A STUDY OF PIAGET'S THEORY OF THE EVOLUTION OF NUMBER IN CHILDREN

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**Introduction**

This thesis is presented as the work of a general practitioner in the field of psychology. The practical research and the reading which it entailed were carried out largely against the background of family life, and here I must thank my very patient wife for putting up with the litter of papers and books all these years, and for frequently urging the children to "leave Daddy in peace, he's doing his Piaget." I am also grateful to the educational and medical authorities of Cumberland in whose schools and occupation centres the subjects were examined, and to the teachers who cooperated so helpfully. The subjects were all examined in the little time that could be occasionally spared from my routine duties. It is for this reason, needless to say, that the thesis has taken the full five years. I wish also to thank my colleague Mrs. Bryce for her heroic task in typing most of the draft, and all of the main copies. Finally I should like to express my gratitude to Dr. Mary Collins who has guided my work since its inception five years ago. I owe so much to her advice and counsel, and I appreciate in particular the very considerable freedom in which she allowed me to pursue my studies.
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PART I. The broad outline of Piaget's theory of mental development

Although this thesis is primarily concerned with Piaget's work on the child's idea of number, this important book cannot be studied adequately unless it is seen against the background of Piaget's extensive theory of mental development in children, and in particular his ideas about the evolution of concepts during childhood. The following brief sketch will therefore attempt to say something about Piaget's method of study, as well as about his long series of investigations.

Piaget began his career as a biologist, and as he tells us himself,

"The way I came to study child psychology was far from orthodox. I had studied natural sciences, and my doctor's thesis dealt, of course with molluscs. What interested me most were the problems of adaptation, of the relation between an organ and its environment and the problem of variation as a function of environment and as a function of structure. But while I was preparing my doctorate in zoology I was taking a very lively interest in the problems of knowledge, of epistemology of logic, and the history of sciences, etc. At the same time I was highly suspicious of philosophers who I thought had treated the problem of knowledge in a fashion that was far too speculative and not sufficiently experimental. I then considered devising a genetic theory of knowledge, studying knowledge as a function of its growth and development, so I felt I should read psychology. I thought I would spend four or five years
studying the development of logic and the intellectual functions in the child and the growth of intelligence during the child's development; these studies have lasted more than thirty years and are not yet finished."

We never lose sight of the biological discipline in Piaget's studies, and we find that certain biological concepts are fundamental to his whole theory of intellectual development, in particular the concepts of accommodation, assimilation and equilibrium.

It is sometimes assumed that Piaget, in his long analysis of mental development in children, was trying to concoct a lengthy, largely unstandardised and somewhat amorphous intelligence test. Piaget, however, was never primarily interested in intelligence as it is normally thought of today, that is in terms of mental efficiency or output expressed by a statistical device, or else as mental components to be inferred from factor analysis. Instead of concerning himself with assessing how much a particular child could do according to a given intelligence scale, Piaget's purpose was to look beyond that aspect of thinking and first of all to reveal the underlying qualities and attributes which are characteristic of a child's thought at a given point in his life, and secondly to demonstrate how those characteristics have emerged from earlier and more immature stages of development, and how they come to form a foundation for further development in the future. In this way Piaget has tried to show the evolution of the structure, as opposed to the

a: Ref. 2 p. 31.
efficiency, of children's thinking.

His method of study was invariably one of interrogation, that is, free and spontaneous questions and answers during which the investigator may ask whatever question he considers may be useful in helping the child to express the true character of his thoughts, without any time-limit being applied, or set form of response being required from the child. In his earlier books the method may appear to be rather too verbal in character, as Piaget himself says, but in time he used pieces of apparatus in order "to study reasoning through objects set up so that the child could make certain experiments". Piaget practises a clinical rather than a test method. "The test method", he says "has its uses, but... it tends to falsify the perspective by diverting the child from his natural inclination. It tends to neglect the spontaneous interests and primitive reactions of the child, as well as other essential problems."

An example from one of Piaget's early books may be helpful at this point, since it illustrates both his psychological method and the general form in which his findings come to be expressed. The problem was to analyse the growth of the concept of "life" in children's thinking. A number of children were interrogated. How large or how small a number Piaget, as is his usual custom, does not say. The question put to the children took the form of "You know what it means to be alive? Is the sun, a tree, the mind, etc, alive? Why? Which is more alive the wind or a bicycle?" ... and so on. A

\begin{references}
  \item Ref. 3 p.4.
  \item Ref. 3 p.
\end{references}
simple "yes" or "no" from the child was not enough: he had to give reasons if possible. "Yes, it makes the sunshine and gives light during the day... No, it's man who makes the engine go." On the evidence of such interrogation Piaget found that the concept of "life" goes through four stages as the child's thinking develops, and these may be summarised briefly as:

**Stage 1**: Life tends to be defined in terms of "an activity in most cases useful to man and always clearly anthropocentric". For instance clouds, the wind and the sun are alive; so too is a gun because it shoots and an oven because it cooks the dinner; but not a stone because "isn't much use".

**Stage 2**: Life tends to be identified more with more movement in itself without such notions as spontaneity or purpose entering into it. For instance a lake is alive "because it is always moving a bit".

**Stage 3**: Life is referred to spontaneous movement only. Thus the wind is alive because it moves itself, but not a cloud since it is moved by the wind.

**Stage 4**: At this stage the concept is restricted to plants and animals, and is defined in every day non-scientific terms. These stages are spread throughout childhood until about the eleventh or twelfth year, and three-quarters of the children do not reach stage 4 before the age of 11 years. One must note that the importance lies not so much in what objects the child declares to be alive, but in his reasons for their being alive. Among other concepts and notions studied in this way were various aspects of physical causality, moral and intellectual judgement, ideas
about the nature of the world around the child, and language
development.

While these and other studies were being pursued at the
Institute of Rousseau at Geneva, Piaget was also observing the
development of thinking at an earlier age, using as subjects his
own children. His findings in this field, to which we shall refer
later, are contained in two books of quite exceptional interest, and
here again he identifies a number of stages of development in
intellectual growth from birth to late infancy.

In his more recent works Piaget records the results of what
many believe to be his most important researches, in which he analyses
the genesis throughout childhood of a variety of notions and concepts
which have a special place in thought as the foundation of scientific
and mathematical thinking. This group of researches includes work
on the child's conception of number, quantity, space, time, chance,
the symbolical function of play, and other topics. It is in relation
to these concepts, moreover, that Piaget develops his theory of mental
operations, which will be discussed in due course.

All these studies are special instances of what for Piaget is
the essence of intelligence, namely adaptation. He defines adaptation
as "an equilibrium between the action of the organism on the environment
and vice versa." Two complementary processes are implied in this
definition. First of all assimilation, when the child acts on the
environment around him, modifying it and "imposing on it a certain
structure of his own". Secondly there is accommodation, when the environment acts on the growing child. Circumstances around the child "modify the assimilatory cycle by accommodating him to themselves". This adaptation is an equilibrium between the processes of assimilation and accommodation, and intelligence or mental adaptation is revealed as a parallel to organic or biological adaptation whose structure is essentially the same. Piaget will go on to point out that equilibrium, whether mental or biological, must if it is to be dynamic at all imply movement two ways, as it were in the correcting of the balance. In other words equilibrium implies reversibility - another key concept in Piaget's theory. Thinking emerges from the simple sensori-motor adaptations of the infant, adaptations which are "rigid and uni-directional", and so thinking is rooted in organic and not psychological functioning. Mental life slowly throughout infancy becomes dissociated from its organic roots, and the equilibrium which adaptation on a mental level demands tends towards a progressively greater degree of reversibility.

Turning from the dynamic forces which direct the growth of intelligence, we find next that for Piaget there emerges an over-all pattern of intellectual development which shows a series of comparatively well marked stages. An interesting statement of Piaget's stages of mental development is given by Inhelder in her contribution to the proceedings of the first meeting of the World Health Organisation in 1953. For a more complete summary however the reader is referred to

Ref. 2.
Piaget's own paper in the 1955 Symposium de l'Association psychologique scientifique de langue francaise. He opens by stating his belief that in displaying stages of growth, intellectual development appears to be in a privileged position. In the evolution of perception or language, for instance, "nous observons une continuité tout autre que sur le terrain des opérations logico-mathématiques, et beaucoup plus grande." On the other hand, in the development of mental operations, "nous voyons des structures se former, que nous pouvons suivre pas à pas dès les premiers linéaments, et que, d'autre part, nous assistons à leur achevément, c'est-à-dire à la constitution de paliers d'équilibre." Piaget makes it clear that before a particular period in a child's mental growth can be regarded properly as a "stage", certain conditions must be fulfilled. Stages are identified by the acquisition of specific mental functions, and it is a necessary condition of having stages at all that the order of succession of stages is more important than the chronological age at which a stage arrives, since the latter is conditioned so much by environment, whereas stages should be thought of as obeying laws of structural growth in a regular and predictable way. Environment cannot alter the order of succession of stages, it can only retard or accelerate their arrival. Moreover, there should be structural integration between the various stages. That is to say, one stage should not in due course be discarded, but must become an integral part of the stage which follows it. Piaget lays great stress on the

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Ref. 16 p. 33-42.
characteristics of stages, because they are evidence that stages are clearly identifiable points of growth, and not merely conventional divisions imposed arbitrarily for the convenience of psychological study.

The various stages which Piaget has found between birth and adolescence are grouped into four main periods. In effect these four periods are identified by the medium of thought which characterises each of them, and the following is a brief outline of their characteristics.

Period I. The sensor-motor period. This extends from birth to about 1½ or 2 years, and during this time the child's thought is carried on largely in terms of motor activity plus sensory perception e.g. in such early habits as sucking the thumb, the co-ordination of vision and prehension, and at a later state contriving means of grasping an object just out of reach. Piaget finds 6 stages in this first period, during which sensor-motor intelligence is growing in complexity until it finally merges into the stages of the second main period. One of the most important acquisitions of the first period is the constitution of the permanent object. In the early stages, the infant does not behave as if it recognised permanency in an object; thus he gives up the attempt to reach an object if it is covered up or obscured from his view, the object having now "ceased to exist" for him. Piaget records many other observations of infants' behaviour.

* The researches which deal primarily with one or other of these periods of development are noted in the bibliography as follows: -
  References for Period I: - 2, 9  ; for Period II: - Stages I, II.  
  for Period III: - 1, 10, 11, 12 etc. ; for Period IV: - 17.
from which he infers this lack of permanency in their comprehension of objects, and he finds that an object becomes a permanent entity in a child's perceptual field only towards the end of the first year. Permanence of an object is therefore the first important "invariant" which the child acquires. He reaches it after an increasingly wide exploration of his spatial field during which time various points in the field gradually cease to be isolated and unique, and become instead recognisable and interrelated in the child's understanding. With this co-ordination of points in the spatial field established, the child is able to move, either bodily or perceptually, from one point to another, and then deliberately to return to his starting point, thus having his first experience of "reversibility".

Period 2. The period of pre-operational thought, extending from approximately 1½ or 2 years to 7 years. This period is subdivided into two stages.

Stage I: From approximately 1½ or 2 years to 4 years. In this stage we have the appearance of a general symbolic function, by means of which the child can think about the world around him in terms of representative symbols, such as images, imaginative games, imitation, and of course language. Piaget distinguishes it further as the stage of a "preconceptual reasoning", meaning by preconcepts "the notions which the child attaches to the first verbal signs he learns to use". The usual characteristic of preconcepts is that
they confuse the generalised concept with the individual and particular one. The child cannot yet distinguish between "all" and "some", and hence cannot appreciate the significance of general classes. At the same time the individuality of an object is not yet fixed in a child's thought but is subject to the influence of space and time. For instance, a mountain is thought by the child at this stage to change its shape as the observer moves past it; or he may think that the cow which he sees in a field as he goes for a walk is the same cow which he sees later on.

Stage 2. From 4 to 7 years approximately. During this period there develops what Piaget calls intuitive reasoning, which can best be illustrated by referring to one of a collection of simple experiments described in his book *Le développement des quantités chez l'enfant*. The child is asked to make two balls of clay the same size and the same weight. If he does this himself, or if he has to have it done for him, a 4 year old child will usually say that each ball has the same amount of clay and that they weigh the same, when however one ball is rolled into a sausage, or flattened like a pancake, he will say that the ball has more clay and weighs more than the sausage or pancake. His thinking shows no idea of conservation of substance or weight. Why does the child's thinking break down in the way that these and other experiments reveal? According to Piaget, it is because at this state of mental development, the child still cannot think in terms of logical operations. During the stages of sensor-motor thought of Period I,
the child indulges in "activity" rather than in thinking as we
normally use this word. His thought is externalised in activity.
With the emergence of the symbolic function, simple mental actions
become possible; that is to say, thought becomes internalised. At
first only relatively simple mental representations play any part
in thinking, and external action on objects as distinct from mental
action must accompany thinking. Thus we have Stage I of this,
the second Period. Gradually, however, the child increasingly
tends to introduce mental action into his thinking; in other words,
the actions which he performed on external objects can now be
performed by representation on his mental symbols. By a mental
action the child can mould two lumps of clay into two balls of the
same size and weight, and will appreciate that the two balls of clay
shown to him will likewise be the same in weight and substance. The
significance of stage 2 of the second Period, however, is that when
one ball of clay is flattened out or rolled into a sausage, the
child cannot mentally reverse the action and "see" the clay in its
original form of a ball. Hence he loses his notion of the
conservation of the similar weights and substances of the balls of
clay once the perceptual correspondence is broken, because his mental
action is irreversible. Piaget uses the word "intuitive" to describe
such mental actions which are as yet irreversible and which are
characteristic of this stage of mental development. He shows in a
vast number of experiments, how intuitive thought is only a preparatory
stage before the emergence of the specific operations which lead to concepts of conservation, equivalence, classification, and several others.

**Period III.** This period, from 7 to 11 or 12 years, is a stage in itself, and occurs when the irreversible mental actions of the previous stage become reversible. The child's understanding of, say, the problem of the balls of clay, is no longer intuitive and stereotyped by perceptual factors. On the contrary, when one ball is flattened out, the child says at once that it "weighs the same and has as much stuff as before" and gives in addition a correct explanation of his answer. Piaget argues that he could not do this unless his thinking had acquired this characteristic of being able to retrace its steps, or reversibility as he calls it, and that it is only when the constraint of irreversible thought is lost, that reversible operations become possible. Operations therefore are "actions which are internalizable, reversible, and co-ordinated into systems characterised by laws which apply to the system as a whole." Concepts of conservation, equivalence, etc., are a consequence of reversible operations. The operations which appear at different points throughout this stage, are not yet however completely internalised, but are closely dependent on objects in the spatial field - hence Piaget calls this the stage of "concrete" operation. He points out at this stage, concrete operations are only fragmentary. They will enable us to classify things with some

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Ref. 18 p. 8.
understanding of the relation of parts to a whole, to set up correspondence between sets of objects or quantities of liquid with full appreciation of necessary conservation, to keep the serial relationship of two sets of objects in mind even when the perceptual view of the series is broken, and so on. But these concrete operations are not yet part of a single psychological structure. They tend until 11 or 12 years to be tied so much to material objects, that they cannot be generalised into an integrated structure. Hence they have a concrete as distinct from a formal character. They cannot yet be the basis of formal logic.

Period IV This period, starting usually at 11 to 12 years, is marked by the gradual weakening of the link between operations and real objects, so that operations tend to become more formal, abstract, and capable of forming the basis of a logical structure. The obvious new characteristic of the period is the ability to reason by hypothesis. The same operations of classifying, constructing equivalence, seriation etc are employed, but the difference is that during the previous stage of concrete operations we were reasoning about external actions on objects themselves, whereas now we are reasoning about operations as such. As Piaget says "Formal thought consists in reflecting on these operations and therefore operating on operations or on their results, and consequently effecting a second-degree grouping of operations". The child is therefore beginning at this stage to reason on the basis of propositions which are laid down for him to

a:Ref. 15 p. 148.
consider. He can start right away to reason from a theory, and
to accept the logical structure of the theory as being the
important thing, rather than the actual content of the propositions
in the theory. Piaget illustrates this point by referring to a
problem from one of Burt's tests "Edith is fairer than Susan; Edith
is darker than Lily: who is the darkest of the three?" This type
of problem is very seldom solved by children under 11 or 12 years,
although these same children, assuming they have reached the mental age of
about 7 years, can solve a problem which is the same in form but
differs in being concrete, such as the serialisation of sticks. In
trying to solve the Burt problem, children between 7 and 11 years
make the same type of formal error as children under 7 who try to
serialise sticks. Moreover, below the age of 11 or 12 years, the
child is often led off the point of the problem by some irrelevancy
touching the content of the propositions themselves - for instance,
getting confused about the whole thing because he happens in real
life to know a blonde Susan. After the age of 12, most children
would accept the propositions as mere hypotheses, without being led
off the trail by secondary considerations such as their truth or
falsehood, conformity with fact, or their moral rightness. Piaget
summarizes the distinction between the periods of concrete and
formal operations in these words "Formal thought reaches its fruition
during adolescence. The adolescent, unlike the child, is an individual
who thinks beyond the present and forms theories about everything,
delighting especially in considerations of that which is not. The child, on the other hand, concerns himself only with action in progress and does not form theories, even though an observer notes the periodical recurrence of analogous reactions and may discern a spontaneous systematization in his ideas. 

It would be interesting to deal at greater length with this notion of "operation", but as a subject for special treatment they are outside the scope of this thesis. The question of whether to some extent they have a parallel in Hebb's learning theory comes to mind, although to the writer's knowledge Piaget and Hebb never refer to each other. Hebb's theory of concept formation as based on a progressively more complex neurological structure - although such a notion in general terms is not of course peculiar to Hebb - appears to parallel in its particular organic framework the logical structure of Piaget's mental operations. It must be remembered that for all the logical symbols by which mental operations are pictured (for instance in the little book "Logic and Psychology"), they are as we have quoted already, "actions which are inter-nalizable etc", the crucial word being "actions".

a: Ref. 15 p. 148.
They are not "mental" in the old sense of being divorced from brain functions; but like all external actions they have a neurological basis. Piaget discusses this aspect of operation in a paper published in 1949, in which he seeks to explain how in the neurological sense "reversibility" becomes possible.

Such are the bare essentials of Piaget's teaching, from which much has obviously been omitted - for instance the notion of ego-centrism has not even been mentioned. A very excellent summary of Piaget's work in its relation to child psychiatry has been given by E.T. Anthony, and in addition he indicates briefly the sources of some of the concepts in Piaget's theory. Anthony calls Piaget "a creative borrower of genius, transposing and amplifying all that he borrows," and this assessment is perhaps a reflection of the growth of Piaget's reputation in this country in recent years. This article is a thought-provoking synoptic treatment of Piaget's work which should be read by every student of Piagetian psychology. A recent paper by D.E. Benjne relates the logical aspect of stages of development to general educational

a:Ref.23.  b:Ref.19.  c:Ref.20.
theory. The writer's short paper on stages of development was intended to introduce this aspect of Piaget's theory to medical officers who have to assess degrees of mental retardation among defectives. Norah Gibbs brings Piaget's work into relation with the difficult problems of backwardness and handicapped children. K. Lovell has also published a study of some aspects of the work of Piaget and Inhelder on the child's conception of space. These papers are mentioned to show how Piaget's psychology is now playing a more significant part in the work of specialists and general practitioners in psychology, and how we are all now fully aware that Piaget thought up other young characters besides the ego-centric child.

a:Ref. 21. b:Ref. 22. c:Ref. 32.
Part II. An outline of the topics considered in "The Child’s Conception of Number"

In the foreword to this book, Piaget indicates clearly its position in the development of his thought. His early books, as we have seen, are concerned with various verbal and conceptual sides of the child’s thinking, covering such fields as language development, judgement and reasoning, ideas about the external world, physical causality, and so on. Then come the studies of the origin of intelligence in children, from the moment of birth, until the end of the sensori-motor period of intelligence at about 2 years of age. This is followed by the book on number, which is the first of a series of volumes devoted to discovering "the mechanisms that determine thought". All these volumes aim at demonstrating how the various schemata of sensori-motor intelligence gradually cast off their sensori-motor dependence, and rise up to the level of relatively free conceptual thinking. As we have seen, Piaget does not consider sensori-motor intelligence to be thinking in the strict sense, but the schemata of this period of mental development do nevertheless become "organised in operational systems on the plane of thought".

The purpose of these later volumes is therefore to trace the development of the operations which give rise to concepts of number, continuous quantities, space, time, etc., through the stage of intuitive pre-logic to that of concrete operations, and on to the final stage of formal operations.

It will be convenient now to outline the contents of the chapters
in his book. *

Chapter I. Piaget opens his book with the proposition "Every notion, whether it be scientific or merely a matter of common sense, pre-supposes a set of principles of conservation either explicit or implicit." (3) He contends that conservation is a necessary condition of all rational activity, and instances the principle of inertia in physics, conservation of matter in chemistry and the schema of the permanent object in perception. In arithmetical thought the same principle can be found. For instance, the concept of a set or collection is possible only if the set remains the same no matter what change may be made in the arrangement of its parts. Thus, that XX XXX is the same set as X XXX X presupposes a principle of conservation. Similarly a continuous quantity such as a specific volume of liquid can be used intelligibly only if it is conceived as a permanent whole, so that a pint of milk remains a pint whether it is contained in one milk bottle or distributed among four tumblers. "Conservation of something is postulated as a necessary condition for any mathematical understanding." Having made this point, Piaget suggests that the psychological problem following from it is to decide (a) whether conservation precedes any numerical or quantifying activities, so as to exist as a sort of "Innate idea" present in the very earliest experience of childhood.

* There will be quite a number of quotations from his book in the following summary. Most of these will be followed by a note in parenthesis of the page on which they will be found in the English edition of "The Child's Conception of Number".
or (b) whether the notion of conservation is a logical
construction which is only gradually built up in the child's
mind. The experimental evidence which Piaget produces suggests
emphatically that the second alternative is the true one. The
chief aim of this part of the book is therefore to analyse the
intellectual mechanism which underlies the evolution of the
concept of conservation.

The first group of experiments are concerned with the
conservation of continuous quantities. Cylindrical glass
containers of various dimensions are used, together with quantities
of water. A typical way of presenting the problem to the child
might be as follows. Two containers, A and B, of equal size and
having the same quantity of water are shown to the child. When the
child has agreed that there is the same quantity of water in each,
the contents of one container are poured into four smaller ones.
The child is then invited to compare the quantity of water in A,
the first of the larger containers, which still lies before him,
with the quantity in the four smaller containers collectively.
Alternatively, the water in B could be poured into one larger, shallow
container and the child asked to make the comparison between this and
A. Obviously there are many ways in which the child's notions of
quantity and conservation can be analysed, according to the variety
of containers used, and it is important to note that Piaget does not
infer a child's conceptual level from one experimental approach only.
The children's responses revealed three stages of development as follows:

Stage 1. Absence of conservation. At this stage there is no evidence that the child has any notion of the conservation of the liquid as it is poured into a variety of containers. A change in the size or the number of containers will invariably suggest to the child a change in the quantity of the liquid itself. Moreover, a given child's response will vary on different occasions. For instance, pouring the water from a large container into four smaller ones may at one moment suggest to the child that there is now less water, while later on he may say that it results in there being more. Similarly, the use of a tall narrow container, or a wide shallow one, will lead to confusion and self-contradiction. The subjects are obviously basing their judgements of quantity on isolated perceptual factors, those usually being the number of containers or the level of the water in a given container - the two are never at this stage coordinated. The children at this stage in short can reason with respect to one dimension only at a time, and they have no concept of multi-dimensional quantity. To take a simple example, comparing A and B quantitatively means for them looking to one or other of two qualities, either the height of the two water levels, or their respective widths, but not to both simultaneously.

The 3-stage development is characteristic of all the pre-number concepts which Piaget studies. The average child is at stage I until approximately 5 years, from which time the operation gradually emerges until by 7½ years most children have reached stage III.
In his interpretation of the reactions of the first stage, Piaget suggests that we examine the way in which, from their earliest perceptual contact with an object, quantity and quality are differentiated from one another. He argues that when objects are perceived, and when concrete judgements (i.e., judgements arising directly out of these perceptions) are made about the same objects, then qualities are necessarily attributed to them, either implicitly or explicitly, and these qualities must necessarily be related to each other. In the present context, the qualities which he has in mind are those which, more than any other, have a bearing on the genesis of number concepts, namely the spatial qualities of shape and size. From the process of relating a group of objects according to a specific quality, two kinds of relationship emerge. We may have symmetrical relations which express resemblance, e.g., glasses $G_1$, $G_2$, $G_3$, $G_4$ are equally small. These lead only to the classification of things which are like one another in a given respect. On the other hand, we may have asymmetrical relations which imply "more" and "less," e.g., $B$ is smaller than $A$ and bigger than $C$, and which thus indicate the beginning of quantification. Thus "quantity" in its simplest form is given at the same time as quality. Perception and judgement lead to awareness of qualities; this in turn implies awareness of relationships between qualities; some of these relationships are asymmetrical and are thus in a limited sense "quantitative". At the level of stage 1, however, quantification is restricted to the
immediate perceptual relationships, and does not extend beyond this point to that of true relationship. Piaget calls this the level of "gross quantity" because of its limited scope, and pairs it with "gross quality", i.e. directly perceived quality which cannot give rise to classification because it is not linked to other qualities by relationships of likeness or difference. Now this primitive system of quantification, based as it is on immediate perceptual relationships, cannot yet lead to systematic quantification (i.e. a true conception of number). Before it can do this, Piaget argues, the perceptual relations must become true relations. Psychologically the chief characteristic of perceptual relationships of gross quantity is that they cannot be composed one with another. The child is asked to compare two quantities from several points of view, height, cross section, number of glasses, etc, but this he cannot do. Instead he considers these relations separately, as though they were quite independent of one another. In other words, he is incapable of the addition and multiplication of relations.

Stage II. Intermediary reactions. There appear to be two types of responses among children at this stage. Some of them seem to realise that there is conservation of the water when it is poured from A into two glasses B1 and B2, but when, say four glasses are used, he falls back to his earlier belief in non-conservation. Secondly, there are some who appreciate the notion of conservation when differences in level, or in cross section, are slight, but when
these differences are increased they will, like the former group revert to non-conservation.

Piaget interprets these reactions as showing that the conditions necessary for the transition from "gross quantity" to true quantification are beginning to be fulfilled. The child at this stage is trying to deal with two relations simultaneously, say level of water and width of containers; in other words, he is trying to coordinate two perceptual relations in a way which would transform them into true operational relations. He attains, however, only partial success, and hesitates between his attempt at coordination and the pressure of the simple perceptual relations. Even if the child at this stage could carry out the operation of logical multiplication of relations, this, Piaget insists, would not in itself lead to conservation of the whole quantity. Given a column of water whose height increases and whose width diminishes with respect to another column; the former column may, in the child's judgement, be as a consequence greater, equal to or less than the latter, and in making his judgement, he may (as these stage II children do) coordinate or multiply two relationships. But such an operation so far gives only intensive quantification i.e. notions of "more" or "less" but without any conception of "by how many parts or units" more or less. In order to be logically certain that there is equality, intensive quantification, possible at this stage, must give place to extensive quantification, i.e. being able to establish a
true proportion between gain in height and loss in width, which would imply if only in a very elementary way, the concept of partition or quantification by specific units. Now during this stage II, the child does begin to understand that a whole remains identical with itself when it is divided into two halves (container A is equivalent to container B1 and B2). Unfortunately this simple instance of extensive quantification, implying also a simple concept of partition, breaks down when more complex perceptual elements are involved (say four small containers instead of two). Piaget's conclusion is that multiplication of relations (e.g. coordinating relations of height, width, etc) must go hand in hand with partition (i.e. concepts of exact or extensive quantity).

Both of them make their appearance and begin to develop during the second stage, and both are subject to the same limitations, namely, neither of them can survive too elaborate a perceptual framework. He asks what is the link that unites these two types of operation, and suggests that the analysis of the third stage would provide the answer.

Stage III. Necessary conservation. At this stage children state immediately, or almost immediately, that the quantities of liquid are conserved. Piaget says that when the child discovers this invariance he "states it as something so simple and natural that it seems to be independent of any multiplication of relations and partition". (17) There arises then the question of whether this independence is genuine or not, and if it is not, what factors link
up the non-conservation of stage I and the apparently direct apprehension of conservation of stage III. Piaget then quotes examples of responses which indicate that conservation does in fact depend on the coordination of relations. In particular he refers us to the responses of some children who, although definitely in stage III, nevertheless hesitate for a moment, and in their verbal responses reveal the mechanisms of their logical constructions. One child, for instance, starts off by thinking that the liquid in A becomes less when poured into P, but immediately afterwards says "It seems as if there is less, because it is bigger (wider), but it is the same." In other words, he corrects his mistake by coordinating the relations of height and width. Although the responses of many children are prompt and immediate, and make it appear as if their notions of conservation were the result of an a priori analytic deduction, Piaget believes that at some point of their development a coordination of relations, such as we have just quoted, must take place, this coordination having the twofold aspect of logical multiplication of relations and mathematical composition of parts and proportions. With regard to the logical multiplication of relations, Piaget declares that in itself it is not enough to ensure the discovery of the invariance of whole quantities, and he proceeds to explain why this should be so. When at stage I a child bases his judgement of two quantities on, say, the level of the water only, the perception is uni-dimensional. At a later point
in his development, he begins to coordinate relationships, and thus to construct multi-dimensional wholes, but these wholes remain "intensive", and are incapable of having any numerical value assigned to them. But if the child is to arrive at the notion of conservation, he must be aware, let us say, not only that similar width and similar water level in two glasses entail a similar amount of water, but that the quantity remains constant even if in one container height increases and width decreases. Such a conclusion as this, however, goes beyond the limits of logical multiplication of relationships, which in itself does not contain the basis of exact numerical inferences. The problem then is how does the child progress beyond this limit; that is to say, how does he effect the transition from intensive to extensive quantification.

It is at this point, according to Piaget, that a second process intervenes, namely, the notion of the "unit", i.e. "extensive quantification in the form of arithmetical partition...... or of proportion." To illustrate the point he refers to the case of a child who was asked to put into B the same amount of liquid as was in A.
The child poured in a higher volume of water and said "It is the same because this one (L) is narrower and this one (A) is wider." Piaget argues that the child could infer that the liquids in A and L were equal only if the height of one and the width of the other were interchangeable. He believes thus that something more is implied in the child's reasoning, i.e. the feeling of a definite proportion, an awareness that the increase in height in liquid in A when it was poured into L was equivalent to the decrease in width. We have seen that mere logical multiplication of relationships does not lead to the division of a given quantity into units that are recognised as equal and yet distinct. Both proportion and numerical partition, however, imply the "fusion of asymmetrical relationships of difference with those of equality", that is to say, two qualitative relationships of difference (increase in level and decrease in width) are seen to be equal, although they continue to preserve their asymmetrical character. The child appears to grasp quite suddenly the notion that the differences compensate one another, and it is through this fusion of equality and asymmetrical relationship that the concept of proportion comes into being, and the step from intensive to extensive quantification is taken. This proportion Piaget says is already a partition, because in equating the increase in level with the decrease in width, the child shows that he now regards the quantity as something capable of division into units. The elements of the whole are equated with one another and
yet remain distinct, and this Piaget says, is the criterion of arithmetical partition. So at this stage the child can see that the liquid in A remains the same when poured into two, four, or any number of smaller glasses of equal size. "Numerical partition", Piaget says, "is therefore essentially an equating of differences, like proportion itself, but in the case of \( A = B_1 + B_2 \), the two halves \( B_1 \) and \( B_2 \) are seen as equal, whereas in the case of \( A = L \), it is only the differences that are equated." (23)

Chapter II. Piaget now goes on to examine the conservation of discontinuous quantities. He uses the same glass containers and pours into them not water but small glass beads. They can serve for the same evaluations as the liquids (i.e. level, cross section etc) but in addition to this it allows the child to make a global quantification on the lines of "Suppose the beads in the two glasses were made into a necklace, which would make the longer necklace?" This enables one to check the quantification of the contents of the glass containers in a way not possible when liquid is used. Operations of correspondence can also be used, for instance the child and the examiner putting beads one by one into two containers which may be the same or different in shape. Such an operation may make more decisive the question of the absence of notions of conservation in the child's thinking. Thus we find that even with the help of one-one correspondence children will continue to be deceived by the apparent variations in quantity which uncoordinated perceptual relations tend to arouse in their minds. In fact, Piaget finds the same three stages
as were revealed in the experiments with water, and they occur at the same ages. "The conflict between one-one correspondence and perceptual relationship", he says "thus comes to an end only during the third stage, with the triumph of correspondence over perception". (37) Piaget stresses the important criterion of stage III, that the perceptual relations have now becomes "operational". For instance, a child will now say "If I emptied this one (P) into this one (L) or that one (L) into this one (P), they would be the same". In this process of thinking, the child is "expressing the reversibility characteristic of any logical mathematical operation."

Chapter III. Assessments of quantity may be made in either of two ways. By comparing the dimensions of two quantities, or by making a one-one correspondence between their elements. The former was considered in Part I, and now Piaget takes up the study of the equivalence of corresponding sets.

His thesis may be summarised as follows. Making a one-one correspondence is the most elementary way of comparing two sets of objects, either in estimating which set is larger, or in building up the two sets to equal each other. Thus two young children (say 4 yrs) are playing marbles: if one lays out 6 marbles, the other will place one of his opposite each of his partner's and thus get an equivalent set, even if he is as yet unable to count. Nevertheless, this one-one correspondence is not in its earliest form accompanied by the concept of lasting equivalence of the two sets. As the experiments will show, a
child may himself build up two equivalent sets, but yet a change in the configuration of one of them will destroy his notion of their equivalence. Furthermore, it is evident that even if the child counts aloud while constructing the sets, his concept of equivalence is not necessarily permanent. The verbal factor, Piaget asserts, plays little part in the development of correspondence and equivalence.

Correspondence may be either between objects of the same kind, e.g. the two sets of marbles or counters, or between objects that are different from each other, but which are qualitatively complementary. This latter, Piaget says, is correspondence which is "provoked by external circumstances"; it is simpler than the first type of correspondence, and he therefore studies it experimentally before the first type.

Provoked correspondence between sets of different objects. A number of experimental techniques are used. For instance, six small bottles are put on the table in front of the child, while at one side there is a collection of small glasses. The child is then asked to "take off the tray just enough glasses for the bottles, the same number as there are bottles, one for each bottle." When the correspondence is established, the glasses are grouped close together and the child is asked "are there as many glasses as bottles?" If he says "no" he is asked to show where there are more and why. The glasses may then be re-arranged opposite each bottle, the bottles grouped, and
the questioning repeated. Other sets of objects are used to make the correspondence, applying the same technique; for instance, setting up a correspondence between flowers and vases, or eggs and eggcups. The technique of exchange may also be used. The child is given pennies to buy sweets. The child puts out pennies one by one for the examiner, who in return gives him one sweet for each penny. The perceptual correspondence between the two sets is at first quite apparent, but it is then broken by grouping one of the sets, and the child is questioned on the same lines as with bottles and glasses. The exchange technique is used twice, once with counting aloud, and once without counting.

All the techniques used give the same three-stage development, which may be summarized as follows:—

Stage I. Here the children's characteristic response is to make a global comparison between the set of objects placed before him, and the sets which he is asked to build up. Usually the comparison is based on total length, and he will make his row of glasses approximately the same length as the two bottles, but with no attempt at getting an equal number of objects in the two sets. The qualities which the child perceives are capable of giving rise only to simple quantitative relationships such as more or less "big", or "long" or "narrow" etc., and are Piaget says "without operation in their true sense". The child's judgements are rigidly stereotyped by perception because the quantities perceived are not multiplied with one another, as we found at the same stage in the study of continuous quantities. At this stage
then, there is no correspondence and no permanent equivalence of sets.

Stage II. The child is now able to make a one-one correspondence. Thus he will place a glass opposite each bottle, place an egg in each egg-cup, or a flower in each vase. This however is only intuitive correspondence, since he no longer recognises the sets as equivalent when the one-one correspondence is broken by one set being closed up or extended. His concept of correspondence at this stage rests on perceptual relationships, and perception, Piaget says, is essentially irreversible. The perception itself can not be "run back" like a film to its original position, and so the child at this stage cannot say "They are still the same because the ones close-up could be spread out to the same places as they had at first." Such a scheme of thought would imply a more advanced operation, namely the equalization of differences, which is the essence of the next stage.

Stage III. The child will reach this stage when he sees that if the objects in one set are, let us say, spaced out, the number of objects for each unit of length is diminished, and that if they are closed up, the relative number increases. In other words, he must be able to see that the differences compensate one another, and this is the operation which Piaget calls "equalization of differences". He defines it as "a coordination of the displacements such that they can offset one another when they become reversible". (58) Thus, when the elements of one set are closed up, they do not become merely something new in
perception, but remain linked to their original configuration by the possibility of the internalised inverse action of going back to their first position, and hence of retaining at one step's remove their relationship of equivalence which was set up by one-one correspondence. Reversibility is thus the source of lasting equivalence, or as Piaget puts it "the triumph of the operation over perceptual intuition is the outcome of the progressive reversibility of thought."

Using the exchange technique, Piaget found that ability to count aloud correctly made no difference to the stages of development of correspondence and equivalence. It may hasten the evolution at the point where correspondence begins to become quantifying, but Piaget is emphatic that the process is not begun by numerals as such.

Chapter IV. In this chapter Piaget goes on to analyse the mechanism of spontaneous correspondence, that is, correspondence which is not suggested to the child by any qualitative relationship between the two sets, as happens between a set of eggs and a set of egg-cups, or flowers and vases. In spontaneous correspondence the elements of the sets are the same, and the problem will be to see how the child spontaneously attempts to estimate the cardinal value of a set. Piaget presented the child with a succession of figures made from counters, and asked him to take the same number of counters as each figure contained. No method of selecting the same numbers is suggested, and one-one correspondence is not suggested to the child by such direction
as "Put an A opposite (or with) each B". In short, whereas in earlier experiments correspondence has been imposed in order that its results could be examined, here the question is to see what the child's procedure will actually be without suggesting any method.

Five types of figures were used, as follows:

1. Badly structured figures: that is to say, a collection of counters distributed at random before the child.
2. Open figures, for instance two parallel rows of counters.
3. Closed figures, in which the shape does not depend on the number of elements used; for instance a circle, a right angle or a house.
4. Closed figures, where the shape depended on the number of counters, such as a square or a cross.
5. More complex closed figures which would be less familiar to the child, such as a rhombus.

The child is shown one of these figures, and then asked to "pick out of the box the same number of counters."

In a supplementary experiment the child is shown a row of six beans, one or two centimetres apart, and is asked to put out the same number.

From the child's responses to all the figures, three stages of development emerge, all of them parallel to the three stages revealed by the various experiments already described.

Stage I. At this stage the child, as we would now expect, does not feel the need for quantitative comparison, but confines himself to global, qualitative assessments, without any coordination of the qualities which he compares, and we find as before that the only quantification of
which he is capable, is expressed through the relationships of "more and less". The child's judgement is still intuitive and perceptual, lacking in mobility and hence irreversible. There would appear to be some small variation in the child's ability to reproduce the different figure, but these do not affect the non-operational and wholly intuitive nature of his thinking, and his inability to effect even simple correspondence.

Stage II. Piaget introduces this stage by classifying certain types of correspondence.

(a) Qualitative correspondence: "based on the qualities of the corresponding elements." This may be either:

(1) intuitive; i.e. entirely based on perception, and therefore not preserved outside the actual field of perception, or at least of clear recollection.

(2) Operational: when the correspondence is based on relationships of an intellectual nature. This implies that it is preserved independently of actual perception and that it is therefore reversible.

(b) Numerical correspondence: each element is considered as a unit, irrespective of the qualities; for instance, n blue counters corresponding to n red counters, whatever their distribution may be. Except for the first three or four numbers, numerical correspondence must be operational.

The responses of Stage II, then, belong to the class of qualitative intuitive correspondence. In the course of Stage I the child becomes more and more adept at copying the models, until he is capable of precise one-one correspondence whatever the shape of the
figures may be, thus signifying his arrival at Stage II. Since however, the correspondence is based on perceptual comparison, it is not numerical, although it may appear to be so. One can prove this by altering the configuration of the corresponding sets, and it will be found that the child no longer accepts the equivalence of the two sets.

Stage III. The correspondence is now operational, either in a qualitative or a numerical sense. The child may or may not copy the model. If he does, he will recognise the lasting equivalence even when one set is altered in shape. He may or may not employ counting. If he does not copy the model, he will recognise the equivalence by counting, and the pattern of the model will be an irrelevant factor.

Piaget now summarizes this chapter at some length. He reminds us of what the experiments described in the three earlier chapters have revealed. We have seen that in comparing both continuous and discontinuous qualities the child does not assume conservation when the perceptual configuration is changed. Next it was found that even correct one-one correspondence of the simplest type did not at first ensure lasting equivalence of sets. The spontaneous processes of quantification were then studied in the present chapter, and three spontaneous procedures were revealed:— (1) global evaluation; (2) correspondence without lasting equivalence, and (3) numerical correspondence with lasting equivalence. These experimental findings lead in their turn to three key questions:—
"A. Why does the child not feel the need, from the beginning, to decompose the global totalities that he thinks he can evaluate?

B. How does the first form of decomposition or intuitive qualitative correspondence make its appearance?

C. In what conditions does operational qualitative correspondence become numerical correspondence?"

Piaget proceeds to answer these questions in two separate analysis, psychological and logical, which we shall now attempt to combine in one summary form.

The experiments described so far reveal that at a first stage of development, terminating at about 5½ years, the child tends to base his quantitative judgement on shape and pattern, and on such relationships as more or less "long", "wide", etc. He evaluates, in other words, discontinuous quantities as if they were continuous. The question now is whether the child does not feel the need to break up the set of objects which he can perceive and which he is trying to evaluate, or whether he is unable to effect this breaking-up process or decomposition through lack of the necessary logical mechanism. Piaget recognises that what we already know about the psychology of thinking at this stage is quite consistent with the child taking this non-analytic view and his not feeling the need to decompose so long as experience does not compel him to do so. In his book "The Language and Thought of the Child" he has already studied this global or syncretic character of the young child's thinking, and this is merely another instance of the same attribute.
He poses the question now, "If the child does not feel the need for decomposition, does this mean that he is incapable of it?" and declares that there must be a clear affirmative answer to this question.

There is only one constructive principle at the disposal of the child at this stage, that of intuitive global perception of the total configuration in question. In making his judgement about relative amounts, size, etc, the child must therefore rely on a number of unrelated perceptions of length, width, density and so on, and in doing this his tendency is to base his evaluation on one criterion only. For instance linear series tend to be evaluated by their total length only, irrespective of their density. "The notion of conservation is lacking" says Piaget "because the elementary relationship inherent in the global perception are merely juxtaposed instead of being coordinated". And there is almost complete irreversibility of thought because the perceptions to which the global judgement is bound are immobile, or one-way only. It is, of course, true that each of the global qualitative relationships established by the child, e.g. "longer", can give rise to an inverse relationship "less long" etc, but these relationships cannot be broken up into units, implying "by how much longer", nor are they coordinated with each other, which would imply, for instance "so much longer at that point, but balanced by so much narrower at this point."
They are merely put together in a non-structured whole and hence cannot form part of a reversible system. Logically, the child at this level of global quantification is incapable of either decomposing, combining or multiplying relationships, and strictly speaking what one calls "relations" at this stage are not yet relations at all. Global relations are described by Piaget as "gross quantities", or relationships between "gross qualities". The word "gross" implies that they are tied rigidly to immediate perceptions, and that mobility is not yet possible.

Global relationships become transformed into relations proper by the appearance of two operations, namely:

1. Additive seriation: this consists in establishing that "the total length of a row is formed by the sum of the intervals separating one element from the next";

2. Multiplication of additive series; this is "the construction of a new row having the same length and same intervals as the first", in other words it is qualitative correspondence.

Given two sets which are in qualitative correspondence, having that is the same total length and the same intervals, then from logical multiplication of the relationships it follows that:

(1) Equal length and equal density

```
0 0 0 0 0 0 0
0 0 0 0 0 0 0
```
the sets are the same. This is simply qualitative correspondence.

(2) Greater length and greater density or smaller length and less density

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}
\]

one set has more or less elements than the other.

(3) Equal lengths and greater density; or equal lengths and smaller density,

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}
\]

one row contains more, or less, elements than the other.

(4) Similarly with equal density and greater length, or equal density and smaller length

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}
\]

(5) Greater length and smaller density or greater density and smaller length, the number of elements in one series may be

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}
\]

larger

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}
\]

or smaller

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}
\]

or equal

The child at the second stage, says Piaget, can easily grasp the first four compositions. None of them requires an "abstract operation" since perception is enough to guarantee equivalence. Although the fifth composition is implied by the other four, the child at Stage II cannot grasp it yet. In the experiments, the composition of five is presented to the child by starting off with composition 1 and then proceeding to alter it to the following variations of 5:-
At stage II the child is still thinking at the intuitive level. From direct perception he sees that the length and density of rows are variable, and from the point of view of perception he is quite right in thinking that this involves variation in the number. He ceases to be logical, however, in failing to understand that in, for instance, a contracted series the decrease in length carries with it increase in density. He cannot yet "logically multiply" and so he makes the mistake of assuming that the number of elements in a row depends on one quality only, either length or density. It is dissociation of length and density which makes his thought at this stage irreversible.

In passing from this to the third stage, when the child discovers the constancy of number, a distinction must be made between qualitative and numerical correspondence. When a series of red counters is placed below a series of blue counters with correct one-one correspondence, the two series are linked by the relationships "above" and "below", and from this point of view they possess
equivalence. A stage III child will often grasp this notion and state, when one row has been closed up, that "they are the same, because the heap can be made into the row again". This however is not numerical correspondence, and does not in itself imply constancy of number. This equivalence depends on the same configuration. The next step is to see that the counters in the heap correspond to those in the row and to any other set of counters of the same number whatever their distribution. Given that each set has six counters, the correspondence is no longer determined by the spatial properties of the counters. Instead each set acquires the new quality of "number six", instead of simply the quality of being a row spaced in a given way. Each counter becomes a unit equal to the other, and distinguished from the other by the order in which they appear in the correspondence - this order being quite relative and varying from operation to operation. Correspondence therefore becomes numerical when the elements are understood to be equivalent in all respects, and "when the differential properties that distinguish them within the set are replaced by the only differences compatible with their equality, that is, their relative position in the order of correspondence. It is once again found that it is the equating of difference that is the origin of the unit and consequently of number". The child now sees that a decrease in, say, length, is compensated for by increase in
density. Thus the length and density of the rows lose their
significance, and it is only this constant relationship of being
a unit equivalent to another unit which holds between the elements.

Chapter V. The various forms of correspondence and
equivalence examined in previous chapters have been concerned
primarily with the cardinal aspect of number. Piaget now asserts
that the making of a one-one correspondence between elements of two
series does not a priori involve primacy of the cardinal over the
ordinal. But when the elements of a set are identical (e.g. a row
of similar counters), the order can be disregarded and the
significance of the correspondence becomes primarily cardinal, since
the equivalence between two such sets can be established irrespective
of the order. On the other hand the ordinal aspect of the
correspondence is more marked when the elements of the sets in
question differ from one another by characteristics capable of
seriation, and when the characteristics determine the position of
the elements in each of the two sets. We may consider a sequence of
wooden figures of varying heights, and a series of sticks of varying
lengths. Three operations are possible:

(1) Simple qualitative seriation, i.e. taking one set at a
time and, from direct observation of the varying sizes,
putting the elements into their serial order.

(2) Qualitative correspondence between two sets: i.e.
matching the smallest doll with the smallest stick,
etc until the two series are complete.

(3) Numerical ordinal correspondence between the two
series: i.e. building up the two series together from
a full understanding of the ordinal concepts: first,
second, etc.
Piaget then examines the reactions of the child with regard to these three operations, and sets out to determine if the "intuitive character" of the series will lead to a more stable correspondence, and also to the notion of permanence of order, that is to say, the understanding that even when the elements are disarranged, a given position in Set A always corresponds to a given position in Set B.

The chief materials used were 10 wooden dolls of clearly differing heights, the tallest being twice as big as the shortest; and 10 sticks similarly varying in size. The child is told that the dolls are going for a walk, and he is then asked to arrange the dolls and sticks so that each doll can find the stick that belongs to it. If necessary help is given, until the child understands the principle of serial correspondence. When the two rows are arranged in correspondence, changes are made while the child watches. The rows are left parallel, but the dolls are brought closer together and the sticks spaced more widely, so that the corresponding elements are no longer opposite one another. The examiner then points to one of the dolls, and asks which stick it will take. This is repeated several times to bring out as clearly as possible that the child has either retained the notion of the two series and can still relate corresponding elements, or that the spatial change has destroyed the notion of seriation. Next the order of one of the series is reversed, so that the largest doll is opposite the
smallest stick, etc. The same questioning then follows. The next test is to disarrange both series, and to ask the child to find the stick which belongs to a specified doll. Finally, all the elements of the two series are mixed, and one of the dolls selected; the child is then told that some of the dolls are going for a walk, namely those that are bigger than the selected one, and he has to find the sticks which belong to the dolls which are going for a walk, and those staying at home.

Three problems emerge from these tests:—(1) that of constructing a serial correspondence; (2) that of determining a serial correspondence when it is no longer perceived; in other words, of making the transition to ordinal correspondence; and (3) that of reconstructing the ordinal correspondence when the intuitive series are destroyed.

In the solution of these problems three stages were observed.

(1) Construction of serial correspondence: Stage I. Global comparison, without either exact seriation or spontaneous one-one correspondence. The children's responses at this stage show furthermore that when the child cannot make the dolls and sticks correspond, he is also unable to construct the single series of dolls or sticks; and that when seriation becomes possible, correspondence also is possible. Stage II. Intuitive, progressive seriation and correspondence. Here the child can himself construct a correct series after some trial and error, and he can solve the
problem of serial correspondence. But this stage differs from the next one in that the child does not yet master the whole set of relations necessary for seriation, but only discovers them by trial and error. He compares the elements of the series by small groups, whilst at the next stage he compares the whole set of data continually and is able to choose, for instance, first "the smallest of all", then "the smallest of those that are left", and so on. This, Piaget says, is because at Stage II there is a primacy of operation over intuition. Stage III. Immediate, operational seriation and correspondence. The child may now operate by immediate correspondence, without finding it necessary to seriate first the dolls and then the sticks.

(2) Finding the correspondence when the intuitive series are disarranged. Stage I. The child cannot find the correspondence between a doll and a stick once the elements in each series are no longer opposite each other. When the series is displaced, the child seems to lose all idea of correspondence, and merely chooses the elements that are opposite one another. Stage II. Here the child may try to solve the problem by counting or by making a new one-one correspondence suggested by the changed appearance of the rows. This can in some instances lead to success, but the child is never certain of his result and he frequently becomes confused. Stage III. He now finds the correspondence by combining ordinal and cardinal notions: that is
to say "he coordinates his estimate of the required position with that of the cardinal value of the sets in question". He no longer hesitates in stating that the number of sticks is still equal to the number of dolls even when the order is inverted or the series displaced. Piaget goes on to say that the two mechanisms of cardination and ordination are now correlated. The cardinal value of an element in a set is now independent of the other parts, and the value in question could be applied to any element in the set. Also the ordinal value is no longer dependent on quality; an element N can have the Nth position simply because of having been counted to position N, irrespective of qualitative factors such as its size.

(3) Reconstruction of a correspondence. Stage I. There is no correspondence and the two series are not constructed, the elements being chosen at random. Stage II. There is an attempt to solve the problem, but without any systematic method of counting or of reconstructing the series. At this stage the child can make only an intuitive attempt to link up the cardinal and ordinal aspects of correspondence, and he does this in four ways. (i) He may simply make a guess at the correspondence, or else order one series and guess at its correspondence with the other. Although he thus knows what is wanted and tries to do it, his solution is only approximate. (ii) He may use cardination but ignore ordination. For instance, he may, in trying to find the
stick for doll 1, correctly seriate dolls 10, 9, 8 and 7, and then pick four large but randomly selected sticks and give one to doll 7. (iii) He may seriate but ignore cardination. For instance, if he wishes to find the stick belonging to doll 6, when the sticks are already seriated, he may seriate the dolls to number 6, but without putting them opposite the sticks, and chooses stick 4 merely because it happens to be opposite doll 6. (iv) He may use cardination and ordination simultaneously, but still not be able to coordinate the position of the element looked for with the cardinal number of the set of elements. For instance, in the case of finding the stick for doll 6, he may count the four preceding dolls (10, 9, 8, 7) and then point to stick 7, the last of four sticks also counting from 10. This method of attacking the problem shows that the child is attempting to reconcile order with cardinal value, but that he cannot keep both in mind at the same time. Stage III. At this stage ordinal correspondence is linked with the process of cardination, both now being fused at an operational level.

Chapter VI. This chapter contains some further experimental material on the problem of a child's attempts at seriation. It is not proposed to summarise it here, since it does not add materially to the findings of Chapter V. The final section of Chapter VI however, is an important milestone in the book, since it comments on and compares the experimental findings concerning
both ordinal number and cardinal correspondence, and links up the data of Chapter V with that of previous chapters.

Piaget first of all indicates how the three stages of seriation and ordinal correspondence are related to the levels of cardinal correspondence.

(1) Two characteristics in common are found in the first stages of seriation and cardination, namely their global nature and the fact that in each case the immediate percept, as distinct from operational, logical composition, controls the child's thought. When the child makes an unseriated row of sticks and dolls, perhaps only distinguishing the large from the small, his reactions are similar to those which he displays when he tries to set up 9 counters as equivalent to 6 by pushing the 9 close together so that they will more or less "look the same" as the 6. In both instances the criterion of truth is perception, and "not a system of operations that can be composed".

(2) Ordination and cardination also correspond in their second stages, when the child no longer reacts globally. He is now capable of correct analysis, but this analysis is limited to perceptual data and has not yet reached the operational stage. Whether he has constructed two series to correspond correctly, or matched two qualitatively similar sets, he only believes in the correspondence as long as the relationship as actively perceived.
(3) Both ordination and cardination now show "the victory of operation over intuition: in both cases the child coordinates beforehand all the relations involved, because operational composition triumphs over perception, or rather, because the latter is from now on controlled by the former".

In brief, then, the same processes and levels are to be found in the development of both ordination and cardination. Piaget next goes on to consider the progressive coordination of cardinal and ordinal processes, and to explain their convergence.

It will be obvious that at the first stage there is no convergence simply because ordination and cardination do not yet exist. The experiments referred to briefly in earlier parts of this summary will make this abundantly clear. Cardinal evaluation is merely global judgement, without conservation or one-one correspondence. Seriation is merely the juxtaposition of one term to another with no governing principles except that of "big" elements and "small" elements. Piaget points out that the two processes are in a sense antagonistic. Seriation requires that each element shall be distinguished as different from the others, whereas classification entails putting together a certain number of elements seen as equivalent. The experiments show that the child at this stage will disregard sets when he is trying to seriate, although he may be able to form simple sets,
and when he is trying to evaluate an assembly of elements, he will disregard order.

In the second stage there is the beginning of a convergence, at a very simple level. The cardinal evaluation of this stage is achieved by means of one-one correspondence which involves a kind of ordination. And on the other hand in the construction of an intuitive series the child grasps the fact that each term can be counted, and that it forms, with the preceding terms, a set capable of being counted. But of course the cardinal correspondence does not yet entail lasting equivalence, because it is still dependent on perception, and the same is true of ordination, since the order is constructed only in so far as the total series is actually perceived. There is then no complete coordination between ordinal and cardinal processes.

The coordination is achieved in the third stage, and as Piaget has said before, it is a result of the triumph of operation over perceptual intuition; or, as he otherwise expresses it, of reversible grouping over static recognition. It is reversibility which gives the reactions of the third stage their operational character. For instance, "to seriate operationally is to coordinate the two inverse relations \( S > R \) and \( S < T \), and this implies the possibility of setting out the series in either
When reversibility is achieved in seriation and classification, it then becomes possible for operations to be assembled into what Piaget calls "groupments", translated as "groupings". For instance, the variously sized dolls may be conceived from two points of view: (1) as similar, by disregarding their differences and considering only their common properties, their "doll-ness". This regard for the equivalence of the elements lead to the concept of logical classes - e.g. the class of "dolls on the table", differing in size perhaps, but dolls nevertheless. (2) In constructing the class the child must at the same time distinguish the elements, i.e. he conceives them as different by other qualities than the common quality of doll; by colour if not by size, or if they are all identical then by position. It is from this idea of the non-equivalence of elements in a set that we get the notion of asymmetrical relations. Piaget now points out that classes and asymmetrical relations are complementary: "It is, he says "impossible to construct classes without relations which distinguish the elements, and to construct relations without classes which define the linked elements". They are, however, no more than complementary; there are no qualitative relationships that are at the same time both relations and classes, since "the class disregards differences, and the asymmetrical
relation disregards equivalences". At first the class is merely a synthesis of qualities, or combination of elements that are qualified but not counted. But the asymmetrical relationship, since it is a relationship between qualities, is essentially quantifying; and in that it distinguishes the elements instead of combining them, it prepares the way for number. When at stage III these operations of correspondence and seriation are composed logically (into a "groupment"), the child can deduce from them the corresponding numerical composition.

In fact, according to Piaget, number is possible "to the extent that the elements of a set are viewed no longer as equivalent or non-equivalent, but as being at the same time equivalent and non-equivalent". In other words, number is neither merely "a uniting class nor merely a seriating relation", but is both at the same time. It has however been noted that we cannot have on the qualitative plane a logical relationship that is both a class and a relation. What has to be done therefore is to eliminate the qualities and to treat each element in the set as a unit equivalent to the other. A cardinal number is therefore a class whose elements are conceived as units that are equivalent, and yet distinct, and thus capable of being seriated and ordered. And conversely, each ordinal number is a series whose terms, although serially ordered, are also units that are equivalent and can therefore be grouped in a
class. Piaget concludes "Finite numbers are therefore necessarily at the same time cardinal and ordinal, since it is of the nature of number to be both a system of classes and of asymmetrical relations blended into one operational whole". (157)

Chapter VII. It has been established that number is the product of class and asymmetrical relation. This does not imply that these come before number. This chapter will try to show that number is necessary for the completion of truly logical structures. Piaget does not derive class from number, nor consider the two as independent, but regards them as developing side by side in complementary fashion. Classes and number have an important common basis, namely the additive operation, which "brings together the scattered elements into a whole, or divides these wholes into parts". In number, however, the parts are homogeneous units, while the parts of a class are still only qualified classes and are united only by virtue of their common qualities. Now additive composition necessarily entails intensive quantification, in that there must be "more" elements in the whole than in any of its parts; and therefore, Piaget says, "the four essential determinants for any combination of classes, namely one, none, some, and all, clearly have a quantitative significance". The problem now is, can the quantitative relationships inherent in the process of additive composition be operated before the constitution of number?
Piaget goes on to describe a series of experiments whose aim was to study the additive composition of classes, and to bring out the element of quantification inherent in any addition either of classes or numbers. The general pattern of the experiments is that B is a set of objects forming a logical class definable in qualitative terms, and A a part of that set forming a sub-class also definable in qualitative terms. The problem put to the child is that of discovering whether there are more elements in B than in A. For instance, Class B might be a set of blue beads, most of them square, A1, but a few of them round A2. The child might then be asked "If you made two necklaces, one with all the blue beads and one with all the square beads, which would be bigger?" The linguistic approach must vary considerably, and the material itself may be flowers (B) in two sub-classes of bluebells (A1) and poppies (A2); or children (B) in sub-classes girls (A1) and boys (A2). In his preliminary questions the examiner must make sure that the child appreciates what is being talked about, e.g. bluebells as distinct from poppies; bluebells and poppies as together being a bunch of flowers. The child may be asked to count the number of elements in the classes and sub-classes. Three stages emerged from the various experiments:

Stage I. At this stage the child is not yet capable of understanding that the B classes will always contain more elements than the A classes. He may say for instance that a necklace of
square beads (A1) would be longer than one of blue beads (B), or that the teacher has more girls than children in her class, and this even after he has counted the classes and sub-classes of beads and children. It appears that as soon as the child is asked to think simultaneously of the whole and the part, he forgets the whole when he thinks of the part, and he forgets the part when he thinks of the whole. He cannot establish a permanent inclusion between the whole and the parts, and Piaget says that "It thus seems to be the relationship of inclusion which is the stumbling block for these children." (171) For these children, a whole is not yet a logical class entailing necessary relationships with the sub-classes, but is merely an "elementary schemata of assimilation or socratic aggregates" held together by merely qualitative participation; there is as yet no quantitative relationship between part and whole. The child knows that the "square beads" are also "blue beads", and that they therefore form part of the same whole as the round ones. But when he is asked to envisage simultaneously the square beads and the blue beads, which means thinking of the inclusion of the two classes, he is then unable to include in the class of blue beads those which he has just classed as square beads. This shows, according to Piaget, that qualitatively the child understands that one bead can be both square and blue, but that from the point of view of quantitative classification he cannot place the same
beads in the two classes simultaneously. The same applies to a class such as "children" with its sub-classes "girls" and "boys". The child will easily point out and count correctly the girls, and the boys, and finally all the children, but yet he will be unable to grasp that "girls" being included in the class of "children" will be smaller than the class. This then is the stage where the child is still incapable of the additive composition of classes, and is unable to handle the relationship of inclusion. He "replaces the nesting of classes by mere intuitive relationships between the sets in question". (174.)

Being intuitive, and dependent on perception, these relationships cannot result in any stable composition, and so Piaget finds on the logical plane the phenomenon common to all the stage I reactions in the numerical problems, non-conservation of wholes. In the numerical problems we have found that once the part is separated from the whole, it is no longer conceived in terms of the initial whole, but only in relation to the parts as they now face the child in the present situation. The parts, although they may be seen as deriving from an original whole, are definitely not regarded as belonging to a logically indestructible whole. Thus at stage I the whole, whether number or class, is not regarded as invariable, but as "varying in qualitative value according to the displacements of its parts".
Stage II. At this stage we have the intuitive discovery of the correct answer, the child achieving it by trial and error and not by immediate insight.

Stage III. Here the discovery is spontaneous and immediate. The children show right away their capacity to think simultaneously of the total class and of one of its parts. Thus they see that the part and the whole are defined by the same attribute, but that the whole also includes the remaining part. Younger children cannot do this because they are still on the plane of perceptual intuition, which is immediate and irreversible. The difficulty, Piaget argues, appears to come about in this fashion. Consider the problem of the wooden beads (Class B) with its two sub-classes of brown beads (A1) and white beads (A2). The child is asked to point to the necklace which could be made by the brown beads, and then the wooden beads. For the latter, most children at stage I would point to the white ones. Their reason for doing so might be something like "because the other girl has taken the brown ones". The reason for such response is that the child can in imagination take the brown beads to make one necklace, but when he has to construct mentally a necklace of wooden beads he disregards the brown ones as if they had already been expended, as no longer available. For the adult, Piaget points out, there is no difficulty here, since adult deduction is characterised by the capacity to construct all possible combinations by returning to the starting point and comparing several possible solutions simultaneously. The child on the other
hand regards his mental experiences as actual and having constructed one set mentally, he cannot mentally make another with the same material. "The irreversibility of the child's thought and representations prevents him from acquiring the power of decomposition that is necessary for combining analysis and synthesis, and therefore for the understanding of inclusions and relations". (179)

The psychological irreversibility can be expressed on the logical plane as follows. If the child is to conceive the parts in terms of the whole, he must be able "to compose simultaneously the two operations $A_1 + A_2 = B$, and $A_1 = B - A_2$ i.e. to carry out both the inverse and the direct operation". The two equations must form part of one operation, which from the logical structure of the equations would necessarily entail reversibility. When the child's thought is irreversible he cannot proceed from one of these operations to the other, which means that he cannot use operations as such. What such a child does, Piaget tell us, is "to replace a two-way mobile operational mechanism by successive static perceptions which it is impossible to synchronise and therefore to reconcile". (180) At the third stage, however, the child is relying on additive composition of classes when, as in the case quoted by Piaget, he makes such a statement as "the wooden ones and the brown ones are the same"; but the necklace "would be longer with the wooden ones because
there are two white ones as well". This is equivalent to saying
that \( B = A_1 + A_2 \), and \( A_1 = B - A_2 \). From the point of view of
logical multiplication, the child can now regard elements in the
part as being at the same time in the part and in the whole.

The point brought out so far in this chapter is that class
and number have a common mechanism, namely their additive and
multiplicative operations. In arithmetic, reasoning consists in
combining objects (numbers) by means of operations, so in
classification, reasoning consists in combining objects by means
of operations on classes, i.e. by logical addition, multiplication etc.

In summing up the position so far, Piaget refers again to
the two equations quoted above. These he says, constitute the
elements of any additive "grouping" of classes, and once they
have been acquired, a number of other logical compositions can
be made. For instance, if \( B \) is included in \( C \), and \( C \) in \( D \), then
\( B \) plus \( B' = C \), and \( C \) plus \( C' = D \), and the inverse operation will
follow, \( D - C' = C \) etc. The important point is that the various
equations which can be made, are "associative with respect to
addition and subtraction" so that in the world of concrete things
they imply a building-up to larger quantities or a reducing to
smaller quantities. But when we consider each term itself, however,
we find that e.g. \( A \) added to \( A \) does not give something quantitatively
new, but results simply in \( A \); \( A + A = A \). Let \( A \) be the "total class
of objects on the table." To add this class to itself is merely to
say that "the total class of objects on the table" is "the total class-
of objects on the table." In the concrete, although you may go on adding new objects to the "objects on the table" you don't affect the status of the assembly of objects as being the "class of objects on the table." You may, in other words, add objects to the class A and make it bigger, but you can't add the class A to itself and get twice the class A, because the class A is qua class something unique. The terms A, or B etc Piaget calls "identical operations". This is the fundamental difference between classes on the one hand in which iteration is disregarded, and on the other hand numbers, of which the main characteristic is iteration. It distinguishes, Piaget says, logical "groupings" from groups of integers, 1 plus 1 = 2 1 plus 2 = 3 etc.

Piaget considers finally how classes are to be transformed into numbers.

The essence of a class is not the number of elements it contains, but the sharing of common qualities by all the elements. One-one correspondence between two classes therefore means qualitative correspondence; it means that the two classes have the same hierarchial structure, the same classification, but nothing is implied as regards their respective numbers. In itself therefore class cannot give rise to number. Correspondence can only become quantifying when the "class" is disregarded, and the elements are regarded as "equivalent from all points of view simultaneously." In such a situation the elements of two sets could correspond
numerically term for term, and this would lead to a numerical definition. As we have seen, however, if we disregard the difference between, say, A and A', i.e. the class B, we arrive not at the number two, but only at a class of objects defined by the common attributes of A and A'. And of course if we try to escape from the dilemma by again taking account of the differences between A and A', these elements will no longer be equivalent. In order that the class B can be equivalent to the number "two", B must be the union of any pair of elements, A and A' or A and E', etc. In such a case, these objects will have lost their differences, and will have become a homogeneous class. "In a word", says Piaget "if we say that A plus A' = two objects, A plus A' plus B' = 3 objects, etc, we are regarding these elements as being so many units equivalent to one another and yet distinct, and this two-fold prerequisite cannot be reduced to the scheme of additive composition of classes without some further operation." (183) When we say that A plus A' = two beads, we are saying that A is any bead, A' is any other bead, but essentially a different one. The differences cannot be of colour, or any other qualitative difference, since this would bring us back to a schema of classification; the difference can only be of position; by "any other bead" we mean "placed next to", "coming after, the first one". And so, in addition to the inclusion of A plus A' = B, we require a principle of seriation. Seriation has already been analysed as
being an addition of differences, differing from addition of classes which in an addition of elements that are equivalent from one specific point of view. These two conditions then, according to Piaget, "are necessary and sufficient to give rise to number. Number is at the same time a class and an asymmetrical relation, the units of which it is composed being simultaneously added because they are equivalent, and seriated because they are different from one another". (184) The fusion of these two processes would be impossible in qualitative logic, as we have seen; but when you have the generalisation of equivalence and then the generalisation of seriation, numbers become possible. In number, a given unit, say the first one, is equivalent to the second, and if their order of enumeration is changed, the second becomes the first, and vice versa.

This analysis, Piaget concludes, shows why the processes of the additive composition of classes, seriation of relationships, and operational generalisation of number appear at approximately the same time, 6 to 7 years. The reason is that class, asymmetrical relation and number are "three complementary manifestations of the same operational construction applied either to equivalences, differences, or both together". The child's intuitive judgments have become mobile, and he has thus reached the level of reversible operations implying that he is now capable of inclusion, seriation and counting.

Such is the explanation of number, "a fusion of class and
asymmetrical relation into a single operational whole”.

Chapter VIII. In the previous chapter Piaget has discussed the problem of how the logical inclusion of one class in another presents systematic difficulties for a child because he cannot consider both the parts and the whole at one and the same time. Now the question arises of whether the same difficulty appears in the numerical field, so that additive composition of units into a number is at first made difficult for a child by an inability to coordinate the relationship of a given whole (e.g. the number 8) to its various parts (e.g. 4 and 4, or 2 and 6 etc).

Three methods are used to study this problem. In the first place a set of 8 beads are presented to the child in two groups of 4 and 4; and a second set of 8 presented in groups of 1 and 7. The problem is of course expressed in a suitable linguistic setting which will lend some realism to the situation for the child’s benefit. The problem for the child consists in deciding whether 4 + 4 consists of more or less sweets than 1 + 7. At stage I the child does not regard the two sets as equivalent. As in previous experiments he is guided by perceptual relationships which he cannot correct by means of operations. Thus he thinks that 1 + 7 is greater or less than 4 + 4 according to whether he concentrates on the 1 or the 7, although he knows that he has to compare the whole set 1 + 7 with the whole set 4 + 4. There is yet no integration
of the whole and part relationships, and his responses are similar in character to those of the child who says that there are more girls than children in the class. In the second stage the child starts by making the same type of response, but he gradually in the course of the questioning comes to see that although 7 is greater than 4, and that therefore 7 + 1 ought to be greater than 4 + 4, at the same time 1 is less than 4, so that these two differences may compensate each other. What is called for here is a double simultaneous comparison, and this leads him to think not only of the relation of parts to parts, but of parts to wholes. It is the principle of compensation, or equalization of differences, which is the important factor in the development of the operation. However, the quantification is still only intensive and does not entail exact numeration. It is complementary to the addition of classes, in which the sub-sets are regarded as having some qualitative individuality. Only when the sub-sets are no longer regarded in this way, but are thought of as units which can be "equated without being identified" can addition as a reversible operation become possible. To quote Piaget in full "All the elements in the various sets and sub-sets become units which are equivalent and distinct and are involved in operations, which at the third stage function automatically and immediately, without the need for preliminary intuitive coordination". So far as addition is concerned, therefore, this becomes possible when the items to be added are united into a
whole, and at the same time when this whole is seen to be quantitatively permanent no matter how its parts are distributed. Thus stage III is reached when "all the relations in question form an operational system such that the whole, which as become invariant, is the result of composition through addition of the parts, and that the relationships between the parts are univocally determined by combined additions and subtractions". (190)

In the second method by which Piaget studies the problem the child is given two unequal sets of counters, say 8 and 14, and he is asked to make them equal. The third method consists in giving the child a heap of counters and asking him to divide it into two equal parts. The two methods involve similar principles as the first one, and the stages of development are complementary.

In the final section of this chapter, Piaget emphasizes his belief that although very young children, say from 2 to 5 years, may 'add' by simple enumeration, there is no true addition without the construction of an invariant whole. More enumeration may, he says, lead to an "awareness of a succession of events, and a more or less vague feeling that the sets in question are exhausted or increased". Such primitive addition however remains global and intuitive. "Addition and enumeration are at first uncoordinated because, for the child, the perception of a figure and the perception of its elements, seen one after the other, have nothing in common". (201) When the stage of true intuitive correspondence is reached, there comes a certain progress in coordination, since the child shows that he can
now in a rough sense quantify the elements of one set by relating them in correspondence to another. As we have seen, however, this does not entail lasting equivalence, and a change in configuration destroys the colligation (i.e. the result of the additive process) of the sets. Finally, with operational correspondence we have complete coordination between enumeration and the process of collecting elements into a quantifiable whole i.e. the additive process: change of configuration is now understood as an irrelevant factor; the process of enumerating leads to a stable and quantified whole which can be split up into a variety of parts, re-assembled and always retain its numerical identity. The child's thinking is now reversible.

Chapter IX. Very briefly, this chapter is concerned with correspondence between several sets. The procedures outlined in Chapter III are used, but in this instance the child must not merely make a correspondence between two sets, but he must compose the relationship of equivalence between three or more sets. He must in other words show an understanding of the formal structure, $X = Y, Y = Z$ therefore $X = Z$. Piaget finds he can effect this composition only if he can already understand one-one correspondence, and in fact that understanding of the latter means that the child can at once compose several relations.

In his concluding note in this chapter, Piaget stresses the need to distinguish, both psychologically and logically, the
various forms of equivalence. In additive equivalence, for instance, the equivalence is in respect to one attribute (e.g., blue and pink flowers are equivalent in being flowers).

In multiplication equivalence on the other hand, two or more attributes will be involved (e.g., the fact of being flowers and of having a certain position in a series). As yet, however, number does not enter into these multiplication operations—it will only do so when "the classes in question shall be 'singular', as we saw in the transition from addition of classes to addition of number in Chapter VIII".

Chapter X. In his final chapter Piaget returns to his technique of water and variously shaped glass containers described in Chapter I. He now puts the following six problems to the child. A. That of conservation of continuous quantities, already studied. B. Spontaneous numerical measurement. The child is given two or more quantities of liquid in containers of different shapes, so that he cannot estimate their relative proportion by direct perception. He is then asked to say whether one of the quantities is equal to, greater than or less than, one or both of the others. He is given some empty containers which he may use as he pleases in solving the problem, and the aim of the experiment is to see if he will adopt for his use a definite unit of measurement. C. This is similar to the previous problem,
but this time a common measure is imposed by the instructions. Using the same containers as a unit of measure each time, the same quantity of water is poured into three containers differing in shape from each other. The same problem as in B is thus put to the child. Problems D, E and F are concerned with composition of relationships.

In the development of the notion of measure (Problems A, B, and C) Piaget declares of the first stage that "there could be no more striking evidence that measure is impossible without conservation of the quantities to be measured, for the very good reason that quantities that are not conserved cannot be composed". By the third stage the child is capable of measuring and making use of a common unit, and he is also successful with all the elementary compositions of Problems, D, E and F. In the solution of the last problem in particular, Piaget finds "a particularly explicit example of the process that takes place in all measurement, the very notion of a common measure depending on that of multiplication equivalence".

And so to his last paragraph, in which he restates finally his thesis -

"We thus find, as at the conclusion of Chapter IX, that while multiplication of classes and multiplication of relations are two distinct operations, the one bringing into correspondence terms that are qualitatively equivalent, and the other asymmetrical relations between non-equivalent terms, all that is necessary in order to make the terms of these relations equivalent and thus to fuse multiplication of relations and that of classes into a single operational whole, is that the differences shall be equalised. We then have multiplication of numbers. Once again, therefore
number is seen to be the synthesis of class and asymmetrical relation. This general conclusion is supported by the whole of our analysis of number in the preceding pages.
PART III. The responses of a group of normal children to some of Piaget's experiments

Section 1.

In spite of the great weight of theory and speculation in Piaget's books, they are all primarily experimental studies, in which the experimental findings are the basis for the theories which follow them. The book on number follows this pattern, and the principal aim of the present writer's own study is to repeat some of the experiments which Piaget has outlined in this book. One hears a good deal about the need for verification of Piaget's experiments. He is criticized for not stating the number of subjects who took part in a given experiment from which he has drawn a specific conclusion; for not giving precisely the age range of the subjects, and for not relating their performance to mental age as well as to chronological age. Up to a point, therefore, our purpose now is simply that of verification, together with a little filling out of the background which Piaget tends to leave rather vague. Do the children of our sample show the same general trends of development of pre-number concepts; do their ages at various points of maturation tally with those quoted by Piaget; how do their mental ages come into the picture; do they give qualitatively the same type of explanation as Piaget's
children? Some further points will be considered as the occasion arises. When Piaget questions a child to determine the presence or absence of a given concept or logical operation, he usually employs two or more tests as a means of probing into the child's mode of thinking. We shall therefore use the word 'experiment' to mean the application of one or more tests to determine the stage of development of an operation; each test will be described, but the results which will be tabulated will refer to the findings of the total experiment, not to individual tests. For instance in Chapter VII the problem of the additive composition of classes is studied by a number of tests in which the material used may be flowers, beads, or drawings of children; the important point is whether or not the various tests all indicate the same stage of development. Whatever the concept or operation under study, Piaget does not like to establish its presence or absence in a child's thinking on the result of one test only. Eight experiments were selected from Piaget's book, and these will be described in due course.

The children who took part in the present study were drawn from schools in the county of Cumberland.

1. School A, supplying 19 boys and 22 girls is an infant and
junior school (5 to 11 years) in a small industrial town. A high proportion of the children are considered by the authorities (medical, social and educational) to come from rather poor homes. The cultural background is usually very low, and many of the children are indifferently cared for - shabbily dressed and somewhat unwashed. This school itself is an antiquated building, but this is off-set by the delightful atmosphere of friendliness which pervades the place. The children talk spontaneously and easily to teachers and visitors. Teaching methods are good within the limits of overcrowding and staff problems.

2. School B, supplying 20 boys and 17 girls, is in a small market town. It is an infant and early junior school for children of 5 to 8 years, and is one of the "show" schools of the county. The building is new, the teaching methods are first rate, and it is rare to see a dirty or neglected looking child. Needless to say the children are friendly and happy.

3. School C, supplying 11 boys and 11 girls, is a rural school, for ages 5 to 11, in a fairly big village. Teaching methods are excellent, and as with most country schools, it is a fair mixture of rather poor children and those who are better off. Extremes at either end of the scale are not so common.
4. School D, supplying 11 boys and 15 girls, is similar to C, but in a very small village. It has two teachers as against the three of School C.

Thus in all 126 children went through the experiments which follow. Their chronological age range was 4:9 to 8:7 and the ages were distributed as follows:

Table 1. Distribution of chronological and mental age

<table>
<thead>
<tr>
<th></th>
<th>Up to 5:0</th>
<th>5:1-6:0</th>
<th>6:1-7:0</th>
<th>7:1-8:0</th>
<th>8:1 Upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chron. age</td>
<td>25</td>
<td>32</td>
<td>34</td>
<td>28</td>
<td>9</td>
</tr>
<tr>
<td>Mental age</td>
<td>17</td>
<td>38</td>
<td>31</td>
<td>26</td>
<td>14</td>
</tr>
</tbody>
</table>

It will be seen that the table also gives the distribution of mental age; for instance, of 126 children tested, 17 had mental ages on Termor Merrill L (1937) of 8 years or lower. In relating Piaget's test results to mental age, the writer is well aware of the objections to the mental age concept which have been put forward in recent years. Without therefore wishing to read too much into the ontological status of a "mental age", the results are quoted in order to give an approximate picture of the comparative maturation of pre-number concepts and a child's general cultural and intellectual development.
All the schools were able to provide that most essential need for an investigation such as this, a private room relatively free from noise. Normally each child underwent three sessions of testing as follows:— (1) The Terman Merrill Intelligence Scale L (1937); (2) the first four or five of the eight Piaget experiments selected for study; (3) the remainder of the experiments. In a few cases a fourth session might be required for checking previous results, or for assessing a child's arithmetical ability. Full testing time averaged about 1½ hours. Perhaps it should be stressed here that the emphasis was on thoroughness of examination rather than on trying to test as many children as possible. Piaget's interrogatory method does not lend itself to high speed testing; it is by Piaget's choice a time-consuming clinical method in which every chance must be given to a child to express subtleties and nuances of thought. The examiner must also try to record as much as possible of what the child says, and without a tape-recorder this inevitably slows up procedure. With very few exceptions, the children appeared to enjoy the session, and cooperation was excellent, even among the youngest. Only one five year old found the situation too much for her, wept bitterly after a few minutes
testing, and could complete satisfactorily only a few tests. A number of the older and brighter children—thorough-going Stage III types—showed unmistakable impatience at the simplicity of the questions, but as a rule these brighter ones appeared to find the test questions rather amusing in spite of being so easy for them to see through. They would justify and explain their answers with great triumph and vigour. On the actual interrogation itself, the questioning may be clothed suitably to the child's age and social maturity, and should always be friendly and conversational. A good example from Piaget is the following: "Have you got a friend? (Yes, Odette.) Well look, we're giving you, Clairette, a glass of orangeade (A1, \( \frac{3}{4} \) full), and we're giving Odette a glass of lemonade (A2, also \( \frac{3}{4} \) full). Has one of you more to drink than the other? (The same.) This is what Clairette does: she pours her drink into two other glasses (B1 and B2, which are thus half filled). Has Clairette the same amount as Odette? " This type of approach seemed to be quite helpful throughout the eight experiments with children of mental ages of 6 years or less; but with older ones the pretence and padding which it involved did not appear to be relished, and the children seemed to favour a much more direct approach, with the problem stated in simple straightforward terms.
When one comes to the crucial question in which the child is asked to make a judgment "Which has the most water?" "Which has the most beads?" "Are there more flowers or more glasses?" etc, the children's responses may have to be accepted with great caution. Children seem to follow the rule "When doubtful, repeat the last thing the man said", so it is better whenever possible to avoid disjunctive questions. Similarly, if the question is put in the form "Are there as many flowers as glasses", the child in doubt will tend to say "yes". The technique should therefore be to present each problem in several ways, if possible varying the form of the crucial question each time, and in this way one can be fairly sure in the end the true state of the child's thinking.

The eight experiments, together with their main findings will now be presented individually.

**EXPERIMENT I.** THE CONCEPT OF THE CONSERVATION OF CONTINUOUS QUANTITIES.

**AIM** (i) To determine whether this concept is an a priori structure or "innate idea" present in the child's mind from its first awareness of its environment; or whether it develops in the child's thought only after environmental contacts and experiences.

(ii) If the latter be true, to reveal the characteristic progress of this development.
A continuous quantity, such as liquid, or clay, is distinguished from a discontinuous quantity whose elements can be separated and counted, such as a quantity of beads. "Conservation" in this context means the recognition that the quantity of, say liquid, remains the same no matter how the shape or appearance of the liquid may be altered. As we have seen, Piaget stresses the idea that any scientific notion presupposes a set of principles of conservation, and the conservation studied here and in later experiments is the prerequisite of the notion of number.

APPARATUS

(i) Supply of water.

(ii) Glass cylindrical containers as follows:

Two large containers of the same size and shape, A1 and A2; similarly two smaller ones B1 and B2; four containers C1 to 4, smaller in size than B; one long narrow container D, a little taller than A.

METHOD

Piaget does not appear to have had a set sequence of presentation of the containers to his subjects, but in this study a fixed sequence was followed, although of course there was no set linguistic formula used in questioning the children. (a) The child was first presented with two containers A1 and A2, each holding the same amount of liquid and being about three-quarters full. The examiner must then establish that the child appreciates that the amount of liquid is the same; it may be necessary to direct his
attention to the identical size and shape of the containers, and to the levels of the water. With some of the younger children it was sometimes thought advisable to pour a little of the water out of A2, let the child see the difference in water level, and then slowly replenish A2 until the levels were once again the same. As one would expect, however, even this procedure made no impression on the few children in whom the concept of equality and inequality had not yet developed, at least as regards linguistic expression. The next step was to pour the liquid in A2 into B1 and B2, and the child was then asked if this liquid was still equal to the liquid in A1. (b) The water in B1 and B2 would then be poured back into A2, and the equality of A1 and A2 confirmed once again. A2 would then be poured into C1, 2, 3 and 4, and the child questioned as before. (c) From the four C containers the water was next poured into D. The water level of D was always appreciably higher than that of A1, and the child was asked to judge which of the two containers A1 and D contained the greater amount of water. (d) A fourth test was sometimes used in doubtful cases, where some further confirmation was wanted. Container B was filled about three-quarters full of water. The child was then asked to pour into A the same quantity of water as B contained. The amount he poured into A was observed, and he was questioned as to why he had poured in just that amount. (e) In some instances some of the
above tests might be repeated two or more times.

RESULTS AND COMMENTS.

1. Some general comments on the children's responses. It very soon became clear that one could not rely on one test alone to reveal a child's level of development. Quite frequently one would find a child giving a correct response to test (a), but failing in (b); or being correct in (a) and (b), and wrong in (c). Sometimes too a child would give inconsistent responses to the same test, being correct in say (b) on one occasion and wrong in it later during the same or a subsequent session. In a few instances this occurred even when the child had given a correct reason when his answer had been right. Rather more common were cases where the child's first response to one or more tests might be wrong, but as the experiment proceeded, understanding seemed to dawn, and he would come finally down on the correct answer with the right reasons. Children whose responses indicated that they understood the principle of conservation always gave a reason for their answers and invariably the right one. But the ones who were in doubt or complete ignorance were frequently unable to justify their attempts at a solution, their answers probably being for the most part sheer guess-work.

2. Piaget's results, he says, seem to prove "that continuous quantities are not at once considered to be constant, and that the
notion of conservation is gradually constructed by means of an intellectual mechanism which it is our purpose to explain. By grouping the children's answers to the various questions he then goes on to distinguish the three stages of development which the children's responses appear to reveal. These stages are already outlined in the summary of the book on number. The responses obtained in the present study undoubtedly support Piaget's findings, and we shall now re-state briefly the three stages of development, and at the same time comment on the responses.

Stage I. When the notion of conservation is absent, the child appears to think that the quantity of liquid increases or diminishes according to the size and number of the containers. One gets such responses as B1 and B2 having more water than A1 "because you've got two glasses and I've only got one". On the other hand another child might give the opposite answer, saying that A1 has more than B1 and B2 "because mine's a bigger tumbler". The problem of containers A and D was invariably answered at this stage by saying that D had more water than A, for instance "because it comes higher up here" (pointing to the level on D).

Stage II. The transitional stage is characterized by uncertainty on the child's part. Piaget refers to two transitional reactions which he believes deserves special note. A child may answer B1 + B2 = A1,
but when B containers are poured into C1, C2, C3 and C4, he then affirms that the C containers altogether have more liquid than A1, for instance "because there are lots of jars". Again, the child may accept the idea of conservation when the differences in level, cross-section etc are not too great; but when these are increased they fall back on notions of non-conservation. The same trends were found in the present sample. For instance, about three-quarters of our Stage II subjects had a steady idea of conservation for $A1 = B1 + B2$, but would contradict themselves repeatedly over the problem of $A1 = C1$.

Piaget states that Stage II is "not necessarily found in all children". This point could be decided conclusively only after a longitudinal study of a large sample of children, but Piaget gives no hint of whether this has been done or not. In this present study the small number of Stage II children suggests simply that for many subjects, probably the majority, the transitional stage is a comparatively short one. The existence of Stage II responses provides Piaget with the answer to a possible objection to his method. It might be argued that the absence of conservation is due to the child not properly understanding the question; instead of regarding the liquid as a whole, he limits his viewpoint to a consideration of the levels only, or the widths. However, the fact that a child may hesitate, and give the correct response when the differences are small, but
fail to assume conservation when the differences in shape are greater, suggests that he does in fact understand the question, that he sees some of the implications of conservation, but that he does not yet see them as necessarily binding under all conditions. As Piaget says, the child at this stage is "trying to take the two relations into account simultaneously, but without success, hesitating continually between this attempt at coordination and the influence of the perceptual illusions".

Stage III. In the stage of necessary conservation, Piaget says, the child states the invariance "as something so simple and natural that it seems to be independent of any multiplication of relations and partition". He quotes responses which suggest that the global comparison between the initial and final state of the liquid is in itself a sufficient basis for conservation, and that conservation seems to be "an a priori analytic deduction". Responses such as "It's still the same, because it always comes from the same glass", illustrate the point. In the present study the majority of responses of this type were given by children over 7 years (chronological). Piaget goes on to argue, however, that on the basis of a second type of response "we are compelled to recognise that the reasoning that leads to the affirmation of conservation essentially consists in the coordination of the relations...." The responses referred to are those in which the
child makes up his mind only after making due allowances for size and width of jar and levels of water; for instance with containers A and D (test C) one of his subjects said "In this one (D) we must put more because it is narrower, and in the other there is more room because it's wider". Piaget appears to accept the fact that since some children reveal the mechanism (of coordination of relations) in their responses, all children therefore arrive at conservation by the same process. Why should this necessarily apply to the children giving the a priori type of responses referred to above? Piaget would probably reply to this objection by saying that the a priori theory does not provide us with an explanatory mechanism to link the initial and final stage; and that since such a mechanism is revealed in some responses, it can reasonably be assumed to be implicitly operating behind all responses which show understanding of conservation. The child giving the a priori responses could if questioned more closely give the coordination of relation type of response. In the present study the opportunity was taken of selecting a dozen children who gave the a priori response, and interrogating them more closely. They were simply told that although they had given the correct answer, it was not the only correct answer, and they were to think carefully what other reasons could be given. Eight of them definitely went on
to give the coordination of relations type of response, while the remaining four gave up the struggle without much show of interest. Such a small sample does not of course prove anything, but it does suggest that on this point, as on many others, a more detailed analysis of responses and some attention to statistics would be of considerable value.

3. As has been already noted Piaget seldom tabulates his results, and in his book on number he does not supply tables showing at what age a given stage of development is reached, or when the child is ready to pass on to the next. Nor does he relate his stages of development to "mental age" as determined by a recognised intelligence scale. When he quotes an individual child's response, he always gives the child's chronological age, and as we have seen before, he states in his various works the approximate ages at which stages of development are reached. Criticism of Piaget on this point has already been discussed. For the moment the following tables and those relating to the remaining seven experiments) are offered as suggestive evidence about the behaviour of a sample of English children in the selected experiments. Basically speaking the tables will aim at answering three main questions:—1. Do the results of the present sample indicate that English children at least (but not all English-speaking children) pass through
the same stages of development of these concepts, and at about the same ages as the Swiss children whom Piaget studied. 2. When the children are classified according to their Terman Merrill mental ages, do the same trends of development appear? 3. And a third important question might be, are the children's responses affected significantly by social background and sex?

Table 2 classifies the children according to five chronological age groups, and shows the percentage in each age group who come within each of the three stages of development. Thus in the age-group 5:1 to 6:0 9% are at Stage I. The lower figure in each cell is the mean mental age of the children who make up the percentage of the cell in question. For instance the 9% children just quoted have a mean mental age of 5:11.

Table 2. (a) Percentage of children at each stage of development according to chronological age group. (b) Average mental age for children in each cell.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>88%</td>
<td>94%</td>
<td>47%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>5:2</td>
<td>5:11</td>
<td>6:2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>-</td>
<td>6%</td>
<td>10%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>5:10</td>
<td>-</td>
<td>6:8</td>
<td>7:0</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>4%</td>
<td>6%</td>
<td>47%</td>
<td>90%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>5:10</td>
<td>7:8</td>
<td>7:3</td>
<td>8:1</td>
<td>8:10</td>
</tr>
</tbody>
</table>
The age limit of the children whose responses Piaget quotes are as follows for each stage of development in the concept of conservation of continuous quantities:

<table>
<thead>
<tr>
<th>Stage</th>
<th>Age Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage I</td>
<td>4:0 to 5:6</td>
</tr>
<tr>
<td>Stage II</td>
<td>5:0 to 7:0</td>
</tr>
<tr>
<td>Stage III</td>
<td>6:6 to 7:2</td>
</tr>
</tbody>
</table>

We have however no indication of the mental level of these children.

The evidence of Table 2 is that the children are pretty solidly at Stage I before 6 years of age. During their 6th year there is it would seem a sharp spell of development when nearly more than 50% of them reach Stage III, but leaving still quite a large proportion at Stage I. By the time 8 years is reached, they have almost all passed on to Stage III. One should notice the small proportion who at any age are at Stage II. As we have seen, Piaget suggests that not all children pass through Stage II. Be that as it may, the present evidence certainly suggests that Stage II is of short duration, and this is not surprising when one recalls that Stage II is not characterised by absence of conservation, but rather by hesitation and confusion in using an operation which is already there. It requires only a short period of normal experience, for the child to use the operation in every situation when it can logically be applied. Looking at the column for children under 5 years of age, one might wonder why close on
100% are not at Stage I, particularly when 96% of 5 to 6 year olds are still at Stage I. The reason which suggests itself is that the Stage II and III children are of high mental age, which would explain their fuller development. So far as the Stage II children are concerned, this is the case, their average mental age being 5:6 and the range 5:6 to 6:3. The 4% at Stage III is in fact only one child who has a mental age of 5:10, and this mental age, as we shall see from a later table, can be associated with Stage III development. In other respects too the mean mental ages quoted in this table reflect the same general trends. For instance the children of 6 to 7 years who are at Stage I have a mean mental age of 6:2, but for those of them at Stage III, it is 7:3. The mental ages of all Stage III children are on the average above 7 years, but if we look at the individual mental ages we find that they range from 5:10 to 12:0, three of them being under 7 years.

In Table 3 the children are classified according to mental age, and it shows the proportion of children found at each stage for the various age groups.

Table 3. Percentage children at each stage of development, according to mental age.

<table>
<thead>
<tr>
<th>Mental age:</th>
<th>Up to 5:0</th>
<th>5:1 - 6:0</th>
<th>6:1 - 7:0</th>
<th>7:1 - 8:0</th>
<th>8:0 upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage I</td>
<td>100</td>
<td>71</td>
<td>64</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>-</td>
<td>12</td>
<td>6</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>17</td>
<td>50</td>
<td>82</td>
<td>100</td>
</tr>
</tbody>
</table>
Below a mental level of about 5 years, all children are at Stage I. Between 5 and 7 years the position seems to be that between 60 and 80% are at Stage I, and between 15 and 30% at Stages II or III. The sudden development into Stage III is again shown by the small proportion of Stage II children, and by the sharp rise from 6% Stage I at 6 to 7 years to 32% Stage III at 7 to 8 years. At this age only about 10% remain at Stage I or II. By the time a mental age of 8 years is reached it seems probable that close on 100% of children have reached Stage III.

In spite of the very positive trends shown by this Table, perhaps the most important finding (particularly from an educational point of view, as we hope to discuss later) is the evidence that a child may have a Terman Merrill mental age of 7 to 8 years, and yet still be at Stage I, and that below a mental age of 6 years a very small proportion may have reached Stage III. It is possible, of course, that larger sampling would show that the true proportions differ from those of Table 3.

**EXPERIMENT II. THE CONCEPT OF THE EQUIVALENCE OF ONE-ONE CORRESPONDENCE**

**AIM** To illustrate the development of the concept of the necessary equivalence of two sets of objects for which a one-one correspondence has been made; the objects in the two sets will be different, but will be functionally related to each other so that
to some extent the correspondence will be provoked or
suggested to the child.

**APPARATUS** Test a. A collection of small bottles (2 or
3 inches in height) and small glasses - about 15 of each is
an adequate number.

Test b. The same collection of glasses and a
bunch of 15 to 20 imitation flowers, all the same colour and
design, and with short enough stems to enable them to stand in
the glasses.

Test c. (Supplementary). The same collection of
flowers, and a handful of pennies.

Test d (Supplementary). Two sets of counters, each
set being of a distinguishing colour.

**METHOD.** Test a. A set of 8 bottles was placed, equally spaced
in a row, before the child. He was given then the collection of
glasses, and instructed to "take the same number of glasses as he
has bottles, so that each bottle will have its own glass." He
was not told to count the bottles. If he made a wrong
correspondence, for example trying to make 12 glasses correspond
to 8 bottles, an attempt was made by suitable questioning to let
him work his way towards a correct correspondence. For instance
he may have been asked "Is there the same number of glasses and
bottles?", and he would then be encouraged to add or subtract from
the bottles until the two sets were matched. If the child succeeded in making the correspondence, one of the sets was closed up into a compact group, while he watched, and he was then asked "Is there the same number of glasses and bottles?" If he said in effect "They are still the same", he was asked to say how he knew this. If on the other hand he said that one set had more or less than the other, he was asked to give a reason for this. The questions, let it be stressed again, were not stereotyped, but were repeated and varied according to the child's age and reactions. The procedure was repeated by making the sets correspond again, usually asking the child to do this, and then closing up the other set and questioning as before. It was found in this study that when the child was making his correspondence he usually placed the glasses opposite or beside the bottles. A few would lift the bottles and place them inside the glasses, and another small group, all of whom of course were Stage III children, merely counted out 8 glasses.

Test b. Using the glasses and the flowers, this was carried out in exactly the same way as the previous test.

Test c. The examiner gave the child a bunch of flowers, saying "Let's pretend that these belong to you. I would like to buy some from you, and I'll give you a penny for every flower you sell
to me". He placed the first penny before the child, and received in exchange a flower, which he placed opposite the penny. This was repeated until there were two rows, 8 pennies with 8 corresponding flowers. The child was then questioned to make sure that he agreed that there were as many pennies as flowers. The flowers were then closed up, and the child again questioned about the size of the two sets. Whatever response he gave, the two sets were made to correspond again, the pennies then closed up and the question put to him as before.

Test d. This technique was not employed by Piaget. A row of 8 red counters was placed before the child. He was told "These red counters are yours". A row of 6 green counters was placed opposite the red set, each green corresponding to a red, with the 7th and 8th red counters standing alone at the end of the red set. "The green counters are mine", said the examiner, and he went on to ask the child, "Who has the most counters, you or me?" If the child said that the examiner had the most, this part of the test was repeated, perhaps using fewer counters, or making a more obvious difference between the two sets, in order to see if the child was able to compare the two sets and make a judgment about their equality or their difference. If he continued to show confusion on this point, it was concluded that he was still
too immature to make any judgment of "more" or "less" or "the same as", and of course he was noted as being at Stage I. However, if the child correctly replied that he himself had the most, a 7th green counter was added to the examiner's set, corresponding to the 7th red counter. Again the child was asked who had the most. If his previous response had been correct, he invariably answered correctly that he still had the most. The 8th green counter was placed opposite the 8th red, and the question repeated. The child was most unlikely to make a mistake at this point if his two previous responses had been correct, but had this occurred, the examiner would have gone over the routine, varying it according to circumstances, in order to see if the two earlier responses were the result of chance rather than understanding. As it happens, in the present study no such instance occurred, and when the child's response showed that he recognised the two sets as now being equal, one set was closed up and the child questioned as in the previous tests.

**RESULTS AND COMMENTS**

1. Tests a and b were used as the primary method of assessing a child's stage of development for "provoked" correspondence. The two tests invariably confirmed one another. In a few cases which were doubtful, and in other cases when it was felt that
supplementary tests might be of special interest, tests c and d were added. Table 4, however, was constructed solely on the result of tests a and b. The children's responses were in line with Piaget's basic thesis that during a child's early years one-one correspondence is not a function of his thinking, and that even when the child can build up a one-one correspondence between two sets, this correspondence does not necessarily entail the notion of true and lasting equivalence between two sets. We have then the characteristic three stages of development which have already been outlined in the summary. Briefly they are as follows:

Stage I. The child is unable to construct a one-one correspondence. For instance, when he is asked to set out as many flowers as there are glasses, he may place opposite the glasses a row of flowers equal in length to the glasses, but of greater density, thus making 8 glasses equivalent to, say, a dozen flowers. In every case when a child made this type of response, the examiner asked him to count aloud the two sets. In a few cases the counting enabled the child to see his error, and such cases went into Stage II; but to the majority it made no difference, and they continued to base the equivalence of the sets on a perceptual comparison of the length of the rows. Clearly with such children the notion of lasting
equivalence is impossible, and even when the examiner made the
correct correspondence for the child, he would then respond to
the closing up of one set in the same way as children of Stage II.
Stage II. This is not merely a transitional stage of confusion
and uncertainty like the Stage II of Experiment I. The child has
now acquired a new mental skill which he can use correctly with
a high degree of reliability - he can, at this stage, make for
himself a correct one-one correspondence. This is probably the
reason why Stage II for discreet correspondence contains many more
subjects than Stage II for correspondence of continuous quantities,
and why as a stage it probably lasts longer. What the Stage II
child cannot do, however, is to realise that, having constructed
a correct one-one correspondence, and agreed that both sets
"are the same... as big as each other, have as many in one as in
the other" etc, a change in the configuration of one set makes no
quantitative difference to either of the sets. Thus, if the
examiner closes up one set, he will now assert that the unclosed
set has more than the closed one. Or he may, but this occurred
in only about 1½ of cases, say that closing up or bunching one set
made that set more than the other. When the test was repeated and
he was asked to count the two sets, this in a very few cases
enabled him to see through the problem, but in most cases it left
the child still believing that closing up a set made a quantitative difference to it. Very surprising responses were obtained from some children and a few will be quoted later. It is obvious however that at this stage the notion of correspondence is still based on factors of configuration (length, density, etc) and not on an appreciation of number as such. Although a child may have a correct idea of "how many", this notion, as a basis for quantitative comparison, is pushed aside by the old and well established habit of basing comparisons on global, perceptual, non-quantitative factors.

Stage III. The child now not only establishes a correct one-one correspondence as in Stage II, but he furthermore realises that once the sets have become equivalent through one-one correspondence, they remain so no matter what alterations are made in their visual appearance. Indeed for most of the mentally older children the step of making a one-one correspondence is by-passed, and the child goes straight to the counting out of 8 flowers to match 8 glasses. Approximately 80% of children with mental ages above 8 years did this, as against 40% of those mentally below 8 years. With a few of the Stage III children (about 8 was all time would allow) the tests a and b were repeated with sets numbering 20 to 25 elements: the increase in the number
of elements did not appear to make any difference to the speed and correctness of their responses.

2. In Table 4 the percentage of children at each stage for the five age-group is shown, together with the mean mental age for the subjects in each cell.

Table 4. (a) Percent. stage of development in C.A. groups. (b) Average M.A. for each cell.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage:</td>
<td>II</td>
<td>II</td>
<td>II</td>
<td>II</td>
<td>II</td>
</tr>
<tr>
<td>20%</td>
<td>6%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4:3</td>
<td>4:6</td>
<td>5:7</td>
<td>6:0</td>
<td>6:3</td>
<td></td>
</tr>
<tr>
<td>64%</td>
<td>72%</td>
<td>44%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5:7</td>
<td>6:0</td>
<td>6:3</td>
<td>100%</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>8%</td>
<td>22%</td>
<td>56%</td>
<td>7:1</td>
<td>7:11</td>
</tr>
<tr>
<td>6:0</td>
<td>6:7</td>
<td>7:1</td>
<td>7:11</td>
<td>8:10</td>
<td></td>
</tr>
</tbody>
</table>

The most clear cut results are: (1) that it is very rare to find a child of more than 5 years who has not reached Stage II; (2) that between 4 and 7 years a large proportion of children are still at Stage II; (3) that from 7 years almost all normal school children may be expected to have reached Stage III. As in all experiments described in this study, the spread of mental age in the sample has tended to blur the outlines somewhat when the results are tabulated according to chronological
age. In Table 5 however, the classification according to Terman Merrill mental age is more sharply defined.

Table 5. Percent. stage of development in M.A. groups.

<table>
<thead>
<tr>
<th>Mental Age:</th>
<th>Up to 5:0</th>
<th>5:1 - 6:0</th>
<th>6:1 - 7:0</th>
<th>7:1 - 8:0</th>
<th>8:1 upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage: I</td>
<td>48%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>52%</td>
<td>35%</td>
<td>44%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>15%</td>
<td>56%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Here we find that no child of mental age above 5 years remains at Stage I; and that from a mental age of 7 upwards they have all reached Stage III. We still have the large proportion of children between 4 and 7 years at Stage II, and in fact nearly 100% of 5 to 6 year olds are at Stage II.

**EXPERIMENT III.** THE COMPOSITION OF RELATIONS OF EQUIVALENCE.

**AIM** Given the following relations of equivalence, \( x = y, y = z \). \( \therefore x = z \), it is necessary to determine whether the child can use this logical structure before he discovers the mathematical notion...
of invariance of sets, seriation, etc.

**APPARATUS**
(1) A collection of small jars.
(2) Two sets of imitation flowers, each a distinguishing colour, say red and blue.

**METHOD**
A set of 10 jars was placed before the child. He was then asked to take as many red flowers as there were jars, one flower for each jar. If the child could not make the correspondence himself, it was done for him by placing a flower in each jar, and the fact noted. (The examiner will usually know if this step is going to be necessary, since the experiment usually follows straight on Experiment II). The flowers were then removed and placed in a bunch on the table in full view of the child. He was then asked to take as many blue flowers as there were jars, assistance again being given where necessary. When the correspondence had been made, and he had agreed that there were as many blue flowers as jars, the blue flowers were removed and placed in a rather more spaced out heap before the child. He was then asked "Are there as many flowers here (pointing to the red) as there?" (pointing to the blue). The test was repeated two or three times, on each occasion arranging the flowers differently as they were taken from jars.

**RESULTS AND COMMENT**

1. Piaget's thesis in brief is that the composition of relations
of equivalence "presupposes that understanding of the equivalence has already been achieved". In other words, before the child can arrive at the equation, "therefore \( x = z \)", he must be at Stage III in the problem of the lasting equivalence of one-one corresponding sets. We therefore do not expect to find a child who cannot appreciate the permanent equivalence of two corresponding sets, and yet is able to argue that since the red flowers are as many as the jars, and the blue flowers as many as the jars, therefore the red flowers are as many as the blue ones - no matter what arrangement the red and blue flowers may have. If a child should in fact give two such responses, it would tend to confirm the view that the comprehension of relations of equivalence is so general a mechanism that it requires logic only, and is independent of whether the thinking refers in its content to number, area, classes, etc. This, of course, is the view which Piaget rejects, and on the contrary he asserts that "formal structure will not be acquired at once, irrespective of its content, but will need to be re-acquired as many times as there are different contents to which it is applied". He quotes a few of his subjects' responses, which reveal a three stage development as follows.

Stage I. Here there is failure both in making a correct one-one
correspondence, and in composing the relations of equivalence. Even when the child was reminded that the red flowers and the blue flowers had both been shared equally among the 10 jars - one flower of each colour to each jar - the child still could not see a relation of equivalence between the two sets of flowers.

Stage II. As in Experiment II, the subject can now make a one-one correspondence, but invariance of sets is only understood as long as the one-one correspondence is observable to the child. When therefore the child has to consider the red and blue flowers at the same time, having recognised them as being each equal to the jar, he can appreciate their equivalence only when he can see the sets opposite one another and having the same perceptual character.

Stage III. This sees the transition from intuitive judgment based on non-numerical factors (of density, length of line etc) to the use of a logical operation. A number of Piaget's Stage III subjects gave responses which he believed clearly demonstrated the "reasonable" character of thinking at this stage; for instance, a child says "I keep on thinking all the time that we are putting them back in the vases." In the present study our records show that a small number of subjects, chiefly from Stage III, did in fact give such telling responses, but by no means all; probably the majority did not have a clear image of the process being reversed.
Reversibility however is not essentially the passing of a series of images through the mind, although in its early functioning there may frequently be a tendency for imagery to accompany an operation. The evidence from the responses of our children at least suggests, let us put it no stronger than that, that after a greater or less period of use, the operation takes the child straight to the solution with implicit rather than explicit reversibility. A small number of Stage III children were asked if they imagined they were putting the flowers back into the jars, set at a time, in order to judge if they were equal or not: all of them answered in the negative, adding, for instance, "I knew because there are 10 jars" - "it's easy, you know at once" - "well, they must be the same, they wouldn't all have gone into the jars unless", etc.

2. In Table 6 we have the results tabulated as in the previous two experiments, basing the classification first on chronological age and secondly on mental age.

Table 6

(a) Percent. stage of development in C.A. groups.
(b) Average M.A. for each cell.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5:4</td>
<td>84%</td>
<td>69%</td>
<td>33%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5:7</td>
<td></td>
<td>6:0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage: II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5:2</td>
<td>8%</td>
<td>13%</td>
<td>6%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6:6</td>
<td></td>
<td>7:0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6:0</td>
<td>8%</td>
<td>13%</td>
<td>61%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>6:7</td>
<td></td>
<td>7:4</td>
<td>7:11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7:0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The most striking difference between the results here and those quoted in Experiment II is that in the present experiment only a small proportion of children are in Stage II. In this respect Experiment III resembles Experiment I, but in the latter Stage II is a period of transition between absence of an operation and a more or less fully developed operation, a temporary unsteadiness pending the moment when the operation becomes completely integrated into its appropriate logical structure. Stage II of Experiment III, on the other hand, is similar in character to that of Experiment II, being the period when a distinct mental activity is achieved (making a one-one correspondence) and one would expect in both instances approximately the same proportion would be found at Stage II. Piaget unfortunately gives no tabulated results to guide one on what his findings were on this point. One may say here in anticipation that Experiment II is the only one which shows a high proportion of children at Stage II, and it may well do this largely because of the simplicity of the activity of making a one-one correspondence - in itself it is a simple almost mechanical type of activity, and in any case was here provoked by the characteristics of the material used. The tables bearing on all the experiments will show that in no other experiment except the second one did such a large proportion of
the children reach Stage II between a mental age of 5 and 6 years - in the other experiments a good many children were still at Stage I up to 6 years of age. In other respects Table 6, and likewise Table 7, is quite straightforward.

Table 7 Percent. stage of development in M.A. groups.

<table>
<thead>
<tr>
<th>Mental age:</th>
<th>Up to 5:0</th>
<th>5:1 - 6:0</th>
<th>6:1 - 7:0</th>
<th>7:1 - 8:0</th>
<th>8:1 upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage: I</td>
<td>100%</td>
<td>60%</td>
<td>44%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>-</td>
<td>11%</td>
<td>11%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>29%</td>
<td>45%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

It seems clear that few if any children with a mental age of less than 5 years can even begin to cope with the problem of composing relations of equivalence, and that after a mental age of 7 years virtually all of them can do so. The emergence of this operation is gradual during the years 5 to 7, but as the 7th year approaches, the child's understanding of the problem seems to dawn inevitably.
EXPERIMENT IV. THE CONCEPT OF MULTIPLE CORRESPONDENCE

AIM We have seen in Experiment III how there comes a stage of development when the child, understanding the equivalence \( x = z \) and \( y = z \), can compose these equivalences into a third one \( x = y \). The present experiment will show that this function can be generalised in the form of one-one correspondence between any number of sets; that is to say, since sets \( x \) and \( y \) each correspond to \( z \) in a one to one relationship, \( x + y \) together will correspond to \( z \) in a two to one relationship, three sets, \( w + x + y \) together will correspond in a three to one relationship etc.

APPARATUS

(a) A collection of artificial flowers of the same design and colour; it is helpful to have 40, although not all these will necessarily be used.

(b) A set of small glasses.

METHOD

This experiment follows straight on after Experiment III. Piaget's description of the experiment is very condensed, and it is probable that the following is a simplified form of his procedure. At the conclusion of Experiment III the two sets of flowers, \( x \) and \( y \), were put in one heap. The child was then asked
how many flowers would be in each glass if they were all put back in the glasses, with the same number of flowers in each glass. If he answered correctly, a third set of 10 flowers was put down with the others, and he was asked how many flowers would each glass contain now. He had also to give if he could a reason for his answers. He was then asked to suppose that he was going to put each individual flower into a small bottle, and to say how many little bottles he would require to take, so that there would be one bottle for each flower. His response was noted, and if it was correct he was asked how many small bottles would be needed for $4$, and then $5$, sets of $10$ flowers.

RESULTS AND COMMENTS

1. Piaget puts this experiment into a very complex setting, in which it appears as one of several experiments which aim at analysing the development of a complicated logical structure from a number of different angles - this point has been brought out in the summary of Chapter 10 in Part II of the thesis. The writer feels now that in rooting this experiment out of its context, he has deprived it of much of its value both as a means of corroborating Piaget's experimental work, and in studying the development of pre-number concepts or operations in the child. The more one gets to grips with Piaget's book on Number, the more one comes to realise that almost every chapter deals with a topic
sufficiently fundamental to provide ample scope in itself for any type of research project. However, for what they are worth, the following results were obtained.

The three stages are again to the fore. Stage I is the response of a child who does not understand that if $x$ and $y$ correspond simultaneously to $z$, then there are two elements, and not merely one, corresponding to each element in $Z$. When asked how many flowers $(x + y)$ would there be in each of the glasses $(z)$ if the flowers were shared equally and without remainder, he does not know. Nor can he say how many small bottles will be needed for all the flowers. Frequently he will say 10, the number of glasses, but he has no understanding of duplication, or triplication, etc. He merely makes an arbitrary estimate of the increase. It is not easy to know what Piaget means by Stage II in this experiment, and we therefore took the simple way out of classifying as Stage II all children who after several trials continued to show confusion of thought together with a limited understanding of the problem involved. In Stage III the child knows at once that if there are two sets of flowers, each glass will contain two flowers, and that 20 small bottles will be needed. He can furthermore show evidence of being able to generalize to 3, 4 or 5, once the 2 to 1 multiplicative relationship
is reached. To what extent the generalization goes beyond
5, Piaget does not make clear.
2. In Table 8 and 9 the results have been tabulated in the
same manner as the previous experimental findings.

Table 8. (a) Percent stage of development in C.A. groups.
(b) Average M.A. for each cell.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>100%</td>
<td>78%</td>
<td>47%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Stage: II</td>
<td>-</td>
<td>14%</td>
<td>29%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>8%</td>
<td>25%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 9. Percent stage of development in M.A. groups.

<table>
<thead>
<tr>
<th>Mental age:</th>
<th>Up to 5:0</th>
<th>5:1 - 6:0</th>
<th>6:1 - 7:0</th>
<th>7:1 - 8:0</th>
<th>8:1 upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>100%</td>
<td>100%</td>
<td>37%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Stage: II</td>
<td>-</td>
<td>-</td>
<td>18%</td>
<td>24%</td>
<td>8%</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>-</td>
<td>45%</td>
<td>76%</td>
<td>92%</td>
</tr>
</tbody>
</table>
Table 9 in particular suggests that we are dealing with a more difficult notion than in the previous experiment. No child with a mental age below 6 years has reached Stage II or III, and it is not until the mental age level of 8 years and over is reached that we approach 100% responses at Stage III. This would agree with the fact that we are concerned with a more elaborate operation than those dealt with in the previous experiments, hence its first appearance should not be contemporaneous with conservation, lasting equivalence of corresponding sets, nor the simple coordination of relations of equivalence.

**EXPERIMENT V SPONTANEOUS CORRESPONDENCE BETWEEN LIKE OBJECTS**

**AIM** To study the mechanism of correspondence when the child is compelled to find the correspondence of his own accord, without having it suggested to him by the characteristics of the material.

**APPARATUS**

(a) A quantity of loose counters.

(b) A set of 5 cards with counters pasted on them making on each card the following pattern:

1. A random scattering of 9 counters.
2. Counters in a circle
3. Forming a right angle
4. Forming a square
5. Forming a cross
METHOD

The first card was put before the child. He was then given the box of loose counters and asked to take out of the box the same number of counters as those on the card. Care was taken to see that his attention was not drawn in any way to the pattern - it was an estimate of number that he was to make. The child's response was noted. If he succeeded in putting out the correct number, he would in most cases have copied the pattern more or less correctly. When therefore he was satisfied that the correct number had been selected, his counters were brought together in a heap as he watched, and he was asked then if there were still as many in each set as before. As in the previous experiments with correspondence, it was necessary to check whether the child had the notion of the permanent equivalence of the two sets once they had been matched either according to pattern or numerically. The procedure was repeated with each card.

RESULTS AND COMMENTS

1. It had been hoped to make a separate analysis and to keep independent statistics for the 5 types of figures, on the assumption that varying characteristics and points of difficulty
of the figures might result in the children giving responses peculiar to the various figures. This, however, was found not to be so, and no differing trends in responses could be observed. The responses to all the figures are therefore considered simultaneously, which in effect is what Piaget himself has done. It may well be of course that a larger sample, and one more homogeneous with respect to age and range of intelligence, might show different reactions to the different types of figure - later studies may clarify this point.

The three stages in the development of the operation - its absence, its transition, and its completion - have already been described in the summary of the book on number. In the present study the children's responses followed much the same pattern as those quoted by Piaget. Tables 10 and 11 show the statistical pattern of responses which we have come to expect in the eight previous experiments. We find again that the crucial time is in the span of mental ages from 5:0 to 7:0.

Table 10. (a) Percent. stage of development in C.A. groups. (b) Average M.A. for each cell.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>72%</td>
<td>40%</td>
<td>21%</td>
<td>7%</td>
<td>-</td>
</tr>
<tr>
<td>I</td>
<td>5:1</td>
<td>5:4</td>
<td>6:0</td>
<td>7:0</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>24%</td>
<td>26%</td>
<td>23%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>5:8</td>
<td>6:2</td>
<td>6:7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>4%</td>
<td>31%</td>
<td>56%</td>
<td>93%</td>
<td>100%</td>
</tr>
<tr>
<td>III</td>
<td>5:10</td>
<td>6:8</td>
<td>7:4</td>
<td>8:11</td>
<td>8:10</td>
</tr>
</tbody>
</table>
Table 11. Percent. stage of development in M.A. groups.

<table>
<thead>
<tr>
<th>Mental age:</th>
<th>Up to 5:0</th>
<th>5:1 - 6:0</th>
<th>6:1 - 7:0</th>
<th>7:1 - 8:0</th>
<th>8:0 upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>100%</td>
<td>50%</td>
<td>10%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Stage:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>-</td>
<td>29%</td>
<td>20%</td>
<td>6%</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>21%</td>
<td>70%</td>
<td>91%</td>
<td>100%</td>
</tr>
</tbody>
</table>

EXPERIMENT VI. THE CONCEPT OF ORDINAL CORRESPONDENCE

AIM. When two sets of similar objects (e.g. a set of red counters with a set of blue counters) are made to correspond, the elements may be arranged in any order, and the significance of the correspondence is primarily cardinal - i.e., concerned with the number itself of the elements in each set. The ordinal aspect (i.e., concerned with position in the set, first, second, etc) comes out more clearly when the elements of the sets differ from each other in a manner which makes them capable of seriation, thus making it necessary that one specific element will be first, another second, and so on.
The aim of this experiment was to study the development of the operation of seriation and ordinal correspondence between two sets, each set consisting of elements which were capable of being seriated according to a specific characteristic of each set, and which when taken together allow of ordinal correspondence.

MATERIAL

1. A series of 10 drawings of small boys, the first one measuring 26 cm, and the tenth one being just over half that size, 14 cm, and the intervening ones become progressively smaller in steps of 1 cm.

2. A series of drawings of hoops, the largest having a diameter of 16 cm, the others decreasing in steps of 1 cm down to the smallest, measuring in diameter 9 cm.

METHOD

The two series were presented in five different ways as follows:

1. The boys and hoops were presented to the child, the sets not being in any order. The child was told that the boys were going out to play, and that each boy was going to take with him his own hoop. "So you must arrange the boys and the hoops so that each boy
will quite easily find his own hoop." In this way the correspondence was suggested to the child, but without any specific reference to size. We wished to find out when the child would see the point of differing size for himself, and when and how he would work on it. The child's success or failure was noted. If he failed to seriate and correspond the two sets, this was done for him, drawing his attention now to the differing sizes, in order that the subsequent problems could be presented.

(2) When the correspondence had been made, the sets formed parallel lines with the elements opposite one another, smallest boy to smallest hoop, etc. The examiner now closed up one set, the hoops for instance, so that they no longer lay directly underneath their corresponding boys, although the two sets still remained in parallel lines beside each other. Pointing to one of the boys, the examiner now asked "Can you show me the boy's hoop?" The child's response was noted, and also how he arrived at it - e.g. by counting from one end of each set, or by sheer guess-work. This was repeated several times, selecting different elements in the sets, until it was known whether the child could recognise and act on the series when the corresponding elements no longer lay beside one another.
(3) The sets were reversed, so that the smallest hoop was under the largest boy, and so on. The child was then questioned as in (2) above.

(4) One series, say the boys, was left seriated in position; the other was disarranged. The examiner pointed to a boy, say number 4, and asked the child to find this boy’s hoop.

(5) Finally, all the elements of the two series were mixed together. The examiner selected, say, the 6th boy, handed it to the child, and said that this boy was going for a walk, together with all the boys who were smaller than he; the child had to find this boy’s hoop, and also all the hoops of the smaller boys.

RESULTS AND COMMENTS

1. From these five questions, Piaget derived three problems which the subjects were required to solve:

(a) that of constructing from scratch a serial correspondence (question 1): (b) that of recognising and actig on a serial correspondence when it is no longer perceptually obvious (questions 2 and 3); this entails making the transition to ordinal correspondence; and (c) that of reconstructing the ordinal correspondence when the intuitive series are destroyed (questions 4 and 5).
The stages of development for each of the three problems are as follows:-

Problem A:— Stage I. Global comparison: No exact seriation or one-one correspondence. In the present study the child tended merely to pick out one of the larger boys, then to select a large sized hoop; if the two matched correctly, it was purely by chance. A roughly smaller boy would be chosen next, accompanied by a chance selection of the corresponding hoop; and so on. Discrepancies in the series did not worry the children at this stage. If one pointed out that here was the biggest boy in the middle of the line, some children would move him and his hoop up to the head of the row, others would merely shift him up one or two places. They showed no precision in their concept of a series, but could construct their series only from the "more or less big to the more or less small", without any appreciation of individual links in the serial chain. Nevertheless they did spontaneously, without prompting from the examiner, try to match the boys and hoops according to the concept of "big to smaller". "Global" describes this reaction very succinctly. On the other hand, some of those with mental ages around 4 to 4½ years merely selected a boy and a hoop at random, without even the slightest understanding of the "large to small"
seriation of the children a little older. One therefore could subdivide Piaget's Stage I into Ia and Ib, the former showing no idea of the "big to small" concept, the latter having a rudimentary awareness of it, and a realisation that it is worth while trying to apply it in this problem.

Stage II. At this level the child can make the two series, and make them correspond, without help from the examiner. He succeeds in doing it, however, only after a certain amount of trial and error, comparing the elements by small groups, and in a somewhat painstaking way building up the series. One assumed from his behaviour that he does not yet apprehend the series, either each one singly or the two together as a whole, but tends still to see them in a fragmentary way.

Stage III. Here, however, the child is all the time considering the relationship between all the elements; for instance, he may start with the biggest of all, the biggest of those that are left, and so on. He is much more systematic and more free from trial and error than those at the previous stage. Problem (B). Stage I. This child loses all idea of correspondence as soon as one of the series is displaced and the elements are no longer opposite one another. The condition
of the elements being opposite to each other is the only guide which he knows to solving the problem of "which is his hoop", and when the guide is lost, there is nothing for him to do but to guess, which he proceeds to do with supreme confidence.

Stage II. When the examiner points to a particular boy, e.g. number 6, and asks the child to "point to this boy's hoop" (the line of hoops of course being closed up), the child tries to use his knowledge of counting to find the correct hoop. He will point to Boy 1, then Hoop 1, then Boy 2, and Hoop 2, and so on. He may sometimes answer correctly using this or similar techniques, but he will certainly make mistakes, either omitting an element or counting it twice. Furthermore, the closing up of the elements of one set may cause some of the children at this stage to think that the closed up set now has fewer elements - and this will obviously make even more uncertain their understanding of the seriation.

Stage III. Here the response is made without any hesitation, and the reason given often shows the complete insight which the child now possesses; e.g. "Well, that's the third biggest boy you are pointing to, so he'll have the third
biggest hoop". "It's the same really, you've just turned
them (the hoop) round:" "You start from that end and that
end now, and you can count up to 4 for the boy and up to
4 for the hoop?"

Problem (C). The stages here follow the same pattern as for
Problem (A).

2. The mechanics of Problem (C) appeared to be different for
a large number of children. Most of those with a mental age
below 8 years required considerable help from the examiner. They
became quite muddled in their organisation of the material. It
was felt that the practical difficulties of this part of the
experiment in many instances prevented the child from
demonstrating whether he understood the seriation or not, and
it was finally decided that the results would not be recorded.

It was evident, as subsequent tables will show, that
Problem (A) was considerably more difficult than Problem (B).
The difference between the two problems is fundamentally this,
that whereas in (B) the child already knows of the seriation
and is asked to solve a problem in which he must use the notion
of seriation as part of the given data, in (A) he must find the
seriation for himself and then proceed to act on it. With a
large element of the unknown in it, the greater difficulty of (A)
is not unexpected. As work on the experiment progressed, however, it was felt that the dimensions of the material may well have constituted an added difficulty to the child's task. Piaget used wooden dolls, but does not specify their size - all he says is that the largest was at least twice as big as the smallest. Our drawings of the boys followed this principle, and it was therefore perfectly obvious that there was a decreasing order of size among the boys, as well as the hoops, so that the material should have suggested the idea of seriation as readily as Piaget's. However, it was when a child had to select between two of the drawings which were next to each other in the series that difficulties appear to have arisen; for instance, the child might be handling the drawing of the third boy, and he would then show unusual uncertainty and hesitation in trying to decide whether number 4 or 5 was the next in size. One had a strong impression that many of the children instead of persisting gave up the effort of making a correct comparison, and so technically failed to seriate, when with more obvious material they might have seriated. It was decided that these children should be regarded as being at Stage II.

In their response to Problem (B), it was noted that very few children indeed succeeded in one part of the test, and
failed in the other. On the face of it, therefore, there seemed to be no difference in difficulty between the two parts. Those who did fail in one and succeed in the other were classed as Stage II. The other Stage II children were those who, irrespective of which part of the test they were working on, would give a number of wrong answers. Clearly some children would jump too hastily to the answer and get it wrong, and it would be absurd to class a child as Stage II for one or two over-enthusiastic slips. Pretty lenient scoring was therefore allowed. Usually four questions were allotted to each part of the test, and a child might get two wrong in each part and yet still be classed as Stage III, provided his other responses were not merely correct in themselves, but were accompanied by satisfactory explanations and evidence that he understood and was using the seriation before him. Among all the experiments, Problem (B) of Experiment VI appeared to be the happiest one for very many of the children, who quickly made it into a game. This may explain why so many exuberant wrong answers were given by children who otherwise gave every indication of knowing what the problem was all about.

3. Tables 12 and 13 give the results for Experiment VI (A), and Tables 14 and 15 for VI (B).
Table 12.  
(a) percent stage of development in C.A. groups.  
(b) Average M.A. for each cell.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>96%</td>
<td>85%</td>
<td>52%</td>
<td>36%</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>5:3</td>
<td>5:10</td>
<td>6:4</td>
<td>7:4</td>
<td>9:2</td>
</tr>
<tr>
<td>Stages:</td>
<td>II</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4%</td>
<td>15%</td>
<td>42%</td>
<td>28%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5:10</td>
<td>6:6</td>
<td>7:4</td>
<td>7:9</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>-</td>
<td>6%</td>
<td>36%</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8:6</td>
<td>9:0</td>
<td>8:9</td>
</tr>
</tbody>
</table>

Table 13. Percent. stage of development in M.A. groups.

<table>
<thead>
<tr>
<th>Mental age:</th>
<th>Up to 5:0</th>
<th>5:1 - 6:0</th>
<th>6:1 - 7:0</th>
<th>7:1 - 8:0</th>
<th>8:1 upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>100%</td>
<td>100%</td>
<td>45%</td>
<td>18%</td>
<td>23%</td>
</tr>
<tr>
<td>Stage:</td>
<td>II</td>
<td></td>
<td>35%</td>
<td>49%</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td></td>
<td>20%</td>
<td>33%</td>
<td>52%</td>
</tr>
</tbody>
</table>
Table 14.  
(a) Percent stage of development in C.A. groups.  
(b) Average M.A. for each cell.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>92%</td>
<td>68%</td>
<td>48%</td>
<td>40%</td>
<td>-</td>
</tr>
<tr>
<td>Stage:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>8%</td>
<td>21%</td>
<td>18%</td>
<td>18%</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>11%</td>
<td>40%</td>
<td>68%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 15.  Percent stage of development in M.A. groups.

<table>
<thead>
<tr>
<th>Mental age:</th>
<th>Up to 5:0</th>
<th>5:1 - 6:0</th>
<th>6:1 - 7:0</th>
<th>7:1 - 8:0</th>
<th>8:1 upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>100%</td>
<td>88%</td>
<td>35%</td>
<td>4%</td>
<td>-</td>
</tr>
<tr>
<td>Stage:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>-</td>
<td>-</td>
<td>40%</td>
<td>25%</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>11%</td>
<td>26%</td>
<td>74%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Looking at the results based on mental age, we see that Problems (A) and (B) are seldom if ever solved with a mental age below 6 years. After that age, however, success in Problem (B) comes away quite quickly, and one is unlikely to find a child who fails in it after reaching a mental age of 8 years. Progress in Problem (A) is considerably slower. In the 7 to 8 year level there are still 50% at Stage I, and among the children above a mental age of 8 only a little over 50% are clearly at Stage III. The difference in the rate of development of Problem (A) and (B) is probably due in part to the sheer difficulty of the mechanics of A. To demonstrate one's understanding of seriation by shuffling among two disordered series of drawings is intrinsically more difficult than merely to point to various parts of the same two series when laid out in order before one. The operational structure which enables a child to seriate will not suddenly appear in a child, so that all at once he finds that he can seriate every type of material in all circumstances. The period of complete absence of the operation will end when the child can, around 6 years, begin to seriate simple material, but fails with harder material whose perceptual qualities make it intrinsically more difficult to work with. The full attainment of Stage III
strictly should come only when the child can seriate any type of material irrespective of its appearance, because understanding should, as Piaget likes to say, "have triumphed over perception". That may be so, but in his study of Number development, he does not appear to stretch his children to their limit in their triumphal assaults on perception, but contents himself by allowing a child to be classed as Stage III when he can seriate successfully and apply the concept of seriation to relatively simple material of the child's daily life. Obviously what has just been said would apply with equal force to all the operations studied in his book and reviewed in this thesis. Piaget nowhere in his experiments is concerned to take his subjects to the point where perceptual factors are finally and irrevocably helpless against understanding expressed through reversible operations. Such a stage is however likely to be reached at some point in the period when concrete operations are becoming consolidated.

**EXPERIMENT VII. THE ADDITIVE COMPOSITION OF CLASSES.**

Piaget's thesis is that "class" and "number" are complementary and develop side by side. Numbers and classes have a common basis, namely the additive operation which...
together the scattered elements in to a whole". Every additive composition entails certain quantitative relationship, "one - none - some - all? The problem is whether "the quantitative relationships inherent in the inclusion of the part in the whole...be manipulated operationally before the constitution of number."

**MATERIAL**

Test 1. A set of 16 wooden beads (Class A): 12 of the beads are round (Class B) and 4 of them are cubes (Class C).

Test 2. A set of 10 flowers (A): 8 of them red (B) and 2 of them blue (C).

Test 3. A card with 8 children drawn on it (A): 6 of them drawn in a line are girls (B); and the remaining 2, shown on a separate group from the girls, are boys (C).

**METHOD**

The questioning for each of the tests followed the same pattern. For instance in Test 1 the child was shown all the beads, both square and round. He was told "Here are a lot of wooden beads: let's sort them out: these ones are round (placed in a line in front of child), and these are square (placed alongside the round ones.)" The question was then put to
the child "Are there more wooden beads or more round ones?"
If he replied that there were more wooden ones, he was asked
to point to the round beads, then to the square ones, and
finally to show the examiner all the wooden beads. The
question was then repeated, possibly changing its form a
little. Piaget allowed some at least of his children to
imagine that they were making a necklace of the beads
followed by the question as to which would then make the
longest necklace, the wooden beads or the round ones. He
also had various other suggestions for using the imagination
in seeking the solution to the problem, thus hoping to leave
no doubt as to whether the child could or could not employ
the operation. When in the present study a child was
experiencing difficulty, he was helped in much the same way,
but the examiner formed little evidence that visualizing a
specific situation (e.g. making a necklace of beads) helped
significantly. The physical presence of the beads, (and of
the flowers and drawings of children) which he can handle if
desired, seems to give the child all the mental props he needs
to demonstrate at what stage of development his thought has
reached in this particular problem. The questioning was
repeated in the same fashion with the flowers and drawings of children. The child would be given a "bunch of flowers", and then told that the red ones were poppies and the blue ones violets. He was asked whether he had more flowers or more poppies, or "a bigger bunch from all the flowers, or from the poppies". For the drawings of the children he was told that the children were all in the same class, "Has teacher more girls or more children in her class?" And as with the beads, he could be asked to point to "all the flowers....all the children" and then to the several parts thereof.

RESULTS AND COMMENTS

The three stages have already been outlined in some detail. It will be remembered that Stage II covered those children who in the course of the examination made an intuitive discovery of the correct answer, more or less by trial and error. In the present study only a very small proportion of subjects, in fact only two children, appeared to do this, and there were quite a number (see Table 16) who fell into Stage II through their inconsistency of responses; that is to say, they might reply correctly to the question on flowers, but wrongly to that on beads. When such an inconsistency occurred, the questioning was always repeated after an interval (anything
from half an hour to a few days according to force of circumstance), and if on this second occasion the child gave correct responses to all the tests, he was graded as Stage III. It was noticeable how confident most of the children were with their answers - they came out emphatically with a right or wrong answer, and they gave one the impression also of being able to resist suggestion more successfully than in the other experiment. As a minor supplement to the experiment the examiner from time to time took a clear Stage I child, and carefully explained to him just why his answer was wrong, and what the correct answer should have been, using the three types of apparatus. Nineteen children were "coached" in this way and were retested after an interval of three to four days. In every case the child on being re-tested was still at Stage I. It seemed pretty evident too that when they were questioned immediately after the working session, although they could usually give the correct response then, this was mainly because they were repeating to the examiner what in effect they had been told to say, and there was little or no evidence of temporary insight into the problem. Clearly, however, the question of to what
extent and at what age a child who gave negative responses can be taught how to solve this problem requires pursuing with a large sample using more controlled questioning and material.

Tables 17 and 18 give the statistical results.

Table 16. (a) Percent stage of development in C.A. groups. (b) Average M.A. for each cell.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>96%</td>
<td>92%</td>
<td>85%</td>
<td>45%</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>5:2</td>
<td>5:11</td>
<td>6:11</td>
<td>7:7</td>
<td>8:0</td>
</tr>
<tr>
<td>Stage: II</td>
<td>-</td>
<td>-</td>
<td>15%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7:2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>4%</td>
<td>3%</td>
<td>-</td>
<td>55%</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>5:10</td>
<td>6:8</td>
<td></td>
<td>8:5</td>
<td>9:2</td>
</tr>
</tbody>
</table>

Table 17. Percent stage of development in M.A. groups.

<table>
<thead>
<tr>
<th>Mental age</th>
<th>Up to 5:0</th>
<th>5:1 - 6:0</th>
<th>6:1 - 7:0</th>
<th>7:1 - 8:0</th>
<th>8:1 upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>100%</td>
<td>87%</td>
<td>84%</td>
<td>54%</td>
<td>42%</td>
</tr>
<tr>
<td>Stage: II</td>
<td>-</td>
<td>-</td>
<td>5%</td>
<td>20%</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>13%</td>
<td>13%</td>
<td>26%</td>
<td>58%</td>
</tr>
</tbody>
</table>
The most evident trend is that most children remain at Stage I until a mental age of about 7 years is reached. During the mental age levels of both 7 to 8 years, and 8+ years, they are spread widely over the three stages, and the experiment did not study far enough at the mental age scale to show at what point one gets virtually 100% response at Stage III. This in fact is the most difficult problem in all these eight experiments to be put to the children. It is clear however that the additive composition of classes does not precede the composition of number.

**EXPERIMENT VIII. THE ADDITIVE COMPOSITION OF NUMBERS.**

**AIM** In the field of number it is the arithmetical union of parts into a single whole which constitutes one of the fundamental operations of number, namely addition. We have to ascertain whether this additive composition of parts into a whole gives rise in the case of number the same sort of difficulties as those of inclusion of classes in a total class. It could be, Piaget suggests, that "the difficulties of inclusion are exclusively logical".

**APPARATUS**

1. A quantity of beads
2. A quantity of counters similar in shape and size.
Piaget used three techniques to study the development of this operation, but in this study there was time only for two of these to be applied as follows:

Test 1. Here we wished to see if the child realized that a whole remains constant irrespective of the various additive composition of its parts. As in Piaget's test, two sets of eight beads were presented to the child, using however a slightly different verbal framework. The examiner asked the child to suppose that teacher is going to give him and his friend John (or whoever the child nominates as his friend) eight sweets. "Let's suppose she gives you four in the morning with your milk (placing 4 beads before the subject) and four after lunch (placing the second four alongside but distinct from the first). Now John did not feel like sweets in the morning, so he has only one sweet in the morning and seven in the afternoon?" (these are placed well away from subject's sweets, first 1 bead and then the 7). The child was then asked to compare the two sets, 4 + 4 and 1 + 7, and to say whether he or his friend would eat the most sweets at the end of the day.

Test 2. A heap of counters, say 18, were placed before the child, and he was asked to divide the heap into two equal parts;
e.g. "Make this big heap of counters into two smaller ones, one for you and one for me, so that we both have exactly the same amount." When the child had done this, whether correctly or not, he was asked "Have we both got the same?" and if he replied that one had more or less than the other, he was urged to make them equal. When he had finally declared the two heaps to be equal, he was asked to explain how he knew they were equal.

RESULTS AND COMMENTS

As we would expect from the results of previous experiments, and numbers II, III and IV in particular, three stages of development emerge, and these have already been outlined in the summary. This experiment demonstrates that although a child may be taught to repeat the formula $2 + 2 = 4$, $2 + 3 = 5$ etc, there is no true insight into the logical structure and content of these statements as long as the child's thinking is dominated by non-operational, intuitive judgments characteristic of Stage I in this and the other experiments. Tables 18 and 19 show that in the present study a surprisingly small number of subjects were classified as being at Stage II. This may be due to inadequate presentation and questioning on the examiner's part - this experiment was always the last to be
presented. On the other hand the relative simplicity of the material used in the procedure may have diminished the tendency of some children from becoming confused and doubtful about their own responses.

Table 18. (a) Percent. stage of development in C.A. groups.
(b) Average M.A. for each cell.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>72%</td>
<td>78%</td>
<td>44%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Stage: II</td>
<td>4%</td>
<td>-</td>
<td>6%</td>
<td>10%</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>24%</td>
<td>22%</td>
<td>50%</td>
<td>90%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>5:10</td>
<td>6:9</td>
<td>7:3</td>
<td>8:1</td>
<td>8:10</td>
</tr>
</tbody>
</table>

Table 19. Percent stage of development in M.A. groups.

<table>
<thead>
<tr>
<th>Mental age:</th>
<th>Up to 5:0</th>
<th>5:1 - 6:0</th>
<th>6:1 - 7:0</th>
<th>7:1 - 8:0</th>
<th>8:1 upwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>70%</td>
<td>83%</td>
<td>62%</td>
<td>9%</td>
<td>-</td>
</tr>
<tr>
<td>Stage: II</td>
<td>6%</td>
<td>-</td>
<td>8%</td>
<td>9%</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>24%</td>
<td>17%</td>
<td>30%</td>
<td>82%</td>
<td>100%</td>
</tr>
</tbody>
</table>
NOTE It will be clear that the percentages quoted in all these tables are subject to a wide margin of error, particularly in those cells which record the collective result of only a few children. Practical considerations made it out of the question for the writer to cope with a larger sample. The tables are presented as merely indicating general trends, and they may be a useful guide to those who wish to obtain more precise statistical validation of Piaget's work.

Section 2. In all his works Piaget gives numerous quotations from his subjects' responses, and indeed in sheer bulk these quotations form a substantial proportion of his books, and they are of undoubted value. Such long extracts from case records would considerably over-load this thesis, and instead there will be presented in this section fairly complete reports on the responses of three subjects. These children are quite characteristic of the others, and give an adequate idea of the qualitative aspects of the responses obtained from the entire sample. In a later section similar reports on the responses of mentally retarded subjects will be presented.


Anne did not display any marked interest in the various pieces of test material as they were put before her. She tended rather to sit quietly with a passive expression on her face waiting to be told what to do. She was at the time in a nursery
class of 3 to 5 year olds and possibly was a little over-awed by the situation. Otherwise, however, she cooperated well and gave every appearance of attending to the questions put to her. It should be noted that with most of the younger children — generally with a mental age below 6:0 years — the examiner found it helpful to let the child inspect the material by handling it if necessary before putting the test questions. For instance, the artificial flowers were a source of curiosity, and many of the subjects were clearly attracted by their appearance and would ask who made them, what were they made of, and so on. All this interest and curiosity had to be worked off to some extent, otherwise it might have interfered with their concentration on the problem.

In Experiment I, when the liquid in A1 and A2 had been shown to be equal to her, it appeared very doubtful if she understood the notion of one thing being equal to another. However, the question was put to her "Now my lemonade is still in this big jar, but yours has been poured into these two (B1 and B2). Has one of us got more lemonade than the other?" She then replied at once "Yes", and left it at that. She was
quite unable to say which of us was supposed to have more lemonade, far less to give a reason why she thought one was more than the other. In the end it was decided that the framing of the question suggested that an affirmative answer was expected. When the question was re-framed, using A1 and A2, as "Have we both got the same amount of lemonade to drink?" she again said "Yes". When a little water was poured out of A2, she still could not decide whether one had more than the other, but persistently followed whatever the framing of the question suggested to her, even although the water levels clearly showed a difference. Even as she looked at her own jar (A2, with its lower level of water) and was asked "Have I got more lemonade or have you got more?", she replied at once "I've got more", but she could give no reason for this, and even when the water levels were pointed out to her it seemed clear that they did not convey to her ideas of more or less. Frequent repetition with different combinations of jars used in the experiment showed this same simplicity of thought. The chief activity of her thinking seemed to be to grope for an element of suggestion in the
examiner's question.

It quickly became evident that in Experiment II the same difficulties would arise. She could not make the correspondence of glasses to bottle or flowers to glasses. The flowers were merely spread along the table in front of her. She was shown how to make the correspondence by putting a flower in each glass. Then the question "Here are your glasses, here are your flowers; which have you got most of?" She then pointed to one flower, but could not elaborate on this to indicate what might be at the back of her mind. Frequent repetition and demonstration using various combinations of the material produced the same type of response. It must be remembered that at this point there had been no bunching or closing-up of the flowers, bottles or glasses. They remained opposite one another with their correspondence and equivalence clearly in view. It was found later that Anne could discriminate correctly between a larger heap of flowers and a small one, but only if the difference between the two was considerable, and if the words "bigger" and "smaller" were used. If she were asked "Where are there more flowers, and less flowers",
her responses were much more uncertain. A return was made to Experiment I, and it was found that she could discriminate A1 and A2 if the quantity of liquid in one was very much less than the other, and again if such an expression as "bigger drink, smaller drink" etc were used. When the difference in amount was small although quite definite (say about one half inch of liquid) her response was very uncertain whatever language one used. She could also discriminate correctly one line being "longer" or "shorter" than one another.

This child, one of the first to be tested, gave two valuable hints for later examinations. First of all, it was realised that one should not rely on the words "more" and "less" exclusively, but that big, more, long, short etc and other more correct expressions of relative quantity, should be used as well. Secondly, the power of suggestibility was well illustrated by her responses, and was found to be present in many of the children subsequently examined. When a straightforward alternative is put to the child "Is it x or y?", the child who is doubtful of the answer will tend to say "y" taking always the second of the alternatives. One can turn the question round and say "Here you have x and y which is bigger?", and this
form of question is much better. In some of the experiments, we have to ask the child to assess the equality or otherwise of two sets or two quantities which are in fact the same but which look different. If we say to the child "Are x and y the same", he will if he is doubtful almost certainly say "Yes they are", and we cannot therefore be sure that the child has given the correct answer through suggestibility or because he has reasoned it out. So we frame the question such that suggestibility would lead him to give the wrong rather than the correct answer, knowing that if he gives the correct response, he is more likely to have arrived at it through understanding of the problem than by mere chance. For instance, having closed up a set of flowers which had corresponded to a set of glasses, we say to the child "Now, have you got more glasses or more flowers?" If the notion of conservation is really operational in the child's thinking, he will disregard linguistic overtones and declare the two sets to be still equal. Since Piaget lays down no set formula for his questions, but on the contrary employs a variety of forms, it is therefore advisable to follow his lead in
maintaining a clinical approach by presenting the problems
in various ways. The forms of questioning discussed here
are only suggested as possible ways of avoiding sources of
unreliability in a somewhat informal method of examination.
Each child presents his own special problems in this respect,
and an examiner must improvise his questioning in the way he
thinks best.

Continuing with Experiment II, it was found that when,
say, a set of 8 flowers was made to correspond to 8 glasses,
the closing-up of one set completely confused her thinking.
In the first place it was doubtful how much understanding was
present when she agreed that the two sets had "the same
number of glasses and flowers". In fact, when the flowers
were closed up, and she was asked "Here are your glasses, and
here are your flowers: have you the same number in each?", or
".....have you as many of one as of the other?", she virtually
gave no response at all, but merely picked up a few flowers and
smiled. There was obviously no point in going on to
Experiment III and IV.

In Experiment V she made no attempt to copy any of the
figures. In fact all she would do was to pick one or two counters from the box and lay them on the table. When she was told "Look at the nice picture which the counters make on this card, you make one just like it with the counters" she simply put a few more counters on the table. Capacity for imitation is obviously very limited in this situation. The expression to do or to make a thing "like" something else did not appear to convey meaning to her in this context. It is interesting however that when a toy dog was made to jump as follows, $X \ldots 1 \ldots 2 \ldots 3 \ldots \ldots 4$, and she was told, "You make the dog jump like this", she did so quite correctly.

There is little one need say about Anne's performance in the remaining experiments. In the second and third parts of Experiment VI, she merely pointed at random to the boys or hoops. Previous to that, the boys were laid out in serial order, and the seriation pointed out to her. She was then told that the hoops also started with a big one, and that each one was smaller than the one before. The hoops were then laid out in serial order to one side of the boys so that no
positional correspondence was shown. She was then asked to pick out the biggest hoop for the biggest boy; she failed twice to do this, and on the third attempt was successful. The characteristics of seriation were not clearly in her mind: unless strongly prompted, she saw the sets as simply two lines of boys and hoops, and not of progressively diminishing boys and hoops. In Experiment VII her response was completely random. When, to avoid the use of "more" or "less", she was asked "Is there a bigger number of poppies or flowers?", or "children or girls", the word "big" suggested physical size, and after some doubt about her slow response, it turned out that she was looking for a big poppy and a big girl.


She was definitely at Stage I in the first experiment, basing her judgment on size of glass, or level of water. "Why do you think these two glasses hold as much as this one?" "Because it's a big one". "How do you know this glass (the tall, thin one) has more?", "Because the water is right up to here" - pointing to level. "Yes, but this one is a big fat jar,
doesn't that make a difference?" No response. In Experiment II she made the correspondence herself, placing the glasses opposite each bottle, and later the flowers inside the glasses. She knew they were the same in number, having counted the sets correctly. When a set was closed up, she always declared the un-closed to be bigger, or to have more, even when the question was expressed as "Are there the same number of $x$ as $y$". She could however respond correctly if after closing up a set she was asked "Are they still the same." At a later session she was shown a row of 8 counters, set A and underneath a second row of 6, set B, these being made to correspond correctly to the first 6 of the larger set. On being asked "Is one bigger than the other?", she agreed that Set A "was bigger" than B. Two counters were added to B, and she was asked again "Is one bigger than the other?", and replied that they were the same. But when one line was closed up, and the same question put, she declared the unclosed line to be bigger. Leaving the two sets before her closed and unclosed, she was asked "If you counted this line, would you get the same number as if you counted that one?" She replied in the affirmative, and when asked why she had said
one was bigger than the other, she answered "Because it's a line." On further questioning it was clear that most quantitative terms such as "big" "more", "longer", etc entailed for her a total, global or bulk assessment, and not a quantity analysed into units. She could it is true count two sets correctly, but her knowledge of concepts entailing "how many" was mixed up with perceptual factors of bulk size and quality. Only at a later stage would quantitative words, when she was applying them to a set of discrete objects, become disengaged from the global assessment, and take on their full numerical significance. In Experiment V she always tended to copy the pattern, with fair success. Usually however she was wrong with the number of counters. She was shown several times how to match the patterns correctly, and seemed to derive satisfaction when two of them were made identical. But it was clear from her responses to questioning that when one pattern was broken and the counters closed up there was no conservation and no numerical identity. "Why aren't they the same now?" "Because you pushed them altogether." "Will they
have the same number of counters still?" "I don't think so." "Why not?" "Well, it's not the same, it's smaller."

Although Linda was not then asked to count them, the case records show that several children were asked to do so at a similar point to this. Most of them would then agree for a while that a pattern and a heap of counters were still the same, but re-testing at a later date show a reversion to their first type of response, with the possible exception of two children who were re-classified as Stage III. It seems that here, and in the other experiments, unless the child spontaneously introduces numerical analysis, it does not help him much if the examiner suggests it to him. The pull of non-numerical perceptual factors invariably reasserts itself. In Experiment VI the seriation, explained carefully to her in Problem B, was quite lost on her. She was quite unable to give any reason for her wrong answers. In Experiment VII she was shown several times the 6 girls, 2 boys, altogether making 8 children. At this stage, she pointed readily enough to the girls and then the boys, but seemed doubtful about all the children, and had to be shown several
times that "the girls and the boys altogether are the children - just like in your class in school." Once the questioning started she was quite lost, and was even unable to distinguish the children from the other two subclasses. Experiment VIII showed the same characteristic as II.

**Example 3.** Ian. Chronological age 7;1. Mental Age 8;2 I.Q. 115

Ian gave quick and alert responses throughout; although he did not have complete success. His performance is quite typical of children around this mental age. In Experiment I he explained why the quantities of water were the same: "Because you haven't put any more water in, so they must be still the same." "Well, it comes up higher here, but this jar is bigger (wider) here and that makes them both hold the same." In short, he was operating the concept of equalization of differences. In Experiment II, III and IV, he had no need to make the correspondence, but merely counted out as many items as were required. The multiplicative element in IV was quite easy to him. A typical response on being asked how he knew the closed up set was equal to the unclosed - "Well, they're just the same number as before." Forms of question seemed immaterial to children at this level, they took
it that "as big as", "more than", "the same as", etc referred to numbers of elements. After Ian had given the above response, it was suggested to him "But surely the heap of flowers is not the same as the line?" He looked puzzled, and then asked "Did you mean, does it look the same?" In Experiment V he copied the pattern carefully, apparently because he had assumed that it was an essential part of his answer. The correct numbers were always put out, since he first counted the number on the cards. Many children around this mental level did not bother to make the pattern, but merely counted out the required numbers. His further responses all implied clear understanding of conservation of number no matter how the pattern might be altered. With other subjects, instead of always making a heap of the counters, they were sometimes laid out in a long impressive line before the child. Only the well-established Stage III children resisted the urge to say that there were more in the long line than in the pattern. Since this was an extra step in the test, and was carried out rather informally, it can not be given a place in assessing stages of development, except on a few occasions to check doubtful cases.
In Experiment VI Ian seriated boys first; he then arranged the hoops, grading them against each other, and finally matched them to the boys. In Problem C of this experiment he seemed to lose his methodical approach and make mistakes through guessing: he could not explain his movements coherently. In Experiment VII the most painstaking identifying of girls, boys and children failed to help him to grasp the problem; and similarly with the flowers and the beads. The problem was put to him slowly, and he was told not to hurry, and it appeared certain that it was not the linguistic trick which was foxing him. It is easy to see that many children will think you are going to ask them "Are there more boys than girls?"; and will hence give the wrong answer to your question, but the correct answer to the question they thought you had asked. One must therefore emphasize clearly that they are aware of "the children, the wooden beads, the flowers". If the child can point to each class, and its two sub-classes, and yet repeatedly gives the wrong answer, the chance that he is falling for a verbal trick is made very improbable.
Section 3. The relation between mental age and stage of development is shown collectively for all the experiments in Table 20. Here we have half year mental age groups, and the table records the number of responses at a given stage of development which were derived from all the experiments. Thus at age 4½ - 4½, the 5 children in this group gave Stage I responses in all the experiments, making 40 such responses in all. It will be seen that the results of this table were obtained from 77 children only, and unfortunately, owing to practical considerations, it was not found feasible to extend the table to include the results of the whole sample. By the age of about 7 years, 50% of the responses are Stage III. It is also clear from this table that, for Terman Merrill mental ages, the great period for the development of concrete operations is between 6½ and 8½, when the proportion of Stage III responses increased by nearly 70%.
Table 20 Relationship between M.A. and frequency of responses at each stage of development

<table>
<thead>
<tr>
<th>M.A. group</th>
<th>N.</th>
<th>No. of responses at Stages:</th>
<th>Expected % at Stage III:</th>
<th>for following ages.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I  II  III</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4:0 - 4:5</td>
<td>5</td>
<td>40  0  0</td>
<td>1</td>
<td>4:6</td>
</tr>
<tr>
<td>4:6 - 4:11</td>
<td>6</td>
<td>46  2  0</td>
<td>2</td>
<td>5:0</td>
</tr>
<tr>
<td>5:0 - 5:5</td>
<td>7</td>
<td>47  9  0</td>
<td>6</td>
<td>5:6</td>
</tr>
<tr>
<td>5:6 - 5:11</td>
<td>11</td>
<td>65  15  3</td>
<td>14</td>
<td>6:0</td>
</tr>
<tr>
<td>6:0 - 6:5</td>
<td>14</td>
<td>74  18  20</td>
<td>28</td>
<td>6:6</td>
</tr>
<tr>
<td>6:6 - 6:11</td>
<td>7</td>
<td>21  15  20</td>
<td>46</td>
<td>7:0</td>
</tr>
<tr>
<td>7:0 - 7:5</td>
<td>10</td>
<td>18  19  43</td>
<td>65</td>
<td>7:6</td>
</tr>
<tr>
<td>7:6 - 7:11</td>
<td>3</td>
<td>0   2  22</td>
<td>81</td>
<td>8:0</td>
</tr>
<tr>
<td>8:0 - 8:5</td>
<td>6</td>
<td>5   1  42</td>
<td>91</td>
<td>8:6</td>
</tr>
<tr>
<td>8:6 - 8:11</td>
<td>5</td>
<td>2   5  33</td>
<td>97</td>
<td>9:0</td>
</tr>
<tr>
<td>9:0 - 9:5</td>
<td>3</td>
<td>0   0  24</td>
<td>99</td>
<td>9:6</td>
</tr>
</tbody>
</table>

The percentage is 50 at about age 7:1.

Dr. D.N. Lawley advised the addition of the last two columns, and he kindly carried out the necessary calculation. The "expected" Stage III responses were fitted by an approximate probit analysis.
Section 4. Sex differences in the stages of development.

In the course of examining the children in the four schools, the examiner formed no impression whatever of any difference in the responses of boys and girls, either qualitatively in their verbal expression, or quantitatively in the relative proportion at any given stage of development. To gain a more exact assessment of any difference which might exist, fourteen pairs of subjects were selected on the basis of mental age. The difference between the mental ages of the boy and girl in any pair was not more than 4 months. The following table shows the stages of development of the children in each pair for three of the experiments. With such a small sample only a tentative conclusion can be drawn, that there is no tendency evident for one sex to be more advanced or retarded than the other in their field of concept development. Certainly if such a sex difference does in fact exist, it is not an obvious one, and until more evidence is accumulated, can be disregarded in teaching practice.
Table 21. Responses of matched pairs of boys and girls in 3 experiments.

<table>
<thead>
<tr>
<th>Stages</th>
<th>Expts: I</th>
<th>Expts: II</th>
<th>Expts: VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BG</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td>3</td>
<td>BG</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>4</td>
<td>BG</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>5</td>
<td>BG</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>6</td>
<td>BG</td>
<td>G</td>
<td>B</td>
</tr>
<tr>
<td>7</td>
<td>G</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>8</td>
<td>BG</td>
<td>G</td>
<td>B</td>
</tr>
<tr>
<td>9</td>
<td>BG</td>
<td>G</td>
<td>B</td>
</tr>
<tr>
<td>10</td>
<td>BG</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>11</td>
<td>BG</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>12</td>
<td>BG</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>13</td>
<td>BG</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>14</td>
<td>BG</td>
<td>B</td>
<td>G</td>
</tr>
</tbody>
</table>
Section 5. Piaget's pre-number concepts and ability in arithmetic. The question which is now to be raised is whether there is any relationship between the presence or absence of pre-number concepts and ability in arithmetic. It will be in vain to ask for a scale of arithmetical ability which will correlate with a given number of passes in the Piaget tests so that it would be possible to predict the one from the other. In the first place, as we have seen, in the present state of our knowledge the concepts assessed by the experiments make their appearance over a broad time belt from at least 6:0 to 8:0 years: nor has it been possible yet to indicate a normal and expected pattern of passes and failures in the Piaget tests for any specific mental age although such a scale may eventually be produced.* Secondly, a valid scale of arithmetical ability is very difficult to obtain with young children. Even among the four schools of this investigation, there was a good deal of variation in teaching methods, time devoted to arithmetic, age of entry to school, and of course such factors as absence from school, change of teacher, etc. In the writer's opinion Piaget would never intend his experiments to be measures of how much arithmetic a child

* A standardisation is in fact in course of preparation. In a letter which I received recently from Professor Inhelder she says: "Je vous remercie beaucoup de l'intérêt que vous témoignez à la standardisation de nos recherches. Le travail est presque achevé mais je crains qu'il faille attendre encore plusieurs mois avant que la publication soit prête. Pour prendre date, M.Vinh Bang a publié quelques pages dans les Études d'épistémologie génétique."
should be able to do. What they try to do is to indicate aspects of mental structure, so that we may know if a child has developed mentally to the point of having the concepts which arithmetic pre-supposes, which are the intellectual foundations of arithmetic, and without which a complete understanding of arithmetic is impossible. Whether he says "it be a matter of continuous or discontinuous qualities, of quantitative relations perceived in the sensible universe, or of sets and numbers perceived by thought, whether it be a matter of the child's earliest contacts with number or of the most refined axiomsations of any intuitive system, in each and every case the conservation of something is postulated as a necessary condition for any mathematical understanding." (a) We will therefore expect that some children, who fail in all the Piaget tests, may nevertheless be drilled so expertly in mechanical arithmetic that they may appear to have a real skill in it, but whose lack of ability becomes evident when they are faced with simple problems, or more advanced mechanical work. Again we will find children who can pass all the tests, but who can do little or no arithmetic, since the former is no guarantee of the latter. Such a child,

(a) Ref. 1 p. 3.
if he has not responded to teaching, is clearly an educational problem, and in need of specific remedial work. We shall in short look for guidance from Piaget's tests when we ask the question "Is this child intellectually equipped to cope with arithmetic?", and the tests do indeed clarify the whole idea of "arithmetic readiness" in its intellectual aspect.

Far from correlating a series of Piaget "scores" with a series of arithmetical scores, the present study could go no further than the following. Class teachers were asked to review each child's work in arithmetic, and to place the child in one of five arithmetic grades.

- **Grade 1.** No number ability of any kind: child may be able to count, but he cannot pick out 5 or more objects from a group, or do a simple sum even with counters.
- **Grade 2.** He can pick out 5 or more objects from a group.
- **Grade 3.** Child can do simple addition and subtraction with or without counters, orally or in writing.
- **Grade 4.** He can do simple problems presented verbally or in writing, and also will have the skills of Grade 2; he can in short use number with some understanding.
- **Grade 5.** Any combination of number skills beyond this point.
Teachers did not find it easy to fit every child clearly into one or other of these grades, and many of them asked for a finer discriminating scale. The above scale, however, was thought to be quite adequate, since its purpose was merely to pinpoint any child who might be able to use number with understanding, and yet be predominantly lacking in the pre-number concepts. By "using number with understanding", perhaps better expressed as "using numbers successfully", we mean the ability to attend to the numerical elements in a number problem, to relate those elements correctly to one another, and to resolve the correct numerical solution to the problem from these relationships. The process must be distinguished from that of "understanding number". For Piaget "number" as such is understood only when the various concepts of conservation, equivalence, equalization of differences, and so on, can be attached to a group of interchangeable units, and this is assessed for a given child not by arithmetical problems, but by the techniques outlined in his book. Usually a negative answer is given to the question whether a child can
use number successfully before he can understand numbers, but our evidence does not support this view. Arithmetic logically entails the concepts of conservation, equivalence, and the rest, and would not make sense without them, although psychologically a degree of arithmetic is possible without them. A child may be trained, not only in mechanical processes, but in problem work, to act as if he understood number. Methods of solving a problem may be skilfully taught, and are moreover facilitated when the child solves the problem by acting on material before him. The material itself helps to keep his thinking in the channels of conservation, equivalence, etc, and the presence or absence of the concepts themselves does not constitute for him an element in the problem. Even a written or orally presented problem given without supporting material may be successfully tackled, if the child knows what has to be done with the number symbols, responding in effect to the pattern of the individual elements in the problem, rather as the car driver who successfully operates the controls without knowing a coil from a cam-shaft. Indeed as we shall see later, the child may
do a problem correctly, and only on being made to reflect on, say, the distribution and size of the elements before him, will he begin to hesitate about the solution. There is no doubt, for instance, that in repeating Piaget's experiments one might on occasion, merely by suggestion and one's implicit authority as an adult, cause a child to revert to an earlier level when he had already ceased to be bothered by misleading perceptual gimmicks.

As yet there is not enough evidence to decide when pre-number concepts become an essential element in a child's thinking for successful arithmetic. The following table, which is only suggestive, may give a rough indication.

Table 22.

<table>
<thead>
<tr>
<th>Grade</th>
<th>I and II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>62</td>
</tr>
</tbody>
</table>

We find no case of a predominant Stage I child being in Grade 5, and although under very favourable circumstances such a
case may be found, it would be fair to state that as a general rule more advanced work than the level of Grade IV will increasingly demand these concepts. There are a fair proportion of Stage I children at Grade IV, and significantly perhaps they all come from the school with good home backgrounds and good teaching methods. Summing up one may say that one should not expect a child of predominantly Stage I responses to go far in arithmetic, and there is a strong case for regarding presence or absence of the concepts as the main factor in weighing up a child's arithmetic readiness on the intellectual plane. On quite other grounds, however, a proportion of Stage III children will be found to be at too low a standard of arithmetic, their backwardness arising from the numerous vicissitudes of school learning and not from mental incapacity.

In addition to accepting the teachers' assessments of ability in arithmetic and relating them to Piaget's stages of development, a small sample of children between 5:0 and 6:0 years was examined by the writer with the same purpose in mind. There were only thirteen children in the sample, varying from predominantly Stage I children to others who were
at Stage III. The aim was to assess their ability to do simple numerical problems, suitable material being provided to help them. They were not required to write down the answers. Some of the problems had a definite slant towards Piaget type situations — for instance when perceptual factors might mislead the child.

There was no set linguistic formula for the problems, but they were approximately as follows:

1. "You have 5 smarties; here they are, let me see you count them. Suppose mother gives you three more, how many would you have then?" Five smarties are placed before him in a line; the box containing plenty more is placed on one side. After stating the problem, and when the child has surveyed the 5, the examiner says, pointing to the box of sweets, "You may help yourself by using some of these if you like", assuming that he has not already done so.

2. "Here are 8 red counters; suppose you lost 5 of them, how many would you have?" The counters are placed in a row before him.

3. "Mother picked 12 flowers, and then asked you to go and pick 5 more. How many flowers would she have then?" Presentation as for No. 1.

4. "John saves up 18 pennies; he spends 6 on sweets. How many will he have left?" Presentation as for No. 2.

5. "May saves up 4 pennies .... here they are 0 0 0 0
Molly saves up 3 pennies .... here they are 0 0 0
Now Jane starts to save up her pennies, and she
saves just as much money as May and Molly together. How many pennies will Jane have?"

6. Examiner has some pencils. He gives child about 10 pennies. "I want you to buy one pencil, and it will cost you 3 pennies.

Here is the pencil

and you put your three pennies here

Now you are going to buy another pencil here it is

How many pennies will you need for the 2 pencils?"

7. "John has 3 marbles and he wins 2 more
Billy has 1 marble but he wins 4 more
Who has the most?"

8. "Here are 9 flowers for you.
Your friend has none, but then
she is given 4 from her mother
and 6 from her teacher.
Who has the most flowers now, you or your friend?"

9. "Tom and Dick are collecting pencils. Tom likes short ones, but Dick likes long ones.
Tom collects 4 red ones, and 3 yellow ones
Dick gets 3 red ones and 3 yellow ones.
Which of them has the most pencils?"

10. "Mary gives her mother for Christmas: 2 flowers, 2 handkerchiefs, 1 pencil.
John gives his father: 1 tin of tobacco, 4 pencils, 1 pipe."
Here they are:

Who is getting the most presents?

Table 23 shows the responses of each child in the 10 problems, together with mental ages and the predominant stage of development for individual children.

<table>
<thead>
<tr>
<th>Subject No.</th>
<th>Predominant Stage</th>
<th>Response to individual Questions</th>
<th>C.A.</th>
<th>M.A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I</td>
<td>- - - - - - - - - - - - - - - -</td>
<td>5:9</td>
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<td>III</td>
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<td>6:1</td>
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Comments on selected individual responses:

Subject 1. He knew in Question 1 that something had to be added, and in fact he added 3 more, but he made no attempt to count, merely declaring the answer to be 9. In Q.2 he did not seem to realise that anything was to be taken away. He responded as a typical Stage I child to Nos.7 to 10, making no attempt to manipulate the numbers.

Subject 2. Similar to above, except that he did not grasp that more sweets should be added to the 5 in Q.1, although he knew that something had to be removed from the 8 counters in Q.2. What he removed seemed however to be quite arbitrary, and his answers were not the result of any sort of counting.

Subjects 3 to 6. All gave virtually no response to Q.1 but they responded as Subject 2 to Q.2.

None of the first 6 subjects showed any idea of the problem situation presented to them by the various questions, particularly Q.5 to 10. Sometimes they were unable to answer simple questions about the problem. For instance in Q.7 when Subjects 3 and 4 were asked "What did I tell you that Billy had?" they pointed to the 4 marbles only, and forgot about the single
one. None of them was even able to answer the question "What is it you've got to find out?", although they all had the question repeated and explained two or three times. The few responses to this question took the form of more or less repeating the factual part of the problem - "I've got these flowers and you've got those ones." The quantitative relationship between the elements of a problem was not realised until directly suggested to them by being asked "Who has the most?" etc.

Subject 7. With much the same mental age as Subject 6, this boy showed a marked advance in arithmetical ability, in spite of being virtually a complete Stage I child. Questions 1 to 8 were all done correctly first time. He seemed to tackle them in a smart business-like way, saying on one occasion, "Well, what you have to do is put out another 3 pennies like that for the other pencil." In Q.9 he was wrong at first, saying that the boy with the "big pencils" would have the most. "You mean, he would have a bigger number of pencils?" "Yes" "Count them just to make sure." He did so correctly and adjusted his answer accordingly. Then in Q.10, he was at first correct, having counted 6 for father and 5 for mother. But when mother's
presents were spread out a little more, he was very
hesitant and finally stated emphatically that mother was
getting the most presents. Unfortunately there was no
opportunity to question this boy further. Within the
limits of what arithmetical skill he had, he seemed to
understand how to operate numbers correctly, although still
at a very early level of pre-number concept development.

Subject 8. Although a complete Stage I child he was correct
with the first four questions and with Q.6. He did not
required actively to add or take away items of the material,
but completed each sum in his head. He failed with Q.5 twice,
on each occasion counting the individual groups of 4 pennies
and 3 pennies, and then declaring right away that the other
girl would save up 6. He did not attempt to count through
the two sets of coins to get 7. In Q.8 he did not attempt
to count, but said at once "I've got most flowers ....because
there past that one" pointing to the wider spread of his 9.
Q.9 and 10 showed the same type of response, even when in
Q.10 he spontaneously counted correctly.

Subject 9. In spite of a mental age of 6:4, this boy was
Stage I even in Experiment II. In Q.1 he added 2 sweets, counted
them and then took one away. When asked he counted to 6.
The question was repeated, with the same failure to grasp it
on his part. In Q.2 he took away 2, having been told that
he had lost 3. He was reminded that 3 counters had been
lost not 2, and he responded by removing another 3. In Q.5
he did not get the idea of adding together the pennies saved
by the first two children - his answer was 3. His responses to
all the other questions (except Q.6 which was the only one
correct) showed the misleading effect of perceptual factors
characteristic of a Stage I child, even after he has counted
items correctly.

Subject 11. This girl seemed to be emerging from the Stage I
level, having reached Stage II for Experiments II, V and VIII,
and Stage III for Experiment VI B, the rest being Stage I. She
was correct in all the sums. In Q.7 to 10 she at first
succumbed each time to the perceptual factors, but on being
prompted to count the items she was then emphatically correct.
Q.10 she corrected spontaneously by counting. After this, no
alteration in spacing etc affected her confidence in her
answers.
Section 6. In this closing section, it should perhaps be mentioned that Piaget anticipates possible objections to his method of study. "It might be argued, "he says, in considering the responses of children at Stage I, "that the mistakes are due to lack of understanding of the words used. May it not be that the child does recognise that the number of bottles and glasses remain the same when one set is grouped together, and that when he says "there are more" he is merely expressing the idea that the shape of the set has changed and the space it occupies is greater." This point presented itself to the writer frequently with great force. For instance many children who gave Stage I types of answers ("there are more here" "this is bigger" etc) were asked a further question "Do you mean that you now have a bigger number of things here than here?" or "If you counted the line of flowers would you now count more than if you counted the heap?" A few children gave a clear answer to this, showing that they knew the number of items in each set to be the same; for instance, "It's only the way they look that's

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a: Ref. 1 p. 46.
different," and they were then passed as Stage III. Many of them however agreed that the number was unchanged, yet when the tests were repeated they persisted in saying that one set was more or bigger than the other. It became a question then of to what extent one should insist on them thinking only of the number in each set by, as it were, helping them by verbal suggestion to set aside in their minds irrelevant perceptual factors. This procedure would have given a somewhat earlier Stage III development, but it would also have meant that the examiner was doing a good deal of the child's thinking for him. The mere fact that a child tends to look at the quantitative problem set to him, not solely in analytic terms of "how many items", but in global, perceptual terms, is evidence surely of immature numerical thinking, however much we may feel that he "really knows all the time that there are still 6 in each set." The fact is that the child, although he may or may not know that the sets are still 6 and 6, cannot prevent himself lumping in with the 6 items the space that lies in between, or the size of the individual items in one set as compared with those in the
Piaget answers the objection by stressing the confidence and clarity of mind which the child displays when he does reach Stage III. "At the third stage the child discovers and states explicitly, that the fact of grouping together or spacing out the elements in no way affects their number," he also makes the important point that even if the child can count the items of the sets correctly, there is no proof that this "verbal enumeration expresses a better quantification, from the child's point of view than the space occupied". Correspondence between numerals and objects at this level may be purely verbal - just as he can make the glasses correspond to the bottles, so he can make the names of the first 6 numerals correspond to the 6 glasses, etc.

While it is certainly true that there is an element of linguistic confusion among Stage I children because their understanding of the words in question is incomplete and limited, it is the eventual appearance of the concept and operation which completes the verbal understanding and removes the limitations. With the notion of permanence and equivalence now a part of his
mental equipment, the child modifies and re-directs his use of expression like "which is bigger, where are there more, etc". Before that point is reached, one may try to teach children the correct quantitative use of such expressions, so that, as some would have it "we implant a sense of number in their minds". With young children such teaching would certainly bear no fruit. From approximately 6 years onwards, one might in many instances help a child on more quickly to Stage III, and certainly the number teaching of young should be directed to this end as will be observed later. But basically Stage III is reached by the normal process of intellectual maturation.
PART IV. RESPONSES OF MENTALLY SUB-NORMAL SUBJECTS TO THE EXPERIMENTS DESCRIBED IN PART III.

Section 1. It has been noted in Part I of this study that in his theory of the growing mind of the child, Piaget gives us a picture of the evolution of a mental structure which, while retaining always the unity of its inter-related parts, increases steadily in size and complexity until the period of formal operations is completed, system following on system and each integrated with the one which preceded it. In his own words “Je conclurai en disant que ces trois grandes périodes, avec leurs stades particuliers, constituent des processus d’équilibration successifs, des marches vers l’équilibre. Des que l’équilibre est atteint sur un point, la structure est intégrée dans un nouveau système en formation, jusqu’a un nouvel équilibre toujours plus stable et de champ toujours plus étendu.” His pre-number concepts are a fragment of this growing system of operations. They emerge over the age range of from 6 to 10 years approximately as concrete operations, and along with other concrete operations, they are the stage on which the final structure of formal operations is built. Knowing therefore their time and position in this procession of mental events among normally developing children, we may now ask whether their

a: Ref. 16 p. 41.
development is affected by the handicap of general mental sub-normality. For instance, a mentally sub-normal boy of 19 years may have an intelligence test mental age of 6 years. Will his thought processes still be stumbling at a pre-concrete operational level, or shall we find that his thinking as expressed in operations has matured along more normal lines because of his contacts and experiences in the society of his home, school or training centre. If we find that he has in effect "learned" the answers to the pre-number problems, then our conclusion must be that the successful achievement of such problems is not so firmly, if at all, pinned down in a system of operations. Persuasive as Piaget's arguments are in his books on number, quantity, space etc., one wonders from time to time - particularly perhaps while in the process of testing the children - if, say, the child who solves the glasses of water problem does so primarily because of his home and school experience, and only in a secondary sense because he "has" or "possesses" a particular concept of conservation. The reaction of chronologically older subjects, however, who have mental ages numerically comparable with the young child, may help to re-inforce one or other point
of view.

At least two studies on these lines have already been undertaken. Inhelder, in her book "Le diagnostic du raisonnement chez les débiles mentaux", sets out with the full intention of demonstrating how Piaget's theory of the system of reversible operations could help materially in the diagnosis of various grades of mental deficiency. The tests which she employed were taken from a work published previously by herself and Piaget, "Le développement des quantités chez l'enfant". She applied the selected tests to about 160 subjects known to be or suspected of being mentally retarded. The subjects' responses were related to their chronological age, I.Q. and scholastic attainment. The results support the hypothesis which Inhelder puts forward as the aim of her research.

"Nous allons chercher à montrer que le raisonnement d'un groupe variable d'arriérés mentaux correspond bien à des arrêts et fixations déterminés par certains des stades établis sur les enfants normaux à propos des notions de conservation de la matière."

Her subjects in fact responded in the way one would expect from normal children of comparable ages. She summarizes her
conclusion as follows:

"La notion de construction inachevée à laquelle nous sommes conduits nous permet par contre de proposer les définitions suivantes ....

1. que l'idiot ne dépasse pas les compositions sensorimotrices (antérieures au langage),
2. que l'imbecile est capable de pensée intuitive (égocentrisme, irréversibilité mais non pas opération),
3. que le débile est capable le construction opératoire mais inachevée, c'est-à-dire d'"opérations concrètes" par opposition aux opérations formelles,
4. enfin que l'arrière simple (l'enfant retardé) parvient aux opérations formelles et rattrape ainsi le normal.

Etre débile signifierait alors: parvenir à penser par opérations concrètes mais non formelles. Le débile se distingue ainsi de l'imbecile par un début de construction opératoire, et de l'individu normal ou simplement retardé, par incapacité à achever cette construction par des raisonnements formels ou hypothético-déductifs. a

In the writer's opinion Inhelder's book gives valuable support to Piaget's whole theory of the development of reasoning, as well as showing how his ideas can find practical use in the assessment of mental retardation.

A recent study of "The behaviour of idiots interpreted by Piaget's theory of sensori-motor development" was undertaken by May Woodward. She applied Piaget's observations on sensori-

a: Ref. 24 p. 273. 

b: Ref. 33.
motor intelligence and the concept of permanent objects in the development of normal infants to 14.7 mentally defective children, and concluded that "sensori-motor activities of severe mental defectives could be classified into the six main types distinguished by Piaget." More important perhaps she found strong evidence to suggest that the sensori-motor development of defectives follows the same sequence as Piaget observed in normal infants. It will be remembered that Piaget has stressed the importance of sequence for his theory of stages in mental development. "Pour qu'il y ait stades, il faut d'abord que l'ordre de succession des acquisitions soit constant."

**Section 2.** In the present study forty mentally sub-normal subjects were examined. These subjects fell into two classes as follows:—

(a) Educationally sub-normal school children. These are defined as "pupils who, by reason of limited ability or other condition resulting in educational retardation, require some specialised form of education wholly or partly in substitution for the education normally given in ordinary
Most of these children fall into the category of those who in other contexts may be referred to as high-grade mental defectives, or feeble-minded persons, of whom E. Tizard writes: "Both the terminology and the definition of this category are confusing. In Great Britain, the "slow learning" children who make up the 1 per cent of the school population lowest in intelligence and educability used to be called feeble-minded, but are now called educationally sub-normal children; only if in addition they suffer from severe behaviour disorder which make it inexpedient for them to be educated with normal children are they today excluded from school and called feeble-minded." In the definition framed for the Education Act the phrase "other condition resulting in educational retardation" suggests that innate mental capacity is not necessarily the sole ground of "educational sub-normality", and it is certainly true that in England and Wales children with I.Q's in the broad band of normality but whose attainment in the basic school subjects is seriously retarded, are on occasion classified by medical officers as educationally sub-normal. Nevertheless, the majority of educationally sub-normal pupils

Ref. 34 p.5.
are in fact mentally retarded, with I.Q. on the Terman Merrill Scale between approximately 55 and 75. The subjects in the present study all had I.Q.'s below 75. All of them were boys attending a residential special school for such pupils. Their ages ranged from 10:3 to 15:7 years, and there were 23 in all.

(b) Mentally defective subjects. Without going into the question of the various categories of mentally defective subjects below the level of the educationally sub-normal, it may be said that in this study the subjects were either

(1) children of school age who had been classified as so defective in intelligence as to be unable to benefit from the education of an ordinary school, or of a special school for educationally sub-normal children; or (2) persons above school age who had been classified as mentally defective within the meaning of the Mental Deficiency Acts. All these subjects attended each day an occupation centre belonging to the local authority. There were seventeen in all, their ages ranging from 9:8 to 41:0 years.

In their general manner of responding to the examination, the whole mentally retarded group tended to differ from the
normal school children in the following ways. (1) The retarded subjects on the average took approximately 10 minutes longer to complete each session of the examination. They moved more slowly, both mentally and physically, and one very seldom found the speedy, alert response which even some normal 5 year olds could display. (2) This slowness was due in part to their poor receptiveness to language. One had to speak to them more slowly and deliberately, with repetition being more frequently necessary. The interpretation of their comments and observations, although they were seldom incoherent, was no easy matter at the time for the examiner, and this was so even when a response was essentially correct. (3) Even among those with higher mental ages, distractibility was commonly in evidence, taking the form mainly of wanting to play with the test material, or breaking-in with quite irrelevant questions or observations. These points were of course more in evidence among the mentally defective than the educationally sub-normal subjects. Neither group showed any evidence of ill-will or resistance to the examinations, and
they gave the impression that as long as one did not hurry them, they would work to the limit of their capacity.

Section 3. We now present Table 26 which tabulates separately the results for the two defective groups. Having given each subject a reference number, the table then gives the subject's mental age (from Terman Merrill Scale I), chronological age, and his stage of development in each of the concepts or operations studied by the eight experiments. An asterisk is placed against the reference number of the few subjects who were females.

This Table makes it clear that, as with young children of normal intelligence, Piaget's pre-number concepts tend to mature as the subject's all-round mental capacity increases. With normally intelligent children any other result than this would of course, be totally unexpected, but among mentally subnormal subjects, the position might have been that general socialization, the effects of schooling, and the training of an occupation centre might have provoked the maturation of these concepts among the chronologically older subjects whose mental development on a standard intelligence
TABLE 2. Showing stage of development in each experiment for  
(a) educationally subnormal, and (b) defective subjects.

<table>
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<tr>
<th>Subject's Ref. No.</th>
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<th>Chron. Age</th>
<th>Experiments:</th>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
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TABLE 24 continued.

(b)

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scale was still theoretically that of a young child. For instance, to take an extreme case, No. 28 was a man of 40 who was attending the occupation centre as an interim measure until more suitable arrangements could be made for him (he was well beyond the usual age limit for the centre). Because of his tendency to down tools when he felt like it, and his limited sense of responsibility in most social matters, he was never a satisfactory worker even in an unskilled job. Nevertheless he had "knocked about a bit", could handle small quantities of money needed for cigarettes, buses etc and in casual conversation did not give the impression of being so low on the mental age scale. But for all his worldly experiences, he remained solidly in Stage I in all the experiments, with no hint at all of even approaching Stage II at any point. He also had the normal 5 year old's frequent difficulty of coping with such a phrase as "Make it the same as.... which one is bigger....have I got more....etc". Similar observations might be made about No. 25, 26 and 27, who are around 15 to 16 years of age, but whose performances in the experiments does not suggest that their development in pre-number concepts is influenced significantly by environmental factors.
Socially these subjects had quite satisfactory home backgrounds, enjoying a normal degree of mental stimulus.

Turning to a higher level on the mental age scale, we find that from about 8½ years upwards there is a preponderance of Stage III responses, exemplified by cases 22, 23 and 40. In nearly every test these subjects gave the quick and confident responses of normal children, including often the same element of surprise and amusement at the obvious simplicity of the problems. Touching on subject number 40, this girl's I.Q., and her general responsiveness both in conversation and while performing the Piaget tests, gave one the impression that her mental capacity was above the level recognised as being normally characteristic of mentally defective subjects. She was among the first of the defective subjects to be examined, and it is interesting to note that on attaining the age of 21 recently, she was discharged as being no longer a person requiring statutory supervision because of mental defect. The association between these subjects' mental retardation and their immaturity in pre-number concepts is brought out again in Tables 25 and 26. From Table 25 one can obtain for each subject the
Table 25 Total of responses at each stage for all the mentally retarded subjects.

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number of responses he gave at each stage of
development from all the experiments taken together.
For instance, subject No.1 had 8 Stage I responses out
of 8. From this one can calculate the "expected"
percentage of Stage III responses at each mental age
level for the mentally retarded subjects, on the same
lines as the last two columns of Table 20. The
original reference numbers of Table 24 are retained on
Table 25, but in the latter the subjects are listed
according to increasing mental age. If we compare
some of the individual responses of Table 25 with the
general pattern of responses recorded in Table 20
anomalous cases will be quickly picked out. For
instance, subjects 9, 20 and 39 show an overloading of
Stage I responses which one would not expect from their
Terman Merrill mental ages. Other cases show the same
trend, if not so strongly. Thus although there is a clear
relation between mental ages and stages of development, the
former is not by any means an invariable guide to the
presence or absence of pre-number concepts. It is not
surprising to find, however, that, when the responses of these three
subjects in the Terman Merrill Scale were examined, it was found that they tended to score well on items which were relatively free of logical content and which instead relied on such factors as memory and social training and experience. On the other hand, they were obviously weak in items calling for the identification of similarities and differences, and in general the entertaining of concepts. The comment made by the occupation centre supervisor of number 39 is typical of the impression these subjects made on those who looked after them: "For a defective boy he is really quite bright, friendly and always ready to do things. But I must say when you do ask him to do something, you must only give him one very simple thing to do, and you must explain it very slowly to him." Her impression was that most of the time he behaved like a normal, sensible 8 to 9 year old, but she recognised all the same that underneath this general retardation, there was a grosser and more intellectually crippling handicap which showed itself only in special circumstances. Much the same opinion was expressed of the others. From Table 26 we find that mentally retarded subjects reach
50% Stage III responses at about mental age 8:8, compared with 7:1 among normal children. Their rate of development, as one would expect, is also much slower; between 6 and 8 years their proportion of Stage III responses increases by only 25% compared with nearly 70% among normal children.

Table 26 Showing "expected" % of Stage III responses for each age for group of mentally retarded subjects. (based on results of Table 24).

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</table>

(extrapolated)

The % is 50 at about age 8:8. The approximate probit analysis was again carried out by Dr. Lawley, who points out that the results are not too reliable since they are based on the results of only 40 subjects.

Section 4. H.C. Gansburg, in his contribution to "Mental Deficiency - the changing outlook", writes, "Piaget's work on number may prove of great help in understanding the difficulties encountered in arithmetical reasoning. He points out.....how

little the ability to count, to add up, and the knowledge of the tables, ensures that the child has grasped the idea of number ....number in the earliest stages is related to size, shape, arrangements, it is perceived and not understood.

An interesting point came to light in the relation of stages of development in pre-number concepts and mental ages among the educationally sub-normal pupils, which had a bearing on their ability in arithmetic. Individual reports on each pupil's ability in arithmetic had been obtained from the class-teacher. These reports were easily given a three-fold classification as follows :

(a) Pupils with virtually no arithmetical ability; some of them could not count, others might be able to do a simple adding sum using counters, but their capacity to do this was shaky and unreliable.

(b) Pupils who could do simple addition and subtraction sums, but whose progress came to a stop with the introduction of carrying and borrowing.

(c) Pupils who although generally about 18 months to 2 years
behind their mental ages in arithmetic, showed definite understanding of and confidence in what they were doing.

The pupils fell into these categories as follows:

Category 1. Nos. 1, 2, 3, 4, 5, 6, 11 and 20
" 2. Nos. 7, 8, 9, 10, 13, 18 and 19
" 3. Nos. 12, 14, 15, 16, 17, 21, 22 and 23

One notices at once that the pupils who failed most heavily on the pre-number concept tests were all very weak in arithmetic. But most of those who passed the Piaget tests successfully were making reasonable progress in number work. There were, however, a few exceptions to this, for instance Nos. 10, 11, and 13, where there was poor arithmetic in spite of good responses in the Piaget tests. There was no instance of success in arithmetic and failure in the Piaget tests being found in the same child. Although the teachers of these boys expected them to be backward in arithmetic compared to normal children of the same chronological age, they did nevertheless expect a boy whose mental age was 7 or more to show some degree of elementary understanding of arithmetic. They had access to test results and knew their pupils' mental ages, and expected to get the
type of responses characteristic of the normal 7 year old. These boys were therefore presented to the writer as something of educational mysteries, particularly since they were found to be considerably more successful in reading than in arithmetic, and therefore "were not by any means fools". The results of the Piaget tests strongly suggest that their failure in arithmetic was due to the absence of pre-number concepts, and that in fact, in spite of their Terman Merrill mental ages, they were mentally not ready for arithmetic. They were still at Piaget's intuitive stage of thinking, and had not yet reached the stage of concrete operations. The irreversibility of their thinking was also apparent as will be gathered from the quotations which follow in the next section.

Section 5. (a) The first boy on the list of Table 24 had a chronological age of 12 years, but a mental age of only 5:8. He gave every impression of being a very retarded boy mentally, with a heavy unresponsive manner. Because his mental age was approaching 6 years, however, it was hoped that concentrated teaching would get something out of him.
When he was examined for this investigation, he had been four terms in school. He could definitely count 4 objects laid out before him, and on occasion would count up as far as 7 objects. He could select only 3 objects from a group (say of a dozen blocks) when asked to do so, succeeding occasionally with 4, but being quite unable to cope with any more than this. When his teacher asked him to write out the numbers up to ten on a blackboard, he wrote 1 2 3 4 5 7 6 and stopped, saying the numbers aloud as he did so. His mistake was pointed out to him. Then he was taken to a second blackboard and asked to write out and recite the alphabet, which he did up to the letter N. Finally he was brought back to the first board and asked to write and recite his numbers again. He wrote the first seven numbers as before, but while writing them recited the first seven letters of the alphabet. It was clear that there was a serious discrepancy between his Terman Merrill mental age and his responses to class-room teaching - his response to teaching being far below that of the normal 5 year old.

In Experiment I this boy was unable to make any sort of intelligent response. Even when shown two large jars filled to
the same level with water, he appeared to be unable to grasp the notion of their equality; and when one of them had some of its liquid poured away while he watched, thus lowering its water level, he was in no way helped to an understanding of what was involved.

In Experiment II he was unable to make a spontaneous correspondence of glasses to bottles, or flowers to bottles, even when only four items were put before him. The examiner then placed six bottles before him, then six glasses in obvious correspondence. The items in each set were slowly counted aloud, 6 in each, with the boy’s finger being held and pointed at each. He was told “There are 6 bottles and 6 glasses, so that you have the same number of each, just as many glasses as bottles”. He agreed, but his look was uncomprehending. The bottles were then closed up and to the question “Are they still the same?” he replied “Yes”. But when the sets were made to correspond again, the items counted once more, and the glasses closed up, to the question “Have you more bottles or more glasses”, he replied simply, “Bottles”. Further questioning showed that his responses were quite arbitrary, and he seemed to be
missing the point of the words "more" and "equal to" etc.
The technique described in Experiment II, (Method ) was then applied.

8 white flowers were laid out in a row before him. "These are your flowers."

6 red flowers were laid out, corresponding to the first 6 of his.

"These are my flowers; who has the most flowers?" "Me"
1 added to the 6 red. "Who has the most?" "Me"
1 more added, making 8 to each set. "Who has most now?" "You have."

The test was repeated but this time when the two sets were brought up to 8 each, he was asked "Are they the same?" and he replied "Yes". He was quite unable to give, even by gestures, any reason for his answer. However, accepting his response at its face value, the white flowers were closed up, and he was asked "Have we got the same number of flowers now?" He gave no response, but merely looked at them. The question was re-framed "Who has the most flowers?" and he replied "You have", but could give no reason.

In Experiment V he could not even approximately copy the patterns, even the simple ones. His counters were placed in a
random, close-set group, without regard to number.

In Experiment VI his responses were quite random selections, and he gave no evidence of grasping the significance of the serial formation.

In Experiment VII the whole verbal framework to the various tests were beyond him.

Following an interval of about four weeks, the boy was re-examined. His responses were all as before, except this one which follows:

For the boy ... 8 white flowers
For examiner ... 6 red flowers "Who has the most" "Me"
    6 + 1 " " " " " "Me"
    6 + 3 " " " " " hesitated, then "You"
    6 + 2 " " " " " "Same"

When the size of the two sets had been doubly checked by further counting, and the red flowers closed up, he affirmed that there were more red ones. Apparently he could not return from the closed-up line to the two rows which he himself had checked as being equal.

Occasional repetition of some of the tests from time to time showed no significant progress. No matter how the question might be varied, lack of comprehension always became apparent, even if at first some responses through mere
suggestibility might be correct. As a final note on this boy, he was eventually discharged from the school as ineducable.

(b) The following notes are from the record sheets of No. 20, who was 12 1/4 years of age at examination, with a mental age of 9 6/12. It will be recalled that according to Piaget's own findings, as well as the result of this investigation, children with a mental capacity of 9 years have invariably passed in all the pre-number concept tests; that is to say, they are well on in the period of concrete operations, and are soon to mature to the level of formal operations. It was therefore a surprise to find this boy, John, having a mental age of 9 1/2 years and yet failing in 5 of the experiments.

According to the class teacher's report John, who had been a full two years in this school, was in Category 1 for arithmetic. He could do simple addition and subtraction without carrying or borrowing, and even this was only achieved with any accuracy when he used counters. He was quite unable to cope with simple problems, whether these were given orally
or in writing - his reading comprehension level at this time being about 8 to $8 \frac{1}{2}$ years. He had no visual or hearing defect, and there was no evidence of any specific memory weakness for oral or visual material. On the face of it, therefore, it seemed unlikely that John's poor progress in arithmetic was due to lack of the essential intellectual equipment, and it had in fact been assumed that emotional factors arising from, e.g. his early schooling, were putting obstacles in the way of his assimilation of arithmetic. This may well have been the case - not enough was known of his early school life to be sure about this - but from his responses to the Piaget tests it was clear that there was an intellectual weakness which was enough in itself to make even simple arithmetic a very difficult subject for him.

He failed in the tests of Experiment I chiefly because he focussed his mind on the water level, and ignored the width of the glasses. He then passed quite easily in Experiment II and V, did not quite make the grade in III, and failed definitely in IV, VI, VII and VIII. His
failure in Experiment VII was a surprise. When the two series had been constructed for him, and the line of hoops or boys closed up, he could be heard talking to himself, saying "A big hoop for a big boy, and a smaller hoop for a small boy." He seemed therefore to have a vague idea of what the serial order entailed, but was unable — through lack of complete reversibility, if we accept Piaget's theory — to re-cast the closed-up set in his mind with sufficient precision for him to identify the various items in serial correspondence to one another. His failure in Experiment VII was very emphatic, even with several repetitions and care being taken to ensure that the dangerous linguistic "catch" in the question should not trip him up. His failure in Experiment VIII had an interesting point. He said he would have more sweets with $1 + 7$ than with $4 + 4$, and when asked why, replied pointing to the 7, "Because you spread them out like that, and that makes them ...", and then followed a clear gesture indicating "more". At a later session the experiment was repeated, but on this occasion the question was put thus "Will you have a bigger number of sweets in the afternoon or in the morning"; he hesitated, was urged to
count them, did so correctly, and replied "They'll be the same." On reconsidering the matter, perhaps he should have been graded Stage II at least in this experiment. His initial failure, however, contrasts unexpectedly with his success in Experiment II, and his success was achieved only after "number" had been suggested to him.

A further short test was added at this point. Two rows of familiar objects were placed in front of him. They were to be Christmas presents for mother and father, and consisted of, for father - 3 boxes of matches, 1 pipe and 2 handkerchiefs; and for mother - 1 bottle of scent, 2 little gold pins, and 3 handkerchiefs. The rows of objects were the same length, with the first and last objects of each row in direct correspondence, but the intervening objects not necessarily opposite one another. He was asked, "Who is getting the most presents, mother or father?", and replied at once "They are getting the same." When asked how he knew this, he pointed to the two rows with what appeared to be an emphasis on their length rather than their number. One line of presents was therefore closed-up, and on being asked again "Who has most presents?" he pointed to the extended line, saying
"That one." This was followed by three or four more simple improvised problems, including the following. Two lines of sticks placed before him as follows:

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The upper row sticks being about 6 ins long, the lower ones about 3 ins. He was told "These (the longer sticks) belong to Harry, and those (the shorter ones) are Tom's sticks. Who has the most sticks?" He pointed at once to the upper row. He was told "Yes, each of these sticks is longer, but who has the bigger number of sticks - you count and see." He counted correctly, but seemed unable to make a decision.

When the two sums were written down for him to add, without using counters, his result was $3 + 3 = 6$ and $4 + 2 = 5$.

From all the evidence, it seemed pretty clear that much of John's thinking at this time was at a purely perceptual level, and that in only a few simple situations could he apply concrete operations. The Terman Merrill mental age may have been due in part to verbal facility and the effects of a good home background, his father being a well-educated professional man who with his wife devoted much time to the boy before he attended this residential school.
(c) This boy, Edward, is No. 6 on Table 2. He had been just over 3 terms in the school, and as the table shows, had a mental age of between 7 and \(7\frac{1}{2}\) at the time of his examination. He was in Category 1 for arithmetic in the teacher's estimation. Apart from the significance as regards conceptual thinking of his failure in all the experiments, the other notable point was the degree of confusion he showed, indicating that sometimes the language of the problems was quite beyond his comprehension. In Experiment II we see clearly the absence of the numerical operation. He made the correct correspondence between bottles and glasses, and agreed that there were the same number of each. When the glasses were bunched he said that there were more glasses, and on being asked why this was so, answered "Because they (the glasses) are bigger." When they were made to correspond again, a glass beside each bottle, he still said there were more glasses, and for the same reason. It seemed that he was thinking not of number, but of the actual size of each glass compared with each bottle, and the glasses were, in fact, larger than the bottles. Glasses and bottles were then counted, and it was stressed that there was
not a bigger number of one than the other, but the same number of each, to which he agreed. Then when the bottles were this time bunched, he said that there were more bottles; and when asked why, could only reply rather hesitatingly, "Because they're glasses!" In Experiment VIII he was again most confused, and some of his responses suggested furthermore what may be purely lack of comprehension of language adding to the confusion. He had answered that he would get the same number of sweets on each day, i.e. on the face of it that he saw that $4 + 4 = 1 + 7$. However, when asked how he knew this he replied, "I've got $14$." He was asked to count how many he would have for each day, and he counted both lines getting 16 in all. He was then shown the sweets for the first day, and then for the second, and told to count those for the first day only. "How many will you have then on the first day?" "8" "Have you the same on the next day?" "Yes" "How many have you on the second day?" (pointing it out clearly) "18" Section 6. It is now a psychological truism to say that although a mentally backward subject of 16 may have the same mental age on an intelligence scale as a 6 year old child,
qualitatively his mental endowments are vastly different; and the difference does not merely apply to such factors as interests and emotional attitudes, but may extend equally to basic processes of thinking. The following notes from the record sheets of mentally defective subjects make this very clear. In each case the subject failed far more profoundly than would normal young children testing out at the same mental age. The linguistic factor in particular appeared to fox these defective subjects much more than the young children, so that one felt that the defective was always more remote from the problem than the children.

(d) This was Henry, subject No. 32, a boy of 13½ years, whose mental age was 5½. He was a pleasant, amenable lad, who talked readily, and who superficially at least did not appear to be more than a somewhat dull school pupil. He would go simple messages, and could be trusted to look after himself on the roads.

In Experiment II he placed the bottles against the glasses counting correctly as he did so - thus he made correct correspondence. He was then asked "Are bottles and glasses the same in number?" to which he gave no reply. The
question was then put in various forms, "Are there more glasses or more bottles?", "Which have you most of, bottles or glasses?" etc, but he would merely echo the second alternative, "More bottles, glasses, etc". When one set was closed-up, the spread line was always said to be more or bigger than the other. Using flowers and bottles, the examiner then said, "I want one flower for each bottle. Take as many flowers as there are bottles." He put a flower into each bottle thus giving spontaneous correspondence. "Which have you got most of S or B?" The response was as before. He appeared to have no idea of the meaning of "more" or "less". Some counters were then taken making two lines consisting of 8 counters and 6 counters. On being asked "Which has the most?", he correctly pointed to 8 counters. "Then make them equal". He made the line of 6 up to 9 counters. "Count the lines for me." He counted 8 and 9 correctly. "Which has most." He pointed to 8. When asked to make them the same he added one to the line of 8. He was then taken back to the flowers and shown a line of 8 flowers and 8 bottles. Although the two sets corresponded accurately,
and the correspondence was pointed out to him, when asked
"Which have you most of, flowers or bottles?", he replied
"Bottles". When two bottles were removed, he saw that there
were more flowers, but when two flowers were taken away, thus
making the sets equal, he then said that the bottles were more.
When the lines were equal he could not see that they were
equal unless the question was put in the form "Are they
the same?", i.e. if one said "Which has the most?", although
they were obviously equal and he had agreed that they were
equal, he could not retain in his mind this equality but
echoed the suggestion implied in the question "Which is most?"

After a rest period, the examiner placed before Henry
a set of 4 counters. The boy was then told, "You put
another line down, just as big as this one." He put a line
of 7. When asked "Is your line the same as mine?" replied
"Yes". He was made to look carefully at the 4 counters,
and he was told to make another line like that. At first
he merely pointed to the set of 4, but on being encouraged,
he put 4 in line close to the first set, and then added 3 more.
Asked how many he had put out, he counted correctly 7. Asked
further "How many are here?" He said "4." On being told to make the two lines the same, he only nodded and smiled.

Finally three sets of 4, 5 and 4 counters were shown to him, and, with the first set being pointed out to him, he was asked to point to the other set which was "like this one." He ignored the other two sets, and pointed to the one which was at the moment being shown to him.

(e) Subject No. 31, a boy Ian. In Experiment I it appeared to be impossible to make him see what each test was about. All the normal 5 year olds knew at least that they were to make a comparison of two or more quantities, although they might invariably make an erroneous comparison. Ian, however, who was a boy who chattered incessantly, showed no sign of making a comparison. He would look at one glass, make a quick gesture with his hands as if estimating its quantity, then make quite irrelevant comments about it. Even using the two larger jars, A three-quarters full, B one-quarter full, he could not be made to consider the two together and to decide which "had most, was bigger, would give him the most to drink, etc". In Experiment II, he put
out 8 glasses to correspond with 6 bottles, and this only after repeated encouragement. When he was shown the two lines of 8 white and 6 red flowers, to see if he could understand that they were equal after 2 had been added to the 6, he could make no comparative judgment at all about their size, bigness or number. A surprising point about this boy, finally, is that he had a reading age on Burt’s Word List Test of between 7 and 7\(\frac{1}{2}\) years; his reading comprehension however appeared to be at about a 6 year old level.

(f) The oldest of the defective subjects was No.28, Willie, who was 41 years. His mental age of 5;2 was possibly a slight under-estimate, since he made two or three doubtful responses for which he was not credited with any points. He was a smiling, friendly little man, who played the part of "teacher’s help" in the centre, running messages and doing a variety of chores. Willie was shown a line of 4 flowers, and asked to make one the same length (the request being made in a variety of linguistic ways). After some hesitation he put out a line of 8. He was then asked "Are they really the same length?", 
to which he replied "No", and on being urged again to make them the same, he took his 3 flowers and added them to the original 4.

These instances illustrate the profound difficulty defectives may have in handling such words and expression as "more, most, bigger than, the same as, equal to, like each other", etc. Cases 24 to 34 all showed the same tendency in varying degrees. In the context of Piaget's experiments, these words imply a quantitative comparison, and it is in making the comparison that the subjects break down. They must be able to hold in the imagination, or actually look at, two objects - say jar A and jar B, or a set of red flowers and a set of blue flowers. The two objects must be linked by a concept of quantity - size, amount, number. The concept becomes active or operational when the mind can pass in attention from one object or image to the other, and then back to the first. Unless it has this reversible mobility along with a distinct orientation towards quantity, it cannot be truly operational, and no comparison of quantity can be made. At this stage of
development, subjects can make only a uni-dimensional comparison, i.e. jars A and B must be identical, with the only difference being level of water: the set of red flowers must be spaced similarly to the blue, the only difference being total length. When, however, two-dimensional differences are introduced, e.g. jar A different in shape from B, or the blue flowers different in density as well as length from the red, a second element of the concept of quantity must be introduced to the first schema, namely the conservation of quantity - as Piaget has elaborated. The responses of the normal 5 year olds in this investigation suggest that although they may not have reached the level of a schema having conservation of quantity, they have reached the simple form of quantitative comparison referred to above. Whereas many defective subjects whose test mental ages are 5 to 6 years, have not yet reached this simple schema, and quantitative comparisons even at a uni-dimensional level, are therefore beyond them.

(Note: Some of the case records quoted here and in Part III suggest that the interrogations may have been rather tortuous and drawn out affairs for the subjects concerned. Let it be stressed again that no interrogation was pursued any further when a subject showed signs of fatigue or boredom. The questioning was not nearly so intensive as these quotations suggest).
PART V. PRE-NUMBER CONCEPTS AND THE TEACHING OF ARITHMETIC

Piaget has not yet entered the British classroom, although his steps can now be heard in the corridor. The following observations will refer to some specific attempts to link Piaget's work to classroom practice, and will also indicate what his "message" should be for the teacher of arithmetic in the junior school.

(1) The first reference is to a book by Hans Aebli, unfortunately not translated, which deals with the application to teaching of Piaget's psychology. Besides summarizing beautifully Piaget's ideas, Dr. Aebli discusses their relevance to the process of education, and outlines specimen lessons in which Piagetian notions are specifically the foundation of the teaching procedure. These lessons deal with a mathematical theme, but suggestions are made on how the underlying principles can be extended to other subjects. The series of lessons was given to an experimental and a control group of pupils, the latter taught on orthodox lines. Needless to say, the experimental group mastered the topic more thoroughly, but the most encouraging point is that the intellectually poorer members of this group showed much more successful learning than their counterparts in the control group. The distinguishing feature of the teaching of the

a:Ref. 3 26.
experimental can only be summed-up briefly here. It endeavoured to provoke a progressive construction of operations by the pupils; they had to discover new operations by personal research carried out either individually using experimental material, or collectively as a result of group discussion. Note that the "new operations" were not merely new practical skills of experimental procedure, but were new logical structures, new pieces of mental equipment. How this orientation of the lesson is achieved is discussed by Aebli in considerable detail.

2. A booklet published by the National Froebel Foundation is concerned solely with Piaget’s book on number, and one of its contributors, T.R. Theakston, discusses the book’s implication for the teacher. Most of his points have been touched on in various sections of this thesis, and in brief Piaget's effect on class-room practice should be as follows:

(a) Few children below a mental age of 6 years are intellectually equipped to cope with number with any real understanding;

(b) the need for more specific attempts to teach the elementary vocabulary of mathematics is still very great - words like "more", "less", "equal to" etc are often

Ref. 27.
relatively meaningless even to 6 year olds; (c) free play
with a wide variety of materials will help both to
develop arithmetical vocabulary, and to give the child
opportunities for judging quantities, making comparisons, and
so on: (d) ability to count does not necessarily mean that
the child understands numbers, nor that he has the notion
of conservation of quantities upon which understanding of
number is based; (e) the idea of "arithmetical readiness needs
to be carefully reconsidered by teachers in the light of
Piaget's teaching. This entire Froebel booklet is
invaluable to teachers who wish to apply Piaget's ideas to
their classroom practice.

3. A most interesting paper on the growth of mathematical
concepts in children through experience was published by
Z.P. Dienes. Mr. Dienes has developed a complete teaching
course which aims at encouraging the understanding of
mathematical concepts in school children, and which is trying
to direct teachers away from the more orthodox, rule-of-thumb
method of teaching both elementary number and advanced
mathematics. One of his sources is Piaget, to whose work he
refers in this paper. Dienes would like mathematics teaching

a. Ref. 31.
to be founded on "individual mathematical discovery", and he would like to see children able to "enjoy abstract games without any tangible material". "Only a very few," he says "are able at the moment to break through the morass of formalism to the beauty of the structure beyond, and those who finally decide to play with this structure are the ones who become mathematicians".

4. Piaget and Cuisenaire.

Perhaps more than any other teaching method, the Cuisenaire technique stands as the counterpart in classroom practice to the number theory of Piaget. Cuisenaire's ideas were developed and to some extent re-interpreted by Gattegno, and glancing through Gattegno's books outlining the method one finds oneself in, as it were, the same "universe of discourse" as Piaget, in which are commonly used the symbols and linguistic expression of mathematical logic and theory - all this being somewhat unusual in the common run of teacher's handbooks. There are at least two cardinal points at which Piaget and Cuisenaire (whether Gattegno-ised or not) meet and re-inforce one another. In both there is the same insistence on the child exploring, experimenting and finding out for himself.
"The child starts from the beginning and is compelled to re-discover arithmetic for himself, at his own pace and according to his own capacity", thus Cuisenaire in opening a summary statement of his method. And on Piaget's side, the element of finding out by the pupil is stressed over and over again in Aebli's book -

"c'est au cours des recherches (d'ordre naturellement très différent) que s'opère chez l'enfant, comme chez l'homme de science, le progrès de la pensée."

Secondly, the Cuisenaire material has certain characteristics which must surely help to crystallize the pre-number concepts as they emerge slowly from the fluid mass of the child's intuitive, pre-logical thinking. The material first of all has great plasticity; numbers are immediately recognisable, are quickly composed and de-composed, and put into a wide variety of relationships with one another. Furthermore, the material provides the child with a set of images which not only have great mobility, but which are mathematically exact and which will enable representationally exact number structure to develop in the child's mind. As Cuisenaire says,

"The child is gradually brought to a certain level of abstraction through repeated practice in seeing mentally. Since it is the child's own thought which

a: Ref. 30 p. 21.
takes material form through his own manipulations and with the active intervention of all his senses, colours and dimensions thus being constructively associated, his analytic capacity is developed through his own calculations and his own experience. He acquires without strain mental flexibility and an attitude of objectivity."

In brief the material concretizes the abstract form of number with complete accuracy.

It may not be impossible for a teacher on occasion to use the Cuisenaire sticks to stir up a pre-number concept in a sluggish child, and it seems a likely speculation that the use of the material in routine number teaching will pin-point such concepts with precision and accuracy at the earliest appropriate moment in the process of maturation. Consider the concept of conservation for instance. The child experimenting with the number 10 rod will discover that it is the same as the 5 and 5, or the 2, 2, 2, 2, and 2 rods put end to end. The equivalence is plainly seen. There will, however, be a stage when the equivalence is not lasting, and when the child will not realize that when the five number 2 rods are spread out they still are collectively as long as the 10 rod. But the moment of passage to Stage III, when the child will understand the lasting equivalence of the five number 2 rods, should be
facilitated, since in his many hours of exercise and experimentation with all the rods, the child has had his mind gradually concentrated on and tied to number as such, with the two irrelevant perceptual factors of "space between" and shape reduced in their potency. Moreover, reversibility is very soon practised by the child as a simple external action (spreading out the five 2s, and putting them back to equal the 10), and is ready to be taken over as an internalised action, or operation, at the moment of maturation. In this way a teacher can create the most favourable conditions for the emergence of an operation.

The operation of equalization of differences can easily be illustrated, at first if necessary without any reference to number. Suppose we build two "houses" with the rods as follows:

![Diagram of two houses built with rods]
Let the child see that they are identical at first, and then alter the second one as shown. "Do the houses still have the same number of rooms?" At first he may be puzzled, but at the appropriate moment the child will see, and understand, that although the second house is now not so broad as the first, it is now taller by the same amount as was taken off its breadth.

Other concepts, for instance seriation and class inclusion, can easily be shown to be capable of simple and vivid demonstration, giving the child an opportunity to realize such concepts in a mathematical situation at an early point in his number development.

It will be remembered that Piaget has defined number as "the fusion of class and asymmetrical relation into a single operational whole....each number is a whole, born of the union of equivalent and distinct terms, it cannot be constituted without inclusion and seriation." The Cuisenaire material seems to give concrete expression to these ideas about the essence of number. Seriation obviously comes in at an early point when the child builds a "stair" of the rods from 1 up to 10, or otherwise orders them in a regular increasing or diminishing series according to size. Furthermore, working and playing with the material, the child will come to appreciate that, for instance, if
he wishes to have "5", he will take a yellow rod; that it doesn't matter which yellow rod he takes, they are all inter-changeable and equivalent, all elements in one class; but that although elements in a class, they are yet distinct, they can be laid out in a series and they can be added. He knows that if he adds any two yellow rods, he will then have something which is equivalent to any member of another class, the class of 10 rods. In brief, the Cuisenaire material seems to play around the conceptual nuances of Piaget's theory of number.

Performing arithmetical operations with the rods, however, is not the primary aim of the Cuisenaire method. The idea is that the child should learn to perform the operations mentally and without relying on the rods. As long as the child relies on the rods, or even too much on the images of the rods, number will tend to be rooted in length. There must come a point when, for instance, "6" will refer not to the rod 6, but to any set of six objects, whatever their size or distribution. In the problem of conservation, moreover, a child might understand conservation when the sets consist of rods, but not if the sets are variously sized objects. In teaching therefore one might
trod to show the child that the set of rods 4, 2, and 3 are equivalent to the 9 rod, just as surely as rods 3, 3 and 3, even although the elements of the first set are variously sized.

Again, suppose a child appreciates conservation of equivalence of rod 6, and rods 1, 1, 1, 1, 1 and 1 put end-to-end are opened out. The insight involved in this can be extended to the situation of a bunch of 6 flowers, and the same flowers considered individually. The child will come to understand each number 1 rod as "standing" for a flower, and the 6 rod as representing the bunch of flowers. The rods will form an accurate foundation from which to generalize numbers and so to make its application universal. The common error in arithmetic teaching today is that when number becomes generalized in the child's mind, as it must and ought to be, it becomes a rather flabby, ill-formed concept with no hard core of precision. The rods will anchor the generalized number concepts to a steady and accurate foundation. Such then is the link between the two most significant faces today in the field of practical number training for children.

Concluding note. In this thesis we have tried (a) to discuss Piaget's theory of the development of number in children, (b) to assess and to some extent to validate his experimental
findings with ordinary school children; (c) to relate these findings to achievement in arithmetic; (d) to study according to Piaget's notions the development of number in mentally retarded subjects; and finally (e) to discuss the relevance of Piaget's theory of number development to current teaching practice. Since our work has perforce been done in bits and pieces spread over five years, we hope that it has not become too shapeless and ragged. Most certainly however we have found the intensive study of Piaget while preparing the thesis involved a very rewarding experience, and speaking as a somewhat elderly student, perhaps we may take the liberty of recommending Piaget to other students who are on the look-out for suitable topics on which to base an academic thesis. He is worthy of much more study than has so far been given to him in this country, particularly in the field of education, and we agree wholeheartedly with Nathan Isaacs' estimate of Piaget in the Froebel booklet:

"Piaget's work, ...provides us with a clearer yardstick of true internal organisation and structuration of thought than we ever possessed before. We can now see far better than before how little we gain by aiming at anything less, as educators, than integrated growth from the start and all the way through. Our achievement will no doubt always remain highly imperfect; but Piaget's researches provide us with an unprecedented wealth
of new information and insight to guide us on the right way. They bring into focus some of the most pivotal points and directions to which our educational efforts need to be addressed."
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NOTE: When the French title of Piaget's works is not given, the date refers to the publication of the English edition only.


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