STUDIES OF THE DETECTION OF OPTICAL COHERENCE USING A

COINCIDENCE COUNTING TECHNIQUE

Thesis

Submitted by

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CHAPTER I

HISTORICAL INTRODUCTION

The historical background to the experiment which is the main subject of this thesis may be traced back as far as 1807, when the first experimental demonstration of the interference of light was performed by Thomas Young\(^{(1)}\). He arranged that sunlight should pass through a pinhole in a screen, and then through two more pinholes in a second screen some distance away. The intermingling of the light propagating from the secondary pinholes gave rise to interference fringes, thereby indicating the wave or "undulatory" nature of light. Various objections to the validity of this experiment were raised, however, delaying the full acceptance of the wave theory, until Fresnel\(^{(2)}\) devised several experiments which incontrovertibly supported the wave theory, by the year 1816. Fresnel also extended Young's interference principle to cover certain examples of diffraction. Thus, the Classical Wave theory of light was born.

The first paper specifically to investigate partial coherence was due to Verdet\(^{(3)}\) (1865), who determined the separation of two pinholes, which would still give some visible interference when illuminated by the sun. Fizeau\(^{(4)}\) (1868), and later Michelson\(^{(5)}\) (1890) suggested that it was possible to measure the angular diameters of stars from the visibility of interference fringes. In a
further paper (1891), Michelson(6) managed to establish the connection between fringe visibility and the energy distribution in a spectral line. He did not interpret his results in terms of correlations, since this concept had not then been evolved, but his work in this field was to form the basis of partial coherence theory much later.

In the course of his researches into the thermodynamics of light-beams, von Laue(7) (1907) formulated the first quantitative measure of the correlation of light vibrations, and two years later, Einstein(8) made a great step forward by applying the energy fluctuation formula to black-body radiation within a closed box. This formula, when combined with Planck's(9) distribution law (1901), yields an expression for the variance of the energy which contains two terms, one of which may be interpreted as the contribution by classical particles, and the other as being contributed by classical waves. The mathematical analysis of radiation fluctuations was pursued by other workers, notably Bothe(10) (1927), and also Fürth(11) (1928) who showed that Einstein's energy variance equation could be obtained directly by the application of Bose-Einstein statistics to photons, irrespective of their spectral distribution. However, neither Einstein nor Fürth was able to generalise their results to a propagating beam of light, only to the somewhat impractical case of a closed-box type of system.

With the publication of a paper by van Cittert(12) in 1934, the theory of partial coherence began to emerge from
the narrow confines of the classical coherence theory into what was later termed the "Second-Order" theory. For the case of a perfectly focused, aberration-free optical system with a rectangular aperture, van Cittert calculated functions expressing the correlation between the complex amplitudes at any two points, defining a "degree of consonance" to assist his somewhat complicated calculations. Van Cittert's complicated and specialised proofs were later generalised and simplified by Zernike's (1938), using a simpler definition of partial coherence. The net result of these two papers has since been termed the "van Cittert-Zernike theorem" - as in Born and Wolf (1959). A refinement of the van Cittert-Zernike theorem was due to Hopkins (1951), who introduced factors, such as transmission functions, which made the application of the theorem to instrumental optics very much easier. However, as Wolf (1954) pointed out, Hopkins' analysis, unlike Zernike's, was applicable only to the limiting case of vanishingly narrow frequency range. Nevertheless, despite occasional errors, the development of Second-Order theory continued apace, viz., Wolf (1955, 1959), Blanc-Lapierre and Dumontet (1955), Pancharatnam (1956, 1957, 1963) et al. As a result, the theory became a well-developed tool for the unified analysis and description of coherence and polarisation phenomena.

Meanwhile, detection of weak radio and radar signals during the Second World War had indicated the necessity of
investigating the fluctuations inherent in electromagnetic radiation. Work in this field was published by Burgess (1941\(^{23}\), 1946\(^{24}\)), and the field was later extended by Lewis\(^{25}\) (1947), Jones\(^{26}\) (1947), and Fellgett\(^{27}\) (1949) to include a variety of radiation detectors, showing that the ultimate limit to the sensitivity of a detector of thermal radiation is set by the inherent fluctuations in the radiation field. These same workers also obtained expressions for the mean squared fluctuation in the number of photon counts recorded by an illuminated photodetector, from the standpoint of thermodynamics. It was claimed by Twiss and Hanbury-Brown\(^{28}\) (1957) that some of the expressions derived in this manner were inapplicable, mainly because the energy fluctuations in the output of a photon detector are, in general, not able to be equated to those of the thermal radiation field. It was shown that the principle of detailed balancing was a macroscopic effect, in that it applied to the average flow of energy, not necessarily to the fluctuations themselves. After some discussion (Fellgett\(^{29}\), 1957, Twiss and Hanbury-Brown\(^{28}\), 1957), the analysis of Hanbury-Brown and Twiss, which agreed with that of Kahn\(^{30}\) (1958), became established. The papers of the earlier workers had pointed out that the fluctuations in the number of photon counts, recorded by a photodetector, departed somewhat from classical counting statistics. Further, they had shown that the mean squared fluctuation in counting-rate could be considered as the sum of two quite separate terms, the "Shot-noise", and what
Hanbury-Brown and Twiss labelled the "Wave-Interaction noise". A later paper by Fellgett, Jones and Twiss (1959) was effective in narrowing the discrepancy between the earlier publications of Jones and Fellgett on the one hand, and Hanbury-Brown and Twiss on the other, demonstrating the limitations in both methods of calculation. Neither method of calculation had properly taken account of the interaction between the incident, emitted and reflected streams of radiation.

Interest in radiation fluctuations had been mainly confined to the limitations they imposed on the accuracy of radiation measurements, until 1952, when Hanbury-Brown, Jennison and Das Gupta (1952) published a paper which proposed a new type of interferometric technique for measuring the angular diameter of radio stars. The technique involved measuring the correlation of fluctuations detected by radio antennae placed at two points in the radiation field, and thence inferring the degree of coherence at those two points. This was a totally new departure, since the observation of the fluctuations was central to the whole concept of the interferometer, whereas previously they had been regarded merely as parasitic background noise. The signals from the two antennae were detected independently, and the correlation between the low-frequency components of the detector outputs was recorded. Thus, all information regarding the relative phases of the two signals was lost; contrast the original Michelson stellar interferometer, which the radio interferometer superficially resembles. Since the detection
process eliminated the high-frequency signal components, the radio-interferometer was able to operate with very long baselines, and proved to be substantially free from the disturbing effects of ionospheric scintillation. These notable advantages prompted Hanbury-Brown and Twiss (33) (1956) to apply the same principle to the measurement of the angular diameter of visible stars, replacing the aerials by mirrors, and the radio-frequency detectors by optical photomultipliers. The correlation between the fluctuations of the photocurrents when illuminated by a star would then be measured as a function of mirror separation. The major drawback to this suggestion was that it was not then known whether the time of arrival of photons at the two photocathodes would be correlated when the light beams incident on the two mirrors were coherent. In addition, it was not even known whether the correlation would be preserved during the process of photoelectric-emission.

In order that their ideas could be put to the test, Hanbury-Brown and Twiss (33) (1956) devised and built a compact laboratory apparatus embodying the same principle as their larger and less manageable stellar optical interferometer. Their light source was a small circular aperture on which the image of a high-pressure mercury arc was focused. The 4358 A line was isolated by means of an interference filter, and the resultant beam was split by a half-silvered mirror to illuminate the cathodes of two photomultipliers, one of which could be traversed normal to the incident light in order to vary the degree of coherence.
of the light beams. The fluctuations in the output currents from the photomultipliers were amplified and multiplied together in a linear mixer. The average value of the product, recorded by means of an integrating motor, gave the measure of the fluctuation correlations. Considerable care was taken to eliminate drifts in the electronic apparatus by the incorporation of synchronous alternating switches and narrow band-pass amplifiers. The experiment proved beyond doubt that photons in two coherent beams of light are correlated, and that the correlation is preserved in the process of photoelectric emission.

The resounding success of this experiment prompted further experimental investigation by correlation techniques (Hanbury-Brown and Twiss\(^{(34)}\), 1957), and also by the slightly more direct pulse-counting technique (Twiss, Little and Hanbury-Brown\(^{(35)}\), 1957, Rebka and Pound\(^{(36)}\), 1957, Brannen, Ferguson and Wehlau\(^{(37)}\), 1958, Twiss and Little\(^{(38)}\), 1959), which proved equally successful. The earlier experiments of Adam, Janossy and Varga\(^{(39)}\) (1955), and Brannen and Ferguson\(^{(40)}\) (1956) were not successful, mainly due to poor sensitivity, which was occasioned by the experimenters' lack of appreciation of the quantum-mechanical aspects of the counting fluctuations and their relationship to the coherence properties of the light, as was pointed out by Purcell\(^{(41)}\) (1956). Following the success of their laboratory experiment, Hanbury-Brown and Twiss\(^{(42)}\) (1956) were able to set up a full-sized astronomical interferometer for the measurement of the angular
diameter of visible stars. Their first test was on the star Sirius, and it gave a very satisfactory result. Apart from the good agreement obtained between the theoretical and experimental correlation curves, the effect of atmospheric scintillation was found to be negligible, which was definitely not the case with the original type of Michelson stellar interferometer.

One of the more widespread misconceptions of the old classical theory of optical coherence was the notion that no interference effects are possible between completely independent light beams. However, in 1955, Forrester, Gudmundsen and Johnson(43) managed to demonstrate the existence of beats resulting from the superposition of two independent light beams. The light beams consisted of the two Zeeman components of a visible spectral line, and the beats were observed by the superposition of the two beams on a photosurface, behind which was a microwave cavity, tuned to the frequency difference between the components. The beats were therefore detected by the excitation of the cavity. This experiment was a considerable achievement, because the theoretical signal to noise ratio was very low indeed. Later workers, such as Javan, Ballik and Bond(44) (1962), and Lipsett and Mandel(45) (1964) were blessed with the newly developed optical Maser, having a considerable coherence length, and were able to use far less sophisticated methods to detect the beats. Indeed, Magyar and Mandel(46) (1964) were even able to photograph fringes produced by the interference of two independent Maser light beams. However, the light produced by such a device has rather different
statistical properties from ordinary thermal light, owing to the process of stimulated emission, and this is outside the scope of this thesis.

It will be recalled that Brannen and Ferguson (1956) had performed an unsuccessful photon correlation experiment. These workers had claimed that a positive correlation would have indicated a violation of fundamental quantum-mechanical concepts, for instance, Dirac's famous dictum that interference between different photons never occurs. Purcell (1956), and Hanbury-Brown and Twiss (1957) were able to show that the correlation effects were quite capable of quantitative explanation in terms of the quantum-statistical behaviour of the photons, and this line of thought was further extended by Mandel (1958, 1959) and Fano (1961). Mandel showed that the counting fluctuations recorded by a photoelectric detector in a beam of thermal light are simply the density fluctuations of a boson assembly. Wolf (1958) proposed a definition of the coherence time of light in terms of the correlation function, while Mandel (1959) proposed a definition of coherence length in terms of the extent of the unit cell of phase-space in the direction of the light beam. In addition, Mandel (1961, 1962) identified the degeneracy as being the number of photons falling on a coherence area in a time of the order of the coherence time, or the number of photons in the same cell of phase-space. It was, therefore, quite definitely established that the fluctuations inherent in a stream of thermal light have a pure quantum-statistical
nature. Nevertheless, it should be remembered that Sillitto (1957) and Hanbury-Brown and Twiss (1957) pointed out that the wave picture of light is by no means superseded, since it is able to account for a good many of the observed effects, apart from those effects which are due to quite discrete quantum events.

It was known that the distribution of time intervals between successive counts is related to the spectral profile in the case of thermal light, and Mandel (1963) suggested a direct method of measuring this time interval by measuring the fluctuations of counts registered by a single photodetector. This experiment was concluded satisfactorily by Morgan and Mandel (1966), their measurements indicating a spectral width of the blue Mercury-198 line of approximately 200 MHz. A somewhat similar type of experiment was performed by Scarl (1966), who utilised a Hanbury-Brown and Twiss arrangement of two photodetectors illuminated by the mauve Mercury-198 spectral line. All pulses in a given time interval were fed to a time-to-height converter and pulse-height analyser. The shape of the resultant peak in the delay spectrum agreed with the theoretical shape for a line width of 1264 MHz, which was also the line width calculated from the Fourier transform of the spectral distribution measured with a special long Fabry-Perot interferometer.

Finally it should be remarked that the union of the classical field theory with modern quantum theory owes much to Glauber (1963), who pointed out that the positive
frequency components of the electric field (annihilation) operator are closely related to the complex fields and the associated complex Fourier amplitudes of the classical field theory.

The groundwork was therefore firmly established on which to base the experiment which is the main subject of this thesis. This experiment was designed to detect and measure a periodic correlation between photoelectron fluctuations as one photodetector was moved, with respect to another, across the optical field due to an incoherently illuminated Young-type double-slit.
CHAPTER 2

THE THEORY OF COHERENCE AND PHOTON FLUCTUATION CORRELATION

Section (a). The Complex Analytic Signal

In developing a classical description of optical interference, it is convenient to let $V(r,t)$ denote a real classical wave-function characterising the field at the point $r$ at time $t$. This function may, perhaps, represent the electric field or the vector potential, but its precise nature need not be specified for this derivation.

In any light beam, $V(r)$ will fluctuate rapidly with time, and it may be regarded as one particular member of a large ensemble consisting of all possible realisations of the field. A beam of light produced by a thermal source may be said to fluctuate because $V(r)$ consists of a large number of independently varying Fourier components, and so it is assumed that $V(r)$ may be represented as a Fourier integral with respect to time:

$$V(r,t) = \int_{-\infty}^{\infty} V(r,v) \exp(-2\pi ivt) dv$$

To ensure that $V(r)$ is square integrable, we may use instead the truncated function:

$$V_T(r,t) = V(r,t) \quad \text{when } |t| < \frac{1}{4T}$$

$$V_T(r,t) = 0 \quad \text{when } |t| > \frac{1}{4T}$$

and the negative frequency components ($v < 0$) may be omitted by the use of the Complex Analytic Signal.
Then
\[ V(r,t) = \int_0^\infty v(r,v) \exp(-2\pi ivt) dv \] (3)

where \( V(r,t) \) is defined as the Hilbert transform of \( v(r,t) \), thus:

\[ v(r,t) = [v(r,t) + iv^{(1)}(r,t)] \] (4)

\[ v^{(1)}(r,t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{v(r,t)}{(t' - t)} \, dt' \] (5)

\[ v^{(r)}(r,t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{v^{(1)}(r,t')}{(t' - t)} \, dt' \]

where it is understood that the principal value of each integral is to be taken.

Section (b). Two Beam Interference and the Coherence Function.

We now consider an interference experiment using a linearly polarised, quasi-monochromatic light beam. It will be appreciated that \( V \) cannot be measured directly by existing photodetectors, if only because it is oscillating far too rapidly. However, it is possible to measure the field correlations at two or more space-time points by proceeding in the following manner;

We consider the light vibrations at two pinholes \( P_1(r_1) \) and \( P_2(r_2) \), in an opaque screen placed across the light beam, and the effect produced when they combine at a second screen on the opposite side from the source.
Let any point of combination on the second screen be \( Q \), distant \( s_1 \) from \( P_1(\mathbf{r}_1) \) and \( s_2 \) from \( P_2(\mathbf{r}_2) \). Then, regarding \( P_1 \) and \( P_2 \) as centres of secondary disturbances, the instantaneous field at \( Q \) is given by:

\[
V(r,t) = K_1 V(r_1, t-t_1) + K_2 V(r_2, t-t_2) \tag{6}
\]

where \( t_1 = \frac{s_1}{c} \) and \( t_2 = \frac{s_2}{c} \) are the times of transit of the light, velocity \( c \), from \( P_1 \) to \( Q \) and \( P_2 \) to \( Q \) respectively. \( K_1 \) and \( K_2 \) are transmission factors which depend on the size of the pinholes and the geometry of the system. Since the diffracted waves from \( P_1 \) and \( P_2 \) are delayed by one quarter of a period behind the primary wave, \( K_1 \) and \( K_2 \) are pure imaginary numbers.

Now, the instantaneous intensity \( I(r,t) \) at point \( P(r) \) at time \( t \) is defined by:

\[
I(r,t) = V^*(r,t)V(r,t) \tag{7}
\]

Hence, from equations (6) and (7), it follows that

\[
I(r,t) = |K_1|^2 I(r_1, t-t_1) + |K_2|^2 I(r_2, t-t_2)
+ 2\text{Re} \left[ K_1^* K_2 V^*(r_1, t-t_1)V(r_2, t-t_2) \right] \tag{8}
\]

where \( \text{Re} \) denotes the real part.

We now take the ensemble average of \( I(r,t) \) over all possible realisations of the field, thus:

\[
\langle I(r,t) \rangle = |K_1|^2 \langle I(r_1, t-t_1) \rangle + |K_2|^2 \langle I(r_2, t-t_2) \rangle
+ 2\text{Re} \left[ K_1^* K_2 \Delta(r_1, r_2, t-t_2, t-t_2) \right] \tag{9}
\]
where we have put:

$$\Gamma(r_1, r_2, t-t_1, t-t_2) = \langle V^*(r_1, t-t_1)V(r_2, t-t_2) \rangle \quad (10)$$

also,

$$\langle I(r_j, t_j) \rangle = \langle V^*(r_j, t_j)V(r_j, t_j) \rangle = \Gamma(r_j, r_j, t_j, t_j) \quad (j = 1, 2) \quad (11)$$

At this point, it is appropriate to note that the quantity $\Gamma(r_1, r_2, t_1, t_2)$ is very important in the theory of partial coherence, since it represents the correlation between the fields at $r_1$ and $r_2$ at times $t_1$ and $t_2$ respectively. It will be shown that this quantity, in the cross-term of equation (9), is normally responsible for a sinusoidal modulation of $\langle I(r, t) \rangle$ with $r$.

Now, since the field is assumed to be stationary, the ensemble average may be replaced by the corresponding time average, and we may shift the origin of time, since the correlation function depends on the time-difference $(t_1-t_2)$. Thus, we put:

$$\Gamma(r_1, r_2, t) = \langle V^*(r_1, t)V(r_2, t+\tau) \rangle$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} V^*(r_1, t)V(r_2, t+\tau)dt \quad (12)$$

and equation (9) becomes:

$$\langle I(r, t) \rangle = \left| K_1 \right|^2 \left( \langle I(r_1, t) \rangle + \left| K_2 \right|^2 \langle I(r_2, t) \rangle + 2 \text{Re} \left( K_1^* K_2 \Gamma(r_1, r_2, \tau) \right) \right) \quad (13)$$
The quantity $\Gamma(r_1, r_2, \tau)$ is now termed the Mutual Coherence Function, or the Second-Order Coherence Function, and it will be found most convenient to normalise it:

$$\gamma(r_1, r_2, \tau) = \frac{\Gamma(r_1, r_2, \tau)}{[\Gamma(r_1, r_1, 0)]^{1/2} [\Gamma(r_2, r_2, 0)]^{1/2}}$$

where $\Gamma(r_1, r_1, 0)$ is obviously the mutual coherence of the light vibrations at point $P_1$, and a similar argument applying to point $P_2$. $\Gamma(r_1, r_1, \tau)$ is termed the Self-Coherence Function of the light at $P_1$ after time $\tau$, and reduces to ordinary intensity when $\tau = 0$:

$$\Gamma(r_1, r_1, 0) = I^{(1)}(r, t), \quad \Gamma(r_2, r_2, 0) = I^{(2)}(r, t).$$

The normalised mutual coherence may then be expressed as:

$$\gamma(r_1, r_2, \tau) = \frac{\Gamma(r_1, r_2, \tau)}{[\langle I(r_1) \rangle]^{1/2} [\langle I(r_2) \rangle]^{1/2}}$$

It then follows that the averaged light intensity at $Q$ may be expressed as:

$$\langle I(r, t) \rangle = \langle I^{(1)}(r, t) \rangle + \langle I^{(2)}(r, t) \rangle + 2 [\langle I^{(1)}(r, t) \rangle]^{1/2} [\langle I^{(2)}(r, t) \rangle]^{1/2} \cdot \text{Re} \left[ \gamma(r_1, r_2, \frac{s_1 - s_2}{c}) \right]$$

Equation (15) indicates that a measurement of the observed intensities in an interferometric experiment will yield a value for the real parts of $\gamma(r_1, r_2, \tau)$, but of course it will not do the same for the imaginary parts.
also. In order to describe the interference process more fully, it is necessary to consider the complex correlation function, rather than just its real part.

If we say \( \nu_0 \) is the effective frequency of the light, assumed nearly monochromatic, we may put:

\[
\gamma(r_1, r_2, \tau) = |\gamma(r_1, r_2, \tau)| \exp\left[i \left\{ a(r_1, r_2, \tau) - 2\pi \nu_0 \tau \right\} \right]
\]

(16)

where \( a(r_1, r_2, \tau) = \arg \gamma(r_1, r_2, \tau) + 2\pi \nu_0 \tau \) \hfill (17)

Then equation (15) becomes:

\[
\langle I(r, t) \rangle = \langle I(1)(r, t) \rangle + \langle I(2)(r, t) \rangle + 2 \langle I(1)(r, t) \rangle^{1/2} \left[ \langle I(2)(r, t) \rangle^{1/2} \left| \gamma(r_1, r_2, \frac{s_1 - s_2}{c}) \right| \right] \times \cos \left[ a(r_1, r_2, \frac{s_1 - s_2}{c}) - \delta \right]
\]

(18)

where \( \delta = \frac{2\pi \nu_0 (s_1 - s_2)}{c} = \frac{2\pi}{\lambda_0} (s_1 - s_2) \) \hfill (19)

and \( \lambda_0 \) represents the effective wavelength of the light.

If we now consider an experimental arrangement such that \( \langle I \rangle \) varies slowly with the position of \( P \), examination of equation (18) will reveal that the cross-term contributes a rapidly oscillating sinusoidal modulation of the intensity as \( Q \) is traversed across the field. The depth of this modulation is an observable in any interference experiment, and following Michelson, we define it as the Visibility at point \( Q(r) \):
\[ U(r) \equiv \frac{\langle I \rangle_{\text{max}} - \langle I \rangle_{\text{min}}}{\langle I \rangle_{\text{max}} + \langle I \rangle_{\text{min}}} \]  

where \( \langle I \rangle_{\text{max}} \) and \( \langle I \rangle_{\text{min}} \) are the intensity maxima and minima near \( Q \).

Inserting the limits of the Cosine in equation (18), to a good approximation, we have:

\[
\langle I \rangle_{\text{max}} = \langle I^{(1)}(r,t) \rangle + \langle I^{(2)}(r,t) \rangle + 2\left[ \langle I^{(1)}(r,t) \rangle \right]^{1/2} \cdot \left[ \langle I^{(2)}(r,t) \rangle \right]^{1/2} \left| \gamma(r_1, r_2, \frac{s_1-s_2}{c}) \right| \\
\langle I \rangle_{\text{min}} = \langle I^{(1)}(r,t) \rangle + \langle I^{(2)}(r,t) \rangle - 2\left[ \langle I^{(1)}(r,t) \rangle \right]^{1/2} \cdot \left[ \langle I^{(2)}(r,t) \rangle \right]^{1/2} \left| \gamma(r_1, r_2, \frac{s_1-s_2}{c}) \right| .
\]

Substituting in equation (20), the Visibility becomes:

\[
U(r) = \frac{2 \left[ \langle I^{(1)}(r,t) \rangle \right]^{1/2} \left[ \langle I^{(2)}(r,t) \rangle \right]^{1/2}}{\langle I^{(1)}(r,t) \rangle + \langle I^{(2)}(r,t) \rangle} \left| \gamma(r_1, r_2, \frac{s_1-s_2}{c}) \right| .
\]

In particular, it may be remarked that when \( \langle I^{(1)} \rangle \) equals \( \langle I^{(2)} \rangle \),

\[
U(r) = \left| \gamma(r_1, r_2, \frac{s_1-s_2}{c}) \right|.
\]

In this case, the visibility of the interference fringes becomes a direct measure of \( \left| \gamma \right| \). The position of the interference fringes in the plane of the locus of point \( Q \) is determined by the argument of \( \gamma \). From equations (17), (18) and (19), we have
\[
\text{arg } \gamma(r_1, r_2, \frac{s_1 - s_2}{\lambda_0}) = \alpha(r_1, r_2, \frac{s_1 - s_2}{\lambda_0}) - \frac{2\pi}{\lambda_0} (s_1 - s_2) = 2m\pi \quad (m = 0, \pm 1, \pm 2, \ldots) 
\]

This is not the only information that a two-beam interference experiment will yield, however. The random process characterised by \( V(r, t) \) may be represented by an autocorrelation Function, \( \Gamma(r, r, t) \), which is related directly to the Power Spectrum of that random process. The Wiener-Khintchine theorem states that the Power Spectrum (or Spectral Density), \( G(r, \nu) \), of the random process, and the Autocorrelation Function \( \Gamma(r, r, t) \) form a Fourier Transform pair, thus:

\[
\Gamma(r, r, t) = \int_{-\infty}^{\infty} G(r, r, \nu) \exp(-2\pi i \nu t) d\nu \quad (25)
\]

\[
G(r, r, \nu) = \int_{-\infty}^{\infty} \Gamma(r, r, \tau) \exp(2\pi i \nu \tau) d\tau \quad (26)
\]

where it should be noted that the transform of \( \Gamma \) does not contain any negative-frequency components, since \( \Gamma \) is analytic.

If the variable \( V(r, t) \) is now identified with the electric field signal, then we see immediately that \( G(r, r, \nu) \) represents the spectrum of the light at point \( P \) in the above interference experiment.

Similarly, in the more general case, we may write:

\[
\Gamma(r_1, r_2, t) = \int_{-\infty}^{\infty} G(r_1, r_2, \nu) \exp(-2\pi i \nu t) d\nu \quad (27)
\]
and \( G(r_1, r_2, v) = \int_{-\infty}^{\infty} G(r_1, r_2, t) \exp(2\pi ivt) dt \) (28)

where \( G(r_1, r_2, v) \) is now known as the Cross-Spectral Density of the light vibrations at \( P_1(r_1) \) and \( P_2(r_2) \).

Section (c). Intensity Correlations in Partially-Coherent Fields

In the foregoing section, we considered the visual effect resulting from the superposition of two partially coherent beams of light from points \( P_1 \) and \( P_2 \) at a point \( Q \). Some degree of partial coherence between the light at \( P_1 \) and \( P_2 \) was ensured by arranging \( P_1 \) and \( P_2 \) to be within, and perpendicular to, a beam of partially coherent light. We will now extend the analysis by considering the effect of placing a photodetector at each of the points \( P_1 \) and \( P_2 \). The output from each photodetector is assumed to be fed into some correlating device, either with or without a delay interposed in one channel. The purpose of this section, therefore, will be to show that if there is some degree of coherence between the light at \( P_1 \) and \( P_2 \), then it will be measurable in terms of a correlation between the fluctuations registered at the outputs of the photodetectors.

From equation (7), the instantaneous intensities of the light at \( P_1 \) and \( P_2 \) respectively are:

\[ I(r_1, t) = V(r_1, t)V^*(r_1, t) \]
\[ I(r_2, t) = V(r_2, t)V^*(r_2, t) \]
and since the cross correlation function is defined as:

\[ \Gamma(r_1, r_2, \tau) = \overline{V(r_1, t+\tau)}V^*(r_2, t), \]

we see that the cross correlation function of the intensities at \( P_1 \) and \( P_2 \) is given by: (Mandel, 1963)

\[
I(r_1, t+\tau)i(r_2, t) = \overline{V(r_1, t+\tau)V^*(r_1, t+\tau)V(r_2, t)V^*(r_2, t)}
\]

\[
= \frac{\overline{V(r)^2(r_1, t+\tau)V(r)^2(r_2, t)}}{
\frac{\overline{V(r)^2(r_1, t+\tau)V(i)^2(r_2, t)}}{
\frac{\overline{V(i)^2(r_1, t+\tau)V(r)^2(r_2, t)}}{
\frac{\overline{V(i)^2(r_1, t+\tau)V(i)^2(r_2, t)}}{}}}
\}
\]

(29)

Now, each of these latter four correlation functions may be evaluated separately, given the probability distributions of each \( V \). For instance, the first term with Gaussian random variates, yields: (Lawson & Uhlenbeck, 1950),

\[
\frac{\overline{V(r)^2(r_1, t+\tau)V(r)^2(r_2, t)}}{\overline{V(r)^2(r_1, t+\tau)V(r)^2(r_2, t)}} = \frac{1}{4} I(r_1)I(r_2) + 2[V(r)(r_1, t+\tau)]^2 V(r)(r_2, t)
\]

(30)

and similarly for the other three terms.

Now, it can be shown that
\[ \bar{V}(r)(r_1, t+\tau)V(r_2, t) = \bar{V}(1)(r_1, t+\tau)V(1)(r_2, t) \]

and

\[ -\bar{V}(r)(r_1, t+\tau)V(1)(r_2, t) = \bar{V}(1)(r_1, t+\tau)V(r)(r_2, t) \]

\[ = \frac{1}{2} \Re \left[ \Gamma(r_1, r_2, \tau) \right] \]

\[ = \frac{1}{2} \Im \left[ \Gamma(r_1, r_2, \tau) \right] \]  \hspace{1cm} (31)

On substituting these quantities back into equation (30), together with the three corresponding expressions, we find that:

\[ \bar{I}(r_1, t+\tau)I(r_2, t) = \bar{I}(r_1)I(r_2) + \left| \Gamma(r_1, r_2, \tau) \right|^2 \]

\[ = \bar{I}(r_1)I(r_2) \left[ 1 + \left| \gamma(r_1, r_2, \tau) \right|^2 \right] \]  \hspace{1cm} (32)

In order to express this important result in terms of fluctuation correlations, we put:

\[ \Delta I(r_1, t) = I(r_1, t) - \bar{I}(r_1) \]

and

\[ \Delta I(r_2, t) = I(r_2, t) - \bar{I}(r_2) \]

then

\[ \Delta I(r_1, t+\tau) \Delta I(r_2, t) = \bar{I}(r_1, t+\tau)I(r_2, t) - \bar{I}(r_1)\bar{I}(r_2) \]

whence, from equation (32):

\[ = \bar{I}(r_1)\bar{I}(r_2) \left| \gamma(r_1, r_2, \tau) \right|^2 \]  \hspace{1cm} (33)

Therefore, provided that \( \left| \gamma(r_1, r_2, \tau) \right| > 0 \), there should certainly exist a correlation between the fluctuations in intensity recorded at the outputs of the photodetectors at
P₁ and P₂. Equations (32) and (33) thus demonstrate the definite possibility of the measurement of the coherence of a light beam by means of correlation interferometry.

Section (d). The van Cittert-Zernike Theorem.

At this point, it is instructive to consider how the expected degree of coherence may be calculated from the geometrical arrangement of an interference experiment.

The source is assumed to be quasi-monochromatic and of finite extent. It is desired to calculate the mutual intensity and the complex degree of coherence at two points, P₁ and P₂, on a screen at some distance from the source, and in a plane parallel to it. We may also assume, with justification in the present case, that the time delay, t, between the light beams is small, by which we mean that:

\[ |t| \ll \frac{1}{\Delta \nu} \]  \hspace{2cm} (34)

It immediately follows that the exponential term in equation (27) may be replaced by unity, in which case:

\[ |\Gamma(r₁, r₂, t)| \sim |\Gamma(r₁, r₂, 0)| \]

similarly

\[ |\gamma(r₁, r₂, t)| \sim |\gamma(r₁, r₂, 0)| \]

and

\[ \alpha(r₁, r₂, t) \sim \alpha(r₁, r₂, 0) \]  \hspace{2cm} .

Simplifying the notation, we will put:

Mutual Intensity = J₁₂ = \(\Gamma(r₁, r₂, 0) = \langle \gamma(r₁,t)\gamma^*(r₂,t) \rangle\)

\hspace{2cm} (35)
Complex degree of Coherence = \( \mu_{12} = \gamma(r_1, r_2, 0) \)

\[
\frac{\Gamma(r_1, r_2, 0)}{\sqrt{\Gamma(r_1, r_1, 0) \Gamma(r_2, r_2, 0)}} = \frac{J_{12}}{\sqrt{I_1 I_2}}
\]

(36)

Phase of Coherence = \( \beta_{12} = a(r_1, r_2, 0) \)

\[
= \text{arg} \gamma(r_1, r_2, 0) = \text{arg} \mu_{12}
\]

(37)

Hence the time difference, \( \tau \), between the two beams may now be disregarded, and we need only consider the quantity, \( J_{12} \), which depends solely on the positions of \( P_1 \) and \( P_2 \) in the plane, parallel to the source.

Let us consider the source, \( \sigma \), divided into elements \( d\sigma_1 \), \( d\sigma_2 \), etc., centred on points \( s_1, s_2 \) etc., where the dimensions of each element are small compared with the mean wavelength \( \lambda \). Let the complex disturbances at \( P_1 \) and \( P_2 \) due to any element \( d\sigma_m \) be \( V_m(t) \) and \( V_m(t) \).

The sum of all the disturbances at these two points will be:

\[
V_1(t) = \sum_m V_m(t), \quad V_2(t) = \sum_m V_m(t).
\]

(38)

Then,

\[
J(P_1, P_2) = \langle \dot{V}_1(t)V_2^*(t) \rangle = \sum_m \langle V_m(t) V_m^*(t) \rangle
\]

\[
+ \sum_m \sum_{n \neq m} \langle V_m(t) V_n^*(t) \rangle
\]

(39)

Now, the source has finite extent, and we cannot assume that \( d\sigma_m \) will be necessarily immediately adjacent to \( d\sigma_n \).
Since each element will radiate independently of the others, \( d_\sigma_m \) and \( d_\sigma_n \) will be mutually incoherent, and their superposed radiations will average to zero, hence:

\[
\langle v_{m1}(t)v_{n2}^*(t) \rangle = \langle v_{m1}(t) \rangle \langle v_{n2}^*(t) \rangle = 0 \quad \text{when} \quad m \neq n.
\]

We now define \( R_{m1} \) and \( R_{m2} \) as the distances of the source element \( d_\sigma_m \) to the points \( P_1 \) and \( P_2 \), then

\[
v_{m1}(t) = A_m(t - \frac{R_{m1}}{c}) \frac{\exp \left[ -2\pi i \nu (t - \frac{R_{m1}}{c}) \right]}{R_{m1}}
\]

and

\[
v_{m2}(t) = A_m(t - \frac{R_{m2}}{c}) \frac{\exp \left[ -2\pi i \nu (t - \frac{R_{m2}}{c}) \right]}{R_{m2}}
\]

where \( |A_m| \) is the strength of the radiation from element \( d_\sigma_m \), and \( c \) is the velocity of light. Substituting,

\[
\langle v_{m1}(t)v_{m2}^*(t) \rangle = \langle A_m(t - \frac{R_{m1}}{c}) \rangle A_m^*(t - \frac{R_{m2}}{c}) \langle A_m(t - \frac{R_{m1}}{c}) \rangle \exp \left[ \frac{2\pi i \nu (R_{m2} - R_{m1})}{c} \right] \frac{1}{R_{m1} R_{m2}}
\]

remembering that the emission of radiation is assumed to be a stationary random process. If \( |(R_{m2} - R_{m1})/c| \ll \tau_o \), equation (41) may be further simplified and substituted in equation (39) to yield:

\[
J(P_1, P_2) = \sum_m \langle A_m(t) A_m^*(t) \rangle \frac{\exp \left[ \frac{2\pi i \nu (R_{m2} - R_{m1})}{c} \right]}{R_{m1} R_{m2}}
\]
Since $\langle A_m(t)A_m^*(t) \rangle$ is the intensity from element $d\sigma_m$, of which there will be a large number, we may denote the source intensity per unit area by $I(s)$, thus:

$$J(P_1, P_2) = \int I(s) \frac{\exp \left[2\pi i (R_1 - R_2) / \lambda \right]}{R_1 R_2} \, ds$$

(43)

Hence, from equation (36), the Complex degree of Coherence will be given by:

$$\mu(P_1, P_2) = \frac{1}{\sqrt{I(P_1) I(P_2)}} \int I(s) \frac{\exp \left[2\pi i (R_1 - R_2) / \lambda \right]}{R_1 R_2} \, ds$$

(44)

Equation (44) is an expression of the van Cittert-Zernike theorem, and it can be seen from this that the Mutual Coherence Function is just the Fourier transform of the normalised intensity distribution across the source. In other words, we may regard one of the points, say $P_2$, as fixed, and then the degree of coherence is simply the normalised complex amplitude at $P_1$ in the diffraction pattern of the source, centred on $P_2$.

Thus, provided that the previously noted assumptions are justified, it is possible to deduce the intensity variation across the source by the measurement of fluctuation correlations at $P_1$ and $P_2$. Conversely, assuming a uniform intensity across the source, the expected fluctuation correlation at $P_1$ and $P_2$ may be calculated.
Section (e) The Theory of Photodetector-Pulse Correlation

We shall now proceed to a consideration of the measurement of the variables involved in carrying out the two experiments which are the main topic of this thesis. The first experiment was a repeat of that performed by Twiss & Little \(^{(38)}\) (1959), and involved the detection of correlations between the outputs of two photodetectors, one of which was scanned, with respect to the other, across the optical field due to a small single aperture illuminated by a Mercury-198 lamp. The second experiment, conceived by Sillitto & Haig \(^{(61)}\) (1966), differed from the first essentially by the use of a double-aperture source. The measuring techniques involved in both experiments were virtually identical. The major difference between the two involves the theoretical evaluation of the Correlation Factors, and these are derived in Sections (f) and (g). This Section, therefore, will be devoted to the derivation of the basic equations, common to both experiments, to which the measured variables will refer and from which the final results will later be calculated.

Let us consider two beams of partially coherent plane polarised light emanating from the same source, having instantaneous intensities \(I_1(t)\) and \(I_2(t)\) at two photodetectors, \(P_1\) and \(P_2\). Let the pulses from \(P_1\) and \(P_2\) be fed into a coincidence circuit of resolving-time \(\tau_r\). If we denote the quantum efficiency of \(P_1\) and \(P_2\) by \(\alpha_1\) and \(\alpha_2\) respectively, then the coincidence rate will be given by:
\[ R = \alpha_1 \alpha_2 \int_{-\tau_r}^{\tau_r} \langle I_1(t)I_2(t+\tau) \rangle \, dt. \]

By a similar argument to that used in deriving equation (32), we then find:

\[ R = 2R_1R_2 \tau_r \left[ 1 + \frac{1}{2} |\gamma_{12}(0)|^2 \zeta(\tau_r)/\tau_r \right] \quad (45) \]

where \( R_1 \) and \( R_2 \) are the single-channel count rates of \( P_1 \) and \( P_2 \) respectively, and:

\[ \zeta(\tau_r) = \int_{-\tau_r}^{\tau_r} |\gamma_{11}(\tau)|^2 \, d\tau \quad (46) \]

The first term on the right-hand side of equation (45) represents the pure random coincidence rate, while the second term represents the correlated coincidence rate. This latter indicates the bunching property of bosons (photons) and is only appreciable if \( \zeta(\tau_r) \sim \tau_r \). From this, it is immediately apparent that the enhanced coincidence effect will be measurable only for circuits having a very short resolving time and for light sources of long coherence time.

In terms of physically measurable quantities, the number of random coincidences expected in time \( T_0 \), \( N_r(T_0) \), is:

\[ N_r(T_0) = 2\alpha_1 \alpha_2 N_1 N_2 \tau_r T_0 \]

where \( N_1 \) and \( N_2 \) are the number of photons at detectors 1 and 2 in time \( T_0 \). For the experiments considered here, we may justifiably put:

\[ \alpha_1 = \alpha_2 = \alpha, \quad N_1 = N_2 = N \]
then, \[ N_r(T_0) = 2a^2N_0^2 \tau_r T_0 \] (47)

The number of correlated coincidences will then be given by
\[ N_c(T_0) = \frac{1}{\pi^2} a^2 N_0^2 \tau_0 T_0 \Delta(v_0) \Gamma_{12}^2 (d, v_0) f_1 f_2 \] (48)

where the Coherence Time, \( \tau_0 \sim \frac{1}{\Delta v} \), the reciprocal bandwidth of the light, and \( f_1 \) and \( f_2 \) are factors representing the loss of correlation in the optical and electronic apparatus, and will be evaluated later. The factor \( \Delta(v_0) \) is termed the Partial Coherence Factor, and represents the degree of coherence over the photodetector apertures. In this case, the apertures were identical in size, shape and position, so that \( |\gamma_{11}| = |\gamma_{22}| \), and we can then say that \( \Delta(v_0) = |\gamma_{11}|^2 \). Similarly, \( \Gamma_{12}(d, v_0) \) is termed the Normalised Correlation Factor, which is a function of \( d \), the lateral separation of the photodetector apertures. Of course, \( \Gamma_{12}(d, v_0) \equiv 1 \) when \( d = 0 \).

Thus, we may now write the total number of coincidences as:
\[ N(T_0) = 2a^2N_0^2 \tau_r T_0 + \frac{1}{\pi^2} a^2 N_0^2 \tau_0 T_0 \Delta(v_0) \Gamma_{12}^2 (d, v_0) f_1 f_2 \]
\[ = 2a^2N_0^2 \tau_r T_0 \left[ 1 + \frac{1}{4} \frac{\tau_0}{\tau_r} \Delta(v_0) \Gamma_{12}^2 (d, v_0) f_1 f_2 \right] \] (49)

If both sides of equation (49) are now divided by \( T_0 \), to obtain the total counting-rate, the direct equivalence of this equation to equation (45) is easily seen.

The quantity actually measured in the experiments was the ratio of correlated coincidence counts to random coincidence counts, \( \rho_c \), which thus becomes:
\[ \rho_c = \frac{N_c(T_0)}{N(T_0)} = \frac{1}{4} \frac{\tau_0}{\tau_r} \Delta(v_o) \Gamma_{12}^2(d,v_o) f_1 f_2 \]  
\( (50) \)

In order to determine the feasibility of the experiments, it is necessary to calculate the expected Signal to Noise Ratio, \( S/N \), and to do this we first have to determine the r.m.s. fluctuation in the number of random coincidences \( n_r(T_0) \), which is defined by

\[ n_r(T_0) = \left[ \left( \frac{N_r(T_0)}{N_r(T_0)} - \frac{N_r(T_0)}{N_r(T_0)} \right) \right]^2 \]

\[ = \alpha N_o(2 \tau_r T_0)^{1/2} \]  
\( (51) \)

assuming that the random coincidences obey a Poissonian distribution. The Signal to Noise Ratio then becomes:

\[ \frac{S}{N} = \frac{N_c(T_0)}{n_r(T_0)} = \frac{1}{2} \alpha N_o \tau_0 \Delta(v_o) \Gamma_{12}^2(d,v_o) f_1 f_2 (2 \tau_r)^{1/2} \]  
\( (52) \)

All the quantities in equations (50) and (52) are measurable, enabling the experimenter to calculate the expected degree of correlation, together with his chances of detecting it, for both experiments.

The partial coherence factor, \( \Delta \), is calculable from the dimensions and geometry of the apparatus, using the van Cittert-Zermike theorem. The correlation factor, \( \Gamma_{12} \), is then calculable using \( \Delta \) and the lateral distance separating the detector apertures. Expressions for the product \( \Delta(v_o) \Gamma_{12}^2(d,v_o) \) will be derived, for each experiment in turn, in the following two Sections of this Chapter. The methods employed for the determination of the instrumental decorrelation factors, \( f_1 \) and \( f_2 \), will be described in a
Fig. 2.1: Diagram to illustrate the coordinate system for the Single Source-Aperture experiment.
later Chapter.

Section (f). The Derivation of \( \Delta(v_0) \) and \( \Gamma^2(d, v_0) \) for the Twiss & Little-type Experiment

We will adopt the style of notation used by Hanbury-Brown & Twiss (1957), and consider a system of rectangular Cartesian coordinates where the origin lies midway between the centres of the two rectangular detector apertures. Both detector apertures lie on the \( x \)-axis, such that the \( z \)-axis passes through the centre of the source aperture. We assume that the surface of the light source, distant \( R_o \) from the \( xy \)-plane, can be divided up into elementary areas, \( dM = dm \, dn \), centred on the points \((m,n,R_o)\). Let \( m \) and \( m' \) be the coordinates of arbitrary points on the surface of the rectangular source (see Fig. 2.1). \( X = (x,y,0) \) are the coordinates of an arbitrary point on the photocathode defined by:

\[-\frac{b}{2} < y < \frac{b}{2}, \quad -\frac{1}{2}(d+a) < x < -\frac{1}{2}(d-a),\]

and \( X' = (x',y',0) \) are the coordinates of an arbitrary point on the other photocathode defined by:

\[-\frac{b}{2} < y' < \frac{b}{2}, \quad \frac{1}{2}(d-a) < x' < \frac{1}{2}(d+a).\]

The sides of the rectangular source are assumed to be parallel to the sides of the detector apertures.

Assuming the Hanbury-Brown & Twiss definition of \( \Delta(v_0) \) and \( \Gamma^2(d, v_0) \), it follows that
\[ \Delta(v_0) r^2(d,v_0) = \frac{1}{\Omega_0^2 A_1 A_2} \left\{ \int \int \int \frac{dm}{R_0^4} \right\} \frac{1}{2}(d+a) \int \int \int dm^* dm^* dy^* dy^* dx^* \quad . \]

where the areas, \( A_1 \) and \( A_2 \), of the photocathodes, and the solid angle, \( \Omega_0 \), subtended by the source at the photo-
detectors are given by:

\[ A_1 = \int dx \quad , \quad A_2 = \int dx' \]

and \[ \Omega_0 = \int \frac{dM}{R_0^2} = \int \frac{dM^*}{R_0^2} \] (54)

(Note: Throughout these successive integrations we will use a compact notation, such that the successive products of Cosine terms involving the independent expressions \((m-m^*),(n-n^*),(x-x^*)\) and \((y-y^*)\) are integrated one at a time, implicitly maintaining the remainder constant.)

Now, if we integrate first over the x-axis, we may put:

\[ \Delta(v_0) r^2(d,v_0) = \frac{1}{\Omega_0^2 A_1 A_2 R_0^4} \left\{ \int \int \int dm \int dm^* \int dy^* \int dy^* \int dx^* \right\} \frac{1}{2}(d+a) \int \int \int \cos \alpha(x-x^*) dx \quad . \]

where \[ \alpha = \frac{2\pi v_o (M-M^*)}{\Omega_0} \]

However, \[ \cos \alpha(x-x^*) = \cos \alpha x \cos \alpha x^* + \sin \alpha x \sin \alpha x^* \].

Substituting in equation (55):
\[ \Delta(v_0) - [2(d, v_0)] = F_1 \left[ \cos a' \int_{-\frac{a}{2}}^{\frac{a}{2}} \cos ax \, dx + \sin a' \int_{-\frac{a}{2}}^{\frac{a}{2}} \sin ax \, dx \right] \]

where

\[ F_1 = \frac{1}{\Omega^2 A_1 A_2 R_0^2} \int \int dM \cdot dM' \int \int dy \cdot dy' \int \int dx \cdot dx' \]

Integrating equation (56), we have:

\[ F_1 \left[ \frac{1}{a} \cos a' \left\{ \sin \frac{a}{2}(d+a) - \sin \frac{a}{2}(d-a) \right\} \right. \]

\[ - \frac{1}{\alpha} \sin ax' \left\{ \cos \frac{a}{2}(d+a) - \cos \frac{a}{2}(d-a) \right\} \]

\[ = F_1 \left[ \frac{1}{\alpha} \cos a' \left( 2 \cos \frac{a}{2} \sin \frac{a}{2} \right) - \frac{1}{\alpha} \sin ax' \left( -2 \sin \frac{a}{2} \sin \frac{a}{2} \right) \right] \]

\[ = F_1 \left( \frac{2}{\alpha} \sin \frac{a}{2} \right) \left[ \cos ax' \cos \frac{a}{2} + \sin ax' \sin \frac{a}{2} \right] \]

Having integrated over \( x \), we now integrate over \( x' \), putting:

\[ F_2 = \frac{1}{\Omega^2 A_1 A_2 R_0^2} \int \int dM \cdot dM' \int \int dy \cdot dy' \int \int dx \cdot dx' \left( \frac{2}{\alpha} \sin \frac{a}{2} \right) \]

Thus

\[ \Delta(v_0) - [2(d, v_0)] = F_2 \left[ \cos \frac{a}{2} \int_{-\frac{a}{2}}^{\frac{a}{2}} \cos ax' \, dx' \right. \]

\[ - \frac{1}{\alpha} \int_{-\frac{a}{2}}^{\frac{a}{2}} \sin ax' \, dx' \left\{ \cos \frac{a}{2}(d+a) - \cos \frac{a}{2}(d-a) \right\} \]

\[ = F_2 \left[ \frac{1}{\alpha} \cos \frac{a}{2} \left( \sin \frac{a}{2}(d-a) - \sin \frac{a}{2}(d+a) \right) \frac{1}{\alpha} \sin \frac{a}{2} \right. \]

\[ \cdot \left\{ \cos \frac{a}{2}(d-a) - \cos \frac{a}{2}(d+a) \right\} \right] \]
\[ F_2 \left[ \frac{1}{a} \cos \frac{a}{2} \left( 2\cos \frac{a}{2} \sin \frac{a}{2} \right) + \frac{1}{a} \sin \frac{a}{2} \left( 2\sin \frac{a}{2} \sin \frac{a}{2} \right) \right] = F_2 \left( \frac{2}{a} \sin \frac{a}{2} \right) \left[ \cos \frac{a}{2} \cos \frac{a}{2} + \sin \frac{a}{2} \sin \frac{a}{2} \right] = F_2 \left( \frac{2}{a} \sin \frac{a}{2} \right) \left[ \cos \frac{a}{2} \right]. \]

Thus, \[ \Delta(v_o) \Gamma^2(d,v_o) = \frac{1}{\Omega_0^2 A_1 A_2 R_0^4} \iint \int dM \cdot dM' \int \int dy \cdot dy' \cdot \left[ \frac{2 \sin \frac{a}{2}}{a} \right]^2 \left[ \cos \frac{a}{2} \right] \] (58)

Having completed the integration over the \(x, x'-axis\), we now perform the similar operation over the \(y, y'-axis\) between the limits:

\[-\frac{b}{2} < y < \frac{b}{2}, \quad \quad -\frac{b}{2} < y' < \frac{b}{2}\]

when we obtain

\[ \Delta(v_o) \Gamma^2(d,v_o) = \frac{1}{\Omega_0^2 A_1 A_2 R_0^4} \iint \int dM \cdot dM' \cdot \left[ \frac{2 \sin \frac{a}{2}}{a} \right]^2 \left[ \frac{2 \sin \frac{ab}{2}}{a} \right]^2 \left[ \cos \frac{a}{2} \right] \] (59)

It will be noted that the Cosine term is a function of the detector separation, \(d\), and is therefore absent from the integrations along the \(y\)-axis.

Remembering that \( A_1 = A_2 = ab \), and

\[ a = \frac{2\pi v_o (M-M')}{cR_0} \], we have
\[ \Delta(n_0) \Gamma^2(d, n_0) = \frac{1}{\Omega R_0^2} \iint dm \cdot dm' \iint dn \cdot dn' \left[ \frac{\sin \frac{\pi v_0 (n-n')}{c R_0}}{\sin \frac{\pi v_0 b(n-n')}{c R_0}} \right]^2 \times \left[ \frac{\sin \frac{\pi v_0 a(m-m')}{c R_0}}{\sin \frac{\pi v_0 b(n-n')}{c R_0}} \right]^2 \cos \frac{2\pi v_0 d(m-m')}{c R_0} \right] \] (60)

Rearranging, and putting \( \Omega = \Theta_1 \Theta_2 \), we have:

\[ \frac{1}{\Omega \Omega_2^2} \iint dm \cdot dm' \iint dn \cdot dn' \left[ \frac{\sin \frac{\pi v_0 b(n-n')}{c R_0}}{\sin \frac{\pi v_0 b(n-n')}{c R_0}} \right]^2 \times \left[ \frac{\sin \frac{\pi v_0 a(m-m')}{c R_0}}{\sin \frac{\pi v_0 b(n-n')}{c R_0}} \right]^2 \cos \frac{2\pi v_0 d(m-m')}{c R_0} \right] \] (61)

In order to facilitate the integrations over the source aperture, we now introduce new variables in terms of the angular diameters of the source:

\[ \phi = \frac{\pi v_0}{c R_0} (m-m') , \quad \phi' = \frac{\pi v_0}{c R_0} \left( \frac{m+m'}{2} \right) \] (62)

\[ \psi = \frac{\pi v_0}{c R_0} (n-n') , \quad \psi' = \frac{\pi v_0}{c R_0} \left( \frac{n+n'}{2} \right) \]

Separating the variables:

\[ m = \frac{c R_0}{2\pi v_0} (\phi + 2\phi') \quad \text{and} \quad m' = -\frac{c R_0}{2\pi v_0} (\phi - 2\phi') \] (63)
Similarly,
\[ n = \frac{c^R_0}{2\pi bv_0} (\psi + 2\psi') \quad \text{and} \quad n' = \frac{-c^R_0}{2\pi bv_0} (\psi - 2\psi') \] (64)

We now invoke the Jacobian method of changing variables, for which the conversion-determinants become:
\[
\frac{\partial(n,n')}{\partial(\psi,\psi')} = \begin{vmatrix} \frac{\partial n}{\partial \psi} & \frac{\partial n'}{\partial \psi} \\ \frac{\partial n}{\partial \psi'} & \frac{\partial n'}{\partial \psi'} \end{vmatrix} = \begin{vmatrix} \frac{c^R_0}{2\pi bv_0} & -\frac{c^R_0}{2\pi bv_0} \\ \frac{c^R_0}{2\pi bv_0} & \frac{c^R_0}{2\pi bv_0} \end{vmatrix}
\]
and
\[
\frac{\partial(m,m')}{\partial(\sigma',\sigma')} = \begin{vmatrix} \frac{\partial m}{\partial \sigma} & \frac{\partial m'}{\partial \sigma} \\ \frac{\partial m}{\partial \sigma'} & \frac{\partial m'}{\partial \sigma'} \end{vmatrix} = \begin{vmatrix} \frac{c^R_0}{2\pi av_0} & -\frac{c^R_0}{2\pi av_0} \\ \frac{c^R_0}{2\pi av_0} & \frac{c^R_0}{2\pi av_0} \end{vmatrix}
\]

Substituting the value of the determinants in equation (61) and changing the variables, we have:
\[
\Delta(v_o) \cdot (2(d_v_o)) = \frac{1}{A_1A_2(\theta_1\theta_2)^2} \int b^2 \left( \begin{array}{c} \text{Sin} \psi \end{array} \right)^2 \left( \begin{array}{c} \frac{c^R_0}{2\pi bv_0} \end{array} \right)^2 d\psi d\psi'
\]
\[
\cdot \int R_o^2 \left( \begin{array}{c} \text{Sin} \sigma' \end{array} \right)^2 \left( \begin{array}{c} \frac{c^R_0}{2\pi av_0} \end{array} \right)^2 \text{Cos} \frac{26d}{a} d\sigma d\sigma'
\]
\[
= \frac{1}{A_1A_2} \left( \frac{c^2}{\pi^2 v_0^2 \theta_1 \theta_2} \right)^2 \int \text{Sin}^2 \psi \int \text{Sin}^2 \sigma' \text{Cos} \frac{26d}{a} d\sigma d\sigma' \] (65)

We now integrate over \( \sigma' \) and \( \psi' \), subject to the inequalities:
\[
\sigma' < \left| \frac{xav_0 \theta_1}{c} - \sigma \right|
\]
\[
\psi' < \left| \frac{xbv_0 \theta_2}{c} - \psi \right|
\]
giving:
\[ \Delta(v_0) \Gamma^2(d, v_0) = \frac{1}{A_1A_2} \left[ \frac{c^2}{2\pi^2 v_0^2 \Theta_1 \Theta_2} \right]^2 \int_0^\infty \frac{\sin^2 \psi}{\psi^2} (\psi - \psi') d\psi \cdot \int_0^\phi \frac{\sin^2 \beta}{\beta^2} \cos \frac{2\beta d}{a} (\beta - \beta') d\beta \]  

(66)

where the angular dimensions of the source aperture, \( \Theta_1 \) and \( \Theta_2 \), are defined by:

\[ \Theta_1 = \left| \frac{m_1 - m_2}{R_0} \right|, \quad \Theta_2 = \left| \frac{n_1 - n_2}{R_0} \right|, \]

and

\[ \phi = \frac{\pi a \nu_0 \Theta_1}{c}, \quad \psi = \frac{\pi b \nu_0 \Theta_2}{c}. \]

This calculation has been carried out for a rectangular source aperture, resulting in equation (66). In fact, a circular source aperture was used by Twiss and Little, and also by the author. The derivation of \( \Delta(v_0) \Gamma^2(d, v_0) \) is slightly simpler in this case, due to the geometrical symmetry, and the final expression becomes:

\[ \Delta(v_0) \Gamma^2(d, v_0) = \frac{1}{A_1A_2} \int_{-\frac{d}{2}}^{\frac{d}{2}} \int_{-\frac{d}{2}}^{\frac{d}{2}} \int_{-\frac{d}{2}}^{\frac{d}{2}} \int_{-\frac{d}{2}}^{\frac{d}{2}} \frac{dx'dy'dx'dy'}{c^2} \left[ \begin{array}{l} \frac{x_0 \Theta_0 v_0}{c} \{ (x-x')^2 + (y-y')^2 \}^{\frac{1}{2}} \\ \frac{x_0 \Theta_0 v_0}{c} \{ (x-x')^2 + (y-y')^2 \}^{\frac{1}{2}} \end{array} \right] \]

(67)

where \( J_1 \) is a Bessel Function of the first order and \( \Theta_0 \) is the angular diameter of the source. A graph of \( \Delta(v_0) \Gamma^2(d, v_0) \) versus \( d \) is shown in Fig. 4.4 of Chapter 4, for which the computer program is given in Appendix (a).
Fig. 2.2: Diagram to illustrate the co-ordinate system used to define the parameters in the Double-Source experiment.
Section (g). The Derivation of \( \Delta(v_o) \Gamma^2(d,v_o) \) for the Sillitto and Haig Experiment

We will now derive the corresponding value of \( \Delta(v_o) \Gamma^2(d,v_o) \) for the Sillitto and Haig experiment, in which the source comprises two rectangular apertures of identical shape, size and orientation, disposed in the same horizontal plane as the rectangular detector apertures. The notation used is similar to that adopted in the previous Section, and is illustrated in Fig. 2.2. It can be seen that the integrations differ from the previous case, in that they are taken in four ways:

(i) From left-hand source to left-hand detector.
(ii) From left-hand source to right-hand detector.
(iii) From right-hand source to left-hand detector.
(iv) From right-hand source to right-hand detector.

Each of the above terms will be treated separately, except in the case of the first few integrations, which are common to all terms.

From the Hanbury-Brown & Twiss definition, equation (53), we have that:

\[
\Delta(v_o) \Gamma^2(d,v_o) = \frac{1}{R^4_o \Omega^2 A_1 A_2} \iint dM.dM'.dX.dX'. \cos \left[ \frac{2\nu_o (X-X')(M-M')}{cR_o} \right] .
\]

Taking any one of the four terms above, we will integrate over the detector apertures, beginning with the \( x \), \( x' \)-direction:
\[ \Delta(v_0) \Gamma^2(d, v_0) = \frac{1}{\Omega_0^2 A_1^2 A_2 R_0} \iiint dM \cdot dM' \int dy \cdot dy' \int dx' \]

\[ \cdot \frac{1}{2} \cos \alpha(x-x') dx \]

where \( a = \frac{2 \pi v_0 (M-M')}{c R_0} \).

(Note: See note, concerning shorthand notation, in Section (f).)

Simplifying, we have

\[ \Delta(v_0) \Gamma^2(d, v_0) = F_1 \left[ \cos \alpha x' \int \cos \alpha x \cdot dx + \sin \alpha x' \int \sin \alpha x \cdot dx \right] \]

\[ \cdot \frac{1}{2} \cos \alpha x + \sin \alpha x' \int \sin \alpha x' \cdot dx' \]

where

\[ F_1 = \frac{1}{\Omega_0^2 A_1^2 A_2^2 R_0} \iiint dM \cdot dM' \int dy \cdot dy' \int dx' \]

and then:

\[ \Delta(v_0) \Gamma^2(d, v_0) = F_1 \left[ \frac{1}{a} \cos \alpha x' (2 \cos \frac{ad}{2} \sin \frac{aa}{2}) + \frac{1}{a} \sin \alpha x' \right] \]

\[ \cdot \left( 2 \sin \frac{ad}{2} \sin \frac{aa}{2} \right) \]

Rearranging, and integrating over \( x' \):

\[ = F_2 \left[ \cos \frac{ad}{2} \int \cos \alpha x' dx' + \sin \frac{ad}{2} \int \sin \alpha x' \cdot dx' \right] \]

\[ \cdot \frac{1}{2} \cos \alpha x + \sin \alpha x' \int \sin \alpha x' \cdot dx' \]

where
\[ F_2 = \frac{1}{\Omega_0^2 A_1 A_2 R_0^2} \int \int dM dM' \int \int dy dy' \left( \frac{2}{a} \sin \frac{ab}{2} \right). \]

Thus,
\[ \Delta (v_o) \Gamma^2 (d, v_o) = F_2 \left( \frac{2}{a} \sin \frac{ab}{2} \right) \left[ \cos \frac{ad}{2} \cos \frac{-ad}{2} + \sin \frac{ad}{2} \sin \frac{-ad}{2} \right] \]

i.e.
\[ \Delta (v_o) \Gamma^2 (d, v_o) = \frac{1}{\Omega_0^2 A_1 A_2 R_0^2} \int \int \int \int dM dM' \int \int dy dy' \left[ \left( \frac{2}{a} \sin \frac{ab}{2} \right)^2 \left[ \cos \frac{ad}{2} \right] \right] \]

Now, so far we have integrated only over the \( x \) and \( x' \)-axis of the detector apertures. The integration over the \( y \) and \( y' \)-axis follows a precisely similar pattern, apart from the obvious omission of the Cosine modulating term, and so, remembering that \( A_1 = A_2 = ab \), we have:
\[ \Delta (v_o) \Gamma^2 (d, v_o) = \frac{1}{(e_1 e_2)^2 R_0^2} \int \int dM dM' \left[ \frac{2\sin \frac{ab}{2}}{ab} \right] \]
\[ \left[ \frac{2\sin \frac{ab}{2}}{ab} \right]^2 \cos ad. \]

Since:
\[ a = \frac{2\nu_o (M-M')}{c R_0}, \]
each term is expressed in full as:
\[ \Delta (v_o) \Gamma^2 (d, v_o) = \frac{1}{(e_1 e_2)^2 R_0^2} \int \int \int \int \int dm dm' dm' dm' dm' \left[ \frac{\sin \frac{\nu_o a (m-m')}{c R_0}}{\nu_o a (m-m')} \right]^2 \]
\[ \left[ \frac{\sin \frac{\nu_o a (m-n')}{c R_0}}{\nu_o b (m-n')} \right]^2 \left[ \cos \frac{2\nu_o d (m-m')}{c R_0} \right] \] (71)
At this point, as in the previous experiment, it is convenient to change the variables. We therefore put:

\[ \phi = \frac{\pi v_0 a(m-m')}{cR_o}, \quad \phi' = \frac{\pi v_0 a(m+m')}{2cR_o} \]

\[ \psi = \frac{\pi v_0 b(n-n')}{cR_o}, \quad \psi' = \frac{\pi v_0 b(n+n')}{2cR_o} \]

whereupon equation (71) becomes:

\[
\frac{1}{(\theta_1 \theta_2)^2 A_1 A_2 R_o^4} \int \int \int b^2 (\sin \psi)^2 \left(\frac{cR}{\pi v_o b}\right)^2 \, d\psi d\psi' \int \int a^2 (\sin \theta)^2 \left(\frac{cR}{\pi v_o b}\right)^2 \, d\theta d\theta' \cos \frac{2\pi d}{a} \, d\phi d\phi' \]

\[
= \frac{1}{A_1 A_2} \left[ \frac{c^2}{\theta_1 \theta_2 \pi v_o^2} \right]^2 \int \int (\sin \psi)^2 \, d\psi d\psi' \int \int (\sin \theta)^2 \cos \frac{2\pi d}{a} \, d\phi d\phi' \]

(72)

Since the variables have been changed, it is now necessary to calculate the new limits of integration in the \( \psi, \psi' \)-plane.

Now, \(-\frac{R e_2}{2} < n < \frac{R e_2}{2}\), where \(n = \frac{cR}{2\pi v_o b} (\psi + 2\psi')\),

also \(-\frac{R e_2}{2} < n' < \frac{R e_2}{2}\), where \(n' = \frac{cR}{2\pi v_o b} (\psi - 2\psi')\).

At the upper limit of \(n\): \(\psi + 2\psi' = \frac{\pi v_o b e_2}{c} = \psi \) (say),

thus \(\psi' = \frac{1}{2}(\psi - \psi)\)  

(73)

At the lower limit of \(n\): \(\psi + 2\psi' = -\psi\)

thus \(\psi' = -\frac{1}{2}(\psi + \psi)\)  

(74)
Fig. 2.3: Diagram to illustrate the area of integration in the $\psi, \psi'$ plane.
At the upper limit of \( n' \):
\[
\psi - 2\psi' = -\psi
\]
thus
\[
\psi' = \frac{1}{2}(\psi + \psi) \quad (75)
\]

At the lower limit of \( n' \):
\[
\psi - 2\psi' = \psi
\]
thus
\[
\psi' = -\frac{1}{2}(\psi - \psi) \quad (76)
\]

Fig. 2.3 illustrates the area of integration defined by equations (73) to (76) in the \( \psi, \psi' \) plane. Summarising, then, we may write each term as:

\[
\Delta(v_0) \Gamma^2(d, v_0) = \frac{1}{A_1 A_2} \left( \frac{c^2}{\epsilon_1 \epsilon_2 \pi^2 v_0} \right)^2 \int_0^\psi (\sin \psi)^2 d\psi
\]

\[
\cdot \frac{1}{2}(\psi - \psi) \int (\sin \psi)^2 \cos \frac{2\phi d}{a} d\phi d\psi',
\]

which, on integrating with respect to \( \psi' \), becomes:

\[
= \frac{1}{A_1 A_2} \left( \frac{c^2}{\epsilon_1 \epsilon_2 \pi^2 v_0} \right)^2 \int_0^\psi (\sin \psi)^2 (\psi - \psi) d\psi \int (\sin \psi)^2 \cos \frac{2\phi d}{a} d\phi d\psi'.
\]

(77)

It is clear, up to this point, that all four terms of

\( \Delta(v_0) \Gamma^2(d, v_0) \)

are identical, whether symmetric or antisymmetric. The differences between the terms are evidently about to arise in the integrations over the \( \phi, \phi' \)-plane. Before carrying out the integrations, the limits have to be recalculated, since the variables have been changed from \( m \) and \( m' \) to \( \phi \) and \( \phi' \):
Thus, in the first symmetric case, we integrate from

\[ m = \frac{R_0 \theta_3}{2} + R_0 \theta_1 \quad \text{to} \quad m = \frac{R_0 \theta_3}{2} \]  

(a)

with

\[ m' = \frac{R_0 \theta_3}{2} + R_0 \theta_1 \quad \text{to} \quad m' = \frac{R_0 \theta_3}{2} \]  

(b)

In the second symmetric case, we integrate from

\[ m = -\left(\frac{R_0 \theta_3}{2} + R_0 \theta_1\right) \quad \text{to} \quad m = -\frac{R_0 \theta_3}{2} \]  

(c)

with

\[ m' = -\left(\frac{R_0 \theta_3}{2} + R_0 \theta_1\right) \quad \text{to} \quad m' = -\frac{R_0 \theta_3}{2} \]  

(d)

In the first antisymmetric case, we integrate from

\[ m = \frac{R_0 \theta_3}{2} + R_0 \theta_1 \quad \text{to} \quad m = \frac{R_0 \theta_3}{2} \]  

(e)

with

\[ m' = -\left(\frac{R_0 \theta_3}{2} + R_0 \theta_1\right) \quad \text{to} \quad m' = -\frac{R_0 \theta_3}{2} \]  

(f)

In the second antisymmetric case, we integrate from

\[ m = -\left(\frac{R_0 \theta_3}{2} + R_0 \theta_1\right) \quad \text{to} \quad m = -\frac{R_0 \theta_3}{2} \]  

(g)

with

\[ m' = \frac{R_0 \theta_3}{2} + R_0 \theta_1 \quad \text{to} \quad m' = \frac{R_0 \theta_3}{2} \]  

(h)

remembering that

\[ m = \frac{cR_0}{2\pi \nu_0 a}(\phi + 2\phi'), \quad \text{and} \quad m' = \frac{cR_0}{2\pi \nu_0 a}(\phi - 2\phi') \, . \]

For the limits (a) and (e):

\[ \frac{R_0 \theta_3}{2} + R_0 \theta_1 = \frac{cR_0}{2\pi \nu_0 a}(\phi + 2\phi') \, , \]

then

\[ \phi + 2\phi' = \phi_3 + 2\phi \, , \quad \text{where} \quad \phi_3 = \frac{\pi \nu_0 a \theta_3}{c} \quad \text{and} \quad \phi = \frac{\pi \nu_0 a \theta_1}{c} \, . \]

thus

\[ \phi' = \frac{1}{2}(\phi_3 + 2\phi - \phi) \]  

(78)
Fig. 2.4: Illustrating the areas of integration in the \( \varphi, \varphi' \) plane.
also, \[ \frac{R_o \Theta_3}{2} = \frac{cR_o}{2 \pi v_0 a} (\phi + 2\phi') \]

thus \[ \phi' = \frac{1}{2} (\phi_3 - \phi) \]  

(79)

For limits (b) and (h): \[ \frac{R_o \Theta_3}{2} + R_o \Theta_1 = \frac{cR_o}{2 \pi v_0 a} (\phi - 2\phi') \]

thus \[ \phi' = \frac{1}{2} (\phi_3 + 2\phi + \phi) \]  

(80)

also, \[ \frac{R_o \Theta_3}{2} = \frac{-cR_o}{2 \pi v_0 a} (\phi - 2\phi') \]

thus, \[ \phi' = \frac{1}{2} (\phi_3 + \phi) \]  

(81)

For limits (c) and (g): \[ -\left( \frac{R_o \Theta_3}{2} + R_o \Theta_1 \right) = \frac{cR_o}{2 \pi v_0 a} (\phi + 2\phi') \]

thus, \[ \phi' = -\frac{1}{2} (\phi_3 + 2\phi + \phi) \]  

(82)

also \[ -\frac{R_o \Theta_3}{2} = \frac{cR_o}{2 \pi v_0 a} (\phi + 2\phi') \]

thus \[ \phi' = -\frac{1}{2} (\phi_3 + \phi) \]  

(83)

For limits (d) and (f): \[ -\left( \frac{R_o \Theta_3}{2} + R_o \Theta_1 \right) = \frac{-cR_o}{2 \pi v_0 a} (\phi - 2\phi') \]

thus \[ \phi' = -\frac{1}{2} (\phi_3 + 2\phi - \phi) \]  

(84)

also \[ -\frac{R_o \Theta_3}{2} = \frac{-cR_o}{2 \pi v_0 a} (\phi - 2\phi') \]

thus \[ \phi' = -\frac{1}{2} (\phi_3 - \phi) \]  

(85)

Now, from the integration limits defined above, it will be seen that the integration is now the sum of eight terms, whose configurations in the \( \phi, \phi' \)-plane may be seen in Fig. 2.4. The lines bounding each area of integration are those defined in equations (78) to (85), with each of the
four major areas divided into two parts along a line 
\( \phi' = \) constant. Let the areas be labelled from \( p \) to \( w \), as 
shown.

From inspection of the diagram, and the good behaviour 
of the integrand, it will be evident that we may double the 
value of the integral over area \( p \), to include area \( q \), and 
perform a similar operation for areas \( r \) and \( s \).

Let \( B = \frac{2}{A_1 A_2} \left( \frac{c^2}{\Theta_2^2 \nu_2^2} \right)^2 \int_0^\psi (\sin\psi)^2 (\psi - \psi)d\psi \),

then, denoting the limits of integration of \( \phi' \) by the 
text-numbers of the respective equations, we have:

\[
\Delta \left( \nu_0 \right) = 2B \int_0^\phi \frac{(\sin\phi)^2 \cos \frac{2\phi a}{a}}{a} d\phi 
+ 2B \int_0^\phi \frac{(\sin\phi)^2 \cos \frac{2\phi a}{a}}{a} d\phi 
+ B \int_{(\phi_3 + \phi)}^{(\phi_3 + 2\phi)} \frac{(\sin\phi)^2 \cos \frac{2\phi a}{a}}{a} d\phi 
+ B \int_{(\phi_3 + 2\phi)}^{(\phi_3 + 2\phi)} \frac{(\sin\phi)^2 \cos \frac{2\phi a}{a}}{a} d\phi 
- B \int_{(\phi_3 + \phi)}^{(\phi_3 + 2\phi)} \frac{(\sin\phi)^2 \cos \frac{2\phi a}{a}}{a} d\phi 
+ B \int_{(\phi_3 + 2\phi)}^{(\phi_3 + 2\phi)} \frac{(\sin\phi)^2 \cos \frac{2\phi a}{a}}{a} d\phi 
+ B \int_0^\phi \frac{(\sin\phi)^2 \cos \frac{2\phi a}{a}}{a} d\phi 
+ B \int_0^\phi \frac{(\sin\phi)^2 \cos \frac{2\phi a}{a}}{a} d\phi 
+ B \int_0^\phi \frac{(\sin\phi)^2 \cos \frac{2\phi a}{a}}{a} d\phi .
\]

(86)

Integrating each term of equation (86) separately,

and rearranging, we have:
Areas \((p + q)\):  
\[2B \int_0^{\phi} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi - \phi)d\phi\]

Areas \((r + s)\):  
\[2B \int_0^{\phi} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi - \phi)d\phi\]

Area \(t\):  
\[-B \int_{\phi_3}^{(\phi_3 + \phi)} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi_3 - \phi)d\phi\]

\[= B \int_{\phi_3}^{(\phi_3 + \phi)} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi - \phi_3)d\phi\]

Area \(u\):  
\[B \int_{(\phi_3 + \phi)}^{(\phi_3 + 2\phi)} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi_3 + 2\phi - \phi)d\phi\]

Area \(v\):  
\[-B \int_{(\phi_3 + \phi)}^{(\phi_3 + 2\phi)} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi_3 + 2\phi + \phi)d\phi\]

\[= B \int_{(\phi_3 + \phi)}^{(\phi_3 + 2\phi)} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi_3 + 2\phi - \phi)d\phi\]

Area \(w\):  
\[-B \int_{(\phi_3 + \phi)}^{\phi_3} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi_3 + \phi)d\phi\]

\[= B \int_{(\phi_3 + \phi)}^{\phi_3} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi - \phi_3)d\phi\]

Collecting terms, we see that:

\[\Delta(v_0) \Gamma^2(a, v_0) = 4B \int_0^{\phi} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi - \phi)d\phi\]

\[+ 2B \int_{\phi_3}^{(\phi_3 + \phi)} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi - \phi_3)d\phi\]

\[+ 2B \int_{(\phi_3 + \phi)}^{(\phi_3 + 2\phi)} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi_3 + 2\phi - \phi)d\phi\]

\[+ 2B \int_{(\phi_3 + 2\phi)}^{\phi_3} \left(\frac{\sin \theta}{\rho}\right)^2 \cos \frac{2\alpha}{a} (\phi_3 + 2\phi - \phi)d\phi\]  

\[(87)\]
Now, this expression is still not yet quite true, since each of the four terms (two symmetric, two antisymmetric) contained implicitly in equation (87) is self-normalised, but \( \Delta(v_o) \mid r^2(d, v_o) \) is not. Therefore, to complete the normalisation of the entire equation, each of the four terms must be divided by four. On re-substituting for B, and normalising the entire expression, the final result becomes:

\[
\Delta(v_o) \mid r^2(d, v_o) = \frac{2}{A_1A_2} \left( \frac{c^2}{\phi_1\phi_2\alpha^2v_o} \right)^2 \int_0^\phi (\sin^2\psi) (\psi - \psi) d\psi
\]

\[
+ \frac{1}{A_1A_2} \left( \frac{c^2}{\phi_1\phi_2\alpha^2v_o} \right)^2 \int_0^\phi (\sin^2\psi) (\psi - \psi) d\psi
\]

\[
+ \frac{1}{A_1A_2} \left( \frac{c^2}{\phi_1\phi_2\alpha^2v_o} \right)^2 \int_{\phi_3}^{\phi} (\sin^2\psi) (\psi - \psi) d\psi
\]

\[
+ \frac{1}{A_1A_2} \left( \frac{c^2}{\phi_1\phi_2\alpha^2v_o} \right)^2 \int_{\phi_3}^{\phi} (\sin^2\psi) (\psi - \psi) d\psi
\]

\[
\int_{\phi_3}^{\phi} (\sin^2\psi) \cos \frac{2\pi d}{a} (\phi - \phi_3) d\phi
\]

\[
\int_{\phi_3}^{\phi} (\sin^2\psi) \cos \frac{2\pi d}{a} (\phi_3 + 2\phi - \phi) d\phi
\]

\[\text{(88)}\]

This expression was evaluated by means of the Edinburgh Regional Computing Centre's KDF-9 computer for a number of different values of the detector separation, d. The resulting graph of \( \Delta(v_o) \mid r^2(d, v_o) \) versus d
may be seen in Fig. 4.6 of Chapter 4, and the computer program is the subject of Appendix (b).

In exactly the same way as for the Twiss & Little experiment, the values of \( \Delta(\nu_0) \) calculated for different detector spacings, are substituted into equations (50) and (52) to find the predicted values of the Correlation Ratio, \( \rho \), and the Signal to Noise Ratio, \( S/N \). Finally, if it is desired to ascertain the value of \( \Delta(\nu_0) \) only, it will be remembered that \( \Gamma^2(d,\nu_0) = 1 \) when \( d = 0 \), and therefore \( \Delta(\nu_0) \) at zero detector separation.
Fig. 3.1: Air-cooled Electrodeless Mercury-198 Isotope Lamp.
Fig. 32: The rear of the Magnetron Unit, showing the Magnetron and heat-sink.
CHAPTER 3

DESIGN AND DEVELOPMENT OF THE APPARATUS

Section (a). The Mercury Lamp, its Excitation, Stabilization and Cooling.

When the decision was taken to employ an electrode-less Mercury-198 discharge tube as the source of light for the experiments, a suitable example was obtained from G.E.C./Osram, as illustrated in Fig. 3.1. Unfortunately, at that time there appeared to be no reasonably-priced commercial oscillator unit on the market which was capable of exciting such a lamp at high power. Originally, therefore, an existing high-powered triode r.f. oscillator was considered, but it was felt that the discharge excitation would be more efficient and less liable to "clean-up" if higher frequencies were used. The Klystron is capable of the higher frequencies but lacks power, and so the oscillator chosen was a Mullard JP2-0.2 Magnetron, a photograph of which is shown in Fig. 3.2, of which the operating frequency is 2.45 GHz.

Work was begun on designing the power supply and regulation unit for the Magnetron, on the basis of a full-wave rectified and partially smoothed H.T. supply. It was soon discovered that no microwave output plug was made commercially for this type of Magnetron, and Mullard therefore supplied a set of working drawings for a plug which had been designed by the Valvo GmbH of Hamburg, and which worked into an RG-14/U cable. As this cable was unobtainable in this country at that time, the drawings were
Fig. 3.3: Circuit diagram of the Magnetron Power Unit. (For connections a and e, see Figs 3.8 and 3.9)
modified to allow the more readily obtainable and electrically similar RG-8A/U cable to be used. A plug, conforming with the modified drawings, was fabricated by the Departmental Workshop.

The circuit of the power unit was essentially very simple indeed (see Fig. 3.3). The H.T. circuit consisted of a Variac variable mains transformer controlling the voltage across a 1.6 kV transformer, in series with a full-wave diode rectifier bridge (4 Texas IN-2900), the output from which was smoothed by an 8 m.f.d. capacitor. The low-tension circuit comprised a mains-powered double-tapped isolating transformer, switched to provide either 4.8 volts or 5.3 volts. This latter provision was necessitated by the characteristics of the Magnetron, which requires a very hot filament at switch-on, and a cooler running filament as soon as the H.T. is applied. The Magnetron, and its associated power supplies, was mounted on a large chassis, capable of fitting into a standard 19" rack. The recommended method of cooling the Magnetron was by free-air convection, and this was accomplished by bolting it to a large bracket, mounted on the rear of the chassis, on the other side of which was a finned aluminium heat-sink and radiator.

The filament voltage switching was performed by means of a three-way double pole wafer switch, having break-before-make contacts to eliminate arcing and reduce voltage transients. The three positions of the switch were arranged to be "Off", "5.3 volts" and "4.8 volts", with
Fig. 3.4: Scale drawing of the modified discharge cavity.
the H.T. power supply switched on only in the "4.8 volts" position. The mains supplies to the H.T. and L.T. transformers were both fused as an added protection against surges.

The waveguide cavity used to excite the source was based on the tapered rectangular type of Broida and Chapman (62) (1950), but modified by the addition of a variable-length quarter-wave tuning-stub, as shown in Fig. 3.4. The waveguide was built in the departmental workshops from brass, and finished with 0.001" of silver. The entry of the RG-8A/U cable into the waveguide was facilitated by the fitting of a Transradio type "N" 50-ohm Microwave Chassis Receptacle and Plug.

Preliminary tests of the equipment, using an old water-cooled discharge tube of natural Mercury, were successful in maintaining a healthy gas discharge, although the Magnetron and RG-cable became rather hot. After several such trials, the glass/metal seal of the Magnetron cracked, rendering it unfit for further use. Investigation revealed that the fabricated coaxial output plug on the Magnetron had become overheated, melting the internal soldered joint. Immediately this occurred, the entire power output had been reflected back into the Magnetron cavity, resulting in prompt and catastrophic failure of the glass envelope. Heated correspondence with the Mullard Company disclosed that a suitable output plug for this Magnetron was produced in small quantities, to order, for Electro-Medical Supplies.
Ltd. of London, whereupon a few plugs were purchased, together with a new Magnetron.

An ebonite collar was made to fit the new output plug and cable very closely, in such a way as to support the cable as it left the plug, thereby preventing the central core from straining the glass/metal seal of the new Magnetron in the event of any further overheating. In addition, the heat sink was replaced by one consisting of a large number of thin brass sheets, carefully spaced out and painted black to improve the radiation and convective dispersion of excess heat.

In order that the Magnetron cavity temperature could be easily monitored, a fine hole was drilled right through the heat-sink, and a Chromel-Alumel thermocouple, connected to a Spot Galvanometer, was placed in contact with the body of the Magnetron. Having been carefully calibrated, this thermocouple was found to be very convenient, for it could be seen, at a glance, how near the Magnetron was operating to the maximum permissible temperature.

The new Magnetron unit performed very well indeed, and the modifications (including an Anode Current meter), helped to ensure that the quoted maximum ratings were not exceeded at any time. The intensity of the resultant Mercury discharge was, as expected, found to be a function both of the waveguide tuning-stub setting and of the position of the discharge tube within the cavity slot. Variation of either of these parameters had a noticeable
effect on the Temperature and Anode Current of the Magnetron.

Investigations into the efficiency of the microwave unit were conducted using water loads, in place of the discharge tube, in the cavity. It was apparent that a considerable amount of power was being lost between the Magnetron and the cavity, as indeed the warm RG-cable implied. As a result, the cable was shortened as much as possible and all terminations and connectors were carefully rebuilt. It had been noted previously that the intensity of the discharge could be varied by running one's hand along the outside of the RG-cable, indicating the existence of strong standing-waves caused by reflections at mis-matches. This effect was removed by the tidying-up of the connections, and further tests using a water-load confirmed the greatly increased microwave output efficiency.

When the light-proof box was being designed to house the optical system (Section (e)), provision was made for the discharge tube to be mounted in Terry-clips screwed into stand-off blocks on a bulkhead. The microwave cavity was then arranged to fit round it, at a small angle to the horizontal, with the tuning stub accessible from above, and the RG-cable entering the light-proof box through a type N bulkhead-connector at the rear. Cooling of the discharge tube was by water, piped through spigots in the box wall, which were connected to the cooling jacket by rubber tubing.

Variations in the cooling water supply pressure led to fluctuations in the discharge intensity, until a
Fig. 35: The Constant-Head Unit, showing the safety-latch and microswitch (left).
constant-head water supply was devised which performed admirably, greatly reducing these fluctuations. At this time, alterations were being made to the pipes supplying water to the building in which the apparatus was housed, and it was deemed advisable to ensure the safety of the Magnetron unit in the event of failure of the water supply. The constant-head bucket was therefore arranged to hinge on its support under the influence of a spring, so that a microswitch was depressed when the bucket was full, and released when it was empty, cutting off the main electrical supply to the Magnetron unit. Of course, restoration of the water supply would then have switched the Magnetron unit back on at full H.T. potential, without the benefit of a warming-up period. Since the results of this might have been disastrous, the constant-head bucket was arranged to catch on a safety-ratchet as soon as it was empty, ensuring that the microswitch could not be depressed again until the safety catch was manually reset. There is reason to think that this device saved the equipment from self-destruction on at least one occasion. A photograph of the constant-head device is shown in Fig. 3.5.

Certain preliminary tests on the Photomultipliers were conducted using the above equipment, but it was apparent that a good deal of microwave power was being devoted to heating up the cooling water instead of exciting the discharge. Accordingly, water cooling was abandoned in favour of a pump forcing air at ambient temperature through
the cooling jacket. The discharge intensity increased considerably, and air cooling has been used ever since. After a long period of running thus, the brilliance of the discharge became a little dimmed through oil mist from the pump condensing on the walls of the cooling jacket. This problem was cured simply by reversing the air lines so that the pump sucks filtered air past the tube.

The stability of the discharge in the short-term was found to be quite good, but long term stability (more than 30 minutes) was found to be somewhat unreliable, particularly at high power outputs. The cause was found to lie in a chain-reaction of events: If the ambient temperature changes, then the heat balance is disturbed and there is a change in power absorbed in the cavity. This change in power alters the effective internal resistance of the cavity, and hence the anode voltage changes. Since the Magnetron operating characteristic is very steep, a small change in anode voltage produces a very large change in anode current, which tends to increase the thermal imbalance still further. At high power outputs, the temperature changes are correspondingly more rapid, and a careful watch had to be maintained on the thermocouple galvanometer to prevent thermal runaway. Since the correlation experiments were obviously going to need a stable light source, the use of some form of servo-control of discharge intensity was imperative.

Using a barrier-layer photocell as a brightness monitor, the feasibility of a servo-control system was
tested with a transistorised Difference Amplifier and Relays. It was found that such a system could be made to work, but two factors militated against the particular circuit under test; firstly, the long-term thermal stability of the transistors was unimpressive, and secondly, the sensitivity of the photocell was very poor. Accordingly, several other circuits were theoretically investigated, and a search was made for more sensitive photodetectors. Calculations indicated that stability was theoretically most easily obtainable in a balanced Wheatstone Bridge configuration. Consequently, this design became the basis for the servo-system. The problem remained of causing the out-of-balance bridge to activate relays which would, by some means, restore the balance. This was to be solved by the simple expedient of using a sensitive moving-coil relay connected across the unbalanced bridge elements. The search for a more sensitive type of photodetector ranged over photomultipliers, phototransistors, photodiodes and Cadmium-Sulphide cells. A photomultiplier would have required an additional stable H.T. supply and associated ancillary equipment, and the available phototransistors were rejected on the grounds of thermal instability. Photodiodes (Texas, L.S.400) were fitted, proving to be a little more sensitive than the barrier-layer cell when tested with a conventional Tungsten lamp. Unfortunately, when they were tested with the microwave unit running, the current passed by them was found to bear no relation to the light level, only to the strength
Fig. 3.6: Circuit diagram of the original Servo Bridge circuit. For connections b, f, g, h, see Figs 3.7 and 3.9.
of the r.f. field. Evidently, the change in resistance due to change in light flux was being swamped by the current induced in the wiring by the r.f. field, which was then being rectified by the diodes. Lastly, a Cadmium Sulphide photocell (Mullard, ORP 93) was obtained and tested on the discharge lamp. Fortunately, this device proved to be immune to the r.f. radiation, was thermally stable, and displayed excellent sensitivity to changes in light flux. On the basis of the operating characteristics of the Cadmium Sulphide cells and of the moving-coil relay, a bridge circuit was designed and built (see Fig. 3.6) ready to receive the relay, for which there was a rather long delivery time. Fortunately, the relay, when it eventually arrived, proved to conform adequately to its specifications, and the servodetector circuit was an immediate success.

Control of the discharge lamp intensity had previously been by manual adjustment of the Variac controlling the Magnetron H.T. voltage. Now, the servomechanism was arranged to actuate the Variac by means of a small electric D.C. reversing motor (Vactric, 11.P.127) through multiple-reduction gearboxes (Vactric, 11.H.76 and 11.H.13). The power to drive the motor was provided by a 24 volt stabilised D.C. supply unit (Farnell, TSV/30) through a potential divider, giving 20 volts at the motor terminals. The direction of rotation of the motor was determined by which of the two auxiliary relays were switched into circuit by the moving-coil relay, as shown in Fig. 3.7.
Fig.3.7: Circuit diagram of the Servo-Bridge Auxiliary Relay system to actuate the Servo Motor. For connections b,c,d,g,h,f, see Figs 3-6, 3-8 and 3-9.
Thus, a complete servo-feedback loop was operating. As soon as the discharge intensity varied, this was sensed by the Cadmium-Sulphide cells, which unbalanced the bridge. This caused the moving-coil relay to close one pair of contacts at the end of its deflection which switched in a relay supplying current of the required polarity to the servomotor. The motor then rotated the Variac, neutralising the original variation in discharge intensity.

This control system, then, possessed the great virtue of simplicity, since the motor operation was a simple binary on/off switching. As pointed out above, the short-term stability of the Magnetron unit was quite good, so that the time-constant of the servo-loop was designed to be of the order of 10 seconds, which avoided positive-feedback oscillation and eliminated the need for the inherently more complex proportional mode.

Stability of the discharge intensity was, in consequence, very good indeed, and the apparatus was used for a large number of minor tests and calibration experiments, including a series of experiments by another research student, giving consistently good service for runs of up to 12 hours or more. However, there was the danger that, if left running unattended for more than a few hours, a minor fault might occur causing the rated operational conditions of the Magnetron to be exceeded, with catastrophic results. To eliminate the possibility of such an unhappy occurrence, several safeguards were incorporated into the system. Firstly, it was important that the
Fig. 3.8: Circuit diagram of the photo-transistorised Thermal Cutout circuit.

For connections a, d, e, see Figs 33, 37, 39.
Magnetron should not be allowed to overheat, so a Mullard OCP 71 phototransistor was fitted inside the thermocouple Spot Galvanometer in such a position that it intercepted the light beam just before the Magnetron heat-sink temperature reached the maximum permitted value of 125°C. The phototransistor was arranged to control a relay by means of the circuit shown in Fig. 3.8. When the light beam illuminated the OCP 71, the relay switched off the mains supply to the Magnetron unit, and in addition it switched off the 24 volt D.C. supply to its own circuit. This latter was a precaution ensuring that power was not re-applied to the Magnetron as the temperature fell again. Resetting of this device was by manual operation of a biased switch.

A further protection for the Magnetron unit was the addition of a low-resistance relay to the earthy end of the Magnetron anode circuit. This relay was shunted by a variable resistor, the setting of which determined the anode current at which the relay pulled in and cut off the L.T. power supply to the thermal cutout circuit. The setting of the variable resistance was normally chosen to correspond to the quoted maximum rated current of the Magnetron, 125 mA, and when the relay cut off power to the cutout circuit, power to the Magnetron unit was locked off until reset with the biased switch mentioned above.

Thus, the Magnetron was fully protected from overheating and from excessive anode current. This system worked
well, but it still left the moving-coil relay vulnerable to full-scale deflection whenever the discharge was extinguished. Now, the Coincidence Circuit and the moving-coil relay circuit were both powered by a small Farnell MSU Stabilised Power Supply Unit giving 8 volts D.C., so it was arranged that the Thermal Cutout unit should also shut off the mains supply to this unit as well as to the Magnetron unit.

All the relays were surge-suppressed by means of diodes, and all inter-chassis cables were properly screened and earthed, thus effectively suppressing extraneous inductive pulses which had previously been found to interfere with the pulse-counting circuitry. A persistent form of this latter interference was traced to surges in the servo-motor supply leads, and was eliminated only by the insertion of a high-frequency choke in each lead.

It may be added that all the above safety features have been made use of through various misadventures and have proved to be completely reliable in more than a year's operation, fully justifying the time and effort spent in developing them.

Thanks to the reliability and stability of the servo-system, and because of the anticipated demands of the later experiments, it was decided that it would be quite safe to increase the sensitivity of the system. This was done by putting the two ORP 93 Cadmium-Sulphide cells in series in one arm of the bridge, while optimising the values of the remaining components to achieve a finer balance condition. The circuit became as shown in Fig. 3.9. The
Fig. 3-9: Circuit diagram of the final Servo Bridge and Power Supply arrangement.

For connections a,b,f,g,h,j, see Figs 3-3, 3-6, 3-7, 3-8 and 3-13.
desired level of discharge intensity was achieved, as in the previous circuit simply by adjusting the setting of the 1 kΩ potentiometer. Amongst other considerations, the resistance values were chosen so that, under normal running conditions, the intensity was continuously variable from almost a flicker up to a painfully dazzling brilliance, with close control of intensity maintained over the entire range.

A relative measure of the discharge intensity was provided by a circuit consisting of a third ORP 93, powered by the 24 volt D.C. supply by way of a 6.3 volt Zener diode, with a 20 mA meter registering the resultant current. This ORP 93 was mounted immediately between the two servo-system ORP 93's, approximately 3½" behind the discharge tube. This monitoring system then provided a completely independent means of checking that the discharge intensity was set to the same level from day to day during the course of any one experiment.

Having used the original Mercury-198 discharge tube for a considerable period, the glass envelope began to darken slowly due to "clean up" of the Mercury. A new discharge tube was purchased, this time of fused quartz, and exchanged for the old. After a short time in use, the rubber tubing connecting the suction pump and the cooling jacket of the discharge tube rapidly began perishing. The cause was the enhanced emission of ultra-violet radiation through the quartz, which caused ozone to be produced in the air stream. The high concentration of ozone then
attacked the rubber on the downstream side of the discharge, opening up cracks and finally rotting it completely. This trouble was immediately eliminated by the substitution of nylon tubing for the rubber. A water-filled glass cell was placed between the discharge and the Cadmium Sulphide cells to protect them from any possible damage by the enhanced ultra-violet radiation. A check on the pressure in the cooling tubes was provided by a U-tube manometer monitoring at a point between the discharge tube and the thick cotton-wool inlet filter. This feature was retained, being a very useful indicator of a clogged filter. The filter itself was fitted inside the lightproof box, near the Cadmium Sulphide cells, so that the air around them was constantly being renewed at room temperature.

Every three or four months, one of the contacts in the moving coil relay used to become oxidised, and the relay arm occasionally stuck in the "on" position. There was no real remedy for this, except to dismantle the relay and clean the contacts carefully. This was done immediately before the two final experiments began, and the adjusting screws were wound close together to reduce the dead-space and increase the sensitivity. In Fig. 4.2 of Chapter 4 are shown graphs of discharge intensity as a function of time, with the servo-system operative and inoperative, for comparison. A full set of instructions for operating the Magnetron servo-system is given in Appendix (c).
Section (b). The Coincidence Unit and Pulse-Counting System.

It was shown in Chapter 2 how necessary it is to use a very fast coincidence system in optical correlation experiments, preferably one having a resolving time of approximately one or two nanoseconds. Accordingly, a promising circuit, due to Bay(63) was constructed. This circuit (see Fig. 3.10) is a two-diode bridge, operated with the difference between the two inputs applied across one of the diodes. Referring to the circuit diagram, the operation is as follows: When pulse-height $x$ is greater than pulse-height $y$, the current through D1 is nearly proportional to $(x-y)$, while the current through D2 is proportional to $x$. The difference between the two currents is thus proportional to $y$. When $y$ is greater than $x$, the current through D1 is effectively zero, while the current through D2 is still proportional to $x$. The difference between these two currents is thus proportional to $x$. Hence, the difference between the currents through each diode is proportional to whichever is the lesser of $x$ and $y$. The output currents from the difference-circuit are injected into the measuring bridge in such a way that current proportional to $(x-y)$ is integrated in $R_1C_1$, and current proportional to $(-x)$ is integrated in $R_2C_2$. The voltages developed across $R_1$ and $R_2$ are then added and amplified in the normal way.

Since the only part of the Bay circuit sensitive to r.f. pickup is the measuring-bridge, this was built inside
Fig. 3-10: Circuit diagram of the Bay Coincidence Unit.
a diecast metal box, with the rest of the circuit mounted on top. The mode of operation of the circuit required that the diodes have well-matched characteristics, and much time was wasted in trying to make it work with the two diodes available (CV 2155, Silicon), which were poorly balanced. Several types of diodes were tested for this circuit, including a batch of Germanium OA 90's, but the best results were achieved using two carefully matched CV 2155's, selected from a batch of sixteen. Very considerable time and patience was expended in tuning the two halves of the measuring-bridge, and on at least one occasion a resolving time (full width at half maximum height) of better than 0.5 nsec. was achieved. However, this circuit displayed a marked instability, and numerous attempts at reproducing cable delay curves met with a notable lack of success. Normally, cable delay curves were measured using one of the photomultipliers as a source of pulses, and at this time it was evident, from the several peaks in the delay curves, that ringing was present. On careful examination of the pulse shapes at different points in the circuit, using a Tektronix 581 oscilloscope, it was evident that the circuit itself was contributing to the ringing, implying that the circuit components were poorly arranged. A second Bay circuit was constructed in which the component layout was carefully arranged to provide maximum symmetry and minimum lead-lengths. All cables used were the 100 ohm type As 50, with
Fig. 3.11: Circuit diagram of the original Franzini Coincidence Unit.

Fig. 3.12: Static Characteristic Curve for a typical 1mA Tunnel Diode.
matching plugs and sockets of P.E.T. manufacture. The cable delay curves measured on this second circuit displayed much less pulse-ringing than previously, and the very short resolving time was still occasionally obtainable. Nevertheless, the tuning of the measuring-bridge was still very critical, and the circuit did not inspire confidence at all. Therefore, instead of attempting to develop the circuit any further with pulse shaping and limiting circuits, it was decided to build some other circuit which would be inherently more stable, while still retaining the required short resolving-time.

A review of the literature revealed a number of possible replacement circuits, but one was conspicuous by virtue of its elegant simplicity and claimed short resolving-time. This was a Tunnel Diode AND-gate designed by Franzini\(^{64}\), shown in Fig. 3.11. The operation of this circuit may be understood by reference to Fig. 3.11, and also to Fig. 3.12, which represents the typical characteristic curve of a Tunnel Diode: If a pulse is applied to, say, input \(I_1\) so that a current of 1 mA flows in \(TD_1\), it will switch to the state represented by point \(C\) on the characteristic. The current in the 15 kΩ load resistor remains substantially constant, and \(TD_2\) will move to point \(B\) on the characteristic. Since the voltage drop across the Silicon switching diodes is independent of current in this state, there will be a small signal at the earthy end of the load resistor (i.e. the output), equal
Fig. 3-13: Circuit diagram of the modified Franzini Coincidence Unit. For connections j, see Fig. 3-9.
to $V_B - V_A$, normally 0.01 volt. In the case of coincident input pulses of greater than 1 mA, both Tunnel-Diodes simultaneously switch to point C on the characteristic, producing a much larger output pulse of 0.5 volt (i.e. $V_C - V_A$).

Thus, from the outset, this circuit displayed several inherent advantages over the Bay circuit. Firstly, in the static case, as we have seen above, the ratio of output pulse amplitudes for coincident input pulses to that for single input pulses was 50/1. Secondly, the Tunnel Diodes were switched only by pulses of more than 1 mA, rendering a separate Discriminator unnecessary. Thirdly, the steeply-rising characteristic of the Tunnel Diodes, after point C, ensured that the output pulses were of relatively uniform amplitude, eliminating the need for a separate Limiter.

As it was intended to use 2 mA Tunnel Diodes instead of the original 1 mA type, the circuit values were altered slightly as shown in Fig. 3.13. In addition, the supply voltage was raised from 6v to 8v and a decoupling filter was added to the supply line to rule out any possibility of "feed-throughs" appearing at the output of the emitter-follower. Since fast switching diodes were being used (Texas IN 914), shorted delay lines were fitted to the photomultiplier outputs in order to reverse-bias the diodes immediately after conduction, thereby switching them off sharply. The value of the input resistors was varied
Fig. 3-14(a): The two input channels of the coincidence circuit.

Fig. 3-14(b): The output stage of the coincidence circuit.
experimentally to achieve optimum matching. As expected, the value at which pulse-ringing was a minimum was $75 \Omega$, and this remained the same for several different types of Tunnel Diode.

The construction and layout of a very fast pulse-circuit is always critical because of the effect of inter-lead capacitance and the possibility of multiple earth-loops due to skin-effect. The circuit was therefore carefully arranged to fit into a small copper box divided into three compartments (see Fig. 3.14). Two compartments in one half of the box contained the two separate and mirror-imaged input channel components, while the third compartment in the other half of the box contained the emitter-follower circuitry. Each input, therefore, was completely shielded from the other, and also from stray inductive coupling at the output. The effect of earth loops was minimised by the standard method of providing a common earth point for the fast input stages together with an entirely separate earth point for the slower output stage.

Preliminary tests of this circuit yielded very encouraging results. The stability, as represented by the reproducibility of results, was excellent, and one of the first cable-delay curves measured, using clipped photomultiplier pulses, gave a resolving time (f.w.h.m.) of 0.7 nsec.

The method employed to obtain such delay curves is described in Chapter 4, but may be briefly summarised here
Fig.3:15: Photographs of Franzini circuit output pulses with, (Upper trace), Single channel input and, (Lower trace), Coincident input pulses. (Horizontal 10ns/cm, vertical 1mV/cm.)
as the branching of a photomultiplier output along two cables which are connected to each input channel of the coincidence circuit. One of the cables is of variable length, giving a variable delay between the identical pulses travelling down each cable. This method is prone to mismatching, manifested as pulse-ringing, but the ringing was always of lower amplitude than the threshold of the Tunnel Diodes and was not troublesome. Some ringing on the output was noticed as double-pulsing in the Scalers. This fault was traced to the unmatched high-impedance input to the Cathode-Follower. A permanent cure was effected by reducing this impedance to $100\,\Omega$, without adversely affecting the counting-rate.

The problem of degradation of resolving-time due to jitter in the different input pulses was not serious, but it was hoped to reduce it by interposing fast-risetime pulse-shapers between the photomultipliers and the Franzini inputs. The most suitable form of circuit appeared to be one using a transistor operating in the avalanche mode, giving a large amplitude pulse of very rapid rise-time. A pair of pulse-shapers, closely following a design by Pardies, Perrin & Soulé (65), was built and each was connected between a photomultiplier and its respective Franzini circuit input channel. After some experimentation to optimise the system, cable delay curves were measured which showed no improvement over the curves measured with the Franzini circuit alone. It was clear that the Tunnel Diodes were performing so well as limiters and discriminators, that there was no need for the avalanche circuit, and
Fig. 3-16: Block diagram of the pulse-counting system.
it was discarded.

On studying a number of cable delay curves obtained with the Pranzini circuit under varying conditions, it was noted that the peaks were not quite centred on zero-delay but were displaced by an average of 0.19 nsec. One of the cables connecting a photomultiplier to the Pranzini circuit was therefore adjusted in length to offset the delay exactly. This cable, together with its slightly shorter partner, was thereafter left permanently connected to the Pranzini circuit. A photograph of typical output pulses from the Pranzini circuit is shown in Fig. 3.15.

Certain anomalous variations in pulse counting rates were traced to the effect on the Pranzini circuit of variations in ambient temperature. The solution to this problem is discussed in Section (d), below. Apart from this trouble the Pranzini circuit has been operating reliably and consistently for nearly three years.

The 8 volts D.C. power supply for the coincidence unit was obtained from a modular Parnell MSU Stabilised Power Unit, which also supplied the power for the Bridge circuit of the servo-control system.

The remainder of the pulse-counting system is shown diagrammatically in Fig. 3.16. The H.T. power supply for the photomultipliers was provided by an Isotope Developments E.H.T. Unit 532D, capable of supplying up to 3 kV at 4 mA. Most experimental work was undertaken in the region of 2.6 kV, when the total current drawn was approximately 3 mA.
Further information relating to the photomultipliers is presented in Section (c) of this Chapter.

The coincidence-output pulses were amplified by a Dynatron Pulse Amplifier, type 1430A, following an associated Cathode-Follower Unit. This amplifier is an Atomic Energy Research Establishment approved design, having a maximum gain of 86 dB, while the Cathode-Follower introduces a loss of 23 dB; the band-width is of the order of 2.8 MHz. The pulses from the amplifier, with suitable adjustment of the integration and differentiation time-constants, were suitable to drive the Scalers. The amplifier is, unfortunately, powered exclusively by thermionic valves, and the power-supply rectifying valves proved to have a very limited life. However, provided these were renewed at suitable intervals the performance was found to be thoroughly reliable.

In the early stages of the experiment it was found necessary to amplify the extremely short rise-time pulses emitted by the photomultipliers. This function was performed admirably by a pair of A.E.R.E. type 2002 A Distributed Amplifiers (built by Fleming Radio), which had a risetime in the region of 2.5 nsec., of the same order as the input pulses. The gain of these amplifiers was approximately 20 dB.

The counting units employed were a Pre-Scaler 1850B and a Scaler 1800B, manufactured by Isotope Developments Ltd. Both were controlled by an I.D.L. Timer 1860A, which
was capable of stopping counts after pre-determined periods of time from 1 sec. up to $10^5$ secs. All three units employed five-stage Dekatron displays. The Pre-Scaler had a pulse-separation resolution of 1 μsec, and comfortably accepted pulses direct from the 1430 Amplifiers. The Scaler provided, in effect, an extension of the counting store of the Pre-Scaler and had a poorer, but quite adequate, time resolution.

Mention should also be made of an abortive pulse counting system which, it was hoped, would prove to be a more sensitive test of coincidence count correlations. This involved counting the number of pulses emanating directly from one photomultiplier and comparing this with the number of coincidence counts recorded in the same period of time. The ratio of the two counts was then to be calculated for different relative positions of the photomultipliers.

Unfortunately, although this was a potentially very sensitive system, the effect of thermal variations on the coincidence counting-rate was found to be markedly different from the effect on the single-channel counting-rate, and so it had to be abandoned in favour of the original arrangement. A Fast-Slow coincidence system was rejected as being likely to suffer from the same defect.

Much of the development of the electronic circuits described in this Section involved the optimisation of pulse shapes and amplitudes to suit all of the operating conditions of the primary detection equipment. Many compromises had to be made in the course of this development,
Fig. 3.17: Circuit diagram of the original carbon-resistor dynode chain.

Mullard 56AVP
and it must be admitted that some of the proprietary equipment was not of the most modern type. Had the funds and equipment been more readily available, a fully automated print-out system would have been incorporated. Nonetheless, it is considered that the available system was developed to its operational limit.

Section (c). The Photomultipliers and Dynode Chains.

The photomultipliers employed throughout this research were of type 56 AVP, manufactured by Mullard. The dynode chain used in preliminary experiments is shown diagrammatically in Fig. 3.17, and was constructed of extra high-stability cracked-carbon resistors. This particular arrangement is that suggested in the manufacturers' Operating Notes for this device, with the omission of the recommended decoupling capacitors over the last three or four stages. It is more usual to include such capacitors for the following reason: The output pulse amplification is critically dependent on the amplification over the last few stages, and the amplification is a direct function of the inter-electrode voltage which is maintained by a constant standing current. If decoupling is not used, the pulses of multiplying electrons travelling down the dynode chain tend to pile up. This behaviour effectively manifests itself as a potential drop which reduces the inter-stage potential, to the detriment of the final amplification. Hence, as the counting rate increases, so the output pulse amplitude decreases.
Initially, identical dynode chains were built to drive the two available photomultipliers, and decoupling was employed over the last three stages of each. Inspection of the resulting output pulses from each dynode chain, by means of the fast Tektronix 581 oscilloscope, revealed pulse ringing of considerable amplitude and duration. Removal of the decouplers immediately cleaned the pulses up but reduced their amplitude at high counting-rates, for the reason given in the previous paragraph. Efforts were made to solve this problem by careful rearrangement of the components and by very careful matching at the output, but to no avail. Correspondence with the Mullard Technical Information Department failed to elicit any advice that had not already been unsuccessfully applied, and so the situation was tolerated, for a while, using low light intensities.

In order that the photomultipliers should perform as satisfactorily as possible, under these circumstances, it was obvious that the Signal/Noise Ratio (S/N) should be as high as possible. In this context we may regard the Noise as being the counting rate recorded with zero illumination of the photocathode. This so-called Dark Count-Rate is composed of three distinct elements:

(1) Thermally emitted electrons, giving rise to white noise.

(2) Insulation leakage, giving rise to Johnson noise.

(3) Electron bursts originating from spurious light effects such as corona discharge, scintillation in the glass envelope caused by cosmic radiation and accelerated ions in the residual gas.
Fig. 318: The later type of photomultiplier assembly, showing the V.D.R.s mounted around the coaxial lead-out cable.
It had been observed that the dark counting rate was rather high, giving a low S/N ratio, and experiments were conducted with a view to reducing this noise. New mountings for the dynode resistor chains were designed, in which the resistors were carefully spaced out from each other and from the earthed container and all soldered joints were made as smooth and rounded as possible. These precautions tended to reduce the possibility of corona discharge by reducing the strengths of the localised electric fields around the resistor chain. Some improvement was observed; but it was not until the basic design of the photomultiplier tube supports came under further scrutiny that the investigation really began to bear fruit. A photograph of the later photomultiplier and resistor chain assembly is shown in Fig. 3.18, and it can be seen that the photomultiplier is arranged to fit into its base socket, next to the resistor chain, at one end, and into a close fitting Paxolin support ring at the cathode end. Both of these supports are mounted on three long screwed brass rods which are firmly fixed to the brass end-plate, all of which is at earth potential. In the original design, the earthed rods extended as far as the plane of the photocathodes, which were at the full H.T. potential of up to 3 kV with respect to earth. It was realised that the strong electric field between the cathode and the rods could lead to a glow discharge within the intervening glass envelope. Consequently, the support rods were shortened until they were
some distance from the photocathode, while still providing the necessary support. In the case of the first modified assembly tested, the reduction in Dark Counting Rate was more than ten-fold. The improvement was a little less for the other assembly as its support rods had been slightly shorter in the first place.

It is well known (Engstrom, 1947) that another method of improving the S/N ratio of a photocathode is to cool it. This course of action was carefully considered, but it was not proceeded with for four reasons: Firstly, the photomultipliers needed to be easily and accurately moved from side to side in order to scan the optical field, implying that the cooling medium would have had to be carried to and from the mountings by flexible tubing. Liquid air temperatures and flexible leak-proof tubing were incompatible. Secondly, if solid CO₂ were to be packed round the photomultiplier cans, the lightproof box would need opening up to replenish it from time to time. This would have meant switching off the E.H.T., which had to be allowed to stabilise for two or three days, and so this was unacceptable. Thirdly, cooling by means of water tubes around the cans would not have improved the S/N ratio enough to warrant the extra complication. Fourthly, the room temperature was eventually stabilised very capably by a thermostatic control, thereby maintaining a steady S/N ratio. This latter was considered to be more desirable than a higher, but less stable, S/N ratio.

A further attempt at reduction of the Dark Current was
made by the application of a high potential to a shield around the cathode of one of the photomultipliers, following a suggestion by Naray (1956). The shield was constructed from thin brass, fitted tightly over the photocathode region of the glass envelope, and was connected to the E.H.T. supply at the same potential as the photocathode. The effect of this shield was found to be slight, and the idea was dropped.

Much time and thought was devoted to the problem of establishing "clean" pulse shapes in the fast circuitry, and at an early stage of development it was decided to standardise exclusively on As 50 coaxial cable (100 Ω), in conjunction with P.E.T. screw connectors, for inter-circuit communication. The only trouble experienced with this combination, in spite of theoretically perfect impedance matching, was in the output pulse across the anode resistor of each photomultiplier. The ringing in this case was found to be independent of the presence or absence of the dynode chain decoupling capacitors. Investigation showed that the disposition of the load resistor with respect to its associated blocking capacitor was very critical indeed. The answer was eventually found to involve mounting the load resistor and its associated ceramic disc blocking-capacitor side by side, directly onto the photomultiplier base anode pin, taking the lead-out straight into a short length of As 50 cable and out through a P.E.T. bulkhead connector. In other words, the stray capacitances were minimised by mounting the load resistor directly onto the
Fig. 3-19(a): Photograph of photomultiplier pulses from the original lead-out, without (Upper) and with (Lower) delay clip.

Fig. 3-19(b): Photograph of photomultiplier pulses from the repositioned lead-out, with (Upper) and without (Lower) delay clip.

(Horizontal, 10 ns/cm, Vertical, 1 V/cm.)
bare end of the cable. This may be seen in Fig. 3.18. The improvement in the pulse shapes is shown in Fig. 3.19.

Since opposite-polarity pulses were required to speed up the diode-switching action of the Franzini circuit, shorted delay cables were fitted to the photomultiplier output cables. The lengths of shorted cable were varied in order to ascertain the delay time most suited to the photomultiplier pulse lengths and risetime. The length of cable which was found to give the best balance between negative and positive pulse amplitudes without introducing ringing was found to be equivalent to a delay of 2 nsec., i.e., the physical length of the cut cable was equivalent to 1 nsec. This figure agreed well with the quoted rise-time for these photomultipliers, which was 2 nsec. The resulting pulse shape is shown in Fig. 3.19.

It has been described above how trouble was experienced with pulse ringing when decoupling capacitors were fitted to the last three stages of the dynode chain, and how the ringing was cured by the omission of these decouplers. The resulting drop in pulse amplitude at high counting-rates was of little consequence for the preliminary tests, for which relatively low intensity illumination sufficed. However, it later became obvious that some improvement was called for, and encouragement was offered by results obtained by Dr. D.G. Vass (68), of this Department, in using Voltage-Dependent Resistors in photomultiplier dynode chains. This work has since been published (69). The original experiments had been conducted on the application of Voltage
Fig. 3-20: Characteristic curve of the Voltage-Dependent Resistors used in the later dynode chains.
Dependent Resisters to E.M.I. "venetian-blind" photomultipliers, but there seemed to be no reason to suppose that this technique would not apply equally well to photomultipliers having Rachman focused dynodes, such as the 56AVP. The main drawback to the use of V.D.R.'s seemed to be that each type was only suitable for use over a limited range of E.H.T. voltage. Experiments indicated that the S/N ratio was highest in the range 2.4 kV to 2.6 kV for the 56AVP, and that the pulse amplitudes in this range were the most suitable for feeding directly into the Franzini circuit. As the gain variation was least affected by the cathode end of the dynode chain, carbon resistors continued to be used for the focus and deflection electrodes, but all inter-stage resistors were replaced by Mullard E299DD/P346 Voltage Dependent Resistors. These gave 150 volts per stage at 1 mA standing current, thus directly replacing the original carbon resistors when operating at 2.5 kV overall H.T. voltage. The characteristic curve for the V.D.R. type used is shown in Fig. 3.20. Inspection of this curve shows that at any given nominal voltage, the current through the V.D.R. may alter considerably with negligible change in this voltage. Hence, even at high counting-rates, when the dynode chain standing current may drop, the interstage voltage remains sensibly constant. Comparison of the pulse-height spectra from one photomultiplier before and after fitting V.D.R.'s proved beyond doubt that the change was beneficial. Indeed, where it had been found that pulse
Fig. 3: Circuit diagram of the later type of dynode chain using Voltage-Dependent Resistors.
amplitudes fell, using carbon resistors, the V.D.R.'s were found to increase the pulse amplitudes at high illumination levels.

It is difficult to quote actual S/N ratios for the photomultipliers using these resistors since there are so many variables, but it is safe to say that the overall S/N ratio was improved by a factor of not less than 2, under normal illumination conditions. The final arrangement of the dynode chains is shown in Fig. 3.21. During early experiments, one photomultiplier was found to be defective and was replaced. The serial numbers of the photomultipliers used in the final experiments are No. 14605 and No. 17140. Fig. 3.22 shows the change in the pulse-height spectrum of No. 14605 when the old carbon resistor dynode chain was replaced by the V.D.R. chain.

**Section (d). The Thermal Stabilisation of the Equipment.**

In the initial stages of this research work, scant attention was paid to the maintenance of a constant ambient temperature in view of the fact that no especially delicate or sensitive measurements were being made. However, as the equipment became more highly developed, it was apparent that variation of room-temperature could well become a significant factor which would require closer control.

The first suspicion of trouble appeared in the form of a variation of counting-rate from day to day, which could
Fig.3-22: Pulse-Height spectra for 56AVP No.14605, showing the improvement when carbon resistors, (A), were replaced by V.D.R.s, (B), in the dynode chain.
not be traced to any other cause but room temperature variation. The count rate observed for any given process was usually higher on a warm day than on a cool day, and so a thorough investigation of the proprietary electronic equipment was undertaken. It appeared that the only equipment whose instability was likely to have any direct effect was the E.H.T. supply unit powering the photomultipliers. A small variation in the H.T. voltage would directly affect the output pulse amplitude and counting rate. Fortunately, this particular unit had a remarkably stable output, with negligible variation of voltage over a very wide temperature range, thereby establishing that the gain of the photomultipliers was constant. The next possibility investigated was the effect of temperature on the number of thermal electrons being emitted from the photocathodes. Now, the photomultipliers were mounted inside heavy brass canisters, and these in turn were contained within the large lightproof box containing a considerable volume of still air. Thus, the thermal inertia of the entire system was formidable, and it was considered to be highly unlikely that the normal changes in room temperature would produce either large or rapid changes in photocathode temperature. Calculations indicated that the observed change in Dark count rate with a given temperature change was greater than that to be expected on the basis of a solely cathodic effect by a factor of at least four.

It was clear that some other variable, or combination
of variables, was to blame, and several experiments were conducted with a view to isolating it. During the course of these experiments, several variables (e.g. Single-channel count-rate, Coincidence count-rate, Magnetron current and temperature, Servo settings, etc.) were monitored simultaneously at frequently intervals, but very little correlation between any of them and room temperature was observed. However, the readings of Single Channel count rate had been taken by feeding the pulses from one photomultiplier simultaneously into both inputs of the Franzini coincidence circuit. The photomultipliers themselves were virtually eliminated as the cause of drift, and so the culprit seemed to be the Franzini circuit. A simple test was devised whereby all other variables were maintained sensibly constant, while the copper box containing the Franzini circuit was alternately heated by a powerful lamp and cooled by a cold damp rag. The result was conclusive, with the graph of circuit box temperature versus time exhibiting a striking similarity to the graph of Single Channel count rate versus time. The thermal inertia of the Franzini circuit was clearly shown by a disparity of four minutes in the time scale from the one graph to the other. The problem therefore, was the well-defined one of maintaining the Franzini circuit temperature constant. Several solutions were considered, such as the fitting of water-tubes, or the attachment of a large block of copper to increase the heat capacity. Finally, the best solution appeared to be the simple one of enclosing the box in heavy
lagging. A large block of expanded polystyrene foam was obtained and cut in half. The centre was hollowed out to accept the box as an intimate fit, and suitable holes were bored to accommodate the input, output and supply cables. The whole was then carefully taped up, and tested by causing the room temperature to fluctuate as widely and as rapidly as possible. The new time-constant of the lagged box was more than twenty minutes, so that the rapid fluctuations of room temperature were considerably smoothed out. For instance, during a three-hour test when the room temperature was caused to vary rapidly, in a random fashion, between 19.3°C and 23.5°C, the Franzini box temperature drifted slowly between 22.9°C and 23.2°C, as measured by a mercury-in-glass thermometer inserted through the lagging and touching the box. This may be seen in the photograph of Fig. 3.23.

The building in which the apparatus was situated was heated by means of a very powerful steam-pipe circuit which was turned on during daylight hours in winter, and completely off during the summer months. In consequence, the room temperature during wintertime often varied between 5°C and 28°C every 24 hours, and in summer the temperature fluctuated according to the changing fortunes of the weather. This state of affairs was obviously not conducive to consistent and stable operation of any of the equipment, least of all the Franzini circuit, however well insulated. A large 3 kW electric convector heater was therefore installed, controlled by a Satchwell thermostat mounted on a
Fig. 3-24: Copies of Thermograph traces showing the variation of Room-Temperature over typical 24-hour periods, with (A) the original room heating only, and (B) the additional heater and Thermostat.
wooden bracket adjacent to the lagging of the Franzini circuit. Initially, the thermostat was set merely to maintain a certain minimum temperature in the room overnight, but it was later appreciated that large variations in temperature could still occur in the course of a very hot summer. Accordingly, the thermostat was set to a relatively high value (nominally 23°C) so that the overall temperature variation throughout the year was minimal.

Fig. 3.24 shows a direct comparison of room temperature between 24 hours with the thermostat on, and 24 hours with the heater switched off entirely. Both of these graphs were replotted directly from the traces given by a 24 hour Thermograph. The effectiveness of the thermostat is clearly shown.

Section (e). The Optical System and the Light-Proof Box.

Before describing the development of the optical equipment, it must be noted that the experimental requirements altered during development. From the outset, it was decided that the arrangement of the optical system, while satisfying the requirements of the experiment in hand, should remain flexible and adaptable to any reasonable modification of the basic experiment. As intimated above, this is precisely what did occur. The initial aim was to undertake the Double Slit/Biprism experiment suggested by R.M. Sillitto (1963), and it was on this basis that the optical dimensions
and layout were designed and built. It was later decided to check the operation of the electronics by performing a repeat of the Twiss and Little (1959) photon-correlation experiment. Due to the difficulty in procuring a sufficiently accurately-made biprism, the initial requirements of the Sillitto experiment could not adequately be met, and this experiment was abandoned in favour of the slightly simpler Double Slit experiment (Sillitto and Haig (1966) using the same electronics as for the Twiss and Little experiment. However, since the initial dimensions of the lightproof box were determined by the requirements of the original Sillitto experiment, we shall briefly consider this experiment first.

The essential equipment for the Sillitto experiment comprised the following: A Mercury-198 discharge lamp, as described in Section (a) of this Chapter, a double slit, a Fresnel biprism, a beam-splitter, and a slit in front of each moveable photomultiplier. The dimensions of the double slits were determined by a number of factors, those with which we are concerned here being the distance from double slit to biprism, the distance from double slit to photomultiplier, and the dimensions of the photomultiplier slits. Preliminary calculations and experiments indicated that the most practicable compromises were to be achieved with a distance from the double slit to the photomultipliers of between 1.5 m. and 2 m. This, of course, was only the optical path length and did not include the space required by the source system at one end and the detector system at the other. Taking this additional length into
325: Top view of the discharge compartment showing the water-cell and tocells. The top of the discharge-tube is in the foreground.
account, it seemed that the overall length of the light-proof box might have to be as much as 3m. Unfortunately, the laboratory space available was insufficient to permit this, and some means had, perforce, to be found by which the box could be shortened while maintaining approximately 2m optical path. It was clear that the waveguide unit and photomultipliers could not be turned to shorten the box, and so the path length would have to be maintained by reflecting it back through $180^\circ$. The use of plane mirrors was rejected on the grounds that any small deviations of the mirrors caused by thermal stresses or mounting flexure would materially affect the positions of the fringes in the plane of the photocathodes. The solution was finally seen to be the use of two pentagonal prisms mounted side-by-side. A prism of this type is exceptionally resistant to either of the previous objections since it is an effective $90^\circ$ constant-deviation prism.

The design of the optical mountings and their positions thenceforth proceeded on this basis, with the light source and detectors at one end, and the pentagonal prisms at the other. A large and substantial table was procured, and faced over with a sheet of Aluminium, $\frac{1}{8}$ in. thick, to serve as a stout platform for the apparatus. On top of this were screwed two standard triangular section optical benches, 1 m. and 1.5 m. long respectively, parallel to one another and to the long side of table, at a separation of 7". The use of such optical benches ensured accurate centering of
of components, facilitated longitudinal adjustments, and allowed the use of standard mountings. The pentagonal prisms were clamped onto a common brass base, by means of phosphor bronze springs, and the base was then firmly fixed to two bench-saddles. The two saddles were placed on each optical bench, allowing the entire prism assembly to be clamped at any point along the benches giving an overall adjustment of the eventual optical path length.

As in the preliminary tests, the waveguide was fixed at one end of the shorter optical bench, lying in a shaped wooden cradle at a small angle to the horizontal. The mount for the slotted-in discharge tube consisted simply of two Terry-clips fixed to two small wooden blocks which were bolted onto a bulkhead of the lightproof box.

Light from the discharge tube was focused onto the double slit by means of a 2" aperture lens of 6" focal length, and passed through a polarising filter and narrow-band interference filter on the way. The function of the polarising filter was to compensate for the polarising effect of the beam-splitter, and to facilitate adjustment it was mounted in a specially built rotatable holder which was arranged to locate positively by click-stops at 5° intervals. The narrow-band filter was an all-dielectric type, made by Barr & Stroud, which transmitted only the 5461 Å line from the Mercury-198 spectrum. Suppression of longer wavelength lines was ensured by a broad-band filter cemented to the first. This compound filter was fitted
Fig. 3.26: The lightproof box, with lids removed.
inside a brass tube mounted on another bulkhead of the lightproof box.

The mountings of the double slit and biprism, in early experiments, were each standard laboratory biprism holders. These could be rotated about the optic axis, and could also be moved to any position along the optical benches.

A beam splitter was necessary to allow superposition of the two photocathode apertures. The problem of "ghost" images, encountered in the use of semi-reflecting plates, was eliminated by the choice of a large glass cube (2" aperture), consisting of two right-angled prisms cemented together along their common hypotenuse. The cube was manufactured by Barr & Stroud, had a Transmission Coefficient of 48%, a Reflection Coefficient of 44%, and was virtually free from striations, bubbles and other defects. Mounting was in the usual manner with king-posts at opposite corners and a large phosphor-bronze strip spring across the top. The baseplate was milled from a thick brass plate which was rigidly screwed into a standard bench-saddle.

The experiments to be undertaken all required the photomultipliers to be able to traverse across the optical field through precisely measurable distances. The problem essentially reduced to the devising of some means by which the light beam and photocathode slits could be moved relative to one another. A large variety of ideas came under consideration for this purpose, including counter-rotating prisms, rotating glass cubes, rotating mirrors, a sliding beam-splitter, and sliding photocathode slits. The latter
idea was rejected as soon as it was realised that transit-
time and sensitivity variations across the photocathodes
would be likely to cause trouble. It was eventually
decided that, despite the possible difficulties of main-
taining accurate tracking, it was preferable to move the
photomultipliers and slits, as complete units, by means of
a micrometer screw.

It is well known that the motion of a kinematic slide
is defined accurately by five constraints, and in the
original design for a moveable photomultiplier carriage
these constraints were applied in the usual manner: The
carriage was to be fitted with two parallel vee-blocks,
at one end, sliding along a fixed transverse cylindrical
metal bar, while the other end was supported by a single
flat bearing surface, sliding along a second cylindrical
metal bar, parallel to the first. The total of bearing
surfaces was therefore five. Due to the relatively high
weight of the photomultiplier canister, rotating slit
assembly, and carriage, the friction at the bearing sur-
faces would have been unduly great, imposing an unaccept-
able strain on the micrometer screws. Each of the bearing
surfaces was therefore replaced by a small high-quality
ball-bearing, mounted at the correct angle. Surface
friction was further reduced by using thick lapped stain-
less steel cylindrical bars for the carriageways. The
micrometer screws were rigidly clamped to one side of the
carriageway mountings.

This equipment was built by the Departmental workshop
Fig.327: The rack of electronic equipment.
staff. The carriageways were bolted in position on the table, elevated on stout wooden blocks to give the correct height. The photomultiplier in the direct beam was fitted with a micrometer screw of 25 m.m. travel, while the one in the reflected beam was fitted with a 50 m.m. micrometer screw. The flat end of each screw was arranged to bear onto a steel ball which was tightly slotted into the side of each carriage. Positive contact between the screw and the steel ball was ensured by tension springs, but it was obvious that the tension would vary with the screw setting, which was most undesirable. Eventually, positive return of each carriage was achieved by the use of a weight attached to one end of a stranded steel wire which ran up through a small hole in the table, over a pulley, and was then fixed to the carriage. The ingress of light through the hole for the wire was prevented by enclosing the hanging weight in a closed coffee-tin screwed to the under surface of the table. Tension was therefore kept constant, ensuring even tracking and a positive return. When the movements were assembled, it was found that the friction was so low that accurate return could be achieved with only 50 gms. weight, while the weight of the entire carriage assembly was nearly 7 kgms. i.e. the Friction Coefficient $\mu \sim 0.007$. Although the fitting of ball bearings meant that the slide was not truly kinematic no detectable shimmying or mal-positioning of the slits ever occurred. The actual photomultiplier slits were clamped rigidly to the front face of the canisters and could be rotated through a small angle about the
optic axis by means of tangential screws fitted to the massive front shroud of the canister.

The light-proof box itself was built up on a framework of Aluminium angle-extrusions, bolted to each other and to the table. The side panels were cut from large sheets of tin-plate, bolted to the framework, and the whole was rendered light-tight by carefully sealing all the joints with long strips of soft black plasticene. The entire top surface of the box was made from sheets of $\frac{1}{8}$ Aluminium, cut into sections lined with baize material, and screwed down with closely spaced 4 B.A. butterfly nuts. Bulkheads and partitions were fitted at intervals inside the box to reduce stray light and specular reflections. The latter were further discouraged by liberally painting the whole box interior matt black.

Provision was made for the occasional insertion of a periscope into the light beam, by way of an aperture cut in a lid section. The position of this aperture was such that the periscope intercepted the light beam just in front of the beam-splitter, enabling a visual check to be made on the appearance and alignment of the source slits or interference pattern. The photomultiplier slits were carefully aligned, by reversing the periscope to observe both of them through the beam-splitter, when they could easily be adjusted to precise coincidence. The usual procedure for aligning the source slits with the photomultiplier slits was to set the source slits at the desired orientation, and then to place a single slit between the discharge
Fig.328: A general view of the entire apparatus.
tube and the double slits, adjusting its orientation to achieve maximum fringe visibility. One of the photomultipliers was scanned across the field with its own slit set at several different angles. The setting giving the greatest variation of single-channel count rate from maximum to minimum was retained, and the other detector slit was aligned with the first using the visual periscope technique described above. The single slit at the source was then removed, and the apparatus was ready for use. The design of the periscope was such that the plane of the eyepiece corresponded optically with the plane of the detector slits, so that a direct visual check could be made on fringe separations, visibility, and so on, as measured by the photomultipliers. The periscope itself was manufactured, from surplus equipment, in the Departmental workshop.

Finally, it is proposed to outline very briefly the methods by which the various types of slits were manufactured. The original Sillitto (biprism) experiments required double slit sources having a very small separation of the order of 0.01 cms. This in turn implied a slit width of less than this value, to give a fine line of opaque material between the slits. Photographic reduction of carefully drawn large-scale slits was unsuccessful because of lack of contrast, combined with noticeable graininess. The only consistently successful method for production of such fine slits was found to be the ruling of fine parallel lines, with a razor blade, in thin Aluminium films deposited on glass plates in vacuo. This
Fig.3-29: The Periscope assembly.
was a very tricky undertaking, as the ruling had to take place under a travelling microscope, and the parallelism of the lines was a function of steady hands and a bevelled steel straight-edge. With practice, however, this method was capable of producing exceedingly accurately-defined slits. The slits for the photomultipliers were much easier to make due to their larger apertures. The method usually employed in this case was to scribe them on blackened photoplates with a knife. When the Sillitto experiment was abandoned, and the biprism removed, it was then possible to work with double slits having a much wider separation. Blackened photoplates were used exclusively thereafter. The double slit actually used in the Sillitto and Haig experiment consisted of a carefully double-ruled photoplate, overlaid with a single-ruled plate at right-angles to the first, with the film surfaces of each in contact. The result was two identical rectangular apertures which were capable of giving excellent interference patterns under coherent illumination conditions.

Photographs illustrating the equipment described in this Section are to be seen in Fig. 3.25 to Fig. 3.29 inclusive.
CHAPTER 4

THE CALIBRATION AND THEORETICAL PERFORMANCE OF THE EQUIPMENT

Section (a). Introduction

It was decided, at an early stage of development, that the Twiss and Little (Single-Slit) Correlation experiment would be performed prior to the Sillitto and Haig (Double Slit) experiment. The reason for this was that the two experiments overlapped one another, in terms of apparatus and measuring techniques, and a successful result for the first would materially assist in determining the feasibility of the second. Because of their similarity in the use of comparative pulse-counting techniques, therefore, the equation for the ratio, $\rho$, of the numbers of correlated to random coincidences (equation (50)) is common to each experiment. In the same way, equation (52), the Signal to Noise Ratio equation, is also applicable to both experiments. For ease of reference, these two equations are reproduced here:

The ratio of Correlated Coincidence Counts to Random Coincidence Counts is:

$$
\frac{N_c}{N_r} = \frac{1}{4} \frac{\tau_0}{\tau_r} \Delta(n_0) \Gamma^2(d, n_0) f_1 f_2,
$$

and the theoretical Signal to Noise Ratio is:

$$
\frac{S}{N} = \frac{1}{2} \alpha N_0 \frac{\tau_0}{\tau_r} \Delta(n_0) \Gamma^2(d, n_0) f_1 f_2 \left(\frac{\tau_0}{\tau_r}\right)^{1/2},
$$
where it will be remembered that:

\[ \tau_0 = \text{Coherence Time of the source.} \]

\[ \tau_T = \text{Resolving Time of the coincidence circuit.} \]

\[ \Delta(v_0) = \text{Partial Coherence Factor.} \]

\[ \gamma^2(d,v_0) = \text{Normalised Correlation Factor.} \]

\[ f_1f_2 = \text{Instrumental decorrelation factors.} \]

\[ aN_0 = \text{Mean Single-Channel counting rate.} \]

\[ T_0 = \text{Total observing time.} \]

The object of this chapter, therefore, is to examine the measured values of the variables in both the above equations, with a view to determining the correlation ratio to be expected in each experiment, together with the probable chances of detecting it with the given apparatus.

Before proceeding with an analysis of the performance of the equipment, it is instructive to pause and ponder the relationship between the Signal to Noise Ratio and the total Observing Time, \( T_0 \). It will be noted that equation (52) tells us that, all other things being equal, the Signal to Noise Ratio will be doubled only by quadrupling the Total Observation Time. Evidently the apparatus must be very drift-free, otherwise the calculated Signal to Noise Ratio will become degraded, as in fact occurred during the Double-Slit experiment. Fortunately, in the case of the Single-Slit experiment, the counting rate was high enough to ensure a significant result within a much
Fig. 4:1 Schematic diagram of the entire apparatus for both experiments.
shorter period. A schematic diagram of the complete apparatus is shown in Fig. 4.1.

(b) The Source System

The methods of counting pulses in both experiments were essentially similar. In both experiments, trains of pulses were sampled in successive time intervals, as opposed to the Multi-Channel Analyser system of sampling the components of a single group of pulses. Thus, an implicit prerequisite for the applicability of equations (50) and (52) was a light-source having a constant intensity during the long periods necessary for this type of experiment. The performance of the Servo-System described in Chapter 3 may be judged from the graphs shown in Fig. 4.2. The general form of these graphs may be regarded as typical of any seven-hour period during the experiment, apart from the initial two hours of warming-up time during which no readings were taken. From inspection of these graphs, it will be seen that the deviation in Single-Channel counting rate over each period of seven hours is approximately:

Servo system inoperative : $\pm 21^0/o$.

Servo system operative : $\pm 3^0/o$.

The success of the servo system is self-evident, but in case it should be thought that even a variation of $\pm 3^0/o$ is too large, it should be remembered that each photomultiplier scan in the case of the long-duration Double-
Fig. 4-2: Graphs showing the variation of Single-Channel Counting Rate vs. Time with (A) Servo off, and (B) Servo on.
Slit experiment took 5 minutes. If we now compare the greatest changes in Single Channel counting rate, over this period, shown in the two graphs of Fig. 4.2, the results are:

Servo system inoperative : 8.0°/o increase.
Servo system operative : 1.6°/o decrease.

Perhaps the greatest advantage of the Servo system was the fact that the desired level of intensity was set at a given value, to which it returned even after the Magnetron had been switched off and on every week-end. Before the Servo system became operational, it was impossible to predict the counting-rate to be expected even when the Magnetron Variac was set to the same value each time.

One of the most important factors influencing the value of the correlation ratio and signal to noise ratio is the Coherence Time, \( \tau_c \), of the light emitted by the source. Indeed, it is clear from the theory derived in Chapter 2 that the two correlation experiments described owe their very existence to a value of coherence time which is capable of being measured by presently available electronic techniques. It is assumed, also, that the light possesses Gaussian random characteristics, i.e. the ensemble distribution of the complex field amplitude must be Gaussian. This at once rules out the possibility of using a Laser, for which the Coherence Time may be as much as several milliseconds, but whose photon probability density distribution is more nearly Poissonian and therefore
gives little, if any, correlation. Direct measurement of \( \tau_0 \) or rather \( c\tau_0 \), the Coherence Length, is difficult without highly specialised equipment, such as a Kösters Interferometer (Bruce, 1956). However, an approximate value of the Coherence Length may be obtained, by measuring the path difference at which interference fringes first become invisible, on a Michelson Interferometer. This measurement was made, for several different laboratory sources, using a Michelson Interferometer that had been specially modified to include large path differences. The results obtained for this measurement with the length of these sources were as follows:

- **Tungsten-filament lamp (white light)** = \( 5 \times 10^{-4} \) cms.
- **High Pressure Mercury lamp** = 8 cms.
- **Low Pressure Mercury lamp** = 14 cms.
- **Electrodeless Mercury-198 lamp** = 27 cms.

The 5461 Å spectral line was selected for the latter three cases. It should be emphasised that these are approximate measurements, but they serve to illustrate the marked superiority of the Low-Pressure Mercury Isotope lamp. Very briefly, the reason for this relatively long Coherence Length (and, therefore, very narrow spectral line) is that the Mercury isotope of atomic weight 198 possesses an even-even nucleus, of zero total angular momentum, so that the emission lines are not effectively broadened by fine-structure. Therefore, if we now assume, with some justification, that the Coherence Length, derived according to Cook (1961),

\[
c\tau_0 \geq 15 \text{ cms,}
\]

it follows that the Coherence Time
\[ \tau_0 \geq 0.5 \text{ nsec.} \quad (1 \text{ nsec.} = 10^{-9} \text{ secs.}). \]

Provided that the temperature and pressure are kept reasonably low to reduce the effects of thermal broadening and self-reversal, the Mercury Isotope lamp is evidently well suited to the photon correlation experiments. In the visible part of the spectrum, the two most prominent lines emitted by this source are at 4358 Å and 5461 Å. The latter line was chosen for the correlation experiments, for two reasons: Firstly, although the sensitivity of the photodetectors was lower at this wavelength than at 4358 Å, the counting-rate was actually greater, which would indicate that the line at 5461 Å was rather more strong. Secondly, the 5461 Å line lay much nearer the peak of the response of the human eye, thereby materially facilitating the operation of setting up and alignment. The exact wavelength of this line, in "standard air", quoted by the National Bureau of Standards, Washington, is:

\[ \lambda = 5460.75 \text{ Å} \quad (1 \text{ Å} = 10^{-8} \text{ cms.}). \]

Selection of this wavelength was performed by a Barr and Stroud all-dielectric narrow-band filter cemented to a broad-band auxiliary filter. The overall Transmission Factor of this compound filter was 72\%.

The Signal to Noise ratio, \( S/N \), is directly proportional to \( N_0 \), the number of photons emitted by the source, in a given time, within the bandwidth of the source. In other words, the source had to be as brilliant as possible.
Fig. 4.3: Scale diagram of the Source and Detector apertures used in the Single-Source experiment.
Unfortunately, the brighter the lamp became, the hotter it became due to r.f. heating of the envelope, in spite of the cooling air-draught. The heating reduced the Coherence Time because of broadening, and the $S/N$ ratio consequently fell. This effect on the Coherence Length was actually measurable using the long-path Michelson Interferometer. When the Magnetron was run at its maximum permissible rating, the Coherence Length was reduced by almost $\frac{1}{2}$ cm., compared with the length quoted above which was obtained under normal running conditions. Nevertheless, even under moderate running conditions, the brilliance of the Isotope lamp was most impressive, even after the inevitable attenuation within the optical system.

**Section (c). The Optical System**

The optical arrangement for the first experiment, the repeat of the Twiss and Little classic, was very simple indeed. The source aperture consisted of a circular pin-hole, and the photodetector apertures were both square in shape and of identical size and orientation. The dimensions of the apertures are shown in Fig. 4.3. A major factor influencing $\rho$ and $S/N$ ratio is the Partial Coherence Factor, $\Delta(v_0)$, which is a function of the size and shape of these apertures, as can be seen in Chapter 2. Since this factor is normalised, it is of course desirable to achieve a value for it as close to unity as possible. However, as previously pointed out, the
Fig. 4.4: Graph of $\Delta(v_o) \Gamma(v_o, d)$ vs. Detector Separation, d, for the Single Source-Aperture experiment.
smaller the source and detector apertures become, the
greater the coherence and the smaller the counting rate.
Hence, rather than simply \( \Delta(v_o) \), the product \( N_0 \Delta(v_o) \)
has to be optimised. Furthermore, the detector apertures
must not be so large that the photoelectron transit-times
differ appreciably. Following the procedure briefly
enunciated above, the final value of \( \Delta(v_o) \) for the
single source aperture experiment became 0.406, which is
the peak value of the correlation curve shown in Fig. 4.4.

With regard to the double source-slit experiment, the
same arguments apply, with the additional constraints that
the width of the source slits must be less than their
separation and the width of the detector slits must not be
greater than half the width of the central maximum of the
correlation pattern. The shape and dimensions of the
source and detector apertures are shown in Fig. 4.5. The
computed value of \( \Delta(v_o) \) in this case was 0.715, and the
entire computed correlation pattern is shown in Fig. 4.6.

For both experiments, the value of the Normalised
Correlation Factor, \( \gamma^2(v_o, d) \), was of course a function of
the detector separation, \( d \), being unity for \( d = 0 \). In
the case of the single source-slit experiment, the require-
ment was simply to detect the difference in correlation
when the detector apertures were exactly superposed
(\( \gamma^2 = 1 \)), and when they were separated by some distance,
\( d(\gamma^2 \ll 1) \). For the double source-slit experiment, the
Normalised Correlation Factor was an oscillatory function
Fig. 4-5: Scale diagram of the Source and Detector Apertures used in the Double Source experiment.
with increasing detector separation. The aim in this case was to compare the difference in correlation between the central maximum and first minimum with the difference in correlation between the next maximum and the first minimum. Inspection of the graph in Fig. 4.6 will show that, in terms of percentages, the computed values of \( \Gamma^2(v_0,d) \) at the central maximum, first minimum, and second maximum were 100\(^0\)/o, 8\(^0\)/o and 32\(^0\)/o respectively. Thus, the expected ratios of the Correlation Factor at these three positions were 12.5/1, 1/1, and 4/1 respectively.

The factor \( f_1 \), in equations (50) and (52), is a reduction factor representing the loss of correlation due to the polarisation effect of the beam-splitter. For the duration of both experiments a sheet of "Polaroid" polarising material was inserted between the discharge tube and the source aperture(s), orientated in such a way as to compensate exactly for the polarisation due to the beam-splitter. The factor \( f_1 \) therefore became unity, although the overall intensity fell due to the elimination of the orthogonally polarised components in each beam which are uncorrelated in any case. With the Polariser set at the correct angle, the transmission factor was measured to be 38\(^0\)/o.

The overall \( S/N \) ratio also depended on the spectral sensitivity of the photodetectors at the relevant wavelength. The spectral response curve of the 56 AVP photomultiplier, as published by the manufacturers, shows that
Fig. 4-6: Graph of $\Delta (v_0) \Gamma^2 (v_0, d)$ vs. Detector Separation, $d$, for the Double Source-Aperture experiment.
the relative spectral energy at 5461 A was greater than 50% of peak value, while the Quantum Efficiency, α, lay between 10% and 15%.

The other optical components, the beam-splitter and pentagonal prisms, were of the highest optical quality, and the overall light loss due to interfacial reflections was not greater than 20%.

In the early stages of development of the Sillitto and Haig experiment, it was envisaged that the envelope of the correlation maxima would be very wide, so that readings of counting rate would be taken at several correlation maxima and minima. When it became clear that this undertaking would require an inordinate amount of time to accumulate sufficient data, it was decided to reduce the number of correlation maxima and take readings at only three detector positions, this being the minimum necessary to check the expected periodic correlation. Therefore, the width of the envelope was reduced to include only three correlation maxima. A considerable advantage of this arrangement was that almost all of the light was concentrated in these three maxima, instead of being dispersed throughout a much larger number, which improved the counting rate to a more practical level.

The theoretical value of the separation of the correlation maxima was checked experimentally by placing a single slit between the source and the double slits, rendering the correlation pattern visible as ordinary fringes. The separation of the maxima was found to lie
within 1% of the predicted value, and the shape of the visibility envelope appeared to conform reasonably well to the predicted shape of the correlation envelope, although this depended largely on the width of the primary slit.

Section (d). The Electronic Apparatus

Both of the experiments described above required the Coincidence Resolving Time, $\tau_r$, to be of the same order as the Coherence Time, $\tau_o$. It can be shown that there is little advantage in making the ratio $\tau_o/\tau_r$ greater than unity, and indeed the theory derived in Chapter 2 is relevant only in the case of this ratio being less than unity. However, if this ratio becomes too small, $\rho$ and $S/N$ become insignificant, and Hanbury Brown and Twiss\(^{(72)}\) (1956) were able to prove that this very fault had been responsible for the failure of several earlier experiments. The Franzini coincidence circuit employed an emitter-follower output stage whose threshold was varied by means of a bias potentiometer. As is often the case, it was discovered that alteration of the setting of this control affected the resolving time of the circuit, as well as the coincidence counting rate. Specifically, as the transistor base-bias was increased, the threshold of triggering was raised, cutting off the smaller coincidence pulses which allowed the larger pulses to trigger the transistor at their narrowest point, i.e. near their peak value. Thus the resolving time became shorter when fewer coincidence pulses were emitted. This, therefore, was yet another case of
Fig. 4.7: Cable delay curve for the Franzini Coincidence Unit.
having to reach an acceptable compromise. In the interests of higher stability, it was decided to run the unit at a higher coincidence count rate and longer resolving-time. Thus, although the circuit was capable of a resolving-time of better than 1 nsec., it was actually used at:

$$2\tau_r = 2.8 \text{ nsec.}$$  (full width at half height).

The cable-delay curve from which this value was obtained is shown in Fig. 4.7. Although the circuit was moderately temperature-sensitive, it was far worse at the shortest resolving-time, and the stability at the bias setting actually used was such that the delay curve illustrated was highly repeatable, with negligible error.

The method used to measure the resolving-time was to divide the output from one photomultiplier, feeding one cable into one input channel of the Franzini circuit, while changing the relative length of the other input cable, thereby altering the relative delay of the identical pulse trains into each input. The pulses from the photomultiplier were clipped in the usual way. This method gave rise to mismatching, but the ringing produced was below the threshold of the Tunnel Diodes and gave no trouble.

Probably the major objection to this method could be said to have been the use of only the one photomultiplier, as the other could give pulses of a different shape and height which would lead to some time-jitter. In fact, as viewed on a Tektronix 581 oscilloscope, the pulses from both photomultipliers were indistinguishable, and this,
coupled with the high discrimination threshold of the Tunnel Diodes, leads us to believe that the time-jitter may be assumed to be negligibly small compared with the time-resolution of the circuit. No better way of measuring the coincidence-resolving-time was found, since the results achieved with a fast-risetime signal-generator gave a good but unrealistically rectangular result, and the results achieved with a radioactive source and plastic scintillator served only to determine the poor properties of the scintillator. Certainly, the significant result for the single source-slit experiment suggests that the quoted coincidence resolving-time cannot be much in error.

It is difficult to quote precise figures for the Signal to Noise ratio of the photomultipliers as this varied with the light intensity among other things, but under normal illumination conditions we may quote an approximate figure by defining the Signal to Noise ratio as:

\[
\frac{S}{N} = \frac{(\text{Illuminated count rate}) - (\text{Dark count rate})}{(\text{Dark count rate})}
\]

Then, for photomultiplier Number 17140, \( \frac{S}{N} = 2.2/1 \), and for photomultiplier Number 14605, \( \frac{S}{N} = 3.1/1 \).

The E.H.T. supply to each was maintained at 2.60 kV at all times. The following characteristics for each 56AVP were quoted by Mullard, the manufacturers:
<table>
<thead>
<tr>
<th>56AVP Number</th>
<th>17140</th>
<th>14605</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photocathode Sensitivity</td>
<td>58 µA/lm</td>
<td>87 µA/lm</td>
</tr>
<tr>
<td>Gain at 2180 volts</td>
<td>$10^8$</td>
<td>$10^8$</td>
</tr>
<tr>
<td>Dark Current at 2180 volts</td>
<td>1.2 µA</td>
<td>1.5 µA</td>
</tr>
</tbody>
</table>

Both of these photomultipliers were carefully selected for their low Dark Current and high Sensitivity to give a better than average $S/N$ ratio under the given conditions. It is almost impossible to find two tubes of precisely similar performance, but these two came much closer to the ideal than several others that were tested. In addition, the dynode chains were found to have virtually identical performances, so that the matching of the 56 AVP's was unimpaired under operational conditions. Photographs of typical output pulses from the 56 AVP's are shown in Fig. 3.19 of Chapter 3. These photographs were taken using a Polaroid-Land camera mounted on a Tektronix 581 oscilloscope, and it will be appreciated that the faintness of the trace is indicative of the difficulty attendant on the photography of such fast pulses with the oscilloscope working at its limit.

It was described in Chapter 3 how the thermal stability of the equipment was excellent, with the exception of the Franzini coincidence circuit. The effectiveness of the polystyrene foam insulation in stabilising the temperature of this circuit may be judged from the
Fig 4.8: Graphs of (A) Coincidence Unit Temperature, and (B), the corresponding Room Temperature over a period of 6 hours, illustrating the effectiveness of the foam insulation.
graphs in Fig. 4.8.

The factor, \( f_2 \), appearing in the equations for \( \rho \) and \( S/N \), represent the loss of correlation arising within the electronic equipment. Twiss and Little\(^{38}\) defined this factor as being the fraction of random counts due to coincidences between pulses in one photomultiplier, and dark current or stray light pulses in the other. This may be stated in experimentally measurable terms thus:

\[
f_2 = \frac{\bar{R}_{12} - \bar{R}_{10}}{\bar{R}_{12}}
\]

where \( \bar{R}_{12} = \) Mean random illuminated coincidence count rate

\( \bar{R}_{10} = \) Mean random illuminated coincidence count rate with one photomultiplier covered.

That the coincidences are random, in this case, is ensured by the insertion of a large (15 nsec.) delay in the output cable from one of the photomultipliers. Averaging the results for each photomultiplier in turn, the result was:

\[
f_2 = 0.814 .
\]

The controls of the 1430 Pulse Amplifier and Cathode Follower were set to enable a wide spectrum of pulses to be accepted from the Franzini circuit, with a short integration time-constant of 0.08 \( \mu \)sec. and a long differentiation time-constant of 250 \( \mu \)sec. The attenuation controls were set at zero to provide the maximum amplification, since the input impedance of the Cathode Follower was lowered to 100 ohms for matching purposes.
The usual counting-rate corrections had to be applied to the Scalers when measuring fast single-channel counts, but were unnecessary at the normal coincidence counting rates which were very low. The counting rate corrections quoted by the manufacturers, Isotope Developments Ltd., were:

<table>
<thead>
<tr>
<th>Pulses recorded by Pre-Scaler, per sec.</th>
<th>% loss in Pre-Scaler</th>
<th>% loss in Scaler</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>$10^4$</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>$10^5$</td>
<td>9.0</td>
<td>0.1</td>
</tr>
<tr>
<td>$2.5 \times 10^5$</td>
<td>23.0</td>
<td>0.1</td>
</tr>
<tr>
<td>$3.0 \times 10^5$</td>
<td>30.0</td>
<td>2.3</td>
</tr>
</tbody>
</table>

In conclusion, therefore, we are now in a position to calculate predicted values of $\rho$ and $S/N$ for each experiment by the insertion of the relevant variables in equations (50) and (52). In the case of the Single Source Aperture (Twiss and Little-type) experiment the total photodetector time at each position was 21,000 secs.

Thus the theoretical Correlation Ratio was:

$$\rho = 0.031$$

and the theoretical Signal to Noise Ratio was:

$$S/N = 3.28.$$  

In the case of the Double Source Aperture (Sillitto and Haig) experiment, the requirement was a little different, as pointed out above. The object was to determine
the difference in Correlation Ratio between the central maximum and the first minimum, and to compare this with the difference in Correlation Ratio between the next maximum and the first minimum. We will therefore quote the Correlation Ratio at the central maximum, first minimum and next maximum. Insertion of the relevant variables, including a total detector time at each position of 120,000 seconds, gives Correlation Ratios of:

At the central maximum, \( \rho_1 = 0.050 \)
At the first minimum, \( \rho_2 = 0.004 \)
At the next maximum, \( \rho_3 = 0.016 \)

Now the experimental procedure is such that \( \rho_2 \) is assumed to be zero, and \( \rho_1 \) and \( \rho_3 \) are calculated with respect to this value. Thus, bearing in mind this experimental procedure, the expected Correlation Ratios become:

At the central maximum, \( \rho_1 = 0.046 \)
At the first minimum, \( \rho_2 = 0.000 \)
At the next maximum, \( \rho_3 = 0.012 \)

The theoretical Signal to Noise Ratios for the three detector positions were:

At the central maximum \( (\text{S}/\text{N})_1 = 1.29 \)
At the first minimum \( (\text{S}/\text{N})_2 = 0.00 \)
At the next maximum \( (\text{S}/\text{N})_3 = 0.57 \)

It was originally intended that the experiment should continue until the value of \( (\text{S}/\text{N})_3 \) became greater than unity, which would have been when \( T_0 = 369,000 \) seconds. However,
for reasons to be discussed in the next chapter, it became evident that no useful purpose would be served by prolonging it beyond \( T_0 = 120,000 \) seconds.

Theoretically, therefore, although both experiments were practical propositions, certain extraneous effects rendered the Double-Source experiment only partially practicable. As may be seen above, the \( S/N \) ratio at the central maximum was the only one to exceed unity. The value of \( (S/N)_2 \) was zero by definition, and the value of \( (S/N)_3 \) was theoretically insufficient to demonstrate conclusively the periodic correlation effect.
Section (a). Measuring Procedure

The procedure adopted for both experiments was, essentially, the comparison of the number of coincidence counts registered when both photodetectors were positioned at theoretical correlation maxima with the number registered when one photodetector was positioned at a theoretical correlation minimum.

Taking the simpler case of the Single Source-Slit experiment first, it can be seen from Fig. 4.4 of Chapter 4 that the correlation is a maximum with the detectors optically superposed, decreasing rapidly as they are separated. The separation, d, chosen for the experiment was 0.4 cms., so that the ratio of detector separation to detector aperture was 1.33, and the ratio of $\gamma_{\text{superposed}}^2$ to $\gamma_{\text{separated}}^2$ was more than 20. The exact procedure for taking readings was as follows: The detector slits were superposed for a period of 5 minutes, during which time, $n_1$, coincidence counts were recorded. The moveable detector was then displaced laterally through 0.4 cms. by means of the micrometer screw, and the number, $n_2$, of coincidence counts registered in a further 5 minute interval was recorded. Now, the intensities of illumination at the two detector positions were not necessarily identical, and so it was necessary to repeat the above
procedure, using a long delay cable in one detector output line. The value chosen for the delay was 15 nsec., which was more than 5 times the resolving-time of the coincidence circuit, ensuring that the coincidences recorded were perfectly random. Thus the decorrelated counts, \( n_1' \) and \( n_2' \), recorded in 5 minute intervals at the superposed and unsuperposed position respectively, constituted an accurate comparison of the intensities at the two points.

This procedure was repeated to give a total of 70 runs, each run consisting of the four measurements, namely:

- Superposed
- Displaced
- Prompt coincidences
- Delayed coincidences

The actual running-time of this experiment was therefore 1400 minutes, excluding the time taken to change delay-cables and move the micrometer screw. Due to other commitments, this length of time was not available continuously, and readings therefore had to be taken in seven separate sessions over a period of a week. The Magnetron was switched off between sessions, but was always allowed at least two hours to warm up before any readings were taken.

With regard to the Double Source-Slit (Sillitto and Haig) experiment, the procedure was very similar, except that readings were taken at three positions of the moveable photodetector, and the counting time at each position was reduced to 100 seconds. It was explained in the previous
chapter that the three readings were to be taken at the central maximum, first minimum and second maximum of the correlation pattern. The reason for the reduction in counting time was to reduce the effect of drifts in the electronics over the period of each run of three readings. Since the effect to be measured was known to be very small and the Franzini coincidence circuit was prone to thermal drift, the additional precaution was taken of noting the temperature of the Franzini unit at the start of each run. In order to save some time, the changing of delay cables was confined to the beginning and end of each group of ten runs, so that ten sets of Prompt coincidence readings were followed by ten sets of Delayed coincidence readings and so on. In this manner, a total of 2400 runs, both Prompt and Delayed, were completed in 48 working days.

Section (b). Readings and Result Obtained for the Single Source-Slit Experiment

The settings of the electronic and optical equipment are given in the previous chapter. Counts were taken with the moveable photodetector at each position for 5 minutes. For ease of inspection, the runs are grouped into tens, so that each figure in Table I gives the total number of coincidence counts registered over a period of 50 minutes. In the "Superposed" position, the photodetectors were in exact optical register, and in the "Separated" position, their centres were 0.40 cms. apart.
Table I

<table>
<thead>
<tr>
<th>Run Numbers</th>
<th>Prompt Coincidences</th>
<th>Delayed Coincidences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\text{Superposed} (n_1))</td>
<td>(\text{Separated} (n_2))</td>
</tr>
<tr>
<td>1-10</td>
<td>2711</td>
<td>2660</td>
</tr>
<tr>
<td>11-20</td>
<td>2082</td>
<td>2132</td>
</tr>
<tr>
<td>21-30</td>
<td>1558</td>
<td>1498</td>
</tr>
<tr>
<td>31-40</td>
<td>2550</td>
<td>2500</td>
</tr>
<tr>
<td>41-50</td>
<td>2410</td>
<td>2399</td>
</tr>
<tr>
<td>51-60</td>
<td>3076</td>
<td>2928</td>
</tr>
<tr>
<td>61-70</td>
<td>3036</td>
<td>2938</td>
</tr>
<tr>
<td>Total Counts for 350 mins.</td>
<td>17423</td>
<td>17055</td>
</tr>
</tbody>
</table>

We now define the total counts thus:

\[
N_1 = \sum_{r=1}^{70} n_{1r}, \quad N_2 = \sum_{r=1}^{70} n_{2r},
\]

and similarly,

\[
N'_1 = \sum_{r=1}^{70} n'_{1r}, \quad N'_2 = \sum_{r=1}^{70} n'_{2r}.
\]

Hence the ratio \(N'_2/N'_1\) will give the true ratio of light fluxes at the "Separated" and "Superposed" positions. This ratio is the correction factor which has to be applied to the "Prompt" coincidences in order to calculate the true ratio of \(N_1/N_2\). We are now able to define the
ratio of correlated to random coincidences thus:

\[ \rho = \frac{N_1 \left( \frac{N_2'}{N_1} \right) - N_2}{N_2} \]

and rearranging:

\[ \rho = \frac{N_1}{N_2} \cdot \frac{N_2'}{N_1} - 1 \]  \hspace{1cm} (a)

Substituting the relevant values from Table I into equation (a), we find that the experimental ratio of correlated to random coincidences becomes:

\[ \rho = 0.0523 \]

Now, this experiment is concerned with the counting of photoelectrons, which obey Poissonian statistics in the absence of correlations. The correlation effect is a relatively small one, and therefore the counts in Table I may be assumed to depart very little from a Poissonian distribution. In fact, this is often tacitly assumed, but in this case it was checked that the distribution of counts was effectively Poissonian by calculating the second and third factorial moments of the expectation values of each count in the first column of readings. It is well known that the Standard Deviation of a series of readings having a Poissonian Distribution is equal to the square-root of the mean value. In addition, since equation (a) is a linear combination of the column totals, we may also say that the mean square deviation in the correlation ratio is equal to
the sum of the squares of the fractional uncertainties in each total, thus:

\[(\frac{\partial \rho}{\rho+1})^2 = (\frac{\partial N_1}{N_1})^2 + (\frac{\partial N_2}{N_2})^2 + (\frac{\partial N_1'}{N_1'})^2 + (\frac{\partial N_2'}{N_2'})^2\]

and then:

\[\sigma^2(\rho) = (\frac{1}{N_1'^2})^2 + (\frac{1}{N_2'^2})^2 + (\frac{1}{N_1'^2})^2 + (\frac{1}{N_2'^2})^2 \quad \text{(b)}\]

On substituting the experimental values into equation (b), we find that:

\[\sigma(\rho) = (\frac{\partial \rho}{\rho+1}) = 1.54 \times 10^{-2}\]

Hence, the final result for the Correlation Ratio of the Single Source–Slit experiment may be written as:

\[\rho = 0.052 \pm 0.015\]

It is thus possible to state that the correlation effect was definitely detected, and that the magnitude of the correlation lay between 6.7°/o and 3.7°/o. This result compares reasonably with the theoretical result of 3°/o (see Chapter 4). The conclusions to be drawn from the experimental result and its comparison with the theoretical result will be discussed later.
Section (c). Readings and Results Obtained for the Double Source-Slit Experiment

The settings and experimental arrangement for this experiment are detailed in the previous chapter. As stated above, the total counts registered in an interval of 100 secs. were recorded with the moveable photodetector in three successive positions to give the number of "Prompt" coincidences. The procedure was repeated with the 15 nsec. delay cable in one photodetector output to give a measure of the random or "Delayed" coincidences. The experiment was terminated when the total number of readings recorded at each position, either "Prompt" or "Delayed", was 1200. This was done because it was found that, contrary to theoretical expectations, the $S/N$ ratio was not increasing with time. The implications of this discovery are considered in the Discussion section following this chapter. It was known that the experiment would take some considerable time, and the possibility of drifts in the electronic equipment had to be countered. Since the coincidence counting rate was known to be affected by temperature variations, the temperature of the Franzini Coincidence Unit was noted, to the nearest 0.1°C, before each scan of the three photodetector positions. It was also known that, whatever the temperature of the Franzini Unit, the coincidence counting rate was steady provided that the temperature was stable. On completion of the experiment therefore, the most meaningful method of extracting the results was
seen to be the rejection of scans during which a relatively large change in temperature occurred. The exact criteria adopted were as follows:

(a) The acceptance of successive scans during which the temperature change was either nil or 0.1°C, provided that the scan with a change of 0.1°C was not succeeded by one with a further change of 0.1°C or more.

(b) The rejection of all scans where it appeared that the thermal variations were random or rapidly cyclic, until a stable temperature was reached and maintained.

Since the readings are so numerous that their reproduction in full would add prohibitively to the bulk of this thesis, they will be grouped into sets of 20 scans, of which merely the totals will be quoted. Unfortunately, this means that the temperatures for the individual scans may not be usefully quoted. Therefore, in order to demonstrate the application of the above two criteria, a set of temperatures taken during 15 actual scans is given below, together with the reasons for acceptance or rejection of those scans.

The actual extremes of temperature registered by the Coincidence Unit thermometer were 28.2°C and 22.9°C over the entire 48-day duration of the experiment.

Following the above procedure for the rejection of thermally unsuitable scans, 1033 "Prompt" scans and 1020 "Delayed" scans were deemed acceptable, out of a total of 1200 original scans for each. To simplify presentation,
### Reasons for acceptance or rejection

<table>
<thead>
<tr>
<th>Scan</th>
<th>Temperature, °C</th>
<th>Acceptance</th>
<th>Reasons for acceptance or rejection</th>
</tr>
</thead>
<tbody>
<tr>
<td>311</td>
<td>23.8</td>
<td>No</td>
<td>Large temperature change, unstable state.</td>
</tr>
<tr>
<td>312</td>
<td>24.0</td>
<td>No</td>
<td>Unstable state.</td>
</tr>
<tr>
<td>313</td>
<td>24.0</td>
<td>Yes</td>
<td>Steady state conditions.</td>
</tr>
<tr>
<td>314</td>
<td>24.0</td>
<td>Yes</td>
<td>Steady state conditions.</td>
</tr>
<tr>
<td>315</td>
<td>24.0</td>
<td>Yes</td>
<td>Steady state conditions.</td>
</tr>
<tr>
<td>412</td>
<td>25.0</td>
<td>Yes</td>
<td>Smooth temperature change in steady state.</td>
</tr>
<tr>
<td>413</td>
<td>25.0</td>
<td>Yes</td>
<td>Smooth temperature change in steady state.</td>
</tr>
<tr>
<td>414</td>
<td>25.0</td>
<td>Yes</td>
<td>Smooth temperature change in steady state.</td>
</tr>
<tr>
<td>415</td>
<td>25.1</td>
<td>Yes</td>
<td>Smooth temperature change in steady state.</td>
</tr>
<tr>
<td>416</td>
<td>25.1</td>
<td>Yes</td>
<td>Smooth temperature change in steady state.</td>
</tr>
<tr>
<td>417</td>
<td>25.1</td>
<td>Yes</td>
<td>Smooth temperature change in steady state.</td>
</tr>
<tr>
<td>1077</td>
<td>23.6</td>
<td>No</td>
<td>Small cyclic variation, unstable state.</td>
</tr>
<tr>
<td>1078</td>
<td>23.7</td>
<td>No</td>
<td>Small cyclic variation, unstable state.</td>
</tr>
<tr>
<td>1079</td>
<td>23.5</td>
<td>No</td>
<td>Small cyclic variation, unstable state.</td>
</tr>
<tr>
<td>1080</td>
<td>23.6</td>
<td>No</td>
<td>Small cyclic variation, unstable state.</td>
</tr>
</tbody>
</table>

A further 13 "Prompt" scans were rejected, having been borderline cases, making a final acceptance total of 1020 scans for both "Prompt" and "Delayed" configurations. The overall utilisation of readings was therefore 85%. The totals of coincidence counts in each block of 20 scans is shown in Table II, together with the final totals. The counting time for each block is 20 x 100 secs. = 2000 secs., so that the total counting time for each column of 51 blocks is 102000 secs.
# Table II

Coincidence Counts Registered in the Course of the Sillitto & Haig Experiment

<table>
<thead>
<tr>
<th>Photomultiplier Separation, d.</th>
<th>Prompt Coincidences</th>
<th>Delayed Coincidences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((n_1))</td>
<td>((n_2))</td>
</tr>
<tr>
<td></td>
<td>0.00 mm.</td>
<td>0.85 mm.</td>
</tr>
<tr>
<td>Block 1</td>
<td>347</td>
<td>315</td>
</tr>
<tr>
<td>Block 2</td>
<td>354</td>
<td>318</td>
</tr>
<tr>
<td>Block 3</td>
<td>324</td>
<td>343</td>
</tr>
<tr>
<td>Block 4</td>
<td>364</td>
<td>331</td>
</tr>
<tr>
<td>Block 5</td>
<td>337</td>
<td>300</td>
</tr>
<tr>
<td>Block 6</td>
<td>331</td>
<td>320</td>
</tr>
<tr>
<td>Block 7</td>
<td>323</td>
<td>263</td>
</tr>
<tr>
<td>Block 8</td>
<td>324</td>
<td>321</td>
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<tr>
<td>Block 9</td>
<td>369</td>
<td>374</td>
</tr>
<tr>
<td>Block 10</td>
<td>436</td>
<td>411</td>
</tr>
<tr>
<td>Block 11</td>
<td>468</td>
<td>410</td>
</tr>
<tr>
<td>Block 12</td>
<td>396</td>
<td>413</td>
</tr>
<tr>
<td>Block 13</td>
<td>385</td>
<td>329</td>
</tr>
<tr>
<td>Block 14</td>
<td>386</td>
<td>387</td>
</tr>
<tr>
<td>Block 15</td>
<td>459</td>
<td>436</td>
</tr>
<tr>
<td>Block 16</td>
<td>441</td>
<td>411</td>
</tr>
<tr>
<td>Block 17</td>
<td>354</td>
<td>390</td>
</tr>
<tr>
<td>Block 18</td>
<td>391</td>
<td>367</td>
</tr>
<tr>
<td>Block 19</td>
<td>438</td>
<td>407</td>
</tr>
<tr>
<td>Block 20</td>
<td>395</td>
<td>355</td>
</tr>
<tr>
<td>Block 21</td>
<td>273</td>
<td>248</td>
</tr>
<tr>
<td>Block 22</td>
<td>312</td>
<td>298</td>
</tr>
<tr>
<td>Block 23</td>
<td>296</td>
<td>316</td>
</tr>
<tr>
<td>Photomultiplier separation, d.</td>
<td>Prompt Coincidence( n_1 )</td>
<td>( n_2 )</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-----------------</td>
<td>--------</td>
</tr>
<tr>
<td>Block 24</td>
<td>361</td>
<td>356</td>
</tr>
<tr>
<td>25</td>
<td>414</td>
<td>409</td>
</tr>
<tr>
<td>26</td>
<td>406</td>
<td>385</td>
</tr>
<tr>
<td>27</td>
<td>608</td>
<td>564</td>
</tr>
<tr>
<td>28</td>
<td>628</td>
<td>569</td>
</tr>
<tr>
<td>29</td>
<td>566</td>
<td>531</td>
</tr>
<tr>
<td>30</td>
<td>478</td>
<td>446</td>
</tr>
<tr>
<td>31</td>
<td>399</td>
<td>335</td>
</tr>
<tr>
<td>32</td>
<td>394</td>
<td>407</td>
</tr>
<tr>
<td>33</td>
<td>441</td>
<td>409</td>
</tr>
<tr>
<td>34</td>
<td>395</td>
<td>375</td>
</tr>
<tr>
<td>35</td>
<td>415</td>
<td>396</td>
</tr>
<tr>
<td>36</td>
<td>391</td>
<td>330</td>
</tr>
<tr>
<td>37</td>
<td>406</td>
<td>368</td>
</tr>
<tr>
<td>38</td>
<td>389</td>
<td>434</td>
</tr>
<tr>
<td>39</td>
<td>492</td>
<td>464</td>
</tr>
<tr>
<td>40</td>
<td>462</td>
<td>455</td>
</tr>
<tr>
<td>41</td>
<td>498</td>
<td>474</td>
</tr>
<tr>
<td>42</td>
<td>523</td>
<td>474</td>
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<tr>
<td>43</td>
<td>647</td>
<td>645</td>
</tr>
<tr>
<td>44</td>
<td>473</td>
<td>423</td>
</tr>
<tr>
<td>45</td>
<td>451</td>
<td>404</td>
</tr>
<tr>
<td>46</td>
<td>488</td>
<td>453</td>
</tr>
<tr>
<td>47</td>
<td>506</td>
<td>487</td>
</tr>
<tr>
<td>48</td>
<td>418</td>
<td>387</td>
</tr>
<tr>
<td>49</td>
<td>457</td>
<td>476</td>
</tr>
<tr>
<td>50</td>
<td>514</td>
<td>502</td>
</tr>
<tr>
<td>51</td>
<td>412</td>
<td>376</td>
</tr>
</tbody>
</table>

Total, \( \Sigma n = \) | 21535 | 20397 | 20652 | 18902 | 18188 | 18248 |

\( \sigma = \sqrt{\Sigma n} = \) | 146.7 | 142.8 | 143.7 | 137.6 | 134.8 | 135.0 |
Following the convention employed in the previous experiment, we define \( n_1, n_2, \) and \( n_3 \) as being the number of coincidence counts registered, per block, in the "Prompt" configuration with photodetector centres separated by: \( d_1 = 0.00 \text{ m.m.}, \ d_2 = 0.85 \text{ m.m.}, \) and \( d_3 = 1.70 \text{ m.m.} \) respectively. Similarly, we define \( n_1', n_2', \) and \( n_3' \) as being the number of coincidence counts registered, per block, in the "Delayed" configuration at the same respective values of \( d \). We may then write:

\[
\begin{align*}
N_1 & = \sum_{r=1}^{51} n_{1r}, \quad N_2 = \sum_{r=1}^{51} n_{2r}, \quad N_3 = \sum_{r=1}^{51} n_{3r} \\
N_1' & = \sum_{r=1}^{51} n'_{1r}, \quad N_2' = \sum_{r=1}^{51} n'_{2r}, \quad N_3' = \sum_{r=1}^{51} n'_{3r}.
\end{align*}
\]

Since the theoretical calculations showed that the ratio of correlated coincidence counts to random coincidence counts should be a minimum at a photodetector separation \( d_2 = 0.85 \text{ m.m.} \), (see Chapter 4), all three correlation ratios are calculated with respect to the total counts, \( N_2 \) and \( N_2' \), registered at this position. Remembering equation (a) of the previous section, we may write the correlation ratio at each photodetector position thus:

At position \( d_1 \) : \( \rho_1 = \frac{N_1}{N_2} \cdot \frac{N_2'}{N_1'} - 1 \)

At position \( d_2 \) : \( \rho_2 = \frac{N_2}{N_2} \cdot \frac{N_2'}{N_2'} - 1 \)

At position \( d_3 \) : \( \rho_3 = \frac{N_3}{N_2} \cdot \frac{N_2'}{N_3'} - 1 \)
It will be noted that, on this basis, \( \rho_2 = 0 \) by definition. On substitution of the totals from Table II, we find that:

\[
\begin{align*}
\rho_1 &= 0.0159 \\
\rho_2 &= 0.0000 \\
\rho_3 &= 0.0092
\end{align*}
\]

The method of estimating the errors attendant on the above correlation ratios is the same as for the previous experiment. Since both experiments employ virtually the same apparatus, performing similar functions in each case, we may justifiably assume that the distribution of the 1020 coincidence counts in each column of Table II is Poissonian. We are then able to state categorically that if we are to take the Standard Deviation, \( \sigma \), as the measure of error, then the error in each total, \( N_i \), will simply be equal to \( \sqrt{N_i} \), where \( i = 1, 2, \) or 3. Hence following the same reasoning as before, we may write for the mean square deviation in \( \rho_1 \):

\[
\frac{(\rho_1)^2}{(\rho_1 + 1)^2} = \left( \frac{\sigma N_1}{N_1} \right)^2 + \left( \frac{\sigma N_2}{N_2} \right)^2 + \left( \frac{\sigma N_1'}{N_1} \right)^2 + \left( \frac{\sigma N_2'}{N_2} \right)^2
\]

\[
= \sigma^2 = \left( \frac{1}{N_1} \right)^2 + \left( \frac{1}{N_2} \right)^2 + \left( \frac{1}{N_1'} \right)^2 + \left( \frac{1}{N_2'} \right)^2
\]

and hence \( \sigma \).

On substitution of the respective values of \( N_i \) into the equation for \( \sigma \), and rounding off to three decimal places, we obtain for the final result that the ratios of correlated to random coincidence counts at the photon-detector separations \( d_1 \), \( d_2 \), and \( d_3 \) are:
\[
\begin{align*}
\rho_1 &= 0.016 \pm 0.014 \\
\rho_2 &= 0.0000 \pm 0.000 \\
\rho_3 &= 0.009 \pm 0.014
\end{align*}
\]

It is therefore only possible to assert that a definite correlation was detected when the photocathodes were superposed, the magnitude of which lay between 3% and 0.2%.

The magnitude of \( \rho_2 \) was bound to be zero by definition, and the magnitude of \( \rho_3 \) does not exceed 0.6\( \sigma \). The theoretically calculated values of the Correlation Ratios (see Chapter 4) are:

\[
\begin{align*}
\rho_1 \text{ theor} &= 0.046 \\
\rho_2 \text{ theor} &= 0.000 \\
\rho_3 \text{ theor} &= 0.012
\end{align*}
\]

It is also pertinent to quote the theoretical Signal to Noise Ratios given in Chapter 4. Calculated on the basis of \( T_o = 120000 \) seconds, they were:

\[
\begin{align*}
(S/N)_1 &= 1.29 \\
(S/N)_2 &= 0.00 \\
(S/N)_3 &= 0.57
\end{align*}
\]

However, as pointed out above, the final experimental Correlation Ratios were calculated from the "smooth" data, for which \( T_o = 102000 \) seconds. If we now recalculate the theoretical Signal to Noise Ratios on this basis, we find that:
\[
\begin{align*}
(S/N)_1 &= 1.20 \\
(S/N)_2 &= 0.00 \\
(S/N)_3 &= 0.53 \\
T_0 &= 102 \text{ ksec.}
\end{align*}
\]

It will be appreciated that these figures show that there is only a small likelihood of detecting the smaller correlation peak, but there is still a good chance that the central peak will be detected. As we have seen, this was borne out by the experimental results. It is encouraging to note that, while \( \rho_2 \) is zero by definition, both the theoretical and experimental values of the correlation ratio agree insofar as \( \rho_1 \) has the largest modulus, and both \( \rho_1 \) and \( \rho_3 \) are positive quantities. It was most unfortunate that the experiment had to be terminated before a conclusive result was achieved, but it was becoming apparent that the effect of thermal variations on the Coincidence Counting-Rate was beginning to mask the very small correlation effect.

It could well be argued that in the case of the "smooth" data, the experiment agreed with the theory, as far as it went, if we regard the experimental Correlation Ratios as the Signals, and the experimental errors as the Noise. Calculating the experimental Signal to Noise Ratios on this basis, we find that:

\[
\begin{align*}
(S/N)_1 &= \rho_1/\sigma_1 = 1.14 \\
(S/N)_2 &= \rho_2/\sigma_2 = 0.00 \\
(S/N)_3 &= \rho_3/\sigma_3 = 0.64
\end{align*}
\]
Comparison of these figures with the theoretical figures for the "smooth" data shows a marked similarity. A fuller discussion of these results and the conclusions drawn from them is contained in the following section.
CONCLUSIONS AND DISCUSSION OF THE RESULTS

The result of the Single Source-Slit experiment was a confirmation of the theory, as was only to be expected following the positive results of other workers (34, 36, 37, 38). As was pointed out in a previous chapter, the experiment was undertaken in order to provide confirmation that the apparatus was functioning properly, and this was undoubted-ly successful. Indeed, the experiment described in this text was only one of several similar experiments which were successfully concluded in the course of optimising the equipment. It happened that the settings for the particular experiment chosen for discussion here were very close to the settings finally chosen for the Double Source-Slit experiment, and its inclusion in this thesis reflects its correspondingly greater relevance to this latter experiment.

It will be remembered that the magnitude of the experimental correlation was found to lie between 6.7% and 3.7%, whereas the theoretical value was 3%. It is relatively easy to argue that the experimental correlation was positive and definite, since the mean value of the correlation ratio lies more than 3.4σ (i.e. 99.97% certainty) above zero. It is far less easy to attempt to justify the accuracy of the predicted value of 3%, since this value was calculated from figures whose accuracy could not be vouched for. The value of calculating the theoretical correlation
ratio lay in estimating the probable result, not in attempting to predict precisely what this would be. There is bound to be some uncertainty in the figures inserted into the theoretical equation. For example, the value of the Partial Coherence Factor, $\Delta(v_o)$, is strongly dependent on accurate measurement of the source and detector apertures, and the accurate measurement of the Coherence Time, $\tau_0$, of the light is very difficult without highly specialised equipment, such as the Kösters Interferometer\(^{(71)}\). In fact, direct visual measurement of $\tau_0$ also showed its variability from day to day, and it was explained in Chapter 4 just how the measured value of the Resolving Time, $\tau_r$, of the coincidence circuit could be in error. In short, then, the theoretical value of the correlation ratio should be regarded as an order of magnitude estimation rather than an accurate prediction, and there is no reason to regard the experimental result as being inconsistent with the well-established theory.

Having established that the apparatus functioned correctly for the preliminary experiment, we will now turn to a consideration of the results obtained in the case of the more important Double Source-Slit (Sillitto and Haig) experiment.

This was clearly a very difficult experiment. It was realised that even if the correlations were detected, their magnitude would be very small, and it was all the more encouraging when the preliminary experiment was a success. It will be appreciated, though, that the predicted correlation pattern for the preliminary experiment (see Fig. 4.4
of Chapter 4) was of a simpler form than that for the main experiment (see Fig. 4.6 of Chapter 4). This new correlation pattern introduced further complexities into an already complicated apparatus which was working very near to the limits of its operational envelope. For instance, the low side-peak meant that a smaller correlation had to be detected and measured, and the counting rate was smaller due to the wider diffraction pattern. Bearing this in mind, the results achieved are perhaps a little less disappointing. The correlation at the central peak was found to be 1.6%, and the probable magnitude of the secondary peak was slightly less than 1%. Taking into account the experimental error of 1.4% in each, it is therefore only possible to state that a definite correlation was detected with the photocathodes superposed, and that the existence of a secondary correlation maximum at the predicted position was (more than) probable. The predicted values were 5% and 1.6% respectively, but here, as in the previous experiment, it is necessary to point out that there exists an indeterminate error in the predicted values which should therefore not be taken too literally. Taking into account the uncertainty in the experimental values, it is possible to speculate that the ratio of the observed peak heights was correct at 3/1, but as the results stand, the actual ratio was only 2/1.

In Chapter 5 it was shown that the results agree with the theory, as far as the experiment was taken. Periodically
throughout the performance of the experiment, provisional calculations of the Correlation Ratios and experimental errors were made as a guide to the overall performance. At a relatively early stage of the experiment, the Correlation Ratios became established at very nearly the predicted values, although with poor statistical significance, of course. It was noticed that as the weeks passed and the statistical significance increased, the values of the Correlation Ratios, which had previously remained steady, began to fluctuate in an apparently random manner. Now, the experiment was begun in July and concluded in September of 1967. Despite the additional heater and thermostat installation, the daily room temperature fluctuations were rather more pronounced in September than in July or August. This is borne out by the fact that proportionally more detector scans were deleted from the later readings than from the earlier ones in the process of selecting "smooth" readings. It would therefore seem that whereas the thermal fluctuations during July and August were small, they increased in size and number during September until they were of comparable size to the photoelectron fluctuations, thereby masking the correlations.

When we begin to consider the modifications which could be made to the apparatus to improve the result, perhaps the most obvious one is to enclose the Coincidence Unit in tubes filled with recirculating water from a constant-temperature bath. Alternatively, if the time had been
available, it might have been better to develop an entirely new and more thermally stable circuit. Looking at the problem from the other end, as it were, would it not be better to reduce the observing time by reducing the Coincidence Resolving Time, \( \tau_r \), and thus improving the value of the Correlation Ratios? Well, to begin with, it has to be remembered that the theory developed in Chapter 2 contains the assumption that \( \tau_o \ll \tau_r \). Thus, if \( \tau_r \) were improved by an order of magnitude, the equations for \( \rho \) and \( S/N \) (equations (50) and (52)) would no longer remain valid. Indeed, if we make \( \tau_o = \tau_r \), then the maximum value of the Correlation Ratio becomes 0.25. This also implies that, for a given observing time, \( T_o \), the Signal to Noise Ratio does not increase without limit as \( \tau_r \) is decreased. In the present case, therefore, it would be of advantage to shorten the Resolving Time by a factor of 3, but it would be of little assistance to shorten it any further.

The other major factor which warrants investigation is the Partial Coherence Factor, \( \Delta(\nu_\alpha) \), as can be seen from the equations for Correlation Ratio and Signal to Noise Ratio, reproduced here for convenience:

\[
\rho = \frac{1}{4} \frac{\tau_o}{\tau_r} \Delta(\nu_\alpha) \Gamma^2(\nu_\alpha, d) f_1 f_2
\]

\[
S/N = \rho (\alpha N_\alpha)(2T_o \tau_r)^{1/2}
\]

The Partial Coherence Factor is primarily a function of the size and shape of the source and detector apertures, and we
will investigate the effect, on the Correlation Ratio and Signal to Noise Ratio, of varying the Partial Coherence Factor by changing the area of the source apertures. In order to calculate the results of this exercise in a meaningful manner, we will make certain assumptions about the relevant variables, and these are the following:

(a) The Coherence Time of the light remains at \( \tau_0 = 0.5 \) nsec.

(b) The Resolving Time of the Coincidence Circuit remains at \( 2\tau_r = 2.8 \) nsec.

(c) The overall Observing Time remains at \( T_o = 12 \times 10^4 \) sec.

(d) The product of the Decorrelation Factors remains at: \( f_1f_2 = 0.814 \).

(e) The lamp intensity remains constant, so that a change of source-aperture area produces a linearly proportionate change in \( dN_o \), the Single-Channel Counting Rate.

(f) The values of the Correlation Ratios, \( \rho_2 \) and \( \rho_3 \), at the first minimum and second maximum respectively, are calculated for the values of Detector Separation, \( d \), at which these occur, since the values of \( d \) will not necessarily be the same as for the original experiment.

(g) The source apertures are perfectly square and in the correct orientation, and their centres are maintained at the original separation of 0.0565 cms.

(h) The between-centres separation of the identical source apertures is greater than their width, to ensure that they do not merge.
On the basis of these assumptions, and keeping all other factors as before, we are now able to set out the theoretical Correlation Ratios and Signal to Noise Ratios we are to be expected when the width and height of the source apertures are changed from 0.010 cms to 0.050 cms in four steps. The values of $\Delta(\nu_0)$ given below were computed by inserting the relevant values of $\psi$, $\phi_1$, $\phi_3$, $\phi_4$, $\phi_5$ and $K$ into the program given in Appendix (b). The figures pertaining to the original experiment are also included in the Table below for the sake of comparison.

It is immediately apparent that $\rho_1$ remains sensibly large throughout the entire range of source apertures, but $\rho_3$ decreases rapidly as the source apertures are widened. On the other hand, $(S/N)_1$, the Signal to Noise Ratio for the central maximum, improves by more than an order of magnitude as the source apertures are widened and the counting rate-increases. It is particularly interesting to note that the Signal to Noise Ratio for the next maximum, $(S/N)_3$, rises to a maximum at approximately $R_0\theta_1 = 0.0325$ cms. In the case of the Single Source Aperture experiment, all that was required was a large ratio of $\rho_1/\rho_2$, together with a reasonable value of $S/N$. In the case of the Double Source Aperture experiment, it is still true that the $(S/N)$ ratio for each peak should be as high as possible, but it is now necessary to stipulate that the secondary maxima should be of comparable magnitude to the central maximum to ensure maximum detectability and most accurate measurement. Bearing this in mind, it is now
<table>
<thead>
<tr>
<th>$R_0\theta_1$ (cms)</th>
<th>$R_0\theta_2$ (cms)</th>
<th>$R_0\theta_3$ (cms)</th>
<th>$\Delta v_0$ (c/sec)</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\rho_3$</th>
<th>$\frac{\rho_1}{\rho_2}$</th>
<th>$\frac{\rho_3}{\rho_2}$</th>
<th>$(S/N)_1$</th>
<th>$(S/N)_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>0.010</td>
<td>0.0465</td>
<td>134</td>
<td>0.8697</td>
<td>0.0602</td>
<td>0.0063</td>
<td>0.0471</td>
<td>1.32</td>
<td>0.150</td>
<td>0.117</td>
</tr>
<tr>
<td>0.020</td>
<td>0.020</td>
<td>0.0365</td>
<td>538</td>
<td>0.8129</td>
<td>0.0489</td>
<td>0.0054</td>
<td>0.0323</td>
<td>1.62</td>
<td>0.488</td>
<td>0.322</td>
</tr>
<tr>
<td>0.030</td>
<td>0.030</td>
<td>0.0265</td>
<td>1208</td>
<td>0.7326</td>
<td>0.0441</td>
<td>0.0041</td>
<td>0.0173</td>
<td>3.03</td>
<td>0.988</td>
<td>0.388</td>
</tr>
<tr>
<td>0.040</td>
<td>0.040</td>
<td>0.0165</td>
<td>2150</td>
<td>0.6452</td>
<td>0.0388</td>
<td>0.0028</td>
<td>0.0073</td>
<td>8.00</td>
<td>1.546</td>
<td>0.291</td>
</tr>
<tr>
<td>0.050</td>
<td>0.050</td>
<td>0.0065</td>
<td>3360</td>
<td>0.5530</td>
<td>0.0333</td>
<td>0.0015</td>
<td>0.0023</td>
<td>39.75</td>
<td>2.073</td>
<td>0.143</td>
</tr>
<tr>
<td>0.0325</td>
<td>0.032</td>
<td>0.0240</td>
<td>1400</td>
<td>0.7150</td>
<td>0.0495</td>
<td>0.0044</td>
<td>0.0159</td>
<td>3.92</td>
<td>1.282</td>
<td>0.412</td>
</tr>
</tbody>
</table>
Fig. 5-1: Showing (Top), the ratio of the Correlation-Peak heights as a function of Source Aperture, and (Bottom), Signal to Noise Aperture. The actual value of \( R_0 \theta \), used was 0.0325 cms.
clear that the most important quantities are those set out in the last three columns of Table III, columns 9, 10 and 11. The final two columns show the variation of the Signal to Noise Ratio, for each peak, with the source aperture, and the previous column shows the ratio of the height of the central maximum to the height of the secondary maximum with respect to the height of the first minimum. This latter parameter is one which is measured experimentally. Graphs illustrating the relationship between this quantity, $\frac{P_1 - P_2}{P_3 - P_2}$ and $R_0^\Theta_1$, and between the two $(S/N)$ ratios and $R_0^\Theta_1$ are shown in Fig. 5.1. To assist in visualising the changes in profile of the correlation patterns for the different values of $R_0^\Theta_1$, graphs of $\Delta(v_0) R^2(v_0, d)$ versus $d$ are shown in Fig. 5.2.

In order to establish the optimum values of source aperture widths for comparison with the ones actually used, it is now necessary to specify two criteria which will delineate the upper and lower bounds. In the first place, we already know that it is preferable to attain the highest possible Signal to Noise Ratios, which in any case should exceed unity. It has already been explained why the value of $(S/N)_2$ did not exceed unity, but if, at this point, we arbitrarily fix the lower bound as being the value of $R_0^\Theta_1$ for which the value of $(S/N)_1$ is equal to unity, this will at least give a standpoint from which to compare the relative merits of the chosen parameters. The upper boundary condition was, in fact, specified in the previous paragraph, where it was said that the height of the secondary maximum
Fig. 5.2: The family of curves of $\Delta(v_d) I^2(v_d)$ vs. Detector Separation, $d$, for varying Source-Apertures in the Double Source experiment.
should be comparable with that of the first maximum. We shall here choose to define the upper bound as being that value of $R_o \Theta_1$ for which the ratio of the peak heights is equal to ten (i.e. an order of magnitude). Inspection of the graphs in Fig. 5.1 will show that the optimum value of $R_o \Theta_1$ lies between:

$$0.030 \text{ cms.} < R_o \Theta_1 < 0.042 \text{ cms.}$$

It will also be seen that the maximum of the $(S/N)_3$ curve lies within these bounds, and in fact it actually occurs at the value chosen for the experimental Source Aperture, $R_o \Theta_1 = 0.0325$ cms. It will be appreciated that this maximum remains at the same value of $R_o \Theta_1$, given that the linear dimensions remain unaltered, and the magnitude of the maximum is only a function of observing time. The most useful conclusion to be drawn from this analysis, then, is that the dimensions chosen for the optical system were the optimum, and that if the thermal fluctuations had not forced the premature conclusion, the experiment would have been a complete success after the necessary 369,000 seconds observing-time. It could be argued that the detector apertures should have been made larger, or that the source aperture separation could have been narrower and so on, but the powerful arguments against such moves have been recounted elsewhere in this text, and there should be little need for such repetition. A modification which would have been worthwhile, given the space, would have been the omission of the pentagonal prisms. The absorption' and interface
reflection removed by this step could well have amounted to 10%, to the resultant benefit of the Signal to Noise Ratios.

Since it has been shown that little could have been done to improve the source and optical system, the failure to achieve complete success in this experiment must be attributed almost entirely to the electronic system. It was pointed out above that the Resolving Time, $\tau_r$, of the Coincidence Unit could usefully be reduced by a factor of not more than 3, but it would then have been imperative to build a new and more stable circuit. It is worthy of note that the first experiment was successfully concluded in less than one week, while the second experiment took a little more than two months to give a less satisfactory result. It is perhaps reasonable to take the view that despite the theoretical assurance that the Signal to Noise ratio improves with measuring-time for this type of experiment, the steadily increasing possibility of drifts in the hard-pressed electronic equipment is a powerful argument against such long runs. It would have been wiser to have chosen a far faster Coincidence-Unit and then to have "de-sensitized" it to give the required Resolving Time with considerably greater stability. With the state of the art as it was when development ceased and production-run experiments began, however, the Franzini Coincidence Circuit was among the fastest and most reliable circuits in use. This is no longer so.
The theoretical reasons for the existence of this correlation effect are sound, and serve to indicate just how small the effect is with presently available thermal-light sources. It is therefore submitted that the experiment was by no means unsuccessful, but indicates the potentialities of this method of correlation detection in partially coherent fields of light.
ACKNOWLEDGEMENTS

I would like to extend my grateful thanks to my Supervisor, Mr. R.M. Sillitto, for his continued interest in my work and for his ready assistance and advice in the course of many stimulating discussions. I would also like to thank Professor N. Feather, F.R.S., for placing the facilities of the Natural Philosophy Department at my disposal.

My thanks are also due to the staff of the Departmental Workshop for their cheerful assistance in the construction of the apparatus, and to Mr. C. McAnna and his staff in the Electronics Workshop for the prompt and efficient repair and servicing of my more wayward electronic equipment.

Finally, I wish to thank the University for the award of a Postgraduate Studentship for the first two years of my research.
APPENDIX A: The Computer Program and Data for Calculating
\[ \Delta(v_o) \Gamma^2(a, v_o) \] for the Single Source Experiment.

```plaintext
***A
JOB
NAP 059/1981 0010/T, AND L. MUTUAL COHERENCE HAYG
EXECUTION 3 MINUTES
OUTPUT 0 LINE PRINTER 150 LINES
COMPIEER AA

begin
cmment to compute the mutual coherence between two rectangular apertures
integer i, sep, d, D, y, z
real x, wvl, diasorc, dist, bred, heit, C, ch, ch, S, parco
array Jco(0:16)
real fn spec I(real x)
cycle i=0, 2, 16
read (Jco(i))
repeat

cycle i=1, 2, 15
Jco(i)=0
repeat
1: read(wvl)
if wvl=0 then -100
read (diasorc, dist, bred, heit, sep)
caption # wavelength $=$ print(1e8*wvl,4,0)
caption # source diameter $=$ print(diasorc,0,3)
caption cm distance $=$ print(dist,3,1)
caption cm detector breadth $=$ print(bred,1,3)
caption cm detector height $=$ print(heit,1,3)
caption cm max displacement $=$ print(sep*bred,0,3)
caption cm displacement/detector breadth $=$ print(sep*bred,0,3)
caption normalised correlation
C=(\pi*diasorc)/(wvl*dist);ch=0.04*(bred)^2;ch=0.04*(heit)^2;D=5*sep
cycle d=0, 1, D
S=0

cycle y=-4, 1, 4
x=C*(sqrt(ch))*(y-d)
S=S+2*(5-yl)*(L(x))^2
repeat ;| end of y cycle for z=0

cycle z=1, 1, 4
cycle y=-4, 1, 4
x=C*(sqrt(ch*(y-d)^2+ch*z^2))
S=S+2*(5-z)*(5-yl)*(L(x))^2
repeat ;| end of y cycle for z>0
repeat ;| end of z cycle
S=0.0016*s
if d=0 then parco=s
```

---

-140-
real fn J(real x); this is 2*J1(x) to 5 decimal places
real ba, bb, bc, p, q, xa, xb
integer j

if x>0 then ->20
result = 1

20: if x>8 then ->21
ba=0; bb=0; bc=0
cycle j=16, -1, 0
bc=bb
bb=ba
ba=0.25*x*bb-bc+Jco(j)
repeat
result =0.125*(ba-bc)

21: xa=x/x; xb=x-2.35610
p=0.00000*(4*xa^2-2)+2.00181; q=-0.00010*(4*xa^2-2)+0.00356
result =0.70708*(p*(cos(xb))-q*(sin(xb)))/(sqrt(x+3))

end
100: end of program

1.20672
-1.10180
1.28700
-0.66144
0.17771
-0.03018
0.00324
-0.00026
0.00002
0.0005461 0.04 180 0.39 0.39 3

***7
APPENDIX B: The Computer Program and Data for Calculating 
\( \Delta(v_o) \Gamma^2(d, v_o) \) for the Double Source Experiment.

### JOB
NAP 053/1001 0011/CORRELATION FUNCTION THESIS N.D.HAIG
EXECUTION 3 MINUTES
OUTPUT 0 LINE PRINTER 150 LINES
COMPILER AA

begin

real PHI, PHI 3, PHI 4, PHI 5, PSI, sum, K, y, tin, ter, hin, her, bin, ber, r
integer k, i
real fn spec twiss(real x)
real fn spec hanbury(real x)
real fn spec brown(real x)
routine spec autoint(real fn f, real a, b, e, real name int, erint, integer k);

PHI=0.4382; PHI3=0.3625; PHI4=0.8487; PHI5=1.3359; PSI=2.245; K=4.01025

!limits of integration and common factor

newlines(4); spaces(20); caption spacing; spaces(8); caption correlation; newline

autoint(twiss, 0, PSI, 0.00001, tin, ter, 10);! vertical integration

cycle i=0,1,60
   r=i/2.35
   k=intpt ((r+20)*0.5)
   y=PHI; autoint(hanbury, 0, PHI, 0.00001, hin, her, k);!areas a,b, and c,d
   y=PHI3; autoint(brown, PHI3, PHI4, 0.00001, hin, her, k);!areas e,h
   sum=sum+hin+bin
   areas a,b,c,d,e,h
   y=PHI5; autoint(hanbury, PHI4, PHI5, 0.00001, hin, her, k);!areas f,g
   sum=sum+bin
   !total area
   !common factors including twiss integral are put in; in print instructions
newline; spaces(20); print((0.0235*r), 1.4); spaces(10); print((K*tin*sum)/4, 1.5)
repeat

real fn twiss(real x)
   ;!depends on vertical displacements
   if x<0.025 then \rightarrow 1
   result =((sin(x))/x)^2*PSI-x
1: result=PSI-x
end
real fn hanbury(real x) ; areas a, b, e, d or f, g according to limits
if x<0.025 then -1
result = ((sin(x))/x)^2*(y-x)*cos(r*x)
1: result = (y-x)*cos(r*x)
end

real fn brown(real x) ; areas e, h
if x<0.025 then -1
result = ((sin(x))/x)^2*(x-y)*cos(r*x)
1: result = (x-y)*cos(r*x)
end

routint autoint(real fn f, real a, b, e, real namex int, erint, integer k)
real h, sa, sh, fa, fb, fc, fd, fe, maxh
real fn spec f(real x)
integer n
int=0; fa=f(a); n=0
maxh=(b-a)/k
h=maxh; n=2
1: h=h-a
2: f=f(a+0.5h); fe=f(a+h)
3: fb=f(a+0.25h); fd=f(a+0.75h)
sa=(fa+4*fe+fb+4*fd+fe)*h/12
-4 if mod(sa-sh)<0
h=sh; fe=fa; f=fb; n=3
4: int=int+sh; n=n+1
-5 if a+h=b; a=a+h
h=2unless2h>maxh; fa=fe
-1 if fa+h>b; n=2
5: erint=n/15
end

end_of_program

***

Note - Areas referred to as a, b, c, d, e, f, g, h in this program are those labelled p, q, r, s, t, u, v, w in the text, respectively.
APPENDIX C

The Procedure for starting up the Magnetron and Servo Unit

1. Switch on Farnell 24 volt supply and press Trip Reset.
2. Press Servo Cutout Reset switch.
3. Switch on 8 volt Servo apply.
4. Ensure that Relay zero switch is up and Sensitivity Control is at Min.
5. Top up water cell in front of Discharge Tube with Dist. water.
6. Switch on Vacuum pump.
7. Switch on Magnetron Temp Galvo lamp.
8. Ensure Magnetron H.T./Filament switch is at off/off.
10. Return Magnetron Variac to Zero (Pull and Turn).
11. Turn Magnetron H.T./Fil switch to Fil 5.3/H.T. off.
12. Wait > 3 mins. for Filament to heat up.
13. Check Vacuum is > 13 cms. of Glycerine.
14. When Filament is heated up, turn H.T./Filament Switch to 4.5/ON.
15. Turn Variac carefully until Anode Current ~ 40 mA, and then carefully re-engage gear teeth.
16. Go round to end of box and light discharge with Tesla coil (from outside).
17. Set Intensity knob to ~ 9.
19. Turn Sensitivity knob carefully up to Max.
20. Turn Set Intensity knob until Discharge Monitor reads the required Current.
21. Ensure Magnetron Cutout Current knob is set at 125 mA.
22. Allow to settle for 2 hours.
Procedure for Running

(1) Ensure that Vacuum never falls below 13 cms of glycerine. If it does, immediately institute shutdown procedure.

(2) Ensure that room temp. is maintained as constant as possible.

(3) Every two or three hours, turn Sensitivity control to Min and top up Discharge Tube Water Cell. Return Sensitivity to Max immediately.

(4) If Magnetron Anode current rises to value set on Cutout Current (≤ 125 mA), or if Mag. Temp. rises to 115°C (see Calibration Curve), the automatic cutout system will function. If this occurs, the apparatus will be quite safe, but it is advisable to
   (a) Switch off the Farnell 24 volt supply,
   (b) Return Mag. Variac to Zero,
   (c) Switch off Mag. Master switch,
   (d) Return H.T./Fil switch to off/off,
   (e) Switch off Mag. Temp. lamp,
   (f) Switch Relay Zero up,
   (g) Turn Sensitivity to Min,
   (h) Switch off Vac. pump.

(5) When changing the position of P/M(l), always screw the Micrometer inwards, in order to eliminate backlash.

(6) Never touch the Coincidence Unit.

(7) Test the Timer and Scalers for correct operation periodically, or if unlikely counts are registered.

(8) If Scalers and Timer are O.K., check the 1430 Amplifier and Cathode Follower.

(9) Always leave E.H.T. at its original setting, checking this value every two or three hours.
Shutdown Procedure

(1) Turn Sensitivity to Min.
(2) Switch Relay Zero up.
(3) Turn Mag. Variac to Zero.
(4) Turn H.T./Fil switch to off/off.
(5) Switch off Mag. Master Switch.
(6) Turn off Mag. Temp. Lamp.
(7) Switch off Vac. Pump.
(8) Leave 24 volt Supply switched on.
(9) Leave 8 volt supply switched on.
(10) Set Timer to off (Free Running).
(11) Reset Scalers and press Start button.
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