Σ⁺ PRODUCTION IN PROTON–PROTON COLLISIONS AT 5 BEV/C

Thesis

submitted by

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INTRODUCTION

The experiment reported in this thesis is concerned with $\Sigma^+$ production in $p - p$ collisions at 5 Bev/c.

The reactions

$$p + p \rightarrow K^0 + \Sigma^+ + p$$  \hspace{1cm} (1)

$$p + p \rightarrow K^+ + \Sigma^+ + n$$  \hspace{1cm} (2)

are of particular interest.

One may expect a contribution to these reactions via the strange particle decay mode of $\Delta^{++}(1920)$ resonance.

There is also a possibility of observing a strange dibaryon resonance\(^{(2)}\) or positive strangeness resonance\(^{(3, 4, 5)}\) should these exist. A detailed study of the reactions (1), (2) provides moreover an opportunity to investigate the validity of one meson exchange models which have been proposed for these reactions\(^{(25)}\).

The $\Sigma^+$ production channels with more than 3 particles in the final state are open at this energy but they have not been studied because of the inherent difficulty of correctly identifying such final states.

In Chapter I the experimental technique used is described. The scanning and measuring procedures as well as the computer
programs used for analysing the data will be described and critically examined. The cross-section computation will be outlined.

In Chapter II the theory used in detailed study of the reactions is outlined.

In Chapter III the experimental data is analysed and compared to the predictions of the theory. The mass spectra are studied in order to detect possible resonances. The mechanism of reaction (2) is discussed. In particular the predictions of the one-pion exchange model and the one K-meson exchange model are computed and compared with the experimental results.
CHAPTER I

EXPERIMENTAL PROCEDURE

The data for the experiment was obtained at the Brookhaven National Laboratory (B.N.L.) in an exposure of the 80 inch hydrogen bubble chamber to a 4.95 BeV/c proton beam from the Alternating Gradient Synchrotron (A.G.S.). The beam transport system and the bubble chamber are described in the B.N.L. reports (6, 7).

1.1 The Beam

The proton beam was essentially pure. A Cerenkov counter placed in front of the chamber indicated at most 1/0 back-ground (due to pions, kaons, etc.). This low level of back-ground was achieved by letting the beam particles pass through two electrostatic velocity separators. A check by E. Bierman et al. (8) on the radius of curvature of delta rays produced by the beam tracks on the film also showed that the contamination of the beam was less than 1/0.

The beam momentum was determined by measurement of the non-interacting beam tracks, using the magnetic field and optical constants as measured at B.N.L. (7). The value obtained was

\[ p = 4.95 \pm 0.03 \text{ BeV/c}. \]
1.2 The Analysis of the Bubble Chamber Photographs and Data Reduction

The analysis of the bubble chamber photographs was done in two steps. The first step was the scanning operation, the purpose of which was to locate the events which may represent the reactions:

\[ p + p \rightarrow p + k^0 + \Sigma^+ \]  \hspace{1cm} (1.1)
\[ p + p \rightarrow n + k^+ + \Sigma^+ \]  \hspace{1cm} (1.2)

with the subsequent decays:

\[ \Sigma^+ \rightarrow p + \pi^0 \]  \hspace{1cm} (1.3)
\[ \Sigma^+ \rightarrow n + \pi^+ \]  \hspace{1cm} (1.4)

The film was scanned at least twice. In each case three stereo views were used. The scanning took place on a special projection table provided with a film transport mechanism. The most common event topologies for reactions (1.1), (1.2) are shown in Fig. 1. Since many reactions other than (1.1), (1.2) may have similar topologies, an attempt was made to eliminate as many of those as possible at the scanning table. For this purpose the expected ranges of different quantities in the processes (1.1), (1.2), (1.3), (1.4) were computed, and appropriate scanning instructions prepared. The scanners were
instructed to look for events where the beam track produces two charged (visible) outgoing tracks, one of which satisfies the following conditions:

1) It decays (with the decay point appearing as a kink) within a specified distance from the production vertex. This distance was made to depend on the ionization of the track and was chosen so that the probability of \( \Sigma^+ \) not decaying within such distance was sufficiently small (\( \frac{\sigma_{\Sigma^+}}{o} \leq 5\% \)). The ionization was estimated with respect to the beam tracks and classified as belonging to one of four visibly differentiable categories.

2) Its angle with the beam track must not exceed 40°. (The computed maximum value of this angle for \( \Sigma^+ \) in reactions (1.1), (1.2) is 36°).

3) The projected sagitta for a given projected length of track must not exceed a specified maximum (i.e., the curvature of the decaying track must not exceed a certain maximum). This maximum was computed from the minimum laboratory momentum of \( \Sigma^+ \).

If an event satisfied these conditions, then scanners were to look for an associated \( V \) due to the decay of a possible \( K_1^0 \).

\* The probability of a particle of a given momentum not decaying within a specified distance is given by equation (A.33) of the Appendix A. The kinematical limits on momentum and angle are given by equations (A.25) and (A.29).
For all such events, the scanners were requested to:

1) record the location of the event;

2) make a sketch of the event showing its approximate location on the frame; for this purpose scanners were provided with forms with the diagram of the bubble chamber region divided into four quadrants and with the positions of the fiducials indicated;

3) list the information required to maintain a record of the current status of the event at every stage of the experiment;

4) take a photograph of the event for use in later analysis;

5) make a note of any unusual conditions making it difficult to identify or to measure the event reliably.

The second step in the analysis of the bubble chamber pictures was the measuring process. All measurable events recorded by the scanners were measured. The measuring was done on a conventional digitised measuring machine (Vanguard) connected on-line to a small digital computer PDP5. The

---

Some events could not be measured because a black spot (due to the illumination of the chamber) was covering one of the vertices. Occasionally an event could not be measured because one view was torn (or missing), or because one of the vertices could not be clearly seen in one or more views (due to overlapping with neighbouring tracks, due to poor ionisation etc.).
least count of the system is 2.5μ on the film. By comparison a typical track bubble diameter would be of the order of 35μ on the film.

After transmitting to the PDP5 the description of the event, including its topology, four fiducial marks were measured in each of the three views. Then the interaction points and the points on the tracks were similarly measured. Only one point was measured on the decaying track (except when the track was visibly curved, in which case four points were recorded). On all other tracks five points were measured in each of the three views, spacing the points approximately equal distances apart along each track. The projected length \( L \) of each track measured was, whenever possible, chosen so that the sagitta \( s \) would be between 2 and 6 cm. This range of values was determined from the following considerations. Since the momentum \( p \) is proportional to the radius of curvature \( R \), and \( s \) can be written as

\[
 s = \frac{p^2}{8R} \tag{1.5}
\]

For each event there were at most four such 5-point tracks, besides the beam track. If any of these had a secondary interaction then it was measured only up to this interaction.

The sagitta \( s \) for a track of length \( L \) is defined by

\[
 s = R(1 - \cos \frac{\theta}{2})
\]

where \( R \) is the radius of curvature and \( \theta = \frac{L}{R} \) is the angle subtended by \( L \). If we let \( \cos \frac{\theta}{2} = 1 - \frac{\theta^2}{4} \), where \( \theta = \frac{L}{R} \), then, if the magnetic field is 20 Kg.,

\[
 s = R(1 - \cos \frac{\theta}{2}) = \frac{L^2}{8R} = \frac{3}{4} \chi^2 \frac{L}{p}
\]

where \( p \) is in MeV/c, \( s \) and \( \chi \) are in cms.
the standard error in determining momentum, $\Delta p$, is related to the standard errors in projected length $\Delta \ell$, and sagitta, $\Delta s$, by

$$\frac{\Delta p}{p} = \frac{\Delta R}{R} = \sqrt{\frac{4(\Delta \ell)^2}{\ell^2} + \frac{(\Delta s)^2}{s^2}}$$

(1.6)

Therefore a small sagitta would lead to a large error in the momentum. On the other hand a large sagitta, corresponding to a low momentum, would lead to a large change in momentum due to energy loss of the particles in traversing the liquid hydrogen. Since the spatial reconstruction program does not correct for this mass dependent loss, the program would not obtain a good fit for the track.

No order of measuring was prescribed, but each type of measurement was accompanied by a four digit label describing its nature, the number of points measured (0 for vertices and the stopping points) and topological inter-connections between vertices and tracks. The view changed automatically when the number of points requested by the label had been measured.

At every stage in the measurement procedure the PDP5 uses all the information about the event being measured to check for logical errors. If it detects an error, then the measurement is not accepted, and the cause is signalled to the measurer via an error light.

The PDP5 puts all the data into the form required by the geometrical reconstruction program THRESH ($^9$) and writes

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* The Cern geometrical reconstruction program THRESH was used in this experiment.
one record for each event directly on a magnetic tape.

This tape is then read by THRESH one event at a time, and the geometrical reconstruction of the event in space is performed.

THRESH transforms all measurements to a common reference system which is defined by the apparent positions of the fiducial marks on the back of the front glass of the bubble chamber.

The following information necessary to make up this reference system was provided for THRESH:

1) Information on the optics of the bubble chamber: the number of media (= 2) between cameras and reference plane, refractive indices and path length of light rays in each medium; the number of cameras (= 3) and their locations;

2) The number of front (= 4) and back (= 0) fiducials and their coordinates.

The tolerances for various coordinates were also specified.

The reconstruction process can be divided into two steps; first the geometrical reconstruction of points along the charged track, and second the fitting of the reconstructed points to an appropriate curve.

The bubble chamber is in a uniform magnetic field perpendicular to the front glass, so that the charged particles can be considered to describe helices, with axes perpendicular to the front glass.

The reconstruction method relies on the fact that if the measured coordinates of a bubble on a track are known in at
least two views, then, tracing the light rays from the two views back through the optics into the bubble chamber, one obtains the coordinates of the bubble in the chamber at the intersection of the two rays. In this manner, the vertices and the stopping points are reconstructed. For the tracks, however, the measurements are not made at corresponding points in the various views, and it is necessary to generate an artificial corresponding point in some view by interpolating between two measured points. The reconstructed points along the track are then fitted by a least squares procedure to a helix, and the parameters of the helix are thus determined. These parameters are: the radius of the helix, the dip angle, and the azimuthal angle of the tangent at the vertex with respect to the X-axis. The parameters of this approximate helix are then iteratively varied, so as to make it intersect the rays from all measured points in all the available views.

The results of the computation, i.e., the parameters of the helix, their variances and covariances as well as the length of the track, are then written out for each track on a magnetic tape.

The results of the geometrical reconstruction were processed through the kinematic program GRIND(10).

The general information on the experiment necessary for these computations was provided. It consists of the mass, charge, and momentum of the beam particles, the magnetic field, a table for range-energy conversion, the range error, the
scattering constant (for Coulomb scattering) the specification of the convergence criteria in fitting, the measurement error on film coordinates, the label and the number of charged tracks for each type of interaction, as well as a list (in code) of possible mass assignments for each type of interaction.

There are three types of interactions to consider in this experiment:

(i) pp interactions producing two charged tracks, one of which decays,

(ii) the two-body decay of a charged particle, and

(iii) the two-body decay of a neutral particle.

For (i) a list of possible reactions was compiled on the basis of the threshold for the reaction (ii) and conservation of quantum numbers. A list of decay products for unstable products of these reactions was also provided.

The program first performs all the mass independent calculations: checks the geometry data; converts the curvature of each track to momentum using the specified magnetic field; and computes the measurement errors for the track variables. (The track variables are the momentum p, the dip angle $\lambda$, and the azimuthal angle $\phi$ of the tangent to the track at the vertex). The program then tests each specified hypothesis in turn to check whether it is compatible with energy and momentum conservation at all vertices. At each vertex, four conservation equations must be satisfied.
by the track variables of all the tracks meeting at the vertex. Of the track variables some might be unknown (e.g., the track variables of a neutral track) or regarded as unknown (e.g., the momentum of a short quasi straight charged track). If the number of unknowns does not exceed the number of conservation equations, these unknowns are first computed. Then, if the number of unknowns is less than the number of conservation equations, the remainder are regarded as constraints. A fitting program adjusts all the track variables so that they satisfy these constraints. In order to modify the values of the track variables as little as possible, the condition

\[ 2 = \text{min.} \]

is imposed. Since the constraints are not linear in the track variables it is necessary to use an iterative procedure.

The computations are carried out first for the secondary vertices and the results are then used in the computations at the production vertex. The following cases arise in this experiment:

1) **The two body decay of a charged particle**

In this case the three track variables of the neutral track are unknown, and we have a one constraint (10) fit, provided the decaying track is visibly curved. However, if the decaying track is almost straight,

---

For a hypothesis with two or more neutral particles the number of unknowns exceeds the number of equations and no fit is possible. The program computes only the missing (neutral) mass for such a hypothesis.
its momentum is regarded as unknown, and we have no constraints or a "CC fit". In the latter case, the program obtains two sets of possible parameters (see Appendix B) for the same hypothesis.

2) **The two body decay of a neutral particle**
   - The momentum of the neutral track is unknown; therefore we have a 3C fit.

3) **Production vertex**
   - a) If there is an associated V, then there are no unknowns and we have a 4C fit.
   - b) If there is no associated V, then the three track variables of the neutral track are unknown, and we have a 1C fit.

The results of kinematic fitting were studied together with the photographs taken at the scanning table. All the good fits for the production vertex were matched with the corresponding fits for the secondary vertices and the predicted ionisations of all the tracks in the combined fit were compared to the bubble density on the photograph. For such a combined fit to be accepted as a possible explanation of the event, the following three requirements had to be satisfied at each vertex:

1) the iterative fitting procedure of THRESH converged,
2) the probability of fit was greater than 1\(^\circ\)/o,
3) the predicted ionisations were consistent with the ionisations observed on the photograph.
All events which had fits satisfying these criteria were copied onto a master tape (each production vertex fit combined with the corresponding charged decay vertex fit) and used in all the subsequent analyses of the data.

About $30\%$ of the events had no acceptable fits or were rejected by THRESH. These were remeasured. In about $10\%$ of cases acceptable fits were obtained after the second measurement and the events added to the master tape.

The events which had no acceptable fits in any of the two measurements were examined to determine if they would be consistent with any hypothesis having at the production vertex two or more neutral particles in the final state. Each such hypothesis was tested to determine the following:

a) whether the missing mass, computed by GRIND, was sufficient;

b) whether there was at least one secondary fit, with correct ionisation for all tracks, for the decay of the decaying charged particle;

c) whether the momentum of the other outgoing charged particle (as given by the mass independent part of the computation in GRIND) was roughly compatible with the ionisation on the photograph.

A third program was written to compute the various quantities (and their distributions) needed to study the dynamics of each reaction and to search for possible resonances.
The outputs desired on a particular computer run of this program and the criteria for selection of fits can be specified by appropriate modifications of the control cards.

The program starts with the fitted values of momentum \((q)\), dip \((\lambda)\) and azimuth \((\phi)\) for each particle \(i\) in the laboratory frame of reference as given by GRIND. From these it computes the components of the 4-momentum for all particles as well as for all pairs of particles according to the equations:

\[
(q_i)_1 = q_i \cos \lambda_i \cos \phi_i \quad (1.7)
\]

\[
(q_i)_2 = q_i \cos \lambda_i \sin \phi_i \quad (1.8)
\]

\[
(q_i)_3 = q_i \sin \lambda_i \quad (1.9)
\]

\[
E_i = \sqrt{q_i^2 + m_i^2} \quad (1.10)
\]

and

\[
(q_{ij})_x = (q_i)_x + (q_j)_x \quad x = 1, \ldots, 3 \quad (1.11)
\]

\[
E_{ij} = E_i + E_j \quad (1.12)
\]

It then computes the dot products of all pairs of momenta vectors and finally the following quantities:

a) the effective-mass squared for all pairs of particles \(q_i\) and \(q_j\) given by the equation

\[
M_{ij}^2 = E_{ij}^2 - q_{ij}^2 \quad (1.13)
\]

b) the 4-momentum transfer and the cosine of the production
angle in the centre-of-mass system given by the equations

\[ \Delta^2 = 2(E_{3^c} - p_2 q_3 \cos(p_{2^c}, q_{3^c})) - m_2^2 - m_3^2 \]

\[ = 2(E_{1^c} - p_1 q_{12} \cos(p_{1^c}, q_{12})) - m_{12}^2 - m_1^2 \quad (1.14) \]

\[ \cos \Theta^* = \cos(p_{1^c}, q_{12}^*) = \cos(p_{2^c}, q_{3^c}^*) \]

\[ = (p_{2^c} \cdot q_{3^c}) / q_{3^c}^* p_{2^c}^* \quad (1.15) \]

where \( p_1, p_2 \) are the initial particles and \( q_1, q_2 \) and \( q_3 \) are the outgoing particles.

Here the particle \( p_1 \) was chosen to be either the target proton or the beam proton, whichever gave the smaller value for \( \Delta^2 \).

c) the Jackson parameters \( \cos \Theta_1^Q \) and \( \Theta_1^Q \) (in the rest frame \( Q \), of particles \( q_1 \) and \( q_2 \)) given by

\[ \cos \Theta_1^Q = \cos(p_{1^Q}^Q, q_{1^Q}^Q) = (p_{1^Q}^Q \cdot q_{1^Q}^Q) / p_{1^Q}^Q q_{1^Q}^Q \quad (1.16) \]

\[ \text{Because of the identity of the two protons in the initial state one expects a forward-backward symmetry for } \cos \Theta^* \text{. If } p_1 \text{ is chosen to correspond to smaller } \Delta^2, \text{ as it is done here, the resulting angular distribution is folded about } 90^\circ \text{.} \]
\[
\cos \theta_1^Q = \frac{(\vec{q}_1^Q \cdot \vec{j})}{q_1^Q |\sin (\vec{p}_1^Q, \vec{q}_1^Q)|} \quad (1.17)
\]
\[
\sin \theta_1^Q = \frac{(\vec{q}_1^Q \cdot \vec{j})}{q_1^Q |\sin (\vec{p}_1^Q, \vec{q}_1^Q)|} \quad (1.18)
\]

where
\[
\vec{j} = \frac{\vec{p}_2^Q \times \vec{q}_3^Q}{|\vec{p}_2^Q \times \vec{q}_3^Q|} \quad (1.19)
\]
\[
\vec{k} = \frac{\vec{p}_1^Q}{p_1^Q} \quad (1.20)
\]
\[
\vec{i} = \vec{j} \times \vec{k} \quad (1.21)
\]

The transformation from the laboratory system to the centre-of-mass system was made using equations (A.4), (A.6) of the appendix A and the transformation to the rest frame of particles \( q_1 \) and \( q_2 \) using equations (A.3) and (A.4).

Finally the quantity \( \cos(q_1^Q, \beta_{12}^-) \) was computed using equation (A.9), where \( \beta_{12}^- = \frac{q_{12}^-}{E_{12}} \).

The relevance of these quantities to a study of the reactions (1.1) and (1.2) is discussed in Chapter II and the interpretation of their experimental distributions in Chapter III.
1.3 Critical Study of the Data

In this chapter a critical study of the experimental data will be made in order to a) correctly classify those events for which GRIND provides more than one interpretation, and b) detect possible systematic biases in the fitting programs. The effects of possible misclassification of events on various distributions will be studied.

The systematic biases will be accounted for by appropriate correction factors in the cross-section computations. Possible effects of these biases on various distributions will also be studied.

A. Ambiguities

In cases when an associated neutral decay was fitted as a $K^0$, there were four constraints in the kinematic fitting, and unambiguous interpretation of the events were obtained. In the majority of cases, however, there was only one kinematical constraint; therefore, many events had more than one fit and could not be separated by the ionisations of the tracks. As a first step if a fit had a $\chi^2$ probability 1/5 or less than that of some other fit, it was ignored. After this, in cases where there were more than one fit for the same hypothesis (there were 36 such cases) the fit with the highest probability was considered to be correct. The ambiguities remaining after this preliminary procedure were resolved in the following manner for the two cases that arise:
1. Ambiguity between a fit to reaction (1.1) or (1.2) with a fit to \( pp \rightarrow K^+ p \Sigma^0 \) or to \( pp \rightarrow K^+ p \Lambda^0 \).

In these cases the former was considered correct because the production cross-sections for \( K^+ \) and \( \Sigma^+ \) are comparable in these reactions while the decay probability for \( \Sigma^+ \) is very much larger than for \( K^+ \), within the limits set for the length of the decaying track (see p. 5).

2. Ambiguity between a fit to (1.1) with a fit to (1.2).

Out of 193 events of type (1.1) and 316 events of type (1.2), there were 62 that fitted both (1.1) and (1.2). In these cases the fit to reaction (1.2) was considered to be correct. In this way 131 events of type (1.1) remained. This classification is plausible in view of the fact that 40 \( K^0 \)-decays were observed, and therefore one expects \( 120 \pm 18 \) events of type (1.1).

When studying various distributions for the reaction of interest (see Chapter III) a check was made on the effects of possible misclassification of events on these distributions.

For this purpose the distributions of the effective mass, the Jackson parameters and the production angle in CM system were plotted for all events corresponding to the reaction of interest, as well as for only those events for which there were unambiguous

---

* Less than 7/0 of \( K^+ \) of length greater than 10 cms., \( p_{K^+} > 0.9 \text{ BeV/c} \) were selected by scanners due to restriction on length vs. ionisation.
interpretations. The general features of the distributions remained unchanged. (See Figs. 22, 24, 26 and 29, 30, 31 of Chapter III).

B. The Scanning Bias

The scanning was done for events consisting of an incident track and two outgoing prongs, one of which has a kink which may represent a decay of $\Sigma^+$. The kink may not be visible if the decay occurs too close to the vertex or outside of the visible volume of the chamber. In order to investigate such a possibility, the decay time $t^*$ in the rest frame of sigma was computed for all measured events, corresponding to reactions (1.1), (1.2) using the expression

$$t^* = \int_0^L \frac{1}{c} \frac{m}{p(l)} \, dl$$

where $p(l)$ is the momentum of sigma after it traverses a distance $l$ from the production vertex and $m$ is the mass of sigma (see eqns. (A.30), (A.31)). Since the momentum loss of sigma is small it was assumed that

$$p(l) = p_0 + \frac{l}{L} (p_L - p_0).$$

Then

$$t^* = \frac{m}{c} \frac{L}{p_0 - p_L} \log \frac{p_0}{p_L}.$$
where the initial momentum $p_0$ (at production), the final momentum $p_L$ (at decay) and the total distance $L$ traversed before decay are given by GRIND.

The experimental distribution of $t^*$ was then plotted for all events and compared to the distribution predicted on the basis of lifetime $\tau$ of sigma. The number $\Delta K$ of sigmas decaying in the time $\Delta t^*$ is

$$\Delta K = -\Delta N(t^*) = \frac{1}{\tau} N(t^*) \Delta t^*$$

where $N(t^*) = N_0 e^{-t^*/\tau}$ is the number of sigmas remaining at the time $t^*$. The number $N_0$ was estimated assuming no loss of sigmas due to this type of bias between the time $t_2^* = 0.25 \times 10^{-10}$ sec. after production, and the time $t_9^* = 2.5 \times 10^{-10}$ sec. With this assumption we have

$$N_0 = \frac{N(t_2^*) - N(t_9^*)}{e^{-t_2^*/\tau} - e^{-t_9^*/\tau}}.$$

The theoretical and experimental distributions are shown in Figs. 2, 3. As expected there is a loss of events with short decay times. A correction for this loss was made in the computation of cross-sections.

The event may also escape the scanner's eye if the kink is small, i.e. if the angle $\theta$ between $\Sigma^+$ and the visible decay product ($p$ or $\pi^+$) is small.
$\Sigma^+$ DECAY TIME

$p + p \rightarrow p + \Sigma^+ + K^0$

129 EVENTS

$t \left(10^{-10} \text{ sec}\right)$

Fig. 2
$\Sigma^+$ DECAY TIME

$\rho+\rho\rightarrow n+\Sigma^++K^+$

$3\Sigma^+$ EVENTS

Fig. 3
Because of the fact that \( \frac{E_{\pi^+}}{p_{\pi^+}} \) is about four times smaller for \( \pi^+ \) than \( p \), it follows, from eqn. (A.28), that for the same momentum of \( \Sigma^+ \), the angle \( \theta \) for \( \pi^+ \) is distributed over a much wider range than for \( p \); therefore far fewer \( \pi^+ \)-decays are lost due to this bias, than \( p \)-decays.

Theoretically the branching ratio is known to be
\[
\frac{\Gamma(\Sigma^+ \rightarrow p\pi^0)}{\Gamma(\Sigma^+ \rightarrow \pi^+n)} = 1.04,
\]
so that the numbers of \( \pi^+ \)- and \( p \)-decays are about equal. Experimentally, however, the number of \( \pi^+ \)-decays found is about twice that of \( p \)-decays. Further, the distribution of the laboratory momentum \( p_{\Sigma} \) of \( \Sigma^+ \) is independent of the eventual mode of decay. Thus the distribution of the laboratory momentum of sigma for those sigma's which decay into \( \pi^+ \) and for those which decay into \( p \) are expected to be similar. However, the experimental distributions of \( p_{\Sigma} \) for \( \pi^+ \)- and \( p \)-decays (Figs. 4, 5) show that there is a large loss for \( p \)-decays for \( p_{\Sigma} > 1.2 \text{ BeV/c}. \)

These facts reveal that there is indeed a large scanning bias.

An estimate of the loss due to this bias can be made from the study of the decay angular distribution (the decay angle is the CM angle \( \Theta^* \) corresponding to \( \Theta \)) which is expected theoretically to be uniform. It can be shown that there is no loss of \( \pi^+ \)-decays for \( |\cos \Theta^*| < 0.4 \). For given C.M. angle \( \Theta^* \) in this range, the laboratory angle \( \Theta \) is smallest when \( p_{\Sigma} \) is maximum (\( p_{\Sigma \text{ max}} = 4.25 \text{ BeV/c} \)). Substitution of this value in

\[\text{For these values of sigma momentum, } \Theta < 11.5^\circ \text{ for } p \text{-decays as can be seen from eqn. (A.29). At such small angles the kink in the decaying track becomes difficult to notice.}\]
\[ \Sigma^+ \text{ MOMENTUM} \]
\[ p + p \rightarrow \Sigma^+ + K^0 + p \]

\[ \text{p-DECAYS} \]
44 EVENTS

\[ \text{\pi}^+ - \text{DECAYS} \]
87 EVENTS

Fig. 4
\[ \Sigma^+ \text{ MOMENTUM} \]

\[ p + p \rightarrow \Sigma^+ + K^+ + n \]

\[ \pi^+ - \text{DECAYS} \]

214 EVENTS

\[ p - \text{DECAYS} \]

102 EVENTS

Fig. 5
the equation (A.26) gives

\[ \theta = \arctan \left[ \frac{0.27 \sin \theta^*}{(\cos \theta^* + g)} \right] \]

with \( g = 1.2 \) for \( \pi^+ \)-decays. It follows that for \( |\cos \theta^*| < 0.4 \),
the smallest angle in the laboratory is greater than 90° for \( \pi^+ \)-decays.
(However for p-decays this angle can be as small as 30°
for the same range of \( \theta^* \)).
Thus one expects very little loss
of \( \pi^+ \)-decays for small \( |\cos \theta^*| \).
Knowing this and the fact
that the distribution is expected to be uniform, the loss at
the edges can be estimated. Once the correct number of \( \pi^+ \)-decays has been estimated, the branching ratio gives an estimate
of the correct number of p-decays. The Figs. 6a, 7a show the
experimental distribution of the cosine of decay angle for \( \pi^+ \)-decays.
The deviation from a uniform distribution in the
middle of \( \cos \theta^* \) distribution is due to a program bias, dis-
cussed in the next section, which has the effect of transferring
events from \( \cos \theta^* = 0 \) to neighbouring bins of the histogram
with positive \( \cos \theta^* \). The remaining deviation from an isotropic
distribution can be explained by the scanning bias discussed
above. It may be remarked that for a given value of \( |\cos \theta^*| \)
the smallest laboratory angle is larger for the negative value
of \( \cos \theta^* \) than for the positive value. Thus more events are
expected to be lost for \( \cos \theta^* > 0 \) than for \( \cos \theta^* < 0 \).
Both p- and \( \pi^+ \)-decays have been included in the distributions
of effective mass, cosine of production angle in CM, and
Jackson parameters, when studying the reaction of interest.
COSINE OF DECAY ANGLE
\[ p + p \rightarrow \Sigma^+ + K^+ + n \]

\[ \Sigma^+ \rightarrow \pi^+ + n \]
214 EVENTS

\[ \Sigma^+ \rightarrow p + \pi^0 \]
102 EVENTS

Fig. 6

COSINE OF DECAY ANGLE
\[ p + p \rightarrow \Sigma^+ + K^0 + p \]

\[ \Sigma^+ \rightarrow \pi^+ + n \]
87 EVENTS

\[ \Sigma^+ \rightarrow p + \pi^0 \]
44 EVENTS

Fig. 7
(see Chapter III). However the same distributions were also plotted without the p-decays. The general features of the distributions remained unchanged. (See Figs. 22, 24, 26 and 32, 33, 34 of Chapter III).

C. The Program Bias

The Decay Vertex

Whenever analysing a sigma decay with an unknown momentum of sigma, this momentum p has to be computed by GRIND from the energy and momentum conservation equations. If the cosine of the decay angle $\cos \Theta^*$ is close to zero, the program may obtain complex values for p due to measurement errors. In such a case, the program uses a least squares procedure to modify the values of the measured quantities so that p becomes real. The condition for this is (see Appendix B)

$$p^T \geq p$$

where

- $p^T$ is the measured transverse momentum and
- $p^*$ is the centre of mass momentum of the decay products.

Actually, the program attempts to reduce $p^T$ to $0.9 p^*$ in order to avoid singularities arising in the error propagation for $p^T = p^*$. This has the effect of transferring events
from $\cos \Theta^* = 0$ to the neighbouring bins of the histogram
($\cos \Theta^* = .43$ for $p_T = .9 p^*$).

All $\Sigma^+$ events for which good fits have been obtained
by GRIND in such a manner (there were 60 such events) were
identified. No significant bias has been found for these
events in any of the distributions studied (such as effective
mass, Jackson parameters etc.).

The Production Vertex

A check was made on possible systematic biases in the
kinematic fitting. For this purpose a study was made of the
amount by which each track variable was shifted by the GRIND
fitting procedure.

The quantity

$$S_x = \frac{x_u - x_f}{\sigma(x_u - x_f)}$$

sometimes called "Pull" (8), was computed for each track
variable $x$. Here

- $x_u$ is the unfitted value of the variable,
- $x_f$ is the fitted value of the variable, and
- $\sigma(x_u - x_f)$ is the standard error of their difference,

given by

$$\sigma^2(x_u - x_f) = \sigma^2(x_u) + \sigma^2(x_f) - 2 R(x_u, x_f) \sigma(x_u) \sigma(x_f).$$

It was assumed that

$$R(x_u, x_f) = \frac{\sigma(x_f)}{\sigma(x_u)}.$$
where $R(x_u, x_f)$ is the correlation coefficient.

The distribution of $S_x$ is expected to be gaussian with a mean of zero and a unit variance if there is no systematic bias in the fitted values of the variable $x$. The experimental distributions of the quantity $S$ for each of the three track variables $\frac{1}{\rho}$, dip angle $\lambda$, and azimuth angle $\phi$ were plotted for all 1C fits corresponding to reactions (1.1), (1.2). The results for all three tracks (at the production vertex) and for both reactions were pooled and the resulting distributions are shown in Figs. (8), (9) and (10). These distributions are in good agreement with the expected gaussian distributions.

1.4 Determination of the Cross Section

To calculate the cross section for a reaction, three quantities are required:

1. The number of events, corresponding to this reaction, within a certain fiducial region of the chamber.
2. The total length of the beam tracks responsible for the production of these events.
3. The density of the hydrogen in the bubble chamber.

For the purposes of the cross-section computation, a rectangular fiducial region 126 cms. by 38 cms. was chosen. In the coordinate system used (a right-handed coordinate
PULL FOR $P^{-1}$ MEASUREMENTS

1050 TRACKS

$\sigma = 4.0$

Fig. 8
PULL FOR AZIMUTH MEASUREMENTS
1054 TRACKS

Fig. 9
PULL FOR DIP MEASUREMENTS

1053 TRACKS

Fig 10

$\sigma = 1.0$
system with origin at the fiducial 4a of Ref. 7; the X-axis parallel to the beam and positive in the beam direction, and the Z-axis negative down into the chamber), this fiducial region is between \( x = 0 \) and \( x = 126 \) cms.; \( y = 0 \) and \( y = 38 \) cms.; with no restrictions on the values of \( z \).

A. The Event Count

In a total of 122,674 frames scanned 755 measurable events were found by the scanners. Of these, 131 fitted reaction (1.1), and 316 fitted reaction (1.2). These numbers apply to the total region scanned. The number of events within the fiducial region were 106 and 222 respectively for the two reactions.

These event counts must be corrected for the following losses: (a) scanning loss due to random causes, (b) loss due to scanning biases, and (c) loss due to non-measurable events. Each of these losses will now be considered and the corresponding correction factor computed. If \( n_0 \) events are found and \( n_1 \) events are estimated as lost due to some cause, then the correction factor \( f_1 \) is the ratio of the sum \( N_1 = n_1 + n_0 \) to the number \( n_0 \) of events found:

\[
f_1 = \frac{n_0 + n_1}{n_0}
\]

It's standard error is given by:

\[
\text{If } n_0 \text{ events out of } N_1 = n_1 + n_0 \text{ are found then the probability } \frac{1}{f_1} \text{ of finding the event is } \frac{n_0}{N_1}, \text{ and the variance of } n_0 \text{ is } \frac{N_1}{N_1} \left( \frac{1}{f_1} \left( 1 - \frac{1}{f_1} \right) \right), \text{ therefore the variance of } \frac{1}{f_1} \text{ is } \frac{1}{N_1} \frac{1}{f_1} \left( 1 - \frac{1}{f_1} \right)
\]
\[
\left( \frac{\Delta f_1}{f_1} \right)^2 = \frac{n_1}{n_0(n_1 + n_0)}
\]

(a) **Scanning loss due to random causes: scanning efficiency**

Assuming all events to be equally visible, the scanning efficiency \( e \), is defined as the ratio of the number of events found to the true number of events on the film. (When there are other losses then it is simply the inverse of the correction factor: \( e = \frac{1}{f_1} = \frac{n_0}{N_1} \)).

A part of our film was scanned twice, the other part three times. The scanning efficiency was computed separately for each part of the film. For the purposes of this computation, only measurable events were considered. Events which were not measurable due to various causes (e.g. a black spot on one of the vertices, missing stereo views, etc.), were considered not equally visible, and were dealt with separately.

For events without a neutral decay, the results, were as follows:

**two scans:** A total of 402 measurable events was identified on this part of the film. The number of events found in the first scan was \( N' = 271 \) while the number found in the second scan was \( N'' = 353 \). The number of events common to both scans was \( N_{12} = 222 \). If we assume that the two scans have efficiencies \( e_1 \) and \( e_2 \), then these can be computed from:

\[ N_{12} = e_1 N'' = e_2 N' \].
The detection efficiency in the two scans is then

\[ e_{1+2} = e_1 + e_2 - e_1 e_2 = (93.3 \pm 1.3)\% \]

and the number of events in this part of the film is

\[ N_{1+2} = \frac{402}{93} = 431. \]

**three scans**: A total of 353 measurable events was identified on this part of the film. The number of events found in the first scan was \( N' = 200 \), the number found in the second scan was \( N'' = 278 \) and the number found in the third scan was \( N''' = 301 \). The number of events common to scans 1 and 2 was \( N_{12} = 167 \), common to scans 1 and 3 was \( N_{13} = 183 \), and common to scans 2 and 3 was \( N_{23} = 233 \).

We have as above:

\[
N_{12} = e_1 N'' = e_2 N'
\]

\[
N_{13} = e_1 N''' = e_3 N'
\]

\[
N_{23} = e_3 N'' = e_2 N'''
\]

\[
e_{1+2+3} = e_1 + e_2 + e_3 + e_1 e_2 e_3 - e_1 e_2 - e_2 e_3 - e_1 e_3
\]

\[
= (99.0 \pm .5)\%
\]

and the number of events in this part of the film is

\[ N_{1+2+3} = \frac{353}{99} = 356. \]

The overall detection efficiency is therefore
The standard error on the overall efficiency can be computed from the relation

$$\frac{\Delta e}{e} = \sqrt{\left[\frac{356 \Delta e_{1+2+3}}{787 e_{1+2+3}}\right]^2 + \left[\frac{1431 \Delta e_{1+2}}{787 e_{1+2}}\right]^2}.$$ 

The result is $\Delta e = 1.5\%$.

For events with an associated $K^0$-decay the results were similar.

(b) **Loss due to scanning biases**

These were discussed in the section 1.3. The correction factor $f_2$ for the loss due to short sigma tracks is $\frac{131 + 22}{131}$ for the reaction (1.1) and $\frac{316 + 39}{316}$ for (1.2). The correction factor $f_3$ for the loss due to small angles is $\frac{131 + 63}{131}$ for the reaction (1.1) and $\frac{316 + 144}{316}$ for (1.2).

(c) **Loss due to non-measurable events**

About $13\%$ of the events found by the scanners were not measurable and it was assumed that the number of events for reactions (1.1) and (1.2) were reduced by the same percentage. The correction factor $f_4$ for this loss is $\frac{755 + 755 \times 0.13}{755} = 1.3$.

The estimate of the true number $N$ of events for each reaction, is the number $n$ found, multiplied by all the
correction factors:

\[ N = n f_1 f_2 f_3 f_4 \]

The standard error on \( N \) is given by

\[ \frac{\Delta N}{N} = \sqrt{\left( \frac{\Delta n}{n} \right)^2 + \left( \frac{\Delta f_1}{f_1} \right)^2 + \left( \frac{\Delta f_2}{f_2} \right)^2 + \left( \frac{\Delta f_3}{f_3} \right)^2 + \left( \frac{\Delta f_4}{f_4} \right)^2} \]

Since 106 events for reaction (1.1) and 222 events for reaction (1.2) were found in the fiducial region, the corrected numbers in the fiducial region become:

216 \( \pm \) 24 events for reaction (1.1)

427 \( \pm \) 34 events for reaction (1.2)

B) Beam count

In order to estimate the total length of track scanned, a beam count was made on every fiftieth frame on the film. In each such frame the number of interacting and non-interacting tracks in the fiducial region was counted and the length of each interacting track was measured up to the interaction point. The number of frames \( n_c \) used for the beam count was 2537. The number of interactions \( N_{\text{int}} \) was 6307 and the scanning efficiency \( e_c \) was \((95 \pm 2)^\circ \). Measured length of interacting tracks was \( 3.81 \times 10^5 \) cms. The number of non-interacting tracks was 30686. Since non-interacting tracks were each 126 cms long, the total track length \( L_c \) found in \( n_c \) frames was \((4.268 \pm 0.033) \times 10^6 \) cms.
The total number $n_s$ of frames scanned in this experiment was 122674, thus the total track length scanned was

$$L = n_s \frac{L_c}{n_c} = (2.064 \pm 0.016) \times 10^8$$

C) Density of Hydrogen

The density $\rho$ of the liquid hydrogen derived from the measurement of hydrogen vapour pressure during exposure was $0.062 \text{ gr/cm}^3$. It can be checked whether this value is consistent with the previously observed cross-section for pp interactions. Given the number of pp interactions per unit track length, one can compute the total cross-section $\sigma_T$ for pp interactions in terms of hydrogen density according to:

$$\sigma_T = \frac{N_{\text{int}} A}{L_c N_0 \rho e_c}$$

where

- $N_0$ is the Avogadro's number,
- $A$ is the atomic weight of hydrogen.

---

The liquid hydrogen conditions were: vapour pressure $P = 60 \text{ p.s.i.}$ and chamber temperature $T = 26.10^\circ\text{K.}$
We have

\[ N_0 = 6.023 \times 10^{23} \text{ mole}^{-1} \]
\[ L_c = 4267685 \text{ cms.} \]
\[ N_{\text{int}} = 6307 \]
\[ A = 1 \]
\[ e_c = 95\% \]
\[ \rho = 0.062 \text{ gr/cm}^3. \]

We obtain \( \sigma_T = 41.7 \text{ mb} \) which is in agreement with the known cross-section, namely \( \sigma_T = 43.5 \pm 1.5 \text{ mb} \).

D) Computation of production cross-section

The cross-section for a reaction \( r \) can be computed according to

\[ \sigma'_r = \frac{N_r}{L} \frac{A}{N_0 \rho} \]

where \( N_r \) is the number of events of type \( r \) and \( L, A, N_0, \rho \) are given above.

The error on the cross-section can be computed according to

\[ \frac{\Delta \sigma'_r}{\sigma'_r} = \sqrt{\left( \frac{\Delta N_r}{N_r} \right)^2 + \left( \frac{\Delta L}{L} \right)^2 + \left( \frac{\Delta \rho}{\rho} \right)^2} \]

The results are \( \text{see Table I} \)

\( \text{The cross-section for reaction (1.1) was obtained also by E. Bierman et al.}^{(8)} \text{, analysing the same film, on the basis of the small number of \( \Sigma^+ \) events with an associated \( K^0 \) decay.} \)
\( \sigma_1 = 28.0 \pm 3.2 \mu b \) for reaction (1.1)
\( \sigma_2 = 55.5 \pm 4.6 \mu b \) for reaction (1.2)

**Remark**

There is previous experimental data based on a very small sample of events for reactions (1.1), (1.2) by Loutit et al. (12) at a lower incident momentum. At 3.7 BeV/c they found 27\( \frac{1}{2} \pm 2\frac{1}{2} \) events of type (1.1), 38\( \frac{1}{2} \pm 2\frac{1}{2} \) events of type (1.2) and obtain
\[
\sigma_1 = 30 \pm 10 \mu b
\]
\[
\sigma_2 = 47 \pm 12 \mu b
\]

I.J. Bloodworth has kindly sent me a preprint of his recent work on \( \Sigma \) production in pp interactions. At 6 BeV/c he finds a total of 37 events of type \( pp \rightarrow K \Sigma N \) and obtains
\[
\sigma_1 = 22 \pm 9 \mu b
\]
\[
\sigma_2 = 40 \pm 11 \mu b
\]
<table>
<thead>
<tr>
<th>Reaction</th>
<th>No. of events in fiducial region</th>
<th>Correction factors</th>
<th>Corrected number</th>
<th>Cross-Section (µb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3\Sigma K^0 p$</td>
<td>106</td>
<td>$f_1$</td>
<td>$f_2$</td>
<td>$f_3$</td>
</tr>
<tr>
<td>$^3\Sigma K^+ n$</td>
<td>222</td>
<td>$f_1$</td>
<td>$f_2$</td>
<td>$f_3$</td>
</tr>
</tbody>
</table>

These values are somewhat larger than previously published values (8, 54) due to more accurate estimate of the scanning losses. The errors shown are purely statistical (one standard deviation).
CHAPTER II

OUTLINE OF THE THEORY

In this chapter a review of the theory required in the detailed study of the reactions (1.1), (1.2) is presented. The topics examined are:

a) Techniques used in detection and study of baryon resonances;

b) Methods of extracting information about the mechanism of reaction.

A preliminary discussion presents the formalism used to describe the strong interactions of baryon resonances. The results are later applied to obtain an expression for the decay rate of the resonance and the production cross-section.

The last section of the chapter examines the consequences of the charge independence for the reactions (1.1) and (1.2).

2.1 Field Formalism for Baryon Resonances

Rushbrooke\(^{(13)}\) has suggested a Rarita-Schwinger formalism to describe baryon resonances and their interactions. According to this formalism a particle of half-integer spin \(s = n - \frac{1}{2}\) (where \(n\) is an integer) can be described by a field operator \(\bar{\Phi}\) which is a tensor of rank \(n-1\), each component of which is a four-spinor satisfying the Dirac equation

\[
(\gamma^\nu \partial_\nu + m) \bar{\Phi}_{\alpha_1 \cdots \alpha_{n-1}} = 0, \quad \alpha_1 = 1, \ldots, 4
\]

(2.1)

and the condition
The field operator $\bar{\mathbf{1}}$ has the following properties: it is a completely symmetric tensor, i.e.,

$$\bar{\mathbf{1}}_{a_1 \ldots a_i \ldots a_j \ldots} = \bar{\mathbf{1}}_{a_j \ldots a_i \ldots a_1} \quad (2.3)$$

and

$$\delta_{a_1} \bar{\mathbf{1}}_{a_1 \ldots a_{n-1}} = 0 \quad (2.4)$$

$$\delta_{a_1 a_2} \bar{\mathbf{1}}_{a_1 a_2 \ldots a_{n-1}} = 0 \quad (2.5)$$

The equations (2.1) - (2.3) reduce the number of independent components of $\bar{\mathbf{1}}$ to $2s + 1$.

Interaction of spinor fields with a pseudoscalar field is assumed to be parity conserving and interaction Lagrangians incorporating derivative couplings of the pseudoscalar field to free spinor fields are used.

For a field $\bar{\mathbf{1}}$ of spin $s$ interacting with a field $\psi$ of spin $\frac{1}{2}$ and a pseudo-scalar field $\phi$ the interaction Lagrangian is

$$L = \frac{G}{(m_\pi)^{n-1}} \bar{\psi} \gamma_\mu_1 \ldots \gamma_{n-1} \bar{\mathbf{1}}_{\mu_1 \ldots \mu_{n-1}} \phi \quad (2.6)$$

where $0$ depends upon the parities $P_s$ of $\bar{\mathbf{1}}$ and $P_{\frac{1}{2}}$ or $\psi$. We have
\[ 0 = 1 \quad \text{if} \quad P_s P_{1/2} (-1)^n = +1 \quad (2.7) \]

and

\[ 0 = i\gamma_5 \quad \text{if} \quad P_s P_{1/2} (-1)^n = -1 \quad (2.8) \]

The \( S \) matrix element can be written in terms of the Lorentz invariant matrix element \( T \) as

\[ S_{fi} = \delta_{fi} - i(2\pi)^4 \delta^4 (P_f - P_i) \frac{T_{fi}}{\sqrt{\text{II} 2p_{0i}}} \quad (2.9) \]

where a factor \( 2p_{0i} \) occurs for every initial and final particle.

Let \( \mu, m_d, m_b \) be the masses of the pseudoscalar particle, particle of spin \( s \) and particle of spin \( \frac{1}{2} \) respectively, and \( k, p_d, p_b \) the corresponding momenta.

For any process represented by the diagram of Fig. 11 the transition amplitude \( T \) squared and summed over initial and final spins is given by (13)

\[ \sum |T|^2 = \frac{\mathcal{E}^2}{(m_\pi)^{2(n-1)}} 4 D_n k^{2(n-1)} (m_b^2 - m_d^2 - p_b \cdot p_d) \quad (2.10) \]

where
\[ g = \frac{G}{\pi} \psi \phi \]

\[ D_n = (n!)^2 \frac{2^n}{(2n)!} \quad (2.11) \]

\[ m' = \pm m \text{ if } P_s P_y (-1)^n = \pm 1. \]

Let \( \omega \) be the total energy of particles \( \mathbf{p}_b \) and \( \mathbf{k} \). In the rest frame of \( \mathbf{d} \) we have

\[ q = \mathbf{k} = \mathbf{p}_b = \sqrt{\frac{\omega^4 - 2\omega^2 (\mu^2 + m_b)^2 + (\mu^2 - m_b^2)^2}{2\omega}} \quad (2.12a) \]

\[ E_b = \frac{\omega^2 + m_b^2 - \mu^2}{2\omega} \]

\[ E_d = m_d = \omega \quad \]

Equation (2.10) becomes

\[ \sum |T|^2 = g^2 \frac{g^2 (n-1)}{m_x} \left[ (m_d + m_b' )^2 - \mu^2 \right] 2D_n \quad (2.12b) \]

since \( m_b' m_d - \mathbf{p}_b \cdot \mathbf{p}_d = \frac{(m_d + m_b')^2 - \mu^2}{2} \).
2.2 The Decay Width of the Resonance

The decay width for the process

\[ p_d \rightarrow q_1 + q_2 \]

is given in the rest frame of \( d \) by

\[
\Gamma_{12}(\omega) = \frac{1}{(2\pi)^2} \int \frac{d^4(q_1 + q_2 - p_d)}{2\omega} \frac{1}{2s+1} \sum |T|^2 \frac{d^3q_1}{2E_1} \frac{d^3q_2}{2E_2}
\]

(2.13)

where \( T \) is the Lorentz invariant amplitude defined by equation (2.9) and the summation is over all polarization states of \( q_1, q_2 \) and \( p_d \). Performing the integration, one obtains

\[
\Gamma_{12}(\omega) = \frac{1}{4\pi} \frac{q_2^2}{2\omega^2} \frac{1}{2s+1} \sum \frac{1}{|T|^2}
\]

(2.14)

If \( q_1 \) is a baryon and \( q_2 \) a pseudoscalar meson then \( \sum |T|^2 \) is given by equations (2.12) with \( m = m_1 \) and \( \mu = m_2 \). The equation (2.14) becomes (13) for \( P_S P_{\frac{1}{2}}(-1)^n = +1 \):

\[
\Gamma_{12}(\omega) = \frac{g^2}{4\pi} \frac{D_n}{n} \frac{q^{2\ell+1}}{2m_\chi} \frac{(\omega + m_1)^2 - m_2^2}{\omega^2}
\]

where \( \ell = n-1 \) is the orbital angular momentum for the decay process.
The decay width can be expressed in terms of its value at \( \omega_0 \) as

\[
\Gamma_{12}(\omega) = \Gamma_{12}(\omega_0) \left[ \frac{\left( \frac{q}{q_0} \right)^{2l+1} \left( \frac{(\omega + m_1)^2 - m_2^2}{(\omega + m_l)^2 - m_2^2} \frac{\omega_0^2}{\omega} \right)}{r(\omega)} \right] (2.15)
\]

where \( q_0 = q(\omega_0) \).

2.3 The Cross-Section for Production of a Resonant State

Jackson proposed a method for computing the cross-section for production of a resonant state when the cross-section for production of a stable particle is known. Let \( d\sigma_s \) be the cross-section for production of a stable particle of mass \( \omega \). The cross-section \( d\sigma_d \) for production of the resonant state \( d \) can be expressed in terms of \( d\sigma_s(\omega) \) by modifying the calculation to include a propagator for \( d \) and the vertex amplitude for the decay of \( d \). Jackson obtains

\[
d\sigma_d = d\sigma_s(\omega) \left[ \frac{1}{\pi} \frac{\omega_0 \Gamma_{12}(\omega)}{(\omega_0^2 - \omega^2)^2 + \omega_0^2 \Gamma^2(\omega)} \right] d\omega^2 (2.16)
\]

Here \( \omega_0 \) is defined as the mass of the resonance, \( \Gamma(\omega) \) is the total decay width of the resonance and \( \Gamma_{12}(\omega) \) is the partial width for the decay of the resonance \( p_d \) into two
2.4 The Effective Mass Distribution

The distribution of the effective mass $M$ of two particles, say 1 and 2, predicted by the invariant phase space for a 3-particle final state $|q_1, q_2, q_3\rangle$ is

$$\frac{d\sigma}{d\omega} = C \frac{q_3^*}{W \omega}$$

according to the equation (A.41b) of Appendix A. Here

- $C$ is a normalisation constant
- $W$ is the total CM energy
- $q_3^*$ is the momentum of particle 3 in CM frame
- $\omega = M$ is the total energy of particles 1 and 2 in their rest frame, and
- $q$ is the momentum of particles 1 or 2 in this frame.

Let $d$ be a resonance produced in a 3-particle final state $|q_1, q_2, q_3\rangle$ and let $q_1$ and $q_2$ be the decay products of the resonance. The effective mass distribution of $q_1$ and $q_2$ can be obtained, in this case, from the equation (2.16). On the
assumption that \( q_3 \) and the resonance are produced according to phase space, the stable particle cross-section \( \sigma_5 \) will be proportional to the two-body phase space \( R_2 \) given by (15)

\[
R_2(w; M_{12}, m_3) = \pi \frac{q_3^*}{W}
\]

Using equation (2.17) we have

\[
R_2(w; M_{12}, m_3) = \frac{1}{C_R} \left( \frac{d\sigma}{d\omega^2} \right) .
\]

Therefore equation (2.16) becomes

\[
d\sigma_d = C_R \left( \frac{d\sigma}{d\omega^2} \right) \frac{\omega_0 \Gamma_{12}(\omega)}{(\omega^2 - \omega_0^2)^2 + \omega_0^2 \Gamma^2(\omega)}
\]

which can be written as

\[
d\sigma_d = R \frac{d\sigma}{d\omega^2} d\omega^2
\]  

(2.18a)

with

\[
R = C_R \left( \frac{d\sigma}{d\omega^2} \right) \frac{\omega_0 \Gamma_{12}(\omega)}{(\omega^2 - \omega_0^2)^2 + \omega_0^2 \Gamma^2(\omega)}
\]  

(2.18b)

where

\[
q = \sqrt{\omega^2 - 2\omega^2 \left( m_1^2 + m_2^2 \right) + \left( m_1^2 - m_2^2 \right)^2}
\]  

(2.18c)
and $C_R$ is a normalisation constant. The total width $\Gamma(\omega)$ can be obtained in terms of its value $\Gamma_0$ at $\omega_0$ assuming that the ratio

$$x = \frac{\Gamma_{12}(\omega)}{\Gamma(\omega)}$$

(2.18d)

does not vary with $\omega$, so that

$$\Gamma(\omega) = \Gamma_0 \left(\frac{q}{q_0}\right)^{2\ell+1} \frac{(\omega+m_1)^2 - m_2^2}{(\omega_0+m_1)^2 - m_2^2} \left(\frac{\omega}{\omega_0}\right)^2$$

(2.18e)

2.5 The Study of Resonances in Multiparticle Final States

The properties of the resonant state are best studied in a reaction in which no other particles are produced, i.e. a "formation experiment". However, this is not always possible, and methods were developed to detect and study resonances in reactions in which other particles are also present in the final state, i.e. "production experiments". The principal techniques consist of evaluating the predictions of the phase space and studying the character of the distortions expected in the presence of a resonance. In a three-particle final state, the resonance can be detected by examining the Dalitz plot. The mass and width of the resonance can be obtained from the effective mass distribution. If the reaction is dominated by production of the resonant state, then the spin and parity of the resonance can be determined from the angular distributions and the

See pp. 122-127 of Appendix A for discussion of phase space.
polarizations of the two body decay products.

Of course, these procedures are exact only in the limit of a small decay width (i.e. long lifetime) and in the limit that the interference with the background (or with possible resonances in other intermediate states) is negligible.

2.6 Mechanism of the Reaction

For scattering experiments resulting in multiparticle final states dynamical information about the process can be obtained from a study of the square of momentum transfer distribution. If the experimental distribution of the four-momentum transfer squared $\Delta^2$ shows a definite preference for small $\Delta^2$, this fact suggests a model for the interaction, the so-called one-particle-exchange (OPE) model, where the interaction is thought to take place through the exchange of a virtual particle $\mu$ between the colliding particles. The essential feature of the model is the presence in the differential cross-section of the factor $\frac{1}{(\Delta^2 + \mu^2)^2}$. The most rapid $\Delta^2$ dependence will come from the exchange of the lowest mass particle. For strong interactions this means one pion exchange; so that one pion exchange can be expected to dominate the process whenever the selection rules permit it. In some cases one can consider also the exchange of a single K-meson.
History of OPE model

Chew and Low\(^{(18)}\) have given a prescription (analogue to the prescription given by Feynman rules of perturbation field theory but formulated directly in terms of the analytic properties of the S-matrix) for the matrix element corresponding to the one-particle intermediate state with the intermediate particle on its mass-shell.

Considering a process of the type shown in Fig. 12

\[ M_1 + M_2 \rightarrow m_1 + m_2 + m_3 \quad (2.19) \]

they define the two variables \( \Delta^2 \) and \( \omega^2 \); \( \Delta^2 \) is defined as the invariant square of the difference of the four-momenta of \( M_2 \) and \( m_3 \) and \( \omega^2 \) as the square of the total energy of all the outgoing particles — excluding \( m_3 \) — in their barycentric system. They then divide all particles into two groups, each of which as the same quantum numbers as some single particle \( \mu \). One of these groups contains two particles and the other contains three particles. They conjecture that the S-matrix has a pole of second order at \( \Delta^2 = -\mu^2 \), the residue of the pole being the product of the S-matrix elements connecting
the two groups of particles to the intermediate particle on its mass-shell.

The $S$-matrix element $\langle q_3 | J | p_2 \rangle$ connecting the group of two particles to the intermediate particle on its mass-shell does not correspond to a physically realizable transition $M_2 \rightarrow m_3 + \mu$ for stable particles but can be defined by a process of analytic continuation. For the case when the two particles are nucleons and the intermediate particle is a pion, Chew and Low find the spin average of the square of this matrix element to be, from the static nucleon model,

$$4\pi f^2 \frac{\Delta^2}{\mu^2} \times \begin{cases} 1 & \text{for } \pi^0 \\ 2 & \text{for } \pi^\pm \end{cases}$$

where $f^2$ is the renormalized, unrationlized pion-nucleon coupling constant, $f^2 = 0.08$.  

The matrix element $\langle q_1, q_2, q_3 | J | p_1 \rangle$, connecting the larger group to the intermediate particle on its mass-shell, is conjectured to be equal to the physical matrix element for the process

$$M_1 + \mu \rightarrow m_1 + m_2$$  

(2.20)

The expression so obtained for the two-dimensional distribution $d\sigma/d\Delta^2d\omega^2$ is strictly valid only when the intermediate particle is on its mass-shell. Various approaches have been tried (18-35) in applying the formula in the physical region $\Delta^2 > 0$. Chew and Low suggested that it can be used to
extrapolate \( \frac{d\sigma}{d\Delta^2 d\omega^2} \) from the physical region of positive \( \Delta^2 \) to the pole at \( \Delta^2 = -\mu^2 \), in order to determine the residue and thus the cross-section \( \sigma_0 \) for the process (2.20). This approach turned out to be difficult to exploit because of the accuracy needed in the experimental data in order to find the correct analytical behaviour for the extrapolation. A more fruitful approach consists of making use of the Chew-Low formula to compute the pole contribution to the cross-section \( \sigma \) for the process (2.19) when the cross-section \( \sigma_0 \) for the process (2.20) is known. The assumption is usually made that in the physical region near the pole, i.e. for small values of \( \Delta^2 \), the pole contribution is dominant. The assumption is justified by the fact that in the physical region near the pole the value of the propagator is large. There have been also attempts to use the Chew-Low formula for larger values of \( \Delta^2 \) in order to obtain some qualitative information about the process studied\(^{24,25,26}\).

More realistic evaluations of OPE model for physical values of \( \Delta^2 \) have been proposed introducing corrections to take into account the virtuality of the intermediate particle.

Salzman and Salzman\(^{21}\) have computed a correction for the off-shell pion-nucleon scattering amplitude from the static nucleon model.

Iizuka and Klein\(^{27}\), Ferrari and Selleri\(^{23,29,30}\), Selleri\(^{31}\), Amaldi and Selleri\(^{32}\) used methods similar to those used by Chew, Goldberger, Low and Nambu\(^{36}\) for the real processes in order to compute the off-shell amplitude for the four particle vertex. Under the assumption that the resonance dominates the dispersion integral they found that the off-shell amplitude is proportional
to the amplitude for the real process, the factor of proportionality consisting of a form factor multiplied by a known function of energy and momentum transfer.

Ferrari and Selleri\(^{33}\) derive the "exact" expression for the cross-section from perturbation theory using the off-shell amplitude for the four-particle vertex and form factors for the three-particle vertex and propagator. For the case when the particles \(M_1, M_2, m_1\) and \(m_3\) are spin \(\frac{1}{2}\) baryons and the intermediate particle is a spinless boson, they take the invariant matrix element to be

\[
M\_{11} = \bar{u}(q_3) 0 G_r u(p_2) K(\Delta^2) \frac{K'(\Delta^2)}{\Delta^2 + \mu^2} M_Q(w^2, \cos \Theta_1^Q, \Delta^2)
\]

(2.21)

where 0 is \(\gamma_5\) or 1 depending upon the parity of the exchanged boson, \(G_r\) is the rationalized, renormalized coupling constant and the \(K\) and \(K'\) are form factors normalised to unity at the pole. For small \(\Delta^2\), Ferrari and Selleri evaluate the residue in the pole approximation, i.e. in the limit \(\Delta^2 \rightarrow -\mu^2\) except for retaining the \(\Delta^2\)-dependence for the scattering angle \(\Theta_1^Q\) entering the off-shell amplitude \(M_Q\). Assuming that the differential cross-section for the virtual process (2.20) has the same dependence on the variables as for the real process they rederive the Chew-Low formula. They obtain

\[
\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2} \left( \frac{G^2_f}{4\pi} \frac{1}{\sqrt{W_p}} \right) \frac{\Delta^2 + (m_3 - M_2)^2}{(\Delta^2 + \mu^2)^2} w^2 \frac{k^r Q}{\mu} d\Omega (w^2, \cos \Theta_1^Q)
\]

(2.22)

\[
\frac{\delta (p_f - p_1)}{E_1 E_2 E_3} \quad \frac{d^3q_1 \ d^3q_2 \ d^3q_3}{w_f w_1 w_2}
\]

For pion-nucleon vertex we have

\[
\frac{G^2_f}{4\pi} = 4 \frac{M^2}{\mu^2}
\]
and, by suitable integrations, the two dimensional distribution of Chew and Low

\[
\frac{d\sigma}{d\Delta^2 d\omega^2}_{\text{pole}} = \frac{1}{8\pi} \frac{g_f^2}{4\pi} \frac{1}{(W_{p_1^*})^2} \frac{\Delta^2 + (m_3 + M_2)^2}{(\Delta^2 + \mu^2)^2} \sigma_0(\omega) k_1^Q \omega
\]  

(2.23)

Here \( W \) is the C.M. energy

\( p_1^* \) is the C.M. momentum of \( M_1 \)

\[ k_1^Q = \frac{\sqrt{\omega^4 - (M_1^2 + \mu^2) 2\omega^2 + (M_1^2 - \mu^2)^2}}{2\omega} \]

\( \frac{d\sigma}{d\Omega}(\omega^2, \cos \theta_1^Q) \) is the differential cross-section for the real process (2.20), and \( \sigma_0(\omega) \) is the total cross-section for this process.

Jackson and Pilkunh evaluate both vertices from the perturbation theory for the special case of a quasi two-body reaction (Fig. 13) when the final system of particles excluding the "recoil" \( m_3 \) consists of a resonance which later decays. In the calculation the resonance is treated as a stable particle of appropriate quantum numbers interacting with a spin \( \frac{1}{2} \) baryon and a virtual pseudo-scalar meson according to (2.6). Form factors are used for both vertices and for the propagator. In the case when \( M_1, M_2 \) and \( m_3 \) are spin \( \frac{1}{2} \) baryons, the exchanged particle is a
pseudo-scalar meson and the resonance has a spin $s$ and parity $P_8$ such that $P_8 P_{1/2} (-1)^{s+1/2} = +1$, the method gives

$$\frac{d\sigma}{d\Delta^2} = \frac{1}{64 \pi n^2} \left( \frac{1}{\Delta^2} + \left( M_2 - m_2 \right)^2 \right) F^2(\Delta^2) \left( \Delta^2 + \mu^2 \right)^2 D_n g^2 \left( \frac{P_1^Q}{m_2} \right)^{2 \ell} \left[ \Delta^2 + (\omega + m_2)^2 \right]$$  \hspace{1cm} (2.24)

where $F(\Delta^2)$ is an empirical function which represents the product of the three form factors,

$\ell = n-1$ is the relative orbital momentum in the formation of the resonance and

$P_1^Q$ is the momentum of $M_1$ in the rest frame of the resonance given by

$$P_1^Q = \sqrt{\omega^4 - (M_1^2 - \Delta^2)^2} \cdot \frac{2 \omega^2 + (M_1^2 + \Delta^2)^2}{2 \omega} \hspace{1cm} (2.25a)$$

Later (17) Jackson modified his expression using equation (2.16) to include the effect of the resonance propagator and the decay vertex. Comparing his result with the "pole approximation" of Ferrari and Selleri (33), Jackson obtains

$$\frac{d\sigma}{d\sigma_{\text{pole}}} = \frac{(\omega + M_1)^2 + \Delta^2}{(\omega + M_2)^2 - \mu^2} \left( \frac{P_1^Q}{m_2} \right)^{2 \ell} F(\Delta^2)^2$$  \hspace{1cm} (2.25b)

which he proves is the same as the "exact" result of Ferrari and Selleri. This formula has been found to agree fairly well
with experimental data for small $l$.

A way of testing the scalar character of the exchanged particle independently of the behavior of $\Delta^2$ distribution has been proposed by Treiman and Yang (22). It follows from the general structure of the OPE diagram that there should be no correlation between the plane of production of particle $q_3$ and the plane of emission of particles $q_1$ and $q_2$ as can be best visualized in the system where the incident particle is at rest.

There have been attempts to modify the OPE model so as to take into account the effect of competing processes. The cross-section for these non-peripheral processes is larger for larger momentum transfers, i.e., for lower angular momenta. The existence of competing channels reduces the low partial waves below the value given by the OPE model while leaving the higher partial waves essentially unchanged.

In the "absorptive model" (38-42), the competition from other inelastic processes is described by absorption in the initial and final states. The scattering amplitude $T$ predicted by the OPE model is decomposed into partial wave amplitudes $T_j$ using the formalism of Ref. 43. The partial wave amplitudes are then multiplied by absorption factors

$$T_j \rightarrow e^{i\delta_b(j)} T_j e^{i\delta_a(j)}$$

(where $\delta_a$ and $\delta_b$ are phase shifts of elastic scattering in initial and final states respectively) and the series is summed.

$\neq$ The angle between these two planes was shown (14) to be equal to the Jackson parameter $\phi^2$ defined by equations (1.17 - 1.21).
However, the procedure is rather arbitrary. The actual $\delta_a$ and $\delta_b$ are not known. The phase shift $\delta_a$ can be deduced from the experimental cross-section for elastic scattering, but only under assumptions that the elastic scattering in the initial state is an exponential diffraction scattering, completely absorptive for the lowest partial waves and that the spin flip amplitude is zero. The phase shift $\delta_b$ is completely unknown and is usually set equal to $\delta_a$. The procedure rests on uncertain theoretical grounds (see Appendix D). The arguments used to justify the absorptive model do not hold for low angular momenta, i.e. precisely in the cases of interest\(^{42}\).

In the "sharp-cut-off-model"\(^{44}\) it is assumed that the absorption completely suppresses the OPE amplitude at low angular momentum ($J < J_c$) and does not affect the partial waves at higher values of $J$. The parameter $J_c$ is then found by comparing the predicted differential cross-section with the observed one. This device however offers no advantage over the form factor approach.

The argument often made in favour of these models is that the lowest partial wave amplitudes predicted by the OPE model exceed the unitarity limit\(^{39}\):

$$6j \leq \pi(j + \frac{1}{2})\lambda^2$$

However it was shown by Selleri\(^{45}\) that the OPE model with an appropriate form factor is compatible with unitarity, provided we make the same assumptions as for the absorptive model.
Application of the OPE model

Since there is no meaningful way to take into account the effect of the competing processes, the unadorned OPE model will be used.

If the reaction turns out to be dominated by a resonance, then the off-shell amplitude can be calculated and the "exact" expression of Ferrari and Selleri or equivalently equation (2.24) can be used. Otherwise, one can only compute the predictions of OPE in the "pole approximation".

The predictions of the model for the various quantities of interest can be obtained from the equation (2.22) in terms of the total cross-section $\sigma_0(\omega)$ for the process (2.20) at C.M. energy $\omega$, i.e. at incident laboratory momentum

$$k_\mu = \sqrt{\left(\frac{\omega^2 - \mu^2 - m^2_1}{2m_1}\right)^2 - \mu^2}$$ (2.26)

In order to obtain the distribution of the effective mass of particles $q_1$ and $q_2$ the expression

$$\frac{d\sigma}{d\omega} = C_\omega \frac{1}{p^4} \omega^2 \sigma_0(\omega) \int_{\Delta^2_{\min}}^{\Delta^2_{\max}} I(\Delta^2) d\Delta^2$$ (2.27)

can be evaluated for a number of equidistant values of $\omega$ between $\omega_{\min} = m_1 + m_2$ and $\omega_{\max} = W - m_3$. For a given value of $\omega$,

---

# The derivation of the formulas and the notation are explained in detail in the Appendix C.
\( k_1^Q \) can be computed (equation C.12), \( \sigma_o(\omega) \) can be obtained by interpolation from a table of values for the process (2.20) and the integral over \( \Delta^2 \) evaluated. The integration limits \( \Delta^2_{\text{min}} \) and \( \Delta^2_{\text{max}} \) (equations C.26, C.18, C.10) are functions of \( \omega \) and the integrand \( I(\Delta^2) \) is given by equation (C.24).

In order to obtain the distribution of the scattering angle in production C.M. system the expression

\[
\frac{d\sigma}{d\cos \theta_3^*} = C_s \frac{1}{p \omega W} \int_{E_3^*_{\text{min}}}^{E_3^*_{\text{max}}} q_3^Q \omega \sigma_o(\omega) I(\Delta^2) dE_3^* \tag{2.28}
\]

can be computed for a number of equidistant values of \( \cos \theta_3^* \) between -1 and +1. For a given value of \( \cos \theta_3^* \) the integral over \( E_3^* \) from \( E_3^*_{\text{min}} \) to \( E_3^*_{\text{max}} \) (equations C.28) has to be computed. In the integrand, \( q_3^* \) (equation C.18a) is a function of \( E_3^* \); if \( \omega \) is expressed as a function of \( E_3^* \) (using equation C.19), then \( k_1^Q \) (equation C.12) and \( \Delta^2 \) (equations C.17, C.18, C.10) for a given value of \( \cos \theta_3^* \) can also be obtained as functions of \( E_3^* \), and \( \sigma_o(\omega) \) can be found by interpolation from the experimental data.

The total cross-section can be computed, integrating the distribution (2.27) over \( \omega \) with the appropriate constant factor according to

\[
\sigma_A = \frac{1}{8\pi} \frac{G^2}{4\pi} \frac{1}{(W p_1^* )^2} \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} 2k_1^Q \omega^2 \sigma_o(\omega) x \int_{\Delta^2_{\text{min}}}^{\Delta^2_{\text{max}}} I(\Delta^2) d\Delta^2 d\omega \tag{2.29}
\]
If the angular distribution for the process (2.20) is known from previous experiments then one can compute the distribution of the effective mass \( u \) of particles \( q_2 \) and \( q_3 \) for the process (2.19). For this purpose the expression
\[
\frac{d\sigma}{du} = C_u \frac{1}{(W^2)^2} q_2^P \int_{t_2}^{t_2\,_{\text{max}}} I(\Delta^2) \, d\cos \theta_3^P
\]
\[
\int_{-1}^{+1} d\cos \theta_2 \, \int_0^{2\pi} d\phi_3^P \, \frac{\Delta^2}{q_2^Q} \frac{d\sigma_1^Q}{d\cos \theta_1^Q} \frac{d\sigma_0}{d\cos \theta_1^Q} \, dt^2
\]
(2.30)
can be computed for a number of equidistant values of \( u \) between \( u_{\text{min}} = m_2 + m_3 \) and \( u_{\text{max}} = W - m_1 \).

For a given value of \( u \) the triple integration has to be carried out. The integration limits \( t^2_{\text{min}} \) and \( t^2_{\text{max}} \) (equations C.31) are functions of \( u \). In order to perform the integrations the quantities \( \cos \theta_1^Q \), \( \Delta^2 \) and \( \omega^2 \) are to be expressed in terms of \( u^2 \) and of the variables of integration \( t^2 \), \( \cos \theta_3^P \), and \( \phi_3^P \). This can be done for \( \cos \theta_1^Q \) by writing the equation (A.16) in the \( Q \) system, and for \( \Delta^2 \) by writing the equation (A.15) in the \( P \) system. The procedure for \( \omega^2 \) is rather involved and can be found in Ref. 33.

It is also of interest to assume OPE for the process (2.19) and study the virtual (off-the-mass-shell) process (2.20). The distribution of the scattering angle \( \theta_1^Q \) for the virtual
process (2.20) obtained in the present experiment can be compared with the angular distributions obtained for the physical (on-shell) process (2.20) in previous experiments performed at different (fixed) values of C.M. energy $\omega$. One can take for this purpose a weighted average of these on-shell distributions at different $\omega$ (the weight of the distribution at each $\omega$ being the number of events with $K\Sigma$ effective mass in the neighbourhood of that $\omega$).

The constants $c_\omega, c_\beta, c_u$ in the equations (2.27, 2.28 and 2.30) above, can be chosen so that all the distributions are normalised to the total number of events.

If
\[ I(\Delta^2) = \frac{\Delta^2}{(\Delta^2 + \mu^2)^2} \]  
then the integral over $\Delta^2$ in equations (2.27 - 2.29) can be computed exactly according to
\[
\int_{\Delta_{\text{min}}^2}^{\Delta_{\text{max}}^2} I(\Delta^2) d\Delta^2 = \int_{\Delta_{\text{min}}^2}^{\Delta_{\text{max}}^2} \frac{\Delta^2}{(\Delta^2 + \mu^2)^2} d\Delta^2
\]
\[ = \frac{\mu^2}{\Delta_{\text{max}}^2 + \mu^2} - \frac{\mu^2}{\Delta_{\text{min}}^2 + \mu^2} + \log \frac{\Delta_{\text{max}}^2 + \mu^2}{\Delta_{\text{min}}^2 + \mu^2}. \]
2.7 Isospin Considerations for Processes $pp \rightarrow K\Sigma N$

We know that the initial state $(p, p)$ has an isospin $T = 1$ and its third component $T_3 = +1$. Thus the final state containing particles $K, \Sigma$ and $N$ (of isospins $1/2, 1/2$ and $1$ respectively) must also be a state of $T = 1$ and $T_3 = +1$, since the isospin and its third component must be conserved in strong interactions. The isotopic spin states of $K, \Sigma$ and $N$ can be obtained from the isotopic spin states of $K \Sigma$ with the aid of the Clebsch-Gordan coefficients. The isotopic spin state of $K, \Sigma$ and $N$ is not uniquely specified by its total isotopic spin $T$ and its third component $T_3$. The notation $|t; T, T_3\rangle$ will therefore be used where the value $t$ of the isotopic spin of $K \Sigma$ is explicitly indicated. There are two cases to consider:

a) If $K \Sigma$ is in $t = 1/2$ state, we can obtain a state of $T = 1, T_3 = +1$ according to (Eqn. E.3)

$$|\frac{1}{2}; 1, +1\rangle = \sqrt{\frac{2}{3}} \Sigma^+ K^0 p - \sqrt{\frac{1}{3}} \Sigma^0 K^+ p \ .$$

b) If $K \Sigma$ is in $t = 3/2$ state we can obtain a state of $T = 1, T_3 = +1$ according to (Eqn. E.6)

$$|\frac{3}{2}; 1, +1\rangle = +\sqrt{\frac{9}{12}} \Sigma^+ k^+ n - \sqrt{\frac{1}{12}} \Sigma^+ K^0 p - \sqrt{\frac{2}{12}} \Sigma^0 K^+ p \ .$$

Taking linear combinations of these we obtain the particle-states:
\[ |\Sigma^+ K^0 p\rangle = -\sqrt{\frac{1}{12}} \left| \frac{3}{2}, 1, +1 \right\rangle + \sqrt{\frac{2}{3}} \left| \frac{1}{2}, 1, -1 \right\rangle \]
\[ |\Sigma^+ K^+ n\rangle = \frac{3}{\sqrt{12}} \left| \frac{3}{2}, 1, +1 \right\rangle \]
\[ |\Sigma^0 K^+ p\rangle = \frac{\sqrt{2}}{\sqrt{12}} \left| \frac{3}{2}, 1, +1 \right\rangle - \sqrt{\frac{1}{3}} \left| \frac{1}{2}, 1, +1 \right\rangle \]  

Therefore the transition amplitudes to each of these states are, letting
\[ A = \langle 1, 1 | T | \frac{3}{2}; 1, 1 \rangle \]
\[ B = \langle 1, 1 | T | \frac{3}{2}; 1, 1 \rangle \]

\[ \langle pp | T | \Sigma^+ K^0 p \rangle = -\frac{1}{\sqrt{12}} B + \sqrt{\frac{2}{3}} A \]
\[ \langle pp | T | \Sigma^+ K^+ n \rangle = +\frac{3}{\sqrt{12}} B \]
\[ \langle pp | T | \Sigma^0 K^+ p \rangle = -\frac{\sqrt{2}}{\sqrt{12}} B - \sqrt{\frac{1}{3}} A \]

We have therefore
\[ \langle pp | T | \Sigma^+ K^+ n \rangle + \langle pp | T | \Sigma^+ K^0 p \rangle \]
\[ = +\frac{3}{\sqrt{12}} B - \frac{1}{\sqrt{12}} B + \sqrt{\frac{2}{3}} A \]
\[ = \frac{2}{\sqrt{12}} B + \sqrt{\frac{2}{3}} A \]
\[ = -\sqrt{2} \langle pp | T | \Sigma^0 K^+ p \rangle \]  

(2.37)
It follows that the cross-sections for the three processes \( pp \rightarrow \Sigma KN \) must satisfy the triangular inequalities:

\[
\begin{align*}
\sqrt{2} \sqrt{\sigma(\Sigma^0 K^+ p)} &\leq \sqrt{\sigma(\Sigma^+ K^+ n)} + \sqrt{\sigma(\Sigma^+ K^0 p)} \\
\sqrt{\sigma(\Sigma^+ K^+ n)} &\leq \sqrt{2} \sqrt{\sigma(\Sigma^0 K^+ p)} + \sqrt{\sigma(\Sigma^+ K^0 p)} \\
\sqrt{\sigma(\Sigma^+ K^0 p)} &\leq \sqrt{2} \sqrt{\sigma(\Sigma^0 K^+ p)} + \sqrt{\sigma(\Sigma^+ K^+ n)}.
\end{align*}
\]

(2.38)
CHAPTER III

ANALYSIS OF THE DATA

3.1 Reaction $pp \rightarrow K \Sigma n$

The data for the reaction, shown in Figs. 14-38, will now be examined.

A. Search for resonances

The Dalitz plot of the effective-mass-squared of $K\Sigma$ vs. that of $\Sigma n$ (Fig. 14) shows an accumulation of events in the band around $M_{K\Sigma}^{2} = (1.85 \text{ BeV})^{2}$. The mass projection of the Dalitz plot, i.e., the $K\Sigma$ effective mass squared distribution (Fig. 15) is clearly not compatible with the phase space prediction (Equation 2.17). The distribution shows a strong enhancement in the region of $(1.85 \text{ BeV})^{2}$. There is a possibility that this enhancement could be identified with the 1.92 BeV resonance $\Delta^{++} (1.48, 49)$ even though the peak appears to be shifted by some 70 MeV from its position observed in $p\pi$ collision experiments. The shift may be due to various causes, such as energy dependence of width, interference effects etc.

In order to investigate this possibility the experimental distribution of $K\Sigma$ effective mass squared was fitted to a linear combination of phase space and resonance, taking into account the energy dependence of the resonance width. The distribution was fitted to a function of the form:

$$F(\omega; \omega_{0}, \Gamma_{0}, a_{1}) = [a_{1} \mathcal{R}(\omega_{0}, \Gamma_{0}) + a_{2}] \frac{d\sigma}{d\omega^{2}}$$

(3.1)
Fig. 14

DALITZ PLOT

\[ p + p \rightarrow n + \Sigma^+ + K^+ \]

316 EVENTS

\[ M^2_{\Sigma N} \text{ (BeV)}^2 \]

\[ M^2_{K\Sigma} \text{ (BeV)}^2 \]
EFFECTIVE MASS SQUARED
OF THE KΣ SYSTEM

\[ p + p \rightarrow K^+ + \Sigma^+ + n \]

316 EVENTS

Fig. 15

23% RESONANCE + 77% PHASE SPACE

PHASE SPACE
where the phase space distribution \( \frac{d\sigma}{d\omega} \) was normalized to the total number of events, the cross-section for production of the resonant state \( R \frac{d\sigma}{d\omega} \) was also normalized to the total number of events and \( a_1 + a_2 = 1 \). The resonant factor \( R \) is given by equation (2.18b) where the energy dependent width \( \Gamma(\omega) \) was expressed in terms of the width \( \Gamma_0 \) at \( \omega = \omega_0 \), according to equation (2.18c), letting \( m_1 = m_\pi \), \( m_2 = m_K \).

An iterative procedure was used to determine the values of the parameters \( \Gamma_0, \omega_0, a_1 \) which minimize

\[
E = \sum_j (F_j \Delta \omega^2 - N_j)^2
\]  

(3.2)

where

\[
F_j = F(\omega_j^2; \Gamma_0, \omega_0, a_1) \quad \text{for} \quad j = 1, \ldots, n \quad (3.3)
\]

are the values of (3.1) at \( n \) equidistant values \( \omega_j^2 \); \( N_j \) is the number of events in the \( j \)th interval of the histogram centered at \( \omega_j^2 \) and \( \Delta \omega^2 \) is the width of each interval. The best fit gave \( \omega_0 = 1.87 \pm 0.04 \) BeV, \( \Gamma_0 = 0.19 \pm 0.29 \) BeV, \( a_1 = 0.23 \pm 0.05 \) and \( a_2 = 0.77 \pm 0.05 \). The fit is shown in Fig. 15.

The standard errors of \( \omega_0, \Gamma_0 \) and \( a_1 \) were computed by assuming \( F \) to be linear in parameters in the neighbourhood of the fitted values and the \( N_j \) to be independent variables with the same standard deviation equal to \( \sqrt{E_{\text{min}}/(n-3)} \).

The shift due to the energy dependence of the width turns out to be small (about 0.025 BeV). However, the standard error
in the estimated value of \( \omega_0 \) is sufficiently large to allow the possibility that the peak in the experimental distribution is due to the \( \Delta (1920) \).

Since the width of the resonance is rather large, the structure of the resonant state may be modified by interaction with the strongly interacting neutron present in the final state. If the width of the resonance is 0.2 BeV, then its mean decay time is \( t = \frac{\hbar}{\Gamma} \) sec. in the rest frame of the resonance or \( t^* = \frac{\hbar}{\Gamma} \gamma_\Delta \) in the C.M. frame. During this time the resonance travels a distance \( t^+ c \beta_\Delta \) and the neutron a distance \( t^* c \beta_n \). The "average" value of \( \gamma_\Delta (\beta_n + \beta_\Delta) \) was calculated to be 1.1. Therefore the distance between the neutron and the resonance is of the order of \( c \frac{\hbar}{\Gamma} \gamma_\Delta (\beta_n + \beta_\Delta) = 1.1 \) F at the time when the resonance decays. This is of the same order as a typical interaction distance \( \frac{\hbar}{m_\pi c} = 1.4 \) F.

Therefore the dynamical interference distorting the isobar can be important. Since the background production is 77% the properties of the resonant state would also be strongly affected by interference with the background. The observed decay angular distributions indicate that interferences may be indeed important. For instance, the decay angular distribution with respect to the \( K \Xi \) direction of motion (Fig. 16) shows a backward forward asymmetry, a feature not possible for a free decay of a pure state through strong (parity conserving) interactions.

There is therefore little hope of verifying the spin and the parity of the resonance from the angular distributions and polarizations of the decay products. The position and the width of the resonance peak in the \( M^2_{K \Xi} \) distribution may also
DECAY ANGULAR DISTRIBUTION
IN REST FRAME OF KS WITH
RESPECT TO KS DIRECTION OF MOTION

\[ p+p \rightarrow K^+ + \Sigma^+ + n \]

316 EVENTS

![Graph showing decay angular distribution distribution](image)

Fig. 16
have been affected by interference effects.

The effective mass distribution for $K^+n$ and $\Sigma^+n$ were also studied (Figs. 17, 37). No evidence was found for resonance production in either case. The $\Sigma^+n$ mass distribution is peaked at about 2.36 BeV/c; however this enhancement appears to be a reflection of $\Sigma^+K^+$ peak. (Removal of the events in the peak of $\Sigma^+K^+$ spectrum (Fig. 26) causes a valley in the $\Sigma^+n$ spectrum as can be seen in Fig. 17.

B. The One Pion Exchange Model

There is evidence that the reaction is predominantly peripheral: the momentum transfer distribution (Fig. 19, 20) and the Chew-Low plot (Fig. 18) show a predominance of events with a low momentum transfer. The most prominent group of events at low $\Delta^2$ which might be expected to correspond to one particle exchange is the group between 1.72 and 2.08 BeV. (The experimental distributions of various physical quantities were therefore plotted separately for this group of events as well as for events with small $\Delta^2$). The angular distribution of $K^+\Sigma^+$ in the production centre of mass (Fig. 21, 22) shows a strong forward-backward peaking. The Treiman-Yang angle distribution (Figs. 23, 24, 25) is consistent with the flat distribution predicted by the one pion exchange model (OPE).

The diagram for OPE is shown in Fig. 28.

At the four particle vertex of this diagram the virtual pion interacts with the "incident" nucleon $p_1$ according to

$\Delta^2 = .8 \text{ (BeV/c)}^2$. 
EFFECTIVE MASS OF $n\Sigma$

\[ p + p \rightarrow n + \Sigma^+ + K^+ \]

316 EVENTS

Events in the peak of $K\Sigma$ spectrum removed

Phase space renormalised to 141 events

Fig 17
A number of experiments have been performed for the physical process (3.4), and there is data available\(^{(1,4,7)}\) for the reaction cross-section and the angular distributions at various (fixed) values of C.M. energy \(\omega\).

The present experiment offers therefore a possibility of comparing the angular distribution for the virtual \(p\pi^+\) process (i.e. the distribution of the Jackson parameter \(\cos \Theta\))^\# with the angular distribution for the physical \(p\pi^+\) process. The two distributions differ significantly as can be seen from Fig. 23. This may be due to the off-mass-shell effect, since the angular distribution for the physical process (3.4) corresponds to 
\[
\Delta^2 = -\mu^2,
\]
while in the virtual process \(\cos \Theta\) depends very strongly on \(\Delta^2\) even for small \(\Delta^2\)^\#.\n
It is of interest to compare the data for reaction (1.2) with the predictions of the OPE model. The computations were made using the pole approximation of Ferrari and Selleri. The outgoing particles \(1, 2, 3\), in the OPE diagram (Fig. 12) were taken to be \(\Sigma, K, n\) respectively and the intermediate particle to be a pion.

The angular distribution in the production centre of mass

---

\# The \(K\Sigma\) effective mass and the Jackson polar angle (the scattering angle of the outgoing \(\Sigma\) or \(K\) with respect to the incident proton in the rest frame of \(K\Sigma\)) in the pp process (1.2) correspond respectively to C.M. energy and C.M. scattering angle in the \(p\pi^+\) process (3.4).

\#\# The angular distribution for the physical process (3.4) was obtained by taking a weighted average of the distributions at different \(\omega\) (see pp. 64-65). The computations were performed for the "mass-cut" only, as there is no data available for \(\omega > 2.08\) BeV. The broken line in Fig. 23 shows the resulting distribution.

\[ p + \pi^+ \rightarrow K^+ + \Sigma^+ \]  (3.4)
Fig. 18

CHEW-LOW PLOT
$p+p \rightarrow n + \Sigma^{+} + K^{+}$
316 EVENTS

$- t (\text{BEV/C})^2$

$M_{K\Sigma}^2 (\text{BEV})^2$
MOMENTUM TRANSFER TO $n$

$p + p \rightarrow n + \Sigma^+ + K^+$

$1.72 \leq M_{K\Sigma} \leq 2.08$

228 EVENTS

Fig. 19
MOMENTUM TRANSFER TO n

\[ p + p \rightarrow n + \Sigma^+ + K^+ \]

316 EVENTS

Fig. 20
$\cos \theta$ IN PRODUCTION CENTER OF MASS

$\mathbf{p} + \mathbf{p} \rightarrow \mathbf{n} + \Sigma^+ + \mathbf{K}^+$

$1.72 \leq M_{K\Sigma} \leq 2.08$

228 EVENTS
COS $\theta$ IN PRODUCTION
CENTER OF MASS

$p + p \rightarrow n + \Sigma^+ + K^+$

316 EVENTS

NO. OF EVENTS

COS $\theta^*$ (C.M.)

WITH FORM FACTOR

ONE

ONE
JACKSON'S PARAMETERS (IN REST FRAME OF $K\Sigma$)

$p + p \rightarrow n + \Sigma^+ + K^+$

$1.72 \leq M_{K\Sigma} \leq 2.08$

228 EVENTS

Fig. 23
JACKSON'S PARAMETERS IN REST FRAME OF $K\Sigma$

$p+p \rightarrow n + \Sigma^+ + K^+$

316 EVENTS

Fig. 24
JACKSON'S PARAMETERS IN REST FRAME OF KΣ

\[ p + p \rightarrow n + \Sigma^+ + K^+ \]

\[ \Delta^2 \leq 0.8 \]

201 EVENTS

---

Fig. 25
predicted by the model was computed according to equation (2.28). The obtained distribution (folded about \( \cos \theta^* = 0 \)) is shown in Figs. 21, 22. It is in qualitative agreement with the experimental distribution but is less sharply peaked.

The effective mass distribution predicted by the model was computed according to equation (2.27) and is shown in Figs. 26, 27. The peak is caused entirely by the peak in the \( px^+ \rightarrow \Sigma^+ K^+ \) cross-section (Fig. 28) corresponding to the 1.92 BeV resonance. Although the shape of our experimental distribution is very similar to the one predicted by the model, the positions of the peaks in the two distributions do not coincide.

The total cross-section predicted by the model can be computed according to equation (2.29), letting \( g^2/4\pi = 1.45 \times 2 \). If both graphs A and B (Fig. 28) contribute the obtained total cross-section is 419 \( \mu \)b which differs almost by a factor of 9 from the experimental value.

Of course, the pole approximation formula is strictly valid for \( \Delta^2 = -m_{\pi}^2 \), and should be used for small \( \Delta^2 \) only. In order to take into account the off-mass-shell effects, Jackson's correction factor (equations (2.25)), which assumes one partial wave \( l \) to be dominant, was tried. This correction (for \( l = 3 \))

Since the errors in the experimental cross-sections for \( px^+ \) between 1.4 and 1.8 BeV/c are large, the predictions of theoretical analysis which includes a \( \Delta \) (1920) resonant term (model V of Ref. 1) were used. These are in reasonable agreement with the experimental data.
EFFECTIVE MASS OF $K\Sigma$

$p + p \rightarrow n + \Sigma^+ + K^+$

316 EVENTS

NO. OF EVENTS
EFFECTIVE MASS OF $\kappa\Sigma$

$p+p \rightarrow n+\Sigma^++K^+$

$\Delta^2 \leq 0.8$

201 EVENTS

Fig. 27
CROSS SECTION FOR 
\[ p + \pi^+ \rightarrow \Sigma^+ + K^+ \]
AS A FUNCTION OF \( P_\pi \) (LAB.)

\[ \sigma_T \text{(mb)} \]

\[ P_\pi \text{(BeV/C)} \]

Fig. 28
COS $\theta$ IN PRODUCTION CENTER OF MASS

$p + p \rightarrow n + \Sigma^+ + K^+$
NON AMBIGUOUS EVENTS
200 EVENTS

Fig. 29
JACKSON'S PARAMETERS IN REST FRAME OF $K\Sigma$

$p + p \rightarrow n + \Sigma^+ + K^+$

NON AMBIGUOUS EVENTS

200 EVENTS

Fig. 30
EFFECTIVE MASS OF $K\Sigma$

$p+p \rightarrow n + \Sigma^+ + K^+$

NON AMBIGUOUS EVENTS

200 EVENTS

Fig. 31
COS $\theta$ IN PRODUCTION CENTER OF MASS

$\rho + \rho \rightarrow n + \Sigma^+ + K^+$

$\Sigma^+ \rightarrow \pi^+ + n$

214 EVENTS

Fig. 32
JACKSON'S PARAMETERS IN REST FRAME OF KΣ

\[ p + p \rightarrow n + \Sigma^+ + K^+ \]

\[ \Sigma^+ \rightarrow \pi^+ + n \]

214 EVENTS

Fig. 33
EFFECTIVE MASS OF $K\Sigma$

\[ p + p \rightarrow n + \Sigma^+ + K^+ \]

\[ \Sigma^+ \rightarrow \pi^+ + n \]

214 EVENTS

Fig. 34
did not significantly change the production angular distribution predicted by \( \Omega \Sigma \) model. The production angular distribution predicted by the model can be brought into better agreement with the experiment by multiplying the pole approximation matrix element by a suitable form factor \( F(\Delta^2) \). Then the expression (2.31) is replaced by

\[
I(\Delta^2) = F^2(\Delta^2) \frac{\Delta^2}{(\Delta^2 + \mu^2)^2}
\]  

(3.5a)

\[
\frac{d\sigma}{d\omega^2 d\Delta^2} = \frac{d\sigma}{d\omega^2 d\Delta^2}^{\text{pole}} F^2(\Delta^2)
\]

Using a form factor of the form

\[
F(\Delta^2) = \frac{a^2 - \mu^2}{a^2 + \Delta^2}
\]  

(3.5b)

with \( a^2 = 40 \mu^2 \), and integrating equations (2.28) and (2.29) numerically over \( \Delta^2 \) one obtains a production angular distribution (Fig. 22) which agrees fairly well with the experimental distribution, and the total cross-section \( \sigma = \sigma_A + \sigma_B \) which is twice the experimental value. The remaining discrepancy between our \( \Omega \Sigma \) computations and our experimental results is not surprising in view of the following approximations made in these computations:
1. It was assumed that all the off-mass-shell effects can be taken into account by multiplying the O\(\Sigma\)E matrix element by an empirical function of \(\Delta^2\) (while it is plausible to assume that the renormalization effects can be taken into account in this manner, this is not so for the correction to the matrix element for the four particle vertex. In the special cases for which the theory has been developed, the matrix element for the virtual process differs from the matrix element for the physical process by a function of \(\Delta^2\) and \(\omega\)).

2. The contributions due to graphs other than O\(\Sigma\)E graphs A and B of Fig. 28 (like K-exchange, etc.) have been neglected.

3. All interferences between graphs have been neglected (including the interference between graphs A and B).

4. The final state interactions were assumed to be negligible.

C. The One K-meson Exchange Model

It is also of interest to investigate the possibility that the reaction may be dominated by one K-exchange (OKE). The diagram for such a process is shown in Fig. 35.

The experimental distributions of the momentum transfer to \(\Sigma^+\) (Fig. 36) appears to be less peripheral than the distribution of momentum transfer to \(n\).

In order to compute the predictions of OKE, the data for the process

\[ K^0 + p \rightarrow K^+ + n \]  \hspace{1cm} (3.6)

are required. However, from charge independence considerations,
Fig. 35

OKE DIAGRAM

K$^+$ LABORATORY MOMENTUM (GeV/C)

CROSS SECTION (mb)

REVISED VALUES

$K^+ n \rightarrow K^0 p$
MOMENTUM TRANSFER TO $\Sigma$

316 EVENTS

$p + p \rightarrow n + \Sigma^+ + K^+$

Fig. 36
the amplitude for the reaction (3.6) can be replaced by the amplitude for the inverse reaction

\[ K^+ + n \rightarrow K^0 + p \]  

(3.7)

for which some experimental data are available \(^{50,51}\). \(^\dagger\)

The Kn effective mass distribution predicted by OKE was computed in the pole approximation using expression (2.27) where particles 1, 2, 3 were taken to be n, K and \( \Sigma \), respectively, and \( \delta_0(\omega) \) was taken to be the cross-section \(^{50}\) for the process (3.7). The obtained distribution, along with the phase-space prediction, is shown in Fig. 37. It would appear that the phase-space distribution is in better agreement with the histogram, than is the OKE.

The total cross-section predicted by OKE was computed according to equations (2.29) letting \( G^2/4\pi = 1.5 \times 2 \). If both graphs A and B (Fig. 35) contribute the value obtained is 180 \( \mu b \), which is again very much larger than the experimental value.

The experimental distributions of Jackson's parameters \( \cos \theta^Q \) (i.e., the angular distributions for the outgoing K or n with respect to the incident proton in the rest frame of Kn) and \( \beta^Q \) are shown in Fig. 36. The distribution of azimuth angle \( \beta^Q \) is flat, as expected for a scalar exchange.

There is no data available for the angular distributions for

---

\(^\dagger\) In a personal communication, Prof. Goldhaber supplied the revised values for some of the cross-sections of Ref. (50) based on later data. These revised values (see Fig. 35) were used in the calculations. Actually all the cross-sections in Ref. 50 and 51 are given for the process \( K + d \rightarrow K^0 + p + p \). These differ from the cross-sections for process (3.7) by about 10\%\( /o \). This difference was ignored in our calculations.
EFFECTIVE MASS OF $K^+n$

$$p + p \rightarrow \Sigma^+ + K^+ + n$$

316 EVENTS

Fig. 37
JACKSON'S PARAMETERS IN REST FRAME OF Kn

\[ p+p \rightarrow \Sigma^+ + K^+ + n \]

316 EVENTS

**Fig. 38**
the process (3.7) at energies higher than $\omega = 1.7$ BeV. Therefore
the distribution of $\cos \theta^Q$ (i.e., the angular distribution for
the virtual $K^0 p$ process) cannot be compared with the
angular distribution for the physical process (3.7). However,
from the general trend of the angular distributions for the
process (3.7) at low energies it is unlikely that these distri-
butions would display characteristics similar to those of $\cos \theta^Q$
distribution for the $pp$ process (1.2).

In conclusion, therefore, the OKE model does not offer any
improvement over the $O\Pi E$ model. This conclusion is also
supported by comparison of the predictions of the two models for
the $Kn$ mass distribution. The prediction of the $O\Pi E$ for this
distribution was computed according to equation (2.63), and is
shown along with the OKE prediction in Fig. 37. The $O\Pi E$
prediction (which is very similar to the phase-space distribution)
is in better agreement with the histogram than is the OKE prediction.

3.2 Reaction $pp \rightarrow K^0 \Sigma^+ p$

The data for the reaction are shown in Figs. 39-45. The
production angular distribution (Fig. 42) indicates that the process
may be dominated by one-pion exchange. The effective mass distri-
butions (Figs. 39, 40, 41) show no evidence of resonance production.
However the statistics are unfortunately too low for any definite
conclusion to be drawn.
EFFECTIVE MASS OF $K\Sigma$

$p + p \rightarrow p + \Sigma^+ + K^0$

131 EVENTS

Fig. 39
EFFECTIVE MASS OF $\Sigma p$

$\rho + \rho \rightarrow \Sigma^+ + K^0 + p$

131 EVENTS

Fig. 40
EFFECTIVE MASS OF $K^0_p$

$p + p \rightarrow \Sigma^+ + K^0 + p$

131 EVENTS

Fig. 41
COS $\theta$ IN PRODUCTION CENTER OF MASS

$p + p \rightarrow p + \Sigma^+ + K^0$

131 EVENTS

Fig. 42
JACKSON'S PARAMETERS (IN REST FRAME OF $K\Sigma$)

$p + p \rightarrow p + \Sigma^+ + K$

131 EVENTS

![Histograms showing the distribution of Jackson's parameters](image)

Fig. 43
Fig. 44

**DALITZ PLOT**

\[ p + p \rightarrow p + \Sigma^+ K^0 \]

131 EVENTS

\[ M_{\Sigma K}^2 (\text{BeV})^2 \]

\[ M_{p \Sigma}^2 (\text{BeV})^2 \]
CHEW - LOW PLOT
\( p + p \rightarrow p + \Sigma^+ + K^0 \)
131 EVENTS

Fig. 45
3.3 A Check on Total Cross-Sections for Processes $pp \rightarrow \Sigma KN$

The experimental cross-sections obtained for the processes (1.1), (1.2) in this experiment and the cross-section for the process $pp \rightarrow \Sigma^0 K^+ p$ obtained in Ref. 8 satisfy the triangular inequalities (2.38) derived from charge independence.

3.4 Conclusion

1) The cross-sections for the reactions $pp \rightarrow \Sigma^+ K^0 p$ and $pp \rightarrow \Sigma^+ K^+ n$ were measured to be $28.0 \pm 3.2 \mu b$ and $55.5 \pm 4.6 \mu b$ respectively.

2) No evidence was found for resonance production in the KN and $N \Sigma$ mass spectra.

3) a) The $K \Sigma$ effective mass spectrum in reaction $pp \rightarrow \Sigma^+ K^+ n$ has an enhancement at 1.86 BeV/c which is most probably due to the $\Delta (1920)$ resonance previously observed in $p \pi$ collisions. A least square fit of the data to a resonance plus phase space gives a value of $1.87 \pm 0.04$ BeV for the mass of the resonance and $0.19 \pm 0.29$ BeV for its width.

b) There is evidence that one pion exchange plays an important role in the reaction. The production angular distribution shows a strong forward-backward peaking characteristic of the one pion exchange. The Treiman-Yang angle is consistent with the flat distribution predicted by the model. However the pole approximation calculation is inadequate. A strong form
factor is needed to improve the agreement between the quantitative predictions of the model and the experimental results for the production angular distribution and the total cross-section.

The shape of the $K\Sigma$ effective mass distribution is similar to the one predicted by the model. However the positions of the peaks in the two distributions do not coincide. The distribution for the scattering angle in the virtual process $p\pi \rightarrow K\Sigma$ at the 4-particle vertex differs significantly from the distribution of the corresponding angle in the physical process.

Whether these disagreements are caused by the off-mass-shell effects, whether they are due to interferences or whether they are mainly caused by contributions from other processes besides one pion exchange can not be established until more sophisticated theoretical models are available.

(a) The OKE model offers no improvement over the one pion exchange model.
APPENDIX A

KINEMATICS

Lorentz Transformation of the Energy Momentum Four-Vector

The Lorentz transformation of the energy momentum four-vector $\mathbf{p} = (\xi, \mathbf{p})$, $p^2 = -m^2$ from a reference frame $R$ to a reference frame $R'$ moving with a velocity $\beta$ with respect to $R$ is given by

$$\mathbf{p}' = \mathbf{p} + \beta \gamma \left( \frac{\gamma}{\gamma + 1} \mathbf{p} \cdot \mathbf{p} - \xi \right) \quad (A.1)$$

$$\xi' = \gamma (\xi - \mathbf{p} \cdot \mathbf{p}) \quad (A.2)$$

where the Lorentz factor $\gamma$ is given by $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$ and $\mathbf{p} \cdot \mathbf{p}' = \sum_x p_x \beta_x \cdot$. Let $R'$ be the rest frame of a particle or system of particles of total four-momentum $\mathbf{P} = (E, \mathbf{P})$, $\mathbf{P}^2 = -M^2$. Then equations (A.1) and (A.2) become

$$p'_x = p_x + \frac{p_x}{M} \left( \frac{\sum p_x}{E/M} p_x - \xi \right) \quad (A.3)$$

$$\xi' = -\frac{1}{M} \left( \sum p_x p_x - E \xi \right) = -\frac{\mathbf{P} \cdot \mathbf{P}}{M} \quad (A.4)$$

where $\mathbf{P} \cdot \mathbf{P}$ is invariant under Lorentz transformation.

If the dot product of two vectors $\mathbf{p}_i$ and $\mathbf{p}_j$ has to be transformed but the directions of momenta are not needed in the new system, then using the invariance of $\mathbf{p}_i \cdot \mathbf{p}_j$ we have
\[ \mathbf{p}_i' \cdot \mathbf{p}_j' = \mathbf{p}_i \cdot \mathbf{p}_j \]

i.e.

\[ \mathbf{p}_i' \cdot \mathbf{p}_j' = \xi_i' \xi_j' = \mathbf{p}_i \cdot \mathbf{p}_j - \xi_i \xi_j \quad (A.5) \]

thus

\[ \mathbf{p}_i' \cdot \mathbf{p}_j' = \mathbf{p}_i \cdot \mathbf{p}_j + \xi_i' \xi_j' \quad (A.6) \]

where \( \xi_i' \), \( \xi_j' \) can be obtained using equation \((A.4)\). If \( i = j \) then \((A.6)\) reduces to

\[ p'^2 = \xi'{}^2 - m^2 \quad (A.7) \]

Sometimes it is convenient to decompose the momentum \( \mathbf{p} \) into components parallel and perpendicular to the velocity \( \mathbf{\beta} \) of the reference frame \( R' \). Calling \( \Theta \) the angle between \( \mathbf{p} \) and \( \mathbf{\beta} \) and \( \Theta' \) the corresponding quantity in the frame \( R' \), the equations \((A.1)\), \((A.2)\) become

\[ p' \sin \Theta' = p \sin \Theta \quad (A.8) \]

\[ p' \cos \Theta' = \gamma (p \cos \Theta - \beta \xi) \quad (A.9) \]

\[ E' = \gamma (\xi - \beta p \cos \Theta) \quad (A.10) \]
Invariants and Kinematic Limits

Invariants

Any kinematical quantity concerning the five particles entering the reaction

\[ p_1 + p_2 = q_1 + q_2 + q_3 \]  \hspace{1cm} (A.11)

can be expressed by means of five independent invariants. Let the masses and the energies of the particles be denoted by \( M_1, M_2 \) and \( \xi_1, \xi_2 \), for the initial particles; \( m_1, m_2, m_3 \) and \( E_1, E_2, E_3 \) for the final particles. The independent invariant necessary to describe the reaction (A.11) can be chosen as

\[ w^2 = - (p_1 + p_2)^2 = 2 \xi_1 \xi_2 - 2 \vec{p}_1 \cdot \vec{p}_2 + M_1^2 + M_2^2 \]  \hspace{1cm} (A.12)

\[ \omega^2 = - (q_1 + q_2)^2 = 2E_1E_2 - 2 \vec{q}_1 \cdot \vec{q}_2 + m_1^2 + m_2^2 \]  \hspace{1cm} (A.13)

\[ u^2 = - (q_2 + q_3)^2 = 2E_2E_3 - 2 \vec{q}_2 \cdot \vec{q}_3 + m_2^2 + m_3^2 \]  \hspace{1cm} (A.14)

\[ \Delta^2 = (q_3 - p_2)^2 = 2 \xi_2 E_3 - 2 \vec{p}_2 \cdot \vec{q}_3 - M_2^2 - m_3^2 \]  \hspace{1cm} (A.15)

\[ t^2 = (q_1 - p_1)^2 = 2 \xi_1 E_1 - 2 \vec{p}_1 \cdot \vec{q}_1 - M_1^2 - m_1^2 \]  \hspace{1cm} (A.16)

Remark

We have \( (q_3 - p_2)^2 = (q_{12} - p_1)^2 \) \hspace{1cm} (A.17)

where \( q_{12} = q_1 + q_2 \).
Kinematic limits

Given the C.M. energy $W$ for the reaction (A.11) the physical ranges for different kinematical quantities can be obtained by imposing two conditions:

a) The energy of each particle must be greater than its mass (so that the 3-momentum of each particle is real).

b) The cosine of the angle between the 3-momenta of any two particles must lie in the interval $(-1, 1)$.

In the following discussion subscripts $a$ and $b$ refer to initial particles and can each have values 1 or 2; $i, j, k$ refer to final particles and can each have values 1, 2 or 3. (If $a$ and $b$ both occur in an equation they must not have the same value; likewise no two of the indices $i, j, k$ can have the same value within any single equation). The physical ranges for various kinematical quantities are determined by the following limits:

1) Limits on the effective mass of two particles $q_i$ and $q_j$

$$M_{ij}^{\text{max}} = W - m_k \quad (A.18a)$$

$$M_{ij}^{\text{min}} = m_i + m_j \quad (A.18b)$$
2) Limits on energy in the C.M. system of $p_1, p_2$

\[ E_{k \text{ min}}^* = m_k \]  \hspace{1cm} (A.19a)

\[ E_{k \text{ max}}^* = \frac{W^2 - m_k^2 - m_{ij \text{ min}}^2}{2W} \]  \hspace{1cm} (A.19b)

3) Limits on the square of momentum transfer

We have

\[ (p_a - q_1)^2 = 2(\xi_a E_i - p_a q_1 \cos (\vec{p_a}, \vec{q_1})) - M_a^2 - m_1^2 \]  \hspace{1cm} (A.20)

Imposing the condition

\[ -1 < \cos(\vec{p_a}, \vec{q_1}) < +1 \]  \hspace{1cm} (A.21)

we obtain for $\cos(\vec{p_a}, \vec{q_1}) = -1$

\[ (p_a - q_1)^2_{\text{max}} = 2(\xi_a E_i + p_a q_1) - M_a^2 - m_1^2 \]  \hspace{1cm} (A.22a)

and for $\cos(\vec{p_a}, \vec{q_1}) = +1$

\[ (p_a - q_1)^2_{\text{min}} = 2(\xi_a E_i - p_a q_1) - M_a^2 - m_1^2 \]  \hspace{1cm} (A.22b)

The limits (A.22a) and (A.22b) depend upon $M_{jk}$ since $E_i$ and $q_1$ depend on $M_{jk}$. To obtain absolute limits, note that as $M_{jk}$ decreases $(p_a - q_1)^2_{\text{max}}$ increases while $(p_a - q_1)^2_{\text{min}}$ also decreases. Therefore the absolute limits are obtained by evaluating both (A.22a) and (A.22b) for $(M_{jk})_{\text{min}}$ (equation (A.18b)).
4) Limits on momentum.

In the C.M. system, obviously

\[ q_i^*_{\text{min}} = 0 \quad (A.23a) \]

\[ q_i^*_{\text{max}} = \sqrt{E_{i_{\text{max}}}^* - m_i^2} \quad (A.23b) \]

where \( E_{i_{\text{max}}}^* \) is given by equation \( (A.19b) \).

The transformation to the laboratory system is given by equations \( (A.8), (A.9) \). Letting \( \vec{\beta}_c^* \) be the C.M. velocity and \( \theta_1 \) the angle between \( \vec{\beta}_c^* \) and \( \vec{q}_i^* \), we have therefore

\[ q_1 \sin \theta_1 = q_i^* \sin \theta_i^* \quad (A.24a) \]

\[ q_1 \cos \theta_1 = \gamma_c(q_i^* \cos \theta_i^* + \beta_c E_{i_{\text{max}}}^*) \quad (A.24b) \]

where \( \gamma_c = 1/\sqrt{1 - \beta_c^2} \) and \( 0 \leq \theta_1 \leq \pi \).

Using equations \( (A.23), (A.24) \) we obtain

\[ q_i^*_{\text{max}} = \gamma_c(\beta_c E_{i_{\text{max}}}^* + q_i^*_{\text{max}}) \quad (A.25a) \]

\[ q_i^*_{\text{min}} = \gamma_c(\beta_c E_{i_{\text{max}}}^* - q_i^*_{\text{max}}) \quad (A.25b) \]

if \( (g_i)_m \equiv \beta_c \frac{E_{i_{\text{max}}}^*}{q_i^*_{\text{max}}} > 0 \). If \( (g_i)_m \leq 0 \) then
q_1 can take all values from 0 to q_1 max.

5) Limits on the angle in the laboratory system

Dividing equation (A.24a) by (A.24b) we have

\[
\tan \Theta_1 = \frac{\sin \Theta_1^*}{\gamma_c (\cos \Theta_1^* + \beta_c \frac{E_1^*}{p_1^*})} = \frac{\sin \Theta_1^*}{\gamma_c (\cos \Theta_1^* + g_1)}
\]

(A.26)

where \( g_1 = \beta_c \frac{E_1^*}{q_1^*} \). Taking the derivative of equation (A.26) with respect to \( \Theta_1^* \) we obtain

\[
\frac{d\Theta_1}{d\Theta_1^*} = \gamma_c \left( \frac{q_1^*}{q_1} \right) (1 + g_1 \cos \Theta_1^*)
\]

(A.27)

If \( g_1 < 1 \), then this derivative is always positive. Also from equation (A.26)

\[
\Theta_1 = 0 \quad \text{when} \quad \Theta_1^* = 0
\]

and

\[
\Theta_1 = \pi \quad \text{when} \quad \Theta_1^* = \pi
\]

Therefore \( \Theta_1 \) can take all values from 0 to \( \pi \), as shown in Fig. 45a.

If \( g_1 > 1 \), then from equation (A.27), \( \frac{d\Theta_1}{d\Theta_1^*} \) vanishes when
\[
\cos \theta_1^* = -\frac{1}{g_1}. \quad \text{At this value of } \cos \theta_1^* \quad \text{equation (A.26) gives}
\]

\[
(tan \theta_1)_{\text{max}} = \frac{1}{\gamma_c \sqrt{g_1^2 - 1}} \quad (A.28)
\]

which may be rewritten as

\[
(sin \theta_1)_{\text{max}} = \frac{p_1^*}{m_1} \frac{1}{\gamma_c \beta_c}
\]

\[
= \frac{p_1^*}{m_1} \frac{m^2}{P^2}
\]

We have, besides,

\[
\cos \theta_1 \geq 0 \quad \text{so that } \theta_1 \text{ max } \leq \frac{\pi}{2}.
\]

Also

\[
\theta_1 = 0 \quad \text{when } \theta_1^* = 0
\]

and

\[
\theta_1 = 0 \quad \text{when } \theta_1^* = \pi
\]

as shown in Fig. 45b.

If \( g_1 = 1 \), equation (A.26) gives

\[
(tan \theta_1) = \frac{1}{\gamma_c} \tan \frac{\theta_1^*}{2}
\]
In this case therefore $\theta_1$ increases from 0 to $\pi/2$ when $\theta_1^*$ varies from 0 to $\pi$ as shown in Fig. 45c.

![Fig. 45c](image)

**Fig. 45c.**

**Lorentz Transformation for Time**

Let $\beta$ be the velocity of a particle with respect to a reference frame $R$. The Lorentz transformation for time $t$ from the reference frame $R$ to the rest frame of the particle is given by

$$
\frac{dt_{\text{proper}}}{dt} = \frac{dt^2}{c^2} - \frac{d\ell^2}{c^2} = dt \sqrt{1 - \frac{v^2}{c^2}} \frac{d\ell^2}{dt^2}.
$$

Now $dt = \frac{d\ell}{\beta c}$

thus

$$
\frac{dt_{\text{proper}}}{dt} = dt(1 - \beta^2) = \frac{dt}{\gamma}.
$$

**Remark.**

If a particle $p$ of lifetime $\tau$ is moving with velocity
in the laboratory system, then its probability \( \mathcal{P} \) not to decay can be expressed as function of the distance \( l \) traversed since the particle was produced. We have

\[
\mathcal{P} = e^{-t/\tau}, \quad \text{where} \quad \langle t \rangle = \langle t \rangle \beta c
\]

\[
= \gamma \langle t_{\text{proper}} \rangle \beta c
\]

\[
= \gamma \tau \beta c
\]

\[
= \tau \frac{E_c}{m} . \quad (A.32)
\]

We can write

\[
\mathcal{P} = e^{-tm/(\tau pc)} . \quad (A.33)
\]

**Phase Space Considerations**

**Phase space definition**

The probability of transition per unit time, per unit volume from an initial state \( |i\rangle \) to a three particle final state \( |f\rangle \) belonging to a set \( F \) in which the final state particles have their 4-momenta between \( q_1 \) and \( q_1 + dq_1 \), \( q_2 \) and \( q_2 + dq_2 \), \( q_3 \) and \( q_3 + dq_3 \) is

\[
P_3(i \rightarrow F) = \int_F \frac{N^2 d^4(P - \sum_j q_j)}{|M_{ij}|^2} \frac{1}{T_j} \delta(q_j^2 + m_j^2) \cdot d^4 q_1 \cdot d^4 q_2 \cdot d^4 q_3 \quad (A.34)
\]

or in the centre-of-mass (C.M.) system
\[ P_3(i \rightarrow F) = \int_{\mathcal{F}} N^2 \delta^3(\sum_j q_j) \left| M_{fi} \right|^2 \delta(W - \sum_j E_j) \frac{d^3q_1}{2E_1} \frac{d^3q_2}{2E_2} \frac{d^3q_3}{2E_3} \]  

(A.35)

where

- \( M_{fi} \) is the transition matrix element,
- \( N^2 \) is a factor depending only upon the normalization factors for the initial and final state particles,
- \( P \) is the total 4-momentum,
- \( W \) is the total C.M. energy, and
- \( E_j, m_j \) are the C.M. energy and mass, respectively, for particle \( j \).

The differential invariant phase space in the C.M. system is

\[ d^9 \mathcal{R}_3(w; m_1, m_2, m_3) = \delta^3(\sum_j q_j) \delta(W - \sum_j E_j) \frac{d^3q_1}{2E_1} \frac{d^3q_2}{2E_2} \frac{d^3q_3}{2E_3} \]  

(A.36)

Thus in the approximation that the matrix element \( M_{fi} \) is independent of the final state momenta \( q_1, q_2, q_3 \), the transition probability to any subunit of volume of the phase space within the constraints imposed by the conservation laws, is proportional to that volume.

The total probability for a three-particle final state

\[ P_3 = \int N^2 \delta^4(P - \sum_j q_j) \left| M_{fi} \right|^2 \prod_j \delta(q_j^2 + m_j^2) d^4q_1 d^4q_2 d^4q_3 \]  

(A.37)

is proportional to
\[ R_3(P; m_1, m_2, m_3) = \int \delta^4(P - \sum_j q_j) \prod_j \delta(q_j^2 + m_j^2) \, d^4q_1 \, d^4q_2 \, d^4q_3 \]  

(A.38)

where the integral is taken over all possible final states. The $R_3$ is called the invariant phase space. In the C.M. system we have:

\[ R_3(W; m_1, m_2, m_3) = \int \delta^3(\sum_j q_j)(W - \sum_j E_j) \, \frac{d^3q_1}{2E_1} \, \frac{d^3q_2}{2E_2} \, \frac{d^3q_3}{2E_3} \]  

(A.39)

Similarly

\[ R_2(a; b, c) = \int \delta(a - E_b - E_c) \delta^3(q_b + q_c) \, \frac{d^3q_b}{2E_b} \, \frac{d^3q_c}{2E_c} \]

is the invariant phase space for two particles.

It can be shown\(^{(15)}\) that

\[ R_2(a; b, c) = \pi \frac{q}{a} \]

where $q$ is the momentum of particles $b$ and $c$ in their rest frame and

\[ R_3(P; m_1, m_2, m_3) = \int_0^\infty dM_{12}^2 R_2(P; M_{12}, m_3) R_2(P_{12}; m_1, m_2) \]

\[ = \int_0^\infty dM_{12}^2 R_2(W; M_{12}, m_3) R_2(w, m_1, m_2) \]

\[ = \int_0^\infty dM_{12}^2 \frac{\pi}{W} q_3^* \frac{\pi}{w} q \]  

(A.40)
where \( q \) is the momentum of particles 1 and 2 in their rest frame,
\( W \) is the total C.M. energy,
\( q_3^\ast \) is the C.M. momentum of particle 3,
\( M_{12} \) is the rest mass of the two-particle system made up of particles 1 and 2,
\( \omega \) is the total energy of particles 1 and 2 in their rest frame, and
\( P_{12} \) is the total 4-momentum of particles 1 and 2.

The Effective mass distribution predicted by phase space

The probability that the effective mass squared lies between \( M_{12}^2 \) and \( M_{12}^2 + dM_{12}^2 \) as predicted by pure phase space is \((15)\), from equation (A.40)

\[
P(M_{12})dM_{12}^2 = C_0 \frac{W}{\omega} q_3^\ast \frac{W}{\omega} q dM_{12}^2 \quad (A.41a)
\]

where \( C_0 \) is such that
\[
P(M_{12})dM_{12}^2 = 1.
\]

The differential cross-section \( \frac{d\sigma}{dM_{12}^2} \) is proportional to \( P(M_{12}) \), i.e.

\[
\frac{d\sigma}{dM_{12}^2} = C_\phi \frac{1}{W} q_3^\ast \frac{1}{\omega} q \quad (A.41b)
\]

where \( C_\phi \) is the normalisation constant, and
\[ q = \frac{1}{2\omega} \sqrt{\omega^2 - 2\omega^2 (m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2} \].

The Dalitz plot

A Dalitz plot for three-particle final states is a scatter plot of C.M. energies (or C.M. kinetic energies) of two of the final state particles. Each point corresponds to a particular event and must lie within a boundary curve which represents the constraints imposed by the conservation laws. (At the boundary the three moments are collinear).

The differential phase space \( d^9R \) depends on nine variables whose range of integration is restricted by four equations:

\[ (q_1)_x + (q_2)_x + (q_3)_x = 0 \quad x = 1, 2, 3, \]

\[ E_1 + E_2 + E_3 = W. \]

Thus it depends on five variables. One can compute the distribution function \( P(E_1, E_2) \) of any two of the energies, say \( E_1 \) and \( E_2 \), by suitable integration over \( d^9R \). We obtain

\[ d^2R = \pi^2 dE_1 dE_2. \]

Thus unit area \( dE_1 dE_2 \) on the Dalitz plot is proportional to unit volume in phase space. The probability of obtaining a

\[ \\]

Occasionally it is more convenient to plot \( M_{23} \) vs. \( M_{13} \)

(instead of \( E_1 \) vs. \( E_2 \)).
final state $|f\rangle$ belonging to a set $\mathcal{F}$ with energies between $E_1$ and $E_1 + dE_1$, $E_2$ and $E_2 + dE_2$ is therefore proportional to

$$|M_{fi}|^2 \, dE_1 \, dE_2 \, .$$

If the invariant matrix element is independent of the final state momenta in the centre-of-mass system (aside from an over-all orientation with respect to the direction of the incoming beam) the density of events plotted on the Dalitz plot is constant and proportional to the square of the matrix element. We have

$$P(E_1, E_2) \, dE_1 \, dE_2 = \text{const.} \, dE_1 \, dE_2 \, .$$

Any clustering of points along a band at some $E_1$ (or $E_2$) is an indication of a resonance or, in general, a preferred mass region for the interaction at the corresponding mass $M_{23}$ (or $M_{13}$).
APPENDIX B

GRID FITTING PROGRAM FOR $\Sigma^+$ - events

(Special procedure for CC fits, at decay vertex)

At the decay vertex when the decaying track is almost straight (as is usually the case) its momentum is considered to be unknown. The three parameters for the neutral track are also unknown to that there are four unknowns at the vertex. These are determined from the momentum-energy conservation equations. Letting $p_2$ be the decaying particle and $p_1, p_3$ the decay products, we have

$$E_2 = E_1 + E_3$$

$$-\vec{p}_2 = \vec{p}_1 + \vec{p}_3$$

Writing these equations as

$$E_1 = E_2 - E_3$$

$$-\vec{p}_1 = \vec{p}_2 + \vec{p}_3$$

and squaring each equation we obtain

$$E_1^2 = E_2^2 + E_3^2 - 2E_2 E_3$$

$$p_1^2 = p_2^2 + p_3^2 + 2p_2 p_3 \cos \theta$$

Subtracting the second equation from the first, we have

$$m_1^2 - m_2^2 - m_3^2 + 2p_2 p_3 \cos \theta = -2E_2 E_3$$
Squaring both sides we get

\[ (m_1^2 - m_2^2 - m_3^2) + 2p_2 p_3 \cos \theta^2 = 4E_2^2 E_3^2 \]

which can be written as a quadratic equation in \( p_2 \):

\[
p_2^2 \left( E_3^2 - p_3^2 \cos^2 \theta \right) + p_2 p_3 \cos \theta \left( m_2^2 + m_3^2 - m_1^2 \right)
\]

\[ + \left[ m_2^2 E_3^2 - \frac{1}{4} \left( m_1^2 - m_2^2 - m_3^2 \right)^2 \right] = 0. \]

This equation has two real roots provided the condition

\[
p_3^2 \cos^2 \theta \left( m_2^2 + m_3^2 + m_1^2 \right)^2
\]

\[
\left( E_3^2 - p_3^2 \cos^2 \theta \right) \left[ 4m_2^2 E_3^2 - \left( m_1^2 - m_2^2 - m_3^2 \right)^2 \right]
\]

is satisfied. The condition can be written as

\[
0 > 4m_2^2 E_3^2 \left( E_3^2 - p_3^2 \cos^2 \theta \right) - \left( m_1^2 - m_2^2 - m_3^2 \right)^2 E_3^2.
\]

Dividing by \( m_2^2 E_3^2 \) we obtain

\[
0 > (E_3^2 - p_3^2 \cos^2 \theta) - \left( \frac{-m_1^2 + m_2^2 + m_3^2}{2m_2} \right).
\]

Using the relations
\[ E_3^2 = p_3^2 + m_3^2 \]
\[ E_3^* = p_3^* + m_3^2 \]
\[ E_3^{**} = \frac{m_2^2 - m_1^2 + m_3^2}{2m_2} \]

the right side of the inequality can be written as

\[ (E_3^2 - p_3^2 \cos^2 \theta) - \left( \frac{-m_1^2 + m_2^2 + m_3^2}{2m_2} \right)^2 \]

\[ = (p_3^2 + m_3^2) - p_3^2 \cos^2 \theta - E_3^{**} \]
\[ = p_3^2 + m_3^2 - p_3^2 \cos^2 \theta - p_3^* \]
\[ = p_3^2(1 - \cos^2 \theta) - p_3^* \]
\[ = p_3^2 \sin^2 \theta - p_3^* \]

The inequality becomes

\[ 0 > p_3^2 \sin^2 \theta - p_3^* \]

thus

\[ |p_3^*| > |p_3 \sin \theta| \]

If this condition is not satisfied, the program uses a least squares procedure to modify the values of the measured
quantities. Actually, the program attempts to reduce
\[ p_T^* = |p_3 \sin \theta| \text{ to } .9 \ p_3^*. \] If this is not possible within
a reasonable number of iterations and a reasonable probability,
then there is no fit for this vertex.

If a satisfactory solution is obtained in the computation
for the decay vertex, both values of the momentum of the decaying
track obtained are used in the computations at the production
vertex. For each of these two values (corrected for energy loss),
the three unknown parameters for the neutral track are first
determined from the momentum conservation equations; the energy
equation is then used as a constraint equation.
The one-particle-exchange diagram for the reaction

\[ P_1 + P_2 = q_1 + q_2 + q_3 \]  \hspace{1cm} (C.1)

is shown in Fig. (47). The process can be described in the following terms. One of the initial particles, \( P_2 \), acts as a "spectator" in the reaction. It simply changes its 4-momentum to \( q_3 \) by emitting a virtual particle of 4-momentum \( k \) and mass \( \mu \). This particle \( k \) interacts with \( P_1 \) according to

\[ k + P_1 = q_1 + q_2 \]  \hspace{1cm} (C.2)

We suppose \( q_2 \) to be a boson, \( P_1, P_2, q_1, q_3 \) to be fermions and the intermediate exchange particle \( k \) to be a spinless boson. The masses and energies of particles occurring in reaction (C.1) will be denoted by: \( m_1, m_2 \) and \( \xi_1, \xi_2 \) for the initial particles; \( m_1, m_2, m_3 \) and \( E_1, E_2, E_3 \) for the final particles.

Defining an invariant matrix element \( M_{f_1} \) in terms of the S-matrix element \( S_{f_1} \) by
the differential cross-section can be written as

$$d\sigma = \frac{1}{(2\pi)^5} \frac{W}{4p_1^*} \delta^4(P_f - P_1) \frac{m_1}{E_1} \frac{m_2}{E_2} \frac{1}{2E_2} \frac{m_3}{E_3} |M_{f1}|^2$$

(C.3b)

where \(\sum\) indicates a sum over final and average over initial spin states. Here \(W\) is the C.M. energy and the asterisk designates momenta (and energies) in the C.M. system.

The S-matrix element for the graph of Fig. 47 is given by

$$s_{f1} = i(2\pi)^8 \delta^4(P_f - P_1) \langle q_3 | j(0) | p_2 \rangle \Delta_F' (\Delta^2)$$

$$\cdot \langle q_1 q_2 | j(0) | p_1 \rangle$$

(C.4)

where \(j\) is the current operator for the virtual intermediate particle, \(\Delta^2 = (p_2 - q_3)^2\) is the four-momentum transfer squared and

$$\Delta_F' = -i \frac{1}{(2\pi)^4} \frac{K'(\Delta^2)}{\Delta^2 + \mu^2}$$

if we assume that the target particle has spin \(\frac{1}{2}\), we have
\[
\langle q_3 \mid j(0) \mid p_2 \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_2}{\xi_2} \frac{m_3}{E_3}} \bar{u}(q_3) \gamma_\mu r\gamma_\nu p_2 K(\Delta^2)
\]

and

\[
\langle q_1 q_2 \mid j(0) \mid p_1 \rangle = (2\pi)^{-9/2} \sqrt{\frac{m_1}{\xi_1} \frac{m_1}{E_1} \frac{1}{2E_2}} M_Q \left(\omega^2, \cos \theta_1^Q, \Delta^2\right).
\]

Here 0 is \( \gamma_5 \) or 1 depending upon the parity of the exchanged particle, \( G_r \) is the rationalised renormalised coupling constant for the vertex \((q_3 p_2 \gamma)\), \( K(\Delta^2) \) is the form factor of the proper vertex part normalised to unity at \( \Delta^2 = -\mu^2 \), \( K'(\Delta^2) \) is the form factor of the propagator (i.e., ratio of the complete propagator to the perturbative propagator) also normalised to unity at \( \Delta^2 = -\mu^2 \), and \( M_Q \) is the analytic continuation, for \((q_1 + q_2 - p_1)^2 > 0\), of the matrix element \( M_o \) for the physical process (C.2). The matrix element \( M_o \) is defined in terms of the S-matrix element \( S_o \) by

\[
S_o = i(2\pi)^4 \delta^4(p_1 + k - q_1 - q_2) \times \\
\sqrt{\frac{1}{(2\pi)^2} \frac{m_1}{\xi_1} \frac{1}{2E_\mu} \frac{1}{2E_2} \frac{m_1}{E_1} M_0(\omega, \cos \theta_1^*)}
\]

so that the cross-section for the process (C.2) can be written in terms of \( M_o \) as

\[
d\sigma_o(\omega^2, \cos \theta_1^*) = \frac{M_o M_1}{4\omega^2} \frac{1}{(2\pi)^2} \frac{\Phi_\omega^{\ast}}{\Phi_{\omega}^{\ast}} \sum_{\text{str}} M_0(\omega, \cos \theta_1^*)^2 d\Omega
\]
where \( \omega \) is the C.M. energy, \( p_{0}^{*} \) is the magnitude of the initial C.M. momentum, \( q_{0}^{*} \) is the magnitude of the final C.M. momentum and \( \theta_{0}^{*} \) is the C.M. scattering angle.

The cross-section for the virtual process (C.2) can be written by analogy with equation (C.6) as

\[
\frac{d\sigma_{Q}}{d\Omega} (\omega, \cos \theta_{1}^{Q}, \Delta^{2}) = \frac{M_{1}m_{1}}{4\omega^{2}} \frac{1}{(2\pi)^{2}} \frac{a_{Q}}{p_{1}^{Q}} \sum |M_{Q}(\omega^{2}, \cos \theta_{1}^{Q}, \Delta^{2})|^{2}
\]

(C.7)

where \( \omega \) is the total energy of particles \( q_{1} \) and \( q_{2} \) in their rest frame and subscript \( Q \) designates momenta and energies in this frame. Here \( p_{1}^{Q} \) is given by

\[
p_{1}^{Q} = \frac{\sqrt{\omega^{4} - (M_{1}^{2} - \Delta^{2})^{2} + 2\omega^{2}(M_{1}^{2} + \Delta^{2})^{2}}}{2\omega}
\]

Using the expression (C.4) for the S-matrix element, one can write the M-matrix element (defined by equation (C.5a)) as

\[
M_{f1} = [\bar{u} (q_{3}) \circ G_{r} u (p_{2}) K(\Delta^{2})] \frac{K'(\Delta^{2})}{\Delta^{2} + \mu^{2}} \times M_{Q}(\omega^{2}, \cos \theta_{1}^{Q}, \Delta^{2})
\]

(C.8)

and therefore the cross-section (equation C.3b) as
\[
\sigma = \int \frac{1}{(2\pi)^4} \frac{M_1 m_1}{4} \frac{1}{W \rho^*} \frac{G_F^2}{4\pi} \left[ \Delta^2 + (m_3 - M_2)^2 \right] K^2(\Delta^2) \frac{K'_2(\Delta^2)}{\left(\Delta^2 + \mu^2\right)^2}
\]

\[
(\Delta^2 + \mu^2)^2
\]

\[
L + \mu
\]

\[
J^4
\]

\[
(m_3 - M_2)^2 \left(\Delta^2 + \mu^2\right)^2
\]

\[
E_1, E_2, E_3
\]

where for identical initial particles

\[
\xi_1 = \xi_2 = \frac{W}{2}
\]

and

\[
\rho^* = \rho_1^* = \rho_2^* = \sqrt{\xi_1^2 - M_1^2}
\]

The pole approximation

For small \( \Delta^2 \) the cross-section can be computed in the pole approximation. Following Ferrari and Selleri we take

\[
K(-\mu^2) = 1
\]

\[
K'(\mu^2) = 1
\]

\[
\lim_{\Delta^2 \to -\mu^2} \rho_1^Q = \rho^Q = \frac{\sqrt{\omega^4 - (M_1^2 + \mu^2)2\omega^2 + (M_1^2 - \mu^2)^2}}{2\omega}
\]

For small \( \Delta^2 \), \( M_Q \) is \( \Delta^2 \) dependent only through \( \cos \theta_{1Q} \) and one can write the differential cross-section \((\Delta^7)\) as
\frac{d\sigma}{d\omega} (w^2, \cos \theta_1^Q) = \frac{m_1 m_1}{4w^2} \frac{1}{(2\pi)^2} \frac{\alpha}{k^2} |M_Q(w^2, \cos \theta_1^Q)|^2 \quad \text{(C.13)}

where the $\Delta^2$ dependence for $\cos \theta_1^Q$ is retained.

The total cross-section can then be written as

$$\sigma_Q(w) = 2\pi \int_{-1}^{1} \frac{d\sigma}{d\cos \theta_1^Q} d\cos \theta_1^Q \quad \text{(C.14a)}$$

which is $\Delta^2$ independent.

It is assumed that

$$\frac{d\sigma}{d\omega} (w^2, \cos \theta_1^Q) = \frac{d\sigma}{d\omega} (w^2, \cos \theta_0^Q) \quad \text{(C.14b)}$$

thus

$$\sigma_Q(w) = \sigma_0(w) \quad .$$

With these approximations the equation (C.9) becomes

$$d\sigma = \frac{1}{(2\pi)^2} \frac{\alpha^2}{4\pi} \frac{1}{\omega \mu^2} \frac{\Delta^2 + (m_3 + m_2)^2}{(\Delta^2 + \mu^2)^2} \ \frac{\omega^2 k^2}{q^2} \frac{d\sigma}{d\omega} (w^2, \cos \theta_1^Q) \quad \text{(C.15)}$$

$$\delta^4(P_f - P_1) \ \frac{d^3q_1}{E_1} \ \frac{d^3q_2}{E_2} \ \frac{d^3q_3}{E_3} \quad .$$

All the distributions of interest can be obtained from (C.15) by suitable integrations.
In order to obtain \( \frac{d^2 \sigma}{d \Delta^2 d \omega^2} \), it is convenient to express
\[
\frac{d^3 q_3}{E_3} \quad \text{in the C.M. system and} \quad \frac{d^3 q_1}{E_1}, \quad \frac{d^3 q_2}{E_2} \quad \text{in the rest frame}
\]
of \( q_1 \) and \( q_2 \) (the Q system) by the relations

\[
\begin{align*}
\frac{d^3 q_3}{E_3} &= q_3^* dE_3^* d \cos \theta_3^* d\phi_3^* \quad \text{(C.16a)} \\
\frac{d^3 q_1}{E_1} &= q_1^Q dE_1^Q d \cos \theta_1^Q d\phi_1^Q \quad \text{(C.16b)} \\
\frac{d^3 q_2}{E_2} &= q_2^Q dE_2^Q d \cos \theta_2^Q d\phi_2^Q \quad \text{(C.16c)}
\end{align*}
\]

We have
\[
\Delta^2 = \frac{1}{2} \xi_2 E_3 + 2p_2 q_3 \cos \theta_3 - m_2^2 - m_3^2 \quad \text{(C.17)}
\]

In the C.M. system, \( p_2^*, \xi_2^* \) are given by equations (C.10), 
\( q_3^* \) by
\[
q_3^* = \sqrt{\frac{2}{E_3^*} - m_3^2} \quad \text{(C.18a)}
\]
and \( E_3^* \) by
\[
E_3^* = \frac{w^2 + m_3^2 - \omega^2}{2W} \quad \text{(C.18b)}
\]

which can also be written as
\[ \omega^2 = W^2 - 2E_3^* W + m_3^2 \]  \hspace{1cm} (C.19)

Therefore

\[
\begin{vmatrix}
\frac{\partial \omega^2}{\partial \cos \theta_3^*} & \frac{\partial \omega^2}{\partial E_3^*} \\
\frac{\partial \Delta^2}{\partial \cos \theta_3^*} & \frac{\partial \Delta^2}{\partial E_3^*}
\end{vmatrix}
= 4W^*_p a_3
\]

so that

\[ q_3^* dE_3^* d \cos \theta_3^* = \frac{d \omega^2 d \Delta^2}{4W^*_p} \]  \hspace{1cm} (C.20)

Using equations (C.16) and (C.20) the expression (C.15) becomes

\[
\begin{align*}
d\sigma &= \frac{1}{(2\pi)^2} \frac{q^2}{4 \pi (W^*_p)} \frac{1}{\Delta^2 + (m_3^2 + m_2^2)^2} \omega^2 \frac{k_1^Q}{q_2^Q} x \\
&\quad \times \frac{d \sigma (\omega^2, \cos \theta_1^Q)}{d \varphi} \left( \frac{P_2 - P_1}{q_1^Q} \right) q_1^Q dE_1^Q d \cos \theta_1^Q d \phi_1^Q x \\
&\quad x q_2^Q dE_2^Q d \cos \theta_2^Q d \phi_2^Q \frac{d \omega^2 d \Delta^2}{4(W^*_p)} d \phi_3^* \\
\end{align*}
\]
Integration over the $\delta$-function gives

$$d\sigma = \frac{1}{(2\pi)^2} \frac{G_r^2}{4\pi} \frac{1}{4(wp^x)^2} \frac{\Delta^2 + (m_3 \pm M_2)^2}{(\Delta^2 + \mu^2)^2} \omega^2 \frac{k_1^Q}{q_2} \times$$

$$\frac{d\sigma_o}{d\omega} (\omega^2, \cos \theta_1^Q) \left[ \frac{q_2}{\omega} \frac{d\cos \theta_1^Q}{d\phi_1} \right] d\Delta^2 d\omega^2 d\phi_3 .$$

(C.21)

The integration over $d\phi_3$ gives

$$d\delta = \frac{1}{8\pi} \frac{G_r^2}{4\pi} \frac{1}{(wp^x)^2} \frac{\Delta^2 + (m_3 \pm M_2)^2}{(\Delta^2 + \mu^2)^2} k_1^Q \omega \frac{d\sigma_o(\omega^2, \cos \theta_1^Q)}{d\omega} \times$$

$$d\cos \theta_1^Q d\phi_1^Q d\Delta^2 d\omega^2 .$$

(C.22)

The integration over $\cos \theta_1^Q$ and $\phi_1^Q$ using equations (C.14) gives

$$d\delta = \frac{1}{8\pi} \frac{G_r^2}{4\pi} \frac{1}{(wp^x)^2} \frac{\Delta^2 + (m_3 \pm M_2)^2}{(\Delta^2 + \mu^2)^2} \delta_o(\omega) k_1^Q \omega d\Delta^2 d\omega^2$$

which can be written as

$$d\sigma = \frac{1}{8\pi} \frac{G_r^2}{4\pi} \frac{1}{(wp^x)^2} I(\Delta^2) \delta_o(\omega) k_1^Q \omega d\Delta^2 d\omega^2$$

(C.23)

where
\[ I(\Delta^2) = \frac{\Delta^2 + (m_3^+ - m_2^0)^2}{(\Delta^2 + \mu^2)^2} \]  \hspace{1cm} (C.24)

Integrating equation (C.24) over \( \Delta^2 \) we obtain

\[ \frac{d\sigma}{d\omega} = \frac{1}{4\pi} \frac{G_F^2/4\pi}{(wp^*)^2} \sigma_o(\omega) k_1^Q \omega^2 \int_{\Delta^2_{\text{min}}}^{\Delta^2_{\text{max}}} I(\Delta^2) d\Delta^2 \]  \hspace{1cm} (C.25)

where

\[ \Delta^2_{\text{max}} = 2(\xi_2 E_3 + p_2 q_3 ) - M_2^2 - m_3^2 \]  \hspace{1cm} (C.26a)

\[ \Delta^2_{\text{min}} = 2(\xi_2 E_3 - p_2 q_3 ) - M_2^2 - m_3^2 \]  \hspace{1cm} (C.26b)

To obtain the angular distribution in the C.M. system, \( d\Delta^2 d\omega^2 \) in equation (C.22) is expressed in terms of \( dE_3^+ \) and \( d\cos \theta_3^+ \) using equation (C.20), we get

\[ \frac{d\sigma}{d\cos \theta_3^+} = \frac{1}{2\pi} \frac{G_F^2/4\pi}{(wp^*)^2} \int_{E_3^+_{\text{min}}}^{E_3^+_{\text{max}}} q_3^+ \omega \sigma_o(\omega) k_1^Q I(\Delta^2) dE_3^+ \]  \hspace{1cm} (C.27)

where

\[ E_3^+_{\text{min}} = m_3 \]  \hspace{1cm} (C.28a)

\[ E_3^+_{\text{max}} = \frac{W^2 + m_3^2 - \omega^2_{\text{min}}}{2W} \]  \hspace{1cm} (C.28b)

Finally we can derive the effective mass distribution \( \frac{d\sigma}{du^2} \) for the particles \( q_2 \) and \( q_3 \). For this derivation, essentially
the roles of \( q_3 \) and \( q_1 \) have to be interchanged in the above equations (C.15) - (C.21). Starting with equation (C.15) the differential invariants \( \frac{d^3q_1}{E_1} \), \( \frac{d^3q_2}{E_2} \) and \( \frac{d^3q_3}{E_2} \) are again expressed in terms of energies and angles by relations analogous to equations (C.16, C.20). However now \( \frac{d^3q_1}{E_1} \) is expressed in the C.M. system and \( \frac{d^3q_2}{E_2} \), \( \frac{d^3q_3}{E_3} \) in the rest frame of \( q_2 \) and \( q_3 \) (the P system). Analogously to equation (C.21), we get

\[
\frac{d\sigma}{dt} = \frac{1}{(2\pi)^2} \frac{G_r^2}{4\pi} \left[ \frac{1}{4(w_p^*)^2} I(\Delta^2) \right] \frac{\omega^2 k_{1Q}^Q}{q_2^P} \frac{d\sigma_0(\omega^2, \cos \theta_1^Q)}{d\Omega} \]

\[
\times \int d\cos \theta_3^P d\phi_3^P d\phi_1^* dt^2 du^2 \quad (C.29)
\]

This expression must now be integrated over all the variables except \( u^2 \) to get the desired result. Carrying out the integration over \( \phi_1^* \) we get

\[
\frac{d\sigma}{du^2} = \frac{1}{2\pi} \frac{G_r^2}{4\pi} \left[ \frac{1}{4(w_p^*)^2} \right] \frac{q_2^P}{u} \int_{t_{\text{min}}}^{t_{\text{max}}} dt^2 \int_{-1}^{1} d\cos \theta_3^P I(\Delta^2) \cdot \int_{0}^{2\pi} d\phi_3^P \omega^2 \frac{k_{1Q}^Q}{q_2^P} \frac{d\sigma_0}{d\Omega}(\omega^2, \cos \theta_1^Q) \quad (C.30)
\]

where

\[
t_{\text{max}}^2 = 2(E_1 \xi_1 - p_1 q_1) - M_1^2 - m_1^2
\]

\[
t_{\text{min}}^2 = 2(E_1 \xi_1 - p_1 q_1) - M_1^2 - m_1^2
\]
APPENDIX E

PURE ISOSPIN STATES OF K\(\Sigma\)N of \(T = 1, T_3 = 1\).

Table of Clebsch-Gordon coefficients

Let the entries in the table\((52)\) be

\[
\begin{array}{ccc}
  t \times s & T \\
  t_3' s_3' & T_3 \\
  t_3'' s_3'' & \pm b'' \\
\end{array}
\]

An isospin state \((T, T_3)\) of a system of two particles can be constructed from the products of isospin states of the individual particles according to\((46)\)

\[
(T, T_3) = \pm \sqrt{b'} (t, t_3') (s, s_3') \pm \sqrt{b''} (t, t_3'') (s, s_3'') 
\]

where \(t\) and \(s\) are the isospins of the two particles.

Isospin states of K\(\Sigma\)N with \(T = 1, T_3 = 1\) constructed from isospin states of K\(\Sigma\) and N.

K\(\Sigma\) can have isospin values ranging from \(t = 1 + \frac{1}{2} = \frac{3}{2}\) to \(t = 1 - \frac{1}{2} = 0\). Thus there are two cases to consider, \((\frac{3}{2}, \frac{1}{2})\)

a) K\(\Sigma\) is in state \(t = \frac{3}{2}\)

A K\(\Sigma\)N isospin state of \(T = 1, T_3 = +1\) can be
constructed from $K\Sigma$-states of isospin $\frac{1}{2}$ and $N$-states of isospin $\frac{1}{2}$ according to

$$\begin{aligned}
|\frac{1}{2}; 1, 1\rangle &= + (\frac{1}{2}, \frac{1}{2})(\frac{1}{2}, \frac{1}{2}) . \\
\end{aligned} \quad (E.1)$$

The $K\Sigma$ isospin states $(\frac{1}{2}, \frac{1}{2})$ can be expressed in terms of isospin states of $K$ (isospin $\frac{3}{2}$) and $\Sigma$ (isospin $1$) according to

$$\begin{aligned}
(\frac{1}{2}, \frac{1}{2}) &= \sqrt{\frac{2}{3}} (1, +1)(\frac{1}{2}, -\frac{1}{2}) - \sqrt{\frac{1}{3}} (1, 0)(\frac{1}{2}, +\frac{1}{2}) \\
&= \sqrt{\frac{2}{3}} \Sigma + K^0 - \sqrt{\frac{1}{3}} \Sigma^0 K^+ \\
\end{aligned} \quad (E.2)$$

Thus

$$\begin{aligned}
|\frac{1}{2}, 1, 1\rangle &= p(\sqrt{\frac{2}{3}} \Sigma + K^0 - \sqrt{\frac{1}{3}} \Sigma^0 K^+) \\
&= \sqrt{\frac{2}{3}} p \Sigma + K^0 - \sqrt{\frac{1}{3}} \Sigma^0 K^+ p \\
\end{aligned} \quad (E.3)$$

b) $K\Sigma$ is in a state $t = \frac{3}{2}$

A $K\Sigma N$ isospin state of $T = 1$ and $T_3 = 1$ can be constructed from $K\Sigma$-states of isospin $\frac{3}{2}$ and $N$-states of isospin $\frac{1}{2}$ according to

$$\begin{aligned}
|\frac{3}{2}; 1, 1\rangle &= \sqrt{\frac{3}{4}} (\frac{3}{2}, +\frac{3}{2})(\frac{1}{2}, -\frac{1}{2}) - \frac{1}{\sqrt{4}} (\frac{3}{2}, +\frac{1}{2})(\frac{1}{2}, +\frac{1}{2}) . \\
\end{aligned} \quad (E.4)$$

The $K\Sigma$ isospin states $(\frac{3}{2}, +\frac{3}{2})$ and $(\frac{3}{2}, +\frac{1}{2})$ can be expressed in terms of isospin states of $K$(isospin $\frac{3}{2}$) and $\Sigma$ (isospin $1$) according to
\[
\left( \frac{3}{2}, \frac{3}{2} \right) = +1 \left( 1, +1 \right) \left( \frac{3}{2}, +\frac{3}{2} \right)
\]
\[
= \Sigma^+ + k^+
\]  \hspace{1cm} (E.5a)

and
\[
\left( \frac{3}{2}, \frac{1}{2} \right) = \sqrt{\frac{1}{3}} \left( 1, +1 \right) \left( \frac{3}{2}, -\frac{1}{2} \right) + \sqrt{\frac{2}{3}} \left( 1, 0 \right) \left( \frac{3}{2}, +\frac{1}{2} \right)
\]
\[
= \sqrt{\frac{1}{3}} \Sigma^+ K^0 + \sqrt{\frac{2}{3}} \Sigma^0 K^+
\]  \hspace{1cm} (E.5b)

Thus
\[
\left| \frac{3}{2}; 1, 1 \right> = \sqrt{\frac{3}{4}} \left( \Sigma^+ K^+ \right) n - \frac{1}{4} \left( \sqrt{\frac{1}{3}} \Sigma^+ K^0 + \sqrt{\frac{2}{3}} \Sigma^0 K^+ \right) p
\]
\[
= \sqrt{\frac{3}{4}} \Sigma^+ K^+ n - \sqrt{\frac{1}{4} \frac{1}{3}} \Sigma^+ K^0 p - \sqrt{\frac{1}{4} \frac{2}{3}} \Sigma^0 K^+ p \]  \hspace{1cm} (E.6)
REFERENCES


7. E.L. Hart, B.N.L. Bubble Chamber report BC-04-3-B.


W. Selove, Rev. Mod. Phys. 37 (1965) 460.


18. G.F. Chew, University of California Radiation Laboratory report 8283.
G.F. Chew and F.E. Low, Phys. Rev. 113 (1959) 1640.


REFERENCES (Contd.)

REFERENCES (Contd.)


Appendix F

Prof. J. V. Major brought to our attention a recent paper by W. Chinowsky et al. (F-1) describing a simultaneous independent experiment. In this experiment the reactions (1.1) and (1.2) were studied at 6 BeV/c incident momentum. The experimental cross section obtained for reaction (1.1) is $29 \pm 5 \mu b$. These values are in very good agreement with the values $28 \pm 3.2 \mu b$ and $55.5 \pm 4.6 \mu b$, respectively, obtained in our experiment.

Of the two reactions (1.1) and (1.2), only (1.2) is discussed in detail in reference (F-1) because the number of events for reaction (1.1) was too small to yield statistically stable distributions. As this was the case in our experiment also, we will make a detailed comparison of the results of the two experiments for reaction (1.2) only.

In reference (F-1) only $\pi^+$-decays were used in the analysis of reaction (1.2) and the distributions of all the physical quantities are presented after weighting for scanning losses. According to the authors this weighting does not significantly alter the distributions. On this assumption their data can be compared to ours:

a) The Chew-Low plot (Fig. 16) of reference (F-1) shows a predominance of events with low momentum transfer to the nucleon, whence the authors conclude that the process is predominantly peripheral.
b) No evidence of the hyperon-nucleon resonance is found in reference (F-1). The Dalitz plot (Fig. 7) shows a concentration of events at low kaon-hyperon effective mass and high hyperon-nucleon mass. The authors attribute this concentration of events to the already well-established kaon-hyperon resonant interaction, namely \( \Delta(1920) \).

These data and their interpretation are in agreement with our findings. In both experiments the peak in the hyperon-nucleon effective mass is interpreted as a reflection of the peak in the kaon-hyperon spectrum. This conclusion is reinforced by the observation that the kaon-hyperon production is predominantly peripheral.

c) The distributions of the Jackson parameters (Figs. 13, 23) and the KE effective mass of reference (F-1) are somewhat different from ours. (In particular, the peak in the effective mass distribution is at 1.97 BeV compared to 1.86 in our experiment.) All these distributions differ from the predictions of the one-pion-exchange model by about the same order of magnitude as do ours. However, the authors conclude that the agreement is satisfactory considering the statistical fluctuations to be expected in the relatively small sample (about 250) of events studied.
The following checks on our experimental data have been made at the suggestion of Prof. J. V. Major.

1. The $\Sigma^+$ decay time (Figs. 2 and 3) have been replotted (Figs. F-1, F-2) on logarithmic graph paper. The straight line corresponding to a lifetime of $0.81 \times 10^{-10}$ sec. has been drawn on each plot and is almost everywhere within one standard deviation of the experimental values. We conclude that the data is consistent with the accepted value of $\Sigma^+$ lifetime.

2. In order to estimate the effect on the Dalitz plot (Fig. 14) of the loss of $p$-decays due to scanning bias, the data was replotted (Fig. F-3) for $\pi^+$-decays only. As compared with Fig. 14 the accumulation of points at low $K\Sigma$ mass was slightly enhanced and the background was slightly weaker in the new plot. No new accumulation of points was observed.

Reference

\[ \Sigma^+ \text{ DECAY TIME} \]

\[ p + p \rightarrow p + \Sigma^+ + K^0 \]

129 Events

\[ \tau = 0.810 \times 10^{-10} \text{ sec} \]

Fig. F-1
\[ \Sigma^+ \text{ DECAY TIME} \]

\[ \rho + \rho \rightarrow n + \Sigma^+ + K^+ \]

311 Events

\[ \tau = 0.810 \times 10^{-20} \text{ sec.} \]

Fig. F-2
$\Sigma^+$ DECAY TIME

$p + p \rightarrow n + \Sigma^+ + K^+$

311 Events

$\tau = 0.810 \times 10^{-20} \text{ sec.}$

Fig. F-2
**DALITZ PLOT**

\[ p + p \rightarrow n + \Sigma^+ + K^- \]
\[ \Sigma^+ \rightarrow \pi^+ + n \]

214 Events

**Fig. F-3**
The experiment is based on 120,000 pictures obtained in an exposure of the Brookhaven 80 inch hydrogen bubble chamber to a 4.95 BeV/c proton beam from the Alternating Gradient Synchrotron. The film was scanned for all events which could represent a \( \Xi^+ \) decay. About 750 such events were measured and a total of 447 events fitting reactions \( pp \rightarrow \Xi^+K^-n \) and \( pp \rightarrow \Xi^+K^-p \) were studied in detail.

In Chapter I the experimental technique used is described. The scanning and the measuring procedures as well as the computer programs used for analysing the data are described and critically examined. The computation of the cross-sections is outlined. The values obtained are \( 28.0 \pm 3.2 \mu b \) for \( pp \rightarrow \Xi^+K^-p \) and \( 55.5 \pm 4.6 \mu b \) for \( pp \rightarrow \Xi^+K^-n \).

In Chapter II the theory used in detailed study of the reactions is outlined.

In Chapter III the experimental data is analysed and compared to the predictions of the theory.

The mass spectra were examined in order to detect possible resonances. No evidence was found either for strange dibaryon resonance (in \( NN \) mass spectra) or for positive strangeness resonance (in the \( KN \) mass spectra) the existence of which was suggested by some earlier experiments.

The \( K \Xi \) effective mass spectrum in reaction \( pp \rightarrow \Xi^+K^-n \) has an enhancement at 1.86 BeV/c which is most likely due to the \( \triangle^{++}(1920) \) resonance previously observed in \( px \) collisions. The mechanism of the reaction is investigated with the help of one meson (K or \( \pi \)) exchange model, proposed by E. Ferrari for this reaction.

These models are based on the following assumptions:

1/
(i) That one-meson-exchange dominates the reaction, i.e., the reaction can be thought to take place through the interaction of the incident nucleon with one meson of the meson cloud surrounding the target nucleon.

(ii) That an S-matrix element for non-physical values of the variables can be obtained by an analytic continuation of the matrix element for the physical values of the variables. This assumption is used to relate the off-shell matrix element for the interaction between the exchanged meson and the incident nucleon to the matrix element for the corresponding physical process.

(iii) That the renormalisation effects can be taken into account by form factors similar to nucleon form factors in the problem of electromagnetic structure of the nucleon.

In order to gain insight into the plausibility of these underlying assumptions the predictions of the models were computed and compared with the experimental results. Since there is no prescription for obtaining the off-shell matrix element in terms of the on-shell matrix element for such processes as \( px^+ \rightarrow K^+ \Lambda^+ \) or \( K^0 p \rightarrow K^+ n \), where many partial waves are present, these computations are made in the pole approximation. The results suggest that the one-pion exchange plays an important role in the reaction. However, the pole approximation calculation is inadequate. A better agreement with the data is obtained if a form factor is used. (This is equivalent to assuming that the off-shell matrix element for \( px \rightarrow K \Sigma \) differs from the on-shell matrix element by a function of \( \Lambda^2 \) only).

Whether the remaining disagreement is caused mainly by the off-mass-shell effects, or by contributions from other processes besides one pion exchange can not be established until more sophisticated theoretical models are available.