schemes to handle the power variations in the array when receiving a series of incoming waves which are continuously varying can be seen from the simulation results presented. The study when the WECs are connected on the AC side is made in the final section of Chapter 5.

Chapter 6 highlights the merits and drawbacks of the proposed control schemes and recommendations for possible improvements. As Chapter 4 and Chapter 5 introduced several strategies for controlling each WEC individually in the array, in this chapter recommendations are made with emphasis on the optimisation of the array’s output power as a group. A proposal to improve the control strategy on the grid side is made based on several criteria for ensuring the optimal power of the array can be delivered. The study of several array topologies is also conducted in this chapter. The purpose of the study is to investigate the flexibility of the topologies when handling faults in the array. The chapter ends with a study of a multi-array or cluster.
Finally the observations and the discussions of the proposed control schemes are the topic for Chapter 7. Conclusions are drawn regarding the subjects and a series of suggestions for future work are discussed.
CHAPTER 2

Components of the Array

2.1 General wave park configuration

The configuration of a future wave park will be similar to that employed for off-shore wind. The concept of a wind park, where several wind turbines can be connected either by AC or DC links could be applied in wave projects by connecting several wave energy converters (WECs) in an array. Figure 2.1 shows the general wave park configuration which includes an array interconnection, collection point, transmission line and the grid connection. The energy from the waves is extracted by the WEC to drive the power take off (PTO) for the generation of the electricity. Depending on the type of PTO, the WEC may or may not require an intermediate power conversion device to convert the AC output power into an electrical form that suits the requirements of the power consolidation in the array. The main function of the power conditioning and control block is to perform the power conversion and achieve the desired power form in an array. It is also controlling the flow of the electrical energy between the power source to the power collecting point (PCP). The power collecting or common point can be considered as the link that connects the WECs together in an array, which can be DC or AC. In the PCP block, the role of the power converter is to convert the input power into the level that is suitable for power transmission: these power converters include the VSC, DC-DC converter, AC-AC converter etc. Finally the transformers and power electronic converters can be a part of the grid side interface block to ensure the power transmitted suits with the requirement of the power grid.

![Figure 2.1: General configuration of wave energy converters in an array](image)
In the following sections, the components of the array are discussed and several possible array configurations are presented. The control principles used in this study are described briefly in the final section to give an overview of the strategies that have been used for the reliable and efficient control of the array.

2.2 PTO for Wave Energy Converters

2.2.1 Introduction to PTO

Harnessing the movement of waves to convert the kinetic energy to electricity can be achieved in numerous ways, as many types of wave energy converter (WEC) have been proposed. Since the concept of power capture for the various wave devices are different, normally the WECs are categorized based on the type of the power take-off (PTO) device. As part of the WEC, the PTO device can be considered as an intermediate mechanical system which is necessary for the extraction of the energy from the wave. The primary function of the PTO is to provide the mechanical linkage to match the motion of the WEC when acted on by the incident wave, with the rotary motion of a conventional electrical generator. Thus the PTO is important to ensure smooth operation between the WEC and the generator that requires to be driven at high speed.

Some types of WEC, such as Pelamis, use a hydraulic system as their PTO to drive a rotational generator using a mechanical-valve system. Meanwhile, the Oscillating Water Column (OWC), a wave device developed by Wavegen, uses a pneumatic system which takes full advantage of the natural crest and trough movement of waves at the coastline. The rise and fall of the wave causes an alteration of pressure and produces bidirectional air flow to drive a Wells turbine and induction machine inside the chamber of the OWC. Other types of PTO system use direct mechanical linkage systems such as a gearbox to provide speed and torque conversions to drive a high speed rotating generator.

Except for the OWC device that is particularly designed for the deployment at the shoreline, the problem with the other aforementioned types of power take off (PTO) is their mechanical interface components require regular maintenance, which is not desirable for offshore deployment. Therefore the direct drive linear generator has been proposed in [10], [22–24] to eliminate the need for the mechanical interface in the WEC. The reciprocating linear machine is claimed to offer the prospect of a low-maintenance generator system [25]. However, there are still maintenance problems with the bearings and end-stop mechanical design that need to addressed, [28] before this particular generator can become the main option for the WEC developers.
2.2.2 Introduction to Direct Drive Linear Generator

In contrast with the conventional PTO, the direct drive system uses the reciprocating movement of the wave to drive a linear generator and converts the wave energy to electrical energy directly. The operating principle of direct drive is shown in Figure 2.2. By eliminating the mechanical interface in the WEC to drive the generator, a robust but simple PTO is anticipated. The task of converting the electrical energy from the waves is done using an electrical generator which is incorporated with the suitable PTO in the WEC system. By having a direct drive PTO system based on an electrical linear generator, the overall maintenance cost for the WECs is expected to reduce due to the absence of the complex mechanical interface in the WEC, thus offering a big advantage.

Due to environmental concerns (from leaking hydraulic oil), as well as for reliability reasons, the direct drive linear generator can be an attractive choice for the WEC developers. However the drawback of the linear generator is the condition of its variable output power where the induced voltage from a single unit will vary, unlike the voltage produced by the rotational machine in a conventional hydraulic system. This problem occurs because of the varying speed experienced by the generator’s translator in the open sea. The magnitude of the voltage depends on the speed of the translator. When the translator reaches its peak/trough, it stops and reverses, resulting in a varying voltage. Also, when it reverses direction the phase rotation reverses. Therefore, a power converter system is required to convert the voltages and currents to constant voltage, constant frequency to suitably feed into the grid network.
2.3 Electrical Power Conversion

Two types of power converters can be used for the power conversion system in the array system studied. These power converters, based on self-commutated converters, are the current source converter (CSC) and the voltage source converter (VSC), and are built with gate-turn-off capability using IGBTs to allow full control of the power converter [30]. This section discusses briefly the unique features of the CSC and the VSC which make them ideally suited for the direct drive WECs system.

Figure 2.3: (a) CSC Topology (b) VSC Topology and its equivalent circuit

Figure 2.3 (a) and (b) shows the circuit diagrams and the single phase equivalent circuits for CSC and VSC respectively. Both these topologies allow these converters to have full control of the amplitude, phase and the frequency of its sinusoidal AC power signal. This makes them not only capable of managing the power from the generator, but they also have the capability to condition the power from the array so that it can be synchronised to the grid. Four quadrant control of AC power operation (real and reactive power), as can be seen in chapter 3 (Figure 3.7), can be obtained by simply generating the appropriate PWM signals to the gate terminals of their six semiconductor switches (S1-S6). The generation of the PWM signals in a CSC is made by comparing triangular signal with the reference phase current, thus it is shown as the phase current source symbol, $I_{SA}$. In a VSC, the converter phase voltage source is used, so it is shown as the phase voltage source symbol, $V_{SA}$.

From the topologies shown in Figure 2.3, the current source converter has an inductor connected in series with the load, where as the voltage source converter has a capacitor...
connected in parallel across the load. The function of the inductor and capacitor in the CSC and VSC topologies is to provide a constant DC link quantity (DC link current, $I_{DC}$ in CSC and DC link voltage, $V_{DC}$ in VSC) [31]. Since the VSC is by far the most widely used, it is chosen in this study. The principle of operation of VSC will be discussed in detail in Chapter 3 and Chapter 4. The CSC is reported as used in high power applications such as large motor drives and some photo-voltaic systems (PV) [32]. Figure 4.7 in Chapter 4 shows the configuration of the VSC in the WEC power line. Both VSCs are required to perform the rectifier and the inverter function in the WEC system. The two control approaches proposed for controlling these VSCs are referred to in this thesis as WEC side and grid side controls.

2.4 Interconnection System

![Figure 2.4: Parallel (Left) and Series WEC Interconnection (Right)](image)

There are two types of interconnection that can be applied to an array of WECs, which are parallel and series interconnections as shown in Figure 2.4. The merit of having series interconnection in the DC link is it could give a higher DC voltage, which is suitable for long distance power transmission. However, due to the irregularity of generated power in the WEC, so far, series interconnection has not been suggested for deployment in wave energy projects. Wave research groups are preferring to adopt parallel interconnection instead. This is due to the fact that the generated power will be zero every time the linear generator’s translator changes direction [33]. In [20], the results show that not only smoother output power can be achieved in parallel, but it can be improved by increasing the number of WECs connected in the farm. Therefore the parallel connection has been used in many offshore wave and wind farm proposal topologies [13], [16], [31-32], and will be considered in this study. Two types of parallel interconnection systems are studied which are DC link and AC link interconnections.
Although seen as less important in wave farm applications, the potential of series interconnection in wave device applications need further investigation due to its capability on cancelling the phase current harmonics of several rotary generators when connected in an array. By simply shifting the phase of the modulating carrier signal in the control power circuit, improved power quality can be obtained from each generator [32]. Thus, the requirement of having a large filter to smooth the output signal can be avoided which will reduce the overall cost. Therefore in addition to parallel interconnection, series interconnection is suggested for future study in order to investigate its full potential for handling the power from a wave farm. In future a comparative study can be made to determine the performance and efficiency of these interconnections.

2.5 System Array Configuration

WEC interconnection schemes must allow for a variety of generation possibilities. Accessibility of power to the grid during a DC fault which will determine the cost effectiveness of different interconnection schemes i.e. radial, star or ring systems for different configurations will be discussed in this thesis. Figure 2.5 shows the conceptual diagram of each configuration.

![Figure 2.5: System Array: (a) Radial, (b) Ring and (c) Star Configurations](image)

The Radial connection is cheaper, but does not offer the same degree of reliability as a ring configuration, which was demonstrated in [21]. In the radial connection, the cable line that connects all wave devices must be rated with the total power capacity of the generators. Due to its poor reliability in the event of a fault, this type of configuration is not suitable for implementation in a large scale wave farm system. Therefore the ring connection is generally proposed for larger scale generation by providing a redundant path for power to flow during a fault period. The diversion of power path is performed by the action of circuit breakers (CB) and disconnector (Dis) which are incorporated within the ring topology. The redundant cable should have the same power rating to allow the full power
to flow within the connection link. However the additional cable and the complexity of the fault detection system adds substantially to the overall cost.

The star connection offers features that reduce the effects of the limitations in the radial and ring connection schemes. It has the advantage of reducing the power rating of the cables since only a single wave device is connected which could reduce the cable cost. Also this type of connection scheme can provide a high level of power security if a fault occurs as the affected wave device can be isolated from the rest of the wave devices in the array. However it is still vulnerable to main cable failure, which will prevent all WECs from delivering their power to the grid. The installation of a second cable can offer greater security for transmitting power to the shore, but will increase the cost.

The simulation studies for these connection schemes in this thesis is based on the DC link interconnection system. The study is to investigate the best arrangement for a fault tolerant array as is discussed and presented in Chapter 6.

### 2.6 Power Conversion and Transmission System

Several power conversion and transmission concepts are presented from the wave farm to the shore line. These topologies include the transmission concept proposed in [33-34] and are discussed in this section and categorized based on their features. Since the output voltage and current from a direct drive WEC varies in amplitude and frequency, each WEC must be connected to a power converter (VSC) before it can be integrated into the single array. Therefore each WEC must consist of at least a linear generator power take-off (PTO) and active rectifier as shown in the figures 2.5 to 2.9.

#### 2.6.1 System Options

From the various power conversion and transmission concepts proposed by the wind and wave offshore farm community [20], [33],[33–35], several concepts can be suggested for WECs in an array system. The simplest arrangement is to connect the WECs in parallel as depicted in the system shown in Figure 2.6, where the WECs are connected on the DC side and the DC power is transmitted directly to the grid. An onshore inverter and transformer station are needed at the end of the transmission line to interface the power coming from offshore with the power grid.
The second option (Figure 2.7) is similar to the previous system, however the inverter station has been moved to offshore to control the power collection from an array before feeding the fixed AC power to the grid. Although AC transmission is recommended for short power transmission, an underground or undersea cable has high capacitance due to the its structure of a conductor which surrounded by the layer of insulation and the ground (seabed). Thus it acts as a long capacitor and its capacitance value increases with the length of the cable. Therefore having an AC transmission would reduce the power at the receiving end (on-shore station) as the alternating current charging this cable capacitance causes the energy losses.

![Figure 2.6: Single DC Transmission line [36]](image)

In an array, the power collected from each WEC can be optimized by maintaining the DC link at a constant voltage. However each WEC will produce a different amount of generated power due to the variations of wave amplitude experienced by each of them. Therefore in order to obtain the optimum power transfer from each WEC, appropriate control of the DC level is required and will vary due to the sea condition. Thus the DC-DC converter can be added to the system shown in the Figure 2.8 to control the output DC voltage before it is fed to the power inverter. This system may include the offshore transformer to reduce the power losses during transmission.

![Figure 2.7: Single HVAC I Transmission line [36]](image)
The last two options presented use a high voltage DC transmission and are shown in Figures 2.9 and 2.10. Both topologies are recommended to be deployed in offshore wave farms which are located more than 50 km from the shoreline. In the HVDC I system, WECs are interconnected in the DC link while in the HVDC II they are interconnected in the AC link.

The main function of the offshore converter is to collect the power from the WECs and to maintain the offshore DC link network between WEC. Meanwhile the function of the onshore converter station is to control the DC voltage level of the transmission line and to ensure the power is converted to the grid AC voltage and frequency. Although the control strategies for both topologies are complex, the amounts of power losses during transmission are low. Table 2.1 discusses these features which categorize each topology on their feasibility, advantages and drawbacks when working in a wave farm.
Table 2.1: Summary of wave farm transmission options

<table>
<thead>
<tr>
<th>Concept</th>
<th>Advantage</th>
<th>Disadvantage</th>
<th>Suggestion Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDC</td>
<td>Simple, fewer components involved and less cost.</td>
<td>Inefficient in power collection due to disappearance of offshore converter</td>
<td>Small wave farm near to the shore</td>
</tr>
<tr>
<td>HVAC I</td>
<td>Efficient offshore power collection and simple control.</td>
<td>High losses with the long AC cable transmission</td>
<td>Small/Medium size wave farm but the distance should be less than 30 km from shoreline.</td>
</tr>
<tr>
<td>HVAC II</td>
<td>More reliable by using the existing transmission technology.</td>
<td>Higher cost of offshore installation and higher power transmission losses.</td>
<td>Medium size wave farm and distance: must be less than 30 km to keep the power losses at a reasonable level.</td>
</tr>
<tr>
<td>HVDC I</td>
<td>Reduced power losses for long power transmission and efficient power management.</td>
<td>Other than HVDC II, the complexity of the system control is the highest.</td>
<td>Medium or large far offshore wave farm.</td>
</tr>
<tr>
<td>HVDC II</td>
<td>Suitable to manage the power collection for a WEC cluster located over several km.</td>
<td>Complex control system management and expensive.</td>
<td>Large far offshore wave farm: the distance can be more than 50 km.</td>
</tr>
</tbody>
</table>
2.6.2 Multi-array Connection Schemes

The previous section focused on the method of transmission and types of connection between WECs in an array. In this section, the discussion is mainly concerned with the configuration of the cluster within the offshore network. Several topologies have been identified from existing schemes proposed for far offshore wind and wave power schemes [33-34]. The different cluster networks can affect the overall system efficiency and the amount of power transmitted. The selection of cluster configuration, however, depends on the wave farm size and the size of the transmission cable. Therefore a brief overview of the system option is given based on the size of the wave farm, the number of WECs connected and the requirements of the transmission line to the grid.

In the first topology, called a single array radial common transmission (SARCT) system, the generated power is transmitted directly to the grid as shown in Figure 2.11. SARCT uses a single transmission line to transmit the collected power from an array of WECs. This transmission line topology offers a simple configuration since it would not be connected with any other array of WECs. Besides that, the sharing power control might be less complex and the numbers of WECs connected within an array determines the rated power of the common link cable. However this topology can increase the installation cost as a transmission cable is required for each array in the wave farm. This concept is not suitable for application in a critical area where environmental concerns may prevent the wave farm developer from increasing the number of transmission lines.

The Group Cluster Radial Common Transmission (GCRCT) is a more likely candidate than the SARCT in offshore wave farm integration. In this topology, several arrays have been connected together as a cluster as shown in Figure 2.12 and the power is collected from the cluster and transmitted using a single transmission line. Even though the control circuit for this system is more complex and the proper control technique must be chosen carefully, the number of transmission lines to the grid and the total installation cost can be reduced. Therefore the GCRCT topology is suitable to be deployed in an offshore wave farm. However the number of clusters which can be connected within the single line transmission is determined by the requirements of the transmission and the network cables. The power cable, rated between medium to high voltage, must be chosen to handle the amount of power collected from the WEC arrays. The main drawback of this topology is the low reliability of the transmission system. If a fault occurs in the transmission line, the power collected from the wave farm cannot be transmitted to the grid.
To avoid the possibility of a transmission fault problem occurring, an extra transmission line should be installed in the system as depicted in Figure 2.13. With the additional transmission cable, the power can still be exported to the grid if one of the transmission cables is defective. Another advantage of group cluster radial separate transmission (GCRST) is the amount of power collected from several arrays can be segregated between the two power transmission lines during normal operation; this will increase the capacity of the exported power to the grid as well as increasing the security of the power transmission. Therefore the cable should have the capability to handle the extra power if a fault occurs in one transmission line. Although this topology offers additional security for the transmission of power to the grid, there is no redundancy path has been installed in current wind projects, due to high cost and the low likelihood of faults occurring [21]. However, this is only related to small/medium farms, but not in far offshore and large scale farms where the consideration of having a redundant path is important due to the difficulty of maintenance work in such a harsh environment.

Figure 2.11: SARCT Integration system

Figure 2.12: GCRCT Integration system

Figure 2.13: GCRST Integration System
The integration system options discussed in the previous topologies can be extended to WEC AC connected systems. Figures 2.14 and 2.15 show the integration connection scheme when the WECs are connected on the AC side. These topologies take full advantage of having a power transformer in the AC system, and may include a tap changing transformer to convert to a suitable voltage level. However, a different winding type of power transformer may define the size of the offshore network. A three winding transformer may be deployed in a high voltage level network while the two-winding type is suitable for medium voltage. These types of topologies will be complex and to control the power sharing between the arrays can be more difficult.

![GC3TCT Integration System](image1)

**Figure 2.14: GC3TCT Integration System [37]**

![GC3TST Integration System](image2)

**Figure 2.15: GC3TST Integration System [37]**
2.7 Introduction of the WEC system control

The concept of the WEC system control made in this study is based on the amplitude and phase control proposed by Falnes [39]. For a situation when the WEC is acted by sinusoidal wave, the system is said to be in the optimum condition if the heaving body (the WEC buoy) is at resonance with the incident wave. Thus if the reciprocal motion of the WEC is controlled so that its natural frequency is the same as the incident wave frequency, the system extracts maximum energy from the waves. As a result the oscillatory velocity of the system (wave device) is in phase with the wave excitation force which acts on the system [40], as can be seen in the following figure.

Figure 2.16: Resonance, phase and amplitude control. The curves indicate the incident wave and the velocity of the heaving body. (a) incident wave (m), and the velocity of the system (m/s) when the control is applied to the heaving body: (b) amplitude control, (c) phase control, (d) amplitude and phase control, and (e) for uncontrolled system.

Figure 2.16 illustrates phase and amplitude control for one mode of oscillation, heave. Curve a represents the elevation of incident waves which is approximately in phase with the excitation force as defined in equations (3.16) and (3.17) respectively. Curves b and c represent the velocity of the heaving body when amplitude and phase control are applied to the system. Curve d is achieved when both control schemes are applied (optimum condition). Curve e is for an uncontrolled system. The concept of the optimum control, called complex conjugate control, is discussed in Chapter 3. Later in Chapter 4 it is shown how these control schemes can be applied in the WEC system.
2.8 Chapter summary

This chapter is intended to give an overview about the various possible configurations for direct drive wave energy converters (WEC) working as an array. The purpose of the first part of this chapter is to briefly describe the components of the array and what is their role in the system. In the second part, several options for the power converters and their interconnection within the system studied has been presented. Also a number of transmissions concepts and multi-array configuration schemes, each with their own merits based on the size, location and suitability of the wave park, are reviewed. Finally, the concept of the point absorber control based on the amplitude and phase control are briefly described and illustrated.
CHAPTER 3

Model Of WEC System Components

3.1 Control of heaving motion of WEC Buoy
3.1.1 Point Absorber of WEC

The definition of a point absorber is a body whose horizontal dimension is much smaller than the predominant wavelength [41]. A heaving buoy is a specific case of a point absorber oscillating in only one degree of freedom, the vertical (heave) oscillation. Thus the heaving point absorber of a WEC is a type of wave energy converter (device) moving in heave motion that could generate a large amounts of power by only using a small size device. The concept of power absorption from the wave to the heaving-type point absorber can be seen when a small floating body is placed on the sea surface. Once the ocean waves travel close to the body, the body will oscillate in the water with a certain amplitude and phase due to the motion of the incident wave. The vertical motion of the body happens because it receives or absorbs energy which has been transferred from the waves to the floating body, where the amplitude (or displacement) of the oscillating body is proportional to the amplitude of the incident wave.

In principle, some of the energy absorbed by the oscillating body is released back to the water by producing a smaller wave. Big and small bodies can produce equally sized waves, provided that the small body oscillates with a larger amplitude [42]. This may be seen when creating symmetric waves through the up and down (heave) movement of the floating body; different displacement amplitudes are required for two bodies of different size. For the purpose of wave energy conversion, the concept of a point absorber is used. Acting as an oscillating buoy placed on the sea surface, its reciprocating motion due to the absorption of energy from the ocean will drive the translator of the linear generator which is directly coupled to the buoy to generate electrical energy. In order to optimise this power conversion, the displacement of the buoy (and translator) needs to be controlled to promote the occurrence of resonance where the conversion energy is at maximum [43].
This happens when the oscillator moves in phase with the incident wave, resulting in a large amount of energy transferred from the wave to the point absorber. There are six modes of motion in wave-body interactions due to the incident wave [44], however for the study of a heaving type point absorber, only heave motion of the oscillator is considered.

### 3.1.2 Optimal Power Extraction

The optimum wave power extraction strategy as proposed by Falnes and Budal [45] is based on amplitude and phase control, which takes the functioning of the oscillating bodies in the water as a mechanical mass-spring damper system. As explained in [44], the system consists of mass \( m \) supported by the spring \( k \) and mechanical damper \( b \). When the force acts upon the mass \( m \), it will oscillate from its original position up to a certain displacement which is proportional to the strength of the external force. In this study, the oscillating body is the buoy directly connected to the WEC’s power take-off; therefore power take-off impedance (load) \( Z_{PTO} \) is added into the mass-spring system (intrinsic impedance) model to represent the linear generator of the WEC as can be seen in Figure 3.1 (a).

The components of the mass-spring damper mechanical system can be represented by the components in an electrical circuit, in which mass and spring are equivalent to the inductor and capacitor in an electrical system and the mechanical damper is analogous with an electrical resistor which dissipates energy. Figure 3.1 (b) shows the mechanical mass-spring damper of the WEC system represented in an electrical equivalent circuit. Taking the concept of an electrical circuit to represent the behaviour of a wave energy converter (WEC) in the water, electrical voltage is analogous with the external force while the electrical current is analogous with the velocity of the oscillating body. Thus summing the voltage over one loop in terms of current is analogous to summing the forces in terms of the velocity as derived in (3.1) to (3.4).

\[
e(t) = V_L(t) + V_R(t) + V_C(t) + V_i(t) \tag{3.1}
\]

\[
e(t) = L \frac{di_L(t)}{dt} + Ri_R(t) + \frac{1}{C} \int i_C(t) dt + i_tZ_l \tag{3.2}
\]

\[
F_e(t) = F_m + F_b + F_k + F_{PTO} \tag{3.3}
\]

\[
F_e(t) = m \frac{dv(t)}{dt} + bv(t) + k v(t)d(t) + Z_{PTO}v(t) \tag{3.4}
\]
In a linear AC circuit, the total power is the sum of all the power (energy) absorbed by the impedance of the circuit, which consists of real (resistance) and imaginary (reactance) components. The power absorbed by the resistance is called active power and the power absorbed by reactance is called reactive power. When the voltage and current waveforms of a circuit are periodic and vary with time, the reactance components will act as an energy storage and no power will be dissipated by them, thus net energy absorbed is zero. However a resistance cannot store energy, instead it dissipates or consumes energy, thus its value determines the average power absorbed in the circuit. As energy storage devices, the reactive components in the circuit will not contribute to the generation of average power, instead they cause energy to oscillate in and out leading to the average power absorbed by the load being reduced [46].

Therefore in order to understand the concept of optimal power extraction in wave energy conversion one should refer to the behaviour of electrical current in a purely resistive circuit, where the power absorbed by the load is at a maximum. In a resistive circuit the electrical current is always in phase with its source voltage. Furthermore, maximum power is received by the load if the resistive load is equal to the resistive impedance of the source voltage. Once reactive components (energy storage elements) such as a capacitor or an inductor are added, the phase and amplitude of the current will change according to the amount of reactance values receives by the load. As a result, the load will deviate from receiving the optimum power. Thus for an optimum power, the adjustment of the load impedance is required so that phase current can be in phase with the input voltage.
3.1.2.1 Complex conjugate control

In wave energy conversion, although there are mechanical energy storage elements such as mechanical spring $k$ and mass $m$ which represent the buoyancy behaviour of the WEC's buoy, optimal wave power extraction can be achieved by controlling the amplitude and the phase of the buoy velocity. This can be done through the adjustment of the damping and stiffness force coefficients in the mechanical load impedance $Z_{po}$.

As shown in Figure 3.1(a), power transfer to the load refers to the rate of removal of energy from the incident wave to the WEC oscillating body, which includes the buoy and the translator of the linear generator. As already described in section 3.1.2, the heaving WEC system can be represented by the electrical circuit shown in Figure 3.1 (b), where the force, $F_e$, is analogous to the voltage source and the velocity of the heaving buoy is analogous to the electric current. The intrinsic impedance elements of the WEC system are equivalent to the electrical elements: where capacitive reactance, $k$ in Figure 3.1 (b) is equivalent to the spring stiffness $k$ in Figure 3.1 (a), inductive reactance, $m$, is equivalent to mass and resistance of the electrical circuit, $b$ is equivalent to the mechanical damper of the system, $b$. Thus the electrical energy stored in the capacitor is analogous to the potential energy in the spring and the magnetic energy stored in the inductor is analogous to the kinetic energy of the mass [40].

In order to ensure optimum power transfer, control action in the load impedance, $Z_{po}$ is required. It is well known that maximum power transfer in the electrical circuit of Figure 3.1 (b) occurs when $Z_{po}$ is the complex conjugate of the source impedance $(b + j(om - k/\omega))$.

The same principle can be used in the mass-spring damper system in Figure 3.14 (a). This so called complex conjugate control is a control function for maximizing power absorption by a wave energy device using the hydrodynamic equation of motion (heave) which is explained in section 3.1.3.

Thus by assuming the velocity and the displacement as a steady state sinusoidal motion, equation (3.4) can be rearranged into the impedance and velocity terms

$$F_e = (j\omega m + b - \frac{j}{\omega} k)v(t) + Z_{pro}v(t) \quad (3.5)$$

The term in the first brackets of (3.5) is known as the intrinsic impedance of the WEC, which is derived from the concept of a mass spring damper system. The second term of
(3.5) consists of damping and stiffness force terms associated with the accelerating mass of the WEC, form the load impedance and is designed so that it conjugates with the intrinsic impedance of the WEC:

\[ Z_{PTO} = j\omega M_L + B_{PTO} - \frac{j}{\omega} K_{PTO} \]  \hspace{1cm} (3.6)

Substituting (3.6) into (3.5) produces the total force equation

\[ F_e = (j\omega m + b - \frac{j}{\omega} k)v(t) + (j\omega M_L + B_{PTO} - \frac{j}{\omega} K_{PTO})v(t) \]  \hspace{1cm} (3.7)

For the maximum power transfer, the impedance matching is performed between the source and the load impedance, \( Z_{PTO} \). The damping force term, which is the real part of \( Z_{PTO} \), is set to equal the damping value of the intrinsic impedance. To cancel the reactive elements in (3.7), the stiffness force term, which represents the imaginary part of \( Z_{PTO} \), is set to be the conjugate of the total mass and stiffness component of the intrinsic impedance, thus:

\[ B_{PTO} = b \]  \hspace{1cm} (3.8)

\[ K_{PTO} = \omega^2 (m + M) - k \]  \hspace{1cm} (3.9)

By controlling the damping and stiffness force using the concept described in (3.8) and (3.9), the system is said to be at resonance where the maximum power is transferred from the wave to the point absorber WEC.

### 3.1.2.2 Reaction Force Control

When a current flows through a conductor in the presence of a uniform magnetic flux density, the conductor will experience a force:

\[ F = i(lxB) \]  \hspace{1cm} (3.10)

where \( i \) is the magnitude of the current

\( l \) is the length of the conductor

\( B \) is the magnetic field vector
The magnitude of the force given by the equation depends on these three elements. Since the magnetic flux density and the length of the coil in a permanent magnet machine is fixed by the geometry and the properties of the permanent magnet in the machine design, the only way to control the generator’s force is through the phase currents.

The maximum force is produced when the current carrying wire is orthogonal to the magnetic flux density vector, which is not always the case for the generator coils in an electrical machine. Thus, the well known Lorentz Force equations (sometimes called Laplace Force) include the angle between the conductor (or generator coils) and the flux density vector into (3.10) in order to determine the force induced by the relative motion of the generator.

The control of the force generated by a linear generator has been proposed in [47] by Shek. The so-called reaction force control, which was inspired by the concept of complex conjugate control, is based on the derivation of the reference power take off force. This reference force, which aims to maximise the power extracted from the wave, is composed of two separate components: damping force coefficient $B_{PTO}$ for the amplitude control, and stiffness force coefficient $K_{PTO}$ for the phase control. Knowing the required reference values of the coefficients $B_{PTO}$ and $K_{PTO}$ as derived in (3.8) and (3.9), these two force terms then form the total desired power take-off force

$$F_{PTO} = B_{PTO} \dot{x} + K_{PTO} x$$

(3.11)

where $\dot{x}$ and $x$ represents the velocity and the displacement of the WEC’s buoy respectively. The force expression given above gives the information for the maximum energy conversion, therefore the generator force needs to be controlled to follow the reference power take-off force as derived in (3.11).

In order to perform reactive control, the generator force [26] is initially derived from the Lorentz Force theory in (3.10) and simplified using trigonometric identities to give the expression given in (3.12), where it has been resolved into two current vector components $i_x$ and $i_x$ so that the phase and amplitude of oscillation can be controlled as two separate entities [48],

$$F_{gen} = K_G (i_x + i_x)$$

(3.12)
where \( i_x = I \cos \varphi_g \cos \omega t \) and \( i_x' = -I \sin \varphi_g \sin \omega t \).

Thus the desired reference phase current needed to be drawn from the linear generator for maximum power conversion can be obtained through the calculation of the current vectors by comparing (3.12) and (3.11), giving the total phase currents:

\[
i_{abc}(t) = (i_x + i_x') \cos \left( \frac{2\pi}{\lambda} \pm \gamma \right)
\]

(3.13)

where \( \gamma = 0, \pm \frac{2\pi}{3} \) for phase a, b and c.

### 3.1.3 Hydrodynamic parameters for Oscillating Heaving buoy

This section is a discussion of the behaviour of waves and oscillating bodies. The previous section discussed how the power is transferred from the wave to the oscillating buoy (point absorber); now, however, the discussion is limited to the components involved in the equation which evaluates the body motion in waves.

The dynamics of the oscillating heaving buoy in waves can be investigated either in the frequency domain or the time domain. An oscillating body that interacts with a monochromatic wave can be described by using the equation of motion which is defined in the frequency domain. The term monochromatic means a single frequency, so a monochromatic wave is a regular wave which has the property of a single frequency.

On the other hand, the equation of motion, which is derived in the time domain is used to describe the behaviour of the oscillating body in both types of waves, regular and irregular. Irregular waves here are defined as waves containing more than one frequency. In this study, only the type of irregular wave that is defined as the superposition of multiple unidirectional monochromatic waves is considered, thus other types of irregular wave were not considered.

#### 3.1.3.1 Body motion in Frequency Domain

The interaction of an oscillating body in a regular wave can be modelled using the mass-spring-damper approximation which can be simplified as:
where \( F \) is the force applied to the body, \( m \) is the body mass, \( b \) is the coefficient of damper friction and \( k \) is the spring stiffness constant. Using the terms expressed in (3.14), the equation of motion of an oscillating body in sea waves can be expanded into the complex amplitudes of the body motions in the frequency domain, as:

\[
F_{\text{ext}}(\omega) = [-\omega^2 (m_{ij} + A_{ij}(\omega)) + j \omega B_{ij} + K_{ij}] x_j(\omega)
\]  

(3.15)

where \( F_{\text{ext}}(\omega) \) is the excitation force due to the result of the incident wave upon the oscillating body \( i \) in one mode of body motion \( j \), which in the point absorber case is in heave mode. The expression derived in (3.15) is valid if the wave is in regular form, where the excitation force and the displacement of the buoy can be expressed as:

\[
F_{\text{ext}}(\omega) = F_o \cos(\omega t)
\]

(3.16)

\[
x_j = X_o \cos(\omega t - \phi)
\]

(3.17)

The force and other components of (3.15) are known as the non-dimensional coefficients for the hydrodynamic parameters of the heaving body and can be computed using a dedicated computer programme like WAMIT [49]. By multiplying the non-dimensional terms (produced by WAMIT) by the water density \( \rho \), acceleration due to gravity \( g \), the amplitude and the frequency of the incident wave \( A \), will give the following dimensional definitions of the body’s motion [50]:

- Added mass
  \[
  A_{33}(\omega) = \rho L^3 A_{33}(\omega)
  \]

- Added damping
  \[
  B_{33} = \rho L^3 \omega B_{33}
  \]

- Force excitation
  \[
  F_{3\text{ext}}(\omega) = \rho g A L^2 F_{3\text{ext}}
  \]

- Displacement coefficient
  \[
  x_3(\omega) = \bar{x}_3 \frac{A}{L}
  \]
where, \( \rho = 1030 \text{ kg m}^{-3} \), \( L = 1 \) is the characteristic length scale. Referring to Figure 3.2, the mass of the body \( m_\Psi \) and hydrostatic spring stiffness \( K_\Psi \) can be calculated as:

\[
\begin{align*}
\rho_\Psi &= 1030 \text{ kg m}^{-3} \\
K_\Psi &= \rho g \pi a^2
\end{align*}
\]  
(3.18)

\[ z = \text{diameter} \]
\[ d = \text{draft} \]

\[ \text{Figure 3.2: Submerged WEC's buoy} \]

### 3.1.3.2 Body motion in Time Domain

The frequency domain equation of motion in (3.15) was derived from two hydrodynamic forces acting upon the oscillating body in the water [51]

\[
\hat{F}_{ij} = \hat{F}_{e,i} + \hat{F}_{r,i}
\]  
(3.19)

where \( \hat{F}_{e,i} \) - excitation force acting on the body which is due to the incident wave

\( \hat{F}_{r,i} \) - force acting on the body which is due to the wave which is radiated as a result of the body’s oscillation.

The second term in (3.19), as (3.5), can be represented as the combination of the complex radiation impedance and complex velocity amplitude

\[
\hat{F}_{r,i} = -Z_\Psi \hat{u}_i
\]  
(3.20)

The complex radiation impedance \( Z_\Psi \) is split into real and imaginary parts known as added damping \( B_\Psi \) and added mass \( A_\Psi \) as shown in (3.21).

\[
Z_\Psi = B_\Psi(\omega) + j\omega A_\Psi(\omega)
\]  
(3.21)

The excitation force \( \hat{F}_{ex} \), complex radiation impedance, \( Z_\Psi \) in (3.21), mass of the body, \( m_\Psi \) and the hydrostatic spring stiffness, \( K_\Psi \) as in (3.18), give a general equation of motion

\[ \text{38} \]
in the frequency domain as defined in (3.15). The derivation of the equation of motion can be found in Appendix B.1.

A system is causal if the output of the system depends only on past and present values of the inputs. A non-causal system is a system where its output response depends on future values of the inputs. As stated in [44], [52], the radiation impedance $Z_0$ is a causal system.

As a consequence of causality, in the time domain the damping term, $B_0$ in (3.21) is a function of present and past values of velocity [18], thus when modelled it should include memory due to the wave radiation. Therefore when converting the equation of motion of (3.15) into the time domain representation, the impulse response corresponding to the damping term is replaced with the memory kernel, $k_0(t)$, which contains the value of damping in the previous moment as defined in (3.22). The impulse response is the output of the system when converting the transfer function in the frequency domain into the time domain using the inverse Fourier Transform [18].

In order to determine the memory kernel term, $k_0(t)$ (so-called radiation impulse response), a memory term of damping coefficient $B_0(\omega)$ must be firstly calculated from the WAMIT programme [51] for several frequencies of interest

$$k_0(t) = \frac{2}{\pi} \int_0^\omega B_0(\omega) \cos(\omega t) d\omega$$  

(3.22)

Since the equation of motion in the time domain is used for describing the behaviour of the oscillating bodies in regular waves (containing several frequencies), the damping coefficients described above are frequency dependent.

By dividing the equation of motion given in (3.15) with $i\omega$ (integrated over frequency) and replacing the added damping term, $B_0(\omega)$, with the new radiation impulse response, $k_0(t)$, in (3.22), taking the inverse Fourier transforms (IFT) of the equation gives the integro-differential equation of motion in the time domain:

$$f_{j, ext} = (m_j + A_j(\infty))\ddot{x}_j(t) + \int_0^t k_0(t-\tau)\dot{x}_j(\tau)d\tau + (K_j)x_j(t)$$  

(3.23)
In a causal system, since the damping term is not directly proportional to the velocity, the force acting on the body due to the radiated wave, $\vec{F}_{r,1}$, cannot be determined by directly multiplied the damping with the velocity as given in (3.15) for the frequency domain and in (4.5) for the time domain: instead the middle term on the right hand side of (3.23), the memory kernel term, $k_q(t)$, is convolved with the velocity as defined in (3.24):

$$k_q(t) \ast \dot{x}(t) = \int_0^t k_q(t-\tau) \dot{x}(\tau) d\tau$$  \hspace{1cm} (3.24)

The integro-differential of motion equation defined in (3.23) is used for theoretical modelling to describe the behaviour of the heaving buoy when incident with the irregular waves. In this study this equation is used for the acquisition of the displacement of the heaving buoy for the irregular wave cases presented in Chapter 5. The control structures diagram that using this equation as their control scheme are shown in Figure 5.3 and Figure 5.11 for sub-optimal and optimal control respectively. The results due to this equation and the PTO impedances defined either in (5.9) for sub-optimal or in (5.13) for optimal control are shown in section 5.4 and sections (5.5.3-5.5.4) respectively.
3.2 Analytical model of Linear generator

A linear generator has two major components, the moving part (translator) and the stationary part (stator). To determine accurately the EMF induced, as well as the power take-off force developed by the generator coils, the electrical system of the linear generator must be modelled using high accuracy tools such as using a finite element model system. However, since this study focuses on transporting the power from an array of linear generators in WECs, it is sufficient to model the linear generator using a simpler analytical model which clearly explains the flux path in the machine, providing the information on the generation of the power take off force and the induced voltage with a reasonable level of confidence.

3.2.1 Air Cored Tubular Permanent Magnet Machine

The linear generator modelled in this research is the linear Air Cored Tubular PM Machine (ACTM). This type of generator, which was firstly proposed by Baker and Mueller [53], does not have iron in its stator in order to reduce the inductance properties of the machine as well as to increase the electrical power factor. By constructing the stator with no iron, it is believed that a lighter and a smaller generator can be made.

![Figure 3.3: Air-Cored Tubular PM Machine](image)

As shown in Figure 3.3, the machine consists of axially magnetised ring magnets mounted alternately with steel. The steel acts as a flux guide and is placed between adjacent magnets [10] to encourage the flux flow through the tubular machine via a stator coil winding. Figure 3.4 shows a cross-section view of the Air-Cored Tubular PM Machine in 2-D [54] where one translator pole consists of a magnet and two flux guide steel sections which is approximately equivalent to the three phase stator coil winding (stator pole). The dimensions of the Air Cored Machine are being investigated by the research group at the University of Edinburgh to optimise the overall performance of this particular machine [55].
3.2.2 Reactive force and estimated induced EMF calculations

To model the equivalent circuit of the Air Cored Tubular Machine (ACTM) as well as to estimate the generator force \( F_{gen} \) and induced voltage, the machine geometry and its properties must be clearly defined as were shown in [23]. Referring to Figure 3.3, mounting the magnets with alternating North and South surface poles would cause flux to be produced as the steel separators and the translator coils provide a path to travel over one pole. The flux linkage distribution in the radial direction could provide the information on the generated force and induced voltage in the coil.

As shown in Figure 3.4(a), the flux flows through one magnet and two separated halves of steel to complete a flux path loop through the stator coils. When the translator is moving, these coils will experience the varying magnetic field with respect to the position of the translator for each phase of the generator's coils. The peak radial flux cutting the stator coil is reached when the phase coils are fully aligned with the steel spacers as shown in Figure 3.4 (a): when this occurs, coil B is fully energized due to the magnetic excitation. The radial flux density then experiences a rapid decline until it is at a minimum in the middle of coil A and C. This pattern is constantly changing with the displacement of the translator.

**Figure 3.4:** (a) Simplified flux flow through ACTM on 2-D dimensions and (b) Derivation of flux linkage cutting coil

Force is developed in current-carrying wire in the presence of a magnetic field, where the force induced on a single coil of wire is given by
The magnitude of the force is determined by the angle between the wire and the direction of the magnetic field. Figure 3.4(b) shows a segment of the coil over the surface of the energised steel. The direction of the force is mutually perpendicular to the current and magnetic flux density vector. The magnetic field is not applied to just a single conductor, so the force expression in (3.25) is the acting force over the width of a coil giving the peak force per coil as stated in (3.26), where $J$ is defined as equivalent current density, $B_g$ is air gap flux density, $l_g$ is equivalent air gap (refer Appendix A in [26]), $g$ is the air gap and $ch$ is coil height ($ch=R_o-R_n$, refer Figure 3.4 (a)).

\[
F_{\text{coil}} = \int_0^\infty \int_0^\infty \int_0^\infty J B_g e^{-\frac{r+R}{l_g}} r \, dz \, dr \, d\theta
\]

\[
= 2\pi(cw)J B_g l_g e^{-\frac{g}{l_g}} [R_m + g + l_g - e^{-\frac{ch}{l_g}} (R_m + g + ch + l_g)]
\]  

(3.26)

The variation of the force magnitude over a stator pole with a magnetic pole wavelength, $\lambda$, for a variety of coil currents, gives the total force for a 3-phase of ACTM as (3.27)

\[
F(x,t)_{\text{gen}} = \sum_{\text{phase-abc}} F_g \cos\left(\frac{2\pi x(t)}{\lambda} + \gamma \frac{2\pi}{3} \right) i_{\text{phase}}(t)
\]

(3.27)

where $\gamma = 0, \pm 1$ for phase a, b and c.

The induced voltage in the stator coil can be determined by estimating the flux linkage variation over the entire stator coil, which occurs due to the variation in magnetic field by the stator coils and is given by (3.28)

\[
\psi_{\text{pole}} = \dot{\phi}_{\text{coil}} \sin\left(\frac{2\pi}{\lambda} x(t)\right)
\]

\[
\dot{\phi}_{\text{coil}} = \frac{F_{\text{coil}} \times N}{J \times ch}
\]

\[
\lambda = 2(W_m + W_c)
\]

(3.28)
where peak magnetic flux linkage of a single coil and the fundamental frequency of air gap flux density are given in (3.28) and denoted as $\hat{\phi}$ and $\hat{\lambda}$ respectively. By applying derivative chain rule on Faraday’s Law to (3.28), the induced phase voltage can be derived as defined in (3.29)

$$v(t) = \frac{d\psi_{\text{pole}}}{dt} = -\left(\frac{d\psi_{\text{pole}}}{dx} \times \frac{dx}{dt}\right)$$

$$v(t) = \dot{V}_p \cos \left( \frac{\pi}{W_m + W_s} x(t) \right) \frac{dx}{dt}$$

(3.29)

where $\dot{V}_p = -\left(\frac{\pi}{W_m + W_s}\right) \hat{\phi}_{\text{coil}}$, and $x(t) = $ translator displacement.

The induced voltage for the ACTM defined in (3.29) as is shown in Figure 4.17 (c) has varying frequency and amplitude which needs to be addressed before power can be delivered to the grid. Interconnecting WECs in arrays will further complicate the conversion of power from waves. However, it is believed that this will reduce the pulsating power produced and significantly smoother output power is expected to be delivered.

### 3.3 Introduction to Power Converters

The power converter acts as a power conditioner to match the electrical power from the source to the power demand in the load. The power conditioning process involves either changing from one type of signal to another type of signal, such as AC to DC or DC to AC, or changing to the same type of signal but with a different voltage and/ or frequency.

In this study, the AC to AC power converter is required to convert the variable voltage and frequency from the linear generator into the constant voltage and frequency of the electrical grid. This type of conversion, can be achieved either directly converting AC to AC or having an intermediate DC link between the conversion. The cycloconverter, for example, converts directly AC signal to another AC signal, by changing the frequency of the output voltage. The cycloconverter normally uses the thyristor as the power switches in its topology. This type of converter is suitable for the application of converting higher AC frequency waveforms to a lower frequency.

Another type of AC-DC-AC power converter uses two power converters, one acting as a rectifier and one as an inverter. The previous study for the direct drive wave energy...
converter done by Shek [47] showed that the power coming from the linear generator can be best synthesized by converting the varying AC power signal to a constant DC signal using a rectifier, then the filtered DC is applied to the inverter to generate, in this case, a 50 Hz AC output power signal.

### 3.3.1 Power Converter Switches

Power converters consist of a network of power semiconductor switches which switch ON and OFF to regulate the output power. The switching function of the semiconductor switches can be controlled in different ways. The most common technique which has full capability of controlling the output power factor and can reduce the harmonic content substantially is pulse width modulation (PWM). The capability to handle medium-high power and as well as turn-ON and OFF in the order of 1µs [56], makes the IGBT to be the common choice for the use in the medium and high power applications.

### 3.3.2 PWM-VSC Converter Topology

The three-phase active power converter as shown in Figure 3.5 is known as a pulse width modulation-voltage source converter (PWM-VSC or VSC), and it is widely used in many power conversion systems. The same topology can act both as a rectifier and as an inverter by simple control of the power semiconductor switches.

![Energy Flow Diagram](image)

**Figure 3.5: Three-phase VSC for active rectifier and inverter**

An attractive feature of the VSC is it has the capability of controlling power in both directions: from AC to DC or from DC to AC. This can be done by controlling the direction of the phase current flowing in the circuit through the adjustment of the amplitude and phase of the voltage at the converter side. To understand how this reversal
process occurs in a VSC, the three-phase VSC is simplified into a single phase equivalent circuit as shown in Figure 3.6.

![Diagram](image)

**Figure 3.6: Single phase representation of VSC power converter (from Figure 12.48 in [32])**

The three-phase VSC can be simplified into a one phase equivalent circuit consisting of two terminal voltages (refer to AC side) which represent the phase voltage of the ac mains, \( V_s \), and the converter side voltage, \( V_{mod} \). The line impedance interconnecting the two voltages, \( Z \) represents the resistance and the inductive impedance of the line circuit. Since the resistance of the VSC is very small, the voltage drop across the line impedance can be considered to be equal to the inductance voltage.

The amplitude and phase of \( I_s \) can be controlled by controlling \( V_{mod} \). To make the power flow from the AC side to the DC side, the voltage at the converter side, \( V_{mod} \), needs to be adjusted so that it lags the voltage at mains side \( (V_s) \) by angle \( \delta \). During this mode, the VSC operates in the rectifier mode. The phasor diagram of Fig. 12 in [30], reproduced here in Figure 3.7, explains graphically the operation of current control in the PWM-VSC.

In contrast, to make the VSC to perform as an inverter, the phase angle \( \delta \) needs to be controlled so that \( V_{mod} \) leads the ac mains voltage, \( V_s \), and the phase current will reverse its direction. Both in rectifier or inverter operation, the amplitude of \( V_{mod} \) and its phase angle \( \delta \) determine the desired phase currents and the reactive components in the line impedance of the circuit. This can be seen from the phasor diagram of Figure 3.7: the magnitude of the inductance in the line impedance and the magnitude and the angle of \( V_{mod} \) and \( \delta \) determine the direction of phase current.
Figure 3.7: Four quadrants VSC operation and phasor diagrams (from Fig. 12 in [30]).
From Top: Unity Power Factor (PF) of rectifier, Unity PF of inverter, lagging PF → inductive zero PF, leading PF → capacitive zero PF

Having the capability of controlling bidirectional power flow as well as managing the active and reactive power allows the VSC to operate in four modes, which is not only
suitable for power control but also for power factor correction as shown in Figure 3.7 (c) and (d).

As described above, the power can flow either from AC to DC or DC to AC as shown in Figure 3.8. The direction and the magnitude of power transferred between the two sides of the power converter can be determined using the expression given in (3.30),

\[ P = \frac{V_S V_{\text{mod}} \sin \delta}{X_L} \]  

(3.30)

By changing the phase of \( V_{\text{mod}} \) relative to \( V_S \), \( \delta \), the power can be delivered from or to the ac source. Furthermore, the amount of power passing through the converter system can be controlled by modifying the magnitude of \( V_{\text{mod}} \).

![Figure 3.8: Direction power control using PWM-VSC](image)

3.3.3 Mathematical description of the PWM-VSC converter

3.3.3.1 Conventional \( abc \) system

For the purpose of analysing the control action of the PWM-VSC it is convenient to develop a mathematical model of the PWM-VSC. This model can then be regarded as a control block in the voltage and current control loops of the PWM-VSC. In this subsection, the mathematical description of the VSC based on two coordinate systems, \( abc \) and \( d-q \) is described to form the necessary background for the control section in Chapter 4. In order to describe the mathematical model for the PWM-VSC, the simplified diagram of the power converter in Figure 3.6 can be used. By referring to the equivalent of the AC
power converter side, the one phase voltage equation for PWM-VSC converter can be calculated as:

\[ V_s = V_l + V_{MOD} \]  
(3.31)

\[ V_s = R_i + L \frac{di_l}{dt} + V_{MOD} \]  
(3.32)

Then the full three-phase equation of the converter can be expanded from the previous two equations as:

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} =
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
\begin{bmatrix}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R
\end{bmatrix}
+ L \frac{di}{dt}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
+ \begin{bmatrix}
V_{MODA} \\
V_{MODB} \\
V_{MODC}
\end{bmatrix}
\]  
(3.33)

The relationship of the three phase ac side of the converter with the dc side is:

\[ C \frac{du_k}{dt} = d_a i_a + d_b i_b + d_c i_c - i_{dc} \]  
(3.34)

where \(d_a, d_b\) and \(d_c\) are the switch conditions of the three phase power converter for each phase (refer to Figure 3.5). The switch condition of each phase can be defined as ‘1’ when the top switch is conducting and ‘0’ when the switch at the bottom is conducting. From the literature [57], the last component of equation (3.33) can also be derived as:

\[
\begin{bmatrix}
V_{MODA} \\
V_{MODB} \\
V_{MODC}
\end{bmatrix} = u_{dc} \begin{bmatrix}
\frac{2}{3} & -\frac{1}{3} & 0 \\
\frac{1}{3} & -\frac{2}{3} & 0 \\
0 & 0 & -1
\end{bmatrix}
\]  
(3.35)

3.3.3.2 Synchronous rotating coordinates \(d-q\)

The mathematical model of the converter can be transformed into synchronous rotating frame coordinates \((d-q)\). The transformation is firstly stated in [57] and also is well explained in [58], [59] and is normally used to study the three-phase converter for control.
purposes. The active and reactive power can be controlled separately when using synchronous reference frame \((d-q)\) coordinates.

The transformation of the three-phases of the power converter to synchronous reference frame coordinates can be expressed in the following equations:

\[
\begin{align*}
    u_{Ld} &= Ri_{Ld} + \frac{di_{Ld}}{dt} - \omega Li_{Lq} + u_{Sd} \\
    u_{Lq} &= Ri_{Lq} + \frac{di_{Lq}}{dt} + \omega Li_{Ld} + u_{Sq} \\
    C \frac{du_{dc}}{dt} &= (i_{Ld} S_d + i_{Lq} S_q) - i_{dc}
\end{align*}
\quad (3.36, 3.37, 3.38)
\]

The derivation of \(abc - dqo\), which is based on the well-known Park’s Transformation is given in Appendix B.2. Park’s Transformation is used to transform the output quantities of the generator, voltage and current from \(abc\) coordinates to the \(d\) and \(q\) reference frame, with the positive \(q\) axis normally defined to lead the \(d\) axis by \(\pi/2\). In general, the transformation of \(abc - dqo\) can be written as in (3.39), where \(\omega_s t\) is the frequency of the currents and the voltages in \(abc\) coordinates and denoted with the general term \(T_{dqo}\) that may represent the currents, voltages or fluxes in the generator.

\[
[T_{dqo}(\omega_s t)] = \frac{2}{3} \begin{bmatrix}
    \sin(\omega_s t) & \sin(\omega_s t - \frac{2\pi}{3}) & \sin(\omega_s t + \frac{2\pi}{3}) \\
    \cos(\omega_s t) & \cos(\omega_s t - \frac{2\pi}{3}) & \cos(\omega_s t + \frac{2\pi}{3}) \\
    \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\quad (3.39)
\]

### 3.3.3.3 \(dqo\) transformation of line impedances

Generally the mathematical transformations are often used to reduce the complexity of the variables involved especially for their control purposes. The advantage of using the \(d-q\)
transformation is that in this formation it clearly shows the way to control the active and reactive power as a separate entities, and it is widely used in line power compensation to improve voltage, increase power transfer and improve the overall system stability [46].

In order to implement the control of PWM-VSC in dq coordinates, the power converter topology shown in Figure 3.6 also needs to be transformed into dko circuit models. This transformation would effect the line impedance calculation since every quantity in the circuit needs to be transformed to suit the dko formation. Figure 3.9, in single phase representation, shows the PWM-VSC for conventional abc and its d and q-axis equivalent circuit model.

By referring to the VSC-PWM dq equivalent circuits above, the dq expressions given in (3.36) and (3.37) are derived.

3.4 Control Strategies of PWM-VSC

There are several control objectives in the PWM-VSC according to their role in the power conversion system. On the DC side, the basic control operation is to keep the DC link voltage at a desired reference value, where the DC current control is used to charge the DC voltage to be at the suitable DC level. On the AC side, the objective is to control the phase AC current to equal the reference value which is calculated from the load power requirement.

Having IGBTs as the power switches allows full control of the VSC where the switches can easily be turned ON and OFF as required. In order to properly control the transferred power, the power switches within the power converter must be switching appropriately by applying PWM signals to each of them.
3.4.1 Carrier Based Sinusoidal PWM

The well known technique of Pulse Width Modulation (PWM) is applied to the IGBT switches to control the current flow and power transfer between the AC side and the DC side of the power converter. In WEC, PWM will be used to switch the VSC switches in order to control the reaction force of the linear generator by controlling the phase currents. There are several methods of constructing the PWM switching technique: the most common is carrier-based sinusoidal PWM.

As was explained in section 3.3.2, converter voltage $V_{mod}$ is the sole manipulated variable in open loop PWM schemes. By comparing the desired $V_{mod}$ with the triangular carrier signal, the PWM modulating signal is formed as shown in Figure 3.10. The generated PWM signal can be controlled by adjusting the amplitude and the phase of its fundamental signal, $V_{mod}$.

![PWM Switching](image)

Figure 3.10: (a) Sinusoidal PWM scheme (b) Generated $V_{MOD}$ due to PWM

3.4.2 Open-loop PWM Schemes

Figure 3.11 (a) depicts the open loop voltage control in a three phase control scheme. Note that the manipulated variable $V_{mod}$ (represent by PWM signal $d_{abc}$) is applied as the voltage control input to the VSC. The system controls the VSC without a feedback control loop, where the desired $V_{mod}$ is calculated from the information on the required phase currents. This involves the prior calculation of the real and reactive power needed by the load before the reference phase current can be known.
In order to generate the PWM signal which is applied to each semiconductor switch, the desired physical control parameters such as the input power, DC link voltage, and phase currents need to be defined. After the reference control value components are chosen, the control signal $V_{mod}$ can be calculated. The first step for calculating the control signal $V_{mod}$ is to identify the power requirement needed by the load. Once the requirement of the load power is known, the reference phase current information can be extracted from it and together with the information on the total impedance between the source voltages and VSC, a control signal $V_{mod}$ can be determined. The general steps for calculating $V_{mod}$ is described in the following section revealing the two control components, $m$ and $\delta$.

### 3.4.2.1 Modulation Index and Phase Modulation

In this section, the analytical approach describing the steps of calculating the modulation index, $m$ and phase modulation, $\delta$ is briefly explained. The voltage control signals at the converter side, $V_{mod}$ can be considered as a composition of the modulation index $m$ and phase modulation $\delta$. In order to determine the required modulation index and phase modulation, as previously explained, prior calculation of the real and reactive power is needed. After the power requirement is calculated, both modulation parameters $m$ and $\delta$ can be derived. This can be done by applying Kirchhoff Voltage Loop (KVL) on the one phase VSC equivalent circuit model as shown in Figure 3.9(a).

As stated in (3.32) the phase current $i_L$ is due to the voltage difference between converter side voltage $V_{MOD}$, and the source voltage $V_S$. The power transfer can be controlled by changing the direction and amplitude of $i_L$ through the circuit. Since the source voltage...
The voltage source is determined by the generator induced voltage, the only option is to control $V_{MOD}$. Thus the phase current can entirely be controlled by changing the phase and amplitude of the converter voltage $V_{MOD}$, which defines the amount of power to be transferred between both sides of the VSC.

Assume the voltage source and the phase currents are single-phase sinusoidal signals (generated from a conventional generator), i.e.

$$V_s(t) = \hat{V}_s \cos(\omega t + \varphi_s) \quad (3.40)$$
$$I_s(t) = \hat{I}_s \cos(\omega t + \varphi_i) \quad (3.41)$$

The desired output power, which consists of real and reactive power, have to be defined. The total impedances, voltage and the currents equations expressed in (3.32), (3.40) and (3.41), are presented in the complex form

$$V_s = \hat{V}_s \angle \varphi_s \quad (3.42)$$
$$I_s = \hat{I}_s \angle \varphi_i \quad (3.43)$$
$$Z_T = \hat{Z}_T \angle \varphi_z \quad (3.44)$$

Multiplying (3.42), $V_s$, with the conjugate of (3.43), $I_s^*$, yields the complex power $S = \hat{V}_s \hat{I}_s \angle \theta$. This power (either supplied or taken by the source) can be decomposed into two complex components, real power $P$ and reactive power $Q$.

$$S = P + jQ \quad (3.45)$$
$$S = \hat{V}_s \hat{I}_s \cos \theta + j \hat{V}_s \hat{I}_s \sin \theta \quad (3.46)$$
Based on the requirement of the real and reactive power expressed in (3.45) and (3.46), the reference currents for the VSC then can be obtained. In (3.45) the phase angle $\theta = \phi_r - \phi_i$ is the power factor. Then the control signal $V_{mod}$ can be calculated using the complex form of (3.32), i.e

$$\hat{V}_{mod} \angle \delta = \frac{V_s}{Z_r \times I^*_s} \angle \phi_s - (\phi_s + \phi_i)$$ (3.47)

After the control signal $\hat{V}_{mod}$ is known, modulation index $m$ and phase modulation $\delta$ can be obtained from the expression given in (3.49) and (3.47), where $m$ is directly proportional to the $\hat{V}_{mod}$:

$$m \angle \delta \propto \hat{V}_{mod} \angle \delta$$ (3.48)

for the three-phase PWM-VSC

$$m = 0.6123 \frac{\hat{V}_{mod}}{V_{dc}}$$ (3.49)

When the components of the control signal $m \angle \delta$ is compared with the triangular carrier signals, the control PWM signals will be generated such as the ones shown in Figure 3.10 (a). The PWM-signals then will be applied to the IGBT switches in the VSC as a manipulated variable $V_{mod}$ and is used for regulating the voltage and/or current for both sides of the converter.

An important feature of a conventional generator is that it delivers constant power: the power produced by a conventional generator does not fluctuate with time as it does in the linear generator of a WEC system. In the next section, the calculation of the open loop control signal of a WEC system is explained. Since the power delivered by the linear generator in a WEC is fluctuating, the required phase current is not derived from the generator power, instead the generator force that defines the maximum power from the waves is used.
3.4.2.2 Calculation of the control signal in WEC

The induced voltage and current are due to the movement of the linear generator's translator when the buoy is interacting with the waves. Thus the voltages and currents induced by the linear generator in a WEC vary with time. Instead of producing constant output power, the generated power from the WEC is constantly varying. Therefore the calculation of input power supplied by the linear generator in a WEC system is slightly different from the power requirement in a conventional rotary generator in a non-renewable power conversion system as discussed in the previous section, where it can be calculated directly from the expression given either in (3.45) or (3.46) which can be derived from the power demand at the load.

Conversely, in renewable power conversion systems such as wave energy, the power from the source is limited, therefore in order to achieve the desired power extraction the power requirement at the load is adjusted to suit the power available from the source. An overview of the theoretical concept of the optimum power transfer from the waves to the WEC was discussed in section 3.1.

In order to calculate the control signal, \( V_{\text{mod}} \), for wave power extraction in the WEC system, the control strategy that best reacts with the incident waves is required. This can be done, by deriving the generator force that is believed to extract the maximum power from the source, or at least which drives the generator to operate at the optimum condition. For the direct drive WEC, the control signal that allows this modification is calculated from the generator force: this type of control is called reaction force control. This force is derived from the hydrodynamic study on the point absorber WEC, which includes the dimensions of the buoy and the conditions of the incident waves, and also due to the capacity of the linear generator in the WEC. Based on these inputs, the desired force can be calculated and the required phase currents can be obtained.

\[
P = Fv
\]

(3.50)

In general, the amount of power extracted from the wave by the WEC can be expressed according to (3.50), where the power is calculated based on the force, \( F \), and the velocity \( v \). For the case of a WEC, by referring to Figure 3.1 (b), the definition of power can be further clarified into two points of view. Firstly, the source defines how much power can be absorbed by the oscillating bodies (buoy of WEC) due to the interaction with the
incident wave. In this case $v$ refers to the velocity of the WEC's oscillating body and $F$ refers to the wave excitation force, $F_{\text{ext}}$. Secondly the load defines how much power captured from the wave can be transferred to the WEC power take off (linear generator). Since the linear generator is connected directly to the oscillating body (buoy), $v$ in this case refers to the velocity of the generator's translator and $F$ refers to the generator force $F_{\text{gen}}$ produced by the movement of the translator.

From (3.50), for both definitions of power described previously, at a certain frequency the maximum value of power, $P$, is calculated when the velocity $v$ is in phase with the force $F$. From the viewpoint of the WEC, this occurs when it operates at resonance and during this period it absorbs maximum energy from the sea. Thus the WEC should be controlled in order to increase the system efficiency while in operation either individually or in arrays. As mentioned, for the direct drive wave energy conversion, the generator force $F_{\text{gen}}$ is used. This control is achieved by forcing the phase generator currents to follow the reference phase currents which are derived from the comparison with the reference power take-off force $F_{\text{PTO}}$.

To derive the reference phase current, the reference force $F_{\text{PTO}}$ needs to be determined by the controller using the expression either given in (3.15) for the regular wave or as given in (3.23) for irregular waves. Once the value of the reference force $F_{\text{PTO}}$ is known, the values of the reference phase current for the linear generator can be obtained by comparing $F_{\text{PTO}}$ with the generator force $F_{\text{gen}}$. After the derivation of the reference currents is made, the methodology of open loop control as described in sections 3.4.2 and 3.4.2.1 can be used, by controlling the voltage control signal $V_{\text{mod}}$ through $m$ and $\delta$ of the WEC side VSC. The details of the control strategy for the WEC to capture energy from the waves will be discussed in the chapter 4.

3.4.3 Closed-loop control Scheme

The block diagram in Figure 3.11 (b) shows the close loop control scheme for the three-phase VSC. The difference between the close loop control and the open loop control is that the control variables are measured and compared with the reference setting, with the controller responsible to bring the control variables towards their reference values using the feedback control loop. Any deviations from the reference values will result in control
action to the inputs of the VSC. As mentioned previously the VSC can control the power to flow in both directions.

For the DC side, the common control objective for the VSC converter is to keep the DC voltage at the reference value. On the other hand, on the AC side, the VSC is controlled to meet two main objectives. First it ensures the phase currents follow the reference currents, and second it ensures the power requirement between the source and the load is fulfilled. If only the real power is considered, the current is controlled to be in phase with the phase voltage, otherwise the current will be controlled according to the requirements of the real and reactive power requirements.

Although their objectives seem to be different, the control schemes for both sides of the converter are directly related to each other, as the power requirements between the two sides of the converter have to be balanced. Furthermore compliance with the limitation of the voltage levels that the VSC can handle is required and must be fulfilled by the controller to ensure the operation of the power converter stays within its range of operation. The relationship between both sides of the VSC [60] is defined in (3.49).

3.4.3.1 DC Voltage Regulator

Since the control scheme for the both sides of the VSC are related to each other, controlling the level of DC voltage to its desired reference value can affect the required AC reference phase current signal. Similarly controlling the AC phase current according to its reference signal will affect the desired level of the DC voltage.

Conventionally the voltage level for the DC side of the VSC is controlled by comparing the measured DC voltage \( V_{dc} \) with the reference setting \( V_{dc}^* \). The error signal is fed into the controller block to generate the amplitude of the AC reference phase current \( I_m \) via (3.51), where \( G_c \) represents the controller gain. In order to ensure the reference phase current has the same shape of waveform as the mains supply, the output of the controller block is multiplied by the mains supply voltage waveform as described in Figure 3.12 and Figure 3.13. For the 3-phase rotary generator, the information of the sinusoidal mains supply waveform is provided by the Phase Locked Loop (PLL) block, which functions to synthesise the frequency of its input three phase sinusoidal signals. As in the open loop
control scheme, the generated control signal is used to produce the PWM input signal for the VSC block.

\[ \hat{I}_m = G_c (V_{DC}^* - V_{DC}) \]  

(3.51)

The literature [30], [32], [56] and [58] describe two different methods for generating the control signals of a VSC control loop scheme to control the DC voltage \( V_{DC} \) and the AC phase currents. The first method, called voltage source current controlled, controls the phase and magnitude of the converter voltage, \( V_{\text{conv}} \), and the second method, called current source current controlled, controls the input phase current. Both methods aim to keep the DC level voltage at the desired value by controlling the AC phase current.

### 3.4.3.2 Voltage Source Current Controlled

If the values of the source impedance, \( Z_s \), is known or can be highly predicted during operation, the generation of the control signal using the voltage source-controlled method can be simpler and less complex than the current source-controlled method. The derivation of the converter voltage, \( V_{\text{conv}} \), which depends on the accuracy of the prediction on the total impedances between the mains supply and the VSC, does not require any extra control except to keep the DC link voltage at its desired reference level. The control scheme for the single phase diagram voltage-controlled method is shown in Figure 3.12.

The expression for the reference phase current amplitude \( \hat{I}_m \) defined in (3.51) can be used to calculate the modulation voltage amplitude \( \hat{V}_{\text{mod}} \) in voltage-controlled method. The control is achieved by multiplying the \( \hat{I}_m \) with the impedance value of the generator. Then the control signal \( V_{\text{mod}} \) is generated as the operating frequency information of the mains voltage is provided by the PLL block. Once this is applied, the resulting control signal \( V_{\text{mod}} \) should have the same frequency as the mains voltage. By changing the amplitude and phase-shift of \( V_{\text{mod}} \) with respect to the mains voltage, the input currents are controlled to fulfil the requirement of the input and the output power. Although the voltage source-control method offers a simpler control technique, the main drawback of this
scheme lies in the dependency on the accurate prediction of the input impedances, $Z_i$, which are known to be varying [61].

![Diagram of Voltage Source Current-Controlled](image1)

**Figure 3.12: Voltage Source Current-Controlled**

### 3.4.3.3 Current Source Current Controlled

![Diagram of Current Source Current-Controlled](image2)

**Figure 3.13: Current Source Current-Controlled**

Figure 3.13 shows a control scheme for the current source-control method. It can be observed that the information of the generator impedance is not required. Instead the control is achieved by measuring the instantaneous phase currents and forcing them to follow the reference phase currents which can be derived from the comparison between the DC voltage and its reference value. In order to implement this scheme, an additional current controller is needed and acts to control the current’s amplitude and phase according to the requirement of the power at both sides of the converter.
Since the values of actual impedance are not required, this control scheme is more stable when compared with the previous scheme described in Figure 3.12, where the impedance parameter of the generator is known to be varying over the operating frequency. Therefore the current source-controlled method is preferred in most applications as this technique provides a robust system that leads to the improvement of the overall control performance [61], although the control scheme is more complicated than the previous one.

Both types of current-control method are used in this study for extracting the power from the wave. In a direct drive WEC system, the control concepts described above are applied to the linear generator to control the extracted power so that it suits with the requirement of the power grid as is discussed in this sub-section.

3.5 Chapter Summary

Chapter 3 has reviewed the theoretical background which is required to model the components of a WEC. The description starts with the introduction of the concept of a point absorber WEC and the equation of motion that can be expressed in both the frequency and the time domain. Both expressions are used to describe the behaviour of the oscillating body due to regular and irregular waves. In sub-section 3.2.2, the analytical derivation of the WEC power take off is discussed. The explanation of the induced voltage and the generator force provides the basic understanding on how these equations of interest are derived due to the motion of the generator translator. The chapter ends with a description of the PWM-VSC working principles and its control strategies, which are important to understand before the controller for managing the power from the WEC can be designed.
CHAPTER 4

Electrical Power Acquisition Of WEC In Arrays For Regular Waves

4.1 WEC Interconnection in Arrays

Collection of the electrical power generated from a wave farm and its transmission to shore depends upon the characteristics of the power take off device, the type of interconnection of individual WECs in the array and the distance to shore. The derivations of the power take-off (electrical linear generator) electrical elements used in this study has been described briefly in the previous chapter. The induced force and its electrical voltage due to the motion of the translator are derived using the theory presented in equation (3.25) and (3.29) respectively.

As seen in Figure 4.1 (a), theoretically the induced voltage for a direct drive WEC varies in amplitude and frequency, which results in pulsating generated power. This unconventional waveform occurs because a linear generator operation in a direct drive WEC system, is different to the conventional rotary generator due to its reciprocating motion. Figure 4.1 (b) illustrates the example of the direct drive WEC concept based on a linear generator where it is connected directly to the buoy which may be semi-submerged or fully submerged below the surface of sea water. To reduce these problems, the direct-drive WEC should to be connected in arrays in order to mitigate the power fluctuations produced by a single WEC.

In this chapter a discussion on the WEC working in arrays is presented. The various array configurations covered in this study and their control strategies for regular waves based on optimal and sub-optimal control will be discussed through case studies to investigate their effectiveness of controlling power flow from an array to the main AC grid.
Figure 4.1: (a) Top - Simulation of a linear generator three phase induced voltage due to the regular wave. (b) Right- Illustration of direct drive Wave Energy Converter (WEC) based on a linear generator (from Fig 1 in [62]).

4.2 Aggregation of Power

In order to understand the ideal concepts of electrical power acquisition for WEC arrays it is helpful to look at the operation of power converters for conventional rotary generators. By applying proper switching control to the IGBT semiconductor switches on the VSC, the sinusoidal phase current on the AC side can be converted into nearly constant amplitude pulses. This transformation process happens when the three phase sinusoidal currents flow through the IGBT switches to complete a loop with the mains voltage. Figure 4.2 shows clearly the way of the generation of DC current using a two level VSC through its IGBT switches and anti-parallel diodes connected in the bridge configuration.

For the diagram shown in Figure 4.2 (a), only one phase is presented and the VSC parts involved are its two IGBT switches \(T_p\) and \(T_N\) and two anti-parallel diodes \(D_p\) and \(D_N\). The currents passing through each of these semiconductor devices can be referred as \(i_{T_p}, i_{T_N}, i_{D_p},\) and \(i_{D_N}\). However only \(i_{T_N}\) and \(i_{D_P}\) are directly involved for AC to DC power conversion, then the following explanation is based on these two semiconductor currents.

Let us consider sinusoidal current \(i_S\), as illustrated in the top plot of Figure 4.2 (c), is the current passing through phase \(A\) as shown in Figure 4.2(a) and (b). At any moment either \(T_p\) or \(T_N\) is conducting with only one of them allowed to conduct at any instant [63], which means when \(T_p\) is switched on, \(T_N\) is always switched off and vice-versa. During
a positive cycle when IGBT \( T_N \) is ON the current \( i_S \) is flowing through it and makes a complete cycle before it returns to the mains of another phase \( (V_{S-phase_B}) \) through another anti-parallel diode which is connected at the bottom terminal of the DC link (ex: \( D_N \) of phase B) as can be seen in Figure 4.2(a). Inversely, in Figure 4.2(b) shows the current \( i_S \) flowing through anti-parallel diode \( D_p \) when \( T_N \) is OFF, helping the generation of DC current \( i_{DC} \) and closing a loop either through capacitor \( C_L \) or the load.

The waveforms of these currents, \( i_{tn} \) and \( i_{dp} \) of phase A can be seen in the second and the third row of Figure 4.2 (c) which shows that the positive cycle of current \( i_S \) is equal to the sum of \( i_{tn} + i_{dp} \); however, only the current \( i_{dp} \) contributes to the \( i_{DC} \) generation. Therefore in the three phase VSC system, adding \( i_{dp} \) from each phase \( (i_{dpA} + i_{dpB} + i_{dpC}) \) generates the constant amplitude pulses of \( i_{DC} \) as shown in the bottom plot of Figure 4.2 (c).

![Figure 4.2: Conversion of sinusoidal input AC currents waveform into the DC direct current (Figure 4.2 (c) from Fig. 13 in [30]).](image)
A similar concept of power conversion would occur if several WECs are connected in parallel as shown in Figure 4.3 (a). However in direct drive wave energy power conversion with a linear generator (LG), the induced voltage and current from a single LG has varying amplitude and frequency, thus the power generated is pulsating and not suitable to meet the requirements of the electric grid. Therefore in order to transmit the power with less fluctuation, the direct drive WECs not only need to be connected in an array as proposed in Figure 4.3 (a), but an appropriate control strategy has to be applied in such a way that the power can be managed efficiently.

![Diagram showing the ideal combination of DC currents from multiple WECs connected to a DC link terminal](image)

**Figure 4.3:** Diagram showing the ideal combination of DC currents from multiple WECs connected to a DC link terminal

The phase current of the linear generator in WEC_1 is shown in the top plot of Figure 4.3 (b). Adding the three-phase semiconductor currents $i_{DA} + i_{DB} + i_{DC}$ (as in the previous case) will produce the DC current $i_{DC1}$ as shown in the second row of the plot. As depicted, the DC current $i_{DC1}$ from one WEC device does not create a constant amplitude like the generated $i_{DC}$ from a conventional rotary generator. However, when the DC current $i_{DCn}$ from each WEC in an array is added, the expected total DC current $i_{DC}$ should imitate the DC current $i_{DC}$ generated in the previous case. This can be achieved only when the voltage level at the DC link is controlled to be at a constant value. Furthermore each VSC connected to the WECs also must be able to produce the same level of DC voltage although each WEC faces a different level of incident wave. If the DC link voltage is kept at a constant value, the DC currents from each WEC can be efficiently converted and added to form a total pulse DC current $i_{DC}$ such as the one shown in the
third row of Figure 4.3 (b). The bottom plot of the Figure 4.3 (b) shows the magnified diagram of ideal $i_{DC}$ which has pulses of almost constant amplitude.

4.3 Array Interconnections

In this study two types of array interconnection are considered, DC and AC link interconnection. The AC link configuration is described later in Chapter 5 with irregular waves (multi-frequency waves). Both configurations have their merits and drawbacks, and the suitability of deployment depend on the types of power converter being used, size, location and the power capacity of a wave farm. However, the control strategies applied in both configurations are the same, but with additional controllers required in the AC configuration.

4.3.1 DC Link Interconnection

In this topology, each WEC is connected in parallel at the DC link terminal as illustrated in Figure 4.4. The VSC system shown is based on two level voltage source converters (VSC) sharing one VSC at the grid side to convert the induced voltages and currents induced by the LGs. The system consists of two sides: the WEC side VSC collects the generated power from the WECs and ensures each WEC operates according to their controller requirement. Meanwhile, the grid side VSC is used to control the level of DC link voltage as well as to control the output voltages and currents to the frequency and the requirement of the grid connection.

Figure 4.4: System Arrays Integration Options: DC link Interconnection
4.3.2 AC Link Interconnection

In the second topology each WEC power line is based on a back-to-back VSC system and are connected at the AC link terminal as depicted in Figure 4.5. This study is necessary when considering the high power levels in a WEC farm. The semiconductor switches of two level VSCs configuration are limited to medium voltage and current blocking capability [64]. Thus the AC link interconnection can be used as an option to avoid high voltages and currents at the semiconductor switches when multiple WECs are connected to a single grid side VSC as proposed in the DC link interconnection. The other option is to connect all the WECs in the DC link with a single multilevel converter at the grid side for operation at voltage levels beyond the rated limit. The advantage of multi-level VSCs in power systems is their high power density and excellent performance [65], thus they are suitable to be used in high power applications. The 3-level VSC has been proposed in a wind farm in [66], where a comparative study is carried out between multi-level VSC (3 levels VSC) and the line-commutated converter HVDC (LCC HVDC) to find the best method of HVDC transmission from the offshore wind farm. The result shows that multilevel VSCs work well up to 500MW. However its suitability for other renewable energy applications, especially for WECs based on a linear generator, needs further investigation [67].

![Figure 4.5: System Arrays Integration Options: AC link Interconnection](image-url)
4.3.3 Evaluation of Interconnection Options

The consideration of the proposed topologies as discussed previously does not indicate the superiority or inferiority of AC interconnection over the DC interconnection, since both topologies are feasible for the low and medium power wave farm [36]. The DC link interconnection offer a simpler configuration and fewer components involved, whereas the AC link interconnection can reduce the power barrier limitation of the IGBT switches by employing a back-to-back converter for each WEC. As a result additional VSC controllers are needed in an AC link interconnection.

A comparison of the feasibility, merits and drawbacks of both types of interconnection in a wave farm is presented in Table 4.1.

Table 4.1: Summary of Interconnection options

<table>
<thead>
<tr>
<th>Type</th>
<th>Advantage</th>
<th>Disadvantage</th>
<th>Suggestion Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td>Simple, fewer components involved, reduced power losses and less cost.</td>
<td>Limited power rating of IGBT switches due to a single grid side VSC</td>
<td>Small and medium wave farm near to the shore</td>
</tr>
<tr>
<td>AC</td>
<td>Reduce the rated current flowing through the self commutated IGBT switches and avoid its limited power rating by employed additional grid side VSCs.</td>
<td>Higher complexity of system control. Increasing harmonic content.</td>
<td>Medium or Large far offshore wave farm.</td>
</tr>
</tbody>
</table>

4.4 Regular or monochromatic waves

In ocean wave terminology, the regular wave is referred to as a monochromatic wave, which is defined as a single frequency wave of unidirectional sinusoidal function. The regular waves considered in this study refer to waves, which have monochromatic behaviour with a constant amplitude and phase for the full period of study. When the monochromatic wave is applied to the WEC, the oscillation of the body (WEC) due to the incident wave can be treated as a sinusoidal motion, therefore the equation of motion in the frequency domain defined in (3.15) can be used to analyse the behaviour of the dynamic movement of the WEC. In [68], the regular wave elevation $\zeta$ is defined as:
\[ \zeta = A \sin (k x - \omega t + \phi) \]  
(4.1)

where \( A \) is the amplitude of the waves, \( \phi \) is the arbitrary phase, \( \omega \) is wave frequency, \( x \) is a distance between neighbouring wave crests and \( k \) is a wave number which is given as 
\[ k = \frac{2\pi}{\lambda}, \lambda \text{ is the magnetic wave length in metres} \]

When evaluating wave propagation in one point of space [18], the wave number \( k \) is normally avoided and the equation (4.1) reduces to:
\[ \zeta = A \sin (\phi - \omega t) \]  
(4.2)

### 4.5 Irregular or multi-frequency waves

For irregular waves, the multi-frequency term normally is used to represent the real waves, but real ocean waves are non-linear and time variant. Thus it is almost impossible to model the exact real sea states just based from the statistical annual wave distribution or by a given input wind speed at a specific given location. In order to model the wave elevation to represent the condition of real sea states, some simplifications are made [68]. Generally the approximation of irregular waves is made by synthesising the wave elevation from a known frequency spectrum, either using measured wave spectra or using artificial spectra such as the Pierson Moskowitz, Jonswap or Bretschneider spectrum [69]. However, the irregular waves generated from the aforementioned spectrum are considered to give an approximation of the frequency content of actual recorded sea waves, thus the reconstruction process of the wave elevation using the Inverse Fourier Transform (IFT) is similar to the sum of a series of sinusoidal waves which may not represent the original signal waves [18].

As mentioned in the previous chapter, irregular waves in this study are modelled as a superposition of regular waves. Consider a few sinusoidal monochromatic waves; each has its own amplitude, frequency and phase, so the superposition of these sinusoidal waves will result in polychromatic or multi-frequency waves. Although by adding a number of sinusoidal signals does not really represent the real irregular waves, in a WEC control scheme, it is very useful for the assessment of the control performance under the influence of time varying incident waves and is a good starting point for the proposed WEC's control
system before further studies can be made for real ocean waves. The irregular waves considered in this study can be derived according to the following equation defined in (4.3):

$$\zeta = \sum_{n=1}^{N} A_n \sin(\phi_n + \omega_n t)$$  \hspace{1cm} (4.3)

where $n$ is the total number of monochromatic wave components.

The following section deals with the first case study of WECs working in an array under regular wave influences. The control schemes applied upon the WECs for several regular wave cases are presented. The derivation of the control algorithm for the WEC system was taken using the concept presented in Chapter 3. In this section, an insight of the principle control operation for the WECs working in an array under monochromatic waves is explained.

### 4.6 Case Study 1: Sub-optimal power extraction from WECs in Regular waves

Before the complete control structure for the WEC in an array can be modelled, it is important to determine the control strategy applied to an individual WEC. However, the control scheme should be valid for each WEC in the array to be controlled individually or collectively.

The section begins with a brief discussion on the hydrodynamic motion of a WEC when interacting with a regular wave. Since this heave mode of motion can be derived analytically using the equation of motion, the following sub-section will review the hydrodynamic behaviour of the heaving buoy in monochromatic waves.

#### 4.6.1 Equation of motion

Theoretically, energy from a known frequency wave can be absorbed by a large or small oscillating body, provided both are tuned [70]. In order to model the point-absorber WEC concept, a simple system of a heaving cylinder buoy which is constrained to move only in heave direction is defined in this model. The buoy dimension used in this study is vertical
and semi-submerged, has a height of 1.5m, a radius of 0.6m and is assumed to be 1.13m under the sea water surface. Based on these dimensions, the hydrodynamic study of the oscillating buoy can be determined using equation (4.4) and is modelled to imitate the buoyancy of the buoy over the incident wave.

### 4.6.1.1 Regular wave

For a regular wave, the complex amplitudes of the body’s motion \( x_j(t) \) can be obtained by applying Newton’s Law in the frequency domain, where \( j \) represents the mode of oscillatory motion of a body in oscillating waves (a body has six modes where \( j = (1) \) for surge, \( (2) \) for sway, \( (3) \) for heave, \( (4) \) for roll, \( (5) \) for pitch and \( (6) \) for yaw) and is induced by an oscillation in the direction of \( i \) [12-13], and can be written as

\[
\sum_{j=1}^{6} \left[ -\omega^2 (m_{ij} + A_{ij}(\omega)) + j\alpha(B_{ij} + B_{PTO}) + (K_{ij} + K_{PTO}) \right] x_j(\omega) = F_{ext}(\omega) \tag{4.4}
\]

The above equation is derived for one degree of freedom (1 DOF), which investigates the heaving motion and is used for an isolated point absorber WEC case \( (i = 3) \), which means the distance between the oscillating bodies is far enough so that the incident wave is not disturbed by the neighbouring pulsating bodies. As seen in (4.4) the damping and the spring stiffness terms \( (B_{PTO} \text{ and } K_{PTO}) \) for the power take-off (PTO-linear generator) are included into the original expression in (3.15) to provide control variables for the heaving body. As variable \( x_{j=3} \) represents the displacement of the heaving body, \( i\omega \) and \( -\omega^2 \) indicate first and second derivatives of the function \( x_j \) for velocity and acceleration in frequency domain representation.

In this section only regular waves are considered, thus coefficients for the WEC and its PTO are constants and as a result conversion of (4.4) into a time domain signal does not involve a convolution process as is normally required for the irregular wave case [71]. Therefore, conversion of the equation of motion into the time domain signal gives:

\[
(m_{ij} + A_{ij}(\omega))\ddot{x}(t) + B_{ij}\dot{x}(t) + K_{ij}x(t) + Z_{PTO} \dot{x}(t) = f_{ext}(t) \tag{4.5}
\]
The first three terms of (4.5) are known as the intrinsic impedance of the WEC which is derived from the concept of the mass-spring-damper system. The PTO of the WEC is represented by the load impedance, $Z_{PTO}$.

### 4.6.1.2 Sub-optimal Control

The control strategies discussed in this study refer to amplitude and phase controls with direct conversion to electrical energy by the use of a linear generator. Refer back to the Figure 3.1 (a) as illustrated in the preceding chapter: the load impedance $Z_{PTO}$ represents the WEC power take off (PTO), which is the linear generator (LG). If $Z_{PTO}$ is controlled through the damping and stiffness coefficients, the amount of extracted energy from the waves is controllable. Conventionally, this is done by controlling both the amplitude and phase of the heaving buoy so that its natural period (and that of the LG's translator) agrees with the wave period (incident wave frequency), by controlling the generator (load spring stiffness) $C_{PTO}$ and (damping coefficients) $B_{PTO}$ respectively [72]. However, in this section only sub-optimal control strategy is considered, which only uses the damping coefficient $B_{PTO}$ as a control variable. Therefore the sub-optimal load impedance at monochromatic frequency $\omega$ as given in (4.6) is composed of the damping coefficients, $B_{PTO}$ and the translator mass, $M$, which is coupled to the buoy, thus by expressing the power take off force as $F_{PTO}(\omega) = Z_{PTO}(\omega) j \omega x_j$ then

$$Z_{PTO}(\omega) = j \omega M + B_{PTO}(\omega) \quad (4.6)$$

The power take off force can be expanded in both domain representations as:

$$F_{PTO}(\omega) = (-\omega^2 M + j \omega B_{PTO}) x_j(\omega) \quad (4.7a)$$
$$f_{pto}(t) = B_{pto} \ddot{x}(t) + M \dot{x}(t) \quad (4.7b)$$

In order to control the buoyant response of the buoy, the phase current drawn from the linear generator has to be controlled to provide the PTO damping force, $B_{PTO}$ in (4.4). Having a damping coefficient will potentially drive the movement of the heaving buoy to sub-optimal power extraction. The control scheme is called sub-optimal because the spring stiffness coefficient, $K_{PTO}$, is not included in the control strategy.
4.6.2 Damping force coefficient on sub-optimal control strategy

For the optimization in the aforementioned control scheme, the PTO damping coefficient, $B_{PTO}$, needs to be set to its optimal value. In regular waves, the optimum damping coefficient can be calculated by using the direct derivative method.

![Graph showing impact of $B_{PTO}$ on absorbed power for an incident sinusoidal wave of frequency $\omega$ [44]]

The graph above shows the general diagram of the effect of applying various damping coefficients on the average power produced by the WEC for a single incident wave frequency where the mean power absorbed by the oscillating bodies is defined as

$$P = B_{PTO}v^2$$

As illustrated in Figure 4.6, if $B_{PTO} = 0$ then $P = 0$ or if $B_{PTO} \to \infty$ (strong damping) then $|v| = 0 \Rightarrow P = 0$. Thus the ideal damping coefficient due to regular wave can simply be determined by finding the best value of $B_{PTO}$ which will give the peak power absorption. Mathematically this can be done by rearranging the above equation with (4.4) and will result in the average power equation given in (4.8). The derivation of the average power for the optimal damping derivation can be found in Appendix B.3.

$$\bar{P} = \frac{1}{2} B_{PTO} \omega^2 x_3^* x_3^*$$

$$\left( (K_{33} - \omega^2 (m_{33} + A_{33}(\infty))^2 + \omega^2 (B_{33} + B_{PTO})^2 ) \right)$$

(4.8)
By determining the average power for a single WEC, the optimal damping coefficient for the optimum power absorption can subsequently be determined by taking the derivative of (4.8)

\[ B_{PTO}^{(optimum)} = \frac{d\bar{P}}{dB_{PTO}} = 0 \]  \hspace{1cm} (4.9)

Once the optimal damping value of \( B_{PTO} \) is known, the reference PTO force can be determined using the equation given in (4.7). The resulting \( F_{PTO} \) then is used as a reference to the generator force, \( F_{gen} \). As defined in (3.27), the force generated in the machine depends on the translator's displacement and the phase current of the generators, however, only phase currents can be used to control this force [47]. Therefore to implement the control scheme on the WEC, the generator phase current must be controlled so that the generator force \( F_{gen} \) is close to its reference power take-off force \( F_{PTO} \). This can be done generally by comparing (4.7) with the generator force \( F_{gen} \), in (3.27), where the error signal generated from this comparison is then fed to the controller. The details about this strategy will be explained in the following section.
4.6.3 Control Strategy for the WEC PTO

Two main approaches are presented to extract power from each of the WECs working in an array: generator side control and grid side control. Figure 4.7 shows the single power line diagram for the proposed WEC model. The components included are: point-absorber, PTO (linear generator-ACTM) and AC-DC-AC converter (PWM-VSC) where the generator-connected converter acts as an active rectifier to provide generator control, while the converter near the medium voltage hub (MV Hub) or grid acts as an inverter to control the power to the grid side.

Figure 4.7: ac-dc-ac PWM-VSC for a single WEC line

4.6.3.1 WECs side control

The main task of the active rectifier is to collect the power from the WEC by controlling the phase current of the linear generator and to set the DC link voltage level. For the case of WECs working in arrays, as previously mentioned the power collected from each WEC can only be collected efficiently by maintaining the DC link voltage at a constant value. Furthermore, a constant DC link shows that the active power exchange between the two converters is balanced [66]. Therefore in order to obtain the optimum power transfer from each WEC, appropriate control of the DC level is required and will vary due to the sea condition.

As can be noted in Fig. 4.7, the input variables for the control block are the reference power take-off force $F_{PTO}$, the information of the translator’s velocity $\dot{x}(t)$ and also the
reference phase current $I_{abc\_\text{wnref}}(t)$, where subscript $\text{wn}$ refers to the WEC number employed in the array. Based from this information, the modulation voltage called $V_{\text{conv}}$ will be generated and used to produce the PWM pattern for triggering the switches of the active rectifier.

As explained in section (4.6.2), in order to determine the required phase current, the required power take off force, $F_{\text{PTO}}$, needs to be compared with the generator force, $F_{\text{gen}}$, to provide the current reference gain coefficient $\hat{I}_p$ in (4.12). For this control scheme, the current is being controlled to be in phase with the induced voltage in order to verify compliance with the sub-optimal force as specified in (4.7). Hence the phase current will be of a similar form to the induced voltage and can be defined as follows:

$$I_{abc\_\text{wnref}}(t) = \hat{I}_p \cos \left( \frac{2\pi}{\lambda} + \gamma \right) \dot{x}(t) ; \text{where } \gamma = 0, \pm \frac{2\pi}{3} \quad (4.10)$$

Substituting (4.10) into the induced force equation for the ACTM given in (3.27) will give the expression of force produced by each phase [47], which can be simplified by

$$F(x,t)_{\text{gen}} = (\vec{F}_g \hat{I}_p) \dot{x}(t) \quad (4.11)$$

Comparing the above equation with the second term of (4.7a) yields

$$j\omega B_{\text{PTO}} x_3(\omega) = \hat{I}_p \vec{F}_g \dot{x}(t) \quad (4.12)$$

The above equation shows that the generator force $F_{\text{gen}}$ is proportional to the reference power take-off force $F_{\text{PTO}}$ generated in (4.4), provided that velocity $\dot{x}(t) = j\omega x_3(\omega)$ and $\vec{F}_g = K_f F_g$, in which $K_f$ is a constant coefficient of linear generator (LG). Thus the amplitude of the reference current can be determined by applying closed-loop control to (4.12):

$$\hat{I}_p = \frac{G_f}{1 - G_f F_{\text{gen}}} \times F_{\text{PTO}} \quad (4.13)$$

where $G_f = K_f + \frac{K_i}{s}$.
$G_f$ represents a controller gain such as P or PI to bring the generator force coefficient, $\hat{I}_p F_g$, closer to the optimal damping coefficient $B_{pro}$. Once the ideal amplitude of the current reference $\hat{I}_p$ is reached through a controller $G_f$, the reference generator phase current can be determined using the expression given in (4.10). A block diagram of reference current coefficient is presented in Figure 4.8.

To indirectly control the LG phase currents to follow its desired reference values, the voltage source current controlled (VSCC) method (discussed in section 3.4.3.2) can be used. By assuming the internal impedance of the linear generator ($L_n$ and $R_n$) are known, the control signal of the PWM-VSC, $V_{conv}$, is generated. In the VSC method, Kirchoff’s Voltage Law, KVL is used to derive the voltage modulation signal $V_{conv}$ as explained below.

### 4.6.3.2 Control signal of WEC side PWM-VSC

Basically the PWM-VSC can be replaced by their 3-phase equivalent voltage sources, thus the equation that links voltage and current through the linear generator can be written as

$$
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
= L_m \frac{d}{dt} \begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} + R_m \begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} + \begin{bmatrix}
V_{conv-a} \\
V_{conv-b} \\
V_{conv-c}
\end{bmatrix}
\tag{4.14}
$$

Equation (4.14) can be simplified into one phase expression as follows:

$$V_{conv}(t) = V(t) - L_m \frac{di}{dt} - R_m i_k
\tag{4.15}$$
Substituting (3.29) and (4.10) into the (4.15), will gives

\[ V_{\text{conv}}(t) = \dot{V}p \cos\left(\frac{2\pi}{\lambda} x(t)\right) - L_m \frac{d}{dt} \left[ I_p \cos\left(\frac{2\pi}{\lambda} x(t)\right) \dot{x}(t)\right] - R_m \left[ I_p \cos\left(\frac{2\pi}{\lambda} x(t)\right) \dot{x}(t)\right] \]

Solving the above equation and rearrange the velocity \( \dot{x}(t) \) and the amplitude of the reference current \( \dot{I}_p \), will give the modulation voltage \( V_{\text{conv}} \), which can be written as

\[ V_{\text{conv}}(t) = \left( \dot{V}p \cos(kx(t)) - \dot{I}_p \left[ (sL_m + R_m) \cos(kx(t)) \right] - kL_m \sin(kx(t) \dot{x}(t)) \right) \dot{x}(t) \quad (4.16) \]

where \( k = \frac{2\pi}{\lambda} \) and \( \lambda = 2(W_m + W_i) \)

The derivation of (4.16) can be found in Appendix B.4 which represented as per-phase control signal block diagram for the WEC side as depicted in Figure 4.9. The inputs for the controller block which are the velocity \( \dot{x}(t) \) and required power take-off force \( F_{\text{pro}} \) can be obtained through (4.4) and (4.6) together with (4.10) which provides the reference current information to the block. The output will be the switching states of the PWM signal, \( S_{abc} \).

**Figure 4.9:** Block diagram of the modulation voltage control for the WEC side converter

As discussed in section 3.3.2, the concept of the conversion power control in WEC can be understood by referring to the equivalent circuit of the PWM-VSC used in this study shown in Figure 3.6. In order to ensure the power extracted from the LG corresponds with the control scheme, the PWM-VSC is used to bring \( F_{\text{gen}} \) closer to \( F_{\text{pro}} \) by manipulating the state of modulation converter voltage \( V_{\text{conv}}(t) \) (previously referred as \( V_{\text{mod}} \)).
controlling the phase and amplitude of $V_{\text{conv}(t)}$ with respect to the induced voltage, $V(t)$, the control variable $I_{\text{abc-wm}(t)}$ (the phase current of linear generator) can be controlled.

Figure 4.10 below shows how the manipulated variable, $V_{\text{conv}(t)}$, is controlled in amplitude and phase to generate the desired phase current (second row) and by doing so it drives the generator force, $F_{\text{gen}}$, to follow the reference power take-off force, $F_{\text{PTO}}$, as can be seen in the third and fourth row of Figure 4.10 below.

![Simulation graph for control variables of WEC 1](image)

**Figure 4.10:** Simulation graph for control variables of WEC 1
(From Top-Bottom): Induced Voltage: $V_a(t)$ vs Manipulated Voltage: $V_{\text{conv}(t)}$, phase current: $I_a$, reference Force: $F_{\text{PTO}}$, and Generator Force: $F_{\text{gen}}$
4.6.3.3 Grid side control

The conversion of energy from an array of direct drive wave energy converters consists of several steps. The primary conversion is the transfer of energy from the incident wave to the point absorber. The sub-optimal control strategy on the oscillating buoy of a point absorber WEC is described in sections (4.6.1) and (4.6.2). The secondary conversion step is made by the electrical generator to convert mechanical power into electrical power. The power from the WEC is controlled close to unity power factor in order to achieve the sub-optimal power extraction from the wave; the control strategy to achieve this is explained in previous section (4.6.3.1). The last stage considers the collection of power from each WEC in an array and transferring it to the grid is the topic discussed in this sub-section.

The primary requirement of the grid side VSC is to control the voltage level of the main DC bus at the desired reference value and at the same time to ensure the power converted corresponds to the required grid AC voltage and frequency. Due to the capability to control active and reactive power as separate control variable entities, the conventional VSC synchronous rotating coordinates \((d_q)\) control is proposed in this study. The discussion on abc-\(d_q\) transformation for a 3-phase circuit system is beyond the scope of this thesis; however the details can be found in [59]. The transformation of abc-\(d_q\) is given in (4.17), where \(\omega_g\) is defined as the electrical operating frequency.

\[
[T_{dqp}(\omega_g t)] = \frac{2}{3} \begin{bmatrix}
\sin(\omega_g t) & \sin(\omega_g t - \frac{2\pi}{3}) & \sin(\omega_g t + \frac{2\pi}{3}) \\
\cos(\omega_g t) & \cos(\omega_g t - \frac{2\pi}{3}) & \cos(\omega_g t + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]  

(4.17)

In order to control the DC link voltage, the main DC bus voltage is measured and compared with its reference value. The error after passing through a conventional PI block is then used as an input parameter to the grid side control block to calculate the reference current \(i_d(I_{inv,q})\) which determines the real power production need to be transferred to the grid. At this side, the controller is set for unity power factor, thus, only real power is transferred to the grid, and the reference current \(i_q(I_{inv,q})\), which provides information on reactive power transmission, is set to zero.
A block diagram for the first control block for grid side control is given in Figure 4.11.

![Block Diagram](image)

**Figure 4.11:** Block diagram of DC voltage control

As can be seen in Figure 3.9(b) and 3.9(c) in the previous chapter, by measuring the voltage for every parameter over one loop in the d-q equivalent circuit, the voltage equation in synchronous d-q coordinates can be obtained.

\[
V_{ds} = V_{d\text{-conv}} + \alpha L_{abc}(i_q^*) - (R_{abc} + j\omega L_{abc})i_d^* \quad (4.18a)
\]

\[
V_{qs} = V_{q\text{-conv}} - \omega L_{abc}(i_d^*) - (R_{abc} + j\omega L_{abc})i_q^* \quad (4.18b)
\]

Since the line impedance of the grid varies with the number of buses which is attached to it and also as there is a possibility of other WEC arrays connecting to this side to create a medium voltage hub, it would be beneficial if line impedances are avoided in the control block. Therefore to determine the appropriate voltage modulation for the grid side control, the output phase currents are measured to become control variables and will be forced to follow the desired current reference value. Thus equation in (4.18) can be further simplified to be:

\[
V_{ds} = V_{d\text{-conv}} + X_{L_{abc}}(i_q^*) - G_{id}(i_{d\text{ref}} - i_d^*) \quad (4.19a)
\]

\[
V_{qs} = V_{q\text{-conv}} - X_{L_{abc}}(i_d^*) - G_{iq}(i_{q\text{ref}} - i_q^*) \quad (4.19b)
\]

In (4.19) another two conventional PI controllers \((G_{id} \text{ and } G_{iq})\) are added to the control block to control the DC voltage as well as the output phase currents of the grid side. The extension control block diagram of the VSC shown previously in Figure 4.11 is shown in Figure 4.12, where the frequency of the output voltage and current will be set by the phase locked loop, PLL block based on the requirement of the grid voltage. The grid side control scheme described above is used for all case studies presented in Chapter 4 and Chapter 5.
4.6.4 Wave Energy Converters in an Array

As mentioned previously, instead of producing a constant output power, the generated power from the LG is constantly varying. Thus in order to generate a smoother electrical power, in this study the WECs are connected in parallel either in the DC or the AC link. This connection will form the WEC array. In the previous section, the control algorithm based on sub-optimal control for an individual WEC is explained. The control strategy for both sides of the WEC is elaborated to introduce the background of the WEC power absorption concept applied in this study.

The simplified control structure for the WECs working in an array (DC link) can be seen in Figure 4.13. Several control strategies for the voltage source converters (VSC) are used to control the power from the array into the grid, providing both generator and grid side control. The WECs are connected to the main DC link with limited energy storage (DC link capacitor) to keep the DC voltage at a constant value. The main purpose of the VSCs on the generator side, which act as active rectifiers are the same as with the single WEC control, to ensure the power generated from each WEC corresponds with the control strategy and to produce the correct phase current control in the corresponding LG. Meanwhile the function of the VSC converter on the grid side is to control the voltage at the main DC bus and at the same time control the power transfer to the grid.

Figure 4.12: Block diagram of inner current control at the Grid side converter
For the optimisation of the WEC farm energy yield, several control structures on the WEC side for controlling parallel connected VSC are presented. The control structures proposed in this study control the power in an array of WECs in which each individual WEC can be controlled either individually or as a group.

Based on the concept of parallel rectifiers made for an array of boost AC-DC high power converters in [73], the approach for controlling multiple conventional rotary generators is extended to wave energy converters (WECs) with direct drive linear generators (LGs). The objectives for the different control structures are: (i) To reduce the complexity of the control structure without affecting the control performance (ii) To maximise the overall efficiency and (iii) To reduce the number of components operating in each WEC power line.

For this reason, three possible parallel control structures are considered, namely:

i) Shifted modulation voltage signal

ii) Mutual current reference control

iii) Independent Damping Force Control

Each control structure proposed for the array and the components involved in it are briefly explained and illustrated in section 4.6.6.
4.6.4.1 Indirect Current Control

As can be seen in Figure 4.13, several control blocks are used to control the two interrelated control variables in the WEC side, the LG phase currents and the generator force, $F_{gen}$. The methods used for both sides are based on the closed-loop control scheme presented in section 3.4.3. The voltage source current controlled (VSCC) method is applied to the WEC side, while on the grid side, current source current controlled (CSCC) is used. The first method applied in this study can also be known as indirect current control since the phase LG currents are controlled by controlling the voltage at the converter side. The VSC’s control signal, $V_{conv}$, is generated by measuring the voltage across each element in the WEC side of the circuit shown in Figure 3.9 (a). As the WECs are connected to the DC link, the control scheme can be used to estimate the value of the reference voltage level at the main DC link.

For the grid side, as with controlling an individual WEC, the array uses a conventional CSCC method called the synchronous rotating coordinates (d-q) scheme. Its function is to control the DC link voltage and the AC phase currents sent to the grid. A useful feature of the VSC is that it is capable of transferring power in both directions, either from the AC to the DC side or from the DC to the AC side, simply by controlling the power switches in the PWM-VSC. This feature is important for the grid side control because there are times when the power generated from an array is not able to produce a constant output power despite having multiple WECs connected in the array, and as a consequence can cause the DC link voltage to drop below its reference level. This happens due to the varying magnitude and frequency of the incident waves. To avoid this event occurring, power in the grid side converter is controlled to flow in both directions.

4.6.4.2 Collected DC link currents

This section deals with the mathematical description of the converted DC currents from the AC phase currents produced by each WEC connected either on the DC link or on the AC link interconnected systems.
The simplified connections of an array of WECs in Figure 4.4 and 4.5 are again shown for reference in Figure 4.14. Neglecting the line impedance, the current equations for the DC link interconnection can be determined by adding the pulse DC current $I_D$ from each WEC:

$$I_{Dn} = S_a I_{an} + S_b I_{bn} + S_c I_{cn} \quad (4.20)$$

where:

$$S_{a,b,c} = \begin{cases} 1 & ; V_{\text{comp}_{-abc}} > \text{carrier signal} \\ 0 & ; V_{\text{comp}_{-abc}} < \text{carrier signal} \end{cases} \quad (4.21)$$

where $n = 1, 2, \ldots, 6$ and $S_{a,b,c}$ are the switching states of the PWM-VSC, whereby

$$I_{DT} = \sum_{n=1}^{6} I_{Dn} \quad (4.22)$$

Substituting (4.20) into (4.22), will give the total DC current in the main link

$$I_{DT} = \sum_{n=1}^{6} S_a I_{an} + S_b I_{bn} + S_c I_{cn} \quad (4.23)$$

where the 3-phase currents for each WEC $I_{abc}$ in (4.23) are defined in (4.10)
The DC link capacitor, $C$, plays an important role to balance the power between the generator and the grid. Thus the capacitor current, $i_c$, must be controlled in order to keep the capacitor voltage at its desired level. From Figure 4.15 the pulse DC currents involved at the junction determine $i_c$:

$$I_{DT} = i_c + i_{DC}$$

$$i_c = C \frac{dV_c}{dt}$$

(4.24)

From (4.24), there are two logical hypotheses; firstly, when the measured DC voltage has reached its reference level, the pulse capacitor current $i_c$ used to charge the capacitor will no longer be needed. Therefore the collective DC current $I_{DT}$ now directly contributes to the generation of the current $i_{DC}$. On the other hand, if the measured voltage is still less than the reference DC level, $i_c$ will be required to support the DC link voltage even when the input power received from the wave is very low. In case of shortage of collective DC current $I_D$, extra current is needed as an auxiliary current source to the DC link capacitor, so it is important for the grid side to have a reversible power flow.

### 4.6.4.3 Reversing Power Flow During Power Shortage

During a power shortage, the current available to charge the DC link capacitor is reduced, and can cause the DC link voltage to drop. If this happens, the individual phase current in the generator side VSCs can reverse. To avoid this, it is necessary to keep the DC voltage at the same level to ensure the total power produced by each WEC is efficiently collected. To overcome the power shortage impact on the array, the PWM-VSC at the grid side is used to control current $i_{DC}$ to enable the current $i_c$ to charge the DC link capacitor and allow the main link to reach the desired DC voltage. As shown in Figure 4.15, during normal operation, the power generated by the WECs is delivered to the grid via both PWM-VSCs. However when a power shortage occurs, the grid side VSC will not act as an inverter anymore, but is controlled to become a rectifier to support the power deficiency in the DC link by taking power from the grid. In the case of a large wave farm, the power is taken from the neighbouring WEC arrays which are at that time delivering power to the
grid. Once sufficient power is once again being extracted, the WEC arrays will return back to the normal operation and resume power generation to the grid via the PWM-VSCs.

![Figure 4.15: DC link- Reversing power flow from the grid](image)

Reversing the direction of power flow is achieved by controlling the reference real current \( i_d (I_{in}) \) via the grid side converter. Simply by changing the direction of the current, the power can be reversed. This case is similar to the conventional operation of a PWM-VSC in abc coordination, where, by reversing the phase angle \( \delta \) with respect to the source voltage phase angle \( \varphi \) (refer Figure 3.6) reverses the power flow. Reversing the real current \( i_d \) in \( d_q \) synchronous control can be achieved indirectly by controlling the \( \delta \) of the \( V_{conv} \). The total power to be delivered to the grid for two-phase synchronous reference frame \( d_q \) can be written as:

\[
P_{abc} = \frac{3}{2} (v_d i_d + v_q i_q) + \frac{1}{3} v_r i_o
\]  

(4.25)

Assuming that the linear generators are balanced 3-phase power sources and only real power is being delivered to the grid, the second and third terms in (4.25) will be zero, hence, the average electrical power is reduced to:

\[
P_{abc} = \frac{3}{2} v_d i_d
\]  

(4.26)

The only difference between the AC link compared to the DC link is the grid side output current, where current \( i_d \) or \( I_{inv} \) in AC link intersection is equivalent to \( \sum_{n=1}^{6} i_{dn} \).
4.6.5 Setup of simulations

4.6.5.1 Simulation types

Three simulation types based on the incident regular waves are conducted in order to observe the feasibility of the proposed system control of WECs working in arrays. All of them use regular waves with a single frequency of a typical wave pattern from a wave frequency spectrum. The regular wave types discussed in this section are as follows:

- **Type 1:** This is the ideal optimal power that WECs can be expected to produce where the buoys are incident with a series of regular waves having the same frequency and amplitude. However, the phase is distributed evenly with respect to the number of WEC in the arrays \((n)\), given by:
  \[
  \xi_n = A \sin(\omega t + \phi_{360/n})
  \]

- **Type 2:** A series of incident waves which synchronously move with the same frequency and amplitude. This study is made for observing the generated electrical power from the array of WECs when they received the synchronous pattern of the incident waves.
  \[
  \xi_n = A \sin(\omega t + \phi)
  \]

- **Type 3:** This case involves a series of incident waves which have different amplitude, frequency and also phase, but are still considered to be regular and have been modelled using a single frequency.
  \[
  \xi_n = A_n \sin(\omega_n t + \phi_n)
  \]

Each of these regular wave type is studied in association with the topologies described in Figure 4.18, 4.20 and 4.23. Figure 4.16 shows simulation of six series of incident waves meeting an array of WECs.
Figure 4.16: Types of monochromatic waves

(a) Ideal Phase Distributed Waves
(b) Synchronized Regular Waves
(c) Varying amplitude, phase and frequency

4.6.5.2 Simulation model of WEC

A six WEC array model has been developed in MATLAB/Simulink, which is used to verify the proposed control strategies and the mathematical model for each component involved in Figure 4.4 and 4.7. In order to acquire the results with an acceptable level of confidence, all the simulations for every case are run with a small sampling time of 2µs. Table 4.2 and Table 4.3 summarises the non-dimensional hydrodynamic parameters
generated by WAMIT, with the dimensions of the buoys and the linear generator characteristics used in the simulation. In this study the buoys are assumed to be excited by waves in water of infinite depth.

Table 4.2: Dimension of the buoys and non-hydrodynamic parameters

<table>
<thead>
<tr>
<th>Non-dimensional Hydrodynamic Parameter (case 1 and case 2, ( \omega = 1.1 \text{ rad/s} ))</th>
<th>( \bar{A}_{33} )</th>
<th>( B_{pro} ) (N/m/s)</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( \phi ) (rad)</th>
<th>( \angle F_j )</th>
<th>( LF_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Added mass ((\bar{A}_{33}))</td>
<td>0.477341</td>
<td>8396.176</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Added damping ((\bar{B}_{33}))</td>
<td>0.0533273</td>
<td></td>
<td>1030</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excitation force coeff. ((\bar{F}_j))</td>
<td>0.9297801</td>
<td></td>
<td></td>
<td></td>
<td>0.41123</td>
<td></td>
</tr>
</tbody>
</table>

| Non-dimensional Hydrodynamic Parameter (case 3, \( \omega = 1.3 \text{ rad/s} \)) | \( \bar{A}_{33} \) | \( B_{pro} \) (N/m/s) | | | | |
|---|---|---|---|---|---|
| Added mass \((\bar{A}_{33})\) | 0.4662479 | 8391.1757 | | | | |
| Added damping \((\bar{B}_{33})\) | 0.06368307 | | | | 0.87266 | |
| Excitation force coeff. \((\bar{F}_j)\) | 0.8596962 | | | | 0.7515863 | |

| Non-dimensional Hydrodynamic Parameter (case 3, \( \omega = 1.5 \text{ rad/s} \)) | \( \bar{A}_{33} \) | \( B_{pro} \) (N/m/s) | | | | |
|---|---|---|---|---|---|
| Added mass \((\bar{A}_{33})\) | 0.4527230 | 5096.12 | | | | |
| Added damping \((\bar{B}_{33})\) | 0.07062172 | | | | 0.43633 | |
| Excitation force coeff. \((\bar{F}_j)\) | 0.7846094 | | | | 1.241187 | |

| Non-dimensional Hydrodynamic Parameter (case 3, \( \omega = 1.8 \text{ rad/s} \)) | \( \bar{A}_{33} \) | \( B_{pro} \) (N/m/s) | | | | |
|---|---|---|---|---|---|
| Added mass \((\bar{A}_{33})\) | 0.4309009 | 3696.183 | | | | |
| Added damping \((\bar{B}_{33})\) | 0.07366829 | | | | 1.2566 | |
| Excitation force coeff. \((\bar{F}_j)\) | 0.6677955 | | | | 2.301610 | |

| Non-dimensional Hydrodynamic Parameter (case 3, \( \omega = 2.0 \text{ rad/s} \)) | \( \bar{A}_{33} \) | \( B_{pro} \) (N/m/s) | | | | |
|---|---|---|---|---|---|
| Added mass \((\bar{A}_{33})\) | 0.4172588 | 3496.15 | | | | |
| Added damping \((\bar{B}_{33})\) | 0.07105643 | | | | -0.2618 | |
| Excitation force coeff. \((\bar{F}_j)\) | 0.5902652 | | | | 3.254127 | |

| Non-dimensional Hydrodynamic Parameter (case 3, \( \omega = 2.2 \text{ rad/s} \)) | \( \bar{A}_{33} \) | \( B_{pro} \) (N/m/s) | | | | |
|---|---|---|---|---|---|
| Added mass \((\bar{A}_{33})\) | 0.4055211 | 2864.1757 | | | | |
| Added damping \((\bar{B}_{33})\) | 0.06549319 | | | | -0.523598 | |
| Excitation force coeff. \((\bar{F}_j)\) | 0.5151713 | | | | 4.426323 | |
Table 4.3: Linear generator characteristics [26]

<table>
<thead>
<tr>
<th>Linear Generator – Air Cored Tubular Machine</th>
<th>Dimensions of the generator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of magnet, $R_m$ (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>Width of magnet, $W_m$ (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>No. of stator poles</td>
<td>4</td>
</tr>
<tr>
<td>Turns / coils</td>
<td>216</td>
</tr>
<tr>
<td>Voltage per coil at linear velocity 1.1 ms^{-1} (V)</td>
<td>47.5</td>
</tr>
<tr>
<td>F/ machine (N)</td>
<td>9760</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Internal impedance of the generator coil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal resistance (Ohm)</td>
</tr>
<tr>
<td>Inductance (mH)</td>
</tr>
<tr>
<td>Power (kW)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak phase (kW)</td>
</tr>
<tr>
<td>Rated power machine (kW)</td>
</tr>
<tr>
<td>Optimum power in the array (kW)</td>
</tr>
</tbody>
</table>
4.6.6 Simulation results

In order to verify the equations of motion used in this study, the simulation series begins by examining the validation of the oscillating of heaving buoy hydrodynamics as defined in (4.4) and (4.5). Figure 4.17 depicts the resultant for the LG's translator displacement and the reference PTO force \( F_{pro} \) of WEC 1 from the frequency and time domain points of view.

4.6.6.1 Frequency vs Time Domain

The displacement of the buoy, the power take-off force \( F_{pro} \) and the induced voltage and current of WEC 1 are shown in Figure 4.17. In the case of regular waves, both types of equation of motion in (4.4) and (4.5) can be used for investigating the dynamic motion of the heaving buoy since they would both eventually converge once the initial transients have died away. By examining Figure 4.17 (a) and (b) for the displacement and power take-off force, it can be seen that the results using the time domain function which was initially formulated to deal with irregular waves are similar to those generated using the frequency domain (when the transient period ends). The reason is that using the time domain function not only gives a valid output response at the steady state but also gives an accurate response signal during the transient period. However, for regular waves, analysing the transient behaviour of the heaving buoy is not as important as in irregular waves since it has a constant amplitude and phase of wave surface over time.

The induced phase voltage and current for WEC 1 are shown in Figure 4.17 (c) and (d) respectively. In the sub-optimal control only the damping coefficient is added into the equation of motion defined in (4.4), thus the phase currents generated by the LG of WEC can be controlled to be in phase with the induced voltage.

![Graph](image-url)
200
Induced Voltage
WEC1
32 34 36 time (s)
38
Phase Currents of WEC 1
0
-50
200
Voltage (V)
Current (A)
50
32 34 36 38 40
time (s)
Figure 4.17: Mechanical and Electrical results of WEC1

4.6.6.2 Type 1: Phase Distributed Waves

For the ideal distributed waves, the WECs are expected to generate the constant electrical power from the wave energy. Due to the control scheme and type of incident waves experienced by the array of WECs, the shifted modulation voltage signal configuration as shown in Figure 4.18 is proposed to control the power from the WEC rectifier. In this control topology, the control signal for each WEC rectifier in the array can be simplified using a common current reference control. This is possible when the series of similar monochromatic waves with different phases shown in Figure 4.16 (a) is applied to the array of WECs. Only one controller is needed to bring the generator force, \( F_{gen} \), towards its reference value, \( F_{pro} \) and will be applied to one of the WEC in the array (ex: WEC 2).

The control signal for other WECs in the array will be shifted with respect to the difference phase of incident waves between the WECs and WEC 2.

The concept of applying constant regular waves but with a different phase over the period of study is similar to power in a 3-phase system in the conventional rotary electrical generator discussed in section 4.2. Even though the induced voltages and currents have variable output levels due to the rotation of the generator’s rotor, constant output power can be produced by shifting the three phase element by 120 degrees. A similar result is expected to occur with WECs working in arrays. Despite the induced voltage and current in Figure 4.17 (c) and (d) having a varying amplitude and frequency due to the oscillation of the translator of WEC, the total power of an array produces nearly constant output power. Similarly for this particular wave device, near constant power can be generated by having at least three direct drive WECs in an array receiving ideal incident waves (as defined in Type 1 of section 4.6.5.1). The more WECs in the array, the smoother will be the resulting output power. It can be clearly seen in Figure 4.19 (c) and (b), where nearly constant current pulses are generated (for the DC current) and delivered (for the output AC
inverter current) by a group of WECs. The top diagram of Figure 4.19(b) shows the constant shape of the inverter/grid current produced over the time study period while the bottom shows it magnified over ten 50 Hz cycles.

Figure 4.18: Shifted modulation voltage signal system

With the PWM-VSCs controlled to obtain the desired DC output voltage, the DC current will be in a pulsed form as depicted in Figure 4.19(c). It should be noted that when the proposed control strategies are applied to the array, the generator force $F_{gen}$ is controlled to be close to the reference $F_{ref}$, which then will force the generator phase currents to follow the desired reference current as seen in Figure 4.19(a) for the WEC1 where the phase of the linear generator currents is forced to follow its reference value in the top diagram. On the grid side, the inverter output voltage and current is regulated to supply unity power factor output power as shown in Figure 4.19(d).
Figure 4.19: Electrical waveforms of WECs in array for DC interconnection (type 1)

4.6.6.3 Type 2: Synchronised Regular waves

In the second case as shown in Figure 4.16 (b), a series of six regular waves oscillating with the same frequency and phase are incident with the array. For this type of incident waves, mutual current reference control as shown in Figure 4.20 is proposed to handle the power from the array. A single control signal is needed to control all the WECs in the array where the control block only needs to observe the incident wave received by the prime WEC (ex: WEC 2) and the same control signals are then applied to every WEC in the array. The study observes the effects of the electrical behaviour of the linear generators due to the synchronous motion of regular waves incident with the array.

As expected although there are several WECs working in the array, the power extracted could not keep the DC link voltage level constant as can be seen in Figure 4.21 (a). The failure to maintain the voltage level at the DC link is because of the periodic lack of power when the array is incident with synchronised waves. The periodic voltage level drop from the reference DC value can be seen due to the synchronized motion of the heaving buoys. The impact of the power shortage becomes obvious in the $i_{DC}$ pulse current shown in Figure 4.21 (b) where the average shape of current is pulsating and is not constant compared to the $I_{pk}$ pulse current shown in Figure 4.19 (c) in the previous case. The effect
of an inconsistent voltage at the main DC link, can cause the generator phase current to deviate from its reference value and will drive the WECs away from its control mode scheme (sub-optimal). The phase current and the induced voltage of WEC1 are shown in Figure 4.21 (c) and (d) respectively.

Figure 4.20: Mutual current reference control

Figure 4.21: (a) DC link Voltage - 1000V (b) \( i_{dc} \) pulse current
The reversing power strategy as discussed previously in 4.6.4.3 is therefore proposed to overcome the power shortage in the array. By assigning a bidirectional power control in the grid side converter, the issue of variations in the DC link voltage can be avoided, whereby the power from the grid is used as an auxiliary source during a shortfall in power. Figure 4.22 shows the electrical waveforms in the array when the reversing control strategy is applied in an array where current is taken from the grid side.
The effect can be seen by observing the $i_{\text{DC}}$ pulse current in Figure 4.22 (a). Looking at the average value of $i_{\text{DC}}$ over the period, the amplitude is pulsating from positive to negative values. The shape of the $i_{\text{DC}}$ pulse current in an array gives two important indications. First, when the signal is positive, the array is generating power to the grid. Second when the $i_{\text{DC}}$ current is in the negative region its power direction is now reversed with power charging the capacitor to keep the voltage at the main DC link to its reference DC level. The action of delivering (inversion) and reversing (rectification) the current at the grid side can be seen clearly in Figure 4.22 (b) through its magnified diagram in Figure 4.22 (c) and Figure 4.22 (d) respectively. It can be assumed here that the need for reactive power is dispensable and therefore only the real power is considered. Therefore the grid side inverter phase current can be seen to be in phase with the output voltage when power is feeding into the grid. Inversely, when the power is taken from the grid the inverter phase current will be out of phase with the grid voltage.

Since the array is now receiving back-up power from the grid, the voltage at the DC link is successfully restored to a nominal value and generator phase current at the WEC side can be controlled to its reference value as shown in Figure 4.22 (f) for WEC 1. The DC link voltage after the power reversal is shown in Figure 4.22 (g).

### 4.6.6.4 Type 3: Regular waves with varying amplitude, frequency and phase

For the third case, the series of incident waves experienced by the WECs is still regular but is varying for each WEC. Since the incident waves vary in frequency, the Independent Damping Force Control structure as depicted in Figure 4.23 is proposed. In this topology each WEC in the array requires a controller on the WEC side to provide generator force and phase current control. By carefully observing the shape of the wave signal shown in Figure 4.16 (c), one can see the varying elevation of the waves overlap each other. For WECs working in arrays, the electrical output power generated can be predicted by observing the inverter output current of the grid side converter as shown in Figure 4.24 (b). With the output grid voltage considered fixed, the shape of the power produced will be relatively similar to the inverter/grid current. Figure 4.24 (c) shows a varying magnified 50 Hz inverter phase current delivered to the grid.

By controlling the DC link voltage at the desired level (as in Figure 4.24 (a)), the objective to control the generator force through its phase current is accomplished. As can be seen in
Figure 4.24(d) the measured phase current of WEC 6 (bottom fig.) follows the reference current (top), so it may be concluded that sub-optimal control scheme is successfully applied in the array. Referring to discussions before, in order to control the generator force as defined in (4.11) to follow the desired power take off force as stated in (4.7), the phase current drawn from the generator must be at the desired reference value. Thus achieving a constant DC link voltage, satisfies two control objectives: first, to force the generator phase current to follow the reference phase current; and second to ensure that the generated power from each WEC is efficiently added at the DC link.

Figure 4.23: Independent Damping Force Control
As expected, by having a varying series of incident waves, the real power delivered to the grid also varies. However, the control strategies applied at the grid side converter are still able to produce almost unity power factor as can be seen in Figure 4.24(e). The phase output current is controlled to be in phase with the grid voltage, despite its amplitude changing over time.

![Figure 4.24: Electrical waveforms produced by WECs (case 3)]

The DC link current when reversing control is applied is depicted in the Figure 4.25. Since the DC link is connected between the two VSCs which are continuously switching ON and
OFF, the current flowing through it is a series of pulses. Figure 4.25 shows the behaviour of the DC current when a power shortage occurs as discussed in section 4.6.4.3.

Figure 4.25: Pulse DC current using sub-optimum control during the power shortage
(Top)- Pulse DC current before the capacitor (I_D). (Second row)- Summation of capacitor current (i_c) and pulse DC current after capacitor (i_{DC}). (Third Row)- Capacitor current (i_c). (Bottom)- Pulse DC current after capacitor (i_{DC}).

During a power shortage period, the DC current I_D is reduced which results in a reduction in the DC link voltage. Therefore in order to overcome this problem power is taken from the grid to re-charge the DC link capacitor. As can be seen from careful examination of Figure 4.25, the second row (i_c + i_{DC}) is similar to the top diagram (I_D). Thus the relation of currents in the DC link defined in (4.24) remains true even when the pulsing DC current (i_{DC}) reverses its direction as seen in the lower diagram of Figure 4.25. The required level of capacitor current (i_c) to charge the capacitor to the desired level of DC link voltage is shown in the third row of Figure 4.25, which is slightly higher than the DC current I_D can supply. Thus when this happens the controller of the VSC at the grid side will perform the control action to reverse the direction of currents (i_{DC}) to help the generation of the required level of capacitor current (i_c).

By looking at the series of incident waves which are applied to the WEC models and the simulation results presented in Figure 4.24 and Figure 4.25, one can conclude that these
control strategies are able to manage the power in an array of WECs for any type of regular waves as long as the incident frequencies are known and the power generated is enough to keep the DC link voltage level constant.

4.7 Case Study 2: Optimal power extraction over WECs in Regular waves

At present, there are several control techniques for maximizing the converted energy which are mostly based on either discrete or continuous control. As stated in [70], optimum mechanical power conversion can be achieved through several optimal control strategies, among these are the slow tuning technique, reactive control and latching control. All of these optimum control techniques are based on the amplitude and phase control originally proposed by Falnes and Budal [44]. Reactive control seems to be suitable for a direct drive WEC, whereas the other two control methods are normally used for WECs with a hydraulic power take-off system.

The control strategy applied in the previous section is based on the sub-optimal control scheme where the generator force $F_{\text{gen}}$ for each WEC is controlled when the stiffness force coefficient term was absent from the reference power take of force $F_{\text{PTO}}$ as defined in (4.7). As discussed in Chapter 3, to ensure maximum (optimal) power transfer in the WEC system, control action in the load impedance is required. If the $Z_{\text{PTO}}$ is controlled so that its value matches the complex conjugate of the intrinsic impedance of the WEC (refer section 3.12), power is transferred at the maximum value. In other words, the control variables must be tuned so that the velocity of the heaving buoy is in phase with the excitation force of the incident wave. During this period the heaving buoy is said to be at resonance. The optimum power control of oscillating bodies (phase control) is discussed in detail by Falnes [43].

The following subsections discuss the concept of optimal power extraction applied to the direct drive WEC in this study.

4.7.1 Reference PTO Impedance

To achieve the optimum power extraction the concept of complex conjugate control is implemented by introducing the load impedance $Z_{\text{PTO}}$ into the system, consisting of
damping and stiffness force term coefficients together with the mass of the WEC, \( M \). In this way \( Z_{PTO} \) consists of a combination of coefficients which is matched with the coefficients of the intrinsic impedance as defined in (4.27):

\[
Z_{PTO} = i\omega M + B_{PTO} + \frac{i}{\omega} K_{PTO}
\]  
(4.27)

Except for the mass of the heaving body \( M \) which is constant within the system, the other reference PTO impedances can be set to the value which satisfy the condition of optimum power extraction. However in this study the parameters chosen for this optimal control are slightly different from the concept discussed previously as the damping coefficient of the PTO in this control scheme is not set to the damping coefficient of intrinsic impedance \( B_j \); instead, it is set to the optimal damping value calculated in (4.9). Thus for the optimal control scheme, the coefficients of the reference PTO impedance are set to be:

\[
B_{PTO} = B_{\text{optimum}}
\]
(4.28a)

\[
K_{PTO} = \omega^2 (M + A_j(\omega)) - K_{ij}
\]
(4.28b)

As defined in (4.28), the reference value of the PTO stiffness coefficient is chosen to be the hydrodynamic spring stiffness of the heaving buoy which can be calculated from the dimensions of the buoy as defined in (3.18). By applying the optimal control scheme (through the damping and stiffness force coefficients), the maximum power transfer from the wave to WEC occurs when the velocity of the heaving bodies (WEC’s buoy) is in phase with the excitation force \( f_{ext} \) which is directly proportional to the wave displacement.

### 4.7.2 Control Scheme

Once the impedance matching defined in (4.28) is applied to every WEC in the array, the reference power take-off force for optimal control (reactive force) can be derived by adding the damping and stiffness force coefficient as defined in (3.11) and repeated here in (4.29). As in the previous control scheme, the linear generator phase currents are directly controlled to control the generator force.

\[
f_{npto}(t) = B_{npto} \dot{x}_n + K_{npto} x_n
\]
(4.29)

\( n \) here represent the numbers of WECs working in the array.
The optimum control is based on amplitude and phase control. By controlling the power take off force through the damping $B_{PTO}$ and spring stiffness coefficient $K_{PTO}$, maximum energy conversion can be achieved. Figures 4.26 and 4.27 are the results of when the optimum and sub-optimum controls are applied by taking their reference PTO force from equations (4.29) and (4.7) respectively. The phase current for sub-optimal control in Figure 4.27 is shown here for comparison with the current in the optimal control scheme as depicted in Figure 4.26. In the top diagram of Figure 4.26, the phase current is shifted corresponding with the generator force control requirement for optimum power extraction. Thus although the control scheme provides both amplitude and phase control, the phase currents seem not to be in phase with the generator phase voltage as can be seen in Figure 4.26 (b) and (c). On the other hand, for sub-optimal control only the damping force is considered in the control strategy so it only provides amplitude control. As a result the linear generator current can be controlled to be in phase with the phase voltage as can be seen in Figure 4.27 for the WEC 3. The same shape of phase current is expected for sub-optimal control in Figure 4.24 (d) for WEC 6 in the array.

Figure 4.26: Simulation graph for the reference phase current, generator phase current and voltage for optimal control (reactive power)
Figure 4.27: Simulation graph for the generator phase current and voltage for suboptimal control of WEC 3

4.7.3 Calculation of the reference phase current

Figure 4.28 shows the proposed control structure of the WEC side in an array. The control structure is the same as that in Independent Damping Force Control discussed in section 4.6.6.4, where the control block sets the PWM signal on the VSC to control the electrical power from every WEC in the array. However, high prediction accuracy for the incoming waves is needed to ensure the objective of optimal control can be achieved. This is due to the fact that the implementation of the optimum control scheme in the array relies on the ability to predict the frequency and phase of incident waves. The control structure Independent Reactive Force Control also requires a calculation of the current vectors, $ix$ and $ix$ (discussed in section 3.1.2.2), prior to the derivation of the reference phase current in the linear generator. The reactive current vector as defined in equation (3.12) previously proposed in [47] can also be used for the phase current control in ACTM machine.

\[ F_{gen} = K_G (i_x + i_x) \]  
\[ (4.30a) \]

\[ i_x = I \cos \phi_g \cos \omega t \quad \text{and} \quad i_x = -I \sin \phi_g \sin \omega t \]  
\[ (4.30b) \]
The above equation shows the relation between $F_{\text{gen}}$ and current vector $i_x$ and $i_x$. The detailed conversion of the reference phase current from the reference current vector is not discussed here since it has been discussed thoroughly in [47]. To examine further how the reference current vector is produced, references [47], [72] are recommended.

As can be seen in Figure 4.28, independent control of the voltage source converters (VSC) is used to control the power from the array into the grid, providing that the VSC on both sides is controlled. To generate the reference phase current, the reference force $F_{\text{PTO}}$ has to be determined by the controller using the expression given in (4.5). As shown in Figure 4.28, each designated $F_{\text{PTO}}$, calculated from (4.29), is different and its values are based on the frequency of the incident waves. Equation (4.27) is used as the first input of the equation of motion (4.5) to calculate the hydrodynamic properties of the WEC upon the incident wave with the information of non-dimensional coefficients provided by WAMIT. The output from the equation of motion in (4.5) gives the power take-off $F_{\text{PTO}}$ which then becomes the reference force to the linear generator.

The generator force $F_{\text{gen}}$ can be calculated from the measured phase current and then compared with the resultant reference power take-off force $F_{\text{PTO}}$. The error after the controller block will then become the amplitude of the current vector $I$ defined in (4.30). Using the expression given in (4.30b), the reference phase current vectors used in amplitude and phase control, $i_x$ and $i_x$, can be formed. Substituting these current vectors back into (4.30a) and then later into (3.26) allows the reference phase current for optimum power extraction to be derived. The phase of $\varphi_g$ in (4.30b) is used to define the phase of current vector $i_x$ and $i_x$, can be calculated as:

$$\varphi_g = \cos^{-1}\left(\frac{\dot{x}(t)}{\cos \omega x}\right)$$

(4.31)

Once the values of the reference phase currents are known, the VSC on the generator side is used to control the phase currents of the linear generator in the WEC by generating the PWM pattern $S_{\text{abc}}$ for triggering the switches of the VSC using the modulation voltage $V_{\text{con}}$ defined in (4.14).
Figure 4.28: Independent Reactive Force Control
4.7.4 Setup of simulation

The simulation study is performed with a series of six regular waves as shown in Figure 4.16c to investigate optimum control on the electrical power generation and how the array responds when the control scheme is applied in an array. As in the previous case study, in order to increase the accuracy of the simulation control setup, a very small sampling rate was set. All the electrical components (including the power converter) used in the simulation are taken with full Simulink models directly from the SimPower Systems in the Power Simulink Block. The hydrodynamic control for each WEC is initially run in a separate window and subsequently stored in the look-up table and used for the derivation of the reference phase current for each WEC in the array. The VSC control system for the array of six WEC power line system were developed in Simulink and were run for 100 seconds.

4.7.5 Simulation Results

Figure 4.29 shows the mechanical feature diagram of WEC 1 when the optimum control strategy is applied. The first two plots show the wave excitation force and the velocity of the heaving buoy. Referring back to the definition on the average power in (3.50), the optimum result occurs when the force and velocity of the heaving body are in phase and this can be seen in Figures 4.29(a) and (b). The generator force is controlled to follow its reference value (PTO force) by generating the reference phase currents which need to be drawn from the linear generator as depicted in Figures 4.29 (c),(d) and (e).
Figure 4.29: Simulation graphs of WEC1 due to optimal control
(a) Force excitation for wave frequency 1.1 rad/s,
(b) velocity, (c) Reference PTO force, (d) generator force and (e) reference phase current of WEC 1
The electrical waveforms of WEC1 with the optimal control scheme is shown in Figure 4.30. Since the optimal control strategy provides amplitude and phase control, the phase current is not in phase with the induced voltage as shown in the top and second plot of Figure 4.30. As process variables, the phase currents $I_a$, $I_b$, and $I_c$ are forced to follow the desired current signals (Figure 4.29 (e)) so that the generator force will drive the WEC to absorb maximum energy from the wave. The instantaneous power and its mean power are shown in Figure 4.30 (c) and (d) respectively.

The controlled phase currents of each WEC in the array are shown in Figure 4.31. Most of the phase currents are generated based from amplitude and phase control except for WEC 4 and WEC 6 where phase control seems to be absent. This can be seen by looking at the Figures 4.31 (d) and (f) respectively, where the phase currents shape drawn from the linear generators of WEC 4 and WEC 6 are very similar to the current with the amplitude control.
as previously shown in Figure 4.27 (a). This is probably due to the response of the WEC upon the incident waves which causes the heaving body (WEC) to oscillate near or close to the natural frequencies of the incident waves. Thus only amplitude control is required to bring the WECs to their optimum operating condition.

![Simulation graph of generator phase current for a single phase a when the optimal control scheme is applied in an array](image)

**Figure 4.31:** Simulation graph of generator phase current for a single phase a when the optimal control scheme is applied in an array

Figure 4.32 shows the electrical waveforms produced in an array after the process of rectification and inversion of the VSCs. Figure 4.32 (b) shows the inverter current represented in the synchronous rotating d axis, $I_{\text{inv},d}$. Since the power on the grid side is set for unity power factor, only $I_{\text{inv},d}$ is shown here. The fluctuation of the amplitude of $I_{\text{inv},d}$ represents the amplitude of the inverter currents since the other synchronous rotating current, $I_{\text{inv},q}$, is set to zero. Thus only real power is sent to the grid. The
reactive power can be considered in future if there is a requirement for reactive power in the network.

It can be seen in Figures 4.32 (c) and (d) that the grid side phase currents are in phase with the phase voltage. Using this control scheme, the array is continuously feeding power to the grid except during a short period at 50s, where power is reversed to support the DC link voltage.

![Graphs of inverter current, inverter current in synchronous rotating d, phase current and voltage at the grid, and magnified diagram of grid voltage and current.](image)

**Figure 4.32:** (a) The inverter current for a single phase
(b) The inverter current in synchronous rotating d, $I_{inv - d}$.
(c) Phase current and voltage at the grid in p.u.
(d) Magnified diagram of grid voltage and current.
4.8 Evaluation of Power Extracted from an arrays of WECs between optimal and sub-optimal control strategies

The assessment of both control schemes is discussed in this section in terms of the total electrical power produced by an array when it is incident with the regular waves shown in Figure 4.16 (c). As the grid voltage is 50 Hz in frequency, both Figure 4.24 (e) for sub-optimal control (PTO damping, $B_{PTO}$, is set to its optimal value but no spring stiffness force is included) and Figure 4.32 (d) for optimal control (both PTO damping and stiffness force, $B_{PTO}$ and $K_{PTO}$, are set to their optimal value) show the magnified 50 Hz inverter side current feeding into the grid. However when the inverter current in sub-optimal control as depicted in Figure 4.24 (a) is compared with the inverter currents shown in Figure 4.32 (a) for optimal control, the inverter current in the former case not only delivers varying current but also a current which is smaller in amplitude when compared to the latter case. This is because in the sub-optimal scheme the control can only provide the damping factor (amplitude control) to the WEC system which cannot bring the velocity of the WEC’s translator to be in phase with the wave excitation force, and as a result reduces the amount of energy being extracted from the incident wave.

![Instantaneous Output Power and mean power for two control schemes](image)

**Figure 4.33:** Instantaneous Output Power and mean power for two control schemes. (a) Optimal Control. (b) Sub-optimal control.

In optimal control the reference phase currents have been derived from the control strategy that provides both amplitude and phase control through the damping and stiffness force to
the WEC system which can bring the WEC to its peak performance. As a result, although the inverter currents sent to the grid are varying due to the incident waves, its amplitude is approximately twice that compared to the previous control scheme.

The instantaneous and the average electrical power over the system period study for the sub-optimal control in array is shown in Figure 4.33 (b). As observed, the peak power for the sub-optimum control is only slightly less than that for optimal control. However the average power for optimal is more than double that in sub-optimum control. This result indicates that sub-optimum control generates higher pulses of power when compared with the optimum control strategy. The average power sent to the grid by the sub-optimum control is 25.8kW, whereas the average power delivered to the grid using the optimal control scheme over the entire time study can be seen in Figure 4.33 (a) to be 57.4 kW.

4.9 Chapter Summary

The concept of extracting the power from an array of direct drive WECs using PWM-VSC based from regular waves is presented in this chapter. The descriptions are given for the hydrodynamic study of heaving buoys through to the handling of power from multiple WECs. The control system for a direct drive WEC, linear generator, VSC is complex, but in return makes the WECs operate at its peak and, as a result, give optimum power production. Although the power collection is made from multiple WECs, the control scheme applied to each WEC is conceptually similar to that applied to a single WEC. Thus the control strategies for an individual WEC were firstly discussed before the management of an array of WECs power capture based on sub-optimal and optimal control schemes were proposed.

Simulation results are presented in order to assess the performance of the two control schemes on the total electrical power generated from the waves. As expected, the optimum control based on the reactive control strategy produces the highest generated power. However, the additional power take-off (PTO) coefficient term \( K_{PTO} \) in the control scheme relies on the accuracy of the information of incoming waves which must be predicted before the control can be efficiently implemented to a WEC array system. Furthermore it is also produces a low generator operating power factor which will increase losses. In contrast, power take-off (PTO) damping force coefficient term \( B_{PTO} \) not only offers a simpler control algorithm, but the generator phase current can be controlled to be in phase with the output voltage resulting in near unity power factor. High accuracy of
wave prediction is also not necessary for the damping force control systems. Although its performance is often considered inferior compared to reactive control, the sub-optimum control based on damping control can offer advantages in terms of a less complicated strategy which is suitable for implementation in real seas. Therefore it would have been good to test all the numerical modelling to test the validity of the simulation results especially for the case studies made in Chapter 4. This can be done either through the scale-modelling or test-rig test. However this was beyond the scope of this project and could be addressed in the future work.
CHAPTER 5

Electrical Power Acquisition of WEC in Arrays For Irregular Waves

5.1 Case Study 3: Suboptimal Power extraction over WECs in Irregular waves (polychromatic waves)

In the previous chapter, several control structures to manage the power from an array of WECs were presented. The equation of motion as a function of acceleration, velocity and displacement of the heaving body defined in (4.5) is used. As a result, two PTO control strategies; optimal and sub-optimal, were proposed by configuring the PTO impedance according to equation (4.28) and (4.7) respectively. However the control schemes and the array control structure discussed so far are for the case of sinusoidal waves (single frequency). In this chapter the discussion on the topic of electrical power acquisition of WECs working in arrays is continued, but this will be based on the control strategy when the waves are irregular. The control schemes that give the ideal solution for power control either for an individual or group of WEC, need to be chosen, therefore analysis on the behaviour of a heaving body (WEC) under the influence of polychromatic waves (multiple frequency) is required. As in regular wave cases, the control scheme presented here focuses on achieving the control objective by controlling the PTO impedance of the WEC (as well as PTO force). This chapter begins with the construction of an irregular wave and discussion on the generalised equation of motion used for the case of waves consisting of more than one frequency.

5.1.1 Polychromatic wave train signal

As discussed in (4.5), the input wave frequency spectrum that is commonly used to reconstruct irregular waves is developed based on the approximation to the frequency content of the original (measured) waves. An inverse Fourier transform time series of
irregular waves can be synthesised for a finite period which generally is made by randomly choosing frequency components in the spectra. These random frequency contents are added with their amplitude and phase to produce an approximation of the original wave [74].

In this study, simplification of the generation of irregular waves is made by considering the irregular wave trains as a superposition of a number of monochromatic wave signals. Figure 5.1 and Figure 5.2 show examples of how the construction of simplified time series irregular waves can be made. The construction of an artificial wave train is based on three types of polychromatic waves:

- Periodic with constant period
- Aperiodic with constant period
- Aperiodic with varying time period

The spectrum sinusoidal components (frequency, amplitude and phase) in each time period of any constructed polychromatic wave, $T_k$, can be assumed to be well known whereby the signal is composed of two or more sinusoids whose amplitude and phase are the inverse Fourier Transforms of the corresponding parameters in the frequency spectrum. Each type of multi-frequency (polychromatic) signal has a fundamental sinusoid, $S_o$, with a frequency which is the fundamental frequency of the signal, $f_o$. The first two polychromatic waves described above have constant time period over the time: this is called the fundamental period based on the fundamental frequency, $T_k = 2\pi / f_o$. The term periodic and aperiodic refer to whether the signals are repetitive or non-repetitive as shown in Figure 5.1. Aperiodic with time varying does not have a fixed time period although it is still constructed using the superposition of sinusoidal signals, where the length of the period may be finite or infinite and is not based on the fundamental frequency, $T_k \neq 2\pi / f_o$ as can be seen in Figure 5.2. Also the time between two successive periods can be varying as $t_1 \neq t_2 \neq t_k$. By assuming the components of these polychromatic waves for each successive time period are known, the control strategy can be designed to collect the power from each WEC in the array.
The figure shows the construction of Polychromatic wave signal based on the composition of multi-sinusoids. (a) Periodic wave signal with constant time period. (b) Aperiodic signal (non-repetitive) wave signal.
5.1.2 Body motion in non-sinusoidal oscillation

The equation of motion is used to study the hydrodynamic motion of the heaving body when incident with the wave train. In the case of regular waves, the WEC is expected to have a sinusoidal oscillation during both transient and post-transient periods. Thus the hydrodynamic study for this oscillating system can be investigated using the concept of a mass-spring damper as discussed previously in section 3.1.2 and section 4.6.1. However, when considering an input signal which is not sinusoid but has amplitude and frequency varying over time, the equation of motion used in the wave energy literature is as previously defined in (3.23). It can be seen that the difference is that the equation of motion when dealing with polychromatic (non-sinusoidal) waves is that there is a memory term, or an impulse response function for the heave motion mode, $k_y(t)$.

The impulse response function corresponds to the inverse Fourier Transform (IFT) of the transfer function and is used to convert the function from the frequency domain to the time domain. When dealing with irregular waves it is convenient to convert the function into the time domain since it is valid for use in both transient and post-transient periods. For
the conversion of a function which is used for studying the oscillation of a heaving body in water, two hydrodynamic force functions need to be considered as defined in (3.19), first due to the incident wave and second due to radiation impedance. However the impulse response for the radiation impedance defined in (3.20) is a causal system (refer section 3.1.3.2) and the consequence of this causal system on the reaction force gives:

\[ \hat{F}_{r,j} = -R_j \hat{u}_j \]  

(5.1)

where \( R_j \) can be considered to be the new radiation impedance (instead of \( Z_{ij} \) in (3.20)) due to the principle of causality, in which a new term \( A_{ij}(\infty) \) is added:

\[ R_j = Z_{ij} - j\omega A_{ij}(\infty) \]  

(5.2)

References [18], [75] describe \( A_{ij}(\infty) \) as an additional added mass due to the entrained water. By referring to the definition of \( Z_{ij} \) in (3.21), one can conclude that the added mass, \( A_{ij}(\omega) \), which is frequency dependent is now hidden into the new radiation impedance function defined in (5.2). As a result, the terms which represent added mass and radiation impedance, previously defined in the frequency domain in (3.15), will be replaced by the terms that can be used to study the oscillation of the heaving body when incident with irregular waves as given in (5.3)

\[ F_{r,ext}(\omega) = [-\omega^2 (m_j + A_{ij}(\infty)) + j\omega R_j + K_j] \hat{x}_j(\omega) \]  

(5.3)

The conversion of equations in (5.1), (5.2) and later (5.3) into the time domain, however, will involve the convolution integral. This is because the effect of the radiation force for the irregular waves required both transient and post transient which are associated with the past and present radiation of waves [18], therefore is given by:

\[ f_{r,j}(t) = A_{ij}(\infty) \ddot{x}(t) + \int_0^t k(\tau) \dot{x}(t-\tau) d\tau \]  

(5.4)

The first term on the RHS of (5.4) is the additional added mass multiplied by the acceleration of the heaving body, which is the post transient effect of the radiation force. The second term, which involves the convolution integral can be considered to be the dynamic response of the radiation force during the transient period. However, the
conversion into the time domain of the function related to the excitation force, which is the first term in (3.19), does not involve the convolution integral and is:

\[ f_{e_x}(t) = m_y \ddot{x}(t) + K_y x(t) \]  

(5.5)

Adding equations (5.4) and (5.5) will give the complete response of the hydrodynamic forces acting upon the oscillating body by the irregular waves as defined in (3.23) and repeated in (5.6) with the additional presence of the PTO impedance, \( Z_{pto} \):

\[ f_{e_x}(t) = (m_y + A_y(\infty)) \ddot{x}(t) + \int_0^t k(\tau) \dot{x}(t - \tau) d\tau + (K_y) x(t) + Z_{pto} \dot{x}(t) \]  

(5.6)

The convolution term

In order to perform the convolution, the radiation impedance needs to be calculated. Since this radiation impulse response function \( R_y(\omega) \), is frequency dependent, the conversion into \( k_y(t) \) can be made using equation (3.22) which essentially is the integration of the added damping function over several frequencies. However, for ease of implementation, the convolution term in (5.6) can be directly solved when performing it as a discrete function for numerical integration by making \( \tau \rightarrow n\Delta t \) as given in (5.7):

\[ \int_0^t k_y(\tau) u(t - \tau) d\tau = \sum_{n=0}^{T-1} k_y(n\Delta t) u(t - n\Delta t) \Delta t \]  

(5.7)

Using the integral approximation method, the trapezium rule, equation (5.7) can be further derived to be:

\[ \sum_{n=0}^{T-1} k_y(n\Delta t) u(t - n\Delta t) \Delta t = \frac{1}{2} k_y(0) u(t) \Delta t + \sum_{n=1}^{T-1} k_y(n\Delta t) u(t - n\Delta t) \Delta t \]  

(5.8)

where \( \Delta t = t / n \)
5.2 PTO Impedances and Control Scheme

The selection of the PTO impedance term, $Z_{PTO}$, for the sub-optimal control scheme in this study is based on two criteria: first, it is constant, and second, the value is chosen according to the fundamental frequency of the polychromatic wave.

As in the regular wave case, the $Z_{PTO}$ in the sub-optimal control scheme (for irregular waves) contains terms including the damping stiffness coefficient, $B_{PTO}$, and the translator’s mass, $M$, as defined in (5.9). The reason for investigating the control scheme without the stiffness force, $K_{PTO}$, is for the optimization of the electrical power generation. Although the presence of $K_{PTO}$ in the control strategy can lead to phase control, which is important for the mechanical power optimization, from the electrical point of view, it can also generate a highly varying power factor in the linear generator which can affect the efficiency and the stability of the generated power. For this reason, in sub-optimal control only the damping force coefficient, $B_{PTO}$, which is based on amplitude control is used as it can lead to the generation of unity power factor in the linear generator, as can be seen in Figure 4.27.

$$Z_{PTO}(\omega) = i\omega M + B_{PTO}(\omega) \quad (5.9)$$

$$B_{PTO} = B_{y}(\omega_o) \quad (5.10)$$

Depending on the fundamental frequency of the polychromatic waves, the value of damping coefficient of the PTO in (5.10) can be a constant or changing over the time period of study. This is because for the aperiodic polychromatic wave the value of $B_{PTO}$ could be varying due to the value of the fundamental frequency in each successive period. Once the value of the damping coefficient is known, using the PTO impedance defined in (5.9) and (5.10), the PTO force can be further derived into time domain as:

$$f_{PTO}(t) = M\ddot{x}(t) + B_{PTO}\dot{x}(t) \quad (5.11)$$

where $f_{PTO}(t) = Z_{PTO}\dot{x}(t)$ and $Z_{PTO} = M\dot{x} + B_{PTO}$

The equation above is similar to (4.7b), but here the dynamic variable $x(t)$ is not a function of a sinusoid as in the regular wave case, instead it is a function of multi-sinusoids. The PTO force equation in (5.11) later will be added into (5.6) as
$(f_{pto}(t) = Z_{pto}x(t))$ to form the complete equation of motion for a WEC under polychromatic wave (irregular) influences. The resulting motion of the WEC is certainly very different to the case of the sinusoidal motion based on the equation derived previously in (4.5).

The calculation of the hydrodynamic motion is made for each WEC in the array due to each incident wave. Figure 5.3 illustrates the three states of the parameter selection in PTO impedance ($B_{pto}$) for each WEC that corresponds to the three different types of irregular waves considered in this study. The chosen PTO impedance is then used for the derivation of the PTO reference force. The task of the control algorithm block is to produce the reference phase current that needs to be drawn from the linear generator in order to fulfill the criteria defined in the control scheme.

![Figure 5.3: WEC side’s Sub-Optimal Control Structure of array for Irregular waves](image-url)
As seen in Figure 5.3, the dimensional hydrodynamic parameters from the WAMIT block, which are frequency-based, are derived for the three types of aforementioned artificial irregular waves. The control structure for WEC 1 is applied to each WEC in the array and the generalised controller block is shown for WEC 6. The fundamental frequency, \( \omega_f \), in the periodic wave signal is constant which is not the same for the aperiodic wave signal, which has a variable fundamental frequency, \( \omega_a \), for each wave period. Therefore the wave measurements (top of Figure 5.3) are taken in aperiodic signals every time the switches which connect to the WAMIT block are closed. When the switches close, the fundamental frequency, \( \omega_f \), of the corresponding wave signal period is sent to the WAMIT block for the calculation of the hydrodynamic parameters.

### 5.3 Setup of Simulations

Simulations of the proposed control structure to demonstrate the control strategy for an array of WECs when incident with a series of irregular waves is shown in Figure 5.4. As in previous simulations, all the analytical models were made in Matlab/Simulink, consisting of the (artificial) wave source, the hydrodynamic WEC model, the linear generator model (ACTM), two voltage source converter blocks (PWM-VSCs), the controller block and the power grid. As can be seen in Figure 5.3, the mathematical models are based on the differential and algebraic equations discussed in the last three chapters together with the elements of Simulink Block to form a complete simulation model of the system. However, in order to reduce the complexity of the simulation model, the wave resource and hydrodynamic model of the WECs were run in separate module.

![Figure 5.4: Irregular waves of varying amplitude, phase and frequency](image-url)
5.4 Simulation Results

The objective of the study on irregular waves is to investigate the effectiveness of the proposed control strategies on the management of power extraction when the movement of the translator is due to waves which constantly vary. Although real seas are more complex than the shape of irregular waves presented in Figure 5.4, it can provide an important indication of how the controller will handle the power from WECs under the influence of varying incident waves. The simulation results for irregular waves are presented in the following figures.

![Graph of current against time for WEC phase current and reference current](image)

**Figure 5.5:** Inverter phase current, $I_{inv}$ and the magnified 50 Hz current

![Graph of current against time for WEC phase current and reference current](image)

**Figure 5.6:** WEC phase current, $I_a$ and the reference current of phase A
Figure 5.5 shows the inverter phase current. The shape of the current varies over time, which can be seen in the top diagram. As the output current at the grid side is controlled to be the same frequency as the grid side voltage, the lower diagram shows the magnified 50 Hz current fed into the grid. Figure 5.6 shows the phase current of WEC 4 with respect to its reference current. It can be seen that the phase currents are successfully controlled to follow its reference value by maintaining the DC link voltage at the desired level of 1000V. The DC voltage at the DC link for the irregular wave is depicted in Figure 5.7, which is less smooth than previously in regular wave cases. A voltage deviation of about 20V can be considered acceptable, significantly greater deviations will result in the phase currents deviation from their reference values.

![DC link voltage - 1000V](image)

Figure 5.7: Main DC link voltage

Figure 5.8(a) shows the induced voltage in WEC 1, whereby the voltage not only has varying frequency and amplitude but also shows variation within each cycle. As expected, \(I_{\text{inv, } d}\) can be used as an indication of how much real power is delivered to the grid. Each time the DC voltage drops from its desired value, the controller at the grid side will respond by taking power from the grid. This happens because at that time the collected total current \(I_{DT}\) does not reach the values in which to charge the capacitor to the desired voltage. When this happens, the control block at the grid side will produce the regulated PWM signal to trigger the VSC to change the direction of the grid current as well as the DC current \(i_{DC}\) in order to help the current \(i_c\) charging the capacitor. At this point, the VSC will no longer act as an inverter but instead will function as a rectifier to restore the DC voltage level.
Figure 5.8: Simulation graphs of the electrical power generation for sub-optimal control (From Top) - Induced voltage of WEC1, $V_{\text{inv}}$ (pu) vs $I_{\text{inv}}$ (pu), magnified 50 Hz signal of $(V_{\text{inv}}$ vs $I_{\text{inv}}$), and $d$-$q$-axis inverter current, $I_{\text{inv} - d}$ and $I_{\text{inv} - q}$. 

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**Induced Voltage - WEC 1**

![Graph showing induced voltage over time (s)]

(a) $V_{\text{inv}}$ (pu) vs $I_{\text{inv}}$ (pu)

(b) $V_{\text{inv}}$ (pu) vs $I_{\text{inv}}$ (pu)

(c) $I_{\text{inv} - d}$ vs $I_{\text{inv} - q}$

(d) Current (A) vs time (s)

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As observed in Figure 5.8 (b) and its expanded diagram in Figure 5.8 (c), the phase output current at the grid side is still controlled for unity power factor but with current flowing in the opposite direction. In Figure 5.8 (d), $I_{\text{inv}.d}$ becomes negative during this. Once the DC voltage reaches its desired value, the grid side VSC will function as an inverter again where power is delivered to the grid. This can be seen in the last few cycles in Figure 5.8 (c), where the grid side output phase current slowly changes to become in phase with the grid voltage. The power delivered to the grid over the entire time study can be seen in Figure 5.9.

### 5.5 Case Study 4: Optimal power extraction from WECs in Irregular waves (polychromatic waves)

The discussion of an array of WECs in irregular waves is continued in this section, and now focuses on the mechanical resonance concept (optimum power absorption) case. As in the regular wave case, control action in the load impedance is required to encourage the velocity of the heaving buoy to be in phase with the excitation force of the incident buoy (resonance effect). Through the PTO impedance coefficients, spring stiffness force, $K_{\text{PTO}}$, and damping coefficients, $B_{\text{PTO}}$, the concept of maximum power extraction can be achieved and therefore the selection of the PTO impedances and the control scheme for optimal power is the topic of the following subsection.
5.5.1 PTO impedances and Control Scheme

From the mechanical point of view, power is the product of force and velocity (refer to section 3.1.2 and equation (3.50)). As the spring stiffness force, $K_{PTO}$, provides the phase control to the WEC, it is included as part of the PTO force in the equation of motion as defined in (5.12)

$$f_{pto}(t) = B_{pto} \dot{x}_n + K_{pto} x_n$$  

(5.12)

Here $n$ represents the number of WECs working in the array.

By taking the concept of complex conjugate discussed previously (3.1.2.1) and from the control strategy proposed in [71], the PTO impedances for polychromatic waves used in this study are controlled according to:

$$B_{PTO} = k(-t) \ast \dot{x}(t) = \frac{1}{0} k(\tau) \dot{x}(t + \tau) d\tau$$  

(5.13a)

$$K_{PTO} = (M + A_{ij}(\omega_j)) \ddot{x}(t) - K_{ij}$$  

(5.13b)

The damping force coefficient defined in (5.13a) according to the reference in [18], is the anti-causal function which is opposite to the definition of the causal function caused by the intrinsic radiation impedance given in (5.4). When performing anti-causal control on the damping force coefficient of the PTO to the system (WEC) which previously was causal as described in section (5.1.2), makes the system becomes an acausal system. An acausal system wave engineering term is defined as a system which requires the prediction of incoming waves to achieve optimum oscillating velocity which can lead to the optimal power absorption [43].
To implement the optimal control scheme, the PTO impedances are matched with the intrinsic impedance of the WEC. The damping coefficients of PTO $B_{PTO}$, is derived based on the reverse impulse response function of $k(t), k(-t)$, while the spring stiffness
coefficient $K_{PTO}$ is defined as in (5.13b). The values are chosen to follow the concept of complex conjugate control. To bring the WEC to its peak condition the value of $B_{PTO}$ has to be a function of a convolution between the reverse impulse response with the velocity, $k(-t) \ast \dot{x}(t)$ and as proposed in [18], the result is expressed by equation (5.13a). The equation indicates that for optimal control, information of the future velocity of the WEC's translator is required. This is because the convolution of the reverse impulse function, $k(-t)$, with the velocity of the heaving body, $\dot{x}(t)$, can be interpreted as an integral function between the impulse response function, $k(\tau)$, and the future prediction of the velocity of the buoy, $\dot{x}(t + \tau)$.

Figure 5.10 illustrates this concept. The top diagram, shows the impulse response function and its reverse function ($k(t)$ and $k(-t)$) which are based on three criteria of the oscillating body: the radius of the buoy, the draft of the immersed body and the depth of the water where the WEC is deployed. The convolution of the intrinsic impedance of WEC and the PTO damping coefficient with velocity are shown in the second and the third row of Figure 5.10. As illustrated in the bottom-right of Figure 5.10, information of future velocity is required when considering the ideal optimal wave energy absorption in irregular waves. Thus future information of the heaving body's velocity is required which can be predicted from the condition of future incident waves and the present velocity of the WEC as can be seen in Figure 5.11. The reference [76] explains some of the methods that can be used in order to generate an approximator of the future sea state conditions for the use of wave energy converters (WEC).

### 5.5.2 Setup of simulations

The WEC control structure shown in Figure 5.11 is an optimal control scheme for an array of WECs when incident with polychromatic waves. The same series of incident waves as depicted in Figure 5.4 is applied to the array of six WECs (although the control structure is only shown for WEC 1). The control model proposed is based on the control...
Figure 5.11: WEC optimal control structure of array for irregular waves
scheme described previously in section (5.5.1). The initial strategy of this control scheme is to control the reference phase current of the linear generator in both amplitude and phase in order to control the generator force, \( F_{\text{gen}} \). The hydrodynamic calculations to determine the value of reference force, \( F_{\text{PTO}} \), that can bring the WEC into the resonance state are done by using the PTO impedances, \( B_{\text{PTO}} \) and \( K_{\text{PTO}} \) as the control components. The error signal generated from the comparison between the actual and desired force will be passed into the PI controller to determine the amplitude of the reference phase current, \( \dot{i}_p \). Using the equation defined in (4.10) and with the information of the velocity of the translator, the reference phase current for the linear generator can be derived. After the derivation of the reference current, \( I_{\text{abc,ref}} \), is made, the converter voltage of the PWM-VSC (WEC’s side), \( V_{\text{conv,abc}} \) can be obtained using the equation (4.16), which is then used to generate the PWM pattern, \( S_{\text{abc}} \).

The unique feature about the optimal control scheme used by the control model shown in Figure 5.11 is the convolution function involved between the impulse response function, \( k(t) \), with both present velocity, \( \dot{x}(t) \) and the future velocity \( \dot{x}(t+\tau) \) of the translator/buoy. In order to perform the convolution function, the incident wave needs to be sampled with the sampling rate of \( T_s \). As a prerequisite for performing the control action, the system should be able to predict the future output of incident wave, \( A[k+1] \), based on information of the current wave discrete pattern, \( A[k] \). From the prediction of the (near) future wave and the present state of the translator’s velocity, the future state of the velocity can be determined and the damping coefficients of the WEC intrinsic impedance, \( B_y \), and the PTO impedance, \( B_{\text{PTO}} \), can be known. The two convolution functions in this control scheme, \( k(t) \cdot \dot{x}(t) \) and \( k(-\tau) \cdot \dot{x}(t) \), can be solved by using equation (5.8). However the performance of the control scheme relies on the accuracy of the prediction on the near future wave and the future state of the translator’s velocity. As the work presented here concerns the performance of the control scheme on handling power from an array, both states of these process variables, \( A[k+1] \) and \( \dot{x}(t+\tau) \), used in the simulation model can be known by applying the derivation of \( A[k] \) and \( \dot{x}(t) \) prior to the simulation. The control model has been developed in MATLAB/Simulink and the simulation is run for 100 seconds with a sampling time of 5\( \mu \text{s} \).
5.5.3 Simulation Results

Figure 5.12 shows the results from the mechanical point of view when the optimal control scheme is applied to WEC 6. To maintain mechanical resonance in the WEC system, the velocity of the buoys is kept in phase with the excitation force, $F_{ext}$ as can be seen in the top plot of Figure 5.12. The velocity, the displacement of the buoy and the incident wave elevation are shown in the last three plots of the figure.

![Figure 5.12: Mechanical characteristics for WEC 6 due to Optimal Control.](image)

(a) Excitation force (p.u) vs WEC velocity, (b) translator velocity, (c) translator's displacement and (d) incident wave for WEC 6.

Figure 5.13 shows the electrical waveforms resulting from the optimization control of the WEC 6. The optimal control scheme can be described as a function of amplitude and phase control of the WEC system. As in previous case studies, the generator force ($F_{gen}$) is
controlled indirectly by the controller through the generator phase current. Controlling the phase currents to their reference values will contribute to the system amplitude and phase control and hence provides appropriate damping and stiffness force to the WEC system.

![Diagram of controlled phase current and voltage](image)

**Figure 5.13:** Simulation graphs of WEC 6 due to optimal control
(a) Reference phase current of WEC 6, (b) Induced voltage,
(c) Reference PTO force, (d) Generator Force, $F_{gen}$ and
(e) Excitation force (p.u) vs translator’s velocity- before and after control scheme.
Figure 5.13 shows the reference phase current \( I_{abc \text{ ref}} \) which is derived from the control scheme employed. The effect on the translator velocity when implementing this control scheme on WEC 6 can be seen by looking at Figure 5.13(e), showing the condition of force excitation \( f_{e,i}(t) \) and the velocity before and after optimum control. It is clear that velocity is stimulated to move more synchronously with the amplitude of the incident wave, \( A \) (as \( f_{e,i}(t) \) is a function of \( A \), refer to section 3.1.3.1), at the time when the control action is applied which is not the case before the resonance effect takes place (during sub-optimal control). The reason for this is the inclusion of \( K_{PTO} \) in the control scheme, providing the phase control to the system which together with the \( B_{PTO} \) can bring the WEC into the resonance state. Also through the regulation of the phase current, the generator force, \( F_{gen} \), is indirectly controlled to follow reference force, \( F_{PTO} \) as can be seen in Figure 5.13(c) and (d). The induced voltage of WEC 6 is shown in Figure 5.13(b).

**Figure 5.14:** Simulation graphs of the electrical properties of WEC 1 due to optimal control for the entire simulation time.
(a) The induced voltage of WEC 1,
(b) Reference phase current of one phase \( (I_{a \text{ ref}}) \)
(c) WEC phase current \( (I_a) \)
Figure 5.14 illustrates the induced voltage and the phase current (reference and actual) when the same control strategy is applied to the WEC1. By carefully looking at the figure, it can be seen that the phase current is not in phase with the induced voltage due to the influence of amplitude and phase control on the generator phase current by the controller.

One of the main objectives of the control scheme is to keep the DC link voltage at a constant value. The reason for having a constant DC link voltage, as explained in chapter 4, is to ensure that the electrical power generated by each individual WEC in the array is efficiently collected before it is sent to the grid. Furthermore, keeping the DC voltage constant is necessary for the input phase current to be controlled to follow the desired generator phase current. Figure 5.15 depicts the DC link voltage deviating by up to 10 V from the desired value. The result shows that although there is a small deviation on the DC link voltage, the control scheme is able to fulfil the control objective by looking at the last two plots of Figure 5.14, which show that the generator phase current is successfully controlled to follow the reference current.

Figure 5.15: DC link voltage in optimal control for irregular waves
The results presented so far for the irregular wave study (optimal control) are based on the control action on the PWM-VSC on the WEC side. The power transferred to the grid, however, is determined through the control on the PWM-VSC on the grid side. The details of the grid side PWM-VSC control structure is illustrated in Figure 4.12 which is simplified in the $abc \rightarrow dq$ block in Figure 5.11. The amount of real and reactive power that is sent or received by the array is decided by the two (d-q) inverter vector $I_{inv_d}$ and $I_{inv_q}$ respectively. At this stage of the study, only real power is considered, thus $I_{inv_q}$ is set to be zero. However, the amount of power to be sent to the grid depends on the condition of the DC link voltage. If the power generated from the array is enough to keep the DC link voltage constant, then the VSC is controlled to deliver the real power to the grid, otherwise the power is taken from the grid. In this study, the grid network interconnection is assumed to be ideal and stable. For a weak network interconnection (which would normally be the case), an additional power converter is incorporated to the system to maintain the voltage stability at the point of common connection. This is not covered in this study and could be explored in future work.

Figure 5.16 shows the inverter phase current in the synchronous d-q frame measured on the grid side as shown in Figure 5.11. The q-axis inverter current, $I_{inv_q}$ is kept at zero all the time, meaning that no reactive power is consumed by grid power line impedances. The amount of real power transmission is controlled directly by acting on the d-axis inverter current $I_{inv_d}$. However, its value is very much dependent on the condition of the DC link voltage level. As can be seen in Figure 5.16, the inverter current, $I_{inv_d}$, varies significantly throughout the simulation period. When the inverter current, $I_{inv_d}$, is positive, the array is sending real power to the grid, otherwise, real power is reversed from the grid to the array.

![Figure 5.16: The d_q-axis inverter current, $I_{inv_d}$ and $I_{inv_q}$](image-url)
By looking at the state of the $I_{inv_d}$ throughout the entire period of study in Figure 5.16, it can be seen that the amount of real power being sent to the grid is far greater than the power which is taken from the grid. The operation of delivering and reversing of power in the grid side is more clearly seen by referring to the grid (inverter) current and voltage for one phase as illustrated in Figure 5.17. Both of these output grid parameters are in per-unit. As the grid frequency is set by the PLL block to be 50 Hz, the last two plots of Figure 5.17 show the magnified phase grid current during the inversion and rectification operation. As can be seen, it is controlled to be in phase with the grid voltage; either power is feeding to the grid or the power is reversed from the grid to support the DC link voltage. It also confirms that the control scheme in the grid side does not provide reactive power support to the grid by looking at the magnified 50 Hz grid voltage and the phase current depicted in Figure 5.17.

![Graphs showing power flow](image)

**Figure 5.17**: (a) Phase current and voltage at the grid in p.u (b & c) - Magnified diagram of grid voltage and current while delivering power (b) and reversing power (c)
5.6 Optimal and Sub-Optimal Control Schemes

The optimal control scheme allows the stiffness spring coefficients to be included and can bring the occurrence of mechanical resonance, enabling optimum power capture. However, the generator phase current that drives the WEC to the optimum condition has an amount of reactive element that leads the phase current not to be in phase with the induced voltage as can be seen in both cases (in Figure 4.26 for regular and in Figure 5.14 for irregular waves). Also, when performing impedance matching (complex conjugate control), the prediction of the near future velocity is required. These are the two main drawbacks of the optimal control scheme, although it can generate more output power than the sub-optimal control scheme.

![Inverter Output Power](image)

Figure 5.18 shows the simulation result of the amount of real output power sent to the grid for the case studies presented so far in this chapter. Both control schemes generate varying
output power due to the oscillations of the incident waves on each WEC in the array. However the VSC power converters not only act as power conditioning devices to convert the varying amplitude, frequency and phase AC supply to a fixed frequency and phase AC grid network, but they also control the power flow in the system. It is demonstrated that the optimal control generates higher real power (kW) flow compared to the second case as can be seen the electrical output power generated from the sub-optimal control has lesser ‘power pulses’ peak power production.

5.7 Case Study 5: Power Collection from WECs connecting in AC link

This simulation study is carried out in order to investigate the feasibility of the proposed control technique applied for an array of WECs connected on the AC side. The same control strategies and WEC models are used for the AC interconnection study, however an additional number of VSCs at the grid side are inserted into the model. This study is equally important for studying faults. If a fault does occur, the faulted line of the WEC can be more easily isolated when the WECs are connected with AC interconnection rather than DC. This is because high power AC breakers are available commercially and considered as a mature technology in power protection systems, which is not the case for DC breakers. This section discussed the simulation results of the AC link Interconnection control model.

5.7.1 WECs system topology

It is essential in order to collect the power from a group of WECs in an array that a power converter is used which regulates the varying voltages and currents generated by each individual WEC into the fixed AC output signal (fixed frequency and voltage) to meet the requirement of the electrical grid. However, instead of using a single PWM-VSC at the grid side, multiple PWM-VSCs can be used to control the power between the array and the grid as illustrated in Figure 4.5 and in Figure 5.19 (b).

As described in section 4.6.3 both WEC side and grid side control schemes were applied to the back-to-back PWM-VSCs of the AC link interconnection. Having multiple power converters (VSCs) at the grid side, AC interconnection can limit the peak current through each IGBT switch every time it turn-ON [77]. Thus it can bring the VSC’s switches (IGBT) to a safe switching operation especially when considering the employment of a high rated power wave energy farm. Before any conclusions can be drawn from the
simulation results presented in the following figures, it is essential to understand the similarity of the configuration between both types of array interconnection performed in the simulation study. The simplified diagrams, in which previously shown in Figure 4.14 are presented here in Figure 5.19.

![Diagram](image)

**Figure 5.19:** Simplified diagram for the connection of WECs in an array (a) DC interconnection (b) AC interconnection

It can be seen that the grid current $i_{d_{-}dclink}$ ($I_{lin}$) in the AC link interconnection is the sum of the outputs of AC current from the inverter (power converter) on the grid side. By connecting the array in AC interconnection, the burden on the semiconductor switches in the inverter upon the peak currents and voltages in (which experienced in DC interconnection) can be reduced as the total grid current of an AC interconnection is such that:

$$i_{d_{-}dclink} = \sum_{n=1}^{6} i_{d_{n_{-}dclink}}$$  \hspace{2cm} (5.14)

$n$ here refer to the number of WEC in array. Thus, the grid current in the DC link ($i_{d_{-}dclink}$) should be the same with the total grid current in AC link interconnection ($i_{d_{n_{-}aclink}}$) after the inversion of the inverter. As controlling the DC link voltage level is the key element in collecting the power from the array in DC link interconnection, the requirement for the AC link interconnection is to control the output current of the grid side inverter ($i_{d_{n}}$) so that they are all in phase for each AC side of the inverters in the array. Therefore to analyse the effectiveness of the grid side control schemes, the same series of
incident waves (Figure 5.4) which was applied in the previous study were also applied in AC link Interconnection.

5.7.2 Simulation Results

As stated in (5.14), the grid current $i_{d_{ac\text{link}}}$ in the AC link interconnection, should correspond with the grid current $i_{d_{dc\text{link}}}$ in the DC link and this as can be seen from the simulation results shown in Figure 5.20. To see some similarity between the grid current in the AC link with the grid current in the DC link, the grid side output phase currents for each link in the AC interconnection $i_{d_{ac\text{ac}}}$ is added and compared with the grid side output current $i_{d_{dc}}$ in the DC link interconnection previously shown in the top of Figure 5.5.

From the figures, it can be observed that the AC link output currents resemble the total current in the DC link but have a more fluctuating shape in comparison with its DC counterpart. This is because the additional number of VSCs placed in the AC link interconnection increase the switching operation of the IGBT and as a result injected the
additional harmonics into the total output current at the grid side. However the result shows high similarities in output grid currents between the two configurations.

Another benefit of studying WECs with AC links is the capability of limiting current flow and the ability to withstand the peak current stress on the IGBT semiconductor switches due to its topology, whereas the protection can be achieved by distributing the current among the VSCs (each link) in the grid side. Thus AC link interconnection is a favourable candidate for the use in medium to high power WEC arrays. Figure 5.21 shows the comparison of IGBT currents flowing in the VSC at the grid side for DC and AC link interconnection, where in the AC link the results presented is for one of the link (WEC 1).

The top diagram in both figures is the pulse phase converter voltage $V_{con}$, while the bottom graph shows the currents in the semiconductor switches (a positive value indicates current flowing through the IGBT and negative value indicates current flowing through the diode). It clearly shows that the current going through the IGBT switches in the VSC is reduced by $1/n$ when AC link interconnection is used. Hence, the IGBTs can be protected from the over current and can create a safe operation area of a switching devices. However there are limitations in the amount of current that can be passed through the IGBT even in an AC link interconnection. Alternative topologies such as multilevel converters are normally another option to be considered because of its suitability for the use in high power applications.

![Figure 5.21: Switching Characteristic between two grid side configuration:](image)

(a) DC link Interconnection and (b) AC link Interconnection (Top) Peak IGBT voltage, (Bottom) Peak IGBT/Diode currents
5.8 Chapter Summary

The control structures and schemes of WECs working in an array in irregular waves are introduced and the control strategy has been described. Simulation results for three cases for both control schemes, optimal and sub-optimal, have been performed. It shows that the proposed control strategies are able to handle the power from a group of WECs, however a small scale test rig of an array of WECs needs to be constructed to verify the proposed control scheme presented in this thesis. Both DC and AC link interconnections have their own strengths which depend on the size, type and location where the WECs are deployed. The results show that an AC link interconnection is more practical for higher and larger power wave farms due to its topology which features medium mode current protection.
CHAPTER 6

Integration Study of WEC Cluster and DC Link Interconnection Issues

The control strategies for handling power from an array of wave energy converters (WECs) was analysed in Chapter 4 and Chapter 5. Each of the control schemes described previously were results from the derivation of the equation of motion due to the condition of the incident waves and based on the power optimisation concept applied for an individual WEC. The main differences between optimal and sub-optimal control strategies were also discussed where the inclusion of a stiffness force coefficient in the governing equations encourages the WECs to achieve mechanical resonance.

Connecting the WECs in an array has the advantage of extracting higher amount of power from the waves and is proven to provide increased power quality [78]. Most oceanic renewable energy configurations which have been proposed currently either for near or far-offshore deployment, typically are connected either as a group (array) or a cluster (multi-group) with at least one transmission cable line to export power to the shore. Certainly it is not practical to have transmission cables for each WEC employed in the array. The topology for integrating WECs that would cut the investment cost of the wave farm most would be more preferable. However the integration of WECs sometimes can also lead to other issues that can affect the amount of electrical power produced by the WEC. Therefore the optimization of the control scheme applied to the array should not be restricted within the context of an individual WEC but should be extended to best suit the array so that maximum power can be transmitted to the grid.

In this chapter some integration issues are discussed which examine the limitations of the control schemes proposed previously and how improvements can be made to optimise the...
wave power collected by the array. Also some integration topologies taken from wind farm technology such as radial, ring and star interconnection are briefly discussed and simulated to observe the opportunities and limitations of these arrangements for wave energy converters (WECs).

6.1 DC Link Interconnection

DC link Interconnection has a common DC link where all WECs are connected. By choosing this configuration, the problem of power variations generated from an individual WEC can be reduced at the lower cost as the number of VSCs required is not as many as in the AC link Interconnection. However, the DC link voltage level must be kept at a constant value so that the energy captured by each WEC can be efficiently collected before it is transmitted to the grid. Under normal operation, the capacitor energy to keep the voltage level of the DC link comes from the WECs where the total DC current generated from the array charges the DC link capacitor to the reference DC level. Whenever a power shortage occurs in the source (when the generated power by the WECs is not enough for charging the capacitor to the reference DC level), the PWM-VSC of the grid side will reverse the power from the grid to the DC link bus as explained previously in Chapter 4.

6.1.1 Impact of DC link voltage level

The capacitor does not consume any power but stores energy at the appropriate time. However, unlike the case of the DC capacitor in a conventional AC-DC-AC electrical converter, the effect of varying amplitude and frequency of the linear generator output voltage results in the power to the DC link capacitor to be constantly varying.

As the energy storage element, the DC link capacitor acts as reservoir to smooth the output of a rectifier. Therefore its primary function in the WEC AC-DC-AC power system in Figure 4.7 is not only to store/release power but to keep the voltage level constant to its reference value. As can be seen through the series of case studies in Chapter 4 and Chapter 5, the energy is borrowed from the grid whenever a power shortage occurs. Setting the reference value of the DC link capacitor to a lower level can reduce the need for borrowing power from the grid.
Equation (6.1) expresses the amount of energy (Joules) received by the DC link capacitor. As the capacitor value is constant, the energy stored is dependent on the level of the capacitor voltage.

\[ E = \frac{1}{2} CV^2 \]  

(6.1)

Due to this fact, the amount of energy stored in the capacitor will increase if the DC voltage is increased. In the direct drive WECs of an array, the level of DC capacitor voltage determines the amount of energy required to hold the DC link voltage level. When the DC link capacitor level of the voltage is kept high, more energy is used to charge the capacitor, thus in the event of power shortage, additional power is needed from the grid as can be seen in Figure 6.1. Conversely, when less energy is required to charge the DC link capacitor, the level of the DC link can be kept constant without the need of additional power from the other auxiliary source. Therefore it is beneficial to have a lower voltage in the DC link.

![Inverter Output Power (P_{inv})](image)

**Figure 6.1:** Power received by the grid for 6 WECS working in array for two levels of DC link voltage
6.1.2 Modulation Index of Grid Side VSC

The advantage of keeping the DC link voltage at a lower level has been discussed. However, the minimum level of the DC link voltage is determined by the modulation index of the grid side VSC. This is because when the DC link voltage is kept beyond its minimal level it affects the VSC’s system stability. Therefore, the selection of the DC link level voltage is very important and should be carefully chosen in order to guarantee the stability limits of the system when the control scheme is applied.

As discussed previously in Chapter 3, the DC link capacitor voltage level can be controlled by modulating the AC voltage of the grid side PWM-VSC. Two important parameters used for this are the modulation index \( m \), and modulation phase, \( \delta \). The converter modulation index relates the fundamental voltage of the modulation voltage in the PWM-VSC, \( V_{\text{conv}} \) to the DC link voltage:

\[
m = \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{\dot{V}_{\text{conv}}}{V_{\text{dc}}} \tag{6.2}
\]

The maximum value of the modulation index \( (m_{\text{max}}) \) in the linear region may differ depending on the method of the PWM generation [60]. The simulation studies presented so far in this thesis are based on the modulating operation of PWM-VSC being in the linear region \( (0 < m < 1) \). Once the modulation index, \( m > 1 \), the PWM-VSC is said to be in the overmodulation mode and \( \dot{V}_{\text{conv}} / V_{\text{dc}} \) is no longer linearly proportional to the modulation index \( m \). The state of the DC link voltage and the modulation index of the array’s inverter when different levels of voltage is used can be seen in Figure 6.2 and Figure 6.3.

Analysing the controller performance over the various level of DC voltage revealed that the stability of the grid side VSC system comes to the limits when the modulation index of the VSC exceed the linear modulation mode \( (m > 1) \). It can be seen in Figure 6.3 that the grid side PWM-VSC remains stable when the operation of the modulation index is kept within its linear region. In the case when the PWM-VSC operates beyond this boundary \( (m > 1) \) as can be seen in the first and second plot of Figure 6.3, the DC link voltage drops well below its reference value. Consequently, control action is taken at the inverter/grid side to provide adequate compensation by bringing the modulation index back to the stable region and within a few seconds the DC link voltage level is brought back to its reference.
value. However the recovery performance of the DC link voltage level is different for every nominal value of the DC link voltage.

As depicted in Figure 6.2, in both critical cases (when the DC voltage is set at 900V and at 950 V), when the overmodulation occurs at 35s, the DC voltage drops to 720V (in the 900V, a drop of about 0.2pu) before recovery action takes place compared to the second case (950V) which is set 50V higher where it exhibits more damped response to be 891V (drop ~0.063pu) with shorter and a smoother recovery oscillation period. In addition to that, as can be seen in Figure 6.2 and its magnified diagram at the top and the middle plot of Figure 6.3, both cases (900V and 950V) produce an under damped oscillatory response and restore the DC level within 8 sec after the voltage drop. However, there are cases when the controller is not able to restore the DC level to its reference value because the system becomes unstable especially whenever the modulation index exceeds its allowable limit. Furthermore, the stable region of the DC link voltage decreases as the voltage level is reduced, thus the reference DC link level should be chosen in which to keep the modulation index operating within its stability boundaries (in the linear mode).

Therefore there is a compromise in selecting the DC link voltage level of the WECs in the array. If $V_{DC}$ is set too high, more energy is required to charge the capacitor. On the other hand, if the $V_{DC}$ is set too low, the overall system response becomes less stable as the modulation index can possibly increase beyond its linear region even though less power is required to maintain the DC link voltage. Simulation studies show that values of $m$ between 0.4 to 0.98 are typical and can give satisfactory results. In this study, the selection of the nominal value of the DC link voltage can be seen to be a trade-off between avoiding
operation of the grid side’s PWM-VSC at the overmodulation mode and limiting the amount of the auxiliary power required by the array.

**Figure 6.3:** The magnified diagram of the DC link voltage with the values of the PWM-VSC’s modulation index.
6.2 Proposed Optimal DC link Voltage Control

Based from the simulation results presented in Figure 6.1 to Figure 6.3, the value of the reference DC voltage level plays a crucial role for the wave power system optimization and the power control stability. In the two cases in Figure 6.1 (1000V and 1300V), the power above 0W is the amount of power received by the grid while the power below 0W is the amount of power borrowed from the grid. For the 1000V case (revised back in Figure 5.8 (d)), except for the first 10s (during the transient period), the array supplies power to the grid for almost the entire simulation period. However, in the 1300V case, there are times where the DC link voltage is kept constant by borrowing the power from the grid. Thus the average power borrowed from the grid during a power shortage is lower for the 1000V case compared to when the DC link level is set at the 1300V.

The above argument and from the discussion made previously in section 6.1.1, shows the advantage of having a lower DC link voltage level in the array. However, as stressed in the section 6.1.2 the value of the modulation index $m$ of the PWM-VSC/inverter should be controlled so that it operates in the linear region ($0 < m < 1$).

Bearing this in mind, Adaptive Optimal DC link Voltage Control (AOVC) is proposed and will be applied to the array consisting of six WECs as shown in Figure 4.4. This control structure changes the reference level of DC link voltage in order to optimize the output power whilst maintaining the linear mode of the WEC modulation index. The proposed AOVC system, as illustrated in Figure 6.5, contains features that can overcome the disadvantages of the conventional grid side control proposed previously in Chapter 4. It is adopted from the grid control structure shown in Figures 4.11 and 4.12 with additional control functions to improve the control performance of the PWM-VSC at the grid side. The proposed adaptive control is a new control scheme for wave energy applications, which could manage highly varying power in the array by setting the DC link voltage to a level that best suits the power extracted from the wave.

In the AOVC system, a deviation reference level voltage (DRLV) unit is added to generate an adaptive DC voltage reference for the DC link. The DRLV unit is based on either a (a) unit step change or (b) unit ramp (Figure 6.4) and is used to set the nominal DC link voltage level according to the condition of the modulation index. During normal operation, the deviation voltage is set to zero by the DRLV unit as the DC link controller generates a current reference ($I_{inv_d_{ref}}$) for the VSC at the grid side. The error between the monitored
DC voltage $V_{DC}$ and initial reference value $V_{DCref}$ is sent to the controller. The control system will decide either to send or receive the power from the grid depending on the state of the inverter reference current. The control system will set $I_{inv_d_{ref}}$ positive when there is sufficient power in the DC link (inverter mode). Whenever a power shortage happens in the array, the control system responds by making $I_{inv_d_{ref}}$ negative with the result that the VSC acts as a rectifier and changes the direction of power by feeding the power from the grid to the DC link.

![Figure 6.4: (a) Step change and (b) Ramp change in the DRLV unit](image)

One objective of implementing the AOVC in the DC link control system is to reduce the amount of power taken from the grid. Therefore in the proposed AOVC control structure, the DRLV unit is used to produce an adaptive variation of the DC link whenever one of these two common events occur:

1. Whenever the modulation index nearly reaches the maximum limit or
2. When there is a power shortage in the array and there is insufficient generating capacity to keep the DC link voltage at its nominal reference value.
As the basic operating principle of the DRLV unit is to change the value of the reference DC link voltage level $V_{DC_{ref}}$, the DC link voltage is fully dependent upon the above conditions. During a power shortage, the DRLV unit will provide a voltage deviation that reduces the voltage level of the DC link so that less power is required to charge the DC link capacitor. However when the modulation index nearly hits the overmodulation region, then the DRLV unit will increase the level of the DC link voltage so that the VSC is modulated within its linear mode range. Otherwise the DC link reference value is kept to its nominal value. The simulation results of the proposed Adaptive Optimal DC link Voltage Control under the influence of both voltage deviations in the DRLV unit can be seen in the following section.

6.2.1 Step Change in DC Link Voltage Level Reference

Two controllers are in operation for the PWM-VSC (inverter) of the array (refer Grid side control in section 4.6.3.3). The performance of the these controllers, DC link voltage controller and the inverter current controller when a step change is applied to the DC link reference voltage can be seen in Figure 6.6. Several step changes are applied at the DC link of the array. There is no voltage drop in the DC link and the overall system is stable.
Figure 6.6: Effect of applying Adaptive Optimal Voltage Control (Step change) in DC link voltage reference (PI regulator: parameters A) and at the array's inverter

The results in Figure 6.6 shows the DC link voltage along with its reference values (dashed line) changed (step up/down) according to the value of the modulation index, \( m \). The critical limit of the modulation index \( m \) will be defined by the range of 0.9 to 1.1: when the ratio of the \( \frac{\dot{V}_{\text{conv}}}{V_{dc}} \) is within this range, the DRLV unit in AOVC will step up the reference voltage so that the modulation index \( m \) returns to the linear region mode. The opposite voltage deviation action will be applied to the control system when the power shortage occurs in the array. As can be seen in the Figure 6.6 (b) and through its magnified diagram, in Figure 6.6 (d), the modulation index is kept within the linear mode area and at the same time the average amount of power delivered to the grid is increased (Figure 6.6(f)).

The results presented in Figure 6.6 are based upon a fine tuning of the gain parameters \( K_p \) and \( K_I \) for all three PI regulators at the inverter end. To test the effect on the control variables (modulation index and the amount of power to the grid) when the values of these control parameters are changed, the same simulation step change is applied but now with different values of the controller gain parameters.
From the simulation results depicted in Figure 6.7, note that seemingly minor changes of the parameters values ($K_p$ and $K_i$ for PI controllers) can have a significant effect on the control variables. It can be seen the overshot response of the DC link voltage when the step change is applied gives an instant spike in the modulation index $m$ (Figure 6.7 (b) and its magnified diagram at Figure 6.7 (d)) and as a result the control parameters $i_d$ and $i_q$, can cause a short power reversal as can be seen at the 34.5s point. In this particular instant, the power is reversed from the grid to the array as the current $i_d$ becomes negative as can be seen in Figure 6.7 (e) and (f).

### 6.2.2 Ramp Change in DC link Voltage Level Reference

The state of the control variables when the DRLV unit applies a ramp function to the DC link Voltage Reference is shown in Figure 6.8. In order to optimise the power sent to the grid, the nominal reference value of the DC link voltage is initially set to the lower level of 900V. However at 29s, the amount of power received by the array causes the ratio of the $\dot{v}_{conv} / V_{dc}$ to rise to 1.1 (Figure 6.8 (d)) and hit its modulation index maximum limit. The
response of the AOVC at the inverter end is to ramp up the DC link reference level to 1000V resulting in the controller smoothly raising the DC link voltage up to the new voltage level (Figure 6.8(c)). A similar ramp up function is applied at 34s; however, when there is insufficient power to charge the DC link voltage to 1200V, the action of the DRLV unit in the AOVC will cause the level of the DC link voltage to ramp back down to 900V. The increment or decrement of the DC link voltage can be seen to be smooth as the reference level rises or falls due the ramp function.

Figure 6.8: Effect of applying Adaptive Optimal Voltage Control (Ramp change) in DC link voltage reference and at the array's inverter
The results shown in Figure 6.8 demonstrate the improvement in the transition between the level of the DC voltages whilst maintaining the objective of the controllers. It can be seen that the deviation of the reference level of the DC voltage by the ramp function is slightly better compared to the two cases of the step change function. The DC voltage does not show any sign of the overshoot response or step delay. As a result, the controlled variables smoothly change and exhibits no oscillations during the transient period as can be seen in the step function cases.

6.2.3 Comparison of the Grid side controls

The variation of the modulation index $m$ when the DRLV unit in the proposed Adaptive Optimal DC link voltage Controller (AOVC) applies a voltage deviation in the DC link is shown in Figure 6.9. It can be seen that the step function (parameter A) and ramp function provide the voltage deviation when the modulation index lies in its critical limit mode region (0.9-1.1), while the step function (parameter B) causes a short spike due to the overshoot response in the DC link voltage. Although this spike does not contribute to the voltage dropped in the DC link since it occurs for a very short period, it can cause short power reversal at the inverter side. The reason is the transition operation in the PWM-VSC at the grid side is affected by the condition of the modulation index and is directly proportional to the current controller $i_d$ and $i_q$, which control the direction of power between the array and the grid.

![Figure 6.9: Modulation index for different characteristics of DC link voltage deviation](image)

As can be seen from the results presented in this section, it can be concluded that the use of the proposed AOVC system control in the array is highly recommended. The main reason for this is because the power received by the array is fluctuating over the time period. As the AOVC provides a continuous monitoring system between the ratio $\dot{V}_{con} / V_{dc}$ and the
amount of output power to the grid, therefore, the best level of DC link voltage can be chosen. In addition, the AOVC offers a flexible and efficient means of controlling the DC link voltage level which ensures the control system operates within the stability limit.

6.3 Arrangement of array for fault tolerant configuration

There are various conceptual interconnection schemes proposed by wind farm researchers [21], [35], [75–77]. In this section some of these interconnection schemes are investigated for use in wave energy schemes in order to find the best configuration for fault handling and system protection.

6.3.1 Types of configurations

Possible array topologies and the faulted link investigated in this study are presented in Figure 6.10. This configuration study looks conceptually at which topology is more tolerant with the DC trips and how the control system handles a fault, thus the simulation model with the PWM-VSC, DC circuit breaker (discussed in 6.3.2) and disconnector as shown in Figure 6.10(a) is considered. These are the protection devices which are essential for the operation of the DC link network if a fault occurs in the array.

The first topology is a radial type configuration. This system shares a cable in the DC link which offers simplicity and can be considered as the most economical configuration because the total length of the DC cable connecting the WECs is the shortest [21]. The main problem with a radial configuration is its poor reliability during the occurrence of a fault in the DC link which can prevent some of the WECs from exporting power as shown in the simulation study in sections 6.3.3 and 6.3.4.

The drawback of the radial configuration, however, can be overcome by connecting the WEC in a star configuration as shown in Figure 6.10(b). If the distances between WECs is considered the same for all configurations, the total length of the DC link cable in this scheme is longer than in the first case since it provides a separate path to DC link for each WEC in the array. However, although it allows other non-faulty WECs to export power to the array, this configuration can be seen to be uneconomical and impractical, especially for longer distances because one DC cable is required to the DC link network for each WEC. Furthermore, it still prevents some of the power generated by the WECs to the grid (at least from the nearest WEC where the fault occurs).
The most critical cases for the two previous configurations would be if the central DC link develop a fault, as illustrated in Figure 6.10 (c) by F2, which can prevent all the WECs in the array from transporting power, thus causing all the power generated by the array to be totally lost. Therefore the ring type configuration might be the best option as it provides a redundant DC link cable for the power flow from the array. In the event of a fault between the WECs or at the central link, the power can still be diverted through the second link. This configuration option is technically more fault tolerant compared with the previous configurations since it can export the power from the WECs to the DC link regardless of any fault in the DC link cables as long as the power path from the array to the DC link is not impeded. However, the cost of this configuration is higher since more circuit breakers and disconnector switches are required and a complex control system needs to be applied to the system for isolating the faulty link and controlling the diversion of power through the second path of the array.

The objective of this study is to gain an understanding of the capability and limitations of these configurations. As can be seen, each of these three topologies have similar DC fault protection devices marked with '□' sign and are equipped with a DC breaker and connector/disconnector switchers in order to provide or disrupt the power path from the array to the DC link.
Figure 6.10: Types of array topologies and DC faults
6.3.2 IGBT DC Circuit Breakers and disconnector for Fault Protection

Only DC line to line faults are considered in this study, denoted by the lighting sign in Figure 6.10. In order to prevent faults in one link of the WEC affecting the rest of the WECs in the array, the IGBT DC breaker illustrated in Figure 6.11 can be used. Protection from a DC fault is provided by the pair of IGBTs and anti-parallel diodes which are placed between the link connecting the WEC side and the DC link, as proposed in [82]. When the fault occurs, the short circuit voltage between the DC link to ground will cause a large current to be drawn from the source to the ground (direction of current is denoted as dashed arrow), thus the IGBT-CB is used to cut-off the flow of the fault current to the DC link. In the case where the direction of fault current is opposite to the dashed arrow (solid arrow) the blocking flow of current will come from the IGBT in the VSC [83]. The reason for this is because this IGBT-CB cannot be used to cut-off the direction of faulted current flow in both directions.

The isolation of the faulty line from the DC network is performed by the disconnectors [38]. However the disconnector can only isolate the faulty DC line once the DC fault current is cleared by the DC circuit breaker, which in this case is the IGBT-CB [84]. The isolation of the faulty DC link is illustrated in Figure 6.12. The operation is begun by triggering the circuit breakers connected between the faulty lines leaving it to be open circuit until the fault currents are totally extinguished. Soon after the fault current reaches zero, the disconnector is used to isolate the faulty line from the DC link network [38]. Once the faulty line is fully separated from the array, the adjacent circuit breaker can be connected again providing a path to the DC link. Theoretically, the disconnector will remain open if the system verifies the fault is permanent, otherwise the disconnector will
close back after the fault parameters are extinguished completely before re-establishing the connection between the WEC and the DC link.

Figure 6.12: Isolation of the line fault and diversion of power path
(a) During normal operation, (b) Fault occurred between the WECs, (CB1 and CB2 open) (c) Disconnector (DC 2) isolating the faulted line, (d) After faulted line being isolated, CB 2 closed for the connection with the second path of the DC link network.

Figure 6.12 shows the example how a diversion path from the WECs to the DC link in ring type configuration can be made through the operation of DC circuit breakers (CBs) and disconnectors (Dis). As can be seen in Figure 6.12 (a), under normal operation, both WECs export the extracted wave power through the main DC link cable. Once the fault occurs (between the WECs), both adjacent CBs (CB 1 and CB 2) will act by opening the connection between the faulty area and the DC link (Figure 6.12 (b)). Then the disconnector DC 2 will isolate (either permanently or temporarily) the faulty cable (Figure 6.12 (c)). Once the fault has been isolated, a path connecting the bottom WEC to the DC link is restored (Figure 6.12 (d)).
link can be provided by closing CB 2 (Figure 6.12 (d)). The function of CB 2 here is therefore twofold; first to clear the fault current and second to provide the diversion path to the DC link.

6.3.3 Array configurations Study under Faulted Conditions

In order to see how the array operates under fault conditions, the least complex type of incident waves (ideal waves) received by the array described in section 4.6.5 is used. The ideal incident waves shown in Figure 4.16 (a) consist of a series of six regular waves having the same frequency and amplitude and are applied to the array configurations shown in Figure 6.10. The detailed circuit of the array and the parameters of interest under fault conditions can be seen in Figure 6.13. The DC link voltages are labelled as \( V_{DC} \), \( V_{DCp} \), \( V_{DCn} \) and \( V_{off} \). The DC link currents are labelled as \( I_{DCp} \) and \( I_{DCn} \). The array DC faults currents are labelled as \( i_{f,p} \) and \( i_{f,n} \). Subscripts \( p \) and \( n \) refer to the positive and negative bus of the DC link. The AC parameters of interest at the inverter side are the instantaneous output AC current and the array's real and reactive power and are labelled as \( I_{inv} \), \( P_{array} \) and \( Q_{array} \) respectively.

Figure 6.13: Details of the array and parameters of interest during fault
Figures 6.14 to 6.17 show the simulated graphs for the DC link voltage, $V_{DC}$, $V_{DCp}$, $V_{Dc}$, and $V_{DCn}$, the array DC link and fault currents $I_{DCp}$, $i_{LP}$, and $i_{LN}$, the grid or PCC terminal voltage and current for a single phase, $V_{inv}$ and $I_{inv}$, and the amount of power sent to the grid by the array, $P$ and $Q$, for three different types of DC array topologies.

Prior to a DC fault, the DC link voltage level is set by the DC voltage reference of the DC link voltage controller in Figure 4.11. After 3s of simulation time, the DC fault is introduced. As can be seen (in Figure 6.14 (a)) the voltage collapses to zero since the DC fault connects the DC link directly to the ground and as a result, large currents are drawn from both sides of the faulty line and quickly discharge the DC capacitors. As shown in Figure 6.14 (b), the DC fault current is limited to 300 A. Because the DC fault brings the impedance between the DC link and the ground close to zero, it causes large currents to flow in the faulty line as can be seen in Figure 6.14 (c) and (d), showing that large currents flow backwards from the grid side. Such currents can flow continually through the anti-parallel diode in the grid side VSC which can lead to equipment damage. Thus proper control action with the CBs and disconnector is required for protection of the devices.
Figure 6.14: DC line to line fault before and under fault condition
(a) DC link voltages,
(b) DC link currents,
(c) voltage and current in p.u at the grid side
(d) Output Power to the grid.

6.3.4 Performance under Fault Isolation and path excursion

Figures 6.15 show the simulated DC link Voltage, DC link currents, output power, and grid side voltage and current before and after a fault in a radial array configuration. The DC fault is applied at the centre of the DC link as was shown in Figure 6.10 (a), which can prevent the three bottom WECs from exporting their power. It should be noted that in Figure 6.15 (a) at 3.2s, the DC link voltage rises back to its nominal value once the DC fault has been fully isolated by the IGBT-CB. However as shown in Figure 6.15 (b) and (d), the amount of positive DC link current, $I_{DCP}$, as well as the power into the grid, $P$, are decreased to half after the fault since the IGBT-CBs block the connection of the other three WECs to the DC link. This explains the phase voltage and current waveforms that are seen in Figure 6.15 (d-f), where the amplitude of the current is reduced to 0.5 p.u.
It can be seen that although radial configuration is a straightforward solution to connect all the WECs in an array, the design has a higher risk of limiting a large amount of power to the grid during the fault. As an example, if the fault occurs in the middle of the WECs’ link (as in the case of Figure 6.10(a)), half of the power cannot be transmitted; however, the worst scenario will be if the fault occurs at the top end of the WEC’s link, which may
sacrifice all the power generated from the array. If no further action is taken, the array would deliver no power to the grid.

Figure 6.16 shows the array in star type configuration under DC link fault conditions (Figure 6.10(b)). This type of configuration is aimed to eliminate the risk of the faulty DC link affecting other WECs in the array. However the star configuration is not really suitable for the larger distance type of array because it is not economically feasible except for multiple arrays or cluster collector system configurations. As can be seen in Figure 6.16 (c), only the power from the faulty WECs is prevented from exporting power to the grid. Since all the WECs in the array receive the ideal series form of incident waves (Figure 4.16(a)), the power loss due to the DC fault in star configuration is a sinusoidal shape, which before the fault is a part of the total power sent to the grid (also can be seen by the shape of the current waveform in Figure 6.16 (d)). Thus in this configuration, the DC fault will only affect the nearest WEC without preventing other WECs from sending power.
The last configuration shown in Figure 6.10 (c) is a ring topology, which can be considered the most reliable of the three configurations studied in this work. The performance of this type of array under fault conditions is shown in Figure 6.17. The disconnector and IGBT-CB can be seen to provide an alternative path for the power to flow from the WECs to the DC link during the fault period. As can be seen in Figure 6.17 (b), after the pair of IGBT-CB fully extinguished the fault current, the disconnector is used to isolate the faulted line from the rest of the WECs in the DC link. The power delivered by some affected WECs
then experience a short disruption before the bottom end IGBT-CB establishes a second path to the DC link to allow the WECs in the array to resume sending power to the grid. The benefits of adopting a ring configuration can be seen through the simulation results presented in Figure 6.17 (a-e). As can be seen, the amount of power received by the grid before and after the fault is the same as shown in Figure 6.17(c-e).
This study demonstrates the power handling capability of three types of array configurations, radial, ring and star topologies during fault conditions. Comparing these three types of array configurations, it can be stated that the ring topology offers good performance and higher reliability under fault conditions compared with the other two configurations. The radial configuration is the best candidate for a small array or any short distance DC link cable where the chances of a fault occurring is lower. The star configuration is considered the second best for fault tolerance and for economical benefits, it is suitable for the medium-large type of array whereby each branch of their DC link may consists of several small number of arrays (WECs in cluster). The IGBT-CBs used in this study are assumed to work with 120kW of WECs in array, other types of DC breakers maybe required for the array with a higher rating DC power interconnection.
6.4 Integration Study for multiple array of WECs

Since the announcement of the plan for the first array of WECs by Pelamis Wave Power [5] in 2008, there have been several proposals for such arrays presented by the wave power community [85]–[89]. As the research in wave power grows (see Figure 14.2 in [9]), integration issues for such arrays have focused on the flexibility and the security of the power delivery and making them economically attractive. In this section the concept of integration of multiple arrays is discussed, and how the control schemes can be derived for a larger size of wave farm working in a cluster.

The configuration of the integrated arrays proposed in this study is shown in Figure 6.18. DC Interconnection is used and each array has VSCs at both sides (WEC and grid side) to transmit power to the onshore grid. As stated in Chapter 4, the main control objective of the VSCs in the array is to optimize the collected wave power at the generator side before the power is transmitted to the shore with the suitable grid voltage and frequency. As can be seen, this integration scheme connects all the arrays in an AC link network and has two AC cables for power transmission to the shore. The ring type configuration is used between the arrays for power security in case a fault occurs within the network to allow a path for power to flow. As can be seen, two transmission cables are proposed for the safe power transmission to the shore. If a fault occurs in the dispatch power line, the power generated can still be transmitted through the second AC transmission cable. However, the diversion of the power path through the actions of AC breakers is not discussed here as it is considered a mature technology in power system protection [90]. The case study is made for control management in Array 1 and Array 2 and the assumed length between WECs, arrays and the shoreline can be seen in Figure 6.18, which represent the potential case of a wave farm off-shore from an island electricity network.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input/Output</td>
<td></td>
</tr>
<tr>
<td>Nominal DC link voltage (V_{DC})</td>
<td>1000 [V]</td>
</tr>
<tr>
<td>Rated output voltage in rms (V_L)</td>
<td>700 [V]</td>
</tr>
<tr>
<td>Rated power (array)</td>
<td>60 [kW]</td>
</tr>
<tr>
<td>Rated output frequency (f)</td>
<td>50 [Hz]</td>
</tr>
<tr>
<td>Impedance</td>
<td></td>
</tr>
<tr>
<td>DC link Capacitor (C_{DC})</td>
<td>80 [mF]</td>
</tr>
<tr>
<td>Cable inductance per km (L_L)</td>
<td>24.5 [mH]</td>
</tr>
<tr>
<td>Cable resistor per km (R_L)</td>
<td>0.125 [Ω]</td>
</tr>
</tbody>
</table>
6.4.1 Simplified model for control design

Due to the complexity of the components in the integrated arrays, the need for a simplified model for control purposes of Figure 6.18 is required. Some assumptions can be made so that an appropriate model can be built in keeping with the objectives of the study. The required model components should be carefully chosen from the actual system so that they fairly reflect the control performance of the proposed system.

In this study the following assumptions are made:

- Ideal incident waves are considered.
- Six WECs in the array are aggregated into a single WEC having a power rating of six WECs with the same control scheme as in 4.6.6.2. The number of WECs have
been aggregated into a single WEC is to remove the averaging effect of multiple WECs driven by the ideal incident waves.

- Only the control scheme for the VSC at the grid side is considered here, thus the VSC at the WEC side is not included in the model.
- The power losses within the system are ignored.
- Due to the similarity of the control schemes between the arrays, the integration study is made between two arrays and the grid.

With these assumptions, the equivalent circuit in the integration arrays study is shown in Figure 6.19. This equivalent circuit is the general model for the power transmission system that can be used in mathematical modelling for the control design.

The equations are derived from Figure 6.19 by dividing it into three parts, which are Array 1, Array 2 and the grid.

\[
L_s \frac{dI_s}{dt} = -R_s I_s + V_s - V_L \quad (6.3)
\]

\[
L_c \frac{dI_c}{dt} = -R_c I_c + V_L - V_c \quad (6.4)
\]

\[
L_o \frac{dI_o}{dt} = -R_o I_o + V_L - V_c \quad (6.5)
\]
However only (6.3) and (6.4) are used for control purposes and can be further developed using a d-qo transformation:

\[ V_{sd} = (I_{sd\_ref} - I_{sd})G_s - X_{Labc} + V_{ld} \]  
(6.6)

\[ V_{sq} = (I_{qd\_ref} - I_{dq})G_s + X_{Labc} + V_{ld} \]  
(6.7)

and

\[ V_{cd} = -(I_{cd\_ref} - I_{cd})G_c + X_{Labc} + V_{ld} \]  
(6.8)

\[ V_{cq} = -(I_{cd\_ref} - I_{cs})G_c - X_{Labc} + V_{ld} \]  
(6.9)

where \( G_s \) and \( G_c \) are the gains of the conventional PI current controllers for the VSC in Array 1 and Array 2. Both of these classical pair of d-q transformation equations then are used to control the power between the arrays as illustrated previously in Figure 4.12.

The next sections 6.4.2 and 6.4.3 discuss the integration control schemes which focus on controlling the DC link voltage of the arrays, two main approaches are presented, primary and secondary voltage control. Although a similar control scheme to control the DC voltage bus has been proposed in wind park [91], in this study a DRLV block unit in the AOVC scheme is added in the primary voltage controller to provide the appropriate reference voltage level of the DC link bus in the arrays.
6.4.2 Primary Central Voltage Coordinator

Figure 6.20 shows the primary voltage coordinator used in this study. The central control system receives all the load current values \((I_L=I_C \text{ and } I_S)\) at the AC link network for the input of the deviation output voltage block, DRLV. As can be seen AC line impedance is used to determine the deviation voltage \(\Delta V\) when setting the DC link voltage level.

\[
\Delta V = \begin{cases} 
0 & ; -0.6 \leq \Delta m \leq -0.1 \\
\Delta m \times 0.2V_{DC} \text{ (p.u)} & ; \text{elsewhere} 
\end{cases}
\]  

(6.10)

The DRLV block in Figure 6.20 moves the DC link voltage level when the modulation index, \(m\) operates, beyond its permissible region as discussed in section 6.1.2. The output voltage from the primary voltage coordinator then is used as the input for the secondary voltage controller to adjust the nominal level of the DC link voltage of the array.
6.4.3 Secondary Voltage Controller

The secondary voltage controller shown in Figure 6.21 provides the function of DC link voltage compensation. The power in the array is now controlled with the new level of DC link voltage set by the primary voltage block. As the deviation voltage $\Delta V$ varies with time, the control action is applied by comparing the DC link voltage, $V_{DC}$, with its reference value. The output of the controllers are the current references for the real and reactive components in the array.

\[
\Delta V + V_{DC\text{ref}} \rightarrow \text{DC Voltage Controller} \rightarrow K_p + \frac{K_i}{s} \rightarrow \text{d-q current controller (Fig. 4.12)} \rightarrow V_{inv} \rightarrow \text{Sabc}
\]

\[
\Delta V + V_{DC\text{ref}} \rightarrow \text{DC Voltage Controller} \rightarrow K_p + \frac{K_i}{s} \rightarrow \text{d-q current controller (Fig. 4.12)} \rightarrow I_{inv_d\text{ref}} \rightarrow \text{Sabc}
\]

Figure 6.21: Secondary voltage controller

6.4.4 Case Study

The model of the system studied is constructed and simulated using the MATLAB SimPower System to validate the proposed control schemes for the integrated arrays shown in Figure 6.18. Assessments of the control performance are made through the results presented in this section.

6.4.4.1 System Characteristics

The power system parameters used in the integration study are shown in Table 6.1. Under normal operation conditions, Array 1 and Array 2 transmit the power to the grid through the local AC bus. When there is a power shortage in Array 2, the Array 1 will deliver some of the power generated to help the power deficiency at Array 2. Therefore the control is made in Array 2 for allowing the current to flow in both directions. Meanwhile, as can be seen in Figure 6.19, the direction of current $I_{SABC}$ in Array 1 is made to flow in one direction thus is assumed to be delivering the power for the full period of study. If the
power shortage in Array 2 cannot be covered by the power generated in Array 1, the deficit should be made by reversing the power from the grid to the Array 2.

The simulation results are presented based on two step variations: first, when the step voltage reference level is applied in Array 1; second, when step load current is applied in Array 2 which changes its power demand. This study is carried out to test the performance of the control schemes over the arrays power management when these demanding variations occurs in the system study.

6.4.4.2 Simulation Results

The simulation results in Figure 6.22 show the control performance when several voltage steps are applied to the Array 1 DC link. As the array receives ideal incident waves, the power generated is assumed constant, thus the deviation reference voltage level is made with no precondition as previously applied in the irregular wave case defined in (6.10). However the system response shows the controllers in Array 1 are able to perform adequate regulatory action when the disturbance is introduced in the DC link by regulating the DC voltage level and the output power to the desired controlled response. This can be seen in Figure 6.22 (a-c) where the DC link voltage level is controlled without affecting its output power to the AC network.

![DC link voltage (Array 1)](image)
Figure 6.22: The system response of the voltage deviation in Array 1
(a) Step voltage deviation in Array 1, (b) modulation index
(c) Real and reactive Power of Array 1 (d) AC Network frequency

The performance of the control schemes for the integration study can be seen by changing the array power requirements. As the power in Array 1 is controlled to remain constant during the step voltage deviation as demonstrated in Figure 6.22 (c), a step change power
demand is made in Array 2. The results due to this power variation are presented in Figure 6.23. As can be seen in Figure 6.23 (a-c), the current controller controls the Array 2 output power based on its power requirement. In the event of a power shortage (during 1-1.5s in Figure 6.23 (a)), control action is made by taking the power from the Array 1 and this can be seen when no power is received by the grid (during 1-1.5s in Figure 6.23 (e)). However when the power required by the Array 2 is more than the power generated in Array 1, as occurs during 2.5 to 3 s, the power deficiency is covered by the power from the grid. The optimum power will be received by the grid when both arrays are delivering power.

(a)

(b)

Figure 6.23. The system response when the power demand in Array 2 is changing. (a) Array 2 output power (b) Array 2 load current. (c) Array 2 load current after power deficiency were applied. (d) AC link voltage to Array 2 load current (e) power is delivered from Array 1 during 1-1.5s period and power exchanged by the grid.
Figure 6.23: The system response when the power demand in Array 2 is changing
(a) Array 2 output power, (b) Array 2 load current, \( I_c \) (c) Array 2 load current, \( I_c \) (magnified) when the step power deviation were applied (d) AC link voltage vs Array 2 load current (no power is delivered from Array 2 during 2-2.5s period) 
(e) Real and Reactive Power received by the grid.
The study shows good control correlation between the power generated from the arrays and the power received by the grid. The simulation results confirm the performance of the control schemes over the power system studied. Although the case study is made based on ideal incident waves, the control procedure made is adequate to validate the performance of both primary control coordinator and secondary voltage controller when handling the power between the arrays.

6.5 Chapter summary

In this chapter the impact of the DC link voltage level over the performance of the system control is discussed and an adaptive control scheme that can determine the best level of the DC link voltage level is proposed. The proposed voltage controller has an additional feature block called the deviation reference level voltage (DRLV) which is included in the present control structure and is applied to the VSC at the grid side. Its function is to change the reference voltage level of the array DC link based on the condition of the modulation index. Also discussed are the general types of array interconnection schemes proposed in wind farms. The performance of these various configurations when applied for WECs in an array was investigated when different line-to-line DC fault locations were applied. The type of array that has advantages in terms of power security and fault tolerance can be identified as the best configuration and this can be seen through the simulation study conducted in section 6.3. The chapter concludes with a control methodology for the integration between arrays of WECs. The control schemes, called Primary and Secondary Voltage Controllers function to synchronize the reference voltage level within the arrays to best suit the power generation from each array in the park. A model of this control structure has been made and the performance of the control schemes were presented in the final section in this chapter.
CHAPTER 7

Discussions And Conclusion

The following summarises the key elements made in this thesis and is followed by a general discussion of the principal work area presented in this study. Areas of future research are suggested and some arising issues that need to be tackled are discussed before the conclusion is given in the final section.

7.1 Evaluation of the Chapters

In chapter 3 the fundamental structure of WECs in an array was explained, and a description was given of the components for the derivation of the mathematical model of WECs working in an array. The models were built with consideration of incoming wave types, interaction of the oscillating body with the sea, geometry of the generator, and the topology and control operation of the power converter circuit. These mathematical models form the main component blocks in the array and can be divided into three sub-groups: Heaving-type Point Absorber, Wave Energy Converter (WEC) and the power converter PWM-VSC. The concept of the Point Absorber is introduced to the reader to provide information on the type of wave energy captured considered in this study. The equations of motion were derived based on the types of incident waves and were used to describe the dynamic behaviour of wave-buoy interactions. In the WEC, the term direct drive describes the method of the WEC power take-off. The generator geometry was shown and due to the movement of its translator the voltage and the generator force equations were derived. Lastly in Chapter 3, the operational principles of a PWM-VSC were explained and how the concept of controlling power in conventional power circuits can be extended to WEC applications.

In Chapter 4, the control schemes for deriving the control signals for WEC when incident with a series of regular waves were presented. Both optimal and sub-optimal control strategies were proposed to control the power from the WEC. To assess the suitability and
the performance of these control schemes, the following three types of regular wave studies were conducted:

1. When WECs receive an incoming series of ideal regular waves,
2. When WECs are incident with synchronous regular waves,
3. When a series of regular waves with different amplitude, phase and frequency are incident with the WECs.

The purpose of the control scheme in a WEC is to optimize the amount of power extracted from the wave by controlling the phase current of the linear generator, and on the grid side to make the output voltage and current conversion suitable for the requirements of the grid. However to ensure the effectiveness of the control schemes, power flow on both sides needs to be controlled so that the voltage level on the DC link is kept at the constant value. In an attempt to increase the accuracy of the simulation results, sample times as small as $5 \times 10^{-6}$ second were used for obtaining reliable results and information from the output signals. For each case study, a reasonable effort is made by carrying out the simulation study as two separate modules.

In the first module, the dynamic behaviour of the oscillating buoy when incident with the incoming regular waves was studied using the power take-off and hydrodynamic parameters produced by WAMIT, a software package used for wave-body interaction analysis. The power take off parameters which could be varied by the control scheme is the load impedance ($Z_{po}$) described in sections (4.6) and (4.7), and is utilised to derive the reference generator force required for the optimal/sub-optimal control strategy. The response of the WEC buoy can be obtained using the equation of motion derived in sections 4.6.1 to 4.6.2 for sub-optimal, sections 4.7.1 to 4.7.3 for optimal control, where the steps taken for the derivation of these control profiles were given. The results of the linear generator translator displacement followed by the derivation of the required phase current were recorded in the Matlab workspace and used as the input profiles in the next simulation session.

In the second module, the complete mathematical model and the component blocks of six individual WECs as partly illustrated in Figure 4.4 and Figure 4.5 are simulated using the input profiles obtained from the first simulation. Several points were concluded based from the results presented for each case study. The first is when the array is acted on by ideal waves as shown in Figure 4.16 (a). This study is very useful as a proof for the concept that
the power generated from the linear generator can be converted into nearly constant output power by using the control scheme configuration shown in Figure 4.18. The process of power conversion of the linear generator's phase current of varying amplitude, phase and frequency when converted into pulses of nearly constant amplitude can be seen in Figure 4.3. The resulting waveform shows that nearly constant output power can be generated when connecting together several direct-drive WECs in the array. Therefore, increasing the number of WECs in the array results in smoother output power. Secondly, the effectiveness of the control schemes (for DC link Interconnection) proposed in this thesis is reliant on maintaining a constant level of DC link voltage in the array. This can be seen when the WECs receive a series of coherent regular waves as shown in Figure 4.16 (b). The effect when an inconsistent DC link voltage level occurs in the array can be seen in Figure 4.21 (c), where the phase current deviates from its reference value potentially causing a reversal of current direction on the generator side. To avoid this, the DC link voltage level needs to be restored to its nominal value by allowing the power in the grid side VSC to flow in both directions. The process of reversing the power flow during the power shortage is well explained in section 4.6.4.3, and the result of the output controlled variables such as the DC link voltage level, generator phase input currents, \(i_{\text{DC}}\) current pulses, and the output inverter phase current, \(I_{\text{inv}}\), feeding into the grid, before and after restoring DC link voltage level can be seen in Figure 4.21 and Figure 4.22 respectively. The simulation results presented in section 4.6.6.3 prove that the control scheme configuration shown in Figure 4.20 is capable of managing the power even for the case when the DC link voltage level drops from its nominal value. However, due to the inevitable shape of the electrical power generated by the linear generator and the inconsistent amount of energy gained from the waves, there is no option except for the array to seek auxiliary power in order to maintain its DC link voltage level during a power shortage. These impacts, however, can be mitigated by lowering the voltage level in the DC link or ‘borrowing’ the power generated from neighbouring arrays as presented and demonstrated in Chapter 6. Finally, the control scheme configuration proposed in Figure 4.23 performs very well even when receiving incident waves of different amplitudes, phases and frequencies as shown in Figure 4.16 (c). Although the generated electrical power from the array of linear generators is highly varying, the resulting output waveform shown in Figure 4.24 fulfils the control objective from the electrical point of view, by maintaining the DC link voltage level and delivering constant frequency and phase output currents to the grid. The results also show that only real power is sent to the grid.

In Chapter 5, the control schemes are applied to WECs in an array when dealing with irregular waves. In a similar approach to the strategies described in Chapter 4, the control
schemes are proposed based on the concept of optimal and sub-optimal power capture. Three types of irregular waves were used to model a time series of wave elevations received by the array. The control structure made in this study was illustrated in Figure 5.3 and Figure 5.11. The selection of the PTO impedance, $Z_{PTO}$, which is used for the derivation of both control schemes was justified and described in detail. The simulation results presented in this chapter confirm the performance of the proposed controller. The stabilisation of power flows in the array can be seen when the DC link voltage is maintained at its nominal level, although the array constantly received a varying input of incoming waves. The impact of optimal and sub-optimal control when successfully applied to the array can be validated through the response of the mechanical and electrical output waveforms in the corresponding WEC. When the optimal power strategy is applied it will lead the WEC operation to its resonant condition, which can be seen when the velocity of the WEC translator is in phase with the excitation Force, $F_{ext}$. Meanwhile, in sub-optimal control the exclusion of the stiffness force coefficients, $C_{PTO}$, in the control schemes results in the WEC not achieving maximum power extraction; however, the control scheme allows the generator phase current to be in phase with the generator output voltage. The total electrical power generated by both control schemes can be seen in Figure 5.18. In the optimal power control scheme, the PTO force, $F_{PTO}$, is derived according to the impedance matching principle. As stated in the equation (5.13), information of future velocity and incoming wave frequency is required for the PTO coefficient values in the control scheme. However since the PTO damping coefficients only provide amplitude control for the WEC device, accurate prediction of the future velocity for the PTO damping coefficient ($B_{PTO}$) is not as important as the prediction of the incoming wave frequency for the derivation of the PTO stiffness coefficient ($K_{PTO}$), which brings the velocity of the WEC device in phase with the excitation force $f_{ext}$, which is vital for optimal power generation. Finally in chapter 5, the benefit of implementing the AC link Interconnection in reducing the current stress on the power converter (IGBTs) of the array can be seen in the simulation results presented in section 5.7.2. The results shows that the current stress on the semiconductor switches (IGBTs) in the VSC at the grid side is reduced by $1/n$ when AC link interconnection is chosen for the array. Thus this configuration is strongly recommended for the implementation of medium or high power direct drive WECs.

The results of keeping the DC link voltage at different levels were shown in Chapter 6. A higher level voltage increases the initial energy required to charge the DC link capacitor, while a lower level voltage results in the control system losing stability. Therefore, an
adaptive reference DC voltage level that suits the power received by the array is proposed. The control action is performed by the PWM-VSC at the grid side of the array. The objective of the control strategy for ensuring power system stability is achieved by making the system operate within the stability margin. This grid side control scheme is then extended for application in a group of arrays (cluster) where the synchronization of the grid side control among the arrays in the wave park is performed using the proposed cascaded control of primary and secondary voltage control. The block structure is described and the effectiveness of the control system is validated through the simulation study confirming its suitability in managing the power sent to the grid. The comparative study of several types of array configuration when subjected to DC link faults is discussed in section 6.3. Various DC link configurations have been studied that consider when the array of direct-drive WECs system experience line-to-line DC faults. The configurations presented represent the general interconnection schemes proposed for wind and wave farms which include radial, star and ring configurations as shown in Figure 6.10. The objective of the study is to investigate the system behaviour under faults and how these different configuration types impact on the security of the power transmission to the grid. Several fault-handling steps through the action of the circuit breakers (CB) and the isolators is made in the simulation study for ensuring the continuity of the output power supply to the grid and for the protection of the power system. Issues such as limiting some of the power generated from the effected WEC when a DC fault occurs can be avoided by choosing the right type of array configuration. The study shows that the power from the non-faulty direct drive WECs can still be transmitted to the grid, as can be seen from the simulation results presented from Figure 6.14 to Figure 6.17. However, this is due to fast and accurate communication systems to coordinate the control action of the circuit breakers and disconnectors in the array. An efficient detection system is required to disconnect only the faulted line and provide flexibility within the array by opening a new pathway to allow the power to flow if the fault occurs in the main cable. Furthermore, a fast coordination time for determining the correct disconnecting and reconnecting action of the circuit breakers is needed in the array to prevent the fault from continuing to extract the power from the arrays.
7.2 Discussions

This section discusses the main work presented in the thesis: control schemes applied, the proposed control structure and DC and AC link interconnection array systems. The four of the WEC side control methods covered in this study can be divided based on regular and irregular wave cases and sub-optimal and optimal control.

7.2.1 Optimal Control Scheme in regular wave

The control strategy applied in this thesis is based on the concept of amplitude and phase control of linear generator developed by Shek [48]. The control scheme was initially designed for controlling the power captured by a single generator, and then was extended in this study for the use of multiple generators working in an array (with parallel interconnection). The strategy to develop this control scheme was explained in section (4.7), which requires the control system to have accurate information of the incoming waves (i.e frequency). The reason for this is because the stiffness force coefficient, \( K_{PTO} \), which is responsible for WEC phase control, is a function of frequency as defined in (4.28). Considering the heave mode in the point-absorber system, the reference value for the stiffness force coefficient depends on the exact details of incoming wave frequency which must be predicted by the control system in order to extract maximum power. Optimum power capture could not be achieved if inaccurate values of the stiffness force coefficient are used in the control scheme, as this would cause the WEC to deviate away from its resonance frequency. Therefore a new adaptive control strategy that converges the \( K_{PTO} \) with its reference value to satisfy the WEC phase control requirements needs to be developed. However the control system still needs to have good prior frequency prediction of the incident waves in order to provide adequate response of the WEC system.

7.2.2 Optimal Control Scheme in irregular wave

A reliable forecast for the incoming wave frequency is required by the array control system as discussed previously for the regular waves. As defined mathematically in (5.13a), in irregular waves, the prediction requires one step further as the future velocity of the heaving buoy/translator has to be predicted by the WEC system in order to perform complex conjugate control. The predictive method as illustrated in Figure 5.11, requires both information of incoming waves prior to incident, \( A(k+1) \), and also the velocity \( \dot{x}(k) \) of the WEC translator due to incoming wave. These two parameters are the inputs to the
predictive block and are used for the damping force coefficient derivation, $B_{PTO}$, which is responsible for WEC amplitude control. As noted in [18] and [92], the term additional added mass, $A_{\omega}(\omega)$, which is required for calculating the stiffness force coefficient, $K_{PTO}$, can be referred to as the added mass at a specific frequency, $A_{\omega}(\omega)$, when studying the sinusoidal motion of waves. In this study an approximation is made by considering additional added mass $A_{\omega}(\omega)$ as the added mass of the fundamental frequency of the polychromatic waves, $A_{\omega}(\omega)$, as defined in (5.13b). These PTO coefficients, $B_{PTO}$ and $K_{PTO}$, form the reference PTO force, $F_{PTO}$, and are used for the derivation of the generator force, $F_{gen}$. The performance of the proposed control schemes in the array when incident with irregular waves can be seen from the simulation results presented in section 5.53-5.54. In general it can be concluded that the control schemes proposed in this study successfully control the power of the array. The good agreement between WEC side control and grid side control can be seen when the DC link voltage level is controlled to its constant value which can be seen in Figure 5.15.

7.2.3 Sub-Optimal Control Scheme

It should be noted that sub-optimal control in this study leads to amplitude control of the WEC buoy/translator. Therefore, without the presence of phase control in the control scheme the WEC could not reached its resonant condition for maximum power absorption. However, the derivation of the reference phase current shows that the current in the sub-optimal scheme can be controlled to be in phase with the generator output voltage as can be seen in Figure 4.27 and Figure 5.8 (a) for the case of regular and irregular waves respectively. Therefore the power factor in the WEC side AC bus can be kept close to unity which is desirable for electrical power generation. Generally, the amplitude control provided in this control scheme imposes damping to the motion of the WEC system. As the damped response in a WEC system is dependent on the damping coefficients in the equation of motion, as defined in (4.5) and (5.6), the impact of the total power absorption in WEC is determined by the damping factor in the PTO system, $B_{PTO}$.

The graph in Figure 4.6 shows a relationship between the absorbed power, $\bar{P}$ and the PTO damping coefficients, $B_{PTO}$, for regular waves: when the PTO damping is completely absent from the system (eg. PTO device disconnected), the WEC buoy is freely floating and no power is captured as the intercepted energy gained by the buoy is returned to the sea. Conversely when the PTO damping, $B_{PTO}$, value is too large the WEC oscillation is impeded resulting in the wave device not capturing power. Therefore, the PTO damping
that can bring the WEC towards its best performance state under the sub-optimal control scheme needs to be calculated from the optimal damping equation as previously derived in (4.8) and (4.9).

7.2.4 Proposed Control Structure in WEC Side Control

The control structures presented in this thesis are aimed to eliminate unnecessary components that are redundant in the array system. Thus in the initial strategy, the same control signal used in the first WEC is also be applied to all WECs in the array. While the control structure on the grid side, used conventional VSC d-q synchronous rotating coordinates, the WEC side control structure was developed by considering the condition of the incident waves. The reason for this is the hydrodynamic parameters in the control scheme are set by the state of incident waves experienced by each WEC in the array. However, the complexity and the number of the components in the array can be reduced if the incident waves received by the array vary in amplitude and phase as was shown for the regular wave types 1 and 2 in Chapter 4. In these both cases, the derivation of the control signal is based on the information of the incident wave received by the primary WEC, and the same control signal is then applied to the rest of the WECs in the array, either with or without appropriate modification as can be seen in Figure 4.18 and Figure 4.20 respectively. However, when the incident waves differ in frequency, the Independent Control Schemes are proposed to control the power in the array as was shown for the regular wave type 3 and the rest of the irregular wave cases studied in this thesis. The reason for the use of a non-mutual control structure in those cases is because the hydrodynamic parameters of the WECs system provided by WAMIT are frequency dependent, thus the control signal block developed in the array due to this type of incident waves could not be simplified and therefore a separate type of voltage source current control is required to generate the control signal for every WEC in the array.
7.2.5 DC and AC link Interconnections

The benefits of employing the WECs in a DC link interconnection can be seen where it employs only a single power converter (inverter) to control the power sent to the grid. Generated power from each of WECs in the array can be controlled by connecting the WECs to the DC link with single/dual DC capacitors whose function is to keep the DC link voltage at a constant level. The benefits of maintaining a constant DC link voltage level is discussed in detail in Chapter 4. However the DC link Interconnection is not suitable for implementation in an array which has a high output power rating. This is due to the very high currents flowing in the IGBTs in the grid side power converter circuit. Thus AC link Interconnection is favoured and is proposed for high power WEC devices in an array where its back-to-back converter configuration reduces the current stress on the converter IGBTs as was demonstrated in the simulation result in Figure 5.21. In the configuration of a larger wave farm, having both DC and AC interconnection in one array may be considered.
7.3 Conclusion

The work presented in this thesis demonstrates that the power from an array consisting of direct drive linear generators can be effectively controlled if the appropriate control strategy is used. By connecting linear generators with IGBT power converters, PWM-VSCs, in parallel as proposed in this study, the problem of receiving varying output power from the direct drive WEC can be mitigated. Different topologies and control schemes were proposed and analyzed in this thesis. The control schemes are designed for controlling the WEC array system, modelled with both DC and AC link Interconnections as discussed in section 4.3. The model for the Air-Cored Tubular Machines (ACTM) developed in this study is based on the simplified models which are used solely for the power system analysis. Thus detailed mechanical modelling is not included in the system model.

The Optimal and sub-optimal control schemes for the array when incident with the regular and irregular waves are described in this thesis. Based on the results presented, it can be concluded that both control schemes proposed in this study are capable of controlling and managing the power for an array of heaving buoys with linear direct-drive WECs. It starts with collecting the power generated, restoring the energy (for the DC link voltage) and transmitting the electrical output power to be fed to the grid. For the case when a power shortage occurs in the array, the direction of the power at the grid side VSC is controlled to flow in the reverse direction for part of the cycle: this can be seen in both regular and irregular type cases in Chapter 4 and Chapter 5 respectively. Moreover, the ability of the grid side control scheme to adapt to dynamic changes in the extracted power by proposing a suitable DC link voltage level as presented in Chapter 6 is a very promising technique for controlling the DC link voltage level. This is particularly important when the modulation index reaches its boundary linear region which could effect the VSC’s controller system stability. As the modulation index value is inversely proportional to the DC link voltage level, the value of the reference DC voltage plays an important role in determining the wave power control stability. Thus the adaptive optimal voltage control (AOVC) method is proposed to set the reference DC link voltage to a level that best suits the power generated from the array. In section 6.12, the study shows that an unstable condition could happen when controlling the output power to the grid; however, a stable operating region can be achieved through the adjustment strategy made in the DRLV unit in the AOVC. The AOVC was initially conceived in the context of controlling the grid side converter for a single array, however, the study made in this thesis has brought this concept one step
further when implemented in multi-arrays as shown in Figure 6.18. Simulation of the integration of arrays shows good control agreement between the controllers (primary central voltage coordinator and secondary voltage controllers of each array) by comparing the power received in the array and the grid.

In conclusion, the work in this thesis not only proposes ways on how to control the power from the direct drive WECs in the array, but it also shows some of the solutions for several problems that occur due to variably in the power extracted or the amplitude and frequency of the incident waves received by the WEC. The effectiveness of the proposed control schemes can be seen from the series of simulation results presented throughout this thesis, where the power generated by the WECs in an array is not only controlled but most importantly the quality of electrical power received by the grid is improved with the help of the power converters in the array system.

7.4 Future Recommendation

The work presented in this thesis can be further extended by taking into account several issues that arise in the controlled system studied. Here the recommendations focus on two work areas that need further attention and is suggested for future work: control improvement in WEC power captured, and the control concept of the WEC working in a cluster.

7.4.1 Recommendation for the control improvement

In irregular waves, the PTO coefficients derived for optimal control schemes were based on the approximation concept. The values for the stiffness force and the damping force coefficients were chosen based on the fundamental frequencies of the irregular wave considered in this study. The previous study suggests that the power absorption would be increased if the PTO coefficients are chosen based on the value which is ideal for each frequency of the excitation. Further studies are required to support this claim by implementing control schemes that comply with the criteria and applying them to the WEC system. The control performance, needs to be assessed.

The study for the WEC topology made in section 6.3 can be further improved if the function of the power converters in the array is extended to include reducing the impact of the DC fault upon the current flowing in the circuit. The fault handling should not only
include the interruption of faults, but more importantly it must have a feature that can diagnose the fault and provide the best solution to mitigate overcurrents and overvoltages in the array. Such a system, previously proposed in wind farms, with fault detection that enables the detection of the fault location and provides an appropriate signal to trip the appropriate CB to isolate the faulty line needs to be developed for the applications of WEC as demonstrated in [82]. However, it could be advantageous if the number of the DC breakers could be reduced, which can cut the overall cost and the complexity of the array system. The development of a fault control strategy is a very relevant topic and could be an interesting area to be addressed in future work.

7.4.2 Further Research for the WEC working in cluster

The study for the integration of multi-arrays made in this thesis is based on ideal incident waves. The control scheme proposed should be extended for the case of irregular waves. In this thesis, the initial work for the control of the array was presented and the work can be continued to investigate how optimum control can be achieved when the array receives a varying input of incoming waves. The control schemes that can manage the power from the wave park needs to elaborated in detail. Also, smarter power system protection for the multi-array needs to be developed so that it can detect faults and provide the way by triggering the appropriate CB to ensure the power continues to flow during the contingency conditions. Finally, the study of multi-array control is very important and should be addressed to provide a greater understanding of which methods that can enhance the wave park overall performance through an efficient control strategy, reliable and low cost maintenance topology, and most importantly, is practicable and economic to implement either near or far offshore.
APPENDIX A

WAMIT Dataset

Non-dimensional coefficients

The output quantities produced by WAMIT [46] was for a cylinder buoy (radius = 0.6m, draft = 1.128m and height = 1.5m). It was simulated for 70 frequencies of the incident wave: from 0.1 rad/s up to 7.0 rad/s, in steps of 0.1 rad/s. The results presented are all non-dimensional: therefore in order there quantities to be included in the equation of motion, they need to be dimensionalised using the equation defined in chapter 3. The values of non-dimensional of added mass, added damping, force excitation coefficients and motion of the body for the WEC buoy are given in Table A.1, Table A.2 and Table A.3 below.

Table A.1: Added mass and damping coefficients

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<td>1.728127E-06</td>
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Table A.2: Exciting force from diffraction potential

| Period          | $|F_3|$ | $\angle F_3$ |
|-----------------|-------|--------------|
| 0.628319E+02    | 1.129184E+00 | 3.363716E-05 |
| 0.314159E+02    | 1.123820E+00 | 5.356431E-04 |
| 0.209440E+02    | 1.114920E+00 | 2.690349E-03 |
| 0.157080E+02    | 1.102506E+00 | 8.409763E-03 |
| 0.125664E+02    | 1.086703E+00 | 2.024420E-02 |
| 0.104720E+02    | 1.067600E+00 | 4.126785E-02 |
| 0.897598E+01    | 1.045389E+00 | 7.494223E-02 |
| 0.785398E+01    | 1.020267E+00 | 1.249772E-01 |
| 0.698132E+01    | 9.924391E-01 | 1.951928E-01 |
| 0.628319E+01    | 9.621915E-01 | 2.893822E-01 |
| 0.571199E+01    | 9.297801E-01 | 4.112320E-01 |
| 0.523599E+01    | 8.955172E-01 | 5.642192E-01 |
| 0.483322E+01    | 8.596962E-01 | 7.515863E-01 |
| 0.448799E+01    | 8.226185E-01 | 9.763327E-01 |
| 0.418879E+01    | 7.846094E-01 | 1.241187E+00 |
| 0.392699E+01    | 7.459426E-01 | 1.548751E+00 |
| 0.369599E+01    | 7.069151E-01 | 1.901438E+00 |
| 0.349066E+01    | 6.779555E-01 | 2.306160E+00 |
| 0.330694E+01    | 6.288276E-01 | 2.751736E+00 |
| 0.314159E+01    | 5.902652E-01 | 3.254127E+00 |
| 0.299199E+01    | 5.523130E-01 | 3.811415E+00 |
| 0.285599E+01    | 5.151713E-01 | 4.426323E+00 |
| 0.273182E+01    | 4.790142E-01 | 5.101814E+00 |
| 0.261799E+01    | 4.439992E-01 | 5.840796E+00 |
| 0.251327E+01    | 4.102587E-01 | 6.646640E+00 |
| 0.241661E+01    | 3.779052E-01 | 7.522508E+00 |
| 0.232711E+01    | 3.470345E-01 | 8.471770E+00 |
| 0.224399E+01    | 3.177103E-01 | 9.497688E+00 |
| 0.216662E+01    | 2.899880E-01 | 1.060340E+01 |
| 0.209440E+01    | 2.638973E-01 | 1.179209E+01 |
| 0.202683E+01    | 2.394485E-01 | 1.306644E+01 |
| 0.196350E+01    | 2.166390E-01 | 1.442894E+01 |
| 0.190400E+01    | 1.954467E-01 | 1.588188E+01 |
| 0.184800E+01    | 1.758363E-01 | 1.742729E+01 |
| 0.179520E+01    | 1.577628E-01 | 1.906639E+01 |
| 0.174533E+01    | 1.411700E-01 | 2.080039E+01 |
| 0.169816E+01    | 1.259917E-01 | 2.263025E+01 |
| 0.165347E+01    | 1.121576E-01 | 2.455649E+01 |
| 0.161107E+01    | 9.959137E-02 | 2.657916E+01 |
| 0.157080E+01    | 8.821499E-02 | 2.869821E+01 |
| 0.153248E+01    | 7.794952E-02 | 3.091398E+01 |
| 0.149600E+01    | 6.871447E-02 | 3.322541E+01 |
| 0.146121E+01    | 6.043205E-02 | 3.563187E+01 |
| 0.142800E+01    | 5.302548E-02 | 3.813299E+01 |
| 0.139626E+01    | 4.642085E-02 | 4.072778E+01 |
| 0.136591E+01    | 4.054755E-02 | 4.341564E+01 |
| 0.133685E+01    | 3.533892E-02 | 4.619516E+01 |
| 0.130900E+01    | 3.073136E-02 | 4.906581E+01 |
| 0.128228E+01    | 2.666645E-02 | 5.202635E+01 |
| 0.125664E+01    | 2.308928E-02 | 5.507636E+01 |
Table A.3: Motion of body (response amplitude operator)

<p>| Period     | (|x_1|) | (&lt;x_3&gt;) |
|------------|----------|-----------|
| 0.628319E+02 | 9.999982E-01 | -1.415315E-10 |
| 0.314159E+02 | 1.000013E+00 | 1.888668E-09 |
| 0.209440E+02 | 1.000084E+00 | -7.521930E-09 |
| 0.157080E+02 | 1.000251E+00 | -5.305500E-08 |
| 0.125664E+02 | 1.000628E+00 | -6.267701E-07 |
| 0.104720E+02 | 1.001298E+00 | -2.227956E-06 |
| 0.897598E+01 | 1.002449E+00 | -9.601452E-06 |
| 0.785398E+01 | 1.004269E+00 | -4.182818E-05 |
| 0.698132E+01 | 1.006981E+00 | -1.285494E-04 |
| 0.628319E+01 | 1.010924E+00 | -3.703889E-04 |
| 0.571199E+01 | 1.016499E+00 | -9.739609E-04 |
| 0.523599E+01 | 1.024209E+00 | -2.335640E-03 |
| 0.483322E+01 | 1.034724E+00 | -5.256128E-03 |
| 0.448799E+01 | 1.048949E+00 | -1.118462E-02 |
| 0.418879E+01 | 1.068106E+00 | -2.272854E-02 |
| 0.392699E+01 | 1.093883E+00 | -4.443620E-02 |
| 0.369599E+01 | 1.128762E+00 | -8.421718E-02 |
| 0.349066E+01 | 1.176435E+00 | -1.558357E-01 |
| 0.330694E+01 | 1.242701E+00 | -2.833836E-01 |
| 0.314159E+01 | 1.337331E+00 | -5.109496E-01 |
| 0.299199E+01 | 1.478012E+00 | -9.232859E-01 |
| 0.285599E+01 | 1.700629E+00 | -1.699036E+00 |
| 0.273182E+01 | 2.090235E+00 | -3.274618E+00 |
| 0.261799E+01 | 2.907108E+00 | -7.023895E+00 |
| 0.251327E+01 | 5.413198E+00 | -2.016287E+01 |
| 0.241661E+01 | 9.943480E+00 | -1.092523E+02 |
| 0.232711E+01 | 3.138350E+00 | -1.540153E+02 |
| 0.224399E+01 | 1.588188E+00 | -1.612229E+02 |</p>
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APPENDIX B

Derivations

7.5 Derivation of the equation of motion

From Newton's Law, the equation of the oscillating body [40] in the monochromatic oscillation:

\[ ma = F_e + F_r + F_k + F_v + F_c \]  \hspace{1cm} (B.1)

Excitation force, \( F_e \), and radiation force, \( F_r \), are the total wave force due to the incident wave as defined in (3.19). When the body is oscillating (after incident with wave), it is subjected to the hydrostatic buoyancy forces (stiffness force), \( F_k \). The second and the last force term in (B.1), \( F_r \) and \( F_c \) represent the viscous effects and the control (load) force from the control mechanism, which is in our case is power take-off force, \( F_{PTO} \). For the equation of motion derivation, the last two terms force \( F_r \) and \( F_c \) would not be included.

If the oscillation of the body is restricted in the heave mode only, thus equation (B.1) becomes:

\[ m_{33} \ddot{x}_3 = F_{e3} + F_{r3} + F_{k3} \]  \hspace{1cm} (B.2)

where subscript 3 represents the mode of the oscillation body, which is heave.

As defined in (3.20) and (3.21), the radiation force, \( F_{r3} \) can be further derived as:

\[ F_{r3} = -\{B_{33}(\omega) + j\omega A_{33}(\omega)\} \dot{x}_3 \]  \hspace{1cm} (B.3)

and the buoyancy stiffness force is

\[ F_{k3} = -K_{33}x_3 \]  \hspace{1cm} (B.4)
Substituting (B.3) and (B.4) into (B.2) gives the equation of motion in the frequency domain:

\[ F_{e3} = m_{33} \ddot{x}_3 + [B_{33}(\omega) + A_{33}(\omega)] \dot{x}_3 + K_{33}x_3 \]  

(B.5)

Having to know that the acceleration, \( \ddot{x}_3 \), and the velocity, \( \dot{x}_3 \), of the body for the single angular frequency are \(-\omega^2\) and \(j\omega\) respectively and rearrange (B.5) will result in the equation of motion previously defined in (3.15):

\[ F_{e3} = [-\omega^2(m_{33} + A_{33}(\omega)) + j\omega B_{33} + K_{33}]x_3 \]  

(B.6)
B.2 Derivation of the abc coordination to synchronous rotating coordinates dqo

Figure B 2.1 shows the relationship between the abc coordination with the stationary d and q coordination.

**Figure B 2.1**: abc coordination and stationary dq coordination

Using the trigonometry rules the equation of stationary dq can be made using the abc coordination as defined in (B.7), (B.8) and (B.9) for the phase current calculation

\[
I_{qi} = I_a + I_b \cos(60) - I_c \cos(60)
\]

\[
= I_a - \frac{1}{2} I_b - \frac{1}{2} I_c
\]

(B.7)

\[
I_{ds} = -\frac{\sqrt{3}}{2} I_b + \frac{\sqrt{3}}{2} I_c
\]

(B.8)

Rearrange (B.7) so that can be defined into (B.9)

\[
I_{qi} = \frac{3}{2} I_a - \left(\frac{1}{2} I_a + \frac{1}{2} I_b + \frac{1}{2} I_c\right)
\]

(B.9)

Therefore the transformation of abc coordination into stationary dq can be shown in matrix form in (B.10)

\[
I_{dq}^s = \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

(B.10)
Normally the scaling factor is added to make the magnitude of vector current the same as
the peak of the phase current, thus

\[
I_{dq}^s = \frac{2}{3} I_{dqo}^s
\]  
(B.11)

\[
I_{dq}^s = \frac{2}{3} [I_{dqo}^s][I_{abc}]
\]  
(B.12)

Converting stationary coordinates \(I_{dqo}^s\) into synchronous rotating Coordinates \(I_{dqo}^r\) can be
seen by looking at their graphical relationship in Figure B 2.2

![Figure B.2.2: stationary dq coordination and rotating dq coordination](image)

The derivation of the rotating coordination \(I_{dqo}^r\) can be made through the relationship with
the stationary coordination \(I_{dqo}^s\) as shown in Figure B 2.2 and as defined in B.13

\[
\begin{bmatrix}
I_q^r \\
I_d^r
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
I_q^s \\
I_d^s
\end{bmatrix}
\]  
(B.13)

Further it can be extended so that it can be relate with abc coordination. Thus

\[
I_{dq}^r = [T_{dqo}][I_{dqo}^r]
\]  
(B.14)

where

\[
[T_{dqo}] =
\begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  
(B.15)
To convert abc coordination into rotating coordination \( I'_{dq} \), it can be made by firstly convert it into stationary \( dq \) coordination \( I^s_{dq} \) before using the equation (B.10) for the fully transformation as defined in the following equation

\[
I^s_{dqo} = [T^s_{dqo}] [I_{abc}]
\]  
(B.16)

Therefore using (B.14) and (B.16)

\[
I'_{dqo} = [T_{dqo}] [T^s_{dqo}] [I_{abc}]
\]  
(B.17)

Substitutes (B.10),(B.12) and (B.15) into (B.17)

\[
\begin{bmatrix}
I'_d \\
I'_d \\
I'_o
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
1 & -1 & -1 \\
0 & \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]  
(B.18)

Equation (B.18) can be further simplified using the Trigonometry theorem to be the general term abc-dqo transformation as previously defined in (3.39)

\[
\begin{bmatrix}
I'_d \\
I'_d \\
I'_o
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos \theta & \cos(\theta - 120) & \cos(\theta + 120) \\
\sin \theta & \sin(\theta - 120) & \sin(\theta + 120) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]  
(B.19)
B.3 Derivation of the average power for optimal damping coefficient

Power extract from the power take off device can be defined as force times with the velocity as defined in (B.20)

\[ P = F_{PTO} \dot{x} \quad (B.20) \]

For sub-optimal control only the damping coefficient is considered which makes

\[ F_{PTO} = B_{PTO} \dot{x} \quad (B.21) \]

Thus

\[ P = B_{PTO} \dot{x}^2 \quad (B.22) \]

Let we defined the velocity as

\[ \dot{x} = |u| \cos (\omega t + \angle u) \quad (B.23) \]

whereas the displacement of the WEC as defined previously in section (4.6.5) as

\[ \xi = |\xi| \sin (\alpha + \angle \xi) \quad (B.24) \]

which means

\[ \dot{x} = \dot{\xi} = -\omega |\xi| \cos (\alpha + \angle \xi) \quad (B.25) \]

Thus

\[ |u| = \omega |\xi| \quad (B.26) \]

As for the regular wave, the power can be further derived by substituting (B.23) into (B.22)

\[ P = B_{PTO} |u|^2 \cos^2 (\alpha + \angle u) \quad (B.27) \]

Using the trigonometry identity,

\[ \cos^2 a = \frac{1}{2} (1 + \cos 2a) \quad (B.28) \]

and complex number rule

\[ zz^* = (x + jy)(x - jy) = x^2 + y^2 = |z|^2 \quad (B.29) \]
average power $\overline{P}$ becomes

$$\overline{P} = \frac{1}{2} B_{PTO} |\xi|^2$$

(B.30)

Substitutes (B.26) into (B.30)

$$\overline{P} = \frac{1}{2} B_{PTO} (\omega |\xi|)^2$$

(B.31)

Using the complex number concept defined in (B.29), then

$$\overline{P} = \frac{1}{2} B_{PTO} \omega^2 \xi \bar{\xi}$$

(B.32)

In order to equate the displacement, $x_j(\omega)$, from the equation of motion defined in (4.4) with the wave elevation, $\xi$, equation (B.33) is introduced

$$\xi = \frac{x_j(\omega)}{-\omega^2(m_{ij} + A_{ij}(\omega)) + i\omega(B_{ij} + B_{PTO}) + K_{ij}}$$

(B.33)

where the power take off stiffness force coefficient, $K_{PTO}$ is being excluded in sub-optimal control. Substitutes (B.33) into (B.32) will give the average power $\overline{P}$ as defined in (4.8), [44]

$$\overline{P} = \frac{1}{2} B_{PTO} \omega^2 x_3 x_3^*$$

$$\frac{1}{2} \frac{(K_{33} - \omega^2(m_{33} + A_{33}(\infty))^2 + \omega^2(B_{33} + B_{PTO})^2)}$$
B.4 Simplification of the equation (4.16)

The reason of the simplification of the equation (4.16) is to develop the control signal based on the information received by the controller block, which are the reference power take force, \( F_{PTO} \) and the velocity of the WEC's translator, \( \dot{x}(t) \), therefore from the equation (4.15), equation (B.34) is derived as previously shown in section (4.6.3.2) as

\[
V_{conv}(t) = \hat{V}_p \cos\left(\frac{2\pi}{A} x(t)\right) \dot{x}(t) - L_m \frac{d}{dt}\left[\hat{I}_p \cos\left(\frac{2\pi}{A} x(t)\right) \dot{x}(t)\right] - R_m \left[\hat{I}_p \cos\left(\frac{2\pi}{A} x(t)\right) \dot{x}(t)\right]
\]

(B.34)

Let \( k = \frac{2\pi}{A} \), thus (B.34) simply becomes

\[
V_{conv}(t) = \hat{V}_p \cos(kx(t)) \dot{x}(t) - L_m \frac{d}{dt}\left[\hat{I}_p \cos(kx(t)) \dot{x}(t)\right] - R_m \left[\hat{I}_p \cos(kx(t)) \dot{x}(t)\right]
\]

(B.35)

Apply the chain derivative function to the middle term of (B.35) would gives

\[
L_m \frac{d}{dt}\left[\hat{I}_p \cos(kx(t)) \dot{x}(t)\right] = \left[L_m \left[\hat{I}_p \frac{d}{dt}(\cos(kx(t)) \dot{x}(t) + k \sin(kx(t)) \dot{x}(t))\right]\right] \dot{x}(t)
\]

(B.36)

By considering \( s = \frac{d}{dt} \dot{x}(t) \) and substituting (B.36) into (B.35) gives

\[
V_{conv}(t) = \left(\hat{V}_p \cos(kx(t)) - \hat{I}_p \left[\left((sL_m + R_m) \cos(kx(t))\right) - kL_m \sin(kx(t)) \dot{x}(t)\right]\right) \dot{x}(t)
\]

(B.37)

Therefore by measuring the velocity of the WEC's translator, \( \dot{x}(t) \) and having an information of power take force, \( F_{PTO} \), the control signal of \( V_{conv}(t) \) can be calculated as an input control signal to the PWM-VSC.
APPENDIX C

Published Paper

References


Chapter 4 of the WAMIT version 6.0 user manual.


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