An Electromagnetic Induction Study of South Cornwall, England.

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Declaration

This thesis has been composed by me and has not been submitted for any other degree. Except where acknowledgement is made, the work is original.

Philip Charles Jones
Abstract

Twenty one magnetotelluric and 16 magnetovariation soundings were taken on or near the Carnmenellis granite over periods in the range of 0.0078 seconds to 3000 seconds. The measured impedance tensors were analysed in detail using decomposition methods. All the data are distorted by galvanic electric charges which build up on the granite country rock contact. The amount of distortion varies between sites with distance from the edge of the batholith. The vast majority of the data are at least two dimensional and the effects of three dimensional induction are increasingly sensed by periods greater than 1s. Short period soundings indicate that the anisotropy in the magnetotelluric field is caused by electric current being channelled along fluid filled cracks. One dimensional modelling of the E-pol response indicates that the bottom of the granite is not flat, but slopes downwards to the south. This finding is evidence to support the theory that the Cornubian granites originated SSE of their present position and were rafted NNW as a thin sheet. Two dimensional modelling suggests that at least a portion of the gradient of this slope is caused by the neglection of 3D induction in models used in the study. The pattern of regional azimuths between 0.1 and 10 seconds is caused by a combination of lateral, near surface, conductivity contrast, such as the surrounding seas, and conductivity contrasts at depth due to the slope of the bottom of the granite.

Mainly due to the effects of conductivity contrasts perpendicular to the regional azimuth, it was found impossible to find a model which fitted the E-pol data at both on and off granite sites. The 2D model indicates that there is a steep rise in the resistivity depth profile of the granite from 800 Ohm-m at the surface to 20000 Ohm-m at 4 km. The closure of fluid filled joints due to the increase in lithological load with depth is interpreted to be the cause of this increase in resistivity. The 2D model indicates the existence of resistive material at depths greater than 20 km and this is evidence to support the theory that the granite may change gradually with depth into a granodioritic crust.

Thin sheet modelling of the measured induction arrows detects the presence of a conductive region north of the Carnmenellis granite and the channelling of current through this region from the south to the north coast of the peninsula. It is proposed that the conductivity of this area is enhanced by a high density of fluid filled fractures.
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Chapter 1

Introduction

It is the aim of geophysics to quantify the properties of the Earth at depth using surface measurements and interpret variations in these properties in terms of geological structure. One such geophysical method is electromagnetic (EM) induction sounding, which indirectly measures the electrical structure of the Earth. Two methods of EM induction sounding will be described in this thesis - the Magnetotelluric method (MT), involves measuring the changes in perpendicular components of the Earth's magnetic and electrical fields and the magnetovariation (MV) method, which utilises changes in perpendicular components of only the magnetic field. Both methods use the fluctuations of the Earth's magnetic field as a source.

This thesis describes an EM induction survey of south-west Cornwall, centring on the Carnmenellis outcrop. Despite the extensive knowledge of the geology of the area, there still remain a number of problems relating to the tectonic history and the structures at depth. The desire to obtain more geological and geophysical information about the area has been given an extra emphasis by the Carnmenellis granite becoming a "hot dry rock" geothermal prospect.

This chapter gives a short description of the source fields used in the magnetotelluric method and the significance of the electrical conductivity of rocks. It proceeds to describe the geological and geophysical knowledge outlined in the literature that is relevant to the work contained in the thesis and finally to outline the aims of the project.

1.2 The Cambourne School of Mines Hot dry Rock Geothermal Project

The Cambourne School of Mines Hot Dry Rock Geothermal Project was set up to research and develop the possibilities of commercially generating electricity from Hot Dry Rock (HDR) technology. It is sited at Rosemanowes Quarry near Falmouth on the Carnmenellis granite outcrop, south-west Cornwall, see figure 1.1. The HDR concept involves drilling a borehole down to a suitable depth and creating a permeable reservoir of rock, then injecting cold water into the reservoir where it extracts geothermal energy from the rock. Finally it is pumped back to the surface using a second borehole (Parker, 1989). The establishment of the HDR project has led to an upsurge in interest in the geology and geophysics of the area.
FIGURE 1.1: THE LOCATION OF THE HOT DRY ROCK PROJECT (HDR) AND THE CARNMENELLIS GRANITE (DOTS AND DASHES)
1.3 The Source fields for the Magnetotelluric and Magnetovariation Sounding Methods

The source field for the magnetotelluric and magnetovariation sounding methods is provided by natural time variations in the Earth's magnetic field. In the period band useful for geophysical prospecting, two types of activity cause these variations in the Earth's magnetic field.

1.3.1 Source Fields with Periods Below 0.2 Seconds

Lightning strokes are the cause of the magnetic field variations which form the source fields with periods below 0.2 seconds. At any one time there is a high probability of a thunderstorm taking place somewhere on the Earth, most likely in the tropics where the vast majority of thunderstorms take place. The signals from the lightning strokes contain a wide spectrum of frequencies and are known as sferics. The formation of these signals is described in many publications for instance Bleil (1964), Ward (1959). The sferics travel around the Earth trapped in a wave guide formed by the Earth and the ionosphere. As they travel in the wave guide some frequencies in the signal are enhanced and some lost, thus producing a number of amplitude peaks in the spectrum, for instance at the Schumann frequencies of 8, 14, 20 and 25 Hz, (Schumann 1957).

The large distances travelled by sferics ensure the formation of a uniform source required by Cagniard M.T. relationships, (Wait, 1954).

1.3.2 Source Fields with Periods Above 0.2 Seconds

Variations in the Earth's magnetic field at periods above 1s are caused by interactions of the Earth's permanent magnetic field (magnetosphere) with ionized particles flowing from the sun (the solar wind). The ions of the solar wind on meeting the Earth's magnetic field flow in opposite directions depending on whether they are protons or electrons, so forming a plasma sheet region with an electric current and a subsequent magnetic field effect. This cancels the Earth's magnetic field where it occurs, thus a boundary to the Earth's magnetosphere is formed and is called the magnetopause.

If a magnetic storm or substorm is to develop, it is essential, that the interplanetary magnetic field (IMF) has a southward component, for only then will the plasma and electric field of the solar wind be able to penetrate the magnetosphere. If the solar wind pressure increases, for instance due to a solar flare, the whole of the magnetosphere can, in a short time, become compressed. This compression squeezes together the lines of force within the magnetosphere, causing the horizontal component of the geomagnetic field to suddenly increase. Compression of field lines on the daylight side occurs first causing the outer portion of the nightside magnetosphere to expand. Under these conditions high energy plasma is pushed along magnetic field lines which now connect the plasma sheet region on the nightside of the earth to the ionosphere over
Figure 1.2
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Figure 1.3
Amplitude spectrum of a magnetic bay, Pc, Pi, ELF and sferics.
After (Matsushita and Campbell, 1967).
the auroral zones. The ionosphere is a portion of the Earth's atmosphere which is ionized to varying degrees. Thus the normal plasma sheet current is short circuited over the auroral zones and the resulting magnetic field causes a net flow of current westwards. This produces a large decrease in the horizontal magnetic field over the auroral regions known on magnetic recordings as magnetic bays. An example of a magnetic bay measured at one of the sites on the Carnmenellis granite is shown in figure 1.2.

Any magnetic effects in the upper magnetosphere due to solar plasma behaviour induce current within the ionosphere. This induction of current leads to a displacement of mass and the interaction of inertial and magnetic forces within the ionosphere produces magnetohydromagnetic waves. These hydromagnetic waves are transferred into electromagnetic waves at the lower limit of the ionosphere (Jacobs, 1970). Thus magnetic effects at the magnetosphere are strongly modified by the time they reach the earth. For further details of the causes and effects of magnetic storms see Parkinson, (1983) and McPheron, (1979).

Figure 1.3 shows that the amplitude spectrum above 0.2s consists of a number of geomagnetic pulsations, including Pc (pulsation continuous), Pi (pulsation irregular), and (pulsation pearls) as well as magnetic bays. Geomagnetic pulsations are small amplitude, almost sinusoidal fluctuations, superimposed on all elements of the Earth's magnetic field. Descriptions of these pulsations are given in Parkinson (1983) and Orr (1973) and the causes of them are discussed in detail in Orr (1973). Protons from solar wind plasma drifting westwards and bouncing within the field lines transfer energy into hydromagnetic waves, thus producing standing Alfven waves along lines of force of the magnetosphere. These standing waves are thought to be the cause of Pc 2, 3 and 4, whilst Pc 1 and pearls are thought to be caused by travelling hydromagnetic waves.

Notice from figure 1.3 that a low in the amplitude spectrum occurs for variations in the magnetic field with periods around 0.3 seconds. This period band will be refered to as the "dead band". It will be shown in later chapters that the low amplitude signal over this period range causes a decrease in the quality of the data recorded.

1.4 The Significance of Conductivity

There is a need to interpret resistivity-depth profiles in terms which have geological interest, such as temperature profile, water content or pressure gradient. This interpretation is facilitated by comparisons with laboratory based petrophysical measurements. However, such interpretations are far from easy due to the great number of parameters that can influence changes in rock conductivity.

Electricity can pass through the Earth by either ohmic flow of free electrons in materials such as metals, or by semi conduction in poor conductors, see Telford et. al. (1981). Although most crustal rocks, when dry, are bad conductors, they contain pores filled with fluid, mainly saline
water, and pass electricity electrolytically via the relatively slow flow of ions in the fluid. The resistivity behaviour of water saturated rocks where electrolytic conduction is dominant is described by the empirical formula, Archie's law, (Archie, 1942).

\[ \rho_r = a \varnothing^m s^n \rho_w \]  

(1.1)

where

- \( \rho_r \) = resistivity of the water saturated rock
- \( a \) = constant \( 0.5 \leq a \leq 2.5 \)
- \( \varnothing \) = porosity
- \( m \) = cementation factor \( 1.3 \leq m \leq 2.5 \)
- \( \rho_w \) = resistivity of the electrolyte
- \( s \) = fraction of pores containing water
- \( n \) = constant \( n=2 \)

In order to have a significant effect on the conductivity of a rock the fluid needs to join along the path of the pores. From equation 1.1 it can be seen that the conductivity varies with the size and geometry of the pores, but more importantly with the content and conductivity of the electrolyte.

1.4.1 Laboratory Experiments Investigating the Electrical Properties of Granites

The water content of crystalline rocks including granites, varies from 0.03% to 3% and is much lower than that of sedimentary rocks, hence the resistivity of crystalline rocks is much greater than that of sediments.

The early work on the electrical properties of granite (Brace et al., 1965; Brace et al., 1966; Brace and Orange, 1968) is reviewed along with the results of new experiments, by Olhoeft (1981) and Parkhomenko (1982). Figure 1.4 shows that water content not only has a drastic effect on the amplitude of resistivity but also on the temperature dependence of resistivity. Indeed water content and temperature are concluded by Olhoeft (1981) to be the dominant factors in determining the resistivity of granite under crustal conditions from room temperature to melting point. Olhoeft (1981) proceeds to show that only free water is significant in terms of resistivity. Volarovich and Parkomenko (1976) in combining Archie's formula (equation 1.1) with other equations describing the resistivity behaviour of the electrolyte, state that the resistivity of rocks decreases exponentially with temperature. Laboratory results show that the effect of hydrostatic pressure is insignificant for temperatures below the melting point, except for rocks saturated with NaCl solutions with salinities less than 0.1 molar. The effect of containing pressure on low salinity water
Figure 1.4
Summary of the available data on electrical resistivity versus temperature for wet and dry granite. Open circles are vacuum dry (10^-10 MPa) Westerly Granite. (Olhoeft, 1981). Closed circles are 0.4 molar NaClaq solution-saturated Westerly Granite (Olhoeft, 1981). Solid lines (with water pressure indicated in MPa) are water-saturated El'dzurta Granite from northern Caucasus (after Lebedev and Khitarov, 1964). Figure taken from (Olhoeft, 1981).

Figure 1.5
The effect of confining pressure on water-saturated Westerly Granite. Figure taken from (Brace and Orange, 1968)
Summary plot of the best available estimate for electrical resistivity versus temperature, pressure, and water content. The dashed lines are for various water pressures in MPa for water salinities less than 0.1 molar NaCl. Above a salinity of 0.1 molar, the wet curve is essentially independent of pressure through granitic melting. Figure taken from (Olhoeft, 1981).

Temperature dependence of the resistivity of lidate and Inada granites. Figure taken from (Llera et. al., 1990)
saturated granites is shown in figure 1.5. A summary of Olhoeft's (1981) conclusions is shown in figure 1.6.

However in discussing the significance of pressure on the bulk resistivity of a granite formation, we must take into account the fact that water filled cracks and joints exist in any batholith. The permeability of such fissures is much greater than the surrounding rock matrix, (Berkold, 1983). Hydrostatic pressure will obviously influence the rate at which these cracks close with depth. Some laboratory evidence for the role of microcracks in the reduction of resistivity of granites is given by Llera et al. (1990). In studying the temperature dependence of the electrical resistivity of water-saturated granites under constant pressure, Llera et al. (1990) found a greater than exponential reduction in resistivity in the range 30° to 200° C, see figure 1.7. The fact that hysteresis effects were found in resistivity behaviour of the granites and other rocks, led Llera et al. (1990) to conclude that the thermal growth of microcracks increases the porosity of the rock and thus contributes to the decrease in electrical resistivity with temperature. Microcracks were found to have an effect after 50° C.

1.4.2 The Electrical Conductivity of Sediments

Sedimentary rocks contain significant amounts of interstitial moisture and the factors effecting electrolytic conduction described in the above section also apply to sediments. The amount of moisture is determined by the porosity, which for sandstones is about 5-20% and for limestones 1.5-15% (Volarovich and Parkhomenko, 1976). The porosity of clays however is 20-50%. The fact that clay absorbs so much moisture that it can become plastic, means that it forms a very good conductor and the presence or absence of clay is also a factor in determining the conductivity of sediments.

1.5 The Geology and Geophysics of South-West England

This section briefly describes the surface geology of south-west Cornwall and reviews the present knowledge gained from geophysical measurements of the structure at depth. There then follows a section detailing how these data inform the various accounts of the tectonic history of the area and finally information on the physical parameters of the Earth which affect electromagnetic induction measurements is presented.

1.5.1 Stratigraphy of South-West Cornwall

Figure 1.8 shows a geological sketch map of south Cornwall based on B.G.S. 1:50 000 maps, (Leveridge et al., 1984).

The Lizard complex, which is in fact south of the survey area, consists of a series of basic/ultrabasic and metamorphic rocks, some of which form the oldest rocks in the area (dated at 370 Ma). They are
Figure 1.8
A Geological sketch map of south Cornwall based on B.G.S. 1:50 000 maps. Taken from Leveridge et al., (1984).
assumed to be a slice of ocean floor thrust over the Devonian sediments during the early stages of the Variscan Orogeny, (Bromley, 1979). The stratigraphy and tectonics of the Lizard has been widely discussed in the literature and only references to a few of the relevant papers will be given here, (Green, 1964; Strong et al., 1975; Styles and Kirby, 1980).

The Gramscatho formation of lower to middle Devonian age consists of dark grey slates interbedded with sandstone turbidites deposited in deep water upon a geosyncline (Sadler, 1973). The Meneage breccias consist of dark grey slates containing macroscopic clasts of volcanic, magmatic, metamorphic and sedimentary rocks. The origin of these rocks is controversial; however recent work suggests they are sedimentary, (Leveridge et al., 1984). The Mylor slate formation is considered to be a basinal sequence formed during the middle to lower Devonian (Wilson and Taylor, 1976). It was metamorphosed to lower grades of regional metamorphism during the Variscan Orogeny to form mainly blue-grey slates with occasional interbedded sandstones. The implantation of the granite has further altered the Mylor slates within the aureole of the granite. The country rock surrounding the Cornish granites is referred to in numerous texts as Killas, and the term will be used in this thesis.

The Carnmenellis, Lands End and Tregonning-Godolphin granite outcrops are all cupolas on the same batholith which extends from Dartmoor to the Scilly Isles and beyond, (Bott et al., 1958). Isochron analysis indicates that the batholith was implanted between 303 to 265 Ma with the most reliable results indicating an age of 295 Ma. (Hawkes and Dangerfield, 1978). The composition of the Cornubian batholith is reviewed by Exeley et. al. (1984). Hawkes and Dangerfield (1978) indicate that 90% of the exposed granite consists of medium to course grained, microcrystic, biotite granite with a few xenoliths of older basic microgranite and small outcrops of fine grained biotite bearing granite. Analysis of cores from the 2.6 km deep HDR drill holes show the granite to be similar in chemical composition and mineralogy to the other Cornubian granites but to be fine to medium grained, (Whittle and McCartney, 1989).

Figure 1.9 shows the mineral lodes in the area along with the elvan dykes. The elvan dykes are granite porphyry and are micrograined and contain large crystals of quartz, alkali feldspar and mica. They mainly strike WSW-ENE and are usually between 1m to 2m wide, (Goode, 1973). The mineralisation has its origins in the fluids and minerals derived from either the granite batholith as it was implanted or fluids released in the surrounding rocks just after implantation. The mineral lodes mainly strike ENE-WSW. The asymmetric distribution of the mineral lodes, the vast majority being on the northern flank of the Carnmenellis granite, as well as the zonation within this distribution, is probably due to the convective fluid flow pattern in the area, (Sams and Thomas-Betts, 1988b).

1.5.2 Geophysical Surveys of South-West England

In interpreting a long line of large negative Bougiere gravity anomalies along the Cornubian peninsula, Bott et al. (1958), confirmed the
Figure 1.9
A Map showing the Mineral Lodes (Thin), Elvan Dykes (Thick) and Cross-Courses in or Around The Carnmenellis Granite Outcrop. Taken from B.G.S. 1:50 000 Geology Map.

Costline shown as a dotted line.
Granite outcrop shown as a line of dots and dashes.

Figure 1.10
A Map Showing the Depth to the Top of the Granite Batholith as Calculated Using a 3D Gravity Model, (Willis-Richards, 1987)

Costline shown as a dotted line.
Depth to the top of the granite batholith measured in (km) shown as a thin solid line.
FIG 1.9: Map Showing Mineral Lodes (Thin), Elvan Veins (Thick) and Cross-Courses (Dotted).
Fig 1.10: Depth to the Top of the Granite (km) From 3D Gravity Survey (Willis–Richards, 1987)
frequently held belief that the granite outcrops from Dartmoor to Lands End were in fact cupolas on a single batholith underlying the whole region. Using magnetic survey information as well, Bott et al. (1958) concluded that the sides of the batholith sloped outwards. Interpretations of gravity measurements at sea indicated that the Lands End and Scilly Isles outcrops were in fact joined, (Holder and Bott, 1971).

A number of three dimensional gravity models of the shape of the granite batholith have been published recently, (Al-Rawi, 1980; Brooks et al., 1984; Tombs, 1977; Willis-Richards, 1987). Willis-Richards (1987), uses a fixed density contrast between the granite and the surrounding Killas and then varies the depth to the top of the granite using 1 km cubes to fit the gravity readings and the shape of the granite outcrops. The model has a batholith thickness of 14 km with a flat bottom sloping slightly upwards in a NNW direction.

The resulting model is shown in figure 1.10. Note the steeply dipping sides of the batholith to the south and east of the Carnmenellis outcrop. In contrast, to the west and north of the outcrop, the model predicts the granite to be under a km below the surface. Although the difference in gravity gradients between the northern and southern edges of the outcrop could be due to a northward increase in density of the granite (Bott and Scott, 1964). Willis-Richards (1987) estimates errors unlikely to exceed 10% of the estimated depth to the top of the granite over the range of 0 to 4 Km depth.

The aeromagnetic map published by the I.G.S. (1965) shows no regional magnetic anomalies over the Carnmenellis outcrop or the surrounding Killas. However there are a series of complicated anomalies ranging from 127-241 gammas over the Lizard complex and extending a few km into the sea. Allan (1960) interprets these anomalies as indicating that the Lizard is limited in its extent off shore.

Reversed refraction lines running along the axis of the granite batholith (Holder and Bott, 1971) found the Mohorovicic discontinuity to be flat, with a crustal thickness of 27 km. No seismic discontinuities were detected in the crust, but the velocity structure indicated a depth to the bottom of the granite of between 10-12 km, with a constant seismic velocity through this crustal section. Below this there was a gradual but definite increase in velocity from 5.85 km/s to 6.9 km/s at the Moho. This velocity increase was interpreted as an increase in density of the lower crust, characteristic of a gradual change of granite into a granodioritic lower crust. This type of lower crust could have been formed by material left behind during partial melting or sunken stoped material (Holder and Bott, 1971).

Brooks et al. (1984) describe a major wide angle seismic reflection experiment, which included a line running from Lizard point in a north easterly direction over the Carnmenellis outcrop. Three crustal reflectors (R1, R2 and R3) were detected along this line and on two other lines across Bodmin Moor and Dartmoor. R3 is interpreted as the bottom of the crust and is placed at a depth of between 27-30 km. R1 lies at a depth of $8 \pm 1.5$ km, with reflection points lying entirely within the boundaries of the
batholith as defined by all published gravity models. Reflector R2 lies at a depth between 12-15 km under the part of the line south of the Carnmenellis outcrop and between 10-13 km under the line north of the outcrop. A low velocity zone was detected between reflectors R1 and R2 and was cited as evidence that the granite was inhomogeneous. However, no reflectors were found within the granite along two normal incidence, seismic reflection lines which also cross the Carnmenellis outcrop in roughly north-south directions. (Cambourne School of Mines, Geothermal Energy Project, 1989). Sams and Thomas-Betts (1988a) find indications from modelling heat flow data taken in the area that there is not only no sign of topography on the bottom of the batholith, but also that the bottom is flat.

A series of deep seismic reflection profiles were obtained in the Celtic sea, across the Cornubian batholith west of the Scilly Isles and in the eastern approaches of the Channel. These profiles are known collectively as the SWAT lines (BIRPS and ECORS, 1986). The two lines which cross the the Cornubian and Haig Fass granite show below a very thin layer of high reflectivity, a non-reflective upper crust with no reflectors that could serve as outlines for the batholiths (BIRPS and ECORS, 1986). The reflection Moho is seen at a depth of 28 km and it is continuous on all the profiles. In the lower crust there are a number of sub horizontal reflectors starting at 6 s two way time (TWT) (18km) on most of the sections and terminating at the Moho. However below the granite there is a greater thickness of lower crustal reflectivity, starting at 5s TWT (15km) below the thickest part of the granite as defined by gravity modelling (Edwards 1984a), and the reflections are of higher amplitude. Schrierer and Hobbs (1990) and Mathews (1987) both argue that the higher reflectivity of the lower crust and Moho below the granites is due to the batholiths acting as a low attenuation zones for seismic energy and that the lower crustal geology is uniform along the length of the profiles.

A magnetotelluric survey of the southern half of the Carnmenellis granite has also been carried out, (Beamish, 1990) and will be discussed in Chapter 5.

1.5.3 The Tectonic History of the Area

The tectonic history is for convenience, though slightly arbitrarily, split into three sections, pre-granite tectonics, the granite emplacement and post granite tectonics.

1.5.3.1 Pre-Granite Tectonics

The Devonian rocks of the area are affected by the Variscan Orogeny. The Variscan Orogeny, which started in the early Palaeozoic in Europe and lasted until the end of the Carboniferous period, is characterised in southern Britain by generally north-south compression and extensive thrusting associated with plate collision. Evidence that thrusting occurred in the area throughout the Devonian period comes
from the structures possessing a dominant slaty cleavage, which is recumbent with a low southerly dip, (Shackleton et al., 1982). Also three thrusts have been traced on land and off shore on seismic profiles, (Leveridge et al., 1984). It is generally agreed (Holder and Leveridge, 1986) that deformation started to the south of the area with up thrusting and erosion forming the Gramscatho group during the lower to middle Devonian. Leveridge et al., (1984) describes a sequence, which accounts for the present stratigraphy, of thrusting and erosions with the deformation gradually moving northwards with time. This thin sheet tectonic model is interpreted as being driven by a subduction zone south of the area with a decollement dipping gently southwards from the Variscan front, (Shackleton et al., 1982).

1.5.3.2 The Emplacement of the Cornubian Batholith

The Cornubian granites were emplaced during the final stages of the Variscan Orogeny. Why they formed during the culmination of the orogeny is uncertain, (Hawkes and Dangerfield, 1978).

The Cornubian granites contain a relatively high proportion of silica indicating that they were formed from the melting of continental crustal material, (Shackleton et al., 1982). It is believed that melting of the continental crust occurs at a depth of between 30 and 40 km. However the present crustal thickness is estimated at only 27 km. For the granites to have formed below their present position, the granite must have formed below the Moho, or the crust was much thicker than at present. There is no evidence that the Moho has been broken (BIRPS and ECORS, 1986). But there is evidence that the crust was thicker during the time of emplacement, due to the compressional tectonics described in section 1.5.3.1, (Shackleton et al., 1982). However Shackleton et al., (1982), go on to state, that from the stratigraphical evidence, it is hard to envisage the crust being thick enough for crustal melt. There is no evidence from the SWAT profiles that the lower crustal reflectors have been disrupted by a rising magma, (Schrierer and Hobbs, 1990). However, the reflectors would only be disturbed if they had formed before the granite was emplaced, Schrierer and Hobbs (1990).

An alternative theory for the history of granite emplacement was proposed by Shackleton et al., (1982) and is summarised in figure 1.11. It envisages the Cornubian Granites originating south of their present position and and then being injected northwards along the south dipping thrust zones in a sheet-like body. This model agrees with the geophysical evidence of the granite having a flat bottom. Also Edwards (1984a, 1984b), in modelling the off shore gravity lows associated with the Haig-Fras and Cornubian granites shows that asymmetric structures with tails to the south fit the data as well, if not better, than symmetrical structures. However, it is unclear why the granite travelled a considerable distance sideways without breaking through the relatively thin crust, (Whittle and McCartney, 1989).
Figure 1.11
Shackleton's Model for Emplacement of The Cornubian Batholith Taken from Shackleton et. al. (1982).
1.5.3.3 Post Granite Tectonics

Following the Variscan Orogeny, a tensional stress regime predominated and led to a series of E-W normal faults in southern Britain. In addition there are a number of large NW-SE strike-slip faults which have displaced the batholith in places. This regime was replaced by north-south rifting, as the Atlantic Ocean opened in the upper Triassic. The Alpine tectonics in the Tertiary period caused compression, like the previous Variscan Orogeny, in a roughly N-S direction. Present day seismic activity in the area is associated with the NW-SE strike slip faults. See Whittle and McCartney (1989) for a summary of post-granite tectonics.

1.5.4 The Carnmenellis Granite

As well as sensing the shape of the granite and the composition of the lower crust, electromagnetic induction measurements will be sensitive to factors such as the temperature and pressure gradients, plus the density, orientation and fluid content of joints, veins and fractures.

The high heat flows associated with the Cornubian batholith are caused by high heat production of radioactive elements within the granite (Wheildon et al., 1980). The extensive heatflow data set that exists for the area has been modelled by a number of authors, using 3D conductive models (Sams and Thomas-Betts, 1988a; Willis-Richards, 1987; Thomas-Betts and Sams, 1991) and 2D convective models (Sams and Thomas-Betts, 1988b). Thomas-Betts and Sams, (1991), find an average temperature gradient of 30 °C/km for measurements down to a depth of 300m. However from borehole logs at the HDR site an average temperature gradient of 35 °C/km was found in boreholes down to a depth of 2.8 km (Pearson et al., 1989). Both Sams and Thomas-Betts, (1988a) and Willis-Richards, (1987) show that the temperature at depth in south-west Cornwall is influenced by the granite shape and the presence of the overlying Killas. From these modelling studies the highest predicted temperatures at depth occur north-east of the Carnmenellis outcrop because this is the position of the centre of the sub-surface batholith and this area is overlain by a thin layer of insulating country rock (Sams and Thomas-Betts, 1988a). A temperature of 82 °C is predicted at a depth of 2 km below the centre of the Carnmenellis outcrop, falling or rising a few degrees, for positions south or north of this point respectively. At 6 km the predicted temperature just north of the Carnmenellis outcrop is 215 °C and in the centre 200 °C, (Sams and Thomas-Betts, 1988a). A temperature of 370 °C is predicted for a depth of 10 km.

Pine et al. (1990) review the available stress data from the Carnmenellis outcrop. Measurements in the HDR boreholes generally agree with measurements taken at other sites on the northern edge of the outcrop. The minimum horizontal stress increases linearly with depth, varying from 6 Mpa at 7m depth to 36 Mpa at 2610m depth. The
maximum horizontal stress varies from 13.75 Mpa at 7m to 73.7 Mpa at 2000m (Pine et al., 1990). As can be seen from these figures, the stress field shows a considerable degree of anisotropy. The mean direction of the maximum horizontal stress from the majority of measurements is 140 degrees east of north, though some measurements indicate a mean direction of 40 degrees west of north (Pine et al., 1990). This direction is coincident with the fault-plane solutions derived from the measured micro-seismicity at the HDR site down to a depth of 3km, (Green et. al., 1988).

The resistivity borehole logs at the HDR site, measured down to 2.8 km, show a very spiky response, (Pearson et al., 1989). There are portions of the logs with resistivities above the tool limit of 20 000 Ωm. But there is also a high, depth integrated density, of spikes ranging between values of a few hundred Ωm and a few thousand Ωm. These spikes can be correlated with fractures in the granite sensed by acoustic televiewers and formation microscanners, (Pearson et al., 1989). These logs emphasis the effect that water filled fractures will have on the bulk resistivity of the granite. These fractures exist throughout the depth of the HDR boreholes and from measurements of the micro-seismicity down to depths of at least 3 km and probably deeper, (Green et al., 1988).

The density and orientation of fractures in the Carnmenellis granite is well documented (Ghosh, 1934; Bergin and Tovey, 1980; Whittle, 1989). The orientation of the fractures will be discussed in detail in section 5.6, but in general there are three main joint sets, two with steep dips and strikes of nearly SSE-NNW and ENE-WSW respectively, and a third set with shallow dips, (Whittle, 1989). Joint spacings at the surface range from 0.5m to 2m. As well as small scale fractures there are a number of faults which cross the granite (Dearman, 1964). These have NW-SE to NNW-SSE strikes, at spacings between about 0.5 km and 2 km. They are known locally as cross courses (see figure 1.9).

As well as the density of fractures, the porosity of the fractures and the salinity of the fluids flowing through them will be important factors governing the bulk resistivity of the granite. It has long been known from mine workings in the north of the granite that water circulates through these fractures, (Burgess et al., 1982; Edmunds et al., 1988). These waters have a salinity of up to 20 g/l, and originally were of meteoric origin, (Edmunds et al., 1985). They have temperatures of 52 °C at 700m, which is in excess of predictions from the thermal gradient in the area. This implies that warmer more saline fluids are upwelling by convective circulation. The majority of the fluids issuing into the mines are from cross-courses, (Edmunds et al., 1985).

Heath (1985) has made a study of the fracture permeability of the granite at a site near to the north western edge of the Carnmenellis outcrop, in boreholes up to 700 m deep. He found that although the fracture density decreased with depth, the flow rates into the borehole stayed constant. Differences in flow rates between fractures were found, with well above average flow rates through the cross-courses and elvan
dykes that were cut by the boreholes. The elvan dykes are highly fractured with the strike of the fractures running along the strike of the dykes.

1.6 The Aims of the Project

The project was designed to give insights into two sets of geological problems.

1) Firstly to obtain information about the conditions within the granite at depth, with emphasis on the role of fluids. The MT method was expected to be sensitive to fluids due to the large resistivity contrast in wet and dry granite outlined in section 1.4. Primary goals of the project were to ascertain at what depth the fluid filled fractures close, to measure any variations in the bulk porosity of the granite, both laterally and with depth and to detect any other zonation within the granite.

2) To obtain information that could help solve the various tectonic problems in the area. Of primary interest was the shape of the bottom of the granite and the structure of the lower crust below the granite, both of which would give clues to the batholith's origin. The previous geophysical knowledge provided little information on the lower crustal structure and only tentative models of the bottom of the batholith exist. The probable large resistivity contrast between the granite and the surrounding rocks would facilitate the mapping of the batholith. Also the resistive granites act as low attenuation windows for electromagnetic energy and thus provided an opportunity for obtaining measurements sampling great depths within the Earth. A number of questions about the Variscan tectonics of the area remain. However, it was expected that only small variations in the electrical properties of the different stratigraphic units would exist, because they were broadly of the same rock type, see section 1.5.1. Hence only a short description of the complicated Variscan tectonics was included in this chapter.

Soundings of resistive structures using EM induction are still relatively rare. This is due to problems such as noise, the details of which will be given in subsequent chapters of this thesis. Therefore it was an aim of the project to find and use methods that would minimise these problems.

1.7 Outline of the Thesis

The outline of the rest of the thesis is as follows. Chapter 2 describes the mathematical background to electromagnetic induction sounding and is followed in chapter 3 by a description of the field survey and data processing. One dimensional and two dimensional models are presented in chapter 5 after a detailed analysis of the impedance tensor in chapter 4. Three dimensional thin sheet modelling of the Carnmenellis data set is described in chapter 6 and finally the geological interpretation of the models and recommendations for further work are outlined in chapter 7.
Chapter 2

The Theory of Magnetotelluric Sounding

This chapter describes the theoretical background to the magnetotelluric method. After a short historical review, there follows a brief description of the derivation, from Maxwell's equations, of the general theory of electromagnetic induction in the earth. The relationships between the horizontal electric and magnetic fields are then described for a homogeneous earth model, an N-layered isotropic earth model, two dimensional and three dimensional earth models.

2.1 Brief Historical Review

The first study of the relationship between telluric and magnetic fields seems to have been carried out by Airy (1868). However the realisation that electromagnetic fields could be used to probe the earth was not made until the 1950's first by Tikhonov (1950), and then Cagniard (1953). Cagniard (1953) was the first to describe methods of producing conductivity depth profiles using surface measurements of telluric and magnetic fields. The assumption of a plane wave source field implicit in Cagniard's theory was first questioned by Wait (1954) and led to further investigations of source field "shape". This avenue of investigation led to a general theory of magnetotelluric sounding for source fields of finite dimensions (Price, 1962; Wait, 1962) and corrections to the simple equations developed by Cagniard were developed. However numerical studies by Madden and Nelson (1964) showed that for a realistic earth models, source fields with periods less than 1000 seconds could be treated as planar. The effects of the earth's curvature were found to be negligible for periods less than a day, (Srivastava, 1965).

Berdichevsky (1960,1963), Tikhonov and Berdichevsky (1966) and Rokityansky (1961) amongst others stated that a two dimensional tensor relationship was needed to describe magnetotelluric fields when the electrical structure of the earth varied laterally. Strangway et al. (1973) were the first to document use of the M.T. method at short periods to investigate shallow structures.

2.2 The General Theory of Induced Electromagnetic fields in the Earth.

The behaviour of an electromagnetic field is described by Maxwell's equations.

\[ \nabla \times E + \frac{\partial B}{\partial t} = 0 \]  

(2.1)
\[ \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \]  
(2.2)

\[ \nabla \cdot \mathbf{B} = 0 \]  
(2.3)

\[ \nabla \cdot \mathbf{D} = \rho' \]  
(2.4)

where
- \( \mathbf{E} \) = electric field in Volts per metre
- \( \mathbf{B} \) = magnetic induction in Webers per square metre
- \( \mathbf{H} \) = magnetic field in Amperes per metre
- \( \mathbf{D} \) = electric displacement in Coulombs per square metre
- \( \mathbf{J} \) = electric current density in Amperes/m²
- \( \rho' \) = electric charge density in Coulombs per cubic metre

Ohm's law states that
\[ \mathbf{J} = \sigma \mathbf{E} \]  
(2.5)

and for an isotropic medium
\[ \mathbf{B} = \mu \mathbf{H} \]  
(2.6)

\[ \mathbf{D} = \varepsilon \mathbf{E} \]  
(2.7)

where
- \( \sigma \) = conductivity of the medium in Ohm-m
- \( \mu \) = magnetic permeability of the medium in Henrys per metre
- \( \varepsilon \) = electrical permittivity of the medium in Farads per metre

If the conductivity of a medium is uniform and non zero, in a very short time
\[ \nabla \cdot \mathbf{D} = 0 \]  
(2.8)

and thus using equations 2.7
\[ \nabla \cdot \mathbf{E} = 0 \]  
(2.9)

Substituting equations 2.5 and 2.7 into equation 2.2
\[ \nabla \times \mathbf{H} - \varepsilon \frac{\partial \mathbf{E}}{\partial t} = \sigma \mathbf{E} \]  
(2.10)

From equations 2.1 to 2.7, it can be shown (see amongst others Price, 1950, Ward et al., 1973, Patra and Mallick, 1980, Grant and West, 1965) that:
\[ \nabla^2 F = \mu \sigma \frac{\partial F}{\partial t} + \mu \varepsilon \frac{\partial^2 F}{\partial t^2} \]  
(2.11)

where
\[ F = E \text{ or } H \]

Assuming the primary field has a harmonic time variation of \( e^{-i \omega t} \)

\[ \nabla^2 F + k^2 F = 0 \]  
(2.12)

The propagation constant \( k \) is given by:
\[ k^2 = \omega \mu (i \sigma - \omega \varepsilon) \]  
(2.13)

where
\[ \omega = \text{angular frequency of the time varying primary field} \]

Let us proceed by considering induction in a simplified earth model. The simplified earth model consists of three layers, the earth, air and source layers. The Earth is assumed to be a source free, conducting half space, occupying the whole region where \( z > 0 \). (Assuming a Cartesian coordinate system, \( z \) is depth and is positive vertically downwards). Electric currents are induced in the Earth by a varying magnetic field occupying a region \( z < -h \). The region \( -h < z < 0 \) is referred to as the air layer and is assumed to be non-conducting and non-magnetic.

Equation 2.12 is a wave equation. However for frequencies used in magnetotelluric soundings (below 1000 Hz), and for the conductivities of Earth materials, \( \sigma \gg \omega \varepsilon \). Therefore we neglect displacement currents, \( \omega^2 \mu \varepsilon = 0 \) and in the earth half space \( (z > 0) \) equation 2.12 becomes a diffusion equation, (Price, 1962).

\[ \nabla^2 F + k_d^2 F = 0 \]  
(2.14)

where
\[ k_d^2 = i \omega \mu \sigma \]  
(2.15)

From henceforth the subscript \( d \) will be dropped and \( k_d \) will simply written as \( k \). \( k \) is referred to as the wavenumber. Equation 2.14 describes electromagnetic energy diffusing through a medium, the amplitude of the energy decaying exponentially with the length of penetration.

In the air layer \( (-h < z < 0) \) the conductivity is zero, and considering
that the field changes are relatively slow, the travel time across the air layer will be small compared with the time taken for field changes to become effective, (Price, 1950). Therefore

$$\mu \varepsilon \frac{\partial^2 F}{\partial t^2} = 0$$

(2.16)

substituting 2.15 and $\sigma = 0$ into 2.11

$$\nabla^2 F = 0$$

(2.17)

Any solution of 2.9, 2.14 and 2.17 or 2.3, 2.14 and 2.17 must meet certain boundary conditions, namely that the tangential components of $E$ and $H$ and the normal component of $B$ must be continuous across the interface between the earth and the air layer.

### 2.3 Electromagnetic Induction in a Homogeneous Earth.

The most rigorous discussion of electromagnetic induction in a uniform half space earth is given in Price (1950). Consider the earth as a uniform half space. There is a plane boundary between the air layer and the earth, a good assumption for periods used in this study (Srivastava, 1965). Price (1950) solved equations 2.9, 2.14 and 2.17 for the $E$ field using the technique of separation of variables and found that two types of solution existed. He showed that only one type of solution is of interest in the induction problem, since only one solution produces an external magnetic field. This solution was shown by Price (1950) to produce currents flowing inside the conducting earth parallel to the air-earth contact and to be either freely decaying from an initial distribution of charge or induced by some external magnetic field. Solutions of this type can be used to solve the problem of induction in the earth for any inducing magnetic field.

Consider a uniform magnetic field in the region $z < -h$ varying harmonically with time. The magnetic field is given by

$$H(t) = H_x e^{-i \omega t} \mathbf{i} + H_y e^{-i \omega t} \mathbf{j}$$

(2.18)

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the $x$ and $y$ directions respectively. With $\sigma$ only varying with $z$ the solution of 2.12 is given by

$$H(t) = (H_x \mathbf{i} + H_y \mathbf{j}) e^{i (-\omega t \pm kz)}$$

(2.19)

and substituting 2.19 into 2.10, the corresponding electric field is given by
\[ E(t) = (\pm H_y \hat{i} + H_x \hat{j}) \frac{\mu_1 \omega}{k_1} e^{i(\omega t \pm k_z)} \] (2.20)

\[ e^{i k_z z} \rightarrow 0 \text{ as } z \rightarrow +\infty \quad \text{and} \quad e^{-i k_z z} \rightarrow 0 \text{ as } z \rightarrow -\infty \] (2.21)

Since in the earth half space \((z > 0)\), the intensities of \(E\) and \(H\) decrease with increasing \(z\), due to the transformation of electromagnetic energy to heat, only the \(e^{i k_z z}\) solution of equations 2.19 and 2.20, meets the conditions of the physical model.

Therefore in the earth half space \((z > 0)\)

\[ H(t) = (H_x \hat{i} + H_y \hat{j}) e^{i(\omega t + k_1 z)} \] (2.22)

\[ E(t) = (H_y \hat{i} - H_x \hat{j}) \frac{\mu_1 \omega}{k_1} e^{i(\omega t + k_1 z)} \] (2.23)

where

\(\mu_1\) = magnetic permeability of the earth half space
\(k_1\) = the wavenumber (defined in equation 2.15) in the earth half space

Whilst in the air layer \((-h < z < 0)\)

\[ H(t) = \left[ (H_x \hat{i} + H_y \hat{j}) e^{-i k_0 z} + (H_x \hat{i} + H_y \hat{j}) e^{i k_0 z} \right] e^{-i \omega t} \] (2.24)

\[ E(t) = \left[ (H_y \hat{i} - H_x \hat{j}) e^{-i k_0 z} - (H_y \hat{i} - H_x \hat{j}) \frac{\mu_0 \omega}{k_0} e^{i k_0 z} \right] e^{-i \omega t} \] (2.25)

\(\mu_0\) = magnetic permeability of free space
\(k_0\) = the wavenumber (defined in equation 2.15) in free space

Equations 2.22, 2.23, 2.24 and 2.25 show that if measurements of amplitude of the electric field in one direction and the amplitude of the magnetic field in a perpendicular direction are taken at the surface of earth, the conductivity of the earth can be calculated. Dividing 2.23 by 2.22 for fields in perpendicular directions.

\[ \frac{E_x}{H_y} = \frac{\mu_1 \omega}{k_1} \] (2.26)

Neglecting displacement currents in the earth and substituting 2.15 into 2.26.

\[ Z = Z_0 = \frac{E_x}{H_y} = \sqrt{\frac{\mu_1 \omega}{\sigma_1}} e^{i \frac{k_1}{4}} \] (2.27)
\( Z_0 \) is the impedance at the surface. Equation 2.27 forms the basis of Cagniard's magnetotelluric method for calculating the earth's conductivity from measurements of impedance at the surface of the earth. Equation 2.27 also shows that there is a 45° phase difference between the oscillating electric and magnetic fields.

Apparent resistivity (\( \rho_a \)) and phase (\( \phi \)) are defined following the notation of Hobbs (1992) as:

\[
\rho_a = \frac{1}{\omega \mu} \left| \frac{E_x}{H_y} \right|^2 = \frac{1}{\omega \mu} |Z|^2
\]

(2.28)

\[
\phi = \arg \left( \frac{E_x}{H_y} \right) = \arg (Z)
\]

(2.29)

Kurtz (1973) showed from Snell's law of refraction, that regardless of the angle of incidence, the low frequency electromagnetic waves travel vertically downwards in the earth.

2.3.1 The Skin Depth

The depth at which \( E \) or \( H \) have an amplitude \( 1/e \) times that at the surface of the medium, is called the skin depth. For a uniform earth it is given by:

\[
\delta = \frac{1}{\Re(k_d)} = \left( \frac{2}{\omega \mu \sigma} \right)^{1/2}
\]

(2.30)

Equation 2.30 contains the important information that the skin depth of a magnetotelluric sounding depends on both the period of the signals being measured and the conductivity of the medium (assuming \( \mu \) to be constant). The longer the period measured or the smaller the conductivity of the medium, the deeper the electromagnetic fields penetrate.

In an inhomogeneous medium, Sims and Bostick (1969) have defined a generalized skin depth \( \delta_w \) from

\[
\Re \left[ \int_0^{\delta_w} \sqrt{i \omega \mu \sigma(z)} \, dz \right] = 1
\]

(2.31)

2.3.2 Electromagnetic Induction in an N-layered half Space

Cagniard (1953) gave equations and master-curves for two and three layered isotropic earth models. The forward problem of calculating the
impedance at the surface of an N layered isotropic half space is discussed in Ward et al (1973). Ward et al (1973) quote the equation of Wait (1962) and Frischknecht (1966) in the form of

\[ Z_0(\omega) = \frac{i \mu \omega}{k_1} \coth \left[ k_i h_i + \coth^{-1} \left( \frac{k_i}{k_{i+1}} \coth \left[ k_{i+1} h_{i+1} + \coth^{-1} \left( \frac{k_{i+1}}{k_{i+2}} \coth \left[ k_{i+2} h_{i+2} + \coth^{-1} \left( \frac{k_{i+2}}{k_{i+3}} \cdots \right) \right] \right) \right] \right) \]  

(2.32)

where

- \( Z_0(\omega) \) = the impedance at the surface
- \( h_i \) = the thickness of layer i
- \( k_i \) = the propagation constant for the i-th layer given by

\[ k_i = \left( \frac{\omega \mu \sigma_i}{2} \right)^{1/2} + \frac{i}{2} \left( \frac{\omega \mu \sigma_i}{2} \right)^{1/2} \]  

(2.33)

2.4 Electromagnetic Induction in a Two-Dimensional Earth.

The structure of many geological bodies such as dykes, faults and rift valleys remains virtually unchanged in one direction, the strike direction, over large distances. If the earth's structure is constant in the strike direction for distances greater than skin depth, the earth can be considered two dimensional.

Consider an earth model where the conductivity varies in one lateral direction as well as with depth \( \sigma(y, z) \). (For two dimensional models the Cartesian coordinate system is defined such that \( x \) is parallel to strike and \( y \) is perpendicular to strike.) Since there is no variation of fields in the \( x \) direction (\( \frac{\partial}{\partial x} = 0 \)) Maxwell's equations 2.1 and 2.2 de-couple into two different modes. In one mode, the E-polarization, the horizontal electric field is parallel to strike and in the other mode, the H-polarization, the horizontal electric field is perpendicular to strike, but the horizontal magnetic field is parallel to strike. Neglecting displacement currents and assuming the primary magnetic field has a harmonic time variation of \( e^{-i \omega t} \), the two sets of equations are given below.

E-polarization

\[ \frac{\partial H_y}{\partial y} - \frac{\partial E_z}{\partial z} = \sigma E_x \]  

(2.34a)

\[ \frac{\partial E_x}{\partial z} = i \omega \mu H_y \]  

(2.34b)

H-Polarization

\[ \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = i \omega \mu H_x \]  

(2.35a)

\[ \frac{\partial H_x}{\partial z} = \sigma E_y \]  

(2.35b)
\[ \frac{\partial E_x}{\partial y} = -i \omega \mu H_z, \quad (2.34c) \]
\[ \frac{\partial H_x}{\partial y} = -\sigma E_z, \quad (2.35c) \]

Equation 2.27 neglects any lateral conductivity variations within the earth. For an anisotropic medium the tangential telluric and magnetic fields are in fact related by a tensor relationship first described in Neves (1957) in terms of admittance and in Berdichevsky (1960,1963), Tikhonov and Berdichevsky (1966) and Rokityansky (1961) in terms of an impedance tensor.

\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix} = 
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
H_x \\
H_y
\end{bmatrix}
\quad (2.36)
\]

For a truly two dimensional earth the tensor relationship in equation (2.20) can be simplified to.

\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix} = 
\begin{bmatrix}
0 & Z_{xy} \\
Z_{yx} & 0
\end{bmatrix}
\begin{bmatrix}
H_x \\
H_y
\end{bmatrix}
\quad (2.37)
\]

As can be seen from equation 2.37 the impedances calculated for the two modes will be different from each other, (O'Brien and Morrison, 1967).

Exact analytical solutions of the equations 2.34 and 2.35 for a two dimensional earth exist for just a few geological structures - for example the dyke (Rankin, 1962), the fault or vertical discontinuity (d'Erceville and Kunetz, 1962, Weaver 1963), dipping beds (Hvozdora, 1968, Geyer 1972) and the layered earth with anisotropy (Sinha, 1969, Praus 1966). Other problems are solved using approximate methods of finite difference (Jones and Price, 1970; Jones and Pascoe, 1971; Brewitt-Taylor and Weaver, 1976), finite element, (Ryu 1971; Coggon, 1971; Wannamaker et. al, 1987), integral equations (Parry and Ward, 1971; Hohmann, 1971) or transmission surface networks, (Madden and Thompson, 1965; Madden and Swift, 1969; and Swift, 1971). Recently, successful and well used inversion methods have been developed (deGroot-Hedlin and Constable, 1990; Smith and Booker, 1988; Smith and Booker, 1990).

2.5 Electromagnetic Induction in a Three-Dimensional Earth.

Many geological environments are complicated and three dimensional. All the elements of the impedance tensor in equation 2.36 measured in such areas will be non-zero. Numerical solutions to simple three dimensional bodies have been obtained, by amongst others, Jones and Pascoe (1973) and Groom and Bailey (1991). They all require large amounts of computational time and space.

Forward modelling of three dimensional bodies can be split into two types.
1) Those using the thin sheet approximation, where a thin inhomogeneous sheet placed on top of a layered half-space (Vasseur and Weidelt, 1977; Dawson and Weaver, 1979)

2) A inhomogeneous body embedded in a layered half-space (Raiche, 1974; Reddy et al., 1977; Ting and Hohmann, 1981; Park et al., 1983; Wannamaker et al., 1984; Park, 1985; Park and Livelybrooks, 1989).

Thin sheet models have mainly been used to study the effects of the conductivity contrast between the sea and the land. Due to computational restrictions, only simple shaped three dimensional bodies to which the thin sheet approximation doesn't apply, have been investigated. These investigations have used forward modelling techniques based on the method of integral equations.
Chapter 3

The Electromagnetic Induction Survey.

3.1 Introduction

In order to produce electrical conductivity models of the earth below south-western Cornwall, broad-band magnetotelluric and magnetovariation soundings (M.V.) were taken at 21 sites on or around the Carnmenellis Granite outcrop.

The data were collected by fieldwork crews led by the author in 3 fieldwork campaigns during the months of April 1988, September to October 1988 and July 1989. Measurements of signals over a period range of 0.0015 seconds to 3000 seconds were taken using two sets of instrumentation. The Short Period AudioMagnetotelluric (S.P.A.M.) Mk II system with CM 11 induction coils was deployed at all sites, in addition to which, at four locations, E.D.A. fluxgate magnetometers measured magnetic field fluctuations for periods greater than 40 seconds. Both sets of equipment used the amplified changes of potential difference between pairs of grounded electrodes to sense the horizontal electric field. All the time series were recorded digitally on magnetic tape or floppy disk and then reprocessed on main frame computers at Edinburgh University.

This chapter describes:

a) The instrumentation and fieldwork procedure used.
b) The sites occupied and reasons behind positioning of soundings
c) The methods used to process the data up to and including rotated impedance tensors and induction arrows.
d) The rotated apparent resistivity and phase curves.

3.2 Audiomagnetotelluric Field Measurements.

Audiomagnetotelluric measurements were made using the S.P.A.M. system developed at the University of Edinburgh by Graham Dawes. A description of the original system is given in Dawes (1984) and the present system is documented in an internal report, Dawes(1987). The equipment is quite portable since it is powered by three 12v car batteries.

Two of the design features of the instrument ensure the optimal signal to noise ratio for measurements of the response functions.

1) Since the strength of the natural signal varies considerably with time the equipment includes auto-ranging amplifiers.

2) Due to large changes in the power spectrum of the magnetic source field over the A.M.T. period range the signals are split into four shorter,
Figure 3.1

Block diagram showing the AMT/MT data acquisition system (S.P.A.M.). From Dawes (1984).
but overlapping, frequency sub-bands using band pass filters and then amplified.

The instrument also has the capability for processing and analysing the digital signals in real time, thus ensuring in-field knowledge of data quality and also providing the ability to set data rejection criteria. An overview of the complete system is shown in figure 3.1 taken from Dawes (1984). As can be seen from the figure, the system can be split into three parts, the sensors, the analogue box and the computer.

3.2.1 The Sensors

The electric field sensors consisted of two pairs of grounded, Cu-CuSO₄, non-polarising electrodes connected to the common sensor box by electric cables. The electrode pairs were always placed in a north-south, east-west, cross configuration, thus ensuring common direction of measurement at all the sites. In general, an increase in separation produces an increase in potential difference between the electrode pairs. But it also produces an increase in noise due to wind moving the electrode connection cables, therefore a compromise distance of between 50-100m was chosen and the cables were usually buried or at least weighted down.

Three induction coils placed with their long axes lying in orthogonal directions of north, east and vertically upwards, were used to measure fluctuations in the magnetic field. The majority of measurements were taken using Eca CM11e induction coils. However, this particular coil model can only measure variations in the magnetic field longer than 0.0078 seconds, hence a second set of induction coils, CM16, were used with the SPAM MkIIa system to sense the high frequency variations. The induction coils and connecting cables were always buried, again to reduce vibration noise.

After entering the distribution box, the very low amplitude telluric signals were passed through pre-amplifiers to increase their signal strength before being sent back to the analogue box along a 50m cable. The sensors are placed 50m away from the rest of the instrumentation to avoid magnetic interference. The signals from the magnetic sensors also passed along the same cable as well as the power needed to supply the induction coils and the telluric pre-amplifiers.

3.2.2 The Analogue Box

Within the analogue box an identical process of amplification, filtering and digitising was performed upon each of the input signals, as follows:

1) All the signals were fed into a fixed gain (x2) input amplifier and a broad band filter. The filter was used to eliminate high frequencies
and DC levels, which were both out of the measurement range, before further amplification.

2) An adjustable input amplifier was applied to all signals. This main amplifier was set at a level just below overload, indicated by a light, and regularly checked to ensure it produced the largest possible amplification, without saturating the filters.

3) Two 'twin T' notch filters were used to eliminate frequencies of 50 and 150 Hz, the mains electricity fundamental and its first odd harmonic.

4) Each signal was then passed into its own band pass filter board. One of four, overlapping, frequency bands could be selected for all channels using a switch. Subsequent post amplification could therefore be adjusted to the signal levels within the smaller band width excluding periods where the power spectrum was widely different. The band pass filter consisted of a single high pass, and a low pass system. The low pass filter had a sharp cut off to prevent aliasing problems during digitizing. The amplifiers had an auto-ranging facility as well as a fixed level for each channel of (x5). The auto-ranging worked by setting the amplification at a level of 128 at the start of window digitisation, and if saturation was imminent, reducing the amplification in steps of 2 before digitizing the next sample.

5) The signals were then converted into digital form by an analogue to digital converter and sent to the computer along with records of post-amplification levels. The digitising rate depended on the period sub-band chosen.

3.2.3 The Computer and In-Field Processing

Within the computer box all the analysis was performed and the data recorded either on floppy disc or magnetic tape cartridge, depending on model of S.P.A.M. used. Two discs or cartridges were loaded into the drives, programs were read from the one and data were written to the other. A thermal printer was used to list output and instructions were fed to the computer using a small Microscribe terminal.

Before any data were recorded the functions of the analogue box were tested using a test program run by the computer. All band and notch filter selections, plus amplifier gains were checked. The amplitude and phase response functions of the band pass filters were measured using a signal generator and compared to previous measurements to check for any change in response. Each of the signals was then viewed using a portable oscilloscope. Coherency between orthogonal magnetic and electric channels, signal strength and possible noise sources were assessed and on this information, the period band to start acquisition was
A flow chart of the data acquisition program is shown in figure 3.2 taken from Dawes (1984). Measurements were taken for each of the four period bands in turn. The time series consisted of 512 samples and were decimated down to windows of 256 samples using a cosine bell function. Conversion into the frequency domain using a Fast Fourier Transform (FFT) routine was performed and tensors calculated from cross and auto spectra averages.

To reduce the sheer volume of data recorded at period bands with high sampling rates and ensure good quality data for in-field modelling, acceptance criteria were selected, and windows not meeting these criteria were rejected. These criteria include a stated number of averaged frequencies within a window having a signal power in the magnetic components, as well as the multiple predicted coherency of impedance tensors, above stated levels, the phase of $Z_{xy}$ and $Z_{yx}$ being in opposite quadrants and the existence of an unpolarised source field.

The multiple predicted coherency is given by Swift (1967) as:

$$\text{Coh}(E_x^{\text{Pred}}E_x) = \frac{\langle E_x^{\text{Pred}}E_x^* \rangle}{[\langle E_x^{\text{Pred}}E_x^{\text{Pred}} \rangle \langle E_xE_x^* \rangle]^{1/2}}$$

* denotes conjugate

Impedance tensors are calculated using Cramer's rule to solve the simultaneous equations (Sims et al., 1971). If the source field is polarised in one of the measurement directions the denominator of this calculation will be zero. Only frequencies where such denominators were non zero for both sets of tensors were accepted.

Acceptance criteria were normally set at 5 frequencies with a predicted coherency of at least 0.9 and a magnetic power level above 0.3 nT for the two M.T. impedance sets. The predicted coherency acceptance criteria for the M.V. tensor set was usually only 0.8, though these could be changed after studying plots of power spectra and time series. For band 3 with the lowest sampling rate, all windows were usually recorded. If the window was accepted the time series were written onto disc or cartridge and tensor elements from the window were stacked with those from other accepted windows.

Results of stacked phase and apparent resistivity could be plotted and the quality of the results assessed. Normally at least 60 windows for each period band were recorded. When data from all bands had been collected a separate modelling program was run and smoothness of the response curves over the whole period range was checked. Also one dimensional resistivity depth profiles were calculated using the Bostick Transformation, (Bostick, 1977).

3.3 Long Period Electromagnetic Induction Measurements.

Two different sets of equipment were used. Both were based
Figure 3.2

The flow chart showing the various steps and options during in field data acquisition and processing. From Dawes (1984).
around an EDA fluxgate magnetometer and incorporated a N.E.R.C. geologger for recording the signals, see Valiant (1976). However one set of instrumentation used included a telluric filter and amplification system designed for Edinburgh University by Graham Dawes (Edin.L.M.T.) and the other was a more compact redevelopment of the whole system by the N.E.R.C. equipment pool (N.E.R.C. L.M.T.).

The magnetic sensors consisted of three fluxgate magnetometers, fixed at right angles to each other in a sealed drum. See Parkinson (1983) for a description of the physical principles used in the design of a fluxgate magnetometer. The magnetometer drum was fixed to a spike driven into the ground at the bottom of a hole and then buried so that the top of the drum was at least 50 cm below the surface. Before burying, the magnetometer was very accurately levelled and aligned so that the fluxgate magnetometers pointed north, east and vertically down. Power was sent to the magnetometers along a 30m cable and the signals sent back along the same cable, which was also placed below the surface. All the equipment was buried to reduce noise from vibration and to try to ensure a stable temperature environment for the magnetometers. The fluxgate drum was placed 30m away to eliminate any magnetic interference with the rest of the equipment. Both sets of magnetic sensors had a total sensitivity of 50 mv/nT.

The electric sensors were the same as those used for A.M.T. measurements except an L-shaped configuration of a common and two other electrodes was used. The telluric signals were passed through an amplifier with a fixed gain of 100 before any filtering was performed and a further gain of 2, 5, 10, 50, or 100 could be applied using the variable post-amplifier in the Edin.L.M.T. equipment and 10, 25 or 100 using the N.E.R.C.L.M.T. system.

Since for induction studies, only measurements of variations in electric and magnetic fields need to be performed, to improve the dynamic range of the recording equipment, the d.c. component of the magnetic field and self potential between the electrodes was backed off using electronic circuitry. A sampling rate of 10s was chosen. Therefore to exclude aliasing effects, a low pass, two pole, Butterworth filter of 40s was applied to both sets of signals. The N.E.R.C.L.M.T. equipment also incorporated a 3000 s high pass filter to exclude any signals produced by diurnal temperature changes affecting the magnetometer circuitry and the slow drift in self potential of the electrodes.

All five signals were sent to a geologger and recorded on magnetic tape. Up to two days of time series could be recorded on one tape. Both systems were powered by 12v car batteries. The time series recorded by the L.M.T. equipment were viewed and analysed in the field using a portable computer. Full analysis up to and including the production of apparent resistivity and phase curves was performed on the good quality sections of data, though emphasis was placed on viewing the time series, looking for magnetic storm 'Events' and instrument problems.
3.4 Testing of Equipment

All the equipment systems were thoroughly tested before being deployed in the field. This consisted of measuring the output from each component of a system in turn when a known signal was used as input. The known signal was supplied by the signal generator and the output measured on an oscilloscope. The only component in the A.M.T. system that was not accurately calibrated was the induction coils. The equipment needed to do this does not exist in this country. However crude tests were performed and the results were as expected.

3.5 Sites Occupied and Reasons Behind the Location of Soundings

Figure 3.3 is a map showing the locations of measurement sites, built up areas and high tension electricity pylons. Table 3.1 gives details of numbers, code names, national grid coordinates and height above sea level of the sounding sites. It also states the instrumentation used at these sites and the range of periods measured.

The sites form three traverses TRE-TRK (810-815), MAR-FOU (820-827) and MAR-GRV (820-836). Four soundings at REL, LAN, MAE and GRV were taken during an initial feasibility study. A six station traverse running north 20 degrees west was then completed (MAR-CAI, 820-825). The direction of the traverse was chosen to run perpendicular both to the long axis of the granite batholith, as described by the gravity model of Willis-Richards (1986), and the coastline to the north of the granite outcrop. This traverse includes the four stations at which long period measurements were taken, MAR, NAP, GAR and CAL. Careful analysis of these data, which will be described in chapter 4, produced a complicated pattern of electrical strikes. However it did indicate that two dimensional modeling could be best performed on measurements taken along traverses running approximately north 20 degrees east and the need for a grid of sounding sites to investigate lateral conductivity contrasts perpendicular to strike. Therefore the subsequent eleven soundings were taken along two other traverses TRE-TRK and MAR-GRV both running 15 degrees east of north, as well as taking more measurements following the original traverse northwards.

An ideal sounding site should be on reasonably flat land, away from lakes, rivers and seas, since topography and large volumes of highly conducting water will cause surface distortion of the impedance tensor. It should also be as far away as possible from cultural noise such as electricity pylons, built up areas and sources of earth vibrations such as main roads and forested areas.

From figure 3.3 it is clear that all the above noise sources exist within the study area and they restricted the siting of soundings. For instance no measurements were taken directly north of the Carnmenellis
Figure 3.3
Locations of Measurement Sites, Centres of Population and Electricity Pylons
Coastline outlined in dots.
Granite outcrops outlined in dots and dashes.
National grid power lines marked as a line of dots running across the land
FIGURE 3.3: A MAP SHOWING LOCATIONS OF MEASUREMENT SITES AND SOURCES OF CULTURAL NOISE
<table>
<thead>
<tr>
<th>Site No.</th>
<th>Site Name</th>
<th>Date of Sounding</th>
<th>National Grid Eastings</th>
<th>National Grid Northings</th>
<th>Height Above Sea Level (m)</th>
<th>Period Range of Measurements (sec)</th>
<th>SPAM MKIIa</th>
<th>SPAM MKIIb</th>
<th>L. M. V. T.</th>
</tr>
</thead>
<tbody>
<tr>
<td>810</td>
<td>TRE</td>
<td>7/89</td>
<td>163325</td>
<td>028350</td>
<td>75.0</td>
<td>0.00781-62.5</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>811</td>
<td>BOS</td>
<td>7/89</td>
<td>165950</td>
<td>031750</td>
<td>162.0</td>
<td>0.00781-62.5</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
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Table 3.1 Showing Site Numbers, Code Names, Locations, Instrumentation Used and the Period Range of Measurements.
granite because of the presence of the towns of Cambourne and Redruth or to the east of the outcrop due to the closeness of the sea and the town of Falmouth. The location of Stithians reservoir and a national grid electricity switching station, positioned where the three electricity pylons meet, ensured no soundings were taken in a 20 square kilometre area at the centre of the granite outcrop. Two soundings were taken close to high tension wires, one (ROS), was at the site of the Hot Dry Rock project and the other TRW, was situated on top of a conductive layer, hence cultural noise travelled shorter distances. In general, since the survey area was relatively highly populated, it was impossible to locate sites on a regular grid and take successful measurements.

3.6 Data Processing of Magnetovariation and Magnetotelluric Soundings.

The data processing of the magnetovariation and magnetotelluric soundings consisted of both classical and robust tensorial estimation. These two methods differ in two important ways, in the suppression of outliers and in the calculation of errors, but they share the same initial time domain processing, FFT program, and frequency and window rejection routines. The robust estimation routines were written by Alan Chave, (Chave et al, 1989). The rest of the processing programs were written by Rooney (1977), and their computational efficiency improved by Dawes (1984).


The time series recorded in windows by S.P.A.M. were transferred onto a mainframe computer for further processing. Figure 3.4 shows a flow chart of the program Amtcross, which takes the raw time series, corrects for the instrument response and produces averaged cross- and auto-spectra.

After initial checks and amplifier gain corrections there are a series of filter and window rejection options. Of these options whole window rejection, plus notch and delay line filters were used. Details of all these procedures are given in Fontes et al. (1988). Before any filtering was performed plots of the time series and FFT amplitudes were produced and studied. If possible, filtering was avoided by rejecting whole windows where the noise was particularly prevalent. This especially applied to windows containing spikes, since the Fourier spectrum of a spike will leak into a wide range of periods and methods of spike removal were found to corrupt impedance estimates, (See Fontes et al.,1988 for methods of spike removal). However if a majority of recorded windows in any one sub-band were contaminated with noise, filtering was performed, but only on the corrupted windows. Identical filtering procedures were performed on all channels used to calculate a tensor group.
Flow Chart of the Program Amtcross
Takes raw time series and produces average cross and auto-spectra
The delay line filter acts upon a fundamental frequency and all the higher harmonics up to the Nyquist frequency. The impulse response of the filter is given by

\[ W(z) = 1 - z^n \]  \hspace{1cm} (3.2)

where

\[ z = e^{i2\pi j/n} \quad j=0,1,2,\ldots,(n-1) \]  \hspace{1cm} (3.3)

\( z_0 \) corresponds to the fundamental frequency to be filtered, and \( z_1,\ldots,z_n \) are the higher harmonics. The delay line filter had an obvious application in suppressing 50 Hz noise and its higher harmonics produced by national grid power lines. For a description of notch filtering see Kanasewich (1981, pp. 247-252).

After setting up band ranges, instrument response correction tables for the magnetic sensors and the telluric pre-amplifiers were produced. Before each fieldwork campaign, the telluric pre-amplifier response was measured using a signal generator as source and these measured values were used to set up the instrument correction tables. The signals were detrended by fitting a straight line by the method of least squares, (Bendat and Piersol, 1971) and tapered using a cosine bell, (Harris, 1978). Either Fourier transform or cross spectral coefficients were produced.

The same procedures were performed on magnetovariation data and that measured using the long period systems. Though obviously instrument response corrections were different for the long period data.

3.6.2 Calculation of Average Response Functions using Classical Procedures

A flow chart of the program used to produce average response functions is shown in figure 3.5.

From this figure it is clear that up and down biased estimates are averaged separately. Sims et al. (1971) showed that six estimates of each element of the impedance tensor can be calculated but two of them are unstable in the one dimensional case with an unpolarized source field. Of the other four, two are biased upwards by uncorrelated noise in the electric field components, but unaffected by random noise on the magnetic channels. Whilst the other two are unaffected by random noise on the telluric channels, but biased downwards by uncorrelated noise in the magnetic field components. The equations for calculating up and down biased estimates of the impedance tensor elements are shown in the figure 3.5, (see Sims et al. (1971) for a full derivation). \(<E_xH_z^*\>\) represents the cross spectrum averaged over some finite band width. An average of the up and down biased estimates weighted by the errors in each of the biased estimates was calculated. This was first proposed by Jones (1977) and is calculated using the equation.
Figure 3.5
Flow Chart of the Program Amtavc.
The program takes cross and auto spectra, rejects estimates and produces average response functions.
Each frequency in each window must satisfy minimum coherency typically 0.8. Calculate best coh. and average coh. for each frequency.
\[ Z_a = \frac{Z_u + Z_d}{(e_u)^2 + (e_d)^2} \]

where \(Z_u, Z_d\) are impedance tensor elements with average, up and down biases, respectively.
\( e_u, e_d\) are the errors on the impedance tensor elements with up and down biases.

Following Sims et al. (1971) three equations can be derived for calculating the magnetic response functions using cross-spectral methods. Two of the equations are given in figure 3.5 and are unaffected by random noise on the vertical magnetic component but are biased downwards by random noise on the horizontal magnetic fields. An equation biased upwards by random noise on the vertical magnetic component can be calculated, Sims et al. (1971). However, since there was a high degree of cultural noise present on the vertical magnetic channel caused by vibration, this equation was not used for the calculation of magnetic response functions.

For each of the response functions, every estimate was tested and then rejected, if it didn’t meet a number of data quality tests. These tests are described in figure 3.5. Estimates were not rejected because the power spectra in \(H_x\) and \(H_y\) fell below a set value. Multiple predicted coherency was calculated using equation (3.1). Estimates were rejected if they had a coherency less than a minimum level, thus suppressing any noise bias, (see Kao and Rankin, 1977). A number was found, above which existed half the coherency estimates at any one frequency. The minimum coherency level was set at this value or a fixed value, usually 0.8, depending on which was the greater.

The preferred sign of the real and imaginary parts of the impedance elements \(Z_{xy}\) and \(Z_{yx}\) was found and those estimates with phases not in the preferred quadrants were rejected. Thus the strict criteria that at all frequencies, the real and imaginary parts of \(Z_{xy}\) are positive, and those of \(Z_{yx}\) are negative, was not implemented. This is because noise biasing of estimates is not the sole reason for the phases of either \(Z_{xy}\) or \(Z_{yx}\) being in different quadrants, at different periods. Distortion of the impedance tensor by near surface inhomogeneities can also cause this phenomenon. Furthermore, after applying decomposition methods described in chapter 4 and removing non-inductive distortions, the phase of \(Z_{xy}\) and \(Z_{yx}\) can be restored to the same opposite quadrants at all periods, (see Groom et al. (1990)). However at frequencies where the data were particularly noisy the above strict criteria were used to reject estimates.

Averaging of the apparent resistivities was done assuming a log-normal distribution (see Bentley, 1973). Outlying estimates were then
rejected if they were greater than 2.2 times the standard deviation away from the mean. The mean and standard deviation of the distribution were recalculated and the process of rejecting outliers repeated. After the whole window rejection procedure described in figure 3.5, mean averaged impedance elements or magnetic response functions were calculated and listed.

### 3.6.3 Robust Estimations of Response Functions

Magnetic response function or impedance tensor estimation using standard methods, briefly described above, assume via the Central Limit Theorem, a stationary, Gaussian model. But considerable departures from this model arise in electromagnetic data due to either non-stationary processes, such as the initial energetic phase of a magnetic storm or the preponderance of “outliers”, caused, for instance, by instrumental errors or spike noise, (Chave et al., 1987), (Chave and Thomson, 1989), (Egbert and Booker, 1986). Therefore in order to gain accurate values of electromagnetic response functions, methods such as robust procedures are needed. These methods are less sensitive to small departures from a Gaussian model.

Traditional methods for calculating electromagnetic response functions are generally based on least squares regression. Taking magnetotellurics as the example, though the theory equally applies to magnetovariation calculations and rewriting equation (2.36) in frequency dependent matrix notation used in Egbert and Booker (1987):

\[
\mathbf{E}(\omega) = \mathbf{H}(\omega) \mathbf{Z}(\omega) + \mathbf{r}
\]

where \( \mathbf{N} \) is the number of estimates and \( \mathbf{r} \) is the residual power.

\[
\mathbf{E}(\omega) = \begin{bmatrix} E_x(\omega) \\ E_y(\omega) \end{bmatrix}, \quad \mathbf{H}(\omega) = \begin{bmatrix} H_x(\omega) \\ H_y(\omega) \end{bmatrix}, \quad \mathbf{Z}(\omega) = \begin{bmatrix} Z_{xx}(\omega) & Z_{xy}(\omega) \\ Z_{yx}(\omega) & Z_{yy}(\omega) \end{bmatrix}
\]

\[
\mathbf{r} = \begin{bmatrix} \text{Re} r_1 \\ \text{Im} r_1 \\ \vdots \\ \text{Re} r_N \\ \text{Im} r_N \end{bmatrix}
\]

where \( E_x(\omega) = \begin{bmatrix} \text{Re} E_{x1} \\ \text{Im} E_{x1} \\ \vdots \\ \text{Re} E_{xN} \\ \text{Im} E_{xN} \end{bmatrix} \), \( Z_{xx}(\omega) = \begin{bmatrix} \text{Re} Z_{xx1} \\ \text{Im} Z_{xx1} \\ \vdots \\ \text{Re} Z_{xxN} \\ \text{Im} Z_{xxN} \end{bmatrix} \), \( H_x(\omega) = \begin{bmatrix} \text{Re} H_{x1} \\ \text{Im} H_{x1} \\ \vdots \\ \text{Re} H_{xN} \\ \text{Im} H_{xN} \end{bmatrix} \)

\( \mathbf{Z} \) can be rewritten using matrix notation by.
\[
\overline{Z}(\omega) = (\overline{H}(\omega)^T \overline{H}(\omega))^{-1} (\overline{H}(\omega)^T \overline{E}(\omega))
\] (3.6)

T denotes Hermitian transpose and \((\overline{H}(\omega)^T \overline{H}(\omega))\) and \((\overline{H}(\omega)^T \overline{E}(\omega))\) are the averaged auto-and cross-power spectral estimates. Equation (3.6) is the unbiased equation in figure 3.5 written in matrix format.

This estimate is a least squares estimate since it minimizes the sum of the squared residuals.

\[
\sum_{i=1}^{2N} r_i^2 = \sum_{i=1}^{2N} (E_i - H_i^T Z)^2
\] (3.7)

Chave and Thomson (1989) suggest using M-estimates regression. This in effect, is a weighted least squares regression, where the weights are chosen to suppress the influence of large residuals.

\[
\overline{Z}(\omega) = (\overline{H}(\omega)^T \overline{W}(\omega) \overline{H}(\omega))^{-1} (\overline{H}(\omega)^T \overline{W}(\omega) \overline{E}(\omega))
\] (3.8)

where \(\overline{W}(\omega)\) is the weighting factor matrix.

They go on to suggest a Huber weighting factor, (Huber, 1964), given by.

\[
w_i = \begin{cases} 
1 & |x_i| \leq a \\
\frac{a}{|x_i|} & |x_i| > a 
\end{cases} 
\]

\[x_i = \frac{r_i}{d}\] (3.9)

These weights are based on a model which is Gaussian at the centre and Laplacian at the tails. \(a\) is a constant. \(d\) is a scale factor and determines when residuals are assumed to be large. See Chave et al. (1987) for the merits of different forms of \(d\).

The program for each frequency calculated a solution of (3.5), using an ordinary least squares regression, and then computed the residuals \(r\). The scale factor \(d\) and hence the Huber weight function could then be found, and a number of iterations were performed using weighted least squares. The residuals from the previous iteration were used to calculate weights for the next iteration and this process continued until the weighted residual power \(r^T w r\) fell below a set value.

The Huber weights in equation (3.8) never fall to zero, hence perform badly when severe outliers that can exist in electromagnetic data are present. So after achieving a good estimate using Huber weights, the scale factor \(d\) was fixed and a few iterations were performed using the more severe weighting factor defined in Thomson (1977).
\[ w_i = \exp\{-e^{\alpha(x_i - \alpha)}\} \] (3.10)

\( \alpha \) like \( a \) determines the residual size at which down-weighting starts.

3.6.3.1 Jackknife Estimation of Variances

The standard processing methods assume independent, normally distributed errors. Estimates of the variance of response functions are especially sensitive to departures from the Gaussian model, (See Chave and Thomson ,1989) for further discussion. A widely used method of calculating statistical parameters and their variances which makes no assumptions about the probability distribution of the data, is the Jackknife. See (Efron, 1982) for an introduction to the jackknife and (Thomson and Chave, 1988) for applications to spectral analysis problems. Only the jackknife calculations of the variance of parameters were used in the processing of the data from this study.

The data are divided into \( N \) groups of size \( N-1 \) by deleting one estimate in turn. Following the notation given in Chave and Thomson (1989), the variance of a statistical parameter (\( \theta \)) is given by.

\[
\hat{s}^2 = \frac{N-1}{N} \sum_{i=1}^{N} (\hat{\theta}_i - \overline{\theta})^2
\] (3.11)

where

\[
\overline{\theta} = \frac{1}{N} \sum_{i=1}^{N} \hat{\theta}_i
\] (3.12)

\( N= \) number of estimates,
\( \hat{\theta}_i = \) the estimate of \( \theta \) where the \( i \)-th datum has been removed.

Therefore in the program a series of "delete one" estimates of \( z \) were calculated. This was done by deleting a row of E and H in turn, and solving either the least squares or the weighted least squares problem.

3.6.3.2 Problems Encountered Using Robust Estimation of Response Functions in the A.M.T. Period Range

When processing A.M.T. data using M-estimate regression as described in 3.6.3.1, it was found that especially for periods in the "dead-band", 0.1-10.0 seconds, the response curves could be erratic. Usually the
amplitude of apparent resistivity was severely underestimated. This is because the signal level over this period range was so low that noise formed the majority of the data recorded. In this case the outliers were the occasional bursts of signal and they were effectively suppressed by the robust regression. Therefore rejection of estimates on the basis of coherency and in the case of the impedance estimates, sign, was performed before any least squares regression. For the impedance tensor calculations up and down biased estimates were also calculated. The combination of estimate rejection, up-down biased estimates and M-estimate regression, produced the smoothest, least biased estimates and the methods were applied to data over the whole period range. Similar ideas to the above have been proposed by Egbert and Livelybrooks, (1990).

Therefore robust data processing consisted of all the steps described in figures 3.4 and 3.5 except for the section in figure 3.5 enclosed in the box with the rounded corners. This was replaced with the robust methods described in section 3.6.3.1.

3.7 The Magnetotelluric and Magnetovariation Data

Apparent resistivity and impedance phase were calculated from the impedance tensor using the equations (2.28) and (2.29)

3.7.1 Rotation of the Impedance Tensors

The principal directions, since they are parallel or perpendicular to the gross strike of the conductivity anomaly, represents a more meaningful coordinate system in which to study the impedance tensor. When the impedance tensor is measured over a perfect two-dimensional structure and then rotated into the direction of the strike or regional azimuth, the diagonal elements vanish and the power in the off diagonal elements is at a maximum. However with measured impedance tensors, three dimensional effects or noise ensure that no angle of rotation \( \theta_0 \), will produce zero diagonal elements. Swift (1967) proposed a method for calculating the angle \( \theta_0 \) for such quasi-two dimensional structures by maximising the off-diagonal elements of the impedance tensor. This is obviously the most desirable method of calculating \( \theta_0 \) for real data. The equation to do this is given by.

\[
\theta_0 = \frac{1}{4} \arctan \left( \frac{(Z_{xx} - Z_{yy})(Z_{xy} + Z_{yx})^* + (Z_{xx} + Z_{yy})^*(Z_{xy} + Z_{yx})}{|Z_{xx} - Z_{yy}|^2 - |Z_{xy} + Z_{yx}|^2} \right)
\]

(3.13)

where * denotes conjugate.

The impedance tensor can be rotated through an angle \( \theta \) using the equation.
\[ Z' = R Z R^T \quad (3.14) \]

where \( R \) is the Cartesian rotation matrix given by:

\[
R = \begin{bmatrix}
\cos \theta & \sin \theta \\
-sin \theta & \cos \theta
\end{bmatrix}
\quad (3.15)
\]

If \( \theta \) is equal to \( \theta_0 \) then \( Z_{xy}^l \) and \( Z_{yx}^l \) are the maximum and minimum impedance estimates respectively. The 90 degree ambiguity in calculating the azimuth was removed by always having \( Z_{xy}^l \) as the maximum.

3.7.2 Rotationally Invariant Parameters

In order to minimize two-dimensional and three dimensional effects, Berdichevsky and Dmitriev (1976) proposed two rotationally invariant averages of the impedance tensor. The arithmetic or Berdichevsky average is given by:

\[
Z_{berd} = 0.5 (Z_{xy} - Z_{yx}) \quad (3.16)
\]

and the Effective impedance, which is the square root of the determinant of the impedance tensor is given by:

\[
Z_{eff} = \sqrt{(Z_{xx}Z_{yy} - Z_{xy}Z_{yx})} \quad (3.17)
\]

3.7.3 The Magnetotelluric Data

Figures 3.6 to 3.26 show the magnetotelluric data measured at each of the 21 sites. This data has been robustly processed using the program written by Alan Chave. Looking at figure 3.6, figures a) and b) are the apparent resistivity and phase curves of the off diagonal elements of the unrotated impedance tensor. The curves plotted are those of the average of the up and down biased estimates calculated using equation (3.4). The error bars are taken from either the up or down biased estimates of the standard error on the impedance elements. They form an envelope giving a true picture of the accuracy of the apparent resistivity and phase by including both statistical errors and noise biasing.

Figure c) is a plot of apparent resistivity and phase calculated from all four components of the impedance tensor after rotation to the principal directions. The values and the standard errors plotted in figure c), d) e) f) and g) are those of the average of the up and biased estimates. The phase of \( Z_{xy} \) in figures b) and d) has been reflected from the third quadrant to the first quadrant. The apparent resistivity and phase plotted in figure e) is derived from the effective impedance, the rotationally invariant average given in equation (3.17).
Figures 3.6 to 3.26
Magnetotelluric Data Measured at Each of the 21 Sites.

This data has been robustly processed using the program written by Alan Chave.

Looking at figure 3.6.
Figures a) and b) are the apparent resistivity and phase curves of the off diagonal elements of the unrotated impedance tensor. The curves plotted are those of the average of the up and down biased estimates calculated using equation (3.4). The error bars are taken from either the up or down biased estimates of the standard error on the impedance elements.
Figure c) is a plot of apparent resistivity and phase calculated from all four components of the impedance tensor after rotation to the principal directions using the methods of Swift (1967).
The values and the standard errors plotted in figure c), d) e) f) and g) are those of the average of the up and biased estimates.
The phase of $Z_{yx}$ in figures b) and d) has been reflected from the third quadrant to the first quadrant.
The apparent resistivity and phase plotted in figure e) is derived from the effective impedance, the rotationally invariant average given in equation (3.17).
The regional azimuth plotted in figure f) is measured positive clockwise from magnetic north.
The coherency plotted in figure g) is a multiple predicted coherency and is the average of all estimates at a particular period.
Figure 3.6 M.T. Data: Site: TREB10
Figure 3.7 M.T. Data: Site: BOS811
Figure 3.8 M.T. Data: Site: REL812
Figure 3.9 M.T. Data Site: BOQ813
Figure 3.10 M.T. Data: Site: CRO814
Figure 3.11 M.T. Data Site: TRK815
Figure 3.12 M.T. Data: Site: MAR820
Figure 3.13 M.T. Data: Site: BIS821
Figure 3.14 M.T. Data: Site: NAP822
Figure 3.15 M.T. Data: Site: SEW823

Unrotated Ryx Average with Up-Down Biased Errors

Major(+), Minor(-) Average of Up-Down Bias

Regional Azim, Rotated By Method of Swift(1967)

Effective Invariant (Average of Up-Down Bias)
Figure 3.16 M.T. Data: Site: GAR824
Figure 3.17 M.T. Data: Site: CAL825
Figure 3.18 M.T. Data: Site: LANB26
Figure 3.19 M.T. Data: Site: FOUB27
Figure 3.20 M.T. Data: Site: EAT830
Figure 3.21 M.T. Data: Site: MAE831
Figure 3.22 M.T. Data: Site: ROS832
Figure 3.23 M.T. Data: Site: LAIB33
Figure 3.24 M.T. Data: Site: PER834
Figure 3.25 M.T. Data: Site: TRW835
Figure 3.26 M.T. Data: Site: GRV836
The regional azimuth plotted in figure f) is measured positive clockwise from magnetic north. The coherency plotted in figure g) is a multiple predicted coherency and is the average of all estimates at a particular period. The average includes those estimates rejected because they had a coherency below the minimum value, hence the sometimes very low values.

3.7.4 Computation of Parkinson Arrows

Following the standard conventions of Hobbs (1992) the vertical magnetic field is related to the horizontal magnetic field components in the frequency domain and at a single station by:

\[ Z(\omega) = A(\omega)X(\omega) + B(\omega)Y(\omega) \]  

(3.18)

where \( X, Y, \) and \( Z \) are the fourier transforms of magnetic field variations in the north, east and downward directions respectively. \( A \) and \( B \) are complex.

\[
A = A_R + iA_I \\
B = B_R + iB_I
\]  

(3.19)

The combination \((A, B)\) forms a two dimensional vector called the magnetic response function. The real and imaginary parts can be combined to form two induction arrows. The induction arrow \((-A_R, -B_R)\) was introduced by Parkinson (1962) and tends to point at areas of relatively high conductivity. The magnitude and phase of the induction arrows are given by:

\[
|K| = \sqrt{A_k^2 + B_k^2} \\
\phi_k = \arctan \left( \frac{B_k}{A_k} \right)
\]  

(3.20)

Figures 3.27 to 3.38 show real and imaginary induction arrows at the sites where a vertical component of the magnetic field was measured. The real arrows take the Parkinson convention of \((-A_R, -B_R)\), whilst the imaginary or quadrature arrows take the convention \((A_I, B_I)\).

3.7.5 Discussion of Data Collected Using Long Period Systems

Although four long period soundings were undertaken, data with periods greater than 80 seconds is presented in figures 3.6 to 3.26 at only two sites, MAR820 and GAR824. The measurements at sites NAP822 and CAL825 were so heavily contaminated by noise that even after extensive data processing, the impedance tensor and induction arrows were
The bold lines are Parkinson Arrows averaged over the period range stated in the title. Thin lines joined by circular segment show range of values over the same period range. The arrow on the circular segment shows the direction of change in the Parkinson arrows as the period increases.

The Coastline outlined in dots.
Granite outcrops are outlined in dots and dashes.
real

imag

magnitude $\cdot 10^{-1}/$tick

FIGURE 3.27: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 0.003 AND 0.01 SECS

FIGURE 3.28: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 0.01 AND 0.03 SECS
FIGURE 3.29: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 0.03 AND 0.1 SECS
FIGURE 3.30: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 0.1 AND 0.3 SECS
FIGURE 3.31: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 0.3 AND 1.0 SECS
FIGURE 3.32: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 1.0 AND 3.0 SECS
FIGURE 3.33: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 3.0 AND 10.0 SECS
FIGURE 3.34: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 10.0 AND 30.0 SECS
FIGURE 3.35: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 30.0 AND 100.0 SECS

FIGURE 3.36: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 100.0 AND 300.0 SECS
FIGURE 3.37: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 300.0 AND 1000 SECS

FIGURE 3.38: AVERAGE PARKINSON INDUCTION ARROWS CALCULATED FOR PERIODS BETWEEN 1000 AND 3000 SECS
meaningless. A large part of the noise recorded at these sites was introduced by faulty recording equipment. An example of such noise is shown in figure 3.39. This shows the same magnetic substorm shown in figure 1.2 recorded this time at site NAP822. Note the unnatural square oscillations recorded on the $H_y$ channel totally obscuring the natural signal.
Figure 3.39
A Magnetic Bay or Substorm Recorded at Site NAP822
Along with Unnatural Noise Recorded on the Hy Channel.
Noise Introduced by the Recording Equipment
Chapter 4

The Analysis of the Measured Impedance Tensors.

4.1 Introduction

It has long been known that electric charge both builds up at conductivity boundaries and subsequently scatters the induction fields within the earth, thus distorting measured impedance tensors. The most well known effect of this galvanic distortion, widely reported in literature, is the so called "static shift" effect, whereby apparent resistivity curves are simply shifted to higher or lower values without a change in the curve shape or any distortion of the phase curve.

Since galvanic distortion is always present in magnetotelluric measurements and its effects can possibly lead to a serious misinterpretation of measured impedance tensors, it is vital to separate the galvanic effects from the inductive response within the tensor. Highly similar methods to achieve this goal have been developed independently by Bahr (1988) and Groom and Bailey (1989). They are both based on decomposing the impedance tensor under a physically realisable model. These methods have been applied to the Carnmenellis data set.

Any modelling of geophysical data is performed by making simplifying assumptions about the earth. In most cases at present, magnetotelluric data is interpreted using three simplified earth models.

1) The one dimensional earth, in which the conductivity varies only with depth
2) The conventional two dimensional model. This only takes into account induction within an earth in which the conductivity varies with depth and in one lateral direction.
3) A model in which a small, inductively weak, three dimensional anomaly lies above the rest of the earth whose conductivity varies in two dimensions. This model will be described in the following sections of this chapter.

In the following chapter these earth models will be referred to as model classes. In interpreting these models it is essential to know how well the assumptions made, are upheld by the data. Methods that do this have been developed, (Groom et. al,1990), (Bahr, 1991) and have also been applied to the data described in this study.

This chapter reviews the causes and effects of galvanic distortion, the theory behind the decompositions of Bahr, and Groom and Bailey and methods for assessing the appropriateness of the available physical approximations of the earth in the interpretation of a data set. It explains
the application of these methods to the data set using an example of the data collected at one site. It then proceeds to describe the pattern of parameters derived from tensor decompositions with emphasis on survey wide interpretations. Finally a strategy is given for subsequent interpretation of the data set based on the analysis described in the chapter.

4.2 The Causes and Effects of Galvanic Distortion

Price (1973) dearly shows that the underlying cause of galvanic distortion is charge build-up over volumes where there is a conductivity contrast. Why this build-up of charge arises can be understood by considering a body made of electrically conducting material placed in an electric field (primary field). By applying the equation of continuity and Ohm's law, it is easy to show (Lajoie and West, 1976) (Kaufmann, 1985) that the volume charge density ($\rho_v$) is given by:

$$\rho_v = -\frac{\varepsilon_0}{\sigma + i\omega\varepsilon_0} \vec{E}.\nabla\sigma$$

(4.1)

where

- $\varepsilon_0$ = the electrical permittivity of free space
- $\omega$ = the angular frequency of the electromagnetic field
- $\sigma$ = the electrical conductivity of the material
- $\vec{E}$ = the primary electric field

Three points arise from this equation.

1) Equation 4.1 will be non zero only if $\vec{E}.\nabla\sigma$ is non zero. In other words charges will accumulate if there is a component of the electric field in the direction of the change in conductivity.

2) The quasi static approach ($\sigma >> \omega\varepsilon_0$) applies to magnetotelluric measurements taken over the period range of this study. Therefore under this assumption, it is clear from equation 4.1 that there is no phase change between the primary field and the galvanic charge produced. Also the volume charge density is independent of the frequency of the primary field.

3) At conductivity boundaries all the volume charge density will reduce to a surface charge density which will be minute due to the small value of the permittivity term, (Price, 1973). However the secondary electric fields produced can be quite large, again due to the small value of the permittivity term in the denominator of Coulomb's Law.
These secondary electric fields will add vectorially to the primary fields thus distorting or scattering the primary fields. Simple examples of this effect are shown in amongst others (Price, 1973) (Jones, 1983a) and (Jiracek, 1988). The subsequent anomalous current produced by the distortion of the electric field, will in turn, produce an anomalous magnetic field.

4.3 Definition of Terms Local and Regional

The factors governing which model class is appropriate in characterising the electromagnetic response of a body are, the scale length of the body and the skin depth within the body.

This can be illustrated by considering a set of broad band M.T. measurements taken at the surface, near to the centre of a generalised body. For short periods, when the skin depth within the body is very much shorter than its smallest dimension, one dimensional induction can be used to explain the body's electromagnetic response. As the period of the source field increases, the skin depth increases, until one edge of the body has an effect on its response. Hence a two dimensional induction model is required to interpret these data accurately. Eventually the period of the source field is such that contacts between the body and the surrounding medium on all sides affect the measurements and a true picture of the body can only be produced if the data are interpreted using a three dimensional induction model. At some period the skin depth will be much greater than the scale length of the body and hence the body will become inductively weak. However, as shown in section 4.2 electric charges will still build up within the body and it will continue to affect the M.T. measurements by acting as a galvanic scatterer of the electromagnetic fields induced within the surrounding medium.

In practise a body is assumed to have no inductive response when the skin depth is more than three times the scale length of the body. Bodies with no inductive response are referred to as local and those with an inductive response are referred to as regional bodies. These terms will obviously apply to bodies of different scale lengths over different period ranges.

4.4 Decompositions of the Impedance Tensor

One dimensional and two dimensional model classes, as given by equations 2.32, 2.34 and 2.35, assume that the diagonal elements of the impedance tensor are zero. Any values present in these elements of a measured impedance tensor are classed as noise. This approximation, as well as excluding half the data collected when calculating earth response functions, is also invalid in many cases, since the diagonal elements of measured impedance tensors are often too large to be dismissed as merely noise.

Therefore a number of tensor decompositions have been proposed
that utilize all four complex impedance elements in the calculation of their parameters whilst making no simplifying assumptions, (Eggers, 1982), (La Turraca et al, 1986), (Spitz, 1985), (Yee and Paulson, 1987), (Cevallos, 1986). However, these decompositions fail to give any physical significance to the decomposition parameters or to separate the local galvanic scattering from the regional inductive response.

Another set of tensor decompositions, (Larson, 1977), (Zhang et al., 1987), (Bahr, 1988) and (Groom and Bailey, 1989), are based on physical models which do make assumptions, the most important of which, restrict any inductive response to one or two dimensional bodies. By making this assumption, the tensor can be decomposed into parameters that are affected only by local galvanic distortion or only by regional inductive effects. The most general decompositions of this type are those proposed by Bahr (1988) and Groom and Bailey (1989). The two approaches of Bahr (1988) and Groom and Bailey (1989) are very similar. The differences lie in extracting parameters from the models. In general the following sections follow the approach of Groom and Bailey (1989), although a large amount of the theoretical background is also to be found in Bahr (1988).

4.4.1 The Physical Model Used In The Decompositions of Bahr and Groom and Bailey

The physical model is that of an inductively small local body of varying conductance overlying a regional two dimensional structure, the local body galvanically distorting only the telluric field produced by the regional structure.

The model therefore makes two simplifying assumptions.

1) The regional electromagnetic field is produced by only one dimensional or two dimensional bodies. Thus the decomposition model neglects any three dimensional inductive effects. This is because these cannot, at present, be parameterised using a credible physical hypothesis. Thus following equation 2.37, and assuming the fields are measured in the principal directions (Ⅰ and Ⅱ), the regional electric field \( E_r \) is related to the regional magnetic field \( H_r \) by the regional impedance \( Z_r \).

\[
E_r = Z_r H_r
\]

(4.2)

where

\[
Z_r = \begin{pmatrix}
0 & Z_⊥ \\
-Z_∥ & 0
\end{pmatrix}
\]

(4.3)
2) The model also neglects distortion of the magnetic field by inductively weak bodies.

Following Bahr (1988) the regional electric field is scattered by a local inductively weak body. A scattering tensor \( \mathbf{C} \) operates on the regional field to form the resulting measured electric field \( \mathbf{E}_m \).

\[
\mathbf{E}_m = \mathbf{C}\mathbf{E}_r \tag{4.4}
\]

where

\[
\mathbf{C} = \begin{pmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{pmatrix}
\tag{4.5}
\]

From the discussion in section 4.2 it is clear that \( \mathbf{C} \) is real and frequency independent.

Groom and Bailey (1991) show that anomalous current density within the earth will produce an anomalous current system and by the Biot-Savart law an anomalous electrostatic magnetic field. The resulting measured magnetic field is given by

\[
\mathbf{H}_m = \mathbf{H}_r + \mathbf{D}\mathbf{E}_r \tag{4.6}
\]

where \( \mathbf{D} \) is a two by two scattering tensor. For proof that four independent elements are needed to represent either magnetic or electrical scattering by an arbitrary 3D homogeneity, see Groom and Bailey (1989).

From equations (4.2), (4.4) and (4.6) it can be shown that the impedance tensor measured in the principal directions \( \mathbf{Z}_m \) is given by

\[
\mathbf{Z}_m = \mathbf{C}\mathbf{Z}_r (\mathbf{I} + \mathbf{D}\mathbf{Z}_r)^{-1} \tag{4.7}
\]

From equation (4.6) it is clear that the magnetic scattering effects are complex and Groom and Bailey (1991) show they are frequency dependant. The resultant scattering matrix is complicated and methods for extracting the parameters from this matrix have not yet been devised. However Groom and Bailey (1991) show that the magnitude of \( \mathbf{D}\mathbf{Z}_r \) tends to zero as \( \sqrt{\omega} \rightarrow 0 \). For the range of periods used in this study the effects of magnetic scattering are very much secondary to those of electric field distortion. However magnetic scattering may become significant at periods below \( 10s \) if strong galvanic scattering is present.

4.4.2 The Theoretical Basis for the Bahr and Groom and Bailey Impedance Tensor Decompositions.

Using the model described in section 4.4.1 the measured impedance
tensor is given by.

\[ Z_m = C \, Z_r \quad (4.8) \]

Using the equations (3.14) and (3.15) we can rotate the measurement axis system by an angle (θ) called the regional azimuth. This gives the measured impedance tensor \((Z_m')\) in a general coordinate system as

\[ Z_m' = R(\theta) \, C \, Z_r \, R^T(\theta) \quad (4.9) \]

Subsequently all measured impedance tensors are assumed to be measured in a general coordinate system and so the dash will henceforth be dropped. Rewriting equation (4.9) using equations (4.3) and (4.5).

\[ Z_m = R(\theta) \begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix} \begin{pmatrix} 0 & Z_\perp \\ -Z_\parallel & 0 \end{pmatrix} \, R^T(\theta) \quad (4.10) \]

From equation (4.10) it can be seen that although the distortion tensor is real and independent of frequency it will mix the elements of the regional impedance tensor, hence impedance phases will also be affected. The equation also shows that factorisation of the measured impedance contains nine distinct parameters, the phase and magnitude of the two principal impedances, the four distortion matrix parameters and the regional azimuth. Yet the measured impedance tensor only contains eight separate pieces of data, namely the four complex elements. Therefore this particular factorisation cannot be performed uniquely for measured data.

Groom and Bailey (1989) proceed to find a useful factorisation by further decomposing the tensor \(C\) into a scalar \(g\) and three tensors \(T, S\) and \(A\). Thus

\[ C = gTSA \quad (4.11) \]

The proof of the uniqueness of this factorisation for most reasonable distortion tensors is shown in Groom and Bailey (1989). \(g\) is known as the scalar gain. The tensor factors are defined as

\[ T = N_2(I + t\Sigma_2) = \frac{1}{\sqrt{1+t^2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + t \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (4.12) \]
\[ S = N_1(1 + e\Sigma_1) = \frac{1}{\sqrt{1+e^2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + e \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \] (4.13)

\[ A = N_3(1 + s\Sigma_3) = \frac{1}{\sqrt{1+s^2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + s \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \] (4.14)

\(N_1, N_2\) and \(N_3\) are normalisation factors used to ensure \(T, S\) and \(A\) remain bounded during their computation. \(t, e, s\) are real numbers.

Multiplying the regional impedance tensor by the splitting tensor \(A\), simply adds to the already existing anisotropy present. The so-called "shear" tensor \(S\) also develops anisotropy but along axes bisecting those of the principal directions of the regional impedance. Figures 4.1.1 and 4.1.2 show the effects of tensors \(A\) and \(S\) on a set of unit vectors. Figure 4.1.2 shows the multiplication by tensor \(S\) where \(e\) is positive. \(S\) deflects a vector running along the \(x\) axis, clockwise through an angle \(\tan^{-1} e\). A vector running along the \(y\) axis is deflected by a similar angle but in an anti-clockwise direction. Therefore the shear \(e\) is defined as an angle, the shear angle, given by

\[ \varnothing_s = \tan^{-1} e \] (4.15)

The tensor \(T\) simply rotates the electric field through a clockwise angle defined as the twist angle.

\[ \varnothing_t = \tan^{-1} t \] (4.16)

Physical constraints restrict the absolute value of the shear angles to the range \(-60^\circ\) to \(+60^\circ\) and the twist angles to the range to \(-45^\circ\) to \(+45^\circ\), (Groom and Bailey, 1989).

Substituting equation (4.11) into (4.9)

\[ Z_m = R(\theta) gTSA Z_r R^T(\theta) \] (4.17)

\(gA Z_r\) is as equally valid a two dimensional regional impedance tensor as \(Z_r\) and therefore \(g\) and \(A\) cannot be calculated separately. Hence, Groom and Bailey (1989) absorb the unknown factors, \(g\) and \(A\), into \(Z_r\), by making them frequency independent scaling factors of the regional impedance tensor \((Z_r)\). Thus the theoretical form of a two dimensional impedance
Figure 4.1.1
A family of unit vectors (a) before and (b) after the application of the splitting tensor $A$. The $x$ axis is up, the $y$ axis to the right. From Groom and Bailey (1989).

Figure 4.1.2
A family of unit vectors (a) before and (b) after the application of the shear tensor $S$. The $x$ axis is up, the $y$ axis to the right. From Groom and Bailey (1989).
tensor is kept. In fact the product $gA$ is the so called "static shift" factor. Therefore equation (4.17) can be rewritten as

$$Z_m = R(\theta)^{T} T S Z_T^r R^{T}(\theta)$$ \hspace{1cm} (4.18)

The dash will be dropped from now on.

Equation (4.18) has seven real parameters which can be calculated from the components of the measured impedance tensor using a set of simple non-linear equations, (Groom and Bailey, 1989).

$$Z_{xx} + Z_{yy} = t \Omega + e \delta$$ \hspace{1cm} (4.19a)

$$Z_{xy} + Z_{yx} = (\delta - et \Omega) \cos 2\theta - (t \delta + e \Omega) \sin 2\theta$$ \hspace{1cm} (4.19b)

$$Z_{yx} - Z_{xy} = - \Omega + et \delta$$ \hspace{1cm} (4.19c)

$$Z_{xx} - Z_{yy} = -(t \delta + e \Omega) \cos 2\theta - (\delta - et \Omega) \sin 2\theta$$ \hspace{1cm} (4.19d)

where

$$\Omega = Z_{\parallel} + Z_{\perp} \quad \text{and} \quad \delta = Z_{\perp} - Z_{\parallel}$$ \hspace{1cm} (4.20)

If an impedance tensor has been measured with no errors and was produced by a physical environment exactly matching that outlined in section 4.4.1, equations (4.19) form a unique decomposition. In reality this never occurs and Groom and Bailey (1989) use an iterative least squares fitting procedure to calculate the model class parameters.

4.4.3 Methods For Assessing How Appropriate a Physical Approximation is For a Particular Data Set

Before producing models or inversions based on a physical approximation of the real earth it is essential to investigate the extent to which the data being modelled or inverted uphold these approximations. In these investigations two factors are important.

1) The assessment should start with the model class based on the simplest approximation to the earth and only consider model classes based on more complex idealised earth, if the data deviates significantly from the model predictions. Parameters in a model class of inappropriate complexity will be both redundant and unstable. For instance if a data set is truly one dimensional then the regional azimuth will not only have no significance but will also have the property that all angles will be equally appropriate. (Bahr, 1991)
2) The appropriateness of any class of model or even the parameters belonging to a model class, will change with period. This was illustrated with the hypothetical example in section 4.2.

### 4.4.3.1 The Normalised Chi-Squared Residual Error of Fit

In the introduction (section 4.1) three model classes were described. A useful chi-squared statistical test of how well any of these three model classes and their parameters fit a set of data at a particular period was introduced in Groom et. al. (1990). This particular chi-squared residual error of fit is normalised using the square of the variances ($\sigma^2_{ij}$) of the four elements of the measured impedance tensor and is given by

$$
\gamma^2 = \frac{1}{4} \sum_{j=1}^{2} \sum_{i=1}^{2} \frac{|\hat{Z}_{ij} - Z_{ij}|^2}{\sigma^2_{ij}}
$$

(4.21)

where

- $\hat{Z}_{ij}$ = modelled data
- $Z_{ij}$ = measured data

If a model is appropriate, it is a reasonable to presume that it will fit the data to within three standard deviations, hence $\gamma^2$ will be between 0-9.

In assessing the significance of the statistic it should be noted that variations in either the noise level or in the accuracy of variance estimate calculations, will obviously have an effect on the value of $\gamma^2$. Therefore models should be assessed over a range of frequencies. However, a more important factor to consider, is the reduction in the normalised residual error of fit caused by an increase in the number of free parameters within the model. Hence in assessing how appropriate a model class is for interpreting a measured impedance tensor, the number of free parameters within the class must also be considered along with the value of the normalised residual error of fit.

### 4.4.3.2 Bahr's Seven Sub-classes of Models.

Bahr (1991) has usefully split the above 3 model classes into 7 sub-classes. He then proceeds to define a set of easily calculated parameters which can be used to decide which sub-class a measured impedance tensor belongs to. Following the notation of Bahr (1991) the parameters are defined using:

- $S_1 = Z_{xx} + Z_{yy}$
- $S_2 = Z_{xy} + Z_{yx}$
- $D_1 = Z_{xx} - Z_{yy}$
- $D_2 = Z_{xy} - Z_{yx}$

(4.22)
The skew ($\kappa$), as defined by Swift (1967), is a parameter indicating the dimensionality of the impedance tensor.

$$\kappa = \frac{|S_1|}{|D_2|}$$  \hspace{1cm} (4.23)

A new parameter directly calculable from the measured impedance tensor is a rotationally invariant measure of the phase differences in the impedance tensor ($\mu$) and is defined by Bahr (1991) as

$$\mu = \frac{\left| \text{Im}(S_2D_1^*) \right| + \left| \text{Im}(D_2S_1^*) \right|}{D_2}$$  \hspace{1cm} (4.24)

A rotationally invariant measure of two dimensionality is ($\Sigma$) defined by Bahr (1991) as

$$\Sigma = \frac{(D_1^2 + S_2^2)}{D_2^2}$$  \hspace{1cm} (4.25)

A rotationally invariant dimensionality parameter ($\eta$) defined by Bahr (1991) as

$$\eta = \frac{\left| \text{Im}(S_2D_1^*) \right| \ - \ \left| \text{Im}(D_2S_1^*) \right|}{D_2}$$  \hspace{1cm} (4.26)

The seven classes defined below emphasise the separation of effects due to galvanic distortion and those due to regional induction.

**Class 1: The simple 2-D anomaly characterised by $\Sigma > 0.1$ and $\kappa < 0.1$**

The methods of Swift (1967) as outlined in section 3.7.1 can be used to produce the earth response functions for impedance tensors bracketed in this sub-class.

Either the methods of Groom and Bailey (1989) or Bahr (1991) need to be applied to produce model parameters for the other sub-classes.

**Class 2: A local 3D anomaly superimposed upon a layered earth characterised by $\eta < 0.05$**

For all other classes $\eta > 0.1$ and the difference between them is the
amount of local distortion as characterised by the values of twist and shear angles.

Class 3: A regional 2D structure weakly distorted by local 3D inhomogeneities, characterised by either \( \varphi_e - \varphi_t < 5.0 \)
and \( \varphi_e + \varphi_t < 20.0 \) or \( \varphi_e - \varphi_t < 20.0 \) and \( \varphi_e + \varphi_t < 5.0 \)

Class 4: A regional 2D anomaly in rotated coordinates characterised by \( \varphi_t = 0 \).

Class 5: A regional 2D anomaly with strong local distortion characterised by \( \eta < 0.3 \) and large twist and shear angles.

Class 6: A regional 2D anomaly with strong channelling characterised by \( \varphi_e = 45.0 \)

Class 7: A regional 3D anomaly characterised by \( \eta > 0.3 \)

4.4.3.3 Analysis of The Carnmenellis Data Set Using Bahr's Model Sub-Classes

Data collected at each station was analysed using the methods described in section 4.4.3.2. From this analysis data at each period were placed in one of the seven model sub-classes described above. Table 4.1 gives a summary of this analysis. A number of points relating to the nature of the induction problem in South-West Cornwall are illustrated in the table.

1) All the data recorded show characteristics of galvanic distortion except at the one site (821) where data recorded at only one period falls into class 1.

2) Very little data recorded show the characteristics of model sub-class 2. Therefore subsequent interpretation of the data at all sites should be performed using 2D models.

3) Data measured at a number of sites is characteristic of electromagnetic induction in a regional 3D anomaly (model sub-class 7). Some of these data were collected over period ranges where the signal to noise ratio was low, for instance at periods in the "dead band", between 1 and 10 seconds or at periods below 0.01 seconds where noise emanating from national grid power lines severely corrupted the data. The fact that these data were contaminated by noise puts into doubt the conclusion that it is characteristic of induction in a 3D structure but does not change the fact
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Table 4.1: Table
that any interpretations of the parameters derived from these data should be treated with caution.

At least a decade of data recorded at all the sites shows characteristics of strong galvanic distortion of induced fields produced in a regional 2D anomaly (model sub-class 5). Bahr (1991) demonstrates that only a decomposition method such as that proposed in Groom and Bailey (1989) will recover the correct model parameters from such data. This gives strong evidence for the conclusion that application of decomposition methods was more than an exploratory course of action. It was necessary for accurate interpretation of the Carnmenellis data set.

A pattern emerges in the period range below 0.1s whereby data recorded at stations close to the edge of the Carnmenellis granite outcrop, such as 823, were strongly distorted by galvanic charges at the short periods, whilst data recorded at stations in the middle of the outcrop, such as 824, continued to be only weakly distorted down to longer periods (10s in the case of 824). This is an early indication that over this period range the cause of galvanic distortion was charge building up on the sides of the granite batholith.

4.5 Methodology Used To Analyse Measured Impedance Tensors

The general methodology used in analysing the Carnmenellis granite data set before producing resistivity depth models is described below using site SEW823 as an example. The computer code used along with initial instruction in its use were kindly provided by Ross Groom.

Firstly, for reasons outlined in section 4.4, the fit of the data to the simplest model, the 1D layered earth, was examined. Plots of the apparent resistivity and phase curves of the off diagonal elements of the impedance tensor as calculated in the measurement coordinate system, plus the chi-squared residual error of fit of the 1D model to the data (r.m.s. error) are shown in figure 4.2. A quick glance at the divergence of the apparent resistivity and phase curves for the two tensor elements clearly indicates the inappropriateness of the 1D model and this is expressed as a large chi-squared residual. (Except for a few frequencies, $\gamma^2 > 400.0$).

Two general points arise from these plots.

1) The error bars are not always centred on the estimate of a parameter. This is due to the non-linear nature of equations 4.19 which retrieve the model class parameters from the data. The error bars in fact show the spread of a parameter when calculated using each of the members of a subset of the total number of estimates used to produce the mean impedance tensor.

2) The sudden drop in $\gamma^2$ at periods between 0.7 and 7 seconds is
due to a difference in the error structure of the data caused by the drop in the signal level, the so called "dead band". This is a good illustration of the sensitivity of $\chi^2$ to the variance of an measured impedance estimate, and the possible need when analysing real data, for comparing the relative values of $\chi^2$ between model classes rather than using a straight comparison with an absolute value.

Figure 4.3 is a set of curves showing the model class parameters for a 2D model. The regional azimuth and rotation of the impedance tensor were performed using the methods of (Swift, 1967), as expressed in equations 3.13 and 3.14. The tensor derived from rotation will be henceforth referred to as the "Swift" impedance tensor. The convention adopted to resolve the 180° ambiguity in azimuth is that the azimuths lie between -90° and +90°. The skew, as defined by equation (4.23), is expressed as an angle.

$$K = \tan^{-1} \kappa$$

(4.27)

An angle is an appropriate form of expression, since the skew is an approximate estimate of the twist and shear angles (Groom and Bailey, 1989). The noise to signal plot shows, for each element of the measured impedance tensor, the square root of the variance divided by the mean.

Figure 4.4 shows a similar plot for parameters derived using the Groom and Bailey decomposition. The twist angle is shown as squares and the shear angle as crosses. The tensor $Z_r$ defined in equation 4.18 will be henceforth referred to as the decomposed tensor. The local channelling angle shown in the azimuth plot as crosses is defined as

$$\theta_L = \theta + \phi_e + \phi_t$$

(4.28)

The local channelling angle is an estimate of the local current direction (Groom and Bailey, 1989).

A contour plot showing the variation of $\chi^2$ with period for different fixed regional azimuths is shown in figures 4.5. Similar plots showing the variations of the shear and twist angle are shown in figures 4.6 and 4.7. These give an overall view of how these model class parameters vary with frequency.

If the model parameters, other than impedance, such as regional azimuth, shear angle and twist angle are all independent of frequency over a range of periods exceeding half a decade, it is indicative of an earth which closely resembles the approximations made by the model, (Groom and Bailey, 1989). Therefore if the above statement is true for a data set, fixing these parameters will still keep a value of $\chi^2$ of about 3 standard deviations.
Figure 4.2
(a) and (b) Variations with period of the apparent resistivity and impedance phase respectively, calculated from the unrotated off diagonal elements of the impedance tensor measured at site SEW823. Squares are Zxy and crosses Zyx. (c) Period dependency of the chi-squared normalised residual error of fit, as defined in equation 4.21. The error of fit has been calculated between two impedance tensors. Tensor one was the unrotated impedance tensor measured at site SEW823. The second impedance tensor has diagonal elements equal to zero and both off-diagonal elements equal to the Zxy element of tensor one.

Figure 4.3
(a) and (b) Variations with period of the apparent resistivity and impedance phase respectively calculated from the unrotated off diagonal elements of the impedance tensor measured at site SEW823. Tensor rotated into the principal directions using the methods of Swift (1967). Squares are Zxy and crosses Zyx. (c) Period dependency of the chi-squared normalised residual error of fit. The error of fit has been calculated between the impedance tensor rotated using the methods of Swift (1967) and the actual measured impedance tensor. (d) Period dependency of the regional azimuth measured in degrees. (e) Noise to signal ratio against the logarithm of period. Straight crosses Zxx, diamonds Zxy, diagonal crosses Zyx, triangles Zyy. (f) Period dependency of the skew angle as defined in equations 4.23 and 4.27. All period scales are logarithmic.

Figure 4.4
(a) and (b) Variations with period of the apparent resistivity and impedance phase respectively calculated from the unrotated off diagonal elements of the impedance tensor measured at site SEW823. Tensor decomposed using equation 4.19. Squares are Zxy and crosses Zyx. (c) Period dependency of the chi-squared normalised residual error of fit, as defined in equation 4.21. The error of fit has been calculated between the elements of Zr in equation 4.18 (the Groom and Bailey (1989) decomposed tensor) and the actual measured impedance tensor. (d) Period dependency of the regional azimuth (squares) and the local channelling angle (crosses), both measured in degrees. Local channelling angle as defined in equation 4.28. (e) Noise to signal ratio against the logarithm of period. Straight crosses Zxx, diamonds Zxy, diagonal crosses Zyx, triangles Zyy. (f) Period dependency of the shear angle (crosses) and twist angle (squares). Shear angle defined by equation 4.16 and twist angle by equation 4.17. All period scales are logarithmic.
SEW823 Unrotated Data & Error Fit to 1D Model
SEW823 2D Model, Rotated by Swift (1967) Method
SEW823 Unconstrained Groom+Bailey Decomposition
Figure 4.5
Contour Plot of the chi-squared normalised residual error of fit, as defined in equation 4.21 for Data Measured at site SEW823. The error of fit was calculated between elements of \( Z_r \) in equation 4.18 and the actual measured impedance tensor. The tensor was decomposed with a fixed regional azimuth using the methods of Groom and Bailey (1989). The regional azimuth measured in degrees is plotted along the X-axis. The period along the Y-axis increases logarithmically. The period values plotted along the Y-axis have been converted back to real numbers and are given in seconds. Values of residual error of fit at each period for fixed regional azimuths increasing in 3 degree steps from 0 to 90 have been contoured. The key gives the Chi-squared value.

Figure 4.6
Contour Plot of the Shear Angle, as defined in equation 4.15 for Data Measured at site SEW823. The fixed regional azimuth measured in degrees is plotted along the X-axis. The period along the Y-axis increases logarithmically. The period values plotted along the Y-axis have been converted back to real numbers and are given in seconds. Values of the shear angle at each period for fixed regional azimuths increasing in 3 degree steps from 0 to 90 have been contoured. The key gives the values in degrees.

Figure 4.7
Contour Plot of the Twist Angle, as defined in equation 4.16 for Data Measured at site SEW823. The fixed regional azimuth measured in degrees is plotted along the X-axis. The period along the Y-axis increases logarithmically. The period values plotted along the Y-axis have been converted back to real numbers and are given in seconds. Values of the twist angle at each period for fixed regional azimuths increasing in 3 degree steps from 0 to 90 have been contoured. The key gives the values in degrees.
From figure 4.5 it is clear that for data recorded at most periods at site SEW823 a range of regional azimuths fit the data to within 3 standard deviations. This is due to either departures of the data from the model class or more importantly noise producing instability within the calculations of the parameter. However further constraints can be placed on the range of suitable regional azimuths by ensuring that the shear and twist angle remain almost constant for any fixed regional azimuth. These further restrictions have the added advantage that from the evidence of model studies, (Groom and Bailey, 1991) twist and shear are more stable in the presence of noise than the regional azimuth.

Graphical representations of how well two of the model classes fit the four components of the measured impedance tensor are shown for the 2D and Groom-Bailey decomposed impedance tensors in figures 4.8 and 4.9.

It is immediately clear from figures 4.8 and 4.9 that the unconstrained, decomposed tensor fits the data much better than the "Swift" tensor and this is confirmed statistically by the plots of normalised chi-squared residual error shown in figures 4.3 and 4.4. This is unsurprising since the unconstrained Groom and Bailey model contains 7N degrees of freedom (where N is the number of periods), whilst the rotation of the impedance tensor using the methods of Swift (1967) contains 5N degrees of freedom. However except at a few isolated periods the degree of fit of the unconstrained, decomposed tensor indicates that the earth sampled by the measurements taken at SEW823 is two dimensional, though strong galvanic distortion exists. This confirms the analysis made using the methods of Bahr (1991) described in section 4.4.3.2.

Figures such as 4.8 and 4.9 contain evidence indicating the reasons for departures of the data from that predicted by a model class. For instance if the misfit of the model class is greatest for elements $Z_{xx}$ and $Z_{xy}$, it indicates that the response of the model doesn't match the $E_x$ component of the data and 3D induction is taking place. Whereas if the majority of the misfit is centred on components $Z_{xx}$ and $Z_{yx}$, it indicates magnetic galvanic distortion of the component $H_x$. From figure 4.9 it can be seen that the majority of the misfit between the measured data and the decomposed tensor exists in the diagonal components, indicating 3D induction. There is no clear evidence of galvanic distortion of the magnetic field.

From figure 4.5 it is clear that between the periods 3.0 and 10.0 seconds the regional azimuth can be constrained to any angle between 26° and 34° with virtually no increase in the residual error of fit between the decomposed tensor and the measured data, ( lowest $\gamma^2$ achieved at a regional azimuth of 32°). The shear and twist angles vary by less than 1° about their means of -11° and -13.5° respectively. By fixing all three
Figure 4.8
A Graphical Representation of the Fit of the 2D, "Swift rotated" Impedance Tensor To the Measured Impedance Tensor.
The 2D impedance tensor, shown as lines is calculated by rotating the measured impedance tensor into the principle directions using the methods of Swift (1967). The actual impedance tensor is shown as squares or triangles. All values plotted have been scaled by the maximum impedance value either real or imaginary. All period scales are logarithmic.

Figure 4.9
A Graphical Representation of the Fit of the Groom and Bailey Decomposed Impedance Tensor To the Measured Impedance Tensor.
The Groom and Bailey decomposed impedance tensor, shown as lines is $Z_r$ in equation 4.18, after decomposition using the methods of Groom and Bailey (1989). The actual impedance tensor is shown as squares or triangles. All values plotted have been scaled by the maximum impedance value either real or imaginary. All period scales are logarithmic.
SEW823 2D Model, Rotated by Swift (1967) Method
FIT TO SCALED IMPEDANCE ELEMENTS

FIT TO ZXX
FIT TO ZXX

FIT TO ZYX
FIT TO ZYX

FIT TO ZXY
FIT TO ZXY

FIT TO ZXY
FIT TO ZXY

Period (seconds)
Period (seconds)
SEW823 Unconstrained Groom-Bailey Decomposition
FIT TO SCALED IMPEDANCE ELEMENTS

FIT TO ZXX  FIT TO ZXY  FIT TO ZYY

- REAL DATA  - IMAGINARY DATA  - REAL FIT - IMAGINARY FIT

Period (seconds) Period (seconds)
parameters at these mean values the average normalised chi-squared residual error of fit over the period range increases but is still below a value of 10. In comparison the average $\chi^2$ between the Swift tensor and the measured data over the same period range can be seen from figure 4.3 to be about 100. Also the constrained Groom-Bailey model contains less free parameters ($4N+3$, where $N$ is 7 in this case) than the unconstrained Swift model ($5N$). The conclusion is that over this period range a local three dimensional anomaly is strongly distorting the electric field generated by induction in a regional 2D body.

A similar process of analysis was performed on all data measured at SEW823. The resulting model parameters derived are shown in figure 4.10. Parameters were constrained only if the following four conditions were met.

1) The fit of the constrained decomposed tensor to the data was better than the fit to the data of a tensor belonging to a model class containing fewer degrees of freedom. (this condition was met for all data recorded at SEW823.)

2) At each period the $\chi^2$ value was kept preferably below 3 standard deviations, or at least less than double the $\chi^2$ value obtained by comparing the unconstrained decomposed tensor with the measured data.

3) Parameters could be constrained to the same values for data collected over a least half a decade without contravening condition 2.

4) The apparent resistivity and phase curves were smoothly varying over the whole range of periods measured.

Comparing figure 4.10 to figure 4.4, it is clear that constraining parameters has little impact on the apparent resistivity and phase curves. This was not the case at other sites, where constraining a parameter not only reduced the number of degrees of freedom, but also stabilised other parameters, resulting in smoother apparent resistivity and phase curves.

4.6 The Pattern of Parameters Derived from the Groom and Bailey Decomposition.

A similar analysis to that described in section 4.5 was performed with the measurements recorded at each of the 21 stations occupied. This section describes the overall pattern of the parameters derived from the Groom and Bailey decomposition at all the stations and the subsequent conclusions that can be drawn about electromagnetic induction in southwest Cornwall.
Figure 4.10

(a) and (b) Variations with period of the apparent resistivity and impedance phase respectively calculated from the unrotated off diagonal elements of the impedance tensor measured at site SEW823. Tensor decomposed with fixed parameters using equation 4.19. Squares are $Z_{xy}$ and crosses $Z_{yx}$. (c) Period dependency of the chi-squared normalised residual error of fit, as defined in equation 4.21. The error of fit has been calculated between the elements of $Z_r$ in equation 4.18 (the Groom and Bailey (1989) decomposed tensor) and the actual measured impedance tensor. (d) Period dependency of the regional azimuth (squares) and the local channelling angle (crosses), both measured in degrees. Local channelling angle as defined in equation 4.28. (e) Noise to signal ratio against the logarithm of period. Straight crosses $Z_{xx}$, diamonds $Z_{xy}$, diagonal crosses $Z_{yx}$, triangles $Z_{yy}$. (f) Period dependency of the shear angle (crosses) and twist angle (squares). Shear angle defined by equation 4.16 and twist angle by equation 4.17. All period scales are logarithmic.
SEW823 "Best" Constrained G+G Decomposition
4.6.1 Regional Azimuth

From figure 6.2 it is clear that a number of lateral conductivity contrasts will affect the data set. In order of increasing distance from the bulk of the measurement sites, these are:

1) For sites on the Carnmenellis outcrop the contact between the granite and the surrounding Mylor series.

2) The steep slope of the granite batholith edge to the east and south of the Carnmenellis outcrop.

3) The surrounding seas.

The effects of these lateral contrasts not only combine with each other but also with the effects associated with the conductivity depth profile and any conductivity anisotropy which may exist within each structural block. Thus these measurements are likely to yield a complex pattern of regional azimuths.

Figures 4.11 to 4.23 show the regional azimuth averaged over period ranges of half a decade. The thick solid lines are the regional azimuths obtained by decomposing the tensor using the methods of Groom and Bailey (1989) and the dotted thin lines are the regional azimuths obtained by using the methods of Swift (1967). Henceforth in this section, when reference is made to a regional azimuth, unless stated, it is the parameter derived from an unconstrained Groom and Bailey decomposition.

Two general points arise from figures 4.11 to 4.23.

1) Assuming "static shift", as defined in section 4.4.2, is negligible, the shorter the minor line, the more anisotropic the measured inductive response.

2) The difference in angle between the regional azimuth obtained by the two methods is an indicator of the degree of galvanic distortion present at a site.

At very short periods, where none of the large scale lateral features discussed above influence it, the regional azimuth will depend on the topography or any electrical anisotropy within the top few kilometres of the granite.

One such source of electrical anisotropy is the macro water-filled crack system which exists within the granite. The latest published survey of in-situ stress measurements within the rock of the Carnmenellis granite, (Pine et. al,1990) states the direction of greatest horizontal stress to be N143°E-N37°W, with a standard deviation of 12°. All angles quoted from true north. This is calculated from measurements all in the northern half of the granite outcrop, down to depths of up to 3 km. Such
Figure 4.11 to Figure 4.23
Average Regional Azimuths.

Dotted lines are regional Azimuths found using equation 3.13 following the methods of Swift (1967). Solid lines are azimuths found using equations 4.19 following the methods of Groom and Bailey (1989). The azimuths are averaged over the period range stated in the title of each figure. The longest line is the average regional azimuth. Lines in the direction of the regional azimuth or alternatively in the direction of the major apparent resistivity are the same length at all periods, for all sites. However, the line at right angles to the regional azimuth (the minor line) is scaled by the ratio of the minor to major apparent resistivities. The spread of azimuths measured at a particular site over a period range is shown by two shorter lines joined by a circular segment. This range includes the errors on the parameter.

The coast is outlined in dots.

Granite outcrops are outlined in dots and dashes.
II: AVERAGE REGIONAL AZIMUTH PLUS ERROR SPREAD CALCULATED FOR PERIODS BETWEEN 0.001 AND 0.003 SECS

II: AVERAGE REGIONAL AZIMUTH PLUS ERROR SPREAD CALCULATED FOR PERIODS BETWEEN 0.003 AND 0.01 SECS

II: AVERAGE REGIONAL AZIMUTH PLUS ERROR SPREAD CALCULATED FOR PERIODS BETWEEN 0.01 AND 0.03 SECS

II: AVERAGE REGIONAL AZIMUTH PLUS ERROR SPREAD CALCULATED FOR PERIODS BETWEEN 0.03 AND 0.1 SECS
FIG 4.15: AVERAGE REGIONAL AZIMUTH PLUS ERROR SPREAD CALCULATED FOR PERIODS BETWEEN 0.1 AND 0.3 SECS

FIG 4.16: AVERAGE REGIONAL AZIMUTH PLUS ERROR SPREAD CALCULATED FOR PERIODS BETWEEN 0.3 AND 1.0 SECS

FIG 4.17: AVERAGE REGIONAL AZIMUTH PLUS ERROR SPREAD CALCULATED FOR PERIODS BETWEEN 1.0 AND 3.0 SECS

FIG 4.18: AVERAGE REGIONAL AZIMUTH PLUS ERROR SPREAD CALCULATED FOR PERIODS BETWEEN 3.0 AND 10.0 SECS
Fig 4.10: Average regional azimuth plus error spread calculated for periods between 10.0 and 30.0 secs.

Fig 4.20: Average regional azimuth plus error spread calculated for periods between 30.0 and 100.0 secs.

Fig 4.21: Average regional azimuth plus error spread calculated for periods between 100.0 and 300.0 secs.

Fig 4.22: Average regional azimuth plus error spread calculated for periods between 300.0 and 1000.0 secs.
FIG 4.23: AVERAGE REGIONAL AZIMUTH, PLUS ERROR SPREAD CALCULATED FOR PERIODS BETWEEN 1000 AND 3000 SECS
a stress system would produce cracks running at N53°E. A study of natural fractures and joint sets of exposed rock at the surface and down mine shafts on the Carnmenellis outcrop, (Whittle and McCartney, 1989) indicated there were essentially two sub-vertical joint sets with strikes varying little from N30°W and N70°E. A series of Cross Courses, large cracks up to 1m wide running for up to a few km and occasionally not closing until depths of 3 km, cross the granite. See figure 1.9. These Cross courses along with fault lines can be split into two sets depending on their strike direction. One set have strike directions between N40°W and N20°W, the other set have strike directions between N50°E and N70°E. Measurements from a network of 3 component seismographs placed around Rosemanowes Quarry (site 832), indicate alignment of shear wave polarisations in the direction N 30°W, for a rock mass down to 2km, (Roberts and Crampin, 1986).

From figure 4.12 it is clear a number of sites both confirm and contradict the above evidence for the orientation of cracks. Obviously only periods at which the effects of the surrounding rocks are negligible can be considered, thus ruling out all measurements at sites 834 and 827. Sites 814, 823, 825, 830, 831 and 832 all have regional azimuths between N56°E and N70°E or between N20°W and N30°W for periods with skin depths less than 1.5 km, thus suggesting that induced currents pass along water filled cracks within the granite. Since all the geological evidence points to two sets of cracks running at right angles to each other, a large difference between the major and minor apparent resistivity curves would not be expected and evidence from these sites supports this hypothesis. However a group of four sites in the south west corner of the granite outcrop all have regional azimuths approximately the same, between N25°E and N35°E, over the period range 0.003 to 0.03 seconds, see figures 4.12 and 4.13. Such regional azimuths bear no relationship to the topography surrounding each site, or the major axes of possible conductors in the area, such as alluvium filled valleys or mineral veins. For half of this period range the skin depths are such that the effects of the surrounding rocks are negligible. Therefore it could be concluded that the crack system in the south-west of the granite is orientated in a different direction, though other factors connected with the conductivity depth profile need to be investigated before this can become a firm conclusion.

Evidence contained in figures 4.11 to 4.14 shows that at thirteen out of the fifteen sites placed on the Carnmenellis outcrop, there is a marked change in the regional azimuth at periods where the skin depth is such that the edge of the granite outcrop is affecting the measurements. Also at these periods the major and minor apparent resistivity curves start to divert. Obviously at these periods the site is on the resistive side of a two dimensional contact and the regional azimuth of the major apparent resistivity will be perpendicular to this contact. However since the
outcrop is very roughly circular, this produces a different azimuth at each site. In addition, the period at which the change in azimuth takes place will vary from site to site, being mainly dependant on the distance from the measurement site to the outcrop edge. This will obviously produce a complex pattern of regional azimuths. Examples of changes in regional azimuth described in the above paragraph are:

1) A contact running at an angle of N76°W is 4.1 km from site GAR824. At 0.1s where the skin depth is roughly 4.2 km the regional azimuth changes from N1°W to N14°E.

2) From figures 4.12, 4.13 and 4.14, the regional azimuth at site PER834 is consistently perpendicular to the granite outcrop edge from the shortest period measured (due to the close proximity of the site to the granite-Kilas contact) to 0.06 seconds. Also throughout this period range there is a high degree of anisotropy between the major and minor apparent resistivity curves measured at this site. The consistency of the regional azimuth over a relatively large period range is due to the contact being almost straight for nearly 5 km.

For periods longer than 0.1 seconds the structures at depth have increasing influence on the regional azimuth. Hence, geological interpretations of the regional azimuth pattern for these periods will be given in the concluding chapter 7. In this chapter the pattern of parameters will be described. From figures 4.15 and 4.16 a pattern of regional azimuths exists for sites upon the Carnmenellis outcrop for the period range 0.1 to 1 second with features as follows:

1) Regional azimuths between N40°E and N50°E in the north east corner of the granite outcrop.

2) Regional azimuths between N5°E and N10°E for sites near the southern edge of the outcrop.

3) Regional azimuths in between these two extremes for sites in between these two areas. The closer the site is to the north east corner of the granite outcrop, the greater the regional azimuth at that site.

4) The influence of galvanic electric fields is strong for the four sites close to the southern edge of the granite. By taking this into account and decomposing the impedance tensor, the magnitudes of the two principal impedances are brought together. This is a good reason for applying the full decomposition, since the shape of the apparent resistivity curves will obviously influence any 2D models.

5) The responses are almost isotropic for sites 812, 813, 814, 824, 826, 832 and 833 in the middle of the granite.
6) Results shown in table 4.1 indicate that sites 812, 813, 824 and 832 are only weakly distorted by 3D inhomogeneities. This is graphically expressed in figures 4.15 and 4.16 by the "Swift" and decomposed regional azimuths being almost coincident. However more detailed analysis indicates that very weak galvanic distortion is present over this period range at these sites.

7) Sites at the northern edge of the granite outcrop have a quite anisotropic response.

This pattern develops between 1 and 10s (figures 4.17 and 4.18). At all sites a split between the principal regional apparent resistivities either develops or continues to widen. Regional azimuths at sites towards the north east corner of the granite outcrop swing towards the north and the regional azimuths at sites nearer the southern edge of the outcrop swing towards the east. Thus sites on lines 82 and 83 form the pattern of lying on radii emanating from a point in the sea off the west coast of the Lizard where the sea floor starts to dip steeply.

Between 10 and 100 seconds (figures 4.19 and 4.20) the regional azimuths at all sites swing to the north, but by different angles ensuring that all azimuths are aligned to within 8° about due north. Regional azimuths found using the methods of Swift (1967) and Groom and Bailey (1989) are no longer coincident at sites in the middle of the granite, indicating that structures - that at shorter periods had an inductive response - are acting as galvanic scatterers at these longer periods.

For the two sites where long period measurements were successfully taken, the regional azimuths rotate to angles between N20°E and N30°E in the period range 100-200 seconds (figure 4.21). The regional azimuths continue to swing clockwise in the period range 300 -1000 seconds to angles of about N40°E (figure 4.22). At longer periods, (figure 4.23) the regional azimuth at site 824 continues to rotate clockwise to angles of N60°E but the regional azimuth at site 820 stays constant at N40°E.

4.6.2 The Local Channelling Angle

The local channelling angle as defined in equation 4.28 is an estimation of the direction of current in the local structure. Except for sites on the southern edge of the granite (sites 811, 823 and 830), at periods below 0.1 seconds, the local channelling angle has virtually the same strike direction as the regional azimuth. The twist and shear angles at these sites over this period range are small in magnitude, thus confirming the evidence in table 4.1 that only weak galvanic scattering is taking place.

This is illustrated in figure 4.24, which shows the comparison
Figure 4.24 to Figure 4.31
Comparison of Local Channelling Angles and Regional Azimuths. In figures 4.24, 4.25, 4.28, 4.29, 4.30, and 4.31 the dotted lines are regional azimuths found using equations 4.19 following the methods of Groom and Bailey (1989). In figures 4.26 and 4.27 the dotted lines are azimuths found using equation 3.13 following the methods of Swift (1967). In all figures the solid lines are the local channelling angle as defined by equation 4.28 and calculated using equations 4.19 following the methods of Groom and Bailey (1989). The azimuths are averaged over the period range stated in the title of each figure. The longest line is the average regional azimuth. Lines in the direction of the regional azimuth are the same length at all periods, for all sites. The spread of azimuths measured at a particular site over a period range is shown by two shorter lines joined by a circular segment. This range includes the errors on the parameter.

The coast is outlined in dots.
Granite outcrops are outlined in dots and dashes.
4.24: LOCAL CHANNELLING ANGLE x 3D REG. AZIMUTH CALCULATED FOR PERIODS BETWEEN 0.1 AND 0.3 SECS  

4.25: LOCAL CHANNELLING ANGLE x 3D REG. AZIMUTH CALCULATED FOR PERIODS BETWEEN 0.3 AND 1.0 SECS  

4.26: LOCAL CHANNELLING ANGLE x 3D REG. AZIMUTH CALCULATED FOR PERIODS BETWEEN 1.0 AND 3.0 SECS  

4.27: LOCAL CHANNELLING ANGLE x 3D REG. AZIMUTH CALCULATED FOR PERIODS BETWEEN 3.0 AND 10.0 SECS
FIG 4.28: LOCAL CHANNELLING ANGLE ±3D REG. AZIMUTH CALCULATED FOR PERIODS BETWEEN 10.0 AND 30.0 SECS

FIG 4.29: LOCAL CHANNELLING ANGLE ±3D REG. AZIMUTH CALCULATED FOR PERIODS BETWEEN 30.0 AND 100.0 SECS

FIG 4.30: LOCAL CHANNELLING ANGLE ±3D REG. AZIMUTH CALCULATED FOR PERIODS BETWEEN 100.0 AND 300.0 SECS

FIG 4.31: LOCAL CHANNELLING ANGLE ±3D REG. AZIMUTH CALCULATED FOR PERIODS BETWEEN 300.0 AND 1000 SECS
between the local channelling angle and the regional azimuth, both obtained from the unconstrained, decomposed tensor at 0.1 to 0.3 seconds, at the edge of this period range, where this pattern is on the point of changing.

It is interesting to note from figures 4.26 and 4.27, that in the decade of data measured between 1 and 10 seconds, the azimuth calculated using the methods of Swift (1967) is indicating the direction of currents in the local body rather than the regional body. The difference between the local channelling angle and the regional azimuth calculated from the decomposed tensor over this period at most sites on the granite is 10° to 20°. This phenomenon has been noted in model studies of a semi-circular local inclusion in a regional structure. Similar results were obtained for impedance tensors calculated on the edge of the local inclusion, (Groom and Bailey, 1991).

From a comparison of local channelling angle to regional azimuth at site GAR824, in the centre of the granite outcrop, in figures 4.24 and 4.25 it is clear that galvanic scattering is only weak and local currents flow in the same direction as the regional azimuth. However at longer periods, (figures 4.28 to 4.31), as the regional azimuth begins to rotate, the local channelling angle stays constant. In fact it is virtually constant throughout the period range 0.1 to 1000 seconds. For periods above 10 seconds the amount of twist and shear starts to rise indicating strong galvanic scattering of the induced fields (table 4.1). All this evidence indicates that a regional 2D structure (in this case part of the granite batholith) becomes at longer periods a local galvanic scatterer of induced currents generated in other regional bodies and continues to distort the impedance tensor for at least two decades.

4.7 Strategy for Subsequent Modelling and Interpretation of the Measured Impedance Tensors.

For virtually all the data measured at all the sites, the chi-squared residual error of fit between the constrained decomposed tensor and the measured data was smaller than that between the Swift tensor and the measured data. Also in most cases the decomposed tensor model class contained fewer degrees of freedom than the conventional 2D model class. Hence the impedance tensors calculated from the "best" constrained decomposed model class were used for subsequent modelling. The "best" constrained tensor is defined as that calculated following the procedure outlined in section 4.5.

The most obvious conclusion to be drawn from the analysis above is the need for two dimensional modelling in order to gain realistic conductivity depth models. However if 1D modelling is to be performed, the invariant used, should be the Berdichevsky average (equation 3.16) of the principal impedances after the tensor has been decomposed using the methods of Groom and Bailey (1989). In this way the galvanic scattering of
the induced electric fields will have been eliminated.

In order to perform two dimensional modelling the principal impedances need to be obtained by rotating the measured impedance by an angle defined here as the regional azimuth. A fixed angle of N19°E was chosen for the following reasons.

1) This is the regional azimuth at sites in the centre of the Carnmenellis outcrop over the period range 0.1 to 10 seconds. These sites are furthest away from the coast line, and hence the least likely to be affected by current channelling in the seas.

2) The data measured over this period range are the most significant collected. The skin depths are such that the data will include information about the granite batholith and the lower crust. Therefore it is vital that the data over this period are rotated into the correct orientation.

3) For reasons stated in section 4.6 the regional azimuth pattern at short periods is complicated, thus ensuring the need for modelling data rotated using a fixed regional azimuth. However, it must be noted that parts of any model obtained using data where the unconstrained regional azimuth differs greatly from the fixed azimuth, should be treated with caution.

4) Evidence for the fact that the regional azimuth N19°E is a facet of a real structure below the centre of the Carnmenellis outcrop comes from the comparison of local channelling angles with the regional azimuth at sites in the centre of the granite, outlined in section 4.6.2. There is good evidence that at long periods when the structure with a regional azimuth N19°E becomes inductively weak, galvanic currents are still flowing in the same direction.

As well as modelling an invariant, 1D models of the principal impedances, the E-pol and H-pol, of the decomposed tensor rotated to the preferred regional azimuth, will provide the best starting models for two dimensional modelling.

4.8 Assessing The "Static Shift" Factor For Soundings in the Carnmenellis Data set

By applying the Groom-Bailey decomposition, some of the effects of galvanic distortion can be separated from the measured impedance tensor. However as stated in section 4.4.2, a linear scaling factor produced by electric charges, cannot be independently calculated from the regional impedances. This so called "static shift" factor, leaves unchanged the principal impedance phases and the shape of the apparent resistivity curves, but multiplies the two apparent resistivities by different factors.
Finding possible "static shifts" involves comparing the sounding curves from nearby sites, or comparing the apparent resistivity levels with measurements using other methods. Both of these methods proved difficult in this case.

The only independent measure of the resistivity of the granite is from borehole logs down drill holes at the HDR site, (Pearson et al., 1989). The borehole logs are so spikey, that estimating a bulk resistivity for the granite with sufficient accuracy to correct for shifts in the M.T. sounding curves was impossible.

A comparison of the sounding curves from adjacent sites also proved difficult. The shape of the sounding curves at most sites differed from those at all the other sites indicating that the inductive structures sensed at each sounding were different. This is not surprising, since the shape of the batholith has a complicated geometry and the survey was conducted on a regional scale, with site spacings of at least a few km. However a comparison of sites in the centre of the granite outcrop (sites 813, 814, 824, 825 and 826) could be performed. Figure 4.32 shows the principal impedances of a decomposed tensor with a regional azimuth constrained to the preferred strike of N19°E. Very similar phase curves exist at all the central granites sites, as well as which, there are great similarities in the shape of the apparent resistivity curves. However the level of the apparent resistivity curves at sites 813 and 825 differs from those recorded at the other sites. The levels at 825 can be accounted for by a high degree of splitting of the two impedances. But from figure 4.32 it is clear that the apparent resistivity levels for both principal impedances at site 813 have been "shifted up", compared to the soundings at other sites. The distance between the three sites which have identical curves is greater than the distance between 824 and 813. It must be stressed that "Static shift" factors may apply to the other soundings away from the central section of the Carnmenellis outcrop, but the existence of possible shifts cannot be ascertained from comparisons within the data set.

Possible methods for removing "static shifts" involve further measurements, either ElectroMagnetic Array Profiling (EMAP), (Bostick, 1986; Word et. al. 1986) or controlled source surveys measuring only the magnetic fields, (Sternberg et. al., 1985). Such measurements are therefore recommended to be included in any possible further work. Another possible method for correcting "static shifts", is to derive an average regional resistivity curve and match this to a global curve using one of a number of methods, (Vanyan et. al., 1983). However this assumes the earth is one dimensional, which in this case, from the analysis contained in this chapter, it clearly isn't. In addition, to take an average of curves of diverging shapes will obviously introduce unacceptable errors.
Figure 4.32
Apparent Resistivity and Impedance Phase Curves for Sites Close to the Centre of the Carnmenellis Granite Outcrop

Curves calculated from impedance tensors decomposed using the methods of Groom and Bailey (1989). Decomposition parameters constrained to meet the conditions set out in section 4.5.
fig 4.32: Curves Calculated From Decomposed Tensor
Chapter 5

One and Two Dimensional Resistivity-Depth Modelling

5.1 Introduction

The two previous chapters describe how the magnetotelluric measurements were made and how the transform functions were calculated. The interpretation of these functions in terms of the earth's physical parameters can be greatly facilitated by their transference into resistivity depth models.

This can be done in two ways - either by comparing the theoretical response of a model to the measured data, referred to here as forward modelling, or by inversion, the direct retrieval of conductivity structure from the data. Both methods have been used in this study.

The limited bandwidth of measurements, coupled with observation errors, biasing effects, both due to cultural noise or the influence of resistivity contrasts outside the measurement area, along with the "smearing out" of sharp conductivity contrasts or thin layers by the diffusing e.m. energy, ensures that any one of an infinite set of conductivity structures will satisfy a finite set of M.T. data. This property of non-uniqueness forms the major problem in the interpretation of resistivity depth models derived from M.T. data. A further problem is posed by the non-linear nature of the M.T. inversion process.

This chapter briefly recounts the theoretical background to the various forward and inverse 1D and 2D modelling schemes used in this study. It proceeds to describe how the overall properties of the measured earth response functions influenced the modelling methodology adopted. Then 1D and 2D models of the conductivity structure beneath the Carnmenellis granite are presented along with a discussion of the possible errors and the extent of the non-uniqueness present in the models.

5.2 One Dimensional Modelling

A great variety of 1D modelling and inversion schemes exist, (Niblett and Sayn-Wittgenstein, 1960; Wu, 1968; Jupp and Vozoff, 1975; Jones and Hutton, 1979; Parker, 1980; Parker and Whaler, 1981; Fischer et al., 1981; Hobbs, 1982; Constable et al., 1982) By comparing the resulting models from a number of these methods, the extent of the possible model space for any data set can be ascertained and in this way the non-uniqueness assessed, (Oldenburg et. al., 1984). Therefore a number of 1D modelling schemes have been used in this study.

This section only presents a brief outline of the theory behind the schemes used. The reasons for using these particular algorithms on the
Carnmenellis data set will be presented in later sections of this chapter.

5.2.1 The Niblett-Bostick Transformation

The Niblett-Bostick transformation takes the values of apparent resistivity at each period and calculates a penetration depth \( h \) at period \( T \) using the following simple formula, (Niblett and Sayn-Wittgenstein, 1960; Bostick, 1977; Jones, 1983b).

\[
h = \sqrt{\frac{p_a(T) \cdot T}{2\pi \mu_0}}
\]  

An expression for the Niblett-Bostick resistivity \( \rho^* \) at depth \( h \), which incorporates impedance phase \( \Phi \) was suggested by Weidelt et al. (1980) and has been used in this study.

\[
\rho^*(h) = \rho_a(T) \left( \frac{\pi}{2\Phi(T)} - 1 \right)
\]  

5.2.2 The D+ Solution to the 1D Inverse Problem

In investigating the existence and construction of solutions to the inverse problem of electromagnetic induction in a 1D earth, Parker (1980) found that the admittance \( Y \) at a frequency \( \omega \) for a very large space of conductivity functions can be written as

\[
Y(\omega) = b_0 + \int_0^\infty \frac{\Omega^2 - i\omega \lambda}{\lambda + i\omega} db(\lambda)
\]  

where \( b \) is a real, bounded non decreasing function and \( \Omega \) is an arbitrary real constant introduced to retain consistent dimensions. Parker proves that the optimal, or best fitting solution, exists in a subset of models \( D^+ \), in which the conductivity \( \sigma \) consists of a finite comb of positive delta functions. In this case equation (5.3) can be written as

\[
Y(\omega) = a_0 + \int_0^\infty \frac{da(\lambda)}{\lambda + i\omega}
\]  

The details of how this equation is applied in practice to a finite data set are given in Parker and Whaler (1981).

Consider a set of L data \( d_1, d_2, \ldots, d_L \), each associated with an error estimate \( \sigma_j \). Forward modelling can predict these data from a discreet
model via a set of functionals $F_j[m]$. The weighted least-squares fit of these model predictions to the data is given by

$$\chi^2 = \sum_{j=1}^{L} \frac{(d_j - F_j[m])^2}{\sigma_j^2}$$

(5.5)

Therefore for a given data set, the $D^+$ model will have the lowest $\chi^2$ misfit. If this misfit is greater than an acceptable value, the conclusion must be drawn that no satisfactory solutions to the inverse problem exist, under the assumption of a 1D earth, (Parker and Whaler, 1981). An acceptable misfit $\chi^2$ is deemed to be when

$$\chi^2 < L + 2\sqrt{2L}$$

(5.6)

(Parker and Whaler, 1981).

The $D^+$ model can be used to find the limiting depth, below which nothing can be ascertained about the conductivity structure, (Parker, 1982). Parker (1982) uses the model of a thin perfectly conducting layer at a depth (H), and finds the smallest H in which the model $\chi^2$ misfit meets the condition in equation (5.6).

### 5.2.3 One Dimensional 'Occam' Inversion

One solution to the problem of non uniqueness associated with magnetotelluric inversion is to place no restrictions on the range of possible values for model parameters, but to restrict the type of model, by ensuring that the model possesses certain characteristics, chosen a priori. In the case of magnetotellurics, where structure tends to be smoothed out, the suppression of complex structure by requiring the smoothest possible model is a sensible constraint. This idea is the basis for the inversion scheme developed by Constable, Parker and Constable (1987).

They propose finding a model which is maximally smooth, but maintains an acceptable level of misfit with the data. In order to explain the idea of smoothness they introduce the converse quantity, roughness. Roughness is defined as the integrated square of the first or second derivative of $m(z)$ with respect to depth ($z$). In the case of magnetotellurics, $m(z)$ is the resistivity or log resistivity.

$$R_1 = \int \left(\frac{dm}{dz}\right)^2 dz$$

(5.7)
\[ R_2 = \int \left( \frac{d^2 m}{dz^2} \right)^2 dz \]  

(5.8)

In the program the smoothly varying function in equations 5.7 and 5.8 is replaced by a piecewise smooth function.

\[ m(z) = m_i, \quad z_{i-1} < z \leq z_i, \quad i = 1, 2, \ldots, N \]  

(5.9)

where \( N \) is large, so that

\[ R_1 = \sum_{i=2}^{N} (m_i - m_{i-1})^2 \]  

(5.10)

\[ R_2 = \sum_{i=2}^{N-1} (m_{i+1} - 2m_i + m_{i-1})^2 \]  

(5.11)

Due to the loss of resolution with depth, the layers increase in size logarithmically and are terminated with a half space.

If \( \chi^2 \) is an acceptable misfit given by equation 5.5, the constraint on the model is given by

\[ \chi^2 - \sum_{j=1}^{L} \frac{(d_j - F_j[m])^2}{\sigma_j^2} = 0 \]  

(5.12)

The optimisation of the model is essentially minimising a functional subject to a constraint. Constable et al. (1987) do this by using Lagrange multipliers. The constraint is multiplied by a parameter, the Lagrange multiplier (\( P^{-1} \)), following the notation of Constable et al., (1987)) and then added to the functional to be minimised. This gives the following equation if \( R_1 \) is used.

\[ U = \sum_{i=2}^{N} [(m_i - m_{i-1})^2] + \mu^{-1} \left[ \sum_{j=1}^{L} \frac{(d_j - F_j[m])^2}{\sigma_j^2} - \chi^2 \right] \]  

(5.13)

Minimising equation 5.13 produces a non-linear equation which is solved iteratively, see Constable et al. (1987) for details.

An ideal model should have the lowest possible misfit between its predicted values and the data whilst keeping a physically realistic model. \( \chi^2 \) will never be zero due to errors in the data, and departures of the data from the ideal 1D case. However the lowest achievable value of \( \chi^2 \) is the
D+ solution described in section 5.2.2. Unfortunately this model is very rough. Constable et. al. (1987) show that for models with misfits close to $\chi^2_{\text{min}}$, the lowest achievable value, a small improvement in fit produces a large increase in roughness. Hence a value of $\chi^2$ 1.4 times the misfit of the D+ solution, was used. The algorithm iterates from a half space. An alternative measure of the model misfit to the data, which will be used in this chapter is the rms misfit given by

$$\text{rms} = \sqrt{\frac{\chi^2}{L}}$$  \hspace{1cm} (5.14)

5.2.4 The Analytical One Dimensional Inversion Scheme of Fischer et al..

This scheme developed by Fischer et al. (1981) is based on the fact that at a given period, the measured surface impedance is influenced only by structure above the maximum penetration depth. Thus at the shortest periods the observed response can be explained with a simple two layer model. By a similar analysis of successively longer periods subsequent layers at progressively greater depths are introduced. The main problems of such a scheme are divergence of the inversion process and the suppression of extraneous structure.

The surface impedance $Z$ is related to the apparent resistivity ($\rho_a$) and phase ($\phi$) in a 1D earth by:

$$Z(T) = \sqrt{i \omega \mu_0 \rho_a(T)} e^{-i \phi(T)}$$ \hspace{1cm} (5.15)

where
- $T$ = period
- $\mu_0$ = magnetic permeability of free space
- $\omega$ = the angular frequency of the source field

The measured impedance $Z(T_2)$ at period ($T_2$) measured at the top of a two layer earth is given by

$$Z(T_2) = z_1 \left( \frac{z_2 + z_1 + (z_2 - z_1) e^{-\gamma_1}}{z_2 + z_1 - (z_2 - z_1) e^{-\gamma_1}} \right)$$ \hspace{1cm} (5.16)

where the impedances $z_1$ or $z_2$ refer to impedances that would be observed at the top of a medium of specific resistivity $\rho_1$ or $\rho_2$ and are given by

$$z_k = \sqrt{i \omega \mu_0 \rho_k}$$ \hspace{1cm} (5.17)

and ($h_k$) depends on the thickness of the layer $k$ ($h_k$) and is given by
\[ \gamma_k = (1 + i) \beta_k = (1 + i) h_k \sqrt{\frac{4 \pi \mu_0}{\rho_k T}} \]  

(5.18)

How the expressions in equation (5.16) are related to the apparent resistivity and phase curves is given in Fischer et. al (1981). For subsequent layers a similar equation to 5.16 for the impedance measured between layers 1 and 2 replaces the quantity \( Z_2 \) in equation 5.16 and thus a recursive scheme builds up when shifting to longer and longer periods introducing a new layer for each period (see Fischer et al. (1981) for details).

To ensure that for a given period the e.m. wave is penetrating deep enough into a layer to warrant the deduction of information about it, one of two conditions must be met or the data are skipped. The skin depth at a period \( j+1 \) given by

\[ \delta_{j+1} = \sqrt{\frac{\rho_{j+1} T_{j+1}}{\pi \mu_0}} \]  

(5.19)

must meet the condition.

\[ \delta_{j+1} > 2 h_j \]  

(5.20)

Alternatively

\[ \delta_a(T_{j+1}) = \sqrt{\frac{\rho_a(T_{j+1}) T_{j+1}}{\pi \mu_0}} > \sum_{k=1}^i h_k \]  

(5.21)

This 1D modelling scheme is quick and was used as a starting model and then refined using the modelling scheme described in the next section.

5.2.5 A 1D Modelling Scheme Which Minimises The Standard Deviation Between the Response of the Model and the Measured Data

The standard deviation between the response of a model and the measured data is a useful test in the 1D situation for the degree of "modelling success". In their paper Fischer et al. (1981) suggest a definition for the standard deviation which takes account of the logarithmic nature of magnetotelluric depth sounding and gives equal emphasis to amplitude and phase responses. The standard deviation between the measured impedances and those calculated from any model is given by Fischer et al. (1981) as

\[ \varepsilon = \frac{1}{\sqrt{2}} \left[ \frac{1}{4N} \sum_{i=1}^N w_{pi} \left( \ln \frac{\rho_{\text{obs}}(T_i)}{\rho_{\text{mod}}(T_i)} \right)^2 + \frac{1}{N} \sum_{i=1}^N w_{\phi i} \left( \phi_{\text{c}}(T_i) - \phi_{\text{a}}(T_i) \right)^2 \right]^{1/2} \]  

(5.22)
where the indices \( m \) and \( c \) refer to measured and calculated respectively. The weights \( w_{pi} \) and \( w_{\bar{O}i} \) are defined as

\[
w_{pi} = \frac{N \left[ \ln \rho_a^{\text{max}}(T_i) - \ln \rho_a^{\text{min}}(T_i) \right]}{\sum_{i=1}^{N} \left[ 1 / (\ln \rho_a^{\text{max}}(T_i) - \ln \rho_a^{\text{min}}(T_i)) \right] > 0}
\]

(5.23)

\[
w_{\bar{O}i} = \frac{N \left[ \bar{\Omega}^{\text{max}}(T_i) - \bar{\Omega}^{\text{min}}(T_i) \right]}{\sum_{i=1}^{N} \left[ 1 / (\bar{\Omega}^{\text{max}}(T_i) - \bar{\Omega}^{\text{min}}(T_i)) \right] > 0}
\]

(5.24)

where \( \rho_a^{\text{min}} \) to \( \rho_a^{\text{max}} \) and \( \bar{\Omega}^{\text{min}} \) to \( \bar{\Omega}^{\text{max}} \) are the confidence ranges of the measured phase and apparent resistivity respectively.

In studying the topography of the standard deviation of a one-dimensional model as the parameters of the model were changed, Fischer and Le Quang (1981) found that \( E \) had a global minimum isolated from local minima. Although the "valley floor" around the absolute minimum can be very flat and long, the minimisation routine is relatively simple, since it doesn't have to jump out of local minima. It was also found that the same final model was obtained from quite different starting models, (Fischer and Le Quang, 1981).

In order to find the model with the lowest \( E \), either a vast range of models needs to be studied or else some restrictions placed on the model space. The number of layers in a model is specified, as well as the ranges of permissible values for each of the model parameters. Although the range can be large enough to cover any realistically possible value, it can still produce a model in a short time. The standard deviation can be calculated for either the phase only, or the apparent resistivity only, (Fischer et al., 1981). Therefore a model which fits only one of the curves can be calculated.

5.2.6 The Damped Most-Squares Inversion Scheme

The non-uniqueness of a data set can be assessed using an extremal iterative or damped most-squares inversion scheme, (Jackson, 1976). The application of such methods to M.T. data has been described by Meju (1988), and Meju and Hutton (1992). Given an optimal solution to the inverse problem of finding the model parameters \( (m_i) \) with a chi-squared misfit of \( \chi^2_0 \), a set of solutions can also be found with a maximum tolerable misfit of \( \chi^2_i \) \( (\chi^2_i > \chi^2_0) \). This is equivalent to extremising the function

\[
M^Tb
\]

(5.25)
under the constraint (using the same notation used in section 5.2.3)

\[
\chi_i^2 = k \sum_{i=2}^{N} [(m_i - m_{i-1})^2] + \left[ \sum_{j=1}^{k} \frac{(d_j - F_j[m])^2}{\sigma_j^2} \right]
\]

(5.26)

where \( M^T \) is the transpose of \( M \), a vector of the model parameters \( m_i \), \( b \) is a vector of zeros except for the \( k \)th element (to be maximised) equal to 1. Therefore \( b^T = (0, \ldots, 0, 1^k, 0, 0) \).

\( k \) is a constant used to stabilise the iterative solution process, (see Meju, 1988) for details). In the interests of simplifying the mathematics the damping factor \( k \sum_{i=2}^{N} [(m_i - m_{i-1})^2] \) is neglected and \( \sigma_j \) \( j=1,L \) are assumed to be 1.

Jackson (1976) and Meju (1988) use a Lagrange multiplier (\( \mu \)) to obtain the most-squares solution.

\[
m_i = \left[ F^T F \right]^{-1} \left[ F^T D - \mu b \right]
\]

(5.27)

where

- \( F \) = a set of functionals \( F_j[m] \) written in matrix form
- \( D \) = a vector containing the data set \( d_1, d_2, \ldots, d_L \)

By substituting 5.27 into 5.26, it can be shown (Meju, 1988) that there are two solutions of \( \mu \) for each model parameter (if \( \mu \) is not equal to 0, the least squares solution) enclosing an envelope of solutions which are maximally consistent with the data. If the errors on the data are assumed to be univariant and uncorrelated \( \chi_i^2 = p \) the number of data points.

5.3 Two Dimensional Modelling Theory

The scheme used to obtain 2D resistivity sections across the Carnmenellis granite was a forward modelling algorithm which uses an analogy with electrical transmission surfaces. A two dimensional inversion using the Occam algorithm was attempted but proved to be unsuccessful due to the large lateral conductivity contrasts.

5.3.1 The Transmission Surface analogy to Perturbations of the Electromagnetic Field in a Two Dimensional Body

There is great similarity between Maxwell’s equations in two dimensions and the relationship between current and voltage in a transmission line or surface. The theory behind transmission surface
analogous, two dimensional, electromagnetic, modelling is contained in Madden and Thompson, (1965); Madden and Swift, (1969); and Swift, (1971).

In a transmission surface, voltage (V) and current (I) along the line are related to each other by the equations:

$$\frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} = -YV$$

(5.28a)

$$\frac{\partial V}{\partial z} = Z I_z$$

(5.28b)

$$\frac{\partial V}{\partial y} = Z I_y$$

(5.28c)

where

\[ Z = \text{distributed impedance per unit length} \]

\[ Y = \text{distributed admittance per unit length} \]

The analogy is clear, if equations 5.28 are compared with Maxwells equations in two dimensions (equations 2.34 and 2.35 reproduced here as equations 5.29 and 5.30)

\[ \begin{align*}
  \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= \sigma E_x \\
  \frac{\partial E_x}{\partial z} &= i \omega \mu H_y \\
  \frac{\partial E_x}{\partial y} &= -i \omega \mu H_z \\
  \frac{\partial H_x}{\partial y} &= \sigma E_y \\
  \frac{\partial H_x}{\partial z} &= -\sigma E_z
\end{align*} \]

(5.29)

(5.30)

If the principles of energy conservation are maintained we can associate V with \( E_x \) (or \( H_x \)), \(-I_z\) with \(-H_y\) (or \( E_y \)), \(I_y\) with \(-H_z\) (or \( E_z \)), \(Z\) with \(-i \omega \mu\) (or \(\sigma\)) and \(Y\) with \(\sigma\) (or \(-i \omega \mu\)) and thus produce an equivalent transmission surface. Bracketed functions refer to H pol, non bracketed to E pol.

The analogy is maintained in the boundary conditions. A constant vertical current is associated with \( H_y \) at the top of the air layer, which is constant in the E-polarization case. A constant voltage at the surface in the H-polarization mode, is equivalent to \( H_x \) being constant at \( z=0 \) and at the
sides a one dimensional transmission line is solved to find the boundary values of \( V \).

The model is covered with a mesh of cells each electrically homogeneous and the cells are linked by Kirchoff's laws of electrical continuity, thus ensuring a set of equations at each node point. These equations are then solved by a numerical method of Gaussian elimination.

### 5.4 One Dimensional Modelling of the Carnmenellis Data Set

For reasons outlined in section 4.7, the E-pol, H-pol and Berdichevsky Average invariant responses were modelled using the assumption that the earth is one dimensional.

The invariant response was obtained from the measured impedance tensor decomposed using the Groom-Bailey decomposition, (equations 4.9), with parameters constrained to values which produced the lowest residual misfit with the measured tensor. The parameters of the decomposed tensor had to meet further conditions set out in section 4.5, which ensured that the regional azimuth was constant over period ranges greater than half a decade, but could vary between the period ranges. This impedance tensor will be henceforth referred to as the "best" decomposed tensor. The Berdichevsky average used as the input to 1D models was the arithmetic average of the principal impedances of this "best" decomposed tensor.

The E-pol and H-pol responses were obtained from the principal impedances of a decomposed tensor with a regional azimuth fixed at the best estimate, N19° E, for all sites, at all periods.

#### 5.4.1 Use of the Parker-Whaler D⁣⁺ Algorithm in Determining Dimensionality and Penetration Depth

The Parker-Whaler D⁣⁺ algorithm was used, firstly to ascertain whether the misfit of the best fitting model, the D⁣⁺, fell below the level acceptable for induction in a one dimensional earth. (Parker and Whaler, 1981). Secondly it was used to find the limiting, or penetration depth, (Parker, 1982). This is defined, such that, no conclusions about the conductivity structure below this depth, can be drawn from the data set.

At all sites, and for all three responses, Berdichevsky Average, E- and H-pol, the misfit of the D⁣⁺ model was well above the acceptable 1D level and could only be brought below such a value by rejecting so many data points that the vast majority of the information contained in the sounding curves was lost. This confirmed the conclusions of chapter 4, that the data sets at all sites are at least two dimensional. However, model misfits could be significantly reduced by rejecting a few data points without impairing the information contained in the responses, and this was done.
The $D^+$ limiting depths for the E-pol and H-pol responses, at all sites are shown in table 5.1. Since an acceptable level of misfit could not be achieved, the limiting depth was estimated from the penetration depth-misfit curve, as a point below which, large increases in penetration depth only resulted in small decreases in misfit, (Parker, 1982). It is clear, the assumptions used in the calculations of values in table 5.1, do not hold for these data sets. Therefore these depths are only approximations and should be treated with a degree of caution. Since the Berdichevsky Average is an arithmetic average of the E- and H-pol responses, (equation 3.16), the limiting depth for this invariant, will be between the values calculated for the two polarisations. Some of the variation in limiting depths seen in table 5.1 is due to differences in the longest period included in the calculations. Limiting depths at a site were taken into account when contouring 1D Occam models and no structure below these depths was included in any of the models.

5.4.2 Analysis of the Berdichevsky Average Invariant

When modelling the Berdichevsky average invariant, the earth is assumed to be one dimensional, with no preferred regional strike direction. Therefore contouring of both the responses and the subsequent models was performed along the best fitting straight lines through the site locations. Figure 5.1 shows the four traverses (1-4) used in the analysis of the Berdichevsky Average invariant.

5.4.2.1 Pseudo Sections of the Berdichevsky Average Invariant

Before any modelling was performed, the sounding curves were studied, in order to find the main features that the response from any model must match. This was mainly done using pseudo sections. The pseudo sections for traverses 1-4 are shown in figures 5.2 to 5.5.

A number of points arise from the study of pseudo sections of the invariant response. Most of these observations equally apply to the E-pol response which will be presented latter in section 5.4.4.1.

1) Except for sites just off the granite outcrop (sites 810, 822, 835) and site 836, the measurements at periods below 1s indicate gradual changes in resistivity with depth rather than sharp contrasts.

2) The main resistivity contrasts are lateral. Figures 5.3.1 and 5.4.1 clearly show both the northern edge of the granite outcrop and the sloping contact between the granite and the more conductive country rocks to the south. This lateral contrast will affect the between site contouring in the pseudo sections. Features that are common to on-granite and off-granite sites will appear at different periods, though they may in reality be generated by structures at the same depth.
<table>
<thead>
<tr>
<th>Site Number</th>
<th>Longest Period Used in E-Pol Calculation (s)</th>
<th>Limiting Depth (km) Calculated for the E-pol Response</th>
<th>Longest Period Used in H-Pol Calculation (s)</th>
<th>Limiting Depth (km) Calculated for the H-pol Response</th>
</tr>
</thead>
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<tr>
<td>810</td>
<td>73.0</td>
<td>550.0</td>
<td>73.0</td>
<td>65.0</td>
</tr>
<tr>
<td>811</td>
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<td>500.0</td>
<td>73.0</td>
<td>240.0</td>
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<td>812</td>
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<td>190.0</td>
<td>14.0</td>
<td>55.0</td>
</tr>
<tr>
<td>813</td>
<td>73.0</td>
<td>500.0</td>
<td>57.0</td>
<td>120.0</td>
</tr>
<tr>
<td>814</td>
<td>7.0</td>
<td>90.0</td>
<td>7.0</td>
<td>35.0</td>
</tr>
<tr>
<td>815</td>
<td>22.0</td>
<td>150.0</td>
<td>22.0</td>
<td>22.0</td>
</tr>
<tr>
<td>820</td>
<td>568.0</td>
<td>350.0</td>
<td>568.0</td>
<td>150.0</td>
</tr>
<tr>
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<td>110.0</td>
<td>73.0</td>
<td>55.0</td>
</tr>
<tr>
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<td>73.0</td>
<td>325.0</td>
<td>47.0</td>
<td>135.0</td>
</tr>
<tr>
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<td>73.0</td>
<td>350.0</td>
<td>73.0</td>
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</tr>
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<td>4551.0</td>
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<td>57.0</td>
<td>40.0</td>
</tr>
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<td>120.0</td>
<td>14.0</td>
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<tr>
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<td>212.0</td>
<td>73.0</td>
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<td>8.0</td>
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<td>832</td>
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<td>60.0</td>
<td>0.89</td>
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<tr>
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<td>425.0</td>
<td>57.0</td>
<td>85.0</td>
</tr>
<tr>
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<td>250.0</td>
<td>73.0</td>
<td>80.0</td>
</tr>
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<td>835</td>
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<td>52.0</td>
<td>50.0</td>
</tr>
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<td>5.5</td>
<td>70.0</td>
<td>8.5</td>
<td>14.0</td>
</tr>
</tbody>
</table>

Table 5.1: Limiting Depths Calculated Using The Parker-Whaler D+ Algorithm For E-pol and H-pol Responses
Figure 5.1
A Map Showing the Locations of the Measurement Sites and the Traverse Lines Used For the Analysis of the Berdichevsky Average Invariant (Ray) of the "Best" Decomposed Tensor.

Coastline shown as dots.
Granite outcrops as dots and dashes.
FIG 5.1: MAP SHOWING LOCATIONS OF MEASUREMENT SITES AND THE TRAVERSE LINES USED FOR 1D RAV CONTOURING
Figures 5.2-5.5
Pseudo sections of Aparrant Resistivity and Phase of the Berdichevsky Average Invariant from the "Best" Decomposed Tensor for Sites Along Traverses 1-4.

The x axis is linear distance, the scale is given in the bottom left hand corner of the plot.
The y scale is log(period) in seconds.
The vertical line of dots below the site locations show the data points used for contouring.
FIG 5.4.2 Traverse & \( \log(n) \) After Decomposition

FIG 5.4.2 Traverse & \( f_{\text{max}} \) After Decomposition
FIG 5.5.1: Traverse 4, $\log(\rho_s \text{ hard})$ After Decomposition

FIG 5.5.2: Traverse 4, $\phi_{\text{hard}}$ After Decomposition
The pattern of apparent resistivities for sites on the granite outcrop is one of gradual increase with increasing period, reaching a peak value at periods just greater than 1s and then falling, again gradually, at longer periods. This peak value is of greatest magnitude for sites on the southern half of the granite outcrop, as well as for site 815. The contoured peak shown at periods just below 1s at sites 825 and 826 is due to the lack of acceptable data at these sites over this period range and therefore the response at site 815 dominates this area of the plot.

4) The phases at the shortest periods recorded are about 40°, indicating a conductive top layer moving into a more resistive material with depth. Over the next decade, phases stay roughly constant or fall a few degrees, indicating a gradual increase in resistivity with depth. From 1s to 10s, there is an increase in phase from 35° to 50° at all sites. However at sites off the granite, this increase in phase occurs at slightly longer periods. Once again it must be pointed out that this may not be a common feature, though it does appear to have a common origin for sites on the granite. The phases below 1s indicate a reduction in resistivity with depth with a steeper reduction below the southern sites.

5) Two sites contradict the above pattern in traverse 1. In contrast to the rise in apparent resistivity with period at sites 811, 813 and 814, between 0.1s and 1s, the apparent resistivity falls at site 812. As the phase response at this site indicates, this fall is not due to galvanic scattering causing a shift down of the curve over this period range, therefore it must be assumed to be real characteristic of the invariant response. The scalar gain or “static shift” of the sounding curves at site 813, described in section 4.8, manifests itself in the pseudo section of traverse 1, as a series of flat lateral changes in apparent resistivity, but of course it has no effect on the phase section.

6) The pseudo sections for traverse 3 shown in figure 5.4, indicate that the top layer below 835 contains the most conductive material encountered in the survey area and that this conductor dips to the north and is below a more resistive overburdened at site 836. At both sites there is a resistive underlying basement. The effects of this conductive structure lying to the north east of the granite outcrop also affects the sounding at site 834 resulting in a significant contrast between the eastern and western halves of the pseudo sections for traverse 4, seen in figure 5.5.

5.4.3 One Dimensional Modelling of the Berdichevsky Average Invariant

Figures A.1 to A.23 in appendix A, show Niblett-Bostick, Occam and Minim 1D models of the Berdichevsky average of the “best” decomposed tensor, for all soundings included in the Carnmenellis data set. Also shown in these figures is a most-squares analysis of the Minim layered model.
5.4.3.1 Occam Smooth One Dimensional Models of the Berdichevsky Average Invariant

It was predicted in section 1.5.4, that the significant factors affecting the resistivity of the granite would be pressure, temperature and water content. Borehole samples and other geophysical work have suggested that the batholith is almost homogeneous in composition. Therefore the physical evidence points to a gradual variation in the resistivity depth profile over the top 20 km. This prediction is reflected in the measured earth response function analysed in section 5.4.2. Except for sites on the granite edge slowly varying changes in the phase and apparent resistivity predominate. Therefore a smoothly varying 1D model such as those produced by the Occam inversion, should reflect more accurately the characteristics of the measurements and the predicted structure of the earth. Emphasis was therefore placed on smooth models.

The fit of the Occam models to the invariant response within the bounds of the errors was generally good. Where the model didn’t fit the apparent resistivity curve, for example at site 811, it fitted the phase response well. It is interesting to note, where the quality of the data was good, for instance at sites 822, 823 and 824, the simple Niblett-Bostick algorithm produced a very similar resistivity-depth profile to the one produced by the much more sophisticated and time consuming Occam inversion.

Figures 5.6 to 5.10 show contoured sections for Occam 1D models of the Berdichevsky Average invariant for sites on traverse 1-4. The thickness of the layers of the models increases logarithmically with depth. Therefore to reduce errors in contouring, depth is plotted logarithmically. Although the thickness of the top layer of the models is only 10m, the conductivity structure at such depths cannot be resolved by these data sets. An estimate of the shallowest depth at which resolution is possible is shown by the point on the Niblett-Bostick resistivity depth profile calculated at the shortest period, (Jones, 1983b). This depth varies between 200m and 1000 for the different soundings, with the shallowest Niblett-Bostick depth of 700m at the majority of sites. The Occam model was extended to both shallower and deeper depths to ensure a smooth profile within the depth range the data are sensitive to.

The resistivity structure shown in figures 5.6 to 5.10 for sites on or just off the Carnmenellis outcrop can be split into six layers.

1) The resistivity increases with depth in the portion of the granite shallower than 2.75 km.

2) Between 2.75 and 7-8 km the resistivity either falls slightly at sites 822, 823 and 810 or more steeply at the other sites.

3) Deeper than 8 km the resistivity increases with depth, reaching peak values at depths between 20 and 25 km. There is no indication from
Figures 5.6-5.10
Contoured Sections of 1D Occam Models of the Berdichevsky Average Invariant from the "Best" Decomposed Tensor for Sites Along Traverses 1-4.

The x axis is linear distance, the scale is given in the bottom left hand corner of the plot.
The y scale is log(depth) in m.
The vertical line of dots below the site locations show the data points used for contouring.
Fig 5.6: Contour plot of ocean 3D models of decomposed bed. Aver. Tanor

$\log(\rho) \ (\text{ohm-m})$

- ABOVE 3.8
- 3.7 - 3.8
- 3.6 - 3.7
- 3.5 - 3.6
- 3.4 - 3.5
- 3.3 - 3.4
- 3.2 - 3.3
- 3.1 - 3.2
- 3.0 - 3.1
- 2.9 - 3.0
- 2.8 - 2.9
- 2.7 - 2.8
- 2.6 - 2.7
- 2.5 - 2.6
- 2.4 - 2.5
- 2.3 - 2.4
- 2.2 - 2.3
- 2.1 - 2.2
- 2.0 - 2.1
- 1.9 - 2.0
- 1.8 - 1.9
- BELOW 1.8
Fig 5.7: Trav 1, Contour Plot of Occam 1D Models of Decomposed Berd. Aver. Tensor

log(ρ) (ohm-m)

- ABOVE 3.8
- 3.7 - 3.8
- 3.6 - 3.7
- 3.5 - 3.6
- 3.4 - 3.5
- 3.3 - 3.4
- 3.2 - 3.3
- 3.1 - 3.2
- 3.0 - 3.1
- 2.9 - 3.0
- 2.8 - 2.9
- 2.7 - 2.8
- 2.6 - 2.7
- 2.5 - 2.6
- 2.4 - 2.5
- 2.3 - 2.4
- 2.2 - 2.3
- 2.1 - 2.2
- 2.0 - 2.1
- 1.9 - 2.0
- 1.8 - 1.9

BELOW 1.8
FIG 5.8: Tray 2, Contour Plot of Occam 1D Models of Decomposed Berd. Aver. Tensor

log(ρ) (ohm-m)

ABOVE 3.8
3.7 - 3.8
3.6 - 3.7
3.5 - 3.6
3.4 - 3.5
3.3 - 3.4
3.2 - 3.3
3.1 - 3.2
3.0 - 3.1
2.9 - 3.0
2.8 - 2.9
2.7 - 2.8
2.6 - 2.7
2.5 - 2.6
2.4 - 2.5
2.3 - 2.4
2.2 - 2.3
2.1 - 2.2
2.0 - 2.1
1.9 - 2.0
1.8 - 1.9
BELOW 1.8

MAR820 NAP822 GAR824 LAN826
BIS821 SEW823 CAL825 TRK815

depth (m)

log(ρ) (ohm-m)

0.0 2.5 5.0 Km.
FIG 5.10: Traverse 4, Contour Plot of Occam 1D Models

$log(\rho) \text{ (ohm-m)}$

- ABOVE 3.8
- 3.7 - 3.8
- 3.6 - 3.7
- 3.5 - 3.6
- 3.4 - 3.5
- 3.3 - 3.4
- 3.2 - 3.3
- 3.1 - 3.2
- 3.0 - 3.1
- 2.9 - 3.0
- 2.8 - 2.9
- 2.7 - 2.8
- 2.6 - 2.7
- 2.5 - 2.6
- 2.4 - 2.5
- 2.3 - 2.4
- 2.2 - 2.3
- 2.1 - 2.2
- 2.0 - 2.1
- 1.9 - 2.0
- 1.8 - 1.9
- BELOW 1.8

Depth (m)

0.0 2.5 5.0 Km.
these Occam models of a systematic pattern in the variation in depth to this second resistivity peak.

4) At sites where there are data at long enough periods to resolve structures at such depths. The resistivity depth profile reaches a second minimum at depths at between 55 and 60 km. Not surprisingly at such depths there is a uniformity in the resistivity structure between sites.

5) Finally there are indications of a rise in resistivity below 60 km, confirmed by the long period measurements taken at site 824, see figure A.12.

Figure 5.7 is a contour plot of Occam models of the Berdichevsky average of the "best" decomposed tensor for all sites on traverse 1 excluding site 813. The model at this site was excluded because of the "static shift" factor affecting the sounding. The northern half of the plot shown in figure 5.7 closely resembles the structure below the northern half of traverse 2, shown in figure 5.8.

It is unclear from these models whether the conductive overburden at site 835 is linked to the conductive layer 800 m below site 836. This is because there are no soundings in the 8 km between the two sites. Finally, either there is a more conductive top layer at site 834 than at other on-granite sites or the invariant is being biased downwards by the nearby conductive region to the north east of the granite outcrop.

5.4.3.2 Minim and Most-Squares One Dimensional Models of the Berdichevsky Average Invariant

For reasons stated in the above section, emphasis has been placed on smooth models. However, in order to verify that all the structure included in the Occam models could be resolved by the data and to ascertain whether any discontinuities in the resistivity-depth profile existed, Minim models were produced along with a most-squares analysis of these layered structures. The Minim and most-squares models for all the sites are also shown in Appendix A, figures A.1-A.23.

Following the procedures outlined in Fischer and Le Quang (1981), a study of the standard deviation (equation 5.22) as a function of the number of layers was performed in order to find the minimum number of layers which adequately fitted the data.

A more conductive overburden is included at all sites except site 814 where a resistive top layer is included down to a depth of 200 m. There is quite a large variation in the depth to the first interface between sites and most-squares analysis indicates this interface is not always resolvable, especially at sites in the middle of the granite outcrop (sites 824, 825 and 826). The depth to the first interface at sites just off the granite will give a measure of the depth of the country rock. This is between 450-650 m at site 810, 450-700 m at site 822 and 350-800 m at site 835. If we assume the first peak in the resistivity profile below site 821 and the second peak at site 820, represent the onset of the granite at each site, the granite is between 1.8 km
and 3.6 km below site 821, but below site 820, the two peaks in the resistivity-depth profile cannot be resolved and are included in the Minim models as only one layer.

The data necessitated the inclusion of a more conductive layer at depths of a few km below only some of the sites, those on the southern half of traverse 1 and sites on traverse 3. Most-squares analysis indicates that only at sites 832, 833 and 834 is this layer barely resolvable by the data and only at site 830 is it definitely resolvable. Therefore the reduction in resistivity at these depths modelled by the Occam inversion may well exist but nothing can be stated quantitatively about this resistivity reduction.

A discontinuity is included in the Minim models at all sites in order to model the fall in resistivity between 25 and 60 km. Despite the quite large resistivity contrast across this discontinuity at many sites, most-squares analysis shows that the resistivity of the bottom layer and the depth to the contrast are not well resolved. This is probably due to the depth of the discontinuity. The contrast is placed at between 35 and 45 km, with no systematic pattern describing the variations in the depth between the sites. The inclusion of a layer below these depths only occurs at sites 824 and 833 and in both cases it is poorly resolved.

5.4.4 Analysis of the E-Pol and H-Pol Responses

Figure 5.11 shows the three traverses (5-7) used in the analysis of the E-pol and H-pol responses. Figures 5.12-5.15 show apparent resistivity and phases of the E-pol and H-pol responses as diamonds and circles respectively. The lines join the principal impedances of the "best" decomposed tensor. Fixing the regional azimuth for all sites at all periods is obviously a simplification of the real structure. Figures 5.12-5.15 are useful in illustrating the validity of this assumption, and show that at the majority of the sites, only small differences exist between the fixed azimuth responses and the principal impedances of the "best" decomposed tensor. Thus the E- and H-pol data at these sites can be inverted without introducing very large uncertainties into the resultant models.

However departures in the two responses do occur, principally at sites close to the southern edge of the granite outcrop (sites 823, 811, 830 and 831) but also at sites 833 and 812. The differences in the regional azimuths at these southern sites from the other sites on the granite outcrop has been documented in section 4.6.1, along with the possible reasons behind such changes.

Figures 5.12-5.15 also show the form of the E- and H-pol responses and are especially useful for illustrating the relationship between the two sounding curves. Both the phases and the apparent resistivities of the principal impedances are split over virtually the whole measured period range, at all sites, except site 822. Thus illustrating the conclusions of chapter 4, that the data set is at least two dimensional.

At all sites on the granite batholith, except at site 825, the E-pol and
Figure 5.11
A Map Showing the Locations of the Measurement Sites and the Traverse Lines Used For the Analysis of the E-pol and H-pol Responses
Coastline shown as dots.
Granite outcrops as dots and dashes.
FIG 5.11: MAP SHOWING LOCATIONS OF MEASUREMENT SITES AND THE TRAVERSE LINES USED FOR E AND H-POL STUDY
Figures 5.12-5.15.
The Apparent Resistivity and Phases of the E-pol (Circles) and H-pol (Diamonds) Responses Along With the Lines Joining the Principal Impedances of the "Best" Decomposed Tensor for All Soundings in the Carnmenellis Data Set.

The diamonds and circles display data obtained from the principal impedances of a decomposed tensor with a regional azimuth fixed at the best estimate N 19° E, for all sites, at all periods. The lines show principal impedances obtained from the measured impedance tensor decomposed using the Groom-Bailey decomposition, with parameters constrained to values which produced the lowest residual misfit with the measured tensor. The parameters of the decomposed tensor had to meet further conditions set out in section 4.5.
Figure 5.12: Data with Fixed Azimuth (points), "Best" Decomposed Data (Lines)
Figure 5.13: Data with Fixed Azimuth (points), 'Best' Decomposed Data (Lines)
Figure 5.14: Data with Fixed Azimuth (points), 'Best' Decomposed Data (Lines)
Figure 5.15: Data with Fixed Azimuth (points), 'Best' Decomposed Data (Lines)
H-pol apparent resistivities cross over. These cross-overs occur just below 1s for sites 823, 824, 811 and 812, but closer to 0.3s at more north-westerly sites, 813 and 814. The cross-over in phases also occurs, but at shorter periods at most of the on-granite sites. Not all the cross-overs in the data joined by lines are real. A large proportion are caused by rotations of 90° in the regional azimuth between consecutive periods.

The split in the apparent resistivities of the two responses at shorter periods, before the cross-over, is greatest at sites close to granite edge (sites 834, 815, 823 and 811) or near to the coastline (site 810). The sounding curves at site 825 also show a large split in the apparent resistivities, but the largest split occurs for periods greater than 1s.

5.4.5 The Case for Modelling the E-Pol, H-Pol or Both Responses

The two dimensional earth model assumes that any changes in structure perpendicular to the traverse are negligible. However from figure 1.10, it is clear potentially large conductivity contrasts exist along lines perpendicular to traverses 5, 6 and 7, including contrasts between the land and the sea and between the batholith and the surrounding country rocks. Therefore it is essential to investigate the possible effects these lateral conductivity contrasts will have on two dimensional models along the three traverses.

Figure 5.16 shows a highly simplified two dimensional forward model along a line perpendicular to traverses 5, 6 and 7 passing through sites 813, 824 and 831. The model is based on the surrounding bathymetry and the gravity model of Willis-Richards (1986) shown in figure 1.10. The responses of the two polarisations at sites on each of the three traverses up to periods of 14 seconds are shown as solid lines in figure 5.17. The dotted line in the figure shows the one dimensional response of the figure 5.16. This would be the response if changes in conductivity structure perpendicular to traverses 5, 6 and 7 were negligible and hence any departures from this response in either the E or H-pol impedances will introduce biases into the data to be modelled by 2D algorithms. From figure 5.17, it is clear that the E-pol response (for the perpendicular traverse \( Z_{xy} \)) is affected by the conductivity contrasts much more strongly than the H-pol response (for the perpendicular traverse \( Z_{yx} \)) at periods below 1s. This means that \( Z_{yx} \) impedances (E-pol for traverses parallel to regional azimuth) recorded on traverses 5, 6 and 7 below 1s are much less affected by conductivity contrasts perpendicular to the traverse than the \( Z_{xy} \) (H-pol for traverses parallel to regional azimuth) impedances. For this reason it has been common practice to model the E-pol response, (Stanley, 1984; Camfield et al., 1989; Ingham, 1992). For periods above 1s the 1D dimensional apparent resistivity and phase curves become sensitive to the more conductive structure below the granite but the nearby sea and thick country rocks affect the H-pol (for the perpendicular traverse \( Z_{yx} \)) response biasing the apparent resistivity curves up and the phase curves
Figure 5.16
Two Dimensional Forward Model Along a Line Perpendicular to Traverses 5, 6 and 7.

The model is based on the surrounding bathymetry and the gravity model of Willis-Richards (1986).
The left hand side of the plot is WNW and the right hand side is ESE.
The y axis is log depth in m.
The y axis is linear distance. The scale in bottom left hand corner of Plot.
Fig 5.16: Forward model of Perpendicular Traverse

LOG (p) (0–m)

- 2600.0
- 400.0
- 0.3
Figure 5.17
E and H-Pol Responses For Sites 813, 824 and 831 on a Traverse Perpendicular to Lines 5, 6 and 7 Calculated From the Forward Model Shown in Figure 5.16 Using The Algorithm of Madden and Thompson (1965)

E-pol response shown as solid line marked with an E at either end of the curves.
H-pol response shown as solid line marked with an H at either end of the curves.
One dimensional response of the structure directly below each of the sites shown as dotted line
E and H Pol + 1D (Dashed) Resps. of Model 5.16
down. In fact at periods shorter than $1s$, neither principal impedances approximate the 1D response, but the E-pol phase is a better approximation to the 1D phase than the H-pol. The divergence between the 1D curve and the E and H-pol curves is greatest at site 831, since this site is the nearest to the sea and the eastern edge of the granite, the two largest conductivity contrasts on this perpendicular traverse.

Wannamaker et. al. (1984) in studying the 1-D and 2-D responses of shallow 3 dimensional bodies produced evidence to contradict this convention. They found that boundary charges, which are ignored by 1 and 2-D models can severely affect the E-pol response of structures below 3-D bodies, but have negligible effect on the H-pol response. However in applying the Groom-Bailey decomposition to the measured impedance tensor, the most severe effects of boundary charges have been isolated from the regional response.

However, figure 5.17 shows that both principal impedances will be biased by lateral conductivity contrasts. Shallower structures sampled by shorter period measurements can be accurately obtained by 2D modelling the $Z_{yx}$ or E-pol (for traverses parallel to regional azimuth) response. Obtaining accurate two dimensional models which fit the longer period data where lateral conductivity contrasts severely affect both responses appears to be impossible, but from figure 5.17, 2D modelling the $Z_{xy}$ data would produce the better approximation. However magnetotelluric measurements at a particular period are affected by all structures shallower than the skin depth. So if erroneous shallow structure is introduced into models, as would be true in the case of 2D modelling the $Z_{xy}$ data, deeper structures will also be inaccurately modelled. Therefore the $Z_{yx}$ or E-pol response was analysed further, with subsequent careful consideration of the effects of the lateral conductivity contrasts on the resultant models.

The resistivity values in the model 5.16 for the granite and the country rocks are average values taken from 1-D models. The resistivity of the granite will therefore be underestimated since both $Z_{xy}$ and $Z_{yx}$ will be biased downwards by lateral contacts with less resistive rocks in all directions. Therefore the conductivity contrast between the granite and country rocks is in reality greater than that shown in figure 5.16, and the effect on the two responses will be even greater.

The fact that resistivity contrasts exist in all directions cause the cross-overs in the principal responses described in the above section. Whether a response is the major or the minor will depend on the direction of the dominant conductivity contrast at a particular period. For on-granite sites at periods below $1s$, the downward biasing effects of conductivity contrasts perpendicular to the traverses, outweigh the effects of more conductive lateral structures along the traverse ensuring $Z_{xy}$ is the minor response. But above $1s$ the normal two dimensional situation for sites on the resistive side of a contact prevails and $Z_{yx}$ becomes the minor response. However the $Z_{xy}$ response is still biased downwards.
5.4.6 Pseudo Sections of the E-pol Responses

Figures 5.18-5.20 show pseudo sections of E-pol responses for traverse 5, 6 and 7.

For sites on the granite outcrop there is a fall in the apparent resistivity with increasing period for periods greater than 1s. This fall is much more marked at sites north of the southern edge of the granite (sites 824, 825, 826, 833 and 834) than for those close to the southern edge (sites 822, 823, 830 and 831). The phase at these northern granite sites starts to rise at shorter periods and rises to higher values before the end of the data set. This all indicates that there is a reduction in resistivity with depth, but resistive structures persist to greater depths to the south of the survey area.

Phases below 45° exist at the shortest periods recorded at all sites, indicating an increase in resistivity with depth close to the surface. However the phase pattern at sites on traverse 7 indicates that the near surface is more conducting on this traverse than elsewhere on the granite. This may be due to the close proximity to these sites of the steeply sloping eastern edge of the batholith.

5.4.7 Occam Smooth One Dimensional Models of the E-pol Response

The resistivity-depth profiles at each site, obtained by modelling the E-pol response using the Occam 1D algorithm are shown in appendix A, figures A.24 -A.43. Contoured sections of these profiles along the three traverses are shown in figures 5.21, 5.22 and 5.23.

Between 1 km and 75 km two types of resistivity profile exist for on or just off granite sites. The resistivity increases with depth down to depths of about 3.5 km at sites 811 and 830 and 7 km at sites 822 and 823. Although the resistivity falls below these depths it rises again to reach peaks at 15 km at 811, 20 km at 830 and 25 km at 822 and 823. At sites 812, 813, 833 and 834 the resistivity increases with depth reaching peak values between 3.5 and 5 km. The resistivity then falls with depth, quite steeply in the case of sites 833 and 834 until about 10 km where the resistivity stays constant for about 5 km, before falling again. The resistivity-depth profile reaches a minimum at these sites at a depth of about 50 km. Site 824 shows characteristics of both types of profiles. The data quality at sites 825, 826, and 814 is poor, though there are indications that they fit the pattern of sites on the northern half of the granite outcrop. The falls in resistivities with depth are quite steep indicating a resistivity contrast rather than a gradual change in resistivity with depth. The differences in the two sets of profiles ensure that more resistive material of about 3000 Ω.m persists to depths of about 30 km below sites close to the southern edge of the granite outcrop, whilst conductive material of resistivities about 250 Ω.m underlies sites further to the north. Therefore one dimensional Occam models of the E-pol response indicate that the bottom of the granite is not flat but slopes upwards towards the north.
Figures 5.18-5.20
Pseudo sections of Apparent Resistivity and Phase of the E-pol Responses for Sites Along Traverses 5-7.

The x axis is linear distance, the scale is given in the bottom left hand corner of the plot.
The y scale is log(period) in seconds.
The vertical line of dots below the site locations shows the data points used for contouring.
FIG 5.18.1: Traverse 5, log($\rho_e$ (ohm-m)) After Decomposition

FIG 5.18.2: Traverse 5, $\phi_{pe}$ After Decomposition
Figures 5.21-5.23
Contoured Sections of 1D Occam Models of the E-pol Response for Sites Along Traverses 5-7.

The x axis is linear distance, the scale is given in the bottom left hand corner of the plot.
The y scale is log(depth) in m.
The vertical line of dots below the site locations show the data points used for contouring.
FIG 5.2: Contour Plot of Occam 1D Models of E-pol Tensor

**log(\rho) (ohm-m)**

- **ABOVE 4.3**
- 4.2 - 4.3
- 4.1 - 4.2
- 4.0 - 4.1
- 3.9 - 4.0
- 3.8 - 3.9
- 3.7 - 3.8
- 3.6 - 3.7
- 3.5 - 3.6
- 3.4 - 3.5
- 3.3 - 3.4
- 3.2 - 3.3
- 3.1 - 3.2
- 3.0 - 3.1
- 2.9 - 3.0
- 2.8 - 2.9
- 2.7 - 2.8
- 2.6 - 2.7
- 2.5 - 2.6
- 2.4 - 2.5
- 2.3 - 2.4
- **BELOW 2.3**
FIG 5.22: Trav 6, Contour Plot of Occam 1D Models of E-pol Tensor

log(ρ) (ohm-m)

MAR820  NAP822  GAR824  LAN826  TRW835

BIS821  SEW823  CAL825  PER834  CRV836

Depth (m)

0.0  2.5  5.0  Km.
The two peaks in the resistivity profiles at sites 821 and 820 at 3.5 and 11 km respectively are poorly resolved by models of the E-pol response. The conductive body lying below 836 lies at a depth of about 800m and has a resistivity of about 50 $\Omega\cdot m$.

5.5 Two Dimensional Modelling of The Carnmenellis Data Set

All two dimensional modelling was performed using the forward modelling algorithm of Madden and Thompson (1965). Two dimensional inversions of the E-pol data sets were attempted using the Occam code of deGroot-Hedlin and Constable (1990). At all periods the responses of the models produced by the inversion failed to satisfactorily fit all the measured apparent resistivity curves. Models fell into two groups. There were models whose apparent resistivity levels were close to measurements taken at the sites off the granite and there were those whose levels matched the data measured on the granite. Unfortunately no model could be found that adequately fitted all the measured apparent resistivity data. Due to the large lateral conductivity contrasts that exist in south-west Cornwall, it appears that the condition of lateral smoothness similar to equation (5.7) and given in deGroot-Hedlin and Constable, (1990), cannot be met whilst maintaining a reasonable degree of misfit.

Emphasis was placed on modelling data from sites on traverse 6 for three reasons.

1) It has already been shown in section 5.4.5 that the effects of conductivity contrasts perpendicular to the traverses are greatest for sites on line 7. Therefore it was preferable to model sites on lines 5 and 6.

2) Figure 5.17 also gives an indication of the effects of the surrounding country rocks on measurements made on the granite. No sites exist north of the granite outcrop on line 5 and so the resistivity of the structure below this area can only be postulated.

3) The data quality for sites on line 6 was superior to sites on the other two traverses.

5.5.1 Two Dimensional Model of E-pol data Recorded at Sites on Traverse 6

After examining an extensive range of models, figure 5.24 was chosen, in the opinion of the author, as the most accurate Madden forward model that could be obtained at the present time, for E-pol data measured at sites on traverse 6. The local topography, which is not shown in the figure 5.24, and the bathymetry of the sea (shown in magenta in figure 5.24) were included in the model. Figures 5.25, 5.26 and 5.27 show the response of the model in figure 5.24 along with the data and the chi-squared misfit between the model response and the data. From figures 5.25, 5.26 and 5.27 it is clear that the fit of the model at some of the sites is
poor, so before the significance of the model is analysed, reasons both for choosing this particular model and for the poor misfit of the model response to the data will be discussed.

In discussing the misfit of the data to a model response, it is important to note that the chi-squared statistical measure of misfit, will depend on the error structure of the data and can also be biased when average values are quoted for an ensemble of measurements. Therefore consideration will also be given in the discussion below to more qualitative factors, such as the relative shapes and levels of the model response and data curves.

From section 5.4.5 it can be easily seen that country rocks and the sea east of the Carnmenellis outcrop will ensure a poor misfit between the E-pol response of any model and the data at longer periods. This will be true certainly for periods greater than 10s and probably for periods greater than 1s. However figures 5.25, 5.26 and 5.27 show the misfit is especially large even at periods below 1s at sites 822, 823 and 834. The data quality of the E-pol measurements at sites 825 and 826 is so poor between 0.01 seconds and 1s that a quantitative comparison with the model response is impossible and therefore discussion of the large misfits will be restricted to sites 821, 822 and 823.

It was found impossible to obtain a model that produced an E-pol response that satisfactorily fitted the data at sites 820 and 821 to the South of the granite outcrop whilst also fitting the data at sites 822 and 823. Similarly a model could not be found to fit the data at 834 and still match the measured responses at sites north of the Carnmenellis outcrop, sites 835 and 836. To fit both the data at site 821 and the shortest periods measured at site 822, it was necessary to include a top layer conductor of 60 $\Omega \cdot m$ or 140 $\Omega \cdot m$ immediately to the South of granite outcrop. The effect of a lateral conductor on the E-pol response, at a site on the resistive side of a contact, is to increase the phase and decrease the apparent resistivity, the greater the conductivity contrast, the greater the effect on the response. However for periods shorter than 1s, the apparent resistivity at sites 822 and 823 increased with increasing period, whilst the phase stayed almost constant. The effects of the lateral conductor could not be counteracted even after dramatic increases in the resistivity-depth gradient below sites 822 and 823. This is because any increase in the resistivity-depth gradient also increased the lateral conductivity contrast, producing greater two dimensional effects on the model response, which in turn, cancelled out the response of an increasing resistivity depth profile. These arguments also apply to site 834 and the off granite sites 835 and 836.

5.5.2 Reasons for the Large Misfit of the Model Response to the Data

One reason for the large misfit between the data at some of the sites and the model response is that the measured $Z_{yx}$ response is in fact three dimensional. Therefore effects on the $Z_{yx}$ response due to conductivity contrasts perpendicular to the regional traverse were investigated using
Figure 5.24
The Most "Accurate" Two Dimensional Forward Model of E-Pol Data Measured Along Traverse 6

The left hand side of the plot is SSW and the right hand side is NNE. The y axis is log depth in m. The X axis is linear distance. The scale is in bottom left hand corner of Plot.
Fig 5.24: Madden toward E-pol model of Traverse 6
Figure 5.25 - 5.27
E-Pol Model Responses and Measured Data at Sites on Traverses 6. Model Responses Calculated From the Forward Model Shown in Figure 5.24 Using The Algorithm of Madden and Thompson (1965)

E-pol response shown as solid line marked with an E at either end of the curves.

Apparent resistivity and Phase of $Z_{yx}$ shown as diamonds
Fig 5.25 Fit of Model 5.24 E-Pol Response To Data
Fig: 5.26 Fit of Model 5.24 E-Pol Response To Data
Fig: 5.27 Fit of Model 5.24 E-Pol Response To Data
Madden's forward two dimensional code. Figure 5.28 shows the traverses used in this investigation. Figures 5.29, 5.31, 5.32, 5.33 and 5.34 show the forward models along each of the traverses. Figures 5.35 and 5.36 show the $Z_{yx}$ (for these traverses the H-pol) and 1D responses of these models at the sites on traverses 8-13 that also lie on traverse 6. The resistivities of layers in the granite were based on a slightly simplified version of the model shown in figure 5.24. The shape of the granite was based on the gravity model of Willis-Richards (1986) and the depth of the sea, shown in the figures as white, was calculated from maps of the local bathymetry. The resistivities of the surrounding country rocks were taken from figure 5.24. The 140 $\Omega$-m top layer conductor in figure 5.24 coincides with the surface outcrops of the Devonian Mylor Slate and Gramscatho groups, therefore areas shown on the B.G.S. geological maps as having a drift layer of these two rock types, were assigned this particular resistivity. With no M.T. soundings east or west of the outcrop and no other geophysical evidence for the thickness of these formations, the thicknesses were just estimated from model results at sites 835 and 820. Therefore the thickness of this top layer conductor was set at 2 km south of Carnmenellis outcrop and 500m north or west of the outcrop. This may represent an over estimation of the volume of this conductor.

Figures 5.35 and 5.36 show that for sites on the resistive side of the contact 822, 823, 824, and 834 there is a divergence between the $Z_{yx}$ response (for these traverses, the H-pol response) and the 1D response at longer periods. Therefore the $Z_{yx}$ data used in 2D modelling of E-pol data on traverse 6 will not be the true 2D responses but will have apparent resistivity curves biased up and phase curves biased down. This is one of the main reasons why no model could be found which satisfactorily fitted all the data at all the sites.

It can be seen from the models that the divergence between the 1D curves and the $Z_{yx}$ curve is greatest at sites on the granite outcrop 823, 824 and 834. The divergence starts at shorter periods at these three sites than the other sites, 822 and 835 and develops into greater differences between the curves. In addition at site 823 the apparent resistivity curve is also biased upwards because the measured regional azimuth at this site diverges from the fixed regional azimuth used to generate the 2D principal impedances at all the sites, see section 5.4.4. In order to fit the data measured at site 822 and the shorter periods at sites 823 and 834, it was necessary to introduce structures into the model that reduced the conductivity contrast across the contact between the granite and the surrounding rock. However for reasons outlined above, a drastic reduction in this contrast that would have produced a good fit at sites 822 and 823, but increased the misfit at the other sites, was not included in the model in figure 5.24.

Figures 5.35 and 5.36 show that a two-dimensional model will be a good approximation for the data measured at site 820 for periods shorter than 14s and at sites 822 and 835 for periods shorter than 0.3s. Most
Figure 5.28
A Map Showing the Traverse Lines Used in the Analysis of The Effects of Conductivity Contrasts Perpendicular to the Regional Azimuth on the $Z_{yx}$ Response and the Locations of the Measurement Sites on These Traverses.

Coastline shown as dots.
Granite outcrops as dots and dashes.
FIG 5.28: MAP SHOWING LOCATIONS OF TRAVERSE LINES PERPENDICULAR TO FIXED REGIONAL AZIMUTH
Figures 5.29 -5.34
Two Dimensional Forward Models Along Traverses 8-13. The Traverses are Perpendicular to Regional Strike.

The left hand side of each of the plots is WNW and the right hand side is ESE.
The y axis is log depth in m.
The y axis is linear distance. The scale is in bottom left hand corner of figures 5.29 and 5.33.
Figure 5.35 - 5.36
H-Pol Responses For Sites on Traverses 8-11 Perpendicular to Regional Azimuth Calculated From the Forward Model Shown in Figure 5.29-5.34 Using The Algorithm of Madden and Thompson (1965)

H-pol response shown as solid line marked with an H at either end of the curves.
One dimensional response of the structure directly below each of the sites shown as dotted line
FIG 5.35: FORWARD 2D MODELS OF ALONG PERPEN. TRAVS.
FIG 5.36: FORWARD 2D MODELS OF ALONG PERPEN. TRAVS.
importantly figures 5.35 and 5.36 indicate that the depth to the bottom of the granite in figure 5.24 is overestimated and the apparent resistivity of the layer below the granite is also overestimated. Since the departures from the 2D approximation at site 834 are much greater than those at site 835, the model shown in figure 5.24 is biased towards fitting the data at site 835.

Additional misfit between the E-pol model response and the data measured at sites on traverse 6 is caused by the fact that sites off the granite outcrop (820, 821, 822 and 835) lie a considerable distance away from the traverse. In projecting sites onto a line, the distance between the edge of the Carnmenellis outcrop and sites off the outcrop was reduced, in the case of site 820 by almost 750 m. Therefore the distance between sites on the resistive and conductive side of the contact was reduced and the two dimensional effects were erroneously increased at both sets of sites.

5.5.3 Reasons for Stating that the Particular Model Presented is the Most Accurate Model of the E-pol Data Recorded on Traverse 6

As indicated in sections 5.5.1 and 5.5.2, there are reasons why no 2D model can adequately fit the data at all sites. In fact the model shown in figure 5.24 represents a compromise between fitting sites 820, 821, 835, 825 and 826 and fitting sites 822, 823, 824 and 836. Figure 5.37 shows a model that achieves a better fit at sites 820, 821, 825, 826, and 835. Figures 5.38, 5.39 and 5.40 show the response of the model in figure 5.37 along with the data and the chi-squared misfit between the model response and the data. The conductivity structure of the granite below sites 822, 823 and 824 remains unchanged between models 5.24 and 5.37. Indeed a comparison of figures 5.24 and 5.37 shows a large improvement in fit between the model and the data at sites 820 and 821 can be achieved by a relatively small extension of the zone of conductivity below these two sites. The improvement in misfit can be made by extending the conductive zone laterally by 1 km towards the granite and by adding between 600m to 1.5 km to the thickness. However from figure 5.38 it is also clear that such an extension also dramatically increases the misfit at sites 822, 823 and 824. This finding provides evidence that errors in the distances between these southerly sites are significant in producing the misfit between the model response and the measured data.

A better fit to the data at sites 825 and 826, especially to the impedance phase can be achieved by extending the 1000 Ω.m zone to greater depths, from 5 km to 20 km. However this again increases the misfit at sites 823, 824 and 834 and the quality of the data at 825 and 826 cannot justify such an extension in the most accurate model. A small improvement in the misfit at site 835 is achieved if the electrical structure of the granite below site 834 is the same as below site 823.

Figure 5.41 shows a model that achieves a better fit at sites 821, 822, and 834. Figures 5.42, 5.43 and 5.44 show the response of the model in figure 5.41 along with the data and the chi-squared misfit between the model response and the data. Section 5.5.2 clearly shows that 3D effects on
Figure 5.37
A Two Dimensional Forward Model of E-Pol Data Measured Along Traverse 6 Which Produces a Better Fit With the Data at Sites 820, 821, 825, 826 and 835.

The left hand side of the plot is SSW and the right hand side is NNE.
The y axis is log depth in m.
The y axis is linear distance. The scale is in bottom left hand corner of Plot.
Fig 5.37: Madden forward E-pol model of Traverse 6
Figures 5.38 - 5.40
E-Pol Model Responses and Measured Data at Sites on Traverses 6. Model Responses Calculated From the Forward Model Shown in Figure 5.37. Using The Algorithm of Madden and Thompson (1965)

E-pol response shown as solid line marked with an E at either end of the curves.

Apparent resistivity and Phase of $Z_{yx}$ shown as diamonds
Fig: 5.38 Fit of Model 5.37 E-Pol Response To Data
Fig: 5.39 Fit of Model 5.37 E-Pol Response To Data
Fig:5.40 Fit of Model 5.37 E-Pol Response To Data
Figure 5.41
A Two Dimensional Forward Model of E-Pol Data Measured Along Traverse 6 Which Produces a Better Fit With the Data at Sites 822, 823, 824 and 834.

The left hand side of the plot is SSW and the right hand side is NNE.
The y axis is log depth in m.
The y axis is linear distance. The scale is in bottom left hand corner of Plot.
Fig 5.4: Madden toward E-pol model of Traverse 8
Figure 5.42 - 5.44
E-Pol Model Responses and Measured Data at Sites on Traverses 6. Model Responses Calculated From the Forward Model Shown in Figure 5.41. Using The Algorithm of Madden and Thompson (1965)

E-pol response shown as solid line marked with an E at either end of the curves.

Apparent resistivity and Phase of $Z_{yx}$ shown as diamonds
Fig: 5.42 Fit of Model 5.41 E-Pol Response To Data
Fig 5.43 Fit of Model 5.41 E—Pol Response To Data
Fig: 5.44 Fit of Model 5.41 E-Pol Response To Data

STA: TRW835 RMS MISFIT = 198.37

STA: GRV836 RMS MISFIT = 7.07
the responses at these sites will ensure the introduction of erroneous structure when fitting data below 1s, therefore this was not attempted. A better fit at sites 822 and 823 can be achieved by reducing the conductive zone below sites 820 and 821 and the 1000 Ω.m structure below sites 825 and 826. If the conductive feature below sites 820 and 821 is to be extended, whilst still achieving a good fit between the data and the model response at sites 822 and 823, then the resistivity of material south of site 820 must be increased considerably. Even if there is no conductive top layer at site 835, a good fit to the apparent resistivity data at site 834 cannot be achieved. This indicates that the E-pol apparent resistivity measured at site 834 has been moved up by a "static shift" factor.

As well as being a compromise between achieving the best fit at two different sets of sites, the model was chosen to meet various restrictions placed upon it by the data. Three characteristics of the measured E-pol response dictate the resistivity structure below sites 822, 823 824 and 834.

1) For reasons set out in section 5.5.1, the resistivity below sites 822, 823, 824 and 834 must increase rapidly with depth.

2) From a comparison of the responses of models 5.24 and 5.37, it is clear that misfits between the model response and the data at sites 820 and 821 are sensitive to rises in the conductivity contrast between the granite and the surrounding rocks at depths between 500m and 4 km.

3) The E-pol phases at sites 822, 823 and 824 indicate that the top layer below these sites is both shallow and less conductive than the second layer.

The resistivity of the layer between 4 and 40 km below these sites in model 5.24 is set at 20000 Ω.m. A decrease in the resistivity of this layer does increase the misfit at sites 822, 823 and 824, even at periods where the data is two dimensional. In addition an increase in the resistivity of this layer only marginally decreases the misfit at sites 822, 823 and 824, but increases the misfit at sites 820 and 821. The second interface between the 8000 Ω.m and the 20000 Ω.m layers is placed at 4 km, the depth to the top of the granite below site 821, thus lateral contrasts between the country rocks and the granite are reduced as far as possible whilst still maintaining a model response close to the data measured on the granite outcrop.

The depth to the bottom of the granite was set so that the model responses fitted the data primarily at site 824 but also at sites 825 and 826. Decreasing the depth to the bottom of the 20000 Ω.m layer does increase the misfit at sites 824, 825 and 826 at periods below 1s, where the data are expected to be interpretable using 2D models.

The E-pol curves at sites 824, 825 and 826 differ with those measured at sites 822 and 823 over the period range (1 to 10s), where the soundings become sensitive to structure below the granite. The E-pol apparent resistivity at site 824 starts to fall at 1s, but at sites 822 and 823 the apparent
resistivity stays constant between 1 and 10s and starts to fall after 10s. On first inspection of the responses at these sites there are indications that the 20000 \( \Omega \cdot m \) layer is thicker below sites 823 and 822 than below sites further to the North. Figure 5.35 shows that although the depth to the bottom of this layer will be overestimated, the curves at 824 and 823 will be equally biased by conductors perpendicular to the traverse. Hence the difference in the relative thicknesses of the granite between sites appears to be true. In addition, figure 5.35 indicates that the apparent resistivity curve at site 822 only starts to become three dimensional after 1s and between 1 and 10s the curves at these sites may be at least partially interpretable using a 2D model.

However a 140 \( \Omega \cdot m \) conducting top layer has been placed east of the granite outcrop on traverse 11 (see figure 5.32). This layer is included in model 5.24 below site 822, and the surface rocks east of the granite belong to the same lithology as those below site 822. The 140 \( \Omega \cdot m \) layer may terminate south of traverse 11. A comparison of figures 5.16 and 5.35 shows that if this conductor does not lie on traverse 11, the assumption of two dimensionality at site 824 will be true for periods of up to 1s or even longer. Therefore the difference in the sounding curves between the sites maybe due to differences in the degree of three dimensionality between the soundings.

In conclusion it can be stated that there are indications that the bottom of the granite slopes downwards to the South, but the soundings are complicated by conductors perpendicular to the traverse and such a slope cannot be resolved using a 2D model.

The 140 \( \Omega \cdot m \) conductive top layer below 835 cannot extend much further north than is shown in figure 5.24 without increasing the misfit between the model response and the soundings at sites 836 and 834 (even taking into account the reservations about the quality of the data at site 834 outlined above). An increase in the misfit at site 834 would again occur, if the conductor lying approximately 1 km below site 836 was laterally increased. However without further measurements, in the area little further can be said about the strike and the extent of this conductive area.

There are no M.T. measurements on the Lizard, therefore the resistivity of the earth south of site 820 had to be estimated from the previously known geology and geophysics of the area. The outcropping rocks of the Lizard series, mainly consist of serpentine along with small outcrops of granite. From laboratory measurements it is known that these rocks are resistive (1000 to 100000 \( \Omega \cdot m \)) (Telford et. al. 1981). However within the Lizard series there are sizeable areas where the outcropping rock is hornblende schist. Positive magnetic anomalies occur in the area and they appear to be roughly centred on the outcrops of this rock type, (I.G.S., 1965; Allan, 1960). It can therefore be assumed that this rock type is electrically conductive. Since two thirds of the Lizard appears to consist of resistive rocks and one third conductive rocks, an average resistivity value of 800 \( \Omega \cdot m \) was assigned to the whole area.
5.6 Previous Electromagnetic Induction Studies of the Area.

Beamish (1990) describes a set of 14 A.M.T. soundings, over a period range of 0.01 seconds to 100 seconds. The measurements were taken at sites along an east-west traverse situated just south of the centre of the Carnmenellis outcrop (approximately along northing 320). The pseudo sections and sounding curves published in Beamish (1990) show data very similar to soundings recorded during this study at sites close to northing 320.

From studies of the skew (equation 4.27) and dimensional weights (Beamish, 1986), Beamish (1990) states that the soundings on the granite outcrop have a high 1D contribution with a much smaller 2D contribution, whilst the two soundings off the granite outcrop contain significant 2D contributions and there is evidence of 3D induction. These findings somewhat contradict the conclusions drawn from the analysis of the impedance tensors measured during this study given in chapter 4. However, Groom and Bailey (1989) and Groom and Bailey (1991) show that skew and dimensional weights are influenced by galvanic scattering, leading to the calculation of erroneous values. Such scattering certainly affects soundings on and around the Carnmenellis outcrop, therefore some doubt must be cast on estimates of dimensionality when they are solely based on the values of skew and the dimensional weights. Beamish (1990) gives strong evidence that the soundings on the granite were affected by "static shift". Indeed subsequent modelling was performed on a 'granite average' impedance tensor formed using all the soundings on the granite.

Beamish (1990) interprets the data primarily using 1D modelling and inversion techniques. Although 2D forward modelling is reported, the responses of only a quite simple model were investigated. The model consists of the granite batholith, of uniform resistivity, with a shape defined using a gravity model (Willis-Richards, 1986) imbedded in a uniform half space. This model is used to show that the anisotropy of the measured data can be attributed, to a first order, to the geometrical form of the granite. A comparison of the 1D models in appendix A with the 2D model of figure 5.24 shows that because the surrounding rocks and seas have a large influence on the soundings taken on the granite, an accurate resistivity depth profile within the granite can only be obtained using a 2D model incorporating such structures. Thus the interpretation of 1D models as performed by Beamish (1990) must be treated with a degree of caution.

One-dimensional Occam smooth models reported in Beamish (1990) show some similar characteristics to Occam models of the Berdichevsky average at central granite stations (see appendix A). They diverge in two ways.

1) The model reported in Beamish (1990) shows no sign of the fall in resistivity between 3 km and 8 km present in the models in appendix A. This is further confirmation of the conclusions drawn from the Minim and Most Squares analysis. Namely that this fall in resistivity may not exist, and if it does, it is unresolvable.
2) Beamish (1990) places a jump discontinuity at 14.6 km, the base of the granite as defined using other geophysical methods. This discontinuity was not included in the models in this study, since the steep fall in resistivity occurs at greater depths, between 30 and 60 km. A similar fall in resistivity between these depths can be seen in the 1D Occam model presented in Beamish (1990).

5.7 Interpretation of the Two Dimensional Models

The content of this section will be restricted to the interpretation of the shape and resistivity structure of the granite batholith shown in the M.T. models presented in this chapter. Interpretations of the resistivity structure of the surrounding country rock will be given at the end of chapter 6 after the results of three-dimensional thin sheet modelling have been discussed. As stated in section 5.5.2, data greater than 1 second may have been affected by 3D induction, therefore interpretation will be restricted to the batholith and not to the structure below the granite.

The depth to the top of the granite, at sites off the outcrop, derived from the 2D M.T. models, is in good agreement with the 3D gravity model of Willis-Richards (1986). In figure 5.24 the top of the granite is marked as the interface between areas shown in green and areas shown in blue. Since the bottom of a batholith is poorly resolved by all geophysical methods including M.T. (see most-squares models in appendix A) it may be misleading to compare actual values of the depth to the bottom of the Cornubian granite. Even so it is clear, that despite reservations about two-dimensional interpretations of data over the period range where the M.T. soundings become sensitive to structure below the granite, the M.T. models place the bottom of the granite at a greater depth than has been presumed from other geophysical measurements, (BIRPS and ECORS, 1986; Willis-Richards, 1987; Sams and Thomas-Betts, 1988a). The considered opinion of geoscientists at Cambourne School of Mines is that the granite is 14 km thick (Willis-Richards, 1986). Highly resistive structures in figure 5.24 persist to a depth of 40 km, although as stated in section 5.5.2 this is certainly an overestimate, though the degree of overestimation is uncertain. This indicates that either granitic rocks exist at depths greater than 14 km or that there is no sharp contrast between the resistivity of the granite and the rocks below.

The presence of resistive material at depths which are normally assigned to be lower crust contradicts the commonly held view that the lower continental crust is electrically conductive (Hyndman and Shearer, 1988; Haak and Hutton, 1986). Also the findings of the SWAT seismic reflection experiment (BIRPS and ECORS, 1986) show that the lower crust in the region exhibits other common characteristics of the lower continental crust, namely its high degree of seismic reflectivity. However, a seismic refraction experiment in south-west Cornwall, described by Holder and Bott (1971), found a gradual increase in seismic velocity between depths of 12 and 27 km. This finding was interpreted by Holder and Bott (1971) as an increase in density of the lower crust, characteristic of
a gradual change of granite into a granodioritic lower crust. The presence of resistive material at depths below 15 km, indicated by the M.T. soundings described in this study, would confirm such an interpretation of the lower crustal structure below south-west Cornwall.

The fact that the 2D M.T. models show the batholith thickening towards the South has major tectonic implications. This thickening maybe due to the effects of 3D induction being neglected, but if genuine, it resembles the shape of the batholith bottom envisaged by Shackleton et. al. (1982) in their model of granite implantation, which is reproduced as figure 1.11. Thus the 2D M.T. model provides evidence to support the theory that the Cornubian granites originated SSE of their present position and were injected NNW in a sheet like body. A number of geophysical models have a flat bottom to the batholith (Sams and Thomas-Betts, 1988a) or even a model in which the granite bottom slopes downwards towards the North (Willis-Richards, 1987), whilst other gravity models of the Cornubian batholith or the nearby Haig Fras batholith confirm such a thickening of granite to the South (Edwards, 1984b; Bott and Scott, 1964).

Both the 1D and 2D M.T. models of the Carnmenellis granite show an increase in resistivity with depth within the top few km. In the case of the 2D model, this is a very steep rise from 800 Ω.m to 20000 Ω.m between the surface and 4 km depth. In the discussion below, the resistivity depth profile was derived from the 2D model beneath all sites on the granite, except sites 825 and 826. Lateral changes in this profile will be discussed at the end of this section.

Laboratory measurements, summarised in section 1.4.1, indicate that the amount of free water and the temperature are the main controls on the resistivity of granite. Olhoeft (1981) points out that the resistivity of granite falls by many orders of magnitude as water is added.

These experiments show that the resistivity of granites decreases with increasing temperature, see figure 1.4 taken from (Olhoeft, 1981). Temperature gradients in the area range from 30 °C/km to 35 °C/km (Thomas-Betts and Sams, 1991; Pearson et.al., 1989) and heat flow models indicate temperatures of 200 °C at 6 km and 370 °C at 10 km (Sams and Thomas-Betts, 1988a). Figure 1.4 indicates that even for 'wet' granite, such a temperature rise with depth would result in a fall of resistivity of at least a decade between the surface and a depth of 10 km. Therefore it is clear that other physical parameters are affecting the resistivity of the Carnmenellis granite and counteracting the effects of temperature.

From the evidence of laboratory experiments, the only change in physical parameters capable of causing the dramatic increase in resistivity of the Carnmenellis granite with depth shown in the 2D M.T. models, is a reduction in the amount of free water. Figure 1.5 taken from Brace and Orange (1968) shows the effects of uniaxial confining pressure reducing the porosity of water saturated granite and thus increasing its resistivity. Olhoeft (1981) in summarising this diagram states that the increase of lithostatic load within the crust can only cause an order of magnitude increase in the resistivity of granite. Thus it appears that the increase in
resistivity cannot be wholly accounted for by a reduction in porosity with increasing pressure. However as Beamish (1990) points out, the role of macro sized fluid filled cracks on the bulk resistivity of a granite batholith is necessarily neglected from laboratory experiments.

Evidence that such cracks and fissures have a significant effect on the bulk resistivity of the Carnmenellis granite comes from the borehole resistivity logs measured in the 2.8 km deep HDR drill holes described in Pearson et. al. (1989) and summarised in section 1.5.4. These show a very spiky response, with portions of the log having resistivities close to and above 20 000 $\Omega\cdot m$ and a high density of resistivity lows of a few hundred to a few thousand $\Omega\cdot m$. These resistivity lows coincide with the detected positions of cracks. In agreement with Beamish (1990) it is proposed that the bulk resistivity of granite at depths less than a few km is dominated by fluid filled joints and that the increase in bulk resistivity of the granite in the top 4 km is caused by closing of these fractures due to the confining pressure of the lithostatic load. Evidence for a reduction in the number of fractures, joints and cracks with depth comes from a number of borehole studies in the area (Heath, 1985; Pearson et. al., 1989).

In contrast to the rapid non-linear increase in resistivity of the Carnmenellis granite in the top 4 km, the resistivity of the granite below this depth stays constant and therefore shows linear behaviour. From the measurements shown in figure 1.5, Brace and Orange (1968) indicate that such a change in behaviour in the resistivity of a rock indicates a transition from crack dominated to pore dominated electrical conduction. Using predictions of the confining pressure from stress measurements in HDR boreholes (Pine et. al., 1990) and the lab measurements shown in figure 1.5, Beamish (1990) predicts that such a transition would take place at depth of about 6 km. However Beamish (1990) also points out that such a comparison of uniaxial pressure in the lab with the complicated, anisotropic, pressures present in the Carnmenellis granite is less than satisfactory. The change in behaviour at 4 km of the resistivity depth profile derived from the 2D M.T. model provides further evidence to indicate there is an absence of joints below such a depth. Here joints are simply defined as a feature that can close and thus not support ionic conduction. The closing of joints at 4 km is in agreement with the interpretations of the microseismicity around the HDR borehole given in Green et. al. (1988).

The persistence of relatively conductive material (1000 $\Omega\cdot m$) down to depths of 4 km below sites 824 and 825 in figure 5.24 reflects the increasing density of joints, cracks and fissures in the northern half of the Carnmenellis outcrop, (Whittle, 1989; Edmunds et. al. 1985). In explaining lateral differences in the resistivity depth profile across the granite, emphasis must be placed on the much higher than average saline fluid flow rates through cross-courses and Elvan dykes, (Heath, 1985; Edmunds et. al. 1985). Figure 1.9 shows a greater density of these types of fractures and veins in the northern half of the Carnmenellis granite. From mine workings and bore holes on the granite it is known that cross-courses are still open down to depths greater than 2 km (Whittle, 1989).
Chapter 6

Three Dimensional Thin Sheet Modelling

6.1 Introduction

From the discussion in chapter 4, it is clear that only three dimensional modelling will successfully characterise the inductive effects of bodies such as the near-by seas, the surrounding country rock and the shallow lying mineral veins. Unfortunately numerical schemes that solve the 3-D induction problem require huge amounts of computer time and storage space and hence they are only used to study highly simplified models.

However the conductivity anomalies mentioned above are all relatively shallow and therefore can be approximated by a thin sheet resting at the surface. By compressing 3-D conductivity anomalies into an mathematically thin sheet of variable integrated conductivity and placing this on top of a half space whose conductivity only varies with depth, numerical calculations need only be performed in the two horizontal directions, thus reducing the problem to a manageable computation.

6.2 The Thin Sheet Algorithm

The boundary conditions for induction in an mathematically thin sheet were developed by Price (1949). Subsequently 3-D numerical algorithms using the thin sheet approximation have been presented, by among others, Vasseur and Weidelt (1977) and Dawson and Weaver (1979). They have been widely used to study regional induction in many parts of the world, (Weaver, 1982), (Agarwal and Weaver, 1989), (Mareschal et al., 1987).

The particular code used in this study is based on the work of Mckirdy, Weaver and Dawson (1985) and Dawson and Weaver (1979). The mathematical model used in the algorithm is shown in figure 6.1. It shows an mathematically thin sheet in the plane z=0 of variable conductance at the surface of an n layered half space (z > 0), with the last layer extending to infinity. The thin sheet and the layered half space are in electrical contact and the thin sheet approaches a 2-D limit at infinity, or in practice the edge of the numerical grid, see figure 6.1.

The boundary conditions for a thin sheet developed by Price (1949) are based on the fact that the tangential (in this case horizontal) magnetic field across the sheet boundary is discontinuous by an amount proportional to the density of the surface current flowing through the sheet. This can be expressed as
Figure 6.1
The Mathematical Model for The Thin Sheet Algorithm from (McKirdy, Weaver and Dawson, 1985)
\[ H_h(r, 0-) - H_h(r, 0+) = 2T(r) \bar{z} \times E_h(r) \] (6.1)

Where:
- \( H_h(r, 0-) \) is the horizontal component of the magnetic field at either the top surface of the thin sheet (\( H_h(r, 0-) \)) or the bottom surface (\( H_h(r, 0+) \))
- \( E_h \) is the horizontal component of the electric field
- \( T \) is the integrated conductivity of the thin sheet
- \( \bar{z} \) is a unit vertical vector

The algorithm works by finding surface integral formulae to express the magnetic field as a convolution of the electric field and an admittance tensor. (Dawson and Weaver, 1979).

Numerical evaluation of these integrals is described in Dawson and Weaver (1979). The region to be modelled is discretised into a set of \( N \) by \( N \) cells which form the numerical grid.

6.3 Design of the Model

Figure 6.2 shows the area of study along with 25, and 50m. contours of the bathymetry. Also shown are the contours of depth to the top of the granite in km., taken from a 3-D gravity model, (Willis-Richards, 1987).

Initially two models were chosen:
1) A model looking only at the effects of the conductivity contrast arising from the land and sea-water of variable depth.
2) A model which also takes into account the shape of the granite and the surrounding country rock, as described by models calculated using other geophysical methods namely gravity and seismics.

Conductivities of 3.2 S/m, 0.00033333 S/m and 0.003333 S/m were chosen for the sea, the granite and the surrounding country rock respectively.

For the first model a thin sheet of only 100m thick could be used, but obviously for the second model the thickness had to be equal to that of the granite batholith. As discussed in Chapters 3 and 6 the bottom of an intrusive body is poorly resolved by any geophysical method including M.T. However as mentioned in chapter 3 the best estimate for the depth to the bottom of the granite is between 14 -16 km. and hence the thickness of the thin sheet was set at 15km.

The thickness of the country rock above the granite in each cell was determined using the 3-D gravity model of Willis-Richards (1987). In the interests of simplicity conductances were calculated for country rock thicknesses of 0, 4, 8, 12 and 15 km. and for sea depths of 0, 12.5, 20, 37.5, 75, and 100m. This resulted in 6 different conductances in model 1 and a further 24 in model 2. The conductivity structure in each of the cells is shown in figure 6.3. The conductances were assigned to the centre of each cell and coded by digital numbers. The conductance values are shown in table 6.1. Figure 6.4 shows the simpler model used to study the sea - land conductivity contrast, figure 6.5 shows the model where the surrounding
Figure 6.2
A Map Showing the Bathymetry of the Seas Around Cornwall and the Depth to the Top of the Granite Batholith as Calculated Using a 3D Gravity Model, (Willis-Richards, 1987).

Coastline shown as a dotted line.
Bathymetry measured in (m) shown as a thick solid line.
Granite outcrop shown as a line of dots and dashes.
Depth to the top of the granite batholith measured in (km) shown as a thin solid line.
DEPTH TO THE TOP OF THE GRANITE (KM) FROM 3D GRAVITY SURVEY (WILLIS—RICHARDS, 1987) AND BATHYMETRY (M)
Figure 6.3 The Conductivity Structure of the Cells
Table 6.1 Showing The Conductance of Cells Used In Thin Sheet Program

<table>
<thead>
<tr>
<th>Cell Number</th>
<th>Conductance (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.03333</td>
</tr>
<tr>
<td>2</td>
<td>40.2917</td>
</tr>
<tr>
<td>3</td>
<td>64.2667</td>
</tr>
<tr>
<td>4</td>
<td>120.2083</td>
</tr>
<tr>
<td>5</td>
<td>240.0833</td>
</tr>
<tr>
<td>6</td>
<td>320.0000</td>
</tr>
<tr>
<td>7</td>
<td>5.0000</td>
</tr>
<tr>
<td>8</td>
<td>17.0000</td>
</tr>
<tr>
<td>9</td>
<td>29.0000</td>
</tr>
<tr>
<td>10</td>
<td>41.0000</td>
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<td>11</td>
<td>50.0000</td>
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<tr>
<td>16</td>
<td>90.0000</td>
</tr>
<tr>
<td>17</td>
<td>68.3333</td>
</tr>
<tr>
<td>18</td>
<td>80.9333</td>
</tr>
<tr>
<td>19</td>
<td>92.9333</td>
</tr>
<tr>
<td>20</td>
<td>125.0000</td>
</tr>
<tr>
<td>21</td>
<td>136.8750</td>
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<tr>
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<td>148.8750</td>
</tr>
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<td>23</td>
<td>160.8750</td>
</tr>
<tr>
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<td>29</td>
<td>289.7500</td>
</tr>
<tr>
<td>30</td>
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<tr>
<td>31</td>
<td>171.5000</td>
</tr>
<tr>
<td>32</td>
<td>254.8333</td>
</tr>
<tr>
<td>33</td>
<td>265.3333</td>
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<td>34</td>
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<td>35</td>
<td>495.3333</td>
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<tr>
<td>36</td>
<td>14.0000</td>
</tr>
<tr>
<td>37</td>
<td>64.0000</td>
</tr>
</tbody>
</table>
FIGURE 6.4: THIN SHEET MODEL USED TO STUDY EFFECTS OF THE SURROUNDING SEAS

FIGURE 6.5: THIN SHEET MODEL USED TO STUDY EFFECTS OF SURROUNDING SEAS AND THE SHAPE OF THE BATHOLITH
country rock has been taken into account. All conductivity contrasts were kept well away from the grid edges to ensure the condition of two dimensionality was kept.

Figure 6.6 shows the layered structure below the thin sheet as used to study the effects of the surrounding seas and figure 6.7 the underlying layered structure which also takes into account the shape of the batholith. Both layered half spaces, which are the same except for the top layer, are taken from layered one dimensional models for sites in the centre of the Carnmenellis outcrop shown in figures A.12 and A.13.

6.4 Restrictions on the Range of Periods a Model Can Investigate

Schmucker (1970) in investigating the conditions in which the thin sheet approximation can be applied, concluded that:

1) The thickness of the surface layer should be small (to the first order) when compared with the skindepth of e.m. fields within the underlying layer.
2) The thickness of any material within the thin sheet should be small (to second order) in comparison with the skindepth of e.m. fields within it.

These two conditions limit any particular model to the study of periods above a certain value.

3) The condition of two dimensionality at the edge of the numerical grid provides the upper boundary to the range of periods, a model can investigate.

If condition 2 was to be in any way met, the thickness and conductivity of the country rock, restricted the model to the study of periods above 3 seconds. The skin depth of e.m. fields within the country rock at 3s is 15.1 km and in some places this rock is 15 km thick. Hence results gained from the model at this period were treated with caution.

Obviously the smaller the distance between each node, the higher the resolution. But this is counter balanced by limited computer storage restricting the thin sheet to a 30 by 30 grid, coupled with the condition of two dimensionality at the grid edge. For this closely spaced survey over a restricted area, resolution was important and the smallest possible node spacing of 5 km. was chosen. This restricted the model to the study of periods below 10s. Therefore parameters were calculated at three periods 3, 5 and 10 s.

6.5 The Parameters Calculated

Calculations were performed for the two modes of induction, by polarizing the primary regional magnetic field in two perpendicular directions, east and north. The latter was produced by rotating the model not the field. With the inducing field due east, an E - polarization limit is reached as x ->\infty. By repeating the calculations with the primary field
Figure 6.6: The Layered Structure Below the Thin Sheet Model used to study the Effects of the Surrounding seas

<table>
<thead>
<tr>
<th>Depth</th>
<th>Conductivity (s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 km</td>
<td>NON UNIFORM THIN SHEET</td>
</tr>
<tr>
<td>0.1 km</td>
<td>$\sigma_1 = 0.00033333$</td>
</tr>
<tr>
<td>15 km</td>
<td>$\sigma_2 = 0.00066667$</td>
</tr>
<tr>
<td>50 km</td>
<td>$\sigma_3 = 0.00125$</td>
</tr>
<tr>
<td>150 km</td>
<td>$\sigma_4 = 0.00025$</td>
</tr>
<tr>
<td>1000 km</td>
<td>HALF SPACE</td>
</tr>
<tr>
<td></td>
<td>$\sigma_5 = 0.001$</td>
</tr>
</tbody>
</table>

Figure 6.7: The Layered Structure Below The Thin Sheet Model Used to Study The Effects of the Surrounding Seas and the Shape of the Batholith

<table>
<thead>
<tr>
<th>Depth</th>
<th>Conductivity (s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 km</td>
<td>NON UNIFORM THIN SHEET</td>
</tr>
<tr>
<td>15 km</td>
<td>$\sigma_1 = 0.00066667$</td>
</tr>
<tr>
<td>50 km</td>
<td>$\sigma_2 = 0.00125$</td>
</tr>
<tr>
<td>150 km</td>
<td>$\sigma_3 = 0.00025$</td>
</tr>
<tr>
<td>1000 km</td>
<td>HALF SPACE</td>
</tr>
<tr>
<td></td>
<td>$\sigma_4 = 0.001$</td>
</tr>
</tbody>
</table>
directed due north (the B-polarization mode), the response for a regional field of any orientation could be obtained.

All the parameters below are described in more detail in Agarwal and Weaver (1991).

a) Anomalous Current Vectors

The anomalous current at a node point \( J_a \) is defined following McKirdy and Weaver (1983) as

\[
J_a = I - J_n
\]  

(6.2)

where

- \( I \) - total current density at that node point
- \( J_n \) - can be defined as either:
  a) The total current density at a 'normal' station, i.e. a station well way from any conductivity anomalies.
  b) The total current density calculated assuming the regional magnetic field to be unity in dimensionless units, i.e. \( H_x=1 \), \( H_y=0 \) for inducing magnetic field directed northwards and \( H_x=0 \), \( H_y=-1 \) for the inducing field directed westwards.

The anomalous current is expressed as real and imaginary arrows defined in a similar way to real and imaginary induction arrows, see equations 3.18-3.21.

b) The Horizontal Disturbance Vector and Equivalent Current Systems

A linear relationship between the horizontal components of the magnetic field \( (H_x, H_y) \) produced by a 'normal' or 1-D conductivity structure and those produced by lateral inhomogeneities \( (H_a^x, H_a^y) \) has been postulated by Schmucker (1970).

Therefore following Agarwal and Weaver (1991)

\[
H_a^x = h_x H_x + h_y H_y
\]  

(6.3)

\[
H_a^y = d_x H_x + d_y H_y
\]  

(6.4)

\( h_x, h_y, d_x \) and \( d_y \) are frequency dependent transfer functions. A time dependence of \( e^{-i \omega t} \) has been assumed in all vector fields. By assuming unit inducing magnetic fields directed northwards and then westwards a horizontal disturbance magnetic field defined as:

\[
H_a = H_a^x x + H_a^y y
\]  

(6.5)

can be calculated. \( x \) and \( y \) are north and east unit vectors respectively.

Using the boundary condition relating the tangential magnetic field across the sheet boundary to the density of the surface current flowing in the sheet, the rotation of vector \( H_a \) anti-clockwise through 90 degrees will produce a vector describing surface current density. This surface current density will be that of an equivalent current system which
would give rise to the anomalous magnetic field $H^a$, when superimposed on the unperturbed west or south flowing currents in the thin sheet. (Schmucker, 1970). This current density is again calculated as real and imaginary current arrows defined in a similar way to real and imaginary induction arrows, see equations 3.18 - 3.21.

c) Induction Arrows Plus Apparent Resistivity and Phase.

Induction arrows, apparent resistivity and phase are all defined in chapter 3. However it should be noted that the model highly simplifies the underlying structure into a layered half space and the irregular coastline and batholith shape into a rectangular grid. Apparent resistivity and phase are especially sensitive to these conductivity contrasts and hence a direct comparison between measured data and model calculated parameters is rendered meaningless and was thus not performed. However trends in the apparent resistivities and phases produced by the thin sheet algorithm can give indications of how conductivity anomalies modelled in the thin sheet affect the measured data.

As similar arguments hold for induction arrows, (see Ramaswamy et al., 1985 and Agarwal and Weaver, 1989), therefore matching the absolute length of arrows was not attempted. Though direction and relative length can be accurately modelled.

d) Comparison of Equivalent and Anomalous Current Arrows.

As equivalent current systems assume there is no electrical contact between the thin sheet and the underlying structure, they are not a true representation of the current system, (Weidelt,1977). As vertical leakage between the thin sheet and the underlying structure exists in the vector field of anomalous currents, the field depicted by these 2-D anomalous current arrows is not solenoidal. (McKirdy and Weaver, 1983).

Equivalent current arrows, by excluding vertical leakage currents which are poloidal and hence produce no external magnetic field, indicate more clearly the conductors in the thin sheet producing the anomalous magnetic field. (Jones, 1983).

6.6 Comparison of Measured Induction Arrows and Those Calculated From Thin Sheet Models

For reasons stated in section 6.5, induction arrows were the principal parameter used to compare the thin sheet model calculations to the measured data.

Figure 6.8 shows real and imaginary Parkinson induction arrows calculated at 5 seconds. In this case, the calculations were performed using the thin sheet model shown in figure 6.4, henceforth referred to as model 1. Only parameters calculated at 5s will be discussed, as the results obtained for 3 and 10 seconds were very similar to those at 5 seconds and the conditions for thin sheet modelling were most easily met at this period. From figure 6.8, it is clear that all the real induction arrows
FIGURE 6.8: PARKINSON INDUCTION ARROWS AT 5 SECONDS MEASURED (THICK), SEA MODEL (THIN)
produced by model 1 at node points adjacent to measurement sites are pointing in directions between west and south. Model 1 only takes account of the lateral conductivity contrast between the land and the much more conductive sea. At 5s the induction arrows appear to be sensitive to directional differences in bathymetry. Off the southern and western coasts of Cornwall the sea floor dips more steeply than the sea floor off the northern coast, see figure 6.2. Even though the thin sheet model is crude, figure 6.8 shows that the pattern of measured induction arrows cannot be explained by bathymetry alone.

Figure 6.9 shows a similar comparison of Parkinson induction arrows, this time for arrows calculated from the thin sheet model shown in fig 6.5 (model 2). This particular thin sheet model also takes into account the shape of the granite batholith as defined by other geophysical methods and represents the best model of the geological structure in south-west Cornwall from previously existing knowledge. The fit between measured induction arrows and those calculated from model 2 is much improved from that shown in figure 6.8 reflecting the fact that the measured arrows at 5s are sensitive to the conductivity contrast between the granite and the much more conducting Killas.

However significant misfits still exist. The real arrows at four sites at the northern edge of the granite batholith, sites 815, 826, 834 and 835 all point in the same direction, N35°W. Yet the arrows derived from model 2 do not match this pattern, pointing towards the nearest coast or the edge of the batholith. This indicates that a region of enhanced conductivity not included in model 2 exists to the north of the Carnmenellis outcrop. This conclusion is confirmed by the real arrows measured at sites 833 and 812 which both point N2°W, yet a model based on existing geophysical knowledge indicates that these arrows should point to the south. Other areas of enhanced conductivity indicated by a comparison of measured and calculated arrows in figure 6.9 are just to the south of the Carnmenellis granite outcrop indicated by real arrows measured at sites 810, 811 and 823, and one to the north of site 836.

Further modelling was performed with the thin sheet model (model 3) shown in figure 6.10 producing induction arrows which most closely matched the pattern of measured arrows. See figure 6.3 for the conductivity structure of the cells. The induction arrows produced by model 3 are shown along with the measured induction arrows in figure 6.11. Taking into account the lack of geographical resolution within the model, all the measured arrows are matched to within a reasonable misfit. A lack of resolution in the model meant that calculated induction arrows adjacent to some measurement sites pointed in different directions to the measured arrow. This most especially applied to site 831, but also to sites 810, 820 and 833. However by projecting the calculated induction arrows at the nearby node points to the measurement location a reasonable match between measured and calculated arrows could be achieved.
FIGURE 6.9: PARKINSON INDUCTION ARROWS AT 5 SECONDS MEASURED (THICK), SEA AND BATHOLITH MODEL (THIN)
FIGURE 6.10: THIN SHEET MODEL THAT PRODUCED 'BEST FITTING' INDUCTION ARROWS TO THOSE MEASURED AT 5S
FIGURE 6.11: PARKINSON INDUCTION ARROWS AT 5 SECONDS MEASURED (THICK), "BEST" FIT MODEL (THIN)
6.7 Interpretation of the Anomalous Current Arrows, Impedances, and Equivalent Current Arrows, Calculated Using the Thin Sheet Models.

Equivalent current arrows calculated from model 2 for the two modes of induction are shown in figure 6.12 and 6.13. A feature of both figures is clearly illustrated by the real arrows in figure 6.12. Real arrows calculated at node points positioned on-land and either west of easting 175 or south of northing 040, point a few degrees west of south, whilst to the north or east of these lines, the real arrows point a few degrees south of west. These geographical lines are close to the eastern and northern edges of the Carnmenellis outcrop. Changes in orientation of the other equivalent current arrows illustrated in figures 6.12 and 6.13 take place along these same geographical lines. Another feature of these plots is the small magnitudes of the imaginary arrows calculated at node points on land. Many of the real, equivalent current arrows calculated at node points in the sea follow the coast, turning around the tip of the peninsula. Here, the arrows have the largest magnitude, due to a sea granite contact at Lands End. This pattern is followed more closely by the E-pol equivalent arrows.

The equivalent current arrows reveal the conducting paths followed by the induced currents that give rise to the observed anomalous magnetic field. Figure 6.12 shows channelling of equivalent current around the coast. Figure 6.12 shows real equivalent current arrows calculated at node points on the resistive structure of the Carnmenellis granite point SSW when the regional field is orientated eastwards. Figure 6.13 shows that with the regional field orientated northwards real equivalent current arrows are all parallel to the long axis of the land except on the Lizard peninsula, where equivalent current is channelled from the west coast to the east. It should be noted that equivalent current flows around resistive areas such as granite outcrops.

The horizontal disturbance vector can be visualised by rotating the equivalent current arrows 90° clockwise. Therefore the real equivalent arrows at node points adjacent to measurement sites on the granite show that the magnetic field produced by the anomalous regions, the surrounding seas and Killas, suppresses any regional field orientated between east and north. This is illustrated in plots of anomalous current arrows shown in figures 6.14 and 6.15.

The extra conductors present in model 3 complicate the pattern of the equivalent current arrows shown in figures 6.16 and 6.17. Equivalent current still flows around the southern coast of Cornwall when the regional field is polarized eastwards. However channelling of equivalent current from the land into the sea at two places on the northern coast interrupts the current flow along this coast. Anomalous current is channelled between the estuary of the river Fall (block [180,35]) and the
FIGURE 6.12: E-POL EQUIVALENT CURRENT ARROWS CALCULATED FROM THE SEA AND BATHOLITH MODEL AT 5S
FIGURE 6.13: H-POL EQUIVALENT CURRENT ARROWS CALCULATED FROM THE SEA AND BATHOLITH MODEL AT 5S
Grid Eastings

Grid Northings

FIGURE 6.14: E-POL ANOMALOUS CURRENT ARROWS CALCULATED FROM THE SEA AND BATHOLITH MODEL AT 5S
FIGURE 6.15: H-POL ANOMALOUS CURRENT ARROWS CALCULATED FROM THE SEA AND BATHOLITH MODEL AT 5S
FIGURE 6.16: E-POL EQUIVALENT CURRENT ARROWS
CALCULATED FROM THE 'BEST FIT' MODEL AT 5S
FIGURE 6.17: H-POL EQUIVALENT CURRENT ARROWS CALCULATED FROM THE 'BEST FIT' MODEL AT 5S
northern Cornish coast, see figure 6.16. The evidence for equivalent current channelling along the same path when the regional magnetic field is orientated northwards, is less clear, but is still present in figure 6.17. The current is obviously being channelled through conducting anomalies that exist to the north of the Carnmenellis outcrop. The flow of current along the coast is interrupted in such a way that equivalent current flows in a circle around Lands End. A further interruption of the flow along the northern coast when the field is polarized easternwards, is provided by equivalent current being channelled through the conductive bodies in model 3 positioned north of site 836.

Both figures 6.16 and 6.17 show that anomalous fields generated at the node point in the centre of the Carnmenellis outcrop and at node points on the southern edge of the outcrop oppose the regional field. This is also illustrated in plots of the anomalous current arrows, figure 6.18 and 6.19. These figures also illustrate the anomalous current channelling, in both induction modes, from the river Fall across the north-eastern edge of the Carnmenellis block to the northern coast. At node points to the west of the northern half of the Carnmenellis outcrop, when the regional field is polarized eastwards, the horizontal disturbance vector adds substantially to the regional field. When the regional field is orientated in a perpendicular direction, the horizontal disturbance has a small positive (points north) magnitude. Hence the magnetic field is enhanced here.

6.8 Interpretation of the Measured Parkinson Arrows

Despite the fact that results from thin sheet modelling could only be obtained for less than a decade of data, the responses of the structures identified using the modelling results, influence measured arrows over a much longer period range.

From a period of about 0.01 seconds the induction arrows measured at sites on the granite are sensitive to the lateral conductivity contrast between the granite and the surrounding rocks. The best illustration of this is figure 3.29. Over the period range of 0.03 to 0.1 seconds all the real induction arrows measured on the granite, point in directions perpendicular to the nearest edge of the Carnmenellis outcrop. Those real arrows measured at sites off the granite outcrop either point towards the sea (site, 810) or a closer lateral conductivity contrast such as a valley, (site 822).

The influence of the conducting anomalies to the north and east of the granite can be seen from periods of 0.1 seconds, (figure 3.30). For instance at sites 831, 834 and 835 the real arrows rotate in an anti-clockwise direction so that between 0.3 and 1.0 seconds (figure 3.31), real arrows at these sites are orientated between directions perpendicular to Carnmenellis outcrop edge and directions pointing to the region of enhanced conductivity. From 0.03 seconds to 10s the real Parkinson induction arrow at site 815 is orientated around 35° west of north, with a
FIGURE 6.18: E-POL ANOMALOUS CURRENT ARROWS CALCULATED FROM THE 'BEST FIT' MODEL AT 5S
FIGURE 6.19: H-POL ANOMALOUS CURRENT ARROWS CALCULATED FROM THE 'BEST FIT' MODEL AT 5S
variation of only $8^\circ$ over the period range. However other lateral conductivity contrasts, such as the sea off the northern coast, and more importantly the shape and position of the granite-Killas contact (figure 6.9), may over parts of this period range reinforce the effects of the conducting anomalies.

For the period range 1-10 seconds the pattern of induction arrows can be explained using model 6.10. Induction arrows measured at sites south of the Carnmenellis granite, or near to its southern edge, are clearly sensitive to the areas of enhanced conductivity south of the outcrop shown in figure 6.10. However at longer periods for sites 820, 821, 822, 824, 831 and 810 the steep dip of the sea floor to the south west of the Lizard peninsula is clearly the strongest influence on the measurements.

6.9 Comparison of Measured Regional Azimuths and Those Calculated From Thin Sheet Models

Due to the simplifications of the thin sheet modelling algorithm, only the comparison of trends in the apparent resistivities and phases are practicle. Therefore an appropriate parameter for such a comparison is the regional azimuth. Figures 6.20 and 6.21 both show the regional azimuths measured at 5s obtained from the tensor decomposed using the methods of Groom and Bailey (1989) with no constraints placed on the parameters. Regional azimuths calculated from the thin sheet models of the batholith and sea and the batholith, sea and extra conductors are shown in figures 6.20 and 6.21 respectively. Azimuths for the thin sheet models were calculated using the methods of Swift (1967).

The agreement between the measured and calculated regional azimuths is generally poor for both thin sheet models. Only at site 821 is there agreement between measured and modelled azimuths and then only for the thin sheet model with extra conductors. Though at sites south of the granite outcrop and at site 822, just north of southern edge of the outcrop, the agreement between measured and modelled regional azimuths shown in figure 6.21 is better than at sites further north.

The modelled regional azimuths in figures 6.20 and 6.21 north east of the Carnmenellis outcrop are a further indication that current is channelled from the Fal river estuary north west to the north coast.

6.10 Conclusions

Unfortunately due to logistical reasons, the thin sheet modelling had to be performed before two-dimensional analysis of the magnetotelluric data was completed. Therefore the resistivity of the granite used in the thin sheet cells and the values in the layered half space below the thin sheet were based on the interpretation of the M.T. data using only one-dimensional models. As stated in chapter 5, 1D modelling underestimates the resistivity of the granite. Induction arrows
FIG 6.20: AVERAGE REGIONAL AZIMUTH, PLUS ERROR SPREAD FOR 5S. MEASURED (THICK), MODEL 2 CALCULATED (THIN)
FIG 6.21: AVERAGE REGIONAL AZIMUTH, PLUS ERROR SPREAD FOR 5S. MEASURED (THICK), BEST MODEL CALCULATED (THIN)
are sensitive to lateral conductivity contrasts, therefore if the resistivity of the granite increases, the conductance of areas of enhanced conductivity need not be as great as indicated in table 6.1. However changes in the resistivity of the granite will not change the locations of these areas of enhanced conductivity.

Figure 6.22 shows areas of enhanced conductivity (thin boxes) that needed to be added to the model based on the geology as already known, to produce a reasonable fit to the measured induction arrows. Each of the three areas of enhanced conductivity correlate with more conductive rocks detected using magnetotelluric measurements. From M.T. data it is postulated that below site 835, an 140 Ω.m surface conductor exists, see figure 5.24. Site 835 is situated within the square, eastings 175 to 180, northings 35 to 40. This is also a block of enhanced conductivity in the best fitting thin sheet model. Surface conductors of 60 Ω.m also exist in the model derived from M.T. measurements below site 821. Surface rocks at this site belong to the same lithological group as the surface rocks within the more conductive block south of the Carnmenellis outcrop. A very conductive block, (less than 10 Ω.m) one kilometre thick has been detected by M.T. soundings at a depth of 1 km below site 836. Site 836 is situated about 2 km south-west of the conductive zone situated in north-east corner of the thin sheet model.

The conductive zones directly to the North of the Carnmenellis outcrop cannot be explained purely as a lithological sequence. The M.T. models imply that these conductors form the top layer. The surface rocks north of the granite are classed as Devonian Mylor slates and sandstones and this same rock sequence surrounds the whole of the granite outcrop, see figure 1.8. However thin sheet modelling indicates that within this rock sequence there are areas of enhanced conductivity.

Aeromagnetic maps (I.G.S., 1965) show a similar signature across all the land north of the Lizard. Therefore the presence of magnetic materials must be ruled out as a cause of the enhanced conductivity of these regions.

However, a highly mineralised zone exists to the North of the Carnmenellis granite, see figure 6.22. It was formed during the implantation of the granite, see chapter 1 for details. Within the area of mineralisation there is a high density of greatly fractured Elvan dykes, along with many cross courses and veins. The work of Heath (1985) described in section 1.5.4 shows that flow rates within cross courses and Elvan dykes are well above average values for the area. Electricity passes through crustal rocks electrolytically and therefore Archie’s law (equation 1.1) can be applied. This equation shows that increasing the porosity of a rock will decrease its bulk resistivity. It is proposed that the bulk resistivity of the mineralised region north of the granite outcrop is controlled by these regions of highly fractured fluid filled rock. The density of Elvan dykes within the mineralised region increases towards the West. In the “best” fitting model, the squares immediately north-west of the Carnmenellis outcrop are twice as conductive as the square directly
FIG 6.22: AREAS OF ENHANCED CONDUCTIVITY (THIN) PLUS ELVAN VEINS (THICK), CROSS-COURSES (DOTTED)
north of the granite outcrop. Hence there is some correlation between the conductivity of the Killas north of the granite and the density of Elvan dykes in the area.

The surface rocks in the two squares of enhanced conductivity to the south of the Carnmenellis granite belong to the Gramscatho formation. This formation consists of dark grey slates interbedded with sandstone turbidites. These two squares cover the bulk of the area of the Gramscatho series to the south of the Carnmenellis outcrop. The M.T. sounding at site 821 is also on the Gramscatho series and the 2D M.T. model indicates a more conductive surface layer of 60 $\Omega\cdot m$ compared to surface layers adjacent to the Gramscatho series, these have resistivities of 140 $\Omega\cdot m$ and 200 $\Omega\cdot m$. Unfortunately data quality of the magnetovariation sounding at site 821 for a period of 5 seconds was contaminated by noise and induction arrows could not be calculated. It is concluded that the cause of the enhanced conductivity of the region south of the Carnmenellis granite is the presence of a more conductive rock type within the Gramscatho formation.

Without further measurements close to site 836, it is impossible to give an interpretation of the conductive region north-east of the Carnmenellis granite.
Chapter 7

Summary And Suggestions for Further Work

This chapter gives a summary of the main conclusions of the thesis and then gives suggestions for further work.

7.1 Summary

Twenty-one magnetotelluric soundings were taken at sites on the Carnmenellis granite or the surrounding Killas. The sites were positioned along two traverses running across the granite outcrop in a SSW-NNE direction and one traverse running in a SSE-NNW direction. At the majority of sites the M.T. soundings were in the period range of 0.0078 seconds to 62.5 seconds, but at two sites long period measurements of up to 3000 seconds were successfully recorded. At 16 sites measurements of the variation of the vertical magnetic field were also taken.

Up, down and average biased impedance tensors along with magnetic field transfer functions were calculated using the robust code of Chave and Thomson (1989). Data quality varied from site to site, but from a number of soundings very smooth estimates of the apparent resistivity and phase curves were calculated. However various forms of cultural noise contaminated many of the soundings. Reasonably smooth earth response functions could only be obtained from these data by modifying the robust code to reject, before any least squares regression was performed, estimates which didn't meet criteria concerning power in the magnetic channels and coherency.

The measured impedance tensors were analysed in detail using the decomposition methods of Groom and Bailey (1989) and Bahr (1987). All the data were found to be galvanically distorted and the vast majority were at least two dimensional. The impedance tensors at a number of sites for periods greater than 1s and all the data recorded at periods greater than 200 seconds show the effects of three dimensional induction. At least a decade of data at each site show characteristics of strong galvanic distortion and therefore the full Groom and Bailey decomposition was applied to each data set in order to recover the correct model parameters.

Emphasis was laid on the study of the impedance tensor parameters on a regional scale. A comparison of the regional azimuths derived from the Groom and Bailey decomposition and the more routine method of Swift (1967) indicates, that between 0.03 seconds and 10 seconds, soundings taken to the east and south of the Carnmenellis outcrop are significantly more distorted by galvanic electric charges than soundings in the middle of the outcrop. This indicates that the main cause of galvanic distortion is the build up of charge on the steeply dipping eastern and southern slopes of the batholith.
Many of the soundings taken at periods below 0.07 seconds indicate that the anisotropy in the M.T. field is caused by electric current being channelled along fluid filled cracks. This conclusion was reached from the evidence that the regional azimuths measured at these periods at many of the on granite sites are nearly parallel to the known direction of the joint system in the granite. Between 0.01 and 0.1 seconds the M.T. fields sense the edge of the Carnmenellis granite. However since the outcrop is nearly circular and the distances from the sites to the edge of the outcrop vary, a complicated pattern of regional azimuths is produced.

Between 0.1 seconds and 10 seconds a definite pattern of regional azimuths for soundings taken on the granite emerges. A regional azimuth of about N 45° E was measured at sites in the north east corner of the granite outcrop, whilst near the southern edge of the outcrop regional azimuths closer to 5° were measured. For sites between these two areas, the regional azimuths are between the two extremes. The regional azimuths calculated from the thin sheet models of the shape of the granite batholith, which also include the surrounding seas, produce a significantly different pattern to the measured azimuths. Therefore structures at depth must be influencing to a large extent the regional azimuths over this period range.

Two dimensional analysis of the M.T. soundings imply that the bottom of the batholith slopes downwards to the south. If this is true and is maintained throughout the length of the batholith, currents would flow along the bottom of the granite in a direction parallel to the axis of the batholith. As the soundings at each site sense the bottom of the granite the regional azimuth will swing towards a direction parallel to the main axis of the batholith. As the period increases, the influence of this lateral contact at the bottom will increase and the off diagonal responses will become increasingly anisotropic. The further north a site is, the shorter the period at which the structure below the granite influences the sounding. The measured regional azimuths between 0.1 and 10 seconds show some of the above characteristics. However at most of the soundings the azimuths swing towards a direction parallel to the axis of the batholith but don’t in fact reach that value. However the soundings are influenced by lateral conductivity contrasts closer to the surface as well as contrasts at depth, and the thin sheet models show that these near surface features act to rotate the regional azimuth in the opposite direction to the deeper structure. Thus these two effects act to produce the pattern of regional azimuths measured over the period range of 0.1 to 10 seconds.

Plots of local channelling angle for sites in the centre of the outcrop show that the channelling angle at periods greater than 100 seconds has the same direction as the regional azimuth at periods between 0.1 and 10 seconds. This indicates that part of the granite batholith which is a regional 2D structure at shorter periods becomes a galvanic scatterer of the fields induced in other regional structures at longer periods. It also indicates that the 2D regional azimuth between 0.1 and 10 seconds is a
facet of real structure for at least part of this period range.

From the analysis of the data using decomposition methods a regional azimuth of N19° E was chosen as the fixed angle through which the impedance tensors were rotated to find the principal impedances for modelling. A fixed angle is needed because of the complex azimuth pattern at short periods. This angle was chosen because it is the average regional azimuth at sites in the middle of the Carnmenellis outcrop over the period range 0.1 to 10 seconds. This period range is significant for modelling the granite batholith and the lower crust and the sites in the middle of the outcrop are less affected by current channelling in the seas.

Comparisons of the shape and levels of apparent resistivity and phase curves for sites in the centre of the Carnmenellis granite show that the soundings at site BOQ813 are "static shifted".

One and two dimensional modelling of the decomposed impedance tensors was performed. After analysis of the pseudo sections of the Berdichevsky average impedance tensors, 1D models were produced using Occam, Minim and Most-Squares modelling routines. The Occam models of the Berdichevsky average for all the on granite sites are quite similar. The Occam models show a generally increasing resistivity from about 600 Ohm-m at the surface to a peak value of 5000 Ohm-m at a depth of 25 km. The resistivity then falls to a minimum value at about 60 km. However between 2.75 km and 8 km the resistivity falls or stays constant. The Minim and Most-Squares models show that this reduction in resistivity cannot be resolved and although a less resistive layer is included below 25 km in all the Minim models, it is also poorly resolved.

Forward 2D modelling shows that the $Z_{xy}$ impedances are affected by structures perpendicular to the regional azimuth at considerably shorter periods than $Z_{yx}$ data. These lateral conductivity contrasts bias the $Z_{xy}$ data downwards and the $Z_{yx}$ upwards. Therefore further modelling was only performed using the $Z_{yx}$ or E-pol data. However at periods greater than 1s both principal impedances will be affected by 3D induction. At virtually all the sites, a crossover in the apparent resistivities of the principal impedances occurs at about 1s. This is caused by resistivity contrasts existing in all directions. One dimensional Occam models of the E-pol response indicate that the bottom of the granite is not flat but slopes downwards to the south.

Two dimensional forward modelling of the data from traverse 6 was performed. Even after the analysis of an extensive range of models, it was found impossible to find a model which reasonably fitted the E-pol measured data at both off and on granite sites. The study of 2D forward models of structures on a number of traverses perpendicular to the regional strike indicated that 3D effects were one of the biggest causes of the large misfit of the traverse 6, 2D model. The magnitude of such three dimensional effects at each site were assessed and this was used in the
bottom of the granite maybe a false deduction caused by differences in the extent of 3D inductive effects at the various sites. Therefore in the 2D model the bottom of the granite was flat. Three dimensional effects will cause an overestimation both of the depth to the bottom of the granite and the resistivity of the layer below. At periods above 10 seconds, 3D effects become so large, that accurate assessment of structures below the granite using 2D models is impossible.

A steep rise in the resistivity-depth profile for depths shallower than 4 km was needed to counteract the effects of the surrounding seas and country rocks and still produce a reasonable fit to the measured response at sites on the granite. Except at sites 825 and 826 where relatively conductive material of 1000 $\Omega$.m persists down to depths of 4 km, the resistivity rises from 800 $\Omega$.m at the surface to 8000 $\Omega$.m at a depth of between 400m and 800 m and finally to 20 000 $\Omega$.m at a depth of 4 km. This 20 000 $\Omega$.m layer persists down to depths of 40 km. Below the granite is a 400 $\Omega$.m layer.

Comparing this model with borehole resistivity logs and laboratory measurements of the resistivity of granites, it is concluded that the closure of fluid filled joints with depth causes the steep increase in the resistivity depth profile. The joints close due to an increase in lithological load with depth and all the cracks are closed by a depth of between 4 and 5 km. The lateral variation of the resistivity depth profile is caused by an increase in the density of joints of sufficient width to stay open down to greater depths in the northern half of the granite.

The depth to the top of the granite in the 2D M.T. model along traverse 6 is similar to the depths calculated using the 3D gravity model of Willis-Richards (1987). If the granite batholith does thicken towards the south, the 2D M.T. model provides evidence to support the theory that the Cornubian granites originated SSE of their present position and were injected NNW in a sheet like body. The persistence of resistive material at depths which are normally assigned to be the lower crust is interpreted as being evidence for a gradual change of granite with depth into a granodioritic lower crust.

In order to study the electromagnetic induction effects of the surrounding seas and country rocks, three dimensional thin sheet modelling was performed. The boundary conditions of the model could only be met between 3 and 10 seconds. However a definite and significant pattern of induction arrows were measured over this period range. Two models were initially investigated - one model consisted of land of uniform resistivity and sea water of variable depth and in the second model the resistivity of the land was varied to take account of the shape of the granite and the surrounding rocks as defined by a 3D gravity model. Large misfits existed between the direction of induction arrows produced by these two models and arrows measured at a period of 5 seconds.

This misfit was significantly reduced until a reasonable fit to the direction of the induction arrows was achieved by adding three areas of enhanced conductivity. By far the most conductive of these areas was to
the north of the Carnmenellis outcrop, a second area was placed just to
the south of the granite outcrop and the third area was about 12 km NE of
the northern edge of the outcrop. Magnetotelluric soundings taken at
sites on these areas also detected conductive layers.

The conductive zone north of the granite cannot be explained
purely as a different lithological sequence or by the existence of
magnetised rocks. However within this area there is a high density of
fluid filled fractures and it is proposed that the enhanced conductivity is
caused by the bulk resistivity of this area being controlled by the fluids
flowing through these joints. The conductive zone to the south of the
granite can be explained as a more conductive lithological sequence
surrounded by more resistive rocks. The most likely reason for the
surface rocks in this zone being more conductive, is that they have a
greater porosity than the rocks in adjacent lithologies. Without further
measurements around site 836, little interpretation can be given for the
conductive zone in the north-east of the thin sheet model.

Anomalous current arrows calculated from the best fit model show
that current is being channelled from the estuary of the river Fall around
the northern edge of the Carnmenellis granite to the north Cornish coast.

7.2 Suggestions for Further Work

Suggestions for further work can be split into three groups.

1) Further field measurements.

2) Further modelling of the present Carnmenellis data set to refine
the interpretation of the measurements.

3) Additional modelling of the present data set to further assess the
usefulness of the methodologies used in study.

7.2.1 Further Field Measurements

Three types of further field measurements could enhance the
understanding of electrical structure of south-west Cornwall:

1) Electromagnetic induction soundings at various different
locations could be performed in order to confirm or disprove
interpretations which can only be tentatively drawn from the present data
set. Since the two dimensional modelling confirms that interpretation of
soundings within the granite crucially depend on knowledge of the
electrical structure of the surrounding rocks, further soundings at sites
off the Carnmenellis granite outcrop would greatly enhance the
understanding of the induction problem. However urban areas surround
the Carnmenellis granite and this was the main reason why soundings
in locations directly north and east of the outcrop were not included in
the present study. Due to high cultural noise levels it is unlikely that successful soundings could be made in these areas. But soundings could be taken west of the outcrop and on the Lizard peninsula. The conductors in the top few km below sites 835 and 836 cannot be understood until further soundings are made around the two sites. This is especially true of the conducting layer below site 836.

3) Measurements of the electrical structure using methods other than M.T. are needed to correct for any possible "static shifts" in the present data set. Details of such measurements are given in section 4.8.

4) Differential geomagnetic soundings as reviewed by Jones (1983a) could confirm the current channelling to the north of the Carnmenellis outcrop.

### 7.2.2 Further Mathematical Modelling

There are good reasons (stated in section 5.5) for a greater emphasis to be laid on the 2D model of data recorded on traverse 6. However, changes in the resistivity structure perpendicular to the regional azimuth could be assessed by 2D modelling the data recorded at sites on traverses 5 and 7.

An understanding of the direction of electric current flow within resistive bodies, such as the Carnmenellis granite, are essential if the parameters of the measured impedance tensors are to be understood. This could be done using three dimensional models of resistive prisms of various shapes and sizes.

A full assessment of the usefulness of the Groom-Bailey decomposition can only be achieved by comparing the 2D models derived from decomposed impedance tensors and 2D models which fitted data rotated using the methods of Swift (1967).

Finally the assessment of the geological implications of resistive material persisting to depths greater than 20 km needs to be continued.
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Appendix A
Figures A.1 - A.23

One Dimensional Models of the Berdichevsky Average of the Principle Impedances of the "Best" Decomposed Tensor for all 21 Sites.

Occam model and response shown as thick line.
Fisher's Minim model and response shown as thin line.
Most-Squares analysis of Minim model shown in bottom plot.
Bostick models shown as crosses. The width of the cross for each measured period shows the depth error bounds. The height of the cross shows the resistivity error bounds.
Fig A.1 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.2 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.3: Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.4 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.5 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.6 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.7 Occam, Minim, Most Squares and Bostick Models of 'Best' Decomposed Tensor
Fig A. B Occan, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.9 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.10 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.11 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
<table>
<thead>
<tr>
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<th>LOG($\rho$) (0-m)</th>
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<td>2.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Fig A.12 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.13 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.14 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.15 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.16 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.17 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.18 Occam, Minim, Most Squares and Bostick Models of “Best” Decomposed Tensor
Fig. A.19 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.20 Occam, Minim, Most Squares and Bostick Models of “Best” Decomposed Tensor
Fig A.21 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.22 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Fig A.23 Occam, Minim, Most Squares and Bostick Models of "Best" Decomposed Tensor
Figures A.24 - A.43

One Dimensional Models of the E-pol Response (Zxy) of the Decomposed Tensor for 20 Sites.

Occam model and response shown as thick line.
Bostick models shown as crosses. The width of the cross for each measured period shows the depth error bounds. The height of the cross shows the resistivity error bounds.
Fig A.24 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.25 Occam and Bostick Models of E–Pol Decomposed Tensor
Fig A.26 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.27 Occam and Bostick Models of E–Pol Decomposed Tensor
Fig A.28 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.29 Occam and Bostick Models of E–Pol Decomposed Tensor
Fig A.30 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.31 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.32 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.34 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.35 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.36 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.37 Occam and Bostick Models of E–Pol Decomposed Tensor
Fig A.38 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.39 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.40 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.41 Occam and Bostick Models of E-Pol Decomposed Tensor
Fig A.42 Occam and Bostick Models of E–Pol Decomposed Tensor
Fig A.43 Occam and Bostick Models of E-Pol Decomposed Tensor