HIDDEN RISKS IN THE CDO – SQUARED MARKET

Rajiv Bhatt
Andrew Adams
James Clunie

September 2005

Working Paper 05.03

ISBN: 1 902850 78 5
ABSTRACT

We show that there are risks (default location risk and overlap risk) unique to CDO-squared structures. These risks may not be well-understood by investors and credit rating agencies. As a result, the tranche of a CDO-squared having the same name and similar rating to the tranche of a CDO may have a very different risk profile, and the returns offered to CDO-squared investors may be unattractive on a risk-adjusted basis. We believe that the hidden risks in CDO-squared structures will be clearly exposed in a distressed credit environment.
Introduction

A Collateralized Debt Obligation (CDO) is a series of obligations that are dependent on the performance of a portfolio of underlying assets (collateral).\(^{1}\) CDOs extend the technology of securitization by tranching the collateral cash flows into tailor-made notes to offer returns to investors with diverse risk/return needs. Since their invention in the 1980s, CDOs have evolved into innovative and complex structured credit products.

A significant recent innovation has been the so-called CDO-squared (CDO\(^2\)), that is a CDO mainly invested in tranches of other CDOs (Cifuentes, 2004). The first CDO\(^2\) was structured in 1999. After a slow start, the CDO\(^2\) market has grown rapidly since 2002, largely due to a benign credit environment, relatively tight credit spreads, and investment banks’ pursuit of fees. Investors’ ‘search for yield’ (in a tight credit spread environment) is widely thought to be the primary motivation behind the structuring of a CDO\(^2\).

Concerns regarding the complexity and lack of understanding of risks in CDOs and CDO\(^2\)s have been expressed frequently in the financial press. However, academic research on CDOs has largely focused on modeling correlated defaults and valuation of CDO tranches. Duffie and Garleanu (2001) do provide a comprehensive risk analysis of CDOs but little, if any, research has been published analyzing the risks in a CDO\(^2\). This paper is a step in that direction. By constructing a model of a simple CDO\(^2\) structure, we aim to provide a better understanding of the known risks, and to explore any hidden risks within these structures. The focus of the paper is on understanding and highlighting the nature of the risks, rather than on tranche valuation or risk quantification.
Structural Characteristics of a CDO\(^2\)

While the collateral pool of a CDO\(^2\) mainly comprises tranches of other CDOs (‘inner CDOs’), asset-backed securities could also constitute part of the collateral pool. A Cash CDO\(^2\) is backed by tranches of existing cash CDOs, whereas a Synthetic CDO\(^2\) is backed by a portfolio of synthetic CDOs. Generally, the underlying CDOs of a synthetic CDO\(^2\) are created specifically for inclusion in the CDO\(^2\), and are merely conceptual structures created to compute cash flows and values of the CDO\(^2\). Losses in the collateral pool first flow into the inner CDO. Typically, losses exceeding the ‘attachment point’ of an invested tranche flow into the CDO\(^2\) structure until the ‘detachment point’ of the invested tranche is reached.

Figure 1: Typical CDO\(^2\) Structure

The highlighted tranches of the inner CDOs are tranches in which the CDO\(^2\) is invested. The lower number in parenthesis indicates tranche subordination and the difference between the numbers indicates tranche size. For example, the equity tranche of the left most inner CDO has no subordination and its size is 5% of the inner CDO’s par value. Similarly, the mezzanine tranche of the CDO\(^2\) has both a subordination and a size of 3% of the CDO\(^2\) par value.
A typical CDO^2 might reference^2 as many as 1000 corporate names (Gilkes and Drexler, 2003). Given a limited universe of investment grade credits^3, it is quite likely that some corporate entities are referenced by more than one inner CDO. This overlapping of reference entities could potentially create new risks in a CDO^2.

**CDO^2 model**

While risks in a CDO^2 are largely a function of the risks in ‘inner CDOs’, there could be some additional and potentially unknown risks, unique to a CDO^2. Figure 2 shows the structure of a model that captures the essential features of a CDO^2. A CDO^2 is invested in tranches of two inner CDOs, namely CDO1 and CDO2. Each inner CDO has three tranches: an equity tranche, a mezzanine tranche and a senior tranche. A CDO^2 could be invested in different tranches of the inner CDOs. For example, the CDO^2 could be invested in the senior tranche of CDO1 and the mezzanine tranche of CDO2.

*Figure 2: Structure of the CDO-Squared Model*

We assume that the inner CDOs are ‘Cash CDOs’ with a collateral pool comprising equally-weighted and similar-rated corporate bonds (‘reference entities’). The
modeled CDO^2 is therefore a ‘Cash CDO^2’. Some reference entities form part of the collateral pools of both inner CDOs. These entities are referred to as *overlaps*.

The CDO^2 model can be segregated into three sub-models which are: Inner CDO collateral model, Inner CDO model, and CDO^2 model. Figure 3 illustrates the linkages between these three sub-models. Interest payments, default losses and maturity proceeds from the collateral pool flow to the inner CDO and are allocated to the tranches of the inner CDO according to the priority rules. Interest payments, tranche losses, and maturity proceeds of all the invested tranches (senior tranche in this illustration) are accumulated and allocated to the CDO^2 tranches according to the priority rules.

*Figure 3: Interaction between the three sub-models*

Each sub-model is described below.

**Inner CDO Collateral Pool Model**

The characteristics of the collateral pool of the *j*th inner CDO, where *j* \( \in \{1,2\} \), are given in Table 1.
Table 1: CDO collateral pool characteristics

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of reference entities</td>
<td>$n(CDO_j)$</td>
</tr>
<tr>
<td>Number of overlapping entities</td>
<td>$n(Overlap)$</td>
</tr>
<tr>
<td>Par value of each bond</td>
<td>$P_j$</td>
</tr>
<tr>
<td>Total initial face (nominal) value</td>
<td>$CDO_j(0) = n(CDO_j) \cdot P_j$</td>
</tr>
<tr>
<td>Weighted average coupon (WAC)</td>
<td>$WAC_j$</td>
</tr>
<tr>
<td>Weighted average life</td>
<td>$T_j$</td>
</tr>
<tr>
<td>Recovery rate</td>
<td>$REC_j$</td>
</tr>
<tr>
<td>Coupon payment frequency per annum</td>
<td>$F_j$</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>$T_j \cdot F_j$</td>
</tr>
<tr>
<td>Duration of each time step (fraction of year)</td>
<td>$S_j = 1/F_j$</td>
</tr>
</tbody>
</table>

Assumptions underlying the collateral pool model are:

- The term structure of interest rates is flat.
- Defaults occur only once during the pool’s weighted average life.
- Defaults occur at the end of a period.
- Recovery occurs in the same time period as the default.
- Default time (t) is chosen randomly between 0 and the collateral’s weighted average life. i.e. $t \in \{0, S_j, 2S_j, 3S_j, ..., T_j\}$
- Defaults occur discretely.
- Default time is the same for collateral pools of all inner CDOs.
- The number of defaults in the collateral pool follows a random distribution.

Reference entities are segregated into 1) those referenced by a particular CDO only ("Unique Pool") and 2) those that are overlapping ("Overlapping Pool"). The number of unique entities in the $j$th inner CDO is given by:

$$UCDO_j = n(CDO_j) - n(Overlaps)$$

Defaults are modeled within the unique pool and the overlapping pool. The number of defaults occurring in the overlapping pool is $DEFLAP_j(k)$ and the number of defaults in the unique pool is $DEFUNI_j(k)$. Total number of defaults at time $k$ in the collateral pool of the $j$th CDO are given by:
\[ DEF_j(k) = DEFLAP_j(k) + DEFUNI_j(k) \]

Interest on the collateral par value outstanding at the beginning of a period \( k \) is received at the end of the period, and is given by:

\[ INT_j(k) = CDO_j(k-1) \cdot WAC_j/F_j \]

When any reference entity defaults, the collateral par value is reduced by the par value of the defaulted entity. The par value of defaulted entities at time \( k \) is given by:

\[ PDEFT_j(k) = DEF_j(k) \cdot P_j \]

and the remaining collateral par after default at time \( k \) is

\[ CDO_j(k) = CDO_j(k-1) - PDEFT_j(k) \]

A fraction \( (REC_j) \) of the par value of defaulting entity is recovered. The amount recovered from defaulting entities is given by:

\[ REC_j(k) = PDEFT_j(k) \cdot REC_j \]

The loss given default (LGD) and cash flows from the collateral pool flow into the inner CDO. Loss given default of a reference entity in the collateral pool of the \( j \)th inner CDO at time \( k \) is

\[ LGD_j(k) = PDEFT_j(k) \cdot (1 - REC_j) \]

and the total cash flow from collateral pool at time \( k \) is given by:

\[ TCF_j(k) = REC_j(k) + INT_j(k) \]

\( TCF_j(k) \) and \( LGD_j(k) \) are inputs to the inner CDO model.
Inner CDO Model

The tranches of the inner CDO have a subordination level (% of CDO value), coupon rate, and size of $SCDO_{ji}$, $Coup_{ji}$, and $CDO_{ji}$ respectively, where

$$i = \begin{cases} 
0, \text{ if Tranche is Equity} \\
1, \text{ if Tranche is Mezzanine} \\
2, \text{ if Tranche is Senior} 
\end{cases}$$

and $SCDO_{j0} = 0$ (i.e, subordination of the equity tranche is zero).

The absolute value of tranche subordination is defined as

$$SUB_{ji} = CDO_{j} \cdot SCDO_{ji}$$

Tranche sizes at deal inception (time = 0) are given by:

$$CDO_{ji}(0) = CDO_{j}(0) \cdot (SCDO_{ji+1} - SCDO_{ji}), \text{ for } i = 0, 1$$

$$CDO_{j2}(0) = CDO_{j}(0) - CDO_{j0} - CDO_{j1}$$

The model assumes a uniform prioritization waterfall, wherein the interest received from the collateral pool is first used to pay interest to the senior tranche and then to the mezzanine tranche. If the interest paid to a tranche is less than the interest due to that tranche, the shortfall is accrued at that tranche’s coupon rate. Default losses are reduced by any excess of interest received from collateral over total interest paid to the tranches (distributable default loss). Distributable default losses are absorbed by tranches in reverse priority, i.e from the equity to the senior tranche. Any excess cash flows (interest income and recovery amounts) from the collateral pool are accumulated in a reserve account earning a risk-free interest rate. Interest earned on
the reserve account is reinvested in the same account. Funds in the reserve account are similar to a capital reserve and are not used to meet any shortfalls in interest payments to tranches during the life of the CDO. At the end of each period, the tranche par value is reduced by the par value lost due to default losses. At CDO maturity, the remaining collateral pool is liquidated at its face value at maturity, and the proceeds transferred to the reserve account. The balance in the reserve account is then used to pay off the senior and mezzanine tranches to the extent of their face values outstanding at maturity, and any residual amount is paid to the equity tranche.

**CDO^2 Model**

The tranches of CDO^2 have subordination (% of CDO value), coupon rate, and size of $SCDO_i^2$, $Coup_i^2$, and $CDO_i^2$ respectively.

Suppose the CDO^2 is invested in the $i^{th}$ tranche of the inner CDOs. Then the total initial nominal value of CDO^2 is

$$CDO^2(0) = \sum_{j=1}^{N} CDO_j(0) \quad \text{where } i \in \{0,1,2\}$$

The size of equity and mezzanine tranche is given by:

$$CDO_i^2(0) = CDO^2(0) \times \{SCDO_{i+1}^2(0) - SCDO_i^2(0)\}, \text{for } i = 0,1$$

The size of senior tranche is given by:

$$CDO_s^2(0) = CDO^2(0) - CDO_0^2(0) - CDO_1^2(0)$$

Total interest received from underlying tranches at time $k$ is given by:
\[ TINT(k) = \sum_{j=1}^{N} PINT_{ji}(k) \quad \text{where } i \in \{0,1,2\} \quad \text{and } PINT_{ji} \text{ is the interest paid to } i^{th} \text{ inner tranche of } j^{th} \text{ inner CDO} \]

Total loss flowing from the underlying tranches at time \( k \) is given by:

\[ TLOSS(k) = \sum_{j=1}^{N} LOSS_{ji}(k) \quad \text{where } i \in \{0,1,2\} \quad \text{and } LOSS_{ji} \text{ is the loss suffered by } i^{th} \text{ tranche of } j^{th} \text{ inner CDO} \]

\( TINT(k) \) and \( TLOSS(k) \) are the inputs required to model the cash flow and loss to \( \text{CDO}^2 \) tranches. With these inputs, the \( \text{CDO}^2 \) model is similar to that of the \( \text{CDO} \) model, and hence not described in detail here.

**Risk Measures**

The measures used to analyze the risks in a \( \text{CDO}^2 \) are:

1. Tranche Loss
2. Tranche Loss Rate
3. Total Loss Rate, and
4. Economic Tranche Loss

Measures 1, 2 and 3 do not consider the timing of default and the time value of any payments made to the tranches prior to default. They implicitly assume that defaults in the collateral pool occur in the first period.

*Tranche Loss* is an absolute measure of loss and is given by:

\[ TLOSS_{ji} = \sum_{k-s_i}^{T} LOSS_{ji}(k) \]
Tranche Loss Rate (also referred to as ‘wipe-out rate’) measures the fraction of the tranche size that is wiped-out due to losses, and is given by:

\[
TLOSSRate_{ji} = \frac{TLOSS_{ji}}{CD_{ji}(0)} \quad \text{for inner CDO tranches}
\]

and

\[
TLOSSRate_{ji} = \frac{TLOSS_{ji}}{2CDO_j^2} \quad \text{for CDO}^2 \text{ tranches}
\]

Total Loss Rate for CDO (CDO^2) is the fraction of total par value of the CDO (CDO^2) that is wiped-out, and is given by:

\[
TOTLOSSRate_{ji} = \frac{\sum_{t=0}^{2} TLOSS_{ji}}{CDO_j}, \text{ for } j = 1,2
\]

The present value of any interest or principal received by a tranche investor before default represents the economic value received by the investor. The later the default, the greater is the economic value. Measure 4 is an economic value measure which considers the timing of default and the time value of any payments made to the tranches prior to default, and hence represent the economic loss to tranches.

Economic Tranche Loss is the difference between initial face value of the tranche and present value of cash flows conditional on default discounted at the tranche coupon rate.\(^5\) It is given by:

\[
X_{ji} = CDO_{ji}(0) - \sum \frac{PINT_{ji}}{(1 + COUP_{ji})^n} + \frac{MAT_{ji}(T)}{(1 + COUP_{ji})^T} \quad \text{for inner CDO tranches, where}
\]

\[
MAT_{ji} \text{ is the amount paid to } i^{th} \text{ tranche of } j^{th} \text{ inner CDO.}
\]

and
\[ X_{ji} = CDO_1^2 (0) - \sum \frac{PINT_{ji}}{(1 + COUP_{ji})^n} + \frac{MAT_{ji}(T)}{(1 + COUP_{ji})^T} \] for CDO^2 tranches

**Monte Carlo Simulation**

Due to its structural complexities, a CDO^2 cannot easily be modeled by a systematic analytical process. But Monte Carlo Simulation (MCS) can be used to model the complexities (such as subordination structures, overlaps, correlations etc) of a CDO^2 in an intuitive way. The behaviour of various tranches under different default scenarios can then be observed. Such observations provide insights into the risks in a CDO^2.

Base parameter values of the modeled inner CDOs and CDO^2 are shown in Table 2 and Table 3 respectively.

**Table 2: Inner CDOs base parameters values**

<table>
<thead>
<tr>
<th></th>
<th>Inner CDO1</th>
<th>Inner CDO2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Collateral Pool</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n(CDO)</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( P )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( WAC )</td>
<td>8%</td>
<td>8%</td>
</tr>
<tr>
<td>( T ) (years)</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>( REC )</td>
<td>40%</td>
<td>40%</td>
</tr>
<tr>
<td>( F )</td>
<td>Semi-annual</td>
<td>Semi-annual</td>
</tr>
<tr>
<td>( n(Overlap) )</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td><strong>Tranches</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mezzanine Subordination</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>Senior Subordination</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Mezzanine Coupon</td>
<td>8.25%</td>
<td>8.25%</td>
</tr>
<tr>
<td>Senior Coupon</td>
<td>8.15%</td>
<td>8.15%</td>
</tr>
</tbody>
</table>
Table 3: CDO^2 base parameter values

<table>
<thead>
<tr>
<th>Invested Tranche</th>
<th>CDO^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDO1 Senior</td>
<td></td>
</tr>
<tr>
<td>CDO2 Senior</td>
<td></td>
</tr>
<tr>
<td>Tranches</td>
<td></td>
</tr>
<tr>
<td>Mezzanine Subordination</td>
<td>5%</td>
</tr>
<tr>
<td>Senior Subordination</td>
<td>10%</td>
</tr>
<tr>
<td>Mezzanine Coupon</td>
<td>8.25%</td>
</tr>
<tr>
<td>Senior Coupon</td>
<td>8.35%</td>
</tr>
</tbody>
</table>

Descriptive charts derived from MCS are used to understand the risks in CDO^2. MCS is combined with scenario analysis to gain better insights into the characteristics unique to a CDO^2 (e.g. overlaps).

Results

The risk measures defined in the previous section are largely functions of the default rate and default timing. To understand the behaviour of (and hence the risks in) the CDO^2, it is important that the simulation captures different combinations of default rates and default timings which are representative of all possible combinations. A 1000-run simulation generates a fairly diverse combination set, which should capture most behavioral characteristics of the CDO^2 and all results that will be presented are based on a 1000-run MCS.

The Tranche Loss Rates (TLRs) for inner CDOs are sequential and monotonic. The mezzanine tranche suffers losses after the equity tranche is fully wiped out, and the senior tranche suffers losses after the mezzanine tranche is fully wiped out (hence sequential). For each tranche, TLR increases with increase in default rate, until that tranche is fully wiped out (hence monotonic). This makes it simple to estimate the TLR of CDO tranches for each additional default in the collateral pool.
Figure 4 shows that the TLRs of the CDO^2 tranches are sequential but non-monotonic. Equity and mezzanine tranches particularly show prominent non-monotonic TLRs. Different TLRs can be observed for a given default rate. It follows that, unlike the TLR of inner CDOs, it is not possible to estimate the TLR of CDO^2 tranches for each additional default in the collateral pool of the inner CDOs. To investigate these non-monotonic TLRs, we examine three data points. Table 4 shows the data underlying these data points, including the CDO^2 default rate, the inner CDO default rate and tranche loss rate for each data point.

Figure 4: Losses to tranches of CDO^2 are non-monotonic

![Graph showing non-monotonic tranche losses](image)

Table 4: Sample data points to examine the non-monotonic behaviour

<table>
<thead>
<tr>
<th>Case</th>
<th>CDO^2 Def Rate</th>
<th>Default Rate</th>
<th>Senior</th>
<th>Mezz</th>
<th>Equity</th>
<th>Total</th>
<th>CDO1 Default Rate</th>
<th>CDO1 Senior</th>
<th>Mezz</th>
<th>Equity</th>
<th>Total</th>
<th>CDO2 Default Rate</th>
<th>CDO2 Senior</th>
<th>Mezz</th>
<th>Equity</th>
<th>Total</th>
<th>CDO^2 Default Rate</th>
<th>CDO^2 Senior</th>
<th>Mezz</th>
<th>Equity</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25%</td>
<td>10%</td>
<td>0%</td>
<td>17%</td>
<td>100%</td>
<td>0%</td>
<td>40%</td>
<td>15%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
<td>51%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
<td>51%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>2</td>
<td>25%</td>
<td>12%</td>
<td>0%</td>
<td>41%</td>
<td>100%</td>
<td>1%</td>
<td>30%</td>
<td>14%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
<td>50%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
<td>50%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>3</td>
<td>25%</td>
<td>25%</td>
<td>5%</td>
<td>100%</td>
<td>100%</td>
<td>15%</td>
<td>25%</td>
<td>5%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
<td>5%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
<td>5%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The CDO^2 default rate is constant at 25% in all the three cases. A total of 50 entities out of the 200 entities of the inner CDO collateral pool default. However, the
distribution of the 50 defaulting entities is different in each case. In case 1, the number of defaulting entities in the inner CDO1 is 10 and in the inner CDO2 is 40. In case 2, the former is 12 and the latter is 38, and in case 3, the defaults are equally distributed. In other words, the concentration of default in CDO2 (CDO1) decreases (increases) from case 1 to case 3. The subordination available to the senior tranche of each inner CDO is 10. In case 1 with 40 defaults, CDO2 bears a loss of 24, whereas with 10 defaults CDO1 suffers a loss of 6. The senior tranche of CDO2 suffers a loss of 14 [i.e. 24 less subordination (10)], whereas that of CDO1 does not suffer any loss because its subordination is not fully exhausted. Hence, total loss of invested tranches is 14, which flows to the CDO^2. The CDO^2 loses about 8% of its par value, and the equity, mezzanine and senior tranches lose 100%, 51%, and 0% respectively of their par values.

**Default location risk**

Table 5 shows total loss suffered by the inner CDOs as a percentage of the total subordination available to the invested (senior) tranche. A value greater than 100% implies subordination is fully exhausted and the invested tranche suffers losses which flow to CDO^2.

<table>
<thead>
<tr>
<th>Case</th>
<th>CDO1</th>
<th>CDO2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>59%</td>
<td>239%</td>
</tr>
<tr>
<td>2</td>
<td>71%</td>
<td>227%</td>
</tr>
<tr>
<td>3</td>
<td>149%</td>
<td>149%</td>
</tr>
</tbody>
</table>

When defaults are concentrated in one inner CDO, the probability of loss reaching the invested tranche in that inner CDO increases. This is because the subordination of invested tranches is not effectively utilized. An effective utilization of subordination would mean that total default loss of all inner CDOs is evenly spread across all inner CDOs (Case 3 in Table 6). A worst-case scenario would be when all defaults occur in one inner CDO only. Figure 5 shows the extent of par value lost by CDO^2 when the 50 defaults are distributed differently in the inner CDOs.
Figure 5: Loss to CDO^2 depends on location of defaults in inner CDOs

<table>
<thead>
<tr>
<th>Case</th>
<th>CDO^2</th>
<th>CDO2</th>
<th>CDO1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>8%</td>
<td>24%</td>
<td>6%</td>
</tr>
<tr>
<td>Case2</td>
<td>7%</td>
<td>23%</td>
<td>7%</td>
</tr>
<tr>
<td>Case3</td>
<td>5%</td>
<td>15%</td>
<td>15%</td>
</tr>
</tbody>
</table>

50 defaults are distributed differently in the inner CDOs. In case 1, 10 defaults occur in CDO1 and 40 in CDO2. In case 2, 12 defaults occur in CDO1 and 38 in CDO2. In case 3, 25 defaults occur in CDO1 and 25 in CDO2. The percentages figures shown on the bars indicate the total loss rate of each inner CDO and the CDO^2, assuming a recovery rate of 40%. For example, total loss rate of CDO1 in case 1 is 10 x (1 – 40%) = 6%. Case 3 is the best-case scenario, optimally utilizing the invested tranche subordination, and hence the CDO^2 loss rate is minimum in that case.

Observation 1: For a CDO^2 investor, the distribution (location) of defaults in the inner CDOs adds a new dimension to default risk. We call this new dimension ‘Default Location Risk’.

Unlike default rate, default location is difficult to model. Figure 6 shows that there is a lower bound to the total CDO^2 loss at a given default rate. This lower bound denotes the best-case default location scenario, i.e., where defaults are evenly distributed within the inner CDOs. The scatter indicates ‘default location risk’. Default location risk explains why there can be different CDO^2 loss rates for a given total default rate in inner CDOs.
Default location risk also suggests that tranches of a CDO^2 and a CDO having similar ratings could have potentially different risk profiles. To investigate this proposition, we simulate a hypothetical CDO^2 using S&P’s CDO Evaluator 2.4.3 (‘CDO Evaluator’). The hypothetical CDO^2 consists of six inner CDOs each of par value 100,000,000. The recovery rate is assumed to be zero. The CDO Evaluator computes the scenario default rate [SDR] for each rating category. The SDR determines the Attachment Point [AP] (i.e., extent of subordination) required by a tranche with desired rating. The higher the default probability, the higher the tranche AP for a given rating category.

Figure 7 shows required tranche APs (as a percentage of notional) for the inner CDOs and the CDO^2 across the rating category spectrum. It compares the APs required by CDO^2 tranches when the CDO^2 is invested in inner CDOs as per each scenario in Table 6.
Table 6: Scenarios of CDO^2 investment in inner CDOs.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Tranche Rating</th>
<th>CDO Evaluator Rating</th>
<th>Tranche Size</th>
<th>Subordination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20% - 35% A-</td>
<td>15,000,000</td>
<td>20,000,000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10% - 25% BBB-</td>
<td>15,000,000</td>
<td>10,000,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5% - 20% B+</td>
<td>15,000,000</td>
<td>5,000,000</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7: Risks of similar-rated CDO^2 and CDO tranches could be different

The four bars for each rating category are (from left to right): Inner CDO, CDO^2 (20-35% :A-), CDO^2(10-25% :BBB-), and CDO^2(5-20% :B+).

Data underlying the chart is generated from S&P’s CDO Evaluator 2.4.3

It can be observed that, for a given rating category, the AP of an inner CDO is different from that of CDO^2.

**Observation 2:** The risk profiles of similar-rated CDO and CDO^2 tranches can be very different.

It can also be observed that when a CDO^2 is invested in ‘A-’ rated tranches of inner CDOs each having an AP of 20% and tranche size of 15,000,000, a CDO^2 tranche to be rated AA+ needs an attachment point of 50.67%. However, when a CDO^2 is
invested in BBB-rated tranches of inner CDOs each having a lower AP of 10% but the same tranche size of 15,000,000, a CDO^2 tranche to be rated AA+ needs an attachment point of 82.33%.

**Observation 3:** A lower attachment point of an invested tranche increases the risk of CDO^2 tranches, despite the invested tranche size being the same.

**Overlap risk**

We now investigate the impact of overlaps on a CDO^2. We create two additional scenarios, one assuming 20% overlap, and another assuming 50% overlap. A 1000-run simulation is performed on each additional scenario. Figure 8, Figure 9 and Figure 10 show the CDO^2 total loss rate at various unique defaults in inner CDOs under the three scenarios. Comparing these charts, it can be observed that as overlap increases, the total loss rate gets more scattered for a given number of defaults. This indicates a greater ‘default location risk’ at higher overlaps. This is because one default in the overlapping pool is equivalent to two defaults - one in each inner CDO. So, for a given number of defaults, the total combined loss of inner CDOs when some defaults occur in the overlapping pool is greater than that when no defaults occur in the overlapping pool, or when there are no overlaps. Figure 8 shows that when there are no overlaps, the CDO^2 total loss rate increases monotonically after a certain level of unique defaults (‘critical default level’). This is because at the critical default level, entire subordination of invested tranches is exhausted, and further loses to CDO^2 would be independent of default location risk. However, when there are overlaps, CDO^2 total loss rate increases non-monotonically across all levels of defaults. Also, the lower bound on total loss rate seen in zero-overlap scenario loses significance as overlaps increase.

Figure 11 shows the standard deviation of CDO^2 total loss rate at different levels of unique defaults under 0%, 20% and 50% overlap scenarios. It can be observed that the standard deviation of total loss rate increases as overlaps increase.
**Observation 4:** Overlaps add a new dimension to the ‘default location risk’. We call this ‘overlap risk’.

Figure 8: CDO$^2$ Total Loss Rate and Unique defaults (Zero Overlap)

Figure 9: CDO$^2$ Total Loss Rate and Unique Defaults (20% Overlap)
Figure 10: CDO^2 Total Loss Rate and Unique Defaults (50% Overlap)

Figure 11: Standard Deviation of CDO^2 Total Loss Rate at different levels of unique defaults under 0%, 20% and 50% overlap scenarios
Analogy with UK Split Capital Investment Trusts

There are similarities between CDO^2s and split capital investment trusts (“splits”), a form of closed-ended fund with more than one class of share capital. Popular in the United Kingdom during the 1990s, a number of these splits invested in one another in a complex web of cross-holdings. In the equity bear market of 2000-2002, the weaknesses inherent in cross-invested, leveraged investment vehicles became exposed, and supposedly ‘safe’ classes of capital (zero dividend preference shares) suffered large losses, prompting an investigation by the Financial Services Authority. A lack of understanding of the true risks by investors, intermediaries and - in some cases - by managers of split capital investment trusts, aggravated the problems [see Adams and Clunie (2006)].

We have found that there are unique risks in CDO^2s, not yet fully understood by investors. During a distressed credit environment, the unique risks in CDO^2s might be exposed, just as the risks in UK splits were exposed in the recent equity bear market. In particular, problems could arise if investors suffer unexpected losses in highly rated, senior tranches of CDO^2s, as a consequence of default location risk and overlap risk.
Summary and Conclusions

To explore risks that could be unique to a CDO^2, we create a simplified CDO^2 model and carry out Monte Carlo Simulations. Using descriptive charts, we observe and compare the behaviour of CDO^2s and CDOs. From these observations, we identify certain risks (default location risk and overlap risk) that are unique to a CDO^2, and which may not be understood by investors. The returns offered to CDO^2 investors may therefore be unattractive on a risk-adjusted basis. Further, a lack of understanding of risks could lead to a misallocation of credit risk. The risks in a CDO^2 may be underestimated and could potentially lead to a crisis if the credit environment becomes distressed.
Notes
1. In the context of this paper, CDO refers to debt obligations collateralized by bonds or credit default swaps.

2. Entities could be referenced through direct investment in bonds (as in a cash CDO^2) or through investments in CDS (as in a synthetic CDO^2).

3. Mahadevan et al (2005) estimates that the global credit investor has access to approximately 1200 investment grade credits.

4. Both cash CDOs and synthetic CDOs generally have similar characteristics related to distribution of cash flow and loss among tranches. Synthetic CDOs have CDS constituting their collateral pool. CDS in turn refer to corporate bonds. Hence insights gained by modeling a cash CDO^2 would also apply to a synthetic CDO^2.

5. Moody’s (Hu and Cantor, 2004) uses coupon rate for discounting cash flows to compute the Loss Severity Rate for RMBS.

6. CDO^2 Default Rate = (Total Unique Loss)/ (Total Unique Entities), where
   Total Unique Loss = Total Losses in CDO1 + Total Losses in CDO2 – Losses in Overlapping pool, and
   Total Unique Entities = Total Reference Entities in CDO1 + Total Reference Entities in CDO2 – Total Overlapping Entities

7. SDR is the default rate that a tranche with a particular rating should be able to withstand under a given cash flow scenario. Refer to Bergman (2001) for details on how CDO Evaluator works.
REFERENCES


