The investigation of underwater acoustic signals using laser Doppler anemometry

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For
Mum, Dad, Lynsey, Laura and Gary
Abstract

Laser Doppler anemometry (LDA) has been used to study underwater acoustic signals both from emitting hydrophones and underwater explosions. A dual-beam LDA arrangement was used to capture Doppler signals arising from light scattered from particles suspended at the point of interest in the flow. These Doppler signals are analysed using either Hilbert transforms or wavelets, both of which allow instantaneous frequency information to be obtained.

When an acoustic signal propagates through a medium it creates refractive index variations within the medium. The apparent motion of the scattering particles, as observed by the detector, which give rise to the Doppler signal, is therefore made up of two components. Firstly, the particles oscillate due to the sound field and secondly, the interference fringes oscillate due to the refractive index variations. This is termed the acousto-optic effect. A theory has been developed to investigate the effect of these refractive index variations on the analysed Doppler signals of an LDA system. Analysis of experimental Doppler signals using the Hilbert transform technique shows close agreement with the theoretical predictions.

LDA has also been used to investigate the acoustic signal emitted by an oscillating explosion bubble. This is generated by an underwater spark which creates a similar situation to an underwater explosion in which a shock wave and an oscillating bubble are produced. Analysis of the Doppler signal using wavelets provides information on the bubble period, radius, energy and particle velocity.

Explosive materials have traditionally been used for investigation of underwater explosions but they have the disadvantage of obscuring the area with explosion debris thus making optical investigation difficult. It is shown in this work that the use of LDA and analysis of Doppler signals using wavelets is an accurate technique for the investigation of acoustic signals from underwater explosions. This allows investigation of the area close to the explosion centre where measurements have been difficult to achieve with traditional techniques.
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Declaration

I declare that this thesis was composed by myself and that the work contained within was executed by myself, unless otherwise stated in the text.
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Chapter 1

Introduction

1.1 Previous research using LDA

Laser Doppler anemometry (LDA) has been successfully used for many years for non-invasive investigation of the physical characteristics of fluid flow. In 1969 Rudd [Rudd, 1969] found that focusing two parallel beams with a single lens, or something equivalent, produces Young's interference fringes. This was fundamental in the development of LDA. The first details on the development of LDA as an optical technique for investigating fluid flow were published in 1964 by Yeh and Cummins [Yeh and Cummins, 1964]. Since then a substantial amount of work has been carried out in developing the technique in terms of both the analysis and the experimental arrangements.
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The basic principle behind LDA is that two laser beams are focussed at a point of interest in a flow in order to create interference fringes. Suspended particles in the flow scatter the laser light which is thereby Doppler shifted. Analysis of this Doppler signal provides information about particle velocities at the point of interest in the flow. Traditional analysis techniques work best if the flow is steady and particles are always present in the signal. The absence of particles at any time can cause problems in the analysis and therefore the flow is usually seeded with particles to prevent this. Chapter 2 presents a detailed explanation of the technique and offers a mathematical treatment of the Doppler signal as well as describing the experimental arrangements, experimental considerations and traditional analysis techniques for LDA.

Very little research has been carried out using LDA to investigate acoustic sources. In 1976 Taylor [Taylor, 1976] extended the LDA technique for use in acoustics by measuring particle velocities with steady state single frequency waves in standing wave and travelling wave tubes in air. Vignola [Vignola et al., 1991] extended this to include sound sources in water. Hanish [Hanish, 1983] has published work on an underwater hydrophone and Hann [Hann and Greated, 1993] published work detailing the use of photon correlation spectroscopy to measure sound in flows. However, most of the signals obtained from the above work have been for single frequency sound waves which can be analysed using traditional techniques and Fourier transforms. Recent research has extended that work to show that it is possible to analyse complex sound fields from Doppler signals [Jack, 1997] using
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Hilbert transforms.

The research detailed in this thesis addresses the use of LDA to analyse acoustic Doppler signals from underwater sources and in particular extend previous research by considering explosive sources. The effect of the refractive index variations of the medium on the Doppler signal is investigated both theoretically and experimentally. LDA is then used to investigate the characteristics of bubbles produced by underwater explosions. Two different methods for analysing these signals are discussed and the design of the underwater spark apparatus is described.

1.2 Analysis of Doppler signals

The research discussed in this thesis involves obtaining the instantaneous frequency of particles suspended at a point in a flow from the Doppler signals and therefore traditional analysis techniques such as Fourier transforms are not suitable for deriving the required information. Instead two relatively new analysis techniques are used in this thesis for analysing the Doppler signals in order to obtain instantaneous frequency information. These techniques use the methods of Hilbert transforms [Grechikhin and Rinkevichius, 1996] and wavelets [Yasin et al., 1999]. More detail on these techniques can be found in Chapter 3.

The Hilbert transform technique is used to analyse data obtained from an under-
water sound source so that amplitude and instantaneous frequency information can be extracted. The acoustic signals produced from underwater sparks are analysed using the wavelet technique to produce a plot of instantaneous frequency as a function of time allowing information such as bubble energy, radius and period of oscillation to be determined.

1.3 Acousto-optic effect on Doppler signals

When LDA is used to measure acoustic sources in water, refractive index variations in the medium can have the effect of varying the path lengths of the laser beams, thus creating an oscillating fringe pattern rather than a stationary pattern as is generally assumed. There are two types of motion which result in scattered light. Firstly, particles oscillate due to the acoustic source and secondly, the fringes oscillate due to the refractive index variations. This is the acousto-optic effect and the detector therefore sees an apparent motion of particles in the fringes. The result of this acousto-optic effect is that although the signal may be easier to detect, the analysed signal may not always present accurate amplitude information about the point of interest in the flow.

The theory of the acousto-optic effect is developed in Chapter 4 for the cases of a travelling acoustic wave and a plane acoustic wave in water. The influences of sound frequency, propagation distance of the laser beams and the angle of
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the wavefronts to the laser beams, on the amplitude of fringe movement, are investigated. The experimental arrangement used to obtain practical results for comparison with the theoretical predictions is detailed in Chapter 5 and shows a comparison between theory and experiment which agrees well within a small error band.

1.4 Underwater explosions and LDA

When an explosion occurs underwater a bubble is created which expands and cools, emitting a shock wave. The bubble continues to expand until the pressure inside reaches a minimum, at which point the bubble has reached its maximum radius, the bubble then starts to contract with the pressure building up inside. Another pressure wave is emitted when the bubble reaches its minimum radius. If the bubble follows a process of spherical collapse then it should begin to expand again thereby setting up an oscillating system.

If a structure is in the vicinity of these events a substantial amount of damage can occur. The shock wave may create localised damage to the structure but the effect of the oscillating bubble can damage a much greater area. The main events which occur in underwater explosions are discussed in Chapter 6 which also details previous research carried out in the field.

A lot of research has been carried out into the effects of explosions. Many
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theories have been developed to describe the dissipation of shock waves including theories proposed by Kirkwood and Brinkley [Kirkwood and Brinkley, 1945], [Brinkley and Kirkwood, 1947] and Taylor [Taylor, 1946]. Extensive investigation of the bubble has also been carried out based on interest in the damage caused by non-spherical collapse. From that work it has been found that the bubble is attracted towards solid surfaces and repelled by free surfaces [Shima et al., 1989]. Also, the collapse of the bubble close to surfaces can create damage pits due to non-spherical collapse [Vogel et al., 1989]. Bubble-bubble interaction [Tomita et al., 1994] and bubble-shock wave interaction [Tomita and Shima, 1986] also cause non-spherical decay and damage to structures.

The ways in which the explosions are generated vary. Some researchers have used explosive materials [Cole, 1948], exploding wires have also been successfully used [Vijayan and Rohatgi, 1988] as have laser pulses [Lauterborn, 1974]. The research detailed in this thesis uses a spark generator [Chahine et al., 1995], a method of producing shock waves and bubbles which is becoming increasingly popular because of its repeatability and relative safety. A further advantage is that the area around the epicentre of the explosion is not obscured by explosive debris which therefore allows optical techniques such as LDA to be used for investigation. Most of the work carried out to date has used explosive materials such as TNT and these results are very well documented.
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It is also of interest to use LDA to investigate bubble characteristics. Previous bubble studies have used high speed photography to calculate the bubble radius and period. Analysis of the LDA signal offers a much quicker, cheaper and more accurate method. The LDA signal allows instantaneous velocity information to be obtained at any point in the water, providing much more detail than can be obtained from a photograph.

The apparatus designed and built to generate the underwater spark and the LDA arrangement are described in detail in Chapter 7. Results obtained from this experimental arrangement are presented and discussed in Chapter 8. The shock wave produced from the underwater spark is shown to have the same decay characteristics as one produced from explosive materials. However, the pulse emitted due to the bubble collapse differs between the two techniques; the one produced from the explosive material has a much longer period and lower peak pressure than the one from the spark.

The wavelet analysis technique is employed in a comparison of the signals derived from a hydrophone and from LDA. The afterflow velocity calculated from the hydrophone signal and from the LDA are shown to agree very well thus verifying the wavelet analysis technique. From the LDA data it is possible to obtain information on bubble energy, radius and period of oscillation. Bubble radius and period are shown to be proportional to the voltage of the spark circuit and the width of the spark gap in such a way that increasing the gap or voltage leads to
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a cubic increase in the radius and period of the bubble.

The pressure in the water generated by the bubble as a function of distance from the bubble centre was found to depend upon the bubble radius and an empirical form obtained based on some 1430 measurements. Experimental data are included on energy transfer from the spark circuit into the bubble. These data show that there is a cubic relationship between energy transfer and gap width. The largest energy transfer obtained in the experiments carried out in this research was 12.8 %. Determining a cubic relationship between maximum bubble radius and circuit energy or gap width is very important for any future research carried out using spark generators in which a bubble of a specific dimension is required.

1.5 Thesis aims

There has been relatively little research carried out using LDA to investigate underwater acoustic sources. The aims of this thesis are to develop this area in the following ways:

- Develop the acousto-optic theory.
- Compare experimental results with the developed acousto-optic theory.
- Verify the use of wavelet analysis for non-stationary signals.
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• Design an experimental arrangement to create underwater explosions.

• Investigate the acoustic emission of underwater bubbles using LDA.
Chapter 2

Basic principles and background of LDA

2.1 Doppler principle

Laser Doppler anemometry is a well established optical method for measuring fluid velocities by detecting the Doppler frequency shift of laser light that has been scattered by small particles moving within the region of the flow of interest.

In any LDA arrangement the Doppler principle indicates that the frequency of the light scattered from a particle, as in figure 2.1, are be shifted by an amount proportional to the particle velocity. The particle is assumed to be accurately following the flow and therefore the particle velocity is equal to the flow velocity.
Assume a plane monochromatic wave is incident upon a particle moving with a velocity \( u \) such that \( |u| \ll c \) where \( c \) is the speed of light.

For a stationary particle, the number of wavefronts, of separation \( \lambda_i \), striking the particle per unit time, \( \nu_i \), would be:

\[
\nu_i = \frac{c}{\lambda_i}
\]  

(2.1)

Similarly, the number of wavefronts per unit time incident upon a moving particle, of velocity \( u \), is:

\[
\nu_i = \frac{(c - u \cdot i)}{\lambda_i}
\]

(2.2)

where \( i \) is the unit vector parallel to the \( k_i \) direction and \( u \cdot i \) is the component of velocity along the \( i \) direction. The wavelength, \( \lambda_p \), apparent to the particle is:

\[
\lambda_p = \frac{c}{\nu_p} = \frac{\lambda_i c}{c - u \cdot i}
\]

(2.3)
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where $\nu_p$ is the number of wavefronts per unit time apparent to the particle, in this case $\nu_p = \nu_i$. For a stationary observer viewing along the direction $-k_s$ the apparent scattered wavelength is:

$$\lambda_s = \left(\frac{c - u \cdot s}{\nu_p}\right)$$ (2.4)

where $s$ is the unit vector parallel to $k_s$ and $u \cdot s$ is the component of velocity along the direction $s$. The frequency, $\nu_s$, of the scattered radiation is:

$$\nu_s = \frac{\nu_i}{\lambda_i} = \frac{c(c - u \cdot i)}{\lambda_i(c - u \cdot s)}$$ (2.5)

The Doppler shift, $\nu_D$, is the frequency difference between the scattered and the incident light:

$$\nu_D = \nu_s - \nu_i = \frac{c}{\lambda_i} \left(\frac{c - u \cdot i}{c - u \cdot s} - 1\right)$$ (2.6)

This frequency can be detected and hence can be used to measure particle velocities. Equation (2.6) indicates that the signal frequency depends upon the wavelength of the light, the geometry of the optical system and the particle velocity.

Although there are many optical arrangements used in LDA, as detailed in Section 2.2.1, it is the dual-beam mode which is used in the experiments detailed within this thesis. Therefore the following discussion is for the dual-beam mode.
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In an LDA system using the dual-beam mode, the particle scatters light from two laser beams. The light scattered from beam one has a frequency:

\[ \nu_1 = \nu_i - \frac{u_y \cdot s_1}{\lambda} \]  

(2.7)

and from laser beam two:

\[ \nu_2 = \nu_i - \frac{u_y \cdot s_2}{\lambda} \]  

(2.8)

The scattered light combines at the detector surface producing an electric field which is proportional to the sum of the two signals \((\sin(2\pi \nu_1 t) + \sin(2\pi \nu_2 t))\). Since detectors are square-law devices, the intensity output from one is proportional to the electric field:

\[ I \propto (\sin(2\pi \nu_1 t) + \sin(2\pi \nu_2 t))^2 \]

\[ \propto \sin^2(2\pi \nu_1 t) + \sin^2(2\pi \nu_2 t) + \cos(2\pi (\nu_1 + \nu_2) t) + \cos(2\pi (\nu_1 - \nu_2) t) \]  

(2.9)

The frequency response of any currently available detector is much lower than the sum of the above components; therefore it is only the final term which can be detected. This is termed optical mixing and the output of the detector oscillates
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at a frequency of \((\nu_1 - \nu_2)\) which corresponds to the Doppler shift frequency:

\[
\nu_1 - \nu_2 = \nu_i - \frac{u_y \cdot s_1}{\lambda} - (\nu_i - \frac{u_y \cdot s_2}{\lambda})
\]

\[
= \frac{u_y}{\lambda} (s_2 - s_1)
\]

\[
= \frac{2u_y \sin \theta}{\lambda}
\]

(2.10)

where \(\theta\) is the angle between the laser beam and the \(x\) axis.

2.2 Experimental considerations

2.2.1 Optical arrangements

There are three main types of optical arrangements for LDA which have been described by investigators: the reference beam mode, dual-beam mode and differential mode. These are represented in figure 2.2.

In the reference beam mode, the beam is split into an intense scattering beam and a weak reference beam [Yeh and Cummins, 1964], [Goldstein and Hagen, 1967], [Welch and Tomme, 1967], [Pike et al., 1968]. The scattered light is mixed with the reference beam which produces beats on the surface of the detector. The beat frequency is equal to the Doppler frequency which is proportional to the particle velocity.
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Figure 2.2: LDA optical arrangements
Chapter 2 — Basic principles and background of LDA

In the differential (two-scattered beam) mode [Mazumder and Wankum, 1970], [Durst and Whitelaw, 1971a] a single beam is focused in the flow and the light scattered by a particle in two directions is collected symmetrically about the system axis. When the scattered beams are combined, the relative phase of their wavefronts depends upon the distances of the particle from each light collecting aperture. As a particle moves across the beam the scattered light produces constructive and destructive interference patterns leading to a light intensity at the detector which fluctuates at the Doppler frequency.

The third optical arrangement is the dual-beam mode in which interference fringes are created from two coherent laser beams [Durst and Whitelaw, 1971b], [Rudd, 1972]. As particles pass across the fringes they scatter the laser light onto a photodetector with a frequency proportional to the flow velocity. This mode has three practical advantages in that it can be used in forward scatter or back scatter mode, there is no path difference introduced between the beams and both beams can be focussed by a single lens.

The main differences between the three optical arrangements are in the signal strengths and the signal to noise ratio (SNR). These differences arise partly due to the differences in the efficiency with which the scattered light is collected and partly due to the way in which mixing between the reference beam and scattered beam is achieved. The size of the scattered light collecting aperture is very important and for the reference-beam and the dual-scatter systems the aperture size
is very restricting. This restriction occurs in the reference-beam system because large apertures cause poor mixing efficiency and in the dual-scatter system the Doppler frequency shift actually varies over the aperture. Therefore the dual-beam mode is the only one which is capable of effective mixing using a large light-collecting aperture to produce strong signals. It is therefore the dual-beam mode which is used in the experimental arrangements described within this thesis.

2.2.2 Gaussian laser beam properties

A laser output generally consists of a Gaussian beam with well defined propagation characteristics which are dependent on the laser wavelength and the cavity configuration. The properties of a focused laser beam with a beam waist $D_e^{-2}$ are shown in figure 2.3. The effect of the lens is to convert the spherical wave diverging from the laser into a converging spherical wave whose radius of curvature first decreases as though the waves are converging at a distance $s_1$ from the lens.
Chapter 2 — Basic principles and background of LDA

and then increases until it is infinite at the point $s_1$, creating a planar wave. At this point, the focal waist, the beam has a minimum diameter of $d_{e-2}$. This focal point is located at [Goldstein, 1996]:

$$s_1 = f + \frac{s_o - f}{(s_o - f)^2 + \left(\frac{\pi D_{e-2}^2}{4f\lambda}\right)^2} \quad (2.11)$$

and the diameter of the focal spot is given by:

$$\frac{1}{d_{e-2}^2} = \frac{1}{D_{e-2}^2} \left(1 - \frac{s_o}{f}\right)^2 + \left(\frac{\pi D_{e-2}^2}{4f\lambda}\right)^2 \quad (2.12)$$

This equation states that the minimum beam diameter does not occur exactly in the focal plane of the lens unless the distance, $s_o$, is equal to the focal length of the lens, $f$, in which case:

$$d_{e-2} = \left(\frac{4f\lambda}{\pi D_{e-2}^2}\right) \quad (2.13)$$

In a dual-beam LDA arrangement the lens causes the parallel beams to cross at the focal point, $f$. Errors occur if the distance, $s_1$, is not equal to the focal length, $f$, as this results in wavefronts which are not planar in the beam intersection. Deviation of the beam waist results in variations in the fringe volume size and broadening of the Doppler spectrum [Dancey and Hetmanski, 1995], [Miles, 1996], [Durst and Stevenson, 1976]. Figure 2.4 shows the resulting fringe
patterns from beams which have their focal waists on the same side (upper case depicted) and on opposite sides (lower case depicted) of the interference region.

Although the properties of a Gaussian laser beam are determined by the actual laser, the deviation in Doppler frequency has been shown to be a function solely of the focal length of the lens [Durst and Stevenson, 1976]. This results in broadening of the Doppler spectrum with the mathematical form:

\[
\frac{f}{\nu_D} \frac{d\nu_D}{dz} = \frac{s_1 - f}{f} \tag{2.14}
\]

where \( s_1 \) is the distance from the beam waist to a focusing lens, \( z \) is the distance from the beam waist to the focal plane and \( f \) is the focal length of this lens.

It can be seen that the broadening is due only to the distance \( s_1 \) and the focal length, it is not dependent on the laser wavelength or the input beam waist size.

However, in typical LDA arrangements the errors are negligible for focal lengths up to several hundred millimetres and the following approximations can be used [Goldstein, 1996]:

\[
s_1 \simeq f \\
d_{e-2} \simeq \frac{4f\lambda}{\pi D_{e-2}^2} \tag{2.15}
\]

In the neighbourhood of the focal spot, equations (2.15) suggest that the focussed beam is essentially a plane wave whose diameter is nearly constant and whose
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Figure 2.4: Fringe model

intensity distribution is Gaussian.

2.2.3 Carrier frequencies

For measurement purposes, it is often advantageous when setting up an LDA system to include a carrier frequency which can be introduced into the system through a phase shifter or a frequency shifter. This is often done if directional information is required about a flow, for example, in a sound field the particles oscillate backwards and forwards through the fringes and it is not possible to differentiate between the two directions without a frequency shift. The introduction
of a frequency shift has the effect of shifting the centre of the power spectrum of
the Doppler signal away from zero to the carrier frequency to allow directional
information to be obtained. Thus, in a sound field the signal resembles a fre-
quency modulated (FM) radio signal, with sidebands on either side of the carrier.
If no carrier frequency is present then the low frequency components could have
a range which overlaps and distorts the Doppler signal, making analysis difficult.
This can also happen if there are very high levels of turbulence or if there are
only a few fringes in the measuring volume.

Frequency shifting is accomplished in practice by splitting the original laser beam,
with a frequency \( \nu_0 \), in two. The frequency of one of the beams say, beam 1 is
shifted by \( \nu_s \) so that \( \nu_{01} = \nu_0 + \nu_s \) and \( \nu_{02} = \nu_0 \) where \( \nu_{01} \) and \( \nu_{02} \) are the
frequencies of the split beams. Frequency shifts can be produced by electro-optic
Pockels cells and Kerr cells, by rotating diffraction gratings or by acoustic-optic
Bragg cells. In a real fringe system which incorporates a frequency shift, the
fringes can be thought of as moving continuously across the measuring region
with a constant velocity. The effect of an electro-optic cell is more correctly
described as a phase shift which has the same overall effect as a frequency shift
although there are large differences in the principles involved. In phase shifting
the fringes are made to move in a cyclical manner over a distance of one fringe
spacing, \( d_f \), as in figure 2.5. The motion in the forward direction is at constant
velocity and the fly back time, \( T_f \), is made as short as possible. The fringes can
again be thought of as moving continuously across the measuring region with a
2.2.4 Particle size and motion

The purpose of LDA is to determine the velocity of a point in a fluid but it is actually the velocity of the suspended particles that is being measured. It is therefore very important to ensure that the suspended particles within the flow accurately conform to the motion of the fluid. Any relative motion between a particle and the surrounding fluid has a significant influence on the signal quality. Signal strength can be increased by up to four orders of magnitude simply by increasing the particle diameter from several tenths of a micron to several microns. However, increasing the particle size reduces the maximum flow velocity which the particle can follow accurately.
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The most important properties of an individual scattering particle are the produced SNR and its hydrodynamic size which is a measure of its ability to conform to the flow. The limiting size for particles of a particular density has been determined by Brandt [Brandt et al., 1937]. Spherical particles, with the force exerted on the particles by the vibrating medium, obey the Stokes-Cunningham law [Batchelor, 1970] which states that the ratio of the particle to medium velocity for a sinusoidal vibration is:

\[
\frac{u}{U} = \left[\frac{4\pi r_p^2 \rho_p \omega F}{9\mu} + 1\right]^{\frac{1}{2}} \tag{2.16}
\]

where \(u\) and \(U\) are the velocity amplitudes of the particle and the medium respectively, \(r_p\) is the particle radius, \(\rho_p\) the particle density, \(\omega\) the sound frequency, \(\mu\) the medium viscosity and \(F\) the Cunningham correction factor. Ideally the ratio would give a value of unity and it is therefore possible to predict the limiting size for particles of a particular density. This allows seeding particles smaller than this limit to be chosen in order that they can accurately follow the flow up to the maximum frequency encountered.

There are a number of other effects which influence the motion of the particles in the flow. These effects include: gravity, shear flow lifting force, spatial averaging of turbulence, Brownian motion, concentration of particles and the Magnus effect which occurs if a rotating spherical particle moves through a fluid and this results in a force being exerted on the particle in a direction perpendicular to the relative
If the particles have a density different from that of the surrounding fluid then gravitational forces cause them to either settle downwards or drift vertically upwards depending on whether they are denser or less dense than the surrounding fluid. The magnitude, $u_t$, of this vertical velocity can be estimated by equating the gravitational force to the Stokes' drag force [Batchelor, 1970]:

$$
\frac{1}{6}\pi d_p^3 (\rho_p - \rho_m)g = 3\pi \mu d_p u_t
$$

(2.17)

where $\rho_m$ is the density of the medium and $g$ is gravitational acceleration. Rearranging produces the Stokes' law terminal velocity, $u_t$, for the descent under gravitational acceleration of a small particle:

$$
u_t = \frac{\rho_p - \rho_m)g d_p^2}{18\mu}
$$

(2.18)

Minimisation of the terminal velocity can be achieved by matching the particle density with that of the fluid and by using small particles.

If a spherical particle moves relative to the surrounding fluid and also has a rotation, then a force is exerted on the particle in a direction perpendicular to the relative velocity and axis of rotation, this is the Magnus effect [Batchelor, 1970]. In the majority of laser Doppler anemometry situations the Magnus effect is extremely small.
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Particles suspended in a shear flow experience a lifting force caused by the velocity gradient. This force is generally small but can be significant in extreme situations such as in the region immediately adjacent to a boundary where velocity gradients are high. This problem has been considered by Saffman (1965) who estimated the lifting force, $F_l$, for a 2D flow field with a uniform velocity gradient $du/dy$ as:

$$F_l = 20(u - U)\rho_p d^2 \left(\frac{\mu\rho}{\mu\rho} \frac{du}{dy}\right)^{1/2}$$

(2.19)

where $u$ and $U$ are the velocities of the particle and the medium respectively. In most LDA situations, this effect is very small.

Spatial averaging of turbulence is caused if a particle is carried by a turbulent flow. Even if the particle and fluid densities are the same, the particle does not respond fully to velocity fluctuations caused by eddies of size less than the corresponding velocity variance of the surrounding fluid; therefore there is a spatial filtering effect which causes the velocity variations of a particle to be less than the corresponding velocity variations of the fluid.

When a small particle is suspended in a fluid, the molecular bombardments cause high frequency random movements which result in the particle diffusing through the fluid. This Brownian motion can be considered as being superimposed on movements caused by macroscopic turbulent fluctuations.

The limits to concentration of scattering particles also affects the relative motion.
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The maximum particle density limit is very rarely reached due to difficulties in seeding flows to very high concentrations. If the flow is seeded to a very high concentration level, the light can be attenuated and damping of turbulence, due to an increased time lag between particles and fluid, can occur. The minimum particle density is set by the limits of the signal capture method to be used. If a continuous signal is required then the minimum concentration must be such that at least one particle is in the measuring volume at any one time.

When choosing the appropriate types and density of seeding particles based on the above equations, a variety of assumptions are usually made. These include assumptions that the particle is rigid and spherical, the particle is small compared to the smallest wavelength of fluid motion, the motion path lines of particle and fluid coincide, the flow is not disturbed by the presence of the particles, Stokes' drag law applies, there is no interaction between particles, the particles are good light scatterers and the particles are chemically inactive.

2.2.5 Doppler signal

For a particles travelling one at a time with a constant velocity across the fringe volume, the Doppler signal reaching the photodetector has the form shown in figure 2.6. When more particles are present in the fringe volume, the individual envelopes overlap and the signal becomes more complex. The low frequency signal variation corresponds to the passage of particles through one or both light
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Figure 2.6: Form of photodetector signal for a mean flow.

Figure 2.7: Form of photodetector signal for an acoustic flow.
beams. The high frequency signal, contained within the envelopes, corresponds to the velocity of individual particles passing through the interference region. However, in acoustic fields the signal frequency contained within the envelopes is not constant (figure 2.7) and in cases without a mean flow there is no amplitude modulation of the signal.

Sources of noise

Noise contributions to the Doppler signal are usually significant as the ratio of the amplitudes of high to low frequency components of the signal is proportional to the SNR. These noise contributions arise from: optical noise sources, photodetection effects and electronic system noise sources.

Optical noise sources may be associated with diffractive and refractive beam perturbations, coherent degradation of the laser beams, light dispersion from lenses and medium and laser hum. Imperfections in optical systems such as angular misalignment can also introduce noise into the signal. These account for noise introduced in the generation and transmission of the laser beams.

Capture of the Doppler signal by the detector can introduce two more noise components: thermal noise (Johnson noise) and shot noise. Thermal movements of conducting electrons in amplifiers and effective load resistors in the detection system cause thermal noise while 'dark currents', which arise due to electron emission by thermal excitation, contribute to internally generated noise. External
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Shot noise arises from the total radiation incident on the detector and is thus contributed to in part by the Doppler light signal and in part by the background environment.

The effect of the various noise sources can be conveniently grouped together and represented as a single additive noise component on the Doppler signal. This noise is white and Gaussian due to the independent nature of the noise sources. The noise level defines the lower limit of signal detectability and determines the sensitivity of the measurements.

Watrasiewicz (1970) and Greated (1971) have both described experimental arrangements which can be used to increase the SNR. Watrasiewicz (1970) overcame the problem by making use of an additional photodetector to record the laser output directly. The signals from the two photodetectors were then fed into a differential amplifier so that only the off-balance fluctuations caused by the passage of particles across the beam were recorded. This gave a considerably improved SNR but required fine adjustment. Greated (1971) showed that by using a mask to block any direct light falling onto the photodetector, a laser velocimeter could be converted into a scattering system, greatly reducing the effects of the laser noise and increasing the signal; hence increasing the SNR.
2.2.6 Spectral broadening

There is a linear relationship between Doppler frequency, $\nu_D$, and velocity. However, it is found that the probability density function of the Doppler frequency is wider than that of the velocity; this is due to broadening.

The main origins of broadening are: finite transit time, velocity fluctuations within the scattering volume, mean velocity gradient, Brownian motion and laser line-width. Experimental arrangements need always to try to minimise broadening.

George and Lumley (1971, 1973) showed that ambiguity broadening (finite transit time) of the spectrum arises because signals from individual scattering particles last only for the time required to cross the scattering volume. The source of the frequency uncertainty for multi-particle scattering can be explained by fluctuations in phase [Lumley et al., 1969], [Edwards et al., 1971] and hence frequency of the combined Doppler signal. If the Doppler signal has a phase $\phi(0)$ at $t = 0$ and a phase $\phi(\tau)$ at $t = \tau$, then $\phi(\tau)$ becomes increasingly uncorrelated with $\phi(0)$ as some particles leave the fringes and others arrive. Eventually when $\tau$ exceeds the transit time, all members from the original set of particles have left and the signal $\phi(\tau)$ is totally uncorrelated with $\phi(0)$. In order to reduce ambiguity broadening it is necessary to keep the number of fringes large and hence increase the transit time. This form of broadening is inherent in LDA whenever light is scattered to the photodetector from several particles simultaneously.
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[George and Lumley, 1973], [Edwards et al., 1971], [Adrian, 1972].

Turbulence broadening of a Doppler signal [Mazuinder and Wankum, 1970], [George and Lumley, 1973], [Berman and Dunning, 1973] is the result of fluctuations of the volume averaged velocity and velocity fluctuations within the scattering volume. The instantaneous velocity is the sum of the mean velocity and fluctuations from the mean. Spatial averaging occurs if turbulent eddies exist which have length scales of the order of, or smaller, than the measuring volume. This causes velocity variations in the particle which are smaller than the velocity variations of the fluid.

Velocity gradient broadening depends only on the fluid flow and the scattering volume dimensions and occurs if a fringe volume of finite size covers a region of the flow where there is a mean velocity gradient. Particles crossing the area therefore have a range of velocities which are independent of any turbulent velocity fluctuations and the probability density function of a velocity component is broadened and skewed relative to that for a point measurement [George and Lumley, 1973]. An approach which accounts for the influence of gradient broadening has been presented by Edwards et al. (1971).

Brownian motion broadening effects are much smaller than those of other effects and can be taken to be negligible as can laser line width [Durst et al., 1976].
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2.3 Mathematics of the Doppler signal

This section describes the mathematical steps in the derivation of the Doppler signal. Although there are three main experimental arrangements which can be used for LDA, it is the dual-beam mode which is used for any experiments detailed within this thesis; therefore the mathematical discussion which follows concentrates on the dual-beam mode.

A plane light wave has an electric field, \( E \), of the form:

\[
E = E_0 \cos(\omega t - kr + \phi).
\]  

(2.20)

where \( E_0 \) is the amplitude, \( \omega \) is the frequency, \( t \) is time, \( k \) is the wavenumber, \( r \) is the distance and \( \phi \) is the phase.

In a dual-beam LDA system there are two laser beams which intersect to form interference fringes. It is therefore necessary to determine the electric field at the point of intersection and the resulting intensity at the detector. It has been shown that for a linearly polarised light beam, the intensity, \( I \), at a point can be expressed as [Hecht, 1987]:

\[
I = \varepsilon_0 c \langle E^2 \rangle
\]  

(2.21)

where \( \varepsilon_0 \) is the permittivity of free space (\( \varepsilon_0 = 8.8542 \times 10^{-12} \text{ Fm}^{-1} \)) and \( c \) is the
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speed of light. In this case the electric field, \( E \), at the point of intersection of two laser beams is due to the separate fields, \( E_1 \) and \( E_2 \):

\[
E = E_1 + E_2 \quad (2.22)
\]

\(< E^2 >\) is the time average of the magnitude of the intensity squared, thus:

\[
E^2 = E \cdot E
\]

\[
= (E_1 + E_2) \cdot (E_1 + E_2)
\]

\[
= E_1^2 + E_2^2 + 2E_1 \cdot E_2 \quad (2.23)
\]

where:

\[
E_1(t) = E_{01} \cdot \cos(\omega t - k_1 r + \phi_1)
\]

\[
E_2(t) = E_{02} \cdot \cos(\omega t - k_2 r + \phi_2) \quad (2.24)
\]

Taking the time average of equation (2.23) produces the intensity:

\[
I = I_1 + I_2 + I_{12} \quad (2.25)
\]

where \( I_1 = < E_1^2 > \), \( I_2 = < E_2^2 > \) and \( I_{12} = 2 < E_1 \cdot E_2 > \). \( I_{12} \) is known as the
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interference term and can be calculated as:

\[ E_1 \cdot E_2 = E_{01} \cdot E_{02} \cos(\omega t - k_1 r + \phi_1) \times \cos(\omega t - k_2 r + \phi_2) \]  
(2.26)

The time average of a function \( f(t) \) taken over an interval \( T \) is:

\[ \langle f(t) \rangle = \frac{1}{T} \int_{t}^{t+T} f(t') dt' \]  
(2.27)

Applying this to equation (2.26) and using:

\[ < \cos(\omega t) > = 0 \]
\[ < \sin^2(\omega t) > = \frac{1}{2} \]  
(2.28)

\[ \langle E_1 \cdot E_2 \rangle = \frac{1}{2} E_{01} \cdot E_{02} \cos(k_1 r - \phi_1 - k_2 r + \phi_2) \]  
(2.29)

The interference term can then be written:

\[ I_{12} = E_{01} \cdot E_{02} \cos \delta \]  
(2.30)

where \( \delta = (k_1 r - \phi_1 - k_2 r + \phi_2) \) is the phase difference. Therefore the intensity
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at the point of intersection of two plane light waves has the form:

\[ I = E_1^2 + E_2^2 + E_{01}E_{02} \cdot \cos \delta. \quad (2.31) \]

It can be seen that the maximum intensity occurs when \( \delta = 0 \) which corresponds to the beams being in phase. A minimum intensity occurs when \( \delta = \pi/2 \) and the beams are out of phase. This general case can be extended to the case of an LDA system.

2.3.1 Photocurrent generated by a single stationary particle in an LDA system

In an LDA system two laser beams are brought to a focus at the point of interest in a flow. Assuming the fringe volume is formed at the focal points of both lenses, as discussed in Section 2.2.2, the wavefronts are parallel and plane. The intensity distribution at the focal point in this situation is Gaussian and can be written as [Kogelnik and Li, 1966]:

\[ I(x,y,z) \approx I_0 \exp \left(-\frac{x^2 + y^2}{d_{x-z}}\right) \quad (2.32) \]

where the origin is taken as the focal point and \( d_{x-z} \) is the focussed beam diameter as in figure 2.3. The electric field distribution of a focussed laser beam at the
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Point \((x, y, z)\) is:

\[
E(x, y, z) = E_0 \exp \left( -\frac{x^2 + y^2}{2d_{e-2}^2} + jkz \right)
\]  

(2.33)

where \(j = \sqrt{-1}\) and:

\[
E_0 = \|i_{1/2}^2 e^{-j2\pi} \]  

(2.34)

The term \(\nu\) is the frequency of the laser light. This exponential factor makes no significant contribution to the following discussion and therefore is neglected. In an LDA system, the field contribution due to laser beam 1 is:

\[
E_1(x_1, y_1, z_1) = E_0 \exp \left( -\frac{x_1^2 + y_1^2}{2d_{e-2}^2} + jkz_1 \right)
\]  

(2.35)

and the field contribution due to laser beam 2 is:

\[
E_2(x_2, y_2, z_2) = E_0 \exp \left( -\frac{x_2^2 + y_2^2}{2d_{e-2}^2} + jkz_2 \right)
\]  

(2.36)

where \(E_0\) is the intensity of the two beams at the focal point.

The field at the photodetector surface is therefore:

\[
E = E_1 + E_2
\]  

(2.37)
Equation (2.21) shows that:

\[ I \propto \langle E^2 \rangle \propto |E|^2 \]  

(2.38)

as \( E \) is not time dependent. Integrating the intensity over the detector surface provides an expression for the photocurrent, \( i_F \), generated by the field:

\[ i_F = \eta \int_S |E|^2 dA \]  

(2.39)

where \( \eta \) is the sensitivity of the photodetector and \( S \) is the surface area of the detector. Using equation (2.31) and assuming that the two laser beams are in phase:

\[ i_F = \eta \int_{-\infty}^{\infty} \int \left| E_1(x, y) + E_2(x, y) \right|^2 dx dy \]

\[ = \eta \left( 2\pi \frac{M \sigma_s}{k^2} \right)^2 \left\{ \left| E_1(x_1, y_1, z_1) \right|^2 + \left| E_2(x_2, y_2, z_2) \right|^2 \right. 

\left. + 2 \Re \{ E_1(x_1, y_1, z_1) E_2^*(x_2, y_2, z_2) \} \right\} \]  

(2.40)

where \( M \) is the magnification factor of the receiving optics which is introduced if the laser beams are not of equal strength, \( \sigma_s \) is the scattering amplitude function and \( k \) is the wavenumber. According to Mie scattering theory [Mie, 1908], for small values of \( \theta \), the half angle between the laser beams and for spherical
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scattering particles:

\[ \frac{\lambda^2 \sigma_s}{\pi} = C_{sc} \]  

(2.41)

where \( C_{sc} \) is the scattering cross section of the particle. Therefore the photocurrent generated by a single stationary particle has the form:

\[
i_F = \eta \left( \frac{1}{2} MC_{sc} \right)^2 \left\{ |E_1(x_1, y_1, z_1)|^2 + |E_2(x_2, y_2, z_2)|^2 + 2\Re[E_1(x_1, y_1, z_1)E_2^*(x_2, y_2, z_2)] \right\}
\]  

(2.42)

![Figure 2.8: Notation in LDA fringe system.](image)

Applying equation (2.42) to the case of a Gaussian beam system and using the following equations from figure 2.8:

\[
x_1 = x \cos \theta - z \sin \theta
\]

\[
x_2 = z \cos \theta - x \sin \theta
\]

\[
z_1 = z \cos \theta + x \sin \theta
\]

\[
z_2 = z \cos \theta - x \sin \theta
\]
Figure 2.9: Fringe volume produced by LDA arrangement.

\[ y_1 = y_2 = y \]

produces a photocurrent generated by a single, stationary particle:

\[
i_F = 2 \left( \frac{1}{2} MC_{te} \right)^2 E_0 \exp \left( -\frac{x^2 \cos^2 \theta + y^2 + z^2 \sin^2 \theta}{d_{e-2}^2} \right) \times \left[ \cosh \left( \frac{x z}{d_{e-2}^2} \sin(2\theta) \right) + \cos \left( \frac{4\pi}{\lambda} x \sin \theta \right) \right]
\]

This current has a maximum value when the particle is at the origin of the coordinate system which corresponds to the focus. The photocurrent drops to 1/e of its peak value if the particle is located on the ellipsoid of figure 2.9 which has a form:

\[
x^2 \cos^2 \theta + y^2 + z^2 \sin^2 \theta = d_{e-2}^2
\]
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This gives the dimensions of the fringe volume as:

\[
\begin{align*}
\Delta x &= \frac{2d_{e-2}}{\cos \theta} \\
\Delta y &= 2d_{e-2} \\
\Delta z &= \frac{2d_{e-2}}{\sin \theta}
\end{align*}
\]  

(2.46)

A maximum value of the photocurrent for a stationary particle at a position \((x, y, z)\) in the fringe volume is obtained when the last cosine term in equation (2.44) has a maximum value. Therefore a maximum current is obtained whenever \(2x \sin \theta\) is equal to an integer number of wavelengths \((N \lambda)\) and a minimum current when \(2x \sin \theta = (N + 1/2)\lambda\). This allows the fringe spacing, \(\Lambda\), to be determined:

\[
\begin{align*}
2x_N \sin \theta &= N \lambda \\
2x_{N+1} \sin \theta &= (N + 1)\lambda \\
\Lambda &= x_{N+1} - x_N \\
&= \frac{\lambda}{2 \sin \theta}
\end{align*}
\]  

(2.47)

The total number of fringes, \(N_f\), in the observation volume is obtained from the width of the fringe volume, \(\Delta x\), and the fringe separation, \(\Lambda\):

\[
N_f = \frac{\Delta x}{\Lambda} = \frac{4d_{e-2}}{\lambda} \tan \theta
\]

(2.48)
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2.3.2 Photocurrent generated by a particle in a constant flow

If the scattering particles are suspended within a flow which has a uniform velocity $U$ in the $x$ direction only, the coordinates of a scattering particle can be defined as $(Ut + x_o, y_o, z_o)$ where the origin is defined as the point $(x_o, y_o, z_o)$. If it is assumed that $x_o = 0$, the equation for the photocurrent generated by a moving particle can be written as:

$$i_F(t) = 2 \left( \frac{1}{2} MC_{sc} \right)^2 E_o^2 \exp \left( -\frac{U^2 t^2 \cos^2 \theta + y_o^2 + z_o^2 \sin^2 \theta}{d_{sc}^2} \right) \times \left[ \cosh \left( \frac{U t x_o}{d_{sc}^2} \sin(2\theta) \right) + \cos \left( \frac{4\pi t}{\lambda} Ut \sin \theta \right) \right]$$

(2.49)

The maximum photocurrent amplitude is obtained when the particle crosses the centre of the fringe volume at time $t = 0$.

2.3.3 Photocurrent generated by a particle in an acoustic field

If a particle is moving in the $x$ direction within an acoustic field of frequency $f_m$, the particle has a displacement of:

$$x = x_o + x_m \sin(2\pi f_m t + \phi).$$

(2.50)
where $\phi$ is the phase and $x_m$ is the sound amplitude. The particle coordinates are therefore $(x_0 + x_m \sin(2\pi f_m t), y_0, z_0)$ where the origin is again defined as the point $(x_0, y_0, z_0)$. In this case, again assuming that $x_0 = 0$, the photocurrent generated using equation (2.42) has the form:

$$i_P(t) = 2 \left( \frac{1}{2} MC_{sc} \right)^2 E_0^2 \exp \left( -\frac{(x_m \sin(2\pi f_m t + \phi))^2 \cos^2 \theta + y_0^2 + z_0^2 \sin^2 \theta}{d_{c-2}^2} \right)$$

$$\times \left[ \cosh \left( \frac{(x_m \sin(2\pi f_m t + \phi)) z_0 \sin(2\theta)}{d_{c-2}^2} \right) + \cos \left( \frac{4\pi}{\lambda} (x_m \sin(2\pi f_m t + \phi)) \sin \theta \right) \right]$$

(2.51)

Again, a maximum value for the photocurrent is obtained when the particle is at the centre of the fringe volume at a time $t = 0$ and with a phase of $\phi = 0$.

### 2.3.4 Changes to fringe spacing due to refractive index variations

If the laser beams pass through a change in medium then their path lengths are affected; therefore when they intersect, the beam angle has changed and the intersection point has moved position by $\Delta y$, as in figure 2.10. This has to be taken into account when calculating the fringe spacing using equation (2.47).

It is known from Snell’s law that [Hecht, 1987]:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

(2.52)
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Figure 2.10: Effect of refractive index changes on position of fringe volume.

where \( n \) is the refractive index of the medium and \( \theta \) is the angle of the laser beam to the normal. Therefore:

\[
\begin{align*}
\sin \theta_3 &= \frac{n_2}{n_3} \sin \theta_2 \\
\sin \theta_2 &= \frac{n_1}{n_2} \sin \theta_1 \\
\sin \theta_3 &= \frac{n_1}{n_3} \sin \theta_1
\end{align*}
\]

(2.53)

In the experiments detailed within this thesis, the medium \( n_1 \) is air, \( n_2 \) is glass and \( n_3 \) is water. This produces:

\[
\sin \theta_3 = 0.752 \sin \theta_1
\]

(2.54)
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The correct formula for fringe spacing is therefore:

\[
\Lambda = \frac{\lambda}{2 \sin \theta_3} = \frac{\lambda}{2 \times 0.752 \sin \theta_1}
\]  

(2.55)

2.4 Traditional analysis of Doppler signal

In order to obtain information about the flow it is necessary to analyse the Doppler signal captured by the photodetector, this has traditionally been done in one of four ways: spectral analysis, photon correlation, frequency tracking or burst Doppler spectrum analysis. Frequency counters, burst Doppler spectrum analysis and frequency tracking all use the fact that, apart from ambiguity noise, the instantaneous frequency of the photodetector signal is a direct measure of the instantaneous velocity of the medium. The instantaneous velocity record can therefore be used to compute the frequency, amplitude and phase of the signal as well as the power spectrum of velocity which cannot be obtained from a direct spectral analysis of the photodetector signal.

However, these analysis techniques are unsuitable for the analysis of complex sound fields or non-stationary signals. The work detailed within this thesis is based on experimental results analysed using the Hilbert transform [Grechikhin and Rinkevichius, 1996] and wavelet [Yesin et al., 1999] techniques.
Figure 2.11: Power spectrum of single frequency sound.

and these are described in Chapter 3.

2.4.1 Spectral analysis

Spectral analysis is a straightforward method involving only the capture and Fourier transform of the signal. Fourier transforming the Doppler signal arising from a sound field generates a frequency spectrum as in figure 2.11 which consists of a set of peaks centred around the carrier frequency with the side peaks spaced at intervals of the sound frequency [Taylor, 1976], [Taylor, 1981], [Vignola et al., 1991].
Comparing the ratio of the first and second peak and the frequency spacing between them it is possible to determine the amplitude and frequency of the sound source. The amplitude is obtained from the ratio of the $n$th to the central peak:

$$J_n^2 \left( \frac{2\pi a_m}{\lambda} \right) / J_0^2 \left( \frac{2\pi a_m}{\lambda} \right)$$  

(2.56)

where $a_m$ is the amplitude of the sound and $J_n()$ is an $n$th order Bessel function.

In figure 2.11 it is clear that the carrier frequency has a value of 200 kHz and the sound frequency a value of 10 kHz. However, the signal obtained from spectral
analysis of a complex sound field is much more complicated. Figure 2.12 shows the power spectrum for a double frequency sound with a carrier frequency of 200 kHz and sound frequencies of 10 kHz and 5 kHz. It is very difficult to determine which side bands correspond to which sound frequency. The spectral analysis technique is therefore not suitable for use in analysing complex fields. Further drawbacks include that this method does not give a real time record of the instantaneous velocity and that the spectrum is seriously distorted by non-uniform input signal levels.

However, spectral analysis has the advantage that in steady state fields reliable spectra can be obtained from signals which have very little Doppler frequency modulation and a high dropout rate which corresponds to time intervals with no signal due to the fluctuating particle concentration.

2.4.2 Photon correlation

The signal captured by the detector in an LDA arrangement is made up of electron pulses. The average number of these pulses is proportional to the intensity of the scattered light. This signal can be correlated with itself to produce an autocorrelation function of the form shown in figure 2.13.

The autocorrelation function contains information on the intensity variations of the scattered light and the velocity with which scattering particles cross the in-
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Figure 2.13: Autocorrelation function

For a constant flow velocity, \( U \), the autocorrelation function is a damped cosine where the damping reflects the intensity distribution across the fringe pattern. Unwanted low frequency components cause the axis of the cosine function to drop with increasing time lag. The number of cycles in the autocorrelation function before complete damping is approximately equal to the number of fringes in the interference region.

The flow velocity can be calculated from the time, \( T_m \), between two successive maxima:

\[
U = \frac{\lambda}{(2T_m \sin \theta)}
\]

(2.57)

where \( \lambda \) is the laser wavelength and \( \theta \) is the half angle between the laser beams.

In the case of an acoustic signal or an acoustic signal plus a mean flow, the autocorrelation function becomes much more complicated. The values of the flow velocity and the acoustic particle velocity are then determined from the positions of the peaks and zeros of the correlogram [Sharpe and Greated, 1987], [Sharpe and Greated, 1987].
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Advantages of photon correlation over other analysis techniques are that the method is especially useful when the intensity of the scattered light is very low or the SNR is very low. This technique also works when there is a low seeding particle concentration or the velocities involved are very small. However, disadvantages include the fact that the autocorrelation function is very complicated if more than one frequency is present.

2.4.3 Frequency counters

Frequency counters measure the frequency of a signal by accurately timing the duration of an integral number of cycles, N, of the signal. This timing is performed with respect to the zero crossings of the Doppler component of the signal, so the first step in the processing system is the removal of the signal pedestal and low frequency components by filtering (figure 2.14), after which the remainder of the data is the Doppler signal plus noise. The signal and noise is then converted to a square wave by a Schmidt trigger whose output changes from a low level to a high level whenever the input increases through zero voltage and vice versa as it decreases through zero. The leading edges of the square wave mark the zero crossings very accurately. In the absence of any signal the zero crossings of the noise would activate the Schmidt trigger and the counter would measure the noise frequency, even if the noise was weak and the SNR high. The time interval in which the signal exceeds the thresholds is called the burst time.
Having defined the burst, the counter can measure the frequency in two ways: N-cycle time and total burst mode. The N-cycle time, $\tau_N$, can be measured and the frequency can be computed from $\nu_D = N/\tau_N$ where $\nu_D$ is the Doppler frequency and $N$ is the number of cycles. The accuracy of this measurement is normally validated by comparing it with the frequency computed from measurements of a smaller number of cycles, say $N/2$. If the difference in the measurements is less than some prescribed error, $\delta$:

$$\left| 1 - 2 \frac{\tau_{N/2}}{\tau_N} \right| \leq \delta$$  \hspace{1cm} (2.58)

the measurement is accepted, otherwise it is rejected. This is called the N-cycle
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mode but does not guarantee one measurement per burst.

The second method of frequency measurement uses the total burst time, $\tau_B$, and the number of cycles in the burst, $N_B$, to compute the frequency:

$$\nu_D = \frac{N_B}{\tau_B} \tag{2.59}$$

This is the total burst mode and provides one measurement per burst. This measurement represents an average of the particle’s velocity during its movement across the measurement volume. The total burst mode is desirable when measuring low bursts of signal. Also, when the SNR is high, small amounts of noise added to the signal cause errors in the signals zero-crossing times at the start and the end of the measurement period. These errors result in the smallest frequency error when the measurement time is large. Hence, total burst mode measurements are more accurate than the N-cycle mode if $N_B > N$ but less accurate if $N_B < N$.

2.4.4 Burst Doppler spectrum

This method of analysis produces a plot of the instantaneous frequency by taking a sample segment of, say, five periods of the signal and analysing them. This is then repeated for the next five periods and so on. Each set of analysed periods gives a value for the frequency and is used to plot the instantaneous frequency.
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This method has the advantages that it is very accurate, provides a measurement of instantaneous frequency and reduces the effects of noise through averaging. However, disadvantages are that in order to analyse a high frequency signal, a very high sample rate is required to give good enough resolution in each period of the sound frequency. Also, a large sample is needed over which the frequency is relatively constant, this provides an upper limit to the frequency measured.

2.4.5 Frequency tracking

Several systems [Fridman et al., 1968], [Mazumder, 1972] have been proposed for processing LDA signals in order to produce the required output. Figure 2.15 shows a suitable arrangement for frequency tracking. The voltage controlled oscillator (VCO) provides a continuous signal which is mixed with the Doppler signal and passed through a narrow band filter. The discriminator provides frequency-to-voltage conversion and the remainder of loop provides SNR enhancement. Variations in frequency of the Doppler signal are compensated for by the VCO and the output signal provides a continuous record of the voltage which is proportional to the instantaneous Doppler frequency and therefore the instantaneous velocity. As a near-continuous input signal is required, a drop out mechanism is used to hold the last known signal until a new signal arrives.

Disadvantages include that the signal quality is low unless the flow is heavily seeded. If the signal is discontinuous as a result of uneven concentrations of
seed particles across the scattering area or if the signals from two particles interfere destructively to produce dropout then the signal is lost for short periods of time. This raises a problem because the output at any point which has dropout is not proportional to the velocity.

\textbf{Figure 2.15:} Arrangement for frequency tracking
Chapter 3

Doppler signal analysis

3.1 Doppler signal

The intensity distribution, $i_F$, at the detector surface generated by a single particle passing across the fringe volume in an LDA system has the form discussed in Chapter 2:

$$i_F = 2 \left( \frac{1}{2} MC_{sc} \right)^2 E_o^2 \exp \left( \frac{-x^2 \cos^2 \theta + y^2 + z^2 \sin^2 \theta}{r^2} \right)$$

$$\times \left[ \cosh \left( \frac{x z}{r_0^2} \sin (2\theta) \right) + \cos \left( \frac{4\pi}{\lambda} x \sin \theta \right) \right] \quad (3.1)$$
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For a Gaussian laser beam with a shift frequency of $\omega_d$, the intensity distribution in one direction, $i(x)$ can be written:

$$i(x) \propto \exp(-\beta^2 x^2) \left[ 1 + M \cos \left( \frac{2\pi}{\Lambda} x + \omega_d t \right) \right]$$  \hspace{1cm} (3.2)

where $M$ is a constant which is introduced if the laser beams are not of equal strength, the beam width $\beta = \cos \theta / r_0$, $r_0$ is the laser beam radius, $2\theta$ is the angle between the beams and $\Lambda$ represents the fringe spacing.

A single particle passing through the fringe volume with a position $u_o t + \chi(t)$ produces an output voltage $V(t)$ from the detector of the form:

$$V(t) \propto \exp \left[ -\beta^2 (u_o \cdot (t - t_n) + \chi(t - t_n))^2 \right]$$

$$\left[ 1 + M \cos \left( \omega_d t + \frac{2\pi}{\Lambda} (u_o \cdot (t - t_n) + \chi(t - t_n)) \right) \right]$$  \hspace{1cm} (3.3)

where $t_n$ is a Poisson distributed random arrival time at the centre of the fringe volume, $u_o$ is a uniform flow velocity and $\chi(t)$ is a summation of sine functions with different amplitudes and frequencies due to multiple particles in the fringe volume.

Assume there are $N$ particles passing through the fringe volume with different positions. If the length scale of the turbulence and the wavelength of the sound field are much larger than the fringe volume then the velocity of all the particles is the same at any point in time and at any position in the fringe pattern. This
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allows \( \chi(t - t_n) \) to be written as \( \chi(t) \) and the intensity at the detector as a function of time, \( i(t) \), can be expressed as:

\[
i(t) = \sum \exp \left[ -\beta^2 (u_o \cdot (t - t_n) + \chi(t))^2 \right] \left[ 1 + M \cos \left( \omega_d t + \frac{2\pi}{\Lambda} (u_o \cdot (t - t_n) + \chi(t)) \right) \right]
\]

For small amplitudes it is necessary to have small fringe separation so \( \chi(t) \) is replaced with \( a_m \sin(\omega_m t + \phi_m) \) to form:

\[
i(t) = \sum_{i=1}^{N} \exp \left[ -\beta^2 (u_o \cdot (t - t_n) + a_m \sin(\omega_m(t) + \phi_m))^2 \right] \left[ 1 + M \cos \left( \omega_d t + \frac{2\pi}{\Lambda} (u_o \cdot (t - t_n) + a_m \sin(\omega_m t + \phi_m)) \right) \right]
\]

This is the form of the Doppler signal. It is this signal which has to be analysed in order to obtain information about the flow. As mentioned in Chapter 2, traditional methods of analysing Doppler signals do not allow complex sound fields or non-stationary signals to be analysed. The Hilbert transform and wavelet transform have the advantages that they do allow analysis of complex and non-stationary signals.
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3.2 Hilbert analysis

3.2.1 Basic Hilbert theory

The Hilbert transform provides a 90 degree phase shift to the input data. If \( x(t) \) is the original data and the signal after Hilbert transforming is \( h(t) \), then the analytic signal, \( z(t) \), is:

\[
    z(t) = x(t) + jh(t) \tag{3.6}
\]

The Hilbert transform consists of generating the fast Fourier transform and shifting the first half of the transform by 90 degrees and the second half by \(-90\) degrees. The shifted vector is then inverse Fourier transformed back into the time domain. The correlation of the Hilbert data with its original data is zero.

The instantaneous phase, \( p(t) \), of the signal is given by:

\[
    p(t) = \arctan \left( \frac{h(t)}{x(t)} \right) \tag{3.7}
\]

and the instantaneous frequency by:

\[
    f(t) = \frac{1}{2\pi} \frac{dp(t)}{dt} \tag{3.8}
\]
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3.2.2 Analysis technique

Figure 3.1 shows a flow diagram of the analysis technique. In discussion of the analysis, the referenced figures correspond to experimental data obtained from a single frequency sound of 10 kHz and a carrier frequency of 200 kHz.

The Doppler signal (figure 3.2) is initially Fourier transformed (figure 3.3) and then multiplied by its complex conjugate to produce the power spectrum (figure 3.4) from which the central peak provides a value for the carrier frequency and the side peaks are spaced at integer values of the sound frequency. However, information is easily obtained from the power spectrum only if the acoustic signal is due to a single frequency as complex sounds give rise to much more complicated spectra. For a complex field the Fourier transformed signal is bandpass filtered to remove the low frequency components and this produces, $i_B$ (figure 3.5):

$$i_B(t) = \sum_{i=1}^{N} \exp \left[ -\beta^2(u_o \cdot (t - t_n) + \chi(t))^2 \right] \cos \left[ \omega_d t + \frac{2\pi}{\Lambda} (u_o \cdot (t - t_n) + \chi(t)) \right]$$

To obtain information about the flow it is also necessary to obtain the orthogonal filtered signal (figure 3.5). In order to do this the Fourier plane is multiplied by:

$$G(\omega) = \exp(-j\frac{\pi}{2}), \quad \omega \geq 0$$
$$= \exp(+j\frac{\pi}{2}), \quad \omega < 0$$

(3.10)
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Figure 3.1: Flow diagram of Hilbert analysis technique.
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and inverse Fourier transformed back into real space to form $i_G$:

$$i_G(t) = \sum_{i=1}^{N} \exp \left[ -\beta^2 (u_0 \cdot (t - t_n) + \chi(t))^2 \right] \sin \left[ \omega_d t + \frac{2\pi}{\Lambda} (u_0 \cdot (t - t_n) + \chi(t)) \right]$$  \hspace{1cm} (3.11)

By multiplying by $G(\omega)$ a Hilbert transform is carried out in which real and imaginary parts are swapped, resulting in a phase change of $\pi/2$. This is visible in figure 3.5.

The form of the amplitude modulation, $I_{am}(t)$ (figure 3.6) can be examined by taking the sum of the square moduli of the signals:

$$I_{am}(t) = |i_B|^2 + |i_G|^2$$

$$= \sum_{i=1}^{N} \exp \left[ -\beta^2 (u_0 \cdot (t - t_n) + \chi(t))^2 \right]$$ \hspace{1cm} (3.12)

The instantaneous phase (figure 3.7) is found from the arctangent of the ratio of the filtered, $i_B(t)$, and orthogonal filtered signals, $i_G(t)$:

$$\Phi(t) = \arctan \left( \frac{i_G}{i_B} \right)$$ \hspace{1cm} (3.13)

$$= \frac{2\pi}{\Lambda} [u_0 \cdot (t - t_n) + \chi(t)] + \omega_d t$$

and the instantaneous velocity from the derivative of the instantaneous phase
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divided by the fringe spacing factor, D:

\[ v_i(t) = \frac{1}{D} \frac{d\phi(t)}{dt} = \frac{1}{D} \left[ \frac{2\pi}{\Lambda} (u_o + \chi'(t)) + \omega_d \right] = u_o + \chi'(t) + \frac{\omega_d}{D} \]  

(3.14)

where

\[ D = \frac{2\pi}{\Lambda} \]  

(3.15)

and

\[ \chi'(t) = \frac{d\chi(t)}{dt} \]  

(3.16)

Multiplying the instantaneous velocity by the fringe spacing factor, D, produces the instantaneous frequency (figure 3.8), the mean value of which gives the carrier frequency and the oscillation frequency corresponds to the sound frequency \( \nu_i(t) \):

\[ \nu_i(t) = D(u_o + \chi'(t)) + \omega_d \]  

(3.17)

Fourier transforming the instantaneous frequency produces the frequency spectrum (figure 3.9) which provides a value for the frequency and amplitude of all
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![Doppler signal](image)

**Figure 3.2:** Doppler signal

the components present.

An analysis program [Hann and Greated, 1999] which uses these steps was written at the University of Edinburgh for the purpose of consultancy work for DERA and has been adapted for the purposes of this work.

### 3.3 Wavelet analysis

Frequency information has traditionally been obtained from signals using many techniques including Fourier transforms. However, this allows only the frequency or time information to be obtained at any one instant. This is usually satisfactory
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Figure 3.3: Real part of Fourier transformed data

Figure 3.4: Power spectrum
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Figure 3.5: Filtered and orthogonal filtered data.

Figure 3.6: Amplitude modulation
Figure 3.7: Instantaneous phase

Figure 3.8: Instantaneous frequency
for stationary signals in which the frequency does not change over time but in contrast, for a "chirp" signal, in which the frequency is constantly changing in time, it is essential to derive information on both time and frequency simultaneously.

### 3.3.1 Wavelet theory

Traditionally, non-stationary signals have been analysed using short time Fourier transforms but these give a fixed frequency resolution and can only provide information on those bands of frequencies that occur within specified time intervals. In contrast, the wavelet transform [Chui, 1992], [Strang and Nguyen, 1996] provides a variable frequency resolution for a given analysis window.
For the continuous wavelet transform (CWT), higher frequencies are more accurately resolved in time and lower frequencies are more accurately resolved in frequency. This can be seen in figure 3.10. The top row shows that at higher frequencies there are more samples at smaller time intervals and the bottom row represents low frequencies where there are fewer samples.

Continuous wavelet transforms are not suitable for analysing data due to the extremes of computation time they require and therefore discrete wavelet transforms are usually used. In the case of discrete wavelet transforms, the time resolution of the signal is the same as for the continuous case but the frequency information changes at each stage, as in figure 3.10. Lower frequencies are more accurately
resolved in frequency than are higher frequencies.

When using wavelet transforms the term frequency is usually neglected and scale is used instead which corresponds to the inverse of frequency. A high scale therefore corresponds to a low frequency and a low scale to a high frequency.

Continuous wavelet transforms

In contrast to Fourier transforms with a fixed analysis window, the width of the analysing window is changed as the continuous wavelet transform is computed for each spectral component in a wavelet transform. Mathematically, the wavelet transform can be expressed as:

\[ CWT^\psi_x(\tau, s) = \Psi^\psi_x(\tau, s) \]

\[ = \frac{1}{\sqrt{|s|}} \int x(t) \psi^* \left( \frac{t - \tau}{s} \right) \]

(3.18)

where the variables \( \tau \) and \( s \) are the translation and scale parameters and \( \psi(t) \) is the transforming function, called the mother wavelet. As a mathematical function, scaling either dilates or compresses a signal. The larger the scale, the more dilated the signal and the smaller the scale the more compressed the signal.

The mother wavelet acts as a prototype for generating other window functions which are scaled and shifted versions of the mother wavelet. In the analysis presented in this thesis, the mother wavelet is a Morlet function although Mexican
hat functions are also popular [Strang and Nguyen, 1996].

The transform starts with the scale $s = 1$ and time $t = 0$. Values of scale less than 1 can be used but for convenience a value of 1 is usually used as a starting point and increased which corresponds to a decrease in frequency. The wavelet function is then multiplied by the signal and integrated over all time. The result is then multiplied by $1/\sqrt{s}$, this ensures that the transformed signal has the same energy at all scales. This provides a value for the wavelet at scale $s = 1$ and time $t = 0$.

The wavelet is then shifted by an amount $\tau$ to the new location $t = \tau$ and the above procedure is repeated to provide a value for the wavelet at $s = 1$ and $t = \tau$ in the time-frequency plane. This is repeated until the wavelet reaches the end of the signal and then the values of $s$ and $\tau$ are increased and the procedure iterated.

As the scale value increases the width of the function increases and as a result lower frequency components begin to be detected. Every computation for a given value of the scale fills the corresponding single row of the time-frequency plane.

The continuous wavelet transform is complete when every value of the scale has been computed. Figure 3.11 shows the wavelet and the signal for various values of scale, $s$ and translation, $\tau$. It can be seen that a wavelet is a localised wave and instead of oscillating forever, it drops to zero.

If the signal has a spectral component which corresponds to the value of the scale then the resulting value of the wavelet has a large value. If the corresponding spectral component is not present then the resulting value is very small or zero.
Figure 3.11: Wavelet and signal for various values of scale, $s$ and translation, $\tau$. 

$s = 1 \quad \tau = 0.0003$

$s = 2 \quad \tau = 0.0020$

$s = 4 \quad \tau = 0.0030$

$s = 8 \quad \tau = 0.0060$
As mentioned above, the short time Fourier transform has constant resolution at all times but the wavelet transform has variable resolution. Figure 3.12 represents the way in which the resolution varies with time and frequency. Every box has the same area corresponding to equal portions of the time-frequency plane. The non-zero size of the boxes implies that the value of a particular point in the time-frequency plane cannot be known. At low frequencies, the boxes are shorter in height but wider, this corresponds to good frequency resolution but poor time resolution. At higher frequencies this reverses to provide poorer frequency resolution (taller box) and better time resolution (thinner box).
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3.3.2 Mathematics of the wavelet transform

Vectors and inner products

A basis of a vector space, \( \mathbf{V} \), is a set of linearly independent vectors. Any vector, \( \mathbf{v} \), in \( \mathbf{V} \) can be expressed as a linear combination of the basis vectors. Irrespective of the number of bases in a vector space they all have the same number of vectors, this number being the dimension of the vector space. For example, in two dimensional space, the basis has two vectors. Any vector, \( \mathbf{v} \), can be written as a linear combination of the basis vectors \( b_k \) and their coefficients \( \nu_k \):

\[
\mathbf{v} = \sum_k \nu_k b_k \tag{3.19}
\]

The vectors can also be replaced by functions. Replacing the vector \( \mathbf{v} \) with the function \( f(t) \) and the basis vectors \( b_k \) with basis functions \( \phi_k(t) \) and their coefficients \( \mu_k \):

\[
f(t) = \sum_k \mu_k \phi_k(t) \tag{3.20}
\]

Let \( f(t) \) and \( g(t) \) be two functions in \( L^2[a,b] \) where \( L^2[a,b] \) denotes the set of square integrable functions in the interval \( [a,b] \). The inner product of these func-
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The continuous wavelet transform, \( CWT \), can be thought of as the inner product of the test signal, \( x(t) \), with the basis functions \( \psi_{\tau,s}(t) \) which are the wavelets:

\[
CWT_x(\tau, s) = \Psi_x(\tau, s) = \int x(t) \cdot \psi_{\tau,s}^*(t) dt \tag{3.22}
\]

where

\[
\psi_{\tau,s} = \frac{1}{\sqrt{s}} \psi \left( \frac{t - \tau}{s} \right) \tag{3.23}
\]

From this it can be seen that the wavelet analysis is a measure of similarity between the basis functions and the signal itself.

Orthogonality and orthonormality

Two functions \( f \) and \( g \) are orthogonal if their inner product equals zero:

\[
\langle f(t), g(t) \rangle = \int_a^b f(t) \cdot g^*(t) dt = 0 \tag{3.24}
\]
A set of vectors is said to be orthonormal if each vector in the set is orthogonal to each other and all have unit length. This is usually expressed as:

$$\langle a_m, b_n \rangle = \delta_{mn}$$  \hspace{1cm} (3.25)

A set of functions $\phi_k(t), k = 1, 2, 3...$ are orthonormal if:

$$\int_b^a \phi_k(t)\phi_l^*(t)dt = 0, \ k \neq l$$  \hspace{1cm} (3.26)

and

$$\int_a^b \phi_k(t)\phi_l^*(t)dt = \delta_{kl}$$  \hspace{1cm} (3.27)

where the Kronecker delta function, $\delta_{kl}$, is defined as:

$$\delta_{kl} = \begin{cases} 1 & \text{if } k = l \\ 0 & \text{if } k \neq l \end{cases}$$  \hspace{1cm} (3.28)

In any vector space there can be more than one set of basis functions. Orthonormal basis functions are particularly useful as they allow computation of analysis coefficients in a straightforward way. For orthonormal bases, the coefficients, $\mu_k$, can be expressed as:

$$\mu_k = \langle f, \phi_k \rangle = \int f(t) \cdot \phi_k^*(t)dt$$  \hspace{1cm} (3.29)
and the function $f(t)$ can be reconstructed using equation (3.20) and the values obtained for the coefficients, $\mu_k$, using:

$$f(t) = \sum_k < f, \phi_k > \phi_k(t)$$  \hspace{1cm} (3.30)

3.3.3 Wavelet synthesis

A continuous wavelet transform is a reversible process. The inverse wavelet transform is created using the reconstruction formula:

$$x(t) = \frac{1}{C_\psi^2} \int_s \int_\tau \Psi_\psi(\tau, s) \frac{1}{s^2} \psi \left( \frac{t - \tau}{s} \right) d\tau ds$$  \hspace{1cm} (3.31)

where $C_\psi$ is a constant that depends on the wavelet used and it is this which determines the success of the reconstruction. This constant is called the admissibility constant and satisfies the admissibility condition:

$$C_\psi = \left\{ 2\pi \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\xi)|^2}{|\xi|d\xi} \right\}^{1/2} < \infty$$  \hspace{1cm} (3.32)

where $\hat{\psi}(\xi)$ is the Fourier transform of $\psi(t)$. If the condition in equation (3.32) is not met then the wavelet transform is not reversible. This condition implies that $\hat{\psi}(0) = 0$, which is:

$$\int \psi(t)dt = 0$$  \hspace{1cm} (3.33)
This can only hold if the wavelet function is oscillatory.

Discretisation

A continuous wavelet transform can not practically be computed and therefore the signal needs to be discretised. The most practical way of doing this is to sample the time-frequency plane. According to Nyquist's rule [Kinsler et al., 1982], a signal must have a sample rate of at least double the signal frequency. Therefore, as the signal frequency decreases, the sampling rate can also be decreased which saves computation time.

It is also still possible to reconstruct the signal if it is discretised. The scale parameter, s, is discretised first on a logarithmic scale. The time parameter can then be discretised with respect to the scale parameter, this allows a different sample rate to be used for each scale value. The base of the logarithm can be selected for the user's application. If the base value chosen is 2, only the scales 2, 4, 8, 16... are computed. Since the scale changes by factors of 2, the sampling rate is reduced for the time axis by a factor of 2 at each scale.

3.3.4 Discrete wavelet transform

The discretised continuous wavelet transform enables computation of the continuous wavelet transform by computers but it is not a true discrete transform. The
wavelet series is basically a sampled version of the continuous wavelet transform and therefore the information obtained from it is not very useful in reconstruction of the signal. The discrete wavelet transform provides information for analysis and synthesis of the original signal with a reduction in computation time.

The main idea is the same as for the continuous wavelet transform. A time-scale representation of the digital signal is obtained using digital filtering techniques. In the discrete wavelet transform, filters of different cutoff frequencies are used to analyse the signal at different scales. The signal is passed through a series of high pass filters to analyse the high frequencies and then passed through a series of low pass filters to analyse the low frequencies.

The resolution of the signal is varied by filtering and the scale is varied through upsampling and downsampling operations. Downsampling refers to reducing the sampling rate; downsampling by two corresponds to dropping every other sample of the signal. Upsampling works in a similar way in that it adds new samples to the signal either by interpolation or adding zeroes.

The initial sequence is denoted by \( x[n] \), where \( n \) is an integer. This signal is initially passed through a half band digital lowpass filter with an impulse response \( h[n] \). The output of a lowpass filter at time \( t = n \) is the average of the input signal, \( x[n] \), at time \( t = n \) and the input signal at the previous time \( t = n - 1 \).
The output signal is given by:

\[ y[n] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1] \]  

(3.34)

Mathematically this corresponds to convolution of the signal with the impulse response of the filter in the time domain:

\[ x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k] \cdot x[n-k] \]  

(3.35)

This removes all frequencies that are above half of the highest frequency in the signal. Since the signal now has a highest frequency of \( \pi/2 \) radians instead of \( \pi \) radians, the signal can then by downsampled by two which halves the number of points and the scale of the signal is now doubled. Mathematically this can be expressed as:

\[ y[n] = \sum_{k=-\infty}^{\infty} h[k] \cdot x[2n-k] \]  

(3.36)

where \( y[n] \) is the filter output.

The discrete wavelet transform analyses the signal at different frequency bands with different resolutions by splitting the signal into a coarse approximation component and a detailed information component. The discrete wavelet transform uses two sets of functions called scaling functions and wavelet functions which are associated with low and high pass filters, respectively. The decomposition of the
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signal into different frequency bands is simply obtained by successive highpass and lowpass filtering of the time domain signal. The original signal, $x[n]$, is passed through a halfband highpass filter, $g[n]$ and a lowpass filter, $h[n]$. Half the samples can then be removed by downsampling. This is one level of decomposition and mathematically is expressed as:

$$y_{\text{high}}[n] = \sum_{n} g[n] \cdot x[2k - n]$$
$$y_{\text{low}}[n] = \sum_{n} h[n] \cdot x[2k - n]$$

(3.37)

where $y_{\text{high}}[n]$ and $y_{\text{low}}[n]$ are the outputs of the high pass and lowpass filters, respectively, after downsampling by two. This decomposition halves the time resolution as only half the number of samples are present but this doubles the frequency resolution since the frequency band of the signal now spans only half the previous band. At every level, the filtering and downsampling results in half the number of samples and half the frequency band spanned. This can be seen in figure 3.13 where $x[n]$ is the original signal, $g[n]$ and $h[n]$ are the highpass and lowpass filters, respectively and the width of the frequency band at each level is marked as $f$.

The frequencies which are most prominent in the original signal appears as high amplitudes in the region of the discrete wavelet transform that includes those particular frequencies. The time localisation has a resolution dependent on the level in which they appear. If the main signal information of interest lies in the
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Figure 3.13: Wavelet analysis procedure
high frequencies then they have good time resolution; good frequency resolution
occurs for low frequencies due to the decreasing number of samples.

3.4 Analysis of Doppler signals using wavelet transforms

The Doppler signals which are analysed using the wavelet transform in this thesis
have the form shown in figure 3.14, in which the frequency can be seen to vary
with time. In order to aid analysis of this signal, a frequency shift is added, as
mentioned in Chapter 2. The form of the wavelet can be seen in figure 3.15, which
shows the superimposed mother wavelet at two different scale values. The final
result of the analysis can be seen in figure 3.16 which represents the instantaneous
frequency. It can be seen that the instantaneous frequency values follow the same
trend as the frequency in figure 3.14. The instantaneous frequency decreases to
zero at the same point as the frequency in figure 3.14 reaches a minimum value
and both then increase again. In the analysed signal the increase is in the negative
direction due to the fact that the particles are moving in the opposite direction
through the fringes.
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Figure 3.14: Form of signal to be analysed.

Figure 3.15: Form of the wavelet used in analysis at two different scale values.
Figure 3.16: Analysed signal

3.5 Comparison of techniques

It can be seen in figure 3.17 that the wavelet transform produces a more distinct signal than the Hilbert transform for non-stationary signals and therefore it is the wavelet technique which is used in the analysis of the spark generated bubble data within this thesis. The Hilbert transform technique is used only for analysis of the acoustic data produced from a hydrophone since it suffers from the limitation that each value of the instantaneous frequency is influenced by each sample in the initial Doppler signal and in consequence, if part of the signal is of low quality this can affect the entire analysed signal.
Figure 3.17: Comparison of Hilbert and wavelet analysis techniques.
Chapter 4

Acousto-optic effect in LDA

4.1 Acousto-optic effect

When laser Doppler anemometry (LDA) is used to measure sound fields in water it is important to take into account the effects of refractive index variations in the water due to the sound wave. These have the effect of creating a path difference between the two laser beams in the LDA arrangement so that when the beams intersect they may create moving fringes rather than stationary ones.

When a sound wave passes through a medium it generates areas of tension and compression. This strain field moves with the acoustic wave through the medium and has the effect of creating both temporal and spatial variations in the refractive index of the medium. When LDA is used to measure a sound field, the two laser
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beams can be affected to differing degrees by these refractive index variations, creating an optical path difference between the two beams. When these laser beams intersect they produce an interference pattern in which the fringes move due to the optical path difference introduced by the sound wave. This is the acousto-optic effect.

There are two types of movement which have to be considered. Firstly, the particles suspended in the interference fringes oscillate due to the sound wave and secondly, the fringes move due to the path difference in the laser beams. Thus, the light scattered by the particles and collected by the detector as the Doppler signal is due to a combination of both the oscillating fringes and the oscillating particles. Using the analysis technique of Grechikhin [Grechikhin and Rinkevichius, 1996] it is possible to determine the instantaneous frequency of the particles suspended in the flow field. However, if the acousto-optic effect is the dominant factor, the instantaneous frequency obtained is mainly due to the oscillating fringes rather than the oscillating particles and any amplitude information obtained may not be an accurate representation of the flow at the point of the beam intersection.
4.2 Travelling waves

4.2.1 Optical path difference in laser beams due to refractive index variations

Stationary fringes are formed only if the difference in optical path lengths between the two laser beams does not vary with time. This requires that both beams are affected by the refractive index changes to the same degree. In the case where the optical path lengths are not equal it is possible to determine the amplitude of fringe movement in the direction perpendicular to the optic axis (i.e. the y direction in figure 4.1) and calculate the ratio of the apparent to the actual motion of the particle in the fringes. The apparent motion of the particle in the fringes as seen by the detector can also be determined. To do this it is necessary to determine the form of the path difference, \( \delta \):

\[
\delta = opl_1 - opl_2
\]  \hspace{1cm} (4.1)

where \( opl_1 \) and \( opl_2 \) are the optical path lengths of the two laser beams. The optical path length of each laser beam is given by:

\[
opl = \int n ds
\]  \hspace{1cm} (4.2)
where the two-dimensional refractive index change, \( n(x, y) \), of the sound wave which affects the laser beams is given by:

\[
n(x, y) = n_0 - \Delta n_0 \sin(k_x x + k_y y + \omega t) \quad (4.3)
\]

Here \( n_0 \) is the refractive index of the medium without the presence of a strain field, \( \Delta n_0 \) is a small change in refractive index, \( \omega \) is the angular sound frequency, \( t \) is the time and \( k_x \) and \( k_y \) are the wavenumbers in the \( x \) and \( y \) directions (\( x \) being in the direction of the optic axis, figure 4.1).

The optical path length of a laser beam propagating through a sound wave can therefore be written as:

\[
opl = \int_c (n_0 - \Delta n_0 \sin(k_x x + k_y y + \omega t)) \, ds \quad (4.4)
\]

In this case, although the contour \( c \) is a curve, it is assumed to be a straight line.
for simplicity. This straight line has equation:

\[ y = mx + y_1 - mx_1 \]  \hspace{1cm} (4.5)

it is therefore possible to substitute for \( x \) in equation (4.4):

\[ opl = \int_{y_1}^{y_2} \left[ n_o - \Delta n_o \sin \left( \frac{k_x}{m} (y - y_1 + mx_1) + k_y y + \omega t \right) \right] \frac{\delta s}{\delta y} dy \]  \hspace{1cm} (4.6)

From figure 4.1 it can be seen that:

\[ \delta s = \sqrt{(\delta x)^2 + (\delta y)^2} \]
\[ \frac{\delta s}{\delta y} = \sqrt{\left( \frac{\delta x}{\delta y} \right)^2 + 1} \]  \hspace{1cm} (4.7)

where \( \delta x/\delta y \) is the inverse gradient of the line, \( 1/m \). Therefore:

\[ opl = \sqrt{1 + \left( \frac{1}{m} \right)^2} \int_{y_1}^{y_2} \left[ n_o - \Delta n_o \sin \left( \frac{k_x}{m} (y - y_1 + mx_1) + k_y y + \omega t \right) \right] dy \]
\[ = \sqrt{1 + \left( \frac{1}{m} \right)^2} \left[ n_o (y_2 - y_1) + \frac{\Delta n_o}{k_x + k_y} \right. \]
\[ \times \left[ \cos \left( \frac{k_x}{m} (y_2 - y_1 + mx_1) + k_y y_2 - \omega t \right) - \right. \]
\[ \left. \cos \left( \frac{k_x}{m} (y_1 - y_1 + mx_1) + k_y y_1 - \omega t \right) \right] \]  \hspace{1cm} (4.8)

Figure 4.2 shows that the wavenumbers \( k_x \) and \( k_y \) have the form:

\[ k_x = k \cos \theta \]
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Figure 4.2: Wavenumbers in the x and y directions

\[ k_x = k \sin \theta \quad \text{(4.9)} \]

where \( k = 2\pi / \lambda \), \( \lambda \) is the wavelength of the sound and \( \theta \) is the angle of the sound wavefront from the y direction. Substituting for \( k_x \) and \( k_y \) into equation (4.8) produces:

\[
opl = \sqrt{1 + \left(\frac{1}{m}\right)^2 \left[ n_o (y_2 - y_1) + \frac{m \Delta n_o}{k \cos \theta} + m k \sin \theta \right]}
\times \left[ \cos \left( \frac{k}{m} (y_2 - y_1 + mx_1) \cos \theta + y_2 k \sin \theta - \omega t \right) - \cos \left( x_1 k \cos \theta + y_1 k \sin \theta - \omega t \right) \right] \quad \text{(4.10)}
\]

In this case beam 1 (figure 4.1) is integrated from the point \((x', y')\) to \((a, -b)\) and beam 2 is integrated from the point \((a, b)\) to \((x', y')\), where \(a\) and \(b\) are arbitrary starting points of the beams. The expressions for the optical path length are only integrated over the \(y\) variable as there is negligible fringe movement in the
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In the $x$ direction, the value of $x$ is therefore taken as a constant, $x'$. Due to the relative negative gradient of beam 2, the integral is taken from $(x', y')$ to $(a, b)$ to compensate for the change in the sign of the differential. Therefore:

\[
\frac{1}{\gamma} = F \bar{I}_1 + \left( n_0 (y' + b) + \frac{m_1 \Delta n_o}{k \cos \theta + m_1 k \sin \theta} \right) - \\
\cos \left( ak \cos \theta - bk \sin \theta - \omega t \right) - \\
\cos \left( x' k \cos \theta + y' k \sin \theta - \omega t \right) \right]
\]

(4.11)

and

\[
\frac{1}{\gamma} = F \bar{I}_2 + \left( n_0 (b - y') + \frac{m_2 \Delta n_o}{k \cos \theta + m_2 k \sin \theta} \right) - \\
\cos \left( b k \cos \theta + bk \sin \theta - \omega t \right) - \\
\cos \left( x' k \cos \theta + y' k \sin \theta - \omega t \right) \right]
\]

(4.12)

where

\[
m_1 = \frac{y' + b}{x' - a} \]
\[
m_2 = \frac{b - y'}{a - x'}
\]

(4.13)

Using equations (4.11) and (4.12) the path difference, $\delta$, can be written as:
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Figure 4.3: Variation of fringe movement $y'$ with time $t$ and path difference $\delta$.

\[
\delta = \text{opl}1 - \text{opl}2
\]

\[
= \sqrt{1 + \left(\frac{1}{m_1}\right)^2 \left[n_o(y' + b) + \frac{m_1 \Delta n_o}{k \cos \theta + m_1 k \sin \theta} \times \left[\cos \left((y' + b + m_1 a) \frac{k}{m_1} \cos \theta + y' k \sin \theta - \omega t\right) - \cos \left(ak \cos \theta - bk \sin \theta - \omega t\right)\right] - \sqrt{1 + \left(\frac{1}{m_2}\right)^2 \left[n_o(b - y') + \frac{m_2 \Delta n_o}{k \cos \theta + m_2 k \sin \theta} \times \left[\cos \left((b - y' + m_2 x') \frac{k}{m_2} \cos \theta + bk \sin \theta - \omega t\right) - \cos \left(x'k \cos \theta + y' k \sin \theta - \omega t\right)\right]\right]}\]

\[(4.14)\]
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There is negligible fringe movement in the $x$ direction and therefore the value of $x'$ is a constant in the calculations. Figure 4.3 shows the form of equation (4.14) from which the variation of the fringe position $y'$ with time $t$ and path difference $\delta$ can be seen. The variables of interest are the fringe movement $y'$ and the path difference $\delta$. It is possible to obtain a 2-dimensional plot of these parameters by noting that when $t = 0$ the path difference has a minimum value and when $t = \pi/\omega$ it has a maximum value. This is due to the fact that when there is no sound wave present the two laser beams form a fringe pattern with the central fringe equidistant in the $y$ direction from the two beam origins, in this case at the position $y = 0$. When a sound wave is present in an LDA system,
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the refractive index variations affect the laser beams in such a way that the path difference between the beams can only be zero when the central fringe is offset in the y direction from the y = 0 position. Figure 4.4 can then be drawn to show the variation of fringe movement y' with path difference δ where the top line is produced for $t = \pi/\omega$ and the lower line for $t = 0$. The value of $y'_{amp}$ gives the amplitude of movement of the central fringe.

Simplification of fringe movement expression

The variable of interest here is the amplitude of the fringe movement, $y'_{amp}$. It is not possible to analytically solve equation (4.14) for the path difference, $δ$, in terms of the fringe movement $y'$. However, it can be simplified and then solved. Simplification occurs by assuming that any movement in the x direction is negligible, the parameters $a$ and $t$ are both set to zero and the quantity $((y' + b)/x')^2$ is assumed to be small, thus:

$$\frac{x'}{y' + b} \sqrt{1 + \left(\frac{y' + b}{x'}\right)^2} \approx \frac{x'}{y' + b} \left(1 + \frac{(y' + b)^2}{2x'^2}\right)$$

(4.15)

Substitution into equation (4.14) produces:

$$\delta = x' \left(1 + \frac{(y' + b)^2}{2x'^2}\right) \times \frac{\Delta n_o (\cos(x'k \cos \theta + y' k \sin \theta) - \cos(\theta k) \sin \theta)}{x' k \cos \theta + k(y' + b) \sin \theta}$$
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\[
+x' \left(1 + \frac{(b - y')^2}{2x'^2} \right)
\times \frac{\Delta n_s (\cos(x'k \cos \theta + y'k \sin \theta) - \cos(bk) \sin \theta)}{x'k \cos \theta + k(b - y') \sin \theta} + \frac{2y'bn_o x'}{x'^2} \tag{4.16}
\]

Setting the path difference to \( \delta = 0 \) corresponds to the case when the two laser beams have both travelled the same distance through the refractive index variations and therefore interference fringes are formed. Thus:

\[
\delta = 0
\]

\[
= 2by'n_o k^2 \left[ b^2 \sin^2 \theta - (x' \cos \theta + y' \sin \theta)^2 \right] + \Delta n_o k [\cos(kx' \cos \theta + ky' \sin \theta) - \cos(kb \sin \theta)] \times [2bx'^2 \sin \theta - 2y'bx' \cos \theta + (y'^2 + b^2)b \sin \theta - 2y'^2b \sin \theta]
\tag{4.17}
\]

If all the terms with \( y' \) greater than first order are neglected then the fringe movement \( y' \) can be written:

\[
y' = \frac{\{\Delta n \cos(kx' \cos \theta + ky' \sin \theta) - \cos(kb \sin \theta)\}(2x'^2 + b^2) \sin \theta}{\{2n_o k(b^2 \sin^2 \theta - x'^2 \cos^2 \theta) - 2\Delta n k \cos(kx' \cos \theta + ky' \sin \theta) - \cos(kb \sin \theta)\} x' \cos \theta}
\tag{4.18}
\]
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Since \( ky' \ll 1 \):

\[
\begin{align*}
\cos(ky' \sin \theta) & \approx 1 \\
\sin(ky' \sin \theta) & \approx 0
\end{align*}
\]

and \( \Delta n_o \ll n_o \)

\[
y' = \frac{\Delta n_o (\cos(kx' \cos \theta) - \cos(kb \sin \theta))(2x'^2 + b^2) \sin \theta}{2n_o k (b^2 \sin^2 \theta - x'^2 \cos^2 \theta)}
\]  
(4.19)

It is shown in the next section that \( \Delta n_o = \sigma \Delta P \) where \( \sigma \) is the acousto-optic coefficient and \( \Delta P \) is the change in pressure. Equation (4.19) can therefore be written as:

\[
y' = \frac{\sigma \Delta P (\cos(kx' \cos \theta) - \cos(kb \sin \theta))(2x'^2 + b^2) \sin \theta}{2n_o k (b^2 \sin^2 \theta - x'^2 \cos^2 \theta)}
\]  
(4.20)

4.2.2 Apparent motion of particles

As stated earlier, when LDA is used in sound fields, particles oscillate due to the sound wave and the interference fringes oscillate due to the refractive index variations, the acousto-optic effect. It is the apparent motion of the particles with respect to the fringes that gives rise to the Doppler signal. It is therefore necessary to determine whether the particles which are moving as a result of the sound wave, with amplitude \( A_m \), move in or out of phase with the motion of the
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fringes, with amplitude $y_{amp}^c$, due to the acousto-optic effect. It is also necessary to determine the apparent position of the particles, $Y(k, x, t)$, in the fringes as seen by the detector.

It is known that the form of the refractive index change in the sound wave is:

$$n = n_0 - \Delta n_0 \sin(k_x x + k_y y + k_z z + \omega t)$$  \hspace{1cm} (4.21)

with [Leigh, 1996]

$$\Delta n_0 = \frac{1}{2} pn^3 \varepsilon_0 = \left(\frac{1}{2} MI_a\right)^{\frac{1}{2}}$$ \hspace{1cm} (4.22)

where $p$ is the photoelastic tensor, $n$ the refractive index, $\varepsilon_0$ the shear strain amplitude, $n_0$ the refractive index without the presence of the strain field, $I_a$ the acoustic intensity, $v$ the speed of the sound wave and $M$ the material parameter, where:

$$M = \frac{\rho^2 n^6}{\rho v^3}$$ \hspace{1cm} (4.23)

with $\rho$ as the density.

For a travelling sound wave it is known that [Kinsler et al., 1982]:

$$I_a = \frac{1}{2} \Delta P \Delta u$$ \hspace{1cm} (4.24)

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\[ \Delta u = \pm \frac{\Delta P}{\rho v} \quad (4.25) \]

where \( \Delta u \) is the magnitude of the acoustic velocity and \( \Delta P \) is a pressure change.

Substituting equations (4.23)-(4.25) into equation (4.22) produces:

\[ \Delta n_o = \frac{m^3}{2v^2 \rho} \Delta P \]
\[ = \sigma \Delta P \quad (4.26) \]

where \( \sigma \) is the acousto-optic coefficient.

The refractive index variation \( \Delta n_o \) can also be written in terms of the acoustic wave amplitude and the wavenumber. The shear strain amplitude, \( \epsilon_o \), in equation (4.22) can be written:

\[ \left| \frac{\delta y}{\delta x} \right| = \epsilon_o \quad (4.27) \]

where

\[ y = A_m \sin(kx - \omega t) \quad (4.28) \]

Therefore:

\[ \epsilon_o = A_m k \quad (4.29) \]
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The acoustic velocity has the form

\[ \Delta u = 2\pi A_m f \]  \hspace{1cm} (4.30)

where \( A_m \) is the amplitude of the sound wave, \( f \) is the sound frequency and \( k \) the wavenumber. Using equation (4.22) produces:

\[ \Delta n_o = \frac{1}{2} \rho m^3 A_m k \]  \hspace{1cm} (4.31)

When LDA is used to measure sound fields in water rather than in air, the refractive index variations are much greater and therefore the acousto-optic effect is much stronger. Using the parameters for water:

\[ \rho = 1000 \text{ kg/m}^3 \]
\[ v = 1500 \text{ m/sec} \]
\[ n = 1.33 \]
\[ M = 160 \times 10^{-15} \text{ sec}^3/\text{kg} \]

and using equation (4.23), produces \( p = 0.312 \). The value of the photoelastic tensor, \( p \), can also be derived from the Lorentz-Lorenz equation [Smith and Korpel, 1965]:

\[ p = \frac{1}{3} \frac{(n^2 - 1)(n^2 + 2)}{n^4} \]  \hspace{1cm} (4.32)
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which produces the same value for $p$; therefore:

$$\Delta n_o = 0.367 A_m k$$  \hspace{1cm} (4.33)

It can be seen from equation (4.33) that the value of $\Delta n_o/A_m$ depends only on the wavenumber $k$ and thus the sound frequency. This can also be written in terms of the acousto-optic coefficient $\sigma$ [Leigh, 1996]:

$$\Delta n_o = \frac{0.367}{\rho v^2} \Delta P$$

$$\Delta n_o = \sigma \Delta P$$ \hspace{1cm} (4.34)

producing a value of $\sigma = 1.63 \times 10^{-10} \text{Pa}^{-1}$ for water where $\sigma = (pn^3)/(2pc^2)$.

To determine whether the particle and fringe movement are in phase it is necessary to examine the relationship between pressure and refractive index. The form of the refractive index variations in a sound wave has been shown to be [Leigh, 1996]:

$$n = n_o - \Delta n_o \sin(k_x x + k_y y + k_z z + \omega t)$$ \hspace{1cm} (4.35)

The refractive index change is proportional to the pressure change, thus:

$$P \propto \Delta P \sin(k_x x + k_y y + k_z z + \omega t)$$

$$\Delta n_o \propto \Delta P$$ \hspace{1cm} (4.36)
and from equation (4.25), the pressure change is proportional to the change in acoustic velocity:

\[ u \propto \Delta u \sin(k_x x + k_y y + k_z z + \omega t) \]

\[ \Delta P \propto \Delta u \quad (4.37) \]

and therefore the acoustic wave has the form:

\[ A \propto \frac{A_m}{\omega} \cos(k_x x + k_y y + k_z z + \omega t) \quad (4.38) \]

From this, the conclusion can be drawn that:

\[ |y'| \propto |\Delta n_o| \propto |\Delta P| \propto |\Delta u| \propto |A| \quad (4.39) \]

in which the fringe movement \( y' \), with amplitude \( y'_{\text{amp}} \), is \( \pi/2 \) out of phase with the movement of the acoustic wave \( A \) with amplitude \( A_m \). Therefore the apparent motion of the particle has the form:

\[
Y(k_x x + k_y y + k_z z + \omega t) = A_m \cos(k_x x + k_y y + k_z z + \omega t) + y'_{\text{amp}} \sin(k_x x + k_y y + k_z z + \omega t)
\]

\[ Y(X) = A_m \cos(X) + y'_{\text{amp}} \sin(X) \quad (4.40) \]
where $X$ is a function of time, position and sound frequency. The ratio of the apparent motion, $Y(X)$, to the actual motion, $A_m$, is:

$$\frac{Y(X)}{A_m} = \cos(X) + \frac{y'_{\text{amp}}}{A_m} \sin(X)$$  \hspace{1cm} (4.41)

### 4.2.3 Dependence of $y'_{\text{amp}}$ on wave number and distance of propagation for low wavenumbers

From figure 4.3 it can be seen that the path difference, $\delta$, varies sinusoidally with time for a constant value of the fringe movement, $y'$, and thus has a maximum and minimum value which determines the amplitude of movement of the fringes, $y'_{\text{amp}}$.

Varying the angle of the sound wave from the $y$ direction, the wavenumber $k$, the distance $x'$ which the laser beams have travelled in the direction of the optical axis, or the value of the change in refractive index $\Delta n_o$ which is proportional to the shear strain amplitude (equation (4.22)), has the effect of varying the maximum and minimum value of the path difference, $\delta$ and thus $y'_{\text{amp}}$.

Figure 4.5 shows the variation of $y'_{\text{amp}}$ with angle from the $y$ direction for various values of $k$ and fixed values of $x' = 1$ m and $P = 500$ Pa. It can be seen that at an angle of $\pm \pi/2$ from the $y$ direction, $y'_{\text{amp}}$ has a maximum value and for an angle of $0, \pm \pi$, the value of $y'_{\text{amp}}$ is a minimum. In the situation for which $\theta = 0, \pm \pi$, both laser beams pass through identical variations in refractive index and therefore
Figure 4.5: Variation of $y_{amp}$ with angle for various wave number values.
Figure 4.6: Variation of $y'_{amp}$ with angle for various values of $x'$. 

their optical path lengths are the same. This leads to zero path difference and thus no variation in the position of the fringes. Figure 4.5 also shows that as the value of the wavenumber increases, the magnitude of the amplitude of the fringe movement increases while the width of the maximum decreases.

The magnitude of $y'_{amp}$ also increases with increasing values of $x'$, the distance the laser beams have travelled in the direction of the optic axis (figure 4.6). In this case the relationship between the angle from the $y$ direction, the distance $x'$ and $y'_{amp}$ appears to have the form of the magnitude of a sinusoidal curve. Again, the maxima and minima occur in the predicted places; in this case $P$ was fixed at 500 Pa and the wavenumber $k = 5 \text{ m}^{-1}$. 

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Figures 4.5 and 4.6 show that for low values of the wavenumber \( k \) and distance \( x' \) and for an angle of \( \pi/2 \) from the \( y \) direction, the value of \( y_{\text{amp}}' \) is a maximum. As most frequencies of interest lie within this low wavenumber limit, the rest of this section assumes a value of \( \theta = \pi/2 \). This reduces equation (4.19) to:

\[
y' = \frac{\Delta n_o (1 - \cos(kb))(2x'^2 + b^2)}{2n_0kb^2}
\]  

(4.42)

assuming \( kb \) is small and \( b^2 << 2x'^2 \):

\[
y' = \frac{\Delta n_o (1 - (1 - \frac{(kb)^2}{2}))(2x'^2 + b^2)}{2n_0kb^2} = \frac{\Delta n_o k(2x'^2 + b^2)}{4n_o} = \frac{\Delta n_o kx'^2}{2n_o}
\]  

(4.43)

Examination of equation (4.43) leads to the conclusion that the ratio \( y_{\text{amp}}'/\Delta n_o \) is dependent only on the wavenumber \( k \) and the distance of propagation \( x' \). Using this conclusion allowed figures 4.7 and 4.8 to be produced. Figure 4.7 shows the variation of \( y_{\text{amp}}'/\Delta n_o \) with wavenumber \( k \) for various values of distance \( x' \) and figure 4.8 shows the variation of \( y_{\text{amp}}'/\Delta n_o \) with distance \( x' \) for various values of wavenumber \( k \).

From figure 4.7 it can be seen that \( y_{\text{amp}}'/\Delta n_o \) varies linearly with wavenumber \( k \) for low wavenumbers. Fitting a straight line to these graphs produces a relationship
between $y'_{amp}/\Delta n_o$, wavenumber $k$ and distance $x'$ of the form:

$$y'_{amp}/\Delta n_o = 0.376kx'^2.$$  \hfill (4.44)

Figure 4.8 shows that $y'_{amp}/\Delta n_o$ is a quadratic function of the distance $x'$ and a quadratic fit to the graphs again produces the relationship in equation (4.44) which holds for an angle of $\pi/2$ and low wavenumbers.

Equations (4.44) and (4.33) can now be used to produce:

$$\frac{y'_{amp}}{A_m} = 0.367k \times 0.376kx'^2$$
Using equations (4.41) and (4.45), the form of the ratio of the apparent motion to the actual motion of a particle in the fringes for a sound wave at an angle of \( \pi/2 \) from the \( y \) direction can be obtained:

\[
\frac{Y(X)}{A_m} = \cos(X) + 0.138x'^2 k^2 \sin(X)
\]  

(4.46)

where \( X \) is a function of position, time and frequency.

This equation only holds for low wavenumber values and for angles of \( \pi/2 \) from the \( y \) direction.
the optic axis. It is obvious from equation (4.46) that varying the value of the wavenumber $k$ or the distance $x'$ affects the apparent particle motion, $Y(X)/A_m$. This is verified in figures 4.9 and 4.10 which show that the ratio of the apparent motion to actual motion of the particle increases for increasing wavenumber $k$ and distance $x'$. Therefore, as the value of the wavenumber increases or the value of the distance of propagation increases, the acousto-optic effect becomes dominant. For very low wavenumbers or short distances, the acousto-optic effect is negligible.

Figure 4.9: Acousto-optic effect with varying wave number $k$ for $x' = 1$. 

Figure 4.9 shows the variation of the apparent particle motion $Y(X)/A_m$ with wave number $k$ for different distances $x'$. The curves illustrate the effect of varying wave number $k$ on the apparent particle motion. The y-axis represents the ratio of the apparent motion to the actual motion, and the x-axis represents the wave number $k$. The different curves correspond to different distances $x'$ and are labeled accordingly. The figure demonstrates that as the wave number $k$ increases, the apparent particle motion also increases, indicating a stronger acousto-optic effect.
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**Figure 4.10:** Acousto-optic effect with varying distance $x'$ for $k = 4$. 

![Graph showing the acousto-optic effect with varying distance](image)

- **$Y(X)/A_m = \text{apparent / actual particle motion}$**
- **$X$ (radians)**

- **Graph labels:**
  - $\cos(X)$, $x' = 0$
  - $\cos(X) + 0.55\sin(X)$, $x' = 0.4$
  - $\cos(X) + 1.24\sin(X)$, $x' = 0.6$
  - $\cos(X) + 2.21\sin(X)$, $x' = 0.8$
  - $\cos(X) + 3.45\sin(X)$, $x' = 1.0$
Variation of fringe movement with angle

Using equation (4.20) it is possible to determine the variation of fringe movement $y'$ with angle $\theta$ for low, medium and high values of the wavenumber $k$. These are shown in figures 4.11-4.13 which were created using the following parameters: propagation distance $x' = 0.5$ m, sound pressure $P = 500$ Pa and initial half beam separation $b = 0.03$ m. In these polar plots, the positive horizontal axis corresponds to an angle of zero radians and the positive vertical axis to an angle of $\pi/2$. It has already been shown that for low wavenumber values there is a linear relationship between $y'_{\text{amp}}$ and $k$ of the form:

$$\frac{y'_{\text{amp}}}{\Delta n_o} = 0.376kx'^2$$  \hspace{1cm} (4.47)

and that the value of $y'_{\text{amp}}$ always occurs when the sound wave is propagating at an angle of $\pi/2$ from the $y$ direction, as shown in figure 4.11. As the value of the wavenumber, $k$, increases (figures 4.12 and 4.13) this linear relationship no longer holds and the angle at which $y'_{\text{amp}}$ occurs begins to change.

It can be seen in figure 4.12 that although there is still a value of $y'_{\text{amp}}$ at an angle of $\theta = \pm\pi/2$, there are also additional angles at which a large value can be obtained. As the wavenumber increases further, figure 4.13, it can no longer be claimed that there is a value for $y'_{\text{amp}}$ at $\theta = \pi/2$. The side bands of figure 4.12 have now increased in magnitude and additional side bands are now also
visible. The angle of these side peaks correspond to integer multiples of the angle of the laser beams. This is examined in figure 4.14. When the value of the wavenumber, \( k \), is low and the sound wave is propagating parallel to the optic axis, the refractive index variations in both the laser beams are the same and therefore there is no path difference. When the sound wave is at an angle of \( \pi/2 \) to the optic axis, the laser beams are affected to differing degrees and the path difference is a maximum. As the wavenumber increases, an angle of \( \pi/2 \) creates similar changes in each beam and therefore the path difference is minimal. When the sound wave is at the same angle as the laser beam, one beam has approximately zero change as all the individual changes cancel each other.
out and the other beam has a maximum change, this results in a maximum path
difference between them.

4.3 Acousto-optic effect from standing waves

The acousto-optic theory developed up to this point has been for a travelling wave,
the theory is now extended for the case of a standing wave. Figure 4.15 illustrates
the situation which is investigated. A pressure standing wave is produced between
the opposite walls of a tank using a hydrophone. The laser beams intersect at a
position half way between a pressure node and antinode as this is the position
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Figure 4.13: Polar plot for $k = 200 \text{ m}^{-1}$

where the ratio of the fringe movement to particle movement should be greatest.

A standing wave may be considered to be two progressive waves travelling in opposite directions and thus the two dimensional refractive index variation, $n(x, y)$, of the standing wave which affects the light beams is given by:

$$n(x, y) = n_o - \Delta n_o \sin(k_x x + k_y y + \phi + \omega t) -$$

$$\Delta n_o \sin(k_x x + k_y y + \phi - \omega t)$$

$$= n_o - 2\Delta n_o \sin(k_x x + k_y y + \phi) \cos(\omega t)$$

(4.48)
where \( n_0 \) is the refractive index of the medium without the presence of a strain field, \( \Delta n_0 \) is the amplitude of the refractive index changes, \( \omega \) is the angular sound frequency, \( t \) is the time, \( \phi \) is the phase and \( k_x \) and \( k_y \) are the wave numbers in the \( x \) and \( y \) directions. For the general case of the light beams propagating through a standing wave which is at an angle \( \theta \) to the \( y \) axis, the optical path lengths can be written as:

\[
opl = \int_{z} (n_0 - 2\Delta n_0 \sin(k_x x + k_y y + \phi) \cos(\omega t)) \, ds \quad (4.49)
\]
Figure 4.15: Standing wave in LDA arrangement
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As in the travelling wave case, the contour $c$ is a straight line with equation:

$$y = mx + y_1 - mx_1 \quad (4.50)$$

Therefore, substituting for $x$, $k_x = k \cos \theta$ and $k_y = k \sin \theta$ in equation (4.49) and using:

$$\delta s = \sqrt{(\delta x)^2 + (\delta y)^2} \quad (4.51)$$

produces:

$$o p l = \sqrt{1 + \left(\frac{1}{m}\right)^2 \left[n_o(y_1 - y_2) + \frac{2m \Delta n_o \cos(\omega t)}{k \cos \theta + mk \sin \theta}ight]} \times \left[\cos\left(\frac{k}{m} \cos \theta (y_1 - y_2 + mx_2) + y_1 k \sin \theta + \phi\right) - \cos\left(x_2 k \cos \theta + y_2 k \sin \theta + \phi\right)\right] \quad (4.52)$$

Integration is carried out in the same way as for the travelling wave; laser beam 1 is integrated from the point $(x', y')$ to $(a, -b)$ and beam 2 is integrated from the point $(a, b)$ to $(x', y')$, as in figure 4.15. Therefore the path difference, $\delta$, is:

$$\delta = o p l 1 - o p l 2 \quad (4.53)$$
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\[ \cos \left( ak \cos \theta - bk \sin \theta + \phi \right) \right] \\
- \sqrt{1 + \left( \frac{1}{m_2} \right)^2 \left[ n_0 (b - y') + \frac{2m_2 \Delta n_0 \cos(\omega t)}{k \cos \theta + km_2 \sin \theta} \right]} \\
\times \left[ \cos \left( ak \cos \theta + kb \sin \theta + \phi \right) - \right] \\
\cos \left( (y' - b + m_2 a) \frac{k}{m_2} \cos \theta + y' k \sin \theta + \phi \right) \right] \]  (4.53)

Equation (4.53) can be solved for the fringe movement, \( y' \), in the same way as for a travelling wave. Again, the value of the distance \( x \) can be taken as a constant, \( x' \).

Equation (4.53) can be simplified by setting \( a = 0, t = 0 \) since the magnitude of a standing wave is not time dependent and \( \delta = 0 \) which corresponds to the situation in which a central fringe is formed. Using the assumption that \((y' + b)/x'\)^2 is small, equation (4.15) can be used. Using the further assumptions that \( ky' \ll 1 \) and \( n_0 \gg \Delta n_0 \) and neglecting all terms with \( y' \) greater than first order produces the expression for the fringe motion due to a standing wave:

\[ y'_{\text{standing}} = \frac{\Delta n_0 (2x'^2 + b^2)}{n_0 k (b^2 \sin^2 \theta - x'^2 \cos^2 \theta)} \times \{ b \sin \theta [\cos(kx' \cos \theta + \phi) - \cos(kb \sin \theta)] \right) \\
+ x' \cos \theta \sin \phi \sin(kb \sin \theta) \} \]  (4.54)
4.3.1 Apparent motion of particles in fringes

Figure 4.16 shows the phase relationships for pressure, displacement and particle velocity in a standing wave. The nodal points for pressure occur at the displacement and velocity antinodes. Thus:

\[ |A_m| \propto |\Delta u| = \frac{j\Delta P}{\rho v} \]  \hspace{1cm} (4.55)

where \( j = \sqrt{-1} \).

It is also known that the pressure variation, \( \Delta P \), is proportional to the variation in refractive index, \( \Delta n_o \), and that \( \Delta n_o \) is proportional to the fringe movement, \( y' \). Thus the motion of the fringes, \( y' \), with amplitude \( y'_{amp} \) is \( \pi/2 \) out of phase with the motion of the particles due to the standing wave, \( A \), with amplitude.
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\( A_m \). There is therefore an apparent motion of particles in the fringes, \( Y(k, x, t) \), as seen by the detector:

\[
Y(k, x, t) = A_m \cos(k, x, t) + y'_\text{amp} \sin(k, x, t) \tag{4.56}
\]

which can be written:

\[
Y(X) = A_m \cos(X) + y'_\text{amp} \sin(X) \tag{4.57}
\]

where \( X \) is a function of position, time and frequency.

The ratio of the magnitude of the apparent motion to the actual motion of the particles has the same form as in the case of a travelling wave:

\[
\left| \frac{Y(X)}{A_m} \right| = \sin(X) + \left| \frac{y'_\text{amp}}{A_m} \right| \cos(X) \tag{4.58}
\]

4.3.2 Ratio of apparent to actual motion of particles

It has been shown that for low wavenumbers and a progressive wave that the maximum value of \( |y'_\text{amp}/\Delta n| \) occurs at an angle of \( \theta = \pm \pi/2 \). For a standing wave, equation (4.54) shows that this is not the case; however this was the angle chosen experimentally as it produced the most practical situation to investigate and maintained the symmetry of the initial conditions required by the theory.
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Using $\theta = \pi/2$, equation (4.54) reduces to:

\[
y'_\text{amp} = \frac{\Delta n_o x'^2 k}{n_o} \cos(\phi)
\]

(4.59)

which corresponds to the amplitude of fringe movement, $|y'_\text{amp}|$.

Using the refractive index of water with no strain field present in equation (4.59) produces:

\[
\left| \frac{y'_\text{amp}}{\Delta n_o} \right| = 0.75 k x'^2 \cos(\phi)
\]

(4.60)

It has been shown that:

\[
\left| \frac{\Delta n_o}{A_m} \right| = 0.367 k
\]

(4.61)

Thus the ratio of the fringe movement to the particle movement is:

\[
\left| \frac{y'_\text{amp}}{A_m} \right| = 0.27 k^2 x'^2 \cos(\phi)
\]

(4.62)

The ratio of the apparent motion to the actual motion of a particle is given by equation (4.58). Substituting equation (4.62) into equation (4.58) produces:

\[
\left| \frac{Y(X)}{A_m} \right| = \sin(X) + 0.27 k^2 x'^2 \cos(\phi) \cos(X)
\]

(4.63)
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It can be seen that for an increase in either the distance of propagation $x'$ or the wave number $k$, the magnitude of the ratio of the apparent motion to the actual motion of the particles must increase.

4.4 Conclusions

When an LDA system is used to measure sound in water both the motion of the particles due to the sound wave and the motion of the fringes due to the refractive index variations give rise to a signal, with the signals from the two effects being $\pi/2$ out of phase. The signal from a vibrating particle dominates at low sound frequencies and at short distances of propagation of the laser beams but at higher frequencies and longer propagation distances the signal from the oscillating fringes due to the acousto-optic effect dominates. This effect is larger in a standing wave than a travelling wave.

For the case of a travelling wave, the relative angle of the sound wave to the laser beams can also have an important effect. When the value of the wavenumber is low and the sound wave is propagating in the direction of the optic axis, the refractive index variations on each laser beam are equal and therefore there is no path difference. If a sound wave propagates at an angle of $\pi/2$ to the optic axis, each beam is affected to a differing degree and the path difference is maximum. As the sound frequency increases at this angle, the path difference reduces and
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tends to a minimum. If the laser beams are propagating at the same angle as the wave fronts, the resulting path difference is again a maximum.

This acousto-optic effect is almost negligible in air due to the acousto-optic coefficient, $\sigma$, (equation (4.34)) for air being 13 times larger than in water [Adler, 1967]. It has been shown that if LDA is being used to investigate underwater sound sources, the acousto-optic effect is the dominant factor if the sound frequencies involved are high or the distance of propagation of the laser beams is long. Although the Doppler signal may be easier to detect, amplitude information obtained from it may not accurately represent the area of the flow of interest.
Chapter 5

Experimental acousto-optic results

5.1 Experimental arrangement

In order to verify the acousto-optic theory developed in Chapter 4, experimental results were obtained for the case of a standing wave in water. The laser Doppler anemometry arrangement used is shown in figure 5.1 which utilises a Uniphase He-Ne, 20 mW laser. The water tank had dimensions $903 \times 283 \times 299$ mm and standing waves were created along the longest side by a 150 mm diameter ball hydrophone attached to one end of the tank. The first and second lenses had focal lengths of 20 mm and 10 mm respectively and the Pockels cell was set at a frequency of 50 kHz. A calibrated Brüel & Kjær 8103 hydrophone was used
Chapter 5 — Experimental acousto-optic results

![Diagram of experimental arrangement]

**Figure 5.1:** Experimental arrangement

to provide a reference measurement for the pressure amplitude due to the sound wave, $|A_m|$. 

Seeding particles with a diameter of 0.196\(\mu\)m were added to the water which allowed frequencies up to 4 MHz to be accurately followed. The light scattered from these particles was collected by the detector, filtered and digitized using a Wavebook 512, 12 bit, 1 MHz data acquisition board and captured using WaveView software. This Doppler signal was then analysed using software to process the Hilbert transform analysis technique [Grechikhin and Rinkevichius, 1996] to provide a value for $Y(X)$, the apparent motion of the particles in the fringes, where $X$ is a function of position, time and frequency. This analysis technique is explained in detail in Chapter 3.

The laser beams propagated through the water before intersecting to form a fringe pattern with a width of 0.68 mm and a fringe separation of $8.43 \times 10^{-6}$ mm at a position half way between a pressure node and antinode. This position choice
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can be explained by examining the ratio of the fringe movement to the particle movement $y'/A$. The pressure $P$ and the particle movement $A$ have the form:

\[
P = \Delta P \cos(X) \cos(\omega t)
\]

\[
A = A_m \sin(X) \sin(\omega t)
\]

The magnitude of a standing wave is not time dependent and so:

\[
|P| = \Delta P \cos(X)
\]

\[
|A| = A_m \sin(X)
\]

(5.1)

It is known that:

\[
|y'_{amp}| \propto |\Delta P|
\]

(5.2)

Therefore:

\[
\frac{|y'|}{|A|} = \frac{\Delta P \cos(X)}{A_m \sin(X)} = \frac{\Delta P}{A_m \tan(X)} = \frac{|y'_{amp}|}{A_m} \frac{1}{\tan(X)}
\]

(5.3)
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for

\[ X = 0, \quad \frac{y'}{A} = \infty \]  \hspace{1cm} (5.4)
\[ X = \frac{\pi}{2}, \quad \frac{y'}{A} = 0 \]
\[ X = \frac{\pi}{4}, \quad \frac{y'}{A} \propto \frac{y'_{\text{amp}}}{A_m} \]

It can be seen that for a value of \( X = 0 \) which corresponds to zero particle displacement and a pressure antinode, the ratio of the fringe to particle movement produces an infinite value. In this case both the laser beams are affected to the same degree by the refractive index variations and so there is no fringe movement.

For \( X = \pi/2 \), the particle displacement is a maximum and there is maximum fringe movement due to a maximum pressure gradient at the pressure node. However, at this position the ratio of the fringe movement to particle movement is zero and this is therefore not a suitable position at which to make measurements to show this acousto-optic effect. The position half way between a pressure node and antinode was therefore chosen since here the ratio of the fringe to particle movement is simply \( y'_{\text{amp}}/A_m \).
5.2 Experimental results

Placing the reference hydrophone in the fringes provides a peak-peak voltage for the standing wave at that point. The voltage amplitude \( V \) is given by half this value and this can be converted into pressure \( P \) by [Kinsler et al., 1982]:

\[
P = \frac{V}{M_e} = \frac{V}{23.6 \times 10^{-6}}
\]

(5.5)

where \( M_e \) is the voltage sensitivity of the hydrophone. The pressure can then be converted into amplitude using:

\[
A_m = \frac{P}{2\pi f \rho c}
\]

(5.6)

where \( \rho \) is the density of the medium, \( c \) is the speed of sound in the medium and \( f \) is the sound frequency. This equation provides a reference value for the amplitude of the actual motion, \( |A_m| \), of the particles in the fringes.

The Doppler signal from the LDA was analysed using the Hilbert transform technique [Grechikhin and Rinkevichius, 1996] to produce a plot of sound frequency against amplitude as shown in figure 5.2. The amplitude obtained from this plot is due to the apparent motion of the particles in the fringes and corresponds to
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\[ |Y(X)| \text{ which has the form:} \]

\[ |Y(X)| = A_m \cos(X) + y'_{\text{amp}} \sin(X) \quad (5.7) \]

Thus both the magnitude of the apparent motion, \( |Y(X)| \), and the actual motion, \( |A_m| \), of the particles in the fringes can be determined experimentally.

### 5.3 Comparison of theory and experiment

The theoretical result for the ratio of the apparent to the actual motion can be obtained from:

\[ \frac{Y(X)}{A_m} = \cos(X) + \left| \frac{y'_{\text{amp}}}{A_m} \right| \sin(X) \quad (5.8) \]

The maximum value of this corresponds to:

\[ \frac{|Y(X)|}{|A_m|} = \left| \frac{y'_{\text{amp}}}{A_m} \right| = 0.27k^2x^2 \cos(\phi) \quad (5.9) \]

Experimental results have been obtained for the 8th, 9th, 10th, 23rd, 32nd and 41st harmonics and a set of experimental data analysed for the six harmonics investigated can be seen in figures 5.2-5.7. These have been produced using the Hilbert transform technique described in Chapter 3. A comparison between the
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Experimental and theoretical values for the ratio of the apparent motion to the actual motion of the particles for these harmonics can be seen in Table 5.1. The value for the distance of propagation of the laser beams used was $x = 210\pm10$ mm and $\phi = \pi/4\pm\pi/9$. The large error in the value of $\phi$ is due to the wavelength in the high frequencies being very short and therefore difficulties arose in determining the exact position for which $\phi = \pi/4$. The values of $Y(X)$ and $A_m$ for each harmonic were obtained by averaging over 6 data sets and the errors correspond to the standard deviation.

Figure 5.8 represents a comparison between the theoretical predictions and the experimental results for the ratio of the apparent to the actual particle motion. The experimental results and their error are represented by points and the solid curve is the theoretical value with the dotted lines as the theoretical error bounds. The results show that the ratio of the apparent to the actual particle motion increases with increasing wave number and therefore frequency. This results in a signal which is easier to detect but may not accurately represent the area of the flow of interest. These results can also be used to verify the theory developed within Chapter 4 as they agree well within experimental error.
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<table>
<thead>
<tr>
<th>harmonic</th>
<th>$k \pm 0.4$ (m$^{-1}$)</th>
<th>$Y(X) \times 10^{-8}$ (m)</th>
<th>$A_m \times 10^{-9}$ (m)</th>
<th>$Y(X) / A_m$ theory</th>
<th>$Y(X) / A_m$ exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>33.3</td>
<td>10.5 ± 3.1</td>
<td>21.3 ± 0.8</td>
<td>9.3 ± 3.2</td>
<td>4.9 ± 2.3</td>
</tr>
<tr>
<td>9</td>
<td>35.8</td>
<td>5.3 ± 2.0</td>
<td>6.3 ± 0.6</td>
<td>10.7 ± 3.5</td>
<td>8.4 ± 4.2</td>
</tr>
<tr>
<td>10</td>
<td>41.8</td>
<td>11.4 ± 1.5</td>
<td>15.8 ± 1.4</td>
<td>14.7 ± 4.8</td>
<td>7.2 ± 2.7</td>
</tr>
<tr>
<td>23</td>
<td>91.8</td>
<td>4.9 ± 4.0</td>
<td>1.4 ± 0.2</td>
<td>71.0 ± 23.3</td>
<td>35.8 ± 29.5</td>
</tr>
<tr>
<td>32</td>
<td>129.0</td>
<td>13.1 ± 4.1</td>
<td>1.2 ± 0.8</td>
<td>140.0 ± 46.0</td>
<td>110.1 ± 83.3</td>
</tr>
<tr>
<td>41</td>
<td>165.6</td>
<td>20.6 ± 7.5</td>
<td>1.8 ± 0.4</td>
<td>230.9 ± 75.8</td>
<td>113.2 ± 50.3</td>
</tr>
</tbody>
</table>

Table 5.1: Experimental and theoretical results
Chapter 5 — Experimental acousto-optic results

5.4 Conclusions

If LDA is being used in the situation where a pressure gradient and hence a refractive index variation is present and the laser beams propagate over long distances or the sound frequency is high then the apparent motion of the particles in the fringes is larger than the actual motion and the acousto-optic effect has to be taken into account in the analysis of the Doppler signal. Theoretical expressions were produced in Chapter 4 for the motion of the fringes and for the ratio of the apparent motion of the particles in the fringes to the actual motion. The experimental results presented here have been used to verify the theory.

Figure 5.2: Frequency and amplitude of 8th harmonic
Figure 5.3: Frequency and amplitude of 9th harmonic

Figure 5.4: Frequency and amplitude of 10th harmonic
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**Figure 5.5:** Frequency and amplitude of 23rd harmonic

**Figure 5.6:** Frequency and amplitude of 32nd harmonic
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Figure 5.7: Frequency and amplitude of 41st harmonic

The theory shows that as the propagation distance or sound frequency increases so does this acousto-optic effect and the apparent motion of the particles in the fringes; this can be beneficial in that the acousto-optic effect produces a much stronger Doppler signal, making detection easier. However, since the Doppler signal represents the apparent motion of the particles in the fringes rather than the actual motion due to the acoustic wave, the analysed signal may not accurately represent the amplitudes of the particles in the flow.
Figure 5.8: Variation of ratio of apparent to actual particle motion with wavenumber, \( k \).
Chapter 6

Underwater explosions

6.1 Equations of fluid motion

6.1.1 Conservation of mass

Consider a volume element, \( V \), within a fluid. Any change in mass of the fluid contained within the volume must equal the net quantity of fluid which flows through the boundary surface. The rate of mass loss from the volume, \( V \) is:

\[
-\int_S p \mathbf{u} \cdot dS
\]  

(6.1)

where \( p \) is the fluid density, \( \mathbf{u} \) is the velocity and \( S \) is the surface of the volume.

The rate of change of mass within the volume can also be determined by local
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changes in density, $\rho$:

$$\frac{d}{dt} \int_V \rho dV$$  \hspace{1cm} (6.2)

Therefore:

$$\frac{d}{dt} \int_V \rho dV = -\oint_S \rho \mathbf{u} \cdot dS$$  \hspace{1cm} (6.3)

It is more useful to know the mass balance at a point rather than over a volume and so the volume, $V$, is shrunk to an infinitesimal size:

$$\frac{\delta \rho}{\delta t} = -\lim_{V \to 0} \int \rho \mathbf{u} \cdot \frac{dS}{V}$$

$$= -\text{div}(\rho \mathbf{u})$$  \hspace{1cm} (6.4)

Therefore:

$$\frac{\delta \rho}{\delta t} + \nabla \cdot (\rho \mathbf{u}) = 0$$  \hspace{1cm} (6.5)

This is the continuity equation.
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6.1.2 Conservation of momentum

Newton's second law says that when a force acts upon a body, the resulting rate of change of momentum of the body is equal to the force. Considering the forces acting on a moving volume of fluid, the acceleration in the $x$ direction of the volume is given by $du/dt$, the total time derivative of the particle velocity. The product of this acceleration and the mass of the volume, $\rho dxdydz$, must equal the force acting on the volume in the $x$ direction. The force is only due to the difference in pressure, $P$, on the faces of area $dydz$ and can be written as:

$$[P_x - P_{x+dz}]dydz = -\frac{\delta P}{\delta x} dxdydz$$

(6.6)

Equating the force and momentum terms:

$$\rho \frac{du}{dt} = -\frac{\delta P}{\delta x}$$

(6.7)

Writing the acceleration in terms of its two parts and expressing all three components of direction:

$$\rho \frac{\delta u}{\delta t} + \rho u \frac{\delta u}{\delta x} + \rho v \frac{\delta u}{\delta y} + \rho w \frac{\delta u}{\delta z} = -\frac{\delta P}{\delta x}$$

$$\rho \frac{\delta v}{\delta t} + \rho u \frac{\delta v}{\delta x} + \rho v \frac{\delta v}{\delta y} + \rho w \frac{\delta v}{\delta z} = -\frac{\delta P}{\delta y}$$

$$\rho \frac{\delta w}{\delta t} + \rho u \frac{\delta w}{\delta x} + \rho v \frac{\delta w}{\delta y} + \rho w \frac{\delta w}{\delta z} = -\frac{\delta P}{\delta z}$$

(6.8)
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These are equivalent to:

$$\rho \frac{d\v}{dt} = \rho \frac{\delta \v}{\delta t} + \rho (\v \cdot \text{grad})\v = -\text{grad}P$$

$$= \rho \frac{\delta \v}{\delta t} + \rho (\v \cdot \nabla)\v = -\nabla P \quad (6.9)$$

This is Euler’s equation.

6.1.3 Conservation of energy

Consider a fluid element moving with the fluid and enclosing a fixed mass of fluid. The total energy per unit mass of the fluid is made up from the kinetic energy and the internal energy, $E$. The internal energy is the sum of the thermal and chemical energy. The change of the volume element, $dx dy dz$, in time $dt$ is:

$$\rho \frac{d}{dt} \left[ E + \frac{1}{2}(u^2 + v^2 + w^2) \right] dx dy dz \quad (6.10)$$

where the total time derivative accounts for the displacement of the fluid element during the time interval.

The change in energy must equal the total work done on the faces of the fluid element. The work done on a face of area $dydz$ while moving in the $x$ direction in a time $dt$ is the product of the force and displacement, $Pu \cdot dt dy dz$. The net
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amount of work done on the two faces of the element is:

\[
[(Pu)_x - (Pu)_{x+dx}]dtdydz = -\frac{\delta}{\delta x}(Pu)dtdx dydz \quad (6.11)
\]

The work done on the faces travelling in the y and z directions is obtained in the same way. Using this it is possible to equate the total to the increase in energy:

\[
\rho \frac{d}{dt} \left[ E + \frac{1}{2}(u^2 + v^2 + w^2) \right] = - \left[ \frac{\delta}{\delta x}(Pu) + \frac{\delta}{\delta y}(Pv) + \frac{\delta}{\delta z}(Pw) \right] \quad (6.12)
\]

This can also be written as:

\[
\rho \frac{\delta}{\delta t} \left[ E + \frac{1}{2}(v \cdot v) \right] = - \nabla \cdot (Pv) \quad (6.13)
\]

This energy equation is more useful when it is solved for variations in internal energy and combined with the equations of continuity and motion which are equations (6.5) and (6.9) respectively:

\[
\rho \frac{dE}{dt} = -P \nabla \cdot v - v \cdot \nabla (P) - \rho v \cdot \frac{dv}{dt} \quad (6.14)
\]

Using equation (6.7) produces:

\[
- \nabla \cdot v = \frac{1}{\rho} \frac{\delta \rho}{\delta t} + \frac{1}{\rho} (v \cdot \nabla) \rho
= \frac{1}{\rho} \frac{d\rho}{dt} \quad (6.15)
\]
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and equation (6.9) gives:

\[ \nabla P = -\rho \frac{d\nu}{dt} \]  \hspace{1cm} (6.16)

which gives on substitution into equation (6.14):

\[ \rho \frac{dE}{dt} = \frac{P}{\rho} \frac{d\rho}{dt} \]  \hspace{1cm} (6.17)

6.1.4 Spherical propagating waves and the afterflow

A spherical wave is one in which the pressure \( P \) is a function of radial distance \( r \) and time \( t \) but not of angular coordinates. The equations of motion and continuity for this case can be written:

\[ \frac{\delta u}{\delta t} = \frac{1}{\rho_0} \text{grad} P \]
\[ \frac{1}{c_o^2} \frac{\delta P}{\delta t} = -\rho_0 \nabla \cdot \mathbf{v} \]  \hspace{1cm} (6.18)

The wave equation for spherical symmetry can be written:

\[ \frac{\delta^2 P}{\delta r^2} + \frac{2}{r} \frac{\delta P}{\delta r} = \frac{1}{c^2} \frac{\delta^2 P}{\delta t^2} \]  \hspace{1cm} (6.19)
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Conservation of energy and the intensity relationship:

\[ I = \frac{P^2}{2\rho_0 c} \quad (6.20) \]

suggests that the pressure amplitude should fall off as \(1/r\) so that the quantity \(rP\) is amplitude independent of \(r\). Treating \(rP\) as the dependent variable:

\[ \frac{\delta^2(rP)}{\delta r^2} = \frac{1}{c^2} \frac{\delta^2(rP)}{\delta t^2} \quad (6.21) \]

Considering the product \(rP\) as a single variable, equation (6.21) has the general solution:

\[ P = \frac{1}{r} f_1(ct - r) + \frac{1}{r} f_2(ct + r) \quad (6.22) \]

The negative sign corresponds to an outgoing wave about the centre.

Spherical, harmonic waves are expressed in complex form by:

\[ P = \frac{A}{r} \exp i(\omega t - kr) \]

\[ = \frac{A}{r} \cos(\omega t - kr) + i\frac{A}{r} \sin(\omega t - kr) \quad (6.23) \]

The relation of the particle velocity \(u\) to the pressure \(P\) can be obtained by
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integrating the first of equations (6.18):

\[
\frac{\delta u}{\delta t} = -\frac{Ak}{\rho_0 r} \sin(\omega t - kr) + \frac{A}{r^2 \rho_0} \cos(\omega t - kr) \tag{6.24}
\]

Integrating again produces:

\[
\int \frac{\delta u}{\delta t} \delta t = u
\]

\[
= \frac{Ak}{\rho_0 r} \frac{1}{\omega} \cos(\omega t - kr) + \frac{A}{r^2 \rho_0 \omega} \sin(\omega t - kr) \tag{6.25}
\]

Differentiating the real part of equation (6.23) produces:

\[
\frac{\delta P}{\delta r} = \frac{Ak}{r} \sin(\omega t - kr) - \frac{A}{r^2} \cos(\omega t - kr) \tag{6.26}
\]

Therefore:

\[
u = \frac{P_k}{\rho_0 \omega} + \frac{1}{r \rho_0} \int_{t_0}^{t} P dt \tag{6.27}
\]

If the time \(t_0 = 0\) is taken to precede any disturbance then it can be seen that the velocity in the fluid at any later time is a function of all the previous changes in pressure before the disturbance reaches that point. In a radial disturbance, the water is left with an outward velocity called the afterflow. This afterflow remains even when the pressure has returned to its equilibrium value and only returns to zero when the pressure falls below the equilibrium value. Further discussion of
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this afterflow is carried out in Section 8.2.5.

In the limit of small pressure differences, the afterflow velocity is simply the velocity of non-compressive flow outward from the expanding gas sphere boundary. When the pressure in the bubble falls below the hydrostatic pressure, the outward flow is brought to rest and the inward flow begins. The kinetic energy of the motion is returned to compression of the bubble.

6.2 Explosion generation

Traditionally, explosive materials such as TNT have been used to produce bubbles and shock waves underwater but this had the disadvantage that photographic studies are difficult to carry out if the bubble is small due to obscuration from the explosion products [Cole, 1948].

In 1974 Lauterborn [Lauterborn, 1974] showed that bubbles could be produced at specific positions in a liquid by a focussed laser pulse. Since then the technique has been used extensively to investigate bubbles [Lauterborn and Bolle, 1975], [Vogel et al., 1989], [Vogel et al., 1996], [Tomita et al., 1994]. Laser produced bubbles have the advantage that they are highly spherical and are free from mechanical distortions.

Exploding wires [Vijayan and Rohatgi, 1988] have also been used to generate shock waves but these are not quickly repeatable as the wire has to be contin-
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ually replaced. The use of an underwater spark to produce bubbles and shock waves is now becoming an increasingly popular method as the bubbles are quickly repeatable and photographic studies would not be obscured by fog and explosion products. The methods mentioned for producing bubbles and shock waves all work in basically the same way. In the case of the underwater spark a high voltage appears in the gap between two electrodes and eventually the water breaks down and allows a spark to form. The discharge of this underwater spark delivers energy to the region between the electrodes and creates a high-pressure bubble in the water which expands rapidly, emitting a shock wave.

Explosively generated bubbles possess the advantage that they are very well documented due to the numerous efforts devoted to their study. This has resulted in well defined scaling laws which are based on testing and observation. Spark generation has not been as extensively used and is therefore not as well documented and the available information is mainly qualitative. Although spark generated explosions are unable to accurately imitate explosive materials due to the lack of explosive components, it is still of great interest to use underwater sparks as they are much quicker to repeat and they use the same amount of energy for each spark unlike explosive materials where some of the material may not detonate. Researchers have criticised spark generated bubbles because they do not oscillate in the same way as those generated by explosive materials but the experimental results detailed in this thesis show that this is not the case.
6.3 Explosion bubble

The motion of the gas sphere is associated with the emission of energy in the form of a shock wave. The initial high pressure of the bubble is considerably decreased once the shock wave is emitted but it is still much higher than the equilibrium hydrostatic pressure. The water around the bubble has a large outward velocity and the diameter of the bubble increases rapidly. The expansion continues for a relatively long time and the internal gas pressure decreases gradually but the motion continues due to the inertia of the outward flowing water. The gas pressure eventually falls below the equilibrium value determined by atmospheric plus hydrostatic pressure; this brings the outward flow to a stop and the bubble begins to contract at an increasing rate. This inward motion continues until the compressibility of the gas abruptly reverses the motion. The inertia of the water together with the elastic properties of the gas and water thus provide suitable conditions for an oscillating system.

The original state of the bubble is usually spherical and the radial nature of the flow results in an asymmetrical oscillation about the mean diameter, the bubble spending most of its time in an expanded condition. The period of oscillation is related simply to the internal energy of the gas and the hydrostatic pressure. Due to the gaseous products the bubble must eventually rise to the surface due to its buoyancy when in equilibrium with the surrounding pressure. Therefore, throughout the expanded phase of the bubble motion very little migration occurs.
but at the minimum bubble radius there is an appreciable upward displacement. The bubble also experiences a repulsive force away from free surfaces and is attracted to any rigid boundaries. The motion of the bubble is therefore affected by its buoyancy and the proximity of the water surface and other boundary surfaces.

6.3.1 The pulsating bubble

A gas bubble in a liquid acts as an oscillator. The inertia is associated with the moving liquid in the bubble system and the restoring force is the elasticity of the gas. The main energy flow in an oscillating bubble is between the potential and internal energies. The natural frequency of a spherical gas bubble in a liquid undergoing oscillations was first calculated by Minnaert [Minnaert, 1933] in which the bubble radius, \( r \), follows the motion:

\[
 r = r_0 - r_e \exp i \omega_0 t
\]

where \( r_0 \) is the mean radius, \( r_e \) is the oscillation amplitude and \( \omega_0 \) is the resonance frequency.

Rayleigh [Rayleigh, 1917] and Watson et al. [Watson et al., 1985] have shown that the bubble expansion and collapse follows a very simple hydrodynamic model. They have assumed that in bubble collapse the bubble maintains its
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Figure 6.1: Oscillating bubble

spherical form which is untrue of most situations in which non-spherical collapse occurs. A model of bubble collapse is very difficult to achieve if spherical collapse does not occur. The assumption is also made that the cavity is empty. Figure 6.1 shows the movement of an oscillating bubble which undergoes oscillations of amplitude $r_e$ and is surrounded by a spherical shell of radius $r_s$ and width $\Delta r_s$.

6.3.2 Bubble expansion

Very early stages

In the early stages of bubble expansion it is possible to ignore the work done by the bubble against external ambient pressure. The bubble can then be treated
as a constant kinetic energy system. For an incompressible fluid:

\[ U r_m = u(r_s) r^2 \]  \hfill (6.29)

where \( U \) is the cavity wall velocity, \( r \) is the bubble radius, \( u(r_s) \) is the shell velocity and \( r_s \) is the shell radius. The kinetic energy surrounding the spherical cavity is therefore given by:

\[ E = \frac{1}{2} \rho \int_r^\infty u^2(r_s) 4\pi r_s^2 dr_s \]

\[ = 2\pi \rho U^2 r^3 \]  \hfill (6.30)

where \( \rho \) is the water density and \( U = dr/dt \). In the early stage of expansion the kinetic energy is constant, \( E_o \), therefore:

\[ \left( \frac{dr}{dt} \right)^2 = \frac{E_o}{2\pi \rho r^3} \]  \hfill (6.31)

Thus, the variation of the bubble radius with time is given by:

\[ r(t) = \left[ \frac{5}{2} \left( \frac{E_o}{2\pi \rho} \right) t \right]^{0.4} + r_o \]

\[ \sim \left[ \frac{E_o}{\rho} \right]^{0.2} t^{0.4} \]  \hfill (6.32)
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Later expansion stages

As the bubble expands it does work against the external pressure $P_o$. The kinetic energy of the system is no longer constant and the initial kinetic energy $E_o$ is represented by:

$$E_o = 2\pi \rho U^2 r^3 + \frac{4}{3} \pi r^3 P_o$$  \hspace{1cm} (6.33)

This can be rearranged to provide an expression relating the velocity to the radius and initial kinetic energy of the bubble:

$$U^2 = \frac{E_o}{2\pi \rho r^3} - \frac{2}{3} \frac{P_o}{\rho}$$ \hspace{1cm} (6.34)

6.3.3 Bubble collapse

The bubble stops expanding when the kinetic energy of the system reaches zero. At this point the internal pressure of the bubble is below ambient pressure and the bubble is fully expanded, the external pressure then causes the bubble to collapse. The work done by the external pressure is equal to the kinetic energy of the fluid. This is found by integrating the energy over the spherical shell of thickness $\Delta r_s$, mass $4\pi r_s^2 \rho \Delta r_s$ and speed $dr_s/dt$:

$$\frac{4}{3} \pi P_o (r_m^3 - r^3) = \frac{1}{2} \rho \int_0^{\infty} \frac{du^2}{dt} \rho 4\pi r_s^2 dr_s$$
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\[ U^2 r^3 = 2\pi \rho U^2 r^3 \]  \hspace{1cm} (6.35)

This provides an equation for the velocity as a function of position:

\[ U^2 r^3 = \frac{2P_o}{3\rho} (r_m^3 - r^3) \]

\[ U^2 = \frac{2P_o}{3\rho} \left( \left( \frac{r_m}{r} \right)^3 - 1 \right) \]  \hspace{1cm} (6.36)

Integrating this provides the time for the bubble collapse. The solution to this collapse is given by Rayleigh [Rayleigh, 1917]:

\[ \tau_{\text{bubble}} = 0.915 r_m \left( \frac{\rho}{P_o} \right)^{1/3} \]  \hspace{1cm} (6.37)

By symmetry, \( \tau_{\text{bubble}} \) is also the time for bubble expansion and so the total lifetime of the bubble is given by:

\[ \tau_{\text{total}} = 2\tau_{\text{bubble}} \]

\[ = 1.83 r_m \left( \frac{\rho}{P_o} \right)^{1/3} \]

\[ = 1.3E_o^{1/3} \rho^{1/2} P_o^{-5/6} \]  \hspace{1cm} (6.38)

where the final term was obtained using equations (6.34) and (6.36).

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6.4 Cavitation damage

During an explosion, the collapse of a bubble close to a solid boundary can cause a significant amount of damage to the boundary surface by deformation and particle removal. Cavitation erosion was first observed by marine and hydraulic engineers at the turn of the century and investigation of this effect is very important in these fields.

Cavitation damage is caused mainly by the non-spherical collapse of a bubble due to boundaries, pressure gradients, shock wave-bubble interactions and bubble-bubble interactions. Very few mathematical models have been developed for bubble collapse due to the lack of spherical symmetry of the problem and only numerical methods seem able to predict bubble behaviour near the final stages of collapse. Rayleigh [Rayleigh, 1917] was the first to theoretically point out that a local high pressure is produced in the final stage of bubble collapse and that this is a dominant factor in cavitation damage.

Zhang et al. [Zhang et al., 1994] and Vogel et al. [Vogel et al., 1989] have also investigated bubble collapse and have shown that the final stage of collapse of a bubble can be characterized by a process of continuous re-entrant jet impact. During this process the bubble is transformed into a toroidal shaped cavity which is attached to an impact interface that represents a shear layer created by the impact. Before initial impact the fluid in the re-entrant jet is travelling towards
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the solid boundary with high speed and the fluid on the other side of the bubble is moving away from the boundary. During impact a high pressure region is generated around the vortex sheet and dramatically decelerates the fluid in the re-entrant jet and forces the fluid on the other side of the bubble to accelerate towards the boundary.

Tomita and Shima [Tomita and Shima, 1986] have carried out detailed experiments to investigate the mechanism of impulsive pressure generation and damage pit formation caused by bubble collapse close to a solid boundary. They found that the modes of bubble collapse are related to the proximity of the boundary. For a bubble very close to a boundary, intensive impulsive pressures occur at the first and second bubble collapse. They have also demonstrated that the interaction of a small bubble and a shock wave can create a damage pit on a solid boundary.

The fact that bubbles are attracted to solid surfaces but are repelled by free surfaces has been the attention of work reported by Shima et al. [Shima et al., 1989]. They investigated the effect of using a deformable coating to protect a rigid wall. They found that bubble migration depends upon the surface properties as well as the bubble size and distance from the surface.

Tomita et al. [Tomita et al., 1994] investigated the interaction of two bubbles created both simultaneously and non-simultaneously. In some combinations of conditions no migration takes place (neutral bubble collapse) in which case bub-
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ble splitting may occur. They found that maximum bubble migration was significantly affected by the size ratio of the two bubbles and that the dependency of bubble migration on the size effect is different for small and large bubbles. They also found that a damage pit can be created by strong bubble-bubble interaction.

6.5 Medical uses

Many researchers have investigated the suitability of using of shock waves in medicine, developing shock waves for a variety of uses which include painless drug delivery [Quinlan et al., 1997], non-invasive treatment of nephrolithiasis [Prieto et al., 1991] (presence of calculi in the kidney) and lithotripsy [Bourlion et al., 1994] (stone fragmentation).

For many medical techniques the shock waves are created by lasers as the plasma formation in a liquid environment is accompanied by a shock wave and this can have advantages for some types of treatment, for example: laser lithotripsy [Bourlion et al., 1994]. However, there are also medical procedures which involve the use of lasers in which the plasma formation and shock wave generation can be a drawback. For example, they have been used in intraocular tissue cutting near sensitive structures of the eye [Vogel et al., 1990] where shock waves and plasma formation can cause unwanted damage.
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6.6 Shock waves

When a disturbance is forced through a medium at a speed greater than the local propagation velocity, the pressure, density and temperature all build up ahead of the disturbance, as in figure 6.2. The resulting wave is a shock wave.

Figure 6.2: Shock wave formation

6.6.1 Rankine-Hugoniot conditions

As the shock wave propagates through the water it develops an increasingly steep front. Fronts of this kind are so steep that they are practically discontinuous and flow parameters (pressure, density, entropy, particle velocity) are all discontinuous over the front. Rankine and Hugoniot derived equations for this discontinuity by considering regions immediately ahead of and behind the front. Rankine considered the equations for mass and momentum [Rankine, 1870] and Hugoniot [Hugoniot, 1887], [Hugoniot, 1888], the equation for the increase in internal energy.

If a shock wave moves with velocity $U$ into an area of pressure $P_o$, particle velocity
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$u_o$ and density $\rho_o$ then the apparent velocity of the fluid moving towards the shock wave is $U - u_o$. The mass of fluid in a time $dt$ entering the front of the shock wave is $\rho_o(U - u_o)dt$. The apparent velocity of the fluid leaving the front is $-(U - u)$ where $u$ is the particle velocity relative to a fixed coordinate. In a time $dt$ the mass of fluid leaving the front is $\rho(U - u)dt$. For a discontinuous front the time $dt$ can be reduced to an infinitesimal value so that the mass flow away from the front must equal that towards it. This produces an equation for the conservation of mass:

$$\rho_o(U - u_o) = \rho(U - u) \quad (6.39)$$

A similar expression can be obtained for the conservation of momentum. The mass flow into the front has momentum $\rho_o(U - u_o)u_o dt$ and the mass flow out has momentum $\rho_o(U - u_o)u dt$. The change in momentum must equal the impulse of the net force per unit area. If the pressure behind and ahead of the front are $P$ and $P_o$ respectively, the conservation of momentum can be written:

$$\rho_o(U - u_o)(u - u_o) = P - P_o \quad (6.40)$$

The net work done by the pressures $P$ and $P_o$ must equal the increase in kinetic plus potential energy when the time increment becomes infinitesimal. The work done per unit area of the front by $P$ is $Pudt$ and by $P_o$ is $P_ou_o dt$. The kinetic
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energies per unit mass are 1/2\textsuperscript{nu}\textsuperscript{2} and 1/2\textsuperscript{u0}\textsuperscript{2} and using \( E \) and \( E_0 \) to denote the internal energies per unit mass produces an equation for the conservation of energy:

\[
P_u - P_0 u_0 = \rho_0 (U - u_0) \left[ E - E_0 + \frac{1}{2} (u^2 - u_0^2) \right]
\] (6.41)

These three equations are obtained for negligible shock front thickness and are equally valid for spherical or plane shock fronts. Simplifying produces:

\[
\begin{align*}
\rho(U - u) & = \rho_0 U \\
P - P_0 & = \rho_0 u u_0 \\
E - E_0 & = \frac{1}{2} (P + P_0) \left( \frac{1}{\rho_0} - \frac{1}{\rho} \right)
\end{align*}
\] (6.42)

These are the Rankine-Hugoniot equations and represent the conditions at a shock front.

6.6.2 Similarity principle

If data have been obtained at a known time and distance from detonation for a shock wave produced from a known size of explosive material it is possible to scale these data to obtain information for another size of explosive material. This is the basis of the similarity principle which states that if the dimensions
of a charge are increased by a factor \( \lambda \) then the pressure and other shock wave properties is unchanged if the length scales and times by which it is measured are also increased by a factor \( \lambda \). This has been shown to be true experimentally for explosive materials [Cole, 1948] and for electrical discharges [Bjorno, 1970].

### Mathematics of the similarity principle

The basic equations describing fluid motion are:

\[
\frac{du}{dt} = -\frac{c^2}{\rho} \text{grad}P \\
\frac{dp}{dt} = -\rho \text{div}u
\] (6.43)

If all the measurements are scaled by a factor \( \lambda \), the first of equations (6.43) becomes:

\[
\frac{d}{dt'} u(r', t') = -\frac{c^2(r', t')}{\rho(r', t')} \text{grad}'P(r', t')
\] (6.44)

where \( r' = \lambda r \) and \( t' = \lambda t \). Therefore:

\[
\frac{d}{dt} u(\lambda r, \lambda t) = -\frac{c^2}{\rho} \text{grad}(\lambda r, \lambda t)
\] (6.45)

The same differential equation is satisfied by \( u(\lambda r, \lambda t) \) as by \( u(r, t) \). The Rankine-Hugoniot conditions satisfy the same scale change.
If a pressure is measured for an explosive with a shell of radius 3 m, a thickness of 2 cm, a weight of 1000 g at a standoff of 9 m then for a 10th scale measurement an explosive of 0.3 m radius, 0.2 cm thickness and 10 g weight with a standoff of 0.9 m would be required.

However, there are limitations to the conditions in which the principle of similarity is applicable. Any situation in which the forces involved do not scale geometrically would cause the principle to fail. It would also fail in any situation in which the chemical reactions behind the detonation front are important.

6.6.3 Pressure-time curves

There has, to date, been no exact expression derived for the decay of pressure with time after arrival of a shock front in terms of the charge characteristics and position of the point of measurement. However, a simple representation by means of a negative exponential curve is convenient as a first approximation.

In this approximation, the pressure $P$ as a function of time $t$ after arrival of the shock front is expressed as:

$$P = P_m \exp(-t/\theta_d)$$  \hspace{1cm} (6.46)

where $P_m$ is the initial peak pressure and $\theta_d$ is the time constant of exponential decay. This approximation has been shown, for an explosive material [Cole, 1948],
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to agree well for the initial decay but at later times the "tail" of the shock wave decays much more slowly than in the exponential approximation.

6.6.4 Shock wave propagation

An extensive amount of work has been carried out developing a theory for the propagation of shock waves as well as the development of topics such as refraction [Taub, 1947] and reflection of shock waves [Dewey and McMillin, 1985a], [Dewey and McMillin, 1985b]. The main shock wave propagation theories have been developed by Kirkwood and Brinkley [Kirkwood and Brinkley, 1945], [Brinkley and Kirkwood, 1947], Taylor [Taylor, 1946] and Osborne and Taylor [Osborne and Taylor, 1946].

The theory of Kirkwood and Brinkley [Brinkley and Kirkwood, 1947] is judged to be superior to that of Osborne and Taylor [Osborne and Taylor, 1946] in that the Osborne and Taylor theory for underwater shock waves is based upon the acoustic approximation and is therefore only strictly valid for small excess pressures at large distances from the source. Osborne and Taylor also reported that their theory did not predict a detectable change in shape or spreading of the pressure wave with propagation. Cole and Coles [Cole and Coles, 1947] disagree with this saying that experimental evidence supports spreading. The theory of Taylor [Taylor, 1946] is also inferior to that of Kirkwood and Brinkley as it is based on ideal gas adiabatics with constant heat capacity, an approximation which fails

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badly close to the explosive source.

More details on these propagation theories can be found in appendix D.

6.7 Conclusion

This chapter has presented the basic theory and work carried out to date in the field of underwater explosions. The basic equations of fluid motion have been examined. The ways in which researchers have traditionally generated explosions for investigation was described and the shock waves and explosion bubbles produced by an explosion have been discussed in detail.

However, the work carried out to date examining the explosion bubble has not used LDA to investigate the situation. In the following chapters, the use of an underwater spark to generate bubbles and the use of an LDA system to capture light scattered from particles in the bubble is detailed. Analysis of the non-stationary Doppler signal allows information on bubble radius, period of oscillation and bubble energy to be obtained. This is of interest as it allows a non-intrusive measurement of the area very close to the bubble centre which is usually obscured by explosive debris.
Chapter 7

Experimental apparatus

The experimental arrangement used to produce an underwater explosion and capture experimental data is shown in figure 7.1. This represents the LDA system as well as the tank and electrodes creating the spark gap.

7.1 Laser Doppler anemometry arrangement

The dual-beam mode (figure 7.2) was chosen for the experimental arrangement as there is no path difference between the beams and so only a single lens was required for focusing the scattered light and it produces a strong signal.

A Uniphase 1135P, 20 mW He-Ne laser (wavelength 633.28 nm) was passed through a polariser and a Malvern RF307 beam splitter to produce two coher-
Figure 7.1: Photograph of experimental arrangement.
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Figure 7.2: Experimental arrangement for LDA.

ent, parallel beams with a separation of 20 mm. These beams were then passed through a Malvern K9023 phase modulator (figure 7.3) in which a sawtooth voltage was applied to the crystals. This advanced the phase of one outgoing beam and retarded the phase of the other. This change in the relative phase of the beams could be set so that the optical fringes moved linearly in space. The fringes were set to move in the opposite direction to the flow which resulted in an increase in Doppler frequency reaching the detector.

Figure 7.3: Pockels cell

The laser beams leaving the phase modulator were passed through a 500 mm
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focal length, bi-covex lens. Thus producing an interference pattern with a fringe separation of $2.11 \times 10^{-5}$ m and a width of 0.0006801 m.

The scattered light from the fringes then passed through a plano-concave 100 mm focal length lens, inclined at an angle of 40° to the beam axis to ensure that no direct laser light reached the detector. A Hamamatsu avalanche photodiode (C5460-01-SPL-S2382-3) was then used to collect the scattered light and store it on a computer. An optical filter was placed over the photodiode to ensure that only scattered laser light was collected. The Doppler signal was captured and digitised by an IMTEC, T3012 12 bit, 30 MHz fast data acquisition board. The software used was IMTEC Insight version 3.0.

The shock wave and bubble were generated using an underwater high voltage spark which ionised the water creating a bubble which emitted a shock wave as it expanded. These events were captured using either the LDA arrangement detailed above or a Brüel & Kjær 8103 hydrophone.

7.2 Spark generator

The underwater spark was generated by the circuit shown in figure 7.4 which consists of an EG&G, HY-61 thyratron, a grid driver for the thyratron and a Brandenburg high voltage power supply. The thyratron is basically a switch which causes the capacitor to discharge rapidly across the spark gap when triggered.
7.2.1 Thyratron

Applying a suitable positive triggering pulse to the thyratron grid creates an electron plasma in the grid-cathode region of figure 7.5. This plasma passes through the aperture of the grid structure and causes electrical breakdown in the high voltage region between the grid and the anode. This begins the process of thyratron switching (commutation).

The plasma that is formed between the grid and the anode diffuses back through the grid into the grid-cathode space. The commutation process is complete when the two sets of plasma meet. The time interval between the trigger breakdown...
of the grid-cathode region and the complete closure of the thyratron is the anode delay time.

During commutation, a high voltage spike appears at the grid of the thyratron. This spike happens in the time it takes for the plasma in the grid-anode space to connect to the plasma in the grid-cathode space. During this time the anode is effectively connected to the grid causing the grid to assume a voltage similar to that of the anode. Although the grid spike voltage is brief it can damage the grid circuit unless measures are taken to prevent it damaging the grid driver circuit (figure 7.6); in this case a Zener breakdown diode is used.

Once the commutation interval has finished, the thyratron conducts with nearly constant voltage drop regardless of the current through the tube. The thyratron then recovers through diffusion of the ions to the inner walls of the tube and the electrode surfaces where the ions can recombine with electrons.
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7.2.2 Capacitors

The thyratron acts as a switch which allows the bank of capacitors to discharge across the spark gap. The bank of capacitors is made up of four 1000 pf capacitors connected in parallel to produce an overall capacitance of 4000 pf. The maximum voltage which can be sustained by the capacitors is 15 kV.

The recharge time of the circuit is given by the value of RC where R is the resistance (1000 MΩ) and C is the capacitance. For the components used in this circuit, RC has a value of 4 seconds which is the time taken for the capacitors to reach $1 - 1/e$ of their maximum value.

7.2.3 Complete circuit

The complete spark controlling circuit can be seen in figure 7.7. The EHT power supply could supply up to 30 kV although the maximum voltage used experimentally was 14 kV which set the maximum available circuit energy at:

$$E_{\text{circuit}} = \frac{1}{2} CV^2$$

$$= 0.392 \text{ J} \quad (7.1)$$

The circuit energy was varied only through the voltage and the total capacitance (4000 pf) was held constant. Varying the circuit energy resulted in a change in
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![Complete spark generator circuit](image)

**Figure 7.7:** Complete spark generator circuit

the bubble size.

### 7.2.4 Spark gap

Figure 7.8 depicts the tank in which the spark was produced. The outer dimensions of the tank are: $400 \times 500 \times 500$ mm and the walls were made from 10 mm thick glass. The tank’s place within the LDA arrangement is depicted in figure 7.2. The laser beams intersected in the middle of the tank and at a depth of 230 mm from the water surface.

The electrodes were connected to a trolley on top of the tank which allowed the gap to be moved laterally and transversely. The height of the gap could also be adjusted by simply lowering the electrodes through the trolley. The electrodes were made of tungsten with a diameter of 1.54 mm and a length of 4 mm. The gap width could be adjusted in increments of 0.1 mm. A coaxial wire carried the charge to above the water surface at which point the wire was split into
two separate wires which were contained within Perspex tubes and lead to the electrodes through the water. This ensured that the wires were well insulated to avoid leakage into the water as this would prevent the gap from sparking. A detailed analysis of the water used can be found in appendix E. This was left for 24 hours in the tank before experiments were carried out; this was to ensure that all gaseous products had been removed from the water.

7.2.5 Earthing

The earthing of the circuit and apparatus was found to be very important in the quality of the Doppler signal and for safety. When the spark discharged, the
individual components of the experimental arrangement detected a high voltage
discharge pulse which appeared in the Doppler signal reducing the quality of the
analysis.

To help reduce the high voltage spike picked up by the components, the EHT
power supply, grid circuit and heater power supplies were placed in a cabinet
covered in wire mesh which acted as a Faraday cage to prevent leakage of the
electric field to the circuits in the other pieces of apparatus in the laboratory.
All wires were earthed and ferro-magnets (low pass RC filters) were added to all
cables to help further reduce the spike. The detector was placed in a screened,
earthed, diecast box. These all helped reduce pickup of the voltage spike which
occurs during discharge of the circuit. It was found impossible to fully eliminate
the spike from the captured Doppler signal but the spike could be advantageous
in that it could be used to trigger the waveform capturing software.
This F/A 18 Hornet is travelling above the Pacific ocean at 330 metres per second, breaking through the sound barrier. On reaching the sound barrier, the Hornet creates shock waves off its tail and decompression waves off the cockpit. These shock waves have the same propagation and decay characteristics as the ones which are discussed in the following chapter.

This picture is included with permission from Boeing.¹

¹http://www.boeing.com/defense-space/military/fa18/images/soundbr.htm
Chapter 8

Experimental results

The experimental results detailed in this chapter were obtained using the apparatus described in Chapter 7. The velocity measurements were produced through analysis of Doppler signals using the wavelet method described in Chapter 3 and pressure measurements were obtained from a Brüel & Kjær 8103 hydrophone suspended at the point of interest.

The data obtained are used to investigate the shock wave and the explosion bubble produced by the underwater spark. A comparison is made between the shock wave decay and acoustic signature obtained from an explosive material and an underwater spark. The wavelet analysis method is shown to produce instantaneous velocity results which agree well with results obtained from a hydrophone. An empirical relationship between the bubble pressure and distance from the bubble centre is determined. The energy transfer between the circuit and the bubble is
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examined as is the effect of the spark gap width and the circuit energy on the bubble radius.

8.1 Doppler signal

It has been found that the spurious signal picked up by the cables and detectors as a result of the capacitor discharge can have both advantages and disadvantages when capturing data. The maximum voltage of the spike was minimised by earthing each piece of equipment and attaching ferro-magnets to all cables. It was not possible to reduce the spike totally but its presence was used to advantage in triggering the software for capturing the data.

The form of the spike captured through the detector with no laser light and no optical filter over the detector is shown in figure 8.1; figure 8.2 shows the signal captured when a filter is used. The low frequency decay of the signal in figure 8.1 is produced by the light from the spark reaching the detector. This can produce problems by overloading the detector and preventing scattered light within the same time interval from being detected. Using an optical filter over the detector removes this problem as can be seen in figure 8.2 which shows the voltage spike from pickup of the various experimental components.

The form of the captured Doppler signal is shown in figure 8.3. This signal has a 20 kHz carrier frequency but the variation in frequency due to the oscillating
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Figure 8.1: Spike captured through detector without an optical filter.

Figure 8.2: Spike captured through detector with an optical filter.
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bubble is still visible as is the voltage spike from the capacitor discharge. The Doppler signals are analysed using the wavelet method described in Chapter 3. The presence of the voltage spike within the Doppler signal influences the analysis as it is treated as though it is due to scattered light rather than pick up of the experimental components. This results in an analysed signal which does not accurately represent the point in the flow at which the measurement was taken. It is therefore beneficial to use the spike to trigger the software in order to produce a more accurate signal for analysis. This would also reduce the length of signal which required to be captured, thereby reducing the analysis time.

Figure 8.4 shows an example of a Doppler signal after analysis. This was triggered using the discharge spike and therefore the analysed signal only contains information from scattered laser light. The velocity can be seen to decay from a maximum value to zero at which point the bubble has reached its maximum radius and has stopped expanding. The bubble then starts to contract and the velocity increases again but in a negative direction due to the fluid flowing in the opposite direction through the fringes.

The bubble period, radius, particle velocity and energy can be calculated from an analysed LDA signal. In figure 8.4 the bubble collapse time, $\tau_{\text{bubble}}$, is calculated as the time between the zero velocity which occurs at the minimum radius and the zero velocity which occurs at the maximum radius. The bubble period, $\tau_{\text{total}}$,
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Figure 8.3: Doppler signal before analysis with a 20 kHz sample frequency.

Figure 8.4: Doppler signal after analysis
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is twice this time:

\[ \tau_{\text{bubble}} = 0.000539027 - 0.000267162 \]
\[ = 0.0002719 \text{ sec} \]

\[ \tau_{\text{total}} = 2 \times \tau_{\text{bubble}} \]
\[ = 0.0005438 \text{ sec} \]  

(8.1)

From this the maximum bubble radius, \( r_m \), can be calculated using:

\[ \tau_{\text{total}} = 1.84 r_m \left( \frac{\rho_o}{P_o} \right)^{1/2} \]  

(8.2)

where \( \rho_o \) is the density and \( P_o \) is the ambient pressure at the depth of measurement, \( h \), given by:

\[ P_o = P + \rho_o g h \]  

(8.3)

where \( g \) is the acceleration under gravity and \( P \) is atmospheric pressure. A value for the maximum bubble radius can therefore be obtained from:

\[ r_m = \frac{\tau_{\text{total}}}{1.84} \left( \frac{\rho_o}{P_o} \right)^{-1/2} \]
\[ = 0.0030078 \text{ m} \]  

(8.4)

The bubble energy can be calculated once the bubble period or maximum radius
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is known using [Leighton, 1994]:

\[
E_{\text{bubble}} = \frac{4\pi}{3} r_m^3 \rho \quad \star
\]

\[
= 0.0116549 \text{ J} \tag{8.5}
\]

8.2 Shock wave results

8.2.1 Pressure measurements

Using a hydrophone it is possible to obtain pressure information about the shock wave. In contrast to the LDA method, the hydrophone is intrusive and it is preferable to use a technique, such as LDA, for investigation which does not influence the area being measured. However, it is convenient here to use hydrophone data for verification of the wavelet analysis technique of LDA signals.

Figure 8.5 represents the data obtained from placing the hydrophone in the tank and generating an underwater spark. There is no spike present in these data as it was used to trigger the software. As the bubble expands and cools it emits a pressure pulse called the expansion pulse, this is the shock wave. The bubble continues to expand and cool and the pressure eventually falls below ambient pressure, this is the rarefaction pulse. Eventually the outward bubble motion is brought to a stop at which point the bubble has reached its maximum radius and
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Figure 8.5: Hydrophone signal showing bubble expansion and collapse pulses.

It then begins to collapse. The pressure inside the bubble increases again until it is high enough to stop the collapse. At this point another pressure pulse is emitted, the collapse pulse. In an ideal situation the bubble would expand again, as in figure 8.6, creating an oscillating system which would continue until the bubble energy is totally consumed. However, in most cases the bubble collapse is non-symmetrical and no further bubble oscillations occur. The bubble period can be determined from the time between the expansion pulse and the collapse pulse. In an oscillating system, dissipation of energy reduces the bubble period with each oscillation.
8.2.2 Shock wave velocity

In order to obtain a value for the shock wave velocity, the hydrophone was placed at three different distances from the spark gap. The discharge spike was used to trigger the capturing software each time. Figure 8.7 presents the hydrophone signal captured for a 14 kV signal with a spark gap width of 0.3 mm.

From this it is possible to determine the shock wave velocity, $U_{\text{shock}}$, using:

\[
U_{\text{shock}} = \frac{\text{distance}}{\text{time}}
\]

where the distance travelled by the shock wave between two hydrophone positions...
Figure 8.7: Varying arrival of shock wave due to distance of hydrophone from the source.
is known and the time is obtained from the difference in arrival time of the shock wave at the two hydrophone positions.

Data has also been obtained for shock waves produced using 12 kV and 13 kV. Between these three voltages it is possible to obtain nine values for the shock wave velocity which can be averaged to produce:

\[
U_{\text{shock}} = 1305 \pm 340 \text{ ms}^{-1}
\]  

(8.7)

where the error is the standard deviation. The large value of the error is due to the large relative error in measuring the distance. Although the hydrophone was placed at three positions separated by 10 mm, there was an error in this measurement of 2 mm due to the size of the hydrophone. These measurements were taken in water at a temperature of 20 °C. The speed of sound in water at this temperature is 1481 ms\(^{-1}\) [Kinsler et al., 1982] which lies within these error bounds.

### 8.2.3 Examination of shock wave decay

To date no equation has been derived which describes the exact form of shock wave decay pressure as a function of time. However, as explained in section 6.6.3, a simple representation of a negative exponential curve has been shown to provide an approximate curve for the decay of explosive materials [Cole, 1948]. Figure 8.8
Chapter 8 — Experimental results

Figure 8.8: Comparison between hydrophone and theory for shock wave decay.

Figure 8.9: Logarithmic pressure
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shows the shock wave decay obtained from a hydrophone for an underwater spark and the fit of a negative exponential curve. It can be seen that the exponential decay fits well to a first approximation for the initial decay. The decay has the form [Cole, 1948]:

\[ P = P_m \exp \left( \frac{-t}{\theta_d} \right) \]  \hspace{1cm} (8.8)

where \( \theta_d \) is the time constant of exponential decay. The accuracy of this approximation can be seen by plotting \( \log P \) as a function of time as in figure 8.9; this is a straight line of slope \(-1/\theta_d\). For the data presented in figure 8.8 the pressure varies as:

\[ P(Pa) = 1.6 \times 10^5 \exp \left( \frac{-t}{0.000013} \right) \]  \hspace{1cm} (8.9)

The initial decay of a spark generated shock wave can be seen to follow the same negative exponential decay as one from an explosive material.

Decay characteristics

As the spark gap width or circuit energy increases, the hydrophone signal would be expected to vary. Figure 8.10 represents how the hydrophone signal varies for four gap widths using a constant voltage of 13 kV. It can be seen that as the gap width increases, the time between the expansion and collapse pulse increases
Chapter 8 — Experimental results

Figure 8.10: Hydrophone measurement of pressures due to bubble oscillation.

and therefore the maximum bubble radius must also increase. As the voltage and gap width vary it is of interest to determine whether the form of the shock wave decay changes and if the characteristics of the oscillations which are visible on the decay in figure 8.8 are characteristic of the system.

Figure 8.11 allows closer examination of the hydrophone data from figure 8.10. It can be seen that the four graphs oscillate in a similar way around hydrostatic pressure. Closer examination of the initial decay also shows similarities in the four curves. The signal oscillations can therefore be attributed to characteristics of the apparatus used. Some explosive materials have also been reported to have produced oscillations on the initial decay curves. It has been mentioned
Figure 8.11: Examination of shock wave decay

[Cole, 1948] that these may be due to internal reflections within the bubble or imperfections in the charge shape. In the case of spark generated bubbles, the oscillations on the initial decay curve may also be due to reflections around the electrode region.

A comparison of shock wave decay for a constant gap width but for three different voltages can be seen in figure 8.12. Although the peak pressure and the bubble
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period vary with voltage, the characteristics of the shock wave decay are visibly similar for each voltage. The change in the shock wave decay with distance from the spark gap is shown in figure 8.13 in which the main difference is the maximum pressure detected; again, the general form of the decay does not vary.

It can therefore be concluded that the oscillations on the shock wave decay are not dependent on voltage, gap width or distance from the bubble centre and must result from the characteristics of the system.

8.2.4 Comparison of acoustic signatures from explosive materials and underwater sparks

The decay of the expansion pulse has been discussed and both explosive materials and underwater sparks produce shock waves which follow a negative exponential decay. However, there is a difference in the form of the collapse pulse. Figure 8.14 represents the acoustic signature of an underwater explosion created from an explosive material, in this case TNT equivalent [El-Deed and Royles, 1999]. The acoustic signature of an underwater spark is shown in figure 8.5. It can be seen that the collapse pulse has a much lower peak pressure than the expansion pulse and a much longer period.

However, in figure 8.6 the collapse pulse has a much lower peak pressure and a longer period than in figure 8.5 and is a much closer representation of the acoustic
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Figure 8.12: Comparison between 12, 13 and 14 kV voltages and a constant gap width of 0.3 mm.

Figure 8.13: Comparison of shock decay at two distances from bubble centre for a gap width of 0.3 mm and a voltage of 14 kV.
signature of the explosive material. The situation represented in figure 8.5 was much more commonly obtained within this research. This may be due to the design of the electrodes in that the casings were too close to the bubble; thereby influencing its motion and preventing oscillations.

The period of oscillation of the bubble in figure 8.5 is much lower than that in figure 8.6 which corresponds to a smaller maximum bubble radius. The larger bubble energy may have overcome the fact that the electrode design influenced the bubble oscillations and an oscillating bubble situation could be created.
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8.2.5 Hydrophone analysis

The effect of afterflow was discussed in detail in section 6.3. After the expanding bubble has emitted the shock wave, it continues to expand until the maximum radius is reached at which point it starts to contract again. The outward motion of the water due to this expansion and contraction is termed the afterflow. Analysis of the afterflow from LDA or hydrophones allows information on the bubble period, radius, energy and particle velocity to be determined.

The particle velocity, \( u \), has the form:

\[
\mathbf{u} = \frac{P - P_o}{\rho_o c} + \frac{1}{\rho_o d} \int (P - P_o) \, dt
\]

where \( P \) is the pressure, \( P_o \) the ambient pressure, \( \rho_o \) the density, \( c \) the speed of sound, \( d \) the distance from the bubble centre and \( t \) is time. The first term of equation (8.10) represents the particle velocity due to the shock wave and the second term represents the particle velocity due to the afterflow. Analysis of the hydrophone pressure produces the instantaneous particle velocity, \( u \), which should agree with the analysed Doppler signal.

Figure 8.15 shows the first and second terms of equation (8.10) obtained from the hydrophone data which represent the particle velocity due to both the shock wave and the afterflow. Adding these terms together produces the total particle velocity as shown in figure 8.16. There is very little difference between the total
particle velocity and the afterflow velocity other than in the very short time scale at the beginning of the signal. The transit time of the shock wave is approximately 0.0000238 seconds and the frequency of the bubble oscillation is approximately 2 kHz and it is this lower frequency acoustic signal which is of interest in this thesis.

8.2.6 Comparison of signals from LDA and hydrophone

As the Doppler signal can be analysed to produce a plot of instantaneous particle velocity with time it is possible to compare the signal from the LDA with that of the analysed hydrophone. Figure 8.17 shows the analysed Doppler signal from
Figure 8.16: Comparison between the total particle velocity, the shock wave velocity and the afterflow.
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the LDA and the afterflow term obtained from a hydrophone placed at the same position as the fringe volume of the LDA arrangement. Very close agreement is visible between them.

Figure 8.18 represents the analysed Doppler signal and the total particle velocity due to the shock wave plus the afterflow from the hydrophone. The LDA data provide information on the afterflow only and therefore no shock wave data is present in the signal. This is due to the refractive index variation caused by the shock wave being so large that the laser beam is diverted in such a way that the detector receives very little scattered light from the shock wave.

A comparison of the afterflow particle velocity from the hydrophone, the particle velocity from the analysed LDA and the raw Doppler signal without a carrier frequency is shown in figure 8.19. From this it can be seen that the hydrophone and the analysed LDA produce a zero velocity at the same place as the Doppler signal has a minimum frequency. This zero in particle velocity occurs when the bubble has reached its maximum radius and expansion has stopped. The bubble then starts to contract and the particles move with an increasing velocity in the opposite direction which causes a negative velocity in the analysed LDA. These three data sets correspond to the same point within the tank but were all captured at different times, taking this into account it is clear that they show close agreement within error.
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Figure 8.17: Comparison between analysed LDA and afterflow.

Figure 8.18: Comparison between total particle velocity and LDA.
Figure 8.19: Comparison between data for the afterflow from hydrophone, analysed LDA and raw Doppler signal with no frequency shift for the same position in the tank.
8.3 Bubble results

8.3.1 LDA measurement of bubble characteristics

LDA without a carrier frequency

When no carrier frequency is present in the Doppler signal, it is possible to observe the varying frequency of the signal due to the expansion and collapse of the bubble. The bubble period, radius and oscillation frequency can be determined from these observations.

As the spark gap is increased, the bubble period increases as expected from observation of the hydrophone data (figure 8.10). This can be seen in figure 8.20 for the 12 kV signal, figure 8.21 for the 13 kV signal and figure 8.22 for the 14 kV signal.

LDA with a carrier frequency

The inclusion of a carrier frequency creates a Doppler signal which is much easier to analyse. Analysis of the Doppler signal is carried out using the wavelet technique outlined in Chapter 3 which produces a plot of instantaneous velocity. Values of the bubble period and maximum radius are obtained using the method outlined in equations (8.1)-(8.4).
Figure 8.20: Variation of Doppler signal with gap width for a voltage of 12 kV.

Figure 8.21: Variation of Doppler signal with gap width for a voltage of 13 kV.
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Figure 8.22: Variation of Doppler signal with gap width for a voltage of 14 kV.

Figure 8.23 shows how the period of the bubble varies with gap width for 12 kV, 13 kV and 14 kV signals. Each value of the bubble period is obtained by averaging over 130 sets of analysed LDA data, the error bars correspond to the standard deviation. It can be seen that at the smallest gap width of 0.1 mm, the three voltages appear to be converging, suggesting a limitation in the energy available for ionisation of the water and bubble production.

Again, analysis of the signals in this way produces an increase in bubble period with increase in gap width. It can be seen that the 12 kV data set does not follow the same trend as the 13 kV and 14 kV sets, probably due to the much lower energy available for ionisation of the water. Figure 8.24 shows a linear, quadratic
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and cubic fit to the 13 kV data set. A cubic fit most accurately represents the relationship between the bubble period and gap width. This is a very useful result for any future research in which a bubble of a specific size is required.

More detail on the relationship between bubble period and radius with gap width and voltage can be seen in table 8.1 and in appendix A.

<table>
<thead>
<tr>
<th>Voltage (kV)</th>
<th>Gap (mm)</th>
<th>Period (sec)</th>
<th>Radius (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.1</td>
<td>0.000237 ± 0.000022</td>
<td>0.002628 ± 0.000246</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.000227 ± 0.000029</td>
<td>0.002510 ± 0.000316</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.000305 ± 0.000029</td>
<td>0.003370 ± 0.000320</td>
</tr>
<tr>
<td>13</td>
<td>0.1</td>
<td>0.000220 ± 0.000026</td>
<td>0.002434 ± 0.000290</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.000251 ± 0.000032</td>
<td>0.002778 ± 0.000360</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.000301 ± 0.000028</td>
<td>0.003330 ± 0.000314</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.000402 ± 0.000037</td>
<td>0.004444 ± 0.000412</td>
</tr>
<tr>
<td>14</td>
<td>0.1</td>
<td>0.000239 ± 0.000029</td>
<td>0.002642 ± 0.000324</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.000299 ± 0.000026</td>
<td>0.003310 ± 0.000288</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.000347 ± 0.000030</td>
<td>0.003834 ± 0.000338</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.000441 ± 0.000037</td>
<td>0.004878 ± 0.000408</td>
</tr>
</tbody>
</table>

Table 8.1: Bubble period and radius obtained from analysed LDA signals for various voltages and gap widths.

8.3.2 Photograph of bubble

To capture an image of the bubble and the shock wave a shadowgraph [Holder and North, 1963] was created of the electrode area. A CCD camera was used to capture the image. Figure 8.25 shows the shock wave produced by the
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**Figure 8.23:** Variation of bubble period with gap width for various voltages.

**Figure 8.24:** Linear, quadratic and cubic fits to variation of bubble period with gap width for 13 kV.
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underwater spark and figure 8.26 shows the bubble. These were both produced from a spark gap of 0.1 mm and a voltage of 13 kV.

It is visible from these shadowgraphs that the bubble and the shock wave are not spherical. This is probably due to the presence of the electrode casings which cause reflections and attract the bubble. Without the casings, the charge dissipated into the water preventing sparking across the gap between the electrodes.

Figure 8.25: Shock wave from underwater spark is visible propagating from electrodes.
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8.3.3 Initial bubble pressure dissipation

Bubble pressure can cause significant damage to structures and materials and it is therefore of interest to determine the form in which bubble pressure decays with distance from the source.

LDA measurements were taken at one millimetre increments from the gap to allow the pressure dissipation of the afterflow to be examined. This was carried out for voltages of 12 kV, 13 kV and 14 kV and for gap widths ranging from 0.1 mm up to 0.4 mm. Higher voltages could not be examined due to limitations of the circuit components. The maximum gap widths were limited by the available energy for plasma formation.
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The pressure was calculated from the particle velocity measurements obtained from wavelet analysis of the LDA Doppler results. The peak velocity measurement, \( u \), was obtained using a peak detection program and converted into pressure, \( P \), using:

\[
P = u \rho_o c
\]

(8.11)

where \( \rho_o \) is the density of the medium and \( c \) is the speed of sound in the medium. This linear relationship is suitable for the pressure conversion due to the fact that only a specific point is of interest and no time information is required. Bubble period and radius have been determined from analysis of the Doppler signal and are known for each plot of pressure decay. Although the LDA is captured at various distances from the spark gap (bubble centre), the bubble radius does not vary with distance, as shown in figure 8.27. However, as the distance from the bubble centre increases the peak pressure of the bubble decreases and any particle motion is more difficult to detect.

The pressure dissipation for each gap width can be seen in figures 8.28 - 8.30 for the 12 kV data, figures 8.31 - 8.34 for the 13 kV data and figures 8.35 - 8.38 for the 14 kV data. These pressure dissipation plots can be seen to have an exponential decay. Each graph was based on some 130 measurements and for the first time an empirical relationship has been established between the pressure decay and
Figure 8.27: LDA captures a constant bubble radius with distance from source.

distance from the source for a spark generated bubble:

\[ P(Pa) \approx 10^5 \times \frac{r_m}{d} \times \exp \left( \frac{0.3\sqrt{r_m}}{d} \right) \]  \hspace{1cm} (8.12)

where \( r_m \) is the maximum bubble radius and \( d \) is the distance from the bubble centre. The factor \( r_m^{1/2} \) controls the gradient of the decay. As the gap increases for each voltage the gradient of the decay also increases. This empirical relationship was based upon the empirical relationship obtained by Penney and Dasgupta [Penney and Dasgupta, 1942] and adapted to fit spark generated data. The values of \( 10^5 \) and 0.3 were obtained from the graphics package xmgr\textsuperscript{1} and allowed

\textsuperscript{1}This is a graph plotting package. More details can be found at http://www.plasma-...
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the best fit to each of figures 8.28-8.38.

The points on the graphs represent the experimental data obtained from averaging over five data sets for each position and the error bars correspond to the standard deviation. The solid line represents the fits from equation (8.12) with the dotted lines representing the associated errors due to the error in the bubble radius. More detail can be found in appendix B.

8.3.4 Bubble energy

It is also of interest to know how much of the circuit energy is transferred into bubble energy. This can be assessed on the basis of the principle of conservation of energy. The electrical energy is converted into a plasma at high temperature and pressure. The spark is eventually extinguished by a decrease in the voltage and an increase in ionisation, temperature and pressure in the plasma. Energy is stored in the bubble in the form of ionisation, dissociation, excitation and kinetic energy of the particles. The energy is dissipated through light radiation, thermal radiation and thermal conduction.

The energy in the circuit, $E_{\text{circuit}}$, is given by:

$$ E_{\text{circuit}} = \frac{1}{2}CV^2 \quad (8.13) $$
Figure 8.28: Pressure decay for a bubble of radius 0.002628 mm using 12 kV and 0.1 mm gap.

Figure 8.29: Pressure decay for a bubble of radius 0.002510 mm using 12 kV and 0.2 mm gap.
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Figure 8.30: Pressure decay for a bubble of radius 0.003370 mm using 12 kV and 0.3 mm gap.

Figure 8.31: Pressure decay for a bubble of radius 0.002434 mm using 13 kV and 0.1 mm gap.
Figure 8.32: Pressure decay for a bubble of radius 0.002778 mm using 13 kV and 0.2 mm gap.

Figure 8.33: Pressure decay for a bubble of radius 0.003330 mm using 13 kV and 0.3 mm gap.
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Figure 8.34: Pressure decay for a bubble of radius 0.004444 mm using 13 kV and 0.4 mm gap.

Figure 8.35: Pressure decay for a bubble of radius 0.002642 mm using 14 kV and 0.1 mm gap.
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Figure 8.36: Pressure decay for a bubble of radius 0.003310 mm using 14 kV and 0.2 mm gap.

Figure 8.37: Pressure decay for a bubble of radius 0.003834 mm using 14 kV and 0.3 mm gap.
where \( C \) is the capacitance and \( V \) is the circuit voltage.

The energy in the bubble, \( E_{\text{bubble}} \), can be determined if the maximum bubble radius, \( r_m \), is known [Leighton, 1994]:

\[
E_{\text{bubble}} = \frac{4}{3} \pi r_m^3 P_o \tag{8.14}
\]

where \( P_o \) is the ambient pressure at the depth of the bubble.

The ratio of the bubble energy to the circuit energy determines the amount of energy which is transferred from the circuit to the bubble. Figure 8.39 shows the variation of the ratio of the bubble energy to circuit energy as a function of

Figure 8.38: Pressure decay for a bubble of radius 0.004878 mm using 14 kV and 0.4 mm gap.
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spark gap width. The variation of transferred energy with gap width is cubic in form. Each value of the maximum radius \( r_m \) is determined from an average of 130 analysed LDA results.

For a constant spark gap width, a constant amount of circuit energy is required to create a spark. As the available circuit energy increases with increasing voltage, the amount of energy which remains to be transferred into bubble energy after the spark is created must also increase. This explains the increase in energy transfer with voltage for a constant gap width.

As the spark gap width increases, the amount of energy transferred also increases. Figure 8.39 displays a cubic increase in energy transfer with spark gap width for a constant voltage of 13 kV and 14 kV. However, the 12 kV data set does not follow this trend. This can be explained by considering the available energy. The available circuit energy at 12 kV is 0.288 J, if this is close to the energy required to ionise the water and create a spark then very little energy remains to be transferred to bubble energy. This would result in a reasonably constant bubble size and energy transfer.

It is also noticeable in figure 8.39 that all three voltages tend to converge towards a point at a gap width of 0.1 mm. This further strengthens the belief that there is a limitation in the energy threshold required for plasma formation. A spark gap of 0.1 mm and a voltage of 12 kV are thus the minimum parameters which could create a spark in this experimental arrangement. The energy transferred from
Chapter 8 — Experimental results

Figure 8.39: Variation of energy transfer with gap width.

The circuit to the bubble was found to reach 12.8% for the largest gap width and highest voltage. Larger energy transfers would be possible for higher voltages and larger gap widths. These results are very important for any future work carried out using spark generators. More detail can be found in appendix C.

8.4 Experimental conclusions

The use of an underwater spark to generate shock waves and explosion bubbles has been shown to be a viable technique which is relatively safe, quickly repeatable and does not obscure the water with explosion debris. The initial decay of shock
waves produced by underwater sparks agrees well with the form of shock waves produced from explosive materials. However, the decay is not smooth and the deviations from the negative exponential curve have been shown to be due to the experimental system and are consistent through varying voltage, distance of measurement from the spark and spark gap width.

It has been shown that the bubble period and therefore radius can be obtained using a hydrophone from the time between the expansion and collapse pulses of the bubble but this involves placing the hydrophone at the point of interest which influences the particle velocities at that point. A pure Doppler signal with no frequency shift has also been shown to be useful in obtaining period and radius measurements by visual inspection of the signal. However, it is not possible to obtain accurate instantaneous frequency information from this. Adding a frequency shift allows a wavelet analysis technique to be used which produces the instantaneous frequency allowing more accurate information to be obtained for the bubble period, radius and energy.

Verification of the wavelet technique has been achieved through comparison of the analysed Doppler signals with data obtained from a hydrophone. The analysed Doppler signals obtained from the LDA agree well with the hydrophone results for the particle velocity of the afterflow due to the movement of the water from the bubble expansion and subsequent contraction and inward flow. This low frequency acoustic emission is of great interest to researchers and therefore a
non-intrusive measurement technique is very useful for investigation.

Analysis of Doppler signals using wavelets has enabled an empirical relationship between bubble pressure and distance from the source to be determined. This was obtained from some 1430 analysed LDA signals.

The amount of energy transferred from the circuit to the bubble was also investigated. It was found that increasing the gap width created a cubic increase in the transferred energy. From the experimental arrangement used for these investigations, the maximum energy transfer from the circuit to the bubble was found to be 12.8%. This is useful for any future investigation in which a bubble of a specific size is required.
Chapter 9

Discussion and conclusion

The principles behind laser Doppler anemometry (LDA) have been reviewed including the mathematical principles and typical experimental arrangements. Two analysis techniques have been used for analysis of the Doppler signals, both of which allow the instantaneous velocity of a point in a flow to be obtained. The use of LDA to measure sound sources has been discussed and the effect on the Doppler signal due to refractive index variations within the medium has been theoretically derived and compared with experimental results. The theory behind underwater explosions has also been detailed and LDA used to investigate the low frequency bubble oscillation produced from such an explosion.

The underwater shock wave and bubble are created from an underwater spark. Traditionally, explosive materials such as TNT have been used and an extensive amount of research has been carried out using explosive materials. However,
the detonation of explosive materials causes the area around the charge to be obscured by explosive debris and optical techniques would not be of any use in investigation of that area. A further drawback is that it is very difficult to ensure that all the explosive material is detonated in each explosion, if some of the material does not detonate then accurate repeatability of the situation is difficult to achieve.

The use of an underwater spark allows optical techniques, such as LDA, to be used to investigate the area close to the centre of the explosion. The research findings discussed here have shown that it is possible to generate a shock wave and bubble from an underwater spark and that the characteristics of the explosion are the same as for one generated by an explosive material. The shock wave decay is of the same form as that of an explosive material and the bubble can oscillate for multiple periods. It has also been shown that it is possible to use LDA to investigate the bubble produced by an underwater explosion and the non-stationary data signal can be accurately analysed using a wavelet technique.

The contributions which this work has made to the field of sound measurement under water are:

- Development of the acousto-optic theory.

- Experimental verification of theoretical acousto-optic predictions.

- Verification of the use of wavelets for analysis of non-stationary signals.
Chapter 9 — Discussion and conclusion

- Investigation of acoustic signal from an underwater explosion using LDA.
- Development of empirical relationship for bubble pressure decay.
- Investigation of energy transfer from circuit to bubble.

9.1 Doppler signal analysis

Traditional methods of analysing Doppler signals do not allow instantaneous frequency information to be obtained about a point in a non-stationary flow. In the case of complex sound fields it is impossible to differentiate between the various frequency components in a power spectrum. It is therefore beneficial to use the Hilbert transform and wavelet techniques to analyse the data as they allow instantaneous frequency information to be obtained and the wavelet technique allows non-stationary signals to be analysed.

The principles behind the two techniques have been described in detail and experimental results obtained using them are presented and discussed. The experimental results obtained for comparison with the acousto-optic theory are analysed using the Hilbert transform technique which produces a plot of amplitude with sound frequency. However, the Hilbert technique has the drawback that each point in the instantaneous frequency is influenced by each point in the Doppler signal so that any low quality part of the signal or dropout has an effect on the analysed signal.
Chapter 9 — Discussion and conclusion

The Doppler signals captured from the underwater spark were analysed using the wavelet technique which produces a plot of instantaneous frequency which could easily be converted into instantaneous velocity. The validity of this technique was verified through comparison of the analysed Doppler signals with results obtained from hydrophone measurements taken at the same position; close agreement was found between them.

9.2 Acousto-optic effect

Various authors have used LDA to investigate sound fields in air and water. However, they have not taken into account the refractive index variations which arise due to the propagating sound wave and their influence on the laser beams within the system. These variations have the effect of creating a phase difference between the two laser beams in the LDA arrangement so that when they intersect they may create moving fringes rather than stationary ones.

Within this thesis, a theory has been developed to investigate the effect of these refractive index variations on the analysed Doppler signal of an LDA system. An expression is obtained for the amplitude of fringe movement for the case of a standing wave and a travelling wave underwater.

It is shown that the movement of the fringes and the movement of the particles are $\pi/2$ out of phase. There is therefore an apparent motion of the particles as
Chapter 9 — Discussion and conclusion

seen by the detector and a theoretical expression has been developed for this. The amplitude of the fringe movement for both a standing wave and a travelling wave is linearly related to the wavenumber for low wavenumbers and quadratically linked to the distance of propagation of the laser beams.

The angle of propagation of the sound wave is also an important factor in determining the amplitude of fringe movement. For the case of a travelling wave, the maximum amount of fringe movement occurs at an angle of $\pi/2$ between the wavefronts and the fringe volume for a low wavenumber. As the wavenumber increases, the maximum value of the fringe movement begins to increase at an angle corresponding to the angle of the laser beams and the amplitude of movement at an angle of $\pi/2$ decreases. If the laser beams propagate through a very low frequency sound field there is no fringe movement as both the laser beams are affected to the same degree and there is no path difference.

The developed theory was verified with experimental results obtained from investigating a standing wave within an LDA arrangement. The amplitude of particle movement was obtained from a hydrophone placed at the point of laser beam intersection. The detector in the LDA arrangement produced a value for the apparent motion of the particle in the fringes. Taking the ratio of the apparent motion of the particles to the actual motion allowed a comparison with the theoretical predictions to be made. Close agreement was found between the theory and the experimental results.
It can therefore be concluded that if LDA is used to investigate a sound field in water then the distance of propagation of the laser beams and the frequency of the sound are very important in determining the amount of fringe movement. The higher the sound frequency, the greater the refractive index variations and the longer the distance of propagation, the greater the influence on the laser beams. These both produce a greater amplitude of fringe movement. This can help to detect the Doppler signal if the amplitudes involved are low but if amplitude information is important, the analysed signal does not accurately represent the area of interest in the flow.

9.3 Explosion bubble

LDA has also been used to investigate the low frequency acoustic signal emitted from an underwater explosion. In this situation the explosion is generated by means of an underwater spark. This creates a bubble which emits a shock wave as it expands and cools. The bubble continues to expand until it reaches a maximum radius at which point it starts to contract again, building up the pressure inside the bubble. An oscillating system has been created which continues until the energy is fully dissipated.

The LDA system is used to detect the particle velocity of the afterflow generated by the expansion and collapse of the bubble. Although the total particle velocity
Chapter 9 — Discussion and conclusion

in the water is due to the particle velocity from the shock wave and from the afterflow, the LDA detects only the afterflow as the large refractive index change across the shock front deflects the laser beams away from the detector so that a negligible amount of scattered laser light due to the shock wave is captured. Analysis of the Doppler signal using wavelets allows the bubble radius, period of oscillation, energy and instantaneous particle velocity to be determined.

Varying the width of the spark gap or the voltage to which the capacitors are charged results in a variation in the size of the bubble. This has been shown from hydrophone results and Doppler signals. The variation of bubble size with gap width and voltage is shown to be cubic in form which is a very useful result for future work if a bubble of a required size is required. Verification of the wavelet analysis technique is obtained from comparison of the analysed LDA signal with data obtained from a hydrophone at the same position.

The peak pressure dissipation of the bubble with distance from the bubble centre is shown to depend upon the bubble radius. An empirical relationship between the pressure and the distance has been formed for the first time for a spark generated bubble from some 1430 measurements. The amount of energy transferred from the circuit to the bubble has also been investigated and is shown to vary as a cubic with spark gap width. As the available circuit energy increases, the percentage of energy transferred to the bubble also increases due to a smaller percentage of the circuit energy being required for evaporation of the water. Also, as the spark
gap width increases for each value of the circuit energy, the percentage of energy transferred also increases. The maximum transfer energy was found to be 12.8 % for a spark gap of 0.4 mm and a voltage of 14kV.

9.4 Future work

There are many ways in which the work detailed within this thesis could be extended.

It would be of great interest to increase the size of bubbles which could be produced by the underwater spark. This could be done by increasing the capacitor values and the maximum working voltage of the thyratron. Redesigning the electrode region would ensure that the bubbles were not affected by surfaces.

Analysis of the LDA signals using the wavelet technique would enable the data obtained from the bubble to be analysed. A larger range of bubble radii due to larger voltages and therefore larger spark gaps which would allow ionisation to occur would allow more accurate determination of the cubic relationship between the bubble radius and the voltage or spark gap. It would also be interesting to investigate the acoustic signature of the oscillating explosion bubble with a new electrode design.

Longer term projects could include using LDA to investigate the influence of the oscillating bubble on structures and boundaries.
## Notation

Where a symbol has more than one meaning, the use in any place should be apparent from the context in which it is used.

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<td>speed of light</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of sound in the medium</td>
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<tr>
<td>$d$</td>
<td>distance from bubble centre</td>
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<tr>
<td>$d_p$</td>
<td>particle diameter</td>
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<tr>
<td>$d_{c-2}$</td>
<td>minimum laser beam diameter</td>
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<td>$f$</td>
<td>focal length of lens</td>
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<td>$g$</td>
<td>gravitational acceleration</td>
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<td>$g[n]$</td>
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<td>$h[n]$</td>
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<td>$i_F$</td>
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<td>$i(x)$</td>
<td>photocurrent in the $x$ direction</td>
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Chapter 9 — Discussion and conclusion

\( k \) wavenumber

\( n \) refractive index

\( n_0 \) refractive index without a strain field

\( opl \) optical path length

\( p \) photoelastic tensor

\( r \) bubble radius

\( r_{\text{max}} \) maximum bubble radius

\( r_o \) mean bubble radius

\( r_o \) laser beam radius

\( r_p \) particle radius

\( s \) scale

\( t \) time

\( u \) particle velocity

\( u \) \( x \) component of velocity

\( x[n] \) original data set

\( x' \) distance of propagation of laser beams along optical axis

\( y' \) fringe movement

\( y_{\text{amp}} \) amplitude of fringe movement

\( y[n] \) filter output

\( A \) acoustic wave
Chapter 9 — Discussion and conclusion

$A_m$ amplitude of acoustic wave

$C$ capacitance

$C_{sc}$ scattering cross section of particle

CWT continuous wavelet transform

$D$ fringe spacing factor

$E$ electric field

$E_{bubble}$ bubble energy

$E_{circuit}$ circuit energy

$E_o$ kinetic energy

$F$ Cunningham correction factor

$F_l$ lifting force

$I$ intensity

$M$ material parameter

$M$ constant introduced in laser beams are not of equal strength

$N_f$ number of fringes

$P$ pressure

$P_m$ peak pressure

$R$ bubble radius

$R$ resistance

$\Re$ real number

$R_o$ mean bubble radius
Chapter 9 — Discussion and conclusion

\( S \) surface area

\( \text{SNR} \) signal to noise ratio

\( T_m \) time between successive maxima

\( U \) flow velocity

\( U_{\text{shock}} \) shock wave velocity

\( U_{up} \) upward velocity of bubble due to buoyancy

\( V \) output voltage

\( W \) weight of explosive material

\( WD \) work done

\( Y(X) \) apparent motion of particle in fringes

\( \delta \) path difference

\( \epsilon_o \) strain amplitude

\( \epsilon_o \) permittivity of free space

\( \eta \) sensitivity of detector

\( \theta \) half angle between laser beams

\( \theta_d \) time constant of decay

\( \lambda \) wavelength

\( \mu \) viscosity

\( \nu \) frequency

\( \nu_D \) Doppler frequency
Chapter 9 — Discussion and conclusion

- \( \nu_s \): shift frequency
- \( \rho_p \): particle density
- \( \sigma \): acousto-optic coefficient
- \( \sigma_R \): Riemann variable
- \( \sigma_s \): scattering amplitude function
- \( \tau \): translation
- \( \tau_{\text{bubble}} \): half lifetime of bubble
- \( \tau_{\text{total}} \): total bubble lifetime
- \( \tau_{\text{vel}} \): velocity relaxation time
- \( \omega \): sound frequency
- \( \omega_o \): resonance frequency
- \( \phi \): phase
- \( \Delta H \): specific enthalpy
- \( \Delta x \): width of the fringe volume
- \( \Lambda \): fringe separation
Appendix A

Bubble Period and Radius

Obtained from LDA
Appendix A — Bubble Period and Radius Obtained from LDA

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<th>Distance (m)</th>
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Table A.1:  
Voltage = 12 kV, Spark Gap = 0.1 mm
Appendix A — Bubble Period and Radius Obtained from LDA

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**Table A.2:**
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Appendix A — Bubble Period and Radius Obtained from LDA

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Table A.3:
Voltage = 12 kV, Spark Gap = 0.3 mm
### Appendix A — Bubble Period and Radius Obtained from LDA

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**Table A.4:**
Voltage = 13 kV, Spark Gap = 0.1 mm

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Appendix A — Bubble Period and Radius Obtained from LDA

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Table A.5:
Voltage = 13 kV, Spark Gap = 0.2 mm
### Table A.6:

Voltage = 13 kV, Spark Gap = 0.3 mm
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**Table A.7:**
Voltage = 13 kV, Spark Gap = 0.4 mm
Appendix A — Bubble Period and Radius Obtained from LDA

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Table A.8: Voltage = 14 kV, Spark Gap = 0.1 mm
## Table A.9:
Voltage = 14 kV, Spark Gap = 0.2 mm
### Table A.10:
Voltage = 14 kV, Spark Gap = 0.3 mm

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Appendix A — Bubble Period and Radius Obtained from LDA

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Table A.11:
Voltage = 14 kV, Spark Gap = 0.4 mm
Appendix B

Velocity and Pressure Obtained from LDA
Appendix B — Velocity and Pressure Obtained from LDA

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Table B.1:
Voltage = 12 kV, Spark Gap = 0.1 mm
### Table B.2:
Voltage = 12 kV, Spark Gap = 0.2 mm

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### Appendix B — Velocity and Pressure Obtained from LDA

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Table B.3:
Voltage = 12 kV, Spark Gap = 0.3 mm
### Table B.4:
Voltage = 13 kV, Spark Gap = 0.1 mm
### Appendix B — Velocity and Pressure Obtained from LDA

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**Table B.5:**
Voltage = 13 kV, Spark Gap = 0.2 mm
Appendix B — Velocity and Pressure Obtained from LDA

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Table B.6:
Voltage = 13 kV, Spark Gap = 0.3 mm
### Appendix B — Velocity and Pressure Obtained from LDA

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<th>Pressure (Pa)</th>
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**Table B.7:**

Voltage = 13 kV, Spark Gap = 0.4 mm
Appendix B — Velocity and Pressure Obtained from LDA

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<th>Pressure</th>
<th>Pressure Error</th>
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<td>(m/s)</td>
<td>(Pa)</td>
<td>(Pa)</td>
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Table B.8:
Voltage = 14 kV, Spark Gap = 0.1 mm
### Appendix B — Velocity and Pressure Obtained from LDA

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<th>Pressure (Pa)</th>
<th>Pressure Error (Pa)</th>
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**Table B.9:**
Voltage = 14 kV, Spark Gap = 0.2 mm
### Table B.10:

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Voltage = 14 kV, Spark Gap = 0.3 mm
### Table B.11:
Voltage = 14 kV, Spark Gap = 0.4 mm
Appendix C

Bubble Energy Obtained from LDA
Appendix C — Bubble Energy Obtained from LDA

Table C.1:

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<th>Cavity / Circuit Error</th>
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Voltage = 12 kV, Spark Gap = 0.1 mm, Circuit Energy = 0.288 ± 0.000125 J
### Appendix C — Bubble Energy Obtained from LDA

Table C.2:

Voltage = 12 kV, Spark Gap = 0.2 mm, Circuit Energy = 0.288 ± 0.000125 J

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<th>Cavity / Circuit Error</th>
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Table C.2:
### Table C.3:
Voltage = 12 kV, Spark Gap = 0.3 mm, Circuit Energy = 0.288 ± 0.000125 J
### Appendix C — Bubble Energy Obtained from LDA

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<th>Cavity / Circuit Energy</th>
<th>Cavity / Circuit Error</th>
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**Table C.4:**

Voltage = 13 kV, Spark Gap = 0.1 mm, Circuit Energy = 0.338 ± 0.000125 J

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### Table C.5:

Voltage = 13 kV, Spark Gap = 0.2 mm, Circuit Energy = 0.338 ± 0.000125 J
### Appendix C — Bubble Energy Obtained from LDA

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**Table C.6:**

Voltage = 13 kV, Spark Gap = 0.3 mm, Circuit Energy = 0.338 ± 0.000125 J
### Appendix C — Bubble Energy Obtained from LDA

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<th>Distance (m)</th>
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<th>Cavity Error (J)</th>
<th>Cavity / Circuit Energy</th>
<th>Cavity / Circuit Error</th>
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| average     | 0.038209          | 0.007061        | 0.113044                | 0.020890               |

**Table C.7:**
Voltage = 13 kV, Spark Gap = 0.4 mm, Circuit Energy = 0.338 ± 0.000125 J

261
### Appendix C — Bubble Energy Obtained from LDA

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Table C.8: Voltage = 14 kV, Spark Gap = 0.1 mm, Circuit Energy = 0.392 ± 0.000125 J

262
### Table C.9:
Voltage = 14 kV, Spark Gap = 0.2 mm, Circuit Energy = 0.392 ± 0.000125 J
### Appendix C — Bubble Energy Obtained from LDA

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| average      | 0.024470          | 0.004253                | 0.062424              | 0.010851            |

**Table C.10:**

Voltage = 14 kV, Spark Gap = 0.3 mm, Circuit Energy = 0.392 ± 0.000125 J
Appendix C — Bubble Energy Obtained from LDA

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Cavity Energy (J)</th>
<th>Cavity Energy Error (J)</th>
<th>Cavity / Circuit Energy</th>
<th>Cavity / Circuit Error</th>
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Table C.11:
Voltage = 14 kV, Spark Gap = 0.4 mm, Circuit Energy = 0.392 ± 0.000125 J
Appendix D

Shock wave propagation

D.1 Kirkwood and Brinkley theory

Kirkwood and Brinkley developed a shock wave propagation theory for one-dimensional waves which holds for either plane, cylindrical or spherical waves [Kirkwood and Brinkley, 1945], [Brinkley and Kirkwood, 1947]. They wanted to be able to express each of the four derivatives:

\[
\frac{\delta P}{\delta t}, \frac{\delta P}{\delta R}, \frac{\delta u}{\delta t}, \frac{\delta u}{\delta R}
\]

and used them to formulate an ordinary differential equation for the peak pressure \( P \) of the shock wave as a function of the distance of propagation, \( R \) in terms of
Appendix D — Shock wave propagation

the pressure $P$ and the time $t$:

$$\frac{dP}{dt} = \delta P + \frac{1}{U} \frac{\delta P}{\delta t} \quad (D.1)$$

and to obtain the initial slope, $1/\theta$, of the Euler pressure-time curve of the shock wave:

$$\frac{1}{\theta} = \frac{1}{P} \frac{\delta P}{\delta t} - \frac{a \rho \delta P}{P \rho_0 \delta R} \quad (D.2)$$

They started by obtaining two conditions from the partial differential equations of hydrodynamics which are valid at any point behind the shock front:

$$\frac{R^2 \delta u}{r^2 \delta t} + \frac{1}{\rho_0} \frac{\delta P}{\delta R} = 0$$

$$\frac{\rho r^2 \delta u}{\rho_0 R^2 \delta R} + \frac{2u}{r} = -\frac{1}{\rho c^2} \frac{\delta P}{\delta t} \quad (D.3)$$

A third relation was obtained from the Rankine-Hugoniot condition for mass concentration at the shock front; the other conditions allow evaluation of the shock front density and velocity as functions only of the pressure $P$. The Rankine-Hugoniot conditions are:

$$P = \rho_0 uU$$

$$u = \left[ P \left( \frac{1}{\rho_0} - \frac{1}{\rho} \right) \right]^{1/2}$$

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Appendix D  —  Shock wave propagation

\[ \Delta H = \frac{P}{2} \left( \frac{1}{\rho_s} - \frac{1}{\rho} \right) \]  

(D.4)

where \( \Delta H \) is the specific enthalpy increment experienced by the fluid in traversing the shock front and \( U \) is the velocity of the shock front.

The derivative in which the shock front is stationary is given by:

\[ \frac{d}{dR} = \frac{\delta}{\delta R} + \frac{1}{U} \frac{\delta}{\delta t} \]  

(D.5)

Applying this operator to the first Hugoniot condition produces the Hugoniot relation:

\[ \frac{\delta u}{\delta t} + U \frac{\delta u}{\delta R} - \frac{g}{\rho_s U} \frac{\delta P}{\delta t} - \frac{g}{\rho_s} \frac{\delta P}{\delta R} = 0 \]  

(D.6)

where \( g = 1 - \rho_0 udU/dP \).

There are now three relations between four partial derivatives. In order to obtain the fourth relation, Kirkwood and Brinkley [Brinkley and Kirkwood, 1947] set a similarity restraint on the energy-time curve. The basis for this came from the fact that the non-acoustical decay of waves of finite amplitude is closely associated with the entropy increment experienced by the fluid in passing through the shock front and the accompanying dissipation of energy. As a shock front passes through a fluid it leaves in its path a residual internal energy increment in each element.
Appendix D — Shock wave propagation

of the fluid determined by the entropy increment produced in it by the passage of
the shock front. The energy propagated ahead of the shock front decreases with
distance from the source.

The total work done may be resolved into the sum of two terms: the increased
internal energy of the fluid at pressure \(P_o\) within a sphere of radius \(R\) and the
work done on this spherical surface. Thus:

\[
W_o = 4\pi \int_{a_o}^R \rho_o r_o^2 E[P_m(r_o)] dr_o + 4\pi \int_{t(R)}^\infty r^2(P_m + P_o)udt \quad (D.7)
\]

where \(E[P_m(r_o)]\) is the increase in internal energy per unit mass of fluid and the
time integral is carried out for the volume element initially at \(R\). The term involving
\(P_o\) in the time integral gives the product of \(P_o\) and the volume displacement
of the fluid element initially at \(R\). This displacement is the sum of the outward
volume displacement \(\Delta V\) of the inner boundary of the fluid and the displacement
of the volume of fluid initially between the shells of radii \(a_o\) and \(R\) to shells of
radii \(a', R'\). It can be written:

\[
4\pi \int_{t(R)}^\infty r^2(P_m + P_o)udt = P_o \Delta V + 4\pi P_o \left[ \int_{a_o}^{R'} r^2 dr - \int_{a_o}^R r^2 dr_o \right]
\]

\[
= P_o \Delta V + 4\pi P_o \int_{a_o}^R \left( \frac{\rho}{\rho_o} - 1 \right) r_o^2 dr_o \quad (D.8)
\]

which follows from the relation \(\rho r^2 dr = \rho_o r_o^2 dr_o\). Combining equations (D.7) and
(D.8), using \(h(p) = E + p_o \Delta(1/\rho)\) and assuming the time integral vanishes at

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Appendix D — Shock wave propagation

\( R = \infty \) produces:

\[
W_0 = 4\pi \int_{\infty}^\infty \rho_0 r^2 h(P(r)) \, dr + P_0 \Delta V
\]

(D.9)

where \( h(P) \) is the specific enthalpy increment of an element of fluid traversed by a shock wave of peak pressure \( P \) after return to pressure \( P_0 \) along its new adiabatic. The term \( P_0 \Delta V \) represents the energy stored in the water by expansion of the gas sphere.

Subtracting equation (D.9) from (D.7) produces:

\[
\int_t^\infty r^2 p' u' \, dt = \int_R^\infty \rho_0 r^2 h(p(r)) \, dr
\]

(D.10)

therefore:

\[
D(R) = \int_{\infty}^R r^2 u' p' \, dt
\]

(D.11)

where

\[
D(R) = \int_R^\infty \rho_0 r^2 h[p(r)] \, dr
\]

(D.12)

Integrating the second term of \( D(R) \) produces an expression in terms of \( p \) and \( R \):

\[
\frac{dD}{dR} = -\rho_0 R^2 h(P)
\]

(D.13)
Appendix D — Shock wave propagation

The benefit of using the energy flux-time curve is that for a given weight of explosive, its shape changes very little with increasing $R$. The value of $D(R)$ does depend on the strength of the source and it is necessary to normalise the time integral to a value which is independent of this factor. This is done by expressing the integrand as a fraction of its initial value of $R^2 Pu$ and choosing a reduced time scale for which the slope of the integrand has unit value. The energy-time curves are approximately of the form $R^2 Pue^{-t/\mu}$ and so it is convenient to use as the time unit the initial logarithmic slope of the curve defined by:

$$\frac{1}{\mu} = -\frac{\delta}{\delta t} \log r^2 Pu$$  \hspace{1cm} (D.14)

Integrating gives:

$$-\frac{1}{\mu} = \frac{1}{P} \frac{\delta P}{\delta t} + \frac{1}{u} \frac{\delta u}{\delta t} + \frac{2u}{R}$$  \hspace{1cm} (D.15)

and so:

$$D(R) = R^2 Pu \nu(R)/\mu$$  \hspace{1cm} (D.16)

where $\nu(R)$ is the normalised integral over reduced time $\tau$ expressed by:

$$\nu(R) = \int_{0}^{\infty} f(R, \tau) d\tau$$  \hspace{1cm} (D.17)
Appendix D — Shock wave propagation

where

\[ f(R, \tau) = \frac{r^2 P u}{R^2 P u} \]
\[ \tau = \frac{t - t_0(R)}{\mu} \]  
(D.18)

The value of the integral \( v(R) \) normalised is an expression of the shape of the shock wave, having a value of unity for an exponential decay and two-thirds for a linear decay. As the shock waves being investigated are in water, the decay is exponential and so the value of \( v \) can be set to unity. Kirkwood and Brinkley used this as their "similarity restraint" on the energy flux-time curves.

There are now four differential equations which can be solved for the four partial derivatives:

\[
\frac{1}{u} \frac{\delta u}{\delta t} + \frac{1}{P} \frac{\delta P}{\delta t} + \frac{2u}{R} = -\frac{R^2 P u}{D(R)} \\
\frac{\delta u}{\delta t} + \frac{1}{\rho_c} \frac{\delta P}{\delta R} = 0 \\
\frac{\rho}{\rho_c} \frac{\delta u}{\delta R} + \frac{1}{\rho c^2} \frac{\delta P}{\delta t} + \frac{2u}{R} = 0 \\
\frac{\delta u}{\delta t} + U \frac{\delta u}{\delta R} - \frac{g}{\rho_c} \frac{\delta P}{\delta R} - \frac{g}{\rho_c U} \frac{\delta P}{\delta t} = 0 
\]  
(D.19)

and hence an ordinary differential equation for \( P \) in terms of \( R \) can be obtained:

\[
\frac{dP}{dR} = \frac{\delta P}{\delta R} + \frac{1}{U} \frac{\delta P}{\delta t} 
\]  
(D.20)
Appendix D — Shock wave propagation

The function $D$ is expressed in terms of $P$:

$$ \frac{dD}{dR} = -\rho_0 R^2 h(P) $$

(D.21)

This pair of ordinary differential equations can therefore be solved by numerical integration. The two constants of integration can be determined in two ways: by the initial conditions at the boundary of the gas spheres following detonation or by the experimental pressure-time curve at a selected distance. The time variation of pressure behind the shock front as measured experimentally at constant $r$ is initially given by the derivative:

$$ \frac{1}{\theta} = - \left[ \frac{\delta}{\delta t} \log P \right] $$

(D.22)

corresponding to the peak approximation to the pressure-time curve $P(t) = P e^{-t/\theta}$. This becomes:

$$ \frac{1}{\theta} = \left[ \frac{\delta}{\delta t} \log P \right] - \left[ \frac{\delta}{\delta R} \log P \right] $$

(D.23)

D.2 Penney and Dasgupta theory

For the case of TNT, Penney [Penney, 1940] and then Penney and Dasgupta [Penney and Dasgupta, 1942] numerically solved the differential equations for fi-
Appendix D — Shock wave propagation

finite amplitude spherical waves which have the form:

\[
\frac{\delta \rho}{\delta t} + \frac{2\rho}{r} u + \rho \frac{\delta u}{\delta r} + u \frac{\delta \rho}{\delta r} = 0
\]

\[
\rho \frac{\delta u}{\delta t} + \rho u \frac{\delta u}{\delta r} + \frac{\delta P}{\delta r} = 0
\]

This was done using the Riemann variable, \( \sigma_R \), defined by:

\[
\sigma_R = \int_{\rho_o}^{\rho} c(\rho) \frac{d\rho}{\rho}
\]

(D.25)

where \( \rho_o \) is the density without a disturbance and \( c \) is the velocity of sound. The variables \( P \) and \( \rho \) in equation (D.24) are replaced by \( \sigma_R \) and \( c \) producing:

\[
\frac{\delta \sigma_R}{\delta t} + \frac{2\rho}{r} u + \rho \frac{\delta u}{\delta r} + u \frac{\delta \rho}{\delta r} = 0
\]

\[
\rho \frac{\delta u}{\delta t} + \rho u \frac{\delta u}{\delta r} + \frac{\delta P}{\delta r} = 0
\]

(D.26)

Adding and subtracting these produces the Riemann form of the equations:

\[
\frac{\delta}{\delta t}(\sigma_R + u) + (c + u) \frac{\delta}{\delta r}(\sigma_R + u) = -\frac{2cu}{r}
\]

\[
\frac{\delta}{\delta t}(\sigma_R - u) - (c - u) \frac{\delta}{\delta r}(\sigma_R - u) = -\frac{2cu}{r}
\]

(D.27)

Numerical integration of these can be carried out from prescribed conditions at a specific time. Considering a small time increment \( dt \) and letting \( N = (\sigma_R + u)/2 \)
and $Q = (\sigma_R - u)/2$:

\[
\begin{align*}
\frac{dN}{dt} &= \frac{\delta N}{\delta t} + \frac{\delta N}{\delta r} dr \\
&= -\frac{cu}{r}dt + [dr - (c + u)dt] \frac{\delta N}{\delta r} \\
\frac{dQ}{dt} &= \frac{\delta Q}{\delta t} + \frac{\delta Q}{\delta r} dr \\
&= -\frac{cu}{r}dt + [dr + (c - u)dt] \frac{\delta Q}{\delta r}
\end{align*}
\] (D.28)

Taking the changes $dr$ for a given $dt$ to be $(c + u)dt$ and $-(c - u)dt$ for the two functions produces:

\[
\begin{align*}
\frac{dN}{dt} &= -\frac{cu}{r} dt, \quad dr = (c + u)dt \\
\frac{dQ}{dt} &= -\frac{cu}{r} dt, \quad dr = -(c - u)dt
\end{align*}
\] (D.29)

Calculation of $dN$ and $dQ$ for a series of values of $u$ and $r$ at time $t$ produces values at a time $dt$ later. Repetition allows the solution to be built up to any desired time. At each stage, the value of particle velocity $u$ is obtained from $(N - Q)$ and the pressure determined from $(N + Q)$. Penney and Dasgupta's method resulted in:

\[
P_m(\text{lb/in}^2) = 14000 \frac{W^{1/3}}{R} \exp(0.274W^{1/3}/R)
\] (D.30)
Appendix D — Shock wave propagation

where $R$ is the shock front radius in feet and $W$ is the weight of explosive material in pounds.
Appendix E

Water analysis

This water analysis was received from the Environment and Quality Report 1998/1999 of East of Scotland water \(^1\) for the Newington area of Edinburgh, zone ED3 7003. It was water from this area which was used in the experiments detailed within this thesis.

\(^1\)http://www.esw.co.uk
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Bibliography


Publications

S. H. Jack, D. B. Hann, C. A. Greated
Acousto-Optic Effect on Laser Doppler Anemometry Systems
IMechE International Conference on Optical Methods and Data Processing in Heat and Fluid Flow, City University, London, April 16-17 1998

S. H. Jack, D. B. Hann, C. A. Greated
The Influence of the Acousto-Optic Effect on Laser Doppler Anemometry Signals

R. I. Crickmore, S. H. Jack, D. B. Hann, C. A. Greated
Laser Doppler Anemometry and the Acousto-Optic Effect

S. H. Jack, D. B. Hann, C. A. Greated
The influence of a standing wave on laser Doppler signals
Measurement Science and Technology, vol. 10, 1999

