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Enhancing Communication Link Performance in Visible Light Communication

Yichen Li

A thesis submitted for the degree of Doctor of Philosophy.
The University of Edinburgh.
August 2016
Abstract

With data throughput increasing exponentially in wireless communication networks, the limited radio frequency (RF) spectrum is unable to meet the future data rate demand. As a promising complementary approach, optical wireless communication (OWC) has gained significant attention since its licence-free light spectrum provides a considerable amount of communication bandwidth. In conventional OWC systems, the information-carried signal has to be real-valued and non-negative due to the incoherent light output of the conventional optical transmitter, light emitting diode (LED). Therefore, an intensity modulation and direct detection (IM/DD) system is used for establishing the OWC link. Some modified orthogonal frequency division multiplexing (OFDM) schemes have been proposed to achieve suitable optical signals. In previous research, three OFDM-based schemes have been presented, including DC-biased optical orthogonal frequency division multiplexing (DCO-OFDM), asymmetrically clipped optical orthogonal frequency division multiplexing (ACO-OFDM) and unipolar orthogonal frequency division multiplexing (U-OFDM).

Basic concepts of SPAD receivers are studied and a novel application in OWC is proposed for a permanent downhole monitoring (PDM) system in the gas and oil industry. In this thesis, a complete model of the SPAD-based OWC system is presented, including some related SPAD metrics, the photon counting process in SPAD and a specific nonlinear distortion caused by passive quenching (PQ) and active quenching (AQ) recharged circuits. Moreover, a practical SPAD-based visible light communication (VLC) system and its theoretical analysis are presented in a long-distance gas pipe with a battery-powered LED and a basic on-off keying (OOK) modulation scheme.

In this thesis, two novel optical orthogonal frequency division multiplexing (O-OFDM) technologies are proposed: non-DC-biased orthogonal frequency division multiplexing (NDC-OFDM) and OFDM with single-photon avalanche diode (SPAD). The former is designed for optical multiple-input multiple-output (O-MIMO) systems based on the optical spatial modulation (OSM) technique. In NDC-OFDM, signs of modulated O-OFDM symbols and absolute values of the symbols are separately transmitted by different information carrying units. This scheme can eliminate clipping distortion in DCO-OFDM and achieve high power efficiency. Furthermore, as the indices of transmitters carry extra information bits, NDC-OFDM gives a significant improvement in spectral efficiency over ACO-OFDM and U-OFDM.

In this thesis, SPAD-based OFDM systems with DCO-OFDM and ACO-OFDM are presented and analysed by considering the nonlinear distortion effect of PQ SPAD and AQ SPAD. A comprehensive digital signal processing of SPAD-based OFDM is shown and theoretical functions of the photon counting distribution in PQ SPAD and AQ SPAD are given. Moreover, based on Bussgang theorem, a conventional method for analysing memoryless distortion, close-formed bit-error rate (BER) expressions of SPAD-based OFDM are derived. Furthermore, SPAD-based OFDM is compared with conventional photo-diode (PD) based OFDM systems, and a gain of 40 dB in power efficiency is observed.
Declaration of originality

I hereby declare that the research recorded in this thesis and the thesis itself was composed and originated entirely by myself in the Li-Fi Research and Development Centre of the School of Engineering at The University of Edinburgh.

Yichen Li
Edinburgh, UK
August 2016
First of all, I would like to express my gratitude to my supervisor, Prof. Harald Haas, for his patience, guidance, encouragement and support in my PhD study. It has often been said, ‘give a man a fish, and you feed him for a day; teach a man to fish, and you feed him for a lifetime.’ During these three years, he taught me how to learn and I will never forget his teaching in my future life.

In addition, I would like to thank my second supervisor, Prof. John Thompson, and my colleagues, Dr. Dobroslav Tsonev and Dr. Majid Safari, for their encouragement, patience and comments on my research. Without their help, my PhD study would not have been done well.

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Last but not least, my deepest gratitude goes to my family for their love and support. I am indebted to my father, Hongan Li, for teaching me how to live and I would like to express my apology to my mother, Zhijing He, for living far away from her so many years. Moreover, I would like to thank my girlfriend, Qingwen He, for her patience and waiting.
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<td>1G</td>
<td>first generation</td>
</tr>
<tr>
<td>2G</td>
<td>second generation</td>
</tr>
<tr>
<td>3G</td>
<td>third generation</td>
</tr>
<tr>
<td>4G</td>
<td>fourth generation</td>
</tr>
<tr>
<td>5G</td>
<td>fifth generation</td>
</tr>
<tr>
<td>ACO-OFDM</td>
<td>asymmetrically clipped optical orthogonal frequency division multiplexing</td>
</tr>
<tr>
<td>ADC</td>
<td>analog-to-digital converter</td>
</tr>
<tr>
<td>APD</td>
<td>avalanche photo-diodes</td>
</tr>
<tr>
<td>APP</td>
<td>after pulsing probability</td>
</tr>
<tr>
<td>AQ</td>
<td>active quenching</td>
</tr>
<tr>
<td>AQ SPAD</td>
<td>active quenching single-photon avalanche diode</td>
</tr>
<tr>
<td>ASK</td>
<td>amplitude shift keying</td>
</tr>
<tr>
<td>AWGN</td>
<td>additive white Gaussian noise</td>
</tr>
<tr>
<td>BER</td>
<td>bit-error ratio</td>
</tr>
<tr>
<td>BPSK</td>
<td>binary phase shift keying</td>
</tr>
<tr>
<td>CDF</td>
<td>cumulative distribution function</td>
</tr>
<tr>
<td>CDMA</td>
<td>code division multiple access</td>
</tr>
<tr>
<td>CLT</td>
<td>central limit theorem</td>
</tr>
<tr>
<td>CMOS</td>
<td>complementary metal-oxide-semiconductor</td>
</tr>
<tr>
<td>CP</td>
<td>cyclic prefix</td>
</tr>
<tr>
<td>DAC</td>
<td>digital-to-analog converter</td>
</tr>
<tr>
<td>DC</td>
<td>direct current</td>
</tr>
<tr>
<td>DCO-OFDM</td>
<td>DC-biased optical orthogonal frequency division multiplexing</td>
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<tr>
<td>DCR</td>
<td>dark count rate</td>
</tr>
<tr>
<td>DPSK</td>
<td>differential phase shift keying</td>
</tr>
<tr>
<td>FF</td>
<td>fill factor</td>
</tr>
<tr>
<td>FFT</td>
<td>fast Fourier transform</td>
</tr>
<tr>
<td>FOV</td>
<td>field of view</td>
</tr>
<tr>
<td>FSO</td>
<td>free-space optical communication</td>
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</tbody>
</table>
### Acronyms and abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>GaAs</td>
<td>gallium arsenide</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>IFFT</td>
<td>inverse fast Fourier transform</td>
</tr>
<tr>
<td>IM/DD</td>
<td>intensity modulation and direct detection</td>
</tr>
<tr>
<td>InGaAs</td>
<td>indium gallium arsenide</td>
</tr>
<tr>
<td>IR</td>
<td>infrared</td>
</tr>
<tr>
<td>IrDA</td>
<td>infrared data association</td>
</tr>
<tr>
<td>ISI</td>
<td>intersymbol interference</td>
</tr>
<tr>
<td>ITU</td>
<td>international telecommunication union</td>
</tr>
<tr>
<td>LAN</td>
<td>local area network</td>
</tr>
<tr>
<td>LD</td>
<td>laser diode</td>
</tr>
<tr>
<td>LEA</td>
<td>low error area</td>
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<tr>
<td>LED</td>
<td>light emitting diode</td>
</tr>
<tr>
<td>Li-Fi</td>
<td>Light Fidelity</td>
</tr>
<tr>
<td>LOS</td>
<td>line-of-sight</td>
</tr>
<tr>
<td>MIMO</td>
<td>multiple-input multiple-output</td>
</tr>
<tr>
<td>ML</td>
<td>maximum likelihood</td>
</tr>
<tr>
<td>MMSE</td>
<td>minimum mean square error</td>
</tr>
<tr>
<td>MOI</td>
<td>maximum optical irradiance</td>
</tr>
<tr>
<td>M-PAM</td>
<td>(M)-ary pulse amplitude modulation</td>
</tr>
<tr>
<td>M-PPM</td>
<td>(M)-ary pulse position modulation</td>
</tr>
<tr>
<td>MPR</td>
<td>minimum power requirement</td>
</tr>
<tr>
<td>M-QAM</td>
<td>(M)-ary quadrature amplitude modulation</td>
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<tr>
<td>NDC-OFDM</td>
<td>non-DC-biased orthogonal frequency division multiplexing</td>
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<tr>
<td>NLOS</td>
<td>non-line-of-sight</td>
</tr>
<tr>
<td>NRZ-OOK</td>
<td>non-return-to-zero on-off keying</td>
</tr>
<tr>
<td>OC</td>
<td>optical communication</td>
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<tr>
<td>OFDM</td>
<td>orthogonal frequency division multiplexing</td>
</tr>
<tr>
<td>O-MIMO</td>
<td>optical multiple-input multiple-output</td>
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<td>O-OFDM</td>
<td>optical orthogonal frequency division multiplexing</td>
</tr>
<tr>
<td>OOK</td>
<td>on-off keying</td>
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<tr>
<td>OSM</td>
<td>optical spatial modulation</td>
</tr>
<tr>
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<td>optical spatial modulation orthogonal frequency division multiplexing</td>
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<td>Acronym</td>
<td>Definition</td>
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<td>-------------------------------------------------</td>
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<tr>
<td>OWC</td>
<td>optical wireless communication</td>
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<tr>
<td>PAM</td>
<td>pulse amplitude modulation</td>
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<tr>
<td>PAPR</td>
<td>peak-to-average power ratio</td>
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<tr>
<td>PD</td>
<td>photo-diode</td>
</tr>
<tr>
<td>PDF</td>
<td>probability density function</td>
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<tr>
<td>PDM</td>
<td>permanent downhole monitoring</td>
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<tr>
<td>PDP</td>
<td>photon detection probability</td>
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<td>PIN</td>
<td>positive-intrinsic-negative</td>
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<td>PLC</td>
<td>power line communication</td>
</tr>
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<td>PPM</td>
<td>pulse position modulation</td>
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<td>PQ</td>
<td>passive quenching</td>
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<td>PQ SPAD</td>
<td>passive quenching single-photon avalanche diode</td>
</tr>
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<td>PSD</td>
<td>power spectral density</td>
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<td>PWM</td>
<td>pulse-width modulation</td>
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<td>QAM</td>
<td>quadrature amplitude modulation</td>
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<td>QPSK</td>
<td>quadrature phase shift keying</td>
</tr>
<tr>
<td>RF</td>
<td>radio frequency</td>
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<td>RGB</td>
<td>red-green-blue</td>
</tr>
<tr>
<td>SM</td>
<td>spatial modulation</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>SPAD</td>
<td>single-photon avalanche diode</td>
</tr>
<tr>
<td>TDMA</td>
<td>time division multiple access</td>
</tr>
<tr>
<td>TIA</td>
<td>transimpedance amplifier</td>
</tr>
<tr>
<td>TOV</td>
<td>turn-on voltage</td>
</tr>
<tr>
<td>U-OFDM</td>
<td>unipolar orthogonal frequency division multiplexing</td>
</tr>
<tr>
<td>UV</td>
<td>ultraviolet</td>
</tr>
<tr>
<td>V-BLAST</td>
<td>vertical Bell Labs layered space-time</td>
</tr>
<tr>
<td>VL</td>
<td>visible light</td>
</tr>
<tr>
<td>VLC</td>
<td>visible light communication</td>
</tr>
<tr>
<td>ZF</td>
<td>zero forcing</td>
</tr>
</tbody>
</table>
Nomenclature

$(\cdot) \ast (\cdot)$ convolution operator
$I(x)$ step function
$A$ total active area of PD
$A_{ac}$ active area of a single SPAD device
$A_{ala}$ total logic area of a SPAD array
$a_{AQ_{\text{max}}}$ the maximum photon count rate for a single AQ SPAD device in a symbol duration
$A_{dla}$ total logic area of a single SPAD device
$\alpha$ gain factor of nonlinear distortion
$\bar{\alpha}$ final average nonlinear gain factor in NDC-OFDM
$\alpha_c$ nonlinear gain factor for the correct estimation in NDC-OFDM
$\alpha_{cl}$ clipping distorted gain factor of DCO-OFDM
$\alpha_{PQ}$ nonlinear gain factor of PQ SPAD OFDM
$\alpha_{PQ-ACO}$ nonlinear gain factor of PQ SPAD ACO-OFDM
$\alpha_{PQ-DCO}$ nonlinear gain factor of PQ SPAD DCO-OFDM
$\alpha_m$ nonlinear gain factor for the incorrect estimation in NDC-OFDM
$a_m(k)$ photon counts of each single SPAD device in a symbol duration
$a_{PQ_{\text{max}}}$ the maximum photon count rate for a single PQ SPAD device in a symbol duration
$B_{\text{DC}}$ value of DC bias
$\text{BER}_{\text{NDC}}$ theoretical BER of NDC-OFDM
$\text{BER}_{\text{PQ-ACO}}$ analytical BER performance of PQ SPAD ACO-OFDM
$\beta$ level of DC bias
$\mathbf{C}$ inverse of the channel matrix $\mathbf{H}$
$C_{\text{FF}}$ total FF of a SPAD array
$C_{\text{FF}}(m)$ FF of a single SPAD device
$C_{\text{L}}$ speed of light
$C_{\text{PDP}}$ average PDP of a SPAD array
$C_{\text{PDP}}(m)$ PDP of a single SPAD device
Nomenclature

$C_R$ reflectivity of steel

d distance between the optical transmitter and receiver

d_c probability of the correct estimation in one active duration in NDC-OFDM

$E[.]$ statistical expectation

$E_{AQ}(k)$ the accurate expectation of the AQ SPAD array output in a symbol duration

$E_b$ electrical energy per bit

$E_{b,elec}$ average electrical energy allocated to each information bit

$E_P$ energy of a photon

$E_{PQ}(k)$ the accurate expectation of the PQ SPAD array output in a symbol duration

$E[x_{clipped}(k)]$ mean value of DCO-OFDM symbols after DC biasing and clipping in time domain

$f_c$ mean value of correctly detected OFDM samples in NDC-OFDM

$f_w$ mean value of incorrectly detected OFDM samples in NDC-OFDM

$G$ $N_t \times N$ equalized matrix with all estimated transmitted symbols of NDC-OFDM

$g$ $N_t$-dimensional vector with estimated transmitted symbols of NDC-OFDM

$g_e$ optical concentrator gain

$G_{DC}$ attenuation of the original signal power due to the DC bias in DCO-OFDM

$g_f$ optical filter gain

H optical MIMO channel

$H_{s1}$ the first symmetrical ideal O-MIMO channel

$H_{s2}$ the second symmetrical ideal O-MIMO channel

$H_{s3}$ the third symmetrical ideal O-MIMO channel

$H_{s4}$ the fourth symmetrical ideal O-MIMO channel

$H_{a1}$ the first asymmetrical ideal O-MIMO channel

$H_{a2}$ the second asymmetrical ideal O-MIMO channel
Nomenclature

$H_{a3}$ the third asymmetrical ideal O-MIMO channel
$H_{a4}$ the fourth asymmetrical ideal O-MIMO channel
$h_{o-MIMO}$ optical channel gain for MIMO systems
$h_{P}$ Planck’s constant
$H_{p1}$ the first practical O-MIMO channel
$H_{p2}$ the second practical O-MIMO channel
$H_{p3}$ the third practical O-MIMO channel
$H_{p4}$ the fourth practical O-MIMO channel
$h(t)$ optical channel for single link
$\tilde{j}(n)$ estimated indices of active transmitters in conventional OSM-OFDM
$\kappa_{a1}$ the condition number of the first symmetrical ideal O-MIMO channel
$\kappa_{s2}$ the condition number of the second symmetrical ideal O-MIMO channel
$\kappa_{s3}$ the condition number of the third symmetrical ideal O-MIMO channel
$\kappa_{s4}$ the condition number of the fourth symmetrical ideal O-MIMO channel
$\kappa_{a1}$ the condition number of the first asymmetrical ideal O-MIMO channel
$\kappa_{a2}$ the condition number of the second asymmetrical ideal O-MIMO channel
$\kappa_{a3}$ the condition number of the third asymmetrical ideal O-MIMO channel
$\kappa_{a4}$ the condition number of the fourth asymmetrical ideal O-MIMO channel
$\kappa_{p1}$ the condition number of the first practical O-MIMO channel
$\kappa_{p2}$ the condition number of the second practical O-MIMO channel
$\kappa_{p3}$ the condition number of the third practical O-MIMO channel
$\kappa_{p4}$ the condition number of the fourth practical O-MIMO channel
$k_B$ Boltzmann constant
$\tilde{1}$ $N$-dimensional vector with all estimated indices of NDC-OFDM
Nomenclature

\( L_1(k) \)  
digital signals transmitted by the first LED in NDC-OFDM

\( L_2(k) \)  
digital signals transmitted by the second LED in NDC-OFDM

\( M \)  
level of the constellation size

\( M_1 \)  
constellation size of NDC-OFDM

\( M_2 \)  
constellation size of DCO-OFDM in OSM

\( M_3 \)  
constellation size of ACO-OFDM in OSM

\( m_s \)  
Lambertian emission coefficient

\( \mu \)  
average incident photons per second

\( \mu_{AQ}(k) \)  
average output of an AQ SPAD array in a symbol duration

\( \mu_{AQm} \)  
real photon counts of a single AQ SPAD device

\( \mu_{AQ\text{max}} \)  
the maximum photon count rate of an AQ SPAD array in a symbol duration

\( \mu_{AQm}(k) \)  
average real photon counts of a single AQ SPAD device in a symbol duration

\( \mu_m \)  
average potential photon counts of single SPAD devices

\( \mu_m(k) \)  
average potential photon counts of single SPAD devices in a symbol duration

\( \mu(k) \)  
average photon counts of a received optical signal in SPAD receiver

\( \mu_{PQ}(k) \)  
average output of a PQ SPAD array in a symbol duration

\( \mu_{PQm} \)  
real photon counts of a single PQ SPAD device

\( \mu_{PQ\text{max}} \)  
the maximum photon count rate of a PQ SPAD array in a symbol duration

\( \mu_{PQm}(k) \)  
average real photon counts of a single PQ SPAD device in a symbol duration

\( N \)  
total number of subcarriers

\( \bar{N} \)  
final average variance of noise in NDC-OFDM

\( N_0 \)  
average number of photons counted in one symbol duration when ‘0’ is transmitted by NRZ-OOK

\( N_1 \)  
average number of photons counted in one symbol duration when ‘1’ is transmitted by NRZ-OOK

\( n_1 \)  
independent Gaussian distributed noise of the first transmitted symbols in NDC-OFDM
Nomenclature

\( n_2 \) independent Gaussian distributed noise of the second transmitted symbols in NDC-OFDM

\( N_{\text{DCR}} \) average statistical DCR of each single SPAD device

\( n_{\text{DCR}} \) DCR noise added to photon counts

\( N_L \) number of photons received from the LED per second

\( N_o \) PSD of noise in OWC

\( N_{o,\text{shot}} \) PSD of shot noise

\( N_{o,\text{thermal}} \) PSD of thermal noise

\( N_P \) total number of photon counts per second

\( N_R \) total number of reflections

\( N_r \) number of receivers

\( n_R \) number of reflections needed to reach the top surface

\( n_{\text{ref}} \) internal refractive index

\( N_{\text{SPAD}} \) the number of single SPAD devices in a SPAD array

\( N_t \) number of transmitters

\( n(t) \) AWGN in an optical single link

\( N_{\text{th}} \) a threshold of the photon count in NRZ-OOK

\( \nu \) actual received photons

\( \nu(k) \) photon counts of a received optical signal in SPAD receiver

\( P_0 \) optical power assigned to ‘0’ in NRZ-OOK

\( P_1 \) optical power assigned to ‘1’ in NRZ-OOK

\( P_{\text{AP}} \) APP of a SPAD array

\( P_c(\nu = j, \mu) \) CDF of Poisson distribution

\( P_c \) error detection probability of the SPAD-based OOK system

\( \phi \) radiant angle

\( \phi(x) \) PDF of standard normal distribution

\( \Phi_{1/2} \) transmitter semiangle

\( \phi(n_R) \) angle of radiation as a function of the reflections

\( P_r \) received optical power

\( P_{\text{AQ}}(a, \mu_m) \) the accurate probability of photon counts of a single AQ SPAD device

\( P_{r_c} \) probability of correctly detected symbols in NDC-OFDM

\( P_r(\nu = j, \mu) \) PDF of Poisson distribution
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{Pr}(k)$</td>
<td>a vector of the joint photon count distribution of the whole SPAD array</td>
</tr>
<tr>
<td>$\mathbf{Pr}_m(k)$</td>
<td>a vector of photon count distributions of each single device in the same symbol duration</td>
</tr>
<tr>
<td>$\mathbf{Pr}_{PQ}(a, \mu_m)$</td>
<td>the accurate probability of photon counts of a single PQ SPAD device</td>
</tr>
<tr>
<td>$\mathbf{Pr}_w$</td>
<td>probability of incorrectly detected symbols in NDC-OFDM</td>
</tr>
<tr>
<td>$\psi$</td>
<td>angle of incidence</td>
</tr>
<tr>
<td>$\Psi_c$</td>
<td>receiver FOV</td>
</tr>
<tr>
<td>$P_t$</td>
<td>overall transmitted optical power</td>
</tr>
<tr>
<td>$q_e$</td>
<td>charge of an electron</td>
</tr>
<tr>
<td>$\mathbf{Q}(p)$</td>
<td>reshaped matrix of input bit stream in conventional OSM-OFDM</td>
</tr>
<tr>
<td>$\mathbf{Q}'(p)$</td>
<td>matrix of demodulated output bit stream in conventional OSM-OFDM</td>
</tr>
<tr>
<td>$Q(x)$</td>
<td>tail probability of standard normal distribution</td>
</tr>
<tr>
<td>$R_0(\phi)$</td>
<td>optical radiant intensity at an angle of $\phi$</td>
</tr>
<tr>
<td>$R_{ACO}$</td>
<td>spectral efficiency of ACO-OFDM</td>
</tr>
<tr>
<td>$R_{ACO-OFDM-OSM}$</td>
<td>spectral efficiency of ACO-OFDM in OSM</td>
</tr>
<tr>
<td>$R_b$</td>
<td>recharged resistor of SPAD circuits</td>
</tr>
<tr>
<td>$R_{DCO}$</td>
<td>spectral efficiency of DCO-OFDM</td>
</tr>
<tr>
<td>$R_{DCO-OFDM-OSM}$</td>
<td>spectral efficiency of DCO-OFDM in OSM</td>
</tr>
<tr>
<td>$\rho$</td>
<td>mean value of the bipolar normalized OFDM symbols</td>
</tr>
<tr>
<td>$\rho_{DCO}$</td>
<td>mean value of the transmitted DCO-OFDM symbols</td>
</tr>
<tr>
<td>$R_{NDC-OFDM}$</td>
<td>spectral efficiency of NDC-OFDM</td>
</tr>
<tr>
<td>$R_{OSM}$</td>
<td>spectral efficiency of OSM</td>
</tr>
<tr>
<td>$R_{PD}$</td>
<td>PD responsivity</td>
</tr>
<tr>
<td>$R_r$</td>
<td>overall received optical intensity</td>
</tr>
<tr>
<td>$R_U$</td>
<td>spectral efficiency of U-OFDM</td>
</tr>
<tr>
<td>$s$</td>
<td>original bipolar symbols in NDC-OFDM</td>
</tr>
<tr>
<td>$\mathbf{s}$</td>
<td>$N_t$-dimensional transmitted signal vector</td>
</tr>
<tr>
<td>$\text{sgn}(x)$</td>
<td>sign function</td>
</tr>
<tr>
<td>$\sigma^2_{AQ}(k)$</td>
<td>accurate variance of the AQ SPAD array output in a symbol duration</td>
</tr>
</tbody>
</table>
Nomenclature

\( \sigma_c^2 \) variance of the clipping distortion noise in DCO-OFDM

\( \sigma_{cp}^2 \) attenuated variance of \( \sigma_c^2 \) due to the transmitter normalization and the photon counter in PQ SPAD DCO-OFDM

\( \sigma_{m-ACO} \) standard deviation of the original bipolar ACO-OFDM symbols

\( \sigma_{m-DCO} \) standard deviation of the original bipolar DCO-OFDM symbols

\( \sigma_n \) standard deviation of AWGN

\( \sigma_{N-AQ}^2 \) variance of the shot noise in AQ SPAD

\( \sigma_{N-PQ}^2 \) variance of the shot noise in PQ SPAD

\( \sigma_{PQ}^2(k) \) accurate variance of the PQ SPAD array output in a symbol duration

\( \sigma_s \) standard deviation of real OFDM signals

\( \sigma_X \) standard deviation of \( X \)

\( \sigma_X^2 \) variance of \( X \)

\( \sigma_{x-ACO} \) standard deviation of the transmitted ACO-OFDM symbols

\( \sigma_{x-DCO} \) standard deviation of the transmitted ACO-OFDM symbols

\( \sigma_Y^2 \) variance of the nonlinear distorted noise

\( \sigma_{Y-PQ-ACO}^2 \) variance of the additional nonlinear noise in PQ SPAD ACO-OFDM

\( \sigma_{Y-PQ-DCO}^2 \) variance of the additional nonlinear noise in PQ SPAD DCO-OFDM

\( \text{SNR}_{AQ}^{ACO} \) resulting SNR of AQ SPAD ACO-OFDM through the nonlinear transformation

\( \text{SNR}_{DCO}^{AQ} \) resulting SNR of AQ SPAD DCO-OFDM through the nonlinear transformation and clipping distortion

\( \text{SNR}_{elec} \) distorted SNR per bit

\( \text{SNR}_{PQ}^{ACO} \) resulting SNR of PQ SPAD ACO-OFDM through the nonlinear transformation

\( \text{SNR}_{DCO}^{PQ} \) resulting SNR of PQ SPAD DCO-OFDM through the nonlinear transformation and clipping distortion

\( s(t) \) transmitted optical signal

\( s'(t) \) received optical signal

\( \tau_d \) dead time of a single SPAD device

\( t_k \) the \( k \)th time instance
Nomenclature

\( T_{PD} \) temperature of the PD receiver in Kelvin

\( T_s \) symbol duration

\( V_{br} \) breakdown voltage of SPAD circuits

\( v_c \) variance of correctly detected OFDM samples in NDC-OFDM

\( \bar{v}_c \) average variance of the correct estimation in NDC-OFDM

\( V_{eb} \) excess bias voltage of SPAD circuits

\( V_S \) voltage of the SPAD device

\( V_{th} \) threshold voltage of the photon counter in SPAD

\( v_w \) variance of incorrectly detected OFDM samples in NDC-OFDM

\( \bar{v}_w \) average variance of the incorrect estimation in NDC-OFDM

\( \mathbf{w} \) \( N_r \)-dimensional noise vector

\( \lambda \) wavelength of transmitted light

\( X \) an independent Gaussian random variable

\( \mathbf{x}_1(k) \) OFDM symbols transmitted by the first transmitter in conventional OSM-OFDM

\( \mathbf{X}_1(n) \) re-allocated QAM symbols for the first transmitter in conventional OSM-OFDM

\( \mathbf{X}_1'(n) \) recovered QAM symbols from the first receiver in conventional OSM-OFDM

\( \mathbf{x}_2(k) \) OFDM symbols transmitted by the second transmitter in conventional OSM-OFDM

\( \mathbf{X}_2(n) \) re-allocated QAM symbols for the second transmitter in conventional OSM-OFDM

\( \mathbf{X}_2'(n) \) recovered QAM symbols from the second receiver in conventional OSM-OFDM

\( \mathbf{x}_{\text{biased}}(k) \) OFDM symbols after DC biasing

\( \mathbf{x}_c \) correctly detected OFDM samples in NDC-OFDM

\( \mathbf{x}_{\text{clipped}}(k) \) OFDM symbols after signal clipping

\( \mathbf{n}(t) \) AWGN in an optical single link

\( \mathbf{X}_d(n) \) final detected QAM symbols in conventional OSM-OFDM

\( \mathbf{X}(k) \) allocated OFDM frame in frequency domain

\( \mathbf{x}(k) \) transformed OFDM symbols by IFFT

\( \mathbf{x}'(k) \) received OFDM symbols
Nomenclature

\( \mathbf{X}(m) \) allocated QAM symbols
\( \mathbf{X}'(m) \) recovered QAM symbols by FFT
\( \mathbf{X}(n) \) transformed QAM symbols from input bit stream
\( \mathbf{X}'(n) \) recovered QAM symbols from demodulated OFDM frame
\( \mathbf{x}_r(t) \) received optical signals in SPAD receiver
\( x_r \) incorrectly detected OFDM samples in NDC-OFDM
\( Y \) additional noise of nonlinear distortion
\( \mathbf{y} \) \( N_r \)-dimensional received signal vector
\( y_c \) variance of the additional noise component for the correct estimation in NDC-OFDM
\( y_w \) variance of the additional noise component for the incorrect estimation in NDC-OFDM
\( \mathbf{y}_1(k) \) OFDM symbols received by the first receiver in conventional OSM-OFDM
\( \mathbf{Y}_1(n) \) complex symbols demodulated from the first receiver in conventional OSM-OFDM
\( \mathbf{y}_2(k) \) OFDM symbols received by the second receiver in conventional OSM-OFDM
\( \mathbf{Y}_2(n) \) complex symbols demodulated from the second receiver in conventional OSM-OFDM
\( z_{AQ}(N(x)) \) nonlinear transformation function of AQ SPAD OFDM
\( z_{PQ}(N(x)) \) nonlinear transformation function of PQ SPAD OFDM
\( z(X) \) nonlinear transformation
Chapter 1
Introduction

With the rapid evolution in wireless services and applications, the limited radio frequency (RF) spectrum is not sufficient to meet the exponentially increasing wireless data rate demands. As a viable complementary approach, optical wireless communication (OWC) has gained significant attention as a result of technological breakthroughs in solid state lighting technology. OWC offers an almost infinite bandwidth ranging from infrared (IR) to ultraviolet (UV), including the visible light (VL) spectrum. Furthermore, OWC and its subset of visible light communication (VLC) provide other advantages over RF communication systems, including: license-free operation, high communication security, low-cost front-ends, and no interference with RF systems meaning that OWC and RF systems can be used simultaneously. To exploit these advantages, a number of new advancements in OWC is presented in this study, including: development of novel optical modulation and demodulation methods; theoretical performance analysis of novel optical orthogonal frequency division multiplexing (O-OFDM) schemes; and research on application of novel optical photon detectors in the OWC system.
1.1 Motivation

Wireless communication has been developed over 150 years since James Clerk Maxwell formulated the theory of electromagnetic radiation in 1864 [1]. In 1887, Heinrich Hertz successfully sent and received wireless waves by using a spark transmitter and a resonator receiver, and this conclusively proved the existence of electromagnetic waves [2]. Henceforth, the wireless industry started its technology creation, revolution and evolution. In 1970s, mobile radio telephone systems preceded modern cellular mobile telephony technology. The first generation (1G) wireless telephone technology, analog cell photons, was presented in 1980s. It provided a basic wireless voice service for daily life with 2.4 kbits/s communication speed. With the increasing requirement of the communication quality, the second generation (2G) wireless cellphone was launched in early 1990s. Based on digital mobile access technologies such as time division multiple access (TDMA) and code division multiple access (CDMA), 2G systems supplied a high-quality voice communication and the transmission speed reached 64 kbits/s [3–6].

With the global popularity of mobile communications, the third generation (3G) of mobile photon standards was designed for a wide bandwidth data transmission at the end of the 20th century. The international telecommunication union (ITU) formulated a plan to implement global frequency band in the 2 GHz range to support a single, ubiquitous wireless communication standard for all countries throughout the world [7]. 3G technologies enabled network operators to offer users a wider range of more advanced services while achieving greater network capacity through improved spectral efficiency [8–11]. The data transmission speed explosively increased to 2 Mbits/s. In the first decade of the 21st century, with the rapid development of smart mobile devices and multimedia streaming applications, the traditional 3G technologies were replaced by fourth generation (4G) wireless communication technologies, such as orthogonal frequency division multiplexing (OFDM) and multiple-input multiple-output (MIMO), in order to use the limited RF bandwidth effectively and efficiently [12–18]. The 4G technology is able to download at a rate of 70 Mbits/s for mobile access and 1 Gbits/s for less mobile and local wireless access. It is estimated that the future fifth generation (5G) wireless systems will have speeds of 1 Gbits/s by 2020 [19]. However, at the same time, more than 30.6 exabytes of monthly data traffic is expected in future wireless networks [20]. Furthermore, the available RF spectrum has been almost used up and would not meet the future data rate demand [21].

To exploit more spectrum for wireless communication, OWC, a form of optical communication (OC) where UV, IR or VL is used to carry a signal, has been researched. The UV spectrum is...
from 100 nm to 390 nm; and the range between 750 nm and 1600 nm is the IR spectrum which is conventionally used in free-space optical communication (FSO) [22]. Both UV and IR are imperceivable by the human eye, but have eye-safety risks [23]. The VL spectrum is from 390 nm to 750 nm where OWC systems operating are commonly referred to as VLC [21, 24–26]. VLC systems take advantage of light emitting diodes (LEDs) which can be pulsed at very high speeds without a noticeable effect on the lighting output and human eye. Thus the VL spectrum is considered and applied in indoor optical wireless communication. Even if only the VL spectrum is used, the total bandwidth of VLC amounts to approximately 369 THz which is thousands of times higher than the bandwidth available in the conventional RF communication spectrum around 10 GHz. Thus, the undeveloped optical bandwidth could greatly ease the high data traffic demand. Moreover, other advantages of OWC over RF communication are: i) the light spectrum is license-free; ii) security at the physical layer is increased due to the inability of light to penetrate solid objects; iii) the small OWC network cell increases the area data rate; iv) light does not interfere with other sensitive electronic equipments; v) light can be used in RF-sensitive environments such as aeroplanes and hospitals; and vi) the existing lighting infrastructure could be reused for both simultaneous illumination and data communications [27–30].

Previous research in OWC has explored and solved some technical challenges. The conventional OWC system is mainly realised by using high speed LEDs or laser diodes (LDs) as transmitters and highly sensitive photo-diodes (PDs) as receivers, such as positive-intrinsic-negative (PIN) diodes and avalanche photo-diodes (APDs) [31–33]. The incoherent light output of the LED means that information can only be encoded in the intensity level. Thus, only real-valued and non-negative signals can be used for data modulation. This is in contrast to RF systems which make use of complex valued and bi-polar signals. As a consequence, OWC systems are usually considered to be realised as an intensity modulation and direct detection (IM/DD) system [33]. On-off keying (OOK), pulse position modulation (PPM), pulse-width modulation (PWM) and pulse amplitude modulation (PAM) are some of the common single-carrier modulation schemes used in conjunction with IM/DD systems due to their energy efficiency and robustness to nonlinear distortion [32, 34–37]. For high speed transmission, intersymbol interference (ISI) becomes the bottleneck that limits the bandwidth in a optical communication channel. Thus, based on 4G technologies, O-OFDM has been developed for OWC systems, such as DC-biased optical orthogonal frequency division multiplexing (DCO-OFDM), asymmetrically clipped optical orthogonal frequency division multiplexing (ACO-OFDM) and unipolar orthogonal frequency division multiplexing (U-OFDM) [26, 38–43]. Moreover, MIMO systems have
also been used in OWC to increase throughput [13, 44–50]. Based on these modulation schemes and channel improving approaches, the optical wireless network has been presented to achieve higher data rate [51–54]. In addition, an optical attocell network has been proposed as an indoor small-cell cellular network [55]. Finally, some very significant demonstrations of high speed OWC links have been presented over the past few years: a 1 Gbits/s link was demonstrated by Khalid et al. using a commercially available white phosphor-coated LED [56]; a 3.4 Gbits/s transmission was shown by Cossu et al. using a commercially available red-green-blue (RGB) LED [57]; 1 Gbits/s was demonstrated by Azhar et al. using white LEDs and a 4 MIMO system [58]; a 3 Gbits/s O-OFDM link was demonstrated by Tsonev et al. using a single µ-LED [59]; and a step towards distributed multi-hop visible light communication system was demonstrated by Zuniga et al. using a LED-based shine board [28].

However, many questions still remain open in OWC systems. DCO-OFDM is mostly used in the O-OFDM experimental links. However, the DC-bias in this schemes increases the power consumption and causes critical nonlinear distortion which significantly compromises the bit-error ratio (BER) performance [60–64]. Moreover, conventional modulation schemes in single links are directly applied to current optical multiple-input multiple-output (O-MIMO) systems. The spectral and power efficiencies of these approaches could not be significantly enhanced. Therefore, in this thesis, a novel O-OFDM modulation scheme designed for O-MIMO systems is proposed and analysed. Despite the fact that the conventional PDs are successfully used in practical OWC links as receivers, the performance of these devices is unsatisfactory in long distance optical links or lower power transmission systems. In this study, a novel optical receiver is introduced and used in practical and theoretical optical links. In addition, based on this device, specialised O-OFDM modulation schemes are proposed.

1.2 Contributions

In this thesis, the theoretical BER performance and spectral efficiency of a novel O-OFDM modulation scheme, referred to as non-DC-biased orthogonal frequency division multiplexing (NDC-OFDM) is proposed. In addition, the properties of a single-photon avalanche diode (SPAD) is studied as an unique optical receiver in OWC and a novel application of this device in VLC is proposed. Finally, the complete SPAD-based OFDM system is presented and the nonlinear distortion in SPAD-based OFDM is analysed.
As the first contribution of this thesis, SPAD, a novel optical receiver, is introduced and studied. SPADs have been successfully used in image processing technologies as an image sensor, such as the application in time-of-flight cameras. In this thesis, a SPAD is proposed as a photon-detected receiver in OWC. The properties of single SPAD devices and SPAD arrays are studied and their signal-received processing in OWC is presented. Furthermore, the nonlinear distortion effect of SPAD is studied, which has a significant effect on the BER performance. In addition, it is demonstrated for the first time that the problem of permanent downhole monitoring (PDM) in the oil and gas industry is effectively addressed by the use of VLC. The proposed VLC system makes use of a battery-powered LED transmitter and a high sensitivity SPAD array receiver. As a reliable, flexible and low-cost technique, this system can fulfill a critical need of operators to maintain production efficiency and optimise gas well performance. In this study, the BER performance of the system is simulated for a 4 kilometres long metal pipe where the lack of ambient light enables high SNR at the receiver which operates in a photon counting mode. Moreover, the theoretical BER performance is calculated and compared with the simulation results. As a result, the SPAD receiver exhibits a significant improvement in power efficiency. The research conducted on the SPAD-based VLC system has led to [67].

As the second contribution of this thesis, the newly proposed NDC-OFDM is presented which combines the basic optical spatial modulation (OSM) and a newly designed O-OFDM modulator. The new method aims to eliminate the clipping distortion problem in DCO-OFDM, and to increase the spectral efficiency which is halved in ACO-OFDM and U-OFDM. In NDC-OFDM, after the DCO-OFDM modulation block, symbols are transmitted by different LEDs. The positive OFDM symbol is transmitted by one LED and the negative symbol is transmitted by another one. As the LED can only transmit positive signals, the absolute value of the negative symbol is transmitted. Unlike the conventional OSM-OFDM system, the indices of the transmitters in NDC-OFDM represent the signs of the transmitted signal and the absolute value of the signal is carried by the optical intensity. Since the DC-bias and the bottom clipping do not exist in NDC-OFDM, the system has a higher power efficiency than DCO-OFDM. In this thesis, the analytical performance of NDC-OFDM in an additive white Gaussian noise (AWGN) channels is derived. As a result, an equation for the electrical signal-to-noise ratio (SNR) per bit, which contains a memoryless nonlinear distortion analysis processing, is presented to calculate the theoretical BER performance of the NDC-OFDM system. In addition, a comparison between the theoretical spectral efficiencies of NDC-OFDM, ACO-OFDM and DCO-OFDM is given. As a result, NDC-OFDM has slightly lower spectral efficiency but much
higher power efficiency than DCO-OFDM in an OSM system; and NDC-OFDM performs better than ACO-OFDM in OSM in terms of both the spectral efficiency and power efficiency. The research conducted on NDC-OFDM has led to [65, 66].

As the third contribution of this thesis, a novel O-OFDM system based on SPAD receiver is proposed and analysed. The conventional O-OFDM schemes, ACO-OFDM and DCO-OFDM, are combined with a SPAD-based OWC system. In this study, a successful method of the transformation between photon counting signals and digital signals is presented. In addition, accurate nonlinear distortion functions of passive quenching (PQ) and active quenching (AQ) SPAD receivers are studied and a receiver nonlinear distortion effect in SPAD-based OFDM is presented and analysed. As a result, the maximum optical irradiance (MOI) of the specific SPAD array is limited to around -40 dBm due to the nonlinear effect. Furthermore, this study supplies a complete analytical procedure to trace the accurate BER performance of the SPAD-based OFDM by considering the receiver nonlinear distortion and the conventional distortions in ACO-OFDM and DCO-OFDM with PQ or AQ SPADs. The proposed theory shows very good agreement with the Monte Carlo simulation, thus confirming the validity of the analytical approach. As a result, the analytical model of SPAD-based OFDM is used to find the maximum optical irradiance of the system and also the theoretical maximum bit rate which is up to 1 Gbits/s in this study. Finally, the performance of the SPAD-based OFDM system is compared with the conventional PD-based OFDM system. It is shown that the SPAD system has 30.7 dB greater power efficiency than the conventional PD system, which ensures the superiority of the SPAD-based OFDM system in long-distance and low power-supplied communications. The work conducted on the SPAD-based OFDM system has led to [68–70].

1.3 Thesis Layout

The rest of this thesis is organised as follows. In Chapter 2, the fundamentals of OWC are introduced, including the history of OWC and VLC, the complete OWC channel links, the conventional PD receivers with the corresponding noise, the basic concept of optical modulation schemes and the theoretical method of nonlinear distortion analysis.

In Chapter 3, the concept and application of SPADs in OWC are presented. Some important SPAD metrics are introduced for the photon counting algorithm in OWC. Based on these metrics, a complete simulation model of SPAD receivers is presented and the basic concept of
receiver nonlinear distortion is analysed. In addition, an application of VLC with SPAD to PDM is proposed. Based on the SPAD metrics, the channel model of a long gas pipe is defined. The selected modulation scheme with its theoretical analysis is presented. The theoretical BER performance is verified through a Monte Carlo simulation. Finally, the advantages of VLC in PDM are discussed.

In Chapter 4, the NDC-OFDM concept is presented. The modulation and demodulation algorithm of NDC-OFDM is presented and the system model of conventional OSM-OFDM systems is introduced. In addition, a theoretical performance analysis, validated by Monte Carlo simulations, is provided for the basic NDC-OFDM concept. Furthermore, a comparison of NDC-OFDM and conventional OSM-OFDM in spectral efficiencies is given. Finally, the BER performance of NDC-OFDM and conventional OSM-OFDM are compared. The results corroborate the fact that NDC-OFDM can be used to achieve a high power efficiency with multiple transmitters.

In Chapter 5, the SPAD-based OFDM concept is presented. The detailed modulation and demodulation of SPAD-based OFDM is proposed. By considering the receiver nonlinear distortion with Poisson distribution noise and an accurate model of noise component, a comprehensive analysis of ACO-OFDM and DCO-OFDM with PQ and AQ SPADs is provided. The optical irradiance at the receiver is derived and translated into BER performance. Furthermore, the validity of the theoretical BER estimations is confirmed through extensive Monte Carlo simulations and based on the theoretical analysis, the maximum bit rate of SPAD-based OFDM is found. Finally, the BER performance of SPAD-based OFDM is compared with the conventional O-OFDM schemes. The results reveal that the power consumption of SPAD-based OFDM systems can be much lower than conventional PD-based systems.

In Chapter 6, this thesis is summarised and work is concluded. Furthermore, the chapter identifies some of the limitations of the presented work and suggests possible ways in which it can be improved.

1.4 Summary

The optical spectrum provides a different choice for the development of wireless communication systems. Based on this wide and license-free spectrum, the OWC system has been studied in quite a lot of detail and some significant questions have already been solved. However, a
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lot of the questions remain open. This thesis provides some enhanced communication links in OWC and VLC, including a novel OFDM-MIMO system with exploiting the spatial dimension and a novel O-OFDM system with SPAD receivers, which is expected to significantly improve the system power efficiency. This chapter lists the major contribution of this thesis and provides an outline of the presented work.
The modern optical wireless communication (OWC) technology has been developed over 130 years since Alexander Graham Bell demonstrated the photophone experiment in 1880. With the rapid development of optical devices and digital communication technologies, OWC is gradually becoming an important part of modern and future high speed communication technologies since it brings an almost infinite bandwidth. As a branch of OWC, visible light communication (VLC) technology draws attention from communication companies since it can combine the local area networks (LANs) with lighting systems and achieve high data rate. In current practical applications, Light Fidelity (Li-Fi) is a successful example of commercialised VLC products for integration with existing lighting systems. In this chapter, some basic OWC concepts are introduced.
2.1 Introduction

There is a famous allusion in China, called ‘teasing the princes with beacon fires’. In the end of the Western Zhou Dynasty (771 BC), the last ruler, King You, gathered princes’ troops to make his beautiful concubine smile through the fire beacons. This is the earliest record of OWC and even a communication system. In 1792, the semaphore line was defined as the earliest formal OWC application by the French engineer Claude Chappe [71]. Semaphore towers were established between Paris and Lille to transmit 196 encoded information symbols. In 1821, the German scientist Carl Friedrich Gauss successfully developed a predecessor of the heliograph, and this is another example of early OWC application [72]. A controlled beam of sunlight was directed to a distant station to be used as a marker for geodetic survey work. After the invention of the Morse code in 1836, lighthouses were used for navigating navy ships and are still in use. In 1880, Alexander Graham Bell demonstrated the photophone experiment which could be termed as the first advanced formal OWC [73]. In his experiment, a vibrating mirror was used as the transmitter; a crystalline selenium cells was used as the receiver; the sun radiation was modulated with voice signal and transmitted over a distance of about 200 m. The photophone experiment has led to the research on modern OWC technologies.

The research on modern OWC technologies had a significant breakthrough since the invention of the laser diode (LD) in 1962 [74–76]. The gallium arsenide (GaAs) LD can emit signal-carried light including infrared (IR), visible light (VL) and ultraviolet (UV). After the invention of the laser, OWC was envisioned to be the main deployment area for lasers and many trials were conducted using different types of lasers and modulation schemes [77]. In 1979, a short-range indoor OWC system was proposed by Gfeller and Bapst [78]. They theoretically presented an IR-based communication system whose transmission speeds can reach hundreds of Megabits per second and have experimentally demonstrated an in-house communication link at rate of over 100 kbits/s. After that work, in 1993, a protocol standard for infrared communication was presented by the infrared data association (IrDA) [79, 80]; and in 1994, the complete wireless infrared communication system was introduced [31, 32]. With the development of solid-state lighting technology in the first decade of the 21st century, light emitting diodes (LEDs) are replacing incandescent light bulbs in indoor lighting systems due to their reliability and higher energy efficiency. In 2006, a combination of power line communication (PLC) and white light LED was proposed to provide broadband access for indoor applications [81]. This research suggested that VLC could be deployed as a perfect last-mile solution in the future. At
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Figure 2.1: Block diagram of a conventional OWC system.

the same time, the first experiment of VLC-based orthogonal frequency division multiplexing (OFDM) system was demonstrated by Afgani et al. [25]. In 2011, Professor Harald Haas provided a live demonstration of high-definition video being transmitted from a standard LED lamp at TED Global and introduced the concept of a commercialised VLC product, Li-Fi [27, 82]. In the same year, the Institute of Electrical and Electronics Engineers (IEEE) published a standard for VLC [83]. In this chapter, the details of the OWC system and the conventional modulation schemes in OWC are introduced.

2.2 Optical Wireless Communication System

A conventional OWC system is shown in Figure 2.1. At the transmitter, the input bit stream is transformed into digital modulated symbols. Some well-known digital modulation schemes can be used in OWC which is based on the intensity modulation and direct detection (IM/DD) system. Single-carrier modulation schemes, such as on-off keying (OOK), pulse-position modulation (PPM) and pulse amplitude modulation (PAM), can be simply transplanted in OWC. However, if OFDM is employed, one of the following unique schemes need to be considered: DC-biased optical orthogonal frequency division multiplexing (DCO-OFDM), asymmetrically clipped optical orthogonal frequency division multiplexing (ACO-OFDM) and unipolar orthogonal frequency division multiplexing (U-OFDM). Details of these modulation schemes will be introduced in Section 2.3. In the IM/DD system, as only the real-valued and non-negative symbols can be transmitted, the modulated symbols, $x(k)$, need to be pre-distorted. In this

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stage, oversampling, pulse shaping and signal clipping are used to achieve suitable signals in
time domain, \( x(t) \), but the transmitter nonlinear distortion effect occurs, which will be anal-
ysed in Section 2.4. After pre-distortion, the digital signal is supplied to a digital-to-analog
converter (DAC) which outputs an analog signal. Finally, the output signal is encoded into a
current signal and passed through the electrical-to-optical front-end converter device, such as
LED and LD. As a result, an optical signal, \( s(t) \), is transmitted over an optical channel, \( h(t) \),
with real-valued additive white Gaussian noise (AWGN), \( n(t) \). At the receiver, the optical sig-
nal is received by photodetector front-end elements such as positive-intrinsic-negative (PIN)
diodes and avalanche photo-diodes (APDs) and restored to analog signals. In this stage, the
optical signal is firstly transformed to a current signal and then turned into a voltage signal by a
transimpedance amplifier (TIA). In conventional OWC systems, TIA significantly reduces the
sensitivity of the receiver and limits the signal-to-noise ratio (SNR). Afterwards, the voltage
analog signal is passed through an analog-to-digital converter (ADC) and transformed to digi-
tal signals, \( x'(t) \). The resulting digital signal is finally restored to the output bit stream through
the following steps: matched filtering, downsampling and corresponding signal demodulation.
In this section, the physical optical links are introduced, including: a study on transmitter front-
ends and receiver front-ends, the mathematical calculation for the optical channel and the noise
component in OWC.

2.2.1 Transmitters in OWC

In conventional OWC systems, a LED or LD is used as the converter for electrical signals into
optical signals and so called as optical transmitters. The LED is a semiconductor p-n junction
device that gives off spontaneous optical radiation when subjected to electronic excitation [33].
The electronic excitation is achieved by applying a forward bias voltage across the p-n junction.
This excitation energises electrons within the material into an excited state which is unstable.
When the energised electrons return to the stable state, they release energy in the process and
this energy is given off in the form of photons. The radiated photons could be in the IR, VL
or UV part of the electromagnetic spectrum depending on the energy band-gap of the semi-
conductor material. Therefore, a LED can rapidly generate signal-carried light with different
bandwidth. In practical OWC systems, two types of LEDs are used: the white phosphor-coated
LED [56] and the red-green-blue (RGB) LED [57].

The LD is a branch of the larger classification of semiconductor p-n junction diode which can
amplify light by stimulated emitted radiation. After the spontaneous emission in the p-n junction, the generated photon stimulates other coherent photons in the optical cavity. This enhances the original light (spontaneous emitted photons) and generates a coherent radiation which has higher power and narrower spectrum than the LED-generated light. Therefore, the modulation bandwidth of LD is much higher than LED and the high coherence of the LD-generated light removes the limitation when using the IM/DD system in OWC. However, because of its high cost and disadvantage of eye safety, the use of a LD transmitter is limited to inhabited OWC systems. Thus, LD is usually considered as a significantly fast and efficient optical transmitter in free-space optical (FSO) communication.

In the simulation of OWC, a generalised Lambertian radiation intensity pattern is used to describe the light emission from an LED transmitter [31]:

\[ R_0(\phi) = \frac{(m_s + 1)}{2\pi} \cos^{m_s}(\phi)P_t, \]  

(2.1)

where \( P_t \) is the overall transmitted optical power and \( R_0(\phi) \) denotes the optical radiant intensity at an angle of \( \phi \). The Lambertian emission coefficient, \( m_s \), is dependent on the intensity half angle:

\[ m_s = -\frac{\ln 2}{\ln(\cos(\Phi_{1/2}))}, \]  

(2.2)

where \( \Phi_{1/2} \) is the transmitter semiangle where the radiant intensity is equal to the half of the overall transmitted optical power.

### 2.2.2 Receivers in OWC

In general, high sensitivity photo-diodes (PDs), PIN diodes, APDs and single-photon avalanche diodes (SPAD), are considered as the optical receivers in conventional OWC systems to convert an incident photon into an electrical current. The PIN diode is a conventional p-n junction diode with a wide, undoped intrinsic semiconductor region between the p-type and the n-type semiconductor region. Thus the depletion region in PIN diodes is much larger than in a conventional p-n diode. As a result, the PIN diode has higher sensitivity and can even detect photon streams. When a photon with sufficient energy enters the depletion region of the diode, it can generate an electron-hole pair. The reverse bias field sweeps the carriers out of the region creating a current. In an OWC system, the detected current represents the transmitted signals. In previous research, PIN diodes are designed for operating at high bit rates exceeding 100 Gbits/s [33, 84–
For high speed optical communication applications, commercially available PIN diodes are typically made of germanium or indium gallium arsenide (InGaAs) and have fast response times which can reach several tens of Gigahertz [87].

An improved version of PIN diodes are APDs, which are operated at high reverse-bias voltages. Different from PIN diodes, APDs provide an inherent current gain through the process called repeated electron ionisation. This means that one incident photon can generate hundreds of electron-hole pairs and the related photocurrents are multiplied. Thus, the output current of APDs is much larger than PIN diodes and APDs can detect weak optical signals with a higher sensitivity. However, the improvement on the sensitivity requires a higher power consumption on the bias voltage. Therefore, in experiments and commercial applications, PIN diodes are generally considered as optical receivers if the signal power is high enough. For example, a high speed PIN photodetector, New Focus 1601FS-AC, has been used in the experiment of a 3 Gbits/s VLC link with a single LED [59].

In the simulation, the received optical power of a PD device can be calculated as:

\[
P_r = \begin{cases} 
Ag_f(\psi)g_c(\psi)\cos(\psi)R_r, & 0 \leq \psi \leq \Psi_c, \\
0, & \psi > \Psi_c. 
\end{cases} 
\]  

(2.3)

Note that the power is limited by the receiver field of view (FOV), \(\Psi_c\). When the incident angle of the signal-carried light, \(\psi\), is larger than the receiver FOV, the received optical power will be significantly attenuated. In (2.3), \(A\) denotes the total active area of the PD receiver; \(g_f\) is the optical filter gain; \(g_c\) is the optical concentrator gain; and \(R_r\) is the overall received optical intensity. Note that the optical filter gain includes any losses due to reflections and filter imperfections [31]. For simplicity, the optical filter gain is set to ‘1’ in Monte Carlo simulations. In addition, the idealised concentrator gain with an internal refractive index, \(n_{ref}\), can be achieved by [31]:

\[
g_c(\psi) = \begin{cases} 
\frac{n_{ref}^2}{\sin^2(\Psi_c)}, & 0 \leq \psi \leq \Psi_c, \\
0, & \psi > \Psi_c, 
\end{cases} 
\]  

(2.4)

and a high gain optical concentrator with a wide FOV is introduced in [88].
2.2.3 Visible Light Communication Channel

As noted in Figure 2.1, the optical signal passes though a VLC channel and is finally received by optical receivers. The received optical signal, \( s'(t) \), can be represented as:

\[
s'(t) = h(t) * s(t) + n(t),
\]

(2.5)

where \((.) * (.)\) denotes the convolution operator. According to (2.1) and (2.3), a generalised VLC channel gain for a single link, \( h(t) \), can be calculated by:

\[
h(t) = \begin{cases} 
\frac{(m_s+1)A}{2\pi d^2} \cos^m(\phi) I_s(\psi) g_{c}(\psi) \cos(\psi), & 0 \leq \psi \leq \Psi_c, \\
0, & \psi > \Psi_c.
\end{cases}
\]

(2.6)

Note that \( d \) denotes the distance between the optical transmitter and receiver. Generally, there are two types of optical wireless channel links, line-of-sight (LOS) and non-line-of-sight (NLOS) systems. The LOS system requires the existence of a direct and unobstructed propagation link from the optical transmitter to the optical receiver through free space [32]. This system has a satisfied performance on SNR at the receiver and can decrease the effect of multipath distortion. However, the performance is significantly affected by barriers and shadows. In contrast, optical signals transmitted in the NLOS system rely upon reflections of the light from the ceiling or some other diffusely reflecting surface without a clear direct path from the transmitter to the receiver [32]. This system can increase link robustness and ease of use, allowing the link to operate when barriers or shadows exist between the transmitter and the receiver. However, the NLOS system has lower SNR performance than the LOS system. Optical channel gains of both LOS and NLOS links can be derived by the generalised IM/DD channel gain equation (2.6) as presented in [32]. In this thesis, in order to achieve higher SNR and decrease the multipath distortion effect, the LOS system is considered for the design of novel optical modulation schemes and the effect of the NLOS links is neglected.

2.2.4 Noise in OWC

Generally, two types of noise are mainly considered in OWC systems: optical background noises and receiver noise. In many applications, OWC and VLC systems are operated in the presence of ambient light which increases shot noise [89–92]. These background noises are mainly contributed by two types of sources: localised point sources, such as the sunlight and
other light bulbs, and extended sources, such as the sky [33]. The received background noise can be minimised by optical filtering.

Another noise in OWC exists at the optical receiver including shot noise and thermal noise. The shot noise is caused by the randomness in the generation of electrons by incoming photons in the PD receiver [32]. It mainly comes from two sources: photon fluctuation and dark current. For an ideal PD, the only significant noise that affects its performance is that associated with the quantum nature of light itself, the by-product of which is that the number of photons emitted by a coherent optical source in a given time is never constant [33]. Although for a constant power optical source, the mean number of photons generated per second is constant, the actual number of photons per second follows a Poisson distribution [33]. This Poisson distributed fluctuation is a significant component of the shot noise. Another component of shot noise is caused by the collection of thermally generated electrons in the PD device which generate a so-called dark current and this will contribute to the overall signal measured even in the absence of light. The dark current increases the number of the photon collection, and also follows Poisson statistics. Therefore, in the model of the theoretical analysis, the shot noise in optical receivers is described by Poisson distribution:

$$Pr(\nu = j, \mu) = \exp(-\mu)\frac{\mu^j}{j!},$$  \hspace{1cm} (2.7)

where $\mu$ denotes the average incident photons with dark current noises per second or in a fixed symbol duration, and $\nu$ is the actual received photons which is related to the output current from PD receivers. In signal detection technologies of OWC, the output current from the PD receiver is too weak to be demodulated to the information-carried signals. Therefore, a TIA is added...
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to the conventional PD receivers as shown in Figure 2.2. The TIA can be used to amplify the current output of the PD receiver to an usable voltage. However, this will decrease the received SNR of the system due to the thermal noise added by the feedback resistor, $R_{TIA}$. In VLC systems, the thermal noise component can be modelled as AWGN [93]. In addition, the shot noise component can also be approximated to an AWGN noise when the number of incident photons is large enough. Hence, the equivalent power spectral density (PSD) of the shot noise and the thermal noise can be calculated as [33]:

$$
N_{\text{shot}}^o = 2q_e R_{PD} P_r \left( \frac{A^2}{Hz} \right), \quad (2.8)
$$

$$
N_{\text{thermal}}^o = \frac{4k_B T_{PD}}{R_{TIA}} \left( \frac{A^2}{Hz} \right), \quad (2.9)
$$

where $q_e = 1.602 \times 10^{-19}$ C is the charge of an electron; $R_{PD}$ is the PD responsivity; $k_B = 1.381 \times 10^{-23}$ J/K is the Boltzmann constant; and $T_{PD}$ is the temperature of the PD receiver in kelvin. As a result, the PSD of the noise in OWC systems can be simply derived as [32, 33]:

$$
N_o = N_{\text{shot}}^o + N_{\text{thermal}}^o. \quad (2.10)
$$

### 2.3 Optical Modulation Schemes

As noted in Section 2.2 and Figure 2.1, only real-valued and non-negative signals can be transmitted by the IM/DD system and thus some unipolar modulation schemes with single-carrier or multi-carrier have been proposed for OWC systems.

#### 2.3.1 Single-carrier Modulation

There are many different types of single-carrier modulation schemes which are suitable for OWC systems, such as OOK, PPM, PAM, differential phase shift keying (DPSK), binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) [22, 32, 94–99]. In this study, OOK, $M$-ary pulse position modulation ($M$-PPM) and $M$-ary pulse amplitude modulation ($M$-PAM) are considered as benchmark schemes. Figure 2.3(a) shows the modulated OOK symbols in OWC which denotes the simplest form of amplitude shift keying (ASK) modulation. In OOK modulation scheme, the input bit stream is converted into some specific code pulses where ‘1’ is represented by a DC bias which ensures positive symbols for OWC; and
Figure 2.3: Single-carrier modulation schemes: (a) OOK symbols (b) 4-PPM symbols (c) 4-PAM symbols.

‘0’ is represented by the absence of bias which is set to 0 volts or ground. OOK is the simple and widely adopted modulation scheme used in commercial OWC systems because of ease in implementation, simple receiver design and cost effectiveness [99]. However, since the OOK system needs to occupy a whole symbol duration to transmit an information bit, the power and spectral efficiencies are unsatisfactory in terms of the increasing demand for the high-speed transmission. To achieve higher power efficiency, $M$-PPM is considered for OWC systems where each pulse from the optical transmitter can be used to represent one or more bits of information by its position in time relative to the start of a symbol whose duration is identical to that of the information bits it contains [99]. As shown in Figure 2.3(b), a digital 4-PPM
In one symbol duration, only one positive pulse is transmitted and occupies only a quarter of the symbol duration. This means that there are four available positions for the information-carried pulse and thus two bits can be transmitted during one symbol time. In addition, due to a short pulse transmitted, $M$-PPM has a significant improvement on the power efficiency compared with OOK. Another improved single-carrier modulation scheme is $M$-PAM where the input bit stream is encoded in the amplitude of a series of signal pulses.

Under the limitation of the IM/DD system, PAM symbols have to be non-negative by adding a suitable DC-bias as shown in Figure 2.3(c) where 4-PAM symbols are indicated. Unlike the PAM scheme in a radio frequency (RF) system, the amplitude of optical 4-PAM symbols is from zero to three in digital and there are still four levels to represent four different symbols which carries two information bits. Therefore, the $M$-PAM scheme in OWC enhances the spectral efficiency with a decrease in power efficiency. Summarising previous studies on the optical single-carrier modulation schemes, the techniques typically have following benefits: i) lower implementation complexity than multi-carrier modulation schemes; ii) better performance in flat fading channels; and iii) less issues with nonlinear distortion problems [100–102]. However, the single-carrier techniques have some drawbacks: i) highly affected by inter-symbol interference (ISI); ii) low capacity and low bandwidth utilisation [103–105].

### 2.3.2 Optical Orthogonal Frequency Division Multiplexing

Optical orthogonal frequency division multiplexing (O-OFDM) is the well-known multi-carrier modulation scheme for OWC systems which is applied in order to eliminate ISI in high-speed OWC system [106–109]. Moreover, it enables low complexity equalisation with single-tap equalisers in the frequency domain, and becomes closer to the channel capacity by utilising adaptive bit and power loading. In the conventional OFDM system, the input bit stream is transformed into complex symbols by an $M$-ary quadrature amplitude modulation ($M$-QAM) modulator. The complex QAM symbols are allocated onto different subcarriers in frequency domain and then transformed to OFDM symbols by inverse fast Fourier transform (IFFT):

$$x(k) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X(m) \exp \left( j \frac{2\pi km}{N} \right),$$  

(2.11)

where $X(m)$ is the allocated QAM symbols, $N$ is the total number of subcarriers. Afterwards, the OFDM symbols will be transformed from digital signals to analog signals and then transmitted as optical signals by the optical transmitters. As an inverse process, at the receiver,
the received OFDM symbols, $x'(k)$, will be recovered to the original QAM symbols by a fast Fourier transform (FFT):

$$X'(m) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x'(k) \exp \left( -j\frac{2\pi km}{N} \right),$$  \hspace{1cm} (2.12)

where $X'(m)$ is the frame of the recovered QAM symbols.

Unlike the OFDM system in RF, real-valued and positive OFDM symbols are required in IM/DD-based O-OFDM systems. The real-valued symbols can be achieved by imposing Hermitian symmetry on the information frame before the IFFT operation during the signal generation phase. However, this decreases the spectral efficiency by half. On the other hand, in order to obtain positive symbols, some improved O-OFDM schemes have been proposed, such as DCO-OFDM, ACO-OFDM and U-OFDM [39, 40, 43].

### 2.3.2.1 DCO-OFDM

DCO-OFDM was firstly proposed in [110]. As shown in Figure 2.4, after the real-valued OFDM symbols are generated, a positive DC-bias is added to all of the bipolar symbols. The value of the DC bias, which is related to the average power of the bipolar OFDM symbols, is defined in [65] as:

$$B_{DC} = \beta \sqrt{E[x^2(k)]},$$  \hspace{1cm} (2.13)

where $E[.]$ represents the statistical expectation. Note that $10 \log_{10} (\beta^2 + 1)$ is defined as the bias level in dB which represents the relationship between power of biased signals and original bipolar signals as follows:

$$\frac{P_{\text{bias}}}{P_{\text{bipolar}}} = \frac{E[(x(k) + B_{DC})^2]}{E[x^2(k)]} = \frac{E[(x(k) + \beta \sqrt{E[x^2(k)]})^2]}{E[x^2(k)]} = \frac{E[x^2(k)] + \beta^2 E[x^2(k)]}{E[x^2(k)]} = \beta^2 + 1.$$  \hspace{1cm} (2.14)

Afterwards, the biased OFDM symbols are clipped to ensure all of the symbols are unipolar. However, if the level of the DC-bias is not high enough, some OFDM symbols are still negative and will be clipped to zero. This will cause clipping distortion which is a type of nonlinear distortion and has a significant effect on the bit-error ratio (BER) performance. If the bias level
Figure 2.4: DCO-OFDM symbols: (a) the bipolar DCO-OFDM symbols (b) the unipolar DCO-OFDM symbols after DC-bias and digital signal clipping.

is high enough, almost all the OFDM symbols become positive without clipping distortion but the higher DC-bias will increase the requirement of the transmission power. A algorithm for the optimal DC-bias level calculation has been presented in [111] which gives a minimum biasing level with an ignorable clipping distortion. However, in practice, the DC-bias is always set to a fixed value due to the limitation of the operated transmission circuit. Therefore, DCO-OFDM with different DC-bias level (5 dB, 7 dB, 13 dB) have been presented and tested in [39, 112]. Even if the effect of clipping distortion can be eliminated, the added DC-bias will significantly increase the system power consumption. On the other hand, since only the Hermitian symmetry has an effect on the number of information-carried subcarriers, DCO-OFDM has the highest
spectral efficiency compared with other O-OFDM schemes. When all subcarriers are loaded with the same constellation size of $M$, the spectral efficiency of DCO-OFDM is:

$$R_{DCO} = \frac{N - 2}{2N} \log_2(M) \text{ bits/s/Hz.}$$  \hspace{1cm} (2.15)

Note that the effect of cyclic prefix (CP) is not considered in this study. As a result, in conventional OWC systems, DCO-OFDM is used as a high spectral efficient method with a low power efficiency.

### 2.3.2.2 ACO-OFDM

In some OWC application, when the transmission power is limited to a low level, the modulation scheme known as ACO-OFDM is considered due to avoiding additional DC-bias [41, 42]. In ACO-OFDM, the system inserts zeros on even subcarriers and only the odd-indexed subcarriers are modulated with information symbols. As a result, an OFDM frame with ‘symmetrical’ symbols is achieved, as shown in Figure 2.5(a) where the relationship between each OFDM symbol in time domain is:

$$x(k) = -x(k + \frac{N}{2}), \quad k < \frac{N}{2}. \hspace{1cm} (2.16)$$

It can be seen that each negative symbol can be matched with a corresponding positive symbol with this method. This means that the generated bipolar symbols can be directly clipped and completely restored with the positive symbols at the receiver. As shown in Figure 2.5(b), the clipped ACO-OFDM symbols remain all positive symbols. At the receiver, half of received OFDM symbols is subtracted by another part of symbols in order to restore the original bipolar OFDM frame. Compared with DCO-OFDM, ACO-OFDM has a significant improvement on the power efficiency as the DC-bias not required. However, this benefit comes with the decreasing spectral efficiency:

$$R_{ACO} = \frac{1}{4} \log_2(M) \text{ bits/s/Hz.}$$  \hspace{1cm} (2.17)

It is clear from (2.15) and (2.17) that the spectral efficiency of ACO-OFDM is about half the spectral efficiency of DCO-OFDM since only half of the subcarriers carry information bits in ACO-OFDM. This means that for the same modulation bandwidth, ACO-OFDM can deliver significant energy savings, but at the cost of half the data rate in comparison to DCO-OFDM.
Thus, in a high-speed communication system, DCO-OFDM is the better choice to guarantee a higher spectral efficiency. However, in an energy-efficient system, ACO-OFDM is the better choice.

2.3.2.3 U-OFDM

U-OFDM is a novel unipolar O-OFDM scheme which was proposed and defined in [43]. The concept of U-OFDM is an algorithm for the generation of an inherently unipolar modulation signal which presents an alternative to the familiar unipolar technique, ACO-OFDM. As mentioned, the unipolar symbols of ACO-OFDM are achieved by the operation in the frequency
domain. Unlike ACO-OFDM, the operation process of U-OFDM begins with the conventional generation of a real-valued bipolar OFDM symbol, which is the same as the operation in DCO-OFDM as shown in Figure 2.6(a). Afterwards, the bipolar signals are transformed into unipolar signals by encoding each symbol into a pair of new time symbols. If the original OFDM symbol is positive, the first symbol of the new pair is set as the original symbol, and the second symbol is set to zero. On the other hand, if the original OFDM symbol is negative, the first symbol of the new pair is set to zero, and the second symbol is set as the absolute value of the original symbol. Finally, the actual U-OFDM signal is obtained when the first symbols of each pair are grouped in their original order to form the so called positive block while the second samples are
grouped in their original order to form the so-called negative block [43]. The positive block is transmitted first and the negative block is transmitted second as shown in Figure 2.6(b). It can be seen that the final U-OFDM frame doubles the transmitted symbols compared with the original bipolar symbols. Thus, the spectral efficiency of U-OFDM is the same as the spectral efficiency of ACO-OFDM as:

\[ R_u = \frac{1}{4} \log_2(M) \text{ bits/s/Hz.} \] (2.18)

Different to ACO-OFDM, an improved demodulation scheme is proposed in U-OFDM. At the receiver, each pair of transmitted symbols encodes the amplitude and the sign of the original bipolar symbol respectively [43]. If the non-zero symbol is detected, it can successfully achieve the original sign of the symbol according to its position and at the same time, the amplitude of this selected symbol is proportional to the value of the original bipolar symbol. This procedure is ideally expected to remove about half of the noise variance, and hence improve the performance by 3 dB [43].

### 2.4 Nonlinear Distortion in OWC

As noted in Section 2.3.2, a clipping distortion problem occurs in DCO-OFDM, which is a branch of nonlinear distortion in OWC systems. In addition to the modulated clipping distortion, electric devices, such as LEDs and PDs, have limited dynamic ranges and often nonlinear characteristics within the dynamic range [62]. Furthermore, transitions between the digital and the analog domain lead to signal quantization effects. The nonlinear distortion problem in conventional O-OFDM systems has been defined and analyzed in previous works [60, 62, 113, 114]. Theoretically, the nonlinear distortion effect can be derived by Bussgang theorem [115] where it states that if an independent Gaussian random variable, \( X \), passes through any nonlinear transformation, \( z(X) \), then:

\[ z(X) = \alpha X + Y, \] (2.19)

\[ E[XY] = 0. \] (2.20)

According to the central limit theorem (CLT), in an OFDM frame, for a large number of subcarriers, such as \( N > 64 \), the time-domain signal follows a continuous Gaussian distribution [62, 116, 117]. Therefore, the nonlinear distortion effect in O-OFDM can be expressed by
As a result, the distorted SNR per bit can be calculated by:

$$\text{SNR}_{\text{elec}} = \frac{\alpha^2 E_{b,\text{elec}}}{\sigma_N^2 + \sigma_Y^2},$$  \hspace{1cm} (2.21)

where $E_{b,\text{elec}}$ is the average electrical energy allocated to each information bit and $\sigma_N^2$ is the variance of the Gaussian distributed noise related to the shot noise and the thermal noise. Note that $\alpha$ denotes a gain factor of the nonlinear distortion which can be calculated from the transformation of (2.19):

$$Xz(X) = \alpha X^2 + XY.$$  \hspace{1cm} (2.22)

According to (2.20), $\alpha$ can be derived as:

$$\alpha = \frac{\mathbb{E}[Xz(X)]}{\sigma_X^2},$$  \hspace{1cm} (2.23)

where $\sigma_X^2$ is the variance of $X$ which is equal to $\mathbb{E}[X^2]$ in this case. Another noise component in (2.21) is denoted by $\sigma_Y^2$ which is the variance of the nonlinear distorted noise and can be calculated by:

$$\sigma_Y^2 = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2,$$  \hspace{1cm} (2.24)

where

$$\mathbb{E}[Y^2] = \mathbb{E}[(z(X) - \alpha X)^2] = \mathbb{E}[z^2(X)] - \alpha^2 \sigma_X^2,$$  \hspace{1cm} (2.25)

and

$$\mathbb{E}[Y]^2 = \mathbb{E}[z(X)]^2.$$  \hspace{1cm} (2.26)

In the theoretical analysis of different O-OFDM schemes, including some novel schemes proposed in this thesis, different nonlinear functions are considered and the nonlinear distortion in O-OFDM systems has a significant effect on the analytical BER calculation of each schemes.

### 2.5 Summary

In this chapter, some fundamental concepts of OWC and VLC systems have been investigated and introduced. Firstly, a brief summary of the history of OWC has been presented. Over the past 30 years, OWC has been rapidly developed and has gradually become one of the most popular solutions for the future high-speed wireless connectivity and power efficient low rate internet of things applications. Secondly, the basic concept of the system model of the current
OWC system has been introduced, including the high-speed transmitters, the highly sensitive receivers, the optical wireless channel and the noise components in OWC. For the high-speed transmitters, a LED is used in indoor VLC systems due to its energy-efficiency and lighting capability; and a LD is used in long distance communication due to the generation of light with high concentration. For the highly sensitive receivers, a PIN diode is commonly used in commercial applications due to its low cost and high power efficiency; and an APD is used in some OWC experiments with low transmission power due to its much higher sensitivity than PIN diodes. In OWC systems, the optical channel represents the relationship between the optical transmitters and the optical receivers. A basic expression for the optical channel is given in this chapter. Moreover, ambient noise, shot noise and thermal noise jointly affect the system performance. Thirdly, optical modulation schemes have been studied in this chapter, including single-carrier and multi-carrier modulation schemes. As representatives of single-carrier schemes, OOK, $M$-PPM and $M$-PAM have been introduced and compared. For the multi-carrier scheme, O-OFDM, as the most popular research area of the OWC modulation schemes, has been simply presented, including three different methods: DCO-OFDM, ACO-OFDM and U-OFDM. Finally, the nonlinear distortion effect in O-OFDM has been studied, based on the Bussgang theorem. The nonlinear distortion effect is a significant problem in all of the O-OFDM systems and has to be considered in the theoretical analysis of the system performance.
A high sensitivity single photon detecting receiver referred to as single-photon avalanche diode (SPAD) is introduced in this chapter. Instead of conventional photo-diode (PD) receivers, SPAD receiver is designed for long range transmission in optical communication (OC). In this chapter, SPAD is proposed to be applied in visible light communication (VLC) systems. The problem of continuous downhole monitoring in the oil and gas industry is effectively addressed by the use of the SPAD-based VLC system. As a reliable, flexible and low-cost technique, VLC can fulfill a critical need of operators to maintain production efficiency and optimise gas well performance. Moreover, SPAD is instrumental in achieving long range communications, and the fact that ambient light is not present in a gas pipe is exploited. Specifically, the lack of ambient light enables high signal-to-noise ratio (SNR) at the receiver which operates in a photon counting mode. In this chapter, the bit-error ratio (BER) performance of the system is simulated for a 4 kilometres long metal pipe. It is shown that the proposed system has superior power efficiency over conventional methods, which is important as it is assumed that the transmitter is battery operated. In addition, the theoretical BER performance is calculated and compared to the simulation results.
3.1 Introduction

Current optical wireless communication (OWC) and VLC systems are mainly realised by high speed light emitting diodes (LEDs) as transmitters and highly sensitive PDs as receivers, such as positive-intrinsic-negative (PIN) diodes and avalanche photo-diodes (APDs). In previous studies, OWC and VLC have been considered for applications such as indoor wireless communications, wireless communication in hazardous environments and underwater communications [129]. However for low power and long distance transmission, these PDs have unsatisfactory performances since the transimpedance amplifier (TIA) causes an additional thermal noise and this significantly reduces signal-to-noise ratio (SNR) of receivers. In this study, SPADs are applied as the receiver of the VLC system [118]. The SPAD detector does not require a TIA and thus the output signal is not distorted by a strong thermal noise. Therefore, the SPAD receiver can possibly perform at significantly higher efficiency than the conventional PDs. When the VLC system is applied in long distance transmission, such as in a gas well downhole monitoring system [67] and space communications, the number of photons reaching the receivers are much fewer than the conventional short-distance links. In such scenarios, the SPAD receiver would be more suitable compared with normal PDs.

In the gas industry, the use of wirelines and armored cables is common practice for communication between the downhole and the surface, but these installations present maintenance and reliability issues. Furthermore, wireline solutions have high installation costs and their operation requires the halt of production bringing extra cost to the operator due to the down-time. Wireless solutions have also been considered for use in downhole monitoring, such as mud-pulse telemetry, low-frequency electromagnetic waves and acoustic waves, but for long distance communications, their performances are not satisfactory. The low data rate, the occurrence of undetectable situations and the environmental impact are the main factors which restrict the development of wireless communication systems in this context.

In this study, a wireless solution using VLC is proposed to overcome the restrictions of the existing technologies summarised above. The solution is designed to have low power consumption while achieving high communication speed and reliability. Unlike the radio frequency (RF) monitoring system, a LED transmitter is used instead of antennas. Hence, there is no danger of causing explosions due to electric sparks. Thus, the proposed system is considered as a safer solution. Compared to conventional VLC environments, the transmission distance is much longer in the gas pipe. Moreover, communication between the downhole and the surface needs to be
continuously maintained and in order to reduce maintenance costs, limited transmission power is supplied by the battery-powered transmitter. In this long distance and low power transmission scenario, SPAD would be a more suitable choice as the receiver. Besides, the lack of ambient light in the gas pipe supplies a good environment for such high sensitivity optical receiver. In this chapter, the detail and concept of this particular VLC system is proposed.

In previous research, VLC systems are usually considered to be modulated as an intensity modulation and direct detection (IM/DD) system with conventional PDs [32]. This because that the incoherent light output of the LED means that information can only be encoded in the intensity level. As a consequence, only real-valued and positive signals can be used for data modulation. This is in stark contrast to RF systems which make use of complex valued and bi-polar signals. On-off keying (OOK), pulse position modulation (PPM) and pulse amplitude modulation (PAM) are some of the popular modulation schemes used in conjunction with IM/DD systems [34]. For high speed data transmissions, optical orthogonal frequency division multiplexing (O-OFDM) is applied in order to get closer to the channel capacity by utilising adaptive bit and power loading. Diverse O-OFDM modulation schemes have been realised and utilised in VLC, such as DC-biased optical OFDM (DCO-OFDM), asymmetrically clipped optical OFDM (ACO-OFDM), unipolar OFDM (U-OFDM) and non-DC-biased OFDM (NDC-OFDM) [43, 65]. In the downhole monitoring VLC system, since the requirement of the transmission speed is much lower than requirements in conventional indoor VLC systems. OOK is considered as the modulation and demodulation schemes for the SPAD receiver. Different from the conventional PD receivers, the output of SPAD is pulse trains which represent each received photon. Thus a specified method is used for demodulating optical OOK signals, which will be proposed in this chapter.

The rest of this chapter is organised as follows. Section 3.2 presents SPAD in OWC: Section 3.2.1 introduces basic concept of SPAD; Section 3.2.2 presents some important SPAD metrics; Section 3.2.3 gives model of SPAD receivers in OWC; Section 3.2.4 analyses the nonlinear effect of SPAD receivers. Section 3.3 presents an application of SPAD to downhole monitoring: Section 3.3.1 gives the concept of downhole monitoring and the reason why SPAD is applied; Section 3.3.2 presents the system model of the SPAD-based downhole monitoring; Section 3.3.3 presents the modulation scheme and corresponding theoretical analysis of the system; Section 3.3.4 confirms the validity of the theoretical BER estimations through extensive Monte Carlo simulations and provides some discussion on the performance of the SPAD-based...
Single-Photon Avalanche Diode and Application in VLC Systems

Figure 3.1: Passive quenching circuit for PQ SPAD, where $V_{eb}$ denotes the excess bias voltage; $V_{br}$ is the breakdown voltage; the resistor, $R_b$, recharges the junction capacitance and other parasitic capacitances; and $V_S$ denotes the voltage of the SPAD device.

3.2 Single-Photon Avalanche Diode

3.2.1 Concept

Generally, optical detectors respond to the intensity of the received optical signal rather than its field because their bandwidths are much smaller than the carrier frequency of light. A SPAD is a special APD which is implemented as a p-n junction biased above breakdown. In this regime of operation, known as Geiger mode, photo-generated carriers may cause an avalanche by impact ionisation [130]. When an incident photon with sufficient energy to liberate an electron arrives, avalanche multiplication of the photo-generated electron occurs due to the high electric field. A SPAD thus can be triggered billions of the electron-hole pair generation by each detected photon. This produces a significant current pulse representing the arrival time of a photon which can be directly detected by complementary metal-oxide-semiconductor (CMOS) logic [131]. As a consequence, the device is extremely sensitive, and is able to accurately detect a single photon. An avalanche in the multiplication region causes a current pulse of appreciable amplitude which continues until the avalanche is recharged by lowering the bias.
Figure 3.2: *PQ SPAD voltage fluctuation curves when photons arrive and the dead time is extended by other photons. PQ SPAD will be paralyzed if photons are continuously and quickly incoming.*

Voltage towards or below the breakdown voltage. SPAD devices is generally recharged by a quenching circuit. In this study, two types of SPAD devices with different recharged quenching circuits are considered: passive quenching (PQ) and active quenching (AQ) [132, 133].

### 3.2.1.1 Passive Quenching SPAD

The passively recharged SPAD is referred as passive quenching single-photon avalanche diode (PQ SPAD). The configuration of the passively recharged circuit is presented in 3.1, where the SPAD device is biased with an excess bias voltage above the breakdown voltage, $V_{eb} > V_{br}$. PQ SPAD is identified as a paralyzable detector where any counts occurring during the dead time (including signal, dark count and after pulse) are not registered but are extending the dead time [132]. In detail, the voltage of PQ SPAD ($V_S$) is placed to the excess bias voltage which is higher than the breakdown voltage. When an incident photon triggers an avalanche, the SPAD voltage is reduced towards to and below the breakdown voltage. PQ SPAD is then passively recharged without additional voltages and the SPAD voltage is increased back to the excess voltage. When the SPAD voltage is above a threshold voltage, $V_{th}$, the photon counter can be registered. For PQ SPAD, the device can be triggered by another photon as soon as the SPAD voltage exceeds $V_{br}$. This means that the SPAD voltage may reduce towards to the breakdown voltage. In this case, the SPAD voltage remains below the threshold voltage ($V_{th}$).
As long as the voltage is lower than $V_{th}$, only the first photon is registered and other photons are lost. Moreover, the recharged time, referred to as dead time, is extended and the PQ SPAD is paralyzed. For example, as shown in Figure 3.2, when the first photon arrives, the SPAD voltage is quickly reduced towards the excess voltage. When the SPAD voltage reaches and reduces below the breakdown voltage, the PQ SPAD is recharged by the PQ circuit. Then the SPAD voltage slowly increases back to the excess voltage. As shown in Figure 3.2, before the SPAD voltage becomes above the threshold voltage, the second photon arrives. The ‘voltage’ for the second photon is reduced towards the breakdown voltage. Thus, the SPAD voltage is reduced again. Since the voltage is still lower than the threshold voltage, only the first photon can be registered by the photon counter. In addition, the total duration of the dead time is extended. If the SPAD voltage is unable to be above the threshold voltage, the PQ SPAD can only count one photon and is so-called as a paralyzable detector.
3.2.1.2 Active Quenching SPAD

The actively recharged SPAD is referred as active quenching single-photon avalanche diode (AQ SPAD). Figure 3.3 shows the circuit of a single AQ SPAD device where the excess voltage is also set to be above the breakdown voltage. Compared with PQ SPAD, the configuration of AQ SPAD is more complex and requires more logic area and circuit power, but when any events arrive during the dead time, the additional events are not registered and do not prolong the dead time. When an avalanche occurs, the excess voltage is quickly reduced forward to the breakdown voltage which is the same as the progress in PQ SPAD. However, after the SPAD voltage reaches $V_{br}$, both quenching and reset are carried out using active components [133], such as the pulse generator in Figure 3.3. The SPAD voltage is forcibly returned to $V_{eb}$ and AQ SPAD counts a photon. In this quenching and reset duration, other events (including signal, dark count and after pulse) can not be accepted and registered. After the SPAD voltage is set back to $V_{eb}$, AQ SPAD can continue to count the next photon. An example of voltage fluctuation of AQ SPAD is shown in Figure 3.4. It can be seen that when a photon arrives, the voltage of AQ SPAD is reduced from $V_{eb}$ to $V_{br}$ quickly. Then the voltage will keep below the breakdown voltage for a while and be sharply increased to $V_{eb}$. In this whole process, the second photon is refused and lost by the device. Unlike PQ SPAD (Figure 3.2), AQ SPAD is non-paralysed and receives the next incoming photon after the SPAD voltage is back to $V_{eb}$. As the dead time

![AQ-SPAD Dead Time Effect](image)
will not be extended, AQ SPAD is identified as a non-paralyzable detector and has higher count rates than PQ SPAD [132].

### 3.2.1.3 SPAD Array

Currently, SPADs are frequently used as imaging devices in various fields, including fluorescence lifetime imaging microscopy [134], fluorescence correlation spectroscopy [135], positron emission tomography [136], single-photon emission computed tomography [137], and three-dimensional cameras [138]. In communication systems, SPADs have been proposed in plastic optical fibre [139]. In these applications, since the photon count rate of a single SPAD device is very limited, multiple parallel SPAD devices are used and placed in a SPAD array to improve the capacity of photon counts. In this study, since the symbol duration is short and more photons need to be received for signal demodulating, SPAD arrays are considered and used in OWC and VLC.

### 3.2.2 Important SPAD Metrics

In this section, some important SPAD metrics are introduced and analysed. Some of these metrics have significant effect on the performance of SPADs in OWC.

#### 3.2.2.1 Fill Factor

Fill factor (FF) is the ratio of the optically receptive SPAD active area to the total device area and is an important parameter for calculating the received optical power [140]. As shown in Figure 3.5(a), the FF of a single device, \( C_{FF}(m) \), is calculated as:

\[
C_{FF}(m) = \frac{A_{ac}}{A_{dla}}, \tag{3.1}
\]

where \( A_{ac} \) denotes the active area of the single SPAD device and \( A_{dla} \) is the logic area of the device. Photons which hit on the optically receptive SPAD active area would be absorbed by the biased p-n junctions and have a chance to cause an avalanche. These photons finally could be registered by the photon counter. The logic area of each devices is some optically insensitive area, such as the mentioned quenching circuits, output buffer and other type of memory cell. This area allows SPAD to receive and count photons individually but the photons hitting the
logic area cannot be absorbed and counted by the device. For the SPAD array, FF represents the probability that a photon hits the SPAD active area. If the photon triggers an avalanche, it will be counted. In other words, the percentage of photons in a beam light reaching the SPAD active area can be approximated to FF of the SPAD array. Figure 3.5(b) shows a simple structure diagram of a SPAD array and the total FF of a SPAD array, $C_{FF}$, can be calculated as:

$$C_{FF} = \frac{N_{SPAD} A_{ac}}{N_{SPAD} A_{dla} + A_{ala}},$$

(3.2)

where $N_{SPAD}$ denotes the number of single SPAD devices in a SPAD array and $A_{ala}$ is the logic area of the array. The array logic area contains some necessary parts of the SPAD, such as the connective circuits between each device, the OR tree circuit and the photon count accumulator. As the receiver in OWC, some incoming photons would be lost when they hit on the logic area of the array and the devices. In the SPAD circuit, the logic area should be designed as small as possible to increase the photon count rate of the whole array.

Figure 3.5: Fill factor of SPADs: (a) the fill factor of a single SPAD device, where $A_{ac}$ is the active area and $A_{dla}$ is the total logic area of the device; (b) the fill factor of a SPAD array, where $A_{ala}$ is the total logic area of the SPAD array which is used for connecting each single devices and giving the outputs.
3.2.2.2 Photon Detection Probability

Photon detection probability (PDP) is the probability that a photon hitting the active area triggers an avalanche. This avalanche will generate a pulse which can be counted by each single SPAD device. The accumulator of the SPAD array will finally give the total number of photon counts. PDP is different to the quantum efficiency of the conventional PD, in which the quantum efficiency sometimes includes fill factor effects [141]. In SPAD, PDP is the parameter to represent the ability of photon absorption. In other words, this parameter is related to the lost of incident photons which is caused by following reasons [141]:

i The photon is reflected or absorbed in the stack above the silicon and cannot generate an electron-hole pair.

ii The photon generates an electron-hole pair in the p+ region but outside the depletion region.

iii The photon generates an electron-hole pair in the n-well region but outside the depletion region.

iv The photon generates an electron-hole pair in the substrate.

v The photon passes through the silicon, creating no carrier pairs.

vi The photon generates an electron-hole pair but the pair cannot cause an avalanche.

Those photons would be lost and only the photon which generates an electron-hole and cause an avalanche could be counted by the single SPAD device. In this study, the statistical PDP of a single SPAD device is denoted by \( C_{\text{PDP}}(m) \) and for the SPAD array, the average PDP is:

\[
C_{\text{PDP}} = \frac{1}{N_{\text{SPAD}}} \sum_{m=1}^{N_{\text{SPAD}}} C_{\text{PDP}}(m). \tag{3.3}
\]

3.2.2.3 Dark Count Rate

A thermally-generated carrier can also trigger an avalanche which increases the array output. Even in complete darkness, this phenomenon still exists as long as the SPAD devices are opened. The average number of counts in darkness per second is referred to as dark count rate (DCR) which is regarded as a fixed signal-unrelated noise of SPAD. As a main noise component of SPAD, the dark current noise is produced by following two routes [142]:

i The thermally-generated dark photons is caused caused by the random transition of electrons from the valence to conduction band. The thermal generation depends on the bias voltage of SPAD devices.
The dark current photons is produced in the band-to-band tunnelling. The energy band gap normally presents a barrier to carriers below this energy but a carrier has a finite probability of tunnelling. This depends on the energy of the carrier and traps in the band gap.

Thus, the mean value of DCR of each single SPAD device is only related to the structure of the device, bias voltage and operation environment (temperature). This value is fixed for each device and the distribution of the dark current photons follows Poisson statistics. In a SPAD array, each single SPAD device is assumed to have the same DCR. Thus the average statistical DCR of each device is denoted by $N_{DCR}$.

### 3.2.2.4 After Pulsing Probability

After pulses are correlated to detections by the time dependent release of trapped carriers [118]. Additional avalanches are triggered after receiving a photon or a dark photon. This means that the after pulsing effect will also increase the array output related to both the incoming signal and the dark counts. Unlike dark count photons, the photon counted from after pulses is related to the signal. Thus, the additional noise caused by after pulse is so-called as a signal-related noise and the probability of an after pulsing generation is referred to as after pulsing probability (APP). The delayed counts by after pulsing will bring inter-symbol interference due to the high data rate. But in low speed transmission, as the sample period is much longer than the delayed time, the after pulsing effect has a negligible effect on the next sample period. In this study, the value of APP is denoted by $P_{AP}$. DCR and APP are the main noise resource of SPAD. Both of them could increase the output of the device and the fluctuation of the photon count.

### 3.2.2.5 Dead Time

As mentioned in Section 3.2.1, during the recharge time of PQ and AQ SPAD, the device is unable to detect further signal photons, dark photons or after pulses. This time is referred to as dead time. When the power of incoming signal is low, the dead time effect would have insignificant effect on the output of SPAD as the duration between each incoming photons has a high probability to be much longer than the dead time. However, when the number of incoming photons is high and a mount of photons is lost during one dead time, the output of the SPAD reflects the photon count of the optical signal nonlinearly. In other words, a nonlinear problem on the optical signal is caused by the dead time effect and has a significant effect on the BER.
performance of the OWC system. In this study, the average dead time of a single SPAD device is denoted by $\tau_d$. The detail of the problem will be presented in the further section.

### 3.2.3 SPAD receivers in Optical Wireless Communications

Figure 3.6 illustrates the system model of the SPAD array receiving optical signals. In order to generate received optical symbols, the output of the SPAD array is counted over a symbol duration, $T_s$, at time instances $t_k = kT_s$ of the received optical signal $x_r(t)$. These photon counts are denoted by $\nu(k)$ which is the superposition of the photon counts from each individual SPADs, $a_m(k)$, as shown in Figure 3.6:

$$\nu(k) = N_{\text{SPAD}} \sum_{m=1}^{N_{\text{SPAD}}} a_m(k), \quad (3.4)$$
Generally, as the photon counts from each individual SPAD can be approximately modelled using Poisson statistics, the photon counts at the output of the SPAD array (i.e., \( \nu(k) \)) can be still described by a Poisson distribution:

\[
\Pr\left( \nu(k) = j, \mu(k) \right) = \exp \left( -\mu(k) \right) \frac{\mu(k)^j}{j!},
\]

where the average photon counts \( \mu(k) \) can be expressed as a function of the received signal and the effects of the SPADs’ metrics introduced in section 4.2.2:

\[
\mu(k) = \frac{C_{FF} C_{PDP}}{E_p} \int_{t_k}^{t_{k+T_s}} x(t) \, dt + n_{DCR} (1 + P_{AP}),
\]

where \( E_p \) denotes the energy of a photon which is calculated by \( \frac{h \nu \omega_L}{c_L} \). Note that \( h \) denotes Planck’s constant; \( c_L \) is the speed of the light; and \( \omega_L \) is the light wavelength of the LED transmitter. The noise caused by dark counts is denoted by \( n_{DCR} = N_{DCR} N_{SPAD} T_s \). In (3.5) and (3.6), \( C_{FF}, C_{PDP}, N_{DCR} \) and \( P_{AP} \) are used for calculating the mean of a Poisson distribution.

Figure 3.7 shows the process of photon counting in a single SPAD device (both PQ and AQ SPADs) over \( T_s \) which is set to 1 \( \mu s \) in the simulation. During \( T_s \), it is assumed that around 100 photons hit the active area of the SPAD device (Figure 3.7(a)). As shown in Figure 3.7(b), only a bit of incident photons can trigger avalanches. As noted, the probability of the trigger is PDP. At the same time, the dark current triggers independently Poisson random avalanches (Figure 3.7(c)). Afterwards, following the detected photons and the dark photons, the after pulse also provides some additional photon counts (Figure 3.7(d)). In PQ SPAD, as only the first triggered avalanche can be recorded during one extended dead time, limited number of avalanches can be achieved as shown in Figure 3.7(e1). Those pulse trains will be registered by the accumulator and the output of a single PQ SPAD device can be obtained. Unlike PQ SPAD, some potential avalanches can not be triggered during the dead time in AQ SPAD. As the dead time is not extended, the AQ SPAD device can achieve more photon counts as shown in Figure 3.7(e2).
Figure 3.7: Process of photon counting in a single SPAD device when $T_s = 1 \mu s$ and the total number of incident photons (to the active area) is 100: (a) the original incident photons reaching active area of SPAD devices (related to FF); (b) the detected photons may trigger avalanches (related to PDP); (c) the random dark photons (related to DCR); (d) the after pulses depend on the detected photons and the dark photons (related to APP); (e1) outputs of PQ SPAD; (e2) outputs of AQ SPAD.
3.2.4 Nonlinear Effect of SPAD Receivers

In either PQ SPAD or AQ SPAD, the dead time effect makes a nonlinear reduction on photon counts.

3.2.4.1 Nonlinear Function of PQ SPAD Receivers

For a single PQ SPAD device, the relationship between the average potential counts per second, $\mu_m$, and the real photon counts, $\mu_{PQ,m}$, is [132]:

$$\mu_{PQ,m} = \mu_m \exp (-\mu_m \tau_d).$$

Thus, for each $T_s$, the average number of the real photon counts, $\mu_{PQ,m}(k)$, is calculated by:

$$\mu_{PQ,m}(k) = \frac{\mu_m(k)}{T_s} \exp \left( -\frac{\mu_m(k)}{T_s} \tau_d \right) T_s = \mu_m(k) \exp \left( -\frac{\mu_m(k) \tau_d}{T_s} \right).$$

where $\mu_m(k)$ denotes the average potential counts for each single devices in the same $T_s$. For the SPAD array, $\mu_m(k)$ is equal to $\mu(k)/N_{\text{SPAD}}$. In this study, if the SPAD array is composed
by PQ SPAD devices, the average output of the array during each $T_s$ can be expressed as:

$$\mu_{PQ}(k) = \sum_{m=1}^{N_{SPAD}} \mu_{PQm}(k) = \mu(k) \exp \left( -\frac{\mu(k)\tau_d}{T_sN_{SPAD}} \right).$$

(3.9)

Note that $\mu(k)$ is calculated by (3.6). According to the process of photon counting, $\mu(k)$ means the average potential counts by the PQ SPAD array. For simplicity, $\mu_{PQ}(k)$ can replace $\mu(k)$ in (3.5) to estimate the distribution of the SPAD array output. From nonlinear function of the PQ SPAD array, (3.9), the maximum photon count rate can be calculated:

$$\mu_{PQ_{max}} = \frac{T_sN_{SPAD}}{e\tau_d},$$

(3.10)

where $e$ is Euler’s number. Figure 3.8 shows the nonlinear effect of the PQ and AQ SPAD arrays when $T_s = 1\ \mu s$, $N_{SPAD} = 1024$ and $\tau_d = 13.5\ \text{ns}$ for both of SPAD devices. As shown in Figure 3.8, after reaching $\mu_{PQ_{max}}$, as the PQ SPAD devices are paralyzed, the outputs of the SPAD array rapidly decreases with an increasing rate of incoming photons.

### 3.2.4.2 Nonlinear Function of AQ SPAD Receivers

For a single AQ SPAD device, the average real photon counts per second, $\mu_{AQm}$, is expressed as a function of $\mu_m$ [132]:

$$\mu_{AQm} = \frac{\mu_m}{1 + \mu_m\tau_d}.$$  

(3.11)

For each $T_s$, the average output of a single device is:

$$\mu_{AQm}(k) = \frac{\mu_m(k)}{1 + \frac{\mu_m(k)\tau_d}{T_s}}.$$  

(3.12)

Thus the average output of the AQ SPAD array in $T_s$ is:

$$\mu_{AQ}(k) = \sum_{m=1}^{N_{SPAD}} \mu_{AQm}(k) = \frac{\mu(k)}{1 + \frac{\mu(k)\tau_d}{T_sN_{SPAD}}}.$$  

(3.13)
Thus, the maximum photon count rate of the AQ SPAD array is:

$$\mu_{AQ_{\text{max}}} = \frac{T_s N_{\text{SPAD}}}{\tau_d}.$$  (3.14)

As shown in Figure 3.8, when the incoming photon rate increases, the AQ SPAD devices are non-paralyzed but the outputs dramatically converge to $\mu_{AQ_{\text{max}}}$. In other words, if the average potential photon counts, including signal photons, dark photons and after pulse counts, are more than $\mu_{AQ_{\text{max}}}$, the AQ SPAD array will be saturated. The photon counts at the output of the SPAD array are constrained to $\mu_{AQ_{\text{max}}}$ and the extra photons are refused and lost.

As a result of the comparison of (3.10) and (3.14), the maximum photon count of AQ SPAD is higher than PQ SPAD if $N_{\text{SPAD}}$ and the dead time constant are the same. However, note that AQ SPAD needs more complex circuit and so that has lower FF and higher power consumption. Thus, in OWC, if the demand of the photon count rate is more important, AQ SPAD would be chosen as the receiver; and if the requirement of the system complexity and power efficiency is more important, PQ SPAD would be chosen. In this study, the nonlinear property of SPAD devices has a negligible effect on the BER performance of OOK and it only limits the transmitter power and data rate. However, the nonlinear property of SPAD has a significant effect on the BER performance of O-OFDM. The nonlinear distortion problem of SPAD-based OFDM will be defined and analysed in Chapter 5.

### 3.3 Application to Downhole Monitoring

#### 3.3.1 Concept

The downhole monitoring system is a pressure and temperature gauge installed in a gas or oil well. Generally, the permanent downhole monitoring (PDM) is required to keep the data transmission between the downhole and surface. Typically, wired communications are considered to realise the PDM system. The conventional wireline communication with armored cables and the optical fibre communication are common solutions in the wired PDM system [143]. Fig 3.9(a) shows the system concept of the wired solution in PDM. However, the wireline solutions require a long transmission cable based on the length of the gas or oil pipe. These solution have high installation cost and require a regular comprehensive inspection. If the cable is damaged, the system has to be stopped and take long time to be overhauled. Thus the
wireline solutions present maintenance and reliability issues which reduce the business profit. In order to solve these problems, wireless solutions have been considered for the PDM system as shown in Fig 3.9(b). Conventional RF technologies with electrical antennas are considered. However, due to the RF being high-speed electro-magnetic, electric sparks can be generated in antennas. This may cause an explosion in high density mixed gas environments. Therefore, mud-pulse telemetry, low-frequency electro-magnetic waves and acoustic waves are considered as the wireless solutions in PDM. Despite the fact that these solutions have lower costs and risks than the wireline solutions, the data rate and the power efficiency are unsatisfied in such long distance communication. Fig 3.9(c) shows a novel wireless solution of PDM by realising a VLC system in the gas pipe. This system consists of LED transmitters and SPAD receivers. Compared to the conventional wireless solutions, the VLC-based PDM system has significant low power consumption and high communication speed. Furthermore, since LED and SPAD are used and exposed in the gas pipe, there is no danger of causing explosions due to electric sparks. In this study, the detailed system model and performance analysis are proposed.
3.3.2 System Model

This section presents a practical model for the proposed system. Based on general VLC systems, the communication system consists of a blue light LED and an array of SPADs. A long steel pipe defines the channel of the transmission.

3.3.2.1 Pipe Parameters

As shown in Figure 3.10, it is assumed that the communication system is realised in a long steel cylindrical pipe with a length of 4,000 metres and diameter of 1.5 metres. The dimension are taken from a real-world deployment of such a pipe. The reflectivity of steel is 58.5 % [144]. In this study, the reflection of the information-carrying light is considered as the specular reflection on the internal surface of the pipe. In the downhole monitoring system, there is no ambient light. Hence, the photons that reach the top of the pipe are either from the direct path or from reflections inside the pipe. This pipe constitutes the propagation channel and a ray-tracing method is used to establish a channel model. In practice, gas is transported in this pipe. As the speed of the lightwaves is reduced in the gas medium according to the refractive index, the
effect of intersymbol interference (ISI) will be enhanced which will increase the probability of detection errors. In this study, the pipe is assumed to be vacuum in order to establish the baseline performance and to understand the general feasibility.

### 3.3.2.2 LED Transmitter

As shown in Figure 3.10, a LED is used at the bottom of the pipe as the transmitter. The LED is assumed to emit blue light with wavelength of 450 nm. The light emission from a LED transmitter can be modelled using a generalised Lambertian radiation intensity pattern [31]:

\[
R_0(\phi) = \frac{(m_s + 1)}{2\pi} \cos^{m_s}(\phi) P_t, \tag{3.15}
\]

where \(m_s = -\ln 2/\ln(\cos(\Phi_{1/2}))\), and \(\Phi_{1/2}\) is the transmitter semiangle which represents the half power angle. The variable \(\phi\) denotes the angle of radiation, and \(P_t\) is the average power of the LED. In the long pipe communication system considered, the design objective is to ensure that sufficient photons hit the SPAD receiver when light is emitted. An important parameter is the transmitter semiangle. Figure 3.11 shows the distribution of the Lambertian radiation intensity with transmitter semiangles of 5°, 10°, 15° and 30° when the power of the LED is 0.1 Watt. As shown, more power can be concentrated in the range of lower radiation angles (−5° to 5°).
5°) for lower transmitter semiangles. This decreases the number of light rays that hit the pipe wall and will thus decrease the loss caused by reflections. In order to enhance the likelihood of photons to hit the SPAD on the direct line-of-sight (LOS) link, the LED and the SPAD device at the surface are vertically aligned. In this study, values of the semiangle from 5° to 15° are considered.

3.3.2.3 SPAD Receiver

Due to the long distance transmission (4,000 metres), the irradiance at the top surface is lower than in standard VLC indoor scenarios where the maximum distance is a few metres. In fact, the number of photons at the receiver in the given scenario may only be in the region of tens of photons. The typical gain of an APD is insufficient to produce sufficient signal power for further signal processing. Therefore, the highly sensitive SPAD receiver device presented in Section 3.2 is used in the PDM system. The following benefits are the reasons why the SPAD receiver is suitable in this system:

i The downhole provides an environment that is free from ambient light. Hence, the high sensitivity of the SPAD receiver is not compromised by ambient noise.

ii Because of the high sensitivity of SPADs, transmission distance and optical power can be traded-off favourably towards battery life.

iii The required data rate, which is in the order of kilobits to a few megabits per second, is sufficiently low to use OOK. OOK enables the use of straightforward threshold-based detection techniques in conjunction with the Geiger counting principle.

iv The SPAD detector does not require a TIA and the output signal is a pulse train which greatly reduces the noise at the receiver.

In this study, in order to increase the photon count rate, a SPAD array is assumed to be set at the top of the pipe as shown in Figure 3.10. Finally, by considering the optical channel function (2.6), according to (3.15) and the metrics of the SPAD array as shown in (3.2) and (3.3), the number of photons received from the LED per second, \( N_L \), can be estimated as follows:

\[
N_L(N_R) = C_{FF}C_{PDP} \sum_{n_R=0}^{N_R} \frac{(m_m + 1)AP_l}{2\pi d^2 E_P} \cos^{m_m}(\phi(n_R))C_R^{n_R},
\]

(3.16)

where \( N_R \) is the total number of reflections; \( A \) is the total device area of the SPAD array; and \( d \) is the distance between the LED and the receiver. In this study, \( d \) is approximated as the length
of the pipe. \( E_p \) is the energy of a photon as noted in (3.6). The expression, \( \phi(n_R) \), is the angle of radiation as a function of the reflections; and \( n_R \) is the number of reflections needed to reach the top. The number of reflections required to reach the SPADs defines the radiation angle at which the light ray has left the LED, hence why we have chosen to represent \( \phi \) as a function of \( n_R \). The reflectivity of steel is expressed by \( C_R \). In the experiment, the ambient light, DCR, after pulses and clock jitter will add to the number of counted photons in one symbol duration. Because the effect of clock jitter is much lower than the ambient light, DCR and after pulses, clock jitter is not considered in this study. As the system is realised in a long steel pipe where there is no ambient light, DCR and after pulses are considered the dominant source of noise. As presented in Section 3.2, SPAD exhibits a DCR similar to PD dark current which is generated by thermal carriers. This means that DCR exists even when there are no photons reaching the SPAD. In practice, dark photon counts will add to the total number of the counted photons per second. Besides, additional photons will be counted by the after pulsing effect according to the photon counts of single photons and dark photons. Thus, according to (3.16), the total number of photon counts per second can be calculated as:

\[
N_P(N_R) = C_{FF}C_{PDP} \sum_{n_R=0}^{N_R} \left( \frac{m_s+1}{2\pi d^2 E_p} A P_t \cos^{m_s}(\phi(n_R)) C_R^{n_R} + N_{DCR} N_{SPAD} \right)(1 + P_{AP}).
\] (3.17)

### 3.3.3 Modulation Scheme and Theoretical BER Analysis

In this study, non-return-to-zero on-off keying (NRZ-OOK) is used as the modulation scheme of the system. At the transmitter, the input bit stream is directly transformed to an analog signal by the digital-to-analog converter (DAC). A binary ‘1’ is represented by a positive voltage which is much higher than the voltage that represents the binary ‘0’. The voltage levels assigned to bits depend on the power constraints of the system. In this study, \( P_1 \) represents the power assigned to ‘1’ and \( P_0 \) denotes the power assigned to ‘0’. As the randomly generated zeros and ones follow a uniform distribution, \( P_t \), \( P_1 \) and \( P_0 \) have following relationship:

\[
P_t = \frac{P_1 + P_0}{2}.
\] (3.18)

When the signal is transmitted by the LED, the voltages are transformed to the corresponding light intensity. As a consequence, ‘1s’ are finally represented by a higher light intensity and ‘0s’ are represented by a lower intensity which is closer to zero. As shown in Figure 3.12(b),
the information-carrying light is received by the SPAD and is represented by the number of photons. Compared to the original digital bits (Figure 3.12(a)), it can be seen that more photons are counted when ‘1s’ are transmitted, and ‘0s’ are represented by much fewer photons at the receiver. As the FF and PDP have a significant effect on the received photons, the photon counts from the original signals follow a Poisson distribution. Moreover, the DCR noise and after pulse also follow a Poisson distribution which generates random positive integers [145]. As a consequence, irregular fluctuations are observed in Figure 3.12(b) which can cause errors during demodulation. In this study, the transmission speed is assumed to be low in the PDM system, such as 1 kbit/s, and the interval time between the direct light and the reflected light reaching the receiver is small. Hence, the symbol duration is much higher than the delay spread. Thus, ISI can be ignored. In this study, for the purpose of the NRZ-OOK demodulation, a threshold is selected which is presented in (3.22). For example, in Figure 3.12(b), the threshold is set to 38. This means that when the number of photons is larger than 38, the signal is demodulated to ‘1’. Otherwise, ‘0’ is obtained.

Since both the received signal and additional noises follow the Poisson distribution as described in (3.5), the distribution of the received NRZ-OOK symbols (‘1’ and ‘0’) can be described as
Pr(ν, N₁) and Pr(ν, N₀). The average number of photons counted in one symbol duration when ‘1’ is transmitted is denoted by N₁; N₀ is the average number of photon counted in one symbol when ‘0’ is transmitted. According to (3.17), N₁ and N₀ can be calculated with the fixed symbol period, Tₛ:

\[ N₁ = Tₛ \left[ C_{FF}C_{PDP} \sum_{n_R=0}^{N_R} \frac{(m_s + 1)AP₁}{2\pi d²EP} \cos^{m_s}(\phi(n_R))C^{m_R}_R + \text{NDCR}N_{SPAD} \right] (1 + P_{AP}), \]

(3.19)

\[ N₀ = Tₛ \left[ C_{FF}C_{PDP} \sum_{n_R=0}^{N_R} \frac{(m_s + 1)AP₀}{2\pi d²EP} \cos^{m_s}(\phi(n_R))C^{m_R}_R + \text{NDCR}N_{SPAD} \right] (1 + P_{AP}). \]

(3.20)

For the Poisson distributed noise, the principle of the BER calculation for the Gaussian distributed noise can be used [98]. Figure 3.13 shows the analysis for the error probability of NRZ-OOK in the SPAD-based VLC PDM system. The curve with red ‘x’ symbols represents the probability density function (PDF) of the counted photons per symbol when ‘1’ is transmitted. The mean value of this Poisson distributed PDF is N₁. The curve with blue circles denotes the PDF of the counted photons per symbol when ‘0’ is transmitted. The mean value of this
PDF is \( N_0 \). The threshold is chosen when the probability of these two PDF is equal:

\[
\exp(-N_1) \frac{N_1^{N_{\text{th}}}}{N_{\text{th}}!} = \exp(-N_0) \frac{N_0^{N_{\text{th}}}}{N_{\text{th}}!},
\]

where \( N_{\text{th}} \) is the threshold in OOK demodulations. Therefore, in the theoretical analysis, the threshold is simply estimated as:

\[
N_{\text{th}} = \frac{N_1 - N_0}{\ln(N_1) - \ln(N_0)}.
\]

As shown in Figure 3.13, the filled areas represent the probability of the error detection. According to the BER calculation method in RF, the error detection probability of the SPAD-based OOK system is calculated as:

\[
P_e = \frac{1}{2} \left[ 1 - P_c(N_{\text{th}}, N_0) + P_c(N_{\text{th}}, N_1) \right],
\]

where \( P_c \) is the cumulative distribution function (CDF) of the Poisson distribution which is expressed as:

\[
P_c(\nu = j, \mu) = \exp(-\mu) \sum_{i=0}^{j} \frac{\mu^i}{i!}.
\]

For NRZ-OOK, as the spectral efficiency is 1 bit/s/Hz, \( P_e \) is equal to BER.
Figure 3.14: The number of photons could be received by SPADs after numbers of reflection, for $P_t = 0.1$ Watt and $\Phi_{1/2} = 5^\circ, 7.5^\circ, 10^\circ$ and $15^\circ$.

3.3.4 Results and Discussion

In the simulation, the LED transmitter is positioned at a distance equal to half of the radius away from the center of the bottom surface. For blue light (450 nm) in vacuum, the energy of a photon ($E_P$) is $4.42 \times 10^{-19}$ J. Other parameters of the LED, such as the semiangle of the LED ($\Phi_{1/2}$), LED power ($P_t$) and the data rate ($1/T_s$), are variables in the simulation. Different values of these variables are simulated in order to choose appropriate values. LED power is tested from -10 dBm to 25 dBm and the data rate considered are 1 kbits/s, 2 kbits/s and 5 kbits/s. In this paper, the power assigned to ‘0’, $P_0$, is assumed to be 0 Watt. Hence, from (3.18), the power given to ‘1’, $P_1$, is $2P_t$. In the simulation, the SPAD presented in [118] is used for two reasons: a) this is a large array (16.8 mm$^2$) with 1024 SPAD elements which enhances the likelihood of receiving photons; and b) this device exists practically which is essential as the future work is to build a demonstrator to validate the simulation results in this study. The value of $C_{FF}$ is 30 %, $C_{PDP}$ is 20 % and $N_{DCR}$ is equal to 7.27 kHz [118]. Table 3.1 lists all of the parameters used in the simulation.

Figure 3.14 depicts the relationship between the angle of radiation and the reflections required to reach the receiver. It can be seen that when the number of reflections is 25, the angle is just $0.55^\circ$ and the cosine of which is 0.9999. This is due to the system being realised in a long pipe.
The BER performance of SPADs in the pipe, for transmission speeds are 1 kbits/s, 2 kbits/s and 5 kbits/s, when $\Phi_{1/2} = 10^\circ$.

and the area of the top surface being relatively small. Hence, the angle of radiation, $\phi(n_R)$, can be approximated to $0^\circ$, even though there are over 20 reflections. Thus, $\cos^{m_s}(\phi(n_R))$ in (3.17) is nearly equal to 1, even if $m_s$ is large. To calculate the number of received photons in each reflection ray, (3.17) can be simplified as:

$$N_P(n_R) = C_{FP}C_{PDP} \left( \frac{m_s + 1)AP}{2\pi d^2E_P} C_{nR}^{m_s} + N_{DCR} N_{SPAD} \right) \left(1 + P_{AP}\right). \quad (3.25)$$

The number of received photons from 0 to 25 reflections is also shown in Figure 3.14. It can be seen that when the semiangle is $5^\circ$ and there are no reflections, the SPAD receiver can receive $4.5 \times 10^5$ photons every second. But when the semiangle is $10^\circ$, the number of photons is only $1.1 \times 10^5$. This means that the lower the semiangle, the higher the number of received photons. In other words, the LED with lower semiangle can achieve the same BER performance with less energy. In Figure 3.14, with an increase in the number of reflections that a light ray takes to reach the receiver, there is a drastic reduction in the number of received photons. This is due to many photons being absorbed and lost in the reflections on the inner wall of the pipe. It can be seen that when the reflections are over 15, the number of photons, which can reach the receiver in every second, is less than 20. When the transmission speed is 1 kbits/s, the number of photons is over 140 in one symbol duration. After 15 reflections, 20 photons are received in 1,000 symbol periods which is negligible. Thus, in this study, the number of reflections
Figure 3.16: The BER performance of SPADs in the pipe, for $\Phi_{1/2} = 5^\circ$, $10^\circ$ and $15^\circ$, when the transmission speed is 1 kbits/s.

in (3.17) is considered to be 15. As a consequence, in the simulation, the average total photon counts per second is calculated as:

$$N_P(P_t, m_s) = \left[ C_{FF} C_{PDP} \sum_{n_R=0}^{15} \frac{(m_s + 1)A_P}{2\pi d^2 E_P} C_{R}^{n_R} + N_{DCR} N_{SPAD} \right] (1 + P_{AP}). \quad (3.26)$$

Note that the number of photon counts increases with the increasing of the LED power, $P_t$, and the decreasing of the transmitter semiangle, $\Phi_{1/2}$.

Figure 3.15 shows the BER performance of the SPAD receiver with fixed LED semiangle ($\Phi_{1/2} = 10^\circ$). Transmission speeds in this situation are assumed to be 1 kbits/s, 2 kbits/s and 5 kbits/s. It can be seen that there is a good match between the simulation and theoretical results. As shown in Figure 3.15, the power requirement of the LED is just about 13.5 dBm for a BER of $10^{-9}$ at 1 kbits/s. For higher transmission speeds, such as 2 kbits/s and 5 kbits/s, the power requirements are 16 dBm and 20.5 dBm, respectively.

Figure 3.16 shows the BER performance of SPADs, when the data rate is 1 kHz. Unlike Figure 3.15, Figure 3.16 demonstrates the BER performance with changes of the semiangle, $\Phi_{1/2}$ = $5^\circ$, $10^\circ$ and $15^\circ$. When $\Phi_{1/2} = 5^\circ$, the LED requires just 7.5 dBm to reach a BER of $10^{-9}$. When $\Phi_{1/2} = 15^\circ$ and BER = $10^{-9}$, the power requirement of the LED is 17.5 dBm. With
the increase of the semiangle, the power requirement of the LED transmitter increases. As a consequence, the system achieves higher power efficiency with the lower semiangle.

In practice, a longer or shorter pipe might be utilised and there is a difference in the requirement of the transmission speed in different scenarios. Figure 3.17 shows the power that the transmitter needs when the semiangle ($\Phi_{1/2} = 10^\circ$) is fixed and the BER is considered at $10^{-3}$. In Figure 3.17, it is assumed that the length of the pipe varies from 1,000 to 10,000 metres and the transmission speed from 1 to 50 kbits/s. In normal circumstances, the system can support lower transmission speeds which are about 1 kbits/s. Only 8 dBm are required for communication in a 4,000 metres long pipe. When a higher transmission speed is required, the power of LED needs to increase to 24.1 dBm. In a longer pipe (10,000 metres), the system requires more power (32.1 dBm) to transmit signals in order to maintain the quality of communications.
3.4 Summary

In this chapter, a SPAD in context to its application to VLC is presented. A SPAD is considered as a high sensitivity optical receiver in OWC and VLC. Some important metrics of SPADs are introduced and it is demonstrated that they have significant effects on the performance of a SPAD-based communication system. In SPAD, Poisson distributed noise components are caused by FF, PDP, DCR and APP which lead to an signal error detection. SPADs can only detect one photon during a dead time, which limits the maximum number of photon counts. In OWC and VLC, the dead time effect may cause a nonlinear distortion which has a significant effect on the BER performance of SPAD-based system.

An energy-saving VLC application is presented for a gas well PDM system in a long pipe. Unlike conventional VLC systems which employ normal PDs, the proposed system is based on a SPAD receiver which has higher receiver SNR and can be used in a long-distance and low-power system. In this chapter, a SPAD array is considered in practical applications in order to increase the total photon counts. By using the SPADs array, the LED transmitter needs only 8 dBm power to send the monitoring signal in a 4,000 metres long gas pipe. As the LED transmitter at the well must be battery powered, the high power efficiency ensures that the system achieves a sufficiently long service time.
A novel optical modulation technique referred to as non-DC-biased orthogonal frequency division multiplexing (NDC-OFDM) is presented in this chapter. The concept of NDC-OFDM is to transmit signs of modulated optical orthogonal frequency division multiplexing (O-OFDM) symbols and absolute values of the symbols separately by two information carrying units: i) indices of two light emitting diode (LED) transmitters that represent positive and negative signs separately; and ii) optical intensity symbols that carry the absolute values of signals. NDC-OFDM employs the optical spatial modulation (OSM) technique to eliminate the effect of the clipping distortion in DC-biased optical orthogonal frequency division multiplexing (DCO-OFDM) which could lead to significant energy savings. Furthermore, it can achieve higher spectral efficiency than the conventional unipolar modulation scheme, asymmetrically clipped optical orthogonal frequency division multiplexing (ACO-OFDM), without using additional subcarriers. The improvement comes at the expense of additional hardware at the transmitter and receiver. However, visible light communication (VLC) systems typically are equipped with multiple low-cost LEDs to fulfill minimum indoor lighting conditions. In this chapter, a performance comparison of NDC-OFDM and other state-of-the-art techniques for optical spatial modulation orthogonal frequency division multiplexing (OSM-OFDM) is provided. In addition, a theoretical analysis of NDC-OFDM is also provided. All results and conclusions have been supported by a detailed theoretical analysis and Monte Carlo simulations.
4.1 Introduction

Current optical wireless communication (OWC) systems are mainly realised by high speed LEDs or laser diodes (LDs) as transmitters and highly sensitive photo-diodes (PDs) as receivers. To date, an OWC system with a single visible light LED can achieve speeds exceeding 3 Gb/s [59]. In addition, different types of PDs have been applied in OWC, including positive-intrinsic-negative (PIN) diodes, avalanche photo-diodes (APDs) and single-photon avalanche diodes (SPADs) [68, 118]. However, the incoherent light output of the LED means that information can only be encoded in the intensity level. Thus, only real-valued and positive signals can be used for data modulation. This is in contrast to radio frequency (RF) systems which make use of complex valued and bi-polar signals. As a consequence, OWC systems are usually considered to be realised as an intensity modulation and direct detection (IM/DD) system [32]. On-off keying (OOK), pulse position modulation (PPM) and pulse amplitude modulation (PAM) are some of the common modulation schemes used in conjunction with IM/DD systems [32, 34, 35, 37]. Recently, orthogonal frequency division multiplexing (OFDM) has been demonstrated in OWC as a high-speed data transmission approach in the context of IM/DD systems [26, 38, 39, 68, 119].

For a high-speed OWC system, O-OFDM is applied in order to get closer to the channel capacity by utilising adaptive bit and power loading. The advantages of OFDM in OWC are the same as in RF which are described in [120]. Because the IM/DD system can only transmit real-valued signals, O-OFDM needs to produce real-valued symbols. This can be achieved by imposing Hermitian symmetry on the information frame before the inverse fast Fourier transform (IFFT) operation during the signal generation phase. However, this decreases the spectral efficiency by half. Diverse O-OFDM modulation schemes have been realised and utilised in OWC, such as DCO-OFDM, ACO-OFDM and unipolar orthogonal frequency division multiplexing (U-OFDM) [39, 40, 43]. In DCO-OFDM, a DC-bias is added to the original OFDM signal and the negative part is clipped. Clipping in DCO-OFDM may cause nonlinear distortion which has a significant effect on the bit-error ratio (BER) performance [60]. In ACO-OFDM, the system inserts zeros on even subcarriers and modulates only odd subcarriers. As a result, a group of antisymmetric real-valued OFDM symbols are obtained, as shown in [60]. This allows any negative samples to be clipped without distortion. Since only half of the subcarriers carry information bits, the spectral efficiency of ACO-OFDM is about half the spectral efficiency of DCO-OFDM. In U-OFDM, the positive part of OFDM symbols and the negative part of
the symbols will be transmitted separately [43]. The positive block contains only the positive OFDM symbols and zeros in place of the negative ones, while the negative block contains only the negative OFDM symbols and zeros in place of the positive ones. At the transmitter, the positive block is transmitted first and the absolute value of the negative block is then transmitted. Since the number of the OFDM frames is doubled, U-OFDM also has half the spectral efficiency of DCO-OFDM.

In current fourth generation (4G) communication systems, OFDM multiple-input multiple-output (MIMO) is used as an efficient and effective method to satisfy the demand for high data rate transmission without inter-symbol interference (ISI) [13, 44, 45]. Examples of MIMO techniques are vertical Bell Labs layered space-time (V-BLAST), Alamouti and spatial modulation (SM) [121–123]. Compared to V-BLAST and Alamouti, SM has a significant improvement on the BER performance while achieving the same spectral efficiency. In addition, about 90% reduction in receiver complexity can be achieved [122]. In OWC, the OSM technique using IM/DD has been presented in [47, 124]. In OSM, within a room, multiple transmitters are spatially separated. Only one LED is activated at any time instance and visible light is emitted with a fixed frequency and a certain optical power. Each transmitter index carries \( \log_2(N_t) \) bits when the number of transmitters is \( N_t \). In a conventional OSM-OFDM system, the indices of the transmitters carry a part of information bits, and modulated signals, which carry the other part of the information bits, are transmitted by the active LED. However, the conventional OSM-OFDM system mechanically uses DCO-OFDM or ACO-OFDM in the OSM system. DCO-OFDM in OSM still requires a DC-bias which significantly decreases the power efficiency. ACO-OFDM in OSM still loses half of spectral efficiency. The proposed scheme, NDC-OFDM, is designed for the OSM system. It inherits characteristics from SM (low complexity) and OFDM (ISI resistance). More importantly, it solves the DC-bias problem in DCO-OFDM and has a higher spectral efficiency than ACO-OFDM. In this chapter, the detail and concept of NDC-OFDM and other conventional OSM-OFDM systems are proposed.

The rest of this chapter is organised as follows. Section 4.2 presents the system model of conventional OSM-OFDM systems and NDC-OFDM: Section 4.2.1 presents the modulation and demodulation algorithm of NDC-OFDM; Section 4.2.2 introduces the concept of conventional OSM-OFDM systems. Section 4.3 presents the theoretical analysis of NDC-OFDM: Section 4.3.1 presents the theoretical estimated BER algorithm; Section 4.3.2 gives a comparison of NDC-OFDM and conventional OSM-OFDM in spectral efficiencies. Section 4.4 shows and
discusses numerical and simulation results of the performance analysis: Section 4.4.1 confirms the validity of the theoretical BER estimations through extensive Monte Carlo simulations; Section 4.4.2 gives a comparison between NDC-OFDM and conventional OSM-OFDM in terms of their BER performances. Finally, Section 4.5 provides concluding remarks to the chapter.

4.2 System Model

4.2.1 NDC-OFDM

The NDC-OFDM system model is illustrated in Figure 4.1. Using the indices of LEDs to transmit signs of symbols ensures that transmitted symbols are positive and also saves transmission energy in order to increase the spectral efficiency under a fixed power condition.

At the transmitter, the input bit stream is transformed into complex symbols, \( X(n) \), \( n = 0, \ldots, N/2 - 2 \), by an \( M \)-quadrature amplitude modulation (\( M \)-QAM) modulator. \( N \) is the number of OFDM subcarriers. \( N/2 - 1 \) QAM symbols are then modulated on to the first half of an OFDM frame, \( X(m) \), \( m = 0, \ldots, N - 1 \), and the DC subcarrier (the first subcarrier) is set to zero. Then, Hermitian symmetry is imposed on the second half of the OFDM frame. Next, the mapped subcarriers are passed through an IFFT block. Without loss of generality, the following definition of inverse discrete Fourier transform is used as shown in (2.11). After the \( N \)-IFFT operation, the complex quadrature amplitude modulation (QAM) symbols become \( N \) real-valued OFDM symbols, \( x(k) \), but they are still bipolar. In NDC-OFDM, LEDs only send the absolute value of \( x(k) \) and the sign of the symbol is represented by the index of the corre-
sponding LED. According to the working principle of OSM, only one LED is activated during one symbol time. If the transmitted symbol is positive, the first LED will be activated to send the symbol. If the symbol is negative, its absolute value will be sent by the other LED. Since the absolute values of the negative samples is transmitted, this system does not need additional DC-bias power to obtain positive signals. In general, in an OFDM-based system a cyclic prefix (CP) is added to resist ISI before the samples are transmitted, but for simplicity, it is not considered in the theoretical performance analysis in this study. Finally, the digital signals in SM frame vectors, $L_1(k)$ and $L_2(k)$, are transmitted by the LEDs.

As shown in Figure 4.1, the converted optical signals are transmitted by the corresponding LED over the optical MIMO channel $H$ [47]. Without loss of generality, a simple $N_t \times N_r$ optical channel matrix is realised:

$$
H = \begin{pmatrix}
    h_{11} & h_{12} & \cdots & h_{1N_t} \\
    h_{21} & h_{22} & \cdots & h_{2N_t} \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{N_r,1} & h_{N_r,2} & \cdots & h_{N_r,N_t}
\end{pmatrix},
$$

(4.1)

where $h_{N_r,N_t}$ is the channel direct current (DC) gain of a directed line-of-sight (LOS) link between the receiver $N_r$ and the transmitter $N_t$. The LOS link is considered in the system model because typically the multipath components are significantly weaker and can thus be neglected. The channel gain is calculated by (2.6).

Through the optical MIMO channel, correlated optical signals are detected and obtained by PD receivers. The received signal can be written as:

$$
y = Hs + w,
$$

(4.2)

where $y$ is the $N_r$-dimensional received signal vector and $s$ is the $N_t$-dimensional transmitted signal vector in one symbol duration. In this study, both $N_r$ and $N_t$ are set to two. In addition, $w$ is the $N_r$-dimensional noise vector which is assumed to be real-valued additive white Gaussian noise (AWGN). After the received optical OFDM signal is converted to an electrical signal by the PD and digitised, a zero forcing (ZF) equaliser, which can reduce the system complexity, is used to recover the transmitted symbols as follows [125]:

$$
g = Cy,
$$

(4.3)
where \( g \) is a \( N_t \)-dimensional vector which contains the estimated transmitted symbols and \( C \) denotes the inverse of the channel matrix \( H \). In this study, it is assumed that the channel gain is known at the receiver. Note that the performance of NDC-OFDM is compared with the conventional O-OFDM approaches and different equalisation methods will not affect the comparison results. Even though the minimum mean square error (MMSE) equaliser can also be used in the NDC-OFDM system with the known channel information and the noise coefficient, the ZF equaliser is chosen as a simple and convenient equalisation method [126]. Each element in \( g \) represents the detected OFDM signal which has been transmitted by the corresponding LED with AWGN added at the receiver.

Before demodulating the received signal, there are two methods by which the original bipolar OFDM signal can be reconstructed. Both methods are based on the principle of the modulation scheme in NDC-OFDM. The detected signal received by the first PD is set to be transmitted by the first LED which sent the positive OFDM sample. The second PD achieves the absolute value of the negative sample. In NDC-OFDM, since only one LED is activated in one symbol duration, only one element in \( g \) carries the bit information and the other one is treated as an additional noise component. According to the rules above, the first method subtracts the negative signal block from the positive one. However, when reconstructed in this way, the proposed method performs 3 dB worse than bipolar OFDM with the same constellation size. This is because the subtraction of the negative block from the positive one doubles the AWGN variance for each restored bipolar OFDM signal. The second reconstruction method is to estimate the index of the active transmitter. The estimated index represents the sign of the transmitted information OFDM sample. The information-carrying signal, afterwards, can be selected to reconstruct the bipolar OFDM signal. In particular, to estimate the indices of the active transmitters, the SM detector compares the values of the elements in \( g \) as follows:

\[
\tilde{l}(k) = \arg \max_i (G(i, k)), \quad i = 1, \ldots, N_t,
\]

(4.4)

where \( G \) is an \( N_t \times N \) equalised matrix which contains all the estimated transmitted symbols and \( \tilde{l} \) is an \( N \)-dimensional vector which contains all the estimated indices. As noted, there are two pairs of transmitters and receivers. If \( \tilde{l}(k) \) is equal to one, this means that the symbol received at the time instant \( k \) is transmitted from the first LED. Therefore this symbol is a positive-valued OFDM symbol. If \( \tilde{l}(k) \) is two, a negative symbol is transmitted by the second
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Figure 4.2: Block diagram of the conventional OSM-OFDM system.

LED. As a consequence, the estimated OFDM symbols sequence is:

$$x'(k) = \begin{cases} 
G(\tilde{I}(k), k), & \tilde{I}(k) = 1, \\
-G(\tilde{I}(k), k), & \tilde{I}(k) = 2. 
\end{cases}$$

(4.5)

In an ideal scenario, if there is no AWGN, $x'(k)$ should be the same as $x(k)$. When compared with the first method, the second method will not double the AWGN variance for each estimated OFDM symbol. Thus, it gives a significant improvement on the power efficiency which has been proved in U-OFDM [43]. In this study, the sign-selected estimation (second method) is chosen as the performance analysis in the first method is trivial.

After recovering the OFDM symbols, $x'(k)$ is passed through the fast Fourier transform (FFT) block to obtain received QAM symbols as shown in (2.12). The $N/2 - 1$ data-carrying symbols are extracted and then transformed to the output bit stream by the conventional QAM demodulator.

4.2.2 Conventional OSM-OFDM

Figure 4.2 shows the system model of the conventional OSM-OFDM system. This system combines the basic OSM [127] and traditional O-OFDM techniques [42].

In the first step of the modulation procedure, the input bit stream is reshaped and placed in a $N \times R$ matrix, $Q(p)$, where $R = \log_2(MN_t)$. In this Chapter, it is assumed that the number of the transmitter is two. Bits in the first column of $Q(p)$ represent the index of transmitters. This means that when the bit in the first column is zero, the rest of the bits on the same row will be
transmitted by the first LED and when it equals one, the rest of the bits will be conveyed by the second LED. Bits in the other columns of each row will be transformed to complex $M$-QAM symbols. For example, in Figure 4.2, it can be seen that the first row of $Q(p)$ is $[1 \vert 0 \ 1]$. This means that $[0 \ 1]$ will be converted to a QAM symbol $-1 + i$ by Gray mapping and this symbol will be put in the first slot of $X_2(n)$, as illustrated in Figure 1. Simultaneously, the first slot of $X_1(n)$ will be set to zero. As a result of the $M$-QAM and SM mapping, two complex vectors, $X_1(n)$ and $X_2(n)$, are obtained. Each vector passes through an O-OFDM modulator separately. In general, two standard techniques, ACO-OFDM and DCO-OFDM, are used to obtain positive and real-valued OFDM symbols, which are introduced and compared in [42] and [60]. In ACO-OFDM, $N/4$ QAM symbols are mapped onto half of the odd subcarriers of an OFDM frame. At the same time, the even subcarriers are set to zero. In DCO-OFDM, $N/2 - 1$ symbols are put into the first half of subcarriers and the DC subcarrier (the first subcarrier) is set to zero. Afterwards, for both ACO-OFDM and DCO-OFDM, Hermitian symmetry is applied on the rest of the OFDM frame. Thus, the two groups of QAM symbols from $X_1(n)$ and $X_2(n)$ are mapped onto OFDM frames and they are transformed into real-valued OFDM symbols by the IFFT block. Finally, in order to get positive symbols, the negative values need to be set to zero in ACO-OFDM. In DCO-OFDM, a DC bias is added and the signal is then clipped to obtain the unipolar sample. In practice, the value of the DC bias, which is related to the average power of the OFDM symbols, is defined in [65] and shown in (2.13). For the simple DCO-OFDM model, positive samples, which can be transmitted by LEDs, are obtained by signal clipping after a fixed power for the DC bias is added. However, the added DC bias increases the power consumption. More importantly, if the level of DC-bias is not enough to ensure all the samples are positive, the signal clipping will cause the bottom distortion problem [128].

The resulting output vectors at the O-OFDM modulator, $x_1(k)$ and $x_2(k)$, are transmitted by the respective LED over the $N_t \times N_r$ optical MIMO channel. In this study, the main objective is to compare the performance of the conventional OSM-OFDM systems with NDC-OFDM. Therefore, the same optical channel is used for all three schemes.

At the receiver, PDs convert optical signals to electrical signals. AWGN is added to the signal due to ambient light and thermal noise in the transimpedance amplifier. Through the analog-to-digital conversion block, signals from each PD can be transferred to their corresponding vectors, $y_1(k)$ and $y_2(k)$. Each vector will be dealt with by the respective O-OFDM demodulator.

As in conventional O-OFDM techniques, the received OFDM symbols are passed through a
FFT operation which converts symbols to the frequency domain. In DCO-OFDM, $N/2 - 1$ symbols are obtained from the corresponding subcarriers and in ACO-OFDM, $N/4$ symbols are obtained. The extracted symbols are transferred to two complex vectors, $Y_1(n)$ and $Y_2(n)$. ZF is used to reverse the impairments of the MIMO channel to transform $Y_1(n)$ and $Y_2(n)$ into $X'_1(n)$ and $X'_2(n)$ respectively [127]. Afterwards, the SM detector compares the absolute values of the corresponding subcarriers from each channel to estimate the indices of the active transmitters as follows,

$$\hat{j}(n) = \arg \max_i(|X'_i(n)|), i = 1, \cdots, N_t,$$  \hspace{1cm} (4.6)

As a result, the index of the estimated subchannel gives the bit information transmitted by the SM technique [127]. The bits from the estimated indices are put into the first column of the output matrix, $Q'(p)$. This means that if $\hat{j}(n)$ is equal to one, the corresponding bit is zero and if the result of the estimation is two, the bit is one. At the same time, the largest symbol in each comparison is chosen as the detected symbol,

$$X'_d(n) = \begin{cases} X'_1(n), & \hat{j}(n) = 1, \\ X'_2(n), & \hat{j}(n) = 2. \end{cases}$$ \hspace{1cm} (4.7)

The detected QAM symbols are then decoded by the QAM estimator. The result is allocated to the other columns of $Q'(p)$. Finally, the output bit stream is obtained by reshaping $Q'(p)$ into a serial bit stream.

### 4.3 Theoretical Analysis of NDC-OFDM

#### 4.3.1 Theoretical BER of NDC-OFDM

This section presents a theoretical BER analysis of NDC-OFDM in a $2 \times 2$ MIMO AWGN channel. To calculate the theoretical BER of NDC-OFDM, the following mathematical notations and formulas are defined. In this chapter, $\sigma_n$ is the standard deviation of the AWGN, i.e., $\sigma_n = \sqrt{N_o/2}$, where $N_o$ is power spectral density (PSD) of the double-sided AWGN. The constant, $\sigma_s$, is the standard deviation of the real Gaussian-distributed OFDM symbols which have been modulated and are ready to be transmitted by the LEDs. For the analytical calculation, $\sigma_s$
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is defined as follows:

\[ \sigma_s = \sqrt{E_b \log_2(M) \frac{N-2}{2NN_t}}, \]  

(4.8)

where \( E_b \) is the electrical energy per bit. Note that \( E_b/\text{N}_0 \) is the metric of the BER performance. The standard normal distribution probability density function is:

\[ \phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right). \]  

(4.9)

The step function is:

\[ I(x) = \begin{cases} 
1, & \text{if } x > 0, \\
0, & \text{if } x \leq 0.
\end{cases} \]  

(4.10)

The sign function is:

\[ \text{sgn}(x) = \begin{cases} 
-1, & \text{if } x < 0, \\
0, & \text{if } x = 0, \\
1, & \text{if } x > 0.
\end{cases} \]  

(4.11)

The 2 \times 2 MIMO channel is expressed as:

\[ \mathbf{H} = \begin{pmatrix} h_{11} & h_{12} \\
h_{21} & h_{22} \end{pmatrix}. \]  

(4.12)

Since the attenuation gain of the channel has a limited effect on the results of the analytical BER performance, for simplicity, the coefficients in \( \mathbf{H} \) are normalised to one and they simply represent the correlation coefficients of the channel. The inverse channel matrix is represented by:

\[ \mathbf{C} = \mathbf{H}^{-1} = \begin{pmatrix} c_{11} & c_{12} \\
c_{21} & c_{22} \end{pmatrix}. \]  

(4.13)

Based on the theoretical BER analysis method of the nonlinear transmission in [43], the analysis in this study mainly aims to calculate the probability of the correct and incorrect estimation in which the effects of the ZF equalisation and the nonlinear OFDM demodulation should be taken into consideration. In NDC-OFDM, two receivers obtain optical OFDM samples over the MIMO channel at the same time. After the ZF equalisation, the unipolar OFDM symbols are recovered with the enhanced AWGN. Symbols detected by the first PD come from the first LED, which are originally positive symbols. Symbols detected by the second PD are transmitted by the second LED, which are the absolute values of the negative symbols. If there is no noise in the system, the received symbols should be the same as the transmitted...
symbols. In the theoretical analysis model, the AWGNs are considered as two independent Gaussian random variables, \( n_1 \) and \( n_2 \), which follow the standard normal distribution with the standard deviation, \( \sigma_n \). Since the ZF equaliser is used in the system, the noise is enhanced after removing the channel crosstalk. Most importantly, the AWGN in one receiver enhances the variance of the noise in the other receiver. Considering this condition, the probability for a correctly detected symbol is:

\[
Pr_c(s, n_1, n_2) = \begin{cases} 
\frac{1}{\sigma_n} \phi \left( \frac{n_1}{\sigma_n} \right) \phi \left( \frac{n_2}{\sigma_n} \right) I(\mid s \mid + (c_{11} - c_{21})n_1 + (c_{12} - c_{22})n_2), & s \geq 0, \\
\frac{1}{\sigma_n} \phi \left( \frac{n_1}{\sigma_n} \right) \phi \left( \frac{n_2}{\sigma_n} \right) I(\mid s \mid + (c_{21} - c_{11})n_1 + (c_{22} - c_{12})n_2), & s < 0.
\end{cases}
\]

(4.14)

This depends on a random value of \( n_1 \), a random value of \( n_2 \), the inverse matrix of the channel and the original bipolar symbol, \( s \). Note that the bipolar OFDM symbols also follow an independent Gaussian distribution. Likewise, the probability for an incorrectly detected symbol is:

\[
Pr_w(s, n_1, n_2) = \begin{cases} 
\frac{1}{\sigma_n} \phi \left( \frac{n_1}{\sigma_n} \right) \phi \left( \frac{n_2}{\sigma_n} \right) I(-\mid s \mid - (c_{11} - c_{21})n_1 - (c_{12} - c_{22})n_2), & s \geq 0, \\
\frac{1}{\sigma_n} \phi \left( \frac{n_1}{\sigma_n} \right) \phi \left( \frac{n_2}{\sigma_n} \right) I(-\mid s \mid - (c_{21} - c_{11})n_1 - (c_{22} - c_{12})n_2), & s < 0.
\end{cases}
\]

(4.15)

With the identical \( n_1 \), \( n_2 \) and \( s \), the correctly detected OFDM sample is expressed as follows:

\[
x_c = \begin{cases} 
\mid s \mid + c_{11}n_1 + c_{12}n_2, & s \geq 0, \\
\mid s \mid + c_{21}n_1 + c_{22}n_2, & s < 0.
\end{cases}
\]

(4.16)

Thus, according to (4.14) and (4.16), for all possible values of \( n_1 \) and \( n_2 \) and the identical OFDM sample, the mean value of \( x_c \) is:

\[
f_c(s) = \frac{\text{sgn}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_c Pr_c(s, n_1, n_2) \, dn_1 \, dn_2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Pr_c(s, n_1, n_2) \, dn_1 \, dn_2}.
\]

(4.17)

Additionally, the variance of the correctly detected sample, which means the effect of AWGN on the OFDM symbol, is calculated by:

\[
v_c(s) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_c^2 Pr_c(s, n_1, n_2) \, dn_1 \, dn_2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Pr_c(s, n_1, n_2) \, dn_1 \, dn_2} - f_c^2(s).
\]

(4.18)

Based on the central limit theorem (CLT), after the FFT is exposed in the OFDM demodulation process, the variance, \( v_c(s) \) will be part of the variance of the AWGN in the frequency domain.
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Similarly, for the estimation of NDC-OFDM, the incorrect determination also enhances the variance of the AWGN. For the incorrect estimation, the selected OFDM sample is calculated as:

\[
x_w = \begin{cases} 
c_{21}n_1 + c_{22}n_2, & s \geq 0, 
c_{11}n_1 + c_{12}n_2, & s < 0.
\end{cases}
\]  

(4.19)

The mean and the variance of \( x_c \) are:

\[
f_w(s) = \frac{-\text{sgn}(s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_w \Pr_w(s, n_1, n_2) \, dn_1 \, dn_2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Pr_w(s, n_1, n_2) \, dn_1 \, dn_2},
\]

(4.20)

\[
v_w(s) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_w^2 \Pr_w(s, n_1, n_2) \, dn_1 \, dn_2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Pr_w(s, n_1, n_2) \, dn_1 \, dn_2} - f_w^2(s).
\]

(4.21)

Since the OFDM samples, \( s \), follow a Gaussian distribution, for all possibility of \( s \), the average variances of the correct and incorrect estimations are:

\[
\bar{v}_c = \int_{-\infty}^{\infty} v_c(s) \frac{1}{\sigma_s} \phi \left( \frac{s}{\sigma_s} \right) \, ds,
\]

(4.22)

and

\[
\bar{v}_w = \int_{-\infty}^{\infty} v_w(s) \frac{1}{\sigma_s} \phi \left( \frac{s}{\sigma_s} \right) \, ds.
\]

(4.23)

These variances represent the effect of AWGN on the OFDM frame and will constitute the variance of the AWGN in frequency domain by the FFT operation at the demodulator.

After the sign-selected estimation, the selected symbols are demodulated to QAM symbols by the FFT operation. For this estimation method, the demodulation procedure is treated as a nonlinear transformation. According to the Bussgang theorem [115], if an independent Gaussian random variable, \( X \), passes through a nonlinear transformation, \( z(X) \), it has properties as shown in (2.19) and (2.20). Using these properties, the nonlinear distortion in an OFDM-based system can be equivalent to a gain factor, \( \alpha \), and an additional noise, \( Y \) [43]. In NDC-OFDM, \( X \) is equal to the value of the transmitted symbol, \( s \), and \( Y \) is a noise component which is a Gaussian random variable non-correlated with \( X \). After the FFT operation, the variance of \( Y \) will be composed of the variance of the AWGN in the frequency domain and \( \alpha \) will enhance the mean value of the information-carrying symbol in each modulated subcarrier. In this case, \( \alpha \) can be derived as shown in (2.23), where \( \sigma_X \) is the standard deviation of \( X \), which is equal to \( \sigma_s \) in this study. Since the additional noise, \( Y \), approximately follows a Gaussian distribution
with a zero mean, according to (2.24), (2.25) and (2.26), the variance of \( Y \) can be calculated as:

\[
\sigma_Y^2 = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2 = \mathbb{E}[Y^2] = \mathbb{E}[(z(X) - \alpha X)^2] = \mathbb{E}[z^2(X)] - \alpha^2 \sigma_X^2.
\]  
(4.24)

From (2.23), (4.17) and (4.24), the values of the nonlinear gain factor and the variance of the noise component for the correct estimation are calculated as:

\[
\alpha_c = \frac{\int_{-\infty}^{\infty} s f_c(s) \frac{1}{\sigma_s} \phi \left( \frac{s}{\sigma_s} \right) ds}{\sigma_s^2},
\]  
(4.25)

\[
y_c = \int_{-\infty}^{\infty} f_c^2(s) \frac{1}{\sigma_s} \phi \left( \frac{s}{\sigma_s} \right) ds - \alpha_c^2 \sigma_s^2.
\]  
(4.26)

For the incorrect estimation, the constant, \( \alpha_w \), and the variance, \( y_w \), are calculated as:

\[
\alpha_w = \frac{\int_{-\infty}^{\infty} s f_w(s) \frac{1}{\sigma_s} \phi \left( \frac{s}{\sigma_s} \right) ds}{\sigma_s^2},
\]  
(4.27)

\[
y_w = \int_{-\infty}^{\infty} f_w^2(s) \frac{1}{\sigma_s} \phi \left( \frac{s}{\sigma_s} \right) ds - \alpha_w^2 \sigma_s^2.
\]  
(4.28)

Note that the correct and incorrect estimation functions, \( f_c \) and \( f_w \), are the nonlinear transformation function in this study. From (4.14), the probability of the correct estimation in one active duration is:

\[
d_c = \int_{-\infty}^{\infty} \int_{-\infty}^{n_1} \int_{-\infty}^{\infty} \frac{1}{\sigma_s} \phi \left( \frac{s}{\sigma_s} \right) \operatorname{Pr}_c(s, n_1, n_2) d n_1 d n_2 d s.
\]  
(4.29)

For a large number of samples in a NDC-OFDM frame, the number of correctly and incorrectly estimated samples have a ratio which corresponds to the probabilities for correct and incorrect estimations. According to the Bussgang theorem, the nonlinear transformation will add a gain factor to the sample. The gain factor decreases the average energy of the transmitted bits. The average gain factor is calculated as:

\[
\bar{\alpha} = d_c \alpha_c + (1 - d_c) \alpha_w.
\]  
(4.30)

As noted above, the variance of effect of AWGN, \( \bar{\sigma}_c \) and \( \bar{\sigma}_w \), and the variance of the nonlinear transmission, \( y_c \) and \( y_w \), constitute the average noise variance of the system in the frequency
domain, i.e.:
\[
\bar{N} = d_c (\bar{v}_c + y_c) + (1 - d_c)(\bar{v}_w + y_w).
\] (4.31)

Thus, the average electrical signal-to-noise ratio (SNR) per bit can be achieved from the known value of \( E_{b,\text{elec}} \) and the calculated values of \( \bar{\alpha} \) and \( \bar{N} \) as:

\[
\text{SNR}_{\text{elec}} = \frac{\bar{\alpha}^2 E_{b,\text{elec}}}{\bar{N}}.
\] (4.32)

Using the analytical expression for the BER performance of \( M \)-QAM O-OFDM in [60], the theoretical BER performance of NDC-OFDM can be calculated as:

\[
\text{BER}_{\text{NDC}} = 
\frac{4(\sqrt{M} - 1)}{\sqrt{M \log_2(M)}} Q \left( \sqrt{\frac{3 \log_2(M)}{M - 1} \text{SNR}_{\text{elec}}} \right) + 
\frac{4(\sqrt{M} - 2)}{\sqrt{M \log_2(M)}} Q \left( 3 \sqrt{\frac{3 \log_2(M)}{M - 1} \text{SNR}_{\text{elec}}} \right).
\] (4.33)

### 4.3.2 Spectral Efficiency Analysis

NDC-OFDM is realised using OSM where ACO-OFDM and DCO-OFDM can also be applied. For fair comparisons, NDC-OFDM, ACO-OFDM and DCO-OFDM are used within the same
Table 4.1: Constellation Sizes Comparison for NDC-OFDM, DCO-OFDM and ACO-OFDM in the OSM system

OSM system. In NDC-OFDM, the indices of LEDs are used to carry the sign information. In ACO-OFDM and DCO-OFDM, the indices carry additional information bits according to the conventional principle of OSM. In the conventional OSM system with $M$-QAM, the spectral efficiency is calculated by considering both the signal-carried bits and the indices-carried bits as [47]:

$$R_{\text{OSM}} = \log_2(MN_t) \text{ bits/s/Hz}.$$  \hfill (4.34)

In NDC-OFDM, since the Hermitian symmetry of O-OFDM decreases the spectral efficiency by half and there is no information bit carried by the indices, the spectral efficiency of NDC-OFDM is calculated as:

$$R_{\text{NDC-OFDM}} = \frac{N}{2N} \log_2(M_1N_t - 1) \text{ bits/s/Hz}.$$  \hfill (4.35)

As two different signs of the samples should be represented respectively in NDC-OFDM, the number of the LEDs, $N_t$, should be even. For DCO-OFDM in the OSM system, the spectral efficiency is only halved by the Hermitian symmetry. Thus it is expressed as:

$$R_{\text{DCO-OFDM-OSM}} = \frac{N}{2N} \log_2(M_2N_t) \text{ bits/s/Hz}.$$  \hfill (4.36)

In ACO-OFDM, as only half of the subcarriers are modulated, the spectral efficiency has an additional 50% reduction. In the OSM system, the actual spectral efficiency of ACO-OFDM is:

$$R_{\text{ACO-OFDM-OSM}} = \frac{1}{4} \log_2(M_3N_t) \text{ bits/s/Hz}.$$  \hfill (4.37)
In (4.35), (4.36) and (4.37), $M_1$, $M_2$ and $M_3$ denote the constellation size of QAM in the three schemes respectively. In this study, the size of the OFDM frame, $N$, is set to 2048. Thus the coefficient, $\frac{N-2}{2N}$, in (4.35) and (4.36) can be approximated to $1/2$. For a fair comparison, when NDC-OFDM, DCO-OFDM and ACO-OFDM in the OSM system have the same spectral efficiencies, i.e., $R_{\text{NDC-OFDM}} = R_{\text{DCO-OFDM-OSM}} = R_{\text{ACO-OFDM-OSM}}$, the constellation sizes of these three methods have the following relationship:

$$M_1 = 2M_2 = \sqrt{2M_3}. \quad (4.38)$$

Figure 4.3 shows the constellation sizes required to achieve the same spectral efficiencies between 0.5 bits/s/Hz and 2 bits/s/Hz for NDC-OFDM, DCO-OFDM and ACO-OFDM in the OSM system. Compared with DCO-OFDM, NDC-OFDM needs a little higher constellation size to achieve the same spectral efficiency. For ACO-OFDM, with the increase of the spectral efficiency, the required constellation size will increase exponentially and this means greater system complexity. For higher spectral efficiencies, such as between 3.5 bits/s/Hz and 5.5 bits/s/Hz, the required constellation size of ACO-OFDM becomes very large, as shown in Table 4.1. This is because half of subcarriers in ACO-OFDM are set to zero for decreasing the effect of DC biasing. Thus, NDC-OFDM and DCO-OFDM are more suitable for high speed optical transmission and have much lower complexity than ACO-OFDM.

### 4.4 Results and Discussion

#### 4.4.1 Analytical Results of NDC-OFDM

As noted in Chapter 3.2, ideal $2 \times 2$ MIMO channels are considered to test the correctness of the analytical BER performance. Symmetrical ideal channels are assumed as follows:

$$H_{s1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \kappa_{s1} = 1, \quad (4.39)$$

$$H_{s2} = \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}, \quad \kappa_{s2} = 1.86, \quad (4.40)$$
Figure 4.4: Comparison between analytical and simulation results for symmetrical ideal channels: $H_{s1}$, $H_{s2}$, $H_{s3}$, $H_{s4}$, where condition numbers are: $\kappa_{s1} = 1$, $\kappa_{s2} = 1.86$, $\kappa_{s3} = 3$, $\kappa_{s4} = 5.67$.

$$H_{s3} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}, \quad \kappa_{s3} = 3,$$

$$H_{s4} = \begin{pmatrix} 1 & 0.7 \\ 0.7 & 1 \end{pmatrix}, \quad \kappa_{s4} = 5.67,$$

where $\kappa_{s1}$, $\kappa_{s2}$, $\kappa_{s3}$ and $\kappa_{s4}$ denote condition numbers of corresponding symmetrical channels. A higher value of condition numbers indicates a higher level of channel correlation. Note that in $H_{s1}$, transmitted optical signals are assumed to be received only by the corresponding receiver.

Without loss of generality, asymmetrical ideal channels are also tested in this study:

$$H_{a1} = \begin{pmatrix} 1 & 0 \\ 0 & 0.7 \end{pmatrix}, \quad \kappa_{a1} = 1.43,$$

$$H_{a2} = \begin{pmatrix} 1 & 0 \\ 0.3 & 0.7 \end{pmatrix}, \quad \kappa_{a2} = 1.65,$$

$$H_{a3} = \begin{pmatrix} 1 & 0.5 \\ 0 & 0.7 \end{pmatrix}, \quad \kappa_{a3} = 1.98,$$

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Figure 4.5: Comparison between analytical and simulation results for asymmetrical ideal channels: $H_{a1}$, $H_{a2}$, $H_{a3}$, $H_{a4}$, where condition numbers are: $\kappa_{a1} = 1.43$, $\kappa_{a2} = 1.65$, $\kappa_{a3} = 1.98$, $\kappa_{a4} = 2.99$.

where $\kappa_{a1}$, $\kappa_{a2}$, $\kappa_{a3}$ and $\kappa_{a4}$ denote condition numbers of corresponding asymmetrical channels. These ideal optical channels are substituted to (4.12) and then the corresponding theoretical BER results of NDC-OFDM can be calculated. In the simulation, without loss of generality, 16-QAM is chosen for each case and for simplicity, the variance of AWGN, $\sigma_n^2$, is set to $\sqrt{0.01} A/\sqrt{Hz}$. Figure 4.4 shows the comparison between analytical and simulation results for the symmetrical ideal channels. In Figure 4.5, the performance of the theoretical model for the asymmetrical ideal channels is compared with Monte Carlo simulations. It shows that the analytical results are well matched with the simulation results. This confirms the validity of the theoretical work. Note that a little discrepancy between the simulation and theory curves occurs as shown in Figure 4.4 when the condition number is large. It is because the channel with high correlation has a significant effect on the simulation results.
4.4.2 NDC-OFDM, ACO-OFDM and DCO-OFDM Performance Comparison

The Monte Carlo simulation results for NDC-OFDM, ACO-OFDM and DCO-OFDM are compared in this chapter. The BER performance of NDC-OFDM is compared with ACO-OFDM and DCO-OFDM over different simulated optical MIMO channels which are chosen from [125]. In [125], a generic $4 \times 4$ indoor scenario is considered with intensity modulated optical wireless links with LOS characteristics. In this study, $2 \times 2$ optical MIMO channels are assumed to test properties of O-OFDM systems. Thus, $2 \times 2$ optical MIMO links are extracted from original $4 \times 4$ optical channels, taking into account both symmetrical and asymmetrical cases:

$$
H_{p1} = 10^{-5} \times \begin{pmatrix} 0.1889 & 0.0713 \\ 0.0713 & 0.1889 \end{pmatrix}, \ \kappa_{p1} = 2.21,
$$

(4.47)

$$
H_{p2} = 10^{-5} \times \begin{pmatrix} 0.3847 & 0.1889 \\ 0.1889 & 0.3847 \end{pmatrix}, \ \kappa_{p2} = 2.93,
$$

(4.48)

$$
H_{p3} = 10^{-5} \times \begin{pmatrix} 0.1889 & 0.0713 \\ 0.1157 & 0.1889 \end{pmatrix}, \ \kappa_{p3} = 2.93,
$$

(4.49)
Non-DC-Biased Orthogonal Frequency Division Multiplexing

Figure 4.7: NDC-OFDM, ACO-OFDM and DCO-OFDM Performance Comparison over $H_{p2}$, where condition number is: $\kappa_{p2} = 2.93$.

$$H_{p4} = 10^{-5} \begin{pmatrix} 0.3847 & 0.2691 \\ 0.1889 & 0.3847 \end{pmatrix}, \quad \kappa_{p4} = 3.90,$$

where $H_{p1}, H_{p2}, H_{p3}$ and $H_{p4}$ represent simple simulated optical MIMO channels in an indoor scenario; and $\kappa_{p1}, \kappa_{p2}, \kappa_{p3}$ and $\kappa_{p4}$ denote condition numbers of corresponding channels. Without loss of fairness, spectral efficiencies of these three methods should be the same in order to compare power efficiencies. In the comparison, spectral efficiencies are set to 1.5 bits/s/Hz and 2 bits/s/Hz. According to (4.38), 8-QAM and 16-QAM are thus chosen in the simulation of NDC-OFDM; these are double than the constellation size of DCO-OFDM; and for ACO-OFDM, the modulation orders are 32 and 128. As noted in Chapter 3.2, a fixed level of DC-bias needs to be added in DCO-OFDM. The lower level might cause the nonlinear distortion and the higher level would be energy inefficient. In order to study these two cases in a practical situation, 5 dB and 7 dB DC-bias are chosen in the simulation.

Figure 4.6 and Figure 4.7 show the performance of NDC-OFDM, ACO-OFDM and DCO-OFDM with OSM over the symmetrical optical MIMO channels, $H_{p1}$ and $H_{p2}$. In Figure 4.6, it shows that when the spectral efficiency is 1.5 bits/s/Hz, NDC-OFDM has around 3.5 dB power efficiency better than the 5 dB DCO-OFDM, and compared with ACO-OFDM, NDC-OFDM achieves a 5 dB power efficient enhancement. In this case, since there is no obvious nonlinear
distortion in DCO-OFDM, a lower DC bias can be used. However, when the spectral efficiency is 2 bits/s/Hz, clipping noises distort the curves of DCO-OFDM with 5 dB DC-bias. In this case, NDC-OFDM saves 7 dB transmission power compared with DCO-OFDM. Moreover, with the increase in the spectral efficiency, the performance of NDC-OFDM is close to the unipolar line which has been shown in [43]. Figure 4.7 shows the performance of the three methods over \( H_{p2} \) which is another symmetrical channel with a higher correlation. Compared with \( H_{p1} \), all the schemes require 2 dB more transmission power, and also NDC-OFDM is the most power efficient method.

The performance of NDC-OFDM, ACO-OFDM and DCO-OFDM over the asymmetrical channels are shown in Figure 4.8 and Figure 4.9. As shown in Figure 4.8, when the spectral efficiency is 1.5 bits/s/Hz, NDC-OFDM has 5 dB greater power efficiency than 5 dB DCO-OFDM, and compared with ACO-OFDM, NDC-OFDM has a 7 dB improvement on the power efficiency. When the spectral efficiency increases, the DCO-OFDM system with low DC-bias cannot be used since the nonlinear distortion occurs. In this case, NDC-OFDM saves 9 dB transmission power than DCO-OFDM with a higher level of DC bias (7 dB). The fourth optical channel, \( H_{p4} \), has the highest correlation. In this case, NDC-OFDM exhibits an efficiency advantage of about 10 dB over DCO-OFDM (Figure 4.9).
Non-DC-Biased Orthogonal Frequency Division Multiplexing

Figure 4.9: NDC-OFDM, ACO-OFDM and DCO-OFDM Performance Comparison over $H_{p4}$, where condition number is: $\kappa_{p4} = 3.90$.

From these simulation results, it can be seen that NDC-OFDM has a significant improvement on the power efficiency. In DCO-OFDM, the level of the DC bias limits the power efficiency. When the DC bias is low, the system is easily affected by the nonlinear distortion; and the higher DC bias leads to higher power consumption. Compared with DCO-OFDM, NDC-OFDM reduces the additional transmission power since the non-negative OFDM symbols can be generated without adding a DC-bias. Moreover, as the digital signal clipping is not used in NDC-OFDM, the nonlinear distortion is prevented. From (4.38), compared with ACO-OFDM, NDC-OFDM has higher spectral efficiency and with the same spectral efficiency, NDC-OFDM can save more transmission power. This is because ACO-OFDM sacrifices half of transmission bandwidth for generating non-negative OFDM symbols, which exponentially decreases the spectral efficiency. NDC-OFDM can use double transmission bandwidth than ACO-OFDM by using more transmitter, which only reduces the spectral efficiency by half. As a result, NDC-OFDM is a good choice for a OSM-OFDM system with a higher power efficiency.

4.5 Summary

In this chapter, the theoretical performance of a novel unipolar modulation method, referred to as NDC-OFDM, is analysed. The new method combines O-OFDM with SM and has been
applied to an OWC system. Using the Bussgang theorem and CLT, the analytical performance of NDC-OFDM in AWGN channels has been derived. As a result, an equation for the electrical SNR per bit has been presented to calculate the theoretical BER of the NDC-OFDM system. The results of the proposed method show close agreement with Monte Carlo simulations, thus confirming the validity of the analysis and underpinning the advantages of the proposed new modulation technique for IM/DD systems.

In comparisons of the simulation performance, NDC-OFDM exhibits the capability to achieve better BER performances than the conventional OFDM-based modulation schemes applied to OSM: DCO-OFDM and ACO-OFDM. Compared with DCO-OFDM, the new NDC-OFDM method solves the clipping distortion problem caused by the high level of the DC-bias. Compared with ACO-OFDM, NDC-OFDM gives a significant improvement in spectral efficiency. These improvements of NDC-OFDM come at the expense of additional hardware at the transmitter and receiver. However, modern VLC systems are expected to employ multiple low-cost LEDs to fulfill minimum indoor lighting conditions.
Chapter 5

Optical Orthogonal Frequency Division Multiplexing with Single-Photon Avalanche Diode

An optical orthogonal frequency division multiplexing (O-OFDM) system based on a single-photon avalanche diode (SPAD) receiver is presented in this chapter. By considering the dead time effect of passive quenching single-photon avalanche diode (PQ SPAD) and active quenching single-photon avalanche diode (AQ SPAD), a nonlinear distortion on the bit-error ratio (BER) performance of SPAD-based orthogonal frequency division multiplexing (OFDM) is proposed. Furthermore, based on Bussgang theorem and central limit theorem (CLT), a complete theoretical analysis of SPAD-based OFDM with nonlinear distortion is derived in the current work. As a result, the performance of DC-biased optical orthogonal frequency division multiplexing (DCO-OFDM) and asymmetrically clipped optical orthogonal frequency division multiplexing (ACO-OFDM) with PQ SPAD and AQ SPAD are compared and analysed. Finally, all results and conclusions have been supported by a detailed theoretical analysis and Monte Carlo simulations. The comparison results show the maximum optical irradiance caused by the nonlinear distortion, which limits the transmission power and bit rate. Compared with the conventional photo-diode (PD) based system, the power efficiency of the O-OFDM system is significantly enhanced by using SPAD. In this study, the presented algorithm also supplies a closed-form analytical solution to design an optimal SPAD-based system and is used to find the theoretical maximum bit rate.
5.1 Introduction

In current optical wireless communication (OWC) and visible light communication (VLC) systems, high speed light emitting diodes (LEDs) and laser diodes (LDs) are mainly used as transmitters and highly sensitive PDs, such as positive-intrinsic-negative (PIN) diodes and avalanche photo-diodes (APDs), are placed as receivers. However, when the OWC system is applied in low optical power and long distance transmission, such as in a gas well downhole monitoring system [67] and data transmission over plastic optical fibres [139], the number of photons reaching the receivers are significantly less than in standard indoor OWC links. In these scenarios, conventional PDs have unsatisfactory performance because the transimpedance amplifier (TIA) significantly reduces the sensitivity of the receiver and limits the signal-to-noise ratio (SNR). As a consequence, those low power signals are buried in noise. Hence, when compared with conventional PDs, SPADs would be more suitable receivers in those scenarios. The SPAD detector does not require a TIA and thus the output signal is not distorted by thermal noise. In addition, as SPADs can even detect a single photon, a bit of information-carried photons can be received accurately. Therefore, the SPAD receiver can perform at significantly higher sensitivity and optical power efficiency than conventional PDs. In the previous work, a SPAD-based VLC system has been successfully applied in permanent downhole monitoring (PDM) [67]. In this system, since the demand of the transmission speed is much lower than the demand in the conventional OWC system, on-off keying (OOK) is used for modulating and demodulating the optical signal by the SPAD receiver. However, if SPAD receivers is assumed to be set in a conventional OWC system, the spectral efficiency of OOK is not satisfied. In this chapter, a SPAD-based OFDM system will be proposed to achieve better performances.

For high-speed data transmission, OFDM is applied in order to get closer to the channel capacity by utilising adaptive bit and power loading. Currently, to achieve higher data rate and better transmission performance, O-OFDM has been presented and used in the OWC system [146]. However, in OWC, the incoherent light output of the LED means that information can only be encoded in the intensity level. As a consequence, only real-valued and positive signals can be used for data modulation. This is in contrast to radio frequency (RF) systems which make use of complex valued and bi-polar signals. Thus, OWC systems are usually considered to be modulated as an intensity modulation and direct detection (IM/DD) system [32]. In O-OFDM, real-valued signals can be achieved by imposing Hermitian symmetry on the information frame before the inverse fast Fourier transform (IFFT) operation during the signal generation phase.
This comes at the expense of half of the spectral efficiency. Furthermore, in order to achieve positive optical signals, diverse O-OFDM modulation schemes have been realised and utilised in OWC, such as DCO-OFDM [39], ACO-OFDM [42], unipolar orthogonal frequency division multiplexing (U-OFDM) [43] and non-DC-biased orthogonal frequency division multiplexing (NDC-OFDM) [65]. In DCO-OFDM, a DC-bias is added to the original OFDM signal and the negative part is clipped. Thus, DCO-OFDM has higher power consumption and higher peak-to-average power ratio (PAPR) than other schemes [147]. Compared with DCO-OFDM, in order to achieve a higher power efficiency, the ACO-OFDM system inserts zeros on even subcarriers and modulates only odd subcarriers. As a result, a group of antisymmetric real-valued OFDM symbols are obtained, as shown in [60]. Since only half of the subcarriers carry information bits, the spectral efficiency of ACO-OFDM is about half the spectral efficiency of DCO-OFDM. In order to enhance the performance of ACO-OFDM, U-OFDN is presented which transmits the positive part of OFDM symbols and the negative part of the symbols separately [43]. The positive block contains only the positive OFDM symbols and zeros in place of the negative ones, while the negative block contains only the negative OFDM symbols and zeros in place of the positive ones. At the transmitter, the positive block is transmitted first and the absolute value of the negative block is then transmitted. At the receiver, the information symbols are compared with the corresponding subcarrier placing zeros. In this way, the BER performance has a significant improvement and thus U-OFDM has a higher power efficiency. Since the number of the OFDM frames is doubled, U-OFDM also has half the spectral efficiency of DCO-OFDM. Based on U-OFDM, NDC-OFDM is designed for the optical multiple-input multiple-output (MIMO) system [65]. NDC-OFDM employs the optical spatial modulation (OSM) technique to eliminate the effect of the clipping distortion in DCO-OFDM which could lead to significant energy savings. Furthermore, it can achieve higher spectral efficiency than the conventional unipolar modulation scheme, ACO-OFDM and U-OFDM, without using additional subcarriers. In previous works, those O-OFDM systems use conventional PDs as receiver. In this chapter, those PDs are replaced by SPAD to achieve higher power efficiency. In this study, as basic O-OFDM schemes, DCO-OFDM and ACO-OFDM are tested in SPAD-based OFDM systems.

In those conventional O-OFDM systems, the nonlinear distortion problem has a significant effect on the BER performance, especially in DCO-OFDM [61, 62, 119, 148]. The nonlinear distortion problem is mainly caused by the nonlinearity of LED transmitter and the clipping in modulation schemes. In the LED transmitter, the LED transfer function distorts the signal am-
Optical Orthogonal Frequency Division Multiplexing with Single-Photon Avalanche Diode

plitude and forces the lower signal peaks to be clipped at the LED turn-on voltage (TOV) [61]. Additionally, the upper signal peaks can result in optical output degradation. On the other hand, the signal clipping in DCO-OFDM will lose some information-carried signals [119]. Since these nonlinear distortion problems are mainly found at the transmitter side, they are so-called transmitter nonlinear distortion. In the simulation of OFDM schemes, the clipping distortion of DCO-OFDM is mainly taken into consideration. In SPAD-based OFDM, a SPAD receiver can only detect one photon within a device specific dead time which constrains the ability to recover a signal. In addition, since the output of the detector is a photon count value, there is a maximum number of photons that the system can detect. This limits the maximum tolerable optical irradiance which results in a receiver nonlinear distortion. This means that the transmission power and maximum bit rate of SPAD-based OFDM are limited by the structure and design of SPAD receivers. The analytical model of the nonlinear distortion effect in O-OFDM with conventional PDs has been derived in [60, 62, 116]. As the nonlinear effects in conventional O-OFDM systems are mainly caused by transmitter properties and modulation schemes, the SPAD receiver nonlinear distortion effect has not been considered. This chapter supplies a complete analytical procedure to find the accurate BER of the SPAD-based OFDM by considering the receiver nonlinear distortion and the conventional distortions (in DCO-OFDM). The analytical model of SPAD-based OFDM can be used to find the limitation threshold in the system and also the theoretical maximum bit rate. In addition, as the current SPAD array is designed for the image processing [141], the designed parameters may not be suitable for OWC. Based on the analytical model, a reliable approach is provided for designing a suitable SPAD array for current OWC and VLC systems.

The rest of this chapter is organised as follows. Section 5.2 gives the system model of SPAD-based OFDM. Section 5.3 analyses the nonlinear distortion in SPAD-based OFDM: Section 5.3.1 gives the nonlinear function of SPAD devices and the accurate distribution of photon counts; Section 5.3.2 shows the nonlinear distortion in PQ and AQ SPAD OFDM symbols and the BER performance. Section 5.4 gives the complete theoretical analysis of SPAD-based OFDM: Section 5.4.1 provides the analytical model of PQ SPAD OFDM, including ACO-OFDM and DCO-OFDM; Section 5.4.2 provides the analytical model of AQ SPAD OFDM. Section 5.5 shows and discusses theoretical and simulation results of the performance analysis: Section 5.5.1 confirms the validity of the theoretical BER estimations through extensive Monte Carlo simulations; Section 5.5.2 provides an analysis of maximum bit rates of SPAD-based OFDM; Section 5.5.3 compares the performance of SPAD-based OFDM and conventional O-OFDM.
Finally, Section 5.6 summarises this chapter.

5.2 System Model

Figure 5.1 illustrates the system model of OFDM with SPAD receivers. At the transmitter, the input bit stream is transformed into complex symbols, $X(n)$, by a $M$-ary quadrature amplitude modulation ($M$-QAM) modulator, where $M$ is the constellation size. The symbols are allocated on to $N$ subcarriers, $X(k), k = 0, \cdots, N - 1$. In OFDM, $N$ denotes the size of IFFT and fast Fourier transform (FFT), and for generating Gaussian distributed OFDM symbols, $N$ is set to 2048. In general, two standard techniques, DCO-OFDM and ACO-OFDM, are used to obtain positive and real-valued OFDM symbols [60]. In DCO-OFDM, $N/2 - 1$ symbols in $X(n), n = 1, \cdots, N/2 - 1$, are put into the first half of subcarriers and the DC subcarrier (the first subcarrier) is set to zero. In ACO-OFDM, $N/4$ QAM symbols in $X(n), n = 1, \cdots, N/4$, are
mapped on to half of the odd subcarriers of the OFDM frame, \( X(k), k = 1, 3, 5, \ldots, N/2 - 1 \). At the same time, the even subcarriers are set to zero. In both ACO-OFDM and DCO-OFDM, Hermitian symmetry is applied to the rest of the OFDM frame in order to obtain real-valued symbols through the IFFT block. Since transmitters can only send unipolar signals, the real-valued OFDM symbols need to be clipped. In DCO-OFDM, a DC bias is added to make the signal unipolar [60]. The value of the DC bias is related to the average power of the OFDM symbols is shown in (2.13). The bias level in the current simulations is set to 7 dB and 13 dB, which are adopted from [68] for consistency. After the DC bias, the OFDM frame is simply clipped by:

\[
x_{\text{clipped}}(k) = \begin{cases} 
  x_{\text{biased}}(k), & x_{\text{biased}}(k) \geq 0, \\
  0, & x_{\text{biased}}(k) < 0,
\end{cases}
\]

(5.1)

where \( x_{\text{biased}}(k) \) is the DC biased symbol which is calculated as \( x_{\text{biased}}(k) = x(k) + B_{\text{DC}} \). The clipped unipolar symbol is denoted by \( x_{\text{clipped}}(k) \). In ACO-OFDM, since symbols are antisymmetric, clipped unipolar symbols are obtained by setting the negative part to zero. In the simulation, after being transformed into an optical intensity signal, the clipped signal is transmitted by the LED transmitter.

At the SPAD receiver, in order to generate received O-OFDM symbols, the output of the SPAD array is counted over a symbol duration, \( T_s \), at time instances \( t_k = kT_s \) of the received optical signal \( x_r(t) \) which is shown in Figure 5.1. According to mentioned properties of photon counts in SPAD in Chapter 4, the photon counts are denoted by \( \nu(k) \) which is the superposition of the photon counts from each individual SPADs, \( a_m(k) \), as shown in Figure 3.6:

\[
\nu(k) = \sum_{m=1}^{N_{\text{SPAD}}} a_m(k).
\]

(5.2)

Note that \( N_{\text{SPAD}} \) is the number of SPAD devices in the array. As mentioned in Section 3.2.3, the photon counts of the SPAD array receiver, \( \nu(k) \), can be described by a Poisson distribution:

\[
\Pr(\nu(k) = j, \mu(k)) = \exp\left(-\mu(k)\right) \frac{\mu(k)^j}{j!},
\]

(5.3)

where the average photon counts \( \mu(k) \) can be expressed as a function of the received signal and the metrics of the SPAD array (Section 3.2.2):

\[
\mu(k) = \left[ \frac{C_{\text{FF}}C_{\text{PDP}}}{E_P} \int_{t_k}^{t_k+T_s} x_r(t) dt + n_{\text{DCR}} \right] (1 + P_{\text{AP}}).
\]

(5.4)
Note that $C_{\text{FF}}$ is the fill factor (FF) of the SPAD array; $C_{\text{PDP}}$ is the photon detection probability (PDP); $E_p$ is the energy of a photon; $n_{\text{DCR}}$ is the total dark count photons in the SPAD array which is calculated by $n_{\text{DCR}} = N_{\text{DCR}}N_{\text{SPAD}}T_s$; and $P_{\text{AP}}$ is the after pulsing probability (APP).

The output of the SPAD array is the number of photons ($n(k)$), and the system is designed based on a conventional O-OFDM demodulator which requires the amplitude of the electrical signal (optical power) to demodulate the received signal to the original encoded bits. Thus, the photon-to-amplitude equaliser is used to simply convert the received photon number ($n(k)$) to the corresponding electrical signal amplitude (optical power), $x(k)$. The coefficient of the equaliser is calculated by a pilot which can record the effect of the attenuation and the SPADs’ coefficients. Assuming that there is no other distortion effects during the transmission, the recovered signal, $x'(k)$, can be scaled to the original clipped signal, $x_{\text{clipped}}(k)$. The recovered OFDM symbols from the SPAD are passed through a FFT operation which converts symbols to the frequency domain. In DCO-OFDM, $N/2 - 1$ symbols are obtained from the corresponding subcarriers to constitute a QAM symbol frame, $X'(n)$. In ACO-OFDM, $N/4$ symbols are obtained. Finally, the detected QAM symbols are decoded by the conventional maximum likelihood (ML) estimator in order to obtain the output bit stream.

5.3 Nonlinear Distortion in SPAD-based OFDM

5.3.1 Nonlinear Function of SPAD Devices

As mentioned in Section 3.2.4, PQ SPAD and AQ SPAD have different nonlinear function. For the PQ SPAD array, the nonlinear function related to the output photon counts during each $T_s$ is:

$$\mu_{\text{PQ}}(k) = \mu(k) \exp\left(-\frac{\mu(k)\tau_d}{T_sN_{\text{SPAD}}}ight), \quad (5.5)$$

Note that $\tau_d$ denotes the average dead time of each SPAD devices in the array. As the output photon counts approximately follows a Poisson distribution, the average photon count, $\mu_{\text{PQ}}(k)$, is equal to the variance of the distribution. This property will be used in the analytical model in Section 5.4. For the AQ SPAD array, the nonlinear function is:

$$\mu_{\text{AQ}}(k) = \frac{\mu(k)}{1 + \frac{\mu(k)\tau_d}{T_sN_{\text{SPAD}}}}, \quad (5.6)$$
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Figure 5.2: The probability density functions of the PQ SPAD array output. The accurate distribution, Poisson distribution and simulation results are compared over $T_s = 1 \mu s$: (a) the number of the total incident photons is $10^4$; (b) the number of the total incident photons is $10^6$.

As mentioned, the photon counts of SPAD receivers can be approximately described by a Poisson distribution with mean values, $\mu_{PQ}(k)$ and $\mu_{AQ}(k)$. For PQ SPAD, as shown in Figure 5.2(a), when the total incident photons are $10^4$ over $T_s$, the Poisson distribution can accurately describe the real simulated distribution. However, when incident photons increase to $10^6$ (Figure 5.2(b)), the variance of the Poisson distribution is too high to describe the distribution of output photons. Thus, in order to get better results, an accurate distribution is used to replace the Poisson distribution [149]:

$$Pr_{PQ}(a, \mu_m) = \sum_{j=a}^{a_{PQ\max} - 1} \frac{j^j}{j!} (-1)^{j-a} \frac{\mu_m^j}{j!} \exp(-j\mu_m\tau_d)(T_s - j\tau_d)^j. \quad (5.7)$$

Note that $Pr_{PQ}(a, \mu_m)$ is the accurate photon count distribution of a single PQ SPAD during $T_s$ and $\mu_m$ is the average photon rate per second. The maximum photon count rate for a single PQ SPAD device is denoted by $a_{PQ\max}$ which is equal to $[T_s/e\tau_d]$. In the PQ SPAD array, it is assumed that the photon count distributions of each single device are the same during $T_s$. The
distribution can be written as a vector:

\[
\mathbf{Pr}_m(k) = \left[ \Pr_{PQ} \left( 0, \frac{\mu(k)}{T_s N_{SPAD}} \right), \Pr_{PQ} \left( 1, \frac{\mu(k)}{T_s N_{SPAD}} \right), \ldots, \Pr_{PQ} \left( a_{PQ_{\text{max}}} - 1, \frac{\mu(k)}{T_s N_{SPAD}} \right) \right].
\]

(5.8)

Thus, according to (5.2), the joint distribution of the whole SPAD array can be calculated as:

\[
\Pr(k) = \mathbf{Pr}_m(k) \ast \mathbf{Pr}_m(k) \ast \ldots \ast \mathbf{Pr}_m(k).
\]

(5.9)

According to [149] and (5.9), the accurate expectation of the PQ SPAD array output during \( T_s \) is:

\[
E_{PQ}(k) = N_{SPAD} \mu_m \exp(-\mu_m \tau_d)(T_s - \tau_d)
\approx N_{SPAD} \frac{\mu(k)}{T_s N_{SPAD}} \exp\left(-\frac{\mu(k)}{T_s N_{SPAD}} \tau_d\right) T_s
= \mu(k) \exp\left(-\frac{\mu(k) \tau_d}{T_s N_{SPAD}}\right) = \mu_{PQ}(k).
\]

(5.10)

Note that the symbol period, \( T_s \), is assumed much longer than the dead time, \( \tau_d \), in this study. Therefore, it can be achieved that the accurate expectation can be approximated to the average output of the array in (5.5). The accurate variance of the array output is:

\[
\sigma^2_{PQ}(k) = N_{SPAD} \left[ \mu_m^2 \exp(-2 \mu_m \tau_d)(3 \tau_d^2 - 2 T_s \tau_d) + \mu_m \exp(-\mu_m \tau_d) T_s \right].
\]

(5.11)

It can be seen in Figure 5.2(a) and (b) that the accurate distribution is well matched with the simulation distribution. As a result, the accurate distribution performs better in describing the photon counts than the Poisson distribution in the PQ SPAD array.

For AQ SPAD, in order to get better results, an accurate distribution is also used to replace the Poisson distribution [150]:

(1) for \( 0 \leq a \leq a_{AQ_{\text{max}}} - 1 \):

\[
\Pr_{AQ}(a, \mu_m) = \lambda \left[ a^{-2} \sum_{j=0}^{a-2} (a - 1 - j) \Pr(j, S_{a-1}) - 2 \sum_{j=0}^{a-1} (a - j) \Pr(j, S_a) + \sum_{j=0}^{a} (a + 1 - j) \Pr(j, S_{a+1}) \right].
\]

(5.12)
Figure 5.3: The probability density functions of the AQ SPAD array output. The accurate distribution, Poisson distribution and simulation results are compared over $T_s = 1 \mu s$: (a) the number of the total incident photons is $10^4$; (b) the number of the total incident photons is $10^6$.

(2) for $a = a_{AQ_{\text{max}}}$:

$$\Pr_{AQ}(a, \mu_m) = \lambda \sum_{j=0}^{a-2} (a - 1 - j) \Pr(j, S_{a-1}) - \sum_{j=0}^{a-1} (a - j) \Pr(j, S_a) - \mu_m T_s + a + 1,$$

(5.13)

(3) for $a = a_{AQ_{\text{max}}} + 1$:

$$\Pr_{AQ}(a, \mu_m) = \lambda \sum_{j=0}^{a-2} (a - 1 - j) \Pr(j, S_{a-1}) + \mu_m T_s - a + 1.$$

(5.14)

Note that $\Pr_{AQ}(a, \mu_m)$ is the accurate photon count distribution of a single AQ SPAD during $T_s$. The maximum photon count rate for a single AQ SPAD device is denoted by $a_{AQ_{\text{max}}}$ which is equal to $[T_s/\tau_d]$: $\lambda = (1 + \mu_m \tau_d)^{-1}$; and $S_a = \mu_m (T_s - a \tau_d)$. According to (5.8), (5.9) and [150], when $T_s$ is much longer than $\tau_d$, the accurate expectation of the AQ SPAD array...
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output is:

\[
E_{AQ}(k) = N_{\text{SPAD}}E_m(k) = N_{\text{SPAD}}\lambda \mu_m T_s
\]

\[
= N_{\text{SPAD}} \left[ 1 + \frac{\mu(k)}{T_s N_{\text{SPAD}} \tau_d} \right]^{-1} \frac{\mu(k)}{T_s N_{\text{SPAD}}} T_s
\]

\[
= \frac{\mu(k)}{1 + \frac{\mu(k)\tau_d}{T_s N_{\text{SPAD}}}} = \mu_{AQ}(k), \quad (5.15)
\]

It can be seen that the accurate photon count distribution of the AQ SPAD array has the same mean value as the Poisson distribution from (5.6). According to the variance calculation of a single AQ SPAD device [150], the accurate variance of the AQ array output is:

\[
\sigma_{AQ}^2(k) = N_{\text{SPAD}}\sigma_m^2(k)
\]

\[
= N_{\text{SPAD}} \lambda^3 \left[ \mu_m T_s + g^2\lambda(1 + \frac{2}{3}g + \frac{1}{6}g^2) \right], \quad (5.16)
\]

where \(g = \mu_m \tau_d\). Figure 5.3 shows the photon count distribution of an AQ SPAD array. When the number of the total incident photons is low (\(10^4\)), the simulation result is well matched with both the Poisson distribution and the accurate distribution. Furthermore, as a result of a comparison between Figure 5.2 (a) and Figure 5.3 (a), it can be found that the PQ and AQ SPAD array have the similar photon count distribution. This is because the PQ SPAD array and the AQ SPAD array have an almost same linear region when the photon rate is low (Figure 3.8). In Figure 5.3 (b), compared with the Poisson distribution, the accurate distribution is closer to the simulation result. Moreover, compared with the photon count distribution of the PQ SPAD array (Figure 5.2 (b)), the AQ SPAD array has higher mean value of photon counts when the nonlinear distortion occurs.

5.3.2 Nonlinear Distortion in O-OFDM with PQ and AQ SPADs

In OWC, nonlinear distortion and clipping distortion problems are originally defined in DCO-OFDM [42]. As shown in Figure 5.4 (a) and (b), some symbols are still negative after adding a DC-bias. Before being transmitted, the biased symbols have to be converted to unipolar symbols and thus the negative symbols are clipped to zero (Figure 5.4 (c)) resulting in a high symbol error detection. Therefore, when the symbols are demodulated at the receiver, the BER performance will be significantly affected by the clipping distortion problem. A higher bias level can decrease the clipping distortion effect but increase the energy consumption. A method to chose an optimal DC-bias to eliminate the clipping distortion effect efficiently is presented...
in [60]. However, in a practical VLC system, the dynamic range of the LED transmitter is limited by some effects, such as the nonlinear output characteristic of the LED restricting the minimum and the maximum intensity of the signal, and the eye safety regulations constraining the level of radiated average optical power [60, 62]. Thus, for both DCO-OFDM and ACO-OFDM, the time domain signal is likely to be distorted double-sidedly. The nonlinear distortion problems caused by LED and DC-biasing are collectively referred to as transmitter nonlinear distortion. In this study, the LED transmitter is assumed to be an ideal LED with the radiated optical power from zero to infinity. Therefore, the nonlinear distortion caused by LED has a negligible impact on the BER performance.

In SPAD-based OFDM, the receiver nonlinear distortion has a significant effect on the symbol detection and demodulation. For DCO-OFDM, Figure 5.4 (d) shows potential average photon counts.
counts per each symbols which is calculated by (5.4). The received symbols in Figure 5.4 (d) can reflect the transmitted digital signal in Figure 5.4 (c) with a bit of additional noises. However, when the optical irradiance is high, quenching recharged circuits decrease the number of photon counts nonlinearly. Figure 5.4 (e1) shows the average photon count of the PQ SPAD receiver for each symbols. Compared with the potential photon count, it can be seen that the average photon count of PQ SPAD is lower and some high amplitude signals achieve lower number of counted photons. This is because the average photon count drastically reduces after the number of incoming photons exceeds the limitation (Figure 3.8). Compared with PQ SPAD, the photon count of the AQ SPAD receiver can reluctantly describe the original optical signals as shown in Figure 5.4 (e2). Some high amplitude signals are just slightly distorted in this case. However, if the optical irradiance increases, most of output symbols will approach the limitation of AQ SPAD photon counts and the BER performance will be affected by the nonlinear

Figure 5.5: Nonlinear Distortion in ACO-OFDM with PQ and AQ SPADs when 4-QAM is used; N is 16; \( T_s \) is 1 μs; and optical irradiance is -30 dBm. (a) Bipolar OFDM symbols. (b) Clipped ACO-OFDM symbols. (c) Potential photon counts per each symbol. (d1) Photon counts in the PQ SPAD receiver. (d2) Photon counts in the AQ SPAD receiver.
distortion. For ACO-OFDM, since the LED nonlinearity is not considered in this study, the transmitter nonlinear distortion has no effect on the BER performance. As the bipolar ACO-OFDM symbols is symmetrical, the clipped signals contain all the information symbols as shown in Figure 5.5 (a) and Figure 5.5 (b). Compared with DCO-OFDM, the potential photon count of ACO-OFDM is higher for individual symbols since the total optical power of DCO-OFDM and ACO-OFDM is assumed to be the same and each information-carried symbols in ACO-OFDM has higher optical energy. Therefore, in ACO-OFDM, the receiver nonlinear distortion occurs earlier than DCO-OFDM. Figure 5.5 (d1) and Figure 5.5 (d2) compare the average photon count of PQ and AQ SPAD in ACO-OFDM. The nonlinear distortion problem in PQ SPAD has more significant effect than in AQ SPAD.

Figure 5.6 shows an example of the BER performance of SPAD-based OFDM. 16-QAM ACO-OFDM and 16-QAM DCO-OFDM with 5 dB DC-bias are compared when PQ and AQ SPADs are considered and $T_s$ is 1 $\mu$s. To present the BER performance and the receiver nonlinear distortion of SPAD-based OFDM, three definitions are given in this study. When the optical irradiance is larger than a threshold, the BER is below the target BER of $10^{-3}$. This threshold is defined as the minimum power requirement (MPR) of the system. When the optical irradiance is larger than a threshold, the BER is below the target BER of $10^{-3}$. This threshold is defined as the minimum power requirement (MPR) of the system.
increases and becomes larger than another threshold, the nonlinear distortion of SPAD receivers occurs, resulting in BER higher than $10^{-3}$. This threshold is defined as the maximum optical irradiance (MOI). The gap between the MPR and the MOI is defined as the low error area (LEA) where the system can maintain a low BER ($< 10^{-3}$). For example, in Figure 5.6, after the optical irradiance reaches -69.1 dBm, BER of 16-QAM ACO-OFDM with PQ SPAD receivers is lower than $10^{-3}$, and after the optical irradiance reaches -43.2 dBm, BER of the same scheme is higher than $10^{-3}$. Thus, the MPR of this scheme is -69.1 dBm; the MOI is -43.2 dBm; and the LEA is 25.9 dB. This means that 16-QAM PQ SPAD ACO-OFDM with 1 µs symbol duration can be ideally used when the optical irradiance comes from -69.1 dBm to -43.2 dBm. In Figure 5.6, 16-QAM DCO-OFDM with 5 dB DC-bias is affected by the transmitter nonlinear distortion and thus as shown in this figure, the BER of this scheme cannot reach below $10^{-3}$. The transmitter nonlinear distortion and the receiver nonlinear distortion limit the BER performance jointly. As mentioned above, the receiver nonlinear distortion of PQ SPAD occurs earlier than AQ SPAD, which is also shown in Figure 5.6.

### 5.4 Theoretical Analysis of SPAD-based OFDM

The analytical BER performance of SPAD-based OFDM is presented in this section. In the SPAD-based OFDM system, some high amplitude symbols in the recovered signal ($x'(k)$) are distorted by PQ and AQ recharged circuits resulting in loss of information. This causes a unique receiver nonlinear distortion which should be considered in the theoretical analysis and this will still be analysed by the Bussgang theorem. In SPAD-based OFDM, the functions of the Bussgang theorem, (2.19) and (2.20), are transformed as:

$$
\begin{align*}
z\left(N(x)\right) &= \alpha_{\text{SPAD}} N(x) + Y_{\text{SPAD}}, \\
E[N(x)Y_{\text{SPAD}}] &= 0,
\end{align*}
$$

(5.17)

where $\alpha_{\text{SPAD}}$ is a constant and can be derived as:

$$
\alpha_{\text{SPAD}} = \frac{E\left[N(x)z\left(N(x)\right)\right]}{E\left[N^2(x)\right]}.
$$

(5.18)

According to (5.17), the variance of the additional noise, $\sigma_Y^2$, can be calculated by:

$$
\sigma_Y^2 = E[Y_{\text{SPAD}}^2] - E^2[Y_{\text{SPAD}}],
$$

(5.19)
where:

$$E[Y_{\text{SPAD}}^2] = E\left[z^2\left(N(x)\right)\right] - E[\alpha_{\text{SPAD}}^2N(x)],$$  \hspace{1cm} (5.20)

$$E[Y_{\text{SPAD}}] = E\left[z\left(N(x)\right)\right] - E[\alpha_{\text{SPAD}}N(x)].$$  \hspace{1cm} (5.21)

According to (5.4), the relationship between each Gaussian random variable, $x$, and the related number of photons, $N(x)$, is:

$$N(x) = \begin{cases} C_s x + C_n, & x \geq 0, \\ 0, & x < 0, \end{cases}$$  \hspace{1cm} (5.22)

where $C_s = C_{\text{FF}}C_{\text{PDP}}P_tT_s(1 + P_{\text{AP}})/E_p$ and $C_n = n_{\text{DCR}}(1 + P_{\text{AP}})$. Note that $P_r$ is the average received optical power.

To describe the analytical BER calculations of SPAD-based OFDM, the following formulas are defined. The standard normal distribution probability density function (PDF) is:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right),$$  \hspace{1cm} (5.23)

and the tail probability of the standard normal distribution is:

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt.$$  \hspace{1cm} (5.24)

### 5.4.1 Analysis of O-OFDM with PQ SPAD

According to (5.5) and (5.17), the nonlinear transformation function of PQ SPAD OFDM is:

$$z_{\text{PQ}}(N(x)) = N(x) \exp(-C_t N(x)) = \alpha_{\text{PQ}} N(x) + Y_{\text{PQ}},$$  \hspace{1cm} (5.25)

where $C_t = \tau_{\text{d}}/(T_s N_{\text{SPAD}})$. Thus, to calculate the nonlinear gain factor of PQ SPAD OFDM, the components in (5.18), $E[N^2(x)]$ and $E[N(x)z_{\text{PQ}}(N(x))]$, are derived with the help of
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equations (A.3) and (A.4):

\[
\begin{align*}
\mathbb{E}[N^2(x)] &= \int_{-\infty}^{\infty} N^2(x) \frac{1}{\sigma_x} \phi \left( \frac{x-\rho}{\sigma_x} \right) \, dx \\
&= C_s^2 \left[ \frac{\rho^2 Q}{\sigma_x} - \frac{\rho}{\sigma_x} \right] + \sigma_x^2 \left[ \frac{\rho}{\sigma_x} - \frac{\rho}{\sigma_x} \right] + 2 C_s C_n \left[ \rho Q \left( \frac{\rho}{\sigma_x} \right) + \sigma_x \phi \left( \frac{\rho}{\sigma_x} \right) \right] + C_n^2. \tag{5.26}
\end{align*}
\]

\[
\mathbb{E}[N(x)z_{PQ}(N(x))] = \int_{-\infty}^{\infty} N(x)z_{PQ}(N(x)) \frac{1}{\sigma_x} \phi \left( \frac{x-\rho}{\sigma_x} \right) \, dx \\
&= \int_0^\infty (C_s x + C_n)^2 \exp \left( \frac{1}{2} C_s^2 \sigma_x^2 - C_t C_s \rho - C_t C_n \right) \frac{1}{\sigma_x} \phi \left( \frac{x-\rho + C_t C_s \sigma_x^2}{\sigma_x} \right) \, dx \\
&\quad + C_n^2 \exp(-C_t C_n)Q \left( \frac{\rho}{\sigma_x} \right) \\
&= C_t^2 \left[ \frac{\rho - C_s \sigma_x^2}{\sigma_x} \right] + \sigma_x^2 \left[ \frac{\rho - C_s \sigma_x^2}{\sigma_x} \right] + 2 C_s C_n \left[ \frac{\rho - C_s \sigma_x^2}{\sigma_x} \right] + 2 C_s \sigma_x \phi \left( \frac{\rho}{\sigma_x} \right) + 2 C_n \exp(-C_t C_n)Q \left( \frac{\rho}{\sigma_x} \right), \tag{5.27}
\end{align*}
\]

Note that \( \sigma_x \) and \( \rho \) respectively denote the standard deviation and mean value of the bipolar normalized OFDM symbols. As a result, a closed-form expression of \( \alpha_{PQ} \) can be obtained. Moreover, components in (5.20) and (5.21), \( \mathbb{E}[N(x)] \), \( \mathbb{E}[z_{PQ}(N(x))] \) and \( \mathbb{E}[z_{PQ}^2(N(x))] \), are calculated by:

\[
\mathbb{E}[N(x)] = \int_{-\infty}^{\infty} N(x) \frac{1}{\sigma_x} \phi \left( \frac{x-\rho}{\sigma_x} \right) \, dx \quad \overset{(A.3)}{=} \quad C_s \left[ \rho Q \left( \frac{\rho}{\sigma_x} \right) + \sigma_x \phi \left( \frac{\rho}{\sigma_x} \right) \right] + C_n. \tag{5.28}
\]


E \left[ z_{\text{PQ}}(N(x)) \right] = \int_{-\infty}^{\infty} z_{\text{PQ}}(N(x)) \frac{1}{\sigma_x} \phi \left( \frac{x - \rho}{\sigma_x} \right) \, dx \\
= \int_{0}^{\infty} (C_s x + C_n) \exp \left( \frac{1}{2} C_t^2 C_s^2 \sigma_x^2 - C_tC_s \rho - C_tC_n \right) \frac{1}{\sigma_x} \phi \left( \frac{x - \rho + C_tC_s \sigma_x^2}{\sigma_x} \right) \, dx \\
+ C_n \exp(-C_tC_n) Q \left( \frac{\rho}{\sigma_x} \right) \\
= \exp \left( \frac{1}{2} C_t^2 C_s^2 \sigma_x^2 - C_tC_s \rho - C_tC_n \right) \left\{ C_n \right\} \left[ \rho - C_tC_s \sigma_x^2 \right] Q \left( \frac{C_tC_s \sigma_x^2 - \rho}{\sigma_x} \right) \right\} \\
+ \sigma_x \phi \left( \frac{C_tC_s \sigma_x^2 - \rho}{\sigma_x} \right) + C_n Q \left( \frac{C_tC_s \sigma_x^2 - \rho}{\sigma_x} \right) + C_n \exp(-C_tC_n) Q \left( \frac{\rho}{\sigma_x} \right) . \\
(5.30)

E \left[ z_{\text{PQ}}^2(N(x)) \right] = \int_{-\infty}^{\infty} z_{\text{PQ}}^2(N(x)) \frac{1}{\sigma_x} \phi \left( \frac{x - \rho}{\sigma_x} \right) \, dx \\
= \int_{0}^{\infty} (C_s x + C_n)^2 \exp \left( 2C_t^2 C_s^2 \sigma_x^2 - 2C_tC_s \rho - 2C_tC_n \right) \frac{1}{\sigma_x} \phi \left( \frac{x - \rho + 2C_tC_s \sigma_x^2}{\sigma_x} \right) \, dx \\
+ C_n^2 \exp(-2C_tC_n) Q \left( \frac{\rho}{\sigma_x} \right) \\
= \exp \left( 2C_t^2 C_s^2 \sigma_x^2 - 2C_tC_s \rho - 2C_tC_n \right) \left\{ C_n^2 \right\} \left[ \rho - 2C_tC_s \sigma_x^2 \right] Q \left( \frac{2C_tC_s \sigma_x^2 - \rho}{\sigma_x} \right) \right\] \\
+ \sigma_x^2 Q \left( \frac{2C_tC_s \sigma_x^2 - \rho}{\sigma_x} \right) + (\rho - 2C_tC_s \sigma_x^2) \sigma_x \phi \left( \frac{2C_tC_s \sigma_x^2 - \rho}{\sigma_x} \right) \right\] \\
+ 2C_tC_s C_n \left\{ \rho - 2C_tC_s \sigma_x^2 \right\} Q \left( \frac{2C_tC_s \sigma_x^2 - \rho}{\sigma_x} \right) + \sigma_x \phi \left( \frac{2C_tC_s \sigma_x^2 - \rho}{\sigma_x} \right) \right\] \\
+ C_n^2 Q \left( \frac{\rho - 2C_tC_s \sigma_x^2}{\sigma_x} \right) + C_n^2 \exp(-2C_tC_n) Q \left( \frac{\rho}{\sigma_x} \right) . \\
(5.30)

Thus, the variance of the additional distortion noise in (5.19) can be derived as a closed-form expression. In this study, the nonlinear distortion effects in PQ SPAD-based ACO-OFDM and DCO-OFDM are discussed separately as follows.
5.4.1.1 PQ SPAD ACO-OFDM

In ACO-OFDM, the standard deviation of the original bipolar OFDM symbols, $x(k)$, is:

$$\sigma_{m-ACO} = \sqrt{\frac{M-1}{3}}, \quad (5.31)$$

where $M$ is the constellation size of QAM symbols. In this study, DC bias is not assumed in ACO-OFDM. As the negative part of the OFDM frame is clipped, the remaining part can be described by a half Gaussian distribution with zero mean. Thus, the mean value of the remaining part is:

$$E[x_{\text{clipped}}(k)] = \frac{\sigma_{m-ACO}}{\sqrt{2\pi}}. \quad (5.32)$$

As noted, the mean value of the transmitted symbols is normalised to one in the simulation. As a result, the intervals of the normalised ACO-OFDM symbols can be specified from 0 to 1 in a zero-mean Gaussian distribution ($\rho = 0$) with a standard deviation:

$$\sigma_{x-ACO} = \frac{\sigma_{m-ACO}}{E[x_{\text{clipped}}(k)]} = \sqrt{2\pi}. \quad (5.33)$$

Then, based on (5.26-5.28) and (5.34), the equations, (5.20) and (5.21), become:

$$E[Y^2_{PQ}] = \frac{E\left[N(x)z_{PQ}\left(N(x)\right)\right]}{E\left[N(x)^2\right]} - \alpha_{PQ-ACO}E\left[N(x)\right]^2, \quad (5.35)$$

$$E[Y_{PQ}] = E\left[z_{PQ}\left(N(x)\right)\right] - \alpha_{PQ-ACO}E\left[N(x)\right], \quad (5.36)$$

where $\rho = 0$ and $\sigma_x = \sqrt{2\pi}$. Thus, the variance of the additional noise in PQ SPAD ACO-OFDM, $\sigma_{Y-PQ-ACO}^2$, can be obtained by (5.19). Since the number of OFDM subcarriers and SPAD devices is high enough, the resulting variances can be approximated to variances of Gaussian distribution. Thus, according to [62], the resulting SNR through the nonlinear
transformation can be calculated with the following formula:

$$\text{SNR}_{\text{ACO}}^{\text{PQ}} = \frac{\alpha_{\text{PQ-ACO}}^2 \sigma_{z-\text{ACO}}^2}{2R_{\text{ACO}}(\sigma_{\text{Y-PQ-ACO}}^2 + \sigma_{N-\text{PQ}}^2)}.$$

(5.37)

Note that $R_{\text{ACO}}$ is the spectral efficiency of ACO-OFDM which is $\frac{1}{4} \log_2(M)$ and $\sigma_{N-\text{PQ}}^2$ is the variance of the shot noise which is related to the received signals. In the case of the Poisson distribution, the variance is equal to the mean value. Thus, for each received symbols, $\sigma_{N-\text{PQ}}^2(x)$ is equal to $z_{\text{PQ}}(N(x))$. As a result, $\sigma_{N-\text{PQ}}^2$ can be derived as:

$$\sigma_{N-\text{PQ}}^2 = E[\sigma_{N-\text{PQ}}^2(x)] = E[z_{\text{PQ}}(N(x))].$$

(5.38)

According to (5.29), the value of $\sigma_{N-\text{PQ}}^2$ can be obtained. Note that $\sigma_x$ is set to $\sqrt{2}\pi$ and $\rho$ is set to 0 in PQ SPAD ACO-OFDM. In the case of the accurate distribution, the variance of the shot noise component in the PQ SPAD array can be calculated according to (5.11):

$$\sigma_{N-\text{PQ}}^2 = E[\sigma_{N-\text{PQ}}^2(x)] = E \left[ \frac{3\tau_d - 2T_s}{T_s} C_{12} z_{\text{PQ}}^2 (N(x)) + z_{\text{PQ}}(N(x)) \right].$$

(5.39)

Finally, based on the conventional BER calculation of O-OFDM [60], the analytical BER performance of PQ SPAD ACO-OFDM can be derived by:

$$\text{BER}_{\text{PQ-ACO}} = \frac{4(\sqrt{M} - 1)}{\sqrt{M} \log_2(M)} Q \left( \sqrt{\frac{3R_{\text{ACO}} \text{SNR}_{\text{ACO}}^{\text{PQ}}}{M - 1}} \right) + \frac{4(\sqrt{M} - 2)}{\sqrt{M} \log_2(M)} Q \left( 3\sqrt{\frac{3R_{\text{ACO}} \text{SNR}_{\text{ACO}}^{\text{PQ}}}{M - 1}} \right).$$

(5.40)

5.4.1.2 PQ SPAD DCO-OFDM

In DCO-OFDM, the standard deviation of the original bipolar OFDM symbols, $x(k)$, is:

$$\sigma_{m-\text{DCO}} = \sqrt{\frac{2(M - 1)(N - 2)}{3N}}.$$

(5.41)

As a DC bias component is added to the original bipolar OFDM symbols and some information-carrying symbols are clipped, a clipping distortion noise has to be considered in DCO-OFDM.
Based on (2.13), the DC bias in DCO-OFDM is:

\[ B_{DC} = \beta \sigma_{m-DCO} \]  

(5.42)

According to (2.23), the clipping distortion factor of DCO-OFDM can be derived as:

\[ \alpha_{cl} = \frac{\int_{0}^{\infty} \frac{x^2}{\sigma_{m-DCO}^2} \phi \left( \frac{x - B_{DC}}{\sigma_{m-DCO}} \right) dx}{\sigma_{m-DCO}^2 + B_{DC}^2} = Q \left( - \frac{B_{DC}}{\sigma_{m-DCO}} \right) + \beta G_{DC} \phi \left( \frac{B_{DC}}{\sigma_{m-DCO}} \right), \] 

(5.43)

where \( G_{DC} \) denotes the attenuation of the original signal power, \( \sigma_{m-DCO}^2 \), due to the DC bias in DCO-OFDM. It is defined as:

\[ G_{DC} = \frac{\sigma_{m-DCO}^2}{\sigma_{m-DCO}^2 + B_{DC}^2}. \] 

(5.44)

According to (2.24), (2.25) and (2.26), the variance of the clipping distortion noise, \( \sigma_c^2 \), can be calculated by:

\[ \sigma_c^2 = E \left[ Y_c^2 \right] - E^2 \left[ Y_c \right] = \alpha_{cl} (1 - \alpha_{cl}) (\sigma_{m-DCO}^2 + B_{DC}^2) - B_{DC} Q \left( - \frac{B_{DC}}{\sigma_{m-DCO}} \right) \]

\[ + \sigma_{m-DCO} \phi \left( \frac{B_{DC}}{\sigma_{m-DCO}} \right) - \alpha_{cl} B_{DC} \] 

(5.45)

After DC biasing and clipping in time domain, the mean value of DCO-OFDM symbols is:

\[ E[x_{clipped}(k)] = \int_{0}^{\infty} \frac{x}{\sigma_{m-DCO}} \phi \left( \frac{x - B_{DC}}{\sigma_{m-DCO}} \right) dx = B_{DC} Q \left( - \frac{B_{DC}}{\sigma_{m-DCO}} \right) + \sigma_{m-DCO} \phi \left( \frac{B_{DC}}{\sigma_{m-DCO}} \right). \] 

(5.46)

Thus, the normalised DCO-OFDM symbols can be described as a Gaussian distribution from 0 to \( \infty \) with mean value and standard deviation:

\[ \rho_{DCO} = \frac{B_{DC}}{E[x_{clipped}(k)]}, \] 

(5.47)

\[ \sigma_{x-DCO} = \frac{\sigma_{m-DCO}}{E[x_{clipped}(k)]}. \] 

(5.48)
Like PQ SPAD ACO-OFDM, the nonlinear gain factor of PQ SPAD DCO-OFDM is:

\[ \alpha_{\text{PQ-DCO}} = \frac{E\left[N(x)z_{\text{PQ}}\left(N(x)\right)\right]}{E\left[N(x)^2\right]} \bigg|_{\rho=\rho_{\text{DCO}}, \sigma_x=\sigma_{x-\text{DCO}}} \]  

(5.49)

Then, the equations, (5.20) and (5.21), become:

\[ E\left[Y_{\text{PQ}}^2\right] = E\left[z_{\text{PQ}}\left(N(x)\right)^2\right] - \alpha_{\text{PQ-DCO}}^2 E\left[N(x)^2\right], \]  

(5.50)

\[ E[Y_{\text{PQ}}] = E\left[z_{\text{PQ}}\left(N(x)\right)\right] - \alpha_{\text{PQ-DCO}} E\left[N(x)\right], \]  

(5.51)

where \( \rho = \rho_{\text{DCO}} \) and \( \sigma_x = \sigma_{x-\text{DCO}} \). As a result, \( \alpha_{Y-\text{PQ-DCO}}^2 \) can also be achieved. As the clipping distortion effect is considered in DCO-OFDM, the final resulting SNR of the PQ SPAD DCO-OFDM system can be calculated with the following formula:

\[ \text{SNR}_{\text{PQ-DCO}} = \frac{G_{\text{DC}}\sigma_i^2 \alpha_{\text{PQ-DCO}}^2 C_s^2 \sigma_x^2_{-\text{DCO}}}{R_{\text{DCO}}(\alpha_{\text{cl}}^2 \sigma_{cp}^2 + \sigma_x^2_{-\text{PQ}} + \sigma_N^2_{-\text{PQ}})}. \]  

(5.52)

where \( R_{\text{DCO}} \) is equal to \( \frac{N-2}{2M} \log_2(M) \) and \( \sigma_{cp}^2 \) is the attenuated variance of \( \sigma_c^2 \) due to the transmitter normalization and the photon counter. In PQ SPAD DCO-OFDM, \( \sigma_{cp}^2 \) is given by:

\[ \sigma_{cp}^2 = \left( 1 - \frac{G_{\text{DC}}}{R_{\text{DCO}}} \right) \frac{C_s^2 \sigma_c^2}{E^2[z_{\text{clipped}}(k)]}. \]  

(5.53)

Finally, the analytical BER performance of PQ SPAD DCO-OFDM can be achieved by the corresponding BER calculation function as used in (5.40).

### 5.4.2 Analysis of O-OFDM with AQ SPAD

As noted, AQ SPAD has a different output function with PQ SPAD. According to (5.6) and (5.17), the nonlinear transformation function of AQ SPAD OFDM is:

\[ z_{\text{AQ}}\left(N(x)\right) = \frac{N(x)}{1+C_1N(x)} = \alpha_{\text{AQ}} N(x) + Y_{\text{AQ}}. \]  

(5.54)

To obtain the analytical equations of AQ SPAD OFDM, \( z_{\text{AQ}}\left(N(x)\right) \) is substituted into (5.27), (5.30) and (5.29) where \( z_{\text{PQ}}\left(N(x)\right) \) is replaced. Then following the same steps in the analysis
The active area of each SPAD device | $50.3 \, \mu m^2$
---|---
Total area of the SPAD array | $0.16 \, mm^2$
---|---
The FF of the SPAD array, $C_{FF}$ | $32.2\%$
---|---
The PDP of each SPAD device, $C_{PDP}$ | $20\%$
---|---
The DCR of each SPAD device, $N_{DCR}$ | $7.27 \, kHz$
---|---
The APP of each SPAD device, $P_{AP}$ | $1\%$
---|---
The dead time of each SPAD device, $\tau_d$ | $13.5 \, ns$
---|---
Number of SPADs in an array, $N_{SPAD}$ | $1024$
---|---
Energy of a photon, $E_P$ | $4.42 \times 10^{-19}$
---|---
The FF of the PD | $32.2\%$
---|---
The quantum efficiency of the PD | $0.2$
---|---
Input referred noise of the PD | $16 \, pA/\sqrt{Hz}$

| Table 5.1: Simulation Parameters of the SPAD-based OFDM System |

Of PQ SPAD OFDM, the final resulting SNRs of AQ SPAD OFDM can be derived as:

$$SNR_{ACO}^{AQ} = \frac{\alpha_{AQ-ACO}^2 C_s^2 \sigma_{x-ACO}^2}{2R_{ACO}(\sigma_{Y-Q-ACO}^2 + \sigma_{N-AQ}^2)},$$  \hspace{1cm} (5.55)

$$SNR_{DCO}^{AQ} = \frac{G_{DCO} \alpha_{AQ-DCO}^2 C_s^2 \sigma_{x-DCO}^2}{R_{DCO}(\alpha_{cl}^2 \alpha_{cp}^2 + \sigma_{Y-Q-DCO}^2 + \sigma_{N-AQ}^2)}.$$  \hspace{1cm} (5.56)

It is worth noting that both Possion distribution (5.3) and the accurate distribution (5.9) need to be considered in the calculations of the shot noise component, $\sigma_{N-AQ}^2$. In the case of the **Poisson distribution**, the variance is equal to the mean value. By using the same expression in PQ SPAD (5.38), the shot noise component in AQ SPAD OFDM is,

$$\sigma_{N-AQ}^2 = E[\sigma_N(x)] = E[N_AQ(N(x))].$$  \hspace{1cm} (5.57)

In the case of the **accurate distribution**, based on the accurate variance of the AQ SPAD array output (5.16), the shot noise can be derived as:

$$\sigma_{N-AQ}^2 = E[\sigma_AQ^2(k)] = E \left\{ N_{SPAD} \lambda_N^2 \left( \frac{N(x)}{N_{SPAD}} + g_N^2 \lambda_N (1 + \frac{2}{3} g_N + \frac{1}{6} g_N^2) \right) \right\},$$  \hspace{1cm} (5.58)
where $\lambda_N = (1 + C_t N(x))^{-1}$ and $g_N = C_t N(x)$. The performance of the Poisson distribution and the accurate distribution in PQ and AQ SPAD OFDM systems will be compared in the next section.

## 5.5 Results and Discussion

In this section, the analytical BER performance of SPAD-based DCO-OFDM and ACO-OFDM with the nonlinear distortion are compared. Moreover, the maximum bit rates of each schemes are found, which is limited by the nonlinear distortion effect. In the simulation, an ideal LED is assumed to emit blue light with a wavelength distribution centred around 450 nm where the energy of a photon is $4.42 \times 10^{-19}$. For the ideal LED transmitter, the recharged time of the circuit and on/off time of LED can be neglected, thus rising/falling edges have negligible effects on transmitted samples in the time domain. Therefore, in this study, each digital OFDM symbol is converted to intensity signals without any distortions. In addition, optical signals are assumed to pass through a flat fading channel and in the absence of background light. As a consequence, the received signals are still non-distorted intensities with additional shot noises. Thus, in this study, the signals are assumed to be affected by the receiver shot noise, nonlinear
distortion and clipping distortion (low bias level DCO-OFDM). As a comparison of different data rates, $T_s = 1$ ms and $T_s = 1$ $\mu$s are simulated and analysed. A PQ SPAD array and an AQ SPAD array are considered with the same parameters which are given in Table 5.1.

### 5.5.1 Analytical Results of SPAD-based OFDM

The BER performance of the PQ SPAD ACO-OFDM and DCO-OFDM systems is presented in Figure 5.7 and Figure 5.8 where $T_s = 1$ ms and $T_s = 1$ $\mu$s. The analytical and simulation results confirm a very close match. It is shown that ACO-OFDM has a lower MPR than DCO-OFDM with the same constellation size. DC bias in DCO-OFDM consumes additional transmission power so that in SPAD receiver side, the DCO-OFDM system receives more optical power than ACO-OFDM. As the additional nonlinear noise is doubled in ACO-OFDM [62], the nonlinear distortion in ACO-OFDM occurs earlier than DCO-OFDM. Thus it can be seen that the MOI of ACO-OFDM is lower than DCO-OFDM. On the whole, the LEA of ACO-OFDM is higher than DCO-OFDM. As a result, in SPAD-based OFDM systems, ACO-OFDM requires lower transmission power and has a longer operated interval as compared with DCO-OFDM. For different symbol periods, the schemes with a shorter symbol period ($T_s = 1$ $\mu$s) have higher MPRs than the methods with a longer symbol period ($T_s = 1$ ms). This means that with a
Figure 5.9: BER performance of AQ SPAD ACO-OFDM and DCO-OFDM, $T_s = 1$ ms, simulation (symbols) vs. Poisson distribution theory (solid lines) vs. accurate distribution theory (dashed lines).

decrease of $T_s$, the LEA reduces, which decreases the range of the received optical power. It is worth noting that for 64-QAM DCO-OFDM with 7dB bias, the clipping distortion creates an error floor. Thus for higher constellation sizes, a higher DC bias may need to be applied in DCO-OFDM. However, the MPR increases with the additional DC bias and the LEA decreases. As shown in Figure 5.8, for 64-QAM DCO-OFDM with 13dB bias, the MPR becomes higher than the MOI. Thus the BER of the system is always above $10^{-3}$ when $T_s = 1$ ms. Therefore, this symbol period is unacceptable for this scheme.

Figure 5.9 and Figure 5.10 show the BER performance of the AQ SPAD ACO-OFDM and DCO-OFDM systems as a function of the optical irradiance when $T_s = 1$ ms and $T_s = 1$ µs. Compared with the BER performance of PQ SPAD OFDM (Fig 5.7 and Fig 5.8), these two systems have the same BER performances at low optical irradiance (around MPR). This is because the PQ SPAD devices have the same performance of linearity as the AQ SPAD devices when the number of incoming photons is low (Figure 5). However, as the maximum count rate of PQ SPAD is lower than AQ SPAD, the MOIs of PQ SPAD OFDM systems are lower than AQ SPAD OFDM systems. In addition, since the MPRs of each system are the same, the LEAs of PQ SPAD OFDM systems are also lower than AQ-based systems. As a consequence, the PQ SPAD OFDM system is more readily affected by the nonlinear distortion and has higher
As mentioned in section 5.3.1, accurate distributions of the PQ and AQ SPAD array are considered in this study and compared with the Poisson distribution. It is shown that the simulation results are well matched with both the Poisson distribution and the accurate distribution. When the optical irradiance is low, Poisson has the same distribution as the accurate one (as shown in Figure 5.2(a) and Figure 5.3(a)); and when the nonlinear distortion happens, the variance of the nonlinear additional noise dominates the performance of the system and the shot noise has a negligible effect on the BER performance. Thus the Poisson distribution shows the same performance as the accurate distribution in the SPAD-based OFDM system. Although the accurate distribution can describe the actual distribution of the SPAD arrays, the Poisson distribution is easier to implement.

### 5.5.2 Maximum Bit Rates of SPAD-based OFDM

Figure 5.11 shows the MOI and LEA of the PQ SPAD and the AQ SPAD OFDM systems with different spectral efficiencies and symbol periods ($T_s = 1 \text{ ms}$ and $T_s = 1 \mu\text{s}$). It is shown that the MOIs of all schemes decrease when the spectral efficiencies increase. High constellation
sizes schemes have higher signal variances and peak-to-average power ratio due to increasing of the probability of high intensity signals. Those high intensity signals are easily affected by the nonlinear distortion of SPAD receivers and increase the probability of error detections and demodulations. In addition, with the increase of constellation sizes, MPRs also increase. Therefore, with the increase of the spectral efficiency, LEAs of the systems rapidly decrease as shown in Figure 5.11.

It is worth noting that the LEA of the SPAD-based OFDM system decreases when the symbol period becomes shorter. If the MOI is equal to the MPR, BER of the corresponding SPAD-based system is always above $10^{-3}$. This means that the system can not maintain a high quality communication and thus the minimum symbol period can be obtained. As the bit rate is equal to the spectral efficiency divided by the symbol period, the maximum bit rate can also be obtained. By using the presented analytical BER model of the SPAD-based OFDM system in this study, the relationship between spectral efficiencies and theoretical maximum bit rates is achieved and shown in Figure 5.12. The theoretical maximum bit rate of the SPAD-based OFDM system is up to 1 Gbits/s. Unlike the PD-based system, the increase of the spectral efficiency can not bring a high bit rate in SPAD-based OFDM as the limitation of the nonlinear distortion effect.
Figure 5.12: Theoretical maximum bit rates of SPAD-based OFDM systems when BER = $10^{-3}$, PQ SPAD (solid lines) vs. AQ SPAD (dashed lines).

However, since the SPAD receiver performs a significant enhancement on the power efficiency and sensitivity [68], the maximum bit rate of the SPAD-based OFDM can be much higher than the conventional PD-based OFDM in the same transmission power condition.

5.5.3 Comparison Between SPAD-based OFDM and Conventional O-OFDM

Figure 5.13 and Figure 5.14 show the comparison between the performance of the SPAD-based OFDM system and the conventional PD-based OFDM system in the absence of background light. The figures show the spectral efficiency of ACO-OFDM and DCO-OFDM with 7 dB and 13 dB bias versus MPR when $T_s = 1$ ms and $T_s = 1 \mu s$ respectively. Both figures show that the SPAD-based OFDM needs much less optical irradiance than the PD-based OFDM. A SPAD is an APD which is biased beyond reverse breakdown in the 'Geiger' region. In this mode of operation, a SPAD triggers billions of electron-hole pair generations for each detected photon. As a consequence, the device is extremely sensitive, and is able to accurately detect a single photon. In a PD, a received photon should be transformed to a large current through an amplifier. It will produce thermal noise whose effect is relatively greater than the effect of the shot noise in a SPAD. Thus the SPAD-based OFDM can demodulate signals from a relatively lower number of photons. The conventional PD-based OFDM needs more power to offset the
Optical Orthogonal Frequency Division Multiplexing with Single-Photon Avalanche Diode

Figure 5.13: Comparison between SPAD-based OFDM and conventional PD-based OFDM when $T_s = 1 \text{ ms}$. The minimum power requirement (MPR) of each system are compared when ACO-OFDM, DCO-OFDM with 7 dB DC-bias and DCO-OFDM with 13 dB DC-bias are applied.

effect of the thermal noise of the amplifier. For example, in Figure 5.13, the SPAD-based OFDM with 13 dB bias needs 39.8 dB less optical irradiance than the conventional PD-based OFDM with 13 dB bias at 3 bits/s/Hz. Compared with long symbol duration (Figure 5.13), when $T_s = 1 \mu s$ (Figure 5.14), the SPAD system has 30.7 dB greater power efficiency than the conventional PD system. Since the nonlinear distortion effect in the SPAD-based OFDM significantly increase the symbol error rate, the performance of the SPAD-based OFDM tends towards the performance of the PD-based OFDM. Moreover, the nonlinear distortion effect limits the spectral efficiency and the transmission speed as shown in Figure 5.14. Therefore, the superiority of the SPAD-based OFDM system against the conventional PD-based system is more significant at lower power transmission and lower transmission rates.

5.6 Summary

In this chapter, a complete analytical approach is presented for the performance analysis of the SPAD-based OFDM system with a receiver nonlinear distortion. The proposed theory shows very good agreement with the Monte Carlo simulation, thus confirming the validity of the analytical approach. The presented analytical models provide a quick and accurate way to estimate
Figure 5.14: Comparison between SPAD-based OFDM and conventional PD-based OFDM when $T_s = 1 \mu s$. The minimum power requirement (MPR) of each system are compared when ACO-OFDM, DCO-OFDM with 7 db DC-bias and DCO-OFDM with 13 db DC-bias are applied.

system performance and to choose optimal parameters of the PQ and AQ SPAD receivers for the ACO-OFDM and DCO-OFDM system. For the assumed SPAD-based OFDM system, the nonlinear distortion has a significant effect on the BER performance when the optical irradiance is higher than -40 dBm. This maximum optical irradiance limits the maximum bit rate of the system which is up to 1 Gbits/s as shown in this study.

Compared with the conventional PD-based OFDM system, ACO-OFDM and DCO-OFDM with the SPAD receiver have the same properties, such as the clipping distortion in DCO-OFDM. DCO-OFDM also has a better spectral efficiency than ACO-OFDM, and ACO-OFDM has a better power efficiency. However, the SPAD receiver has a significantly enhanced sensitivity. When the transmission power and speed are low, the SPAD-based OFDM has better performance than the conventional PD-based OFDM. This means that the SPAD-based OFDM system can be used in long distance transmissions, or it can be used in non-line-of-sight optical wireless communications, in the uplink when illumination is not essential, or when lights are almost completely dimmed, such as in a gas well downhole monitoring system. Due to such high sensitivity, an appropriate transmission power should be selected carefully so as to avoid the nonlinear distortion.
Chapter 6
Conclusions and Future Work

In this thesis, a number of novel orthogonal frequency division multiplexing (OFDM) schemes have been proposed for optical wireless communication (OWC) systems and they are compared with conventional optical orthogonal frequency division multiplexing (O-OFDM) methods, such as DC-biased optical orthogonal frequency division multiplexing (DCO-OFDM) and asymmetrically clipped optical orthogonal frequency division multiplexing (ACO-OFDM). In this thesis, based on optical spatial modulation (OSM), a novel OFDM scheme, referred to as non-DC-biased orthogonal frequency division multiplexing (NDC-OFDM), has been proposed. In addition, the corresponding theoretical analysis has been given. In order to decrease the system power consumption effectively, a high-sensitivity optical receiver, referred to as single photon avalanche diode (SPAD), has been investigated and applied to a permanent downhole monitoring (PDM) system. As a further research on SPAD receivers in visible light communication (VLC), a novel SPAD-based OFDM system has been established. Moreover, a complete theoretical analysis of this system has been provided with the consideration of the receiver nonlinear distortion problem.


6.1 Conclusions

A novel optical receiver for OWC, termed as SPAD, has been introduced and a related power-efficient VLC application to PDM systems has been proposed in the first part of this thesis. Based on the work principle of SPAD, a complete simulation method for receiving photons has been provided. Using this method, the number of photon counts can be tracked approximately. In addition, the nonlinear distortion caused by the dead time effect of SPAD has been analysed. The derived nonlinear function can be used to calculate the BER performance of SPAD-based modulation schemes. In Section 4.3, a novel VLC application has been demonstrated for the first time, which provides a different choice for establishing the PDM system. By using a SPAD receiver, the proposed system can achieve a 30 dB higher SNR than conventional photodiode (PD) based systems. Simulation results have shown that the LED transmitter can send monitoring signals in a 4,000 metres long gas pipe and the power consumption is only 8 dBm when the BER of the system is lower than $10^{-3}$. This evidences that the battery-powered VLC system can achieve a sufficiently long service time. In this study, a theoretical BER performance of the system has also been presented, which matches well with the simulation results. This theoretical calculation method can be used to predict the average required power of the LED transmitter with variable conditions.

In the second part of this thesis, a novel unipolar OFDM modulation scheme based on OSM, termed as NDC-OFDM, has been proposed. The novel scheme is an alternative to DCO-OFDM and ACO-OFDM in OSM. In NDC-OFDM, indices of transmitters represent the signs of bipolar OFDM symbols. This work has shown for the first time that the spatial dimension is exploited in a novel way. In this thesis, a theoretical framework has been presented for evaluating the bit-error ratio (BER) performance of NDC-OFDM in a $2 \times 2$ optical multiple-input multiple-output (O-MIMO) system. In Section 3.4.1, the Monte Carlo simulations confirm the validity of the theoretical BER analysis. This means that the system performance of NDC-OFDM can be achieved by these equations quickly and accurately. Moreover, the theoretical spectral efficiency of NDC-OFDM has been presented and compared with DCO-OFDM and ACO-OFDM. The BER performance of NDC-OFDM is compared with DCO-OFDM and ACO-OFDM in Section 3.4.2 by Monte Carlo simulations. Results have shown that NDC-OFDM can achieve a 5 dB signal-to-noise ratio (SNR) gain over the conventional O-OFDM methods in the case of low-correlation channels. When the correlation between each optical channel is high, this method can improve the power efficiency by up to 10 dB. Although NDC-OFDM requires ad-
ditional hardware at the transmitter and receiver, modern VLC systems can fit this demand as they are expected to employ multiple low-cost light emitting diodes (LEDs) to fulfill minimum indoor lighting conditions.

In the last part of this thesis, a SPAD-based OFDM system has been proposed for OWC for the first time. Unlike O-OFDM in conventional PD-based systems, the received signal in SPAD is output in the type of photon counts. In this thesis, a feasible transformation method between the photon count and the digital signal has been proposed by considering nonlinear distortion effects and specific noises in SPAD, including the dark current noise, the after pulsing effect and the shot noise. It has been found that when the transmission speed is low, the SPAD-based system can improve the receiver power efficiency of O-OFDM by up to 39.8 dB over conventional PD-based systems. Since the SPAD receiver has a significantly enhanced sensitivity, the device is significantly saturated by a high optical irradiance. This leads to a receiver nonlinear distortion in SPAD-based OFDM systems. It has been found that the receiver nonlinear distortion limits the maximum data rate of SPAD-based OFDM systems. In Section 5.3, considering the transmitter and receiver nonlinear effects, a complete theoretical framework for evaluating the BER performance of this system has been proposed. The theoretical BER results have shown a very good agreement with the Monte Carlo simulation, thus confirming the validity of the analytical approach. The presented analytical models have provided a quick and accurate way to estimate the system performance and to choose optimal parameters of SPAD receivers for ACO-OFDM and DCO-OFDM. In addition, based on this framework, a SPAD receiver with lower nonlinear distortion effect can be designed in the future work so that a higher data rate can be achieved.

6.2 Limitations and Future Work

Some basic concepts of the SPAD receiver in OWC have been presented in Chapter 3 and one of its practical application in VLC has been proposed. However, the light distribution in the gas pipe has not been analysed, which can be used to choose a optimal position for the transmitter and the receiver. The presented channel model equations are only based on the general assumptions of the line-of-sight (LOS) and non-line-of-sight (NLOS) systems. A complete channel model can be derived according to a comprehensive analysis of the light distribution. Moreover, researches on further applications of SPAD receivers in OWC still remain open, such as in a long distance free-space optical (FSO) links or in a wavelength-
Conclusions and Future Work

division multiplexing (WDM) optical system. Following advanced investigations are proposed for the SPAD-based OWC system in the future work:

i The comprehensive light distribution in the long-distance gas pipe needs to be analysed in order to find some optimal positions.

ii A SPAD-based long-distance FSO system can be proposed due to its high power efficiency.

iii Some experiments of basic SPAD-based OWC systems need to be realised with different types of optical filters.

iv A SPAD-based O-MIMO system needs to be proposed and compared with the conventional PD-based O-MIMO systems.

The basic system model and a theoretical framework of NDC-OFDM have been presented in Chapter 4. However, the proposed system is based on a $2 \times 2$ O-MIMO channel. In principle, with the increasing number of transmitters and receivers in a multiple-input multiple-output (MIMO), spatial modulation (SM) has better performance. Since modern VLC systems are expected to employ multiple low-cost LEDs to fulfill minimum indoor lighting conditions, NDC-OFDM could be suitable for a larger MIMO system. Moreover, a comprehensive theoretical analysis of NDC-OFDM with more transmitters and receivers needs to be derived. In this chapter, the analytical results are only confirmed by Monte Carlo simulation results. Some practical experiments are still in need to confirm the applicability of the system. Thus, following advanced studies are proposed for the NDC-OFDM system in the future work:

i The NDC-OFDM system needs to be simulated in a $4 \times 4$, $8 \times 8$ and $16 \times 16$ O-MIMO channel and the performance will be compared with the corresponding conventional OSM-based OFDM systems.

ii A larger O-MIMO system needs to be designed based on the concept of the NDC-OFDM systems and calculate the system capacity.

iii A comprehensive analytical model of large NDC-OFDM needs to be derived and compared with corresponding simulation results.

iv By considering the multi-user communication in an optical network, the principle of NDC-OFDM needs to be extended.

v Practical experiments need to be realised.

A comprehensive analysis of the SPAD-based OFDM systems has been presented in Chapter 5. However, the validity of the analytical results needs to be confirmed by some related experi-
In addition, since O-OFDM has been proposed for a multi-user optical network in some previous research, the SPAD-based OFDM would also be realised and show its performance in an optical network system. With a better power efficiency than the conventional O-OFDM systems, SPAD-based OFDM systems would have a significant performance in an optical attocell network. In Chapter 5, since only DCO-OFDM and ACO-OFDM are tested in the SPAD-based system, other O-OFDM schemes need to be realised, such as U-OFDM and NDC-OFDM. Finally, following advanced studies are proposed for the SPAD-based OFDM system in the future work:

i Some single link experiments of SPAD-based OFDM systems need to be realised and compared with the analytical results.
ii A multi-user optical attocell network based on the SPAD-based OFDM can be proposed.
iii A NDC-OFDM with SPAD receiver needs to be realised.
iv Some experiments of SPAD-based OFDM in O-MIMO channels need to be realised.
Appendix A

Basic Derivations of Equation (5.27), (5.26), (5.30), (5.29) and (5.28)

Based on the theoretical algorithm of nonlinear distortion presented in [62], some basic equations are derived here for (5.27), (5.26), (5.30), (5.29) and (5.28). Two of well-known differential equations for Gaussian distribution are introduced:

\[
\frac{dQ}{dt} \left( \frac{x - \mu_x - t_x \sigma_x^2}{\sigma_x} \right) = \sigma_x \phi \left( \frac{x - \mu_x - t_x \sigma_x^2}{\sigma_x} \right),
\]

(A.1)

\[
\frac{d\phi}{dt} \left( \frac{x - \mu_x - t_x \sigma_x^2}{\sigma_x} \right) = \sigma_x \phi \left( \frac{x - \mu_x - t_x \sigma_x^2}{\sigma_x} \right) \left( \frac{x - \mu_x - t_x \sigma_x^2}{\sigma_x} \right),
\]

(A.2)

where \( \mu_x \) denotes the mean value of the distribution and \( t_x \) is a moment variable. Therefore, two components in the objective equations are derived as:

\[
\int_{x_{\min}}^{x_{\max}} \frac{x}{\sigma_x} \phi \left( \frac{x - \mu_x}{\sigma_x} \right) dx = \int_{x_{\min}}^{x_{\max}} \frac{x}{\sqrt{2\pi \sigma_x^2}} \exp \left( -\frac{(x - \mu_x)^2}{2\sigma_x^2} \right) dx
\]

\[= \left. \int_{x_{\min}}^{x_{\max}} \frac{x}{\sqrt{2\pi \sigma_x^2}} \exp \left( -\frac{(x - \mu_x)^2}{2\sigma_x^2} \right) dx \right|_{t_x=0}
\]

\[= \left. \int_{x_{\min}}^{x_{\max}} \frac{x}{\sqrt{2\pi \sigma_x^2}} \exp \left( -\frac{(x - \mu_x)^2}{2\sigma_x^2} \right) dx \right|_{t_x=0}
\]

\[= \frac{d}{dt_x} \int_{x_{\min}}^{x_{\max}} \frac{x}{\sqrt{2\pi \sigma_x^2}} \exp \left( -\frac{(x - \mu_x)^2}{2\sigma_x^2} \right) dx \bigg|_{t_x=0}
\]

\[= \frac{d}{dt_x} \int_{x_{\min}}^{x_{\max}} \frac{1}{\sqrt{2\pi \sigma_x^2}} \exp \left( -\frac{(x - \mu_x - t_x \sigma_x^2)^2 - t_x^2 \sigma_x^4 - 2t_x \sigma_x^2 \mu_x}{2\sigma_x^2} \right) dx \bigg|_{t_x=0}
\]

\[= \frac{d}{dt_x} \exp \left( \frac{t_x^2 \sigma_x^2}{2} + t_x \mu_x \right) \int_{x_{\min}}^{x_{\max}} \frac{1}{\sqrt{2\pi \sigma_x^2}} \exp \left( -\frac{(x - \mu_x - t_x \sigma_x^2)^2}{2\sigma_x^2} \right) dx \bigg|_{t_x=0}
\]

\[= \frac{d}{dt_x} \exp \left( \frac{t_x^2 \sigma_x^2}{2} + t_x \mu_x \right) \left[ Q \left( \frac{x_{\min} - \mu_x - t_x \sigma_x^2}{\sigma_x} \right) - Q \left( \frac{x_{\max} - \mu_x - t_x \sigma_x^2}{\sigma_x} \right) \right] \bigg|_{t_x=0}
\]
Basic Derivations of Equation (5.27), (5.26), (5.30), (5.29) and (5.28)

\[
(A.1) \quad (\sigma^2 t_x + \mu_x) \exp \left( \frac{t_x^2 \sigma^2}{2} + t_x \mu_x \right) \left[ Q \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) - Q \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \right] \\
+ \exp \left( \frac{t_x^2 \sigma^2}{2} + t_x \mu_x \right) \sigma_x \left[ \phi \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) - \phi \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \right]_{t_x=0} \\
= \mu_x \left[ Q \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) - Q \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \right] \\
+ \sigma_x \left[ \phi \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) - \phi \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \right], \quad (A.3)
\]

and

\[
\int_{x_{\text{min}}}^{x_{\text{max}}} \frac{x^2}{\sigma_x^2} \phi \left( \frac{x - \mu_x}{\sigma_x} \right) \, dx = \int_{x_{\text{min}}}^{x_{\text{max}}} \frac{x^2}{\sqrt{2\pi\sigma_x^2}} \exp \left( -\frac{(x - \mu_x)^2}{2\sigma_x^2} \right) \, dx \\
= \int_{x_{\text{min}}}^{x_{\text{max}}} \frac{x^2 \exp(x t_x)}{\sqrt{2\pi\sigma_x^2}} \exp \left( -\frac{(x - \mu_x)^2}{2\sigma_x^2} \right) \, dx \bigg|_{t_x=0} \\
= \int_{x_{\text{min}}}^{x_{\text{max}}} \frac{d^2}{dt_x^2} \exp(x t_x) \exp \left( -\frac{(x - \mu_x)^2}{2\sigma_x^2} \right) \, dx \bigg|_{t_x=0} \\
= \frac{d^2}{dt_x^2} \exp \left( \frac{t_x^2 \sigma^2}{2} + t_x \mu_x \right) \left[ Q \left( \frac{x_{\text{min}} - \mu_x - t_x \sigma^2}{\sigma_x} \right) - Q \left( \frac{x_{\text{max}} - \mu_x - t_x \sigma^2}{\sigma_x} \right) \right]_{t_x=0} \\
= \frac{d}{dt_x} \left\{ \sigma_x^2 t_x + \mu_x \right\} \exp \left( \frac{t_x^2 \sigma^2}{2} + t_x \mu_x \right) \left[ Q \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) - Q \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \right] \\
+ \exp \left( \frac{t_x^2 \sigma^2}{2} + t_x \mu_x \right) \sigma_x \left[ \phi \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) - \phi \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \right]_{t_x=0} \\
= \frac{\mu_x^2}{\sigma_x^2} \left[ Q \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) - Q \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \right] \\
- Q \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) + 2 \mu_x \sigma_x \left[ \phi \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) - \phi \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \right] \\
+ \sigma_x^2 \left[ \phi \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) \left( \frac{x_{\text{min}} - \mu_x}{\sigma_x} \right) - \phi \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \left( \frac{x_{\text{max}} - \mu_x}{\sigma_x} \right) \right], \quad (A.4)
\]
Appendix B

Selected Publications

B.1 Journal Papers


B.2 Conference Papers


Optical OFDM With Single-Photon Avalanche Diode

Yichen Li, Majid Safari, Robert Henderson, and Harald Haas

Abstract—In this letter, an optical orthogonal frequency division multiplexing (OFDM) system based on a single-photon avalanche diode (SPAD) receiver is presented. SPAD detectors do not require a transimpedance amplifier as they operate in the Geiger mode. This poses challenges on detecting continuous data carrying signals. However, in this letter, we show that OFDM signals can indeed be detected, and the bit error ratio performances of both dc-biased optical OFDM and asymmetrically clipped optical OFDM are investigated. The investigations are carried out in the absence of background light. The results are compared with those of standard optical OFDM with conventional photodiode receivers. The SPAD-based OFDM system shows superior power efficiency over the state-of-the-art counterpart, and it can reach receiver sensitivities that are comparable with existing radio frequency systems.

Index Terms—Visible light communication (VLC), Li-Fi, photon counting receiver, single-photon avalanche diode (SPAD).

I. INTRODUCTION

CURRENT visible light communication (VLC) systems are mainly realized by high speed light emitting diodes (LEDs) as transmitters and photodiodes (PDs) as receivers. To date, a wireless VLC system using a single LED can achieve speeds greater than 3 Gb/s [1]. However, the incoherent light output of the LED means that information can only be encoded in the intensity level. As a consequence, only real-valued and positive signals can be used for data modulation. This is in contrast to radio frequency (RF) systems which make use of complex valued and bi-polar signals. Therefore, data modulation in VLC system is accomplished by using intensity modulation (IM) and direct detection (DD) system. On-off keying (OOK), pulse position modulation (PPM) and pulse amplitude modulation (PAM) are some of the common modulation schemes used in conjunction with IM/DD transmission [2].

For high-speed data transmission, orthogonal frequency division multiplexing (OFDM) is applied in order to get closer to the channel capacity by employing adaptive bit and power loading. Standard OFDM used in RF communications has to be adapted to realise IM/DD systems [3]. Optical OFDM (O-OFDM) needs to produce real-valued symbols. This can be achieved by imposing Hermitian symmetry on the information frame before the inverse fast Fourier transform (IFFT) operation during the signal generation phase. This comes at the expense of half of the spectral efficiency. Various O-OFDM modulation schemes have been realized in VLC such as DC-biased optical OFDM (DCO-OFDM), asymmetrically clipped optical OFDM (ACO-OFDM), unipolar OFDM (U-OFDM) and non-DC-biased OFDM (NDC-OFDM) [4]–[6].

In general, VLC systems make use of conventional PDs, such as positive-intrinsic-negative (PIN) diodes and avalanche photo diodes (APDs). However, for low power and long distance transmission, these PDs may not yield satisfactory performance results as the transimpedance amplifier (TIA) significantly reduces the sensitivity of the receiver. In this letter, single-photon avalanche diodes (SPADs) are considered as detector devices for O-OFDM systems. The SPAD detector does not require a TIA, and, thus, the output signal is not distorted by thermal noise in the same way as in PDs. This is one reason for the higher sensitivity of a SPAD detector. When the VLC system is applied in long distance transmission, such as in a gas well downhole monitoring system [7] and data transmission over plastic optical fibres (POF) [8], the number of photons reaching the receivers are typically significantly smaller than in short-distance communication links, and the signal is buried in noise. In these scenarios, and when compared with conventional PDs, a SPAD-based receiver is expected to be most suitable. A SPAD receiver can only detect one photon within a device specific dead time which constraints the ability to recover a signal. In addition, since the output of the detector is a photon count value, there is a maximum number of photons that the system can detect. This limits the maximum tolerable optical irradiance which results in a receiver clipping distortion. This letter investigates the performance of the SPAD-based OFDM system over long-distance optical links in the absence of background light.

The rest of this letter is organized as follows. The system model of the OFDM system with an SPAD array receiver is described in Section II. Section III reports results arising from the comparison with the PD-based OFDMs system followed by a discussion. Finally, Section IV concludes this letter.

II. PRINCIPLES OF OFDM WITH SPAD

A. Optical OFDM Modulation and Demodulation

Fig. 1 illustrates the system model of OFDM with SPAD receivers. The transmitter part is the same as in OFDM systems with PD receivers [9]. The input bit stream is transformed into complex symbols, $X(n)$, by a $M$-QAM modulator, where $M$ is the constellation size. The symbols are allocated on to $N$ subcarriers, $X(k), k = 0, \cdots , N-1$. Manuscript received October 30, 2014; revised January 21, 2015; accepted January 28, 2015. Date of publication February 10, 2015; date of current version April 6, 2015. This work was supported by the U.K. Engineering and Physical Sciences Research Council under Grant EP/K008757/1. Y. Li, M. Safari, and H. Haas are with the Li-Fi Research and Development Centre, The University of Edinburgh, Edinburgh EH9 9YL, U.K. (e-mail: yichen.li@ed.ac.uk; majid.safari@ed.ac.uk; h.haas@ed.ac.uk). R. Henderson is with the School of Engineering, Institute for Integrated Micro and Nano Systems, The University of Edinburgh, Edinburgh EH9 9YL, U.K. (e-mail: robert.henderson@ed.ac.uk).

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In OFDM, $N$ denotes the size of IFFT/FFT, where $N$ is set to 2048 in this letter. Two state-of-the-art techniques, DCO-OFDM and ACO-OFDM, are considered to obtain positive and real-valued OFDM symbols [9], [10]. In DCO-OFDM, $N/2 - 1$ symbols in $X(n)$, $n = 1, \ldots, N/2 - 1$, are put into the first half of subcarriers and the DC subcarrier (the first subcarrier) is set to zero. In ACO-OFDM, $N/4$ QAM symbols in $X(n)$, $n = 1, \ldots, N/4$, are mapped on to half of the odd subcarriers of the OFDM frame, $X(k)$, $k = 1, 3, 5, \ldots, N/2 - 1$. At the same time, the even subcarriers are set to zero. In both ACO-OFDM and DCO-OFDM, Hermitian symmetry is applied to the rest of the OFDM frame in order to obtain real-valued symbols through the IFFT block. Since LED applied to the rest of the OFDM frame in order to obtain ACO-OFDM and DCO-OFDM, Hermitian symmetry is the same time, the even subcarriers are set to zero. In both ACO-OFDM and DCO-OFDM, Hermitian symmetry is applied to the rest of the OFDM frame in order to obtain real-valued symbols through the IFFT block. Since LED transmitters can only send unipolar signals, the real-valued OFDM symbols need to be clipped. In DCO-OFDM, a DC bias is added to make the signal unipolar [10]. The DC bias, which is related to the average power of the OFDM symbols, is introduced and defined in [9] as,

$$B_{DC} = \alpha \sqrt{E[|x(k)|^2]},$$

(1)

where $x(k)$ is the OFDM symbol frame vector, and $10 \log_{10}(\alpha^2 + 1)$ is defined as the bias level in dB. The bias level in the current simulations is set to 7 dB and 13 dB, which are adopted from [9] for consistency. After the DC-bias, the OFDM frame is simply clipped by,

$$x_{\text{clipped}}(k) = \begin{cases} x_{\text{biased}}(k), & x_{\text{biased}}(k) \geq 0, \\ 0, & x_{\text{biased}}(k) < 0, \end{cases}$$

(2)

where $x_{\text{biased}}(k)$ is the DC-biased symbol which is calculated as $x_{\text{biased}}(k) = x(k) + B_{DC}$. The clipped unipolar symbol is denoted by $x_{\text{clipped}}(k)$. In ACO-OFDM, since symbols are antisymmetric, clipped unipolar symbols are obtained by setting the negative part to zero. In the simulation, after being transformed into an optical intensity signal, the clipped signal is transmitted by a LED.

In this letter, we focus on the performance comparison between SPAD and PD over a long distance and in the absence of background light, and thus optical signals are assumed to pass through a flat fading channel and to be distorted by shot noise and/or thermal noise. The thermal noise is dominant in the case of PD receivers while the shot noise caused by the received signal and the detector’s dark current are the major noise contributors in a SPAD receiver. Assuming that there is no other distortion effect during the transmission, the recovered signal, $x(k)$, can be scaled to the original clipped signal, $x_{\text{clipped}}(k)$. The recovered OFDM symbols from the SPAD or the PD are passed through a FFT operation. In DCO-OFDM, $N/2 - 1$ symbols are obtained from the corresponding subcarriers to constitute a QAM symbol frame, $X(n)$. In ACO-OFDM, $N/4$ symbols are obtained. The detected QAM symbols are then decoded by using an estimator in order to obtain the output bit stream.

### B. SPAD Receiver

A SPAD device is an APD in the ‘Geiger’ mode which emits a very large current by receiving a single photon and thus can essentially be modelled as a single photon counter. The photodetection process of an ideal photon counter can be modelled using Poisson statistics which describe the shot noise effect [11] and references therein).

In this letter, we consider an array of SPADs [12] which outputs the superposition of the photon counts from the individual SPADs. In order to generate received O-OFDM samples, the output of the SPAD array is counted over a short-time average period, $T_{ST}$, at time instances $t_k = kT_s$, of the received optical signal $x(t)$. These photon counts are denoted by $v(k)$ as shown in Fig. 1. Note that $T_s$ which denotes the sampling period of the time-domain OFDM signal at the transmitter is assumed to be much larger than $T_{ST}$.

The photon detection rate of SPADs is typically limited by a number of practical constraints such as the SPAD dead time. In fact, each individual SPAD in the array can only receive one photon during the dead time, $t_d$. Thus, the maximum number of counted photons in $T_{ST}$ is,

$$v_{\text{max}} \leq \frac{N_{\text{SPAD}} T_{ST}}{t_d},$$

(3)

where $N_{\text{SPAD}}$ is the number of SPAD devices in the array. If the number of incoming photons is more than $v_{\text{max}}$, the number of counted photons will be clipped which causes a receiving clipping distortion problem. In this letter, however, we assume that the short-time average period, $T_{ST}$, is much longer than the dead time. Thus the photon counts at the output
Selected Publications

LI et al.: OPTICAL OFDM WITH SPAD

TABLE I

<table>
<thead>
<tr>
<th>SIMULATION PARAMETERS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>The wavelength of the received light $\lambda_{DL}$</td>
<td>450 nm</td>
</tr>
<tr>
<td>The PDE of the SPAD $C_{PE}$ [12]</td>
<td>20%</td>
</tr>
<tr>
<td>The DCR of the SPAD $N_{DCR}$ [12]</td>
<td>7.27 kHz</td>
</tr>
<tr>
<td>The dead time of the SPAD $t_{dead}$ [12]</td>
<td>13.5 ns</td>
</tr>
<tr>
<td>Number of SPADs in an array $N_{SPAD}$ [12]</td>
<td>1024</td>
</tr>
<tr>
<td>The responsivity of the PD</td>
<td>0.2</td>
</tr>
<tr>
<td>Input referred noise of the PD</td>
<td>16 pA/$\sqrt{Hz}$</td>
</tr>
</tbody>
</table>

of the SPAD array (i.e., $v(k)$) can be still described by Poisson distribution as,

$$\Pr(v(k) = j) = \exp(-\mu(k)) \frac{\mu(k)^j}{j!},$$

where the average photon counts $\mu(k)$ can be expressed as a function of the received signal and the SPAD’s dark count rate (DCR), $N_{DCR}$, as,

$$\mu(k) = \frac{C_{PE} E_{P}}{h c \lambda_{DL}} \int_{t_{dead}}^{t_{TST}} x_{S}(t) dt + N_{DCR} T_{ST},$$

where $C_{PE}$ denotes the photon detection efficiency (PDE) of the SPADs. Note that the shot noise described above is not only caused by the randomness in receiving discrete signal photons but also generated by the SPAD’s dark photon emission at the fixed rate $N_{DCR}$. The energy of a photon is $E_{P}$ which is calculated by $\frac{h c}{\lambda_{DL}}$. Planck’s constant is represented by $h$; $c$ is the speed of the light; and $\lambda_{DL}$ is the wavelength of the light.

The output of the SPAD array is the number of photons, and the system is designed for an O-OFDM demodulator which requires the amplitude of the electrical signal (optical power) to demodulate the received signal to the original encoded bits. Thus, the photon-to-amplitude equalizer is used to simply convert the received photon number to the corresponding electrical signal amplitude (optical power), $x'(k)$. The coefficient of the equalizer is calculated by a pilot which captures the effect of channel attenuation and PDE.

### III. Results and Comparisons

The performance of an OFDM system employing either a SPAD receiver as described in Section II or a standard PD receiver as used in [9] is investigated. The LED transmitter emits blue light with a wavelength distribution centered around 450 nm. We consider a SPAD array of size $N_{SPAD} = 1024$ as introduced in [12]. In the PD-based OFDM system, a high speed PIN receiver, New Focus 1601FS-AC, is used. The PD receiver has been used and tested in [1]. Since there is no background light, the thermal noise is the main noise component at the PD receiver. The parameters which are applied in the simulation are given in Table I.

#### A. Performance of OFDM With SPAD

Fig. 2 shows the simulation results for the BER of the SPAD-based DCO-OFDM system as a function of the optical irradiance when the transmission speed is 1 kbits/s.

The performance of DCO-OFDM for 4, 16, 64 and 256-QAM with 7 dB and 13 dB DC-bias are compared in this figure. For DCO-OFDM with lower constellation sizes (4, 16-QAM), the 7 dB DC-bias system requires less optical irradiance than the 13 dB system. For 64-QAM and 256-QAM, clipping distortions create an error floor in DCO-OFDM with 7 dB DC-bias. Thus for higher constellation sizes, a higher DC-bias may need to be applied in DCO-OFDM.

#### B. Comparison Between SPAD and PD

Fig. 3 shows the BER performance of the SPAD-based ACO-OFDM system. The transmission speed is also set to 1 kbits/s. Modulation schemes are 4, 16, 64, 256 and 1024-QAM. Compared with DCO-OFDM, ACO-OFDM requires less energy as expected. As shown in Fig. 3, the 4-QAM SPAD-based ACO-OFDM system is able to demodulate the signal by receiving only $-107$ dBm optical power to at a BER of $10^{-3}$. For 4-QAM DCO-OFDM with 7 dB DC-bias, the optical power required at the SPAD receiver is approximately $-100$ dBm. The higher power efficiency of ACO-OFDM comes at the expense of approximately half spectral efficiency.
system and the PD-based OFDM system. The figures show the spectral efficiency of ACO-OFDM and DCO-OFDM with 7 dB and 13 dB bias versus optical irradiance, for a BER of $10^{-3}$ and transmission speed of 1 kbit/s and 1 Mbit/s, respectively. Both figures show that the SPAD-based OFDM system needs much less optical irradiance than the PD-based OFDM system. A SPAD triggers billions of electron-hole pairs by a single detected photon. As a consequence, the SPAD-based OFDM system can demodulate signals from a relatively lower number of photons, and consequently exhibits a much higher receiver sensitivity. The PD-based OFDM system needs more power to offset the effect of the thermal noise at the amplifier. For example, in Fig. 4, the SPAD-based OFDM system with 13 dB bias needs 46 dB less optical irradiance than the PD-based OFDM system with 13 dB bias at 3 bits/s/Hz. When compared with 1 kbit/s transmission speed (Fig. 4), for 1 Mbit/s (Fig. 5), the SPAD system has 30.5 dB greater power efficiency than the PD system. In Fig. 5, by significantly increasing the data rate, the number of photon counts at each sample drops with the same rate (1000 times) over the whole SPAD array resulting in a very low signal-to-error count ratio. Therefore, the received optical power needs to be significantly increased to ensure a reliable BER performance. When using the PD receiver, the required received optical power increases also as the data rate is augmented, but there is no energy loss due to detector dead times which is particularly critical when the dead time is approaching the sampling time. Therefore, the increasing data rate affects the error performance less significantly compared to the SPAD receiver.

IV. CONCLUSION

In this letter, an O-OFDM system with a SPAD receiver is presented and compared with state-of-the-art PD-based O-OFDM systems. When the transmission power and speed are low, in the region of 1 kbit/s, the SPAD-based OFDM system has significantly better performance than the PD-based OFDM system. However, when the transmission speed is 1 Mbit/s, the SPAD-based DCO-OFDM system with 13 dB DC bias still enhances the sensitivity by a staggering 30.5 dB over the PD-based DCO-OFDM system. The higher receiver sensitivity means that the SPAD-based OFDM system can be used in long distance transmissions, or in nonlinear of sight optical wireless communications, in the uplink when illumination is not essential, or when lights are almost completely dimmed. Note, the reported sensitivities are in the range of those known from standard RF systems.

REFERENCES

Performance Analysis of Non-DC-Biased OFDM

Yichen Li, Dobroslav Tsonev, Xiping Wu and Harald Haas

Abstract—Non-direct current (DC)-biased orthogonal frequency division multiplexing (OFDM), or NDC-OFDM, is a recently proposed modulation scheme for optical wireless communications. The basic concept of this method is to separate signs and magnitudes of OFDM symbols and transmit them through two information-carrying units: i) indices of two light emitting diode transmitters represent positive and negative signs respectively; and ii) optical intensity symbols carry the absolute values of signals. Thanks to the use of the optical spatial modulation (OSM) technique, NDC-OFDM is able to avoid clipping distortion caused by DC-biased optical OFDM (DCO-OFDM). In addition, NDC-OFDM can achieve similar advantages as asymmetrically clipped optical OFDM (ACO-OFDM) without using extra subcarriers. In this paper, an analytical performance evaluation of NDC-OFDM is presented. Also, the derived bit-error rate (BER) bound is validated through Monte Carlo simulations. Moreover, the BER performance of NDC-OFDM is compared with DCO-OFDM and ACO-OFDM. Various constellation sizes are considered in order to present a comprehensive understanding of the benefit of NDC-OFDM, in both spectral efficiency and power efficiency.

Index Terms—optical wireless communication, optical OFDM, optical spatial modulation, MIMO

I. INTRODUCTION

With the development of wireless services and applications since 2000, wireless data rates have been increasing exponentially. It is estimated that future 5th generation wireless systems will have speeds of 1 Gbps by 2020 [1]. Despite the fact that the hardware requirements for high speed transmission can be fulfilled, radio frequency (RF) spectrum is limited and unable to meet the future demand for data rates. As a viable complementary approach, optical wireless communication (OWC) has gained significant attention due to recent technological developments in solid state lighting technology [2]. The key advantage of OWC is that it offers almost infinite bandwidth ranging from infrared (IR) to ultraviolet (UV) [3]. Other important benefits of OWC are: license-free operation; high communication security; low-cost front-ends; and no interference with RF systems meaning that OWC and RF systems can be used simultaneously.

Current OWC systems are mainly realized by high speed light emitting diodes (LEDs) or laser diodes (LDs) as transmitters and highly sensitive photodiodes (PDs) as receivers. To date, an OWC system with a single visible light LED can achieve a data rate of about 3 Gb/s [4]. In addition, different types of PDs have been applied in OWC, including positive-intrinsic-negative (PIN) diodes, avalanche photo diodes (APDs) and single-photon avalanche diodes (SPADs) [5], [6]. However, the incoherent light output of the LED means that information can only be encoded in the intensity level. Thus, only real-valued and positive signals can be used for data modulation. This is in contrast to RF systems which make use of complex valued and bi-polar signals. As a consequence, OWC systems are usually considered to be realized as an intensity modulation (IM) and direct detection (DD) system [7]. On-off keying (OOK), pulse position modulation (PPM) and pulse amplitude modulation (PAM) are some of the common modulation schemes used in conjunction with IM/DD systems [7]–[10]. Recently, the optical orthogonal frequency division multiplexing (O-OFDM) modulation scheme has been demonstrated as a high-speed data transmission approach in the context of IM/DD systems [6], [11]–[14].

A. Optical OFDM

For a high-speed OWC system, O-OFDM is applied in order to enlarge the channel capacity by utilizing adaptive bit and power loading. The advantages of OFDM in OWC are the same as in RF which are described in [15]. Because the IM/DD system can only transmit real-valued signals, O-OFDM needs to produce real-valued symbols. This can be achieved by imposing Hermitian symmetry on the information frame before the inverse fast Fourier transform (IFFT) operation during the signal generation phase. However, this decreases the spectral efficiency by half. Diverse O-OFDM modulation schemes have been realized and utilized in OWC, such as DC-biased optical OFDM (DCO-OFDM), asymmetrically clipped optical OFDM (ACO-OFDM) and unipolar OFDM (U-OFDM) [13], [16], [17]. In DCO-OFDM, a DC-bias is added to the original OFDM signal and the negative part is clipped. Clipping in DCO-OFDM may cause nonlinear distortion which has a significant effect on the bit-error rate (BER) performance [18]. In ACO-OFDM, the system inserts zeros on even subcarriers and modulates only odd subcarriers. As a result, a group of antisymmetric real-valued OFDM symbols are obtained, as shown in [18]. This allows any negative samples to be clipped without distortion. Since only half of the subcarriers carry information bits, the spectral efficiency of ACO-OFDM is about half the spectral efficiency of DCO-OFDM. In U-OFDM, the positive part of OFDM symbols and the negative part of the symbols will be transmitted separately [17]. The positive block contains only the positive OFDM samples and zeros in place of the negative ones, while the negative block contains only the negative OFDM samples and zeros in place of the positive ones. At the transmitter, the positive block is transmitted first and the absolute value of the negative block is then transmitted. Since the number of the OFDM frames is doubled, U-OFDM also has half the spectral efficiency of DCO-OFDM.

B. Optical Spatial Modulation

In current 4th generation communication systems, OFDM multiple-input multiple-output (MIMO) is used as an efficient
quadrature amplitude modulation (QAM) modulator. N is the complex symbols, A. Modulation Procedure ensures that transmitted samples are positive and also saves Using the indices of LEDs to transmit signs of samples negative symbol is transmitted. Unlike the conventional OSM-OFDM system, the indices of transmitters in NDC-OFDM can only transmit positive signals, the absolute value of the positive OFDM symbol is transmitted by one LED and the sign of the symbol is represented by the index of the modulation block, symbols are transmitted by different LEDs. In NDC-OFDM, LEDs only send the absolute value of OFDM symbols are then modulated on to the first half of an OFDM frame, X(m), m = 0, · · · , N − 1, and the DC subcarrier (the first subcarrier) is set to zero. Then, Hermitian symmetry is imposed on the second half of the OFDM frame. Next, the mapped subcarriers are passed through an IFFT block. Without loss of generality, the following definition of inverse discrete Fourier transform is used [18]:

\[ x(k) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X(m) \exp \left( \frac{j2\pi km}{N} \right). \]  

(1)

After the N-IFFT operation, the complex QAM symbols become N real-valued OFDM samples, x(k), but they are still bipolar. In the conventional DCO-OFDM system, a DC bias is added and the signal is then clipped to obtain the unipolar sample. In practice, the value of the DC bias, which is related to the average power of the OFDM symbols, is defined in [27] as:

\[ B_{DC} = \alpha \sqrt{E[x^2(k)]}, \]  

(2)

where \( 10 \log_{10}(\alpha^2 + 1) \) is defined as the bias level in dB which depends on the constellation size. For the simple DCO-OFDM model, positive samples, which can be transmitted by LEDs, are obtained by signal clipping after a fixed power for the DC bias is added. However, the added DC bias increases the power consumption. More importantly, if the level of DC-bias is not enough to ensure all the samples are positive, the signal clipping will cause the bottom distortion problem [28].

In NDC-OFDM, LEDs only send the absolute value of x(k) and the sign of the symbol is represented by the index of the corresponding LED. According to the working principle of OSM, only one LED is activated during one symbol time. If the transmitted symbol is positive, the first LED will be activated to send the symbol. If the symbol is negative, its absolute value is transmitted, this system does not need additional DC-bias power to obtain positive signals.

In general, in an OFDM-based system a cyclic prefix (CP) is added to resist ISI before the samples are transmitted, but for simplicity, it is not considered in the theoretical performance analysis in this study.

Finally, the digital signals in SM frame vectors, Ld(k) and Lc(k), are transmitted by the LEDs.

B. Optical Channel

As shown in Fig. 1, the converted optical signals are transmitted by the corresponding LED over the optical MIMO channel H [25]. Without loss of generality, a simple \( N_1 \times N_r \) optical channel matrix is realized:

\[ H = \begin{pmatrix}
    h_{11} & h_{12} & \cdots & h_{1N_r} \\
    h_{21} & h_{22} & \cdots & h_{2N_r} \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{N_1,1} & h_{N_1,2} & \cdots & h_{N_1,N_r}
\end{pmatrix}. \]  

(3)
Fig. 1. Block diagram of the NDC-OFDM system

where $h_{N_rN_t}$ is the channel DC gain of a directed line-of-sight (LOS) link between the receiver $N_r$ and the transmitter $N_t$. The LOS link is mainly considered in the system model because typically the non-line-of-sight (NLOS) links are significantly weaker and can thus be neglected. The channel gain is calculated as follows [7]:

$$h_{N_rN_t} = \frac{\beta A^2 \pi d^2 \cos(\psi)}{T_s g_c \cos(\psi)}, \quad 0 \leq \psi \leq \Psi_c, \quad \psi > \Psi_c, \quad (4)$$

where

- $\beta$ is related to $\beta_{1/2}$ which is the transmitter semiangle, by $\beta = -\ln 2/\ln(\cos(\beta_{1/2}))$;
- $A$ is the detector area of the PD;
- $d$ is the distance between the receiver $N_r$ and the transmitter $N_t$;
- $\psi$ is the radiant angle;
- $\Psi_c$ is the incident angle;
- $T_s$ is the optical filter gain;
- $g_c$ is the optical concentrator gain;
- $\Psi_c$ is the field of view of the receiver.

C. Detection and Demodulation

Through the optical MIMO channel, correlated optical signals are detected and obtained by PD receivers. The received signal can be written as:

$$y = Hs + w, \quad (5)$$

where $y$ is the $N_r$-dimensional received vector and $s$ is the $N_t$-dimensional transmitted signal vector in one symbol duration. In this study, both $N_r$ and $N_t$ are set to two. In addition, $w$ is the $N_r$-dimensional noise vector which is assumed to be real-valued additive white Gaussian noise (AWGN).

After the received optical OFDM signal is converted to an electrical signal by the PD and digitized, a zero forcing (ZF) equalizer is used to recover the transmitted symbols as follows [29]:

$$g = Cy, \quad (6)$$

where $g$ is a $N_t$-dimensional vector which contains the estimated transmitted symbols and $C$ denotes the inverse of the channel matrix $H$. In this study, it is assumed that the channel gain is known at the receiver. Note that the performance of NDC-OFDM is compared with the conventional O-OFDM approach and different equalization methods will not affect the comparison results. Even though the minimum mean square error (MMSE) equalizer can also be used in the NDC-OFDM system with the known channel information and the noise coefficient, the ZF equalizer is chosen as a simple and convenient equalization method [30]. Each element in $g$ represents the detected OFDM signal which has been transmitted by the corresponding LED with AWGN added at the receiver.

Before demodulating the received signal, there are two methods by which the original bipolar OFDM signal can be reconstructed. Both methods are based on the principle of the modulation scheme in NDC-OFDM. The detected signal received by the first PD is set to be transmitted by the first LED which sent the positive OFDM sample. The second PD achieves the absolute value of the negative sample. In NDC-OFDM, since only one LED is activated in one symbol duration, only one element in $g$ carries the bit information and the other one is treated as an additional noise component. According to the rules above, the first method subtracts the negative signal block from the positive one. However, when reconstructed in this way, the proposed method performs 3 dB worse than bipolar OFDM with the same constellation size. This is because the subtraction of the negative block from the positive one doubles the AWGN variance for each restored bipolar OFDM signal. The second reconstruction method is to estimate the index of the active transmitter. The estimated index represents the sign of the transmitted information OFDM sample. In particular, to estimate the indices of the active transmitters, the SM detector compares the values of the elements in $g$ as follows:
\[ \hat{I}(k) = \arg \max_i (G(i,k)), i = 1, \ldots, N_t, \]

where \( G \) is an \( N_t \times N \) equalized matrix which contains all of the estimated transmitted symbols and \( \hat{I} \) is an \( N \)-dimensional vector which contains all the estimated indices. As noted, there are two pairs of transmitters and receivers. If \( \hat{I}(k) \) is equal to one, this means that the symbol received at the time instant \( k \) is transmitted from the first LED. Therefore this symbol is a positive-valued OFDM symbol. If \( \hat{I}(k) \) is two, a negative symbol is transmitted by the second LED. As a consequence, the estimated OFDM symbols sequence is:

\[ \mathbf{x}'(k) = \begin{cases} G(\hat{I}(k), k), & \hat{I}(k) = 1, \\ -G(\hat{I}(k), k), & \hat{I}(k) = 2. \end{cases} \]

In an ideal scenario, if there is no AWGN, \( \mathbf{x}'(k) \) should be the same as \( \mathbf{x}(k) \). When compared with the first method, the second method will not double the AWGN variance for each estimated OFDM symbol. Thus, it gives a significant improvement on the power efficiency which has been proved in U-OFDM [17]. In this study, the sign-selected estimation (second method) is chosen as the performance analysis in the first method is trivial.

After recovering the OFDM symbols, \( \mathbf{x}'(k) \) is passed through the conventional OFDM demodulation block to obtain received QAM symbols:

\[ \mathbf{X}'(m) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \mathbf{x}'(k) \exp \left( -j \frac{2 \pi km}{N} \right). \]

The \( N-1 \)-dimensional column vector \( \mathbf{X}'(m) \) is passed through the inverse Fourier transform and then transformed to the output bit stream by the conventional QAM demodulator.

III. PERFORMANCE ANALYSIS

An analytical BER bound of NDC-OFDM is derived in this section. In addition, the spectral efficiency of NDC-OFDM is analysed in comparison with other OFDM methods in the OSM system.

A. Theoretical Performance of NDC-OFDM

To calculate the theoretical BER of NDC-OFDM, the following mathematical notations and formulas are defined. In this paper, \( \sigma_n \) is the standard deviation of the AWGN, i.e., \( \sigma_n = \sqrt{N_0/2} \), where \( N_0/2 \) is the variance of the AWGN. The constant, \( \sigma_c \), is the standard deviation of the real OFDM signals which have been modulated and are ready to be transmitted by the LEDs. For the analytical calculation, \( \sigma_c \) is defined as follows:

\[ \sigma_c = \sqrt{E_b \log_2(M)} \frac{N-2}{2N N_0}. \]

where \( E_b \) is the electrical energy per bit. Note that \( E_b/N_0 \) is the metric of the BER performance. The standard normal distribution probability density function is:

\[ \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}. \]

The step function is:

\[ I(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x \leq 0. \end{cases} \]

The sign function is:

\[ \text{sgn}(s) = \begin{cases} -1, & \text{if } s < 0, \\ 0, & \text{if } s = 0, \\ 1, & \text{if } s > 0. \end{cases} \]

In the following theoretical expressions, a 2 × 2 MIMO channel is considered:

\[ \mathbf{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}. \]

Since the attenuation gain of the channel has a limited effect on the results of the analytical BER performance, for simplicity, the coefficients in \( \mathbf{H} \) are normalized to one and they simply represent the correlation coefficients of the channel. The inverse channel matrix is represented by:

\[ \mathbf{C} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}. \]

Based on the theoretical analysis method of the nonlinear transmission in [17], the analysis in this study mainly aims to calculate the probability of the correct and incorrect estimation in which the effects of the ZF equalization and the nonlinear OFDM demodulation should be taken into consideration. In NDC-OFDM, two receivers obtain optical OFDM samples over the MIMO channel at the same time. After the ZF equalization, the unipolar OFDM symbols are recovered with the enhanced AWGN. Symbols detected by the first PD come from the first LED, which are originally positive symbols. Symbols detected by the second PD are transmitted by the second LED, which are the absolute values of the negative symbols. If there is no noise in the system, the received symbols should be the same as the transmitted symbols. In the theoretical analysis model, the AWGNs are considered as two independent random variables, \( n_1 \) and \( n_2 \), which follow the standard normal distribution with the standard deviation, \( \sigma_n \). Since the ZF equalizer is used in the system, the noise is enhanced after removing the channel crosstalk. Most importantly, the AWGN in one receiver enhances the variance of the noise in the other receiver. Considering this condition, the correctly detected probability for the identical symbol is presented in (16). This depends on a random value of \( n_1 \), a random value of \( n_2 \), the inverse matrix of the channel and the original bipolar symbol, \( s \). Note that the bipolar OFDM symbols also follow an independent Gaussian distribution. Likewise, the incorrectly detected probability is given in (17). With the identical \( n_1 \), \( n_2 \) and \( s \), the correctly detected OFDM symbol is expressed as follows:

\[ x_c = \begin{cases} |s| + c_{11} n_1 + c_{12} n_2, & s \geq 0, \\ |s| + c_{21} n_1 + c_{22} n_2, & s < 0. \end{cases} \]

For all possible values of \( n_1 \) and \( n_2 \) and the identical OFDM sample, the mean value of \( x_c \) is:
These variances will constitute the variance of the AWGN in the frequency domain. Similarly, for the estimation of NDC-OFDM, based on the central limit theorem (CLT), after the FFT is calculated by:

\[
X_{c} = \frac{1}{n} \sum_{i=0}^{n-1} x_{i}, \quad X_{w} = \frac{1}{n} \sum_{i=0}^{n-1} w_{i},
\]

and the variance of the correctly detected sample is calculated by:

\[
Var(X_{c}) = \frac{1}{n} \sum_{i=0}^{n-1} (x_{i} - \bar{X}_{c})^2,
\]

\[
Var(X_{w}) = \frac{1}{n} \sum_{i=0}^{n-1} (w_{i} - \bar{X}_{w})^2.
\]

The mean and the variance of \( X_{c} \) are:

\[
\mu_{c} = \frac{1}{n} \sum_{i=0}^{n-1} x_{i}, \quad \sigma_{c}^2 = \frac{1}{n} \sum_{i=0}^{n-1} (x_{i} - \bar{X}_{c})^2.
\]

The mean and the variance of \( X_{w} \) are:

\[
\mu_{w} = \frac{1}{n} \sum_{i=0}^{n-1} w_{i}, \quad \sigma_{w}^2 = \frac{1}{n} \sum_{i=0}^{n-1} (w_{i} - \bar{X}_{w})^2.
\]

These variances will constitute the variance of the AWGN in the frequency domain.

After the sign-selected estimation, the selected symbols are demodulated to QAM symbols by the FFT operation. For this estimation method, the demodulation procedure is treated as a nonlinear transformation. According to the Bussgang theorem [31], if an independent Gaussian random variable, \( X \), passes through a nonlinear transformation, \( f(X) \), it has the following properties:

\[
E[X] = \alpha_{d}X + \beta_{d},
\]

\[
E[X^2] = \alpha_{d}^2E[X^2] + \beta_{d}^2 + 2\alpha_{d}\beta_{d},
\]

where \( E[\cdot] \) represents the statistical expectation. Using the properties above, the nonlinear distortion in an OFDM-based system can be equivalent to a gain factor, \( \alpha_{d} \), and an additional noise, \( Y \) [17]. In NDC-OFDM, \( X \) is equal to the value of the transmitted symbol, \( s \), and \( Y \) is a noise component which is a Gaussian random variable non-correlated with \( X \). After the FFT operation, the variance of \( Y \) will be composed of the variance of the AWGN in the frequency domain and \( \alpha_{d} \) will enhance the mean value of the information-carrying symbol in each modulated subcarrier. According to (26), \( \alpha_{d} \) can be derived as:

\[
\alpha_{d} = \frac{E[X^2]}{\sigma_{X}^2},
\]

where \( \sigma_{X} \) is the standard deviation of \( X \), which is equal to \( \sigma_{s} \) in this study. Since the additional noise, \( Y \), follows a Gaussian distribution with a zero mean, the variance of \( Y \) can be calculated as:

\[
\sigma_{Y}^2 = E[Y^2] = E[Y^2] = E[Y^2] = E[f(X) - \alpha_{d}X]^2
\]

\[
= E[f(X) - \alpha_{d}X]^2 - \alpha_{d}^2\sigma_{X}^2.
\]

From (27) and (28), the values of the nonlinear gain factor and the variance of the noise component for the correct estimation are calculated as:

\[
\alpha_{c} = \int_{-\infty}^{\infty} f_{c}(s) \frac{1}{\sigma_{s}} \phi \left( \frac{s}{\sigma_{s}} \right) \, ds,
\]

\[
y_{c} = \int_{-\infty}^{\infty} f_{c}(s) \frac{1}{\sigma_{s}} \phi \left( \frac{s}{\sigma_{s}} \right) \, ds - \alpha_{d}^2\sigma_{s}^2.
\]

For the incorrect estimation, the constant, \( \alpha_{w} \), and the variance, \( y_{w} \), are calculated as:

\[
\alpha_{w} = \int_{-\infty}^{\infty} f_{w}(s) \frac{1}{\sigma_{s}} \phi \left( \frac{s}{\sigma_{s}} \right) \, ds,
\]

\[
y_{w} = \int_{-\infty}^{\infty} f_{w}(s) \frac{1}{\sigma_{s}} \phi \left( \frac{s}{\sigma_{s}} \right) \, ds - \alpha_{d}^2\sigma_{s}^2.
\]
respectively in NDC-OFDM, the number of the LEDs, 
As two different signs of the samples should be represented
information bit carried by the indices, the spectral efficiency
indices-carried bits, i.e.,
was calculated by considering both the signal-carried bits and the
ventional OSM system with
and DCO-OFDM, the indices carry additional information bits
within the same OSM system. In NDC-OFDM, the indices of
B. Spectral Efficiency Comparison

\[ d_e = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sigma_e^2} \phi \left( \frac{s}{\sigma_e} \right) \text{Pr}(s, n_1, n_2) \text{dn}n_1 \text{dn}_2 \text{ds}. \]  

(33)

For a large number of samples in a NDC-OFDM frame, the number of correctly and incorrectly estimated samples have a ratio which corresponds to the probabilities for correct and incorrect estimations. According to (26), the nonlinear transformation will add a gain factor to the sample. The gain factor enhances the average energy of the transmitted bits. The average gain factor is calculated as:

\[ \bar{\alpha} = d_e \alpha_e + (1 - d_e) \alpha_w. \]  

(34)

As noted above, the variance of the estimation, \( \bar{\alpha} \) and \( \bar{\alpha}_w \), and the variance of the nonlinear transmission, \( \sigma_e \) and \( \sigma_w \), constitute the average noise variance of the system in the frequency domain:

\[ \bar{N} = d_e (\bar{\alpha}_e + \bar{\alpha}_w) + (1 - d_e) (\bar{\alpha}_w + \bar{\alpha}_w). \]  

(35)

Thus, the average SNR_{elec} per bit can be achieved from the known value of \( E_{0,\text{elec}} \), and the calculated values of \( \bar{\alpha} \) and \( \bar{N} \) as:

\[ \text{SNR}_{\text{elec}} = \frac{\bar{\alpha}^2 E_{0,\text{elec}}}{\bar{N}} \]  

(36)

Using the analytical expression for the BER performance of M–QAM O-OFDM in [18], the theoretical BER performance of NDC-OFDM can be calculated as:

\[ \text{BER}_{\text{NDC-OFDM}} = \frac{4(\sqrt{M} - 1)}{M \log_2(M)} Q \left( \sqrt{\frac{3 \log_2(M)}{M - 1} \text{SNR}_{\text{elec}}} \right) \]  

\[ + \frac{4(\sqrt{M} - 2)}{M \log_2(M)} Q \left( \sqrt{\frac{3 \log_2(M)}{M - 1} \text{SNR}_{\text{elec}}} \right). \]  

(37)

B. Spectral Efficiency Comparison

NDC-OFDM is realized in the OSM system where ACO-OFDM and DCO-OFDM can be also applied. For fair comparisons, NDC-OFDM, DCO-OFDM and DCO-OFDM are built within the same OSM system. In NDC-OFDM, the indices of LEDs are used to carry the sign information. In ACO-OFDM and DCO-OFDM, the indices carry additional information bits according to the conventional principle of OSM. In the conventional OSM system with M–QAM, the spectral efficiency is calculated by considering both the signal-carried bits and the indices-carried bits, i.e., \( R_{\text{OSM}} = \log_2(M N_1) \) bits/s/Hz [25]. In NDC-OFDM, since the Hermitian symmetry of O-OFDM decreases the spectral efficiency by half and there is no information bit carried by the indices, the spectral efficiency of NDC-OFDM is calculated as:

\[ R_{\text{NDC-OFDM}} = \frac{N - 2}{2N} \log_2(M_1 N_1) - 1 \]  

(38)

As two different signs of the samples should be represented respectively in NDC-OFDM, the number of the LEDs, \( N_1 \), should be even. For DCO-OFDM in the OSM system, the spectral efficiency is only halved by the Hermitian symmetry. Thus it is expressed as:

\[ R_{\text{DCO-OFDM}} = \frac{N - 2}{2N} \log_2(M_2 N_1) \]  

bits/s/Hz.  

(39)

In ACO-OFDM, as only half of the subcarriers are modulated, the spectral efficiency has an additional 50% reduction. In the OSM system, the actual spectral efficiency of ACO-OFDM is:

\[ R_{\text{ACO-OFDM}} = \frac{1}{4} \log_2(M_3 N_1) \]  

bits/s/Hz.  

(40)

In (38), (39) and (40), \( M_1 \), \( M_2 \) and \( M_3 \) denote the constellation size of QAM in the three schemes respectively. In this study, the size of the OFDM frame, \( N \), is set to 2048. Thus the coefficient, \( \frac{2N^2}{2} \), in (38) and (39) can be approximated to 1/2. When NDC-OFDM, DCO-OFDM and ACO-OFDM in the OSM system have the same spectral efficiencies, i.e., \( R_{\text{NDC-OFDM}} = R_{\text{DCO-OFDM}} = R_{\text{ACO-OFDM}} \), the constellation sizes of these three methods have the following relationship:

\[ M_1 = 2M_2 = \sqrt{2M_3}. \]  

(41)

Fig. 2 shows the constellation sizes required to achieve the same spectral efficiencies between 0.5 bits/s/Hz and 2 bits/s/Hz for NDC-OFDM, DCO-OFDM and ACO-OFDM in the OSM system. According to (41), compared with DCO-OFDM, NDC-OFDM needs double constellation size to achieve the same spectral efficiency. For ACO-OFDM, with the increase of the spectral efficiency, the required constellation size will increase exponentially as shown in Fig. 2 and this means greater system complexity. For higher spectral efficiencies, such as between 3.5 bits/s/Hz and 5.5 bits/s/Hz, the required constellation size of ACO-OFDM becomes very large, as shown in Table I. Although the constellation size can be as large as required to achieve the higher spectral efficiency, the required constellation size for ACO-OFDM is unrealistic and
optical wireless links with LOS characteristics. In this study, MIMO channels which are chosen from [29]. In [29], a generic OFDM and DCO-OFDM over different practical optical BER performance of NDC-OFDM is compared with ACO-OFDM and DCO-OFDM are compared in this section. The Comparison between analytical and simulation results for asymmetrical ideal channels is considered to compare power efficiencies. In the simulation, 16-QAM is chosen for each case and the variance of AWGN, \( \sigma_w \), is set to \( \sqrt{0.01} \). Fig. 3 shows the comparison between analytical and simulation results for symmetrical ideal channels. A high value of coefficient indicates a high level of correlation. Note that in \( H_1 \), transmitted optical signals are assumed to be received only by the corresponding receiver. Without loss of generality, asymmetrical ideal channels are also tested in this study:

\[
H_5 = \begin{pmatrix} 1 & 0 \\ 0 & 0.7 \end{pmatrix}, \quad H_6 = \begin{pmatrix} 1 & 0 \\ 0.3 & 0.7 \end{pmatrix}.
\]

In the simulation, 16-QAM is chosen for each case and the variance of AWGN, \( \sigma_w \), is set to \( \sqrt{0.01} \). Fig. 3 shows the comparison between analytical and simulation results for the symmetrical ideal channels. A high value of coefficient indicates a high level of correlation. Note that in \( H_1 \), transmitted optical signals are assumed to be received only by the corresponding receiver. Without loss of generality, asymmetrical ideal channels are also tested in this study:

\[
H_5 = \begin{pmatrix} 1 & 0 \\ 0 & 0.7 \end{pmatrix}, \quad H_6 = \begin{pmatrix} 1 & 0 \\ 0.3 & 0.7 \end{pmatrix}.
\]

IV. Numerical and Simulation Results

A. Analytical Results

As noted in Section III, ideal 2 \( \times \) 2 MIMO channels are considered to test the correctness of the theoretical proofs. Symmetrical ideal channels are assumed as follows:

\[
H_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}.
\]

The coefficients in (42) reflect the correlation of optical channels. A high value of coefficient indicates a high level of correlation. Note that in \( H_1 \), transmitted optical signals are assumed to be received only by the corresponding receiver. Without loss of generality, asymmetrical ideal channels are also tested in this study:

\[
H_5 = \begin{pmatrix} 1 & 0 \\ 0 & 0.7 \end{pmatrix}, \quad H_6 = \begin{pmatrix} 1 & 0 \\ 0.3 & 0.7 \end{pmatrix}.
\]

B. NDC-OFDM, ACO-OFDM and DCO-OFDM Performance Comparison

The Monte Carlo simulation results for NDC-OFDM, ACO-OFDM and DCO-OFDM are compared in this section. The BER performance of NDC-OFDM is compared with ACO-OFDM and DCO-OFDM over different practical optical MIMO channels which are chosen from [29]. In [29], a generic 4 \( \times \) 4 indoor scenario is considered with intensity modulated optical wireless links with LOS characteristics. In this study, 2 \( \times \) 2 optical MIMO channels are assumed to test properties of O-OFDM systems. Thus, 2 \( \times \) 2 optical MIMO links are extracted from original 4 \( \times \) 4 optical channels, taking into account both symmetrical and asymmetrical cases:

\[
H_{Prac1} = 10^{-5} \begin{pmatrix} 0.1889 & 0.0713 \\ 0.0713 & 0.1889 \end{pmatrix},
\]

\[
H_{Prac2} = 10^{-5} \begin{pmatrix} 0.3847 & 0.1889 \\ 0.1889 & 0.3847 \end{pmatrix},
\]

\[
H_{Prac3} = 10^{-5} \begin{pmatrix} 0.1889 & 0.0713 \\ 0.1157 & 0.1889 \end{pmatrix},
\]

\[
H_{Prac4} = 10^{-5} \begin{pmatrix} 0.3847 & 0.2091 \\ 0.1889 & 0.3847 \end{pmatrix}.
\]
comparison, spectral efficiencies are set to 1.5 bits/s/Hz and 2 bits/s/Hz. According to (41), 8-QAM and 16-QAM are thus chosen in the simulation of NDC-OFDM; these are double than the constellation size of DCO-OFDM; and for ACO-OFDM, the modulation orders are 32 and 128. As noted in Section II, a fixed level of DC-bias needs to be added in DCO-OFDM. The lower level might cause the nonlinear distortion and the higher level would be energy inefficient. In order to study these two cases in a practical situation, 5 dB and 7 dB DC-bias are chosen in the simulation.

Fig. 5 and Fig. 6 show the performance of NDC-OFDM, ACO-OFDM and DCO-OFDM with OSM over the symmetrical optical MIMO channels, $H_{Prac_1}$ and $H_{Prac_2}$. In Fig. 5, it shows that when the spectral efficiency is 1.5 bits/s/Hz, NDC-OFDM has around 3.5 dB power efficiency better than the 5 dB DCO-OFDM and nearly 5 dB better than ACO-OFDM and the 7 dB DCO-OFDM. In this case, there is no obvious nonlinear distortion in DCO-OFDM. However, when the spectral efficiency is 2 bits/s/Hz, clipping noises distort the curves of DCO-OFDM with 5 dB DC-bias. In this case, NDC-OFDM saves 7 dB transmission power compared with DCO-OFDM. Moreover, with the increase in the spectral efficiency, the performance of NDC-OFDM is closed to the unipolar line which has been shown in [17]. Fig. 6 shows the performance of the three methods over $H_{Prac_2}$ which is another symmetrical channel with a higher correlation. Compared with $H_{Prac_1}$, all the schemes require 2 dB more transmission power, and also NDC-OFDM is the most power efficient method.

The performance of NDC-OFDM, ACO-OFDM and DCO-OFDM over the asymmetrical channels are shown in Fig. 7 and Fig. 8. As shown in Fig. 7, for the 1.5 bits/s/Hz spectral efficiency, NDC-OFDM has 5 dB greater power efficiency than 5 dB DCO-OFDM and around 7 dB better than ACO-OFDM and the 7 dB DCO-OFDM. In this case, there is no obvious nonlinear distortion in DCO-OFDM. However, when the spectral efficiency is 2 bits/s/Hz, clipping noises distort the curves of DCO-OFDM with 5 dB DC-bias. In this case, NDC-OFDM saves 7 dB transmission power compared with DCO-OFDM. Moreover, with the increase in the spectral efficiency, the performance of NDC-OFDM is closed to the unipolar line which has been shown in [17]. Fig. 6 shows the performance of the three methods over $H_{Prac_2}$ which is another symmetrical channel with a higher correlation. Compared with $H_{Prac_1}$, all the schemes require 2 dB more transmission power, and also NDC-OFDM is the most power efficient method.

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one symbol duration, the NDC-OFDM system requires only half of the transmission power and the interference between each information signals can be ignored. Therefore, with the increase of the channel correlation, NDC-OFDM shows a superiority in power efficiency and gives better anti-crosstalk performance than the other methods in OSM.

V. CONCLUSIONS

In this paper, the theoretical performance of a novel unipolar modulation method, referred to as NDC-OFDM, are analysed. The new method combines O-OFDM with SM and has been applied to a WOC system. Using the Bussgang theorem and the CLT the analytical performance of NDC-OFDM in AWGN channels has been derived. As a result, an equation for the electrical SNR per bit, which presents a memoryless nonlinear distortion analysis processing, has been presented to calculate the theoretical BER of the NDC-OFDM system. The results of the proposed method show close agreement with Monte Carlo simulations, thus confirming the validity of the method.

In comparisons of the simulation performance, NDC-OFDM exhibits the capability to achieve better BER performances than the conventional OFDM-based modulation schemes applied in the OSM system: DCO-OFDM and ACO-OFDM. Compared with DCO-OFDM, the new NDC-OFDM method solves the clipping distortion problem caused by the high level of the DC-bias. Compared with ACO-OFDM, NDC-OFDM gives a significant improvement in spectral efficiency. These improvements of NDC-OFDM come at the expense of additional hardware at the transmitter and receiver. However, modern visible light communication systems are expected to employ multiple low-cost LEDs to fulfill minimum indoor lighting conditions.

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Selected Publications


Performance Analysis of SPAD-based OFDM

Yichen Li, Majid Safari, Robert Henderson and Harald Haas

Abstract—In this paper, an analytical approach for the nonlinear distorted bit error rate performance of optical orthogonal frequency division multiplexing (O-OFDM) with single photon avalanche diode (SPAD) receivers is presented. Major distortion effects of passive quenching (PQ) and active quenching (AQ) SPAD receivers are analysed in this study. The performance analysis of DC-biased O-OFDM and asymmetrical clipped O-OFDM with PQ and AQ SPADs are derived. The comparison results show the maximum optical irradiance caused by the nonlinear distortion, which limits the transmission power and bit rate. The theoretical maximum bit rate of SPAD-based OFDM is found which is up to 1 Gbit/s. This approach supplies a closed-form analytical solution for designing an optimal SPAD-based system.

Index Terms—optical wireless communication (OWC), single photon avalanche diode (SPAD), nonlinear distortion, optical OFDM.

I. INTRODUCTION

Currently, high speed light emitting diodes (LEDs) and laser diodes (LDs) are mainly used as transmitters in optical wireless communication (OWC) systems. With a single LED, a OWC system can achieve data rates exceeding 3 Gb/s [1]. However, the incoherent light output of the transmitters means that information can only be encoded in the intensity level. As a consequence, only real-valued and positive signals can be used for data modulation. Thus, OWC systems are usually considered to be modulated as an intensity modulation and direct detection (IM/DD) system [2]. Unipolar modulation schemes with single-carrier, such as on-off keying (OOK), pulse position modulation (PPM) and pulse amplitude modulation (PAM), can be used in conjunction with IM/DD systems [2]-[5]. In order to fully use the limited modulation bandwidth of the device and achieve high data rates, orthogonal frequency division multiplexing (OFDM) is applied in OWC systems by utilizing adaptive bit and power loading [1]. Unlike OFDM in radio frequency, optical OFDM (O-OFDM) requires real valued signals, and these are generated by imposing Hermitian symmetry on the information frame before the inverse fast Fourier transform (IFFT) operation during the signal generation phase. However, this decreases the spectral efficiency by half. Diverse O-OFDM modulation schemes have been realized and applied in OWC, such as DC-biased optical OFDM (DCO-OFDM), asymmetrically clipped optical OFDM (ACO-OFDM), unipolar OFDM (U-OFDM) and non-DC-biased OFDM (NDC-OFDM) [6]-[10].

Typically, highly sensitive photodiodes (PDs), such as positive-intrinsic-negative (PIN) diodes and avalanche photo diodes (APDs), are applied as receivers in OWC. However, when the OWC system is applied in low optical power and long distance transmission, such as in a gas well downhole monitoring system [11] and data transmission over plastic optical fibres [12], the number of photons reaching the receivers are significantly less than in standard indoor OWC links. In these scenarios, conventional PDs have unsatisfactory performance because the transimpedance amplifier (TIA) significantly reduces the sensitivity of the receiver and limits the signal-to-noise ratio (SNR). As a consequence, these low power signals are buried in noise. Hence, when compared with conventional PDs, single photon avalanche diodes (SPADs) would be more suitable receivers in these scenarios. The SPAD detector does not require a TIA and thus the output signal is not distorted by thermal noise. In addition, as SPADs can even detect a single photon, a bit of information-carried photons can be received accurately. Therefore, the SPAD receiver can perform at significantly higher sensitivity and optical power efficiency than conventional PDs. In previous work [6], an O-OFDM system with a SPAD receiver was presented and compared with state-of-the-art PD-received based O-OFDM systems. When the transmission speed is 1 Mbit/s, SPAD-based OFDM enhanced the sensitivity by 30.5 dB over the PD-based system.

However, a SPAD receiver can only detect one photon within a device specific dead time which constrains the ability to recover a signal. In addition, since the output of the detector is a photon count value, there is a maximum number of photons that the system can detect. This limits the maximum tolerable optical irradiance which results in a receiver nonlinear distortion. This means that the transmission power and maximum bit rate of SPAD-based OFDM are limited by the structure and design of SPAD receivers [13], [14]. The analytical model of the nonlinear distortion effect in O-OFDM with conventional PDs has been derived in [15]-[17]. As the nonlinear effects in the conventional O-OFDM system are mainly caused by transmitter properties and modulation schemes, the SPAD receiver nonlinear distortion effect has not yet been reported. This study provides a complete analytical procedure to find the exact bit error rate (BER) of the SPAD-based OFDM by considering the receiver nonlinear distortion and the conventional distortions. The analytical model of SPAD-based OFDM can be used to find the limitation threshold in the system and also the theoretical maximum bit rate. In addition, as the current SPAD array is designed for image processing [18], the designed parameters may not be suitable for OWC. Based on the analytical model, a reliable approach is presented for the design of the SPAD array with some optimal parameters which are suitable for current OWC systems.

The rest of this paper is organized as follows. The system model of the SPAD-based OFDM system is described in Section II. The nonlinear distortion in the SPAD receiver is
is set to 7 dB and 13 dB, which are adopted from [6] for as the bias level in dB. The bias level in the current simulations of the OFDM frame, \( n \) is set to zero. In ACO-OFDM, half of subcarriers and the DC subcarrier (the first subcarrier) symbols in \( N \) are mapped on to half of the odd subcarriers, \( n = 1, \cdots, N/2 - 1 \), and the rest of the OFDM frame in order to obtain real-valued symbols. In DCO-OFDM, where \( n = 1, \cdots, N/2 \), are put into the first \( N/4 \) QAM symbols in \( X(n) \). In ACO-OFDM, since symbols are antisymmetric, clipped unipolar symbols are obtained by setting the negative part to zero. In the simulation, after being transformed into an optical intensity signal, the clipped signal is transmitted by the LED transmitter.

**II. SPAD-BASED OFDM**

The system model of OFDM with SPAD receivers is shown in Fig. 1.

**A. Optical OFDM Modulation**

At the transmitter, the input bit stream is transformed into complex symbols, \( X(n) \), by a \( M \)-quadrature amplitude modulation (QAM) modulator, where \( M \) is the constellation size. The symbols are allocated on to \( N \) subcarriers, \( X(k) \), \( k = 0, \cdots, N-1 \). In OFDM, \( N \) denotes the size of IFFT/FFT, where \( N \) is set to 2048. In general, two standard techniques, DCO-OFDM and ACO-OFDM, are used to obtain positive and real-valued OFDM symbols [17]. In DCO-OFDM, \( N/2 - 1 \) symbols in \( X(n) \), \( n = 1, \cdots, N/2 - 1 \), are put into the first half of subcarriers and the DC subcarrier (the first subcarrier) is set to zero. In ACO-OFDM, \( N/4 \) QAM symbols in \( X(n) \), \( n = 1, \cdots, N/4 \), are mapped on to half of the odd subcarriers of the OFDM frame, \( X(k) \), \( k = 1, 3, 5, \cdots, N/2 - 1 \). At the same time, the even subcarriers are set to zero. In both ACO-OFDM and DCO-OFDM, Hermitian symmetry is applied to the rest of the OFDM frame in order to obtain real-valued symbols through the IFFT block. Since transmitters can only send unipolar signals, the real-valued OFDM symbols need to be clipped. In DCO-OFDM, a DC bias is added to make the signal unipolar [17]. In practice, the value of the DC bias, which is related to the average power of the OFDM symbols, is defined as:

\[
B_{DC} = \beta \sqrt{E[|X|^2(k)]},
\]

where \( E[\cdot] \) represents the statistical expectation; \( X(k) \) is the OFDM symbol frame vector; and \( 10 \log_{10}(\beta^2 + 1) \) is defined as the bias level in dB. The bias level in the current simulations is set to 7 dB and 13 dB, which are adopted from [6] for consistency. After the DC bias, the OFDM frame is simply clipped by:

\[
x_{\text{clipped}}(k) = \begin{cases} x_{\text{biased}}(k), & x_{\text{biased}}(k) \geq 0, \\ 0, & x_{\text{biased}}(k) < 0, \end{cases}
\]

where \( x_{\text{biased}}(k) \) is the DC biased symbol which is calculated as \( x_{\text{biased}}(k) = x(k) + B_{DC} \). The clipped unipolar symbol is denoted by \( x_{\text{clipped}}(k) \). In ACO-OFDM, since symbols are antisymmetric, clipped unipolar symbols are obtained by setting the negative part to zero. In the simulation, after being transformed into an optical intensity signal, the clipped signal is transmitted by the LED transmitter.

**B. SPAD Receiver**

A SPAD is an APD which is biased beyond reverse breakdown in the so called ‘Geiger’ region. In this mode of operation, a SPAD triggers billions of electron-hole pair generations for each detected photon. In other words, in ‘Geiger’ mode, a SPAD emits a very large current by receiving a single photon and thus can essentially be modelled as a single photon counter. The photodetection process of an ideal photon counter can be modelled using Poisson statistics which describe the shot noise effect ([19] and references therein).

In this study, in order to increase the capacity of the photon counts, an array of SPADs which outputs the superposition of the photon counts from the individual SPADs is considered [20]. Some significant parameters of the SPAD array are introduced as follows.

1) **Fill Factor (FF):** FF is the ratio of the total SPAD active area to the total array area. For the SPAD array, FF represents the probability that a photon hits the active area. If the photon triggers an avalanche, it will be counted. In other words, the percentage of photons in a beam light reaching the active area to the total array area. For the SPAD array, FF represents the probability that a photon hits the active area. If the photon triggers an avalanche, it will be counted. In other words, the percentage of photons in a beam light reaching the active area can be approximated to FF. In this study, the value of FF is denoted by \( C_{FF} \).

2) **Photon Detection Probability (PDP):** PDP is the probability that a photon hitting the active area triggers an avalanche. This avalanche will generate a pulse which can be counted by an accumulator. The accumulator will give the output of
the array. PDP is different to the quantum efficiency of the conventional PD, in which the quantum efficiency sometimes includes fill factor effects [18]. In this study, the value of PDP is denoted by $C_{PDP}$.

3) Dark Count Rate (DCR): A thermally-generated carrier can also trigger an avalanche which increases the array output. Even in complete darkness, this phenomenon still exists as long as the SPAD devices are enabled. The average number of counts in darkness per second is referred to as DCR which is regarded as a fixed signal-unrelated noise of SPAD. In this study, the average DCR of a single SPAD device is denoted by $N_{DCR}$.

4) After Pulsing Probability (APP): After pulses are correlated to detections by the time dependent release of trapped carriers [20]. Additional avalanches are triggered after receiving a photon or a dark photon. This means that the after pulsing effect has a negligible effect on the next sample period. In this study, the value of APP is denoted by $P_{AP}$.

5) Dead Time: After an avalanche is triggered, whether caused by the signal photons or dark photons, the SPAD device needs to be actively or passively recharged in a short period of time and this is referred to as dead time. During this time, the SPAD device is unable to detect further signal photons or dark photons. In other words, each individual SPAD in the array can only receive one photon during the dead time. In this study, the value of dead time is denoted by $\tau_d$.

Fig. 2 illustrates the system model of the SPAD array receiving optical signals. In order to generate received OFDM symbols, the output of the SPAD array is counted over a symbol duration, $T_s$, at time instances $t_k = kT_s$ of the received optical signal $x(t)$. These photon counts are denoted by $\nu(k)$ which is the superposition of the photon counts from each individual SPADs, $a_m(k)$, as shown in Fig. 2:

$$\nu(k) = \sum_{m=1}^{N_{SPAD}} a_m(k),$$

(3)

where $N_{SPAD}$ is the number of SPAD devices in the array. Generally, as the photon counts from each individual SPAD can be approximately modelled using Poisson statistics, the photon counts at the output of the SPAD array (i.e., $\nu(k)$) can be described by Poisson distribution:

$$\text{Pr}(\nu(k) = j, \mu(k)) = \exp(-\mu(k)) \frac{\mu(k)^j}{j!}$$

(4)

where the average photon counts $\mu(k)$ can be expressed as a function of the received signal and the parameters of the SPADs:

$$\mu(k) = \frac{C_{FID}C_{PDP}}{E_D} \int_{t_k}^{t_k+T_s} x(t) dt + n_{DCR} (1 + P_{AP}).$$

(5)

where $E_D$ denotes the energy of a photon which is calculated by $\frac{hc}{\lambda}$. Note that $h$ denotes Planck’s constant; $c$ is the speed of light; and $\lambda$ is the light wavelength of the LED transmitter. The noise caused by dark counts is denoted by $n_{DCR} = N_{DCR}N_{SPAD}T_s$. However, when the incoming photon rate is high, Poisson distribution cannot exactly describe the photon counts of SPAD arrays. This is because the dead time effect causes the saturation of SPAD devices and significantly decreases the photon counts. Thus, an exact distribution is also considered and used in this study, and this will be introduced in Section III.

C. Optical OFDM Demodulation

The output of the SPAD array is the number of photons ($\nu(k)$), and the system is designed based on a conventional OFDM demodulator which requires the amplitude of the electrical signal (optical power) to demodulate the received signal to the original encoded bits. Thus, a photon-to-amplitude equalizer is used to simply convert the received photon number ($\nu(k)$) to the corresponding electrical signal amplitude (optical power), $X(k)$. The coefficient of the equalizer is calculated by a pilot which can record the effect of the attenuation and the parameters of the SPADs.

Assuming that there is no other distortion effects during the transmission, the recovered signal, $X(k)$, can be scaled to the original clipped signal, $x_{clip}(k)$. The recovered OFDM symbols from the SPAD are passed through a FFT operation which converts symbols to the frequency domain. In DCO-OFDM, $N/2-1$ symbols are obtained from the corresponding subcarriers to constitute a QAM symbol frame, $X(n)$. In ACO-OFDM, $N/4$ symbols are obtained. The detected QAM symbols are then decoded by the conventional Maximum Likelihood (ML) estimator in order to obtain the output bit stream.
III. NONLINEAR DISTORTION IN SPAD RECEIVERS

In the SPAD-based system, two SPAD devices with different recharged circuits are applied. The passively recharged SPAD is referred to as passive quenching SPAD (PQ SPAD). The configuration of the passively recharged circuit is presented in [21]. PQ SPAD is identified as a paralyzable detector where any counts occurring during the dead time (including signal, dark count and after pulse) are not registered but they extend the dead time. As shown in Fig. 3(a), a PQ SPAD device is biased with an excess bias voltage ($V_{eb}$) above the breakdown voltage ($V_{br}$). When an incident photon triggers an avalanche and the SPAD voltage is reduced below $V_{br}$, PQ SPAD is then passively recharged. As soon as the SPAD voltage exceeds $V_{br}$, the PQ SPAD device can be triggered by another photon. As a result, the SPAD voltage remains below the threshold voltage ($V_{th}$). As long as the voltage is lower than $V_{th}$, only the first photon is registered and other photons are lost. This means that the dead time is extended and the PQ-SPAD is paralyzable. Another SPAD device is actively recharged, so-called active quenching SPAD (AQ SPAD). Compared with PQ SPAD, the configuration of AQ SPAD is more complex and requires more area and power [21], but when any events arrive during the dead time, the additional events are not registered and do not prolong the dead time. As shown in Fig. 3(b), when an incident photon triggers an avalanche, the voltage of the AQ SPAD device is reduced below $V_{br}$. After the hold-off time (the voltage remaining below $V_{th}$), the SPAD voltage is forcibly returned to $V_{th}$ by the active recharged circuit. As a consequence, AQ SPAD cannot be triggered by other incident photons during the dead time. As the dead time will not be extended, AQ SPAD is defined as a non-paralyzable detector and has higher count rates than PQ SPAD.

Fig. 4 shows the process of photon counting in SPAD devices over $T_s$ which is set to 1 µs in the simulation. During $T_s$, it is assumed that around 100 photons hit the active area of the SPAD device (Fig. 4(a)). As shown in Fig. 4(b), only a bit of incident photons can trigger avalanches. As noted, the probability of the trigger is PDP. At the same time, the dark current triggers independently Poisson random avalanches (Fig. 4(c)). Afterwards, following the detected photons and the dark photons, the after pulse also provides...
some additional photon counts (Fig. 4(d)). In PQ SPAD, as only the first triggered avalanche can be recorded during one extended dead time, a limited number of avalanches can be achieved as shown in Fig. 4(e1). The pulse trains will be registered by the accumulator and the output of a single PQ SPAD device can be obtained. Unlike PQ SPAD, some potential avalanches cannot be triggered during the dead time in AQ SPAD. As the dead time is not extended, the AQ-SPAD device can achieve more photon counts, as shown in Fig. 4(e2).

In either PQ SPAD or AQ SPAD, the dead time effect makes a nonlinear reduction on photon counts. For a single PQ SPAD device, the relationship between the average potential counts per second, \( \mu_m \), and the real photon counts, \( \mu_{PQ_m} \), is [21]:

\[
\mu_{PQ_m} = \mu_m \exp(-\mu_m \tau_d). \tag{6}
\]

Thus, for each \( T_s \), the average number of the real photon counts, \( \mu_{PQ_m}(k) \), is calculated by:

\[
\mu_{PQ_m}(k) = \frac{\mu_m(k)}{T_s} \exp\left(-\frac{\mu_m(k) \tau_d}{T_s}\right) T_s = \mu_m(k) \exp\left(-\frac{\mu_m(k) \tau_d}{T_s}\right). \tag{7}
\]

where \( \mu_m(k) \) denotes the average potential counts for each single device in the same \( T_s \). For the SPAD array, \( \mu_m(k) \) is equal to \( \mu(k)/N_{SPAD} \). In this study, if the SPAD array is composed by PQ SPAD devices, the average output of the array during each \( T_s \) can be expressed as:

\[
\mu_{PQ}(k) = \sum_{m=1}^{N_{SPAD}} \mu_{PQ_m}(k) = \mu(k) \exp\left(-\frac{\mu(k) \tau_d}{T_s N_{SPAD}}\right). \tag{8}
\]

Note that \( \mu(k) \) is calculated by (5). According to the process of photon counting, \( \mu(k) \) means the average potential counts by the PQ SPAD array. For simplicity, \( \mu_{PQ}(k) \) can replace \( \mu(k) \) in (4) to estimate the distribution of the SPAD array output.

From the nonlinear function of the PQ SPAD array, (8), the maximum photon count rate can be calculated:

\[
\mu_{PQ_{max}} = \frac{T_s N_{SPAD}}{e \tau_d}, \tag{9}
\]

where \( e \) is Euler’s number. As shown in Fig. 5, after reaching \( \mu_{PQ_{max}} \), the PQ SPAD devices are paralyzed, the outputs of the SPAD array rapidly decreases with an increasing rate of incoming photons.

For a single AQ SPAD device, the average real photon counts per second, \( \mu_{AQ_m} \), is expressed as a function of \( \mu_m \) [21]:

\[
\mu_{AQ_m} = \frac{\mu_m}{1 + \mu_m \tau_d}. \tag{10}
\]

For each \( T_s \), the average output of a single device is:

\[
\mu_{AQ}(k) = \sum_{m=1}^{N_{SPAD}} \mu_{AQ_m}(k) = \frac{\mu(k)}{1 + \frac{\mu(k) \tau_d}{T_s N_{SPAD}}} \tag{12}
\]

Thus, the maximum photon count rate of the AQ SPAD array is:

\[
\mu_{AQ_{max}} = \frac{T_s N_{SPAD}}{\tau_d}. \tag{13}
\]

As shown in Fig. 5, when the incoming photon rate increases, the AQ SPAD devices are non-paralyzed but the outputs dramatically converge to \( \mu_{AQ_{max}} \). In other words, if the average potential photon counts, including signal photons, dark photons and after pulse counts, are more than \( \mu_{AQ_{max}} \), the AQ SPAD array will be saturated. The photon counts at the output of the SPAD array are constrained to \( \mu_{AQ_{max}} \), and the extra photons are refused and lost.

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![Fig. 5. Nonlinearity of the PQ SPAD array and the AQ SPAD array.](image)
As mentioned in Section II, the photon count distribution of SPAD receivers can be approximately described by Poisson distribution. For PQ SPAD, as shown in Fig. 6(a), when the total incident photons is $10^4$ over $T_s$, the Poisson distribution can accurately describe the real simulated distribution. However, when incident photons increase to $10^6$ (Fig. 6(b)), the variance of the Poisson distribution is too high to describe the distribution of output photons. Thus, in order to get more accurate results, an exact distribution is used to replace the Poisson distribution [22]:

$$\Pr_{PQ}(a, \mu_m) = \sum_{j=0}^{a_{PQ_{\text{max}}}} \frac{(j/a)!(-a/a)!\mu_m^j}{j!} \exp(-\mu_m\tau_d)(T_s - j\tau_d)^j.$$

Note that $\Pr_{PQ}(a, \mu_m)$ is the exact photon count distribution of a single PQ SPAD during $T_s$. The maximum photon count rate for a single PQ SPAD device is denoted by $a_{PQ_{\text{max}}}$, which is equal to $[T_s/\tau_d]$. In the PQ SPAD array, it is assumed that the photon count distributions of each single device are the same during $T_s$. The distribution can be written as a vector:

$$\Pr_m(k) = \Pr_{PQ} \left(0, \frac{\mu(k)}{N_{SPAD}} \right), \Pr_{PQ} \left(1, \frac{\mu(k)}{N_{SPAD}} \right), \ldots, \Pr_{PQ} \left(a_{PQ_{\text{max}}}, \frac{\mu(k)}{N_{SPAD}} \right).$$

Thus, according to (3), the joint distribution of the whole SPAD array can be calculated as:

$$\Pr(k) = \Pr_m(k) \ast \Pr_m(k) \ast \ldots \ast \Pr_m(k).$$

According to [22] and (16), the exact expectation of the PQ SPAD array output during $T_s$ is:

$$E_{PQ}(k) = N_{SPAD} \mu_m \exp(-\mu_m\tau_d)(T_s - \tau_d)$$

$$\approx N_{SPAD} \frac{\mu(k)}{N_{SPAD}} \exp(-\frac{\mu(k)}{T_s N_{SPAD}})(T_s - \tau_d)$$

$$= \mu(k) \exp(-\frac{\mu(k)\tau_d}{T_s N_{SPAD}}) = \mu_{PQ}(k).$$

Note that the symbol period, $T_s$, is assumed to be much longer than the dead time, $\tau_d$, in this study. Therefore, the exact expectation can be approximated to the average output of the array in (b). The exact variance of the array output is:

$$\sigma_{PQ}^2(k) = N_{SPAD} \mu_m^2 \exp(-2\mu_m\tau_d)(3\sigma_d^2 - 2T_s\tau_d)$$

$$+ \mu_m \exp(-\mu_m\tau_d)T_s.$$
are distorted by PQ and AQ recharged circuits resulting in some high amplitude symbols in the recovered signal (presented in this section. In the SPAD-based OFDM system, the AQ SPAD array has a higher mean value of photon counts compared with Poisson distribution, the exact distribution is Furthermore, as a result of a comparison between Fig. 6(a) with both Poisson distribution and the exact distribution. Of an AQ SPAD array. When the number of the total incident

\[ E[N(x)\mid Q(N(x))] = \int_{-\infty}^{\infty} N(x)\nu_{\frac{1}{2\sigma_x}} \phi \left( \frac{x-\mu}{\sigma_x} \right) dx \]

\[ = \int_{0}^{\infty} (C_x + k)^2 \exp \left[ \frac{1}{2} C_x^2 \sigma_x^2 - C_x \mu - C_x \mu \right] \frac{1}{\sigma_x} \phi \left( \frac{x-\mu + C_x \sigma_x^2}{\sigma_x} \right) dx + C_n \exp(-C_n)Q \left( \frac{\mu}{\sigma_x} \right) \]

\[ = \exp \left[ \frac{1}{2} C_x^2 \sigma_x^2 - C_x \mu - C_x \mu \right] \left\{ C_x^2 \left[ \left( C_x \sigma_x^2 - \mu \right) + 2C_x \mu \right] Q \left( \frac{C_x \sigma_x^2 - \mu}{\sigma_x} \right) + C_x^2 Q \left( \frac{\mu}{\sigma_x} \right) + C_n \exp(-C_n)Q \left( \frac{\mu}{\sigma_x} \right) \right\} \]

\[ + C_x Q \left( \frac{\mu}{\sigma_x} \right) \]

\[ E[N^2(x)] = \int_{-\infty}^{\infty} N^2(x) \frac{1}{\sigma_x} \phi \left( \frac{x-\mu}{\sigma_x} \right) dx \]

\[ = C_x^2 \left[ \sigma_x^2 + C_x \sigma_x^2 \right] + C_x Q \left( \frac{\mu}{\sigma_x} \right) + C_n \exp(-C_n)Q \left( \frac{\mu}{\sigma_x} \right) \]

is:

\[ E_{AQ}(k) = N_{SPAD}E_m(k) = N_{SPAD}\lambda_m T_x \]

\[ = N_{SPAD} \left[ 1 + \frac{\mu(k)}{T_x E_{\text{SPAD}}} \right] \left[ T_x E_{\text{SPAD}} \right] \]

\[ = \frac{\mu(k)}{T_x E_{\text{SPAD}}} = \mu_{AQ}(k). \]

It can be seen that the exact photon count distribution of the AQ SPAD array has the same mean value as the Poisson distribution from (12). According to the variance calculation of a single AQ SPAD device [23], the exact variance of the AQ array output is:

\[ \sigma_{AQ}^2(k) = N_{SPAD} \sigma_{AQ}^2(k) \]

\[ = N_{SPAD} \left[ T_x + g^2 \lambda (1 + \frac{2}{r} + \frac{1}{g^2}) \right]. \]

where \( \sigma = \mu_{AQ} T_x \). Fig. 7 shows the photon count distribution of an AQ SPAD array. When the number of the total incident photons is low (10^3), the simulation result closely matches with both Poisson distribution and the exact distribution. Furthermore, as a result of a comparison between Fig. 6(a) and Fig. 7(a), it shows that the PQ and AQ SPAD array have the same photon count distribution. This is because the linear region of the PQ SPAD array is almost coincident with the AQ SPAD array when the photon rate is low (Fig. 5). In Fig. 7(b), compared with Poisson distribution, the exact distribution is closer to the simulation result. Moreover, when compared with the photon count distribution of the PQ SPAD array (Fig. 6(b)), the AQ SPAD array has a higher mean value of photon counts when the nonlinear distortion occurs.

IV. THEORETICAL ANALYSIS OF SPAD-BASED OFDM

The analytical BER performance of SPAD-based OFDM is presented in this section. In the SPAD-based OFDM system, some high amplitude symbols in the recovered signal (\( \alpha'(k) \)) are distorted by PQ and AQ recharged circuits resulting in loss of information. This causes a unique receiver nonlinear distortion which should be considered in the theoretical analysis. According to [15], [16], a nonlinear distortion in an OFDM based system can be described with a gain factor (\( \alpha \)) and additional noise (\( Y \)), both of which can be explained and quantified with the Bussgang theorem. It states that if an independent Gaussian random variable, \( X \), passes through a nonlinear transformation, \( z(X) \), then [24]:

\[ \begin{align*}
    z(X) &= \alpha X + Y,
    E[XY] &= 0,
\end{align*} \]

where \( \alpha \) is a constant which can be derived as:

\[ \alpha = \frac{E[Xz(X)]}{E[X^2]} \]

According to (24), the variance of the additional noise, \( \sigma_Y^2 \), can be calculated by:

\[ \sigma_Y^2 = E[Y^2] - E^2[Y] \]

where:

\[ E[Y^2] = E[z^2(x)] - E[\alpha^2 X^2] \]

\[ E[Y] = E[z(x)] - E[\alpha X] \]

To describe the analytical BER calculations of SPAD-based OFDM, the following formulas are defined. The standard normal distribution probability density function (PDF) is:

\[ \phi(x) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{x^2}{2} \right). \]

According to (5), the relationship between each Gaussian random variable, \( x \), and the related number of photons, \( N(x) \), is:

\[ N(x) = \begin{cases} 
    C_x x + C_n, & x \geq 0, \\
    0, & x < 0.
\end{cases} \]

where \( C_x = C_{FF} C_{PDP} P T_x (1 + P_{AP}) / E_P \) and \( C_n = n_{DC59}(1 + P_{AP}) \). Note that \( P \) is the average received optical power.
Table of Contents

1. Introduction
2. System Model
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1. Introduction

In this chapter, we present the design and implementation of a novel OFDM-based radio system for indoor wireless communication. The system utilizes a combination of analog-to-digital conversion (ADC) and digital-to-analog conversion (DAC) to achieve high data rates and robust performance in challenging indoor environments. The system architecture is based on a hybrid analog/digital approach, which allows for efficient power and energy consumption while maintaining high data rates. The system design is validated through comprehensive simulation studies, which demonstrate its superior performance compared to existing OFDM-based systems.

2. System Model

The system model consists of several key components: the transmitter, the channel, and the receiver. The transmitter modulates the input data using a combination of ADC and DAC, resulting in a complex-valued OFDM signal. The channel introduces impairments such as frequency-selective fading, multipath propagation, and noise. The receiver demodulates the received signal using a combination of ADC and DAC, followed by symbol recovery and channel equalization. The system model is implemented using a discrete-time simulation framework, which allows for accurate modeling of the system's behavior under various operating conditions.

3. Performance Analysis

The performance of the system is evaluated using a variety of metrics, including bit error rate (BER), signal-to-noise ratio (SNR), and delay spread. The simulation results demonstrate the system's robustness against various channel impairments and its potential for high data rates in indoor environments. The performance analysis is conducted using a combination of numerical simulations and theoretical analysis, which provides insights into the system's behavior under different operating conditions.

4. Simulation Results

The simulation results showcase the effectiveness of the proposed OFDM-based radio system in indoor wireless communication. The system's performance is evaluated using a variety of simulation scenarios, including indoor environments with different levels of multipath propagation and noise. The simulation results demonstrate the system's ability to maintain high data rates and low BER, even in challenging indoor environments. The simulation results are compared with existing OFDM-based systems, which highlights the system's potential for high data rates and robust performance.

5. Conclusion

In conclusion, the proposed OFDM-based radio system demonstrates superior performance in indoor wireless communication, thanks to its hybrid analog/digital approach and efficient power and energy consumption. The simulation results demonstrate the system's robustness against various channel impairments and its potential for high data rates in indoor environments. The proposed system is a promising candidate for future indoor wireless communication applications, where high data rates and robust performance are critical.
defined as:

$$\text{SNR}_{\text{ACO}} = \frac{\alpha_{\text{PQ-ACO}}^2 C^2 \sigma_{\text{SNR-ACO}}^2}{2R_{\text{ACO}}(\sigma_{\text{SNR-ACO}}^2 + \sigma_{\text{PQ-ACO}}^2)}.$$  \hfill (43)

Note that $R_{\text{ACO}}$ is the spectral efficiency of ACO-OFDM which is $\frac{1}{2}\log_2(M)$ and $\sigma_{\text{SNR-ACO}}^2$ is the variance of the shot noise which is related to the received signals. In the case of Poisson distribution, the variance is equal to the mean value. Thus, each received symbol, $\sigma_{\text{SNR-ACO}}^2(x)$ is equal to $z_{\text{PQ}}(N(x))$. As a result, $\sigma_{\text{SNR-ACO}}^2$ can be derived as:

$$\sigma_{\text{SNR-ACO}}^2 = E[\sigma_{\text{SNR-ACO}}^2(x)] = E[z_{\text{PQ}}(N(x))] = \frac{3\alpha^2 - 2E_c}{2} C_{\text{DCO}} z_{\text{PQ}}(N(x)) + z_{\text{PQ}}(N(x)).$$  \hfill (44)

According to (35), the value of $\sigma_{\text{SNR-ACO}}^2$ can be obtained. Note that $\sigma_{\text{SNR}}$ is set to $\frac{3E}{2}$ and $\rho$ is set to 0 in PQ SPAD ACO-OFDM. In the case of the exact distortion, the variance of the shot noise component in the PQ SPAD array can be calculated according to (18):

$$\sigma_{\text{SNR-ACO}}^2 = E[\sigma_{\text{SNR-ACO}}^2(x)] = E[z_{\text{PQ}}(N(x))] = \frac{3\alpha^2 - 2E_c}{2} C_{\text{DCO}} z_{\text{PQ}}(N(x)) + z_{\text{PQ}}(N(x)).$$  \hfill (45)

Finally, based on the conventional BER calculation of OFDM [17], the analytical BER performance of PQ SPAD ACO-OFDM can be derived by:

$$\text{BER}_{\text{PQ-ACO}} = 4\sqrt{\frac{M-1}{M}} \rho \left( \frac{3\alpha^2 - 2E_c}{2} C_{\text{DCO}} z_{\text{PQ}}(N(x)) + z_{\text{PQ}}(N(x)) \right)^2 + 4\sqrt{\frac{M-2}{M}} \left( \frac{3\alpha^2 - 2E_c}{2} C_{\text{DCO}} z_{\text{PQ}}(N(x)) + z_{\text{PQ}}(N(x)) \right)^2.$$  \hfill (46)

2) PQ SPAD DCO-OFDM: In DCO-OFDM, the standard deviation of the bias factor of the OFDM symbols, $\mathbf{b}(k)$, is:

$$\sigma_{\mathbf{b}} = \sqrt{\frac{2(\mathbf{M}-1)(\mathbf{N}-2)}{3\mathbf{N}}}.$$  \hfill (47)

As a DC bias component is added to the original OFDM symbol, the clipping distortion noise, $\sigma_{\mathbf{b}}$, has to be considered in DCO-OFDM. Based on (1), the DC bias in DCO-OFDM is:

$$B_{\text{DC}} = \beta \sigma_{\mathbf{b}}.$$  \hfill (48)

According to (25), the clipping distortion factor of DCO-OFDM can be derived as:

$$\alpha_{\mathbf{b}} = \int_0^\infty \frac{\sigma_{\mathbf{b}}^2 + B_{\text{DC}}}{\sigma_{\mathbf{b}}^2 + 2B_{\text{DC}}} \sigma_{\mathbf{b}} \phi \left( \frac{\sigma_{\mathbf{b}}^2 + B_{\text{DC}}}{\sigma_{\mathbf{b}}^2 + 2B_{\text{DC}}} \right) \sigma_{\mathbf{b}}^2.$$  \hfill (49)

where $G_{\text{DC}}$ denotes the attenuation of the original signal power, $\alpha_{\mathbf{b}}$, due to the DC bias in DCO-OFDM. It is defined as:

$$G_{\text{DC}} = \frac{\sigma_{\mathbf{b}}^2}{\alpha_{\mathbf{b}} + B_{\text{DC}}^2}.$$  \hfill (50)

According to (26), (27) and (28), the variance of the clipping distortion noise, $\sigma_{\mathbf{b}}^2$, can be calculated by:

$$\sigma_{\mathbf{b}}^2 = E(Y^2) - E(Y^2)$$

$$= \alpha_{\mathbf{b}}(1 - \alpha_{\mathbf{b}})(\sigma_{\mathbf{b}}^2 - \sigma_{\mathbf{b}}^2) + B_{\text{DC}}^2 \left( \frac{B_{\text{DC}}}{\sigma_{\mathbf{b}}^2 - \sigma_{\mathbf{b}}^2} \right)^2 + 2 \sigma_{\mathbf{b}}^2 \phi \left( \frac{B_{\text{DC}}}{\sigma_{\mathbf{b}}^2 - \sigma_{\mathbf{b}}^2} \right).$$  \hfill (51)

After DC biasing and clipping in time domain, the mean value of DCO-OFDM symbols is:

$$E[X_{\text{DCO}}(k)] = \int_0^{\infty} \frac{x}{\sigma_{\mathbf{b}}^2} \phi \left( \frac{x - B_{\text{DC}}}{\sigma_{\mathbf{b}}^2} \right) dx$$

$$= \frac{B_{\text{DC}}}{\sigma_{\mathbf{b}}^2} \phi \left( \frac{B_{\text{DC}}}{\sigma_{\mathbf{b}}^2} \right) + \sigma_{\mathbf{b}}^2 \phi \left( \frac{B_{\text{DC}}}{\sigma_{\mathbf{b}}^2} \right).$$  \hfill (52)

Thus, the normalized DCO-OFDM symbols can be described as a Gaussian distribution from 0 to $\infty$ with mean value and standard deviation:

$$\rho_{\text{DCO}} = \frac{B_{\text{DC}}}{E[X_{\text{DCO}}(k)]},$$

$$\sigma_{\mathbf{b}} = \frac{\sigma_{\mathbf{b}}}{E[X_{\text{DCO}}(k)]}. \hfill (53)$$

Similar to PQ SPAD ACO-OFDM, the nonlinear gain factor of PQ SPAD DCO-OFDM is:

$$\alpha_{\text{PQ-DCO}} = \frac{E[X_{\text{DCO}}(k)]}{E[X_{\text{DCO}}(k)]} - \rho_{\text{DCO}} \sigma_{\mathbf{b}}.$$  \hfill (54)

Then, the equations, (27) and (28), become:

$$E(Y^2) = E[z_{\text{PQ}}(N(x))^2] - \sigma_{\mathbf{b}}^2 \rho_{\text{DCO}} E[X_{\text{DCO}}(k)^2],$$

$$E[Y] = E[z_{\text{PQ}}(N(x))] - \rho_{\text{DCO}} \sigma_{\mathbf{b}} E[X_{\text{DCO}}(k)].$$  \hfill (55)

where $\rho = \rho_{\text{DCO}}$ and $\sigma_{\mathbf{b}} = \sigma_{\mathbf{b}}$. As a result, $\sigma_{\mathbf{b}}^2$ can be obtained. As the clipping distortion factor is considered in DCO-OFDM, the final resulting SNR of the PQ SPAD DCO-OFDM system can be calculated by:

$$\text{SNR}_{\text{DCO}} = \frac{G_{\text{DCO}} \alpha_{\mathbf{b}}^2 z_{\text{PQ}}(N(x))^2}{R_{\text{DCO}}(\alpha_{\mathbf{b}}^2 \sigma_{\mathbf{b}}^2 + \sigma_{\mathbf{b}}^2 \sigma_{\text{SNR-DCO}}^2)}$$

$$= \frac{1}{R_{\text{DCO}}} \frac{G_{\text{DCO}} \alpha_{\mathbf{b}}^2 z_{\text{PQ}}(N(x))^2}{2E[X_{\text{DCO}}(k)^2]}.$$  \hfill (56)
B. OFDM with AQ SPAD

As noted, AQ SPAD has a different output function with PQ SPAD. According to (12) and (24), the nonlinear transformation function of AQ SPAD OFDM is:

\[
z_{AQ}(N(x)) = \frac{N(x)}{1 + C_{AQ}N(x)} = \sigma_{AQ}N(x) + Y_{AQ}. \quad (60)
\]

To obtain the analytical equations of AQ SPAD OFDM, the shot noise component, \(\sigma_{AQ}^2\), is substituted into (32), (34) and (35) where \(z_{AQ}(N(x))\) is replaced. Then following the same steps as in the analysis of PQ SPAD OFDM, the final resulting SNRs of AQ SPAD OFDM can be derived as:

\[
SNR_{AQ}^{ACO} = \frac{\alpha_{AQ}^2 \cdot \sigma_{AQ}^2 \cdot C_{ACO}}{2H_{ACO}(\sigma_{AQ}^2 - C_{ACO} + \sigma_{N}^2 - AQ)}.
\]

\[
SNR_{AQ}^{DCO} = \frac{G_{DCO} \cdot \alpha_{AQ}^2 \cdot \sigma_{AQ}^2 \cdot C_{DCO}}{N_{DCO}(\sigma_{AQ}^2 + \sigma_{DCO}^2 + \sigma_{N}^2 - AQ)}.
\]

It is worth noting that both Poisson distribution (4) and the exact distribution (16) need to be considered in the calculations of the shot noise component, \(\sigma_{AQ}^2\). In the case of Poisson distribution, the variance is equal to the mean value. By using the same expression in PQ SPAD (44), the shot noise component in AQ SPAD OFDM is:

\[
\sigma_{AQ}^2 = E[\sigma_{AQ}^2(x)] = E[z_{AQ}(N(x))]. \quad (63)
\]

In the case of the exact distribution, based on the exact variance of the AQ SPAD array output (23), the shot noise can be derived as:

\[
\sigma_{AQ}^2 = E[\sigma_{AQ}^2(k)] = E\left[N_{SPAD} \cdot \lambda_{N} \cdot N(x) + \sigma_{N}^2 \cdot \lambda_{N} \left(1 + \frac{1}{6} \cdot \sigma_{N}^2 \right)\right]. \quad (64)
\]

where \(\lambda_{N} = (1 + C_{AQ}N(x))^{-1}\) and \(\sigma_{N} = C_{AQ}N(x)\). The performance of Poisson distribution and the exact distribution in PQ and AQ SPAD OFDM systems will be compared in the next section.

V. RESULTS AND DISCUSSION

In this section, the analytical BER performance of SPAD-based DCO-OFDM and ACO-OFDM with the nonlinear distortion are compared. Moreover, the maximum bit rates of each scheme are found, which are limited by the nonlinear distortion effect. In the simulation, an ideal LED is assumed to emit blue light with a wavelength distribution centred around 450 nm. For the ideal LED transmitter, the recharged time of the circuit and on/off time of LED can be neglected, thus rising/falling edges have negligible effects on transmitted samples in the time domain. Therefore, in this study, each digital OFDM symbol is converted to intensity signals without any distortions. In addition, optical signals are assumed to pass through a flat fading channel and in the absence of background light. As a consequence, the received signals are still non-distorted intensities with additional shot noises. Thus, in this study, the signals are assumed to be affected by the receiver shot noise, nonlinear distortion and clipping distortion (low bias level DCO-OFDM). As the additional nonlinear noise is doubled larger than another threshold, the nonlinear distortion of SPAD receivers occurs, resulting in BER higher than \(10^{-3}\). This threshold is defined as the maximum optical irradiance (MOI). The gap between the MPR and the MOI is defined as the low error area (LEA) where the system can maintain a low BER (< \(10^{-3}\)). For example, in Fig. 8, after the optical irradiance reaches \(-90.7\) dBm, the BER of 4-QAM ACO-OFDM with PQ SPAD receivers is lower than \(10^{-3}\), and after the optical irradiance reaches \(-39.6\) dBm, the BER of the same scheme is higher than \(10^{-3}\). Thus, the BER of this scheme is \(-90.7\) dBm; the MOI is \(-39.6\) dBm; and the LEA is \(51.1\) dB. This means that 4-QAM PQ SPAD ACO-OFDM can be ideally used when the optical irradiance ranges from \(-90.7\) dBm to \(-39.6\) dBm.

The BER performance of the PQ SPAD ACO-OFDM and DCO-OFDM systems is presented in Fig. 8 and Fig. 9 where \(T_s = 1\) ms and \(T_R = 1\) µs. The analytical and simulation results confirm a very close match. It is shown that ACO-OFDM has a lower MPR than DCO-OFDM with the same constellation size. DC bias in DCO-OFDM consumes additional transmission power so that at the SPAD receiver side, the DCO-OFDM system receives more optical power than ACO-OFDM. As the additional nonlinear noise is doubled in ACO-OFDM [16], the nonlinear distortion in ACO-OFDM occurs earlier than DCO-OFDM. Thus it shows that the MOI of ACO-OFDM is lower than DCO-OFDM. On the whole, the LEA of ACO-OFDM is higher than DCO-OFDM. As a result, in SPAD-based OFDM systems, ACO-OFDM requires lower transmission power and has a longer operated interval when compared with DCO-OFDM. For different symbol periods,

<table>
<thead>
<tr>
<th>TABLE I SIMULATION PARAMETERS</th>
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<tbody>
<tr>
<td>The active area of each SPAD device</td>
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<tr>
<td>Total area of the SPAD array</td>
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<tr>
<td>The FF of the SPAD array, C_FF</td>
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<tr>
<td>The PDP of each SPAD device, C_PDP</td>
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<tr>
<td>The DCR of each SPAD device, N_DCR</td>
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<tr>
<td>The APP of each SPAD device, P_APP</td>
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<tr>
<td>The dead time of each SPAD device, τ_D</td>
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<tr>
<td>Number of SPADs in an array, N</td>
</tr>
<tr>
<td>The wavelength of the received light, λL</td>
</tr>
</tbody>
</table>
the schemes with a shorter symbol period ($T_s = 1 \mu s$) have higher MPRs than the methods with a longer symbol period ($T_s = 1 \text{ ms}$). This means that with decreasing $T_s$, the LEA reduces, which decreases the range of the received optical power. It is worth noting that for 64-QAM DCO-OFDM with 7 dB bias, the clipping distortion creates an error floor. Thus for higher constellation sizes, a higher DC bias may need to be applied in DCO-OFDM. However, the MPR increases with the additional DC bias, and at the same time, the LEA decreases. As shown in Fig. 9, for 64-QAM DCO-OFDM with 13 dB bias, the MPR becomes higher than the MOI. Therefore, this symbol period is unacceptable for this scheme.

Fig. 10 and Fig. 11 show the BER performance of the AQ SPAD ACO-OFDM and DCO-OFDM systems as a function of the optical irradiance when $T_s = 1 \text{ ms}$ and $T_s = 1 \mu s$. Compared with the BER performance of PQ SPAD OFDM (Fig 8 and Fig 9), these two systems have the same BER performances at low optical irradiance (around MPR). This is because the PQ SPAD devices have the same performance of linearity as the AQ SPAD devices when the number of incoming photons is low (Fig. 5). However, as the maximum count rate of PQ SPAD is lower than AQ SPAD, the MOIs of PQ SPAD OFDM systems are lower than AQ SPAD OFDM systems. In addition, since the MPRs of each system are the same, the LEAs of PQ SPAD OFDM systems are also lower than AQ-based systems. As a consequence, the PQ SPAD OFDM system is more readily affected by the nonlinear distortion and has higher limitation of the optical irradiance.

As given in Section III, exact distributions of the PQ and AQ SPAD array are considered in this study and compared with Poisson distribution. It is shown that the simulation results are well matched with both the Poisson distribution and the exact distribution. When the optical irradiance is low, Poisson has the same distribution as the exact one (Fig. 6(a) and Fig. 7(a)); and when the nonlinear distortion occurs,
the variance of the nonlinear additional noise dominates the performance of the system and the shot noise has a negligible effect on the BER performance. Thus Poisson distribution shows the same performance as the exact distribution in the SPAD-based OFDM system. Although the exact distribution can describe the actual distribution of the SPAD arrays, the Poisson distribution is easier to implement in the simulation.

### B. Maximum Bit Rates

Fig. 12 shows the MOI and LEA of the PQ SPAD and the AQ SPAD OFDM systems with different spectral efficiencies and symbol periods ($T_s = 1\, \text{ms}$ and $T_a = 1\, \mu\text{s}$). It is shown that the MOIs of all schemes decrease when the spectral efficiencies increase. High constellation size schemes have higher signal variances and peak-to-average power ratio due to increasing of the probability of high intensity signals. Those high intensity signals are easily affected by the nonlinear distortion of SPAD receivers and increase the probability of error detections and demodulations. In addition, with the increase of constellation sizes, MPRs also increase. Therefore, with the increase of the spectral efficiency, LEAs of the systems rapidly decrease, as shown in Fig. 12.

Note that the LEA of the SPAD-based OFDM system decreases when the symbol period becomes shorter. If the MOI is equal to the MPR, the BER of the corresponding SPAD-based system is always above $10^{-5}$. This means that the system cannot maintain a high-quality communication and thus the minimum symbol period can be obtained. As the bit rate is equal to the spectral efficiency divided by the symbol period, the maximum bit rate can also be obtained. By using the presented analytical BER model of the SPAD-based OFDM system in this study, the relationship between spectral efficiencies and theoretical maximum bit rates is obtained and is shown in Fig. 13. The theoretical maximum bit rate of the SPAD-based OFDM system is up to 1 Gbit/s. Unlike the PD-based system, the increase of the spectral efficiency cannot bring a high bit rate in SPAD-based OFDM due to the limitation of the nonlinear distortion effect. However, since the SPAD receiver performs a significant enhancement on the power efficiency and sensitivity [6], the maximum bit rate of the SPAD-based OFDM can be much higher than the conventional PD-based OFDM in the same transmission power condition.

### VI. CONCLUSION

In this paper, a complete analytical approach is presented for the performance analysis of the SPAD-based OFDM system with a receiver nonlinear distortion. The proposed theory shows very close agreement with the Monte Carlo simulation, thus confirming the validity of this analytical method. The presented analytical models provide an effective and accurate way to estimate system performance and to choose optimal parameters of the PQ and AQ SPAD receivers for the ACO-OFDM and DCO-OFDM system. For the assumed SPAD-based OFDM system, the nonlinear distortion has a significant effect on the BER performance when the optical irradiance is higher than - 40 dBm. This maximum optical irradiance limits the maximum bit rate of the system, which is up to 1 Gbit/s, as shown in this study.

The SPAD receiver has a significantly enhanced sensitivity. This means that the SPAD-based OFDM system can be used in long distance transmissions, or it can be used in non-line-of-sight OWC links, in the uplink when illumination is not essential, or when lights are almost completely dimmed. However, due to such high sensitivity, an appropriate transmission power should be selected carefully so as to avoid the nonlinear distortion.

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Selected Publications


Non-DC-Biased OFDM with Optical Spatial Modulation

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Abstract—In this paper, a novel optical Orthogonal Frequency Division Multiplexing (OFDM) modulation approach is presented. This method uses the Optical Spatial Modulation (OSM) technique to obtain positive and real-valued signals which are required by an Optical Wireless Communication (OWC) system. In comparison to existing OFDM methods applied to the OSM system, the new scheme, Non-DC-biased OFDM (NDC-OFDM), has significant advantages. Compared to DC-biased Optical OFDM (DCO-OFDM), NDC-OFDM avoids DC-biasing and, thus, improves the power efficiency. Moreover, the spectral and power efficiency of the new approach are better than the well-known unipolar optical modulation scheme, Asymmetrically Clipped Optical OFDM (ACO-OFDM). The bit-error ratio (BER) performances of these three methods are compared. Compared to ACO-OFDM and DCO-OFDM, NDC-OFDM has an energy saving gain of at least 5 dB for the same spectral efficiency. The improvement comes at the expense of additional hardware at the transmitter and receiver. However, visible light communication (VLC) systems typically are equipped with multiple low-cost Light Emitting Diodes (LEDs) to fulfill minimum indoor lighting conditions.

Index Terms—optical spatial modulation, optical OFDM, optical wireless communication, MIMO.

I. INTRODUCTION

With the rapid increase in wireless services and applications, the limited radio frequency (RF) spectrum may not be sufficient to cope with future data rate demands. As a viable complementary approach, Optical Wireless Communication (OWC) has gained significant attention as a result of technological breakthroughs in solid state lighting technology [1]. The momentous advantage of OWC is that it offers a very large bandwidth for each transmitting LED.

In current OWC systems, Light Emitting Diodes (LEDs) are used as transmitters to convert the modulated electrical signal to an optical signal. At the receiver, the optical signal is detected by photodiodes (PDs) and demodulated using digital signal processing techniques. Off-the-shelf LEDs and PDs can be used to realize a low-cost visible light communication (VLC) system which can achieve high bit rates of at least 500 Mb/s [2]. However, LEDs disallow the use of phase information for data transmission. As a consequence, only real-valued and positive signals can be used for data modulation. This is in stark contrast to RF systems which make use of complex valued and bi-polar signals. Thus, OWC using incoherent light sources as described can only be realized as an intensity modulation (IM) and direct detection (DD) system [3]. For IM/DD, standard digital modulation techniques are conceived, such as On-Off Keying (OOK), Pulse Position Modulation (PPM) and Pulse Amplitude Modulation (PAM) [4].

For high-speed data transmission, Intersymbol Interference (ISI) becomes an issue and computationally complex equalization techniques are required. In the fourth-generation (4G) wireless communication, Orthogonal Frequency Division Multiplexing (OFDM) is used, as it is better equipped to handle severe ISI. In optical communications, OFDM can also be applied in the context of IM/DD systems [5]. Because the IM/DD system can only transmit real-valued signals, Optical OFDM (O-OFDM) needs to produce real-valued symbols. This can be achieved by imposing Hermitian symmetry on the information frame before the inverse fast Fourier transform (IFFT) operation during the signal generation phase. This comes at the expense of half of the spectral efficiency. In general, standard techniques to ensure positive optical signals, which are required by LEDs, are DC-biased Optical OFDM (DCO-OFDM) and Asymmetrically Clipped Optical OFDM (ACO-OFDM) [6][7]. In DCO-OFDM, a DC-bias is added. In ACO-OFDM, the system inserts zeros on even subcarriers and modulates only odd subcarriers. As a result, a group of antisymmetric real-valued OFDM symbols are obtained, as shown in [8]. This allows any negative samples to be clipped without distortion. DC-bias and clipping noise in DCO-OFDM have an impact on the bit-error ratio (BER) performance [9]. When high-power signals are required, the effect becomes significant. In ACO-OFDM, although there is negligible DC-bias, the scheme sacrifices 50% spectral efficiency compared to DCO-OFDM for the same Quadrature Amplitude Modulation (QAM) constellation size.

As introduced in this paper, the original O-OFDM modulator can be combined with Spatial Modulation (SM) [10] to result in a new method, Non-DC-biased OFDM (NDC-OFDM). This system inherits characteristics from SM (low complexity) and OFDM (ISI resistance). More importantly, it...
solves the DC-bias problem in DCO-OFDM and has a higher spectral efficiency than ACO-OFDM.

The rest of this paper is organized as follows. The system model of traditional OSM combined with DCO-OFDM and ACO-OFDM is described in Section II. Section III presents the system model of NDC-OFDM. Section IV shows the result of a comparison between NDC-OFDM and conventional OSM-OFDM in terms of their BER performances, power efficiency and spectral efficiency. Finally, Section V concludes this paper.

II. CONVENTIONAL OSM-OFDM SYSTEM MODEL

Fig. 1 shows the system model of the conventional OSM-OFDM system. This system combines the basic SM-OFDM [10] and traditional O-OFDM techniques [6].

In the first step of the modulation procedure, the input bit stream is reshaped and placed in an \( N \times n \) matrix, \( \mathbf{Q}(p) \), where \( N \) is the number of OFDM subcarriers and \( m = \log_2(MN) \). Moreover, \( M \) is the QAM constellation size and \( N_t \) denotes the number of transmitters. This paper compares NDC-OFDM with OSM-OFDM when \( N_t \) is set to two. Under this assumption, bits in the first column of \( \mathbf{Q}(p) \) represent the index of transmitters. This means that when the bit in the first column is zero, the rest of the bits on the same row will be transmitted by the first LED and when it equals one, the rest of the bits will be conveyed by the second LED. Bits in the other columns of each row will be transformed to complex \( M \)-QAM symbols. For example, in Fig. 1, it can be seen that the first row of \( \mathbf{Q}(p) \) is \([1\,0\,1]\). This means that \([0\,1] \) will be converted to a QAM symbol \(-1 + i\) by Gray mapping and this symbol will be put in the first slot of \( \mathbf{X}(n) \), as illustrated in Fig. 1. Simultaneously, the first slot of \( \mathbf{X}(n) \) will be set to zero. As a result of the \( M \)-QAM and SM mapping, two complex vectors, \( \mathbf{X}_1(n) \) and \( \mathbf{X}_2(n) \), are obtained. Each vector passes through an O-OFDM modulator. In general, two standard techniques, ACO-OFDM and DCO-OFDM, are used to obtain positive and real-valued OFDM symbols, which are introduced and compared in [6] and [9]. In ACO-OFDM, \( N/4 \) QAM symbols are mapped onto half of the odd subcarriers of an OFDM frame. At the same time, the even subcarriers are set to zero. In DCO-OFDM, \( N/2 - 1 \) symbols are put into the first half of subcarriers and the DC subcarrier (the first subcarrier) is set to zero. Afterwards, for both ACO-OFDM and DCO-OFDM, Hermitian symmetry is applied on the rest of the OFDM frame. Thus, the two groups of QAM symbols from \( \mathbf{X}_1(n) \) and \( \mathbf{X}_2(n) \) are mapped onto OFDM frames and they are transformed into real-valued OFDM symbols by the IFFT block. Finally, in order to get positive symbols, the negative values need to be set to zero in ACO-OFDM. In DCO-OFDM, before clipping, a DC-biased power is added to the bipolar OFDM symbols.

The resulting output vectors at the O-OFDM modulator, \( \mathbf{x}_1(k) \) and \( \mathbf{x}_2(k) \), are transmitted by the respective LED over an \( N_r \times N_c \) optical multiple-input multiple-output (MIMO) channel, \( \mathbf{H} \), where \( N_r \) is the number of receivers. In this paper, the main objective is to compare the performance of the conventional OSM-OFDM system with NDC-OFDM. Therefore, the same optical channel is used for all three schemes. This channel is presented in Section III.

At the receiver, PDs convert optical signals to electrical signals. Additive white Gaussian noise (AWGN) is added to the signal due to ambient light and thermal noise in the transimpedance amplifier. Through the analog-to-digital conversion block, signals from each PD can be transferred to their corresponding vectors, \( \mathbf{y}_1(k) \) and \( \mathbf{y}_2(k) \). Each vector will be dealt with by the respective O-OFDM demodulator. As in conventional O-OFDM techniques, the received OFDM symbols are passed through a fast Fourier transform (FFT) operation which converts symbols to the frequency domain. In DCO-OFDM, \( N/2 - 1 \) symbols are obtained from the corresponding subcarriers and in ACO-OFDM, \( N/4 \) symbols are obtained. The extracted symbols are transferred to two complex vectors, \( \mathbf{Y}_1(n) \) and \( \mathbf{Y}_2(n) \). Zero forcing (ZF) is used to reverse the impairments of the MIMO channel to transform \( \mathbf{Y}_1(n) \) and \( \mathbf{Y}_2(n) \) into \( \mathbf{X}_1'(n) \) and \( \mathbf{X}_2'(n) \) respectively [10]. Afterwards, the SM detector compares the absolute values of the corresponding subcarriers from each channel to estimate...
the transmitted symbol is positive, the first LED will be acti-
the indices of the active transmitters as follows,
\[
\hat{j}(n) = \arg \max(|X_i(n)|), \quad i = 1, \cdots, N_z,
\] (1)
As a result, the index of the estimated subchannel gives the bit information transmitted by the SM technique [10]. The bits from the estimated indices are put into the first column of the output matrix, \(Q'(p)\). This means that if \(\hat{j}(n)\) is equal to one, the corresponding bit is zero and if the result of the estimation is two, the bit is one. At the same time, the largest symbol in each comparison is chosen as the detected symbol,
\[
X_{d}(n) = \begin{cases} X_1(n), & \hat{j}(n) = 1, \\ X_2(n), & \hat{j}(n) = 2. \end{cases}
\] (2)
The detected QAM symbols are then decoded by the conventional Maximum Likelihood estimator. The result is allocated to the other columns of \(Q'(p)\). Finally, the output bit stream is obtained by reshaping \(Q'(p)\) into a serial bit stream. The described SM detection algorithm is not the optimal one according to [11]. However, it has been selected in this work for its low computational complexity.

III. NDC-OFDM SYSTEM MODEL

The system model of NDC-OFDM is illustrated in Fig. 2. The input bit stream is transformed into complex symbols, \(X(n), \quad n = 1, \cdots, N/2 - 1\), by an \(M\)-QAM modulator. As in DCO-OFDM, \(N/2 - 1\) QAM symbols are modulated onto the first half of an OFDM frame, \(X(k), \quad k = 1, \cdots, N\), and Hermitian symmetry is imposed on the second half of the OFDM frame. After the \(N\)-IFFT operation, the complex QAM symbols become \(N\) real-valued OFDM samples, \(x(k)\), but they are still bipolar.

In NDC-OFDM, LEDs only send the absolute value of \(x(k)\) and the sign of the symbol is represented by the index of the corresponding LED. According to the working principle of OSM, only one LED is activated during one symbol time. If the transmitted symbol is positive, the first LED will be activated to send the symbol. If the symbol is negative, its absolute value will be sent by the other LED. This principle constitutes the most significant difference between the traditional OSM-OFDM and the NDC-OFDM. An additional difference is that QAM symbols go through an OFDM modulator first and then pass through the SM mapping block in NDC-OFDM. In conventional OSM-OFDM, the order is reversed.

As shown in Fig. 2, after SM mapping, the converted optical signals, \(L_1(k)\) and \(L_2(k)\), will be transmitted by the corresponding LED over the optical MIMO channel \(H\) [12]. The \(N_1 \times N_2\) optical channel matrix is
\[
H = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1N_2} \\ h_{21} & h_{22} & \cdots & h_{2N_2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_1,1} & h_{N_1,2} & \cdots & h_{N_1,N_2} \end{pmatrix}
\] (3)
where \(h_{N_1,N_2}\) is the channel DC gain of a directed line-of-sight (LOS) link between the receiver \(N_1\) and the transmitter \(N_2\). The LOS link is considered in the system model, because the multipath components are significantly weaker and can thus be neglected. The channel gain can be calculated as follows [3]:
\[
h_{N_1,N_2} = \left\{ \begin{array}{ll} (2A/\pi)^{1/2} \cos^2(\phi)T_e(\psi_0) \cos(\psi), & 0 \leq \psi \leq \psi_e \\ 0, & \psi > \psi_e \end{array} \right.
\] (4)
where \(\beta = -\ln 2/\ln(\Phi_{1/2})\) and \(\Phi_{1/2}\) is the transmitter semiangle. Moreover, \(A\) denotes the detector area of the PD and \(d\) is the distance between the receiver \(N_1\) and the transmitter \(N_2\). The radiant angle and the incident angle are modelled respectively by \(\phi\) and \(\psi\). The optical filter gain \(T_e\) and the optical concentrator gain \(g_e\) depend on the properties of the receiver.

Thus, optical MIMO signals can be obtained as [13],
\[
y = Hs + w.
\] (5)
In an ideal scenario, if there is no AWGN, defined in [6] as the average power of the OFDM symbols, is introduced and

\[ g = H^{-1} y, \quad (6) \]

where \( g \) is an \( N_r \)-dimensional vector which contains the estimated transmitted symbols and \( H^{-1} \) denotes the inverse of the channel matrix \( H \). To estimate the indices of the active transmitters, the SM detector compares the values of the elements in \( g \) as follows,

\[ \hat{\mathbf{l}}(k) = \arg \max_i (G(i, k)), i = 1, \ldots, N_t, \quad (7) \]

where \( G \) is the \( N_t \times N \) equalized matrix which contains all the estimated transmitted symbols and \( i \) is an \( N \)-dimensional vector which contains all the estimated indices. As mentioned, there are two transmitters and two receivers. If \( \hat{\mathbf{l}}(k) \) is equal to one, this means that the symbol received at the time instant \( k \) is transmitted from the first LED. Therefore this symbol is a positive-valued OFDM symbol. On the contrary, if the result of \( \hat{\mathbf{l}}(k) \) is two, a negative symbol is transmitted by LED2. As a consequence, the estimated OFDM symbols sequence is

\[ \mathbf{x}'(k) = \begin{cases} G(\hat{\mathbf{l}}(k), k), & \hat{\mathbf{l}}(k) = 1, \\ -G(\hat{\mathbf{l}}(k), k), & \hat{\mathbf{l}}(k) = 2. \end{cases} \quad (8) \]

In an ideal scenario, if there is no AWGN, \( \mathbf{x}'(k) \) should be the same as \( \mathbf{x}(k) \). After recovering the OFDM symbols, \( \mathbf{x}'(k) \) is passed through the conventional OFDM demodulation block and the \( M \)-QAM demodulator in order to obtain the output bit stream.

IV. RESULTS AND COMPARISONS

This section analyses the spectral efficiency and the BER performance of NDC-OFDM. In addition, this section compares the new technique with DCO-OFDM and ACO-OFDM. Thus, a simple practical optical MIMO channel realized in [13] has been selected for the numerical simulations.

\[ \mathbf{H} = 10^{-5} \begin{pmatrix} 0.1889 & 0.0713 & 0.1889 \\ \end{pmatrix}, \quad (9) \]

where the values in this matrix describe the path loss between the LEDs and the PDs.

A. NDC-OFDM versus DCO-OFDM

In DCO-OFDM, the DC bias is added and the signal is clipped [9]. In practice, the value of the DC bias, which is related to the average power of the OFDM symbols, is introduced and defined in [6] as

\[ B_{DC} = \alpha \sqrt{E(\mathbf{x}'(k))}, \quad (10) \]

where \( \mathbf{x}(k) \) is the OFDM symbol frame vector and \( 10 \log_{10}(\alpha^2 + 1) \) is defined as the bias level in dB. The bias level in the current simulations is set to 7 dB and 13 dB, which are adopted from [6] for consistency. For the simple DCO-OFDM model, signal clipping is used to eliminate the negative part of the DC-biased OFDM symbol frame as,

\[ \mathbf{x}_c(k) = \begin{cases} \mathbf{x}(k), & \mathbf{x}(k) \geq 0 \\ 0, & \mathbf{x}(k) < 0 \end{cases} \quad (11) \]

where \( \mathbf{x}_c(k) \) is the DC-biased symbol which will be transmitted by an LED.

The spectral efficiencies of NDC-OFDM and DCO-OFDM in an OSM system are defined as,

\[ R\text{NDC-OFDM} = \frac{N - 2}{2N} \log_2(M_1N_t) \text{bits/s/Hz}, \quad (12) \]

where \( N_t \) is even,

\[ R\text{DCO-OFDM} = \frac{N - 2}{2N} \log_2(M_2N_t) \text{bits/s/Hz}, \quad (13) \]

where both \( M_1 \) and \( M_2 \) are the order of the QAM modulation in NDC-OFDM and DCO-OFDM respectively. From (12) and (13), when \( R\text{NDC-OFDM} \) is equal to \( R\text{DCO-OFDM} \), the constellation sizes of NDC-OFDM and DCO-OFDM have the following relationship,

\[ M_1 = 2M_2. \quad (14) \]

Fig. 3 shows the comparison between NDC-OFDM and DCO-OFDM for the same spectral efficiency, where \( M_1 = 8, 32, 128 \) and \( M_2 = 4, 16, 64 \). At BER=10^-4, the power requirement of NDC-OFDM is at least 5 dB less than the power requirement of DCO-OFDM for all the presented cases in Fig. 3. This indicates that NDC-OFDM can allocate system power more efficiently than DCO-OFDM while maintaining the same spectral efficiency. In DCO-OFDM, signal clipping has a considerable effect on the BER performance when the constellation size is larger than 64. For instance, when using
OFDM becomes approximately such as 2048, the formula for the spectral efficiency of NDC-OFDM if the number of OFDM subcarriers is chosen to be large, should satisfy the following relationship, compared to that of ACO-OFDM for the same spectral efficiency, to compare the BER performance between NDC-OFDM and ACO-OFDM.

When $a_{\text{BD}}$ and $b_{\text{A}}$ are set to 13 dB and 6 dB, respectively, then nonlinear distortion and signal clipping from below in DCO-OFDM, it eliminates the need for a DC-bias. As a consequence, it exhibits a considerable power efficiency gain for the same spectral efficiency.

### ACO-OFDM Versus NDC-OFDM

As a well-known unipolar optical modulation method, ACO-OFDM has notable energy efficiency at the expense of a reduction in spectral efficiency. In an OSM system, the spectral efficiency of ACO-OFDM is

$$R_{\text{ACO-OFDM}} = \frac{1}{4} \log_2(M_3 N_3) \text{bits/s/Hz},$$

where $M_3$ denotes the order of the constellation. In (12), if the number of OFDM subcarriers is chosen to be large, such as 2048, the formula for the spectral efficiency of NDC-OFDM becomes approximately $\frac{1}{4} \log_2(M_1)$. Thus, in order to compare the BER performance between NDC-OFDM and ACO-OFDM for the same spectral efficiency, $M_1$ and $M_3$ should satisfy the following relationship,

$$M_1 = \sqrt{2} M_3.$$  \hfill (16)

A BER comparison of NDC-OFDM and ACO-OFDM for the same spectral efficiency is shown in Fig. 4, where $M_1 = 4, 8, 16$ and $M_3 = 8, 32, 128$. It can be seen that NDC-OFDM can use lower order QAM schemes to achieve higher power efficiency than ACO-OFDM. For instance, when $M_1 = 4$ and $M_3 = 8$, NDC-OFDM could save about 5 dB in energy. When $M_1 = 16$ and $M_3 = 128$, ACO-OFDM requires 9 dB higher $E_b/N_0$ than NDC-OFDM to achieve the same BER performance.

### V. Conclusion

In this paper, a novel unipolar modulation method for OWC based on OSM is introduced. Results show that the new approach improves the power efficiency of the OSM-OFDM scheme. For the same spectral efficiency, it exhibits 5 dB to 9 dB higher energy efficiency than the conventional unipolar OFDM technique, ACO-OFDM. When compared to DCO-OFDM, it eliminates the need for a DC-bias. As a consequence, it exhibits a considerable power efficiency gain for the same spectral efficiency.

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Selected Publications

Single Photon Avalanche Diode (SPAD) VLC System and Application to Downhole Monitoring

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Abstract—In this paper, it is demonstrated for the first time that the problem of continuous downhole monitoring in the oil and gas industry is effectively addressed by the use of visible light communication (VLC). As a reliable, flexible and low-cost technique, VLC can fulfill a critical need of operators to maintain production efficiency and optimize gas well performance. The proposed VLC system makes use of a light emitting diode (LED) transmitter and a high sensitivity single photon detecting receiver referred to as single-photon avalanche diode (SPAD). The latter is instrumental in achieving long range communications, and the fact that ambient light is not present in a gas pipe is exploited. Specifically, the lack of ambient light enables high signal to noise ratio (SNR) at the receiver which operates in a photon counting mode. In this study, the bit error ratio (BER) performance of the system is simulated for a 4 kilometres long metal pipe. It is shown that the proposed system has superior power efficiency over conventional methods, which is important as it is assumed that the transmitter is battery operated. In addition, the theoretical BER performance is calculated and compared to the simulation results.

Index Terms—optical wireless communication (OWC), visible light communication (VLC), photon counting receiver, single-photon avalanche diode (SPAD).

I. INTRODUCTION

With the rapid increase in wireless services and applications, the limited radio frequency (RF) spectrum may not be sufficient to deal with future data rate demands. As a viable complementary approach, optical wireless communication (OWC) has gained significant attention in part due to recent technological advances in solid state lighting technology [1]. The advantage of OWC is that it offers almost infinite bandwidth ranging from infrared (IR) to ultraviolet (UV) including the visible light spectrum in the range of 400-790 terahertz (THz) [2]. Other important benefits of OWC include: licence-free operation, high communication security, low-cost front-ends and no interference to RF systems. The last benefit means that OWC and RF systems can be used simultaneously.

In current visible light communication (VLC) systems, light emitting diodes (LEDs) are mainly used as transmitters. At the receiver, highly sensitive photodiodes (PDs), such as positive-intrinsic-negative (PIN) diodes, avalanche photo diodes (APDs) and single-photon avalanche diodes (SPADs) are used [3]. To date, the fastest wireless VLC system using a single LED can achieve speeds exceeding 3 Gb/s [4]. However, the incoherent light output of the LED means that information can only be encoded in the intensity level. As a consequence, only real-valued and positive signals can be used for data modulation. This is in stark contrast to RF systems which make use of complex valued and bi-polar signals. Thus, VLC systems are usually considered to be modulated as an intensity modulation (IM) and direct detection (DD) system [5]. On-off keying (OOK), pulse position modulation (PPM) and pulse amplitude modulation (PAM) are some of the popular modulation schemes used in conjunction with IM/DD systems [6]. For high speed data transmissions, optical orthogonal frequency division multiplexing (O-OFDM) is applied in order to get closer to the channel capacity by utilizing adaptive bit and power loading. Diverse O-OFDM modulation schemes have been realized and utilized in VLC, such as DC-biased optical OFDM (DCO-OFDM), asymmetrically clipped optical OFDM (ACO-OFDM), unipolar OFDM (U-OFDM) and non-DC-biased OFDM (NDC-OFDM) [7], [8].

In previous studies, VLC has been considered for applications such as indoor wireless communications, wireless communication in hazardous environments and underwater communications [9]. The focus of this study is on the application of VLC in the gas extraction industry, and in particular, downhole monitoring communication systems. In the gas industry, the use of wirelines and armored cables is common practice for communication between the downhole and the surface, but these installations present maintenance and reliability issues. Furthermore, wireline solutions have high installation costs and their operation requires the halt of production bringing extra cost to the operator due to the down-time. Wireless solutions have also been considered for use in downhole monitoring, such as mud-pulse telemetry, low-frequency electromagnetic waves and acoustic waves, but for long distance communications, their performances are not satisfactory. The low data rate, the occurrence of undetectable situations and the environmental impact are the main factors which restrict the development of wireless communication systems in this context.

In this paper, a wireless solution using VLC is proposed
to overcome the restrictions of the existing technologies summarized above. The solution is designed to have low power consumption while achieving high communication speed and reliability. Unlike the RF monitoring system, a LED transmitter is used instead of antennas. Hence, there is no danger of causing explosions due to electric sparks. Thus, the proposed system is considered as a safer solution.

The rest of this paper is organized as follows. The system model of the downhole communication system with visible light is described in Section II. Section III presents the theoretical bit error ratio (BER) analysis. Section IV introduces simulation results for the system performance. Finally, Section V concludes this paper.

II. SYSTEM MODEL

This section presents a practical model for the proposed system. Based on general VLC systems, the communication system consists of a blue light LED and an array of SPADs [3]. A long steel pipe defines the channel of the transmission.

A. Pipe Parameters

As shown in Fig. 1, it is assumed that the communication system is realized in a long steel cylindrical pipe with a length of 4,000 metres and diameter of 1.5 metres. The dimension is taken from a real-world deployment of such a pipe. The reflectivity of steel is 58.5 % [10]. In this study, the reflection of the information-carrying light is considered as the specular reflection on the internal surface of the pipe. In the downhole monitoring system, there is no ambient light. Hence, the photons that reach the top of the pipe are either from the direct path or from reflections inside the pipe. This pipe constitutes the propagation channel and a ray-tracing method is used to establish a channel model. In practice, gas is transported in this pipe. As the speed of the lightwaves is reduced in the gas medium according to the refractive index, the effect of inter-symbol interference (ISI) will be enhanced which will increase the probability of detection errors. In this study, the pipe is assumed to be vacuum in order to establish the baseline performance and to understand the general feasibility.

B. LED Transmitter

As shown in Fig. 1, a LED is used at the bottom of the pipe as the transmitter. The LED emits blue light with wavelength of 450 nm. The light emission from a LED transmitter can be modelled using a generalized Lambertian radiation intensity pattern [11].

\[
R(\phi) = \frac{(\beta + 1)}{2\pi} \cos^\beta(\phi) P_t, \tag{1}
\]

where \(\beta = -\ln 2/\ln(\cos(\Phi_{1/2}))\), and \(\Phi_{1/2}\) is the transmitter semiangle which represents the half power angle. The variable \(\phi\) denotes the angle of radiation, and \(P_t\) is the average power of the LED. In the long pipe communication system considered, the design objective is to ensure that sufficient photons hit the SPAD receiver when light is emitted. An important parameter is the transmitter semiangle. Fig. 2 shows the distribution of the Lambertian radiation intensity with transmitter semiangles of 5°, 10°, 15° and 30° when the power of the LED is 0.1 Watt. As shown, more power can be concentrated in the range of lower radiation angles (-5° to 5°) for lower transmitter semiangles. This decreases the number of light rays that hit the pipe wall and will thus decrease the loss caused by reflections. In order to enhance the likelihood of photons to hit the SPAD on the direct line-of-sight (LOS) link, the LED and the SPAD device at the surface are vertically aligned. In this study, values of the semiangle from 5° to 15° are considered.

C. SPAD Receiver and Photon Counting

Due to the long distance transmission (4,000 metres), the irradiance at the top surface is lower than in standard VLC indoor scenarios where the maximum distance is a few metres. In fact, the number of photons at the receiver in the given
scenario may only be in the region of tens of photons. The typical gain of an APD is insufficient to produce sufficient signal power for further signal processing. Therefore, a highly sensitive SPAD receiver device is proposed for use in the downhole monitoring system. This diode is an APD which is biased beyond reverse breakdown in the so called 'Geiger' region. In this mode of operation, a SPAD triggers billions of the electron-hole pair generation for each detected photon. As a consequence, the device is extremely sensitive, and is able to accurately detect a single photon. Also, SPADs are considered for use in the downhole monitoring system for the following reasons:

i. The downhole provides an environment that is free from ambient light. Hence, the high sensitivity of the SPAD receiver is not compromised by ambient noise.

ii. Because of the high sensitivity of SPADs, transmission distance and optical power can be traded-off favourably towards battery life.

iii. The required data rate, which is in the order of kilobits to a few megabits per second, is sufficiently low to use on-off keying (OOK). OOK enables the use of straightforward threshold-based detection techniques in conjunction with the Geiger counting principle.

iv. The SPAD detector does not require a transimpedance amplifier (TIA) and the output signal is a pulse train (as shown in Fig. 3(b)) which greatly reduces the noise at the receiver.

In this study, an array of SPADs, which is presented in [3], is assumed to be set at the top of the pipe as shown in Fig. 1. The number of photons received per second can be estimated as follows:

\[ N_P(N_R) = C_{PDE} \sum_{n=R}^{N_R} \left( \frac{(\beta + 1)A}{2\pi d^2E} \right) \cos(\phi(n_R))C_{R}^{n_R}, \]  

where \( C_{PDE} \) is the value of the photon detection efficiency (PDE); \( N_R \) is the total number of reflections; \( A \) is the area of the SPAD receiver; and \( d \) is the distance between the LED and the receiver. In this study, \( d \) is approximated as the length of the pipe. \( E_0 \) is the energy of a photon which can be calculated by \( \frac{hc}{\lambda} \). Planck’s constant is represented by \( h \); \( c_0 \) is the speed of the light in vacuum; and \( \lambda_0 \) is the wavelength of the light. The expression, \( \phi(n_R) \), is the angle of radiation as a function of the reflections; and \( n_R \) is the number of reflections needed to reach the top. The number of reflections needed to reach the SPADS defines the radiation angle at which the light ray has left the LED, hence why we have chosen to represent \( \phi \) as a function of \( n_R \). The reflectivity of steel is expressed by \( C_R \).

In the experiment, the ambient light, dark count ratio (DCR), extinction ratio, relative intensity noise and clock jitter will add to the number of counted photons in one time slot [3]. Because the effect of the extinction ratio, relative intensity noise and clock jitter is much lower than the ambient light and DCR, they are not considered in the simulation [3]. As the system is realized in a long steel pipe where there is no ambient light, DCR is considered the dominant source of noise. SPAD exhibits a DCR similar to PD dark current which is generated by thermal carriers. This means that DCR exists even when there are no photons reaching the SPAD. In practice, DCR will add to the number of the counted photons in each time slot.

### D. Modulation Scheme

In this study, non-return-to-zero OOK (NRZ-OOK) is used. At the transmitter, the input bit stream is directly transformed to an analog signal by the digital-to-analog (D/A) converter. A binary ‘1’ is represented by a positive voltage which is much higher than the voltage that represents the binary ‘0’. The voltage levels assigned to bits depend on the power constraints of the system. In this study, \( P_T \) represents the power assigned to ‘1′ and \( P_0 \) denotes the power assigned to ‘0’. As the randomly generated zeros and ones follow a uniform distribution, \( P_T \) and \( P_0 \) have following relationship:

\[ P_T = \frac{P_0 + P_0}{2}, \]  

When the signal is transmitted by the LED, the voltages are transformed to the corresponding light intensity. As a consequence, ‘1s’ are finally represented by a higher light intensity and ‘0′s are represented by a lower intensity which is closer to zero. As shown in Fig. 3(b), the information-carrying light is received by the SPAD and is represented by the number of photons. Compared to the original digital bits (Fig. 3(a)), it can be seen that more photons are counted when ‘1s’ are transmitted, and ‘0′s’ are represented by much fewer photons at the receiver. As the received photons and the DCR noise follow a Poisson distribution which generates random positive integers [12], both increase the number of counted photons. As a consequence, irregular fluctuations are observed in Fig. 3(b) which can cause errors during demodulation. In this study, the transmission speed is assumed to be low (1 kbit/s), and the interval time between the direct light and the reflected light reaching the receiver is small. Hence, the symbol duration is

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**Fig. 3.** Example: modulation and demodulation of NRZ-OOK for SPADs.
with the fixed system bandwidth, error counted photons caused by DCR. They can be calculated when ‘0’ is transmitted; and the number of the received photons when ‘1’ is transmitted is denoted $P_{r1}$. The Poisson distribution is, 

$$N_e = N_{e0} + N_{e1}$$  \tag{9}$$

As noted, the threshold of the NRZ-OOK demodulation is the mean value of the counted photons per transmission interval. Therefore, in the theoretical analysis, the threshold is

$$N_{t} = \frac{N_{t0} + N_{t1}}{2} = \frac{N_{e0} + N_{e1}}{2} + N_{e}$$  \tag{10}$$

As shown in Fig. 4, the filled areas represent the probability of the error detection. Combined with the threshold in (9) and the mean values in (7) and (8), this probability is calculated by using the CDF of the Poisson distribution,

$$P_e = \frac{1}{2}[1 - P_e(N_t, N_{t0}) + P_e(N_t, N_{t1})]$$  \tag{11}$$

IV. SIMULATION RESULTS

In the simulation, the LED transmitter is positioned at a distance equal to half of the radius away from the center of the bottom surface. For blue light (450 nm) in vacuum, the energy of a photon ($E_0$) is $4.42 \times 10^{-19}$ J. Other parameters of the LED, such as the semigle of the LED ($\Phi_{1/2}$), LED power ($P_L$) and the bandwidth ($f_0$), are variables in the simulation. Different values of these variables are simulated in order to choose appropriate values. LED power is tested from -10 dBm to

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<th>TABLE I SIMULATION PARAMETERS</th>
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<tbody>
<tr>
<td>The length of the pipe $d$</td>
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<tr>
<td>The reflectivity of the steel $C_R$</td>
</tr>
<tr>
<td>The size of the SPAD $A$</td>
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<tr>
<td>The PDE of the SPAD $C_{PDE}$</td>
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<tr>
<td>The DCR of the SPAD $N_{DCR}$</td>
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<tr>
<td>The energy of a photon $E_0$</td>
</tr>
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</table>

where both $N_{t1}$ and $N_{t0}$ are calculated using (2) with the transmission power of $P_L$ and $P_e$. $N_{DCR}$ denotes the value for DCR.

In standard VLC and RF systems, additive white Gaussian noise (AWGN) is usually the most prominent noise component. The equations to calculate the BER performance of different modulation schemes over an AWGN channel are presented in [13]. For the Poisson noise, a similar principle can be used. Fig. 4 shows the analysis for the error probability. The curve with blue circles represents the PDF of the counted photons when ‘0’ is transmitted. As the receiver counts the photons from the transmitter and the DCR noise process, the mean value of this PDF is,

$$N_0 = N_{t0} + N_e$$  \tag{7}$$

The curve with red ‘x’ symbols denotes the PDF of the counted photons when ‘1’ is transmitted. The mean value of the PDF is,

$$N_1 = N_{t1} + N_e$$  \tag{8}$$

As noted, the threshold of the NRZ-OOK demodulation is the mean value of the counted photons per transmission interval. Therefore, in the theoretical analysis, the threshold is

$$N_t = \frac{N_{t0} + N_{t1}}{2} = \frac{N_{e0} + N_{e1}}{2} + N_e$$  \tag{9}$$

As shown in Fig. 4, the filled areas represent the probability of the error detection. Combined with the threshold in (9) and the mean values in (7) and (8), this probability is calculated by using the CDF of the Poisson distribution,

$$P_e = \frac{1}{2}[1 - P_e(N_t, N_{t0}) + P_e(N_t, N_{t1})]$$  \tag{10}$$

For NRZ-OOK, $P_e$ is equal to BER.
to 25 dBm and the bandwidths considered are 1 kHz, 2 kHz and 5 kHz. In this paper, the power assigned to ‘0’, $P_0$, is assumed to be 0 Watt. Hence, from (3), the power given to ‘1’, $P_1$, is $2P_0$. In the simulation, the SPAD presented in [3] is used for two reasons: a) this is a large array (2.4 $\times$ 2.1 mm) with 1024 SPAD elements which enhances the likelihood of receiving photons; and b) this device exists practically which is essential as the goal is to build a demonstrator to validate the simulation results in this study. The value of $C_{PDE}$ is 20% and $N_{SCR}$ is equal to 7.27 kHz [3]. Table I lists all of the parameters used in the simulation.

A. Number of Received Photons

Fig. 5 depicts the relationship between the angle of radiation and the reflections needed to reach the receiver. It can be seen that when the angle is just 0.55° and the cosine of which is 0.9999. This is due to the system being realized in a long pipe and the area of the top surface being relatively small. Hence, the angle of radiation, $\phi(n_R)$, can be approximated to 0°, even though there are over 20 reflections. Thus, $\cos^2(\phi(n_R))$ in (2) is nearly equal to 1, even if $\beta$ is large. To calculate the number of received photons in each reflection ray, (2) can be simplified as,

$$N_P(n_R) = C_{PDE} \frac{(\beta + 1)AP}{2\pi d^2 ER} C_{R}^{n_R}.$$  \hspace{1cm} (11)

The number of received photons from 0 to 25 reflections is also shown in Fig. 5. It can be seen that when the semiangle is 5° and there are no reflections, the SPAD receiver can receive $4.5 \times 10^5$ photons every second. But when the semiangle is 10°, the number of photons is only $1.1 \times 10^3$. This means that the lower the semiangle, the higher the number of received photons. In other words, the LED with lower semiangle can achieve the same BER performance with less energy.

In Fig. 5, with an increase in the number of reflections that a light ray takes to reach the receiver, there is a drastic reduction in the number of received photons. This is due to many photons being absorbed and lost in the reflections on the inner wall of the pipe. It can be seen that when the reflections are over 15, the number of photons, which can reach the receiver in every second, is less than 20. When the transmission speed is 1 kbits/s, the number of photons is over 140 in one symbol period. After 15 reflections, 20 photons are received in 1,000 symbol periods which is negligible. Thus, in this study, the number of reflections in (2) is considered to be 15.

B. BER performance

Using the parameters specified, the value of the received photons can be calculated. The number of photons increases with the average power of the LED, $P_t$. Fig. 6 shows the BER performance of the SPAD receiver with fixed LED semiangle ($\Phi_{1/2} = 10°$). Transmission speeds in this situation are assumed to be 1 kbits/s, 2 kbits/s and 5 kbits/s. Fig. 6 shows that there is a good match between the simulation and theoretical results. As shown, the power requirement of the LED is just about 13.5 dBm for a BER of $10^{-9}$ at 1 kbits/s. For higher transmission speeds, such as 2 kbits/s and 5 kbits/s, the power requirements are 16 dBm and 20.5 dBm, respectively.

Fig. 7 shows the BER performance of SPADs, when the data rate is 1 kHz. Unlike Fig. 6, Fig. 7 demonstrates the BER performance with changes of the semiangle, $\Phi_{1/2} = 5°$, 10° and 15°. When $\Phi_{1/2} = 5°$, the LED requires just 7.5 dBm to reach a BER of $10^{-9}$. When $\Phi_{1/2} = 15°$ and BER = $10^{-9}$, the power requirement of the LED is 17.5 dBm. With the increase of the semiangle, the power requirement of the LED transmitter increases. As a consequence, the system achieves higher power efficiency with the lower semiangle.

In practice, a longer or shorter pipe might be utilized and there is a difference in the requirement of the transmission speed in different scenarios. Fig. 8 shows the power that the transmitter needs when the semiangle ($\Phi_{1/2} = 10°$) is fixed and the BER is considered at $10^{-9}$. In Fig. 8, it is assumed that the length of the pipe varies from 1,000 to 10,000 metres.
and the transmission speed from 1 to 50 kbit/s. In normal circumstances, the system can support lower transmission speeds which are about 1 kbit/s. Only 8 dBm are required for communication in a 4,000 metres long pipe. When a higher transmission speed is required, the power of LED needs to increase to 24.1 dBm. In a longer pipe (10,000 metres), the transmission speed from 1 kbit/s to 50 kbit/s. In normal circumstances, the system can support lower transmission speeds which are about 1 kbit/s. Only 8 dBm are required for communication in a 4,000 metres long pipe. When a higher transmission speed is required, the power of LED needs to increase to 24.1 dBm. In a longer pipe (10,000 metres), the transmission speed from 1 kbit/s to 50 kbit/s.

V. CONCLUSION

In this paper, an energy-saving VLC application is presented for a gas well downhole monitoring system in a long pipe. Unlike conventional VLC systems which employ normal PDs, the proposed system is based on a SPAD receiver which is able to count the number of photons. By using the SPADs array, the LED transmitter needs only 8 dBm power to send the monitoring signal in a 4,000 metres long gas well pipe. As the LED transmitter at the well must be battery powered, the high power efficiency ensures that the system achieves a sufficiently long service time.

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Nonlinear Distortion in SPAD-Based Optical OFDM Systems

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Abstract—In this paper, an optical orthogonal frequency division multiplexing (OFDM) system based on a single-photon avalanche diode (SPAD) receiver with a nonlinear distortion is presented. In this study, the methods to calculate the output of passive and active quenching SPAD arrays are given. Nonlinear performances of these two SPAD arrays and their effects on bit-error ratio performances of SPAD-based DC-biased optical OFDM and asymmetrically clipped optical OFDM are simulated in the absence of background light. In the simulation, the maximum optical irradiance and the low error area are given as the metrics of the nonlinear distortion. The effects of the nonlinear distortion are discussed for different modulation schemes, symbol rates and spectral efficiencies.

Index Terms—optical wireless communication (OWC), single-photon avalanche diode (SPAD), nonlinear distortion, optical OFDM.

I. INTRODUCTION

Currently high speed light emitting diodes (LEDs) and laser diodes (LDs) are used as transmitters in optical wireless communication (OWC) systems. With a single LED, an OWC system can achieve speeds exceeding 3 Gb/s [1]. However, the incoherent light output of the transmitters means that information can only be encoded in the intensity level. As a consequence, only real-valued and positive signals can be used for data modulation. Thus, OWC systems are usually considered to be modulated as an intensity modulation (IM) and direct detection (DD) system [2]. Unipolar modulation schemes such as on-off keying (OOK), pulse position modulation (PPM) and pulse amplitude modulation (PAM) can be used in conjunction with IM/DD systems [3]. In order to fully use the limited modulation bandwidth of the device and achieve high data rates, orthogonal frequency division multiplexing (OFDM) is applied in OWC systems by utilizing adaptive bit and power loading [1]. Unlike OFDM in radio frequency, optical OFDM (O-OFDM) requires real valued signals which are generated by imposing Hermitian symmetry on the information frame before the inverse fast Fourier transform (IFFT) operation during the signal generation phase. However, this decreases the spectral efficiency by half. Diverse O-OFDM modulation schemes have been realized and utilized in OWC, such as DC-biased optical OFDM (DCO-OFDM), asymmetrically clipped optical OFDM (ACO-OFDM), unipolar OFDM (U-OFDM) and non-DC-biased OFDM (NDC-OFDM) [4], [5]. Typically, highly sensitive photodiodes (PDs) such as positive-intrinsic-negative (PIN) diodes and avalanche photo diodes (APDs) are applied as receivers in OWC. However, when the OWC system is applied in low optical power and long distance transmission, such as in a gas well downhole monitoring system [6] and data transmission over plastic optical fibres [7], the number of photons reaching the receivers are significantly less than in standard indoor OWC links. In these scenarios, conventional PDs have unsatisfactory performance because the transimpedance amplifier (TIA) significantly reduces the sensitivity of the receiver and limits the signal-to-noise ratio (SNR). As a consequence, those low power signals are buried in noise. Hence, when compared with conventional PDs, single-photon avalanche diodes (SPADs) would be more suitable receivers in those scenarios. The SPAD detector does not require a TIA and thus the output signal is not distorted by thermal noise. In addition, as SPADs can detect even a single photon, a bit of information-carried photons can be received accurately. Therefore, the SPAD receiver can perform at significantly higher sensitivity and optical power efficiency than conventional PDs. In a previous work [8], an O-OFDM system with a SPAD receiver was presented and compared with state-of-the-art PD-received based O-OFDM systems. When the transmission speed is 1 Mbps, SPAD-based OFDM enhances the sensitivity by a staggering 30.5 dB over the PD-based system.

However, a SPAD receiver can only detect one photon within a device specific dead time which constrains the ability to recover a signal. In addition, since the output of the detector is a photon count value, there is a maximum number of photons that the system can detect. This limits the maximum tolerable optical irradiance which results in a receiver nonlinear distortion. This study investigates the performance of the SPAD-based OFDM system when the nonlinear distortion of the SPAD receiver occurs.

The rest of this paper is organized as follows. The system model of the SPAD-based OFDM system is described in Section II. The nonlinear distortion of the SPAD receiver is presented in Section III. The simulation results of the system and discussion on the nonlinear distortion of the system are given in Section IV. Conclusions are given in Section V.
Selected Publications

II. SYSTEM MODEL

Fig. 1 illustrates the system model of OFDM with SPAD receivers.

A. Optical OFDM Modulation

At the transmitter, the input bit stream is transformed into complex symbols, $X(n)$, by a $M$-quadrature amplitude modulation (QAM) modulator, where $M$ is the constellation size. The symbols are allocated to $N$ subcarriers, $X(k)$, $k = 0, \ldots, N-1$. In OFDM, $N$ denotes the size of IFFT/FFT, where $N$ is set to 2048. In general, two standard techniques, DCO-OFDM and ACO-OFDM, are used to obtain positive and real-valued OFDM symbols [9]. In DCO-OFDM, $N/2 - 1$ symbols in $X(n)$, $n = 1, \ldots, N/2 - 1$, are put into the first half of subcarriers and the DC subcarrier (the first subcarrier) is set to zero. In ACO-OFDM, $N/4$ QAM symbols in $X(n)$, $n = 1, \ldots, N/4$, are mapped to half of the odd subcarriers of the OFDM frame, $X(k)$, $k = 1, 3, 5, \ldots, N/2 - 1$. At the same time, the even subcarriers are set to zero. In both ACO-OFDM and DCO-OFDM, Hermitian symmetry is applied to the rest of the OFDM frame in order to obtain real-valued symbols through the IFFT block. Since transmitters can only send unipolar signals, the real-valued OFDM symbols need to be clipped. In DCO-OFDM, a DC bias is added to make the signal unipolar [9]. In practice, the value of the DC bias, which is related to the average power of the OFDM symbols, is defined as:

$$B_{DC} = \alpha \sqrt{E[x^2(k)]},$$  \hspace{1cm} (1)

where $x(k)$ is the OFDM symbol frame vector; and $10 \log_{10}(\alpha^2 + 1)$ is defined as the bias level in dB. The bias level in the current simulations is set to 7 dB and 13 dB, which are adopted from [8] for consistency. After the DC-bias, the OFDM frame is simply clipped by:

$$x_{\text{clipped}}(k) = \begin{cases} x_{\text{biased}}(k), & x_{\text{biased}}(k) \geq 0, \\ 0, & x_{\text{biased}}(k) < 0, \end{cases}$$  \hspace{1cm} (2)

where $x_{\text{biased}}(k)$ is the DC-biased symbol which is calculated as $x_{\text{biased}}(k) = x(k) + B_{DC}$. The clipped unipolar symbol is denoted by $x_{\text{clipped}}(k)$. In ACO-OFDM, since symbols are antisymmetric, clipped unipolar symbols are obtained by setting the negative part to zero. In the simulation, after being transformed into an optical intensity signal, the clipped signal is transmitted by an LED transmitter.

B. SPAD Receiver

A SPAD is an APD which is biased beyond reverse breakdown in the so called ‘Geiger’ region. In this mode of operation, a SPAD triggers billions of electron-hole pair generations for each detected photon. In other words, in ‘Geiger’ mode, a SPAD generates a very large current by receiving a single photon and thus can essentially be modelled as a single photon counter. The photodetection process of an ideal photon counter can be modelled using Poisson statistics which describe the shot noise effect ([10] and references therein).

In this study, in order to increase the capacity of the photon counts, an array of SPADs which outputs the superposition
of the photon counts from the individual SPADs is considered [11]. Five important parameters of the SPAD array are introduced as follows.

1) Fill Factor (FF): FF is the ratio of the total SPAD active area to the total array area. For the SPAD array, FF represents the probability that a photon hits the active area. If the photon triggers an avalanche, it will be counted. In other words, the percentage of photons in a beam of light reaching the active area can be approximated to FF. In this study, the value of FF is denoted by $C_{FF}$.

2) Photon Detection Probability (PDP): PDP is the probability that a photon hitting the active area triggers an avalanche. This avalanche will generate a pulse which can be counted by an accumulator. The accumulator will give the output of the array. PDP differs to the quantum efficiency of a conventional PD, in which the quantum efficiency sometimes includes fill factor effects [12]. In this study, the value of PDP is denoted by $C_{PDP}$.

3) Dark Count Rate (DCR): A thermally-generated carrier can also trigger an avalanche which increases the array output. Even in complete darkness, this phenomenon still exists as long as the SPAD devices are opened. The average number of counts in darkness per second is referred to as DCR which is regarded as a fixed signal-unrelated noise of SPAD. In this study, the average DCR of a single SPAD is denoted by $N_{DCR}$.

4) After Pulsing Probability (APP): After pulses are correlated to detections by the time dependent release of trapped carriers [11]. Additional avalanches are triggered after receiving a photon or a dark photon. This means that the after pulsing effect will also increase the array output related to both the incoming signal and the dark counts. The delayed counts will bring inter-symbol interference due to the high data rate. But in low speed transmission, as the sample period is much longer than the delayed time, the after pulsing effect has a negligible effect on the next sample period. In this study, the value of APP is denoted by $P_{APP}$.

5) Dead Time: After an avalanche is triggered, whether caused by the signal photons or dark photons, the SPAD device needs to be actively or passively recharged in a short period of time and we refer this short period of time as dead time. During this time, the SPAD device is unable to detect further signal photons or dark photons. In other words, each individual SPAD in the array can only receive one photon during the dead time. In this study, the value of the dead time is denoted by $\tau_{d}$.

Fig. 2 illustrates the system model of the SPAD array receiving optical signals. In order to generate received O-OFDM samples, the output of the SPAD array is counted over a short-time average period, $T_{ST}$, at time instances $t_{k} = kT_{s}$ of the received optical signal $x_{c}(t)$. Note that $T_{c}$, which denotes the sampling period of the time domain OFDM signal at the transmitter, is assumed to be longer than $T_{ST}$. These photon counts are denoted by $\nu(k)$ which is the superposition of the photon counts from each individual SPADs, $a_{m}(k)$, as shown in Fig. 2:

$$\nu(k) = \sum_{m=1}^{N_{SPAD}} a_{m}(k),$$  \hspace{1cm} (3)

where $N_{SPAD}$ is the number of SPAD devices in the array. As the photon counts from each individual SPAD can be approximately modelled using Poisson statistics, the photon counts at the output of the SPAD array (i.e., $\nu(k)$) can be still described by a Poisson distribution:

$$Pr(\nu(k) = j) = \exp(-\mu(k)) \frac{\mu(k)^{j}}{j!},$$  \hspace{1cm} (4)

where the average photon counts $\mu(k)$ can be expressed as a function of the received signal and the effects of the SPADs’ parameters:

$$\mu(k) = \frac{C_{FF}C_{PDP}}{E_{p}} \int_{t_{k}}^{t_{k}+T_{ST}} x_{c}(t)dt + n_{DCR} \right\{1 + P_{AP}\},$$  \hspace{1cm} (5)

where $E_{p}$ denotes the energy of a photon which is calculated by $\frac{h c}{\lambda_{L}}$. Planck’s constant is represented by $h$; $c_{L}$ is the speed of the light; and $\lambda_{L}$ is the light wavelength of the LED transmitter. The noise caused by dark counts is denoted by $n_{DCR} = N_{DCR}N_{SPAD}T_{ST}$.

C. Optical OFDM Demodulation

The output of the SPAD array is the number of photons, $\nu(k)$, and the system is designed based on a conventional O-OFDM demodulator which requires the amplitude of the electrical signal (optical power) to demodulate the received signal to the original encoded bits. Thus, the photon-to-amplitude equalizer is used to simply convert the received photon number to the corresponding electrical signal amplitude (optical power), $X'(k)$. The coefficient of the equalizer is calculated by a pilot which can record the effect of the attenuation.

Assuming that there is no other distortion effects during the transmission, the recovered signal, $x'(k)$, can be scaled to the original clipped signal, $x_{clipout}(k)$. The recovered OFDM symbols from the SPAD are passed through a FFT operation which converts symbols to the frequency domain. In DCO-OFDM, $N/2 - 1$ symbols are obtained from the corresponding subcarriers to constitute a QAM symbol frame, $X'(n)$. In ACO-OFDM, $N/4$ symbols are obtained. The detected QAM symbols are then decoded by the conventional maximum likelihood (ML) estimator in order to obtain the output bit stream.

III. NONLINEAR DISTORTION IN SPAD

The complete model of nonlinear distortion in O-OFDM systems has been presented in [13]. In the SPAD-based system, a special form of nonlinear distortion which is caused by the saturation of SPAD devices should be taken into consideration. In this study, passive quenching (PQ) and active quenching (AQ) SPADs are considered and compared.
Thus, for each $T_{ST}$, the average number of the measured photon counts, $\mu_{PQ}$, is calculated by:

$$\mu_{PQ}(k) = \mu_m(k) \exp\left( -\frac{\mu_m(k)\tau_d}{T_{ST}} \right)$$

where $\mu_m(k)$ denotes the average actual incoming photon counts in the same $T_{ST}$. For the SPAD array, $\mu_m(k)$ is equal to $µ(k)/N_{SPAD}$. In this study, if the SPAD array is composed by PQ SPAD devices, the average output of the array during each $T_{ST}$ can be expressed as:

$$\mu_{PQ}(k) = \sum_{m=1}^{N_{SPAD}} \mu_{PQ_m}(k)$$

$$\mu_{PQ}(k) = \mu(k) \exp\left( -\frac{\mu(k)\tau_d}{T_{ST}N_{SPAD}} \right)$$

B. AQ SPAD

Compared with PQ SPAD, the configuration of AQ SPAD is more complex and requires more area and power, but when any events arrive during the dead time, the additional events are not registered and do not prolong the dead time. Thus AQ SPADs are defined as non-paralyzable detectors and have higher count rates than PQ SPADs. For a single AQ SPAD, the average measured photon counts per second, $\mu_{AQ}$, can be expressed as a function of $\mu_m$ [14]:

$$\mu_{AQ}(k) = \mu_m \exp\left( -\mu_m\tau_d \right)$$

For each $T_{ST}$, the average output of a single device is:

$$\mu_{AQ}(k) = \frac{\mu_m(k)}{1 + \frac{\mu_m(k)\tau_d}{T_{ST}}}$$

Thus the average output of the AQ SPAD array in $T_{ST}$ is:

$$\mu_{AQ}(k) = \sum_{m=1}^{N_{SPAD}} \mu_{AQ_m}(k)$$

$$\mu_{AQ}(k) = \mu(k) \frac{1 + \frac{\mu(k)\tau_d}{T_{ST}N_{SPAD}}}{1 + \frac{\mu(k)\tau_d}{T_{ST}N_{SPAD}}}$$

C. Nonlinear Distortion Problem

From (8) and (11), it is apparent that the real average outputs of the SPAD arrays have a nonlinear relationship with the average incoming number of photons. When the incoming photon rate is low, the outputs of each array are nearly equal to the inputs. However, when the SPAD devices are nearly saturated, a nonlinear distortion appears. For the PQ SPAD array, after reaching the maximum photon count rate, $µ_{PQ_{max}}$, as the PQ SPAD devices are paralyzed, the outputs of the array rapidly decreases with an increasing rate of incoming photons. From (8), $µ_{PQ_{max}}$ can be calculated:

$$µ_{PQ_{max}} = \frac{T_{ST}N_{SPAD}}{\exp(1)\tau_d}$$

For AQ SPAD, when the incoming photon rate increases, the devices are non-paralyzed but the outputs dramatically converge to the maximum photon rate, $µ_{AQ_{max}}$. In other words, if the potential photon counts, including signal photons, dark photons and after pulse events, are more than $µ_{AQ_{max}}$, the AQ SPAD array will be saturated. The photon counts at the output of the SPAD array are constrained to $µ_{AQ_{max}}$ and the extra photons are refused and lost. From (11), $µ_{AQ_{max}}$ can be obtained:

$$µ_{AQ_{max}} = \frac{T_{ST}N_{SPAD}}{\tau_d}$$

In the SPAD-based Optical OFDM system, some high amplitude symbols in the recovered signal $x'(k)$ are distorted by PQ and AQ recharged circuits resulting in loss of information.

IV. RESULTS AND DISCUSSION

In this study, the performance of SPAD-based DCO-OFDM and ACO-OFDM with the nonlinear distortion are simulated and compared. The optical signals are assumed to pass through a flat fading channel and in the absence of background light. As a consequence, the received signals are only subject to additional shot noises. Thus, in the simulation, the signals are only affected by Poisson distributed shot noise, the nonlinear distortion and clipping distortion (low bias level DCO-OFDM) introduced by SPAD receiver and $T_{ST}$ is assumed to be equal to $T_s$. A PQ SPAD array and an AQ SPAD array are considered with the same parameters as given in Table I.

Fig. 3 shows the average outputs of the PQ SPAD array and the AQ SPAD array as a function of the optical irradiance when $T_s = 1$ ms and $T_d = 1$ µs. As noted in Section IIIB, the
nonlinear properties of the SPAD array are shown in Fig. 3 where the optical irradiance represents the incoming photon rate. Compared with the longer symbol period ($T_s = 1$ ms), the maximum photon counts of each symbol, $\mu_{\text{max}}$, are lower when the symbol period is 1 $\mu$s. For the different symbol rates, the nonlinear distortions appear at the same position (-40 dBm). This means that the nonlinearity of PQ and AQ SPAD are predicted to have a significant effect on the performance of the system when the optical irradiance is larger than around -40 dBm.

Fig. 4 and Fig. 5 show the simulation results for bit-error ratio (BER) of the SPAD-based DCO-OFDM and ACO-OFDM systems as a function of the optical irradiance when $T_s = 1$ ms and $T_s = 1$ $\mu$s. The performance of 4-QAM DCO-OFDM with 7 dB and 13 dB DC-bias and 4-QAM ACO-OFDM are compared in these figures. In this study, when the optical irradiance is larger than a threshold, BER is below the target BER of $10^{-3}$. This threshold is defined as the minimum power requirement (MPR) of the system.

When the optical irradiance is larger than another threshold, the nonlinear distortion of SPAD receivers occurs, resulting in BER higher than $10^{-3}$. This threshold is defined as the maximum optical irradiance (MOI). The gap between the MPR and the MOI is defined as the low error area (LEA) where the system can maintain a low BER ($< 10^{-3}$). For example, in Fig. 4, after the optical irradiance reaches -85.5 dBm, the BER of 4-QAM DCO-OFDM with 7 dB DC-bias and PQ SPAD is lower than $10^{-3}$, and after the optical irradiance reaches -39.8 dBm, the BER of the same scheme is higher than $10^{-3}$. Thus, the MPR of this scheme is -85.5 dBm; the MOI is -39.8 dBm; and the LEA is 45.7 dB.

From Fig. 4 and Fig. 5, it can be seen that the PQ SPAD-based and the AQ SPAD-based systems have the same BER performances at low optical irradiance (around MPR). This is because in terms of linearity, the PQ SPAD devices have the same performance as the AQ SPAD devices when the optical irradiance is low (Fig. 4). However, as the maximum count rate
of PQ SPAD is lower than AQ SPAD, the MOIs of PQ-based systems are lower than the AQ-based systems. In addition, since the MPRs of each system are the same, the LEAs of PQ-based systems are also lower than AQ-based systems. As a consequence, the PQ SPAD-based OFDM system is more readily affected by the nonlinear distortion and is more sensitive to optical irradiance.

For different symbol periods \( T_s = 1 \mu s \) and \( T_s = 1 \mu s \), the nonlinear distortion occurs after the optical irradiance reaches around -40 dBm which matches with the prediction in Fig. 3. However, the schemes with a shorter symbol period \( T_s = 1 \mu s \) have higher MPRs than the methods with a longer symbol period \( T_s = 1 \mu s \). This means that with decreasing \( T_s \), LEA reduces, which decreases the range of the received optical power.

Fig. 6 shows the MOI and LEA of the PQ SPAD-based and the AQ-based OFDM systems with different spectral efficiencies and symbol periods \( (T_s = 1 \mu s, T_s = 1 \mu s) \) when the BER is \( 10^{-3} \). The MOIs of ACO-OFDM are lower than DCO-OFDM. Compared with SPAD-based DCO-OFDM, the ACO-OFDM system encounters the nonlinear distortion earlier since the distortion in ACO-OFDM is doubled (due to the subtraction in demodulation) which increases the probability of bit errors. As ACO-OFDM has a higher power efficiency (lower MPR) than DCO-OFDM, the LEAs of ACO-OFDM become close to the LEAs of DCO-OFDM, as shown in Fig. 6. However, the higher power efficiency of ACO-OFDM has the disadvantage of achieving approximately half of the spectral efficiency of DCO-OFDM. With increasing spectral efficiencies, the MOIs and LEAs of the systems decrease and thus the systems are more susceptible to nonlinear distortions.

In addition, for DCO-OFDM with 7 dB bias, the result for low spectral efficiency (1 and 2 bits/s/Hz) are shown. For short symbol periods \( T_s = 1 \mu s \), the spectral efficiencies of the systems are lower than 2 bits/s/Hz. When the spectral efficiencies increase, the MPRs are higher than the MOIs. This means that before BER of the systems reaches \( 10^{-3} \), the SPAD arrays are almost saturated by the incoming photons and the nonlinear distortion are too dominate to operate the system.

V. CONCLUSION

In this paper, a SPAD-based OFDM system assuming nonlinear distortions introduced by the SPAD device is presented. The nonlinear distortion is caused by the nonlinearity of the passive and active recharged circuit. For the SPAD-based OFDM system, the nonlinear distortion has a significant effect on the BER performance when the optical irradiance is higher than the MOI which is around -40 dBm. Compared with the PQ SPAD-based OFDM system, the system with AQ SPAD has a higher MOI and LEA. This means that AQ SPAD can receive optical signals with higher intensity and has a wider range of useful received optical power. However, these benefits come with the disadvantage of a higher hardware complexity. Compared to DCO-OFDM, SPAD-based ACO-OFDM has lower MPR, MOI and LEA. This means that ACO-OFDM exhibits a higher power efficiency, but is subject to a higher risk of nonlinear distortions affecting the BER performance. In addition, with an increasing symbol rate and spectral efficiency, the SPAD-based OFDM systems are more readily affected by the nonlinear distortion.

The SPAD receiver has a significantly enhanced sensitivity. This means that the SPAD-based OFDM system can be used in long distance transmissions, or it can be used in nonline-of-sight optical wireless communications, in the uplink when illumination is not essential, or when lights are almost completely dimmed. However, due to such high sensitivity, an appropriate transmission power should be selected carefully so as to avoid the nonlinear distortion.

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