LONG FLOATING CYLINDERS IN THREE-DIMENSIONAL RANDOM SEAS

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The behaviour of a long elastic cylinder floating in three-dimensional random seas is examined in a wide wave basin.

Numerical and analytic predictions of the response of the cylinder are performed and the results compared with experimental measurements. The wave forces in these predictions are assumed to be given by a modified form of Morison's Equation and the empirical constants required are determined in narrow tank tests.

Random seas are modelled in the experiments by the linear superposition of regular waves. The generation of pseudo random waves using such an additive approach is discussed and an alternative form of random wave generation is suggested.

The analytic approach to the prediction of spine responses is found to give good agreement between predicted and measured horizontal bending moments but the agreement for vertical moments is found to be less satisfactory. The numerical approach, which allows certain non-linear effects to be considered, is shown to produce useful predictions of both vertical and horizontal bending moment.
DECLARATION

THIS THESIS HAS BEEN COMPOSED BY MYSELF AND,
EXCEPT WHERE STATED, THE WORK CONTAINED
IS MY OWN.
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CONTENTS

Abstract
Declaration
Acknowledgements
Contents

CHAPTER 1 INTRODUCTION

1.1 History
1.2 Linear Wave Theory
1.2.1 Assumptions
1.2.2 Determination of Dispersion Relationships
1.2.3 Energy Density
1.3 Wave Forces on a Fixed Object
1.3.1 Linear Diffraction Theory
1.3.2 Morison's Equation
1.3.3 Modifications to Morison's Equation
1.4 Moving Bodies
1.5 Scaling and Similarity
1.6 Aims of the Current Work

CHAPTER 2 GENERATION OF THREE DIMENSIONAL RANDOM SEAS

2.1 Introduction
2.2 Existing Methods of Generation
2.2.1 'Mixed Frequency Snake'
2.2.2 The 'Wallingford' Method
2.3 White Noise Filtration
2.4 Spectral Relationships
2.5 Implementation
2.6 Analysis of Results
2.6.1 Spectrum Used
2.6.2 X-Correlation
2.6.3 'Total' Power Spectrum
2.7 Visualisation of Filters, Spectra and Records

CHAPTER 3 EXPERIMENTAL METHODS AND TECHNIQUES

3.1 The Wide Tank
3.1.1 Description
3.1.2 Wave Height Variations Over the Wide Tank
3.2 Determination of Hydrodynamic Parameters
3.2.1 Inertial Wave Forces
3.2.2 Radiation Tests
3.3 Measurement of Bending Moment

CHAPTER 4 ANALYTIC SPINE ANALYSIS

4.1 Introduction
4.2 Structural Treatment of Pipe
4.3 Wave Force Assumptions

continued...
CHAPTER 1

INTRODUCTION
CHAPTER 1
INTRODUCTION

1.1 History

The human race has been using the oceans for many thousands of years. Men have fished the water for food and travelled across the surface of the seas in vessels ranging from crude dug-out canoes to the latest nuclear powered aircraft carriers. This intimate relationship has resulted in a necessity for those who live and work with the sea to gain an understanding of its behaviour.

Mariners' lives and livelihoods depend on a knowledge of winds, waves and currents along the routes their ships travel. Four hundred years ago charts were produced using the observations and experience of travellers but now detailed studies of the oceans can be made using satellites and sophisticated electronic buoys. Before the development of the science of hydrodynamics the response of a ship to waves and winds could only be discovered, perhaps disastrously, by experience. Ships now have their behaviour in the water accurately determined by model tests and theoretical analysis even before they are built.

The effect of the sea on coastal areas can be potentially destructive. Tsunamis can cause great devastation and loss of life when they strike land. Coastal roads, even in the British Isles, can become dangerous in comparatively frequent storms. This has resulted in the construction of sea walls to protect vulnerable areas. The effect of waves on coastal structures such as these walls and on harbours has, therefore, become well known.
Research into ocean waves and their capacity to exert forces has, until recently, been restricted to coastal or ship related problems. The last three decades have, however, seen an increase in the number of offshore structures being constructed around the world. The oil industry, for example, requires large stable platforms from which exploratory drilling or oil extraction can be conducted. Semi-permanent structures such as these must be capable of surviving harsh conditions for up to thirty years. In addition to continual buffeting the longer an oil platform is to be in position the greater is the possibility of a large 'freak' wave hitting it. The oil industry has, therefore, prompted greater investigation into the behaviour of offshore structures some of which is reviewed by Hogben (1974).

Recent proposals for the extraction of energy from ocean waves have produced new problems in hydrodynamics. Wave energy devices such as the 'Duck' (Salter, 1974) and the 'Clam' (Bellamy, 1982) must be situated in energetic seas if they are to produce economic quantities of electricity. It is not possible, therefore, to situate them in quiet areas of sea as might be possible for an oil platform. The shape of a wave energy device must be such that it maximises energy absorption, yet it must do so without putting an unbearable strain on its structure. These requirements are difficult to realise simultaneously. The energy extraction of a Salter Duck, for example, can be maximised by mounting it on an individual compliant axis. Such an axis would, however, be unlikely to survive heavy seas. If wave-power is ever to become a competitive source of energy then it is necessary that problems concerning the survival of the devices be overcome.

If offshore structures are to be studied, then it is necessary to have some knowledge of water waves themselves such as their speeds and energy contents. The simplest useful description is the 'Linear Wave Theory'
as described by Lamb (1952).

1.2 Linear Wave Theory

1.2.1 Assumptions

The ocean is assumed to consist of an incompressible, non viscous fluid. The motion of this fluid can be expressed in terms of a velocity potential \( \phi(t, \mathbf{r}) \). The fluid velocity can be retrieved from this potential to be given by

\[
\mathbf{U}(t, \mathbf{r}) = \nabla \phi(t, \mathbf{r})
\]  

where \( \mathbf{U} = \) velocity 
\( t = \) time 
\( \mathbf{r} = \) position.

The incompressibility of the fluid results in the velocity being non-divergent. Hence

\[
\nabla \cdot \mathbf{U} = 0 \quad \text{therefore} \quad \nabla^2 \phi = 0 \quad \text{(Laplace Equation)}
\]  

The Bernoulli equation holds at all points in the fluid and so

\[
\frac{|\mathbf{U}|^2}{2} + \frac{P_t}{\rho} + gh + \frac{\partial \phi}{\partial t} = \text{CONST}
\]  

where \( g = \) gravitational acceleration
\( h = \) elevation
\( P_t = \) pressure
\( \rho = \) fluid density.

These equations can easily be investigated further if all surface waves are assumed to be infinitely small. This means that the \( |\mathbf{U}|^2 \) term becomes negligible. The solutions become even simpler if pressures are
assumed to be measured relative to the atmospheric pressure. So, if the fluid air boundary is described by \( z = 0 \), equation [1.3] can be simplified and expressed in Cartesian coordinates as

\[
g(t,x,y) + \frac{\partial \phi}{\partial t} = 0 \quad \text{when} \quad z = 0 \tag{1.4}
\]

No fluid can flow across the free surface so it is assumed that the velocity of the rising surface \( \frac{\partial \eta}{\partial t} \) is equal to the vertical fluid velocity at \( z = 0 \). Hence

\[
\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} \quad \text{if} \quad z = 0 \tag{1.5}
\]

Equation [1.2] can be expressed, in Cartesian coordinates, as

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{1.6}
\]

### 1.2.2 Determination of Dispersion Relationship

Equation [1.6] has an infinite number of solutions. It is convenient to examine those solutions representing a travelling surface wave given by

\[
\eta(t,x,y) = A\cos(\omega t - k_x x - k_y y) \tag{1.7}
\]

where \( k_x = \frac{|k|\cos \theta} \)

\( k_y = \frac{|k|\sin \theta} \)

\( k \) = surface wave vector, \( A \) = wave amplitude, and

\( \theta \) = angle between \( k \) and the \( x \)-axis.

The substitution of [1.7] into [1.4] and [1.5] yields

\[
gA\cos(\omega t - k_x x - k_y y) = \frac{\partial \phi}{\partial t} \tag{1.8}
\]
and

\[-A\omega \sin(\omega t - k_x x - k_y y) = \frac{\partial \phi}{\partial y}\]  \[1.9\]

Equation [1.6] can be solved as follows. Try

\[\phi(t,x,y,z) = \Re\{Q(t,x,y,z)\}\]

and

\[Q(t,x,y,z) = e^{i\omega t}X(x)Y(y)Z(z)\]

assuming an oscillation of frequency \(\omega\).

Equation [1.7] suggests that \(X(x)Y(y)\) has the form

\[X(x)Y(y) = e^{i(k_x x + k_y y)} = e^{-i(k_x x + k_y y)}\]  \[1.10\]

If [1.10] is substituted into [1.6] then

\[k_x^2 X(x)Y(y)Z(z) - k_y^2 X(x)Y(y)Z(z) + X(x)Y(y) \frac{\partial^2 Z(z)}{\partial z^2} = 0\]  \[1.11\]

which implies

\[-Z(z)(k_x^2 + k_y^2) + \frac{\partial^2 Z(z)}{\partial z^2} = 0\]  \[1.12\]

This equation has two possible solutions, these being \(A_1 e^{-kz}\) and \(A_2 e^{+kz}\) where \(k^2 = k_x^2 + k_y^2\) and \(A_1\) and \(A_2\) are constants. In addition to the conditions already discussed it is also necessary to consider the fluid on the sea bed, as there can be no flow through the bottom boundary. Hence

\[\nabla \phi \cdot n = 0 \quad \text{if} \quad z = -h\]  \[1.13\]
where \( n = \) normal to sea bed
\( h = \) water depth.

In our coordinate system the bottom condition can be written as

\[
\frac{\partial \phi}{\partial z} = 0 \quad \text{when} \; z = -h \tag{1.14}
\]

Hence

\[
\frac{\partial}{\partial z} \left\{ (A_1 e^{-kz} + A_2 e^{kz}) e^{i(\omega t - kx - ky)} \right\} = 0 \quad \text{if} \; z = -h
\]

This implies

\[
-kA_1 e^{-kz} + kA_2 e^{kz} = 0 \quad \text{if} \; z = -h \tag{1.15}
\]

and if

\[
A_2 e^{kh} = A_1 e^{-kh} = \text{Const} = C/2
\]

then

\[
A_2 = \frac{C}{2} e^{-kh} \quad \text{and} \quad A_1 = \frac{C}{2} e^{kh}
\]

Hence

\[
Q(t,x,y,z) = i \frac{C}{2} \left( e^{-k(z+h)} + e^{k(z-h)} \right) e^{i(\omega t - kx - ky)} \tag{1.16}
\]

therefore

\[
Q(t,x,y,z) = i C \cosh(k(z+h)) e^{i(\omega t - kx - ky)}
\]

and
\[ \phi(t,x,y,z) = -C\cosh(k(z+h))\sin(\omega t - k_x x - k_y y) \]  

[1.17]

The dispersion relationship can be determined from equations [1.17], [1.5] and [1.4]. Hence

\[ g + \frac{\partial \phi}{\partial t} = 0 \implies g + \frac{\partial \phi}{\partial t} + \frac{\partial^2 \phi}{\partial t^2} = 0 \quad \text{where} \quad z = 0 \]

and

\[ \frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} \implies g + \frac{\partial \phi}{\partial z} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{where} \quad z = 0 \]

Hence

\[ g\cosh(kh) - \omega^2 C\cosh(kh) = 0 \]

therefore

\[ \frac{\omega^2}{g} = \tanh(kh) \]  

[1.18]

1.2.3 Energy Density

It is convenient, in this section, to consider a travelling wave described by

\[ \eta(t,x,y) = A\cos(\omega t - k_x x - k_y y) \]

as the sum of two standing waves. For example

\[ \eta(t,x,y) = A\cos(\omega t)\cos(k_x x + k_y y) + A\sin(\omega t)\sin(k_x x + k_y y) \]

The energy density due to the travelling wave is equal to the sum of the energy densities due to each of the component standing waves. These can be evaluated by considering the maximum potential energy of
each sub-wave. For example

\[ f(t,x,y) = A \sin(\omega t) \sin(k_x x + k_y y) \]

has a maximum when \( \omega t = \frac{\pi}{2} \) at which point all of the wave energy is potential. If a change of axis is considered, so that the axis \( \hat{x}' \) lies along the vector \( \hat{k} \), then we can write

\[ f(X') = A \sin(kX') \]

To get the potential energy over one wavelength then we must consider the energy of 'wave slices' of length \( dx' \), height \( A \sin(kx') \) and unit width. The mass of such a slice is given by \( M(x',dx') \) where

\[ M(x',dx') = \rho A \sin(kx') dx' \quad [1.19] \]

The height of the COM of a slice is \( \frac{1}{2} A \sin(kx') \). Hence

\[ PE(x',dx') = \rho g \frac{A^2}{2} \left( \sin^2(kx') \right) dx' \quad [1.20] \]

The potential energy of the entire wave is therefore given by \( P \), where

\[ P = \rho \frac{A^2}{2} \int_{k}^{2\pi} g \sin^2(kx') dx' \]

This implies

\[ P = \frac{\pi \rho g A^2}{2k} = \rho g \frac{A^2}{4} \lambda \]

where \( \lambda = \) wavelength.

The potential energy per unit area is therefore

\[ P(\text{unit area}) = \rho g \frac{A^2}{4} \]
This is the density for one of the standing waves, each of which has the same energy density. The total energy per unit area for a wave of amplitude $A$ is, therefore, given by

$$E = \rho g A^2/2 \quad [1.21]$$

1.3 Wave Forces on a Fixed Object

1.3.1 Linear Diffraction Theory

This theory, like linear wave theory, requires the fluid to be irrotational and incompressible. The waves are assumed to be small and all other linear wave assumptions are assumed to hold.

The incoming wave needs to be described by a velocity potential such as

$$\phi_I = \text{Re}\{\phi_0 \cosh(k(z+h))e^{i(\omega t-kx)}\} \quad [1.22]$$

In addition there are waves scattered by the presence of the object in the fluid. These waves have a potential $\phi_s$. The total potential is given by $\phi$ where

$$\phi = \phi_I + \phi_s$$

$\phi$, $\phi_I$ and $\phi_s$ must all satisfy the Laplace equation. Hence

$$\nabla^2 \phi = \nabla^2 \phi_I = \nabla^2 \phi_s = 0$$

They must also satisfy the boundary equation at the free surface given by

$$g \frac{\partial \phi}{\partial z} + \frac{\partial^2 \phi}{\partial t^2} = 0 \quad \text{where} \quad z = 0 \quad [1.23]$$
The scattered wave potential \( \phi_s \) must also satisfy a radiation condition. This means that far away from the object \( \phi_s \) must represent an outgoing wave. There must also be no flow of fluid through the object boundary. Hence

\[
\mathbf{n} \cdot \nabla \phi = 0
\]

where \( \mathbf{n} \) = normal to the fluid/object boundary.

Therefore

\[
\mathbf{n} \cdot \nabla \phi_I = -\mathbf{n} \cdot \nabla \phi_s \quad [1.24]
\]

The Laplace equation must be solved while satisfying all of the boundary equations. When this has been done, the resultant wave forces on the object can be determined. The force \( \mathbf{F} \) on the object can be given by utilising a linearised form of the Bernoulli equation, i.e.

\[
\text{Pressure (} P \text{) } = \rho \frac{\partial \phi}{\partial t} \quad [1.25]
\]

The force on the object is given, therefore, by

\[
\mathbf{F} = -\int_\mathbf{s} \mathbf{P} \mathbf{n} \cdot d\mathbf{s} \quad \text{where } \mathbf{s} = \text{surface of object}
\]

The force can be conveniently divided into that produced by the incident wave, given by

\[
\mathbf{F}_I = -\int_\mathbf{s} \mathbf{P}_I \mathbf{n} \cdot d\mathbf{s} \quad \text{where } \mathbf{P}_I = \rho \frac{\partial \phi_I}{\partial t} \quad [1.26]
\]

and that resulting from the scattered wave given by

\[
\mathbf{F}_s = -\int_\mathbf{s} \mathbf{P}_s \mathbf{n} \cdot d\mathbf{s} \quad \text{where } \mathbf{P}_s = \rho \frac{\partial \phi_s}{\partial t} \quad [1.27]
\]
These equations can, in general, be solved numerically.

1.3.2 Morison's Equation

This equation was first suggested by Morison et al (1950) as a simple equation describing the wave forces on a vertical pillar. The horizontal force acting on a slice of the pillar of thickness \(dz\) is given by

\[
F(z)dz = \left[C_m \rho V_U X + \frac{1}{2} C_d \rho A U X |U_X| \right]dz
\]

[1.28]

where 
- \(C_m\) = inertial coefficient
- \(V\) = volume/unit length
- \(A\) = surface area/unit length
- \(C_d\) = drag coefficient
- \(U_X\) = horizontal fluid element velocity.

The term \(C_m \rho V_U X\) is the 'inertial' term. This can be expressed in the form \(\rho V_U X + k \rho V_U X\) where \(C_m = 1 + k\). The parameter \(k\) is known as the added mass coefficient. The inertial part of equation [1.28] implies certain assumptions about the pillar and the waves. The pillar must be sufficiently small that the velocity field, in the absence of the pillar, would not vary greatly over the object's position otherwise the incoming waves might be heavily scattered by the object. If this condition is obeyed then the terms \(U_X\) and \(U_X\) can be assigned as the velocity and acceleration of the fluid which would have been present at the central axis of the pillar. The term \(\rho V_U X\) is the force obtained by integrating the pressure field of the undisturbed waves over the surface of the pillar (equation [1.26]). This is known as the Froude-Krylov force. The term \(k \rho V_U X\) is an attempt to represent the effect of the pressure field due to localised disturbance of the field by the
object. This is only valid, as previously stated, when D/\lambda is small.

It is possible to summarise the wave and body assumptions required to validate the inertial component of equation [1.28] by stating that when D/\lambda is greater than about 0.2 diffraction theory must be used but for D/\lambda less than 0.2 the inertial component of [1.28] will suffice although diffraction theory can, if required, be used to determine the value of C_m. This is discussed in greater depth by Standing (February 1981).

The term \( \frac{1}{2} C_d \rho A U_x |U_x| \) is the drag component due to the viscous nature of the fluid. Diffraction and linear wave theory require the assumption of a non-viscous fluid and, as such, there can be no diffraction theory equivalent of this term. The drag term becomes appreciable when the ratio of wave amplitude to body diameter becomes 'large'. If the ratio of amplitude to diameter is greater than 0.5 (approximately) then the drag term of Morison's equation must be included. Keulegan and Carpenter (1958) parameterised their results of force measurements in an oscillating fluid using an equivalent ratio known as the Keulegan-Carpenter number, given by

\[
N_K = \frac{U T}{D}
\]

where

- U = typical fluid velocity = \( 2\pi \frac{A}{T} \)
- T = period of oscillation
- A = wave amplitude
- D = object diameter.

Hence

\[
N_K = 2\pi \frac{A}{D}
\]
In addition to the in-line drag forces described by Morison's equation, flow past a cylinder can also create transverse forces due to vortex shedding and these cannot be simply described but the Keulegan-Carpenter number can describe the relative importance of the inertial forces and the drag/shedding forces. If $N_K$ is less than 3 (equivalent to the ratios of amplitude to diameter being less than 0.5) only inertial forces need be considered but if $N_K$ is greater than 3 then the drag forces need to be included and the transverse shedding forces may require consideration. The Keulegan-Carpenter inertial/drag condition and the $\frac{D}{\lambda}$ inertial/diffraction condition are demonstrated graphically in Figure 1.1.

1.3.3 Modifications to Morison's Equation

Equation [1.28] represents the horizontal force acting on a vertical cylinder. Dixon et al (1979) suggested modifications to the basic Morison equation so that it could be applied to partially submerged horizontal two dimensional cylinders. The main modification to the equation was to allow the volume term to vary with time. The force on a cylinder is, therefore, given by

$$F(t) = C_p V(t) U + Mg - \rho V(t)g$$

where $C_p =$ inertial force tension

$$V(t) = \text{volume of fluid displaced by the cylinder (Figure 1.2)}.$$ 

If the wave steepness is small and the wave elevation at the axis of the cylinder is $\eta(t)$ then the displaced volume $V(t)$ is given by

$$V(t) = \frac{D^2}{8}(\pi + \frac{4}{D}(\eta(t) + td)(1 - \frac{4}{D^2}(\eta(t) + td)^2)^{\frac{1}{2}} + 2\sin^{-1} \frac{2(\eta(t) + td)}{D})$$

where $td =$ distance of cylinder axis below the free surface.
Regions of Validity for Morison's Equation

**Figure 1.1**

Regions of Validity for Morison's Equation
DESCRIPTION OF TERMS RELATING TO A HORIZONTAL CYLINDER IN WAVES

FIGURE 1.2
Description of Terms Relating to a Partially Submerged Cylinder in Waves

AXIS CONVENTION

FIGURE 1.3
Axis Convention for a Horizontal Cylinder
The inertial coefficient is now replaced by the inertial tensor $C$. If the coordinate system is chosen so that the cylinder lies along the $x$-axis, the waves propagate along the $y$ direction (surge) and the $z$ axis is up (heave), as shown in Figure 1.3, then the inertial tensor is diagonal, e.g.

$$\begin{bmatrix} C_H & 0 \\ 0 & C_S \end{bmatrix} = \begin{bmatrix} K_H & 0 \\ 0 & K_S \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

if we are only interested in the surge and heave forces.

Easson (1983) suggested that it is more appropriate to replace equation [1.29] by

$$F(t) = C_p \frac{\partial}{\partial t} (v(t)u) + Mg - \rho V(t)g \quad [1.30]$$

This is more satisfactory than equation [1.29] as it relates the force acting upon the cylinder to the rate of change of momentum of the displaced fluid.

1.4 Moving Bodies

The wave force analysis in Section 1.3.1 can be adapted for a body which is free to move. The motion of the body must, however, be assumed to be infinitesimal so that the assumptions made already about the input and scattered potential remain valid. An object is assumed to respond in one or more of its degrees of freedom, each of these oscillations producing a radiation potential given by

$$\phi = \phi_I + \phi_s + \sum_{j=1}^{N} \phi_{Rj} \quad [1.31]$$

where $\phi_{Rj} =$ velocity potential produced by oscillations in the $j^{th}$ degree of freedom,
Each of the radiation potentials must satisfy a surface boundary condition expressed as

\[ \mathbf{n} \cdot \nabla \phi_{Rj} = -U_j n \quad \text{for} \quad j = 1, N \]  

[1.32]

where \( \mathbf{U} \) = body velocity.

The radiated potential must satisfy the Laplace Equation expressed as

\[ \nabla^2 \phi_{Rj} = 0 \quad \text{for} \quad j = 1, N \]

The linearity of the system means that the body can only respond at the same frequency as the input waves. This means that if

\[ \phi_I = \phi_I e^{i\omega t} \]

then [1.31] can be written as

\[ \phi = \{ \phi_I + \phi_s + \sum_{j=1}^{N} \phi_{Rj} \} e^{i\omega t} \]  

[1.33]

The motion dependent forces relating to the radiation potentials given by

\[ F_j = -\rho \frac{d}{dt} \int_{\mathbf{s}} \phi_{jn} ds \]  

[1.34]

can be conveniently expressed in the form

\[ -A \ddot{\delta} - B \dot{\delta} = F_R \]  

[1.35]

where \( \delta \) = body displacement,
\[ A = \text{added mass tensor} \]
\[ B = \text{damping tensor}. \]

The added mass tensor can be expressed, in the same coordinate system as was used in Section 1.3.2, as

\[ A = \rho V K = \rho V [ \begin{array}{cc} K_H & 0 \\ 0 & K_S \end{array} ] \]

The values of \( K_H \) and \( K_S \) are the same as those discussed in the wave force section. The similarity of equations [1.27] and [1.34] show that, other things being equal, it is the body shape and not whether waves are scattered or radiated that decides the value of \( K \).

The damping effect described by \( B \) is caused by the generation of outward going waves which carry away energy. The importance of the damping term decreases, therefore, as the distance below the free surface increases and for an unbounded fluid \( B = 0 \).

The usual approach to a linear response problem is to solve the scattering problem for waves of the required frequency and to obtain the forcing vector \( F_0 \) so that

\[ F(t) = F_0 e^{i\omega t} \]

The radiation problem must be solved to determine the values of the tensors \( A \) and \( B \). In certain special cases this can be done analytically. Ursell (1949) solved the equations relating to a heaving semi-submerged cylinder but in general the problem must be treated either experimentally as in this work, or numerically as did Yue et al (1976) who developed a finite element approach which could be used for either radiation or diffraction problems.
Once the response matrices have been determined the equation of motion [1.33] can be described by

\[(M + A)\ddot{\delta} + B\dot{\delta} + R\delta = F\]  

where \(R\) = linear spring tensor.

This linear equation can be readily solved using analytic techniques for the response \(\delta_i\) in each degree of freedom.

1.5 Scaling and Similarity

In order to achieve similarity between systems of different scales certain parameters should be identical.

(a) Keulegan Carpenter Number \(N_K = \frac{U}{D}\)

where \(U\) = typical fluid velocity.

If, for waves, \(U = \omega D\) (\(A\) = wave amplitude), then \(N_K\) takes on the form discussed in Section 1.3.2 of \(N_K = 2\pi \frac{A}{D}\). This is a reasonable assumption to make as all that is required is a constant ratio of wave amplitude to body diameter.

(b) Reynolds Number \(R_e = \frac{UD}{\nu}\)

where \(\nu\) = kinematic viscosity.

This determines the similarity between systems where viscosity is the most important factor. The relevance of this number to wave related work is under continual debate. The impossibility of matching Reynolds numbers between large and small scale wave problems has forced much of the work on cylinders to be conducted in wind tunnels as described by Miller B L (1977) who investigated the change in the drag coefficient.
$C_D$ with Reynolds number or in oscillatory flows as described by Sarpkaya (1975) who confirmed that the drag coefficient changes dramatically over a critical range of Reynolds numbers. Typical ocean systems are above the critical Reynolds number but at a small scale the Reynolds number in a wave system is well below the critical value. If however the system is within the inertial/diffraction range of Morison's equation/diffraction theory then it would not appear that Reynolds number is a critical parameter for similarity.

(c) Ratio of Diameter to Wavelength ($D/\lambda$)

This ratio describes the nature of the flow field about the object (still or moving) and as such determines, in the appropriate regimes, the added mass parameters $k$ (or inertial force parameters $C$) or the nature of the scattered or radiated potentials. This is the most important scaling parameter in inertial dominated wave behaviour and the one most used in this work. In 1977 the University of Edinburgh tested the validity of the diameter/wavelength ratio between 1:150 and 1:15 scale models of their 'Duck' devices and confirmed that the resulting scaling laws worked well. This experiment is briefly discussed by Jeffrey et al (July 1978).

(d) Steepness Parameter, $S$

where $S = \frac{A}{gT^2}$.

This similarity condition is satisfied automatically if the Keulegan Carpenter and wavelength to diameter ratio are constant.

1.6 Aims of the Current Work

It has already been stated that one of the problems facing wave energy researchers is that of maximising the power extraction while minimising
FIGURE 1.4

Photograph of the 1:150 Scale Model of a String of 'Salter Ducks'
the strain felt by the device used. The most promising solution to this problem in the case of the 'Salter Duck' is to mount the ducks on a long compliant spine (Figure 1.4). This type of mounting would prevent side to side collisions between ducks and should, by virtue of its own motion, spread the strain due to localised forces over several ducks, thus minimising the risk of failures caused by 'freak' waves. The proposed size of such a spine is 12 metre average diameter and up to 3Km in length. The length is, therefore, going to be much longer than the wavelength of typical ocean waves and elastic waves will be induced by the wave excitation. It is necessary to determine the magnitudes of the bending moments induced in a spine if its required strength is to be evaluated. This requires a knowledge not just of spine structure but of the random waves the spine is likely to encounter and the resultant wave forces. The problems concerning long elastic structures floating in random seas are not confined to wave energy research. The oil industry has recently investigated the possibility of towing pipelines at sea and some of the related research has been described by Guilloud and Vignat (1979).

The aim of this work is to use the relationships and equations discussed in this chapter, making modifications and additions where necessary, in order to analyse the behaviour of long compliant spines in three-dimensional random seas. The theoretical analysis will be compared with experimental work performed on a 1:150 scale model of a duck spine in the wide wave tank at Edinburgh University.

Chapter 2 is an investigation into the modelling of random seas for generation in tanks and for inclusion in numerical simulations of structural responses to waves. Existing methods of wave generation are discussed, including that used in the Edinburgh tank, and an alternative
method is suggested by the author.

Chapter 3 describes the experimental layout and methods used in determining the hydrodynamic tensors $C$ and $B$ (equations [1.29] and [1.35]). The results in certain cases are presented and discussed. The energy content of waves across the working area of the wide tank is mapped as a preliminary to the experimental work done on spines which is discussed in detail with particular attention being paid to hardware.

Chapters 4 and 5 describe alternative approaches to the solutions of the equations governing spine responses to random waves. Chapter 4 discusses the analytic solutions of the linearised response equations while Chapter 5 describes the numerical treatment of the spine problem when certain non-linear effects are incorporated. This chapter contains references to the wave generation systems described in Chapter 2.
CHAPTER 2

GENERATION OF THREE DIMENSIONAL RANDOM SEAS
2.1 Introduction

It is important that certain offshore and coastal structures, such as oil rigs and breakwaters, be tested experimentally, at laboratory scale in three-dimensional random seas. There is, therefore, an increasing interest in methods of generating complex seas in the laboratory.

It is common to assume that the wave elevation is a Gaussian process (Pierson 1955). This considers the sea as a linear summation of an infinite number of wavelets with random phases, and amplitudes related to the energy content of the waves in that direction and at that frequency. This assumption enables a random sea to be statistically described in terms of an energy spectrum (Longuet Higgins 1957) which can be expressed in the form

\[ S(\omega, \theta) \]

where \( \omega \) = angular frequency

\( \theta \) = angle of propagation.

The energy density (per unit area of sea surface) apportioned to wavelets having frequencies between \( \omega_1 \) and \( \omega_2 \) and directions between \( \theta_1 \) and \( \theta_2 \) is, therefore, given by

\[ E(\omega_1, \omega_2, \theta_1, \theta_2) = \int_{\omega_1}^{\omega_2} \int_{\theta_1}^{\theta_2} S(\omega, \theta) d\omega d\theta \quad [2.1] \]

or if

\[ \cdots \]
\[\omega_2 = \omega_1 + \Delta\omega \quad \text{and} \quad \theta_2 = \theta_1 + \Delta\theta \quad \text{as} \quad \Delta\omega \quad \text{and} \quad \Delta\theta \to 0\]

then

\[E(\omega_1 + \Delta\omega, \theta_1 + \Delta\theta) = S(\omega_1, \omega_2)\Delta\omega\Delta\theta\]

[2.2]

It is common to express the directional spectrum in the form of a frequency dependent term multiplied by a spreading term, e.g.

\[S(\omega, \theta) = S_T(\omega)H(\omega, \theta)\]

where

\[\int_{-\pi}^{\pi} H(\omega, \theta)d\theta = 1\]

\[H(\omega, \theta) = \text{spreading term}\]

\[S_T(\omega) = \text{'total' spectrum.}\]

The 'total' spectrum expresses the energy density apportioned to a certain frequency irrespective of direction. Figure 2.1 shows an example of a directional spectrum measured off South Uist.

There are many theoretical spectra. That described by Pierson and Moskowitz (1964) is intended to represent a fully developed wind created sea, i.e. a sea over which a steady wind has been blowing for an infinitely long time over an infinitely large fetch.

The 'total' spectrum is expressed in the form, for a PM sea,

\[S_T(\omega) = ag^2\omega^{-5}\exp(-\beta(\omega_\alpha/\omega)^b)\]

where \(\omega = \text{angular frequency}\)

\[\alpha = 0.0081\]
FIGURE 2.1
Directional Spectrum Measured Off South Uist

FIGURE 2.2
Schematic Diagram of Edinburgh Tank
\[ \beta = 0.74 \]
\[ g = \text{gravitational acceleration}. \]

The parameter \( \omega_0 \) specifies the spectrum uniquely. The sea is more usually described by either the wind velocity \( U_0 \) or the energy period \( T_E \). These parameters are all related by

\[ \omega_0 = g/U_0 \quad \text{and} \quad gT_E/2\pi = 0.9773U_0 \]

The spreading term is often described as a cosine power, i.e.

\[ H(\omega, \theta) = \cos^s(\theta - \theta_0) \]

where \( s \) might be 2, 4 or any number required to match the observed spectrum.

Mitsuyasu et al (1975) suggested that the directional spread depends on the period (T) in the manner

\[ H(\omega, \theta) = C_m \cos^m(\frac{1}{2}(\theta - \theta_0)) \]

where

\[ m = 15.85 \left( \frac{T}{T_0} \right)^{-5} \quad \text{if} \quad \omega < \omega_0 \]
\[ m = 15.85 \left( \frac{T}{T_0} \right)^{2.5} \quad \text{if} \quad \omega > \omega_0 \]
\[ T_0 = \omega_0/2\pi, \quad T = \omega/2\pi \]

and \( C_m \) is chosen so that \( \int_{-\pi}^{\pi} H(\omega, \theta) d\theta = 1. \)

There are already in existence various methods for the generation of random seas whose spectra can be specified \textit{priori}.

2.2 \quad \textbf{Existing Methods of Generation}

2.2.1 'Mixed Frequency Snake'

This method requires a linear array of wavemakers such as in the 'Wave Power Project' tank at Edinburgh University (Jeffrey et al 1978) which has outside dimensions of 25.5m x 11m. There are eighty wavemaking
flaps of width 30cm generating waves in a working area of 25m x 7.3m approximately. This arrangement is shown schematically in Figure 2.2.

If waves of a frequency $\omega$ are to be generated propagating at an angle $\theta$ to the array of flaps, then it is necessary to introduce a phase difference between signals to adjacent flaps. The value of this phase difference can easily be evaluated using the dispersion relationship from linear wave theory, i.e.

$$\omega^2 = g \frac{2\pi}{\lambda} \quad \text{(deep water)} \quad [2.3]$$

where $g =$ gravitational acceleration

$\lambda =$ wavelength.

If $D$ is the distance between successive crests measured along the wave-makers ($x$-axis), then

$$\lambda = D\sin\theta \quad \text{(Figure 2.3)}$$

Hence

$$D = \frac{\lambda}{\sin\theta}$$

So, if the flap separation is $d$ then the phase difference between successive flaps ($\Delta\phi$) is given by

$$\Delta\phi = \frac{2\pi}{\lambda} d\sin\theta \quad [2.4]$$

Hence the signal to wavemaker $I$ should be given by

$$\text{Sig}(I) = A_0\cos(\omega t - I \frac{2\pi}{\lambda} d\sin\theta + \phi_0) \quad [2.5]$$

where $A_0$ and $\phi_0$ are constants.
RELATIONSHIP BETWEEN WAVELENGTH AND CRESTLENGTH

FIGURE 2.3

Relationship Between $\theta$, $\lambda$ and $D$
If we wish to add wave fronts to produce a mixed sea then the linearity of the system enables us to add the signals. Hence

$$\text{Sig}(t) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} \cos(\omega_m t - I \frac{2\pi}{\lambda_m} dsin\theta_n + \phi_{mn})$$

[2.6]

for M frequencies and N angles

where $A_{mn}$ are amplitudes

$\phi_{mn}$ are phases, where

$$m = 1, M$$

$$n = 1, N.$$

Three dimensional random seas can be simulated using this technique by making $\phi_{mn}$ a random variable between 0 and $2\pi$ and choosing $A_{mn}$ to match a chosen spectrum.

One method of implementing this technique is to use fronts having equally spaced frequencies and directions given by

$$\omega_m = \omega_0 + m\Delta\omega$$

and

$$\omega_n = \omega_0 + n\Delta\theta$$

where $\omega_0, \theta_0 = \text{constant}$

and

$$A_{mn} = \sqrt{S(\omega,\theta)\Delta\omega\Delta\theta}$$

[2.7]

This method has the disadvantage that it results in many fronts of the same frequency but different angles existing in a tank simultaneously.
This results in interference which affects the RMS height over the tank area.

The usual manner in which fronts superposition is utilised, is to add equally sized fronts of varying frequency separation. The density of fronts represents the size of the spectrum at a particular frequency and direction. Figure 2.4 demonstrates this approach for a unidirectional sea. Figure 2.5 demonstrates the discrete front representation of a Pierson-Moskowitz spectrum with \( \cos^2 \theta \) spreading. Directional seas generated in this manner contain no two wave fronts having either the same frequency or direction. This, therefore, minimises any stationary effects in the tank.

The summation approach to random wave generation gives the user the option of choosing starting phases at random, or so as to produce freak effects which occur in nature very occasionally. If all the fronts produce a crest at the same time, and in the same place, then a large 'freak' wave such as might be seen in nature only once in a hundred years, arises. This enables designers to prepare an offshore structure for such a possibly disastrous event. The spectra produced are not, however, continuous and it is possible that certain fine structural resonances might be missed if this method is used.

2.2.2 The 'Wallingford' Method (Hydraulics Research Station 1973)

This uses an arc of large independently controlled flaps (Figure 2.6). These flaps produce unidirectional trains of pseudo-random waves aimed at a central region of the tank. The spreading function is controlled by varying the amplitudes of the waves from each flap. This can be done by altering the gains of the amplifiers driving each flap motor. This method of generation is obviously not suitable for testing large structures as correct directional reproduction is only achieved in a
FIGURE 2.4

Unidirectional Comb Spectrum
Directional Comb Spectrum

FIGURE 2.5

1 sec PM sea, Mitsuyasu spread (17)
75 wavefronts, 204.8 sec repeat

Amplitude (cm)

Size 1380 m/s m 1.34 cm
Period 1.003 sec 27%
Angle -2.6° 33.5°
FIGURE 2.6

Schematic Diagram of Wallingford System
small central area. The very small number (10) of directional components limits the kind of spectra that can be generated. It is impossible, for example, to generate an experimentally determined spectrum as in Figure 2.1.

2.3 White Noise Filtration

The author has suggested an alternative method of random wave generation in tanks such as that in Edinburgh.

The array of wavemakers is considered, initially, as an infinitely long, continuous, linear system lying along the x-axis. The power spectrum of the array's motion must match that of the required sea.

The response of such a system to a unit impulse at time $t = 0$ and position $x = 0$ is given by the unit impulse response function $h(t,x)$.

The response $z(t,x)$ to a general force $A(t,x)$ is given by

$$z(t,x) = \int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h(t-T,x-X)A(T,X)dTdX \quad [2.8]$$

or, alternately,

$$z(t,x) = \int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h(T,X)A(t-T,x-X)dTdX \quad [2.8']$$

2.4 Spectral Relationships

In order to produce output signals ($Z(t,x)$) with the required power spectrum, we need to evaluate the required form of the unit impulse response function. It is necessary to determine a relationship between the spectrum of the input $A$ and the output $Z$. This may be obtained as follows.

Multiply both sides of [2.8'] by the complex conjugate of
Z(t-\tau,x-\chi) \quad \text{i.e.} \quad Z^*(t-\tau,x-\chi)

Hence

\[ Z(t,x)Z^*(t-\tau,x-\chi) = \int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h(T,X)A(t-T,x-X)Z^*(t-\tau,x-\chi) \, dT \, dX \quad [2.9] \]

Take the expected value of both sides over the x,t domain so that

\[ E\{Z(t,x)Z^*(t-\tau,x-\chi)\} = E\{ \int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h(T,X)A(t-T,x-X)Z^*(t-\tau,x-\chi) \, dT \, dX \} \quad [2.10] \]

E\{Z(t,x)Z^*(t-\tau,x-\chi)\} is the two dimensional auto correlation of Z and is expressed as R_{zz}(\tau,\chi). Similarly E\{A(t-T,x-X)Z^*(t-\tau,x-\chi)\} over the x,t domain is a cross correlation between A and Z with arguments (\tau-T) and (x-\chi) and is expressed in the form

R_{AZ}(\tau-T,X-\chi)

Hence

\[ R_{zz}(\tau,\chi) = \int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h(T,X)R_{AZ}(\tau-T,X-\chi) \, dT \, dX \quad [2.11] \]

If a fourier transform of both sides is taken, then

\[ F\{R_{zz}(\tau,\chi)\} = F\{ \int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h(T,X)R_{AZ}(\tau-T,X-\chi) \, dT \, dX \} \quad [2.12] \]

F\{R_{zz}(\tau,\chi)\} is the spectrum of the response of the system expressed in terms of the angular frequency and the x-component of the wave vector K_x and is expressed as S_{zz}(\omega,K_x).

The RHS can be simplified, using the convolution theorem, to

\[ F\{h(T,X)\}F\{R_{AZ}(\tau,\chi)\} \]
\( F\{R_{AZ}(\tau, \chi)\} \) is the two dimensional cross power spectrum between \( A \) and \( Z \) and is expressed as \( S_{AZ}(\omega, K_x) \). Hence

\[
S_{ZZ}(\omega, K_x) = F\{h(T, X)\}S_{AZ}(\omega, K_x) \quad [2.13]
\]

If we take the complex conjugate of both sides of equation \([2.8']\) we obtain

\[
Z^*(t, x) = \int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h^*(T, X)A^*(t-T, x-X)dTdx \quad [2.14]
\]

Multiplying both sides of this by \( A(t+\tau, x+\chi) \) gives

\[
A(t+\tau, x+\chi)Z^*(t, x) = \int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h^*(T, X)A(t+\tau, x+\chi)A^*(t-T, x-X)dTdx \quad [2.15]
\]

Taking the expected values of both sides over the \( x, t \) domain gives

\[
E\{A(t+\tau, x+\chi)Z^*(t, x)\} = E\{\int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h^*(T, X)A(t+\tau, x+\chi)A^*(t-T, x-X)dTdx\} \quad [2.16]
\]

\( E\{A(t+\tau, x+\chi)Z^*(t, x)\} \) is the cross-correlation between \( A \) and \( Z \) and is expressed as \( R_{AZ}(\tau, \chi) \).

Similarly \( E\{A(t+\tau, x+\chi)A^*(t-T, x-X)\} \) is the auto-correlation of \( A \) with arguments \( T+\tau \) and \( X+\chi \) and is expressed in the form \( R_{AA}(T+\tau, X+\chi) \). Hence

\[
R_{AZ}(\tau, \chi) = \int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h^*(T, X)R_{AA}(T+\tau, X+\chi)dTdx \quad [2.17]
\]

If the fourier transform of each side is taken, we obtain

\[
F\{R_{AZ}(\tau, \chi)\} = F\{\int_{T=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h^*(T, X)R_{AA}(T+\tau, X+\chi)dTdx\} \quad [2.18]
\]

\( F\{R_{AZ}(\tau, \chi)\} = S_{AZ}(\omega, K_x) \) is the cross power spectrum of \( A \) and \( Z \) and the
\[ \text{RHS} = F\{h^*(T,X)\}F\{R_{AA}(\tau,X)\} \]  
\[ = F\{h^*(T,X)\}S_{AA}(\omega,K_x) \]

where \( S_{AA}(\omega,K_x) \) is the power spectrum of the input signal \( A \). Hence

\[ S_{AZ}(\omega,K_x) = F\{h^*(T,X)\}S_{AA}(\omega,K_x) \]  
\[ \text{[2.20]} \]

The Fourier transform of the response function \( h(t,x) \) is known as the system function \( H(\omega,K_x) \) so

\[ S_{ZZ}(\omega,K_x) = H(\omega,K_x)S_{AZ}(\omega,K_x) \]

and

\[ S_{AZ}(\omega,K_x) = H^*(\omega,K_x)S_{AA}(\omega,K_x) \]

Hence

\[ S_{ZZ}(\omega,K_x) = |H(\omega,K_x)|^2S_{AA}(\omega,K_x) \]  
\[ \text{[2.21]} \]

If it is known that the power spectrum of the input signal is that of two-dimensional white noise, i.e.

\[ S_{AA}(\omega,K_x) = \text{const} \]

then we may evaluate, from the output spectrum, the required response function.

2.5 Implementation

The power spectrum which is to be produced will probably, as discussed, be expressed in terms of \( \omega,0 \) but this can readily be expressed in terms
of $\omega$ and $K_x$ by using the relationship

$$K_x = \frac{\omega^2}{g} \sin \theta \quad \text{(deep water)}$$

Power spectra give no phase information and so it is assumed that, as the indeterminate nature of the waves arises from the input signal $A$, then we need only use the imaginary component of the inverse fourier transform of $H(\omega,K_x)$. Hence

$$h(T,X) = \frac{1}{4\pi^2} \int_{\omega=-\infty}^{\infty} \int_{K_x=-\infty}^{\infty} H(\omega,K_x) \sin(\omega T + K_x X) d\omega dK_x$$

[2.22]

This produces a convenient form of the response function for computational purposes and produces signals with no DC offset.

Figure 2.7 shows how, for a real spectrum, the infinite integrals of equation [2.8] are not necessary as the value of the integral [2.22] falls to zero for large values of $T$ and $X$. Hence we can replace [2.8] by

$$Z(t,x) = \int_{T=t-\tau}^{t+\tau} \int_{X=x-L}^{x+L} h(t-T,x-X) A(T,X) dT dX$$

[2.23]

provided $\tau$ and $L$ are sufficiently large.

In a wave basin we are not concerned with a continuous time and space domain as, obviously, wavemakers have a finite width and digital computers require a finite digitising frequency. Hence if

$$t = a \Delta t$$

and

$$x = b \Delta x$$
FIGURE 2.7
Visualisation of Digital Filter

FIGURE 2.8
Shift Register Operation
for \(a, b = 0, \pm 1, \pm 2, \ldots\), then

\[
Z(a \Delta t, b \Delta x) = \int_{T=a \Delta t - \tau}^{a \Delta t + \tau} \int_{X=b \Delta x - L}^{b \Delta x + L} h(a \Delta t - T, b \Delta x - X) A(T, X) dT dx \quad [2.24]
\]

If \(\tau = M \Delta t\) and \(L = N \Delta x\) then

\[
Z(a, b) = \int_{T=(a-M)\Delta t}^{(a+M)\Delta t} \int_{X=(b-N)\Delta x}^{(b+N)\Delta x} h(a \Delta t - T, b \Delta x - X) A(T, X) dT dx \quad [2.25]
\]

If the forcing function is discrete, i.e.

\[
A(T, X) = 0 \quad \text{for} \quad T \neq (0, \pm 1, \ldots) \Delta t \\
X \neq (0, \pm 1, \ldots) \Delta x
\]

then

\[
A(T, X) = A'(p, q) \delta(T-p \Delta t, X-q \Delta x) p, q = 0, \pm 1, \ldots
\]

Hence

\[
Z(a, b) = \int_{T=(a-M)\Delta t}^{(a+M)\Delta t} \int_{X=(b-N)\Delta x}^{(b+N)\Delta x} h(a \Delta t - T, b \Delta x - X) A'(p, q) \delta(T-p \Delta t, X-q \Delta x) dT dx \quad [2.26]
\]

Hence

\[
Z(a, b) = \sum_{p=a-M}^{a+M} \sum_{q=b-N}^{b+N} h(a-p, b-q) A'(p, q) \quad [2.27]
\]

The 'white' noise source used was an array of shift registers, one for each possible value of \(q\), i.e. \((2N+1)\), each of length \(2M+1\). Each location of a register being in either a logic high or low state. The operation of each one of these being as is shown in Figure 2.8. The repeat time of this system depends on how far apart the registers used as inputs to the NAND gate are, i.e. number of steps before repeat = \(2^r-1\) where \(r\) is the separation of the utilised locations.
FIGURE 2.9
Spectral Analysis of Shift Register Output

FIGURE 2.10
Schematic Representation of Filtering System
This sort of system must be sampled over an entire repeat time if a totally white pseudo-random sequence is to be produced. The power spectrum of the output of such a shift register is shown in Figure 2.9. A schematic diagram of the system is shown in Figure 2.10. The program written to perform the filtering calculations (equation [2.27]) is called QOS82. This program is written in Fortran 77 and is running on an ICL 2972 mainframe.

2.6 Analysis of Results

2.6.1 Spectrum Used

In all of the outputs of QOS82 tested or drawn the spectral shape being simulated was of the general form

$$S(\omega, \theta) = \exp \left( \frac{(\omega - \omega_0)^2}{2\sigma^2} \right) \cos^S \theta$$

[2.28]

The input parameters to the program INFOR used to generate the filters are:

1. the spreading term S;
2. MEAN = $\omega_0/2\pi$;
3. SD = $\sigma/2\pi$.

The terms MEAN, SD and S are used in some graphs to identify the particular sea state. This spectrum has, unlike the PM spectrum, no theoretical backing, nor has it been observed experimentally. It does, however, have three easily varied parameters. Some of the effects of varying these parameters are observed and discussed.
2.6.2 X-Correlation

The theoretical correlation between wave records as a function of separation in the x-dimension is given by taking the inverse fourier transform of the spectrum so

\[ R_{zz}(\tau, x) = F^{-1}(S_{zz}(\omega, k_x)) \]  \[2.29\]

and, as we are only interested in the correlation as a function of \( X(R_{zz}(0, x)) \), this calculation can be performed very rapidly. If the spectrum is symmetrical about \( k_x = 0 \), then \( R_{zz}(0, x) \) is a real function again saving computer time.

The correlation of the signals output by the filter program were evaluated using the equation

\[ R_{zz}(0, x) = E\{z(t, x+X)z^*(t, x)\} \]  \[2.30\]

(E represents expected value over x,t domain).

Figures 2.11 to 2.18 show, for a variety of theoretical spectra, the effect of varying the utilised width of the digital filter (\( N \) in equation [2.27]) upon the correlation between signals along the x-axis. The continuous line in all of these graphs show the theoretical correlation evaluated using equation [2.29]. The situation where \( N = 0 \) shows the case when, theoretically, the signals are totally uncorrelated due to their independence from each other. This gives an indication of the likely statistical fluctuations of measured correlations and as such gives us a value of the minimum significant variation of value as a function of separation. In all of the cases indicated the deviation of the \( N=0 \) correlation from zero increases with separation. This is a result of the analysis technique used. The data analysed consisted of
CORRELATION AGAINST SEPARATION
MEAN=1.2; SD=0.25; S=2

FIGURE 2.11
Correlation Along X-Axis

CORRELATION AGAINST SEPARATION
MEAN=1.0; SD=0.25; S=2

FIGURE 2.12
Correlation Along X-Axis
CORRELATION AGAINST SEPARATION
MEAN=1.5; SD=0.25; S=2

0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
SEPARATION (M)

CORRELATION

FIGURE 2.13
Correlation Along X-Axis

CORRELATION AGAINST SEPARATION
MEAN=1.5; SD=0.25; S=4

0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
SEPARATION (M)

CORRELATION

FIGURE 2.14
Correlation Along X-Axis
Correlation Against Separation

Mean=1.5; SD=0.25; S=6

Figure 2.15

Correlation Along X-Axis

Correlation Against Separation

Mean=1.7; SD=0.25; S=2

Figure 2.16

Correlation Along X-Axis
CORRELATION AGAINST SEPARATION
MEAN=1.7; SD=0.25; S=4

○ N=0
+ N=2
○ N=4
△ N=6

CORRELATION AGAINST SEPARATION
MEAN=1.7; SD=0.25; S=6

○ N=0
+ N=2
○ N=4
△ N=6
▲ N=8

FIGURE 2.17
Correlation Along X-Axis

FIGURE 2.18
Correlation Along X-Axis
FIGURE 2.19
Schematic Representation of Data Series

POWER SPECTRA (THEORY & SIMULATED)
MEAN=1.2; SD=0.25; S=2

FIGURE 2.20
'Total' Spectrum
POWER SPECTRA (THEORY & SIMULATED)
MEAN = 1.5; SD = 0.25; S = 2

FIGURE 2.21
'Total' Spectrum

POWER SPECTRA (THEORY & SIMULATED)
MEAN = 1.5; SD = 0.25; S = 4

FIGURE 2.22
'Total' Spectrum
CENTRAL AXIS OF FILTER

MEAN=1.0; SD=0.25; S=2

FIGURE 2.23

Demonstration of a Shaping Function
8 streams of numbers are shown in Figure 2.19. These streams each contain 1024 numbers.

If the correlation for a separation of Δx is to be evaluated, then the correlation between \( S_1 \) and \( S_2 \), \( S_2 \) and \( S_3 \), etc, is calculated and the average of these correlations used. This involves 7168 data pairs. The correlation for a separation of 7Δx can only be evaluated using \( S_1 \) and \( S_8 \) and as such only uses 1024 data pairs. It might, therefore, be expected that the statistical error for the 7Δx case to be 7 times greater than for the Δx case.

It can be seen in the graphs that, as might be expected, the value of \( N \) required for accurate reproduction depends on the sea spectrum used. The case in Figure 2.12 shows correlations just outside the significant deviation from theory for \( N=4 \) but within for \( N=6 \). The case in Figure 2.13, however, is within significant deviation for \( N=4 \).

2.6.3 'Total' Power Spectrum

A wave power spectrum is, as described earlier, conveniently expressed in the form \( S(\omega, \theta) = S_T(\omega)H(\omega, \theta) \). The total spectrum expresses the energy density solely as a function of \( \omega \) and includes the contribution from waves from all directions, i.e.

\[
S_T(\omega) = \frac{1}{\pi} \int_{-\pi}^{\pi} S(\omega, \theta) d\theta
\]

This spectrum can be easily evaluated by using fast fourier transform techniques applied to wave records at a single point. In our case the spectrum is obtained by averaging the spectra calculated from each of the eight records.

Figures 2.20 to 2.22 show for sample spectra the reproduction obtained.
These graphs are of unsmoothed data and, as such, demonstrate the continuous nature of the spectra of waves generated using the filtration technique. The spectral peaks are correctly positioned but there is a tendency for the simulation spectra to be rather broader than the theoretical case.

2.7 Visualisation of Filters, Spectra and Records

Figure 2.23 shows the central axis (x=0) of a typical filter. This shows how, should it be thought necessary, the filter can be tapered with a window function. The use of such a function has a detrimental effect on spectral reproduction but if as shown, the filter does not fall away to zero at its furthest point, then high frequency effects caused by the truncation of the filter can be minimised.

Figures 2.24 to 2.35 show corresponding spectra, filters and wave records. The filter diagrams, in particular, demonstrate why only the central region needs to be used in calculations. Filters, even those of relatively long crested seas (Figure 2.31) tend to decay rapidly as x increases.

The noise streams used to produce all of the wave records were the same. This means, for example, that although the records in Figure 2.26 and Figure 2.32 have different spreading functions, the phase relationship in the seas are exactly the same. The records, therefore, look very similar, although the increased crest length of Figure 2.32 can be easily seen. This effect has important consequences in the testing of structures which have non-linear responses to waves implying that response spectra are sensitive to the phase distribution of the waves. The effect of varying spectra, without altering the phase relations can, therefore, be measured. The 'snake' method, using the equal size
teeth ('comb') technique, cannot reproduce this property, as changing a spectrum involves altering the component frequencies.
DIRECTIONAL SPECTRUM

SPECTRAL PEAK AT 1.2 Hz.
SPECTRAL WIDTH=0.25 Hz.
COS^2 Z SPREADING

FIGURE 2.24

Visualisation of Directional Spectrum (k max = -k min = 11.6m⁻¹)
POSITIVE TIME HALF OF DIGITAL FILTER
SPECTRAL PEAK AT 1.2 Hz.
SPECTRAL WIDTH = 0.25 Hz.
COS**2 SPREADING

FIGURE 2.25
Visualisation of Digital Filter
WAVE RECORDS

SPECTRAL PEAK AT 1.2 Hz.
SPECTRAL WIDTH = 0.25 Hz.
COS^2 SPREADING

FIGURE 2.26
Visualisation of Wave Records
DIRECTIONAL SPECTRUM
SPECTRAL PEAK AT 1.2 Hz.
SPECTRAL WIDTH=0.25 Hz.
COS²θ SPREADING

FIGURE 2.27
Visualisation of Directional Spectrum (k max = -k min = 11.6m⁻¹)
FIGURE 2.28

Visualisation of Digital Filter
WAVE RECORDS
SPECTRAL PEAK AT 1.2 Hz.
SPECTRAL WIDTH=0.25 Hz.
COS**x SPREADING

FIGURE 2.29
Visualisation of Wave Records
DIRECTIONAL SPECTRUM

SPECTRAL PEAK AT 1.2 HZ.
SPECTRAL WIDTH = 0.25 HZ.
COS**12 SPREADING

Figure 2.30
Visualisation of Directional Spectrum (k max = -k min = 11.6 m⁻¹)
Positive Time Half of Digital Filter

Spectral Peak at 1.2 Hz
Spectral Width = 0.25 Hz
Cos x12 Spreading

FIGURE 2.31
Visualisation of Digital Filter
WAVE RECORDS

SPECTRAL PEAK AT 1.2 Hz.
SPECTRAL WIDTH=0.25 Hz
COS**12 SPREADING

FIGURE 2.32
Visualisation of Wave Records
DIRECTIONAL SPECTRUM

SPECTRAL PEAK AT 1.5 HZ.
SPECTRAL WIDTH=0.25 HZ.
COS**b SPREADING

FIGURE 2.33

Visualisation of Directional Spectrum (k max = -k min = 16.1m⁻¹)
POSITIVE TIME HALF OF DIGITAL FILTER

SPECTRAL PEAK AT 1.5 Hz.
SPECTRAL WIDTH = 0.25 Hz.
COS**b SPREADING

FIGURE 2.34
Visualisation of Digital Filter
WAVE RECORDS

SPECTRAL PEAK AT 1.5 HZ.
SPECTRAL WIDTH=0.25 HZ.

FIGURE 2.35
Visualisation of Wave Records
CHAPTER 3

EXPERIMENTAL METHODS AND TECHNIQUES
3.1 The Wide Tank

3.1.1 Description

The experimental work relating to spine responses was performed in the wide wave tank at Edinburgh University. The technique currently used to simulate random seas in this tank was described in Chapter 2. This section is an attempt to describe the hardware of the tank and some of its properties which affect experimental work. A more detailed description is given by Jeffrey et al (1978, vol 3). The external dimensions of the tank are, length 11.0m, width 27.5m and depth 1.2m. The eighty wavemaking flaps are each of width 30cm and are arranged along one side of the tank with a dry sump between them and the outside of the tank. This arrangement minimises the amount of energy required to generate waves of a particular size as there is no need to move water behind the flaps. There is also no possibility of generating troublesome standing waves behind the flaps. The flaps act as absorbers to any waves incident upon them. Each flap incorporates a transducer measuring its velocity and the resultant voltage, proportional to the velocity, is amplified and fed into the flap motor. This produces a force proportional to the velocity, equivalent to damping, and the gain can be varied to absorb waves of a given frequency range. This helps to prevent large standing waves within the working area of the tank. External signals can be added to the force signal to produce waves without affecting the absorption. The problems caused by the non-absorption of waves present in the tank can be critical.
FIGURE 3.1
Plan of Edinburgh Wide Tank
Reflections off the tank sides can cause stationary effects especially with monochromatic waves. The tank has, therefore, two sides lined with 'beaches' made from a material known as 'Expamet'. This is a dense mesh of metal foil which, if suitably packed, can absorb much of the incident wave energy upon it. The remaining side of the tank is a glass window which is used for the observation of models under test. This arrangement was shown schematically in Figure 2.2, but a more detailed plan of the tank is given in Figure 3.1.

As stated in Chapter 1 the model duck spine to be tested in this tank is 1:150 the size of the anticipated full size device. The typical energy period of waves in the North Atlantic, where a wave power installation might be sited, is about 10 seconds. If the similarity condition of constant diameter/wavelength is to be satisfied then the energy period at tank (1:150) scale must be about 1 second. The tank was, therefore, designed for optimal performance at frequencies near to one hertz.

3.1.2 Wave Height Variations Over the Wide Tank

If the wide tank is to provide a useful model of the real sea then it must be capable of reproducing seas which have constant wave height over the entire working area. There will be variations in the energy density near to the tank boundaries due to reflections off the glass and imperfect beach and wavemaker absorption. It is also anticipated that waves will be attenuated as they pass along the side of the tank lined by beaches. These and other effects may limit the useful working area of the tank and it has become clear that a mapping of the energy density, or RMS wave height, is necessary before further experimental work can be performed.
As is shown in Figure 3.1 there are walkways in front of the wavemakers and over the opposite beaches. It was assumed that the area of the tank under each of these walkways was unlikely to be used for experimental purposes and therefore the length of the tank was limited to 5m. The areas within 1m of the glass and 1m of the far beaches was also assumed to be unused which left a working area of 23m wide by 5m long.

The walkways over the beaches and wavemakers were marked off in 1m lengths using the grid which was marked on the tank bottom during construction as a guide. This involved dropping a plumb line from the walkway down to a grid marker and marking the corresponding position on the walkway with adhesive tape. Ten wave gauges were fitted onto a 5m long dexion angle strip mounted on a moveable bridge which spanned the tank. This arrangement is shown in Figure 3.2. The gauges used were of the 3-wire conductivity compensated type, a diagram of which is shown in Figure 3.3. A calibrated current ($I_{\text{ref}}$) is applied to wire (2), establishing the voltage ($V_{\text{ref}}$) needed to give a fixed current per unit length of wire immersed. $V_{\text{ref}}$ is then applied to wire (1) through a buffer and the current flow is measured. This can be simply shown as follows

\[ V_{\text{ref}} \alpha \frac{I_{\text{ref}}}{W_c} \]  

[3.1]  

where $W_c$ = conductivity of water.

Hence

\[ I_{\text{mea}} \alpha V_{\text{ref}} \times W_c \times D_{\text{im}} \]  

[3.2]
FIGURE 3.2

Photograph of Wavegauge Bridge
FIGURE 3.3

Diagram of a Conductivity Gauge
where $\text{Dim} = \text{depth of immersion}$

$I_{\text{mea}} = \text{the measured current.}$

Therefore

$$I_{\text{mea}} \propto \frac{I_{\text{ref}}}{W_C} \times W_C \times \text{Dim} \quad [3.3]$$

Hence

$$I_{\text{mea}} \propto \text{Dim} \quad [3.4]$$

The wave height can therefore be calculated from the measurement of the current $I_{\text{mea}}$. It is convenient to remove the DC component of the measurement of $I_{\text{mea}}$ and thus avoiding having to examine the still water in order to determine the wave height. This cannot, however, be done when the gauges are being calibrated as the gauges are progressively lifted through known distances and the DC measurement of $I_{\text{mea}}$ noted. The constant of proportionality between $I_{\text{mea}}$ and $\text{Dim}$ can therefore be determined. It is possible to adjust $I_{\text{ref}}$ in these gauges to keep the constant of proportionality near some normal figure but it is usually easier to note the value of the constant and use this value to relate $I_{\text{mea}}$ to $\text{Dim}$. The outputs from the ten gauges mounted upon the bridge were multiplexed to avoid excessive quantities of cable being required and the calculations to determine the measured wave heights were performed on a PDP11/60 mini-computer.

The experimental procedure involved placing the wave gauge bridge at one of the metre marks on the walkways. The wavemakers would then be started and, after a period of 30s to allow transient effects to decay, the sampling began. The waves were sampled over the repeat time of the sea state or, if regular waves were used, over an integral number
PLAN OF TESTING AREA

FIGURE 3.4

RMS Wave Height Distribution for 0.8Hz Waves
of wave periods. The wave gauge bridge was then moved to the next metre mark on the walkway and the process repeated. It was found that continually moving the bridge caused the calibration of the gauges to drift and continual re-calibration was necessary if the potential errors in measurement were to be kept below 5%.

Figure 3.4 shows the RMS wave height in the working area for waves of a frequency 0.8Hz, nominal amplitude 2cm, propagating normally to the wavemakers. The sampling time in this test was 51.2 seconds. The variation of RMS wave height is large in this case, ranging from less than 1cm in the top left hand corner, where the waves have been attenuated by the far beaches, to over 1.55cm at isolated points in the tank. There is evidence in the graph of standing wave effects caused by reflections by the beaches opposite the wavemakers. The peaks which can be seen 3m from the bottom of the testing area have a separation of about 1.2m which compares with a wavelength at this frequency of 2.4m (approximately). Similar stationary peaks can be seen about 2m from the far side of the working area. It should be pointed out that the near side standing wave corresponds to the position in the tank at which the so-called '50 year' freak wave is frequently demonstrated and that it appears likely that the resulting battering of the beaches has diminished their effectiveness.

The average RMS wave height over the testing area is 1.32cm with an average deviation from this mean of 1mm. This is about 6% lower than the expected RMS of 1.41cm. This deviation is at the limit of the expected drift of the gauges and modifications of the wavemaker transfer functions would be a lengthy process and unlikely to produce a more accurate reproduction.
Figure 3.5 shows the RMS wave height in the working area for waves of frequency 1.0Hz, nominal amplitude 2cm and propagating normally to the wavemakers. The sampling time was 51.0 seconds. The variation in this case is lower than the 0.8Hz situation ranging from 1.15cm to 1.55cm. The extreme values are once again in the top left hand corner, demonstrating attenuation by the beach, where the RMS falls to 1.15cm and 3m from the bottom where peaks can be seen. The indication in this case is that the peaks are, once again, half a wavelength apart but the wavelength is now 1.57m and a peak separation of 0.78m is less easily resolved with gauges separated by 0.56m than were the 0.8Hz peaks.

The average RMS height over the testing area is 1.40cm with an average deviation from this mean of 0.8mm. This corresponds much better to the nominal RMS height of 1.41cm than the 0.8Hz distance. This is to be expected as Salter (1981) stated that the wavemakers are tuned for optimal performance at 1.0Hz.

Figure 3.6 shows the RMS wave height in the working area for waves of a frequency of 1.2Hz, nominal amplitude 2cm and propagating normally to the wavemakers. The sampling time was 51.6 seconds. The variation in wave height is from 1.25cm to 1.45cm but shows certain features in common with the earlier graphs. The lowest RMS heights are once again in the top left hand corner and the highest three metres from the bottom. The wavelength at this frequency is 1.08m so that any expected peaks due to reflections would be closer together than are the wave gauges and so the finite nature of the variation is difficult to evaluate. The figure does however show less variation overall which suggests that the beaches are more effective at high frequencies. This is to be expected as, for effective absorption, the beaches must be at least as
FIGURE 3.5

RMS Wave Height Distribution for 1.0Hz Waves
FIGURE 3.6
RMS Wave Height Distribution for 1.2Hz Waves
wide as the incident wavelength otherwise scattering as discussed in Section 1.4.1 will become significant. The beaches are 2m wide so it would appear that their efficiency might be expected to be low for 0.8Hz as was suggested by Figure 3.4.

The average RMS height is 1.41cm with a mean deviation of 0.8mm. This is in good agreement with the nominal height and confirms the accuracy of the wavemaker transfer functions at this frequency.

Figure 3.7 shows the RMS wave height in the working area for waves of frequency 1.4Hz, nominal amplitude 2cm and propagating normally to the wavemakers. The variation in RMS wave height is very similar to the 1.2Hz case. The minimum value of 1.25cm is, as usual, in the far left hand corner of the tank. The maximum RMS of 1.5cm still appears 3m from the bottom although, because of the wavelength at this frequency is 0.80m, little fine detail of the amplitude variation can be seen.

The average RMS height is 1.35cm with a mean variation of 0.8mm. This is somewhat lower (4%) than the nominal value of 1.41cm but is within the expected 5% error of the gauges.

Figure 3.8 shows the RMS wave heights in the working area for 'random' waves possessing a 1.0 second energy period Pierson-Moskowitz 'total' spectrum with Mitsuyasu spreading about the normal to the wavemakers. The spectrum was simulated using the equal sized tooth technique, described in section 2.2, consisting of 72 fronts with the frequency spacing chosen so that the sea repeated after 51.2 seconds. This being the sampling time at each bridge position.

The distribution of RMS height has its minimum, as expected, in the far left hand corner of the working area but other variations in the wave
FIGURE 3.7

RMS Wave Height Distribution for 1.4Hz Waves
FIGURE 3.8
RMS Wave Height Distribution for a 1.0 Second Pierson Moskowitz Sea With Mitsuyasu Spreading
height are minimal being very much less pronounced than those for regular waves of frequency close to the mean frequency of the mixed sea (e.g. Figure 3.5). The average RMS wave height is 1.32cm with a mean variation of 0.6mm over the tank. This is much smoother than the 1.0Hz case which had a mean variation of more than 1mm. The theoretical RMS wave height for a 1.0 second PM sea is 1.36cm so the measured waves are 3% lower which is within the expected error of the wave gauges.

The mixed frequency 'snake' generation system involves the superposition of 72 wave fronts each of which has a different starting phase (equation [2.6]). The starting phase is a random number between 0 and 2π which is usually generated using a linear congruential technique which is demonstrated in Figure 3.9. The initial value of the input to the generation function RAND(X) which is typically of the form

\[ \text{RAND}(X) = \text{Fractional Part}\{(A \times X + B)/C\}, \]

where A, B and C are constants, is called the seed. The effect on the RMS wave height distribution of changing the random number used in the sea state of Figure 3.8 is shown in Figure 3.10. It is difficult to compare 3.8 and 3.10 but Figure 3.11 shows the difference between them. It can be seen how similar the two distributions are. The change in seed produces at no place a change of more than 5% and the average RMS wave height in Figure 3.10 is only 0.1mm greater than in Figure 3.8.
FIGURE 3.9

Flowchart of Linear Congruential Pseudo Random Number Generator
FIGURE 3.10
RMS Wave Height Distribution for a 1.0 Second Pierson Moskowitz Sea With Mitsuyasu Spreading After a Change of Random Number Seed
PLAN OF TESTING AREA

FIGURE 3.11
Percentage Difference Between Figure 3.8 and Figure 3.10
FIGURE 3.12
Photograph of the University of Edinburgh Physics Department Narrow Tank

FIGURE 3.13
Wave Force Measurement Rig
Variation in Horizontal Forces Due to Wave Amplitude

where $a =$ wave amplitude, $t_d =$ hub depth, $\lambda =$ wavelength, $D =$ diameter and $f =$ frequency
HORIZONTAL FORCE (Experimental)
VARIATION WITH FREQ.

\[ \frac{a}{D} = 0.125, \quad \text{td} = 0 \]

FIGURE 3.15a
Variation in Horizontal Forces Due to Wave Frequency

HORIZONTAL FORCE (Theoretical)
VARIATION WITH FREQ.

\[ \frac{a}{D} = 0.125, \quad \text{td} = 0 \]

FIGURE 3.15b
Variation in Horizontal Forces Due to Wave Frequency

where \( a \) = amplitude, \( \text{td} \) = hub depth, \( \lambda \) = wavelength, \( D \) = diameter and \( f \) = frequency
where $a$ = amplitude, $td$ = hub depth, $D$ = diameter

$\lambda$ = wavelength and $f$ = frequency
where $a = \text{amplitude}$, $t_d = \text{hub depth}$, $D = \text{diameter}$

$\lambda = \text{wavelength}$ and $f = \text{frequency}$
### Horizontal

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### Vertical

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**TABLE 3.1**

Empirically Determined Values of CH and Cs in Regular Waves

where a = amplitude, td = hub depth, D = diameter, λ = wavelength and f = frequency
3.1.3 Conclusions of Wave Height Tests

1. Irregular seas give a more even RMS wave height distribution than do regular ones.
2. The RMS wave height distribution is more even at the centre of the tank than near the glass or the far beaches.
3. The change of a random number seed appears to have little effect on the RMS distribution of a 72 component sea.

3.2 Determination of Hydrodynamic Parameters

3.2.1 Inertial Wave Forces

The cylinders examined in this section have a diameter of 9cm and the wavelengths of the waves range from 3.2m down to 0.8m. The maximum ratio of diameter to wavelength is therefore 1:8. This means that diffraction theory does not need to be used as an examination of Figure 1.1 shows this to be well within the limit of 1:5. The waves had an amplitude of up to 2cm which means that the extreme ratio of amplitude to diameter is 2:9 which Figure 1.1 shows to be in the region where the drag terms in Morison's equation (equations [1.28], [1.29] and [1.30]) can be neglected.

The values of the inertial parameters used in equation [1.27] were determined empirically from experimental data. The method used by the author was to find, using a least squares fit program, the values of $C_H$ and $C_S$ which minimised the deviation of the forces predicted by equation [1.28], over one wave cycle, from the wave forces measured in the narrow tank belonging to the fluid dynamics group in the University of Edinburgh's Department of Physics (Figure 3.12). Monochromatic waves were produced by an absorbing wavemaker at one end of the tank. A rig (Figure 3.13) held the cylinder in position at
different hub depths and forces were measured using strain gauges. This procedure is described in greater detail by Dixon et al (1979). As in Dixon's work no attempt is made to vary the values of the force coefficients over a cycle. However, Keulegan and Carpenter (1958) did attempt to analyse the variation of the force coefficients with time. Figures 3.14a to 3.17a show, for a variety of situations, the form of the experimentally determined forces and show trends observed with increasing hub depth and frequency. In particular the heave force can be seen to include components at twice the wave frequency especially when the hub depth approaches the totally submerged case. Figure 3.14b to 3.17b show the corresponding predicted forces using the appropriate values of $C_H$ and $C_S$ to produce the closest possible fit between experimental forces and those predicted by the inertial parts of equation [1.28]. Table 3.1 displays the required values of $C_H$ and $C_S$ over a wide range of frequencies and hub depths.

3.2.2 Radiation Tests

In Section 1.4 it was stated that a body oscillating near to the free surface experiences a retarding force proportional to its velocity and that the energy subtracted from the motion in this way goes into generating waves travelling out from the body. The radiation condition for two dimensional radiation problems results in the amplitude of the outgoing waves, when far from the cylinder, being given by

$$\eta(t,y) = A \cos(\omega t - ky)$$

where $y$ = distance along the wave tank.

If the displacement (heave, say) is
\[ z(t) = A_0 \cos(\omega t) \]

then the velocity is

\[ z(t) = -A_0 \omega \sin \omega t \]

If the cylinder experiences a damping force

\[ F_o = -Q \nu \]

where \( \nu = \) velocity, then the energy lost in one cycle is

\[ E = \int_{0}^{T} Q A_0^2 \omega^2 \sin^2(\omega t) \, dt \]

Hence

\[ E = Q A_0^2 \omega^2 \int_{0}^{T} \sin^2(\omega t) \, dt \]

Hence

\[ E = Q A_0^2 \omega^2 \left[ \frac{t}{2} - \frac{1}{2\omega} \sin(2\omega t) \right]_{0}^{T} \]

Hence

\[ E = Q A_0^2 \omega^2 \left\{ T \cdot \frac{1}{2\omega} \sin(2\omega T) \right\} \]

Hence

\[ E = Q A_0^2 \omega^2 \frac{2\pi}{\omega} = Q A_0^2 \omega \]

The power dissipation is, therefore,

\[ F_D = QA_0^2 \omega^2 \]
FIGURE 3.18
Oscillating Cylinder Rig
The energy density of waves of amplitude $A_1$ is given by equation [1.18], i.e.

$$\bar{E} = \rho g A_1^2 / 2$$

The energy dissipated during one cycle is $\bar{E} \times \lambda$, where $\lambda = \text{wavelength}$.

In deep water

$$\lambda = 2\pi g / \omega^2$$

Hence

$$\bar{E} / \text{cycle} = \rho g^2 A_1^2 \frac{\pi}{\omega^2}$$

The power dissipation due to the waves is

$$P_1 = \frac{\rho g^2 A_1^2}{2\omega}$$

Finite sized oscillations tend to produce waves of more than one frequency and if the wave train consists, therefore, of a summation of waves each specified by a frequency $\omega_i$ and an amplitude $A_i$ then the power dissipated by the waves is given by

$$\bar{P}_T = \sum_{i=1}^{N} \frac{\rho g^2 A_i^2}{2\omega_i}$$

[3.5]

The value of the damping coefficient required to produce this dissipation can be found using the value of $\bar{P}_T$ (equation [3.5]) and $\bar{P}_0$ from equation [3.4].

The experiments to determine the damping on a cylinder were performed in the narrow tank belonging to the University of Edinburgh Wave Power Project. A 2.5cm diameter cylinder was held in a rig (Figure 3.18) so
that it could be driven electrically in either surge or heave. The
symmetry of the cylinder ensures that it will radiate energy away
from itself equally to both the left and to the right (Figure 3.19).
The waves need only be analysed on one side and the power of a single
side doubled to give the total power radiated. The wave trains were
analysed using an FFT routine which returned the amplitudes and
frequencies of the component waves for use in equation [3.5]. The
amplitude of the cylinder oscillation was measured electrically by
integrating the velocity signal produced by transducers in the driving
mechanism. It was found that 97% of the total power of the waves was
carried by waves of the driving frequency $\omega$ and its second harmonic
$2\omega$. This appears to be analogous to the mechanism producing the
double harmonic wave forces as demonstrated in Figure 3.17. The power
in the second harmonic does however decrease as the amplitude of
oscillation decreases and the system becomes more linear. A variety of
the cases studied are shown in Tables 3.2 and 3.3 which covers
variations in hub depths and frequencies.

3.3 Measurements of Bending Moments

The University of Edinburgh Wave Power Group have built a long flexible
spine which is designed to be the backbone of an array of wave power
devices known as ducks (Section 1.6). This model is not continuous
but consists of jointed segments as is shown in Figures 3.20 and 3.21.
The spine segments are each 40cm long and orientated so that heave
joints are separated by surge joints. This means that the separation
between joints flexing in the same direction is 0.8m. The response of
the spine will not, therefore, match that of a continuous elastic pipe
for very short crested waves. Figure 3.22 shows a cut-away section of
a spine segment and Figure 3.23 is a schematic representation. The
FIGURE 3.19

Radiation of Waves from a Heaving Cylinder
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Amplitude (cm)</th>
<th>td/D</th>
<th>$Q_H \ (\text{N}\cdot\text{s/m}^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.63</td>
<td>2.01</td>
<td>0.5</td>
<td>16.16</td>
</tr>
<tr>
<td>0.63</td>
<td>2.37</td>
<td>0.5</td>
<td>18.16</td>
</tr>
<tr>
<td>0.83</td>
<td>0.836</td>
<td>0.5</td>
<td>17.28</td>
</tr>
<tr>
<td>0.83</td>
<td>0.515</td>
<td>0.5</td>
<td>20.15</td>
</tr>
<tr>
<td>0.83</td>
<td>0.773</td>
<td>0.5</td>
<td>20.62</td>
</tr>
<tr>
<td>0.83</td>
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<td>0.5</td>
<td>21.05</td>
</tr>
<tr>
<td>0.83</td>
<td>1.575</td>
<td>0.5</td>
<td>21.33</td>
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<tr>
<td>0.83</td>
<td>1.94</td>
<td>0.5</td>
<td>19.27</td>
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<td>0.88</td>
<td>0.87</td>
<td>0.5</td>
<td>29.27</td>
</tr>
<tr>
<td>0.88</td>
<td>1.07</td>
<td>0.5</td>
<td>32.39</td>
</tr>
<tr>
<td>0.88</td>
<td>1.24</td>
<td>0.5</td>
<td>32.15</td>
</tr>
<tr>
<td>0.88</td>
<td>1.43</td>
<td>0.5</td>
<td>27.15</td>
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<td>0.88</td>
<td>1.94</td>
<td>0.5</td>
<td>30.56</td>
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<tr>
<td>0.9</td>
<td>0.675</td>
<td>0.5</td>
<td>29.22</td>
</tr>
<tr>
<td>0.9</td>
<td>0.831</td>
<td>0.5</td>
<td>39.18</td>
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<td>0.9</td>
<td>0.911</td>
<td>0.5</td>
<td>41.15</td>
</tr>
<tr>
<td>0.9</td>
<td>1.15</td>
<td>0.5</td>
<td>43.03</td>
</tr>
<tr>
<td>0.9</td>
<td>1.69</td>
<td>0.5</td>
<td>41.49</td>
</tr>
<tr>
<td>1.1</td>
<td>1.49</td>
<td>0.5</td>
<td>38.82</td>
</tr>
<tr>
<td>1.1</td>
<td>1.09</td>
<td>0.5</td>
<td>48.55</td>
</tr>
<tr>
<td>1.25</td>
<td>0.487</td>
<td>0.5</td>
<td>38.82</td>
</tr>
<tr>
<td>1.25</td>
<td>1.24</td>
<td>0.5</td>
<td>48.55</td>
</tr>
</tbody>
</table>

**TABLE 3.2**

Experimentally Determined Values of $Q_H$
<table>
<thead>
<tr>
<th>Frequency</th>
<th>Amplitude (cm)</th>
<th>td/D</th>
<th>Qs (Ns/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.63</td>
<td>0.638</td>
<td>0.5</td>
<td>18.78</td>
</tr>
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<td>27.84</td>
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<td>0.5</td>
<td>28.01</td>
</tr>
<tr>
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<td>0.5</td>
<td>26.99</td>
</tr>
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<td>0.88</td>
<td>0.787</td>
<td>0.5</td>
<td>153.4</td>
</tr>
<tr>
<td>0.88</td>
<td>1.010</td>
<td>0.5</td>
<td>154.0</td>
</tr>
<tr>
<td>0.88</td>
<td>1.256</td>
<td>0.5</td>
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<td>1.789</td>
<td>0.5</td>
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</tr>
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<td>0.5</td>
<td>502.1</td>
</tr>
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<td>0.384</td>
<td>0.5</td>
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</tr>
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<td>0.693</td>
<td>0.5</td>
<td>497.3</td>
</tr>
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<td>0.5</td>
<td>493.3</td>
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<td>0.5</td>
<td>452.3</td>
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<td>1.15</td>
<td>0.5</td>
<td>433.3</td>
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<td>0.5</td>
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<td>0.5</td>
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<td>0.83</td>
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<td>0.83</td>
<td>252.3</td>
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<td>0.83</td>
<td>239.0</td>
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<tr>
<td>0.63</td>
<td>1.15</td>
<td>0.83</td>
<td>23.3</td>
</tr>
</tbody>
</table>

**TABLE 3.3**

Experimentally Determined Values of Qs
FIGURE 3.20
Photograph of the Edinburgh 'Wave Power' Spine

FIGURE 3.21
Close Up Photograph of a Spine Joint
FIGURE 3.22

Photograph of a Cut Away Spine
FIGURE 3.23

Diagram of Spine Components
rotation of spine (1) about the hinge (2) moves the torque arm (3). The torque about the hinge is measured using the strain gauge bridges (4) and (5). The far end of the torque arm is connected to a toothed rubber belt (6) which converts the reciprocating motion of the arm into rotational motion. The rotating shaft is passed through a transducer (7) which sends an electric signal proportional to the angular velocity of the shaft to an analogue to digital converter and then to a micro-computer. This computer integrates the velocity signal to get a signal proportional to the displacement of the joints about the central position. A signal is then sent back to the motor (8) which is proportional to the negative displacement and as such produces a force analogous to a torsional spring. The constant of proportionality between the displacement and the restoring force can be set by the experimenter to make the spine as floppy or as stiff as he chooses. It is also possible to simulate torsional damping about the joints, which can be used to model power extraction from the hinges in a full scale duck string. This is done by driving the joint motors with a signal proportional to the velocity signal from the transducers. The stability of the spine with this kind of feedback has been found however to be very low.

The spine was moored using a variety of weights and floats so that drift forces could be countered and the spine could not drift out of position. These moorings also ensured that the spine, which is very slightly buoyant, was kept just submerged with the outside tangential to the free surface. The mooring arrangement is shown in Figure 3.24.
MOORING ARRANGEMENT

FIGURE 3.24
Diagram of Spine Moorings
CHAPTER 4

ANALYTIC SPINE ANALYSIS
CHAPTER 4

ANALYTIC SPINE ANALYSIS

4.1 Introduction

In Chapter 3 the hydrodynamic forces on fixed and moving objects were examined experimentally and attempts were made to fit theoretical force equations to the results. In this chapter it is intended that the experience gained on fixed cylinders and electrically driven cylinders will be drawn upon to develop an analytic model of a long floating pipe.

The object of interest is a long jointed structure described in Chapter 3 which is intended to be the back-bone of a string of Salter Duck wave energy devices. The spine is ballasted and moored so that it lies just submerged in still water.

It is hoped that certain gross effects of wave excitation on the spine can be predicted by a linear analytic model as described in this chapter. The properties which are of greatest interest to engineers developing wave power devices are the maximum and RMS bending moments. The maximum moments might occur when the spine is struck by a freak wave as discussed in Section 2.2.1 or described by Dawson (1977). RMS moments indicate the likelihood of fatigue fractures as described by Fairbairn (1864) and Hardrath et al (1958). The earlier investigation used a mechanism driven by a water wheel to apply, repeatedly, a load to the centre of a 6.7m long wrought iron girder. The girder was found to break statically under a central load of 120kN but a repeated load of 30kN would eventually cause fracture. The 1958
investigation involved subjecting an alloy beam to a repeated bending moment and observations were made of the growth of fractures.

4.2 **Structural Treatment of Pipe**

The experimental spine model is, as described in Chapter 3, a jointed one in which the torsional stiffness of the joints can be varied. It is, however, of more general interest to treat the spine as a continuous elastic beam which can flex vertically and horizontally. The properties of such a beam are well known and are discussed by many authors, such as Timoshenko (1945).

If a force density \( W(x) \) is applied to a pipe lying along the x-axis then, if \( E \) is the Young's Modulus of the pipe material and \( I \) is the moment of inertia of the cross section, the pipe will deform according to the equations

\[
W_z(x) = -EI \frac{\partial^4 z}{\partial x^4} \quad \text{(vertical displacement)} \tag{4.1}
\]

and

\[
W_y(x) = -EI \frac{\partial^4 y}{\partial x^4} \quad \text{(horizontal displacement)} \tag{4.2}
\]

The bending moments induced by such forces are given by

\[
M_z(x) = -EI \frac{\partial^2 z}{\partial x^2} \tag{4.3}
\]

and

\[
M_y(x) = -EI \frac{\partial^2 y}{\partial x^2} \tag{4.4}
\]

Similarly the shear forces are given by
\[ Sh_z(x) = -EI \frac{\partial^3 z}{\partial x^3} \]  \hspace{1cm} [4.5] 

and

\[ Sh_y(x) = -EI \frac{\partial^3 y}{\partial x^3} \]  \hspace{1cm} [4.6]

At a free end of a pipe there can be neither bending moments or shear forces and so the end conditions can be expressed as

\[ \frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 y}{\partial x^2} = \frac{\partial^3 z}{\partial x^3} = \frac{\partial^3 y}{\partial x^3} = 0 \]  \hspace{1cm} [4.7]

The link between the flexural rigidity (EI) of a continuous beam and the stiffness of a joint in a spine as described in Chapter 3 is given by the Universal Beam Equation. If \( M \) = bending moment and \( R \) = radius of curvature of the beam then

\[ \frac{M}{I} = \frac{E}{R} \]

If the stiffness of a joint is \( S \), so that an angle of \( \theta \) produces a moment of \( M = -\theta S \), then

\[ \frac{S\theta}{I} = \frac{E}{R} \]

but

\[ \theta = \frac{L}{R} \]

(Figure 4.1), where \( L \) = joint separation, so

\[ \frac{S\theta}{I} = \frac{E\theta}{L} \]

Hence
The spine segments are 40cm long but the joints are arranged to flex alternately in surge and heave resulting in an effective joint separation of 0.8m. It is therefore possible to equate results evaluated with a continuous spine to those measured in a jointed one.

4.3 Wave Force Assumptions

4.3.1 Three Dimensional Nature of Pipe and Waves

The force equations discussed in Section 1.3.3 related to 2 dimensional waves such as those in a long narrow wave flume. In a real sea, however, the waves and structures are three dimensional. The hydrodynamic assumptions required for Section 1.3.3 are limited and exclude effects such as diffraction around the body. The radiation from a body in three dimensions is also rather more complicated than discussed in Section 3.3. The pipes in this work are, however, assumed to be long in comparison to wavelengths and are constrained to respond only in the y and z directions. Deflections of the pipe in the y-direction are assumed to be small so that the angle of incidence of an incoming regular wave to the pipe is constant along its length. These assumptions limit any large three dimensional effects to near the pipe ends. It is assumed in this analysis that effects caused by three dimensional diffraction and radiation are small in comparison to structural end effects.

Regular waves incident on the pipe are considered to have wave elevations given by

$$\eta(t, x) = \text{Amp} \cos(\omega t - k_x x + \phi)$$
FIGURE 4.1

Description of Terms Relating to a Jointed Spine

FIGURE 4.2

Description of Terms Relating to a Semi-submerged Cylinder
where \( t \) = time
\( x \) = distance along pipe
Amp = wave amplitude
\( \omega \) = frequency
\( k_x \) = \( x \)-component of wave vector
\( \phi \) = starting phase.

If \( \theta \) is the angle between the direction of wave propagation and the normal to the pipe then

\[
k_x = \frac{\omega^2}{g} \sin \theta
\]

according to deep water linear wave theory where \( g \) = gravitational acceleration.

4.3.2 Linearity Assumptions

The movements of the pipe are assumed small so that the response is linearised. The total force on a section of pipe can be separated into two types, the fluid forces and the external forces. The fluid forces can be separated into the forces which act on a moving object in still water and the forces on a fixed object in waves. In a linear system, as is assumed here, these forces can be added to obtain the total force as is discussed by Standing (1979) who derived equation [4.8] for the response of an object to wave excitation.

\[
(M + A) \ddot{\mathbf{r}} + B \dot{\mathbf{r}} - (G + S) \mathbf{r} = \mathbf{F} + \mathbf{Ext} \tag{4.8}
\]

where
\( A \) = added mass tensor
\( B \) = added damping tensor
\( G \) = added spring tensor
\[ S = \text{restoring spring tensor} \]
\[ F = \text{fluid forces} \]
\[ \text{Ext} = \text{external forces} \]
\[ M = \text{mass}. \]

It is usual, as described in Section 1.4, to express the added mass using an added mass coefficient tensor \( k \). Hence

\[ A = \rho k V \]

where \( \rho \) = \text{fluid density}
\[ V = \text{displacement volume}. \]

If the surge/heave coordinate system is used then, if we are considering a partially submerged horizontal cylinder, the added spring will be purely in the heave direction and related to the depth of submergence of the hub depth or, more precisely, the 'effective diameter' \( (d' \) in Figure 4.2) by

\[ G_H = -\rho g d' \] \[ [4.9] \]

The added damping is treated exactly as in Section 1.4 by linear damping parameters determined as in Section 3.3.

The wave force equation is a linear version of [1.27]

\[ F(t,x) = C_D A(t,x) - \rho g d' \eta(t,x) \] \[ [4.10] \]

= force/unit length

where \( U(t,x) = \text{local fluid element velocity} \)
\[ \eta(t,x) = \text{wave elevation} \]
\[ A_0 = \text{volume of water displaced by a unit length of cylinder in still water.} \]

It will be assumed here that the values of the inertial and damping parameters are determined by the free floating hub depth.

### 4.3.3 Differential Equations

If the surge/heave coordinate system is used then the surge/heave problems can be assumed to be separate. The validity of this assumption is tested in Chapter 5. The external forces acting on an infinitesimal section of pipe of length \( dx \) include mooring forces, which are neglected in this analysis, and the flexural forces discussed in Section 4.2. The equations of motion are, therefore, given by

\[
\text{Surge: } \left\{ \left( \rho_p + k_s A_0 \right) y + Q_s y + EI \frac{\partial^4 y}{\partial x^4} \right\} dx = C_{s} A_0 U_s dx \tag{4.11}
\]

and

\[
\text{Heave: } \left\{ \left( \rho_p + k_h A_0 \right) z + Q_h z + \rho g z^\prime + EI \frac{\partial^4 z}{\partial x^4} \right\} dx = C_{h} A_0 U_h dx - \rho g \ddot{n}(t,x) dx \tag{4.12}
\]

where \( \rho_p = \text{density of pipe.} \)

### 4.3.4 Adjustments for Scale

The narrow tank tests on fixed cylinders to determine inertial force parameters were conducted on cylinders of diameter 9cm. In order to satisfy the similarity of the wavelength to diameter ratio the frequencies must be considered before the narrow tank tests can be applied to the 12.5cm diameter duck spine. There is similarity between a wide tank test with frequency \( f_1 \) and a narrow tank test of frequency...
The modification to the frequency scale required for similarity can be seen in Figure 4.3 and Figure 4.4 which show the surge and heave inertial coefficients as a function of frequency for the wide tank spine diameter of 12.5cm lying in its usual tangentially submerged state.

The adjustments in model and small scale simulation results required for prediction of results at the full scale can be made according to the following scaling conditions:

**Flexural Rigidity (EI) at tank scale = \( \left( \frac{1}{150} \right)^5 \) Full Scale rigidity**

**Bending Moment at tank scale = \( \left( \frac{1}{150} \right)^5 \) Bending Moment at full scale**

These are arrived at using the normal scaling laws as discussed by Jeffries et al (1978, Vol 1).

### 4.4 Prediction of Spine Responses

#### 4.4.1 Solution of Differential Equations for Regular Waves

The equations governing the motion can be expressed in the forms

\[
\gamma_H \frac{\partial^2 Z}{\partial t^2} + Q_H \frac{\partial Z}{\partial t} + EI \frac{\partial^4 Z}{\partial x^4} = F_{OH} \cos(\omega t - k_x x) \quad \text{(Heave)} \tag{4.13}
\]

where \( \gamma_H = \rho p A_0 + \rho A_1 k_H \)

\( F_{OH} = C_{H \rho A_1} \omega^2 - \rho g d' \)

and

\( f_2 \) if

\[
f_1^2(0.125) = f_2^2(0.09)
\]
FIGURE 4.3

Experimentally Determined Inertial Force Coefficients at
Wide Tank Scale
URGE COEFFICIENTS WIDE TANK SCALE

see Table 3.1

FIGURE 4.4

Experimentally Determined Inertial Force Coefficients at Wide Tank Scale
\[
\gamma_s \frac{\partial^2 y}{\partial t^2} + Q_s \frac{\partial y}{\partial t} + EI \frac{\partial^4 y}{\partial x^4} = F_{os} \sin(\omega t - k_x x) \quad \text{(Surge)} \quad [4.14]
\]

where \( \gamma_s = \rho p A_0 + \rho A_1 k_s \)

\[ F_{os} = C_s p A_1 \omega^2 \]

The solution of both the surge and heave problems will be similar as only the values of the parameters and phase of the force and not the equation forms differ. The heave case is to be solved here.

It is convenient to adopt the complex exponential approach for this problem, i.e.

\[ F_0 \cos(\omega t - k_x x) = \text{Re}\{\phi(t,x)\} = \text{Real part of } \phi(t,x) \]

where

\[ \phi(t,x) = F_0 e^{i(\omega t - k_x x)} \]

Similarly

\[ Z = \text{Re}\{\beta(t,x)\} \]

The equation

\[
\gamma_h \frac{\partial^2 \beta(t,x)}{\partial t^2} + Q_h \frac{\partial \beta(t,x)}{\partial t} + EI \frac{\partial^4 \beta(t,x)}{\partial x^4} = F_0 e^{i(\omega t - k_x x)} \quad [4.15]
\]

can, therefore, be solved to find \( \beta(t,x) \) and then the real part taken to find \( Z \).

It is assumed that the spine to be tested has reached a steady state. This means that the oscillation is such that
\[ \beta(t,x) = X(x)e^{i\omega t} \quad [4.16] \]

If [4.16] is substituted into equation [4.15] then we obtain

\[ Y_H \frac{\partial^2}{\partial t^2} (X(x)e^{i\omega t}) + Q_H \frac{\partial}{\partial t} (X(x)e^{i\omega t}) + EI \frac{\partial^4}{\partial x^4} (X(x)e^{i\omega t}) = F_0 e^{i\omega t} e^{-ikx} \quad [4.17] \]

Hence

\[ -Y_H \omega^2 X(x)e^{i\omega t} + iQ_H \omega X(x)e^{i\omega t} + EI \frac{\partial^4}{\partial x^4} (X(x)e^{i\omega t}) = F_0 e^{i\omega t} e^{-ikx} \quad [4.18] \]

This equation can be simplified to

\[ -Y_H \omega^2 X(x) + iQ_H \omega X(x) + EI \frac{\partial^4}{\partial x^4} (x) = F_0 e^{-ikx} \quad [4.19] \]

This is a non-homogeneous complex equation of the fourth order. The equation now needs to be solved for its particular integral and for the independent solutions to the homogeneous equation, i.e.

\[ -Y_H \omega^2 X(x) + iQ_H \omega X(x) + EI \frac{\partial^4}{\partial x^4} (x) = 0 \quad [4.20] \]

**Particular Integral**

Assume a \( X(x) \) to be given by

\[ X(x) = A e^{-ikx} \]

Hence

\[ \frac{\partial^4}{\partial x^4} (x) = A k^4 e^{-ikx} \]

The substitution of these equations into [4.19] yields

\[ -Y_H \omega^2 A e^{-ikx} + iQ_H \omega A e^{-ikx} + EIA k^4 e^{-ikx} = F_0 e^{-ikx} \quad [4.21] \]
Hence

\[-\gamma \omega^2 A + iQH\omega A + EI\Delta k_x^4 = F_0\]

Hence

\[A \{(EIk^4 - \gamma \omega^2) + iQH\omega\} = F_0\]

Hence

\[A = \frac{F_0}{\{(EIk^4 - \gamma \omega^2) + iQH\omega\}} \quad [4.22]\]

It is convenient to express this in terms of a real and imaginary component, i.e.

\[A = F_0 \frac{\{(EIk^4 - \gamma \omega^2) - iQH\omega\}}{\{(EIk^4 - \gamma \omega^2)^2 + QH^2\omega^2\}} \quad [4.23]\]

**Independent Solutions to the Homogeneous Equation**

\[-\gamma \omega^2 X(x) + iQH\omega X(x) + EI \frac{d^4 X}{dx^4}(x) = 0 \quad [4.24]\]

This equation, being of the fourth order, will have four independent solutions. They can be found by making the initial assumption that their form is

\[\phi_j(x) = B_j e^{ij\lambda x} \quad \text{where } j = 0, 3 \quad [4.25]\]

The possible values of \(\lambda_j\) can be found by substituting into equation [4.20]. Hence

\[-\gamma \omega^2 B e^{ij\lambda x} + iQH\omega B e^{ij\lambda x} + EI\lambda^4 B e^{ij\lambda x} = 0 \quad [4.26]\]

therefore
\[-\gamma_H\omega^2 + iQ_H\omega + EI\lambda^4 = 0\]

therefore

\[\lambda^4 = \frac{(\gamma_H\omega^2 - iQ_H\omega)}{EI}\]  \[\text{[4.27]}\]

The four possible roots of this equation can be found as follows. First convert the RHS of equation [4.27] to complex exponential form. Hence

\[\lambda^4 = C^4e^{i\theta} = C^4(\cos\theta + isin\theta)\]

where

\[C = \left\{\left(\gamma_H\omega^2 + Q_H^2\omega^2\right)/EI\right\}^{1/4}\]

and \(\theta\), in this case, is

\[-\arctan\left(\frac{Q_H}{\gamma_H\omega}\right)\]

\[\text{[4.28]}\]

\(\lambda\), when expressed in this form, is given by

\[\lambda = |\lambda|e^{i\rho}\]

therefore

\[\lambda^4 = |\lambda|^4e^{i4\rho}\]

Hence, by equating angles

\[4\rho = \theta + n2\pi \quad \text{where } n = 0, 3\]

Hence
when \( n = 0 \) \( \rho = \theta/4 \) and \( \lambda_0 = Ce^{i\theta/4} \)

when \( n = 1 \) \( \rho = \theta/4 + \frac{\pi}{2} \) and \( \lambda_1 = i\lambda_0 = iCe^{i\theta/4} \)

when \( n = 2 \) \( \rho = \theta/4 + \pi \) and \( \lambda_2 = -\lambda_0 = -Ce^{i\theta/4} \)

when \( n = 3 \) \( \rho = \theta/4 + \frac{3\pi}{2} \) and \( \lambda_3 = -i\lambda_0 = -iCe^{i\theta/4} \)

Hence

\[ \lambda_0 = a - ib \]

\[ \lambda_1 = ia + b \]

\[ \lambda_2 = -a + ib \]

\[ \lambda_3 = -ia - b \]

where \( a = C\cos\theta/4 \) and \( b = -C\sin\theta/4 \).

The angle \( \theta \) is in the fourth quadrant so \( a \) and \( b \) are both positive. The total solution of equation [4.19] can, therefore, be expressed in the form

\[ X(x) = Ae^{-ikx} + \sum_{j=0}^{3} B_j e^{i\lambda_j x} \tag{4.29} \]

The values of the constants \( B_j \) can only be evaluated by utilising the boundary conditions at each end of the pipe.

Case 1 - The Infinite Spine

The pipe extends to \( \pm \infty \). The independent solutions are given by
\[ \phi_j(x) = B_j e^{i\lambda j x} \]

Consider

\[ \lambda_0 = Ce^{i\theta/4} \]

The value of \( \theta \), from equation \( [4.30] \), is \(-\frac{\pi}{2} < \theta < 0\) so the imaginary component of this is negative. This means that \( \lambda_0 \) can be written as \( a - ib \), where \( a, b > 0 \). Hence

\[ \phi_0(x) = B_0 e^{iax} e^{bx} \]

This term obviously tends to infinity as \( x \to \infty \) and as such is not physically realistic unless \( B_0 = 0 \). Similarly

\[ \lambda_1 = ia + b \]
\[ \lambda_2 = -a + ib \]
\[ \lambda_3 = -ia + b \]

All suggest solutions which tend to infinity as \( x \) tends to either \( +\infty \) or \( -\infty \). This suggests that the solution of equation \( [4.19] \) for an infinite spine is given by the particular integral as \( B_0, B_1, B_2 \) and \( B_3 = 0 \). Hence

\[ X(x) = Ae^{-ikxx} \]

The parameter \( A \) is, however, complex and defined by equation \( [4.23] \). This can be written in complex exponential form as

\[ A = |A|e^{i\delta} \]
where

$$|A| = \frac{F_0}{\sqrt{(EIk_x - \gamma H\omega^2)^2 + (QH\omega)^2}} \quad [4.30]$$

and

$$\delta = -\arctan\left(\frac{QH\omega}{(EIk_x - \gamma H\omega^2)}\right) \quad [4.31]$$

This enables the solution $X(x)$ to be written as

$$X(x) = |A|e^{-ik_x x + i\delta}$$

The solution of the time dependent equation becomes

$$\beta(t,x) = |A|e^{i(\omega t - k_x x + \delta)}$$

The displacement of the spine as a function of time and position is, therefore, given by

$$Z(t,x) = |A|\cos(\omega t - k_x x + \delta)$$

The displacement follows the force on the spine but with a phase lag of $-\delta$. The displacement is a maximum when the frequency is given by

$$\omega = \frac{EIk_x^4}{\gamma H}$$

**Single Ended Spine**

The spine extends from $x = -\infty$ to 0. This model is useful for consideration of end effects in a very long spine where each end is not likely to affect the other.

The boundary conditions of this case are:
1. The displacement and all subsequent derivatives of the displacement must remain finite as $x \to -\infty$.

2. The bending moments at $x = 0$ must be zero, i.e.

$$\frac{\partial^2 z}{\partial x^2} = 0 \quad \text{at} \quad x = 0 \quad [4.32]$$

3. The shear forces at $x = 0$ must remain zero, i.e.

$$\frac{\partial^3 z}{\partial x^3} = 0 \quad \text{at} \quad x = 0 \quad [4.33]$$

Condition 1. results in

$$\lambda_2 = -a + ib$$

$$\lambda_1 = ia + b$$

being invalid as they both predict infinitely large solutions at $x = -\infty$ unless $B_1, B_2 = 0$.

The solution of equation [4.21] for a single ended spine is, therefore,

$$X(x) = Ae^{-\lambda_0 x} + B_0 e^{i\lambda_0 x} + B_3 e^{i\lambda_3 x} \quad [4.34]$$

or

$$X(x) = Ae^{-\lambda_0 x} + B_0 e^{i\lambda_0 x} e^{bx} + B_3 e^{i\lambda_3 x} e^{-ibx} \quad [4.35]$$

The boundary conditions 2. and 3. must be used to determine $B_0$ and $B_3$.

Boundary condition 2. implies

$$\frac{\partial^2 X(x)}{\partial x^2} = -k_x^2 A - \lambda_0^2 B_0 - \lambda_3^2 B_3 = 0 \quad \text{for} \quad x = 0$$
Boundary condition 3. implies

\[ \frac{\partial^3 X}{\partial x^3}(x) = i k_x^3 A - i \lambda_o^3 B_0 - i \lambda_3^3 B_3 = 0 \quad \text{for } x = 0 \]

Hence

\[ B_0 = \frac{A k_x^2 (k_x + \lambda_3)}{\lambda_o^2 (\lambda_o - \lambda_3)} \tag{4.36} \]

and

\[ B_3 = \frac{A k_x^2 (k_x + \lambda_3)}{\lambda_3^2 (\lambda_o - \lambda_3)} \tag{4.37} \]

\[ \lambda_3 = -i \lambda_o \quad \text{so} \]

\[ B_0 = \frac{A k_x^2 (k_x - i \lambda_o)}{\lambda_o^3 (1 + i)} \tag{4.38} \]

and

\[ B_3 = \frac{-A k_x^2 (k_x + \lambda_o)}{\lambda_o^3 (1 + i)} \tag{4.39} \]

These terms are best written in exponential terms as

\[ B_0 = |B_0| e^{i \alpha} \]

and

\[ B_3 = |B_3| e^{i \beta} \]

where

\[ |B_0| = \frac{|A| k_x^2}{2|\lambda|^3} \left( (k_x - |\lambda| \cos \frac{\theta}{4})^2 + (|\lambda| \sin \frac{\theta}{4})^2 + (|\lambda| \sin \frac{\theta}{4} + |\lambda| \cos \frac{\theta}{4} + k_x)^2 \right)^{\frac{1}{2}} \tag{4.40} \]

and
\( \alpha = \delta - 3 \frac{\theta}{4} + \varepsilon \)

where

\[
\epsilon = -\arctan \frac{(|\lambda|\cos \frac{\theta}{4} + |\lambda|\sin \frac{\theta}{4} + k_x)}{(k_x - |\lambda|\cos \frac{\theta}{4} + |\lambda|\sin \frac{\theta}{4})}
\]

Similarly

\[
|B_3| = \frac{-|A|k_x^2}{|\lambda|^3} \left( (k_x + |\lambda|\cos \frac{\theta}{4} + |\lambda|\sin \frac{\theta}{4})^2 + (k_x - |\lambda|\sin \frac{\theta}{4} + |\lambda|\cos \frac{\theta}{4})^2 \right)^{\frac{1}{2}}
\]

and

\[
\beta = \delta - 3 \frac{\theta}{4} + \xi
\]

where

\[
\xi = -\arctan \frac{(k_x - |\lambda|\sin \frac{\theta}{4} + |\lambda|\cos \frac{\theta}{4})}{(k_x + |\lambda|\cos \frac{\theta}{4} + |\lambda|\sin \frac{\theta}{4})}
\]

The solution of equations [4.19] is, therefore,

\[
X(x) = |A|e^{-ikx+i\delta} + |B_0|e^{i\lambda_0x+i\alpha} + |B_3|e^{i\lambda_0x+i\beta}
\]

Hence

\[
X(x) = |A|e^{i(\delta-kx)} + |B_0|e^{i((a-i\beta)x+\alpha)} + |B_3|e^{(a-i\beta)x+i\beta}
\]

Hence

\[
X(x) = |A|e^{i(\delta-kx)} + |B_0|e^{i(ax+\alpha)} e^{bx} + |B_3|e^{i(\beta-bx)} e^{ax}
\]

The solution of equation [4.15] can therefore be written as
\[ \beta(t,x) = |A|e^{i(\omega t-k_x x+\delta)} + |B_o|e^{i(\omega t+ax+\alpha)}e^{bx} + |B_3|e^{i(\omega t-bx+\beta)e^{ax}} \]

The equation for the displacement of the spine is, therefore,

\[ Z(t,x) = |A|\cos(\omega t-k_x x+\delta)+|B_o|\cos(\omega t+ax+\alpha)e^{bx}+|B_3|\cos(\omega t-bx+\beta)e^{ax} \]

The amplitude of oscillation varies with the distance from the free end and tends towards the infinite length solution as \( x \to -\infty \).

**Double Ended Spine**

In this case the moments and shears must disappear for \( x = 0 \) and \( x = -L \), i.e.

\[ \frac{\partial^2 Z}{\partial x^2} = 0 \quad \text{if} \quad x = 0, -L \]

and

\[ \frac{\partial^3 Z}{\partial x^3} = 0 \quad \text{if} \quad x = 0, -L \]

The solutions of these boundary conditions yield \( B_j, j = 0, 3 \) for the equation

\[ X(x) = Ae^{-ikx} + \sum_{j=0}^{3} B_j e^{i\lambda_j x} \]

Hence

\[ \frac{\partial^2 Z}{\partial x^2} = -k_x^2 Ae^{-ikx} - \lambda_0^2 B_0 e^{i\lambda_0 x} - \lambda_1^2 B_1 e^{i\lambda_1 x} + -\lambda_2^2 B_2 e^{i\lambda_2 x} - \lambda_3^2 B_3 e^{i\lambda_3 x} \]

and
\[
\frac{\partial^3 z}{\partial x^3} = i k_x^3 A e^{-i k_x x} - i \lambda_0^3 B_0 e^{i \lambda_0 x} - i \lambda_1^3 B_1 e^{i \lambda_1 x} - i \lambda_2^3 B_2 e^{i \lambda_2 x} - i \lambda_3^3 B_3 e^{i \lambda_3 x}
\]

if \( x = 0, L \).

These conditions can be expressed as a matrix equation:

\[
\begin{pmatrix}
-\lambda_0^2 & -\lambda_1^2 & -\lambda_2^2 & -\lambda_3^2 \\
-\lambda_0^2 e^{-i \lambda_0 L} & -\lambda_1^2 e^{-i \lambda_1 L} & -\lambda_2^2 e^{-i \lambda_2 L} & -\lambda_3^2 e^{-i \lambda_3 L} \\
-\lambda_0^3 & -\lambda_1^3 & -\lambda_2^3 & -\lambda_3^3 \\
-\lambda_0^3 e^{-i \lambda_0 L} & -\lambda_1^3 e^{-i \lambda_1 L} & -\lambda_2^3 e^{-i \lambda_2 L} & -\lambda_3^3 e^{-i \lambda_3 L}
\end{pmatrix}
\begin{pmatrix}
B_0 \\
B_1 \\
B_2 \\
B_3
\end{pmatrix}
= \begin{pmatrix}
k_x^2 \\
\lambda_x^2 e^{-i k_x L} \\
-k_x^2 \\
-k_x^3 e^{-i k_x L}
\end{pmatrix} \tag{4.48}
\]

This can best be solved by a computer program as an algebraic solution as in the single ended spine it would be rather lengthy. The basic approach is however similar.

**Determination of Bending Moments**

The heave bending moments are given by

\[
M_z = EI \frac{\partial^2 z}{\partial x^2}
\]

This means that, for a single ended spine, the instantaneous moments are given by

\[
M_z(t,x) = \frac{\partial^2 z}{\partial x^2} \text{Re}\{EI \beta(t,x)\}
\]

where

\[
M_z(t,x) = -k_x^2 |A| \cos(\omega t - k_x x + \delta)
\]
\[- a^2 B_0 \cos(\omega t + ax + \alpha) e^{bx} \]
\[+ b^2 B_0 \cos(\omega t + ax + \alpha) e^{bx} \]
\[- b^2 B_3 \cos(\omega t - bx + \beta) e^{ax} \]
\[+ a^2 B_3 \cos(\omega t - bx + \beta) e^{ax} \]

The most useful form for the response of the spine to waves is the RMS bending moment. This gives engineers a good indication of how strong a structure needs to be.

\[
\text{RMS moment}(x) = \left( \frac{\omega}{2\pi} \int_0^{2\pi} M_z^2(t,x) dt \right)^{\frac{1}{2}}
\]

[4.49]

4.4.2 Analysis of a Spine in Random Waves

Added mass and damping parameters are frequency dependent as was shown in Section 3.3. This implies further non-linearities in addition to those discussed in Section 4.3 but, if the response of the system to irregular waves is assumed to have little frequency spread and be peaked about the same frequency as the wave spectrum, then it might be possible to determine useful values for the added mass and damping from the peak frequency of the sea spectrum. The hydrodynamic parameters, therefore, for a spine in a sea whose spectral peak is at 1Hz are assumed to match those of a regular 1Hz sea. If these assumptions are made then the linearity of the system allows the response of the spine to random waves, whose spectral shape is known, to be calculated from the calculated response to single frequencies. It is convenient to express a random sea as a sum of wavelets with different frequencies and amplitudes. This can be done using either of the methods described in Section 2.2. There are, however, no real time calculations to be performed and so the number of
wavelets can be made arbitrarily large. The mean square response (displacement, bending moment or shear force) of the spine can be calculated for each wavelet and the results added to obtain the mean square response for the complete sea. This method of analysis was used in preference to simply performing a numerical integration over the amplitude spectrum as it gives the user the opportunity of simulating the response of the spine in exactly the same sea state as used in wide tank experiments. The unequally spaced, equally sized teeth approach was used due to its superiority over the equally spaced teeth system in physical tank tests.

4.5 Results, Comparisons and Discussion

As was stated in Chapter 3 the spine floats in still water with its hub submerged, as closely as possible, by one spine radius. The inertial and radiation parameters as a function of frequency in this configuration are shown in Figures 4.3 to 4.6. A least squares fitting program was applied to these curves to obtain the values of the parameters used in calculations. The displacement mass of a spine segment was 9.65kg/m. This is somewhat lower than that which might be expected by a simple application of \( M = \pi \frac{D^2}{4} \times \rho \) but an examination of Figures 3.21 and 3.22 shows that although the outer diameter is 12.5cm water can get into large parts of the spine around the joints.

The problem was treated by considering an effective length/m. This involved calculating the length of spine which would displace 9.65kg of water when totally submerged. Thus the proportion of the spine which is considered to be effective in determining buoyancy, inertial and radiation forces is 0.79. This means that the inertial forces (\( C_{\text{IAU}} \) and \( k_{\text{IAz}} \)), the buoyancy forces (\( \rho g A \)) and the radiation forces \( Q_z \) should all
SURGE RADIATIONS WIDE TANK SCALE

FIGURE 4.5

Experimentally Determined Radiation Damping/Unit Length
HEAVE RADIATIONS WIDE TANK SCALE

FIGURE 4.6

Experimentally Determined Radiation Damping/Unit Length
be scaled by a factor of 0.79 in an attempt to model the nature of the 'leaky' ends of spine segments.

The mismatch of 0.275kg/m between mass and displacement results in the spine being not quite tangentially submerged but the difference is not sufficient to affect the hydrodynamic parameters and the resultant hydrodynamic spring was neglected as being insignificant compared to the elastic properties of the spine.

The solution given in equation [4.32] is the maximum displacement (surge or heave depending on parameter values) for an infinitely long spine of flexural rigidity EI, in waves of frequency $\omega$ with an $x$-component of the wave vector $k_x$. The corresponding maximum bending moment is given by

$$M = \frac{F_0 k_x^2 EI}{\sqrt{(E I k_x^4 - Y_m^2 \omega^2)^2 + (Q_H \omega)^2}} \quad [4.50]$$

If, as expected, the force due to the waves follows the Morison form then $F_0$ can be expected to be proportional to $\omega^2$ and so

$$M_h \propto \frac{\omega^2 k_x^2 EI}{\sqrt{(E I k_x^4 - Y_m^2 \omega^2)^2 + (Q_H \omega)^2}} \quad (\text{Heave}) \quad [4.51]$$

and

$$M_s \propto \frac{\omega^2 k_x^2 EI \sin \theta}{\sqrt{(E I k_x^4 - Y_s \omega^2)^2 + (Q_s \omega)^2}} \quad (\text{Surge}) \quad [4.52]$$

where $\theta = \text{angle between spine and wave crests}$.

Figures 4.7 to 4.12 show lines of equal predicted bending moments as calculated using equations [4.51] and [4.52]. These show clearly how, at high frequencies, there are sharp peaks in the predicted moments for
FIGURE 4.7

Lines of Equal Theoretical Surge Bending Moments in an Infinitely Long Spine (EI = 800Nm²)
Expressed Over the Period/Angle

$C_s = 3.14$, $Q_s = 401.5$Nsm² (figures for period = 1.0s)
FIGURE 4.8

Lines of Equal Theoretical Heave Bending Moments in an Infinitely Long Spine (EI = 800Nm²)
Expressed Over the Period/Angle Domain

$C_H = 1.22$, $Q_H = 42.0$Nsm⁻² (figures for period = 1.0s)

contour values are relative to the largest surge moments at 1Hz.
Lines of Equal Theoretical Surge Bending Moments in an Infinitely Long Spine ($EI = 2800$Nm$^2$) Expressed Over the Period/Angle Domain

$Cs = 3.14$, $Qs = 401.5$Nsm$^{-2}$ (Figures for period = 1.0s)
FIGURE 4.10

Lines of Equal Theoretical Heave Bending Moments in an Infinitely Long Spine (EI = 2800Nm²)

$C_H = 1.22, Q_H = 42.0\text{Nsm}^{-2}$ (figures for period = 1.0s)

Contour values are relative to the largest surge moments at 1 Hz.

Period (s)

- 1.0
- 1.2
- 2.0

$\theta$

-70°

0°

70°
FIGURE 4.11

Lines of Equal Theoretical Surge Bending Moments in an Infinitely Long Spine (EI = 4000Nm²) 

$C_S = 3.14$, $Q_S = 401.5\text{Nsm}^{-2}$ (figures for period = 1.0s)

contours are relative to the largest surge moments at 1 Hz

Period (s)

2.0

1.0
Lines of Equal Theoretical Heave Bending Moments in an Infinitely Long Spine (EI = 4000Nm²)

$C_H = 1.22$, $Q_H = 420Nsm^{-2}$ (figures for period = 1.0s)
waves incident almost normally to the spine. The frequencies at which these occur are, however, very high (typically 5Hz) and it is unlikely that the wide tank will contain large amounts of energy due to such waves.

The diffraction conditions that \( \frac{D}{\lambda} < 0.2 \) is no longer satisfied at 5Hz and it is difficult to assess the validity of the simple linearised model. The size of the peaks does suggest, however, some sensitivity at these points in the frequency/direction domain.

At more easily tested frequencies the region of maximum response for a given frequency can be deduced from equations [4.51] and [4.52] bearing in mind the different hydrodynamic properties in surge and heave. In 1Hz regular waves, for example, the heave 'peak' for a spine of stiffness 1000Nm/rad (EI = 800Nm²) should occur, according to Figure 4.7, for \( \theta \) between 13° and 27°. This corresponds to a crest length of between 6.9m and 3.4m. It is difficult to be more accurate than this from the graph because of the nature of the contours. The peaks in the surge distribution should, from Figure 4.8, occur between \( \theta = 12° \) and \( \theta = 22° \). The corresponding crest lengths being 7.5m and 4.1m. Although Figures 4.7 and 4.8 were both evaluated using only the hydrodynamic parameters valid for 1.0Hz excitation it can still be deduced that the maximum heave moments and surge moments will occur with larger incidence angles as the period increases. Figures 4.13 and 4.14 show the experimentally determined surge and heave moments on a 44 segment spine of joint stiffness 1000Nm/rad in regular waves of periods 0.8, 1.0 and 1.2 seconds and crest lengths of 2, 4, 5, 6, 7, 8, 10 and 12m. As can be seen the largest moments occur in the 1 second sea at crest lengths of 6m for heave and 5m for surge which is well within the range deduced from the contour plots.
Figures 4.15 and 4.16 show superimposed, predicted and measured bending moments in heave and surge over a variety of random sea states. In each case the frequency envelope was of a one second energy period Pierson-Moskowitz spectrum (see Chapter 2). The sea state in each of the sub plots differ, however, in the cosine spreading power \( s = 2, 8, 32, 128, 512 \) and in the mean direction of waves \( \theta_0 = 0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ \) as defined by

\[
S(\omega, \theta) = S_T(\omega) \cos^{s/2}(\theta, \theta_0)
\]  \[4.56\]

The experimental model consisted of fortyfour 40cm segments (see Section 3.5) while the analytic model was of a semi-infinite spine. This means that only the behaviour of one end of the spine can be compared with theory.

All of the sub plots show maxima near the end of the spines which are similar, but less pronounced, than those observed at the 'tail' or down wave end of the spines depicted in Figures 4.13 and 4.14. The reduction in size of the peaks compared to the regular waves is probably due to the multitude of wavelets in a mixed sea each trying to impose its own pattern on the spines behaviour. The results of this being an averaging of the distributions to produce reduced and more localised end effects. It can be seen from the figures that this analytic model greatly over-estimates the size of the heave moments (approximately double) but the surge predictions are considerably more accurate (less than 10% average deviation between experiment and simulation once the inability of the single ended spine model to evaluate the 'up-wave' end behaviour is considered).

The inability of the analytic model to predict accurately heave moments
FIGURE 4.13

Experimental RMS Surge Bending Moments in a 17.6m Long Spine of Joint Stiffness 1000Nm/rad Excited by Regular Waves of Amplitude 2cm, Periods 0.8s, 1.0s and 1.2s and Crest Lengths of 2m, 4m, 5m, 6m, 7m, 8m, 10m, and 12m
FIGURE 4.14

Experimental RMS Heave Bending Moments in a 17.6m Long Spine of Joint Stiffness 1000Nm/rad Excited by Regular Waves of Amplitude 2cm, Periods 0.8s, 1.0s and 1.2s and Crest Lengths of 2m, 4m, 5m, 6m, 7m, 8m, 10m, and 12m
FIGURE 4.15

Superimposed Theoretical and Experimental RMS Surge Bending Moments in a 17.6m Long Spine of Joint Stiffness 1000Nm/rad (EI = 800Nm²) Excited by a Pseudo Random Sea With a 1.0 second Pierson-Moskowitz 'Total' Spectrum and Spreading Given by

\[ H(\theta) = \cos^{\frac{1}{2}}(\theta - \theta_0) \]

where \( S = 2, 8, 32, 128 \) and 512 and \( \theta_0 = 0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ \) and 50°

The continuous lines are the theoretical moment distribution and the broken line the experimental measurements. C=3.14, Q=401.5Nm⁻²s
FIGURE 4.16
Superimposed Theoretical and Experimental RMS Heave Bending Moments in a 17.6m Long Spine of Joint Stiffness 1000Nm/rad ($EI = 800Nm^2$) Excited by Pseudo Random Seas With 1.0 second Pierson-Moskowitz 'Total' Spectra and Spreading Given by

$$H(\omega) = \cos^{\frac{3}{2}}(\theta_0 - \theta)$$

where $s = 2, 8, 32, 128$ and 512 and $\theta_0 = 0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ$ and $50^\circ$.

The continuous lines are the theoretical moment distributions and the broken lines are the experimental measurements. $C_H = 1.22$, $Q_H = 42.0Nsm^-2$
is possibly due to the interaction of the spine with the fluid-air interface. This is likely to have some effect on the inertial forces and the buoyancy force is likely to be more complicated than the simple treatment according to equation [4.9]. The non linear nature of this force needs further consideration. Figure 3.17 shows the effects of these interactions on the forces experienced by a cylinder which is almost tangentially submerged. In Chapter 5 the effects of allowing the displaced volume term in the equations of motion to vary will be incorporated into a numerical spine model and the resulting predictions of spine behaviour will be discussed and compared with experimental measurements.
CHAPTER 5

NUMERICAL SPINE ANALYSIS
5.1 **Introduction**

In Chapter 4 a long spine in three dimensional random waves was simulated by linearising the equations of motion and then solving them analytically. Random seas were treated as linear sums of equally sized wave fronts, the response to each of which was calculated. The mean square responses to each component of the sea were added to give an estimate of the spine response in the whole sea. This method has been seen to produce some useful indications of spine behaviour but has been found to be inaccurate in the determination of bending moment amplitudes especially in heave. This is probably due to the interaction between inertial and buoyancy forces which was discussed in detail by Dixon (1980) and briefly in Section 1.3.3.

In this chapter the effects of the varying volume are introduced to the equations of motion and the resulting problem is solved numerically. Allowing the inertial forces and the buoyancy forces to vary non-linearly with position and wave elevation is seen to allow more accurate estimation of RMS moments than the purely analytic model could.

5.2 **Assumptions Made**

In this chapter the volume term in equation \[1.27\] is allowed to vary with the wave elevation and with the axis position. The assumption that wave forces and forces due to the motion of the object can be
added, as in Chapter 4, is kept even though non-linearities have been introduced into the system.

The displaced volume/unit length is given by

\[ A(\eta, t, z) = \frac{D^2}{8} \left( \pi + \frac{4}{D}(\eta(t, x) + td - z)(1 - \frac{4}{D^2}(\eta(t, x) + td)^2)^{\frac{1}{2}} \right. \]

\[ \left. + 2\sin^{-1} \left( \frac{\eta(t, x) + td - z}{D} \right) \right] \]  \text{[5.1]}

where
- \( D \) = spine diameter
- \( t \) = time
- \( x \) = position along the spine
- \( \eta(t, x) \) = wave elevation
- \( td \) = distance of free floating spine axis below surface in still water
- \( z \) = vertical displacement of axis from free floating position.

The wave elevation is given by linear wave theory either as a summation of fronts as in Section 2.2 or as filtered white noise as in Section 2.3. If the first approach is used, then if \( t = a\Delta t \) and \( x = b\Delta x \)

\[ \eta(a, b) = \sum_{m=1}^{M} A_m \cos(\omega_m a\Delta t - K_m b\Delta x + \phi_m) \]  \text{[5.2]}

where
- \( K_m \) = \( x \) component of wave vector of \( m^{th} \) front
- \( \phi_m \) = starting phase of \( m^{th} \) front (\( -\pi < \phi_m < \pi \))
- \( A_m \) = amplitude of \( m^{th} \) front as calculated to match the desired sea state.

This approach has the attraction of being the same as is used to generate waves in the Edinburgh wide tank and as such allows direct
comparisons between simulated and experimental measurements.

An alternative approach to the wave simulation problem involves the use of the white noise filtration technique discussed in Section 2.3. In this case the wave elevation is given by

\[ \eta(a,b) = \sum_{\rho=a-\tau}^{a+\tau} \sum_{q=b-L}^{b+L} h(a-\rho,b-q)A(\rho,q) \]  

where \( h \) = digital filter array
\( A \) = white noise array
\( \tau, L \) = filter dimensions.

The advantages and disadvantages of the two systems of wave simulation were discussed in Chapter 2. The summation technique was used in this work because of the direct comparisons with experimental work that this made possible.

The inertial forces in surge and heave are given by

\[ F_S = C_S \rho A(t,z) \ddot{U}_S \]  
\[ F_H = C_H \rho A(t,z) \ddot{U}_H \]

where \( C_S, C_H \) = the surge and heave inertial force coefficients
\( \rho \) = fluid density
\( \ddot{U}_S, \ddot{U}_H \) = the horizontal and vertical fluid element accelerations.

As in equation [1.28] \( \ddot{U}_S \) and \( \ddot{U}_H \) are the fluid accelerations which would have been present at the spine axis position if the spine were not present. If deep water linear wave theory is used then the fluid element displacement at the axis position is given by
\[ d_y(t,x) = \sum_{m=1}^{M} A_m \sin(\omega_m t - K_m x + \phi_m) e^{\frac{\omega_m^2}{g} (z-td)} \quad \text{(Surge)} \quad [5.6] \]

\[ d_z(t,x) = \sum_{m=1}^{M} A_m \cos(\omega_m t - K_m x + \phi_m) e^{\frac{\omega_m^2}{g} (z-td)} \quad \text{(Heave)} \quad [5.7] \]

The fluid element acceleration at the axis is given, therefore, by

\[ \dot{u}_H = \sum_{m=1}^{M} -\omega_m^2 A_m \cos(\omega_m t - K_m x + \phi_m) e^{\frac{\omega_m^2}{g} (z-td)} \]

and

\[ \dot{u}_S = \sum_{m=1}^{M} -\omega_m^2 A_m \sin(\omega_m t - K_m x + \phi_m) e^{\frac{\omega_m^2}{g} (z-td)} \]

5.3 Differential Equations

If the assumptions discussed in the previous section are utilised then the surge equation of motion is

\[ \frac{\partial}{\partial t} \left\{ (\rho_p A_o + K_S \rho A(z,t)) \frac{\partial y}{\partial t} \right\} + Q_S \frac{\partial y}{\partial t} + R_S y + EI \frac{\partial^2 y}{\partial x^2} = C_S \rho A(z,t) \sum_m -\omega_m^2 A_m \sin(\omega_m t - K_m x + \phi_m) e^{\frac{\omega_m^2}{g} (z-td)} \cos \theta_m \quad [5.8] \]

and the heave equation of motion is

\[ \frac{\partial}{\partial t} \left\{ (\rho_p A_o + K_H \rho A(z,t)) \frac{\partial z}{\partial t} \right\} + Q_H \frac{\partial z}{\partial t} + EI \frac{\partial^2 z}{\partial x^2} = C_H \rho A(z,t) \sum_m -\omega_m^2 A_m \cos(\omega_m t - K_m x + \phi_m) e^{\frac{\omega_m^2}{g} (z-td)} + \rho g A(z,t) - \rho_p g A_o \quad [5.9] \]

where \( K_S, K_H \) = the surge and heave added mass coefficients \( \rho_p \) = density of spine \( \theta_m \) = angle between spine and mth wave front \( Q_S, Q_H \) = the surge and heave damping coefficients \( R_S \) = the linear spring in surge.
If the spine is floating very low in the water, as in this case, then

\[ \frac{3A}{\partial t} = 0 \]

and equations [5.8] and [5.9] can be simplified. The surge equation is now

\[
(pA_0 + K_{pA}(z,t)) \frac{\partial^2 y}{\partial t^2} + Q_s \frac{\partial y}{\partial t} + R_{sy} + EI \frac{\partial^4 y}{\partial x^4} = C_{0pA}(z,t) \sum \omega_m^2 (z-t)d
\]

\[ -\omega_m^2 A_m \sin(\omega_m t - K_m x + \phi_m) e^{-g} \]

and the heave equation is

\[
(pA_0 + K_{HpA}(z,t)) \frac{\partial^2 z}{\partial t^2} + Q_H \frac{\partial z}{\partial t} + EI \frac{\partial^4 z}{\partial x^4} + g(pA_0 - pA(z,t)) \]

\[ = C_{HpA}(z,t) \sum -\omega_m^2 A_m \cos(\omega_m t - K_m x + \phi_m) e^{-g} (z-t)d \]

The linear spring term \( R_{sy} \) is included in the surge equation as a crude model of the horizontal铭oring forces experienced by the spine.

5.4 Numerical System

5.4.1 Difference Scheme

Equations [5.10] and [5.11] contain non-linear terms such as the buoyancy term of [5.11] and are linked by the non-linear z dependence of the displaced volume \( A(z,t) \). This means that an analytic approach as in Chapter 4 cannot be applied. The use of finite differences in the time and x-domain has however enabled an examination of the solution to be made. If the position along the spine \( x \) is given by

\[ x = qAx \quad \text{where} \ q = 0, 1, \ldots \]
and the time $t$ is given by

$$t = p\Delta x \quad \text{where} \quad p = 0, 1, \ldots$$

then the derivatives in equations [5.10] and [5.11] can be approximated by finite difference relationships.

$$\frac{\partial^4 z}{\partial x^4} \approx \frac{z(x+2\Delta x) - 4z(x+\Delta x) + 8z(x) - 4z(x-\Delta x) + z(x-2\Delta x)}{(\Delta x)^4} \quad [5.12]$$

$$\frac{\partial^4 y}{\partial x^4} \approx \frac{y(x+2\Delta x) - 4y(x+\Delta x) + 8y(x) - 4y(x-\Delta x) + y(x-2\Delta x)}{(\Delta x)^4} \quad [5.13]$$

$$\frac{\partial^2 z}{\partial t^2} \approx \frac{z(t+\Delta t) - 2z(t) + z(t-\Delta t)}{(\Delta t)^2} \quad [5.14]$$

$$\frac{\partial^2 y}{\partial t^2} \approx \frac{y(t+\Delta t) - 2y(t) + y(t-\Delta t)}{(\Delta t)^2} \quad [5.15]$$

$$\frac{\partial z}{\partial t} \approx \frac{z(t+\Delta t) - z(t-\Delta t)}{2\Delta t} \quad [5.16]$$

$$\frac{\partial y}{\partial t} \approx \frac{y(t+\Delta t) - y(t-\Delta t)}{2\Delta t} \quad [5.17]$$

**5.4.2 End Conditions**

In Chapter 4 the boundary conditions at a free end were expressed as

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 y}{\partial x^2} = \frac{\partial^3 z}{\partial x^3} = \frac{\partial^3 y}{\partial x^3} = 0$$

moments shears

In finite differences, for a spine of length $L$, these become

(Moments)

$$z(x + \Delta x) - 2z(x) + z(x - \Delta x) = 0 \quad x = 0, L \quad [5.18]$$
\[ y(x + \Delta x) - 2y(x) + y(y - \Delta x) = 0 \quad x = 0, L \]  
\[ z(x + 2\Delta x) - 2z(x + \Delta x) + 2z(x - \Delta x) + z(x - 2\Delta x) = 0 \quad x = 0, L \]
\[ y(x + 2\Delta x) - 2y(x + \Delta x) + 2y(x - \Delta x) + y(x - 2\Delta x) = 0 \quad x = 0, L \]

5.4.3 The Computer Program

The program written to implement the principles described is included in Appendix 2 and a detailed flowchart is included in Appendix 3. A simplified chart is given in Figure 5.1.

In the program the spine is considered as consisting of \( MM \) segments each of length \( DX \) and the sea is considered as a summation of \( N \) independent fronts, the amplitudes of which are calculated in a separate computer program. The flexural rigidity of the spine can have different values in surge and heave but in this program the rigidity is not allowed to vary over the length of the spine.

Once the program has received the input data it sets the positions and velocities of the spine segments to be compatible with the initial conditions specified in the input. The program then enters a loop (the time loop) over the number of time steps which are required to simulate the response of the spine over the repeat time of the sea (usually 409.6 seconds). Before this loop is entered the time variable 'T' is set to zero and after each circuit of the loop is increased by the time interval 'DT' which corresponds to the \( \Delta t \) terms in [5.14] to [5.17].

The first action within the loop is to calculate the wave forces at each
Input data

Calculate constants and set time to zero ($T=0$)

Calculate wave forces on spine segments

Use end conditions to calculate positions of imaginary segments beyond each end of spine

Use finite differences to calculate elastic forces on spine and so determine the total forces on segment

Use finite differences to determine what position of segment will be after time $DT$

Loop over spine segments

Use finite differences to calculate bending moments along the spine

Increase time by $DT$

Output RMS bending moments

FIGURE 5.1

Simplified Flowchart for the Numerical Spine Simulation
segment of the spine using the current positions of the spine segments. This involves first finding the wave amplitude at the centre of each segment according to equation [5.2] and then using these values with the current heave positions of the segments to discover the displaced volume at each position along the spine. Once this has been done equations [5.4] and [5.5] are used to determine the wave forces on each segment.

The program uses the difference equations [5.12] and [5.13] to evaluate the flexural forces on the segments but it is clear from these equations that the difference calculations require a knowledge of the positions of the segments to either side. This is not possible for the segments at either end of the spine. It is for this reason that the next action in the time loop is the use of equations [5.18] to [5.21] to set the positions of imaginary segments beyond each end so as to be compatible with the boundary conditions. Once these end conditions have been set the program enters a loop over the MM individual segments, within this loop the program calculates the total forces on each segment and uses the finite difference representations of the velocity and acceleration of the segments given by equations [5.14] to [5.17] to determine what the positions of the segments will be after the time interval DT. The bending moments along the spine are then calculated from the positions using finite differences.

Once moments have been calculated the time variable T is increased by DT and then starts the time loop again using the newly calculated spine positions in its calculations.

The principles of this program can be viewed in the simplified flowchart in Figure 5.1 and a detailed chart is given in Appendix 3.
TIME INCREMENTS=0.015 SECONDS

FIGURE 5.2
Simulated Segment Displacement for $dt = 0.015$ seconds
TIME INCREMENTS = 0.001 SECONDS

![Graph for time increments = 0.001 seconds]

SIMULATED SEGMENT DISPLACEMENT FOR DT = 0.001 SECONDS

TIME INCREMENTS = 0.0005 SECONDS

![Graph for time increments = 0.0005 seconds]

SIMULATED SEGMENT DISPLACEMENT FOR DT = 0.0005 SECONDS
FIGURE 5.5
Simulated RMS Bending Moment Distributions for $dx = 0.2$, $0.4$, $0.8$ and $1.6$ m
FIGURE 5.6

Experimental RMS Heave Moments in a 17.6m Long Spine With Surge Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad and Heave Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad Subjected to a 1.0 second Pierson-Moskowitz Sea with Mitsuyasu Spread
Experimental RMS Surge Moments in a 17.6m Long Spine With Surge Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad and Heave Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad Subjected to a 1.0 second Pierson-Moskowitz Sea With Mitsuyasu Spread
5.4.4 Stability of Simulation Program

Figures 5.2 to 5.4 show the effect of the time step size \( dt \) on the output of the numerical simulation. These graphs show the predicted surge displacement in a central part of a 16m long spine of joint stiffness 1000Nm/rad (surge and heave) in a regular wave sea of frequency 1Hz, amplitude 0.01m and with an angle of incidence to the spine of 40°. Figure 5.2 shows unstable behaviour when the time steps are large. If the simulation had been allowed to continue an overflow would certainly have occurred. Figures 5.3 and 5.4 show, however, the predicted response at the same position in the spine for time steps of 0.001s and 0.0005s. The differences between these predictions are minimal indicating convergence of outputs as \( dt \) decreases. It is likely, however, that instability will re-occur due to rounding errors when \( dt \) is very small but in this section \( dt \) was set to 0.001 seconds.

Figure 5.5 shows the calculated RMS surge moments along a 16m long spine excited by a 1 second energy period Pierson-Moskowitz spectrum with Mitsuyasu spreading about the normal to the spine. The values of the spatial difference sizes are 0.2, 0.4, 0.8, 1.6m. The resolution is obviously higher for a smaller difference but the solutions appear to be converging as \( dx \to 0 \). It is likely that, as in the time steps, rounding errors will become crucial for very small space steps.

5.5 Results and Comparison Between Simulation and Experiment

5.5.1 Bending Moment Variation With Surge and Heave Stiffnesses

The analytic model presented in Chapter 4 assumes that the surge and heave properties of the spine are totally independent. The equations used, however, in this chapter assume that the heave response of the
spine will have an influence on the response of the spine in surge. This is because of the appearance of the varying, heave dependent, volume term in equation [5.10]. The purpose of this section is to find, both in experiment and simulation, the effect of varying the surge stiffness on both surge and heave bending moments and similarly the effect of heave stiffness on surge and heave moments. Figure 5.6 shows the experimental heave moments in a 17.6m long spine subjected to a 1sec Pierson-Moskowitz sea with Mitsuyasu spreading. The rows show the variations with surge stiffness (joint stiffness = 250, 500, 1000, 2000, 4000 Nm/rad) and columns show the variation with heave stiffness (joint stiffness = 250, 500, 1000, 2000, 4000 Nm/rad) so that the sub plot marked say 009 represents a spine with surge stiffness of 1000Nm/rad and a heave stiffness of 500Nm/rad. The un-numbered plots on the right hand side show, superimposed, the sub-plots representing surge stiffness variation and the plots on the bottom show, superimposed, the sub-plots representing heave stiffness variation. The surge stiffness can be seen to have a small effect on the heave moments. However, the variation over a range of stiffnesses from 250Nm/rad to 4000Nm/rad is of the order of 8% which, when the repeatability of the tank (about 3%) and the expected deviation of the spine stiffness from the nominal setting of about 7% is noted, suggests that further experiments are required, with a more continually calibrated spine, to determine the relevance of the variations. This would be impractical with the current equipment. Figure 5.7 shows the corresponding surge moment distributions displayed according to the same conventions as Figure 5.6. Once again, it can be seen that the distribution is almost independent of the stiffness in the alternative direction which, in this case, is demonstrated by a variation in surge moments of less than 7% over a range of heave stiffnesses of 250Nm/rad.
to 4000Nm/rad. This case is particularly surprising when equation [5.10] is recalled as it suggests a heave stiffness dependence in surge. The variation in heave moments with heave stiffness can also be seen in Figure 5.6 just as the variation of surge moments with surge stiffnesses can be seen in Figure 5.7. As would be expected the higher the stiffness the larger are the bending moments. The relationship is by no means linear and an examination of equations [4.50] and [4.51] suggests that the moments will reach a limiting value as the stiffness tends to infinity. This limit will correspond to a totally inflexible spine.

Figure 5.8 shows the variation in the heave moment distribution, over the same domain as Figure 5.6, as predicted by the numerical simulation program. The overall properties are similar to those shown in Figure 5.6 but no surge stiffness dependence whatsoever is suggested because equation [5.11] contains no reference to the y direction. The heave stiffness, as displayed by the variation in the columns, is however similar in form to the experimental case. Figure 5.9 shows, superimposed, the experimental and simulated heave moment plots over the surge/heave stiffness domain. The agreement between the two ranges from a mean deviation of 11% between experimental and simulated moments for the 250Nm/radian (surge and heave) case to 18% for the 4000Nm/rad (surge and heave) case. This is outside of the estimated experimental error in the spine model but may be close enough to be a useful tool for engineers. Figure 5.10 shows the variation in the surge moment distribution, over the surge/heave stiffness domain, as predicted by the numerical simulation program. As in the heave case the trends are similar to the experimental case but here a very slight heave dependence can be observed. The surge moments rise very slightly with the heave stiffness. The moments for a heave stiffness of
4000Nm/rad are only 4-5% higher than for a heave stiffness of 250Nm/rad and, as such, likely to be experimentally difficult to observe in the mechanical model. The relationship between the surge moments and surge stiffness is very similar to the experimental case and displays the same overall form. Figure 5.11 shows, superimposed, the experimental and simulated surge moment plots over the surge/heave stiffness domain. The average percentage deviation of the simulated moments from the experimental moments ranges from less than 9% for the 250Nm/rad plot numbered 001 to 17% for the 4000Nm/rad plot numbered 029. These deviations are, like the heave case, outside of experimental error but still likely to be useful for engineers.

5.5.2 Bending Moment Variation With Spreading, Direction and Energy Period

Figures 5.12 and 5.13 show, superimposed, experimental and simulated RMS bending moment distributions in a spine of length 17.6m. The seas used for each sub-plot all had a 1.0sec Pierson-Moskowitz form with cosine spreading given by

\[
S(\omega, \theta) = S_{PM}(\omega) \cos^s \frac{1}{2} (\theta - \theta_0)
\]

where \(S_{PM}\) is described in detail in Chapter 2.

The \(s\) term of the cosine spreading could take the values \(s = 2, 8, 32, 128, 512\) and the principal angle could take the values \(\theta_0 = 0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ\). These spectra are displayed in a 'dot map' form in Figure 5.14. These show the distribution of the equally sized spikes in the spectrum in the period/angle domain. The density of spikes indicates the size of the directional spectrum. As can be seen in Figure 5.12 the surge moment distributions along the length of the
FIGURE 5.8.
Simulated RMS Heave Moments in a 17.6m Long Spine With Surge Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad and Heave Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad Subjected to a 1.0 second Pierson-Moskowitz Sea With Mitsuyasu Spread.

\[ C_H = 1.22, \quad C_S = 3.14, \quad Q_H = 42.0 \text{Nsm}^{-2}, \quad Q_S = 401.5 \text{Nsm}^{-2} \]
FIGURE 5.9

Superimposed Simulated and Experimental RMS Heave Bending Moments in a 17.6m Long Spine With Surge Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad and Heave Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad Subjected to a 1.0 second Pierson-Moskowitz Sea With Mitsuyasu Spread

The continuous lines are the simulated moment distributions and the broken lines are the experimental measurements.

\[ C_H = 1.22, \quad C_S = 3.14, \quad Q_H = 42.0 \text{ Nsm}^{-2}, \quad Q_S = 401.5 \text{ Nsm}^{-2} \]
Simulated RMS Surge Moments in a 17.6m Long Spine With Surge Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad and Heave Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad Subjected to a 1.0 second Pierson-Moskowitz Sea With Mitsuyasu Spread

\[ C_H = 1.22, \ C_S = 3.14, \ Q_H = 42.0 \text{Nsm}^{-2}, \ Q_S = 401.5 \text{Nsm}^{-2} \]
Superimposed Simulated and Experimental RMS Surge Bending Moments in a 17.6m Long Spine With Surge Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad and Heave Joint Stiffnesses of 250, 500, 1000, 2000 and 4000 Nm/rad Subjected to a 1.0 second Pierson-Moskowitz Sea With Mitsuyasu Spread. The Continuous Lines Are the Simulated Moment Distributions and the Broken Lines Are the Experimental Measurements

\[ C_H = 1.22, \quad C_S = 3.14, \quad Q_H = 42.0 \text{Nsm}^{-2}, \quad Q_S = 401.5 \text{Nsm}^{-2} \]
Superimposed Simulated and Experimental RMS Surge Moments in a 17.6m Long Spine of Joint Stiffnesses 1000 Nm/rad
Excited by Pseudo Random Seas With 1.0 second Pierson-Moskowitz 'Total' Spectra and Spreading Given by

\[ H(\omega) = \cos^{\frac{s}{2}}(\theta - \theta_0) \]

where \( s = 2, 8, 32, 128 \) and 512 and \( \theta_0 = 0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ \) and 50°

The continuous lines are the theoretical moment distributions and the broken lines are the experimental measurements

\[ C_H = 1.22, C_S = 3.14, Q_H = 401.5 \text{ Nsm}^{-2}, Q_S = 42.0 \text{ Nsm}^{-2} \]
Superimposed Simulated and Experimental RMS Heave Moments in a 17.6m Long Spine of Joint Stiffnesses 1000Nm/rad Excited by Pseudo Random Seas With 1.0 second Pierson-Moskowitz 'Total' Spectra and Spreading Given by

$$H(\omega, \theta) = \cos^{8\frac{1}{2}}(\theta-\theta_0)$$

where $s = 2, 8, 32, 128$ and $512$ and $\theta_0 = 0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ$ and $50^\circ$. The Continuous Lines are the Theoretical Moment Distributions and the Broken Lines are the Experimental Measurements

$C_H = 1.22$, $C_S = 3.14$, $Q_H = 42.0 \text{ Nm}^{-2}$, $Q_S = 401.5 \text{ Nm}^{-2}$
FIGURE 5.14

'Dot' Map Representations of the Pseudo Random Seas Used to Generate the Results Displayed in Figures 5.12 and 5.13
spine are predicted with a high degree of accuracy over all of the $S, \theta_0$ domain and, in particular, the lop-sided nature of the RMS moment distribution in an asymmetric sea can be clearly seen. Overall the average deviation between the predicted and measured moments is less than 16%. The heave moments shown in Figure 5.13 also show a high degree of correlation between the experimental and predicted moments although the sub-plots $h500$ and $h600$ do show considerable disagreement, in both magnitude and form. These are both highly directional seas with $S = 128$ and 512 and an investigation of the corresponding sub-plots in Figure 5.13 shows that there are at least two component wave fronts near the peaks which can be seen in Figure 4.8. The tank does not respond well to frequencies of more than 2Hz and is likely to have filtered out these fronts before they reached the spine. The numerical model is not, however, limited by such problems and it would appear likely that the simulation has responded to these high frequency components. This hypothesis is substantiated by the increased correlation between simulated and experimental RMS heave moments for $S = 128$ and 512 once the seas have been offset by a non zero value of $\theta_0$. Figure 5.13 shows that for $\theta_0 = 10^\circ$ or greater the component fronts all lie outside the sensitive area suggested by Figure 4.8.

Figures 5.15 and 5.16 show, for the same spine as in the previous section, the simulated and experimental RMS moment distributions along the spine length in a Pierson-Moskowitz spectrum with Mitsuyasu spreading of three energy periods ($TE = 0.8, 1.0, 1.2s$). For all three periods the average deviation of the simulated heave RMS moments from the experimental heave moments is less than 20% which, although outside of the expected experimental error, is useful for engineers.
Superimposed Simulated and Experimental RMS Heave Moments in a 17.6m Long Spine of Joint Stiffnesses 1000 Nm/rad Excited by Pseudo Random Seas With Mitsuyasu Spreading and Pierson-Moskowitz 'Total' Spectra With Energy Periods of 0.8, 1.0 and 1.2 seconds

For $T_E = 0.8s$, $C_S = 2.15$, $C_H = 1.22$, $Q_S = 450.0\text{Nsm}^{-2}$, $Q_H = 52.0\text{Nsm}^{-2}$
For $T_E = 1.0s$, $C_S = 3.15$, $C_H = 1.38$, $Q_S = 401.5\text{Nsm}^{-2}$, $Q_H = 42.0\text{Nsm}^{-2}$
For $T_E = 1.2s$, $C_S = 3.20$, $C_H = 1.51$, $Q_S = 140.5\text{Nsm}^{-2}$, $Q_H = 18.6\text{Nsm}^{-2}$

The continuous lines are the simulated distribution and the broken lines are the experimental measurements.
FIGURE 5.16
Superimposed Simulated and Experimental RMS Surge Moments in a 17.6m Long Spine of Joint Stiffnesses 1000Nm/rad Excited by Pseudo Random Seas With Mitsuyasu Spreading and Pierson-Moskowitz 'Total' Spectra With Energy Periods of 0.8, 1.0 and 1.2 seconds

For $T_E = 0.8\,\text{s}$, $C_S = 2.15$, $C_H = 1.22$, $Q_S = 450.0\text{Nms}^{-2}$, $Q_H = 52.0\text{Nms}^{-2}$
For $T_E = 1.0\,\text{s}$, $C_S = 3.15$, $C_H = 1.38$, $Q_S = 40.5\text{Nms}^{-2}$, $Q_H = 42.0\text{Nms}^{-2}$
For $T_E = 1.2\,\text{s}$, $C_S = 3.20$, $C_H = 1.51$, $Q_S = 140.5\text{Nms}^{-2}$, $Q_H = 18.6\text{Nms}^{-2}$

The continuous lines are the simulated distributions and the broken lines are the experimental measurements
The surge RMS moments are predicted to within 15% average error along the spine for $T_E = 0.8$ and 1.0 seconds but the average deviation between simulation and experiment for the $T_E = 1.2$ second case is almost 30%. This is approaching the limit for useful forecasting but, as it over rather than under predicts, useful observations about material strengths could still be made. The positions and forms of the peaks in the RMS moment distribution are, in all of the cases, predicted with considerable accuracy and the peaks at the ends of a spine are still observable.

5.5.3 Bending Moment Variation With Spine Length

Parameters such as the stiffness of the experimental spine model can be varied electronically by the experimenter but the length of the model can only be changed by adding or removing spine segments. This is not difficult to do but would be very time consuming. The numerical model, however, does not suffer from this limitation and, for this reason, it was decided to use the simulation program to investigate the effect of the spine length on the maximum RMS moment. All of the work displayed so far, either experimental or simulated, shows there to be peaks in the RMS moment distribution along a spine near the ends. This suggested that the RMS moments might be very large in the centre of a spine which is of such a length that the two end peaks coincide. Figure 5.17 shows the maximum predicted RMS surge moments in a spine of stiffness 1000Nm/rad (surge and heave) subjected to a one second Pierson-Moskowitz sea with Mitsuyasu spreading. The largest moments are found in a 4.6m long spine and there is a slight dip in the curve at about 5m. The curve flattens off for higher spine lengths. This is probably because the spine behaviour is approaching the single ended behaviour discussed in
Figure 5.17

Maximum Simulated RMS Surge Moments, as a Function of Spine Length, When Spines With Joint Stiffnesses of 1000N/m/rad Are Excited by a 1 second Pierson-Moskowitz Sea With Mitsuyasu Spreading

\[ C_S = 3.15, \quad C_H = 1.38, \quad Q_S = 401.5\text{Nsm}^{-2}, \quad Q_H = 42.0\text{Nsm}^{-2} \]
Maximum Simulated RMS Heave Moments, As a Function of Spine Length, When Spines of Joint Stiffnesses of 1000Nm/rad Are Excited by a 1 second Pierson-Moskowitz Sea With Mitsuyasu Spreading

$$C_s = 3.15, \ G_H = 1.38, \ Q_s = 401.5 \text{Nsm}^{-2}, \ Q_H = 42.0 \text{ONsm}^{-2}$$
Maximum Simulated RMS Surge Moments, as a Function of Spine Length, When Spines of Joint Stiffnesses 3500Nm/rad Are Excited by a 1 second Pierson-Moskowitz Sea With Mitsuyasu Spreading

\[ C_s = 3.15, \quad C_H = 1.38, \quad Q_s = 401.5 \text{Nsm}^{-2}, \quad Q_H = 42.0 \text{Nsm}^{-2} \]
Figure 5.20

Maximum Simulated RMS Heave Moments, As a Function of Spine Length, When Spines With Joint Stiffnesses of 3500 Nm/rad Are Excited by a 1 second Pierson-Moskowitz Sea With Mitsuyasu Spreading

\[ C_s = 3.15, \quad C_H = 1.38, \quad Q_s = 401.5 \text{Nsm}^{-2}, \quad Q_H = 42.0 \text{Nsm}^{-2} \]
Chapter 4.

Figure 5.18 shows the corresponding changes in the maximum heave moments with spine length. The peak in this case is more pronounced and occurs for a rather longer spine of 5.0m in length. In the heave case the largest moments in a 5m spine are over 50% higher than the largest moments predicted for a very long spine.

Figure 5.19 shows the change in the maximum RMS surge moments with length for a spine of joint stiffness 3500Nm/rad. The peak in this case occurs for a length between 6m and 6.5m. The same overall form as Figure 5.17 can be seen. Figure 5.20 shows the variation in the maximum RMS heave moments for a joint stiffness of 3500Nm/rad. As in the 1000Nm/rad case the peak is more pronounced in heave than in surge but in this stiffer situation there is less evidence of a shift in the peak position between surge and heave.

No experimental work has been done to verify these findings but if the results are valid then the implications to engineers designing and building full scale structures such as duck strings are that certain lengths of spine should be avoided. In a full scale duck power scheme the critical lengths would be 400-700m which would be likely to be the size of a prototype scheme.

5.5.4 Bending Moment Correlations

A modified version of the numerical simulation was written which, in addition to outputting RMS surge and heave bending moments, could output the correlation between the bending moments at one point with those at any other. The correlation between the surge moments at \( x = x_1 \) with those at \( x = x_2 \) could be examined for example, as could the
correlation between heave moments at \( x_1 \) and \( x_2 \). The correlation \( C \) between two variables, \( A_1 \) and \( A_2 \) for example, which vary in time is given by

\[
C = \frac{E\{A_1(t)A_2(t)\}}{E\{A_1(t)\}E\{A_2(t)\}}
\]

where \( E\{ \} \) represents expectation over the time domain.

Figure 5.21 shows the predicted correlation between heave moments in a 4m long spine of joint stiffness 1000Nm/rad subjected to a 1sec PM sea with Mitsuyasu spread. The correlation between any point and itself is obviously 1 and this shows itself as a ridge from the (0,0) near corner to the (4,4) far corner. In this short spine the correlation between any two points is always greater than 0. The correlation between the surge moments as shown in Figure 5.22 is very similar to the heave case. Figures 5.23 and 5.24 show the heave (Figure 5.23) and surge (Figure 5.24) correlations in a 14.4m long spine with the same joint stiffnesses and in the same sea as in Figures 5.21 and 5.22. In these cases the unit ridge from (0,0) to (14.4,14.4) is still apparent but the correlation between certain points in both surge and heave is negative. The nature of the correlations can be seen in an alternative form in Figure 5.25 which shows, superimposed, predicted and measured correlations. These correlograms are arranged, like the physical spine model, alternately in surge and heave. The top left plot shows the correlation between the surge moments at the end with the surge moments at 0.8m intervals down the spine. The next plot shows the correlation between the heave moments 0.4m from the end with those at 0.8m intervals down the spine. This sequence is continued till the last plot which shows the correlation between the heave moments at the end of the spine with those from points each
Correlations Between Predicted Heave Moments at Different Positions Along a 4m Long Spine, With Joint Stiffnesses of 1000Nm/rad Excited by a 1 second Pierson-Moskowitz Sea With Mitsuyasu Spreading

\( C_s = 3.15, C_H = 1.38, Q_s = 401.5 \text{Nsm}^{-2}, Q_H = 42.0 \text{Nsm}^{-2} \)

Correlations Between Predicted Surge Moments at Different Positions Along a 4m Long Spine With Joint Stiffnesses of 1000Nm/rad Excited by a 1 second Pierson-Moskowitz Sea With Mitsuyasu Spreading

\( C_s = 3.15, C_H = 1.38, Q_s = 401.5 \text{Nsm}^{-2}, Q_H = 42.0 \text{Nsm}^{-2} \)
Correlations Between Predicted Heave Moments at Different Positions Along a 14.4m Long Spine With Joint Stiffnesses of 1000Nm/rad Excited by a 1 second Pierson-Moskowitz Sea with Mitsuyasu Spreading

\[ C_S = 3.15, C_H = 1.38, Q_S = 401.5 \text{Ns}^{-2}, Q_H = 42.0 \text{Nsm}^{-2} \]
Correlations Between Predicted Surge Moments at Different Positions Along a 14.4m Long Spine With Joint Stiffnesses of 1000Nm/rad Excited by a 1 second Pierson-Moskowitz Sea With Mitsuyasu Spreading

\[ C_S = 3.15, \quad C_H = 1.38, \quad Q_S = 401.5\text{Nsm}^{-2}, \quad Q_H = 42.0\text{Nsm}^{-2} \]
FIGURE 5.25

Correlations Between the Moments at Each Joint Along a 14.4m Long Spine With the Moments at Each Other Like Orientated Joint (the Even Numbers are Surge Joints and the Odd Numbers are Heave Joints). Each Joint Had a Stiffness of 1000Nm/rad and the Waves had a Pierson-Moskowitz Spectrum With Mitsuyasu Spreading. The Continuous Lines Show the Predicted Moments and the Broken Lines Show the Experimental Measurements.

\[ C_s = 3.15, \quad C_H = 1.38, \quad Q_s = 401.5 \text{Nsm}^{-2}, \quad Q_H = 42.0 \text{Nsm}^{-2} \]
separated by 0.8m. This arrangement is convenient because it means that alternate plots are surge or heave and so it can easily be seen that the surge moments become much more negative than the heave moments which is probably because of the more linear nature of the surge responses. The agreement between the simulated and experimental correlations is very good, differing only at the spine ends where theory predicts zero moments and therefore the correlation between moments at either end of the spine with any other point in the spine is undefined. The physical model is likely to show certain non-zero results because of noise or drift in the unloaded strain gauges at the ends.

5.6 Observations and Comments

The numerical simulation has proved to be successful in predicting a number of important spine properties. The Wave Power Group at Edinburgh University have used many of the numerical predictions as an indication of where spine research should be concentrated. This has led, in particular, to simulation and experimental evidence that RMS moments do not vary greatly over the central regions of long spines in random seas.
CHAPTER 6

DISCUSSION
6.1 Summary and Conclusions

In Chapter 1, the theoretical background for the treatment of a long floating cylinder in random waves was presented. Linear wave theory was used throughout this work and a brief outline of the necessary conditions and predictions was given. Wave forces on cylinders were approximated in all of the analytic and numerical treatments of cylinder behaviour by a modification to Morison's equation. This modification, which is given in equation [1.29], allowed the effect of variations in the volume of water displaced by a cylinder to be included. It was assumed, for the purposes of calculating the volume of water displaced, that the wave steepness was sufficiently small that a simple equation which neglected wave slope could be used. Small wave steepness is in any case an assumption underlying Morison's equation and so no further loss of generality was introduced.

A modified Morison equation was used in this work because it had been found in earlier investigations by Dixon et al (1979) to be simple to apply yet capable of predicting certain wave effects on partially submerged cylinders which could not be explained using a straight application of Morison's equation or by using diffraction theory, a brief description of which was included in Chapter 1. The most obvious anomalous wave force effects are wave forces at double the wave frequency which can be observed when cylinders are very nearly totally submerged. This effect was observed in some of the results in Chapter 3.

All of the results contained in this thesis related to scale models of large wave power devices. In order that small scale results could be related to large scale effects, some knowledge of the scaling laws for
wave forces on cylinders was required. In many fluid scaling problems the Reynolds number is of great importance but, as in this work an initial assumption of non-viscous fluid was made, Reynolds number, which contains viscosity as one of its terms, would not appear to be a relevant choice of scaling term. The Keulegan Carpenter number, which consists of the product of a typical fluid velocity and an oscillation period divided by a body dimension, is of great importance. If the waves are regular and the cylinder diameter is considered as the typical body dimension, the Keulegan Carpenter number becomes a measure of the ratio of wave amplitude to body diameter. In purely two dimensional hydrodynamics equivalence of this ratio is an obvious requirement for similarity between different systems. In cases such as those outlined in this thesis where inertial wave forces are dominant the ratio of body diameter to wavelength is another ratio which must be equivalent for similarity between systems.

In Chapter 2 a brief description of the method used to generate random waves in the Edinburgh wide tank was given. This method has the serious disadvantage that it produces waves which have discontinuous energy density spectra. In many situations this is not a great problem but there are resonances in some structures which may have such fine frequency sensitivities that they may not be excited by waves in the tank because they slip between the 'teeth' of the simulated spectrum. This prompted an investigation into an alternative method for generating random waves in wide tanks. A physical feel for the method suggested can be had by considering the array of wavemakers as an infinitely long structure. If this 'structure' were to be struck by a hammer at one point in time and space then an outgoing response wave would be generated whose form could be described by an impulse response
function. If the structure were to be struck at random along its length then the power spectrum of the response would be simply related to the response function. In the wide tank the required form of the spectrum of the wavemaker motion would be known. The problem is to make the wavemaker array behave as a structure with the required impulse response function. Once the form of the response function had been calculated from the required spectrum, the required responses of the wavemakers to random signals could be produced. A computer program was written which could determine the required form of the response function from the spectral shape and store the digital representation as a two dimensional 'digital filter' in the computer memory. A second program, included in Appendix I, was written which used these digital filters to filter numerically generated white noise in order to generate wavemaker signals. The wave signals generated in this manner were shown to be accurate representations of the theoretical wave conditions when their total power spectra and correlations were compared with theoretical values.

In Chapter 3 some of the experimental equipment and preliminary experimental results were described. The energy density (or RMS wave height) distribution over the working area is of particular importance for the following reason. If the mean square wave height varied much over the tank then this would imply that the tank was not a good model of the real sea and that account would have to be taken of the variations when considering measurements taken in the tank. In experimental measurements of the RMS wave heights in the tank it was discovered that, although regular waves did produce large variations in wave height measurements, pseudo random seas did not. This was due to irregular seas not being able to build up large standing waves across
the tank. In irregular seas the RMS wave height varied by less than 5% (the drift observed in a stationary gauge) over the working area of the tank. The wave height variation over the tank in random seas was, therefore, considered to be negligible in future wide tank tests in random seas. The experiments which were performed in coming to these conclusions involved several week's continuous work due to the amount of data involved. This, and the fact that few if any of the multi-directional tanks in use elsewhere can match the Edinburgh tank in performance and reliability, is probably the reason why, to the best of the author's knowledge, no comparable measurements have been made in other wide wave tanks.

The subsequent theoretical analysis of the behaviour of long cylinders required input values for the empirical constants in the modified form of Morison's equation. These were determined using a least squares fitting program, which adjusted the value of the empirical constant in the modified equation, so that the mean square difference, over an integral number of wave cycles, between experimental wave force measurements and the forces predicted by the force equation, was minimised.

Surge force measurements produced no surprises. The forces increased with wave frequency as might be expected because of the larger accelerations in a rapidly oscillating fluid and with the depth of submergence because of the greater volume of water displaced by the cylinder. Heave forces were found to be rather more surprising. The buoyancy forces present in heave tend to oppose the inertial wave forces and when a cylinder is very nearly totally submerged and the ratio of effective diameter (d' in Figure 4.2) to wave height is small, the combination of forces acting on the cylinder can result in the total
wave force containing components at twice the wave frequency. The modified Morison equation was shown to be capable, with the right coefficients, of predicting this effect, as in Figure 3.17.

The empirical coefficients required to make the modified Morison equation fit experimental data were found to be frequency dependent, tending to decrease as frequencies increase, but were roughly constant over the range of frequencies used in wide tank tests. The coefficients for surge forces were found to be roughly twice those for heave forces.

The cylinders used in the wide tank had a slightly larger diameter than those used for the least square experiments and so it was necessary to use the diameter to wavelength ratio to relate the empirical coefficients to the frequencies at which they were used for predicting spine behaviour in Chapters 4 and 5.

Any object which oscillates in a fluid near the free surface generates surface waves which carry energy away from the object. This energy loss can be considered as being due to a damping force on the object. In order to determine the damping due to wave radiation a series of tests was conducted on an electrically oscillated cylinder in a narrow wave tank. The actual forces on the cylinder were not measured but the energy content of the radiated waves was measured and the damping forces deduced. The damping on a cylinder which lies just submerged was found to be highly frequency dependent, a property which is discussed further in Chapter 4 and in Section 6.2.

The theoretical and numerical considerations of the behaviour of floating long cylinders were compared with measurements taken on a spine consisting of rigid segments joined elastically. This was actually intended as a scale model of the backbone of a string of 'Salter Duck'
devices and its properties are of great interest to the engineers of the Edinburgh Wave Power Group.

Chapter 4 outlined a linear theory to determine the responses of a wave power spine to waves over a wide range of conditions. Linearised versions of the wave force equations discussed in Chapter 1 were applied to the three dimensional hydrodynamic problem of the long flexible spine in random seas. The two dimensional equations discussed in earlier sections are not strictly applicable to a three dimensional problem but it was assumed that the spine is sufficiently long, compared with the wavelength, that any three dimensional hydrodynamic effects were confined to a small region near the spine ends. Structural end effects were assumed to be of greater importance than hydrodynamic end effects.

The response of an infinitely long spine to regular wave excitation was considered, analytically, as an elastic wave travelling down the spine at the same speed as the wave crests. This wave was described by the particular solution of the differential equation governing the response of the linearised system. The response becomes a maximum when the forced elastic waves match the natural elastic waves of the spine in wavelength at the particular wave frequency. This principle was demonstrated graphically in contour plots (Figures 4.7 to 4.11) of theoretical bending moments which show how, for particular wave frequencies, certain wave directions produced particularly large moments. The plots show that at high frequencies the Morison equation predicted very high narrow resonances for waves almost incident to the spine. Experimental measurements of bending moments in a long spine showed that high moments did occur close to the frequencies and directions predicted. The particular solution given in equation [4.23] is now
being used by wave power engineers as an aid in testing alternative wave force equations which may eventually replace Morison's equation in spine analysis.

Analytic consideration of the interactions between forced travelling waves in the spine with spine ends involved complex treatments involving particular solutions and independent solutions of the linearised differential equations. The principles involved can however be visualised by considering reflections of waves at free ends. When the elastic wave strikes a spine end it is reasonable to consider that it would be reflected but, although the reflected waves would have the same frequency as the incident waves, the wavelength of the reflected wave would be determined by the elastic and inertial properties of the spine. The reflected waves are not forced by the incident water waves, as are the travelling waves which would be present in an infinite spine, and so decay within a few cycles. The interaction between the forced and reflected waves was found to result in large RMS moment predictions at the down wave end of spines in regular waves.

The hydrodynamic parameters required for the prediction of spine responses in regular waves were assumed to be specified as functions of the wave frequency. The actual values were presented in Chapter 3. In the case of random wave excitation this was not so easy. In Chapter 4 and in Chapter 5 the hydrodynamic parameters were defined as functions of the peak frequency of the sea state in use. This means that for prediction purposes the added mass and damping parameters were assumed to be the same for a spine in 1Hz regular waves as for a spine in a random sea with a peak frequency of 1Hz. This assumption put serious limitations on the applicability of the analysis. A very broad band spectrum or a double peaked spectrum would require a somewhat more
complex treatment, such as suggested in Section 6.2.

The equations which were derived for the response of a single ended spine showed that the response of a spine to random waves showed peaks in the RMS bending moment distribution near the end but end effects in random seas did not extend as far along the spine as for regular waves. This is caused by a smoothing out of the responses of the spine to the multitude of component wavelets in the sea. Experimental measurements taken on the wave power spine confirmed these predictions but, although the surge (horizontal) predictions were close to experimental measurements, the magnitude of the heave (vertical) moments were over predicted in the simple linear spine model by a factor of about two.

In Chapter 5 the effects of allowing the volume of water displaced by the spine to vary in the calculations were examined. The principal effect of varying the displaced volume with the vertical position of the spine and with the wave elevations is to produce buoyancy forces opposing the inertial wave forces. This tends to reduce wave forces in heave.

The equations which resulted from including the volume variations were non linear and required a numerical approach. The Fortran program which was written to solve the spine equations showed that the non linear equations were more successful in predicting the heave moments in a spine than were the linear equations of Chapter 4. Heave moment predictions were found to agree, within experimental errors, with experiments to measure moments in a spine subjected to a one second Pierson-Moskowitz sea.

One useful prediction of the program was that although the non linear equations relating to surge responses contained heave dependent terms,
the surge moments predicted in a spine had no noticeable dependence upon the stiffness of the spine to heave deflections. This independence of surge and heave responses is of interest to wave power engineers as it simplifies their work on spine behaviour to two separable problems. The program also backed up experimental evidence that in any particular sea the bending moments in a spine increase with spine stiffness but eventually reach an asymptotic limit defined by the sea conditions.

The numerical program has made some useful predictions of how the maximum RMS bending moments along the length of a spine vary with length. Numerical and experimental measurements on long spines indicated that the largest moments occur near the ends. The actual position of the peaks is a complicated function of sea state and spine properties which is not readily definable. The numerical program has shown that, when the spine is sufficiently short and the two end peaks coincide, the maximum moments in the spine increase from those in a long spine by up to a factor of two. The program also showed that, at these critical lengths, the correlations between moments at any position in the spine were greater than zero. An increase in length produced negative correlations between certain points. The predictions of critical lengths have been used by wave power engineers to describe potentially dangerous lengths of duck string for use in the Atlantic. A full scale Duck String of 400m to 600m in length should be avoided by engineers interested in constructing prototype wave power stations.

One unexpected result from both numerical and experimental tests on spine behaviour was the discovery that even random seas with directional spectra symmetrical about the normal to the spine produced asymmetric bending moment distributions, such as can be seen in Figures 5.9 and
5.11. This was unlikely to be caused by variations in the sea state over the tank as tests described in Chapter 3 showed that the variation in wave height over the tank was small for random seas. The asymmetry is thought to be an artefact of the discrete wave front method of generation of pseudo random seas. The spines were shown in Chapter 4 to possess some rather fine resonances, as shown in Figures 4.7 to 4.11, and it is thought that some of these are being excited by teeth in the discontinuous wave spectra to produce the asymmetry. This is an argument in favour of the white noise filtration method of wave generation described in Chapter 2.
6.2 Future Work

Both of the techniques used in this work to predict spine responses involved the use of hydrodynamic properties known as functions of frequency. This is inappropriate for a time domain simulation, as described in Chapter 5, and it would be more satisfactory to use properties expressed in the time and space domain. It would, therefore, be a useful exercise to determine, possibly empirically, hydrodynamic properties in space and time (i.e. t, r, r, etc). This would then not require the sweeping assumption that radiation parameters can be determined from the mean frequency of the waves. It should also be possible to adapt the numerical simulation to problems such as the mooring cables of tension leg platforms by changing the force equations and end conditions. This would involve the introduction of a drag term because of the small cable diameters.

The use of filtered white noise in a numerical spine analysis would be a useful exercise but would only be really worthwhile if the wide tank itself were to be converted to this system of wave generation and this remains a major aim of future work. The software for white noise filtration is likely to involve large quantities of machine code if a conventional micro or mini computer is to be used to drive the tank in real time. This problem could be overcome if each wavemaker were to be controlled by its own microprocessor which would have access to a common memory containing the digital filter. It is likely, however, that within the next four years desk top array processors will be introduced which are ideally suited to the multiple array multiplications described in Chapter 2. The testing of spectral reproduction is also of
great importance if further work on wave generation is to be performed. It would, therefore, be of great practical interest to develop a simple but effective technique for spectral analysis in wide tanks. The moveable wave gauge array approach poses problems of calibration drift which must be overcome if a good degree of accuracy is required. The Edinburgh 'Wave Power' engineers are, however, currently investigating the possibility of using lightweight sonic wave gauges which should in the near future facilitate the analysis of multi-directional seas.
APPENDIX I

WAVE GENERATION PROGRAM 'QOS82'
C PROGRAM QOS82 3/8/82
C THIS PROC. WAS WRITTEN BY IAN BRYDEN (CASE PROJECT)
C THE THEORY OF TWO DIMENSIONAL DIGITAL FILTERING
C CAN BE SEEN IN A REPORT WRITTEN BY HIM, DR STANDING
C HAS A COPY OF THIS REPORT
C THE FILTER REQUIRED MAY BE CREATED USING A PROGRAM
C CALLED INFOR

INTEGER COLF, M, I, K, AMAX, NUMWAV, S
REAL X, H, MEAN, SD
LOGICAL A

C A IS AN ARRAY OF SHIFT REGISTERS USED TO SIMULATE
C WHITE NOISE
C H IS THE DIGITAL FILTER
DIMENSION H(-15:15, -64:64), A(-9:25, -64:65)
COMMON/CONSTS/AMAX, DT, NUMWAV
COMMON H, COLEN, M, X, A

C INPUTTING THE MEAN STD. DEV. AND COS**S TERMS OF
C SPECTRUM
C READ(4, 761) MEAN, SD, S
WRITE(7, 761) MEAN, SD, S
761 FORMAT(2F8.4, 12)
C WRITE(8, 917) MEAN, SD, S
917 FORMAT(' CAUSSIAN(MEAN=', F6.3, ' ;SD ', F6.3, ' ;SPREAD=', I2, ' )')
READ(4, 1) COLEN, DF
WRITE(7, 118) DF
118 FORMAT(F8.4)
WRITE(6, 987)
987 FORMAT(' RUN TIME: ') READ(5, *) DF
WRITE(6, 876)
876 FORMAT(' NUMBER OF FLAPS: ') READ(5, *) NUMWAV

C INPUTTING FILTER
DO 2 I=0, M
  DO 3 K=-COLEN, COLEN
    READ(4, *) H(K, I)
    H(K, -I) = -H(K, I)
  3 CONTINUE
2 CONTINUE
WRITE(6, 17) M
17 FORMAT(' UTILISED LENGTH OF FILTER(MAX=', I2, ' ): ')
READ(5, 18) M
DO 79 I = M, K
  H(K, I) = H(K, I) * SQRT(COS(3.14 / 2 * I / M))
79 CONTINUE
C HANNING WINDOW FUNCTION COULD BE FITTED IN HERE
C ALSO BE USED IN THE DIRECTIONAL DOMAIN BUT THAT
C WOULD BE WALKING ON SOME THIN ICE AS NO WORK HAS
C TO MY KNOWLEDGE, BEEN DON^ ON THAT SUBJECT
CALL GENE
STOP
END

C LINEAR CONGRUENTIAL PSEUDO RANDOM NUMBER FUNCTION
REAL FUNCTION RANY(X)
A=2.134
B=1.876
C=0.874
Y=(A*X+B)/C
RANY=Y-INT(Y)
RETURN
END

C INITIALISING SHIFT REGISTERS
SUBROUTINE NOISE
INTEGER P, Q, AMAX, NUMWAV, COLEN, M
REAL X, H
LOGICAL A
COMMON H, COLEN, M, X, A
DIMENSION H(-15:15, -64:64), A(-9:25, -64:65)
COMMON/CONSTS/AMAX, DT, NUMWAV
WRITE(6, 172)COLEN
172 FORMAT(' COLEN(MAX=', 12, ')
READ(5, 173)COLEN
WRITE(7, 173)COLEN, M
173 FORMAT(212)
C RANDOM NUMBER SEE
X=0.10345
DO 1 P=-M, M+1
   DO 2 Q=-1-COLEN, NUMWAV+COLEN
      X=RANY(X)
      A(G, P)=X.GT.0.5
2 CONTINUE
1 CONTINUE
RETURN
END

C BUSINESS PART OF PROGRAM
SUBROUTINE GENER
INTEGER AA, BB, P, Q, WAV, TIM, COLEN, M, AMAX, NUMWAV
REAL SIGNAL, X, H
LOGICAL A
DIMENSION H(-15:15, -64:64), A(-9:25, -64:65)
DIMENSION SIGNAL(1:15)
COMMON/CONSTS/AMAX, DT, NUMWAV
COMMON H, COLEN, M, X, A

C WRITE(8, 37)
37 FORMAT(' SIGNALS TO WAVEMAKERS--SEPERATION=2M')
C WRITE(8, 38)
       '7', 6X, '8', 2X, 'TIME')
C TIME STEPS
   DO 1 AA=0, AMAX
      T=AA*DT
1 CONTINUE

```fortran
SIGNAL(BB)=0.0
C CONVOLUTION OVER FILTER
DO 3 P=-M,M
  DO 4 Q=B-B-COLEN, B+COLEN
ċH(BB-Q,P)
  IF(A(Q,P)) SIGNAL(BB) = SIGNAL(BB) +
  FORMAT(4I4)
  CONTINUE
  CONTINUE
3 CONTINUE
  FORMAT(6F8.4)
  CONTINUE
WRITE(7,99)(SIGNAL(BB), BB=1, NUMNAV)
WRITE(8,111)(SIGNAL(BB), BB=1, NUMNAV), T
C C C
C SOFTWARE SHIFTING OF REGISTERS
C CONTINUE
3 CONTINUE
111 FORMAT(6F8.4, ', ', F8.4)
C DO 7 WAV=(1-COLEN), NUMNAV+COLEN
  DO 8 TIM=-M, M
    A(WAV,TIM)=A(WAV,TIM+1)
    CONTINUE
    A(WAV,M+1)=A(WAV, 10), NEG. A(WAV, 0)
  CONTINUE
7 CONTINUE
1 CONTINUE
RETURN
END
```

**EMAS 2972 EMA5>>> EGNP40 I. Bryden**
**EMAS 2972 EMA5>>> EGNP40 I. Bryden**
**EMAS 2972 EMA5>>> EGNP40 I. Bryden**
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APPENDIX II

SPINE SIMULATION PROGRAM 'SPINE'
This programme was the state of the art on 06/05/1981.

```fortran
common zrms, yrms
common/sink/zz
common/consts/pi, g, row, alen, akv, akh, td
common/waves/n, a, b, c, bg, cg, gc
common/store/for, far
dimension zrms(204), yrms(204)
dimension zz(204), yy(204), ssz(204), ssy(204), wzw(204)
dimension wwy(204), bz(204), by(204)
dimension el(202)
dimension a(75), b(75), c(75), bg(75), cg(75), gc(75)
dimension for(202), far(202)
dimension ccos(3000)
logical*1 hname(30)
logical*1 sname(30)
call assign(10, 'sm:[300,333]spinein.dat ')
call assign(10, 'sm:[300,333]wav.dat ')
10 read(10, *, end=20) amas, alen, akv, akh, d, td, mm, ei, sz, sy, o, p,
  &al, rz, ry, q, qss
read(10, 314) hname
read(10, 314) sname
314 format(30a1)
c setting of other constants
  nn=mm+2
  g=9.81
  row =1000.0
  alen=alen/mm
  amas=amas*alen
  rz=rz*alen
  ry=ry*alen
  q=q*alen
  qss=qss*alen
  pi=3.141593
  dx=alen
call initco(sy, sz, o, p, nn, ssy, yy, ssz, zz)
call intab(ccos)
call waves
dt=.001
  alt=al
al=al/dt
  limit=int(int(al/50)
t=0
c starting time loop
c write(5, 785)
785 format(’ ’)
do 999 jkj=1, limit
  write(5, 786) t, alt
call elevat(nn, t, ccos, elar, elor, el)
c write(5, 3436) for(3), for(52)
3436 format(’ end moments are: ’, 2f10.4)
do 101 kjk=1, 50
  t=t+dt
c setting of end conditions
  zz(2)=2*zz(3)-zz(4)
  zz(1)=zz(5)-2*zz(4)+2*zz(2)
  yy(2)=2*yy(3)-yy(4)
  yy(1)=yy(5)-2*yy(4)+2*yy(2)
  zz(nn+1)=2*zz(nn)-zz(nn-1)
  zz(nn+2)=zz(nn-2)-2*zz(nn-1)+2*zz(nn+1)
  yy(nn+1)=2*yy(nn)-yy(nn-1)
```

yy(n+2) = yy(n-2) - 2*yy(n-1) + 2*yy(n+1)

```
c starting spatial loop element by element
  do 202 ki=3,nn
    k=ki
  c working out displaced volume
    v=vol(ey(k),zz(k),d)
  c working out effective mass
    scrub=amas+(akv)*row*v
  c heave force
    f0=for(k)*row*v
  c surge force
    fl=far(k)*row*v
  c heave flexural force
    call elasti(zz,k,ei,dx,elas,nn)
  c total heave force
    fo=f0-elas+row*g*v-amas*g
  c heave time step
    www(k)=waz(scrub,q,rz,fo,zz(k),ssz(k),alen)
  c surge flexural force
    call elasti(yy,k,ei,dx,elas,nn)
  c total surge force
    fo=fl-elas
  c surge time step
    scrub=amas+(akh)*row*v
    wwy(k)=waz(scrub,qss,ry,fo,yy(k),ssy(k),alen)
  202 continue
  do 409 kj=3,nn
    ssz(kj)=zz(kj)
    zz(kj)=wwz(kj)
    ssy(kj)=yy(kj)
    yy(kj)=wwy(kj)
  409 continue
101 continue
  call bendin(ei,zz,yy,dx,limit,nn)
999 continue
  do 94 jjk=1,mm
    zrms(jjk)=sqrt(zrms(jjk+2))
    yrms(jjk)=sqrt(yrms(jjk+2))
  94 continue
  call fputf(1,mm,zrms,hname)
  call fputf(1,mm,yrms,sname)
  go to 10
20 continue
  call close(10)
  call close(15)
  stop 'moments computed'
end
```

```subroutine elasti(zz,k,ei,dx,elas,nn)
  dimension zz(204)
  elas=ei*(zz(k+4)-4*zz(k+1)+6*zz(k)-4*zz(k-1)+zz(k-2))/(dx*dx*dx)
  return
end subroutine elasti```

```subroutine initco(sy,sz,p,n,ssy,yy,ssz,zz)
  common zrms,yrms
  dimension zz(204),ssy(204),yy(204),ssz(204)
  dimension zrms(204),yrms(204)
  do 100 i=3,nn
    jji=i
    ssy(jji)=sy```

```
ssz(jjj)=sz
yy(jjj)=sz+0.001
zz(jjj)=sz+p*0.001
zrms(jjj)=0
yrms(jjj)=0
100 continue
return
end

realfunction vol(ei,z,d)
  common/consts/pi,g,row,alen,akv,akh,td
  x=td+ei-z
  if(x.ge.d/2)goto204
  if(x.le.(-d/2))goto105
  vol=alen*d*z/8*pi*(1+sin(pi*x/d))
  goto106
204 vol=d*d*alen*pi/4
  goto106
105 vol=0
106 continue
return
end

realfunction waz(a,q,r,f,z,sz,alen)
dt=.001
  c=a/(dt*dt)+q/(2*dt)
  waz=(f+a*(2*z-sz)/(dt*dt)+q*sz/(2*dt)-r*z)/c
  return
end

subroutine bendin(ei,zz,vy,dx,al,nn)
  integer al
  common zrms,yrms
  dimension zz(204),yy(204),bz(204),by(204)
  dimension zrms(204),yrms(204)
    bz(3)=0,0
    by(3)=0,0
    bz(nn)=0,0
    by(nn)=0,0
    nnj=nn-1
  do 33 j=4,nnj
    bz(j)=ei*((zz(j+1)-2*zz(j)+zz(j-1))/(dx*dx))
    by(j)=ei*((yy(j+1)-2*yy(j)+yy(j-1))/(dx*dx))
    zrms(j)=(zrms(j)+bz(j)*bz(j)/al)
    yrms(j)=(yrms(j)+by(j)*by(j)/al)
 33 continue
return
end

subroutine waves
  common/ways/n,a,b,c,bg,cg,gc
  dimension a(75),b(75),c(75),bg(75),cg(75),gc(75)
  read(15,1)n
  format(i2)
  do 2 i=1,n
    read(15,*)a(i)
    read(15,*)b(i)
    read(15,*)c(i)
    read(15,*)bg(i)
    cg(i)=b(i)*b(i)
    gc(i)=sqrt(1-(9.81*c(i)/cg(i))**2)
 2 continue
return
subroutine eev(t, k, ccos, el)
common/store/for, far
common/sink/zz(204)
common/wavs/n, a, b, c, bg, cg, gc
common/consts/pi, g, row, alen, akv, akh, td
dimension for(202), far(202)
dimension ccos(3000)
dimension a(75), b(75), c(75), bg(75), cg(75), gc(75)
dimension el(100)
el(k)=0.0
for(k)=0.0
do i=1, n
ib=mod(b(i)*t+c(i)*k-1.5*alen+bg(i), 6, 283)
jb=ib+502
elar=a(i)*ccos(jb)
elor=a(i)*ccos(jb+256)
el(k)=el(k)+elar
stv=-(1+akv)*cg(i)*exp(cg(i)*(zz(k)-td)/9.81)
sth=-(1+akv)*cg(i)*exp(cg(i)*(zz(k)-td)/9.81)
for(k)=for(k)+stv*elor
far(k)=far(k)+sth*elar+gc(i)
continue
return
end

subroutine elevat(nn, t, ccos, el)
common/consts/pi, g, row, alen, akv, akh, td
common/store/for, far
dimension ccos(3000)
dimension el(202)
dimension for(202), far(202)
do i=1, nn
k=i
call elev(t, k, ccos, el)
continue
return
end

subroutine intab(ccos)
dimension ccos(3000)
do i=1, 3000
ccos(i)=cos(i*6.283/1024)
continue
return
end
APPENDIX III

FLOWCHARTS FOR SPINE SIMULATION PROGRAM
START

READ

AMAS, ALEN, AKV, AKH, D, TD, MM
EIV, EIHI, SZ, SY, O, P, AL, RZ, RY, Q, OSS

G=9.81
ROW=1000.0
ALEN=ALEN/MM
AMAS=AMAS*ALEN
RZ=RZ*ALEN
RY=RY*ALEN
O=O*ALEN
QSS=QSS*ALEN
P1=3.141593
DX=ALEN

CALL INITCO
SY, SZ, O, P, NN, SSV, YY, SSZ, ZZ

CALL INTAB
CCOSS

CALL WAVES

DT=0.001
ALT=AL
AL=AL/DT
LIMIT=INT(AL/50)
T=0

JKJ=1

CALL ELEVAT
NN, T, CCOS, ELAR, ELAR, EL

KJK=1

T=T+DT

ZZ(2)=2*ZZ(3)-ZZ(4)
ZZ(1)=ZZ(5)-2*ZZ(4)+2*ZZ(2)
YY(2)=2*YY(3)-YY(4)
YY(1)=YY(5)-2*YY(4)+2*YY(2)
ZZ(NN+1)=2*ZZ(NN)-ZZ(NN-1)
ZZ(NN+2)=2*ZZ(NN-1)+2*ZZ(NN+1)
YY(NN+1)=2*YY(NN)-YY(NN-1)
YY(NN+2)=YY(NN-1)+2*YY(NN+1)
CALL ELASTI
ZZ, K, E1, DX, ELAS, NN

\[ \text{ELAS} = E1 \times \left( (ZZ(K+2) - 4 \times ZZ(K+1) + 6 \times ZZ(K) - 4 \times ZZ(K-1) + ZZ(K-2)) / DX \times DX \times DX \right) \]

RETURN

CALL INITCO
SY, SZ, 0, P, NN, SSY, YY, SSZ, ZZ

I = 3

JJJ = I
SSY(JJJ) = SY
SSZ(JJJ) = SZ
YY(JJJ) = SY + 0 \times 0.001
ZZ(JJJ) = SZ + P \times 0.001
YRMS(JJJ) = 0.0
YRMS(JJJ) = 0.0

I = I + 1

RETURN
CALL WAVES

READ

N

I = 1

END

READ

A(I), B(I), C(I), DG(I)

CC(I) = SQRT(1 - (9.81 * C(I) / CC(I))^2)

I = I + 1

I = N

RETURN

CALL ELEV

T, K, CCOS, EL

EL(K) = 0.0

FOR(K) = 0.0

FAR(K) = 0.0

I = 1

AB = AMOD(B(I) + (K - 1.5) * ALEN + BC(1), 6.283)

IB = INT(AB / (2 * PI) * 1023)

JB = IB * 1023

ELAR = A(I) * CCOS(JB)

ELOR = A(I) * CCOS(JB + 256)

EL(K) = EL(K) + ELAR

STV = -(1 + AKV) * CC(I) * EXP(CC(I) * ZZ(K) - TD / 9.81)

STH = -(1 + AKH) * CC(I) * EXP(CC(I) * ZZ(K) - TD / 9.81)

FOR(K) = FOR(K) + STV * ELOR

FAR(K) = FAR(K) + STH * ELAR * CC(I)

I = I + 1

I = N

RETURN
V = VOL(EL, Z, D)

X = TD + EL - Z

\[ X : D / 2 \]

\[ \text{RETURN} \]

VOL = D * D * ALEN * PI / 4

\[ \text{RETURN} \]

VOL = 0

\[ \text{RETURN} \]

\[ \text{RETURN} \]

VOL = ALEN * D / 8 * PI * (1 + SIN(PI * X / D))

CALL WAZ A, Q, H, F, Z, SZ, ALEN

DT = 0.001

C = A / (DT * DT) + Q / (2 * DT)

WAZ = (F + A * (2 * Z - SZ) / (DT * DT) + Q * SZ / (2 * DT) - R * Z) / C

\[ \text{RETURN} \]
CALL BENDIN
EIV, EIH, ZZ, YY, DX, AL, NN

BZ(3) = 0.0
BY(3) = 0.0
BZ(NN) = 0.0
BY(NN) = 0.0
NNJ = NN - 1

J = 4

BZ(J) = EIV * ((ZZ(J+1) - 2*ZZ(J) + ZZ(J-1)) / (DX*DX))
BY(J) = EIH * ((YY(J+1) - 2*YY(J) + YY(J-1)) / (DX*DX))
ZRNS(J) = (ZRNS(J) + BZ(J) * BZ(J) / AL)
YRNS(J) = (YRNS(J) + BY(J) * BY(J) / AL)

J = J + 1

J: NNJ

RETURN
CALL ELEVAT
NN, T, CCOS, ELAR, ELOR, EL

II = 3

K = II

CALL ELEV
T, K, CCOS, EL

II = II + 1

II: N

RETURN

CALL INTAB
CCOS

I = 1

CCOS(I) = COS(I * 6.283 / 1024)

I = I + 1

I: 3000

RETURN
APPENDIX IV

REFERENCES
REFERENCES


APPENDIX V

NOTATION
NOTATION

a    wave amplitude
A    wave amplitude; cross sectional area
A    wave amplitude
A_1  a constant
A_2  a constant
C    inertial force tensor
C    a constant
C_D  drag coefficient
C_H  inertial heave force coefficient
C_S  inertial surge force coefficient
d'   effective diameter
D    diameter; general dimension
E    Youngs modulus
F    force vector
G    spring tensor
h    water depth; elevation
H(ω,θ) spreading function
I    moment of inertia
K    wave vector
L    length
L    general dimension; a constant
M    mass
n    normal unit vector
Q    damping tensor
r    position vector
R_{AA} : R_{zz} correlation function
R_s  spring constant
$R_{AZ}$  cross correlation function

$S_{AA}; S_{ZZ}$  power spectrum

$S_{AZ}$  cross power spectrum

$t$  time variable

$T$  wave period

$U$  water particle velocity vector

$V$  volume

$x$  coordinate

$y$  coordinate

$z$  coordinate

$\delta$  dirac delta function

$\eta$  wave elevation

$\theta$  an angle

$\lambda$  wave length

$\pi$  PI

$\rho$  density of water

$\phi$  velocity potential

$\phi$  velocity potential

$\omega$  angular frequency
APPENDIX VI

PUBLISHED PAPERS
Abstract

The purpose of this work was to develop a technique for the generation of multi-directional random waves, which could be used for driving wave-makers in a wave basin or for the simulation of elongated structures in a real sea.

The method used was an extension of the idea of digital filtration of white noise, which has been used successfully for the generation of waves in unidirectional tanks.
1. Introduction

There are several existing methods for the generation of 3-dimensional random waves. The Wallingford Method [1] consists of an arc of flaps all generating one dimensional spectra along different axes focussing to produce a required directional spread within a central working area of a square basin. This is not suitable for work involving structures which are longer than the small central region of the tank.

The snake [2] superimposes discrete wave fronts, each having a random starting phase. This method is not continuous in either direction or frequency but does produce flexible control of the types of spectra used. The diffraction technique [3] utilising large independently driven wavemakers which produce spreading by the natural diffraction of the waves from the paddles. This produces waves whose directionality is continuous, rather uncontrolled but known.

There is therefore a need for a method of generation which gives controllable continuous spectra over a large area of the wave basin. The technique presented here is an extension of a method already used to generate waves in 2-d wave flumes [4]. A random Boolean series is passed through a filter corresponding to the desired spectrum to produce the signal record.

2. Theory

An array of wave-makers can be thought of as a structure, the power spectrum of whose motion matches the spectrum of the required sea.

The response of an infinitely long, regular linear system, lying along the x-axis, to a driving force $A(t,X)$ can be expressed as

$$ R(t,x) = \int_{\tau=-\infty}^{\infty} \int_{X=-\infty}^{\infty} h(\tau, X)A(\tau, X) d\tau dX \tag{1} $$

where $h(\tau, X)$ is the unit impulse response function. The power spectrum, $S_0(\omega, C)$ of the response of the system to a driving spectrum $S_1(\omega, C)$ is given by [5]

$$ S_0(\omega, C) = |H(\omega, C)|^2 S_1(\omega, C) \tag{2} $$

$\omega$ = angular frequency
$C$ = X-component of the wave vector
$H(\omega, C)$ is the system function, or inverse fourier transform of the response function.

If $S_1(\omega, C) = \text{const.}$

Then assuming an anti-symmetric phase distribution:

$$ h(\tau, X) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(\omega, C) \sin(\omega t + cX) d\omega dc \tag{3} $$
The full integral (1) is not required as any real sea has only a finite coherence time and correlation distance so:

\[ R(t,x) = \int_{\tau=(t-T)}^{(t+T)} \int_{x=(x-L)}^{(x+L)} h(t-\tau, x-X) A(\tau,X) d\tau dX \quad (4) \]

T > coherence time.
L > correlation length.

We are only interested in the response of the wavemakers at discrete times and places i.e. \( t = a\Delta t \) and \( x = b\Delta x \)

hence \( T = M\Delta t \) and \( L = N\Delta x \)

so that

\[ R(a,b) = \int_{\tau=(a-M)\Delta t}^{(a-M)\Delta t} \int_{x=(b-N)\Delta x}^{(b-N)\Delta x} h(a\Delta t-\tau, b\Delta x-X) A(\tau,X) d\tau dX \quad (5) \]

and if the driving force is also discrete

i.e. \( A(\tau,X) = A'(p,q) \delta(\tau-p\Delta t, X-q\Delta x) \)

\[ A'(p,q) = 0 \text{ or } 1 \]
and \( p,q = 0,1,2,3, \ldots \)

(5) becomes

\[ R(a,b) = \int_{\tau=(a-M)\Delta t}^{(a-M)\Delta t} \int_{x=(b-N)\Delta x}^{(b-N)\Delta x} h(a\Delta t-\tau, b\Delta x-X) A'(p,q) \delta(\tau-p\Delta t, X-q\Delta x) d\tau dX \quad (6) \]

\[ R(a,b) = \sum_{p=(a-M)}^{(a-M)} \sum_{q=(b-N)}^{(b-N)} h(a-p, b-q) A'(p,q) \quad (7) \]

where \( h'(m,n) = h(m\Delta t, n\Delta x) \)

3. Method

The programme used to generate the waves was written in P.A.S.C.A.L. on an ICL 2972 mainframe. An array of shift registers containing 1's and 0's was convolved with the digital filter to produce the wave records. The input to the shift registers was produced using a linear congruential random number generator.

Figure 1 shows an example of a 2-dimensional spectrum which has a Gaussian form in the frequency domain and \( \cos^2 \) directional spreading. Notice that the
The drawing is drawn in C space rather than O space. The corresponding digital filter is shown in figure 2. To simplify the diagram only the positive time part of the filter is plotted.

4. Results

The results were tested in two ways.

The total power spectrum of the wave signals was compared with the model power spectrum. Then cross-correlations of the time records at various x-positions were compared with the predicted cross-correlation.

Figure (3) shows the superimposition of the signal spectrum on the desired Gaussian form. These show very good agreement and the conclusion is that this method provides excellent spectral reproduction at the wavemakers. Only one tank test has been run to date; the resulting power spectrum is shown in figure (4). The high frequency shift of the spectrum is due to the transfer function of the tank which had not been accurately determined.

Figure (5) is a 3 dimensional plot of the wave signals on which the analysis was carried out. The wave direction is indicated. The diagram displays, admirably, the short crested nature of the wave field.

The correlation of the wave signals is plotted on figure 6. The separation of the wave-makers was assumed to be 0.3 m. This is to be compared with the correlations obtained between wave gauges positioned at a separation distance of 0.35 m. The inaccuracies in the latter may be accounted for by diffraction effects at the edges of the tank (note that only 7 wavemakers were used in the preliminary test).

5. Conclusions

The method of 2 dimensional filtering of random noise has been successful in producing the desired sea states. However several tests still need to be performed before full implementation. The method lends itself for use on small tanks by direct signal production from a micro-computer. Larger tanks may require a somewhat faster machine although the possibility of calculating the signals before running the sea state should be borne in mind.

Acknowledgements

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References


COMPARISON OF THEORETICAL AND SIGNAL SPECTRA

CENTRAL FREQ. = 1.5HZ
WIDTH = 0.25HZ

Figure 3:

POWER SPECTRUM FROM TANK TEST

Figure 4
Wave Signals Using Digit 0 as Filter

Figure 5
Correlation Between Wave Signals

Figure 6

Correlations Between Wave Measurements

Figure 7
Abstract

The behaviour of long floating pipes in random seas is studied and two methods of prediction of induced bending moments are presented. This work has been mainly orientated towards wave energy extraction but can be applied to certain problems relating to the offshore oil industry.

Introduction

Any structure which is to be placed in the sea must be capable of surviving any conditions it is likely to meet. This has resulted in large amounts of theoretical and experimental work being performed this century into the response of various structures to wave forces. Much of this work has been reviewed by Hogben(2).

This paper outlines work which has been carried out by the authors into the behaviour of long floating pipes in three dimensional random seas. Wave energy research (6) and the possibility of towing long constructed pipelines by sea (3) are two applications of the result.

A two dimensional force equation for fixed cylinders is modified and used for a long flexible pipe. The resultant differential equations are examined by two methods. Initially the equations are linearised and the analytic solution is outlined. This treatment is discussed and results presented. A numerical solution still possessing non-linear properties is also given. This method is used to predict the bending moments in a cylindrical spine under study by the Edinburgh University wave energy group and comparisons between experimental and simulation made.

Theoretical Model

Dixon(1) described a force equation which predicted wave forces on fixed partially submerged horizontal cylinders in two dimensional finite sized waves.

\[ F = -C_D \frac{\partial}{\partial t} \{ A(z,t)U \} + \rho g A_0 - \rho A(z,t)g \]  

\[ \text{inertia} \quad \text{gravity} \quad \text{buoyancy} \]  

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C.A. Greated, Director of Fluid Dynamics Unit, Edinburgh University.
\( C \) = non dimensional inertial force tensor  
\( \rho \) = fluid density  
\( A_0 \) = cross sectional area of cylinder  
\( A(z,t) \) = cross sectional area under the water line (dependent on \( \eta(t) \) the wave height and \( z \) the cylinder height)  
\( \rho \) = density of cylinder  
\( g \) = gravitational acceleration  
\( U \) = local fluid element velocity at C.o.M. of cylinder assuming that cylinder does not perturb the wave field  
\( z \) = displacement of cylinder centre above a reference level.

**DESCRIPTION OF TERMS RELATING TO A HORIZONTAL CYLINDER IN WAVES**

If the wave steepness is small and the wave elevation at the central position of the cylinder is given by \( \eta(t) \), then the displaced area \( A(z,t) \) is given by:

\[
A(z,t) = \frac{D^2}{8} \left[ \pi + \frac{4}{D}(\eta(t)+td-z)\left(1 - \frac{4}{D^2}(\eta(t)+td-z)^2\right)^{\frac{1}{2}} + 2\sin^{-1}\frac{2(\eta(t)+td-z)}{D} \right]
\]

where \( D \) = cylinder diameter.

Equation (1) implies several assumptions:

The diameter of the cylinder is small in comparison with the wave length. This is necessary as otherwise there would be considerable scattering of the incoming waves with the result that \( U \) would vary greatly over the volume displaced by the cylinder. The body diameter is large in comparison with the wave amplitude otherwise drag forces due to the viscosity of the fluid must be incorporated.

Equation (1) strictly refers only to a fixed cylinder. If the cylinder is free to move, other forces must be incorporated. These forces are generally coupled with the forces of (1). If, however, we assume that wave amplitudes are small then these forces can be uncoupled from those due to incoming waves.

**Added Mass (\( m \))**

An object oscillating in a fluid with frequency \( \omega \) is assumed to have its normal mass increased by
\[ M_A = K z A(z, t) \quad \Rightarrow \quad F_A = -\frac{\partial}{\partial t} \{ M R + M_{A R} \} \]  

\( M_A \) is the added mass tensor
\( K \) is the non dimensional added mass tensor

Added Damping

An oscillation object near to the free surface generates waves which carry energy away from that object. This power loss may be treated as a damping force.

\[ F_{DAM} = Q \mathbf{R} \]  

It is usual to choose the coordinate system so that the tensors in \( C, K \) and \( Q \) are non zero only along the leading diagonal if possible. In this case surge (y dim.) and heave (z dim.) are chosen. In this frame the cross components of the tensors are minimised.

The added mass and damping can be determined experimentally for any object and theoretically for some special cases, as functions of frequency. This is not, in general, appropriate for prediction of the response when non linearities are incorporated, as use of these parameters implies a previous knowledge of the object behaviour. If, however, the response is assumed to have little frequency spread and to be peaked about the same frequency as the sea spectrum, then it may be possible to determine useful values for the added mass and damping from the energy period of the sea state. If this assumption is made then theory can be used to predict that \( C = I + K \) where \( I \) is the unit diagonal tensor. This assumption minimises the amount of input data required.

It would be most appropriate for time-domain simulation to know the effects of body motion in terms of variables in the time and space domain i.e. \( (z, \tilde{z} \text{ etc.}) \) but this has not been attempted here.

Three Dimensional Nature of Pipe and Waves

The equations discussed here have been relating to 2-dimensional waves such as those in a wave flume. In the real sea however the waves and structures are three dimensional. This strictly requires somewhat different hydrodynamic assumptions to be made. The pipes examined in this work are assumed to be constrained so as to be free to respond only in the \( y \) and \(-z\) plane and that the lengths of the pipes are long enough for 2-dimensional hydrodynamics of response to suffice.

The waves incident on the pipes are considered to have wave elevations as a function of \( t \) and \( x \) of the form for regular waves.

\[ \eta(t, x) = A \cos(\omega t - K x + \phi) \quad \{ \text{deep water linear wave theory} \} \]  

where \( K_x = \frac{\omega^2}{g} \sin \theta, \phi = \text{phase} \)
\( \phi \) = angle between angle of propagation of the wave and the normal to the pipe.

In the case of regular waves the waves incident on the pipes are assumed to have the form

\[ \eta(t,x) = A \cos(\omega t - K_x x + \phi) \]

where \( K_x = \frac{\omega^2}{g} \sin \theta \) and \( \phi \) = phase

Random waves are considered to be summations of fronts as described by equation (5).

**AXIS CONVENTION**

**Structural Treatment of Pipe**

The pipe was treated as a continuous elastic beam which could flux vertically and horizontally. (5).

If a force \( W(x) \) is applied to the pipe then, if \( EI \) is the flexural rigidity, the pipe will deform according to the equation

**Vertical**

\[ W_z(x) = -EI \frac{\partial^4 z}{\partial x^4} \] (6)

**Horizontal**

\[ W_y(x) = -EI \frac{\partial^2 y}{\partial x^2} \] (7)

If the ends of the pipe are assumed to be free, the end conditions of the pipe are

\[ \frac{\partial^2 y}{\partial x^2} = \frac{\partial^3 y}{\partial x^3} = \frac{\partial z}{\partial x} = \frac{\partial^3 z}{\partial x^3} = 0 \quad x = 0, L \] (8)
In addition it is assumed that the pipe is held in position by weak horizontal springs:

$$F_{\text{spring}} = - R \frac{\partial Y}{\partial y}$$

This has the effect of countering any drift of the pipe due to non-linear wave effects and, as such, models any moving systems that the pipe may be equipped with. The value of the spring constant which is chosen $R$ does not have a significant effect on bending moments but does, as required, give control over how far the pipe drifts.

**Equation of Motion**

Given the assumptions made previously the response of the pipe to wave excitation is given by:

**Vertical**

$$\rho_p A_0 \ddot{Z} + K_p \rho \frac{\partial}{\partial t} (A(z,t)\dot{Z}) + Q_{hs} \dot{Z} + EI \frac{\partial^4 \gamma}{\partial x^4} = C_H \rho \frac{\partial}{\partial t} \{A(z,t)\dot{U}_H\} A_0 \rho g + \rho A(z,t)g$$  \hspace{1cm} (9)

**Horizontal**

$$\rho_p A_0 \ddot{Y} + K_s \rho \frac{\partial}{\partial t} (A(z,t)\dot{Y}) + Q_s \dot{Y} + EI \frac{\partial^4 \gamma}{\partial x^4} = C_s \rho \frac{\partial}{\partial t} \{A(z,t)\dot{U}_s\}$$  \hspace{1cm} (10)

$U_s$ is now considered to be a function of $x$ as well as $t$ and $z$ where $x$ is, of course, the position along the pipe.

**Analytic Linear Model**

Linearised totally by assuming $A(z,t) = \text{const} = A_1$. Then the resulting uncoupled equations can be solved as follows.

Surge:

$$\left(\rho_p A_0 + K_s \partial A_1 \right) \ddot{Y} + Q_s \dot{Y} + EI \frac{\partial^4 \gamma}{\partial x^4} = C_s \partial A_1 \dot{U}_s$$  \hspace{1cm} (11)

$$M_1 = \rho_p A_0 + K_s \partial A_1$$

hence

$$M_1 \ddot{Y} + Q_s \dot{Y} + EI \frac{\partial^4 \gamma}{\partial x^4} = C_s \partial A_1 \dot{U}_s = F_s(x,t)$$  \hspace{1cm} (12)

Assume $F_s(x,t) = F_0 \cos(\omega t - \eta x)$

$\omega = \text{angular frequency of wave}$

$\eta = \text{x component of wave vector}$
hence \( F_s(x,t) = R \{ F_0 e^{i(-t-x)} \} \)

Let us assume that all transients have decayed, i.e.

\( Y(x,t) = R \{ \phi(x)e^{i\omega t} \} \)

hence

\[
-\omega^2 M_1 \phi(x) + i\omega Q \phi(x) + EI \frac{\partial^4 \phi(x)}{\partial x^4} = F_0 e^{-inx}
\]  (13)

**Homogeneous Soln.**

\[
-\omega^2 M_1 Q(x) + i\omega Q \phi(x) + EI \frac{\partial^4 \phi(x)}{\partial x^4} = 0
\]  (14)

\( \phi(x) = e^{\lambda x} \)

hence

\[
-\omega^2 M_1 + i\omega Q + \lambda^4 EI = 0
\]  (15)

\( \lambda \ i = 1 \ldots 4 \) are the four complex roots of this equation.

**Particular Soln.**

Assume a soln. of the form \( \phi(x) = \phi_o e^{-inx} \) and substitute into

\[
-\omega^2 M_1 \phi_o + i\omega Q \phi_o + \eta^4 EI \phi_o = F_0 e^{inx}
\]  (16)

hence

\[
\phi_o = \frac{F_0}{(\eta^4 EI - \omega^2 M_1 + i\omega Q)}
\]  (17)

**Complete Solution**

\[
\phi(x) = \phi_o e^{-inx} + \frac{4}{\lambda_1 x} \sum_{i=1}^{4} A_i e^{\lambda_i x}
\]  (18)

The values of the constants \( A_i \) can be determined by using eqns. (8)

Once the function \( Y(t,x) \) has been found, the bending moments \( M(t,x) \) can be determined using

\[
M = EI \frac{\partial^2 Y}{\partial x^2}
\]  (19)

and the mean square bending moment as a function of position.

If the response to random waves is needed then the sea should be represented as a sum of discrete wave fronts each specified by an amplitude, frequency \( \omega \), and wave vector \( \eta \). The linearity of equation

6

Bryden
(11) allows us to simply add the mean square moments due to each wave front. The examples given of solutions are for a semi-infinite pipe. This is a situation where a numerical simulation would be very costly.

**Non Linear Numerical Model**

If the assumption is made that the pipe is lying low in the water then, for a circular cross section, terms involving \( \frac{\partial A(z,t)}{\partial t} \) become negligible and the equations of motion can be simplified to.

**Vertical**

\[
(\rho A_o + A(z,t)\rho K_H)\ddot{Z} + Q_H\dot{Z} + EI \frac{\partial^4 z}{\partial x^4} = C_H\dot{A}(z,t)\dot{U}_H + A(z,t)\rho g - A_o\rho g
\]

(20)

**Surge**

\[
(\rho A_o + A(z,t)\rho K_s)\ddot{Y} + Q_o\dot{Y} + EI \frac{\partial^4 z}{\partial x^4} = C_s\dot{A}(z,t)\dot{U}_s
\]

(21)

If finite differences are utilised and the position of the pipe at

\[
x = q\Delta x \quad q = 0, 1, \ldots
\]

\[
t = \rho\Delta t \quad p = 0, 1, \ldots
\]

are considered only.

Then \( \frac{\partial^4 z}{\partial x^4} \) can be approximated by

\[
\frac{Z(x+2\Delta x) - 4Z(x+\Delta x) + 6Z(x) - 4Z(x-\Delta x) + Z(x-2\Delta x)}{(\Delta x)^4}
\]

(22)

Similarly for \( \frac{\partial^4 y}{\partial x^4} \)

Also

\[
\frac{\partial^2 z}{\partial t^2} = \frac{z(t+\Delta t) - 2z(t) + z(t-\Delta t)}{(\Delta t)^2}
\]

(23)

and

\[
\frac{\partial x}{\partial t} = \frac{z(t+\Delta t) - z(t-\Delta t)}{2\Delta t}
\]

(24)

The end conditions can be expressed as

\[
z(x+\Delta x) - 2z(x) + z(x-\Delta x) = 0 \quad x = 0, L
\]

(25)

and

\[
y(x+\Delta x) - 2y(x) + y(x-\Delta x) = 0 \quad x = 0, L
\]

(26)

\[
z(x+2\Delta x) - 2z(x+\Delta x) + 2z(x-\Delta x) + z(x-2\Delta x) \quad x = 0, L
\]

(27)
\[ y(x+2\Delta x) - 2y(x\Delta x) \cdot 2y(x-\Delta x) + y(x-2\Delta x) \quad x = 0, L \quad (28) \]

These equations enable the response of the pipe to regular and random waves to be simulated once the hydrodynamic parameters \( C_H, C_S \) etc have been evaluated in tank tests.

**Comparisons with Experiment**

The wave power group at Edinburgh University (6) have built a long flexible spine which is designed to be the back-bone of an array of wave energy devices known as ducks. This model is not continuous as has been discussed but consists of jointed segments. The stiffness of the joints can be varied electronically so its response can be examined over a wide range of conditions.

The link between the flexural rigidity (EI) of a continuous beam and the stiffness of a joint (5) where \( L \) is the distance between joints is given by the universal beam equation.

\[ \frac{M}{I} = \frac{E}{R} \]

i.e. if \( M \) = bending moment and \( R \) = radius of curvature

now an angle of \( \theta \) will give a moment of \( M = \theta S \)

hence \( \frac{S\theta}{I} = \frac{E}{R} \) and \( \theta = \frac{L}{R} \)

so \( \frac{S\theta}{I} = \frac{E\theta}{L} \), so \( EI = S\theta \)

The experiments were performed in the Edinburgh University wide wave tank and the following diagrams show a comparison of the experimental results with the output of the computer simulations and of the analytic linear model.
Results

Figs (1) - (3) show total RMS bending moments as calculated using the analytic linear model, on a semi-infinite single ended pipe in a 1 sec Pierson-Moshowitz sea with Mitsuyasu Spread. The stiffness of the pipe were:-

- Figs. 1, 4 & 5 \( EI = 800 \text{Nm}^2 \)
- Figs. 2, 6 & 7 \( EI = 2800 \text{Nm}^2 \)
- Figs. 3, 8 & 9 \( EI = 4000 \text{Nm}^2 \)

These all predict peaks in the distributions near the end of the pipe.
Figs. (4) - (9) show, separated into surge and heave, comparisons between the numerical model predictions and the results of experiments done on a 16m long spine (121cm diameter) in the Edinburgh Tank, using the same sea state as previously. These comparisons show that, although exact agreements are not found, overall trends are being predicted successfully.

Conclusions

Several important spine properties have been predicted using the numerical model and subsequently verified experimentally.

These include the prediction of critical lengths of spines of certain stiffness in particular sea states e.g. (1). In a 1 sec P.M.
sea with Mitsuyasu spread (10 sec at full scale for the duck spine), if the spine has scale concrete stiffness then very large moments are observed for spines of 4 metre length (400m full scale). (2) In general the bending moment distribution does not change greatly over much of the central region of a spine but there are peaks in the distributions at a distance, dependent on the pipe stiffness and the crest length of the sea, from the ends.

These and other predictions indicate the usefulness of simulation work concerning long structures and it is considered that there is still very much to be discovered in this field both experimentally and theoretically.

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