DESIGN AND APPLICATIONS OF FOURIER TRANSFORM PROCESSORS USING SURFACE ACOUSTIC WAVE AND CHARGE COUPLED DEVICES.

A thesis submitted to the Faculty of Science of the University of Edinburgh, for the degree of Doctor of Philosophy

by

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DESIGN AND APPLICATIONS OF FOURIER TRANSFORM PROCESSORS USING
SURFACE ACOUSTIC WAVE AND CHARGE COUPLED DEVICES.

The current availability of analogue surface acoustic wave (SAW) and charge coupled devices (CCD) permits the hardware realisation of real time Fourier transform processors as an alternative to use of the digital fast Fourier transform (FFT). This thesis demonstrates how such analogue Fourier transform processors have been designed, developed and applied to engineering systems since the initial work in the years 1974-1975.

A rigorous mathematical analysis of the operation of the SAW (chirp) Fourier transform is presented. This demonstrates that recovery of transform components at baseband demands high tolerance components and circuit design. However, by holding the outputs on a carrier, considerable hardware simplification is possible.

Specific applications in spectrum analysis, cepstrum analysis and signal correlation are considered since these permit operation with the transform components on a carrier.

In addition, the design and application of CCD Fourier transform processors, based on the chirp-z-transform and prime transform algorithms is presented. The performance of these analogue processors is compared to that of the digital FFT. This work leads directly to consideration of the design and application of Fourier transform processors which incorporate combinations of SAW and CCD devices. Experimental results are included throughout to demonstrate the operation of the systems discussed in the thesis.
DECLARATION OF ORIGINALITY.

This thesis, composed entirely by myself, reports work conducted in the Department of Electrical Engineering at the University of Edinburgh exclusively by myself as part of Science Research Council grant B/RG/6550.9.
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LIST OF AUTHOR'S PUBLICATIONS
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GLOSSARY OF TERMS.

B_{opt}  Optimum IDT bandwidth (SAW)
CCD   Charge coupled device
C-M-C Convolve-multiply-convolve (SAW transform)
CTD Charge transfer device
CW   Continuous wave
CZT Chirp-z-transform (algorithm)
D/A Digital-to-analogue (conversion)
DFT Discrete Fourier transform
dB Decibel
DCPT Discrete cosine prime transform (algorithm)
ECM Electronic countermeasures.
FFT Fast Fourier transform (algorithm)
FH Frequency hopped
FILO First-in-last-out
FIR Finite impulse response (filter)
FM Frequency modulation
FT Fourier transform
IDT Interdigital (SAW) transducer
IF Intermediate frequency
k^2 Coupling coefficient (SAW substrate)
MDAC Multiplying digital-to-analogue (converter)
MOS Metal oxide silicon (semiconductor)
MEM Maximum entropy method (spectrum analysis)
M-C-M Multiply-convolve-multiply (SAW transform)
RAC Reflective (SAW) array compressor
ROM Read-only-memory
SAW Surface acoustic wave
SNR Signal-to-noise (ratio)
TB Time-bandwidth (product)
TDM Time domain metrology
V_{gg} CCD reset gate voltage
W_{50} Optimum SAW IDT aperture (50Ω)
CHAPTER 1: INTRODUCTION

1.1 ANALOGUE SIGNAL PROCESSING USING CCD AND SAW DEVICES

Finite impulse response (FIR) analogue transversal filters, realisable in both surface acoustic wave (SAW) and charge coupled device (CCD) technologies play an increasingly important role in the area of analogue signal processing. The CCD is intrinsically a baseband device which has variable signal propagation delay controlled by an external clock waveform. In contrast, the SAW device has a bandpass frequency characteristic and a fixed signal propagation delay determined by substrate properties.

Since the CCD is a baseband device it can process only real signal values whereas the bandpass SAW device which possesses a signal phase reference relative to the carrier phase permits simultaneous processing of complex (amplitude and phase) signal information. This point will be developed to a large extent throughout this thesis.

SAW and CCD analogue transversal filters form the basic signal processing elements considered in this thesis which demonstrates how they may be configured to realise real time Fourier transform processors.

The FFT algorithm, developed in 1965, which permits computation of the Fourier transform using a digital computer instigated the use of the Fourier transform as a signal processing operation. Such
transform signal processing techniques have been increasingly applied with the recent development of several alternative algorithms which now permit the Fourier transform to be computed using analogue CCD and SAW devices.

The aim of this thesis is to demonstrate how analogue Fourier transform processors using CCD and SAW devices have been designed, developed and applied to engineering systems since the initial work in the years 1974-75. The coincidence of the start of this thesis with these initial developments permits the original material presented here to be accurately collated with developments in the field by other workers.

In broad terms the thesis shows that, although Fourier transform processors using CCD and SAW devices have been widely applied to systems, these analogue processors exhibit distinct performance limitations. In specific applications, however, such as airborne radar and torpedo-borne sonar, which demand low power consumption coupled with small physical size, the CCD and SAW processors have found immediate application. In addition, the complementary nature of CCD and SAW devices permits direct interfacing of these technologies to realise more sophisticated transform signal processing functions.

The fact remains, however, that the more versatile, higher accuracy digital FFT processor can perform any of the functions of the analogue CCD and SAW Fourier transform processors - at the expense of increased power consumption, volume, weight and component cost.
1.2 LAYOUT OF THESIS

The basic principles of CCD and SAW devices introduced in section 1.1 are expanded in chapter 2, with details on device design, materials and fabrication. This material cross-references the many excellent review and tutorial papers on these subjects and considers, in detail, the SAW linear FM, or chirp, filter since this device is concerned with many of the signal processing functions discussed in this thesis.

Chapter 3 provides a concise introduction to the various algorithms and techniques which exist for computation of the Fourier transform. These concepts are considered in detail in chapter 4 which presents a rigorous mathematical analysis of Fourier transform techniques using SAW chirp filters - the chirp transform. The original analysis contained in chapter 4 was developed in conjunction with Professor E G S Paige, Oxford University.

In chapter 5, the practical design of SAW chirp transform processors is expanded, and several signal processing applications are discussed with section 5.2, which considers real time network analysis using SAW devices, being original work. In contrast to these applications, which require a single SAW transform processor, chapter 6 considers the applications of combined chirp transform processors. Here section 6.1, which details cepstrum analysis techniques using SAW devices, and section 6.3, which considers the SAW chirp transform memory correlator, are original work.
In chapter 7 which considers the design of Fourier transform processors based on CCD devices, two algorithms, the chirp-z-transform and the prime transform, are developed and discussed. Section 7.2 includes details of the author's work with the CCD prime transform and section 7.3 compares and contrasts results of a computer simulation of processor performance with previously published work. Chapter 7 also includes a comparison of analogue CCD transform processors with the digital FFT.

Techniques and systems applications which exist for combined CCD and SAW Fourier transform processors are discussed in chapter 8. Here, section 8.2.4, which considers time compression beamforming techniques for sonar applications is original work. Chapter 8 represents the areas of development which are currently being investigated at the time of submission of this thesis, and in conjunction with the conclusions of chapter 9, offers scope for further extensive work in the field of CCD and SAW Fourier transform processors.
CHAPTER 2 : SAW AND CCD DEVICES

2.1 SURFACE ACOUSTIC WAVE DEVICES

2.1.1 Basic Principles

The surface acoustic wave (SAW) is a longitudinal wave which can propagate on the surface of elastic solids with a velocity some five orders of magnitude lower than the velocity of electromagnetic waves. Also known as a Rayleigh wave\(^{(1)}\), the surface acoustic wave is non-dispersive and most of the wave energy (usually more than 95%) is confined within a depth equal to one wavelength. In piezoelectric materials, the surface strain produced by the acoustic wave is associated with an induced electric field and, since the wave energy is confined near the surface, the wave can be sampled, or otherwise modified, while it is propagating. Practical SAW devices became possible with the development of the interdigital transducer\(^{(2)}\) which permits efficient transduction of electrical and acoustic energy and which forms the basis of SAW devices which have application in sophisticated signal processing\(^{(3,4)}\).

The interdigital transducer (IDT) consists of a set of interleaved electrodes made from a metal film deposited on a piezoelectric substrate. In its simplest form, the width of the inter-electrode gap is equal to the width of the metallised electrode and is constant over the whole transducer, figure 2.1.1. Electrical excitation of the transducer produces, through the
piezoelectric effect, a strain pattern on the substrate surface of periodicity equal to the transducer electrode periodicity. At resonance, there is a strong coupling from electrical into acoustic wave energy and two surface acoustic waves, figure 2.1.1, are launched in opposite directions normal to the IDT. The unwanted surface waves generated by the IDT can either be absorbed at the ends of the crystal, by means of wax or adhesive tape, or they may be reversed in direction in a unidirectional IDT design. By suitable choice of piezoelectric substrate material and direction of propagation, table 2.1.1, it is possible to minimise the attenuation and diffraction of the wave.

The impedance of the transducer as seen at the electrical input can be represented by a series equivalent circuit. In the device passband, the transducer can be electrically matched by an inductor to cancel the reactive component so that all electrical energy is converted to acoustic energy. The value of electrode overlap required to make the resonant impedance of the device equal to 50Ω ($W_{50}$) can be easily achieved in practice, see table 2.1.1.

The insertion loss of the SAW device can be minimised by increasing the number of electrodes in the IDT. However, this has the effect of narrowing the device bandwidth and in practice an optimum value ($N_0$) is chosen which provides maximum bandwidth for minimum insertion loss at the device centre frequency. Full details of SAW device design can be found in published reviews.
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<th>CUT</th>
<th>VELOCITY (km/s)</th>
<th>$k^2$ (%)</th>
<th>$N_o$</th>
<th>$W_{50}$</th>
<th>$B_{opt}$ (%)</th>
<th>TEMP. COEFF. (ppm/°C)</th>
<th>ATTENUATION. (dB/μs)</th>
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<td>LiNbO$_3$</td>
<td>Y,Z</td>
<td>3.458</td>
<td>4.3</td>
<td>4</td>
<td>108</td>
<td>22</td>
<td>85</td>
<td>1.6 (1GHz)</td>
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<td>Quartz</td>
<td>ST,X</td>
<td>3.158</td>
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<td>4.5</td>
<td>3</td>
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<td>Bi$<em>{12}$GeO$</em>{20}$</td>
<td>(100),(011)</td>
<td>1.681</td>
<td>1.2</td>
<td>8</td>
<td>26</td>
<td>14</td>
<td>122</td>
<td>1.5 (1GHz)</td>
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<td>4</td>
<td>-</td>
<td>23</td>
<td>-</td>
<td>5.0 (40MHz)</td>
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<tr>
<td>AIN/A$_2$O$_3$</td>
<td>X,Z</td>
<td>6.170</td>
<td>0.63</td>
<td>11</td>
<td>60</td>
<td>10</td>
<td>40</td>
<td>1.7 (200MHz)</td>
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Table 2.1.1 Properties of SAW substrate materials
Non-uniform IDT structures where either the electrode periodicity, or the aperture - or both - are allowed to vary, form the basis of many SAW devices\(^{(11,12)}\). Such devices belong to the general class of transversal filters\(^{(13,14)}\) as discussed in section 1.1. Figure 2.1.2 shows schematically a SAW dispersive delay line\(^{(11)}\) or "chirp" filter. Here, the transducer electrode periodicity varies linearly along the structure and the resulting impulse response is a swept frequency (chirp) pulse. If required, the impulse response of the chirp filter can be amplitude weighted by varying the electrode overlap along the device.

A range of piezoelectric substrate materials is available for SAW device fabrication\(^{(15)}\). The choice of material depends on the type of device being made, frequency of operation, bandwidth, time delay and operating conditions. Trade-offs exist in respect of strong piezoelectric coupling which permits high percentage bandwidth device operation - and low acoustic velocity for long signal time delay. Other factors include temperature coefficient of velocity, spurious signal rejection, acoustic loss and substrate cost. Table 2.1.1 presents the parameters of some of the available substrate materials and details optimum uniform IDT design considerations. Techniques and materials for SAW device fabrication have been widely reviewed\(^{(10,16,17,18)}\).

2.1.2 The SAW Chirp Filter

As an example of SAW device design and performance, it is
Figure 2.1.1 Basic SAW delay line

Figure 2.1.2 SAW dispersive delay line
(Single dispersive, in-line geometry)
useful to consider the design of the SAW chirp or linear dispersive filter\(^{(19)}\) which is a delay line whose group delay is a function of the instantaneous frequency of the input signal and whose amplitude characteristics are shaped for some specific application. In particular, the linear dispersive filter is designed to have a linear group delay vs instantaneous frequency characteristic. The primary application of such dispersive filters is in long range, high resolution radar systems for the detection, identification and tracking of high speed airborne targets\(^{(20,21,22)}\). The linear FM dispersive filter is by far the most highly developed SAW device and is widely used due to the simplicity of filter design and the inherent tolerance of the linear FM waveform to Doppler-shifted radar returns. Detecting and tracking high speed airborne targets requires\(^{(23)}\) a range resolution of 1-2m and a velocity resolution of the order of 1 kms\(^{-1}\). For a typical 3 GHz (\(\lambda = 10\) cm) radar the above resolutions require a chirp pulse of duration 50 \(\mu\)s and swept bandwidth 100 MHz.

The impulse response of a linear FM waveform of centre frequency \(f_0\), pulse length \(T\) and bandwidth \(B\) is given by\(^{(19)}\)

\[
h(t) = \cos \{2\pi(f_0 t + B/T \cdot t^2/2)\}, \quad -T/2 \leq t \leq T/2
\]

(2.1.1)

From this equation several important design limitations can be seen. The pulse length \(T\) of a single filter is limited by the length of piezoelectric crystal (typically 50 \(\mu\)s) and the centre frequency \(f_0\) is limited to about 300 MHz by current photolithography.
In order to implement the SAW filter, it is necessary to employ a scheme where two transducers are used to produce the desired frequency response. One method, the single dispersive design, figure 2.1.2, employs an unapodised, non-dispersive transducer whose spectral response is constant over the desired bandwidth $B$, and the other transducer has the desired overall transfer function of the filter. The main disadvantage of this configuration is the inherent bandwidth limitation of the non-dispersive transducer (4% for quartz). By using a double dispersive design\(^{(19)}\) - figure 2.1.3(a) - wideband operation can be achieved by dividing the differential time delay $T$ between two identical transducers of bandwidth $B$. The fractional bandwidth of the device must be less than 100% to prevent low frequency electrodes (synchronous at frequency $f_0/2$) from generating interfering third harmonic signals at frequency $3f_0/2$.

Consideration must be made of several interactive and distortive effects\(^{(19)}\), such as surface wave-to-bulk wave mode conversion, which can be eliminated by ensuring that high frequency SAW waves do not propagate under long period electrodes as would happen in an up-chirp filter. This can readily be ensured by using an inclined transducer geometry\(^{(19)}\), figure 2.1.3(b) where the effects of SAW-SAW conversion produced at electrode edges can be minimised by using low $k^2$ materials (quartz), table 2.1.1, or by using split electrode IDT geometries\(^{(24)}\). Here again the inclined geometry minimises the number of electrodes in the wavefront path, reducing the generation of spurious signals.
Figure 2.1.3  SAW chirp filter geometries

(a) Double dispersive, in-line geometry

(b) Double dispersive, inclined geometry
Reciprocal ripple designs of SAW chirp filters\(^{(25,26)}\) permit the realisation of low compression sidelobe levels\(^{(25)}\), for low time bandwidth product devices. These employ a compensating ripple in the compressor spectrum which corresponds to the Fresnel ripple in the expander spectrum. In this way sidelobe levels some 14 dB lower than produced by normal device design have been achieved.

A further technique which permits the designer to eliminate the 1 dB compression loss experienced with a Hamming weighted compressor is termed "non-linear chirp"\(^{(27)}\). Here, the amplitude responses of expander and compressor are flat, however, the phase responses of the two filters are conjugate with Hamming phase weighting and reciprocal ripple incorporated.

Two alternative methods to the non-uniform metallised electrode IDT design, so far considered, have recently been developed in the USA for the fabrication of SAW chirp filters. The first of these employs a reflective array of grooves with graded periodicity to produce a reflective array compressor or RAC\(^{(28,29)}\). The main advantage of the RAC structure lies in the fact that second order effects such as mode conversion and tap reflections present in metallised IDT designs, are eliminated. In the RAC, the reflective elements are made as shallow etched grooves, which reflect the propagation direction through approximately \(45^\circ\) and a second symmetrically placed mirror image grating sends the wave to the output transducer, figure 2.1.4(a).
(a) Reflective array geometry

(b) Reflective dot array geometry

Figure 2.1.4 Alternative implementations of SAW chirp filters
The etched grooves introduce no propagation loss and negligible dispersion and because the reflectivity of shallow grooves is linear with groove depth, accurate amplitude weighting can be incorporated.

The RAC device is etched by means of ion-beam sputter etching\(^{(18)}\) and RAC devices with dispersive delay of 40 μs and bandwidth 250 MHz have been fabricated\(^{(29)}\).

Although attempts were made to realise RAC geometries using metallised strip reflectors, thereby eliminating the need for sophisticated ion-beam etching, this technique is limited by the fact that reflectivity from the strips cannot be varied in a simple controllable manner. However, a technique for producing a metallised RAC called the reflective dot array has been recently developed\(^{(30)}\). Here the reflecting grooves of the RAC are replaced by a row of metallic dots, figure 2.1.4(b). Reflections from each dot create a nearly circularly symmetrical wavefront, part of which intersects an interdigital transducer, and the response from a given row is proportional to the number of dots in the row.

2.2 **CHARGE COUPLED DEVICES**

The charge coupled device (CCD), first demonstrated in 1970\(^{(31,32)}\) is an analogue shift register which is capable of storing and transferring charge packets. The CCD is fabricated by means of standard MOS technologies\(^{(33)}\) and for this reason,
the development of the CCD was rapid since the new designs could be realised on existing production lines. The CCD is a member of the charge transfer device (CTD) family which also includes MOS bucket brigade devices\(^{(34)}\) and buried channel, or peristaltic CCD\(^{(35)}\). The BBD is more simply made using standard MOS processing whilst the CCD requires a more sophisticated technology to achieve the required performance. It is for this reason that the BBD is presently more readily available than the CCD. The peristaltic CCD has been developed primarily for high speed shift register applications\(^{(34)}\).

The CCD can be considered to be a multi-gate MOS transistor, figure 2.2.1. When positive voltages greater than the threshold voltage (~0.6 V) are applied to the gate electrodes, a depletion region whose depth is dependent on the gate voltage is formed. This depletion region can be regarded as a potential well\(^{(36)}\). A three phase clocking scheme is normally used to control charge transport within the device, figure 2.2.1. A charge packet located in a potential well under a \(\phi_1\) electrode will transfer into the potential well under the adjacent \(\phi_2\) electrode as the voltage on \(\phi_1\) falls. Normally, for efficient charge transfer, the phases are operated with overlap. Other electrode arrangements are widely discussed in the literature\(^{(34)}\).

The transfer of charge from one potential well to the next is non-ideal, and a small fraction (\(\epsilon\)) of the signal charge is left behind. The effect of charge transfer inefficiency (\(\epsilon\)) is to produce a reduction in device bandwidth\(^{(37)}\), figure 2.2.2. For a
Figure 2.2.1 Basic CCD using three phase clock electrodes
N stage CCD, the product $N_e$ provides a measure of device performance. The effects of charge transfer inefficiency become significant at high clock rates\(^{(34)}\), figure 2.2.3.

Other device imperfections limit the CCD performance. A small amount of charge recombination takes place, however, in most applications, the net attenuation along the device is negligible. Several noise sources are present in the CCD, however, the device noise performance is normally limited by peripheral circuitry. Clock breakthrough, caused by the proximity of the transfer electrodes and the high voltage (30 V) clock signals, must be removed at the output by means of sample-and-hold circuits. The low frequency behaviour of the CCD is limited by leakage current (dark current), generated during the time when the charge packet is within the device. This restricts the CCD storage time to less than 1 s. Much longer times can be achieved by cooling the device.

Various techniques for charge input to the CCD have been developed\(^{(34,38,39)}\) and these are summarised in table 2.2.1.

Two important signal detection techniques have been developed for charge output sensing in CCD transversal filter applications. These are the split-electrode-gate\(^{(39)}\) and the floating-gate-reset technique\(^{(40,41)}\). The split-electrode weighting technique is illustrated in figure 2.2.4(a) where the signal charge is detected on the $\phi_2$ clock lines by a differential current integrator. The signal charge is determined by integrating the current flow in
Figure 2.2.2 CCD signal bandwidth as a function of charge transfer inefficiency.

Figure 2.2.3 CCD charge transfer inefficiency ($\epsilon$) as a function of clock frequency.
<table>
<thead>
<tr>
<th>TECHNIQUE</th>
<th>ADVANTAGES</th>
<th>DISADVANTAGES</th>
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</thead>
<tbody>
<tr>
<td>DYNAMIC CURRENT INJECTION&lt;sup&gt;(34)&lt;/sup&gt;</td>
<td>BASIC</td>
<td>POOR SIGNAL LINEARITY</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FREQUENCY DEPENDENT</td>
</tr>
<tr>
<td>DIODE CUT-OFF&lt;sup&gt;(34)&lt;/sup&gt;</td>
<td>IMPROVED LINEARITY</td>
<td>SIGNAL NON-N-LINEARITY</td>
</tr>
<tr>
<td></td>
<td>HIGHER STABILITY</td>
<td>STILL EXISTS</td>
</tr>
<tr>
<td>FILL-AND-SPILL&lt;sup&gt;(34)&lt;/sup&gt;</td>
<td>LINEAR CHARGE INPUT</td>
<td>INPUT SAMPLE-AND-HOLD REQUIRED</td>
</tr>
<tr>
<td>FEEDBACK LINEARISATION&lt;sup&gt;(38)&lt;/sup&gt;</td>
<td>IMPROVED SIGNAL LINEARITY AND GAIN STABILITY</td>
<td>REQUIRES OPERATIONAL AMPLIFIER (&lt;10MHz)</td>
</tr>
</tbody>
</table>

Table 2.2.1 CCD charge input techniques
each $\phi_2$ clock line during each charge transfer. Amplitude weighting the output of each CCD tap is achieved by splitting the $\phi_2$ electrode and integrating and subtracting the currents flowing in each section. This weighting scheme is readily implemented on very long (800 stage) CCD delay line transversal filters.

The floating-gate-reset tap output arrangement permits the implementation of more than one absolute tap weight per tap output. Tap weights are not generally included on-chip and must be added by external circuitry. Each tap output therefore demands one package pin and so this method is limited to devices with less than 50 taps. The technique does, however, offer the attractive possibility of electrically programmable tap weighting coefficients\(^{(42)}\). In figure 2.2.4(b) which illustrates the floating-gate-reset tapping method, when a charge packet appears under the sense electrode, the electrode voltage changes and this voltage change is transmitted to an output terminal by an MOST transistor connected in either the common-source or common-drain configuration. A voltage $V_{gg} \approx 15$ V is applied by a reset transistor, during $\phi_1$, to establish a reference level for subsequent charge sensing operations.

In addition to transversal filter and analogue delay applications, CCD devices can perform imaging and memory functions\(^{(43)}\).
Figure 2.2.4 CCD charge sensing techniques.
2.3 COMPARISON OF CCD AND SAW DEVICES

Section 1.1 outlined the basic points of comparison between CCD and SAW devices. However, several, more detailed comparisons can be drawn, table 2.3.1. The two technologies can be considered to be complementary since at frequencies between 1 MHz and 10 MHz CCD and SAW offer mutual operation. Further, the linear dynamic range of both technologies is comparable, in the range 40-60 dB. In the CCD this is limited by the sensitivity of the tapping structure whereas in the SAW device it is limited by spurious excited waves.

The SAW device which offers large time bandwidth processing by virtue of its wide bandwidth possesses a significant temperature coefficient of delay\(^{10}\). In contrast the CCD, where large time bandwidth processing is achieved by virtue of long signal delay, exhibits a temperature dependence dominated by the stability of the clock source.

For signal processing applications, CCD and SAW devices tend naturally to be used in distinct regions of the frequency spectrum, figure 2.3.1. CCD devices have impacted video or post detection signal processing whilst SAW devices have impacted IF or pre-detection signal processing\(^{44}\). More detailed applications of the technologies have been widely reported\(^{45,46}\) and these are summarised in table 2.3.2.
<table>
<thead>
<tr>
<th>SIGNAL</th>
<th>S A W</th>
<th>C C D</th>
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<tbody>
<tr>
<td>SIGNAL</td>
<td>SAMPLED ANALOGUE OR CONTINUOUS ANALOGUE OR DIGITAL REAL VALUE SIGNALS</td>
<td>SAMPLED ANALOGUE OR DIGITAL COMPLEX SIGNALS (PHASE REFERENCE)</td>
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<tr>
<td>FREQUENCY RESPONSE</td>
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<td>BASEBAND</td>
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<td>BANDWIDTH DETERMINED BY</td>
<td>TRANSUDER DESIGN</td>
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<td>CLOCK RATE</td>
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<tr>
<td>TYPICAL DELAY</td>
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<td>LESS THAN 10 MHz</td>
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<tr>
<td>TIME-BANDWIDTH PRODUCT</td>
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<td>LESS THAN 100 ms</td>
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<tr>
<td>TRANSFER RATE</td>
<td>FIXED BY MATERIAL</td>
<td>UP TO 800</td>
</tr>
<tr>
<td>SIGNAL TRANSFER</td>
<td>BI-DIRECTIONAL</td>
<td>CLOCK VARIABLE</td>
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<td>OUTPUT TAPPING</td>
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<td>NON-LINEARITY</td>
<td>DIFFRACTION AND ATTENUATION</td>
<td>NON-DESTRUCTIVE</td>
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<td>TEMPERATURE COEFFICIENT OF DELAY</td>
<td>CHARGE TRANSFER INEFFICIENCY</td>
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<td>PASSIVE</td>
<td>DARK CURRENT</td>
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<td>POWER CONSUMPTION</td>
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<td>100 mW</td>
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Table 2.3.1  Main Features of SAW and CCD Devices
Figure 2.3.1 CCD and SAW device time-bandwidth comparison
<table>
<thead>
<tr>
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<th><strong>CCD</strong></th>
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<tr>
<td><strong>RADAR</strong></td>
<td><strong>MTI FILTER</strong></td>
</tr>
<tr>
<td>PULSE COMPRESSION</td>
<td>DOPPLER FILTER</td>
</tr>
<tr>
<td>BEAM FORMING</td>
<td>SIGNAL INTEGRATION</td>
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<td>COMPRESSIVE RECEIVER</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>SONAR</strong></td>
</tr>
<tr>
<td></td>
<td><strong>SONOBUOY</strong></td>
</tr>
<tr>
<td></td>
<td>BEAM FORMING</td>
</tr>
<tr>
<td></td>
<td>BEAM STEERING</td>
</tr>
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<td><strong>INSTRUMENTATION</strong></td>
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<td>SPECTRUM ANALYSIS</td>
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<td>TIME COMPRESSION OF SPEECH</td>
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<td>TV IMAGE CODING</td>
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**Table 2.3.2**  Signal Processing Applications of CCD and SAW Devices
CHAPTER 3: FOURIER TRANSFORM PROCESSORS

3.1 INTRODUCTION TO FOURIER TRANSFORMS AND FOURIER TRANSFORM PROCESSORS

In systems analysis, the Laplace transform, $F(s)$, is used to map a continuous time function $f(t)$ into the complex s-plane by means of the relationship

$$F(s) = \int_{-\infty}^{+\infty} f(t) \cdot \exp(-st)dt \quad (3.1.1)$$

where $s = \sigma + j\omega$

The $j\omega$ axis of the s-plane corresponds to the frequency axis and values of the Laplace transform on this axis correspond to values of the Fourier transform. The Fourier transform, $F(\omega)$, defined as $^{(47,48)}$

$$F(\omega) = \int_{-\infty}^{+\infty} f(t) \cdot \exp(-j\omega t)dt \quad (3.1.2)$$

identifies or distinguishes the different frequency components which combine to form an arbitrary continuous time waveform $f(t)$.

The z-transform $^{(49)}$ performs the same function on a sampled data record as the Laplace transform performs on a continuous function. For a sampled data function with sample values $x_n$, $1 \leq n \leq N$, the z-transform is defined
\[ X(z) = \sum_{n=0}^{N-1} x_n \cdot z^{-n} \]  \hspace{1cm} (3.1.3)

where \( z = \exp(-sT) \)  \hspace{1cm} (3.1.4)

and \( T \) is the data sample interval. The unit circle in
the \( z \)-plane corresponds to the \( j\omega \) axis in the \( s \)-plane and
the discrete Fourier transform (DFT) is defined \( ^{(50)} \)

\[ X(m) = \sum_{n=0}^{N-1} x_n \cdot \exp(-j2\pi mn/N) \quad 0 \leq m \leq N-1 \]  \hspace{1cm} (3.1.5)

A linear transformation such as equation (3.1.5) which maps a
sequence of \( N \) data points into a sequence of \( N \) transform
points may be regarded as a multiplication of the data vector
by a square matrix of order \( N \). A transform processor
implementation which employs a single multiplier element
requires \( N \times N \) multiplication times plus associate shift and
add operations. The delay line time compressor (DELTIC)
configuration \( ^{(51,52)} \) permits fast computation of the DFT
power spectrum. Here the data is time compressed, or
expanded in bandwidth, by a factor \( N \) to permit a single, fast
multiplier element to perform the required \( N \times N \) multiplications
in a time equal to \( N \) input sample periods. However, the
multiplier element must operate at a rate of \( N \) times the input
sample rate. Thus it is only applicable to narrowband analysis.

Algorithms have been developed \( ^{(53)} \) for the efficient
computation of the \( N \) point DFT in \( N \log_2 N \) multiplication times,
for \( N \) a power of 2, or \( 2 \sum m_i \) multiplication times if the integers \( m_i \) are the prime factors of \( N \). Any algorithm which achieves such a reduction in computational time is called a fast Fourier transform (FFT), however the term FFT is now popularly regarded\(^{50,54}\) as being the Cooley-Tukey algorithm\(^{55}\) (decimation-in-time) of radix 2. The constraint on the use of the FFT is that the number of transform points, \( N \), must be a power of 2. However, the reduction in DFT computation time possible with the FFT, figure 3.1.1, has permitted extensive application of the algorithm\(^{56}\).

The development\(^{57}\) of the chirp-z-transform (CZT) algorithm yielded a further dimension in the calculation of the DFT. The CZT is derived from equation (3.1.5) by using the substitution

\[
2mn = m^2 + n^2 - (m - n)^2 \tag{3.1.6}
\]

to form

\[
X(m) = \exp(-j\pi m^2/N) x
\]

\[
\sum_{n=0}^{N-1} \{x_n \cdot \exp(-j\pi n^2/N)\} \exp(j\pi(m - n)^2/N) \tag{3.1.7}
\]

This algorithm is called the chirp-z-transform because, for increasing \( m \), a function of the form \( \exp(-j\pi m^2/N) \) represents a chirp\(^{58}\).
The CZT algorithm thus converts the calculation of the Fourier transform into a pre-multiplication by a discrete (complex value) chirp function, followed by convolution by a discrete chirp and subsequent post-multiplication by a discrete chirp. Further, the CZT permits direct computation of the DFT by transversal filter techniques\(^{(59,60)}\) using SAW\(^{(61,62)}\) and CCD\(^{(63,64)}\) devices.

The CZT implemented by transversal filter techniques permits computation of the DFT in a time proportional to \(N\), the number of transform points, in comparison to \(N/2 \cdot \log_2 N\), the processing time for the FFT, figure 3.1.1. The ability to compute an \(N\) point DFT in a time proportional to \(N\) is termed real time operation. The term "real time" processing has been loosely applied\(^{(65,66)}\) to high speed Fourier analysers. A more rigorous definition requires that the processor bandwidth (or data rate) is equal to or greater than the signal bandwidth (or data rate). Thus real time operation can be defined as "on-line" processing. If the processor demands signal storage at any point in the computation, the analysis is more correctly termed "off-line" or non-real time processing. Figure 3.1.1 compares the relative processing times of the DFT, FFT and CZT. SAW CZT processors have been demonstrated\(^{(61)}\) with 5 MHz real time signal bandwidth and 32 transform points. To date CCD CZT processors are limited to data rates below 500 kHz. However, processors with 500 transform points have been reported\(^{(67,68)}\). The real time signal bandwidth of the
Figure 3.1.1 Relative computation times of DFT, FFT and CZT.
digital FFT processor is typically less than 10 kHz with 1024 transform points.

In 1975, several publications\(^{(69, 70, 71)}\) simultaneously reported the chirp transform technique\(^{(72, 73)}\). The significant features of this algorithm are firstly that it permits use of SAW IF chirp filters, widely developed as radar components, and secondly, it can result in a hardware implementation which does not involve parallel processing. The chirp transform\(^{(73)}\) is similar in nature to the CZT in that it permits decomposition of the Fourier transform into pre-multiplication, convolution and post-multiplication with chirp waveforms. Two essential differences exist, however, between the chirp transform and the CZT. Firstly, the chirp transform is capable of computing directly the continuous Fourier transform or Fourier integral, in addition to computing the DFT, see chapter 4. Secondly, because all the signal processing functions are performed at IF, real and imaginary (amplitude and phase) components can be computed in a single 'complex' operation. Since, with CCD devices only real sampled values are employed, the simple (single channel) processor of the chirp transform can only be realised in SAW devices. Chapter 4 of this thesis presents a rigorous analysis of the operation of the SAW chirp transform processor.

In an attempt to eliminate the multiplication elements required in the CZT architecture, a further DFT algorithm
known as the prime transform\(^{(74)}\) has been applied to SAW\(^{(75)}\) and to CCD\(^{(76)}\) processors. The architecture required in the prime transform is a simple data correlation with associated permutation or re-ordering of data, and for real data significant hardware simplification is possible. To date, the data permutation process has not been achieved with an acceptable accuracy level and future developments may tend towards digital techniques for data permutation with analogue correlation performed in CCD or SAW filters.

The techniques considered thus far in this section have realised Fourier transform processors for computation of the Fourier integral or DFT. However, several other techniques have been developed specifically for power spectrum analysis. The most basic of these consists of a bank of contiguous bandpass filters each yielding the power density for a specific frequency component\(^{(47)}\). Alternatively a single narrowband filter could be employed with a swept local oscillator possibly with time compression to permit operation with fast sweep rates\(^{(77)}\).

SAW panoramic receivers\(^{(78)}\) or compressive intercept receivers\(^{(79)}\) which in fact pre-dated the development of the chirp transform have been developed for electronic countermeasure (ECM) applications and similar SAW processors (with suitable interface circuitry) have been developed\(^{(80)}\) for radar target Doppler frequency measurement. An alternative
SAW processor based on the coherent memory filter technique\(^{(81,82)}\) has been applied to Doppler frequency measurement\(^{(83)}\). Here, successive radar returns from a specific radar range bin are coherently integrated on successive memory circulations to yield frequency measurement information.

Finally, a technique widely used in geophysics and sonar applications is the maximum entropy method of spectrum analysis\(^{(84,85,86)}\). This technique has application where short duration signals compromise the frequency resolution of a Fourier transform processor.

### 3.2 MULTIDIMENSIONAL FOURIER TRANSFORMS

It is important to note that the Fourier transform definitions of section 3.1 are not limited to temporal transformations. Space-time Fourier transformation is characterised by the computation of multidimensional Fourier transforms involving both temporal and spatial functions. In the most general case, one temporal and three spatial transform dimensions exist. In television video bandwidth reduction by means of redundancy elimination\(^{(87)}\), there are two spatial dimensions and one temporal dimension. For a line array, there is one spatial and one temporal dimension. The last case is considered in detail in chapter 8 of this thesis.

In two dimensions the DFT can be computed, using auxiliary
memory, by computing the DFT of rows of the input signal matrix, using the auxiliary matrix as a row-to-column transformation, ie, transposing the partial Fourier transform matrix and computing the two dimensional Fourier transform with a second processor. In general, the accuracy requirements, as well as the number of transform points is different for the temporal and spatial dimensions. The temporal dimension is characterised by large transform size (1000 points) for good frequency resolution whereas the accuracy requirements of the spatial transform are limited by the number of sensors (100).

Two dimensional Fourier transforms can be performed by a temporal transformation of the rows of the two dimensional data field followed by spatial transformation of the resultant transform data. The temporal transform could be performed in a CCD serial in/serial out CZT and the spatial transform performed in a parallel in/serial out SAW device\(^{62}\) such as the SAW diode convolver\(^{88,89}\). An alternative technique involves time compression of the data field\(^{90}\).

Optical Fourier transforms\(^{91}\) offer a simple technique for spatial Fourier transformation. These techniques are not considered in this thesis.

3.3 COMPARISON OF FOURIER TRANSFORM PROCESSOR CAPABILITIES

When Fourier transform processors are compared, care must be taken to differentiate between those which yield amplitude
and phase (real and imaginary) components of the transform and those which yield only the power spectral density. Section 3.1. indicated those algorithms and techniques which yield the Fourier transform components and hence in this section, the processors will be compared in terms of power spectral density computation capabilities only, figure 3.3.1.

Scanning filter techniques are universally employed in swept frequency spectrum analysers which cost in the region of £5,000 and cover wide bandwidths (20 Hz - 40 GHz) with high dynamic range (70 dB display).

SAW spectrum analysis has been rapidly developed for compressive receiver applications where by matching the characteristics of filter and swept oscillator fast scan rates can be used thereby increasing the probability of intercept for transient signals. Compressive receivers cost in the region of £10,000 and can analyse signal bandwidths up to 100 MHz in 10 μs with a CW frequency resolution of 100 kHz. Alternative SAW processor designs are commercially available which analyse signal bandwidths up to 4 MHz with frequency resolution of 40 kHz and cost approximately £5,000.

Although CCD based spectrum analysers are currently being developed in several laboratories (67, 68, 92), no commercially available CCD spectrum analyser exists at present. Current prototype analysers are capable of operation over bandwidths up to 200 kHz with 500 points and 2 MHz bandwidth.
is projected. CCD chirp filters are commercially available which contain 4 x 500 stage filters for the CZT processor at a cost of approximately £500. Projected costs for a CCD spectrum analyser vary between £500 and £5,000.

The digital FFT is the most widely accepted method for spectrum analysis. The range of samples processed is typically in the range 512 to 8192 and bandwidths up to 50 kHz. FFT processor systems which are software based normally have a cycle time of several ms, hence are limited to slow data rates.

Faster processors employ FFT hardware and, more specifically, can use parallel computation. Commercially available dedicated hardware processors have real time bandwidths up to 20 kHz (2048 points, £25,000) and microprocessor based FFT processors have recently been developed with 1 kHz real time bandwidth (1024 points, £5,000). For real time signal bandwidths in excess of 20 kHz, custom designed FFT processors are required. For 100 kHz real time bandwidth the processor would cost approximately £100,000 (1024 points). At bandwidths in excess of 1 MHz the processor cost is very much in excess of £1,000,000.

Several optical Fourier transform techniques have been developed for spectrum analysis over bandwidths up to 100 MHz (10^6 points). Figure 3.3.2 indicates the main areas of application in relation to the several types of Fourier transform processors.
Figure 3.3.1 Comparison of Fourier transform processor capabilities

Figure 3.3.2 Main application areas for Fourier transform processors
CHAPTER 4: MATHEMATICAL THEORY OF THE SAW CHIRP TRANSFORM

4.1 INTRODUCTION

In chapter 3, it was indicated that frequency components of a signal can be displayed using a scheme based on linear frequency modulated (chirp) filters and the following chapters consider such Fourier transformation and derived signal processing functions using SAW chirp filters (97,98,99). The purpose of this chapter is to develop formally, the conditions under which the SAW chirp transform using physically realisable devices can perform a valid Fourier transform of a complex input function, within the operational limits of the SAW devices.

The basic form of the chirp transform (100,101) can be derived from the Fourier integral (equation 3.1.2) by using the substitution (98)

$$-2\Omega t = (t - \Omega)^2 - t^2 - \Omega^2$$  \hspace{1cm} (4.1.1)

and expanding to yield

$$F(\Omega) = F(\mu t) = \exp(-j\mu t^2) \times$$

$$\int_{-\infty}^{+\infty} f(\tau) \exp(-j\mu \tau^2) \cdot \exp(j\mu [t - \tau]^2) \, d\tau \hspace{1cm} (4.1.2)$$
which suggests that pre-multiplication of an input signal $f(t)$ with a chirp waveform, followed by convolution in a chirp filter and subsequent post-multiplication with a chirp will yield the Fourier transform $F(\Omega)$. This configuration of multiply, convolve, multiply (M-C-M) will be shown to produce a valid Fourier transform using SAW chirp filters with impulse response of the form shown in equation (2.1.1), which represents the real part of the functions required in equation (4.1.2).

Two alternative arrangements of the M-C-M scheme must be distinguished; one in which the time duration of the multiplying chirp signal is short compared with the duration of the impulse response of the filter - the M(S) - C(L) - M scheme, and one in which the multiplying chirp duration is long, the M(L) - C(S) - M. Here (S) denotes shorter duration and (L) denotes longer duration. It will be shown that whereas the M(L) - C(S) - M scheme has a role to play in spectrum analysis, it is the M(S) - C(L) - M scheme which is best suited to Fourier transformation.

An alternative scheme which can be considered as the 'dual' arrangement of the M-C-M scheme is analysed in section 4.3. This configuration involves convolution of the input signal $f(t)$ with a chirp, followed by multiplication with a chirp and subsequent convolution with a chirp.
4.2 FOURIER TRANSFORMATION USING THE M-C-M SCHEME

4.2.1 The M(S) - C(L) - M Arrangement

In the SAW implementation of the M-C-M chirp transform, the chirp waveforms used for signal multiplication are assumed to be generated by impulsing SAW chirp filters. The convolution filter is also assumed to be a SAW chirp filter. The impulse responses of the pre-multiplying chirp $C_1(t)$, the post-multiplying chirp $C_2(t)$ and the convolution filter $H(t)$ are given by

\[ C_1(t) = \Pi \left\{ \frac{t - \frac{1}{2}T_1}{T_1} \right\} W_1(t) \cdot \cos \left\{ \omega_1 t - \frac{1}{2} \mu t^2 + \phi_1 \right\} \quad (4.2.1) \]

\[ C_2(t) = \Pi \left\{ \frac{t - \frac{1}{2}T_2}{T_2} \right\} W_2(t) \cdot \cos \left\{ \omega_2 t + \frac{1}{2} \mu t^2 + \phi_2 \right\} \quad (4.2.2) \]

\[ H(t) = \Pi \left\{ \frac{t - \frac{1}{2}T_0}{T_0} \right\} W_0(t) \cdot \cos \left\{ \omega_0 t + \frac{1}{2} \mu t^2 + \phi_0 \right\} \quad (4.2.3) \]

where $\Pi \left\{ (t - t_n)/T_n \right\}$ defines a rectangular gating function of duration $T_n$ centred on time $t = t_n$. In these equations the delay between the applied impulse and commencement of the response has been neglected. Here $W_n(t)$ is an arbitrary weighting function and $\phi_n$ is a phase term which will be assigned
the values 0 or $\pi/2$ later. Note that these responses have been defined such that the angular instantaneous frequency $(d\phi/dt)$ at the commencement of each waveform is given by $\omega_1$, $\omega_2$ and $\omega_0$ for $C_1(t)$, $C_2(t)$ and $H(t)$ respectively. The factor $\mu$ defined as the magnitude of the rate of change of angular instantaneous frequency is the same for all three waveforms. A discrepancy exists between the sign of $\mu$ in the post-multiplier (equation 4.2.2) and that shown in equation 4.1.2. This arises because a difference, as distinct from a sum term will ultimately be selected at the post-multiplication stage to yield the output Fourier transform components at baseband. The alternative case of choosing the transform components to appear modulated on a carrier would not normally arise since it will be shown that it is often convenient to eliminate the post-multiplication operation and execute subsequent signal processing of the amplitude and phase components of the transform relative to the carrier, see section 6.2.

The input to the processor must carry the real and imaginary signal components, $f(t)$ and $g(t)$ respectively, to perform complex arithmetic. Assume that both these quantities are available and can be read from a memory, say, on demand. These components may form two separate real functions in separate input channels, or they may be combined into a single channel by modulation on to a carrier, $\omega_{c1}$, giving

$$s(t) = a(t) \cdot \cos \{\omega_{c1} t + \phi(t)\} \quad (4.2.4)$$
where \( f(t) = a(t) \cos \{ \phi(t) \} \)

\[ g(t) = a(t) \sin \{ \phi(t) \} \]

All three signals, \( f(t) \), \( g(t) \) and \( s(t) \) are purely real. Here the passage of one of them, \( f(t) \), during a single cycle of operation of the processor is considered. It will be shown that the analysis based on the function \( f(t) \), considered as the real component of the input, is also valid for the function \( g(t) \), considered as the imaginary part of the input and hence the analysis shows that the SAW processor is capable of performing complex data analysis.

The processor cycle is initiated by, and takes its time origin from, the impulse signal used to generate the pre-multiplying chirp waveform. The signal, \( f(t) \), appears at the input to the SAW processor and, after pre-multiplication by \( C_1(t) \), the signal is given by

\[
S_1(t) = \pi \left\{ \frac{t - \frac{1}{2}T_1}{T_1} \right\} C_1(t) x f(t) \quad (4.2.5)
\]

After passage through the filter, \( H(t) \), the signal is given by the convolution integral

\[
S_2(t) = \int_{-\infty}^{\infty} dt f(\tau) \cdot \pi \left\{ \frac{\tau - \frac{1}{2}T_1}{T_1} \right\} C_1(\tau) \times H(t - \tau) \quad (4.2.6)
\]
Consider first the consequence of only the gating functions in this convolution integral. The product of these functions is zero for \( t < 0 \) then increases linearly with \( t \) to a maximum value at \( t = T_1 \). It then remains at this value until \( t = T_0 \), after which it decreases linearly to zero for \( t > T_1 + T_0 \). In this way, the maximum value of this product corresponds to the input \( S_1(t) \) being wholly within the filter.

In performing Fourier transformation, only this part of the signal will be considered. This is arranged to be selectively gated by the post multiplier operating in the time interval

\[
\Pi \left\{ \frac{t - t'}{T'} \right\} = \Pi \left\{ \frac{t - \frac{1}{2}(T_0 + T_1)}{T_0 - T_1} \right\}
\] (4.2.7)

At this stage assume the weighting functions \( W_n(t) \) take the value of unity. The relevant output from the filter is

\[
S_2(t) = \Pi \left\{ \frac{t - t'}{T'} \right\} \times \int_{-\infty}^{+\infty} d\tau \hat{f}(\tau) \cdot \cos \left\{ \omega_1 \tau - \frac{1}{2} \mu \tau^2 + \phi_1 \right\} \times \cos \left\{ \omega_0(t - \tau) + \mu(t - \tau)^2/2 + \phi_0 \right\}
\] (4.2.8)

where \( \hat{f}(t) = f(t) \Pi \left\{ \frac{t - \frac{1}{2}T_1}{T_1} \right\} \) (4.2.9)
Multiplication of the cosine terms gives sum and difference arguments of which only the former is equal to or is close to zero. The difference term gives rise to a rapidly oscillating integrand which may be neglected. This approximation is discussed in section 4.2.4. Equation (4.2.8) can then be written

\[ S_2(t) = \frac{i}{4} \Pi \left( \frac{t - t'}{T'} \right) \times \]
\[ \int_{-\infty}^{+\infty} d\tau \cdot f(\tau) \cdot \left[ \exp(-j\Omega \tau) \times \right. \]
\[ \exp(j(\omega_o t' + \frac{1}{2} \mu t^2 + \phi_o + \phi_1) \right) + C.C \]
\[ (4.2.10) \]

where C.C. stands for complex conjugate and the variable \( \Omega \) is defined

\[ \Omega = \omega_o - \omega_1 + \mu t \]  
\[ (4.2.11) \]

Equation (4.2.10) can be expanded in the form

\[ S_2(t) = \frac{i}{4} \Pi \left( \frac{t - t'}{T'} \right) \times \]
\[ \left[ \int_{-\infty}^{+\infty} d\tau \cdot x(\tau) \cdot \exp(-j\Omega \tau) + \int_{-\infty}^{+\infty} d\tau \cdot x^*(\tau) \exp(+j\Omega \tau) \right] \]
\[ (4.2.12) \]
where

\[ x(t) = \hat{f}(t) \exp(j\omega t + j2\mu t^2 + \phi_0 + \phi_1) \]  \hspace{1cm} (4.2.13)

and here the symbol * denotes the complex conjugate, since for \( f(t) \) a real function,

\[ f^*(t) = f(t) \]  \hspace{1cm} (4.2.14)

Thus, equation (4.2.12) can be written

\[ S_2(t) = \Im \left\{ \frac{t - t'}{T'} \right\} [X(\Omega) + X^*(\Omega)] \]  \hspace{1cm} (4.2.15)

where \( X(\Omega) \) is the Fourier transform of \( x(t) \)

\[ S_2(t) = \Im \left\{ \frac{t - t'}{T'} \right\} x \]

\[ \text{Re}[F(\Omega) \cdot \exp(j\omega t + j2\mu t^2 + \phi_0 + \phi_1)] \]

\[ (4.2.16) \]

where \( \text{Re} \) denotes the real part.

Splitting the transform into real and imaginary components,

\[ F(\Omega) = F_R(\Omega) + jF_I(\Omega) \]  \hspace{1cm} (4.2.17)
\[ S_2(t) = \Pi\left( \frac{t - t'}{T'} \right) \]

\[ \frac{1}{2} \left( F_R(\Omega) \cos[\omega_0 t + \frac{1}{2} \mu t^2 + \phi_0 + \phi_1] \right. \]

\[ \left. - F_I(\Omega) \sin[\omega_0 t + \frac{1}{2} \mu t^2 + \phi_0 + \phi_1] \right) \]  \hspace{1cm} (4.2.18)

This signal is multiplied by \( C_2(t) \) in the post-multiplier.

As stated previously, \( C_2(t) \) is used to perform the gating operation on \( S_2(t) \) hence

\[ T_2 = T_0 - T_1 \]  \hspace{1cm} (4.2.19)

and the delay time required for the impulse to \( C_2(t) \) is

\[ \Delta_2 = T_1 \]  \hspace{1cm} (4.2.20)

Two choices are available in selecting the value \( \omega_2 \): it may either be equated to \( \omega_0 \), in which case a baseband output is obtained, or the output may automatically be modulated onto a carrier \( \omega_{c2} \) by setting

\[ \omega_2 = \omega_0 - \omega_{c2} \]  \hspace{1cm} (4.2.21)

Consider the latter case, retaining the option to set \( \omega_{c2} \) to zero when desired.

The output after post-multiplication is given by
\[ S_3(t) = \Pi \left( \frac{t - t'}{T'} \right) \times \]
\[
\left\{ \frac{1}{4} F_R(\Omega). \left[ \cos(2\alpha - \omega_2 t + \phi_0 + \phi_1 + \phi_2) + \cos(\omega_2 t + \phi_0 + \phi_1 - \phi_2) \right] \right. \\
- \left. \frac{1}{4} F_I(\Omega). \left[ \sin(2\alpha - \omega_2 t + \phi_0 + \phi_1 + \phi_2) + \sin(\omega_2 t + \phi_0 + \phi_1 - \phi_2) \right] \right\}
\]

(4.2.22)

where \( \alpha = \omega_0 t + \frac{1}{2} \mu t^2 \)  

(4.2.23)

and \( \omega_2(t) \) has been set to unity.

Combinations of transform components can be selected by choice of \( \phi_0, \phi_1, \phi_2 \) and \( \omega_c \). Setting \( \omega_c \) equal to zero yields the components shown in table 4.2.1.

Figure 4.2.1 shows an arrangement of multipliers and filters suggested by table 4.2.1, organised to yield \( \hat{F}_R \) and \( \hat{F}_I \) outputs from a single real input. This is identical to the arrangement required for the CZT(61), figure 7.1.2, discussed in section 7.1 and demonstrates the validity of the theory.

Considerable simplification in hardware requirements can be achieved by the introduction of low pass filters on the outputs, together with the choice of \( \omega_c \ll 2\omega_0 \) such that the sum terms in equation (4.2.22) are rejected. The corresponding outputs for combinations of \( \phi_0, \phi_1 \) and \( \phi_2 \) are shown in table 4.2.2. Here \( \omega_c \) has not been set to zero and the relations
\[ \phi_2 = 0 \quad \phi_2 = \pi/2 \]

<table>
<thead>
<tr>
<th>( \phi_1 + \phi_0 = 0 )</th>
<th>( \phi_1 + \phi_0 = \pi/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{F}/2 \cos^2 \alpha )</td>
<td>( \hat{F}/4 \sin 2 \alpha )</td>
</tr>
<tr>
<td>( \hat{F}/4 \sin 2 \alpha )</td>
<td>( \hat{F}/2 \sin^2 \alpha )</td>
</tr>
</tbody>
</table>

\[ \alpha = \omega_0 t + i \mu t^2 \quad \omega_{c2} = 0 \]

Table 4.2.1. Transform component selection table.

<table>
<thead>
<tr>
<th>( \phi_2 = 0 )</th>
<th>( \phi_2 = \pi/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{F} \cos(\omega_{c2} t + \phi) )</td>
<td>( \hat{F} \sin(\omega_{c2} t + \phi) )</td>
</tr>
<tr>
<td>( \hat{F} \sin(\omega_{c2} t + \phi) )</td>
<td>( \hat{F} \cos(\omega_{c2} t + \phi) )</td>
</tr>
</tbody>
</table>

Table 4.2.2. Transform components after low pass filtering.
Figure 4.2.1. Basic chirp transform arrangement.
\[ \hat{F}_R = \hat{F} \cos \phi \] (4.2.24)

\[ \hat{F}_I = \hat{F} \sin \phi \] (4.2.25)

have been introduced.

It can be seen that the true Fourier transform components of the truncated input signal, \( f(t) \), can be obtained, for example, by choosing \( \phi_0 + \phi_1 = 0, \phi_2 = 0 \), with quadrature multiplication by \( \omega_c \). \( \hat{F}_R \) and \( \hat{F}_I \) can be obtained directly by setting \( \omega_c = 0 \), \( \phi_0 + \phi_1 = 0 \) and using quadrature chirp post-multipliers \( (\phi_2 = 0; \phi_2 = \pi/2) \) in two output channels. Figure 4.2.2(a) shows one of several arrangements in which real and imaginary data channels of baseband information feed directly into the SAW transform processor with baseband output of the real and imaginary Fourier transform components. In contrast, figure 4.2.2(b) shows a processor for which the input is modulated onto a carrier, \( \omega_c \). Here single sideband mixers are used throughout and the output is modulated onto a carrier \( \omega_c \). In this case, the analysis is essentially the same as outlined previously except that equation (4.2.11) becomes

\[ \Omega' = \omega_0 - \omega_1 + \mu t - \omega_c \] (4.2.26)

Figure 4.2.2 indicates that a major simplification is possible over the CZT scheme.
Figure 4.2.2. SAW chirp transform architectures.

(a) Baseband input and output signals.

(b) Input and output modulated on a carrier.
In section 4.2.4 the performance of the SAW chirp transform processor is discussed. Note that the processor is limited not only in the duration of signal which may be input without truncation (T_1), but also the transform display is limited in frequency range by the gating window in equation (4.2.7).

4.2.2 Inverse Fourier Transform Processor

The arrangement required for computation of the inverse Fourier transform using SAW chirp filters can be derived in a similar manner to equation (4.1.2). The inverse Fourier transform is defined

\[ f(t) = \mu \cdot \int_{-\infty}^{+\infty} F(\mu \tau) e^{j\mu \tau t} d\tau \]  \hspace{1cm} (4.2.27)

and substitution of

\[ 2\tau t = t^2 + \tau^2 - (t - \tau)^2 \]  \hspace{1cm} (4.2.28)

yields

\[ f(t) = \mu \cdot \exp(j \frac{\mu}{2} t^2) \times \int_{-\infty}^{+\infty} d\tau \cdot \{F(\mu \tau) \exp(+j\frac{\mu}{2} \tau^2)\} \cdot \exp(-j\frac{\mu}{2}(t - \tau)^2) \]  \hspace{1cm} (4.2.29)

which is an identical configuration to that shown in equation (4.1.2) except that the sign of each chirp is reversed. It is
apparent that the basic chirp transform arrangement is capable of generating either the Fourier transform or the inverse Fourier transform of a given input signal. The definitions of equation (4.1.2) and equation (4.2.29) are therefore mutually inter-dependent and apply only when the output from a given chirp transform processor configured as equation (4.1.2) is required to be transformed back to the time domain when an inverse chirp transform configured as equation (4.2.29) is required.

The mathematical proof that a signal processor based on this structure and dispersive character performs an inverse Fourier transformation is exactly as given in section 4.2.1.

4.2.3 The M(L) - C(S) - M Arrangement

This section considers the case where the duration \( T_1 \) of the chirp pre-multiplier \( C_1(t) \) is greater than the duration \( T_0 \) of the impulse response of the convolution filter \( H(t) \). It is shown that this is not an optimum arrangement for a Fourier transform processor.

The output from the chirp filter, \( S_2(t) \), described in equation (4.2.6) is independent of the relative magnitudes of \( T_0 \) and \( T_1 \). It is therefore acceptable to revert to this equation, introducing the condition \( T_1 > T_0 \) and consider the time interval defined by
\[ \pi \left\{ \frac{t - t''}{T''} \right\} = \pi \left\{ \frac{t - \frac{1}{2}(T_0 + T_1)}{T_1 - T_0} \right\} \quad (4.2.30) \]

when convolution is taking place in the window defined by

\[ \pi \left\{ \frac{\tau - t - \frac{1}{2}T_0}{T_0} \right\} \]

Thus, equation (4.2.6) may be written

\[ S_2(t) = \pi \left\{ \frac{t - t''}{T''} \right\} \times \]

\[ \int_{-\infty}^{+\infty} d\tau \cdot [f(\tau) \pi \left\{ \frac{\tau - t - \frac{1}{2}T_0}{T_0} \right\}] \times C_1(\tau) \cdot H(t - \tau) \]

\[ (4.2.31) \]

Here, when \( W_1(t) \) and \( W_2(t) \) are set to unity, the Fourier transform of the function

\[ f(\tau) \cdot \pi \left\{ \frac{\tau - t - \frac{1}{2}T_0}{T_0} \right\} \quad (4.2.32) \]

can be determined, as shown in section 4.2.1. The significant difference here is that the gating function on \( f(t) \) is itself a function of \( t \). As a consequence, the process of Fourier transformation of the gated function \( f(t) \) is not taking place on a fixed gated function, ie, \( \hat{f}(\tau) \) as defined previously, but the gating function is sliding across \( f(t) \) presenting different samples of the signal whilst at the same time performing
Fourier transformation in a time ordered fashion.

The variation of samples caused by this sliding window can be prevented by pre-gating $f(t)$ itself, or in some way choosing $f(t)$ to be sufficiently limited in time. It is necessary for the signal duration to be less than $T_0$. This will be recognised as simply reducing the duration of the signal emerging from the pre-multiplier to less than the duration of the impulse response of the convolution filter. As a consequence, the $M(L) - C(S) - M$ arrangement is effectively converted to the $M(S) - C(L) - M$ scheme, but with very inefficient use of the large time bandwidth product of the pre-multiplying chirp. However, by repetitively reading the input waveform from a memory, the full swept bandwidth of this processor can be employed.

4.2.4 Operation of the SAW Chirp Transform Processor

This section considers the operation and performance limitations of the chirp transform processor implemented with typical SAW devices. The $M(S) - C(L) - M$ scheme is considered almost exclusively, and, unless otherwise stated, it is assumed $T_1 < T_0$.

The SAW Fourier transform processor displays frequency components in the time interval specified by equation (4.2.7). Substitution in equation (4.2.11) shows the range of frequencies analysed by the processor to be
\[ \omega_0 - \omega_1 - \mu T_1 \leq \Omega \leq \omega_0 - \omega_1 + \mu T_1 \]  \hspace{1cm} (4.2.33)

and hence the maximum bandwidth of the processor is

\[ B_{\text{max}} = \mu (T_0 - T_1)/2\pi \]  \hspace{1cm} (4.2.34)

The maximum signal duration which the processor can analyse without truncation is \( T_1 \). Taking the ratio of \( T_1/T_0 = c \),

\[ B_{\text{max}} \cdot T_{\text{max}} = \mu T_0^2 c(c - 1)/2\pi \]  \hspace{1cm} (4.2.35)

which is a maximum for \( c = 0.5 \), \( T_0 = 2T_1 \). The optimum processor thus has a time bandwidth product of \( B_0 T_0/4 = B_1 T_1 \).

With current technology, section 2.1.2, SAW dispersive delay lines can be made with time bandwidth products in excess of 10,000, hence, SAW Fourier transform processors with time bandwidth product of 2500 can be designed\(^\text{(103)}\).

In describing the performance of the SAW processor it is important to realise that the expected Fourier transform output must include negative and positive frequency components. To ensure the display of both components, the centre instantaneous frequencies of the chirp pre-multiplier and convolution filter must be equal.
\[ \omega_f - \frac{1}{4} \mu T_1 = \omega_0 + \frac{1}{4} \mu T_0 \] (4.2.36)

In this case, the angular frequency range displayed is given by equation (4.2.11).

\[-\frac{\mu T_0}{4} \leq \Omega \leq +\frac{\mu T_0}{4} \] (4.2.37)

for \( T_0 = 2T_1 \). Thus the required positive and negative frequency components are displayed over a frequency range \(|\mu T_0/4|\). Zero frequency occurs at the centre of the display. Figure 4.2.3 represents the chirp waveforms involved in convolution assuming input frequencies of zero and \( \mu T_1/2 \).

The time-separated outputs are shown on a separate time axis.

The performance of a SAW chirp transform processor is shown in figure 4.2.4. The parameters of the SAW devices used here were \( T_0 = 5 \mu s \), \( T_1 = 2.5 \mu s \), \( \mu = 2\pi \cdot 5 \text{ MHz/\mu s} \) with all device centre frequencies 60 MHz. In figure 4.2.4(a), the Fourier transform of a (real) 7.5 MHz cosine input (defined even relative to the centre of the pre-multiplier chirp) is seen to be purely real and even. In figure 4.2.4(b), the Fourier transform of a (real) 7.5 MHz sine input is seen to be purely imaginary and odd. These results exhibit the required positive and negative frequency components at \( \pm 7.5 \text{ MHz} \) and also display the sinc function output with nulls at 400 kHz produced by the 2.5 \( \mu s \) rectangular input window.
Figure 4.2.3. Convolution in the chirp transform showing responses at zero frequency and at window edges.
(a) Real and even (cosine) input.

(b) Real and odd (sine) input.

*Figure 4.2.4. Operation of SAW chirp transform processor.*
As discussed in section 4.2.1, the simplicity of the SAW chirp transform processor shown in figure 4.2.2(a) was derived by neglecting the difference term generated by convolution in the filter $H(t)$, see equation (4.2.8). To check the validity of this assumption, a comparison of the peak of the difference term with the peak of the sum term was undertaken. This revealed that the difference between the two terms increases with increasing processor time bandwidth product and also with increasing centre frequency. Illustrative examples given in table 4.2.3 show that for modest centre frequencies and time-bandwidth products, the difference term may in fact be neglected for normal dynamic range operation.

Another feature which affects performance arises from device phase errors and control circuitry timing errors. Referring to equation (4.2.22), it is clear that phase errors in $\phi_0$, $\phi_1$ and $\phi_2$ will tend to accumulate in the arguments of the cosine and sine terms. Lumping these together as $\delta \phi$ we obtain an error function by the derivative of equation (4.2.22)

$$\delta S_3(t_1, \delta \phi) = \frac{1}{2} \Pi \left\{ \frac{t - t'}{T'} \right\} x$$

$$\{- F_R(\Omega)\sin(\omega_c t + \phi_0 + \phi_1 - \phi_2) - F_I(\Omega)\cos(\omega_c t + \phi_0 + \phi_1 - \phi_2)\}$$

(4.2.38)

Similarly, if there is an error due to the impulse timing of $C_2$
relative to $C_1$ by time $\delta t$, there will be a minor error due to
displacement of the output transform window, and a significant
error in phase given by

$$\delta S_3(\delta t_1, \phi) = \frac{1}{2} \pi \left\{ \frac{t - t'}{T'} \right\} x (\omega_0 + \mu t)$$

$$x \{ -R(\Omega) \sin(\omega_c t + \phi_0 + \phi_1 - \phi_2) - F_I(\Omega) \cos(\omega_c t + \phi_0 + \phi_1 - \phi_2) \}$$

(4.2.39)

From these results, table 4.2.4 can be constructed which is
equivalent to table 4.2.2 but displays error contribution.
The overall effect is to produce cross-talk between real and
imaginary outputs. Using the ratio of leakage in the unwanted
channel to signal in the wanted channel as a measure of per-
formance degradation, $\varepsilon$, say. For 40 dB rejection ($\varepsilon = 10^{-2}$)
from equation (4.2.38) a phase error of 0.5 degree or less is
required. Further taking a centre frequency of 100 MHz and
bandwidth 50 MHz, a timing precision of 0.01 ns is required to
achieve 40 dB rejection. This high sensitivity arises because
in determining the phase component in the transform domain,
$\Phi(\Omega)$, it is necessary to avoid phase errors which are produced
relative to the chirp centre frequencies in the processor. In
terms of Fourier transformation, this is the price paid for the
simplicity introduced in moving from baseband to bandpass
operation.
<table>
<thead>
<tr>
<th>CENTRE FREQUENCY</th>
<th>SLOPE</th>
<th>TIME-BANDWIDTH</th>
<th>ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>30MHz</td>
<td>5MHz/µs</td>
<td>32</td>
<td>-38.6dB</td>
</tr>
<tr>
<td>60MHz</td>
<td>5MHz/µs</td>
<td>32</td>
<td>-44.9dB</td>
</tr>
<tr>
<td>60MHz</td>
<td>10MHz/µs</td>
<td>64</td>
<td>-57.7dB</td>
</tr>
</tbody>
</table>

Table 4.2.3. Ratio of contributions due to difference and sum terms for equation (4.2.8)

\[
\begin{align*}
\phi_2 &= 0 \\
\phi_2 &= \pi/2 \\
\phi_1 + \phi_0 &= 0 \\
\phi_1 + \phi_0 &= \pi/2
\end{align*}
\]

<table>
<thead>
<tr>
<th>(\phi_1 + \phi_0 = 0)</th>
<th>(\phi_1 + \phi_0 = \pi/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{F}\sin(\omega_c t + \phi))</td>
<td>(\hat{F}\cos(\omega_c t + \phi))</td>
</tr>
<tr>
<td>(\hat{F}\cos(\omega_c t + \phi))</td>
<td>(\hat{F}\sin(\omega_c t + \phi))</td>
</tr>
</tbody>
</table>

Table 4.2.4. First order error coefficients in chirp transform.
There are specific applications in which it is unnecessary to establish the phase of the transformed signal, for example, in spectrum analysis\(^{(104,105)}\). Several such applications are considered in the following chapters.

**4.2.5 Spectrum Analysis**

If the power spectrum, as distinct from the Fourier transform, of a signal is required the chirp post-multiplication stage of the SAW chirp transform processor can be dispensed with\(^{(104)}\). In the case of the M(S) - C(L) - M scheme, this may be shown explicitly by reference to equation (4.2.18), the output after convolution in the chirp filter. If this signal is squared and low pass filtered, the output is

\[
S_3(t) = \Pi \left\{ \frac{t - t'}{T'} \right\} \frac{1}{8} \left[ \hat{F}_R^2 + \hat{F}_I^2 + \hat{G}_R^2 + \hat{G}_I^2 + 2(\hat{F}_I \hat{G}_R - \hat{F}_R \hat{G}_I) \right]
\]

(4.2.40)

for \(\phi_0 = 0\) and \(\phi_1 = 0\) in the 'real' input channel and \(\phi_1 = \pi/2\) in the 'imaginary' input channel. Equation (4.2.40) is the required power spectrum of the input signal. The output gating normally performed by the chirp post-multiplier must be introduced here as a separate operation. Note that impulse timing is no longer of importance and constant phase errors in \(\phi_1\) and \(\phi_0\) are of no importance if input data is confined to one channel.
Consider the case where the SAW spectrum analyser is required to give an indication of the presence and relative strengths of frequency components when the signal duration ($T_s$) is long in comparison with $T_1$ and $T_0$ and the variation of frequency components during a time interval $T_0$, $T_1$ is negligible. Clearly here, the true power spectrum cannot be found because of the gating action implicit in equation (4.2.6). However, spectral components can still be estimated using the processor since the distortion introduced by the gating action may be reduced by use of the weighting functions $W_1(t)$ and $W_0(t)$.

In the $M(S) - C(L) - M$ scheme, with $W_0(t)$ and $W_1(t)$ set to unity, the power spectrum of

$$f_{W_1(t)} = f(t)W_1(t)\Pi \left\{ \frac{t - \frac{1}{2} T_1}{T_1} \right\}$$

(4.2.41)

is found. The Taylor weighting function (101) has been used with SAW chirp filters to reduce truncation distortions below 40 dB relative to the spectral peak, at the expense of broadening the peak response width. A practical problem is introduced here because of the limited dynamic range of the mixer. This becomes a severe limitation if the inputs to both ports of the mixer have wide dynamic range. It is therefore preferable to operate with $W_1(t)$ set to unity and to separately introduce amplitude weighting to the input signal.
In the M(L) - C(S) - M scheme, equation (4.2.31) shows that at the output of the convolution filter, the power spectrum of the function

$$f_{w2}(t) = f(t) \cdot W_0(t - \tau) \cdot \Pi\left\{\frac{t - \tau - \frac{1}{2} T_0}{T_0}\right\}$$

(4.2.42)

is found. Here it is possible to introduce the necessary spectral weighting using $W_0(t)$. This permits use of a switching mixer with high dynamic range whilst still achieving low spectral sidelobes and represents a distinct advantage of the M(L) - C(S) - M scheme as a processor for the analysis of spectral components for long duration signals.

4.3 FOURIER TRANSFORMATION USING THE C-M-C SCHEME

The alternative Fourier transform scheme shown in table 4.3.1 - the convolve-multiply-convolve scheme (C-M-C) - can be obtained from the M-C-M arrangement by interchanging the operations of multiplication and convolution, i.e., the input signal is now first convolved in a chirp filter, then multiplied by a chirp of opposite dispersive slope and finally convolved in a chirp filter. The dualised nature of convolution and multiplication in the time domain and frequency domain makes it possible to analyse the operation of the C-M-C arrangement using the theory of section 4.2 as a dual in the frequency
Maximum signal duration $T_1$
Maximum signal bandwidth $B_2 = B_0 - B_1$
For $B_s > B_0 - B_1$ truncation of output spectrum
For $T_s > T_1$ truncation of input signal

Maximum signal bandwidth $B_1$
Maximum signal duration $T_2 = T_0 - T_1$
For $T_s > T_0 - T_1$ truncation of input signal
For $B_s > B_1$ truncation of output spectrum

Table 4.3.1. Dualism of M-C-M and C-M-C chirp transform arrangements.
domain. Table 4.3.1 demonstrates the main equivalences involved in the dualism.

The Fourier transforms of the chirp waveforms can be defined (101)

\[ C_1(\omega) = \pi \frac{\omega - \omega_1}{2\pi B_1} \exp \left( +j\left( \frac{\omega - \omega_1}{2\mu} \right)^2 \right) \]  
\[ (4.3.1) \]

\[ C_0(\omega) = \pi \frac{\omega - \omega_0}{2\pi B_0} \exp \left( +j\left( \frac{\omega - \omega_0}{2\mu} \right)^2 \right) \]  
\[ (4.3.2) \]

\[ C_2(\omega) = \pi \frac{\omega - \omega_2}{2\pi B_2} \exp \left( +j\left( \frac{\omega - \omega_2}{2\mu} \right)^2 \right) \]  
\[ (4.3.3) \]

This approximation is valid for the chirp spectrum since Fresnel terms (101) are minimised for large time-bandwidth product devices with extended leading and trailing edges on the chirp filter impulse response, figure 4.3.1.

The Fourier transform of the product obtained after multiplication is given by

\[ S_1(\tilde{\omega}) = \pi \left\{ \frac{\omega - (\omega_b + \omega_1)}{2\pi (B_0 - B_1)} \right\} \]

\[ + \int_{-\infty}^{+\infty} d\sigma \cdot F'(\sigma) \exp\left( j\left( \frac{\sigma - \omega_1}{2\mu} \right)^2 \right) \cdot \exp\left( -j\left( \frac{\omega - \sigma - \omega_0^2}{2\mu} \right) \right) \]  
\[ (4.3.4) \]
Figure 4.3.1. Effect of cosine extension on chirp spectrum. (Device timebandwidth product 250, impulse response having $\sqrt{TB}$ cosine extension)
where \( F'(\sigma) = F(\sigma) \Pi \left\{ \frac{\sigma - \omega_1}{2\pi B_1} \right\} \) \hfill (4.3.5)

\[
S_1(\omega) = \Pi \left\{ \frac{\omega - (\omega_0 + \omega_1)}{2\pi (B_0 - B_1)} \right\}
\]

\[
x \exp \left( j\left( \frac{\omega_1^2 - (\omega - \omega_0)^2}{2\mu} \right) \right) f(\omega - \omega_1 - \omega_0)/\mu \) \hfill (4.3.6)
\]

where \( f(\tau) \) is the inverse Fourier transform of \( F'(\sigma) \)

\[
\tau = (\omega - \omega_1 - \omega_0)/\mu \] \hfill (4.3.7)

The Fourier transform of the output of the second filter is written

\[
S_2(\omega) = \Pi \left\{ \frac{\omega - (\omega_0 + \omega_1)}{2\pi (B_0 - B_1)} \right\}
\]

\[
\exp( j\{ (\omega_1^2 - \omega_0^2 + \omega_2^2)/2\mu \} ) \times \exp( -j\omega_1/\mu ) \times f(\omega - \omega_1 - \omega_0)/\mu ) \hfill (4.3.8)
\]

Taking the inverse Fourier transform

\[
S_2(t) = \Pi \left\{ \frac{\omega - (\omega_0 + \omega_1)}{2\pi (B_0 - B_1)} \right\} \exp( j(\omega_1^2 - \omega_0^2 + \omega_2^2)/2\mu ) \times
\]

\[
F(\mu t + \omega_1) \times \exp \{ \omega_2 t \} \) \hfill (4.3.9)
\]

or, assuming \( S_2(t) \) is purely real
\[
S_2(t) = \Pi \left\{ \frac{\omega - (\omega_0 + \omega_1)}{2(B_0 - B_1)} \right\} \times \\
F_R(\mu t + \omega_1) \cos(\omega_2 t + \Phi) - F_I(\mu t + \omega_1) \sin(\omega_2 t + \Phi) \quad (4.3.10)
\]

In a manner similar to the M-C-M arrangement, the Fourier components can be obtained by demodulation with frequency \( \omega_2/2\pi \) setting \( \Phi = 0 \) or \( \Phi = \pi/2 \).

In hardware terms, the C(S) - M(L) - C scheme offers one advantage over the equivalent M(S) - C(L) - M arrangement since here the longest duration chirp waveform is used simply as a multiplying signal. This permits use of frequency doubling in the multiplier chirp permitting analysis over the full SAW bandwidth with the full frequency resolution possible with the SAW devices. Further, the accuracy of this scheme is dependent mainly on the accuracy of demodulation frequency \( \omega_2 \).

One disadvantage of the C(S) - M(L) - C arrangement is that three SAW devices are always required, even for power spectrum analysis in comparison to the two devices required in the M(S) - C(L) - M spectrum analyser.
CHAPTER 5: DESIGN AND APPLICATIONS OF SAW CHIRP TRANSFORM PROCESSORS

INTRODUCTION

In this chapter, applications of the SAW chirp transform which require only one SAW processor are considered. These applications are inherently restricted to signal analysis for visual display or electronic level detection/selection. The applications of the SAW chirp transform considered here are in spectrum analysis, where determination of only the power density spectrum of a signal is required, and network analysis where the phase spectrum of a signal - in this case the impulse response of a network under test - is determined.

The basic conventional techniques used in spectrum and network analysis are considered prior to a description of the respective SAW realisations. A detailed comparison is drawn between the conventional and the new SAW approaches in terms of bandwidth, accuracy and speed of operation.

In contrast to these signal analysis techniques, the use of the (inverse) chirp transform processor is considered in section 5.3 as a waveform synthesis technique.

5.1 REAL TIME SPECTRUM ANALYSIS

5.1.1 Basic Types of Spectrum Analyser

Spectrum analysers can be divided into two main classes - those which indicate component frequencies simultaneously, and
those which provide a time-sequential read-out\(^{(47)}\). The simultaneous type of spectrum analyser consists of a bank of frequency contiguous, narrowband filters connected in parallel. These achieve very fast operation at the expense of a large number of channels. The sequential type of spectrum analyser either incorporates a single time-varying filter, or it employs a heterodyne mixing technique to present different parts of the input signal spectrum, in time sequence, to a fixed frequency (IF) narrowband filter. Such spectrum analysers are generally slow in operation since the rate of inspection of the input signal spectrum must be low enough to permit the build-up of energy within the IF filter corresponding to the instantaneous frequency being analysed.

In radar systems applications it is desirable that a spectrum analyser should be fast in operation and provide a time-sequential output to permit direct modulation of a radar display. The principle of pulse compression for a linear frequency modulated waveform\(^{(57)}\) offers a convenient technique for such applications\(^{(78,106)}\). The development of SAW device technology has permitted the full exploitation of this technique in compressive receiver design\(^{(79)}\). The use of SAW chirp filters (see section 2.1.2) eliminated the major problem which existed in early pulse compression spectrum analysis\(^{(106)}\), namely matching and maintaining a
sweeping local oscillator linear frequency/time characteristic to be the inverse of that of the pulse compression filter. By deriving the local oscillator from a SAW chirp filter, used as a pulse expansion line, which is identical to the SAW compression filter, matching of frequency/time characteristics can be automatically achieved. This corresponds to a passive sweeping local oscillator configuration.

5.1.2 SAW Compressive Receiver Design

The prime requirement of a SAW compressive receiver design is to permit rapid spectrum analysis over a broad frequency range for (normally) CW input signals. As such these systems have naturally employed the long pre-multiplier chirp arrangement, M(L) - C(S) - M, discussed in chapter 4, which has been shown to be valid for the power spectrum analysis of CW signals. For a given frequency resolution and RF bandwidth, the SAW compressive receiver will perform a spectrum analysis 500 to 10,000 times faster than a superheterodyne receiver. For the same frequency resolution capability, a comb filter bank would require 500-10,000 parallel channels. As an example(79), using a SAW compressive receiver, a 100 MHz band can be swept in 20 µs with a frequency resolution of 100 kHz (SAW device time-bandwidth product 1,000). Using parallel filters to achieve the same performance would require 1,000 contiguous channels and a superheterodyne receiver would require a sweep duration in excess of 10 ms. The block diagram of a SAW compressive receiver is
shown in figure 5.1.1(a). Here a conventional wideband receiver front end transmits received signals to a mixer. The LO waveform is a high-level signal which linearly sweeps frequency with time. A received CW signal is thus converted to a linear FM signal at the mixer output which is pulse compressed in the chirp filter. Normally, the centre frequencies of the LO \( f_{LO} \) and (SAW) chirp filter \( f_o \) are offset such that the chirp filter of bandwidth \( B_0 \) MHz only accepts difference terms in the mixer output produced from input signals occupying a bandwidth \( B_{in} \) centred at frequency \( f_c \) where

\[
B_{in} = B_{LO} - B_o
\quad \text{(5.1.1)}
\]

\[
f_c = f_{LO} + f_o
\quad \text{(5.1.2)}
\]

Here, \( B_{LO} \) is the swept bandwidth of the LO signal. In this way input frequencies which occur at higher or lower frequencies relative to the frequency \( f_c \), are displayed as compressed output peaks at positive or negative times relative to time of occurrence of the output peak corresponding to input frequency \( f_c \), figure 5.1.1(b). If the output pulse occurs at time \( t_0 \) for input frequency \( f_c \), the output pulse occurring for frequency \( f_c \pm f_d \) will occur at time

\[
t_d = t_0 \pm 2\pi f_d / \mu
\quad \text{(5.1.3)}
\]
(a) SAW compressive receiver schematic.

(b) Principle of operation.

Figure 5.1.1 Saw compressive receiver
and since the system is linear a number of different input frequencies will produce a corresponding number of such pulses.

Since the maximum signal length which can be processed at any instant in time is limited by the impulse response duration \( T_0 \) of the SAW pulse compression filter, the output power spectrum must necessarily exhibit truncation effects. For a CW input this truncation effect corresponds to a sinc function \( \text{sinc} \) envelope at the output. The nulls of this sinc function are spaced in time by an interval \( (B_0)^{-1} \) where \( B_0 \) is the chirp filter bandwidth and the -3 dB pulse width is approximately \( (B_0)^{-1} \). The time and frequency domains are related by the relationship \( (5.1.3) \), hence the interval \( (B_0)^{-1} \) is equivalent to a frequency resolution for CW signals of \( \mu(2\pi B_0)^{-1} = (T_0)^{-1} \). This is the minimum frequency difference which can be detected between input CW signals.

To reduce the peak sidelobe level of -13 dB relative to the mainlobe intrinsic in the sinc function envelope, spectral amplitude weighting techniques \( (101) \) can be employed with sidelobe levels of the order of -40 dB being achievable using SAW devices. As was demonstrated in chapter 4, using the long pre-multiplier chirp arrangement, \( M(L) - C(S) - M \), amplitude weighting must necessarily be achieved by amplitude weighting the response of the SAW chirp filter. An alternative (less attractive) technique would be to weight the filter output
in a separate weighted frequency filter.

Two different definitions can be made for the dynamic range of the compressive receiver design. The first of these considers the ability to distinguish between two (or more) time coincident signals. Here, since the minimum sidelobe level achievable is limited to -40 dB, this level must set the limit of dynamic range for time coincident (CW) signals as 40 dB. However, when only a single signal of arbitrary power level is being analysed, a second definition of dynamic range is possible in terms of the linear dynamic range and signal-to-noise ratio of the components used in the compressive receiver. The critical factor here is found to be spurious emission in the SAW device. As has been shown previously by appropriate selection of the SAW chirp filter bandwidth, signal breakthrough and intermodulation components can be eliminated. Figure 5.1.2 shows the transfer characteristic of a microwave double balanced mixer as a function of LO power level, and demonstrates that for an LO level of +7 dBm (rated) a linear dynamic range (±1 dB) is achieved over a 100 dB range of variation of the input signal. Note however that the mixer characteristic is strongly non-linear with respect to the LO input port, figure 5.1.2. Thus, the linear dynamic range of the compressive receiver (single CW signal) can be shown to be in excess of 60 dB.
5.1.3 General SAW Spectrum Analysis

As discussed in chapter 4, spectrum analysis of short duration or time variant signals demands use of the short chirp pre-multiplier configuration, M(S) - C(L) - M. In this case, however, attention must be paid to ensure that the signal falls entirely within the pre-multiplier duration. In applications where the signal to be analysed is read from a memory store\(^9\)\(^0\)\(^1\)\(^0\)\(^7\) this short chirp multiplier configuration must be used since the memory store must have finite time-bandwidth product (TB). The short chirp pre-multiplier signal of time-bandwidth product TB is therefore matched to the signal whereas the long chirp pre-multiplier configuration, where the pre-multiplier is of time bandwidth product 4TB, would fail to intercept the stored output signal.

Two significant disadvantages exist with this configuration. Firstly, the processor duty cycle is limited to 50% and secondly, spectral weighting must be performed by amplitude modulation of either the input signal or the pre-multiplier chirp\(^8\)\(^0\). The difficulty which arises in weighting the input or chirp multiplier is that the multiplier element requires signal dynamic range on two ports simultaneously. Figure 5.1.3. shows the transfer characteristic of a linear four quadrant analogue multiplier (Motorola MC 1495L) and demonstrates an output linear dynamic range of 60 dB. Thus using Hamming weighting which requires an envelope of dynamic
Figure 5.1.2 Double balanced mixer characteristic

Figure 5.1.3 Analogue multiplier characteristic
range 20 dB, the signal dynamic range is limited to 40 dB and the dynamic range defined by output sidelobes is again 40 dB.

3.1.4 Comparison of the Two SAW Spectrum Analysers

For fast wideband spectrum analysis of CW input signals the long chirp pre-multiplier configuration, \( M(L) - C(S) - M \), which offers >60 dB linear dynamic range with sidelobe levels of -40 dB would be used. This configuration can operate with 100% duty cycle and, in principle, it is possible to employ an actively generated LO signal permitting fast spectrum analysis over bandwidths wider than are realisable in SAW technology.

For general (short duration) signals the short chirp pre-multiplier configuration would be used since this minimises the intercept problems present with the alternative configuration, chapter 4. In applications which require spectral weighting the processor dynamic range is reduced to 40 dB by the limitations of the analogue multiplier, however, -40 dB sidelobe levels are again achievable.
5.2 REAL TIME NETWORK ANALYSIS

5.2.1 Conventional Network Analyser Design

Conventional network analyser designs which employ swept frequency techniques such as the Hewlett Packard 8410S system are limited in frequency sweep rate, and hence speed of operation, by the network impulse response time, when high frequency resolution is required. This is because signal energy corresponding to the specific instantaneous frequency at which any given measurement is being made must be present for a time in excess of the impulse response time of the network under test to effectively present a CW response for this network. In its basic form the swept frequency network analyser can be described as a dual channel receiver which performs the function of a ratio meter between two signals and then displays these complex ratios on an output display, figure 5.2.1.

Here the frequency of the oscillator is swept uniformly at a rate which is variable between 15 and 150 kHz/µs and this signal is divided, one part passing through the network under test (the test channel) and the second part serving as a reference signal for the network analyser. The signals from the test and reference channels enter the harmonic frequency converter where they are down-converted to a fixed IF (20 MHz). This constant IF is maintained by an internal
Figure 5.2.1 Conventional swept frequency network analyser design
phase lock loop - figure 5.2.1 - which keeps the first LO tuned to the reference frequency from the sweep generator. The two resultant IF signals at 20 MHz retain the same relative amplitudes and phases as the input test and reference channels. In this way, using a fixed IF, the network analyser can operate over an extremely wide input frequency range, eg, 110 MHz to 12.4 GHz.

A second down-conversion process to a low IF (278 kHz) is employed prior to amplitude and phase detection/display in the network analyser. The characteristics of the network under test can then be conveniently displayed either as a dual trace of amplitude/phase vs frequency or alternatively using a polar (Smith chart) display. Display dynamic ranges of 100 dB (logarithmic scale) are achievable with amplitude accuracies of 0.01 dB, phase accuracy 0.1° and group delay 0.1 ns. Typical cost of such a system is £15,000.

The swept frequency network analyser does represent an industrial standard in terms of frequency resolution, accuracy and acceptability in terms of military/commercial test requirements and also in terms of computer interface for control and error compensation. However, to gain advantage from the high measurement accuracy, great care must be taken to minimise the impedance mismatch between the various components used. Further, the very wide (100 dB) dynamic range is only reasonable when used in narrow band
analysis. In wideband analysis, noise limited dynamic range is restricted to 60 dB, especially for accurate phase measurements.

5.2.2 Time Domain Techniques for Network Analysis

Time domain techniques have been reported\(^{109,110}\) which permit small signal two port characterisation for frequencies up to 10 GHz. With conventional swept frequency network analysers operating at such high frequencies it is essential to take into account the various reflection and mismatch mechanisms present in the test arrangement. The application of time domain methods to network analysis avoids many of these problems by virtue of the ability to "window out" extraneous system reflections.

In time domain metrology (TDM)\(^{111}\), the required frequency response data is derived from the network impulse response (or transient response) produced by an impulse-like excitation signal using Fourier transform and deconvolution techniques. Figure 5.2.2 shows a simple block diagram of a TDM arrangement which consists of a baseband (unipolar) pulse generator coupled to a fast sampling oscilloscope through the network under test. The generator produces typically a 60 ps impulse and also a synchronising trigger pulse for the sampling oscilloscope. By sampling the appropriate interval of the
Figure 5.2.2 Time domain metrology network analysis
network impulse response, \( h(t) \), it is possible to Fourier transform this data by means of the FFT algorithm. To complete the measurement procedure, the network under test is removed and the impulse response of the remaining components, \( h_0(t) \), is sampled prior to FFT processing. The ratio of the two Fourier transforms can therefore yield the desired network characteristic. Since both impulse excitation and network response are of limited duration, the FFT can be computed with little or no truncation error provided a suitable time window is chosen. This window duration must however be less than the delay time to the first reflection - figure 5.2.2(b).

Recent improvements in technique, in particular, the introduction of sub-nanosecond sampling equipment coupled with computer-controlled scanning procedures and FFT systems have advanced the capabilities of TDM. Table 5.2.1 summarises the state-of-the-art in pulse generation techniques.

TDM techniques can be considered as a means of exchanging laboratory time for computer time. The fact that the entire data collection process over the frequency range from 100 MHz to 10 GHz can be accomplished in a time-span of 5-20 minutes makes it feasible to investigate aspects of device behaviour - such as temperature sensitivity - which would be too time-consuming if undertaken by swept frequency techniques. Whilst the discrete frequency resolution obtainable with this
<table>
<thead>
<tr>
<th>TYPE</th>
<th>RISE TIME</th>
<th>AMPLITUDE</th>
<th>p.r.f. or p.r.f. &gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>MERCURY SWITCH</td>
<td>STEP 70ps</td>
<td>100V</td>
<td>&lt;200Hz</td>
</tr>
<tr>
<td>AVALANCHE TRANSISTOR</td>
<td>PULSE 150ps</td>
<td>12V</td>
<td></td>
</tr>
<tr>
<td>TUNNEL DIODE</td>
<td>STEP 100ps</td>
<td>1V</td>
<td></td>
</tr>
<tr>
<td>STEP RECOVERY DIODE</td>
<td>STEP 60ps</td>
<td>20V</td>
<td></td>
</tr>
<tr>
<td>AVALANCHE DIODE</td>
<td>PULSE 400ps</td>
<td>125V</td>
<td>&gt;1GHz</td>
</tr>
</tbody>
</table>

*Table 5.2.1 State-of-the-art waveform generators*
method cannot approach the capabilities of the continuous frequency resolution achievable with narrowband swept frequency techniques, bandwidths of up to two or even three decades can be spanned with a single computation. The TDM technique is thus most suited to wideband components whose properties change relatively slowly with frequency such as material complex permittivity and permeability.\(^{(112)}\)

TDM systems operate with pulse widths of less than 60 ps and pulse amplitude of typically 1 volt. The sampling interval is typically 10 ps with 256 sample points (one sample being measured on each impulse cycle) resulting in a time window of 2.5 \(\mu\)s and an FFT fold-over frequency of 50 GHz. In this way FFT truncation and aliasing effects become small in comparison with other sources of error. Typical scanning times for the 256 point waveform is about 1 minute.

Performance limitations do exist with TDM systems, some being basic to the technique and others due to state of the present technology. One fundamental limitation is that incorrect measurement occurs for non-linear elements such as large signal amplifiers. This is due to the fact that the FFT is not valid for non-linear functions. A second restriction limits the accuracy of narrowband measurements since the spectral energy contained in a narrow band of the spectrum of the excitation signal is low, the measurement signal-to-noise ratio tends to be very low. With present
technology the dynamic range of TDM measurements is limited to about 50 dB. This figure will improve as higher energy impulse generators are developed, table 5.2.1.

Thus, in view of these intrinsic limitations, it is more natural to measure very wideband characteristics with TDM whilst using swept frequency network analysis techniques for high dynamic range, narrowband measurements.

5.2.3 SAW Network Analyser Design

In the following sections, the theory, design and performance of three distinct network analyser configurations employing SAW components are described. These network analyser designs use different arrangements of the chirp transform algorithm\(^{(99)}\) implemented with SAW chirp filters. To simplify differentiation between the three individual designs, these are classified, using the definitions of chapter 4, into network analysers based on the M(S) - C(L) - M arrangement\(^{(69)}\) with the short chirp pre-multiplier configuration; the C(S) - M(L) - C arrangement\(^{(102)}\), and the M(L) - C(S) - M arrangement\(^{(97)}\) which describes the long chirp pre-multiplier configuration.

In the first network analyser design, the M(S) - C(L) - M scheme, the network under test is energised with an accurately generated line spectrum produced by a repetitive narrow IF
pulse train. This technique, which is an extension of the impulse response analysis, used in TDM eliminates synchronisation and window problems with the SAW chirp transform. Subsequent signal processing in the SAW processor where the signal is pre-multiplied with a short duration chirp, convolved with a second, longer duration chirp and finally post-multiplied with a third chirp yields the network transfer function amplitude and phase characteristics as two individual (sampled) time outputs.

In the second SAW network analyser design, the C(S) - M(L) - C scheme is implemented. Here, as an alternative to using three SAW devices to realise the convolve (SAW filter) - multiply (chirp) - convolve (SAW filter), an actively generated sweeping local oscillator, matched to the frequency/time characteristic of the SAW chirp filters, can be used as the multiplying signal. Further, as an alternative to impulsing the network under test and filtering this impulse response in a SAW chirp filter, the same sweeping oscillator signal can be used to energise the network under test. Thus, only a single SAW device is required. In this way, energising the network under test with a long duration chirp waveform, followed by multiplication with a similar, long duration chirp and convolution in a shorter chirp filter yields the network transfer function amplitude and phase characteristics as two continuous outputs. This arrangement is an extension of conventional swept frequency
network analysis techniques which permits significantly higher sweep rates to be employed.

The third network analyser design is broadly similar to the first except that it uses the sliding transform \( M(L) - C(S) - M \) where the pre-multiplier and post-multiplier are of longer duration than the convolution filter. Here the network transfer function is again displayed as two sampled outputs. This configuration incorporates features of the other two designs in that it uses line spectrum excitation of the network under test and also permits analysis of bandwidths greater than that achievable with SAW chirp filters.

5.2.4 \( M(S) - C(L) - M \) Network Analyser Realisation

In an arrangement similar to that described in section 5.2.2 for time domain metrology techniques in network analysis, the SAW chirp transform using the short pre-multiplier arrangement, \( M(S) - C(L) - M \), can perform network analysis. In essence, the SAW processor replaces the computer based FFT processor for calculation of the Fourier transform of the impulse response of the network under test. The SAW \( M(S) - C(L) - M \) scheme thus offers a possible alternative to the computer based FFT processor used in TDM permitting real time operation of TDM systems. The SAW TDM system would obviously still be subject to the inherent limitations of
accuracy, frequency resolution and dynamic range already discussed in section 5.2.2 for conventional TDM systems.

In comparison to computer based FFT processors the SAW chirp transform processor has the fundamental limitation that the transform "time-window", equal to the pre-multiplier chirp duration is fixed whereas the FFT "time-window" is programmable. The performance of the SAW processor is therefore restricted by the finite duration of SAW chirp filters which introduces conflicting bandwidth and resolution constraints. As was shown in chapter 4, compromise is normally selected where the pre-multiplier chirp duration is equal to one half of the impulse response duration of the (convolution) chirp filter. This results in a processor bandwidth equal to the pre-multiplier chirp bandwidth with a frequency resolution ("time-window") governed by the pre-multiplier chirp duration. Due to the finite duration of the pre-multiplier chirp waveform, (see chapter 4), unless the signal to be transformed is time limited to less than the pre-multiplier chirp duration and lies wholly within the pre-multiplier waveform interval, signal truncation will occur and errors will be produced in the output spectrum. Thus to analyse a waveform of arbitrary time duration and arbitrary time of occurrence, attention must be paid to processor truncation effects. This problem is of particular relevance to narrowband network analysis when network impulse response durations may exceed the pre-multiplier chirp duration and also in remote
testing of systems - such as microwave link analysis\(^{113}\) where synchronisation of the chirp pre-multiplier interval to the time of the received impulse response would still present technical problems.

One type of signal where these truncation effects are entirely predictable is the CW signal which yields the characteristic sinc function response as described in section 5.1.2 for the SAW chirp transform. By making use of this known truncation effect a novel technique has been developed\(^{69,114}\) to eliminate truncation and synchronisation errors in the SAW processor. The network under test, figure 5.2.3, is excited with a narrow pulse train. In this case the pulses are gated sections of a microwave carrier as an alternative to baseband pulses, to improve signal-to-noise performances. The pulses occur with period \(T_1/2\) where \(T_1\) is the duration of the chirp pre-multiplier. This impulse train corresponds to a line spectrum with frequency interval \(\Delta_f = 2/T_1\). With short duration pulses the input spectrum is essentially flat over the bandwidth of the SAW processor and by adjusting the pulse repetition interval equal to \(T_1/2\), the network under test is effectively excited by a line spectrum whose components satisfy a condition of orthogonality such that the frequency interval of the input spectral lines equals the frequency resolution of the SAW processor.
Figure 5.2.3 Basic SAW chirp transform network analyser
If the repetition rate of the pulse train is $f_0 = \omega_0/2\pi$, the input to the SAW Fourier transform processor is

$$h(t) = \sum_{n=-N}^{+N} A(n\omega_0) \cos \{n\omega_0 t + \theta(n\omega_0)\} \quad (5.2.1)$$

where the summation limits are defined by integer

$$N = T_1 \times B_1/4 \quad (5.2.2)$$

where $T_1$ and $B_1$ are the pre-multiplier chirp duration and bandwidth. The terms $A(\omega)$ and $\theta(\omega)$ are the amplitude and phase transfer characteristics of the network under test. The envelope of the SAW filter output can then be represented by

$$S(t) = \sum_{n=-N}^{+N} A(n\omega_0) \frac{\sin \{(n\omega_0 + \mu t)T_1/2\}}{(n\omega_0 + \mu t)T_1/2} \quad (5.2.3)$$

where $\mu$ and $T_1$ are the sweep rate and duration of the pre-multiplier chirp. This waveform envelope has amplitude peaks at intervals $t_{pk}$ given by

$$t_{pk} = \frac{n\omega_0}{\mu} \quad -N \leq n \leq +N \quad (5.2.4)$$

For the condition $\omega_0 = 4\pi/T_1$ this reduces to

$$t_{pk} = \frac{4n\pi}{\mu T_1} \quad -N \leq n \leq N \quad (5.2.5)$$
For any particular value of \( n \), a complete sinc response exists for that frequency component. All other values of \( n \), figure 5.2.4, give time separated sinc responses, the time between successive amplitude peaks being \( (4\pi/\mu T_1) \).

Amplitude nulls of the sinc function for a given \( n \) occur at values of \( t = t_n \) corresponding to

\[
(n\omega_0 + \mu t_n)T_1/2 = \pm k\pi \quad (k = 1, 2, 3) \quad (5.2.6)
\]

and for the condition \( \omega_0 = 4\pi/T_1 \)

\[
t_n = 2(2n \pm k)\pi/\mu T_1 \quad -N \leq n \leq N \quad (k = 1, 2, 3) \quad (5.2.7)
\]

Figure 5.2.4 shows that by selection of \( \omega_0 = 4\pi/T_1 \) the output peak for any given frequency component \( n\omega_0 \) occurs at the nulls of all other responses and ensures that truncation effects from the various input components are non-additive. Hence an accurate output can be obtained by sampling at instants where each individual response is not distorted by the other responses.

The amplitude/frequency characteristic \( A(n\omega_0) \) is given directly by the SAW chirp filter output waveform envelope at the sample instants \( t_{pk} \). Extraction of the network phase/frequency characteristic requires separate amplitude limiting of the filter output to yield a signal of the form
\[ S_e(t) = \sum_{n=-N}^{+N} \cos(\omega_c t + \frac{1}{2}ut^2 + \theta(n\omega_0)) \] (5.2.8)

which can be phase detected by multiplying with a post-multiplier chirp and low pass filtering. Subsequent sampling of this signal in synchronism with the amplitude channel yields the phase/frequency characteristic \( \theta(n\omega_0) \). Figure 5.2.5 simulates this performance under short circuit conditions (ie, no network under test) and demonstrates that by sampling the amplitude and phase responses at the dictated instants the flat transfer characteristics of the short circuit test are obtained. If the input test signal is now distorted by the network under test prior to chirp pre-multiplication, the sampled outputs represent directly the amplitude \( A(n\omega_0) \) and phase \( \theta(n\omega_0) \) transfer characteristics of the network.

In the practical realisation, figure 5.2.6, the input test signal was obtained by gating a 145 MHz reference oscillator with pulses of width 25 ns and repetition period 1.25 \( \mu \)s. The line spectrum of this signal, centred at 145 MHz, was flat (±0.5 dB) over 20 MHz bandwidth. A short (8 ns) impulse was used to excite the SAW (pre-multiplier) chirp filter \( C_1(t) \), producing a 2.5 \( \mu \)s burst of up-chirp signal, centred at 60 MHz which swept linearly over 12.5 MHz. After spectral inversion and bandpass filtering, the resulting down-chirp, centred at 85 MHz was used to multiply the input signal. Lower side-band filtering was performed after this multiplication.
Figure 5.2.4  Principle of interlaced sinc functions

Figure 5.2.5  Simulation of M(S) - C(L) - M network analyser.
(Short circuit input conditions)
Figure 5.2.6. SAW network analyser block diagram. (M(S) - C(L) - M)
stage permitting the convolution, and post-multiplication to be performed with down-chirp SAW filters. These were centred at 60 MHz with bandwidths of 25 MHz and 12.5 MHz respectively. Figure 5.2.7 shows the response of this SAW network analyser to a 145 MHz CW input (no gating or network under test). Figure 5.2.7(a) shows the predicted truncation effect in the sinc function response and figure 5.2.7(b) shows the amplitude limited, phase detected output which demonstrates the expected phase reversals in alternate side-lobes of the sinc function. Figure 5.2.8 records the short circuit characteristic of the SAW network analyser. In the upper trace, figure 5.2.8(a) the amplitude/frequency characteristic is presented as a series of overlapping time separated sinc function responses similar to that shown in figure 5.2.5. Only the main lobe of each response is visible and with the input pulse repetition interval fixed at 1.25 μs \((T_1/2)\) the output peaks appear at intervals

\[
t_{pk} = \frac{4\pi}{\mu T_1} = 160 \text{ ns} \quad (5.2.9)
\]

which corresponds in the transform domain to a frequency interval \((\mu = 2\pi \times 5 \text{ MHz/μs})\)

\[
\Delta f = \frac{\mu t_{pk}}{2\pi} = \frac{2}{T_1} = 800 \text{ kHz} \quad (5.2.10)
\]

Figure 5.2.8(b) shows the network analyser phase/frequency
Figure 5.2.7. $M(S) - C(L) - M$ network analyser performance (145 MHz CW input).

Figure 5.2.8. $M(S) - C(L) - M$ network analyser performance (Short circuit input conditions)
characteristic to be flat over the processor bandwidth since sampling at times corresponding to the peaks of the amplitude response would yield a constant level.

Measurements performed on a 5-section 145 MHz centre frequency wideband filter (±7.5 MHz, 3 dB bandwidth) are shown in figure 5.2.9. Transfer function characteristics obtained with a conventional swept frequency network analyser are compared in figure 5.2.9(a) with the SAW network analyser in figure 5.2.9(b). Comparison of the upper traces in figure 5.2.9(b) and figure 5.2.8 shows that the amplitude/frequency characteristic of the SAW Network Analyser has been modified by the transfer function amplitude characteristic of the filter shown in the upper trace of figure 5.2.9(a). The phase/frequency characteristic in the SAW network analyser, detailed in the centre trace of figure 5.2.9(b) shows the same form as the conventional network analyser, changing by 360° over the centre 12 MHz. The phase detector used in the SAW processor was non-linear (cosine law). The network delay can be compensated in the SAW processor to give the flat phase characteristic shown in the lower trace of figure 5.2.9(b). This permits more detailed investigation of phase linearity with frequency, ie, group delay measurement.

Figure 5.2.10 shows the performance of the M(S) - C(L) - M network analyser scheme when output sampling is incorporated.
Figure 5.2.9. M(S) - C(L) - M network analyser performance. (Wide band filter)
(a) Conventional swept frequency network analyser.

(b) SAW network analyser.

Figure 5.2.10. M(S) - C(L) - M network analyser. Performance with output sampling. (Wide band filter)
The network under test was a wideband filter (±5.0 MHz, 3 dB bandwidth) at 145 MHz centre frequency. The filter transfer function characteristics obtained with the SAW analyser are compared in figure 5.2.10 (b) with a conventional swept frequency network analyser in figure 5.2.10(a). The response for the SAW network analyser has a 15 point display.

5.2.5 C(S) - M(L) - C Network Analyser Realisation

As described in chapter 4, it is possible to perform a Fourier transform using SAW chirp filters configured in the arrangement C(S) - M(L) - C. This arrangement permits realisation of an alternative design of SAW network analyser which can be derived from the M(S) - C(L) - M processor design by interchanging the operations of multiplication and convolution. In this second design of network analyser, it is possible to perform the first step of the processing, ie, convolution of the network impulse response with the first chirp filter by directly exciting the network under test with a chirp waveform. The same chirp waveform can also be used as the multiplying chirp which yields the simplified configuration shown in figure 5.2.11. Here the network is excited by the reference chirp, previously time-gated and down-converted by mixing with a local oscillator operating at the centre frequency of the SAW chirp filter. The response of the
Figure 5.2.11. SAW network analyser block diagram. (C(S) - M(L) - C)
network, after multiplication by the reference chirp, is processed in the SAW chirp filter, of opposite frequency/time slope. This selects the difference frequency component and equalises the quadratic phase distortion of the signal present after multiplication.

Using this configuration with the component parameters as shown in figure 5.2.11, the following conditions (102) apply:

\[ B_1 > B_s \]  \hspace{1cm} (5.2.11)
\[ T_0 > T_s + T_1 \]  \hspace{1cm} (5.2.12)
\[ T_2 = T_0 - T_1 \]  \hspace{1cm} (5.2.13)

where \( T_s \) and \( B_s \) are the duration and bandwidth of the input signal. The maximum signal bandwidth which can be processed is \( B_1 \) and the maximum signal duration is \( T_2 \) corresponding to a maximum frequency resolution \( (T_2)^{-1} \).

Excitation of the network under test with a chirp and subsequent multiplication with a chirp is the basic arrangement used in a conventional swept frequency network analyser. For slow sweep rates in the C(S) - M(L) - C network analyser, ie,

\[ \mu / 2 \pi \ll (T_s)^{-2} \text{ Hz/s} \]  \hspace{1cm} (5.2.14)

where \( T_s \) is the impulse response duration of the network under
test, the quadratic phase distortion present after multiplication can be neglected. Equation (5.2.14) corresponds to the condition for correct operation of a conventional swept frequency network analyser. The SAW C(S) - M(L) - C network analyser design therefore offers a technique for converting conventional swept frequency network analyser designs to high sweep rate operation, simply by the incorporation of a single SAW chirp filter. The reference chirp can be passively or actively generated. Passive generation from a SAW chirp filter while ensuring coherent operation and maintaining an exact match of dispersive slope with the final SAW chirp filter, requires SAW chirp filters with a long duration impulse response and high dispersive slope for wideband analysis with high frequency resolution. Thus given a maximum impulse response for the SAW chirp as \( T_0 \), the optimum compromise is to make \( T_2 = T_0/2 \) which maximises both analyser bandwidth, \( B_0/2 \), and analyser frequency resolution \( (2/T_0)^{-1} \) and the useful time-bandwidth product of the network is \( (T_0B_0/4) \). The more preferrable, active generation of long duration chirp waveforms allows wide bandwidths to be analysed with moderate sweep rates. The single SAW chirp filter, which determines the processor frequency resolution can now possess the full duration attainable with SAW devices. In this way using a SAW chirp with a maximum impulse response of \( T_2 \) the analyser bandwidth is now \( B_1 > B_2 \). With active generation it is therefore possible to analyse wide bandwidths
using a single (narrowband) SAW chirp filter, thus avoiding the need for sophisticated large time-bandwidth SAW devices. Note that phase incoherence and frequency drift of the actively generated chirps do not affect the system performance as they are cancelled in multiplication. Problems can arise, however, in matching the dispersive slopes of the active chirp and SAW filter.

Figure 5.2.12 shows experimental results obtained with the C(S) - M(L) - C network analyser. The SAW chirp filter employed in the processor, which was fabricated on ST-Quartz operated at 9 MHz centre frequency with 2.7 MHz bandwidth and 33 µs dispersive delay. The reference chirp of duration 100 µs sweeping linearly over the frequency range 34.5 - 43.5 MHz was generated by a sweep oscillator with phase lock facility. The corresponding chirp used to excite the network under test was derived by mixing with a 9 MHz LO and low pass filtering to produce a chirp sweeping 25.5 MHz - 34.5 MHz in 100 µs. The system is therefore capable of analysing a bandwidth of 6.3 MHz centred at 30 MHz with a frequency resolution of 33 kHz. Figure 5.2.12(a) shows the amplitude/frequency and phase transfer characteristics of a narrowband filter centred at 30 MHz with 500 kHz 3 dB bandwidth. Figure 5.2.12(a) - (iii), shows the amplitude response without the SAW chirp filter and illustrates the intrinsic distortion produced when a high sweep rate is used with a narrowband network in a conventional
Figure 5.2.12  C(S) - M(L) - C network analyser performance (Narrow band filter)

(a) SAW network analyser

(i) AMPLITUDE

500kHz/division

(ii) PHASE

(iii) WITHOUT FILTER

(b) Conventional network analyser

(i) AMPLITUDE

10dB/division

(ii) PHASE

90°/division

Figure 5.2.12  C(S) - M(L) - C network analyser performance (Narrow band filter)
swept frequency network analyser. Figure 5.2.12(b) compares the performance of the SAW C(S) - M(L) - C network analyser with a conventional swept frequency network analyser.

5.2.6 M(L) - C(S) - M Network Analyser Realisation

A third configuration of SAW Network Analyser has been suggested based on the M(L) - C(S) - M scheme. In principle, this configuration is very similar to the M(S) - C(L) - M scheme, but overcomes the bandwidth and frequency resolution constraints imposed by SAW chirp filter fabrication techniques.

In chapter 4, it was shown that the configuration considered here (M(L) - C(S) - M arrangement) performs a valid Fourier transform when the input signal is periodic with period less than the impulse response duration of the SAW chirp filter. This arrangement can be achieved in a manner similar to that of the M(S) - C(L) - M network analyser scheme considered in section 5.2.4 where the inherent problem of synchronisation was overcome by producing a periodic input.

Figure 5.2.13 shows results obtained from computer simulation of the M(L) - C(S) - M network analyser configuration. The simulated pre-multiplying and post-multiplying chirps were defined as being 50 μs duration and 5 MHz bandwidth and the convolution filter having 20 μs duration and 2 MHz bandwidth.
Figure 5.2.13. Simulation of M(L) - C(S) - M network analyser. (Short circuit input conditions)
The simulated processor is capable of analysing 3 MHz bandwidth with 100 kHz frequency resolution. The upper trace of figure 5.2.13 shows the amplitude response, under short circuit test conditions, at the output of the convolution filter. The lower trace shows the phase characteristic after post-multiplication. The required sampling instants are shown in these responses.

5.2.7 Comparison of the Three SAW Network Analyser Designs

All three SAW network analysers can be fully realised employing three SAW chirp filters. The C(S) - M(L) - C and M(L) - C(S) - M processors, however, can use actively generated chirp multiplying signals, with one SAW chirp filter performing convolution. This permits these processors to analyse bandwidths in excess of the SAW chirp filter bandwidth. In the C(S) - M(L) - C processor, a single actively generated chirp can serve two waveform functions, automatically providing phase coherence. In comparison, with the M(L) - C(S) - M scheme, separate, actively generated chirps are, in principle, required for both multipliers, unless a suitable time delay can be incorporated. The M(S) - C(L) - M processor normally employs three identical SAW chirp filters, where the multiplying chirps are derived by synchronous time gating.

A comparison of analyser bandwidth and frequency resolution
is given in table 5.2.2 for the three SAW processors. This shows that in the C(S) - M(L) - C and M(L) - C(S) - M processors, the analyser bandwidth is restricted only by the single sideband filtering required after chirp multiplication, assuming the availability of a suitable linear, wideband sweep oscillator. Hence, both the centre frequency $f_0$ and bandwidth $B$ of the SAW chirp filter control the analyser bandwidth. In the M(S) - C(L) - M processor, the analyser bandwidth is determined by the difference in bandwidth between the convolution chirp filter and the multiplying chirp, normally $B/2$. The frequency resolution of the network analysers based on the M(S) - C(L) - M and M(L) - C(S) - M processors is governed by the discrete (sampled) output responses. These correspond to frequency intervals of $(2T)^{-1}$ where $T$ is controlled by the time duration of the pre-multiplier and the convolution filter impulse response in the M(S) - C(L) - M and M(L) - C(S) - M processors respectively. The C(S) - M(L) - C processor provides continuous outputs similar to conventional swept frequency network analysers. Here the frequency resolution is controlled by the impulse response duration of the SAW convolution chirp filter.

The speed of operation of the three SAW network analysers is assessed by comparison with a swept frequency network analyser in figure 5.2.14. In the context of real time operation defined by the condition that the processor signal
<table>
<thead>
<tr>
<th></th>
<th>M(S) - C(L) - M</th>
<th>C(S) - M(L) - C</th>
<th>M(L) - C(S) - M</th>
<th>CONVENTIONAL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FREQUENCY RANGE</strong></td>
<td>10MHz - 1GHz</td>
<td>10MHz - 1GHz</td>
<td>10MHz - 1GHz</td>
<td>20Hz - 20GHz</td>
</tr>
<tr>
<td><strong>ANALYSER BANDWIDTH</strong></td>
<td>200MHz</td>
<td>1GHz</td>
<td>1GHz</td>
<td>40GHz</td>
</tr>
<tr>
<td><strong>FREQUENCY RESOLUTION</strong></td>
<td>40kHz</td>
<td>20kHz</td>
<td>20kHz</td>
<td>PROGRAMMABLE</td>
</tr>
<tr>
<td><strong>SAW DEVICES</strong></td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td><strong>ACTIVE GENERATION</strong></td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>SLOW SWEEPS</td>
</tr>
<tr>
<td><strong>COMPATABLE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>OUTPUT FORMAT</strong></td>
<td>DISCRETE</td>
<td>DISCRETE</td>
<td>CONTINUOUS</td>
<td>CONTINUOUS</td>
</tr>
<tr>
<td><strong>ERRORS</strong></td>
<td>SAMPLING</td>
<td>μ MISMATCH</td>
<td>μ MISMATCH</td>
<td>REFLECTIONS</td>
</tr>
<tr>
<td><strong>DYNAMIC RANGE</strong></td>
<td>40dB</td>
<td>40dB</td>
<td>60dB</td>
<td>100dB</td>
</tr>
</tbody>
</table>

*Table 5.2.2. Comparison of SAW and conventional network analysers.*
Figure 5.2.14. Performance bounds of SAW network analysers.
bandwidth be greater than or equal to the bandwidth of the signal to be analysed, only the $M(S) - C(L) - M$ SAW processor is real time over its full bandwidth. Both the $C(S) - M(L) - C$ and $M(L) - C(S) - M$ processors, which exhibit an instantaneous real time bandwidth equal to the SAW convolution filter bandwidth, effectively sweep this real time bandwidth over a wider range of centre frequencies. However, the sweep rate and hence operating speed of the SAW network analysers are considerably faster than conventional swept frequency network analysers, figure 5.2.14. Note that swept frequency network analysers are typically limited in sweep rate to $\mu < 2\pi(\Delta f)^2/10$ where $\Delta f$ is the required frequency resolution. In the SAW network analysers, the effective parallel processing in the convolutional filter permits the use of higher sweep rates without compromising the analyser frequency resolution. In figure 5.2.14, the $C(S) - M(L) - C$ and $M(L) - C(S) - M$ processors are assumed to incorporate wideband actively generated chirps which are typically limited to accurate linear sweep rates of less than 100 kHz/µs.

In the measurement of network group delay (deviation from linear phase) it is necessary to employ some means to compensate for the inherent time delay through the network. In the conventional swept frequency network analyser and in the $C(S) - M(L) - C$ SAW network analyser, this requires the use of a separate phase-locked oscillator for phase detection.
In comparison, in the M(S) - C(L) - M and M(L) - C(S) - M SAW chirp transform processors, this time delay compensation can be simply achieved by adjusting the timing of the post-multiplier chirp.

5.3 WAVEFORM SYNTHESIS USING THE (INVERSE) CHIRP TRANSFORM

The inverse chirp transform processor, discussed in chapter 4, was shown to be identical in arrangement to the chirp transform, but employing SAW chirp filters with the opposite frequency vs time slope. The existence of such a signal processor permits generation of a time waveform by defining the Fourier transform of this waveform as the input to the processor (the frequency domain).

In the case of a sinusoidal waveform the concept offers considerable reduction in hardware since here the Fourier transform of a sinusoid of angular frequency \( \omega_s \) consists of two spectral lines located \( \omega = \pm \omega_s \), separated by \( 2\omega_s \). The equivalent time domain version is then given by two impulses separated by

\[
\Delta t_s = \frac{2\omega_s}{\mu}
\]  

and the corresponding output signal is a pulse of frequency \( \omega_s \). By varying the mutual delay \( \Delta t_s \) between the two input pulses, the frequency of the output pulse can be programmably changed.
This technique has been employed for the generation of coherent frequency-hopped (FH) waveforms (115) for use in spread spectrum communications, figure 5.3.1. Here, because the input (frequency domain) consists of impulse signals the pre-multiplier chirp can be dispensed with. Further, for generation of baseband signals where both chirp filters have the same sign, the post-multiplier chirp can be removed, such that two impulses occurring at $t = t_o$ and $t = t_o + \Delta t$ applied to the single chirp filter produce an output frequency $\mu \Delta t/2\pi$, after square law amplification and low pass filtering. When the output is required on a carrier, the chirp post-multiplier dispersive slope is opposite to that of the filter and the sum frequency is chosen.

In baseband (difference frequency) operation, a bandwidth of $\mu T_0/4\pi$ can be synthesised since the absence of the pre-multiplier effectively defines an equivalent input spectral representation over the frequency range zero to $\mu T_0/4\pi$. With sum frequency generation (116) the possibility of interchanging the functions of the two (opposite) chirps permits definition of frequencies in the range $(f_0 + f_2)$ to $(f_0 + f_2) + \mu T_0/4\pi$ in one case and $(f_0 + f_2)$ to $(f_0 + f_2) - \mu T_0/4\pi$ in the other case.

This technique permits extension to generate M-ary phase-shift key sequences. Bi-phase coding can be simply achieved by properly reversing the polarity of both impulses of the input pair. Quadrature phase coding can be achieved by reversing the
Figure 5.3.1 Waveform synthesis by inverse SAW chirp transform
polarity of one impulse of the input pair.

The maximum duration of output pulse is however limited to $T_0/2$ - to achieve the optimal time bandwidth product in the processor - and for this reason, the single synthesiser is limited to 50% duty cycle. In practical applications of frequency hopped waveform synthesis, two interlaced channels are required to produce a continuous output.
CHAPTER 6: APPLICATION OF COMBINED CHIRP TRANSFORM PROCESSORS

INTRODUCTION

Numerous applications which employ combined SAW chirp transform processors have been developed. Access to the frequency domain characteristics of a signal permit frequency domain editing or conditioning to be employed in the configuration of figure 6.1.1. The various conditioning functions which have been used are summarised in table 6.1.1 and suitable reference made to appropriate publications.

6.1 REAL TIME CEPSTRUM ANALYSIS

6.1.1 Introduction to Cepstrum Analysis

The cepstrum (kepstrum) relates to the frequency domain in the same way as the spectrum relates to the time domain. The use of spectrum analysis and network analysis of time waveforms has been considered in chapter 5 for SAW devices. These analysis techniques permit the display of time periodic effects as frequency functions in the spectrum. In this section, the use of cepstrum techniques to analyse periodic effects in the frequency domain is developed. Such frequency domain periodic waveforms are associated with time repetitive or "echo" waveforms.
Figure 6.1.1. Signal processing using combined SAW chirp transform processors

<table>
<thead>
<tr>
<th>EDITING SIGNAL</th>
<th>FUNCTION</th>
<th>APPLICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWITCH ON/OFF WITH EXTERNAL PULSE</td>
<td>BANDPASS/BANDSTOP FILTER (70,71,72,73)</td>
<td>INTERFERENCE SUPPRESSION</td>
</tr>
<tr>
<td>MIX WITH SINEWAVE</td>
<td>VARIABLE DELAY (117) VARIABLE CHIRP (118)</td>
<td>TARGET SIMULATION ECM</td>
</tr>
<tr>
<td>LOGARITHM</td>
<td>CEPSTRUM (119)</td>
<td>SIGNAL CLASSIFICATION</td>
</tr>
<tr>
<td>MULTIPLY WITH CONJUGATE FOURIER TRANSFORM</td>
<td>PROGRAMMABLE MATCHED FILTER (120)</td>
<td>SIGNAL CORRELATION</td>
</tr>
<tr>
<td>MULTIPLY WITH CHIRP</td>
<td>TIMEBASE CHANGE (121)</td>
<td>INTERFACE</td>
</tr>
</tbody>
</table>

Table 6.1.1. Combined SAW Fourier transform processor applications
The need for cepstrum analysis techniques originated for seismological analysis where the arrival times for the various waves - which are essentially multipath echoes - yield information on earthquake focii. The required time separations of the various waves were not sufficiently obvious from time records and hence frequency domain techniques were developed. Although mathematics have been aware of the potential of cepstrum analysis for more than a decade\(^{(122)}\), it is only recently that cepstrum analysis has been applied by system engineers to waveform identification for target classification, signal extraction in multipath or reverberation limited environments\(^{(123,124)}\) and in speech processing\(^{(125)}\). To date, signal processing based on cepstrum techniques has been largely confined to computer based systems which operate on sampled data and, in general, are not real time processors. These software systems have demonstrated the application of the power cepstrum\(^{(123,126)}\), the complex cepstrum\(^{(123)}\) and the phase cepstrum\(^{(127)}\). This section illustrates how the SAW chirp transform processor can be used to implement real time wideband cepstrum analysis with projected application in sophisticated signal processing for radar, sonar and communications systems\(^{(119)}\).

Cepstrum analysis is, by definition\(^{(122)}\), performed by a serial arrangement of two Fourier transform processors. The first processor transforms from the time domain to the frequency domain, yielding the spectrum of the input waveform. After signal amplification (in a true logarithmic amplifier), the second
processor transforms from the frequency domain to a pseudo-time (quefrency)\(^{(122)}\) domain. This yields the cepstrum of the input time domain waveform. For certain waveforms, particularly time periodic and superimposed time coincident waveforms, spectrum analysis can yield an ambiguous display. The deconvolution achieved by the logarithmic processing used in the cepstrum analyser permits detailed examination of the features of the spectrum, to identify constituent elements of these input waveforms.

A simplified description of the operation of the cepstrum analyser is given in figure 6.1.2 for a signal which consists of a pulse of duration \(T\) with an echo at epoch \((\text{delay}) \, \tau'\) relative to the main pulse, figure 6.1.2(a). The effect of this echo in the time domain can be interpreted as the time domain convolution of the basic pulse \(f(t)\) with two impulses separated by an interval \(\tau'\), figure 6.1.2(b). In the frequency domain this corresponds to the product of the spectra of the two signals, figure 6.1.2(c). Taking the logarithm in the frequency domain reduces this frequency domain multiplication to a linear summation of two signals which are approximately periodic in the frequency domain, figure 6.1.2(d). The log power spectrum is a frequency function and it seems reasonable to apply Fourier transform techniques to this function to develop the cepstrum.

The power cepstrum\(^{(122)}\), \(C(\tau)\) has been defined as the power spectrum of the logarithmic power spectrum of an input signal \(s(t)\).
Figure 6.1.2. Cepstrum analysis of pulse waveform.
\[ C(\tau) = |FT(\log|FT(s(t))|^2)|^2 \] (6.1.1)

where FT denotes Fourier transformation. The second Fourier transform effectively analyses the frequency periodic components of the log power spectrum to yield the cepstrum. The term quefrency \(^{122}\) is used in the cepstrum and corresponds to frequency in the spectrum. Quefrency, which is effectively a measure of time period, has units of cycles/Hz (seconds). Figure 6.1.2(e) shows the cepstrum of the waveform under consideration with responses at three different quefrencies.

The zero quefrency response results from the constant dc offset in the log power spectrum while the other two responses correspond to the quefrencies produced by the basic input pulse and echo. If required, the zero quefrency term can be removed by using a dc offset or, alternatively, high pass filtering in the frequency domain before the second transform. Similarly the sum quefrency term occurring at \((T + \tau)\) can be removed by suitable low pass filtering before the second transform. The cepstrum decomposition effect is valid irrespective of the waveform characteristics, even when the waveform and echo (or echoes) are time coincident.

The power cepstrum has been demonstrated \(^{123}\) as an effective analysis technique in the determination of waveform arrival times for a basic waveform distorted by echoes. The technique is effective for echoes of amplitude greater than or less than the basic waveform amplitude. Ambiguity does however exist in respect of the amplitude of the quefrency response due to an echo \(^{123}\).
This can be either a fraction or a multiple of the amplitude of the basis waveform quefrency. With multiple echoes, the number of echoes and their relative delays cannot easily be estimated from the power cepstrum. This is because the cepstrum peaks occur at sum and difference quefrencies of the echo delays, in addition to the predicted quefrencies.

The complex cepstrum is defined as the complex inverse Fourier transform of the complex log Fourier transform. Note that the previous definition of power cepstrum as the modulus of the Fourier transform of the log power spectrum does not conflict with this definition since in both the power spectrum and power cepstrum only real functions are considered hence taking the Fourier transform and inverse Fourier transform are equivalent. The significance of the complex cepstrum is that this procedure permits recovery of the basic wave shape from the contaminating echoes. The algorithm is complicated, however, consisting of taking the complex Fourier transform of the input signal and computing the log of the modulus. Subsequent inverse Fourier transformation of the recombined log modulus and phase yields the complex cepstrum. The complex cepstrum exhibits sharp discontinuities (theoretically delta functions) at positions determined by the echo delay times. These discontinuities can be removed by low pass filtering the complex cepstrum function or alternatively by numerical interpolation. Subsequent inverse cepstrum processing, by taking the Fourier transform followed by amplitude exponentiation and complex inverse Fourier
transformation yields the basic wave shape. This recovery is effective for a wide range of echo amplitudes and in the presence of a large number of echoes although special weighting techniques (128) are required to eliminate the masking effect of echoes at short delays over longer delay echoes. The complex cepstrum, however, does not easily yield the echo arrival times since the peaks in the complex cepstrum are easily masked. The power cepstrum is a more appropriate technique for the determination of echo arrival times. In the detection of signal phase in the presence of distorting noise the phase cepstrum (127) derived from the complex cepstrum, has been demonstrated. The complex cepstrum evaluated in two dimensions (129) extends the one dimensional complex cepstrum to the field of image processing (130) where the deconvolution properties of the cepstrum permit elimination of certain types of image blurring. The cepstrum in two dimensions can also be used for image classification where specific features such as rectangles or cylinders appear as distinct features in the cepstrum (130).

The main application of the cepstrum lies in the deconvolution of speech, where by determining the pitch period of speech (131), high quality speech transmission is possible using reduced bandwidth. In addition the cepstrum has been applied to the measurement of signal amplitude (132) and in adaptive decomposition of signals (133).
6.1.2 SAW Cepstrum Analyser Operation

Cepstrum analysis can be performed by coupling two SAW chirp transform spectrum analysers through a true logarithmic amplifier, figure 6.1.3. The chirp filters used in the demonstration cepstrum analyser were fabricated on ST,X cut quartz at 60 MHz centre frequency. The signal used to excite the multiplying chirp was an impulse of duration 8 ns (corresponding to one-half cycle at 60 MHz). The SAW filter impulse response duration of 5 μs comprises a linear frequency sweep over 25 MHz bandwidth with a corresponding dispersive slope of 5 MHz/μs. After amplification, synchronous time gating was used to obtain the required 2.5 μs pre-multiplier chirp sweeping 12.5 MHz. The two chirp multipliers and the associated gating were all derived synchronously from the master clock employed in the timing circuitry. As discussed in chapter 4, post-multiplication was not employed in the SAW spectrum analysers. The logarithmic amplifier used was a four stage Plessey SL530C amplifier. The first chirp transform processor (figure 6.1.3) transforms from the time domain to the frequency domain and is capable of analysing a bandwidth of 12.5 MHz in 2.5 μs with a CW frequency resolution of 400 kHz determined by the width of the sinc function response. The second chirp transform processor permits cepstrum analysis of quefrequencies up to 2.5 μs with a resolution of 80 ns, again determined by the sinc function obtained in the cepstrum.
Figure 6.1.3. Block diagram of SAW cepstrum analyser.
6.1.3 Determination of Pulse Duration

The power spectrum of a pulse of duration $T$ which is obtained as a time function at the output of the SAW processor is given by

$$|\phi(\omega)|^2 = |\phi(\mu t)|^2 = \frac{\sin \mu t T}{\mu t}$$ (6.1.2)

where $\mu$ corresponds to the dispersive slope of the SAW chirp filters. Logarithmic amplification yields

$$\log |\phi(\mu t)|^2 = 2 \log (\sin \mu t T) - 2 \log (\mu t T)$$ (6.1.3)

which is approximately periodic in the frequency domain with period $\omega_T = \mu t = 1/T$. Since the log spectrum approximates to a sinusoid further analysis yields a single cepstrum response whose quefrency is determined by $T$, the basic pulse duration.

Figure 6.1.4 shows the operation of a demonstration SAW cepstrum processor with such a pulse waveform. The spectrum of a $T = 1$ $\mu$s pulse is shown in figure 6.1.4(a) as measured at the output of the first SAW chirp transform processor. This demonstrates the characteristic sinc function envelope with nulls spaced at 200 ns which corresponds to a frequency interval of 1 MHz (1/T) for this SAW chirp transform processor with dispersive slope of 5 MHz/$\mu$s. The waveform envelope after logarithmic amplification, figure 6.1.4(b) shows the spectrum envelope to be
Figure 6.1.4. SAW cepstrum analyser operation
(1μs pulse waveform)
more nearly cosinusoidal, with period 200 ns. The second SAW chirp transform processor therefore effectively sees a 5 MHz CW signal and performs a spectrum analysis of this waveform. The output of the second chirp transform processor, figure 6.1.4(c), demonstrates sinc function response at $t = \pm 1.0 \mu s$, corresponding to the power spectrum of a truncated 5 MHz CW signal. Figure 6.1.4(c) corresponds to the cepstrum of the input signal to the first chirp transform processor with responses of $\tau = \pm 1 \mu s$ and can be used to directly measure pulse width. Note the presence of the zero quefrency response in figure 6.1.4(c) which could be removed by incorporating a dc offset in the log spectrum. Here, the variation in amplitude between the positive and negative cepstrum peaks is attributable directly to a non-uniform amplitude response in the second SAW convolution chirp filter.

The cepstrum responses identifying pulse durations of $T = 0.8 \mu s$ and $T = 1.5 \mu s$ are shown in figure 6.1.5. The described SAW filter parameters $T = 5 \mu s$, $B = 25 MHz$, permit this cepstrum analyzer to determine pulse durations up to the 2.5 $\mu s$ duration of the pre-multiplier chirp. In practice, the zero quefrency term present in the cepstrum limits the minimum resolvable pulse duration to typically 150 ns. The SAW processor, with a limited time bandwidth product of 32, is therefore capable of determining pulse duration over a range in excess of 10 to 1. As discussed in chapter 4, the chirp transform processor operates in a synchronous mode thus the
Figure 6.1.5. Cepstrum determination of pulse length.
accurate determination of pulse duration requires the input pulse to be time coincident with the pre-multiplier chirp. As might be expected, however, variation of the pulse timing relative to the pre-multiplier chirp results simply in a phase change in the spectrum with no effect on the power spectrum or power cepstrum. The responses shown in figure 6.1.4 and figure 6.1.5 were in fact obtained with a pulse operating synchronously with the cepstrum processor.

In comparison to alternative techniques for the determination of pulse length, such as signal differentiation, the cepstrum processor provides signal enhancement over noise in a noisy environment. Consider a chirp pre-multiplier with bandwidth $B_1 \text{ MHz}$ and duration $T_1 \text{ ms}$ with an input pulse of duration $T \text{ ms}$ and signal strength 0 dBm contaminated by noise of bandwidth $B_1 \text{ MHz}$ and level 0 dBm. The maximum signal-to-noise-ratio (SNR) in the spectrum display which occurs at the peak of the sinc response is given by

$$\text{SNR}_p = 10 \log(\mu T^2/2\pi) = 10 \log(B_1 T_1 T^2/T_1^2) \quad (6.1.4)$$

As the input duration $T$ is reduced, the SNR improvement reduces. However, the lobes of the sinc response widen, maintaining good average SNR over the output waveform. The number of sidelobes in the transform window is given by

$$N = 2 T_1(\mu T/2\pi) = 2 B_1 T_1(T/T_1) \quad (6.1.5)$$
and the SNR enhancement of the $N_{th}$ sidelobe is

$$\text{SNR}_n = \text{SNR}_p - L$$  \hspace{1cm} (6.1.6)

and the value $L$ is shown in figure 6.1.6(a). The effect of the logarithmic amplifier is to force the SNR enhancement to unity since low level signals are preferentially amplified. A typical log amp characteristic is given by

$$V_{\text{out}} = 1.0 + 0.025 \{P_{\text{in}} \text{ (dBm)} - 40 \}$$  \hspace{1cm} (6.1.7)

for amplifier input power, $P_{\text{in}}$, in the range

$$-80 \text{ dBm} < P_{\text{in}} < 0 \text{ dBm}$$

where the noise level at the log amp input is assumed -40 dBm.

The SNR at the output of the log amp is given by, figure 6.1.6(b),

$$\text{SNR}_{1p} = [10 \log 1.0 + 0.025 \{\text{SNR}_p - 3\}]$$  \hspace{1cm} (6.1.8)

$$\text{SNR}_{1n} = [10 \log 1.0 + 0.025 \{\text{SNR}_n - 3\}]$$  \hspace{1cm} (6.1.9)

where a 3 dB loss in SNR is assumed for the square law detector. Using figure 6.1.6(b) the SNR at the output of the second SAW chirp transform processor can be shown to lie in the range
Figure 6.1.6. Signal-to-noise performance of SAW cepstrum analyser
Thus, in the cepstrum analyser, the second chirp transform processor operates with an input SNR close to unity which is subject to the full SNR improvement of $10 \log \frac{\mu T}{2\pi}$ independent of input pulse duration $T$. Here $T$ is the duration of the chirp pre-multiplier of the second processor. Note that increasing the time bandwidth product $(\mu T/2\pi)$ of the second transform processor would tend to increase the overall processing gain of the cepstrum analyser and also permit investigation of the wave-shape surrounding the nulls of the sinc response which are determined by the rise time and fall time of the input pulse.

### 6.1.4 Decomposition of a Pulse with Distorting Echoes

A signal, $f(t)$, distorted by an echo of amplitude $\alpha$ and relative delay $\tau'$ can be expressed as

$$s(t) = f(t) + \alpha \cdot f(t - \tau') \quad (6.1.10)$$

The spectrum of this signal is given by

$$S(\omega) = F(\omega) + \alpha \cdot F(\omega) \cdot \exp(j\omega \tau') \quad (6.1.11)$$

and the relevant power spectrum can be written
\[ |S(\omega)|^2 = \phi \cdot \{1 + 2\alpha \cos \omega \tau' + \alpha^2\} \]  \hspace{1cm} (6.1.12)

where \( \phi = |F(\omega)|^2 \)

Equation (6.1.12) is seen to be a product of two terms, i.e., a frequency domain multiplication or a time domain convolution. These terms can be decomposed or deconvolved by taking the logarithm

\[ \log |S(\omega)|^2 = \log \phi + \log \{1 + 2\alpha \cos \omega \tau'\} \]  \hspace{1cm} (6.1.13)

and it has been shown\(^{(123)}\) that for \( \alpha < 1 \) and \( \alpha > 1 \), equation (6.1.13) reduces to a single cosinusoidal quefrency response of amplitude \( 2\alpha \) and periodicity \( 1/\tau' \), corresponding to the echo.

In the case of a pulse of duration \( T \) with a distorting echo of amplitude equal to the basic pulse amplitude and at epoch \( \tau' \), relative to the basic pulse, equation (6.1.13) can be written \((\alpha = 1)\)

\[ \log |S(\omega)|^2 = \log (\sin \mu \tau' \cdot \alpha) + \log (\cos \mu \tau') \]  \hspace{1cm} (6.1.14)

considering only the frequency dependent terms.

The cepstrum of a pulse signal of duration \( T = 800 \) ns, distorted by echoes is illustrated in figure 6.1.7 for echoes
Figure 6.1.7. Cepstrum decomposition of pulse and echo.
of amplitude equal to the basic pulse \((a = 1)\) arriving at epochs of \(\tau' = 1000\) ns, figure 6.1.7(b) and \(\tau' = 400\) ns, figure 6.1.7(d). The cepstrum shows one response corresponding to the basic pulse duration at 800 ns with a further response at the echo epoch. Information on basic pulse length and echo epoch are not directly obvious in the signal spectra. The 400 ns echo epoch condition corresponds to a self-distortion of the waveform since the pulse and echo are partially time coincident.

**6.1.5 Measurement of Pulse Repetition Period**

The spectra of two 100 ns pulse trains are shown in figure 6.1.8 with repetition frequency (p.r.f.) of 1 MHz and 2 MHz. The corresponding cepstrum responses are also shown. The spectra exhibit the expected sinc function envelope with nulls at \(\pm 10\) MHz (\(\pm 2\) μs x 5 MHz/μs), corresponding to the pulse width of 100 ns. The lines within this sinc envelope have separations determined by the input p.r.f. The power cepstrum of the 1 MHz waveform, figure 6.1.8(b) exhibits peaks at quefrency \(\tau = \pm 1\) μs, which correspond to the repetition period. The spectrum of an input p.r.f. of 2 MHz is shown in figure 6.1.8(c) while figure 6.1.8(d) shows unambiguous cepstrum peaks at \(\tau = \pm 0.5\) μs.
Figure 6.1.8. Cepstrum determination of repetition rate.
6.1.6 Determination of Binary Code Length and Bit Rate

In a manner similar to the measurement of repetition period, the power cepstrum can also be used to determine the code length of a binary code and the bit rate.

The spectrum of a 7 bit pseudo noise code at 5 MHz bit rate is shown in figure 6.1.9(a). The first nulls of the sinc envelope are spaced at ±5 MHz (±1 ns x 5 MHz/ns) and the number of spectral lines to the first null is equal to the code length (7 bits). The centre response (dc) is due to the bit disparity of unity in the code. The power cepstrum, figure 6.1.9(b) of this code is shown to consist of peaks at $\tau = \pm 1.4 \mu s$. This corresponds to the quefrency of a pseudo noise code of length $n$ bits given by

$$\tau = \frac{2n}{R} \quad (6.1.15)$$

where $R$ is the bit rate. In figure 6.1.9(b), $\tau = 7/5 = 1.4 \mu s$ as predicted. The spectrum and power cepstrum of a 15 bit code at 10 MHz bit rate are shown in figure 6.1.9(c) and figure 6.1.9(d) with cepstrum peaks at $\tau = \pm 15/10 = 1.5 \mu s$. Note that the bit rate information is not displayed in these traces due to the fact that this is beyond the resolution capability of the demonstration processor with time bandwidth product 32.
Figure 6.1.9. Cepstrum analysis of PN codes.

(a) SPECTRUM
2.5MHz/division

(b) CEPSTRUM
500ns/division

(INPUT 7 BIT PN CODE AT 5MHz RATE)

(c) SPECTRUM
2.5MHz/division

(d) CEPSTRUM
500ns/division

(INPUT 15 BIT PN CODE AT 10MHz RATE)
6.1.7 Determination of Chirp Slope

One potential application of the SAW cepstrum analyser lies in the determination of the unknown dispersive slope of a radar chirp signal. Practical difficulties have prevented experimental studies hence computer simulation has been undertaken to determine the capability of the SAW cepstrum analyser in determining the unknown dispersive slope of a chirp.

The power spectrum of a chirp waveform centred at frequency $f_0$ and sweeping over a bandwidth $B_0$ in time $T_0$ is given by \[(6.1.16)\]

$$|F(\omega)|^2 = \pi/4 \mu \cdot \left\{ \left[ C(X_1) + C(X_2) \right]^2 + \left[ S(X_1) + S(X_2) \right]^2 \right\}$$

where $C(X)$ and $S(X)$ are respectively the Fresnel cosine and Fresnel sine integrals and

$$X_1, X_2 = (T_0 B_0 / 2)^{1/2} (1 \pm n): \quad n = (f - f_0) / B_1$$

The form of the Fresnel integrals has the basic shape of a chirp and the power spectrum of any chirp signal of finite duration displays these characteristic Fresnel ripples. For an input chirp of duration greater than the duration of the first chirp pre-multiplier, an increase in dispersive slope (and hence $B$) will increase the rate of variation with frequency of the Fresnel ripples. This effect results in an increase in the quefrency of
the unknown chirp spectrum. Hence the cepstrum of an unknown chirp waveform is predicted to exhibit a linear variation of quefrency with input chirp slope.

The flow diagram of the computer program used in the simulation is shown in figure 6.1.10. Basically the program defines the SAW chirp waveforms as complex values and, in effect, performs a CZT. However, it was demonstrated in chapter 4 that the chirp transform and CZT were equivalent. Complex signal convolution is performed using an IBM subroutine HARM to implement the FFT which achieves efficient computation. Figure 6.1.11 shows simulated results for the power cepstrum analysis of two input chirps with dispersive slope 7.0 MHz/μs and 10.0 MHz/μs in a cepstrum analyser employing the previously described SAW chirp filters with dispersive slope 5.0 MHz/μs. Analysis of a range of input chirp slopes from 6 to 12 MHz/μs has confirmed that the position of the largest peak in the cepstrum response is linearly dependent on the dispersive slope of the unknown chirp over this range, figure 6.1.12.

6.1.8 Comment on SAW Cepstrum Analyser Operation

This chapter has described the principles and demonstrated the performance of a wideband real time SAW cepstrum analyser. The waveforms used here were restricted to the pulse waveforms normally encountered in radar and communication systems, however, the cepstrum analyser operation is independent of wave shape.
Figure 6.1.10. Flow chart of SAW cepstrum analyser simulation.
Figure 6.1.11. Cepstrum analysis of chirp waveforms. (Simulation results)
Figure 6.1.12. Cepstrum analysis of chirp slope.
The SAW chirp filters used with modest time bandwidth product of 125 permit the computation of a 32 point transform on 12.5 MHz signal bandwidth.

The processor was shown to be effective in determining pulse duration over a 10:1 range from 250 ns to 2.5 μs, this being limited by the time bandwidth product of the SAW devices. With currently available SAW chirp filters of impulse response durations ranging from 2 μs to 50 μs and time bandwidth products up to 10,000, SAW cepstrum analysers\(^{134}\) have the projected capability of measuring pulse durations in the range 50 ns to 50 μs. In comparison to conventional threshold detection or differentiation methods for pulse duration measurement, the SAW cepstrum analyser suffers the limitation of being a synchronous processor with 50% duty cycle. However, the ability of the cepstrum processor to decompose a basic waveform and interfering echoes was demonstrated by determination of relative epochs. In this context, the SAW processor offers the capability of isolating basic radar returns from distorting echoes. In a dense, multiple echo environment, the cepstrum display will exhibit responses at the relevant quefrencies as well as at sum and difference points. However, given that the quefrequency due to the basic pulse duration is constant, it is possible to determine this pulse duration, since assuming motion of the transmitter or space diversity techniques in the receiver, all other quefrencies due to echoes would be changing. Further, it is conceivable that a real time signal processor capable of
determining the complex cepstrum, could be designed, permitting the extraction of an unknown radar signal from a distorted signal produced by echoes.

The experimental SAW processor was effective in determining the pulse repetition period of a waveform over a 10:1 range from 250 ns to 2.5 μs. Again, using currently available SAW chirp filters this capability can be extended to measurement of repetition period in the range 50 ns - 50 μs. The same approach has been used in the cepstrum analysis of binary codes at bit rates up to 10 MHz and code lengths up to 15. This performance can be extrapolated to permit measurement of bit rates up to 200 MHz and code lengths up to 511.

Computer simulation of the system performance in the determination of unknown chirp slope has demonstrated a capability of measuring chirp slope up to 2 μ where μ is the dispersive slope of the SAW chirp filters used in the processor. In principle cepstrum processors implemented with SAW devices can be designed for the determination of chirp slopes in the range 40 kHz/μs to 40 MHz/μs.

An alternative technique has been reported which can be used to measure unknown chirp slope. This employs two SAW filters with a quadratic frequency vs time characteristic interconnected by a programmable local oscillator whose frequency determines the chirp slope. In comparison to this technique, the SAW cepstrum analyser offers real time - as
opposed to swept - measurement of chirp slope. However, the
programmable system offers the possibility of operation as a
matched filter, permits measurement over very wide (10:1) range
of dispersive slope and is asynchronous.

6.2 PROGRAMMABLE CORRELATOR

6.2.1 Basic Principles

The convolution theorem represents one of the most
important relationships in signal processing, giving complete
freedom to implement the convolution of two signals, \( s(t) \) and
\( r(t) \), either as a time domain convolution

\[
x(t) = s(t) * r(t) = \int_{-\infty}^{+\infty} s(\tau) \cdot r(t - \tau) d\tau
\]

(6.2.1)

or as a multiplication, in the frequency domain, of the Fourier
transforms of these two signals, followed by inverse transformation

\[
x(t) = \text{FT}^{-1} [S(\omega) \times R(\omega)]
\]

(6.2.2)

Similarly, time domain correlation,

\[
y(t) = s(t) * r(t) = \int_{-\infty}^{+\infty} s(\tau) \cdot r(t + \tau) d\tau
\]

(6.2.3)

can be performed as a frequency domain multiplication of two
Fourier transforms followed by inverse transformation, although
in this case, one transform must incorporate phase conjugation.
\[ y(t) = \text{FT}^{-1} [S(\omega) \times R^*(\omega)] = \text{FT}^{-1}[S^*(\omega) \times R(\omega)] \quad (6.2.4) \]

The output of any filter is the convolution of the input waveform with the impulse response of the filter. Thus, a variable filter can be made by transforming the input and reference signal to the frequency domain, multiplying the signals and transforming back to the time domain. The signal phase conjugation required to implement correlation of the input and reference can be achieved in three ways:

(i) \( r(t) \) may be time reversed since if \( r(t) \) is a real function, \( \text{FT}[r(-t)] = R^*(\omega) \).

(ii) selection of the difference frequency terms after multiplication of the two transforms - assuming a suitable frequency offset, implements conjugation(73).

(iii) using an inverse Fourier transform processor to produce the inverse transform of the reference signal which corresponds to the Fourier transform of the time reverse version of the reference. This scheme requires dissimilar Fourier transform processors for the signal and reference which operate with equal, but opposite dispersive slopes.

The main advantage of this technique is that it can perform matched filtering for a library of signals using a design which employs well-characterised SAW devices. An additional advantage lies in the fact that access to the spectra of the signals permits
sophisticated equalisations and compensations to be implemented for the purposes of time-sidelobe suppression and interference rejection. Further, the matched filter is not limited to any particular class of input waveform (in comparison to programmable tapped delay line designs\(^\text{(135)}\)) and the only signal limitations are that the signals fall within a specified bandwidth, time window and dynamic range.

6.2.2 Realisation with SAW Devices

Figure 6.2.1 shows a block diagram of a basic programmable matched filter using the SAW chirp transform\(^\text{(120)}\). Two identical chirp transform processors generate the Fourier transforms of signal and reference. The LO in the reference branch translates that signal in frequency and, by selecting the difference frequency sideband, forms its complex conjugate. The multiplication of the transformed signals is a critical step in the SAW processor. Figure 6.2.1 shows a double balanced mixer employed to perform this multiplication. However, as was shown in chapter 5, for a mixer to provide true signal multiplication, it is necessary that one signal be at sufficiently high level to keep the mixer in saturation, and in general a true bilinear multiplier is required to perform correlation. For the particular case when the reference signal is a flat, linear FM (chirp), the reference transform permits multiplication to be performed in a mixer\(^\text{(120)}\).
Figure 6.2.1. Basic SAW chirp transform correlator.
The availability of the signal spectrum using chirp transform techniques permits additional signal processing, since video modulation of the transform output corresponds to modification (filtering) of the input signal spectrum. Narrowband interference is a common problem in radar and spread spectrum systems and using the filter techniques described in table 6.1.1(71) it is possible to time-gate out the effect of such a CW interference in the signal spectrum(70, 71). An alternative technique which corresponds to a pre-whitening filter consists of amplitude limiting the signal transform(71). These two techniques have been shown to yield a reduction of 10 dB in the effect of CW interference on the correlation output.

An important practical consideration in the design of the SAW programmable matched filter, lies in the fact that the effect of device errors can be minimised by balanced cancellation, thereby reducing the correlation errors. For example, it is possible to use the reference information, say a chirp waveform derived as the impulse response of a SAW chirp filter, as the transmitted signal in a radar application. Thus any errors in the SAW chirp filter would be automatically compensated. Also since errors in the chirp pre-multiplier are divided equally between signal and reference transform channels and since the convolution filters in each are matched, the errors introduced in the correlation output are minimised and only errors which occur in the SAW chirp filters used in the inverse Fourier
transform cause problems. In fact, it has been shown\(^{(120)}\) that the error performance of such a programmable matched filter with chirp waveforms is superior to the error performance of a straight pulse compression loop.

6.3 MEMORY CORRELATOR

6.3.1 Basic Principles

In the previous section the design of a programmable correlator using SAW chirp filters was discussed. The correlator operates by multiplying the complex Fourier transforms of input and reference signals. Subsequent inverse Fourier transformation displays the conventional time domain correlation function.

Addition of a memory capability to this design permits the storage of reference signal information for correlation with delayed replica signal returns. This offers an extension to the error reductions discussed in section 6.2.2, in that by storing a version of the transmitted signal—actually derived at the transmitting antenna—errors and distortions introduced in the up conversion/power amplification/down conversion process can be correlated-out.

This realisation of memory correlator\(^{(136)}\) offers a possible alternative to semiconductor-based storage convolver designs\(^{(137,138)}\). One advantage of the new approach is that access to the frequency domain permits spectral weighting to be introduced. One disadvantage lies in the synchronous nature of the processor
which requires either, use of chirp multipliers longer than the expected signal durations or time-interlaced processor channels. In terms of component hardware, a single SAW processor can be used, figure 6.3.1 to perform the Fourier transforms of both the reference - in the "STORE" cycle (when the transformed information is stored) - and the signal - in the "CORRELATE" cycle when correlation takes place. This reduces the number of chirp filters required to four, figure 6.3.1.

6.3.2 Memory Correlator Using Recirculating SAW Delay Line Memory

The memory correlator design shown in figure 6.3.1 employs a recirculating SAW delay line for reference waveform storage. The input signal at 34 MHz is pre-multiplied by a chirp waveform, generated by impulsing a SAW chirp filter, which sweeps linearly over a bandwidth $B_1$ in time $T_1$. At the output of the convolution chirp filter, the switch labelled "STORE/CORRELATE" permits the first transformed input signal sample to be stored as the reference waveform for correlation against an identical time delayed signal return. The quadratic phase term does not require to be removed before the memory input.

The memory used here consists of a SAW delay line of delay $T_m$ and bandwidth $B_m > B_1$. By recirculating the stored waveform $M$ times, the memory storage time can be extended to $M \times T_m$ and by multiplying this recirculating data with the Fourier transform of subsequent input signals, followed by
Figure 6.3.1. Basic SAW chirp transform memory correlator.
inverse Fourier transformation, the required correlation function can be displayed.

Figure 6.3.2 shows experimental results for a chirp transform correlator using SAW chirp filters at centre frequency 17 MHz with bandwidth 2 MHz and impulse response duration 20 μs. The shorter duration chirp pre-multipliers are derived by synchronous time gating, to provide the correlator with a time-bandwidth product of 10. Figure 6.3.2(b) shows the autocorrelation function of 13 bit Barker code at a bit rate of 1.5 MHz which was bi-phase modulated on to a 34 MHz carrier. The "bright-up" region in figure 6.3.2(b) corresponds to the output time window when the correlation output is valid. Some distortion, due to the signal bandwidth exceeding the correlator bandwidth is evident in broadening of the compressed pulse and reduction in sidelobe amplitudes. Note that these results were obtained without memory, to demonstrate the processor operation.

Figure 6.3.3 shows the operation of the correlator when a SAW recirculating memory of delay $T_m = 25.5 \mu s$ and 2 MHz bandwidth is employed. The signal and reference waveforms were both 6 μs bursts of 34 MHz carrier. In figure 6.3.3(a) the memory can be seen performing 32 recirculations of the transformed reference. This corresponds to a storage time of 0.8 ms before the memory loop is cleared. Figure 6.3.3(b) shows the correlation output when input signals occur synchronous with every second memory recirculation. Note that the build-up of
Figure 6.3.2. SAW chirp transform correlator operation. (No memory circuits)
Figure 6.3.3. SAW chirp transform memory correlator operation.
spurious signals and noise in the recirculating memory signal, figure 6.3.3(a), produce only a minimal degradation effect on the correlation function, figure 6.3.3(b). Figure 6.3.3(c) shows in greater detail the correlation function after 0, 5, 10, 15 recirculations. The relative 10 μs interval between each response in figure 6.3.3(c) is used to produce a convenient display, the absolute interval being 127.5 μs.

Figure 6.3.4 shows results from a second memory correlator design which operated on a $T_1 = 2.5$ μs sample window with a $B_1 = 12.5$ MHz bandwidth centred at 120 MHz, producing a correlator time bandwidth product of 32. The memory used here incorporated a ST,X quartz substrate SAW delay line of delay $T_m = 15$ μs operating at 140 MHz centre frequency with 32 MHz, 3 dB bandwidth. In figure 6.3.4(a) the correlation function of a 20 bit subsequence, derived from a 127 bit maximal length sequence, bi-phase modulated at 8.65 MHz rate on a 120 MHz carrier is shown after 4 and 5 recirculations within the memory. In figure 6.3.4(b) the correlation response is distorted when in-band CW interference at 120 MHz whose amplitude is 5 times greater than the coded signal, is introduced at the input, coincident with the 5th memory cycle. Incorporation of a limiting amplifier in the signal transform channel is seen, figure 6.3.4(c), to reduce by approximately 6 dB, the distortion of the output correlation function.
6.3.3 SAW Memory Correlator Design Parameters

Figure 6.3.5 shows the timing diagram of the system relative to the impulse excitation time, \( t = 0 \), of chirp \( C_1 \). The dispersive delay, \( T_1 \), \( T_2 \) and non-dispersive delay, \( \Delta_1 \), \( \Delta_2 \) of chirp filters \( C_1 \), \( C_2 \) produce a valid Fourier transform (bright-up in figure 6.3.3) over the time interval

\[
\Delta_1 + \Delta_2 + T_1 \leq t \leq \Delta_1 + \Delta_2 + T_2
\]  

(6.3.1)

This time interval must match the memory data block duration \( T_B \) and have a corresponding repetition period \( T_m \). Further, the second chirp pre-multiplier, \( C_3 \), in the inverse Fourier transform must also occur within this interval. For optimum processor design, operation of the chirp pre-multiplier, \( C_1 \), with 50\% duty cycle results in the memory and correlator operating on a signal sample of duration \( T_B \) where

\[
T_B = T_2 - T_1
\]  

(6.3.2)

which equals \( T_1 \) when \( T_2 = 2T_1 \), and \( T_m = T_2 \). When the condition \( T_m = T_2 \) is not satisfied, this also reduces the processor duty cycle.

Optimum correlator design thus dictates that \( T_m = T_2 \) to permit operation with 50\% duty cycle with four chirp filters.
Figure 6.3.4. SAW memory correlator operation.
(Suppression of CW jamming signal)

(a) NO INTERFERENCE
(b) WITH INTERFERENCE
(c) WITH LIMITING

Figure 6.3.5. Saw memory correlator design parameters.
CHAPTER 7: DESIGN OF CCD FOURIER TRANSFORM PROCESSORS

7.1 THE CHIRP-Z-TRANSFORM

7.1.1 Derivation of the Chirp-Z-Transform Algorithm

The discrete Fourier transform (DFT) is defined

\[ F_k = \frac{1}{N} \sum_{n=0}^{N-1} f_n \cdot \exp(-j2\pi kn/N) \]  

(7.1.1)

where the N samples of the input sequence \( f_n \) and the N samples of the transform \( F_k \) can be complex values. The range of values of \( k \) can be conveniently taken as 0 to N-1 since the exponent in equation (7.1.1) is cyclicly periodic in N. It is possible to rewrite equation (7.1.1) using the substitution

\[ -2nk = (k - n)^2 - k^2 - n^2 \]  

(7.1.2)

and the result is, (omitting the constant)

\[ F_k = \exp(-j\pi k^2/N) \times \]  

\[ \sum_{n=0}^{N-1} \{ f_n \cdot \exp(-j\pi n^2/N) \} \exp(j\pi(k - n)^2/N) \]  

(7.1.3)

Thus, equation (7.1.3) can be considered as consisting of three basic processing steps which involve discrete chirp waveforms of the form \( \exp(-j\pi n^2/N) \).
The term "chirp" is applied here since the function can be written

\[ \exp(-j\pi n^2/N) = \exp(-j2\pi \cdot \frac{1}{2}m^2) \]

\[ = \cos (2\pi \cdot \frac{1}{2}m^2) - j \sin (2\pi \cdot \frac{1}{2}m^2) \]

(7.1.4)

where \( n^2 = N \mu t^2 \), and \( \mu \) is a constant. This equation is seen to contain the type of terms previously defined in chapter 2 as a "chirp" waveform. The algorithm defined in equation (7.1.3) is therefore known as the chirp-z-transform (CZT) in that it performs a z-transform (the discrete Fourier transform being a special case of the z-transform, see section 3.1), using sampled chirp values.

The calculation of the discrete Fourier transform coefficients \( F_k \), can therefore be performed by the following algorithm: (1) premultiplication of the input sequence \( f_n \) with discrete (complex) samples of a chirp waveform; (2) convolution of the product with discrete chirp samples; (3) post-multiplication with discrete chirp samples. The CZT therefore effectively reduces the computation of the discrete Fourier transform to a convolution process (with attendant multiplications). For digital implementation this offers no advantage over the FFT algorithms (section 3.1). However, the availability of CCD analogue transversal filters now permits efficient real
time convolution, and computation of the CZT(63).

For simplicity, equation (7.1.3) can be re-written in a form which can be directly related to the impulse response of a CCD transversal filter by using the substitution

\[ m = k + N - n \]  

(7.1.5)

where \( m \) is taken as the index denoting the \( m^{th} \) CCD output tap. The CZT can then be written

\[
F_k = \exp(-j\pi k^2/N) \sum_{m=k+1}^{k+N} \exp(j\pi(m - N)^2/N) \times \{f_{k+N-m} \cdot \exp(-j\pi(k+N-m)^2/N)\}
\]  

(7.1.6)

By blanking the input data \( f_n \) in the range

\[ N < n < 2N - 1 \]

(which demands a 50% duty cycle for the CZT processor), the summation limits in equation (7.1.6) can be more conveniently written

\[
F_k = \exp(-j\pi k^2/N) \sum_{m=1}^{2N-1} \exp(j\pi(m - N)^2/N) \times \{f_{k+N-m} \cdot \exp(-j\pi(k+N-m)^2/N)\}
\]  

(7.1.7)

since for \( m \) in the range \( 1 \leq m \leq k+1 \), \( f_{k+N-m} = 0 \) as defined by the blanking function. The upper limit \( m = 2N - 1 \) is the
minimum required CCD filter length to yield transform coefficients over the range 0 \( \leq k \leq N-1 \) for \( N \) input data points, 0 \( \leq n \leq N-1 \).

It is possible to directly implement equation (7.1.7) using CCD filters by multiplying the input data \( f_n \) by the coefficient \( P_n \) where

\[
P_n = \exp(-j\pi n^2/N)
\]

(7.1.8)

However, each CCD tap output can represent only a single valued (real) filter coefficient and so therefore cannot contain both the real and imaginary (amplitude and phase) weightings demanded by equation (7.1.8). This equation must therefore be decomposed into real and imaginary parts and the required data multiplication must be performed in separate operations. The separate chirp pre-multiplier values are thus given by

\[
P_n(\text{real}) = \cos \{\pi n^2/N\} \quad 0 \leq n \leq N-1
\]

(7.1.9)

\[
P_n(\text{imag}) = -\sin \{\pi n^2/N\} \quad 0 \leq n \leq N-1
\]

and for complex input data \( f_n \), four multiplications are required.

The impulse response of the CCD filter is given by equation (7.1.7) as

\[
h_m = \sum_{m=1}^{2N-1} \exp(j\pi (m - N)^2/N)
\]

and the CCD tap weights can therefore be written
\[ h_m(\text{real}) = \cos \left\{ \pi (m - N)^2 / N \right\} \quad 1 \leq m \leq 2N-1 \]
\[ h_m(\text{imag}) = \sin \left\{ \pi (m - N)^2 / N \right\} \quad 1 \leq m \leq 2N-1 \]

The convolution operation therefore demands four parallel CCD filters each of 2N-1 stages. Note that the CCD filter tap weights can be written

\[ h_m = \exp(jN\pi) \cdot \exp(jm^2/N) \quad 1 \leq m \leq 2N-1 \]
with \[ h_m = h_{m+N} \quad 1 \leq m \leq N-1 \]
and

\[ h_m(\text{real}) = P_m(\text{real}) \quad \text{N even} \]
\[ h_m(\text{imag}) = -P_m(\text{imag}) \]
\[ h_m(\text{real}) = -P_m(\text{real}) \quad \text{N odd} \]
\[ h_m(\text{imag}) = P_m(\text{imag}) \]

which means that identical numerical values for the sequences employed in the pre-multiplier and filter are required. This fact is demonstrated in figure 7.1.1(a) for N=10. In systems where the pre-multiplier waveforms are derived as the impulse response of a CCD filter, this permits the CZT to be realised using 12 CCD filters of length N stages, 6 weighted as \( h_m(\text{real}) \) and 6 weighted as \( h_m(\text{imag}) \) using two cascaded sections for each CCD filter.
Figure 7.1.2 shows a schematic diagram of the architecture required for computation of the CZT using CCD filters. The schematic relates to computation of a transform with an even number of points N. For N odd, the signs employed in the post-convolution summation are reversed.

For applications where only the power spectral density output is required, the chirp post-multiplication can be replaced by modulus (square-and-add) circuitry. Alternative linear modulus approximation techniques have also been reported$^{(92)}$ as possessing dynamic range advantages.

In addition to the use of CCD filters, two alternative techniques exist for the generation of the chirp pre-multiplier sequence. The first stores the required sequences in a digital read-only-memory (ROM) combined with either a D/A converter and four quadrant multiplier$^{(92)}$ or a multiplying D/A converter$^{(64)}$ (MDAC). The other technique uses actively generated analogue chirp waveforms$^{(67)}$. Using this second technique it is more convenient to employ chirp waveforms which sweep through zero frequency, figure 7.1.1(b). This can be incorporated into the CZT algorithm by a shift in indices by N/2 such that

$$F_k = \exp(j\pi N/4) \exp(-j\pi(k + N/2)^2/N).$$

$$\sum_{m=1}^{2N-1} \exp(j\pi(m - N/2)^2/N).\{f_{k+N-m}\exp(-j\pi(k+N/2-m)^2/N)\}$$

(7.1.11)
Figure 7.1.1. Symmetry of CZT filter responses.
Figure 7.1.2. CCD chirp-z-transform architecture.
Note, however, that this modification results in a constant phase offset in the output transform. Obviously for spectrum analysis applications (139) this modification is acceptable. In this context, for weighted spectrum analysis, in a similar manner to the concepts discussed in chapter 4, the amplitude weighting $W_n$ must be incorporated in the chirp pre-multiplier as

$$|F_k|_W^2 = \sum_{n=0}^{N=1} (W_n \cdot f_n) \exp(-j2\pi nk/N)^2 \quad (7.1.12)$$

7.1.2 The Sliding Chirp-Z-Transform

Two disadvantages of the CZT were identified in section 7.1.1. These are firstly the requirement to blank the input data to the processor restricting its operating duty cycle to 50%, and secondly, the fact that CCD filters of length $2N-1$ stages are required to provide an $N$ point transform. The sliding CZT offers a technique whereby only an $N$-stage filter is required to implement an $N$-point transform such that for every $N$ points of data an equal number of transform points is generated.

The sliding DFT has been defined (139)

$$F^s_k = \sum_{n=k}^{k+N-1} f_n \exp(-j2\pi nk/N) \quad (7.1.13)$$

Here since the indicies on the summation are different for each Fourier coefficient, the calculation of each $F^s_k$ occurs
over a slightly different data set. By direct comparison of equations (7.1.13) and (7.1.1), the sliding CZT can be derived

\[
F_k^S = \exp\left(-j\pi k^2/N\right) \sum_{n=k}^{k+N-1} \{f_n \cdot \exp\left(-j\pi n^2/N\right)\} \exp\left(j\pi (k-n)^2/N\right).
\]

(7.1.14)

From equation (7.1.14) it can be seen that the sliding CZT only yields the true DFT when \( f_n \) is periodic in \( N \) (\( f_n = f_{n+N} \)). This is the case when lines or frames of video data are to be transformed and here the sliding CZT yields the true DFT since the line-to-line (or frame-to-frame) signal variation is very small (139). Other applications of the sliding CZT are concerned with the calculation of spectral density functions for signals whose spectra are unchanging over a data record of length \( 2N \) samples. (Note that this requirement is identical to that required in the M(L) - C(S) - M configuration with SAW devices discussed in chapter 4.) Several applications for spectrum analysis of such data exist in speech processing where voiced spectrum components remain constant for intervals of approximately 100 ms in comparison to the normal "window" period of 20-25 ms. Here also the 100% duty cycle of the sliding CZT is of importance. Other applications for the sliding CZT exist in Doppler processing for radar and sonar.

Two additional advantages of the sliding CZT exist. The first is that the sliding CZT is significantly less affected
by charge transfer inefficiency (see section 2.2), than is the conventional CZT, since here the same filter stages are involved in the production of each output point and consequently the effect of charge transfer inefficiency is the same for all output points\(^{(139)}\). Secondly, for power spectrum analysis, the sliding CZT can be written

\[
|F_k^S|^2 = \left| \sum_{n=0}^{N-1} \{f_{n+k} \exp(-j\pi(n+k)^2/N) \exp(j\pi n^2/N)\}^2 \right|^2 \tag{7.1.15}
\]

and in consequence, the weighted power spectrum

\[
|F_k^S|^2 = \left| \sum_{n=0}^{N-1} \{f_{n+k} \exp(-j\pi(n+k)^2/N)\}[w_n \exp(j\pi n^2/N)]\right|^2 \tag{7.1.16}
\]

demands that the weighting function be applied to the filter. This result is also paralleled by chapter 4 for the M(L) - C(S) - M SAW configuration.

### 7.2 THE PRIME TRANSFORM

#### 7.2.1 Derivation of the Prime Transform Algorithm

The prime transform algorithm\(^{(74)}\) is based on the result derived from number theory that if \(N\) is a prime number, there exists at least one integer \(R\), said to be a "primitive root" of \(N\)\(^{(140)}\), which will produce a unitary (one-to-one) mapping of the integers \(n'\), \(1 \leq n' \leq N-1\) to the integers \(n\), \(1 \leq n \leq N-1\) according to the relationship
\[ n = R^{n'} \pmod{N} \quad 1 \leq n \leq N-1 \]  

(7.2.1)

This concept was applied to the computation of the discrete Fourier transform (equation 7.1.1) which has the property \( F_k = F(k+aN) \) and can be written

\[ F_k = f_0 + \sum_{n=1}^{N-1} f_n \cdot \exp(-j2\pi n (k+aN)/N) \]  

(7.2.2)

for an input series \( f_n \) of length \( N \) points.

Equation (7.2.2) can be written

\[ F_k = f_0 + \sum_{n=1}^{N-1} f_n \cdot \exp(-j2\pi/R \cdot n + (n'+k')) \]  

(7.2.3)

where \( n' = \log_R(n) \) and \( k' = \log_R(k+aN) \)  

(7.2.4)

Rearrangement of these definitions results in

\[ n = R^{n'} \quad 1 \leq n \leq N-1 \]

or \( n = R^{n'} \pmod{N} = (((R^{n'}))) \)

and \( k = R^{k'} - aN \)

or \( k = R^{k'} \pmod{N} = (((R^{k'}))) \)

which shows that the substitutions \( n, n' \) and \( k, k' \) are one-to-one permutations of the form of equation (7.2.1) when \( N \) is prime and \( R \) is a primitive root of \( N \). Here the double parenthesis
notation \(((R_n^\prime))\) denotes modulo N truncation.

The summation variable \(n\), may be replaced by \(n'\) since \(n\) and \(n'\) have a one-to-one relationship and this substitution therefore merely alters the order in which the terms are added in the summation. Further, the subscripts \(k\) and \(n\) may be replaced by their equivalent in equation (7.2.4)

\[
F_{(R_k')} = f_0 + \sum_{n'=1}^{N-1} f_{n'} \exp(-j2\pi/N ((R_n^\prime)))
\]

(7.2.5)

Note that since \(((R_k^\prime)) \neq 0\), the zeroth Fourier coefficient must be computed separately as

\[
F_0 = \sum_{n=0}^{N-1} f_n
\]

(7.2.6)

The presence of the summation term \((n' + k')\) in the exponent of equation (7.2.5) implies that the Fourier transform coefficients \(F_{(R_{k'})}, 1 \leq k' \leq N-1\), can be derived as a discrete correlation of two sequences. The sequence \(f_{(R_{n'})}, 1 \leq n' \leq N-1\) is a permuted version of the input data sequence \(f_n, 1 \leq n \leq N-1\). This is correlated with the sequence \(\exp(-j2\pi/N((R_{n^\prime}))), 1 \leq n' \leq N-1\) which is a sequence of permuted samples of a complex sinusoid. The sequence \(F_{(R_{k'})}, 1 \leq k' \leq N-1\) is a permuted sequence of the transform coefficients \(F_k, 1 \leq k \leq N-1\).
As in the case of the CZT discussed in section 7.1.1, the computation has been reduced to three principle operations which can be realised using CCD transversal filters. In the case of the prime transform, however, both the $f_0$ data sample and the $F_0$ transform coefficient require special consideration. To implement the prime transform, it is necessary to permute a prime number of data samples according to the rule $n = R^{|n'}| \mod N$, $1 \leq n' < N-1$ to form the sequence $f_{n'}$. Next correlation of this sequence $f_{(R^n)}$ with the sequence $\exp(-j2\pi((R^n'|)/N)$ is performed and $f_0$ added to the result. Then, the inverse permutation of the output samples $F_{k'}$, $1 \leq k' < N-1$ is taken to obtain the transform coefficients $F_k$, $1 \leq k < N-1$. Since in general, the data samples are complex, a hardware implementation of the prime transform requires two input permutations, four correlators and two output permutations. Techniques for reducing the required number of operations are considered in section 7.2.4.

In the same way as for the CCD CZT arrangement, computation of an $N$ point prime transform, as defined by equation (7.2.5) requires permutation of $N-1$ data samples and correlation in a CCD filter of length $2N-3$ stages with impulse response

$$h_m(\text{real}) = h_{m+N-1}(\text{real}) = \cos(2\pi((R^m))/N), \quad 1 \leq m \leq N-2$$

$$h_m(\text{real}) = h_{m+N-1}(\text{imag}) = -\sin(2\pi((R^m))/N), \quad 1 \leq m \leq N-2$$

(7.2.7)
7.2.2 Development of Permutation Codes

Consider the value \( N = 13 \) with permutation code defined by equation (7.2.1)

\[
n = R^{n'} \pmod{N} \quad 1 \leq n < N-1 \quad (7.2.8)
\]

where \( R \) is a primitive root of 13. A computer program was used to establish the primitive roots of \( N = 13 \) and these are shown in table 7.2.1. The primitive roots are the numbers \( R = 2, 6, 7, 11 \). Investigation of table 7.2.1 shows that little advantage can be gained from the choice of a particular primitive root. It is interesting to note that the sequence for \( R = 11 \) is the reverse (except for \( n' = 12 \)) of the sequence for \( R = 6 \). Similarly for \( R = 7 \) and \( R = 2 \). Also for \( R = 11 \) and \( R = 2 \) the 10th element remains non-permuted. In general, the possible permutation codes exhibit such minimal numbers of non-permuted elements. Consider an arbitrary choice \( R = 11 \). Since a data correlation is required, the permuted data sequence must be time-reversed prior to entering the filter. The data permutation process can include this time reversal, resulting in the time reverse code shown in table 7.2.1. Inverse permutation follows the direct (non-time reversed) sequence of table 7.2.1.
**OUTPUT SAMPLE NUMBER** \( (x_{out}) \)  & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12  \\
**CORRESPONDING INPUT SAMPLE NUMBER** \( (x_{in}) \)  & \( R=2 \) & 2 & 4 & 8 & 3 & 6 & 12 & 11 & 9 & 5 & 10 & 7 & 1  \\
 & \( R=6 \) & 6 & 10 & 8 & 9 & 2 & 12 & 7 & 3 & 5 & 4 & 11 & 1 \\
\( x_{in} = R(x_{out}) \mod 13 \)  & \( R=7 \) & 7 & 10 & 5 & 9 & 11 & 12 & 6 & 3 & 8 & 4 & 2 & 1  \\
 & \( R=11 \) & 11 & 4 & 5 & 3 & 7 & 12 & 2 & 9 & 8 & 10 & 6 & 1  \\
**TIME REVERSE CODE**  & \( R=11 \) & 1 & 6 & 10 & 8 & 9 & 2 & 12 & 7 & 3 & 5 & 4 & 11  \\

**Table 7.2.1.** Permutation codes for 13 point prime transform
7.2.3 CCD Prime Transform Processor

This section describes a CCD prime transform processor\(^{(141)}\) designed to demonstrate the operation of an \(N = 13\) point prime transform processor configured for the power spectrum analysis of CW input signals.

The block diagram of the CCD prime transform processor, figure 7.2.1, shows the two operations of permutation and correlation. In the permuter design used here, \(N = 13\) input data samples are stored in an analogue CCD tapped shift register. Hardwire connections are made from each CCD tap output to the inputs of a CMOS 16 line-to-1 line analogue multiplexer. The relevant CCD tap output can then be selected by means of the digital address held in a CMOS counter, figure 7.2.1. As an alternative to the hardwire cross-overs to achieve re-ordering, the connections could have been made directly and the multiplexer addressed from a permuted sequence of addresses held in a read-only-memory (ROM). This is in fact the technique used in commercially available permutation filters\(^{(142)}\).

The operation of the CCD permuter is demonstrated in figure 7.2.2 for a sinusoidal input, figure 7.2.2(a). In figure 7.2.2(b) the permuter taps are arranged to produce a first-in last-out sequence (FILO) and figure 7.2.2(c) shows the operation when connected in the time-reverse permutation code presented in table 7.2.1. The performance of the permuter is
Figure 7.2.1. CCD prime transform processor schematic.
more clearly demonstrated by the use of short duration pulse inputs. Figure 7.2.3(b) shows the output of the permuter in response to a pulse of length equal to four sample periods, i.e., input samples number 1, 2, 3, 4, figure 7.2.3(a). These input samples are then permuted to appear as samples number 1, 6, 9, 11, respectively, figure 7.2.3(b) as predicted by table 7.2.1. The CCD permuter accuracy was measured as ±3% and the maximum operating frequency was 1 MHz.

The sequence of permuted data samples is correlated with permuted samples of a complex sinusoid calculated from equations (7.2.7) and each filter requires $2N - 3 = 23$ stages, with the tap weights as shown in figure 7.2.4. Using a tapped CCD register employing floating gate reset tap outputs (see section 2.2), both the required tap weights $h_m(\text{real})$ and $h_m(\text{imag})$ can be incorporated on each tap output. The respective impulse responses of the two transversal filters are compared in figure 7.2.4 with the ideal responses. It can be seen that the practical results show close agreement with theory.

Figure 7.2.5 shows the output (in permuted order) of this 13 point CCD prime transform processor for two CW inputs of 4 kHz and 6 kHz. Here the input sample rate was 13 kHz and the expected positive and negative spectral components can be distinguished for each CW tone. The tap weight accuracy of ±5% for the CCD filter, together with an accuracy of ±3% in the CCD permuter produced an output transform accuracy of 2% rms.
Figure 7.2.2. CCD permuter operation (SINE inputs)

Figure 7.2.3. CCD permuter operation. (Pulse signal)
Figure 7.2.4. Prime transform filter responses.

(a) Permuted cosine filter impulse response

(b) Permuted sine filter impulse response.
Figure 7.2.5. CCD prime transform spectrum analyser
(CCD clock 13kHz, N=13)
Note that the necessary circuitry for the inclusion of the $f_0$ and $F_0$ terms has not been shown.

7.2.4 Reductions in Transform Hardware

In common with the chirp-z-transform (figure 7.1.2), the prime transform (figure 7.2.6) requires four transversal filters when the input data possesses real and imaginary terms. When the input data is purely real, two of the pre-multiplier chirps can be dispensed with in the CZT. With the prime transform, for purely real input data, since the data permutation stage involves only a real operation, only two transversal filters are required (figure 7.2.6(b)). In both of these cases, only $(N-1)/2+1$ valid output coefficients are available due to fold over at the Nyquist rate. With the prime transform, a transform of length $N' = (N-1)/2+1$ requires processing of $N$ input samples. In this case, the requirement is that $2N'-1$ be prime, $N'$ itself need not be prime.

A very important simplification is possible with the prime transform when the input data is real and even over the input block of $N$ samples. By using the symmetry of an even extension of a block of length $N$, the number of filters can be reduced to one. The discrete cosine prime transform (DCPT) is defined
(a) Input complex.

(b) Input real.

(c) Input real and even.

Figure 6.2.6. Simplification of prime transform hardware.
\[
F_k = \frac{1}{2} \sum_{n=-(N-1)}^{(N-1)} f_n \exp(-j(2\pi kn/2N)) \quad 0 \leq k \leq N-1
\]  
(7.2.9)

where \( f_n = f_{-n} \) and implies that the DCPT is based on an even extension of the sequence \( f_n \) of length \( N \). Here again, \( 2N-1 \) must be prime, and must provide the modulus base for the permutation codes. Using the modular notation previously defined in section 7.2.2, equation (7.2.9) can be written

\[
F_{k'} = f_0/2 + \frac{1}{2} \sum_{n'=-(N-1)}^{N-2} f_{n'} \exp(-j2\pi((R'_{n}+k')/2N-1)) \quad n' \neq 0
\]  
(7.2.10)

with \( F_0 = \frac{1}{2} \sum_{n=-(N-1)}^{N-1} f_n \)  
(7.2.11)

which may be written

\[
F_{k'} = f_0/2 + \sum_{n'=1}^{N-1} f_{n'} \cos \{2\pi((R'_{n}+n'))/2N-1\} \quad 1 \leq n' \leq N-1
\]  
(7.2.12)

and \( F_0 = f_0/2 + \sum_{n=1}^{N-1} f_n \)  
(7.2.13)

Thus equation (7.2.12) represents a correlation of the permuted sequence \( f_{n'} \), \( 1 \leq n' \leq N-1 \) with the sequence \( (R'_{n'}) \).
\cos(2\pi((R^n))/2N-1)\) where

\[ n' = N-2, N-1, ..., 1, 0, 1, ..., N-1, N-2 \]

and thus requires \(2N-3\) taps. Therefore, based on this arrangement, the scheme required for computing the DCPT coefficients \(F_k\) \(1 \leq k \leq N-1\) of a data sequence \(f_n\) \(1 \leq n \leq N-1\) where \(N\) is prime is as shown in figure 7.2.6(c). Here permutation of the data sequence \(f_n\) \(1 \leq n \leq N-1\) (with associated time reversal) is followed by filtering in a transversal filter with tap weights according to \(\cos(2\pi((R^s))/2N-1)\) \(s = N-2, ..., 1, 0, 1, ..., N-2\). Finally inverse permutation yields the transform coefficients in serial order.

7.3 COMPARISON OF ANALOGUE CCD TRANSFORMS WITH THE DIGITAL FFT

In general terms, the comparison presented in this section can be summarised in the following way: the analogue CCD transforms have performance limitations when compared with the digital FFT. However, CCD processors do possess distinct cost advantages for high volume production and in addition, they offer advantages in smaller physical size, lighter weight, significantly lower power and improved reliability. Although the analogue CCD transforms exhibit modest performance, a great many systems exist in radar
and sonar signal processing which can accept such performance
to achieve the power/size premiums available with CCD. The
analogue SAW transform processors will not be included in
this comparison, since they generally operate over different
signal bandwidths in comparison to the FFT and CCD processors.

The digital FFT(55) is currently the most widely used
technique for Fourier transformation and spectrum analysis,
this importance deriving from the fact that it requires
only N/2 \cdot \log_2 N complex multiplications to perform an N-point
DFT, as compared with N^2 complex multiplications required to
directly implement the DFT. This represents a significant
reduction in the number of computations to be performed.
For example for N=1024 it produces a 100-fold reduction.
FFT algorithms can be classified as decimation-in-time or
decimation-in-frequency. The most common FFT algorithm is
the radix 2 decimation-in-time algorithm which implies that
the entire FFT is performed by sequential operations
involving only pairs of elements.

An 8 point radix 2, decimation-in-time FFT algorithm is
illustrated in figure 7.3.1. The input sequence is first
split into its even and odd parts by a bit reversal technique(56),
and the first set of butterflies combines the 2 point transforms
using "twiddle" factors(56) to achieve two 4 point transforms
on the even and odd numbered inputs. The final set of
"butterflies"(56) combines the 4 point transforms using
twiddle factors to achieve the final 8 point DFT. The
Figure 7.3.1. Digital FFT flow diagram.

\[ W_n = \exp(-j2\pi/N) \]
flow graph notation used in figure 7.3.1 is used to represent pictorially the algorithm. A node represents an add/subtract operation with the addition appearing in the upper branch and the subtraction in the lower branch. An arrow represents multiplication by the value written above the arrow.

Several important facts concerning radix 2 FFT's are apparent in that: (i) $N$ must be an integral power of 2; (ii) there are $\log_2 N$ stages each requiring $N/2$ butterfly operations; and (iii) each Fourier coefficient is processed through $\log_2 N$ butterfly operations so that quantisation errors are cumulative.

Comparison of figure 7.3.1 with figure 7.1.2 and figure 7.2.6 shows the vastly reduced hardware requirements of the CCD processors.

In comparing the analogue CCD and the digital FFT perhaps the most important criterion is transform accuracy. The comparison reduces to the question of how many bits are necessary for the digital words in the FFT in order to achieve superior accuracy than the analogue transforms. The accuracy of the CCD processors is limited by several error sources:

CCD errors:

(i) charge transfer inefficiency;

(ii) thermal noise;
(iii) accuracy of the filter weighting coefficients; 
(iv) linearity of the CCD filters.

other sources:

(v) multiplier or permuter errors;
(vi) amplifier gain and non-linearity.

7.3.1 Accuracy of the CCD Prime Transform and CZT

The prime transform and chirp-z-transform were both modelled by computer simulation programs. Error sources, inherent in hardware implementations, were included and the error produced in the transform coefficients was measured. The sensitivity of the transform coefficient error as a function of hardware error sources was investigated and the relative accuracy of the two transforms was compared to the digital FFT.

Similar work has been published elsewhere. Wrench et al\(^{(76)}\) have performed a statistical analysis of the sensitivity of these algorithms to hardware imperfections, errors being introduced as random variables at suitable points in the transform architecture. Another study by Campbell et al\(^{(143)}\) employed computer simulations to investigate the practical implications of device error sources.

In section 7.2.4, the possible architectures for transform implementation were discussed, highlighting the
fact that when both real and imaginary input data values exist, both the CZT and prime transform require the full complex arithmetic processors shown in figure 7.1.2 and figure 7.2.6. It is these configurations which have been modelled here. In contrast Campbell considered the processing of real data only where the prime transform (unlike the CZT) permits considerable hardware simplification, and Wrench considered the computation of the discrete cosine transformation where the prime transform arrangements requires only one filter. The results of these studies must therefore be accepted in the light of these facts. Wrench (76) has estimated that the prime transform offers an order of magnitude improvement in accuracy, over the CZT for real and even inputs, to achieve 0.5% transform accuracy with ±1% accuracy in the analogue devices. Campbell's results demonstrate an improvement by a factor of 2 in the prime transform over the CZT for real input data. The computer simulations presented here modelled the full (four filter) arrangement of the two types of processor and were therefore anticipated to show little variation in the error performance of the prime transform and CZT.

Various definitions of the accuracy of a set of transform coefficients are possible. For example, signal-to-noise ratio may be defined as the ratio of the peak error term to the peak output coefficient. In the work described here,
the figure of merit is defined as the rms noise-to-rms signal ratio. This is defined as follows. Let the coefficients of an ideal, errorless, transform be contained in two arrays \( F_r \) and \( F_i \) each of \( N \) elements. The output of a computer simulation which models device errors is contained in two similar arrays \( F'_r \) and \( F'_i \). The rms coefficient error is then

\[
e_{\text{rms}} = \sqrt{\frac{\sum_{r=0}^{N-1} (F_r - F'_r)^2 + \sum_{i=0}^{N-1} (F_i - F'_i)^2}{2N}} \tag{7.3.1}
\]

The rms signal is written

\[
s_{\text{rms}} = \sqrt{\frac{\sum_{r=0}^{N-1} F_r^2 + \sum_{i=0}^{N-1} F_i^2}{2N}} \tag{7.3.2}
\]

The ratio of the rms signal to the rms error is therefore a measure of the signal-to-noise ratio based on the properties of the DFT, normalised to both transform length and signal amplitude.

The input signals used in the simulation were chosen to be "basis vectors" of the transform, i.e., the "sample window" contains an integer number of complete cycles of a complex sinusoid. Such an input produces a single, non-zero ideal transform coefficient of magnitude equal to \( N \).
Figure 7.3.2 shows simulated predictions of the sensitivity of CCD analogue transform processor accuracy as a function of component error. Results show that for CCD tap weight accuracy in the range 1-10% (curve A) and CCD charge transfer inefficiency in the range $10^{-4}$ to $10^{-3}$ (curve C), the prime transform and chirp-z-transform present no detectable difference in transform accuracy. When the tap-weight accuracy ($\pm 5\%$) and permuter accuracy ($\pm 3\%$) measured in the experimental CCD prime transform processor are included in figure 7.3.2, the measured transform accuracy of 2% rms is in close agreement with the simulation results. Figure 7.3.2 also demonstrates that the 3-9% accuracy of present analogue permuter designs is the dominant error source in the prime transform. Permuter development should achieve $\pm 1\%$ accuracy resulting in a projected overall transform accuracy of 0.5% rms.

The most important source of error in the digital FFT is usually overflow and round-off of data words during butterfly computation. If the data words are carried with $b$ bits plus sign, the error level is given by \[ \Delta_B = 0.85 \times \sqrt{N} \cdot 2^{-b} \] assuming overflow occurs at every butterfly.

The errors of a 13 bit FFT (fixed point) processor are typically below 0.3% for transforms of length less
than $N = 100$, figure 7.3.3.

The accuracy performance of the prime transform and CZT algorithm have therefore been shown to be similar. In applications where reductions in hardware complexity are possible, other publications have indicated improved accuracy using the prime transform. In general, however, the analogue CCD processors offer reduced accuracy in comparison to the digital FFT.
Figure 7.3.2. Accuracy of CCD CZT and prime transform.

Figure 7.3.3. Comparison of CCD transforms and digital FFT.
CHAPTER 8: APPLICATION OF COMBINED CCD AND SAW PROCESSORS

8.1 CCD-SAW INTERFACE TECHNIQUES

Fourier transform processors based on the chirp-z-transform (CZT) algorithm implemented with CCD are more flexible, in terms of real time signal bandwidth, than those based on SAW devices. This feature arises from the externally programmable CCD clock waveforms which permit a single CCD processor to process, within the bounds depicted in figure 8.1.1, several octaves of signal bandwidth. In comparison, the parameters of the SAW Fourier transform processor are dictated by the SAW device design. However, within the bounds shown in figure 8.1.1, the SAW processor can achieve high performance with transform lengths in excess of 2500 points. The disadvantage of the CCD CZT processor is that this arrangement requires four accurately gain-matched parallel baseband channels which are required to implement complex data processing. This necessitates $8 \times N$ tapped CCD register stages to implement an $N$ point convolution and thus restricts the total number of transform points in the CCD processor to less than 250.

This chapter discusses the marriage of untapped CCD analogue shift registers, which are easier to fabricate than the tapped CCD devices required in the CZT, with high performance SAW chirp transform processors. These combined CCD-SAW Fourier transform processors have two main application areas. The
Figure 8.1.1. Performance bounds of CCD and SAW Fourier transform processors.
temporal Fourier transform permits their use as spectrum analysers realising Doppler filter banks for radar and sonar systems applications. The spatial Fourier transform allows the synthesis of a beam pattern from an array of sonar hydrophone transducers by controlling the transmitted, or received, power density across the array. In sonar systems the combined CCD-SAW processor capabilities closely match the system requirements.

Three distinct regions of operation can be identified in figure 8.1.1 for individual CCD and SAW processors with the region of mutual CCD/SAW performance, Region II currently occupying bandwidths of 1-8 MHz with time-bandwidth products up to 200. The modes of interfacing signals between these three regions can be summarised:

Mode A (Region I - Region II). Since an overlap region of mutual operation for CCD and SAW components exists, and since the CCD is clock frequency programmable, signals which occupy bandwidths in Region I can be linearly compressed in time (expanded in bandwidth) using analogue CCD shift registers. The time compressed signals can then be interchanged with SAW devices operating in Region II. In this way, signals which occupy a (kHz, ms) region of time vs bandwidth space can be interfaced with a SAW processor which demands signals with a (MHz, µs) characteristic, the fixed parameters of the SAW
processor being made flexible by the clock programmable CCD buffer store\(^{(147)}\).

**Mode B (Region I - Region III).** By employing special techniques such as parallel input - serial output SAW devices, it is possible to interface directly between CCD operating in Region I and SAW operating in Region III. Obviously, those systems which do not directly interconnect CCD and SAW devices but which choose the appropriate technology for each specific component operate in mode B.

**Mode C (Region III).** Here both CCD and SAW devices operate at fixed bandwidth. Although mode C operation is possible using CCD alone, the more mature, higher accuracy, SAW device offers distinct advantages - especially for signal correlation which can be performed in a single SAW transversal filter as compared to the four devices necessary with the baseband CCD.

### 8.2 Applications in Mode A Interface

By time compression of analogue signals by factors of up to 10,000 in analogue CCD registers, a fixed parameter SAW Fourier transform processor can be made to serve four decades of signal bandwidth which are normally inaccessible to SAW techniques\(^{(148)}\). With such a large time compression factor, it is possible to time-share many data channels with one sophisticated high speed SAW processor.
8.2.1 Radar Pulse Doppler Spectrum Analysis

A radar pulse Doppler spectrum analyser based on a CCD time compressor in combination with a SAW spectrum analyser \((80, 149, 150)\) is shown in figure 8.2.1. In this system the CCD performs not only a signal time compression to match the radar kHz Doppler spectrum to the MHz bandwidth of a SAW real time spectrum analyser, but also produces a matrix transposition or 'corner turning' of the radar video to arrange the radar returns in range bins. Here, a radar bipolar video signal of 2 MHz bandwidth is clocked into a CCD tapped shift register at 4 MHz for a period of 2.5 \(\mu\)s (10 range bins) every 250 \(\mu\)s (radar p.r.i.). These samples are then transferred at a 4 kHz rate into the vertical CCD stacks, to generate a history of 100 returns in each range bin. The returns, with Doppler bandwidth of 2 kHz are stored in the range bins for a period of 25 ms (to permit a Doppler frequency resolution of 40 Hz). The CCD stacks are emptied in 25 \(\mu\)s (time compression \(\times 1000\)) to match to the 40 kHz resolution of the SAW spectrum analyser. For a single channel CCD time compressor, only 50 Doppler cells of 40 Hz spacing can be identified. With double sideband operation, to differentiate between approaching and receding targets, in-phase and quadrature CCD channels are required to achieve the full 100 Doppler cells over \(\pm 2\) kHz bandwidth. In the SAW spectrum analyser used in this system \((80)\) the input signal is mixed with a 25 \(\mu\)s chirp signal of 4 MHz bandwidth and input to the second SAW chirp of duration 50 \(\mu\)s and bandwidth
Figure 8.2.1. Radar pulse Doppler CCD-SAW spectrum analyser.
8 MHz. Spectral weighting to reduce sidelobes is performed by multiplying the input with a raised cosine waveform derived from a ROM. This high performance processor achieves -40 dB sidelobe levels with the facility for in-phase and quadrature operation.

8.2.2 Combined CCD-SAW Variable Resolution Spectrum Analyser

Figure 8.2.2 shows a schematic diagram of a CCD-SAW variable resolution spectrum analyser which permits programmable increase of frequency resolution to \((ST)^{-1}\) Hz over a bandwidth \((B/S)\) Hz. Dual channel synchronous CCD analogue time compressors are employed with demodulation of the input signal using local oscillators in phase quadrature to permit identification of positive and negative frequencies relative to the demodulation frequency, \(f_1\). Here, \(f_1\) sets the analyser centre frequency and progressive variation of the synthesised LO frequency \(f_1\) and time compression factor, \(S\), permits the processor to 'zoom-in' for detailed analysis of spectral lines over the complete SAW analyser bandwidth. After analogue time compression, the CCD output is modulated on quadrature carriers operating at the sum frequency of the chirp multiplier and SAW filter to minimise signal breakthrough problems.

The performance of a combined CCD-SAW processor when analysing a modulated signal in the short wave band is demonstrated in figure 8.2.3. The analyser is based on SAW chirp filters fabricated on ST,X quartz at centre frequency 17 MHz with
Figure 8.2.2. High resolution CCD-SAW spectrum analyser.
Figure 8.2.3. Operation of CCD-SAW spectrum analyser.
2 MHz chirp bandwidth and 20 μs dispersive delay. This permits analysis of 1 MHz signal bandwidth in 10 μs with a frequency resolution of 100 kHz. The input test signal was 15.432 MHz carrier, tone modulated at a 400 Hz rate. Figure 8.2.3(a) shows the operation of the basic SAW spectrum analyser without time compression ($S = 1$) and $f_1 = 15.000$ MHz. Here the display centre line is 15.000 MHz and the frequency scale is 100 kHz/division, ie, full-scale coverage ±500 kHz. The input signal frequency is measured as $(15.000 + 4.3 \times 0.1)$ MHz, ie, 15.43 MHz. Resetting the synthesiser to 15.430 MHz and using a time compression factor $S = 100$ produces the output shown in figure 8.2.3(b). Here the display centre line is 15.430 MHz and the frequency scale is 1 kHz/division. The input signal can now be read more accurately as 15.432 MHz. After resetting the input synthesiser to 15.432 MHz and $S$ to 1000, the modulation sidebands can be seen at ±400 Hz in figure 8.2.3(c).

8.2.3 Combined CCD-SAW, High Resolution Spectrum Analyser Design

Figure 8.2.4 shows the principle by which this time compression technique can be extended to permit real time, high resolution spectrum analysis over the full bandwidth of the SAW analyser. Thus, the processor time-bandwidth product exceeds that of the individual CCD and SAW components extending its capabilities to the higher time-bandwidth limits shown in figure 8.1.1. The input signal is demodulated by $S$ offset local oscillators (LO) to generate contiguous frequency bands which cover the whole SAW bandwidth. The individual down-
Figure 8.2.4. CCD-SAW wideband, high resolution spectrum analyser.
converted signals are input to $S$ parallel CCD time compressors which sample at a rate $B/S$ Hz and are read out at a rate $B$ Hz such that a SAW spectrum analyser of bandwidth $B$ Hz and chirp duration $T$ s can sequentially access the $S$ channels to yield a frequency resolution of $(ST)^{-1}$ over the SAW bandwidth $B$ Hz.

The required frequency offset LO signals can conveniently be generated by fabrication of a SAW mode-locked oscillator (151) on the same quartz substrate as the SAW chirp filters. This SAW oscillator generates a comb spectrum providing all the offset frequencies. Individual LO signals can be obtained with a series of narrowband SAW transducers which select the required frequencies, each separated by $(B/S)$ Hz.

With presently available SAW chirp filters and CCD shift registers, this technique permits an increase in the effective time bandwidth product of a SAW spectrum analyser by 1-2 orders of magnitude at the expense of a bank of input time compression channels. In this way a SAW spectrum analyser with a 4 MHz real time bandwidth and 40 kHz frequency resolution ($BT = 100$) can operate with an effective 2 kHz frequency resolution ($BT = 2000$) using 20 time compression channels (each incorporating two 100 stage CCD shift registers) and 20 LO frequencies each offset by 200 kHz.

8.2.4 Two-Dimensional Transform Processing for Sonar Beamforming

Beamforming refers to the process by which an array of spatially separate elements achieves directional sensitivity.
The array possesses an intrinsic sensitivity characteristic called a beam pattern, which can be modified by weighting, delaying or phase shifting and summation of the signals received at each element. In sonar systems the direction of maximum sensitivity (the look angle) can be controlled, as in radar, in one of two ways: firstly, by the mechanical rotation of the array itself and secondly, by the insertion of appropriate phasing or equivalent time delay networks in series with each element to effect electronic scanning. This effect is known as beamsteering. The former of these two techniques obviously proves a much more difficult task for underwater applications than its equivalent in radar, due to the difference in operating medium. The inertial effects of water on large array structures creates a considerable drive power requirement. For wide bandwidth operation beamsteering can be achieved electronically by variable time delay networks inserted in series with each array element. These effectively compensate for the difference in arrival times between each hydrophone. For narrowband operation beamsteering can be achieved by phase shift networks or by modulation scanning\(^{(152)}\). Alternatively, beamsteering can be achieved by performing a Fourier transform on each hydrophone signal and modifying the transform by a constant phase factor\(^{(153,154)}\).

Each of the above techniques effectively sweeps over the field of view to achieve coverage. The combined CCD-SAW system described here\(^{(90)}\) forms a series of permanent beams and
gives simultaneous outputs for each. In essence the system operates by Fourier transformation of the wideband signals received at each hydrophone to create a series of narrow bandwidth signals, a temporal transform. These narrowband signals are then in a suitable form for target bearing analysis which is performed by a second Fourier transform, as a spatial transform. The technique considered here was originally designed\(^{(62, 155, 156)}\) to use computer based Fourier transform techniques to perform the temporal transform on each hydrophone followed by a SAW chirp-z-transform (CZT) processor to achieve the spatial transform. It was proposed that CCD CZT elements would be used on each hydrophone to perform the temporal transform and a parallel-in-serial-out SAW diode convolver for the spatial transform. The development and application of analogue CCD time compression techniques\(^{(157)}\) have suggested the present arrangement to perform such a two dimensional analysis.

Instead of employing parallel Fourier transform processors operating at data bandwidths up to 20 kHz on each hydrophone to achieve temporal transformation, it is possible to sequentially transform the record from each hydrophone by employing time compression with attendant bandwidth expansion to a few MHz and SAW Fourier transformation. Figure 8.2.5 shows a block diagram of the proposed system. Each hydrophone channel has two analogue time compressors corresponding to in-phase and quadrature data permitting asynchronous operation of the system. In an active sonar application a quadrature demodulation scheme
Figure 8.2.5. CCD-SAW two dimensional sonar signal processor
would be used to transfer the input data to baseband. On the read cycle the CCD shift registers on each hydrophone channel would be clocked in parallel at a rate $R\text{Hz}$ corresponding to the maximum frequency component to be analysed. After reading $M$ samples into each CCD shift register of length $M$ stages the clock rate is increased by a factor $S$ and the quadrature registers are read out in parallel, one channel at a time. In the example of figure 8.2.5, the input clock rate was chosen at 16 kHz with 16 stages in each register. With a system of 16 hydrophones this requires $16 \times 2$ CCD registers of 16 stages each. The write cycle clock was taken as 1 MHz and each channel is assumed to be accessed at 10 ms intervals, for input to the SAW processor. The SAW chirp transform processor has the form discussed in chapter 4 with quadrature chirp waveforms on input and output to perform complex processing. The SAW chirp filter parameters of 1 MHz bandwidth and $16\mu s$ duration are matched to the output waveform of the CCD time compressor. The SAW processor performs the Fourier transform in $32\mu s$ and is then available for data from the next hydrophone channel. Figure 8.2.6 shows results from a computer simulation of the system. The time bandwidth product of the CCD registers and pre-multiplier chirps were chosen as 16 hence the temporal transform provides a resolution of $1/16$ of the CCD read clock frequency, ie, 1 kHz. Figure 8.2.6(a) shows the power spectrum obtained after temporal transformation with input test data of 3 kHz Doppler at 22 degrees bearing. The power spectra of
Figure 8.2.6. Simulation of 2D processor output.
successive hydrophone records are all identical since the successive Fourier transforms only have relative phase shift. Figure 8.2.6(b) shows the phase response of each of the 3 kHz Doppler samples of the successive hydrophone responses. This shows a linear phase variation with hydrophone number. The temporal transform data is read simultaneously into two 16 stage CCD registers at 1 MHz rate and when 16 samples have been read into real and imaginary channels these are transferred vertically. Successive transformed hydrophone records build up a waveform in each vertical stack which corresponds to one specific Doppler frequency. The vertical registers are clocked at 10 kHz rate. After the temporal transforms of all the hydrophones are complete, each vertical stack contains a waveform which has constant amplitude and a linear phase variation dependent on the bearing of the target, i.e., effectively a CW waveform. Subsequent Fourier transformation of each Doppler stack in a second SAW processor (or re-use of the first) yields the target bearing. Figure 8.2.6(c) shows simulated output results for a simulated target with 3 kHz Doppler at 22° bearing.

8.3 APPLICATIONS IN MODE B INTERFACE

8.3.1 Large Time-Bandwidth Fourier Transform Processors

Figure 8.3.1 shows a large time-bandwidth product Fourier transform system using CCD and SAW components (158, 159). The scheme uses a programmable SAW diode convolver which operates by
non-linear mixing in diodes positioned on taps along the SAW substrate. The diode dc bias currents are used to weight the tap signals before summation. By propagating contra-directed chirp waveforms under the tap array, the (spatial) Fourier transform of the tap weights is obtained. In this way the diode convolver is a parallel input-serial output transform processor [159].

The CCD/SAW system, figure 8.3.1, is composed of a serial input-parallel output CCD transform section followed by the parallel input SAW diode convolver. The input section uses \( P \) identical CCD chirp-z-transform (CZT) processors of length \( N \). These operate at a rate \( 1/P \) times the input data rate since they are sequentially demultiplexed from the serial input signal. The \( P \)-line input SAW diode convolver operates at the input data rate \( P \) Hz to provide a real time transform output. The processor therefore permits high resolution transforms of length \( PN \) to be achieved using moderate time bandwidth SAW and CCD devices. A diode convolver with 64 taps and 25 MHz bandwidth interfaced with 32 stage CCD processors operating at a clock rate of 400 kHz could therefore perform analysis of 25 MHz bandwidth in real time with 2048 points, and dynamic range up to 60 dB.

8.3.2 High Resolution Spectrum Analysis

A high resolution spectrum analyser [44] with CCD and SAW operating in mode B, is shown in figure 8.3.2. This design
Figure 8.3.1. Large time bandwidth CCD-SAW Fourier analyser.

Figure 8.3.2. CCD-SAW spectrum analyser.
uses a programmable SAW oscillator operating as a cheap, compact synthesised local oscillator mixing a chosen signal bandwidth into the passband of a fixed high Q (possibly SAW) filter. Further down-conversion permits analysis in a narrowband CCD CZT spectrum analyser. Using a programmable SAW local oscillator the CCD/SAW system covers 4 MHz bandwidth in 25 kHz steps. The CCD CZT processor resolves 100 frequency cells within the 25 kHz band to yield the full 4 MHz bandwidth with 250 Hz resolution. This system is however non-real time.

8.4 APPLICATION IN MODE C INTERFACE

8.4.1 Maximum Entropy Spectrum Analysis

Maximum entropy spectrum analysis\(^{(84)}\) offers a technique which eliminates the finite data window problems associated with Fourier transform processing. The finite duration convolution process intrinsic to such Fourier transform techniques results in a smoothing of the anticipated spectrum with reduced frequency resolution. Fourier transform spectrum analysis is therefore fundamentally limited in frequency resolution by the processing window duration, or alternatively by the data window duration. The possibility of achieving super-resolution in spectrum analysis has been demonstrated using the maximum entropy method (MEM)\(^{(85,86)}\) for resolution enhancement.
In simple terms, the maximum entropy method can be envisaged as an adaptive filter such that, at the conclusion of the adaptive process, the filter has reduced the input signal to white noise. Subsequent Fourier transformation of the final filter coefficients yields the power spectrum of the original input signal. This resultant power spectrum is consistent with the data segment being analysed, however, it is non-committal with respect to data outwith this segment.

One limitation of the maximum entropy method is that it is an approximate technique, involving errors in frequency measurement up to 16%. Further, it applies only for sinusoidal input signals and requires high input signal-to-noise ratio.

A schematic diagram of a CCD-SAW implementation to perform maximum entropy spectrum analysis is shown in figure 8.4.1. Here, the adaptive algorithm is computed using CCD registers and the subsequent Fourier transform is performed in a SAW processor. In this way the MEM technique would permit the full frequency resolution of the SAW transformer to be achieved even for short duration input signals, figure 8.4.2.
Figure 8.4.1. CCD-SAW maximum entropy spectrum analyser.
Figure 8.4.2. Maximum entropy spectrum analyser performance.
CHAPTER 9: CONCLUSIONS

This thesis has developed the theory, design and application of Fourier transform processors based on SAW and CCD devices, and has demonstrated techniques whereby these processors can be incorporated in future signal processing systems.

Chapter 4 detailed a rigorous analysis of the operation and performance limitations of SAW chirp transform processors with chapter 5 developing this theory in terms of processor design and application in spectrum analysis and network analysis. Further, chapter 6, which effectively considered the SAW chirp transform processor as a signal processing module, discussed the signal processing applications of combined transform processors in cepstrum analysis and signal correlation.

In chapter 7, the theory and design of CCD Fourier transform processors was presented with chapter 8 describing the logical extension to systems applications for signal processing modules employing combination of the CCD and SAW processors.

Throughout this work, emphasis has been placed on the possibility of hardware simplification in the progression from processors based on digital technology to those based on CCD technology and finally to those based on SAW technology.
However, as has been detailed in chapter 4 and chapter 7, this reduction in processor hardware is associated with a corresponding reduction in transform accuracy. Indeed, chapter 4 demonstrates that the high tolerances required in the SAW Fourier transform processor preclude accurate recovery of the transform components. For this reason, chapter 5 and chapter 6 concentrate on those designs and applications for SAW Fourier transform processors which do not require baseband recovery of the transform components.

Chapter 7 compares the performance of the CCD Fourier transform processors with the digital FFT algorithm. Here it was shown conclusively that the accuracy performance of the CCD processors is inferior to that of the digital processor, although these CCD designs do offer engineering premiums in terms of low power consumption, low volume, low cost, low weight and fast operation. In fact, it was shown that under certain conditions, the prime transform algorithm (section 7.2) permits operation with a single channel in a comparable scheme to that of the simple SAW arrangement.

Finally, chapter 8 addressed the possible systems applications of combined CCD and SAW processors. Here simple CCD analogue shift registers can be employed to permit programmable interface of narrow band signals directly with SAW devices. The significance of this technique lies in the fact that it overcomes the fixed parameters of the SAW device
in signal processing applications. Further, it permits one fast, sophisticated SAW processor to access several low bandwidth CCD channels. This principle was considered in chapter 8 for radar pulse Doppler analysis and sonar hydrophone array signal processing.

Finally, the application of combined CCD-SAW systems to signal processing represents the main area of possible future research work, resulting from this thesis. The combination of technologies is important here because fully engineered SAW spectrum analysers are only now becoming commercially available and therefore the techniques of chapter 5 and 6 will require several years of industrial development before their engineering implementation. Further, CCD spectrum analysers have not yet become available commercially therefore the combination of readily available CCD analogue delay lines with commercial SAW spectrum analysers will by-pass these problems of industrial inertia permitting the engineering development of CCD-SAW systems in sonar hydrophone array signal processing, radar pulse Doppler spectrum analysis and speech signal processing.
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Network analysers employing swept-frequency techniques are limited in sweep rate by the network impulse response time when high-frequency resolution is required. An alternative approach is to excite the network under test with a baseband pulse. The desired network parameter is then calculated from the ratio of the Fourier transforms of the digitised input and output signals, using computer based subroutines. The network analyser we have considered uses surface-acoustic-wave (s.a.w.) chirp filters. It operates by exciting the network under test with a narrow i.f. pulse. The output signal is subsequently analysed in real time by a s.a.w. discrete Fourier transform (d.f.t.) processor. Thereby the desired network parameter is displayed on a conventional oscilloscope.

A Fourier transform may be implemented with s.a.w. devices in real time by a mathematical expansion of

\[ F(\omega) = \int f(t) \exp(-j\omega t) dt \]

in the form

\[ F(\omega) = \exp(j\mu t^2) \int f(t) \exp(j\mu t^3) \left( - \exp(-j\mu t) + j \right) dt \]

Thus, if the function \( f(t) \) is premultiplied by a chirp of dispersive slope \( \mu \), Fresnel transformed (convolved) with a chirp and postmultiplied by a chirp, its Fourier transform \( F(\omega) \) is obtained; this requires three s.a.w. chirp filters.

**Fig. 1** Block diagram of real-time network analyser employing surface-acoustic-wave discrete-Fourier-transform processor

A variant of eqn. 2, the chirp z transform (c.z.t.), has been used by Whitehouse to design an s.a.w. Fourier-transform processor. Discrete baseband data are processed by multiplication and convolution with discrete samples of chirp functions using weighted s.a.w. tapped delay lines. Our system differs from the c.z.t. implementation by operating on i.f. input waveforms with conventional unweighted s.a.w. chirp filters and yields, for the first time, both amplitude and phase information.

**Fig. 1** shows in bold outline our practical realisation of the Fourier-transform processor in relation to eqn. 2. Premultiplication of the signal \( f(t) \) with a chirp, followed by convolution in a chirp filter with opposite dispersive slope, yields directly the amplitude spectrum \( |F(\omega)| \). Postmultiplication with a third chirp yields the phase spectrum. The use of a special signal with the Fourier-transform processor permits extension into a network analyser. The input to the network under test is an i.f. pulse train of period \( T \), corresponding to a line spectrum spaced at \( 1/T \). The network modifies these spectral lines in amplitude and phase by the network transfer function. Further, the Fourier-transform processor using s.a.w. chirps of duration \( nT \), where \( n \) is an integer, yields the line spectrum as a series of individual \( \sin x/x \) responses. Sampling at the peaks of individual responses implements the required d.f.t., since at these instants all other responses are at nulls. Thus our processor differs from the related s.a.w. compressive receiver in embodying a special test signal, sampling and an additional s.a.w. chirp filter to obtain phase information. This extends the capability from spectrum analysis to network analysis.

In the practical realisation (Fig. 1), the special test signal was obtained by gating a 145 MHz oscillator with 25 ns pulses of \( 1-25 \mu s \) p.r.p. This line spectrum centred at 145 MHz was flat \((\pm 0.5 \text{ dB})\) over \(>20 \text{ MHz} \) band. The master-timer output was also divided by eight before generating a short \((8 \text{ ns})\) impulse to excite the first s.a.w. chirp filter. This produced a 2-5 \( \mu s \) burst of up-chirp signal, centred at 60 MHz, and swept linearly over 12.5 MHz. After spectral inversion and bandpass filtering the resulting down-chirp centred at 85 MHz was used as the premultiplying signal. Convolution and postmultiplication were both performed by down-chirp filters, centred at 60 MHz, with 25 MHz bandwidth and 5 \( \mu s \) dispersive delay.

**Fig. 2a** shows the unsampled response of the s.a.w. d.f.t. processor to a 145 MHz c.w. input (without gating or network under test). The upper trace shows the characteristic compressive receiver (\( \sin x/x \) amplitude characteristic). The lower trace shows the soft-limited and phase-detected output, which demonstrates the theoretical phase reversals in alternate 'sidelobes'. **Fig. 2b** records the short-circuit characteristic of the network analyser. In the upper trace the amplitude/frequency characteristic is presented as a series of discrete responses. The horizontal scale is directly related to the \( \mu = -5 \text{ MHz}/\mu s \) dispersive slope of the chirp filters. The input line-spectrum spacing of 800 kHz is resolved into...
discrete outputs every 167 ns. The lower trace shows the analyser phase/frequency characteristic.

Fig. 3 shows measurements performed on a 5-section 145 MHz centre-frequency Telonic bandpass filter, with ±7.5 MHz, 3 dB bandwidth. Transfer-function characteristics obtained with a Hewlett-Packard 8410/2 network analyser are compared in (a) with the s.a.w. network analyser in (b). Comparison of the upper traces in Figs. 2b and 3b shows that the amplitude/frequency characteristic of the s.a.w. network analyser has been modified by the transfer-function amplitude characteristic of the filter, shown in the upper trace of Fig. 3a. The phase/frequency characteristic in the s.a.w. analyser, which is detailed in the centre trace of Fig. 3b, shows the same characteristic as the conventional analyser, changing by 360° over the centre 12 MHz. The network delay can be compensated in the s.a.w. analyser to give the flat phase characteristic shown in the lower trace of Fig. 3b for more detailed investigation of phase linearity with frequency.

This letter has described the principles and demonstrated the performance of a new realisation of network analyser. It employs a s.a.w. d.f.t. signal processor which yields, for the first time, both amplitude and phase information. Attractive features are real-time operation and portability for rapid checkout of communication and radar modules. Further development is in progress to incorporate a time-compression module employing charge-coupled devices at the s.a.w. d.f.t. processor frontend. Potential application areas will thereby be extended to include sonar signal processing, speech and vibration analysis.

Acknowledgement: This research was carried out under a UK Science Research Council grant, and was facilitated by discussions with their Panel. Acknowledgments are also due to R. Coackley, D. P. Morgan, B. J. Darby, J. D. Maines, G. L. Moule and M. B. N. Butler.

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Fig. 1 C.C.D. s.a.w. 2-dimensional Fourier-transform processor
of 16 stages each. The write-cycle clock operated at 1 MHz, matching the s.a.w. processor bandwidth, and each channel is assumed to be accessed at 10 ms intervals, for input to the s.a.w. processor. The s.a.w. chirp-transform processor has the form shown in Fig. 2. This has quadrature chirp waveforms on input and output to perform complex processing. The centre frequencies of the premultiplier chirp and the chirp filter parameters are offset to eliminate signal breakthrough in the multipliers. Thus an offset I.o. operating in phase quadrature is required at the input to the s.a.w. processor. The s.a.w. chirp-chirp parameters of 1 MHz bandwidth and 16 µs duration are matched to the output waveform of the c.c.d. time compressor. The s.a.w. processor performs the Fourier transform in 32 µs and is then available for data from the next hydrophone channel. Fig. 3 shows results from a computer simulation of the system. The time—bandwidth product of the c.c.d. registers and premultiplier chirps was chosen as 16; hence the temporal transform provides a resolution of 1/16 of the c.c.d. read clock frequency, i.e. 1 kHz. Fig. 3a shows the power spectrum obtained after temporal transformation (switches at position T in Fig. 1) with input test data of 3 kHz Doppler at 22° bearing. The power spectra of successive hydrophone records are all identical, since the successive Fourier transforms only have relative phase shift. Fig. 3b shows the phase response of each of the 3 kHz Doppler samples of the successive hydrophone responses. This shows a linear phase variation with hydrophone number. The real and imaginary components of the temporal transform are read simultaneously from the s.a.w. processor into dual 16-stage serial c.c.d. registers (Fig. 1). Here the input clock rate is 1 MHz, and, when 16 samples have been read into the real and imaginary channels, these samples are transferred vertically into the c.c.d. memory stack. Successive transformed hydrophone records build up a waveform in each vertical stack which corresponds to one specific Doppler frequency. The vertical registers are clocked at 10 kHz rate. After the temporal transforms of all the hydrophones are complete, each vertical stack contains a waveform which has constant amplitude and a linear phase variation dependent on the bearing of the target, i.e. effectively a c.w. waveform. Subsequent Fourier transformation of each Doppler stack in a second s.a.w. processor yields the target bearing. Alternatively, it is possible to further time share the first s.a.w. processor to perform both the temporal transform (switches in the T position in Fig. 1) and the spatial transform (switches in the S position). The limitation in this case is that a maximum of 5/4 hydrophone channels can now be used, where 5 is the time compression ratio. Fig. 4 shows results for a simulated target with 3 kHz Doppler at 22° bearing, yielding a single output in the 3 kHz Doppler cell on the Y-axis at n = 3, i.e. 22° on the X-axis.

Conclusion: This letter has demonstrated how existing c.c.d. and s.a.w. processors can be configured in a system to perform 2-dimensional analysis in sonar environments. The fast processing speed of the s.a.w. Fourier transform permits multichannel c.c.d. access for efficient Fourier transformation. The common capabilities of analogue c.c.d. and s.a.w. for signal processing over bandwidths of 1 to 5 MHz, and their similar dynamic range (50 dB) matches the two technologies in this system. The alternative digital time compression technique, which offers narrower bandwidth and improved dynamic range, is less attractive here, owing to the practical problems of performing the data transposition after the temporal transform. The B-scan output is dependent on the ratio of input wavelength to hydrophone spacing, and, consequently, for passive sonar application, microprocessor control of the axis scale factor may be necessary.

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C.C.D. SPECTRUM ANALYSER USING PRIME TRANSFORM ALGORITHM

Indexing terms: Charge coupled device circuits, Transforms, Spectral analysers

The design and performance of a spectrum analyser based on the prime transform algorithm and implemented with charge coupled devices is reported. Detailed consideration is included of the hardware reductions possible with this algorithm. Computer simulations are presented that critically compare the performance of the prime transform, chirp-Z-transform and fast Fourier transform algorithms.

Introduction: New algorithms, such as the chirp-Z-transform, and the prime transform have been developed as an alternative to the established digital fast Fourier transform (f.f.t.) algorithm. The significance of these new algorithms is that they permit direct analogue computation of the discrete Fourier transform using charge coupled device (c.c.d.) transversal filters, fabricated in m.o.s. technology.

This letter considers the design principles of prime transform processors. The operation of a prototype 13 point processor is demonstrated when analysing c.w. input signals, and its performance is related to computer-simulation results which compare the accuracy of c.c.d. analogue transforms with the digital fast Fourier transform.

Prime transform: The prime transform can be computed by means of three separate operations. The first is a permutation (re-ordering) of the input data with a code derived by defining a primitive root relative to the prime number of input-data samples. The second operation is correlation of the permuted input data with permuted discrete cosine and sine samples. The third stage, repermutation, which yields the discrete Fourier transform components in the conventional order of linearity frequency may be dispensed with for applications which involve inverse Fourier transformation. The additional circuitry required for computation of the d.c. transform component and the constant transform offset are not considered here.

In our permuter design, N input-data samples are stored in an analogue c.c.d. tapped shift register. Hardwire connections are made from each output tap to the inputs of a c.m.os. analogue demultiplex circuit. The relevant c.c.d. tap output can then be selected by the digital address held in the c.m.os. counter, Fig. 1. More sophisticated permutation filters, which use an analogue random access memory (a.r.a.m.), are available commercially, but they require the use of a separate digital memory. The four possible codes which can be used in the design of a prime transform processor of length N = 13 are shown in Table 1. These codes were computed using the relationship:

\[ x_{\text{out}} = R^{x_{\text{in}}} \pmod{N} \]

so that the input sample number \( x_{\text{in}} \) appears as output sample number \( x_{\text{out}} \). The permutation code chosen for our processor was that corresponding to \( R = 11 \). Fig. 2b shows the c.c.d. permuter operating in a first-in last-out mode with the sine input shown in Fig. 2a. Fig. 2c shows the operation when connected in the time reverse permutation code presented in Fig. 1. Fig. 2e shows the permuter output when a '1' input, Fig. 2d, occurs at input samples number 1, 2, 3, 4. The outputs appear in samples number 1, 6, 9, 11 respectively, see Table 1.

![Fig. 2 C.C.D. permutation operation](image)

<table>
<thead>
<tr>
<th>Output sample number ((x_{\text{out}}))</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding input sample number ((x_{\text{in}})) (R = 2)</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>3</td>
<td>6</td>
<td>12</td>
<td>11</td>
<td>9</td>
<td>5</td>
<td>10</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>(R = 6)</td>
<td>6</td>
<td>10</td>
<td>8</td>
<td>9</td>
<td>2</td>
<td>12</td>
<td>7</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>(R = 7)</td>
<td>7</td>
<td>10</td>
<td>5</td>
<td>9</td>
<td>11</td>
<td>12</td>
<td>6</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>(R = 11)</td>
<td>11</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>12</td>
<td>2</td>
<td>9</td>
<td>8</td>
<td>10</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Time reverse code (R = 11)</td>
<td>1</td>
<td>6</td>
<td>10</td>
<td>8</td>
<td>9</td>
<td>2</td>
<td>12</td>
<td>7</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 1 PERMUTATION CODES \(N = 13\)
The tap weight accuracy of ±5% for the c.c.d. filter together with a c.c.d. permuter accuracy of ±3% produced an output transform accuracy of approximately 2% r.m.s.

Fig. 3 C.C.D. prime transform spectrum analyser operation (c.c.d. clock 13 kHz, N = 13)

a: Input 4 kHz b: Input 6 kHz

Revisions in transform hardware: Considerable reductions in hardware are possible under certain conditions with the prime transform (p.t.). This is in contrast to the chirp-Z-transform (c.z.t.) where complex multiplication of the input signal always produces real and imaginary components necessitating use of four c.c.d. filters. In the prime transform however, for purely real input data, the number of c.c.d. filters is reduced to two. Furthermore, for even symmetry of the input data, the discrete cosine prime transform (d.c.p.t.) permits operation with a single c.c.d. filter. Table 2 indicates the relative number of c.c.d. stages required in each case, highlighting the significant hardware reduction in the discrete cosine prime transform.

Table 2 REQUIRED NUMBER OF C.C.D. STAGES TO COMPUTE AN N-POINT POWER SPECTRUM OF A REAL INPUT SIGNAL

| C.C.D. stages (split gate taps) | 20 N | 12 N | 4 N |
| C.C.D. stages (floating gate taps) | 10 N | 8 N | 4 N |

Computer simulation of transform accuracy: Fig. 4 shows simulated predictions of the sensitivity of c.c.d. analogue Fourier transform processor accuracy as a function of component error, when processing complex data. Results show that for c.c.d. tap weight accuracy in the range 1-10% (curve a) and c.c.d. charge transfer inefficiency in the range 10^-2-10^-3 (curve c) the prime transform and chirp-Z-transform present no detectable difference in transform accuracy. Furthermore, when the effect of permuter accuracy in the range 1-10% (curve b) is included, it is found that the prime transform employing reduced hardware, Table 2, achieves no significant improvement in transform accuracy. When the tap-weight accuracy (±5%) and permuter accuracy (±3%) measured in our experimental c.c.d. prime transform processor (two c.c.d. filters) are included in Fig. 4, the measured transform accuracy of 2% r.m.s. is in close agreement with the simulation results. Fig. 4 also demonstrates that the 3-9% accuracy of present analogue permuter designs is the dominant error source in the prime transform. Permuter development should achieve ±1% accuracy, resulting in a projected overall transform accuracy of 0.5% r.m.s. In comparison, the accuracy of a 13-bit f.f.t. (fixed point) processor is typically 0.1% for transform lengths less than N = 100. However, the hardware, power consumption and cost of the f.f.t. processor is considerable.

Conclusions: The design and performance of a prototype c.c.d. prime transform processor, which gave an output accuracy of 2% r.m.s., have been reported. The simplification in hardware with the prime transform and discrete cosine prime transform, Table 2, has been shown to offer distinct advantages over the alternative chirp-Z-transform. In addition, for monolithic implementation the prime transform does not require high-performance onchip bipolar analogue multipliers. Although the accuracy of c.c.d. transforms is inferior to the digital f.f.t., they offer lower power, compact analogue processors which can operate at bandwidths up to 5 MHz in real time. It is, therefore, in systems applications which require such engineering advantages at the expense of transform accuracy e.g. sonar and speech analysis, that the analogue c.c.d. transforms will find application.

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Design and performance of a programmable real-time charge-coupled-device recirculating delay-line correlator

J. Mavor, M.A. Jack, D. Saxton and P.M. Grant

Abstract: The design and performance of a recirculating delay-line correlator employing a charge-coupled-device high-density digital memory are reported. The operation of the processor is demonstrated when arranged as a matched filter and spectrum analyser for typical sonar waveforms. The processor permits rapid evaluation of system performance with changes in signal parameters, and results are presented which simulate the effects of target Doppler on linear frequency modulation and linear period modulation.

1 Introduction

The development of compact lightweight low-power hardware to perform real-time, programmable correlation permits the realisation of many sophisticated signal-processing functions such as matched filtering and spectrum analysis. In sonar and seismic systems, there exists a requirement for low-cost signal processors with time-bandwidth products up to 1000 at bandwidths below 20 kHz. Integrated circuit charge-coupled-device (c.c.d.) correlators employing analogue and digital reference signals have been reported. These custom designed devices offer low-cost real-time programmable correlation at bandwidths up to 1 MHz, but they are presently limited to time-bandwidth products of 64.

Dedicated digital hardware correlators are based on the Cooley-Tukey fast Fourier transform (f.f.t.) algorithm. These correlators make use of the property that correlation of a signal and reference is equivalent to the inverse Fourier transform of the product of the Fourier transforms of the signal and reference. As such, this correlate requires three f.f.t. processors. Real-time f.f.t. based correlators are commercially available at present.

In contrast, the ready availability of high-density semiconductor memory permits efficient data correlation to be achieved with the delay-line time compressor (Deltic) configuration. Here, stored data samples are processed during multiple recirculations through a high-speed memory. This permits the design of large time-bandwidth product programmable narrowband correlators. The prototype Deltic processor reported here, which operates over a real-time bandwidth of 500 Hz with 1024 samples, is demonstrated when operating as a programmable correlator. Use of the processor as a simulator to aid sonar waveforms and system design is also demonstrated. The advantages of this processor are as follows:

(a) fully programmable operation
(b) low-power c.c.d. memory (50 mW recirculation)
(c) high-density memory (1024 x 9 bits in one device)
(d) real-time correlation
(e) lightweight

2 Correlator principles

Discrete correlation of two data vectors A and B, each of length N points, requires multiplication of data vector A by a matrix of \(N \times N\) elements where the rows of this matrix consist of increasing time-shifted versions of the data vector B. In a system employing a single multiplier, element correlation requires a total of \(N \times N\) multiplications plus associated shift and add operations. In correlators based on transversal filter techniques\(^4\), the parallel processing of \(N\) simultaneous multiplications in an \(N\)-stage transversal filter reduces the processing time by a factor \(N\). This permits real-time wideband (1 MHz) correlation. Transversal filters for analogue and digital operation have been realised in both charge-coupled device (c.c.d.)\(^5\) technology and in surface-acoustic-wave (s.a.w.)\(^6\) technology.

An alternative technique which permits real-time correlation is the Deltic processor. Here, the data is time compressed, or expanded in bandwidth, by a factor \(N\) to permit a single, fast multiplier element to perform the required \(N \times N\) multiplications in a time equal to \(N\) input sample periods. However, the multiplier must operate at a rate of \(N\) times the input sample rate, thus, it is only applicable to narrowband systems. Further, the data contained in memory must be recirculated at a rate \(N\) times the input sample rate. For a fixed data record, the memory information is held for \(N\) complete recirculations before being replaced by a new record. With a varying input signal, after each recirculation the oldest memory sample is replaced by a new input sample. Since all the data samples are available for subsequent processing, multiplying each sample of the recirculating data with a fast reference signal and integrating over \(N\) samples provides one point of the correlation function. Further points are obtained on successive recirculations. Since the reference is a stored digital signal, the correlator is infinitely programmable.

3 C.C.D. memory technology

C.C.D. shift registers, which are available for either analogue or digital operation, are suited to the nondestructive fast recirculating data storage required in the Deltic correlator. The c.c.d. charge-storage element is dynamic and it is necessary, especially in long shift operations, to periodically refresh or regenerate the stored signal charge to compensate for effects intrinsic to the c.c.d. such as signal attenuation and smearing due to charge-transfer inefficiencies. With digital c.c.d. operation, on-chip refresh and switching functions can readily be provided, say, every 128 bits, if
required. However, with analogue c.c.d. operation, the stored charge must be accurately regenerated for efficient analogue signal transfer.

The additional requirements of data recirculation in the Deltic processor can be readily met using digital c.c.d. storage and recirculation. With analogue recirculation, it becomes necessary to define and stabilise the feedback gain accurately against drift and thermal variation, limiting the achievable number of recirculations to, typically, less than 50. Further, inaccuracies produced by charge-transfer inefficiency in the c.c.d. become magnified on each recirculation. Techniques for improving the performance of a recirculating store were considered in the correlator design.

These include the following:

(i) precoding or predistorting the input data to minimise the major effects of charge-transfer inefficiency;
(ii) operation of the recirculating memory with loop gain less than unity, with subsequent compensation;
(iii) using alternative memory organisations, such as a serial-parallel-serial arrangement to minimise the number of charge transfers.

While not being subject to the stability problems of an analogue recirculating memory, digital recirculation does require associated analogue-digital (a.d.) and digital-analogue (d.a.) conversion. However, inexpensive high-precision (12-bit) convertors for moderate data rates (1 MHz) are currently available. Although the digital c.c.d. requires many parallel channels per sample, the analogue c.c.d. requires a larger chip area per storage element to achieve adequate signal/noise performance. In practice, both techniques require comparable chip areas.

Fig. 1 Recirculating c.c.d. delay-line correlator

The correlator design reported here employs digital memory since c.c.d. high-density memory components are at present available which can transfer data efficiently at clock rates of a few megahertz. This maximum rate is at present limited not by the c.c.d. element, which is capable of efficient operation at clock rates in excess of 100 MHz, over a limited number of stages, (see Reference 7), but by peripheral circuitry such as on-chip clock drivers and input/output amplifiers.

4 Prototype correlator

The digital store used in the prototype Deltic correlator design was a Fairchild CCD450 serial-storage memory consisting of 9216 bits which are organised into a format of 1024 bytes by 9 bits. This architecture is realised by the use of 9-bit bytes stored or retrieved in a byte-serial mode. In this application only 8-bit words were used, the ninth register being blanked. The correlator, see Fig. 1, operates by reading 1024 samples of the input signal at a slow sample rate. Each sample is a.d. converted into an 8-bit word and input to memory. The memory contents are recirculated at a rate 1025 times the input sample rate with one new data sample added each recirculation, displacing the oldest memory sample, see schematic in Fig. 2. In the time interval between input samples, the memory recirculates all 1024 stored samples, reading out (non-destructively) a complete history of the input data with a time compression of 1025 times. Although the maximum memory clock rate is 3 MHz, the correlator is currently operated with a 500 Hz input signal bandwidth which requires a memory clock frequency of 1.025 MHz. The correlator can therefore process 1000 samples of a waveform of 1-s duration and bandwidth 500 Hz. This corresponds to a time-bandwidth product of 500.

Correlation is achieved by storing 1024 x 8-bit samples of the required reference waveform in the reference store using the a.d. converter, see Fig. 1. In practice, this is an n.m.o.s. r.a.m. where the samples are accessed in time-reversed sequence in synchronism with the recirculating signal store. Multiplying each of the recirculating memory samples with the corresponding stored reference sample and summing all 1024 such products, yields one sample of the correlation waveform. The adjacent correlation waveform sample is obtained on the subsequent memory recirculation after updating the memory with a new sample. Multiplication is performed in an 8 x 8-bit digital multiplier. Here, the 16-bit output is digitally integrated to form a 24-bit word, whose upper 12 bits represent the value of one correlation sample. The output can be displayed in either unipolar or bipolar format using a 12-bit resolution d.a. convertor.

Fig. 2 Principle of delay-line time compression

5 Correlator applications in matched filtering

In an active sonar system, signal-design considerations are important for target detection. For long-range operation the transmitted energy must be high, requiring either high output power or long-duration signals. However, a long-duration signal gives poor target-range resolution. It can be shown, however, that it is in fact the frequency characteristic of the transmitted waveform which governs the range resolution. Thus, by transmitting a coded signal of the same basic duration, the system range resolution can be increased by a factor equal to the code time-bandwidth product. To achieve this performance, however, autocorrelation or matched filtering is required in the receiver.

Our prototype correlator has been successfully used to correlate digital pseudonoise codes with time-bandwidth products up to 255. However, this paper details the correlator performance with the more attractive linear frequency and linear period modulation waveforms.

ELECTRONIC CIRCUITS AND SYSTEMS, JULY 1977, Vol. 1, No. 4
5.1 Linear frequency modulation

A signal design which offers high time-bandwidth products with readily predictable sidelobe levels is the linear frequency modulation or 'chirp' waveform, characterized by the unique property that the instantaneous frequency of the signal varies linearly with time.

Fig. 3a shows the correlator response when matched to a chirp sweeping 100–125 Hz in 10s. The displayed correlation function shows close agreement with the theoretical \((\sin x)/x\) response. The amplitude of the first time sidelobe is \(-13.3\) dB relative to the main lobe, with the second sidelobe at \(-20\) dB. The width of the main lobe is governed by \(t_0 = 2/B = 2/25 = 80\) ms.

Fig. 3b shows the autocorrelation response for a narrowband chirp sweeping 100–110 Hz in 10s which demonstrates a degradation from the \((\sin x)/x\) response due to the low time-bandwidth product.

It is clear from Fig. 3 that the high sidelobe levels of the correlation function limit the simultaneous detection of two signals, separated in time, which possess widely differing amplitudes. However, it is possible to reduce these sidelobes by weighting the reference signal. This suppression of the correlation sidelobes is obtained at the expense of broadening the main lobe width. For example, with 'raised-cosine' weighting on an 8% pedestal, the near-in correlation sidelobes are suppressed to a level of better than \(-40\) dB relative to the main lobe. The main lobe is, however, broadened by 50% relative to the unweighted form, at the \(-4\) dB points.

Fig. 4 shows the correlator response for two similar input chirp signals with a time separation of 100 ins. The filter characteristic is 200–250 Hz linear sweep in 10s; i.e. time-bandwidth product of 50. In Fig. 4, the central peak which is of amplitude 5 V, exceeds the displayed vertical scale of 100 mV/division, and is offscreen. One half division on this increased gain scale represents a signal of \(-40\) dB relative to the main response. The sidelobe response and spurious levels are seen to be of the order of \(-40\) dB relative to the main lobe as predicted by theory. The delayed chirp signal is detected with amplitude 200 mV.

The effect of reducing the quantisation accuracy of the correlator reference waveform has been simulated using the Deltic correlator. It was found that reducing the reference accuracy from eight bits to four bits degraded the sidelobe level to \(-32\) dB. In this way, our correlator permits simulation of the trade offs which exist between system performance and hardware complexity.

5.2 Linear period modulation

One of the major limitations in the application of linear frequency modulation waveforms to sonar systems lies in the fact that Doppler shifts can be of the same order as the modulation bandwidth. Here, the simplified narrowband model of Doppler transformation, which assumes that target motion results simply in a frequency shift of the received signals is no longer accurate, and the true effect of signal time expansion or compression must be considered. For wideband linear frequency modulated waveforms the Doppler effect produces a mismatch between received signal and matched filter which results in a distortion in the correlation response. For this reason the technique of linear period modulation or hyperbolic frequency modulation has application in sonar. Wideband linear period modulation exhibits a high Doppler tolerance since, here, Doppler produces only a time shift and does not modify the basic hyperbolic frequency law of the waveform. Thus, whereas a Doppler-shifted linear-frequency-modulation waveform is mismatched to the receiver, a Doppler-shifted linear period modulation waveform remains matched with only a small amplitude change in the correlation waveform.

The Deltic correlator permits simulation of the effects of Doppler on a proposed signal design.
Fig. 5 demonstrates the superior Doppler tolerance of linear period modulation over linear frequency modulation for a wideband signal. Here the correlator input signal was obtained by storing the ideal 'matched' signal in a 1024 x 8-bit memory. By varying the read-cycle clock frequency of the memory it is possible to expand or contract the duration of the 'matched' waveform, thus simulating the true effect of Doppler. Time expansions and contractions up to 30% were simulated as representing the effect of Doppler in a sonar system with wide percentage bandwidth at low frequency. Fig. 5a shows the simulated effect of Doppler on a linear frequency modulated signal sweeping 50–100 Hz in 10 s. It will be seen that the correlation response is destroyed for Doppler changes as low as 10%. In contrast, Fig. 5b shows the performance of a corresponding linear period modulation signal sweeping 50–100 Hz in 10 s. Here, it can be seen that for Doppler changes up to 30% correlation is still effective.

As a further simulation of signal design, Fig. 6 shows the effect of Doppler on a narrow band signal. Here, the input waveform sweeps 230–270 Hz in 10 s with linear frequency modulation (see Fig. 6a) or with linear period modulation (see Fig. 6b). It is down-converted using a 200 Hz local oscillator then input to the correlator which is matched to a sweep of 30–70 Hz in 10 s. The performance of linear frequency modulation and linear period modulation in such an effectively narrowband system is seen to be similar, for expansion and contraction ratios up to 9%. This can be attributed directly to the fact that for small percentage sweeps the difference between hyperbolic and linear frequency modulation is negligible since the hyperbolic law tend to a linear law in the limit.

Thus, we have demonstrated, using the simulator, that for narrowband operation, linear period modulation provides a negligible improvement in Doppler tolerance. For wideband modulation, however, linear period modulation is superior to linear frequency modulation with respect to Doppler tolerance.

6 Correlator applications in spectrum analysis

Passive operation of a sonar system implies that no transmitted signal is employed, the system being used to monitor signals generated by targets themselves. Increased operational capability can be achieved by displaying received signals in both the time and the frequency domains since this aids target classification. For this purpose, real-time spectrum analysers operating at low-signal bandwidths, up to 20 kHz, have been developed. The ability to Fourier transform signals also permits other signal processing operations such as beamforming10 to be performed.

Spectrum analysis also has application in active sonar systems, i.e. where a known waveform is transmitted, for
the determination of target Doppler. As we have shown, for
a narrowband modulation, target motion can be considered
to result in a direct frequency translation of the modulation
waveform spectrum. By measuring such a Doppler fre-
quency shift the velocity of the target can be determined.
The ability of our programmable correlator to perform
chirp waveform correlation makes it suitable for application
as a spectrum analyser using chirp-transformation tech-
niques. The chirp-transform algorithm is derived from the
Fourier transform and permits computation of the Fourier
transform by a process consisting of three basic steps.13
The first step requires multiplication of the input data with
a chirp followed by convolution in a filter with an impulse
response which is matched to the chirp multiplier. The
third step, postmultiplication with a chirp waveform
performs phase compensation. In power-spectrum analysis
this final chirp multiplication can be replaced by appropriate
square and sum circuitry at the output of the chirp filter to
yield the power spectrum.

Fig. 7 Spectrum analyser configurations
a 4-channel operation
b Dual-channel operation
c Single-channel operation

6.1 Spectrum analyser design
Several distinct variations are possible for the realisation of
a chirp transform spectrum analyser depending on the
percentage bandwidth of the waveform to be analysed.

Fig. 7 shows three different arrangements for chirp-
transform spectrum analysis. Fig. 7a shows the basic form
required for a spectrum analyser operating with both
positive and negative frequencies; i.e. zero centre frequency.
Such a configuration would be required in a sonar target
Doppler analysis system where the received signal was
demodulated to baseband and where negative frequencies
exist owing to the movement of targets away from the
transmitter. Since no phase reference is available here, it
becomes necessary to demodulate the input signal using
local oscillators in phase quadrature to retain signal phase
information. The quadrature demodulation can be con-
sidered to generate real and imaginary components of the
waveform. Correlation at baseband requires four parallel
filters

\[
(A_R + jA_I)(B_R + jB_I) = (A_R B_R - A_I B_I) + j(A_R B_I + A_I B_R)
\]

as shown in Fig. 7a, the power spectrum being obtained by
squaring and adding the appropriate terms.

Fig. 8 4-channel spectrum-analysers operation (chirp 50 Hz d.c.
50 Hz sweep in 1.0 s)
a 4-channel output waveforms (zero Doppler) Scale: 200 ms/
division
b Resultant output spectrum (zero Doppler) Scale: 200 ms/
division

The main advantage of the configuration of Fig. 7a is
that the maximum clock rate required in the filter corre-
sponds to the Nyquist rate for the signal being analysed.
This arrangement therefore corresponds to the conventional
chirp-Z-transform13 which performs a discrete Fourier
transform on signals sampled at the Nyquist rate. The dis-
advantage is that it requires four parallel channels whose
gain and phase characteristics must be matched accurately.
Consequently, as seen in Fig. 8, the accuracy of the spec-
trum analyser is dependent on the equalisation of the four
channels. Fig. 8 simulates the four correlation products of
eqn. 1 for zero Doppler. Here, the chirp filter and chirp
premultiplier are V-chirps sweeping +50 Hz to d.c. to
+50 Hz in 1.0 s. The sin (sine) chirp waveform and sin chirp filter were produced by 90° phase shift of chirp multiplier and reference waveforms relative to the cos (cosine) chirp versions. Note that the sin chirp to sin chirp filter and cos chirp to cos chirp filter responses show maxima in the same sense with sidelobe structures in the opposite sense. Addition of this pair of responses results in enhancement of the peak with the sidelobes tending to null. Similarly, the responses of sin chirp to cos chirp filter and cos chirp to sin chirp filter when subtracted, tend to null. Fig. 8b shows errors in the spectrum analyser output due to quantisation errors in the four channels. Note that these responses have been synthesised in our correlator system, which has been realised with only one channel. Each channel response of 1.0 s duration was therefore stored in a 1024 point memory and the responses added to form the output shown.

The cause of the large sidelobe response in the filters of Fig. 8a is simulated in Fig. 9. Here, correlation is shown for a down-chirp filter with an input chirp of opposite sense, (an up-chirp) and for the same down-chirp filter with an input chirp sweep of the same sense. The correlation of a down-chirp signal in the down-chirp filter results in an output which is down-chirp with one-half the frequency time slope. This type of waveform is seen superimposed symmetrically on the sidelobes of Fig. 8a. This added component appears in V-chirp operation since there are periods when the premultiplier and the filter have identical sweep directions.

Fig. 9 Matched and mismatched chirp correlation (chirp filter 50–0 Hz sweep in 1.0 s) Scale: 200 ms/division
(i) Input chirp 0–50 Hz sweep in 1.0 s (ii) Input chirp 50–0 Hz sweep in 1.0 s

Fig. 10 Effect of negative Doppler on dual-channel spectrum analyser (chirp filter 50–0 Hz sweep in 1.0 s) Scale: 200 ms/division

Using the simulator it is possible to show that translation of the chirp sweep by a frequency offset to exclude the V-chirp condition, eliminates the production of this sidelobe component. This further removes the requirement for quadrature chirp filters permitting the simpler, dual channel approach of Fig. 7b to be adopted. Fig. 10 shows the operation of the resulting dual-channel system as a spectrum analyser. Here, the chirp multiplier and reference waveform swept linearly over 50–0 Hz in 1.0 s. The simulation shows, however, that for a negative Doppler frequency this system still produces the large sidelobe levels previously discussed. However, only negative Doppler shift introduces degradations in this processor. Care must therefore be taken in such a system design to ensure that the demodulated signal cannot pass through zero frequency.

Fig. 11 Single-channel spectrum analyser operation (chirp filter 250–200 Hz sweep in 1.0 s) Scale: 200 ms/division

Table 1: Comparison of spectrum analyser configurations

<table>
<thead>
<tr>
<th></th>
<th>4-channel system</th>
<th>2-channel system</th>
<th>1-channel system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of filters</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Filter bandwidth</td>
<td>-25 to +25 Hz</td>
<td>0 to +50 Hz</td>
<td>200 to 250 Hz</td>
</tr>
<tr>
<td>Minimum sample rate</td>
<td>50 Hz</td>
<td>100 Hz</td>
<td>500 Hz</td>
</tr>
<tr>
<td>Number of samples</td>
<td>4 × 50 = 200</td>
<td>2 × 100 = 200</td>
<td>1 × 500 = 500</td>
</tr>
<tr>
<td>Number of multipliers</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

As the minimum processing frequency increases, the correlation response appears at i.f. Here, the single cos chirp filter can be considered as also being the sin chirp filter with one-quarter period delay at the centre frequency. Thus, the processor can be realised with a single filter, see Fig. 7c. As the ratio of bandwidth to centre frequency increases, the accuracy of the spectrum analysis is shown in Fig. 11 also increases. For a percentage bandwidth of 50%, it has been shown that the error in an unweighted response is only 1%. In the single-channel case the power spectrum is obtained by square-law envelope detection.

Note that since the dual-channel and single-channel case employ sampling at a rate greater than the Nyquist rate for the signal under analysis, these two configurations do not perform a true Z-transformation, and are therefore termed simply chirp-transform processors.

We can therefore summarise the design criteria for the three different cases when analysing a 50 Hz signal bandwidth, see Table 1. The single-channel configuration has the

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obvious disadvantage of requiring operation at 10 times the Nyquist rate, with a total filter length of 2.5 times greater than the other configurations. However, there is considerable difference in the degree of complexity of the circuitry associated with the systems. The 2-channel and 4-channel configurations require accurate matching between channels to achieve acceptable performance. The absolute frequency employed in the filter design is governed to a great extent by the technology used to implement them. Although it may be easier to make four filters of length \( N \) than one of length greater than \( 4N \), as, for example with serial analogue c.c.d. storage, our spectrum analyser is based on the single-channel case because of the ease of implementation with a digital c.c.d. store and the simplicity of the peripheral components.

![Fig. 12 Single-channel spectrum-analysers performance](image)

**Fig. 12 Single-channel spectrum-analysers performance**

(a) Chirp-transform spectrum analyser

(i) Weighted premultiplier chirp (200 ms/division)

(ii) Reference chirp (200 ms/division)

(iii) Spectrum-analysers output (40 Hz/division)

(b) Sliding chirp-transform spectrum analyser

(i) Premultiplier chirp (200 ms/division)

(ii) Weighted reference chirp (200 ms/division)

(iii) Spectrum-analysers output (40 Hz/division)

### 6.2 Spectrum-analysers performance

Fig. 12a shows the performance of the single-channel spectrum analyser using the chirp transform. To achieve accurate spectrum analysis in this configuration, it is necessary for the filter bandwidth to be twice the multiplier bandwidth to ensure uniform frequency resolution and sensitivity over a signal bandwidth equal to the multiplier bandwidth. As the filter and multiplier have the same chirp slope, the filter impulse duration must be twice the multiplier chirp duration. Here, amplitude weighting to reduce frequency sidelobes must be implemented on the multiplier chirp. In consequence, the multiplier element must possess high dynamic range on two ports simultaneously. analogue multipliers (such as Motorola MC 1595L) can provide limited dynamic range of up to 40 dB simultaneously on two ports. This severely limits the usable dynamic range of the input signal. In Fig. 12b the length of the multiplying chirp (0.5 s) is half the length of the filter (1.0 s) and the multiplying chirp bandwidth of 200 Hz, centred at 900 Hz, controls the analyser bandwidth. The upper trace of Fig. 12a shows the 'raised-cosine' weighting on the multiplier chirp and the centre trace shows the chirp filter coefficients in the reference store. The lower trace demonstrates spectrum-analysers performance on a scale of 40 Hz/division for simultaneous c.w. inputs of 600, 700 and 800 Hz.

In comparison, Fig. 12b shows the performance of the single-channel spectrum analyser implementing a sliding chirp transform. Here, the multiplier chirp duration is twice that of the filter. Since the filter now controls the signal bandwidth which can be analysed, the bandwidth weighting can be achieved within the filter. In consequence, a switching multiplier can be used. With such a multiplier element, noise-limited, linear dynamic ranges of >70 dB can be achieved.

In Fig. 12b the multiplying chirp of duration 1.0 s has a bandwidth 400 Hz centred at 1000 Hz. The upper trace shows the unweighted premultiplier chirp and the centre trace shows the raised-cosine weighting on the filter coefficients stored in the reference memory. The lower trace shows the spectrum-analysers performance for simultaneous c.w. input frequencies of 600, 700 and 800 Hz on a scale of 40 Hz/division.

### 6 Conclusions

This paper has discussed the design parameters and demonstrated the performance of a programmable correlator system based on a fast recirculating c.c.d. memory, designed both as an engineering prototype and as a simulator for sonar system design.

The performance of the correlator has been demonstrated in a simulation of the effects of target Doppler on linear frequency modulation and linear period modulation to assess their relative trade offs in sonar waveform design. It has been shown that, for wide fractional bandwidth operation, linear period modulation offers significant advantages over linear frequency modulation. The programmable correlator is in principle capable of fast switching between the two types of modulation to optimise correlator performance in variable resolution sonars. The application of the correlator to spectrum analysis has also been demonstrated, using a single-channel configuration in comparison with the more complex multichannel approaches previously reported. The advantages of the sliding chirp transform architecture in the analysis of signals with high dynamic range have been illustrated.

In contrast with analogue programmable correlators, based on transversal filters, the correlator described here permits hardware extension to achieve high dynamic range (70 dB) operation with increased number of data samples. The maximum bandwidth of our correlator is ultimately limited by the memory clock rate. C.C.D. memory technology promises commercial memory clock rates up to 50 MHz in the near future to permit real-time correlation at bandwidths up to 25 kHz (1024 samples).

Although other faster semiconductor memories are applicable to the Deltic correlator design, in practice the correlator speed is limited by the projected near term operating speed of the digital multiplier. Thus it is realistic
to predict only 25 kHz for the operating bandwidth of a 1024 point correlator using low-power c.c.d. memory.

Compared with f.f.t. based processors, the Deltic correlator involves more multiplication operations, but its architecture is much simpler. For spectrum-analysis applications both techniques are broadly comparable in overall complexity. However, for programmable correlation and matched filtering, f.f.t. based processors cannot directly compete since they require three f.f.t. operations. Thus, we consider the programmable Deltic correlator has considerable attractions for low-data-rate (<25 kHz) correlation of coded waveforms with time-bandwidth products between $10^2$ and $10^6$.

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8 References

Waveform Detection and Classification With SAW Cepstrum Analysis

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Abstract

A prototype real time cepstrum analyzer incorporating surface acoustic wave (SAW), Fourier transform processors is reported. This system offers sophisticated wideband signal processing for radar, sonar, and communications applications. Practical results demonstrate its capabilities when analyzing bandwidths in excess of 10 MHz in a few micro seconds with simulated pulsed RF waveforms in the presence of multipath echoes. Pulse duration, repetition interval, and binary code length are resolved and the potential to characterize unknown chirp waveform is briefly reported.

Although mathematicians have been aware of the existence of the cepstrum (kepstrum) [1] for more than a decade, it is only recently that cepstrum analysis has been applied by engineers to waveform identification for target classification, signal extraction in multipath, or reverberation limited environments [2], [3], and in speech processing [4]. To date, signal processing based on cepstrum techniques has been largely confined to computer based systems which operate on sampled data and, in general, are not real time processors. These software systems have demonstrated the application of the power cepstrum [2], [5], the complex cepstrum [2], and the phase cepstrum [6]. This paper illustrates how recent developments in surface acoustic wave (SAW) device technology [7], [8] can be used to implement real time wideband cepstrum analysis which projected application in sophisticated signal processing for radar, sonar, and communications systems.

The design and performance of a prototype real time cepstrum analyzer are described. The system uses two SAW Fourier transform processors based on the chirp transform algorithm [9]. Practical results are included which demonstrate the operational capabilities of the SAW based cepstrum analyzer when processing either IF or baseband waveforms.

Principles of Cepstrum Analysis

Cepstrum analysis is, by definition [1], achieved by a serial arrangement of the Fourier transform processors. The first processor transforms from the time domain to the frequency domain, yielding the spectrum of the input waveform. After signal amplification in a true logarithmic amplifier, the second processor transforms from the frequency domain to a pseudotime (quefrency) domain [1]. This yields the cepstrum of the input time domain waveform. For certain waveforms, particularly time periodic and superimposed time coincident waveforms, spectrum analysis can yield an ambiguous display. The deconvolution effect achieved by the logarithmic processing used in the cepstrum analyser system permits detailed examination of the features of the spectrum to identify constituent elements of these input waveforms.

A simplified description of the operation of the cepstrum analyser is given in Fig. 1 for a signal which consists of a pulse of duration $T$ with an echo at epoch (delay) $r'$ relative to the main pulse, Fig. 1(a). This signal may be interpreted as the time domain convolution of the basic pulse $f(t)$ with two impulses separated by an interval $r'$, Fig. 1(b), which corresponds to the product of the spectra of the two signals, Fig. 1(c). Taking the logarithm reduces this frequency domain multiplication to a linear summation of two signals which are approximately periodic in the frequency domain, Fig. 1(d).

The power cepstrum [2], $C(r)$, has been defined as the power spectrum of the logarithmic power spectrum of an input signal $s(t)$:
where $FT$ denotes Fourier transformation. The second Fourier transform effectively analyzes the frequency periodic components of the log power spectrum to yield the cepstrum. The term quefrency [1] is used in the cepstrum and corresponds to frequency in the spectrum. Quefrency, which is effectively a measure of time period, has units of cycles per hertz (seconds). Fig. 1(e) shows the cepstrum of the waveform under consideration with responses at three different quefrencies. The zero quefrency response results from the constant dc offset in the log power spectrum, while the other two responses correspond to the quefrencies produced by the basic input pulse and echo. If required, the zero quefrency term can be removed by using a dc offset or, alternatively, high pass filtering (liftering) [10] before the second transform. Similarly the sum quefrency term occurring at $(T + \tau')$ can be removed by suitable low pass filtering before the second transform. The cepstrum decomposition effect is valid irrespective of the waveform characteristics, even when the waveform and echo or echoes are time coincident.

The power cepstrum [2] has been demonstrated as an effective analysis technique in the determination of waveform arrival times for a basic waveform distorted by echoes. The technique is effective for echoes of amplitude greater than or less than the basic waveform amplitude. Ambiguity does, however, exist with respect to the amplitude of the quefrency response due to an echo. This can be either a fraction or a multiple of the amplitude of basic waveform quefrency. With multiple echoes the number of echoes and their relative delays cannot easily be estimated from the power cepstrum. This is because the cepstrum peaks occur at sum and difference quefrencies of the echo delays, in addition to the predicted points.

The complex cepstrum [2] is defined as the complex inverse Fourier transform of the complex log Fourier transform. For this cepstrum processing, signal phase information must be retained throughout. The complex cepstrum of a time waveform exhibits sharp discontinuities whose positions are determined by the echo delays. These can be removed by low pass filtering in the frequency domain or by interpolation. Subsequent inverse processing by taking the forward Fourier transform followed by amplitude exponentiation and complex inverse transformation yields the basic waveform recovered from its distorting echoes. Other related waveform decomposition techniques include the phase cepstrum [6] which has application in the analysis of signal phase in the presence of distorting noise.

**SAW Fourier Transform Processors**

The present availability of SAW linear frequency modulated (chirp) filters [7], [8] permits efficient, real time, wideband computation of the chirp transform. This algorithm [11] is derived from the Fourier transform

$$ F(\omega) = \int f(t) \exp(-j\omega t) \, dt $$

(2)

\[ C(\tau) = |FT\{\log|FT\{x(t)\}|^2\}|^2 \]
The maximum input signal duration which can be processed is $T_m$, the duration of the premultiplier chirp, and consequently unless the input signal is time limited to less than $T_m$ and lies wholly within the premultiplier waveform interval, signal truncation will occur and errors will be produced in the output spectrum. The effect of signal truncation is to reduce the frequency resolution of the processor. In the limit the frequency resolution for CW input signals is $1/T_m$. The condition for highest frequency resolution over the widest bandwidth is given by [13]

$$T_m = \frac{1}{2} T_f$$

(7)

For power spectrum analysis [14], where phase information is not required, the final chirp postmultiplier can be replaced by a square law detector at the output of the convolution chirp filter to yield a very close approximation [15] to the true power spectrum.

SAW technology now permits, through the ready availability of compact chirp filters, the realization of a Fourier analyser using three identical filters plus associated amplifiers and timing electronics. The effective parallel processing during convolution in the SAW chirp filter permits real time analysis over signal bandwidths exceeding 10 MHz. Many applications of chirp transform techniques have been proposed including programmable filtering [16], signal detection [17] and analysis [18], and in the design of a programmable correlator [19], [20]. Fully engineered modules employing SAW components are currently used for both coherent and incoherent radar pulse compression [21], compressive receivers [14], and radar pulse Doppler spectrum analysis [22], Fig. 2.

**SAW Cepstrum Analyser: Design**

Cepstrum analysis can be performed by coupling two SAW chirp transform spectrum analyzers through a true logarithmic amplifier, Fig. 3. The four SAW chirp filters employed in this demonstration cepstrum analyser system were fabricated on ST, X cut quartz at 60-MHz center frequency. The signal used to excite the multiplying chirp was an impulse of duration 8 ns (corresponding to one-half cycle at 60 MHz), and amplitude 5 V. The SAW filter impulse 25-MHz bandwidth with a corresponding dispersive slope $\mu$ of 5 MHz/µs. After amplification, synchronous time gating was used to obtain the required 2.5 µs premultiplier chirp sweeping over 12.5 MHz. The two chirp multipliers and the associated gating were all derived synchronously from the master clock employed in the timing circuitry, Fig. 3. Postmultiplication was not employed in the spectrum analyzers. The logarithmic amplifier used was a four stage Plessey SL530C amplifier. The first chirp transform processor (spectrum analyser) transforms from the time domain to the frequency domain and is capable of analyzing a bandwidth of 12.5 MHz in 2.5 µs with a CW frequency resolution of 400 kHz [13] determined by the width of the sinc function response. The second chirp transform processor permits cepstrum analysis of frequencies up to 2.5 µs with a resolution of 80 ns, again determined by the sinc function obtained in the cepstrum.

**SAW Cepstrum Analyser: Performance**

**A. Determination of Pulse Duration**

The power spectrum of a pulse of duration $T$ which is obtained as a time function at the output of the SAW processor is given by

$$|\Phi(\omega)|^2 = |\Phi(\mu)|^2 = |\sin(\mu T) / (\mu T)|^2$$

(8)

where $\mu$ corresponds to the dispersive slope of the SAW chirp filters, employed in the chirp transform processor. Thus the spectrum of the input signal is obtained at the output of the spectrum analyzer through the relationship $\omega = \mu T$. Logarithmic amplification yields

$$\log |\Phi(\mu)|^2 = 2 \log[\sin(\mu T)] - 2 \log[\mu T]$$

(9)

which is approximately periodic in the frequency domain with period $\omega = \mu T = 1/T$. Since the log spectrum approximates to a sinusoid, further analysis yields a single cepstrum response whose quefrency is determined by $T$, the basic pulse duration.

Fig. 4 shows the operation of our demonstration SAW cepstrum processor with such a pulse waveform. The spectrum of a $T = 1$ µs pulse is shown in Fig. 4(a) as measured at the output of the first SAW chirp transform processor. This demonstrates the characteristic sinc function envelope with nulls spaced at 200 ns which corresponds to a frequency interval of 1 MHz (1/T) for this SAW chirp transform processor which employs filters with dispersive slope $\mu = 5$ MHz/µs. The waveform envelope after logarithmic amplification, Fig. 4(b), shows the spectrum envelope to be more nearly sinusoidal, with period 200 ns. The second chirp transform processor therefore effectively sees a 5-MHz CW signal and performs a spectrum analysis of this waveform. The output of the second chirp transform processor, Fig. 4(c), demonstrates sinc

...
function responses at \( t = \pm 1 \mu s \), corresponding to the power spectrum of a truncated 5-MHz CW signal. Fig. 4(c) corresponds to the cepstrum of the input signal to the first chirp transform processor with responses at \( t = \pm 1 \mu s \) and can be used to directly measure pulse width. Note the presence of the zero quefrency response in Fig. 4(c), which could be removed by incorporating a dc offset in the log spectrum. The variations in amplitude between the positive and negative cepstrum peaks is attributable directly to a nonuniform amplitude response in the second SAW convolutional chirp filter. Fig. 5 shows the cepstrum responses identifying pulse durations of \( T = 0.8 \mu s \), Fig. 5(a), and \( T = 1.5 \mu s \) in Fig. 5(b). The described SAW filter parameters permit our cepstrum analyzer to determine pulse durations up to the 2.5-\( \mu s \) duration of the premultiplier chirp. In practice, the zero quefrency term present in the cepstrum limits the minimum resolvable pulse duration to typically 150 ns. Our SAW processor, with a limited time bandwidth product of 32, is therefore capable of determining pulse duration over a range in excess of 10 to 1. Accurate determination of pulse duration requires the input pulse to be time coincident with the premultiplier chirp. However, variation of the pulse timing relative to the premultiplier chirp results simply in a phase change in the spectrum with no effect on the power spectrum or power cepstrum. The responses shown in Figs. 4 and 5 were in fact obtained with a pulse operating synchronously with the cepstrum processor.

In comparison with alternative techniques for the determination of pulse length (such as differentiation), the cepstrum processor provides signal-to-noise ratio (SNR) enhancement. The first Fourier transform processor provides a maximum SNR in the spectrum display at the peak of the sinc response. Here the pulse compression provides a processing gain equal to \( 10 \log_{10} \frac{\mu T^2}{\mu T^2} \), where \( T \) is the input pulse duration. As the input duration \( T \) is reduced, the SNR improvement reduces. However, the lobes of the sinc response widen, maintaining good average SNR over the output waveform. The effect of the logarithmic amplifier is to force the SNR to unity since low level signals are
preferentially amplified. Typically, an SNR of \(-20\) dB at the log amp input is reduced to \(-3\) dB at the output. Thus in the cepstrum analyser, the second chirp transform processor operates with an input SNR close to unity which is subject to the full SNR improvement of \(10 \log_{10} \mu T^2\), independent of \(T\). Here \(T_m\) is the premultiplier duration. Note that increasing the time bandwidth product of the second chirp transform processor would tend to increase the processing gain of the cepstrum analyser and also permit investigation of the waveforms surrounding the nulls of the sinc response which are determined by the rise time and fall time of the input pulse. Further detailed analysis is in progress to establish precisely the limits to SNR improvement in the cepstrum processor.

B. Decomposition of Pulse With Distorting Echoes

A signal \(s(t)\), distorted by an echo of amplitude \(\alpha\) and relative delay \(\tau'\), can be expressed as

\[
s(t) = f(t) + \alpha f(t - \tau').
\]

The spectrum of this signal is given by

\[
S(\omega) = F(\omega) + \alpha F(\omega)e^{j\omega \tau'}
\]  

Equation (12) is seen to be a product of two terms, i.e., a frequency domain multiplication, or time domain convolution. These terms can be decomposed or deconvolved by taking the logarithm

\[
\log |S(\omega)|^2 = \log \Phi + \log(1 + 2\alpha \cos \omega \tau' + \alpha^2)
\]  

and it has been shown \([2]\) that for \(\alpha \ll 1\) or \(\alpha \gg 1\), (14) reduces to a single sinusoidal queue frequency response of amplitude \(2\alpha\) and periodicity (repiod) \(1/\tau'\), corresponding to the echo.

In the case of a pulse of duration \(T\) with a distorting echo of amplitude equal to the basic pulse amplitude and epoch \(\tau'\) relative to the basic pulse, (14) can be written \((\alpha = 1)\)

\[
\log |S(\omega)|^2 = \log[\sin(\mu T)] + \log[(1 + \cos \mu \tau')] \tag{15}
\]  

calculating only the frequency periodic terms.

The cepstrum of a pulse signal of duration \(T = 800\) ns distorted by echoes is illustrated in Fig. 6 for echoes of amplitude equal to the basic pulse \((\alpha = 1)\) arriving at epochs of \(\tau' = 1000\) ns, Fig. 6(b), and \(\tau' = 400\) ns, Fig. 6(d). The cepstrum shows one response corresponding to the basic pulse duration at 800 ns, with a further response at the echo epoch. Information regarding basic pulse length and echo epoch are not directly obvious in the signal spectra, Fig. 6(a) and (c). The 400-ns echo epoch condition corresponds to a self-distortion of the waveform since the pulse and echo are partially time coincident.

C. Measurement of Pulse Repetition Period

The spectra of two 100-ns pulse trains are shown in Fig. 7

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\*AU: Do you mean exp(j\omega') here, not epsilon?
Fig. 7. Capstrum determination of repetition rate. (a) Spectrum and (b) cepstrum of 100-ns pulse, 1-MHz PRF. (c) Spectrum and (d) cepstrum of 100-ns pulse, 2-MHz PRF.

with pulse repetition frequency (PRF) of 1 MHz, Fig. 7(a), and 2 MHz, Fig. 7(c). The corresponding cepstrum responses are shown in Fig. 7(b) and (d). The spectra exhibit the expected sinc function envelope with nulls at ±10 MHz (±2 μs X 5 MHz/μs), corresponding to the pulsewidth of 100 ns. The lines within this sinc envelope have separations determined by the input PRF. The power cepstrum of the 1-MHz waveform, Fig. 7(b) exhibits peaks at quefrency $\tau = \pm 1 \mu s$, which correspond to the repetition period. The spectrum of an input PRF of 2 MHz is shown in Fig. 7(c), while Fig. 7(d) shows unambiguous cepstrum peaks at $\tau = \pm 0.5 \mu s$. The achieved measurement range of the processor was greater than the 250 ns - 2.5 μs pulse repetition period (PRP).

D. Determination of Binary Code Length and Bit Rate

In a manner similar to the measurement of PRP the power cepstrum can also be used to determine the code length of a binary code or the bit rate given a priori knowledge of one of these parameters. In principle it is possible to determine both parameters simultaneously. However, in practice, the limited processor time bandwidth product (32) and the fact that the input code was asynchronous relative to the processor precluded this operation.

The spectrum of a 7-bit pseudonoise code at 5 MHz bit rate is shown in Fig. 8(a). The first nulls of the sinc envelope are spaced at ±5 MHz (±1 μs X 5 MHz/μs) and the number of spectral lines to the first null is defined by the code length (7 bits). The center response (dc) is due to the bit disparity of unity in the code. The power cepstrum, Fig. 8(b), of this code is shown to consist of peaks at $\tau = 1.4 \mu s$. This corresponds to the quefrency of a pseudonoise code of length $n$ bits given by

$$\tau = (2^n - 1)/R$$

(16)

where $R$ is the bit rate. In Fig. 8(b), $\tau = 7/5 = 1.4 \mu s$ as predicted by (16). The spectrum and power cepstrum of a 15-bit code at the 10-MHz bit rate are shown in Fig. 8(c) and (d) with cepstrum peaks at $\tau = 7/5 = 1.5 \mu s$.

The spectrum and power cepstrum of two time coincident codes both of length 7 bits at slightly different bit rates of 7 MHz and 8.5 MHz are shown in Fig. 9(a) and (b). The cepstrum display resolves the ambiguous spectra response into two identifiable responses as defined by (16) at $\tau_1 = 7/7 = 1 \mu s$ and $\tau_2 = 7/8.5 = 0.82 \mu s$. In principle, our cepstrum analyzer is capable of classifying codes of bit lengths up to 31 and bit rates up to 10 MHz and of decomposing codes with a frequency difference greater than 500 kHz.

D. Determination of Chirp Slope

A potential application of the SAW cepstrum analyzer lies in the determination of the unknown dispersive slope of a radar chirp signal. Practical difficulties have prevented experimental studies, hence computer simulation has been undertaken to determine the capability of the SAW cepstrum analyzer in determining the dispersive slope of an unknown chirp.

The power spectrum of a chirp waveform, centered at frequency $f_0$, sweeping over bandwidth $B$ in time $T$ is given by [23]

$$|F(\omega)|^2 = (\pi/4\mu)[(C(X_1) + C(X_2))^2 + [S(X_1) + S(X_2)]^2]$$

(17)
Fig. 8. Cepstrum analysis of binary codes. (a) Spectrum and (b) cepstrum of 7-bit PN code at 5-MHz rate. (c) Spectrum and (d) cepstrum of 15-bit code at 10-MHz rate.

Fig. 9. Cepstrum decomposition of binary codes. (a) Spectrum. (b) Cepstrum.

where $C(X)$ and $S(X)$ are, respectively, the Fresnel cosine and Fresnel sine integrals and

$$X_1, X_2 = (TB/2)^\frac{1}{n} (1 \pm n), \quad n = (f - f_0)/B.$$  \hspace{1cm} (18)

The form of the Fresnel integrals has the basic shape of a chirp. The power spectrum of any chirp signal of finite duration displays these characteristic Fresnel ripples. For a fixed duration chirp, an increase in dispersive slope, $\mu$ (and hence $B$) will increase the variation with frequency of the Fresnel ripples. This effect results in an increase in the quefrcency of the unknown chirp spectrum. Hence the cepstrum of an unknown chirp waveform is predicted to exhibit a linear variation of quefrcency with input chirp slope.

The flow diagram used in the simulation is shown in Fig. 10. The program is written in Fortran and is executed on an IBM 370/168 computer. Basically the program defines the SAW chirp waveforms as complex values and, in effect, performs a chirp-Z-transform (CZT). However, the chirp transform and the CZT can be shown [12], [15] to be equivalent. Complex signal convolution is performed using an IBM subroutine HARM to implement the fast Fourier transform which achieves efficient computation. Fig. 11 shows simulated results for the power cepstrum analysis of two input chirps with dispersive slope 7.0-MHz/µs, Fig. 11(a), and 10.0 MHz/µs, Fig. 11(b), in a cepstrum analyzer employing the previously described SAW chirp filters with dispersive slope $\mu = 5$ MHz/µs. The implication of the precise shape of the cepstrum response is currently under investigation. However, analysis of a range of input chirp dispersive slopes from 6 to 12 MHz/µs has shown that the position of the largest peak in the cepstrum response is linearly dependent on the dispersive slope of the unknown chirp, Fig. 12, over this range.

These preliminary experiments have been reported here as it is considered that detection, identification, and classification of an unknown chirp waveform represents a potentially important application area for the SAW Cepstrum processor.
This paper has described the principles and demonstrated the performance of a wideband (megahertz) real time SAW cepstrum analyzer when processing waveforms typical of those encountered in radar and communication systems. The prototype analyzer computes the cepstrum with two serial Fourier transforms based on the chirp transform algorithm and implemented with existing SAW chirp filters. The modest design parameters, $B = 25$ MHz, $T = 5 \mu$s ($TB = 125$), of these devices permits the computation of a 32-point transform on 12½-MHz signal bandwidth. However, it demonstrates clearly the potential of SAW devices to implement wideband real time cepstrum analysers.

Our prototype analyzer was shown to be effective in determining pulse duration over a 10:1 range from 250 ns to 2.5 µs, this being limited by the $TB$ product of the SAW chirp filter. With currently available SAW chirp filters of impulse response durations ranging from 2 µs to 50 µs and $TB$ products up to 10 000 [8], SAW cepstrum analyzers have the projected capability of measuring pulse durations in the range 50 ns to 50 µs.

Further, the ability of the cepstrum processor to decompose a basic waveform and interfering echoes was demonstrated by determination of relative epochs. In this context the processor offers the capability of isolating basic radar returns from distorting echoes. Further, it is conceivable that a signal processor capable of determining the complex cepstrum could be designed, permitting the extraction of a radar return from a distorted signal produced by echoes.

The paper has also demonstrated that our prototype SAW cepstrum analyser is effective in determining the PRP of a waveform over a 10:1 range from 250 ns to 2.5 µs. Again, using currently available SAW chirp filters this capability can be extended to measurement of PRP in the range 50 ns to 50 µs. The same approach has been used in the cepstrum analysis of binary codes at bit rates up to 10 MHz and code lengths up to 15. Results have been presented which demonstrate the determination of code length and bit rate. This performance can be extrapolated to permit measurement of bit rates up to 200 MHz and code lengths...
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up to 511. Computer simulation of the system performance in the determination of unknown chirp slope has demonstrated a capability of measuring chirp slope up to 100 Hz per 1000 ns, where $\mu$ is the processor dispersive slope. The system performance outwith this range is presently under consideration. In principle, cepstrum processors implemented with SAW devices can be designed for the determination of chirp slopes in the range 40 kHz/µs to 40 MHz/µs. Further, by employing frequency offsets in the premultiplier chip and/or input chip, to perform spectral inversion, the basic SAW cepstrum processor can measure a wide range of absolute values of chirp slope.

Real-time wideband SAW cepstrum analyzers, which are an extension of existing compressive receiver designs, are seen as providing an additional technique for waveform classification and signal processing in ECM and ELINT applications to determine signal duration period and dispersive slope. Further, the signal deconvolution properties of the cepstrum are of potential application in multipath in multipath environments such as low elevation search radar systems. The analyzer offers possible application in radar/sonar target classification [23] and to reverberation limited sonar systems by employing time compression techniques [22], [24] which permit the analysis of narrowband (kilohertz) waveforms, with a resolution which is also improved by the time compression factor. In principle, target classification and identification might now be possible through the cepstrum analysis of target spectral characteristics such as engine sidebands.

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References


