THE BEHAVIOUR OF PERIODIC DISTURBANCES IN THE
LAMINAR BOUNDARY LAYER ON A FLAT PLATE

Thesis submitted by

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The following symbols are used to describe the quantities indicated in this list, unless otherwise stated in the text.

\begin{align*}
\begin{array}{ll}
x & \text{distance from leading edge of the flat plate.} \\
y & \text{distance from surface of flat plate.} \\
z & \text{distance from centre line of plate perpendicular to } x \text{ and } y. \\
U_0 & \text{free stream velocity.} \\
U & \text{mean velocity at a point in the boundary layer.} \\
u & \text{x component of fluctuation velocity.} \\
v & \text{y component of fluctuation velocity.} \\
w & \text{z component of fluctuation velocity.} \\
u', v', w' & \text{root mean square values of } u, v, w. \\
\beta_r & = 2\pi f \text{ where } f \text{ = oscillation frequency.} \\
c & \text{wave velocity.} \\
\alpha_r & = 2\pi/\lambda \text{ wave number.} \\
\alpha_i & = \text{complex part of complex wave number } \alpha_r + 1\alpha_i. \\
\rho & \text{density of air.} \\
\nu & \text{kinematic viscosity.} \\
\delta & \text{boundary layer thickness.} \\
\delta_i & \text{boundary layer displacement thickness.} \\
\delta_i & = 1.7208 \sqrt{\frac{x}{U_0}} \text{ for Blasius distribution.} \\
F & \text{non-dimensional frequency } = 2\pi \beta_r / U_0^2. \\
R_s & = \frac{U_0 \delta}{\nu} \text{ boundary layer Reynold's number.}
\end{array}
\end{align*}
SYMBOLS (Contd.)

- $R_{b_{o}}$ = minimum ribbon amplitude in any series of experiments.
- $H$ = rate of heat loss from a heated wire. 
  In Chapter 4, $H$ is used to represent the $z$ component of vorticity.
- $h$ = $z$ component of disturbance vorticity.
- $R_{w}$ = resistance of hot wire at temperature $T_{w}$.
- $R_{a}$ = 'cold' resistance of wire at temperature $T_{a}$.
- $i$ = hot wire current.
CHAPTER 1

HISTORICAL REVIEW AND INTRODUCTION

1.1 The Boundary Layer

Until the end of the 19th century the theory of fluids was almost entirely based on the Euler equations of motion. The formulation of the Navier-Stokes equations in 1820 had done little to remove the vast discrepancies between theoretical and experimental results, such as the prediction of zero drag and lift on solid bodies in an airstream, D'Alambert's paradox. Little use was made of the newly formulated equations, owing to the complexity of their solution.

Although, in 1851, Stokes, studying the small oscillations of a pendulum in air, had introduced the concept of zero relative velocity between the air and the bob of the pendulum, progress was only made when Prandtl (1904) introduced the concept of the boundary layer. Prandtl postulated that there was no slip between a moving fluid and a solid boundary, and that there existed a narrow region of fluid close to the boundary, in which the velocity of the fluid relative to the boundary changed asymptotically, from the no slip value at the boundary, to the free stream value at the edge of the region. The effect of viscosity was limited to this region, the boundary layer, the inviscid Euler equations being valid elsewhere in the flow field.
The proposed existence of the boundary layer allowed the Navier-Stokes equations to be reduced to the much simpler boundary layer equations (Appendix 1). These equations were first solved by Blasius (1908), for the case of flow over a semi-infinite flat plate with zero pressure gradient, the solutions being in the form of an infinite series. Howarth (1938), has refined the solution, tabulating the solution throughout the boundary layer.

The existence of the boundary layer was first verified by Burgers and van der Hegge Zijen (1925). Further verification was given by Nikaradse (1933), who produced velocity distributions in extremely good agreement with the Blasius distribution.

1.2 Hydrodynamic Stability

Reynolds (1883), recognised the existence of two types of flow, from the results of experiments on the flow of water through pipes. He found that a smooth streamlined flow could abruptly change to a disordered one, and that the transition between the laminar and turbulent flows could be characterised to some extent by the non-dimensional quantity \( \frac{Ud}{\nu} \), now known as the Reynolds Number, where \( U \) = velocity, \( d \) = diameter of pipe, and \( \nu \) = kinematic viscosity. By varying the parameters \( U \), \( d \), and \( \nu \) he found that a critical Reynolds Number, \( R_{\text{crit}} \), existed, below which the flow was laminar and above which turbulent

Prandtl in the course of experiments on the boundary layers on bodies of different shapes, found transition
from laminar to turbulent flow in the boundary layer.

The theory of instability of a perfect fluid was investigated by Rayleigh in a series of papers between 1880 and 1915. He studied the effect on the flow of small perturbations, and postulated that amplification of the disturbances classified an unstable flow whereas damping of the disturbances classified a stable flow. By considering the eigenvalue problem of the linearised equations of motion of the disturbances, he found that a necessary condition for instability in an inviscid fluid was the existence of an inflexion in the mean flow profile.

Orr (1907) viewed the small disturbance theory of instability as an energy exchange process between the mean flow and the disturbance. From studies of the balance between energy transferred to the disturbance from the mean motion under the action of Reynolds stresses, and energy transferred from the disturbance to the mean flow by viscous damping, he was able to work out a critical Reynolds number for Couette flow. Not all of the allowed disturbance velocities, however, were hydrodynamically possible, and the critical Reynolds number of 177 was much lower than the experimentally observed value of 1940.

1.3 Stability of the Laminar Boundary Layer

Two theories have been developed to explain the transition of a laminar boundary layer, namely the separation theory of G.I. Taylor and the theory of small disturbances.
It was proposed by Taylor (1936) that a laminar boundary layer is inherently stable to small disturbances, but that the pressure gradients associated with large fluctuations, which are taken up by the boundary layer from the free stream, cause intermittent flow reversal and separation, leading immediately to breakdown. He postulated that disturbances were taken up more readily, the thicker the boundary layer, so that a given amount of disturbance in the free stream can only cause transition at a certain minimum Reynolds number. The analysis is based on the Karman-Polhausen theory of velocity distribution in a boundary layer, and shows how the level of turbulence in the free stream can be related to the Reynolds number of transition. Good agreement was found between Taylor’s theory and experimental data taken by Dryden, on the transition of the boundary layer on spheres of differing size, in air flows with differing levels of turbulence.

Many authors have contributed to the theory of small disturbances, which deals with the exponential growth of fluctuations so small, that the equations of motion may be linearised. Tietjens attempted to study the stability of a boundary layer, approximating the mean flow to a series of straight lines. The solution, however, broke down because of the discontinuities in velocity gradient of the assumed mean flow. Important advances were made by Tollmien (1929), who removed the discontinuities in the
velocity gradient of the mean flow, by approximating the Blasius profile to a straight line and a parabola. He was thus able to consider the effect of viscosity in the region of the critical layer, having first shown that the effect of viscosity is only predominant in this region.

The results show that disturbances within a certain frequency range will be amplified and all others will be damped. The most convenient way of representing Tollmien's results is by a 'neutral stability diagram', Fig. 1.1 in the \((F, R_p)\) plane. This is a two branched curve, and it divides the \((F, R_p)\) plane into three separate regions; in one a small disturbance will be amplified, and in the other two it will be damped. The boundaries of the curve are neutral conditions, under which a disturbance is neither amplified nor damped. Tollmien's work was extended by Schlichting (1935), who calculated the distribution of the r.m.s. disturbance velocities across the boundary layer, Fig. 1.2, at branches I and II of the neutral curve. A modified neutral stability curve was calculated by Lin (1945), Fig. 1.1. From Lin's results Shen (1954) calculated curves of constant amplification within the neutral curve, and the growth curves of a disturbance passing from branch I to branch II. Osborne (1965) has recently used a high speed digital computer to recalculate the neutral stability curve Fig. 1.1 and the growth curves for disturbances of different non-dimensional frequency.

A summary of the method of small disturbances is given
Figure 1.2 Distribution Of Fundamental Disturbance Through The Boundary Layer (Shlichting)
in Chapter 4, but it is relevant, at this point, to look at one of the basic assumptions of the method. It is assumed that any disturbance existing in the flow can be Fourier analysed into single sinusoidal components, and that any one of these components can be described by a stream function \( \psi = \phi(y) e^{i(\alpha x - \beta t)} \). The problem may be treated in two ways, either \( \alpha \) or \( \beta \) being complex and the other real, i.e. the disturbance frequency or the wave number may be complex. In the work summarised, \( \beta \) has been taken as complex \( (= \beta_r + i\beta_i) \) and the amplification or damping of the disturbance governed by terms of the form \( e^{+\beta_1 t} \), where \( \beta_i \) may be positive or negative. The amplification of the disturbance has thus been considered as a function of time. Theoretically it is equally valid to choose \( \alpha = \alpha_r + i\alpha_i \), keeping \( \beta \) real, the amplification being governed by terms of the form \( e^{-\alpha_1 x} \).

This approach has been adopted by Jordinson (1968). His neutral curve calculation, based on spatial amplification is not significantly different from the temporal curve of Osborne, the critical Reynolds number in each case being 525. However, the latter theory is of prime importance to the experimentalist, as only the spatial growth of small disturbances may be studied experimentally.

The boundary layer differs from the model used in published work in that no account has been taken of the \( y \) component of local mean velocity. At any one Reynolds number, the boundary layer has been taken to be non-thickening with a velocity profile of the form \( U = U(y) \),
and $V = 0$, where $U$ and $V$ are the velocity components in the direction of flow and perpendicular to the boundary. Jordinson's calculations have been modified by Barry and M.A.S. Ross (1969) to include the local mean velocity component, perpendicular to the boundary. The effect is a destabilising one, the critical Reynolds number being reduced from 525 to 500.

1.4 Experimental Work on Linearised Disturbance Theory

Disturbances in the boundary layer were first found experimentally by Dryden (1936), but his results were inconsistent and could not be related to linearised theory. For the next decade it was believed that the separation theory of G.I. Taylor (1936) controlled the transition process and that the selective amplification process of Tollmein-Schlichting theory took no part in the transition process.

Using a very low turbulence wind tunnel, Schubauer and Skramstad (1947), showed the existence of disturbances of the Tollmein-Schlichting type, in the boundary layer on a flat plate. With a hot-wire anemometer they found bands of low frequency fluctuation velocities, the predominant non-dimensional frequency at any Reynolds number lying near to Branch II of the neutral stability curve of Schlichting. They concluded that small disturbances present in the boundary layer were selectively amplified as they passed downstream, and a disturbance of a particular frequency reached its maximum amplitude at a Reynolds...
number near to that of Branch II of the neutral curve. Artificial, controlled disturbances of small amplitude were then introduced into the boundary layer, by means of an electrically excited vibrating ribbon placed close to the plate. Using the controlled disturbances they were able to establish the existence of a neutral stability curve, in reasonable agreement with the theoretical curves of Tollmein and Lin.

Further confirmation of the validity of the linearised theory of hydrodynamic stability was obtained from the agreement of the measured distribution of disturbance amplitude across the boundary layer at branches I and II of the neutral stability curve with the distributions worked out by Schlichting (1935). Perhaps the most striking verification of the theory was the existence of the $180^\circ$ phase shift in the $u$ component of disturbance velocity in the outer region of the boundary layer, which had been predicted by Schlichting (1935).

1.5 Non Linear Work

The classic work of Schubauer and Skramstad showed that the behaviour of small disturbances was as predicted by the linearised theory, but it soon became apparent that before transition the behaviour of the disturbance was radically different from that predicted by linearised theory. This change in behaviour was believed to occur when the non-linear terms in the equations of motion ceased to be negligible.
The effects of finite perturbations have been examined theoretically, but usually for Poiseuille flow. It has been suggested, however, that despite the differences in mean profiles the results obtained for Poiseuille flow might be qualitatively valid for boundary layer flow.

Meksyn and Stuart (1951) considered the non-linear effects of two dimensional disturbances, and gave an approximate method for the solution of the equations of motion including the non-linear terms responsible for mean flow distortion and distortion of the fundamental disturbance. They showed the existence of a threshold above which a disturbance would amplify and below which a disturbance would decay, in a region which was stable according to linearised theory. They have also examined the variation of the critical Reynolds number with disturbance amplitude, and have shown that as the amplitude increases $R_{crit}$ falls.

Stuart (1958) using an energy balance method and considering only the effects of mean flow distortion derived the non-linear equations of motion, and has shown the possibility of a stable finite disturbance. He has also postulated that disturbance levels of about 10% of the free stream velocity are required to produce appreciable mean flow distortion. The effects of finite disturbances are, according to Stuart (1960a), the distortion of the mean flow by the action of the Reynolds stress, the generation of higher harmonics by the extraction of energy from the
fundamental disturbance, and the distortion of the fundamental with respect to its dependence on the coordinate $y$. In this paper Stuart showed also that the mean flow distortion and the generation of second harmonic are of the same order, but that the higher harmonics are of much smaller order. If the fundamental is of small order $|A|$, he suggests that any harmonic of order $n$ is developed to the order $|A|^n$.

The work described has all been based on the temporal growth of the disturbances. Bradshaw, Stuart and Watson (1960) have extended the calculations to consider the case of spatial amplification, and have obtained results similar to those in the case of temporal amplification.

Lin (1958) considered the case of Poiseuille flow, and concluded that in the region of the critical layer all harmonics of the fundamental disturbance become important whilst the disturbance levels are too low to cause appreciable distortion. It was pointed out by Stuart (1960a) that Lin's disturbance levels were much higher than those considered by himself.

Stimulated by the experimental work of Klebanoff et al. (1959) and (1962), described in the next section, which showed that the transition process in the boundary layer was essentially a three dimensional process, several workers have made theoretical investigations of the non-linear effects of three dimensional disturbances.

Lin and Benney (1960) and Stuart (1960b) considered Poiseuille flow perturbed by both two and three dimensional disturbances, and considered the interaction of the two
types of disturbance to the second order in amplitude. They predicted harmonic generation, modification of mean flow and the appearance of new harmonics non-periodic in time. Benney (1961) extended the earlier work to show that those components were non-periodic in time gave rise to streamwise vorticity, which could lead to a spanwise transfer of energy as indicated in the experiments of Klebanoff and Tidstrom (1959). He also considered the behaviour of this longitudinal vorticity in relation to the relative magnitude of the three dimensional wave component, and showed that as the three dimensional wave increased in amplitude, the spanwise period doubled. In 1964 Benney extended his calculations to a boundary layer type profile consisting of two straight lines.

Finally the approach of Görtler and Whitting (1958) to the existence of streamwise vorticity is worth mentioning. They suggested that streamwise vorticity would be developed due to centrifugal forces, at positions where the streamlines of the total mean flow are concave relative to an observer moving with the wave speed. However, there is experimental evidence (Klebanoff, Tidstrom and Sargent, 1962), to show that the longitudinal vorticity which has been observed experimentally is not generated in this way.

1.6 Experimental Work on the Non-Linear Region

At the time when the various non-linear theories were being derived, there was much activity in the experimental
field, sometimes stimulated by theoretical predictions and at other times posing questions for the theorist.

Emmons (1951) was the first to observe the random nature of natural transition. During experiments with a water table he observed the random appearance in the boundary layer of turbulent spots, which travelled downstream whilst at the same time growing laterally. Each spot was observed to merge with others some distance downstream of the position where it initially appeared. By this mechanism the whole width of the table was eventually covered with turbulent flow. He proposed a statistical theory of transition, characterised by an intermittancy factor ($\gamma$) for each position on the table. $\gamma$ was equal to the ratio of the time the flow was turbulent to the total time.

Schubauer and Klebanoff (1955), using a hot-wire anemometer, verified the spot like nature of the transition process in the boundary layer on a flat plate. In a low turbulence wind tunnel they created turbulent spots, by means of an electrical discharge, and were able to show that the artificially induced spots were essentially the same as naturally occurring spots. They then investigated the structure of the spots.

As the aim of stability theory is to isolate the effects which lead to transition and to predict when transition will occur, much interest has been shown in non-linear events prior to actual breakdown.

Fales (1955) used dye to visualise the transition to turbulence on a flat plate in a stream of water. He found
that the vortices cast off from a trip wire near the leading edge of the plate concentrated the initially uniform dye into discrete lines, which he identified as vortex lines. Hama, Long and Hegarty (1957) found that these vortex lines became warped when the induced vortices were strong. The warping was so great that vortex loops, which were constantly stretching in the streamwise direction, were quickly formed. From near the shoulders of these loops, bursts of turbulence were seen to arise, the succession of turbulent spots being concentrated along fixed lines. They explained the stretching of the vortex loops as due to the rotation of the warped vortex lines, in accord with a theorem developed by Lord Kelvin, in which the head of each loop was raised to a region of higher mean velocity, whilst the tail of the loop was lowered to a region of lower mean velocity.

Hama (1960) repeated the work of Hama, Long and Hegarty using the vibrating ribbon technique of Schubauer and Skramstad for the production of disturbances, following a suggestion by Schubauer (1953) that the behaviour of the vortices induced by the trip wire might not be the same as that of the sinusoidal ribbon disturbances. He was able to show that the sinusoidal induced vorticity rolled up into discrete vortices, and that for larger disturbances the behaviour was the same as that of the disturbances introduced by the trip wire. Hama (1962) did concede, however, after he had carried out a theoretical study of the streaklines near the critical layer, that the concentration of
the dye into discrete vortices could have been caused by
the nature of the streaklines and not by the rolling up of
the disturbances. At the same time he pointed out that
similar behaviour would be observed if in fact the dis-

In the context of their work, Hama et al. have concluded that
the formation of discrete vortices is a necessary prereq-
quisite to boundary layer transition. Their work, however,
does not give any indications as to the cause of the initial
three-dimensionality leading to the vortex loop configura-
tion or any indication of why the final turbulent burst
suddenly breaks away from the heads of the vortex loops.

A hot-wire anemometer study of the transition process
has been made in the low turbulence wind tunnel at the
National Bureau of Standards in Washington. Klebanoff and
Tidstrom (1959) used the vibrating ribbon technique to in-
troduce large disturbances into the boundary layer on a
flat plate. They found that the disturbance amplification
showed a pronounced spanwise variation and that turbulent
spots were observed only along discrete lines at the span-
wise positions where the disturbance amplification was
greatest. They found evidence of spanwise energy transfer
from the 'valleys' to the 'peaks' of the amplitude distribu-
tion. It was concluded that the turbulent spots form at
the 'peaks' of the disturbance amplitude distribution and
then spread into the neighbouring 'valleys'. Spanwise
variations of up to ± 10% in the boundary layer thickness
were suspected to have caused the periodic spanwise
variation in disturbance amplification.
Klebanoff, Tidstrom and Sargent (1962), by modifying the smoothing screens in the wind tunnel reduced the spanwise variation in boundary layer thickness to within their accuracy of measurement. They found that the three-dimensionality in disturbance amplitude still appeared. Having concluded that the transition process was inherently a three-dimensional process, they modified the ribbon system so that controlled three dimensional disturbances were introduced into the essentially two dimensional boundary layer. They showed the existence of longitudinal counter rotating vortices on either side of each 'peak' in the amplitude distribution, and showed that the development of these vortices was in qualitative agreement with the work of Benney and Lin (1960). An evaluation of the various theoretical approaches to the transition problem was made from the experimental results. Phenomena such as the generation of harmonics, the distortion of mean flow, and the Görtler-whitling mechanism of concave stream line curvature were considered in turn. Each of these was found inadequate in describing the observed phenomena. They did, however, find close agreement with the theoretical results of Lin and Benney, in that a longitudinal eddy system was found and they inferred a doubling of this system as the three dimensionality of the system increased.

Tani (1960) and Tani and Kamoda (1962) studied the three dimensional transition process by injecting two dimensional disturbances into a three dimensional boundary layer. Their results, though not as extensive, were in
broad agreement with those of Klebanoff et al.

1.7 Reviews

The problems associated with, and the work done on boundary layer instability have been reviewed by Dryden (1955), Schlichting (1960) and Stuart (1960c).

Relevant material is also contained in the books by Lin (1955), Schlichting (1955) and in that edited by Rosenhead (1963).

1.8 Conclusion

The behaviour of small disturbances in the boundary layer is, broadly, as predicted by linearised theory. However, few measurements in this region have been made, and no thorough experimental investigation has been made to find out quantitatively how well the observed behaviour over a wide range of conditions is fitted by theory. The reason for this may be due to the lack of theoretical data over a wide range of conditions. The complexity of the calculations is such that until recently they have been restricted to small regions of disturbance behaviour near the two branches of the neutral stability curve.

The observed behaviour of disturbances beyond the linear region is not in good agreement with any of the theories put forward to date. Perhaps the fact that nearly all calculations performed on the non-linear region have been made for Poiseuille flow, or rather inadequate
models of the boundary layer flow, has some relevance to this discrepancy.

With the present availability of high speed digital computers a numerical solution of the complete equations of motion for a perturbed boundary layer would seem to be a possibility. This task is at present being undertaken by several theoreticians at the University of Edinburgh. Concurrently a detailed experimental investigation of the behaviour of periodic disturbances in a boundary layer on a flat plate at all stages of development from the linear region to final breakdown, is being undertaken in a low turbulence wind tunnel.

It was felt that both the theoretical and experimental investigations should begin with the treatment of the linear region and proceed gradually, stage by stage, to the point of breakdown and beyond.

The work in this thesis compares the experimental and theoretical behaviour of disturbances in the region of linearised theory, and presents preliminary results of the investigation of the non-linear region, between the linear region and the breakdown of the laminar boundary layer, which it is hoped will indicate the most profitable lines along which the investigation should continue.
CHAPTER 2

DESCRIPTION OF APPARATUS

2.1 The Wind Tunnel

A closed circuit wind tunnel, specifically designed for low turbulence work was used. An airline diagram is shown in Fig. 2.1(a). The contraction area ratio of the settling chamber to the working section was 15:1. Three smoothing screens were fixed in the rapid expansion section in front of the settling chamber, and two more were situated 2 ft. apart at the beginning of the settling chamber. The screens were made from 38 mesh 36 s.w.g. phosphor-bronze wire and had a blockage coefficient of 0.506.

The working section was ten feet long and of basic 4 ft. square section. Perspex fillets were set diagonally into each corner, making the actual cross-section octagonal. Each of the four fillets could be flexed a distance of ± 1 inch by means of twelve screws fixed along their length. This allowed a variation of ± 2% in the 13 sq. ft. area of the working section. A series of breather holes at the downstream end of the working section ensured that the static pressure inside the working section was always equal to that in the sealed central room, thus eliminating the need for elaborate sealing of holes made in the walls for the introduction of probes. The total turbulence level

\[ \frac{100}{u_0} \sqrt{\frac{2}{3}(u'^2 + u'^2 + w'^2)} \]
Figure 2.1(a) Airline Diagram Of Tunnel
has been measured by Barnes (1966) and was 0.027% at 65 ft./sec., dropping to 0.006% at 30 ft./sec.

2.2 Tunnel Drive and Control

The tunnel was powered by a hydrostatic servo system, based on an N.E.L. design, which provided a complete hydraulic analogue of the Ward-Leonard system. A 35 H.P. squirrel cage induction motor drove the pump at a constant speed and this in turn drove the hydrostatic motor coupled to the fan. The fan was designed to absorb 30 H.P. at 750 revs. per minute, giving a top speed of 140 ft. per second.

Coarse power, and hence speed, control was obtained by altering the angle of the swash plate in the pump. The 230V. swash drive motor was operated by means of a pair of relays. A switch in the control room allowed either of two such relay pairs to be chosen, one set of relays being activated manually from the control panel, and the other remotely, under program control by a P.D.P.8 computer.

Fine speed control was achieved by bleeding a small amount of oil between the high pressure delivery and the low pressure return pipe of the pump. This was regulated by an electrically driven moog valve of variable aperture. Current to the moog valve was controlled by a 2000 SL variable resistance. Either of two identical resistors could be switched into the circuit. One of the resistors
was varied, through a 50:1 right angled gear box, by a 12V. stepper motor, which was turned by pulses provided under program control by the P.D.P.8 computer. The other resistor was adjusted manually from the control panel. Thus both coarse and fine control of speed was possible, either manually or remotely under program control. The fine control mechanism allowed the fan speed to be adjusted to within $\pm 0.1$ r.p.m. of any given value. A block diagram of the tunnel drive and control is shown in Fig. 2.1(b).

A full description of the design and the testing of the wind tunnel is given by Barnes (1966).

2.3 The Flat Plate

The flat plate was a $\frac{1}{8}$ inch thick sheet of perspex, 9 ft. long and 4 ft. wide. The leading edge was symmetrically tapered over the front six inches to a $\frac{1}{32}$ inch diameter circular tip by means of two intersecting milled faces. Shoulders left by the milling process were removed by polishing. The trailing edge was tapered, to a $\frac{1}{64}$ inch diameter circular tip by a single milled face 6 inches long on the reverse side.

The plate entered the working section through a slit in the floor. The plate was placed along the centre line of the working section and was bolted at the top, to one side of a $\frac{1}{4}$ inch angle iron fixed to the roof and offset
Figure 2.1(b) Block Diagram of Tunnel Drive
by $\frac{1}{4}$ in. from the centre line. The bottom of the plate was wedged against the slit in the floor, with small aluminium brackets screwed to the floor. At regular intervals during the work the plate was polished and coated with anti-static paste.

2.4 The Traversing Mechanism

Hot wire probes were carried on a carriage with shaped mahogany leading and trailing edges. The basic carriage was 3 ft. 6 ins. high, $9\frac{5}{8}$ ins. long and $1\frac{3}{8}$ ins. wide. The addition of the streamlining leading and trailing edges made the total length 1 ft. 4$\frac{1}{2}$ ins.

This traversing mechanism has been described by Barnes (1966) and was used for most of the work in this thesis. A new carriage was designed and used in the latter stages of the work. This is described later in this chapter. Both carriages were designed to move a hot wire accurately to $\pm 0.05$ ins. in the $x$ direction, 0.02 ins. in the $z$ direction, and .001 ins. in the $y$ direction. The original traversing carriage is shown in Fig. 2.2.

The $x$-movement

The carriage moved on ball races along a 1 in. diameter steel rod, fixed to the floor of the tunnel, four inches from the flat plate, and was sprung against a T-section rail at the top. A loop capstan drive, powered by a 24v. D.C. motor moved the carriage in the $x$-direction.
The Traversing Mechanism

Figure 2.2 The Traversing Mechanism
The carriage could be moved a distance of 9 ft. in 3
minutes.

The z-movement

The z traverse unit moved between two ½ inch di-
meter steel rods set into the main carriage frame and was
spring loaded against them. The unit was counterweighted
as shown in Fig. 2.3, and was moved by winding a steel
cord, fixed to the unit, round a ½ inch diameter cable
drum. The drum was driven by a size 11 Vactric D.C.
motor through a 1000:1 reduction gear box, giving a move-
ment of 0.13 inches per second at a motor speed of 5000
r.p.m.

The y-movement

The drive mechanism for y-movement was situated inside
the z-traverse unit. A size 11 Vactric D.C. motor with an
80:1 reduction gear head was fixed rigidly in a vertical
position and coupled to a 50:1 anti-backlash right angled
worm reduction gear. The output shaft of the worm gear
protruded through the front cover of the z-traverse unit
and was coupled to a micrometer screw 2 inches long and
of pitch 1 m.m. The other end of the screw was mounted
in a ballrace in the front of the boom-mounting bracket,
Fig. 2.4, which was fixed rigidly to the z-carriage. On
turning, the thread drove an anti-backlash copper nut along
its length. The nut was prevented from turning by the sides
of the boom-mounting bracket. The boom on which the hot
wire was mounted was pivoted at the end of the support
boom, a \( \frac{1}{2} \) inch square section dural rod, 1 ft. 2 ins. long, which was fixed to the z-traverse unit. It was terminated as shown in Fig. 2.5, and the two ballraces ran freely on the parallel sides of the mounting bracket. A ball-bearing tipped pin in the terminator of the boom was sprung against the copper block on the micrometer screw. The rate of movement of the boom was 0.008 inches per second.

2.5 On Line Experimentation

It was decided to adopt the wind tunnel and associated equipment so that experiments could be carried out either manually or under the programmed control of a P.D.P.8 computer. The motors controlling the movement of probes in the x, y and z-directions were to be replaced by stepper motors. Consequently a new traversing carriage was built, and the opportunity was taken of improving some of the design features.

2.6 The Modified Traversing Carriage

The x-movement

This was basically unchanged. The drive motor was a SLO-SYN type SS25/1011 24v stepping motor. The friction drive capstan was 3 inches in diameter and was driven through a 12:1 reduction gear.
Main Frame and Z-Traversal Carriage

Figure 2.3 The Z Traverse Unit
Figure 2.4 Boom Mounting Bracket
Figure 2.5  End Of Boom On Original Carriage
The y-movement

A direct drive was employed as shown in Fig. 2.6. The 2 inch long micrometer screw of pitch .05 inches, was driven by a Muirhead 24v. stepping motor type 11M30G4 through a 100:1 right angle worm reduction gear box. The end collar of the boom was pivoted at the top and bottom of the drive nut. The end collar terminated in a steel rod 6 inches long, which fitted inside the boom and which was free to slide on two points, as shown in Fig. 2.6, so as to allow for the necessary change in length as the drive nut moved from one end of the micrometer screw to the other. The changing length of the boom was small enough to preserve the linear relationship between the number of revolutions of the motor and the distance moved by the hot wire at the end of the boom.

The z-movement

The unit containing the y-traverse gear was mounted in the same way as on the old carriage. The counterweights were discarded and a direct drive from above was employed. A 24v. Muirhead stepping motor type 11M30G4 turned a 1 inch diameter threaded drum, through an 80:2:1 Vactric reduction gearhead and a 12½:1 worm reduction gear box. A \( \frac{1}{2} \) inch diameter steel cable connected the top of the y-drive housing to one end of the drum. Movement in the z direction was obtained by winding the cable on or off the drum. The mounting of the z-movement drive is shown in Fig. 2.7. Cover plates, which were free to
Figure 2.6 End Of Boom On New Carriage
to slide over the fixed cover plates at the top and bottom of the carriage were attached to the housing of the $y$ traverse gear and allowed a $z$-movement of $\pm 3$ inches about the centre line of the plate.

2.7 The Carriage Control Unit

The carriage control unit was designed so that the three 24V. stepping motors could be controlled either manually or by computer.

The main component of the manual drive circuit was an electronic switch, consisting of two pairs of transistors functioning as bistable multivibrators. Each bistable multivibrator controlled the current to one stator. The diodes in part 1 and the similar ones in part 2 of the switch shown in Fig. 2.8 functioned as gates to control the switching sequence. The switch was assembled on a Vero-board standard part number 243/2504. The board layout is shown in Fig. 2.9.

Negative pulses applied to the switch at A gave motor rotation in one direction and when applied to B gave rotation in the opposite direction. The 15 V. negative pulses were supplied from the free running multivibrator shown in Fig. 2.10. The variable resistance $R_8$ controlled the frequency of these pulses between limits of 110 pulses per second and 20 pulses per second set by the resistors $R_7$ and $R_9$.

The 24 V. collector outputs of the switch were each
Figure 2.8 Electronic Switch of Stepping Motor Control Unit
Figure 2.9 Layout Of Electronic Switch
Figure 2.10 Multivibrator In The Carriage Control Unit
passed to the base of an OC 22 power transistor and each stator was supplied with a current of up to 500 mA. from the emitters of the OC22 transistors. The diodes associated with the motor windings provide short circuit paths for the high transient voltages generated during switching.

The unit was arranged so that a rotary switch selected a particular motor and two push button switches provided rotation in either direction. A two-way switch provided the option of manual or computer control. With the switch in the computer control position, the electronic switch was disconnected from the circuit and 24 volts applied directly to the centre tapping of each stator winding. The 24V supply voltage and ground reference level were passed to the interface of the P.D.P.8 computer, which under program control completed the circuit to the ends of the stator windings in the correct sequence, to give either clockwise or counter-clockwise rotation of each motor. The switching diagram of the unit is shown in Fig. 2.11.

2.8 Measurement of the Hot Wire Coordinates x, y and z

The x-position of the hot wire was measured from a potentiometer slide wire 0.02 inches in diameter and length \( L = 98.34 \pm 0.3 \) inches. This was stretched between supports \( \frac{1}{4} \) inch from the rail along which the traversing carriage moved. The slide contact was attached to the traversing carriage and the wire voltage was supplied.
Figure 2.11 The Switching System Of The Stepping Motor Unit
from a 2V accumulator. The wire could be accurately placed at the \( x = 1 \) ft. position which was marked. At other positions the \( x \) coordinate of the wire was found from the equation

\[
x = \frac{v_x}{v_T} x L - (\frac{v_x}{v_T} x L - 12) \text{ inches}
\]

where \( L \) = the length of the wire in inches, 
\( v_x \) and \( v_s \) are the fractional voltages at the position \( x \) and at the \( x = 1 \) ft. position, and \( v_T \) is the total voltage along the wire. The \( x \) position could be measured to an accuracy of \( \pm 0.05 \) inches.

A similar system was used to measure the \( z \)-coordinate of the wire when required. The reference position was the centre line of the plate \( z = 0 \) and the coordinate \( z \) given by

\[
z = \frac{v_s}{v_T} x L_1 - \frac{v_z}{v_T} x L_1
\]

where \( L_1 \) is the length of the wire, and the other variables are similar to those above. The \( z \) coordinate could be measured to \( \pm 0.05 \) inches.

Direct measurement of the distance \( y \) of the hot wire from the plate could not be made. With the original traversing carriage distances proportional to the wire movement, were obtained from a clock gauge mounted on the fixed support boom, with the feeler pressing on the boom terminating collar. The movement was calibrated by measuring the distance moved by the hot-wire with a microscope and comparing it with the clock gauge measurement. As described in Chapter 5 Section 8, the Blasius boundary layer
profile was used to find the true distance of the wire from the plate via the clock gauge readings. The clock gauge could record wire movements to an accuracy of \( \pm 0.005 \) inches.

With the new traversing carriage the distances moved by the wire in the \( y \)-direction were measured by counting, with an electronic counter, the number of pulses given to the \( y \) stepping motor. The number of steps moved by the motor was proportional to the distance moved by the wire. 16 motor steps moved the wire \( .001 \) inches. The actual distances of the wire from the plate were calculated in the same way as with the original traversing carriage.

2.9 The Measurement of Windspeed

Windspeeds were found by measuring, with a Chattock gauge, the dynamic pressure from a pitot tube fixed in the free stream. The pitot tube was stationed 5 ft. from the beginning of the working section, 8 inches from the tunnel wall and 2 ft. 9 inches from the floor of the tunnel. The air temperature was measured by a thermometer in the working section. The atmospheric pressure at the time of each velocity measurement was noted, so that the density and the kinematic viscosity of the air could be calculated.

The windspeed was obtained from the relationship

\[
U_o = 1.778 \left[ \frac{a(t + 273)}{p} \right]^{\frac{1}{2}}
\]

which was obtained from the geometry of the gauge where

- \( p \) = atmospheric pressure in \( \text{mm. Hg.} \)
- \( t \) = air temperature in \( ^\circ\text{C} \)
and \( a \) = Chattock differential in \( \frac{1}{180} \times \frac{1}{16} \) inches.

The accuracy of the windspeed measurement was limited by the accuracy to which the gauge could be read. At windspeeds around 10 feet per second the gauge could be read accurately to \( \pm 3 \) divisions giving the windspeed accurate to \( \pm 4\% \). As the windspeed increased the accuracy of the gauge increased. At windspeeds of around 40 ft. per second the accuracy was better than \( \pm 1\% \).

2.10 Introduction of Disturbances into the Boundary Layer

The vibrating ribbon technique for the introduction of controlled disturbances into the boundary layer was developed by Schubauer and Skramstad (1947). Since then, this method has been used extensively by other workers, notably, Klebanoff and Tidstrom (1959), Tani (1960), Hama (1960) and Kersley (1965).

The ribbon system used in this work is shown in Fig. 2.12. A phosphor bronze ribbon 0.001 inches thick and 0.1 inches wide, was mounted on the working side of the plate and transverse to the direction of flow. The top of the ribbon was clamped to the roof of the tunnel at the junction with the plate, and the bottom end passed through a slot in the floor of the tunnel. Two 0.0065 inch diameter glass bridges, fixed to the plate 4 inches above and below the centre line, raised the centre span of the ribbon from the plate. The portion of the ribbon above the bridges was sellotaped to the plate, and that below the
Figure 2.12 The Ribbon System
bridges was covered with a paper strip and then sellotaped to the plate, thus preventing sideways movement but allowing tension to be applied to the centre span. The ribbon was held against the bottom of the plate by a small brass rod screwed to the bottom of the tunnel. The ribbon was tensioned by weights hung from a small hook soldered parallel to its length. Three 1\(\frac{1}{2}\) inch wide permanent horse shoe magnets were mounted on a perspex former, which was screwed to the reverse side of the plate, in such a way that the magnets were symmetrical about the centre of the ribbon. When A.C. current was passed through the ribbon it oscillated, with the forcing frequency, in the presence of the magnetic field.

A block diagram of the ribbon circuit is shown in Fig. 2.13. A Hewlett-Packard oscillator type 207 provided a signal at a given frequency, which was checked by a Venner frequency meter type T.S.A. 3336/2. The signal was passed through a decade attenuator, amplified by a Mullard 10 watt power amplifier and then used to excite the ribbon. The ribbon current could be adjusted in 5 db steps by the attenuator or continuously by the amplitude control in the oscillator. A Solartron true R.M.S. voltmeter type VM 1484 measured the voltage across a 1\(\Omega\) standard resistor in series with the ribbon, and hence the ribbon current. For ease of reading the output from the true R.M.S. voltmeter was fed to a Solartron digital voltmeter type L M 1420.

The ribbon could be placed either 8 inches or 12 inches from the leading edge of the plate. The tension on
Figure 2.13  Block Diagram Of The Ribbon System

Oscillator  Decade Attenuator  Power Amplifier  Ammeter  Ribbon

Frequency Meter
the ribbon was adjusted such that the working frequency was less than $\frac{5}{6}$ of the resonant frequency and at least 10 cycles per second more than $\frac{1}{3}$ of the resonant frequency. This criterion has been shown by Kersley (1965) to ensure that ribbon amplitude is proportional to ribbon current.

The ribbon amplitude could be measured at the 12 inch position by the method described by Kersley (1965). A brass plug was fitted into a hole in the flat plate at the centre of the vibrating segment of the ribbon. The surface of the plug was flush with the plate. A wire was attached to the plug on the reverse side of the plate, and taken to a cathode ray oscilloscope. The ribbon was earthed and the shorting of the oscilloscope input once per cycle, when the ribbon just touched the plate, gave a method of estimating when the ribbon amplitude was 0.0065 inches. Thus knowing the ribbon current giving an amplitude of 0.0065 inches, the amplitude for any smaller ribbon current could be calculated.

All the work on small disturbances described in Chapters 5 and 6 was done with the ribbon in the 8 inch position and it was thus not possible to measure the ribbon amplitude.
2.11 The Measurement of Disturbance and Mean Velocities in the Boundary Layer

a) The Hot Wire Method

Both mean and disturbance velocities were measured with a hot-wire anemometer.

The hot wire is a fine electrically heated wire, which is cooled convectively when placed in an airstream.

King (1914) showed that for a wire placed normal to an air stream the heat loss \( H \) is related to the velocity \( U \) by an equation of the form

\[
H = A + BU^{\frac{1}{2}}
\]

where \( A \) and \( B \) are constants depending on the physical properties of the wire.

For a wire operated at constant current \( i \) this equation takes the form

\[
\rho = \rho_0 + mU^{\frac{1}{2}}
\]

where \( \rho = \frac{R_w}{(R_w - R_A)} \), and \( R_w \) is the resistance of the heated wire, and \( R_A \) is the resistance of the unheated wire. \( \rho_0 \) and \( m \) are constants which must be determined from the calibration of the wire.

Collis and Williams (1959) have shown that for hot wires with Reynold's numbers, based on wire diameter, in the range \( 0.02 < Re < 44 \) the equation

\[
\rho = \rho_0 + mU^{0.45}
\]
gives better agreement with experimental results than the equation using $U^{\frac{1}{3}}$. This was the relation used in this work.

The hot-wires were calibrated against the Chattock gauge in the windspeed range $10 < U_o < 40$ ft. per second. The relation between $p$ and $U^{0.45}$ was found to be linear, thus allowing the calibration to be used in the low speed region of the boundary layer.

b. The Hot-Wire Heads

The wire heads were designed to screw into a coaxial plug set into the end of the boom of the traversing carriage. A diagram of the construction of the head is shown in Fig. 2.14. The prongs were nickel plated number 6 sewing needles. One prong was hard soldered to the stainless steel case of the head and the other to the central pin, which was insulated from the case. The perspex top was made to fit tightly into the case. The whole assembly was araldited in place.

c. The Mounting of the Wire

A piece of Wollaston wire was soldered across the tips of the prongs. The silver coating of the Wollaston wire was then removed by electrolytic etching in a fine jet of 10% nitric acid. The length of wire etched was the effective length of the hot wire, and this was typically 1 m.m.
Figure 2.14 Construction of Hot Wire Head
d. The Hot-Wire Control Unit

The hot-wire control unit was that designed and built by Kersley (1965). The two channel unit with 12 Volt D.C. power supply was designed to supply currents of between 1 mA and 100 mA to wires with resistances between 2 ohms and 100 ohms. The stability of the system was such that a 1% change in hot-wire voltage resulted in a $\frac{1}{300}\%$ change in hot wire current. The hot-wire voltage was measured directly with a Solartron digital voltmeter type number LM 1420. The hot-wire current was found by measuring, with the digital voltmeter, the voltage across a standard 10 ohm resistor in series with the hot wire.

The fine current control in one channel of the unit was altered to allow the adjustment of hot wire current under computer control. The potentiometer providing fine current adjustment could be switched out and replaced by an identical one which was driven by an Implex 12 volt stepping motor, type AU5105/80. This stepping motor was powered from the same supply as the instrument carriage stepping motors. Current variation of up to 3mA was available under computer control.

d. Measurement of Mean and Fluctuating Velocities

Mean velocities were measured directly from the mean hot-wire voltages with the aid of the hot-wire calibration. Small fluctuation velocities were measured from the
r.m.s. level of the fluctuating part of the hot-wire voltage. The D.C. mean velocity was removed by passing the hot wire output through a choke-capacitance coupled amplifier. The relation between r.m.s. fluctuation velocity and r.m.s. voltage is

\[ \frac{u'}{U_0} = \frac{U}{U_0} \frac{(\rho - 1)^2 e'}{0.451 R_A (\rho - \rho_0)} \]

where \( u' = \sqrt{u^2} \) is the r.m.s. fluctuation velocity and \( e' \) is the r.m.s. fluctuation voltage. The derivation of this formula can be found in either Kersley (1965) or Schubauer and Klebanoff (1945).

2.12 The Recording Equipment

a) The Data Logging System

The Solartron data logging system, incorporating the digital voltmeter type LM 1420, the command range unit type EC 1475, scanning unit type LU 1461, and punch encoder type LU 1467, was used to measure all D.C. voltages.

The voltmeter was accurate to \( \pm 0.05\% \) or \( \pm 1 \) digit on any range and could measure D.C. voltages between 10 \( \mu \)V and 200V. The command unit allowed the voltmeter to be used independently or as part of the data logging system. Up to 20 voltages could be scanned. Any single channel could be scanned continuously. The scanning unit could also be set so that any of the 20 channels could be addressed from the P.D.P.8 computer. The punch encoder was modified.
to feed directly to the computer the voltage on any selected channel.

b) The Recording of Hot Wire Signals

The hot-wire output was amplified by either a Tektronix type FM 122 low level preamplifier with a gain of either 100 or 1000, or a Solartron type AA900 low noise amplifier with a gain variable in steps up to 1000. Both amplifiers served to remove the mean hot-wire voltage and amplify the fluctuating voltage.

The amplified hot-wire output was passed to a Brüel and Kjaer type 2107 frequency analyser, which had a range from 20 to 20,000 cycles per second. This was normally operated on its narrowest bandwidth of 45 db octave selectively. Amplifiers in the analyser allowed further amplification of the signal in steps of either 10 or 20 db. The amplifier was in general set at the frequency of the vibrating ribbon, but could be used to find the total signal level or as a frequency rejection filter. R.m.s., peak, or average voltages could be measured.

The output of the analyser was fed to a Brüel and Kjaer type 2305 pen recorder which recorded the hot-wire output. For ordinary voltage recording a linear 10-110 MV range was used. The recorder could be used on a logarithmic scale in synchronism with the frequency analyser to carry out continuous spectrum analysis.
CHAPTER 3

PRELIMINARY EXPERIMENTS

3.1 The Importance of the Free Stream Pressure Gradient

The stability of a laminar boundary layer is influenced strongly by the static pressure gradient in the undisturbed free stream.

The theoretical analysis of a two dimensional flow with zero pressure gradient is a particular case of the analysis of the same flow with a non-zero pressure gradient. The only effect of a pressure gradient is to alter the shape of the velocity profile \( U(y) \), from that with zero pressure gradient. Flows with \( \frac{dp}{dx} < 0 \) generate velocity profiles with negative curvature everywhere, that is \( U''(y) < 0 \), whereas flows with \( \frac{dp}{dx} > 0 \) generate profiles exhibiting a reversal of sign in \( U''(y) \). For inviscid flow an inflexion in the velocity profile is necessary for instability to small disturbances, and it might be expected that a boundary layer with positive pressure gradient will be less stable than one with zero or negative pressure gradient.

The effect of a pressure gradient on the velocity profile has been described in terms of a single parameter \( \Lambda \) by Polhausen,\(^{(Rosenhead (1963))} \). \( \Lambda = \frac{2}{\infty} \frac{dU}{dx} \) and is known as the Pohlhausen shape factor. Schlichting (1935) has calculated neutral stability curves for various
Figure 3.1 The Effect Of A Pressure Gradient On The Neutral Curve (Schlichting)
Figure 3.2 Static Pressure Along The Working Section
values of the shape factor. His results are shown in Fig. 3.1.

Schubauer and Skramstad (1947) have investigated the effect of both positive and negative pressure gradients on the stability of a laminar boundary layer on a flat plate. Their results show that a positive pressure gradient does indeed make the boundary layer less stable, whereas a negative pressure gradient makes the boundary more stable.

3.2 The Pressure Gradient Along the Working Section

The pressure gradient was measured using two static head tubes. The reference tube was positioned 4 inches behind the leading edge of the flat plate, 4 inches from the plate on the reverse side, and 1 ft. 6 inches from the floor of the tunnel. The other tube was attached to the traversing carriage so that it moved down the middle line of the plate and 4 inches from it. A Chattock gauge was used to measure the difference between the static pressures and measurements were made at six inch intervals. The results for wind speeds of 20, 40 and 50 ft. per second are shown in Fig. 3.2. For convenience the value of $p - p_o/\frac{\rho u_o^2}{2}$ has been made zero at $x = 3$ ft.

The amount by which the perspex fillets should be adjusted to remove the large pressure increase at $x > 6$ ft. was calculated, and it was found that the available movement was not sufficient. A closer investigation of this region was made, the wind speed being 40 ft. per second unless otherwise stated.
The static head tube was removed from the carriage and mounted at \( x = 3 \) ft. and the carriage was moved about the trailing edge of the plate. The static pressure at the 3 ft. position, for each position of the carriage relative to the trailing edge of the plate is shown in Fig. 3.3. It appears that as the carriage passes the trailing edge of the plate a pressure rise occurs.

The plate was extended by 4 ft., and it was found that the maximum pressure rise in the region \( 6 < x < 8 \) ft. was reduced by about one third, Fig. 3.4(a). The breather holes on the working side of the plate were blocked with the extension retained and the maximum pressure rise was reduced by about 50\%, Fig. 3.4(b).

The plate extension was removed and the breather holes unblocked. The static tube was then mounted on a small Meccano carriage which could be moved along the working section. The pressure was measured along the working section with the main carriage in four different fixed positions. These results are shown in Fig. 3.5 and the four different carriage positions are also indicated in Fig. 3.3.

When the reference static tube was moved to the working side of the plate the pressure rise along the working section dropped to half the original value, indicating that when the pressure rose on the working side of the plate the pressure dropped on the reverse side. The pressure rise on the working side was accompanied by a corresponding velocity decrease.
Figure 3.3 Static Pressure Variation In Working Section With Carriage Position
Figure 3.4 Static Pressure Along Working Section With Plate Extended And Breather Holes Blocked
Figure 3.5 Static Pressure Along The Working Section For Different Fixed Carriage Positions
Conclusions

Fig. 3.5 shows that the pressure variation due to the proximity of the carriage to the end of the flat plate occurs over the length of the working section. The pressure gradient over the length of the plate, at all times is small.

It is concluded that the changing pressure in the working section is caused by changing blockage in the tunnel as the instrument carriage passes the trailing edge of the plate. Similar results were obtained on creating extra blockage on the working side, by fixing perspex fins of differing size to the trailing edge of the plate.

Part of the additional blockage is probably due to the interference of the wakes of the plate and the carriage, although pitot tube traverses made at 1 ft. 1 inch downstream of the plate show only small differences in the wake patterns, with the carriage near the trailing edge of the plate and the carriage well upstream of the trailing edge, Fig. 3.6. This effect is shown in Fig. 3.4(a) by the reduction in $\frac{p-p_0}{\frac{1}{2} \rho U_o^2}$ when the plate is extended by 4 ft. Some of the extra blockage appears to arise from an interaction of air entrained through the breather holes, due to imperfect sealing of the control room, with the carriage. This effect is shown by the reduction in magnitude of $p-p_0/\frac{1}{2} \rho U_o^2$ in Fig. 3.4(b). It has been found that the pressure change with carriage position can be
Figure 3.6 Wakes Behind Plate and Carriage
magnified by running the tunnel with the control room at atmospheric pressure, on leaving the door of the control room open, thereby increasing the flow of air into the tunnel through the breather holes.

The true pressure gradient in the working section was measured with a static head tube mounted on a Meccano trolley with the instrument carriage 12 ft. from the beginning of the working section, which was as far downstream as possible. Readings were taken at 6 inch intervals. The pressure gradient is shown in Fig. 3.7 and Table 3.1. Between 1 and 4 ft. the pressure gradient is zero within experimental error. A small positive pressure gradient exists between 4 ft. and 6\frac{1}{2} ft., the maximum Pohlhausen shape factor \( A \) being \(-0.30\).

The first sign of pressure change due to carriage position occurred when the trailing edge of the carriage was 6 inches in front of the trailing edge of the plate. Work in the boundary layer therefore could not be carried out with the carriage further back than this point. Hot wire probes were situated 3 ft. 0 inches upstream of the rear of the carriage so that the permissible working length of the plate was restricted to 5 ft. 6 inches.

**The Effect of the Unexcited Ribbon on the Boundary Layer**

Barnes et al. (1966) have shown that in the present wind tunnel natural transition of the boundary layer on a flat plate occurs at a Reynolds number \( R_T \) of
3.6 \pm 0.1 \times 10^6. An investigation of the position of natural transition with the ribbon in place, but not vibrating was made.

A surface pitot tube of external diameter 0.024 inches and internal diameter 0.0165 inches was traversed along the centre line of the plate. The end of the tube was flattened to an oval shape with internal axes of 0.03 and 0.007 inches.

The dynamic pressure was measured with the Chattock gauge. As the tube was moved downstream the pressure fell as the laminar boundary layer became thicker. It rose steeply as the mean flow became distorted at the start of the transition region. The point of minimum pressure was taken as the onset of the transition region.

It was found that at windspeeds below 35 feet per second the boundary layer was laminar along the whole length of the plate. At higher windspeeds the onset of transition, Table 3.2, was observed to shift to progressively smaller Reynolds numbers. $R_T$ was reduced when the span of the ribbon was increased.

Conclusion: The length of the plate on which boundary layer studies could be made was limited by early transition due to the ribbon. For this reason the span of the ribbon was limited to 8 inches, which provided an acceptable width in the $z$ direction, over which the injected disturbances were essentially two dimensional, and at the same time gave a laminar boundary layer over a considerable length.
Figure 3.7 Pressure Gradient In The Working Section
TABLE 3.1
The Pressure Gradient in the Working Section

\[ U_0 = 41 \text{ feet per sec.} \]

<table>
<thead>
<tr>
<th>Distance from Leading Edge in Feet.</th>
<th>( \frac{P-P_0}{\frac{1}{2} U_0^2} )</th>
<th>Pohlhausen Shape Factor ( \Lambda = \frac{2 \frac{dU_0}{dx}}{v} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+0.027 ( \pm ) 0.002</td>
<td>+0.13</td>
</tr>
<tr>
<td>0.25</td>
<td>+0.007 ( \pm ) 0.002</td>
<td>+0.11</td>
</tr>
<tr>
<td>0.5</td>
<td>+0.001 ( \pm ) 0.002</td>
<td>0.00</td>
</tr>
<tr>
<td>1.0</td>
<td>+0.001 ( \pm ) 0.002</td>
<td>+0.09</td>
</tr>
<tr>
<td>1.25</td>
<td>-0.001 ( \pm ) 0.002</td>
<td>-0.06</td>
</tr>
<tr>
<td>1.75</td>
<td>0.000 ( \pm ) 0.002</td>
<td>0.00</td>
</tr>
<tr>
<td>2.0</td>
<td>0.000 ( \pm ) 0.002</td>
<td>-0.10</td>
</tr>
<tr>
<td>2.25</td>
<td>+0.003 ( \pm ) 0.002</td>
<td>-0.22</td>
</tr>
<tr>
<td>2.5</td>
<td>+0.003 ( \pm ) 0.002</td>
<td>0.00</td>
</tr>
<tr>
<td>2.75</td>
<td>+0.005 ( \pm ) 0.002</td>
<td>-0.27</td>
</tr>
<tr>
<td>3.0</td>
<td>+0.007 ( \pm ) 0.002</td>
<td>-0.30</td>
</tr>
<tr>
<td>3.25</td>
<td>+0.007 ( \pm ) 0.002</td>
<td>0.00</td>
</tr>
<tr>
<td>3.5</td>
<td>+0.007 ( \pm ) 0.002</td>
<td>0.00</td>
</tr>
<tr>
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<td>0.00</td>
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<tr>
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<td>+0.007 ( \pm ) 0.002</td>
<td>0.00</td>
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<td>0.00</td>
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<td>4.5</td>
<td>+0.007 ( \pm ) 0.002</td>
<td>0.00</td>
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<td>0.00</td>
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<tr>
<td>5.5</td>
<td>+0.007 ( \pm ) 0.002</td>
<td>0.00</td>
</tr>
<tr>
<td>5.75</td>
<td>+0.007 ( \pm ) 0.002</td>
<td>0.00</td>
</tr>
<tr>
<td>6.0</td>
<td>+0.007 ( \pm ) 0.002</td>
<td>0.00</td>
</tr>
</tbody>
</table>
### Table 3.1 (Contd.)

<table>
<thead>
<tr>
<th>Distance from Leading Edge in Feet</th>
<th>( \frac{p-p_0}{\frac{1}{2} u_o^2} )</th>
<th>Pohlhausen Shape Factor ( \frac{2 du_o}{dx} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.25</td>
<td>+0.008 ± .002</td>
<td>-0.16</td>
</tr>
<tr>
<td>6.5</td>
<td>+0.005 ± .002</td>
<td>+0.54</td>
</tr>
<tr>
<td>7.0</td>
<td>+0.005 ± .002</td>
<td>0.00</td>
</tr>
<tr>
<td>7.25</td>
<td>+0.005 ± .002</td>
<td>+0.40</td>
</tr>
<tr>
<td>7.75</td>
<td>+0.003 ± .002</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>-0.011 ± .002</td>
<td></td>
</tr>
<tr>
<td>8.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3.2

The Onset of Transition in the Presence of the Stationary Ribbon

<table>
<thead>
<tr>
<th>Windspeed ((u_o))</th>
<th>x Position</th>
<th>Reynolds Number ( (R_x = \frac{U_o x}{v}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 ft./sec.</td>
<td>&gt; 9 ft.</td>
<td>&gt;1.97 x 10^6</td>
</tr>
<tr>
<td>40 ft./sec.</td>
<td>7 ft. 6 inches</td>
<td>1.88 x 10^6</td>
</tr>
<tr>
<td>50 ft./sec.</td>
<td>4 ft. 9 inches</td>
<td>1.48 x 10^6</td>
</tr>
<tr>
<td>55 ft./sec.</td>
<td>3 ft. 9 inches</td>
<td>1.29 x 10^6</td>
</tr>
</tbody>
</table>
of the plate. It is observed that for windspeeds below 40 ft. per second, where most of this work was done, the working length of the plate was limited by the pressure effects described in Section 3.2 rather than by the transition due to the presence of the ribbon.

The Disturbances Provided by the Ribbon

A large part of the present work consists of an experimental evaluation of the theory of the boundary layer with small disturbances present.

The theory of small disturbances, or linearised theory, is built on the premise that products of disturbance velocities in the equations of motion are small enough to be neglected. Ideally such disturbances should be infinitesimally small. Lin (1945) has suggested that a disturbance may be considered small, if by doubling or halving the input disturbance, the disturbance some distance downstream is doubled or halved. This criterion has been used throughout this work to define the linearised region. The range of disturbance levels for which this type of behaviour is observed, has been investigated.

The ribbon was vibrated with different amplitudes and the r.m.s. disturbance velocities in the boundary layer were measured at various x stations with the hot wire probe at $\frac{y}{\delta} = 0.15$. The ribbon amplitude was measured as described in Chapter II. It was found that disturbance velocities were related linearly to ribbon amplitude for
disturbances up to 1% of the free stream velocity, a result in agreement with the work of Klebanoff and Tidstrom (1959) and Kersley (1965). The relation between disturbance amplitude and ribbon amplitude is shown in Tables 3.3 and 3.4 and in Figs. 3.8 and 3.9 for two different ribbon frequencies.

The theoretically infinitesimally small disturbance will be nearly realised in practice if the disturbances injected into the boundary layer are made as small as possible. Consequently the ribbon amplitudes were chosen so as to produce disturbance velocities much smaller than the 1% 'maximum' of the linear region. It was found that disturbance levels as low as 0.03% of the free stream velocity at $\frac{y}{\delta} = 0.2$ were large enough to produce hot wire signals some twenty times greater than the noise level due to natural turbulence and the electronics. Disturbance levels giving a signal to noise ratio of at least 5:1, except in regions of the boundary layer where the disturbance velocities decreased to zero were used throughout.

It might appear that some of the disturbance velocities measured are comparable with the natural turbulence level in the tunnel. However, the turbulence levels quoted in Chapter 2 are total levels, whereas measurements were made of the $u$ component of disturbance velocity at a single frequency. The natural turbulence level in this case is of course much smaller than the total level and difficult to distinguish from the noise level of the hot wire unit.
Care was taken not to use windspeeds requiring a fan speed such that the fan blade frequency was near to the frequency of the injected disturbances, as the components of natural turbulence are considerably increased at frequencies near to the fan blade frequency.
Figure 3.8 Variation Of Disturbance Velocity With Ribbon Amplitude
Figure 3.9 Variation Of Disturbance Velocity With Ribbon Amplitude
<table>
<thead>
<tr>
<th>Disturbances injected at x = 12 inches</th>
<th>Disturbances measured at x = 16.6 inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ribbon Amplitude in inches</td>
<td>u' / U₀ ^ o/o</td>
</tr>
<tr>
<td>0.0007</td>
<td>0.067</td>
</tr>
<tr>
<td>0.0014</td>
<td>0.130</td>
</tr>
<tr>
<td>0.0021</td>
<td>0.200</td>
</tr>
<tr>
<td>0.0028</td>
<td>0.270</td>
</tr>
<tr>
<td>0.0035</td>
<td>0.340</td>
</tr>
</tbody>
</table>

**TABLE 3.4**

Variation of Disturbances in the Boundary Layer with Ribbon Amplitude

U₀ = 41.6 ft./sec.  \( \frac{u}{U₀} = 0.34 \)  \( \frac{y}{δ} = 0.21 \)

Frequency = 150 c/s

<table>
<thead>
<tr>
<th>Disturbance Amplitude in inches</th>
<th>u' / U₀ ^ o/o</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>0.37</td>
</tr>
<tr>
<td>0.0006</td>
<td>0.44</td>
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<tr>
<td>0.00066</td>
<td>0.51</td>
</tr>
<tr>
<td>0.0007</td>
<td>0.54</td>
</tr>
<tr>
<td>0.00086</td>
<td>0.68</td>
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<tr>
<td>0.0011</td>
<td>0.85</td>
</tr>
<tr>
<td>0.0014</td>
<td>1.00</td>
</tr>
</tbody>
</table>
CHAPITE 4

AN OUTLINE OF THE SMALL DISTURBANCE THEORY OF HYDRODYNAMIC STABILITY

4.1 The Small Disturbance Theory of Hydrodynamic Stability

An outline is given here of the small disturbance theory of hydrodynamic stability, for two dimensional flow perturbed by two dimensional disturbances, the main results of which have been given in Chapter I.

The mean velocity of the flow is assumed to be only a function of y and the pressure a function of x and y. The basic flow is represented by $U = U(y)$, $V = 0$, $W = 0$ and $P = P(x,y)$. Two dimensional disturbances of the form,

$u(x,y,t)$, $v(x,y,t)$ and $p(x,y,t)$

are superimposed on the mean flow, so that the resultant velocity components and pressure are $U + u$, $v$, and $P + p$. It is assumed that the disturbances are small and that squares and products of disturbance velocities and their differential coefficients may be neglected.

The hydrodynamical equations of the disturbance velocities are found by subtracting from the Navier-Stokes equations for the whole flow field, the Navier-Stokes equations for the undisturbed mean flow, that is with $u = v = p = 0$. These equations which form the basis for all calculations on the behaviour of small perturbations
in a large scale mean flow are:

\[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + v \frac{\partial U}{\partial y} = v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{\rho} \frac{\partial p}{\partial x} \tag{1}
\]

\[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} = v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{1}{\rho} \frac{\partial p}{\partial y} \tag{2}
\]

The pressure terms can be eliminated from (1) and (2) by differentiating the equations with respect to \( y \) and \( x \) respectively and subtracting the second from the first and the resulting equation is:

\[
\frac{\partial^2 u}{\partial y \partial t} + u \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial U}{\partial y} \frac{\partial u}{\partial x} + \frac{\partial U}{\partial y} \frac{\partial U}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial U}{\partial y} - \frac{\partial v}{\partial x} \frac{\partial v}{\partial t} - U \frac{\partial}{\partial x} \frac{\partial^2 v}{\partial x \partial y} = v \left( \frac{\partial^3 u}{\partial x^2 \partial y} + \frac{\partial^3 u}{\partial x^3} - \frac{\partial^3 v}{\partial x^3} - \frac{\partial^3 v}{\partial y^2 \partial x} \right) \tag{3}
\]

The disturbance velocities may be expressed in terms of a stream function \( \psi \) such that

\[
u = \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x},
\]

It is wished to find a periodic solution to equation (3) and as this can be represented by a Fourier series, the solution may be investigated by using any single term of the series. This is done by supposing that the disturbances are periodic so that the stream function becomes

\[
\psi = \phi(y) e^{i(\alpha x - \beta t)} \tag{4}
\]

where \( \phi(y) \) describes the \( y \) dependence of the stream function, \( \alpha = \alpha_r + i\alpha_i \) is the complex wave number, and
\[ \beta = 2\pi f \] is the angular frequency of the disturbance. \( \psi \) may be written as

\[ \psi = \phi(y) e^{-a_1 x} e^{i(\alpha_0 x - \beta t)} \]

and the term \( e^{-a_1 x} \) is a term representing damping or amplification of the disturbance, depending on the sign of \(-a_1\).

If equation (3) is made non-dimensional in terms of the free stream velocity \( U_0 \) and the boundary layer displacement thickness \( \delta_1 \), and \( u \) and \( v \) as obtained from (4) are substituted into (3), the non-dimensional form of the Orr-Sommerfeld equation is obtained

\[ (\alpha U - \beta)(\phi - \alpha_0^2 \phi') - \alpha U''\phi = \frac{1}{R \delta_1} (\phi''' - 2\alpha_0^2 \phi'' + \alpha_0^4 \phi') \] (5)

where the primes represent differentiation with respect to \((y/\delta_1)\). This is a homogeneous linear equation with the general solution

\[ \phi = c_1 \phi_1 + c_2 \phi_2 + c_3 \phi_3 + c_4 \phi_4 \]

The form of the particular solutions depends on the mean velocity profile and the boundary conditions.

Equation (5) is the fundamental equation on which, until recently, all stability calculations have been based.
4.2 The Solution of the Orr-Sommerfeld Equations for the Boundary Layer

Equation (5) is non-dimensionalised in terms of the free stream velocity \( U_0 \) and the boundary layer thickness \( \delta \). The boundary conditions are

\[
\begin{align*}
y = 0 : & \quad u' = v' = 0 : \quad \phi = 0, \quad \phi' = 0 \\
y = \infty : & \quad u' = v' = 0 : \quad \phi = 0, \quad \phi'^2 = 0.
\end{align*}
\]

As the viscous terms in equation (5) are of order \( R \) down in the inertial terms, two solutions can be obtained from the non-viscous equation

\[ (U-c)(\phi'' - a^2 \phi) - U'' \phi = 0. \]

One of these solutions is found to have a singularity at the level in the boundary layer where the wave velocity \( c \) is equal to the local mean velocity \( U_0 \), the region in which the viscous terms have their largest effect. The singularity is removed by including in the initial equation the largest of the viscous terms. Solution of the new equation in terms of the variable \( \eta \) defined by

\[
y - y_c = (a R U_0^2)^{-1/3} \eta = \xi \eta
\]

where the subscript \( c \) refers to the value of the parameters \( y \) and \( U \) at the critical layer, not only removes the singularity in \( \phi_2 \) but yields another two solutions \( \phi_3 \) and \( \phi_4 \). As \( \phi_4 \) is found to increase rapidly with \( y \) the constant \( c_4 \) in the general solution is set to zero and the general solution becomes
\[ \phi = c_1 \phi_1 + c_2 \phi_2 + c_3 \phi_3. \]

The stability problem is one in which the boundary conditions give sufficient equations between the solutions at the boundaries to determine the values of the parameters for which equation (4) is satisfied. The approximation is made that the solutions \( \phi_1, \phi_2 \) and \( \phi_3 \) are small enough to be made zero at \( y = \delta \), instead of at \( y = \infty \). The problem is basically that of finding the complex eigenvalue \( \alpha_r + ia_1 \) for given values of \( \beta \) and \( R \).

Most authors have calculated the locus of \( \alpha_1 = 0 \), and Shen and Jordinson have also calculated curves of constant \( \alpha_1 \) when \( \alpha_1 \neq 0 \). These loci are usually expressed in a \( (\frac{\delta y}{U}, R) \) plane or in an \( (\alpha, \delta_1, R) \) plane. The locus of \( \alpha_1 = 0 \), which divides either of these planes into amplifying and damping regions is called the neutral stability curve, or neutral curve, and has been illustrated in Chapter 1. It is a curve on which a disturbance is neither amplified nor damped, and the lowest Reynolds number on the curve is the smallest Reynolds number at which a small disturbance can be amplified.

Corresponding to each eigenvalue \( \alpha_r + ia_1 \) there is an eigenfunction \( \phi(y) = \phi_r + i\phi_1 \). If the constants in the general solution of (5) are found then \( \phi_r \) and \( \phi_1 \) can be obtained and the amplitude distribution of the disturbance across the boundary layer obtained.
4.3 Extension of Theory to Include the $V$ Component of Boundary Layer Velocity

If the $V$ component of velocity is taken into account, i.e. $V \neq 0$, then equation (3) becomes

$$\frac{\partial h}{\partial t} + U \frac{\partial h}{\partial x} + U \frac{\partial h}{\partial y} + V \frac{\partial h}{\partial y} + V \frac{\partial H}{\partial y} = v V^2 h$$ \hspace{1cm} (6)

where $H = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}$ and $h = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$.

This leads to a modified Orr-Sommerfeld equation containing five additional terms.

Barry and M.A.S. Ross (1969) have considered the modified equation and have shown that three of the five terms, introduced as a consequence of making $V \neq 0$, are of the same order as the viscous terms, while two are of smaller order. They then investigated the eigenvalue problem of the modified Orr-Sommerfeld equation, including the additional terms of the highest order, by numerical methods. The equation they considered was

$$(\alpha U - \beta)(D^2 - \alpha^2) \phi - i V D (D^2 - \alpha^2) \phi + i \frac{\partial^2 V}{\partial y^2} D \phi - \alpha \frac{\partial^2 U}{\partial y^2} \phi$$

$$= \frac{1}{R_s} (D^2 - \alpha^2)^2 \phi$$ \hspace{1cm} (7)

where all variables are non-dimensionalised in terms of the boundary layer thickness $\delta$, and $D = \frac{\partial}{\partial y}$.

Using equation (7) Barry and M.A.S. Ross have repeated Jordinson's work, and have shown that the effect of the $V$ component of velocity is to make the boundary layer less stable.
4.4 Predictions Made by Theory

It is worthwhile to consider briefly the implications of the theory just outlined from the point of view of the experimentalist.

Let us assume firstly that a disturbance is injected into the boundary layer at a position where $R_S$ is less than the Reynolds number on the first branch of the neutral curve, corresponding to the non-dimensional frequency $F = \frac{\beta v}{U}$ of the disturbance. If the disturbance is traced in the $x$ direction, then one would expect the disturbance to be damped at first, $(a_1 > 0)$, and then to be amplified after branch I of the neutral curve is crossed, $(a_1 < 0)$, and finally damped once more after branch II of the neutral curve is crossed, $(a_1 > 0)$. The minimum and maximum points on a disturbance growth curve represent conditions for which $a_1 = 0$, i.e., they are neutral points.

The disturbance being derived from the stream function

$$\psi = \phi(y) e^{-\alpha x} e^{i(ax-\beta t)}$$

must, if the theory of small disturbances is valid, grow exponentially between the two branches of the neutral curve. If $A_0$ is the disturbance amplitude at the start of the amplifying region, then the amplitude at any $x$ position in the amplifying region should be

$$A = A_0 \exp \int_{x_0}^{x} \alpha \, dx,$$

where $x_0$ is the $x$ position of branch I of the neutral curve.
Measurement of the amplitude distribution of the disturbance velocity components across the boundary layer, that is in the y direction, should provide a measurement of the r.m.s. of the derivatives of the eigenfunction 
\[ \phi_{ri} + i\phi_1, \quad \sqrt{\phi_r^2 + \phi_1^2} \] at any given eigenvalue \[ \alpha_{ri} + i\alpha_1. \]
CHAPTER 5

EXPERIMENTAL WORK WITH SMALL DISTURBANCES

5.1 Object of Work on Small Disturbances

The object of this work is to provide experimental data to test in detail the most recent theoretical investigations of the behaviour of small disturbances in the boundary layer on a flat plate. The investigation was comprised of three parts, (a) the streamwise growth of small disturbances, (b) the distribution of disturbance velocities across the boundary layer, and (c) the location of the neutral stability curve.

5.2 The Streamwise Development of Small Disturbances

The growth curves for small disturbances in the boundary layer have been calculated for various non-dimensional frequencies $F$ by Shen (1954), for the amplifying region of the neutral stability curve. Osborne has used a digital computer to recalculate these curves and has extended them to the regions outside the neutral curve. The growth curves based on spatial amplification, the kind of amplification that is measured experimentally, have been computed by Jordinson (1968). These curves are given as graphs of $\ln \frac{A}{A_0}$ against $R$, where $A$ is the r.m.s. amplitude of the $u$ component of disturbance velocity and $A_0$ is the r.m.s. amplitude on Branch I of the neutral stability curve.

The variation in amplitude of the $u$ component of the
disturbance velocity was measured by tracking a hot wire along the centre line of the plate. During each experiment the sensitivity of the hot wire was held constant by adjusting the $y$ position of the wire so that the mean voltage across the wire remained constant. Thus the amplitude ratio $\frac{A}{A_0}$ was simply equal to $\frac{e}{e_0}$, the ratio of the r.m.s. output of the fluctuating hot wire voltage at any $x$ position, to that at branch I of the neutral curve. No measurements were made within one disturbance wavelength of the ribbon.

Let us consider the implication of working with constant wire sensitivity. The wire, set normal to the free stream and parallel to the plate is sensitive to both $U$ and $V$ components of velocity in the boundary layer. Consequently the voltage across the wire depends on the local mean velocity $\sqrt{U^2 + V^2}$, and so the wire was tracked along the plate on a line of constant mean velocity. The ratio $\frac{V}{U}$ is greatest near the leading edge of the plate, and a typical value at $x = 1$ ft. for $U_0 = 20$ ft./sec. is $2.5 \times 10^{-3}$, at a $\frac{V}{U}$ position of 0.5. It is therefore apparent, that to a good approximation $\sqrt{U^2 + V^2} = U$ and that a wire operated at constant sensitivity is essentially operated at a constant $\frac{V}{U}$ position in the boundary layer. The measured growth curves show therefore the amplification of a small disturbance along a path fixed at the same relative depth in the boundary layer.

It was intended to study the downstream change in $\phi'(y)$, the distribution of the disturbance velocity across
the boundary layer, in relation to the growth curves, and as these are given theoretically as functions of the non-dimensional \( \frac{y}{\delta} \), which is directly proportional to \( \frac{y}{\delta} \), it was thought desirable to carry out x-traverses at a constant \( \frac{y}{\delta} \) position.

Experimental amplification curves at \( \frac{y}{\delta} \) positions \( 0.10 < \frac{y}{\delta} < 0.15 \) are compared with the theoretical curves of Jordinson in Figs. 5.1 and 5.2 with the theoretical curves of Shen in Figs. 5.3 and 5.4. At each \( x \) station the noise was subtracted from the hot wire output signal.

The error in the Reynolds number is largely due to the error in the measurement of \( U_0 \). The error in \( U_0 \) is determined by the sensitivity of the Chattock gauge and is found to decrease from \( \pm 4\% \) at a speed of 10 ft. per second to \( 1\% \) at a speed of 40 ft. per sec. The maximum error in Reynolds number, due to error in \( U_0 \) is therefore \( \pm 2\% \). The accuracy of the ratio \( \frac{A}{A_0} \) is limited by the resolution of the pen recorder used to measure the hot wire output signals. The error is greatest at values of \( \frac{A}{A_0} \) near 1, and for higher values of non-dimensional frequencies where the output signals are smallest and tend to be unsteady. At \( F \) numbers < 125 the error in \( \frac{A}{A_0} \) near the minimum of the growth curve is \( \pm 5\% \), and decreases as higher values of \( \frac{A}{A_0} \) are reached. Work on the higher non-dimensional frequencies was in general done with low velocities, so that the higher the \( F \) number the
Figure 5.1 Growth Curves, Comparison Of Theory (Jordinson) And Experiment
Figure 5.2 Growth Curves, Comparison Of Theory And Experiment
Figure 5.3 Growth Curves, Comparison of Theory (Shen) And Experiment
Figure 5.4 Growth Curves, Comparison Of Theory And Experiment
greater was the error in both $R_\xi$ and $\frac{A}{A_0}$.

The experimental curves all show considerably greater total amplification than that predicted by Jordinson. The discrepancy in $\ln \frac{A}{A_0}$ for $F = 175 \times 10^{-6}$ is 0.42, a difference of 53% in $\frac{A}{A_0}$. As $F$ decreases the difference between the theoretical and experimental results becomes smaller until at $F = 52.5 \times 10^{-6}$ it is only 28% in $\frac{A}{A_0}$ when compared with the theoretical curve for $F = 50 \times 10^{-6}$. Considerably better agreement is found with the theoretical curves of Shen. The measured total amplification is in general less than the predicted values and the maximum discrepancy is 0.25 in $\ln \frac{A}{A_0}$ or 28% in $\frac{A}{A_0}$ for $F = 125 \times 10^{-6}$.

5.3 The Extrapolation of the Theoretical Growth Curves to the Experimental Conditions

In general the growth curves given by Shen and Jordinson do not correspond exactly to the experimental conditions. Unless the experimental value of $F$ lay very close to a theoretical value it was felt desirable to extrapolate from the given theoretical growth curves the growth curve for the experimental conditions.

The calculation was done as shown in Fig. 5.5. $\ln R$ was plotted against $\ln F$ for the maxima and minima of the given theoretical curves and gave two straight lines. Contours of constant $\ln \frac{A}{A_0}$ from the theoretical curves were then superimposed. Growth curves for any value of $F$ could then be built up.
Figure 5.5 Extrapolation of Theoretical Growth Curves (Shen) to Experimental Values
5.4 The Variation in Total Amplification with $\frac{y}{\delta}$

A closer investigation of the growth curves was made by tracking the hot wire downstream at different constant $\frac{y}{\delta}$ positions between 0.09 and 0.45. As shown in Figs. 5.6 to 5.9 there are striking differences, not predicted by theory, in the growth curves at different $\frac{y}{\delta}$ positions for the same $F$ number. The total amplification of a disturbance is found to decrease rapidly as $\frac{y}{\delta}$ increases. The decrease in total amplification with increasing $\frac{y}{\delta}$ is largest for low values of $F$. The drop in total amplification between $\frac{y}{\delta} = 0.09$ and $\frac{y}{\delta} = 0.405$ for $F = 82 \times 10^{-6}$ is 1.2, whereas between $\frac{y}{\delta} = 0.09$ and $\frac{y}{\delta} = 0.417$ for $F = 159 \times 10^{-6}$ it is 1.6. Upstream of the amplification region disturbances are more readily damped at large values of $\frac{y}{\delta}$. This effect is clearly seen in three of the four sets of results shown in Figs. 5.6 to 5.9.

The theoretical growth curves of Jordinson shown with the experimental curves at different $\frac{y}{\delta}$ positions do not at first sight agree well with any of the measured curves. The maxima and minima of all the experimental curves are at lower Reynolds numbers than the maxima and minima of the theoretical curves. If the theoretical curves are shifted so that the minima of the theoretical and experimental curves coincide, then there is good agreement, between Jordinson's curves and the second largest of the amplification curves in each set. For $F = 107 \times 10^{-6}$,
Figure 5.6 Growth Curves At Differing y/δ Positions
Figure 5.7 Growth Curves At Differing \(\frac{y}{\delta}\) Positions

GROWTH CURVES
- \(\frac{y}{\delta} = 0.15\)
- \(\frac{y}{\delta} = 0.20\)
- \(\frac{y}{\delta} = 0.30\)
- \(\frac{y}{\delta} = 0.415\)

DUE TO OVERLAPPING OF POINTS THE GROWTH CURVES ON STREAMLINES ARE NOT SHOWN AS STATED IN TEXT
Figure 5.8 Growth Curves At Differing $y/\delta$ Positions
Figure 5.9 Growth Curves At Differing $y/\delta$ Positions
132 x 10^{-6}, and 154.9 x 10^{-6} the curves at $\gamma/\delta = 0.20$ agree well with theory, and for $F = 81.3 x 10^{-6}$ the curve at $\gamma/\delta = 0.15$ agrees well with theory. In the region of the critical layer the growth curves at constant $\gamma/\delta$ are in good agreement with theory, if the curves are shifted so that the minima coincide.

5.5 The Variation of Amplification Factor with $R$ and $\gamma/\delta$

If $A_o$ is the amplitude of a disturbance at branch I of the neutral stability curve, then following the method described in Chapter 4, the amplitude $A$ at any point downstream will be given by

$$\frac{A}{A_o} = \exp \int_{x_0}^{x} (-\alpha_1) dx$$

or

$$\ln\left(\frac{A}{A_o}\right) = \int_{x_0}^{x} (-\alpha_1) dx$$

(1)

which can be written in terms of $R$ as

$$\ln\left(\frac{A}{A_o}\right) = \frac{2}{m^2} \int_{R_0}^{R} \left(-\alpha_1 \delta_1\right) dR$$

(2)

where $m = 1.7208$, and $(-\alpha_1 \delta_1)$ is the amplification factor of the disturbance.

Theory defines $\alpha_1$ as the imaginary part of the complex wave number of the disturbance given by

$$\alpha = \alpha_r + i\alpha_1$$

It will be realised that the total amplification of a disturbance can only vary with $\gamma/\delta$ if the amplification factor $(-\alpha_1 \delta_1)$ also varies with $\gamma/\delta$, a contradiction
of the theoretical assumption that the disturbance may be
described by a stream function of the form

$$\psi = \rho(y)e^{(ax-\beta t)}$$

An attempt to look more closely at the variation of
the amplification factor was made. Least square poly-
nomials of fifth order were fitted through the growth
curves shown in Figs. 5.6 to 5.9, \( \ln \frac{A}{A_0} \) being expressed
in terms of \( R_5 \). The amplification factor at any
Reynolds number could then be found from the gradients of
the best fit curves and was given by

$$(-a_1 \delta) = \frac{m^2}{2} \frac{d}{dR} \left( \ln \frac{A}{A_0} \right).$$

Shen (1954) assumed a cubic form for his growth curves
but in this case a fifth order curve was chosen as it gave
a better 'least squares' fit.

The fifth order polynomials did not give a good fit
near the minima of the growth curves, but were satisfactory
over the remainder of the curves. Perhaps this is due to
the small number of experimental points upstream of the
minima.

The variation of \((a_1 \delta)\) with \( R_5 \) is shown in Figs.
5.10 to 5.12 for the growth curves \( F = 154.9 \times 10^6, \)
107 \( \times 10^{-6} \) and 81.4 \( \times 10^{-6} \).

The variation of \((a_1 \delta)\) with \( \gamma / \delta \) at different
Reynolds numbers is shown in Figs. 5.13 and 5.14 with
\( F = 154.9 \times 10^{-6}, 107 \times 10^{-6} \) and 81.4 \( \times 10^{-6} \).
Figure 5.10 Variation Of Amplification Factor With $R_g$
Figure 5.11 Variation Of Amplification Factor With $R_g$
Figure 5.12 Variation Of Amplification Factor With $R_g$
Figure 5.13 Variation Of Amplification Factor With $\gamma/\delta$
Figure 5.14 Variation Of Amplification Factor With $y$
increases, as \( \frac{y}{\delta} \) increases, along a curved path such that the change in \((a_1s_1)\) is greatest near the plate.

5.6 The Wave Number of the Disturbances

It has been shown that the imaginary part of the complex wave number \( a = a_r + ia_1 \) of the disturbances, based on amplification along a line of constant \( \frac{y}{\delta} \), is not constant in the \( y \) direction. The wavelength, and hence \( a_r = \frac{2\pi}{\lambda} \) of the disturbances was measured for different values of \( F \) and at different \( \frac{y}{\delta} \) positions to see if \( a_r \) as well as \( a_1 \) varied with \( \frac{y}{\delta} \).

The ribbon and hot wire signals were fed to the X and Y plates of a Solartron CD 1400 oscilloscope respectively. As the hot wire was moved downstream, at a fixed \( \frac{y}{\delta} \) position, the Lissajou figure changed from an ellipse to a straight line at intervals of about 1 inch. The distance moved by the hot wire between two positions producing straight line traces, was taken as half of the wavelength of the disturbance. In this way the disturbance wavelength could be found to an accuracy of ± 0.05 inches when the disturbance signal was steady. At higher values of \( F \) the accuracy is less than this as the hot wire output was unsteady, and there was difficulty in seeing exactly where the Lissajou figure became a straight line.

No variation of wavelength with \( \frac{y}{\delta} \) was detected. The wavelength of the disturbance was found to be nearly constant in the \( x \) direction. Possibly there was a small
increase in \( \lambda \) in the \( x \) direction, but only in the case of \( F = 81.3 \) was the total increase greater than the accuracy of the measurement.

Assuming that the wave velocity of the disturbance is given by \( f \lambda \), the wave velocities at \( F = 48.5, 81.3, 116.6 \) and \( 158.9 \times 10^{-6} \) are equal to the local mean velocities at \( \frac{y}{\delta} = 0.210, 0.217, 0.229 \) and \( 0.243 \), where \( \lambda \) has been taken as the average value for each \( F \) number. A small increase in the \( \frac{y}{\delta} \) position of the mean velocity corresponding to the wave velocity with \( F \) is noted. These values agree well with the results of Schubauer and Skramstad (1947) and Tani and Komoda (1962).

The most convenient way of comparing the measured wavelengths with the results of the theoretical eigenvalue problem is in a graph of \( a^t R^{-1} \) against \( R \), where \( a^t = \frac{2\pi}{\lambda} \delta \). Barry and M.A.S. Ross (1969) have computed the values of \( a^t R^{-1} \) for various values of \( F \). In this calculation account has been taken of the \( V \) component of velocity in the boundary layer.

The experimental and theoretical results are shown in Fig. 5.15. The four sets of results are in good agreement with the theoretical results. At \( F = 116.6 \times 10^{-6} \) and \( 158.9 \times 10^{-6} \) there is a considerable scatter of the experimental points, because at higher values of \( F \) the hot wire output signal was unsteady, making it difficult to decide where a particular wavelength ended. The results agree with theory to within 7\%, in general being slightly lower than the predicted values.
Figure 5.15 Comparison of $\alpha_r R^*$ with Theory

- $F = 158.9 \times 10^{-6}$
- $F = 116.6 \times 10^{-6}$
- $F = 81.4 \times 10^{-6}$
- $F = 48.5 \times 10^{-6}$
5.7 The Amplitude Distribution of Disturbances Through the Boundary Layer

Schubauer and Skramstad (1947) have measured the amplitude distribution of a disturbance across the boundary layer at branch I of the neutral curve, for \( F = 61.4 \times 10^{-6} \) and at branch II of the neutral curve, for \( F = 40.6 \times 10^{-6} \). They found that the experimental distributions were in reasonable agreement with the theoretical calculations of Schlichting, notably in that the peak of the distribution at branch II occurs at a smaller \( \frac{y}{\delta} \) position than the peak of that at branch I, and that there was a phase change in the fluctuation velocity in the region \( 0.65 < \frac{y}{\delta} < 0.85 \).

Tani and Komoda (1962) and Kersley (1965) have shown that the distribution is considerably broadened and that the peak moves away from the plate as the size of the disturbance is increased beyond the linear region. Jordinson (1968) has computed the linear amplitude distributions at various \( F \) numbers and at various points between branches I and II of the neutral curve, and has found that the peak moves from its branch I position to its branch II position gradually. No detailed experimental investigation of the changing shape of \( \delta'(y) \), the amplitude distribution function, has been previously made.

5.7(a) Measurement of the Amplitude Distribution

At the start of any distribution measurement the hot wire voltage was noted when the wire was in the free stream
The wire was then set at the desired $x_0$ station and moved into the boundary layer as far as the buoyancy region, (Collis (1956)). This is the region very close to the plate where the air velocity is so low that free convective cooling of the wire becomes comparable with the forced convective cooling. The edge of this region was taken as the point at which the hot wire voltage ceased to increase, and sometimes even decreased, as the wire was moved even further into the boundary layer. The wire was then moved out of the boundary layer in steps measured with the clock gauge. The steps were initially of the order 0.005 inches and were gradually increased, until at the outer edge of the boundary layer they were of the order 0.02 inches. After each step the wire current was set to the chosen value, the wire voltage, step length and r.m.s. disturbance level were noted. When the wire reached the free stream the voltage across it was compared with the original free stream voltage. If it was found to be more than 2 parts in 1000 different from the original free stream voltage, the results were discarded because of excessive drift in the hot wire calibration during the traverse. Finally the hot wire was calibrated.

5.8 The Location of the Initial Distance of the Wire from the Plate

In order to plot the disturbance intensity through the boundary layer, it was necessary to know the distance
of the hot wire from the plate after every hot wire step.

Fitting a small pressure transducer to the hot wire head as a means of indicating when the wire was a known distance from the plate was considered, but rejected when it was found that small enough transducers were not commercially available.

The boundary layer profile was used to locate the initial distance of the wire from the plate. It was found that $U/U_o$ varied linearly with $y$ up to $U/U_o = 0.3$, and that the initial $y$ distance of the wire from the plate could be found by extrapolating this line to $U/U_o = 0$. Each $y$ position measured with the clock gauge could then be corrected to give the true $y$ position relative to the plate. A typical example of this procedure is shown in Fig. 5.16. It is seen that the first few points do not lie on the straight line. These points are at positions where the hot wire is in the buoyancy region described in Section 5.7(2). It was found that by fitting a least squares straight line to the graph of $y$ against $U/U_o$ that the $y$ zero could be located to an accuracy of $\pm 0.002$ inches.

5.9 The Reduction of the Boundary Layer Traverse Results

The results were processed on the P.D.P.8 computer in the Department of Natural Philosophy.

Firstly the program worked out the calibration constants of the hot wire. These were then used to work out the local mean velocity and the disturbance intensity at
Figure 5.16 Location Of Initial Hot Wire Position
each hot wire station through the boundary layer. After printing out the twelve mean velocities corresponding to the first twelve clock gauge intervals the program halted. The points through which the $y$ against $U$ straight line was to be fitted, were then given to the P.D.P.8 from a teletype, and the program restarted. The $y$ zero position was then found and each clock gauge reading turned into the true $y$ value. A numerical integration of

$$\delta_1 = \int_0^\infty (1 - \frac{U}{U_0}) dy$$

was carried out and the boundary layer thickness $\delta$ found from the relation $\delta = \delta_1 / 0.350$.

Finally a print out of $y/\delta$, $U/U_0$ and $u'/u_0$ was given.

It is pointed out that the relation between $\delta$ and $\delta_1$ differs from the relation $\delta = \delta_1 / 0.341$ used by Schubauer and Skramstad (1947) and Kersley (1965). However, the present formula has been derived from the most modern data available, Rosenhead (1963), and is believed to be more accurate than that used by previous workers.

5.10 The Measured Amplitude Distribution Through the Boundary Layer

The amplitude distributions through the boundary layer at various $x$ stations between branches I and II of the neutral curve for the same windspeeds and frequencies as the amplification curves of Figs. 5.6, 5.8 and 5.9 are shown in Figs. 5.17 to 5.19 for $F = 80 \times 10^{-6}$, in Figs 5.20 to 5.22 for $F = 107 \times 10^{-6}$ and in Figs. 5.23 and 5.24 for $F = 155 \times 10^{-6}$. The ribbon current
was different for each distribution, and is given in terms of $Rb_0$, the smallest ribbon amplitude used in any set of distributions. The distributions can be scaled linearly to any one ribbon amplitude. The theoretical distributions at the same conditions as each experimental distribution have been evaluated by Jordinson. These are shown by the solid lines in Figs. 5.17 to 5.24.

In order to compare the experimental and theoretical results it was necessary to scale the normalised theoretical results to each experimental distribution. It has been shown by Jordinson (1969) that the r.m.s. disturbance velocity $\sqrt{u'^2}$ is related to the derivatives of the real and imaginary parts of the eigenfunctions by

$$\sqrt{u'^2} = \frac{1}{\lambda \omega_1} e^{-(a_1 \delta_1)x} \sqrt{\frac{\beta_1^2 r^2 + \beta_2^2}{\lambda \omega_1 \delta_1}},$$

where $\frac{1}{\lambda \omega_1} e^{-(a_1 \delta_1)x}$ is the constant required to compare the theoretical calculations of $\sqrt{u'^2}$ with the experimental values of $\sqrt{u'^2}$. This constant was found by equating the values of $\frac{1}{\lambda \omega_1} \int_0^1 \sqrt{u'^2} \, d(Y/\delta)$ and

$$\frac{1}{\lambda \omega_1} \int_0^1 \frac{\beta_1^2 r^2 + \beta_2^2}{\lambda \omega_1 \delta_1} \, d(Y/\delta).$$

It is difficult to estimate the errors associated with the experimental distribution. The boundary layer thickness is accurate to $\pm 8\%$ as discussed in the next chapter and this is likely to be the main source of error. However in the region very near the plate the sensitivity of the hot wire changes very rapidly with the resistance of the wire, and small errors of the order of 1 part in 1000 in
Figure 5.17 Distribution Of Fundamental Disturbance Through The Boundary Layer
Figure 5.18 Distribution Of Fundamental Disturbance Through The Boundary Layer
Figure 5.19 Distribution Of Fundamental Disturbance Through The Boundary Layer

- $F = 83.5 \times 10^{-4}$
- $R = 1396$
- $U = 29.0$ Ft/Sec
- $\theta = 164.4 \times 10^2$
- THEORETICAL
- RIBBON = $1.67R$

- $F = 80.7 \times 10^{-4}$
- $R = 1555$
- $U = 29.0$ Ft/Sec
- $\theta = 159.0 \times 10^2$
- THEORETICAL
- RIBBON = $1.67R$
Figure 5.20 Distribution of Fundamental Disturbance Through The Boundary Layer
Figure 5.21 Distribution Of Fundamental Disturbance Through The Boundary Layer

- $F = 111.6 \times 10^{-6}$
- $R = 770$
- $U = 19.4$ Ft/Sec
- $\omega = 158.3 \times 10^{-6}$
- THEORETICAL RIBBON 1.27$Re_0$

- $F = 110.2 \times 10^{-6}$
- $R = 913$
- $U = 19.5$ Ft/Sec
- $\omega = 158.6 \times 10^{-6}$
- THEORETICAL RIBBON 1.14$Re_0$
Figure 5.22 Distribution of Fundamental Disturbance Through The Boundary Layer

- Theoretical RIBBON B
- Theoretical RIBBON 1.05/9

Parameters:
- F = 112.0 \times 10^{-6}
- R = 1025
- U = 19.32 Ft/Sec
- \nu = 158.4 \times 10^{-6}
Figure 5.23 Distribution Of Fundamental Disturbance Through The Boundary Layer

- \( F = 106.2 \times 10^{-6} \)
- \( R = 1146 \)
- \( U = 19.9 \text{ Ft/Sec} \)
- \( \delta = 159.4 \times 10^{-6} \)

Theoretical Ribson 1.05Rb₀

- \( F = 111.7 \times 10^{-6} \)
- \( R = 1240 \)
- \( U = 19.4 \text{ Ft/Sec} \)
- \( \delta = 159 \times 10^{-6} \)

Theoretical Ribson 1.05Rb₀
Figure 5.24 Distribution Of Fundamental Disturbance Through The Boundary Layer
Figure 5.25 Distribution Of Fundamental Disturbance Through The Boundary Layer
the reading of the mean hot wire voltage can lead to relatively large errors in the magnitude of $u'$. It is thought that this may be the reason for the considerable scatter in the experimental points near the peaks of the amplitude distributions.

All the distributions show the characteristic peak near to the plate and a reversal of phase in the region $0.65 < \frac{y}{\delta} < 0.8$. The inwards shift of the peak between branches I and II of the neutral curve takes place gradually and moves from $\frac{y}{\delta} = 0.3$ to $\frac{y}{\delta} < 0.2$. The shift is greater at the lower values of $F$. At $F = 80 \times 10^{-6}$ the peak moves inwards to a $\frac{y}{\delta}$ position of 0.11, whereas for $F = 160 \times 10^{-6}$ the peak moves to a $\frac{y}{\delta}$ position of 0.18. The inwards shift of the phase change in general follows the same pattern as that of the peak.

The agreement between theory and experiment in most cases is excellent. The shifts in the position of the peaks and the phase changes are predicted quite accurately. Where there is disagreement, the tendency is for the experimental curves to be slightly to the right of the theoretical curves. This tendency may perhaps be explained if the results of the next section are anticipated. The inward shifting of the general shape of the distribution will be shown to depend on the position of the distribution relative to the neutral stability curve rather than on the value of Reynold's number. Further it will be shown in Chapter 6 that the experimental curve of neutral stability
is wider in the Reynolds's number range than the theoretical curve. Thus the amplitude distribution at any $R$ value is further upstream from branch II of the neutral curve than would be predicted by theory. As the distribution shape moves nearer the plate as branch II is approached, it might be expected that the experimental results would tend to be to the right of the theoretical.

5.11 The Variation of Amplitude Distribution with $F$ at Constant Reynolds Number

The amplitude distributions at a Reynolds number of 728 for the same windspeed = 19.5 ft. per sec. and for $F = 107.5, 140.4, 191.9$ and 230.3 are shown in Figs. 5.25 and 5.26. The distribution for $F = 107.5$ lies near branch I of the neutral stability curve and that for $F = 230.3$ lies near branch II.

As the value of $F$ is increased the shape of the distribution changes in the same way as it does with increasing Reynolds's number at a constant value of $F$. It thus appears that the change in shape of the amplitude distributions depends on the position relative to the neutral stability curve rather than Reynolds number. It is in fact likely that the value of $a_1 \delta_1$, the amplification factor, and the distribution shape are correlated, this being the main parameter which varies appreciably, apart
Figure 5.26 Distribution Of Fundamental Through The Boundary Layer
Figure 5.27 Distribution Of Fundamental Through The Boundary Layer
from Reynolds number, between the two branches of the neutral curve.

5.12 The Relationship Between Growth Curves and Amplitude Distributions

In each of the three sets of amplitude distributions shown in Figs. 5.17 to 5.24 one of the distributions is at a Reynolds number very close to the minimum of the corresponding set of growth curves, Figs. 5.6 to 5.9. It was thus possible to calculate from the amplitude distributions the values of $\ln \frac{A}{A_o}$ at any $Y/\delta$ position at the Reynolds numbers where we have an amplitude distribution. As the distributions were not all measured with the same ribbon amplitude, it was necessary to scale all the distributions of each set to the same ribbon amplitude.

The values of $\ln \frac{A}{A_o}$ from the distributions are shown, with the relevant growth curves, in Figs. 5.6 to 5.9. For clarity the $\ln \frac{A}{A_o}$ values from the distributions and growth curves are shown in Tables 5.1 to 5.3, for $F = 81.3 \times 10^{-6}$, $107 \times 10^{-6}$ and $154.9 \times 10^{-6}$. The small variations in the value of $F$ between the distributions of any one set have been neglected.

In all three cases the results are in remarkable agreement with the experimental growth curves.
TABLE 5.1
Comparison of Experimental Growth Curve with Growth Curves Built from Amplitude Distributions

\[ F = 107 \times 10^{-6} \]

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<thead>
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<th>( R_\delta )</th>
<th>( \ln \frac{A}{A_0} ) from distributions</th>
<th>( \ln \frac{A}{A_0} ) from growth curves</th>
<th>( \ln \frac{A}{A_0} ) from distributions</th>
<th>( \ln \frac{A}{A_0} ) from growth curves</th>
<th>( \ln \frac{A}{A_0} ) from distributions</th>
<th>( \ln \frac{A}{A_0} ) from growth curves</th>
<th>( \ln \frac{A}{A_0} ) from distributions</th>
<th>( \ln \frac{A}{A_0} ) from growth curves</th>
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<td>2.33</td>
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<td>1.66</td>
<td>1.55</td>
<td>0.87</td>
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<tr>
<td>1555</td>
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</table>
### TABLE 5.2

Comparison of Experimental Growth Curve with Growth Curves Built from Amplitude Distributions

\( F = 107 \times 10^{-6} \)

<table>
<thead>
<tr>
<th>( R )</th>
<th>( \ln \frac{A}{A_0} ) from distributions</th>
<th>( \ln \frac{A}{A_0} ) from growth curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>658</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>770</td>
<td>0.40</td>
<td>0.37</td>
</tr>
<tr>
<td>913</td>
<td>1.05</td>
<td>1.02</td>
</tr>
<tr>
<td>1025</td>
<td>1.43</td>
<td>1.41</td>
</tr>
<tr>
<td>1110</td>
<td>1.65</td>
<td>1.64</td>
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<tr>
<td>11116</td>
<td>1.03</td>
<td>1.40</td>
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<tr>
<td>12110</td>
<td>1.75</td>
<td>1.75</td>
</tr>
<tr>
<td>110</td>
<td>0.40</td>
<td>0.37</td>
</tr>
<tr>
<td>913</td>
<td>1.05</td>
<td>1.02</td>
</tr>
<tr>
<td>1025</td>
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<td>1.41</td>
</tr>
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<td>1110</td>
<td>1.65</td>
<td>1.64</td>
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<tr>
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<td>1.75</td>
<td>1.75</td>
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<tr>
<td>110</td>
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<td>0.37</td>
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<tr>
<td>913</td>
<td>1.05</td>
<td>1.02</td>
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<tr>
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<td>1.43</td>
<td>1.41</td>
</tr>
<tr>
<td>1110</td>
<td>1.65</td>
<td>1.64</td>
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<td>11116</td>
<td>1.03</td>
<td>1.40</td>
</tr>
<tr>
<td>12110</td>
<td>1.75</td>
<td>1.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( y/\delta )</th>
<th>( \ln \frac{A}{A_0} ) from distributions</th>
<th>( \ln \frac{A}{A_0} ) from growth curves</th>
</tr>
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<tbody>
<tr>
<td>0.125</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.200</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>0.300</td>
<td>0.14</td>
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<tr>
<td>0.415</td>
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</table>
## TABLE 5.3

Comparison of Experimental Growth Curve with Growth Curves Built from Amplitude Distributions

\[ F = 155 \times 10^{-6} \]

<table>
<thead>
<tr>
<th>( R_x )</th>
<th>( y/\delta = 0.09 )</th>
<th>( y/\delta = 0.200 )</th>
<th>( y/\delta = 0.317 )</th>
<th>( y/\delta = 0.417 )</th>
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</thead>
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<td>( \ln \frac{A}{A_0} ) from distributions</td>
<td>( \ln \frac{A}{A_0} ) from growth curves</td>
<td>( \ln \frac{A}{A_0} ) from distributions</td>
<td>( \ln \frac{A}{A_0} ) from growth curves</td>
</tr>
<tr>
<td>547</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>659</td>
<td>0.39</td>
<td>0.36</td>
<td>0.23</td>
<td>0.22</td>
</tr>
<tr>
<td>783</td>
<td>0.97</td>
<td>0.93</td>
<td>0.68</td>
<td>0.57</td>
</tr>
<tr>
<td>827</td>
<td>0.99</td>
<td>1.06</td>
<td>0.44</td>
<td>0.26</td>
</tr>
</tbody>
</table>
5.13 Growth Curves Along the Streamlines of the Boundary Layer

As the fluid particles in the boundary layer follow the streamlines given by $\psi = (2\omega U_0)^{\frac{1}{2}} \frac{1}{\gamma} f(\gamma) = \text{const}$, as defined for the Blasius integration of the equations of motion, it was felt that consideration of the growth curves along lines of constant $\psi$, might yield further information on the variation of the parameter $\frac{\gamma}{L}$. It was possible to calculate several points on such growth curves from the experimental growth curves at constant $\frac{y}{\delta}$ and the relevant amplitude distributions.

The calculation was carried out as follows. For given values of $F_0$, growth curves have been measured at four different $\frac{y}{\delta}$ positions. The value of $\psi$, was calculated, at the first point on each curve, $\gamma$ and hence $f(\gamma)$ being found from Rosenhead (1963) p. 224. At every experimental point on each of the curves the value of $f(\gamma)$ required to keep $\psi$ equal to its initial value was calculated and again from Rosenhead the corresponding values of $\gamma$ and $\frac{y}{\delta}$ were found. Thus a series of $\frac{y}{\delta}$ positions in the downstream direction were found which traced out lines of constant $\psi$, which at the first point of each experimental curve corresponded to the $\frac{y}{\delta}$ position of that curve.

In order to construct the growth curve at constant $\psi$, the value of $\frac{\delta n A/A_0}{\text{Reynolds number} (R_\delta)^2}$ was found by taking the value of $A_0$ from the measured
amplitude distribution at the Reynolds number \((R_\delta)_1\), of the minimum of the experimental curve, at the initial \(y/\delta\) position and the value of \(A\) from the amplitude distribution for \((R_\delta)_2\), at the \(y/\delta\) position calculated at \((R_\delta)_2\) to keep \(\psi\) equal to its value at \((R_\delta)_1\). The points calculated in this way are shown with the growth curves in Figs. 5.6 to 5.9 and in Tables 5.4 to 5.6. It is clear that the amplification of a disturbance, and hence the amplification factor, varies with \(\psi\), in much the same way as with \(y/\delta\).

The same type of analysis was used to extrapolate the experimental growth curves to growth curves along lines at fixed distances \(y\) from the plate. In keeping \(y\) constant the \(y/\delta\) positions for initial values of \(y/\delta < 0.25\) became so small that they lay on the innermost part of the amplitude distributions \(y/\delta < 0.1\), where the shape of the distribution is uncertain and of very large gradient, so that in general the extrapolation could only be performed for the outer two experimental growth curves in each set. However, the indications were that the growth curves at constant \(y\) position varied with \(y\).

5.14 Discussion of Results

It is convenient to look at the various aspects of this work in a different order to that in which it has been described.
### TABLE 5.4

Growth Curves on Constant Streamline $\psi_1$ for $F = 81.4 \times 10^{-6}$

<table>
<thead>
<tr>
<th>$R_\delta$</th>
<th>$\psi_1 \times 10^2$</th>
<th>$y/\delta$</th>
<th>$\delta n/A_0$</th>
<th>$\psi_1 \times 10^2$</th>
<th>$y/\delta$</th>
<th>$\delta n/A_0$</th>
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<tr>
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<td>0.215</td>
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<td>0</td>
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<td>0.593</td>
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<tr>
<td>1210</td>
<td>0.215</td>
<td>0.067</td>
<td>1.99</td>
<td>0.593</td>
<td>0.11</td>
<td>1.99</td>
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<td>0.593</td>
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<td>0.593</td>
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<tr>
<td>1555</td>
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<td>2.29</td>
<td>0.593</td>
<td>0.10</td>
<td>2.03</td>
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</table>
TABLE 5.5

Growth Curves on Constant Streamline $\psi$ for $F = 107 \times 10^{-6}$

<table>
<thead>
<tr>
<th>$R_s$</th>
<th>$\psi \times 10^2$</th>
<th>$y/\delta$</th>
<th>$\epsilon$/A/A$_o$</th>
<th>$\psi \times 10^2$</th>
<th>$y/\delta$</th>
<th>$\epsilon$/A/A$_o$</th>
<th>$\psi \times 10^2$</th>
<th>$y/\delta$</th>
<th>$\epsilon$/A/A$_o$</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>1.957</td>
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<td>0</td>
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<tr>
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<td>0.346</td>
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<td>0.17</td>
<td>1.957</td>
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<td>0.13</td>
</tr>
<tr>
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<td>0.77</td>
<td>1.957</td>
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<td>0.69</td>
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<td>0.086</td>
<td>0.92</td>
<td>0.874</td>
<td>0.13</td>
<td>0.72</td>
<td>1.957</td>
<td>0.20</td>
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</tbody>
</table>
### TABLE 5.6

*Growth Curves on Constant Streamline $\psi$, for $F = 154.9 \times 10^{-6}$*

<table>
<thead>
<tr>
<th>$R_\omega$</th>
<th>$\psi, x 10^2$</th>
<th>$y/\delta$</th>
<th>$\ell nA/A_0$</th>
<th>$\psi, x 10^2$</th>
<th>$y/\delta$</th>
<th>$\ell nA/A_0$</th>
<th>$\psi, x 10^2$</th>
<th>$y/\delta$</th>
<th>$\ell nA/A_0$</th>
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</thead>
<tbody>
<tr>
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<td>0.090</td>
<td>0</td>
<td>0.813</td>
<td>0.20</td>
<td>0</td>
<td>0.22</td>
<td>0.29</td>
<td>0.25</td>
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<tr>
<td>659</td>
<td>0.167</td>
<td>0.081</td>
<td>0.25</td>
<td>0.813</td>
<td>0.18</td>
<td>2.04</td>
<td>0.32</td>
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<tr>
<td>783</td>
<td>0.167</td>
<td>0.074</td>
<td>0.81</td>
<td>0.813</td>
<td>0.16</td>
<td>2.04</td>
<td>0.26</td>
<td>0.64</td>
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</tr>
<tr>
<td>827</td>
<td>0.167</td>
<td>0.072</td>
<td>0.77</td>
<td>0.813</td>
<td>0.16</td>
<td>0.43</td>
<td>2.04</td>
<td>0.25</td>
<td>0.35</td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>
The variation of wavelength at constant non-dimensional frequency $F$ with Reynolds number, Fig. 5.16 shown as the variation of $a'R^{-1}$ is in close agreement with theory. It is significant that where the experimental results differ from theory they lie below the theoretical results. Now the calculations of Barry and M.A.S. Ross, as pointed out in Section 4.3, take into account the $V$ component of velocity in the boundary; as a result of this their values of $a'R^{-1}$ are smaller than the values computed by neglecting the $V$ component of velocity. Thus it would appear that the latest calculations, taking into account the largest of the terms containing $V$ in the equations of motion, give better agreement with experiment than older calculations which neglect the $V$ component of velocity.

It has been shown that along lines of constant $\frac{y}{\delta}$ and constant value of stream function $\psi$, no unique amplification curve can be defined. The amplification factor $a_1 \delta_i$ has been shown to be a function of $\frac{y}{\delta}$. This variation can be predicted from the various sets of amplitude distributions which are shown and are in excellent agreement with theory.

Let the situation be viewed firstly from the point of view of a simple disturbance amplification phenomenon. The amplification factor is greatest near to the plate and falls off as $\frac{y}{\delta}$ increases. As a disturbance is amplified downstream the amplification is greatest at $\frac{y}{\delta}$ values near the plate. Consequently the inward shift of
the peak of the amplitude distribution curves, initially at a \( \frac{y}{\delta} \) value = 0.3, is to be expected. If one tries to look at the inwards shift of the phase change positions of the amplitude distribution in the same way, a very peculiar result is obtained. Some line of constant \( \frac{y}{\delta} \) initially just nearer the plate than the phase change will eventually become further away from the plate than the phase change, which moves to smaller values of \( \frac{y}{\delta} \) as \( x \) increases. Thus along that line of constant \( \frac{y}{\delta} \) the amplification factor \( a_1 \delta_1 \) must at first be positive, the disturbance being initially damped and then becoming negative, within the neutral curve, as the line of constant \( \frac{y}{\delta} \) passes through the phase change position, when the disturbance is once more amplified. As the direction of the disturbance velocities is reversed on moving through the phase change position in the \( y \) direction, the disturbance velocity must at some \( y \) position be zero. The point where the amplification factor changes sign along a line of constant \( \frac{y}{\delta} \) must in fact be a singularity in \( a_1 \delta_1 \). If the idea of simple streamwise disturbance amplification is valid, there must be a line of singularity in \( a_1 \delta_1 \) which follows the path traced by the phase change positions in the \( x \) direction.

Now let us look at the problem as an eigenvalue problem. From the amplitude distributions shown in Figs. 5.17 to 5.25, it is evident that the derivatives of the eigenfunctions, and presumably the eigenfunctions, of the
equations of motion are accurately predicted by theory for any eigenstate \((a_1 R_\delta)\). It is strange that the amplitude distributions 

\[ u' = A' \sqrt{2} e^{-\frac{(a_1 \delta_1)}{\sqrt{\beta^2 + \beta^2_1}}} \]

are accurately predicted by theory if \(a_1\) is not independent of \(y\).

It would appear that as the small disturbance problem is basically an eigenvalue problem and as the correct velocity distributions are predicted from the eigenfunctions calculated by Jordinson that \(a_1\) must be independent of \(y\) at any \(x\) station. The only obvious way of making \(a_1\) independent of \(y\) is to assume that \(a_1\) can only be defined along the fractional height contours of the velocity distributions. Thus if the problem is thought of as disturbance amplification in the \(x\) direction, the amplification can only be considered along contour lines of \(\rho(y)\). The line following the highest point of \(\rho'(y)\) at varying \(R\) is the simplest example of such a contour. Viewed in this way the line of phase change is a nodal line of the eigenvalue problem. A boundary layer map, Fig. 5.31, illustrates the valid amplification paths.

From the experimental amplitude distributions the growth curves along the amplitude distribution peaks have been calculated. These are shown in Figs. 5.28, 5.29 and 5.30. The results for \(F = 81.3 \times 10^{-6}\) and \(107 \times 10^{-6}\) are in quite good agreement with the theoretical curves of Jordinson. The result for \(F = 154.9 \times 10^{-6}\) is at first sight not in good agreement with the theoretical curve. It
Figure 5.28 Final Growth Curves Compared With Theory
Figure 5.29  Final Growth Curves Compared With Theory
Figure 5.30 Final Growth Curves Compared With Theory
Figure 5.31 Boundary Layer Map, Showing Paths Along Which Amplification Factor Is Defined
should be remembered, however, that the total amplification at any given point on a growth curve is the cumulative amplification in the streamwise direction from the first neutral point, where $\alpha_1 S_1 = 0$. Now the growth curve at $F = 154.9 \times 10^{-6}$, Fig. 5.30, lies considerably to the left of the theoretical curve, and as the amplification is small, it is not surprising that good agreement with theory is not obtained. Discrepancies due to the neutral points not coinciding with those of theory at the other two values of $F$ are masked by the much larger total amplifications. The dashed curve of Fig. 5.30 shows the theoretical curve of Jordinson shifted to the left so that the branch I point coincides with the experimental branch I point. The agreement between the shifted theoretical curve and experiment is as good as for the other two values of $F$.

As the disturbance behaviour in the $y$ direction is in very good agreement with theory, whilst it is difficult to obtain agreement with theory in respect of the streamwise behaviour, it is interesting to look at the expression which has been used to derive the theoretical growth curves. It has been assumed that the separable solution $\phi(y)e^{i(\alpha x - \beta t)}$ of the Orr-Sommerfeld equation adequately represents the behaviour of small disturbances in a boundary layer. It is evident from the changing shape of the experimental velocity distributions and from the theoretical eigenfunctions calculated by Jordinson that $\phi'$ is a function of both $x$ and
If $\phi$ is taken as $\phi(x,y)$ rather than $\phi(y)$, then the relation

$$\ln \frac{A}{A_0} = \frac{2}{m^2} \int_{R_0}^{R} (-a_1 \delta_1) dR$$

used in the derivation of the theoretical growth curves becomes

$$\ln \frac{A}{A_0} = \frac{2}{m^2} \int_{R_0}^{R} (-a_1 \delta_1) dR + \ln \frac{\phi'}{\phi_0}.$$  

Thus the theoretical growth curves given in terms of $\ln \frac{A}{A_0}$ do not strictly represent the growth of the disturbance in the streamwise direction.

It would therefore seem unlikely that complete agreement between theory and experiment in respect of the streamwise behaviour of the disturbances can be obtained, unless a solution of the Orr-Sommerfeld equation in the form $\phi(x,y)e^{i(\alpha x - \beta t)}$ is sought.

5.15 Conclusion

It has been shown that the behaviour of small disturbances, growing spatially in a boundary layer, is not completely described by the existing numerical solutions of the Orr-Sommerfeld equation. The dependence of the disturbances on the coordinate $y$ is accurately predicted by theory. It has been found, however, that there is difficulty in defining the amplification factor $(\alpha_1 \delta_1)$. 
A definition of the amplification factor so that it is independent of the coordinate $y$, as required by the basic assumptions of theory, has been given. If $\alpha_1$ is only defined along the fractional contour lines of the distribution of disturbance velocity across the boundary layer, then growth curves, in fair agreement with the theoretical growth curves of Jordinson are obtained. Complete agreement between the experimental and theoretical streamwise behaviour of the disturbances cannot be expected in view of the dependence of the eigenfunction derivatives $\phi'$ on the coordinate $x$.

The results of the latest theoretical calculations, taking into account the $V$ component velocity in the boundary layer, give slightly better agreement with experiment than the previous calculations.
CHAPTER 6

THE NEUTRAL STABILITY CURVE

6.1 The Neutral Stability Curve

One of the best ways of comparing theory and experiment is undoubtedly in a study of the neutral stability curve. In this way one views, at once, the whole range of linearised theory in the \((F, R)\) plane, rather than small sections as is necessary with the phenomena already studied.

The neutral stability curve is the locus of points such that \(\frac{dA}{dR} = 0\) or \(c_1 \delta_1 = 0\) in the \((F, R)\) plane. As explained in Chapter 4, the points on this curve can be obtained from individual growth curves.

The r.m.s. amplitude \(A\) of a small disturbance was measured at twelve or more positions in the region of either branch of the curve, the interval between any two measurements being not greater than 0.75 inches. A cubic curve was fitted to the \((A, x)\) points assuming no error in \(x\), which has been shown in Chapter 3 to be \(\pm 0.05\)" and is considerably less than the error in the measurement of \(A\), and the maximum or minimum of this curve was taken as a point on the neutral stability curve. Experience showed that it made little difference to the results if a cubic or fifth order polynomial was fitted to the results.

The location of branch II was relatively straightforward. The main difficulty was in deciding the highest value of \(F\) at which amplification of the disturbance was observed.
because of the large error in the measurement of high values of F. Branch I proved more difficult to locate. It was necessary to use low windspeeds to obtain Reynolds numbers low enough, that the minimum point on any growth curve was at least 5 inches downstream of the ribbon. For all values of $F > 150 \times 10^{-6}$ on branch I, it was necessary to vibrate the ribbon at $<20$ cycles per second, which was below the lower cut-off of the frequency analyser. The hot wire output was therefore prerecorded on E.M.I. 16 track recording tape at a speed of $1\frac{7}{8}$ inches per second and then played through the frequency analyser, which was set to 4 times the injection frequency, at a speed of $7\frac{1}{2}$ inches per second.

The neutral stability curve is shown in Fig. 6.1 compared with the theoretical curve of Shen (1954), in Fig. 6.2 compared with the theoretical curve of Barry and M.A.S. Ross (1969), and in Fig. 6.3 compared with the experimental results of Schubauer and Skramstad (1947). The neutral stability points of both branches I and II are tabulated in Table 6.1.

The error in the measurement of $F = \frac{\beta r}{U_0} \rho / U_0^2$ is almost entirely due to the error in the measurement of $U_0$. It has been pointed out in Chapter 5, Section 2, that the error in $U_0$ varies between $\pm 4\%$ and $<1\%$ as $U_0$ varies between 10 and 40 ft. per second. This leads to errors of $\pm 8\%$ in the value of $F$ near the top of the curve but only of some $2\%$ at most at the lowest values of $F$. The
Figure 6.1 Neutral Stability Curve Compared With Theory
Figure 6.2 Neutral Stability Curve Compared With Theory
Figure 6.3 Neutral Stability Curve Compared With That Of Schubauer And Skramstad
TABLE 6.1(a)

Branch I of the Neutral Stability Curve

<table>
<thead>
<tr>
<th>$F \times 10^6$</th>
<th>$R_9$</th>
<th>$F \times 10^6$</th>
<th>$R_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>288</td>
<td>374</td>
<td>97.9</td>
<td>720</td>
</tr>
<tr>
<td>253</td>
<td>383</td>
<td>97.2</td>
<td>689</td>
</tr>
<tr>
<td>240</td>
<td>381</td>
<td>91.2</td>
<td>693</td>
</tr>
<tr>
<td>238.1</td>
<td>397</td>
<td>91.2</td>
<td>689</td>
</tr>
<tr>
<td>260</td>
<td>376</td>
<td>93.8</td>
<td>720</td>
</tr>
<tr>
<td>230</td>
<td>417</td>
<td>85.4</td>
<td>767</td>
</tr>
<tr>
<td>212</td>
<td>451</td>
<td>85.4</td>
<td>753</td>
</tr>
<tr>
<td>193.6</td>
<td>426</td>
<td>76.6</td>
<td>781</td>
</tr>
<tr>
<td>188.3</td>
<td>449</td>
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<td>885</td>
</tr>
<tr>
<td>187</td>
<td>457</td>
<td>62.8</td>
<td>869</td>
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<tr>
<td>179</td>
<td>446</td>
<td>61.3</td>
<td>864</td>
</tr>
<tr>
<td>162</td>
<td>485</td>
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<td>877</td>
</tr>
<tr>
<td>148.9</td>
<td>537</td>
<td>43</td>
<td>1061</td>
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<tr>
<td>142.9</td>
<td>547</td>
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<td>129.9</td>
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<td>638</td>
<td>34.9</td>
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<td>115.6</td>
<td>653</td>
<td>31.9</td>
<td>1334</td>
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<tr>
<td>112</td>
<td>.9492</td>
<td></td>
<td></td>
</tr>
<tr>
<td>99.82</td>
<td>.9492</td>
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</table>
# TABLE 6.1(b)

Branch II of the Neutral Stability Curve

<table>
<thead>
<tr>
<th>$F \times 10^6$</th>
<th>$R_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>401.3</td>
<td>400</td>
</tr>
<tr>
<td>394.8</td>
<td>404</td>
</tr>
<tr>
<td>382.8</td>
<td>412</td>
</tr>
<tr>
<td>382.5</td>
<td>471</td>
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<td>313.8</td>
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<td>277.5</td>
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<td>568</td>
</tr>
<tr>
<td>263.2</td>
<td>589</td>
</tr>
<tr>
<td>260.2</td>
<td>606</td>
</tr>
<tr>
<td>245.3</td>
<td>716</td>
</tr>
<tr>
<td>240.6</td>
<td>771</td>
</tr>
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<td>205.7</td>
<td>951</td>
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</tr>
<tr>
<td>125.8</td>
<td>1123</td>
</tr>
<tr>
<td>108.5</td>
<td>1211</td>
</tr>
<tr>
<td>98.6</td>
<td>1256</td>
</tr>
<tr>
<td>95.1</td>
<td>1476</td>
</tr>
<tr>
<td>75.2</td>
<td>2063</td>
</tr>
<tr>
<td>48.6</td>
<td>1.0089</td>
</tr>
<tr>
<td>1023</td>
<td>1.0504</td>
</tr>
<tr>
<td>98.6</td>
<td>1.0831</td>
</tr>
<tr>
<td>95.1</td>
<td>1.0990</td>
</tr>
<tr>
<td>75.2</td>
<td>1.1671</td>
</tr>
<tr>
<td>48.6</td>
<td>1.3145</td>
</tr>
</tbody>
</table>

Note: The values in the table are approximate.
error in $R$ due to error in $U_0$ varies between $\pm 2\%$ and $\pm \frac{1}{2}\%$ for windspeeds between 10 and 40 ft. per second. In most cases it is also necessary to consider the uncertainty in the location of the maximum or minimum of the growth curves. At the higher regions of the curve the hot wire signals were unsteady, and it was in general not possible to locate the minimum to more than an accuracy of $\pm 1$ measurement interval, i.e. $\pm \frac{3}{4}$ inch. At the lower regions of the curve it was found that the growth curves had broad turning points where the signal varied slowly and it was again only possible to locate the turning point to an accuracy of $\pm \frac{3}{4}$ inch. This represents an error in Reynold's number of $\pm 3\%$ where the turning point of the growth curve is at $x = 1$ ft. dropping to $\pm \frac{3}{4}$ where the turning point is at $x = 4$ ft. The two sources of error in Reynold's number are additive. Both the error in $F$ and $R$ vary with position in the $(F, R)$ plane, and to some extent, with the windspeed used and the turning point location for each point. The errors in each section of the curve are shown to scale in Figs. 6.1, 6.2 and 6.3.

The neutral stability curve when compared with the theoretical curve of Barry and M.A.S. Ross shows some large deviations. The most obvious is the difference in the upper limit of the curves, the theoretical curve rising only to $F = 260 \times 10^{-6}$ whereas amplification was observed experimentally up to $F = 400 \times 10^{-6}$. Branch 1 generally lies considerably to the left of the theoretical results, at high values of $F$. At low values of $F$ theory and experiment are in good agreement. As $F$ increases the
experimental results steadily diverge from the theoretical results. A critical Reynolds number of about 370 has been found from experiment as opposed to a theoretical value of 498. Branch II is in much better agreement with theory than Branch I. In the lower regions agreement is good. At higher values of \( F \), because of the much higher total height of the experimental curve, the theoretical curve lies at lower Reynolds numbers than the experimental results.

When the experimental results are compared with the neutral curve of Shen, better agreement is obtained with branch I, but the agreement with branch II is not as good, the experimental results now lying further away on the low Reynolds number side. The critical Reynolds number is still too low, 370 as opposed to a predicted 420 and the top of the curve is still too high.

6.2 The Influence of \( y \) Position on the Neutral Curve

The neutral stability curve shown in Figs. 6.1 to 6.3 was located at a \( y/\delta \) position of 0.15.

As the growth of disturbances along lines of constant \( y/\delta \) has been shown to vary with \( y/\delta \), a search was made for any variation in the neutral curve with the \( y/\delta \) position at which it was located.

Fig. 6.4 shows the growth curves in the region of branch I of the neutral curve for \( F = 37.9 \times 10^{-6} \) at \( y/\delta = 0.175, 0.30 \) and \( 0.375 \), where \( A_o \) is the amplitude at the minimum of the curve. Fig. 6.5 shows the growth curves in
Figure 6.4 Branch I Of The Neutral Stability Curve At Different $y/\delta$ Positions
Figure 6.5 Branch II Of The Neutral Stability Curve At Different $y/\delta$ Positions
the region of branch II of the neutral curve for 
\[ F = 103.9 \times 10^{-6} \text{ at } \frac{y}{\delta} = 0.157, 0.220, \text{ and } 0.285, \]
where \( A_0 \) is the amplitude at the maximum of the curve. The points on both branches of the neutral curve, within the experimental error do not appear to vary with the \( \frac{y}{\delta} \) position used for their location.

6.3 The Neutral Curve at Constant Distance \( y \) from the Plate

The neutral curve of Schubauer and Skramstad (1947) was found by tracking a hot wire at a constant distance \( y \) from the plate.

An investigation to find out if the neutral curve found in this way differs from that found at constant \( \frac{y}{\delta} \) position has been carried out. This was done by locating the maxima and minima of growth curves without adjusting the position of the wire at each reading. The movement of the carriage could not be guaranteed to run accurately parallel to the plate, but as the distance traversed was not large, it was felt that the accuracy was sufficient to show up any difference in the location of the neutral points found by tracking at constant \( \frac{y}{\delta} \), and at constant \( y \).

It was found that if the hot wire was tracked at constant \( y \) position in the region of the peak of the disturbance amplitude distribution, neither points on branch I or branch II were different from those found by tracking along lines of constant \( \frac{y}{\delta} \).
When the wire was tracked at constant $y$ such that it traced a path through the steeply sloping regions of the amplitude distributions on either side of the peak, the location of the neutral points depended on whether the wire was tracked inside or outside the distribution peak.

Consider first the region further from the plate than the peak. At branch I, as the wire is tracked downstream, $y/\delta$ decreases, and the wire moves nearer to the peak. The effect of this was to move the neutral point upstream or to a lower Reynolds number, than that found by tracking at constant $y/\delta$. If the wire is tracked upstream, the $y/\delta$ position in increased. The effect on the neutral position was again to push the neutral points to a lower Reynolds number. The situation is reversed if the wire is tracked nearer the plate than the amplitude distribution peak, the neutral points moving to a higher Reynolds number. At branch II the shifts in position of the neutral points were in the opposite directions to those at branch I. When the wire was tracked outside the distribution peak there was a shift to higher Reynolds numbers, and when it was tracked inside the peak a shift to lower Reynolds numbers.

In any case the magnitude of the shift depended on the steepness of the slope of the amplitude distribution in the region of tracking.

Fig. 6.6 shows a branch II point found by tracking in the region of the distribution peak, outside the distribution peak, and at a constant $y/\delta$ position.

In view of the work described in Chapter 5 it would
Figure 6.6 Branch II Of The Neutral Stability Curve At Constant $y/6$ Position Compared With Constant $y$ Position
appear that the correct neutral stability would be obtained by tracking along contours of the amplitude distributions. However as the distance tracked in the region of branch I and branch II was in general not more than 1 ft., and the change in distribution shape over this distance is small, traverses at constant $\frac{y}{\delta}$ are very close to being traverses along contours of the distributions. Differences in the neutral position when $\frac{y}{\delta}$ positions are used would thus not be expected. It is concluded that the neutral curve found at constant $\frac{y}{\delta}$ positions should be more accurate than one found at constant $y$ position.

6.4 The Boundary Layer Thickness $\delta_1$

The Reynolds numbers of the points on the neutral curve have been evaluated from the theoretical formula $1.7208 \sqrt{\frac{U_o x}{\nu}}$. The validity of this expression is based on the boundary layer displacement thickness $\delta_1$ being accurately given by $1.7208 \sqrt{\frac{x}{\nu}}$.

In view of the fact that the experimental results disagree with the theoretical, especially at branch I, the boundary layer thickness was measured directly, by means of boundary layer traverses near the leading edge of the plate.

The displacement thickness is difficult to measure accurately owing to the number of ways in which error may be introduced and the difficulty in combining the various
sources of error into some reasonable estimate of the likely error. It was found that the calibration of the hot wires used was accurate in both gradient and intercept to \( \pm 1\% \), yet experience showed that this error lead to an error in the mean velocity measurements, which gave up to \( \pm 4\% \) error in boundary layer thickness. The error of only \( \pm 1\% \) part in 1000 in the hot wire voltage readings used in the measurement of mean velocities was found to give rise to a possible \( \pm 2\% \) error in boundary layer thickness. An error in the final result of up to \( \pm 1\% \) could also arise from the \( \pm 0.002 \) inch error in the fixing of the \( y \) zero in the integration of the boundary layer thickness. Finally an error of \( \pm 1\% \) might arise from calibration of the hot wire movement against the clock gauge movement.

It is realised that these are the maximum errors in each case and that all are unlikely to contribute to their maximum values, but it is difficult to combine realistically the various sources of error, to give an estimate of the error in the final results. Thus associated with the measured boundary layer thickness \( S_i \) there is a maximum error of \( \pm 8\% \).

The measured boundary layer thicknesses are compared with the theoretical values of \( S_i \) in Table 6.2. Whilst all the measured thicknesses are smaller than the theoretical values, none lie outside the limits of experimental error. It is possible that the boundary layer on the flat plate is slightly thinner than the theoretical value due to
<table>
<thead>
<tr>
<th>$x \times 10^{6}$</th>
<th>$U_0$ ft./sec.</th>
<th>$\delta_1 = 1.72 \frac{x}{U_0}$ inches x $10^3$</th>
<th>Measured Boundary Layer Thickness inches x $10^3$</th>
<th>Difference between Theoretical and Measured $\delta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.81</td>
<td>162.0</td>
<td>16.58</td>
<td>58.1</td>
<td>56.1</td>
</tr>
<tr>
<td>0.83</td>
<td>157.3</td>
<td>16.36</td>
<td>58.4</td>
<td>54.4</td>
</tr>
<tr>
<td>1.08</td>
<td>157.7</td>
<td>16.04</td>
<td>67.3</td>
<td>63.7</td>
</tr>
<tr>
<td>1.42</td>
<td>157.7</td>
<td>16.04</td>
<td>77.2</td>
<td>74.1</td>
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<tr>
<td>1.92</td>
<td>157.7</td>
<td>16.04</td>
<td>89.6</td>
<td>86.5</td>
</tr>
<tr>
<td>2.37</td>
<td>162.0</td>
<td>16.58</td>
<td>99.3</td>
<td>94.9</td>
</tr>
<tr>
<td>2.87</td>
<td>162.0</td>
<td>16.58</td>
<td>109.2</td>
<td>104.9</td>
</tr>
</tbody>
</table>
the positive angle of attack of the flow at the leading edge of the plate.

The discrepancies between the measured and theoretical displacement thicknesses are not great enough to affect significantly the Reynolds numbers of the points on the neutral stability curve.
CHAPTER 7

WORK ON THE NON-LINEAR REGION

7.1 Introduction

The experimental study of small disturbances in the boundary layer on a flat plate has been completed and this chapter describes the results of an initial investigation of non-linear disturbances.

At present theorists at the University of Edinburgh are engaged in the development of the theory of small disturbances, to include larger non-linear disturbances.

7.2 The Non-Linear Region

When the r.m.s. level of a disturbance in the boundary layer exceeds a level of about $1\%$ of the free stream velocity $U_0$, it can no longer be described as small. The product terms in the equation of motion cease to be negligible, and higher harmonics of the fundamental disturbance are generated along with distortion of the mean flow profile.

This region was first investigated by Klebanoff and Tidstrom (1959) and by Klebanoff, Tidstrom and Sargent (1962). They concluded that even in the absence of measurable three-dimensionality in the mean flow, the process of breakdown was three-dimensional, and they therefore injected controlled three-dimensional disturbances. The non-linear region has also
been studied by Tani and Komoda (1961) who injected two-dimensional disturbances into a boundary layer with induced three-dimensionality.

In the present investigation, at least initially, interest is in the two-dimensional processes occurring in the non-linear region, as the initial theoretical results will be derived from the modified two-dimensional equation of motion, expanded to take into account the non-linear terms. Consequently no attempt was made to control the three dimensional process of transition.

7.3 The Spanwise Variation in Mean Flow

Klebanoff and Tidstrom (1959) found that the boundary layer thickness \( \delta \) varied by as much as \( \pm 10\% \) in a periodic manner in the z direction. Bradshaw (1965) has shown that a possible origin of this phenomenon lies in quite small perturbations of the flow through the smoothing screens upstream of the working section.

A surface pitot tube was traversed in the z-direction, to check if the velocity near the plate varied, which would indicate a variation of the boundary layer thickness in the z-direction. The velocities used in the non-linear work were always between 25 and 30 ft. per second and the mean velocities measured by the surface pitot tube were of order 5 ft. per second. From the sensitivity of the Chattock gauge it was calculated that only differences in boundary layer thickness of \( \pm 5\% \) could be detected. Within the
limits of experimental accuracy no variation in boundary layer thickness could be detected.

7.4 The Spanwise Variation of Disturbance Velocity

Fig. 7.1 shows the spanwise variation of disturbance amplitude at a fixed $x$ position = 3 ft., for two ribbon amplitudes. In one case the ribbon amplitude was chosen so that the disturbance was linear and in the other case, so that the disturbance was in the non-linear range.

In the linear case no variation of amplitude in the spanwise direction was detectable when the wire was tracked at a constant $y/\delta$ value. The signal levels could be read from the pen recorder to an accuracy of $\pm 2\%$.

With the larger signal a spanwise variation, up to $36\%$, in disturbance amplification was observed. This is in agreement with the work of Klebanoff, Tidstrom and Sargent (1962). The variation of the amplitude was periodic in the $z$-direction with wavelength about 0.8 inches.

All the work in this chapter was done at the $z$-position + 0.125 inches, where positive $z$ is above the centre line of the plate. At this $z$ station the disturbance amplitude was a maximum, or a 'peak' in the terminology of Klebanoff and Tidstrom (1959).
Figure 7.1 Spanwise Variation Of Amplification
Measurement of Ribbon Amplitude

Although the facility for measurement of ribbon amplitude was available as described in Chapter 2, it was not possible to measure the actual ribbon amplitude in this work. The disturbance frequency chosen was 60 cycles per second. The resonance frequency of the ribbon was about 140 cycles per second, and it was found that the power amplifier used to amplify the ribbon current could not supply enough power to make the ribbon strike the brass stud while oscillating at 60 cycles per second.

Consequently only the relative values of each ribbon amplitude are given.

The Streamwise Growth of the Disturbance

The streamwise growth of a disturbance for three ribbon amplitudes is shown in Figs. 7.2, 7.3 and 7.4. The smallest ribbon amplitude \( R_b_0 \) was small enough to ensure linear behaviour all the way downstream, and the linear growth curves are shown on a different scale from the non-linear ones. The other two amplitudes \( 3.2 R_b_0 \) and \( 6.4 R_b_0 \) produce non-linear growth curves, showing rapid amplification quickly leading to breakdown of the laminar boundary layer. The non-dimensional frequency \( F \) in each case was approximately \( 90 \times 10^{-6} \), the exact value being given with each of the curves. For each ribbon amplitude growth curves were obtained by tracking the hot wire at different
Figure 7.2 Streamwise Growth Of Fundamental And Total Disturbances
Figure 7.3 Streamwise Growth Of Fundamental And Total Disturbances
Figure 7.4 Streamwise Growth of Fundamental and Total Disturbances
$\gamma/\delta$ positions. $\gamma/\delta$ positions of nearly .14, .26 and .37 were used, the exact positions again being shown in the appropriate figures. The position of the wire was set at each $\gamma/\delta$ station with the ribbon switched off, as the mean velocity profile becomes distorted in the presence of large disturbances. The disturbance level is plotted against $R_e$ rather than $R_5$, as the boundary layer changes from the laminar to the thicker turbulent type during the transition process, making $R_5$ difficult to define.

The points where the first signs of turbulent bursts were observed are shown in each case. These bursts in the form of sharp negative spikes on a C.R.O. trace of the hot-wire output were most pronounced in the outer regions of the boundary layer, $\gamma/\delta \approx 0.6$, but could still be observed at the innermost $\gamma/\delta$ positions used in this work. This result is in agreement with the work of Schubauer and Klebanoff (1955) and Klebanoff, Tidstrom and Sargent (1962), who have shown that the first signs of turbulence appear in the outer regions of the boundary layer.

It is evident that the behaviour of the $u$ component and of the fundamental disturbance is different from that of the $u$ component of the total disturbance. Initially the fundamental and total disturbances are all but indistinguishable from each other and follow the behaviour of linearised theory. When the disturbance level is in excess
of 10/\% of the free stream velocity, the disturbance begins to grow more rapidly and components at other frequencies than the input frequency begin to appear. The growth patterns of the total signal at all three \( \frac{y}{\delta} \) positions are similar in shape to that observed by Klebanoff and Tidstrom (1959), exhibiting sharp growth prior to the transition region and falling to an almost constant level as fully developed turbulence is obtained. The fundamental disturbance at the outer two \( \frac{y}{\delta} \) positions rises to a much sharper peak, at a lower amplitude than the total signal, and immediately falls off sharply. The fundamental growth at the innermost \( \frac{y}{\delta} \) position is much less than that at the other two positions and exhibits two peaks. The first sign of turbulent spots at the outer \( \frac{y}{\delta} \) positions were found very close to the peak in the fundamental growth curve. In the case of the innermost \( \frac{y}{\delta} \) position the spots first appeared near the downstream peak.

Compared with the results of Klebanoff et al. (1959) and (1962) some differences are immediately apparent. The maximum levels of the total disturbance signal in the present case are lower. Klebanoff et al. found the maximum r.m.s. levels of total signal had an average value of 14\% of the free stream velocity, whereas the average value of the maximum in this case is 7.2\%. They also found that the signal continued to grow for about 4 inches downstream of the position at which the turbulent spots first appeared, and that the total transition region was
about 7 inches in length. In the present case the total signal level is found to increase very little after the appearance of the first turbulent spot, although the total transition length is comparable with that found by Klebanoff et al.

Also, if the results of the next section may be anticipated, large differences in the harmonic content of the disturbances prior to breakdown are found when compared with the results of the above workers. In the next section it will be shown that the differences in the levels of the total and fundamental signals can be largely accounted for by the development of higher harmonics of the fundamental. From the curves shown in Figs. 7.2, 7.3 and 7.4 it is apparent that by the first sign of breakdown the harmonic content of the disturbance is at least as great as the fundamental itself. In fact for the middle \( \frac{y}{s} \) value the harmonic content is almost equal to that of the fundamental, whilst in the other two cases the harmonic content is considerably greater than the fundamental. Klebanoff, Tidstrom and Sargent (1962) found that at transition the content of the second harmonic was only 20\% of the fundamental level and the content of third harmonic only 8\% of the fundamental level.

7.7 The Development of the Turbulent Spectrum

For the growth curve with ribbon amplitude 3.4 \( R_b \) and at the \( \frac{y}{s} \) position 0.365, shown in Fig. 7.4, the spectrum of the \( u \) component of disturbance velocity was
recorded at several x-stations with the frequency analyser set to the automatic analysis mode.

The frequency calibrated pen recording traces are reproduced in Figs. 7.5(a) to 7.5(e), where
\[ R_x = 25.5 \times 10^4, \ 51.8 \times 10^4, \ 62.4 \times 10^4, \ 70.2 \times 10^4 \text{ and } 78.4 \times 10^4. \]

The positions where the five spectra were observed were as follows.

a) In the linear region.
b) Near the end of the linear region.
c) In the non-linear region, near the position of maximum growth rate.
d) Downstream of the first appearance of turbulent spots.
e) In a position where the boundary layer was fully turbulent.

The results are shown on a logarithmic scale, referred to a standard signal of 100 M.V. which is shown in each trace. The hot-wire signals recorded on each trace have been amplified by different amounts by the frequency analyser before being fed to the pen recorder.

The traces illustrate the development of the completely turbulent spectrum, Fig. 7.5(e), from the initially linear disturbance, Fig. 7.5(a), which is nearly free of all frequencies other than the fundamental of 60 cycles per second. The small peak at 27 c/s is due to vibration of the boom. Figs. 7.5(b) and 7.5(c) show that as the disturbance proceeds to the non-linear region higher
harmonics of the fundamental disturbance are generated.

Fig. 7.5(b) shows that even by the end of the linear region there is evidence of harmonic development, the second harmonic amplitude is about 22 db down on or 7\% of the fundamental and the third harmonic 32 db down on or about 2\% of the fundamental. In the non-linear region the harmonic development is very strong and in 7.5(c) all harmonics up to the fifth are clearly visible, and there are indications of the sixth and seventh harmonics. The harmonic content is as follows:

2nd Harmonic 10 db down on Fundamental
3rd Harmonic 14 db down on Fundamental
4th Harmonic 16 db down on Fundamental
5th Harmonic 21 db down on Fundamental.

Trace 7.5(d) was obtained about 3 inches downstream of the appearance of the first turbulent spot. The spectrum is nearly that of a completely turbulent flow, except that a peak of about 5 db above the general level exists at the fundamental frequency. The trace in 7.5(e) shows a fully turbulent spectrum.

The development of the turbulent spectrum would appear to occur in a distinct manner. Firstly the initially single frequency disturbance generates harmonics of the fundamental. Just before breakdown the spectrum is still made up largely of the fundamental disturbance and its higher harmonics. It is only with the appearance of the first turbulent spots that the random frequencies associated with a turbulent spectrum make their appearance.

It is not clear why the abundant development of harmonics was not observed by Klebanoff, Tidstrom and Sargent.
Figure 7.5(a) Spectrum Of Hot Wire Signal
Figure 7.5(b) Spectrum Of Hot Wire Signal
Figure 7.5(c) Spectrum Of Hot Wire Signal
Figure 7.5(d) Spectrum Of Hot Wire Signal
Figure 7.5(e) Spectrum Of Hot Wire Signal
7.8 Distortion of the Mean Flow Profile and the Distribution of the Disturbance Through the Boundary Layer

The mean velocity profiles at a fixed Reynolds number $R_e = 1330$, based on the undisturbed boundary layer for $F = 90 \times 10^{-6}$, using ribbon amplitudes $R_b_0, 2R_b_0, 32R_b_0$, and $6.4R_b_0$ are shown in Figs. 7.6(a), 7.7(a), (7.8(a) and 7.9(a)). $R_b_0$ is the same ribbon amplitude as in Section 7.6. The traverses were done in basically the same way as the linear traverses described in Chapter 5, but at each y-station the ribbon was switched off and the undisturbed mean voltage was noted as well as the disturbed mean voltage.

The distribution of the 60 cycles per second fundamental disturbance frequency through the boundary layer is shown in each case, Figs. 7.6(b) to 7.9(b).

In Fig. 7.6 the ribbon amplitude was small enough to ensure that the disturbance was linear. The mean velocity profile, Fig. 7.6(a), is in good agreement with the Blasius profile, and the disturbance distribution is as expected from the predictions of linearized theory.

With the ribbon amplitude of $2R_b_0$, Fig. 7.7, the disturbance is just at the upper limit of the linear region. A distortion of the mean velocity profile in the region $0.15 < y/\delta < 0.5$ in the direction indicating a velocity defect, is apparent when compared with the undisturbed
Figure 7.6(a) Disturbed Boundary Layer Profile
Figure 7.6(b) Distribution Of Fundamental Through The Boundary Layer
Figure 7.7(a) Disturbed And Undisturbed Boundary Layer Profiles
Figure 7.7(b) Distributions Of Fundamental And Total Disturbances Through The Boundary Layer
Figure 7.8(a) Disturbed And Undisturbed Boundary Layer Profiles
Figure 7.8(b) Distribution Of Fundamental Through The Boundary Layer
Figure 7.9(a) Disturbed and Undisturbed Boundary Layer Profiles
Figure 7.9(b) Distributions Of Fundamental And Total Disturbances Through The Boundary Layer
profile, which is also shown in Fig. 7.7(a). The peak of the amplitude distribution has shifted outward relative to its position in the linear case. Also shown in Fig.
7.7(b) is the distribution of total signal through the boundary layer, showing that the difference in levels between the fundamental and total disturbance amplitudes, observed in the work of Section 6, is just becoming apparent.

In Fig. 7.8 the disturbance has entered the non-linear region, and the distortion of the mean flow profile, Fig.
7.8(a), is quite marked in the region $0.2 < \frac{y}{\delta} < 0.5$. The distortion is in the same direction as in Fig. 7.7(a) but considerably greater. The distribution of the fundamental disturbance has become broader than in the previous two cases and the peak has shifted to a $\frac{y}{\delta}$ position of 0.30.

With the largest ribbon amplitude the disturbance development was such that the first turbulent bursts were observed about 2 inches downstream of the working station. The mean flow profile, Fig. 7.9(a), has developed a strong inflexion, and in the region $0 < \frac{y}{\delta} < 0.25$ the velocity is greater than that of the undisturbed profile, and for $\frac{y}{\delta} > 0.25$ the velocity is less than that of the undisturbed profile. The peak of the fundamental distribution has now moved to a $\frac{y}{\delta}$ position $>0.4$, and the distribution has broadened considerably. No phase change was observed in this distribution, but any change of phase might have been obscured by the extreme unsteadiness of the hot wire signals. The total disturbance
7.9 The Systematic Error in the Disturbance Velocities Due to the Non-Linearity of the Hot Wire

The disturbance levels used in the work of this chapter are such that an appreciable systematic error in their measurement, due to the non-linearity of the hot wire is introduced. The analysis of this effect follows that of Schubauer and Klebanoff (1945). Hot-wire theory assumes a linear relationship between small changes in velocity and small changes in hot-wire voltage. This implies that the gradient $G$ of the hot wire voltage versus velocity curve is given by

$$G = \frac{\Delta V_{\text{max}}}{\Delta U_{\text{max}}}$$

where $\Delta V_{\text{max}}$ is the voltage change corresponding to a velocity change $\Delta U_{\text{max}}$. In order to estimate the error introduced into the measured r.m.s. disturbance velocities it is necessary to assume some relationship between $u'$ and $\Delta U_{\text{max}}$. In the non-linear region the disturbance waveform is a superposition of several frequencies, and it is difficult to know exactly how $u'$ is related to $\Delta U_{\text{max}}$. However, if it is assumed that the waveform is a pure sine wave and that $\Delta U_{\text{max}} = \sqrt{2} u'$, then it is possible to calculate the order of the systematic error involved.

Fig. 7.10 shows a typical hot wire voltage versus velocity curve, and the tangent to this curve at a velocity of 6.2 ft. per second, which corresponds to a $\gamma/8$ position.
of 0.125 for a free stream velocity of 25.5 ft. per second. For various disturbance levels the maximum voltage changes, assuming a linear relationship between voltage and velocity, were found from the tangent, and from the hot-wire curve, the voltage changes that would actually occur in the measurement of each disturbance level were found. The difference in the theoretical and actual voltage changes gave the percentage error in the disturbance velocity at any given level. The results are shown in Table 7.1.

It is observed that as the disturbance velocity increases beyond 2/0 of the free stream velocity that the error due to the non-linearity of the hot-wire increases rapidly. For any given free stream velocity the error is found to be nearly independent of the local mean velocity. The error is such that the measured fluctuation velocities will always be too high. This error in the measurement of disturbance velocities is inherent and apart from noting its presence, little can be done about it.

7.10 Discussion of Results

Many workers have contributed to the theory of the non-linear region, but the work most relevant to the experimental work described in this chapter is that of Lin (1958) and Stuart and his collaborators in a series of papers since 1951.

Lin (1958) studied the non-linear effects of a two-dimensional perturbation and predicted that in the vicinity of the critical layer all higher harmonics of the fundamental
TABLE 7.1

Non-Linearity of the Hot Wire

<table>
<thead>
<tr>
<th>$u'/U_0$ $^\circ/\circ$</th>
<th>$u'/U$ $^\circ/\circ$</th>
<th>Estimated $^\circ/\circ$ Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>41</td>
<td>28</td>
</tr>
<tr>
<td>8</td>
<td>33</td>
<td>19</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>too small to estimate</td>
</tr>
</tbody>
</table>
disturbance became important, and that this should occur for a disturbance level lower than that required to produce mean flow distortion, through the action of the Reynolds stress. He did not, however, clarify the meaning of 'important'. Furthermore, he predicted that outside the critical layer the harmonic development would be smaller and comparable with the mean flow distortion.

The results of this chapter are in some agreement with the work of Lin, in that strong development of harmonics up to the fifth have been observed. It is also evident from Section 7.6, 7.7 and 7.8 that strong harmonic development is not limited to the region within the critical layer. Perhaps it is of some significance that the results obtained nearest to the plate show by far the largest harmonic to fundamental ratio. Mean flow distortion, Section 7.9, was first observed at the same time as the first signs of strong harmonic development, a contradiction to Lin's prediction that harmonic development should occur for lower disturbance levels than mean flow distortion. On closer examination, however, partial agreement in this respect can be obtained with Lin. The mean flow distortion discussed in Section 7.8, except in the case of the largest ribbon amplitude, exists primarily outside the critical layer, little distortion being evident in the region 0 < \( \frac{y}{\delta} < 0.2 \), whereas the harmonic development is as large in this region as in any of the other regions studied. Thus, inside the critical layer it would appear
that harmonic development does occur before mean flow distortion. The work of Lin appears to agree with the present experimental work inside the critical layer, but as it is to a great extent qualitative, it cannot be compared fully with experimental results. Outside the critical layer, Lin's prediction of smaller quantities of harmonics is not in good agreement with experimental evidence.

Most other workers who have considered the non-linear effects of two-dimensional disturbances have concluded that harmonic generation should be small compared with mean flow distortion. Plane Poiseuille flow has usually been considered, but it has been conjectured that the results might have general validity.

Meksyn and Stuart (1951) have predicted that harmonic generation should be small compared with mean flow distortion. Stuart (1958) using an energy balance method has concluded that disturbance amplitudes of some 10% of the free stream velocity are necessary for appreciable mean flow distortion. Stuart (1960) has shown that if the fundamental disturbance is of order $|A|$ then higher harmonics will be developed to order $|A|^n$ where $n$ is the harmonic number. At the same time he points out that the work of Lin would require higher disturbance levels than those considered by himself. The various energy exchange processes involving the time rate of change of the fundamental disturbance have been defined by Stuart (1968) in the form of the equation
\[
\frac{d |A_1|^2}{dt} = 2 a C_1 |A_1|^2 + (k_1 + k_2 + k_3) |A_1|^4 
\] (1)

where \( A_1 \) is the amplitude of the fundamental disturbance, the term involving \( k_1 \) representing mean low distortion, the term involving \( k_2 \) representing the generation of the second harmonic of the fundamental, and the term involving \( k_3 \) representing the distortion of the fundamental with regard to its dependence on \( y \).

The experimental results indicate that mean flow distortion is appreciable at disturbance levels less than \( 10^0/0 \) of the free stream velocity. Harmonic generation is not in accord with the predictions of Stuart. The second harmonic generation, Section 7.79, is less than the fundamental, but the second harmonic to fundamental ratio would appear to depend on how far the non-linear processes have proceeded. The generation of higher order harmonics is too large to agree with Stuart’s hypothesis.

It seems likely that all three energy exchange processes postulated by Stuart do in fact take place. Mean flow distortion, and the distortion of the fundamental, as shown by the outward shift and the broadening of the disturbance distribution across the boundary layer, certainly occur. As energy is imparted into the boundary layer at the fundamental frequency it is reasonable to assume that the energy needed for harmonic generation comes from the fundamental. It seems likely that at any time the fundamental extracts energy from the mean flow so that it grows,
but at the same time it gives energy to the higher harmonics, resulting in a growth which is not as large as might be expected. The term in equation (1) involving energy transfer to the harmonics would appear to increase rapidly as the transition region is approached, and could conceivably be proportional to some power of the fundamental intensity.

No comment has been made here about the important contributions to non-linear theory on three-dimensional aspects of the process, such as the generation of longitudinal vortices, made by Lin and Benney (1960), Benney (1961) and Stuart (1960b). It is appreciated that before the final breakdown of the boundary layer there is evidence of three dimensionality, Section 7.2. However the present investigation has been confined to two-dimensional phenomena arising from the initially two-dimensional disturbances, whereas in the above work the effect of three-dimensional disturbances on the boundary layer has been considered.

7.11 Conclusion and Suggestions for Further Work

It is clear that at present there is no one theory which adequately describes the non-linear behaviour of disturbances in a boundary layer. Part of the reason for the poor agreement of experiment and theory may lie in the fact that much of the theory has been done for the case of Poiseuille flow rather than boundary layer flow.

The experimental results of this investigation are
not in complete agreement with the earlier work of Klebanoff et al. (1958) and (1962) particularly in regard to the size of disturbances reached in the boundary layer and the magnitude of the harmonic generation.

These preliminary results in non-linear behaviour, rather than providing answers, indicate avenues for further work, both theoretical and experimental.

As the present results on the generation of harmonics are not in agreement with theory and not entirely in agreement with previous experimental results, a detailed investigation of harmonic development over a wide range of conditions is desirable. This long and tedious project can be speeded by considerably by the methods of power spectrum analysis, the facilities for which will shortly be available on line to a P.D.P.8 computer.

Detailed results on the various stages of mean flow distortion would also be useful. Such results would provide starting data for a numerical investigation of the disturbance behaviour, which is critically dependent on the mean flow profile.
CHAPTER 8

PRELIMINARY TEST OF ON LINE RUNNING

8.1 Mean Flow Measurements

As a preliminary check on the on line running of the tunnel, some measurements of the mean flow in the boundary layer have been made. It was felt that the successful measurement of the mean flow profile of the undisturbed boundary layer would vindicate the on line running system.

A typical profile is shown in Fig. 8.1. The traverse was performed as follows. The hot-wire was first set very close to the flat plate manually, and control was then switched to the program mode. At each hot wire interval the number of steps moved by the $y$-stepping motor (stepping motor no. 2) was recorded. The hot wire current was read from the digital voltmeter and if it was different from the working value of 50.0 m.A., then stepping motor no. 6 was activated to correct the current. Finally after recording the wire voltage from the digital voltmeter, the wire was moved to its next position. When the hot wire had moved a total distance of $\frac{1}{2}$ inch, the cumulative distance moved by the wire at each interval and the hot wire voltage at each interval was punched out.

The data was processed as described in Chapter 5, Section

A method of measuring the windspeed in digital form has been developed by Mr. T. Robertson, but has not yet
Figure 8.1 'On-Line' Mean Flow Profile Of The Boundary Layer
been implemented. It was thus not possible to check the
windspeed under program control. This was done manually
in the normal manner.

The program controlling the data collection is shown
in Section 8.3, and its flow diagram in Fig. 8.2. The
program was written in Atlas Autocode form, and a compiler
written for the purpose by Mr. S. Hayes, was used to
translate the autocode program into PAL 3, the assembler
language used by the P.D.P.8 computer.

8.2 Conclusion

The agreement between the mean flow profile of Fig.
8.1 and the theoretical Blasius profile shows that all the
equipment used so far in the on-line running works well.
In the near future it should thus be possible to do much
of the work with the tunnel on-line.
Figure 8.2 Flow Diagram of Data Collection Program
6.3 Atlas Autocode Version of Mean Flow Data

Collection Program

\[
\begin{align*}
\% & \text{ BEGIN} \\
\% & \text{ INTEGER } I, J, K, COUNT, STEP, FIXCUD, TIMES \\
\% & \text{ INTEGER } CUR, VLTS, ARRAYTOP, MAXSTP \\
\% & \text{ INTEGERARRAY } A(1:200) \\
\% & \text{ ROUTINESPEC } HW \\
\% & \text{ ROUTINESPEC AJUST} \\
\% & \text{ OPENHSROCTAL} \\
\% & \text{ READHSR (FIXCUR)} \\
\% & \text{ READHSR (STEP)} \\
\% & \text{ READHSR (MAXSTP)} \\
\% & \text{ READHSR (TIMES)} \\
\% & \text{ CLOSEHSR} \\
\text{ COUNT} & = 0 \\
J & = 1 \\
A(J) & = \text{ COUNT} \\
J & = J + 1 \\
\text{ HW} \\
A(J) & = \text{ VLTS} \\
J & = J + 1 \\
\% & \text{ CYCLE } K = -3, 1, -1 \\
\% & \text{ CYCLE } I = -10, 1, -1 \\
\text{ AJUST} \\
\text{ COUNT} & = \text{ COUNT} + \text{ STEP}/2 \\
A(J) & = \text{ COUNT} \\
J & = J + 1 \\
\text{ HW} \\
\text{ A(J) = VLTS} \\
J & = J + 1 \\
\% & \text{ REPEAT} \\
\text{ STEP} & = \text{ STEP} + \text{ STEP}/2 \\
\% & \text{ REPEAT} \\
\text{ STEP} & = \text{ STEP} + \text{ STEP}/2 \\
4 : \text{ AJUST}
\end{align*}
\]
COUNT = COUNT + STEP/2
A(J) = COUNT
J = J + 1
HW
A(J) = VLTS
J = J + 1
% IF COUNT > MAXSTP % THEN \rightarrow 6
4
6 : J = J - 1
ARRAYTOP = J
J = 1
% OPENPUNCH
5 : % PUNCHSYMBOL (13)
% PUNCHSYMBOL (10)
% PUNCHOCTAL (A(J))
J = J + 1
% PUNCHOCTAL (A(J))
J = J + 1
% IF J > ARRAYTOP %THEN \rightarrow 7
\rightarrow 5
7 : % CLOSEPUNCH
% ROUTINE HW

% INTEGER DIFF
1 : % READVMZERO (VLTS)
% READVMEXT (CUR)
DIF = FIXCUR - CUR
% IF DIF = 0 % THEN \rightarrow 2
% STEPPINGMOTORSELECT (6)
% STEPPINGMOTORDRIVE (DIF TIMES)
\rightarrow 1
2 : END

% ROUTINE ADJUST
% STEPPINGMOTORSELECT (2)
% STEPPINGMOTORSELECT (-STEP)
% END

% ENDPREGRAM
APPENDIX 1

The Boundary Layer Equations

The Navier-Stokes equations for two dimensional flow are:

\[
\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right)
\]

\[
\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right)
\]

If the equations are non-dimensionalised by substituting the non-dimensionalising relations

\[
x = \frac{t}{L}, \quad y = R^{\frac{1}{2}} \frac{U}{L}, \quad U = \frac{U}{U_0}, \quad V = R^{\frac{1}{2}} \frac{U}{U_0}, \quad V = R^{\frac{1}{2}} \frac{U}{U_0} \quad \text{v}
\]

\[
p = \frac{p}{P}, \quad t = \frac{L}{U_0} t'
\]

where \( R = \frac{U_0}{v} \) then the Navier-Stokes equations become:

\[
\frac{\partial U'}{\partial t} + U'\frac{\partial U'}{\partial x} + V'\frac{\partial U'}{\partial y} = -\frac{\partial p'}{\partial x} + \frac{1}{R} \frac{\partial^2 U'}{\partial x'}^2 + \frac{\partial^2 U'}{\partial y'}^2
\]

\[
\frac{1}{R} \left( \frac{\partial V'}{\partial t} + U'\frac{\partial V'}{\partial x} + V'\frac{\partial V'}{\partial y} \right) = -\frac{\partial p'}{\partial y} + \frac{1}{R} \frac{\partial^2 V'}{\partial x'}^2 + \frac{1}{R} \frac{\partial^2 V'}{\partial y'}^2
\]

(1)

For large values of \( R \) these equations reduce to:

\[
\frac{\partial U'}{\partial t} + U'\frac{\partial U'}{\partial x} + V'\frac{\partial U'}{\partial y} = -\frac{\partial p'}{\partial x} + \frac{\partial^2 U'}{\partial y'}^2
\]

\[
\frac{\partial p'}{\partial y} = 0
\]

which are the Prandtl boundary layer equations.
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