MAGNETOTELLURIC MEASUREMENTS ACROSS THE KENYAN RIFT VALLEY

By
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Thesis presented for the degree of Doctor of Philosophy of the University of Edinburgh in the Faculty of Science.

1976.
DECLARATION

I hereby declare that the work presented in this thesis is my own unless otherwise stated in the text, and that the thesis has been composed by myself.
Prior to a discussion of the experimental results, a review of regional induction studies is presented, which emphasises the strengths and limitations of the magnetotelluric technique.

The fundamental theory and equation of the magnetotelluric data are discussed.

Instrumentation used for recording magnetotelluric variations in Kenya is described.

Magnetotelluric measurements were made at fourteen stations in Kenya, between July 1973 and May 1974. The main group formed a 300 km long east-west line crossing the Rift Valley at the geographic equator. Telluric field and magnetic field variations were recorded in the period range 10 to 1000 seconds.

Data from all sites have been analysed to obtain principal impedances and single station induction vectors. Possible sources of systematic error involved in the data reduction process have been investigated. In particular, the biasing effect of strong source field polarisation has been taken into account.

Transfer functions expressed in maximum- and minimum- response form indicate a current concentration at shallow depths flowing along the axis of the Rift Valley and are in accord with the results of Banks and Ottey (1974).

Maximum apparent resistivities measured at stations inside the Rift Valley and on its escarpments ('Rift stations') are more than one
order of magnitude smaller than the maximum apparent resistivities measured at stations outside the Rift Valley. In addition, the azimuths of maximum resistivity axes are oriented approximately parallel to the main rift faults at 'Rift stations' and approximately perpendicular to the main rift faults at stations away from the Rift Valley. The apparent resistivity data suggest that some tens of kilometers of conductive material lie under the rift valley and that the western and eastern edges of this conductor lie outside the region defined by the major rift faults.

Only data which is believed to be relatively unaffected by sources of systematic error have been selected for interpretation in terms of conductivity models. This eliminates the highly anisotropic apparent resistivity data obtained at all stations away from the Rift Valley, and also induction arrows calculated at stations to the east of the rift valley, where the inductive response is complex. Greatest weight has been placed on the high quality isotropic data collected at Ol Joro Orok, a station on the eastern escarpment of the Rift Valley.

Simultaneous magnetic variation measurements made at stations 200 km apart suggest that the horizontal dimensions of the inducing field do not significantly affect the main conclusions of the interpretation.

The magnetotelluric data provide the first independent evidence for the existence of high conductivities at depths corresponding to the upper mantle below the Kenyan Rift Valley. The conclusions of previous Geomagnetic Depth Sounding in the region were heavily dependent upon other geophysical and geological data. An explanation of the
upper mantle conductor in terms of partial melting is in accord with
gеological evidence of recent volcanic activity in the area.

The depth to the top of and the thickness of the upper mantle
conductor can not be resolved by the MT data since the mantle conductor
is obscured by the presence of a good conductor at depths (< 10 km)
corresponding to the upper crust. The high conductivity of upper
crustal material can only reasonably be explained in terms of high
temperatures and water saturation of the crust under the rift valley.

The conductivity model and its geophysical implications are similar
to those suggested by Hermance (1973b) and Berktold (1974) in their
interpretations of magnetotelluric data collected in Iceland and
Ethiopia, respectively.

The possible existence of large quantities of water in the crust
below the Kenyan Rift Valley and in the Sub-Icelandic and Sub-Ethiopian
crusts suggests a relationship between high water concentration, high
temperatures and tectonic activity. The presence of water in rocks at
high temperatures is of considerable interest in the understanding of
the petrology of rocks in the region of the Rift Valley.
I thank my supervisor Dr. Rosemary Hutton for suggesting the magnetotelluric project in Kenya. I am indebted to her not only for her advice and guidance but also for the kindness she has shown me during my stay in Edinburgh. I am grateful to the Department of Geophysics for providing facilities for my research, and also to the members of the department who have assisted me in innumerable ways. In particular, I should like to thank Mr. Alex. Jackson who sacrificed many of his evenings and weekends in the frantic rush to prepare the MT instrumentation for fieldwork in Kenya. The MT measuring system was designed and built by Mr. Ian M. Brazier to whom I owe my entire knowledge of instrumentation and whose wordly advice helped me to cope with fieldwork disappointments.

I thank the Government of Kenya for permission to work in the country. It is a pleasure to express my gratitude to all the Kenyan people who provided me with such warm hospitality and whose fields, cricket pitches and playing fields formed the experimental laboratory. The members of the Physics Department of the University of Nairobi provided generous facilities and a great deal of help with local problems, for which I am very grateful. I am especially indebted to Professor Neville Skinner who acted as my overseas supervisor and gave me much moral encouragement during the first dark days of the fieldwork period. Also, I should like to express my gratitude to Dr. Bhatt and Professor Akazuki of the Department of Geology, University of Nairobi for allowing me to borrow the Department's conventional resistivity equipment.
The purchase of equipment for the magnetotelluric project was made possible by a grant from the Royal Society of London, for which I am grateful. I thank the Leverhulme Trustees for awarding me a Royal Society Leverhulme Studentship which supported me throughout the fieldwork period.

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1. INTRODUCTION

1.1 Fundamentals of Geomagnetic Induction Studies

1.1.1 General Remarks

All geomagnetic induction studies attempt to estimate the electrical conductivity distribution within the earth. The energy source for probing geoelectrical structure is magnetic time variations with frequencies less than 0.5 c/year which have their sources external to the solid earth.

The physical phenomenon involved is that of electromagnetic induction. External magnetic variations induce eddy currents in the conducting Earth. The resultant magnetic field measured at the earth's surface represents the interference between the external inducing field and an 'internal' field set up by these earth currents. Induction studies infer the electrical conductivity distribution by comparing a suitably defined induction response parameter derived from measured data with that obtained from calculations using an electrical model. When a good fit is obtained between measured and calculated values, it is implied that the best fitting electrical model represents a close approximation to the actual conductivity distribution within the earth.

Parameters describing the induction response of the earth may be defined in various ways according to the induction method employed, but they always consist of the ratio of one field variation to another. Global induction methods measure the ratio of internal to external
magnetic fields. The magnetotelluric response is a function of the ratio of the surface electric field to the horizontal magnetic field measured in the orthogonal direction. Geomagnetic depth sounding techniques try to separate the 'anomalous' magnetic field caused by lateral variations in conductivity from the 'normal' magnetic field which would have been measured in the absence of any lateral conductivity changes. The parameter for interpretation can then be defined in terms of ratios of combinations of anomalous, normal and total magnetic fields.

The selection of the type of model to be used in the comparison of induction responses is guided by the nature of the data, by the availability of or lack of a suitable analytical or numerical treatment of the model, and by independent limits set by other geophysical considerations.

Depth control is achieved naturally through the skin depth effect. More rapid variations in the inducing field induce stronger eddy currents which act to reduce the amplitude of the disturbing variation. In any conductor low frequency variations penetrate deeper into the conductor, and thus surface measurements of different frequencies record variations which have sampled different depth regions of the conductor. Because of the direct relationship between the frequency of the variation and the depth region probed, response data is usually transformed into the frequency domain.

While the skin depth effect enables the induction phenomenon to be used for conductivity depth sounding, it also ensures that the vertical resolution of the technique is poor when the frequency
bandwidth of available data is restricted. A measurement at one frequency gives a conductivity estimate which represents a 'smeared' average of all conductivities penetrated to a depth of the order of a skin depth of the variation. To increase resolution it is essential to make measurements at as wide a range of frequencies as possible. This can often be difficult to achieve owing to the character of the spectrum of natural variations.

1.1.2 Natural Variation Fields

Since all induction studies probing depths of more than a few kilometers use the energy provided by natural varying magnetic fields, it is the character of these fields which, by controlling the quality of data obtained, ultimately limits the inferences which can be drawn about the earth's internal electrical properties.

A representative spectrum of natural variation amplitudes in the horizontal magnetic field at geomagnetic mid-latitudes is shown in figure 1.1. Beyond the annual spectral line, the spectrum rises sharply with decreasing frequency as the large secular variation of the earth's internal field swamps external magnetic variations. At the present time it is not possible to extract the external component, and the spectrum of frequencies less than 0.5 c/year can not be exploited by conventional induction methods. The spectral peaks are related to specific types of magnetic disturbance.

It has long been suspected that the main contribution to the spectral peak at one day comes from currents flowing in the ionosphere.
Fig. 1.1  Amplitudes of natural variations in the horizontal magnetic field at mid-latitudes.

Fig. 1.2  Types of geomagnetic disturbance
(a)  Equatorial $D_{st}$ - July 8, 9, 10, 1958
(b)  Quiet daily variation - Sept. 16, 17, 18, 1973
    recorded at Kampi Ya Moto, Kenya
(c)  Geomagnetic Bay
(d)  Micropulsation - Jan. 20 1974
    recorded at McCall's Siding, Kenya
Gammas

[Graph showing frequency vs. time with various labeled periods such as 1 Day, 27 Days, 1 Hour, and ELF.]
Tidal motions in the upper atmosphere force charged particles to cross the geomagnetic field lines and hence set up large scale current systems. The magnetic variation observed on a geomagnetically quiet day is depicted in figure 1.2b. The variation has significant harmonics at periods of 24, 12, 8 and 6 hours and is often referred to as Sq variation, although much argument centres on the precise definition of Sq.

The 27 day variation and harmonics can be related to the rotation period of the sun. There is evidence (Chapman & Bartels 1940) that certain magnetic storms show a 27 day cycle, and these storms are believed to be caused by the emission of high velocity plasma from specific regions of the sun (Matsushita 1975). When the plasma stream reaches the Earth, it causes an increase in the energy of particles trapped in the Van Allen radiation belts. These particles drift around the Earth under the influence of the main dipole field, creating a ring current which acts to decrease the dipole field. This constitutes the main phase of the storm and occurs over an interval of about one day. The recovery phase of the storm may take 1-20 days as the ring current decays (fig. 1.2a).

At periods of about one hour most energy comes from fluctuations associated with moderate magnetic storms and from geomagnetic bays of the type illustrated in fig. 1.2c. Bays mostly occur at night, and usually last 1-2 hours. They may be the result of leakage of particles from Van Allen belts.

Below a period of 1 hour, micropulsations make up the main contribution to the variation signal. Micropulsations have their source in the variability of the solar wind, the name given to the
plasma stream continuously emitted by the sun. In both magnetically stormy and magnetically quiet times solar wind velocity and density is variable, and pulsations of solar plasma are often able to penetrate the earth's protective magnetosphere. It is believed (Burch and Green 1963) that plasma injected into the earth's field high above the ionosphere can propagate in various modes across and down the lines of the earth's field as hydromagnetic waves to the base of the ionosphere. Below this, hydromagnetic wave propagation is no longer possible and the wave is transformed into an electromagnetic wave. Orr (1973) has reviewed the morphology of and possible mechanisms for geomagnetic micropulsations.

A knowledge of the mechanisms of different geomagnetic disturbances is not essential in order to use the variations for induction studies. However, an understanding of the variation mechanisms allows the researcher to anticipate the type of energy which will be available for induction probing also its spatial character. Micropulsation activity may be expected to occur in bursts even on the 'quietest' days and the spectrum will be sharply peaked at certain periods characteristic of the geomagnetic latitude of the station.

However, because of the irregular occurrence of storms, a recording system may have to operate for weeks to be certain of registering useable energy in the period range 30 minutes to 6 hours. Such requirements may mean that the available data is limited in frequency content and signal to noise ratio. The use of data which has a limited bandwidth, restricted frequency content and high noise level greatly increases the ambiguity of interpretation.
The shape of the natural spectrum and its variability with time also has important consequences for the construction of magnetometers. Instruments designed to measure long period variations have to be very stable in view of the long recording intervals involved, while magnetometers for recording short period fluctuations (less than 30 minutes) require a very high sensitivity to detect low signal levels.

Fig. 1.1 represents a spectrum of typical amplitudes likely to be analysed at a station in geomagnetic mid latitudes. Different parts of the spectrum vary in amplitude with geographical position in different ways. There is a general decrease in the signal amplitude in going towards low magnetic latitudes. It is well established that the amplitudes of daily variations as well as those of fast daytime fluctuations are greatly enhanced near the line of zero dip ('dip equator'). This 'electrojet' effect is produced by the ionospheric current systems on the day-lit side of the earth concentrating within a narrow latitude zone just above the dip equator. From observations made in Peru (Forbush & Casaverde 1961) and Nigeria (Ogbuehi and Onwumechilli 1965) it was found that the enhanced variation amplitudes dropped off within ±10° dip to their normal levels outside the electrojet zone. Another electrojet caused by pinching of the worldwide current system is found in auroral latitudes (Akasofu et al 1965, Kisabeth 1975).

The spatial variation in the amplitude of the source field affects the response parameters measured in all induction studies. In investigations which involve simultaneous operation of several magnetometers the effect of the source field can be separated from that of the earth's electrical structure. In the analysis of data recorded
at single sites it is necessary to make assumptions regarding the spatial uniformity of the source field. Madden and Nelson (1964) have shown that in magnetotelluric (MT) studies the source field effect can be neglected if the depths being probed are small compared with the wavelength characterising the spatial variation of the source field.

Swift (1967) concluded that for MT studies in magnetic mid-latitudes using periods greater than 15 minutes, the source wavelength is of the order of 1000 km. or more. Schmucker (1959) found a high degree of uniformity for bay disturbances which would support this view. Smith (1964) has reported the high degree of uniformity of activity in the range 1-6 seconds observed at low and mid-latitudes. However, experimental studies in the 10-600 second period band are lacking. The MT results of Morrison et al (1968) for this period range imply a significant source field effect. On the other hand, MT measurements by Caner & Auld (1968) suggest a spatial wavelength of greater than 8000 km.

The finite dimensions of the source field have to be considered in any MT study looking deeper than 100 km. In regions near the electrojet even crustal investigations are severely limited by the source effect.

Magnetic variations are geographically variable not only in amplitude but also in horizontal polarisation characteristics (Fowler et al 1967). To estimate the effect of lateral variations in conductivity, data processing techniques require signals to possess different directions of polarisation. This presents a difficulty in certain geographic locations where energy in some period bands is often strongly polarised in a preferred direction.
1.1.3 Inversion

The inversion procedure attempts to infer the electrical properties of the earth from available data. The usual inversion strategy is essentially, to construct a conductivity model, to calculate its inductive response, and, if this is compatible with measured data, to conclude that this model is a likely approximation to the earth's actual conductivity structure. Refinements to this method can be made; by varying different parameters of the model a 'best' fit to the data can be obtained and the sensitivity of the data to certain parts of the model can be investigated.

There is no guarantee of uniqueness in the conductivity distribution determined in this fashion. Not only may the parameters of a certain form of model be poorly resolved, but there may also be the possibility that the data is compatible with a completely different type of distribution which requires different geophysical, physical, chemical and tectonic explanations. In the latter case we will say, for the sake of brevity, that the data support geophysically distinct solutions.

The basic problem of inversion is not simply to find a model which fits the data, but to find all the geophysically distinct models which can satisfy the data. The refinement of a particular type of model to obtain a data fit in some 'best' sense, and a study of the resolution of the data with respect to that type of model loses most of its geophysical significance if no consideration has been given to the possible existence of other geophysically distinct models compatible with the observed data.

An ideal inversion scheme for induction data has been suggested by Anderssen (1975). The basic steps in the scheme are (1) define a
criterion for geophysically distinct solutions; (2) test for the existence of geophysically distinct solutions; (3) (a) if there are no solutions then there is an inconsistency in the inversion scheme or in the data which must be removed; (b) if there are many geophysically distinct solutions, only qualitative information may be resolvable. Greater effort should be made in this case to improve the quality and quantity of data or to acquire new data capable of imposing independent constraints; (c) it is only when there exists a single geophysically distinct solution that there is a geophysical case for finding 'best fit' models or for determining the resolution of the data associated with models of that type.

In practice, however, the mathematical complexity of induction theory and the great expense of implementing numerical techniques has meant that most effort has concentrated simply on determining models consistent with available data.

Indirect modelling techniques require a human operator to attempt to fit the data by successively changing the parameters of a certain form of model, and can be very laborious. Practical procedures for calculating conductivity distributions directly from the data are very desirable, but so far these have been limited to the treatment of models in which conductivity is a function only of one spatial coordinate (one dimensional distributions). Progress in two-dimensional direct inversion has been reported by Weidelt (1975) but the stage of routine implementation of such techniques has not yet been reached.

Most direct inversion schemes start with a guessed model and refine this to obtain better fit to the data. The success of the inversion usually depends on how close to a possible solution the
The inversion technique of Parker (1971) has attracted a great deal of interest. By using a modification of the method of Backus & Gilbert (1968), involving a linearisation of the mathematics, Parker was able not only to find models which fitted global induction data, but also to calculate the resolution 'window' associated with the model. This 'window' corresponds with the vertical extent over which estimated conductivities represent a smeared average of actual conductivity. As an inversion method, Parker's linearisation approach has been criticised by Anderssen (1975) on the grounds that a poor initial guess may lead to improper convergence or even to divergence; but the approach is of considerable value in providing an insight into the resolution of induction data.

It is important to remember that the resolution calculated by Parker's method corresponds to a particular type of model, and that the data may support geophysically distinct models. At the present time no deterministic method is known for establishing the existence of geophysically distinct solutions to the non-linear induction problem (Anderssen 1975).

The practical approach to the non-uniqueness modelling problem varies widely among induction researchers. Vozoff (1972) proposed that in the light of the non-uniqueness of the method, any independent geophysical or geological data should be used as early as possible in the inversion scheme. Rokityanski (1975) has criticised the usual approach of finding a model consistent with the data and proceeding to draw all kinds of geophysical and tectonic conclusions without consideration of other possible models. He suggested that all possible conductivity models should be considered solely on the basis of
induction data, so as to reduce the subjective element of interpretation. Only then should independent evidence be used to restrict the range of model solutions.

The former approach seems to be the more popular, and it is often difficult to estimate the significance of published results because it is not obvious which parts of a model are fixed by the induction data and which are arbitrary.

Tozer (1969) has urged that an attempt should be made simply to discuss possible systematic errors when it is not possible to evaluate their effect on interpretation.

Probably the most difficult systematic error to investigate is that of assuming N-dimensionality for the earth's conductivity structure when its inductive characteristics more closely approximate (N + M)-dimensionality.

1.1.4 Parameters affecting electrical conductivity in Rocks & Minerals

The electrical properties of the earth are important because, apart from their intrinsic interest, they can be related to other physical and compositional parameters, which are often more geophysically interesting. A knowledge of this relationship can sometimes be used in reverse to restrict the range of conductivity models which fit induction data, by considering independent geological and geophysical evidence.

Laboratory measurements have provided most information about the parameters governing electrical conductivity in rocks and minerals. However, although large compilations of data have existed for some time,
it is only fairly recently that refinements in laboratory techniques have permitted satisfactory isolation and control of the most important variables involved.

In porous rocks, the conductivity has been found to depend most strongly on the shape and interconnectivity of fluid-filled pore spaces and upon the conductivity of the pore fluid. The conductivity of individual minerals is most affected by temperature, the partial pressure or fugacity of oxygen, the content of transition metal ions (in practice this usually means iron) and the phase of the mineral.

There is a great deal of experimental evidence (for references see Rikitake 1966 p.226 and Shankland 1975) which shows that the temperature dependence of the conductivity, \( \sigma \), of rocks and minerals can be expressed as

\[
\sigma = \sum \sigma_i e^{-E_i/kT}
\]

where \( \sigma_i \) is a constant, \( k \) is Boltzmann's constant, \( T \) is the absolute temperature in K, \( E_i \) is the activation energy, and the summation is over all possible conduction mechanisms. The mechanisms usually considered are of three types: impurity or mineral, intrinsic or electronic, and ionic. Because of the exponential temperature dependence, a single conduction mechanism predominates within a given temperature range, and plots of \( \log \sigma \) versus \( 1/T \) often show straight line segments (fig. 1.3).

The extreme sensitivity of electrical conductivity to temperature suggested that geomagnetic induction studies might be used to determine the temperature distribution within the earth, particularly within the upper mantle and lower crust. To calibrate induction results, there was a great increase in laboratory studies to investigate the temperature-
conductivity dependence of possible upper mantle constituents.

There is overwhelming geophysical and geochemical evidence that olivine is the major component of the upper mantle (Kennedy & Higgins 1972), with a composition of about 90% forsterite \((\text{Mg}_2 \text{Si}_4 \text{O}_{10})\) and 10% fayalite \((\text{Fe}_2 \text{Si}_4 \text{O}_{10})\) (Wager 1958, Ringwood 1966).

The major continuous phase in any mixture of minerals determines the conductivity of the mixture (Duba 1972); so the upper mantle conductivity should be close to that of its olivine constituent. Mizutani and Kanamori (1967), Kobayashi and Maruyama (1971), Duba (1972) and Duba et al. (1974) studied natural olivine, whereas Bradley et al. (1964) and Hamilton (1965) studied the forsterite-fayalite solid solution series.

With available data for olivines alone it is possible to virtually cover a log \(\sigma\) versus \(\frac{1}{T}\) plot (Duba and Lilley 1972). However, a major step forward to reducing this scatter was made by Duba and Nicholls (1973) who, by demonstrating the influence of oxygen fugacity on conductivity, stressed the importance of controlling the oxidation state of olivine in experiments.

As well as depending heavily on fugacity, the conductivity of olivine is also very sensitive to the concentration of iron-bearing fayalite. Both Hamilton (1965) and Duba (1972) have pointed out that in going from fayalite-poor to fayalite-rich compositions, the absolute conductivity may change by several orders of magnitude.

Pressure effects on conductivity are obviously important especially in the deep mantle (Tozer 1959), but experiments have failed to clarify the magnitude of the pressure effect. Measurements on single crystals
Fig. 1.3 Summary of conductivity (σ) data (ohm\(^{-1}\) cm\(^{-1}\)) for Forsterite single crystals. For significance of numbers and symbols see Fig. 4. Duba et al. (1974).

Fig. 1.4 Generalised diagram of how effective pore pressure, $\bar{P}$, affects the conductivity of rocks saturated with aqueous solutions which remain at room temperature and pressure (after Brace 1971).
of olivine (Duba et al. 1974) exhibit pressure shifts comparable with temperature shifts caused by temperature uncertainties. However, polycrystalline samples often show strong pressure effects (Mao & Bell 1972) particularly at the olivine-spinel phase. This phase change was first demonstrated by Akimoto and Fujisawa (1965) to produce a hundred-fold increase in the conductivity of a synthetic fayalite at a pressure of 60 Kb and a temperature of 900°C. Though no similar experiment has been made on forsterite, there is no reason to suppose that a similar transition could not happen.

The great range in temperature-conductivity curves makes it difficult to place useful limits on absolute temperatures in the earth from conductivity estimates alone. However, as Canér et al. (1967) and Hermance (1973) have noted, most of the curves tend to run parallel. It appears that the change in temperature determined from a change in conductivity is not so sensitive to the composition and oxidation state of the mantle material, and useful limits can be placed on temperature gradients within the earth.

Beyond the melting temperature, and provided that the degree of melting is greater than a critical value, the bulk conductivity of the mantle depends on the properties of the liquid phase. If the melt is in the form of isolated pockets, the bulk conductivity remains close to that of the mother material (Waff 1974). However, if the melt is interconnected, even very small melt fractions may change the bulk conductivity by several orders of magnitude. Waff estimated on the basis of network model calculations that the bulk conductivity would be 2/3 C \( \sigma \) where C is the melt fraction and \( \sigma \) is the conductivity of the melt. Experimental evidence has been provided by Presnall et al. (1972)
that melting can produce a two orders of magnitude increase in the conductivity of basalt across its melting interval.

Fortunately, magmatic liquid conductivities are known to be extremely sensitive to temperature and relatively insensitive to composition, oxygen fugacity and probably total pressure and water content (Waff et al. 1974). It is possible that magma conductivities could be combined with melt fraction limits to interpret earth conductivity profiles in terms of temperature for regions of active partial melting (Waff 1974).

In contrast with the conductivity of mantle rocks, the conductivity of crustal rocks is determined primarily by the amount of water present, the salinity of the water, and the way in which it is distributed through the rock.

In water-bearing rocks, resistivity may be connected with water content through Archie's empirical law (Keller 1966)

\[ \rho = \alpha \rho_w \phi^m S^n \]

where \( \rho_w \) is the resistivity of the water in the rock, \( \phi \) is the fractional pore volume, \( S \) is the fraction of the pore volume filled with water, and \( \alpha, m, n \) are constants.

In the earth rocks below the water table are saturated with aqueous solutions to at least 4 or 5 km. based on observations in deep wells, and possibly to even greater depths (Brace 1971).

Data from numerous wells (White et al. 1963, Keller 1966) show that resistivity of waters found in igneous and metamorphic rocks is almost always within the range 0.5 to 100 ohm m. with average values from 1 to 30 ohm m.
Brace (1971) attempted to calculate the conductivity-depth distribution within a normal crustal region based on a model of an igneous or metamorphic crust saturated with likely aqueous solutions. Conductivity is controlled by the effect of pressure on the porosity of crustal rocks. Natural rocks have three types of porosity - fracture, crack and pore. Fractures and cracks close at low pressures and the conductivity of saturated rocks decreases rapidly with increased depth of burial. Brace quoted Snow's observations that fractures are nearly closed at 100 metres. Though varying widely between different rock types, conductivity typically decreases by a factor of 5 to 10 in the first 5-10 Kms. of burial as crack porosity is eliminated. Changes in pore porosity are much slower and conductivity decreases by a factor 10 in the depth range 5-50 Kms.

Fig. 1.4 illustrates Brace's crustal model. The model agrees well with available data. Crystalline rocks near the surface range in resistivity from $10^2 - 10^3$ ohm m. (Keller et al. 1966). For highly resistive rocks at depth, resistivities can be as high as $10^4 - 10^5$ ohm m. (Anderson and Keller 1966).

Very few controlled experiments have been done to estimate the effect of high temperatures on rocks buried in the lower crust. Over a temperature of a few hundred degrees, it is known that mineral conduction dominates conduction through pore fluids. The magnitude of the temperature effect is difficult to assess from the large scatter of experimental data (Brace 1971).

If water is present in the lower crust and temperatures are high the conductivity may show an increase of orders of magnitude. Lebeder and Khitarov (1964) have shown that for temperatures above 600°C and water pressures greater than 500 Atmospheres, the conductivity of granite can be well in excess of 0.1 mho/m.
1.2 Regional Induction

1.2.1 Regional Induction – Methods

A simple approach to the determination of conductivity at depth appears to be offered by the magnetotelluric (MT) method, first presented in a practical form by Cagniard (1953). Simultaneous measurements are made of time variations in the horizontal magnetic field and the surface electric (telluric) field in mutually orthogonal directions. The response parameter, called apparent resistivity because of its direct relation to actual resistivity in a homogeneous conductor, is defined as a function of the ratio of electric to magnetic field amplitudes at a specific frequency, normalised by the frequency. For historical reasons, it is more conventional to use the parameter resistivity expressed in ohm m. rather than its reciprocal conductivity (mho/m), though in most publications the parameters interchange freely. Curves of apparent resistivity versus frequency for a single station may be used to infer conductivity distribution down to depths of over 700 km, on the assumption that conductivity is a function of depth only. The use of this method permits great savings in cost and time over the use of geomagnetic methods, which usually involve operation of three component magnetometers at a number of sites.

Price (1962) has shown that MT and geomagnetic methods are theoretically comparable and that the reduction in the number of stations occupied for a MT sounding is accomplished by ignoring spatial variations in the external inducing field. Wait (1962) and Madden and Nelson (1964) have established that the MT method can be reliably used to probe the crust and upper mantle down to depths of more than 100 km except in
areas near the auroral and equatorial electrojets. Even here, modifications of Cagniard's theory can be made to take account of the dimensions of the electrojet field (Hutton 1969). The validity of Cagniard's MT method depends on the conductivity being a function of depth only. Measurements at many sites have shown that the measured apparent resistivity depends on the orientation of the telluric measuring line. Simultaneous recording of two orthogonal horizontal magnetic components and two orthogonal telluric components permits the mathematical rotation of the measuring axes so that apparent resistivity can be estimated for any orientation of the telluric line. After the paper by Bostick and Smith (1962), refined data processing techniques came into common use and the values and directions of maximum and minimum apparent resistivities were calculated at many stations. It became clear that the vast majority of sites are anisotropic; in addition, minimum and maximum apparent resistivity versus frequency curves often appear to be quite dissimilar in form.

Cantwell (1960) has interpreted the apparent anisotropy as being due to actual anisotropy in rocks or to lateral variations in conductivity. From published data, it is apparent that most apparent anisotropy is caused by near surface inhomogeneities (Dowling 1970). Laboratory measurements show that coefficients of anisotropy are much too low to explain the observed anisotropy of MT curves.

From the study of the general properties of 2-dimensional and 3-dimensional conductivity models, it is apparent that a very great proportion of MT data can be fitted by 2-dimensional structures (Sims and Bostick 1969). Practical 2-dimensional modelling techniques have
only been available since about 1969, and so it is only fairly recently that it has become possible to obtain a physical insight into the behaviour of apparent resistivity curves in the presence of a lateral discontinuity in conductivity.

Fig. 1.5a illustrates the variation of apparent resistivity at one frequency across a discontinuity. $\rho_L$, the apparent resistivity corresponding to the telluric line at right angles to the strike of the discontinuity (H-polarisation) behaves discontinuously, while $\rho_{11}$ corresponding to telluric measurements parallel to the strike (E-polarisation) varies smoothly and bears a relation to the actual resistivities involved. $\rho_L$ is the maximum apparent resistivity, $\rho_{\text{max}}$, on the resistive side of the contact and the minimum apparent resistivity, $\rho_{\text{min}}$, on the conductive side. Fig. 1.5b shows that a Cagniard 1-dimensional interpretation of either the $\rho_{\text{max}}$ or $\rho_{\text{min}}$ curves at a station near the discontinuity would be quite misleading.

Computations of Patrick and Bostick (1969) indicate that a 1-dimensional analysis of curves obtained over 2-dimensional structures yields different and grossly incorrect results in the immediate region of the inhomogeneity. However, Vozoff et al. (1968) report that a check of many 2-dimensional models of layered structures in the presence of a lateral discontinuity shows that interpretation of each $\rho$ curve often yields approximately correct depths to horizontal interfaces. Rankin (1972) shows that for a 2-dimensional model corresponding to a sedimentary basin a 1-dimensional interpretation of the $\rho_{11}$ curve gives acceptable results.

Clearly, at this stage, establishing the validity of a 1-dimensional
Fig. 1.5 (a) Variation of apparent resistivity with distance across a vertical contact at a single frequency. Horizontal scale is distance.

Fig. 1.5 (b) 1-dimensional interpretation of principal resistivity versus period curves measured at station B in fig. 1.5a.
interpretation of 2-dimensional data demands a knowledge of the form of the actual conductivity structure involved. Normally this is not possible, but the situation could be improved with the availability of 'master curves' for 2-dimensional models. These could be used qualitatively to obtain a better physical insight into the nature of the induction problem.

Of the two apparent resistivity curves measured in the 2-dimensional situation \( \rho_{11} \) is least affected by lateral discontinuities and so it is more meaningful to use \( \rho_{11} \) in a 1-dimensional interpretation. However in most situations this corresponds to \( \rho_{\text{min}} \), the minimum measured resistivity. Because of signal to noise considerations, most researchers prefer to interpret \( \rho_{\text{max}} \) (e.g. Everett and Hyndman 1967, Rankin 1973, Nienaber et al. 1974). This means that published results may be biased on the side of large resistivities, and that lengths may have been interpreted as depth values which are more meaningfully related to horizontal distances to discontinuities.

To be able to confirm the existence and strike of a regional anomaly and to be able to relate \( \rho_{11} \) reliably to \( \rho_{\text{max}} \) or \( \rho_{\text{min}} \), MT measurements must be made in traverses. In addition, because \( \rho_{\perp} \) will be discontinuous across a surface discontinuity of the smallest width, care must be taken in a regional survey to avoid as far as possible measurements near faults, dikes, hills, valleys etc. (Wescott and Hessler 1962).

Unfortunately for the MT method local conductors seem to be of almost universal occurrence, and traverse results, more often than not, reveal azimuths of maximum resistivity which do not correlate with the probable regional strike.
For this reason the geomagnetic depth sounding (GDS) method is preferred for mapping lateral variations in conductivity. Measurements are made of the three magnetic field components at a number of sites occupied either consecutively or simultaneously. Because magnetic fields represent a volume integrated effect of eddy currents flowing within a skin depth of the measuring point, GDS measurements at periods over one second are little affected by localised conductors.

The method of processing GDS data has been described by Frazer (1974) and consists of spectral analysis of time series followed by separation of 'anomalous' fields, caused by the effect of lateral discontinuities from the 'normal' fields which would be measured in the absence of an inhomogeneity.

Problems involved in estimating a normal field have been discussed by Cochrane and Hyndman (1970) and Gough (1973). Fortunately in most areas the normal vertical field is very small so that can be accurately equated to the measured vertical field. The parameters for quantitative interpretation, the GDS responses, are either (a) the anomalous vertical field and anomalous horizontal field normalised with respect to the normal horizontal field measured in a preferred direction, or (b) the anomalous vertical field normalised with respect to the total horizontal field, again in a specified direction.

Since the vertical field component is almost entirely anomalous, the behaviour of this component is most diagnostic of the existence of lateral discontinuities. The vertical field shows a high correlation with the horizontal field component inducing current flow in a lateral inhomogeneity. In the case of an elongated conductor, the inducing
component is directed perpendicular to the strike of the body. By finding the azimuth of the horizontal variation with which the vertical field shows highest correlation, a line may be drawn perpendicular to the strike of the elongated conductor. An arrowhead can be added to this line, after considering the phase relationship between vertical and horizontal components, so that the resulting 'induction arrow' points towards the region of current concentration.

In general, GDS surveys are excellent for delineating the horizontal extent and, in the case of elongated structures, the strike. Quantitative interpretation in terms of depths and conductivities is viable only for elongated structures, since the computational treatment of 3-dimensional structures is prohibitively expensive. In the case of elongated conductors, the GDS method can accurately estimate the width, and the maximum depth to the top of the conductor, as well as conductivity contrasts (Rokityanski 1975). However the depth resolution is very poor. This is illustrated strikingly by Gough (1973) who shows that GDS data are consistent with two conductivity distributions under the Colorado plateau. One distribution corresponds to a good conductor (5 ohm m.) at 350 km depth; the second represents a conductor (2 ohm m.) at 80 km depth.

The geophysical interpretation of a conductivity anomaly depends primarily upon its depth of burial. GDS models are often heavily dependent on the results of other geophysical techniques to provide vertical control. Vertical resolution of resistivities and depths can only be provided by the MT method.

A pertinent point to note here is that GDS data are usually
frequency limited to periods of greater than 3 or 4 minutes. MT data, in contrast usually extend down to periods of the order of 10 seconds. This contrast is the consequence of the different instrumentation used, multistation measurements necessitating the use of magnetometers of high stability and low sensitivity. Because the skin depth of a variation with period T in a homogeneous material, resistivity \( \rho \), is of the order of \( 0.5 \sqrt{T \rho} \) kilometers, the shortest period GDS variations will penetrate 250 kms in normal crustal and upper mantle material (resistivity 100 ohmm) compared with only 50 kms for the shortest period MT variations.

The complementary nature of the GDS and MT methods is obvious and the type of induction technique used must be selected to correspond with the kind of information it is desired to acquire.

GDS surveys are excellent reconnaissance techniques, mapping lateral variations in conductivity and indicating if a region is suitable for MT measurements at a single site or in a linear traverse. If the region has a complex 3-dimensional inductive appearance, it is probably unsuitable for MT measurements (e.g. Atlantic Canada, Cochrane and Hyndman 1973).

1.22 Regional Induction Studies - Results

Within the last 20 years large areas of North America, Europe, Russia and Southern Australia have been studied using regional induction techniques. The most notable result of these studies is the discovery that large conductivity anomalies occur in the crust or upper mantle in most of the regions studied and in almost every type of geological and tectonic environment.
Most early Mt data have very limited bandwidth, show large scatter,

...
and lack adequate spectral and coherency analysis so that the effect of noise on measurements is difficult to estimate. In addition, the lack of a horizontal network of stations makes it impossible to relate any observed anisotropy to a regional variation in conductivity. Data of this sort can not be interpreted meaningfully.

Because of the profusion of regional conductivity anomalies, the regional results discussed here will only relate to those areas where (a) MT measurements have been made in a traverse, and acceptable data processing techniques have been used, (b) the anomaly appears to be elongated, (c) the area has been studied by the GDS technique, so that comparisons can be made of the relative contributions of the two methods. More detailed reviews of conductivity anomalies have been written by Porath and Dziewonski (1971), and Schmucker (1973), with the accent mainly on GDS results.

Most of the reported induction data come from North America. The anomaly near Alert in Arctic Canada has been investigated using linear magnetovariational and MT profiles. Analysis of anomalous magnetic field components led Rikitake and Whitam (1964) to postulate a highly conducting zone at a depth of 25 km corresponding to an upwelling of the 1500°C isotherm. Other geophysical evidence shows a normal regional heat flow and MT apparent resistivities increase with period (Whitham and Anderson 1965) so that an interpretation in terms of enhanced temperature is physically unrealistic. The azimuth of anisotropy of the telluric field and phase relation between magnetic and electric fields convinced Dyck and Garland (1969) that the Alert anomaly can not be explained by induction in a 2-dimensional structure, but is caused by local channelling of telluric currents which are
induced elsewhere. The conductor which channels the currents is located in the crust and has been associated with either a hydrated lower crust (Hyndman & Hyndman 1968) or with the Robeson sea channel (Porath and Dziewonski 1971). The recognition of the possibility of current channelling has radically altered the philosophy of interpretation of conductivity anomalies.

Western Canada has probably been more intensively studied using induction techniques than any other area of the world. Geomagnetic depth sounding has been done in profiles from the coast to the Alberta plains (Hyndman 1963, Caner et al. 1967). Vertical field variations are consistently smaller to the west of a narrow boundary zone lying close to the Rocky Mts Track than they are to the east. This observation suggests that the crust and upper mantle have different physical properties under the eastern and western regions and that the two structural regimes are separated by a narrow transition zone. Caner et al. (1969) have used the data of a large scale MT experiment to show that there is a conducting layer, resistivity 1000 Ωm beginning at a depth of 10 km below the western region which is not present under the eastern region. A more refined analysis of GDS data by Cochrane and Hyndman (1970) gave results for the depth to the western conductor in good agreement with the MT model.

GDS work in the Great Plains by Camfield et al. (1971) has revealed one of the largest known local geomagnetic anomalies. This was interpreted as being due to a long narrow body of graphite schist in the crust beneath the Black Hills. A series of MT soundings was carried out by Rankin and Reddy (1973) to obtain a more detailed interpretation. However, the appearance of the MT data is inconsistent
with a 2-dimensional geometry, and their conductivity models based on a 1-dimensional interpretation of strongly 3-dimensional data must be regarded with some reserve.

A large scale magnetometer array study in S.E. Australia (Gough et al. 1974) revealed a major anomaly extending northwards from Adelaide. The conductor was traced along its strike for about 700 km and strong anomalies in vertical field amplitude and reversal of induction arrows were observed across the feature. A MT traverse (Tammemagi & Lilley 1973) across the feature was able to locate the lateral extent of the anomaly, and to locate the top of the conductor within the top 10 km of the crust. Absolute values for the resistivity of the conductor were found to be astonishingly low—of the order of 0.1 ohm m. The contrast between direction of GDS induction arrows and azimuth of maximum apparent resistivity demonstrated that anisotropy was caused by local conduction and was not related to the regional feature. Taking into account the depth and high conductivity of the structure, Tammemagi and Lilley suggested that the body is a remnant of a geosyncline still containing salt saturated sedimentary rocks.

The use of the MT technique to probe sedimentary basins has proved that some sedimentary sequences can have very high integrated conductivity. Prior to the middle 1960's magnetic variation anomalies were generally attributed to lateral variations in the electrical conductivity of the mantle produced by temperature effects on semiconducting silicates (Rikitake 1959, 1966). A variation anomaly striking east—west in Northern Germany was at first interpreted in terms of a conductor in the upper mantle at about 100 km depth, corresponding to a zone of enhanced temperature (Schmucker 1959). MT measurements made across
the strike of the anomaly by Vozoff and Swift (1968) are consistent with a 2-dimensional model of a surface layer of 1 ohmm resistivity decreasing in thickness from north to south, overlying a resistive substratum. The thickness of the conducting layer corresponds closely with geological estimates of the thickness of sediment in the German basin. With the thickness and extraordinarily high conductivity of the sediment to be found in this area, little effect could be seen at the surface from a conductor in the upper mantle.

Further south, a second conductivity anomaly has been observed associated with the Rhinegraben. MT results have been discussed by Haak (1970). Perhaps due to the simple geological structure and the small thickness of highly conducting sediment, telluric fields show an 'ideal' behaviour. Anisotropies and azimuths of major resistivities can be explained by a very simple 2-dimensional model of sedimentary infill. Because of the simplicity of behaviour of the telluric currents, the $\rho_\perp$ curves can be used simply and directly together with Ohm's Law and the condition of current continuity to estimate the conductivity contrast between the sub-surface layers inside and outside the rift. The Rhinegraben telluric study is one of the very few where the $\rho_\perp$ data have proved to be of any value. In fact the MT data corresponds more closely with a simple 2-dimensional structure than do the GDS data; the spread of induction arrows indicates the superposition of two anomalies (Untiedt 1970), perhaps the effect of the Rhinegraben and that of the North German basin. The sensitivity of MT data to localised structures can be regarded in this case as a merit of the method.

In some geophysically interesting areas of the world a large scale GDS or MT traverse is not feasible. Such an area is Iceland, which
represents the culmination of the mid-Atlantic ridge rift system. Hermance and Garland (1968) analysed geomagnetic variation data from three sites on a profile perpendicular to the neovolcanic zone in northern Iceland. The small range of stations in logistically difficult conditions forced them to attempt a single station geomagnetic depth sounding. By comparing vertical to horizontal field ratios between one of the Iceland stations and a station in Greenland, Hermance and Garland were able to show that the electrical conductivity beneath Iceland is anomalously high. The conductivity and depth to the conducting zone were determined by the MT method to be 300\(\Omega\)m and 10 km respectively (Hermance and Grillot 1970, 1974).

An important class of anomaly is that linked with continent-ocean boundaries. Ocean bottom MT measurements (Filloux 1967, Cox et al. 1970) strongly support a rise in the conductive mantle off the Californian coast. Most studies of continental edge anomalies have involved GDS measurements on the continental side. Results have been discussed in detail by Gough (1973b).

1.2.3 Regional Studies - Geophysical significance

The geophysical significance of conductivity anomalies depends upon the ability to relate high conductivity reliably to other physical or compositional parameters of geophysical interest.

The conductivity of rocks of the upper mantle is a sensitive function of temperature, oxidation state, degree of melting, composition, and perhaps pressure (Section 1.1.4). At first, the effect of enhanced temperature was thought to be the most plausible explanation for regions
Fig. 1.6  Apparent resistivity curves obtained in the Paris Basin, and multilayered model interpretation. After Fournier and Rossignol, 1974.

Fig. 1.7  Temperature curves and melting curves for the upper mantle.

I.  2.1  HFU
II. 1.9  HFU
III. 1.7  HFU
of high conductivity in the mantle (Rikitake 1966 p.228). Duba (1974) has shown that mantle temperatures in excess of 1000°C are required to account for the observed high conductivities of greater than 0.1 mho/m (See Fig. 1.3). In several areas of the world, anomalies with possible mantle sources have been detected at depths of much less than 50 km. An explanation of conductivity based on enhanced temperature alone requires the anomalous body to have a temperature which is physically unrealistic.

Wyllie (1971) has shown that at the low pressures of the crust and upper mantle material will undergo melting before a temperature of 1000°C is reached. The presence of small amounts of water considerably reduces the melting temperature. Waff (1974) and Presnall et al. (1972) have shown that the conductivity of molten basalt, a possible partial melt product of mantle material, is so high (\( > 5 \) mho/m) that beyond a critical degree of melting any observed conductivity contrast can be explained simply by adjusting the proportions of solid and liquid fractions.

Isotherms for different heat flow provinces are shown in fig. 1.7. The solidus curve of gabbro (from Wyllie 1971) is intersected by isotherms only in regions of high heat flow; the effect of increasing pressure upon the solidus curve means that melting will be restricted to a thin layer, a few tens of kms thick.

A zone of partial melting in the upper mantle is the most likely explanation for the seismic low velocity zone (Anderson & Sammis 1970) observed in the depth range 50-150 kms. Since magmatic activity is believed to be the major process in the chemical evolution of the earth's crust and upper mantle (Green 1972), the mapping of zones of
partial melt by seismic and induction means is of paramount importance. Partial melt zones are also predicted in regions of spreading or subduction of lithosphere plates. So delineating the regions of melting near those features is of considerable significance to the formulation of theories of global tectonics.

Deductions from induction surveys concerning temperature enhancement and partial melting can only be reliably accepted if the results of other geophysical methods are consistent with high temperatures or melting. A general correlation should be expected between high conductivity and seismic wave attenuation and velocity diminution. Anomalies in the gravity field may also be anticipated if effects of temperature or melting change the density of mantle material. Correlations would be expected with surface heat flow measurements provided sufficient time has elapsed for heat to reach the surface. Law et al. (1963) have estimated the time constant for diffusion of a temperature anomaly through a distance of 100 km is about $10^7$ years. An attempt by Uyeda and Rikitake (1970) to correlate heat flow results to conductivity anomalies in the upper mantle seems to have been a partial success. A comparison of heat flow results with conductivity anomalies at greater than 100 km depth can only be used to demonstrate a negative correlation.

Garland (1974) has reviewed the correlation between conductivity and seismic attenuation and velocity reduction for those areas where mantle conductors are known to exist.

Anomalies have been found in Japan (Rikitake 1969) and Peru (Schmucker 1973) above active descending lithospheric plates. There is not a simple one-to-one correspondence between conductivity and low Q, low
seismic velocities and high heat flow. The probability is that the conductivity anomaly is not simply a region of frictionally heated material but is as much an effect of the poorly conducting subducted slab interrupting the normal mantle conductivity structure.

Iceland is a summit of the mid-Atlantic ridge rift system. The crust-mantle interface has been determined seismically (Palmason 1971) at a depth of 10 km, and low upper mantle velocities of 7.4-7.6 km/sec. have been recorded. Highly conducting material, resistivity 30 ohm m, has been located in the mantle at 10 km depth by MT methods (Hermance & Grillot 1974). A similar depth has been determined to a conducting layer under the Baikal Rift in Russia by Gornostaev et al. (1970). In the Baikal Rift zone heat flow is 2-3 times higher than normal (Lubimova & Shelyagin 1966). The highly conducting layer placed by MT sounding at 15 km corresponds well with the depth of earthquake foci in the Baikal area (Florensov 1969).

Intermediate velocities of 7.6-7.7 km/sec have been found at 30 km depth under the Rhinegraben and may represent a zone of enhanced temperature or partial melting in the upper mantle. This zone is in the same depth range as the conducting layer of resistivity 30 ohm m placed between 25 and 75 km depth by Winter (1973) in order to fit his GDS data.

Conductivity anomalies have been determined by GDS at upper mantle depths in the Western United States. One is associated with the southern Rocky Mts. and one with the Wasatch Fault zone and they are separated by the Colorado Plateau. Gough (1973) has stressed the non-uniqueness of interpretation and has shown that the results are consistent either with undulations of 100-400 km in the depth to highly conducting
mantle or with variations in thickness of a conducting layer in the upper mantle, separated from the deep mantle. If the latter model is considered it may be correlated with variations in thickness of the seismic low velocity zone. Heat flow in the west also shows good correlation with the postulated thickness of conductor, and the pattern of high heat flow in the east and low heat flow in the centre corresponds with the qualitative picture of the conducting layer.

Results of a MT survey in the South Western United States indicate that upper mantle conductivities may be 50-100 times higher under Arizona than those of the upper mantle under New Mexico (Madden and Swift 1969). The high conductivity values correlate with high heat flow and low seismic velocities.

It is possible that high mantle conductivities could be due to factors other than temperature and partial melting, such as an increase in fayalite content of the mantle or to water saturation caused by hydrothermal alteration of amphibole and phlogopite (Wyllie 1971). Until a clearer understanding of the nature of conductivity anomalies emerges, it is probably unwise to make inferences concerning the temperature of mantle material from induction studies in the absence of independent geological or geophysical data.

The significance of conductivity anomalies in the crust lies primarily in the way they screen mantle anomalies. Conductivity in crustal rocks is controlled chiefly by the amount of interstitial water (Section 1.1.4). Induction workers are now aware that sedimentary basins can often be of such great depths and high conductivity that their effect is comparable with that of the ocean and is sufficient to
conceal mantle anomalies. For this reason, surveys of lateral variations in the mantle must be carried out in areas with little sedimentary cover.

In metamorphic zones anomalies may be due to extensive mineralisation resulting in massive sulphate or graphite bodies. The huge local anomaly, observed around the Black Hills, Dakota has been related to a metamorphic zone containing highly conductive (> 1 mho/m) graphite schists (Camfield and Gough 1972).

Away from metamorphic zones and thick sedimentary cover, large conductivity anomalies have been found, and their sources have been placed in the lower crust at depths below 10 km (Whitham & Anderson 1965, Caner et al. 1967, Berdichevsky et al. 1969).

Experimental data (fig. 1.8) obtained by Lebedev and Khitarov (1964) for wet granitic rocks in the base of the crust, show that conductivities can be as high as 1 mho/m at temperatures less than 800°C. This represents a 3-4 orders of magnitude increase in conductivity over that of dry rocks.

Hyndman and Hyndman (1968) showed that an increase in temperature at the bottom of young geosynclines can cause a release of water in the lower crust. As a result low density granite melt is formed and rises to the upper levels of the crust, carrying water which is highly soluble in the melt, and leaving a dehydrated lower crust. Hyndman and Hyndman suggested that GDS and MT measurements should provide evidence of the present stage of development of geosynclines.

A close association between inland geomagnetic variation anomalies and tectonic features such as rift-fault zones and young fold belts has been noted by Law and Riddihough (1971).
First Melting

1 atm. (Water pressure)
The prospect of using induction techniques to map present and ancient boundaries between sub plates appears attractive. However, before this can be attempted, a detailed study is required of how water is produced in the lower crust and of the relative importance of interstitial, bound or adsorbed water, and the water vapour pressure on the conductivity of the lower crust.

High conductivities in the lower crust have also been observed in areas where there is no geosynclinal evidence. Mitchell and Landisman (1971) explain the conductive region encountered at 20 km depth under western Texas by a zone of water saturation of granite. Saturation is attributed to an increased availability of water produced by hydrothermal reactions in certain crustal minerals.

High crustal conductivity may arise for a variety of other reasons. Serpentinised basalts, perhaps typical of oceanic crustal material, have shown high electrical conductivity (Cox 1971). Conductivity anomalies could be associated with ancient oceanic crustal material incorporated but not chemically assimilated into a continental margin (Cochrane and Hyndman 1974).

There is evidence from MT studies (Dowling 1970, Kurtz (1973)) of a decrease in resistivity by a factor of 10 at depths which correspond closely with an increase in seismic P velocity from 6.1 to 6.5 km/sec. The interface is presumably a more basaltic part of the crust, perhaps the Conrad discontinuity. Pecova et al. (1970) have correlated a conductive layer under the Carpathan region of Czechoslovakia with the Conrad discontinuity.

The profusion of explanations for lower crustal conductors, rather
reduces their geophysical significance. Until more work has been done in working out in greater detail the relative effects of hydration, composition and temperature on crustal rocks, the main interest of induction workers will be mantle anomalies.

1.3 Geology of the Kenyan Rift Valley

The Kenyan Rift System is known to be connected to the global mid-ocean ridge system (Heezen and Ewing 1963) via the Ethiopian Rift and the Afar triple junction (fig. 1.9). The possible existence of a tectonic connection between the Kenyan Rift and the mid-oceanic ridge system is of special interest in the light of current theories of plate tectonics.

The physiography of the Kenyan Rift Valley has been described by Baker et al. (1972). The East African Plateau, which is 1500 km long and about 1000 km wide has a height of 1-2 kms above sea level. The plateau is bordered by the eastern and western rifts and the intervening region is occupied by the shallow Lake Victoria. Superimposed on the plateau is the Kenya dome which reaches 3 km above sea level. This dome is bisected by the eastern rift valley. The Gregory Rift (Fig. 1.10) is in the central part of the Kenya rift zone and is a complex graben 60-70 km wide bounded by major normal faults arranged 'en echelon'. Elevation of the rift floor is highest in the central part of the dome where it reaches 2 km above sea level, compared with 600 metres in South Kenya and 400 metres at Lake Rudolph.

Little drainage enters the rift valley due to upwarping of the plateau margins overlooking the rift. So the accumulation of volcanic and lake sediments has been little disturbed by erosion, and the rift
Fig. 1.9 Structural pattern of the Afro-Arabian Rift System (after Baker and Wohlenberg, 1971).

Fig. 1.10 Structure of the Kenya Rift Zone (after Baker & Wohlenberg, 1971).
ETHIOPIA

Grobén floor with fault swarms

Floors of anti-
thetically faulted expressions

Edge of step-fault platforms

Edge of main plateaux

Major faults

Monoclinal flexures

Isobases-sub-Miocene erosion surface—feet

(Eyegpt, Red Sea, Arabia, Gulf of Aden, Sudan, Somalia, Indian Ocean)
The rift floor is relatively flat. The rift floor is divided by volcanic ranges and broad transverse arches into a series of isolated internal drainage basins, and many of the resulting lakes are brackish.

The highest summits in Kenya, Mts. Elgon, Kenya and Kilimanjaro are situated 100-150 km east or west of the rift.

The geological history of the area has been summarised by Baker (1965), Baker and Wohlenberg (1971), Baker et al. (1972) and Logatcher et al. (1972).

The foundation of eastern Africa is a complex of metamorphic and igneous rocks of Precambrian and lower Palaeozoic age. The process of rifting can be subdivided into four time-stratigraphic stages. Volcanism and faulting associated with each of these stages is illustrated in Fig. 1.11.

Stage I \( [23-12 \text{ my B.P.}, \text{ Lower-Upper Miocene}] \) started with the subsidence of the Turkana depression in north western Kenya and uplift of the area along the Uganda-Kenya border. The Turkana depression was partly filled by flood basalts, and on the uplifted area a linear arrangement of volcanoes was formed, coinciding with the future strike of the Kavirondo Rift.

In the upper Miocene events happened to determine the location and development of the Gregory Rift structure. The area of volcanic activity shifted to the Kenya domal uplift. The central rift area was uplifted and was the site of immense fissure eruptions of phonolites. (For an elementary discussion of the mineralogical composition of igneous rocks, the reader is referred to Cox 1971b) During this first stage of rift development there was higher magmatic productivity than during any
later stage. Logatchev et al. (1972) regard the large volumes and petrochemical uniformity of phonolite eruptives as evidence of an easy connection between the interior and the surface of the earth. Stage II [10–5 my B.P., Lower-Middle Pliocene] is the most complex being characterised by a radical change in the character of volcanic activity. There were relatively minor phonolite and trachyte fissure eruptions in the central part of the rift. Most volcanic material was produced by eruptions of the central type. More than 19 volcanoes were situated along the floor or over the scarps of the future rift valley, and voluminous flows of basalts were produced.

The first extensive rift faults developed during this stage - either at its beginning (Baker & Wohlenberg 1971) or at its end (Logatcher et al. 1972). On the western side of the valley, faulting affected almost the whole length of the structure, but displacements on the eastern side occurred only in a few places.

Stage III [5–1 my B.P., Upper Pliocene – Middle Pleistocene] was again marked by a change of eruptive type. Massive fissure eruptions of trachytes began in the central sector of the rift, locally filling the rift and overflowing its flanks. The Upper Pliocene saw the last major uplift of the marginal plateaux, and this was accompanied by several phases of graben faulting which outlined the Gregory rift.

Simultaneous with immense basalt activity, autonomous melting centres appeared to the east of the rift, forming the great volcanoes of Kenya, Kilimanjaro and Kulal. Since that time volcanic activity has been localised inside and to the east of the rift valley, revealing clearly the asymmetrical character of the deep magma generating process.
Further local rejuvenations of the main graben and step faults continued until mid-Pleistocene times, and at the same time the floor of the rift was shattered by a closely spaced network of grid faults which in some places reach densities of 2 to 3 faults per km.

In Stage IV $110$ my B.P. the eastward migration of volcanic activity continued. Eruptions from Kenya, Kilimanjaro and Kulal almost ceased, and multicentre eruptions appeared to the east of the large cones, forming large basaltic ridges.

In the rift floor volcanicity was confined to a series of central volcanoes occupying the middle segment of the rift. Recent volcanic activity has formed a complex succession of lavas and pyroclastics on the rift floor interbedded with local lake sediments. A typical cross section through the Gregory Rift near its centre is shown in figure 1.12.

The composition and location of volcanic products indicates that there was maximum heating in the central section of the rift valley. Melting in eastern magmatic chambers was deeper, as testified by the occurrence of basalts of a more differentiated nature.

The central part of the Gregory rift is peculiar in many respects. (1) It is the geometric centre of the rift valley and of the Late Miocene phonolite layer; (2) it coincides with the topographic culmination of the rift floor; (3) it is a zone of the highest volcanic productivity throughout the whole history of rift development.

A natural conclusion is that the central section of the rift is the part at which the disturbed and thermally activated upper mantle comes closest to the surface of the earth.
Fig. 1.12  Geological section across the Gregory Rift Valley near the Equator.

Fig. 1.13  Composite gravity and seismic model for a cross-section through the Gregory Rift near the Equator.
PreCambrian gneisses
Mesocene volcanics and sediments
Plio-Pleistocene volcanics and sediments

$V_p = 7.0 \text{ km/sec}$

Depth (km)

S.G.

2.7 3.0 3.2 3.3

km
The variation with age of the positions of faults and volcanoes has been discussed by Girdler et al. (1969) and is thought to indicate progressive thinning of the lithosphere.

From the geological evidence, Baker and Wohlenberg (1971) and Logatchev et al. (1972) conclude that the Kenya Rift Valley should not be regarded as a typical example of the first stage of continental break up. The geological data are consistent with maximum crustal disruption and basic igneous injection during the Upper Miocene. The Kenyan Rift Valley is characterised by exceedingly slow rates of crustal extension and by long continued interaction between crust and thermally activated mantle.

One important factor which distinguishes the Kenyan Rift from mid-ocean ridge rifts is the highly alkaline character of its lavas. An explanation for the abundance of alkaline volcanics in terms of crustal assimilation and differentiation at depth is inconsistent with the intimate intermingling in space and time of lavas of different alkalinity. McCall and Hornung (1972) suggest that a chain of discrete cupola reservoirs, quite separate from the main basaltic reservoirs, underlie the Rift Valley reaching up close to the Rift floor. In these cupolas a complex differentiation process is operating on a mantle-derived basalt parent.

1.4 Previous Geophysical Studies in the Kenyan Rift Valley

The Gregory rift is one of the minor seismic zones of the earth. Only one major earthquake (intensity 7.0 on the Richter scale) has been recorded in the area and occurred in 1928 at a point to the east of
the Laikipia escarpment east of Lake Hannington. Minor shocks are fairly frequent in the area, being most commonly felt on the rift floor just to the north east of Menengai Caldera.

Fairhead and Girdler (1972) have plotted epicentres for the whole of Africa for the period 1963–1970. Epicentres for East Africa show a high degree of scatter compared with those of the Gulf of Aden and the Red Sea. The scatter is attributed to the long history and complex nature of the continental crust in East Africa.

Away from the rift zone, Gumper and Pomeroy (1970) found compressional, $P_n$, and shear, $S_n$, wave velocities typical of stable shield areas. Their study of travel time delays revealed that rays travelling near or beneath the rift system are slowed, particularly for paths crossing the rift north of $5^\circ$ S. This finding correlates with the observed change from south to north in the nature of rifting. Going from south to north a region of block faulting merges with a more complex system of grid faulting and volcanism. From the apparent failure of $S_n$ to propagate across the rift north of the equator, Gumper and Pomeroy inferred the existence of a low Q zone in the mantle and postulated that it is the low velocity material which has low Q.

The results of a seismic refraction experiment carried out on the rift floor between Lakes Rudolf and Hannington have been reported by Griffiths et al. (1971). It was found that rocks with velocity 6.4 km/sec or slightly higher were encountered directly below the 3 km or so of volcanic cover. Velocities of this magnitude are on the high side for a normal continental area, and were explained in terms of crustal intrusions, perhaps in the form of basic dykes. To obtain information for greater depths, Griffiths et al. were forced to use a plane
horizontal layer interpretation. It appeared from this interpretation that the high velocity crustal rocks were underlain at 20 km by a medium with velocity 7.5 km/sec and minimum thickness 100 km. Such a velocity is intermediate between that of normal crust and normal mantle and it was postulated that for this northern section of the rift valley, the asthenosphere is penetrating the lithosphere to within 20 km of the earth's surface.

Sundaralingham (1971) has studied the dispersion of Rayleigh waves along reversed paths between Addis Ababa and Nairobi. Though limited in resolution, the long period data indicate that the average shear velocity over a depth range from 50 to more than 120 kms takes the value of 4.25 km/sec which is anomalously low compared with the value of 4.8 km/sec which Gumper and Pomeroy (1970) calculated for an African shield model. These surface wave data strongly support the existence of low velocity material in the topmost mantle. However, the short period dispersion data do not require a crust of the type suggested by Griffiths et al. for the rift valley, but are more consistent with normal African shield crust. The great circle path between Addis Ababa and Nairobi passes to the east of the rift axis, and the surface wave data are consistent with a model of near normal crust and topmost mantle within 50 km of the Rift axis.

Long et al. (1972) have studied the apparent velocities of earthquake arrivals using a seismic array sited just to the west of the Gregory rift at Kaptagat. Those regional earthquake arrivals whose paths crossed the Rift showed velocities of 7.1 km/sec. The discrepancy between this value and the 7.5 km/sec of Griffiths et al. may be due to structural complexity. Regional arrivals from the west of the rift
exhibited an apparent velocity of 7.9 km/sec, independent of the azimuth of their epicentres. The lack of an azimuthal variation was interpreted by Long et al. as representing a horizontally uniform crust immediately to the west of the Gregory Rift.

When the apparent velocities of first arrivals at Kaptagat from teleseismic events were studied, they were found to show a continuous variation in azimuth, Long et al. demonstrated that this could be explained by a wedge of ultra low velocity material in the upper mantle, thinning to the west. A value of 7.0 km/sec was considered to be an upper limit to the minimum velocity, and it is possible that the 7.5 km/sec zone below the Rift axis determined by refraction could be a high velocity upper surface of the wedge. Backhouse (1972) has estimated that the upper face of the wedge dips at an angle of about 40° away from the rift, and is a mirror image of the lower wedge face. (See fig. 1.13)

The empirical Nafe-Drake relationship (Talwani et al. 1959) suggests that the low and high seismic velocity regions may be linked with low and high density material respectively. Bouguer gravity profiling might then be able to map the lateral extent of the two anomalous regions.

A Bouguer profile across the Rift north of 5° S shows a positive anomaly limited to the Rift Valley superimposed upon a negative anomaly about 1000 km wide and reaching a minimum under the rift axis. The axial positive anomaly has been intensively mapped by Searle (1969, 1970). His most favoured model represents a mantle-deprived intrusive zone about 20 km wide reaching to within 2 km of the rift in places. However, the gravity data could also be fitted by a body only 10 km wide (Baker & Wohlenberg 1971).
Delineation of the horizontal limits of the low density mantle region has proved difficult, because of the unknown contribution to the negative Bouguer anomaly by low density surface volcanics. The model of Girdler and Sowerbutts (1972) indicates low density material beginning at just over 30 km depth under the rift axis, and extending more than 200 km to east and to west. Baker and Wohlenberg (1971) have a low density body of width less than 100 km, beginning at 30 km depth.

A composite model, broadly consistent with the bulk of seismic and gravity data has been presented by Banks and Ottey (1974) and is illustrated in fig. 1.13. The model represents a cross section through the rift at the equator.

Most geological and geophysical evidence indicates that the low velocity, low density region in the mantle is one of abnormally high temperature. It might then be expected that a zone of high electrical conductivity exists under the rift due either to the direct effect of enhanced temperature on conductivity or to the effects of partial melting brought on by temperature and low overburden pressure.

Geomagnetic depth sounding results for a traverse across the Gregory Rift Valley at the equator have been reported by Banks and Ottey (1973). Induction arrows show a strong concentration of current flow along the rift parallel to its strike. The induction responses measured at stations to the west of the rift have a simple form and give a strong indication that the western edge of the zone of high electrical conductivity lies under the western escarpment of the rift. Stations to the east of the rift show complex induction responses indicating that the conductor extends eastwards and has a complicated structure. The reversal of induction arrows from west to east is
evidence for a good conductor directly beneath the rift floor. Because of the small lateral scale of the induction anomaly, the depth to the top of the rift conductor must be less than a few tens of kilometers.

Numerical modelling of the induction responses using the Jones-Pascoe (1971) program revealed that two geophysically distinct models could fit the data equally well at a period of 25 minutes. These models are illustrated in fig. 1.14. Both models consist of a sub-rift conductor at shallow depth and an eastern conductor at greater depth.

Banks and Ottey consider the lateral position of the two bodies to be well defined, but that there is considerable uncertainty about the depth, thickness and conductivity of the bodies. They used independent geophysical and geological evidence to assign depths to the top of each conductor. The top surface of the eastern conductor was placed at 50 km to correspond with the start of the low surface wave velocity zone of Sundaralingham (1971). In the light of geological estimates of the thickness of volcanic infill in the rift, the possibility of a highly conducting rift infill being responsible for the induction anomaly was considered unlikely. A sub-rift conductor at depth corresponding to a zone of partial melting was thought more plausible, and the depth to the top surface was chosen to be 20 km to correspond with the depth to the seismic refraction low velocity interface.

As Banks and Ottey made clear, the modelling of the eastern body depended heavily upon the complex phase characteristics of the induction responses measured at the two easternmost stations on their traverse. However, little mention is made in their paper of the complete phase
Fig. 1.14 Conductivity profile across the Gregory Rift near the Equator compatible with Geomagnetic Depth Sounding results.

(a) Deep sub-rift conductor: comparison of the computed response (circles) with the observed response (crosses) at 25 min. period. Figures give conductivity in ohm\(^{-1}\)m\(^{-1}\). Arrows indicate lateral extent of the rift valley (After Banks and Ottey, 1974).

Maximum response, \( G_p = |G_p| \exp(i \phi_p) \)

(b) Shallow sub-rift conductor: comparison of the computed and observed responses at 25 min. period.
mismatch between observed and modelled phase data for these two stations.

Models fitting the geomagnetic traverse data contrast sharply with models obtained from magnetotelluric measurements by Odera (1974) for Nairobi, a station only 100 km away from the traverse line. A one-dimensional interpretation of data in the period range 10 minutes to 1 day yielded a three-layer conductivity model. The top layer of thickness 6 km and conductivity 0.2 mho/m is of the type discounted by Banks and Ottey as being geologically unreasonable. An intermediate layer of conductivity 0.002 mho/m lies above a deep highly conducting layer which is located at a depth of 170 km.

As Odera points out, his MT interpretation is meaningful only if the Nairobi site is laterally homogeneous. Because of the proximity of Nairobi to the Rift Valley, and the almost universal occurrence of anisotropy even in the absence of obvious geological boundaries, Odera's results must be regarded as preliminary.

1.5 The Purpose of the Magnetotelluric experiment

A knowledge of the physical properties and state of the crust and upper mantle beneath large scale rift zones is of paramount importance in understanding the physical, chemical and tectonic processes responsible for the evolution of the earth's crust and mantle.

In order to obtain a fuller and more reliable picture of subsurface conditions, measurements should be made by as many independent geological, geophysical and geochemical techniques as possible.

In the Kenyan Rift Valley, the section of the Gregory Rift near
the equator has been most intensively studied, because it appears to be the volcanic centre of the area. It is also the most easily accessible area for traverse measurements.

The GDS survey across the Gregory Rift located the lateral positions of two good conductors, one under and one to the east of the Rift, but there is considerable ambiguity concerning the depth, thickness and conductivity of these conductors. It was hoped that a MT survey would reduce the ambiguity of interpretation and resolve which of two geophysically distinct conductivity models is more appropriate. In the event that the two regions represent zones of partial melting, it was of particular interest to obtain estimates of the depth to the top of each conductor to help explain petrochemical differences observed between lavas in the centre and to the east of the Rift.

To obtain good depth resolution, measurement of short period variations is essential. The magnetic and telluric instrumentation available was capable of recording periods down to 10 seconds compared with the lower useable limit of about 300 seconds of the instrumentation used in the GDS traverse. This represents a reduction by a factor of five in the skin depth of the shortest periods in a homogeneous medium.

Previous GDS work had shown that the region has a 2-dimensional inductive behaviour. The area is sufficiently far from the ocean for the 'ocean-edge effect' to be negligible. Lying 15° south of the equatorial electrojet, MT data are probably unaffected by the spatial variations of the source inducing field. Absence of any significant thickness of sediment appeared to ensure that upper mantle conductivity could be determined. So the area appeared to be very suitable for MT measurements.
Because of the low micropulsation amplitudes near the equator, and in view of the high conductivities known to exist at shallow depth, it was expected that telluric signal levels would be extremely low. The area, however, was known to be relatively free from noise generated by leakage of industrial electricity, and no problem was expected concerning signal to noise levels.

By conducting the MT survey in a region which has been the subject of multidisciplinary studies, the number of variables involved in interpretation could be minimised; in addition, the relationship between induction results and those of other disciplines could help in the understanding of conductivity anomalies.
CHAPTER 2

THEORY

2.1 General Remarks

The basic magnetotelluric (MT) problem is to infer the electrical conductivity distribution within the earth from measurements of natural transient magnetic and electric fields at its surface.

With a limited set of measured data, it is possible to gain information only about very broad features of the earth's actual conductivity. As with other geophysical techniques, it is necessary to represent the real earth by a model which is characterised by a very small number of parameters. The type of model considered, (defined by the number and nature of its parameters) must be selected carefully so that (a) for certain values of its parameters the model is consistent with the measured data and (b) the parameters satisfying the data are restricted to a range which is of geophysical significance.

In some cases, these two requirements cannot be met simultaneously. Data may be severely limited in quality or quantity, or the model may have to be very complex to fit the measurements. In either case, the range of acceptable parameters is too large to be of much interest. When a model is found which does satisfy the two conditions above, it is regarded as representing the actual conductivity of the earth.

The problem of interpretation (i.e. the investigation of possible model solutions) is made easier by transforming the data into a more convenient form. In MT studies it is assumed that the magnetic and electric field variations at the earth's surface represent the earth's
response to a magnetic inducing field which has its source external to 
the solid earth. This assumption allows electromagnetic induction 
theory to be used to express magnetotelluric data concisely in terms 
of a response measure called impedance, which has a more direct 
relationship to electrical conductivity.

Interpretation can then proceed by comparing experimental impedance 
estimates with the calculated impedances of selected conductivity 
models.

At the present time techniques to calculate MT impedance are 
available only for very special conductivity distributions, and observed 
data must necessarily be interpreted in terms of highly idealised 
models. It might be suspected that the response of the real earth 
would prove to be too complex to allow an adequate or meaningful model 
representation. Observational studies show, however, that in many 
cases the response of the earth can be approximated by that of a simple 
conductivity model.

To see how MT impedance may be defined and how it may be calculated 
for a particular conductivity distribution, we must investigate solutions 
to Maxwell's equations of electromagnetism.

2.2 Basic equations

2.2.1 The Inhomogeneous wave equation

Maxwell's equations are expressed here in M.K.S. units and through- 
out this chapter the same convention will be used unless stated to the 
contrary.
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]  
\[ \nabla \cdot \mathbf{B} = 0 \]  
\[ \nabla \cdot \mathbf{D} = \varepsilon \]  
\[ (2.1) \]
\[ (2.2) \]
\[ (2.3) \]
\[ (2.4) \]

where \( \mathbf{E} \) is the electric field vector, \( \mathbf{H} \) is the magnetic field vector, \( \mathbf{B} \) is the induction field vector, \( \mathbf{D} \) is the displacement vector, \( \mathbf{J} \) is the current density vector, and \( \varepsilon \) is the electric charge density.

We employ the constitutive relations

\[ \mathbf{D} = \varepsilon \mathbf{E}, \]  
where \( \varepsilon \) is the dielectric constant \( (2.5) \)

\[ \mathbf{B} = \mu \mathbf{H}, \]  
where \( \mu \) is the magnetic permeability \( (2.6) \)

and Ohm's Law

\[ \mathbf{J} = \sigma \mathbf{E}, \]  
where \( \sigma \) is the conductivity of the medium \( (2.7) \)

This reduces the complexity of Maxwell's equations. In general, \( \mu, \varepsilon \) and \( \sigma \) may be tensor quantities.

It is appropriate at this stage to consider the magnitudes of the electrical parameters encountered in MT studies.

Electrical conductivity, \( \sigma \), may range from about \( 10^{-5} \) mho/m for granite to near \( 0.2 \) mho/m for sea water. In contrast to this enormous spread of values, relative permeability, \( \mu \), keeps very close to its free air value, \( \mu_0 \), of \( 4 \times 10^{-7} \) H/m throughout the earth, even when ferromagnetic minerals are present. For the most magnetic sulphide ore, \( \mu \) is less than \( 1.25 \times \mu_0 \) (Ward 1967 p.230). The dielectric constant, \( \varepsilon \), in most geological materials is usually of the order of \( 9 \times \varepsilon_0 \) where \( \varepsilon_0 \) is the value for free space i.e. \( 8.85 \times 10^{-12} \) F/m.
(Grant & West 1965, p.469). The maximum value which \( \varepsilon \) can attain is \( 81 \times \varepsilon_0 \) corresponding to the dielectric constant of water.

It is easy to show that a coupling exists between the space charge distribution and the electromagnetic fields in a conductor unless either \( \frac{\sigma}{\varepsilon} \) is constant or \( \mathbf{J} \) is perpendicular to \( \nabla (\frac{\sigma}{\varepsilon}) \). (Rikitake 1966 p.126). A general theory of induction has thus far been developed only for the special case of an infinitesimally thin sheet of conductor (Price 1949).

We shall confine our attention here to the solution of Maxwell's equations for media where \( \varepsilon \) is constant and \( \sigma \) is either constant or changes only in a direction at right angles to the direction of current flow; \( \mu \) is so close to \( \mu_0 \) for all materials that it may safely be equated to its free space value at all points.

The decay of volume space charge with time within such a conductor is then given by

\[
K = K_1 e^{-\left(\frac{\sigma}{\varepsilon}\right) t}
\]

(2.8)

where \( K_1 \) is the initial volume space charge distribution, (Rikitake 1966 p.127). In a dielectric \( K_1 \) is zero and in rocks \( \sigma/\varepsilon \) is so large that \( K \) may be assigned zero everywhere. Consequently we replace (2.4) everywhere by

\[
\nabla \cdot \mathbf{E} = 0
\]

(2.9)

Taking the curl of 2.1 and substituting from (2.2) we get

\[
\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial}{\partial t} \left( \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \right)
\]

(2.10)

Expanding the left hand side and using equation (2.9) leads to the inhomogeneous wave equation
\[ \nabla^2 E - \mu_0 \sigma \frac{\partial E}{\partial t} - \mu_0 \epsilon \frac{\partial^2 E}{\partial t^2} = 0 \] (2.11)

Solution of this equation subject to appropriate boundary conditions is the fundamental problem in establishing expressions for magnetotelluric impedance. Once a solution for \( E \) has been found, \( H \) may easily be determined using equations (2.1) and (2.6).

2.2.2 Boundary Conditions

Maxwell's equations from which (2.11) was derived are valid only for points of a medium where the physical properties of the medium vary continuously. However, across any surface which bounds two media sharp changes in \( \epsilon, \mu \), or \( \sigma \) may be expected and discontinuities may appear in the field vectors. To relate the field vectors on either side of the surface of discontinuity we need to know the boundary conditions. These conditions are derived by Stratton (1941 p.34) and may be stated as:

(a) the component of electric field parallel to a boundary is continuous across that boundary;

(b) the component of magnetic induction parallel to a boundary is continuous across that boundary;

(c) the component of current density normal to the boundary is continuous across the boundary.

In addition to these boundary conditions, it is usually necessary to have two further conditions in order to make the problem determinate. The first describes the behaviour of the field near the source and the second forbids the field to increase without limit as it goes to infinity.

In calculating the MT impedance of a specific conductivity
distribution, the ease of solution depends on the ease with which boundary conditions can be applied. Elementary solutions of (2.11) can be found by the method of separation of variables and, since the equation is linear, the general solution is simply the superposition of all possible elementary solutions. However, matching the general solutions on either side of a boundary remains simple only so long as the surfaces of equal phase or equal amplitude can be described by an equation involving only a single coordinate of a suitable coordinate system. Plane waves incident normally on a horizontally layered medium is a simple case to treat. But if the layers are not exactly parallel or if the incident field is not strictly uniform, the mathematics may prove intractable.

2.2.3 Plane wave propagation

In the dielectric \( \sigma = 0 \), and elementary solutions of (2.11) are of the form

\[
E_{x,y,z} = A e^{i(ax + by + cz)} e^{iwt}
\]

(2.12)

where \( A, a, b, c \) and \( w \) are constants, and

\[
a^2 + b^2 + c^2 = \frac{w^2}{\mu_0 \varepsilon_0}
\]

(2.13)

If \( a, b, c \) and \( w \) are all real, expression (2.12) represents a real plane wave, that is, a wave whose surfaces of constant phase are planes coincident with the surfaces of constant amplitude.

Because of the ease of applying boundary conditions, many authors have expressed MT theory in terms of real plane wave propagation in a layered conducting medium. This permits boundary conditions to be.
introduced in the form of Snell's laws of reflection and refraction, and MT impedances are derived in terms of transmission and reflection coefficients at plane boundaries (Cagniard 1953, Sims & Bostick 1969, Word et al. 1970).

Alternatively, analogies with transmission line theory may be exploited and magnetotelluric relationships can be established directly from transmission line results (Madden & Swift 1969).

Unfortunately, real plane waves may not even approximately represent the source fields produced by realistic current distributions in the ionosphere or magnetosphere. For example, Wait (1962) has shown that the field due to a uniform line current bears little similarity to a real plane wave unless the observer is more than $10^6$ km from the line current.

It is obvious from the elementary solution (2.12) that even if a real $w$ is assumed, any two of the constants could be imaginary. Expression (2.12) would then represent a plane wave at a complex angle of incidence on the earth's surface, sometimes called an evanescent wave. So although any oscillating field at the earth's surface can be built up from a superposition of plane waves, an adequate representation would have to include evanescent waves.

Wait (1954, 1962) has extended the plane wave approach to account for an arbitrary source field. But, since evanescent waves do not represent real waves in the physical sense, their inclusion in a general treatment makes it difficult to relate the mathematical problem to the physical reality.

The plane wave approach has been accused of fostering misconceptions and producing misleading results in MT studies (Wait 1954, Price 1962, Rikitake 1966, Bullard & Parker 1970).
2.2.4 Electromagnetic Field Diffusion

To obtain some physical insight into the problem, Price's approach seems preferable in discussing magnetotelluric theory. Price (1962) stressed that the electromagnetic variations we are concerned with are so slow that physically the problem is one of field diffusion and not of wave propagation into the conductor.

In the conductor, \[ \sigma \frac{\partial E}{\partial t} \gg \epsilon \frac{\partial^2 E}{\partial t^2} \]
if \( T \gg \epsilon / \sigma \), where \( T \) is the period of variation of the oscillating field.

This condition evidently holds in the MT range of periods (i.e. periods greater than 1 second) even in materials with conductivities as low as \( 10^{-5} \) mho/m. So in the conductor, equation (2.11) can be replaced by

\[ \nabla^2 \vec{E} = \mu_0 \sigma \frac{\partial \vec{E}}{\partial t} \] (2.14)

In the free space region where \( \sigma = 0 \), the time of propagation across any part of the region can be neglected in comparison with the time taken by field changes at a point to become effective. This permits the quasi static approximation (Lahiri & Price 1939 p.511)

\[ \nabla^2 \vec{E} = 0 \] (2.15)
2.3 Magnetotelluric relationships for One Dimensional Structures

2.3.1 General Theory for a conducting half-space

A \( n \)-dimensional model is one in which the parameters describing the model are functions of \( n \) space coordinates. The model considered in this section is one in which conductivity is a function of depth only.

Since the depths being probed in a MT study are a small fraction of the earth's radius, the earth may be treated as a semi-infinite conductor occupying the half space \( z > 0 \) of Cartesian coordinates (Wait 1962). The \( z \)-axis is taken as vertically downwards from the surface.

We assume that an arbitrary oscillating magnetic field outside the conductor is inducing electric currents to flow in the conductor. These currents themselves produce a magnetic field at the earth's surface which interferes with the source field. To calculate the quantity called impedance which is the fundamental measurement in the MT method, we must find the horizontal components of the magnetic and electric fields at the surface of the conductor.

The way we proceed is first, (following Price (1962)) to find elementary solutions of (2.14) by the method of separation of variables. These solutions are expressed as functions of a separation constant. The general solution then consists of a summation or integration of elementary solutions over all possible values of the separation constant.

Let \( E(t,x,y,z) = T(t)Z(z)E(x,y) \) \( (2.16) \)

Then equation (2.14) may be written.
\[ \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = \left[ \mu \sigma \frac{1}{T} \frac{dT}{dt} - \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} \right] F \quad (2.17) \]

The coefficient of \( F \) on the right is independent of \( x \) and \( y \) and so is a constant, say \(-\nu^2\). (2.16) thus decomposes into two equations

\[ \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = -\nu^2 F \quad (2.18) \]

and

\[ \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = \nu^2 + \mu \sigma \left( \frac{1}{T} \frac{dT}{dt} \right) \quad (2.19) \]

To reduce the complexity of equation (2.19) we assume a field varying harmonically in time with period \( 2\pi/\omega \). Thus \( T(t) = e^{i\omega t} \) where \( i = \sqrt{-1} \), and (22) can be replaced by

\[ \frac{\partial^2 Z}{\partial z^2} = \left[ \nu^2 + \mu \sigma i \omega \right] Z \quad (2.20) \]

It follows from equations (2.9) and 2.16 that

\[ Z \left( \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} \right) + F_z \left( \frac{\partial Z}{\partial z} \right) = 0 \]

So either

\[ \left( \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} \right) = - \frac{1}{Z} \frac{\partial Z}{\partial z} \quad (2.21) \]

or

\[ F_z = 0 \quad \text{and} \quad \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = 0 \quad (2.22) \]

Price (1950) showed that solutions corresponding to (2.21) had associated with them varying electric currents which produced no magnetic field outside the conductor. These solutions are required only for the study of free decay of current distributions in a conductor and are of no interest to induction studies.

Taking (2.22) as the solution required for induction problems, it is obvious that \( F \) is of the form \( \left( \frac{\partial P}{\partial y}, -\frac{\partial P}{\partial x}, 0 \right) \) where \( P \) is a scalar function of \( x \) and \( y \).
In order that $\mathcal{E}$ may satisfy (2.18) $P$ must also satisfy

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \nu^2 P = 0 \quad (2.23)$$

Hence the electric field inside the conductor has an elementary solution

$$\mathcal{E} = e^{iwt} Z(z) \left( \frac{\partial P}{\partial y}, - \frac{\partial P}{\partial x}, 0 \right) \quad (2.24)$$

where $Z(z)$ and $P(x,y)$ are given by solutions of the second order differential equations (2.20) and (2.23) respectively.

The corresponding elementary solution for $\mathcal{H}$ is, by virtue of (2.1), (2.6) and (2.24)

$$\mathcal{H} = -\frac{e^{iwt}}{\mu^2 w} \left\{ \frac{dZ}{dz} \frac{\partial P}{\partial x}, \frac{dZ}{dz} \frac{\partial P}{\partial y}, \nu^2 ZP \right\} \quad (2.25)$$

Although we now have elementary solutions for $\mathcal{E}$ and $\mathcal{H}$ which can be used to build up the complete solution, they are expressed in terms of the unknown functions $P$ and $Z$. To identify these functions we have to relate them to source fields present in the free space region.

Equation (2.15) describes the field behaviour in free space, and may be solved by separation of variables to obtain the relation for $Z$.

$$\frac{d^2 Z}{dz^2} = \nu^2 Z$$

So that $Z$ has a solution of the form

$$Z = Ae^{-\nu Z} + Be^{\nu Z} \quad (2.26)$$

$A, B$ constants

Expanding equation (2.9) we have

$$Z \frac{\partial F_x}{\partial x} + Z \frac{\partial F_y}{\partial y} + F_z \frac{dZ}{dz} = 0 \quad (2.27)$$
But we know the tangential components of $\mathbf{E}$ are continuous at $z = 0$ so that
\[
\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = 0 \quad \text{which implies} \quad F_z = 0 \quad (2.28)
\]
Free space solutions for $\mathbf{E}$ and $\mathbf{H}$ are thus
\[
\mathbf{E} = Z(z) \left\{ \frac{\partial P}{\partial y}, -\frac{\partial P}{\partial x}, 0 \right\} \quad (2.29)
\]
\[
\mathbf{H} = \frac{1}{\mu} i \omega \left\{ \frac{dZ}{dz} \frac{\partial P}{\partial x}, \frac{dZ}{dz} \frac{\partial P}{\partial y}, \mu^2 P \right\} \quad (2.30)
\]
with $Z$ and $P$ satisfying (2.26) and (2.23) respectively.

Recognising that \( -\frac{d}{dz} (dZ/dz) \) in free space, (2.30) can be rewritten as
\[
\mu i \omega \mathbf{H} = \nabla \left( P \frac{dZ}{dz} \right)
\]
or
\[
\mathbf{H} = -\nabla \Omega
\]
where
\[
\Omega = \left\{ -\frac{A}{\mu} i \omega \ e^{-\mu Z} + \frac{B}{\mu} i \omega \ e^{\mu Z} \right\} P(x,y,\mu) \quad (2.31)
\]
Then the elementary magnetic field in the non conducting region may be characterised by the scalar potential function $\Omega$.

If $\mu$ is taken to be real and positive, the term in $e^{-\mu Z}$ represents the inducing field, while that in $e^{\mu Z}$ corresponds to the field of the induced currents. We can now determine the physical significance of the parameter, $\mu$. The reciprocal of $\mu$ is related to the horizontal dimensions of the source field. For example if $P(x,y,\mu)$ assumes the simple form $\cos \mu x$, the first term of (2.31) corresponds to an inducing field having a spatial wavelength of $2\pi/\mu$ in the $x$ direction.

An interesting point to note from the elementary solution is that the $\mathbf{E}$ and $\mathbf{H}$ vectors are everywhere orthogonal since
\[
E_x H_x + E_y H_y + E_z H_z = 0.
\]
This property of the field vectors means that the ratio of orthogonal horizontal components of $E$ and $H$ is independent of measuring axes except for a possible phase change of $\Pi'$. 

2.3.2 Surface impedance for a $n$-layered half space

Equations (2.20), (2.23), (2.24) and (2.25) hold for every layer of the model illustrated in Fig. 2.1. Each layer is homogeneous and the bottom layer extends to infinity.

In the $m$th conducting layer

$$Z(z) = A e^{\theta_m z} + B e^{-\theta_m z}$$ (2.32)

where $\theta_m^2 = \omega^2 + i \frac{w}{\mu} \sigma_m$ (2.33)

and

$$\frac{E_x}{H_y} = \frac{\frac{\partial P}{\partial y} Z(z)}{\frac{1}{iw/\mu} \frac{\partial P}{\partial y} \frac{dZ}{dz}} = -i\omega/\mu \frac{\theta_m}{A e^{\theta_m z} + B e^{-\theta_m z}} \frac{\theta_m}{A e^{\theta_m z} - B e^{-\theta_m z}}$$ (2.34)

This ratio is called the impedance and we will follow convention and denote it by $Z$. Henceforth, the $Z$ notation will be used exclusively for impedance and the separation function $Z(z)$ will be replaced by its explicit form $(A e^{\theta_m z} + B e^{-\theta_m z})$.

Analytic solutions for the surface impedance take the form of recursion relations that satisfy in sequence the boundary conditions at each interface. The approach we follow here is that suggested by Srivastava (1965).

In the $m$th layer at a depth $z = z_1$ from the surface

$$\frac{E_x}{H_y} = -i\omega/\mu \frac{\theta_{m1}}{A e^{\theta_{m1} z_1} + B e^{-\theta_{m1} z_1}} \frac{\theta_{m1}}{A e^{\theta_{m1} z_1} - B e^{-\theta_{m1} z_1}}$$

$$= -i\omega/\mu \frac{\cosh \theta_{m1} z_1}{\theta_{m1}} - \log \left( \frac{A}{B} \right)^{\frac{1}{\theta_{m1}}}$$ (2.35)

$$= Z_1$$
$V_Y^z IC_0 Z = h_1 Z = h_2 Z = h_m Z = h_{n-1}$
At a depth \( z_2 < z_1 \) in the \( m \)th layer

\[
Z_2(z_2) = -\frac{i \omega \mu}{\varepsilon_m} \coth \int \theta_m z_2 - \log \left( \frac{A}{B} \right)dz
\]

\[
= -\frac{i \omega \mu}{\varepsilon_m} \coth \int \theta_m(z_2 - z_1) - \coth^{-1} \left( \frac{z_1 \theta_m}{i \omega} \right)
\]  

(2.36)

Equation (2.36) can be used to relate the impedance at the top of a layer to the impedance at the bottom of a layer. So we can start with the \( Z \) value at the top of the lowest layer and work upwards to determine \( Z \) at the surface.

In the bottom layer, which is imagined to extend to infinite depth, \( A \) must be zero since the field must vanish as the depth tends to infinity. The impedance, denoted \( Z_n \), at the top of the lowest layer is given by (2.34)

\[
Z_n = \frac{i \omega \mu}{\varepsilon_n}
\]  

(2.37)

Relation (2.36) is employed to calculate the impedance \( Z_{n-1} \) at the top of the \( (n-1) \)th layer

\[
Z_{n-1} = -\frac{i \omega \mu}{\varepsilon_{n-1}} \coth \int - \theta_{n-1} h_{n-1} - \coth^{-1} \left( \frac{\theta_{n-1}}{\varepsilon_n} \right)
\]

where \( h_{n-1} \) is the thickness of the \( (n-1) \)th layer.

This procedure can be continued upward to obtain the surface impedance relation

\[
Z(0) = Z_1 = \frac{i \omega \mu}{\varepsilon_1} \coth \left\{ \theta_1 h_1 + \coth^{-1} \frac{\theta_1}{\varepsilon_2} \coth \int \theta_2 h_2 + \coth^{-1} \left( \frac{\theta_2}{\varepsilon_3} \theta_3 h_3 + \ldots + \coth^{-1} \frac{\theta_2}{\varepsilon_n} \right) \right\}
\]

(2.38)
2.3.3 Source field considerations

The orthogonality of \( \mathbf{E} \) and \( \mathbf{H} \) vectors and the impedance relationship have been shown for only a single value of \( \nu \). However, a natural source field may be built up from a number of elementary source fields each characterised by a different value of \( \nu \).

For the sake of illustration we will represent the inducing field of a horizontal line current in terms of its elementary sources. A line current \( J \) at height \(-z\) flowing parallel to the \( y \) axis at \( x = 0 \) produces a horizontal magnetic inducing field at the earth's surface \( z = 0 \) given by

\[
H_x = \frac{Iz}{\sqrt{2\pi (x^2 + z^2)}}
\]

which, with the aid of the standard integral transform,

\[
\int_0^\infty e^{-\nu z} \cos \nu x \, d\nu = \frac{z}{(x^2 + z^2)}
\]

can be rewritten

\[
H_x = \frac{I}{2\pi} \int_0^\infty e^{-\nu z} \cos \nu x \, d\nu
\]  

(2.39)

Comparing this expression with (2.31) we can identify the function \( P(x,y,z) \) and the constant \( A \) in the elementary solution. For the line current \( P(x,y,\nu) = \sin \nu x \), and \( A = -\frac{I}{2\pi} \). To obtain a full solution of \( \mathbf{E} \) and \( \mathbf{H} \) we need to calculate the contribution to the fields by the field of the induced currents. This is given by \( B \) which is a function of the source geometry and the conductivity structure of the earth.

A complete solution for the electromagnetic field of a line current over a layered earth has been obtained by Hermance and Peltier (1970), who also calculated the fields of a Gaussian distribution of line
currents (Peltier and Hermance 1971). The problem of induction in a layered earth was reformulated by Summers and Weaver (1973) in terms of one scalar component of the magnetic Hertz vector, and they were able to find complete solutions for the fields associated with line currents and magnetic dipoles.

These complete solutions are very rarely used in MT interpretation (except when artificial source fields have been used). The introduction into the interpretation of another unknown in the form of the source field distribution usually makes the interpretation prohibitively non-unique. Complete solutions for particular source field distributions are only used to calculate the effect on interpretation of neglecting the source field distribution.

We now assume that the source field is uniform i.e. \( \nu = 0 \) and examine the effect this has on the values for surface impedances as calculated by equation (2.38). It is first helpful to introduce the concept of skin depth.

It is apparent from (2.32) that in a homogeneous medium with conductivity \( \sigma \), a uniform electric or magnetic field is attenuated exponentially as it diffuses into the conductor. The depth at which the amplitude of the field is \( 1/e \) its surface value is called the skin depth, denoted \( \delta \), and represents the effective penetration depth of the field. From (2.32) \( \delta \) may be calculated to be

\[
\delta = \sqrt{\frac{2}{\mu \omega \sigma}} \tag{2.40}
\]

In a medium with conductivity \( \sigma \) mho/m and magnetic permeability \( 4 \pi \times 10^{-7} \) H/m, a field variation of period \( T \) seconds has a skin depth of

\[
\delta = 0.504 \sqrt{T/\sigma} \text{ km}. \tag{2.41}
\]
Returning to equation (2.38) and noting relationship (2.33), it may be seen that the assumption of a uniform field has negligible effect on \( Z(o) \) if \( \nu^2 \ll \frac{w}{\mu} \sigma_m \) for all \( m \). Thus one criterion for ignoring the source field effect is that the wavelength \( 2 \pi / \nu \) of the source field is very much less than the skin depth of a uniform field in each of the \( n \) layers. Although this is a sufficient condition, a less restrictive criterion can be applied which takes into account the effect on the field of successive layers.

Sims and Bostick (1969) show that if a generalised skin depth for a layered medium is defined to be

\[
\Re \int_0^l \left( \frac{s'(w)}{\sqrt{\frac{iw}{\mu} \sigma(z)}} \right) \, dz \, J = 1 \quad (2.42)
\]

then \( S \) will be a good measure of the depth of penetration of the MT fields.

Source fields may be treated as horizontally uniform if

\[
\frac{2 \pi}{\nu} \gg S(w) \quad (2.43)
\]

i.e. if the horizontal wavelength of the source field is much greater than the depth of penetration in the earth.

In order to be able to apply this criterion, we require knowledge of the range of values which \( \nu \) can take. Price (1962) suggests that the greatest value of \( 2 \pi / \nu \) is of the order of the circumference of the earth and corresponds to a first order spherical harmonic. This gives \( \nu > 10^{-4} \) \text{ km}^{-1} \). The smallest values of \( 2 \pi / \nu \) likely to be encountered are probably related to very local fields like the field of an ionospheric line current. For a line current at a height of 100 km (2.39) shows that the contributions to the surface field by
elementary sources drops off sharply for \( \frac{1}{\nu} > 100 \text{ km} \). This effectively places \( \nu \) in the range \( 10^{-4} \text{ km}^{-1} < \nu < 10^{-2} \text{ km}^{-1} \).

So when MT soundings are being made to depths of less than 100 km, the effects of the source field may be ignored unless measurements are being made close to a highly localised current system like the auroral or equatorial electrojets.

2.3.4 Interpretation of single site magnetotelluric data

Magnetotelluric data is most commonly interpreted in terms of a layered conductivity distribution, since this type of model is simplest in terms of the number of parameters. The concept of apparent resistivity has proved to be of great usefulness in interpretation. Apparent resistivity is defined as

\[
\rho_a = \frac{1}{\mu_0 w} \left\| \frac{E_x}{H_y} \right\|^2 = \frac{1}{\mu_0 w} \left| z(0) \right|^2 \tag{2.44}
\]

If the amplitudes of \( E_x \) and \( H_y \) are measured in working units of MV/km and gammas, respectively, then

\[
\rho_a = 0.2 T \left| \frac{E_x}{H_y} \right|^2 \tag{2.45}
\]

where \( \rho_a \) is in ohm m and \( T \) is in seconds.

For a homogeneous half space, expression (2.37) can be substituted into (2.44) to yield

\[
\rho_a = \frac{1}{\sigma}
\]

So in this case the apparent resistivity is directly related to the actual resistivity of the medium.
For a two-layered conductivity distribution, apparent resistivity is a function of angular frequency, $\omega$, and values of $\rho_a$ at certain frequencies may show no similarity to the actual resistivities present in the distribution. However, even here, it is possible to derive simple interpretation rules by looking at how the curve of apparent resistivity values behaves at extreme values of frequency. Figure 2.2a shows apparent resistivity curves for several two-layered distributions of the same form as that in Figure 2.2c. The curves all have the characteristic that at high frequencies, short periods, $\rho_a$ approximates the actual resistivity of the top layer, and at low frequencies, long periods, $\rho_a$ is very near the resistivity of the substratum.

The physical explanation for this behaviour lies in the variation of the skin depth with period. At very short periods the skin depth is very small according to (2.40). Effectively all of the induced current flows in the top layer and the two-layer distribution is indistinguishable from a homogeneous distribution. Conversely, for very long periods the magnetic field penetrates into the second layer with little attenuation and the induced current is concentrated in the substratum.

In the case of a two layered distribution it may be possible using the simple interpretation rules mentioned above to estimate the resistivities of the two layers directly. However, there is no simple way to estimate directly the thickness of the top layer.

For a complete interpretation of the data it is necessary to match the experimental apparent resistivity curve with synthetic curves obtained for all possible two layer model distributions. Cagniard
Apparent Resistivity ($\rho_a$) in Ohm-metres

Period ($T$) in Seconds

$\rho_1 = 1\,\text{ohm m.}$

$z = 1\,\text{km.}$

$\rho_2 = \rho\,\text{ohm m.}$
showed that all possible two layer curves could be represented by a single family of curves obtained for the simple model shown in Figure 2.2c. By matching the field curve with one of his master curves all the parameters of the two layer distribution could be found.

Cagniard's method is based on the law of electric similitude which describes how the apparent resistivity curve for a layered model changes with a given change in the model. When a model with parameters $\rho_1$, $\rho_2$ and $h$ is changed by scaling to a model with parameters $\rho$, $\rho_2$ and $\tilde{h}h$, each point $(\rho_a, T)$ on the resistivity curve is transformed to the point $(\tilde{\rho}, \rho_a, \tilde{T}T)$ where

$$\tilde{T} = \frac{\tilde{h}^2}{\tilde{\rho}}$$

If the apparent resistivity curve is drawn on log - log paper, changes in the model will not change the shape of the curve but will merely displace it along the vertical axis by an amount $\log \tilde{\rho}$ and along the horizontal axis by an amount $\log \tilde{T}$. By comparing the field curve with one of Cagniard's master curves, we can find the displacements of one relative to the other and hence find the way in which the Cagniard model should be changed to obtain the model corresponding to the field curve.

The curve matching technique can be extended to consider more than two layers. But there is only one variable parameter in the two layer model and only one set of master curves, while there may be many independent parameters in a multi layered master model and a very large number of sets of master curves are needed to represent the full range of resistivity combinations.

When models of more than two layers are being considered and they
do not correspond to special cases where a catalogue of curves exists, it is simpler to employ a direct inversion method and derive a satisfactory model directly from the data.

All direct inversion procedures for MT data (apart from random search methods) involve three steps. Firstly the form of the model is specified, usually the number of layers in it. Then a 'first-guess' model is generated from the data. Thereafter, algorithms are used to modify the first guess so that it fits the data in some optimum way. It is hoped that the first guess is reasonably close to the best fitting model, for the iteration to converge.

The method most commonly used to invert MT data is the method of least square error, and a full discussion of several variations of this technique is given by Laird and Bostick (1970).

Until now we have considered interpreting only apparent resistivity data and have ignored the fact that impedance data also contains phase information. Master curves of phase variation with period have been constructed (see Figure 2.b) and phase data can often be used to reduce the ambiguity of interpretation. Direct inversion methods are available to invert complex impedance data. However, many authors ignore phase data, apparently because of the large errors in their estimates.

When phase data are reliable, the complete complex impedance estimate may be used in a very simple but effective inversion scheme (Schmucker 1970).

In the two layer case of a resistive surface layer overlying a conductive substratum, $\Theta_1$ may be approximated by the wavenumber $\lambda$ in the top layer, and $\Theta_2$ may be replaced by $(1+i)\delta_2$ in the conductor, where $\delta_2$ is the skin depth defined by (2.40).
If it is assumed further that the thickness, \( h \), of the upper layer is small compared with the dimensions of the source i.e. \( h \ll 1 \) then to a first order approximation, equation (2.38) may be reduced to

\[
\frac{Z(0)}{2w/\mu_0} = h + \frac{\delta 2}{2} - i \frac{\delta 2}{2}
\]

Replacing the left hand expression by \( C \), it follows that

\[
\rho_2 = \frac{2w/\mu_0}{\text{Im} \{C\}^2} \quad (2.47)
\]

and \( h = \text{Re} \{C\} - \text{Im} \{C\} \quad (2.48) \)

Thus, if impedance estimates have been determined above a multilayered substratum, (2.48) and (2.47) can be applied to obtain for each frequency component the depth to the top of and the resistivity, respectively, of a uniform substitute conductor. Such an interpretation is physically meaningful only when the phase of \( Z \) lies between \( \pi/4 \) and \( \pi/2 \).

When \( h \) is positive, the multilayered substratum may be replaced by a uniform conductor at depth \( h \), as far as its response to one frequency is concerned.

It can be proved quite generally (Weidelt 1972) that the real part of \( C \) is the mean depth \( \delta^* \) of the in-phase eddy currents flowing in a multilayered conductor.

In this simple two layer case, \( \delta^* \) is given by

\[
\delta^* = \text{Re} \{C\} = h + \frac{\delta 2}{2} \quad (2.49)
\]

Schmucker has argued that a multilayered conductor can be represented in its response at any frequency by a two-layered conductor, and that the depth \( \delta^* \) at which current flow is concentrated is in a region, where resistivity \( \rho_2 \), can be determined.
His inversion procedure consists simply of calculating $\rho_2$ from (2.47) and $h^*$ from (2.49). A plot of the depth of current flow, $h^*$, versus the resistivity, $\rho_2$, of a uniform substitute conductor represents a good approximation to the actual conductivity distribution as Schmucker has demonstrated (Schmucker 1970, p.70, Schmucker 1975).

2.4 Magnetotelluric relationships for two dimensional structures

2.4.1 Solution of the differential equations

Even after source effects have been discounted, it is often found that the results obtained for layered media can not explain field data. Earth currents at a single site are generally found to predominate in a certain direction and this kind of apparent anisotropy has usually been explained in terms of a lateral variation in conductivity around the observation point. In such situations the MT impedances are very different from those of layered structures. However, there are great mathematical and numerical difficulties in treating structures with lateral inhomogeneities, principally because all six magnetotelluric field components are coupled to each other. Fortunately, many interesting geologic structures can be well described by two-dimensional models. The criterion for two dimensionality is that the structure keeps the same form for a distance of a few skin depths along strike. Theoretical results for the one dimensional case (section 2.3.4) demonstrate that measurements are unaffected by features occurring more than one skin depth away from the measuring point.

For the case of a two-dimensional distribution $\sigma$ is a function of only two coordinates, say $x$ and $z$, and the mathematical analysis
separates into two distinct modes. It is assumed that all field quantities are independent of the y direction, which defines the direction of strike, and also that displacement currents may be neglected.

Then, assuming a time variation of the form \( e^{i \omega t} \), Maxwell's equations (2.1) and (2.2) may be expressed in terms of the field components

\[
- \frac{\partial H_y}{\partial z} = \sigma E_x
\]

(2.50)

\[
\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \sigma E_y
\]

(2.51)

\[
\frac{\partial E_y}{\partial x} = i \omega \mu H_x
\]

(2.52)

\[
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = i \omega \mu H_y
\]

(2.53)

\[
\frac{\partial E_y}{\partial x} = i \omega \mu H_z
\]

(2.54)

\[
E_y, H_x \text{ and } H_z \text{ appear only in equations (2.51), (2.53) and (2.55) while equations (2.50), (2.52) and (2.54) contain only } H_y, E_x \text{ and } E_z. \]

Obviously the six equations decouple into two distinct sets, and each set may be solved independently. The first set (2.51), (2.53), (2.55) corresponds to E polarisation since for this set the electric field is polarised parallel to the strike of the conductor. By eliminating \( H_x \) and \( H_z \) we reduce the problem to one of solving

\[
\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} = i \omega \mu \sigma E_y
\]

(2.56)

\( H_x \) and \( H_z \) are then obtained from (2.53) and (2.55).
Similarly, the set of equations corresponding to H polarisation can be reduced to

\[ \frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} = \frac{iw}{\mu \sigma} H_y \]  

(2.57)

A solution for \( H_y \) allows \( E_x \) and \( E_z \) to be found from (2.50), (2.52).

Despite the similarity in appearance between (2.56) and (2.57) the field distributions corresponding to their respective solutions show important differences. For the H polarisation case, we can see from (2.50) and (2.52) that in the non conducting region \( H_y \) is independent of \( x \) and \( z \) and so is uniform throughout this region. In E polarisation, on the other hand, none of the field components is uniform in any region and field perturbations can extend far into the non-conductor.

Because of the difficulties encountered in applying boundary conditions to the general solutions of (2.56) and (2.57), analytic solutions exist only for very special conductivity distributions. These analytic solutions are reviewed by Hobbs (1975). They are sometimes only approximate solutions and they are invariably expressed in such a complicated form that they have to be evaluated numerically. Since numerical methods can be used from the start to solve (2.56 and 2.57) for a very much wider range of conductivity structures, analytic solutions have little relevance to MT interpretation. However, they do provide a useful check on the numerical methods of solution.

In the pure numerical procedures the free space-conductor model is enclosed within a mesh of grid points. The boundaries of the mesh must be sufficiently far away from vertical discontinuities that the perturbations in the field at the boundaries may be assumed to be insignificant. Usually the mesh lines are taken as boundaries of homogeneous conducting regions so that all conductivity changes are
A variety of numerical methods exists, such as the finite element method used by Coggan (1971) and Reddy and Rankin (1972), and the transmission line analogy applied by many authors including Madden and Swift (1969). The method most widely discussed in the literature is the finite difference method in the form developed by Jones and Price (1969). In this method, equations (2.56) and (2.57) are replaced by a set of finite difference equations for each point on the mesh, and this set is solved simultaneously by the Gauss-Seidel iterative method.

In all the numerical methods, the field variation between the grid points is assumed to have some special form – usually linear. For this assumption to be valid, the grid spacing must be chosen with care. For example, in the free space region Jones and Price (1969) show that the field variation is nearly linear in both directions and a large grid spacing can be used. However, in the conducting region the field varies approximately linearly in the horizontal direction but exponentially near horizontal discontinuities and in the vertical direction so that very close grid spacing is required in the vertical direction, and near horizontal boundaries.

There is still considerable debate as to the acceptability of solutions obtained by numerical methods. Different numerical techniques sometimes produce solutions which differ from each other and also from approximate analytic solutions. For instance, Weaver and Thomson (1972) used an approximate analytic solution to evaluate magnetic field above a vertical discontinuity for the E-polarisation case. These results are not in agreement with those of Jones and Price (1970) for the same model. Jones (1972) using a more refined numerical technique was able to reduce but not eliminate the discrepancy.
2.4.2 Behavior of MT response parameters

To get some indication of how apparent resistivity and other MT response parameters might vary in the general two dimensional case it is helpful to study results obtained for the simplest two dimensional model. This model illustrated in Figure 2.3a consists of a uniform source field inducing currents in a semi-infinite half space which is separated into two quarter spaces of homogeneous conducting media by a vertical discontinuity.

For this particular model, sometimes called the fault model, analytic and numerical techniques of solution have the same general characteristics. The behaviour of three parameters widely used in MT studies is illustrated diagrammatically in Figures 2.3b,c,d. \( \rho_\perp \) and \( \rho_\parallel \) represent apparent resistivities calculated from equation (2.44) using the electric field components perpendicular and parallel, respectively, to the strike. The tipper parameter is defined to be

\[ \text{Tipper} = \frac{|H_\parallel|}{|H_\perp|} \]

where \( H_\parallel \) is the vertical component of the magnetic field variation and \( H_\perp \) is the horizontal component of the field perpendicular to strike.

Figures 2.3b and 2.3c show the variation in tipper and apparent resistivity values at different positions on a traverse crossing the discontinuity. We may note immediately that tipper is a maximum near the discontinuity or fault, decreasing to zero in either direction. \( \rho_\parallel \) varies smoothly but \( \rho_\perp \) varies discontinuously along the traverse. As the measuring point recedes from the fault, both \( \rho_\parallel \) and \( \rho_\perp \) approach the actual resistivity of the substratum. At sites near the fault apparent resistivity shows apparent anisotropy with major axis parallel to the fault on the conducting side, and perpendicular to the fault on the resistive side.
Magnetotelluric responses near a vertical conductivity discontinuity (After Vozoff, 1972). In (a), (b), (c) horizontal scales are distance, and response curves are given for a single frequency.

(a) Vertical contact model
(b) Variation of apparent resistivity with distance
(c) Variation of tipper with distance
(d) Apparent resistivity versus frequency curves at two stations near the discontinuity.
The explanation for the behaviour of tipper, $\rho_\perp$ and $\rho_\parallel$, lies in the way the induced currents react to the presence of the fault.

In the H polarisation mode all currents flow through the fault and current densities are the same immediately on either side of the contact. Knowing (2.7), this means that the electric field in medium 2 at the contact is \( \left( \frac{\rho_2}{\rho_1} \right)^2 \) times the electric field in medium 1. At a great distance from the fault current densities are appropriate to the uniform medium and apparent resistivities are in the ratio $\rho_2/\rho_1$.

In the E-polarisation mode, the converse argument is true. Electric field is continuous across the contact, and current density changes abruptly. This sudden change in current distribution produces a magnetic field which is responsible for the appearance of a vertical magnetic component, $H_z$, incorporated into the tipper. Since both $E$ and $H$ are continuous, apparent resistivity is continuous across contact.

Figure 2.3d illustrates how apparent resistivity varies with period at two sites on either side of the fault. The anisotropy of measurements decreases with decreasing period. When the period of variation decreases so that its skin depth is less than the distance to the contact, the apparent resistivity approaches that for a homogeneous conductor.

It emerges from our discussion of the simple fault model that apparent resistivity values $\rho_\parallel$ derived from the electric field component parallel to the strike will provide a first approximation to the change of true resistivity with distance. Apparent resistivities, $\rho_\perp$, corresponding to the electric field component perpendicular to the strike may not in the least resemble the true resistivities present.
The discontinuous behaviour of $\rho_\perp$ is seen not only in the fault model but wherever vertical or sloping boundaries reach the surface because the induced current flow must always be continuous across the boundary.

Jones (1971) has considered models of fault, sloping shelf and step, and has shown that $\rho_\perp$ is very sensitive to the shape of the boundary. This apparent instability in $\rho_\perp$ would indicate that $\rho_\perp$ is not a particularly useful parameter to employ in interpretation.

2.4.3 Rotation of measuring axes and the effect on measured impedance

Thus far, the 2-dimensional problem has been discussed relative to a special set of axes. When the measuring axes are rotated additional features of the 2-dimensional situation become evident.

Consider how impedance changes for a clockwise rotation of the measuring axes though the angle $\gamma$ from a coordinate system $\{x,y\}$ with $x$ along strike, to a new coordinate system $\{x',y'\}$.

Define

$$Z_\perp = -\frac{E_y}{H_x}$$

$$Z_{\parallel} = \frac{E_x}{H_y}$$

With a clockwise rotation the field components transform to

$$E_x' = E_x \cos \gamma + E_y \sin \gamma$$

$$E_y' = -E_x \sin \gamma + E_y \cos \gamma$$

$$H_x' = H_x \cos \gamma + H_y \sin \gamma$$

$$H_y' = -H_x \sin \gamma + H_y \cos \gamma$$

Equations (2.58) through (2.63) can be combined to obtain
\[ \mathbf{E}_x = \int (Z_u - Z_\perp) \sin \gamma \cos \gamma \mathbf{H}_x + \int (Z_u \cos^2 \gamma + Z_\perp \sin^2 \gamma) \mathbf{H}_y \]
\[ = Z_{xx} \mathbf{H}_x + Z_{xy} \mathbf{H}_y \]
\[ \mathbf{E}_y = Z_{yx} \mathbf{H}_x + Z_{yy} \mathbf{H}_y \]

where
\[ Z_{xx} = \left( \frac{Z_u + Z_\perp}{2} \right) \sin 2 \gamma \]
\[ Z_{xy} = \left( \frac{Z_u - Z_\perp}{2} \right) \cos 2 \gamma \]
\[ Z_{yx} = -\left( \frac{Z_u + Z_\perp}{2} \right) \sin 2 \gamma \]
\[ Z_{yy} = \left( \frac{Z_u - Z_\perp}{2} \right) \cos 2 \gamma \]

It follows that, in the 2-dimensional situation an electric field component measured in any direction other than parallel or perpendicular to strike is affected by the magnetic component parallel to it.

Another interesting feature can be observed in equation (2.64)
\[ \mathbf{E}_y \mathbf{H}_x = Z_{xy} + Z_{xx} \mathbf{H}_x \mathbf{H}_y \]

This means that the Cagniard impedance changes value when the polarisation of the magnetic field changes.

In the practical situation, the impedance values \( Z_{xx}, Z_{xy}, Z_{yx}, Z_{yy} \) are known for the measuring axes, and we wish to find the direction of strike and the impedances measured parallel and perpendicular to it.

Choose measuring coordinates \((x,y)\) and consider how the impedances change for a clockwise rotation through angle, \( \theta \), to the coordinate system \((x', y')\).
Start with the tensor equation

\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
= 
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
H_x \\
H_y
\end{bmatrix}
\] (2.71)

A rotation through \( \theta \), gives the relations.

\[
\begin{bmatrix}
E_x^1 \\
E_y^1
\end{bmatrix}
= 
\begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
\] (2.72)

and

\[
\begin{bmatrix}
H_x^1 \\
H_y^1
\end{bmatrix}
= 
\begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
H_x \\
H_y
\end{bmatrix}
\] (2.73)

where \( \begin{bmatrix}
T
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} \) (2.74)

For simplicity write (2.71) as \( \begin{bmatrix}
E
\end{bmatrix} = \begin{bmatrix}
Z
\end{bmatrix} \begin{bmatrix}
H
\end{bmatrix} \) and use this convention throughout.

Then

\[
\begin{bmatrix}
E^1
\end{bmatrix}
= 
\begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
E
\end{bmatrix}
= 
\begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
Z
\end{bmatrix}
\begin{bmatrix}
H
\end{bmatrix}
= 
\begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
Z
\end{bmatrix}
\begin{bmatrix}
T
\end{bmatrix}^{-1}
\begin{bmatrix}
H^1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
Z^1
\end{bmatrix}
\begin{bmatrix}
H^1
\end{bmatrix}
\]

where \( \begin{bmatrix}
Z^1
\end{bmatrix} = \begin{bmatrix}
T
\end{bmatrix}
\begin{bmatrix}
Z
\end{bmatrix}
\begin{bmatrix}
T
\end{bmatrix}^{-1} \) (2.75)

(2.75) can be expanded and simplified to give

\[
2Z_{xx}^1(\theta) = (Z_{xx} + Z_{yy}) + (Z_{xx} - Z_{yy}) \cos 2\theta + (Z_{xy} + Z_{yx}) \sin 2\theta \quad (2.76)
\]

\[
2Z_{xy}^1(\theta) = (Z_{xy} - Z_{yx}) + (Z_{xy} + Z_{yx}) \cos 2\theta - (Z_{xx} - Z_{yy}) \sin 2\theta \quad (2.77)
\]

\[
2Z_{yx}^1(\theta) = -(Z_{yx} - Z_{xy}) + (Z_{yx} + Z_{xy}) \cos 2\theta - (Z_{xx} - Z_{yy}) \sin 2\theta \quad (2.78)
\]

\[
2Z_{yy}^1(\theta) = (Z_{xx} + Z_{yy}) - (Z_{xx} - Z_{yy}) \cos 2\theta - (Z_{xy} + Z_{yx}) \sin 2\theta \quad (2.79)
\]

From (2.77) it is possible to find the maximum and minimum values of \( Z_{xy}^1(\theta) \), called the principal impedances, by maximising the right hand side with respect to \( \theta \). Equation (2.67) shows that \( Z_\parallel \) and \( Z_\perp \).
are the maximum and minimum impedances though not necessarily in that order. The value of \( \theta \) which maximises \( Z_{xy}^1(\theta) \) corresponds to the orientation of the principal axes. One of these principal axes is in the direction of strike.

2.4.4 Interpretation of Magnetotelluric data

According to equations (2.66) and (2.69), \( Z_{xx} = Z_{yy} = 0 \), when \( Z_{xy} \) is maximised or minimised. When experimentally determined impedances are rotated they are never found to vanish. This may be attributed not only to measurement noise but also and more importantly to the fact that no geophysical structure is ever exactly two dimensional.

Fortunately, in three dimensional situations, equations (2.71) to (2.79) are still valid, assuming induction by horizontal magnetic components only (Sims and Bostick 1969). Thus the tensor elements always have the properties

\[
\begin{align*}
Z_{xx}^1(\theta) + Z_{yy}^1(\theta) &= \text{constant} \\
Z_{xy}^1(\theta) - Z_{yx}^1(\theta) &= \text{constant} \\
Z_{yx}^1(\theta) &= -Z_{xy}^1(\theta + 90^\circ)
\end{align*}
\]

Behaviour of empirically determined tensor elements is illustrated for a single frequency in Figure 2.4.

Since interpretation of measured impedances is possible now only for 2-dimensional structures, interpretation cannot proceed meaningfully when the impedances have a 3-dimensional character. It is necessary to define a parameter which can provide a measure of two dimensionality. The most commonly used dimensionality indicator is the skew (Swift 1967),
defined to be

\[ \text{skew} = \frac{|z_{xx} + z_{xy}|}{|z_{xy} - z_{yx}|} \]  

(2.83)

which is independent of the orientation of measuring axes by virtue of (2.80) and (2.81).

In an exact 2-dimensional situation, skew would be zero in the absence of measurement noise. Most authors agree that impedances can be accepted for 2-dimensional interpretation if the skew value is less than 0.4.

If impedances are accepted, the principal impedance axis can be determined by maximising \( \left\{ |Z_{xy}(\theta)|^2 + |Z_{yx}(\theta)|^2 \right\} \) (Swift, 1967). The result is

\[ 4\theta_o = \tan^{-1}\left\{ \frac{(z_{xx} - z_{yy})(z_{xy} + z_{yx})^* + (z_{xx} - z_{yy})^*(z_{xy} + z_{yx})}{z_{xx} - z_{yy}} - \frac{z_{xy}^* + z_{yx}}{z_{xx} + z_{yy}} \right\} \]  

(2.84)

Principal impedances derived from substituting \( \theta_o \) for \( \theta \) in (2.77) and (2.78) are converted to apparent resistivity values using (2.44).

If the principal apparent resistivities are not markedly different, they may be interpreted in terms of one dimensional layered models as described in section 2.3.4.

In general, the principal apparent resistivities show anisotropy, and if the anisotropy is due to the presence of a lateral inhomogeneity, the strike of the structure may be estimated using the vertical component \( H_z \).

Equation (2.51) shows that a vertical magnetic component appears only when there is a horizontal magnetic component perpendicular to strike. Hence finding the horizontal direction in which the magnetic field shows highest correlation with \( H_z \) will give the direction
STATION  JOR1
EVENT  907 24/ 2/74 TO 1136 24/ 2/74

AMPLITUDE OF TENSOR ELEMENTS.

PERIOD = 69.0 SECONDS

\[ |Z_{xy}| \]

\[ |Z_{xx}| \]

\[ |Z_{yx}| \]

\[ |Z_{yy}| \]

ROTATION ANGLE, DEGREES
perpendicular to strike. In nearly 2-D structures tipper can be used by itself for interpretation. Tipper measured at various points on a traverse perpendicular to strike may be compared with tipper computed for various 2-dimensional model distributions. Because of rapid changes in tipper near a vertical discontinuity it is very effective in revealing the edge of an anomaly (See Figure 2.3c).

Interpretation of principal apparent resistivities can in principal be carried out using a 2-dimensional numerical procedure like that of Jones and Price (1969) in combination with an iterative program for applying changes in an optimum way to the model. It is believed that such a procedure would converge to a best-fitting solution. However, the number of calculations involved would be prohibitive when considered in relation to the significance of the final model. Practical methods of 2-D interpretation are many and are reviewed by Vozoff (1972).

Basically, the inversion methods all involve making a 1-dimensional interpretation of the data at each site on a traverse and then extending the models half way to each adjoining site. The parameters of this 2-dimensional model are then modified in a systematic way until the model results match the field data.

An alternative interpretation of apparent resistivity anisotropy has been based on horizontal layering with anisotropic conductivity. Rikitake and Sawada (1962) advanced a theory of electromagnetic induction in an anisotropic thin sheet overlying a thick uniformly resistive substratum, and interpreted field data in terms of their model. Rankin and Reddy (1969) argued that it is valid to treat a multilayered anisotropic system where principal axes of anisotropy in each layer are the same as two independent isotropic layered systems.
However, as Wright (1970) has pointed out, it is difficult to imagine any plausible geologic structure whose conductivity could have different values in different directions in the plane of bedding. In terms of geophysical significance, models of laterally inhomogeneous structures are preferred in interpreting anisotropic data.

For the case where a 2-dimensional induction anomaly is caused by a non-uniform surface layer, an elegant inversion scheme has been proposed by Schmucker (1971).

Take a set of coordinate axes with z down, x parallel to strike and y perpendicular to the strike of the anomalous conductor. Then, assuming that the substructure underlying the non uniform thin surface layer exhibits no lateral conductivity changes, and provided that the surface layer is of uniform, known integrated conductivity, $\tau_n$, on either side of the anomalous region, the integrated conductivity $\tau_a(y)$ at any point of the anomalous thin layer can be calculated from a horizontal profile of E - polarisation impedances across the anomaly. Integrated conductivity $\tau = \int_0^d \sigma(z) \, dz$ where $d$ is the thickness of surface cover.

Starting with impedance values $Z(w,y)$ corresponding to E - polarisation for all stations on the profile, the first step is to subtract the 'normal' impedance $Z_n(w,y)$ measured at stations far from the anomalous region, to obtain the parameter $\gamma_H$ where

$$\gamma_H (w,y) = \frac{1}{i} \frac{Z - Z_n}{\mu_o}$$

Then $c^+$ and $c^-$ are derived.

$$c^+ (w) = \tau_n \frac{Z_n}{i} \mu_o$$

$$c^- (w) = \frac{c^+}{1 - i} \frac{c^+}{\tau_n \mu_o}$$
Values for the parameter $q_H$ can be determined for different points on the profile from the relation

$$q_H(w,y) = -(K^+ + K^-) \ast \left( \frac{\partial C_H}{\partial y} \right)$$  \hspace{1cm} (2.88)

where

$$K^+(y) = \frac{1}{\pi y}$$  \hspace{1cm} (2.89)

and

$$K^-(w,y) = \frac{1}{\pi} \int_0^\infty \left\{ \sin(ky) / \int kC^-(w,k) \right\} dk$$  \hspace{1cm} (2.90)

The symbol $\ast$ represents the operation of convolution.

Then, finally, an estimate for $\tau_a(y)$ can be calculated from the equation

$$\tau_a(y) = \frac{q_H(w,y) /iw/\mu_o - \tau n C_H(w,y)}{C^+(w,0) + C_H(w,y)}$$  \hspace{1cm} (2.91)

A test of the assumptions involved in the method is provided by the fact that the non-uniformity $\tau_a(y)$ found by the inversion must be real and independent of frequency. Hence the interpretation should be carried out with more than one frequency.

If impedance data is unreliable, the inversion can be carried out using a profile of tipplers, $\left| \frac{H_Z}{H_L} \right|$, together with the phase difference data, $\theta(w,y)$ between vertical field and correlated horizontal field. In this case, the one dimensional conductivity distribution below the non-uniform surface layer must be assumed.

If $Z_H$ is defined to be

$$Z_H(w,y) = \left| \frac{H_Z}{H_L} \right| e^{i\theta}, \text{ then } q_H \text{ is given by}$$

$$q_H(w,y) = -(K^+ + K^-) \ast Z_H$$  \hspace{1cm} (2.92)

When both tipper data and impedance data are reliable for all stations on the profile, a further test that the anomaly is caused by a thin surface layer is provided by combining equations (2.93) and
and (2.88). For a thin surface layer, tippers and impedances should
be related by

\[ Z_H(w, y) = \frac{\partial c_H}{\partial y} \] (2.94)

Schmucker's treatment can be used for interpretation in cases where
two conditions are satisfied. They are (a) the variation with angular
frequency, \(w\), has a mean skin depth in the surface layer which is
several times the layer thickness, \(d\), and (b) the depth of penetration
of the variation field into the underlying crust and mantle is much
greater than \(d\) (Schmucker 1970 p.71).
3.1 Choice of recording sites

Results from a previous geomagnetic investigation in Kenya (Banks 1972) were considered together with other geophysical and geological evidence in order to select suitable sites for the MT investigation. The line of traverse across the Rift Valley was chosen because it crosses the apparent centre of tectonic activity and all parts along it are easily accessible by road. The line coincides with that chosen by the University of Lancaster for their 1969 GDS studies.

A preliminary interpretation of Lancaster's data involved an L-shaped conductivity model of the shape shown in Fig. 3.1 (Banks, 1973). To find how apparent resistivity, $\rho_a$, might be expected to vary with position across the Rift Valley, the finite difference program of Jones and Pascoe (1971) was used to calculate E-polarisation resistivities for points along a line across the L-shaped model, for three different periods, 10, 100 and 1000 seconds. 10 seconds represented the minimum period of variation which could be recorded by the instruments available for the MT study, and 1000 seconds was the order of the maximum period of interest in investigating the local structure of the rift.

Then, assuming that eight stations could be occupied in the time available for fieldwork, eight points were chosen on the traverse line so as best to define the curves of apparent resistivity. It was intended to occupy sites as close to the selected ones (Fig. 3.1) as local conditions would allow, and to record magnetotelluric variations in the period range 10 seconds to 1 day. Measurements in the range 10 to 1000 seconds were expected to provide information about the local variation in conductivity in and around the rift. It was hoped that
very long period measurements obtained at the eight sites could be averaged to produce a significant model of conductivity deep within the upper mantle for comparison with results obtained from other tectonic regions of the earth.

When recording commenced in Kenya, it became apparent that the intended fieldwork program would have to be modified because of the difficulty of obtaining useful long period records. Periods as long as one day were recorded at two sites, but the main effort was expended in acquiring good quality data in the range 10 to 1000 seconds at as many stations as time would allow. A description of MT instrumentation and recording is contained in the following sections together with a discussion of the influence of experimental findings on the final choice of sites.

3.2 Telluric Measurements

3.2.1 General Principles

The principles of telluric field measurements are very simple. The tangential electric field, \( E \), at the earth's surface is given by

\[
E = - \nabla \cdot \nabla \text{ potential at any point on the surface.}
\]

To measure the surface electric field in any direction, all that is required is to measure the potential gradient in that direction. Sensors for detecting one component of the telluric field consist simply of two electrodes placed in the earth at a known distance apart along the direction of interest.
Apparent resistivity (ohm m.)

T = 10 secs
T = 100 secs
T = 1000 secs

Distance (km)

Depth (km)

rift axis

\[ \rho_1 = 1000 \text{ ohm m} \]
\[ \rho_2 = 10 \text{ ohm m} \]
Potential differences detected between the sensors are amplified filtered, if necessary, and recorded on chart paper, photographic film or magnetic tape.

To measure the total surface field, field components must be recorded in two independent directions. A minimum of three electrodes is required for this purpose. Usually, for ease of comparison between telluric and magnetic records, the electrodes are arranged in an L shape and oriented so that components of telluric field are measured along magnetic north and east axes. However, if conditions at the site prevent the use of this standard set of telluric axes, any two lines of different azimuths can be used (Rooney 1939).

Although the principles of telluric field measurements are straightforward, it is often difficult to obtain useable telluric records.

Most of the problems associated with telluric recording can be traced to the electrodes. When a metal is placed in the ground, a contact potential is produced between the metal and electrolytic solutions in the earth. A contact potential difference exists between two electrodes if the electrodes differ from each other chemically or physically, or if soil conditions are not identical at both sites.

This potential difference fluctuates whenever changes occur in the physical and chemical environments at the electrodes. Variations in temperature and solution concentration appear to have most influence on contact potentials (Garland 1966). The effect of temperature can be minimised by burying the electrode so that it is thermally insulated by 1 or 2 metres of top soil. Solution concentration can be kept at a steady level by using a 'reversible' or 'non-polarising'
electrode consisting of a metal dipping into a solution of its own salt contained in a porous pot. Because electrical contact with the earth is made by the salt solution diffusing out through the porous pot, the use of reversible electrodes results in a gradual change in the character of the surrounding soil and this may modify the telluric field to a significant extent over a period of time.

Plain metal electrodes are often preferred to reversible electrodes because of cheaper construction and ease of maintenance. Metals which show very small contact potentials must be used, and of these lead has become the most popular electrode material. Lead plates function very satisfactorily as telluric field sensors except in very wet weather when soil moisture content considerably affects solution concentrations and hence contact potential differences.

Large electrode separations can be used to enhance the telluric potential difference relative to the contact potential difference, but they have the disadvantage of averaging out local effects, as well as presenting great problems of cost and of maintenance. Significant measurements have been reported for an electrode spacing of only 15 metres (Simon & Rossigol 1974), but most MT investigators employ electrode separations of the order of 50 to 500 metres.
3.2.2 Electrodes and Lines

For this experiment an L-shaped array of three lead electrodes was used. Orientation and spacing of the electrodes depended on the conditions prevailing at each site, but rectangular geomagnetic axes were used at the majority of sites, and electrodes were approximately 100 metres apart.

Three core domestic wire, buried in shallow trenches, was used to carry the signal from each electrode to the measuring equipment. Joints between lead electrodes and copper lines were insulated with a thick layer of pitch to avoid copper-ground contact - potential effects.

Since the amplifier and filter circuit drew a small current from the ground, it was necessary to ensure that the contact resistance between the electrodes was very much smaller than the circuit resistance. To reduce the contact resistance, flat lead sheets with large surface area (0.5 metres x 0.5 metres) were used. When each electrode was being installed, the procedure was to press it flat against the bottom of a 2 metre deep hole, cover it with a thick layer of top soil, and then saturate the top soil with water before completely burying the electrode.

Contact resistance was usually in the region of 100 $\Omega$ and at none of the sites did it exceed 1 k $\Omega$. This represented an insignificant fraction of the circuit resistance which was always at least 1 M $\Omega$.

Incorporation of a very large resistance in the measuring circuit also meant that problems of electrode polarisation were avoided. The electrode polarisation effect is often observed at very short periods and involves the accumulation of ions at the surface of the electrodes.
this can be modelled as an electrical impedance. In common rocks, the resistance measured between electrodes is frequency dependent in the frequency range 0.01 to 10000 Hz (Ward 1967). However any change in resistance between electrodes has negligible effect on measurements, if the electrode resistance is a very small fraction of the measuring circuit resistance.

An assumption which is implicit in accepting manufacturer's specification for the measuring circuit resistance is that the insulation resistance of the lines is very high. The magnitude of the error introduced into potential difference measurements by faulty insulation can easily be shown to be proportional to the ratio of contact resistance to insulation resistance. To satisfy the requirement of high insulation resistance, it was necessary to check the leakage resistance of the lines in situ. Before and after recording at each site, each line was disconnected and taped at the electrode end and the resistance was measured between the instrument end of the line and a grounded electrode. Lines were checked for faults if the measured leakage resistance had a value below 1 MΩ.

3.2.3 Telluric Measuring Equipment

Each telluric component had to be preamplified and filtered before it was recorded on a Watanabe 5-channel paper chart recorder. For this purpose a high performance telluric amplifier and filter system was built by I. M. Brazier following the design and specifications of D. Trigg (1972). A circuit schematic of the Trigg unit is given in Fig. 3.2.
The primary stage A - B is a differential amplifier with amplification factor $-2$ and common mode rejection ratio (CMRR) of, typically, 100 dB. Because the impedance at the non-inverting input of each operational amplifier is in excess of $10^9 \ \Omega$, a 1 M resistor can be added in series with each of the inputs to protect the operational amplifiers from damage arising from lighting transients.

Section B - C is a low pass filter rejecting signals with periods shorter than 10 seconds. It is essential to use such a filter in areas near a mains power supply. As the mains supply is loaded and unloaded, modulated electric signals with a wide range of periods leak into the ground, and these signals have to be rejected otherwise they may degrade the record. The low pass filter shown here has the advantages that (a) it has very high input impedance and zero output impedance and so can be easily matched to other stages, (b) the cut off frequency can be changed without affecting the gain of the pass band, simply by changing values of resistors, and (c) cut off is very sharp since it is a Butterworth second order low pass filter with $-3$ dB roll off occurring at frequency

$$f_0 = \frac{1}{\pi \sqrt{R_L C_L}}$$

Following this stage comes a Butterworth second order high pass filter C - D of similar design to the low pass stage. This filter has a very large time constant and can pass signals of up to $10^4$ seconds; because of the enormous resistor values employed in the high pass circuit, the operational amplifier needs to have the extraordinarily large input impedance of $10^{11} \ \Omega$. A high pass filter with cut off period of 1 day would be ideal for long period measurements in order to reject long period drift, but components with the required resistance
values are outwith the reach of existing technology.

The last section D–E of the Trigg unit is a variable gain stage which consists simply of a standard inverting amplifier with a gain varying from 0.5 to 50.

When measurements commenced in the Rift Valley, it was found that the signal to noise ratio of the Trigg circuit was unsatisfactory. With an electrode separation of 100 metres, short period micropulsation activity showed amplitudes of less than 5\(\mu\)V, while the Trigg circuit noise level was greater than 2–3\(\mu\)V at short periods.

To overcome this noise difficulty, a Keithley Model 155 null detector microvoltmeter was used as a preamplifier. This instrument has an extremely low noise level (0.15\(\mu\)V peak to peak on its most sensitive range), high input impedance, (\(>1\) M\(\Omega\)) and almost zero drift (0.1\(\mu\)V/°C).

Because the d.c. contact potential difference between the electrodes was usually in the region 10 to 100 mV, the most sensitive setting which could be used on the Keithley instrument without saturating its transistors was 100 mV which represented a preamplification factor of 10. This meant that the Trigg variable gain stage had to be kept in the circuit for a suitably high overall amplification; but, since the Keithley amplifier has its own common mode rejection stage with CMRR of 120 dB, the primary differential input stage of the Trigg unit, could be discarded.

Due to transients coming from the P511C analog devices in the high pass filter stage, the filter had to be modified to use available LM308D operational amplifiers. Because LM308's have a much smaller
input impedance than the analog devices, the resistor values in the filter circuit had to be reduced, resulting in a lower cut off period of 1200 seconds.

The telluric recording circuit which emerged after modifications based on fieldwork experience is shown in Fig. 3.3a.

To extend the spectral range of the data, long period recording was attempted at two sites, with attention being concentrated on obtaining measurements in the period range 200 seconds to 1 day. The circuit used was similar to the one described above, but with the high pass filter replaced by a resistor, potentiometer and battery circuit to back off the 'd.c.' contact potential difference. It was found that the signal level was sufficiently high that the Trigg amplifier could be dispensed with. Fig. 3.3b illustrates the long period recording circuit.

3.2.4 Appearance of the Telluric Records

Long period telluric recording started at Thomson's Falls (TFL) and coincided with the wet season there. The telluric traces were very noisy and drifted considerably, but their most noticeable feature was a succession of impulsive events occurring at intervals of anything from 5 minutes to 3 hours (Fig. 3.4a). The complex nature of the noise resembles that reported by Mitrafanov (1965) and Odera (1974) who attribute the impulsive noise to the instability and electrical breakdown of chemical films produced by electrolytic action on the electrode surface. Although recording continued for more than two weeks, no useful telluric records were obtained, such was the magnitude of the
Fig. 3.3  Block diagrams of magnetotelluric measuring circuits.

(a) Short period recording circuit and frequency response
(b) Long period recording circuit and frequency response.
noise, and the site was abandoned.

Recording was restarted at Longonot (LON) and at Kampi Ya Moto (KYM) both near the rift axis. Conditions at Longonot were very dry and no noise was observed; there appeared to be very good correlation between electric and magnetic events (3.4b). At Kampi Ya Moto occasional rainstorms produced sudden changes in soil moisture levels and contact potentials drifted rapidly. Hot dry winds quickly restored the soil moisture content to its former level in a matter of a day or so and there was little problem in obtaining useful traces. There was no sign of impulsive noise like that seen at Thomson's Falls.

During the dry season recording was made near towns to take advantage of the mains power supply. At Kericho (KER), noise which was evidently leaking from the mains appeared on the telluric traces and reached a maximum at times of peak consumption (3.4c). Measurements made between 7p.m. and 8a.m. were usually so contaminated with mains noise as to be useless, but telluric amplitudes were so large at Kericho that the day-time traces were perfectly satisfactory.

A second attempt was made to record at Thomson's Falls during the dry season, on the opposite side of town from the first site. Though no impulsive noise was present, there was a great deal of unnatural signal which showed no correlation with the magnetic records. It appeared as a 'rectified' wave (Fig. 3.4d). Telluric amplitudes were so small compared to noise that the site was again abandoned, and from that time on measurements were made as far from towns as was feasible.
Fig. 3.4 Appearance of different noise effects on magnetotelluric traces.

(a) Impulsive telluric trace recorded at Thomson's Falls (TFL).

(b) Clean N-S magnetic (H) and N-S telluric (N) signals recorded at Longonot (LON).

(c) Telluric trace measured at Kericho (KER) at 6p.m. local time showing mains leakage noise.

(d) Noisy telluric trace at T. Falls (TFL) showing 'rectified wave' appearance.

(e) Parallel recording of E-W magnetic variation with two parallel Jolivet magnetometers.
3.3 The Magnetic Sensors

3.3.1 General Remarks

The amplitude of natural magnetic signals is strongly dependent on the period of variation of the signal. As illustrated in Fig.1.1a, the variations show characteristic spectral peaks superimposed on a continuum whose power level drops very sharply going towards shorter periods.

It has proved difficult to design a magnetic sensor which demonstrates a high signal to noise level for all periods of interest in magnetotelluric studies, and sensors based on different physical principles have been produced to make measurements in different period bands.

The two types of magnetometers operated in Kenya were of the fluxgate and torque types. Fluxgates have good stability and drift characteristics and they were used to measure long periods up to daily variation. Because of the drop in signal to noise ratio, the fluxgate could not be satisfactorily used to record periods of much less than 5 minutes. The torque magnetometer used was designed by Jolivet (1966) and modified by Albouy et al. (1971). The magnetometer has a very high sensitivity and its frequency response is flat for the range of periods greater than about 5 seconds. Because it is based on the moving magnet principle, the Jolivet magnetometer's response and hence signal/noise ratio, falls off rapidly for periods shorter than 1 second, due to mechanical inertia. For recording periods longer than 1000 seconds, the fluxgate was preferred to the Jolivet magnetometer, because of the latter's uncertain thermal drift characteristics.
3.3.2 The Jolivet Magnetometer

This magnetometer measures a single component of the earth's varying magnetic field. The principle of operation is very simple (Albouy et al. 1971), and is illustrated diagrammatically in Figure 3.5. In essence, the Jolivet instrument consists of a moving magnet and feedback system. The magnet is immersed in a liquid of its own density so as to make it insensitive to shocks and vibrations.

A fine ribbon of wire constrains the magnet to lie between two coils whose axis determines the direction of the component to be measured. A mirror attached to the magnet reflects a light beam on to a photoresistant cell, which is connected into an amplifier and feedback circuit.

Any movement of the magnet is translated by the photoresistor, into an electrical signal which is amplified and fed back as a current to the coils in such a direction that the resultant magnetic field balances the magnetic change producing the signal.

The voltage drop across a 1 k resistor produced by the flow of the feedback current is the output from the device. Since the coils are constructed so as to produce a compensating field of 1 γ when a current of 0.5 μA flows through them, the output voltage from the magnetic sensor is 0.5 mV/γ. The frequency response of the instrument is constant above a period of about 5 seconds. At a period of approximately 2 seconds, the magnet has a resonance and the frequency response peaks as shown in Fig. 3.5b. Because of its mechanical inertia, the magnet responds slowly to variations below 1 second period and the magnetometer sensitivity decreases sharply.
Fig. 3.5  Principle of operation of the Jolivet magnetometer.

(a) Diagram illustrating the principle of operation of the Jolivet magnetometer.

(b) Diagrammatic frequency response of the Jolivet magnetometer.
1 Moving magnet immersed in fluid.
2 Mirror
3 Feedback coils.
4 Photoresistor.
5 Operational amplifier LM 308B
To set up the Jolivet instrument as a magnetic variometer, the instrument must be levelled and oriented so that the axis of the coils is along the direction of the desired magnetic component. To measure any horizontal component, the instrument is first oriented with the coil axis along the magnetic east west line. In this position the magnet lies in its natural rest position and there is near zero output from the instrument.

When the 'zero' position has been found, the instrument may be rotated into the direction of the component it is wished to measure and compensating magnets may be screwed into ports on the side of the magnetometer to back-off the d.c. magnetic field in the direction of interest. For example, to measure the magnetic north south component the magnetometer is first oriented with axis approximately east west and then slowly rotated until a minimum is observed in the output. The instrument is then rotated through 90° with the help of a suitably calibrated base plate, and a compensating magnet is screwed in from the north side of the magnetometer until output voltage is minimised. To measure the vertical magnetic field component, the magnetometer is supported by a stand so that its axis is vertical, and a compensating magnet is inserted into the top of the instrument to 'zero' the output.

Because it is a dynamic feedback device, output from the magnetometer is insensitive to changes of intensity of the lamp bulb. In addition, because the magnet moves very little from its null position (magnet axis perpendicular to coil axis) a change in the torsion constant of the suspension wire due to the effects of temperature produces a negligible error of measurement.

Though not particularly sensitive to changes of temperature the
magnet is highly sensitive to the rate of change of temperature. Turbulence can be induced in the liquid which surrounds the magnet, and this creates noise problems. It is necessary to insulate the Jolivet instrument well against thermal effects, and for this reason the sensors were operated inside polystyrene-lined double-walled wooden boxes, which were buried under more than 0.5 metres of soil.

Tests conducted with two Jolivet magnetometers measuring parallel components revealed that the noise level was quite satisfactory in the period range at which the instruments were operated i.e. 10 to 1200 seconds ($3.4e$).

Output from the Jolivet sensor was filtered with a 10 second Trigg low pass filter to remove the resonance signal and then passed through a 1200 second high pass filter before being recorded on the Watanabe multicorder.

Use of identical filters for telluric and magnetic recording had the advantage that the traces could be compared directly, and the ratio of magnetic and telluric spectral amplitudes was independent of the filters used. However, the disadvantage of using a 1200 second filter was that, since the longest periods in the pass band had amplitudes typically 50 times as great as those of the shortest periods, a chart sensitivity which kept long periods on scale was often inadequate for recording short periods. Fortunately at most sites, very strong bursts of short period signal appeared on the record with the 1200 second filter in the circuit. At sites where the level of short period energy was low throughout the whole time of recording, it was necessary to change to a 300 second cut off high pass filter and to increase the sensitivity of the Watanabe recorder.
A Keithly amplifier had to be used before the filter stages because of the d.c. level of the filter. Because magnetic variations of less than 0.1 \( \gamma \) were being recorded, representing a voltage output of less than 0.05 mV, the most sensitive range of the Watanabe, i.e. 1 mV full scale deflection, had to be used. On this range, the Watanabe potentiometer could not back off the d.c. level of 1.5 mV or so coming from the filter. So the Jolivet sensor output was preamplified by a factor of about 10 before passing through the filter. The entire magnetic recording system is shown in diagrammatic form in Fig. 3.3.

3.3.3 The Fluxgate Magnetometer

Because of its sensitivity to temperature variations, the Jolivet could not be used to record long periods up to diurnal variation. A three-component fluxgate magnetometer of the type described by Trigg et al. (1971) was used to record variations of period greater than 200 seconds.

The fluxgate has a sensitivity of 1V per 100 \( \gamma \); there is an inbuilt temperature compensation device and the manufacturer's specify a temperature drift of only 1 \( \gamma / ^\circ C \). To reduce the daily temperature variation, the magnetometer was placed in a basin and buried under 1 metre of top soil. At this depth temperature variation as monitored by a max-min thermometer was about 2\(^\circ\) C/day. Since the amplitude of the geomagnetic daily variation in Kenya is of the order of 100 \( \gamma \), the error caused by temperature drift was within the limits of experimental accuracy.

A low pass filter was required to remove short period (\( \sim 2 \) seconds) jitter in the fluxgate output, and the filter cut off period was selected to be 200 seconds to match that of the telluric measuring system.
3.4 Recorder and Power Supply

A Watanabe 5-channel paper chart recorder was used to record simultaneously the two telluric field components and three magnetic field components. A range of chart speeds were available but values of 30 mm/hour and 7.5 mm/minute were chosen to record long periods (200 second to 1 day) and short periods (10 to 1200 seconds) respectively. Timing marks were placed on the chart manually from radio network time signals with an accuracy of 30 seconds. The 5 liquid ink pens were spaced so that each could move over the whole width, 250 mm, of the chart paper. With a noise level corresponding to less than 0.5 mm of chart width, this represented a recording dynamic range of 54 dB for the recorder.

In field operation the Watanabe recorder proved to be very reliable and the only source of annoyance was that, with slow chart speeds, the pens required cleaning every couple of days to prevent their drying up.

A 250 V a.c. power supply was required to operate it. Because the chart speed was synchronised with the supply frequency, a stable power supply was required and this was provided by an inverter. The entire power generating system is shown in Figure 3.6.

A 3 H.P. 1 kw Briggs and Stratton electric generator provided power to four battery chargers connected to the inverter with four 12-Volt 60 Amp Hrs batteries in parallel, acting as a 'dummy' load. The inverter could not be driven directly by the battery chargers; batteries were required in parallel to satisfy impedance matching requirements. Apart from this function, the batteries were redundant in the circuit while the generator was running. They provided power to drive the inverter when the generator was being filled.
The power supply for the filter units and telluric amplifier was derived from the inverter power supply using standard transformer, rectifier and voltage regulator circuits.

3.5 Calibration

The filter units were calibrating using a Hewlett Packard 3310B function generator before commencing the fieldwork program, and directly afterwards. At each site, after recording had ceased, the telluric and magnetic sensors were disconnected from the measuring circuit and the entire measuring circuit was calibrated by passing a 60 second sine wave through it and monitoring input and output signals on the recorder. A circuit diagram of the calibration oscillator is shown in Figure 3.7. The period of oscillation was chosen to be within the pass band of the short period filters. The system used to measure periods of greater than 200 seconds was calibrated with and without the 200 second low pass filter in the circuit.

The Jolivet and fluxgate sensors were approximately calibrated at each site using a bar magnet rotating on a synchronous motor a known distance away from the sensor. By bringing the north pole of a magnet up to the magnetic sensor from different directions, and by noting the pen deflections on the chart, the sense of the magnetic changes could be determined.

After completion of the fieldwork, the Jolivet and fluxgate sensors were calibrated accurately in the laboratory in Edinburgh using a Helmholtz coil system. Alternating currents of different periods were passed through the Helmholtz coils and these produced a time varying
magnetic field which was uniform within a small volume near the centre of the coils. The magnetic field at the centre could be monitored with a Förster Oerstedmeter, and the frequency response, measured in mV/\gamma, of all the magnetometers used in the field could be calculated quite simply.

3.6 Sites occupied and data collected

Recordings were obtained of periods up to 1 day at two sites, Kampi Ya Moto (KYM), and Longonot (LON). As well as quiet day variation, a few substorms were recorded. Long period recording had to be abandoned at Thomson's Falls (TFL) and Kericho (KER) because of interference effects.

For long period investigations, two or three weeks of continuous recording is usually required. The strain of long intervals of continuous operation wrecked the small 3 HP generator engine which was the source of power for the measuring system. Because the replacement engine and other components of the power supply unit had to be treated with greater consideration, the maximum period of continuous operation was limited to about 4 to 5 days.

It was decided that available resources could be more profitably used by concentrating on measurements in the period range 10 to 1200 seconds. Between January and April 1974, magnetotelluric recordings were made at Longonot (LON), McCall's siding (CAL), Ol Joro Orok (JOR), Solai (SOL), Molo (MOL), Kericho (KER), Nanyuki (NAN), Mutara (MUT), Marigat (MAR), and Isiolo (ISI). In addition, measurements were made of the east and north components of the magnetic variations in the
period band 10 to 300 seconds at Nairobi (NAI), simultaneously with MT recording at Isiolo. From a comparison of these records, it was hoped to estimate the horizontal dimensions of the magnetic source field.

Operations had to be discontinued at two sites - Thomson's Falls and Molo (site 2) - because of mains interference. The location of sites at which recordings were made can be found in Figure 3.8.
CHAPTER 4

Data Processing and Presentation

4.1 The General Problem

In Chapter 2 it was shown that magnetotelluric relationships could be simplified by considering sinusoidal variations of the magnetic field. For such a time variation, characterised by angular frequency $\omega$, complex valued tensor impedances $Z_{ij}$ relate the magnetic and electric fields according to

$$
\begin{align*}
E_x &= Z_{xx}(\omega) H_x + Z_{xy}(\omega) H_y \\
E_y &= Z_{yx}(\omega) H_x + Z_{yy}(\omega) H_y
\end{align*}
$$

(4.1)

(4.2)

$E_x$, $E_y$, $H_x$, $H_y$ are complex and contain the amplitude and phase information of the corresponding sinusoid.

The starting point of MT processing and interpretation is the estimation of the tensor impedances from records of the time variations of magnetic and electric fields.

The most direct way to proceed is to decompose the original time variations into a series or continuum of sinusoids. Assuming that techniques exist to perform this operation, equations (4.1) and (4.2) can be solved provided that at least two independent sets of spectral coefficients are available.

$$
e.g. \quad Z_{xx}(\omega) = \begin{vmatrix} E_{x1}(\omega) & H_{y1}(\omega) \\ E_{x2}(\omega) & H_{y2}(\omega) \\ H_{x1}(\omega) & H_{y1}(\omega) \\ H_{x2}(\omega) & H_{y2}(\omega) \end{vmatrix}
$$

(4.3)

conditional to

$$
H_{x1} H_{y2} - H_{x2} H_{y1} \neq 0
$$

(4.4)
A statement of expression (4.4) would be that solution of (4.1) and (4.2) requires records containing different horizontal magnetic field polarisations.

All measurements of natural signals contain noise, which is defined here to be any contribution to the spectral estimate which degrades the relationships (4.1) and (4.2). To help reduce the effect of the noise, it is desirable to make more than two independent measurements and to employ some form of averaging of the spectral estimates.

For example, to minimise the effect of noise on telluric records, a least squares approach can be followed (Sims, Bostick & Smith 1971).

Defining $\Psi$ to be

$$
\Psi = \sum_{i=1}^{N} (E_{xi} - Z_{xx} H_{xi} - Z_{xy} H_{yi})(E_{xi}^* - Z_{xx}^* H_{xi}^* - Z_{xy}^* H_{yi}^*) \tag{4.5}
$$

and setting to zero derivatives of $\Psi$ with respect to the real and imaginary parts of $Z_{xx}$ yields

$$
\sum_{i=1}^{N} E_{xi} H_{xi}^* = Z_{xx} \sum_{i=1}^{N} H_{xi} H_{xi}^* + Z_{xy} \sum_{i=1}^{N} H_{yi} H_{xi}^* \tag{4.6}
$$

$$
\sum_{i=1}^{N} E_{xi} H_{yi}^* = Z_{xx} \sum_{i=1}^{N} H_{xi} H_{yi}^* + Z_{xy} \sum_{i=1}^{N} H_{yi} H_{yi}^* \tag{4.7}
$$

The solutions of $Z_{xx}$ and $Z_{xy}$ in (4.6) and (4.7) will give those estimates least affected by noise on the telluric records.

By defining

$$
\langle E_x H_y^* \rangle = \frac{1}{N} \sum_{i=1}^{N} (E_{xi} H_{yi}^*) \tag{4.8}
$$

equations (4.6 and 4.7) can be replaced by the more concise form

$$
\langle E_x H_x^* \rangle = Z_{xx} \langle H_x H_x^* \rangle + Z_{xy} \langle H_y H_x^* \rangle \tag{4.9}
$$
\[ \langle E_x H_y^* \rangle = Z_{xx} \langle H_x H_y^* \rangle + Z_{xy} \langle H_y H_y^* \rangle \quad (4.10) \]

Sims and Bostick (1969) show that the comparable relationships which minimise noise on the magnetic components are

\[ \langle E_j E_j^* \rangle = Z_{xx} \langle H_j H_j^* \rangle + Z_{xy} \langle H_y H_j^* \rangle \quad (4.11) \]
\[ \langle E_x E_y^* \rangle = Z_{xx} \langle H_x E_y^* \rangle + Z_{xy} \langle H_y E_y^* \rangle \quad (4.12) \]

In principle, any two of the equations (4.9), (4.10), (4.11), (4.12) may be solved simultaneously to obtain estimates of \( Z_{xx} \) and \( Z_{xy} \). The six possible solutions are given by Sims et al. (1971).

In practice, however, the nature of the data usually results in five of the solutions becoming unstable. The equation commonly used among MT workers and one which is unstable only when the magnetic field is highly polarised is, in the case of \( Z_{xy} \)

\[ Z_{xy} = \frac{\langle H_x H_x^* \rangle \langle E_x H_y^* \rangle - \langle H_x H_y^* \rangle \langle E_x H_x^* \rangle}{\langle H_x H_x^* \rangle \langle H_y H_y^* \rangle - \langle H_x H_y^* \rangle \langle H_y H_x^* \rangle} \quad (4.13) \]

In the derivation of equations (4.9) to (4.12) it was implicitly assumed that the impedance tensors are constants, and strictly speaking, terms like (4.8) represent averages at one fixed frequency over an assemblage of events. However, if impedance values change slowly with frequency, then the average may be taken over a frequency bandwidth, provided that estimates within the bandwidth show some degree of independence.
4.2 Estimation of Spectral Coefficients

A variety of techniques is available for estimating quantities of the form \( \langle E_x H_x^* \rangle \) appearing in equation (4.13); these methods have been reviewed by Sims and Bostick (1969) and by Hermance (1973a).

The most popular method and the most efficient as far as computation time is concerned is the power spectral analysis of an entire time sequence. This method involves calculating Fourier transforms for each complete time series using the Fast Fourier Transform algorithm of Cooley and Tukey (1965). The products \( E_x H_x^* \) etc. are then formed for each harmonic and finally the products are averaged over several neighbouring frequencies to obtain the desired bandwidth.

Fourier transforms represent estimates averaged over the entire time sequence, and it has been argued (Hermance 1973a) that low signal to noise ratios may result from spectral analysis of an event where short bursts of signal are averaged with long quiescent intervals containing only system noise. It is possible however, to subdivide the event into subsections and perform a spectral analysis of each subsection, and then to discriminate against subsections with high noise content using the cross spectral techniques discussed later.

The Fourier transform of a continuous function \( x(t) \) is given by

\[
X(w) = \int_{-\infty}^{\infty} x(t) e^{-iwt} dt \quad (4.14)
\]

Since data processing is performed digitally, the raw analogue records have to be digitised into the form of time series.

If a record contains no frequencies higher than \( f_N \), the Nyquist frequency, it can be shown (Bendat and Piersol 1971 p.228) that the record
is uniquely determined by values sampled at intervals $\Delta t$ equal to $1/2. f_N$. The occurrence in the record of frequencies higher than $1/2. \Delta t$ produces noise in the spectral estimate. This so called 'aliasing' effect is well documented and will not be considered here further. Care must be taken to ensure that MT records are digitised at a rate corresponding to a higher frequency than the Nyquist frequency.

In the case of a time series of $N$ points, the discrete representation of the Fourier spectrum is (Bendat & Piersol 1971 p.327)

$$X_k = \sum_{n=1}^{k} x_n \exp \left(-j \frac{2\pi k n}{N}\right)$$  \hspace{1cm} (4.15)

with Fourier coefficients given at frequencies

$$w = \frac{2\pi k}{N \Delta t} \quad k = 0, \ldots, N - 1$$  \hspace{1cm} (4.16)

Coefficients need to be calculated only as far as $k = \frac{N}{2}$ since the Nyquist frequency occurs at that point.

The Cooley-Tukey algorithm is a special case of the general Fast Fourier Transform and applies to time series in which the number of points, $N$, is some integral power of 2. To satisfy this requirement the data set has to be augmented by the addition of zeros. A very readable account of the operation of the Cooley-Tukey algorithm is given by Jenkins and Watts (1968 p.313).

Only finite sections of data can be spectrally analysed and the act of truncating a time series introduces a distortion into the Fourier coefficients. Bendat and Piersol (1971 p.323) show that the finite Fourier coefficient at a specific frequency $f$ is not identical to the ideal Fourier transform for an infinite series. Instead, it is a
weighted average of the ideal infinite transforms at all frequencies, with greatest weight being given to coefficients near f. The weighting function is often referred to as the 'spectral window' because it affects the way in which the ideal spectrum is observed.

Truncation of a time series in the time domain may be represented by multiplication of the series by the 'box car' function \( b(t) \) where

\[
\begin{align*}
  b(t) &= 1 & -\frac{T}{2} < t < \frac{T}{2} \\
  &= 0 & t < -\frac{T}{2} \quad t > \frac{T}{2}
\end{align*}
\]  

In the spectral domain this window has a shape described by the function \( \left\{ \frac{\sin \pi f T}{\pi f T} \right\} \). The side lobes of this spectral window die out slowly and significant contributions to the estimate at frequency, \( f_0 \), may come from coefficients at frequencies far removed from \( f_0 \), particularly if those coefficients are large.

This 'leakage' of power from side lobes into the main lobe may be minimised by employing a spectral window which has reduced side lobes. A very popular spectral window is the cosine taper suggested by Tukey (1967), which is applied to 10% of each end of the raw time series.

Whatever window is used, significant leakage invariably occurs from the coefficients at low frequencies, due to the high power at the low frequency end of the spectrum. The high power level is caused not only by a rise in the natural magnetotelluric variation spectrum but also by the existence of long period noise trends in the data.

Leakage of d.c. components may be reduced by removing the mean of the data and its linear trend by the least squares method of Bendat and
Piersol (1971 p.291). Leakage of long periods may be minimised by high pass filtering the time series to obtain a flat spectrum (Blackman & Tukey 1959 p. 125). Such a process is called prewhitening. Spectral coefficients calculated for prewhitened time series may be corrected for the high pass filtering operation, providing the frequency response of the filter is known.

When the spectrum is spiky, little can be done to avoid leakage and in this case the effective width of the spectral window may be much greater than its main lobe.

Such leakage effects are not particularly important in the estimation of MT impedance relations in the frequency ranges where impedance changes slowly from one resolved frequency to the next. However, at the long period end of the spectrum, impedance changes rapidly and leakage caused by poor spectral windows and the effect of truncation are significant. Sims and Bostick (1969) from tests with a natural magnetic time series and a synthetic telluric signal concluded that the first 4 to 6 harmonic estimates were corrupted by leakage due to truncation. Distortion of these harmonics was reduced if the time series was prewhitened before spectral analysis.

Because of the cyclic nature of the Fourier transform there is leakage from one data interval to the next. Leakage may be minimised by spacing the 'images' of the time series further apart by the addition of zeros. This is usually done anyway to augment the data series to a number permitting the use of the Cooley-Tukey algorithm.

Most MT time series show a certain degree of randomness in their amplitudes and phases at any frequency. Fourier transforms of such
series display highly erratic behaviour irrespective of the length of
the time series. This is due to the fact that conventional Fourier
analysis assumes a fixed frequency amplitude and phase content.

Jenkins and Watts (1968 p.231) show that spectral estimates of a
random time series are mutually independent and are distributed as \( \chi^2 \)
random variables with two degrees of freedom. The standard deviation
of the spectral estimator is as great as its expected value. The variance
of the estimate may be reduced by band averaging over \( N \) neighbouring
frequencies. Resulting estimates are approximately independent and
their variance is proportional to \( N^{-1/2} \) (Jones 1965).

4.3 Coherency analysis

The frequency band averaging spectral analysis discussed in the
last section can be used to measure the degree of linear relationship
between time series.

For example, consider two signals \( x(t) \), \( y(t) \) which have Fourier
coefficients related by the equation

\[
X(f) = A(f) \cdot Y(f)
\]

(4.18)

Suppose now that the measured coefficients \( X_M \), \( Y_M \) are contaminated
by random noise contributions \( x \), \( y \)

\[
X_M = X + x
\]

\[
Y_M = Y + y
\]

(4.19)

The cross power between the measured signals is defined to be

\[
\phi_{X_M Y_M} = \langle X_M Y_M^* \rangle
\]

\[
= \langle X Y^* \rangle + \langle X y^* \rangle + \langle y X^* \rangle + \langle x y^* \rangle
\]

(4.20)
where averaging is over a frequency bandwidth.

When the phases of two series are random to each other, a vector addition of the cross powers of a group of neighbouring frequencies tends to cancel. If sufficient terms are averaged in (4.20) the last three terms tend to zero, and the crosspower of the measured signals is equal to the crosspower of the clean signals.

The autopower of \( X_M \) is defined to be

\[
\phi_{X_M X_M} = \langle X_M X_M^* \rangle \\
= \langle X X^* \rangle + \langle x x^* \rangle + 2 \text{Re} \langle xx^* \rangle
\]

(4.21)

When many terms are involved in the average i.e. the smoothed powers have many degrees of freedom, autopowers are biased up by random noise while crosspowers are unbiased.

This property can be used to estimate the ratio of random noise to signal in the linear relationship (4.18). The most obvious way to do this is to define a parameter, coherence \( C_{XY} \), to be the ratio of the modulus of crosspower to the product of the root mean autopowers

\[
C_{XY} = \left( \frac{\phi_{XY}}{\phi_{X_M X_M} \phi_{Y_M Y_M}} \right)^{1/2}
\]

(4.22)

In the simple example discussed here, \( C_{XY} \) is given by

\[
C_{XY} = \left( \frac{1}{1 + \langle xx^* \rangle} \right)^{1/2} \left( \frac{1}{1 + \langle yy^* \rangle} \right)^{1/2}
\]

(4.23)

A high value of coherence implies a high signal to random noise ratio on both measured components, assuming again a large number of degrees of freedom.
The coherence between orthogonal components of telluric, $E$, and magnetic, $H$, fields has been used frequently as a means of determining the random noise contaminating the Cagniard relation

$$E(w) = Z(w). H(w) \quad (4.24)$$

High coherencies indicate that estimates of impedance $Z$, are biased very little by random noise on $E$ and $H$. Coherence can also be used to construct confidence limits for $Z(f)$. It can easily be shown (Sims & Bostick 1969) that the effect of random noise on $E$ can be minimised in a least squares sense by calculating $Z$ from

$$Z_E = \frac{\langle E^* H \rangle}{\langle H^* H \rangle} = \frac{\langle E^* E \rangle}{\langle H^* H \rangle}^{\frac{1}{2}} \cdot C_{EH} \quad (4.25)$$

Similarly, the effect of noise on $H$ is minimised by calculating

$$Z_H = \frac{\langle E^* H \rangle}{\langle H^* E \rangle} = \frac{\langle E^* E \rangle}{\langle H^* H \rangle}^{\frac{1}{2}} \cdot C_{EH} \quad (4.26)$$

Ward et al. (1971) have shown that $Z_E$ and $Z_H$ represent $(1 - C_{EH}) \times 100\%$ confidence limits for $Z$.

Care has to be taken in applying these error bars since estimated coherence approximates the theoretical coherence between two series only when a large number of terms has been averaged in the cross-power.

Jenkins and Watts (1968 p.397) show that if the theoretical coherency between two series is zero, the expected value of coherency is

$$E[C_{XY}] \approx N^{-\frac{1}{2}} \quad (4.27)$$

where $N$ is the number of frequencies in the averaging bandwidth.

Consider now the case where $X$ is a linear combination of the spectral
components of two time series $y(t)$ and $z(t)$

i.e. \[ X(f) = A(f)Y(f) + B(f)Z(f) \] (4.28)

A low coherence value between $X$ and any one of the components does not now imply that the relationship (4.28) is heavily contaminated by noise. For example, low coherence between $X$ and $Y$ could be explained by $A \ll B$.

In this case it is more meaningful (Kurtz 1973) to define a coherence between the measured $X_M(f)$ and the value predicted by the relation

\[ \hat{X}(f) = \bar{A}(f)Y_M(f) + \bar{B}(f)Z_M(f) \] (4.29)

where $\bar{A}$ and $\bar{B}$ are estimates of $A$ and $B$ respectively.

The predicted coherency is given by

\[ C_{\hat{X}X} = \frac{\bar{A} \langle X_M Y_M^* \rangle + \bar{B} \langle X_M Z_M^* \rangle}{\sqrt{\langle X_M X_M^* \rangle \left\{ |\bar{A}|^2 \langle Y_M Y_M^* \rangle + |\bar{B}|^2 \langle Z_M Z_M^* \rangle \right.}} \]

\[ + 2 \text{ Real } (\bar{A}^* \bar{B} \langle Y_M Z_M^* \rangle) \] (4.30)

High values of predicted coherence in general indicate that relation (4.28) is little contaminated by random noise. Implication of a high predicted coherence are considered in greater detail in section 4.8.
4.4 **Digital Filters**

Before undertaking routine spectral analysis of events recorded during this project computer programmes were written to implement different types of digital filtering operations. These operations are now discussed.

All spectral analyses involve averaging over an entire record section. The use of digital band pass filters permits the investigator to view the behaviour of the spectral components with time. Sections of the record with high signal to noise ratios and desirable polarisation characteristics can then be determined; these sections can be selected for subsequent spectral analysis.

Long period spectral estimates can be derived only from a record section of very great length. If short period energy is present in the record, the digitisation frequency must be high to avoid contamination by aliasing. A low pass filter can be used to attenuate short period energy; this procedure, followed by decimation to lower the Nyquist frequency, reduces the number of points of the record section to a manageable number.

Behannon and Ness (1966) and Hermance (1973a) have reviewed the use of convolution filters in geomagnetic data analysis. To reduce the cost of processing, the Kenyan data was filtered with recursive filters of the type reviewed by Shanks (1967). This kind of filter produces exactly the same output as a convolution filter but the number of filter coefficients involved in its implementation is usually much smaller than the equivalent convolution filter.

The recursive operation involves computing each output point as a
weighted sum of input points plus a weighted sum of previously computed output points. The speed of the algorithm is dependent on special properties of the $z$-transform of a sampled time series.

Shanks (1967) has shown that the response of a digital filter may be expressed as the ratio of two polynomials in $Z$.

\[ F(Z) = \frac{A(Z)}{B(Z)} = \frac{a_0 + a_1 z + a_2 z^2 + \cdots + a_N z^N}{1 + b_1 z + b_2 z^2 + \cdots + b_M z^M} \quad (4.31) \]

If $X(Z)$ is the input to the filter, then the output $Y(Z)$ is

\[ Y(Z) = F(Z) X(Z) = \frac{A(Z)}{B(Z)} X(Z) \quad (4.32) \]

Using the $Z$-transform theory given by Jury (1964), Shanks has derived a feedback or recursion algorithm which may be implemented by a digital computer. This algorithm is

\[ y_n = \sum_{i=0}^{N} a_i x_{n-i} - \sum_{j=1}^{M} b_j y_{n-j} \quad (4.33) \]

Equation (4.31) may be written

\[ F(Z) = \frac{A_1(Z) A_2(Z) \ldots A_N(Z)}{B_1(Z) B_2(Z) \ldots B_M(Z)} \quad (4.34) \]

The roots of the numerator and denominator are called 'zeros' and 'poles' respectively. The desired filter response can be approximated by suitable location of the poles and zeros.

The band pass filter chosen to analyse data from Kenya was the filter whose poles and zeros are given by Hermance (1972). It has the response function

\[ F(Z) = H_0 \frac{(Z+1)(Z-1)}{(Z-C)(Z-C^*)} \quad (4.35) \]
where \( C = (1+a)Z_0 \)

\[ Z_0 = \exp \left[ -2 \pi j f_o \Delta t \right] \]

\[ a = 1.8394 f_o \Delta t \cdot S \]

and \( S \) is the selectivity of the filter, \( f_o \) is the centre frequency of the pass band and \( \Delta t \) is the digitisation interval of the time series.

\( H_o \), the normalisation factor, ensures unit amplitude response at the centre frequency and is given by

\[ H_o = \frac{(Z_0 - C)(Z_0 - C^*)}{(Z+1)(Z-1)} \]

Equation (4.35) may be written

\[ F(Z) = H_o \frac{(1 - Z^2)}{(1+a)^2 - 2 \cos f (1+a)Z + Z^2} \]  \hspace{1cm} (4.36)

By comparing (4.36) with (4.31) and (4.33) it can be seen that only four coefficients are needed to implement the algorithm (4.33).

Fig. 4.2 illustrates the results of operating a recursive narrow band filter of selectivity 0.2 on data from station JOR for various centre periods. The convention used here and in subsequent sections to denote magnetotelluric components is \( H \) and \( D \) for the horizontal North and East magnetic variations respectively and \( N \) and \( E \) for the North and East telluric variations respectively. Correlations between orthogonal electric and magnetic components which are not apparent in the unfiltered record are clear in the band pass filtered output. A high signal to noise ratio is obvious at short periods on examining the intervals of quiescence on the 20 second band pass filtered data. In addition, on the same filtered section, bursts of activity on the horizontal magnetic components \( H \) and \( D \) can be seen which have clearly different polarisations at times of 40 minutes and 2 hours after the beginning of the event.
The low pass digital filter employed to permit decimating time series prior to spectral analysis was of the recursive type and approximated the frequency response of a Butterworth second order low pass filter. The choice of a Butterworth digital filter seemed appropriate in view of the fact that analogue Butterworth second order high pass and low pass filters had been used in field recording.

Poles and zeros of the Butterworth polynomial were obtained from Kanasewich (1973 p.177). By expanding the numerator and denominator of the right hand side of (4.34), the recursive filter coefficients were evaluated for use in algorithm (4.33). Only five coefficients were involved in the Butterworth filtering operation.

Fig. 4.2d depicts the raw event of fig. 4.1 after being low pass filtered (-3db cut off point at 200 seconds) by the Butterworth digital filter.

Kanasewich (1973 p.179) shows that the frequency response of a Butterworth second order high pass filter is given by

\[ F_H(w) = \frac{1}{1 - i \sqrt{2} \frac{w}{w_c} - \frac{w}{w_c}^2} \]  

(4.37)

where \( w_c \) is the angular frequency corresponding to the -3db cut off point. This response value was used in the spectral analysis of events to correct the spectral coefficients for the effects of different analogue filters being used in fieldwork for the recording of different magnetotelluric components.

Prior to spectral analysis, the Kenyan data were 'prewhitened' using
Fig. 4.2 Examples of recursively filtered data. Event illustrated in fig. 4.1 is the input to the recursive filters.

(a) Band pass filtered, selectivity = 0.2, Centre period = 25 s
(b) do.
(c) do.
(d) Butterworth low pass filtered, -3db cut-off period = 200 s
the difference equation

\[ y_i = x_{i+1} - k x_i \quad |k| < 1.0 \] (4.38)

The z-transform theory shows that this is equivalent to the operation

\[ Y(Z) = Z X(Z) - k X(Z) \]
\[ = X(Z) \sum z - k \cdot \mathcal{I} \]

In the frequency domain, this becomes

\[ Y(f) = X(f) \left( \cos 2\pi f \Delta t - k \right) + j \sin 2\pi f \Delta t \mathcal{J} \] (4.39)

The prewhitening operation multiplies the amplitudes of the Fourier coefficients by the factor \((1 + k^2 - 2k \cos \omega \Delta t)^{1/2}\) and so attenuates the low frequency portion of the spectrum.

4.5 Transfer function Estimation

As pointed out in section 4.1 the estimation of transfer function coefficients depends on the availability of record sections containing magnetic fields displaying different directions of polarisation. This requirement presents great difficulty in the processing of data recorded at low latitudes, since the horizontal magnetic field is very strongly polarised in a north south direction. Magnetic disturbances in Kenya originate from the east–west flow of ionospheric currents concentrated near the magnetic equator.

The degree of polarisation is observed to increase towards short periods (< 100 seconds), and for most of the recording time in Kenya very little short period activity was observed on D, the east west magnetic component. Over a recording period of about 5 days at each site an average of only two or three 2-hour sections were recorded which
had significant short period signal on the D component. Even then the signal appeared invariably to be highly polarised at an angle close to the North - South axis.

Morrison et al. (1968), Berdichevskiy et al. (1973) Hermance (1973a) and Grillot (1973) have described algorithms for digitally filtering MT data, and selecting different parts of the record which show high signal to noise ratio and different directions of polarisation. Fourier amplitudes and phases can be calculated from the filtered output and substituted into equation (4.3) to obtain tensor estimates. The procedure is rather more complicated than conventional techniques of spectral analysis and is valid only when the filtered oscillations are quasi-sinusoidal (Berdichevskiy et al. 1973) Long period (>100 secs) data which exhibit this steady state quasi-sinusoidal behaviour are difficult to find, so that filter techniques are normally restricted to the treatment of short periods.

The Fourier amplitudes and phases of short period data can be much more easily determined using spectral analysis techniques on short data sections. If desired the choice of suitable data sections for spectral analysis may be guided by visual inspection of filtered data.

The use of equation (4.3) with substitution of spectral estimates derived from two independent events gives most satisfactory results when the events have as widely divergent polarisations as possible. However, it is unclear how accurate the results will be for any given difference in the two polarisations. An application of the method to two events recorded at Ol Joro Orok (JOR) in Kenya, which displayed a difference of 20° in azimuth of polarisation produced apparent resistivity values which scattered badly.
The least squares approach of Hyndman (1967) and Sims et al. (1971) described in section 4.1, may be expected intuitively to give more accurate results since the spectral estimates are averages over a wide range of disturbances. For convenience the power spectral approach of Sims and Bostick (1969) has been used in processing the Kenyan data. This method substitutes the averages of Fourier coefficients over a frequency bandwidth into equation (4.13).

An equivalent of (4.13) is widely used to determine vertical magnetic field transfer functions. Experience has shown that this least squares estimator produces reliable repeatable results even when the magnetic field is strongly polarised (e.g. Banks and Ottey 1974). However, very little discussion has appeared in the literature concerning the degree of polarisation beyond which the polarisation of the horizontal source field begins to bias impedance and transfer function estimates.

Berkoldt (1974) has made MT measurements in the period range 30 to 2000 seconds in the Afar Delta region of Ethiopia. Magnetic field polarisations in Ethiopia are similar to those in Kenya, and are strongly constrained to within a few tens of degrees of the north-south axis. Berkoldt considered the use of least squares spectral techniques unsuitable for application to the Ethiopian MT data and was unable to obtain transfer function estimates. He did not feel that the least squares approach could give equal weight to all polarisations of the external field and he suspected that resultant estimates would be biased.

However, as Banks (1975) has made clear, the least squares estimator does involve a very subtle weighting of individual polarisations. This was first indicated by Everett and Hyndman (1967), who compared the
results of the least squares method with those of their 'unit vector'
method. The latter method involves linearly combining the horizontal
magnetic fields of individual events using suitable weighting coefficients
to produce two unit horizontal vectors polarised North and East
respectively. A superposition of vertical fields, using the same
weighting coefficients as the horizontal linear combination yields
unbiased estimates of the ideal transfer functions.

The estimators obtained by the unit vector approach are found to be
exactly the same as the least squares estimators.

When noise is added to the magnetic field components, the least
squares estimator is no longer an unbiased estimate of the ideal transfer
function. The relationship between degree of polarisation, signal to
noise ratio and biasing of estimates has received little attention in
the literature. Most workers prefer to reject highly polarised data
or, if forced to include such data by the lack of unpolarised data,
estimate the polarisation effect by the amount of scatter in the data.
To examine the validity of the latter approach it is necessary to
determine quantitatively the relation between the ideal transfer function
and its least squares estimate.

A rigorous analytic treatment of noise effects on computed transfer
function estimates is not attempted here nor is it thought practical.
To obtain some insight into the phenomenon of biasing, only the effect
of a random uncorrelated noise component is investigated.

The process of averaging spectral coefficients over a narrow band-
width is equivalent to performing a narrow band filtering operation on a
time series. The output of the filter represents a quasi-monochromatic
signal i.e. one in which $\Delta \omega / \bar{\omega} \ll 1$ where $\Delta \omega$ is the bandwidth of the signal and $\bar{\omega}$ is the mean signal frequency. Fowler et al. (1967) have shown that such a signal may be regarded as the sum of a totally polarised signal and a completely unpolarised signal.

Thus the spectral coefficients $H(f)$, $D(f)$ of the horizontal magnetic signals along the North and East axes may be represented by

$$
H = P + h \\
D = CP + d
$$

(4.40)

where $P$ and $CP$ are complex Fourier coefficients of the polarised signal (C complex) and $h$ and $d$ are coefficients of the random unpolarised signal.

The measured coefficients $H_M$ and $D_M$ represent the sum of clean signal plus noise i.e.

$$
H_M = P + h + a \\
D_M = CP + d + b
$$

(4.41)

It is assumed that the component $X(f)$ is related to $H(f)$ and $D(f)$ through the equation

$$
X = AH + BD + \varepsilon
$$

(4.42)

where $\varepsilon(f)$ represents the random error involved in the linear relation.

The measured Fourier coefficient $X_M(f)$ is a combination of signal and random noise terms i.e.

$$
X_M = X + x
$$

(4.43)

The least squares estimator, $A^1$, of the ideal transfer function $A$ is given by (4.13)
i.e. \[ A^1 = \frac{\langle X_M H^*_M \rangle < D_M D^*_M \rangle - \langle X_M D^*_M \rangle < D_M H^*_M \rangle}{\langle H_M H^*_M \rangle < D_M D^*_M \rangle - \langle H_M D^*_M \rangle < D_M H^*_M \rangle} \] (4.44)

It is assumed that the spectral estimates contain enough terms in the average so that cross terms between noise components and between noise and signal components may be neglected. Then using equations 4.41, 4.42 and 4.43 the measured signals may be replaced by their signal plus noise representations.

The numerator of (4.44) becomes

\[ A \sim (d^2 + b^2)(h^2 + h^2) + |c|^2 h^2 H^2 + b^2 h^2 \]

and the denominator is

\[(d^2 + b^2)(p^2 + a^2) + |c|^2 h^2 (h^2 + a^2) \]

For simplicity of notation, the smoothing brackets have been dropped, so that terms such as \( h^2 \) replace the more explicit \( \langle h(f) h^*(f) \rangle \) and represent smoothed autopowers.

Thus the estimated value \( A^1 \) of \( A \) is

\[ A^1 = \frac{(d^2 + b^2)(p^2 + h^2) + |c|^2 h^2 p^2}{(d^2 + b^2)(p^2 + h^2)\sqrt{1 + \frac{a^2}{p^2 + h^2}} + |c|^2 h^2 p^2(1 + \frac{a^2}{h^2})} \] (4.45)

\[ + \frac{C}{(1 + \frac{d^2}{b^2})(1 + \frac{h^2}{p^2} + \frac{a^2}{p^2})} + |c|^2 \frac{h^2}{b^2} + \frac{a^2}{b^2} \]

Similarly \( B^1 \) can be determined to be

\[ B^1 = \frac{C^* A}{(\frac{d^2}{a^2} + \frac{b^2}{a^2})(1 + \frac{h^2}{p^2} + \frac{a^2}{p^2}) + |c|^2 (1 + \frac{h^2}{a^2})} \] (4.46)

\[ + \frac{b^2}{d^2 + |c|^2 p^2(h^2 + a^2)} \frac{1}{(p^2 + h^2 + a^2)} \]
As already discussed, these formulae describe the behaviour of the least squares estimator when only random noise is present in the magnetotelluric components and in the linear relationship (4.42), provided that each of the smoothed cross and autopowers has many degrees of freedom. This rather idealised situation demonstrates several interesting features of the least squares estimator.

Firstly, appreciable leakage occurs between orthogonal transfer function estimates, especially when \( C \) the parameter describing the degree of ellipticity of the horizontal field is large. Reliable estimates can not be obtained of either \( A \) or \( B \) when the magnetic field is strongly elliptically polarised.

If the magnetic field is strongly linearly polarised along one of the measuring directions, say the north south axis (corresponding to \( C=0 \)), one transfer function, \( A \), is relatively unbiased by noise and polarisation effects, provided that the ratio of polarised signal to noise is high. \( B \), the other transfer function is biased down by the ratio \( \left( \frac{1}{1 + \frac{b^2}{d^2}} \right) \) where \( \frac{b^2}{d^2} \) represents the ratio of noise to random signal on the east west component. As the degree of polarisation increases, \( B \) becomes biased further down.

The case of coherent noise existing on the MT components can be treated in a way similar to that of random noise, but the equations become, algebraically, more complex.

Cross spectral estimates fail to remove coherent noise components. The direction of the bias in the least squares estimator depends on the nature of the noise sources and on the relative contributions of random and coherent noise.
An idea of the relative biases produced in an estimate by the presence of random and coherent noise may be obtained by considering the simple case of a magnetic field strongly linearly polarised in the North South direction i.e. \( C = 0 \) and \( \langle H D^* \rangle = 0 \).

The measured \( X_M \) and \( D_M \) are assumed to have noise components which correlate with the signals \( X \) and \( D \) respectively.

\[
\begin{align*}
X_M &= X + x + m X \\
D_M &= d + b + k d
\end{align*}
\]

where \( x \) and \( b \) are random noise components, and \( m \) and \( k \) are complex constants.

The least squares estimator, \( B^1 \), of the ideal transfer function, \( B \), is then

\[
B^1 = \frac{\langle X_M D_M^* \rangle}{\langle D_M D_M^* \rangle}
\]

\[
= \frac{(1 + m)}{(1 + k) + \frac{b^2}{d^2(1 + k^*)}}
\]

Coherent noise on \( D_M \) biases the estimate further down than the same power level of random noise, and can also produce phase changes in \( B^1 \). However many noise sources are principally random in character. If the random noise to signal ratio is \( S^2 \) then to a good approximation, random noise produces a greater bias in \( B^1 \) than coherent noise when

\[
k^2 s^2 > 1
\]

In the estimation of vertical magnetic field transfer functions it may reasonably be expected that the coherent noise levels are similar on both \( X \) and \( D \), and that transfer function bias is caused primarily by the presence of random noise on \( D \).
4.6 Polarisation Analysis

A knowledge of the polarisation parameters of the magnetic field is useful in assessing the distortion of transfer function estimates determined by the least squares spectral analysis method. In addition, the polarisation of the telluric field is a sensitive indicator of lateral inhomogeneity near the recording station. For these reasons the polarisation analysis suggested by Fowler et al. (1967) was incorporated as a routine part of the data reduction.

The polarisation of any wave field can be described in terms of the coherency matrix elements of the wave field. For the case of the magnetic field this matrix is defined to be

\[
J = \begin{bmatrix}
\langle HH^* \rangle & \langle HD^* \rangle \\
\langle DH^* \rangle & \langle DD^* \rangle
\end{bmatrix} = \begin{bmatrix}
J_{xx} & J_{xy} \\
J_{yx} & J_{yy}
\end{bmatrix} (4.51)
\]

where \( H \) and \( D \) are the Fourier coefficients of the orthogonal magnetic field components measured along the \( x \) and \( y \) axes respectively, and the matrix elements represent smoothed auto and cross power estimates.

Fowler et al. have shown that any coherency matrix can be expressed as the sum of a polarised signal \( P \) and random signal \( U \).

\[
J = \begin{bmatrix}
J_{xx} & J_{xy} \\
J_{yx} & J_{yy}
\end{bmatrix} = \begin{bmatrix}
P_{xx} & P_{xy} \\
P_{yx} & P_{yy}
\end{bmatrix} + \begin{bmatrix}
U_{xx} & U_{xy} \\
U_{yx} & U_{yy}
\end{bmatrix} (4.52)
\]

For the random signal

\[
U_{xy} = U_{yx} = 0
\]

and \( U_{xx} = U_{yy} = U \) otherwise a degree of polarisation would exist.
Algebraic manipulation yields

\[ U = \frac{1}{2} \left[ J_{xx} + J_{yy} \right] - \frac{1}{2} \left[ \left( J_{xx} + J_{yy} \right)^2 - 4 |J| \right]^{1/2} \]  

(4.53)

where \(|J|\) is the determinant of \(J\).

Also \( P_{xx} = J_{xx} - U \), \( P_{xy} = J_{xy} \), \( P_{yx} = J_{yx} \), and \( P_{yy} = J_{yy} - U \).

The intensity of the total signal is

\[ \phi \left( \sum J \right) = J_{xx} + J_{yy} \]  

(4.54)

and the intensity of the polarised portion is

\[ \phi \left( \sum P \right) = P_{xx} + P_{yy} = \left( J_{xx} + J_{yy} \right)^2 - 4 |J| \]^{1/2}  

(4.55)

The degree of polarisation, \( R \), is defined as the ratio of polarised intensity to the total intensity i.e.

\[ R = \frac{P_{xx} + P_{yy}}{J_{xx} + J_{yy}} = \left[ 1 - \frac{4 |J|}{\left( J_{xx} + J_{yy} \right)^2} \right]^{1/2} \]  

(4.56)

Fowler et al. have demonstrated that the polarisation parameters of the wave field are completely specified by the matrix \(P\).

The angle \( \theta \) which the principal axis of polarisation makes with the \(X\) - axis is given by

\[ \tan 2\theta = \frac{2 \text{Re} P_{xy}}{P_{xx} - P_{yy}} \]  

(4.57)

The parameters describing the ellipticity and sense of the polarisation ellipse are expressed in terms of the angle \( \beta \) where
$$\sin 2\beta = \frac{i (P_{yx} - P_{xy})}{\sqrt{(P_{xx} - P_{yy})^2 + 4 P_{yx} P_{xy}}}$$

The ellipticity or ratio of minor axis to major axis is defined by \(\tan \beta\) and the sense of polarisation is given by the sign of \(\beta\) i.e. \(\beta\) positive or negative as the polarisation is clockwise or anticlockwise respectively when looking in the direction of propagation.

4.7 Processing of Single Events

Fortran programmes were written by the author to implement the least squares spectral analysis techniques discussed in the preceding sections. Several data sections or 'events' were processed to obtain estimates of tensor impedances and vertical magnetic field transfer functions. The data was processed in the following stages:

1. Single events of duration corresponding to a chart length of 1 metre were visually selected for digitising if they exhibited a high degree of activity on the east west magnetic component, D. On fast speeds (7.5 mm/min) the event corresponded to about 2 hours of record and on slow speeds (30 mm/minute) to about 30 hours of record section.

   Usually only one or two events could be found per station, which displayed short period (<50 secs) energy on D. About four or five events with long period (>50 secs) activity were also chosen at each station. The choice of consecutive events permitted splicing of the data sections and the estimate of responses at very long periods (> 3000 seconds).

2. The analogue traces were digitised using a Ferranti Freescan digitiser,
and data was output to magnetic tape. A digitising interval of 0.5 mm was used, corresponding to a time interval of about 4 and 60 seconds respectively for the fast and slow chartspeeds employed. This high sampling rate avoided any aliasing problem.

Digitised data were edited and digitising errors corrected. Then the data were scaled to mV/km or gammas and interpolated at 5 or 50 second intervals depending on the chartspeed. Finally, each time series was replotted on a CALCOMP plotter to the same scale as the original analogue record for purposes of comparison.

3. Each component of the event, consisting of approximately 1800 data points was Fast Fourier transformed in the following sequence of operations.

a) The least squares best fit line was subtracted from the time series using the formula of Bendat and Piersol (1971 p.289).

b) Then the data series was prewhitened using the difference equation (4.38) with a prewhitening factor of 0.96.

c) The prewhitened data were normalised with respect to their standard deviations, tapered 10% at each end with a Tukey cosine bell, and augmented to 2056 points by addition of zeros. Subroutines for performing this data conditioning operation were kindly supplied by C. Green of I.G.S. (Edinburgh).

d) Fast Fourier transformation of the data was carried out by the IBM SSP routine RHARM which implements the Cooley - Tukey algorithm. Spectral coefficients were corrected only for the normalising operation in (c).

4. Auto- and cross spectral estimates were averaged over 8 neighbouring frequencies except for the first and last estimates which were averaged over only 4 frequencies. Tests indicated that the first 4 harmonics
were usually distorted by leakage effects; so the first smoothed estimate could not be used in interpretation. Smoothed estimates were approximately independent because of the small overlap of the spectral window (Jones 1965) and had 16 degrees of freedom associated with them, except for the highest frequency estimate which had only 4 degrees of freedom.

5. All components had been passed through identical 10 second low pass analogue filters in the recording stage. So the ratios of spectral components were assumed unchanged by the low pass filtering operation. Auto- and cross-powers had to be compensated for the effect of recording different components with different high pass filters. This correction was made on the smoothed power estimates using (4.37).

6. Polarisation parameters of the telluric and magnetic fields were calculated from equations (4.56) (4.57) and (4.58).

7. Cagniard apparent resistivities and single coherencies between orthogonal electric and magnetic components were calculated from equations of the type (4.26) and (4.22) respectively.

8. Transfer function estimates for the vertical magnetic field was calculated using the equivalent of equation (4.13), with \( Z \) substituted for \( \xi \). The transfer functions \( A \) and \( B \) were expressed as a pair of 'real' and 'imaginary' induction arrows having magnitudes

\[
M_R = (A_R^2 + B_R^2)^{1/2}, \quad M_i = (A_i^2 + B_i^2)^{1/2}
\]

(4.59)

The angles \( \Theta_R \) and \( \Theta_i \) made by the arrows with the local geomagnetic meridian were calculated from

\[
\tan \Theta_R = B_R/A_R, \quad \tan \Theta_i = B_i/A_i
\]

(4.60)
9. Tensor impedances were determined from relations of the type (4.13).

10. Equation (4.30) was used to calculate the coherence between the measured vertical magnetic field and that predicted by the linear transfer function relation (4.42).

    Predicted coherencies were similarly calculated for each of the telluric components.

11. The azimuth of the major principal impedance axis was calculated using equation (2.84).

12. Principal impedance values were determined from equations (2.76) to (2.79), and then the principal apparent resistivities from (2.45).

13. The Skew factor defined by equation (2.83) and the ellipticity factor, BO, suggested by Swift (1967) were next calculated.

14. The electric field induced in large 2-dimensional structures may be diverted by the presence of small conducting anomalies in the neighbourhood of the station. To test for the existence of this effect Haak (1972) suggested that the orthogonal telluric components should be mathematically rotated until a minimum of coherency is obtained between them. This is followed by a mathematical rotation of the orthogonal magnetic fields. The object is to determine the direction of the magnetic field which is most coherent with each of the two telluric components. In the case of local current channelling the two directions of the magnetic field need not be orthogonal.

    This 'Haak' analysis was carried out as a routine part of the data reduction programme, but the strong magnetic field polarisations usually produced unstable estimates of the relative orientation of
135

coherent telluric and magnetic components.

15. All the parameters calculated in the programme were printed out, and most of the parameters were plotted as a routine step. Fig. 4.3 illustrates a CALCOMP plotter presentation of the results obtained by processing a single event recorded at station MAN.

Apparent resistivities and phases were plotted only if the predicted coherence for one of the telluric components was greater than 0.9.

4.8 Sources of Noise and Systematic Error

For our purposes noise may be regarded in the broadest sense as any influence which causes the measured signal relationships to depart from the linear form predicted by

\[ X(f) = A(f) H(f) + B(f) D(f) \]  \hspace{1cm} (4.61)

where \( X \) represents one of the telluric components or the vertical magnetic field component, \( H \) and \( D \) represent the orthogonal magnetic field components and \( A \) and \( B \) are functions only of frequency and the conductivity of the earth.

The noise may be categorised as either coherent or random or a superposition of these two. Random noise shows no consistent phase relationship with either signal or noise components. This property can be used to estimate the proportion of random noise present in the relationship (4.61). Smoothed cross spectral estimates and coherencies are the devices used in power spectral analysis to guard against random noise contamination of estimates. Their effectiveness increases with the number of degrees of freedom contributing to each estimate.
Fig. 4.3  Example of output from single event analysis

(a) Maximum apparent resistivity and phase data
(b) Minimum apparent resistivity and phase data
(c) Azimuth of maximum resistivity axis, skew factor and ellipticity factor, BO.
(d) Telluric field polarisation parameters
(e) Magnetic field polarisation parameters
(f) Real and imaginary induction vectors
(g) Predicted coherencies for Z, E and N, the vertical magnetic field, the east-west, and north-south telluric fields respectively
(h) Results of the 'Haak' analysis
   (i) coherence between minimum telluric field component and magnetic field component whose azimuth is given in (ii)
   (iii) coherence between maximum telluric field component and magnetic field component whose azimuth is given by (iv)
(i) Power spectra of north-south, Hi, and east-west, Dl, magnetic components normalised with respect to the short period power estimate in each component.
(j) Normalised power spectra of north south, Nl, and east-west, El, telluric components.
EVENT 900 2/374 TO 1140 2/374

MAGNETIC FIELD POLARISATION PARAMETERS

PERIOD IN SECONDS

INDUCTION VECTORS

PERIOD IN SECONDS

(e)

(f)
EVENT  900  2/3/74 TO 1140  2/3/74

PREDICTED COHERENCIES

Z

E

N

PERIOD IN SECONDS

(i)

(ii)N

(iii)

(iv)N

PERIOD IN SECONDS

EMEM
Coherent noise, on the other hand, is undetectable in the course of a routine spectral analysis and must be kept to a minimum. It is impossible to catalogue all sources of coherent noise and the analyst must hope that an influence which appears as coherent noise on one event may appear as random noise when averaged over a number of events.

It is important to be aware of the major sources of coherent noise, bias and systematic error involved in the processing of MT data so that attempts may be made to control and reduce distortion due to noise, or, alternatively, to realise the limitations of the available data and to appreciate the fact in any subsequent interpretation.

The distortion of spectral estimates due to aliasing has already been mentioned. Elimination of the aliasing problem is straightforward.

Because impedance and transfer functions are insensitive functions of frequency, leakage from spectral windows is not a significant source of error except at long periods, where the truncation effect is most noticeable. Spectral windows may be tailored to reduce leakage, but the first one or two smoothed estimates are usually degraded by noise and must be discarded.

Digitising error is a major source of noise. A test was carried out to determine the amplitude of digitising noise and to estimate the coherence between digitiser noise and the digitised trace. For this purpose, the analogue record of a natural magnetic variation signal was digitised, replotted by CALCOMP plotter to the same scale as the analogue record, and finally the CALCOMP record was digitised.

In this way two data sets were obtained, one representing 'clean' signal and the other representing signal plus digitiser noise.
The power of the digitiser noise and the coherence between noise and signal could be calculated from the smoother autopower and cross-power estimates of the two data series.

Fig. 4.4 illustrates the power of the clean signal and the digitising noise power, together with the estimated coherence between signal and noise, for 16 degrees of freedom in each estimate. The digitising noise power level corresponds to a root mean square digitising error of about 0.3 to 0.5 mm for periods between 30 and 300 seconds. Estimated coherence does not depart significantly from the value $\sqrt{\frac{2}{16}}$, theoretically expected for coherence between two completely random series (Equation 4.27). Thus the proportion of digitiser noise which is coherent with the signal is very small. In view of the discussion of section 4.4 and the relation (4.50), it may be expected that the biasing effect due to digitiser noise is caused principally by its random part.

It is rather more difficult to estimate the relative amplitudes of random and coherent noise in system noise (defined here to be the difference between the analogue signal recorded on the paper chart and the signal actually existing at the sensor). A comparison was made of the magnetic signals sensed by two parallel Jolivet sensors and recorded on a Watanabe multicorder. A visual inspection of the record indicated that system noise at short periods ($< 100$ secs) was of the same order or less than digitiser noise (of Fig. 4.5a). Of course this relation need not hold for all sensors and all channels at every point in time of the data collection stage.

In order to test how well the least squares estimates could cope with highly polarised magnetic fields, two sensors were used to record
Fig. 4.4  Comparison of signal power and digitising noise power. Insert is the coherence between signal and digitising noise. Theoretically expected value for coherence between two random time series is 0.36.
POWER SPECTRA

STATION MOL3 EVENT 2000 24/4/74 TO 2230 24/4/74

POWER

COMPONENT SIGNAL

PERIOD IN SECS

COMPONENT DIGITISER NOISE

PERIOD IN SECS
the D component at station MOL while the third sensor recorded H. The
digitised time series were processed routinely, with one of the D traces
taking the place of a Z component. Transfer functions were calculated
in the normal way.

Results showed that in three out of five events selected to be
processed, the amplitude and direction of the 'induction arrow' were
estimated very well for periods greater than 100 seconds, even when the
horizontal field was more than 90% polarised in the north south direction
(e.g. Fig. 4.6). However, 'induction arrow' amplitudes dropped off
with decreasing period for all three events.

It was believed at first that this was caused by the biasing
downwards of the transfer function, B, as predicted by (4.46), by an
increasing ratio of random noise to randomly polarised signal. However
random noise alone could not explain the pronounced phase shifts introduced
into the 'induction arrows'.

Inspection of the original records revealed that in all of the
three events, the 'Z' component was in a less sensitive range than the
D component. 'Z' signal amplitudes were small, and coherent noise had
been introduced into the 'Z' signal by the Watanabe recorder pen's
dragging over the chart paper (cf Fig. 4.5b). This type of coherent
noise could be discriminated against by analysing only spectral
coefficients with a power level several times greater than the power of
a random white noise series with root mean square amplitude of about
0.5 mm.

Another annoying source of coherent noise in the system is the
uncertainty in the relative spacing between pens of different Watanabe
Fig. 4.5 Comparison of parallel recording of the east west magnetic components. Components were sensed by parallel Jolivet variometers recorded on a Watarnabe recorder, digitised on a Ferranti Freescan digitiser and replotted to the same scale as the original analogue records.

(a) Illustration that system noise level is comparable with digitising noise level of approximately 0.5 mm.

(b) Illustration of drag in the pen recording the 'Z' component. Short period amplitudes are noticeably larger on the D trace.
Fig. 4.6  Test of induction arrow calculations for a synthetic event. Even when the magnetic field is more than 90% polarised in the north south direction, 'maximum response' amplitudes, phases and azimuths are well estimated at periods greater than 100 seconds.

Bias in short period estimates is caused by the presence of noise on horizontal magnetic components.
recorder channels. This uncertainty introduced systematic timing errors between the MT components. While considering the effect of this timing error, $b$, it is convenient to consider also the effect of a constant scaling error, $S$. The procedure follows that of Schmucker (1970).

In the frequency domain $S$ becomes the proportional amplitude error of the Fourier transforms and $b$ the phase error $\phi = 2 \pi fb$ for each estimated frequency component $f$. Let $\tilde{Z}$ be the unbiased transform of $z(t)$. Then the measured transform $Z$ is given by

$$Z = \tilde{Z} (1 + S) \exp \left( \frac{i}{s} \phi \right) = \tilde{Z} (1 + S) (1 + i \phi) \text{ for small } \phi.$$  \hspace{1cm} (4.62)

Consider the relation

$$\tilde{Z} = A \tilde{H} + B \tilde{D}.$$  

Then, replacing the undistorted values $\tilde{Z}$, $\tilde{H}$ and $\tilde{D}$ by their measured values, and assuming that $H(t)$ and $D(t)$ are in perfect synchronisation, the worst case result is

$$\tilde{Z} = A \frac{(1 + h)(1 + i\phi)H}{(1 + S)} + B \frac{(1 + d)(1 + i\phi)D}{(1 + S)}.$$  \hspace{1cm} (4.63)

where $H$ and $D$ are the scaling errors in $H$ and $D$ respectively.

The relative position of the Watanabe pens can not be determined to better than 0.5 mm. For the chart speeds used in Kenya, this corresponds to a relative timing error of about 4 seconds, on the fast speed setting, which represents a considerable phase change in the transfer functions and tensor impedances at short periods. The effect of timing errors is discussed further in the next section.

In the case of estimating the vertical magnetic field transfer...
functions, the 'clean signal' on the left hand side of (4.6.1), i.e. the quantity of interest to the induction worker, represents the 'anomalous' vertical field $Z_a$, resulting from the effect of a lateral inhomogeneity distorting the flow of 'normal' eddy current. The measured $Z$ field consists of the sum of anomalous and normal parts. For the purposes of the present discussion the normal part, $Z_N$, is regarded as a noise term. If $Z_N$ shows no correlation with the horizontal field components $H$ and $D$ it appears as random noise; and its effect can be estimated and, usually, eliminated. The persistence of external current systems may mean that $Z_N$ is correlated with $H$ and $D$ and $Z_N$ appears as coherent noise to contaminate the transfer function estimates.

4.9 Acceptance Criteria

Spectral analysis offers a powerful means through cross-power and coherence estimation of assessing the contamination of the linear relation (4.61) by random noise.

When cross spectral terms have been averaged over a large number of neighbouring frequencies, the predicted coherence, $\chi$ defined by (4.30) satisfies the relation

$$\frac{1}{\chi^2} = (1 + \frac{e^2}{E^2})^{-1} + \frac{|\tilde{A}|^2 a^2 + |\tilde{B}|^2 b^2}{|\tilde{A} H + \tilde{B} D|^2}$$

(4.64)

where $\tilde{A}$ and $\tilde{B}$ are estimates of $A$ and $B$ and $a$, $b$ and $e$ are random noises on the signals $H$, $D$ and $E$.

Generally, a large value of $\chi$ may be taken to indicate a high ratio of total signal to random noise. In the case of strong linear polarisation of the magnetic field like that observed in Kenya, one
transfer function, $\bar{A}$, is relatively unbiased when there is a high total signal to noise ratio. However, the accuracy of the second estimate, $\bar{B}$, depends on the ratio of random signal to random noise power, $\frac{d^2}{b^2}$ (equation 4.46). This ratio can not be estimated from the predicted coherence alone, but depends on the degree of polarisation of the magnetic field.

Assuming many degrees of freedom, the minimum value of $\frac{d^2}{b^2}$ can be estimated as a function of $R$, the degree of polarisation of the magnetic field, $\chi$ the predicted coherence, and $A/B$, the ratio of the transfer functions. Alternatively, a value for random noise power, $b^2$, may be assumed, (say a multiple of the digitising noise), and $\frac{d^2}{b^2}$ may be estimated using $R$ together with the autopower estimates of $H$ and $D$.

However, in the practical application of both these estimates it was found that with a finite number of degrees of freedom, the statistical fluctuations in the estimates of both predicted coherence, and degree of polarisation caused the estimate of $\frac{d^2}{b^2}$ to become unstable.

In the absence of a satisfactory estimator for $\frac{d^2}{b^2}$ the only available criterion for discriminating against bias by random noise was the criterion of high predicted coherence. It was hoped that response estimates with significant random noise to random signal bias but passing the acceptance criterion of high coherence, would show a tell-tale scatter which would be reflected in a large error bar.

In the case where the geoelectric structure is 2-dimensional and its strike is known, $B$ can be expressed in terms of $A$. Banks and Ottey (1974) have shown that a more straightforward way of interpreting the vertical field transfer functions is to consider referring the transfer
functions $A$, $B$, to a new set of axes $H^1$ and $D^1$ such that the modules of the response is a maximum along the $H^1$ axis and a minimum along the $D^1$ axis.

So \[ Z = G_P H^1 + G_L D^1 \] (4.65)

and

\[ G_P = A \cos \theta + B \sin \theta \] (4.66)

\[ G_L = B \cos \theta - A \sin \theta \] (4.67)

The angle which maximises $|G_P|$ is the angle $\theta_0$ given by

\[ \tan 2\theta_0 = \frac{A \overline{B} + \overline{A} B}{|A|^2 - |B|^2} \] (4.68)

In the case of a 2-dimensional body $G_L$ should be zero, and the azimuth of $H^1$ should be at right angles to the conductor. The ratio of minimum to maximum responses $|G_L|/|G_P|$ provides a measure of how closely the station approximates to 2-D. When 2-dimensionality may be assessed and the strike of the conductor is known, (4.67) implies that $B = A \tan \theta_0$ and so $G_P = A/\cos \theta_0$. This relationship was used at two of the stations KER and MOL which appeared highly 2-dimensional to check that $B$ was not biased by random noise.

Methods and criteria for rejecting estimates biased by coherent noise depend on the nature of the coherent noise. It can be expected that averaging transfer function estimates over several individual events will reduce the problem of coherent normal $Z$ field, since the phase difference between $Z$ and the horizontal magnetic components may be expected to vary in a random manner over a sufficient number of different disturbances. In any case, the normal $Z$ field appears to be very small in Kenya (Chapter 5) so that its noise contribution is small.

Again, effects of consistent timing errors may be minimised by averaging the transfer function estimate of different events recorded
preferably on different charts. Unfortunately, only one or two events containing desirable short period activity were recorded at each station. So in this case the systematic error caused by improper synchronisation of pens is unavoidable. This is particularly unfortunate since short periods are most distorted by timing errors. Though the effect could not be removed, the magnitude of the distortion of transfer estimates could be assessed by assuming reasonable values for the timing error (Section 4.10), changing the transfer functions according to (4.63) and observing the effect on induction vectors and apparent resistivities.

Noise effects caused by pen drag were minimised by neglecting spectral values which had a power level less than five times the power of a random time series with root mean square amplitude of 0.5 mm. The adoption of this power level criterion also helped to discriminate against bias due to random digitiser and system noise.

4.10 Calculation of Confidence Limits

Error estimates are inherent in the least squares analysis leading to equation (4.13). However relevant equations do not appear to have been published, probably because of the complexity of the distribution functions corresponding to this equation (Jenkins & Watts 1968 p.477). Schmucker (1970 p.23) suggests that a circle of confidence for each transfer function may be defined in terms of the uncorrelated part of the vertical field, Z. Thus high coherency between Z and a horizontal component of the magnetic field implies a small error in the transfer function. Sims and Bostick (1969) also use the coherence between telluric and magnetic fields to define confidence limits for tensor impedances. These confidence intervals are satisfactory for data which
have been derived from many degrees of freedom from events displaying random polarisation of the horizontal magnetic field and when coherent noise sources can be neglected. However, the method may be inappropriate when the magnetic field is strongly polarised. It has already been pointed out (section 4.8) that high coherence alone does not imply accurately determined transfer functions.

In the case of data which may be systematically biased from one event to another it seems more reasonable to examine the variance in response estimates over a large collection of events in the hope that systematic errors might vary between events and that they might be reduced by event averaging.

One way to estimate confidence limits for the data would be simply to plot the response estimates for all events, and then their reliability could be judged by the scatter of data points. However, as the number of data points increases their range of scatter generally increases and this makes it difficult for the eye to assess their reliability.

The alternative is to assume some form of distribution and assign theoretical confidence limits. The validity of the percentage confidence intervals so determined can be judged by how well a smooth curve can be drawn and remain within the confidence limits on more than a specified percentage of points. This method presents a more pleasing picture to the eye, and has been the method applied to the calculation of confidence intervals for the Kenyan data.

Because the results were plotted on a log period scale, response estimates have been averaged over frequency bandwidths as well as over different events to produce values which are equispaced on a log period
scale. The response was assumed to be relatively constant over the averaging bandwidth. The error bar was usually found to be relatively large at short periods where there are many single event estimates but a low signal to noise level, large at long periods where single event estimates are few, and relatively small at intermediate periods where there is a high signal to noise ratio together with several single event estimates. Within some frequency ranges only one single event value was available so that no error bar could be assigned. The reliability of these points may only judged by their relation to a smooth hypothetical curve drawn through the error bars on the other data points.

The confidence limits were calculated in the following stages. Firstly, the arithmetic mean, $\mu$, and the variance $\sigma^2$ of the real and imaginary parts of the individual event transfer functions were calculated by averaging over constant log-period bandwidth and over separate events.

The variances of the transfer function estimates were converted to confidence intervals assuming a normal Gaussian distribution. Bentley (1973) has presented a histogram of apparent resistivities and phases and they appear to indicate a normal distribution for both the real and imaginary parts of the logarithm of the tensor impedance. The assumption of a normal distribution leads to larger error bars than would result from the assumption of a lognormal distribution, so that confidence limits on transfer functions and tensor impedances may be regarded as conservatively estimated.

Error 'bars' for the real and imaginary parts of the transfer function and tensor impedance values were calculated on the basis of a Student $t$ distribution with $\nu$ degrees of freedom where $\nu = N - 1$ and $N$ is the number of single event estimates contributing to the mean.
\[ (3.77) \]

\[ I + \frac{2}{2I} = \theta \]

\[ (3.78) \]

\[ z(\theta \pm \theta_1) = I_S \]

\[ \frac{\theta}{\sqrt{2\theta_1}} = I_V \]

The data by the lognormal distribution of the corresponding logarithmic mean and the mean error of the root mean square of the lognormal distribution for the treatment errors and their errors. The treatment and their errors are represented by the lognormal distribution. The parameters are known. The error is a function of the independent variables and errors where \( I \) is a function of the lognormal distribution. The equation for propagation of error which does not depend on a parameter with functions and tensor treatments, using the lognormal distribution of the parameters and asymptotes, etc. were calculated from their functions. The bars for induction error are determined from the asymptotes, principal confidence intervals.

The error bars in the results presented in Chapter 5 are given by

\[ (3.79) \]

\[ \frac{2}{2N} + I \]

\[ \frac{2}{2N} \left[ \sum_{i=1}^{N} \frac{(x_i - x)^2}{N} \right] \]

\[ \left( \frac{\sigma}{\theta} - I \right) \]

\[ (1-\alpha) \times 100\% \] confidence intervals were constructed using the formulae.
Principal apparent resistivity $\rho_{ij}$ is given by

$$\rho_{ij} = 0.2 T |z_{ij}|^2$$  \hspace{1cm} (4.74)

where $T$ is the period. So the final result to be plotted on a log scale is

$$\log (\rho_{ij}) = \log (0.2 T) + 2^{S_L}$$  \hspace{1cm} (4.75)

$$S (\rho_{ij}) = 2 S_L$$  \hspace{1cm} (4.76)

This results in a simple error bar presentation.

4.11 Algorithm for Event Averaging

A FORTRAN programme was written to average response estimates calculated for several individual events recorded at each station. The programme performed the following sequence of operations.

1. Transfer functions and tensor impedance estimates were read in from each event, together with predicted coherencies and the power estimates of all components.

2. Transfer functions were accepted for further processing if two criteria were met: (a) all magnetic components possessed a power level five times that of the random noise power corresponding to a root mean square signal amplitude of 0.5 mm. (b) Predicted coherence for measured $Z$ was greater than 0.8. At sites SOL, KYN, ISI, JCR and MAR where $Z$ amplitudes were small this criterion was reduced to - predicted coherence $> 0.7$.

Tensor impedances were accepted if all horizontal LT components satisfied the power level requirement described in (a), and if the
predicted coherence for one of the measured telluric components was
greater than 0.9.

3. The available period range represented by the accepted response
values was divided into bands equispaced on a log period scale in such
a way as to provide 10 estimates per decade.

4. Transfer functions and tensor impedances were averaged over events
and over the appropriate log period bandwidth. Confidence intervals
were calculated from the variance of the estimates using equation (4.69).

5. Real and imaginary induction arrows were determined from equations
(4.59), (4.60) and maximum and minimum response arrows were calculated
using equations (4.66), (4.67), (4.68). Confidence intervals were
calculated for amplitudes, phases, azimuths etc. using equation (4.70)
together with the known functional relationship.

6. The azimuth and error of the principal impedance value were determined
from (2.84) and (4.70) respectively. Principal impedances and phases
were calculated using (2.76) to (2.79) and errors calculated in the normal
way.

    The means and error bars calculated for the amplitudes of the
principal impedances on the basis of normal statistics were converted to
logarithmic means and error bars with the help of (4.71) and (4.72).

    Final apparent resistivities and errors were obtained from (4.74).

7. The Skew factor was determined using (2.83). No error bar was
calculated as this was thought unnecessary.

8. The principal impedance values were inverted using the Schmucker
inversion scheme described in Chapter 2 section 2.3.4. Confidence
intervals were assigned to the resistivity - depth estimates using the functional relationship and the propagation of error as before.

9. All the results were plotted together with the number of degrees of freedom, $\nu$, associated with each estimate.

10. An 'error' in timing of 4 seconds was introduced into the Z pen and the scaling factor was increased by 5%. Transfer function estimates were calculated using equation (4.63) and the program was run again to estimate the systematic effect that timing errors and scaling errors might have on the results.

Similarly tensor impedance estimates were distorted by introducing into the telluric east and west pens timing errors of +4 and -4 seconds respectively and scaling errors of +5% and -5% respectively.

All results were again plotted.

Preliminary runs of the event averaging program revealed a lack of useable short period data. Recursive filters were used to select short period data sections to extend the short period end of the estimates.

It has been mentioned in Chapter 3 that long period activity was recorded at station KYM in the form of storm activity and diurnal variation. Storms were analysed in the same way as micropulsation activity and response estimates were calculated at periods up to 10000 seconds.

Telluric diurnal records were found to be heavily contaminated by noise. In contrast to the well defined harmonic appearance of the magnetic power spectra, the telluric spectra displayed a continuum of power. It was felt that apparent resistivities were too distorted to be considered in the interpretation.
In the presentation of the magnetotelluric results the order in which the stations are discussed is based upon the similarity in the appearance of their apparent resistivity curves.

Mutara (MUT) and Kericho (KER)

Kericho is located about 50 km. west of the crest of the western escarpment of the Rift Valley while Mutara lies about 30 km. east of the highest point of the eastern escarpment of the Rift (Fig. 5.1). Minimum and maximum apparent resistivities are high at both stations and the resistivity curves show little frequency dependence. There is a slight tendency for resistivity to decrease with increasing period but the error bars obscure the extent of this decrease (Fig. 5.2b & 5.3b). The ratio of maximum to minimum resistivities at both stations is approximately 10 to 1. The low signal to noise ratio along the minimum axis is reflected in the large error bars on minimum apparent resistivities and in the large values of phase estimates.

Maximum and minimum resistivities at Kericho are not significantly different from those at Mutara and the maximum resistivities are nowhere higher than 800 ohm m.

The azimuth of the maximum apparent resistivity at Kericho varies very little from a value of about 80° East of North for all periods. Maximum apparent resistivity at Mutara is strongly constrained to an azimuth
of about 60° East of North. Both these major resistivity axes are approximately perpendicular to the strike of the major faults of the Rift, which has a strike direction of about 30° West of North.

Skew values at both MUT and KER lie predominantly between 0.2 and 0.6 indicating, on the basis of Swift's (1967) Criterion that the geoelectric structure is "weakly" 2-dimensional.

Maximum response Induction arrows at KER are larger than at MUT. (Figs. 5.2a and 5.3a) but at both stations the maximum response arrows are much larger than the minimum response arrows and the arrows lie along the line perpendicular to the observed Rift strike. Induction arrows are approximately parallel to the major resistivity axes at both stations.

KER shows very large maximum response arrows at all periods: again, the variation is a weak function of frequency, but the response appears to peak at about 100 seconds when its value is of the order of 0.6. The azimuth of the maximum vector undergoes a slight change from its constant value of 60° East of North at periods above 100 seconds to a value of 40° East of North at 30 secs. period. This change coincides with a change in the character of the response arrows from a 2-dimensional to a 3-dimensional behaviour. It should be noted that the phase of the maximum response arrows departs little from ± 180°.

At station MUT very short period activity (< 18 second) was apparent for most of the recording time, necessitating an increase in chartspeed to 30 mm per minute. Short period estimates are available of induction vectors down to periods of less than 15 seconds and these are little affected by relative timing errors in recorder pens. Maximum response peaks at a period of about 25 secs. to a value 0.4, double its long period value. Below 30 secs. period the phase of the
response arrows shows a systematic change from $0^\circ$ to $-60^\circ$. The direction of the maximum response arrow rotates only slightly from a value of $120^\circ$ West of North at 300 seconds period to $140^\circ$ West of North at short periods.

Nanyuki (NAN)

Nanyuki lies 40 km. west of Kutara and 70 km. from the Rift. The most noticeable feature of the resistivity curves for NAN (5.4b) is the very high degree of anisotropy, with the ratio of maximum to minimum resistivities being more than 500 to 1. The anisotropy of the resistivities was quite apparent during the recording stage since the North-South telluric trace appeared to be an exact reduced mirror image of the East-West telluric signal. Each telluric line was cut in turn, to check that the signals did not represent an instrument effect such as coupling or leakage between the telluric components. Tests showed that the recorded signals represented real anisotropy of the telluric currents.

The maximum resistivity curve displays small error bars and the insensitivity of the data values to period is obvious. Phases between the electric field along the major resistivity axis and the orthogonal magnetic field decrease steadily from about $90^\circ$ at 30 secs. period to less than $45^\circ$ at long periods.

Skew is low ($<0.3$) at most periods (Fig. 5.4c) indicating 2-dimensionality, and the azimuth of the major resistivity axis is close to $80^\circ$ west of north.

The error bars on the major resistivity azimuths are very small for NAN and this is also the case for the error bars on the azimuths of the maximum induction arrow. Fig. 5.4 shows that the former and
latter azimuths are very accurately parallel to each other for all periods. Maximum response amplitude at NAN changes little with period but shows a small peak at 100 seconds period. Minimum response arrows have amplitudes which indicate strong 2-dimensional structure under NAN. An important feature of the maximum response arrow is the change in phase from a value of +60° at 1500 seconds to less than +20° at 100 seconds.

Isiolo (ISI)

Isiolo is the furthest of the recording stations from the Rift Valley and is almost 140 km. east of it. It is also the last of the stations showing high apparent resistivity values.

Skew factors of greater than 1 were calculated for Isiolo though they are plotted as values of 0.8 in Fig. 5.5c to retain them within the plotting boundary. The exact value of skew is of little consequence when it is high. Skew is only used as a criterion for judging the validity of a 2-Dimensional interpretation of MT Data. In the case of Isiolo it is obvious that the behaviour of apparent resistivities is much more complex than that relating to a 2-Dimensional structure. The large value of skew indicates coupling between the telluric fields and magnetic fields measured along the same principal axis.

Despite the 3-Dimensional behaviour of ISI, the maximum and minimum resistivity curves (5.56) have approximately the same amplitude and frequency dependence as those for KER. However the maximum at ISI shows a distinct trough at 50 second period which is related to a change in direction of the major impedance axis. The direction of this axis varies between about +10 and +30° East of North.
The diagram shows the induction vectors for two stations, ISI and STATION 1. The graphs display the magnitude of the real and imaginary vectors, as well as their phase, over a period of seconds ranging from 10 to 10,000 seconds. The data is presented in a scatter plot format, with each point representing a measurement at a specific period.
APPARENT RESISTIVITY vs PHASE vs PERIOD IN SECONDS

STATION  ISI

APPARENT RESISTIVITY IN OHM METERS

PERIOD IN SECONDS

PHASE

PERIOD IN SECONDS
Maximum induction response errors are in general small compared to those of the sites discussed previously. Minimum responses are comparable with maximum responses at many periods. The representation of the transfer functions in the form of minimum and maximum responses seems inappropriate for this site. In a 2-dimensional situation the transfer functions $A$ and $B$ should be in phase with each other and so the ratio of their real parts should be in phase with the ratio of their imaginary parts

\[ \frac{A_R}{A_I} = \frac{B_R}{B_I} \implies \frac{A_R}{B_R} = \frac{A_I}{B_I} \]

This is clearly not the case at Isiolo as is evident from the contrast in the azimuths of real and imaginary vectors (depicted in fig. 5.5a).

Both tensor impedances and induction arrows indicate that Isiolo is a Complex site.

Molo (MOL) and Ol Joro Orok (JOR)

Molo and Ol Joro Orok are situated on the western and eastern escarpments of the Rift respectively. Apparent resistivities at both stations (Figs. 5.6 and 5.7) are much smaller than the resistivities described previously for stations outside the Rift. In addition, the azimuth of the maximum resistivity is quite differently related to the strike of the rift axis and to the direction of induction arrows. The degree of anisotropy is smaller at these stations, especially at short periods, than the anisotropy observed at other stations outside the Rift Valley.

Maximum apparent resistivities at MOL show a peak at a period of about 100 seconds, but the minimum resistivities at MOL and both maximum and minimum resistivities at JOR show little frequency dependence.
Phases are well defined at JOR and lie close to $45^\circ$ for maximum impedance and $50^\circ$ for minimum impedance.

The skew factor is less than 0.2 at JOR and generally less than 0.3 at MOL indicating the presence of a 2-dimensional structure near each station.

At JOR the azimuth of the maximum resistivity is close to $25^\circ$ West of North at periods above 100 seconds but is less well defined at periods less than 100 seconds due to the decrease in anisotropy. The major impedance azimuth at MOL scatters around $30^\circ$ West of North at periods of greater than 100 seconds but decreases at shorter periods to lie along the North South axis.

The amplitudes and phases of the maximum induction responses at JOR and MOL are quite dissimilar although both maximum response arrows point in the same line i.e. along the line defined by the direction $60^\circ$ East of North. MOL has a maximum response arrow of magnitude 0.5 at 1000 secs. which slowly decreases to about 0.3 at 30 seconds period. Fig. 5.6b shows that the extent of the decrease at the first three short period estimates could be obscured by the effect of a timing error in the Z pen and by a possible scaling error of 5%. Fig. 5.6c shows the results of calculating the induction arrow from only one transfer function, $A$, assuming a two dimensional structure and a strike of $30^\circ$ West of North. Induction arrows calculated in this way show very little change with period and suggest that the decrease in amplitude of maximum response at short periods could be due to bias of the East West transfer function $B$ as described in Chapter 4 section 4.4.

All induction arrow estimates display minimum responses close to
STATION JOR

PERIOD IN SECONDS

AZIMUTH

SKEW

NU
zero and maximum response phases of approximately $180^\circ$ and indicate a very strong two-dimensional structure in the region of MOL.

The maximum response amplitude at JOR is less than 0.1, but high signal to noise levels permit the strong 2-dimensional behaviour of the arrows to be observed. In contrast to MOL, the phase of the maximum response at JOR increases continuously with period from less than $-120^\circ$ at 30 seconds to $+90^\circ$ at 1500 secs. As the phase increases through $-90^\circ$ with increasing period, the azimuth of the maximum response arrow changes discontinuously from $90^\circ$ East of North to $120^\circ$ West of North.

Solai (SOL)

The induction responses at this Rift Valley station are similar in general form (Fig. 5.8a) to those calculated for JOR. Values of apparent resistivity are of the same order of magnitude as those of JOR, but estimates scatter a great deal.

The scatter of apparent resistivity values (Fig. 5.8b) is greater than is consistent with the assumption of a lognormal distribution of the data points, and suggests some form of noise contamination of the MT records.

The most probable explanation for the scatter is a high noise level on the telluric signals recorded at SOL. Telluric current measurements were made unavoidably near a power line carrying high voltage current and leakage noise is evident in the raw analogue records. The results displayed in fig. 5.8b indicate that visual selection of events has not been successful in discriminating against leakage noise.
Induction response estimates are relatively unscattered and indicate that there has been little contamination of the magnetic recordings. Amplitudes of the maximum responses are of the same order as those obtained for JOR i.e. about 0.1. The azimuth of the maximum response is close to 130° West of North for periods between 100 and 1000 seconds but below 50 seconds estimates show large error bars and the azimuth is poorly defined.

The phase variation of the maximum response is remarkably similar to that at JOR.

Kampi Ya Moto (KYM)

Because this station lies closer to the rift axis than the other stations on the line of traverse, an effort was made to obtain response estimates at as wide a range of periods as was possible. It has already been mentioned (Section 4.10) that recording of the diurnal variation in telluric currents appeared to be too noisy to be used. Storm occurrences were analysed to give estimates up to 10000 secs. period. KYM was the first site at which MT measurements were made successfully. However the results of the analysis of data from KYM are disappointing.

The filters used at KYM for recording short periods were those constructed originally for the MT system i.e. 10 to 300 secs. filters. During the 5 day short period recording interval, very little energy was recorded at periods greater than 100 seconds. Results of spectral analysis of individual events displayed a high degree of scatter and when events were averaged by the algorithm of section 4.10, very few points passed acceptance criteria outside the 25-50 second period band. Fig. 5.9b represents the average of results after the power level
acceptance criteria has been discarded but the coherence criteria of section 5.10 has been retained.

The short period results are poor and scatter widely in resistivity, azimuth and skew factor.

Long period results obtained from analysis of magnetic storms are smoother in appearance than short period results but are band limited to the period range 1000 to 10000 seconds. Unfortunately therefore there is no overlap of results from short period and long period recording. The long period data illustrated in Fig. 5.9 are those which have passed both power level and coherence acceptance criteria. Relaxation of the power level criterion introduced a large number of points into the averaging algorithm due simply to the statistical fluctuation in coherence and to the availability of a large number of short period estimates from the spectral analysis of individual storms. Such low power points present what is believed to be a misleading picture of apparent resistivity decreasing with period from 1000 seconds downwards.

Marigat (MAR)

Marigat is situated 65 km north of the main line of traverse and measurements were made there to obtain an indication of the northward extent of the Rift Conductor.

Apparent resistivities at MAR (Fig. 5.10b) are the lowest of all stations operated in Kenya. Resistivities show no obvious frequency dependence and fluctuate around 1 or 2 ohm metres. Maximum resistivity rises rapidly with increasing period from 1 or 2 ohm metres at short periods to about 20 ohm m. at periods of over 1000 seconds. Error bars are small on all estimates because very clean telluric signals were
The diagrams show the apparent resistivity in ohm-meters as a function of period in seconds for two different stations, STATION and MAR. The data is plotted on logarithmic scales for both the x-axis and y-axis. The phase is also shown for each period, with markers indicating the range of values. The graphs demonstrate the variation in resistivity over different periods, highlighting the stability or variability of the resistivity measurements at these stations.
recorded at this station. The azimuth of the direction of maximum resistivity is close to 10° East of North, a value which is 40° different from the azimuth of the major axis at JOR and MOL.

Strong 2-dimensionality is suggested by the low skew factor at MAR (Fig. 5.10c).

Maximum induction responses are of the order of 0.15 and peak at 500 seconds (Fig. 5.10a). At periods longer than 1000 seconds the maximum response arrow points at 60° East of North i.e. along the same line as the arrow at KER, MOL, KYM, SOL, JOR and NUT.

A change of direction in the arrow is noticeable as period decreases from 1000 secs to 200 secs. From 200-600 seconds the induction arrow is accurately defined and points in a direction of 100 to 110° East of north i.e. perpendicular to the axis of major impedance. Below 100 secs. period the arrow fluctuates between -40° and +40°.

Longonot (LON)

To measure the southward extent of the rift conductor measurements were also made at Longonot in the Rift Valley 110 km south of the traverse line.

The azimuth of the major impedance direction is about 30° west of North (Fig. 5.11c) in accordance with the azimuth at MOL, JOR and KYM. However large skew factors are observed at LON indicating that apparent resistivities can not be interpreted in terms of a 2-dimensional structure. Maximum and minimum resistivities (Fig. 5.11b) are of the same order of magnitude as at the Rift Valley and Rift escarpment stations, but phase values are much lower at LON - generally less than 30°.
STATION LON

INDUCTION VECTORS

PERIOD IN SECONDS

STATION LON

INDUCTION VECTORS

PERIOD IN SECONDS
Induction vectors show considerable frequency dependence (5.11a). The maximum response arrow increases from near zero at 30 seconds to more than 0.4 at 1000 seconds. This is accompanied by an increase in phase from an undefined value at short periods to 180° at 1000 secs. At periods less than 300 seconds the azimuth of maximum response is poorly resolved. At periods longer than 300 secs, the maximum vector points at an angle of 30° East of North.

Simultaneous measurements at Nairobi and Isiolo

Because the MT method is dependent on assumptions concerning the wavelength of the source inducing field and because Kenya is relatively near the equatorial electrojet (15° south) an attempt was made to estimate the horizontal wavelength of the source field. Since the electrojet flows east west, the quantity of interest in the experiment is the variation in amplitude of the north south horizontal magnetic component $H$, with distance measured in a direction south from the dip equator. Simultaneous measurements were made at Nairobi and Isiolo of both horizontal magnetic components $H$ and $D$ in the period range 10-300 secs. The line through Isiolo parallel to the dip equator is about 160 km from Nairobi.

Four 40 min. long events recorded at different times of the day were selected for spectral analysis. Both magnetic components recorded at Nairobi Geomagnetic Observatory showed a higher noise level than those recorded at Isiolo. On the $D$ trace a regular pulse was observed at about 100 second intervals for the whole recording period.

The ratio of power at Nairobi, $\langle H_{NAI}^* H_{NAI} \rangle$ to power at Isiolo $\langle H_{ISI}^* H_{ISI} \rangle$ was calculated using equations similar to (4.25) with
$H_{NAI}$ substituted for $E$ and $H_{ISI}$ replacing $H$. This is equivalent to finding the power ratio by minimising the effect of random noise on $H_{NAI}$.

The square root of the power ratio i.e. the ratio of the amplitude of $H_{NAI}$ to $H_{ISI}$ is given in Fig. 5.12 for the four events processed. Also shown is the amplitude ratio of the D component and the coherence between parallel components.

The results indicate that the quantity of interest in the MT experiment i.e. $\frac{H_{NAI}}{H_{ISI}}$ fluctuates between 0.9 and 1.1 for data with coherence greater than 0.9, irrespective of the time of occurrence of the event. At periods greater than 100 seconds, the amplitude of $H$ recorded at Nairobi seems to be consistently greater than $H$ at Isiolo.
CHAPTER 6

Interpretation and Conclusions

6.1 Qualitative interpretation

Figures 6.1 to 6.4 illustrate the behaviour and the relationship between induction arrows and principal impedance amplitudes at periods 30, 100, 300 and 1000 seconds respectively.

The following discussion will concentrate first upon the results obtained for stations on the line of traverse. The most obvious feature of the principal impedance diagrams is the large difference in amplitudes of major impedances at all periods between stations outside the rift valley and stations within the rift valley and on its shoulders.

The azimuth of the major impedance axis shows very little change with period at all sites except at SOL where all MT parameters show very great scatter. Because of this scatter, MT impedance results at SOL are not considered further in this discussion. Major resistivity axes at all other sites which have low maximum apparent resistivities lie approximately parallel to the strike of the main rift faults.

At stations KER and MUT which lie outside the rift the azimuth of the major impedance axis is directed approximately perpendicular to the rift.

Maximum response arrows at stations along the traverse line from KER in the west to MUT in the east are very accurately parallel to each other and perpendicular to the strike of the rift at periods of 100 seconds and above. Thus they are oriented approximately parallel to the major impedance axes at KER and MUT outside the rift and perpendicular
to major axes at stations inside the main rift faults. Further east at NAI, the maximum response arrow is again parallel to the major impedance axis but the axes of both these responses show a difference in orientation of about 40° from the other stations on the traverse. At KER, MOL, KYM, JOR, MUT and NAI the ratio of minimum to maximum induction response and the MT skew factor indicate that all these stations are relating to a 2-dimensional geoelectric structure.

At ISI, the station furthest from the rift valley, the maximum response arrow is small and shows considerable variation in azimuth as well as in dimensional behaviour. Skew values are high though the direction of maximum resistivity is approximately constant at all periods. Neither induction response nor impedance behaviour can be interpreted in terms of a simple 2-dimensional model.

Qualitative interpretation of the induction response data is based on the following considerations (a) maximum response arrows are normal to and point in the horizontal direction of current concentrations. (b) the magnitude of the maximum response is proportional to the intensity of anomalous current concentrations. This intensity will depend upon the steepness of conductivity interfaces and the conductivity contrast across an interface.

It appears from the reversal of induction response arrows that there is a current concentration within the region of the rift valley flowing approximately parallel to the strike of the main rift faults. It is highly improbable that this current could be external in nature: the direction of flow of the equatorial electrojet is predominantly east west and it is difficult to imagine a persistent ionospheric current system which would coincide with the position of the rift valley.
throughout the 12 months recording period. The current must be concentrated in a good conductor lying beneath the rift valley.

The large amplitudes of the maximum response at KER and MOL are evidence that the western edge of the conductor lies somewhere between these stations. MUT and NAN have comparable response amplitudes at a period of 300 seconds, but at 100 seconds the response is larger at MUT. This appears to place the eastern edge of the conductor in the region of MUT.

Maximum response amplitudes are smaller at the eastern stations MUT and NAN than those measured at the most westerly stations KER and MOL. This indicates that the volume integrated rate of change of conductivity is more gradual towards the east. This would occur if the conductivity contrast is less at the eastern edge of the conductor or if the conductor - resistor interface has a more gradual slope towards the east. Another possibility is that there is a second conducting body lying beneath the area to the east of the rift. This latter possibility has been suggested by Banks and Ottey (1974) to explain some of the results of their GDS survey.

MT impedances provide independent evidence of the lateral extent of the rift conductor. The consistently low apparent resistivities at all stations within the rift valley are strong evidence for the existence of a very good conductor underlying it.

In an ideal 2-dimensional case, stations positioned near a conductor - resistor interface have the maximum resistivity axis oriented parallel to the interface if the conductor is on the conductive side of the contact and perpendicular to the interface if the station lies over
the resistor (e.g. See fig. 2.3b). The orientation of major impedance axes measured at stations on the traverse locate the western edge of the conductor between MOL and KER and the eastern edge between JOR and MUT.

The character of the induction arrow is significantly different at NAN from that of any of the other stations on the traverse, especially at long periods. At 1000 seconds the phase of the arrow is about +50° which suggests that conductivity structure is complex to the east of the rift.

The ratio of minimum to maximum response at all periods at NAN is consistent with the existence of a 2-dimensional structure. However, the orientation of the induction arrow at NAN shows a significant difference in direction (∼40°) from the direction of arrows at the other six stations to the west of NAN. It is puzzling why MUT and NAN, stations only 45 km apart, should have responses with such a strong 2-dimensional character and yet relate apparently to structures with a 40° difference in strike direction. The GDS survey by the University of Lancaster did not include any stations between NAN and the rift valley, and the relative orientation between their induction arrows at NAN and those at other stations on their traverse were not discussed (Banks & Ottey 1974).

Error bars on both the azimuths of the response arrows and the azimuths of major impedance axes at NAN are very small. It seems a remarkable coincidence that these two directions should be so closely parallel to each other, while reasonable physical explanations for the behaviour of each are quite unrelated. Anisotropy of the magnitude observed at NAN can only reasonably be explained in terms of current
channelling in a local conductor (Nienaber 1973, Lilley & Tammennagi 1971). Magnetic variations are insensitive to such local conductors.

Measurements at the two satellite stations MAR and LON display interesting features. Low apparent resistivities at LON suggest that the station lies on the conductor. Skew factor is high and is not consistent with a simple 2-dimensional conductor. The maximum response arrow is not perpendicular to the major impedance axis. This response arrow increases in magnitude from 0.15 at 30 seconds to 0.45 at 1000 seconds and displays complex 3-dimensional behaviour. The current concentration indicated by the large response arrow suggests that the southern edge of the rift conductor lies in the vicinity of LON.

Apparent resistivities at MAR are the smallest measured in the MT project. At short periods both maximum and minimum resistivities are less than 4 ohm m. Since MAR lies between Lakes Baringo and Harrington and on top of lake sediments, these low resistivities are not surprising. The direction of the major impedance axis corresponds at all periods with the direction of the strike of the major rift faults. North of the equator, the rift changes direction turning clockwise through 40°. It is thought likely that the major impedance axis at MAR is determined by the strike of conducting sediments and thus only indirectly by the strike of the main rift conductor. At long periods the maximum resistivity, corresponding to the E polarisation resistivity appears to increase and then level off at a value of about 15 ohm m, i.e. in the same range of values as the maximum resistivities for other stations in the rift. The main rift conductor thus appears to extend further north beyond MAR, and this evidence is supported by the small response arrow at MAR.
The values of maximum resistivities measured at stations in the rift valley and on its shoulders show very little geographical variability. Maximum resistivities at JOR, MOL and LON and the values at KYM and SOL with smallest error bars all lie in the region of 10 to 30 ohm m. The relative frequency independence of the E-polarisation apparent resistivity curves at all rift stations except MAR suggest the presence of a conductor several tens of kilometers thick directly beneath the rift.

6.2 Discussion of assumptions implicit in modelling

Present modelling techniques are adequate only for the treatment of 1-dimensional and 2-dimensional conductivity structures. This fact has important implications for the interpretation of the MT data obtained in Kenya. Firstly, only data having pronounced 1-dimensional or 2-dimensional character may be used in quantitative interpretation. Secondly, it must be assumed that the 2-dimensional character of the data used in interpretation is caused by a regional 2-dimensional geoelectric structure, and not by several local 2-dimensional structures. Thirdly, the induction problem is assumed to be one of induction in a 2-dimensional regional structure by a uniform inducing field.

The assumption that the response data used in interpretation relates to a single regional conductor seems reasonable in view of the remarkable uniformity of the maximum resistivity curves obtained at all the stations within the Rift and on its escarpments.

It is necessary to examine the validity of the uniform source field assumption. Source field dimensions are unimportant when the depths
being probed are much less than the spatial wavelength of the inducing field (Chapter 2 section 2.3.3). Simultaneous measurements of magnetic variations in the period range 10-300 seconds at Nairobi and Isiclo (Chapter 5) indicate a change in amplitude of less than 10% for a separation distance of about 200 km. The shortest spatial wavelengths implied from these results is about 2000 km, assuming that anomalous geoelectric structures do not significantly distort the amplitude of the horizontal magnetic field. It will be seen later that the principal information provided by the MT data relates to depths of a few tens of kilometers; so the spatial wavelength of the inducing field could be much less than the estimated 2000 km without materially affecting results based on an assumption source field uniformity.

Although the response parameters used in the subsequent interpretation have a strongly 2-dimensional character, the possibility can not be ignored that the problem is one of 3-dimensional induction over a very wide area, accompanied by current channelling of the induced currents by the elongated rift conductor.

For the phenomenon of current channelling to produce a significant effect in the Rift Valley, there must be a highly conducting pathway connecting currents induced in the ocean with the Rift conductor. The Indian Ocean lies 500 km from the Rift. Though sediments, which might offer a conductive pathway, extend 200 and 300 km inland from the Ocean, there are 200 kms or so of metamorphic and igneous rocks creating a high resistance barrier between the Rift and the Ocean.

There is the unlikely possibility that currents are channelled into the Rift Valley from the Indian Ocean via the northward extension of the Rift through Ethiopia. However, if currents are somehow channelled
into the Kenyan Rift Valley, which have been induced by sources a large distance from the recording site, there should be poor coherence between the channelled currents and the local magnetic field variation in Kenya. In fact, the data which is interpreted here is derived from records showing high coherence between electric and magnetic components. This suggests that if leakage currents are present, their contribution to the anomaly is small.

For the 3-dimensional case of a good conductor immersed in a resistive medium, Weidelt (1975) has shown that the inductive response measured across a central profile approximates a 2-dimensional behaviour if the length of the conductor exceeds three times its width. Low apparent resistivities within the rift at MAR and LON suggest that the rift conductor satisfies this criterion. This evidence of an elongated conductivity structure is quite independent of that provided by the strong 2-dimensional character of the MT impedance and induction responses at stations on the traverse.

6.3 Selection of data for interpretation

The great disadvantage of the MT method is that normally several stations have to be operated in order to obtain a few sites providing meaningful and interpretable data. As has been discussed before, this is due to the unstable behaviour of apparent resistivity curves when measurements are made near lateral discontinuity charges.

Because MT responses are contaminated by the presence of lateral inhomogeneities located within approximately one skin depth of the measuring station, responses at stations located over a resistor usually
appear more distorted than the corresponding curves for a station overlying a conductor. Even assuming a low value of 300 ohm m for the resistive basement in Kenya, the shortest recording period of about 30 seconds will react to lateral conductivity changes within 50 kms of the recording station. It is debatable whether 2-D MT responses measured at stations on the resistor relate solely to the regional conductor or whether they are also influenced significantly by local elongated conductors.

Apart from the possibility of contamination by local lateral conducting changes, the usefulness of response data collected at stations on the resistor is limited also by simple instrumental and data processing considerations. As discussed in Chapter 2, the apparent resistivity curve least sensitive to distortion by lateral inhomogeneities is the E-polarisation curve, which in the case of stations on a resistor corresponds to the minimum apparent resistivity curve.

A low signal to noise level on the telluric component along the minor impedance axis means that the minimum resistivity estimates are more scattered than the corresponding maximum resistivities. In addition, any systematic bias introduced into the tensor impedance estimates is amplified in the minimum principal impedance estimate. As has been discussed (Chapter 4), the problem of obtaining unbiased estimates of the tensor impedances is particularly acute at stations near the dip equator. Bias in minimum resistivities may be expected to increase with an increase in anisotropy.

As a result, it was not thought reasonable or meaningful to interpret quantitatively the apparent resistivity curves obtained at stations off the rift conductor. MT impedance measurements at these stations have
been used in the present analysis only to provide information about the lateral extent of the rift conductor. Induction arrow estimates for the 'resistor' stations are regarded as reliable and can be used in interpretation.

The poor E- polarisation apparent resistivity estimates for stations outside the Rift mean that the standard approach to 2-dimensional interpretation i.e. construction of resistivity pseudo sections (Chapter 2.4,4) can not be applied to the interpretation of the Kenyan data.

Data collected at stations above the Rift conductor are superior in quality and more informative than those collected at stations off the conductor. The E- polarisation resistivities, i.e. the responses theoretically predicted to be least affected by lateral conductivity contrasts, in this case correspond to the maximum resistivity curves and so are much less biased by system noise and systematic errors than E-polarisation values at stations on the resistor. Also, because the skin depth in the conductor is very much less than in the resistor the scale length over which lateral inhomogeneities may contaminate results is greatly diminished.

6.4 Objects of the Interpretation

The original aim of the MT experiment was to locate the top of the rift conductor in order to determine its geophysical significance. The basic aim of the ensuing interpretation is to establish which types of geophysically distinct models are compatible with the MT data.

It is believed that an approach which aims at constraining the range of possible conductivity models and assigning approximate dimensions to
the conductor is more satisfactory than one which attempts to fit the data in some 'best' way using highly idealised conductivity models and 2-dimensional modelling techniques of questionable reliability.

In the subsequent discussion the emphasis is placed upon the results of a 1-dimensional interpretation of the E-polarisation apparent resistivities at stations within the rift valley. 2-dimensional modelling techniques are employed in a semi-qualitative way, to show that E-polarisation curves are closely similar in shape and amplitude to 1-dimensional curves and hence to support the validity of the 1-dimensional interpretation. In a similar fashion the single station induction responses are used to provide qualitative support for the main conclusions which are based on an interpretation of the MT data alone.

In terms of geophysical significance, the most important parameter to estimate in the data interpretation is the distribution of conductivity with depth below the rift. It is felt that a quantitative interpretation in terms of a 2-dimensional model introduces so many additional parameters into the model that it becomes unclear which of the geophysically interesting parameters are fixed by the MT data and which parameters can vary.

6.5 One dimensional Modelling

All stations with the Rift show very low resistivities at the shortest periods for which estimates are available. Of the four Rift stations KYM and SOL show considerable scatter in their response estimates and telluric records at these stations indicate contamination by noise.
JOR has the cleanest MT response estimates, very low anisotropy at short periods and a low skew factor at all periods. JOR thus appears to be ideal for the purposes of a 1-dimensional interpretation. The maximum resistivity curve is the one selected for interpretation. Corresponding to the E-polarisation case, the maximum resistivity estimates should be less affected by lateral conductivity changes than the minimum resistivity curve.

The short period apparent resistivity data may be used to estimate a maximum depth to the zone of high conductivity below JOR. Short period estimates at JOR suggest that there is very little coupling at these periods between parallel electric and magnetic components. Fig. 6.5 shows the rotated tensor impedance elements for a period of 69 seconds. The diagonal elements of the impedance tensor are much smaller than the non diagonal elements for all rotation angles. This suggests a very low degree of contamination by lateral inhomogeneities.

Keller and Frischknecht (1966 p.223) show that for the case of a 2-layer model consisting of a conductor underlying a relatively resistant layer, the maximum depth to the top of the conductor is \(0.8 \left( \frac{\rho_{app}}{w}\right)^{1/2} \) km. This 'method of asymptotes' suggests that the top of the conducting body lies less than 8 km under JOR.

Using the phase data together with the resistivity data places more severe limits on the depth to the top of the conductor. Schmucker's method (described in Chapter 2 section 2.3.4) of fitting a model of a resistor overlying a conductive substratum uses the complex impedance i.e. both resistivity and phase information. The impedance value at every period can be used to obtain an estimate of the depth to and
STATION JOR1
EVENT 907 24/2/74 TO 1136 24/2/74

AMPLITUDE OF TENSOR ELEMENTS
PERIOD = 69.0 SECONDS

ROTATION ANGLE, DEGREES

AMPLITUDE Zx

AMPLITUDE Zy

ROTATION ANGLE, DEGREES
resistivity of a uniform substitute conductor. All major impedances for JOR suggest a conductor at a depth not significantly different from zero. A Schmucker inversion of both maximum and minimum impedance data (described in section 2.3.4) is illustrated in fig. 6.6 and shows a uniform conductor extending from the surface down to depths of more than 20 km.

Two layer master curves of the form shown in Fig. 6.7 were constructed using equation 2.38 to fit the E- polarisation resistivity curves at JOR. A two layer model consisting of a conductor overlying a resistor with conductivity contrast of x 10 or more could be fitted to the JOR data only if the conductor had a minimum thickness of 35 km.

It is possible to fit the JOR data within the error bars by a 3-layer model consisting of a surface conductor and deep conductor both of 10 ohm m resistivity with a layer of resistive material (> 100 ohm m) sandwiched between them. However, for a 5 km thickness of the surface conductor the top of the deep conductor can not lie more than 15 km below the surface.

A 1- dimensional interpretation of the maximum resistivity curve at JOR indicates the existence of a uniform conducting layer extending from a few kms below the surface to a depth of more than 35 kms. The data however can not resolve the existence of a 5-10 km thick resistant layer at a depth of more than 5 km.

The E- polarisation and H- polarisation curves at MAR are very similar in shape to those obtained by Reddy and Rankin (1972) for a 2- dimensional model corresponding to a sedimentary section 20 km long and 1 km thick of resistivity 10 ohm m. and surrounded on all sides by
Fig. 6.6 Schmucker inversion of impedance data obtained at Ol Joro Orok (JOR).

(a) Inversion of maximum impedances

(b) Inversion of minimum impedances
Reddy and Rankin showed that the E- polarisation curve in this case is close to the 1- dimensional curve at all periods.

Mirror images of the master curves of fig. 6.7 were used to fit the E- polarisation curve. The 2 layer model shown in fig. 6.8 fits the data well and represents a layer of conducting material of resistivity 2 ohm m. and thickness 1-2 km overlying a uniform conductor of resistivity 15 ohm m.

MAR lies between Lakes Baringo and Hannington and on top of lake sediments. The results of Schlumberger and electromagnetic soundings near Lake Hannington indicate a minimum thickness of 500 metres of material with resistivity between 1 and 5 ohm m. (Harthill 1975). The consistency of MT and conventional resistivity findings is gratifying.

6.6 The Two-dimensional modelling programme

The Pascoe - Jones (1971) finite difference programme in its modified form was used to calculate apparent resistivities for 2- dimensional conductor models.

The programme begins by dividing the conductor and free space region into rectangular grids of variable size, each characterised by a constant value of conductivity. The derivation of the boundary values on the outer boundaries of the grid is based on the discussion by Jones and Price (1970). After the outer boundary values have been calculated the diffusion equations are replaced by the appropriate finite difference equations at each point of the grid and these are solved simultaneously by the Gauss-Seidel iterative method. The only feature
STATION M AR

APPARENT RESISTIVITY IN OHM METERS

PERIOD IN SECONDS

PHASE

PERIOD IN SECONDS

2 ohm m.  h = 1-2 km
15 ohm m.
of the Pascoe-Jones programme which pertains specially to the electromagnetic induction problem is the establishment of outer boundary values. Apart from this, the solution of the electromagnetic diffusion equation and the application of internal boundary conditions is a standard exercise in the numerical solution of partial differential equations (e.g. See Smith 1965).

There are four types of error involved in the finite difference programme.

(a) error arising as a result of replacing differential equations by difference equations.

(b) error due to the transfer of the outer boundary conditions on to the boundary of the mesh space

(c) error arising because the set of difference equations can generally be solved only approximately.

(d) error involved in calculating the surface values where a finite difference approximation is used to replace first order derivatives.

Errors involved in using finite difference approximations to analytic derivatives can be minimised only by very close grid spacing. There is the additional requirement that to reduce the error in (b) the edges of the grid must be several skin depths away from any lateral conductivity changes so that outer boundary conditions are accurately met. These two requirements mean that a very large number of grid points is required to approximate the 2-dimensional model in a satisfactory way.

In their many papers Jones and Pascoe have given little discussion of how the finite number of grid points should be distributed in relation
to the skin depths and dimensions of the conductivity model being considered. The answer is not simply to use an extremely large number of grid points because, apart from the prohibitive increase in computer time needed to run the programme, the field values for grids of large size tend to become unstable (Fox & Mayers 1968 p.193). In general the grid size in the vertical direction and in the horizontal direction near lateral discontinuities should be a small fraction of the skin depth in the medium for the finite difference equation to be a good approximation to the partial differential equations. This criterion would require an extremely large number of grids in the vertical direction, because the Pascoe-Jones programme requires that the bottom boundary of the grid should be several skin depths below any lateral discontinuity. In theory, surface values are insensitive to conductivity changes at depth and so should be insensitive to poor approximations made in the finite difference equations at deeper grid points. This appears to be the rationale behind Pascoe and Jones' choice of vertical grid points (e.g. Jones & Pascoe 1971).

To obtain some systematic control over the choice of grid sizes a computer programme was written to choose grid sizes for the particular types of conductivity models considered in the interpretation. Grid sizes in the vertical direction were calculated to be a specified fraction of the skin depth in the medium weighted by the amplitude of the field at the top of the grid space subject to a maximum of one skin depth. Grid sizes in the horizontal direction near discontinuities were chosen according to the same procedure with depth from the surface being replaced by horizontal distance from a lateral discontinuity.

Another problem connected with the use of the Pascoe-Jones
programme is the determination of when the iterative scheme has converged to a solution. Pascoe and Jones take as a convergence criterion the field value deviation between consecutive iteration passes. Müller and Losecke (1975) have suggested that it is more appropriate to consider the deviation at the end of each group of 50 iteration passes in order to eliminate intermediate oscillations. Since there is no relation between these deviations and the accuracy of the solution (because the final values are unknown) Müller and Losecke insist that iterations must be continued until the deviations reach zero or slightly oscillate around zero due to the limited computer precision. For the model which Müller and Losecke considered a mesh of 8000 grid points was used and 5600 iterations were necessary to satisfy the defined convergence criterion. This represented a computer time of 34 minutes for each period in each mode on a very efficient CDC computer.

2-dimensional model fitting of MT data involves the consideration of several different conductivity models. Müller and Losecke's approach, while providing results of high reliability would be prohibitively expensive to apply to the situation of practical interpretation. For this reason, Jones and Pascoe's convergence criterion was used in model fitting of the Kenyan data; a deviation of 0.01% between iterative passes was taken to indicate convergence.

In view of the possible inaccuracy of the model results it was decided to use them only as qualitative support of the results of a 1-dimensional MT interpretation. The convergence criterion is very probably (on the basis of Müller & Losecke's investigations) quite adequate to define the general form of the 2-dimensional apparent resistivity curves and the correct value of resistivity to within better than 50% of its 'true' value.
6.7 Two dimensional Models

Attempts were made to obtain apparent resistivity curves similar in amplitude and shape to those measured at JOR and MOL, by considering only 2-dimensional models corresponding to (a) a conductive infill of the rift, and (b) a highly conducting upper mantle. E-polarisation, H-polarisation and 1-dimensional curves were constructed for 'stations' at different positions across the surface of the model.

The simplest 2-dimensional model consisting of a conducting rectangle immersed in a resistive medium involves 5 variable parameters—the width, the top surface depth and the thickness of the conductor and the resistivities of the conductor and the surrounding medium.

On the basis of the MT traverse data discussed in section 6.1, the width of the rift conductor was estimated to be 60 - 100 km. A width of 80 km was assumed for all the conductivity models considered in the 2-dimensional model fitting discussed below. It was established that for several conductivity models, a reduction in width of the conductor to 60 km did not significantly affect the model results but merely contracted the horizontal scale of the anomaly.

It has been mentioned before that the maximum resistivities at all periods for all stations in the rift valley are restricted to between 10 and 25 ohm m. It thus seems reasonable to postulate that assuming a value of 16 ohm m for the resistivity of the conductor will not lead to a loss in generality of the conclusions of subsequent 2-dimensional interpretations.

The first model considered in an attempt to fit the MT data at MOL and JOR was that corresponding to a conductive infill of the rift.
All attempts to fit the resistivity curves with a 10 ohm m. surface conductor of thickness less than 10 km. and resistivity contrasts of greater than 10 failed. Figure 6.9 illustrates the response curves obtained for a conductor of 10 km thickness. Figure 6.10 shows the apparent resistivity curves calculated at points 10 km and 2 km from the discontinuity on the conductive side. Even at points so close to the discontinuity, an interpretation of the E- polarisation curve in terms of a 1-dimensional model would not be significantly in error, and, if anything, would underestimate the conductivity and thickness of the conductor. Apparent resistivities and the frequency dependence of impedance and induction responses are similar to those at JOR and MOL at short periods, but above 100 seconds, there is a large divergence of model curves and field data. Both E- polarisation and 1- dimensional apparent resistivity curves for the model (Fig. 6.10) rise sharply at 45° on a log-log plot.

The second type of model considered was a rectangular model of resistivity 10 ohm m. at a depth corresponding to a conducting upper mantle. It was found that the resistivity assumed for the resistive region did not significantly influence the results provided the value was greater than 100 ohm m. Even when the conductor was 40 km thick and raised to within 20 km of the 'rift floor', E- and H- polarisation resistivities and phases were found to be much too high to fit the Kenyan data (Fig. 6.11). In addition, both resistivity and phase, together with induction response amplitudes (6.12) show a considerable frequency dependence especially between 30 and 300 seconds, which is the region where the MOL and JOR data are best defined.

Models were next considered corresponding to combinations of a
Fig. 6.9  E- polarisation response curves over a surface conductor thickness 10 km. Model resistivities are in ohm m.

- 30 secs period
- 300 secs
- 3000 secs
Fig. 6.10 Apparent resistivity curves calculated for the model of Fig. 6.9 at points 10 km (station D) and 2 km (station E) from the discontinuity on the conductive side.

- E - polarisation
- H - polarisation
- 1 - dimensional
Fig. 6.11  E- and H- polarisation resistivity and phase curves over a deep conductor. Model resistivities are in ohm m.

\( \nabla \) 30 secs period
\( \circ \) 300 secs
\( \square \) 3000 secs
Fig. 6.12 Induction response amplitude and phase over a deep conductor. Model resistivities are in ohm m.

\[ V \quad 30 \text{ secs} \]
\[ O \quad 300 \text{ secs} \]
\[ D \quad 3000 \text{ secs} \]
E-POLARISATION

AMPLITUDE AND PHASE OF MAGNETIC FIELD RATIO Z/H

DISTANCE KM.
surface conductor resistivity 10 ohm m and a deep conductor, thickness 40 km and resistivity 10 ohm m, both immersed in a medium with resistivity of 300 ohm m. The apparent resistivities and induction responses approach those of JOR and MOL in form and magnitude only when the surface conductor is more than 5 km thick and the top surface of the deep conductor is less than 20 km beneath the rift floor.

The MT data can not distinguish between this twin conductor model and a single conductor model representing 10 ohm m. material extending from the rift floor down to depths of greater than 40 km (See Fig. 6.13). Apparent resistivities and phases calculated for both of these models display values similar to those of the resistivities and phases measured at JOR and MOL.

Figures 6.14 and 6.15 - display E- and H- polarisation and 1-dimensional apparent resistivity calculated at 'stations' 40 km, 10 km and 2 km from the lateral discontinuity on the conductive side. The rise in E- polarisation and 1-dimensional resistivity at periods greater than 300 seconds suggests that the base of the conductor extends to a depth greater than 40 km. For this model the E- polarisation curve shows significant deviation from the 1-dimensional curve only for periods greater than 300 seconds or for stations closer than 10 km to the discontinuity. The anisotropy measured at JOR indicates that JOR is more than 10 km from a discontinuity; so a 1-dimensional interpretation of the JOR maximum resistivities should be valid up to at least 300 seconds. It can be observed, however, that for this type of model, the 1-dimensional curve is always bracketed by the E- polarisation and H- polarisation curves. An interpretation of the E- polarisation curve at JOR in terms of a 1-dimensional structure should
Fig. 6.14 and Fig. 6.15  Apparent resistivity curves calculated for the model of Fig. 6.13 at points 40 km (station A), 10 km/(station D) and 2 km (station E) from the discontinuity on the conductive side.

- E– polarisation
- H– polarisation
- 1– dimensional
give an under estimate of the thickness of the conductor. A 1-
dimensional interpretation of the long period data at JOR provides
strong evidence that conducting material is present at depths (> 40 kms)
corresponding unambiguously to the upper mantle. It should be remarked
that induction responses calculated for the model (Fig. 6.13b) show
the type of frequency independence characteristic of stations on the
MT traverse.

The results of the modelling studies may be summarised as follows:

(a) Neither a surface conductor (< 10 km thick) alone nor a single
conductor at upper mantle depths (> 15 kms) can fit the apparent
resistivity data or the frequency behaviour of the induction response
data.

(b) A composite model consisting of a conductor at 20 km depth repre-
senting conducting mantle and a surface conductor representing conductive
infill of the rift can not satisfy the MT data unless the surface
conductor is more than 5 km thick. Geological estimates (McCall 1967)
put a maximum estimate of 2-3 km on the thickness of rift infill and the
thickness is much less on the rift shoulders at MOL and JOR.

(c) The MT data is broadly consistent with a model of high conductivity
extending from the surface to depths of more than 40 kms. The data
is not sufficient to resolve the presence of a resistive zone 5 km thick
near the surface or 10-15 km thick at depth below 5-10 km. However,
the data are sufficient to indicate the presence of a highly conducting
zone in the upper crust (0-15 km), which can not be explained in terms
of a conductive infill of the rift. The MT data can be fitted only by
a model involving high conductivities at depths greater than 30 km.
Thus the data require the existence of a conductor in the upper mantle
under Kenya. The vertical extent of this mantle conductor can not be resolved.

The conductivity model consistent with the data collected in the Kenyan Rift region is similar in appearance to those fitting MT results obtained in Iceland (Hermance 1973b) and in the Afar region of Ethiopia (Berktoldt 1974). Hermance and Berktoldt have fitted their MT data with conductors of resistivity 30 ohm m and 10 ohm m respectively, extending from depths of less than 5 km down to upper mantle depths.

6.8 Geophysical significance of the conductivity model

The most probable explanation for high conductivities in the upper mantle under the rift valley is the effect of partial melting caused by enhanced temperatures under the rift. The conductivity - temperature data published by Duba (1974) suggest that temperatures high enough to produce resistivities of the order of 10-25 ohm m in olivine are probably sufficiently high to induce partial melting of mantle material at these shallow depths (< 50 km) particularly if the mantle is wet (Wyllie 1971). Using the relation derived by Waff (1974) and discussed in section 1.4, and assuming a value of 1 ohm m for the resistivity of basalt, it can be calculated that a partial melt fraction of less than 7% of olivine is sufficient to produce a bulk resistivity of 10 ohm m.

It is unreasonable to suppose that the low resistivities encountered at upper crustal depths below Kenya are caused by solid conduction in dry rocks or by partial melting since temperatures of over 700° C are required at depths of a few km. It is also improbable that such a large conductor as that underlying the Rift can be explained in terms of basement mineralisation. The only reasonable explanation for high conductivity in such a large volume of upper crustal rocks is the
presence of electrolytic fluids in the rock pores and fissures.

Brace (1971) has pointed out on the basis of evidence from deep wells that rocks below the water table are saturated with aqueous solutions to depths of more than 5 kms. The contribution to bulk rock conductivity from electrolytic conduction through rock pores can be estimated from Archie's Law in simple form (Brace et al. 1965; Brace and Orange 1968), and is simply the conductivity of the fluid times the porosity squared. Increasing effective pore pressure tends to close pore spaces and raise the bulk rock resistivity. However, little is known about the relationship between effective pore pressure for crustal rocks and lithostatic and hydrostatic pressures. Effective pore pressure could vary from zero to the difference between lithostatic and hydrostatic pressures.

The variation of the electrolytic conductivity with depth can be treated to a very good approximation (Hermance 1973b) in terms of an equivalent concentration of NaCl using the relation of Dunlap and Hawthorne (1951).

Hermance (1973b) has presented data from Quist and Marshall (1968) which shows the variation of conductivity of different molal solutions of NaCl in terms of depth, under conditions of hydrostatic and lithostatic pressure for geothermal gradients of 60 and 120°C/km. At shallow depths the conductivity of the electrolyte increases due to the effect of increased temperature reducing the viscosity of the fluid. At greater depths and higher temperatures ionic association acts to inhibit conductivity.

Fig. 6.16 is reproduced from Hermance (1973b) and illustrates the
Fig. 6.16  Rock resistivity as a function of depth for the electrolytic conducting component only. The pore fluid is at lithostatic pressure and results for a range of Na Cl concentrations (0.01, 0.1 and 0.5 molals) are shown. Two geothermal gradients are considered. For comparison, the component due to solid conduction in basalt is shown (After Hermance 1973b).
variation of rock resistivity as a function of depth, for various equivalent concentrations of NaCl. It is assumed that rock porosity is 0.05 and that lithostatic pressure corresponds to a specific rock density of 2.8 gm/cm$^3$. The figure illustrates the point that a large temperature gradient is capable of reducing the resistivity of saturated rocks by up to one order of magnitude, with the minimum in resistivity being reached at depths of the order of 4 km.

A high equivalent concentration of NaCl can also explain a reduction in resistivity of saturated crustal rocks. Little is known concerning the salinity of deep crustal waters, but it is not inconceivable that salinities in the deep crust ($>5$ km) might greatly exceed that of sea water.

The existence of considerable quantities of water in the crust under the rift valley could provide an explanation for high conductivities at lower crustal depths ($>10$ kms). Lebedev and Khitarov (1964) have presented data for the electrical conductivity of granite as a function of temperature and water pressure. Their graph is reproduced in fig. 6.17a. Beside their graph is a plot (fig. 6.17b) of water pressure versus depth for a geothermal gradient of $60^\circ$ C/km as calculated by Hermance (1973b). Assuming a temperature gradient of $60^\circ$C/km for the crust below Kenya, water pressures at depths of more than 10 kms are sufficient to lower the resistivity of granite to below 10 ohm m. It may be inferred that rocks of similar composition to the granite measured will also display high conductivity under high water vapour pressure.

Reasonable explanations for the existence of high conductivities within the crust under the rift valley require that large amounts of
Fig. 6.17

(a) Electrical conductivity of granite as a function of temperature and water vapour pressure. After Lebedev and Khitarov (1964).

(b) Variation of water vapour pressure with depth, assuming a geothermal gradient of 60°C/km and a specific rock density of 2.8 g/cm³.
water are present in the crust and that geothermal gradients are high. These conditions are also consistent with the existence of a region of partial melting in the upper mantle below the Kenyan Rift Valley.

6.9 Conclusions

Data collected in the Kenyan Magnetotelluric project have been carefully screened to obtain only data of high reliability for use in interpretation. This greatly reduces the possibility of ambiguities in the final interpretation.

All data which may possibly be biased significantly by systematic errors have not been used in quantitative interpretation. This eliminates the results from stations outside the rift and also from stations with a high noise level on the telluric records. Greatest weight has been placed on the results obtained at Ol Joro Orok (JOR) since the data is of high quality and is less influenced by the presence of lateral inhomogeneities than the data acquired at other stations.

Because of inaccuracies involved in the practical application of the Jones - Pascoe 2-dimensional modelling programme, and so as not to obscure the basic findings of the interpretation by the introduction of extra parameters, two dimensional models have been used only in a limited way. Results of 2-dimensional modelling indicate that, for the simple one - and twin - conductor models considered in the interpretation, the existence of lateral conductivity changes does not significantly alter the main conclusions.
The MT data provide the first independent evidence for the existence of high conductivities at depths corresponding to the upper mantle below the Kenyan Rift Valley. The conclusions of previous GDS surveys in the region were heavily dependent upon other geophysical and geological data. An explanation of the upper mantle conductor in terms of partial melting is in accord with geological evidence of recent volcanic activity in the area.

The depth to the top of and the thickness of the upper mantle conductor can not be resolved by the MT data since the mantle conductor is obscured by the presence of a good conductor at depths (<10 kms) corresponding to the upper crust. The high conductivity of upper crustal material can only reasonably be explained in terms of high temperatures and water saturation of the crust under the rift valley.

The possible existence of large quantities of water in the crust below the Kenyan Rift Valley and in the Sub-Icelandic and Sub-Ethiopian crusts suggests a relationship between high water concentration, high temperatures and tectonic activity. The presence of water in rocks at high temperatures is of considerable interest in the understanding of the petrology of rocks in the region of the Rift Valley.
6.10 **Suggestions for Future Work**

Greater constraints on the maximum depth to the top of the upper crustal conductor could be fixed by obtaining shorter period (<10 seconds) MT measurements. This might help to eliminate certain geophysical interpretations of the crustal conductor. Longer period (>1000 seconds) MT measurements would enable superior estimates to be placed on the depth to which conductive material extends and would provide valuable information concerning the extent of partial melting within the upper mantle.

The most appropriate location for such MT measurements would be in the vicinity of Ol Joro Orok (JOR) since this site shows least anisotropy of all the stations; short period MT measurements, especially, are very little contaminated by the effect of lateral inhomogeneities.

The MT method could be used to map the northward and southward extent of the rift conductor. To obtain good resolution it would be necessary to record variations within a wide period range and to make measurements away from the highly conducting lake sediments. It would be of considerable interest to determine whether a conductor in the mantle is always overlain by a highly conducting upper crust.

Experience has shown that in view of the very low amplitudes of telluric signals recorded in the Kenyan Rift, it is advisable to make MT measurements during the dry season and as far from towns as possible in order to avoid contamination of the telluric records.

Single station induction arrows were obtained for every site at which telluric measurements were made. This data was used only in a qualitative way, to support the results of an interpretation based on
the magnetotelluric data alone. However, the induction response data could be used in a future analysis together with an improved version of existing 2-dimensional programmes to provide further constraints on the conductivity distribution below the rift region.

In addition, the induction data could be used to provide information concerning the conductivity structure to the east of the rift where MT measurements are particularly susceptible to distortion by lateral conductivity changes. However, inherent in any 2-dimensional modelling of the induction response is the assumption that all the responses relate to the same regional 2-dimensional model. The validity of this assumption with regard to the response estimates at Nanyuki (NAN) is questionable, since both the Nanyuki and Mutara (MUT) arrows are strongly 2-dimensional and yet apparently relate to conductors with significantly different strike directions. It would be interesting to examine more closely the inductive behaviour of the area around NAN by operating a few satellite stations in the vicinity of NAN.

The present MT experiment was directed at determining the electrical properties of the deep crust and upper mantle. However, the MT technique could be profitably applied to the estimation of electrical properties at very shallow depths under the Rift floor. Electrical resistivity is a very sensitive indicator of the presence and temperature of pore fluids within rocks. In an area of hydrothermal activity such as the Rift Valley it is of considerable practical interest to map zones of suppressed resistivity within the upper few kms of crust, using very short period (<1 sec) MT variations. To obtain good resolution, sites would have to be selected carefully to avoid contamination by lateral conductivity changes.
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