REGENE TRANSITIONS IN TWO-PHASE FLOW

by

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Thesis submitted for the degree of Ph.D.

October 1964.

UNIVERSITY OF EDINBURGH.
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ACKNOWLEDGMENT.

The author wishes to record his deeply felt gratitude to Mr. Roy Jackson for his encouragement and guidance throughout this work. He would also like to thank Dr. D. M. Wilson for his advice, and Mr. C. McLeod and Mr. L. Morris for their help, in the construction of the experimental apparatus. Thanks are also due to Mr. D. Nisbet for his help with electronics, Dr. J. F. Davidson for permission to refer to Ref. (18), Dr. G. F. Hewitt for permission to include reference to Ref. (19), to the Atomic Energy Research Establishment, Harwell, for an equipment grant and to D.S.I.R. for a maintenance grant.
CHAPTER 1.

Introduction.

The problems of two-phase gas-liquid flow have become of greater concern to engineers in recent years. This type of flow is encountered in an increasing number of important situations. Some of its applications are to be found in the fields of the production and transport of crude petroleum and its products, the operation of heat transfer equipment such as steam generators, refrigeration equipment, evaporators and condensers. Heat removal from nuclear reactors is an important recent application of two-phase heat transfer. It is the last named application, more than any other that has led to the recent great increase in research on two-phase flow.

Owing to the great complexity of the subject and the large number of variables involved, most workers have confined their studies to one regime at a time. It seems surprising therefore, that very little work has been done to determine the boundaries of the different regimes, and the mechanisms of transition from one regime to another. This work has therefore been carried out in an attempt to improve knowledge of these matters.

In the ensuing discussion only flow in vertical tubes will be considered. In cocurrent upwards two-phase flow, five different flow patterns are found to exist viz., Bubble, Slug, Churn, Annular and Mist flow. These are shown diagrammatically in fig. 1. The present study is restricted to the slug, churn and annular regimes, and in particular, the transitions from slug to churn, churn to annular and slug to annular flow.
Survey of Literature.

Two-phase flow, unlike single phase flow, has only relatively recently been studied in any detail. One possible reason for this is the great complexity of the subject. The presence of a second phase in a tube introduces many more variables for consideration. In addition to the increased number of variables it is found that there are several widely different flow patterns possible, and in studying the subject of two-phase flow it is necessary to consider the different regimes separately. It is hardly conceivable that a correlation of pressure drop, say, would fit the flow of small air bubbles as well as annular flow of water up the wall of a tube with air flowing up the core.

Some workers have approached the subject by considering the two phases as one homogeneous medium: an intimate mixture of the two phases. As would be expected, this approach has not led to much success, as at times the two phases flow together in a completely different manner e.g. in annular flow, the liquid flows up the wall and the gas flows in the core.

More recently the trend has been to study the different regimes separately.

As might be expected, different workers have given different names to the flow patterns. Govier et al. (1) have given a table of the nomenclature used up to 1957. The names for the regimes that will be used in this work, in order of increasing air flow are:-

bubble, slug, churn, annular and mist flow.
Although two-phase flow as a whole or the different regimes separately have received quite a large amount of attention, there has been very little work done on the mechanism of transition from one regime to another or at what flow rates one regime gives way to another.

One of the first attempts to break the flow down into regimes was that of Martinelli et. al. (2). They considered four different possible flow conditions namely,

1) Liquid viscous, gas viscous.
2) Liquid viscous, gas turbulent.
3) Liquid turbulent, gas viscous.
4) Liquid turbulent, gas turbulent.

The problem then presented is:
At what flow rates does the flow change from viscous to turbulent?

Criteria based upon Reynolds Number were proposed, but again a problem arises, of how to calculate the Reynolds Number for two-phase flow.

The first systematic attempt to separate the regimes is that of Govier et. al. (1). They give a regime map based on visual observations, which is shown replotted in fig. 2. At moderate and low water flow rates, the expected transitions from slug to churn and churn to annular flow are found (considering Govier's ripple flow to be an extension of annular flow). However, at high water flow rates they found that the churn flow regime narrows until, at a water superficial velocity of approximately 5.7 ft./sec, the churn flow regime disappears altogether, giving way to a direct transition from slug to annular flow.
At these superficial water velocities they first of all observed normal slug flow, the slugs were seen to move faster and become more frequent in occurrence as the air flow rate was increased, until finally the slugs merged to give way to annular flow.

It is noted also that the gas flow rate for the churn/annular transition is virtually independent of the water flow rate as is the gas flow rate for the slug/churn transition at low water flow rates, (i.e. superficial water velocity less than 1 ft./sec.).

In addition to visually noting the regimes and regime transitions, Govier et al. attempted to separate them by using curves of pressure drop and hold-up at constant water flow rate. A typical plot of pressure drop vs. superficial gas velocity is shown in fig. 3. This breaks the flow down into four different regimes, the three transition points being the three turning points of the curve, namely two minima and a maximum. One great disadvantage of this is that at high water flow rates the curve smooths out to give only one minimum and no maximum. It is found that the locus of the second minimum of the pressure drop and the transition from churn to annular flow lie very close together. However, the first minimum and maximum pressure drop loci do not correspond closely with any visible regime transition. The attempted separation into regimes according to curves of hold-up did not prove to be very satisfactory.

Although data on regime transitions is not plentiful in the literature, there are a
few reports on transitions from one regime to another. Wallis (3) gives a line showing the transition from churn flow to annular flow. This covers only very low water flow rates (up to a superficial velocity of 0.13 ft./sec.). It is reported that up to this superficial water velocity the transition is independent of water flow rate and occurs at a superficial air velocity of 39 ft./sec. This is a very much higher gas superficial velocity than that reported by Govier et al.

Calvert and Williams (4) have studied upward co-current annular flow, and present a method for predicting liquid hold-up and pressure drop. They divide the film into two layers and consider a boundary layer at the tube wall to be in laminar flow and the layer from the gas/liquid interface to the wall boundary layer to be in turbulent flow. Differential equations expressing velocity gradient are first integrated to give a description of velocity distribution, then once the velocity distribution is known, the rate of flow is determined by further integration. This is done separately for the two layers.

From their equations, they have produced a set of curves relating water feed to film thickness with the interfacial shear as a parameter. The set of curves is shown with termination points indicating that annular flow is no longer possible. Now if the termination points are joined by a curve, this would be the curve of a regime boundary - presumably the boundary between annular and churn flow.
The parameter used in these curves is the interfacial shear. Now in order to use this line as part of a regime map it would be necessary to relate the interfacial shear to pressure drop and hence gas flow rate. The authors do give a pressure drop correlation but unfortunately, the notation in presenting this correlation is so obscure as to make it most difficult to use.

In the development of their equations Calvert and Williams do not take into account the curvature of the film. This would be a valid approximation for very thin films but one would expect it to introduce appreciable errors for the thick films, one would expect to encounter near the transition from annular to churn flow.

The authors obtain an expression for the velocity distribution in the film in terms of pressure drop and film thickness, this they have expanded according to the binomial theorem, and then integrated, neglecting terms of a power greater than unity. In effect they have assumed that

\[ \frac{\tau_b}{\rho \cdot g} (I - x) \]

Where \( \tau_b \) is the interfacial shear stress,
\( \rho \) the density of the liquid,
\( g \) acceleration due to gravity,
\( x \) the film thickness

and \( x \) is the laminar boundary layer thickness.
Now, from the authors' figures it can be shown that $\frac{\gamma b}{\rho g}$ is of the same order as $X$, and when one considers thick films which are on the point of collapse, the portion of the film $x$, probably extends down to quite small values of $X$, and so over quite a large part of the integration it is not a good approximation to take

$$\frac{\gamma b}{\rho g} \gg (X - x)$$

and hence the authors' expansion of the integrand with neglect of higher order terms is not a good approximation. It would only be expected to be a good approximation in the thin film region well away from the region of instability of the annular flow regime, and so it would not really be expected to form the basis for a theory of transition.

Wallis (3) proposes that there should be a relationship between the phenomenon of flooding in countercurrent flow and a transition between regimes in co-current flow. In the conclusions of his paper he states:

"The transition to co-current upwards annular flow in a vertical tube at low liquid velocities is the same as the flooding limit for zero liquid downflow in a tube in which the wall is fully wetted along its length and the limiting process occurs in the tube."

Experiments were carried out to determine this flooding limit for two different water flow rates. The results indicate that at the water
flow rates used, the flooding limit is independent of water flow rate and that it occurs at a superficial air velocity of

\[(0.8 \pm 0.9)/g^{1/2}[D \cdot g \cdot (\rho_f - \rho_g)]^{1/2}\]

where \(\rho_g\) is the density of the gas,

\(\rho_f\) is the density of the liquid,

\(D\) is the diameter of the tube

and \(g\) is the acceleration due to gravity.

This superficial gas velocity is found to agree well with the transition given by Hewitt and Hall-Taylor in the same paper. It should be noted that this transition criterion was only tested for very low liquid feed rates, to a liquid superficial velocity of 0.26 ft./sec. Wallis also states that in qualitative experiments carried out at Winfrith it has been observed that higher air velocities are needed to support annular flow at higher water velocities.

Shearer and Davidson (18) have developed a theory predicting the flooding point in a wetted wall column. The basis of their theory is to assume that, at the limiting gas flow, a standing wave forms on the liquid surface, the amplitude being several times the mean film thickness. Their calculations show that the wave amplitude is very sensitive to gas flow so that within a narrow range of increasing flows, the amplitude becomes very large indeed, and they find the upper limit of this range to agree with the measured gas flow for flooding of the wetted wall column.
The above theory takes no account of the effect of the length of the falling film on the flooding point. It has recently been shown (19) that the flooding point occurs at lower gas velocities for longer columns. This is a variable that has been neglected until now, and further investigation would appear to be merited.

Galegar, Stovall and Huntington (5), (6), (which are the same paper) give a regime map. This is reproduced in Fig. 4. Their semi-annular flow is taken to be annular flow in the present notation. As can be seen, their slug/annular transition is very largely dependent upon the water flow rate. They also report the annular/annular transition to be dependent upon the water flow rate. This paper is purely a report of experimental observations and attempts no explanation of the regimes or the mechanism of transition between them.

Nicklin (7) has proposed a mechanism for the direct transition from slug to annular flow, from consideration of the slug flow regime. He states that as the gas rate is increased, the voidage fraction will increase, but the limiting film thickness will also increase. Hence liquid will be transferred from the wakes to the film, producing progressively longer slugs. This process continues, and the transition to annular flow is complete when all the liquid in the wakes has been transferred to the film. However, Nicklin points out that in practice, churn flow appears
between slug and annular flow. He suggests that churn flow is probably caused by instability in the film, corresponding to flooding in wetted wall columns. He states further, that flow in the liquid film of a slug is identical with the flow in the liquid film of a wetted wall column and hence whether or not the flow is likely to be stable, can be determined from flooding data for wetted wall columns. From this, Nicklin predicts that for a 1.02 in. tube, at atmospheric pressure and for zero liquid flow, slug flow is no longer a stable flow pattern at superficial gas velocities greater than about 5.5 ft. per second.
Experimental Apparatus.

A general flow diagram of the rig is shown in fig. 5. The test section was 1 inch I.D. perspex tube having a bore tolerance of 0.004 inches. This was firmly mounted on a length of 6 in. steel channel which was fixed to both the floor and the roof of the laboratory. It was ensured that the tube was mounted vertically. All joins and flanges in the test section were very carefully machined so as to offer no obstruction to the flow within the tube. The test section used was 13 ft. 8 ins. long and had a calming section of 3 ft. 6 ins. below it.

Before building the rig, the wetting properties of the perspex tube to be used, were checked as Clayton (8) found it necessary to specially treat the tube in order to improve the wetting properties. This was found to be unnecessary as very thin falling films were found to be quite stable.

The air was fed to the test section through a 2 in. BSP Tee leading to a cone which reduced the bore to 1 in.

Water was fed to the test section through an annular slot having a 45° bevel, from a feed box containing a weir to ensure a uniform feed of water around the annular slot. (fig. 6). Feed boxes were placed at both the bottom and the top of the tube to allow both co-current and countercurrent flow experiments to be carried out.

The top feed box was constructed in such a way that different feed sections could be put in position. In experiments with falling films
the 45° bevel, a belled feed piece (fig. 6), and a cylinder of porous sintered phosphor bronze were used.

Very accurately machined full-bore cocks were positioned at each end of the test section. These cocks were made with 2 3/4 in. brass barrels in 4 in. cubes of phosphor bronze. The barrels were lapped into the blocks using a very fine abrasive, and the flow bore was machined to an accuracy of 0.0002 in. (fig. 7). The two cocks were linked together by a steel rod so that they could be closed with one fast movement of a lever. There was a Bowden cable connected to this lever so that, as the cocks were closed, a valve was opened to permit the air to by-pass the test section in order to prevent hammer effects and pressure build-up. The cocks had adjustable stops fixed to the actuating arms in order that they could always be reset in exactly the same position to offer no obstruction to the flow in the tube.

The air supply used was a Sihl liquid ring compressor rated at 50 Std. cu. ft. per min. This compressor was chosen because it would deliver a pulse-free, essentially water-saturated supply of oil-free air. The water pump used was a 2 H.P. Worthington-Simpson centrifugal pump.

Downstream from the test section was placed a tank for separating the air and water mixture. As the pressure control of the system was to depend upon throttling the air exhaust, this separating tank had to work efficiently and no air could be permitted to escape through the water exit. This was accomplished by
fitting the tank with a liquid level controller which would maintain a constant head of liquid. The two-phase mixture entered the tank through the side, the air left from the top and the water exit was at the bottom. From the separating tank the water flowed to a header supply tank for the pump.

Air and water fed to the system were controlled by valves placed between the supplies and the test section. Control of the mean pressure in the test section was effected by throttling the exhaust air with a valve situated after the separating tank.

Measuring Equipment.

High air flow rates were measured by measuring the pressure drop across a very accurately made orifice plate with corner taps. A water manometer was used to measure this pressure drop. For measuring low air flow rates a rotameter which was calibrated against a special orifice plate was used. Both orifice plate and rotameter were placed in the compressor inlet line. This was done in order to eliminate fluctuations in reading that might be caused by fluctuations in pressure in the test section.

As in the case of air, the water flow rate was measured by either an accurate orifice plate or by rotameters for the low flow rates. All orifice plates were carefully made to British Standard Specifications, as laid down in (9). The pressure drop across the water metering orifice plate was measured by a manometer filled with n-heptane (density 0.68 gm./c.c.) on water.
The hold-up was measured by suddenly closing the cock, then allowing the water to settle in the tube. The height of the water was then measured by using a set-square (to obviate parallax) and a steel tape that was fixed to the steel channel behind the test section.

Two pressure measurements were to be made. One was the mid-point pressure of the test section and the other the pressure drop across the test section. It was decided to measure only the total pressure drop across the test section as Bennett and Thornton (10) have shown that pressure drop varies linearly with distance along the tube. The pressures were measured by means of two mercury manometers, one measuring gauge pressure and the other differential pressure. A diagram of the pressure tappings is shown in fig. 8. The four holes had a diameter of 0.0465 ins, and were carefully drilled so that there would be no 'rags' to obstruct the flow within the tube. These holes led into a piezometer ring, which in turn led to nylon tubing leading to the manometers. There were facilities for a small water feed into this tubing. This water flow was measured by small rotameters and was controlled by needle valves. The reason for the water feed was to prevent air bubbles entering the pressure circuit and so causing errors in the pressure readings. This flow of water was at all times kept
very low in order not to have any effect on the pressure readings and also not to have any effect on the flow in the test section. Photographs of the apparatus are shown in figs. 9, 10.
Regime Map for Co-current Upwards Flow.

The first experimental work to be carried out was the plotting of a regime map from visual observations. This map was to cover the regimes of slug, churn and annular flow for a wide range of water flow rates.

In order to eliminate one variable of the system, namely the density of the gas phase, it was decided to conduct all experimental work at a constant mid-point, test section pressure. The figure of 30 p.s.i.a. was chosen as it would be an easily attainable pressure, and it would fall within the range covered by most other workers. All experimental work, therefore, was carried out at a mid-point test section pressure of 30 p.s.i.a. and all gas volumes and flow rates presented in this work are calculated for this pressure.

In plotting the regime map, sets of runs at a constant water flow rate and varying air flow rates, were carried out. This was done, both for ease of operation and also in order to facilitate the plotting of curves of pressure drop and hold-up against air flow rate at constant water flow rate. Subsequently, for each run, air flow rate, water flow rate, hold-up and mean pressure drop were measured, and the flow pattern was noted from visual observation.

Before starting a test run, the small water feeds into the pressure tapping lines were established and a check made to ensure
that the flow had no effect on the manometer zeros. To carry out a run, an air flow somewhat above that required was first established, the water flow rate was set to the desired value and the exhaust valve was adjusted to give an approximate mid-point pressure of 30 p.s.i.a. The flows were then carefully adjusted to the required values, the required readings were taken and the flow pattern was noted visually. Finally the full-bore cocks were suddenly closed to trap the two-phase mixture in the test section. After settling, the height of the water in the tube was measured and gave an estimate of the voidage.

As expected, it was found to be very difficult to distinguish visually between the different flow patterns in the regions near the regime transitions. The criteria used in distinguishing between the regimes were as follows:

**Slug flow.** The regime was called slug flow as long as regular pistons of liquid could be seen to rise the full length of the tube. The characteristic round nose of the slug flow regime could not always be observed. This was not deemed necessary as at some of the high water flow rates considered, with slug velocities of up to 18 ft./sec., there were a large number of small entrained air bubbles in the water, which made the nose of the slug difficult to see. In any case, it is difficult for the eye to follow a slug moving at a velocity of over 10 ft./sec.
Annular flow. The regime was called annular flow, so long as the water was flowing on the walls of the tube, and there was no bridging of the tube by the water.

Churn flow. The regime was called churn flow if it did not satisfy either of the above criteria. In other words, if no regular slugs could be seen travelling up the tube, and yet the tube was intermittently bridged with water, then the regime was deemed to be churn flow. In many cases there were local reversals in the direction of flow of the water in churn flow, but this was not the case when the water flow rate was very high (superficial water velocity greater than approximately 4 ft./sec.), so this was not used as a criteria for churn flow. Photographs of typical slug, churn and annular flow are shown in fig. 11.

In presenting the regime map (fig. 12), some comment must be made on the validity of the concept of flow regimes in two-phase flow. At fairly low water flow rates there are certainly finite ranges of air flows over which the pattern can visually be assigned quite unambiguously, to one of the three types defined above. Thus the concept of separate and well defined regimes is a valid one, but it is less certain that the customary representation of the demarcation between adjacent regimes by a line, has any validity. Between adjacent regimes there is always a range of air flows over which the allocation of the flow pattern
to one or other is uncertain; in other words there is a transitional range rather than a transition point as the air flow rate is varied. Rather than attempting to conceal this by drawing a sharp line, it has been decided to indicate the transition regions by shaded zones in the present regime map. The boundaries of these zones are the points at which there is no longer any doubt about the nature of the flow, and all points at which there is any real uncertainty at all, have been included in the transitional regions.

The slug to churn transition was reasonably well defined and hence this transition band is very narrow. On the other hand, it was often found to be very difficult to distinguish between churn and annular flow over a relatively large range of air flow rates and correspondingly the transition band is rather wide. It is interesting to note that the gas velocity for the slug to churn transition is independent of the water flow rate over the full range of water flow rates considered. The same feature is exhibited by the churn to annular regime transition for water superficial velocities of less than 0.5 ft./sec., but for water superficial velocities greater than this, the air velocity at the transition becomes dependant upon the water flow rate.

For water superficial velocities of less than 0.5 ft./sec., well defined slug, churn and annular regime flow patterns were observed. However at higher water superficial velocities,
the flow patterns were not nearly so clearly distinguishable, except at the extremes of the slug and annular regimes. In fact, at very high water superficial velocities (greater than about 3 ft./sec.) it would possibly be better to use a descriptive term other than churn flow to describe the regime.

The sequence of events observed for an increasing gas flow rate at a constant water superficial velocity of, say, 6 ft./sec. was as follows:

First of all, at very low air flow rates, slug flow with very fast moving slugs, was seen. The slugs then increased in velocity until the eye could no longer easily follow an individual slug as it rose up the tube. The flow then appeared as a regular pulsing white foam, as there was a large amount of entrainment of air in the water. As the gas flow was further increased, the pulsing appeared to become less regular for a time, but then with further increase in air flow rate, the pulsing became more and more regular again.

The initial regular pulsing was deemed to be slug flow but at higher gas flow rates it would appear inappropriate to apply the terms churn or annular to the flow pattern.

In order to learn more about this high flow rate pulsing flow, the Atomic Energy Research Establishments' Photographic Department undertook to take a series of high speed cine films on the A.E.R.E. 'Lotus' two-phase flow rig. These films were taken at a speed of 6,000 frames per second, at water superficial
velocities varying from 4.2 to 6.5 ft./sec., and air flow rates up to a superficial velocity of 15 ft./sec.

In all cases, the flow pattern was a very disturbed one, exhibiting a high degree of entrainment, both of air in water and of water in air. For air superficial velocities of less than 8 ft./sec., the flow resembled very disturbed slug flow, although the term 'finger flow' would possibly be more appropriate. Fingers of gas with thick (of the order of \( \frac{1}{4} D \)), very wavy films around them, were seen to rise up the centre of the tube. The fingers were followed by regions of water with such a high proportion of entrained air, that they appeared more as a foam. These foamy regions moved considerably faster than the fingers, and in fact appeared to destroy them by projecting a rod of foam up the centre of the finger.

As the air flow rate was increased above a superficial velocity of 8 ft./sec., the foamy regions increased in frequency and the fingers became less distinct and regular, but otherwise the general appearance of the flow pattern remained much the same. The rods of foam which apparently destroyed the fingers appeared in the films at all gas and water flow rates. Some still photographs were also taken in this high flow rate region, and are reproduced in figs. 13, 14.

In the light of the above observations, it was decided to present the regime map as shown (fig. 12) with the churn/annular transition band widening with increasing water flow rate,
up to a water superficial velocity of 2.2 ft./sec. At water superficial velocities greater than this, there are no longer three visually distinguishable regimes, but a more continuous variation in appearance of the contents of the tube with increasing air flow rate.

The present regime map may now be compared with those given by other workers. Govier et al. (1) have given a very comprehensive map, based on visual observations. This is superimposed on the present map in Fig. 15. It is seen that the slug/churn transitions agree extremely well up to a water superficial velocity of 1.0 ft./sec. For water superficial velocities greater than 1.0 ft./sec, Govier's transition line rises upwards to give an increased range of slug flow. This was not observed in the present work. Govier's churn/annular transition is seen to occur at a superficial gas velocity somewhat higher than that of the present work. On the same figure is shown the churn/annular transition line of Hewitt and Hall-Taylor (given in (3)), which is seen to lie at a very much higher air flow rate than that given in the present work. Also superimposed on Fig. 15 is the map of Galegar, Stovall and Huntington (5). Their regime boundaries do not appear to agree with any of the other boundaries given, except at very low water flow rates, when their slug/churn transition agrees with that of Govier et al., and with the present work. Fig. 15 also shows a curve indicating the transition from slug to churn flow predicted from a theory of Nicklin. This curve is derived in a later section and indicates a transition at lower
superficial air velocities than found in the present work.

It should be borne in mind that the regime boundaries of Govier et al., Hewitt and Hall-Taylor, Galegar, Stovall and Huntington, and Nicklin were all obtained at different air pressures and therefore at different densities. The work of Govier et al. was carried out at a mean pressure of 36 p.s.i.a., Hewitt and Hall-Taylor and Nicklin's work are for essentially atmospheric pressure, while the curves of Galegar, Stovall and Huntington were obtained at varying pressure. Govier et al. (11) have shown that regime transitions occur at lower superficial air velocities when the density of the gas phase is increased.
CHAPTER 4.

Pressure Drop and Hold-Up in Co-current Upwards Flow.

As already mentioned, the runs used in plotting the regime map were carried out in sets at constant water flow rate. This was done in order to facilitate the plotting of curves of pressure drop and hold-up against superficial air velocity, at constant water flow rate.

These curves were plotted to obtain a comparison with the work of Govier et. al.(1), and to explore further their proposed separation into regimes, based on the form of the pressure drop curve. A typical pressure drop curve at low water flow rate is shown in fig. 3. Govier et. al. have proposed that, instead of separating two-phase flow regimes by visual methods, the pressure drop maximum and minima should be used as regime boundary criteria. This method is not unsuccessful at low water flow rates. Unfortunately as the water flow rate is increased, the pressure drop curve smooths out, and at water superficial velocities greater than approximately 2.0 ft./sec. the curve shows only one minimum and no maximum. It is mainly because of this smoothing out of the curve, that this proposed method of separating the regimes is of doubtful value.

Typical pressure drop curves obtained are shown in figs. 16, 17, 18, 19. Superimposed on figs. 16, 18, 19, are pressure drop curves
obtained by Govier et al. at a mean pressure of 36 psia. It is noted that the form of the curves obtained is the same as that obtained by Govier et al. However, in the present work, the pressure drops obtained are a little higher than those by Govier et al. This difference cannot be entirely explained by the difference in pressures at which the curves were obtained as Govier et al. (11) found very little pressure drop dependence on absolute pressure. The general trend of the curves with increasing water flow rate, is found to be the same as in Govier's work. At low water flow rates, the curves exhibit a double minimum, and then as the water flow rate is increased, the double minimum disappears and the curves exhibit only one minimum and no maximum. As the present results were found to agree so closely with Govier's work, it was decided not to pursue this approach to the separation into regimes any further.

It should be noted that the smoothing of the pressure drop curve provides additional reason for the presentation of the regime map in the form already discussed. As stated, the pressure drop curves are found to smooth out to a monotonic curve at a value for the superficial water velocity of about 2 ft./sec. and this corresponds extremely closely with the visual observation that the flow regimes become smoothed together at water superficial velocities greater than about 2.2 ft./sec.

Curves showing the relationship between hold-up \((1 - \epsilon)\), where \(\epsilon\) is the voidage fraction in the test section) and superficial air velocity at constant water flow rate, are
shown in figs. 16, 17, 18, 19, 20, and are compared with those obtained by Govier et al. in the same figures. As can be seen, the agreement with Govier's work is excellent. In view of this agreement, it was again decided not to pursue the possibility of there being a correlation between hold-up and regime transitions.
CHAPTER 5.

Instantaneous Pressure Drop Measurements in Co-Current Upwards Flow.

Owing to the lack of success in separating two-phase flow regimes by visual observation and by means of pressure drop and hold-up curves, it was decided to explore the possibility of finding some other parameter on which to base a regime map. From consideration of the appearance of the flow patterns in the tube, it was decided to explore the possibility of measuring fluctuations in pressure drop and hold-up. This in turn reduces itself into the problem of making instantaneous measurements of these variables.

Calderbank and Rennie (12) have developed a method of measuring hold-up utilising a radioactive technique with a Caesium 137 source. This was investigated and its use appeared to be feasible until the size of the necessary radioactive source was calculated to be of the order of 4 curies. A source of this strength would obviously create many problems in handling, and in addition, the expense involved in the counting and measuring equipment would be very large. In view of these problems, and taking into account the limited time available for developing the technique, it was decided to abandon this method for measuring hold-up. Other methods for the instantaneous measurement of hold-up, such as conductance probes, were considered, but as no method appeared to be entirely satisfactory, it was decided to
abandon this approach in order to concentrate on the measurement of instantaneous pressure drop.

On investigation, it was decided that the instantaneous measurement of pressure drop and pressure would be feasible using pressure transducers and suitable electronic equipment for measuring the electrical output from the transducers. The Chemical Engineering Division of AERE, Harwell, built the electronic equipment and supplied the pressure transducers and ancillary equipment for this purpose.

The basic test rig has already been described, this was modified for the present work as follows.

Solartron pressure transducers, Type NT4 - 313, were used for the actual pressure measurement. These were mounted in solid perspex blocks, carefully reamed out and flanged to fit into the original test section without causing any disturbance to the flow. The transducer face was connected to the test section by means of a 0.040 ins. diameter hole. Transducers were placed at the mid-point, three-quarters of the way up, and at the top of the test section, immediately below the top full-bore cock. Unfortunately, due to supply problems, transducers of different ranges had to be used, which meant that the effective sensitivity of some of the transducers had to be reduced. The sensitivities of all the transducers could be matched by suitably adjusting their supply voltages. The transducer supply circuit is
shown in fig. 21. The output from the transducers was fed to a long-tailed pair amplifier giving a gain of up to 1,300. The circuit diagram, shown in fig. 23, is obtained from the Mullard Reference Manual of Transistor Circuits p.271. The output from the amplifier could be fed to one of three units:

a) A Solartron Cathode Ray Oscilloscope (Solarscope AD 557). The trace on the Oscilloscope could be photographed with a Cossor oscillograph camera (Model 1428).

b) The mean voltage of the output could be measured with a D.C. voltmeter having a capacitor across its terminals.

c) The output could be fed to a circuit to measure the R.M.S. value of the signal fluctuations. This circuit is shown in fig. 24 and is a transistorised differential amplifier designed to measure only the fluctuating component of the signal for frequencies greater than 0.05 c/s. The mean value of the signal was subtracted before the R.M.S. fluctuation was measured by means of a hot wire voltmeter.

A photograph of the measuring equipment is shown in fig. 22.

The design of the pressure transducer supply circuit was such that, in order to measure the pressure drop between two transducers,
it was only necessary to connect their outputs in opposition. Then so long as their sensitivities and zero's were the same, the algebraic sum of the voltages was proportional to the pressure difference. Some trouble was experienced with this measurement as originally the transducers were supplied by a common circuit and, as the transducers are wheatstone bridge networks, when the outputs were connected in opposition to measure the pressure difference, the balance of the bridges was destroyed and an incorrect reading was recorded. This was overcome by building a separate supply for each transducer.

The above method of measuring the pressure drop was adopted in preference to a differential transducer because the differential transducer entails a hydraulic link which would introduce transmission delays and consequent errors. The system used has no such errors.

The complete transducer system was designed to cope with frequencies up to 3 KHz, with an accuracy of 3%.

The runs in this phase of the work were carried out in exactly the same manner as those described for the plotting of the regime map, with the difference that, before starting, the amplifier was balanced and the zero's of the transducers were set to the atmospheric pressure prevailing. The sensitivities of the transducers were then set equal and then the transducers were calibrated by filling the test section with water only. Hold-up was not measured in this phase of the work as this had already been done in plotting the regime map.
The parameters measured in any run were:

- water flow rate,
- air flow rate,
- mean pressure drop, and the R.M.S.
- value of the pressure drop fluctuations.

Absolute pressure measurements were not made as it was decided that pressure fluctuations occurring outside the test section would have too great an effect on the absolute pressure measurement made within the test section.

If one considers the fluctuations in pressure drop occurring in a tube during two-phase flow at a relatively low constant water flow rate and an increasing air flow rate, then the sequence of events expected would be as follows:

At very low air flow rates the fluctuations would be minimal, then as slug flow started one would expect the fluctuations to increase rapidly and to continue to increase into the churn flow regime, and to reach a maximum somewhere within the slug or churn regimes. As the air flow was increased still further one would expect the fluctuations to decrease as the flow pattern changes to annular flow, and to continue to decrease until in mist flow the pressure should not fluctuate at all. One would therefore expect a curve such as fig 25 when plotting the R.M.S. value of the pressure drop fluctuations against superficial air velocity for a constant water flow rate. In addition, it would be reasonable to expect the R.M.S. value of the fluctuations to increase with increasing water flow rate.
The above phenomena are in fact found to occur except that the expected falling off of the R.M.S. value of the fluctuations at high air flow rates was not observed. This is possibly due to the fact that the air flow rate was not high enough to attain the mist flow regime. The experimentally obtained curves are shown in fig. 26 as a plot of the R.M.S. value of the pressure drop fluctuations against the superficial gas velocity for different constant water flow rates.

As can be seen, for water superficial velocities below 2.19 ft./sec., the curves reach a maximum at a superficial gas velocity between 5 and 8 ft./sec. and then decrease with increasing air flow rate. The curves for a superficial water velocity roughly between 1 and 2.19 ft./sec. then show a slight increase in fluctuations. It should be noted that the maximum of these curves occur in the neighbourhood of the observed transition from churn to slug flow, i.e. at an air superficial velocity of 8 ft./sec.

For higher water superficial velocities than 2.19 ft./sec., there is seen to be a great difference in the form of the curves. The maximum disappears and the curves increase uniformly with increasing air flow rate.

As expected, the height of the curves increases steadily with increasing water flow rate up to a water superficial velocity (\(\nabla\)) of 2.19 ft./sec., there is then a
sudden large increase in the R.M.S. value of the fluctuations to the curve for 

\[ \bar{v}_f = 2.60 \text{ ft./sec.} \]

the curves then again increase steadily to the highest water flow rate used.

Fig. 27 shows a plot of R.M.S. fluctuations against \( \bar{v}_g \) as percentages of the mean pressure drop. Although these curves are not as uniform as the plots of the R.M.S. fluctuations themselves, they illustrate, even more markedly than the previous curves, the great difference in the fluctuations when the superficial water velocity is increased above 2.19 ft./sec.

Up to 

\[ \bar{v}_f = 2.19 \text{ ft./sec.} \]

the curves increase steadily, they then make a large jump to the curve for 

\[ \bar{v}_f = 2.60 \text{ ft./sec.} \]

and then they decrease with increasing water flow rate.

In Chapter 3, when discussing the form of the regime map it was stated that for superficial water velocities greater than about 2.2 ft./sec., there was no obvious change in flow pattern from one regime to another but that the flow pattern appeared to change uniformly from slug flow, through a pulsing flow to annular flow. This observation was found to fit in very well with the change in the form of the pressure drop curves (Chapter 4). They smoothed out and lost the double minimum exhibited at low water flow rates).

The present curves are found to again verify the above observations. There is a very definite
change in the form of the curves of the R.M.S. value of the pressure drop fluctuations for water superficial velocities greater than 2.19 ft./sec. In fact the change in the form of the curves is so marked as to suggest a change in the mechanism of flow, and it would appear that one is not justified in using the terms slug and churn flow when the superficial water velocity is greater than about 2.2 ft./sec.

During the course of the above work, photographs were taken of the trace of the instantaneous pressure drop on the cathode ray oscilloscope. Typical traces are shown in figs. 28, 29. Nothing much can be said about these traces from visual scrutiny apart from noting the different characteristics exhibited by the different regimes. Correlation methods of analysing the traces were considered with a view to finding out if there are any characteristic frequencies present for the different regimes, but were abandoned for lack of time and suitable data processing facilities.
Nicklin (7) has developed a theory of slug flow which agrees very well with his experimental results. It is proposed to restate this theory in a slightly different form and then to extend it and so obtain a curve predicting the transition from slug to churn flow.

If the condition of instantaneous continuity of volume flow is applied to a cross-section occupied by a film of stable thickness well below the nose of a slug, we obtain:

\[ L_f + v_b \cdot \epsilon \cdot A \cdot \frac{dA}{dL} = G + L \]

where

- \( L_f \) = upwards volume flow of water in the film.
- \( v_b \) = rate of rise of the slug.
- \( \epsilon \) = voidage fraction at the cross-section under consideration.
- \( A \) = cross-sectional area of tube.
- \( G \) = volumetric gas flow.
- \( L \) = volumetric liquid flow.

Introducing the gas and liquid feed superficial velocities

\[ \bar{v}_g = \frac{G}{A} \]

and \( \bar{v}_l = \frac{L}{A} \),

this can be written:

\[ \frac{L_f}{A} = (\bar{v}_g + \bar{v}_l) - v_b \cdot \epsilon \]
or \[ \frac{L_f}{A} = v_b(1 - \xi) - \left[v_b - (\bar{v}_g + \bar{v}_l)\right] \] ... (6:1)

where the bubble velocity is given by:

\[ v_b = 1.2(\bar{v}_g + \bar{v}_l) + v_b^o \] ... (6:2)

where \[ v_b^o = 0.35 \sqrt{gD} \]

and is the rising velocity of a Dumitrescu bubble, (13)

\( g \) is the acceleration due to gravity,

and \( D \) the diameter of the tube.

Thus given values of \( \bar{v}_g \) and \( \bar{v}_l \) determine a value of \( v_b \) through (6:2) and hence determine a straight line (6:1) when \( L_f/A \) is plotted against \( (1 - \xi) \).

The curve showing the relationship between \( L_f/A \) and \( (1 - \xi) \) can be determined by assuming that the flow in a wetted wall column is the same as the flow in the falling film around a slug flow bubble and then conducting experiments on a wetted wall column. Nicklin's results are replotted in fig. 30, where the upwards superficial liquid velocity in a falling film, \( L_f/A \) is plotted as a function of \( (1 - \xi) \) for different constant mean velocities of gas in the central core, \( \bar{v}_g \). It is noted that, although this is a family of curves they all lie on top of one another within the experimental accuracy, though they terminate at different points as
shown. This means, rather surprisingly, that for a constant liquid flow rate the liquid film does not thicken as the gas flow rate is increased.

Now the intersection of the line (6:1) with the curve of $L_{f/A}$ against $(1 - \xi)$ corresponding to the value $v_b$ for the parameter $v_g$, gives a pair of values for $L_{f/A}$ and $(1 - \xi)$ consistent with slug flow. However since the curve of $L_{f/A}$ against $(1 - \xi)$ terminates at a point depending on the value of $v_b$, the line and curve may not intersect at all indicating that there can be no stable slug flow pattern. The above procedure can be followed through to obtain a series of points in the $\bar{v}_g - \bar{v}_l$ plane representing conditions of limiting stability, the resultant curve is shown in Fig. 31. It is seen to predict that, as the water feed rate is increased, the value of the gas superficial velocity at which the slug flow regime becomes unstable, decreases until at a water superficial velocity of approximately 5.1 ft./sec. a slug flow regime can no longer exist, no matter what the superficial gas velocity.

If this transition curve is compared with that of Govier et al. it is seen that they disagree entirely. Whereas the present curve predicts that the critical superficial gas velocity decreases with increasing liquid flow rate, Govier's curve predicts an increase. In addition, the present curve is seen to be well below both Govier's curve and that obtained in the present work.
As a result of these discrepancies it was decided to carry out experiments on falling films to obtain a comparison with Nicklin's work, and the experimental apparatus was accordingly adapted as follows:

A \( \frac{3}{4} \) in. I.D. copper tube which rose up the centre of the lower calming section was fitted to the air inlet. (See fig. 32). This arrangement permitted the withdrawal of water from the bottom of the tube in such a manner that there would be no interference between the water exit and air inlet. The water was fed to the tube via a feed-box placed above the top full-bore cock and made in such a way that the geometrical design of the feed-piece could easily be changed. In the falling film experiments the feed-pieces used were:

- a 45° Bevel, a belled feed-piece
- and in later work a porous brass sintered cylinder (fig. 6).

The experiments using the porous brass sinter were done as part of a later series where water was fed to the test section at its mid-point. This means that the falling film for this feed-piece was only half the length of that for the other feed-pieces. This was the only difference between this and the other sets of results.

In these falling film experiments, the required water flow rate was first established, then the air flow was started. In order to prevent any air from leaving the test section via the water exit, a liquid seal was maintained.
at the bottom of the tube by adjusting the valve on the water exit pipe. Hold-up was again measured by suddenly closing the full-bore cocks once steady state conditions within the tube had been achieved. The flooding point was taken as the air velocity at which some water just began to flow upwards from the feed-box instead of all the liquid flowing down the tube and out at the bottom. To determine the flooding points, a falling film was first established and then the air flow rate was very gradually increased until flooding just occurred.

The first experimental work on falling films determined the relationship between hold-up and air and water flow rates. The results are shown in fig.33. It was found that, at constant water flow rate, the hold-up was independent of the air flow rate up to the flooding point. This somewhat surprising result confirms Nicklin's observations, and the results agree very well with Nicklin's, as shown in fig.34. The agreement is so good it would appear that, not only is the hold-up independent of gas flow rate, but it is also independent of the gas density, as Nicklin's work was carried out at atmospheric pressure and the present work at a pressure of 30 p.s.i.a.

The next series of experiments was the determination of the flooding points in the falling films. Results were first obtained using the 45° Bevel feed-piece. These and the later results using different feed-pieces are shown in fig.35. In the experiments with
the 45° bevel feed-piece it was found that, except at the lowest water flow rates, flooding always occurred at the water inlet. At the feed-piece, the water formed a lip that extended further into the tube than the rest of the falling film and presumably this was the seat of flooding, (a diagrammatic sketch of this is shown in fig. 36).

With the belled feed-piece, and also with the porous sinter feed-piece, this lip did not form and the flooding started well down the tube. For the belled feed-piece, the belling permits the acceleration, and consequent thinning of the water film before it meets the air. For the porous sinter feed-piece the liquid is introduced into the tube over a length of the test section and so again, there would be no lip formed. At flooding, the tube was seen to bridge with water well down from the water inlet, after which the bridge would rise some distance up the tube and then break up to leave a falling film only. This process would be repeated until one of the bridges reached and passed the water inlet. Once this had occurred, an entirely falling film could not be re-established without reducing the air flow rate. As can be seen in fig. 35 the flooding points are very largely dependant upon the method of feeding the water to the tube. This confirms the observations of Wallis (14) who found that flooding for a nozzle feed-piece occurred at higher gas rates than for a 'sharp' feed-piece. It is seen also that the flooding velocities determined in the present work are much lower than those reported by Nicklin. This may be due to the dependence of the flooding velocity on the density of the gas phase, when
the flooding velocity would be expected to decrease with increasing gas density, as appears to be the case. Another factor to be considered, when comparing the present work with that of Nicklin is that the definitions of the flooding points may well be different, as Nicklin does not clearly define the criterion for flooding in his work. Nicklin's feed-piece appears to have had the edge rounded, which would approximate to the 45° Bevel feed-piece used in the present work and hence, one would expect some of his flooding to occur at the water inlet. Another factor to be considered with the present results is the presence of joins in the test section which could possibly affect the flooding point as Nicklin found the flooding velocities to be higher in a single tube than in one formed from jointed sections.

Shearer and Davidson (18) have developed a theory predicting the flooding point in a wetted wall column. Their predicted flooding curve for a pressure of 30 p.s.i.a. is shown in fig. 35. This curve is seen to predict flooding to occur at lower gas velocities than the present experimental results.

Recently obtained results of Hewitt (19) have shown a dependence of the flooding point on the length of the falling film. He has shown that flooding occurs at lower gas velocities when the length of the falling film is increased. His flooding curves at pressures of 20 and 40 p.s.i.a. for a 9 ft. long 1.25 ins. I.D. tube are shown in fig. 37. These are compared with the results obtained using the porous sintered feed-piece where the length of the falling film was also approximately
9 ft. Our results were obtained at a pressure of 30 p.s.i.a., and again show that the flooding points in the present work tend to occur at somewhat lower air velocities than those of other workers. Hewitt's results illustrate very well, the effect of pressure on the flooding point.

One must now consider Nicklin's slug/churn transition theory described earlier in this chapter, in the light of the present results. In order to determine the proposed transition curve it is necessary to plot the curves showing the relationship between the liquid flow in the falling film and the hold-up. This is obtained from fig. 33 and is shown in fig. 38. The figures shown on the curves are the termination points determined from the flooding points obtained with the porous sinter feed-piece. Superimposed on fig. 38 is the curve obtained from Nicklin's results. As expected from the agreement between the results shown in fig. 34, the two curves agree very well except that the termination points differ widely. This is to be expected as the curves of flooding velocities differ so greatly, Nicklin's values being much higher than those obtained in the present work. If one now uses the present curve to obtain the limit of stability of the slug flow regime it is found that, because of the very much lower values of the termination points on fig. 38, the transition is predicted to occur at even lower values of gas flow rate than those obtained from Nicklin's results.

The conclusion to be drawn from the above
observations is that the proposed theoretical determination of the limit of stability of the slug flow regime is of very doubtful value without further investigation of the mechanism of flooding of a falling film and the factors which affect it.
Rising Film Slug Flow.

In his thesis, Nicklin (7) proposes a mechanism for a direct transition from slug to annular flow with no intervening churn flow regime. He suggests that the liquid in the pistons separating the slugs is transferred progressively to the films surrounding the slugs. Now with such a mechanism, just before the transition to annular flow, the films round the slugs would have to be rising, rather than falling films and must carry the greater part of the liquid fed to the tube. The possibility of flow patterns consisting of slugs surrounded by rising films has not previously been explicitly mentioned. Nevertheless, there seems to be no reason why such slugs should not exist under certain conditions, and they may possibly explain the direct slug/annular transition observed by Govier et al. (1) at high liquid feed rates.

It is, of course, only possible to satisfy the hydrodynamic conditions for a round nosed slug if the liquid in the film flows downwards relative to the slug, but since the slug is moving relative to the tube wall, at quite a high velocity in some cases, there is no reason why the liquid flow in the film should not be upwards relative to the tube wall, even though it is downwards relative to the slug. The condition of continuity well down a slug is still embodied.
in equation (6:1) so slugs surrounded by rising films would be represented by the intersection of this line with the appropriate curve

\[ v_g = v_b \]

chosen from the rising film set in a plot of \( \frac{L_f}{A} \) against \((1 - \ell^2)\). Given values of \( \bar{v}_g \) and \( \bar{v}_b \) determine a value for the slug velocity \( v_b \), and this in turn, determines a straight line (6:1) and a unique curve from the rising film set. If this line and this curve do not intersect, a slug flow regime with rising films round the slugs is certainly impossible. If they do intersect, such a regime would not violate the continuity condition, but there is a further condition to be satisfied, namely that the film flow rate \( L_f \) shall not exceed the liquid feed rate \( L_f \). If \( \frac{L_f}{A} \) at the intersection of the curve and the straight line is smaller than \( \bar{v}_b \), not all the liquid fed to the tube is carried up it by rising film, and the remainder forms pistons between the slugs.

On the other hand, no pattern is possible in which the tube is bridged by liquid pistons, and a purely annular flow solution must be found.

It is not difficult to see under what conditions slugs with rising films (which we shall abbreviate to PFS, to distinguish them from slugs with falling films, abbreviated to FFS) may occur. Consider the limiting condition in which the gas flow has been reduced
as far as possible without the annular regime becoming unstable in the manner already described. The horizontal line \( \frac{L_g}{A} = \bar{V}_L \) will then cut the rising film curve corresponding to a core gas velocity \( V_g \), where

\[
V_g \cdot \bar{\epsilon}_i = \bar{V}_g
\]

or

\[
V_g = \frac{\bar{V}_g}{\bar{\epsilon}_i}
\]

where \( \bar{\epsilon}_i \) is the voidage at the intersection of the line and the curve. This is shown as point A on the curve OA in fig. 39. At this point we have

\[
\frac{L_g}{A} = \bar{V}_L
\]

and

\[
V_g \cdot \bar{\epsilon}_i = \bar{V}_g
\]

Adding these and rearranging, we obtain

\[
\frac{L_g}{A} = \bar{V}_g + \bar{V}_L = V_g \cdot \bar{\epsilon}_i \quad \quad \quad \quad \quad (7:1)
\]

This equation represents a straight line of gradient \( V_g \), and is similar in form to the equation

\[
\frac{L_g}{A} = \bar{V}_g + \bar{V}_L = V_b \cdot \bar{\epsilon} \quad \quad \quad \quad \quad (6:1)
\]

This is shown as the line PQ in fig. 39.

Now compare this line (7:1) with the straight line (6:1) representing the continuity condition for slug flow.

\( V_b \) is given by

\[
V_b = 1.2 (V_g + V_L) + V_0 \quad \quad \quad \quad \quad (6:2)
\]

while \( V_g \) in (7:1) is given by

\[
V_g = \frac{\bar{V}_g}{\bar{\epsilon}_i}
\]
For small values of $\bar{V}_L$ the climbing film will be thin even at its stability limit, so $(\bar{V}_g - \bar{V}_b)$ will be small, and consequently $V_g$ will not be very much larger than $\bar{V}_g$. However, $V_b$ is certainly greater than $1.2 \bar{V}_g$, so $V_b$ will be greater than $V_g$ and the rising film curve for the parameter $V_b$ will therefore look like the curve $OA'$ in fig. 39. In addition it is seen that the change from $V_g$ to the larger $V_b$ in changing from line (7:1) to line (6:1) displaces this line and therefore its intersection with the horizontal, $L_f/A = \bar{V}_L$, to the right. Thus the straight line (6:1) looks like the line XY sketched in fig. 39. This line may, or may not, intersect the curve $OA'$, but if it does, their intersection point will lie in $L_f/A > \bar{V}_L$ and, as seen above, this cannot represent a physical RFS flow pattern. Thus we should not expect a RFS regime to be a possible alternative flow pattern at the point of collapse of the annular regime when $\bar{V}_L$ is small.

If $\bar{V}_L$ is large, on the other hand, the rising film will be thick at its stability limit, and $(\bar{V}_g - \bar{V}_b)$ will therefore be large. Under these conditions, $V_g$ will be substantially larger than $\bar{V}_g$, and at very high liquid feed rates it is perfectly possible for $V_g$ to be larger than $V_b$. In this case the curve and the straight line whose intersection determines the
possibility of slug flow are as represented by OA'' and UV respectively in fig. 39. If these intersect, their intersection point lies in $\frac{L_f}{\Delta}<\overline{V}$ and therefore represents a perfectly possible slug flow pattern.

Without quantitative information on the form of the rising film curves, we cannot decide whether or not this actually happens, but it is at least possible to say that a stable RFS pattern may exist at low gas flow rates and high liquid flow rates.

The next step is to obtain quantitative results for the form of the rising film curves. Unfortunately, the figures required are in the region of high liquid loading and the majority of work carried out in the annular regime has been at very much lower liquid loadings than the present work requires. Calvert and Williams (4), have analysed the stress at the gas/liquid interface for rising film flow which leads to a correlation between the interfacial shear stress and liquid flow and film thickness. It should then be possible to relate the interfacial stress to $v_g$ by means of a pressure drop correlation given by Calvert and Williams, but their notation is so obscure as to make it virtually impossible to use. In their analysis, Calvert and Williams have neglected the curvature of the film, and before integrating their equation 11, which is their expression for the velocity profile in the film, they have expanded it according to the binomial theorem and neglected all but first order terms. It can be shown that this is not a good approximation except
at very low water flow rates. In view of the above, the analysis was reworked in exact form, taking into account the curvature of the film (see Appendix I). The resulting equations were then programmed for a mercury computer. The results of this, a set of curves showing the relationship between liquid flow in the film and \((1 - \epsilon)\) at constant interfacial shear stress are shown in fig. 40. Now these curves are labelled with the shear stress, whereas we need to have them labelled with the gas velocity in the core, \(v_g\). The shear stress is related to the pressure drop by

\[
P' \cdot \tau_b = - A' \left( \frac{\delta P}{\delta z} \right)
\]

where
- \(P'\) = perimeter of gas core.
- \(\tau_b\) = interfacial shear stress.
- \(A'\) = area of gas core.
- \(\left( \frac{\delta P}{\delta z} \right)\) = pressure drop per unit length of tube.

and we can now plot the curves of \(L_f/A\) against \((1 - \epsilon)\) at constant pressure drop. The resulting curves are shown in fig. 41. It should now be possible to use these in conjunction with the experimental curves described in chapter 4, relating the pressure drop and superficial air velocity at constant liquid flow rate, to derive curves relating liquid flow in the film to hold-up at constant air velocity in the core. This was attempted but was found impossible, as when considering a fixed liquid flow it was found that the experimental pressure drops were so
much lower than the theoretically obtained pressure drops that the required pressure drop curve on fig.41 did not cut the line of constant $L/A$ under consideration. A comparison between the theoretically obtained curves and a curve derived from the experimental results discussed in chapter 4 is shown on fig.41. The dotted curve labelled (E, 1.0) is the experimentally obtained curve for a constant pressure drop of 1.0 ft. $H_2O/ft$. As can be seen, the experimental results in no way agree with the theory and we must conclude that the theory does not adequately describe annular flow at high liquid loadings.

It is possible, however, to obtain the required curves relating liquid flow in the film and hold-up at constant core superficial velocity from the experimental curves relating pressure drop and hold-up to superficial air velocity at constant water flow rate, described in chapter 4. The set of curves obtained is shown in fig.42. Superimposed on this figure are the extremes of the regime boundaries described in chapter 3. It should be remembered however, that the curves shown within the slug and churn regions cannot be interpreted in terms of climbing films. The analysis described for predicting the occurrence of the rising film slugs can now be carried out with the aid of this figure. The predicted limits of stability of the rising film slugs are shown in fig.43. A FPS pattern is not prohibited by continuity considerations in the region to the right of the curve.

While plotting the regime map the rising film slugs were looked for and could be reasonably
clearly distinguished as part of the slug flow regime at superficial gas velocities greater than about 4 ft./sec. (see fig.13). The region in which they were observed lay below the lower part of the curve shown in fig.43, and their occurrence was not therefore in conflict with the theory given above. It would appear though, that without more information on the mechanism of transition from rising film slugs to annular flow, that this theory is of very doubtful use in predicting the occurrence of these slugs.

* It should be noted that in plotting the curve in fig.43, the intersection on fig.42 to obtain point (a) (fig.43) lay in the slug flow region, and that, for point (b) lay on the boundary of the slug region. One is therefore doubtful of the validity of plotting these two points on fig.43.
Flooding in a Falling Film and Simultaneous Co-current and Counter-current Flow.

Wallis, in the conclusions of his paper (3) states: "The transition to co-current upwards annular flow in a vertical tube at low liquid velocities is the same as the flooding limit for zero liquid downflow in a tube in which the wall is fully wetted along its length and the limiting process occurs in the tube." He states also, that this transition is independent of the liquid flow rate and that it occurs at a gas superficial velocity given by

$$\bar{v}_g = (0.8 \rightarrow 0.9) \sqrt{\frac{\rho_f}{\rho_g}} \left[ \frac{D \rho_g (\rho_f - \rho_g)}{g} \right]^{\frac{1}{2}}$$

where, $\rho_f$ is the liquid density.

$\rho_g$ is the gas density.

Wallis has compared this theory with the work of Collier and Hewitt (15) and that of Govier et al. (1), and has also carried out experiments of his own, and has found the three sets of results to agree well with the theory. Wallis' experimental work was carried out in a 1.5 ins. I.D. tube with the water feed at the mid-point of the test-section. He started with a high air velocity in the tube and gradually decreased this, finding the sequence of events in the tube to be as follows. First an agitated liquid film appeared below the injector. With no further decrease in air velocity, this film would hang on the tube wall for long periods,
but with further decrease of air velocity, the film would descend until, at a certain velocity, it fell more catastrophically and the general overall motion was downwards. Once this had occurred the tube became filled with a violently agitated two-phase mixture resembling churn flow. On increasing the air velocity the churn flow changed to a pulse-annular pattern. On further increase of the air flow rate the pattern became less agitated and the waves changed to small ripples until, at a certain flow rate, the film became very thin and the general upwards motion of the liquid led to the drying out of the tube and the retreat of the film.

In this experimental work there was no facility for separating the air and water at the bottom of the tube, so once a downwards motion of the film had been established this water would collect at the bottom, interfere with the air inlet, and so cause the observed churn flow. One would therefore not expect this churn flow to be a regime in its own right but merely a result of the geometry of the experimental apparatus. With an increasing air flow Wallis finds that annular flow only starts once all the water causing this churn flow has been removed from the tube. Now if the geometry of the bottom of the tube were to be changed in such a manner that any water flowing down the tube was drained away without interfering with the air inlet, the churn flow pattern could not start and one would expect to find only a falling film in the bottom half of the tube. With this geometry of tube it would seem quite reasonable
to expect that, for some range of air flow rates, there could be a falling film below the water inlet and some entirely rising flow pattern above the water inlet. If this does occur one would again expect that, for some range of air and water flow rates within the above range, there could be co-current upwards annular flow above the water inlet and counter-current annular flow below the water inlet. This would obviously be expected to occur at air flow rates lower than that at which Wallis predicts the transition to upwards co-current annular flow. In fact his predicted transition point is that at which the entire water feed flows upwards in co-current annular flow irrespective of the method of feeding the air and water to the tube. From the above argument it would appear that co-current upwards annular flow could occur at lower air velocities than the minimum predicted by Wallis so long as any water leaking downwards from the liquid feed point could be removed without interference with the air inlet. It was therefore decided to change the experimental rig so that the above transitions could be investigated more closely. In the previous set of experiments (those with falling films) the water exit at the bottom of the tube had been changed so that the water could be removed from the tube with no interference with the air inlet. It was felt that this arrangement would be suitable for the present work, so it was left unaltered as shown in fig.32. The feed-box, previously above the test section was moved to the mid-point, and
the actual water feed into the tube was via a 5 ins. length of porous sintered brass tube having an I.D. exactly the same as the test section, namely 1.0 ins. It was felt desirable to measure the hold-up in both the top and bottom halves of the test-section, so two more full-bore cocks, exactly the same as those already described, were manufactured and placed immediately above and below the feed-box. These cocks were connected to the same bar that connected the two original cocks, so all four cocks could now be closed simultaneously. The previously described pressure transducers and ancillary equipment were again used to measure pressure. The three transducers available were placed, one immediately below the top full-bore cook, one immediately above the cook above the water inlet and the third immediately below the cook beneath the water inlet. A small hole (0.040 ins. diameter) was drilled through the transducer block just above the water inlet. Through this hole a length of stainless steel hypodermic tubing was inserted in such a way that its depth of projection into the hole could be varied. To the hypodermic tubing, a syringe could be fitted in order to allow the injection of dye into the test section.

The measurements to be made in this phase of the work were air feed rate, total water feed rate, water flowing from the bottom of the tube, hold-up in both halves of the test section and pressure drop in the top half of the test section. The mid-point pressure was again to be held steady at 30 p.s.i.a. The water flowing from the bottom of the tube was collected for
a timed period and weighed. The water flowing in the top half of the tube would then be the difference between the total water feed rate and the rate of flow from the bottom. For this work the method of operating the rig was the same as described for the falling film experiments.

The first phase of this experimental work was a qualitative exploration of the phenomena occurring in the tube. At a fairly low water superficial velocity (of say, 0.3 ft./sec.) and with an increasing air flow rate the sequence of events observed was as follows:

i) All the liquid flows down the tube as a falling film.

ii) On increasing the air flow rate the tube floods (the 'flooding point' as defined in chapter 6). The flow patterns are now a falling film below the water feed and churn flow above the water feed.

iii) With further increase in air flow rate the film below the water feed begins to thin as more water flows upwards. The flow above the water feed changes from churn to annular. At this point there are annular films in both halves of the test section, the upper half being co-current and the lower half being counter-current.
iv) With further increase in air flow rate a point is reached where water no longer flows out of the bottom of the tube, and a hanging film is left in the lower half of this tube. This film has upwards moving waves on its surface, and presumably there is a downwards motion within the film. The film appeared to be quite stable in length.

v) With yet further increase in air flow rate the film in the lower half appeared to thin out somewhat (although the film remained the same length), and the waves diminished in size until at a certain air flow rate the waves had faded out to leave only rising ripples, when the film retreated up the tube to the water inlet. This was termed the dry-out point. In the top half of the tube, once the flow had changed from churn to annular, this pattern persisted right through and beyond the dry-out point.

If the same procedure is carried out for a very low water flow rate of, say, \( \bar{v}_w = 0.01 \) ft./sec, the sequence of events is the same except that there is no churn flow observed in the top half of the tube; immediately flooding occurs annular flow develops in the top half of the tube.

If the above procedure is now reversed, and we start with a high gas flow rate and gradually
decrease it, the observed sequence of events is as follows. Again considering a water superficial velocity of 0.3 ft./sec.

i) There is first an entirely rising co-current annular flow pattern.

ii) A short hanging film then appears below the water feed-box. This occurs at approximately the same air flow rate at which the hanging film became established while increasing the air flow rate.

iii) This film gradually descends in short occasional surges that appear to have no regular pattern.

iv) With continuing decrease of the air flow rate this film continues to descend until at some point it falls more catastrophically right through the length of the tube to below the air inlet. Until this point the flow pattern in the top half of the tube is annular, but at approximately the same time as the film falls to the bottom, the pattern in the top half changes to churn flow. This state of affairs continues until at approximately the flooding point for increasing air flow all the water falls out of the top half of the tube and there is a falling film only.
With a water superficial velocity of about 0.01 ft./sec., the observed sequence of events is the same as with the higher water flow rate except that an entirely falling film is established at approximately the same air flow rate as that at which the hanging film falls to the bottom of the tube, consequently the churn flow regime was not observed at all at this water feed rate.

Quantitative experiments were now carried out in order to chart the above mentioned occurrences. The results are shown in fig.44. Superimposed upon this map is the churn/annular transition band for co-current upwards flow described and presented in chapter 3. Line A is the flooding curve. Line B is the line where the hanging film is first established with an increasing air flow rate i.e. where liquid flow out of the bottom of the tube ceases. Line C shows the dry-out points for the hanging film. D is the line at which a hanging film first reappears with a decreasing air flow rate. Line E is where the hanging film first reaches the bottom of the tube and water first begins to emerge at the bottom end. Line F is where all the water falls out of the top half of the tube to leave only a falling film. It is noted that lines B, C, D and E are all horizontal, i.e. these transitions are independent of the water feed rate. Now at these lines all the water feed is flowing upwards from the water inlet and so this means that the water flow in the top half of the tube has no effect on what is happening in the lower half of the tube.
It is interesting to note the relationship between lines B and D. It would appear that these two lines are really coincident and that the separation is caused by the difficulty in observing exactly when the hanging film appears. A similar state of affairs would appear to be the case with lines A and F, for water flow rates above 0.045 ft/sec.

When considering the areas of stability of the hanging film, it is noted that there is apparently a large amount of hysteresis between the curves for increasing air flow and decreasing air flow. On consideration, this is not entirely unexpected as in the area above line B, for an increasing air flow rate, the flow rate has been high enough to prevent any net downflow of water, and so when decreasing the air flow rate, one would expect this flow still to be great enough to prevent any downflow of water, so a hanging film would not be able to appear in this area. Now between lines B and E, and considering a decreasing air flow rate, it is seen that there is a hanging film where, during an increasing air flow, there were simultaneous rising and falling films. This would lead one to expect that line E would be dependent on the wetting properties of the tube. The close proximity of line E and the churn/annular transition for co-current flow should be noted. This close proximity would lead one to suspect that the hanging film does not finally fall the full length of the tube until an instability occurs in the top half of the tube which, one would expect, would cause a
leakage of water to the bottom half of the tube. In other words it appears possible that line E might occur at a lower air flow rate if it were not for the fact that the instabilities of churn flow occur at this flow rate to cause the leakage of water to the bottom half of the tube.

It has already been stated that it would appear as if lines A and F are coincident, and this is true except at extremely low water flow rates of less than about 0.05 ft/sec. As one would expect, lines A and E have no bearing on one another. However one would expect lines E and F to be related in a region of similar superficial air velocities. This is the case, and line F is seen to bend away from the flooding curve A to follow line E at water feed rates below 0.05 ft/sec. Although lines A and E cross at a total water feed superficial velocity \( \dot{V}_w \) of 0.045 ft/sec, one would not expect line F to do the same, as line F must be lower, or at highest, coincident with line E.

It should be noted therefore, that there is a triangular area on fig.44, at very low water feed rates where there is a very large apparent hysteresis, so that either all the water flows out of the bottom of the tube if the air flow is increasing, or all the water flows out of the top if the air flow is decreasing.

One should now consider churn flow in the light of the above experimental work. It was noted that the churn flow pattern was observed in the top half of the tube only while there was a falling film in the bottom half. In this experimental work the gas inlet was not being
interfered with by the falling film, so it would appear that the churn flow regime is itself a stable regime at certain values of $\bar{v}_f$ and $\bar{v}_g$, and is not merely a product of the geometry of the apparatus, although it is obvious that it could be caused to occur by closing the bottom of the tube as it is seen that when there was churn flow in the top half of the tube there was always a downwards leakage of water which could build up in the bottom half of the tube. In considering a decreasing air flow rate one would expect churn flow to be caused by the annular flow pattern becoming unstable and then collapsing, then building up to the annular pattern and then collapsing again.

Another point to be noted is the fact that the churn/annular transition band observed in this work in the top half of the tube (as shown with double cross-hatching in fig. 44), is exactly the same as that observed in the co-current flow experiments described in chapter 3. This is rather unexpected, as when churn flow was observed in the top half of the tube, there was a leakage of water downwards from the water inlet. Now one would expect this leakage, in co-current flow experiments, to affect the position of the transition band. The only explanation for this that presents itself is that at the transition band the leakage downwards is very small (as shown later in this chapter) and so has only a very small effect on the position of the transition band, this being masked by the width of the band. We must conclude that the churn/annular transition cannot be caused by leakage of water to
the bottom of the tube and that it is caused by a genuine instability of the rising film.

An important feature of the map when considering the transition between churn and annular flow is the triangle marked (a). This shows two contrasting situations depending upon the water flow rate. For water superficial velocities less than approximately 0.045 ft./sec., no churn flow was observed in these experiments, there was either a rising film or a falling film, depending upon the air velocity and whether it was decreasing or increasing. Now for these water velocities and for co-current flow with the bottom of the tube closed, churn flow can only be caused by interference between the water and the air inlet. For water flow rates greater than 0.045 ft./sec., churn flow could be caused either by instability of the rising annular film or by the tube bottom being closed. It should be noted that Wallis (3) predicts that the churn/annular transition should occur when line A meets the \( \bar{v}_g \) axis, but this is not found to be the case in the present work.

One must now compare the above results with those of other workers. They will first be compared with the work of Wallis (3). He gives a table of results of an experiment which was carried out at an average pressure of 31 psia and a water superficial velocity of 0.01 ft./sec.

The sequence of events observed was as follows:

With a decreasing air flow rate a wet patch below the water inlet was observed at a superficial air velocity of 30.4 ft./sec., this had formed a hanging film when

\[
\bar{v}_g = 25.3 \text{ ft./sec.}
\]
This fell to the bottom of the tube in the region of

\[ \overline{v} = 20.3 \text{ to } 18.7 \text{ ft./sec.} \]

On increasing the air flow rate, annular flow was seen to start at

\[ \overline{v} = 26.1 \text{ ft./sec.} \]

and dry-out at 31.4 ft./sec.

As Wallis states that the transition to annular flow is the same as the point at which the hanging film is formed for increasing air flow, we can take his figure of

\[ \overline{v} = 26.1 \text{ ft./sec.} \]

to correspond to line B in fig. 44, and his figures are seen to agree very well with the present work.

Calculating Wallis' expression for the different lines of fig. 44 at a pressure of 30 p.s.i.a. gives figures of

\[ \overline{v} = 26.3 - 29.5 \text{ ft./sec. for line B,} \]

\[ \overline{v} = 28.0 \text{ ft./sec. for the dry-out point} \]

and \[ \overline{v} = 18.1 \text{ ft./sec. for conditions immediately above line E.} \] Apart from his dry-out point these also agree very well with the present work.

Bashforth, Fraser, Hutchison and Nedderman (16) give figures for points at zero water flow corresponding to lines C and E of fig. 44. They worked at atmospheric pressure and found the
points to occur at

\[ \bar{v}_g = 41 \text{ ft./sec.} \]

and \[ \bar{v}_g = 31 \text{ ft./sec.} \]

respectively. These figures are much higher than the present work, but one would expect this as the work was carried out at a lower pressure. It should be noted that Wallis' results agree with the present work in finding the transitions to be independent of the water flow rate.

It has already been mentioned that during this work, facilities were provided to inject dye into the top half of the tube. The results of this work showed that there was very little back mixing of water in the top half of the tube. This was found to be the case under widely differing conditions of flow. For a wavy annular pattern the dye was seen to move back only about \( \frac{1}{2} \) ins, and for a violently agitated churn flow pattern the back mixing was seen to be no more than 2 ins. It had been suspected that in a wavy annular pattern there may be a circulation of water within the film, but this was not observed.

Experiments were carried out in the region where there was flow both up and down the tube in order to determine the flow rate and hold-up in the two sections of the tube. Fig. 45 shows a plot of flow in the top half of the test section against superficial gas velocity for different total water feed rates. These are seen to be a family of curves, all having
much the same form. They were obtained by measuring the total feed to the test section, then collecting and weighing the water flowing from the bottom of the tube. The flow upwards is then obtained by difference. Fig. 46 shows the plots of the flow down the tube for the same total water feed rates as above. These curves, as expected, also form a regular family of curves all meeting at zero flow at an air superficial velocity of 26.6 ft./sec. They show a greater spread than might be expected, indicating the effect of the flow of water in the top half of the tube on the flow in the bottom half. If the flow in the top had no effect on the bottom then all the curves would coincide. Fig. 47 shows an isometric plot of these conditions which illustrates, perhaps better, the liquid flowing in the lower half of the tube.

The results of the measurement of hold-up in the two halves of the tube are shown in fig. 48. The curves of hold-up in the bottom half have the expected behaviour, with the hold-up increasing with increasing flow of water in the bottom half. (It has already been shown in chapter 6 that the thickness of a falling film is independent of the air flow rate). For the top half of the tube the hold-up decreases with increasing air flow rate. Fig. 49 shows an isometric plot of the hold-up in the lower half of the test section. This illustrates well the independence of the film thickness and the air flow rate up to the flooding point, and also the fall off in hold-up with the decreasing liquid flow after flooding has started.
CHAPTER 9.

Suggestions for future work.

During the course of the foregoing work certain questions have arisen, the answers to which, it is felt, would be of value in the field of two-phase flow in vertical tubes.

One section in particular that appears to merit further investigation is the subject of counter-current flow and the effect of different conditions on the phenomenon of flooding in a falling film. It has recently been suggested (19) that the length of a falling film has an effect on the flooding point. It is felt that this question warrants further investigation, and it is suggested that care be taken to avoid allowing pressure drop changes (caused by changes in the length of the falling films) to mask any genuine effect of the length of the falling films on the flooding point. It is felt also, that an investigation into the relationship between pressure (and hence density) of the gas phase and flooding would prove informative. Other questions in connection with falling films and their flooding points that appear to warrant investigation are the effects of:

irregularities in the tube,
the geometry of the air supply and
the geometry of the liquid take off.

It is felt also, that a further investigation into the effect of different water feed arrangements on the flooding point, would be worthwhile.
NOMENCLATURE.

A cross-sectional area of tube.
A' cross-sectional area of gas core.
D diameter of tube.
G volumetric gas flow.
g acceleration due to gravity.
L volumetric liquid flow.
L' upwards volumetric flow of water in film.
l Prandtl mixing length.
P wetted perimeter of tube.
P' perimeter of gas core.
r_o radius of tube.
u velocity in axial direction.
v_1 variable of integration
v_2 variable of integration.
v_rate of rise of a slug.
v_o rate of rise of a Dumitrescu bubble.
v_g gas velocity in core.
v_o superficial gas velocity.
v_l superficial liquid velocity.
X film thickness
x thickness within film
x thickness of laminar sublayer.
\( \frac{\Delta P}{\Delta z} \) pressure drop per unit length of tube.
In chapter 8, when discussing the different flow patterns observed, an apparent anomaly was revealed in that co-current annular flow was observed in a region where experiments with the mid-point feed showed that there was a leakage below the water inlet which one would expect to cause churn flow in an arrangement of the tube to deal with co-current flow only. It is suggested that the leakage downwards is so small that the rising film can carry this extra amount of water, but it is felt that further investigation of this point would be worthwhile.

A question that arises indirectly from the above is whether or not the churn/annular transition in co-current flow depends significantly upon the feed arrangement of the air and water. It has been suggested (19), that an arrangement where the air and water are fed together to a very wide tube which gradually narrows to the test section diameter, might be of help in answering this question.

It is suggested that an analysis similar to that of Shearer and Davidson (18), for flooding in a falling film, could be used to predict the churn/annular transition by considering churn flow to be caused by the annular regime becoming unstable.
\( \xi \) voidage fraction.
\( \rho \) density of liquid.
\( \rho_f \) density of liquid.
\( \rho_g \) density of gas.
\( \tau \) stress within film.
\( \tau_b \) stress at air liquid interface.
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APPENDIX I.

Analysis of Co-current Upwards Annular Flow.

The Calvert and Williams analysis of co-current upwards annular flow is reworked, taking into account the curvature of the film.

Calvert and Williams (4) assume that there are two layers in the liquid film, a laminar sublayer, and a turbulent layer.

Consider a force balance over the shaded part of the film as shown in fig. 50.

Then:

\[ \tau_b - 2\pi (r_o - x) = \tau - 2\pi (r_o - x) + \rho g \left( \pi (r_o - x)^2 - \pi (r_o - x)^2 \right) \]

\[ + \frac{dP}{\delta z} \left( \pi (r_o - x)^2 - \pi (r_o - x)^2 \right) \]

... (A.1.)

where \( \tau_b \) - stress at air/liquid interface.

\( \tau \) - stress between turbulent and laminar films.

\( r_o \) - radius of tube.

\( x \) - film thickness.

\( x \) - thickness within film.

\( \rho \) - density of liquid.

\( g \) - acceleration due to gravity.

\( \frac{dP}{\delta z} \) - pressure drop per unit length of tube.
Now \( \tau = \rho l^2 \left( \frac{\partial u}{\partial x} \right)^2 \) or \( \tau = \rho l^2 \frac{\partial u}{\partial x} \frac{\partial u}{\partial x} \)

where \( l = \) Prandtl mixing length.
\( u = \) velocity in axial direction.
\( x \) = distance \( x \) from wall of tube.
also \( l = \) as evaluated by
Nikuradse (17)
\[ \tau = (0.4x)^2 \frac{\partial u}{\partial x} \frac{\partial u}{\partial x} \]

and \( \frac{d}{dx} = -\frac{P_0 \tau}{A_0} \)

where \( P_0 = \) perimeter of gas core.
\( A_0 = \) area of gas core.

hence

\[ \frac{\partial u}{\partial x} = \pm \frac{\sqrt{P_0 \tau}}{2A_0} \left[ \frac{r_0 - x + \frac{P_0}{A_0} \left( \frac{x^2 - x^2 - 2x (r_0 - x) + 2x \partial x}{2(r_0 - x)} \right)}{0.4r x} \right] \]

\[ = f(x) \]

The total liquid flow \( L \) is obtained from
\[ L = \int_{x=0}^{x=X} 2\pi(r_0 - x) \partial x \partial u. \]
\[ 75 \]

\[ = \int_0^1 2\pi (r_0 - x) u_* dx + \int_{\delta}^X 2\pi (r_0 - x) u_* dx. \]

where \( \delta \) is the thickness of the laminar sublayer

and

\[ P = 2\pi r_0 \]

\[ \therefore \frac{L}{P} = \int_0^\delta (1 - \frac{x}{r_0}) u_* dx + u(\delta) \int_{\delta}^X (1 - \frac{x}{r_0}) dx + \int_{\delta}^X (1 - \frac{x}{r_0}) \left\{ u - u(\delta) \right\} dx \]

We shall use the following notation -

\[ \frac{L_{\text{lam}}}{P} = \int_0^\delta (1 - \frac{x}{r_0}) u_* dx + u(\delta) \int_{\delta}^X (1 - \frac{x}{r_0}) dx \]

\[ \frac{L_{\text{c}}}{P} = \int_{\delta}^X (1 - \frac{x}{r_0}) \left\{ u - u(\delta) \right\} dx \]

and

\[ \frac{L}{P} = \frac{L_{\text{lam}}}{P} + \frac{L_{\text{c}}}{P} \]

In conventional pipe flow theory, the quantity \( \frac{L}{P} \sqrt{\frac{c(\delta)}{\rho}} \), which is dimensionless, is introduced and it is assumed that the laminar boundary layer ends when this has some value \( N \).
From this we obtain -

\[
\delta = \frac{N}{\rho \sqrt{\left( \frac{r_o - \delta}{r_o} \right) - \rho \left( \frac{2 - x^2 - 2r_o \delta + 2r_o \delta \delta}{2 (r_o - \delta)} \right)}}
\]

We now consider \( \frac{L_t}{f} \).

We introduce a new variable \( v_1 \) such that

\[
v_1 = u - u(\delta) = \int_{\delta}^{x} f(x) \, dx
\]

then \( \frac{dv_1}{dx} = f(x) \)

with \( v_1 = 0 \) at \( x = \delta \).

Now

\[
\frac{L_t}{f} = \int_{\delta}^{x} \left( 1 - \frac{x}{r_o} \right) \{ u - u(\delta) \} \, dx
\]

We introduce a second variable \( v_2 \) such that

\[
v_2 = \int_{\delta}^{x} \left( 1 - \frac{x}{r_o} \right) \{ u - u(\delta) \} \, dx
\]

\[
= \int_{\delta}^{x} \left( 1 - \frac{x}{r_o} \right) \cdot v_1 \, dx
\]

differentiating

\[
\frac{dv_2}{dx} = \left( 1 - \frac{x}{r_o} \right) \cdot v_1
\]

with \( v_2 = 0 \) at \( x = \delta \).

\( \frac{L_t}{f} \) is \( v_2(x) \).
Hence we need to solve

\[
\frac{dv_1}{dx} = f(z)
\]

\[
\frac{dv_2}{dx} = (1 - \frac{z}{r_o}) v_1
\]

with initial conditions

\[v_1 = v_2 = 0 \text{ at } z = \delta\]

we solve out to \( z = X \)

It remains to determine \( \frac{L_{_{a_n}}}{P} \)

The velocity profile in the laminar film can be deduced from the basic force balance by the use of the viscous stress

\[
\text{I.e. } \tau = \mu \frac{du}{dx}
\]

and \( \int \frac{P}{\delta^2} = -\frac{F'}{A'} \)

Hence

\[
\tau b.2. (r_o - x) = \mu \frac{du}{dx} 2. (r_o - x)
\]

\[
+ \rho g_b (z^2 - x^2 - 2r_o x + 2r_o X)
\]

\[- \frac{F'}{A'} \gamma_b (z^2 - x^2 - 2r_o x + 2r_o X)
\]

and this can be integrated to give \( u(x) \).
This can now be integrated and so the total liquid flow in the film can be found,
FIG. 1. TWO-PHASE FLOW PATTERNS IN VERTICAL TUBES.
FIG. 2. REGIME MAP OF GOVIER et. al.
FIG. 3. TYPICAL PRESSURE DROP CURVE
FIG. 4. REGIME MAP OF GALEGAR, STOVALL & HUNTINGTON.
A full-bore cock
B pressure tappings
C water feed-box
FIG. 6. DIAGRAMS OF WATER FEED-PIECES.
FIG. 7  ISOMETRIC VIEW OF FULL-BORE COCK.
FIG. 9. GENERAL VIEW OF APPARATUS.
FIG. 11. PHOTOGRAPHS OF TYPICAL REGIMES.
FIG. 12. REGIME MAP.
FIG. 13. PHOTOGRAPHS OF FLOW IN TUBE.

\[ \nabla_g = 4.6 \\
\nabla_l = 3.98 \\
\nabla_g = 7.2 \\
\n\nabla_l = 5.17 \\
\nabla_g = 13.4 \\
\n\nabla_l = 4.49 
\]
FIG. 14. PHOTOGRAPHS OF FLOW IN TUBE.
FIG. 15. REGIME MAPS.
FIG. 16. PRESSURE DROP & HOLD-UP AT CONSTANT $\bar{V}_l$.

constant $\bar{V}_l = 0.31$ ft./sec.

$\frac{dP}{dz}$ ft. H$_2$O/ft.

$1 - \varepsilon$

$\bar{V}_g$ - FT./SEC.

$\bar{V}_l$ - FT./SEC.

--- curves of Govier et al.
for $\bar{V}_l = 0.30$ ft./sec.
constant $\bar{V}_i = 1.03$ ft./sec.

FIG. 17 PRESSURE DROP & HOLD-UP AT CONSTANT $\bar{V}_i$. 
constant $\bar{V}_1 = 2.01$ ft./sec.

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FIG. 18. PRESSURE DROP & HOLD-UP AT CONSTANT $\bar{V}_1$

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curves of Govier et al.
for $\bar{V}_1 = 2.06$ ft./sec.
FIG. 19. PRESSURE DROP & HOLD UP AT CONSTANT $\bar{v}_f$.

Constant $\bar{v}_f = 3.52$ ft./sec.

Curve of Govier et al. for $\bar{v}_f = 3.47$ ft./sec.
**FIG. 20. HOLD-UP AT CONSTANT $\bar{v}_f$.**

\[ \text{constant } \bar{v}_f = 7.79 \text{ ft./sec.} \]
FIG. 21. TRANSDUCER SUPPLY CIRCUIT.
FIG. 23. AMPLIFIER CIRCUIT.
FIG. 24. R M S. CIRCUIT.
FIG. 25. EXPECTED FORM OF P.D. FLUCTUATION CURVE
FIG. 26. PRESSURE DROP FLUCTUATIONS AT CONSTANT $\bar{V}_1$. 

R.M.S. VALUE OF PRESSURE DROP FLUCTUATIONS - FT. $\, H_2O$ / FT. 

$\bar{V}_g$ FT./SEC. 

$\bar{V}_1 = 2.19$ 

$\bar{V}_1 = 1.66$ 

$\bar{V}_1 = 1.25$ 

$\bar{V}_1 = 0.89$ 

$\bar{V}_1 = 0.53$ 

$\bar{V}_1 = 0.41$ 

$\bar{V}_1 = 0.22$ 

$\bar{V}_1 = 0.13$ 

$\bar{V}_1 = 6.41$ 

$\bar{V}_1 = 5.20$ 

$\bar{V}_1 = 4.52$ 

$\bar{V}_1 = 3.68$ 

$\bar{V}_1 = 2.60$
FIG. 27  % PRESSURE DROP FLUCTUATIONS.
\[ \bar{v}_g = 7.5 \]
\[ \bar{v}_l = 2.60 \]

\[ \bar{v}_g = 26.9 \]
\[ \bar{v}_l = 3.12 \]

FIG. 28. TYPICAL PRESSURE DROP TRACES.
FIG. 29. TYPICAL PRESSURE DROP TRACE.

\[ \bar{V}_g = 62.8 \]

\[ \bar{V}_l = 0.53 \]
FIG. 30. FLOW IN A WETTED WALL COLUMN
NICKLIN'S RESULTS.

\[ (1 - \varepsilon) \]

\[ \frac{L}{A} = \text{FT./SEC.} \]

\[ 20 = \bar{V}_g \] (flooding points)

Points labeled: 20, 16, 12, 10, 8
FIG. 31. LIMIT OF SLUG FLOW FROM NICKLIN'S THEORY.
FIG. 32. AIR INLET FOR FALLING FILM WORK.
FIG. 33. HOLD-UP IN A WETTED WALL COLUMN.

$\bar{V}_l = 0.082$ FT/SEC.
**FIG. 34.** HOLD-UP IN A WETTED WALL COLUMN.
FIG. 35. FLOODING IN FALLING FILMS.
FIG. 36. FLOODING WITH A 45° BEVEL FEED-PIECE.
FIG. 37  FLOODING CURVES FOR 9 FT. COLUMNS.
FIG. 38. FLOW IN A WETTED WALL COLUMN
FIG. 39. RISING FILM CURVES (envisaged form).
FIG. 40. THE CALVERT & WILLIAMS ANALYSIS FOR FLOW IN A RISING FILM.
FIG. 41. FLOW IN A RISING FILM.
CURVES OF CONSTANT PRESSURE DROP
FIG. 42. FLOW IN A RISING FILM AT CONSTANT GAS CORE VELOCITY.
FIG. 43. LIMIT OF STABILITY OF RFS REGIME.
FIG. 44.8. MID-POINT FEED. $\dot{V}_f > 0.045$ FT./SEC.
FIG. 44. MID-POINT FEED. MAP OF PHENOMENA IN LOWER HALF OF TUBE.
FIG. 45. MID-POINT FEED. FLOW UPWARDS.
FIG. 46. MID-POINT FEED. FLOW DOWNWARDS.
FIG. 47 MID-POINT FEED. ISOMETRIC PLOT OF FLOW IN LOWER HALF OF COLUMN.
FIG. 48. MID-POINT FEED. HOLD-UP IN COLUMN.
FIG. 49. MID-POINT FEED. ISOMETRIC PLOT OF HOLD-UP IN LOWER HALF OF COLUMN.
FIG. 50. SECTION OF LIQUID FILM IN ANNULAR FLOW.