OBSERVATIONS OF THE TRANSITION TO TURBULENCE IN THE
BOUNDARY LAYER ON A FLAT PLATE

Thesis submitted by

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SYMBOLS

The following symbols are used to describe the quantities indicated in this list unless otherwise stated in the text.

\[ x = \text{distance from the leading edge of the flat plate} \]
\[ y = \text{distance from the surface of the flat plate.} \]
\[ z = \text{distance from the centre line of the plate perpendicular to } x \text{ and } y. \]
\[ U_0 = \text{free stream velocity} \]
\[ U = \text{mean velocity at a point in the boundary layer.} \]
\[ u = x \text{ component of fluctuation velocity.} \]
\[ v = y \text{ component of fluctuation velocity.} \]
\[ w = z \text{ component of fluctuation velocity.} \]
\[ c = \text{wave velocity.} \]
\[ c_g = \text{group velocity.} \]
\[ \beta_r = 2nf, \text{ where } f = \text{oscillation frequency.} \]
\[ a = \frac{2\pi}{\lambda} = \text{wave number.} \]
\[ \lambda = \text{wavelength of oscillation.} \]
\[ \rho = \text{density of air.} \]
\[ \nu = \text{kinematic viscosity of air.} \]
\[ \delta = \text{boundary layer thickness.} \]
\[ \delta^* = \text{boundary layer displacement thickness.} \]
\[ \delta^* = 1.72 \frac{\nu \alpha}{U_0} \text{ for Blasius distribution.} \]
\[ \delta^* = 0.3418 \delta \] relation used by Tollmien and Schlichting (19-22)
\[ R = \frac{U_0 \delta^*}{\nu} = \text{Reynolds number.} \]
\[ R_x = \frac{U_0 \alpha}{\nu} = x - \text{Reynolds number.} \]
The research in this thesis was carried out in the Heriot-Watt College, Edinburgh, under the joint supervision of Professor W.H.J. Childs, Professor of Physics in the Heriot-Watt College, and Dr. M.A.S. Ross of the Department of Natural Philosophy, University of Edinburgh. The major part of the experimental work was carried out in conjunction with Mr. J. Morgan, of the Department of Natural Philosophy, University of Edinburgh.
CHAPTER I

INTRODUCTION AND HISTORICAL SURVEY OF BOUNDARY LAYER RESEARCH

I.1. Introduction

The aim of this work was to gain further insight into the transition from laminar to turbulent flow in the motion of a fluid past a solid surface. The present experimental work has been confined to the most simple example; the flow of an incompressible fluid along a smooth flat plate at zero angle of incidence and with zero pressure gradient in the direction of flow. Although the aspects of transition which are of greatest importance in practice are those of flow over aerofoils and in closed channels, it was hoped that an experimental investigation of transition on a flat plate would lead to a better understanding of the mechanism of fluid flow in an unstable boundary layer, which can in turn be applied to more complex problems.

I.2. Historical Survey

Many extensive summaries of the literature on different branches of boundary layer research have been published. The most notable of these have been by Prandtl in 1935(1), Dryden in 1955(2), Morkovin
in 1958(3), Dryden in 1959(4) and Schlichting in 1959(5), and 1960(6), (7).

Due to the extent of the work which has been carried out on boundary layer flows in the past, the following survey will include in detail only those papers with particular relevance to the present research. Most emphasis will thus be laid on investigations of the flow in the flat plate boundary layer with zero longitudinal pressure gradient, with particular reference to stability and the transition to turbulence.

In the year 1820 the Navier-Stokes equations were first formulated. As a development of the Euler equations, which were at that time well known, they gave for the first time the general equations of motion for the flow of a viscous fluid. These equations are now taken as the basis for most theoretical work on fluid motion but at that time the mathematical problems associated with their general solution were insuperable.

Use of Euler's equations, neglecting viscosity, led to a number of solutions of practical value but in other cases gave absurd results. The dilemma confronting the theoretical hydrodynamicist of that time can be summed up in d'Alembert's Paradox, namely that although the viscosity of air is extremely
small, to neglect it gives rise to the theoretical result that air can offer no resistance to a body passing through it. As Lord Rayleigh remarked, "On this theory the screw of a submerged boat would be useless but, on the other hand, its services would not be needed."

Empirical laws of hydraulics were meanwhile being formulated by engineers. In 1883 Osborne Reynolds investigated experimentally the two distinct types of flow, laminar and turbulent, in round pipes running full. He found that a certain value of a dimensionless parameter $R$ (i.e. velocity times length divided by kinematic viscosity) now known as the Reynolds number, roughly characterised the transition from the laminar regime to turbulent flow and vice versa.

At the Mathematical Congress in Heidelberg in 1904, Prandtl introduced the concept of the boundary layer which proved a major contribution in the fusion of theory and experiment. He showed that it was possible, for the purposes of theoretical calculation to consider separately the flow in two distinct regions. In the first region, well away from a solid wall, the fluid behaves as if it were inviscid and the velocity gradient perpendicular to the direction of flow is negligible. Making the
assumption that the velocity of the fluid at a solid boundary will be equal to that of the solid itself, a second region can be recognised in which the velocity of the fluid changes rapidly from the velocity of the main flow to that of the wall. In this region, now known as the "boundary layer", the velocity gradient has a finite value and the viscosity of the fluid becomes an important factor. Prandtl showed that by making these assumptions the Navier-Stokes equations could be greatly simplified for flow in boundary layers of most kinds.

Prandtl's boundary layer equations were solved for the case of flow along a flat plate by Blasius in 1908\(^{(10)}\), in terms of an infinite power series. The solution was later improved by Howarth who tabulated the results in 1938\(^{(11)}\).

In 1914 Prandtl\(^{(12)}\) carried out experiments with spheres and showed that flow in a boundary layer can be either laminar or turbulent and that the problem of flow separation is closely related to this transition. At this time the techniques available for the study of boundary layer flows were limited to flow visualisation and measurement of overall effects such as drag which was found to increase when the boundary layer became turbulent.

The first detailed measurement of velocities in
the boundary layer was made by van der Hegge Zijnen in 1928. He confirmed the distribution of velocities across the boundary layer which had been theoretically predicted by Blasius twenty years previously. Similar confirmatory experiments have also been made by Burgers(13) and Nikuradse(14).

A hypothesis was formulated by Reynolds and Lord Rayleigh that the laminar boundary layer becomes turbulent when a critical Reynolds number is reached. Experiments showed that the boundary layer on a flat plate could remain laminar up to a Reynolds number \( R_\infty = 3 \times 10^6 \), but that the critical Reynolds number was dependent on the degree of turbulence in the main stream of the flow, rarely falling below \( 3 \times 10^5 \). In a study of the effect of free stream turbulence on the stability of a laminar boundary layer in 1936(15), Taylor assumed that separation at a given point is determined by the local pressure gradient and boundary layer thickness. From the theory of isotropic turbulence he calculated the form of a parameter upon which the critical Reynolds number was dependent. His theory showed good agreement with experiment for the case of flow round spheres and elliptic cylinders when the value of the free stream turbulence was greater than 0.2%. The level of free stream turbulence is defined as the root mean square of the three mutually perpendicular fluctuating velocities, as a percentage of the mean flow velocity. This
theory, however, predicted the point of reversal or separation of the laminar boundary layer flow and not the point of transition to turbulence. Experiments using hot wire anemometers showed that transition could occur without previous separation of the flow, particularly in streams with a low degree of turbulence. Taylor's theory did thus not fully solve the problem of the breakdown of the laminar boundary layer.

The most fruitful approach to this problem was suggested by Lord Rayleigh in 1887$^{16}$. This was to study the effect of small perturbations on the laminar boundary layer. It was considered that if a perturbation was damped with time the boundary layer was stable and would remain laminar, instability occurring when the perturbations became amplified.

In 1933 it was shown by Squire$^{17}$ that a two-dimensional infinitesimal perturbation has a greater disturbing effect on the boundary layer than a similar three dimensional perturbation, that is the boundary layer becomes unstable at a lower Reynolds number when in the presence of a two dimensional disturbance. Following this conclusion most of the theoretical work on boundary layer stability, except that carried out very recently, has been concerned with the effect of a two dimensional perturbation on the boundary layer flow.
Tietjens in 1925\(^{18}\) produced a solution of the characteristic value problem posed by the Orr-Sommerfeld equation, which was derived by considering the conservation of mass and momentum in a laminar flow with a superposed infinitesimal periodic disturbance. His solution, however, was of an approximate nature due to his assumption that the velocity distribution across the boundary layer could be approximated by a set of straight lines.

The papers published by Tollmien in 1931\(^{19}\) and 1936\(^{20}\) and those by Schlichting in 1933\(^{21}\) and 1935\(^{22}\) gave more rigorous solutions to this problem, though their original solutions have in turn been extended and modified by many workers.

This work predicted the conditions under which the boundary layer was unstable and infinitesimal disturbances would be amplified in time. It also showed that under certain conditions a disturbance of a certain frequency would be amplified and travel down the boundary layer in the form of a wave. Waves of this type have become known as "Tollmien-Schlichting waves". The results of these solutions can be summarised by a curve in the \(\left( \frac{\delta_x v}{u^2}, R \right)\) plane which separates the "damping" and "amplifying" regions of the boundary layer. \(R\) denotes the Reynolds number based on the boundary layer thickness.
and main stream velocity, $u_0$, and $\beta_T$ and $\nu$ are
the angular frequency of the disturbance and the
kinematic viscosity of the air. This neutral
stability curve shows the points in the boundary
layer where a disturbance will be neither amplified
nor damped.

Figure 1.1 shows the neutral stability curve
calculated by Schlichting and also that calculated
by Lin$^{(23)}$ and Shen$^{(24)}$ in 1954.

Experimental attempts to test this theory at
that time failed to discover these disturbances due
to high levels of free stream turbulence which
effectively masked the waves.

It was not until 1947 that Schubauer and
Skramstad$^{(25)}$, working at an exceptionally low level
of the free stream turbulence, were able to confirm
this theory by observing Tollmien-Schlichting waves
in the boundary layer on a flat plate. They also
found, using artificially introduced disturbances, an
experimental neutral stability curve which was in
good agreement with that calculated by Shen. A
further confirmation of the theory lay in their
observations of the distribution of the amplitude and
phase of the disturbance across the boundary layer.

The minimum Reynolds number for neutral stability
was shown to be of the order $R = 420$ while
transition did not occur until a Reynolds number of around 2,000, thus the boundary layer does not become turbulent at the point where the infinitesimal disturbances become amplified. Thus this theory, although of great intrinsic value does not enable the position of transition to be predicted.

As the theory assumes small perturbations it is no longer applicable when the disturbance becomes amplified and cannot be considered infinitesimally small. The problem of the effect on the boundary layer of disturbances of a finite size is very complex as the initial equations cannot in this case be approximated to a linear form. There is a further complication in that the Reynolds number increases as the disturbance moves down the boundary layer, that is the boundary layer becomes thicker.

In all physical problems, there are two main processes associated with oscillations of finite amplitude. These are the generation of higher harmonics and the change induced in the mean conditions by dissipation of the disturbance energy. The second of these effects was considered by Meksyn and Stuart (26) for the case of a two dimensional disturbance in parallel flow. They considered the non-linear effects to be most important in the distortion of the mean flow, this distortion in turn altering
the amplitude distribution of the disturbance across the flow. They concluded that the inclusion of the non-linear terms in the equations showed the flow to be less stable.

More recently Stuart\(^{(27)}\) developed a similar analysis by considering the energy balance for oscillations of finite amplitude. He considered the distortion of the mean flow but showed that the resulting change in the velocity distribution of the oscillation can be neglected. The non-linear terms were found to affect the mean flow, only when the amplitude of the disturbances becomes of the order of 10\% of the maximum velocity, as is observed in fully developed turbulent flow in channels and pipes.

The analysis of Meksyn and Stuart\(^{(26)}\) which showed that the production of higher harmonics plays only an unimportant role has been shown by Lin\(^{(28)}\) to be incorrect. He proved that for disturbances in a parallel flow, all the harmonics of the oscillation become important around the critical layer, i.e. the point where the wave velocity equals the local flow velocity, before the amplitude of the fundamental component becomes large enough to cause any significant distortion to the mean flow.

The suggestion of Von Karman that a solution should be sought which includes both effects seems to
be well founded. This problem has been tackled by Benney\(^2\), for the case of parallel flows.

The theoretical problem of disturbances of finite amplitude in the laminar boundary layer remains unsolved, though the work which is being carried out on parallel flows may indicate a suitable approach to the problem.

Experimental work on the transition to turbulence in the laminar boundary layer has thus proceeded with very little theoretical guidance. However a fairly clear picture of the physical phenomena which accompany this transition has been gained, by work both with wind tunnels and with water tables.

From studies of boundary layer velocity profiles it appeared that the onset of turbulence occurred slowly over a finite distance, the region between the fully laminar and fully turbulent zones being known as the "transition region". While investigating the boundary layer in an inclined water table Emmons\(^3\) in 1953 noticed bursts of turbulence occurring during transition. He noticed what he called turbulent "spots" which were swept downstream with the flow, spreading out as they moved.

A technique for artificially producing turbulent spots in a gas boundary layer by an electrical discharge was developed by Michner in 1954\(^3\). This
technique was later used by Schubauer and Klebanoff in their investigation of the mechanics of boundary layer transition. They found that hot wire oscillograms of natural transition were similar in nature to those produced by an artificially produced turbulent spot. It was thus shown that the transition region was not due, as had been previously supposed, to the continuous change of position of an irregular line denoting an abrupt transition from laminar to turbulent flow. The transition region was found to consist of turbulent spots in the basic laminar flow, these spots spreading and becoming more numerous until the boundary layer becomes fully turbulent.

By studying oscillogram traces of flow in the boundary layer, Schubauer and Klebanoff were able to measure the fraction of the time for which the flow at any point in the transition region was turbulent. This fraction, known as the intermittency factor \( Y \), was shown to be distributed across the transition region in a form very similar to a Gaussian integral curve, indicating near randomness of the formation of the turbulent spots. The spots were shown to grow transversely in a nearly linear manner forming wedges whose angle agreed well with that calculated by Charters for transverse turbulent contamination. Each spot was followed by a stable region where
turbulence could not occur.

Dhawan and Narasimha in 1958(33) suggested that the Gaussian curve which gives the best agreement with the observed intermittency factors has a standard deviation approaching zero. This implied that the rate of production of spots has a distribution very near that of the Dirac \( \delta \) function.

They found the extent of the transition region was approximately governed by a law of the form:

\[
R_\lambda = \alpha R_t^\beta
\]

where \( R_\lambda \) and \( R_t \) are Reynolds numbers based on the extent of the transition zone and the position of the onset of transition. The functions of \( \alpha \) and \( \beta \) were found to be constants of value 5.0 and 0.8 respectively.

It was also shown by Dhawan and Narasimha that the distribution of \( \gamma \) across the transition region was given by the equation:

\[
\gamma(x) = 1 - \exp \left\{ -A \left( \frac{x - x_t}{\frac{x_{\gamma=75} - x_{\gamma=25}}{y_{\gamma=75}} - x_{\gamma=25}} \right)^2 \right\}
\]

where \( x_t \) is the position of the onset of the transition region and \( A \) is a constant of value 0.412.

The remaining problem is the immediate cause of the production of turbulent spots. What phenomena
occur in the boundary layer between the point of amplification of a small disturbance and the onset of transition which has been described above? This is now the outstanding problem in the field of transition to turbulence on a flat plate and it is currently being tackled by many experimental and theoretical workers.

One very important result has so far emerged. Namely that when small disturbances become amplified they lose their two dimensional character and spanwise variations in the behaviour of the boundary layer must then be considered.

The dye-study investigations of Hama, Long and Hagerty\(^{34}\) in 1957 showed that amplified two dimensional vortices, Tollmien-Schlichting waves, have a strong tendency in shear flow to form three dimensional vortex loops with a marked transverse periodicity. The formation of these loops was shown to be an essential feature preceding the creation of turbulent spots, which takes place near the top of the loop and at the outer edge of the boundary layer. It was suggested that a strong vortex line amplifies any small three dimensional disturbances in the boundary layer and intensifies their effect. The spanwise variations in vorticity were thought to originate in imperfections in the trip-wire used to disturb the boundary layer flow. Hama\(^{35}\) in 1960
observed very similar phenomena in an experiment using an oscillating ribbon instead of a trip wire.

A study of the spanwise distribution of boundary layer properties has been made recently by Klebanoff and Tidstrom (36). They measured the transverse variations in a disturbance introduced into a flat plate boundary layer. They found that the amplitude of the disturbance varied almost sinusoidally with a constant wavelength. Associated with these variations of disturbance intensity they found also variations in boundary layer thickness which did not appear to be produced by imperfections of the flat plate. The only factor which affected the position of the peaks and valleys in the boundary layer thickness was the position and cleanliness of the smoothing screens of the wind tunnel. This gives rise to the idea that the spanwise variation of the boundary layer flow may be connected with the scale or intensity of the residual turbulence in the tunnel which is controlled by these screens.

Turbulent spots were found to originate at transverse positions where the amplitude of the disturbance was a maximum. Turbulence only appeared in the regions where the amplitude of the disturbance was small, when the spots had spread transversely into these regions. There was found to be a strong
correlation between the variations in boundary layer thickness and the intensity of the disturbance. The phase of the correlation depended on the frequency of the disturbance.

It was shown by Görtler in 1940(37) and Liepmann(38) in 1945 that concave curvature of the wall has a destabilising effect on the boundary layer flow. It was later postulated by Görtler and Witting(39) that large disturbances in the laminar flow on a flat plate would cause a curvature of the streamlines which would in turn lead to the formation of longitudinal "Görtler" vortices. This theory shows a different approach to the problem from that by Stuart, but it does succeed in predicting the spanwise variations in the flow which have been found in recent experiments.

I.3. Some details of the Theoretical Approach to the problem of boundary layer stability.

In this section the mathematical approach to the problem will be considered in more detail, with special emphasis on the physical assumptions that are made.

Throughout this work a coordinate system has been used as shown in the following sketch:
With the laminar boundary layer velocity distribution thus:

Following the theorem of Squire\(^{(17)}\) which showed that the laminar boundary layer is less stable in the presence of a two dimensional disturbance than a three dimensional one, flow in a two dimensional boundary layer is considered.

The Navier-Stokes equations for two dimensional flow are:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \tag{1}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] \tag{2}
\]
These equations are derived by applying Newton's second law of Motion to a small element, stationary in space, containing unit mass of an incompressible Newtonian fluid. The terms on the left hand side of the equations represent the rate of accumulation of momentum within the element and the rate of convection of momentum out of the element due to the changes in the velocities with \( x \) and \( y \). The terms on the right hand side of the equations represent the forces on the fluid within the element due to normal (i.e., pressure) and shear (i.e., viscous) stresses in the fluid.

The assumption may be made that a fluid is incompressible so long as its velocity is well below the velocity of sound in that fluid. The fluid is also considered to be Newtonian, that is the shear stress on the fluid is proportional to its rate of strain.

A consideration of the conservation of mass in the element leads to the equation of continuity:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(3)

For the undisturbed laminar boundary layer flow, with zero pressure gradient, these equations can be simplified, making Prandtl's boundary layer assumptions, to:

\[
\omega \frac{\partial u}{\partial x} + \omega \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}
\]  

(4)
\[ \frac{\partial u^{(0)}}{\partial x} + \frac{\partial v^{(0)}}{\partial y} = 0 \]  

(5)

with the boundary conditions \( \{ \infty = 0 \}, u = 0, v = 0; \) \( \{ y = 0 \} \) and \( \{ y = \infty \}, u = u_0 \). The solution of these equations has been calculated by Blasius(10) and Howarth(11). The distribution of velocity across the boundary layer is shown in Figure 1, the values being taken from Howarth's calculation.

Let the disturbed flow be described by the equations:

\[ U(\infty, y, t) = u^{(0)}(\infty, y) + \varepsilon u^{(1)}(\infty, y, t) + \varepsilon^2 u^{(2)}(\infty, y, t) \cdots \]

\[ V(\infty, y, t) = v^{(0)}(\infty, y) + \varepsilon v^{(1)}(\infty, y, t) + \varepsilon^2 v^{(2)}(\infty, y, t) \cdots \]  

(6)

\[ P(\infty, y, t) = p^{(0)}(\infty, y) + \varepsilon p^{(1)}(\infty, y, t) + \varepsilon^2 p^{(2)}(\infty, y, t) \cdots \]

where \( u^{(0)}, v^{(0)} \) and \( p^{(0)} \) are the parameters of the undisturbed flow and thus themselves satisfy equations (4) and (5). \( u^{(1)}, u^{(2)} \) etc. are the perturbation velocities and pressures which may include terms which modify the mean flow, and \( \varepsilon \) is a small constant quantity.

By substitution of equations (6) into (1), (2) and (3) the equations of the perturbed flow are obtained. Comparing coefficients of the zero-th power of \( \varepsilon \) gives a set of equations which reduce,
making the boundary assumptions to equations (4) and (5). If the series expansions of \( u \), \( v \) and \( p \) in powers of \( \varepsilon \) (equations (6)) can be shown to be converging, then the coefficients of the higher powers of \( \varepsilon \) can be equated separately.

By equating the coefficients of the first power of \( \varepsilon \) the following equations are obtained:

\[
\begin{align*}
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \varepsilon \frac{\partial^2 u}{\partial y^2} &= -\frac{1}{\varepsilon} \frac{\partial p}{\partial x} + \nu \nabla^2 u \\
\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} + \varepsilon \frac{\partial^2 v}{\partial y^2} &= -\frac{1}{\varepsilon} \frac{\partial p}{\partial y} + \nu \nabla^2 v \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0
\end{align*}
\]

where \( \nabla^2 \) denotes the operator \((\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\).

By neglecting the equations for the second and higher powers of \( \varepsilon \) the effect of the mean secondary flow and second and higher harmonic oscillations are neglected. With this assumption the equations are linear in the first order perturbation terms.

If a perturbation stream function, \( \psi^{(0)} \), is now introduced which satisfies equation (9), i.e.

\[
\begin{align*}
\frac{\partial^2 u}{\partial x^2} = \frac{\partial \psi^{(0)}}{\partial y} ; & \quad \frac{\partial^2 v}{\partial x^2} = -\frac{\partial \psi^{(0)}}{\partial x}
\end{align*}
\]
and eliminating the term $p^{(0)}$ the equations reduce to:

$$\left[ \frac{\partial}{\partial t} + U^{(0)} \frac{\partial}{\partial x} + v^{(0)} \frac{\partial}{\partial y} - u \nabla \right] \nabla \psi^{(0)} = \frac{\partial \psi^{(0)}}{\partial y} \frac{\partial u^{(0)}}{\partial x} + \frac{\partial \psi^{(0)}}{\partial x} \frac{\partial u^{(0)}}{\partial y}$$

(10)

This is the equation for the stream function $\psi^{(0)}$ of a small perturbation superimposed on the basic laminar flow described by the velocities $u^{(0)}$ and $v^{(0)}$.

Considering the order of magnitude of the terms in equation (10), $U^{(0)}$ and $V^{(0)}$ are the velocities of the basic laminar flow and it was shown by Blasius that

$$\frac{u^{(0)}}{u_0} = \hat{f}(\eta)$$

where $U_0$ is the free stream velocity and $\eta = \frac{y}{\delta}$

where $\delta \sim 5 \times (R_{\infty})^{1/3}$

thus

$$\frac{V^{(0)}}{U^{(0)}} \approx \frac{2.5}{\sqrt{R_{\infty}}} \left[ \eta - \frac{\int_0^\eta f(\eta') d\eta'}{f(\eta)} \right]$$

where $R_{\infty} = \frac{U_0 x}{V}$

The value of the term in brackets can be shown to have a maximum value of approximately 0.35 where $y > \delta$. Considering a value of $R \approx 6 \times 10^4$ for the conditions $U_0 = 20$ f.p.s. and $x = 6$ ins. the largest value of this ratio is given by:

$$\frac{V^{(0)}}{U^{(0)}} = 0.0035$$

This value decreases for larger values of $U_0$. 
and \( x \) and smaller values of \( y/s \), thus under most circumstances \( u^{(e)} \) is negligible compared with \( u^{(o)} \).

If the assumption is made that \( \frac{\partial}{\partial y} \nabla^2 \psi^{(o)} \) is not of the order of 300 times greater than \( \frac{\partial}{\partial x} \nabla^2 \psi^{(o)} \) then the term \( u^{(e)} \frac{\partial}{\partial y} \nabla^2 \psi^{(o)} \) will be negligible compared with \( u^{(o)} \frac{\partial}{\partial x} \nabla^2 \psi^{(o)} \). Similarly the term \( \frac{\partial \psi^{(o)}}{\partial y} \nabla^2 u^{(o)} \) is considered to be negligible compared with the term \( \frac{\partial \psi^{(o)}}{\partial x} \nabla^2 u^{(o)} \).

If the function \( \frac{\partial u^{(o)}}{\partial y} \) is neglected, then so also must be the term \( \frac{\partial u^{(o)}}{\partial x} \), by equation (5).

These assumptions are equivalent to assuming a basic flow of the form:
\[
U = u(y) \\
V = 0
\] (11)

The equation for the stream function of the perturbed flow is thus reduced to:
\[
\left[ \frac{\partial}{\partial t} + u^{(o)} \frac{\partial}{\partial x} - u \nabla^2 \right] \nabla^2 \psi^{(o)} = \frac{\partial \psi^{(o)}}{\partial x} \frac{\partial^2 u^{(o)}}{\partial x \partial y^2} \] (12)

The approximation of the basic laminar flow to the form of equation (11) means that only a small section of the boundary layer is under consideration, where the thickening of the boundary layer can be neglected. The region of the boundary layer under consideration is defined by the form of the function
This assumption is valid if it is wished to calculate the effect of certain perturbations at different points in the boundary layer, the correctness of the assumption is less certain if the passage of a disturbance along the boundary layer is to be considered.

The form of the function \( \psi^{(\omega)} \) must now be considered. The stream function representing a single oscillation of the disturbance is usually assumed to be of the form:

\[
\psi^{(\omega)}(x, y, t) = \phi^{(\omega)}(y) \exp \left\{ i (\alpha x - \beta t) \right\}
\]

(13)

Any arbitrary two dimensional disturbance is assumed expanded in a Fourier series, each of whose terms represents such a partial oscillation. In equation (13) \( \alpha \) is taken to be real where \( \lambda = \frac{2\pi}{\alpha} \) is the wavelength of the disturbance. The quantity \( \beta \) is taken to be complex with \( \beta = \beta_r + i\beta_i \), where \( \beta_r \) is the angular frequency of the oscillation and \( \beta_i \) determines the degree of amplification or damping with time. It is convenient to introduce the complex function \( c = \beta/\alpha \) where \( c_r \) is the phase velocity of the wave and \( c_i \) is another form of the amplification factor.

It should be pointed out that by assuming \( \alpha \) to be real and \( \beta \) complex, a wave is considered which
is amplified with time but not with distance. For the purposes of finding the point where a wave is neither amplified nor damped this assumption is immaterial. In an experimental verification of the theory, however, it is most convenient to measure amplification with distance when the disturbance has reached a steady state with respect to time. The problem of distance amplification can be tackled most simply after finding the shape of the neutral curve from the time amplification theory.

By substituting equation (13) into equation (12), the following is obtained

\[
(u - c)(\phi'' - \alpha^2 \phi) - u'' \phi = -\frac{i}{\nu} \left( \phi''' - 2\alpha^2 \phi'' + \alpha^4 \phi \right)
\]

where dashes denote differentiation with respect to \( y \).

Now making the variables dimensionless by introducing the functions

\[ u = \frac{u^{(u)}}{u_0} \]

and

\[ \phi(y^*) = \phi'(x) \]

also

\[ R = \frac{u_0 \delta^*}{\nu} \]

then (14) becomes:

\[
(u - c)(\phi'' - \alpha^2 \phi) - u'' \phi = -\frac{i}{\nu} R \left( \phi''' - 2\alpha^2 \phi'' + \alpha^4 \phi \right)
\]
where dashes now denote differentiation with respect to $\gamma/\xi$. This is the Orr-Sommerfeld equation for small disturbances.

The boundary conditions require that the perturbation velocities are zero at the wall (no slip) and are bounded at a large distance from the wall. Thus:

\[
\begin{align*}
y = 0, & \quad u^\omega = v^\omega = 0, \quad \phi' = 0, \quad \phi = 0 \\
y \to \infty, & \quad u^\omega = v^\omega = 0, \quad \phi' = 0, \quad \phi = 0
\end{align*}
\]

When the mean flow $U(y)$ is specified the equation contains the parameters $\alpha$, $R$ and $c$, of which $c$ is complex, thus for each pair of values for $\alpha$ and $R$ the equation defines one eigenfunction $\phi(y)$ and one complex eigenvalue, $c$. There are four fundamental solutions of equation (15) for $\phi$ analytic in the complex variable $y$ and of the parameters $c$, $\alpha$ and $R$.

The different approximate, asymptotic methods of solution of equation (15) have been fully discussed by Lin\(^{40}\) in his book on Hydrodynamic Stability, so only an outline of one method of solution will be given here.

By inspection of equation (15) it can be seen that when $U = c$ the viscous terms, i.e. those in $(dR)^{-1}$, are predominant. The point where the wave
velocity is equal to the local stream velocity is thus termed the critical point and a new variable introduced thus:

$$\eta = \frac{(y - y_c)}{e} = (y - y_c)(\alpha R)^{-\frac{1}{3}}$$

where \( y = y_c \) at the critical point.

A solution of equation (15) is sought of the form

$$\phi\left(\frac{y}{\varepsilon}\right) = \chi^{(\omega)}(\eta) + \varepsilon \chi^{(\omega)}(\eta) + \varepsilon^2 \chi^{(\omega)}(\eta)$$

(16)

Four solutions are obtained when (16) is substituted into (15) and the coefficients of various powers of \( \varepsilon \) are compared. These solutions are all regular and are expressed in terms of the function

$$J^{\frac{1}{2}} H^{\omega,\omega}_J \left[ \frac{a}{3} (i J)^{\frac{3}{2}} \right]$$

where

$$J = \left( \frac{U_y}{u_x} \right)^{\frac{1}{2}} \eta$$

This method of solution assumes that as \( y \to y_c \) then \( \alpha R \to \infty \).

The solution of the Orr-Sommerfeld equation for real values of \( c \) (i.e. \( c_1 = 0 \)) enables the points of neutral stability to be plotted in both the \((\alpha \delta^*, R)\) and the \(( \beta_1 v / u_x, R)\) planes. No direct solution of the equation for finite values of \( c_1 \) has been found although interesting mathematical problems have arisen (cf. Lin^{(20)}). Curves of constant \( c_1 \) have however been calculated by Shen using
an indirect method. He considered small values of $c_1$ as a perturbation to the value of $c_R$, already calculated, and found values of the function $\frac{2c_i}{\partial R}$ at the neutral curve. He was thus able by extrapolation to find the points for larger values of $c_1$ in the $(\alpha \delta^*, R)$ plane. A graph of these results is shown in Figure 1.3.

Schubauer and Skramstad postulated that when a disturbance of the form of equation (13) moves down the boundary layer, its amplitude at a given point $x_2$ in terms of that at an earlier point $x_1$ is given by the equation:

$$\frac{A_2}{A_1} = \exp \int_{t_1}^{t_2} \frac{\partial c}{\partial t} dt$$

where the time integral covers the interval for the disturbance to travel from $x_1$ to $x_2$.

Assuming the relationship:

$$R = m \int R$$

where $R = \frac{u_0 \delta^*}{v}$ and $R = \frac{u_0 x}{v}$

this equation can be rewritten in the form

$$\log \frac{A_2}{A_1} = \frac{2}{m} \int_{R_1}^{R_2} \frac{\alpha \delta^* c_i}{c_g} dR$$

where $c_g$ is the group velocity of the disturbance.

Using this equation and the values of $c_1$ shown
in Figure 1.3 Shen\(^{24}\) calculated the amplitude of disturbances with different values of \( \frac{\rho_r v}{u^2} \) as they cross the amplifying region from Branch I to Branch II of the neutral stability curve. A graph of these results is shown in Figure 1.4.

These values are only approximate, however, due to the approximations in the calculation of \( v_1 \) and the fact that the theory of small disturbances has been amplified.

These graphs could be used in conjunction with experimental readings to find the point where the behaviour of a disturbance diverges from that predicted by the infinitesimal theory and also the amplitude of the disturbance at this point.

Although the solution of the problem of infinitesimal disturbances in the laminar boundary layer has been found and verified with a fair degree of accuracy, the theory of finite disturbances in the boundary layer is still being investigated and though the mathematical equations can be formulated they have not yet been solved. This theoretical problem, however, is aimed at finding the stream functions for the disturbed flow. It is unlikely that, even when the equations are solved, this will lead to a prediction of the point of transition to turbulence as there is as yet no mathematical way to

*been used, irrespective of the amount the disturbance has
describe the production of a turbulent spot. As the production of these spots in the transition region has been shown to occur in a random manner, with a certain statistical probability at each point, it would seem that a statistical approach to the theoretical problem will have to be introduced before the position of the transition region can be predicted.
Figure 1.1

Neutral stability curves calculated by Schlichting and Shen and Lin.
Figure 1.2
Graph of the "Blasius" velocity distribution across a laminar boundary layer.
Figure 1.3

Contours of different values of $c_1$ in the $(\alpha \delta^*, R)$ plane, after Shen.
Graph showing the amplitude of a disturbance, A, in terms of its value at the neutral curve Ao, as it crosses the amplifying region, after Shen.
CHAPTER II

DESCRIPTION OF APPARATUS AND PRELIMINARY INVESTIGATIONS

II.1. Description of the Wind Tunnel

The wind tunnel used in this work was a modification of the N.P.L. Design No. A 155. This is an open circuit tunnel with an 18 in. octagonal working section and a 3.16:1 contraction ratio. Extra settling lengths had been added to the original design, before and after the contraction, to ensure that the flow in the working section was as smooth as possible.

A diagram and photograph of the tunnel are shown in Figure 2.1a, b.

The tunnel was built with its supporting angle iron girders firmly embedded in concrete blocks. These blocks were insulated from the floor of the room by felt pads and held in place by rubber-covered iron pins set into the floor. This arrangement was used to reduce the transmission of vibrations to the tunnel.

The inlet section was fitted with a honeycomb straightener made of 4 in. long hexagonal brass tubes measuring 3/8 in. from face to face. Behind the honeycomb were two mesh screens of 32 S.W.G. wire with 30 wires/inch. Burns et al. (41) whose work was
carried out using the same tunnel found that the best arrangement of these screens was to place them 15 ins. and 27 ins. downstream from the honeycomb. This gave a value of the free-stream turbulence, in the working section, of 0.3%. Although this value is a factor of ten greater than that for the tunnel used by Schubauer and Skramstad\(^{(25)}\) in their work it has been shown by Burns et al. that Tollmien-Schlichting waves can be detected in the boundary layer under these conditions.

The air in the tunnel was driven by a four bladed fan powered by a 7.5 H.P. D.C. motor. A generator driven by an A.C. motor produced the current for the armature of the fan motor. The field coils of this generator were supplied from a variable D.C. source, providing a sensitive speed control.

A diagram of this arrangement is shown in Figure 2.2.

A full description of the design and construction of this tunnel can be found in the work by Burns et al.\(^{(41)}\).

II.2. The Flat Plate and Traversing Mechanism

A flat perspex plate, on which the boundary layer measurements were made, was built into the tunnel, spanning it from floor to ceiling in the
working section. The plate was 6 ft long, 18 ins. high and 0.25 ins. thick. Its leading edge, which was four feet down stream from the contraction, was smoothly tapered over a length of 4 ins. to a symmetrical knife edge. The centre line of the flat plate was graduated in feet and inches from the leading edge to facilitate the accurate positioning of instruments.

If the boundary layer on the flat plate is to be initially laminar the stagnation point of the flow should be on the working side of the plate. To ensure that this condition was complied with, a narrow aluminium fin, making an adjustable angle to the air flow, was placed at the downstream end of the plate. This was arranged to slightly increase the blockage on the working side of the plate, thus preventing the onset of turbulence at the leading edge.

The pressure drop along the tunnel was counteracted by false walls which were fitted downstream from the contraction. These were of perspex 0.125 ins. thick and were parallel to the vertical walls of the tunnel, diverging to the end of the working section. The position of the wall on the working side of the tunnel could be varied slightly by means of small adjusting screws at various positions along
the wall. In this way a fine control of the pressure gradient in the tunnel was achieved.

To prevent any vibrations being transmitted to the measuring instruments in the tunnel, a traversing mechanism was used which allowed the instruments to be moved from outside the tunnel without any contact with the walls.

A framework of 2 inch angle iron girders was built round the working section of the tunnel. This was set in concrete blocks in a similar manner to the tunnel itself, to cut down the transmission of vibrations. Two collars were screwed to the vertical girders on a level with the centre line of the flat plate. Two cylindrical shafts were placed through the collars and through clearance holes in the tunnel wall and false wall. These shafts held rigidly the ends of two brass beams, one inside and one outside the tunnel. The beam inside the tunnel was of 0.25 in. square cross section and tapered at the upstream end to reduce the disturbance to the air flow. This acted as a rail for the movement of instruments parallel to the direction of the air flow. On the beam outside the tunnel were mounted two micrometers, whose probes bore against the angle iron frame, thus affording accurate movement of the inner rail perpendicular to the air flow and the flat plate.

A close fitting square sleeve running on the inner rail could be moved by means of a length of
screwed rod, passing through a nut on the sleeve and attached at the downstream end of the rail to a small gearbox which could be operated manually from outside the tunnel. The screwed rod had a pitch of $\frac{1}{32}$ inch and the gearbox a ratio of $1:1$, so the distance of the sleeve from the leading edge, could be accurately adjusted.

A similar type of vertical traversing mechanism could be attached to the square sleeve and was used only when a vertical traverse was required. This enabled two narrow beams of streamlined cross section supporting horizontally a light hollow rod, to be moved in the vertical direction by moving a chord turning a small pulley in the tunnel.

Where a vertical traverse was not required the hollow rod was supported by similar beams fixed to the square sleeve itself. This rod was horizontal and about an inch from the flat plate.

All instruments used to investigate the air flow could be mounted on the end of this rod so that the measurements were taken at least 12 ins. upstream of the supporting beams.

Diagrams of these traversing mechanisms are shown in Figures 2.5 and 2.4. All the parts were kept as thin as possible without loss of rigidity to cut down the blockage and disturbance to the air flow.
II.3. The Control of the Wind Speed

A coarse speed control was provided by a potentiometer which varied the direct current in the field coils of the generator producing current for the fan motor. This potentiometer was only variable in discrete steps and the external direct voltage source of nominally 240 volts, across which it was connected, was liable to slight fluctuations. In order that the rate of air flow in the tunnel might be continuously variable and as constant as possible a fine control and feedback unit was built.

A 100 ohm continuously variable rheostat was connected in series with the windings of the field coils of the generator providing a fine control of the current in the coils and thus of the speed of the fan. The contact of the rheostat was driven by a small reversible, variable speed motor. The windings of this motor were connected to two relays which triggered when the pressure in the tunnel 4 ft. upstream from the leading edge of the plate increased or decreased, destroying the equilibrium of an inverted cup pressure balance connected at that point.

The design for this servomechanism was a simplified form of that described by Salter\(^{(42)}\). Diagrams of the pressure balance and relay system are shown in Figures 2.5 and 2.6.
It was essential that the speed control should compensate as quickly as possible for any variation in the electrical power supply which would change the windspeed. If the system was made too sensitive the control unit tended to oscillate or "hunt" and aggravate rather than minimise the effect of any slight change. The related elements in the system may be shown diagrammatically, thus:

If the fan speed increases the static pressure in the tunnel decreases, this is detected by the pressure balance and contact is made which causes a relay to start the motor in the appropriate direction. This drives the contact along the theostat, changing the current through the field coils of the generator which in turn reduces the voltage produced and thus the speed of the fan. Each step in this chain will take a finite time but only the mechanical delays need be considered as these will be much greater than those produced by the time of travel of an electrical
or pressure pulse. The sensitivity of the system is thus controlled by three main factors.

a) The time of swing of the pressure balance between the two contacts.

b) The time taken for the servomotor to drive the contact along the rheostat.

c) The time taken for the fan motor to reach a steady speed after an adjustment in the field coil current.

The time of swing of the pressure balance was reduced to a minimum by reducing the distance between the electrical contacts until a swing of the balance of 0.2° changed the connection from the lower to the upper contact.

The rate of traverse of the rheostat contact could be varied by varying the speed of the driving motor. It was found that at high speeds the contact moved faster than the increase in speed of the fan motor, causing hunting. As the inertia of the fan could not be changed, the speed of the servomotor was reduced until the system just ceased to oscillate, this was taken to be the most sensitive setting. The time taken for the speed control to increase the velocity of the air in the tunnel by 1 per cent of its value was measured to be approximately ten seconds. The control thus reduces low frequency fluctuations in wind speed but has little effect on high frequency fluctuations.
II.4. **Calibration of Wind Speed**

Different windspeeds were obtained by placing different weights on the scale pan of the pressure balance, setting the coarse control rheostat appropriately and allowing the control mechanism to run until it reached equilibrium. In order that the value of the wind speed should be known the pressure balance was calibrated, using a pilot-static tube in the wind tunnel.

A pilot-static tube of N.P.L. design was used for the calibration. The tube was inserted through the tunnel wall in the central region of the working section, 2 ft. from the leading edge of the flat plate. The difference between the total head and static pressures was measured using a simple manometer.

The manometer used in this work was of the sloping tube type, containing paraffin. It was fixed rigidly to the side of the tunnel and calibrated carefully both directly and indirectly. A diagram of the manometer is given in Figure 2.7. The surface area of the paraffin in the reservoir was approximately 1500 times the cross section area of the sloping tube. Thus any correction for the fall in height of the paraffin level in the reservoir would be of the order of .07% and was taken to be negligible.
A direct measurement of the slope of the tube was carried out using a travelling microscope. The tube was found to be at an angle of \( \sin^{-1} 0.1589 \) to the horizontal at all points along its length to an accuracy of 0.13%. The manometer readings were also compared with readings from a calibrated Chattock manometer and a value for the slope of the manometer obtained which agreed with the previous calibration. The Chattock manometer itself was not used for the measurements as although it was slightly more sensitive than the sloping tube manometer it was found impossible to maintain a constant zero reading, the accuracy of all readings thus being seriously reduced.

By comparing the weights, placed on the scale pan of the pressure balance, with the reading on the manometer, connected to the pilot-static tube in the working section, a calibration of the pressure balance was obtained. This calibration was checked from time to time and found to remain constant unless the inlet screens became extremely dusty. As these screens were removed and cleaned regularly this caused little trouble.

The calibration curve for the pressure balance, giving the relationship between weight and windspeed, is shown in Figure 2.8.

Fluctuations in wind speed which could not be
controlled by the feed-back mechanism were noted, however over a period of three hours the wind speed variation was only of the order of $\pm 0.5$ ft. per second, which is an acceptably low rate.

II.5. Measurement and Adjustment of the Pressure Gradient along the Flat Plate

As the rate of amplification of a disturbance in the boundary layer is very sensitive to the local pressure gradient an effort was made to reduce the variations in static pressure along the plate to a minimum.

A small static pressure tube of 0.2 in. diameter and British Standard design was made for measurement of the static pressure in the working section of the tunnel. This tube was traversed downstream 3 ins. from the flat plate and parallel to its centre line. At 3 inch intervals the static pressure was compared with that at a fixed tapping in the tunnel wall, using the sloping tube manometer. Slight temporal fluctuations of the pressure were noticed and it was thought that these might be due to changes in wind speed caused by oversensitivity of the speed control. These fluctuations were found to increase in magnitude when the speed control was disconnected and it was thus concluded that they were due to variations in wind speed more rapid than could be controlled by the feed back system.
As these fluctuations were apparently unavoidable a reading of the mean value of the pressure difference was taken in each case.

After each traverse the position of the false wall was adjusted until it was found that further adjustment did not improve the constancy of the pressure.

A graph of the final readings of pressure versus x position is given in Figure 2.9. On the same graph is drawn the pressure distribution in the tunnel used by Schubauer and Skramstad\(^{25}\) during their work on fluctuations on the boundary layer with "zero" pressure gradient.

A relevant parameter in determining the stability of boundary layer flows in the presence of a pressure gradient is the Pohlhausen shape factor \( \Lambda \), which is defined by the equation:

\[
\Lambda = \frac{S_l^2}{\nu} \frac{dU_o}{dx}
\]

where \( S_l \) is the boundary layer thickness, \( \nu \) is the kinematic viscosity of air, and \( \frac{dU_o}{dx} \) is the local gradient of the free stream velocity along the plate.

Graphs of the neutral stability curves for small disturbances in terms of \( dS^\infty \) and \( R \), and also the shape of the velocity profile across the boundary layer, have been calculated by Schlichting and
Ulrich\(^{43}\) for various values of \(\Lambda\), both positive and negative. These graphs are shown in Figures 2.10 and 2.11.

Figure 2.12 shows a graph of the parameter \(\Lambda\) calculated from a smooth curve drawn through the experimental points. The value of \(S\) used for this calculation was that given by Blasius for the boundary layer with zero pressure gradient. It has been assumed that the main effect of the pressure variations on the value of \(\Lambda\) will lie in the value of \(\frac{d\mu}{dx}\) rather than the value of \(S\).

It can be seen that the parameter \(\Lambda\) lies between +0.1 and -0.25. It can thus be assumed that the pressure variations will have a negligible effect on the boundary layer velocity profiles and a very small effect on the shape of the neutral stability curve.

If it was found necessary to reduce the pressure variations further, the tunnel would have to be dismantled and false walls installed of a material more flexible than perspex, with a more sensitive method of adjustment.
II.6. Preliminary Investigation of the Boundary Layer Flow

To discover the region of the flat plate over which the boundary layer flow is laminar a preliminary study of the undisturbed boundary layer flow was carried out.

A total-head and static pressure pilot tube were mounted on the light boom in the tunnel. The tubes were both made from hypodermic tubing with an outside diameter of 0.04 ins. The end of the total-head tube was flattened and tapered so that the aperture was a slit 0.004 ins. wide surrounded by a wall of thickness 0.005 ins. This tube was bent so that it entered the boundary layer at a slight angle and then pointed upstream parallel to the flat plate. The static pressure tube was of similar hypodermic tubing with a closed hemispherical end and six holes 0.35 ins. from the end. The difference between the pressure in these tubes was not expected to give a direct reading of $\frac{1}{2}\rho u^2$, as with a standard pitot-tube, however the tubes could be simply calibrated against a standard tube were a direct reading required. The static tube was arranged to remain always outside the boundary layer, approximately two inches from the flat plate. The assumption was made throughout this work that the static pressure remains constant across the boundary layer.
Pressure leads were connected from the hypodermic tubes to the sloping tube manometer. The pitot tubes were then traversed downstream at various z positions with the total head tube just in contact with the flat plate. Readings were taken of the pressure difference at various x positions. A graph of such a traverse is shown in Figure 2.13. It can be seen that on increasing the x position the pressure difference at first remains at a low constant value, it then rises sharply to a higher value after which it falls slightly to a new constant level. The point where the pressure difference starts to rise is taken as the start of the transition region and the point of maximum pressure difference is taken as the start of fully developed turbulence. This interpretation can be seen to be valid by comparison with the graphs of the boundary layer velocity profile through the transition region. As the degree of turbulence increases the velocity close to the flat plate increases, this is indicated by an increase in pressure difference between the total head and static tubes.

It should perhaps be emphasised that the pressure difference measured when the total head tube is touching the plate has not been taken to be a measurement of the velocity near the plate. The true total head pressure will not be measured when the air stream is blocked by the flat plate at one side of the tube.
No conclusions have been drawn from the actual values of the pressure difference measurements but it has been assumed that measured pressure difference will rise when the velocity near the wall rises.

A plan of the laminar, transition and turbulent regions of the flow is shown in Figure 2.14. The lines marking the onset of the transition region cross the plate at an angle of approximately $7.5^\circ$ to the horizontal. This is in fair agreement with the work by Charters who reported the spread of turbulence in the wake of a disturbance to be in the form of a wedge with a half angle of between $8.5^\circ$ and $11.5^\circ$. The disturbances causing turbulent wedges in the present case are at the points where the leading edge of the plate meets the roof and floor of the tunnel and also the weight used to tension the ribbon as described in the following chapter.

Figure 2.14 shows that on the centre line of the flat plate the boundary layer was laminar for a distance of 3 ft. 5 ins. from the leading edge. Although the height of the plate was 18 ins. only in a comparatively narrow region at the centre of the plate does the boundary layer remain laminar; a region only 6 ins. wide at a point 2 ft. 6 ins. from the leading edge. An investigation of the behaviour of the laminar boundary layer can thus be carried out over a distance of 3 ft. 5 ins. on the centre
line of the plate, where \( z = 0 \). For values of \( z \) not equal to zero the distance over which the boundary layer is laminar reduces rapidly with distance from the centre line. The apparatus is thus seen to be most suited to boundary layer investigations carried out on the centre line of the plate. For investigations of the three dimensional properties of the laminar boundary layer a wider flat plate would be preferable.

II.7. Measurements of the Boundary Layer Velocity Profiles

Using the same pitot-tubes as in the previous experiment, measurements were made of the velocity distribution across the boundary layer at different distances from the leading edge of the plate.

At the desired \( x \) position the total-head tube was placed so that it was slightly sprung against the flat plate. The carriage was then moved away from the flat plate, manometer readings being taken at intervals of 0.010 or 0.020 ins. It was found that the pressure difference remained constant for two or three readings and then began to rise sharply. This rise indicated that the total-head tube was no longer touching the plate and had entered a region of higher velocity. The readings were continued until the pressure difference remained constant showing
that the edge of the boundary layer had been reached. The pressure difference was taken to be proportional to $U^2$, with the maximum value proportional to $U_0^2$ and thus a graph could be drawn of $U/U_0$ versus $y$ with an arbitrary zero on the $y$-axis. Figure 2.15 shows a graph of this kind. The zero on the $y$-axis was found by extrapolation of the lower part of the curve to the axis.

When the total-head tube broke contact with the flat plate a slight vertical oscillation occurred and with great care the tube could be placed just in contact with the plate with an accuracy of $\pm 0.001$ ins. When this was done the micrometer reading agreed with that found by extrapolation of the curves, giving a verification of this method of positioning the $y$ axis of experimental points. The distance from the centre of the aperture to the edge of the tube which touched the flat plate was known, from examination of the tube through a microscope, to be 0.007 ins.

A series of graphs of the readings taken at different $x$ positions is shown in Figure 2.16. The vertical axis has been normalised to $U/U_0$ where $U$ is the measured velocity and $U_0$ the free stream velocity, in this case 38 feet per second. The horizontal axis is $y/\delta$ where $y$ is the distance from the flat plate and $\delta$ the calculated boundary layer thickness. The boundary layer thickness was calculated from the formula:-
Also in Figure 2.16 is shown the theoretical velocity profile as calculated by Blasius.

It can be seen that the measured velocity profiles are in fair agreement with the calculated profile for values of \( x \) up to 3'4". For values of \( x \) greater than 3'4" there is a marked difference which is an indication that the boundary layer has started to become turbulent.

The greatest difference between the laminar and turbulent velocity profiles occurs at \( y/s = 0.1 \) and \( y/s = 0.7 \). At either of these positions a single reading of the velocity will indicate the nature of the flow in the boundary layer. When the total head tube is touching the flat plate the pressure difference measured will be an indication of the velocity at approximately \( y/s = 0.1 \), and for this reason readings taken with the total-head tube touching the flat plate give a sensitive and quick method of determining the limits of the laminar boundary layer flow.
Figure 2.1a

Diagram of wind-tunnel, plan view.
Figure 2.1b
Photographs of the wind tunnel.
Upper: general view from the inlet end.
Lower: the working section.
Figure 2.2
Sketch of power generator system, with coarse speed control.
Figure 2.3

Sketch of horizontal traversing mechanism.
Figure 2.4

Sketch of vertical traversing mechanism.
Figure 2.5
Diagram of inverted cup pressure balance.
Figure 2.6

Key

c = contact on pressure balance.
P₁ & P₂ = relays.
R₁ = 1000 ohms.
R₂ = 470 ohms.
M = Servo-motor.
R₃ = rheostat for adjusting motor speed.
S₁ & S₂ = switches on rheostat driven by motor, to prevent over-running.
S₃ = on-off switch.
⊕ = 24 volts D.C.
Figure 2.6
Circuit diagram for relay system.
Figure 2.7
Sketch of sloping tube manometer.
Figure 2.8

Pressure balance calibration curve.
Figure 2.9

Graph of the pressure distribution along the flat plate.

\[ \frac{p-p_0}{\frac{1}{2} \rho U^2} \]

Key
- present work
- work of Schubauer and Skramstad.

Graph:
- \( x - x_T \) iseari
- \( x_T \) is position of the vibrating ribbon.

Notes:
- Slightly unstable with the data.
Figure 2.10

Neutral stability curves for different values of $\Lambda$, after Schlichting.
Figure 2.11

Graphs showing the boundary layer velocity profile for different values of $\Lambda$, after Schlichting.
Figure 2.12

Graph of $\Lambda$ versus $x - x_r$, where $x_r$ is the position of the ribbon.
Figure 2.13

Graph of pressure distribution across the transition region due to the turbulent wedges on the plate.

\[ \Delta p = -2.5 \text{ ins.} \quad U = 40 \text{ f.p.s.} \]
Figure 2.14
Plan of the flat plate showing the laminar, transition and turbulent regions.
Figure 2.15

Graph of boundary layer velocity readings showing method of zero extrapolation.
Figure 2.16

Boundary layer profiles for different values of $x$

$U_o = 38$ f.p.s.
CHAPTER III

THE INTRODUCTION OF ARTIFICIAL DISTURBANCES INTO THE BOUNDARY LAYER

1. The Oscillating Ribbon Technique

In order to study the effect of different types of disturbances on the laminar boundary layer, oscillations of known amplitude and frequency were introduced into the boundary layer at one point. This was achieved by using a vibrating ribbon suspended in the boundary layer, a technique first used by Schubauer and Skramstad.

A thin phosphor-bronze ribbon is placed in the boundary layer, parallel to the flat plate and perpendicular to the direction of the air flow. This ribbon is held away from the plate by two spacers or "bridges" and kept under tension. A small magnet is placed on the other side of the plate so that the ribbon vibrates in the magnetic field when a suitable alternating current is passed through it. The amplitude of the ribbon vibration depends on the tension in the ribbon, the distance between the bridges, the current through the ribbon and the strength of the magnetic field. As the amplitude of the disturbance introduced into the boundary layer is of fundamental importance in this work all these factors must be carefully controlled.
In their experiments Schubauer and Skramstad used an elastic band to maintain the tension in the ribbon. It was felt, however, that this was not an altogether satisfactory method as the tension will vary with temperature and the ageing of the rubber. In the present work small streamlined lead weights were used which could be hooked onto the bottom of the ribbon.

To examine the behaviour of the ribbon in detail a duplicate was set up on a vertical strip of perspex outside the wind tunnel. In the first instance bridges similar to those used by Schubauer and Skramstad were employed. Two strips of sellotape were stuck to the perspex, six inches apart. Over these was placed a strip of phosphor-bronze with cross section 0.001 x 0.1 ins. This strip was secured to the plate at the top and a weight of 100 gm. hung on the bottom, two more strips of sellotape were then placed over the first, securing the ribbon in position. A small permanent horseshoe magnet was placed level with the centre of the ribbon on the other side of the flat plate, and connections made so that current from an audiofrequency signal generator could be passed through the ribbon.

The motion of the ribbon was examined through a travelling microscope. By viewing light reflected from the edge of the ribbon the amplitude of its
motion was measured in terms of the graduations in the microscope eyepiece.

While attempting to calibrate the ribbon amplitude for different frequencies and voltages it was found that the resonant frequency was highly dependent on temperature and an accurate calibration could not be made. This temperature dependence can be explained by remembering that the ribbon is fixed at the bridges, and the tension, initially applied by the weight, does not necessarily remain constant.

The resonant frequency of the ribbon, \( f \), is given by the equation

\[
 f = \frac{1}{2S} \sqrt{\frac{T}{s}}
\]

(19)

where \( T \) is the tension in the ribbon, \( s \) the length of the ribbon between the bridges and \( \rho \) the mass of the ribbon per unit length.

The length \( S \) may be considered to be made up of two parts, \( L \) the length of the ribbon when not under tension and \( \ell \) the extension produced by the tension \( T \). \( L \) and \( T \) are related by the equations:

\[
 S = L + \ell
\]

(20)

and

\[
 \frac{Tg}{A} = Y \frac{L}{L}
\]

(21)

where \( A \) is the area of cross section of the ribbon, \( Y \) the Young's modulus of phosphor-bronze and \( g \) the acceleration due to gravity.
Taking logarithms of (19), (20) and (21) and differentiating with respect to temperature, \( \Theta \), gives:

\[
\frac{1}{f} \frac{df}{d\Theta} = -\frac{1}{s} \frac{ds}{d\Theta} + \frac{1}{2T} \frac{dT}{d\Theta} - \frac{1}{2} \frac{d\phi}{d\Theta}
\]  

(22)

\[
\frac{ds}{d\Theta} = \frac{dL}{d\Theta} + \frac{d\ell}{d\Theta}
\]  

(23)

and \( \frac{1}{\tau} \frac{d\tau}{d\Theta} = \frac{1}{A} \frac{dA}{d\Theta} + \frac{1}{Y} \frac{dY}{d\Theta} + \frac{1}{\tau} \frac{d\ell}{d\Theta} - \frac{1}{\tau} \frac{dL}{d\Theta}
\]  

(24)

Eliminating \( \ell \), \( \frac{d\ell}{d\Theta} \) and \( \frac{1}{\tau} \frac{d\tau}{d\Theta} \) from these equations leaves the relationship:

\[
\frac{1}{f} \frac{df}{d\Theta} = -\frac{1}{s} \frac{ds}{d\Theta} + \frac{1}{2A} \frac{dA}{d\Theta} + \frac{1}{2Y} \frac{dY}{d\Theta} + \frac{1}{2} \frac{dL}{d\Theta} \left[ \frac{YA}{13} + 1 \right] - \frac{1}{2} \frac{d\phi}{d\Theta}
\]  

(25)

where \( \alpha \) and \( \beta \) are the coefficients of linear expansion of perspex and phosphor bronze, and \( Y \) is the coefficient of temperature dependence of the Young's Modulus of phosphor-bronze.

Representative values for the variables may be taken thus:

\[
\alpha = 1 \times 10^{-4} \degree C \quad Y = 12 \times 10^6 \text{ dynes/cm}^2
\]

\[
\beta = 0.17 \times 10^{-4} \degree C \quad A = 6.5 \times 10^{-4} \text{ cm}^2
\]

\[
\gamma = 4 \times 10^{-4} \degree C \quad \text{T.g.} = 10^5 \text{ dynes}
\]

\[
f = 100 \text{ cycles/sec.}
\]

hence \( \frac{df}{d\Theta} = 32 \text{ cycles per second per degree centigrade.} \)
It can thus be concluded that owing to the high coefficient of thermal expansion of perspex the resonant frequency of the ribbon will increase by 32% on a rise of room temperature of 1°C.

This unfavourable situation can be overcome by changing the design of the bridges. If the ribbon is allowed to slip over the bridges the tension in the ribbon will remain constant and the above calculation reduces to:

\[
\frac{df}{d\theta} = \frac{f}{2} \left\{ -\frac{2}{s} \frac{ds}{d\theta} + \frac{1}{\rho L} \frac{dL}{d\theta} \right\}
\]  \hspace{1cm} (26)

and using the same values as previously

\[
\frac{df}{d\theta} = \frac{f}{2} \left\{ -2\alpha + \beta \right\}
\]  
\[
= -0.0075 \, \text{c/s/°C}
\]

The temperature dependence is seen to be negligible with this arrangement.

When the sellotape above the bridges was removed the ribbon tended to lift off the bridges during the vibration and in the final design the flat bridges were replaced by short lengths of glass capillary tubing with an outside diameter of 0.008 ins., giving a line contact rather than an area. To ensure that the ribbon touched the bridges at all times the ribbon was secured to the plate just above the upper bridge. Below the lower bridge a strip of metal was
placed over the ribbon, a groove in the strip ensured that though the ribbon could move freely in a vertical direction it could not lift more than a few thousandths of an inch from the plate.

A diagram of the method of ribbon mounting is shown in Figure 3.1.

2. Calibration of the Ribbon Motion

The behaviour of the duplicate ribbon set up outside the tunnel was carefully studied under different conditions of driving current and ribbon tension. On viewing the ribbon by the light of a stroboscope set at a frequency only slightly different from that of the forced vibration of the ribbon, it was possible to examine the form of the ribbon motion and ensure that it was vibrating in a single uniform loop.

It was found that for amplitudes greater than 2/3 of the bridge height, i.e. 0.0053 ins., and frequencies within 20 c/s of its resonant frequency, the ribbon did not vibrate in a uniform manner and these conditions were carefully avoided. The ribbon also showed a tendency to twist while vibrating, probably caused by a slight difference in tension between the two edges of the ribbon. This effect could be obviated by ensuring that the ribbon was
hanging symmetrically.

Although the movement of the duplicate ribbon could be studied visually through a microscope this method was not convenient for a ribbon mounted on a flat plate in the tunnel. A method of calibration was sought which complied with the following conditions:

a) It should not interfere with the ribbon movement.

b) It should not disturb the air flow in the boundary layer, or near the ribbon.

c) It should give a continuous reading of the amplitude of the ribbon vibration.

An electronic method of calibration was found most suitable.

A small metal plate of dimensions 0.5 x 0.25 ins. was placed underneath the magnet on the reverse side of the flat plate, level with the midpoint of the ribbon. This plate and the ribbon can be considered to act as a parallel plate condenser whose capacitance is inversely proportional to the distance apart of the plates. A knowledge of the capacitance between the fixed plate and the vibrating ribbon at any time allows their separation to be calculated.

The electronic device used for measuring the small changes in capacitance between the ribbon and
the static plate was designed by Burns et al.\(^{(41)}\) to measure the oscillations of the vane discussed in the following chapter.

A diagram of the electronic capacity meter is shown in Figure 3.2.

The plate, the capacity of which is to be measured, is incorporated in an oscillator circuit, from a design by Clapp. A variable capacitor enables the circuit to be tuned to a desired frequency, this frequency is modulated by small changes in the varying capacitance. The frequency modulated wave is amplified, passed through a crystal filter and detected as an oscillating voltage which can either be displayed on the screen of a cathode ray oscilloscope or measured on a valve voltmeter. The oscillator is tuned to a frequency corresponding to the midpoint of the steepest side of the response peak of the crystal filter. The tuning is monitored by a valve voltmeter measuring the voltage developed across the filter. A constant check was kept on this output so that any slight drift in the tuning of the oscillator could be corrected.

Full details of the design of this electronic capacity meter can be found in the work by Burns et. al.\(^{(41)}\).

This instrument was used to measure the changes in capacitance caused by the ribbon oscillations and
the output voltage was continuously displayed on an oscilloscope. A typical photograph of the oscilloscope trace is shown in Figure 3.3. The lower trace shows the alternating voltage applied to the ribbon and the upper trace the capacity meter output voltage used to measure the ribbon oscillations.

It was noticed that under some conditions the trace was not a smooth curve of the same frequency as that of the current driving the ribbon, but included extraneous bumps and ripples. An examination of the movement of the ribbon by the light of a stroboscope showed that on these occasions the ribbon was not vibrating uniformly, due either to the fact that the amplitude was too great or that the forcing frequency was too near the resonant frequency of the ribbon. Thus by ensuring that the oscilloscope trace was a smooth curve it was possible to avoid the conditions where the ribbon was not vibrating uniformly.

A valve voltmeter connected in parallel with the oscilloscope gave a reading of the root mean square voltage of the output from the capacity meter.


In order that a correct interpretation be made of the output voltage from the capacity meter produced
by the ribbon vibrations a calibration of this instrument was carried out using an oscillating capacitor of known characteristics.

A metal plate of dimensions 0.5 ins. x 0.25 ins. x 0.02 ins. was attached by an insulating rod to a small vibrator unit in such a manner that it could be vibrated in a direction perpendicular to its plane. A similar plate was mounted parallel to the first with their separation variable by means of a micrometer screw.

The two plates were connected by coaxial cable to the electronic capacity meter so that the change in capacity between them could be measured. The separation between the plates was varied from 0.07 ins. to 0.33 ins. in intervals of 0.02 ins. The current driving the vibrator which was proportional to the amplitude of the oscillations was varied from 10 to 60 milliamps, and the frequency of the current driving the vibrator was kept at a constant value of 80 cycles per second. For each combination of these conditions a reading was taken of the root mean square output voltage from the capacity meter.

A graph of these readings is shown in Figure 3.4.

The capacitance of an ideal parallel plate condenser is given by the equation:

\[ C = \frac{A k}{4 \pi D} \]
where \( A \) is the area of the plates, \( D \) their separation and \( k \) the dielectric constant of the medium between them.

The change in capacitance \( \Delta C \) produced by a small change in the separation \( \Delta D \) is thus given by

\[
\Delta C = \frac{A k}{4 \pi} \cdot \frac{\Delta D}{D^2}
\]

If \( \Delta D \) is taken to be the amplitude of the oscillation of the vibrating plate, which is proportional to \( I \), the current driving the vibrator, then \( \Delta C \) is the maximum change in capacitance produced. Assuming that the root mean square output voltage from the capacity meter is proportional to this maximum capacitance change, the equation can be written:

\[
V \propto -\frac{A k}{4 \pi} \frac{i}{D^2}
\]

or

\[
\frac{D^2}{i} = \frac{K}{V}
\]

where \( K \) is a constant.

If the above assumption is valid then a graph of \( D^2/i \) versus \( 1/V \) should be a straight line through the origin. Such a graph has been plotted in Figure 3.5 and, although there is a certain amount of scatter due to experimental error, these conditions are seen to be complied with. The conclusion can thus be drawn that the assumption made was valid and hence that

\[
V \propto \frac{\Delta D}{D^2}
\]
Thus the output voltage from the capacity meter is proportional to the amplitude of the oscillation of the vibrating plate and inversely proportional to the square of the distance apart of the plates.

A direct calibration of the amplitude of the ribbon vibrations cannot be obtained from these results as the exact area of the ribbon which acts as the moving plate of the capacitor cannot be found. It has been shown however that the capacity meter gives a reading directly proportional to the amplitude of the ribbon.

A quantitative interpretation of the voltage from the capacity meter was made by measuring the amplitude of the ribbon motion visually.

The duplicate ribbon was set up on a vertical perspex plate outside the tunnel in an exactly similar manner to the one of the flat plate. The electronic method was set up to monitor the motion of the ribbon and simultaneously its amplitude was measured visually in terms of the graduations in a microscope eyepiece. This eyepiece was later calibrated. The amplitude of the current driving the ribbon was controlled so that the output voltage from the capacity meter was constant for a range of frequencies from 90 to 130 cycles per second. The amplitude of the ribbon was measured for five frequencies within this range and values of the output
from 0.25 to 1.25 volts. Higher values of the frequency and voltage were not used because the resonant frequency of this ribbon was 150 c/s and only in this range did the ribbon vibrate uniformly in a single loop.

As the experimental results are not suitable for graphical display a table of the measured ribbon amplitude for the different values of the capacity meter output voltage and frequency of the driving current is given in Figure 3.6. These figures show that the amplitude is constant with frequency for constant values of the output voltage from the capacity meter, within a limit of ±0.0001 ins. The graph of mean amplitude versus output voltage is shown in Figure 3.7, and may be seen to be a straight line passing through the origin. The calibration of the capacity meter is given by the slope of this graph and found to be $360 \pm 120$ volts per inch.
Figure 3.1

Sketch showing method of mounting the ribbon.
**Figure 3.2**

**Key**

- $c_1$ = alternating capacity to be measured
- $c_2$ = variable tuning capacitor
- $c_3$ = 75 p.f.
- $c_4$ = 500 p.f.
- $c_5$ = 0.001 µf
- $c_6$ = 0.01 µf
- $c_7$ = 0.01 µf
- $c_8$ = 0.01 µf
- $c_9$ = 0.01 µf
- $c_{10}$ = 0.1 µf
- $c_{11}$ = 220 p.f.
- $c_{12}$ = 220 p.f.
- $c_{13}$ = 0.01 µf
- $c_{14}$ = 220 p.f.
- $c_{15}$ = 220 p.f.
- $c_{16}$ = 0.05 µf
- $c_{17}$ = 0.01 µf
- $c_{18}$ = 0.1 µf
- $c_{19}$ = 1 µf

- $R_1$ = 33 K ohms
- $R_2$ = 1 M
- $R_3$ = 330
- $R_4$ = 1 M
- $R_5$ = 220
- $R_6$ = 10 K
- $R_7$ = 10 K
- $R_8$ = 47 K
- $R_9$ = 1 M
- $R_{10}$ = 470
- $R_{11}$ = 10 K
- $R_{12}$ = 10 K
- $R_{13}$ = 100 K
- $R_{14}$ = 47 K
- $R_{15}$ = 1 M
- $R_{16}$ = 1.5 K
- $R_{17}$ = 0.25 M
- $R_{18}$ = 150 K

- $V_1$ = valve 12 A.T.7
- $V_2$ = 6.BR.7
- $V_3$ = 12 A.T.7
- $V_4$ = 6.BR.7

- $F$ = 1 megacycle crystal filter

- C = 0.01 µf
- M = 0.1 µf
- F = 1 megacycle crystal filter
Figure 3.2

Circuit diagram of capacity meter.
Figure 3.3

Photograph of oscilloscope traces for observing ribbon motion.

Upper trace: output from capacity meter.

Lower trace: voltage applied to the ribbon.
Figure 3.4

Graph of capacity meter voltage, V, versus the distance apart of the capacitor plates D, for different values of the current to the vibrator driving one plate.
Figure 3.5
Graph of \( \frac{1}{V} \) versus \( \frac{D^2}{I} \) for different values of the vibrator driving current. \( f = 80 \text{ c/s} \).
Table of measured ribbon amplitude, in thousandths of an inch, for different values of the forcing frequency and output voltage from the capacity meter measuring the ribbon motion. Also given are the mean values for each voltage.
Figure 3.7

Graph of the capacity meter output voltage versus the amplitude of the ribbon vibration.
CHAPTER IV

THE USE OF A VANE TO MEASURE BOUNDARY LAYER VELOCITY FLUCTUATIONS

1. The Measurement of Fluctuating Velocities

Schubauer and Skramstad\textsuperscript{(25)}, in their work, assumed the velocity fluctuations in the boundary layer to be two dimensional, i.e. to involve only two components of velocity $u$ and $v$. In their measurement of the distribution of these fluctuations across the boundary layer using a hot wire anemometer they only measured the $u$ component, as the $v$ component could be calculated from this using the equation of continuity. In recent work, however, it has been shown that the boundary layer fluctuations are three dimensional, involving three mutually perpendicular velocity components. The continuity equation thus involves three terms, from which one velocity can be calculated if the distribution of the other two are found experimentally.

The fluctuating $u$ velocity can be measured using a single hot wire\textsuperscript{(25)}, but to measure the $v$ and $w$ velocities by this method entails the use of two wires in the form of a cross or a vee, in conjunction with a complex electronic correlation network. The disadvantage of this method for measurements in the
boundary layer is that the hot wires cannot be placed very close to the plate because of the finite area they occupy, also they measure an average value of the velocity over this area.

A vane device was developed by Burns et al.\textsuperscript{(41)} to respond to fluctuations of the $v$ component of velocity in the boundary layer. An advantage of their device over hot wires is that readings can be taken with the vane to within 0.005 ins. of the flat plate. An investigation has been made into the possibility of using this instrument for more detailed boundary layer measurements than have hitherto been made.

**Description of the instrument**

The vane consists of a strip of aluminium foil 0.001 ins. thick, stiffened with a leaf of mica, and of dimensions approximately 0.3 x 0.3 x 0.002 ins. This is freely hinged on to a piece of nichrome wire of 0.001 ins. diameter, under tension between two supporting prongs of hypodermic tubing. The prongs are set in a small streamlined ebonite block. Another prong held by the ebonite block rigidly supports a static metal plate, of approximate dimensions 0.3 x 0.3 x 0.02 ins., parallel to the hypodermic tubing and about 0.3 ins. from the vane.

The vane head is mounted in the boundary layer
with the prongs pointing upstream and the static plate on the opposite side of the vane to the flat plate. The hypodermic tubes make a small angle to the flat plate so that the ebonite block is always outside the boundary layer even when the tip of the tubes touch the plate.

Electrical connections are made to the static plate and the nichrome wire which is in electrical contact with the vane itself through the hinge. The vane is at earth potential and as it moves causes changes in the capacity of the static plate. These changes in capacity are measured by the electronic capacity meter designed for this purpose and described in the previous chapter.

Full details of the design and construction of the vane head are given by Burns et al. (41). A photograph of the vane head is given in Figure 4.1.

3. **The Use of the Vane as a Measuring Device**

In the work by Burns et al. the vane was traversed downstream in the boundary layer on a flat plate at a distance from the plate which was a constant fraction of the boundary layer thickness. Measurements were taken of the amplitude of an oscillating ribbon injecting disturbances into the boundary layer which forced the vane to oscillate
with the same frequency and at a predetermined amplitude. In this way it was found whether the disturbance was being damped or amplified as it travelled down the boundary layer and the points of neutral stability at the change from one regime to the other were determined. The frequency under consideration was chosen to be the resonant frequency of the vane, different frequencies being investigated by using vanes of different dimensions.

The analysis of the vane motion showed that it was a resonant system with a resonant frequency, $f_0$, given by the equation

$$f_0^2 = \frac{0.3 C_{La} s U^2}{4 \pi^2 m}$$

where $U$ is the local wind speed, $\rho$ the density of the air, $m$ the mass of the vane, $s$ the span of the vane and $C_{La}$ the coefficient of the lift force per radian angle of incidence.

The analysis assumes that the lift force acting on the vane is directly proportional to the angle it makes with the incident air stream at any instant, even when the vane is rotating and the form of the boundary layer flow over the vane will be continuously changing. The above equation was experimentally verified, however, to within 10%, the expected limits of the experimental error, so the assumption that $C_{La}$ is constant is apparently valid.
The resonant frequency of the vane is thus seen to be proportional to $U$ for a given set of vane parameters. If the vane is used to measure the distribution of the $v$ velocity fluctuations across the boundary layer it will be moved through a region of changing wind speed. The velocity $U$, in the boundary layer, increases from zero at the wall to a maximum value in the free stream, according to the velocity distribution calculated by Blasius. Thus from the above theory the resonant frequency of the vane will increase from zero to a maximum value, $f_{\text{max}}$, in a similar manner. If the vane is traversed across the boundary layer under the influence of a fluctuating $v$-velocity of constant amplitude and frequency, the amplitude of the vane response will vary as its resonant frequency changes. In order to measure an unknown distribution of the fluctuating velocity the response of the vane to a given velocity distribution must be known in detail.

Without a knowledge of the response of the vane at different windspeeds to different frequencies and amplitudes of the transverse component of the incident air flow, the only comparable measurements that can be made are those with the vane at positions where the wind speed is the same. It was felt that measurements of this nature had been fully covered
by Burns et al. so the problem of calibrating the vane was undertaken.

4. The Calibration of the Vane Motion

It is required to know the amplitude of the fluctuating component of the wind velocity perpendicular to the vane which causes a given amplitude of vibration of the vane, for various frequencies and wind speeds.

When the vane is in the boundary layer measurements are made under the following conditions:

1) The hinge of the vane is stationary.
2) The impinging wind velocity has a known \( U \) component and a \( v \) component of unknown amplitude but known frequency.

In order to calibrate the vane the following conditions must be created:

1) The hinge of the vane is stationary.
2) The impinging wind velocity has a known \( U \) component and a \( v \) component of known amplitude and frequency.

A possible, though less favourable, set of conditions for calibration are thus:

1) The hinge of the vane oscillates with known amplitude and frequency.
2) The impinging wind velocity has a known \( U \) component and a zero \( v \) component.

In this case the amplitude of the vane oscillations relative to the hinge must be measured.
The first set of calibrating conditions are most desirable as they give a direct reading of the vane response to any amplitude of the fluctuating transverse velocity component. How then can these conditions be achieved? The most obvious case for preliminary investigation was that in which the air in the free stream is forced to oscillate transversely by a rigid oscillating ribbon. This method is similar to that used to excite the vane in the boundary layer. There is one fundamental difference however, the wake behind a ribbon in the boundary layer does not become turbulent because of the stabilising effect of the boundary layer. The wake behind a ribbon in the free stream will, on the other hand, always become turbulent as the residual turbulence in the free stream will cause breakdown in the wake due to the inherent instability of its inflected velocity profile.

To investigate this possibility more fully a ribbon was set up in the free stream, it was mounted between two rigid supports attached to a small vibrator outside the tunnel. The ribbon could be vibrated perpendicular to its own plane and also to the direction of the air flow. The resonant frequency of the ribbon was at least three times the frequency at which it was vibrated, so that it could be considered to vibrate with almost uniform amplitude along its length.
The vane was placed just down-stream from this ribbon and it was found possible, by bringing it to within about 0.2 ins. of the ribbon to place it up-stream of the point where the wake became turbulent. It was found however that the vane would only respond noticeably to large amplitudes of the vibrating ribbon. Under these circumstances the hinge of the vane was moving across the wake of the ribbon to such an extent that the impinging U velocity could not be considered constant. For small amplitudes of the ribbon motion the vane oscillations were not distinguishable from the flutter caused by the free stream turbulence.

The vane cannot be calibrated by placing it downstream from the ribbon in the boundary layer as unless it is very close to the ribbon the amplitude of the forcing disturbance is not known. If the vane is very close to the ribbon in the boundary layer the windspeed is again uncertain due to the ribbon wake.

An alternative method of calibrating the vane would be to place it in an oscillating jet of air. This would have been experimentally quite straightforward but here again we have the problem of the variation in velocity across the jet. If a wide jet was used to obviate this difficulty it would be an extremely complex problem to calculate how the air itself would respond when the walls of the jet were
oscillated. In any method of calibration the velocity components of the air impinging on the vane must be accurately known.

The possibility was then considered of using the second set of calibration conditions mentioned above. The vane head was mounted on a vibrator with the hypodermic tubes perpendicular to the direction of the air flow. The static plate was replaced by another which was parallel to the air flow and again 0.3 ins. from the vane. It was found that small oscillations of the vane head caused the vane to vibrate. The amplitude of these vibrations was of the same order of magnitude as the amplitude with which the vane vibrated when under the influence of boundary layer oscillations.

These conditions do not give a direct calibration of the vane motion, however, as vibrating the vane head imparts momentum to the hinge of the vane which is not present under the conditions of measurement in the boundary layer. The equation of motion of the vane in the boundary layer when acted upon by a fluctuating transverse velocity component is given by Burns et al.\(^{(41)}\). Assuming the hinge of the vane to be stationary the equation simplifies to:

\[
C_{l\alpha} \alpha s u^2 \left( \alpha + \frac{v}{u} \right) (1 - l) + r_\alpha + \frac{4}{3} ma \ddot{\alpha} = 0
\]

where \(2a\) is the chord of the vane, \(\alpha\) is the angle
the vane makes with the direction of the undisturbed air flow, $U$ and $v$ are the steady and fluctuating velocity components of the air flow in the $x$ and $y$ directions, $l$ is the distance from the hinge to the centre of pressure of the vane, $r_\alpha$ is the force acting at the hinge resisting angular motion of the vane and dots denote differentiation with respect to time.

By a similar analysis the equation of motion of the vane in a steady air stream with the hinge vibrating sinusoidally in a direction perpendicular to the air stream, is given by the equation:

$$C_{\alpha} \rho \sigma U^2 \left( \frac{q}{u} \right) \left( 1 - l \right) + r_\alpha + \frac{4}{3} m \ddot{\alpha} = m \ddot{\eta}$$

where $\eta$ is the velocity of the movement of the hinge of the vane and all the other terms have already been defined.

The solution of these two equations is different, but a knowledge of the experimental solution of the second equation should enable the value of the term $r_\alpha$ to be calculated. This should allow a theoretical solution of the first equation to be found.

It was found however that consistent experimental readings could not be obtained using this set-up, owing to the sensitivity of the vane to all spurious vibrations. The range of frequencies under investigation, 50 to 200 cycles per second, includes
the resonant frequency of a large number of mechanical structures. The distance from the vibrator to the vane was necessarily of the order of eight inches as the vibrator was outside the tunnel and the vane could not be placed too near the tunnel wall or the flat plate. The shaft holding the vane head could not be so massive that it caused a disturbance to the air flow in the tunnel. Although the mounting of the vibrator was made as nearly as possible vibrationless it was found impossible to prevent the lateral vibrations of the vane head in the tunnel. This added an unknown disturbance to the vane which could not be allowed for as it did not remain exactly the same when the vane head was removed and replaced.

As the step from one equation of motion to the other would necessarily be approximate in nature, the experimental solution of the second equation must be found to some accuracy if the final calibration of the vane motion is to be sufficiently accurate. It was thus decided that this method of calibration was not suitable.

The final method of calibration which was considered was that of comparing the vane response with that of another, calibrated instrument under exactly the same conditions. Hot wire anemometers which measure the $v$ component of the wind velocity have
been described by Schubauer and Skramstad(25) and many other workers. Although these instruments are not suitable for measurements very close to the flat plate they can be used at the edge of the boundary layer on in the free stream, that is at distances greater than 0.2 inches from the flat plate.

A possible method of calibration would thus be to measure the \( v \) component of the velocity at a certain position at the edge of the boundary layer under different conditions of windspeed and amplitude and frequency of the ribbon vibration, using a hot wire anemometer. The vane could then be placed in the same position and its response measured under the same conditions.

Although hot wire anemometer equipment is at present under construction it was not ready for use at the time this work was carried out.

5. **The Use of the Vane to Measure the Frequency of Boundary Layer Oscillations.**

It has been shown that the vane cannot be used directly to measure the amplitude of velocity fluctuations in the boundary layer. It can be used however to measure the frequency of these fluctuations. A resonant system under forced vibration always oscillates at the forcing frequency, though the
amplitude and phase of the oscillation depends on the resonant frequency and damping of the system. The frequency of the signal produced by the capacity meter monitoring the vane motion may thus be taken as the frequency of the \( v \) component of the air velocity acting at the hinge of the vane.

Comparisons were made of the frequency of the oscillations of the vane and the ribbon for different positions of the vane in the boundary layer and under different conditions of wind speed and amplitude and frequency of the ribbon motion. No detectable difference in the frequency was found under any conditions where the forced vane motion was distinguishable from the random flutter produced by the free stream turbulence.

It has been suggested by Shen\(^{24} \) that when a disturbance is amplified as it passes down the boundary layer the frequency of the disturbance \( \omega \) changes according to the relationship

\[
2 \pi f = \alpha_r c_r \left( 1 + \frac{\alpha_i}{\alpha_r^2} \right)
\]

where \( \alpha_i \) is the amplification rate with respect to distance and a function of \( x \), \( \alpha_r / 2 \pi \) is the wavelength of the disturbance and \( c_r \) is the wave velocity of the disturbance. These changes in frequency would be very small unless the amplification became very strong.
No experimental evidence of this change in frequency was found, even at a point just prior to the breakdown of the laminar boundary layer where the amplification might be expected to be a maximum.
Figure 4.1.
Diagram and photograph of the vane head.
CHAPTER V

MEASUREMENTS OF THE TRANSITION TO TURBULENCE

1. Measurements of the Start of the Transition Region

Although many investigations have been made of the properties of the transition region in the boundary layer on a flat plate there appears to be a gap in the present knowledge of boundary layer flow. What effect do the various parameters such as wind-speed and the amplitude and frequency of the disturbance have on the position of the onset of the transition region? A study of this problem has been carried out, within the limits imposed by the apparatus available.

It has been shown by many workers, whose experiments are discussed in Chapter I, that the detailed flow in the transition region cannot be satisfactorily studied using instruments with a slow response and which thus measure a time average of the property under consideration. The flow in the transition region has been shown to consist essentially of rapid changes between laminar and turbulent flows.

If the position of the onset of the transition region is defined as the point where the boundary layer ceases to be entirely laminar, this point will be indicated by a change in the shape of the velocity
profile across the boundary layer. A measurement of the position of the start of the transition region can thus be made using a total head pressure tube, which although it is a time averaging instrument, can be used to measure the shape of the velocity profile with some accuracy.

The method used for measuring the position of the transition region was similar to that described in Chapter II for finding the extent of the laminar boundary layer on the flat plate. The hypodermic total head and static pressure tubes were traversed along the centre line of the flat plate with the total head tube in contact with the plate. At two inch intervals the difference between the pressures in the two tubes was measured on the sloping tube manometer. In order to obtain the greatest possible accuracy in the manometer readings the position of the paraffin surface in the sloping tube was measured through a microscope, enabling changes in position of 0.01 cm. to be detected. This corresponds to a change in pressure of $0.48 \times 10^{-3}$ ins. of water, this is not a measure of the final accuracy of the readings however owing to the fluctuations in pressure which occurred.

Measurements were taken over a range of $x$-positions from before the onset of the transition region to an $x$ position of 3'2". Readings were not
taken beyond this point because transition at the centre line due to the turbulent wedges on the plate commenced at an x position of 3.5", and transition beyond this point might be due to two different causes. It was found that in the laminar region and at the start of the transition region small pressure fluctuations occurred which had an amplitude of the order of $0.48 \times 10^{-3}$ ins. of water. In the centre of the transition region these fluctuations became much more pronounced, the amplitude rising to the order of $2.4 \times 10^{-3}$ ins. of water, the amplitude dropped to its previous value when the boundary layer became fully turbulent. These variations in pressure were thought to be due to changes in the position of the transition region caused perhaps by fluctuations in the free stream wind speed which would not be compensated for by the speed control mechanism owing to their high frequency.

Pressure readings were taken across the transition region under many different combinations of conditions. The ribbon was situated 13 ins. from the leading edge of the plate, and the frequency at which it vibrated was varied from 40 to 160 c/s. in steps of 10 c/s. In order that the frequency should always be within the permitted range, with respect to the resonant frequency of the ribbon, for uniform vibration, three different ribbon tensions were used
to cover the range of frequencies. Tensions of 50, 100 and 150 gm. were used for the ranges 40-70, 80-120 and 130-160 cycles per second respectively.

For each frequency five different ribbon amplitudes were used. These were the amplitudes which gave a root mean square output voltage from the capacity meter monitoring the ribbon motion, of 0.25, 0.50, 0.75, 1.00 and 1.25 volts. These voltages corresponded to amplitudes of the ribbon motion of 0.07, 0.14, 0.21, 0.28, 0.35 x 10^-2 ins., the details of this calibration being given in Chapter III. A constant check was made on the ribbon amplitude, in terms of the output voltage, throughout the experiment. A constant watch was also kept on the shape of the output signal as displayed on an oscilloscope screen. Readings were only taken when the shape of the curve was smooth and of the correct frequency, with no spurious bumps or shoulders which would indicate an uneven motion of the ribbon.

The experiment was repeated for windspeeds of 25, 30, 35 and 40 feet per second. At the lower windspeeds the number of readings was limited by the fact that under some conditions induced transition had not started before transition due to the turbulent wedges occurred. For a more comprehensive set of results a wind tunnel with a wider flat plate is
indicated.

A graph showing the type of readings obtained is given in Figure 5.1. Smooth curves were drawn through the experimental points which showed a certain amount of scatter in the centre of the transition region due to the oscillations in the pressure difference. The point of the onset of the transition region was read as the point where the two curves diverge, the upper curve indicating disturbed flow and the lower curve undisturbed laminar flow. The slight curvature of the lower curve is due to the thickening of the boundary layer, causing the pressure at any constant distance from the flat plate to decrease.

Each set of readings was repeated after an interval of a few days and except for a few isolated cases the position of the onset of transition agreed to within $\pm 1$ inch. There are so many different factors which affect the position of transition that this was as good an accuracy as could be expected. Although every effort had been made to control the windspeed, degree of turbulence, ribbon amplitude and ribbon frequency, a slight change in any of these factors could give rise to this discrepancy.

Tables of the position of the onset of turbulence for each set of readings, together with the mean value of each pair and the corresponding Reynolds number are given in Figures 5.2a - 5.2d.
2. **Discussion of Results**

A convenient method for representing the regions of amplification and damping in the boundary layer is to plot the curve of neutral stability in the \((\frac{\beta r \nu}{u_0^2}, R)\) plane. The present results were plotted in terms of these variables in order to investigate their relation to the neutral stability curve. These graphs are shown in Figures 5.3a - 5.3d, also on these graphs are plotted the part of the neutral stability curve calculated by Shen. The vertical line on the left-hand side of these figures represents the position of the vibrating ribbon; it appears in a different position on each graph due to the axes which have been used.

The first important feature of these graphs is that although for lower values of \(\frac{\beta r \nu}{u_0^2}\) the transition region starts in the amplifying region, for higher values of this parameter transition starts in the damping region. All the graphs which reach high enough values of \(\frac{\beta r \nu}{u_0^2}\) show this effect. It is also noticeable that the curves tend to cross Branch II of the neutral stability curve at right angles, this is true for ten of the twelve curves drawn.

The interpretation of the shape of these curves cannot be made in a precise way as there are no
theoretical or experimental results with which they can be compared. The graphs indicate the final event in a sequence which starts with the injection of small disturbances into the boundary layer by the ribbon. The ribbon is in some cases in the amplifying region and in others in the damping region, accordingly the disturbance is either immediately amplified or first damped and then amplified when the disturbance reaches a point further down stream. The initial amplification will be according to the linear theory of small perturbations. At some stage in the amplification the amplitude of the disturbance reaches such a magnitude that the theory of infinitesimal disturbances no longer applies, new effects now appear such as the distortion of the mean flow, the introduction of higher harmonics and strong spanwise variations in the amplitude of the disturbance. Further down stream the production of turbulent spots commence which marks the beginning of the transition region which has been measured.

Although the change from one regime to the next in the passage of the wave along the boundary layer is likely to be gradual rather than abrupt, a point can perhaps be defined as that where the wave ceases to behave according to the theory of infinitesimal disturbances. It seems possible that this point will
occur when the wave either reaches a certain critical amplitude (with respect to particle displacement) or a certain critical energy (i.e. amplitude times frequency reaches a critical value). In all cases it would be expected that this critical amplitude or energy would be reached at some point in the amplifying region. After this point the behaviour of the wave changes in an unknown manner. The experimental results do show however that a curve similar to branch II of the neutral curve, does have some significance. Although breakdown does occur in the region where small disturbances would be damped it appears to be delayed in this region as indicated by the change in direction of the experimental curves when they cross branch II of the neutral curve. It was not experimentally possible to ascertain whether there are some disturbances which are not amplified to a sufficient extent to reach this critical amplitude and would thus be damped after crossing the neutral curve and so that they could not be the cause of breakdown. If a disturbance of this type could be found to exist then it might be concluded from the present results that once the injected disturbance had been amplified to the critical value then it was bound to cause breakdown at some point further down-stream. The distance the wave travels between the point where
it reaches its critical value and the start of the transition region appears to depend on whether it is passing through an amplifying or damping region of the boundary layer, the wave travelling further through the damping region before breakdown occurs.

It should be remembered, however, that the experimental conditions are far from ideal, especially in the fact that there is a certain amount of turbulence in the free stream. The possibility of an interaction between the free stream turbulence and the amplified wave leading to the breakdown of the laminar boundary layer cannot be ruled out. This possibility is usually neglected in a theoretical approach to the problem. The question of whether the boundary layer would become turbulent due to the influence of an injected disturbance of a single frequency in the complete absence of free stream turbulence is an interesting though academic problem as these conditions could never be realised experimentally.

Although the full interpretation of these experimental results is difficult, they raise a number of questions which could be verified by further experiment with a hot wire anemometer. Is the hypothesis of a critical amplitude or energy valid? This could be investigated by following the disturbance along the boundary layer until it ceases to
obey the theory of small disturbances, and noting the energy or amplitude of the disturbance at this point. Is there any empirical relationship which governs the distance the disturbance covers after the critical point before breakdown occurs? An empirical relationship is suggested as the calculation of a theoretical one appears at the moment to be impossibly complex.

Figures 5.3a - d also show the point of breakdown produced by disturbances of different amplitude. The boundary layer remains laminar longer when the disturbance has a lower amplitude, as might be expected, but the curves denoting the point of breakdown show the same characteristic shape. No direct conclusion can be drawn from the spacing between the curves for different amplitudes, the spacing between the curves of higher amplitude is certainly less than between the low amplitude curves though the difference does not appear to be a simple function of the ribbon amplitude. It is probable that the spacing of these curves is again related to the rate of amplification of the disturbance which is unknown for disturbances of finite amplitude.
3. Measurements of the Extent of the Transition Region.

Although the primary aim of the readings described in the previous section was to discover the position of the start of the transition region, in many cases they also provided a measurement of the extent of this region. In graphs like those shown in Figure 5.1 the point where the two curves diverge was taken to be the start of the transition region and the point where the upper curve reaches its maximum value was taken to be the end of this region and the start of fully developed turbulence.

A table showing the extent of the transition region measured in this way, $\lambda$, together with the parameters describing the flow and disturbance introduced into the boundary layer, is given in Figure 5.4.

The length of the transition region was found to be between 7 and 12 ins., with the values lying almost randomly in this range. No correlation could be found between the length of the transition region and its distance from the leading edge of the plate.

An empirical formula was calculated by Dhawan and Narasimha (33) relating these two quantities thus:

$$ R_\lambda = 5 R_t^{0.6} $$

where $R_\lambda$ and $R_t$ are Reynolds numbers based on the
extent and position of the transition region respectively. This formula predicts the extent of the transition region, for the values of $R_t$ found in this experiment, to be of the same order of magnitude as the present experimental values, around 10 ins., but it is in no way confirmed in detail by the present results.

This lack of agreement between the empirical equation and the present experimental readings could be due to two factors. Either the equation is incorrect in that all the relevant parameters have not been taken into account, or the present results contain a very high degree of random error.

Figure 5.5 shows a graph of this equation together with points denoting the experimental readings of many workers which were collected together by Dhawan and Narasimha. Also on this graph are plotted points denoting the results of the present experiment. It can be seen that the formula can only be said to describe the experimental results very approximately and the present readings lie well within the scatter of the results of previous experiments.

As the pressure readings during this experiment were made when the total head tube was touching the flat plate they could not be expected to be directly proportional to the square of the velocity at any point in the boundary layer. It was thus not possible to calculate from them the distribution of the intermittancy factor $\delta$ across the transition region for comparison with the equation calculated by Dhawan and Narasimha.
Measurements of the Transverse Distribution of Boundary Layer Properties

During a measurement of the velocity profile across the boundary layer it was noticed that the boundary layer thickness did not correspond exactly to that calculated from the formula:

\[ S = \frac{1.72}{0.341} \sqrt{\frac{v_x}{u_0}} \]

where \( S \) in this case is the point where the velocity reaches 99\% of its maximum value. Klebanoff and Tidstrom\(^\text{(36)}\) had found during their work that the thickness of the boundary layer varied across the plate, in a direction perpendicular to the air flow. Although in the present work the region suitable for laminar boundary layer measurements away from the centre line of the plate was severely limited, the variation in boundary layer thickness over a region \(-1.5 \text{ ins} < z < 1.5 \text{ ins.} \) was measured.

The most accurate method found for measuring the boundary layer thickness was to take readings of the complete velocity profile by the method described in Chapter II. The point where the velocity reached 99\% of its maximum value was then read off the graph.

Graphs of the variation in \( S \) are shown in Figure 5.6 for two different values of \( x \), the distance from the leading edge. The boundary layer
thickness was found to vary periodically about a value which corresponds approximately to that given by the above formula, the wavelength of the variations being about one inch. An alternative method to measure the variations in the boundary layer thickness is to measure the changes in velocity at a constant distance from the flat plate within the boundary layer. A graph of some readings of this kind are shown in Figure 5.7, also on this graph are plotted values of $u/U_0$ calculated from the measured boundary layer thickness assuming a velocity distribution of the form calculated by Blasius. It can be seen that the measured and calculated values of $u/U_0$ are in all cases very close, indicating that although the growth of the boundary layer differs from that theoretically predicted, the velocity distribution across the boundary layer is similar in form. The value of $y$ chosen for this comparison is approximately half the boundary layer thickness, a point where the velocity is very sensitive to variations in the value of $\delta$.

The wavelength of the variations in boundary layer thickness measured by Klebanoff and Tidstrom (36) was also one inch. They found this value to be unchanged by changing the windspeed or replacing the leading edge of the plate. The magnitude and position
of the variations were found to change when the smoothing screens of the wind-tunnel were cleaned, though their wavelength remained unaltered.

As the object of this work was to measure the position of the start of the transition region an experiment was carried out to see whether there were spanwise variations in this position corresponding to the variations in boundary layer thickness.

The hypodermic total head tube was placed touching the centre line of the flat plate and in the middle of the transition region caused by the ribbon oscillation. In this position the pressure in the tube increases rapidly with a small increase in $x$ and thus a movement of the transition region up- or down-stream will be immediately detected by a rise or fall in the pressure measured. The pitot tubes were then traversed in the $z$ direction with the total head tube touching the plate. Pressure readings were taken at intervals of $\frac{1}{4}$ inch, with and without the ribbon vibrating.

Figure 5.8 shows a graph of a set of readings of this kind. It can be seen that the slight variations in the pressure measured when the ribbon was not vibrating correspond to the previously measured variations in boundary layer thickness.
When the boundary layer increases in thickness the velocity, and thus pressure, at a constant distance from the plate falls. The upper curve which shows the pressure when the ribbon was vibrating in a single smooth curve with a peak at \( z = 0 \). This indicates that the start of the transition region occurs at its lowest value of \( x \) on the centre-line of the plate. The transition region moving downstream as the distance from the centre line increases.

The fact that the spanwise distribution of the position of the transition region is of this form is due to the shape of the ribbon disturbance. The amplitude of the ribbon vibration is zero at the bridges (i.e. at \( z = \pm 3 \) ins.), and has a maximum at the centre of the ribbon which is also the centre of the plate where \( z = 0 \). It has been shown already that the larger the amplitude of the ribbon, the further upstream the boundary layer starts to become turbulent.

In order to measure the effect of the variations in boundary layer thickness on the position of the transition region it would be necessary to introduce a disturbance which had a constant amplitude for all values of \( z \). To do this using the oscillating ribbon technique would involve forcing the ribbon to vibrate with a centre section always parallel to the plate.
This cannot be done with a magnetic field which has a maximum at the centre of the ribbon, as produced by the single permanent magnet in the present work. A uniform disturbance could be produced by replacing the single magnet by a series of electromagnets, along the length of the ribbon, whose power could be varied individually, so that their combined effect was a magnetic field of the required form.
Figure 5.1

Graph of one set of pressure readings versus x position, for transition induced by the vibrating ribbon.

\[ U = 40 \text{ f.p.s.}, \quad f = 160 \text{ c/s}. \]
Tables of results

Figure 5.2a  \( U_0 = 40 \text{ f.p.s.} \)

5.2b  \( U_0 = 35 \text{ f.p.s.} \)

5.2c  \( U_0 = 30 \text{ f.p.s.} \)

5.2d  \( U_0 = 25 \text{ f.p.s.} \)

Key

\( x_1 \) = first reading of the start of the transition region.

\( x_2 \) = second reading of the start of the transition region.

\( x_m = \frac{x_1 + x_2}{2} \)

\( R = 1.72 \sqrt{\frac{a_0 x_m}{v}} \)

\( a \) = ribbon amplitude of 0.0007 ins.

\( b \) = " " " 0.0014 ins.

\( c \) = " " " 0.0021 ins.

\( d \) = " " " 0.0035 ins.

\( f \) = frequency of ribbon oscillation.
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Figure 5.2d
Graphs showing the start of the transition region under different conditions.

**Key**

---

neutral stability curve calculated by Shen.

*•••••••••••*

position of the oscillating ribbon.

○○○○○○○○○○○

start of transition when ribbon amplitude = 0.0035 ins.

□□□□□□□□□□□□

start of transition when ribbon amplitude = 0.0028 ins.

△△△△△△△△△△△

start of transition when ribbon amplitude = 0.0021 ins.

△△△△△△△△△△△

start of transition when ribbon amplitude = 0.0014 ins.

△△△△△△△△△△△

start of transition when ribbon amplitude = 0.0007 ins.

**Figure 5.3a** $U_o = 40 \text{ f.p.s.}$

5.3b $U_o = 35 \text{ f.p.s.}$

5.3c $U_o = 30 \text{ f.p.s.}$

5.3d $U_o = 25 \text{ f.p.s.}$
Figure 5.3a

Graph showing the start of the transition region under different conditions. \( U_0 = 40 \text{ f.p.s.} \) (Key earlier).
Figure 5.3b

Graph showing the start of the transition region under different conditions. \( U = 35 \text{ f.p.s.} \) (Key earlier).
Graph showing the start of the transition region under different conditions. $U_0 = 30$ f.p.s. (Key earlier)
Figure 5.3d
Graph showing the start of the transition region under different conditions. \( U_0 = 25 \text{ f.p.s.} \) (key earlier)
**Figure 5.4**

**Key**

\( U_o \) = free stream windspeed in feet per second  
\( A \) = amplitude of ribbon vibration.

\[ \begin{align*} 
  b &= 0.0014 \text{ ins.} \\
  c &= 0.0021 \text{ ins.} \\
  d &= 0.0028 \text{ ins.} \\
  e &= 0.0035 \text{ ins.} 
\end{align*} \]

\( x_t \) = distance from leading edge of start of transition region  
\( R_t = \frac{U_o x_t}{v} \)

\( \lambda \) = length of transition region in ins.  
\( R_{\lambda} = \frac{U_o \lambda}{v} \cdot 12 \)
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Figure 5.5

Graph of \( \log R_\lambda \) versus \( \log R_\infty \); after Dahwan and Narasimha, showing also the results of the present work.

\[
R_\infty = \text{Reynolds number for the start of the transition region.}
\]

\[
R_\lambda = \text{Reynolds number for the extent of the transition region.}
\]
Graphs showing the variations in boundary layer thickness with $z$, for two different $x$ positions. $U_o = 40$ f.p.s.
Figure 5.7. Graph showing the variation of $\frac{u}{u_0}$ with $z$, also values of $\frac{u}{u_0}$ calculated assuming a Blasius velocity distribution.

Figure 5.8. Graph of manometer readings, $l$, for a $z$ transverse with and without the ribbon vibrating.

$U_0 = 40$ f.p.s., $f = 80$ c/s, $x = 216$".
CHAPTER VI

CONCLUSIONS

A picture of the phenomena occurring in the boundary layer on a flat plate, prior to the onset of fully developed turbulence, has been obtained by previous workers.

Small disturbances of the laminar boundary flow, due perhaps to free stream turbulence or slight imperfections in the plate, are amplified at certain positions, depending on their frequency, the free stream velocity of the flow and the kinematic viscosity of the fluid. When these disturbances become sufficiently amplified their distribution transversely across the plate becomes noticeably periodic and amplification occurs most rapidly at certain spanwise positions. At a certain distance from the leading edge of the plate turbulent "spots" are formed at the positions of greatest amplification. These spots travel down stream and spread transversely into the regions of minimum amplification, until the boundary layer becomes fully turbulent. The spanwise periodicity of the disturbance amplitude appears to be associated with variations in the boundary layer thickness, whose origin is not yet fully understood.

In making a study of such boundary layer flow it
is desirable to measure the effect of each factor separately with all others carefully controlled. In the present work the effect of the amplitude and frequency of the disturbance has been investigated for different values of the free stream velocity. The disturbance was introduced into the boundary layer by an oscillating ribbon whose movement was carefully measured, while other disturbing factors such as free stream turbulence and surface roughness were of a very low level in the wind tunnel used. The ribbon could be forced to oscillate with a frequency from 40-160 c/s., with the amplitude at its centre fixed in the range 0.0007 to 0.0035 ins. to an accuracy of 0.0001 ins. The mean position of the ribbon was 0.008 ins. from the flat plate.

As the longitudinal pressure gradient affects the points in the boundary layer where a disturbance starts to be amplified, this was reduced as near as possible to zero downstream from the oscillating ribbon. The Pohlhausen shape factor describing the pressure gradient along the plate was shown to lie between the values + 0.1 and - 0.25.

It has previously been shown that the boundary layer flow varies transversely across the plate when a disturbance becomes amplified. Thus a full quantitative picture of the flow on the plate cannot be
found unless two perpendicular components of the velocity are measured at many points in the field of flow. The third component can then be calculated from the continuity equation. The u-velocity component could be measured with a single hot wire, although such equipment was not available at the time of this work. The measurement of the v or w velocity component is a more difficult problem, especially at positions very close to the flat plate.

An instrument for measuring the fluctuating v component of the velocity by using a small, freely-hinged vane, was devised by Burns et al. (41). An investigation was made into the possibility of using this vane for more detailed boundary layer measurements. It was shown that although this instrument is inherently very useful, due to its simplicity and the fact that it can be placed very close to the plate, since it is a resonant system with a response varying with windspeed, a calibration must be made before any conclusion may be drawn from its measurements. Several methods of calibration have been discussed but none proved very satisfactory and it is suggested that a direct and accurate method of calibration would be to compare the responses of the vane and a hot-wire anemometer under identical conditions. The vane could then be used for measurements where a hot-wire anemometer is not suitable.
Measurements were also made of the point at which turbulent spots start to occur, the start of the transition region, under different conditions of windspeed and amplitude and frequency of the ribbon motion. Such measurements, when plotted on the \((\beta^{1/4} u^1, R)\) plane for comparison with the neutral stability curve, show that the onset of transition may occur in the damping region, confirming that this transition is not solely governed by the theory of small disturbances.

The shape of the neutral stability curve can be predicted by the linear perturbation theory and the results have been well confirmed by Schubauer and Skramstad\(^{(25)}\). There are many assumptions made in the theory, however, which are only valid for the neutrally stable disturbance. When a disturbance travels down the boundary layer its amplitude, at each \(x\) position, reaches a steady state with respect to time, and thus in the theoretical approach amplification with distance and not with time should be considered. Also when a disturbance is considered to move down the boundary layer the increasing thickness of the layer should no longer be neglected. The present theoretical approach includes a further approximation in that the disturbance is considered to extend to infinity in space and time whereas in
practice the boundary layer will be disturbed by a finite wave train.

Thus the linear theory cannot be expected to give very good agreement with experiment, even when the disturbance is very small, except under the conditions of neutral stability. The curves calculated by Shen, for finite values of $c_1$, by assuming the linear theory to apply in the amplifying region are thus not very meaningful, except for very small values of the amplification, and cannot be used in an interpretation of the present experimental results.

When a disturbance becomes amplified the linear theory is certainly not valid as the disturbances can no longer be considered infinitesimal, however a satisfactory solution to the non-linear boundary layer equations has yet to be found.

If a detailed experimental study could be made of the disturbed boundary layer with the velocities measured very accurately it should be possible to discover what theoretical assumptions can be made to simplify the solution of the non-linear boundary layer equations. For example, Stuart\(^{(27)}\) has developed a solution of the non-linear equations for parallel flow which requires a knowledge of the distribution of the amplified disturbances across the boundary layer. His critical assumption is that this
distribution is of the same form as that for the neutral disturbance.

Another field for further experimental work would be the search for the factors affecting the spanwise variations in boundary layer thickness found by Klebanoff and Tidstrom\(^{36}\) and also in the present work.

It can thus be concluded that although a fairly clear qualitative picture of the phenomena accompanying transition to turbulence in the laminar boundary layer has been found, much further work, both experimental and theoretical, must be carried out before a quantitative picture is found which enables the occurrence of the various phenomena to be predicted.
ACKNOWLEDGEMENTS

I should like to thank Dr. M.A.S. Ross for her continued suggestions and encouragement during this work and Professor W.H.J. Childs for providing the laboratory accommodation and workshop facilities and for encouraging the project in every way.

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