COMPARATIVE FORECASTING PERFORMANCE OF SYMMETRIC AND ASYMMETRIC CONDITIONAL VOLATILITY MODELS OF AN EXCHANGE RATE

Abstract

The relative out-of-sample forecasting quality of symmetric and asymmetric conditional volatility models of an exchange rate differs according to the symmetric and asymmetric evaluation criteria as well as a regression-based test of efficiency. Both symmetric and asymmetric forecast competitors of currency volatility are biased and systematically overpredict volatility.

Key words: Volatility clustering, symmetric/asymmetric volatility, forecasting, exchange rates, forecast evaluation

JEL classification: C22, C53, F31

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1. INTRODUCTION

The use of conditional heteroscedastic models has been a common tool for modelling and forecasting volatility of asset and currency returns following the introduction of the ARCH model by Engle (1982) and its generalized version, the GARCH model, by Bollerslev (1986). An extensive survey of the literature of such volatility clustering is given by Bollerslev et al. (1992). This family of models has been applied to exchange rate returns using different frequencies, from intraday to monthly data. See the studies using intraday data (Andersen and Bollerslev, 1997; and 1998a,b; Andersen et al., 1999), daily data (Engle and Bollerslev, 1986, Bollerslev, 1987, Milhoj, 1987, Hsieh, 1988 and 1989, Baillie and Bollerslev, 1989, and Jorion, 1995), weekly data (Engle and Bollerslev, 1986, Diebold, 1988, and Baillie and Bollerslev, 1989), and monthly data (Domowitz and Hakkio, 1985, and Baillie and Bollerslev, 1989), among many others. The previous research has generally focused on fitting a conditional variance model to exchange rate data with different distributional assumptions and investigating their in-sample properties. While these investigations provide useful insights into volatility, the predictive ability of competing models needs to be examined out-of-sample. This is of particular importance at least to options traders that require volatility forecasts to price options, central banks that require interval forecasts whether an exchange rate will fluctuate within a target zone, international traders for export and import decisions, international investors that require portfolio diversification beyond national borders, or risk managers using internal models such as value-at-risk applications.

An early contribution with respect to the evaluation of out-of-sample forecasting performance of exchange rate volatility is due to Taylor (1987). Lee (1991) concludes that
out-of-sample performance of exchange rate volatility model depends on the criteria used to measure it. More recently, West and Cho (1995) and Brooks and Burke (1998) investigate the out-of-sample predictive ability of several models of exchange rate volatility, using a weekly data set for the period March 1973 to September 1989 and the forecast horizons of 1, 12 and 24 weeks. The former study employs six models including a homoscedastic one, two GARCH specifications, two autoregressions, and a nonparametric one. They find that for one-week horizon GARCH models tend to make slightly more accurate forecasts and for longer horizons it is difficult to select an outperforming model based on the mean squared prediction errors. They conclude that none of the models perform well in a conventional test of forecast efficiency. The latter study uses modified information criteria to select the appropriate model order from the GARCH family (from AR(0)-GARCH(0,0) up to AR(5)-GARCH(5,5). They report that on mean squared error grounds the GARCH(1,1) model always outperforms those selected by the new information criteria, irrespective of the forecast horizon. However, the mean absolute forecast errors favour their new criteria. Jorion (1995) claims that implied volatility derived from option prices provides a better forecast compared to a moving average model and an ex-post GARCH model. Andersen and Bollerslev (1998a) investigate the predictive ability of the GARCH(1,1) model for one-day-ahead volatility forecasts of two exchange rates. They claim that the effective use of frequently sampled data leads to constructing more accurate ex-post volatility measurements and conclude that the ARCH models do provide good volatility forecasts. See also Christodoulakis and Satchell (1998).

This study aims at testing the relative quality of exchange rate volatility forecasts generated not only by the symmetric but also the asymmetric conditional variance models, using a larger data set of the US dollar-Deutsche mark exchange rate than the other studies and a monthly forecast horizon. Using calendar month as the forecast horizon is missing in the previous research. In addition, it is useful to analyze further the relationship between the expected volatility and macroeconomic variables since the latter are often made publicly
available in monthly announcements. Furthermore, in contrast to the earlier research on forecasting currency volatility using the conventional error statistics, a mean mixed error statistic is also calculated to penalize under(over)-predictions more heavily.

2. DATA AND METHODOLOGY

The data obtained from the Deutsche Bundesbank are a set of daily average of closing bid-ask prices of the US dollar-Deutsche mark (FX$_t$) at the Frankfurt Exchange at 14.00 German time. The sample covers the 24-year or 288-month period 2 January 1974-30 December 1997, making a total of 6,012 daily continuously compounded exchange rate returns ($R_t$):

$$ R_t = \ln \left( \frac{FX_t}{FX_{t-1}} \right) \quad (1) $$

We model daily exchange rate returns as an AR(1) process, a common practice in the literature of this field (see, for example, Andersen and Bollerslev, 1997 and 1998b), where forecast errors ($\varepsilon_t$) are conditionally normally distributed with zero mean and variance $h^2_{t-1}$:

$$ R_t = c + \theta R_{t-1} + \varepsilon_t \quad (2) $$

$$ \varepsilon_t \mid I_{t-1} \sim N(0, h^2_{t-1}) \quad (3) $$

The AR(1) term accounts for the economically minor but statistically significant first order autocorrelation in the daily exchange rate returns.

The following symmetric (ARCH(p) and GARCH(1,1)) and asymmetric (GJR-GARCH(1,1) and EGARCH(1,1)) conditional volatility models are employed. The first model is the Engle’s (1982) ARCH(p) specification where today’s expected volatility depends on the squared forecast errors of the previous $p$ days:

$$ h^2_{t} = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon^2_{t-i} \quad (4) $$

We find that an ARCH(5) model fits the empirical distribution of data well.

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$^1$ See Coppes (1995) for the distributional properties of exchange rates of different frequencies.
The Bollerslev's (1986) GARCH(1,1) model predicts that today's volatility is a weighted average of yesterday's squared forecast error and yesterday's conditional variance:

\[ h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}^2 \]  

Note that the standard ARCH/GARCH models are symmetric; i.e., they evaluate positive and negative forecast errors of the same magnitude equally. However, good news and bad news regarding exchange rate developments may have a different impact on expected volatility. For example, Hsieh (1989) find that the EGARCH model fits the daily exchange rate data well. To model asymmetry in volatility, we run the models that allow for different impacts on conditional volatility by positive and negative forecast errors of equal magnitude. First, the GJR-GARCH(1,1) model of Glosten et al. (1993) is given by

\[ h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 D_{t-1} + \beta h_{t-1}^2 \]  

where \( D_{t-1} \) is a dummy variable taking the value of 1 if \( \varepsilon_{t-1} < 0 \), and 0 otherwise.

The Nelson's (1991) EGARCH(1,1) model reads as follows:

\[ \ln (h_t^2) = \alpha_0 + \gamma \left( \varepsilon_{t,1}/ h_{t-1} \right) + \lambda \left[ \left( \frac{1}{h_{t-1}} \right) - \frac{(2/\pi)^{0.5}}{\beta} \right] \ln (h_{t-1}^2) \]  

It should be noted that both symmetric and asymmetric conditional volatility models have different news impact curves which measure how new information is incorporated into volatility estimates (see Engle and Ng, 1993, and Hentschel, 1995, for a detailed discussion and comparison).

The measure of volatility is the within-month standard deviation of daily returns in each calendar month. A 72-month rolling estimation procedure is adapted; i.e., each month's forecast is based on the estimated daily model parameters in the last 72 calendar months, on the average, 1,500 observations. All models are estimated using quasi-maximum likelihood (see, for example, Lopez, 1995, West and Cho, 1995, Brailsford and Faff, 1996, Andersen and Bollerslev, 1998a, and Brooks and Burke, 1998). The whole procedure requires estimation of 864 equations of conditional volatility, for 216 calendar months and 4 models,
from which multi-step ahead forecasts of 18,036 daily variances are constructed. These out-of-sample forecasts of daily variances are summed up to obtain monthly total variance. Dividing the last figure by the number of trading days in each month and then taking its square root gives each model’s forecast \( \sigma_{t,m} \) of within-month standard deviation which is compared to the realized standard deviation \( \sigma_{r,m} \). A similar methodology has been employed by West and Cho, (1995), Brailsford and Faff (1996), Andersen and Bollerslev (1998a), and Brooks and Burke (1998), among others. Note that our approach is free from the criticism that realized squared returns are not a good approximation of future volatility.

The forecast performance of each model is evaluated by symmetric and asymmetric statistical loss functions or error statistics. The four commonly used symmetric error statistics employed herein are the mean error (ME), the mean absolute error (MAE), the mean squared error (MSE), and the mean absolute percentage error (MAPE). The relative error statistic is the ratio between the actual error statistic of a model and that of the worst-performing model. These conventional error statistics are symmetric; i.e., they give an equal weight to under-and-over-predictions of volatility of similar magnitude. However, under(over)-prediction of volatility has important implications for option pricing. For example, an under(over)-prediction of volatility will lead to a downward (upward) biased estimate of a call option price. The under-estimate of volatility is more likely to be of grater concern to a writer than a buyer of a call option. Hence following Brailsford and Faff (1996), to penalize

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2 All the estimated parameters for the ARCH(5), GARCH(1,1) and GJR-GARCH(1,1) models are always consistent with the standard parameter restrictions for stationarity and non-negativity of conditional variance series. Since the EGARCH(1,1) model is in logarithmic form, there is no parameter restriction. The standardized residuals \( \frac{z_t}{h_t} \) and their squared values from all models always obey the standard assumptions of no autocorrelation and no heteroscedasticity although the \( \frac{z_t}{h_t} \)'s are not normally distributed. The daily forecast variance converges its unconditional value fast in all models.

3 The utility-based loss functions (West et al., 1993) and profit-based loss functions (Engle et al., 1993) are out of the scope of the paper. See Lopez (1995) for a discussion and transformation of volatility forecasts into probability forecasts.
under(over)-predictions more heavily, the following mean mixed error (MME) statistics are constructed:

\[
MME(U) = \frac{1}{216} \left[ \sum_{m=1}^{O} |\sigma_{t,m} - \sigma_{r,m}| + \sum_{m=1}^{U} |\sigma_{t,m} - \sigma_{r,m}| \right]^{0.5} \tag{8}
\]

\[
MME(O) = \frac{1}{216} \left[ \sum_{m=1}^{O} |\sigma_{t,m} - \sigma_{r,m}| + \sum_{m=1}^{U} |\sigma_{t,m} - \sigma_{r,m}| \right]^{0.5} \tag{9}
\]

where \(O\) (\(U\)) is the number of over-(under)-predictions. MME(U) and MME(O) penalize the under-predictions and over-predictions more heavily, respectively.

Another asymmetric evaluation criterion (see Pagan and Schwert, 1990) is logarithmic error (LE) statistics:

\[
LE = \frac{1}{216} \sum_{m=1}^{216} \left[ \ln(\sigma_{t,m}) - \ln(\sigma_{r,m}) \right]^2 \tag{10}
\]

Following West and Cho (1995), Andersen and Bollerslev (1998a,b), among others, we run the following regression to test the unbiasedness or rationality of each model, using the Newey-West heteroscedasticity and autocorrelation consistent standard errors:

\[
\sigma_{r,m} = \alpha + \beta \sigma_{t,m} + u_m \tag{11}
\]

If a model is unbiased, \(\alpha\) and \(\beta\) should be equal to zero and one, respectively. Our approach is consistent with Andersen and Bollerslev (1998a) who show that using high-frequency data in construction of an ex-post measure of volatility of lower-frequency data is more appropriate for such a test of forecast efficiency (see also Christodoulakis and Satchell, 1998; and Balaban, 1999).

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4 Since the absolute values of all forecast errors are smaller than one, taking their square root introduces a heavier penalty.
3. RESULTS

Tables 1a,b present the results according to the conventional symmetric error statistics. All models overpredict monthly volatility as shown by the ME criterion. According to the other error statistics, there is consistency to choose among the models. The EGARCH model always outperforms the other forecast competitors. According to the MAE, MSE and MAPE criteria, its performance is respectively 54%, 86% and 51% better compared to the GJR-GARCH model, which is consistently the worst performing model. The MAPE of the EGARCH model is 29%. This is similar to the results reported by Balaban (1999) and Balaban, Bayar and Faff (2002) but considerably lower than those given by Brailsford and Faff (1996). The GARCH and the ARCH models rank second and third, respectively. However, it is particularly difficult to choose between the EGARCH and the GARCH models. Note that the maximum superior performance of the former compared to the latter is according to the MSE criterion and only 1%. The corresponding figure for the ARCH model is, however, 11% and according to the MAPE criterion.

Table 2 provides the results for a regression-based evaluation. All models are biased as all the estimated $\beta$’s are significantly different from unity and all the $\alpha$’s are non-zero. Similar results are also reported by West and Cho (1995), Jorion (1995), Figlewski (1993), and others. However, all models have some predictive power for future volatility. On the other hand, Balaban (1999) reports that both symmetric and asymmetric conditional volatility models are unbiased predictors of future volatility in an emerging market setting. The GARCH model has the highest $R^2$, 15.6%, and the GJR-GARCH model has the lowest one.

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5 We also calculated the median error statistics. The rankings do not change but the performance differences between the best and worst performing models are lower compared to the corresponding mean error statistics (the minimum is 33.1% and according to the MAPE). However, the performance difference of the best performing model compared to the second and third ranking models are higher leading to the ranges between 4.1% and 5.3%, and between 17.1% and 34.3%, respectively.

6 The rolling estimation results show that the asymmetric conditional volatility parameters of the GJR-GARCH and the EGARCH models are significant in 46 and 60 months out of 216 or 21.3% and 27.3% of the forecast sample, respectively. The rankings do not change if we calculate the mean and median error statistics for those months with the significant asymmetric parameters. This strongly suggests that reliance on measures of in-sample fit are clearly insufficient.
5.5%, leading to a 65% performance difference. The ARCH and the EGARCH models rank second and third, respectively, and have an $R^2$ half of that of the GARCH model. Their performance is 25% to 35% better than the GJR-GARCH model. The results are consistent with the previous findings based on a similar regression. Note that the previously reported $R^2$'s are, however, lower than those reported here (see West and Cho, 1995, Jorion, 1995, and Figlewski, 1993). On the other hand, Balaban (1999) reports an $R^2$ 27% for the ARCH(p) model. This suggests that using daily data to construct ex-post measure of monthly volatility leads to a higher explanatory power (see Andersen and Bollerslev, 1997a, Balaban, 1999).

Table 3 shows the results of asymmetric error statistics that penalize under-(over)predictions more heavily. All models tend to overpredict volatility two-third to three-fourth of the time. The ARCH and the EGARCH models are chosen respectively the best and the worst models if the underpredictions are more heavily penalized. The relative performance of the former is about 25% better compared to the latter. If the overpredictions are more heavily penalized, the EGARCH model turns out to be the best whereas the GJR-GARCH model is the worst. The former is more than 40% better than the latter. The GARCH model consistently ranks second and there is at most 8% performance difference compared to the best model.

Table 4 presents the results according to the LE statistic. The GJR-GARCH model is again the worst performing model. The GARCH and the EGARCH models are about 65% better than the benchmark and about 10% than the ARCH model.

Engle and Ng (1993) and Glosten at al. (1993) find that the GJR-GARCH model fits the in-sample stock market data very well and Brailsford and Faff (1996) report that it is the best performing model for out-of-sample forecasting of monthly stock market volatility. However, our results suggest that although the GJR-GARCH model fits in-sample exchange rate data well, its out-of-sample forecasting performance is poor based on all evaluation criteria. This is consistent with Balaban (1999) who report that the GJR-GARCH model has a
poor performance of out-of-sample volatility forecasting in an emerging market setting as well.

4. SUMMARY AND CONCLUSION

This study employs symmetric and asymmetric error statistics as well as a regression-based test of efficiency to evaluate the monthly out-of-sample forecasting accuracy of symmetric and asymmetric conditional volatility models of a daily exchange rate. The symmetric error statistics consistently choose the EGARCH and the GJR-GARCH models respectively as the best and the worst with at least 50% performance difference, although both have an asymmetric news impact curve. The standard symmetric GARCH model always ranks second and its performance is negligibly less than the EGARCH model (1% at most). A regression-based approach clearly favours the GARCH model. On the other hand, the GJR-GARCH model has the worst performance. The asymmetric error statistics respectively rank the ARCH and the EGARCH models as having the highest and the lowest performance with a 25% difference, if the underprediction of volatility is undesired. Otherwise, the EGARCH model is the best whereas the GJR-GARCH model is the worst (40% difference). The GARCH model always rank the second regardless of the symmetric error statistics and the MME statistics employed. However, all models are biased and most of the time overpredict the volatility. According to the LE statistics the GARCH model ranks first with a negligible difference compared to the EGARCH model.

The results suggest that forecasting exchange rate volatility is a clearly difficult task. However, an overall evaluation indicates that the standard symmetric GARCH(1,1) model provides relatively good forecasts of monthly exchange rate volatility whereas the asymmetric GJR-GARCH(1,1) model seems to be a poor alternative. In contrast to the previous research findings, these results can be regarded as robust across different error statistics.
### Table 1a. Forecast evaluation: symmetric error statistics

<table>
<thead>
<tr>
<th>Model</th>
<th>ME Actual</th>
<th>MAE Actual</th>
<th>Relative</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH</td>
<td>0.001003</td>
<td>0.002159</td>
<td>0.532</td>
<td>3</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.000651</td>
<td>0.001907</td>
<td>0.469</td>
<td>2</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>0.002777</td>
<td>0.004062</td>
<td>1.000</td>
<td>4</td>
</tr>
<tr>
<td>EGARCH</td>
<td>0.000255</td>
<td>0.001861</td>
<td>0.458</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: The mean error (ME), the mean absolute error (MAE), the mean squared error (MSE), the mean absolute percentage error (MAPE). *Relative* is the ratio between the actual error statistic of a model and that of the worst performing model.

### Table 1b. Forecast evaluation: symmetric error statistics (continued)

<table>
<thead>
<tr>
<th>Model</th>
<th>MSE Actual</th>
<th>MAE Actual</th>
<th>Relative</th>
<th>Rank</th>
<th>MSE Actual</th>
<th>MAE Actual</th>
<th>Relative</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH</td>
<td>0.000008</td>
<td>0.168</td>
<td>3</td>
<td>3</td>
<td>0.358857</td>
<td>0.594</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.000007</td>
<td>0.149</td>
<td>2</td>
<td>2</td>
<td>0.298104</td>
<td>0.494</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>0.000045</td>
<td>1.000</td>
<td>4</td>
<td>4</td>
<td>0.604060</td>
<td>1.000</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>EGARCH</td>
<td>0.000006</td>
<td>0.136</td>
<td>1</td>
<td>1</td>
<td>0.293399</td>
<td>0.486</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: The mean error (ME), the mean absolute error (MAE), the mean squared error (MSE), the mean absolute percentage error (MAPE). *Relative* is the ratio between the actual error statistic of a model and that of the worst performing model.
Table 2. Regression-based forecast evaluation

<table>
<thead>
<tr>
<th></th>
<th>α</th>
<th>β</th>
<th>R²</th>
<th>Relative</th>
<th>Rank</th>
<th>Wald test</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH</td>
<td>3.532</td>
<td>0.432</td>
<td>0.086</td>
<td>0.640</td>
<td>3</td>
<td>37.9</td>
</tr>
<tr>
<td>GARCH</td>
<td>3.253</td>
<td>0.488</td>
<td>0.156</td>
<td>0.352</td>
<td>1</td>
<td>20.9</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>6.027</td>
<td>0.097</td>
<td>0.055</td>
<td>1.000</td>
<td>4</td>
<td>720.2</td>
</tr>
<tr>
<td>EGARCH</td>
<td>3.084</td>
<td>0.538</td>
<td>0.074</td>
<td>0.743</td>
<td>2</td>
<td>15.7</td>
</tr>
</tbody>
</table>

Notes: The numbers in parentheses are the Newey-West heteroscedasticity and autocorrelation consistent standard errors. \( \alpha \) and its standard error must be multiplied by \( 10^{-3} \). The Wald test gives the \( \chi^2 \) statistics for the difference of \( \beta \) from unity. All \( \alpha \)'s and \( \beta \)'s are different from zero at the 1% level. All \( \beta \)'s are also different from unity at the 1% level. Relative is the ratio between the \( R^2 \) of a model and that of the worst performing model.

Table 3. Forecast evaluation: asymmetric error statistics

<table>
<thead>
<tr>
<th></th>
<th>MME(U) Actual</th>
<th>Relative Rank</th>
<th>MME(O) Actual</th>
<th>Relative Rank</th>
<th>U(%)</th>
<th>O(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH</td>
<td>0.012161</td>
<td>0.758</td>
<td>1</td>
<td>0.032716</td>
<td>0.755</td>
<td>3</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.013239</td>
<td>0.825</td>
<td>2</td>
<td>0.028512</td>
<td>0.658</td>
<td>2</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>0.014950</td>
<td>0.932</td>
<td>3</td>
<td>0.043335</td>
<td>1.000</td>
<td>4</td>
</tr>
<tr>
<td>EGARCH</td>
<td>0.016040</td>
<td>1.000</td>
<td>4</td>
<td>0.025205</td>
<td>0.582</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: MME(U) [MME(O)] is the asymmetric error statistic that penalizes the underpredictions [overpredictions] more heavily. Relative is the ratio between the actual error statistic of a model and that of the worst performing model. U(%) and O(%) give the percentage of underpredictions and overpredictions.

Table 4. Forecast evaluation: logarithmic error (LE) statistic

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Relative</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH</td>
<td>0.140538</td>
<td>0.420</td>
<td>3</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.111610</td>
<td>0.333</td>
<td>1</td>
</tr>
<tr>
<td>GJR- GARCH</td>
<td>0.333716</td>
<td>1.000</td>
<td>4</td>
</tr>
<tr>
<td>EGARCH</td>
<td>0.117564</td>
<td>0.351</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: Relative is the ratio between the actual error statistic of a model and that of the worst performing model.
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