AN INVERTER DRIVE

FOR TRACTION AND INDUSTRIAL APPLICATIONS

by

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1974
DECLARATION

I declare that this Thesis has been composed by myself and that the research work described is my own.
ABSTRACT

Several solutions have been proposed for the problem of providing a direct current motor without a commutator. A three-phase synchronous machine with reactance compensation and supplied from a shaft-controlled inverter appears to be the nearest equivalent of the d.c. commutator motor. Separate or series modes of excitation can be provided, the former requiring two rotor windings at 90°. The interaction between the compensated synchronous type of motor and the three-phase thyristor bridge inverter leads to maximum inverter efficiency at normal running speeds; although commutation demands are minimal at most speeds, full load current commutation will be necessary on starting. The inverter has been designed to remove commutation duty from the load-carrying thyristors.

Prediction of the system performance has been experimentally verified and 180° thyristor conduction was found necessary to give the desired characteristics. Applications of the system clearly exist in traction and industrial drives and estimated costs compare most favourably with present induction motor drives. Further development work is suggested.
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<tr>
<td>C, Cl, etc.</td>
<td>Capacitor</td>
</tr>
<tr>
<td>Dl, etc.</td>
<td>Diode</td>
</tr>
<tr>
<td>di/dt</td>
<td>Rate of change of current</td>
</tr>
<tr>
<td>dv/dt</td>
<td>Rate of change of voltage</td>
</tr>
<tr>
<td>E, Em</td>
<td>Peak generated back e.m.f.</td>
</tr>
<tr>
<td>Es</td>
<td>Average back e.m.f. of equivalent d.c. machine</td>
</tr>
<tr>
<td>Ed</td>
<td>Peak direct axis back e.m.f.</td>
</tr>
<tr>
<td>Eq</td>
<td>Peak quadrature axis back e.m.f.</td>
</tr>
<tr>
<td>ed</td>
<td>Instantaneous direct axis back e.m.f.</td>
</tr>
<tr>
<td>eg</td>
<td>Instantaneous total back e.m.f.</td>
</tr>
<tr>
<td>eq</td>
<td>Instantaneous quadrature axis back e.m.f.</td>
</tr>
<tr>
<td>ed, eq, etc.</td>
<td>Instantaneous back e.m.f.s. for axis and harmonic order shown</td>
</tr>
<tr>
<td>I, Im</td>
<td>Peak stator phase current</td>
</tr>
<tr>
<td>Ia</td>
<td>Average armature current of equivalent d.c. machine</td>
</tr>
<tr>
<td>Icom(pk)</td>
<td>Peak commutation current</td>
</tr>
<tr>
<td>Id, Irq</td>
<td>Average control field current</td>
</tr>
<tr>
<td>Idc</td>
<td>Average d.c. link current</td>
</tr>
<tr>
<td>If</td>
<td>Average rotor field current</td>
</tr>
<tr>
<td>Iload(pk)</td>
<td>Peak load current</td>
</tr>
<tr>
<td>Iq, Irq</td>
<td>Average compensating field current</td>
</tr>
<tr>
<td>iq</td>
<td>Instantaneous compensating field current</td>
</tr>
<tr>
<td>i_y, i_yb, i_br, i_s</td>
<td>Instantaneous stator phase currents</td>
</tr>
<tr>
<td>i_1, i_5, i_7, etc.</td>
<td>Instantaneous stator phase current for harmonic order shown</td>
</tr>
<tr>
<td>i_4</td>
<td>Instantaneous positive d.c. line current</td>
</tr>
<tr>
<td>Jsd, Jsq</td>
<td>Stator direct and quadrature axis currents in rotor reference frame</td>
</tr>
<tr>
<td>J</td>
<td>Moment of inertia (kgm²)</td>
</tr>
<tr>
<td>j</td>
<td>√(-1)</td>
</tr>
<tr>
<td>K</td>
<td>Field constant (V/rad/sec)</td>
</tr>
<tr>
<td>Kd</td>
<td>Torque constant (Nm/rad/sec)</td>
</tr>
<tr>
<td>k</td>
<td>Compensation constant (=1 for full compensation)</td>
</tr>
<tr>
<td>L, Ll, etc.</td>
<td>Inductor</td>
</tr>
<tr>
<td>La</td>
<td>Armature inductance of equivalent d.c. machine</td>
</tr>
<tr>
<td>Lp</td>
<td>di/dt protection inductance</td>
</tr>
<tr>
<td>Lss</td>
<td>Stator phase self inductance</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
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<td>--------</td>
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<tr>
<td>$M, M_{sf}$</td>
<td>Peak rotor to stator phase mutual inductance</td>
</tr>
<tr>
<td>$M_{sd}$</td>
<td>Peak mutual inductance between stator phase and control field</td>
</tr>
<tr>
<td>$M_{sq}$</td>
<td>Peak mutual inductance between stator phase and compensating field</td>
</tr>
<tr>
<td>$n$</td>
<td>Compensation ratio ($= 0.604$ for full compensation, 180° conduction, separate excitation)</td>
</tr>
<tr>
<td>$p'$</td>
<td>$d/dt$</td>
</tr>
<tr>
<td>$R, R_l, etc.$</td>
<td>Resistor</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Armature resistance of equivalent d.c. machine</td>
</tr>
<tr>
<td>$R_s$</td>
<td>Stator phase resistance</td>
</tr>
<tr>
<td>$s$</td>
<td>Laplace operator $d/dt$</td>
</tr>
<tr>
<td>$T$</td>
<td>Shaft torque</td>
</tr>
<tr>
<td>$T_l, etc.$</td>
<td>Thyristor</td>
</tr>
<tr>
<td>$t, T_{tl}, etc.$</td>
<td>Transistor</td>
</tr>
<tr>
<td>$t$</td>
<td>Time variable (suffix indicates particular instant)</td>
</tr>
<tr>
<td>$V, V_m$</td>
<td>Peak stator phase voltage (fundamental)</td>
</tr>
<tr>
<td>$V_a$</td>
<td>Average armature voltage of equivalent d.c. machine</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Capacitor voltage</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>d.c. link voltage</td>
</tr>
<tr>
<td>$V_{oc}$</td>
<td>Stator terminal voltage in rotor reference frame</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Peak inverter terminal voltage (half voltage between d.c. rails)</td>
</tr>
<tr>
<td>$v_r, v_y, v_b$</td>
<td>Instantaneous inverter terminal voltage</td>
</tr>
<tr>
<td>$v_{ry}$</td>
<td>$(v_r - v_y)$</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Instantaneous stator phase terminal voltage</td>
</tr>
<tr>
<td>$v_1, v_5, v_7, etc.$</td>
<td>Instantaneous stator phase terminal voltage for harmonic order shown (red-yellow phase)</td>
</tr>
<tr>
<td>$U_{sd}$</td>
<td>Stator direct axis terminal voltage in rotor reference frame</td>
</tr>
<tr>
<td>$U_{sq}$</td>
<td>Stator quadrature axis terminal voltage in rotor reference frame</td>
</tr>
<tr>
<td>$X_{ss}$</td>
<td>Stator phase synchronous reactance</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Angle (phase displacement)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Angle (between stator and rotor axes)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle (between fundamental current and voltage vectors)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Angle (variable)</td>
</tr>
<tr>
<td>$\delta_{d}$</td>
<td>Damping factor</td>
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\( \tau \)  
Time constant

\( \tau_m \)  
Mechanical time constant

\( \tau_e \)  
Electrical time constant

\( \omega \)  
Radian frequency

\( \omega_0 \)  
Theoretical no-load value of \( \omega \)
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CHAPTER 1

INTRODUCTION

Variable-speed electric drives require several types of motor characteristic and the d.c. commutator motor with variable voltage supply is sufficiently versatile for most applications. However, increasing maintenance expenditure has turned attention towards the use of more simply constructed a.c. machines where appropriate. In this chapter, an outline is given of typical characteristics required of variable-speed drives and of principal a.c. motor systems with their limitations. From these systems is developed the present scheme which basically consists of an inverter and synchronous type of a.c. motor and it is shown that this system most nearly corresponds to the d.c. motor.

1.1 Variable-speed Drives.

The term "variable-speed drive" can cover various forms of connection between motor shaft and rotational load and not all of these methods involve continuous control of the motor speed. As an example, a typical machine tool drive will use a constant-speed motor with a selection of output gears or pulleys to select the output shaft speed. Continuous variation of output speed may be obtained from a constant-speed shaft with an eddy current coupling. Further types of drive, e.g. for some fans and pumps, may require only discrete steps in operating speed and in these cases a pole-changing or pole-amplitude-modulated machine can avoid mechanical gear changing. Only the most general case of a motor operating from standstill over a continuous speed
range and connected directly to its load is considered here.

The conventional d.c. motor drive uses a variable-voltage d.c. supply; although the thyristor chopper is now a common voltage controller, the Ward-Leonard system (Fig. 1.1) finds use for heavy regenerating drives or short-term overloads where supply capability is limited. Development of both a.c. and d.c. systems continues but power semi-conductor technology has removed many limitations on uses of a.c. motors; this increased design freedom has rearranged the logical allocation of particular motors to particular types of drive. Barring large changes in semi-conductor costs, it is unlikely that the usage pattern will change as rapidly in future. In succeeding sections, the capabilities of a.c. synchronous and asynchronous motor systems are examined in relation to the requirements of industrial drives.

1.2 Industrial Drive Requirements.

Metal process industries are extensive users of variable-speed machinery both for processing and material handling. Particularly harsh environments may exist in cases where metallic dust, heat and corrosive gases are present. A rolling mill drive for steel strip, consisting basically of two tension rollers, mill stand motor and screw-down motors, is a good example of severe motor duty. Since tension reel drive horsepower is proportional to metal speed at constant tension, the horsepower must remain constant despite rotational speed changes due to reel diameter during rolling. If strip speed or tension are to be altered, then the horsepower level must change and a measure of tension
FIGURE 1.1  WARD-LEONARD SPEED CONTROL SYSTEM

3-ph. Induction Motor  d.c. Generator  d.c. Drive Motor
is available from monitored motor torque and speed. Acceleration or deceleration may have to occur at a constant rate and deceleration may be so rapid that regeneration is necessary.

The mill stand motor similarly must withstand heavy load changes. Screwdown of the rollers requires a fast-response drive with stalling involving an overload of, say, 300% of full load current. A roller table drive is another example of a system in which stalling must be allowed and again rapid stopping and reversal require regeneration facilities. Here many separate motors are involved, each group of motors being separately controlled. If semi-conductor power control is used, overload requirements must satisfy commutation ability as well as thermal rating.

Most process industries make some use of variable-speed drives. In some winding duties in the man-made fibre and paper industries, extremely accurate speed holding is required. Speed range is, however, often small but long-term continuous running requires high reliability. Again, tension control is critical for synthetic materials.

Reliability of the drive must extend in some cases to process maintenance during temporary mains failure, for example in the glass industry. If interruptions longer than circuit breaker reclosing times have to be catered for, the drive must be capable of operation from a standby battery supply.
1.3 **Traction Drives.**

A traction vehicle powered by an internal combustion or gas turbine prime mover will be limited to a constant horsepower range as indicated by Fig. 1.2. The field of the mechanically coupled generator will be regulated to suit this characteristic within its current and voltage rating and the motor characteristic will approximately follow these limits. Although a.c. induction and commutator motors have successfully been used in traction, the series d.c. commutator motor remains the most popular type. The series motor speed on load is limited by the generator excitation and field weakening is usually arranged to increase the speed for a given horsepower.\(^{(11)}\) Electrical drives supplied from a battery or external source depend on thermal rating of components for their load characteristic. Traction duty requires a high starting and accelerating torque, i.e., starting from a stalled condition is part of the motor duty. Hence past practice included a "\(\frac{1}{2}\)-hour" or "1-hour" rating for acceleration or work on gradients.

The inertia of the system is high and in addition resilient shaft coupling is frequently employed. Pulsating torques therefore have little effect but can cause excessive wear on rigid mechanical couplings. Future increases in traction speeds place further strain on the mechanical commutator unless the gearing ratio is altered, causing the motor to operate at a higher torque.
FIGURE 1.2  SIMPLIFIED TRACTION GENERATOR REGULATION
Braking by regeneration is common in traction systems and the series motor becomes a separately excited generator in this case. Unlike industrial drives, traction drives do not require a rapid changeover from motoring to regeneration. Field regulation of a separately excited motor to provide a smooth changeover, correct field control for constant horsepower and rapid recovery from wheelslip appears to be the system of the near future.

Constant speed operation of a traction drive is limited to slow-speed cases in which manual control is insufficiently precise.

Traction external supplies in the past were mainly d.c. at a voltage to suit the maximum motor terminal voltage and many such systems are still being extended today. Later systems used high-voltage a.c. with stepped transformer ratios. (12)

1.4 Inverter/Induction Motor Drives.

The three-phase squirrel cage induction motor combines cheapness, ruggedness and reliability and is therefore most attractive for industrial drives. Hence much attention has been paid to its combination with a variable-frequency a.c. source as a competitor for the d.c. motor. The latter can cost up to four times as much as the equivalent horsepower squirrel cage machine at present.

The wound-rotor induction motor is more expensive to manufacture and less rugged than the squirrel cage machine but permits rotor control of speed. Non-resistive methods of effective rotor impedance variation improve the efficiency of variable-speed
operation from a fixed-frequency stator supply and include the use of back-to-back thyristor pairs in the rotor output lines, chopping of rectified rotor currents and a.c. rotor supply from a variable-frequency regenerating source. An effective control method is the "Static Kramer" system in which energy is injected into or abstracted from the rotor at slip frequency.

If a variable-frequency supply is provided, the cage-rotor induction motor becomes sufficiently versatile for most applications. The supply may be provided by a cycloconverter, working at between 10 and 90 per cent of mains frequency, though efficiency can be low; a d.c. link inverter is more commonly used. For a variable-voltage d.c. link input, thyristor conduction angle would normally be 120° or 180°. A pulse width modulated inverter may be used to improve the harmonic content of the voltage output and can also produce variable r.m.s. output voltage from a fixed d.c. link input. When the more common rectangular voltage waveshape is applied to the induction motor, harmonic currents will flow in the stator and corresponding harmonic currents will be induced in the rotor conductors. Some of the harmonic torques produced may be negative.

A typical torque-speed curve for fixed frequency operation of an induction motor is shown in Fig. 1.3(a). With a variable frequency supply at constant flux, an infinite family of such curves can be produced, as in Fig. 1.3(b). From the simple circle diagram of Fig. 1.4, it can be seen that the induction motor produces its greatest output at a power factor of 0.7.
FIGURE 1.3(a)  INDUCTION MOTOR TORQUE-SLIP CURVE

FIGURE 1.3(b)  FAMILY OF INDUCTION MOTOR TORQUE-SPEED CURVES
FOR SEVERAL FREQUENCIES AT CONSTANT FLUX
FIGURE 1.1  SIMPLE CIRCLE DIAGRAM FOR INDUCTION MOTOR
(NEGLECTING COPPER AND IRON LOSSES)
Maintaining a constant relationship between motor terminal voltage and frequency produces approximately constant flux; if very low speeds are required, some correction may be applied for stator resistance. Under these conditions, a characteristic similar to that of the d.c. shunt motor is produced.\(^{(13)}\) If, in addition, the slip frequency is held constant, the characteristic becomes that of a d.c. series motor. Constant horsepower operation is also feasible with an appropriate control system.\(^{(14)}\)

Derating of induction motors for operation from non-sinusoidal supplies is estimated at 5 - 10\%.\(^{(15,16)}\)

1.5 \textbf{Synchronous Drives.}

If the speed accuracy of an induction motor is inadequate for a particular application, a synchronous motor fed by a stable frequency may be used. For operation over a variable power factor range, an inverter with 180\(^{\circ}\) conduction should be employed. Voltage phase changes with smaller angles, e.g. 120\(^{\circ}\), produce a potentially unstable system. Nonaka and Okada\(^{(17)}\) and Sato and Seki\(^{(18)}\) have considered such systems. The machine operates at each frequency in the same way as a mains frequency synchronous motor and field regulation can determine the power factor. However, harmonic stator currents are present and appreciable harmonic currents will be induced in the rotor windings if the series inductance is not sufficiently large. More seriously, harmonic currents induced in the damper windings will give rise to harmonic torques.
Alston and Hayden\textsuperscript{(19)} have described a closed-loop system in which excitation is controlled for unity power factor; in effect the motor is "compounded". Slemon, Dewan and Wilson\textsuperscript{(20)} have described a synchronous motor drive with "constant current" operation.

At present, conventional synchronous motor systems controlled by free running oscillators appear to be little used except in low power mechanisms.

1.6 "Commutatorless" Motors.

The stability of an inverter-fed synchronous machine is assured if the rotor angle remains at a fixed value. If a shaft position sensor controls the phase of the applied voltage, and hence also its frequency, change in rotor angle is impossible. The motor, however, loses its synchronous characteristics since an increase in load causes a corresponding drop in speed. The quantities in this machine are then similar to those of a d.c. machine, i.e., the frequency is internally generated by the relationship between supply and load with d.c. field quantities. The frequency of armature winding voltages is the commutation frequency. Normally, the armature windings are on the stator and the field windings on the rotor.

This type of machine has received considerable attention and has been termed a "commutatorless motor" by Japanese authors in particular. Research has been carried out on single-phase motors of this type\textsuperscript{(21)} although these are not basically self-starting. Gaede\textsuperscript{(22)} has described both a semi-four-phase, four-thyrister
system using a permanent-magnet rotor and a three-phase reluctance motor. Three-phase machines are commercially available and the AEI-3L motor (23) is an example of a brushless type with transformer rotor excitation. Operation direct from a.c. mains through a valve cycloconverter was described as early as 1943 by Holters (24) and later using static devices by Eckert (25) and Tsuchiya et al. (26). Interest in this type of motor in Germany dates back to the 1930's. (27) Most descriptions have, however, been of a conventional three-phase synchronous machine with stator fed by an inverter and d.c. supply to the rotor via sliprings, the only addition being the shaft position transducer. (28) (Fig. 1.5) It is interesting to note that both the "commutatorless motor" and the d.c. motor supplied from a three-phase rectifier have been described as the static equivalent of the Ward Leonard system shown in Fig. 1.1.

Sato (29) has analysed the operation of the three-phase motor-inverter scheme, considering the motor as an a.c. machine. Miyairi and Tsunehiro (30) on the other hand, use an equivalent d.c. machine with few commutator segments as a basis for analysis. Kollen- sperger (31,32) has used a rotor angle setting to produce leading power factor for natural commutation of thyristor currents. Similar work has been done by other authors (25, 33,34) and Magureanu (35) has fully analysed the case of a salient-pole machine. In an attempt to minimise the harmonic content of the applied voltage, the author applied a double three-phase system to a machine with two separate three-phase stator windings (36) (see Appendix 1).
FIGURE 1.5  TYPICAL "COMMUTATORLESS MOTOR" ARRANGEMENT

FIGURE 1.6  PHASE EQUIVALENT CIRCUIT FOR FUNDAMENTAL SINUSOIDAL QUANTITIES IN "COMMUTATORLESS MOTOR"
The maximum starting torque of a "commutatorless" motor is produced when the rotor back e.m.f. would directly oppose the forward e.m.f. of the fundamental component of a stator phase voltage. Appropriate setting of the shaft position transducer will provide this angle. When the machine is operating at full speed, the stator phase impedance is largely reactive and peak torque can only be obtained by advancing the applied voltage phase through nearly 90° in order that the back e.m.f. and current will be in phase. Thus a continuous transducer angle shift with speed is necessary for optimum performance; alternatively, some intermediate angle may be selected as a compromise. If the cylindrical synchronous machine equivalent circuit per phase of Fig. 1.6 represents a machine phase, Figs. 1.7(a) and 1.7(b) show vector diagrams of motor operation at 0° and 90° angles, for full frequency operation. In practice, sinusoidal theory was found inadequate even for the double three-phase system. The use of 120° conduction caused stator voltage phase changes of ±30° with power factor independent of transducer variation. Poor r.m.s. content of current waveforms made determination of a "power factor"* difficult and reduced overall efficiency. The principal drawback was, however, the manual control of the shaft transducer angle. At first, angle alteration by electronic displacement of thyristor firing signals was considered, but rejected in favour of the present experimental system.

* Footnote: The term "power factor" loses its meaning when non-sinusoidal quantities occur. Where used in this thesis, "power factor" refers only to fundamental current and voltage components.
FIGURE 1.7 "COMMUTATORLESS MOTOR" PHASOR DIAGRAMS

(a) VOLTAGE AND BACK E.M.F. IN PHASE

(b) VOLTAGE AND BACK E.M.F. IN QUADRATURE
The Compensated Machine.

Compensated d.c. machines have been commercially available for a considerable time and their advantages have been described by Harris. On the quadrature axis, i.e. that of the interpole, a series winding provides an m.m.f. opposing that of armature reaction. Effective removal of the armature inductance improves the motor response. Krug has described a compensation scheme for the "commutatorless" motor using the d.c. link current into the inverter to supply a quadrature axis rotor winding (see Fig. 1.8). Sato has also suggested improvement of operation of a salient-pole machine with compensation.

Taking this system further for a cylindrical machine, compensation can be made complete and, in the ideal case, no armature inductance will exist. Considering the case of Fig. 1.7(a) again, the same phasor diagram will apply at all speeds if all reactance is cancelled, as in Fig. 1.9. This presupposes that the "synchronous reactance" equivalent circuit of Fig. 1.6 is valid. If the quadrature axis e.m.f. is added, the new phase equivalent circuit is shown in Fig. 1.10 and this reduces to a resistive impedance alone in Fig. 1.11.

The ideal case is not, however, entirely realisable with a rectangular-wave supply. Since the applied voltage per phase will contain voltage harmonics, the phase current will not be a perfect sine wave. The quadrature axis e.m.f. will not represent a sine wave unless the d.c. link current is very smooth. Fortunately, filtering of the link current can produce an effectively smooth
FIGURE 1.8  BLOCK DIAGRAM OF COMPENSATED MOTOR SYSTEM
WITH SEPARATE CONTROL EXCITATION

FIGURE 1.9  PHASOR DIAGRAM FOR FULLY COMPENSATED MOTOR
FIGURE 1.10  FUNDAMENTAL SINE-WAVE EQUIVALENT CIRCUIT PER PHASE OF FULLY COMPENSATED MOTOR

FIGURE 1.11  REDUCTION OF FIG. 1.10 FOR IDEAL COMPENSATION
quadrature field current and, if the fundamental component of the phase current is large, harmonic currents will be negligible. These will also be attenuated by the phase inductance.

It has been found necessary to use thyristor conduction angles of almost 180° in order to maintain a constant phase relationship between the transducer angle and inverter output voltages.

Since resistance drop alone determines speed regulation in the ideal case, the speed-torque characteristic will conform to that of the d.c. shunt machine. With slight additional direct-axis field control, absolutely constant speed could be obtained.

Not all drives require a "shunt" characteristic and the series characteristic, as suggested by Krug (39) can be reproduced by replacement of the direct axis and quadrature axis field windings with one series field winding, at an intermediate angle, whose quadrature m.m.f. component is equal to the required quadrature axis m.m.f. This can be realised by exciting an appropriate winding configuration and arranging the transducer angle as necessary. Operation is now completely analogous to that of a d.c. series machine. In addition to quadrature axis considerations, the direct axis field component must be arranged to suit the field strength required. If direct and quadrature axis fields are separately connected into the d.c. link, control of the direct axis field alone will allow the motor torque-speed curve to follow the constant horsepower characteristic previously described for a traction or reel drive application.

A further benefit of reactance cancellation is the reduction or removal of the electrical time constant $L/R$ but series field
and filter inductance will delay response to a step voltage input.

The principal advantage of sinusoidal m.m.f's. is the provision of uniformly rotating magnetic axes, which should give the same result as stationary axes; a d.c. commutator machine does not have perfectly stationary axes unless the commutator has an infinite number of segments.

1.7.1. Advantages of the Compensated Machine.

It can be seen that the compensated synchronous motor conforms more closely to the equivalent d.c. motor than do the preceding schemes and the large effect of armature reaction on "commutatorless motor" operation is overcome. The machine air gap can be larger than that of an induction machine but a compromise must be reached; a smaller air gap means more reactance to cancel but a larger reactance further attenuates harmonic currents without affecting the fundamental current. On full load, the machine phase current can be assumed sinusoidal and in phase with both the fundamental applied voltage and direct axis back e.m.f. Output torque for the three-phase machine will then be uniform.

A unity power-factor sine wave current will be self-commutating between inverter thyristors. Harmonic currents will be small compared with the full-load fundamental current and will require only minimal commutation ability. Unity power factor allows optimum utilisation of the inverter and the absence of reactive power demand on full load permits the inverter rating to be described as "watts" rather than "VA". The induction machine, which cannot provide its full torque at unity power factor, will require an
inverter whose rating is increased by a factor \( \frac{1}{\cos\phi} \) above the value of power delivered. In addition, higher ratings of inverter feed-back diodes and commutation components will be required. In common with all machines using a rectified supply, regeneration requires an inversion facility and in this case feed-back diodes would have to be of the full current rating. For short regeneration periods resistive loading of the d.c. link is possible. The d.c. link supply can, of course, be provided by rectified mains a.c. with any number of phases, or by mains d.c. In each case, some form of voltage control would be required.

The machine itself has sliprings and would be more reliable than a commutator machine. On the other hand, a cage-rotor induction motor has the advantage of having no brushes at all. Comparison between a.c. and d.c. brush wear is difficult since the brush material is usually different. Although advances have been made in commutator construction,\(^{(41)}\) the d.c. machine is limited to a lower speed than the a.c. machine. No problem exists with the eddy-current probe shaft transducer described in the next chapter and, given suitable covering, it is expected to be reliable in any environment in which the motor can operate. The probe itself is an industrially proven device.\(^{(42)}\)

For high-power machines, a series field current proportional to the average d.c. link current could supply a high-resistance field. Thus the sliprings and brushes would not require dimensioning for the full link current. If only a limited speed range was required, a brushless excitation system might be developed but there is little evidence that brushgear is unreliable in most applications.
1.7.2 Reasons for Selection of Three-phase.

Self-starting of the basic machine requires a multi-phase winding and two, three and four-phase connections were considered. Larger numbers of phases and double three-phase were rejected due to the large numbers of thyristors employed. A two-phase machine gives poorer utilisation of thyristors than four-phase and has a second harmonic component in the torque with sine wave currents. The corresponding four-phase connection, like the three-phase, has uniform torque; the "4-R" system (22) is not a true four-phase system and cannot carry the sine wave currents required. Three-phase provides similar winding and thyristor utilisation to four-phase with a lower d.c. line current ripple, and hence was preferred. There is the practical advantage that three-phase stator windings are commercially standard.
CHAPTER TWO
THE THYRISTOR INVERTER

The compensated machine is supplied with variable-frequency a.c. from a three-phase thyristor bridge inverter, the design of which is described in this chapter. The operation of a basic three-phase bridge inverter on various types of load has been analysed by previous authors. (43-48)

2.1 Design Requirements

The basic three-phase bridge inverter has six main (load-carrying) thyristors and six feed-back diodes to return reactive energy from the load to the d.c. supply rails. Each thyristor is individually commutated to give continuity of load current in other thyristors. In Fig. 2.1, thyristors T1 - T6 are numbered in firing order.

2.1.1 Conduction Interval

$120^\circ$ conduction was at first applied to the compensated machine and Fig. 2.2 shows the power and commutation components for the $120^\circ$-conduction inverter originally built. During the interval in which T6 and T1 would be conducting, say, C1 would receive the charge polarity shown. When T3 is subsequently fired, $60^\circ$ after extinction of T6, the hitherto negative terminal of C1 is swung through the d.c. rail voltage and a resonant half cycle of current will flow in the loop C1 - L1 - D1' - D1 - T3. Some current will flow to the load will
FIGURE 2.1  BASIC THREE-PHASE BRIDGE INVERTER

FIGURE 2.2  POWER AND COMMUTATION CIRCUITS FOR 120° INVERTER
continue until either the load current drops to zero or until D4 is forward-biased.

Although the $120^\circ$-conduction inverter requires only six thyristors, it has the disadvantage that a half-sine wave of output line current cannot be carried by a thyristor alone. Fig. 2.3 indicates how an output terminal potential will vary due to feedback diode conduction when a sinusoidal current is carried at unity power factor. Appendix 2 compares the harmonic contents of this voltage waveform with that of approximately $180^\circ$ conduction; the poor results obtained with the former waveform and discussed in Chapter 4 necessitated a change to $180^\circ$ thyristor conduction. The term "$180^\circ$ conduction" is used loosely since a commutation margin must be allowed between the conduction intervals of two thyristors in the same inverter leg. The actual conduction angle is variable, as the margin allowed is a constant time, irrespective of frequency.

Pulse-width modulation (p.w.m.) techniques\(^{(49)}\) afford a variable-voltage facility from a fixed d.c. rail voltage and a typical $180^\circ$ p.w.m. voltage waveform is shown in Fig. 2.4. The major advantage of this waveform is the removal of low-order harmonics as required. However, application of p.w.m. to the inverter circuit of Fig. 2.1 will again require feedback diode conduction for continuous currents and the voltage waveform will depend on load power factor. A further disadvantage of p.w.m. is the frequency limit for an inverter of specified rating due to the increased number of commutations per cycle; only at low speeds and moderate loads might p.w.m. be of use in the system under consideration.
FIGURE 2.3  TERMINAL WAVEFORMS FOR 120° CONDUCTION

FIGURE 2.4  TYPICAL P.W.M. VOLTAGE WAVEFORM FOR SINUSOIDAL CURRENT
2.1.2 Design Data

The inverter was designed as a versatile item of laboratory equipment under the assumption that a variable-voltage d.c. supply would be available.

The output line current would be 55 amps r.m.s. (sine wave) and the output voltage would be a 120° or 180° rectangular wave from a d.c. rail at 600 volts maximum. Operation on full load current at low d.c. rail voltages was required and the maximum thyristor commutation frequency would be 400 Hz.

2.2 Selection of Commutation Circuit

For a given turn-off time, the r.m.s. current carried by a commutation capacitor increases with both inverter rating and frequency. Therefore, although three-phase inverter commutation circuits with 1, 2 or 3 capacitors have been developed, a 6-capacitor circuit is more appropriate for 400 Hz and allows the full time between commutations for recharging against losses. Various circuit arrangements of this type have been used for some time in choppers and p.w.m. inverters.

The circuit of Fig. 2.5 provides 180° conduction with the same commutation mechanism as used in the 120° inverter; as all inverter phases are identical, only one has been shown. Gate drives to T1 and T1A, for example, are mutually exclusive and, to allow commutation to take place, gate drive to T4 is delayed behind that of T1A. When the d.c. supply voltage is low, full commutation may still be required and auxiliary supplies equal to the maximum supply voltage are connected as shown in Fig. 2.5. R1 carries charging current for C1.
FIGURE 2.5  COMMUTATION CIRCUIT FOR 180° CONDUCTION BASED ON 120° INVERTER PRINCIPLE

FIGURE 2.6  IMPROVED COMMUTATION CIRCUIT FOR 180° CONDUCTION
when T1 is conducting but also a loss current when T1A is conducting. R1A is a charging resistor for the latter interval but will be a much higher resistance than R1 as only a small capacitor charge is required to turn off T1A. The charging losses, which may approach 20% of inverter power at 400 Hz, can be reduced by switching on the auxiliary supply to each capacitor only during the charging period and R1A etc. would then be unnecessary.

Carrying of commutation pulses by the main thyristor is a major disadvantage of the above method and the circuit shown in Fig. 2.6 was developed to minimise main thyristor duty. The turn-off thyristor is now placed across the L-C circuit whose capacitor is normally charged as indicated.

Consider that T1 is conducting: when T1A is fired with a short pulse, a resonant current flows for a half-period of \( \frac{\pi \sqrt{L/C1}}{2} \) round the loop C1 - T1A - L1 and the voltage on C1 is reversed. D1 and D1A are now forward-biased and a second half-cycle of the commutation current will flow from C1 through L1, D1 and D1A. T1 is reverse-biased during this half-period, as in the previous commutation circuits, and T1A will similarly turn off. The mechanism of commutation is explained by Fig. 2.7 in which voltages relative to the positive d.c. rail are shown. Assume that the capacitor C1 is charged to the full d.c. rail voltage +Vdc. At the start of the commutation cycle the cathode of D1A (point X) is at +2 Vdc, the other capacitor terminal being at +Vdc while T1 is conducting. Firing of T1A brings points X and Z to the same potential and hence point Y drops to below +Vdc as L1 accepts current at a rate
31.

Load inductance may cause point Z to drop to the negative d.c. rail potential earlier if D4 conducts after the commutation current has dropped below the load current (see Fig. 2.9)

A safety margin will be allowed in a practical case between completion of commutation and firing of T4.
\[ \frac{\text{di}}{\text{dt}} = \frac{V_c}{L_1} \]. A complete cycle of oscillation then continues as described and in the ideal case point Y returns to the zero voltage level of the lower d.c. rail. When current ceases at the end of the cycle, the voltage on L1 disappears and points Z and Y resume the same potential. During the commutation of T4 and firing of T1, these points return to + Vdc and X returns to +2 Vdc.

Since the commutation capacitor is charged in one direction for at least 96% of the cycle, an isolated, single-polarity charging circuit can be employed as shown. Capacitor charging current can flow for almost the entire cycle but the R-C time constant must be sufficiently long to prevent excessive energy removal from the circuit during the reverse-voltage interval. Effective commutation here also depends on a high Q-factor but despite these restrictions the latter circuit is preferable for high-frequency inverters.

2.3 Inverter Component Design

2.3.1 Main Thyristor

The rated average current (half sine wave) per thyristor was 25 amps and a type T527 by Westinghouse Electric S.A. was selected. The principal data for the device used are as follows:

- Peak repetitive reverse voltage: 1200 V
- Max. repetitive rate of rise of forward current: 100 A/\mu s.
  
  " " " voltage: 200 V/\mu s.
- Max. forward current (average half-sine): 60 A
- Turn off time: 40 \mu s
Since no commutation pulse is carried by the main thyristor, an estimate of conduction loss can be taken from the highest average power dissipation with an allowed forward voltage of 2.5 V. If 1.5 times full load is accepted as overload, then the average dissipation may reach 100 watts.

A temperature rise of 40°C is taken as the maximum and the listed thermal impedance of the thyristor junction-sink is 0.14°C/W. Hence the heat sink must have an impedance of 0.26°C/W or less, and the thyristors were therefore acquired mounted with adequate double-sided cooling. Forced ventilation was not required. A photograph of the disc-type thyristor with double-sided cooling is shown in Fig. 2.8.

2.3.2 Commutation Components

The peak voltage obtainable from a 415 volt, three-phase supply is 585 volts. It is assumed that an auxiliary supply with a large reservoir capacitor is available and that the commutation capacitors will be charged to 550 volts.

In Fig. 2.9, the half sine wave of commutation current is shown, commencing at time \( t_0 \). At time \( t_1 \), this current exceeds the load current, which is assumed constant throughout this interval. Reverse-bias of the main thyristor lasts until time \( t_2 \).

Hence \( (t_2 - t_1) \approx 40 \mu s \).

To reduce the peak commutation current, \( (t_3 - t_0) \) is made as long as possible without forming an excessive proportion of the total cycle time, \( (2500 \mu s \text{ at } 400 \text{ Hz}) \). 100 \( \mu s \) was taken as suitable.
Figure 2.8  T527 DISC THYRISTOR AND HEATSINKS

Figure 2.9  COMMUTATION CURRENT PULSE

Note: With an inductive load, the load current may be partly taken over by a feed-back diode after time $t_2$. The commutation current will then reach zero at some time $t_4$ determined by the load time constant.
\[ \sqrt{L/C} = \frac{t_3 - t_0}{t_3 - t_0} \]

116A, i.e. 1.5 times peak load current, is taken as the maximum commutation requirement.

With \( \frac{(t_2 - t_1)}{(t_3 - t_0)} \approx 0.4 \)

\[ I_{\text{load(pk)}}/I_{\text{com(pk)}} \leq 0.81 \] (from Fig. 2.9)

\[ \therefore I_{\text{com(pk)}} \geq 143A \]

Now

\[ I_{\text{com(pk)}} = V_c/\sqrt{L/C} = \frac{V_c(t_3 - t_0)}{(\pi L)} \] (from 2.1)

\[ \therefore L = \frac{V_c(t_3 - t_0)}{(\pi I_{\text{com(pk)}})} \]

\[ \therefore L \leq 122 \mu H \]  

From 2.1 \[ C = 8.3 \mu F \]

In practice, a shortfall in capacitor voltage of up to 15% was allowed for by making \( L = 115 \mu H \) and \( C = 10 \mu F \).

At 400 Hz, the approximate commutation thyristor and diode current will be 5 amps average, 25 amps r.m.s. and the capacitor and inductor current will be 35 amps r.m.s.

The commutation thyristor turn-off time must be less than 100 \( \mu \)s.

Stud-mounting devices (type T507) were selected as the commutation thyristors. The principal data for the version obtained are:-

Peak repetitive reverse voltage 1200 V.

Max. repetitive rate of rise of forward current 100 A/\( \mu \)s.

" " " voltage 750 V/\( \mu \)s.

Max. forward current (average half-sine) 50 A

Turn off time 50 \( \mu \)s
In estimating the value of the auxiliary charging resistor, it is assumed that for full load current conditions frequency will be proportional to the d.c. rail voltage. As the capacitor voltage after commutation is difficult to determine at the design stage, it is set at the d.c. rail voltage only. The charging circuit must then raise the voltage to 95% of the auxiliary voltage, i.e. 550 volts. Hence the charging time will be 3 RC. In Fig. 2.10, the variation of R with frequency is shown, the minimum value being approximately 200 ohms. With this value, the calculated voltage loss to the auxiliary circuit during the 200μs commutation cycle is 10%.

The r.m.s. current in the 200-ohm resistor increases with voltage difference and frequency. Assuming triangular current pulses of 3 RC (6 ms) duration, and d.c. rail voltage initially on the capacitor, Fig. 2.11 shows the variation of r.m.s. current with frequency; the peak dissipation is 160 watts. This dissipation, however, would only be brief in practice and the anticipated load for 25% voltage loss, say, would be about 60 W.

Although the load currents in the experimental system will be virtually self-commutating after starting the machine, the commutating circuit does not provide for reduction of charge on the capacitor and further circuitry would be required to accomplish this. Preferably the method used should not involve dissipation of energy already stored in the capacitor.
FIGURE 2.10  VARIATION OF MAXIMUM AUXILIARY CHARGING RESISTANCE WITH FREQUENCY

FIGURE 2.11  VARIATION OF R.M.S. CURRENT LOADING ON 200-OHM AUXILIARY CHARGING RESISTOR WITH FREQUENCY
2.3.3 Feed-back Diodes

The current rating of the diode in antiparallel with each main thyristor is defined by the load current and power factor and by the commutation pulse also carried.

At zero power factor, the feed-back diode duty will exceed that of the thyristor, as the load components would then be equal. In this extreme case, the required diode would be rated at 60 amps average, 80 amps r.m.s., say. For normal duties at higher power factors, this rating would be reduced and would be equal to that of the commutation diode at unity power factor.

2.3.4 Protection Components

Fig. 2.12 is a complete schematic diagram of the inverter, showing the protection components. The small chokes, labelled Lp, protect each thyristor from high rate of rise of current \( \frac{di}{dt} \) on "fire-through" faults. Each choke is 4.5\(\mu\)H, restricting \( \frac{di}{dt} \) to 66A/\(\mu\)s.

A thyristor may be triggered by excessive rate of rise of forward voltage \( \frac{dv}{dt} \) across it. The R-C combination between anode and cathode of each thyristor slows the \( \frac{dv}{dt} \) from a step voltage applied across the R-C network and Lp. The values of R and C were selected using a nomogram\(^{51}\) and are 8 ohms and 1\(\mu\)F respectively.

2.4 D.C. Link Filter

An L-C filter will reduce a particular frequency \( \omega \) in the ratio \( \frac{1}{\omega LC-1} \). A large reduction in the predominantly 6th
FIGURE 2.12  COMPLETE SCHEMATIC DIAGRAM OF INVERTER
harmonic ripple of rectified three-phase a.c. would be, say, 90%. Hence LC will be 2.54 μs.

To minimise the cost of, and dissipation in, the choke, C is made large, say 400μF. L will then be 6.35 mH.

2.5 Performance on Passive Loads

Basic inverter operation is illustrated by output waveforms on passive loads. Load power factor does not affect the voltage on 180° conduction and Fig. 2.13 shows phase currents for this mode in a "delta" configuration at leading, lagging and unity power factors.

The voltage waveform of 120° conduction changes with power factor up to a load phase angle of 56°32' (52) and a selection of 120° waveforms is shown in Fig. 2.14.

2.6 Inverter Construction

The inverter is housed in the steel cabinet seen in Fig. 2.15. All panels are removable and the upper left side panel visible in the photograph is of "Perspex" and gives access to cable terminals. The upper front panel allows access to the negative rail thyristors, T2, T4 and T6, and also carries the firing circuit. Fig. 2.16 shows this panel open for inspection. The flexible steel conduit carrying the sheathed gate cables to the positive rail thyristors is also visible.

For Fig. 2.17, the rear panel has been removed and shows, top to bottom, 4.5μH chokes, positive rail thyristors, turn-off thyristors with commutation and feed-back diodes, commutation
FIGURE 2.13  180°-CONDUCTION VOLTAGE AND CURRENT WAVEFORMS
FIGURE 2.14  120°-CONDUCTION VOLTAGE AND CURRENT WAVEFORMS FOR RESISTIVE AND LAGGING LOADS WITH PHASE ANGLE < 56° 32'
FIGURE 2.16  INVERTER CABINET WITH FIRING CIRCUITS
OPEN FOR ACCESS

FIGURE 2.17  INVERTER CABINET WITH BACK REMOVED
capacitors and inductors, auxiliary charging resistors and auxiliary supplies. The heatsink on the lower left is half of a low-power main d.c. supply in use pending completion of a fully-controlled three-phase bridge rectifier. The auxiliary supplies shown are also temporary.

2.7 Control Electronics

2.7.1 Control Requirements

The inverter control equipment must record rotor position and translate it into appropriate firing signals for 12 inverter thyristors. During the interval of almost 60 degrees (electrical) between inverter commutations, it is necessary to specify the 60-degree interval in which the rotor reference position is located but not the exact position within that interval. Thus discrete rather than continuous monitoring of the rotor position is required. Since the machine must be self-starting, the shaft position transducer must produce the correct firing signal at standstill and furthermore the transducer output level must be independent of rotor speed. The most convenient way of producing a separate "message" for each 60-degree interval is to use a 3-digit code, as 3 digits can provide up to $2^3$ unique combinations; 3 tracks, each with its own transducer, would then be employed.

The second stage of the rotor position translation is the construction of pulses of appropriate lengths to drive main and turn-off thyristors, allowing for both 120° and 180° modes of operation. Apart from the difference in main thyristor gate-
drive periods, different phasing of turn-off thyristor drives is necessary. When a commutation is taking place in the 180° mode, the thyristor due to turn on must be delayed in firing to allow the opposite-polarity thyristor on the same phase to turn off; if insufficient time is allowed, a "fire-through" could occur. Fig. 2.18 illustrates the intervals within the commutation time.

This constraint does not of course occur with 120° conduction in which a 60° gap between thyristor conduction periods on the same phase exists.

For testing and other experimental purposes, a local oscillator built into the firing circuits provides an alternative manual control of inverter frequency. This oscillator is free-running, i.e. it is not linked to the rotor speed or position.

Fig. 2.19 shows how the various types of gate pulses are segregated within the logic unit in block diagram form.

Data for the T507 and T527 thyristors require a recommended gate drive of 20 volts through 80 ohms; the output circuit must supply this drive throughout the conduction interval of the main thyristors but in addition a higher gate drive is provided initially. Turn-off thyristors receive only a short gate pulse of about 50 μs duration. Further requirements are that the continuous gate drive should be effectively d.c., that each gate must be isolated from all other gates, that the gate voltage should not reverse and that the initial rise time should be less than 1 μs.
FIGURE 2.18  COMMUTATION PERIOD (RED PHASE) ON RESISTIVE LOAD

FIGURE 2.19  THYRISTOR FIRING CIRCUIT PULSE SYSTEM
2.7.2 Shaft Transducer

The three-element shaft transducer can be selected from several means, for example mechanical, optical, magnetic (Hall-effect) or eddy-current.

A mechanical system\(^{(29)}\) was ruled out on the grounds of reliability; contact bounce and susceptibility to dirt would be difficult to overcome and frequent maintenance would be required. The transducer used in the earlier "commutatorless motor" experiments\(^{(36)}\) was of the optical type, using M.E.S. light bulbs and photo-transistors with a collimator to give a very narrow light beam; more modern devices would use light emitting diodes. In both cases, there is still the problem of faulty operation in an industrial environment due to dirt and, particularly with light bulbs, there is a strictly limited operational life. The Hall-effect probe operates by detection of a magnetic field and hence is more suitable for general use\(^{(31)}\) provided that strong external fields are not present. The Hall-effect probe is not a switching device and its output is approximately a sine wave. Therefore a precise trigger or zero level detector is required on both rising and falling edges of the output to obtain a rectangular pulse suitable for operating successive circuitry.

An eddy-current proximity switch produces a high-frequency oscillation in a small coil at its tip. Presence of sufficient metal within the small conical detection area causes a change in frequency. Detection of this change causes a swing of output voltage. The probe is not affected by stray fields or dirt and
only by metallic particles above a certain size.

As part of an undergraduate project the feasibility of building a Hall-effect or eddy current transducer was examined. The latter type appeared more suitable and was also the more readily available, as these probes are used as metal detectors in industry for automatic machine travel control. Although normal usage does not require fast response, the probes obtained switched between 0 and 24 volts in approximately 300 ns. Unfortunately, the fast leading edge had to be slowed down in practice by suppression capacitors to remove stray spikes picked up in the cabling.

Metallic tape, as used in recording tapes, was found to be satisfactory for operating the eddy-current probe. A disc of approximately 4 mm. diameter was the smallest size which could operate the probe and any piece of metal swarf, say, small enough to pass through a 1 mm. gap between probe and surface would be unlikely to cause mal-operation. Collections of small pieces of swarf likewise had no effect. The non-metallic drum which supports the metallic strips was laminated from 'C' sections of the form shown in Fig. 2.20, with alternate pairs mounted at right angles. The transducer could thus be mounted on the motor shaft without removal of bearings. The three tracks, formed by strips and spaces, were \( \frac{1}{8} \)" in width, with \( \frac{1}{4} \)" spacing; the probes themselves were over \( \frac{3}{8} \)" in outside diameter and were therefore staggered round the circumference to reduce the overall length of the transducer.

To illustrate the code in use, Fig. 2.21 shows a developed view of the drum surface as it would appear with the probe axes
FIGURE 2.20  LAMINATION FOR TRANSDUCER DRUM

FIGURE 2.21  DEVELOPMENT OF DRUM SURFACE (HATCHED LINES SHOW POSITIONS OF METALLIC STRIPS)
concurrent. As in the Gray code, each transition involves switching on one track only, and a unity mark-space ratio for each track allows simple translation for $180^\circ$ conduction. As the experimental machine is 4-pole, the code is repeated on the circumference of the drum.

Fig. 2.22 is a photograph of the complete transducer assembly as mounted on the machine. The transducer angle may be altered by rotation of the entire mounting relative to the stator frame and the scale of movement, calibrated in degrees (mechanical), can be seen in the photograph.

The transducer has performed entirely reliably throughout all experiments and it is considered that this type of transducer will be suitable for industrial and traction environments where dust (metallic and otherwise), oil, water and stray fields might be present.

2.7.3 Logic Circuits

The code of Fig. 2.21 can be expressed in logical form, with presence of a metallic strip representing a logic "1" as indicated in Table 2.1. The input signals to the logic elements are clipped from 0-24 volts to 0-5 volts using zener diodes. Three TTL AND gates then restore the fast leading edge removed by the suppression capacitors.

$180^\circ$ Gate Drive

Matching of the gate drive pattern of Table 2.2 to the logic plan of Table 2.1 is obvious and no logic elements are required. Logic "1" on tracks A, B and C fires thyristors 1, 2 and 3 respectively. Inversion of the signals causes logic "0" on these tracks.
**Figure 2.22** Shaft transducer as mounted on machine

**Figure 2.23** Delay formed by monostable and NAND gate
TABLE 2.1  LOGICAL EQUIVALENT OF TRANSDUCER DRUM CODE

<table>
<thead>
<tr>
<th>A</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE 2.2  180° CONDUCTION PATTERN

<table>
<thead>
<tr>
<th>T1</th>
<th>T1</th>
<th>T1</th>
<th>T4</th>
<th>T4</th>
<th>T4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T5</td>
<td>T2</td>
<td>T2</td>
<td>T2</td>
<td>T5</td>
<td>T5</td>
</tr>
<tr>
<td>T6</td>
<td>T6</td>
<td>T3</td>
<td>T3</td>
<td>T3</td>
<td>T6</td>
</tr>
</tbody>
</table>

TABLE 2.3  120° CONDUCTION PATTERN

<table>
<thead>
<tr>
<th>T1</th>
<th>T1</th>
<th>T3</th>
<th>T3</th>
<th>T5</th>
<th>T5</th>
</tr>
</thead>
<tbody>
<tr>
<td>T6</td>
<td>T2</td>
<td>T2</td>
<td>T4</td>
<td>T4</td>
<td>T6</td>
</tr>
</tbody>
</table>
to fire thyristors 4, 5 and 6 similarly. On each of the 6 outputs, a leading edge delay of over 200μs must be provided as commutation of thyristor 1, for example, will only commence when track A goes to logic "0" and thyristor 4 must not be fired until thyristor 1 has been turned off.

A simple delay which depends on a fast-rising input pulse, is provided by the monostable and NAND gate shown in Fig. 2.23; this figure also depicts the operation of the delay, which inverts the driving pulse.

The output stages are driven through 6 open collector drivers, conduction of which indicates the "off" condition.

120° Gate Drive

No delay units are required for the 120° gate drive, the pattern for which is shown in Table 2.3. A small amount of logic is, however, required and is performed by NAND gates; Fig. 2.24 describes the circuits. The gates which invert digits A and B also act as the inversion gates for 180° conduction, but the open collector drivers for the two modes are entirely separate; selection of the conduction angle is carried out at the output stage.

Gate Drive for Turn-off Thyristors

A turn-off thyristor requires a gate drive only long enough to initiate conduction and a differentiating network is incorporated in the output stage for each of these thyristors. Therefore the output stage can be supplied by a much longer pulse, say 180°. For 180° gate drive, therefore, the drive for T4A, for example,
FIGURE 2.24  LOGIC DIAGRAM FOR 120° CONDUCTION

FIGURE 2.25  SELECTION OF 120° OR 180° CONDUCTION FOR TURN-OFF THYRISTORS
will be taken from track A, and that for T1A will be taken from track \( \overline{A} \), the inverse of A. Commutation will occur correctly even if the inverter is run in reverse by opposite rotation of the machine. For 120° conduction, T4A will now be fired 60° earlier, i.e. by track \( \overline{C} \), but in this case commutation for the reserve sequence will not automatically be correct; a reverse running facility with 120° conduction has not been built in, due to the limited use expected of this mode. Again, selection of conduction angle is performed at the output stage. Fig. 2.25 indicates how these gate drives are provided.

**Local Oscillator**

For manual frequency control of the inverter, a local oscillator provides a pulse train variable from 6 to 600 Hz (1 - 100 Hz inverter frequency). Fig. 2.26 shows the oscillator and its succeeding stages. The 6 pulses per cycle, pulse length 90\( \mu \)s, are counted by a TTL 4-bit binary converter, the binary output of which is gated as shown to produce a code identical to that supplied by the shaft transducer. A mechanical 3-pole 4-way rotary switch selects either the oscillator or the transducer as the signal source and also selects the conduction angle.

**2.7.4 Output Circuits**

Power for the thyristor gate drives is supplied on two lines by a 50 kHz square wave oscillator. The voltages on these two lines are in antiphase, i.e. the sum will be d.c. and the amplitude is 24 volts. LA and LB are these output lines in Fig. 2.27, which

* from a design by M.D. Brown
Variable-frequency Oscillator

Binary Generator

Binary to transducer code convertor

FIGURE 2.26 FREE-RUNNING SIGNAL GENERATION
FIGURE 2.27  GATE DRIVE SUPPLY
shows a typical output stage for a main thyristor.

Both of the open-collector transistors in the integrated circuits which supply controlling pulses to the output stage will be receiving switched base drive; one, Tr1A, say, will be receiving 180° drive while the other receives 120° drive. If, for example, 180° drive is selected, the collector supply to Tr1A will be at 24 volts while that for Tr1B will be at zero volts. The converse will apply for 120° conduction. When Tr1A receives no base drive, the Darlington pair formed by Tr1C and Tr1 will conduct and current will flow in both windings of the output transformer. Energy stored in these transformers can be returned to the positive rail via diode D1 when Tr1 cuts off.

The output transformers are wound on toroidal cores with the primary winding encapsulated on the core and the secondary then wound on top. The voltages of the two transformers for each gate are summed by the "OR" gate formed by D2 and D3. With the output capacitor Cl initially uncharged, the full transformer output voltage is applied to the gate. As Cl charges, the gate voltage then falls exponentially to a value limited by the voltage drop in the 60-ohm resistor R1. The oscilloscope trace of Fig. 2.28 shows the full gate drive pulse. Additional components at the output are D4; which prevents a reverse gate voltage of over 0.6 volts, and a 1000-ohm resistor, which helps to prevent spurious thyristor triggering by gate leakage current.

In addition to the 6 circuits described above, 6 similar output stages feed the turn-off thyristors. From Fig. 2.29, it
FIGURE 2.28  THYRISTOR GATE DRIVES

FIGURE 2.29  PART OF GATE DRIVE CIRCUIT FOR TURN-OFF THYRISTOR
can be seen that the only difference is the addition of a differentiating network within the Darlington pair to produce a short output pulse, as seen from Fig. 2.28. The pulse is in fact so short that the 60-ohm output resistor is unnecessary. However, omission of the differentiator would allow the same firing circuit to be used in a parallel-thyristor type of commutating circuit, as shown in Fig. 2.5. Only six open-collector driver stages feed the 12 transistors for the turn-off output circuits; for example, the 120° drive to the output stage of thyristor 1A will have the same phase as the 180° drive to the output stage of thyristor 6A.

2.7.5 Firing Circuit Improvements

The present firing circuit is in "breadboard" form and a modular type of firing circuit is under development for future use. The intention is that facilities should be provided on a basic firing circuit for addition of anticipated future control schemes, for example free-running, closed loop speed control, or shaft control. Other desirable features of a versatile firing system may include availability of pulse width modulation or variable conduction angle modes. The existing circuit has, however, performed reliably although the "breadboard" construction encourages pick-up of interference spikes from other equipment.
CHAPTER THREE

THEORETICAL PERFORMANCE

The phase voltages produced by the thyristor inverter operating on 120° and 180° conduction have been described in Appendix 2. Analysis of the system is simplified by considering performance obtained with only the phase voltage fundamental component present, the effect of voltage harmonics being considered separately.

3.1 Steady State Sinusoidal Approximation

Consider a three-phase synchronous machine with cylindrical stator and rotor. The stator carries a conventional three-phase delta-connected winding and the rotor windings are excited with direct current via sliprings. It is assumed that no harmonic e.m.f.'s are present and that there is no magnetic saturation.

A machine phase can be represented by the equivalent circuit of Fig. 3.1 in which \( v_s \) is the terminal voltage, \( i_s \) is the phase current, \( R_s \) and \( L_{ss} \) are the phase resistance and inductance respectively and \( e_g \) is the e.m.f produced in the stator phase by the rotor current; \( L_{ss} \) includes leakage inductance. Conventional synchronous motor operation at a rotor angle \( \delta \) is described by Fig. 3.2, where -

\[
\begin{align*}
    v_s &= V_m \sin \omega t \\
    e_g &= E_m \sin (\omega t - \delta) \\
    i_s &= I_m \sin (\omega t - \gamma)
\end{align*}
\]

As described in Chapter 1, shaft control of the phase of \( v_s \) fixes \( \delta \) at a pre-set value. Maximum efficiency is attained at unity
FIGURE 3.1 EQUIVALENT CIRCUIT FOR MACHINE PHASE
(SINUSOIDAL QUANTITIES)

FIGURE 3.2 PHASOR DIAGRAM FOR CONVENTIONAL SYNCHRONOUS MOTOR OPERATION
power factor \((\gamma = 0)\) as shown by Fig. 3.3 and the equations for this condition are:

quadrature axis:
\[ E_m \sin \delta = \omega L_{ss} I_m \]  
(3.4)

direct axis:
\[ E_m \cos \delta = V_m - R_s I_m \]  
(3.5)

\(E_m\) is proportional to \(\omega\) and to the d.c. field current \(I_f\), the constant being a mutual inductance \(M_{sf}\)

i.e. \[ E_m = \omega M_{sf} I_f \]  
(3.6)

Equation 3.4 now becomes
\[ M_{sf} I_f \sin \delta = L_{ss} I_m \]  
(3.7)

which is independent of speed. Unity power factor will always be maintained provided that
\[ I_f = I_m L_{ss} / (M_{sf} \sin \delta) \]  
(3.8)

If \(R_s\) is neglected, dividing equation 3.4 by equation 3.5 gives
\[ \omega = V_m \tan \delta / (L_{ss} I_m) \]  
(3.9)

i.e. for constant applied voltage, the speed is inversely proportional to the armature current with resistance neglected. This corresponds to the characteristic of the ideal series d.c. motor.

3.1.1 Separate Excitation

Consider now a rotor with direct and quadrature axis windings producing e.m.f.'s \(e_d\) and \(e_q\) respectively in the stator; the relevant phasor diagram is now as shown in Fig. 3.4 and the equations are:

\[ e_d = E_d \sin \omega t \]  
(3.10)

\[ e_q = -E_q \cos \omega t \]  
(3.11)
FIGURE 3.3  PHASOR DIAGRAM FOR UNITY POWER FACTOR

FIGURE 3.4  PHASOR DIAGRAM FOR UNITY POWER FACTOR
WITH TWO FIELD AXES
Hence \( E_d = V_m - R_s I_m \) \( (3.12) \)

and \( E_q = \omega L_{ss} I_m \) \( (3.13) \)

If \( E_q = \omega M_{sq} I_q \) \( (3.14) \)

then unity power factor will be maintained if

\[ I_q = L_{ss} I_m / M_{sq} \] \( (3.15) \)

Similarly \( E_d = \omega M_{sd} I_d \) \( (3.16) \)

and if \( R_s \) is neglected in equation 3.12,

\[ \omega = V_m / (M_{sd} I_d) \] \( (3.17) \)

\( \omega \) is now proportional to the applied voltage and inversely proportional to an independent control current \( I_d \), corresponding to the case of an ideal d.c. separately excited motor.

3.1.2 Average Supply Current

Equation 3.15 defines the relationship between \( I_q \) and \( I_m \).

With an inverter source, \( I_m \) can be related to \( I_{dc} \), the average value of the inverter d.c. supply current. If each inverter thyristor conducts for up to \( 180^\circ \), each output line is always connected to one of the supply lines and at unity power factor with sinusoidal output currents no feedback diode conduction is necessary. Consider the average value of the positive line current with the reference of \( 0^\circ \) at the peak value \( (\sqrt{3} I_m) \) of the red line current, as in Fig. 3.5. The average value of \( i_+ \) over the interval from \( -\pi/6 \) to \( \pi/2 \) is

\[ \overline{i_+} = 3\sqrt{3} I_m / 2\pi \int_{-\pi/6}^{\pi/2} \cos \theta d\theta + \int_{\pi/6}^{\pi/2} (\cos \theta + \cos(\theta - 2\pi/3)) d\theta \]

\[ \therefore i_{dc} = 3\sqrt{3} I_m / \pi = I_m / 0.604 \] \( (3.18) \)

If each thyristor conducts for up to \( 120^\circ \), unity power factor with sine wave currents can only be obtained with feedback diode
FIGURE 3.5  180° CONDUCTION AT UNITY POWER FACTOR, SINE-WAVE CURRENTS

FIGURE 3.6  120° CONDUCTION AT UNITY POWER FACTOR, SINE WAVE CURRENTS
conduction for a $30^\circ$ interval on either side of the thyristor con-
duction period. From Fig. 3.6, it can be seen that the positive d.c.
rail carries the following currents over a typical $120^\circ$ interval:

- $0^\circ - 30^\circ$ \(i_r\) forward via \(T_1\)
- $30^\circ - 60^\circ$ \(i_r\) forward via \(T_1\)
- $60^\circ - 90^\circ$ \(i_y\) forward via \(T_3\)
- $90^\circ - 120^\circ$ \(i_y\) forward via \(T_3\)
- $+i_r$ reverse via \(D_1\)

The average current over this interval is then

\[
\bar{i}_r = 3\sqrt{3}I_m/2\pi \left[ \int_{\pi/6}^{\pi/3} (\cos \Theta + \cos(\Theta - 2\pi/3))d\Theta + \int_{\pi/3}^{\pi/2} \cos(\Theta - 2\pi/3) + \cos \Theta)d\Theta \right]
\]

\[
\therefore I_{dc} = 3\sqrt{3}(\sqrt{3}-1)I_m/\pi = I_m/0.825 \tag{3.19}
\]

These values establish a link between peak phase current and
average d.c. link current. Equations 3.8 and 3.15 can now be
expressed as

\[
I_r = n\frac{L_{ss}}{M_{sf} \sin \delta} \tag{3.20}
\]

and

\[
I_q = n\frac{L_{ss}}{M_{sq}} \tag{3.21}
\]

where \(n\) takes the value 0.604 or 0.825 as appropriate.

3.1.3 Effect of Power Factor

The voltage waveform for $180^\circ$ conduction is unaltered in shape or
phase by feed-back diode conduction. If the current remains sinus-
oidal but is shifted in phase by an angle \(\phi\) relative to the voltage
waveshape, then feed-back diode conduction occurs over an angle \(\phi\).

Replacing \(\Theta\) by \((\Theta - \phi)\) in the expression for \(\bar{i}_x\) yields
The corresponding case for 120° conduction is less simple since the voltage waveshape alters as feed-back diode conduction periods change with power factor. For example, over the interval from -30° to +90° the positive d.c. rail now carries currents as follows, where $\phi \leq 30°$

-30° to -\phi $i_r$ forward via T1
   +i_b reverse via D5
-\phi to +30° $i_r$ forward via T1
   +i_y reverse via D3
30° to (60° - \phi) $i_r$ forward via T1
(60° - \phi) to 90° $i_y$ forward via T3

Taking the average of $i_+$ as before, for $\phi \leq 30°$,

$$I_{dc} = I_m (\sqrt{3} \cos \phi -1)/0.825$$  \hspace{1cm} (3.23)

If $\phi$ exceeds 30°, the voltage waveform no longer alters with a further increase in $\phi$ and $I_{dc}$ then decreases with $\cos \phi$, with an initial phase shift of 30°.

3.1.4 Supply Current Ripple

The presence of a.c. components in the d.c. link current will affect the quality of the e.m.f. produced by the series winding. Consider first 180° conduction. From -30° to +30° the positive d.c. rail is, say, connected to the red line alone.

i.e. $i_+ = \sqrt{3} I_m \cos(\theta - \phi)$  \hspace{1cm} (3.24)

From 30° to 90° both red and yellow output lines are connected to the positive rail.
i.e. \[ i_+ = \sqrt{3} I_m \left[ \cos(\theta - \phi) + \cos(\theta - \phi - 2\pi/3) \right] \]
\[ = \sqrt{3} I_m \left[ 2\cos(\theta - \phi - \pi/3)\cos\pi/3 \right] \]
\[ = \sqrt{3} I_m \cos(\theta - \phi - \pi/3) \] (3.25)

Equations 3.24 and 3.25 represent the same waveform, shifted in phase by 60°; hence \[ i_+ \] has a period of 60° and the predominant ripple in the supply current is the 6th harmonic.

Evaluating the 6th harmonic Fourier series terms from \[ i_+ \] over a 60° interval for 180° conduction,
\[ 3\sqrt{3} I_m/\pi \left[ \cos \omega t \int_{-\pi/6}^{\pi/6} \cos(\theta - \phi)\cos 6\omega t d\theta + \sin \omega t \int_{-\pi/6}^{\pi/6} \cos(\theta - \phi)\sin 6\omega t d\theta \right] \]
\[ = 3\sqrt{3} I_m/\pi \left[ 0.057 I_m \cos \phi \cos 6\omega t + 0.344 I_m \sin \phi \sin 6\omega t \right] \]
\[ = 0.094 I_m \cos \phi \cos 6\omega t + 0.568 I_m \sin \phi \sin 6\omega t \] (3.26)

Similarly, the expression for the 6th harmonic component with 120° conduction is
\[ 0.351 I_m \cos \phi \cos 6\omega t + 2.11 I_m \sin \phi \sin 6\omega t \] (3.27)

It is evident that the much larger ripple content of a 120° waveform is one reason for rejection of this inverter mode. The high harmonic content of the "castellated" voltage waveform previously mentioned and the variable voltage phase with power factor are additional reasons for employing 180° conduction. Further theory will be concerned with 180° conduction alone.

The actual harmonic content will be considerably reduced by filtering and the series field winding itself will form part of the L-C filter in the d.c. link. In Chapter 2, suitable filter values were found for reduction of 6th harmonic ripple by 90%.

The above theory also indicates that the d.c. link current for 180° conduction is reduced by \[ \cos \phi \] when the power factor is
less than unity. This effect of power factor acting back on the value of the compensating field current increases the sensitivity of the system to changes in compensation.

3.1.5 Leading Power Factor Operation

Operation of the compensated machine at leading power factor might offer several advantages. Inverter currents which were almost self-commutating at unity power factor would be entirely so; speed in the separately excited mode would remain almost constant or even increase with load. However, this condition in which a "negative inductance" exists is only conditionally stable. The phasor diagram of Fig. 3.7 describes leading power factor operation and the following equations are obtained:

\[ E_q = I_m X_{ss} \cos\phi + I_m R \sin\phi \] (3.28)

\[ V_{m_d} - E_q = I_m R \cos\phi - I_m X_{ss} \sin\phi \] (3.29)

If \( k \) denotes a compensation constant which is unity for perfect reactance cancellation, and \( E_q \) is proportional to \( \cos\phi \), (from section 3.1.4), then

\[ E_q = kI_m X_{ss} \cos\phi \] (3.30)

Equations 3.28 and 3.29 can now be rearranged as follows

\[(k-1)X_{ss} = R_s \tan\phi \] (3.31)

and

\[ I_m = (V_{m_d} - E_q) / [R_s \cos\phi (1 - X_{ss} \tan\phi / R_s)] \] (3.32)

From 3.31,

\[ I_m = (V_{m_d} - E_q) / [R_s \cos\phi (1 - (k-1)X_{ss}^2 / R_s^2)] \] (3.33)

If \( k = 1 \), from 3.31, \( \phi = 0 \) and from 3.33, \( I_m = (V_{m_d} - E_q) / R_s \)

Consider a typical ratio of \( X_{ss} : R_s \) of 10 : 1. Equation 3.33 now contains a term \( (1-100(k-1)) \) and an increase in \( k \) to 1.01
FIGURE 3.7 PHASOR DIAGRAM FOR LEADING POWER FACTOR OPERATION
makes $I_m$ theoretically infinite. The phase angle is
\[ \tan^{-1}\left(\frac{(k-1)X_{ss}/R_s}{5}\right) = 5.7^\circ \]

Similarly, for a 1% decrease in $k$, the phase current is halved.

Since $X_{ss}$ is proportional to frequency, the maximum increase in $k$ is also affected by machine speed. Increasing $\phi$ in equation 3.29 can be satisfied by increasing $I_m$. It may also be satisfied by increasing $\omega$ alone if both sides of the equation decrease at the same rate. Generally, both $I_m$ and $\omega$ will therefore increase as $\phi$ increases. Sensitivity of the system to changes in $k$ depends on the size of $L_{ss}$; a smaller value of $L_{ss}$ reduces power factor change with $k$.

3.2 Response to Inverter Waveform

From Appendix 2, the phase voltage between red and yellow terminals is given by
\[ v_{ry} = v_r - v_y = 4 \sqrt{3} V_p/\pi \left[ \cos(\omega t + \pi/6) + 1/5 \cos(5\omega t - \pi/6) - 1/7 \cos(7\omega t + \pi/6) + 1/11 \cos(11\omega t - \pi/6) + 1/13 \cos(13\omega t + \pi/6) + \ldots \right] \]  

(3.34)

where $V_p$ is the peak square wave voltage and is equal to half the voltage between d.c. lines. Since the peak phase voltage is $V_m$,
\[ V_m = 4 \sqrt{3} V_p/\pi \]

The voltage expression in equation 3.34 can be dealt with in three parts.

1. The fundamental component is $v_1 = V_m \cos(\omega t + \pi/6)$
   in which the phase subscript is omitted for clarity.
   This component obeys the sinusoidal approximation described previously. $v_1$ will be opposed by a fundamental back e.m.f. and the difference will drive a fundamental current round a purely resistive circuit if the reactance voltage drop is
perfectly cancelled by the fundamental quadrature axis e.m.f.
The equivalent circuit is shown in Fig. 3.8.

(ii) The 5th and 7th harmonic voltages are

\[ v_5 = V_m \cos(5\omega t - \pi/6) \] (3.35)

and \[ v_7 = -V_m \cos(7\omega t + \pi/6) \] (3.36)

Harmonics of the order \((3n+2)\), e.g. the 5th, rotate backwards and those of the order \((3n+1)\), e.g. the 7th, rotate forwards. Hence both of these components rotate at a speed of \(6\omega\) relative to the rotor. 6th harmonic rotor currents from the a.c. component of the d.c. link current will therefore produce 5th and 7th harmonic e.m.f.'s. in an equivalent circuit as shown in Fig. 3.9. These e.m.f.'s. may amplify or attenuate the harmonic currents depending on the relative phasing and this is considered in section 3.2.1.

(iii) The remaining harmonic voltages (11th, 13th, 17th, etc.) are relatively small and their effect is negligible. In the equivalent circuit of Fig. 3.10, larger values of \(L_{ss}\) or \(\omega\) aid attenuation of currents produced by these voltages.

3.2.1 5th and 7th Harmonic Components

The effect of 5th and 7th harmonic quantities depends on the magnitude and relative phasing of stator currents and e.m.f.'s. The phase of stator e.m.f.'s. produced by rotor currents is determined by that necessary to produce cancellation of the fundamental reactance voltage drop.

The fundamental current will be \([i_1 = I_m \cos(\omega t + \pi/6)]\)

The reactance voltage drop will then be \([j \omega L_{ss} i_1 = \omega L_{ss} I_m \cos(\omega t + 2\pi/3)]\)
FIGURE 3.8  CIRCUIT PER PHASE FOR FUNDAMENTAL

FIGURE 3.9  CIRCUIT PER PHASE FOR 5th AND 7th HARMONICS

FIGURE 3.10  CIRCUIT PER PHASE FOR 11th, 13th etc. HARMONICS
and the cancelling voltage for the fundamental will be

\[ e_{iq} = \omega L_{ss} I \cos(\omega t - \pi/3) \]  

(3.37)

Consider separate excitation at unity power factor. From equations 3.18 and 3.26 the unfiltered d.c. link current is

\[ i_r = 3\sqrt{3}I_m/\pi (1 + 0.057\cos 6\omega t) \]  

(3.38)

Using the total link current in equation 3.21, with \( n = \pi/3\sqrt{3} \),

\[ i_q = (\pi/3\sqrt{3})i_r L_{ss} / M_{sq} \]

The correct phase for the compensation voltage is

\[ e_q = \omega M_{iq} \cos(\omega t - \pi/3) \]

\[ = (\pi/3\sqrt{3})\omega L_{ss} i_r \cos(\omega t - \pi/3) \]

\[ = \omega L_{ss} I_m [1 + 0.057\cos 6\omega t] \cos(\omega t - \pi/3) \]  

(3.39)

The first term of \( e_q \) provides fundamental compensation as required. The second term can be split into 5th and 7th harmonic components. i.e.,

\[ e_{5q} + e_{7q} = 0.057\omega L_{ss} I \cos 6\omega t \cos(\omega t - \pi/3) \]

\[ = 0.028 \omega L_{ss} I_m [\cos(5\omega t + \pi/3) + \cos(7\omega t - \pi/3)] \]  

(3.40)

\( e_{5q} \) and \( e_{7q} \) lead \( v_5 \) and \( v_7 \) respectively by \( \pi/2 \). If the machine is operated in the series mode, the 6th harmonic component of the supply current will also flow in the winding producing direct axis e.m.f. components and 5th and 7th harmonic e.m.f's. in phase with \( v_5 \) and \( v_7 \) will exist.

If the power factor is less than unity, the harmonic e.m.f's. are further split between "direct" and "quadrature" axes.

Appendix 3 yields details of these e.m.f's. for the general case of no d.c. link filtering, series excitation at an angle \( \delta \) (see
equations 3.4 and 3.5) and a power factor for the fundamental component of $\cos \phi$.

5th harmonic "direct axis" voltages (all multiplied by $\cos(5\omega t-\pi/6)$):

$$V_m/5 - \omega L_{ss} I_m (0.172\sin \phi + 0.028\cos \phi / \tan \delta)$$

5th harmonic "quadrature axis" voltages (all multiplied by $\cos(5\omega t-2\pi/3)$):

$$\omega L_{ss} I_m (0.172\sin \phi / \tan \delta - 0.028\cos \phi)$$

7th harmonic "direct axis" voltages (all multiplied by $\cos(7\omega t+\pi/6)$):

$$-V_m/7 + \omega L_{ss} I_m (0.172\sin \phi - 0.028\cos \phi / \tan \delta)$$

7th harmonic "quadrature axis" voltages (all multiplied by $\cos(7\omega t-\pi/3)$):

$$\omega L_{ss} I_m (0.028\cos \phi + 0.172\sin \phi / \tan \delta)$$

These results can be interpreted as follows:

(i) Although a high inductance attenuates harmonic currents, the higher quadrature axis e.m.f. required raises the level of induced harmonic e.m.f.'s. in the same ratio. With adequate filtering, the predominant harmonic e.m.f.'s. will be $V_m/5$ and $V_m/7$ which are unaffected by current and a high inductance is then beneficial. A similar consideration applies to the value of $\omega$ in use.

(ii) The presence of the factor $1/\tan \delta$ can lead to inferior waveshapes in the series mode compared with separate excitation. This applies particularly at low values of $\delta$. 
Extreme values can be taken as $\omega L_{ss} I_m = 500$ volts peak, tan $\delta = 1/\sqrt{3}$ and $\cos \phi = 1$. In this case an unfiltered harmonic current could produce a peak voltage of 24.2 volts, which is of the same order as $V_m/5$ or $V_m/7$. The harmonic current produced, however, will be less than 1A for an $X_{ss}$, say, of 30 ohms. At lower frequencies and fundamental currents, the induced harmonic voltages will be reduced but the harmonic currents will have a comparatively larger effect. $V_m/5$ and $V_m/7$ will therefore be the predominant harmonic voltages in any range where harmonic currents are appreciable. Hence

$$ i_1 = \frac{V_m - E_d}{R_s} \cos(\omega t + \pi/6) - \frac{V_m}{25 \omega L_{ss}} \sin(5 \omega t - \pi/6) + \frac{V_m}{49 \omega L_{ss}} (7 \omega t + \pi/6) \quad (3.41) $$

The product of a harmonic current with the corresponding harmonic e.m.f. induced by rotor currents will give a constant power component of positive or negative value. The product of the harmonic current with another harmonic or the fundamental e.m.f. will, however, produce a time-varying torque. For example, the product of the 5th harmonic current and fundamental e.m.f. for each phase produces 6th and 4th harmonic torques. For all three phases, the 4th harmonic torques sum to zero. Similarly, the 7th harmonic currents and fundamental e.m.f's. produce a 6th harmonic torque, the 8th harmonic components summing to zero.
Harmonic currents lag their driving voltages and will not necessarily be zero when a unity power factor fundamental current passes through zero. These currents may therefore have to be commutated by the inverter at all speeds.

Equation 3.41 provides an estimate of the magnitude of harmonic currents; using these ratios and different values of the fundamental component due to variation of $E_d$, Figs. 3.11 to 3.14 show anticipated current waveforms.

These waveforms are calculated from fundamental and 5th and 7th harmonic components only, as described by equation 3.41. The fundamental component is expressed as a percentage of $V_m/\omega L_{ss}$. Thus Fig. 3.11 shows the fundamental component set at 50% and the resulting current waveform is a good approximation to a sine wave. A decrease in fundamental to 15%, corresponding to a smaller load, is shown in Fig. 3.12. This component is still obviously predominant as the 5th and 7th harmonic amplitudes correspond to 4% and 2% respectively. In Fig. 3.13 the fundamental has been lowered to 7.5% and the current is now passing through zero at intermediate points during the conduction interval; this corresponds to, say, light running of a machine. Finally, the complete absence of a fundamental component is illustrated by Fig. 3.14 and the maxima and minima of the sum of 5th and 7th harmonics are seen where the current peaks are respectively in phase and in anti-phase.

The harmonic currents in fact aid commutation in the case shown since the current passes through zero before its fundamental component does so.
FIGURE 3.11  CURRENT AT 50% FUNDAMENTAL

FIGURE 3.12  CURRENT AT 15% FUNDAMENTAL

FIGURE 3.13  CURRENT AT 7½% FUNDAMENTAL
FIGURE 3.14  SUM OF 5th AND 7th HARMONIC CURRENTS

FIGURE 3.15.  BLOCK DIAGRAM FOR D.C. SHUNT MOTOR
3.3 Transient Performance

The response of a machine to a change in input variables depends on both mechanical and electrical time constants. In a perfectly compensated machine, the presence of the series quadrature field e.m.f. should allow constant flux linkages to be maintained over a transient interval and transient reactance should therefore not exist. At unity power factor, the inverter and compensated machine with separate excitation should operate as a compensated d.c. shunt motor.

3.3.1 Shunt Motor Response

A d.c. shunt motor without complete compensation is described by the following equations, using the Laplace notation $s = d/dt$.

$$
V_a = I_a (R_a + sL_a) + E_a
$$

$$
E_a = K \omega
$$

$$
T = K I_a
$$

$$
\omega = T/(K_d + sJ)
$$

From these equations, the transfer function

$$
\frac{\omega}{V} = \frac{K/JL_a}{s^2 + s(\frac{K_d}{J+R_a/L_a} + (K^2+R_aK_d)/JL_a)}
$$

is derived. The corresponding block diagram is shown in Fig. 3.15. If the mechanical and electrical time constants are denoted respectively by

$$
\tau_m = J/K_d \quad \text{and} \quad \tau_e = L_a/R_a;
$$

$$
\omega = \frac{K/K_d R_a \tau_m \tau_e}{s^2 + s \left(\frac{\tau_e + \tau_m}{\tau} \right) + \left(\frac{K^2 + K_d R_a}{K_d R_a \tau_m \tau_e} \right)}
$$

(3.43)
This is a second order system and may have an oscillatory response to a step voltage input if the damping factor is less than 1.

The expression for the damping factor is

$$\zeta = \frac{(\tau_e + \tau_m) \sqrt{K_dR_a}}{2\sqrt{(\tau_e \tau_m) \sqrt{K^2 + K_dR_a}}}$$

(3.44)

With perfect compensation, $$L_a = 0 \quad \therefore \tau_e = 0$$

The transfer function is now reduced to the simple lag

$$\frac{\omega}{V} = \frac{K}{K^2 + K_dR + \tau R \tau}$$

(3.45)

Over-compensation produces an effective "negative inductance" and the term $$\tau e$$ would be negative. The Routh-Hurwitz criterion can be applied to equation 3.43. The coefficient of $$s$$ will be negative in the case of over-compensation and the system may therefore be unstable with certain values of the system constants.

If compensation of the inverter-machine system is incorrect, however, the power factor will not be unity and the system cannot be described by the simple analogy to a d.c. shunt machine.

In the original "commutatorless motor" analysis described in Appendix 1, the three-phase machine was converted to a two-phase equivalent, assuming that all stator currents and voltages were sinusoidal. Referring stator quantities to the rotor reference frame produces d.c. currents and voltages in direct and quadrature axes. In this case, the compensating field current appears in the direct axis and the control field current in the quadrature axis.

In Appendix 1, the following equations are defined,
\( V_{oc} = R_3^s_{sd} + Ld/dt(3_{sd}) - \omega L_3^q_{sq} + M/d/dt(I_{rd}) + \omega M_3^r_{rq} \)  \hspace{1cm} (3.46)

\( 0 = R_3^q_{sq} + \omega L_3^q_{sd} + Ld/dt(3_{sq}) + \omega M_3^r_{rd} - M/d/dt(I_{rq}) \)  \hspace{1cm} (3.47)

\( I_{rq} \) is the separately excited field current and its derivative will be zero. Correct compensation is arranged by putting \( M_3^r_{rd} = -L_3^q_{sd} \).

Equation 3.47 now reduces to

\( 0 = R_3^q_{sq} + Ld/dt(3^q_{sq}) \)

i.e. \( 3^q_{sq} = \text{constant} \times \exp(-Rt/L) = 0 \) at \( t = \infty \)

In steady state, no d.c. current should alter and \( 3^q_{sq} \) is assumed to be zero. Hence equation 3.46 becomes

\( V_{oc} = R_3^q_{sd} - \omega M_3^r_{rq} \)

as for a perfectly compensated machine.

During a transient interval, stator quantities and compensating field current will change and it is possible that \( 3^q_{sq} \) may exist. Hence all terms in equations 3.46 and 3.47 may exist and, in addition, the presence of \( 3^q_{sq} \) causes a change in power factor which in turn reduces the value of the compensating e.m.f. by \( \cos \phi \).

i.e. \( M_3^r_{rd} = -L_3^q_{sd} \cos \phi \)

\( \cos \phi \) is not constant with time, as it varies with changes in \( 3^q_{sq}/3^q_{sd} \). Thus calculation of the system response is no longer possible from the simple block diagram and a digital computer simulation has instead been employed. Simulation allows a direct comparison of the inverter-machine system performance with performance of an equivalent d.c. shunt machine at different levels of compensation. Results of this analysis are considered in section 3.4.2.
3.4 Computer Simulation

As an adjunct to the simple theory, a dynamic simulation of machine performance with waveform demonstration was performed by digital computer. Programming was carried out in IMP (modified Atlas Autocode) with the facilities of the Edinburgh Regional Computing Centre. At various times between 1971 and 1974, IBM 360/50, 370/155 and 370/158 and ICL 4-75 machines were available.

The lumped resistance and inductance equivalent circuit per phase for the cylindrical machine representation was adhered to for the simulation. The programming may be divided into two sections: in constant-speed programs the input variables were speed, d.c. link voltage and field strength; currents and torques were computed. In variable-speed programs, d.c. link voltage, torque and field strength were input variables and speed and currents were calculated; for the latter case, the torque might be speed-dependent. With separate excitation, the field strength inputs were the compensation value and direct axis field current; with series excitation, the field strength variables were the amount of compensation and the angle $\delta$.

3.4.1 Constant-speed Programming

The flow-chart of Fig. 3.16 describes the calculation of current and torque for various inputs. Approximately 29 cycles were required to allow the values to stabilize with one calculation per degree. 180° conduction was assumed ideal in that commutation between positive and negative thyristors on the same phase was instantaneous. 120° conduction was also represented, with feed-back diode conduction as required by the current direction. Inverter voltage drops and com-
86.

**START**

SET SPEED, D.C. VOLTAGE
DIRECT AXIS E.M.F.
COMPENSATION RATIO

SET INITIAL VALUES OF
CURRENTS, ANGLE, TIME,
SINE FUNCTIONS

SET QUADRATURE E.M.F.

INCREMENT TIME, ANGLE,
SINE FUNCTIONS

CALC. PHASE VOLTAGES
DETECT WHETHER A PHASE
IS CONNECTED TO A SUPPLY
LINE

CALC. INCREMENTS IN
PHASE CURRENTS

CALC. NEW QUADRATURE
E.M.F.

CALC. LINE CURRENTS, INPUT
POWER, MECH. POWER, COPPER
LOSS, D.C. CURRENT

PRINT QUADRATURE E.M.F.

NO

FINAL CYCLE?

YES

PRINT FULL RESULTS

NO

END?

YES

PRINT MEAN INPUT POWER, MEAN
MECH. POWER, MEAN COPPER LOSS
MEAN LINK CURRENT

STOP

See Note (1)

See Note (ii)

---

**NOTES**

(1) For series operation, the direct axis e.m.f. is set to zero and the quadrature axis e.m.f. is displaced in phase.

(11) 120° operation may involve feedback diode conduction. A capacitance at each terminal simulates decay of current in the phase inductance.
mutation transients were neglected.

Constant-speed programming is particularly suitable for full series operation in which the speed range is larger, whereas use of excessive speed increments in the separately excited mode may lead to unrealistic currents and torques. In the series programs, $\delta$ was set at $60^\circ$ and the ratios of 0.604 and 0.825 in equations 3.18 and 3.19 were increased to 0.7 and 0.955 respectively. Fig. 3.17 demonstrates series operation with theoretically correct compensation and $180^\circ$ conduction; d.c. link voltages of 200, 150 and 100 volts are shown. The performance follows closely the ideal case of $\omega \sqrt{T} = \text{constant}$, with a small additional loss of speed due to phase resistance. For example, the speed loss at 200 V, 500 r.p.m. is about 10% below the 550 r.p.m. expected from measuring $\omega \sqrt{T}$ at smaller loads. This point also corresponds to a total computed average power of 8897 W, from which the direct-axis back-e.m.f. is estimated at 134 volts. This is in fact 10% below 156 Volts r.m.s. which is the phase voltage corresponding to 200 Volts d.c. Reproduction of the d.c. machine characteristic is therefore achieved and harmonic currents have not had a noticeable effect on performance although the computed current waveforms show the expected amount of harmonic distortion at small loads.

For comparison, Fig. 3.18 shows three similarly computed curves with $120^\circ$ conduction at the same voltage and transducer angle values; the correct compensation constant of 0.955 is applied. In this case the general shape of the torque-speed curve is similar but the additional speed loss is about 30% at 200 Volts, 500 r.p.m.; this is not accounted for by copper loss alone, which was about 15% of total power in the case of the largest load shown. With current waveform
FIGURE 3.17  SERIES COMPUTED CHARACTERISTICS FOR $\delta = 60^\circ, 180^\circ$ CONDUCTION
FIGURE 3.18 SERIES COMPUTED CHARACTERISTICS FOR $\delta = 60^\circ$, $120^\circ$ CONDUCTION
variation, the 120° voltage waveform also varies and alteration of the fundamental component requires adjustment of the factor 0.955. This was, however, constant throughout these runs. The large waveform variation encountered with 120° conduction is shown by Figs. 3.19 and 3.20. In Fig. 3.19 the load is small and the current is largely harmonic with corresponding distortion of the phase voltage. Fig. 3.20 is taken from a program at very heavy load with harmonic currents negligible in comparison to the fundamental current; the voltage waveform in this case is castellated and symmetrical.

Corresponding current and voltage waveforms from the programs for 180° conduction are shown in Figs. 3.21 and 3.22. In Fig. 3.21 the average torque is small and the current is largely harmonic. Fig. 3.22 is on normal full load and shows an approximately sinusoidal current.

The anticipated instability with excessive over-compensation appeared in constant-speed programs as rapidly increasing current. Instability is, however, more readily studied by variable-speed programming.

3.4.2 Variable-speed Programs

In the variable-speed computations, an initial speed was set, usually 90% of the theoretical no-load speed \( \omega_0 \). Fig. 3.23 is a flow chart for this type of program and in this case 20 cycles were allowed for machine quantities to approach a steady state. Calculation of the e.m.f. produced by the d.c. link current flowing in the compensating windings did not include a term for the voltage induced by the rate of change of this current.
FIGURE 3.19  PHASE VOLTAGE AND CURRENT FOR 120° CONDUCTION, SMALL LOAD

FIGURE 3.20  PHASE VOLTAGE AND CURRENT FOR 120° CONDUCTION, LARGE LOAD
FIGURE 3.21  PHASE CURRENT AND VOLTAGE FOR 180° CONDUCTION, SMALL LOAD

FIGURE 3.22  PHASE CURRENT AND VOLTAGE FOR 180° CONDUCTION, LARGE LOAD
NOTE (1) Increment in supply current may be used to calculate e.m.f. due to rate of change of d.c. current for next cycle.

NOTE (11) This program was not used for series operation or 120° conduction.

FIGURE 3.23
From a selection of variable-speed programs, with theoretically correct compensation, the torque-speed characteristics shown in Fig. 3.24 were obtained for separate excitation. The characteristics are taken to large load torques and remain linear. However, the actual speeds obtained do not agree with the estimated final values calculated from copper loss alone. If the peak value of the fundamental component of the inverter line to line voltage is 220 Volts, the direct axis back e.m.f. will have a peak value of 220 Volts at a speed of $\omega_0$. The phase resistance was taken as 1 ohm and hence the phase current will have a peak value $220 (1-\omega/\omega_0)$. The average torque supplied by all three phases is then given by

$$T = 72900(1-\omega/\omega_0)/\omega_0$$

(3.48)

$T$ may include a term proportional to $\omega$.

In one program, $T$ is constant at 20Nm. If $\omega_0 = 100$ rad/sec, the speed for this torque should be 97.75 rad/sec, but is computed at 93 rad/sec. This speed, applied to equation 3.48 gives $T = 51$Nm.

A speed-dependent component in the torque making $T = 100 + \omega$ should produce a steady-state speed of 75.8 rad/sec but the simulation settles at 72 rad/sec. Similarly $T = 200 + 0.5\omega$ should produce a final speed of 67.9 rad/sec but the computed result gives 55 rad/sec. Setting of the torque in the simulation at zero produced a steady speed of 97 rad/sec rather than the 100 rad/sec expected. Only a 6th harmonic torque of zero average value was obtained and Fig. 3.25 shows the red line current for the 20th cycle of this program.

Line currents in the heavily-loaded simulations were good sine waves but were obviously lagging by about 5°. That inadequate compensation was not the main cause was confirmed by raising the
FIGURE 3.24  COMPUTED "SHUNT" CHARACTERISTIC (180°)

FIGURE 3.25  NO-LOAD TERMINAL CURRENT AND VOLTAGE (180°)
compensation ratio "n" from 0.604 to 0.61 for T = 20Nm. The final speed only rose to 95 rad/sec. A further increase to n = 0.62 brought a continuous speed oscillation.

The greater amount of compensation did improve the power factor but did not restore unity power factor at n = 0.61, even with larger loads. A further unexpected result was the tendency of the speed to overshoot its eventual value at n = 0.604; this contradicts the prediction that perfect compensation should produce a simple lag response. Although harmonic currents are entirely negligible at high loads, the effect of harmonics was demonstrated by repetition of a program with only the fundamental component of voltage present. The speed in this case was closer to the ideal value and unity power factor was approximately attained.

It can be seen that linear theory does not explain the operation of the machine in a dynamic situation. Possibly a "sustained transient" state exists continuously and the quadrature-axis current described in equations 3.46 and 3.47 is not zero, even under steady-state conditions. The response of the simulation is apparently underdamped whether the load torque is constant, speed dependent, or a combination of both. Fig. 3.26 shows a typical result for a constant load torque of 2Nm, compared with the equivalent response using a sine wave source.

Since an electrical time constant was apparently present, an attempt was made to restore the system to a first order response by setting the speed at a constant value; the mechanical constants would therefore disappear. The response of the machine currents again showed an overshoot and thus indicates that linear analysis cannot explain performance with the rectangular voltage waveform input.
FIGURE 3.26  COMPARISON OF SINE AND SQUARE WAVE SUPPLIES

FIGURE 3.27  COMPARISON OF SQUARE WAVE SUPPLY WITH D.C. MOTOR
Simulation therefore remains the most straightforward method of performance prediction.

A check was carried out on the effect of the very small harmonic currents; these should be reduced by an increase in inductance or speed. However, an increase of inductance to 250 mH in the simulation produced a further speed loss compared with 100 mH. Similarly results obtained at \( \omega_o = 333.33 \) were markedly inferior in final speed to those for \( \omega_o = 166.67 \) or \( \omega_o = 100 \). Conversely, a reduction of inductance improves the speed towards the expected value but the machine current waveforms deteriorate as a result of the lower inductance.

An additional check on program operation is the parallel simulation of a d.c. shunt machine without compensation and with parameters equivalent to those of the a.c. machine seen from the d.c. side of a perfect inverter. The inverter is assumed to be operating at unity power factor with sine wave currents. From equation 3.18, the ratio

\[
\frac{I_m}{I_{dc}} = \frac{\pi}{3\sqrt{3}}
\]

For power invariance \( V_{dc} I_{dc} = \frac{3}{2} V_m I_m \)

\[
\therefore \frac{V_m}{V_{dc}} = \frac{2\sqrt{3}}{\pi}
\]

If the d.c. machine armature resistance \( R_a \) is the ratio of d.c. voltage to current while \( R_s \) is the ratio of a.c. voltage to current, then

\[
R_a = R_s \frac{\pi^2}{18}
\]

The equivalent d.c. inductance must store the same energy as its a.c. three-phase counterparts

\[
i.e. \quad L_a I_{dc}^2 = L_{ss} (i_y^2 + i_y^2 + i_y^2)_{av} = \frac{3}{2} L_{ss} I_m^2
\]

\[
\therefore L_a = L_{ss} \frac{\pi^2}{18}
\]
Using the same mechanical constants as in the a.c. machine simulation, the equivalent d.c. machine demonstrated that the system is less than critically damped, as seen from Fig. 3.27. The d.c. machine speed approaches the final value dictated by its copper loss alone. Alteration of the a.c. machine mechanical constants, which were scaled down to shorten computing time, did not affect the final speed.

A number of the above computations were repeated with \( n = 0.606 \). The principal effect was a larger overshoot of current in constant speed cases and of speed in the general case. When the load torque was set at \( T = 200 + 0.5\omega \), the speed still only settled at approximately 56 rad/sec. For the same speed as with \( n = 0.604 \), the currents were larger in this case but remained below the level expected from a resistive voltage drop alone. An increase in \( n \) to 0.607 produced currents closer to the expected value and an improved power factor.

The operation of the machine is therefore improved with the increased compensation, although this is not a suitable solution due to the possibility of unstable operation.

The absence of the compensation e.m.f. due to rate of change of d.c. link current was previously mentioned. Addition of the full value of this term produced an unstable result in the program and rising currents were obtained as the coupling of this voltage into the stator windings was increased. A fixed speed of \( \omega = 300 \) rad/sec at \( \omega_0 = 333.3 \) rad/sec gave the following results
Proportion of additional term  | Peak a.c. line current
---|---
0  | 7.9 A
20% | 8.8 A
50% | 11.5 A

The expected line current for perfect compensation was 38 A. Power factor did not appear to improve with increasing current. Sine wave programming also showed instability with the entire term due to rate of change of link current present. Use of only 50% of this term produced results similar to those of the equivalent d.c. machine. Programming at constant speed with the sine wave input does not instantaneously establish full current since some inductance is still apparently present; the current rise was nevertheless more rapid than in the d.c. equivalent case without compensation.

Specimen listings for both of the basic types of program are given in Appendix 5 and some additional results are shown.

3.5 Conclusion

The basic theory which has been developed in earlier sections indicates that a close approximation to steady-state d.c. series and separately excited commutator motors should be attainable. Harmonic currents are negligible at high speed and load and their effect is readily calculable for commutation duty at crawl speed, for example. Similarly, constant torque components produced by harmonic currents are negligible but a small 6th harmonic time-varying torque will exist with the 180° voltage waveform described. A small additional no-load speed loss due to the currents produced by the phase voltage harmonics has been noted.
Operation of the system at a leading power factor is seen to be very restrictive and unity power factor must be taken as the normal operating condition and also the limit of stability. With a high phase magnetizing inductance, the machine is extremely sensitive to compensation variations and a high quadrature voltage must be produced. A small inductance decreases the necessary quadrature field strength and aids stability with small compensation discrepancies, for example from a proportional field control, if used. Nevertheless, the higher inductance is desirable for attenuation of harmonic currents.

Dynamic performance of the system has been found stable but not first-order and less than critically damped in computer simulations; a simple analogy to the compensated d.c. machine would present a first-order system at unity power factor. Since the system is stable, application to the more heavily-damped case of an experimental machine is feasible and will be considered in Chapter 4. Provided that the mechanical time constant is sufficiently large compared to the electrical time constant, the effect of the quadrature current will appear as a reduction in damping factor, together with a speed loss. On load, machine currents are sinusoidal but lagging and the d.c. link current shows the corresponding asymmetric ripple. Steady-state theory therefore breaks down for this dynamic system since equation 3.31 indicates that \( k = 1 \) (i.e. \( n = 0.604 \)) can only be satisfied by \( \phi = 0 \).

Experimental verification of the simulated performance on a rectangular wave input is necessary to confirm that the machine
does not attain the ideal speed and power factor obtainable with a variable-frequency sine wave source.
4.1 The Laboratory Machine

The theoretical analysis and simulations described in Chapter 3 were tested experimentally where possible on a Multiform Experimental Set (manufactured by Mawdsley's, Ltd.). In this equipment, the inverter-fed machine is rigidly coupled to a conventional d.c. machine which acts as a dynamometer load. Some details of the former machine are as follows:

**Stator Winding**: 48 axial coils in 48 slots of a double-layer winding with coil pitch 1 - 12; all coil ends brought out separately to a terminal board.

**Rotor Winding**: 3-phase 4-pole double layer winding in 18 slots; each phase connected separately via sliprings; coil pitch 1 - 5.

Average air gap = 1 mm.
Core length = 108 mm.

The stator coils were connected to form a 4-pole delta winding with a series-parallel coil arrangement to suit the series rotor winding current rating. In Appendix 4, a trapezoidal approximation (54, 55) is used to estimate the harmonic content of the stator m.m.f. when both layers are identically connected. The estimated harmonic amplitudes vary with angle, but harmonics of order 3n (3rd, 9th, etc.) do not appear. Displacement of the layers through an angle $\gamma$ reduces the nth harmonic by the factor $\cos (n \gamma /2)$ and thus selected harmonics can be reduced. It is evident from Chapter 3 that 5th and 7th harmonics in particular should be
suppressed if possible. Clearly if $\gamma = \pi/5$, the 5th harmonic would disappear altogether and the 7th harmonic term would be changed in sign and reduced to 58.8% of its previous value.

A null of the 5th harmonic appears at $\gamma = n\pi/5$ and of the 7th harmonic at $\gamma = n\pi/7$. If the 5th, 7th, 11th and 13th harmonics are present in the percentages of the fundamental shown in Table 4.1, an optimum reduction can be obtained by finding the minimum value of the sum of the harmonics as $\gamma$ increases.

Fig. 4.1 shows that the best value of $\gamma$ is 28° but that the range $25^\circ < \gamma < 38^\circ$ provides a good all-round reduction. For 5th, or 5th and 7th harmonic components, 36° remains the optimum value of $\gamma$. Displacement of the standard layer winding configurations by 2 slots gives $\gamma = 30^\circ$ and this position was considered satisfactory. In this case, $\cos(5\gamma/2) = 0.259$ and $\cos(7\gamma/2) = -0.259$. Similarly, $\cos(11\gamma/2) = 0.966$ and $\cos(13\gamma/2) = -0.966$.

Since individual rotor coils cannot be reconnected, harmonic reduction is impossible. Two rotor phases provide the quadrature field winding as shown in Fig. 4.2 while the remaining phase carries constant excitation. When the machine is entirely series-excited, the direct-axis winding is open-circuited and the shaft transducer angle is altered to decrease $\delta$ from 90°.

As described in Appendix 4, the rotor phase windings have asymmetrical positive and negative half-cycles in the m.m.f. with a consequently complicated harmonic content.
TABLE 4.1

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>One phase only %</th>
<th>All three phases %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>3rd Harmonic</td>
<td>22.22</td>
<td>0</td>
</tr>
<tr>
<td>5th</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>7th</td>
<td>2.04</td>
<td>2.04</td>
</tr>
<tr>
<td>9th</td>
<td>2.47</td>
<td>0</td>
</tr>
<tr>
<td>11th</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>13th</td>
<td>0.59</td>
<td>0.59</td>
</tr>
<tr>
<td>15th</td>
<td>0.89</td>
<td>0</td>
</tr>
</tbody>
</table>
% of harmonic content @ $\gamma = 0$

**FIGURE 4.1** REDUCTION OF HARMONIC M.M.F. BY DISPLACEMENT OF WINDING LAYERS

Compensating field axis

Control field axis

**FIGURE 4.2** ROTOR PHASE ARRANGEMENT
4.1.1 Machine Constants

The required reactance cancellation can be estimated from open and short circuit characteristics of the machine operating as a separately excited generator without compensation. If stator resistance is neglected, the open circuit voltage for a particular speed and field current will supply the reactance voltage drop due to the short circuit current at the same speed and field current. For example, Fig. 4.3 indicates that the quadrature axis field would have to produce a phase voltage of 123 volts r.m.s. to cancel the reactance voltage drop due to a line current of 4 amps r.m.s. Since the series field voltage constant is $\sqrt{3}$ times the separately excited field voltage constant, the series field must carry 2.8 amps. With the ratio of r.m.s. a.c. line current to d.c. link current of $3\sqrt{2}/\pi$, the ratio of quadrature field winding current to d.c. link current is 1:1.92 for separate excitation. This ratio reaches unity at $\delta = 31.7^\circ$.

If the known phase resistance of 1.65 ohms is taken into account when using the measured voltage drop due to short circuit current, the ratio of currents becomes 1:2.03 and minimum $\delta$ is 29.5°.

4.2 Generation Tests

As a further check on correct reactance cancellation, quadrature field excitation was applied to the machine working as an alternator. If the equivalent circuit per phase is as shown in Fig. 4.4, the open circuit condition allows $e_d$ to be in phase with $v_s$ with $e_q = 0$. Once a load current $i_s$ is established,
FIGURE 4.3  OPEN AND SHORT CIRCUIT TESTS WITH MAIN FIELD EXCITATION
FIGURE 4.4  EQUIVALENT CIRCUIT PER PHASE OF ALTERNATOR
WITH QUADRATURE EXCITATION

FIGURE 4.5  UNITY POWER FACTOR CONDITION FOR LOADED ALTERNATOR
$v_s$ decreases due to the phase impedance voltage drop and $i_s$ lags behind $e_d$. In the alternator, presence of the load-dependent e.m.f. $e_q$ does not affect the steady-state power factor since $e_d$ will simply be replaced by the vector sum of $e_d$ and $e_q$, which will be the new open-circuit value of $v_s$.

$v_s$ is free to shift in phase relative to $e_d$ and thus $i_s$ will still lag $e_d$. However, the value of $v_s$ will have increased for the same value of load current and, for the theoretically correct reactance compensation, the drop in $v_s$ should be due to $R_s$ alone. With both fields excited, the magnitude of the internal e.m.f. will be

$$E = \sqrt{E_d^2 + E_q^2} \quad \text{If} \quad E_q = I_s X_{ss} \quad \text{and} \quad I_s = \frac{E}{\sqrt{(R_s + R)^2 + X_{ss}^2}}$$

then

$$E_d^2 + I_s^2 X_{ss}^2 = I_s^2 [(R_s + R)^2 + X_{ss}^2]$$

Hence the regulation of the alternator supplying a resistive load depends on the phase resistance alone in this case. The essential difference between the motor case and the alternator case is the freedom of $v_s$ to shift in phase relative to $e_d$. The shaft transducer fixes this phase relationship in the inverter-supplied motor and power factor variation is directly controllable.

Theoretically, there might exist a state defined by the vector diagram of Fig. 4.5 in which $v_s$ and $i_s$ are in phase. It has been found that two-axis excitation of an alternator does give improved transient performance but steady-state performance is not affected. Direct-axis compounding of alternators is well-known, but quadrature-axis compounding is of more recent development.
If an alternator supplies a constant d.c. load current via a bridge rectifier, the alternator reactance causes a delay in conduction changeover between phases.\textsuperscript{(57,58)} This overlap cannot be overcome by quadrature axis excitation for the reasons previously mentioned.

Using externally applied compensation of the theoretical value, voltage regulation tests were performed. In Fig. 4.6, the regulation of terminal voltage with load current is shown to be linear for the theoretically correct value of quadrature-axis excitation, i.e., the impedance \( \sqrt{R_s^2 + X_{ss}^2} \) has been reduced to the value \( R_s \). Also in Fig. 4.6, the effect of over-excitation of the quadrature field is shown and it can be seen that the curve of terminal voltage against current does not reach its minimum value at zero current.

Let \( E_q = k I_s X_{ss} \) and \( V_s = I_s R \). If \( k > 1 \), it can be found from equation 4.1 that \( V_s < E_d \) provided that \( (k^2 - 1)X_{ss}^2 < R_s^2 + 2RR_s \).

As \( R \) decreases, i.e., the current rises, \( V_s \) may become equal to or greater than \( E_d \).

4.3 Separate Excitation Tests

The machine was operated here as a motor with a constant inverter terminal voltage and constant direct-axis field current. The instrument arrangement is shown in Fig. 4.7. All d.c. quantities are measured with moving coil meters. The moving iron meter which measured a.c. current should give a true r.m.s. reading at its designed frequency and hence the value of the fundamental component is a variable fraction of the actual reading \textsuperscript{(59,60)}. In accurate assessment of the a.c. voltage, however, a standard dynamometer...
FIGURE 4.6 ALTERNATOR VOLTAGE REGULATION CURVES
FIGURE 4.7 EXPERIMENTAL INSTRUMENT ARRANGEMENT
voltmeter had to be used as a moving iron meter was insufficiently precise. Provided that the power factor is close to unity, and some load is applied to the machine, a reasonable estimation of the a.c. fundamental current will be given. For 180° conduction, the voltage waveshape has a constant ratio of 0.96 between r.m.s. value and the fundamental r.m.s. value. Dynamometer wattmeters give a good estimate of total power and should give approximately equal readings at unity power factor when the fundamental current is high compared to harmonic currents. The mechanical torque balance attached to the d.c. load machine was insufficiently accurate and an improved estimate of shaft torque was obtained from the dynamometer output power; a no-load torque estimated from operation of the dynamometer as an unloaded motor was added to the results. Speed was measured with a mechanical tachometer.

Fig. 4.8 illustrates the effect of voltage variation on the machine speed with constant field current; the direct axis field current was maintained at 4 amps and speed-torque lines are shown for inverter d.c. terminal voltages of 75, 100 and 125 volts. As expected, speed remains nearly constant and the characteristics are linear. The speeds also reduce in roughly the same ratio as the applied voltages, again corresponding to the d.c. motor case. In the computer simulations described in Chapter 3, a speed drop at no load was noted and this can also be seen in the experimental case. Projection of the torque-speed characteristics back to zero torque produces speeds below the values at which the fundamental voltage and the direct axis back e.m.f. would be equal. The no-load speed loss may be as high as 20% rather than 10% as expected from programming.
FIGURE 4.8 SEPARATE EXCITATION LOAD CHARACTERISTICS FOR 4A CONTROL FIELD CURRENT
Also in agreement with the simulations, the currents obtained are below the values which would be expected from subtraction of the open-circuit generated phase voltage from the inverter-supplied phase voltage. If these voltages were considered equal at the experimentally estimated no-load speed, then the actual currents obtained are close to the expected values with phase resistance alone present. For example, the d.c. voltage of 125 volts produces an a.c. phase voltage of 93 volts, 5 volts being dropped in the inverter. If the machine is running at 990 r.p.m., and its no-load speed is 1080 r.p.m. then the a.c. line current should be 8.3 A r.m.s. but is found to be 7.2 A r.m.s. This current is of the correct order of magnitude and the result of this relationship is that the torque-speed characteristic will be approximately linear as desired. The speed will nevertheless be lower than in the ideal machine with a purely resistive phase impedance. Additional load-varying effects may occur due to saturation and inverter impedance variation.

If the separately excited machine is operated at constant voltage, speed should be inversely proportional to the direct axis field current. Fig. 4.9 shows three characteristics for a constant d.c. voltage of 125 V and field currents of 3, 4 and 6 A were selected. The speed/field current relationship is approximately as expected.

A comparison of the compensated and uncompensated cases is shown in Fig. 4.10. The speed of the uncompensated machine can be seen to fall off rapidly with increasing torque. This is not of course a true performance comparison since the uncompensated
FIGURE 4.9  SEPARATE EXCITATION LOAD CHARACTERISTICS FOR 125V D.C.
FIGURE 4.10 COMPARISON OF COMPENSATION - SEPARATE EXCITATION, 125V D.C., 4A FIELD
machine would normally operate at some displacement of the shaft transducer, as described in Chapter 1. The torque-speed curve for 50% compensation is also shown on Fig. 4.10. This has the same form as the zero-compensation curve but has improved speed regulation.

Over-compensation has been described as a conditionally stable state, more easily obtained with lower speed. A d.c. voltage of 50 V and direct axis field current of 4A produce the torque-speed curve shown on Fig. 4.11. Figs. 4.10 and 4.11 can be compared in shape with Fig. 4.6 and in particular it can be seen that both of the over-compensated cases show a drooping curve; in Fig. 4.11 the speed falls below the no-load value before rising again.

Referring back to equation 3.29, it can be seen that

\[ V_m > E_d \text{ provided that } R_s \cos \phi > X_{ss} \sin \phi \]

From equation 3.31,

\[ (k-1)X_{ss} = R_s \tan \phi \]

Hence

\[ R_s^2 > (k-1)X_{ss}^2 \]

This is the condition for which the speed will be below the no-load speed and does not occur at zero load. As k increases, or the approximate value of \( \omega \) increases, the point of minimum speed moves towards the zero load condition. This in itself forms a stability criterion, indicating that the minimum speed point may represent a limit of dynamic stability. Beyond such a point, the speed will continue to increase with load.

For comparison, Fig. 4.12 shows a torque-speed curve for separate excitation with 120° conduction. The theoretical compensation value was again employed and the result is very much inferior to the 180° case.
FIGURE 4.11  OVER-COMPENSATION SHOWING SPEED DROOP.
SEPARATE EXCITATION 50V D.C., 4A FIELD
FIGURE 4.12  120° CONDUCTION SEPARATE EXCITATION CHARACTERISTICS
4.4 Series Excitation Tests

The rotor phase which had carried direct axis control field current was an open circuit during series operation. Rotation of the transducer mounting ring lowered $\delta$ from $90^\circ$ and produced a direct axis e.m.f. The compensation was similarly increased by a factor $1/\sin\delta$ up to the value of $\delta = 29.5^\circ$ when the total link current flowed through the quadrature field. Characteristics were obtained at $\delta = 75^\circ$ (weak main field), $60^\circ$, $45^\circ$ and $29.5^\circ$ (strong main field).

Fig. 4.13 shows speed-load curves for $\delta = 75^\circ$ and $\delta = 60^\circ$; the d.c. voltages were 60 and 80 Volts respectively and these curves are almost identical. Comparison of $\omega\sqrt{I}$ at various points along the curves shows a close approximation to the ideal series motor characteristic. Similar curves for $\delta = 45^\circ$ and $\delta = 29.5^\circ$ are shown in Fig. 4.14 at 100 and 80 Volts d.c. respectively. From the known compensation constant, d.c. link current and open circuit characteristic, the value of the direct axis back e.m.f. component can be estimated. Comparison of phase currents with the calculated values from $I_m = (V_m - E_d)/R_s$ shows good agreement in this case.

Fig. 4.13 also shows the effect of leading power factor on series performance. The correct compensation has been exceeded by 34% yet there is little difference in the speed-torque curve. Since speed falls sharply with load in the initial stages of the series characteristic, leading power factor operation will be stable over a larger range than with separate excitation. Current wave-shapes improve with increasing fundamental component and with increasing speed. Poorer wave-shapes may therefore be obtained with series excitation since the speed falls sharply with increasing load.
Figure 4.13  Series excitation characteristics, 180° conduction. $\delta = 60^\circ$ and $\delta = 75^\circ$.
FIGURE 4.14  SERIES EXCITATION CHARACTERISTICS, 180° CONDUCTION.  δ = 45° AND δ = 29.5°
4.4.1 Waveshape Variations at $\delta = 29.5^\circ$

Alteration of the waveshape of phase current when full compensation is applied gives a slight variation in the experimental results not apparent from the torque-speed curve. The phase current waveform is a poor representation of a sine wave as shown in Fig. 4.15(a). A further waveshape change occurs over an intermediate section of the operating range and the altered shape is shown in Fig. 4.15(b). As the open-circuit characteristic indicates, saturation may affect the value of the compensation e.m.f. but a small increase in $\delta$ gave an apparent phase shift rather than a waveshape improvement. In the torque speed characteristic of Fig. 4.14, the altered waveshape appeared in the approximate speed range from 550 to 450 r.p.m.

4.5 System Efficiency

The inverter efficiency, as would be expected, is high and experimentally determined values lie in the range 88 - 97%.

The speed loss of up to 20% occasioned by the voltage harmonics is not accompanied by a large loss in efficiency in either the simulations or the experimental tests; the effect is similar to that of slightly reduced compensation. The machine efficiency is reasonably constant when fully compensated but rather below the value expected from deduction of copper loss alone. The no-load loss of the set is high (mainly mechanical losses) and the overall efficiency is therefore around 50%. 2 Volts brush drop and copper loss in the d.c. load machine are allowed for. An estimate of the no-load loss in the machine was made for the purpose of projecting
(a) Normal waveforms

(b) Anomalous waveforms appearing over part of range

FIGURE 4.15 PHASE CURRENT AND VOLTAGE FOR $\delta = 29.5^\circ$
torque-speed curves for separate excitation back to the true no-load condition. Using these approximations, the system efficiency for full compensation rises to around 80%. The power lost in converting the d.c. input power into “airgap power” has three main components (i) Inverter conduction and commutation loss; (ii) Machine copper loss; (iii) Loss due to apparent drop in power factor at full compensation. All three components have similar magnitudes at full load (around 7% of input power).

A further machine efficiency loss occurs due to the apparent drop in power factor over the small range previously described for \( \delta = 29.5^\circ \).

4.6 Experimental Waveforms

The effect of harmonic currents at small fundamental current is seen in Fig. 4.16 in which the upper trace is the phase current through a shunt of 0.05 ohms. The resemblance to Fig. 3.13 indicates that the 5th and 7th harmonic currents are lagging 90° behind the 5th and 7th harmonic components of the voltage waveform also shown. The 11th and 13th harmonic currents are small in magnitude but will be almost in antiphase with the 5th harmonic at the instant of peak fundamental current. Hence their presence in the oscilloscope trace causes slight lowering of the second current peak. A higher level of fundamental current is illustrated by Fig. 4.17 and the close approximation to a sine wave is evident.

As expected, the current is noticeably less sinusoidal with full series excitation and saturation does not appear to contribute to this effect. The speed is relatively low on full load and
FIGURE 4.16  PHASE CURRENT AND VOLTAGE (NO-LOAD), 180° CONDUCTION

FIGURE 4.17  PHASE CURRENT AND VOLTAGE (FULL-LOAD), 180° CONDUCTION
reactance will therefore have a smaller effect on harmonic attenuation. Fig. 4.18 shows phase current and voltage at $\delta = 45^\circ$ and this can be compared with Fig. 3.21. Fig. 4.19 depicts typical waveforms for $\delta = 29.5^\circ$ and comparison of Figs. 4.18 and 4.19 shows the large difference in waveshape occurring at the smaller angle.

An almost sinusoidal current can be provided by $120^\circ$ conduction if the load is sufficiently large, as shown by Fig. 4.20, which was recorded during separately excited operation with a small field current. Fig. 4.21 on the other hand shows the phase current on light load and the fundamental current is almost undetectable within the very large 5th and 7th harmonic currents. The symmetrical castellated voltage waveform of Fig. 4.20 has been almost completely converted to a rectangular waveform in Fig. 4.21.

The waveform of d.c. link current indicated a small lag up to the compensation ratio at which instability occurred. This further confirmed the results of the computer simulation.

4.7. Transient Performance

The response of the separately excited machine to a step change in a control signal is severely limited in the experimental case by the supply impedance. The series field and filter impedances are included in this. Examination of the response has therefore been limited to the effect of a step in d.c. link voltage caused by the sudden short circuit of a large additional d.c. link resistance. As mentioned in Chapter 3, the response at non-unity power factor does not correspond with an equivalent d.c. machine; however, the correctly compensated machine was compared with slightly over and
FIGURE 4.18  PHASE CURRENT AND VOLTAGE ON LOAD, 180° CONDUCTION
($\delta = 45^\circ$)

FIGURE 4.19  PHASE CURRENT AND VOLTAGE ON LOAD, 180° CONDUCTION
($\delta = 29.5^\circ$)
under-compensated cases.

Fig. 4.22(a) shows the response to a voltage rise of 25 volts from an initial value of 75 volts d.c. The upper trace shows the d.c. tachometer output rising with a speed change from 695 to 870 r.p.m. Normal compensation was applied here. The lower trace is the d.c. link current over the transient period, indicating from the steady ripple content that no power factor change occurs during acceleration. The experimental machine constants are as follows:

L_{ss} (full value) = 54mH  \quad R_s = 1.65 \text{ ohms}  \quad J = 0.1175 \text{ kgm}^2

K is variable by main field control and is difficult to estimate. Previous results have shown that the estimated no-load speeds are below the calculated values and that inverter voltage drops should be allowed for. The d.c. voltage of 75 volts should create a fundamental phase voltage of 55.5 volts r.m.s. At 695 r.p.m., the calculated back e.m.f. is 51.5 volts r.m.s. On a percentage basis, the corresponding d.c. back e.m.f. would be 70 volts, giving

\[ K = \frac{E_a}{\omega} = 0.48 \text{ V/} \text{rad/sec.} \]

K_d is normally controlled by d.c. load machine excitation but in this case the machine was unexcited. From the torque required to rotate the set, K_d is estimated at 0.0035 Nm/\text{rad/sec.} The d.c. equivalent of R_s is 0.9 ohms and hence the overall time constant estimated from equation 3.45 is

\[ \tau = \frac{R_s J}{(K_d^2 + K R_s)} = 460 \text{ ms.} \]

In Fig. 4.22(a), the speed rises from approximately 79.8% (i.e. 1.6\tau) to about 100%. An estimation of the time constant is
FIGURE 4.22(a) SPEED AND D.C. CURRENT RESPONSE TO STEP VOLTAGE (FULLY COMPENSATED)

FIGURE 4.22(b) SPEED AND D.C. CURRENT RESPONSE TO STEP VOLTAGE (UNDER-COMPENSATED)
therefore 2.4 cm., i.e. 480 ms. This is extremely close to the estimated value, indicating that the practical response does conform to that of an equivalent compensated d.c. shunt motor. This does not necessarily mean that the machine is operating in the ideal fashion described by steady-state theory. The practical machine constants are such that the mechanical time constant is dominant and damping is very much larger than in the simulation. The speed overshoot is then absent and the response approximates to the desired characteristic.

A 5% decrease in the ratio of compensation current to total d.c. current produced the response shown in Fig. 4.22(b). The voltage rise in this case was from 78.5 to 104 volts d.c. and was accompanied by a speed rise from 670 to 830 r.p.m. The time constant has now increased to about 960 ms, and the effect of the power factor change can be seen in the large ripple content of the d.c. current in the lower trace.

Fig. 4.22(c) shows the effect of 8% over compensation and is the limit to which compensation could be increased at this voltage setting. The principal effect is the overshoot in speed, which rose from 720 to 970 r.p.m. before returning to 920 r.p.m. This shows that the system appears to change from an overdamped second order system through a single lag to an underdamped second order system as the compensation rises through the correct value. It may be seen that the d.c. current ripple appears to be lower at a leading power factor. This effect is again due to the lagging power factor obtained at correct compensation.
FIGURE 4.22(c) SPEED AND D.C. CURRENT RESPONSE TO STEP VOLTAGE (OVER-COMPENSATED)
4.8 Discussion of Results

The experimental system has been tested and the predicted characteristics are effectively verified. The speed loss with separate excitation between light running and full load is small and is linear with respect to speed. Similarly, the series characteristic is as close to the ideal case as that expected from a comparable d.c. commutator motor. The large speed loss from the theoretical no-load speed with separate excitation is in agreement with the results of the simulations and does not appear to be due to current harmonics; respectable sine waves were obtained from both simulated and experimental cases. Filtering may therefore not affect this speed loss. As the no-load speed decreases, theoretical and experimental characteristics become closer although the current waveforms deteriorate.

Harmonic currents are larger than expected in the series mode, as it was assumed that filtering would dispose of appreciable rotor harmonic currents. The factor \( \frac{1}{\tan \delta} \) which appears in the expression for induced e.m.f's. due to these currents may have an effect on the case of \( \delta = 29.5^\circ \), where there is an additional wave-shape change over a restricted range of the torque-speed curve with a corresponding loss of efficiency. The characteristic is apparently unaffected but the r.m.s. a.c. current is increased over this interval. In spite of the poorer current waveforms, series operation gives satisfactory results and the relatively low speed at full load is very close to the theoretical value.
The system response to a step voltage change is an excellent replica of its theoretical d.c. equivalent. The less satisfactory nature of over and under-compensation has been seen, although these cannot be compared with a d.c. equivalent due to the presence of quadrature currents.
5.1 Achievements of the Project

The variable-speed drive which has been described operates most effectively at its maximum load and thus the use of discontinuous voltage waveforms does not increase the frame size or call for special modes of construction. A power factor of unity allows an increase in rating of up to 41% for a given inverter supply over the rating for a comparable induction motor drive and a substantial reduction in the duty cycle of the inverter commutation circuit may also be effected.

The experimental behaviour of the compensated machine and inverter conforms to that of a compensated d.c. commutator motor in both shunt and series modes. Compound excitation has not been examined but will be entirely stable provided that, as in all cases, the correct compensation is not exceeded. As with a d.c. commutator motor, there is a limit to commutation but in this case the limit is independent of speed. The machine has also been tested with a step voltage input and found entirely suitable for a system requiring a fast response. Both the experimental system and the compensated d.c. machine will possess an external d.c. line inductance formed by the series winding; for complete compensation, these inductances will be of comparable size.

The advantage of a high phase inductance in attenuating harmonic currents has been described but the extreme sensitivity of the power factor to compensation changes at high values of $X_{ss}$ was also noted.
The method of controlling the d.c. supply voltage depends on the particular application and the experimental system has been tested on rectified single-phase a.c. Operation is basically similar but waveforms are considerably less sinusoidal than with three-phase since the filter size was not increased. If a single-phase a.c. supply was used in practice, a large reservoir capacitance would be necessary.

5.2 Principal System Uses

Industrial usage of the inverter and compensated motor is likely to be limited to a narrow range of power requirements. The inverter and induction motor with appropriate control loops may do the work of a d.c. motor in an environment where commutator maintenance would be a problem. Beyond this range, the available inverter rating is exceeded with an induction motor but an extension of the rating is available with the compensated synchronous type of motor. This machine still uses brushes, but it will be remembered that the entire armature current need not flow in the compensating field, whereas the d.c. machine brushes carry (and commutate) the armature current. An example of a duty where a great deal of d.c. motor brush replacement occurs is in railway traction. Starting at full torque and continuous slow-speed running have all but eliminated squirrel cage induction motors from such duty. Machine dimensions are strictly limited, making an armature speed increase desirable for greater horsepower, and high full-load efficiency is of paramount importance. Present traction motor speeds are not high; for example, a 200 km/hr unit with 1.65 m. wheel diameter and a 4:1 gear ratio is typical.
The frequency for a 4-pole machine would be 130 Hz and a 500 kW inverter drive might be feasible. However 500 kW is still only about half the rating of the largest d.c. traction motors in "straight" (i.e. mains-supplied) electric traction. Hence present thyristor technology has not advanced to the supply of such units without the use of multiple devices in parallel. A diesel-electric unit is, on the other hand, much heavier than a "straight" electric unit and at present requires at least four axles to carry a 2500 h.p. unit at a maximum of 20 tons per axle. The horsepower per axle now comes down to approximately the limit of present inverters. Nevertheless, application of the system to a lower horsepower unit, for example in a rapid transit power car, is more probable; motor speeds will be as high as in main line operation and motor dimensions are even more restricted in such a vehicle.

Traction systems operating from fixed supply voltages require some form of continuous voltage control but a chopper or controlled rectifier produces considerable harmonic distortion in the supply current. Prevention of interference with adjacent circuits is essential and multiple converters provide one solution. As mentioned earlier, a rectifier and p.w.m. inverter form a suitable system for lower speeds but the standard 180° inverter has the possible advantage of commutation spike reduction as speed increases, thus limiting a further source of interference.

Grant has proposed a list of factors involved in selecting the drive for a particular application. These factors are

(a) First cost
(b) Efficiency
The selection of priorities from this list depends on the purpose and about half of these factors might constitute constraints in any industrial drive. The remaining factors would be matters of choice or, more probably, a function of equipment availability. Almost all the listed considerations will, however, apply to a traction drive and the d.c. commutator motor has given the best "all-round" performance of any machine. The compensated machine and inverter cannot at present compete on first cost but should be at least the equal of a standard d.c. motor in every other factor.

5.3 Economic Comparisons

The squirrel cage induction motor is the most cheaply constructed of the more common types of three-phase machine. The synchronous motor with two rotor windings will be less expensive than a wound
rotor induction motor which will in turn be cheaper than a commutator motor of the same rating.

The ratio of commutator motor cost to squirrel cage motor cost is variable but can be taken as four to one for larger machines still within the power range of semiconductor control gear. A typical price for an industrial three-phase squirrel cage induction motor is £5 per kW. Thus a d.c. commutator motor will cost about £20 per kW in larger sizes (over 100 h.p., say). Wound-rotor slipring machines will come somewhere between these values, depending on the number of separate windings and the slipring current loading. £12 per kW is a reasonable estimate for the compensated machine.

Inverter prices continue to reflect development costs and at an estimated price of £100 per kVA represent about three times the cost of individual components. Without a voltage controller, a price of £75 per kVA would be typical. Brown and Jones\(^{(65)}\) suggest that the provision of the d.c. supply would be about 40% of total cost but this is presumably based on component costs alone; 25% is a more realistic estimate based on the difference of price between an inverter and the more common rectifier plus chopper or controlled rectifier. These costs are represented in Fig. 5.1.

The economy of the compensated system depends on peripheral benefits such as reduction of commutation circuit size. Comparative costs produced by Brown and Jones and by Ludbrook\(^{(66)}\) give commutation component costs as around 30% of total inverter cost. If the problem of zero-frequency starting (see section 5.4) is otherwise overcome, and component cost is taken as proportional to rating,
FIGURE 5.1  COMPARATIVE COSTS

A. Inverter + Voltage controller at p.f. 1.0
B. Inverter + Induction motor at p.f. 0.7 (standard system)
C. Inverter alone at p.f. 1.0
D. Inverter + Compensated machine (estimated standard system)
   - not including voltage or field controllers
E. Voltage Controller + d.c. Motor (standard system)
F. D.c. Motor alone
G. Compensated machine alone (estimated)
H. Cage-rotor induction motor alone
then the commutation component cost may be around 1% of total cost. If the induction motor drive operates at a minimum power factor of 0.707, the system cost of inverter plus machine becomes around £111 per kW compared with the £20 per kW of the d.c. machine alone. However, complete drive systems can be obtained at substantially lower prices than the individual components; for the three-phase squirrel cage induction motor and fixed voltage inverter, £80 per kW is an approximate price.

The compensated system operating around unity power factor requires an inverter costing around £53 per kW if the commutation component cost is taken as 1% instead of 30%. The system cost is therefore £65 per kW and is 58.5% of the cost of the corresponding induction motor system if the latter has to operate at maximum torque.

Comparison with the d.c. commutator motor is of course less favourable. Adding £14 per kW to allow for supply of variable voltage d.c. from three-phase mains a.c., the comparative costs are £34 per kW for the d.c. system and £79 per kW for the compensated system. The ratio of 2.3 however, compares favourably with the induction motor system cost difference and it must be remembered that the induction motor system was 28% cheaper than the sum of individual item prices. Checking of d.c. system figures does in fact show that the commutator motor plus controller as a complete system is about £24 per kW, a 29% reduction on the above figure. Applying the same reduction to the compensated system, the total cost is now £56 per kW.

In conclusion, abolition of the mechanical commutator in favour of sliprings gives an estimated 130% increase in system cost.
Removal of the sliprings by using an induction motor drive represents a 290% cost increase on the d.c. machine system with the added disadvantages that efficiency may be lower and that further control loops may be required to replicate the d.c. motor characteristics.

Reduction of slipring current loading may be achieved by load-dependent separate excitation of the compensating field. The voltage controller required might add only £5 per kW to the cost of a small air-gap machine.

A summary of system costs is given by Table 5.1.

5.4 Suggestions for Further Work

An extension of the theoretical investigation is of primary importance. No cause has yet been established for the no-load speed loss evident in both computer output and experimental tests. Although a power factor deficiency is obviously present and cannot be corrected by a compensation increase, the linear characteristic of the d.c. shunt motor has been reproduced by the compensated machine. Simulation of a sine-wave variable frequency source has produced improved results but the ideal speeds and currents defined by an equivalent d.c. shunt motor have not been obtained. More detailed simulation may provide the answer to the above discrepancies. A further anomaly occurring only in the experimental case at $\theta = 29.5^\circ$ with maximum compensation will also require further study.

At effectively zero speed, the machine reactance can be neglected and the current waveform will be a replica of its driving voltage. Hence on starting the machine, the inverter must commutate blocks of d.c. current. A traction or screwdown drive operates on full load.
**TABLE 5.1**

<table>
<thead>
<tr>
<th>Motor Type</th>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
<th>Column D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Induction Motor</td>
<td>0.04</td>
<td>0.82</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>p.f. = 0.707</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Induction Motor</td>
<td>0.03</td>
<td>0.61</td>
<td>0.73</td>
<td>-</td>
</tr>
<tr>
<td>p.f. = 0.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.C. Motor</td>
<td>0.16</td>
<td>-</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>Compensated Motor</td>
<td>0.1</td>
<td>*0.48</td>
<td>*0.58</td>
<td>*0.62</td>
</tr>
</tbody>
</table>

* Allowing for reduction in commutation capability

Column A: Motor alone

B: Motor operating from d.c.

C: Motor operating from controlled d.c.

D: Motor operating from controlled d.c. with compensating field controller

Base cost = £136 per kW

**FIGURE 5.2  PHASE CURRENT AND VOLTAGE, 120° CONDUCTION, δ = 45°**

(CURRENT FORCING)
current from standstill and a commutation circuit capable of turning off peak load current must be provided. At the low frequency of starting, the commutation duty in terms of power output would be small and at higher speeds, when the circuit duty cycle would be greater, much smaller commutation pulses would be needed. One line of approach is the development of a commutation circuit capable of large currents at a low duty cycle with automatic reduction of pulse size with increasing frequency. It is preferable that some energy should be prevented from reaching the commutation capacitor as opposed to dissipation of part of the stored charge between commutations. At very low frequencies, commutation might be performed by removal of the d.c. link current, a method which was previously rejected for higher frequencies.

Two possible control systems might be investigated. The first, for industrial purposes, involves an asynchronous run-up followed by "locking in" to a high-stability oscillator to give synchronous performance at a given speed. The shaft transducer provides an indication of rotor angle and loss of synchronism may be rapidly overcome since the inverter can relapse into the asynchronous mode as required.

Secondly, a traction drive with one field winding providing excitation and compensation will require field control to overcome wheelslip and provide stages of field-weakening. Combined electronic phase shifting of the inverter voltage waveform and maintenance of correct compensation will give an effective variation of direct axis field strength.
One basic assumption made for the foregoing work was that the voltages generated by the machine would be sinusoidal and that inverter currents would have to basically fundamental sine waves to provide the desired operation. The inverter would be operating in a "constant voltage" mode. If a machine provided a more trapezoidal e.m.f. for reactance cancellation, the corresponding stator current might in fact be a rectangular wave and the inverter might operate in a "constant current" mode. Farrer and Miskin\(^{(67)}\) have described current forcing in an induction motor drive with a high series d.c. link inductance. It is interesting to note that current forcing was recorded during compensated series operation with \(\delta = 45^\circ\) at \(120^\circ\) conduction when the system was first being studied in that mode. The result shown in Fig. 5.2 can be compared with those provided by Farrer and Miskin and further investigation of this occurrence might prove valuable.

Improved performance in the lower speed range may be obtained by conversion of the inverter to p.w.m. operation. This also allows the voltage on the d.c. link to be uncontrolled but the frequency range is restricted. At full speed the additional commutation duty of the p.w.m. system is wasteful and there is little advantage over the performance of the square wave system; at standstill, the same commutation problem of starting on a d.c. current still exists. Perhaps by a reduction in commutation frequency with speed, culminating in a \(180^\circ\) waveform, p.w.m. might be applicable for a fixed bus system; voltage reduction for starting might be accomplished by free-running chopping of the thyristor current.
A return to the double three-phase configuration (Appendix 1) might be contemplated and the behaviour of this type of inverter has been analysed. The greater number of thyristors involved could only be justified by a return to 120° conduction, provided that a suitable inverter connection could be devised to make use of the improved voltage harmonic content without recourse to "castellated" waveforms. The "self-commutating" nature of sine wave currents with 180° conduction is, however, lost at smaller conduction angles.

Apart from inverter developments, work remains to be done on the machine and on the overall system. The machine used in the experimental work is part of a laboratory demonstration set and as such can operate in many modes without being typical of any particular kind of machine. In a fully designed system, the form of inverter to be used would be first selected. On the basis of harmonic content over the operating speed range, the machine air gap size and lamination thickness would then be decided; for wide range operation, a small air gap, high-inductance machine appears desirable. If the machine available has a high-impedance series rotor winding, a proportional and integral controller would have to be designed to create a field current proportional to the average d.c. link current. With such a controller, the series field inductance will be eliminated.

The experimental machine seemed unaffected by the unsymmetrical rotor conductor configuration but a machine with a true two-phase rotor winding is obviously preferable in further system evaluation. More sophisticated instrumentation and a "stiff" mains supply would be further aids. A machine with three-phase stator and two-phase rotor has recently been located and it is hoped that the machine will shortly become available to extend the work of this thesis.
APPENDIX 1

A.1.1. Double Three-phase Inverter Systems

The voltage waveforms produced by three-phase inverters can be improved by coupling of the outputs of two inverters with suitably displaced firing. For example, the sum of two six-pulse systems (e.g. three-phase bridge inverters) will give a twelve-pulse system if the firing of corresponding thyristors is displaced by 30° between inverters (see Fig. A.1.1). Combination of the waveforms may be achieved in a machine or transformer in which the three-phase systems have a 30° spatial displacement.

Both series and parallel connections of the inverters are possible. Wilson (52) has analysed the performance on passive loads of both arrangements as shown in Fig. A.1.2; an interphase transformer was used to improve operation in the parallel case. The voltage waveform for 120° conduction altered with the load phase angle. Constant voltage waveforms were found to occur with conduction angles of 150° or larger. On resistive or inductive loads, the double three-phase configuration produced no line voltage harmonic lower than the 11th.

The waveform obtainable from a highly inductive load on 120° conduction or any load on 150° conduction is shown in Fig. A.1.3.

The phase change in voltage with 120° conduction is a maximum of 15° in double three-phase compared with 30° in single three-phase; the corresponding r.m.s. voltage change is similarly reduced from a maximum of 15.5% to 3.5%. The supply current ripple frequency is also doubled and its amplitude halved in a double three-phase system.
FIGURE A.1.1  TYPICAL DOUBLE-THREE-PHASE CONFIGURATION

FIGURE A.1.2  SERIES AND PARALLEL INVERTER ARRANGEMENTS

FIGURE A.1.3  VOLTAGE WAVEFORM FOR 120° CONDUCTION, HIGH-INDUCTANCE LOAD OR 150° CONDUCTION, ALL LOADS (DOUBLE 3-PHASE)
Short-circuiting of the machine or transformer windings with $180^\circ$ conduction is mentioned\(^{(69)}\) as a disadvantage of this conduction angle. There may not be a similar problem with an active load.

A.1.2. Double Three-Phase Commutatorless Motor System.\(^{(36)}\)

The work from which the compensated machine system arose was carried out in 1968-69 at the University of Edinburgh under the supervision of Mr. J. Shepherd. The two 120°-conduction three-phase bridge inverters were arranged to supply two separate "star" windings on the machine as shown in Fig. A.1.1. The windings were arranged to provide maximum coupling between systems. Each inverter had feed-back diodes and commutation components as described in Chapter 2. The twelve thyristors required a 4-digit binary code produced by a photo electric transducer for translation into twelve unique firing signals. Current sharing between the inverters is aided by coupling of the d.c. supply chokes in an interphase transformer. The subsequent analysis of the double three-phase system\(^{(52)}\) did not include performance with a machine load.

Mathematical Analysis.

A theoretical prediction of machine performance was based on the assumption that currents and voltages were sinusoidal and the three-phase machine quantities were converted to a two-phase equivalent.\(^{(68)}\) By referring all stator quantities to the rotor reference frame, all currents and voltages become d.c.

\[
\begin{align*}
(U_{sd}) & = R (I_{sd}) + L \left( \frac{d}{dt} \right) (I_{sd}) + M \left( (p' - \omega) I_{rd} \right) \\
(U_{sq}) & = R (I_{sq}) + L \left( \frac{d}{dt} \right) (I_{sq}) + M \left( (p' - \omega) I_{rq} \right)
\end{align*}
\]

where $p' = \frac{d}{dt}$. 
$U_{sd}$ and $U_{sq}$ are the stator voltages referred to the rotor; since
the applied voltage is being used as a reference, there is no quad-
rature component and $U_{sd}$ is a constant denoted by $V_{oc}$. $R$, $L$ and $M$
are the resistance, inductance and stator-rotor mutual inductance
respectively of each phase of the two-phase machine, $I_{sd}$ and $I_{sq}$
are the stator currents referred to the rotor. The transducer angle
is variable to cope with reactance voltage drop and, depending on the
reference used, either rotor current or stator voltage components
will exist in both axes.

Experimental Results.

The agreement between predicted results and experimental results
with the sinusoidal theory was poor. No computer simulation was
performed in this case. By suitable transducer setting and field
control, the machine could run at lagging, leading and unity power
factors.

The most relevant results were the speed-torque curves for
various values of transducer setting. The transducer angle is
considered zero when the starting torque is a maximum, i.e. when
applied and generated voltages are in opposition. Speed-torque
curves were obtained for various excitation levels at several different
values of transducer angle. As the angle was moved further from
zero, these curves bore a greater resemblance to the characteristic
of a series d.c. motor and similar results were published in 1972
by Magureanu.\textsuperscript{(35)}

Results obtained at a constant speed showed that a peak torque
was obtained at a particular value of excitation and that increase
or decrease of excitation about this point produced a reduction in torque.

Machine phase voltages exhibited both six and twelve-step waveforms as shown in Fig. A.1.4. Leading, lagging and unity power factor conditions could be obtained and the effect of leakage inductance can be seen in the minor current peaks on the current trace of Fig. A.1.4(b).
(a) Leading p.f.

(b) Lagging p.f.

(c) Unity p.f.

FIGURE A.1.4  (Note: Voltage traces reversed)
180° and 120°-Conduction Voltage Waveforms with Unity Power Factor Sinusoidal Currents.

The current produced in an active load by an inverter output voltage at unity power factor does not necessarily have the same waveform as the current in a resistive load. The compensated machine, for example, has a power factor of one for the fundamental component and a power factor of almost zero for harmonic components. Harmonics are also severely attenuated at higher speeds and the resulting current is a sine wave rather than a replica of the voltage waveform. This does not affect the voltage waveform for 180° conduction, as shown in Fig. A.2.1.

The voltage between an inverter terminal and a point midway between d.c. line potentials of unit voltage, as in Fig. A.2.1, has the Fourier cosine expansion.

\[ F(\omega t) = \frac{4}{\pi} \left[ \cos \omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \frac{1}{7}\cos 7\omega t + \frac{1}{9}\cos 9\omega t - \frac{1}{11}\cos 11\omega t + \frac{1}{13}\cos 13\omega t - \ldots \right] \]

A delta-connected winding has the inverter line to line voltage applied to each phase, as shown in Fig. A.2.2. The series for the delta phase voltage is then

\[ F(\omega t) = \frac{4\sqrt{3}}{\pi} \left[ \cos \omega t + \frac{1}{5}\cos (5\omega t - \pi/6) - \frac{1}{7}\cos (7\omega t + \pi/6) + \frac{1}{11}\cos (11\omega t - \pi/6) + \frac{1}{13}\cos (13\omega t + \pi/6) + \ldots \right] \]

The voltage between an inverter terminal and the d.c. voltage midpoint for a resistive load is shown below (Fig. A.2.3). As previously described in Chapter 2, this waveform is converted to
the castellated shape also shown when the line current is sinusoidal. The resistive-load waveform obviously has the same frequency spectrum as the 180° conduction line-to-line voltage. The castellated equivalent, however, is expressed by

\[ F(t) = \frac{2}{\pi} [1.464\cos\omega t + 0.667\cos3\omega t - 1.093\cos5\omega t + 0.781\cos7\omega t - 0.222\cos9\omega t - 0.133\cos11\omega t + 0.113\cos13\omega t + 0.133\cos15\omega t + \ldots] \]

The magnitudes of the 5th and 7th harmonics are particularly notable. The line-to-line voltage formed by the difference between two castellated voltages is shown in Fig. A.2.4. The Fourier series for the phase voltage is now

\[ F(\omega t) = 2\sqrt{3}/\pi [1.464\cos(\omega t + \pi/6) - 1.093\cos(5\omega t - \pi/6) - 0.781\cos(7\omega t + \pi/6) + 0.133\cos(11\omega t - \pi/6) + 0.113\cos(13\omega t + \pi/6) + \ldots] \]

Both the waveforms shown in Fig. A.2.3 alter in shape with changing power factor, tending towards the 180° equivalent as the load phase angle increases. At unity fundamental power factor with sine wave currents, the 5th harmonic voltage component is 75% of the fundamental value with 120° conduction; this compares most unfavourably with the 20% obtained with 180° conduction.

In Appendix 1, double three-phase inverter operation was described. Application of symmetrical castellated waveforms displaced by 30° to a coupling device produces the "90° conduction" waveform shown in Fig. A.2.5. This will, with load phase angle change, move to the waveform shown in Fig. A.2.6, with a 30° phase shift. In this case, both waveforms have harmonic contents in the ratio 1/n, where n is odd.
APPENDIX 3

Consider first the 5th harmonic quantities; using the same reference as for the fundamental case,

\[ v_5 + e_{5d} = R_s i_5 + j5\omega L_{ss} i_5 + e_{5q} \]

For almost all the speed range, \( R_s \) can be neglected in comparison with \( 5\omega L_{ss} \).

In this case,

\[ V_m/5\cos(5\omega t - \pi/6) = j5\omega L_{ss} i_5 + 0.028\omega L_{ss} \ I \cos(5\omega t + \pi/3) \]

\[ i_5 = 1/(5\omega L_{ss}) \left[ -jV_m/5 - 0.028\omega L_{ss} \ I \right] \cos(5\omega t - \pi/6) \]

\( e_{5q} \) opposes reactance cancellation and will tend to increase the lag angle of \( i_5 \) with respect to \( v_5 \).

A similar expression can be obtained for the 7th harmonic current under the same conditions.

\[ i_7 = 1/(7\omega L_{ss}) \left[ jV_m/7 + 0.028\omega L_{ss} \ I \right] \cos(7\omega t + \pi/6) \]

Additional components of the harmonic e.m.f.'s produced by rotor currents will occur with power factor change and series excitation.

Effect of Power Factor

When the power factor is non-unity, equation 3.39 is modified to

\[ e_q = \omega L_{ss} \ I \left[ 1 + 0.057\cos\phi\cos6\omega t + 0.344\sin\phi\sin6\omega t \right] \cos(\omega t - \pi/3) \]

\( e_{5q} \) and \( e_{7q} \) are now reduced by a factor \( \cos\phi \) and \( e_{5d} \) and \( e_{7d} \) now exist.

\[ e_{5d} = 0.344\omega L_{ss} \ I \sin\phi\sin(5\omega t + \pi/3) \]

\[ = 0.344\omega L_{ss} \ I \sin\phi\cos(5\omega t - \pi/6) \]

and \( e_{7d} = -0.344\omega L_{ss} \ I \sin\phi\cos(7\omega t + \pi/6) \)
These terms therefore act against \( v_5 \) and \( e_5 \) with lagging power factor and aid \( v_5 \) and \( e_5 \) with leading power factor. As large excursions from unity power factor are not desired, the terms are unlikely to exceed the values of \( e_{5q} \) and \( e_{7q} \).

**Effect of Series Excitation**

When the machine is operating with full series excitation, additional 5th and 7th harmonic e.m.f's. will be induced in the stator due to the 6th harmonic component in the direct axis field current. At an angle \( \delta \),

\[
e_d = \omega L_{ss} I_m / \tan \delta [1 + 0.057 \cos \phi \cos \omega t + 0.344 \sin \phi \sin \omega t] \cos (\omega t + \pi/6)
\]

This contains terms

\[
0.5 \omega L_{ss} I_m / \tan \delta [0.057 \cos \phi \cos (5 \omega t - \pi/6) - 0.344 \sin \phi \cos (5 \omega t + \pi/3)]
\]

and

\[
0.5 \omega L_{ss} I_m / \tan \delta [0.057 \cos \phi \cos (7 \omega t + \pi/6) + 0.344 \sin \phi \cos (7 \omega t - \pi/3)]
\]
If each layer of each phase of the stator winding has a 60° spread, the m.m.f. per phase may be represented by the trapezoidal approximation shown in Fig. A.4.1. Fourier series expansion of this waveform gives harmonic quantities as defined in the first column of Table A.4.1.

If three phases comprising one layer contribute to the total m.m.f., the sum of the trapezoids will vary over the cycle. For example, Figs. A.4.2 and A.4.3 show the results of two different positions of the phase vectors; the harmonics vary in sign but not in magnitude over the cycle, as seen from Table A.4.1 (2nd and 3rd columns).

Evidently, addition of an identical second layer does not alter the harmonic magnitudes. However, if the layers are displaced through an angle $\gamma$, as in Fig. A.4.4, a new total waveform is produced. Clearly, addition of two identical waveforms separated by an angle $\gamma$, e.g. $F(\theta-\gamma/2)$ and $F(\theta+\gamma/2)$, produces a result in which the nth term of $F(\theta)$ is multiplied by $2\cos(n\gamma/2)$.

Hence a sine series, describing, for example, Fig. A.4.4, is

$$F(\theta) = \sin\theta \cos \gamma/2 + 1/25\sin5\theta \cos5\gamma/2 - 1/49\sin7\theta \cos7\gamma/2$$
$$- 1/121\sin11\theta \cos11\gamma/2 + 1/169\sin13\theta \cos13\gamma/2 \text{ etc.}$$

The rotor coil configuration is less straightforward and is not symmetrical. Figs. A.4.5 and A.4.6 describe coil configurations used to supply control and compensation fields. Sine and cosine terms will be present in the sum of two phases and even harmonics are expected due to the asymmetry. A rigorous analysis has not been carried out since the trapezoidal shapes are tolerable and the coil arrangement is not variable, unlike that of the stator.
FIGURE A.4.1  
Phase A

FIGURE A.4.2  
\[ A = \sqrt{3}/2 \]
\[ B = -\sqrt{3}/2 \]
\[ C = 0 \]

FIGURE A.4.3  
\[ A = 1 \]
\[ B = -\frac{1}{2} \]
\[ C = -\frac{1}{2} \]

FIGURE A.4.4  SUM OF TWO DISTRIBUTIONS AS A.4.3 DISPLACED BY \( \gamma \)
TABLE A.4.1

<table>
<thead>
<tr>
<th>Ratio A:B:C</th>
<th>$1:0:0$</th>
<th>$\sqrt{3}/2:1-\sqrt{3}/2:0$</th>
<th>$1:1/2:1-1/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harmonic order</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$3/2$</td>
<td>$3/2$</td>
</tr>
<tr>
<td>3</td>
<td>$2/9$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>$1/25$</td>
<td>$-3/50$</td>
<td>$3/50$</td>
</tr>
<tr>
<td>7</td>
<td>$-1/49$</td>
<td>$3/98$</td>
<td>$-3/98$</td>
</tr>
<tr>
<td>9</td>
<td>$-2/81$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>$-1/121$</td>
<td>$-3/242$</td>
<td>$-3/242$</td>
</tr>
<tr>
<td>13</td>
<td>$1/169$</td>
<td>$3/338$</td>
<td>$3/338$</td>
</tr>
<tr>
<td>15</td>
<td>$1/225$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Rotor coil arrangement

FIGURE A.4.5
Control field (C)

FIGURE A.4.6
Compensating field (A & B)
APPENDIX 5

Program Listings and Additional Results

The following two listings are typical of programs used for system simulation. Both are in IMP language.

The first program corresponds with the flow chart of Fig. 3.16 and is arranged for 180° conduction, separate excitation. 120° conduction can be obtained by switching of VA etc. at the angles at which VAT etc. are ± 0.5 rather than zero. Series excitation is accomplished by increasing PH, adjusting COMPK and setting VP_MAX at zero.

The second listing is of a dynamic simulation for 180° conduction with separate excitation, as in Fig. 3.23. DRY etc. define terms due to quadrature axis voltages produced by the rate of change of link current. Since the full values of DRY etc. are used, the result of this particular program will be unstable as described in Chapter 3.

Further results from the latter type of program follow the listings.
Constant-speed Program

BEGIN
REALSLONG
PRINTTEXT' CONSTANT VOLTAGE SYNCHRONOUS INVERTER'
PRINTTEXT' 180 DEGREES CONDUCTION'
NEWLINES(2)
PRINTTEXT' PRESET LINK VOLTAGE = 2*VLINK ='
REAL VLINK
VLINK = 40
PRINT(VLINK,4,0)
NEWLINE
REAL IAB, IBC, ICA, LPH, AL, RPH, AR, CN, AC, VPMAX, PH, ACOSPH, ASINPH, VQMAX
REAL COMFK, IA, IB, IC, ILINK, AD, AE, VAT, VBT, VCT, VFAB, VFBC, VFCA, VQAB
REAL VQBC, VQCA, VQFAB, VQFAB, PM, PF, PC, IS
REAL PERIOD, PULSATANCE, T, DT, O, DO, TIMING, AA, ASIN, ACOS, VA, VB, VC
INTEGER X, Y, Z, M, L
REAL N
N=1000
80:
PRINTTEXT' PRESET MACHINE SPEED 4-POLE R-P-M( = N='
PRINT(N,4,0)
PERIOD=30000/N
PULSATANCE=0.209406*N
T=0; O=0
DT=PERIOD/600; DO=360/600
TIMING=PERIOD/6.28318
AA=DT/TIMING
ASIN=0; ACOS=1
NEWLINES(2)
PRINTTEXT' DELTA WINDINGS NODE VOLTAGES VA, VB, VC'
VA=0; VB=0; VC=0
NEWLINE
PRINTTEXT' PHASE CURRENTS IAB, IBC, ICA'
IAB=0; IBC=0; ICA=0
NEWLINE
%PRINTTEXT' INDUCTANCE PER PHASE = LPH (MH) ='
LPH=100
PRINT(LPH,5,0)
AL=DT/LPH
NEWLINE

%PRINTTEXT' RESISTANCE PER PHASE = RPH ='
RPH=1
PRINT(RPH,5,2)
AR=RPH*DT
NEWLINE

%PRINTTEXT' NODE CAPACITANCE = CN (MILLI F) ='
CN=0.03
PRINT(CN,1,4)
AC=DT/CN
NEWLINES(2)

%PRINTTEXT' MAX. E.M.F. FOR FIXED FIELD ='
%PRINTTEXT' PULSATANCE*FIELD CONSTANT = VMAX ='
VMAX=VLINK*2.2
PRINT(VMAX,4,0)
NEWLINES(2)

%PRINTTEXT' PHASE (DEGREES) OF VPAB W.R.T. COS'
%PRINTTEXT' (+30 IS NORMAL) = PH ='
PH=30
PRINT(PH,4,1)
ACOSPH=COS(PH*E/180) : ASINPH=SIN(PH*E/180)
NEWLINE

%PRINTTEXT' MAX. E.M.F. FOR VARIABLE QUAD FIELD'
%PRINTTEXT' PULSATANCE*LPH ILINK*CONSTANT/1000=VQMAX'
ILINK=0
AD=0.866 ; AE=0.5
Y=0
COMPK=0.604
IS=5
IA=0 ; IB=0 ; IC=0
VQMAX=PULSATANCE*LPH*IS*COMPK/1000
NEWLINE
%PRINTTEXT FOR FIRST CYCLE VQMAX ='
PRINT(VQMAX,6,0)
PC=0
PP=0
ILINK=0
%CYCLE M=1,1,1802
%CYCLE L=1,1,10
T=T+DT; 0=0+DO
ASIN=ASIN+ACOS*AA; ACOS=ACOS-ASIN*AA
IS=0
VAT=ACOS
VBT=-AE*ACOS+AD*ASIN
VCT=-(VAT+VBT)
VA=VA-I*AC
VB=VB-IB*AC
VC=VC-IC*AC
->11 %IF VAT>0
->12 %IF VAT<0
->11 %IF VA>VLINK
->12 %IF VA<VLINK;->13
11: VA=VLINK; IS=IA; ->13
12: VA=-VLINK
13: ->21 %IF VBT>0
->22 %IF VBT<0
->21 %IF VB>VLINK
->22 %IF VB<VLINK;->23
21: VB=VLINK; IS=IS+IB;->23
22: VB=-VLINK
23: ->31 %IF VCT>0
->32 %IF VCT<0
->31 %IF VC>VLINK
->32 %IF VC<VLINK;->33
31: VC=VLINK; IS=IS+IC;->33
32: VC=-VLINK
33: ILINK=ILINK+IS*DT
VQMAX=(PULSATANCE*LPH*ILINK*COMPK)/(T*1000)
VPAB=VPMAX*(ACOSPH*ACOS*ASINPH*ASIN)
\[ VPQAB = VPMA \times (\sin \phi \cos + \cos \phi \sin \phi) \]
\[ VPBC = -(VPAB \times AE + VPQAB \times AD) \]
\[ VPQA = -(VPAB + VPBC) \]
\[ VQAB = VQMAX \times (\sin \phi \cos + \sin \phi \cos \phi) \]
\[ VQPAB = VQMAX \times (-\cos \phi \cos + \sin \phi \sin \phi) \]
\[ VQBC = -(VQAB \times AE + VQPAB \times AD) \]
\[ VQCA = -(VQAB + VQBC) \]
\[ IAB = IAB + (VA - VB - VPAB - VQAB - IAB \times RPH) \times AL \]
\[ IBC = IBC + (VB - VC - VPBC - VQBC - IBC \times RPH) \times AL \]
\[ ICA = ICA + (VC - VA - VFCA - VQCA - ICA \times RPH) \times AL \]
\[ IA = IAB - ICA \]
\[ IB = IBC - IAB \]
\[ IC = ICA - IBC \]
\[ PM = (VPAB + VQAB) \times IAB + (VPBC + VQBC) \times IBC + (VFCA + VQCA) \times ICA \]
\[ PP = PP + PM \times DT \]
\[ PC = PC + (IAB \times IAB + IBC \times IBC + ICA \times ICA) \times AR \]
\[ \rightarrow 41 \text{ %IF } \cos < 0.95 \]
\[ X = 1; \text{ X = 1 %IF } \sin < 0; Z = X - Y \]
\[ \rightarrow 42 \text{ %IF } Z < 2 \]
\[ \rightarrow 60 \text{ %IF } M < 1730 \]
\[ \text{NEWLINES}(2) \]
\[ \%PRINTTEXT'ACTUAL PERIOD = '; PRINT(T, 4, 2) \]
\[ \text{NEWLINE} \]
\[ \%PRINTTEXT'MEAN LINK CURRENT ILINK =' \]
\[ PRINT(ILINK/T, 4, 2) \]
\[ \text{NEWLINE} \]
\[ \%PRINTTEXT'MEAN MECHANICAL POWER ='; PRINT(PP/T, 6, 0); \text{PP = 0} \]
\[ \text{NEWLINES}(2) \]
\[ \%PRINTTEXT' COPPER LOSSES ='; PRINT(PC/T, 6, 0); \text{PC = 0} \]
\[ \text{NEWLINE} \]
\[ \%PRINTTEXT' INPUT POWER =' \]
\[ PRINT((2*VLINK*ILINK/T); 4, 0) \]
\[ \text{NEWLINE} \]
\[ \%PRINTTEXT' VQMAX FOR NEXT CYCLE =' \]
\[ PRINT(VQMAX, 6, 0) \]
\[ \text{NEWLINE} \]
NOTE: \[ \mathcal{E} = \pi \]
Variable-speed Program

$\%$BEGIN
$\%$REALS LONG
$\%$REAL R,L,DTL
$\%$REAL VDC,IDC,WDC,EMTORDC,AEDC,LTORDC,EMTOR,LTOR
R=1
L=100
DTL=L/(R*10)
$\%$REAL VS,VP,VR,VM,VB,AVDF,AVGF,WO,W
VS=100
VDC=2*VS
VP=VS
VR=VP
VM=-VP
VB=-VP
WO=333.3
W=300
WDC=W
AVDF=2.2*VS/WO
AEDC=VDC/WO
$\%$REAL IS,IP,IR,IY,IB,IRY,IYB,IBR,II,DRY,DRY,DER
II=0
IS=0
IP=0
IDC=0
IRY=0;IYB=0;IBR=0
$\%$REAL ACOS,ASIN,A,A,AY,AB,ARY,ABR
ACOS=1;ASIN=0;A=2*$\pi$/360
$\%$INTEGER J,K,M,N,Z,Q,S,U
$\%$REAL ADT,DTR,DT,AL,ACF,ALF,T
T=0
ADT=2*$\pi/1000/360$
$\%$REAL VDF,VDFR,VDYB,VDJFR
$\%$REAL AVDFR,AVDFYB,AVDFR
$\%$CYCLE N=1,1,20
S=0;U=0;NEWLINE
%CYCLE \theta=1,1,360
ACOS=ACOS-ASIN*A
ASIN=ASIN+ACOS*A

->1 %IF ACOS<0; J=1; VR=VP; \rightarrow2
1: J=-1; VR=-VP
2: AY=ACOS*0.5+ASIN*0.866

->3 %IF AY<0; K=1; VY=VP; \rightarrow4
3: K=-1; VY=-VP
4: AB=(ACOS+AY)

->5 %IF AB<0; M=1; VB=VP; \rightarrow6
5: M=-1; VB=-VP
6: ARY=ACOS*0.866-ASIN*0.5

ABR=(ARY+ASIN)

VDF=W*AVDF

VDFRY=VDF*ARY

VDFYB=VDF*ASIN

VDFBR=VDF*ABR

Q=1

DTR=ADT/W

10: DT=DTR/Q

->11 %IF DT<DTL
Q=Q+1; \rightarrow10

11: AL=DT/L; ACF=DT/CF; ALF=DT/LF

AVCF=W*L*0.0006046

AVCFRY=(-AB)*AVCF

AVCFYB=(-ACOS)*AVCF

AVCFBR=(-AY)*AVCF

12:

DRY=0.6046*II*ARY

DYB=0.6046*II*ASIN

DBR=0.6046*II*ABR

IRY=IRY+(VR-VY-IRY*R-VDFRY-IS*AVCFRY)*AL+DRY

IYB=IYB+(VY-VB-IYB*R-VDFYB-IS*AVCFYB)*AL+DYB

IBR=IBR+(VB-VR-IBR*R-VDFBR-IS*AVCFBR)*AL+DBR

IDC=IDC+(VDC-AEDC*WDC-IDC*\$*\$*R/18.0)*AL*18.0/(\$*\$)

IR=IRY-IBR
IY=IYB-IRY
IB=IBR-IYB
II=IP
IP=0
IP=IP+IY %IF K=1
IP=IP+IB %IF M=1
IS=IP
II=IP-II
T=T+DT
Q=Q-1
->12 %IF Q>0
S=S+1
EMTOR=IRY*(VDFRY+IS*AVCFRY)
EMTOR=EMTOR+IYB*(VDFYB+IS*AVCFYB)
EMTOR=EMTOR+IBR*(VDFBR+IS*AVCFRB)
EMTOR=EMTOR/W
LTOR=2
W=W+(EMTOR-LTOR)*DTR*0.018
EMTORDC=AEDC*IDC
LTORDC=2
WDC=WDC+(EMTORDC-LTORDC)*DTR*0.018
->20 %UNLESS S=10
S=0; U=U+10
NEWLINE
PRINT(U,3,0)
PRINT(T,6,2)
PRINT(IP,6,1)
PRINT(IR,6,1)
PRINT(W,6,2)
PRINT(EMTOR,6,0)
PRINT(((EMTOR*W),6,0)
PRINT(((IRY*IRY+IYB*IYB+IBR*IBR)*R),6,0)
PRINT((2*VS*IS),6,0)
PRINT(IDC,6,1)
PRINT(WDC,6,2)
20: %REPEAT
%REPEAT
%ENDOFPROGRAM
ACKNOWLEDGEMENTS

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The following pages contain the text of a Paper describing portions of the Thesis work. This Paper has been accepted for a Conference on "Power Thyristors and their Applications" to be held at the Institution of Electrical Engineers, Savoy Place, London, in December, 1974.
AN INVERTER DRIVE SUITABLE FOR TRACTION

B D McLeod, A Renfrew, and J Shepherd.

1. INTRODUCTION

The rising costs of manufacture and of maintenance for d.c. machines directs attention toward the competitive development of thyristor inverters coupled to a.c. machines. Although inverter/induction motor drives are well established these combinations may not be ideal. The lagging power factor of an induction motor load adds costs and losses to the supply inverter while frequent starting and slow speed running give thermal problems in induction machines.

A load current of sinusoidal form at unity power factor despite a supply voltage of rectangular waveform reduces inverter commutation and smoothing problems. A synchronous motor may be adapted to provide an inverter with this load current (1,2) and to be thus significantly more suitable for inverter coupling than an induction motor. The synchronous motor requires a shaft-position transducer to control thyristor triggering; this transducer may be made maintenance free and completely reliable by the use of proximity detectors with no sliding contacts. Slip-rings and rotor d.c. windings are required but these disadvantages are countered by the synchronous machine copper losses being around half the losses in an induction machine providing the same torque at low speed.

The inverter/synchronous motor unit has the self-starting properties and the shunt or series characteristics of the d.c. machine to which it is so closely equivalent.

2. INVERTER

For unity power factor operation, 'sine' waves and the near natural commutation, each thyristor must be arranged to give 180° conduction. This requirement necessitates that each commutation circuit is not connected to or controlled by the firing of another main thyristor. This can be achieved by employing well known circuit arrangements as used in chopper or pulse-width modulated inverters.

A d.c. link inverter supplied by a controlled rectifier is employed in this case, Fig.1. The thyristors in each leg of the inverter bridge are fired 180° apart with a small delay to ensure satisfactory thyristor turn-off, and to prevent short circuit conditions in the inverter.

In high frequency circuits it is good design to arrange for no commutating current flow in the load carrying thyristors, thus obviating high switching losses in these thyristors. A circuit satisfying the requirement is shown in Fig.2. Here the commutation capacitor C is initially fully charged by the action of

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the auxiliary supply circuit and charging resistor R. During starting and slow speed running the current will not naturally fall to zero after 180° conduction. The charged circuit must be capable of satisfactory commutation of say 1.5 times full load current.

Consider thyristor THₘ to be conducting, capacitor C is charged with the polarity shown. When the auxiliary thyristor THₐ is fired the charge on the capacitor is reversed. The charged capacitor is now presented across D₁, L and THₘ and normal forced commutation of the main thyristor will take place. The charge polarity on the commutating thyristor returns to the initial direction. Any reduction in charge due to losses incurred in supplying the motor load during commutation and to commutating circuit losses will be replaced by the firing of the other main thyristor and by the charging circuit. Typical waveforms of thyristor voltage and current are shown in Fig.3. These demonstrate the in-phase relationship of the voltage and current and the almost natural commutation of thyristor current at the 180° point. A full commutation capacitor charge is therefore not required once the motor is rotating and operating at unity power factor. At low frequencies, i.e. at start, full charge is required whereas at the higher frequencies little charge is required. The duty of the commutation circuit of D₁, L, C and THₐ is low and the power requirement minimal.

Commutation component cost and loss are reduced in comparison with lagging power factor systems since the factor of safety between rated current and peak commutation current may be considerably reduced in the unity power factor system. Further work is planned to ascertain whether, without loss of
reliability, the commutation current may not actually be reduced to less than peak rated current as the motor speed and frequency increase.

This reduction in commutation circuit and main thyristor requirements does not hold for induction motor drives where at any speed it will be required to commutate a significant multiple of full load current.

With unity power factor operation the kVA rating of the inverter is optimal for a given kW load. 180° operation also results in low d.c. rail current ripple, minimal voltage fluctuations and d.c. rail filter components.

3. SYNCHRONOUS MACHINE

Unity power factor operation of an ideal synchronous motor with synchronous reactance $wL_{ss}$ requires an excitation $wL_{sf} I_f$ with the phase relationship $\delta$ of Fig.4. $L_{ss}$ is the idealised total inductance of a stator phase with balanced currents and $L_{sf}$ maximum mutual inductance between a stator phase and the rotor field winding assuming that this mutual inductance varies cosinusoidally with respect to the angle of displacement between a stator phase axis and the rotor field axis. $V_m$ is the maximum of the fundamental component of the inverter output voltage for each phase of the machine; $I_m$ is the maximum current for each phase and $I_m$ and $V_m$ are in phase for unity power factor operation.

![Fig. 4](image)

There must be a shaft position transducer to arrange the required phase relationship $\delta$ between the open-circuit excitation $wL_{sf} I_f$ and the fundamental applied voltage $V_m$ from the inverter. With this transducer Fig.4 shows that at unity power factor:

\[
L_{ss} I_m = L_{sf} I_f \sin \delta \quad (1)
\]
\[
V_m = wL_{sf} I_f \cos \delta \quad (2)
\]

(1) is independent of speed showing that for unity power factor it is required only that:

$$I_f = I_m \frac{L_{ss}}{(L_{sf} \sin \delta)} \quad (3)$$

Now the inverter supply current $I_{dc}$ is proportional to the peak phase current $I_m$

$$I_{dc} = 3 \sqrt{3}/\pi I_m \quad (4)$$

for a delta connected stator winding carrying sensibly sinusoidal current. A fraction, $nI_{dc}$ say, of the supply current flowing in the rotor field winding gives a field current $I_f = n I_{dc}$ and unity power factor for

$$n = (\pi/3\sqrt{3}) \times \left(\frac{L_{ss}}{L_{sf} \sin \delta}\right) \quad (5)$$
Substitution for $I_f$ from (3) in (2) gives:
\[ \omega = \frac{V_m \tan \delta}{L_{ss} I_m} \]
i.e. the speed is inversely proportional to the current - the classic series motor characteristic.

By equating the fundamental a.c. power $3 V_m I_m / 2$ to the input power $V_{dc} I_{dc}$ gives:
\[ V_m = 2 \sqrt{3} \frac{V_{dc}}{\pi} \]
for delta connection, from (4) and neglecting inverter voltage drop.

The series characteristic is suitable for traction where regeneration is not required. When a constant speed or 'shunt' characteristic is required it may be obtained by setting the shaft position transducer to give $\delta = 90^\circ$ and using a rotor with an additional quadrature field winding which carries a direct current $I_C$. This winding has not necessarily the same number of turns as the first rotor field and $L_{sc}$ may be taken as the maximum mutual inductance between this winding and any one of the stator phase windings.

For the machine with quadrature fields operating at unity power factor Fig.5 applies.

\begin{align*}
L_{ss} I_m &= L_{sf} I_f \\
V_m &= \omega L_{sc} I_C
\end{align*}

Unity power factor operation may be achieved with an a delta connected stator by using a fraction $n'$ of the supply current $I_{dc}$ to form the main field current $I_f$; $n'$ is given by equation (5) with $\delta = 90^\circ$.

The speed $\omega$ is set for a given supply voltage $V_m$ by the relations of equation (7) i.e. a constant speed inversely proportional to the independent control current $I_C$ - the classic shunt motor characteristic.

4. SHAFT POSITION TRANSDUCER

The inverter conduction pattern must change with rotor position at intervals of $60^\circ$ (electrical). The transducer which supplies the required logic information to the firing circuit has three stationary solid-state proximity switches detecting the presence or absence of metallic strips on a non-metallic drum or disc which turns with the rotor. The proximity switches each have a high frequency current in a coil adjacent to the detection area and a metallic object coming within this area causes a magnetic coupling change to switch the output.

Each metallic strip is $180^\circ$ (electrical) in length to give an
equal mark-space ratio in the output. The strips lie circumferentially in 3 tracks round the non-metallic drum on the rotor shaft; the drum surface covers 720 electrical degrees when the machine is 4-pole. The strips are arranged such that only one detector will switch at 60° interval change.

The proximity switches are mounted on a manually-rotatable ring for easy alteration of the rotor angle when series operation is required (section 3).

As this type of transducer involves neither mechanical nor photoelectric devices, it should, with reasonable protection be adequate for industrial or traction environments.

5. SYSTEM PERFORMANCE

Fig. 6 shows the equipment arrangement; for 'constant-speed' operation, both fields were excited as shown with the angle δ (as in section 3) set at 90°, while for 'series' operation, δ was decreased to 45° (electrical) and the direct-axis field was not excited.

![Fig. 6. System layout for inverter/motor drive.](image)

Fig. 7 gives curves of experimental results, (a) with 'constant speed' connection and (b) with 'series' connection, the latter being 'constant-power'. Fig. 3 shows the waveform of voltage across and current through a thyristor. The current is a near perfect sine wave. \( I_m \), determined by equation (1), for series operation and equation (6) for shunt operation, is sinusoidal; however a small harmonic current flows since \( V_m \), equation (2) or (7), is the peak of the fundamental component of a rectangular voltage wave whereas the r.h.s. represents a sine wave only and the difference is a harmonic voltage producing a small harmonic current in the inductive winding.
6. CONCLUSION

The motor drive which has been described will produce characteristics similar to those of the shunt and series d.c. motors but no mechanical commutation is required. As traction speeds and powers increase, there will be a need for a practical alternative to the series d.c. commutator motor to avoid an increase in mechanical gearing. The above system will provide a 'constant power' characteristic for matching to a prime mover without additional regulating equipment.

Although this system does require sliprings, it has several advantages over an inverter and cage-rotor induction motor: a power factor of approximately unity and near-sinusoidal currents improve the matching of motor to inverter and the smaller losses give economic utilisation of equipment. Torque pulsations will also be negligible. The shaft transducer which controls the inverter is considered adequate for most environments.

7. REFERENCES
