Workshop on Reconstruction Schemes for MR data
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Multi-frame Super Resolution based on Sparse Coding

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Outline

1. Sparse coding: Introduction and Mathematical Formulation
2. Super Resolution via Sparse Representation: An overview
3. Our algorithm
4. Results
5. Final comments
Sparse Representation Theory, (Donoho-Candes, 2006)

- Sparse coding consist of writing a signal $x$ as:

$$x = \alpha_1 D_1 + \alpha_2 D_2 + \ldots + \alpha_n D_n$$

where $\alpha$ is a coefficient vector and $D$ is a matrix whose columns are called atoms of a dictionary.

- The idea of expressing a signal in a compact form is an old strategy in Digital Signal processing.
  It is convenient because $\alpha$ exhibits desired properties of the signal.

- Traditionally, $D$ is taken as an orthonormal basis (Fourier basis, Wavelets basis, DCT basis).
  It is convenient because: $\alpha = D^T x$
Where is the novel idea of Sparse Coding?

- The set $D_1, \ldots, D_n$ is **not** a mathematical basis.

- The dictionary D is an overcomplete set that should guarantee the sparseness of $\alpha$.

- The existence of such D for some signals is a fact statistically proved. For example:
  Image patches can be well-represented as a sparse linear combination of elements taken from a finite and not too big bag.

- Advantage over orthonormal basis:
  Most of the coefficients in $\alpha$ are zero if D is properly selected.
Basic Formulation of Sparse Coding

- We assume any signal of our interest has a sparsity far less the total number of atoms in D.

- First formulation:

\[
\min_\alpha \|\alpha\|_0 \quad s.t. \quad x = D\alpha
\]

Main inconvenience: It is a intractable combinatorial problem.

- Second formulation (Lasso model):

\[
\min_\alpha \|x - D\alpha\|_2 + \lambda \|\alpha\|_1
\]

A variety of optimization methods have been proposed:
LARS, Coordinate descent, Feature-sign search algorithm
Dictionary Learning

- The calculation of a good overcomplete dictionary is a challenging problem that has evolved a lot in the last years.

- The Dictionary $D$ is usually learned from a set of training examples $X$:

$$D = \arg \min_{D,Z} \| X - DZ \|_2^2 + \lambda \| Z \|_1$$

The problem is not convex in the two variables.

But, it is convex in one of them with the other fixed.

If $D$ is fixed $\implies$ Nonnegative quadratic linear programming

If $Z$ is fixed $\implies$ LASSO model

- The learning procedure depends on the applications.
Multi-frame Super-resolution problem: Definition

Create a clear image from low resolution LR images

Downscaling: Easy problem

Upscaling: Ill-posed problem

We are here
They assume each high resolution patch and the corresponding low resolution patch share the same sparse linear coefficients assuming the HR and LR Dictionaries are defined properly.
Super-resolution via Sparse Representation, Yang et al. 2011

• The algorithm has two phases:
  
  **Learning phase:**
  Construction of the bilateral dictionaries $D_h$ and $D_l$

  **Testing phase:**
  Calculation of the sparse representation coefficients $\alpha$ of each LR patch
  Reconstruction of the HR patch using the coefficients $\alpha$

• An additional step: It consists of a back projection strategy to diminish the discrepancy produced by noise and blur in the LR image.
Super-resolution via Sparse Representation, Yang et al. 2011

Learning phase: Construction of the bilateral dictionaries $D_h$ and $D_l$

- Form the training samples

  - $X_h$: Set of $N$ high resolution sampled patches
  - $Y_l$: Set of $M$ low resolution sampled patches

- Calculate $D_h$ and $D_l$ solving the minimization problem

\[
\min_{D_h, D_l, Z} \left\| X_c - D_c Z \right\|_2^2 + \lambda \| Z \|_1
\]

where

\[
X_c = \begin{bmatrix}
\frac{1}{\sqrt{N}} X_h \\
\frac{1}{\sqrt{M}} Y_l
\end{bmatrix}, \quad D_c = \begin{bmatrix}
\frac{1}{\sqrt{N}} D_h \\
\frac{1}{\sqrt{M}} D_l
\end{bmatrix}
\]
Super-resolution via Sparse Representation, Yang et al., 2011

**Testing phase**: Reconstruction of the HR image

• Calculate the coefficients $\alpha$ of the sparse representation of each LR patch $y$ solving the minimization problem:

$$\alpha^l = \arg\min_{\alpha} \| y - D_1 \alpha \|_2^2 + \lambda \|\alpha\|_1$$

• The HR patch $x$ is taken as:

$$x = \alpha^l \ D_h$$
Super-resolution via Sparse Representation, Kato, et al., 2015

• The strategy is an extension of the algorithm proposed by Yang et al. to the case of **multi-frame** case (videos).

• The main idea is to incorporate into the model the subpixel differences between the target frame and the neighbor frames.

• **Learning phase:**
  
  The training set $X$ is formed by **only** HR patches

  The **unilateral** dictionary $D_h$ is calculated solving the problem

  $$\min_{D_h, \alpha} \{ \| X - D_h \alpha \|_2 + \delta \| \alpha \|_1 \}$$

  The LR dictionary $D_l$ is generated by atoms in $D_h$ according to the assumed blur, down-sampling and estimated translation.
Super-resolution via Sparse Representation, Kato, et al., 2015

**Testing phase**: Reconstruction of the HR image

Calculate the coefficients $\alpha$ of the sparse representation of each LR patch $y$ solving the minimization problem:

$$
\bar{\alpha} = \arg \min_{\alpha} \sum_{j=1}^{K} \| \tilde{D}' \alpha - \tilde{y} \|_2 + \lambda \| \alpha \|_1
$$

$$
\tilde{y} = \begin{bmatrix} y_1 \\ y_K \end{bmatrix}, \quad \tilde{D}' = \begin{bmatrix} DBW_1 D^h \\ DBW_K D^h \end{bmatrix}
$$

where $D$, $B$ and $W_i$ are the down-sampling, blur and motion estimation operator.

The HR patch $x$ is taken as $x = D^h \ast \bar{\alpha}$
Our algorithm:

- It is a **multi-frame + Unilateral Dictionary strategy**.
- It is basically the algorithm proposed by Kato.
- We include an additional step: a previous bicubic interpolation.

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**Step 1:**

(Patch by patch)
Registration:
Calculate motion estimation by Block Matching

**Step 2:**

(Patch by patch)

a) Construct the LR Dictionary.
b) Perform the Sparse Coding (calculate $\alpha$)
c) Reconstruct the patch:
   \[ x = D * \alpha \]

a) Form $X_0$ using the patches $x$

**Step 3:**

Enforce global reconstruction constrain by back projection:

\[ \overline{X} = \arg \min_X \|DBX - Y\|^2_2 + c\|X - X_0\|^2_2 \]
Experiments

• Lower resolution size: 120 x160, Scale factor: 2
• Matlab implementation
• Block matching: Sub-pixel algorithm proposed by Shimizu et al., 2006
Final Comments

• A C++ implementation of the algorithm is in progress. The experimental results will be important to evaluate the proposed algorithm.

• Sparse coding is a promising way to improve the present multi-frame super resolution algorithms.

• Existing super resolution algorithms typically run offline. There is no real possibility for online processing. It is a big challenge.