RESPONSE OF AN UNDERWATER STRUCTURE
OF OPTIMUM SHAPE
TO GENERAL LOADING

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PREFACE

This thesis is the result of three years' research work for the degree of Doctor of Philosophy in the Department of Civil Engineering and Building Science, University of Edinburgh.

Papers published by the author during that period are included in an appendix of this thesis and are as follows:


It is declared that this thesis has been composed by the author himself and the works and results reported were carried out solely by him under the supervision of Dr. Rodney Royles, unless otherwise stated.

Edinburgh, July 1985

John M. Llambias
ABSTRACT

An investigation was undertaken of the response of an underwater enclosure of optimum form to a variety of loadings to which it could be subjected during its construction, tow-out, installation and operational stages.

The use of the finite element method for the linear static stress analysis under both axisymmetric and non-axisymmetric loads, linear and non-linear buckling analyses and free vibration analysis of a small prototype was examined. Several different finite elements were employed and for each type of analysis a particular finite element was recommended for use in design.

These recommendations were substantiated by a series of experimental investigations on the small prototype.

In addition to this, a comparison of the finite element method with the membrane theory for thin shells was made for the linear static stress analysis of the structure under axisymmetric loading to determine whether the classical approach could be of any use in the initial stages of design.

Finally, based on the findings and recommendations from the work done on the prototype, an underwater oil storage tank was designed and its responses to some of the important loadings it could be subjected to was examined.
To my parents
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NOTATION

A list of symbols used in this thesis is given below:

A = cross sectional area

Am = amplitude for a particular value of m

a = maximum wave particle acceleration

Cd = drag force coefficient

Cm = inertia coefficient

Cp = pressure coefficient

Cp(θ) = pressure coefficient as function of θ

C = wave celerity in still water

D = width of structure

d = depth of water

d_m = mean diameter of structure

E = Young's modulus of elasticity

F_i = inertia force

F_d = drag force

F(θ) = load as a function of θ

G = dynamic gust factor

H = wave height

H_c = wave height in a current

k = constant

L = total meridional length of shell

L = wave length (chapter 7)

M = mass

M_{am} = added mass

M_{eff} = effective mass
m = meridional wave

N_\phi = stress resultant in meridional direction

N_\theta = stress resultant in circumferential direction

P' = single vertical load at the apex

P(\theta) = load as a function of \theta

P(\theta,Z) = load as a function of \theta and Z

Q = magnification factor

R_e = Reynold's number

r_0 = radius of parallel circle

r_1 = meridional radius of curvature

r_2 = circumferential radius of curvature

S = meridional distance

s = meridional coordinate

T = wave period

t = shell wall thickness

t = time (Chapter 7)

U = displacement in global X direction (LUSAS)

U_c = current velocity (Chapter 7)

U_x = displacement in global X direction (PAFEC)

U_y = displacement in global Y direction (PAFEC)

U_z = displacement in global Z direction (PAFEC)

U_{xm} = displacement in global X direction for a particular value of m (PAFEC)

U_{ym} = displacement in global Y direction for a particular value of m (PAFEC)

U_{zm} = displacement in global Z direction for a particular value of m (PAFEC)

\bar{U}(Z) = mean wind speed at a height Z

u = meridional displacement (Mistry)

u_{max} = max. horizontal water particle velocity
\( \dot{u}_{\text{max}} \) = max. horizontal water particle acceleration

\( \bar{u} \) = mean velocity of flow

\( V_d \) = volume of liquid displaced

\( V \) = displacement in global Y direction (LUSAS)

\( V_s \) = design wind speed

\( v \) = circumferential displacement (Mistry)

\( W \) = displacement in global Z direction (LUSAS)

\( w \) = normal displacement (Mistry)

\( x, y, z \) = rectangular coordinates

\( Z' \) = radial load intensity (Chapter 3)

\( Z \) = vertical height (Chapter 7)

\( \alpha \) = power law exponent of the mean wind speed profile

\( \beta \) = rotation about nodal ring (Mistry)

\( \beta' \) = \( 2\pi/L \) (Chapter 7)

\( \phi \) = angle of inclination of the meridian

\( \phi_z \) = rotation about global Z axis (PAFEC)

\( \Omega \) = frequency ratio

\( \omega \) = natural frequency

\( \nu \) = Poisson's ratio (Chapter 2)

\( \nu \) = kinematic viscosity

\( \rho \) = mass density

\( \gamma \) = specific weight

\( \theta \) = circumferential coordinate

\( \theta_1, \theta_2 \) = loop rotations

\( \xi, \eta \) = curvilinear coordinates

\( \sigma_\phi \) = meridional stress

\( \sigma_\theta \) = circumferential stress

\( \zeta \) = damping ratio
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CHAPTER ONE

GENERAL INTRODUCTION
1.1 Introduction

The underwater world has been a source of food and wealth to many developing civilisations over the centuries. The earliest records of underwater activities date back to 4500BC in Mesopotamia and in ancient history, great writers such as Homer and Aristotle referred to divers hunting for food and going in search of pearls, sponges and sunken treasures.

This exploration of the submarine environment and the exploitation of its resources has continued ever since and over the years some progress has been made in underwater technology in order to support it.

However, up to twenty years ago, this progress had been slow and had been mainly in response to military needs, civil engineering works or salvage operations. But in the last two decades, with the intensified search for and discovery of offshore oil and gas deposits in response to the world’s ever increasing energy demands, subsea technology has developed dramatically.

More recently, the need to explore the seabed and exploit marine resources in deeper waters remote from land has led to the examination of underwater structures either fixed to the seabed or floating just above it.

1.2 Underwater structures

The concept of one atmosphere dry enclosures eliminates most of the problems associated with deepwater production and exploration and the extreme environmental conditions experienced at the air/sea interface in remote offshore sites. Hence it offers an attractive alternative to the conventional platforms operating from above the surface.

Typical seabed complexes for deepwater production have already been proposed. These would consist of several underwater structures or ‘modules’ each performing a different function such as accommodating the personnel, housing the production equipment and storing the product.
However, although detailed feasibility studies have been undertaken, it is unlikely that these complexes would be in operation before the end of the century but the use of underwater structures solely as storage vessels has more immediate applications.

In the oil and gas industry, offshore storage has received much attention in the past few years. Some storage vessels have already been built and one has been particularly useful as a temporary storage facility in a field where production had commenced before the pipeline had been laid.

Similar underwater storage tanks could be employed in remote sites where pipeline export may be uneconomical and the use of tankers for the export of oil or gas was essential. In these cases underwater storage vessels near the production sites for the temporary storage of their products would be very beneficial, especially during extreme environmental conditions when the tankers were unable to load up.

Nearer land the storage of liquid gases, such as LNG and LPG, in surface and submerged floating tanks has been proposed in an effort to overcome the hazards associated with the storage of such volatile liquids on land near centres of population and at the same time keeping them away from sight.

Outside the oil industry, the use of underwater structures as storage vessels also has great attractions. In places like Gibraltar, for example, where there is a shortage of fresh water and a shortage of land in which to build storage reservoirs, the use of underwater vessels for the storage of fresh water in the surrounding sea as part of the water supply system could be advantageous. Especially if it were combined with some form of desalination plant.

Also in such places, similar underwater storage vessels could be employed as tanks to hold flow in excess of that which could be treated in a sewage or waste water treatment plant during a storm. When conditions abated and demand on the plant dropped, the stored liquid could be passed back for treatment. In the absence of such storm tanks, the only option is to pass the excess untreated liquid straight into the nearest open
water – adding to the pollution of the seas and oceans of the world.

With regards to undersea living, the use of one atmosphere underwater enclosures as habitats has received much interest since the first Man in the Sea and Conshelf projects in 1962 and many experimental habitats have been built to date. These experimental enclosures have served mainly as research centres and laboratories and many have played a leading role in the study of the sea and its environment.

More so in the future, as work is carried out in progressively deeper waters, subsea habitats may be essential to accommodate the personnel involved in the task as surface controlled operations either by divers or manned submersibles may be impossible.

Also it should be noted at this point that although the use of underwater habitats to relieve the housing problem in a world with an ever increasing population is still too much in the future, the idea of cities in the sea with underwater farms, refuges for divers working on the seabed and underwater museums has been considered.

1.3 Structural forms for underwater structures

Various different structural forms have been examined in connection with these underwater structures. These structural forms have ranged, in most cases, from spheres and cylinders to combinations of both but other geometric shapes such as toroids and ellipsoids have been considered.

All these structural forms are feasible but the choice of a geometric shape usually results from a compromise between structural efficiency, internal functional layout and cost.

For great depths, the sphere is the most favoured shape because of its capability to resist the high hydrostatic pressure with the least membrane stresses. But the inefficient usage of space in spherical structures has led to the consideration of cylindrical structures with hemispherical ends or cylindrical type structures formed by the intersection of several spheres.
On the other hand a compromise in terms of space usage can be provided also by a prolate spheroid which also has the advantage of the efficient use of the material.

With regards to costs, the toroidal structures are the most expensive to construct and the cylindrical structures are the least expensive \(^4\). However support and maintenance costs and personnel costs, in the case of manned stations, are very much higher than the construction costs \(^4\) and whereas in shallow waters cylindrical structures still offer the most economical solution, as the depth of water increases the overall costs of spherical and cylindrical structures become very similar \(^4\).

In shallower waters of depths less than 200m, the most efficient form in terms of stressing and enclosing a maximum volume for a minimum weight is a spheroidal type structure with the profile of a shell of revolution of constant or uniform strength \(^17,18\).

1.4 The drop shaped shell

The shell of revolution of constant or uniform strength, see Fig. 1.1, is one in which under a given applied hydrostatic pressure the membrane forces at all points are equal. It has been shown that it has the same shape as a drop of liquid resting on a flat surface, where the applied pressure is the internal hydrostatic pressure and the surface tension forces containing the pressure are equal at all points \(^19,20,21,22\).

This shape is very dependent on the applied hydrostatic pressure and the design stress of the shell. For a given design head (pressure head at the apex) and strength parameter (design stress x thickness) there is only one shape that would ensure uniform stressing provided the thickness is constant throughout.

The actual profile of the shell can be derived from the solution of the differential equations of the drop-shaped shell \(^19,20,21,22\). But these equations, which are obtained from the membrane theory, are rather complex and do not lend themselves to any closed form or analytical
solution.

Various methods of solution have been suggested by previous workers, some based on graphical methods $^{19,20}$ and others on numerical methods using the explicit Euler approach $^{21,22}$ and a survey and discussion of these methods of solution has been presented by Sofoluwe $^{18}$. From a practical point of view, the graphical methods were considered slow and inaccurate and the numerical methods unreliable $^{18}$. Consequently this led to the examination of several other numerical methods of solution and the explicit improved or modified Euler method was recommended for use in design $^{18}$.

Based on this method of solution, a shape prediction program was developed $^{17,18}$ which could generate a whole range of shapes corresponding to various design heads for a range of design stresses and thicknesses as an aid to designers.

This shape prediction program was therefore employed in this work and it is interesting to note that for high hydrostatic pressures, such as those encountered in depths greater than 200m, the shape of the shell tends towards that of a sphere. This emphasises the fact that for deepwater use, the sphere is the most structurally efficient shape.

However, in shallower waters of depth less than 200m, the drop shaped shell offers an attractive alternative and in this work the suitability of shells of revolution of constant strength for use as underwater enclosures in shallow and intermediate depths is examined.

In addition it is encouraging to note that nature has provided the common sea urchin with a test whose shape conforms with the shell of revolution of uniform strength $^{17}$, supporting the suitability of this structural form for underwater applications. These sea urchins belong to the phylum Echinodermata and hence the generic term for this type of structure — The Echinodome $^{17}$.

Above ground oil storage drop shaped tanks have already been designed and built by the Chicago Bridge and Iron Company $^{23}$ among others. The first one was constructed and tested by the company in 1928 and since
then some development has taken place in Holland $^{24}$ and France $^{25}$. These structures were designed to withstand tensile stresses resulting from the internal pressure due to the contained liquid but an unintentional test subjected one of those tanks to an equivalent external pressure of 9 times its design head without disastrous effects $^{23}$. Thus indicating the suitability of the drop shaped shell under external pressure and hence supporting its use in underwater applications.

However, prior to the use of the Echinodome as an underwater enclosure a detailed investigation, both experimental and theoretical, of its behaviour in the submarine environment is essential. There are many different types of load to which it could be subjected during its life, not only during its operational stage but during the construction, tow out, and installation stages as well and previous work has only examined the behaviour of the Echinodome under axisymmetric hydrostatic pressure $^{17,18}$.

The work reported in this thesis therefore aims to further the understanding of the behaviour of the Echinodome under some of the most important loadings to be expected and perhaps contribute towards the realisation of the underwater drop-shaped enclosure.
1.5 **Objectives of thesis**

In order to further the understanding of the behaviour of the Echinodome under various types of load, the objectives of this thesis are as follows.

1. To examine the suitability of several finite elements and finite element programs for the following types of analysis on the Echinodome:
   (i) Static stress analysis under both axisymmetric and symmetric loads;
   (ii) Buckling analysis; and
   (iii) Free vibration analysis.

2. To recommend a particular finite element for use in design for each of the above types of analysis.

3. To confirm the validity of the results of the finite element analysis experimentally.

4. To examine whether the membrane theory or other classical approach could be used for the above types of analysis in the initial stages of design to give a quick and reliable solution.

5. To propose a procedure for the design of an underwater oil storage drop-shaped tank.

6. To assess the different types of load likely to be expected during various stages in the life of an oil storage drop-shaped tank in the North Sea and examine its behaviour under some of those loads using the recommended finite elements.
Fig. 1.1 The drop-shaped shell
CHAPTER TWO

FINITE ELEMENTS

FOR

UNDERWATER SHELL STRUCTURES
2.1 Introduction

In the previous chapter, the need for underwater enclosures was highlighted and an optimum shape was proposed. However before this form of structure can become a reality a thorough investigation of the behaviour of such structures in the submarine environment needs to be undertaken in order to ensure its safety during its serviceable life.

This chapter discusses the theoretical methods available for the analysis of thin shells and in particular the finite element method. Three finite element programs, each utilising different finite elements, are used to analyse the behaviour of the drop-shaped shell under axisymmetric hydrostatic loads. The different finite elements are compared and discussed and one is recommended for use in design. The results obtained from this numerical analysis are subsequently compared with those obtained from the membrane theory.

The use of the recommended finite element for analysing the drop shaped shell under symmetric but non-axisymmetric loads, such as current drag and wind loading, is then outlined.

2.2 Types and methods of shell analysis

Analysis forms an important part of structural design and is necessary in order to ensure the integrity of a complete structure. It consists, basically, of the determination of stress and displacement distributions under environmental and other forms of load, both static and dynamic, to which the structure may be subjected, but it also embraces many other areas which affect the performance of shell structures. These areas include structural stability, natural frequencies and mode shapes, dynamic response, plasticity, creep, stress concentration and fatigue, to name but a few.

The methods of structural analysis can be classified into two groups, analytical and numerical methods.
Analytical methods for the analysis of thin shell structures were developed over a century ago, with Lamé and Clapeyron \(^{27}\) establishing the membrane theory in 1826 and Aron \(^{28}\) introducing the bending theory in 1874. These so-called classical continuum methods were advanced by Love\(^{29}\) who, in 1888, developed the first general shell theory. These analytical methods produce differential equations which are subsequently solved by employing classical techniques and using arbitrary constants to satisfy the boundary conditions. However, available analytical solutions are limited in scope and cannot deal with complex structures nor with many other aspects of practical design such as cut-outs, irregular stiffening and arbitrary load conditions. Even for simple problems, the differential equations produced may sometimes have no closed form or analytical solution and approximate solutions need to be employed.

As a result, numerical methods were developed for use in structural analysis and there are two types of such methods. The first type is based on a mathematical approximation of the differential equations formed from the equations of elasticity, and its solution by direct numerical integration or by using the finite difference method. Again, this type of numerical analysis involves the differential equations and is therefore restricted to simple problems because of the difficulties in obtaining differential equations for complex structures.

The second type of numerical methods is the matrix method, in which a structure is idealised into an assemblage of discrete structural elements connected at the nodes and suitably orientated to approximate the overall geometric shape of the shell. Using this type of analysis, arbitrary shapes and load conditions can easily be accommodated. The complete structural problem is developed in matrix algebra and is well suited for solution by computer.

Amongst the different matrix methods, one method has emerged from the aeronautical industry providing a powerful, reliable and efficient approach to the analysis of thin shell structures. This approach, namely, the finite element method is the one employed in this research.
2.3 The finite element method

The application of the finite element method to structural continua problems originated in the 1950s for use in the analysis of complex aircraft frames and bodies. In the 1960s the aerospace industry, with a need to accurately predict the behaviour of space vehicles, advanced this method considerably. Since then much progress has been made in response to the need for the detailed analysis of complex structural systems. Nowadays, the method has not only proved valuable in aeronautical and structural engineering but has been used successfully for the solution of problems in other fields of engineering including rock and soil mechanics, thermodynamics, fluid mechanics and biomechanics.

Essentially, the finite element analysis of a continuum such as a thin shell consists of three steps:

(i) Structural idealisation;
(ii) Evaluation of the stiffness of the elements; and
(iii) Structural analysis of the assemblage of discrete elements.

In the first step, the structure is divided into a finite number of discrete elements connected only at the nodes. The material properties of these elements were identical to those of the original structure whether linear elastic, non-linear elastic or elasto-plastic. The second step is the most critical phase and involves the evaluation of the stiffness of the individual elements. This is done in matrix form after assuming an interpolation function, which represents the displacement of any point within the element.

The third step, the structural analysis, begins with the formation of the overall stiffness matrix. This is achieved by the superposition of the individual element stiffness matrices whilst satisfying the equilibrium and compatibility conditions and the force-deflection relationships at each node. The rest of the analysis is common with the displacement matrix method and involves the solution of the force-deflection relationship.
It should be noted that the force (flexibility) method which is in effect the reverse of the displacement (stiffness) approach could also be used in a finite element analysis. However, for general purpose finite element programs, the stiffness method is preferred since it is easier to write, requires a minimum of input data and is entirely automatic, enabling a user with little knowledge of structural mechanics to analyse structures efficiently.

For a more detailed explanation of the finite element method of analysis, the reader is referred to the numerous texts and articles published on the subject.

Various different types of finite elements have been developed for the analysis of shell structures, ranging from flat plate elements to curved ring elements. For the analysis of thin shell structures, such as the drop-shaped shell, the faceted plate elements have not proved to be very successful and will not be considered in this research. On the other hand, ring elements have produced favourable results and such a type of element will be employed in this chapter. A third type of element, the semi-loof element, which has received considerable attention in the past few years and is one of the most complex isoparametric elements available is also used.

A number of general and special purpose finite element programs and packages have been developed for stress analysis. In this work two general purpose packages, i.e. PAFEC and LUSAS are employed together with a finite element program for the analysis of shells developed by Mistry. The displacement method is used in all three programs for the structural analysis step and a simplified flow diagram for the linear static stress analysis of a shell structure using any of these programs is given in appendix 1.1.

The next three sections give a general outline of each of these programs and describe the elements used for the analysis of the drop-shaped shell under hydrostatic pressure.
2.3.1 The Mistry program

The Mistry program is a finite element computer program, developed in Liverpool, for the analysis of thin axisymmetric shell structures and is capable of performing:

(i) vibration analysis;
(ii) linear buckling analysis;
(iii) nonlinear elastic-plastic stress analysis using given load history;
(iv) nonlinear buckling analysis; and
(v) linear stress analysis.

The shell structure is first idealised into a number of axisymmetric segments which could be conical, cylindrical spherical, toroidal or general axisymmetric segments. A combination of any of these segments is also possible. Each segment is then divided into a number of finite elements.

The type of finite element used is a ring element with ring nodes at the top and bottom and with either straight sides or curved with constant curvature. Each element has eight degrees of freedom, four at each node and they refer to the middle surface of the element. These degrees of freedom are meridional displacement (u), circumferential displacement (v), normal displacement (w) and rotation about the nodal ring (\( \theta \)). see Fig. 2.1.

The global X axis of the shell elements lies along the axis of symmetry of the shell and the Z axis is perpendicular to it and radially outwards. In this case, the Y axis is redundant since it is an axisymmetric problem. The positional coordinates of the elements are the meridional length, s, and the circumferential coordinate, \( \theta \).

The assumed interpolation functions for the element are linear functions of s for u and v and a cubic function of s for w. The element stiffness matrix is formulated using the principle of stationary total potential energy and the element mass matrix is derived from the kinetic energy equation for the shell element.
2.3.2 Program for Automatic Finite Element Calculations (PAFEC)

PAFEC is a general purpose package developed in Nottingham and based entirely on the finite element method of analysis. The version of PAFEC employed in this work was PAFEC 75, which was designed to facilitate the input of data. This version exists at various 'levels' according to the range of facilities offered and in this chapter, level 3.4, which can perform the following types of analysis, was employed:

(i) linear static stress analysis;
(ii) nonlinear static stress analysis;
(iii) dynamic and vibration analysis; and
(iv) thermal analysis.

There are about 75 different finite elements available in this level, ranging from simple beam elements to 20 noded orthotropic and isoparametric brick type elements, together with special elements for heat transfer problems.

The two types of finite element used in this work were (i) the three noded thin shell of revolution element (42130) and (ii) the eight noded isoparametric element for axisymmetric Fourier applications (36610).

The thin shell of revolution element has three nodes as shown in Fig. 2.2, and each node has four global degrees of freedom which are \( U_x \) in the direction of the global X axis, \( U_y \) in the direction of the global Y axis, \( U_z \) in the direction of the global Z axis and \( \phi_z \) which is the rotation about the global Z axis. The sides of the element can be either straight or curved with constant curvature.

The global X axis lies along the axis of symmetry of the shell and the global Y axis is perpendicular to it, see Fig. 2.2.

The interpolation functions are functions of \( s \), the meridional length and are polynomials of the second order for \( U_x \) and \( U_y \) and a polynomial of the fifth order for \( U_z \). The stiffness matrix is formed using the same variational principle as in the Mistry program and the mass matrix is also formed from the equation for the kinetic energy of the shell element.
The second type of element used was the eight noded isoparametric element for axisymmetric Fourier applications, which has 24 degrees of freedom, three at each node. These degrees of freedom are: \( U_x \) in the direction of the global X axis; \( U_y \) in the direction of the global Y axis and \( U_z \) in the direction of the global Z axis. The element is input as a two-dimensional curvilinear quadrilateral in the global XY plane and the program rotates it through 360° about the global X axis, see Fig. 2.3.

The analysis of this type of element, i.e. isoparametric element, starts off with the transformation of the curvilinear elements into very simple shapes in the \( \xi-\eta \) domain, where \( \xi \) and \( \eta \) are the curvilinear coordinates.

The interpolation functions are eight term cubic polynomials containing the two variables \( \xi \) and \( \eta \) and is of the second order. The stiffness and mass matrices are formed for the whole ring element using the same method as for the previous two elements but in the \( \xi-\eta \) domain.

### 2.3.3 London University Stress Analysis System (LUSAS)

LUSAS was the second general purpose finite element package used for the analysis of the drop-shaped shell under hydrostatic pressure. The package was developed by Finite Element Analysis Ltd, (FEAL)\(^{34}\) and the version of LUSAS employed was 83/1A. This version can perform the following types of analysis:

(i) linear static stress analysis;
(ii) nonlinear static stress analysis; and
(iii) thermal analysis.

The finite element library of version 83/1A consists of 62 elements including a series of semi-loof elements. The elements used in this work were the 4 noded axisymmetric solid element (QAX4) and a combination of the 6 noded triangular semi-loof element (TSL6) and the 8 noded quadrilateral semi-loof element (QSL8).

The axisymmetric solid element used is one of a family of isoparametric elements. It has eight global degrees of freedom, two at each node, and
these are $U$ in the direction of the global $X$ axis and $V$ in the direction of the global $Y$ axis. In this case, the global $Y$ axis lies along the axis of symmetry of the shell and the $X$ axis is perpendicular to it, see Fig. 2.4.

The formation of the element stiffness matrix and the element mass matrix is done in a similar way to the isoparametric element in PAFEC in the $\xi-\eta$ domain except that only a one radian section of the ring element is considered.

The second type of element used from LUSAS was the semi-loof element, both triangular and quadrilateral. These elements are doubly curved and their thicknesses can vary within them. The triangular element has 24 degrees of freedom, 3 at each corner node and 5 at the midside nodes, whilst the quadrilateral element has 32 degrees of freedom, 3 at each corner node and 5 at each midside node, see Fig. 2.5. These degrees of freedom are $U$ in the direction of the global $X$ axis, $V$ in the direction of the global $Y$ axis and $W$ in the direction of the global $Z$ axis. The rotational degrees of freedom, $\theta_1$ and $\theta_2$ refer to the loot rotations about the edge of the element at the loot points. The loot points are located at $1/\sqrt{3}$ of the distance from a midside node to a corner node.

As with the previous isoparametric elements, the curvilinear elements are transformed into simple shapes in the $\xi-\eta$ domain and the element stiffness and mass matrices are obtained for these simple elemental shapes.
2.4 Analysis of the drop shaped shell under hydrostatic pressure

The behaviour of the drop shaped shell under hydrostatic pressure has been investigated, both experimentally and numerically, in previous work \(^1\),\(^2\),\(^3\). However, the numerical work was limited to the use of the particular type of ring element employed in the Mistry program. It was therefore necessary to analyse the shell using other types of finite elements before being able to recommend a particular type of element for use in design.

With this view in mind, a test shell was analysed using the elements described above under two different hydrostatic heads, one of which was the design head.

2.4.1 The test shell

The shell analysed was of the same geometrical and material characteristics as a fibreglass prototype which was employed in previous work \(^1\),\(^2\),\(^3\) and which will be used in the experimental sections of this thesis. The prototype was constructed in two halves with randomly layered chopped strand mat fabric with a glass fraction of 0.26. Both halves were bonded together using a general purpose araldite adhesive and then mounted symmetrically over a rectangular tufnol base, see Fig. 2.6.

It was designed for a head of water of 1.525m and a design stress of 0.46MN/m\(^2\). The average thickness of the shell was 3.8mm with a standard deviation of 0.506mm (a table showing the variation in thickness over the shell is given in appendix 3). The meridional profile of the shell was determined from the numerical integration of the differential equations for the shell. It had a maximum diameter of 450mm and a height of 380mm. The material properties of the shell were determined from material control tests and were as follows:

- Modulus of elasticity \((E)\) = $0.88 \times 10^4$ MN/m\(^2\)
- Poisson’s ratio \((\nu)\) = 0.36
- Mass density \((\rho)\) = 1100 kg/m\(^3\)
- Ultimate tensile strength = 54.2 MN/m\(^2\)
2.4.2 Idealisation of the shell

The numerical integration of the differential equations of the shell was carried out using a shape prediction program \textsuperscript{17,18}. This program generated a set of coordinates for the centreline of the shell wall which were subsequently employed in the idealisation of the shell.

The three noded thin shell of revolution element (42130) in PAFEC and the ring element in the Mistry program only required the coordinates of the centreline of the shell wall and so the coordinates from the shape prediction program could be used directly. However, for the isoparametric elements in PAFEC (36610) and LUSAS (QAX4) the coordinates of the inner and outer surfaces of the shell wall were both required. These coordinates were obtained from a subroutine, PAFCOORD, which was added to the shape prediction program. A flow chart of the modified shape prediction program is given in appendix 1.2. It should be noted that this modified program produces a complete data file for input into PAFEC.

For the ring elements mentioned above, the shell was divided into a mesh of 65 elements in each case. The same number and size of elements were used so that a comparison of the results obtained when using the different elements could be made.

The idealisation of the shell using the semi-loof elements was somewhat different. Only one quarter of the shell was modelled and the appropriate boundary conditions were applied at the edges. In this case, 45 elements were employed, 15 down the meridian and the coordinates of the centreline of the shell wall were utilised. However, some approximations were required to determine the coordinates of the element's midside nodes.

In this investigation, the shell was fixed at the base and free at the apex and the external hydrostatic pressure was applied at the nodes, acting normal to the surface of the shell. The analysis was carried out for a pressure head at the apex of 1.525m (the design head) and 25m.

Typical data files for each of the elements are given in appendices 2.1 to 2.5.
2.4.3 Numerical results

The results obtained from this numerical analysis are given in tables 2.1 to 2.4. These tables show the meridional variations in the meridional and circumferential stresses on the inner and outer surfaces of the shell. From these results, it is possible to make a comparison of the different finite elements employed to idealise the shell.

A more detailed analysis giving the stress resultants and displacements for each element was performed previously.

Meridional stress on outer surface
(MN/m²)

<table>
<thead>
<tr>
<th>Meridional distance</th>
<th>Mistry</th>
<th>PAFEC</th>
<th>LUSAS</th>
</tr>
</thead>
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<td></td>
<td>36610</td>
<td>42130</td>
<td>QAX4</td>
</tr>
<tr>
<td></td>
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Table 2.1(a)

Meridional stress on inner surface
(MN/m²)

<table>
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<th>Meridional distance</th>
<th>Mistry</th>
<th>PAFEC</th>
<th>LUSAS</th>
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Table 2.1(b)

Meridional stresses at the design head
### Circumferential stress on outer surface (MN/m²)

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<th>LUSAS</th>
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</table>

**Table 2.2(a)**

### Circumferential stress on inner surface (MN/m²)

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<th>PAFEC</th>
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<td>-0.25</td>
</tr>
</tbody>
</table>

**Table 2.2(b)**

Circumferential stress at the design head
### Table 2.3(a)

**Meridional stress on outer surface**

<table>
<thead>
<tr>
<th>Meridional distance</th>
<th>Mistry</th>
<th>PAFEC</th>
<th>LUSAS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>36610</td>
<td>42130</td>
<td>QAX4</td>
</tr>
<tr>
<td>0.1L</td>
<td>-8.95</td>
<td>-9.22</td>
<td>-9.13</td>
</tr>
<tr>
<td>0.2L</td>
<td>-7.26</td>
<td>-7.64</td>
<td>-7.43</td>
</tr>
<tr>
<td>0.3L</td>
<td>-7.25</td>
<td>-7.52</td>
<td>-7.51</td>
</tr>
<tr>
<td>0.4L</td>
<td>-6.93</td>
<td>-7.23</td>
<td>-7.10</td>
</tr>
<tr>
<td>0.5L</td>
<td>-6.78</td>
<td>-7.11</td>
<td>-6.91</td>
</tr>
<tr>
<td>0.6L</td>
<td>-7.26</td>
<td>-7.46</td>
<td>-7.42</td>
</tr>
<tr>
<td>0.7L</td>
<td>-7.44</td>
<td>-7.71</td>
<td>-7.64</td>
</tr>
<tr>
<td>0.8L</td>
<td>-8.93</td>
<td>-9.16</td>
<td>-9.31</td>
</tr>
<tr>
<td>0.9L</td>
<td>-3.79</td>
<td>-3.31</td>
<td>-4.25</td>
</tr>
<tr>
<td>1.0L</td>
<td>-54.13</td>
<td>-63.71</td>
<td>-54.57</td>
</tr>
</tbody>
</table>

### Table 2.3(b)

**Meridional stress on inner surface**

<table>
<thead>
<tr>
<th>Meridional distance</th>
<th>Mistry</th>
<th>PAFEC</th>
<th>LUSAS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>36610</td>
<td>42130</td>
<td>QAX4</td>
</tr>
<tr>
<td>0.1L</td>
<td>-8.58</td>
<td>-8.79</td>
<td>-8.41</td>
</tr>
<tr>
<td>0.2L</td>
<td>-7.62</td>
<td>-7.80</td>
<td>-7.46</td>
</tr>
<tr>
<td>0.3L</td>
<td>-7.03</td>
<td>-7.49</td>
<td>-6.78</td>
</tr>
<tr>
<td>0.4L</td>
<td>-6.97</td>
<td>-7.16</td>
<td>-6.80</td>
</tr>
<tr>
<td>0.5L</td>
<td>-6.88</td>
<td>-7.08</td>
<td>-6.75</td>
</tr>
<tr>
<td>0.6L</td>
<td>-7.17</td>
<td>-7.43</td>
<td>-7.02</td>
</tr>
<tr>
<td>0.7L</td>
<td>-7.45</td>
<td>-7.68</td>
<td>-7.26</td>
</tr>
<tr>
<td>0.8L</td>
<td>-7.26</td>
<td>-7.16</td>
<td>-6.89</td>
</tr>
<tr>
<td>0.9L</td>
<td>-18.11</td>
<td>-17.43</td>
<td>-17.74</td>
</tr>
<tr>
<td>1.0L</td>
<td>22.96</td>
<td>39.95</td>
<td>23.30</td>
</tr>
</tbody>
</table>

Meridional stresses at a hydrostatic head = 25 metres
Circumferential stress on outer surface
(MN/m²)

<table>
<thead>
<tr>
<th>Meridional distance</th>
<th>Mistry</th>
<th>PAFEC</th>
<th>LUSAS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>36610</td>
<td>42130</td>
<td>QAX4</td>
</tr>
<tr>
<td></td>
<td>QSL8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1L</td>
<td>-8.81</td>
<td>-8.87</td>
<td>-8.83</td>
</tr>
<tr>
<td>0.2L</td>
<td>-7.33</td>
<td>-7.48</td>
<td>-7.36</td>
</tr>
<tr>
<td>0.3L</td>
<td>-6.95</td>
<td>-7.03</td>
<td>-7.01</td>
</tr>
<tr>
<td>0.4L</td>
<td>-6.45</td>
<td>-6.51</td>
<td>-6.48</td>
</tr>
<tr>
<td>0.5L</td>
<td>-6.03</td>
<td>-6.10</td>
<td>-6.04</td>
</tr>
<tr>
<td>0.6L</td>
<td>-5.92</td>
<td>-5.95</td>
<td>-5.95</td>
</tr>
<tr>
<td>0.7L</td>
<td>-5.42</td>
<td>-5.50</td>
<td>-5.47</td>
</tr>
<tr>
<td>0.8L</td>
<td>-4.63</td>
<td>-4.79</td>
<td>-4.75</td>
</tr>
<tr>
<td>0.9L</td>
<td>-3.98</td>
<td>3.95</td>
<td>3.85</td>
</tr>
<tr>
<td>1.0L</td>
<td>-21.81</td>
<td>-24.75</td>
<td>-22.02</td>
</tr>
</tbody>
</table>

Table 2.4(a)

Circumferential stress on inner surface
(MN/m²)

<table>
<thead>
<tr>
<th>Meridional distance</th>
<th>Mistry</th>
<th>PAFEC</th>
<th>LUSAS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>36610</td>
<td>42130</td>
<td>QAX4</td>
</tr>
<tr>
<td></td>
<td>QSL8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1L</td>
<td>-8.59</td>
<td>-8.81</td>
<td>-8.57</td>
</tr>
<tr>
<td>0.2L</td>
<td>-7.38</td>
<td>-7.59</td>
<td>-7.36</td>
</tr>
<tr>
<td>0.3L</td>
<td>-6.84</td>
<td>-7.15</td>
<td>-6.79</td>
</tr>
<tr>
<td>0.4L</td>
<td>-6.44</td>
<td>-6.65</td>
<td>-6.42</td>
</tr>
<tr>
<td>0.5L</td>
<td>-6.06</td>
<td>-6.24</td>
<td>-6.05</td>
</tr>
<tr>
<td>0.6L</td>
<td>-5.89</td>
<td>-6.03</td>
<td>-5.86</td>
</tr>
<tr>
<td>0.7L</td>
<td>-5.44</td>
<td>-5.61</td>
<td>-5.39</td>
</tr>
<tr>
<td>0.8L</td>
<td>-4.14</td>
<td>-4.11</td>
<td>-4.00</td>
</tr>
<tr>
<td>0.9L</td>
<td>-0.85</td>
<td>-0.84</td>
<td>-0.61</td>
</tr>
<tr>
<td>1.0L</td>
<td>9.60</td>
<td>15.30</td>
<td>9.75</td>
</tr>
</tbody>
</table>

Table 2.4(b)

Circumferential stresses at a hydrostatic head = 25 metres
2.4.4 Discussion of results

The results of this numerical investigation show that there is good agreement in the meridional variation in stresses along 4/5th of the shell, both at the design head and at 25m. However, there are some discrepancies along the bottom 1/5th of the shell.

In this 'critical' zone, the four types of ring element show a similar trend but it varies with that produced when using the semi-loof elements. This inconsistency can be attributed to the fact that the four types of ring elements have straight sides whilst the semi-loof elements have curved sides and are doubly curved.

The effect of using elements with straight sides is not as apparent over the top 4/5th of the shell because the radius of curvature of the shell's profile around that region is large and the change in curvature is small. As a result the elements are nearly coplanar. In the 'critical' zone, this is not the case. The radius of curvature of the shell's meridional profile is smaller and the change in curvature is large, resulting in big discontinuities of slope between adjacent elements. This would produce bending moments which would not be present in the actual shell.

It is interesting to note that along the whole profile of the shell, the scatter band is greater for the meridional stresses than for the circumferential stresses. This is to be expected since the discontinuities exist only in the meridional plane.

Nevertheless, the outcome of this investigation is encouraging. At the design head, the stresses are all close to the design stress (0.46MN/m²). The difference in the values of the stresses were due to the variation in thickness down the meridian and is to be expected.

Any of these five finite elements could be utilised successfully for the linear static stress analysis of the drop-shaped shell. However, in the design of such shells many factors would have to be considered before choosing a particular finite element and program to employ in the structural
analysis. A comparison of these five finite elements and their corresponding programs, leading to the recommendation of a particular element and program to be used in design is given in the next section.

2.4.5 Comparison of the different finite elements

The above investigations were carried out at the Edinburgh Regional Computing Centre (E.R.C.C.). An engineer in a design office may only have limited access to such a centre and the use of such centres for commercial purposes could be very expensive.

It is therefore necessary to choose a finite element which would be cost effective in terms of computer run time and storage and at the same time be reliable and accurate. A finite element program which could be implemented also on a minicomputer or even a microcomputer would be advantageous.

In order to make such a choice, the following factors need to be considered:

(i) the storage space required;
(ii) the preparation of input data;
(iii) the accuracy of the solution;
(iv) the interpretation of the results;
(v) the time used for analysis; and
(vi) other capabilities available.

Of the five elements used, the ring element in the Mistry program was the most cost-effective in terms of computer run time and storage. A cpu time of 6.4s for the linear static stress analysis of the Echinodome compares very favourably with the cpu times required for the other elements.

As for the storage space required, 30 Kbytes would be sufficient for both the input and output files for the ring element in the Mistry program whereas, at least 67 Kbytes would be necessary for the OAX4 element in LUSAS. The other elements requiring over 100 Kbytes.
A detailed summary of the CPU times and file sizes required for the linear static stress analysis of the Echinodome is given in Table 2.5.

<table>
<thead>
<tr>
<th>Element</th>
<th>Input file (Kbytes)</th>
<th>Output file (Kbytes)</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mistry</td>
<td>2.1</td>
<td>24.0</td>
<td>6.4</td>
</tr>
<tr>
<td>PAFEC 36610</td>
<td>3.5</td>
<td>224.7</td>
<td>102.3</td>
</tr>
<tr>
<td>PAFEC 42130</td>
<td>3.0</td>
<td>93.6</td>
<td>42.2</td>
</tr>
<tr>
<td>LUSAS QAX4</td>
<td>5.1</td>
<td>61.4</td>
<td>74.9</td>
</tr>
<tr>
<td>LUSAS TSL6 &amp; QSL8</td>
<td>4.2</td>
<td>207.0</td>
<td>139.9</td>
</tr>
</tbody>
</table>

Table 2.5 Computer run time and file sizes for the different finite elements used

* based on the linear static stress analysis of the Echinodome at the design head (no graphics)

The preparation of the input data, however, is simpler for the general purpose packages. Both LUSAS and PAFEC require the data file in a free format field and in a modular form making it quick to prepare and easy to understand. In this case element 42130 in PAFEC necessitated the smallest and easiest input file to prepare. The datafile for the Mistry program was slightly smaller but it was required in a specified format and was therefore not as straightforward to prepare. However, it is important to note that the format statements in the Mistry program could be changed so that the input data could be presented in a free format field as in the general purpose packages if required. The data file for the semi-loof elements, on the other hand, was the hardest to prepare since the coordinates of the nodes could not be obtained directly from the shape prediction program.
and some approximations were required to determine them.

The interpretation of the output results is straightforward in all five cases. Enough information is provided to enable the engineer to make a quick and accurate assessment of the stresses and displacements in the structure. The stresses for elements 36610 in PAFEC and QAX4 in LUSAS are given in the global axes direction at the nodes and the stresses for the semi-loof elements are given in the direction of the local element axes at the Gauss points - the points within the element where the element stiffness is evaluated. Although easy to understand it is more useful to the engineer designing shell structures to obtain the circumferential and meridional stresses straight away from the computer output. This is done by the thin shell of revolution elements in PAFEC (42130) and Mistry, which give the circumferential, meridional and inplane shear stresses on the outer and inner surfaces of the shell. In addition to this, the latter element gives a listing of the stress resultants, equivalent stresses and moments for each element.

Once again the Mistry program provides the smallest output file and the most useful information. It should be noted that the two general purpose packages provide information regarding the stiffness matrices, degrees of freedom and restraints which are not of much use to the designer except as a check.

With regards to the accuracy of the solution, the semi-loof element would produce the most accurate one as it is doubly curved and is the most complex element used. However errors could arise in the application of the boundary conditions since only a small section of the shell can be modelled at any one time for reasons of economy. The two elements in PAFEC and the ring element in the Mistry program produced similar results as can be seen in tables 2.1 to 2.4 but the isoparametric element in PAFEC (36610) with 24 degrees of freedom and the most complex interpolation function of the three would be expected to produce the most accurate solution. Yet it should be noted that this element has no rotational degrees of freedom whereas the other two have and hence the results obtained in the critical region of the shell, where bending is significant would not be as accurate as those produced by the ring element in the Mistry program and
Element QA4 in LUSAS, on the other hand, has only 2 degrees of freedom at each node and hence would be the less accurate element to use. This is clearly indicated in the results obtained from the numerical analysis.

The interpolation functions and number of degrees of freedom for each element are compared in appendix 4. This appendix also shows what other types of analysis the element is capable of performing.

In all, the Mistry finite element program seems to provide the most efficient solution for the linear static stress analysis of the drop-shaped shell. It is small enough to be implemented on most microcomputers and is the only program available at the E.R.C.C. capable of performing linear and non-linear buckling analyses. With a cpu time very much smaller than that required for a run using the semi-loof elements, the greater accuracy achieved by using that curved element is by far outweighed.

As a result, in the design section of this research the Mistry program was employed for the linear static stress analysis of the shell under axisymmetric hydrostatic loads.

However, the one main limitation with Mistry’s program is that only axisymmetric loads can be applied to the shell. For non-axisymmetric loads element 42130 in PAFEC, which seems to be the next best element when considering the factors listed above, is recommended. This element is almost identical to the element in the Mistry program, see appendix 4. Therefore the idealisation of the structure using these two elements would be very similar and some uniformity in modelling the behaviour of the shell would be present in the different analyses. Section 2.5 discusses the use of this element for the analysis of non-axisymmetric loads such as current drag, wind loading and point loading. Nonetheless it should be noted that modifications could easily be made to the Mistry program to deal with these non-axisymmetric loads.26
At this stage it is important to note that if the structure has openings or intersections which render the shell non-axisymmetric then none of the ring elements can be utilised to model the shell and doubly curved shell elements such as the semi-loof elements in LUSAS need to be employed.

2.4.6 Comparison of the finite element method with the membrane theory

The previous sections have shown that the finite element method could be used for the analysis of the drop-shaped shell. However, this method of analysis is expensive in terms of computer storage and run time and there is thus a need for a simpler and quicker method which could be used in the initial stages of design.

The membrane theory could provide a feasible alternative and an algorithm for dealing with the differential equations produced by this theory has already been applied to the drop-shaped shell 38.39.

This approach was followed in this section to analyse the test shell, described in section 2.4.1, under a hydrostatic head of 1.525m and 25m. The results obtained from this investigation have been tabulated alongside the results obtained from the Mistry program in tables 2.6(a) and 2.6(b) on page 32.

These tables show that there is some agreement in the stress resultants over 4/5th of the shell, but a region exists near the base of the shell where there are some discrepancies. These discrepancies are expected because in this 'critical' zone the effects of bending and shear are at a maximum and whilst the finite element analysis takes these effects into account, the membrane theory ignores all bending and shear effects completely.

Nevertheless, a cpu time of 0.33s for a linear static stress analysis of the drop-shaped shell makes it a very attractive proposition for the designer. In the initial stages, at least, the membrane theory could give an accurate indication of the magnitude of the stresses that might be expected over most of the shell.
**Table 2.6(a) Stress resultants at the design head**

<table>
<thead>
<tr>
<th>Meridional distance</th>
<th>Nφ Mistry prog. Membrane analysis</th>
<th>Nθ Mistry prog. Membrane analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1L</td>
<td>-1.75  -1.97</td>
<td>-1.75  -1.53</td>
</tr>
<tr>
<td>0.2L</td>
<td>-1.75  -1.87</td>
<td>-1.76  -1.63</td>
</tr>
<tr>
<td>0.3L</td>
<td>-1.75  -1.84</td>
<td>-1.77  -1.66</td>
</tr>
<tr>
<td>0.4L</td>
<td>-1.75  -1.82</td>
<td>-1.75  -1.67</td>
</tr>
<tr>
<td>0.5L</td>
<td>-1.75  -1.81</td>
<td>-1.75  -1.68</td>
</tr>
<tr>
<td>0.6L</td>
<td>-1.75  -1.81</td>
<td>-1.74  -1.68</td>
</tr>
<tr>
<td>0.7L</td>
<td>-1.75  -1.82</td>
<td>-1.74  -1.66</td>
</tr>
<tr>
<td>0.8L</td>
<td>-1.75  -1.86</td>
<td>-1.77  -1.59</td>
</tr>
<tr>
<td>0.9L</td>
<td>-1.74  -2.07</td>
<td>-1.66  -1.01</td>
</tr>
<tr>
<td>1.0L</td>
<td>-2.02  1.67</td>
<td>-0.81  34.67</td>
</tr>
</tbody>
</table>

**Table 2.6(b) Stress resultants at a hydrostatic head = 25 metres**

<table>
<thead>
<tr>
<th>Meridional distance</th>
<th>Nφ Mistry prog. Membrane analysis</th>
<th>Nθ Mistry prog. Membrane analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1L</td>
<td>-28.7   -32.2</td>
<td>-28.5   -24.9</td>
</tr>
<tr>
<td>0.2L</td>
<td>-28.4   -30.3</td>
<td>-28.1   -25.9</td>
</tr>
<tr>
<td>0.3L</td>
<td>-28.1   -29.4</td>
<td>-27.2   -25.4</td>
</tr>
<tr>
<td>0.4L</td>
<td>-27.9   -28.9</td>
<td>-25.8   -24.5</td>
</tr>
<tr>
<td>0.5L</td>
<td>-27.6   -28.5</td>
<td>-24.4   -23.2</td>
</tr>
<tr>
<td>0.6L</td>
<td>-27.5   -28.4</td>
<td>-22.5   -21.4</td>
</tr>
<tr>
<td>0.7L</td>
<td>-27.8   -28.9</td>
<td>-20.3   -18.7</td>
</tr>
<tr>
<td>0.8L</td>
<td>-29.3   -31.2</td>
<td>-15.9   -12.8</td>
</tr>
<tr>
<td>0.9L</td>
<td>-37.7   -42.7</td>
<td>5.4     21.1</td>
</tr>
<tr>
<td>1.0L</td>
<td>-53.6   153.1</td>
<td>-21.0   1858.7</td>
</tr>
</tbody>
</table>
2.5 Analysis of the drop shaped shell under non-axisymmetric loads

So far in this chapter, only axisymmetric loads have been applied to the drop shaped shell and the application of such loads on axisymmetric shell elements has been quite simple. However, an underwater structure could be subjected to a wide range of non-axisymmetric loads such as current loading, wave loading and point loading. The simulation of these types of loads acting on shell of revolution elements such as element 42130 in PAFEC, is not as simple and requires special attention.

2.5.1 Fourier series for the representation of non-axisymmetric loads

In order to apply non-axisymmetric loads on axisymmetric shell elements, it is first necessary to represent the loading as a series of Fourier components of the form:

\[ F(\theta) = \sum_{m=0}^{\infty} A_m \cos m\theta \]  

(2.1)

where \( F(\theta) \) is the load as a function of \( \theta \), \( \theta \) is the circumferential coordinate, \( A_m \) is the amplitude of the load and \( m \) is the harmonic number.

Each term of the resulting Fourier series is then applied to the structure separately with the appropriate harmonic number and boundary conditions.

Unfortunately, most of the loadings in the marine environment would require an infinite series and hence an infinite number of runs would be necessary in order to simulate the load correctly. In this work the Fourier series used to represent the loads were truncated after six terms. Examination of the 7th, 8th and 9th terms showed that the resulting stresses and displacements were decreasing rapidly and were negligible compared to the first few components.

Examples of the representation of loads as a Fourier series can be seen in appendices 5.1 to 5.3.
It should be noted that three dimensional semi-loof elements could be used for these load cases and avoid the Fourier approximations for the loads. However, in order to apply a non-axisymmetric load on the prototype, the whole shell might need to be modelled using the semi-loof elements and in terms of computer cpu times, six runs using the recommended finite element (i.e. element 42130) would still take less time than a single run with the semi-loof elements.

2.5.2 Interpretation of results

The output from element 42130 in PAFEC contains the nodal displacements and stresses for each harmonic number.

The total displacement of the structure can be found by superimposing the displacements obtained from each run as follows:

\[ U_x(x,y,\theta) = U_{x0}(x,y) + \sum_{m=0}^{\infty} U_{xm}(x,y,\theta) \cos m\theta \tag{2.2} \]

with similar terms for \( U_y \) and \( U_z \).

\( U_x(x,y,\theta) \) is the total displacement in the global X direction at the point \( x,y,\theta \); \( U_{x0} \) is the displacement in the global X direction for harmonic number \( m=0 \); and \( U_{xm} \) is the displacement in the global X direction for harmonic number \( m \).

From these equations, it is possible to determine the total displacement at any point on the shell surface.

The stresses at any point on the shell can be obtained in a similar manner by the superposition of the stresses obtained from each particular run.

This procedure for analysing the shell under non-axisymmetric loads using element 42130 in PAFEC was computerised by the development of a masterfile, PAFMAS. A flow diagram of which is shown in appendix 1.3.
An experimental investigation to verify the results obtained from such a procedure is described in the next chapter.

2.6 Summary and Conclusions

This chapter was concerned with the structural analysis of the Echinodome. A brief introduction to the different types and methods of structural analysis was presented and an outline of the method used in this work, namely the finite element method, was given.

A number of different finite elements were subsequently employed to model the behaviour of the drop-shaped shell under axisymmetric hydrostatic loads and some agreement was evident in the results.

On comparing the relative merits of each type of element, the Mistry program was recommended for use in the structural analysis section of design.

Then the membrane theory was compared with the finite element analysis and the results showed that it provided an efficient and reliable method which could be used in the initial stages of design, as long as the designer was aware of the limitations of the theory.

Finally, since the Mistry program cannot be used when the loading is non-axisymmetric a second element, element 42130 in PAFEC, was chosen for this purpose and a method for dealing with such types of loads was proposed.

It should be noted that this chapter considered only the linear static stress analysis of the shell. Two other types of analysis, namely buckling and vibration analysis, are dealt with in detail in chapters 4 and 5 respectively and a finite element is recommended for use in design in each case.
Fig. 2.1 The Mistry finite element
Fig. 2.2 Element 42130 in PAFEC

Fig. 2.3 Element 36610 in PAFEC
Fig. 2.4 Element QAX4 in LUSAS

Fig. 2.5 Semi-loof elements in LUSAS
Fig. 2.6 The prototype
CHAPTER THREE

EXAMINATION OF THE RESPONSE OF

THE DROP-SHAPED SHELL

TO CONCENTRATED LOADS
3.1 Introduction

A structure in the submarine environment would be subjected to many different types of loading other than axisymmetric hydrostatic pressure. Some of these loads could be non-axisymmetric, such as current drag, wave loading and concentrated loads due to nozzle connections or due to impact. It is important that the structure could withstand those non-axisymmetric loads safely and therefore a reliable method for predicting the behaviour of the Echinodome under such loads is required.

To this end, the response of the drop-shaped shell under concentrated loads is examined, both numerically and experimentally, in this chapter.

A linear static stress analysis of the Echinodome under both axisymmetric and non-axisymmetric point loading is first carried out using the finite element method. Then the resulting stress distribution induced by axisymmetric point loading is compared with an approximate solution obtained from the membrane theory.

Finally, an experimental investigation of the behaviour of the drop-shaped shell under concentrated loads is carried out in order to verify the results obtained from the numerical analysis and give support to the use of the finite element method in design.

3.2 Theoretical investigation

The previous chapter indicated that the finite element method of analysis is one of the most reliable and efficient methods available to analyse the behaviour of thin shell structures under general loading. For non-axisymmetric loads, in particular, the thin shell of revolution element in PAFEC, namely element 42130, was recommended and a technique for carrying out such an analysis was outlined.

In this theoretical investigation, that procedure was followed to examine the response of a prototype to both axisymmetric and non-axisymmetric point loading.
3.2.1 Finite element analysis

The test shell analysed in chapter two was discretised using element 42130 in PAFEC and the input file was prepared as detailed in section 2.4.2.

Two loading conditions were examined, see Figs 3.1(a) & 3.1(b), and for each case the load was simulated in a different manner.

3.2.1.1 Point load at the apex

This was the simpler loading condition of the two, as it was an axisymmetric problem. The load was applied directly at the apex (node 1) and acting downwards along the axis of symmetry of the shell, as shown in Fig. 3.1(a).

The linear static stress analysis followed the same form as that for hydrostatic pressure (section 2.4) and was carried out for loads from 50N to 300N in step of 50N, in order not to over-stress the shell.

3.2.1.2 Point load at the maximum diameter

In this case, the load was applied normal to the surface of the shell at its maximum diameter as shown in Fig. 3.1(b) and it was therefore a non-axisymmetric problem. Consequently, the point load was represented as a series of Fourier components of the form specified in section 2.5.1. The resulting Fourier series is shown in equation 3.1 below and for the interested reader, the numerical calculations involved in obtaining this equation are given in appendix 5.1.

\[ P(\theta) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{m=1,3}^{\infty} \cos m\theta \]  

(3.1)

Where \( \theta \) is the circumferential coordinate measured anticlockwise from the point of application of the load.
Following the recommendations made in the previous chapter, this infinite series required to simulate the point load was truncated after 6 terms and the following equation was employed in the finite element analysis:

\[
P(\theta) = \frac{1}{\pi} \left\{ \frac{1}{2} + \cos \theta + \cos 2\theta + \cos 3\theta + \cos 4\theta + \cos 5\theta \right\}
\] (3.2)

The effects of truncating the series shown in equation 3.1 has been studied by previous workers \(^{37}\) and Fig. A5.2 in appendix 5.1 shows the effect of stopping the series after 9 and 20 terms.

Again a linear static stress analysis was carried out for loads of 50N to 300N in increments of 50N and followed the procedure outlined in section 2.5 which was computerised by the masterfile PAFMAS (see appendix 1.3).

### 3.2.1.3 Numerical results

The results obtained from the stress analysis of the test shell under axisymmetric point loading are shown in Figs 3.2(a) & 3.2(b) whilst the results obtained under non-axisymmetric point loading are shown in Figs 3.3 to 3.9 at different values of \(\theta\), the circumferential coordinate. These results are for a point load of 300N in each case. For all the other loads the resulting stresses varied in a linear fashion, i.e. for a load of 150N, the stresses obtained were half those obtained under a load of 300N.

The corresponding deflected shape of the test shell under these two loading conditions are shown in Figs. 3.10 & 3.11, again under a point load of 300N.

The finite element analysis took a CPU time of 57.5s for the axisymmetric load case and a total of 344.8s for the six runs required for the non-axisymmetric load case.
The input file in each case was similar to that shown in appendix 2.2 and was 3.0 kbyte in length whilst the output file was 93.6 kbyte for each PAFEC run.

3.2.1.4 Discussion of numerical results

The results for the axisymmetric point loading confirm the fact that the optimum location for penetrating the shell is at its apex. Figs 3.2(a) & 3.2(b) show that the effects of the axisymmetric point load were very localised, i.e. within 10% of the total meridional length (L) from the apex, with a peak tensile circumferential stress at a distance of about 0.09L and a peak tensile meridional stress at a distance of about 0.07L from the apex.

Over the rest of the prototype, except very near the base, the circumferential and meridional stresses were very low and remained constant throughout; the tensile circumferential stresses being approximately equal in magnitude to the compressive meridional stresses. Also in that area, the outer surface stresses were of the same sign and magnitude as the corresponding inner surface stresses indicating a membrane type behaviour and further suggesting that a membrane analysis could be suitable for examining the behaviour of the shell under this particular load case, see section 3.2.2.

A second peak in the surface stresses is evident in the 'critical zone' near the base of the prototype but the stresses were lower than those near the apex.

The deformed shape of the prototype under this loading, see Fig. 3.10, also shows these localised effects in a region near the point of application of the load. The development of a dimple at the apex conforms with the meridional variation of surface stresses near the apex and verifies the localised bending indicated by the inner and outer surface stresses.

Very little deformation is apparent around the shell's maximum diameter, although a second peak in the normal deflection is evident in the 'critical
zone' near its base. The deflections involved were very much smaller than those near the apex as is obvious from Fig. 3.10 but nevertheless it is indicative of the localised bending indicated by the outer and inner surface stresses along the bottom tenth of the prototype.

The application of a point load at the shell's maximum diameter, on the other hand, had a much greater effect on the structure. There were three zones covering almost a third of its surface area in which the surface stresses were high and bending was significant.

The first of these zones covered the area near the apex, and extended down to about 0.05L from it, in which the circumferential stresses were in general three times bigger than the meridional ones. In this zone the outer and inner surface stresses were of approximately the same magnitude but of different signs. Considering only the meridional variation in these stresses, the outer surface stresses started off as compressive and then rapidly changed sign to give peak tensile stresses at a distance of 0.03L from the apex whereas the inner surface stresses started off as tensile and then changed to compressive to peak at 0.03L.

However this was only true on the section of the shell facing the load i.e.

\[-90^\circ < \theta < +90^\circ\]

where \(\theta\) is the circumferential coordinate, see Figs 3.3 to 3.5. At \(\theta = \pm 90^\circ\) the surface stresses were quite low in comparison and no peak was obvious, see Fig. 3.6. Thereafter on the other side of the prototype the reverse occurred, i.e. the outer surface stresses started off tensile and then changed to compressive to peak at 0.03L and vice versa, see Figs 3.7 to 3.9.

This was an indication of the presence of bending both in the meridional and circumferential planes.

A similar pattern was evident in the 'critical zone' except that the meridional stresses predominated in this area and that the outer surface stresses were tensile on the side of the prototype facing the load and compressive on the side facing away from the load - the reverse was the
case for the inner surface stresses. Also in this zone the maximum stresses on the structure were induced.

At this point it is interesting to note that, in these two zones, the outer surface stresses at $\theta = 0^\circ$ were approximately equal in magnitude to the corresponding inner surface stresses at $\theta = 180^\circ$ but of different signs and vice versa. Again this was indicative of a considerable circumferential variation in surface stresses and bending stresses as would be expected from the application of a non-axisymmetric point load.

The third zone, where bending was significant, encompassed the point of application of the load. However the direct and bending stresses in this region were not as high as in the other two zones and they reduced significantly a small distance away from the load indicating the presence of a small dimple.

Over the rest of the shell, the stresses were low reaching a minimum at $\theta = \pm 90^\circ$ where the stresses were negligible compared with the maximum stresses induced at $\theta = 0^\circ$. Nevertheless the inner and outer surface stresses were still of different signs indicating a departure from the membrane type behaviour exhibited by the prototype when subjected to an axial point load and therefore rendering a membrane analysis unsuitable for this particular load case.

These variations in surface stresses were substantiated by the deflected form of the prototype under this non-axisymmetric load, see Fig. 3.11. This figure shows that there was much deformation in the plane of application of the load, i.e. at $\theta = 0^\circ$ but very little deformation in a plane at right angles to it. This conforms with the circumferential variation in stresses shown in Figs 3.3 to 3.9. In some areas, especially near the base of the prototype, a considerable amount of rotation was evident thus accounting for the high bending stresses expected in the critical zone and at the apex.

Also shown clearly in Fig. 3.11(a) is the presence of a dimple around the point of application of the load which is consistent with the variation in surface stresses induced around that area.
In general this numerical investigation has shown that the application of a point load at the maximum diameter of the Echinodome is the more critical load case and although it is envisaged that any penetrations of the shell would be at the apex, the possibilities of impact loading of an accidental nature at the maximum diameter cannot be ignored if structural integrity of the complete design is to be ensured.

3.2.2 The membrane theory

An alternative and simpler approach to the analysis of the Echinodome under axisymmetric point loading could be provided by the membrane theory for thin shells. This method, which gave reasonable results when analysing the shell under hydrostatic pressure (see section 2.4.6), could be used in the initial stages of design when a quick and reliable solution is required.

In this section the method is outlined and a comparison of the results obtained from this classical approach is made with those obtained from the finite element method.

3.2.2.1 Point load at the apex

Fig. 3.12 shows the drop shaped shell subjected to a point load, \( P' \), acting down the axis of symmetry of the shell.

The two equilibrium equations \(^{22}\), based on the membrane theory, for a thin shell of revolution are:

\[
2\pi r_0 N_\phi \sin \phi + P' = 0
\]

and

\[
N_\phi/r_1 + N_\theta/r_2 = -Z'
\]
where \( N_{\phi} \) and \( N_{\theta} \) are the meridional and circumferential stress resultants; \( P' \) is a single vertical load at the apex; \( Z' \) is the radial load intensity; \( r_0 \) is the radius of the parallel circle; \( r_1 \) and \( r_2 \) are the meridional and circumferential radii of curvature and \( \phi \) is the angular coordinate.

Since the shell is only under a single point load at the apex, then the radial load intensity, \( Z' \), is zero and equation 3.4 becomes:

\[
\frac{N_{\phi}}{r_1} = -\frac{N_{\theta}}{r_2} \quad (3.5)
\]

From equation 3.3,

\[
N_{\phi} = -\frac{P'}{2\pi r_0 \sin \phi} \quad (3.6)
\]

and therefore from equations 3.5 and 3.6,

\[
\frac{P'}{2\pi r_0 r_1 \sin \phi} = \frac{N_{\theta}}{r_2} \quad (3.7)
\]

However from Fig. 3.12, \( r_2 \sin \phi = r_0 \), thus,

\[
N_{\theta} = \frac{P'}{2\pi r_1 \sin^2 \phi} \quad (3.8)
\]

Using equations 3.6 and 3.7, the stress resultants can be calculated at any point along the meridian so long as the appropriate \( r_0, r_1 \) and \( \phi \) are known. \( P' \) is the vertical load acting above the point in consideration and hence in the absence of any other vertical load, as is the case here, it would be constant down the meridian of the shell.

The values of \( r_0, r_1 \) and \( \phi \) can be obtained directly from the shape prediction program (see appendix 1.2) for any point on the surface of the test shell.
3.2.3 Comparison of the membrane theory with the finite element method

The above equations were used to analyse the shell under an axial point load of 300N. The resulting stress resultants were then converted into stresses using equations 3.9 and 3.10 below in order to facilitate the comparison with the finite element method.

\[ \sigma_\phi = \frac{N_\phi}{t} \]  

(3.9)

\[ \sigma_\theta = \frac{N_\theta}{t} \]  

(3.10)

where, \( \sigma_\phi \) and \( \sigma_\theta \) are the meridional and circumferential stresses on the middle surface of the shell and \( t \) is the thickness of the shell wall.

The results obtained are given in table 3.1 alongside the results obtained from the finite element analysis (F.E.A).

<table>
<thead>
<tr>
<th>Meridional distance</th>
<th>Circumferential stress</th>
<th>Meridional stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Membrane theory (MN/m²)</td>
<td>F.E.A. (MN/m²)</td>
</tr>
<tr>
<td>0.1L</td>
<td>0.95</td>
<td>1.08</td>
</tr>
<tr>
<td>0.2L</td>
<td>0.28</td>
<td>0.23</td>
</tr>
<tr>
<td>0.3L</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>0.4L</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>0.5L</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>0.6L</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>0.7L</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>0.8L</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>0.9L</td>
<td>0.50</td>
<td>0.48</td>
</tr>
<tr>
<td>1.0L</td>
<td>6.54</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

Table 3.1 Comparison of the membrane theory and the finite element method for a point load of 300N at the apex (middle surface stresses)
The table shows that there is very good agreement over most of the shell except near the base, where the membrane theory predicted much higher stresses than the finite element method. However, this could be expected since the membrane theory ignores all bending and shear effects which would be present in the critical region near the base. Therefore for a detailed analysis of the stresses in this critical zone the finite element method would be necessary.

Nonetheless, the results show that in the initial stages of design, the membrane theory could be useful in obtaining an accurate overall representation of the behaviour of the Echinodome under axisymmetric point loading, without having the burden of high computing costs.

It should be noted, though, that the analysis of the Echinodome under non-axisymmetric point loading using the membrane theory is not as straightforward as that described above. This is mainly due to the general form of the drop shaped shell and the fact that Fourier series approximations have to be included in the equilibrium equations to represent the point load, rendering the solution of the shell equations difficult and time consuming. Under such loading conditions, therefore, the finite element method offers the simplest and most efficient procedure for analysing the Echinodome.
3.3 Experimental investigation

The comparison made in the preceding section was fruitful since it showed that there was good agreement between the results obtained from the two theoretical approaches. However complete reliance would not be placed on a theoretical method until it had been verified by experimental work and to this end a series of tests to substantiate the use of the finite element method was undertaken.

The response of the G.R.P. prototype, see Fig. 2.6, was examined experimentally under two different loading conditions - point load at the apex and point load at the maximum diameter - as in the finite element analysis and the details of this investigation are presented in the following sections.

3.3.1 Loading arrangement

The prototype was first bolted via its tufnol base onto a slotted steel circular base plate which was subsequently secured to a rotating table positioned in a loading frame as shown in Fig. 3.13. The slotted base plate allowed the shell to be rotated through 90° in the horizontal plane whilst the rotating table, in turn, allowed the prototype to be rotated through 90° in the vertical plane so that non-axisymmetric loads could be applied at any point on the surface of the shell.

The load was applied via a stiff circular wooden strut, positioned normal to the shell's surface, by means of a balanced dead load lever system. A different length of strut had to be employed for each of the two load cases, as can be seen in Figs 3.14(a) & 3.14(b).
3.3.2 Strain Gauging

The surface strains induced on the shell were measured using electrical resistance foil strain gauge rosettes of the $45^\circ$ type (type GFRA 3–350) with the following characteristics:

- gauge resistance $= 350 \pm 1.0 \Omega$;
- gauge factor $= 2.15$ (average); and
- gauge length $= 3$mm.

High resistance foil strain gauges were used in an attempt to minimise the heating effects due to the poor conductivity of the G.R.P.

The rosettes were bonded onto the outer surface of the prototype at the intersection of three symmetrically arranged meridians (at $120^\circ$ spacing) and four parallel circles. Two of these parallel circles were inside the ‘critical zone’, described in section 2.4.4, and the other two were fairly evenly spaced out as shown in Fig. 3.15.

A further two strain gauge rosettes were bonded onto the inner surface of the shell at the fourth parallel circle, at similar locations to those rosettes on the outer surface at the same parallel circle.

Each rosette was orientated with one gauge along the meridian and the other two arranged in a clockwise sense so as to monitor both circumferential and meridional strains.

The outer surface strain gauge rosettes were mounted following the usual procedure for bonding strain gauges \textsuperscript{40,41} but the mounting of the inner surface strain gauges required a special technique which has been described in previous work \textsuperscript{26}.

The rosettes were bonded onto a prepared surface - clean and lightly abraded - using a general purpose strain gauge adhesive, namely cyanoacrylate. This bonding agent had a curing time of 1 min at room temperature and was compatible with both the foil strain gauge rosettes and the material of the prototype, i.e. glass reinforced plastic.
However, this adhesive did not have a very good resistance to humidity and some form of moisture proofing was recommended when employing it in strain gauging. As a result the strain gauges and terminal tags were covered with a thin layer of microcrystalline wax, which formed an excellent moisture and water resistant coating. This soft and flexible wax coating, which was brushed on lightly from the molten state, offered no mechanical protection and consequently a coating of silicone rubber (room temperature vulcanising) had to be applied over the wax for this protection.

The strain gauges were energised by a Farnel E30 bench power supply providing a steady 2.5V d.c.. The gauges were wired in a half bridge configuration with similar gauges mounted in corresponding positions of curvature on a dummy half shell for temperature compensation. This dummy shell was placed near the loading rig so as to be exposed to similar environmental conditions.

3.3.3 Displacement transducers

The displacements were measured using spring loaded potentiometric displacement transducers (Tokyo Sokki Kenkyujo type S30FLP100A). These rectilinear potentiometers produced an electrical signal directly proportional to the linear mechanical movement of its shaft, with a full scale output equal to the applied volts and a linearity of 0.25%. They had a full scale resistance of 2kΩ and a resolution of ± 1 micron was possible.

A constant voltage of 5V d.c. (1 Amp) was applied by the transducer power supply (Techni Measure type TPU-30) to the potentiometers, which all had a mechanical stroke of 100 mm. Hence an output voltage of 5V would correspond to a movement of the shaft (i.e. displacement) of 100 mm.

The displacements were monitored normal to the surface of the shell at the apex and at three other parallel circles, one of which was at the maximum diameter. The transducers were symmetrically arranged along these parallel circles. However the number of transducers used and their distribution was different for each of the two load cases.
Eight potentiometers were employed for the axisymmetric point load. Six of these were placed at the intersection of two meridians (at $120^\circ$ spacing) and the three parallel circles, one was placed at the apex and the remaining one was placed at the maximum diameter on a third meridian at $120^\circ$ to the other two meridians, see Fig. 3.16(a).

For the non-axisymmetric load case, ten transducers were employed. Six of these were positioned at the intersection of two meridians (at $180^\circ$ spacing) and the three parallel circles, one was placed at the apex and the remaining three were symmetrically arranged along half of the circumference at the maximum diameter (at $45^\circ$ spacing) so as to monitor the circumferential variation in the normal displacements, see Fig. 3.16(b).

In both cases, the potentiometric displacement transducers were held in position by means of a rigid dexion frame built around the prototype and secured to the loading frame, see Figs 3.14(a) & 3.14(b).

### 3.3.4 Instrumentation

The general arrangement of the instrumentation employed in this experimental investigation is presented in a block diagram form in Fig. 3.17.

The scanning and logging system was controlled by a 32 kbyte CBM microcomputer with a back up storage of 1 Mbyte provided by a dual floppy disk drive (CBM type 8050).

The signals from the strain gauges and the displacement transducers were all directed to a 32 channel purpose built scanner. This scanner was connected to the microprocessor by means of the parallel user port and accepted signals from any of the 32 channels when instructed to do so.

The voltages were measured using a Keithley programmable multimeter (model 192) which was in turn connected to the microprocessor via the IEEE 488 general purpose interface bus (GPIB). This digital multimeter had a resolution of $5^{1/2}$ digits which corresponded to a resolution of $1\mu$V when reading the voltages from the strain gauges (0.2V range) and a $100\mu$V.
resolution when reading the voltages from the displacement transducers (20V range).

The multimeter was programmed to output a single reading, with a line cycle integration period of 20ms, only when addressed by the microprocessor.

3.3.5 Computer program

A computer program, STRAIN, was developed for controlling this scanning system and a flow diagram of it is shown in appendix 1.4.

This program was divided into two main sections. The first section controlled the scanning and the second section processed the results.

In the first section, the program instructed the scanner to switch open a particular channel and addressed the multimeter to output a reading of the voltage in that channel. This voltage was then displayed on the screen of the microcomputer and listed on a line printer (CBM type 4020).

Using this system the datum readings could be stored in the computer's memory so that the net strains and deflections could be determined in subsequent runs.

The second part of the program calculated the principal stresses and strains, circumferential stress, meridional stress, Von Mises stress and deflections and thereafter listed them on the line printer.

All the results of the experiment were sent also to the disk drive for permanent storage.
3.3.6 **Experimental procedure**

Once the prototype was firmly in position on the loading rig, a few scans were made with no load applied to ensure that all the equipment was connected and functioning correctly and that the datum readings were stable and consistent.

The load was applied via the dead load balanced lever arm system using a ratio of 5:1 for the point load at the apex and 2.25:1 for the point load at the maximum diameter. The load was applied in increments of 50N from 50 to 300N. A maximum load of 300N was considered so as not to exceed the ultimate strength of the G.R.P..

The loading rate was kept constant throughout the series of tests - an increment every 2 mins - and the scanning was done by the computer program, STRAIN, on the load increasing part of the loading cycle immediately after applying the load. Each scan took approximately 7.5s to read the 30 channels of the scanner when only strain gauges were monitored and 9.85s to read the 30 channels when 10 displacement transducers and 6 strain gauge rosettes were monitored together.

A total of 10 runs were carried out for the point load at the apex and a total of 20 runs for the point load at the maximum diameter, each time monitoring a different combination of strain gauges and displacement transducers in the 30 channels. Fewer runs were required for the point load at the apex as it was possible to average all the displacements and strains on a particular parallel circle due to the axisymmetric behaviour of the shell. Whereas for the point load at the maximum diameter, the non-axisymmetric nature of the shell's response to this load, meant that this was not possible.

A period of at least 5 mins was allowed between runs to allow for creep recovery.
### 3.3.7 Experimental results

The results obtained from this series of tests are tabulated in table 3.2 for the point load at the apex and table 3.3 for the point load at the maximum diameter.

<table>
<thead>
<tr>
<th>Parallel circle</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.028</td>
<td>-0.066</td>
<td>-0.107</td>
<td>-0.152</td>
<td>-0.198</td>
<td>-0.242</td>
</tr>
<tr>
<td>2</td>
<td>-0.013</td>
<td>-0.023</td>
<td>-0.036</td>
<td>-0.047</td>
<td>-0.065</td>
<td>-0.072</td>
</tr>
<tr>
<td>3</td>
<td>-0.016</td>
<td>-0.033</td>
<td>-0.058</td>
<td>-0.074</td>
<td>-0.091</td>
<td>-0.117</td>
</tr>
<tr>
<td>4(outer)</td>
<td>0.050</td>
<td>0.139</td>
<td>0.201</td>
<td>0.284</td>
<td>0.353</td>
<td>0.425</td>
</tr>
<tr>
<td>4(inner)</td>
<td>-0.088</td>
<td>-0.224</td>
<td>-0.363</td>
<td>-0.482</td>
<td>-0.594</td>
<td>-0.736</td>
</tr>
</tbody>
</table>

Table 3.2(a) Average meridional stresses for point load at the apex (MN/m²)

<table>
<thead>
<tr>
<th>Parallel circle</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.026</td>
<td>0.043</td>
<td>0.065</td>
<td>0.089</td>
<td>0.112</td>
<td>0.136</td>
</tr>
<tr>
<td>2</td>
<td>0.018</td>
<td>0.029</td>
<td>0.032</td>
<td>0.049</td>
<td>0.056</td>
<td>0.075</td>
</tr>
<tr>
<td>3</td>
<td>0.017</td>
<td>0.033</td>
<td>0.043</td>
<td>0.066</td>
<td>0.085</td>
<td>0.101</td>
</tr>
<tr>
<td>4(outer)</td>
<td>0.096</td>
<td>0.175</td>
<td>0.280</td>
<td>0.397</td>
<td>0.506</td>
<td>0.619</td>
</tr>
<tr>
<td>4(inner)</td>
<td>0.034</td>
<td>0.062</td>
<td>0.100</td>
<td>0.156</td>
<td>0.203</td>
<td>0.252</td>
</tr>
</tbody>
</table>

Table 3.2(b) Average circumferential stresses for point load at the apex (MN/m²)

<table>
<thead>
<tr>
<th>Parallel circle</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>apex</td>
<td>0.065</td>
<td>0.288</td>
<td>0.466</td>
<td>0.632</td>
<td>0.806</td>
<td>0.923</td>
</tr>
<tr>
<td>a</td>
<td>0.009</td>
<td>0.020</td>
<td>0.021</td>
<td>0.026</td>
<td>0.027</td>
<td>0.029</td>
</tr>
<tr>
<td>b</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.003</td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td>c</td>
<td>-0.005</td>
<td>-0.013</td>
<td>-0.028</td>
<td>-0.043</td>
<td>-0.058</td>
<td>-0.073</td>
</tr>
</tbody>
</table>

Table 3.2(c) Average normal displacements under a point load at the apex (mm)
<table>
<thead>
<tr>
<th>Meridian Parallel Load</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.006</td>
<td>0.011</td>
<td>0.017</td>
<td>0.021</td>
<td>0.023</td>
<td>0.038</td>
</tr>
<tr>
<td>2</td>
<td>-0.006</td>
<td>-0.014</td>
<td>-0.018</td>
<td>-0.023</td>
<td>-0.030</td>
<td>-0.034</td>
</tr>
<tr>
<td>3</td>
<td>0.008</td>
<td>0.014</td>
<td>0.015</td>
<td>0.021</td>
<td>0.029</td>
<td>0.041</td>
</tr>
<tr>
<td>4(outer)</td>
<td>0.012</td>
<td>0.025</td>
<td>0.025</td>
<td>0.032</td>
<td>0.040</td>
<td>0.023</td>
</tr>
<tr>
<td>4(inner)</td>
<td>0.038</td>
<td>0.096</td>
<td>0.152</td>
<td>0.206</td>
<td>0.270</td>
<td>0.318</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Meridian Parallel Load</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.005</td>
<td>0.008</td>
<td>0.009</td>
<td>0.013</td>
<td>0.016</td>
<td>0.017</td>
</tr>
<tr>
<td>3</td>
<td>0.025</td>
<td>0.056</td>
<td>0.078</td>
<td>0.142</td>
<td>0.199</td>
<td>0.225</td>
</tr>
<tr>
<td>4(outer)</td>
<td>-0.176</td>
<td>-0.354</td>
<td>-0.614</td>
<td>-0.773</td>
<td>-0.998</td>
<td>-1.163</td>
</tr>
<tr>
<td>4(inner)</td>
<td>0.083</td>
<td>0.191</td>
<td>0.299</td>
<td>0.425</td>
<td>0.596</td>
<td>0.651</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Meridian Parallel Load</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-0.011</td>
<td>-0.019</td>
<td>-0.027</td>
<td>-0.034</td>
<td>-0.039</td>
<td>-0.038</td>
</tr>
<tr>
<td>4(outer)</td>
<td>-0.176</td>
<td>-0.354</td>
<td>-0.614</td>
<td>-0.773</td>
<td>-0.998</td>
<td>-1.163</td>
</tr>
<tr>
<td>4(inner)</td>
<td>0.083</td>
<td>0.191</td>
<td>0.299</td>
<td>0.425</td>
<td>0.596</td>
<td>0.651</td>
</tr>
</tbody>
</table>

Table 3.3(a) Average meridional stresses for point load at the maximum diameter (MN/m²)

<table>
<thead>
<tr>
<th>Meridian Parallel circle</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.008</td>
<td>-0.016</td>
<td>-0.026</td>
<td>-0.032</td>
<td>-0.033</td>
<td>-0.041</td>
</tr>
<tr>
<td>2</td>
<td>0.009</td>
<td>0.019</td>
<td>0.019</td>
<td>0.027</td>
<td>0.030</td>
<td>0.038</td>
</tr>
<tr>
<td>3</td>
<td>-0.012</td>
<td>-0.020</td>
<td>-0.026</td>
<td>-0.035</td>
<td>-0.041</td>
<td>-0.048</td>
</tr>
<tr>
<td>4(outer)</td>
<td>-0.064</td>
<td>-0.143</td>
<td>-0.219</td>
<td>-0.307</td>
<td>-0.387</td>
<td>-0.469</td>
</tr>
<tr>
<td>4(inner)</td>
<td>-0.073</td>
<td>-0.148</td>
<td>-0.222</td>
<td>-0.300</td>
<td>-0.364</td>
<td>-0.450</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Meridian Parallel circle</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.007</td>
<td>0.014</td>
<td>0.011</td>
<td>0.027</td>
<td>0.035</td>
<td>0.040</td>
</tr>
<tr>
<td>3</td>
<td>-0.029</td>
<td>-0.043</td>
<td>-0.066</td>
<td>-0.079</td>
<td>-0.101</td>
<td>-0.119</td>
</tr>
<tr>
<td>4(outer)</td>
<td>-0.125</td>
<td>-0.246</td>
<td>-0.382</td>
<td>-0.533</td>
<td>-0.692</td>
<td>-0.838</td>
</tr>
<tr>
<td>4(inner)</td>
<td>-0.065</td>
<td>-0.129</td>
<td>-0.198</td>
<td>-0.282</td>
<td>-0.344</td>
<td>-0.402</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Meridian Parallel circle</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-0.005</td>
<td>-0.006</td>
<td>-0.011</td>
<td>-0.015</td>
<td>-0.019</td>
<td>-0.021</td>
</tr>
<tr>
<td>2</td>
<td>-0.029</td>
<td>-0.065</td>
<td>-0.084</td>
<td>-0.144</td>
<td>-0.172</td>
<td>-0.195</td>
</tr>
</tbody>
</table>

Table 3.3(b) Average circumferential stresses for point load at the maximum diameter (MN/m²)
Table 3.3(c) Average normal displacements under a point load at the maximum diameter (mm)

<table>
<thead>
<tr>
<th>Parallel circle</th>
<th>Meridians</th>
<th>50N</th>
<th>100N</th>
<th>150N</th>
<th>200N</th>
<th>250N</th>
<th>300N</th>
</tr>
</thead>
<tbody>
<tr>
<td>apex</td>
<td>I &amp; V</td>
<td>-0.026</td>
<td>-0.061</td>
<td>-0.094</td>
<td>-0.121</td>
<td>-0.159</td>
<td>-0.190</td>
</tr>
<tr>
<td>a</td>
<td>III</td>
<td>0.164</td>
<td>0.336</td>
<td>0.502</td>
<td>0.669</td>
<td>0.833</td>
<td>1.000</td>
</tr>
<tr>
<td>b</td>
<td>II &amp; IV</td>
<td>0.075</td>
<td>0.158</td>
<td>0.239</td>
<td>0.322</td>
<td>0.401</td>
<td>0.479</td>
</tr>
<tr>
<td>b</td>
<td>I &amp; V</td>
<td>0.007</td>
<td>0.013</td>
<td>0.022</td>
<td>0.032</td>
<td>0.042</td>
<td>0.051</td>
</tr>
<tr>
<td>c</td>
<td>I &amp; V</td>
<td>0.007</td>
<td>0.017</td>
<td>0.026</td>
<td>0.034</td>
<td>0.045</td>
<td>0.052</td>
</tr>
</tbody>
</table>

3.4 Discussion of experimental results and comparison with the numerical results

The results obtained from this experimental investigation show that the responses to both the axisymmetric and non-axisymmetric point loading had been approximately linear in nature at most measurement locations.

For the axisymmetric point loading the maximum deflection was obtained at the apex of the shell, just under the point load, as would be expected, and of the four parallel circles monitored the maximum surface stresses were found along the fourth parallel circle. The strain gauges in this parallel circle, which lies in the ‘critical zone’ near the base of the prototype, also indicated the presence of localised bending, especially in the meridional plane as the difference between the outer and inner surface meridional stresses was the greatest in this zone.

Very little deformation was evident around the middle zone of the prototype and this was substantiated by the stresses obtained from the rosettes in parallel circles 2 and 3.

A comparison of these experimental results with those obtained from the finite element method for an axial point load of 300N is given in Figs
3.2 & 3.10. Good agreement is demonstrated for the measurement locations except for the experimental deflection at the apex which was twice the numerical one.

However the presence of a seam bonding the two halves of the prototype together and the localised thinning expected near this seam could account for this difference. Fig. 3.18 shows the results of a finite element analysis to determine the effects of local thinning around the apex on the deflection of the prototype under an axial point load. Only the deflection at the apex was affected significantly and this figure suggests that a reduction in thickness of about 38% would be enough to give a numerical deflection at this point comparable to the experimental one obtained.

For the non-axisymmetric point loading the maximum deflection was obtained also at a location just under the point load and the minimum deformation was recorded along the two meridians at 90° to the meridional plane of the load.

The maximum stresses measured were compressive and were obtained from the strain gauges at the intersection of the fourth parallel circle and meridian 2 (θ = 90°) on the outer surface. Also at this point, which lies in the 'critical zone', the results for the outer and inner surface stresses demonstrated the greatest maximum bending stresses.

In agreement with the measured deflections, the minimum stresses were obtained along the meridian at 90° to the direction of the load hence indicating the type of non-axisymmetric, but symmetric, response that could be expected under this loading.

A comparison of these results with the numerical ones is shown on Figs 3.4, 3.6, 3.8 & 3.11. This comparison is for the maximum point load of 300N and good agreement is demonstrated for most of the measurement location although some discrepancies are evident in the critical zone where the experimental results, especially those from the inner surface strain gauge rosettes, were generally higher than the numerical ones.
However it is worth noting that the finite element analysis used an approximation in the form of a Fourier series to define the point load and that since this infinite series was curtailed after 6 terms for reasons of economy some differences could be expected in such a comparison.

An important factor which also might have influenced these results was the circumferential variation in the thickness of the prototype's wall, see appendix 3. This variation could not be accounted for in the finite element idealisation since axisymmetric ring elements were employed and an average value for the thickness had to be used for each ring finite element - the meridional variation in shell wall thickness was however taken into account. For the axial loading this was of no significance since, because of the axisymmetric nature of its response, the results obtained from all the rosettes on each parallel circle could be averaged out whereas for the point load at the maximum diameter this could not be done because of the non-axisymmetric response of the shell. Consequently local thickening in some areas could well explain the low experimental results obtained from some of the strain gauges.

A common source of error that could have accounted for the discrepancies obtained under both loading conditions could be that due to gauge misalignment or mislocation. Although great care was taken to ensure that the gauges were bonded exactly in position and correctly orientated, it was possible that the inner surface strain gauges, which required a special technique for their installation \[26\], were slightly misaligned and thus could account for the difference in results obtained for that location.

Similarly the doubly curved form of the prototype made the positioning of the displacement transducers very difficult and a slight deviation from the required normal position might have been present.

Yet despite all these error sources which are inherent in strain gauging and model testing \[41\], the results of this experimental investigation and the comparison with the numerical results were encouraging. Although more measurement locations for both strains and displacements would be
required for a detailed investigation of the Echinodome under a point load, the results obtained from the few locations measured were enough to confirm the suitability of the finite element method for that purpose.

Also, since the point load at the maximum diameter is a form of non-axisymmetric load then this investigation substantiates the use of the finite element method, and in particular, element 42130 in PAFEC, for the examination of the behaviour of the drop-shaped shell under non-axisymmetric loads of a general nature such as wind loading and current drag.
3.5 Summary and Conclusions

The response of the Echinodome to axisymmetric and non-axisymmetric concentrated loads was examined in this chapter both theoretically and experimentally.

In the theoretical investigation the finite element method was employed to analyse the behaviour of the drop-shaped shell under a point load normal to the shell's surface at the apex and at its maximum diameter.

For the axisymmetric point loading, the numerical results were compared with the analytical results obtained from the membrane theory for thin shells and very good agreement was evident indicating the suitability of the membrane theory for that purpose.

Then a series of tests was carried out on the prototype, employing both strain gauges and displacement transducers, so as to examine its response to concentrated loads experimentally with the view of confirming the numerical results.

A comparison of the experimental and numerical results showed that there was good agreement between the results for both axisymmetric and non-axisymmetric point loads. Thus, gave some confidence to the use of the thin three noded shell of revolution finite element (42130) in PAFEC in the analysis the Echinodome under concentrated loads and under non-axisymmetric loads in general.
Fig. 3.1 Form of loading on the Echinodome

(a) axisymmetric point load

(b) non-axisymmetric point load

dimensions in mm
Fig. 3.2 Stress distribution under an axial point load (load = 300N)
Fig. 3.3(a) Stress distribution under a point load at the maximum diameter ($\theta = 0^\circ$) - Circumferential stresses
Fig. 3.3(b) Stress distribution under a point load at the maximum diameter ($\theta = 0^\circ$)
- Meridional stresses
Fig. 3.4(a) Stress distribution under a point load at the maximum diameter (θ = 30°) - Circumferential stresses
Fig. 3.4(b) Stress distribution under a point load at the maximum diameter ($\theta = 30^\circ$)
- Meridional stresses
Fig. 3.5(a) Stress distribution under a point load at the maximum diameter (θ = 60°)
- Circumferential stresses
Fig. 3.5(b) Stress distribution under a point load at the maximum diameter ($\theta = 60^\circ$) - Meridional stresses
Fig. 3.6(a) Stress distribution under a point load at the maximum diameter (θ = 90°)
- Circumferential stresses

LOAD = 300 N
Fig. 3.6(b) Stress distribution under a point load at the maximum diameter (θ = 90°)
- Meridional stresses
Fig. 3.7(a) Stress distribution under a point load at the maximum diameter (θ = 120°)
- Circumferential stresses
Fig. 3.7(b) Stress distribution under a point load at the maximum diameter (θ = 120°)
- Meridional stresses
Fig. 3.8(a) Stress distribution under a point load at the maximum diameter (θ = 150°) - Circumferential stresses
Fig. 3.8(b) Stress distribution under a point load at the maximum diameter ($\theta = 150^\circ$) - Meridional stresses
Fig. 3.9(a) Stress distribution under a point load at the maximum diameter (θ = 180°) - Circumferential stresses
Fig. 3.9(b) Stress distribution under a point load at the maximum diameter (θ = 180°)
- Meridional stresses
Fig. 3.10  Deflected shape under an axial point load of 300N
(displacements relative to the original shape X 100)
Fig. 3.11 Deflected shape under a point load at the maximum diameter of 300N (displacements relative to the original shape X 100)
Fig. 3.12 Point load on the Echinodome - membrane theory
Fig. 3.13 Connection of base to loading rig
Fig. 3.15 Arrangement of strain gauge rosettes
Fig. 3.16 Arrangement of displacement transducers

(a) point load at the apex

(b) point load at the maximum diameter

Dimensions in mm
Fig. 3.17 Block diagram of instrumentation
Fig. 3.18 - Effect of local thinning at the apex on the deflection
CHAPTER FOUR

EXAMINATION OF THE BUCKLING BEHAVIOUR

OF THE DROP-SHAPED SHELL

UNDER HYDROSTATIC PRESSURE
4.1 Introduction

Buckling under hydrostatic pressure is a major problem area facing an engineer designing underwater shell structures. Structural instability in such shells can occur well before the material is highly stressed and could result in a catastrophic implosive failure. Therefore it is very important that the buckling behaviour of any particular form of structure is investigated carefully before employing it in underwater applications.

In this chapter, the buckling behaviour of the drop-shaped shell is examined both theoretically and experimentally.

A linear and non-linear elastic buckling analysis of a small prototype is carried out, using the finite element method, to determine the critical buckling pressures and their corresponding mode shapes. These are subsequently compared with an approximate solution obtained using the classical shell theory.

The numerical results are then verified by experimental work based on the Southwell technique for predicting the critical buckling loads of structures.

4.2 Theoretical methods for shell buckling analysis

Considerable work has been done, over the past fifty years, to develop a theoretical model for predicting the buckling loads and mode shapes of shells of revolution \(^{22,44,45,46}\).

Most of this work was based on Love's general shell theory \(^{29}\) and was restricted to simple problems such as spheres and cylinders under external pressure, as the differential equations for these shapes lent themselves to exact analytical solutions. The results obtained from this classical approach were well above observed experimental values and further work was necessary to lower the predicted buckling pressures by taking into account initial imperfections \(^{46,47}\).
It was not until the development of numerical methods for structural analysis and the advances in the electronic computer, that the buckling behaviour of general shells of revolution under arbitrary loading conditions was first studied. These numerical methods, especially the finite element method, predicted loads which agreed very closely with experimental results for both axisymmetric and non-axisymmetric buckling modes. Consequently, the finite element method was chosen for the examination of the buckling behaviour of the Echinodome.

Of the three finite element programs readily available, see appendix 4, only the Mistry program was capable of performing a buckling analysis and consequently it was used in this work.

4.2.1 The Mistry program

The Mistry finite element program can be used to calculate the minimum critical load corresponding to either snap-through collapse or non-axisymmetric bifurcation buckling of axisymmetric shell structures.

The shell is first discretised into finite elements, as described previously in section 2.4.2, and the axisymmetric pre-buckling behaviour of the structure is then modelled, using either linear or geometric non-linear static stress analysis. A typical pressure vs deflection curve showing the non-linear axisymmetric pre-buckling path, OAB, is given in Fig. 4.1.

As this pre-buckling fundamental path is generated, the determinant of the stability matrix is calculated and the stiffness matrix is examined at each pressure step. A condition of non-axisymmetric bifurcation buckling would exist if the determinant of the stability matrix were zero whilst axisymmetric snap-through collapse would occur if the stiffness matrix were non-positive definite.

A detailed explanation of the theory behind this procedure for calculating the critical load of axisymmetric shells can be found in numerous papers and a flow diagram of the subroutines carrying out the buckling analysis in the Mistry program is given in appendix 1.5.
4.2.1.1 Finite element analysis

The small prototype used in the previous investigations was discretised into a number of ring elements as described in section 2.4.2.

Elastic buckling analyses of the prototype under hydrostatic pressure were performed using both linear and non-linear pre-buckling stress resultants and the results are shown in table 4.1 below.

<table>
<thead>
<tr>
<th>Type of buckling</th>
<th>Buckling pressure head $(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>linear</td>
</tr>
<tr>
<td>Snap through $(n=0)$</td>
<td>103.74</td>
</tr>
<tr>
<td>Bifurcation $(n=1)$</td>
<td>40.26</td>
</tr>
</tbody>
</table>

Table 4.1 Theoretical buckling pressure heads $(n = \text{harmonic number})$

The corresponding buckling mode shapes are shown in Fig. 4.2.

The cpu time required for a buckling analysis using linear pre-buckling stress resultants was 22.5s whilst the cpu time required for a buckling analysis using non-linear pre-buckling stress resultants was 193.9s.

The input files for these problems were very similar to that shown in appendix 2.1 and were 2.1 Kbytes long. The size of a typical output file from the buckling analysis was 28 Kbytes.

It is also interesting to note that in this particular case, the ultimate strength of the shell material would be exceeded in the bottom tenth of the shell wall before bifurcation buckling occurred. A table showing the maximum pre-buckling Von Mises stresses for the non-linear buckling analysis is shown on the next page.
<table>
<thead>
<tr>
<th>Type of buckling</th>
<th>Max. Von Mises stress (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snap through</td>
<td>(n=0) 178.2</td>
</tr>
<tr>
<td>Bifurcation</td>
<td>(n=1) 78.7</td>
</tr>
</tbody>
</table>

Table 4.2 Maximum pre-buckling Von Mises stresses for the non-linear buckling analyses (ultimate strength of the material = 54.2MN/m², see section 2.4.1)

4.2.2 The classical approach

The cost in terms of computer run-time and storage is quite high for a buckling analysis based on the finite element method. As a result a classical approach to buckling was examined to determine whether it could be used in the initial stages of design to give an accurate and quick approximation of the critical buckling pressure of the Echinodome.

The differential equations for the buckling of the drop-shaped shell under hydrostatic pressure were developed from the general shell theory following the procedure described by Flügge. However, the equations produced did not have any exact analytical solution and could not be taken any further.

As an alternative, the equation for the buckling of a sphere was considered. An equivalent sphere with a diameter of $\frac{1}{2}(0.45+0.38) = 0.415\text{m}$ and an average thickness of 3.8mm was examined under external pressure.

Using Von-Karman and Tsien's modified linear equation for the buckling of the sphere, the critical buckling pressure head was found to be 99.8m for axisymmetric snap through collapse. Although this agrees well with the critical buckling pressure head for snap through collapse obtained from a linear buckling analysis using the finite element method (see table 4.1), the more critical bifurcation buckling pressure head is about 32% of this value. Thus indicated that the classical approach could not be used safely in the initial stages of the design of the Echinodome.
4.2.3 Discussion of theoretical results

The results of this numerical work showed that the linear elastic buckling analysis predicts higher critical loads than the analysis based on non-linear pre-buckling stress resultants. For the most critical condition - bifurcation buckling (see table 4.1) - the values were within 22% of the greatest one.

However, because of the large number of iterations required for a non-linear buckling analysis, the cpu time for a linear buckling analysis was only about $\frac{1}{9}$th of the cpu time required for the non-linear run. Thus, in the absence of any quicker and reliable method of predicting the critical buckling pressure of the Echinodome, a linear elastic buckling analysis of the shell could be useful in the initial stages of the design.

The results also showed that the critical region, i.e. the bottom tenth of the shell wall, was highly stressed and that the ultimate strength of the G.R.P. would be reached in that region before buckling occurred. This indicated that for this particular shell profile, wall thickness and material, buckling was not the criterion in design and that the material would fail before the shell became structurally unstable.

Nevertheless, it should be noted that the profile of the drop-shaped shell varies with the design head, material’s design stress and wall thickness - it could well be flatter or more spherical depending on these variables. Hence an examination of the buckling behaviour would be required in each particular case before an indication could be obtained as to whether the criterion for design was buckling or material failure. A linear elastic buckling analysis would suffice in most cases to obtain this indication.

In general, it is encouraging to note that the buckling capabilities of the Mistry finite element program have compared well with other numerical methods and programs such as the finite difference program, BOSOR5 \(^{48}\). It has also been used successfully for analysing the buckling behaviour of cone - cylinder and nozzle - torisphere combinations \(^{48,49}\). But before using it in the design of underwater drop shaped enclosures, experimental verification is necessary to examine the suitability of the program for the buckling analysis of such forms of structures.
4.3 Experimental investigation of shell buckling

The experimental determination of the buckling loads of shells is of great importance to the structural designer. It provides an indication of how the 'real' structure behaves under load and allows a comparison to be made with the theoretical model used to predict the failure loads.

Traditionally, this has been done by testing a large number of identical specimens to failure and then using statistical techniques to evaluate the critical load for that particular form of structure. However, in these tests, the critical loads can sometimes be very difficult to measure and several factors which may affect it such as the level of geometric imperfections and the influence of the boundary conditions are indeterminate. In other cases, material failure can occur well before buckling and the critical buckling load is not identifiable accurately - as is the case with the prototype used in this work.

As a result, large discrepancies between theoretical and experimental critical loads have been obtained in the past.

This form of buckling test can be very expensive, especially if a large number of specimens with awkward shapes have to be built. It is also impossible to predict the buckling loads for different mode shapes. Consequently other techniques for determining the buckling loads of shell structures, using non-destructive principles and without catastrophic buckling have come to the forefront in recent years. Amongst these techniques is the Southwell Plot, which was initially developed in 1932 for determining the buckling loads of columns.

4.3.1 The Southwell Plot

The Southwell Plot is a simple technique for interpreting the results of a non-destructive buckling test on a structure. It could only be applied to structures exhibiting an approximately hyperbolic load \( v \) deformation curve, passing through the origin, as long as the deflections were small and that the elastic limit of the material was not exceeded. Using the Southwell
technique, this hyperbolic curve is transformed into a straight line by plotting deformation/load against deformation and the critical load is equal to the reciprocal of the gradient of this line 53.

In the past, the Southwell Plot has been successful in predicting the critical loads in neutral buckling problems, such as the buckling of columns 52,53 and is nowadays a well accepted technique for solving those type of problems. However the use of this technique in unstable buckling problems, such as shell buckling, has been received with mixed criticism 52,54.

Nonetheless it should be noted that careful use of this technique has predicted critical loads which have agreed very closely with the 'actual' collapse load and numerical results for a wide range of shell structures including cylinders 55,56,57,58, pipe elbows 59, buried pipes 60, spheres and spherical caps 61 under different load conditions. It has also been used to predict the critical load of structures from the results of a non-linear static stress analysis using the finite element method 62. In some cases it has even been possible to identify different buckling mode shapes of shell structures 58. Consequently, the Southwell technique was chosen for interpreting the results of this work.

4.3.2 Buckling tests on the Echinodome

In order to gain a better understanding of the buckling behaviour of the Echinodome and assess the suitability of the finite element method in predicting the critical loads, a series of buckling tests was performed on a small prototype. The prototype was the same one that was used in the previous experimental work and the reader is referred to section 2.4.1 for details of the shell's characteristics.

The surface strains were measured in three directions at 10 locations using electric resistance strain gauge rosettes and again the reader is referred to section 3.3.2 for details of the strain gauge arrangement together with the instrumentation since these were the same as for the examination of the behaviour of the shell under concentrated loads.
However, it should be noted that due to the limited space inside the pressure chamber, it was not possible to monitor displacements.

4.3.2.1 Pressure chamber test arrangement

The pressure chamber was a copper autoclave, specially adapted for testing the prototype under hydrostatic pressure up to a head of 20 m. It was cylindrical with an internal diameter of 465mm. It had a torispherical bottom and a spherical removable lid giving it an overall height of 750mm. The lid had a sealable bleed hole at its apex to allow air to be expelled as the chamber filled. The chamber was pressurised, through a hole in its side, directly from the water mains.

The pressure was monitored by both a water manometer and a digital pressure gauge for heads less than 1 metre and by the digital gauge alone for greater heads. The digital pressure gauge was a Setra Systems pressure transducer (model 205-2) capable of reading up to 0.17 MN/m² with a full scale accuracy of 0.11% at constant temperature. It was calibrated against the water manometer and adjusted to read directly to ± 0.005m.

The tufnol base of the shell was bolted onto a dural platform and a cylindrical aluminium strut was placed around the shell in order to counteract the buoyancy when the pressure chamber was flooded. The whole arrangement was then placed inside the chamber as shown in Fig. 4.3.

The leads from the strain gauges were passed through an opening in the wall of the chamber which was subsequently filled with silicone rubber (room temperature vulcanising) to keep it watertight.

A view of the whole test apparatus is shown in Fig. 4.4.
4.3.2.2 Test procedure

Once the prototype was in position inside the chamber, the strain gauges were tested using a gauge installation tester and a few scans were made to ensure that all the equipment was connected and functioning correctly.

The pressure chamber was then closed, keeping the bleed valve on the lid opened and water was allowed in slowly. When the chamber was completely full and after inspecting for leaks and air bubbles in the system, the bleed valve was closed and the pressure was increased gradually using a control valve.

The pressure head, over the apex of the shell was raised to 1000mm in increments of 100mm and then up to 3500mm in increments of 250mm. The pressurisation was done at a uniform rate throughout - approximately every 40s.

At each pressure level a scan was made of the strain gauges in the two meridians containing the inner surface strain gauges (i.e. meridians 1 & 2 in Fig. 3.15). The scans were controlled by a Commodore Pet computer using the program STRAIN (with modified prompts) and took approximately 7.5s to read the 30 strain gauge channels at each increment. A flow diagram of STRAIN can be seen in appendix 1.4. These readings were taken on the pressure increasing part of the loading cycle as soon as the required pressure was reached in each case.

A total of five runs were made allowing 5mins between runs for creep recovery.
4.3.2.3 Experimental results

Using the numerical output obtained from the program STRAIN, the pressure – strain curve for each strain gauge was drawn and the resulting relationship for a typical strain gauge at each of the four parallel circles monitored can be seen in Figs 4.5 to 4.9.

All these curves, which were fitted by eye, showed a linear relationship between the pressure and strain up to a pressure head of about 2.0m. Above this pressure head, the relationship became non-linear and approximated to a hyperbolic curve.

It should be noted, though, that not all the strain gauges exhibited a hyperbolic pressure – strain relationship. Ten strain gauges located on the second and third parallel circles gave values of strain that were either very low or erratic. This was partly due to the region being one of low stress under the loading employed and therefore these gauges were not used in predicting the critical load.

4.3.2.4 Analysis of experimental results

The approximately hyperbolic nature of the pressure – strain curves shown in Figs 4.5 to 4.9 suggested that the Southwell technique would be suitable to interpret the results of these buckling tests. Also, since any deformation parameter could be used for obtaining a Southwell plot, the measured strains were employed directly for that purpose.

A Southwell plot was drawn for all the strain gauges exhibiting a hyperbolic pressure – strain relationship (20 out of 30) and the plots corresponding to the gauges in Figs 4.5 to 4.9 are shown in Figs 4.10 to 4.14.

Further pressure – strain relationships for the internal surface strain gauges in the critical zone together with their corresponding Southwell plots are given in appendix 8. 63,64
In all these Southwell plots, the straight lines were fitted using a curve fitting package based on the least squares approximation – CURVEFIT, which was available on the ICL 2900 at the E.R.C.C.. The critical loads were obtained directly from the equations of the straight lines, i.e. the inverse of the slope of the line, and a value for the standard deviation of all the points used in obtaining this equation was obtained from CURVEFIT. A typical output from this package is given in appendix 2.6.

The average critical buckling load predicted by the gauges at each parallel circle was then determined and a table showing the resulting meridional variation in the predicted critical buckling load is shown in table 4.3 below.

<table>
<thead>
<tr>
<th>Parallel circle</th>
<th>Direction of gauge</th>
<th>Pressure head (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>mer</td>
<td>36.3</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>44.4</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>38.5</td>
</tr>
<tr>
<td>2</td>
<td>mer</td>
<td>44.2</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>25.1</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>mer</td>
<td>46.6</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>29.6</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>-</td>
</tr>
<tr>
<td>4(outer)</td>
<td>mer</td>
<td>42.5</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>43.3</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>43.7</td>
</tr>
<tr>
<td>4(inner)</td>
<td>mer</td>
<td>44.7</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>46.4</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>45.0</td>
</tr>
</tbody>
</table>

Table 4.3 Mean predicted buckling pressure heads at each parallel circle
(mer = meridional gauge; 45 = gauge at 45° to the meridian; cir = circumferential gauge)
4.3.2.5 Discussion of experimental results

Table 4.3 above shows that there is some variation in the predicted critical load depending on the position and orientation of the strain gauge on the surface of the shell.

All the strain gauges in the critical zone gave values for the critical load which were within 6% of each other. The outer surface strain gauges predicted an average pressure head of 43.2m whilst the inner surface strain gauges predicted a value of 45.9m, giving an overall mean value for the critical pressure head of 44.3m for that region.

It is interesting to note that on each of the other three parallel circles, some of the strain gauges predicted a buckling pressure head very close to this value, indicating the possibilities of global buckling taking place around a pressure level equal to an overall mean hydrostatic head of 41.7m. The fact that around the shell's maximum diameter (i.e. parallel circles 2 & 3) only the meridional gauges predicted a global type buckling further suggested that in this mode shape there was much more deformation in the meridional plane than in any other and therefore indicated an axisymmetric buckling mode.

However, some of the other strain gauges in the upper part of the shell predicted values considerably less than 41.7m, indicating that local buckling was occurring at a lower head. This lower head was predicted to be 36.3m by the meridional strain gauges in parallel circle 1 and 27.4m by the non-meridional strain gauges in parallel circles 2 and 3. Suggesting that the effects of the local buckling were more pronounced around the shell's maximum diameter than near the apex. The region near the apex of the shell being influenced to a greater extent by the global snap-through buckling, i.e. dimpling, at the higher head would account for a predicted critical load halfway between that predicted by the gauges near the maximum diameter and the global buckling load.

The strain gauges indicating this local buckling around the shell's maximum diameter were orientated at 45° to the meridian implying deformation in a plane other than the meridional plane and hence
suggesting a non-axisymmetric mode at the lower hydrostatic head of 27.4m.

4.4 Comparison of numerical and experimental results

In general, the experimental results compared well with the numerical ones obtained previously in this chapter, see table 4.4 below.

<table>
<thead>
<tr>
<th>Mode of buckling</th>
<th>critical pressure head</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exp. (m)</td>
</tr>
<tr>
<td>Axisymmetric</td>
<td>41.7</td>
</tr>
<tr>
<td>Non - axisymmetric</td>
<td>27.4</td>
</tr>
</tbody>
</table>

Table 4.4 Comparison of experimental and numerical results

The non-linear results from the finite element analysis were used for this comparison since the Southwell plot predicts the non-linear buckling loads of structures.

The table above shows that for both axisymmetric and non-axisymmetric buckling the experimental critical load was lower than the numerical one and within 4% and 12% of it respectively. The greater discrepancy between the values for the non-axisymmetric buckling could be accounted for by the fact that very few of the strain gauges were predicting that mode shape. The translational deflections involved with that particular mode of buckling were very small and consequently some of the strain gauges that could have predicted it (i.e. those on parallel circles 2 and 3) gave values of strain that were too low and could not be used to construct a Southwell plot.

However, some difference could be expected between the experimental and numerical results. Considering the variation in thickness over the surface of the shell, it was only possible to model a meridional variation in
thickness in the finite element analysis whilst in actual fact there was also a circumferential variation, see appendix 3. Nonetheless an attempt was made to deal with this variation by averaging the results obtained from each parallel circle so as to enable a comparison with the finite element method.

Other factors, such as the level of geometric imperfections and the degree of fixity at the support also could have influenced the experimental values obtained. But, it should be noted that a Southwell plot would be predicting the critical load of the actual prototype together with all its imperfections, variations in thickness etc. and it is the numerical finite element simulation which has to approximate its actual behaviour as good as possible.

Only a test to destruction could give the actual buckling load of any particular form of shell together with all its imperfections and irregularities and ultimately a buckling test to destruction should be carried out as it would be the only way of confirming the Southwell plot.

In this work, the non-destructive test based on the Southwell plot was employed to verify the results obtained from the finite element method as only one prototype was available for testing and a whole series of future tests on it had been planned. The results of this work substantiates the use of the Mistry finite element program for buckling analyses and adds some confidence to its use in the design of Echinodomes for underwater applications.
4.5 **Summary and Conclusions**

This chapter investigated the buckling behaviour of the Echinodome under hydrostatic pressure both experimentally and theoretically.

The finite element method was used to determine the linear and non-linear critical buckling loads of a prototype and the corresponding mode shapes and the results were compared with an approximate solution obtained from the classical theory for shell buckling. The lowest critical buckling load being associated with a non-axisymmetric (i.e. translational) bifurcation buckling mode.

Following this an experimental investigation was described based on the Southwell technique and careful interpretations of the results led to prediction of two buckling loads and their mode shapes.

Quite good agreement was obtained between experimental and theoretical results which gave an indication of the suitability of the Mistry finite element program for use in the design of underwater drop-shaped enclosures.
Fig. 4.1 Pressure v deflection curve showing bifurcation, buckling and snap through collapse
Fig. 4.2 Critical buckling mode shapes

(a) Snap through buckling
   \((n=0)\)

(b) Bifurcation buckling
   \((n=1)\)
Copper chamber
Tufnol base (200 x 200 x 20)
Water manometer
Gland for strain gauge leads
Cylindrical anti-buoyancy strut (Aluminium)
Tufnol base (200 x 200 x 20)
Dural base (460 dia. x 20)

Fig. 4.3 Pressure chamber test arrangement
Fig. 4.4 General layout of apparatus
Fig. 4.5 Pressure vs strain curve for circumferential strain gauge at parallel circle No. 1
Fig. 4.6 Pressure v strain curve for strain gauge at 45° to the meridian at parallel circle No. 2
Fig. 4.7 Pressure v strain curve for strain gauge at 45° to the meridian at parallel circle No. 3

**KEY**

- ---
- □ run 1
- △ run 2
- ○ run 3
- ● run 4
- x run 5

Microstrain
Fig. 4.8 Pressure v strain curve for circumferential strain gauge at parallel circle No. 4 (outer surface)
Fig. 4.9 Pressure v strain curve for strain gauge at 45° to the meridian at parallel circle No. 4 (inner surface)
Fig. 4.10 Southwell plot corresponding to Fig. 4.5
Fig. 4.11 Southwell plot corresponding to Fig. 4.6
Fig. 4.13 Southwell plot corresponding to Fig. 4.8
Fig. 4.14 Southwell plot corresponding to Fig. 4.9
CHAPTER FIVE

DETERMINATION OF THE NATURAL FREQUENCIES AND

MODE SHAPES OF THE

DROP-SHAPED SHELL
5.1 Introduction

The natural frequencies of an underwater structure are the most important parameters influencing its dynamic response to the loads encountered in the submarine environment. It is important that the frequencies of those environmental loads do not coincide with one of the natural frequencies of the structure in order to avoid resonance and the subsequent magnification of its response. Hence an accurate assessment of the natural frequencies and mode shapes of a particular form of underwater enclosure is a prerequisite before it is launched.

In this chapter, the theoretical methods available for the dynamic analysis of general shells of revolution are first outlined. Then the natural frequencies and mode shapes of an Echinodome are determined using two different finite elements programs.

Following this an experimental investigation is described, based on the method of resonance testing, to determine the natural frequencies and mode shapes of a prototype.

The experimental and numerical results are compared and a particular finite element is recommended for use in design.

5.2 Dynamic analysis of shells

A vibrating shell structure is a complex dynamic system with an infinite number of degrees of freedom and an infinite number of natural frequencies which do not lie in any order. The doubly curved form of these structures together with the coupling of the membrane and bending behaviour gives way to a wide variety of vibrating mode shapes ranging from pure extensional modes, flexural modes and torsional modes to combinations of the three.

Consequently, the derivation of a universally accepted general shell theory for dynamic analysis becomes a very difficult task and over the years
various theories have been proposed.66,67

As with the static analysis of thin shells, most of these classical theories were based on Love's general shell theory 29 but with different simplifying assumptions and expressed the motion of the shell as an 8th order differential equation. However because of the complexity of this equation, exact analytical solutions were obtainable only for certain simple cases such as cylinders, shallow spherical shells and conical shells 67,68 and in most cases resort had to be made to approximate solutions using the Rayleigh–Ritz 69 or other approximate methods 69.

For general shells of revolution these approximate methods could not be applied because of the general nature of their profiles and as a result numerical methods had to be employed.

These numerical methods 70, which were mainly of the matrix type (see section 2.2), were based on the principle of conservation of energy and involved the solution of the general equation of motion by using either the determinant method 69 or by converting the equation into an eigenvalue problem in an attempt to obtain a more efficient solution 69.

Of the different numerical methods available, the finite element method provides the most efficient and reliable approach for the dynamic analysis of doubly curved shells such as the Echinodome and therefore was chosen for this investigation.

Two different finite element programs, PAFEC and the Mistry program, see section 2.3, were used in this dynamic analysis. Both these programs employed the eigenvalue process for determining the natural frequencies and mode shapes of thin shells but whereas PAFEC followed the eigenvalue economisation method 69.70 to reduce the size of the problem and hence save on computer time and storage the Mistry program used the Sturm count technique 70 for the same purpose. The stiffness matrix and mass matrix were formulated as described in section 2.3 and for a detailed explanation of the procedure involved in the free vibration analysis of shells, using the finite element method, the reader is referred to the numerous publications on the subject 30,37.
5.2.1 Finite element analysis

The natural frequencies of the prototype used in the previous investigations were determined using both the Mistry finite element program and PAFEC. When using the latter, the two elements described in chapter 2 i.e. the three noded thin shell of revolution element (element 42130) and the eight noded isoparametric element for axisymmetric Fourier applications (element 36610) were employed.

The structure was discretised into the same number of elements, 65, in each case and the same boundary conditions and material properties were input as in the previous analyses – all the data files were very similar to those shown in appendices 2.1 to 2.3.

The results obtained from this numerical investigation are tabulated in tables 5.1 to 5.3 and their corresponding mode shapes are shown in Figs 5.1 to 5.6.

<table>
<thead>
<tr>
<th>Meridional waves (m)</th>
<th>Circumferential waves (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250.4 66.8 1467.5 1768.1 1886.9 1969.4</td>
</tr>
<tr>
<td>2</td>
<td>1459.4 979.0 1716.6 1852.6 1950.7 2039.5</td>
</tr>
<tr>
<td>3</td>
<td>1688.5 1569.0 1808.3 1918.3 2032.2 2121.1</td>
</tr>
<tr>
<td>4</td>
<td>1784.8 1734.4 1876.1 1970.3 2087.6 2198.7</td>
</tr>
<tr>
<td>5</td>
<td>1843.3 1815.7 1939.7 2032.8 2158.7 2288.2</td>
</tr>
<tr>
<td>torsional</td>
<td>542.5</td>
</tr>
</tbody>
</table>

Table 5.1 Natural frequencies (Hz) of the prototype using the Mistry finite element program
Table 5.2 Natural frequencies (Hz) of the prototype using element 36610 in PAFEC

<table>
<thead>
<tr>
<th>Meridional waves \ (m)</th>
<th>Circumferential waves \ (n)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>254.6</td>
<td>68.1</td>
<td>1467.7</td>
<td>1767.6</td>
<td>1884.9</td>
<td>1965.8</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1465.0</td>
<td>982.0</td>
<td>1719.6</td>
<td>1850.4</td>
<td>1946.2</td>
<td>2033.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1693.1</td>
<td>1573.9</td>
<td>1811.1</td>
<td>1917.7</td>
<td>2019.0</td>
<td>2114.8</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1786.6</td>
<td>1739.2</td>
<td>1878.0</td>
<td>1970.1</td>
<td>2084.7</td>
<td>2193.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1842.9</td>
<td>1820.0</td>
<td>1941.4</td>
<td>2035.0</td>
<td>2160.7</td>
<td>2281.6</td>
<td></td>
</tr>
<tr>
<td>torsional</td>
<td>542.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3 Natural frequencies (Hz) of the prototype using element 42130 in PAFEC

<table>
<thead>
<tr>
<th>Meridional waves \ (m)</th>
<th>Circumferential waves \ (n)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250.6</td>
<td>66.7</td>
<td>1538.1</td>
<td>1799.7</td>
<td>1892.6</td>
<td>1976.7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1546.3</td>
<td>1002.1</td>
<td>1795.2</td>
<td>1924.5</td>
<td>1968.7</td>
<td>2060.9</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1741.1</td>
<td>1627.8</td>
<td>1875.4</td>
<td>2060.8</td>
<td>2074.8</td>
<td>2188.9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1828.2</td>
<td>1778.3</td>
<td>1941.6</td>
<td>2090.7</td>
<td>2167.0</td>
<td>2254.9</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1885.3</td>
<td>1874.7</td>
<td>1987.1</td>
<td>2149.6</td>
<td>2241.0</td>
<td>2376.1</td>
<td></td>
</tr>
<tr>
<td>torsional</td>
<td>543.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.2.2 Discussion of numerical results

The results show that the fundamental frequency of the prototype corresponds to a non-axisymmetric but symmetric, translational mode as shown in Fig. 5.2(a). The second natural frequency corresponds to an axisymmetric mode, see Fig. 5.1(a) and the third natural frequency corresponds to a torsional mode, see Fig. 5.1(b).

These three natural frequencies are well spaced out as indicated in tables 5.1 to 5.3, but it should be noted that for the higher modes some of the natural frequencies are very close together and lie within narrow
frequency bands. Although, this is of no significance for the prototype because the level of the frequency bands are so high (>1kHz), it is envisaged that in larger Echinodomes, these frequency bands could be much lower and may lie within a critical zone.

A comparison of the results obtained for these three modes, using the different finite elements is given in table 5.4 below.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mistry</th>
<th>PAFEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>symmetric (n=1,m=1)</td>
<td>66.77</td>
<td>68.09</td>
</tr>
<tr>
<td>axisymmetric (n=0,m=1)</td>
<td>250.39</td>
<td>254.61</td>
</tr>
<tr>
<td>torsional</td>
<td>542.50</td>
<td>542.55</td>
</tr>
</tbody>
</table>

Table 5.4 Comparison of the lowest three natural frequencies (Hz) using the different finite elements

The table above shows that there is very close agreement between the results obtained by the different finite elements for the first three natural frequencies. The frequencies obtained for the torsional mode were all within 0.1% of each other whilst the frequencies obtained for the first symmetric and first axisymmetric mode were all within 2% in each case. However, as the order of the frequency increases, a greater discrepancy is evident from tables 5.1 to 5.3. This discrepancy is due to the results obtained from the three noded thin shell of revolution element in PAFEC as the other two elements agree well with each other throughout.

Nonetheless this investigation shows that any of the three finite elements could be used to predict the fundamental and higher natural frequencies of the Echinodome.

In design, though, consideration would have to be given to the factors listed in section 2.4.5 before choosing a particular element to use in the free vibration analysis of the drop-shaped vessel. Most of the points
discussed in that section also apply to a free vibration analysis, except that the cpu time required for a dynamic analysis would be different. In this investigation, the Mistry program took a cpu time of 72.6s to obtain the natural frequencies whilst PAFEC took 92.7s when using element 42130 and 176.3s when using element 36610. Both input and output files were of the same size as in the previous analyses.

Again, as in the static stress analysis, the Mistry program seemed to provide the most efficient solution to the free vibration problem especially with a cpu time much smaller than the cpu time required for the two elements in PAFEC and consequently was chosen for use in the design section of this work.

However it should be noted that the present version of the Mistry program cannot examine the dynamic response of shells to a given load history and for such analysis one of the elements in PAFEC would have to be used.

In general, it is encouraging to note that the Mistry program also has compared well with PAFEC and other numerical methods such as the finite difference method in the free vibration analysis of a cylinder-cone combination. But, as with the previous types of analyses performed in this work, experimental verification is necessary before complete reliance can be placed on the Mistry program for the determination of the natural frequencies and mode shapes of the Echinodome.
5.3 **Experimental Investigation**

In this second part of the chapter, the fundamental natural frequency and the first axisymmetric natural frequency of the prototype were determined experimentally in order to confirm the result obtained from the finite element analysis. This was done using a method of resonance testing, namely the Peak – Amplitude method. 

5.3.1 **Simple extension of the Peak – Amplitude method**

The Peak – Amplitude method of resonance testing is the simplest and most widely used approach for obtaining the natural frequencies and mode shapes of a structure. The method involves the excitation of the structure and the measurement of the amplitude of its response at several points on the surface of the structure at the various excitation frequencies. Using this information, a response curve showing the total amplitude against the excitation frequency can be drawn for each point measured, see Fig. 5.7, and the natural frequencies can then be calculated as the excitation frequency at which the total amplitude reaches a peak, e.g. point A in Fig. 5.7.

The principal mode shapes can subsequently be derived from the ratios of the amplitudes of the response at the recorded points when the structure is resonating, i.e. excited at its natural frequency.

The theory behind this method of resonance testing has been described in detail by Bishop and Gladwell.

In this simple extension of the Peak – Amplitude method, the amplitude of the response at the base of the structure is also measured and the natural frequency is calculated as the excitation frequency at which the ratio of the amplitude of the response of the structure to the amplitude of the response at the base is a maximum.

Using this approach, it is possible to take account of any resonance of the base plate or the configuration connecting the structure to the vibrating table.
5.3.2 Test set-up

The prototype, described in chapter 2, was bolted onto a stiff wooden base plate which was in turn fixed to a vibrating table. The prototype was excited in two directions as shown in Fig. 5.8 in order to obtain both symmetric and axisymmetric modes and the method of fixing the wooden base plate onto the table was different in each case.

For the axisymmetric mode, the base plate was fixed directly onto the table top as shown in Fig. 5.9(a) but for the symmetric mode, the vibrating table had to be rotated by 90° and a supporting L-shaped configuration had to be employed as shown in Fig. 5.9(b).

5.3.3 Instrumentation

The excitation was applied to the prototype by an electromagnetic shaker (vibrating table) driven by a 2KVA solid state power oscillator and amplifier (Ling Dynamic Systems model TPO 2K). The amplitude of the oscillations was controlled by the oscillator itself but the excitation frequency was controlled externally and recorded by a digital frequency meter (Feedback type FM 610) which had a resolution to ± 0.001Hz.

The responses of the prototype and of the supporting structure were monitored by piezo-electric accelerometers whose signals were subsequently amplified by charge amplifiers.

These amplified signals were then directed to a dual channel fast Fourier transform spectrum analyser (Hewlett Packard model 3582A) in order to be processed. This low frequency spectrum analyser converted the analogue signals from both accelerometers, simultaneously, into discrete digital data through a sampling process and then performed a narrow band frequency analysis for the required frequency span. Three such frequency spans were used in this work: 0 to 50Hz span with a band width of 1.2Hz; 0 to 250Hz span with a band width of 3.0Hz; and 0 to 500Hz span with a band width of 6.0Hz.
These spectra were displayed on the screen of the analyser and if necessary were stored and sent to a plotter if a hard copy of the spectra was required.

The two spectra were then divided one by the other by means of the amplitude transfer function of the analyser and the result was presented as a logarithmic graphical display on the screen. From this display, it was also possible to determine, numerically, the amplitude ratio in decibels and the frequency at any point of interest within the display.

A block diagram of the instrumentation is presented in Fig. 5.10 and an overall view of the experimental apparatus and instrumentation can be seen in Fig. 5.11.

5.3.4 Arrangement of accelerometers

Two different piezo-electric transducers were employed in this investigation. One of them was fixed to the wooden base plate to record its response to the excitation and had the following characteristics:

<table>
<thead>
<tr>
<th>Type:</th>
<th>Bruel &amp; Kjaer type 4370</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge sensitivity:</td>
<td>98.5 pc/g</td>
</tr>
<tr>
<td>Voltage sensitivity:</td>
<td>77.1 mV/g</td>
</tr>
<tr>
<td>Weight:</td>
<td>52.3 grams</td>
</tr>
<tr>
<td>Natural frequency:</td>
<td>26 kHz</td>
</tr>
</tbody>
</table>

The second transducer was used to map the surface of the prototype and record the amplitude of its response at sixteen predetermined points, see Fig. 5.12. This accelerometer had the following characteristics:

<table>
<thead>
<tr>
<th>Type:</th>
<th>Bruel &amp; Kjaer type 4366</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge sensitivity:</td>
<td>46.2 pc/g</td>
</tr>
<tr>
<td>Voltage sensitivity:</td>
<td>35.6 mV/g</td>
</tr>
<tr>
<td>Weight:</td>
<td>27.9 grams</td>
</tr>
<tr>
<td>Natural frequency:</td>
<td>39 kHz</td>
</tr>
</tbody>
</table>
Both transducers were attached to the structure by a thin layer of microcrystalline wax, which was spread, whilst in its molten state, over the surface of the prototype at the predetermined points.

5.3.5 Experimental procedure

Once the prototype had been firmly fixed in position on the exciter and the base accelerometer attached securely, the whole system was switched on and allowed to warm up for a period of approximately 30 mins.

The second accelerometer was then placed on the prototype at a position where the maximum response was expected so that a detailed examination of the dynamic response could be undertaken. For the axisymmetric vibrations, the accelerometer was placed at the apex and for the non-axisymmetric but symmetric vibrations, the accelerometer was placed at the maximum diameter.

In the axisymmetric case, after the amplitude had been set to a predetermined level, the frequency of the excitation was increased from 5 to 200Hz in increments of 10Hz and thereafter up to 300Hz in increments of 5Hz, noting the ratio of the amplitude of the response of the prototype to the amplitude of the response at the base at each frequency level. Following this the excitation frequency was increased in increments of 0.5Hz in the close vicinity of the resonant frequency (i.e. the excitation frequency at which the amplitude ratio reaches a maximum) in order to obtain a precise value for the natural frequency. On reaching this natural frequency the spectrum was stored and subsequently plotted by the graph plotter.

The accelerometer was then removed and attached at another point on the surface of the prototype, see Fig. 5.12, and the procedure repeated. The only difference being that instead of increasing the frequency in steps of 5 and 10Hz as in the previous test, 25Hz increments were used.

A similar approach was adopted for the symmetric vibrations. However, resonance was expected at a much lower frequency and therefore the excitation frequency was increased only up to 100Hz and 5Hz increments.
were used throughout in the initial detailed examination. For the subsequent tests with the accelerometers in the different positions the frequency was only increased to 20 Hz as resonance was occurring below that level. In all cases, increments of 0.1 Hz were employed to home in on the precise natural frequency.

Each test took approximately 12 mins for the axisymmetric mode and approximately 10 mins for the symmetric mode and was performed at a mean ambient temperature of 15.5°C.

5.3.6 Experimental results

Typical variations in the amplitude ratio with excitation frequency are shown in Figs 5.13(a) & 5.13(b).

Fig. 5.13(a) corresponds to the axisymmetric vibrations and in particular to the response obtained when the accelerometer was fixed to the apex of the prototype. This graph or ‘response curve’ shows that the maximum amplitude ratio, expressed in decibels, occurs at an excitation frequency of 221.10 Hz. Hence according to the simple extension to the Peak – Amplitude method of resonance testing, the natural frequency predicted from this response was 221.10 Hz.

Similarly Fig. 5.13(b) corresponds to the symmetric vibrations and in particular the response obtained from the accelerometer when attached to the prototype’s maximum diameter. In this case the response predicts a maximum amplitude ratio and hence resonance at an excitation frequency of 15.55 Hz.

The response curve from all the other measured points were analysed in the same manner and the natural frequency predicted from each of the sixteen locations for both vibrating modes are summarised in tables 5.5 and 5.6. Several points on the prototype did not predict any resonance frequency and thus were considered to be the nodal points for that particular vibrating mode shape. These points are marked as ‘np’ in the table below.
<table>
<thead>
<tr>
<th>Parallel circle</th>
<th>meridian 1 (Hz)</th>
<th>meridian 2 (Hz)</th>
<th>meridian 3 (Hz)</th>
<th>mean (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>apex</td>
<td>220.0</td>
<td>-</td>
<td>-</td>
<td>220.0</td>
</tr>
<tr>
<td>A</td>
<td>219.5</td>
<td>222.5</td>
<td>221.5</td>
<td>221.2</td>
</tr>
<tr>
<td>B</td>
<td>221.0</td>
<td>np</td>
<td>np</td>
<td>np</td>
</tr>
<tr>
<td>C</td>
<td>np</td>
<td>np</td>
<td>np</td>
<td>np</td>
</tr>
<tr>
<td>D</td>
<td>222.0</td>
<td>222.5</td>
<td>221.0</td>
<td>221.8</td>
</tr>
<tr>
<td>E</td>
<td>221.0</td>
<td>222.0</td>
<td>220.5</td>
<td>221.2</td>
</tr>
</tbody>
</table>

Table 5.5 Natural frequency at the measured points - axisymmetric mode (np = nodal point)

<table>
<thead>
<tr>
<th>Parallel circle</th>
<th>meridian 1 (Hz)</th>
<th>meridian 2 (Hz)</th>
<th>meridian 3 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>apex</td>
<td>15.70</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A</td>
<td>15.85</td>
<td>15.70</td>
<td>15.45</td>
</tr>
<tr>
<td>B</td>
<td>15.55</td>
<td>np</td>
<td>15.40</td>
</tr>
<tr>
<td>C</td>
<td>15.75</td>
<td>15.55</td>
<td>15.40</td>
</tr>
<tr>
<td>D</td>
<td>15.55</td>
<td>15.55</td>
<td>15.30</td>
</tr>
<tr>
<td>E</td>
<td>15.55</td>
<td>15.60</td>
<td>15.40</td>
</tr>
</tbody>
</table>

Table 5.6 Natural frequency at the measured points - symmetric mode (np = nodal point)

Typical response spectra of the prototype and the wooden base when excited at these frequencies are presented in Figs 5.14(a) & 5.14(b).

The mode shapes corresponding to these natural frequencies were calculated from the ratios of the amplitude of the response at the sixteen measured points when the structure was resonating and the recorded nodal points and are shown in Figs 5.15(a) & 5.15(b).
5.3.7 Discussion of experimental results and comparison with numerical results

The results tabulated in tables 5.5 & 5.6 indicate that the fundamental natural frequency of the prototype corresponds to a symmetric mode of vibration as shown in Fig. 5.15(a) and has an average value of 15.55Hz. Also, since no other symmetric mode was evident with a natural frequency less than 250Hz, then the second natural frequency corresponds to an axisymmetric mode of vibration as shown in Fig. 5.15(b) and has an average value of 221.10Hz.

A comparison of these experimental frequencies with those obtained from the Mistry program is shown in table 5.7 below and a comparison of the mode shapes is presented in Fig. 5.15(a) & (b).

<table>
<thead>
<tr>
<th>mode</th>
<th>F.E.M. (Hz)</th>
<th>Exp. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>symmetric</td>
<td>66.77</td>
<td>15.55</td>
</tr>
<tr>
<td>axisymmetric</td>
<td>250.39</td>
<td>221.10</td>
</tr>
</tbody>
</table>

Table 5.7 Comparison of numerical and experimental results

Unfortunately, it was not possible to excite the first torsional mode of vibration with the available apparatus and hence no experimental value for the torsional natural frequency could be obtained.

These comparisons show that although the mode shapes and natural frequencies for the axisymmetric mode agree well, a much greater discrepancy is evident in the results obtained for the symmetric mode.

This difference in the results can be partly explained by a consideration of the boundary conditions at the shell - base connection. In the finite
element model, the shell wall was assumed to be totally fixed to the base and have no degrees of freedom whereas in the actual prototype, total fixity at the base was not possible.

Consequently, further finite element analyses were performed using the Mistry finite element program with different combinations of degrees of freedom at the base and the results obtained are summarised in table 5.8 below.

<table>
<thead>
<tr>
<th>mode</th>
<th>degrees of freedom free at the base</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>none</td>
</tr>
<tr>
<td>symmetric</td>
<td>66.77</td>
</tr>
<tr>
<td>axisymmetric</td>
<td>250.39</td>
</tr>
</tbody>
</table>

Table 5.8 Effect of the fixity at the base on the natural frequencies of the prototype (frequencies in Hz)

As can be seen from table 5.8, some of the natural frequencies of the Echinodome drop if total fixity at the support was not achieved. There is a considerable drop in the fundamental frequency if the axial degree of freedom (u) was free and this could well explain the low experimental value obtained. However this is very unlikely since at the same time it increases the first axisymmetric frequency by about 600% and this does not agree with the lower experimental value obtained.

It is more likely that the experimental results were affected by a combination of the radial, circumferential and rotational degrees of freedom and in particular the rotational one. If in the construction of the prototype 80% fixity had been achieved in all three degrees of freedom then, by interpolation, the fundamental frequency would have been reduced to 62.37Hz and the first axisymmetric frequency to 222.50Hz. This modified value for the axisymmetric natural frequency agrees well with the experimental result but the same could not be said for the modified fundamental frequency which is still 400% higher than the experimental
It is worth noting, at this stage, that the experimental value for the fundamental frequency may have been influenced by the location of the applied excitation. Whereas for the axisymmetric load the excitation was applied in line with the centroid of the prototype via its base, see Fig. 5.9(a), for the symmetric vibrations the excitation was applied via an L-shaped configuration and was not in line with the centroid of the shell, see Fig. 5.9(b). This may account for the slight difference in mode shape obtained for the symmetric vibrations, especially at the apex, and may have reduced the fundamental frequency.

Also during the vibration tests, it was possible that the continuous oscillations, especially at excitation frequencies close to or at the natural frequencies, may have fatigued the araldite bond between the shell wall and the base reducing the level of fixation even further but the repeatability of results obtained throughout the testing indicated that this had not occurred.

It is also important to note that the ambient temperature, which was carefully monitored throughout, increased by about $10^\circ C$ during each series of tests and at the same time a decrease in the predicted natural frequencies and an increase in the temperature of the prototype was noted. Thus indicating some frequency and thermal effects on the dynamic characteristics of glass reinforced plastic, as has been observed by other researchers $^{73}$, which may have influenced the results.

With regard to the accuracy and reliability of the instrumentation employed, it is encouraging to note that other dynamic investigations have been undertaken using the same setup for a different structure and good results had been obtained $^{74}$. The mass of the accelerometers were negligible compared with the prototype – approximately 2% of it and therefore would have had no effect on the vibrating modes.

In general the outcome of this investigation is encouraging even though some discrepancy was to be expected between the results. The level of geometric imperfection, the circumferential variation in shell wall thickness and the presence of a seam bonding the two halves of the prototype together could not be accounted for in the finite element model and could
have easily affected the results. The orientation of the seam in relation to
the excitation, in particular, may have had an effect on the symmetric mode
of vibration, as the seam would be less stiff in shear than in
torsion/compression. Apart from that it shows that the finite element
method and in particular the Mistry finite element program can be used
successfully to predict the natural frequencies and mode shapes of the
Echinodome provided an accurate discretisation of the structure and the
correct boundary conditions were used.

5.4 Summary and Conclusions

This chapter was concerned with the determination of the natural
frequencies and mode shapes of an Echinodome.

Three different finite elements were employed in the numerical
investigations to evaluate the axisymmetric, symmetric and torsional natural
frequencies of a prototype and their corresponding mode shapes. Similar
results were obtained from all three elements and indicated that the
fundamental frequency corresponded to a non-axisymmetric, but symmetric,
mode of vibration. The second natural frequency corresponded to an
axisymmetric mode and the third to a torsional mode.

On comparing the relative merits of each of the three elements, the
Mistry finite element was recommended for use in design.

Following this an experimental investigation was undertaken to obtain
the first symmetric and first axisymmetric natural frequencies of the
prototype. A simple extension of the Peak - Amplitude method of
resonance testing was used and the results obtained agreed with the
numerical ones in predicting a fundamental frequency corresponding to a
symmetric mode and a second natural frequency corresponding to an
axisymmetric mode.

Hence confirms the suitability of the finite element method and in
particular the Mistry finite element program for determining the natural
frequencies and mode shapes of a drop-shaped tank.
Fig. 5.1 Vibrating mode shapes for $n = 0$
Fig. 5.2 Vibrating mode shapes for $n = 1$
Fig. 5.3 Vibrating mode shapes for $n = 2$
Fig. 5.4 Vibrating mode shapes for \( n = 3 \)
Fig. 5.5 Vibrating mode shapes for $n = 4$
Fig. 5.6 Vibrating mode shapes for $n = 5$
Fig. 5.7 Response curve for the Peak-amplitude method
Fig. 5.8 Direction of excitation of prototype
Fig. 5.9 Connection of base to vibrating table
Fig. 5.10 Block diagram of instrumentation
Fig. 5.11 Overall view of instrumentation
Fig. 5.12 Arrangement of accelerometers
(a) axisymmetric mode

(b) symmetric mode

Fig. 5.13 Variation of amplitude with frequency
Fig. 5.14(a) Response spectra - axisymmetric mode
Fig. 5.14(b) Response spectra - symmetric mode

- - -  response of prototype
- - -  response of base
Fig. 5.15 Comparison of experimental and numerical mode shapes

(a) symmetric mode
\( (n=1, m=1) \)

(b) axisymmetric mode
\( (n=0, m=1) \)

--- Numerical
○ Experimental
CHAPTER SIX

DESIGN OF AN UNDERWATER OIL STORAGE TANK

- STRUCTURAL FORM
6.1 Introduction

The work described in the previous chapters has been restricted to the examination of the behaviour of a small prototype which was built for the purpose of model testing in the laboratory. The results obtained were encouraging and gave some confidence to the use of the finite element method in the analysis of the Echinodome.

In this and the next chapter, the findings and recommendations from this work on the prototype are used to design an underwater storage vessel.

This present chapter is only concerned with the structural form of such a tank whilst the next chapter is concerned with the assessment of the loads likely to act on it during its life and its response to those loads.

A structural form for the primary structure is first selected and a choice of materials for the walls of the vessel is made. The procedure for determining the meridional profile of the tank walls is then described and an initial check on the stability of the vessel is carried out.

6.2 Design brief

An underwater drop-shaped tank for the storage of crude oil is designed in this chapter. The tank was to be fixed to the seabed at a position 57° 25'N and 0° 20'E in the North Sea where the depth of water is approximately 80.0 metres. A storage capacity of approximately 45000m³ of North Sea crude oil, with a specific gravity of 0.84 ⁷⁵, was required.

In the absence of a specific code of practice for the design of underwater structures, the guidance provided by BS 6235 ⁷⁶, the code of practice for fixed offshore structures, was followed.
6.3 **Structural form**

A double skin arrangement was chosen for this design. The outer jacket being the primary structure and capable of resisting the external loads whilst the inner tank contains the stored liquid. An air gap of constant thickness separating both shells was provided and in order to ensure uniform stressing in the outer structural shell no connections were made between the inner tank and outer jacket except at the base.

A double walled structure was selected as opposed to a single walled one for safety purposes and to prevent contamination of the surrounding water should a leakage in the containment vessel occur.

6.3.1 **Choice of materials**

In this configuration, the inner tank and outer jacket both fulfil different functions and operate in different environments. Consequently the choice of material for each shell was considered separately.

6.3.1.1 **Inner tank**

The inner tank operates inside the outer jacket and would thus not be exposed to the harsh marine environment. It would only be subjected to an internal hydrostatic pressure due to the liquid stored within it and would therefore be totally in tension.

In this tensile state, it would be comparable with above-ground oil storage tanks and it should be noted that drop-shaped oil storage tanks have been previously designed and constructed. These tanks were constructed of steel and proved to be very successful – one such tank was unintentionally subjected to a vacuum equal to nine times its design pressure without rupture.

With this history in mind, steel was chosen as the material for the inner tank.
6.3.1.2 Outer jacket

The outer jacket is the main protective enclosure and would be subjected directly to the severe marine environment throughout its working life. It should be capable of resisting the external hydrostatic loads and at the same time maintain a dry one atmosphere environment within it. Therefore the choice of material for the outer jacket requires careful attention.

The two main materials for use in such underwater structures are concrete and steel, although in the recent years considerable attention has been directed towards the development of alloys with a high strength/weight ratio for such purposes.

However, a fixed underwater tank requires little or no positive buoyancy and therefore materials with a low strength/weight ratio are beneficial. Concrete is such a material and its use for underwater structures has many inherent advantages. From a strength point of view, thicker walls are required for a concrete tank than for a steel one and hence the critical buckling pressure of a concrete tank would be higher. Needless to say the drop-shaped shell under hydrostatic pressure (at its design head) would be totally in compression as can be seen in tables 2.1 and 2.2 in chapter 2 and the good compressive behaviour of concrete makes it an attractive proposition.

Concrete also shows excellent durability when permanently submerged in seawater and hence fewer inspections and dry-dockings would be required during its operational life.

As a result concrete was chosen in this design for the outer shell of the storage vessel. Its ability to be formed doubly curved makes it particularly suitable for the construction of a drop shaped tank and the fact that concrete has already been employed successfully in the construction of other similar offshore structures was very encouraging.

A high grade concrete with a characteristic strength of 60MN/m$^2$ was selected for the design of the outer jacket of the storage tank.
6.3.2 Shape prediction

A procedure for predicting the profiles of the two walls in a double skin drop-shaped tank has been outlined previously. That procedure was followed in this chapter to determine the shape of the outer jacket and inner tank of the storage vessel and a more detailed description of that method is reported in the next two sections.

6.3.2.1 Profile of the outer jacket

A range of wall thicknesses varying from 230mm to 250mm and a range of design stresses varying from 20MN/m$^2$ to 30MN/m$^2$ were considered for the outer jacket of the storage tank. A shape prediction program was employed to generate a series of design curves corresponding to the range of wall thicknesses and design stresses mentioned above. A flow diagram of this shape prediction program is given in appendix 1.2 and a typical set of such design curves, for a wall thickness of 240mm is shown in Fig. 6.1.

From these design curves three profiles were selected for the outer jacket and their basic characteristics are summarised in table 6.2 on the next page.
Table 6.2 Profiles of outer jacket

Previous experimental work \(^{18,26}\) indicated that a critical zone exists near the base of the shell where the effects of bending are significant and that buckling in this region was likely to be a criterion in design.

As a result, a linear elastic buckling analysis was performed on the three profiles using the Mistry program and the resulting critical buckling pressure heads are tabulated in table 6.3 below.

<table>
<thead>
<tr>
<th>thickness of shell (mm)</th>
<th>elastic buckling pressure head (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>230</td>
<td>72.60</td>
</tr>
<tr>
<td>240</td>
<td>79.39</td>
</tr>
<tr>
<td>250</td>
<td>85.96</td>
</tr>
</tbody>
</table>

Table 6.3 Linear elastic buckling pressure heads of outer jackets

Unfortunately, these critical buckling pressure heads were very low. Higher buckling pressures would be required in order to achieve a reasonable factor of safety against buckling. Consequently, the critical zone, i.e. the bottom tenth of the shell, was strengthened by linearly increasing its thickness down its length from its initial uniform thickness, \( t \), to a modified thickness, \( t' \), given by equation 6.1.

\[
t' = k \cdot t
\]  

(6.1)
The value of the constant $k$ was varied from 1.0 to 2.25 and the corresponding critical buckling pressure was obtained for each case, see Fig. 6.2.

This figure shows that a big increase in the buckling pressure is attainable by a relatively small increase in the value of $k$, for values of $k$ between 1.0 and 2.0. The buckling pressure more than doubles within that range. However, the rate of increase of buckling pressure with respect to $k$ decreases considerably for values of $k$ greater than 2.2.

A value of $k$ around 2.0 was considered adequate, providing an increase in the buckling pressure of approximately 2.5 in each case as shown in table 6.4 below.

<table>
<thead>
<tr>
<th>thickness of shell (mm)</th>
<th>$k$</th>
<th>elastic buckling pressure head (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>230</td>
<td>2.0</td>
<td>182.48</td>
</tr>
<tr>
<td>240</td>
<td>2.0</td>
<td>200.18</td>
</tr>
<tr>
<td>250</td>
<td>2.0</td>
<td>218.79</td>
</tr>
</tbody>
</table>

Table 6.4 Linear elastic buckling pressure head modified outer jackets

This table indicates that any of the three profiles could be used safely in this design.

However, tables 6.2 and 6.4 show that the higher the buckling pressure head, the more material is required for the shell wall, but on the other hand the lower the design stress. A shell with a wall thickness of 240mm and a design stress of 25MN/m² seemed to provide the best compromise and was the one chosen for this work. The bottom tenth of the wall was increased linearly to 500mm ($k=2.08$) in this particular case.

A listing of the coordinates of the centreline profile of this shell was then obtained and is presented in appendix 6.1.
6.3.2.2 Profile of inner tank

The centreline profile of the inner tank is determined directly from the profile of the outer jacket as it is only dependent on the width of the air gap in between the two walls.

An air gap of 1.0 m was considered adequate for maintenance purposes and services and was used in this design to obtain the coordinates of the profile of the inner tank.

Three different wall thicknesses were considered for this inner tank, 40mm, 45mm and 50mm, and as buckling was expected to be a criterion in the design, a linear elastic buckling analysis of the three alternatives was carried out. As for the outer jacket, the buckling pressure head of the tanks with uniform thickness were very low. In fact the tank would buckle under the weight of the liquid stored within it, see table 6.5.

<table>
<thead>
<tr>
<th>Thickness of shell (mm)</th>
<th>Elastic buckling pressure head (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>12.9</td>
</tr>
<tr>
<td>45</td>
<td>17.2</td>
</tr>
<tr>
<td>50</td>
<td>22.2</td>
</tr>
</tbody>
</table>

Table 6.5 Critical buckling pressure heads of inner tank

Hence, it was also necessary to strengthen the critical zone by increasing its thickness linearly by a factor k.

Further buckling analyses were carried out on these modified shapes with k ranging from 1.0 to 2.25 and the resulting variations in buckling pressure with k are shown in Fig. 6.3.

In this case, the optimum value of k seemed to be around 1.75 and table 6.6 below shows the buckling pressure heads for the three modified profiles.
<table>
<thead>
<tr>
<th>thickness of shell (mm)</th>
<th>k</th>
<th>elastic buckling pressure head (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>1.75</td>
<td>35.2</td>
</tr>
<tr>
<td>45</td>
<td>1.75</td>
<td>45.3</td>
</tr>
<tr>
<td>50</td>
<td>1.75</td>
<td>58.9</td>
</tr>
</tbody>
</table>

Table 6.6 Critical buckling pressure heads of modified inner tanks

This table shows that the critical buckling pressure heads are still much lower than those for the concrete outer jacket but this was to be expected since the wall thickness/maximum diameter ratio for the outer jacket is greater than that for the steel tank. Nevertheless, in order to maximise the buckling pressure of the inner container, the 50mm shell was chosen for this work. The bottom tenth of the shell increasing linearly to 88mm (k=1.75).

A listing of the coordinates of the centreline profile of this shell was subsequently obtained and is presented in appendix 6.2.
6.3.3 Dimensions of the double skin drop shaped tank

A summary of the physical dimensions of the outer jacket, inner tank and base of the drop-shaped oil storage vessel is given below in tables 6.7(a) and 6.7(b) below and a cross sectional elevation of the final design is shown in Fig. 6.4.

<table>
<thead>
<tr>
<th>outer jacket</th>
<th>inner tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>material</td>
<td>grade 60 concrete BS 1501-223 steel</td>
</tr>
<tr>
<td>thickness (mm)</td>
<td>240 50</td>
</tr>
<tr>
<td>k</td>
<td>2.08 1.75</td>
</tr>
<tr>
<td>height (m)</td>
<td>34.67 32.19</td>
</tr>
<tr>
<td>maximum diameter (m)</td>
<td>52.02 49.54</td>
</tr>
<tr>
<td>height/max. diameter</td>
<td>0.67 0.65</td>
</tr>
<tr>
<td>surface area (m²)</td>
<td>6342 6048</td>
</tr>
<tr>
<td>cross sectional area (m²)</td>
<td>1480 1322</td>
</tr>
<tr>
<td>volume of material (m³)</td>
<td>1606 295</td>
</tr>
<tr>
<td>volume enclosed (m³)</td>
<td>51968 44975</td>
</tr>
</tbody>
</table>

Table 6.7(a) Dimensions of the drop shaped oil storage tank

<table>
<thead>
<tr>
<th>material</th>
<th>grade 60 concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>diameter (m)</td>
<td>30.00</td>
</tr>
<tr>
<td>height (m)</td>
<td>3.63</td>
</tr>
</tbody>
</table>

Table 6.7(b) Dimensions of the base
6.4 Summary

This chapter was concerned with the initial stages of the design of an underwater oil storage vessel for the North Sea.

A double skin drop-shaped tank with a concrete outer jacket and a steel inner tank was chosen for this work.

The meridional centreline profile of both walls were determined using a shape prediction program. Then, after an initial check on the stability of the structure was made, the thicknesses of both shell walls were increased towards the base in order to provide a high factor of safety against buckling.
Fig. 6.1(a) Design curve - volume enclosed v operating depth

Fig. 6.1(b) Design curve - volume enclosed v design head
Fig. 6.1(c) Design curve - material volume v design head

Fig. 6.1(d) Design curve - maximum diameter v design head
Fig. 6.1(e) Design curve - height/maximum diameter v design head
Fig. 6.2 Linear elastic buckling of outer concrete jacket

Fig. 6.3 Linear elastic buckling of inner steel tank
Crude oil in/out

capacity of storage tank = 44975 m³

(a) cross sectional elevation

(b) bottom tenth of shell walls

Fig. 6.4 Design details of oil storage tank
CHAPTER SEVEN

DESIGN OF AN UNDERWATER OIL STORAGE TANK

- ASSESSMENT OF LOADS

AND

STRUCTURAL ANALYSIS
7.1 Introduction

This chapter deals with the analysis of the underwater oil storage tank proposed in the previous chapter under some of the important loadings expected during its construction, tow-out, installation and operational stages.

An assessment of the different types and magnitudes of the loads expected on such a structure is carried out and the responses of the tank to those loads are examined.

This is followed by an examination of the dynamical behaviour of the tank and a further check on the stability of the structure in the submarine environment.
7.2 Structural analysis of the oil storage tank

The analysis of the double skin drop shaped tank was performed using the finite element method and employed the particular finite element recommended earlier for each type of analysis, see table 7.1 below.

<table>
<thead>
<tr>
<th>Type of analysis</th>
<th>Recommended finite element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear static stress analysis</td>
<td></td>
</tr>
<tr>
<td>(i) hydrostatic pressure</td>
<td>Mistry ring element</td>
</tr>
<tr>
<td>(ii) other loads</td>
<td>PAFEC 42130</td>
</tr>
<tr>
<td>Linear and non-linear buckling analyses</td>
<td>Mistry ring element</td>
</tr>
<tr>
<td>Dynamic analysis</td>
<td></td>
</tr>
<tr>
<td>(i) free vibrations</td>
<td>Mistry ring element</td>
</tr>
<tr>
<td>(ii) forced vibrations</td>
<td>PAFEC 42130</td>
</tr>
</tbody>
</table>

Table 7.1 Finite elements recommended for use in design

The responses of both the inner containment vessel and the outer jacket were examined. The response of the concrete base was not considered to be as critical in this design and was therefore not included in this work.

7.2.1 Finite element idealisation

The inner tank and outer jacket, see Fig. 6.4, were both divided into the same number of ring elements for the finite element analysis. A total of 102 elements of approximately equal length were employed each time in order to obtain the overall response of the tank to the different loads expected. The use of more elements in the idealisation produced little differences in the results at the expense of a greater cpu time. The coordinates of these elements were obtained from the modified shape
prediction program, see appendix 1.2, and are given in appendices 6.1 & 6.2.

The physical characteristics of both shells were as shown in table 6.7(a) and the material characteristics employed are given in table 7.2 below.

<table>
<thead>
<tr>
<th></th>
<th>outer jacket</th>
<th>inner tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus (MN/m²)</td>
<td>30 X 10⁵</td>
<td>210 X 10⁵</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.20</td>
<td>0.30</td>
</tr>
<tr>
<td>Mass density (kg/m³)</td>
<td>2400</td>
<td>7860</td>
</tr>
</tbody>
</table>

Table 7.2 Material characteristics of the double skin oil storage tank

The degrees of freedom at the base of both shells were all fixed to simulate the 'built-in' condition that would exist in the actual tank and all those at the apex were free.

The format of the data files for both the inner tank and outer jacket were very similar to those shown in appendices 2.1 & 2.2 for the Mistry finite element and element 42130 in PAFEC respectively.

7.3 Static stress analysis

The first stage in the analysis was the static stress analysis of both the inner tank and outer jacket under the various loads expected during the lifetime of the structure, which can be classified as follows [BS 6235:cl.3.3.1]⁷⁶:

(i) dead loads;
(ii) live loads;
(iii) hydrostatic loads; and
(iv) environmental loads.
An assessment of these different categories of loads together with an examination of the responses of the storage tank to them are given in sections 7.3.1 to 7.3.4 respectively.

7.3.1 Dead loads

The dead loads are the permanent static loads which can only vary as a result of a major alteration on a structure [BS 6235:cl.3.3.2]⁷⁶. In this design, the only contribution was that due to the self weight of the structure and its base together with the weight of the ballast and permanent machinery stored within it.

However, the ballast and machinery would be located within the concrete base and would therefore have no effect on the tank walls. But, on the other hand, the effect of the self weight of both shells must be considered.

From table 6.7(a), the volume of the material required for the outer jacket of the storage tank is 1606 m³. Therefore, assuming an average mass density of 2400 kg/m³, for concrete, the mass of the shell is 3.85 X 10⁶kg. Similarly the volume of the inner tank is 295 m³ and assuming a density of 7860 kg/m³ for steel, the mass of the inner shell is 2.32 X 10⁶kg.

An examination of the response of both tanks under their own weight was undertaken using the GRAVITY module in PAFEC ³³ and the results are presented in Figs 7.1 & 7.2.

7.3.2 Imposed loads

The imposed loads should include any load apart from dead loads, hydrostatic loads and environmental loads acting on a structure [BS 6235:cl.3.3.3]⁷⁶. They can be either static or dynamic and can vary in magnitude and direction.
Therefore, the imposed loads considered in this design were:

(i) the buoyancy load; and
(ii) the thermal loads.

7.3.2.1 Buoyancy loads

A structure submerged in a liquid would be subjected to an upward force due to the volume of liquid displaced by it. In this case, the volume of sea water displaced by the shell structure is 53574 m$^3$ and the volume of sea water displaced by the base is 3267 m$^3$. Therefore the total mass of water displaced is $56841 \times 10^2 = 5.826 \times 10^7$ kg, which is equivalent to a load of 571.5 MN.

However this force would be taken up by the concrete base of the structure and therefore would not have an effect on the outer jacket or inner tank.

7.3.2.2 Thermal loads

The storage of hot crude oil at a temperature of 35°C would produce a thermal differential across the walls of the vessel. This thermal load would be influenced by the air and sea temperatures around the location of the structure and hence an indication of the maximum and minimum temperatures to be expected is necessary.

The maximum air temperature for the U.K. sector of the North Sea, north of 54°N is 18°C and the minimum is −10°C [BS 6235:cl.2.9.1]. Little information is available regarding the sea temperature at depth and in most cases the design sea surface temperatures can be used as the design temperature at all depths. The suggested design minimum temperature is −2°C and the design maximum temperature is 18°C [BS 6235:cl.2.9.3].

The maximum thermal load would arise when the inner tank was full of crude oil at a temperature of +35°C and the surrounding seawater was at
its minimum design temperature of \(-2^\circ C\). However, if a leakage occurred either from the inner steel tank or outer concrete jacket, then the temperature gradient of \(37^\circ C\) would exist across a single wall. This would produce a worse thermal load than that mentioned above and therefore must be considered in the design.

The stress distributions induced as a result of this temperature differential were obtained using PAFEC and are shown in Figs 7.3 & 7.4 for the outer and inner tank respectively.

### 7.3.3 Hydrostatic loads

Hydrostatic loads act in a direction normal to the contact surface and can be either external or internal (i.e. axisymmetric pressure) [BS 6235:cl.3.3.4]76.

An underwater storage vessel would be subjected to two different hydrostatic loads:

(i) external hydrostatic pressure; and

(ii) the load due to the liquid stored within it.

#### 7.3.3.1 External hydrostatic pressure

In the design of fixed underwater structures, the head of water above it is likely to exert the worst load expected on the structure and it is therefore important to determine the maximum water level expected during its working life. The depth of water at a specific location is always changing, as a result of tidal variations, even in calm conditions and hence an accurate assessment of the maximum tidal range and the maximum change in the water level due to wind and pressure induced storm surges needs to be made. This is shown in table 7.3.
This table shows that the mean spring tidal range is 2.0m and the mean water level is 78.3m, hence the depth of water at the required location is 78.3 ± 1.0m. The height of the outer jacket is 34.67m (table 6.7(a)) and the height of the base is 3.63m (table 6.7(b)). Therefore the overall height of the structure above the seabed is 38.3m. This implies that under normal environmental conditions the head of water above the structure is 40.0 ± 1.0m which is equal to the design head of the outer shell. Under extreme environmental conditions, this depth of water would be further increased by the design wave and the wind and pressure induced storm surges, see section 7.3.6.

The pressure distribution due to this load case is shown in Fig. 7.5 and the resulting stress distributions on the outer jacket and the displaced shape are given in Figs 7.6(a) & 7.6(b).

### 7.3.3.2 Loads due to stored liquid

The storage vessel was to be operated full of liquid at all times, whether it be crude oil, sea water or a combination of both, with a rubber membrane separating the different liquids, in order to ensure maximum negative buoyancy.
The crude oil was to be stored at a pressure of 1.5 bar above ambient to facilitate the pumping to the surface and the resulting pressure distribution on the inner shell would be as shown in Fig. 7.7. It should be noted that this would be the worst load case. When the tank was full of sea water, it would be at ambient pressure and hence would be less critical than that due to the crude oil.

The stresses induced on the inner tank by the pressure due to the crude oil are shown in Fig. 7.8.

### 7.3.4 Environmental loads

This fourth category of loads includes all those loads imposed on the structure by the wind, sea, weather and other natural effects [BS 6235:cl.3.3.5]\(^76\). These loads are usually dynamic and random in nature. They can act in any direction and be of any magnitude.

A probabilistic approach is normally followed to determine the most severe magnitudes of these loads and the resulting loads are specified in terms of a recurrence interval or return period.

This return period is defined as the average length of time in which, statistically, the magnitude of the load will be equalled or exceeded only once. The Department of Energy \(^84\) recommends the use of a 50 year return period for the determination of the extreme environmental loads.

For a structure, designed on the basis of this recommendation and with an expected lifetime of 30 years, the probability that the design loads will be exceeded during its lifetime is 0.45. Whereas a probability of 0.26 would be obtained if the structure had been designed to withstand environmental loads on a return period of 100 years\(^93\).

However, a design based on the 100 year return period may well be over designed and uneconomical - the 100 year wave, for example, may never occur during its lifetime. Hence a compromise between acceptable
risk that the design loads will be exceeded and the cost of repairing or strengthening the structure should be reached.

In this design, the recommendation of the Department of Energy was followed and the tank, with an expected life of 30 years, was designed on the 50 year criterion. It should be noted, though, that the application of factors of safety in the design would ensure that even under the most severe load combinations, the structure should be able to withstand a load at least 20% greater than the 50 year design load [BS 6235:cl.3.2.2.1]76.

The different types of environmental loads considered in this design were those due to:

(i) wind;
(ii) waves;
(iii) currents;
(iv) sea ice and icebergs;
(v) marine growth; and
(vi) earthquakes.

The first four types of environmental loads are discussed in sections 7.3.4.1 to 7.4.4.4 respectively whereas the other two are discussed in the dynamic analysis section of this chapter, i.e. section 7.4.

7.3.4.1 Wind load during construction

During the construction period and whilst waiting to be towed out to site, the structure could be subjected to loads due to the wind. It would be of extreme importance that the structure could withstand such loads safely so that it could commence its working life in good conditions. The economic cost of a structure failing before it started to fulfil its design purpose could be very high.

The pressure exerted by the wind at any point on the surface of a structure is given by [CP3:Chap.V:cl.4.3]88.
\[ P = \frac{1}{2} C_p \rho V_s^2 \]  

(7.1)

where \( C_p \) is the pressure coefficient, \( \rho \) is the density of air and \( V_s \) is the design wind speed.

However, this pressure varies around the tank and its distribution is determined by the pattern of airflow over the surface of the structure. For an axisymmetric drop-shaped tank, the pressure distribution along a parallel circle would be dependent on \( \theta \), the circumferential coordinate and the above equation becomes:

\[ P(\theta) = \frac{1}{2} C_p(\theta) \rho V_s^2 \]  

(7.2)

This equation represents the load due to a steady wind but in practice, dynamic response to turbulent winds is also evident and needs to be taken into account.

Hence a further modification needs to be included in the above equation to take account of the fluctuating wind forces arising from speed fluctuations of turbulent winds. This is achieved by multiplying the pressure by a dynamic gust factor, \( G \), \(^{85,86,87}\) resulting in equation 7.3 below.

\[ P(\theta) = \frac{1}{2} G C_p(\theta) \rho V_s^2 \]  

(7.3)

The next three sections deal with the determination of the design wind speed, the pressure distribution over the tank and the dynamic gust factor. Once these three variables have been determined an assessment of the magnitude of the load due to the wind can then be made.

### 7.3.4.1.1 Design wind speed

The construction of the storage tank is to take place in a dry dock in the Firth of Forth, where the maximum three second gust speed at a height of 10 metres above the ground with a recurrence period of 50 years is 50 m/s \(^{88}\).
It should be noted, though, that the vessel is only likely to be out of the water and directly exposed to the wind for a short period of time and the probability that this wind speed is exceeded in a short period of time such as 1 year is only 2%. The corresponding mean hourly wind speed at a height of 10 metres above the ground with a recurrence period of 50 years is 50/1.5 = 33.34 m/s, where 1.5 is the static gust factor 88.

7.3.4.1.2 Pressure distribution over the tank's surface

In the absence of numerical or experimental data on the distribution of pressure across the surface of the Echinodome, the distribution of pressure across the surface of an equivalent sphere was considered.

Considerable work has been done on the flow across spherical objects but most of the work has been based on sub-critical and super-critical flow. Unfortunately, Reynolds number for wind flow across the tank is rather high.

i.e. \[ R_e = \frac{\ddot{u} d}{v} = 50 \times 43.37 \div 1.51 \times 10^{-5} = 1.4 \times 10^8 \]

where \( \ddot{u} \) = mean velocity of flow = 50 m/s
\( d \) = average diameter of the tank
\[ d = \frac{1}{2} (34.67+52.06) = 43.37 \text{ m} \]
\( v \) = kinematic viscosity of air = \( 1.51 \times 10^{-5} \text{ m/s} @ 20^\circ C \)

Thus the flow is transcritical and very little information is available on the pressure distribution for such flow.

The potential flow theory 89 predicts a pressure distribution across spheres which is independent of Reynolds number. But various researchers have found that this was far from the truth. Experiments 90,91 have shown that separation takes place at different places along the surface depending on the velocity of flow and that the distribution was influenced by many
other factors such as turbulence and the roughness of the surface. The potential flow theory would result in an overdesigned structure as can be seen from the comparison of experimental and theoretical pressure distributions in Fig. 7.9.

On the other hand, the Swiss code of practice \(^{92}\), the only existing code of practice providing guidance on the pressure distribution across spheres, recommends a distribution which lies in-between the experimental and theoretical distributions, see Fig. 7.9. This seemed to provide the best compromise and was employed in this work.

It should be noted that the pressure distributions in the Australian and American codes of practice are based on this Swiss code \(^{92}\) and have stood the test of time.

The meridional variation of the pressure on the surface of the Echinodome was assumed to vary according to the power law for the wind profile, see equation (7.5) et seq..

7.3.4.1.3 Dynamic gust factor

The dynamic response of the tank to the randomly varying wind loads imposed by the turbulent winds can be taken into account by the use of a dynamic 'gust factor', \(G\).

The British code of practice \(^{88}\) to date, provides no guidance relating to such response but a number of simplified gust procedures for the assessment of the gust factor have been suggested for inclusion in the code. In this design three of these simplified procedures were considered, namely those proposed by Davenport \(^{85}\), Vickery \(^{86}\) and Kanda \(^{87}\).

All these procedures are based on structures with a rectangular cross section but in the absence of any information based on other shapes these methods had to be used. The tank was thus assumed to have a rectangular cross section. A structural damping ratio of 2% was assumed for the outer concrete shell which has a predicted fundamental frequency of 2.2 Hz in air, see section 7.4.1.
The wind characteristics parameters were those proposed by Kanda\textsuperscript{87} for an open site.

The results obtained from these methods are tabulated in table 7.4 below. For a detailed description of the procedures the reader is referred to the published literature\textsuperscript{85,86,87}.

<table>
<thead>
<tr>
<th>Method</th>
<th>Gust Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Davenport</td>
<td>2.16</td>
</tr>
<tr>
<td>Vickery</td>
<td>2.00</td>
</tr>
<tr>
<td>Kanda</td>
<td>2.01</td>
</tr>
</tbody>
</table>

Table 7.4 Dynamic Gust Factor for the tank

All three methods gave similar results for the dynamic gust factor as can be seen in the table above. Davenport's approach seems to be conservative but this can be attributed to the fact that it assumes some of the wind characteristic parameters and various other characteristics such as a dynamic drag coefficient equal to the static drag coefficient. The two other methods gave gust factors which were within 0.5\% of each other. Both these methods allow the designer to make use of meteorological data available for a particular site and are more flexible to use than Davenport's approach.

In this work, Kanda's method was used. Although based on Vickery's method, it allowed for more flexibility and the availability of a computer program RESPONSE\textsuperscript{87} for determining the dynamic response (including the dynamic gust factor) of a structure to turbulent winds made it a very attractive proposition. Therefore, a value of 2.01 for the gust factor was employed in this design.

A typical input file for use with the program RESPONSE and its corresponding output file is given in appendix 2.7.
7.3.4.1.4 Finite element analysis of wind loading

Wind loading is a form of symmetric loading and hence the recommended finite element for such loadings, i.e. element 42130 in PAFEC, was employed for this purpose.

The pressure distribution recommended by the Swiss code \(^92\) was first divided into a series of Fourier components as given by equation 7.4 below.

\[
C_p(\theta) = -0.2 + 0.2035 \cos \theta + 0.9513 \cos 2\theta \\
+ 0.1374 \cos 3\theta - 0.05 \cos 4\theta \\
- 0.018 \cos 5\theta
\] (7.4)

The numerical calculations involved in obtaining the above Fourier series are shown in appendix 5.2.

The meridional variation in the wind load was idealised as shown in Fig. 7.10.

The velocity of the wind was evaluated from equation 7.5 at each of the five steps.

\[
\bar{U}(Z) = (Z/H)^\alpha \bar{U}(H)
\] (7.5)

where \(\bar{U}(H) = 50\text{m/s}\) is the mean wind speed at a reference height; \(H = 10\text{m}\) is the reference height at which the mean wind speed is determined; \(\bar{U}(Z)\) is the wind speed at a height \(Z\) and \(\alpha = 0.15\) is the power law exponent for the mean wind speed profile.

On substituting all these values in Eq. 7.5 the following is obtained:

\[
\bar{U}(Z) = 33.34 (Z/10)^{0.15}
\] (7.6)

Hence, from Eqs 7.3 & 7.6 the pressure at any height, \(Z\), on the tank can be expressed as:

\[
P(\theta,Z) = \frac{1}{2} \, G \, C_p(\theta) \, \rho \, V_s(Z)^2
\]
i.e. \[ P(\theta,Z) = 1340 \ C_p(\theta) \ (Z/10)^{0.3} \] (7.7)

where the density of air, \( \rho \), is 1.2 kg/m\(^3\).

From equation 7.4, the maximum pressure would occur when \( \theta = 0 \).

In which case \( C_p(\theta) = 1.02 \) and the meridional pressure distribution is given by:

\[ P(Z) = 1366.8 \ (Z/10)^{0.3} \] (7.8)

Further more equation 7.7 can be rewritten as

\[ P(\theta,Z) = 1340 \ (Z/10)^{0.3} \sum_{m=0}^{5} A_m \cos m\theta \] (7.9)

This equation is in a form suitable for PAFEC and can be input as a pressure module in PAFEC.

The results from the finite element analysis of the outer concrete jacket under this wind load are shown in Fig. 7.11.

7.3.4.2 Wave loading

The waves are likely to have a substantial influence on the design of an underwater structure. They will produce a dynamic load on the tank whether by increasing the head of water above it or by exerting a drag force and an inertia force on the structure as the wave crest moves over it.

There are two basic methods for evaluating the load due to the wind-generated waves on a fixed offshore structure, the design wave method and the wave spectrum method \(^93\). Both these methods are semi-empirical. Theoretical considerations are involved in the determination of the wave characteristics but empirical drag and inertia coefficients are required to predict the loads.
The deterministic design wave method is the one most often used in designing offshore structures and was the method used in this work. This static method has been found satisfactory in shallow and moderate depth and is hence well suited for the specific location considered in this design.

7.3.4.2.1 The design wave

The design wave is the wave that causes the worst loading on a structure. It is usually specified in terms of a wave height, direction and a range of possible periods.

As for the wind loading, a return period of 50 years was used on the recommendation of the Department of Energy. For the location considered, the 50 year storm wave height for a fully developed storm lasting 12 hours is 28 metres [BS 6235: Fig.2.3] with a corresponding wave period of 15 seconds [BS 6235: Fig.2.4].

However the maximum forces on large volume structures, such as storage tanks, are dependent on the wave period selected and the wave period corresponding to the maximum wave height might not produce the worse load. The Department of Energy therefore recommends the use of a range of wave periods up to a maximum of 20 seconds. As a result, the design wave used in this design had the characteristics listed below,

- Wave height, (H) = 28 m
- Range of wave periods, (T) = 14 to 20 s
- Direction of wave = any direction

7.3.4.2.2 Wave Theories

Numerous wave theories are available to model the wave profile and evaluate the wave particle velocities and accelerations. They range from a simple linear wave theory to complicated higher order theories. Each theory is valid for a specific range of water depth, wave height and wave
period and Fig. A7.2 in appendix 7.2 shows the range of validity for various wave theories.

For this particular design $H/T^2$ ranges from 0.07 to 0.12 m/s$^2$ and $d/T^2$ ranges from 0.20 to 0.36 m/s$^2$ where $d$ is the depth of water from the seabed to the mean still water level. Hence from Fig. A7.2 in appendix 7.2, Stokes 5th order wave theory is the best suited for this location. However, this theory involves the solution of a 5th order polynomial which requires the use of a computer and is time-consuming in terms of cpu time. A quicker and more economical solution could be provided by the linear Airy theory and consequently the use of this theory was also examined in this work to determine whether it could be employed in the initial stages of design.

An outline of these two theories can be found in appendix 7.1.

7.3.4.2.3 Comparison of the two wave theories

Two computer programs were written to determine the wave characteristics when using the linear theory (Airy) and the 5th order theory (Stokes) and a flow diagram of these programs can be seen in appendix 1.6. These programs were subsequently used to determine the characteristics of the design wave and the results obtained for two periods are compared in table 7.5, with the difference between them expressed as a percentage of the Stokes 5th order theory values.
<table>
<thead>
<tr>
<th></th>
<th>Airy</th>
<th>Stokes</th>
<th>%</th>
<th>Airy</th>
<th>Stokes</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear</td>
<td>5th</td>
<td>diff</td>
<td>Linear</td>
<td>5th</td>
<td>diff</td>
</tr>
<tr>
<td>d/L</td>
<td>0.25</td>
<td>0.235</td>
<td>+6.4</td>
<td>0.167</td>
<td>0.159</td>
<td>+5.0</td>
</tr>
<tr>
<td>L (m)</td>
<td>322.5</td>
<td>343.9</td>
<td>-6.2</td>
<td>483.2</td>
<td>508.9</td>
<td>-5.1</td>
</tr>
<tr>
<td>( \overline{c} ) (m/s)</td>
<td>21.5</td>
<td>22.9</td>
<td>-6.1</td>
<td>24.6</td>
<td>25.5</td>
<td>-3.5</td>
</tr>
<tr>
<td>u (m/s)</td>
<td>2.89</td>
<td>2.92</td>
<td>-1.0</td>
<td>3.73</td>
<td>3.87</td>
<td>-3.6</td>
</tr>
<tr>
<td>a (m/s²)</td>
<td>1.21</td>
<td>1.18</td>
<td>-4.7</td>
<td>1.17</td>
<td>1.09</td>
<td>-12.0</td>
</tr>
</tbody>
</table>

Table 7.5 Comparison of the two wave theories
(where \( L \) = wave length; \( \overline{c} \) = wave celerity; \( u \) = maximum wave particle velocity - horizontal component; and \( a \) = maximum wave particle acceleration - horizontal component)

### 7.3.4.2.4 Wave induced forces

The most commonly used approach for the calculation of the wave induced loads on a rigid structure is that proposed by Morison. However, Morison's equation can only be used if the motion of the water particles is unaffected by the presence of the structure itself. Hence there is a limit to the size of structure for which this method is applicable. The generally accepted limit is:

\[
D/L \leq 0.2
\]

where \( D \) is the width of the structure

In this case \( D = 52.06 \)m and from table 7.5, the value of \( L \) which gives the highest ratio is \( L = 322.5 \)m resulting in:

\[
D/L = 52.06/322.5 = 0.16
\]

Thus Morison's equation was valid for use in this design. It should be noted that for larger structures the diffraction theory should be used.
Morison's equation states that the total wave induced force on a submerged object can be expressed as the sum of the drag force due to the flow velocity and the inertia force due to the acceleration of the water flowing past the object.

\[ F = F_i + F_d \]
\[ = C_m V_d \frac{du}{dt} + \frac{1}{2} C_p \rho A |u| u \]

where \( C_m \) = coefficient of inertia; \( C_d \) = coefficient of drag; \( V_d \) = volume of fluid displaced; \( \rho \) = density of the fluid; \( A \) = cross sectional area of structure; \( u \) = horizontal water particle velocity; and \( \frac{du}{dt} \) = horizontal water particle acceleration.

The horizontal water particle velocities and accelerations are obtained from the appropriate wave theory.

**7.3.4.2.5 Drag coefficient**

The drag coefficient is obtained from the consideration of Reynolds number. Reynolds number is in turn dependent on the velocity of the water particles and will thus vary down the meridian of the tank. However in this work, the maximum velocity, which occurs at the top of the tank, was used to determine Reynolds number and the drag coefficient. This drag coefficient was taken to be constant down the meridian of the tank.

Again in the absence of any experimental work based on the flow past the Echinodome, an equivalent sphere was used in the evaluation of Reynolds number.

From table 7.5 the maximum velocity is 3.87 m/s. Assuming an equivalent sphere of diameter 43.47m, then using a kinematic viscosity of sea water, \( \nu = 1.3 \times 10^{-6} \text{m/s} \), Reynolds number, \( R_e = 1.3 \times 10^8 \).

Limited information is available on transcritical flow across a sphere, but experimental work carried out by Achenbach \(^{90,91}\) predicted a drag coefficient of 0.2 for this Reynolds number and this value was employed in this work.
7.3.4.2.6 Mass coefficient

Similarly an equivalent sphere was used to determine the mass coefficient for the Echinodome. This mass coefficient is independent of the type of flow but is related to the added mass due to the amount of water moving with the structure. Experimental and theoretical work predicts a value of $C_m = 0.2$ for a spherical object and this value was used in the design.

7.3.4.2.7 Evaluation of the maximum wave-induced forces

In order to obtain the wave-induced forces acting on the tank, the structure was divided down the meridian into a number of segments. The velocity and acceleration of the flow was determined at each segment and the corresponding elemental forces were evaluated. The total load on the tank was then determined by the summation of all these elemental forces.

The two computer programs, mentioned earlier, were extended to deduce these forces. The total maximum drag and inertia loads on the structure were computed independently for the design wave using both theories and a comparison of the results obtained is shown below in table 7.6 with percentage differences related to the Stokes values.

<table>
<thead>
<tr>
<th>Wave Period (s)</th>
<th>Drag force</th>
<th>Inertia force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Airy</td>
<td>Stokes</td>
</tr>
<tr>
<td>14</td>
<td>1.003</td>
<td>0.994</td>
</tr>
<tr>
<td>15</td>
<td>1.184</td>
<td>1.213</td>
</tr>
<tr>
<td>16</td>
<td>1.418</td>
<td>1.426</td>
</tr>
<tr>
<td>17</td>
<td>1.565</td>
<td>1.631</td>
</tr>
<tr>
<td>18</td>
<td>1.754</td>
<td>1.827</td>
</tr>
<tr>
<td>19</td>
<td>2.021</td>
<td>2.016</td>
</tr>
<tr>
<td>20</td>
<td>2.049</td>
<td>2.195</td>
</tr>
</tbody>
</table>

Table 7.6 Maximum wave-induced forces on the tank
The table above shows that the inertia loads were predominant and that the inertia forces predicted by the linear wave theory were higher than those obtained from the non-linear theory. Yet, the values obtained for the drag force from the linear theory were lower than those obtained from the non-linear theory. This suggests that, in cases such as this one where inertia loads predominate, the linear wave theory could be used satisfactorily in the initial stages of design but the subsequent use of the non-linear wave theory would be essential to obtain a more accurate solution and avoid an overdesigned structure.

It is interesting to note, nevertheless, that both theories agreed in predicting a maximum drag force for a wave with a period of 20s. But, whilst the linear theory predicted a maximum inertia load for a wave period of 19s the non-linear theory predicted a maximum load for a period of 16s.

However, both these forces vary with time, according to the position of the wave with respect to the structure. Appendix 7.2 indicates that both forces are trigonometrical functions and are out of phase. Fig. 7.12 shows the variation of these forces with respect to time.

As this figure suggests the peak force occurs at

$$\beta(x-ct) = 90^\circ$$

where $$\beta = 2\pi/L$$ and is entirely an inertia force. This implies that the maximum force leads the wave crest (or trough) by 90°.

A diagram showing this position of the wave with respect to the structure together with the meridional load distribution is given in Fig. 7.13 for the design wave.
7.3.4.2.8 Finite element analysis of wave loading

The wave loading is another form of symmetric but non axisymmetric load and hence a Fourier series is also required to describe the circumferential load distribution. Appendix 5.3 shows that the required Fourier series is

\[
P(\theta) = 0.5 - 0.6366 \cos \theta + 0.2122 \cos 3\theta \\
- 0.1273 \cos 5\theta + 0.0909 \cos 7\theta \\
- 0.0707 \cos 9\theta
\] (7.11)

where \( \theta \) is the circumferential coordinate measured from the upstream point.

The meridional load distribution shown in Fig 7.13 is input as a load module in PAFEC and is applied at each of the three ring nodes of the three noded thin shell of revolution element.

The stress distributions induced by the design wave both with a period of 16s and 20s using the non-linear Stokes theory are given in Figs 7.14 & 7.15.

7.3.4.3 Current loading

The presence of a current may alter the propagation of the waves or even change the wave profile. However, the most significant contribution due to the current with respect to submerged structures is the drag force it will exert on the structure.
7.3.4.3.1 Design current velocity

The maximum current velocity to be used in design should take account of the tidal current and wind generated current. The maximum tidal current for the location considered is 0.375 m/s [BS 6235:Fig. 2.5] \(^7^6\) and during strong sustained winds this velocity can be increased by 0.51 m/s. The resulting design current current is therefore 0.89 m/s. However most offshore structures in the North Sea are designed to withstand a maximum current of 1.5 m/s \(^9^8\) and hence in this design also this maximum current velocity was considered.

7.3.4.3.2 Effect of the current on the wave height

The current can increase or decrease the wave height depending on whether it is opposing or following the wave. The modified wave height in a current can be obtained from the following equation \(^8^4\):

\[
\frac{H_c}{H} = \frac{2}{1 + 4U_c/c + \sqrt{1 + 4U_c/c}} 
\]

(7.12)

where \(H\) = wave height in still water; \(H_c\) = wave height in a current; \(c\) = wave celerity in still water; and \(U_c\) = current velocity ( +ve if following the wave and -ve if opposing)

This equation was used to determine the wave height of the range of design waves in a current and the results are given in table 7.7.
Wave Height (m)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>26.38 30.02</td>
<td>26.47 29.88</td>
<td>25.42 31.75</td>
<td>25.58 31.46</td>
</tr>
<tr>
<td>15</td>
<td>26.43 29.93</td>
<td>26.52 29.82</td>
<td>25.51 31.61</td>
<td>25.64 31.33</td>
</tr>
<tr>
<td>17</td>
<td>26.51 29.82</td>
<td>26.58 29.71</td>
<td>25.63 31.35</td>
<td>25.74 31.13</td>
</tr>
<tr>
<td>18</td>
<td>26.54 29.79</td>
<td>26.61 29.68</td>
<td>25.68 31.26</td>
<td>25.78 31.06</td>
</tr>
</tbody>
</table>

Table 7.7 Modified wave height in the design current using both linear and non-linear wave theory

This would result in a slight increase or decrease in the hydrostatic pressure due to the passage of the wave over the tank, depending on whether the current is opposing the wave or following the wave respectively.

7.3.4.3.3 Current drag

The current will exert a constant drag force on the structure. In the absence of waves, this current drag can be evaluated straight from the vortex flow component of Morison's equation:

\[ F_d = \frac{1}{2} \rho C_d A U_c |U_c| \]

Since little information is available regarding the variation of the current with depth, the surface current was assumed to be constant all the way down to the seabed.

Following the same principle as for the wave-induced drag force, the total load on the tank was calculated. This was found to be 0.1212 MN for the design current and 0.3443 MN for a current with a velocity of 1.5 m/s.
7.3.4.3.4 Combined current and wave drag

The most common way of dealing with the combined current and wave drag is by summing vectorially the wave and current induced water particle velocity and by subsequently employing Morison’s equation to determine the total force. This was done using the two programs mentioned previously and the results are shown in the table below for the different wave and current combinations. It should be noted that this table shows the maximum load which occurs when the current is following the wave. When the current is opposing the waves, the combined water particle velocity would be lower and the resulting drag force would also be smaller.

<table>
<thead>
<tr>
<th>wave period (s)</th>
<th>Linear current 0.89m/s</th>
<th>Linear current 1.5m/s</th>
<th>Non-linear current 0.89m/s</th>
<th>Non-linear current 1.5m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>1.819</td>
<td>2.518</td>
<td>1.807</td>
<td>2.505</td>
</tr>
<tr>
<td>15</td>
<td>2.054</td>
<td>2.794</td>
<td>2.099</td>
<td>2.847</td>
</tr>
<tr>
<td>16</td>
<td>2.367</td>
<td>3.158</td>
<td>2.378</td>
<td>3.170</td>
</tr>
<tr>
<td>17</td>
<td>2.557</td>
<td>3.376</td>
<td>2.641</td>
<td>3.473</td>
</tr>
<tr>
<td>18</td>
<td>2.797</td>
<td>3.652</td>
<td>2.889</td>
<td>3.757</td>
</tr>
<tr>
<td>19</td>
<td>3.131</td>
<td>4.032</td>
<td>3.125</td>
<td>4.025</td>
</tr>
<tr>
<td>20</td>
<td>3.167</td>
<td>4.073</td>
<td>3.347</td>
<td>4.277</td>
</tr>
</tbody>
</table>

Table 7.8 Maximum current and wave induced drag force using both linear and non-linear wave theories

7.3.4.3.5 Finite element analysis of current drag

The finite element analysis of the current drag is carried out in the same manner as for the wave load and has been described previously in section 7.3.4.2.8. The resulting stress distributions on the outer jacket induced by the design current and maximum current are shown in Figs 7.16 & 7.17.
7.3.4.4 **Sea ice and icebergs**

Sea ice only forms along the coastline of the North Sea [BS 6235:cl.2.11] and only in the most severe conditions. Therefore it would not be a problem around the location of the structure.

Similarly, icebergs would not cause any concern since the area under consideration is well away from the iceberg plough lines.

7.3.5 **Loads during normal operating conditions**

The analyses carried out in the previous sections have shown that the greatest load on the outer jacket during normal operating conditions was the hydrostatic load. This is encouraging since the design of the drop shaped outer jacket was based on a hydrostatic pressure head and therefore supports the use of such a structure in the submarine environment.

Fig. 7.6(a) shows the membrane forces and stresses induced on the outer shell by a depth of water corresponding to the mean sea water level, i.e. the design head. As expected, the stresses induced were all compressive and remained uniform and equal to 25MN/m$^2$ (the design stress) over most of the structure except near the base.

In this critical zone near the base, there was a considerable variation in stress but no tension was evident. The maximum induced stress was meridional and equal to 30.6MN/m$^2$, which provided a factor of safety of approximately 2 with respect to the characteristic strength of the concrete selected.

The displaced shape under this load is given in Fig. 7.6(b) and shows that not much deformation was evident except in the critical zone.

During normal environmental conditions this level of water would be expected to change by ± 1m with the tide but this change would make little difference on the stressing of the shell.
The loads due to the 50 year return wave and the maximum current were small compared with this hydrostatic load, see tables 7.6 & 7.8, and from that it is reasonable to assume that any wave or current loading experienced during normal environmental conditions would be insignificant and could be neglected.

With regards to the inner tank during normal operating conditions the stresses induced by the crude oil and the thermal differential across its wall are shown in Figs 7.4 & 7.8.

These figures show that over most of the tank the maximum stresses were induced as a result of the internal pressure and were tensile. The meridional stresses were uniform at around 160MN/m² and the circumferential stresses started off at a maximum of 165MN/m² at the apex and then decreased gradually down the meridian; hence providing a factor of safety of at least 3 with respect to the ultimate tensile strength of the steel i.e. 490MN/m².

However, in the critical zone near the base, the stresses induced are of the same order as that of the ultimate tensile strength of the material and hence a steel with the maximum ultimate tensile strength of 610MN/m² would be essential in this region.

The effect of the thermal loading on the inner tank was small compared to this internal pressure loading and hence should not cause any problems to the inner tank.

7.3.6 Loads during extreme environmental conditions

The design of underwater structures, such as this storage tank is governed mainly by the severe environmental loadings that can be exerted on it as a result of extreme storm conditions at the surface.

Section 7.3.4.2 indicated that waves not only exert a drag and an inertia force on a structure but also increase the head of water above it as the wave crest moves over it. The maximum drag force and inertia force due
to the design wave were small compared with the hydrostatic load applied as it moves over the structure. The former, in particular, could be neglected safely since the stresses induced by it were less than 2% of the design stress, see Figs 7.14(b) & 7.15(b) whilst on the other hand the inertia loads induced stresses in the critical zone which were close to the design stress, see Figs 7.14(a) & 7.15(a) and hence cannot be ignored. However the worst condition would arise as a result of the increase in head of water above the structure as the crest of the wave passes over the tank and more so if the current at that time was opposing the wave.

Consequently the worst load combination on the outer jacket due to severe environmental conditions was considered to occur when the maximum current of 1.5m/s was opposing the design wave with a period of 14s, resulting in a wave height of 31.46m (see table 7.7) and the crest of this wave was directly above the centreline of the tank. In addition to this the tide was assumed to be at its highest (i.e. mean high water spring) and coincided with the maximum 50 year wind and pressure induced surges, as shown in table 7.3. This would result in a hydrostatic load due to a head of water equal to 58.1m and the membrane forces and stresses induced under this load are shown in Fig. 7.18(a).

This figure shows that over most of the structure, the meridional surface stresses were compressive and approximately uniform around 36MN/m$^2$. Whilst the circumferential surface stresses, which also were compressive, started off at the apex with a maximum stress of 36MN/m$^2$ and gradually decreased down the meridian. Although these stresses were above the design stress, they provided a factor of safety of 1.67 with respect to the characteristic strength of the concrete and hence is acceptable in order to ensure the integrity of the structure during extreme environmental conditions.

However, within the critical zone the meridional surface stresses were of the order of the characteristic strength and thus a higher grade concrete would be required. A concrete with a characteristic strength of 80MN/m$^2$ would give a factor of safety of approximately 1.2 and this would suffice to ensure the safety of the structure.
Also in this region, there were significant tensile stresses induced and hence some prestressing would be required to counteract it.

With regards to the maximum inertia load due to the design wave, this would only arise when the centreline of the tank was directly below the midpoint between the wave crest and the trough, see Fig. 7.13. In this case there is no increase in the hydrostatic load acting on the structure as a result of the wave.

Nonetheless, assuming that the sea water level was at its highest, as in the previous case, and that the maximum current was flowing then the resulting membrane forces and stresses were as shown in Fig. 7.18(b).

This figure shows that this load case was not as critical as the one examined above and therefore could be withstood safely by the outer jacket with the modified critical zone.

7.3.7 Loads prior to launching

On completion and prior to the initial floatation the structure would be subjected to a dead load due to its self weight and wind loading.

Fig. 7.11 shows the maximum stress distribution on the outer jacket as a result of the wind loading and indicates that the stresses were all less than 1MN/m², i.e. 4% of the design stress and hence could be neglected safely.

On the other hand the effect of the self weight on both the outer jacket and inner tank were a bit more critical especially within the critical zone. Figs 7.1 & 7.2 indicate that there was considerable bending in the critical zone of the outer jacket and inner tank respectively under their own weight. However, all the stresses were below the design stress and therefore should not present any problems. Although it is important to note that some tension was evident in the critical zone of the outer jacket and the prestressing required in the previous section also would be beneficial for this load case.
7.3.8 **Loads during initial floatation**

In order to avoid excessive loads on the structure during the launching process, the usual method of initial floatation for large sea structures would be followed.\(^{101}\)

The inner tank would first be filled slowly with sea water to a predetermined level so as to enable the vessel to float with its maximum diameter at sea level. This level was determined by the consideration of Archimedes principle and was found to be 1.4m. Thus part of the inner tank would be subjected to a small hydrostatic pressure due to this amount of water contained within it.

Once the inner tank had been filled to the required level, the dry dock in which the structure had been constructed would be flooded so as to lift the vessel off the bottom of the ground. The sea water would be admitted into the dock slowly and at a constant rate to avoid any sudden loads on the structure. At the same time air would be admitted into the air gap in between the two walls to maintain the air pressure at atmospheric pressure.

On completion of the flooding, the tank would be floating with the submerged section of the outer jacket subjected to a hydrostatic pressure. It should be noted that the level of water in the dock would be such that in the event of a low tide, the base of the structure would still be floating clear of the bottom.

7.3.9 **Loads during the tow-out**

The tow-out would only be undertaken when the long term weather forecast was favourable and preferably during the months of July and August when gales were infrequent, occurring only 2% of the time\(^ {84}\) and there were no dangers of collision with sea ice and icebergs. A suitable tow route should be chosen to take account of the size of the structure, direction of tidal streams and navigational hazards.
During tow-out in calm conditions, the structure would continue to be subjected to the hydrostatic pressures mentioned in the previous section. However, an extra load case must be considered, i.e. the load due to the towing wires, which would be distributed into the shell by means of local thickening of the outer jacket around the point of attachment.

A towing configuration suitable for towing out large structures drop-shaped structures was proposed previously \(^{12}\) (see Fig. 9 in appendix 8.1).

In these calm conditions any loads due to the waves, wind, sea spray etc. would be minimal and were not considered in this design.

Nonetheless the possibilities of a sudden change in environmental conditions cannot be ignored, even in periods of calm weather. To this end it was envisaged that in such deteriorating environmental conditions the structure would be temporarily submerged to a depth less than or equal to its design head in order to override the storm.

This would be achieved by introducing more seawater into the inner tank. Hence the inner tank would be subjected to an increased hydrostatic pressure depending on the level of water within it. The hydrostatic loads on the outer jacket would also be increased according to the submerged depth.

However, these loading conditions would be equal to or less than those described in sections 7.3.3 and 7.3.4 and therefore would not be critical. It should be noted that in this configuration, the towing wires would not be taking any loads since the structure would be neutrally buoyant and can therefore be safely neglected.
7.3.10 Loads during installation

On reaching the required location, the structure would be lowered slowly onto the seabed, see Fig. 7.19. This lowering would be achieved by introducing sea water into the inner tank and would take approximately 2 hours.

The structure would be neutrally buoyant at all times in order to ensure a controlled descent to the seabed and hence no load would be taken by the towing wires. During the installation, the towing wires and tugs would only be used for positioning the vessel at the correct location.

The only loads experienced during this stage would be the increased hydrostatic load in the inner tank due to the water contained within it and the load on the outer jacket due to the head of water above it. Again both these hydrostatic loads would be less critical than that examined in section 7.3.3. and will not be considered here.

7.4 Dynamic analysis

The second stage in the analysis of the oil storage tank was the examination of its dynamic behaviour both in air and in the submarine environment.

This dynamic behaviour is dependent to a great extent on the natural frequencies of the structure. If its fundamental frequency was greater than the frequencies of the environmental forces then the dynamic amplification of those loads would be small. However, if one of the natural frequencies of the structure coincided with the frequency of an environmental load then resonance could occur together with a significant dynamic amplification of that load.

Hence the first step in this dynamic analysis was the determination of the natural frequencies of the drop-shaped tank.
7.4.1 Free vibration analysis

The natural frequencies of both the inner tank and the outer jacket in air were calculated using the Mistry program and the results are tabulated in tables 7.9 & 7.10 below.

<table>
<thead>
<tr>
<th>Meridional Modes (m)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Torsional Mode (n=0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.43</td>
<td>2.20</td>
<td>18.99</td>
<td>19.97</td>
<td>20.76</td>
<td>13.68</td>
</tr>
<tr>
<td>2</td>
<td>16.41</td>
<td>13.61</td>
<td>19.32</td>
<td>20.79</td>
<td>21.69</td>
<td>41.74</td>
</tr>
<tr>
<td>3</td>
<td>18.18</td>
<td>18.00</td>
<td>19.96</td>
<td>21.61</td>
<td>22.63</td>
<td>64.14</td>
</tr>
<tr>
<td>4</td>
<td>18.97</td>
<td>18.84</td>
<td>20.62</td>
<td>22.36</td>
<td>23.51</td>
<td>85.32</td>
</tr>
</tbody>
</table>

Table 7.9 Natural frequencies (Hz) of the outer jacket

<table>
<thead>
<tr>
<th>Meridional Modes (m)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Torsional Mode (n=0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64.58</td>
<td>2.36</td>
<td>27.34</td>
<td>28.84</td>
<td>29.30</td>
<td>21.65</td>
</tr>
<tr>
<td>2</td>
<td>23.25</td>
<td>19.69</td>
<td>28.22</td>
<td>29.35</td>
<td>29.87</td>
<td>63.12</td>
</tr>
<tr>
<td>3</td>
<td>25.54</td>
<td>25.34</td>
<td>28.38</td>
<td>29.86</td>
<td>30.44</td>
<td>96.82</td>
</tr>
<tr>
<td>4</td>
<td>27.30</td>
<td>27.12</td>
<td>28.77</td>
<td>29.98</td>
<td>30.94</td>
<td>128.83</td>
</tr>
<tr>
<td>5</td>
<td>27.93</td>
<td>27.93</td>
<td>29.16</td>
<td>30.35</td>
<td>31.73</td>
<td>160.09</td>
</tr>
</tbody>
</table>

Table 7.10 Natural frequencies (Hz) of the inner tank

It is interesting to note that the natural frequencies obtained for both the inner tank and outer jacket followed the same pattern as those obtained for the prototype in chapter 5. The fundamental frequency corresponding to a symmetric translational mode as shown in Fig. 5.2(a) and the second
natural frequency corresponding to an axisymmetric mode as shown in Fig. 5.1(a).

However these natural frequencies were only for the tanks in air. When submerged the effects of two other factors such as the added mass of water and the marine growth on the outer jacket has to be taken into account.

\[ 7.4.1.1 \text{ Effect of added mass on the natural frequencies} \]

The added mass is the amount of water that becomes entrained and moves with a structure oscillating in the marine environment. This mass of entrained water is dependent on the geometrical shape of the structure and is difficult to calculate.

In this work, and in the absence of any information regarding the added mass for the drop-shaped shell, the added mass of an equivalent sphere enclosing the same volume was considered.

The entrained mass of water for a sphere is given by \(96^\):

\[ M_{am} = 0.5 \times M \]

where \( M \) is the mass of the volume of water displaced by the sphere.

Hence the added mass for the tank is

\[ M_{am} = 0.5 \rho V_d \quad (7.13) \]

\[ M_{am} = 0.274 \times 10^8 \text{ kg} \]

where \( \rho = 1025 \text{ kg/m}^3 \) is the density of sea water and \( V_d = 53490\text{m}^3 \) is the volume of water displaced by the tank.

The total effective mass of the outer jacket therefore becomes:

\[ M_{\text{eff}} = \text{Mass of outer jacket} + \text{Added mass} \]
\[ M_{\text{eff}} = 0.385 \times 10^7 + 2.74 \times 10^7 \]
\[ M_{\text{eff}} = 3.125 \times 10^7 \]

This effective mass was then used to determine the natural frequencies of the outer jacket. As an approximation, an equivalent density was used in the Mistry program in order to increase the mass of the outer jacket without increasing its stiffness and the results of the finite element analysis are shown in table 7.11 below.

<table>
<thead>
<tr>
<th>Meridional Modes (m)</th>
<th>Circumferential Modes (n)</th>
<th>Mode (n=0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1.52</td>
<td>0.76</td>
</tr>
<tr>
<td>2</td>
<td>5.63</td>
<td>4.67</td>
</tr>
<tr>
<td>3</td>
<td>6.24</td>
<td>6.18</td>
</tr>
<tr>
<td>4</td>
<td>6.51</td>
<td>6.47</td>
</tr>
<tr>
<td>5</td>
<td>6.76</td>
<td>6.71</td>
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</table>

Table 7.11: Natural frequencies (Hz) of the outer jacket with the added mass of water

It should be noted that these values for the natural frequencies are likely to be low because of the approximations employed and thus any further analysis based on these values would be on the conservative side.

7.4.1.2 Effect of marine growth on the natural frequencies

After a structure has been underwater for a while, there would be a build up of marine growth producing an increase in mass without any significant change in stiffness. It is therefore necessary to predict the amount of marine growth, as an increase in mass would cause a further reduction in the fundamental frequency of the structure. The marine growth on a structure, submerged at a depth 30m below the mean water level or
greater, accounts for an extra mass of 80kg per unit surface area and this corresponded to a total mass due to fouling of $80 \times 6342 = 0.51 \times 10^8$ kg in this particular case.

The increase in geometric dimensions due to this growth would also produce a corresponding increase in the added mass of water moving with the structure and the effect of this increase needs to be considered. Thus assuming a total depth of growth equal to 100mm throughout the outer jacket, then the total volume of water displaced by the storage tank and the marine growth would be $54124m^3$ and from Eq. 7.13 this would correspond to an added mass of $0.275 \times 10^8$ kg.

The effective mass of the outer jacket therefore would be the sum of the added mass of water, the mass of the marine growth and its actual mass totalling $3.19 \times 10^7$ kg.

As in the previous section, a finite element analysis was carried out using this effective mass in order to take into account the marine growth and the results obtained are shown below.

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<th>Meridional Modes (m)</th>
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<th>Torsional Mode</th>
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<td>6.61</td>
<td>7.16</td>
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</tbody>
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Table 7.12 Natural frequencies (Hz) of the outer jacket together with the added mass and marine growth
7.4.2 Dynamic response

The previous section has shown that the fundamental frequency of the outer jacket, which is the part of the structure withstanding the environmental loads, was 2.20 Hz when in air and would reduce to 0.74 when submerged and the effects of the marine growth were evident. In this section the dynamic response of the outer jacket to the wave forces is examined and a brief assessment of the seismic loads expected is presented.

7.4.2.1 Wave loading

As mentioned earlier in this thesis, the waves are likely to exert the most critical dynamic load on a structure operating in the submarine environment. It is therefore important to consider the dynamic effect of the waves on the drop-shaped tank.

In section 7.3.4.2.1, the design wave chosen for this work had a range of wave periods ranging from 14 to 20s, corresponding to excitation frequencies in the range 0.05 to 0.07 Hz. Hence the ratio of the excitation frequency to the fundamental frequency of the outer jacket, i.e. the frequency ratio, \( Q \), was between 0.095 to 0.067.

The wave force amplification due to this frequency ratio is given by the magnification factor, \( Q \), which is given by\(^93\):

\[
Q = \frac{1}{\left[(1-\Omega^2)^2 + (2\zeta\Omega)^2\right]^{1/2}}
\]

where \( \zeta \) is the damping ratio.

Now, \( Q \) would be a maximum when \( \zeta = 0 \) and thus substituting for \( \Omega \), the maximum magnification factor becomes 1.0091.

This value for the magnification factor indicates that there is negligible force amplification and that a static analysis as that carried out in section...
7.3.4 would suffice in this design.\textsuperscript{93,100}

Also, it could be noted that based on the recommendations of the American Concrete Institute\textsuperscript{102} and other publications\textsuperscript{93,100}, a dynamic analysis was required only if the fundamental frequency of the marine structure is less than 0.5Hz.

### 7.4.2.2 Seismic loading

Earthquake resistant design is very important in regions where seismic effects may cause structural damage to the structure.

However due to the low probability of earthquakes occurring in the seas around Great Britain, BS 6235 cl.3.3.5.10\textsuperscript{76} states that seismic loading need not generally be considered and as a result was not examined in this thesis.

Nonetheless recommendations have been made\textsuperscript{103} on the 100 year ground motion for use in the design of structures for the North Sea and future work on the drop shaped-tank could examine its behaviour under that dynamic load.

A 100 year design peak ground acceleration of 1m/s\textsuperscript{2} is recommended and the corresponding response spectrum is shown in Fig. 7.20.

### 7.5 Buckling analysis

In the preceding chapter an initial check was made on the stability of both the inner tank and outer jacket. However this check was based on a linear buckling analysis in order to obtain a quick and approximate indication as to whether buckling would be a criterion in design.

A better indication of the buckling pressure head of both shells could be obtained from a non-linear buckling analysis which took into account the change in geometry of the structure as it was loaded. To this end non-linear buckling analyses were performed and the results showed that
the critical buckling pressure head for the outer jacket was 246m and that for the inner tank was 80m. Both corresponding to axisymmetric collapse.

Under extreme environmental conditions the head of water above the outer jacket could be expected to increase to 58.1m, see section 7.3.6, and hence the factor of safety against buckling would be 4.2. Whereas, the maximum pressure expected on the inner tank could be equivalent to a pressure head at the apex of 56.24m, see section 7.3.3.2, and hence a factor of safety against buckling of 1.4 would be obtained.

These factors of safety were considered to be adequate for this design.

7.6 Discussion

The examination carried out in this chapter has covered all the important loads that could arise during the life of the structure. However, other unexpected or accidental loads might occur and further work is necessary to identify and determine the magnitude of such loads. It is important that this type of structure does not fail catastrophically in such an event and a check would be made on the design so that the structure would be capable of absorbing most of the energy from these loads, even if there were considerable local damage.

In this design it is intended that the outer jacket would take all the load and that the inner tank would remain intact, thus avoiding a major oil spillage. Consequently, the response of the outer jacket to those loads, such as impact by a submersible, should be examined in future work on this design.

The internal pressure that could be expected on the outer jacket if the inner tank leaked would be balanced out by the external hydrostatic pressure and hence no problems would be expected with regard to oil spillage in this event. Also, with respect to the thermal loading arising as a result of this leakage, Fig. 7.3 shows that the maximum stresses induced would be below the design stress of the outer jacket and therefore would not be a cause for concern. The same would be the case if the outer jacket
leaked since the external pressure on the inner tank would balance out some of the internal pressure due to the crude oil and the thermal stresses, see Fig. 7.4, would all be well below the ultimate tensile strength of the material.

Also this assessment has not considered the fact that during the tow-out and installation, the structure would behave as a semi-submersible and dynamic loads would be present. In this work it was assumed that these loads would be negligible compared with those arising during the structure's operational life and thus were not considered. Nonetheless it is necessary that any future work on the tank examines these dynamic effects in some detail.

It should be noted, though, that this analysis has not taken into account any openings in the structure for pipeline connections. However the effect of a 1m hole at the apex of the outer jacket as shown in Fig. 6.4 has been examined briefly and the results show that the difference in the overall behaviour of the structure was negligible. The membrane forces and stresses under the design head are shown in Fig. 7.21 which compares favourably with those shown in Fig. 7.5.

The non-linear buckling pressure head was reduced by 5% to 234m of water providing a factor of safety of just over 4 but the fundamental frequency of the shell in air did not change.

In general this analysis has shown that the greatest loads were due to the hydrostatic pressure and this is very encouraging since the drop-shaped shell is the optimum form for a structure subjected to hydrostatic pressure
7.7 Summary and Conclusions

This chapter has dealt with the structural analysis of the underwater oil storage tank proposed in chapter 6.

An assessment was made of the different types of load which might be expected on the structure during its construction, tow-out, installation and operational stages and the responses of the structure to those loads were examined. This examination showed that the main loading was that due to hydrostatic loads and that the other loads, including the environmental loads, were rather low compared with them. However a higher strength concrete and some prestressing would be essential in the critical zone of the outer jacket and a higher strength steel would be required in the critical zone of the inner tank.

Then free vibration analyses of both the inner tank and outer jacket were carried out and the dynamic responses of the latter to wave loading was investigated. Negligible force amplification was predicted indicating that a static analysis would suffice in this design.

Following this non-linear buckling analyses were carried out and the results predicted adequate factors of safety against buckling.

Finally, other loadings were considered and recommendations were made for future work on this design.
TYPE OF ANALYSIS = LINEAR STATIC
GRAVITY LOADING
TOTAL MERIDIONAL LENGTH (L) = 55.813m
MERIDIONAL LENGTH = SL, APEX= A, BASE= B.

Fig. 7.1 Stress distribution in the outer jacket due to the self weight
TYPE OF ANALYSIS = LINEAR STATIC
GRAVITY LOADING
TOTAL MERIDIONAL LENGTH (L) = 51.752m

MERIDIONAL LENGTH = SL, APEX = A, BASE = B.

MAX/\(R\) ORIGINATE VALUE = (R=ORIGINATE LENGTH)/S

\[ \begin{array}{ccc}
0 & 0.5 & 1.0 \\
MIN/\(R\) & SL/L \\
\end{array} \]

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Fig. 7.2 Stress distribution in the inner tank due to the self weight
Fig. 7.3 Stress distribution in the outer jacket due to thermal loading.
**Fig. 7.4 Stress distribution in the inner tank due to thermal loading**

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**Type of analysis** = Linear Static

**Thermal loading**

**Total meridional length (L) = 51.752m**

**Meridional length = SL, Apex = A, Base = B.**

**Fig. 7.4 Stress distribution in the inner tank due to thermal loading**
Fig. 7.5 Hydrostatic pressure distribution on the outer jacket at the design head
TYPE OF ANALYSIS = LINEAR STATIC
DEPTH OF END A = 40.000m
TOTAL MERIDIONAL LENGTH (L) = 55.813m

MERIDIONAL LENGTH = SL, APEX = A, BASE = B.

Fig. 7.6(a) Stress distributions in the outer jacket at the design head
DISPLACEMENT OF SHELL AT DEPTH OF END A = 40.000m
TYPE OF ANALYSIS = LINEAR STATIC

Fig. 7.6(b) Displaced shape of the outer jacket at the design head
(displacements relative to original shape x 136)
Fig. 7.7 Pressure distribution on the inner tank due to the crude oil
TYPE OF ANALYSIS = LINEAR STATIC
DEPTH OF END A = 56.240m
TOTAL MERIDIONAL LENGTH (L) = 51.248m
MERIDIONAL LENGTH = SL, APEX = A, BASE = B.

Fig. 7.8 Stress distribution in the inner tank due to the crude oil
Fig. 7.9 Theoretical and experimental static pressure distribution across a sphere due to wind flow (θ is the circumferential coordinate measured from the upstream point).
Fig. 7.10 Meridional distribution of wind loading
TYPE OF ANALYSIS = LINEAR STATIC
WIND LOADING
TOTAL MERIDIONAL LENGTH (L) = 55.813m

MERIDIONAL LENGTH = SL, APEX = A, BASE = B.

MAX/R
ORDINATE VALUE = (ORDINATE LENGTH) / 5

MIN/R

0.0

0.5

1.0

SL/L

GRAPH

TITLE

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3 MERIDIONAL STRESS (INSIDE SURFACE) (N/m²) 594651.4 -193420.1 784879.8
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8 EQUIV. STRESS (OUTSIDE SURFACE) (N/m²) 0.0 -836225.3 836225.3

Fig. 7.11 Stress distribution in the outer jacket due to wind loading
Fig. 7.12 Variation of wave force with time

(a) Inertia force

(b) Drag force

$\beta'(x - \xi t)$ (degrees)

key

- - linear

- Stokes V
Fig. 7.13 Meridional distribution of wave loading
(a) linear wave theory
(b) Stokes 5th order wave theory
Fig. 7.14(a) Stress distribution in the outer jacket due to wave loading (T=16s) - inertia component
**TYPE OF ANALYSIS** = **LINEAR STATIC WAVE LOADING**

**TOTAL MERIDIONAL LENGTH (L) = 55.813m**

**MERIDIONAL LENGTH = SL, APEX= A, BASE= B.**

**MAX/R ORIGIRATE VALUE = (R/ORDINATE LENGTH)/S**

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*Fig. 7.14(b) Stress distribution in the outer jacket due to wave loading (T=16s) - drag component*
TYPE OF ANALYSIS = LINEAR STATIC
WAVE LOADING
TOTAL MERIDIONAL LENGTH (L) = 55.813m

MERIDIONAL LENGTH = SL, APEX = A, BASE = B.

MAX/R ORDEinate VALUE = (R=ORDINATE LENGTH)/S

MIN/R

Fig. 7.15(a) Stress distribution in the outer jacket due to wave loading (T=20s)
- inertia component
**TYPE OF ANALYSIS** = LINEAR STATIC

**WAVE LOADING**

**TOTAL MERIDIONAL LENGTH (L) = 55.813 m**

**MERIDIONAL LENGTH = SL, APEX= A, BASE= B.**

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**Fig. 7.15(b) Stress distribution in the outer jacket due to wave loading (T=20s) - drag component**
### Fig. 7.16 Stress distribution in the outer jacket due to current loading (vel. = 0.89m/s)

**TYPE OF ANALYSIS** = LINEAR STATIC  
**CURRENT LOADING**  
**TOTAL MERIDIONAL LENGTH** (L) = 55.813m  
**MERIDIONAL LENGTH** = SL, APEX = A, BASE = B.

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Fig. 7.17 Stress distribution in the outer jacket due to current loading (vel. = 1.5m/s)
Fig. 7.18(a) Stress distribution in the outer jacket under the most severe environmental conditions.
**TYPE OF ANALYSIS** = LINEAR STATIC
**DEPTH OF END A** = 42.400m
**TOTAL MERIDIONAL LENGTH (L)** = 55.813m

**MERIDIONAL LENGTH** = SL, APEX = A, BASE = B.

MAX/R

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MIN/R

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**Fig. 7.18(b)** Stress distribution in the outer jacket under severe environmental conditions (midpoint between wave crest and trough directly above centreline of tank)

<table>
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<tr>
<th>GRAPH IIIIF</th>
<th>MAX (COMPRESSION)</th>
<th>MIN (TENSION)</th>
<th>MAX-MIN (R)</th>
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<td>1 MERIDIONAL FORCE/LENGTH [N/m]</td>
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<td>5 CIRCUM. STRESS (INSIDE SURFACE) [N/m²]</td>
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<td>7 EQUIV. STRESS (INSIDE SURFACE) [N/m²]</td>
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<tr>
<td>8 EQUIV. STRESS (OUTSIDE SURFACE) [N/m²]</td>
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<td>-26035502.5</td>
<td>26035502.5</td>
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</table>
tugs controlling the descent

attachment points

inner tank full of seawater

tank during installation

tank in operating position

prepared seabed

Fig. 7.19 Installation process
Fig. 7.20 Design earthquake for the North Sea

Magnitude, $M = 5.5$
Distance from source, $R = 24.4\text{ km}$
Max. peak ground accel., $a = 1.0\text{ m/s}^2$

Spectral acceleration ($\text{m/s}^2$)

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<th>4</th>
<th>5</th>
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</table>
TYPE OF ANALYSIS = LINEAR STATIC
DEPT OF END A = 40.000m
TOTAL MERIDIONAL LENGTH (L) = 55.313m

MERIDIONAL LENGTH = SL, APEX = A, BASE = B.

MAX/R

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Fig. 7.21 Stress distribution in the outer jacket with a 1 metre hole at the apex under the design head
CHAPTER EIGHT

GENERAL DISCUSSION, CONCLUSIONS

AND

RECOMMENDATIONS FOR FUTURE WORK
8.1 General discussion

This thesis was concerned with the examination of the behaviour of the drop-shaped shell under various types of load in order to support its use in underwater applications.

The study was undertaken in response to the need for underwater enclosures, whether as storage vessels in the immediate future or as habitats and seabed production centres in the years to come, with a view to proposing an alternative structural form for use in intermediate depths of water of less than 200m.

However, prior to employing the Echinodome in underwater applications, an understanding of its behaviour in the submarine environment is essential and therefore a reliable analytical or numerical method of analysis is required.

To this end the behaviour of a small prototype under various loading conditions was examined, both theoretically and experimentally, in the first part of this thesis. The theoretical investigations were based mainly on the finite element method but in some cases the membrane theory for thin shells also was considered.

Three different finite element programs were used in the numerical work and the prototype was modelled using five different finite elements, which included ring elements and the semi-loof element.

The use of these finite elements for the linear static stress analysis under both axisymmetric and non-axisymmetric loads, linear and non-linear buckling analyses and the free vibration analysis of the Echinodome was examined. Then, based on several factors such as the accuracy of the solution, the cpu time and storage space required for the analysis, and the capabilities of the element, a particular finite element was recommended for use in design for each type of analysis.
The results obtained from the numerical static stress analysis were compared with results from an analytical approach based on the membrane theory for thin shells to assess the suitability of the analytical method for that purpose. A similar comparison was made also between the numerical and analytical values for the critical buckling head of the prototype.

The good comparison obtained in the theoretical investigation between the results from the different finite elements was encouraging – as was the comparison of the results obtained from the membrane approach and the finite element method in the static stress analysis of the prototype under axisymmetric loads.

However, complete reliance could not be placed on a numerical or analytical approach until it had been verified by experimental work and consequently a series of tests to substantiate the use of the finite element method for the types of analysis mentioned above was undertaken.

Previous experimental work \(^{17,18,26}\) on the small prototype had confirmed already the finite element approach to the analysis of the Echinodome under hydrostatic pressure and so in this work an experimental investigation was undertaken of the static response of the prototype to both axisymmetric and non-axisymmetric concentrated loads.

This was followed by a series of non-destructive buckling tests on the prototype to predict its critical buckling load and the resonance testing of the shell to determine its natural frequencies and corresponding mode shapes.

Good agreement was evident between experimental and theoretical results in all three cases hence confirming the suitability of the finite element method for the analysis of the Echinodome and substantiating the recommendations made for design.

Based on these recommendations, in the second part of this thesis, an illustrative design of an underwater oil storage tank for the North Sea was carried out. A procedure for selecting its structural form was described and
its response to some of the important loadings expected was examined. The worst loading effects predicted were those due to hydrostatic pressure and this was very encouraging since the Echinodome is the optimum form for a structure subjected to hydrostatic pressure\textsuperscript{16,17,26}, and hence supports its use in underwater applications.

8.2 Conclusions

From this work on the prototype and the illustrative design example, and in line with the objectives of this thesis, the following conclusions may be drawn.

(a) The finite element method could be used satisfactorily for the structural analysis of the drop-shaped shell and this was confirmed by experimental work.

(b) The Mistry finite element was recommended for the static stress analysis under axisymmetric pressure, linear and non-linear buckling analyses, and free vibration analysis of the Echinodome.

(c) The three-noded thin shell of revolution element in PAFEC (element 42130) was recommended for the static stress analysis of the Echinodome under loads of a general nature, both axisymmetric and non-axisymmetric.

(d) The semi-loof element would be required for the analysis of the drop-shaped shell if openings, other than an axisymmetric hole at the apex, were present.

(e) The membrane theory for thin shells could be employed satisfactorily in the static stress analysis of the Echinodome under axisymmetric loads of a general nature in the initial stages of design, as long as the designer was aware of the limitations of the theory. The use of the classical shell theory for predicting its critical buckling load, on the other hand, could lead to a serious overestimation of the critical load.
(f) The experimental and theoretical investigation of the response of the prototype to concentrated loads confirmed that the optimum location for penetrating the shell for pipeline connections was at the apex and that the effect of impact loading at the maximum diameter could not be ignored.

(g) The examination of the buckling behaviour of the prototype predicted material failure in the bottom tenth of the shell wall before buckling occurred - the lowest buckling load being associated with a non-axisymmetric buckling mode. Careful use of the Southwell technique led to the experimental verification of the first two buckling loads and their corresponding mode shapes.

(h) The profile of the shell is dependent on the design head, material design stress and wall thickness therefore the buckling behaviour of a bigger Echinodome would be different and an examination of its buckling characteristics would be essential so as to determine whether buckling or material failure was the criterion in design.

(i) The experimental and numerical determination of the natural frequencies of the prototype indicated a fundamental frequency corresponding to a symmetric translational mode. The second natural frequency corresponded to an axisymmetric mode and the third to a torsional mode. A simple extension to the Peak-Amplitude method was suitable for the experimental verification.

(j) As in (h), the free vibration characteristics of the Echinodome would be dependent on its particular profile.

(k) An illustrative design example was undertaken in which a procedure for selecting the structural form for the primary structure of an underwater storage tank was proposed. In this design example buckling was the criterion and it was recommended that the shell wall's thickness be increased along the bottom tenth of the tank to provide a high factor of safety against buckling.
(l) An assessment of the types and magnitudes of loads likely to be expected on an underwater structure indicated that the worst loading effects were those due to the hydrostatic pressure. This was encouraging since the drop shaped shell is at its optimum when under hydrostatic pressure.

(m) The dynamic response of the storage tank during its operational stage was negligible and could be neglected safely.

8.3 Recommendations for future work

The work reported in this thesis has covered a consideration of most of the important loadings expected during the lifetime of an underwater drop shaped tank. However, as mentioned earlier, unexpected or accidental loads such as impact or blast loading might occur and it is important that the structure could withstand those loads safely. Future work on the prototype should therefore examine its response to impact and blast loading, both experimentally and numerically, with a view to obtaining a reliable method for predicting its behaviour under such loadings.

Another area worthy of examination is the dynamic response of the prototype to the different types of dynamic loads likely to be encountered by the Echinodome during its launching, tow-out and installation stages, when it would be behaving as a semi-submersible. In addition, although the dynamic response of the structure to waves during its operational stage was found to be negligible, a fatigue analysis of the tank should be included in future work to ensure the integrity of the structure throughout its working life.

It should be noted that all the work carried out up to now has considered only the shell wall but before such a structure can become a reality the design of the base and the seabed soil/structure interaction would need to be investigated fully. Similarly in the case of floating submerged tanks the design of the moorings would require some
attention.

In the near future the design, construction and in-situ testing of a bigger prototype, preferably in concrete, in about 10 to 20m of water would be advantageous as it would highlight any unforeseen problems that could arise in the 'real life' case and encourage further research in that particular direction. Also it would be an ideal opportunity to examine different methods of construction and the launching, tow-out, installation and recovery procedures.

Nevertheless, the work carried out on the small prototype to date, although limited to laboratory conditions has shown that the Echinodome is suited functionally to underwater applications and offers an aesthetic and attractive structural form which is at its optimum under the main type of load encountered in the submarine environment of shallow to medium water depths.

In closing, it is hoped that the work described in this thesis has contributed towards the realisation of the Echinodome as an underwater enclosure and serves as a basis for future work.
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64. ROYLES, R. & LLAMBIAS, J.M., "Examination of the buckling behaviour of an underwater storage vessel", Experimental Mechanics, (to be published).


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APPENDIX ONE

FLOW DIAGRAMS

A1.1 Basic flow diagram of the finite element method (1-2)
A1.2 Flow diagram for the shape prediction program (1-3)
A1.3 Flow diagram for 'PAFMAS' (1-4)
A1.4 Flow diagram for 'STRAIN' (1-5)
A1.5 Flow diagram for the buckling analysis (1-6)
A1.6 Flow diagram for the wave loading program (1-7)
A1.1 BASIC FLOW DIAGRAM OF THE FINITE ELEMENT METHOD

START

read input data and store it
(N = no. of elements)

1 = 1

form element stiffness matrix

is 1 = N

yes

form overall stiffness matrix

invert overall stiffness matrix

compute nodal displacements

print nodal displacements

STOP

compute stresses

is 1 = N

yes

print stresses

STOP
A1.2 FLOW DIAGRAM FOR THE SHAPE PREDICTION PROGRAM

START

Main shape prediction program

determine centreline coords. of shell wall

are shell's characteristics required

no

yes

determine shell's characteristics

are coords. of inner and outer surfaces required

no

yes

determine coords of inner and outer surfaces of shell's wall

are characteristics to be printed

no

yes

print out characteristics

are coordinates to be printed

no

yes

print out coordinates

is a data file for Mistry or PAFEC required

no

yes

print out data file

is a double-walled tank required

no

yes

are the coords./characts. required

no

yes

calculate distance between centrelines of walls

STOP

* For a flow diagram for the main shape prediction program the reader is referred to 1-3
A1.3 FLOW DIAGRAM FOR 'PAFMAS'

START

input no. of harmonics, N

I = 1

run PAFEC for N = 1

store stresses in s(I)

store disp in d(I)

I = I + 1

are tables required

yes

list tables

are graphs required

yes

prepare data file for EASYGRAPH

run EASYGRAPH

print out graphs

STOP

are tables required

no

no

I = N = 1

yes

add stresses

add disp
A1.4 FLOW DIAGRAM FOR 'STRAIN'

START

send instructions to multimeter

read voltage in channel 1 (No. of channels = N)

compute strain (or deflection) and display it on the screen

is $N=1$

yes

evaluate and display the time taken for the scan

read in the details of the test

select output device: printer (P), cassette (C) or neither (N)

(P) or (C) (N)

print out results on selected output device

is this the datum run

yes

is another run required

no

STOP
A1.5 FLOW DIAGRAM FOR THE BUCKLING ANALYSIS

START

- current pressure $P_i$ (mode=m)
  - form the stiffness matrix, $S_{ei}$
    - is $S_{ei}$ non positive definitive
      - no
        - increase $P_i$ by $dp$
      - yes
        - axisymmetric collapse
          - STOP
    - yes
      - determine the axisymmetric prebuckling stresses
        - calculate the stability matrix $S_{ii}$
          - is $S_{ii}=0$
            - no
              - is $S_{ii}$ of a different sign to $S_{ii-1}$
                - yes
                  - repeat cycle with smaller increments to narrow down range
                    - set up eigenvalue problem $s=0$
                      - solve eigenvalue problem $s_{ii}=0$
                        - search for lower value of m (mode shape)
                          - bifurcation buckling
                            - STOP
                - no
                  - STOP
The flow diagrams for both programs are very similar. The only difference being the introduction of an extra step to determine a number of constants required for the evaluation of the wave profile and water particle velocities and accelerations (see appendix 7) when using the non-linear wave theory. As a result one flow diagram is shown here, with the extra step required by Stokes non-linear wave theory indicated by dotted lines.

START

input coords. of shell and wave parameters

---

determine the constants for the particular wave

compute the cross sectional area of each element

---

determine velocity of water particles at the centroid of each element (u)

---

is there a current (vel=cu)

---

yes

modify the velocity of the wave induced water particle motion (u+cu)

---

compute the volume enclosed by each element

determine the acceleration of the water particles at the centroid of each element

compute the inertia force on each element

compute the total inertia force on the structure

print the inertia force on each element and the total inertia force on the structure

---

no

determine the drag force on each element

compute the total drag force on the structure

print drag force on each element and the total drag force on the structure

---

STOP
APPENDIX TWO

TYPICAL COMPUTER FILES

A2.1 Typical data file for the Mistry program (2-2)
A2.2 Typical data file for PAFEC (element 42130) (2-4)
A2.3 Typical data file for PAFEC (element 36610) (2-7)
A2.4 Typical data file for LUSAS (element QAX4) (2-11)
A2.5 Typical data file for LUSAS (elements QSL8 & TSL6) (2-14)
A2.6 Typical output file from 'CURVEFIT' (2-17)
A2.7 Typical input and output file from 'RESPONSE' (2-19)
A2.1 TYPICAL DATA FILE FOR THE MISTRY PROGRAM

1
LINEAR STATIC STRESS ANALYSIS OF ECHINODOME
  6  10
  0.88000E+10 0.36000E+00 0.00001.1000E+03 0.98100E+04 0.98100E+04 0.39240E+06
  0.00000E+00 0.00000E+00 0.98100E+04 0.15250E+01 0.00000E+00 0.15250E+01
  0.00000E+00 0.00327E+00 0.00382E+00 0.00394E+00 0.00401E+00 0.00404E+00 0.00390E+00
  0.00381E+00 0.00374E+00 0.00362E+00 0.00344E+00 0 0 0 0
  0 0
  0 0
  3  12
  0.13672E-03 0.80000E-02 0.54761E-03 0.16000E-01 0.12344E-02 0.24000E-01
  0.21995E-02 0.32000E-01 0.34468E-02 0.40000E-01 0.49812E-02 0.48000E-01
  0.68091E-02 0.56000E-01 0.89382E-02 0.64000E-01 0.11378E-01 0.72000E-01
  0.14139E-01 0.80000E-01 0.17236E-01 0.88000E-01 0.20684E-01 0.96000E-01
  3  7
  0.24502E-01 0.10400E+00 0.28713E-01 0.11200E+00 0.33344E-01 0.12000E+00
  0.38427E-01 0.12800E+00 0.44004E-01 0.13600E+00 0.50124E-01 0.14400E+00
  0.56851E-01 0.15200E+00 3  5
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A2.2 TYPICAL DATA FILE FOR PAFEC (ELEMENT 42130)

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CONTROL.END

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TOPOLOGY

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3  20  21
R4, 0  1  1
4  25  26
R4, 0  1  1
5  30  31
R4, 0  1  1
6  35  36
R4, 0  1  1
7  40  41
R4, 0  1  1
8  45  46
R5, 0  1  1
9  51  52
R4, 0  1  1
10  56  57
R9, 0  1  1

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1.7767E4         45      51      1
1.8133E4         51      56      1
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END OF DATA
### A2.3 TYPICAL DATA FILE FOR PAFEC (ELEMENT 36610)

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**CONTROL.END**
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 .88E10 .36 1.1E3
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PLATE  MATERIAL  THICKNESS  RAD1
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NODE.NUMBER  PLANE  DIRECTION
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PRESSURE.VALUE  START  FINISH  STEP
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112 0.16413 0.34310
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114 0.15613 0.34951
115 0.14813 0.35914
116 0.14813 0.35507
117 0.14013 0.36380
118 0.14013 0.35988
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120 0.13213 0.36399
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122 0.12413 0.36746
123 0.11613 0.37391
124 0.11613 0.37031
125 0.10813 0.37609
126 0.10813 0.37255
127 0.10013 0.37769
128 0.10013 0.37420
129 0.09213 0.37869
130 0.09213 0.37524
131 0.08413 0.37908
132 0.08413 0.37564

MATERIAL PROPERTIES
1 65 1 0.88E10 0.36 1.1E3

SUPPORT NODES
131 132 1 R R 0 0

LOAD CASE
BFP
1 31 2 0 0 0 2.4544E5
33 41 2 0 0 0 2.4580E5
43 51 2 0 0 0 2.4617E5
53 61 2 0 0 0 2.4657E5
63 71 2 0 0 0 2.4696E5
73 79 2 0 0 0 2.4731E5
81 89 2 0 0 0 2.4766E5
91 99 2 0 0 0 2.4806E5
101 107 2 0 0 0 2.4842E5
109 131 2 0 0 0 2.4878E5

END
PROBLEM TITLE LINEAR STATIC STRESS ANALYSIS OF ECHINODEMME
OPTION 13.58
UNITS N M
TSL6 ELEMENT TOPOLOGY
FIRST 1 1 164 4 3 2 163
INC 1 0 1 2 2 1 3
QSL8 ELEMENT TOPOLOGY
FIRST 4 4 10 15 14 13 9 2 3
INC 1 2 1 2 2 1 2 2 3
INC 3 11 11 11 11 11 11 11 14
NODE COORDINATE
  1 0.0 0.0 0.0
  167 1.0 0.0 0.0
  168 0.0 1.0 0.0
LOCAL CYLINDRICAL COORDINATE 1 167 168
FIRST 163 0.0000683 0.004 0
INC 1 0 0 30 4
FIRST 2 0.0001367 0.008 0
INC 1 0 0 15 7
FIRST 9 0.0017290 0.0281 0
INC 1 0 0 30 4
FIRST 13 0.0049812 0.048 0
INC 1 0 0 15 7
FIRST 20 0.011704 0.0724275 0
INC 1 0 0 30 4
FIRST 24 0.020684 0.096 0
INC 1 0 0 15 7
FIRST 31 0.0277996 0.1102647 0
INC 1 0 0 30 4
FIRST 35 0.035885 0.124 0
INC 1 0 0 15 7
FIRST 42 0.0458622 0.138396 0
INC 1 0 0 30 4
FIRST 46 0.05685 0.152 0
INC 1 0 0 15 7
FIRST 53 0.066381 0.162060412 0
INC 1 0 0 30 4
FIRST 57 0.076474 0.17153 0
INC 1 0 0 15 7
FIRST 64 0.0862487 0.1796333 0
INC 1 0 0 30 4
FIRST 68 0.096478 0.18725 0
INC 1 0 0 15 7
FIRST 75 0.1063160 0.1936024 0
INC 1 0 0 30 4
FIRST 79 0.11648 0.19942 0
INC 1 0 0 15 7
FIRST 86 0.1261962 0.204328055 0
INC 1 0 0 30 4
FIRST 90 0.13648 0.20888 0
INC 1 0 0 15 7
FIRST 97 0.15648 0.2157 0
GLOBAL CARTESIAN COORDINATES

TSL6 GEOMETRIC PROPERTIES

| 1  3  1  | 0.00327 | 0.00327 | 0.00327 | 0.00327 | 0.00327 | 0.00327 |

QSL8 GEOMETRIC PROPERTIES

| 4  9  1  | 0.00327 | 0.00327 | 0.00327 | 0.00327 | 0.00327 | 0.00327 | 0.00327 | 0.00327 |
| 10 15 1  | 0.00382 | 0.00382 | 0.00382 | 0.00382 | 0.00382 | 0.00382 | 0.00382 | 0.00382 |
| 16 21 1  | 0.00394 | 0.00394 | 0.00394 | 0.00394 | 0.00394 | 0.00394 | 0.00394 | 0.00394 |
| 22 27 1  | 0.00401 | 0.00401 | 0.00401 | 0.00401 | 0.00401 | 0.00401 | 0.00401 | 0.00401 |
| 28 30 1  | 0.00404 | 0.00404 | 0.00404 | 0.00404 | 0.00404 | 0.00404 | 0.00404 | 0.00404 |
| 31 33 1  | 0.00390 | 0.00390 | 0.00390 | 0.00390 | 0.00390 | 0.00390 | 0.00390 | 0.00390 |
| 34 36 1  | 0.00381 | 0.00381 | 0.00381 | 0.00381 | 0.00381 | 0.00381 | 0.00381 | 0.00381 |
| 37 39 1  | 0.00374 | 0.00374 | 0.00374 | 0.00374 | 0.00374 | 0.00374 | 0.00374 | 0.00374 |
| 40 42 1  | 0.00362 | 0.00362 | 0.00362 | 0.00362 | 0.00362 | 0.00362 | 0.00362 | 0.00362 |
| 43 45 1  | 0.00344 | 0.00344 | 0.00344 | 0.00344 | 0.00344 | 0.00344 | 0.00344 | 0.00344 |

MATERIAL PROPERTIES

| 1  45  1  | 0.88E10 | 0.36  | 1.1E3 |

SUPPORT NODES

| 1  0  0  F  R  R  R  R  R  R  0  0  0  0  |
| 156 162 1  R  R  R  R  R  R  R  0  0  0  0  |
| 12 166 11 F  R  F  R  F  0  0  0  0  |
| 8  151 11 F  R  F  R  F  0  0  0  0  |
| 9  163 11 F  F  R  R  F  0  0  0  0  |
| 2  145 11 F  F  R  R  F  0  0  0  0  |

LOAD CASE

BFP

| 1  23  1  0  0  -1.5145E4 |
| 24  45  1  0  0  -1.5508E4 |
| 46  67  1  0  0  -1.5883E4 |
| 68  89  1  0  0  -1.6276E4 |
| 90  100 1  0  0  -1.6668E4 |
| 101 111 1  0  0  -1.7020E4 |
| 112 122 1  0  0  -1.7374E4 |

2-15
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LEAST SQUARES APPROXIMATION OF DISCRETE FUNCTIONS BY A POLYNOMIAL OF DEGREE

COEFFICIENTS (IN ASCENDING ORDER)

A(0) = 2.53494095580165@ -1
A(1) = 2.50015888770655@ -2

STANDARD DEVIATION: 7.5102@ -1

52 DATA POINTS USED IN ABOVE CALCULATION

END OF INPUT DATA
A2.7(a) TYPICAL INPUT FILE FOR 'RESPONSE'

OIL STORAGE TANK
37.67 52.02 52.02 1.7 116. 1.0 2.0
33.33 .15 .190 .080 440. 0.35 2.00 16.30 14.80 0.38 10.00
0.20 0.80 3.50

A2.7(b) TYPICAL OUTPUT FILE FROM 'RESPONSE'

**** DYNAMIC WIND RESPONSE ANALYSIS RESULTS ****

NAME : OIL STORAGE TANK
SIZE OF STRUCTURE : H = 38.M B = 52.M D = 52.M F0=1.70HZ ZETA=2.00%
WIND CHARACTERISTICS AT Z=H ( () SHOWS POWER EXPONENT OF PROFILE ) :

\[ U(H) = 41.M/S \quad T.I. = 14.\% \quad L(H) = 854.M \quad BETA = 2.0 \]
\[ (0.15) \quad (-0.08) \quad (0.35) \]

DECAY FACTOR KY = 12.0 \quad KZ = 10.9 \quad (-0.38)

COEFFICIENTS :
\[ C = 0.20 \quad C = 0.80 \quad C = 3.50 \]
DST  DQS  DDR

RESULTS: DEFLECTION X & ACCELERATION A AT Z=H
(MODAL SHAPE POWER EXPONENT = 1.0 )

\[ XMEAN = 0.0004M \quad AERO.D.R. =0.025\% \]

DAMPING RATIO  2.00%

AERODYNAMIC DAMPING EFFECT
\[ X RMS (M) \quad 0.00009 \quad 0.00009 \]
\[ X MAX (M) \quad 0.00076 \quad 0.00076 \]
\[ A RMS (G) \quad 0.00072 \quad 0.00072 \]
\[ A MAX (G) \quad 0.00312 \quad 0.00310 \]

GUST FACTOR  2.01  2.01

PEAK FACTOR X  4.23  4.23

\[ A \quad 4.31 \quad 4.31 \]

ROUGHNESS FACTOR  0.311  BACKGROUND EX.F.  0.491

GUST EX. FACTOR  0.044  SIZE RED. FACTOR  0.014

2-19
APPENDIX THREE

VARIATION IN WALL THICKNESS OF PROTOTYPE
### Table A3.1 Variation in wall thickness of prototype

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Table A3.1 Variation in wall thickness of prototype
APPENDIX FOUR

COMPARISON OF FINITE ELEMENTS
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<th>TYPES OF ANALYSIS POSSIBLE</th>
<th>OUTPUT OF STRESS RESULTS</th>
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<td>N₁, N₂, N₁₂</td>
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<td>v = a₃ + a₄ s</td>
<td>- linear</td>
<td>M₁, M₂, M₁₂</td>
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<td>σ₁, σ₂, σ₁₂</td>
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<td></td>
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<td>- non-linear</td>
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<td>36610</td>
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<td>Uₓ, Uᵧ, U₂z</td>
<td>static stress</td>
<td>σₓₓ, σᵧᵧ, σ zza</td>
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<td>+ a₇ s² + a₈ s²</td>
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<td>dynamic response</td>
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<td>thermal</td>
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<td>+ a₁₀ s³ + a₁₁ s⁴</td>
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<td>+ a₁₂ s⁵</td>
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<tr>
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<td></td>
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<td></td>
<td></td>
<td>- non-linear</td>
<td>σₘₓₙ, σₘᵦ, β</td>
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<td>(top &amp; bottom)</td>
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<td></td>
<td>U, V, W</td>
<td>static stress</td>
<td>Nₓ, Nᵧ, Nₓᵧ</td>
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<td></td>
<td>- non-linear</td>
<td>σₓₓ, σᵧᵧ, σ zza</td>
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<td>in local axes</td>
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<td></td>
<td>(top &amp; bottom)</td>
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A5.1 Point loading (5.2)
A5.2 Wind loading (5.5)
A5.3 Wave loading (5.7)
In order to idealise a horizontal point load acting at the maximum diameter of the shell (see Fig. A5.1(a) above), a Fourier series was required.

This load, which is represented in a graphical form in Fig. 5.1(b) can be expressed mathematically as:

\[
\begin{align*}
f(\theta) &= \begin{cases} 
\infty & (\theta = 0) \\
0 & (\theta \neq 0)
\end{cases} 
\end{align*}
\]  

(A5.1)

where \( f(\theta) \) is the load intensity per circumference

The function \( f(\theta) \) has one singularity, i.e. it is zero everywhere except at the origin, where it is infinity, and hence is a Delta function (or unit impulse function), \( \delta(\theta) \)\(^{104}\)

i.e. \[ f(\theta) \equiv \delta(\theta) \]  

(A5.2)
Expressing the function as a Fourier series of the form,

\[ F(\theta) = A_0 + \sum_{m=1}^{\infty} A_m \cos m\theta \]  

(A5.3)

where

\[ A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) d\theta \]  

(A5.4)

and

\[ A_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\theta) \cos m\theta d\theta \]  

(A5.5)

Substituting in (A5.2) & (A5.4),

\[ A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \delta(\theta) d\theta \]

Now, since \( \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \delta(\theta) d\theta = 1 \), by definition of a Delta function \(^{104}\) for any \( \xi > 0 \), then this equation becomes,

\[ A_0 = \frac{1}{2\pi} \left[ 1 \right] = \frac{1}{2\pi} \]  

(A5.6)

Similarly substituting (A5.2) in (A5.3),

\[ A_m = \frac{1}{\pi} \int_{-\pi}^{\pi} \delta(\theta) \cos m\theta d\theta \]  

(A5.7)

Now, since another property of a Delta function is that it acts as a 'reproducing kernel' \(^{104}\), then for any \( \xi > 0 \),

\[ \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \delta(\theta) x(\theta) d\theta = x(0) \]  

(A5.8)

Thus from (A5.7) & (A5.8),

\[ A_m = \frac{1}{\pi} \left[ \cos m\theta \right] = \frac{1}{\pi} \]  

(A5.9)

Therefore from (A5.3), (A5.6) & (A5.9),

\[ F(\theta) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{m=1}^{\infty} \cos m\theta \]
i.e. \[ F(\theta) = \frac{1}{\pi} \left[ \frac{1}{2} + \cos \theta + \cos 2 \theta + \cos 3 \theta + \ldots \ldots \cos m \theta \right] \]

It should be noted that when \( m \) is sufficiently large, \( F(\theta) \) approximates a point loading at \( \theta = 0 \).

Fig. A5.2 Effect of truncating the Fourier series for a point load after 9 and 20 terms
A5.2 FOURIER SERIES APPROXIMATION OF WIND LOADING

The Swiss code of practice defines the circumferential pressure distribution over a sphere, $C_p$, by tabulating its value at a finite number of points, as shown in Table A5.1 below.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$C_p(\theta)$</th>
<th>$\theta$</th>
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</thead>
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<tr>
<td>0</td>
<td>1.0</td>
<td>180</td>
<td>0.4</td>
</tr>
<tr>
<td>15</td>
<td>0.9</td>
<td>195</td>
<td>1.3</td>
</tr>
<tr>
<td>30</td>
<td>0.5</td>
<td>210</td>
<td>0.1</td>
</tr>
<tr>
<td>45</td>
<td>-0.1</td>
<td>225</td>
<td>-0.2</td>
</tr>
<tr>
<td>60</td>
<td>-0.7</td>
<td>240</td>
<td>-0.6</td>
</tr>
<tr>
<td>75</td>
<td>-1.1</td>
<td>255</td>
<td>-1.0</td>
</tr>
<tr>
<td>90</td>
<td>-1.2</td>
<td>270</td>
<td>-1.2</td>
</tr>
<tr>
<td>105</td>
<td>-1.0</td>
<td>285</td>
<td>-1.1</td>
</tr>
<tr>
<td>120</td>
<td>-0.6</td>
<td>300</td>
<td>-0.7</td>
</tr>
<tr>
<td>135</td>
<td>-0.2</td>
<td>315</td>
<td>-0.1</td>
</tr>
<tr>
<td>150</td>
<td>0.1</td>
<td>330</td>
<td>0.5</td>
</tr>
<tr>
<td>165</td>
<td>0.3</td>
<td>345</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table A5.1 Pressure distribution across surface of a sphere

Several methods exist for calculating the Fourier series coefficients of such a function numerically. In this work the method presented by Kufner and Kadlec based on Runge's twenty four point scheme was used. The reader is requested to follow the description given in that work when studying this appendix.

Setting up Runge's 24 point scheme for the above function

| 1.0 0.9 0.5 -0.1 -0.7 -1.1 -1.2 -1.0 -0.6 -0.2 0.1 0.3 0.4 |
| 0.9 0.5 -0.1 -0.7 -1.1 -1.2 -1.0 -0.6 -0.2 0.1 0.3 |

sum 1.0 1.8 1.0 -0.2 -1.4 -2.2 -3.0 -1.2 -0.4 0.2 0.6 0.4

diff 0 0 0 0 0 0 0 0 0 0 0 0

5-5
\[ \begin{array}{cccccc}
1.0 & 1.8 & 1.0 & -0.2 & -1.4 & -2.2 & -2.4 \\
0.4 & 0.6 & 0.2 & -0.4 & -1.2 & -2.0 \\
\end{array} \]

\[ \begin{array}{cccccc}
\text{sum} & 1.4 & 2.4 & 1.2 & -0.6 & -2.6 & -4.2 & -2.4 \\
\text{diff} & 0.6 & 1.2 & 0.8 & 0.2 & -0.2 & -0.2 \\
\end{array} \]

\[ \begin{array}{cccccc}
1.4 & 2.4 & 1.2 & -0.6 \\
-2.4 & -4.2 & -2.6 \\
\end{array} \]

\[ \begin{array}{cccccc}
\text{sum} & -1.0 & -1.8 & -1.4 & -0.6 \\
\text{diff} & 3.8 & 6.6 & 3.8 \\
\end{array} \]

From which the following constants can be obtained

\[ K_0 = -1.0 \quad q_0 = 0.6 \]
\[ K_1 = -1.8 \quad q_1 = 1.2 \]
\[ K_2 = -1.4 \quad q_2 = 0.8 \]
\[ K_3 = -0.6 \quad q_3 = 0.2 \]
\[ q_4 = -0.2 \]
\[ L_0 = 3.8 \]
\[ L_1 = 6.6 \]
\[ L_2 = 3.8 \]

Substituting these values into equations 6.12 in reference 102.

\[ a_0 = -0.2000 \]
\[ a_1 = 0.2035 \]
\[ a_2 = 0.9513 \]
\[ a_3 = 0.1374 \]
\[ a_4 = -0.0500 \]
\[ a_5 = -0.0180 \]

Hence the Fourier series is:

\[ C_p(\theta) = -0.20 + 0.2035\cos \theta + 0.9513\cos 2\theta + 0.1374\cos 3\theta - 0.05\cos 4\theta - 0.018\cos 5\theta \]
The circumferential distribution of the wave loading is shown in Fig. A5.3 below.

![Diagram of wave loading distribution](image)

**Fig. A5.3 Distribution of wave loading**

This force can be expressed mathematically as shown in Eq. (A5.10) below.

\[
F(\theta) = \begin{cases} 
0 & (0 < \theta < \frac{\pi}{2}, \quad \frac{3\pi}{2} < \theta < 2\pi) \\
1 & (\frac{\pi}{2} < \theta < \frac{3\pi}{2})
\end{cases} \quad (A5.10)
\]

In order to represent this load as a Fourier series of the form,

\[
F(\theta) = \sum_{m=0}^{\infty} A_m \cos m\theta \quad (A5.11)
\]

the values of the constants \( A_0, A_1, \ldots, A_m \) need to be calculated. These values can be obtained from Eqs (A5.12) & (A5.13) shown below\(^{106}\).

i.e. \[
A_0 = \frac{1}{2\pi} \int_{0}^{2\pi} f(\theta) \, d\theta \quad (A5.12)
\]
From (A5.12) and considering Eq. (A5.10), the following can be obtained:

\[
A_0 = \frac{1}{2} \pi \int_{\pi/2}^{3\pi/2} d(\theta) = \frac{1}{2} \pi [\theta]_{\pi/2}^{3\pi/2} = \frac{1}{2}
\]

and similarly from Eqs (A5.10) & (A5.13),

\[
A_m = \frac{1}{\pi} \int_{\pi/2}^{3\pi/2} \cos m\theta \, d\theta = \frac{1}{m\pi} [\sin m\theta]_{\pi/2}^{3\pi/2} = \frac{2}{m\pi} \cos m\pi \sin m\pi/2
\]

Hence, substituting in (A5.11),

\[
F(\theta) = \frac{1}{2} + \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} \cos m\pi \sin m\pi/2 \cos \theta
\]

Now when \( m \) is even, \( A_m \), from Eq. (A5.15) is zero since \( \sin m\pi/2 = 0 \).

Also, when \( m \) is odd, \( \cos m\pi = -1 \), and \( \sin m\pi/2 = \pm 1 \).

Hence, Eq. (A5.16) can be rewritten as:

\[
F(\theta) = \frac{1}{2} + 2 \frac{(-1)^{m+1}}{m} \sum_{m=1}^{\infty} \cos m\theta
\]
APPENDIX SIX

DESIGN DETAILS

A6.1 Centre line coordinates of outer jacket (6-2)
A6.2 Centre line coordinates of inner tank (6-3)
### A6.1 CENTRE LINE COORDINATES OF OUTER JACKET

<p>| Axial (m) | Radial (m) | 0.13472E+02 | 0.13972E+02 | 0.14472E+02 | 0.14972E+02 | 0.15472E+02 | 0.15972E+02 | 0.16472E+02 | 0.16972E+02 | 0.17472E+02 | 0.17972E+02 | 0.18472E+02 | 0.18972E+02 | 0.19472E+02 | 0.19972E+02 | 0.20472E+02 | 0.20972E+02 | 0.21472E+02 | 0.21972E+02 | 0.22472E+02 | 0.22972E+02 | 0.23472E+02 | 0.23972E+02 | 0.24472E+02 | 0.24972E+02 | 0.25472E+02 | 0.25972E+02 | 0.26472E+02 | 0.26972E+02 | 0.27472E+02 | 0.27972E+02 | 0.28472E+02 | 0.28972E+02 | 0.29472E+02 | 0.29972E+02 | 0.30472E+02 | 0.30972E+02 | 0.31472E+02 | 0.31972E+02 | 0.32472E+02 | 0.32972E+02 | 0.33472E+02 | 0.33972E+02 | 0.34472E+02 | 0.34972E+02 | 0.35472E+02 | 0.35972E+02 | 0.36472E+02 | 0.36972E+02 |
|-----------|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|</p>
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APPENDIX SEVEN

WAVE THEORIES

A7.1 Outline of wave theories (7.2)

A7.2 Range of validity of wave theories (7.6)
A7.1 OUTLINE OF WAVE THEORIES

Both the linear Airy wave theory and the non-linear Stokes wave theory are developed from solutions to Laplace's equation of continuity,

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0
\]  

(7.1)

where \( \phi \) is the velocity function; \( x \) is the axis in the direction of propagation of the wave; and \( y \) is the axis normal to the water surface and measured positively upwards, see Fig. A7.1 below.

![Wave Profile Diagram](image)

**Fig. A7.1 Wave Profile**

A different velocity function is assumed in each case to satisfy the boundary-value problem with three boundary conditions:

(i) the bottom boundary condition requiring that the water particles at the seabed remain in contact with it at all times,

\[
\frac{\delta \phi}{\delta y} = 0 \quad \text{at} \quad y = -d
\]

(7.2)

where \( d \) is the depth of water below the still water level;

(ii) the kinematic free-surface boundary condition requiring that the
water particles at the free surface remain at the free surface at all times,

\[ v = u - \frac{\delta y}{\delta x} + \frac{\delta y}{\delta t} \]  

(7.3)

where \( u \) and \( v \) are the components of the water particle velocity in the \( x \) and \( y \) directions respectively; and

(iii) the dynamic free surface boundary condition requiring that the pressure at the free surface is zero.

\[ P = 0 \text{ at } s = d + y \]  

(7.4)

**Linear Airy wave theory**

The linear Airy wave theory is developed following the linearisation of the non-linear free surface boundary condition (i.e. (ii)). An assumed velocity potential of the form,

\[ \phi = Pe^{i(\sigma t - kx)} \]  

(7.5)

is employed to solve the boundary value problem resulting in an equation for the free surface, \((\eta(t))\), of the form,

\[ \eta(t) = \frac{H}{2} \cos(\frac{\pi}{L} - \frac{t}{T}) \]  

(7.6)

where \( H \) is the wave height; \( L \) is the wave length; and \( T \) is the wave period.

The corresponding horizontal water particle velocity and acceleration due to the waves are then given by:

\[ \frac{\delta u}{\delta t} = \frac{\pi}{T} \left( \frac{\cosh 2\pi \frac{(y+d)/L}{2\pi d/L}}{\sinh 2\pi d/L} \right) \cos 2\pi \left( \frac{x/L-1}{T} \right) \]  

(7.7)

and
\[
\frac{\delta^2 u}{\delta t^2} = \frac{2\pi^2 H}{T^2} \left( \frac{\cosh \frac{2\pi (y+d)}{L}}{\sinh \frac{2\pi d}{L}} \right) \sin \frac{2\pi \left( \frac{x}{L} - \frac{t}{T} \right)}{T} \quad (7.8)
\]

With similar expressions for the vertical water particle velocity and acceleration.

The derivations of these velocities and accelerations can be readily appreciated in numerous references.\(^{93,96}\)

**Stokes 5th order wave theory**

Stokes 5th order wave theory is based on the non-linear free surface boundary condition and assumes a potential function of a trigonometrical form as shown in equation (7.9) below.

\[
\phi = \frac{L^2}{2\pi} \left\{ (\lambda A_{11} + \lambda^3 A_{13} + \lambda^5 A_{15}) \cosh \beta s \sin \theta \\
+ \lambda^2 A_{22} + \lambda^4 A_{24} \cosh 2\beta s \sin 2\theta \\
+ \lambda^3 A_{33} + \lambda^5 A_{35} \cosh 3\beta s \sin 3\theta \\
+ \lambda^4 A_{44} \cosh 4\beta s \sin 4\theta \\
+ \lambda^5 A_{55} \cosh 5\beta s \sin 5\theta \right\} \quad (7.9)
\]

Where \(A_{11}, A_{12}, \text{ etc.}\) are constants for the particular wave; \(\bar{c}\) is the wave celerity; \(\lambda\) is \(\frac{2\pi a}{L}\) (\(a = \text{constant}\)); \(\beta\) is \(\frac{2\pi}{L}\); and \(\theta\) is \(\frac{2\pi (x/L - t/T)}{T}\).

The resulting profile is

\[
\gamma = \frac{1}{\beta} \left\{ \lambda \cos \theta + (\lambda^2 B_{22} + \lambda^4 B_{24}) \cos 2\theta \\
+ (\lambda^3 B_{33} + \lambda^5 B_{35}) \cos 3\theta \\
+ \lambda^4 B_{44} \cos 4\theta \\
+ \lambda^5 B_{55} \cos 5\theta \right\} \quad (7.10)
\]

where \(B_{22}, B_{24}, \text{ etc.}\) are constants for the particular wave.

7-4
Hence the horizontal water particle velocity is given by:

\[ u = \tilde{c} \left\{ (\lambda A_{11} + \lambda^3 A_{13} + \lambda^5 A_{15}) \cosh \beta s \cos \theta \\
+ 2(\lambda^2 A_{22} + \lambda^4 A_{24}) \cosh 2\beta s \cos 2\theta \\
+ 3(\lambda^3 A_{33} + \lambda^5 A_{35}) \cosh 3\beta s \cos 3\theta \\
+ 4\lambda^4 A_{44} \cosh 4\beta s \cos 4\theta \\
+ 5\lambda^5 A_{55} \cosh 5\beta s \cos 5\theta \right\} \quad (7.11) \]

And, the corresponding horizontal water particle acceleration is given by:

\[ \frac{\partial^2 u}{\partial t^2} = -2\tilde{c}^2 \left\{ (\lambda A_{11} + \lambda^3 A_{13} + \lambda^5 A_{15}) \cosh \beta s \sin \theta \\
+ 4(\lambda^2 A_{22} + \lambda^4 A_{24}) \cosh 2\beta s \sin 2\theta \\
+ 9(\lambda^3 A_{33} + \lambda^5 A_{35}) \cosh 3\beta s \sin 3\theta \\
+ 16\lambda^4 A_{44} \cosh 4\beta s \sin 4\theta \\
+ 25\lambda^5 A_{55} \cosh 5\beta s \sin 5\theta \right\} \quad (7.12) \]

For a complete derivation of this theory, the reader is referred to \textsuperscript{107}
A7.2 RANGE OF VALIDITY FOR THE DIFFERENT WAVE THEORIES

Fig A7.2 below shows the range of validity of numerous wave theories including the linear Airy and non-linear Stokes wave theories.  

- $H$ = Wave height
- $d$ = Depth below still water level
- $T$ = Wave period

The diagram illustrates the range of validity for different wave theories, with shaded areas indicating the range of applicability for each theory.
APPENDIX EIGHT

PUBLISHED PAPERS


transport and storage of LPG & LNG

PROCEEDINGS
Volume 2 (poster session)

Brugge, 7 - 10 May 1984

KONINKLIJKE VLAAMSE INGENIEURSVERENIGING
Technologisch Instituut (Kommissie Metaalbouw)
Jan van Rijswijcklaan 58, 2018-Antwerpen (Belgium)
Aspects of the problems associated with the storage of liquid gases such as LPG and LNG are discussed and the question of using underwater constructions for such purposes is considered. The relative merits of different forms of underwater storage such as the use of existing spent gas fields and specially built vessels are examined.

In particular, regarding storage vessels, an optimum structural form is proposed and its suitability under many different types of loading is examined.

Benefits that could be gained by using underwater storage vessels in inshore and more open waters within a supply chain for liquid gases are put forward.

Keywords: Underwater storage, liquid gases, structural implications.

INTRODUCTION

The storage of liquid gases such as liquid natural gas (L.N.G.) and liquid petroleum gas (L.P.G.) on land is quite space consuming and can present a hazard threat to the surrounding neighbourhood.

The surface storage of these liquids is the most risky and has to be carried out remote from urban areas. Consequently distribution costs to centres of demand are increased.

Enhanced protection can be achieved using underground storage tanks with the containment of LNG by ground freezing around suitably excavated holes and in buried pre-stressed concrete chambers. [1,2] The feasibility of constructing such tanks depends a lot on the level of local seismicity, geological and soil factors.

When large quantities of natural gas are obtainable far in excess of immediate demand, steps can be taken to store it in exhausted underground gas fields [3]. In this approach to storage use is made of an already established distribution system and has appeal as a more energy conserving procedure than flaring off at source.

As an alternative to land based surface and underground storage of volatile liquids consideration could be given to storing them offshore from distribution centres and underwater to benefit from a calmer environment than that existing at the air-sea interface.

The need for some form of back-up storage facility during adverse weather conditions could exist at offshore gas and oil production facilities feeding to tanker transportation. Some underwater storage capacity sufficient to allow production to continue during storms would be useful.

In this work a particular form of structure suitable for the underwater storage of hazardous liquids is considered and the design for an L.N.G. vessel of quite large capacity is considered in some detail with particular reference to the types of loading it would have to endure.

NOTATION

\[ D = \text{maximum diameter of shell} = 2R \]
\[ d = \text{design head} \]
\[ H = \text{height of shell} \]
\[ H_w = \text{wave height} \]
\[ R = \text{maximum radius of shell} \]
\[ t_g = \text{width of gap between inner and outer shells} \]
\[ t_i = \text{thickness of inner shell} \]
\[ t_o = \text{thickness of outer shell} \]
\[ V_i = \text{capacity or volume of inner shell} \]
\[ V_o = \text{capacity or volume of outer shell} \]
\[ Z = \text{operating depth} = d + H \]
\[ z = \text{hydrostatic head} \]
\[ \rho_L = \text{mass density of contained liquid} \]
\[ \rho_w = \text{mass density of sea water} \]
\[ \sigma_d = \text{design stress} \]

STRUCTURAL FORM

The type of storage facility to be used in conjunction with gas and oil production offshore has received much attention in recent years. Mostly these facilities have been associated with offshore gas and oil fields.
but one of the early uses of large underwater tanks was for extending the collection point for onshore produced oil to a location where large tanker ships could approach without the need for an expensive harbour (4). In this case a series of three steel tanks, pinned to the sea bed, were employed and supported a mooring platform above the sea surface.

One of the first large underwater storage tanks for offshore oil production was that built for the Ekofisk field in the North sea in 1973 (5). This was constructed in cellular form using pre-stressed, post-tensioned concrete and relied on gravity for its stability. Subsequently a number of gravity production platforms were developed and built in concrete with base storage facilities resting on the sea bed (6,7,8).

The concept of floating surface storage vessels with associated production facilities has been developed for liquefied gase using pre-stressed concrete as the material of the hull supporting steel storage tanks (9).

The conditions at the air/sea interface are demanding and more recently underwater storage tanks in proximity to the production platforms have been proposed (10-13) with the emphasis on concrete as the main structural material.

In all these cases the predominant structural form was cylindrical in single units or cellular groups often of circular section with hemispherical ends or conical caps. The shell of revolution has stable characteristics under external or internal pressure and can be stiffened easily against buckling. The common shapes particularly suitable for great water depths (>200m) are circular cylinders and spheres (13,14).

For shallow and intermediate depths of water (<200m) there is at least one other shape of shell of revolution which could be considered and would provide economy of material and lend itself to fabrication readily. In particular the spheroidal or drop shape, Fig.1, is worthy of examination since it has been shown that uniform stressing or strength can be achieved within it under certain external pressure design conditions (15-17). It is interesting to note also that there are apparent precedents within the marine animal kingdom for using such a shape underwater, (15), giving rise appropriately to the term Echinodome for this generic structural form.

A design for a fully submerged L.N.G. tank is outlined below based on the concept of the Echinodome or shell of revolution and uniform strength.

STORAGE REQUIREMENT FOR L.N.G. - GENERAL BRIEF

A requirement for an underwater storage facility to contain nominally 45000m$^3$ of L.N.G. at a location 57º25' 30" N and 000º00' 00" W was examined, the mean water depth being 90m (18).

In order to minimise fabrication and installation costs a single vessel was desired. The structure was to be recoverable and reusable in other similar situations, and could be either free floating submerged or resting on the sea bed.

A double skin design was envisaged with the outer structural shell resisting external forces and the inner shell, separated by a constant air gap from the outer one, containing the liquid at nominally atmospheric pressure.

The form selected was that of the drop shape or Echinodome in an effort to achieve uniform stressing under the operating mean hydrostatic head and economise on materials as well as construction costs.

DESIGN PROCEDURE

Choice of Materials:

For the outer structural shell concrete was preferred because it offered the prospect of greater stability under external pressure since a thicker shell, from a strength point of view, would be required in comparison with a steel alternative. In zones where high tension might arise concrete could be pre-stressed in compression without much difficulty. A grade 60 concrete was chosen for the tank.

The inner shell was to sustain only internal hydrostatic pressure from the liquid but had a severe thermal loading with the inner surface being at -162ºC under fully operational conditions. The characteristics of 9% nickel steel were felt to be most suitable for this task.

Shape Selection:

The procedure for selecting the centre line profile of a shell of uniform strength has been described elsewhere (15,16) and in the case of a double skin structure it is necessary only to apply the procedure to one or other of the two layers when the air gap profile is specified. In this case it was decided to determine the shape of the outer shell by the prediction procedure, an outline of which is given by the following steps.

1. Select a range of values for the outer shell of,
   (a) the design head, $d$, (see Fig.1);
   (b) the design stress, $a_d$, for the material; and,
   (c) the material thickness $t_0$.

2. Run the shape prediction program to obtain a set of design curves corresponding to the range of parameter values in 1. above.
A typical set of curves is shown in Fig. 2 giving the variation of capacity, \( V_0 \), with operating depth, \( Z \).

3. For the actual operating depth establish from the design curves the combination of \( d \), \( t_0 \) and \( \sigma_d \) that would give the required volume.

4. Select (a) the width of gap/thickness of insulation, \( t_g \), between inner and outer shells; and, (b) the thickness of the inner tank, \( t_1 \).

5. Run the shape prediction program to obtain, (a) the shapes of the inner and outer shell; (b) the enclosed volumes of the inner and outer shells; (c) the volume or amount of material used in each shell.

The particular location considered here had a total mean water depth of 90 m. For a design head \( d = 40 \) m with \( t_0 = 240 \) mm and \( \sigma_d = 25 \text{MN/m}^2 \) the above procedure gave a capacity \( V_0 = 52800 \text{m}^3 \) at an operating depth of \( Z = 75 \text{m} \).

An adequate gap between shells was felt desirable for maintenance purposes and \( t_g = 1000 \text{mm} \) was chosen, and with \( t_1 = 50 \text{mm} \) this gave an inner container capacity, \( V_1 = 44975 \text{m}^3 \).

The operating depth of \( 75 \text{m} \) was consistent with a maximum shell height \( H = 35 \text{m} \) and a maximum diameter of \( 52 \text{m} \).

A floating submerged structure was feasible, and allowing for a base to the actual shell of \( 3 \text{m} \) depth a nominal clear distance remained between sea bed and underside of structure of \( 12 \text{m} \).

Stability of Shells

Experimental investigations [17] on a small prototype and computer studies [17, 19] on both large and small shells, using finite element analysis, indicated the existence, even under axisymmetric loading, of high hoop and meridional stresses near the base. Evidence was present also of bending in this region suggesting that localised buckling could occur, Figs 3 and 4. The first of these figures shows the variation of the membrane forces and the inner and outer surface stresses over the meridian, and the second gives the deflected shape. Both correspond to the design head of \( 40 \text{m} \) on the outer shell.

In consequence, although stress limits were not exceeded in the material at the design head it was felt necessary to strengthen the shell over the bottom 10% of the meridional length, to raise the critical buckling pressure to more than twice the design head. The modification to the shape in the lower region is as shown in Fig. 5. Buckling analysis of the modified shape under external pressure revealed a critical pressure corresponding to an external head of more than three times the design value.

A similar approach was used for the inner steel shell resulting in a critical buckling pressure of more than 3.6 times the design head under normal operating conditions. The greater improvement in buckling performance from the outer shell compared with the inner was due to the greater thickness of the former.

The final design shape is shown in Fig. 6.

RESPONSES TO LOADINGS

The main loadings on the tank would arise during launch and installation as well as operation. The following loading effects have been examined:

1. mean static head - axisymmetric;
2. fluctuating static head due to waves and tidal variations - axisymmetric;
3. current drag - symmetric;
4. hydrodynamic effects due to currents and waves - symmetric;
5. wind on the completed structure - symmetric; and
6. installation and towing.

The worst load combination was found to be the sum of 1. to 4. except that in the case of the hydrodynamic loading the drag component alone gave the worst effect.

The design load characteristics are given in Table 1 and the critical wave position with respect to the structure is illustrated in Fig. 7a for the maximum hydrodynamic load, and Fig. 7b for the worst load combination.

<table>
<thead>
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<th>Location</th>
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<tr>
<td>Mean water depth</td>
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<tr>
<td>Design head</td>
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<tr>
<td>Design wave*</td>
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<tr>
<td>Design Current</td>
<td>velocity = 0.89 m/s</td>
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<td>Design wind*</td>
<td>mean hourly speed = 33.33 m/s</td>
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*Design wave and design hourly wind speed based on a 50 year return period

The stress distributions and displacements arising from this load combination are shown in Figs 8a and 8b.

Seismic and impact effects are receiving attention as are the thermal loadings from the storage of cryogenic liquid in the inner tank.
INSTALLATION

The structure would be provided with a series of towing points at the maximum diameter arranged in two symmetrical diametrically opposed groups, Fig. 9, the localised forces being transferred into the outer shell via distribution plates on its inner surface.

Two tugs would be required for the towing operation, one fore and aft. The structure would be ballasted down to the level of the maximum diameter using water in the lower part of the inner shell. This water would be contained beneath an insulated and impervious flexible membrane attached to the inner periphery at the maximum diameter. Also ballasting chambers would be provided in the concrete base and use made of power controlled valves. The integral base, concentric with the flat bottom of the shell structure, would be nominally 30 m diameter and 3 m deep.

The nominal mass of the shell structure and its base are listed in Table 2.

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<th>Portion of Tank</th>
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<tr>
<td>Inner steel shell</td>
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</tr>
<tr>
<td>Concrete base</td>
<td>1000</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>7035</strong></td>
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Approximately 1.9 x 10^4 m³ of water is required to ballast the tank to the towing level. The tugs would control stability in the direction of tow with the partially submerged tank being reasonably stable in moderate seas suitable for towing.

At a towing speed of 2 m/s the cable forces could be transferred adequately into the outer shell without any localised thickening of the shell wall.

The towing points would be provided with universal bearing heads and once the tank was located over the station sinking would proceed with the power controlled valves, operated from the surface, permitting a very gradual descent on to a previously located anchor block at the sea bed. The tugs would guide the descent taking very little load in the attached cables, see Fig. 10a.

OPERATION

The sea bed anchor block of nominal size 50 m x 50 m x 6 m, which is required to hold the tank full of L.N.G. at its operating depth, would be placed near to a riser from a buried pipeline to the production platform.

A flexible connection would be made from the riser to the tank, via its apex, and the tank filled with L.N.G. with the aid of pumping. The ballasting water would be exhausted to the sea. As filling with L.N.G. took place the tank would rise from the anchor block because of the nominal 50% reduction in contained liquid mass density compared with sea water, tensioning the cables in the process, see Fig. 10b.

Outlet from the tank would be through the apex using a flexible line via a submerged buoy to a surface pick-up buoy close to a tanker (20). The tanker would uplift the hose from the pick-up buoy in order to receive the L.N.G., Fig. 11. A return line to the production platform via the same route is required to take care of boil-off from the tanker. The hoses would be insulated to reduce the risk of external freezing (20).

The removal of the underwater tank for maintenance or re-location would be facilitated by disconnecting the hoses from the tank and linking them directly. This would allow the continuation of a supply of L.N.G. from the rig to surface tankers.

DISCUSSION AND CONCLUSIONS

The shape of the tank proposed is capable of being designed to sustain the loadings arising from its functions and various environmental situations. However, the probability of impact effects from objects such as underwater trawls could be minimised by the use of surface markers defining a zone prohibited to fishing vessels.

The flat bottomed shell of spheroidal form has weight advantages over similar capacity cylindrical or spherical tanks that might be used for the same purpose. A spherical shell would require a supporting girdle around its horizontal diameter from which its cables/tension legs would extend. The stress distributions in a spherical shell would be far from uniform at the design head, except at very great depths. The equivalent cylindrical tank would be non-uniformly stressed and require ring stiffening against buckling.

In deep water the advantage at first might appear to lie with the cylinder or sphere on or close to the sea bed but in fact the tension leg arrangement would permit an Echinodome to be located in the upper 200 m water layer above the anchor block on the sea bed. The sea bed anchor block could comprise part of a deep sea production facility and remove the need for a storage vessel on or near the sea bed.

The tethering arrangement would have benefits for the storage tank from the point of view of seismic disturbance compared with the situation where the tank was founded on the sea bed.

The high stresses near the base, shown in Fig. 8a for the worst load combination including the 50 year design wave, are of the order of strength
of the concrete selected. Consequently it was prudent to increase the quality of the concrete for the bottom one tenth portion of the outer shell to a grade 80. Some pre-stressing in this portion would be required but in the upper portion the lower grade 60 material would be more than adequate to cope with the purely compressive stresses arising. The deflected shapes given in Figs 4 and 8b indicate clearly the incidence of bending near the base and the need for extra precautions in this region.

The membrane separator is an essential feature of the tank to prevent mixing of L.N.G. and water, as was pointed out previously for the case of oil and water (10, 11). The same design approach could apply to the storage of L.P.G. under water. The thermal loadings are not as great but as it is a light liquid (similar in mass density to L.N.G.) an anchor block would be required to overcome buoyancy.

ACKNOWLEDGEMENTS

Professor A.W. Hendry, Head, Department of Civil Engineering and Building Science, University of Edinburgh for his support and encouragement, and his secretarial and technical staff for their kind assistance. The financial support of the Vans Dunlop Fund is appreciated greatly.

REFERENCES


Fig. 1 The drop shaped shell - Echinodome
Design head = 30 m

Design stress
- 20 MN/m²
- 22.5 MN/m²
- 25 MN/m²
- 27.5 MN/m²
- 30 MN/m²

Operating depth (m)

Volume enclosed ($\times 10^3$ m$^3$)

Thickness = 240 mm

Fig. 2 Variation of capacity with operating depth
Fig. 3 Membrane forces and stresses in the shell at the design head.
DISPLACEMENT OF SHELL AT DEPTH OF END $a = 40.000m$

TYPE OF ANALYSIS = LINEAR STATIC

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Fig. 4 Deflected form of shell at the design head
Fig. 5 Modification to shell walls in lower region
Fig. 6 Design details of double skin tank
Fig. 7(a) Location of wave with respect to structure for greatest hydrodynamic load.

Fig. 7(b) Location of wave with respect to structure for worst load combination.
**TYPE OF ANALYSIS** = LINEAR STATIC

**DEPTH OF END A** = 56.400m

**TOTAL MERIDIONAL LENGTH (L)** = 55.813m

**MERIDIONAL LENGTH = SL, APEX = A, BASE = B.**

---

**GRAPH**

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<td>4. MERIDIONAL STRESS (OUTSIDE SURFACE)</td>
<td>60615914.3</td>
<td>PSI</td>
</tr>
<tr>
<td>5. CIRCUM. STRESS (INSIDE SURFACE)</td>
<td>35156529.0</td>
<td>PSI</td>
</tr>
<tr>
<td>6. CIRCUM. STRESS (OUTSIDE SURFACE)</td>
<td>35433055.0</td>
<td>PSI</td>
</tr>
<tr>
<td>7. EQUIV. STRESS (INSIDE SURFACE)</td>
<td>0.0</td>
<td>PSI</td>
</tr>
<tr>
<td>8. EQUIV. STRESS (OUTSIDE SURFACE)</td>
<td>0.0</td>
<td>PSI</td>
</tr>
</tbody>
</table>

---

**Fig. U(a)** Membrane forces and stresses in the shell under worst load combination.
DISPLACEMENT OF SHELL AT DEPTH OF END A = 56.400 m
TYPE OF ANALYSIS = LINEAR STATIC

Fig. 8(b): Deflected form of shell under worst load combination
PLAN

towing points

towing wire

to tug

direction of travel

to tug

ELEVATION

level of seawater in tank

towing wire to tug

to tug

Tank ballasted to position using seawater

dimensions in metres

Fig. 9 Towing out configuration
support vessels

structure during installation

structure resting on the anchor block

Fig. 10(a) Installation process

tank ballasted down with seawater

attachment points

seabed

anchor block

Fig. 10(b) Operating configuration
Fig. 11  Tanker loading via underwater storage vessel
Proceedings of the
V International Congress
On Experimental Mechanics

Society for Experimental Stress Analysis
EXAMINATION OF THE BUCKLING BEHAVIOUR OF AN UNDERWATER STORAGE VESSEL

K. Royles and J. H. Llambias
Department of Civil Engineering and Building Science, University of Edinburgh

ABSTRACT
Optimum design considerations for an underwater storage vessel to contain liquid gases and oils led to the assessment of an axisymmetric shell of revolution - the Echinodome or drop shape.

Analytical treatment of the various types of loading, to which the shell could be subjected, indicated that buckling was the more critical design criteria.

A small G.R.P. spherical shell under hydrostatic pressure was investigated for its buckling behaviour both experimentally and theoretically. In the experimental approach surface strains were measured using electric resistance strain gauge rosettes on the inner and outer surfaces. Predictions of critical buckling pressure were made from the experimental results using a Southwell technique and numerically by the finite element method.

Comment is made upon the influence of the results on design procedures.

Key words: Buckling, underwater, shells of revolution, electric resistance strain gauges, G.R.P.

INTRODUCTION:
The storage of hazardous liquids, including liquid gases and oils, underwater has attractions from the point of view of safety and economy of space on dry land. The environment beneath the surface is calmer and in many respects less demanding than at the air/sea interface.

An optimum design approach for such storage vessels, based on the minimum weight concept, resulted in an axisymmetric shell of revolution - the Echinodome or drop shape being proposed1,2. Analytical and numerical considerations of the various types of loading that could occur on this kind of structure such as current drag, hydrodynamic and hydrostatic forces, seismic and wind effects revealed that symmetric buckling was the more critical failure mode to be expected.

In order to gain a better understanding of the buckling behaviour of this form of shell a small scale prototype, representing the outer pressure hull of a vessel was examined under various external pressure heads. The details of the experimental and theoretical studies are reported in this paper and comparisons made between them. The implications of the results for full scale design are discussed.

NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>maximum diameter of shell</td>
</tr>
<tr>
<td>d</td>
<td>design head</td>
</tr>
<tr>
<td>E</td>
<td>Young's modulus of elasticity of shell wall</td>
</tr>
<tr>
<td>H</td>
<td>height of shell</td>
</tr>
<tr>
<td>R</td>
<td>maximum radius of shell</td>
</tr>
<tr>
<td>t</td>
<td>mean shell wall thickness</td>
</tr>
<tr>
<td>z</td>
<td>hydrostatic head</td>
</tr>
<tr>
<td>γ</td>
<td>mass density of sea water</td>
</tr>
<tr>
<td>γ_s</td>
<td>mass density of shell wall</td>
</tr>
<tr>
<td>s_d</td>
<td>design stress</td>
</tr>
<tr>
<td>ν</td>
<td>Poisson's ratio of shell wall</td>
</tr>
</tbody>
</table>
THE TEST STRUCTURE

An axisymmetric drop shaped shell structure was designed for a static external pressure head of 1.525 m of water with a mean thickness of 3.8 mm and a uniform design stress of 0.46 MN/m². The meridional profile of the shell was determined using a shape prediction program based on the membrane theory and is of the form shown in Fig. 1. The shell was constructed in two halves using glass reinforced plastic (G.R.P.) with randomly layered chopped strand mat fabric with a glass fraction of 0.26. The bonding together of the two halves was carried out using a general purpose araldite adhesive and similarly the shell was mounted symmetrically on a rectangular base of tufnol.

The actual thickness of the shell was established by mapping with an ultrasonic thickness tester. The general appearance of the structure is shown in Fig. 2 and its dimensional and material characteristics are given in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Shell Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design head, d</td>
</tr>
<tr>
<td>Mean wall thickness, t</td>
</tr>
<tr>
<td>Standard deviation on t</td>
</tr>
<tr>
<td>Height, H</td>
</tr>
<tr>
<td>Maximum diameter, D</td>
</tr>
<tr>
<td>Tufnol base dimensions</td>
</tr>
<tr>
<td>Design stress, σd</td>
</tr>
<tr>
<td>Young’s modulus of elasticity, E</td>
</tr>
<tr>
<td>Poisson’s ratio, ν</td>
</tr>
<tr>
<td>Ultimate tensile strength</td>
</tr>
<tr>
<td>Mass density of wall, ρ</td>
</tr>
</tbody>
</table>

STRAIN GAUGING

The locations of electric resistance strain gauges were chosen at the intersection of three symmetrically arranged meridians and four parallel circles on the outer surface. The more critical zones were indicated theoretically to be near the base and one parallel circle was chosen in this region and the others were fairly evenly distributed over the shell height with one near the apex. In order to detect bending as well as membrane effects in the more critical zone, two locations on the inner surface were selected corresponding to similar points on the outer surface at the parallel circle nearest the base, Fig. 3.

The strain gauges selected were foil type 45° rectangular rosette form, having the following characteristics.

- Gauge resistance = 350 ± 1.0 Ω
- Gauge factor = 2.15
- Gauge length = 3 mm

High resistance foil gauges were chosen to minimise heating effects due to bridge excitation arising from the poor conductivity of GRP.

Each rosette was orientated with one gauge along a meridian and the other two disposed in a clockwise sense. The gauges were bonded using a cyanoacrylate adhesive to a clean and lightly abraded surface. The gauges were given a thin waterproof coating of wax which was brushed on lightly from the molten state. This coating was extended over the terminal tags and extended up the insulation of the electrical leads. In addition a coating of silicone rubber (room temperature vulcanising type) was applied as a further protection.

A special bonding technique was developed for attaching the two rosettes to the inner surface of the shell using a hand manipulated polystyrene former shaped to the localised curvature. The tufnol base was bonded to the shell after attachment of the inner rosettes and electrical leads which passed through a central hole in the base sealed with silicone rubber, Fig. 4.

LOADING ARRANGEMENT

An autoclave was adapted as a pressure chamber for testing the shell. The chamber was a vertical circular cylinder of 465 mm internal dia. and 550 mm length having a spherical bottom and lid, with overall height = 750 mm. It was suitable for a safe working pressure of 2 atmospheres (0.2 MN/m²) and was fitted with a digital pressure indicator as well as a water manometer system.

The shell was mounted via its tufnol base on a dural platform and its positive buoyancy within the flooded chamber was counteracted by a circular cylindrical aluminium strut. The strain gauge leads passed through a
gland in the wall of the chamber which was sealed with silicone rubber, see Fig. 5.

INSTRUMENTATION

Temperature compensation for the active strain gauges was provided by mounting similar rosettes on a dummy half shell of the same material in corresponding positions of curvature. The dummy shell, see Fig. 2, was placed in an open water tank so that it floated with its external surface gauges immersed and the internal surface ones dry.

The gauges were wired in half bridge configuration and connected to a micro-processor controlled data collection system, a schematic representation of which is given in Fig. 6.

The four digit read-out pressure gauge, Setra Systems type (0.17 MN/m² capacity), on the pressure chamber was calibrated against the water manometer and adjusted to read directly to ± 0.005 m.

TESTING PROCEDURE

The test shell was placed in the empty chamber and with the lid sealed and bleed valve, Fig. 5, open gradual filling with water commenced. When all air was expelled and the bleed valve closed, pressure was raised slowly from a gravity head water supply via a control valve. Pressure heads over the apex were applied in increments of 100 mm from zero to 1000 mm, and thereafter in increments of 250 mm up to 3500 mm. The two meridians complemented by strain gauges on the inner surface at the base were monitored for each pressure head. A single scan of the 30 strain gauge channels was carried out in approximately 7.5 secs. for each increment in head.

Using the 100 mm head readings as a datum a computer program calculated the strain changes for each head relative to the datum. These strains were printed out and stored on disk file. Circumferential and meridional stresses were also computed although not required for the buckling investigation described here.

The strain readings were taken on the pressure increasing part of the loading cycle and after unloading a rest period was allowed for creep recovery before repeating the cycle. Five loading cycles were carried out in all.

BUCKLING EXAMINATION

A linear static stress analysis treatment of the axisymmetric loaded shell at its design head was carried out using a finite element simulation based on ring elements. A representation of the displaced shape is given in Fig. 7 from which it can be seen that a buckle might arise near the base of the shell. The associated circumferential and meridional stresses in the shell are given in Fig. 8 and these displayed a mainly uniform variation over the upper 90% of the meridian in conformity with membrane theory. A sharp change in the stresses occurred near the base.

Consequently it was decided to examine the buckling behaviour near the base using the two strain gauge rosettes on the inner surface near the bottom of the shell, Fig. 9. In particular the strain gauges orientated along the meridians in these positions were studied in detail. The pressure-strain relationships at the two positions are shown in Figs 10 and 11 directly and plotted in Figs 12 and 13 respectively. As can be seen there is some scatter in the results but using a least squares approximation the best straight line through the data points was obtained along with the standard deviation. The critical buckling pressure head was determined in each case from the slope of the straight line and is quoted in Figs 12 and 13 from which a mean predicted critical buckling pressure of 46.35 m is found.

A finite element analysis of the axisymmetric loading problem using a non-linear elastic treatment based on the Mistry7 program yielded a first axisymmetric critical buckling pressure of 43.26 m with a node shape as shown in Fig. 9.

DISCUSSION

The comparison of the experimentally predicted critical buckling pressure with that derived from a finite element analysis is good although it must be stated that the finite element approach did predict a lower critical buckling pressure in the first symmetric mode, see Fig. 14. In the experimental approach described here the choice of meridional strain near the base as the deformation parameter for use in the Southwell plot could be expected to yield information mainly about the major deformation mode in that region, i.e. first axisymmetric mode. Contributions to deformation from the first symmetric mode could be anticipated at levels near the maximum diameter. This phenomenon is being investigated from the data obtained at parallel circle 2, see Fig. 3, about which it is hoped to report later.
Overall the experiments have verified the type of finite element simulation employed and given some confidence to its use with thin axisymmetric shells of revolution.

The distribution of stresses in the shell at the design head, Fig. 8, was fairly uniform. The minor fluctuations in the upper 90% of the shell arose from variations in the wall thickness. These results tend to justify the use of a simple membrane analysis in the initial design stages.

However a design based on the membrane approach must be modified for the bottom 10% of the shell in order to counter buckling tendencies in this region. Some gradual thickening of the shell wall in this region could have very beneficial effects.

CONCLUSION

The Southwell plot approach to buckling in spheroidal type structures is quite feasible and offers a non-destructive means of predicting buckling pressures.

ACKNOWLEDGEMENTS

The authors wish to thank Professor A.W. Hendry, Head, Department of Civil Engineering and Building Science, University of Edinburgh for his encouragement and support, and also for the kind assistance of members of his secretarial and technical staff. The advice of Mr. John Mistry, Department of Mechanical Engineering, University of Liverpool, regarding the finite element approach is appreciated greatly. Lastly, but not least, gratitude is expressed for the financial support of the Vans Dunlop Scholarship scheme.

REFERENCES

Fig. 1 The drop shaped shell
Fig. 3 Location of strain gauge rosettes
Fig. 4 Base of Shell
Fig. 5 Pressure chamber test arrangement
to strain gauges on the tank and dummy

Fig. 6 Block diagram of instrumentation
DISPLACEMENT OF SHELL AT DEPTH OF END A= 1.525m
TYPE OF ANALYSIS = LINEAR STATIC

Fig. 7 Displaced shape of shell at the design head
TYPE OF ANALYSIS = LINEAR STATIC
DEPTH OF END A = 1.525m
TOTAL MERIDIONAL LENGTH (L) = 0.575m

MERIDIONAL LENGTH = SL, APEX = A, BASE = B.

\[ \frac{m}{a} \quad \text{ORDINATE VALUE (ORDINATE LENGTH)/S} \]

\[ \frac{a}{b} \quad \text{ORDINATE VALUE} \]

\[ \frac{b}{a} \quad \text{ORDINATE LENGTH} \]

\[ \frac{m}{a} \quad \text{ORDINATE VALUE} \]

\[ \frac{a}{b} \quad \text{ORDINATE VALUE} \]

\[ \frac{b}{a} \quad \text{ORDINATE LENGTH} \]

Fig. 8 Stress distributions in the shell at the design head
Fig. 9 Displaced shape of shell at critical buckling
Fig. 10 Variation of pressure head with strain for meridional strain gauge - rosette II (see Fig. 3)
Fig. 11 Variation of pressure head with strain for meridional strain gauge - rosette 12 (see Fig. 3)
Fig. 12 Southwell plot corresponding to Fig. 10

Buckling press. head = 43.1 m

Scatter band
(Standard deviation = 0.78)
Fig. 13 Southwell plot corresponding to Fig. 11

SOUTHWELL PLOT
(STRAIN GAUGE No. 28)

Scatter band
(Standard deviation = 1.19)

Buckling presse. head = 49.6m

Microstrain

Fig. 13 Southwell plot corresponding to Fig. 11
Fig. 14 Displaced shape of shell at first symmetric buckling mode
BUCKLING ASPECTS OF THE BEHAVIOUR OF AN
UNDERWATER PRESSURE VESSEL

by
R Royles* and J M Llambias*

SYNOPSIS

The overall buckling behaviour under external hydrostatic pressure of
a small prototype spheroidal shell of revolution - an Echinodome - is
examined both experimentally and theoretically.

The analysis is approached numerically using the finite element method
and in addition a simplified classical approach is considered.

An experimental investigation is described relating to a glass
reinforced plastic shell structure with electric resistance strain
gauges bonded on the outer and inner surfaces on several meridians and
parallel circles.

The pressure - strain response from the various locations were
analysed using the Southwell method and good agreement was found
regarding critical pressures and mode shapes with the theoretical
treatment.

*University of Edinburgh
1. **INTRODUCTION**

Buckling under hydrostatic pressure is a major problem area facing an engineer designing underwater shell structures. Structural instability in such shells can occur well before the material is highly stressed and could result in a catastrophic implosive failure. Therefore it is very important that the buckling behaviour of any particular form of structure is investigated carefully before employing it in underwater applications.

The shell examined here is one of uniform strength, the Echinodome (1). It is axisymmetric and for a certain loading condition - mean hydrostatic head - it is an optimum form. This type of vessel could be used for storage of liquids or as a one atmosphere enclosure in a sub-sea environment.

The buckling behaviour of this shell form is examined here both experimentally and theoretically.

A linear and non-linear elastic buckling analysis of a small prototype is carried out, using the finite element method, to determine the critical buckling pressures and their corresponding mode shapes. These are subsequently compared with an approximate solution obtained using the classical shell theory.

Then the numerical results are verified by experimental work based on the Southwell technique for predicting the critical buckling loads of structures.

2. **ANALYTICAL APPROACH**

Considerable work has been done, over the past fifty years, to develop a theoretical model for predicting the buckling loads and mode shapes of shells of revolution (2,3,4,5).

Most of this work was based on Love's general shell theory (6) and was restricted to simple problems such as spheres and cylinders under external pressure, as the differential equations for these shapes lent themselves to exact analytical solutions. The results obtained from this classical approach were well above observed experimental values and further work was necessary to lower the predicted buckling pressures by taking into account initial imperfections (5,7).

The development of the finite element method permitted the buckling examination of general shells of revolution (8,9,10).

In the present work the Mistry finite element program (8,9,11) was adopted for predicting critical buckling loads corresponding to either snap through collapse or non-axisymmetric bifurcation buckling.

A detailed explanation of the theory behind this procedure has been presented previously (9,11).
A small prototype of glass reinforced plastic having the dimensions as shown in Fig 1, was simulated using ring elements taking into account the variation in mean thickness down the meridian.

The material has a Young's modulus of $0.88 \times 10^4 \text{ MN/m}^2$, an ultimate tensile strength of $54.2 \text{ MN/m}^2$ and a Poisson's ratio of 0.36.

Elastic buckling analyses of the prototype under external hydrostatic pressure were performed using both linear and non-linear pre-buckling stress resultants and the results are shown in Table 1.

<table>
<thead>
<tr>
<th>Type of buckling</th>
<th>Buckling pressure head (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>linear</td>
</tr>
<tr>
<td>Snap through (n=0)</td>
<td>103.74</td>
</tr>
<tr>
<td>Bifurcation (n=1)</td>
<td>40.26</td>
</tr>
</tbody>
</table>

Table 1 Theoretical buckling pressure heads (n = harmonic number)

The corresponding buckling mode shapes are shown in Fig 2.

The cpu time required for a buckling analysis using linear pre-buckling stress resultants was 22.5s whilst the cpu time required for a buckling analysis using non-linear pre-buckling stress resultants was 193.9s.

It is interesting to note that in this particular case the ultimate strength of the shell material would be exceeded in the bottom tenth of the shell wall before bifurcation buckling occurred. The maximum pre-buckling Von Mises stresses for the non-linear buckling analysis are shown in Table 2.

<table>
<thead>
<tr>
<th>Type of buckling</th>
<th>Max Von Mises stress (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snap through (n=0)</td>
<td>178.2</td>
</tr>
<tr>
<td>Bifurcation (n=1)</td>
<td>78.7</td>
</tr>
</tbody>
</table>

Table 2 Maximum pre-buckling Von Mises stresses for the non-linear buckling analyses

The cost in terms of computer run-time and storage is quite high for a buckling analysis based on the finite element method. As a result a classical approach to buckling was examined to determine whether it could be used in the initial stages of design to give an accurate and quick approximation of the critical buckling pressure of the Echino dome.
The equation for the buckling of a sphere was considered. An equivalent sphere with a diameter of \( \frac{1}{2}(0.45+0.38) = 0.415 \text{m} \) and an average thickness of 3.8 mm was examined under external pressure.

Using Von-Karman and Tsien's (5) modified linear equation for the buckling of the sphere, the critical buckling pressure head was found to be 99.8 m for axisymmetric snap through collapse. Although this agrees well with the critical buckling pressure head for snap through collapse obtained from a linear buckling analysis using the finite element method (see Table 1), the more critical bifurcation buckling pressure head is about 32% of this value. Thus indicating that the classical approach could not be used safely in the initial stages of the design.

3. EXPERIMENTAL INVESTIGATION

Test Arrangement

In order to gain a better understanding of the buckling behaviour of the Echinodome and assess the suitability of the finite element method in predicting the critical loads, a series of buckling tests was performed on the small prototype, Fig 1.

Employing an epoxy matrix and randomly orientated chopped strand glass fibre mat the shell was manufactured in two halves using a mould consisting of an inner and an outer part. The shell was layerd up gradually on the inner surface of the outer part of the mould, the inner part being used to gauge the thickness. The two halves of the shell were bonded together along the meridional seam using araldite and likewise the complete shell was fixed to its tufnol base. The shell wall thickness was determined by mapping the whole surface with an ultrasonic device capable of measuring to better than ± 0.01 mm in 10 mm. The final mean shell thickness was 3.8 mm and the glass fraction was 26% by weight.

The surface strains were measured using electric resistance strain gauges in a rectangular rosette form at 10 locations on two meridians at 120° spacing. A typical meridional arrangement of the gauges is indicated in Fig 4. The gauges were of 350 ± 1.0, gauge factor = 2.15 and gauge length = 3mm. More details of the strain gauging and associated instrumentation have been presented elsewhere (12,13).

The pressure chamber was a copper autoclave, specially adapted for testing the prototype under hydrostatic pressure up to a head of 20 m of water. It was cylindrical with an internal diameter of 465 mm. It had a torispherical bottom and a spherical removable lid giving it an overall height of 750 mm. The lid had a sealable bleed hole at its apex to allow air to be expelled as the chamber filled. The chamber was pressurised, through a hole in its side, directly from the water mains.
The pressure was monitored by both a water manometer and a digital pressure gauge for heads less than 1 metre and by the digital gauge alone for greater heads. The digital pressure gauge was a Setra Systems pressure transducer (model 205-2) capable of reading up to 0.17 MN/m² with a full scale accuracy of 0.1% at constant temperature. It was calibrated against the water manometer and adjusted to read directly to ± 0.005m.

The shell inside the test chamber was held in position via its base.

The leads from the strain gauges were passed through an opening in the wall of the chamber which was subsequently filled with silicone rubber (room temperature vulcanising) to keep it watertight.

A view of the whole test apparatus is shown in Fig 3.

**Test Procedure**

Once the prototype was in position inside the chamber, the strain gauges were tested using a gauge installation tester and a few scans were made to ensure that all the equipment was connected and functioning correctly.

After closing the pressure chamber and keeping the bleed valve on the lid open, water was allowed in slowly. When the chamber was completely full and after inspecting for leaks and air bubbles in the system, the bleed valve was closed and the pressure was increased gradually using a control valve.

The pressure head over the apex of the shell was raised to 1000mm in increments of 100mm and then up to 3500mm in increments of 250mm. The pressurisation was done at a uniform rate throughout - an increment approximately every 40s.

At each pressure level a scan was made of the strain gauges in the two meridians. The scans were controlled by a Commodore Pet computer and took approximately 7.5s to read the 30 strain gauge channels at each increment. These readings were taken on the pressure increasing part of the loading cycle as soon as the required pressure was reached in each case.

A total of five runs were made allowing 5 mins between runs for creep recovery.

**Test Results**

The pressure - strain curve for each strain gauge was drawn using the results from all five test runs. Typical relationships from outer surface strain gauges near the apex, maximum diameter and near the base are shown in Figs 5 to 7.

All these curves, which were fitted by eye, showed a linear relationship between the pressure and strain up to a pressure head of about 2.0m. Above this pressure head, the relationship became non-linear and approximated to a hyperbolic curve.
It should be noted, though, that not all the strain gauges exhibited a hyperbolic pressure - strain relationship. Seven gauges located on the second and third parallel circles gave values of strain that were either very low or erratic. This could be attributed in part to the region being one of very low stress under the loading employed. These gauges were ignored in predicting critical loads.

Analysis of Results

The approximately hyperbolic nature of the pressure - strain curves shown in Figs 5 to 7 suggested that the Southwell technique (14,15,16) would be suitable to interpret the results of these buckling tests. Since any deformation parameter could be used for obtaining a Southwell plot, the measured strains were employed directly for that purpose. A Southwell plot, in the form of strain per unit pressure head against strain, was drawn for all the strain gauges exhibiting a hyperbolic pressure - strain relationship (21 out of 30) and the plots corresponding to the outer surface data in Figs 5 to 7 are shown in Figs 8 to 10. Typical representations from the internal surface data were made earlier (13).

In all these Southwell plots, the straight lines were fitted using a curve fitting package based on the least squares approximation (17). The critical loads were obtained directly from the equations of the straight lines, i.e the inverse of the slope of the line, and a value for the standard deviation of all the points used in obtaining this equation was obtained from the package.

The average critical buckling load predicted by the gauges at each parallel circle was then determined and a listing of the resulting meridional variation in the predicted critical buckling load is shown in Table 3.
### Table 3  Mean predicted buckling pressure heads at each parallel circle
(mer = meridional gauge; 45 = gauge at 45° to the meridian; cir = circumferential gauge)

<table>
<thead>
<tr>
<th>Parallel circle</th>
<th>Direction of gauge</th>
<th>Pressure head (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>mer</td>
<td>36.3</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>44.4</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>38.5</td>
</tr>
<tr>
<td>2</td>
<td>mer</td>
<td>44.2</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>25.1</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>mer</td>
<td>46.6</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>29.6</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>-</td>
</tr>
<tr>
<td>4(outer)</td>
<td>mer</td>
<td>42.5</td>
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<tr>
<td></td>
<td>45</td>
<td>43.3</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>43.7</td>
</tr>
<tr>
<td>4(inner)</td>
<td>mer</td>
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<tr>
<td></td>
<td>45</td>
<td>46.4</td>
</tr>
<tr>
<td></td>
<td>cir</td>
<td>45.0</td>
</tr>
</tbody>
</table>

4. **DISCUSSION**

Table 3 shows that there is some variation in the predicted critical load depending on the position and orientation of the strain gauge on the surface of the shell.

All the strain gauges near the base gave values for the critical load which were within 6% of each other. The outer surface strain gauges predicted an average pressure head of 43.2m whilst the inner surface strain gauge predicted a value of 45.9m, giving an overall mean value for the critical pressure head of 44.3m for that region.

It is interesting to note that on the other three parallel circles, some of the strain gauges predicted a buckling pressure head very close to this value, indicating the possibilities of global buckling taking place around a pressure level equal to an overall mean hydrostatic head of 41.7m. The fact that in the vicinity of the shell's maximum diameter (i.e. parallel circles 2 and 3) only the gauges orientated along the meridian predicted a global type of buckling further suggests that in this mode shape there was much more deformation in the meridional plane than in any other and therefore indicated an axisymmetric buckling mode.
However, some of the other strain gauges in the upper part of the shell predicted values considerably less than 41.7m, suggesting that local buckling was occurring at a lower head. This lower head was predicted to be 36.3m by the meridional strain gauges in parallel circle 1 and 27.4m by the non-meridional strain gauges in parallel circles 2 and 3. The implications of these results are that the effects of local snap through buckling were more pronounced at the apex, i.e. dimpling, and that around the shell’s maximum diameter a non-axisymmetric buckling mode was exhibiting itself.

In general, the experimental results compared well with the numerical ones, see Table 4.

<table>
<thead>
<tr>
<th>Mode of buckling</th>
<th>critical pressure head</th>
<th>Exp. (m)</th>
<th>F.E.M. (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axisymmetric</td>
<td>41.7</td>
<td>43.2</td>
<td></td>
</tr>
<tr>
<td>Non-axisymmetric</td>
<td>27.4</td>
<td>31.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 Comparison of experimental and numerical results

The non-linear results from the finite element analysis were used for this comparison since the Southwell plot predicts the non-linear buckling loads of structures.

The table above shows that for both axisymmetric and non-axisymmetric buckling the experimental critical load was lower than the numerical one, the experimental value being within 4% and 12% of the numerical one for the axisymmetric and non-axisymmetric modes respectively. The greater discrepancy between the values for the non-axisymmetric buckling could be accounted for by the fact that very few of the strain gauges were predicting that mode shape. The translational deflections involved with that particular mode of buckling were very small and consequently some of the strain gauges that could have predicted it (i.e. those on parallel circles 2 and 3) gave values of strain that were too low and could not be used to construct a Southwell plot.

However, some difference could be expected between the experimental and numerical results in view of the thickness variation over the shell surface. It was possible only to model a meridional variation in thickness in the finite element analysis whilst in actual fact there was also a circumferential variation, see Table 5. Nonetheless an attempt was made to deal with this variation by averaging the results obtained from each parallel circle so as to enable a comparison with the finite element method.

Other factors, such as the level of geometric imperfections and the degree of fixity at the support also could have influenced the experimental values obtained. But, it should be noted that a Southwell plot would be predicting the critical load of the actual
prototype together with all its imperfections, variations in thickness etc. and it is the numerical finite element simulation which has to approximate the actual behaviour as best as possible.

Only a test to destruction could give the actual buckling load of any particular form of shell together with all its imperfections and irregularities and ultimately a buckling test to destruction should be carried out as it would be the only way of confirming the Southwell plot.

In this work, the non-destructive test based on the Southwell plot was employed to verify the results obtained from the finite element method as only one prototype was available for testing and a whole series of future tests on it had been planned. The results of this work substantiates the use of the Mistry finite element program for buckling analyses and adds some confidence to its use in the design of Echinodomes for underwater applications.

5. CONCLUSIONS

For the Echinodome under external hydrostatic pressure two buckling modes were predicted analytically and confirmed experimentally, the lowest critical load being associated with a non-axisymmetric (i.e. translational) bifurcation buckling mode.

The Southwell approach to the buckling of spheroidal shells of revolution was found to be extremely relevant in its ability to predict both global and local modes of buckling.
### Table 5  Thickness variation over the shell.

<table>
<thead>
<tr>
<th>PARALLEL</th>
<th>Distance from apex (millimetres)</th>
<th>AVERAGE THICKNESS (millimetres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIRCLE</td>
<td>10</td>
<td>3.27</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>3.82</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>3.94</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>50</td>
<td>4.04</td>
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<tr>
<td></td>
<td>60</td>
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<td></td>
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<td>3.62</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>3.44</td>
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</table>

**Total meridional length = L**

<table>
<thead>
<tr>
<th>MERIDIAN</th>
<th>AVERAGE THICKNESS (millimetres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.46</td>
</tr>
<tr>
<td>2</td>
<td>3.93</td>
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<tr>
<td>3</td>
<td>3.40</td>
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<tr>
<td>10</td>
<td>3.37</td>
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<td>11</td>
<td>3.89</td>
</tr>
<tr>
<td>12</td>
<td>4.21</td>
</tr>
</tbody>
</table>

6. ACKNOWLEDGEMENTS

The authors are most grateful for the encouragement and support of Professor A W Hendry, Head of Department of Civil Engineering and Building Science, University of Edinburgh, and of his secretarial and technical staff. Thanks are also due to Mr John Mistry, Department of Mechanical Engineering, University of Liverpool for permission to use his finite element program. Finally, the financial support of the Vans Dunlop Scholarship Fund is appreciated greatly.
7. REFERENCES


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Fig 1 The Enchinodome
(\(\gamma\) = mass density of water)

Fig 2 Numerical critical buckling modes

Fig 3 General layout of apparatus

Fig 4 Typical meridional arrangement of strain gauge rosettes

Fig 5 Pressure – strain curve for a gauge near the apex (parallel circle 1)

Fig 6 Pressure – strain curve for a gauge near the maximum diameter (parallel circle 3)

Fig 7 Pressure – strain curve for a gauge near the base (parallel circle 4 – outer surface)

Fig 8 Southwell plot corresponding to Fig 5

Fig 9 Southwell plot corresponding to Fig 6

Fig 10 Southwell plot corresponding to Fig 7
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Fig. 2
Fig. 4
GRAPH OF PRESSURE V. STRAIN

(STRAIN GAUGE No. 3)

Microstrain

Fig. 5
GRAPH OF PRESSURE HEAD VS STRAIN
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Microstrain

Fig. 6
Graph of Pressure Head vs Strain

(Stain Gauge No. 24)

Fig. 7
Fig. 8

SOUTH WELL PLOT

(STRAIN GAUGE No. 3)

Scatter band
(Standard deviation = 0.75)

Buckling press. head = 40.0

Microstrain

Microstrain/metre pressure head

0 10 20 30 40 50 60 70 80 90 100
SOUTHWELL PLOT
(STRAIN GRUGE No. 13)

Scatter band
(Standard deviation = 1.48)

Buckling press. head = 29.1k

Microstrain/metre pressure head

Fig. 9
Fig. 10