"A THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE STATIC AND DYNAMIC LATERAL RESISTANCE OF BRICKWORK PANELS WITH REFERENCE TO DAMAGE BY GAS EXPLOSIONS"

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by

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April 1972.
TO THE MEMORY OF MY FATHER.
ABSTRACT

The objective of this thesis was to investigate the strength of brickwork panels when subjected to lateral pressure, within the more general framework of accidental loading.

The project has been divided into three parts:

(i) brickwork panels with precompression subjected to static lateral pressure.

(ii) fully restrained brickwork panels under dynamic lateral pressure.

(iii) correlation between the strength of brickwork panels with precompression subjected to dynamic and static lateral pressure.

After an historical review of the events which caused the awakening of interest in lateral pressures these three topics are dealt with, in order, in the three main chapters. Chapter 4 attempts to discuss the problem of gas explosions with reference to the more general category of accidental damage and thus to place the main chapters of the thesis into true perspective.

Chapter 1 therefore examines a theoretical explanation of the strength of laterally loaded brickwork panels supported on two edges with precompression. Although basically concerned with solid walls it is extended to cover cavity walls when the inner leaf only is loaded. The lateral strength of three and four sided panels is empirically related to the strength of panels supported on only two sides.
The strength of fully restrained brickwork panels when subjected to dynamic lateral pressure is investigated in Chapter two. Although basically concerned with panels supported on two edges, it is extended to panels restrained on all four sides and the implications of the investigation are discussed.

An experimental investigation to establish the effect of rate of loading on the lateral strength of brickwork panels supported along two edges with precompression is described in Chapter 3.

Finally the problem of designing structures to withstand accidental damage is discussed with particular reference to damage caused by gas explosions, and the implications of the previous work is discussed in perspective and within the framework of present legislation.
ACKNOWLEDGEMENTS

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A major part of this project involved experimental work and my thanks go to all the Technical Staff who were involved.

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General Introduction.
INTRODUCTION

The main advantage of loadbearing brickwork over other forms of construction lies in the fact that the structural element can also fulfil other important roles; the good thermal and acoustic properties of brickwork are utilised when the loadbearing wall is used either externally, as cladding to the structure, or internally, as spatial dividers. Because of this, in suitable structures, the use of loadbearing brickwork has achieved structural cost reduction of up to 30% of its nearest rival. The suitability referred to is concerned with architectural plan form; loadbearing brickwork having the advantage when spans are relatively small and where the plan form is identical in all or most of the storeys. Occasionally use is made of a concrete podium for the first one or two storeys where a large unobstructed floor area is required for aesthetic or functional reasons.

The main concern of designers and research workers in the field of brickwork has, until recently, been the overall structural behaviour of loadbearing brickwork structures and the behaviour of wall elements within these structures when loaded vertically or horizontally in the plane of the wall: the behaviour, in fact, of the structure under the action of normal dead, superimposed and wind load. This was the situation until May 16th 1968 when a high-rise, system-built, concrete-panel block of flats at Ronan Point was severely damaged (Plate 1/
PLATE 1

Ronan Point following Gas Explosion
(Plate 1) following, what was later established to be a gas explosion in one of the flats. Four people died and seventeen others were injured as a result of the collapse, which demolished a corner of the block throughout its height of 22 storeys.

This collapse not only highlighted a certain inadequacy in the particular form of system-building, but also reinforced the need for designers to widen their horizons and to consider at least two further conditions of which account was not necessarily taken in the structural design. The first condition concerned the behaviour of the structure and its overall stability should a local load bearing member fail following accidental damage whilst the second concerned the ultimate strength of such load bearing members when subjected to accidental loading.

In October 1968 the Tribunal which headed the inquiry into the collapse of the flats at Ronan Point submitted its report to the Minister of Housing and Local Government. The possibility of a more general collapse ensuing in multi-storey structures, as a result of accidental local damage, led the Tribunal to recommend in its Report that the Building Regulations should be amended to include provisions to deal with progressive collapse. This Report was subsequently published in November and that month, acting on the Tribunal's recommendation, the Ministry of Housing and Local Government issued a circular concerning the design of new blocks and the strengthening of existing blocks of flats constructed with precast concrete panels. This circular was essentially of a stop/
stop gap nature and was followed in December 1968 and subsequently by three publications (3, 4, 5) from the Institution of Structural Engineers. The first (3) was basically extended guidance notes on the Ministry's circular, whilst the second (4) contained general recommendations to assist in the structural design of concrete panel structures although, in principle, the recommendations were applicable to loadbearing brickwork structures. The third publication (5), however, in May 1969 dealt with "Guidance on the design of Domestic Accommodation in Loadbearing Brickwork and Blockwork to avoid collapse following an Internal Explosion".

The final result of the Tribunal's recommendation was "The Building (Fifth Amendment) Regulations 1970", (6) which was approved by Parliament and became law on 1st April 1970. In essence it specifies certain requirements deemed necessary to minimise and restrict the total damage resulting from local accidental damage and extends the scope of the Ministry of Housing and Local Government's original circular to all types of construction over 4 storeys high. Two methods, or a combination of the two, are available to the designer. He must either design his structure in such a way that it will remain stable following the loss of any load bearing member, or he must design the local loadbearing member for a static uniformly distributed load of 5 lbf/sq in. Reduction in superimposed and wind loads are permitted and the resultant total load must not exceed 95% of the collapse load. A "deemed to satisfy" provision specifies/
specifies the maximum permissible floor area per storey to be affected by the local structural failure.

The designer of loadbearing brickwork, as of other forms of construction, had these two 'new' criteria to consider:—
(a) the strength of local structural members
(b) the overall stability of the structure after accidental damage to a local member.

The questions therefore arose, "How strong are loadbearing brickwork panels under transverse load?" "What type of brickwork wall, if any, will be able to resist 5 lbf/sq in?" The problem can be generally divided into two parts:—

the strength of walls
(a) under normal precompression
(b) with rigid restraint

The first classification refers to walls commonly seen in loadbearing brickwork structures; walls which are normally under compressive stress. These can be seen in houses, low-rise residential buildings and high-rise blocks of flats. The second is concerned with non loadbearing brickwork; with walls placed within a concrete, steel or similar rigid frame. Such walls can be seen as partition walls and barriers against the natural environment in buildings designed using a structural frame. The basic difference between the two classifications is one of end conditions.

Under sufficient lateral pressure both types of wall will fail, but whilst failing, the former will usually develop sufficient thrust to raise the floor slab supported on it, whilst the latter is unable/
unable to 'stretch' the frame in which it stands.

When considering the wall system with the simplest end conditions, namely where the wall panel is not restrained along both its vertical edges, both type of panels fail by "arching action". A hinge forms at the top, bottom and midheight mortar joints, and thrusts develop at these hinges. It is because of the similarity between this type of failure and the action of a three pinned arched, that the failure mechanism has become known as arching action. When wall panels are also restrained along one or both of their vertical edges the form of failure differs slightly from the simple case: arching is still present in the vertical direction but a horizontal arching component also develops. It has been shown from experimental work that such walls exhibit a failure pattern similar to that exhibited by concrete slabs under similar support conditions.

With this understanding of the salient points which subdivide the problem into more easily manageable units and with a general understanding of the form of failure exhibited only one more question remains to be asked. What effect, if any, results from changing the rate of loading? Are the effects of applying a load, similar in character to a pressure pulse generated from a gas explosion (a high rate of loading) greater, less, or the same as would result from applying a basically static load (very slow rate of loading)?

Answers/
Answers to, and comments on, these questions form the major part of the work here presented; the latter part is concerned with the considerations which should be given to structural solutions, having particular reference to gas explosions, within the present framework of legislation.

Plates

1. Block of Flats at Ronan Point following structural collapse.

By courtesy of London Express News & Features Services
A Theoretical Investigation of the Lateral Strength of Brick Walls with Precompression
ABSTRACT

Many tests have now been carried out on brick walls under precompression to establish the maximum lateral pressure at which such walls fail.

This chapter introduces a theoretical method which is capable of satisfactorily explaining these results. Based on a work method a relationship is established between slenderness ratio, the material properties, and maximum lateral pressure for solid walls under precompression but without returns. This is extended to cover cavity walls where the inner leaf only is loaded.

An empirical relationship between the lateral strength of three and four sided panels and panels supported on only two sides is shown to exist and an empirical design curve is suggested; a comparison of experimental and theoretical results demonstrates the accuracy which this empirical method affords.

Finally, some of the implications resulting from experimental evidence are discussed.
INTRODUCTION

The Building (Fifth Amendment) Regulations (6), which was conceived as a consequence of the partial collapse at Ronan Point, concerns the behaviour of a structure when accidentally loaded. The philosophy contained in this piece of legislation can be thought of as an added constraint on the freedom of the designer. He must now design his structures in such a way that local accidental damage will not result in a more serious collapse. Within the framework of the Fifth Amendment, two possibilities exist.

either (a) an 'alternative path of support' approach
or (b) a strength criterion for individual loadbearing members.

A combination of these alternatives is possible within a structure and consequently a satisfactory design may well make use of both approaches. The 'loadbearing member' referred to above may be taken as

1. a beam between supports
2. a column between supports
3. a wall between supports or between an extremity and a support
4. a floor between supports or between an extremity and a support

where all the above constitute part of the structural machine. The strength criterion has been laid down by legislation and individual loadbearing members must be deemed to be structurally absent if they/
they cannot withstand a normal pressure of 5 lbf/sq in, with a reduced load factor. Further definitions and details need not concern us here and are dealt with later and well documented elsewhere (7, 8, 9, 10, 11).

It has been demonstrated (12) that it is possible for a loadbearing brickwork structure to sustain local structural damage without a more general collapse resulting (plates 3, 4, Chapter 4). Nevertheless, within the scope of present legislation it could be considered foolish not to make use of the second approach, namely the strength of individual structural members. Designers, however, lacked knowledge regarding the strength of brickwork walls when subjected to a static uniform pressure. Whilst some work has been done on this topic (13, 14, 15, 16, 17, 18, 19, 20, 21, 22), it was deemed necessary to test brickwork walls under a combination of vertical precompression and lateral static pressure to ascertain the maximum lateral pressure which these walls can withstand and to compare the strength of such walls to the new requirements in the Building Regulations. To this end experimental work was undertaken by the British Ceramic Research Association and also by the Structural Ceramics Research Unit at Edinburgh University.

In the tests conducted by British Ceramic Research Association, walls were built using various types of brick and the common mortar mixes. These walls were all 100 inches high, but the wall thickness varied depending on the brick used and whether the wall was of single or double brick thickness. These walls were placed in a
900 ton testing frame where the precompressive stress was applied from an hydraulic ram. Lateral pressure was then brought to act on the wall by slowly increasing the air pressure within an air bag which was placed between one side of the wall and a solid abutment. At a certain level of lateral pressure the wall cracked along the mid-height mortar bed and along the mortar bed at both the top and bottom edge of the panel. The two half-walls so formed were then slowly rotated by the lateral pressure until a position was reached which resulted in the failure of the wall. While the two half-walls rotated to failure, the precompressive force applied to the wall from the ram of the testing frame was kept constant by bleeding the hydraulic system as and when this was required.

The tests conducted at Edinburgh University took place on the ground level walls of a five storey test structure built, previous to the development of active interest in lateral pressures, in order to study the effects of wind loading on the whole structure. These tests could therefore be said to be more representative of the behaviour of laterally loaded walls in practice. In the five storey structure the wall was, of course, already under a compressive stress; the lateral pressure, however, was still applied to the wall using an air bag sandwiched, in this case, between the wall under test and the quarry face next to which the test structure had been built. In some of the earlier tests a system of jacks was used, instead of the airbag, the load from each jack being spread onto the wall by plates in an attempt to more closely simulate uniform loading.

Much/
Much work has been done by both research teams and many results have now been obtained (12, 23). This chapter is an attempt to explain theoretically the strength of walls with precompression when laterally loaded and consequently produce design criteria for loadbearing brickwork walls with precompression.

Mode of Failure.

The mode of failure seen in experimental work was remarkably consistent. When the wall under test was restrained only along the top and bottom edges, the two vertical side edges being free a horizontal tension crack developed along the mortar joint at the top and bottom edges and at the midheight when a certain level of lateral pressure was acting, this value of pressure being dependent on the vertical load being carried per unit length of wall. (Fig. 1) With further increase in lateral pressure, the tension crack opened throughout the full wall thickness and the crack continued to open wider as the rotation of the two half-walls progressed. During this rotation the load causing precompression, either the floor slab or the ram of the test frame, was lifted slightly by the wall as it finally failed.
Fig. 1. WALL and ROTATION MODELS.
Theoretical Approach.

The theoretical approach was based on a work method equating work done by the lateral pressure on the wall to the work done by the wall in lifting the precompressive load. A mathematical model of the wall system and the test rig is shown in Fig 1b. A basic theory was first developed. This theory assumed a mode of failure similar to that already described but neglected the elastic deformations of the brickwork as the wall arched to failure. The mode of failure associated with the basic theory is shown in Fig 1c. Two halfwalls of material of infinite compressive strength rotate to failure and whilst rotating the precompressive load is lifted. In an attempt to establish more compatibility between the failure associated with lateral loading and other failure criteria, the basic theory was later modified. The modifications took into account the elastic properties of the material and its self weight. The type of failure associated with the modified theory in which account is taken of elastic deformations is shown in Fig 1d, and it is seen that the precompressive load is not lifted as far in this case as it was in the basic theory.

Basic Theory

The assumptions were first made.

(1) the material is infinitely strong in compression and under the most severe precompression will not deform as the wall fails.

(2) no tensile bond exists between the two half-walls.

(3) the/
the self weight of the wall is negligible in comparison to the precompressive forces.

(4) the top of the wall is supported against lateral movements such that vertical movement is not impeded.

(5) the precompression applied to the wall remains constant throughout uplift.

With these assumptions and considering a section of wall of unit length, height $H$ and thickness $t$, under a precompressive stress $\sigma$, the total work done in lifting the precompressive force, $\sigma t$, can be written

\[
\text{Work} = \sigma t \times 2\alpha
\]

where $2\alpha = \text{maximum lift of precompressive force during rotation.}$ (Fig 2)

From considerations of the geometry of the problem $\alpha$ can be written

\[
\alpha = \sqrt{(\frac{H}{2})^2 + t^2} - \frac{H}{2}
\]

and the work done can then be expressed as

\[
\text{Work} = 2\sigma t \left[ \sqrt{(\frac{H}{2})^2 + t^2} - \frac{H}{2} \right]
\]

\[\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{
Fig. 2. **HALF BLOCK ROTATION—BASIC THEORY.**

Fig. 3. **STRESS STRAIN CURVE.**
the air bag (or the oil reservoir in the hydraulic system - in the case where jacks were used) increases and the load on the wall will consequently begin to fall. Taken to its conclusion, when the wall fails the pressure applied to it by the air bag or jacking system is zero. The assumption is made that this decrease in pressure is linear. Consequently while the wall will support a maximum lateral pressure \( p_c \), this diminishes to zero at the instant of failure. The average pressure acting on the wall over the period during which the wall rotates to failure is therefore \( p_c/2 \) and the lateral load moves through a distance approximately equal to half the wall thickness. The work done by the lateral pressure on the wall as it is loaded to failure can then be expressed as

\[
\text{Work} = \frac{p_c H t}{4} \quad \text{.....(2)}
\]

Equating equations 1 and 2

\[
p_c = \frac{8 \sigma}{H} \left[ \sqrt{\left(\frac{H}{2}\right)^2 + \frac{t^2}{2}} - \frac{H}{2} \right] \quad \text{.....(3)}
\]

and on adopting a slenderness ratio notation, \( S = \frac{H}{t} \), this equation can be rewritten

\[
p_c = \frac{4 \sigma}{S} \left[ \sqrt{1 + \frac{t^2}{S^2}} - 1 \right] \quad \text{.....(4)}
\]

Equation 4 is represented in Graph 1 which is a plot of maximum lateral pressure versus slenderness ratio. This illustrates both the increase in strength with increase in precompression for a fixed value of slenderness ratio and the decrease in strength for an/
an increase in slenderness ratio with a fixed value of precompression. Another way of presenting Equation 14 is shown in Graph 2 where linear curves are produced for different values of slenderness ratio when maximum lateral pressure is plotted against precompression. The formula is relatively simple and it is probably more convenient to use than the graphs when obtaining a maximum lateral pressure for a particular wall system. The basic analysis as represented by equation 14 is capable of giving fair correlation with experimental results at low values of precompression. This can be seen in the theoretical and experimental comparison which are made in Graphs 3, 4 and 5. At higher values of precompression, however, the strength of walls is considerably overestimated since the equation is limitless. This overestimation can be partly explained by the fact that at higher precompressions the corners of the two rotating blocks will either deform elastically or crack and chip. The precompression will then not be lifted as far as the basic theory predicts. Although accurate at low values of precompression the basic theory predicts zero strength when no precompression acts on the wall. As can be seen from practical results, this is not correct; walls of various slenderness ratios do exhibit some degree of resistance to lateral pressure under zero precompression.

The discrepancy between the basic theory and the experimental evidence can be explained partly by the assumption that the material is infinitely strong in compression, partly by the assumption that the self weight is negligible and partly by the assumption that no/
Basic Theory

\[ p_e = 4\sigma'[\sqrt{1 + \frac{4}{S^2}} - 1] \]
Graph 2.
Graph 4.
Graph 5.

MORTAR TEST RESULTS

- 1:1/4:3 MORTAR
- 1:1:6 MORTAR
- FONDU

KEY

B.C.R.A. TEST RESULTS
8 5/8 IN. WALL

E = 1.25 x 10^6 psi
E = 1.00 x 10^6 psi

Maximum Lateral Pressure (psi)

Precompression (psi)
no tensile bond exists within the wall where the three hinges form. The work done in lifting the precompression should include a term to account for both the elastic deformation of the brickwork and the slight lifting of the wall's self weight; the solution should cater for the tensile bond in the brickwork. A more accurate theoretical explanation of the strength of brickwork walls with precompression might therefore be obtained by taking into account these three factors.

**Modified Theory**

The half-wall rotation is assumed to be similar to that in the basic theory but the deformation of the rotating wall blocks is taken into account assuming the material exhibits a classical elasto-plastic property as shown in the stress-strain curve in Fig 3. The salient feature of this assumed relationship is that once the stress has reached its ultimate value, it remains constant at that value with increasing strain. As in the basic theory a work method is used and the work done by the lateral pressure is equated to the work done in lifting the precompression and the self weight of the wall. Whilst the work done by the lateral pressure is still as stated in Equation 2, the work done by the wall in failing is now given by

\[
\text{Work} = 2 \delta \sigma t + \frac{W \delta^2}{2} + \frac{W \delta^3}{2} \quad \ldots \ldots (5)
\]

assuming the weight of each half-wall acts at its centre of gravity where/
where \( W \) = weight/unit length of wall

\[ 2 \delta = \text{the maximum distance through which the precompression is lifted as the wall fails.} \]

Equating equation 5 and 2

\[
P_C = \frac{H}{ht} \left[ \sigma t + \frac{W}{2} \right] \times 2 \delta \]

\[ .... (6) \]

and this equation will give the general solution if \( 2\delta \) is expressed in terms of the known parameters. The distance through which the precompression is lifted can be expressed as the geometric lift of the precompression (already derived in the basic theory as \( 2\alpha \)) minus some shortening contribution due to the elastic deformation of the corners of the half-walls. To evaluate the shortening due to elastic/plastic deformation, expressions for the stress patterns developed at the ends of the half-walls are derived as functions of rotation. This necessitates expressing the angle of eventual rotation, associated with the point of instability, as a function of the known parameters \( \sigma, \sigma_c \) and \( S \). To find this relationship some basic rotation equations must first be established.

With reference to Fig 14

\[ B = \frac{H}{2} \sin \Theta + \frac{t}{2} (1 - \cos \Theta) \]

where \( \Theta \) = angle of rotation

\( B \) = horizontal displacement of the wall centre line at midheight.

Using/
$$B = \frac{t}{2}(1 - \cos \Theta) + \frac{H}{2}\sin \Theta$$

Fig. 4. **ROTATION GEOMETRY.**

Fig. 5. **INITIAL STRESS CHANGE.**
Using dimensionless parameters this can be expressed as

\[ u = \frac{S}{2} \sin \theta + \frac{1}{2} (1 - \cos \theta) \]  

\[ u = \frac{B}{t} \]  

where \( u \) = \( \frac{B}{t} \)

There is as yet little experimental information regarding the extent of wall deflections at the point of instability. A reasoned argument was therefore applied to give an assumed expression for the value of \( u \). Two conditions are at present known: when the precompression is equal to \( \sigma_c \) the wall is already unstable and \( u = 0 \). When there is no precompression instability does not occur until \( B \approx t \) (\( u = 1 \)). Lacking evidence of the form of curve joining these limits a straight line relationship is postulated of the form

\[ u = 1 - \frac{\sigma}{\sigma_c} \]  

Equating equations 7 and 8 and rearranging to solve for \( \theta \), a quadratic in \( \cos \theta \) develops

\[ (1 + S^2) \cos^2 \theta + 2(1 - \frac{2\sigma}{\sigma_c}) \cos \theta + [(1 - \frac{2\sigma}{\sigma_c})^2 - S^2] = 0 \]  

Which on solving for various values of \( S \) and \( \frac{2\sigma}{\sigma_c} \) provides two roots; the positive root giving the value of \( \cos \theta \) which defines the position of instability - it effectively establishes the position at which the returning moment is zero.
The stress patterns are assumed to develop as shown in Fig. 5, the slope of the stress patterns being equal to the block rotation. The conditions which differentiate between one stress pattern and another were found by equating the forces developed at the ends of the wall to the precompressive force acting.

Considering a small rotation (Fig 5)

\[
\begin{bmatrix}
\sigma_y = 0 + \sigma y = t \\
2
\end{bmatrix} \Rightarrow t = \sigma t
\]

\[
\sigma_y = t = \sigma y = 0 + \frac{t \tan \theta}{2}
\]

giving \( \sigma_y = 0 = \sigma - \frac{t \tan \theta}{2} \)

\[
\sigma y = t = \sigma + \frac{t \tan \theta}{2}
\]

But for this stress pattern

\[
\sigma_y = t \leq \sigma_c \Rightarrow \tan \theta \leq \frac{2}{t} (\sigma_c - \sigma)
\]

\[
\sigma_y = 0 \geq 0 \Rightarrow \tan \theta \leq \frac{2}{t} \sigma
\]

Similar rotation constraints were found for the other possible stress patterns and these are shown in Fig 6. The derivation of the stress pattern constraints is contained in Appendix 1. Once the stress pattern is identified in terms of the angle of rotation and other relevant parameters, the shortening of the material can be found. Hence the lift \( 2 \delta \), can be calculated

\[ \text{Lift/} \]
Lift = 2\delta = 2\left[\sqrt{\left(\frac{H}{t}\right)^2 + t^2} - \frac{H}{2}\right] - \Delta

where \Delta = \text{elastic shortening of the wall.}

In case 2 and 3 Fig 6 the average strain can be written

$$\varepsilon_{av} = \frac{\sigma}{E}; \quad E = \text{Young's Modulus}$$

$$= \frac{\sigma}{c}\frac{c}{c}$$

giving \Delta = \varepsilon_{av} H = \frac{\sigma}{E} H

For case 1 the assumption is made that the stress distribution along the outer fibre of the wall varies linearly to zero at the mid height section for this end stress condition; the average stress is therefore half of the stress at one end. Based on this assumption the shortening of the wall for case 1 stress pattern (derived in Appendix 2) is expressed as

$$\Delta = \frac{H}{2E} \sqrt{\frac{2}{\sigma_t^2} - 2t \sigma \tan \Theta}$$

With the corner deformations now found the work done in lifting the precompression during rotation of the wall is now defined in terms of the known parameters and a solution of a particular wall system is now possible. A more rigorous analytical approach, however, is somewhat complex and the analysis is more readily amenable to a numerical solution.

Theoretical Results.
Fig. 6. **ROTATION CONSTRAINTS FOR STRESS PATTERNS.**
Theoretical Results

A computer program was used to obtain the results from this theory. In these solutions the ultimate strain was assumed to be 0.001, the density of brickwork 110 lbf/ cubic ft, and the ultimate tensile stress 50 lbf/ sq in, although the latter value was found to be of small significance. Since elastic deformations are to be accounted for during rotation, the value for wall height was altered within the program for original shortening due to the precompression. Results of the analysis are shown in Graphs 6, 7 and 8. In these graphs the maximum lateral pressure is drawn against slenderness ratio which produces a family of curves for the various precompressive loads - there being one such family of curves for each E value. These graphs are for E values of 1, 1.5 and 2 (x 10^6 lbf/sq in) respectively. Extrapolation can then be used for differing E values, precompressions and slenderness ratios.

The general trend of an increase in wall strength with a corresponding decrease in slenderness ratio can be seen, the rate of strength development increasing with smaller values of slenderness ratio. Similarly the maximum pressure which a wall system can sustain increases with precompression. The solutions can be presented for a particular wall configuration as a plot of maximum lateral pressure versus precompression; a family of curves result, one for each unique value of Young's Modulus. Each separate wall system will have such a family of curves, two such families being presented in Graphs 9 and 10. Both are for storey height walls, the former of single/
Graph 8.

Maximum Lateral Pressure (psi)

Slenderness Ratio (H/t)

$E = 2 \times 10^6$ psi
single brick thickness whilst the latter is of double brick thickness. These curves demonstrate the ability of brickwork to withstand high values of lateral pressure at high precompressions, which are so high as to be outside the practical limits. The strength of these walls can be seen to increase with precompression to a maximum at about one half of the ultimate stress and then decrease to zero at a precompression approximately equal to the ultimate stress. All the graphs are based on a wall height of 100 inches, the slenderness ratio being the height divided by the actual brick thickness. A slight discrepancy appears between the maximum lateral pressure for a 100" high wall with a specific slenderness ratio and the maximum lateral pressure of a wall of similar slenderness ratio but different height. With a 30" high wall this discrepancy is of the order of 10%.

The results from this modified theory are compared with practical results and the solution of the basic theory in Graphs 3, 4 and 5. The modified theory differs from the basic theory by "tailing off" at higher values of precompression and thus gives better correlation with experimental results. Similarly, it better agrees with the experimental work by producing a positive value of maximum lateral pressure at zero precompression. This effect is due more to the influence of the wall's self weight than to the tensile properties of the brickwork. The effect of accounting for the self weight of the wall improves the strength predicted from the basic theory within the scale of precompression where the self weight could be considered/
4\,^{1/8}\text{IN. STOREY HEIGHT WALL.}

![Graph 9](image-url)

- Maximum Lateral Pressure (psi)
- Precompression (psi)
- E VALUE (MO psi): 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50
$8^{5/8}$ IN. STOREY HEIGHT WALL.

Graph 10.

Maximum Lateral Pressure (psi)

Precompression (psi)

$E$ VALUE ($\times 10^6$ psi) = 0.75

0 1000 2000

1.00 1.25 1.50 1.75 2.00 2.25 2.50

0 20 40 60
considered important - namely at small values of precompression. At higher values of compression the deformation of the elastic/plastic brickwork predominates and the modified theory produces theoretical predictions of strength which are lower than those produced from the basic theory. The way in which the solutions from the two theories intersect can therefore be explained. In Table 1 test nos 1 - 4 compare theoretical predictions with experimental results obtained by Hendry, Sinha, and Maurenbrecher (12) from walls tested in the full scale five storey structure at Edinburgh University.

Cavity Walls

Both the theories presented in this Chapter deal with the rotation of solid walls. The use of cavity wall construction is, however, normal practice for the outer walls of buildings. Cavity walls would appear not to be amenable to the form of analysis presented here. Where perforated bricks are used a modified value of the weight per unit volume may be used at low values of precompression but this approach cannot be applied to cavity walls where the ties joining the two leaves cannot transmit vertical shear. Where only the inner leaf of a cavity wall is used to transmit the structural load, however, it would appear that the cavity wall may be thought of as a wall of only one leaf thickness. Test nos 5 and 6 (Table 1) compare experimental results from tests on 11 inch cavity walls, where only the inner leaf is loaded, with theoretical predictions of maximum lateral pressure obtained using the modified theory but neglecting altogether the presence of the outer/
## TABLE 1

### Test Results

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<th>Max. Lateral Pressure (psi)</th>
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<td>10.5</td>
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<td>0</td>
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<td>200+</td>
<td>12.2</td>
<td>12.8</td>
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<td>1.1</td>
<td>1.0</td>
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<tr>
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<td>0.97</td>
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<td>2.0</td>
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<tr>
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<td>3.7</td>
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</tr>
</tbody>
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---

- Theoretical Results Based on Equivalent Panel Basis.

* Precomp. Not accurately known

- Structural Ceramics Research Unit, Edinburgh University

* British Ceramic Research Association

X Cavity wall-inner leaf only loaded.
outer unloaded leaf and basing the value of slenderness ratio on the actual brick thickness of the inner leaf only. There is a strong argument for supposing that when both leaves are loaded the strength of such walls will be equal to twice the strength of each leaf. This is based on the fact that the cavity wall fails in much the same way for both leaves and it would not seem unreasonable to suggest a factor of two on the strength of one leaf. To date little experimental work has been done on this type of wall system and the comments made here regarding cavity walls loaded in each leaf should be regarded as tentative only.

Panels supported on more than Two Edges

So far, only panels with two unrestrained vertical edges have been considered. In practice it is often necessary to incorporate panels, restrained along three or four edges within a loadbearing brickwork structure; the restraints referred to are usually provided by returned walls. From experimental work (12, 23) it has been found that in general when a panel is restrained along one or both of its vertical edges, the failure pattern closely follows the yield line patterns associated with concrete slabs under similar boundary conditions (Fig 7) although the actual pattern developed depends on the type of edge restraint provided. In these tests a return wall was used to provide the edge restraint and the wall was loaded from the same side as the return in an attempt to more closely simulate what could occur in practice. Although Fig 7a is typical of the failure associated with a three sided panel where the/
Fig. 7. **GENERAL FAILURE PATTERN**

(a) THREE SIDED

(b) FOUR SIDED
the length is greater than the height the mode of failure changed as the length decreased. When the length/height ratio was small a tension failure occurred between the wall and its supporting return wall; the wall then consequently failed as for a two sided panel with three horizontal cracks, one at the top, bottom and mid height mortar joints, resulting in a simple arching failure. Where the restraint afforded to the vertical edge/s was more complete, when for example, panels were loaded on the opposite side of the wall to the return/s, the yield line patterns of Fig 7 were seen to hold true for panels with smaller length/height ratios.

For the type of failure associated with three or four sided panels to be fully explained it would appear more than probable that some form of analysis based on yield line principles should be adopted. So far this has not been successfully completed and therefore an attempt has been made to correlate the performance of three and four sided panels to that of a simple one-way panel using an empirical approach combined with the preceding modified theory. This approach is basically of a stop gap nature until such times as a more exact theory is presented. The basic philosophy behind this approach is quite straightforward: the three/four sided panel can be thought of as an equivalent two sided panel of such dimensions as to represent the strength of the better restrained panel. The strength of a panel restrained along more than two edges will be dependent on its two-dimensional proportions which can be quantified as the aspect ratio of the wall (length/height) and also on the thickness of the panel. If an equivalent panel hypothesis were valid
valid, and the equivalent panel were of the same thickness there would surely be some relationship between the aspect ratio of the original panel and the aspect ratio of the equivalent panel.

**Equivalent Panels**

From the results of experimental work undertaken by the British Ceramic Research Association & the Structural Ceramic Research Unit at Edinburgh University (8, 9, Table 1) on panels supported on more than two sides a graph of the results was plotted with the ordinate being effective aspect ratio, the abscissae the original aspect ratio. All results used to plot the graph were based on maximum lateral pressure values obtained when the wall was loaded from the same side as the return/s. It has been shown (23) that when a panel is loaded on the opposite face from the returns it can sustain a higher pressure. The test results, therefore, constitute the weakest form of wall systems with three/four edges restrained and the empirical relationship would therefore refer to the worst condition. From the plot of results on Graph 11, it can be seen that an empirical relationship would appear to exist between the strength of two sided, three sided and four sided panels with precompression. This graph was formed on the basis that the maximum lateral pressure of the equivalent one way panel obtained from the modified theory was equal in magnitude to the value of the maximum pressure, recorded experimentally on tests with three or four sided panels. Each point on the Graph represents one test result and demonstrates that/
Graph II.

Effective Aspect Ratio $L_E/L_1$

- **Key**
  - △ Edinburgh S.C.R.U.
  - □ B.C.R.A.
  - ○ Three Sided
  - □ Four Sided

Both curves based on $E$ value = $1.25 \times 10^6$ (psi)
that a relationship exists between three/four sided panels and two sided panels when experimental results are married with the modified theory. A best fit curve through the relevant points is suggested as an empirical design aid and the degree of accuracy which this curve affords is demonstrated by a comparison in Table 1 of the experimental results (on which the curve is formed) and the theoretical results obtained using the best fit curve and the modified theory. A third curve is shown in Graph 11. This curve is the conversion curve for two sided panels; the datum curve, as it were, for the three/four sided panels. It is in fact, the locus of all points with effective aspect ratios equal to original aspect ratios.

It can be seen, then, that a fairly well defined relationship exists with little scatter from the best fit curves. The three curves exhibit the same general trend which is remarkably similar at values of original aspect ratio less than about 0.9. With values of original aspect ratio greater than 0.9 the two best fit curves tend to move toward the datum curve, the curve for three sided panels intersecting the datum curve at a value of original aspect ratio approximately equal to 1.6 whilst the curve for four sided panels would appear to intersect the datum curve when the original aspect ratio was of the order of 2.3 - 2.5. The crossing of the datum line is of significance; the three/four sided panel is then weaker than the same panel supported on two sides only. This apparent contradiction of the very basic concepts regarding the strength of structural membranes under various boundary conditions is/
is as yet only experimental fact and remains unanswered.

One possible argument which could explain this peculiar loss of strength is based on the concept of stability. If the behaviour of walls under lateral load is viewed as a stability problem there is, considering a two sided panel, an unique position during the rotational failure which could be described as critical. Such a position of unstable equilibrium can be thought of in terms of returning moments, the critical position being that position at which the returning moment equals zero. For two sided panels the form of failure is extremely well balanced and no part of the broken panel is under the influence of a resultant force within the plane of the wall. There is, in fact, no shear stress acting along the hinge. This is not so, however, for three sided panels. With reference to Fig 7a when the cracked wall begins to rotate, the movement of section A immediately induces shear at the hinges at the top boundary of section B and the bottom boundary of section C. This infers that sections B and C are both subjected to a force in such a direction as to suggest that both these sections might undergo translatory movement in the direction indicated by the arrows. On considering the equilibrium of the wall in a deflected position, it can be seen that two more forces exist as a result of the interaction of section A on sections B and C. The first of these is due to the fact that the resultant force at the yield line tends to act nearer the inner face - on which the lateral pressure acts - than the outer face. This would suggest a rotational movement of both sections B and C about the axis AA. When considering the/
the wall three dimensionally in its deflected position the second effect can be seen - namely a tendency for the interaction of section A with sections B and C to produce a resultant force the direction of which would imply rotational movement of sections B and C about the axis BB and CC respectively (Fig 7a). Furthermore, when discussing the movement of one section relative to another the problem can be seen to contain one final out of balance criterion. The fact that line BB is not totally restrained but can be lifted a small amount by the action of the wall failing, implies an added lack of symmetry for three and four sided panels. Section B can be lifted bodily by a small amount, whilst the very nature of the problem completely restrains section C from an equal and opposite downward movement. When this is considered in terms of a vertically stationary section A and in conjunction with the nature of the failure and of the material and the consequent zig-zag yield lines, it is extremely unlikely that the shear along BB is of the same magnitude as the shear along CC. It is extremely probable then that a resultant shear acts along the horizontal mid-height hinge.

When viewed from a stability point of view, if any of these resultant in plane forces were to effect any form of relative movement between the three sections, it is likely that the unique point of instability would be reached sooner than if relative movement were either inhibited or excluded. In reinforced concrete slabs such movement is excluded at both the supports and along the yield lines by the presence of tensile reinforcement; this effect is/
is not present in brickwork. In essence, then, this three dimensional geometric interplay of forces which is exhibited across and along the yield lines of a brickwork panel can be quite distinctly seen to differ from the conventional yield line theory when dealing with brickwork panels. The three sided panel has the added disadvantage of being unsymmetrical and therefore the behaviour of this type of panel exhibits the most difference between yield line theory pertaining to brickwork and the 'conventional' theory. Furthermore, when viewed as a stability problem it is more likely that the effect of the interplay of forces along the yield lines will increase with an increase in the original aspect ratio, the effect being more pronounced for the unsymmetrical three sided panel than the symmetrical four sided case. This argued hypothesis could well explain the apparent weakness of the more restrained panels, which is seen from Graph 11, by differentiating qualitatively between the basic nature of the failure mechanism of brick panels with precompression and the conventional yield theory developed for reinforced concrete.

Conclusions

A theory to determine the lateral resistance of brickwork panels supported on two sides with precompression has been presented; good correlation exists between theoretical predictions and the experimental evidence. A basic theory is first postulated which is capable of fair correlation with observed results at low precompression, whilst a modified theory developed later, affords a better fit with/
with both the experimental results and the expected general trend and is applicable within the range of precompression and slenderness ratio likely to occur in practice. Cavity walls, with only the inner leaf loaded appear from strength considerations to act as solid walls of only one leaf thickness. Experimental results are not however available for smaller values of slenderness ratio and this theory may not be applicable to walls with slenderness ratios of less than six. Except at high precompression the effect of Young's Modulus of the brickwork is seen to be of no practical significance to the lateral resistance of one way panels. An empirical relationship has been developed to facilitate the determination of maximum lateral pressure for wall panels restrained along one or both of their vertical edges. This empirical relationship is shown to exist for solid walls, and for walls of cavity construction where only the inner leaf, on which the pressure acts, is loaded with precompression. The nature of the failure of such three and four sided panels, and the implications, have been discussed.

Experience to date suggests that the form of brickwork failure described in the preceding pages is relatively consistent and not as susceptible to the degree of scatter usually associated with some other forms of brickwork failure.

The analysis and all the results apply only when the precompressive force is applied in such a way that it may be raised to allow the wall to fail. No account, however, has been taken of the increase in the precompressive force with lifting; such a phenomenon is inherent/
inherent in brickwork structures and is due to the general stiffness of the structure. In practice, therefore, the theory should prove somewhat conservative
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<td>Thickness of wall</td>
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<td>( S )</td>
<td>Slenderness Ratio ( \frac{H}{t} )</td>
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<td>( p )</td>
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<td>( p_c )</td>
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**IMPERIAL UNITS IN TERMS OF SI UNITS**

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LIFT OF FIGURES

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Graph 7  Slenderness Ratio v Maximum Lateral Pressure - Modified Theory \( E = 1.5 \times 10^6 \) psi
Graph 8  Slenderness Ratio v Maximum Lateral Pressure - Modified Theory \( E = 2.0 \times 10^6 \) psi
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APPENDICES

1. Derivation of rotational constraints for stress pattern development

2. Derivation of Shortening Effect based on Case 4 Stress Pattern
APPENDIX 1

Case 2  Derived previously

Case 3

\[ \sigma_y = t = (1 - a)t \tan \theta \]
\[ \sigma_y = 0 \quad \text{at} \tan \theta \]
\[ \frac{(1 - a)t}{2} \sigma_y = 0 - \frac{a t}{2} \sigma_y = 0 = \sigma t \]

Solving for \( a \) gives:
\[ a = \frac{1}{2} - \frac{\sigma}{t \tan \theta} \]

which on substituting into main equations:

\[ \sigma_y = t = \frac{t \tan \theta}{2} + \sigma \]
\[ \sigma_y = 0 = \frac{t \tan \theta}{2} - \sigma \]

Conditions:

\[ \sigma_y = t < \sigma_c \Rightarrow \tan \theta < \frac{2}{t}(\sigma_c - \sigma) \]
\[ \sigma_y = 0 < \sigma_t \Rightarrow \tan \theta < \frac{2}{t}(\sigma_t - \sigma) \]

Case 4

\[ \sigma_y = t = (b - a)t \tan \theta \]
\[ \sigma_y = 0 = 0 \]
\[ \frac{(b - a)t}{2} \sigma_y = t - \frac{a t}{2} \sigma_t = \sigma t \]
\[ \sigma_t = at \tan \theta; \quad a = \frac{\sigma_t}{t \tan \theta} \]
Eliminating \( \sigma_y = t \) leads to a quadratic equation:

\[
\begin{align*}
\left( b - 2ab + a^2 \right) + \left( \frac{a}{t \tan \theta} \right) - \left( \frac{2 \sigma}{t \tan \theta} \right) &= 0
\end{align*}
\]

which has roots

\[
b_{1,2} = a + \sqrt{\left( \frac{a}{t \tan \theta} \right) + \frac{2 \sigma}{t \tan \theta}}
\]

to ascertain which root is relevant the boundary condition of case 3 is used. When \( \tan \theta = \frac{2}{t} \left( \frac{\sigma}{t} + \sigma \right) \) \( b \) should be +1. On substituting this value of \( \tan \theta \) into the quadratic solution provides the two roots

\[
b_1 = 1 \quad \text{(with +ve sign)}
\]

\[
b_2 = -\frac{2 \sigma}{2(\sigma + \sigma)} \quad \text{(with -ve sign)}
\]

therefore

\[
b = a + \sqrt{\left( \frac{a}{t \tan \theta} \right) + \frac{2 \sigma}{t \tan \theta}}
\]

Condition 1. \( \sigma_y = t \leq \sigma_c \)

\[
(b - a) \tan \theta \leq \sigma_c
\]

\[
t \tan \theta \sqrt{\left( \frac{a}{t \tan \theta} \right) \sigma_t + \frac{2 \sigma}{t \tan \theta}} \leq \sigma_c
\]

On squaring and substituting for \( a \):-

\[
\frac{2}{\sigma_t}
\]
\[
\sigma_t^2 + 2\sigma_t t \tan \Theta \leq \frac{1}{2 \sigma_t} \left( \frac{2}{\sigma_c} - \frac{2}{\sigma_t} \right)
\]

which on simplifying gives

\[
\tan \Theta \leq \frac{1}{2 \sigma_t} \left( \frac{2}{\sigma_c} - \frac{2}{\sigma_t} \right)
\]

Condition 2. \( b < 1 \)

After substituting \( \frac{\sigma_t}{t \tan \Theta} \) for \( a \) in the equation for \( b \)

\[
\frac{\sigma_t}{t \tan \Theta} + \sqrt{\left( \frac{\sigma_t}{t \tan \Theta} \right)^2 + \frac{2 \sigma_t}{t \tan \Theta}} < 1
\]

On squaring and simplifying

\[
2\left( \sigma + \sigma_t \right) < t \tan \Theta
\]

\[
\tan \Theta \geq \frac{2}{t} \left( \sigma + \sigma_t \right)
\]
APPENDIX 2

Derivation of Shortening Effect based on Case \( \text{b} \) Stress Pattern

Assuming a linear stress distribution along the outer fibres of the half wall the average stress can be written as half the stress as derived in the Case \( \text{b} \) stress pattern (Appendix 1)

\[
\sigma_{\text{average}} = \frac{t \tan \Theta}{2} \sqrt{\frac{\sigma}{t \tan \Theta}} + \frac{2 \sigma}{t \tan \Theta}
\]

\[
\varepsilon_{\text{average}} = \frac{\sigma_{\text{average}}}{E} ; \quad \Delta = H \varepsilon_{\text{average}}
\]

and on substituting and simplifying:

\[
\Delta = \frac{H}{2E} \sqrt{\sigma^2 - 2t \sigma \tan \Theta}
\]
A Theoretical Investigation into the Strength of Fully Restrained Brickwork Panels when subjected to Lateral Pressure from a Gas Explosion.
ABSTRACT

In this chapter a theoretical method is described to ascertain the strength of brickwork panels rigidly restrained at the top and bottom edges and subjected to a dynamic lateral load such as may result from a gaseous explosion. A differential equation of motion is set up which describes the behaviour of the wall under this lateral impulse load. This equation requires the pressure pulse associated with gas explosions to be expressed as a function of time. Such an empirical expression is shown to exist which can describe the pressure at any point in time for any arbitrary gas explosion. The equation of motion is based on the well established mode of failure known as arching action, and the use of this in the equation is described. Solutions obtained from this analysis are shown for the various relevant parameters, and a method of equating the strength of panels rigidly restrained on all four edges to that of panels restrained along the top and bottom edges only is outlined. After a comparison of the theoretical results to the experimental evidence, implications resulting from this investigation are discussed. Specimen design calculations are appended as is a correlation of the theory presented with the experimental evidence available to date.
INTRODUCTION

In this Chapter the strength of infil fully restrained brickwork panels, under a dynamic lateral load, is examined, the dynamic load being similar in character to the load produced by a gas explosion. In practice such panels are often seen as partition walls within a structural frame, the frame acting as the structural element and resisting all the applied (dead + super+wind) forces whilst the brickwork panels are used either as spacial dividers or barriers against the natural environment. Whilst a study of this problem should not raise any serious doubts about progressive collapse, it represents not only a starting point for the dynamic response of panels with precompression but also provides reassurance, or perhaps raises doubts, regarding the amount of restraint available within a frame and whether the beams or columns or both may be damaged due to the failure of an infil panel. Little experimental work has been done on this type of panel when subjected to rapidly increasing load such as is produced during a gaseous conflagration although work has been done on similar panels which have been subjected to the blast loading from atomic devices and consequently this chapter which is mainly theoretical has few experimental results with which the theory can be correlated.

Basis of Investigation.

In order to analyse wall panels restrained along their top and bottom edges, and consequently to determine the maximum lateral pressure which such walls are able to resist, a theory based on arching/
arching action (24) is adopted. This theory assumes a mode of failure, substantiated by experimental work, similar to that shown in Fig 1b, where the rigid supports do not move. Movement of the supports produce a mathematical model (Fig 1a) dealt with in the previous chapter. With ever increasing lateral pressure the wall cracks and forms two half-walls which rotate slowly to failure. Whilst rotating, the walls deform elastically and/or plastically at the corners of the brickwork as shown. Consequently a force P develops at opposite corners of both half-walls, the force P being some function of rotation and of the other relevant parameters. Basic assumptions are made regarding the material properties of brickwork and from these assumptions and geometric considerations the magnitude of the force P can be evaluated. The use of arching action as a theoretical tool has been substantiated by experimental evidence (25, 26). In the majority of cases seen in practice, however, panels are restrained on all four edges or along the top and bottom edges and one vertical side edge. For such cases an empirical approach has been adopted from which the maximum lateral pressure for such panels can be found from its relationship to the maximum lateral pressure of a panel supported along only the top and bottom edges, but similar in all other respects. Because the loading on a wall produced by a gas explosion is of a pulse form (i.e. time dependent) the analysis was based on the dynamic response of the panels and a dynamic equation representing the motion of the wall was used. This equation required the pressure loading to be expressed as a function of time and a general empirical expression for the pressure produced from a gaseous/
gaseous conflagration of any magnitude was derived. Using this
general equation expressing the motion of the wall when transversely
loaded with a pressure pulse a solution was obtained based on a
pulse of any arbitrary severity which indicated whether the wall
was stable or not. The type of differential equation used and the
type of problem in general, proved particularly amenable to a form of
numerical solution and a computer was used to obtain the solution.
On obtaining this solution for an arbitrary loading, a new solution
was found for the same wall system but this solution used the
empirical expression for a gas pressure of greater or less severity
than in the first solution, depending on whether the wall proved to
be stable or was shown to fail. This approach of successive
approximations was continued to a point where the maximum lateral
pressure (to within a specified accuracy) just caused the wall to
fail. Such then was the general approach and with this in mind a
more thorough study of the main features can be made.

Arching Theory.

In 1956 McDowell, McKee and Sevin presented a theory of arching
action (24) in order to explain the apparent high strength of
masonry walls when rigidly restrained on all edges and loaded
transversely with a dynamic load. Their approach represented a
rather radical departure from the techniques usually assumed for this
type of problem. Their interest lay in the ability of certain
types of construction to withstand a blast from an atomic device
within certain limits of range from, and size of, the atomic device.
The/
The rigid restraints in their investigation were provided by placing the masonry panel within a steel framework. Whilst fuller details of the theory are to be found in the reference it is advantageous to discuss the concept briefly here.

The wall of height $H$ and thickness $t'$ restrained along only the top and bottom edges, is assumed to act as two identical half-wall blocks with the bond broken at the top, bottom and mid height. When lateral pressure is applied the wall is assumed to deflect into a position shown in Fig 2, each half wall rotating about the first point in contact with the support. From the geometry of Fig 2:

$$a = \frac{H}{l} \frac{1 - \cos \theta}{\sin \theta}$$

where $a$ is as shown in Fig 2 and the centre deflection $B$, can be expressed as

$$B = \frac{H}{l} \frac{1 - \cos \theta}{\sin \theta}$$

giving the decrease in contact length as

$$a = \frac{B}{l}$$

Introducing non dimensional parameters $u$ and $S$ for centre deflection and slenderness ratio (height/thickness) respectively:

$$u = \frac{B}{t'}; \quad S = \frac{H}{t'}$$

the trigonometrical functions can be obtained:

$$\sin \theta$$
Fig. 2
\[
\sin \Theta = \frac{2u}{S(1 + (\frac{u}{S})^2)} \quad ; \quad \cos \Theta = \frac{1 - (\frac{u}{S})^2}{1 + (\frac{u}{S})^2}
\]

and the fraction of the half depth in contact with the support can be given by

\[
\alpha = \frac{1 + (\frac{u}{S})^2}{1 - (\frac{u}{S})^2} \left( 1 - \frac{u}{2} \right)
\]

The shortening of the material at any position \( y \) can then be expressed,

\[
\delta_y = \frac{u t' (1 - \frac{2y}{t'} - \frac{u}{2})}{S \left( 1 - (\frac{u}{S})^2 \right)}
\]

The average strain along a fibre of the beam at a distance \( y \) from the bottom surface can be defined as

\[
\varepsilon_{av} = \frac{2}{H} \delta_y
\]

Each fibre of the half beam is unstressed at one end, where the crack develops, and the assumption is made that the strain varies linearly to zero at this end. The strain at the contact end is then given by

\[
\varepsilon_y = 2 \varepsilon_{av} = \frac{H}{H} \delta_y
\]

\[
= \frac{Hu}{S^2} \left( 1 - \frac{2y}{t'} - \frac{u}{2} \right) \left( 1 - (\frac{u}{S})^2 \right)
\]

The/
The non dimensional parameter, $R$, is then defined

$$ R = \frac{\varepsilon_c}{4} s^2 $$

whence, to a good approximation

$$ \varepsilon_y = \frac{u \varepsilon_c}{R} \left( 1 - \frac{2y}{t'} - \frac{u}{2} \right) \quad \ldots \ldots (1) $$

This equation provides a means of determining the distribution of strain along the contact area at both the supports and the mid height of the wall, and from this equation the distribution of stress can be evaluated. The arching force, $P$, developed is evaluated as the resultant of the stress distribution along the contact area at both the supports and the mid height section, and is obtained by integrating the stress condition across the depth of the wall. The stress condition can, then, be obtained from Equation 1, but first the properties of the brickwork must be defined. The following assumptions are made:

- the brickwork material
  1. has no tensile strength
  2. behaves in a classical elasto-plastic way and
  3. exhibits no strength recovery properties beyond the elastic range.

The assumed stress strain curve is shown in Fig 3. Initially the strain at any point within the wall increases linearly with centre deflection, until the stress reaches the maximum elastic stress, whereupon the stress remains constant with increasing strain.

Furthermore,
Fig. 3
Furthermore, the assumption is made that the material exhibits no strain recovery properties on unloading from the plastic range and consequently a slight decrease in strain once the material is in the plastic range effects both an instantaneous drop in stress, from the ultimate stress to zero, and a permanent set defined by the maximum strain the material has suffered. When these assumed material properties are combined with the strain development, as defined by Equation 1, the stress patterns shown in Fig 4 develop. Since the force P, developed is dependent on Equation 1 which is itself dependent on the dimensionless centre deflection u, the force developed is also a function of u and is more correctly designated as P(u).

When the state of stress is defined, and the resultant force P(u) evaluated and positioned, the returning moment can be found using

\[ M(u) = P(u) r(u) \]

where \( r(u) \) is the lever arm, as shown in Fig 2, which can be expressed approximately as

\[ r(u) = t'(1 - u - \frac{2v}{t'}) \]

The algebraic expressions for P(u) and M(u), both expressed in dimensionless form, are shown in Fig 4. With this understanding of arching action, it is now possible to use this concept to form an equation of motion for the wall.
<table>
<thead>
<tr>
<th>Range of $R$</th>
<th>Range of $u$</th>
<th>Stress Patterns</th>
<th>$\frac{8}{\sigma_c t^*} P(u)$</th>
<th>$\frac{16}{\sigma_c (t^*)^2} M(u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R \geq \frac{1}{2}$</td>
<td>$u \geq 0$</td>
<td>$u &gt; 0$</td>
<td>$\frac{2u}{R} \left(1 - \frac{u}{2}\right)$</td>
<td>$\frac{8u}{3R} \left(1 - \frac{5u}{4}\right) \left(1 - \frac{u}{2}\right)$</td>
</tr>
<tr>
<td>$R &lt; \frac{1}{2}$</td>
<td>$0 \leq u \leq 1 - \sqrt{1 - 2R}$</td>
<td>$u \leq 1 - \sqrt{1 - 2R}$</td>
<td>$4(1 - \frac{u}{2} - \frac{R}{2u})$</td>
<td>$4\left(1 + \frac{R}{2} + \frac{3u^2}{4} - 2u - \frac{R^2}{3u^2}\right)$</td>
</tr>
<tr>
<td>$R &lt; \frac{1}{2}$</td>
<td>$1 - \sqrt{1 - 2R} \leq u &lt; \sqrt{2R}$</td>
<td>$u &gt; 1 - \sqrt{1 - 2R}$</td>
<td>$4(1 - u) + \frac{u}{2R} \left(2\sqrt{2R} - u\right)^2$</td>
<td>$4(1-u)^2 + \frac{u}{6R} \left(2\sqrt{2R} - u\right)^2 \left(5u - 4\sqrt{2R}\right)$</td>
</tr>
<tr>
<td>$\frac{1}{8} \leq R &lt; \frac{1}{2}$</td>
<td>$\sqrt{2R} \leq u &lt; 1$</td>
<td>$u \leq \sqrt{2R}$</td>
<td>$\frac{u}{2R} \left(2\sqrt{2R} - u\right)^2$</td>
<td>$\frac{u}{6R} \left(2\sqrt{2R} - u\right)^2 \left(5u - 4\sqrt{2R}\right)$</td>
</tr>
<tr>
<td>$\frac{1}{8} \leq R &lt; \frac{1}{2}$</td>
<td>$1 \leq u &lt; 2\sqrt{2R}$</td>
<td>$u \leq 1$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$u \geq 2\sqrt{2R}$</td>
<td></td>
<td></td>
<td>$0$</td>
<td>$0$</td>
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</tr>
<tr>
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<td>$2\sqrt{2R} \leq u &lt; 1$</td>
<td>$u \leq 1$</td>
<td>$4(1 - u)$</td>
<td>$4(1 - u)^2$</td>
</tr>
<tr>
<td>$u \geq 1$</td>
<td></td>
<td></td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Fig. 4
Dynamic Equation of Motion.

The equation of motion is derived from the Lagrange Equation:

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}} \right) - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = Q_\theta \]

where:
- \( T \) = kinetic energy
- \( V \) = potential energy
- \( t \) = time parameter
- \( Q \) = Angle of rotation
- \( Q_\theta \) = work done in moving through unit rotation

Dealing with equation 2:

Kinetic Energy:

\[ T = \frac{1}{2} I \dot{\theta}^2 \]

where \( I \) = second moment of mass

\[ \frac{\partial T}{\partial \theta} = I \ddot{\theta} \]

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \theta} \right) = I \dddot{\theta} \]

\[ \frac{\partial T}{\partial \dot{\theta}} = 0 \]

Work Done: \( Q_\theta \) requires to be evaluated for the different loading conditions.

Uniformly Distributed Load:

Considering/
Considering a half length of wall and neglecting the wall thickness, $Q_0$ can be evaluated for time dependent load $p(t)$ with reference to Fig 5.

Work done $= Q_0 \, d\phi$

\[
\text{Work done} = \int_0^{H/2} p(t) \cos \phi \, h \, d\phi \, dh
\]

where $p(t)$ is the pressure function acting on the wall.

Equating these two expressions:

\[
\frac{H}{2} = \int_0^{H/2} p(t) \, h \, dh
\]

assuming small rotations

\[
= \frac{H^2}{2} \, p(t)
\]

**Line Loading**

Using similar considerations it can be shown that for a line load applied horizontally onto the wall at mid height,

\[
Q_0 = \frac{H}{2} \, p(t)
\]

The Lagrange Equation then becomes on substituting

\[
\dddot{\phi} + \frac{3}{2} \frac{V}{\phi} = k \, p(t)
\]

where

\[
I = \frac{H^3 \gamma}{24g}
\]

$g = \text{gravitational constant}$

$\gamma = \text{specific gravity of brickwork}$

and/
Fig. 5

Fig. 6
and where \( k = \frac{H^2}{8} \) for uniform loading

\( = \frac{H}{2} \) for line loading at mid height.

Following the dimensionless parameter presentation as used to describe arching action, the main equation can be rewritten in the form

\[ G \ddot{u} + f(u) = k' \ p(t) \]

where \( G \) and \( k' \) are constants

\( f(u) \) is a function of \( u \)

and when this is done and the equation multiplied a factor \( N \)

where

\[ N = \frac{16}{\sigma_c(t')^2} \; ; \; \sigma_c = \text{ultimate compressive stress} \]

a dimensionless differential equation develops

\[ \left( \frac{4}{3} \frac{H^2 y}{\sigma_c G} \right) \ddot{u} + \frac{16}{\sigma_c(t')^2} \ M(u) = cp(t) \]

where \( c = \frac{2}{\sigma_c} \ S^2 \) for uniform loading

\[ = \frac{8}{\sigma_c} \ \frac{S}{t'} \] for line loading applied at the mid height.

The function \( \frac{16}{\sigma_c(t')^2} \ M(u) \) is a multiple of the returning moment function as derived by McDowell et alia and shown in Fig 4. It represents the derivative of the potential energy in the system, the potential energy being in fact the strain energy within the wall.
wall. The solution of this equation describes the behaviour of a wall fully restrained along the top and bottom edges when loaded transversely with a dynamic pulse p(t). McKee and Sevin (27) used this form of analysis to investigate the strength of wall panels when loaded by a pulse generated from an atomic blast. The peak of pressure associated with atomic devices is extremely quickly developed, and McKee and Sevin assumed a pressure pulse model shown in Fig 6. The pressure was assumed to act instantaneously on detonation producing an impulse and thereafter a constant over-pressure, $p_0$, was assumed within the time interval of practical interest. When this function was used as the function $p(t)$ in equation 3, the differential equation of motion was solved using an analytical approach, first making some simplifying assumptions, and the results of this investigation appear elsewhere (25, 27, 28). The assumption that an atomic blast could be idealised into an initial impulse and a steady overpressure was not felt to be applicable when gas explosions were being considered, since the time scale involved to reach peak pressure is known to be greatly different. When an inflammable mixture of gas and air is ignited there follows what must be described, in explosive terms, as a slow conflagration, and when compared to the rate of reaction of atomic fission, there is a factor of the order of $10^3$ involved in the time taken to reach peak pressure. It was therefore desirable to find an analytical expression to describe the pressure vs time relationship for gas/air conflagrations.
In the aftermath of Ronan Point, the brick industry conducted a series of tests to investigate the effect of gas explosions on loadbearing brickwork and in particular to attempt to establish the strength of wall panels when subjected to a lateral dynamic load. The British Ceramic Research Association conducted this experimental work which, in relation to this Chapter, could be subdivided into two parts. The first concerned experimental work done on fully restrained brickwork panels whilst the second phase of the test program investigated the strength of panels restrained only partially along their edges. The tests involving fully restrained panels were conducted by subjecting panels built within the mouth of a concrete bunker to gas explosion forces, the brick panel forming the weakest structural member of the pressure vessel. The second phase involved the testing of loadbearing brickwork walls which formed part of a three storey building specifically built as a test structure. The walls in this test structure were not as well restrained as those in the bunker experiments, as the floors could be lifted by the wall as it arched to failure (Fig 1a). In both types of experiment, gas was fed into an enclosed space behind the wall under test, and the resulting gas/air mixture was ignited. In some of the tests a balloon was used to contain the gas/air mixture. The structural implications of what followed are not of interest here and are well documented elsewhere (9,29). The rate of development of pressure was measured in all these tests using rapid response pressure transducers which were strategically positioned/
PLATE 1

General View of Bunker with wall ready for testing.
PLATE 2

Single Leaf Wall at point of failure showing

yield line type failure
positioned within the pressure vessel in which the explosion was centred. From pressure profiles produced during these tests an analytical expression in terms of time was derived. The original traces are shown in Fig 7, and relate to the series of tests which were concerned with panels placed in the mouth of the concrete bunker. In these experiments there was no opportunity for "preventing" which happens in normal practice, when the rate of pressure development may be reduced by the venting of certain weaker materials such as glass and light cladding: in the bunker no weak materials were present and venting could only take place once the brick wall under test had failed. Figs 7a and 7b show the traces produced from bunker experiments when the explosion was large enough to cause the brick wall to fail. In these traces, therefore, we have a short decay period, similar, it was thought to the decay times which exist in normal buildings. Fig 7c however, shows a trace from a bunker explosion when the wall did not totally fail, but was severely cracked and damaged. In this case venting could not take place to the same extent and the pressure had to dissipate in part through the cracks and fissures of the damaged wall and in part by the gases cooling. As a consequence the decay time was much greater than in the two cases where venting occurred. Because of the difference in the type of pressure profile it was desirable to decide on one form or another and it was decided to consider an upper bound solution where the pulse caused the wall to fail as opposed to the lower bound solution where the wall just withstands the explosion; thus only pressure profiles similar to Figs 7a and 7b are to be considered. An advantage of the upper approach lies in the fact that the pressure/
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Fig. 7

(a) Times in milli seconds

(b) P.S. 400

(c) Times in milli seconds

1.4 p.s.i.
pressure profile can simulate a situation where pre-venting can take place. Similarly an upper bound solution allows the effects of refracted and negative pressure waves to be neglected as the wall is already structurally broken prior to these waves acting. Thus the pulse need only be considered through its first positive pressure peak and up to a time when this pressure peak decays to zero; any further pressure behaviour is of no consequence.

From the evidence of the curves shown (Fig 7), the peak appeared much later than may have been expected, and the length of the peak appeared to be related to the maximum pressure. The pressure rise appeared somewhat parabolic and the drop in pressure seemed to be independant of maximum pressure and more related, perhaps, to the quality of venting offered to the expanding gases. As a consequence of these observations a profile is suggested in Fig 8. Here the rise is assumed to be of a parabolic form rising to a maximum pressure at time $t_1$. This maximum pressure is maintained for a time which is related to the peak pressure after which it decays to zero linearly in a time which is independant of any previous characteristics of the profile. The experimental pressure profile which the analytical form is attempting to simulate is shown dotted in Fig 8.

The algebraic equations which best fit the experimental values are:

\[
\begin{align*}
p &= 157.4t^2 & 0 \leq t < t_1 \\
p &= 157.4t_1^2 & t_1 \leq t < t_2 \\
p &= 157.4t_1^2 (t_3-t)/0.04 & t_2 \leq t < t_3
\end{align*}
\]

where/
Fig. 8
where $t_2 = t_1 + 0.035 \times 157.4t_1^2$

$t_3 = t_2 + 0.04$

where all pressure values are in lbf/sq inch units and the time parameters are expressed in seconds.

In Fig 7a and 7b, this analytical expression is compared with the experimental curves obtained from the bunker experiments, the former being shown dotted. Expressing the pressure pulse in this form, only one unknown remains, namely the value of $t_1$ which causes the pressure pulse to be large enough to break the wall. In adopting this expression from the pressure pulses generated by only three gas explosions, doubts remained as to its general suitability to describe the pressure development with time for all gas explosions. Such doubts can be erased in part by comparing the theoretical peak pressures and the time taken for them to develop with the corresponding values obtained from the explosions which were generated in the series of tests conducted by the British Ceramic Research Association on the typical three storey test structure (9). This comparison is made in Fig 9, and from this figure it can be seen that good correlation exists between the theoretical and practical pulses when maximum pressure and its time of occurrence are compared for town gas. The theoretical pulse would, however, seem rather more severe than those found in explosions resulting from the ignition of a natural gas/air mixture. Nevertheless while both gases remain in common use the most severe form of loading must be/
Mean Actual Pressure (p.s.i.)

Mean Time to First Peak (m. secs.)

Fig. 9
be assumed when tackling the problem in general. Furthermore the theoretical curve shown in Fig 9 is based on a value of the time taken to reach peak pressure where this is assumed to be at Z (Fig 8) midway between $t_1$ and $t_2$. The fact that the pressure reaches its maximum at $t_1$ introduces a slightly conservative element into the results of an analysis using this theoretical pressure profile. Although the mathematical expression for gas pressures generated was originally based on bunker experiments, subsequent tests carried out by the British Ceramic Research Association on the three storey test structure, indicate that where sufficient venting exists, this form of profile describes well the gaseous pressures generated within modern buildings with good venting characteristics when the wall remains stable, the rapid final decrease in pressure being attributable to the venting of weaker materials such as glass, light cladding, doors and partitions. When the strength of infil panels is being examined under conditions where good venting characteristics apply the mathematical expression for gas pressures generated remains equally relevant when the wall remains stable; the idea of an upper bound solution therefore becomes less important.

**Solution of Dynamic Equation of Motion**

The differential equation of motion, now incorporating the analytical expression for the pressure pulse, $p(t_1)$, as the forcing function $p(t)$, was more readily amenable to a numerical form of solution and a computer program based on a Runge Kutta method was used to obtain solutions. The data input included a first approximation to the likely/
likely ultimate pressure expressed as an initial value of $t_1$. Because the pressure profile adopted was of an upper bound nature, the initial input value of $t_1$ was chosen to be of such a value as to produce a pressure which it was thought the wall could not sustain. The program therefore solved the equation basing failure on the criterion that centre deflection, $u$, is ever increasing with time and produced a solution indicating wall failure. An iterative procedure in the program then reduced the initial value of $t_1$ and with this new smaller value a second solution was obtained. Depending on whether this second solution indicated failure or stability of the wall system, a third estimate of $t_1$ was evaluated and used in a third solution of the equation; the third value of $t_1$ being either decreased, if the second solution indicated wall failure, or increased if the previous solution indicated wall stability. This iterative process was continued until the solution indicated the wall to be unstable to the required accuracy. Thus it was possible to solve the equation iteratively using an ever decreasing pressure profile, specified as a value of $t_1$, until the wall proved to be unstable. This iteration was written into the program and in one run the wall system under investigation was solved. A general library program for solving differential equations was used and other parameters which required to be given as input data included certain limits relevant to the type and order of the equation to be solved, the required accuracy of solution, and initial boundary conditions. The other parameters given as input data were those necessary to specify fully the wall system which required to be analysed; these parameters were the wall height and thickness, the material properties of density, ultimate compressive stress/
stress and ultimate strain and also the gravitational constant. With this data, solutions for any unique wall system rigidly restrained along the top and bottom edges could be obtained for the theoretical pulse loading applied uniformly over the full transverse area of the wall. The computer output included a resume of the data input specifying the problem and was followed by the results of each individual iteration completed. The ultimate value of $t_1$ was then output as was the corresponding value of critical pressure for the wall system under investigation. Finally a graphical representation of the deflection of the wall centre line at mid-height versus time was output and a similar trace was drawn for the arching force developed, $P(u)$, again drawn against time. This graphical output proved useful in establishing at a glance whether the wall proved stable or not. This graphical output was then repeated for values of $0.25t_1$, $0.5t_1$ and $0.75t_1$. Although the last three graphical output curves were indicative of wall stability and therefore not truly relevant for the upper bound solutions for which the analytical pressure profile was derived, they again give a quick indication as to the behaviour of the wall system.

**Theoretical Results**

From initial results of this analysis, it was found that the value of Young's Modulus of the material and the slenderness ratio $S$, of the wall were critical factors, as had been expected. Furthermore the ultimate pressure of walls both with the same slenderness ratio and value of Young's Modulus but of different height, proved to be the same within practical accuracy. Solutions were therefore considered to/
to be independent of height and were dependent solely on the
slenderness ratio of the wall. The same, however, could not be said
about the value of Young's Modulus used, since a solution based on a
critical stress of $\sigma_1$ and critical strain of $\varepsilon_1$ (giving a Young's
Modulus of $E_1$), produced a solution different from a solution of the
same wall problem using a similar value of Young's Modulus ($E_1$) but
using different values of critical stress and strain
of $\sigma_2$ and $\varepsilon_2$ respectively where
\[
\sigma_2 = k \sigma_1 \\
\varepsilon_2 = k \varepsilon_1
\]
k any real number $> 0$

Solutions therefore were not only dependent on the $E$ value of the
material but also on the value of the ultimate stress. This
behaviour was not altogether surprising when account is taken of the
basis of the analysis; the stress patterns are not only dependent on
$E$ but also vary with the value of ultimate stress, thus affecting
both the arching force $P(u)$ developed and the value of the returning
moment.

Fig 10 shows a typical solution for a particular wall system with
certain material properties; the displacement of the centre line of the
wall at mid-height, expressed non dimensionally as $u$, is plotted versus
time. The individual forcing functions on which the four curves of
Fig 10 are based are shown in Fig 11. Curve D1(Fig 10) is due to
the pressure profile $p_1$ (Fig 11) with a time characteristic $t_1$ of such
a value that the pressure profile is severe enough to just cause the
wall to fail. It is the upper bound solution and the pressure
 corresponding/
corresponding to this value of $t_1$, or a greater pressure, will cause
the wall to fail whilst the wall will remain stable when a pressure
pulse based on a characteristic value of less than $t_1$ acts uniformly
on it. Curve D2 (Fig 10) demonstrates this stability being the
deflection v time curve when a pulse $p_2$ (Fig 11) with a characteristic
of $0.75 t_1$ acts on the wall. Similarly curves D3 and D4 are
deflection v times curves pertaining to the pressures profiles of $p_3$
and $p_4$ respectively where $p_3$ and $p_4$ have characteristics of $0.5t_1$ and
$0.25t_1$ respectively.

The pressure profile varies as the time characteristic squared, and
halving the value of $t_1$ implies a reduction of 75% in the value of
maximum pressure; this can be seen in Fig 11. From Fig 10 and 11 the
increase in the severity of the pulse acting on the wall can be seen to
effect an increase in the wall deflections and the time taken for the
wall to reach maximum deflection can be seen to increase with an
increase in the magnitude of central deflection and hence with an
increase in the severity of the loading pulse. The frequency of
vibration when the pressure acting on the wall returns to zero after
the pulse has acted remains fairly constant. The initial frequency
for the first half cycle is seen, then, to be affected whilst the
pressure pulse still acts, the positive value of pressure effectively
impeding the return of the wall to its position of minimum energy.

One or two points, nonetheless, are perhaps not fully expected.
Firstly the initial displacements of all four curves differs little
except in amplitude until the wall nears its maximum displacement.
Fig. 10

Unstable

Stable

D1

D2

D3

D4

Time (secs.)

0.1

0.05

0.2

0.3

Fig. 11

Maximum Lateral Pressure (p.s.i.)

2.0

1.0

P1

P2

P3

P4

Time (secs.)

0.1

0.2
If the forcing term is defined, however, in such a way that the curve of pressure development with time is not dependent on the amplitude of the pressure profile, as has been done, whilst all other parameters, including the inertial force, remain constant, the similarity in initial wall displacement is then not surprising. Secondly, no significant damping is seen to take place within the wall system and consequently the stable solution has vibrations the amplitudes of which remain constant with time, once the pressure applied has returned to a zero value. The equation of motion however does not include any provision for damping either explicitly or in terms of material behaviour and consequently no damping can be expected.

**General Solutions**

General solutions in terms of maximum lateral pressure and other relevant parameters are shown in Figs 12 and 13, the solutions being for wall systems loaded uniformly over the full area.

Fig 12 demonstrates the influence of ultimate stress on the solutions and is a plot of maximum lateral pressure versus slenderness ratio - the latter being based on a value of $H$ equal to 100 inches. The three curves shown are members of a family of curves which forms the solution to the differential equation of motion equ. 3, all curves pertaining to a wall built of material with a value of Young's Modulus of $1 \times 10^6$ psi, there being one such unique curve for every value of ultimate stress. The three curves differ in the values assigned to their properties of ultimate stress and ultimate strain and the three/
Fig. 12

- $\sigma_c = 2000$ p.s.i.
- $\varepsilon_c = 0.002$
- $\sigma_c = 1000$ p.s.i.
- $\varepsilon_c = 0.001$
- $\sigma_c = 500$ p.s.i.
- $\varepsilon_c = 0.0005$

Maximum Lateral Pressure (p.s.i.) vs. $s$
Fig. 13

Maximum Lateral Pressure (p.s.i.)

C = 2000 psi
C = 1750 psi
C = 1500 psi
C = 1250 psi
C = 1000 psi
C = 0

S
three shown here are for values of ultimate stress equal to 500, 1000 and 2000 p.s.i. In general, the ultimate lateral pressure can be seen to increase with a decrease in slenderness ratio. The relationship, however, is not linear; the rate of development of strength increases with a decrease in slenderness ratio. It can be seen that an increase in ultimate stress effects an increase in maximum lateral pressure.

Fig 13 shows five curves belonging to the family of curves which represent the solution to the differential equation of motion when the parameter of ultimate strain is kept constant - in this case 0.001. The influence of the value assigned to the Young's modulus of the material on the solution can be seen. The general trend of increasing pressure with decreasing values of slenderness ratio can again be seen, as can the resulting increase in ultimate stress when the value of maximum lateral pressure is increased. There exists a family of curves similar to that in Fig 13 for every unique value of critical strain.

Fig 14 compares the results of this analysis to the results of a similar type of analysis completed by McKee and Sevin using an approximation to atomic loading as the forcing term and solving the equation of motion by an approximate mathematical method. The graph is a plot of maximum lateral pressure versus slenderness ratio for a wall with material properties of $E = 1 \times 10^6$ p.s.i, $\sigma_c = 1000$ p.s.i, $\varepsilon_c = 0.001$, $L = 100"$. Curve 1 is a solution to the differential equation of motion (eq. 3) using for the forcing function/
function the analytical expression for a gas explosion. Curve 2 is based on the same basic theory but the forcing term used is an approximation to the loading caused by air movement following the detonation of an atomic device, as shown in Fig 6. It can be seen from Fig 14 that both solutions follow the same general trend, but the magnitude of the pressure which the wall can sustain differs slightly.

With a value of $S$ equal to 10, the maximum lateral pressure which this particular wall system can sustain when analysed using the computer solution, with the load produced from a pulse generated from a gas explosion, is reduced by about 30% when the wall system is analysed using the approximate solution with a forcing function which is theoretically the pulse generated by an atomic explosion. The percentage reduction appears to decrease with an increase in the value of $S$ and when $S$ lies between 15 and 20 the reduction is of the order of 25% while with $S = 25$ the reduction is about 20%. Whilst some of this reduction may be due to the different accuracy involved in the two solutions it would appear fair to postulate that the majority of the reduction is due to the different nature of the two loading pulses involved; the severity of the loading pulse assumed to be representative of an atomic blast consisting of both a step function and an initial impulse, is much greater than the pulse assumed to represent the pressure generated by a gas explosion.

In order to sustain lateral pressures of the magnitude illustrated the wall system must be fully restrained; the forces $P(u)$ can then be/
Fig. 14

Maximum Lateral Pressure (p.s.i.)

Curve 1 Gas
Curve 2 Atomic

σ = 1000 p.s.i.; ε = 0.001
be developed and these forces, the resultants of the energy stored in the wall as strain energy, provide the conditions required to allow stability at high lateral loads. A quantitative understanding of the value of $P(u)$ developed during failure is, therefore, necessary.

Fig 15 is a plot of the arching force developed, expressed non-dimensionally, versus the dimensionless parameter $R \left( = \frac{c}{l} \frac{s^2}{l} \right)$ and illustrates the development of the returning force $P(u)$. Thinking in terms of a fixed value for $c$, $\epsilon_c$ and $t'$, an increase in $R$, implying an increase in slenderness ratio, effects a decrease in the value of $P(u)$. Again the relationship is not linear and the development of the force $P(u)$ increases with a decrease in slenderness ratio.

Panels Restrained along all 4 Edges

The analysis presented so far has dealt only with simple one way arching, the failure mechanism of a brickwork panel supported on two opposite edges. The geometry required to apply this type of analysis to panels restrained along all four edges is both extremely tedious and complicated and a rigorous solution would therefore prove somewhat difficult.

A simple empirical approach was, however, postulated by McKee and Sevin (27) and this approach would appear to be valid. They suggested that any two way panel can be converted to an equivalent one way panel; this one way system can then be analysed by the solutions already/
already derived. Fig 16 shows the suggested conversion curve for four sided panels. The basic approach is one of obtaining a modified slenderness ratio for the equivalent panel which describes the stiffness of the two way panel. This modified slenderness ratio is obtained by evaluating an effective length for the equivalent panel whilst leaving unaltered the thickness of the panel. The conversion curve enables this to be done; it is a plot of original aspect ratio (expressed as a fraction with $L_1 < L_2$) versus the effective length of the equivalent panel expressed as a fraction of the shorter side. Thus from the original aspect ratio a modified slenderness ratio can be obtained using this curve.

**Material Properties**

The value of the parameters $\sigma_c$ and $\epsilon_c$ require to be found before the theory can either be validated or used as a design tool. These parameters are best evaluated from experimental work, little of which has been executed on this particular topic. Recent work (Chapter 1) along similar lines on brickwork panels with precompression suggests that values of $E = 1.00 - 1.25 \times 10^6$ lbf/sq in, using a value of critical strain $\epsilon_c = 0.001$, produce good correlation between experimental and theoretical work.

**Other Modes of Failure**

There exists at present some, albeit little, evidence of other modes of failure should the wall be of great stiffness. Such high stiffness may/
Fig. 16
may be due to a very small slenderness ratio or due to the presence of a uniform surcharge on the wall acting in the opposite direction to the lateral gas pressure or to some other force which inhibits deflection. In such stiff walls, initial small deflections may not be sufficiently large for the arching force along the edges to develop to a significant magnitude, and consequently arching action may not occur; in this situation, the panel may shear along its edges and move bodily through a distance in the direction of the applied lateral force. When dealing with extremely small slenderness ratios of 1 and 2, it could be strongly argued that arching failure will not occur and the most probable mode of failure would be a punching shear where the whole wall is either bodily ejected or moved a distance towards the position of full ejection. This type of behaviour has been witnessed during the Bunker experiments (9), when the back wall of the bunker (a brick wall against which a large quantity of back fill material was placed) was observed to have moved bodily a small distance under the action of the pressure generated by one of the explosions. This particular anomaly in the failure pattern can be explained on the above basis assuming the wall to be effectively stiffened by the earth pressure acting on it; if such a pressure had not acted on the wall to restrain the magnitude of deflections. Some yield like cracking would doubtless have occurred. This shear failure of the whole wall is therefore another mode of failure which must be considered. A true general solution to the failure pressure of unreinforced brickwork panels when subjected to a dynamic type of load, should therefore be a combination of arching action analysis and shear considerations/
considerations and if this is so Figs 12 and 13, do not tell the full story. On to each of these curves a shear type failure solution should be superimposed and the interaction curve so formed should be used. No experimental or theoretical work to the author's knowledge, has been done on this topic and from the point of view of practical interest such work is of restricted significance to the practicing engineer. It is however possible to postulate the interaction curve.

Considering a wall of height $H$, thickness $t'$, and of unit length rigidly supported along the top and bottom, the equilibrium equation can be written

$$p_{\text{max}} H = 2 \tau_u t'$$

where $\tau_u = \text{ultimate shear stress of mortar or the mortar/support interface.}$

$$p_{\text{max}} = \frac{2 \tau_u t'}{H} \quad \text{and using the notation } S = \frac{H}{t'}$$

$$p_{\text{max}} = \frac{2 \tau_u}{S}$$

This equation is plotted in Fig 17 as a plot of maximum lateral pressure versus slenderness ratio, $S$. The shear failure criterion is compared in Fig 17 with an arching solution based on a value of Young's Modulus of $1 \times 10^6 \text{ psi}$ and an ultimate compressive stress of 1000 psi. It would appear therefore that for walls rigidly restrained along two opposite edges the nature of the failure is dependent/
Shear Failure \( p_{\text{max}} = \frac{2\tau}{S} \)

Arching Failure

Fig. 17
dependent on the value of slenderness ratio; at high values of $S$, the failure will be of an arching action type while at low values of $S$ the mechanism of failure will be predominantly shear. When considering the shear failure of panels fully restrained along all four edges the equilibrium equation requires modification:

\[ P_{\text{max}} \frac{H}{L} = 2 \frac{\tau_u t'}{(H + L)} \]

where $L = \text{length of panel}$

\[
giving \quad P_{\text{max}} = \frac{2\tau_u}{S\left(\frac{L}{H + L}\right)} = \frac{2\tau_u}{S'}
\]

$S'$ being the effective slenderness ratio, defined by

\[
S' = S \left(\frac{L}{H + L}\right)
\]

Using this effective slenderness ratio the curves in Fig 17 can be used for shear considerations; for arching failure, however, the equivalent panel method should still be used (Appendix 1 and 2)

The shear failure of brickwork infill panels is as yet only a hypothesis and should therefore be treated accordingly. Nevertheless it is of interest to note that the 9 inch wall tested in Round 27 of the Bunker Experiments (9) was found to have moved forward approximately 0.25 inches in addition to being severely cracked in the /
the manner associated with arching failure (Appendix 1).

**Implications and General Discussion**

The theory presented here suggests that walls when fully restrained (Fig 1b) are capable of withstanding much greater lateral pressures than identical panels where the restraint provided is not as complete (Fig 1a), and there are certain implications inherent in this fact.

Walls of low slenderness ratio rigidly restrained will not fail at low pressures and consequently cannot be thought of terms of vents. In the event of a high lateral pressure acting on such a wall, loads of a severe magnitude will be induced in the surrounding framework. When this is viewed in the light of present design practice since the Introduction of the Fifth Amendment, two points are significant. Firstly, in a load bearing brickwork structure which incorporates some degree of reinforced concrete beam and column framing designed to operated in the event of a local structural failure of a wall or pier, the strength of the infil panel will be increased and it will fail at a correspondingly higher value of lateral pressure than would occur if the framing was not present. In a building which for architectural reasons already has poor venting characteristics, the effect of strengthening the wall panels may give rise to more widespread and more severe damage following an explosion. Secondly, the beams and columns introduced to support the remaining structure in the event of local damage may well themselves be structurally damaged by the large arching force developed when the wall deflects. An anomalous situation/
situation may well arise when the lateral pressure is of sufficient magnitude to cause the wall to deform and crack but not to fail. This wall although damaged will still have structural strength: the structural condition of the reinforced concrete frame would however be doubtful, because of the magnitude of the arching force developed. There exists a probability, then, that the framing provided solely for this type of emergency will suffer greater structural damage than the wall for which it is required to be a structural substitute. Great care must therefore be taken when use is made of a structural reinforced concrete frame to satisfy the requirements of the Fifth Amendment. (Appendices 2 and 3).

The inherent strength of infill panels when properly designed could be of use where a pressure barrier is required to isolate areas, where the risk of a rapid development of pressure is deemed unacceptable, from areas where such a high pressure wave is undesirable or may prove fatal. Many situations like this exist in practice; two examples are the boilerhouse used to service adjoining office or domestic accommodation and the areas in industrial premises, where volatile and potentially dangerous chemicals are stored, when this area is close to the shop floor. Any use of structural pressure barriers in such situations would necessitate the incorporation of a high degree of venting in such a position as to be acceptable, on a safety basis, to the surrounding neighbourhood.

Experimental evidence on the lateral strength of brickwork walls with precompression (Chapter 3), suggests that the rate of loading has little/
little effect on the maximum lateral pressure which such wall systems can sustain. It would therefore be in order to suggest that the strength of infil panels is independent of the rate of load applied. This would imply that the theoretical presentation could be used for determining the strength of infil panels when wind forces, derived from codes of practice, were assumed to act on the panel (30)

Conclusions.

A theory based on the well established concept of arching action has been presented to determine the values of maximum lateral pressure which a wall system can sustain when supported rigidly along two opposite edges, and loaded uniformly with a pressure pulse similar to that generated during a gas explosion. An empirical relationship based on an equivalent panel approach, has been suggested to deal with panels supported along all four edges.

Other possible modes of failure are discussed and a general solution incorporating two failure modes is suggested. The implications of this work, when viewed in conjunction with other recent research, are discussed and some practical applications are suggested; specimen design calculations and a comparison of theoretical results to the few experimental results available to date are appended to this Chapter.

Whilst the material properties to be used in the interpretation of this theory may be deduced from other work, there would appear to be scope for some experimental work to both validate the theory and to establish the relevant material properties of the brickwork.
Plates

Courtesy British Ceramic Research Association.

2. ¼" Wall breaking at the bunker.

Appendices

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**Notation**

- \( H \): Height of Wall
- \( t' \): Thickness of Wall
- \( S \): Slenderness Ratio
- \( S' \): Effective slenderness ratio
- \( p \): Lateral pressure
- \( P, P(u) \): Arching force developed
- \( a \): Section of half wall not in contact with support
- \( B \): Deflection of centre line of wall at mid-height
- \( u \): Dimensionless horizontal displacement (\( B/t' \))
- \( \alpha \): Fraction of half depth in contact with support.
- \( \delta_y \): The shortening of the material at position \( y \).
- \( y \): Coordinate throughout wall thickness.
- \( \varepsilon_c \): Elastic limit strain of brickwork
- \( \sigma_c \): Ultimate stress of brickwork
- \( \tau_u \): Ultimate shear stress of mortar/support interface
- \( R \): Dimensionless parameter \( \varepsilon_c S^2 / h \)
- \( r(u) \): Lever/
\( r(u) \)  
Lever arm of arching force developed

\( M(u) \)  
Returning moment  
\( P(u) r(u) \)

\( T \)  
Kinetic energy

\( V \)  
Potential Energy

\( t \)  
Time parameter

\( \Theta \)  
Angle of rotational

\( Q_\theta \)  
Work done in moving through unit rotation

\( I \)  
Second moment of mass

\( i_0 \)  
Initial impulse associated with nuclear blasts.

\( \gamma \)  
Density of brickwork

\( g \)  
Gravitational constant

\( N \)  
Multiplying factor

\( \dot{u} \)  
First derivative of \( u \) with respect to time

\( \ddot{u} \)  
Second derivative of \( u \) with respect to time.

\( P_o \)  
Steady overpressure associated with nuclear blasts

\( t_1, t_2, t_3 \)  
Time characteristics of pressure profiles

\( L_1, L_2 \)  
Dimensions of four sided wall panels \((L_1 \leq L_2)\)

\( L \)  
Length of equivalent two sided panel

\( k, k', c \)  
Forcing function constants

\( G \)  
Constant
Appendix 1

Experimental Verification

Virtually no work has been done in the experimental field which could be directly compared with this theoretical analysis. Only one result exists, to the authors' knowledge which is of direct relevance, namely the failure pressure of the $1\frac{3}{16}$" wall tested in the bunker opening by British Ceramic Research Association. Under similar conditions a 9" wall was tested, but though the wall's stability was shown within a certain range of lateral pressure, the experiments were not continued to conclusion and consequently no ultimate pressure, associated with failure, was found. The one result, then, and the stability range for another wall system are the only available means of correlating theory with experiment. When the curve correlating 4 sided to 2 sided panels and the theory postulated is used assuming the height of the bunker is the max height i.e. to the top of the arch roof then the theoretical results shown in the following table are obtained.

<table>
<thead>
<tr>
<th>Experimental</th>
<th>Theoretical*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.11$ psi (failure)</td>
<td>$\sigma_c = 1,000$ psi $\sigma_c = 1,250$ psi</td>
</tr>
<tr>
<td>$13-15$ psi (sustained with cracking but no failure)</td>
<td>$4.2$ psi $5.3$ psi</td>
</tr>
<tr>
<td></td>
<td>$25$ psi</td>
</tr>
</tbody>
</table>

* based on a similar calculation as is demonstrated in Appendix 2
Bunker Results

4 inch Wall

Arching Failure
Taking the maximum height of the arch since this will produce the weakest effect

\[
\frac{L_1}{L_2} = \frac{87}{120} = 0.725
\]

From Fig 16:-
\[
\frac{L}{L_1} = 0.84
\]
when \[
\frac{L_1}{L_2} = 0.725
\]
giving the equivalent panel length, L, equal to

\[
L = 0.84 \times 87 = 73.1 \text{ inches}
\]

\(S\), the slenderness ratio, of the equivalent panel can then be found

\[
S = \frac{73.1}{L \times 125} = 17.7
\]

From Fig 13 (with \(\sigma_c = 1000 \text{ psi}, \varepsilon_c = 0.001\))

\(S = 17.7\) gives \(P_{\text{max}} = 4.2 \text{ psi}\)

From Fig 13 (with \(\sigma_c = 1250 \text{ psi}, \varepsilon_c = 0.001\))

\(S = 17.7\) gives \(P_{\text{max}} = 5.3 \text{ psi}\)
Shear failure

\[ S = \frac{H}{t'} = \frac{87}{4.725} = 21.1 \]

Assuming, for purposes of evaluating the length of the shear surface, a square opening 120" x 78.5"

\[ S' = S \left( \frac{L}{L + H} \right) \]

\[ = 21.1 \left( \frac{120}{120 + 78.5} \right) = 12.74 \]

From Fig 17 assuming \( \tau_u = 70 \) p s i

\( p_{\text{max}} = 10 - 11 \) p s i

Arching failure will therefore occur at \( p_{\text{max}} = 14 - 5 \) p s i

9 inch Wall

Arching Action

Using a similar approach to the one used for the single leaf calculation

\[ \frac{L_1}{L_2} = \frac{84}{120} = 0.7 \]

From conversion graph:

\[ \frac{L}{L_1} = 0.85 \]
giving the equivalent length \( L = 0.85 \times 84 = 71.5 \) inches

\[
S = \frac{71.5}{8.625} = 8.25
\]

From Fig 13 (using \( \sigma_c = 1000 \text{ psi}, \varepsilon_c = 0.001 \))

\[ P_{\text{max}} = 25 \text{ psi} \]

(using \( \sigma_c = 1250, \varepsilon_c = 0.001 \))

\[ P_{\text{max}} = 31 \text{ psi} \]

**Shear Failure**

\[
S = \frac{8h}{8.625} = 9.7
\]

Assuming a square opening of 120" x 77"

\[
S' = 9.7 \times \left( \frac{120}{120 + 77} \right) = 5.9
\]

From Fig 17, assuming \( \tau_u = 70 \text{ psi} \)

\[ P_{\text{max}} = 24 \text{ psi} \]

The failure would therefore be partly due to shear and partly due to arching action

**NOTE**

Whilst the panel was not loaded to failure it was severely loaded and this resulted in both severe cracking, of a yield-line-pattern type, and also lateral movement of the whole panel.
Appendix 2

Design Procedure

Example 1  One Way

It is required to build a wall within a structural frame to withstand a gaseous pressure of 5.5 psi - adequate venting will be available to the expanding gases. The panel is to be 10 ft high and 4 ft in length, restrained only along the top and bottom edges.

A value of $c_c = 1000$, and $c_e = 0.001$ is assumed and with these values using Fig 13 with $p_{max} = 5.5$ psi

$$S = \frac{120}{15.9} = 7.55\text{ inches}$$

The wall must be thicker than this value. Adopt a 9 inch (nominal) wall. Assuming the actual thickness is 8.00 inches then

$$S = \frac{120}{8} = 15$$

and this wall will fail by arching action at an actual pressure of 6.3 psi.

Now, $R = \frac{e_c}{l_i} \cdot S^2 = \frac{0.001}{l_i} \cdot 15^2 = 0.0562$

From/
From Fig 15 with this value of \( R \)

\[
\frac{P(u)}{\frac{P(u)}{c}} = 0.382
\]

giving \( P(u) = 0.382 \times 1000 \times 8 = 3,060 \text{ lbf/inch run of wall.} \)

The rigid structural abutments will have to be designed to withstand a maximum load equal to this load, which will act eccentrically. Furthermore the lateral stability of the wall and the frame would require to be examined.

**Example 2**  
**Two Way**

It is required to design a brickwork panel 12'-0" long by 10'-0" high with rigid restraints along all edges. This wall is to withstand a maximum lateral pressure of 15 psi.

**Arching Failure**

The original aspect ratio \( \frac{L_1}{L_2} = \frac{10}{12} = 0.833 \)

which, from Fig 16, gives an effective aspect ratio

\[
\frac{L}{L_1} = 0.81
\]

giving \( L = 0.81 \times 120 = 95.2 \text{ inches} \)

From Fig 13, assuming \( \sigma_c = 1000, \epsilon_c = 0.001 \) a value of \( P_{\text{max}} = 15 \text{ psi} \) gives a slenderness ratio of

\[
S = 10.4
\]
Equating the slenderness ratio to the equivalent panel length

\[ S = 10 \cdot l = \frac{95 \cdot l}{t'} \; ; \; \; t' = \frac{95 \cdot l}{10 \cdot l} = 9.15 \text{ inches} \]

A nominal 9 inch wall is required with the actual thickness required to be greater or equal to 9.15 inches.

**Shear Failure**

Assuming \( \tau_u = 70 \text{ psi} \) from Fig 17 a maximum lateral pressure of 15 psi requires an effective slenderness ratio

\[ S' = 9.3 \]

\[ S = S' \left( \frac{H + L}{L} \right) = 9.3 \times \left( \frac{10 + 12}{12} \right) = 17.05 \]

\[ t' = \frac{H}{S} = \frac{120}{17.05} = 7.038 \]

Thus to prevent shear failure a wall of thickness greater than 7 inches is required. Such a wall, however would fail by arching action with a lateral pressure of less than 15 psi and a nominal 9" wall is required to safeguard against an arching failure
APPENDIX 3

Emergency Framing Provided to comply with Building Regulations

One method which is practiced to safeguard loadbearing brickwork structures against a general collapse is to incorporate some degree of beam and column framing, strategically positioned in the structure.

When columns and beams are incorporated into loadbearing brickwork structures to fulfill requirements defined by the Building Regulations, the basic nature of the problem is changed; the presence of vertical structural members tied to the floors above and below a brickwork panel changes the problem from one of panels with precompression to one of infill panels between rigid supports. When lateral pressure acts on the wall arching forces will develop which put the columns provided into tension. When the wall has failed, the columns then fulfill the role of carrying the load which the wall originally carried. Columns provided specifically as emergency compression members, will therefore require to first act as tension members before being utilised for the role for which they were designed. Such columns should therefore be checked to ensure that they will neither be damaged nor fail due to tension.

Example

With reference to Example 1, Appendix 2, the maximum arching force developed was 3,060 lbf/in run of wall.

\[ p(u)_{\text{max}} / \]
\[ p(u)_{\text{max}} = 36,720 \text{ lbf/ft run}. \]

If the precompression on the 8 inch wall were greater than about 9,000 lbf/ft run no columns would be incorporated since at this level of precompression the wall is capable of withstanding a lateral pressure of 5 psi.

Considering a wall with a precompressive load of only 6,000 lbf/ft run, an out of balance force of 30,720 lbf/ft run acts forcing the floors apart and tensioning the column.

Were this wall say 10 ft in length the maximum force developed by this wall would be 307,200 lbs. It would be fair to assume that the reinforced concrete columns which are often incorporated in the higher storeys of brickwork structures are not capable of withstanding this magnitude of force in tension.

In fact, the force of 3,060 would only be developed if 6.3 psi acted on the wall (Appendix 2), and consequently if only 5 psi were assumed to act laterally, the force developed would be somewhat less than this maximum figure.
An Experimental Investigation into the Lateral Strength of Brickwork Panels with Precompression under Dynamic and Static Loading
Introduction

At the time of planning the experimental work which is described in this Chapter, little had been published on the lateral strength of loadbearing brickwork under precompression, (13, 14, 15, 16, 17 18, 19, 21) when uniformly loaded dynamically over the full wall. A series of tests had been planned by the British Ceramic Research Association with the intention of investigating the strength of panels when loaded transversely with a static load (23). The effect of rate of loading had, however, still to be investigated if dynamic pressure pulses generated from gaseous explosions were to be represented for design purposes as a static design pressure. The effect of the material properties of the brickwork on the lateral strength of walls was similarly unknown. It was with this background that the experimental work was planned and the objectives of the experimental work could be summarised:-

"To investigate the strength of brickwork panels with precompression when subjected to dynamic loading and

(1) to correlate the effect of dynamic loading to that of static loading, and

(2) to ascertain the effect of the material properties of the brickwork on the lateral strength.

With these general objectives in mind and the awareness that the experimental work should simulate, as closely as possible, the practical problem in loadbearing brickwork structures, the experimental work was planned.

Experimental Design.
Experimental Design.

Preliminary costing and the available facilities immediately suggested that full scale wall tests were not feasible; it was consequently decided to use either one-third or one-sixth scale walls. The difference in end restraint between infil panels and panels with precompression, and the consequent difference in lateral strength which walls can sustain under the different end conditions had been appreciated prior to the experiment being designed. The walls were therefore to be compressed using dead weight, which could be freely lifted by the wall as it arched to failure, and not by the use of jacks which would necessitate extremely careful manipulation to avoid the likelihood of the panels behaving more like fully restrained infil panels. Thus the end conditions would more closely resemble those associated with loadbearing brickwork panels as seen in practice at the time of experimental design. A further constraint, of which account was taken, was the form of dynamic loading which was to be adopted for these tests. It was envisaged that a type of loading similar to that produced from a gas explosion should be used. Two possibilities were explored.

1. The use of some explosive chemical such as an explosive mixture of gas and air or an explosive charge similar to that used for ammunition propulsion.

2. A more simple approach of generating an impulsive load by means of a falling weight and spring/damper system.

Both/
Both approaches were technically feasible. The first, however, proved both costly and somewhat impractical since it immediately excluded the use of existing facilities on grounds of safety. The alternative approach was amenable to a laboratory environment, and was in comparison, much cheaper than the first. After more detailed investigation it proved to be as technically practical as the first, although it removed to some extent the similitude between the experimental and practical model. This, however, would not invalidate the main objectives and on further considerations of manpower resources, safety and the time scale involved, it was decided to adopt a 'drop weight' system and plan the experiment within the laboratory using a one-third scale factor.

To facilitate the application of vertical compressive stress, it was decided that the wall would stand vertically in the test rig and the precompressive dead weight load would then be applied directly. This necessitated taking the force derived from the drop weight system to the wall via a rope and pulley system, and any modifications required to the drop weight system, to make the loading pulse more closely resemble the pulse associated with gas explosions, could then be incorporated in the rope and pulley system. The type of modification envisaged was the inclusion of a spring and damper system.

**Line Loading**

The practical difficulties of applying onto the wall a uniformly distributed/
PLATE 1
Front View of Test Rig

PLATE 2
Rear View of Test Rig
distributed dynamic load generated by a method based on the dropping of a load proved quite severe. It was therefore decided to adopt a line form of loading across the full width of the wall at mid-height. This approach facilitated the design of the test rig but posed a complication regarding the type of pulse required. It was therefore decided to adopt the mathematical model shown in Fig 1 where the pulse $P(t)$ was the analytical expression for pressures generated from a gas explosion (Chapter 2), it was assumed that this pulse acted over an area equal to the area of the wall but was brought onto the wall by a line load applied at mid-height. This expedient approach immediately removed the similitude between the laboratory and practical model but nevertheless did not invalidate the basic experimental objectives, and it was consequently adopted.

The Test Rig.

After a more detailed appraisal of the problems already exposed, a test rig as illustrated in Plates 1 and 2 was built. The one-third scale brick wall panel stood in the rig on a concrete foundation centrally positioned between four twelve feet vertical channels connected to the strong floor and braced about eight feet above floor level. On top of the wall a system of spreaders was placed and on top of this rested an I beam with its longitudinal axis along the length of the wall. The ends of this I beam protruded beyond the 4 vertical channels and on the underside of the bottom flange two small pieces of angle were welded at either end, the angle pieces having been first drilled to take a one-inch bolt (Plate 3).
Fig. 1

Diagram showing a column with an applied load $P(t)$ and a deformation $t'$. The symbol $\sigma'$ appears on the left side of the column.
From these bolts the dead weight was hung and the wall thus put under a compressive stress. The I beam carrying the dead load onto the wall was attached to four vertical ball races positioned one in each of the four vertical channels. The dead weight was therefore restrained from any other form of motion other than vertical and the amount of precompressive load carried by the wall was constant and independent of the vertical movement of the dead weight (Plate 3). Small lengths of railway line were used as the dead weight each piece being approximately 200 lbf. The web section was drilled in two places and the rails were then laid, with the web horizontal, on top of each other in such a way that the holes were aligned. Tension rods were then placed through the holes and on to the top railway line a lug, drilled for a one-inch bolt, was welded (Plate 3). Two sets of rails were made and these were hung, one at either end of the main precompression I beam, using the lugs and the angle sections attached to the lower flange and a bolt (Plate 3). By varying the number of pieces of rail in each set, the amount of precompression applied was adjusted, and in some instances extra small pieces of rail had to be added by attaching them to the set with hooks.

The impulse load was generated by dropping a weight attached to a steel rope, in which a spring and damper system was incorporated, which then took the load onto the wall via a system of pulleys; the rope was attached to a structural yoke which lay horizontally around the wall and was pulled towards the wall when the rope, was tensioned. Two I beams connected by tension rods formed the structural/
PLATE 3

Side View of Wall in position and the dead weight connection
PLATE 4

Vertical Linear Races
structural yoke which in turn transmitted the load to a load cell which was positioned between the yoke and a system of horizontal spreaders. (Plate 5) A length of $\frac{3}{8}$ inch diameter bar was connected to the horizontal spreaders and this bar applied the line load to the wall through a piece of hard rubber matting which took up the irregularities inherent in the brickwork surface. When the rope was tensioned, then, the bar was pulled onto the wall. Both the yoke and the horizontal spreader system were mounted in linear races (Plate 5) which were fixed horizontally to two steel tables, one either side of the wall. Linear races were used since the line load was required to move through a distance of between one to three inches to cause failure and it was desirable that friction losses should be minimised if not excluded.

**Spring Damper System**

The three variables which were potentially controllable and which could affect the character of the loading pulse were:

(a) the stiffness of the spring

(b) the coefficient of the damper

(c) the magnitude of the drop weight.

Having regard to the experimental mathematical model and the type of pulse to be applied, the salient features of the loading pulse were then clearly defined. The rope required to be tensioned in such a way that a time dependent pulse, with characteristics similar to those pertaining to a true gaseous conflagration pressure profile, would be produced. The rope also required to be tensioned to a magnitude/
PLATE 5

Load Cell, Horizontal Spreader System and horizontal linear races.
magnitude equal to the product of the area of the wall and the maximum pressure required. The time characteristics of the pressure profile varied with the size of pressure profile and it was therefore necessary to have direct control in the laboratory of the three parameters mentioned above. A more detailed account of the design of this spring damper system is included as Appendix 1. Based on these more detailed considerations, two identical springs were purchased each spring having an adjustable constant lying within the range 80 - 150 lbf per inch, and capable of working at 200 to 1000 lbf respectively. The spring constant could be adjusted by effectively removing a certain number of coils from the spring and this could be done by altering the position of a brass slug which was supplied with the spring. To increase the stiffness the slug, which was positioned closely fitting inside the coils, was screwed deeper inside the spring and after adjustment, the slug acted as a fixing for one end of the spring. A damper was also purchased with characteristics lying between 25 - 100 lbf per inch per second, and with a working stroke length of up to 18 inches. The characteristics of this damper could be fairly easily adjusted within this range. These three components were placed on the end of the rope using two sections of 2 inch x 2 inch mild steel bars as anchor points. (Plate 6). By altering the constants of the springs and damper and by choosing an appropriate drop weight, a pressure pulse could be generated which simulated closely a pressure pulse similar to that produced by a gas explosion of a certain severity. To balance the self weight of the spring and damper system, a small counterbalance weight (Plate 1) was attached to the horizontal yoke/
Spring and Damper System with Drop Weights attached.

Rear view of test rig showing rope and pulley system.
yoke in such a way as to leave the yoke in a position of equilibrium when no drop weight was attached to the end of the rope.

**Load Measurements.**

Using one-third scale walls, it was expected that the range of load which would be required during the test programme would vary from approximately 100 lbf up to approximately 1 tonf. Of the load cells which were already available none were suitable for the smaller range of loads on the grounds of sensitivity. A 1 tonf load cell was therefore built for this series of tests. (Plates 7 and 8). It was constructed on a basic strain bridge principle, using a hollow cylinder of duralumin as the central unit. The driving voltage could be altered and the load cell was calibrated at intervals of driving voltage between 10 and 70 volts. This variable voltage allowed for greater sensitivity as and when this was required, and because of the variable voltage and in order to interface the load cell with the equipment measuring the output voltage, a variable potentiometer was included in the circuit and the driving voltage was supplied using a common earth.

**Instrumentation**

Since the design load was to be measured by a load cell previous to being applied to the wall, and since the load was a transient, the load measuring equipment required to be capable of a time dependent recording. The rise time from zero load up to maximum load was expected/
PLATE 7

1 Ton Load Cell
PLATE 8

Exploded view of load cell and cover.
expected to vary up to a maximum value of approximately 200 - 300 milliseconds, whilst the majority of tests were expected to have much smaller rise times. Because of this factor and after considering the potential of graph plotters and ultra violet recorders, it was decided to use an oscilloscope to record the pulse load as measured by the load cell. In an attempt to reduce the complexity of laboratory procedure, a Telequipment DM 53A Dual Beam Storage Oscilloscope was used, thus removing the need to photograph the load pulse as it occurred. The load cell was driven by a dual channel Advance 0 - 50V stabilised D.C. power supply. The instrumentation is shown in Plate 9.

Quick Release Mechanism.

The spring damper system had been designed in such a way that the load required to be released on to it from some other point of support. The fact that the loading was to be time dependent required this release to be both quick and efficient. The instrumentation for measuring the load applied required synchronization between the oscilloscope and the instant at which the load was released. Both these problems were solved by a quick release mechanism which is shown in Plates 10,11. The mechanism, suspended from an overhead travelling crane, carried all the force of the drop weight required for the particular wall under test. The drop weight was also attached by steel ropes to the spring and damper system to the rope connecting the drop weight to the structural yoke, and hence on to the wall. As the mechanism was operated an electric switch/
PLATE 9

Instrumentation

D.C. power supply
Digital Volt Meter
Storage Oscilloscope
D.C. supply to synchronisation switch
Quick release mechanism in closed position
PLATE 11

Quick release mechanism in open position showing the lead to the instrumentation synchronisation switch, and counterbalance weight.
switch was closed and this produced an electric pulse which provided the synchronization required to start the instruments and thus record the transient load.

Materials.

The third scale bricks which were used were supplied by the British Ceramic Research Association and were of different strengths; they were nominally 3000, 4000, 5000, 6000 and 8000 lbf/sq. in. bricks. The sand used was a Leighton Buzzard sand passing a 25, 52 sieve. The cement used was Ferrocrete Quick Set and no lime or other additives were used. The mortar mixes used were 1:3 and 1:6 by weight.

Wall Building

All the walls were built by the one mason working under laboratory conditions. These walls were built away from the test rig into which they were moved prior to testing. Steel channels (5 x 2.5 inches) partly filled with a concrete mix, were used as the foundations for the wall, the channels being 36 inches long whilst the wall was 30 inches in length. A frame was used to assist in the wall building and this provided vertical support at each end of the wall. No support, however, was given to the wall along its front or back face. The walls were built of 28 courses giving a height of 31.5 inches, and were topped by a similar channel, again part filled with concrete, to the one used for the base. Walls of single and
double thickness were built and these were built in the space of two or three days. To facilitate easy movement of the walls between their building position, storing position and the test rig without breaking the bond, the walls were post compressed by tensioning two rods which connected the top and bottom steel channels. These rods were threaded at either end and by applying torque to the nuts the rods were tensioned. After adopting this system no wall was damaged accidentally in transit. The rods were removed, however, before the test, but not until the precompression, supplied by the rail sets, had been transferred to the wall.

During the wall building process, control specimens were made. These included one-inch and four-inch mortar cubes, three-inch brickwork cubes and five course brickwork piers. Sample bricks were also taken from the brick stock used to build the wall. All the wall building process and the control specimen and their manufacture were executed within the framework laid down by the British Ceramic Research Association in their Model Specification (31). Walls were tested at a convenient time after their completion but under no circumstance were walls less than five days old tested. The mortar and brickwork cubes and the specimen sample of bricks were tested the same day using the specification as laid down in (31).

Method of Testing

Once the wall was positioned in the test rig, and when it had been vertically aligned, and the precompression had been applied, it was ready/
ready for testing. The basic approach was to successively shock load the wall with ever increasing loads until failure occurred. Consequently, a drop weight was first chosen, the effects of which it was thought the wall could sustain. This weight was hung from the overhead travelling crane via the quick release mechanism; the weight being attached also to the spring and damper system. By slowly raising and lowering the hook of the travelling crane, a small load was applied to the wall via the knife edge. This load was measured by the load cell which was being driven by a voltage, suitable to the sensitivity required, under the condition that zero load was equivalent to zero output. After applying the small load, the crane was slowly raised taking load off the wall until a zero reading was obtained from the load cell output. In this position and after suitably adjusting the instruments the release mechanism was operated and the wall suffered a pulse loading, the form and magnitude of which was displayed on the oscilloscope. If the wall remained stable (Plate 12) the procedure was repeated, often several times, using a more severe loading pulse each time until the wall failed (Plates 13, 14). The load required to cause failure was then taken as the maximum load from that particular pulse. This approach was of course recognised as an upper bound solution and the increase in severity of the pulse was restricted to within the degree of accuracy which the experimental system permitted. In these conditions the load causing failure was deemed to be the failure load of that particular wall system.

Static Tests.
Oscilloscope trace of pressure pulse when wall remains stable
OSCATLSCOPL OF PRESSURE PULSE causinK wall KALURE.
PLATE 14

Oscilloscope trace of Pressure Pulse causing wall failure.
Static Tests.

To compare the behaviour of walls when loaded dynamically and statically a small number of walls were tested using a slowly applied line load. Essentially the wall and test rig remained the same, but the spring and damper system and the steel rope were replaced by two sets of jacks inserted between the four vertical channel members of the test rig and the I beam of the yoke on the opposite side of the wall from the line load rod. These jacks were connected to the Losenhausen test machine and on increasing the pressure of the fluid within the machine and the jacks the yoke was moved relative to the wall and the line load was pulled onto the wall in the same basic manner as occurred in the dynamic tests. The one ton load cell for this series of tests was connected to a Dynamco DM 2022S digital volt meter which had the facility of automatically measuring the maximum positive or negative signal generated by the load cell. In all other respects the tests conducted statically using the jacking system were similar to the dynamic load series.

Measurement of Lift.

Midway through the test series it was decided to measure the uplift of the precompression due to the wall arching to failure, and two linear displacement transducers were positioned to measure this vertical movement. The transducers were fixed to a frame which stood around the rig but was not attached to it and consequently the/
the vibration of the wall and test rig due to the dynamic loading
in no way affected the measurements recorded. A pen chart recorder
was used to record the magnitude of the uplift, but this machine
did not have a sufficient rate of paper feed to allow a closer
examination of the relationship of uplift with time. The instrument-
ation is shown in Plate 15.

Test Results

The number of solid walls tested totalled twenty seven, seventeen of
which were of single brick thickness (1.5\text{"
})\), the other ten being
3 inch thick. No cavity walls were tested. Of the total of
twenty seven all but four were tested under dynamic loading. Results
were obtained for twenty four of the tests; three walls produced no
results either because of instrumentation difficulties or because
the wall broke unexpectedly. Of the three walls which yielded no
results, two were 1.5\text{"
} thick. The result of one 3\text{"
} wall which was
under static test, was accidentally lost; the other two fruitless
tests were under dynamic loading. The full results of this series
are shown in Table 1, which also gives the average values obtained
from the control specimen. Values of the maximum uplift - the
vertical distance through which the precompressive force was moved
by the wall failing - are also recorded in Table 1 for the tests
during which it was measured.

Experimental Observations

The/
PLATE 15
Instrumentation for Measuring Uplift of Precompressive Force.
<table>
<thead>
<tr>
<th>Wall No</th>
<th>Thickness (ins)</th>
<th>Nominal Brick Strength (p s i)</th>
<th>Mortar Mix</th>
<th>Precompressive Stress (p s i)</th>
<th>Max. Line Load (lbs)</th>
<th>Equivalent Max. Pressure (p s i)</th>
<th>Lift (ins)</th>
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</table>

* Tested Statically
f Wall failed without a result.

**Table 1**

**WALL TEST RESULTS**

**CONTROL SPECIMENS**

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<th>Mortar Mixes</th>
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<th>1:6</th>
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**BRICK STRENGTHS (p s i)**

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<th>Nominal</th>
<th>Actual</th>
</tr>
</thead>
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<tr>
<td>3000</td>
<td>2887</td>
</tr>
<tr>
<td>4000</td>
<td>3908</td>
</tr>
<tr>
<td>5000</td>
<td>5149</td>
</tr>
<tr>
<td>6000</td>
<td>6787</td>
</tr>
<tr>
<td>8000</td>
<td>9544</td>
</tr>
</tbody>
</table>

**BRICKWORK STRENGTHS**

<table>
<thead>
<tr>
<th>Brick Strength</th>
<th>1:3</th>
<th>1:6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>2523</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>3535</td>
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<tr>
<td>5000</td>
<td>4012</td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td>4432</td>
<td>3327</td>
</tr>
<tr>
<td>8000</td>
<td>5733</td>
<td>4430</td>
</tr>
</tbody>
</table>
Experimental Observations

The mode of failure seen during this series of tests was remarkably consistent and followed the mode of failure known as arching action. The behaviour of the half-walls after they rotated to failure depended on the type of wall tested and also on the properties of the brickwork. Whilst in some cases the two half-walls remained virtually intact, in others (Plate 17) they were shattered into small pieces. Occasionally the cracks did not appear at the top and bottom mortar joints but at the one next to it, but in all the tests, however, local damage to the bricks in the courses between which the hinge formed, was noticeable to a degree which appeared to increase with increasing values of precompression. (Plates 18, 19, 20).

Attempts were made to record a dynamic wall failure photographically, using high speed filming techniques, for various reasons. One important reason was to look at the way in which the top constraint of the wall behaved, as the speed of movement during testing precluded human observation; this attempt did not reach fruition. In the static tests, however, the behaviour of the part-filled concrete channel which capped the wall was observed. In all four static tests this top channel was seen to rotate slightly until this rotation was impeded by the presence of the four vertical heavy channels which formed the main members of the test rig. Only after this rotation had taken place did the top hinge form in the brickwork. No doubts arose about the bottom edge constraint on the panel, however,
PLATE 16

Central Hinge formed prior to failure
PLATE 17

WALL AFTER FAILURE
PLATE 18
Bottom Channel after Failure

PLATE 19
Crushing of Brickwork associated with arching action
PLATE 20

Detail of crushing associated with arching action.
however, as the channel in which the wall stood had been firmly fixed within the test rig.

**Experimental Conclusions.**

The test results are shown as a plot of maximum lateral equivalent pressure versus precompressive stress in Fig 2, the ordinate referring to the equivalent pressure as defined in Fig.1. It can be seen from these results that a general trend exists and the experimental results form two linear or nearly linear curves, the results pertaining to one curve pertain also to a unique value of slenderness ratio. The two curves formed by the experimental data refer to slenderness ratio values of 10.5 and 21. Although results exist only for these two values of $S$, there exists a family of curves for this graph, one for each unique value of slenderness ratio.

From these test results, certain conclusions can be drawn:-

The strength of walls under transverse loading is not significantly affected by the number of shock loads to which it is subjected.

The failure of brick walls with precompression by arching action occurs in a consistent manner relative to the scatter associated with other failure modes of brickwork.

The strength of transversely loaded walls with precompression is not/
Fig. 2

Maximum Lateral Pressure (psi)

Precompressive Stress (p.s.i.)

\[
P = \frac{4\sigma}{S^2} \quad S = 10.5
\]

\[
P = \frac{3\sigma}{S^2} \quad S = 21
\]

Static Results
Results

Fig. 3

Uplift (in.)

Maximum

Precompressive Stress (p.s.i.)

S = 10.5

S = 21

Static Results

0
not significantly influenced by the rate at which load is applied.

The strength of transversely loaded walls with precompression increases with an increase in the applied precompressive stress.

Fig. 3 is a plot of maximum vertical uplift of the precompressive force versus the precompressive stress applied. Two suggested best fit curves are shown. A similar trend can be seen to that present in Fig 2; two different curves resulting for the two unique values of slenderness ratio used in the test series. Unlike the pressure $v_{precompressive}$ stress relationship an increase in the precompressive stress results in a reduction in the uplift. It can be seen that the uplift for statically and dynamically loaded panels is similar in magnitude.

**Theoretical Validation.**

To explain the experimental results in theoretical terms a simple static approach is used.

The assumptions are first made that:
(a) The wall is infinitely strong in compression
(b) No tensile bond exists at the top, bottom or middle hinges.
(c) The self weight of the brickwork is negligible in comparison to the precompression applied.
The free body diagram for one-half wall is shown in Fig 4.

For equilibrium \[ \frac{H}{P} \cdot \frac{H}{2} = \sigma t' \]

Giving \[ p = \frac{4 \sigma}{S^2}, \quad S = \frac{H}{t'} \]

This equation describes the relationship between the pressure applied to a wall (through a line load applied at mid-height) of slenderness ratio \( S \) and under a compressive stress \( \sigma \).

This curve is shown in Fig 2, and has been drawn for the two values of \( S \) which pertain to the experimental results. Notwithstanding the essential differences between the assumptions adopted and the properties of brickwork, the correlation between the theory and the experimental evidence is not convincing.

Taking cognisance of what has already been said regarding the rotation of the channel which capped the wall, the theoretical approach suggested should perhaps be modified; the precompressive force of the top of the wall will not act at the corner as assumed but will act centrally as in Fig 5a. The free body diagram for the
bottom half wall will then be as shown in Fig 5b.

and for equilibrium \( \frac{H}{F_2} \frac{H}{2} = \frac{3}{4} \sigma t^2 \)

giving \( p = \frac{3 \sigma}{S^2} \)

This expression is shown in Fig 2 for the two relevant values of slenderness ratio. This theory provides better correlation between the theory and the experimental evidence.

It has been shown (Chapter 1) that when more accurate assumptions are made regarding the elastic properties and self weight of brickwork the theoretical prediction is modified. At low values of precompressive stress a basic theory, such as above, underestimates the maximum lateral pressure because of the assumption of negligible self weight, whilst the omission of elastic properties in the assumptions makes a basic theory overestimate the maximum lateral pressure at higher values of precompression. Seen in context with what has already been established, the experimental results are validated.

Conclusions

An experimental investigation into the strength of brickwork panels with precompression when loaded transversely with both dynamic and static line loading has been described.
The test results, which can be theoretically validated, suggest that the strength of such walls,

(a) is not significantly affected by the brickwork properties at low values of compressive stress.

(b) is not adversely affected by previous shock loads.

(c) is not affected by the rate of loading at the range associated with gas explosions.

TABLES

1. Experimental Wall Results.

PLATES

1. Front View of Test Rig.
2. Rear View of Test Rig.
3. Side View of Test Rig showing connection between the dead weight sets and the I Beam.
4. Vertical Linear Ball Races.
5. Horizontal Spreader Beams, Load Cell, and Horizontal Ball Races.
6. Spring and Damper System and rope and pulley system.
7. 1 Ton Load Cell
8. Exploded view of load cell and cover
9. Instrumentation to record loading pulses.
12. Trace of Load versus time for stable wall.
13,14. Traces of Load versus time for unstable walls.
15. Instrumentation for recording uplift of precompressive force showing Losenhausen control panel for the jacking system.
16. Central Hinge prior to wall instability.
17. Wall after failure showing jacks in position.
18,19,20. Local crushing of the brickwork found in the courses adjacent to the hinges formed during failure.

FIGURES

1. Mathematical Model.
2. Experimental Results and Theoretical Validation
3. Uplift versus precompressive stress
4. Free body diagram of half-wall.
5. Free body diagram of wall with modified end conditions.
The motion of a weight with time when suspended on a spring damper system is known to be:

\[ mx'' + cx' + kx = 0 \quad (1) \]

and a solution can be obtained assuming that a root of this equation is

\[ x(t) = e^{st} \quad (2) \]

substituting this root in Eq. (1) a quadratic is formed:

\[ (s^2 + \frac{c}{m} s + \frac{k}{m}) e^{st} = 0 \quad (3) \]

which is satisfied for all values of \( t \) by the roots

\[ s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \quad (4) \]

and the general solution Eq. 2 then becomes

\[ x(t) = A e^{s_1 t} + B e^{s_2 t} \quad (5) \]

A and B being arbitrary constants depending on the way and at what time the motion starts.
By defining critical damping as that value of $C$ which provides Eq. 3 with equal roots ($S_1 = S_2$) then

$$\frac{c_c}{2m} = \sqrt{\frac{k}{m}} = \omega_n$$

$$c_c = 2m\omega_n$$

The amount of damping present can then be specified non-dimensionally in terms of critical damping

$$\xi = \frac{c}{c_c}$$

The roots of Eqn 3 can then be written

$$S_{1,2} = \{ -\xi \pm \sqrt{\xi^2 - 1} \} \omega_n$$

since

$$\frac{c}{2m} = \frac{c_c}{c} \frac{c_c}{2m} = \xi \omega_n$$

The general solution (Eq 5) can then be written

$$x(t) = A e^{(-\xi + \xi^2 - 1)\omega_n t} + B e^{(-\xi - \xi^2 - 1)\omega_n t}$$

$$= A e^{\gamma t} + B e^{\lambda t}$$

$$\gamma = (-\xi + \xi^2 - 1) \omega_n$$

$$\lambda = (-\xi - \xi^2 - 1) \omega_n$$

To find /
To find A and B let

\[ x(0) = x_0 \]
\[ x(0) = 0 \]

and on substituting these conditions in Eq 6

\[ A + B = x_0 \]
\[ \gamma A + \lambda B = 0 \]

which give

\[ A = -\frac{\lambda}{\gamma - \lambda}x_0 \]
\[ B = \frac{\gamma}{\gamma - \lambda}x_0 \]

Substituting these values of A and B into the general solution gives

\[ x(t) = \frac{x_0}{\gamma - \lambda} \{ \gamma e^{\lambda t} - \lambda e^{\gamma t} \} \]

and the derivatives become

\[ \dot{x} = x_0 (\frac{\gamma \lambda}{\gamma - \lambda}) \{ e^{\lambda t} - e^{\gamma t} \} \]
\[ \ddot{x} = x_0 (\frac{\gamma \lambda}{\gamma - \lambda}) \{ \lambda e^{\lambda t} - \gamma e^{\gamma t} \} \]

Now the tension in the rope can be written

\[ F = mg + kx + cx \]
\[ = m(g - \ddot{x}) \text{ since } kx + cx = -m\ddot{x} \]
\[ = m \left( g + x_0 \frac{\gamma \lambda}{\gamma - \lambda} \{ \gamma e^{\gamma t} - \lambda e^{\lambda t} \} \right) \]
If \( x_0 \) is defined as the position, above the final equilibrium position, where the spring is untensioned then

\[
x_0 = \frac{-mg}{k} = -\frac{x}{\omega^2}
\]

giving

\[
F = mg \left\{ 1 - \frac{1}{\omega^2} \gamma \frac{\lambda}{\gamma - \lambda} \left( \gamma e^{\gamma t} - \lambda e^{\lambda t} \right) \right\}
\]

If \( \gamma \) and \( \lambda \) are now redefined as

\[
\gamma = -\xi + \sqrt{\xi^2 - 1}
\]
\[
\lambda = -\xi - \sqrt{\xi^2 - 1}
\]

then the force in the wire can be expressed as:

\[
F = mg \left\{ 1 - \frac{1}{\gamma - \lambda} \left( \gamma e^{\gamma \omega n t} - \lambda e^{\lambda \omega n t} \right) \right\}
\]

This equation describes the force in the wire when a load is released at time \( t = 0 \), from a position where the spring is neither tensioned nor compressed.

The solution is, however, only directly amenable to a value of \( \xi \) such that \( \xi^2 - 1 > 0 \) and is a general solution only, in terms of \( \gamma \) \( \lambda \) and \( m \). This general solution required to be closely fitted to the theoretical gas pressure profile liable to be encountered during experimental work.

By/
By specifying that at time $t_1$ the force in the wire was to be some fraction of the drop-weight force, the spring constant $k$ could be evaluated, and since $\xi$ was to be specified the damper constant could then be evaluated.

Thus the boundary condition

$$F(t_1) = r \cdot mg \quad \ldots (8)$$

where $r = \text{a constant < 1}$

The solution required the use of a recurrence formula and an iterative process. The iteration equation used was obtained by incorporating Eq 8 into the general solution Eq 7 giving

$$\omega = \frac{1}{\lambda t_1} \log_e \left\{ \frac{\lambda - \gamma}{\lambda} (1 - R) + \frac{\gamma}{\lambda} e^{\gamma \omega t_1} \right\}$$

and the first approximation - the initial value with which to start the iterative process was taken as

$$\omega = \frac{1}{\lambda t_1} \log_e \left\{ \frac{\lambda - \gamma}{\lambda} (1 - R) \right\}$$

A value of $R = 0.95$ was arbitrarily chosen as was a value of $\xi = 1.1$

The solution was obtained using a computer, and the program incorporated/
incorporated a sub routine which solved the wall systems with various degrees of precompression. It was, therefore, possible to analyse the wall using a method based on arching theory and to obtain a hypothetical value of critical pressure. This value could then be used in conjunction with the analytical expression for a gas profile and the equation representing the force generated by the weight/spring/damper system, finally arriving at a value for spring stiffness and damping constant required.

This solution was completed for the full range of wall systems and precompression likely to be used in the laboratory, and from these solutions the type of spring and damper required was established.

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Structural Considerations for Design against Accidental Damage with particular reference to Gas Explosions.
Accidental Damage

The unfortunate collapse of the corner of a block of flats at Ronan Point in 1968 has resulted in a great awakening of the construction industry to both the dangers resulting from and the probability of occurrence of gas explosions. The vigorous interest which has been shown by both the popular press and some technical journals has resulted in a "gas explosion event" being elevated to a position where rightly it does not belong; it is to the engineer and to the construction industry, so we are led to believe, a thorn in the flesh, an insoluble problem. At this point in time such a view could not be further from the truth; whilst it could be considered fair criticism to state that not sufficient thought had been given in the past to the structural aspects of accidental damage and, in particular, the structural effects of gas explosions the state of knowledge is now such that structural safety in multi-storey buildings following a gas explosion should no longer be a cause of great concern. It is necessary, however, to remove the event of a gas explosion from the position to which it has been elevated by the events of the last three years and the publicity these events received; the occurrence of a gas explosion must be seen in true perspective, and placed in the general category of accidental damage.

The word 'accident' is unfortunately highly subjective, and is defined in terms of 'unforeseen' and 'unexpected'. The question then arises, "Unforeseen by whom?". The Oxford dictionary is more specific and uses the phrase "irregularity of structure" as a definition quoting the latin/
latin root as meaning 'to fall'! Whilst the latter interpretation is perhaps too near the truth, the word "accident" is still ill defined and it is necessary to define it more exactly before it can be used in further discussion. If the loadings which are usually adopted in structural design considerations are defined as the normal loading conditions, then any type of non-normal loading can be defined as accidental. For the purposes of this discussion we will consider normal loading to incorporate only the dead load plus the superimposed loading plus the natural load of wind and snow; any other form of loading is then defined as accidental. Accidental damage can be subdivided into two categories, that which is due to natural events and that which is not. Natural accidental events could arise from earthquakes, earth slips, the shedding of snow or ice masses, the erosion of foundations by water, high pressures generated by flood waters and many other occurrences, whilst unnatural accidental events could result from material faults such as honeycomb concrete or weakness induced by fire, the improper loading of floors by the user, impact of high energy masses, explosions of gases, liquids, solids, and dusts, and many others. The list is meant to indicate the width of the spectrum of events which can now be regarded as accidents and is nothing if not incomplete. Gas explosions are therefore to be seen as only one aspect of the unnatural accident spectrum. Having established this, however, and without contradicting what has earlier been said, it is necessary to look closer at the cause, the event and the effect of a gas explosion.

**Gas Explosions**

A/
Gas Explosions

A gas explosion is basically a chemical reaction between the oxygen in the air and the gas; the gases involved are usually town or natural gas as supplied by mains for domestic and industrial use, but may also be the gases which are now commonly supplied in bottles. When the volume of gas present is greater than a certain percentage of the total volume of the chamber in which it collects, a mixture exists which on initiating the chemical reaction will produce an explosive effect. For town gas the percentage must lie between about $\frac{5}{14} \%$ of the total volume for an explosive mixture. Reaction is not spontaneous and ignition of the gases is necessary, after which a flame front advances and the reaction takes place on the interface of this flame front. The rate of chemical reaction is extremely fast in comparison to natural human events, the time involved being of the order of $0.25 - 0.5$ seconds. In terms of reaction times of high explosives and atomic fission this is extremely slow and the reaction of a gas and air mixture should more correctly be referred to as a conflagration. The reaction produces hot exhaust gases of a greater volume than the initial constituents and if no path of escape is afforded to these exhaust gases, a rise in pressure occurs within the vessel in which the reaction takes place. Four constituents are therefore necessary in order that an explosion will take place; there must be a source of gas, a vessel into which the gas can percolate and in which it can collect, a sufficient volume of accumulated gas to provide an explosive mixture, and a source of timely ignition. Dealing with each point in turn, the source of gas is usually connected with faulty mains or appliances but may/
may also be due to a bottle of gas. The vessel in which the gas accumulates is of great significance. Gas being lighter than air in general, under most favourable conditions, settles in rooms with poor ventilation in a layer on the ceilings and the thickness of this layer increases with time as more gas escapes. It is possible that the volume present will be sufficient for the gas to leak into another room when the base of the gas layer reaches the top of a door of similar opening while a large volume is retained in the first room. The second room will then build up a gas layer and under suitable conditions the gas may leak into several compartments in this way, each compartment still retaining a large volume of gas. If this volume is sufficient to provide an explosive mixture, as is highly probable in many situations, the accidental ignition of the mixture will have an explosive effect. Such ignition could be caused by the flame or the heat generated from many domestic appliances, by a lit cigarette, by a spark produced from electrical appliances or switches, by a spark produced mechanically, and by any one of many other similar causes. Two other conditions also have a considerable effect on the effect of gas explosions, namely the effect of both turbulence, before and after ignition, and the venting characteristics of the chamber in which the explosion takes place.

When poor ventilation exists and a gas layer forms in an empty chamber, ignition will result in a uniform motion of the flame front. If, however, conditions are such that the majority of gas is not removed but is mixed with the air in the empty chamber, a more violent reaction results generating a correspondingly higher pressure. This effect also occurs when a well layered quantity of gas is ignited in a room in which/
which obstacles have been placed, since such obstacles effect an irregular motion of the wavefront and consequently the unignited gases are subjected to the turbulence induced by the irregular motion of the flame front, and are therefore better mixed when ignition occurs. This effect is present in most domestic rooms and offices where furniture and equipment act as the obstacles, inhibiting the smooth motion of the flame front. Furthermore the turbulence effect will occur when the reaction travels through door and hatch openings into other rooms. This cascading effect has been seen to produce much increased values of pressure generated. On ignition of an explosive mixture a pressure rise is generated. If the pressure vessel has inherent weaknesses, these weak points will fail before the reaction is complete and this failure is called venting. If venting does occur it has two main effects. The first is that the rate of development of pressure will be reduced; the second is that unburnt gas can also be expelled through the vent if the vent is opened by a pressure small enough to be developed before the reaction is complete. The potential of the explosion can therefore be reduced.

The pressures generated by the ignition of an explosive mixture of gas and air belong to a different scale of loading to the scale used by engineers for normal design purposes. In a closed compartment pressures of up to 100 psi can be developed if the pressure vessel is capable of withstanding this pressure without venting; the normal design load for domestic floors is 30 lbf/ft² and for shop floors is 80 lbf/ sq ft (ie = 0.2 and 0.5 psi respectively). It is therefore apparent that under adverse conditions, pressures generated by gas explosions/
explosions can result in spectacular damage around the area of the vessel in which the explosion occurs. If the gas leaks into several compartments or into stair or lift wells, the explosion may cascade throughout the building causing damage to many areas. Fire can also be induced by gas explosions and this adds another danger for any person trapped or dazed by the initial blast. The events which followed the gas explosions at both Ronan Point and Clarkston are dramatic reminders of the effect of gas explosions on structural behaviour.

What, then, can be done to cope with this particular problem of gas explosions having regard to the fact that it is but one small bandwith within the accidental damage spectrum?

Non Structural Solutions

Where particular problems can be forseen, non structural solutions can be introduced to reduce or eliminate the probability of occurrence, and hence the structural damage which may be occasioned by that particular problem, if the probability of occurrence necessitates such action. Each case should, of course, be dealt with on its merits but in general an eye to detail and an awareness of the cost effectiveness should be among the governing criteria. The probability of vehicular impact, for example, could be countered by strategically positioning energy absorbers such as deep gravel beds or concrete obstacles Plates 1, 2. Similar precautions can be taken when dealing with the problems associated with gas. Natural or artificial ventilation can be/
PLATE 1

View of a barrier protecting a building against vehicular impact

PLATE 2

Detail of Barrier
be used to ensure that large gas volumes do not accumulate. Whilst widespread use of this involves extra installation, maintenance and running costs, discreet use of ventilation in spaces where there is a high risk of gas escaping and accumulating can reduce the probability of an explosive mixture being formed. Reliable gas alarms can similarly be used and the use of such devices in zones of high risk, such as central boilerhouses, can again reduce the probability of an explosion.

The venting characteristics of rooms should be carefully assessed, having regard to the inherent human element. Results of tests conducted by The British Ceramic Research Association indicate that panels of single glass act as vents at pressures of between 0.33 and 1.8 psi, depending on the type of glass, and the size of panel. In general venting relief should be afforded on external walls where any unburnt gas can be expelled from the building and not deeper inside it. Care in detail can also have effects; small windows will vent at lower pressures if a perimeter hinge is used with weak latches instead of a central pivot type hinge, as the former mechanism is more easily unbalanced. Certain types of sliding doors may for similar reasons be thought unsuitable. The exclusion of gas from new structures will provide the ultimate non structural solution, if such action is feasible; much wider issues are, however, involved in such a step.

Where the circumstances are such that a particular type of accidental loading may be foreseen, a suitable course of action should therefore be taken using non structural methods to reduce the probability of occurrence/
occurrence to an acceptable level. In the real world, the immediate limitations of this approach can, however, be seen, Furthermore, such actions are usually associated with only one particular bandwidth of the accident spectrum. What can be done to minimise the effects of an incident due to other causes? The answer to this question lies with the soundness of the structural solution.

Structural Solutions.

In the field of civil engineering, situations do arise where the probability of accidental loading is considered to be of sufficient order that account of it is taken in the structural design. In zones where earth tremors have regularly occurred, the structural design is such that account of this will be taken. The same philosophy can be adopted for accidental loading in general, within the framework of present legislation.

Since April 1st 1970, the structural considerations which must be given to the design of new buildings have been defined both in type and magnitude by the Fifth Amendment to the Building Regulations. No mention is given to gas explosions. The requirements stipulated are designed to deal with incidents, and although the amendment was conceived as a direct result of the partial collapse at Ronan Point, all thoughts regarding the specific cause must be put aside and the basic concept considered in terms of a general accidental event. The regulations offer an alternative to designers both of which define a certain standard which the structure must exhibit when accidentally loaded. These
These alternatives are

either (a) to design each loadbearing member to withstand a pressure of 5 psi acting normally on it.

or (b) should the local member not be capable of withstanding 5 psi, it must be considered to be removed by the incident and the load which it was carrying must be transmitted downwards through the structure to the foundations using an alternative load path, within prescribed conditions which stipulate the maximum permitted damage.

In suitable cases, the best solution may result from a combination of the alternative methods. In general, accidental damage occurs infrequently, as does the full design wind and gravity loads, and a reduced load factor (already applicable for wind loading) may be used, as is consistent with limit state philosophy.

With structures which incorporate masonry panels, the strength of individual members likely to be encountered in practice can now be ascertained. (Chapters, 1, 2 and 3) Reinforced concrete can be similarly dealt with for transverse static and dynamic loading using either ordinary yield line analysis or a solution of the dynamic equation of motion (32) respectively. Within the framework of present legislation, the statutory requirements can be satisfied using the strength of individual members and designing these members accordingly.

It/
It may be argued that the ability of a local structural member to withstand one type of incident, implies a high probability of it performing in a similar manner for other types of incidents. When such incidents are viewed having regard to the full spectrum of accidental loading, the validity of this argument must be questioned. Severe as the present design pressure stipulated by the Building Regulations may be, it can never be said to deal satisfactorily with every type of incident, nor, more important, with every type of incident the probability of occurrence of which is high; the very concept of stipulating a loading value at all is an implicit contradiction of the interpretation of the events with which the regulations are designed to deal. It can therefore be strongly argued that the soundest approach when attempting to minimize the effects of a local accidental load lies in the alternative load path approach, in the structural integrity of a part damaged structure. If this is so, how well can buildings adjust their structural behaviour following local accidental damage?

This topic was investigated by Hendry et al. (33, 34) with reference to loadbearing brickwork structures. A number of high and low rise structures were selected which were believed to incorporate most of the design features found in modern loadbearing brickwork design and this sample was studied to evaluate their susceptibility to a more general collapse following accidental damage to a major loadbearing wall or pier. Conclusions were reached which suggested that systematic and easily applied checks can be made to guard against the possibility of a partial collapse of a structure throughout its height. Examination of the eight low rise and eleven high rise structures suggested that there/
there were three situations which required close investigation in relation to partial collapse:-

Case A: where there is an outside wall without returns or with only one internal return.
Case B: where there is an internal wall without returns.
Case C: where the removal of a section of wall imposes high local bearing stresses on a return wall or walls.

Typical illustrations of these cases are shown in Figs 1, 2 and 3 respectively.

Calculations for evaluating a particular case were based on the approach that a partial collapse did not occur simultaneously throughout the height, but occurred progressively floor by floor. Each floor slab would therefore require to be capable of carrying the wall resting on it if the wall under the floor slab were accidentally removed. Calculations were therefore based on either bearing stresses in orthogonally placed walls or on yield line principles, and although some of the structures examined were deemed to be unacceptable on grounds that stresses were induced which exceeded the increased permissible value, the remaining structures which appeared to be satisfactory demonstrated that it is quite feasible to achieve a loadbearing brickwork structure which is not liable to a more general collapse following the removal of one major loadbearing element, without having to resort to special measures. Where precast floor units were used, however, special considerations must be given in order that they may support/
FIG. 2.
FIG. 3.
support sections of wall by bending action (8,35).

Further proof of the structural integrity approach to loadbearing brickwork structures and validation of the theoretical method of predicting the safety factor of partially damaged structures was obtained by Hendry, Sinha and Maurenbrecher (36). In this experimental work ground storey walls in a five storey test structure were tested to destruction under a transverse load. The theoretical prediction, based on an alternative load path, that the structure would remain stable after the removal of the wall (Appendix 1) was seen to occur in practice. The test structure was built previous to the event at Ronan Point and no special precautions had been taken to deal with such a situation, the design of the building was based on good practice before the awakening of interest to the possibility of partial collapse. Plates 3, 4 show the test structure with a ground wall removed and is photographic evidence of the inherent advantages of a highly indeterminate structure when accidental damage occurs. In general the behaviour of structures of steel and concrete construction could be similarly viewed; whilst high structural indeterminacy implies good resistance to accidental damage, each case should be carefully examined. There are basic inherent differences however between loadbearing brickwork structures and those constructed with steel or concrete frames when considering the effects of accidental loading, and these differences can be of great significance. In general a loadbearing brickwork wall or pier is not directly tied to either the floor which it supports or the floor on which it stands, and consequently, as has been seen in experimental work, the failure of a loadbearing brickwork wall causes no direct damage to the surrounding floors. Similarly, whilst the/
PLATE 3

Test Structure at Torphin Quarry with end wall removed.
PLATE 4

Test Structure at Torphin Quarry with middle wall removed.
the presence of a return wall enhances the strength of brickwork panels, the material is such that failure of the panel causes no more than slight localised damage to the return wall. With reinforced concrete panels, however, the higher tensile strength, the ability to resist load by bending and the high degree of continuity which steel reinforcement affords, all imply that a degree of rotation and bending of the supporting beams or columns will be induced. Methods are available to the designer to deal with reinforced concrete structural members under rapidly applied high loads but such considerations must be tempered with the effect which is induced in surrounding elements while the local member fails. High tension and shear stresses may occur at beam and column junctions and these should be accounted for. The high strength of reinforced concrete members may in adverse conditions have a dramatic effect on the venting characteristics, and great care should be exercised that no strong pressure vessels are inadvertently incorporated within a structure.

Expert opinion given during the enquiry into the Gas Explosion at Clarkston indicated that the explosion occurred in a relatively strong pressure vessel and consequently a higher pressure was generated than would have developed had there been an area of weaker material incorporated in the wall of the pressure vessel. Such an area of weakness would have acted as a vent at lower pressures and consequently the maximum pressure developed would have been less severe.

By the very nature of things, accidental loading of such magnitude as/
as to severely damage the structure, perhaps inducing total collapse, will from time to time occur. Such severe accidental damage, which will occur most infrequently cannot be catered for in a structural design procedure and should be treated by the engineer and by society as a catastrophe. Nevertheless, the effects on a structure of accidental loading on a smaller scale and of a higher probability of occurrence can, and have been shown to be, minimised by adopting a strongly indeterminate solution. Perhaps the most striking evidence available to date of a concrete structure resisting accidental damage is shown in Plate 5. In this plate, taken after a considerable explosion occurred at the base of a newly erected concrete panel block of flats in Algeria in 1966 the severe local damage which resulted, including the removal of loadbearing walls, can be clearly seen; the remaining structure can also be clearly seen to have remained stable by internally adjusting its structural behaviour.

Plates 3, 4 & 5 therefore serve as constant reminders that local accidental damage need not result in a more general collapse.
PLATE 5

Block of flats in Algeria damaged by sabotage.
PLATES

1. View of Barrier protecting a building against vehicular impact.

2. Detail of Barrier.

3,4. Five Storey test structure with one ground level loadbearing wall removed.
   Courtesy Structural Ceramics Research Unit, Edinburgh University.

5. Damaged Block of Flats in Algeria.
   Courtesy of Tracoba S.A. and Gilbert Ash Ltd.
   (As submitted in evidence to the Inquiry into the Collapse of flats at Ronan Point, Canning Town, London (Plate 9) )

FIGURES

1. Case A

2. Case B
   Situations requiring investigation for partial collapse

3. Case C

APPENDIX

1. Theoretical Prediction for the structural stability of the five storey structure at Torphin Quarry
"Explosions in Domestic Structures"


"Explosions in Domestic Structures - Discussion"


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(b) Gas Explosions in Loadbearing Brick Structures.

Appendix 1

Stability Calculations for Quarry Structure.

WALL PLAN

Floor Steel for shaded area

TOP STEEL

Design midspan moment = 11,340 lbf in
By using standard yield line techniques \((1,2)\) and by using calculations to estimate the bearing stress \((3)\) the stability of the quarry structure was calculated before the removal of the walls.

**Wall 1**

No bottom steel was carried through under wall 1.

External Work

\[
P = hW + w(43 - 1.33x)
\]

Loads

\[
W = \frac{1}{2} x 4.1 x 8 x \frac{120}{12} = 165 \text{ lbf/ft run}
\]

\[
w = 60 \text{ lbf/sq ft}
\]

No live load to be on floors at time of testing

This gives \(P = 3240 - 80x\)

Dissipation of Energy along Yield lines

\[
D = \frac{8}{x} + \frac{x}{6} + \frac{10 \cdot 75}{8} + \frac{2.5}{8} + 2
\]

On/
On equating $P$ to $D$

$$m = \frac{3240 - 80x}{x^2 + \frac{x}{5} + \frac{15.25}{8}}$$

To evaluate $x$, \( \frac{dm}{dx} = 0 \),

$$x^2 + 2.296x - 46.5 = 0$$

Valid root is \( x = 5.77 \)

Substituting back into expression for $m$:

$$m = 8306 \text{ lbf in}$$

which is less than both the design moment and therefore the ultimate moment and this is consequently safe.

Wall 2

This can be dealt with two ways

(a) Bearing pressure on orthogonal return wall - wall Z.

The load to be carried is due to $h_1$ walls above

plus 5 floor slabs

(the four slabs which rest on the walls above Wall 2 are assumed to act such that only $\frac{1}{3}$ of their dead weight is carried by Wall 2. The floor slab directly above the opening created by the removal of wall 2 is assumed to act such that $\frac{1}{2}$ of its dead weight is distributed along the return wall).

Stress due to Walls /
Stress due to walls

\[
4 \times \frac{4 \times 1}{12} \times \frac{8 \times 8 \times 120}{4 \times 1 \times (4 \times 1 + 5)} = 281 \text{ lbf/in}^2
\]

Stress due to first slab

\[
\frac{1}{2} \times \frac{8 \times 10 \times 75 \times 60}{10 \times 75 \times 4 \times 1} = 59 \text{ lbf/in}^2
\]

Stress due to remaining slabs

\[
4 \times \frac{10 \times 75}{3} \times \frac{8 \times 60}{4 \times 1 \times (4 \times 1 + 5)} = 185.0
\]

Total \( 525 \text{ lbf/in}^2 \)

This is less than the ultimate stress and can be considered both permissible and safe.

(b) Had this not proved satisfactory, the walls could still have been carried by bending action of the floors.

\[
W = 1 \times \frac{4 \times 1}{12} \times 8 \times 120 = 330 \text{ lbf/ft run}
\]

\[
w = 60 \text{ lbf/sq ft}
\]

Giving
Giving \( P = 3900 - 80x \)

Dissipation of Energy along the yield lines:

\[
D = m \left( \frac{8}{x} \times 1.553 + \frac{3}{x} + \frac{8}{x} + \frac{10.75}{8} + \frac{2.5}{8} + \frac{2}{8} \right)
\]

\[
D = m \left( \frac{23.52}{x} + \frac{x}{8} + \frac{15.25}{8} \right)
\]

Equating \( P \) and \( D \) gives

\[
m = \frac{3900 - 80x}{\frac{x}{8} + \frac{23.52}{x} + \frac{15.25}{8}}
\]

The condition \( \frac{dm}{dx} = 0 \) gives valid root of \( x = 9.37 \)

which gives \( m = 6779 \) in lbf

which is less than the design moment and is consequently safe.

In both these calculations it was unnecessary to find the ultimate moment of the floor slab; if the moment calculated from the above calculations had exceeded the design moment it would have been necessary to check that it was smaller than the ultimate moment of the floor slab for it to be deemed safe.

From these calculations it is therefore possible to predict the structural integrity/
integrity of the building after removing one wall.

These calculations have been repeated using a superimposed load for the floor in (h).

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