Determination of the branching ratio
\[ \frac{\Gamma(K_L \rightarrow \pi\mu\nu)}{\Gamma(K_L \rightarrow \pi\mu\nu)} \]
and a Study of
\[ \frac{\Gamma(K_L \rightarrow \pi^+\pi^-\pi^0)}{\Gamma(K_L \rightarrow \text{all charged})} \]

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ABSTRACT

Using preliminary data from NA48, the new direct $CP$-violation experiment at the CERN SPS, the relative branching ratios of the semileptonic decays of $K_L$ have been determined. The value measured is:

$$\frac{\Gamma(K_L \rightarrow \pi^+\nu)}{\Gamma(K_L \rightarrow \pi e\nu)} = 0.700 \pm 0.007^{+0.010}_{-0.009}$$

(0.1)

The first error is statistical and the second systematic.

A preliminary study of the ratio:

$$\frac{\Gamma(K_L \rightarrow \pi^+\pi^-\pi^0)}{\Gamma(K_L \rightarrow \text{all charged})}$$

(0.2)

is also presented.
Declaration

This work represents the efforts of many members of the NA48 Collaboration at CERN, the European Centre for Particle Physics; the analysis and final results presented here are, however, entirely my own.
Acknowledgements

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Finally, I would like to thank my parents for their support, both financial and otherwise, while working on my PhD.
To my parents.
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One of the fundamentals upon which modern physics is based is that of symmetry. Symmetry, and its associated methodology of group theory, has allowed seemingly intractable problems to be simplified and solved in all branches of physics. For example, by applying a symmetry group to a physical system it is often possible to infer something of the underlying physics. A good example of this was the application of the unitary group $SU(3)$ to try to understand the large number of observed hadronic states. Indeed this classification led to the idea of quarks, in that the then known hadronic states could all be built out of the fundamental u, d and s quarks and antiquarks.

A symmetry (i.e. the invariance of the physical equations describing a system under a particular group of transformations) leads, in both classical and quantum mechanics, to a conserved quantity (Noether’s Theorem). Invariance with respect to Lorentz space-time translations and rotations leads to the conservation of momentum-energy and angular momentum respectively. In quantum mechanics a conserved quantity is one which commutes with the Hamiltonian of the system.

In addition to continuous transformations it is also possible, in quantum mechanics, to define discrete transformations. Three such transformations are represented by the operators $C$, $P$ and $T$.

The $C$ operator is known as the charge conjugation operator. It converts particles to their antiparticles thus reversing the sign of charge and of baryon number, $B$, or lepton number, $L$. Antiparticles and particles have the same space-time properties in classical electromagnetism. This requires that $C$ operator commutes with the Poincaré group extended to include the inversions $P$ and $T$. Under the operation of $C$ the single particle state, $|\alpha;\beta\rangle$ transforms as

$$|\alpha;\beta\rangle \rightarrow |\alpha';\beta'\rangle = C|\alpha;\beta\rangle = \eta_c |\alpha;\beta\rangle$$

(1.1)
where $\alpha$ denotes all the labels corresponding to observables with operators which anti-commute with $C$ (e.g. charge, $B$, $L$) and $\beta$ corresponds to those which commute with $C$ (e.g. $x_\mu$ - the space-time coordinates of the particle). $\eta_e$ is a phase factor introduced by the transformation. A subsequent operation of $C$ on the antiparticle state returns the original particle. By a suitable choice of the phase factor $C^2$ may be identified with the identity operator, $I$. With this phase convention the eigenvalues of $C$ are $\pm 1$. Only particles which are their own antiparticle, like the $\pi^0$ and the $\eta$, can be eigenstates of $C$ as they are required to have $Q = B = L = 0$.

The $P$ operator is the parity operator. It is defined by the coordinate transformation
\[(x_0, x) \rightarrow (x'_0, x') = (x_0, -x)\] (1.2)
Under the action of $P$ the description of a physical event in a right-handed frame is converted to a description of the same event in a left-handed frame. As for $C$ the action of $P$ twice on the same single particle state results in the original state. Thus $P^2$ must be a multiple of the identity. The undetermined phase factor is chosen such that $P$ is Hermitian and unitary, i.e.
\[P = P^\dagger = P^{-1}\] (1.3)
With this convention the eigenvalues of $P$ are $\pm 1$.

The $T$ operator is the time reversal operator. It is defined by the coordinate transformation
\[(x_0, x) \rightarrow (x'_0, x') = (-x_0, x)\] (1.4)
The operator $T$ is anti-linear and anti-unitary. Since it is anti-unitary it cannot represent an observable. Two consecutive time reversal operations leave the original state unchanged and therefore $T^2$ is a multiple of the identity
\[T^2 = \lambda I\] (1.5)
or
\[T = \lambda T^{-1} = \lambda T^\dagger\] (1.6)
By Hermitian conjugation of Equation 1.6 we find
\[T^\dagger = \lambda^* T\] (1.7)
which, inserted in Equation 1.6, gives $|\lambda|^2 = 1$. If we multiply Equation 1.6 by $T$ from the left we obtain (using the anti-linearity of $T \Rightarrow T a_i |\psi_i\rangle = a^*_i T |\psi_i\rangle$)

$$T^2 = \lambda^* I$$

(1.8)

Comparing this with Equation 1.5 we see that $\lambda^* = \lambda$ and so $T^2$ has the eigenvalues $\lambda = \pm 1$. Note that compared with the $P$ and $C$ cases there is no choice of phase convention.

According to the $CPT$ theorem of Lüders and Pauli any local field theory which is invariant under Lorentz transformations is invariant under the combined operation of $CPT[1]$. For the strong and electromagnetic interactions $C$, $P$ and $T$ are found to be exact symmetries. Initially it was thought that the weak interaction would also be invariant under the action of $C$, $P$ and $T$ separately. The $\tau - \theta$ paradox, in which two otherwise identical particles decay to different parity states, led Lee and Yang to postulate that parity is not conserved in weak decays and that the $\tau$ and $\theta$ particles are in fact the same particle (now identified as the positively charged kaon)[2]. Parity violation in the weak interaction was confirmed, in 1957, in an experiment by Wu et al. in which it was shown that the spatial distribution of electrons emitted by $\beta$ decay of $^{60}$Co nuclei was aligned with the spin of the nuclei[3]. The independent violations of $C$ and $P$ can be understood, within the framework of the Standard Model, as the coupling of the $W^\pm$ to only left handed fermions or right handed antifermions. This is illustrated by the $\beta$ decay of the neutron. This process contains a right handed antineutrino in the final state. If either of the operations of $C$ or $P$ are applied separately a right handed neutrino or a left handed antineutrino are obtained. Neither of these states are observed. However, if the combined operation of $CP$ is applied a left handed neutrino is obtained. This state is observed. Thus it seems that $CP$ could be a good symmetry within the weak interaction.

In 1964 $CP$ violation was discovered, by Christenson et al., in the decay of the long lived kaon ($CP$ odd state) into two pions ($CP$ even state)[4]. Since then considerable experimental and theoretical effort has been devoted to understanding its origin. After thirty years no evidence of $CP$ violation has been observed in particle physics outside the neutral kaon system. However, in “baryogenesis” – the science of explaining the preponderance of matter over matter in the observed
universe – CP-violation is of vital importance.

As described in Chapter 2, there are two quark level mechanisms for CP violation within the theoretical framework of the Standard Model: “indirect” CP violation and “direct” CP violation. Indirect CP violation occurs when the physical states are not pure CP states, the contamination of the state is given by the parameter \( \varepsilon \). Direct CP violation arises from decays in which CP odd states decay to CP even states with no intermediate states involved in the transition. The magnitude of this effect is measured by a parameter \( \varepsilon' \).

Although the magnitude of \( \varepsilon \) is well established there is still some uncertainty in the value of \( \varepsilon' \). The Superweak Model, postulated by Wolfenstein in 1964, introduces a new force with a small coupling constant of the order \( 10^{-3} G_F \) which gives rise to physical states with mixed CP which then decay via CP conserving weak interactions[5]. In this model \( \varepsilon \neq 0 \) and \( \varepsilon' = 0 \). Later, in 1973, Kobayashi and Maskawa were able to incorporate CP violation within the Standard Model by extending the number of quark generations to 3[6]. Within their model CP violation is a result of flavour mixing. This model predicts both direct and indirect CP violation although the calculation of \( \varepsilon' \) is not without theoretical problems.

Initial measurements of \( \varepsilon' \) have all been based on a measurement of the double ratio

\[
\left| \frac{A(K_L \to \pi^0\pi^0)/A(K_S \to \pi^0\pi^0)}{A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-)} \right|^2 \approx 1 - 6 \text{Re}(\varepsilon'/\varepsilon) \tag{1.9}
\]

Two recent high statistics experiments, E731 at Fermilab and NA31 at CERN, have measured \( \varepsilon' \) with a precision better than \( 10^{-3} \). The NA31 experiment has obtained a value of \( (2.0 \pm 0.7) \times 10^{-3} \)[7] while E731 has has found a value of \( (0.74 \pm 0.59) \times 10^{-3} \)[8]. The CERN measurement is almost three standard deviations from zero while the Fermilab result is one standard deviation from zero. Forming the standard deviation of the combined result the means of the two experiments differ by 1.4\( \sigma \) which is reasonably acceptable. However, comparing the two results with the hypothesis that \( \text{Re}(\varepsilon'/\varepsilon) = 0 \) shows that E731 can be reconciled with this result while NA31 cannot. To clarify the situation requires a new generation of experiments. At CERN, NA48, and at Fermilab, KTeV, aim for a more precise determination of \( \varepsilon'/\varepsilon \) with an accuracy on \( \text{Re}(\varepsilon'/\varepsilon) \) of \( \sim 2 \times 10^{-4} \).

The construction of the NA48 experiment is nearing completion during 1996 and
1. Introduction

This thesis is concerned with preliminary data recorded during 1995. A precision study of the three-body decays of $K_L$ is made as a thorough test of the detector, reconstruction software and the Monte Carlo simulation of the experiment. A measurement of the relative branching ratios:

$$\frac{\Gamma(K_L \rightarrow \pi \mu \nu)}{\Gamma(K_L \rightarrow \pi e \nu)}$$

(1.10)

plus a preliminary study of the ratio

$$\frac{\Gamma(K_L \rightarrow \pi^+ \pi^- \pi^0)}{\Gamma(K_L \rightarrow \text{all charged})}$$

(1.11)

are presented.

The structure of the thesis is as follows:

- introduction to kaon phenomenology and CP violation,
- description of the experiment: beams and subdetectors,
- overview of the trigger system and the dataflow,
- data reconstruction and reduction of data sample,
- Monte Carlo simulation of the experiment,
- measurement of $\Gamma(K_L \rightarrow \pi \mu \nu)/\Gamma(K_L \rightarrow \pi e \nu)$,
- study of $\Gamma(K_L \rightarrow \pi^+ \pi^- \pi^0)/\Gamma(K_L \rightarrow \text{all charged modes})$,
- discussion of the results,
- description of the muon veto monitoring system (appendix).
2. THEORETICAL BACKGROUND

This chapter introduces the phenomenology of CP violation in the neutral kaon system. Models of CP violation are briefly discussed.

2.1 Introduction to the Kaon System

Neutral kaons are produced by associated production in interactions such as:

\[ pp \rightarrow \Lambda^0 K^0 \pi^+ p \]  \hspace{1cm} (2.1)

In strong interactions strangeness is conserved so that at the time of creation neutral kaons have definite strangeness. The neutral kaons, \(| K^0 \rangle\) and \(| \bar{K}^0 \rangle\), have the valence quark composition \(| d\bar{s} \rangle\) and \(| \bar{d}s \rangle\) so that their strangeness is \(S = +1\) and \(S = -1\) respectively. Since the neutral kaons are the lightest strange particle they may only decay by the weak interaction which does not conserve strangeness. The lowest order decay processes are shown in Figure 2.1 and both lead to a \(u\bar{u}d\bar{d}\) final state.

![Fig. 2.1: The lowest order decay processes for \(K^0\) and \(\bar{K}^0\).](image)

We know, from muon decay and other processes, that the \(W\) only couples to left-handed fermions or right-handed antifermions. The operation of charge con-
jugation C turns a left-handed quark into a left-handed antiquark, which does not couple to the W. Subsequent operation of the parity operator P gives a right-handed antiquark which can couple to the W. Thus it was expected, in the 1950’s, that the physical states which decay are eigenstates of the combined operation CP. \( K^0 \) and \( \bar{K}^0 \) are not eigenstates of CP but are transformed into one another by this operation. Taking the convention that

\[
\text{CP} \mid K^0 \rangle = \mid \bar{K}^0 \rangle, \quad \text{CP} \mid \bar{K}^0 \rangle = \mid K^0 \rangle,
\]

(2.2)

the following linear combinations of \( K^0 \) and \( \bar{K}^0 \), which are CP eigenstates, can be formed

\[
\left| K^0_1 \right> = \frac{1}{\sqrt{2}} \left[ \left| K^0 \right> + \left| \bar{K}^0 \right> \right] = \frac{1}{\sqrt{2}} \left[ \left| d\bar{s} \right> + \left| s\bar{d} \right> \right], \quad \text{CP} = +1,
\]

(2.3)

\[
\left| K^0_2 \right> = \frac{1}{\sqrt{2}} \left[ \left| K^0 \right> - \left| \bar{K}^0 \right> \right] = \frac{1}{\sqrt{2}} \left[ \left| d\bar{s} \right> - \left| s\bar{d} \right> \right], \quad \text{CP} = -1.
\]

(2.4)

Alternatively the strangeness eigenstates can be expressed in terms of states of definite CP

\[
\left| K^0 \right> = \left| d\bar{s} \right> = \frac{1}{\sqrt{2}} \left[ \left| K^0_1 \right> + \left| K^0_2 \right> \right],
\]

(2.5)

\[
\left| \bar{K}^0 \right> = \left| s\bar{d} \right> = \frac{1}{\sqrt{2}} \left[ \left| K^0_1 \right> - \left| K^0_2 \right> \right].
\]

(2.6)

As the two pion system, with zero angular momentum, has a CP eigenvalue of +1 the \( K^0_1 \) will decay to this state while the \( K^0_2 \) will decay predominantly to three body final states which have a CP eigenvalue of -1. Due to the more limited phase space available for three body decays, compared to that for decays to two pions, the \( K^0_2 \) is longer lived than the \( K^0_1 \).

The discovery in 1964 by Christenson et al. that the long lived neutral kaon decays to two pions with a branching ratio of \( \sim 2 \times 10^{-3} \) implies that CP is violated. The CP violation can arise from two sources within the Standard Model:

- The physical eigenstates which decay are not pure CP eigenstates but are an admixture of \( K^0_1 \) and \( K^0_2 \). This is known as "indirect" CP violation and arises from state-mixing between \( K^0 \) and \( \bar{K}^0 \) via \( \Delta S = 2 \) weak interactions. Such interactions proceed via "box" diagrams as shown in Figure 2.2.
• The $K^0$ decays directly to the CP even two pion state. This is known as "direct" CP violation and involves $\Delta S = 1$ interactions. A mechanism for direct CP violation is via so-called "penguin" diagrams as shown in Figure 2.3.

Since the discovery of CP violation considerable experimental effort has been expended to find out whether CP violation is present only in the CP impurity of the mass eigenstates, or if there is a contribution from direct CP violating amplitudes in which the $K^0$ decays to two pions without an intermediate state.
2. Phenomenology of CP Violation in the Neutral Kaon System

2.2.1 Indirect CP Violation

Representing the short and long lived states, \( K_L \) and \( K_S \), as an admixture of \( K^0 \) and \( \bar{K}^0 \) gives

\[
| K_S \rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}} \left[ | K_1^0 \rangle + \epsilon | K_2^0 \rangle \right] \tag{2.7}
\]

\[
= \frac{1}{\sqrt{2(1 + |\epsilon|^2)}} \left[ (1 + \epsilon) | K^0 \rangle + (1 - \epsilon) | \bar{K}^0 \rangle \right] \tag{2.8}
\]

\[
| K_L \rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}} \left[ | K_2^0 \rangle + \epsilon | K_1^0 \rangle \right] \tag{2.9}
\]

\[
= \frac{1}{\sqrt{2(1 + |\epsilon|^2)}} \left[ (1 + \epsilon) | K^0 \rangle - (1 - \epsilon) | \bar{K}^0 \rangle \right] \tag{2.10}
\]

as the physical eigenstates. The magnitude of the violation of CP is characterised by the complex parameter \( \epsilon \).

In addition to the CP violation observed from the two pion decay of \( K_L \) the unequal mixture of \( K^0 \) and \( \bar{K}^0 \) implies a charge asymmetry in the semileptonic decay channels of \( K_L \). The following decay channels are observed:

\[
K^0 \rightarrow \pi^- l^+ \bar{\nu}
\]

\[
\bar{K}^0 \rightarrow \pi^+ l^- \nu
\]

as shown in Figure 2.2.1. So, if the \( K_L \) state contains an enhancement of \( K^0 \) compared to \( \bar{K}^0 \) more positive leptons will be observed in the decays \( K_{\nu\overline{\nu}} \) (\( K_L \rightarrow \pi e\bar{\nu} \)) and \( K_{\mu\overline{\nu}} \) (\( K_L \rightarrow \pi \mu\bar{\nu} \)). Measurements of the charge asymmetry yield the real part of \( \epsilon \).

2.2.2 Direct CP Violation

Information on direct CP violation can also be obtained from analysis of \( K_S \) and \( K_L \) decays to two pion final states.
Bose statistics demand that two pions with zero angular momentum have isotopic spin $I=0$ or $I=2$. There are therefore four possible amplitudes:

$$A(K_S \to \pi\pi; I = 0) \quad A(K_L \to \pi\pi; I = 0)$$
$$A(K_S \to \pi\pi; I = 2) \quad A(K_L \to \pi\pi; I = 2)$$

Three quantities are defined in terms of these amplitudes:

$$\varepsilon_0 = \frac{A(K_L \to \pi\pi; I = 0)}{A(K_S \to \pi\pi; I = 0)}$$
$$\varepsilon_2 = \frac{1}{\sqrt{2}} \frac{A(K_L \to \pi\pi; I = 2)}{A(K_S \to \pi\pi; I = 0)}$$
$$\omega = \frac{1}{\sqrt{2}} \frac{A(K_S \to \pi\pi; I = 2)}{A(K_S \to \pi\pi; I = 0)}$$

In addition the direct CP violation term $\varepsilon'$ is defined to be:

$$\varepsilon' = \frac{1}{\sqrt{2}} A(K_2 \to \pi\pi; I = 2)/A(K_1 \to \pi\pi; I = 0)$$

Decomposing the charged and neutral two pion states into their isospin components using Clebsch-Gordan coefficients results in:

$$| \pi^+\pi^- \rangle = \sqrt{\frac{2}{3}} | \pi\pi; I = 0 \rangle + \sqrt{\frac{1}{3}} | \pi\pi; I = 2 \rangle$$
$$| \pi^0\pi^0 \rangle = \sqrt{\frac{1}{3}} | \pi\pi; I = 0 \rangle - \sqrt{\frac{2}{3}} | \pi\pi; I = 2 \rangle$$

The reason for introducing the pure isospin states is that the matrix elements of the transitions from $K^0$ and $\bar{K}^0$ states to the same $|\pi\pi; I \rangle$ state can be related by CPT-invariance plus Watson's theorem on final state strong interactions[9]. The relation is

$$e^{-2i\delta_I \langle I \mid T \mid K^0 \rangle} = (\langle I \mid T \mid \bar{K}^0 \rangle)^*,$$
where $\delta_I$ denotes the appropriate $J = 0$, isospin $I \pi \pi$ phase-shift.

To prove this relation one needs to look at the $S$-matrix of the interaction. With $S = 1 + iT$, the unitarity of the $S$-matrix, $SS^\dagger = 1$, implies

$$TT^\dagger = i(T^\dagger - T)$$  \hspace{1cm} (2.20)

If one takes matrix elements of this operator relation between an initial state $K^0$, and a final $2\pi$-state with isospin $I$, we have

$$\sum_F \langle I \mid T^\dagger \mid F \rangle \langle F \mid T \mid K^0 \rangle = i\langle I \mid T^\dagger \mid K^0 \rangle - i\langle I \mid T \mid K^0 \rangle,$$  \hspace{1cm} (2.21)

where a complete set of states, $\sum | F \rangle \langle F | = 1$, has been inserted between $T$ and $T^\dagger$. In the strong interaction sector of the $S$-matrix, which must conserve isospin, only the state $F = I$ can contribute to the $T^\dagger$-matrix element. All the other states are suppressed by selection rules; e.g. the $3\pi$-states have opposite $G$-parity than the $2\pi$-states. Then, introducing the $\pi \pi$ phase-shift definition:

$$\langle I \mid S \mid I \rangle = e^{2i\delta_I},$$  \hspace{1cm} (2.22)

results in the relation

$$i(e^{-2i\delta_I} - 1)\langle I \mid T \mid K^0 \rangle = i\langle I \mid T^\dagger \mid K^0 \rangle - i\langle I \mid T \mid K^0 \rangle,$$

$$= i\langle K^0 \mid T \mid I \rangle^* - i\langle I \mid T \mid K^0 \rangle.$$  \hspace{1cm} (2.23)

CPT-invariance then implies: $\langle K^0 \mid T \mid I \rangle^* = (\langle I \mid T \mid K^0 \rangle)^*$. The result in Equation 2.19 then follows.

The amplitudes of the decays of $K^0$ to the final isospin states are:

$$\langle \pi \pi; I = 0 \mid K^0 \rangle = A_0 e^{i\delta_0}$$  \hspace{1cm} (2.25)

$$\langle \pi \pi; I = 2 \mid K^0 \rangle = A_2 e^{i\delta_2}$$  \hspace{1cm} (2.26)

where $\delta_0$ and $\delta_2$ are phases arising from final state strong interactions between the pions for, $I=0$ and $I=2$ respectively. By convention (due to Wu and Yang) $A_0$ is taken to be real [10]. From the relations proved above we can deduce the amplitudes for the decays of $\overline{K}^0$:

$$\langle \pi \pi; I = 0 \mid \overline{K}^0 \rangle = A_0 e^{i\delta_0}$$  \hspace{1cm} (2.27)

$$\langle \pi \pi; I = 2 \mid \overline{K}^0 \rangle = A_2^* e^{i\delta_2}$$  \hspace{1cm} (2.28)
From the previous equations it can be seen that:

\[
\varepsilon = \varepsilon_0 \tag{2.29}
\]
\[
\varepsilon' = \frac{i}{\sqrt{2}} (Im A_2/A_0) e^{i(\delta_2-\delta_0)} \tag{2.30}
\]

Thus \(\text{Re}(\varepsilon')\) is non-zero if there is a phase difference between \(A_0\) and \(A_2\). From the definition of \(\varepsilon'\) in Equation 2.16 it can be seen that this indicates direct CP violation. If all CP violation had state-mixing as its source \(\varepsilon'\) would be zero.

Experimentally the parameters which can be measured are:

\[
\eta_{+-} \equiv |\eta_{+-}| e^{i\phi_{+-}} = A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-) \tag{2.31}
\]
\[
\eta_{00} \equiv |\eta_{00}| e^{i\phi_{00}} = A(K_L \to \pi^0\pi^0)/A(K_S \to \pi^0\pi^0) \tag{2.32}
\]

The current fitted values for these parameters are[11]:

\[
|\eta_{+-}| = (2.285 \pm 0.019) \times 10^{-3} \tag{2.33}
\]
\[
\phi_{+-} = 43.7 \pm 0.6^\circ \tag{2.34}
\]
\[
|\eta_{00}| = (2.275 \pm 0.019) \tag{2.35}
\]
\[
\phi_{00} = 43.5 \pm 1.0^\circ \tag{2.36}
\]

From the definitions of \(K_S, K_L, \varepsilon_0, \varepsilon_2, \omega, \varepsilon'\) and the isospin decomposition of the pion final states it can be shown that:

\[
\eta_{+-} = \frac{\varepsilon_0 + \varepsilon_2}{1 + \omega} \approx \frac{\varepsilon + \varepsilon'}{1 + \omega} \approx \varepsilon + \varepsilon' \tag{2.37}
\]
\[
\eta_{00} = \frac{\varepsilon_0 - 2\varepsilon_2}{1 - 2\omega} \approx \frac{\varepsilon - 2\varepsilon'}{1 - 2\omega} \approx \varepsilon - 2\varepsilon' \tag{2.38}
\]

It is observed experimentally that the transition \(|K_S \to \pi\pi; I = 2\) \((\Delta I = 3/2)\) is strongly suppressed relative to \(|K_S \to \pi\pi; I = 0\) \((\Delta I = 1/2)\), so the factor \(\omega\) is small \((\omega = (4.48 \pm 0.02) \times 10^{-2})[12]\) and may be neglected. These equations form the “triangle relations” which are best illustrated in the so-called “Wu-Yang” diagram (see Figure 2.2.2).

The experimental values obtained for \(\eta_{+-}\) and \(\eta_{00}\) are very similar in magnitude and phase. This implies that \(|\varepsilon'|\) is small compared to \(|\varepsilon|\). The near equality of \(\eta_{+-}\) and \(\eta_{00}\) means that the current measurements cannot determine the phase of any non-zero \(\varepsilon'\). The phase of \(\varepsilon\) is given by

\[
\phi(\varepsilon) \approx \tan^{-1} \frac{2(m_{KL} - m_{KS})}{\Gamma_{KS} - \Gamma_{KL}} = 43.49 \pm 0.08^\circ \tag{2.39}
\]
while the phase of $\varepsilon'$ can be determined from an analysis of the $\pi\pi$ phase shifts[12, 13]

$$\phi(\varepsilon') = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 48 \pm 4^\circ$$  (2.40)

Thus it is expected that the phase difference between $\varepsilon$ and $\varepsilon'$ is small ($\phi(\varepsilon') - \phi(\varepsilon) \approx 4 \pm 4^\circ$)[11]. In the Chell and Olsson evaluation of $\delta_2 - \delta_0$, they take measurements of the $s$-wave $\pi - \pi$ scattering lengths from threshold pion production and use them to calculate $\delta_2 - \delta_0$ at the kaon mass. This is done using forward dispersion relations and assuming the analyticity of the scattering amplitude. Their evaluation compares quite well with phase shifts calculated in the kaon mass region[12]

$$\phi(\varepsilon') = 48.6 \pm 8.1$$  (2.41)

The Chell and Olsson error is determined from the effect of changing the input parameters to their dispersive sum rules within their assigned errors. This does not take into account any inaccuracies in their model which could contribute to the predicted phase of $\varepsilon'$.

Using the fact that $\omega \ll 1$ and $\varepsilon' \ll \varepsilon$ the measured quantity $| \eta_{00}/\eta_{+-} |^2$ is given
to a good approximation by:

\[ \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \approx 1 - 6 \text{Re}(\varepsilon'/\varepsilon) \]

\[ = 1 - 6(\varepsilon'/\varepsilon) \cos [\Phi(\varepsilon') - \Phi(\varepsilon)] \] (2.42)

Since the \( \cos \) in Equation 2.42 is expected theoretically to be very close to unity

\( \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \) determines \( \varepsilon'/\varepsilon \).

All current measurements of \( \text{Re}(\varepsilon'/\varepsilon) \) are based on Equation 2.42. This requires the measurement of the double ratio:

\[ \frac{A(K_L \rightarrow \pi^0\pi^0)/A(K_S \rightarrow \pi^0\pi^0)}{A(K_L \rightarrow \pi^+\pi^-)/A(K_S \rightarrow \pi^+\pi^-)} \] (2.43)

to very high accuracy.

### 2.3 CP Violation Within the Standard Model

In the standard model of electroweak interactions it appears that the massive vector bosons \( (W^+, W^-) \) couple directly to the physical eigenstates of the leptons but not to the mass eigenstates of the quarks. Instead they couple to superpositions of the quark mass eigenstates.

This idea was first introduced by Cabibbo (1963)[14] to explain the difference in the decay rates for \( \Delta S = 0 \) and \( \Delta S = 1 \) transitions. The \( \Delta S = 1 \) transitions are supressed by a factor of about 20 compared to the \( \Delta S = 0 \) transitions. Instead of introducing an additional coupling constant for decays involving strange quarks Cabibbo introduced a mixing angle, \( \theta_C \), which redefines the physical eigenstates participating in the weak interactions:

\[ \left( \begin{array}{c} u \\ d_c \end{array} \right) = \left( \begin{array}{c} u \\ d \cos \theta_C + s \sin \theta_C \end{array} \right) \] (2.44)

However this model predicted the existence of \( \Delta S = 1 \) neutral currents in contradiction to experimental measurements of

\[ \frac{K^+ \rightarrow \pi^+\nu\bar{\nu}}{K^+ \rightarrow \pi^0\mu^+\nu_\mu} < 10^{-5} \quad (\Delta S = 1). \] (2.45)

In 1970, Glashow, Iliopoulos and Maiani (GIM) proposed the introduction of a new quark with flavour labelled \( c \) for "charm"[15]. The Cabibbo theory was then
2. Theoretical Background

extended to two weak quark doublets:

$$\begin{pmatrix} u \\ d_c \end{pmatrix} = \begin{pmatrix} u \\ d \cos \theta_C + s \sin \theta_C \end{pmatrix} \quad \begin{pmatrix} c \\ s_c \end{pmatrix} = \begin{pmatrix} c \\ s \cos \theta_C - d \sin \theta_C \end{pmatrix}$$

(2.46)

Extra terms from the second doublet cancel the flavour changing neutral current terms from the first doublet. This cancellation is known as the GIM mechanism.

In this model there is still no scope for CP violation as it is always possible to redefine the phases of the quark fields such that the fields are real. In 1972 Kobayashi and Maskawa extended the $2 \times 2$ Cabibbo matrix to a $3 \times 3$ matrix known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix[6]. At that time there was no evidence to suggest a third quark generation. They found that, of the three Euler angles and six phases which characterised their matrix, five of the phases could be absorbed by a redefinition of the quark fields. One phase, $\delta$, remains. The CKM matrix, in the parameterisation of Kobayashi and Maskawa, is given by

$$V_{CKM} = \begin{pmatrix} c_1 & c_3 s_1 & s_1 s_3 \\ -c_2 s_1 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_3 s_2 e^{i\delta} \\ s_1 s_2 & -c_1 c_3 s_2 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_3 c_2 e^{i\delta} \end{pmatrix}$$

(2.47)

where $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$. $\theta_1$, $\theta_2$ and $\theta_3$ are the three mixing angle replacing the single angle $\theta_C$ of the $2 \times 2$ matrix. Since under time reversal $e^{i\delta} \to e^{-i\delta}$, the phase angle $\delta$ introduces the possibility of CP violating amplitudes.

In the standard model the main contribution to $\epsilon$ comes from $K^0 - \bar{K}^0$ mixing. The CP violating part of the $(K^0 - \bar{K}^0)$ mass matrix has been calculated from the second-order box diagram (Figure 2.2) by Ellis et al., 1976[16].

The main contributions to $\epsilon'$ come from the "penguin" diagrams (Figure 2.3). There are large theoretical uncertainties in the calculation of $\epsilon'$. In the past there was an uncertainty due to the mass of the $t$ quark which was then unknown. There are also large cancellations between the gluonic and electroweak penguins which therefore need to be calculated with an accuracy which is not presently attainable[17]. Future generations of computational facilities purpose built for lattice calculations should allow predictions of $\epsilon'$ with an accuracy of $10^{-4}$ – similar to the accuracy of the present generation of direct CP violation experiments.
2.4 Phenomenology of the Semileptonic Decays of Kaons

The phenomenology of CP violation in the neutral kaon system is outlined in the previous section as it is relevant to the ultimate goal of NA48: to measure the direct CP violation parameter $\varepsilon'$ with better accuracy than that achieved in the previous generation of experiments. With an understanding of the method of the measurement (via Equation 2.42, described on page 14) the motivation for the layout of the combined $K^0_S$ and $K^0_L$ beams and the system of sub-detectors described in Chapter 3 can be more easily appreciated.

However, the experimental apparatus is versatile enough that it can be used to measure many other parameters of neutral kaon decays. For example: branching ratios, searches for rare decays and measurements of the form-factors of various decay matrix elements. The measurement described in this thesis (Chapter 7) concerns the relative branching ratios of the semileptonic decays of $K^0_L$.

Assuming a $V-A$ interaction for the semileptonic decays the matrix element can be written as [18]

$$M = \frac{G}{\sqrt{2}} \sin \theta C \langle \pi | V^\mu | K \rangle \bar{l} \gamma_\mu (1 - \gamma_5) \nu$$  \hspace{1cm} (2.48)

Since the kaon and pion both have parity $(-)$ the axial vector part of the hadronic matrix element must be zero. The matrix elements of the strangeness changing vector current, $V^\mu$, can be expanded in the form

$$(\pi | V^\mu | K) \propto f_+(t)(P_{K^0} + P_\pi) + f_-(t)(P_{K^0} - P_\pi)$$  \hspace{1cm} (2.49)

where $P_{K^0}$ and $P_\pi$ are the four-momenta of the $K^0$ and $\pi$ mesons and $f_+$ and $f_-$ are dimensionless form factors depending only on $t = (P_{K^0} - P_\pi)^2$, the square of the four-momentum transfer to the leptons. Inserting (2.49) into (2.48) and using the Dirac equation, $(P_{K^0} - P_\pi)\bar{l} \gamma_\mu = (p + P_\nu)\bar{l} \gamma_\mu = m_\ell$, one obtains [18, 19]

$$M \propto f_+(t) \left[ (P_{K^0} + P_\pi)\bar{l} \gamma_\mu (1 - \gamma_5) \nu \right] + f_-(t) \left[ m_\ell (1 - \gamma_5) \nu \right]$$  \hspace{1cm} (2.50)

where $m_\ell$ is the lepton mass. An induced scalar term is introduced which is proportional to the lepton mass, $m_\ell$.

For $K_{\mu3}$ the Dalitz plot density is normally parametrised as [20]:

$$\rho(E_\pi, E_\nu) \propto f_+^2(t) \left[ A + B \xi(t) + C \xi(t)^2 \right]$$  \hspace{1cm} (2.51)
where

\begin{align*}
A &= m_{K^0} \left( 2E_\mu E_\nu - m_{K^0} E_\pi \right) + m_\mu^2 \left( \frac{1}{4} E'_\pi - E_\nu \right), \\
B &= m_\mu^2 \left( E_\nu - \frac{1}{2} E'_\pi \right), \\
C &= \frac{1}{4} m_\mu^2 E'_\pi, \\
E'_\pi &= E_{\pi \text{max}} - E_\pi = \frac{\left( m_{K^0}^2 + m_\mu^2 - m_\pi^2 \right)}{2m_{K^0}} - E_\pi.
\end{align*}

$E_\pi$, $E_\mu$, and $E_\nu$ are, respectively, the pion, muon and neutrino energies in the kaon centre of mass. $\xi(t)$ is the ratio of the two form factors $f_-(t)/f_+(t)$.

For $K_e3$ the form factor is a simpler expression as $f_-(t)$ in Equation 2.50 is suppressed by the small lepton mass. Here the Dalitz plot is parametrised as

\[ \rho(E_\pi, E_\nu) \propto f_+^2(t) \left[ \frac{4E_\nu E_\nu}{m_{K^0}^2} - \frac{2(E_{\pi \text{max}}^\text{max} - E_\pi)}{m_{K^0}} \right], \]

which is identical to Equation 2.51 except that all terms involving $m_\nu^2$ are assumed to be negligible\(^1\).

The variation of the form factors $f_+$ and $f_-$ is normally assumed to be linear in $t[18, 21, 22]$:

\[ f_\pm(t) = f_\pm(0) \left[ 1 + \lambda_\pm(t/m_\pi^2) \right]. \]

The ratio of the form factors $\xi(t)$ can also be approximated by a linear form:

\[ \xi(t) = f_-(t)/f_+(t) = \xi(0) + \Lambda t/m_\pi^2 \]

where, assuming $\lambda_+$ is small,

\[ \Lambda \simeq \xi(0)(\lambda_- - \lambda_+). \]

Re-examining the matrix element in Equation 2.50 it can be seen that $f_+(t)$ is related to the pure vector term while $f_-(t)$ is related to the induced scalar term proportional to the lepton mass. To fully separate scalar and vector parts of the matrix element another form factor, $f(t)$, is defined. $f(t)$ is associated with the matrix element of the current divergence:

\[ f(t) = \sqrt{2}(2\pi)^3 \frac{\sqrt{4E_K E_\pi}}{m_{K^0}^2 - m_\pi^2} \langle \pi | \partial_\mu V^\mu | K \rangle \]

\(^1\) The equation also been multiplied by $2/m_{K^0}^3$ to make the Dalitz plot density dimensionless.
How this is related to $f_+$ and $f_-$ can be seen by examining the most general matrix element for pseudoscalar to pseudoscalar vector transitions:

$$P_1 \rightarrow P_2 \ell \nu.$$  \hspace{1cm} (2.61)

From translation invariance and Lorentz covariance the matrix element must be of the form:

$$\sqrt{4E_1E_2} \langle P_1|V^\mu|P_2 \rangle = \exp(i(p_2-p_1)z) \left[ f_+(t)(p_1 + p_2)_\mu + f_-(t)(p_1 - p_2)_\mu \right]$$  \hspace{1cm} (2.62)

Current conservation then gives the condition

$$\langle P_2|\partial_\mu V^\mu|P_1 \rangle = i(p_2 - p_1)\langle P_2|V^\mu|P_1 \rangle$$  \hspace{1cm} (2.63)

or

$$(m_1^2 - m_2^2)f_+(t) + tf_-(t) = 0.$$  \hspace{1cm} (2.64)

Thus in the symmetry limit of exact current conservation knowledge of either of $f_+$ or $f_-$ determines the other.

For strangeness changing $K_{\ell 3}$ decays the current is not conserved and we have:

$$\sqrt{4E_KE_\pi} \langle \pi|V^\mu|K \rangle \equiv (m_K^2 - m_\pi^2)f_+(t) + tf_-(t) \equiv (m_K^2 - m_\pi^2)f(t)$$  \hspace{1cm} (2.65)

so that $f(t)$ is defined to be:

$$f(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2}f_-(t).$$  \hspace{1cm} (2.66)

(Note that $f(0) = f_+(0)$ unless $f_-(t)$ diverges as $t \rightarrow 0$.)

If Equation 2.49 is evaluated in the centre-of-mass of the dilepton system:

$$\langle \pi|V^0|K \rangle \propto \frac{m_K^2 - m_\pi^2}{\sqrt{t}}f(t)$$

$$\langle \pi|V^i|K \rangle \propto 2(p_{K^0} + p_\pi)f_+(t).$$  \hspace{1cm} (2.67)

So $f(t)$ and $f_+(t)$ are seen to be related to the transition amplitudes $0^+$ and $1^-$, respectively, for the final lepton pair.

The form factor $f(t)$ is also assumed to be linear in $t$ and is written

$$f(t) = f(0)(1 + \lambda_0 t/m_\pi^2)$$  \hspace{1cm} (2.68)
In the $f(t)$, $f_+(t)$ parametrisation $\xi(t)$ is given by:

$$
\xi(t) = \frac{m_{K^0}^2 - m_\pi^2 (f(t) - f_+(t))}{f_+(t)}
= \frac{m_{K^0}^2 - m_\pi^2 (\lambda_0 - \lambda_+)}{m_\pi^2 (1 + \lambda_+ t/m_\pi^2)}
$$

(2.69)

with

$$
\begin{align*}
\xi(0) &= \frac{m_{K^0}^2 - m_\pi^2 (\lambda_0 - \lambda_+)}{m_\pi^2} \\
\Lambda &= -\xi(0) \lambda_+
\end{align*}
$$

(2.70)

for small $\lambda_+$. Note that if $f(0) \neq f_+(0)$ there is a divergence of $\xi(t)$ at $t = 0$.

It is usually assumed that the form factors $f_+$ and $f$ satisfy at most once subtracted dispersion relations[18]. The absorptive part of $f_+$ is determined by spin one intermediate states and the absorptive part of $f$ by spin zero intermediate states. The dispersion integrals are therefore uncoupled. If only $K\pi$ intermediate states are taken into account the once subtracted dispersion relations reduce to Omnes' equations[23] with solutions:

$$
\begin{align*}
f_+(t) &= f_+(0) \exp \left[ \frac{1}{\pi} \int_{(m_{K^0} + m_\pi)^2}^{\infty} dt' \frac{\delta_1(t')}{t' (t' - t - i\epsilon)} \right] \\
f(t) &= f_+(0) \exp \left[ \frac{1}{\pi} \int_{(m_{K^0} + m_\pi)^2}^{\infty} dt' \frac{\delta_0(t')}{t' (t' - t - i\epsilon)} \right]
\end{align*}
$$

(2.71) (2.72)

where $\delta_1$ and $\delta_0$ are respectively the p-wave and s-wave phase shifts in the $\Delta I 1/2$ $K\pi$ scattering channel. However the solutions to Equations 2.71 and 2.72 are not unique and may be multiplied by a polynomial of order $\leq L + 1$ (or $\leq L$ if $L$ is an integer) where

$$
\lim_{t \to \infty} \delta(t) = L\pi
$$

(2.73)

and with $P(t = 0) = 1$. Unless it is known that $L = 0$ nothing can be said about the behaviour of the form factors in the decay region.

A more common procedure is to assume that the $f_+(t)$ and $f(t)$ amplitudes are unsubtracted. In this case the dispersion integrals may be approximated by simple poles (zero-width resonances):

$$
\begin{align*}
f_+(t) &= f_+(0) \frac{m_\pi^2}{m_\pi^2 - t} \\
f(t) &= f_+(0) \frac{m_\pi^2}{m_\pi^2 - t}
\end{align*}
$$

(2.74) (2.75)
where \( m_1 \) and \( m_0 \) are the resonance masses of the 1\(^-\) and 0\(^+\) spin-parity states respectively. The graph for the pole contribution to \( K^0 \rightarrow \pi l\nu \) is shown in Figure 2.4. Figure 2.4(a) shows the graph in the current algebra picture, where the interactions at the quark level are unknown, while 2.4(b) illustrates a possible mechanism for the same interaction at the quark level\(^2\).

Pole dominance gives slopes which are positive and determined by the masses of the exchanged particles:

\[
\lambda_+ = \frac{m_2^2}{m_*^2} \quad \lambda_0 = \frac{m_0^2}{m_*^2}.
\]

Assuming that \( f_+ \) is dominated by the lightest \( J^P = 1^- \) \( K\pi \) resonance, \( m_* \) is the mass of the \( K^*(892) \). This leads to a prediction for \( \lambda_+ \) of \( m_2^2/m_*^2 = 0.024 \). Similarly, for the scalar form factor, the pole model predicts \( \lambda_0 = m_0^2/m_*^2 = 0.01 \) where \( m_0 \) is the mass of the \( K^*(1430) \), the lightest \( J^P = 0^+ \) strange meson. Birulev et al have measured the parameters \( \lambda_+ \) and \( \lambda_0 \) for \( K_{\mu3} \) decays and \( \lambda_+ \) for \( K_{e3} \) decays. By fitting the \( K_{e3} \) data to the pole expressions they obtain the resonance mass \( m_* = (835 \pm 40) \text{ MeV}/c^2 \) in rough agreement with the \( K^*(892) \) mass. For \( K_{\mu3} \) decays they found \( m_* = (680 \pm 40) \text{ MeV}/c^2 \) and \( m_0 = (760 \pm 70) \text{ MeV}/c^2 \) which is inconsistent with the simple pole model.

\(^2\) Note that although we can write a Feynman diagram for the interaction it cannot be easily evaluated, apart from in the spectator model case. Due to the lightness of the \( K \) meson perturbative QCD techniques are inapplicable and the hadronic matrix element can only be calculated using phenomenological chiral Lagrangians.
Referring back to Equations 2.51 and 2.56 for the Dalitz plot density for $K_{\mu 3}$ and $K_{e3}$ respectively, and denoting by $T$ the dimensionless momentum transfer

$$T = \frac{t}{m^2},$$

(2.78)

the branching ratios $\Gamma_{\mu 3}$ and $\Gamma_{e3}$ can be expressed as integrals over the Dalitz plot:

$$\Gamma_{\mu 3} = K \int [A + 2AT\lambda_+ + AT^2\lambda_+^2 + B\xi(0) + BT\xi(0)\lambda_+ + C\xi(0)^2] (2.79)$$

$$\Gamma_{e3} = K \int [A + 2AT\lambda_+ + AT^2\lambda_+^2]$$

(2.80)

If we assume $\mu - e$ universality then $K$ is the same for both modes. By integrating these equations a theoretical expectation for $\Gamma(K_{\mu 3}^0)/\Gamma(K_{e3}^0)$ can be obtained in terms of the parameters $\lambda_+$ and $\xi(0)$.

By measuring the branching ratio $\Gamma(K_{\mu 3}^0)/\Gamma(K_{e3}^0)$ and comparing it with the theoretical ratio[20]:

$$\Gamma(K_{\mu 3}^0)/\Gamma(K_{e3}^0) = 0.6452 + 1.3162\lambda_+ + 0.1264\xi(0) + 0.0186\xi(0)^2 + 0.0064\lambda_+\xi(0)$$

(2.81)

$$\simeq 0.6452 - 0.1429\lambda_+ + 1.4591\lambda_0$$

(2.82)

a quadratic relationship can be obtained between the parameters $\lambda_+$ and $\xi(0)$ or $\lambda_+$ and $\lambda_0$. Since these parameters are expected to be $\ll 1$ (c.f. Eqns. 2.76-2.77) Equation 2.81 can be simplified and written in a linear form.

If $\mu - e$ universality holds then the form factor slopes should be equal for both $K_{\mu 3}$ and $K_{e3}$ decays. The current PDG values[20], $\lambda^\mu_+ = 0.0300 \pm 0.0016$ and $\lambda^\mu_+ = 0.034 \pm 0.005$ (where the superscript denotes $K_{e3}$ and $K_{\mu 3}$ respectively), are consistent with $\mu - e$ universality. Similarly, the PDG value for $\lambda_0 = 0.025 \pm 0.006$[20]. If we insert these values in Equation 2.82 then we find that the expectation for $\Gamma(K_{\mu 3}^0)/\Gamma(K_{e3}^0) = 0.677 \pm 0.009$. 

---

2. Theoretical Background
3. THE EXPERIMENT

This chapter outlines the design and operation of the NA48 experiment. The production of the kaon beams is described along with the associated tagger, anti-K$_S$ (AKS) and beam monitor detectors. The operation of the other sub-detectors is also described.

A description of the multi-level trigger scheme and the dataflow is deferred to the next chapter.

It is important to note that, during August and September 1995, NA48 was in a state of partial completion. The electromagnetic calorimeter and one of the four drift-chambers in the magnetic spectrometer were missing entirely and, in addition, the hadron calorimeter and the muon veto were readout using a temporary scheme due to the unavailability of the final electronics. Nevertheless the apparatus was sufficiently complete to allow meaningful physics studies to be made.

The NA48 detector is situated in the high intensity hall in the CERN North Area and, when taking data, receives spills of $\sim 10^{12}$ protons per pulse (ppp) every 14.4 s with a spill duration of 2.6 s. The momentum of the protons is 450 GeV with $1.5 \times 10^{12}$ ppp on the K$_L$ target and $3.7 \times 10^7$ ppp on the K$_S$ target. This should produce a total of $\sim 3 \times 10^5$ K$_L$ decays and $\sim 2 \times 10^2$ K$_S$ decays into the detector per second [24].

The experiment is designed with the aim of measuring the direct CP-violation parameter, $Re(\epsilon'/\epsilon)$, with an accuracy of $2 \times 10^{-4}$. The error on this measurement is a factor of 3 smaller than the previous generation of experiments. To achieve the required statistics for such a precise measurement requires a high rate data-acquisition system plus a trigger with a very high rejection of background from the 3-body decays of K$_L$. During 1995 a much simpler trigger system was implemented. The data with which this thesis is concerned was recorded using this,
3. The Experiment

Tab. 3.1: K$_S$, K$_L$ beam definition.

so-called, “minimum bias” trigger. This trigger is described in the next chapter.

3.1 The Kaon Beams And Associated Detectors

3.1.1 The K$_L$ And K$_S$ Beams

In order to eliminate all sources of systematic differences, such as time dependence in the flux or detection efficiencies, the experiment is designed with two nearly collinear K$_S$ and K$_L$ beams which are produced concurrently. The K$_S$ and K$_L$ decays are distinguished by a tagging detector which is positioned in the proton beam producing the K$_S$ component. A schematic of the experiment is shown in Figure 3.1.

Both the K$_S$ and K$_L$ beams are produced in 2 mm diameter, 400 mm long beryllium targets, followed by collimators that define the acceptance. The quantities involved are shown in Figure 3.2 and Table 3.1$^1$.

A primary proton beam is used to produce the K$_L$ beam at a production angle of 2.4 mrad. This angle is chosen to give the optimal ratio of kaons to neutrons (there are approximately $10^7$ neutrons and $10^8 \gamma$'s per burst in the K$_L$ beam). The remaining protons in the beam are deflected further from the K$_L$ beam-line by a sweeping magnet. A bent silicon crystal is placed in the beam in such a way as to channel a small fraction ($\sim 5 \times 10^{-5}$) of the remaining protons back towards the beam-line$^2$. The majority of the protons, not channeled by the crystal or

\begin{table}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
 & HORIZ. RMS & VERT. RMS & PROD. ANGLE & LENGTH & COLL. DIAM. \\
 & x & x' & y & y' & $\theta$ & L & $\phi$ \\
\hline
K$_L$: & \approx 0.2 & \approx 0.4 & \approx 0.3 & \approx 0.1 & 2.4 & 40.62 & 12.2 \\
K$_S$: & \approx 0.2 & \approx 0.02 & \approx 0.3 & \approx 0.1 & 4.2 & 4.80 & 3.6 \\
\hline
\end{tabular}
\end{table}

$^1$ The information which is shown in this table was obtained from the URL http://www1.cern.ch/NA48/BeamLine/k0prod.html

$^2$ If a beam of charged particles is incident on a crystal at an angle which is close to a
interacting in the holder, continue undeviated and are absorbed in the tungsten-crystallographic direction (axis or plane), coherent scattering from the lattice nuclei forces the particles to follow the lattice direction. This phenomenon is known as "crystal channeling".
alloy inserts of a dump-collimator assembly. The KL beam is further defined by three collimators: the “defining”, “cleaning” and “final” collimators, positioned downstream of the KL target at a distance of 40, 100 and 125 m respectively. The “defining” collimator defines the beam profile whilst the later collimators are used to remove particles produced by the edges of the preceding collimators.

Downstream of the dump the protons channeled by the silicon crystal are deflected back towards the KL axis, passing through a series of “tagging” counters, before being deflected along the KL axis. 100 m downstream of the crystal the protons are deviated, by dipole magnets, to a point 72 mm above the KL axis where the KS target is situated. The KS target is of similar dimensions to the KL target and the beam is arranged so that the production angle is 4.2 mrad. This angle produces a KS beam in which the momentum spectrum is as similar as possible to that of the KL beam (in the range 70–170 GeV/c). The KS target is followed by a dipole sweeping magnet, packed with tungsten-alloy inserts, in which all remaining protons are finally absorbed. After the dipole magnet there is a collimator which ends 6 m after the KS target and defines the direction and divergence of the KS beam. The KS beam axis converges on the KL axis at the electromagnetic calorimeter 122 m downstream of the KS target. The angle between the KS and KL beams is 0.6 mrad.
The proton and kaon beams are transported in vacuum throughout the 250 m from the $K_L$ target to the last subdetector apart from a region of 4.6 m length at the crystal and a short region at the tagging station [25].

In addition to the proton tagger, in the beam transported to the $K_S$ target, there are two other subdetectors associated with the beam: the anti-$K_S$ (AKS) and the beam monitor. The AKS defines the beginning of the fiducial region for $K_S$ decays and is also used to determine the neutral energy scale of the electromagnetic calorimeter (see section 3.1.3). The beam monitor is used for measuring the beam profiles at the end of the experiment, immediately before the beam dump (the position is shown in Figure 3.1).

### 3.1.2 The Proton Tagger

The $K_S$ or $K_L$ assignment for each decay is done by measuring the time difference between the passage of a proton in the tagging counter upstream of the $K_S$ target and the event time in the detector. Events with a time difference inside of a given interval $\Delta t$ will be called "$K_S$", any other events will be called "$K_L$". The performance requirements are:

- a high rate capability ($3 \times 10^7$ ppp) with deadtime-less readout,
- a time resolution $\sigma_t < 100$ ps,
- double pulse resolution down to 7 ns,
- light material to preserve the proton beam quality,
- radiation hardness (1 SPS burst corresponds to 1 Gy),
- high efficiency.

Inefficiencies in the tagging counter such as misalignment and deadtime will cause $K_S$ decays to be incorrectly identified as $K_L$ decays, whereas accidental hits will cause a $K_S$ decay to be misidentified as a $K_L$ decay. These misidentifications are decay-mode independent. They lead to a correction of the double ratio but cannot generate an artificial non-zero value of $Re(\epsilon'/\epsilon)$ provided that the time window
generated by the charged hodoscope, for charged events, and the electromagnetic calorimeter, for neutral events, has the same mean time and jitter.

To ensure that the rate in each counter is less than 1 MHz the tagger is designed with two sets of staggered scintillation foils arranged alternately in the horizontal and vertical planes. The depth of the foils in the beam direction is 4 mm and the width varies from 200 μm on the beam-axis to 3000 μm at the beam edges. Thus the entire beam profile is covered. The arrangement of the scintillation counters is shown in Figure 3.3. The scintillators are attached to a carbon-fibre support structure leaving a central beam passage of 8 mm × 8 mm free of material except for the scintillators. The tagger is aligned such that scintillator foils are parallel to the proton beam. The alignment ensures that the multiplicity of hits for each proton passing through the detector is low and that there are no geometrical inefficiencies.

Anticounters (A1 and A2) are fixed on both sides of the structure to identify beam halo particles and special trigger counters (T1 and T2) are mounted in front of and behind the tagging counter for efficiency studies.

The tagger is read out using a custom built 8-bit 1 GHz FADC. The time resolution of this system is better than 50 ps. This system comprises a pipe-lined readout which is fed into an optical link transmitter and merged with the dataflow from the other subdetectors [26]. Due to the longitudinal dimensions of the tagger there is a time difference of 1 ns between the two counters at each end. The time-of-flight of the proton in the tagger must be taken into account in the offline analysis to determine whether a hit in one of the foils corresponds with an event in the other subdetectors.

### 3.1.3 The Anti-K$_S$ (AKS) Detector

The AKS counter is situated in the K$_S$ beam immediately after the K$_S$ collimator. It defines the beginning of the fiducial region for K$_S$ decays. It is comprised of a scintillator assembly, read out by photomultipliers, with two radiation lengths of tungsten/iridium converter upstream. Undecayed K$_S$ pass through a hole in the center of a veto counter, then through the converter, followed by three thin scintillators, as shown in Figure 3.4.
Decays upstream of the AKS will be detected in the scintillators. The charged pions from $K_S \rightarrow \pi^+\pi^-$ are detected directly in the thin scintillators while the tungsten/iridium is used to convert gammas which are produced by the neutral decays of $K_S \rightarrow \pi^0\pi^0 \rightarrow 4\gamma$. It has been proposed to use a tungsten/iridium crystal as the converter in preference to amorphous metal as a strong enhancement of pair production from photons is observed when the photon direction is aligned with a crystal axis. The factor by which the effective radiation length, $X_0$, is reduced in comparisons between $X_0^{\text{Amorphous}}$ and $X_0^{\text{Crystal}}$ is between 1.5 and 5 depending on photon momentum and the angle it makes with the crystal axis [27]. It should thus be possible to use a thinner converter in the AKS while maintaining high efficiency for photon conversion and also reducing the scattering of kaons in the converter due to nuclear interactions (elastic and inelastic). During 1995 an
amorphous converter was used while in 1996 a motor driven mounting on the AKS allowed the choice of either an amorphous or crystal converter.

For charged decays scattered kaons can be observed from the reconstruction of the decay vertex, using the spectrometer information. Such events will show up as having a vertex outwith the normal $K_S$ beam profile. However for neutral decays only the $z$-coordinate of the decay vertex is known accurately so it is difficult to determine if the kaon has been scattered or not. Hence it is desirable to keep the amount of material in the $K_S$ beam as small as possible.

The purpose of the AKS is in defining the beginning of the fiducial region for $K_S$ and thus also in determining the absolute energy scale of the electromagnetic calorimeter. Using the relation between the neutral length and energy scales:

\[
Z_0 = Z_{cat} - d_0 \tag{3.1}
\]

\[
d_0 = \frac{\sqrt{\sum_{i,j=1}^4 E_i E_j ((x_i - x_j)^2 + (y_i - y_j)^2)}}{m_{K^0}} \tag{3.2}
\]
3. The Experiment

where \( d_0 \) is the distance of the decay from the calorimeter, \( Z_{\text{cal}} \) is the \( z \)-position of the calorimeter, \( Z_0 \) is the \( z \)-vertex of the neutral decay, \( E_{i/j} \) are the energies of the showers in the electromagnetic calorimeter, \( x_{i/j} \) and \( y_{i/j} \) are the coordinates of the centres of the showers and \( m_{K^0} \) is the kaon mass, the decay distribution of \( K_S \rightarrow \pi^0\pi^0 \rightarrow 4\gamma (Z_0) \) can be obtained. After corrections for acceptance this distribution should correspond to an exponential, \( Z_0 \propto e^{-z/\gamma m_{K_S}} \) (for fixed momentum), with the beginning of the distribution corresponding to the \( z \)-position of the AKS. Fine tuning of the energy scale of the calorimeter can be made by fitting the edge of the \( Z_0 \) distribution to the AKS position.

3.1.4 The Beam Intensity Monitor

The beam monitor, shown in Figure 3.6, is used to measure the beam profile and intensity of either \( K_S \) or \( K_L \) if the beams are operated separately. If both beams are measured concurrently it is dominated by the \( K_L \) component, the \( K_S \) component being of the order \( 10^{-3} \). It is situated at the end of the hall, immediately before
the beam dump, as shown in Figure 3.1.

The beam intensity monitor is designed as a scintillating fibre matrix. Two planes of scintillating fibres are oriented in the vertical and horizontal direction. In each projection there are 24 fibre bundles each separated by 7.8 mm. Three fibre bundles are read out by a common photomultiplier. The overall dimensions of the detector are 180 mm \times 180 mm of which 7\% is active material. This sampling was necessary to ensure that the rate in each counter did not exceed a few MHz.

![Fig. 3.6: The Beam Monitor.](image)

In addition to providing measurements of the beam intensity the beam monitor signals are prescaled and delayed by 69 \(\mu s\) to produce random triggers. The 69 \(\mu s\) delay is introduced to remove any correlation between real events in the detector and hits in the beam monitor and, as this delay corresponds to three times the SPS RF period, the instantaneous intensity \(I(t_0 + 69 \mu s)\) should be equal to \(I(t_0)\) [28].

### 3.2 The Decay Region

The following sections describe the fiducial region of NA48 and the associated detectors upstream of the spectrometer and the central detector.
3. The Experiment

3.2.1 The Vacuum Tank

The decay fiducial region, shown in Figure 3.7, is contained in a 330 m$^3$ cylindrical steel vacuum tank. The inside diameter is 1.9 m over the first 40 m and 2.4 m over the remaining 48 m of its length. It is intended that the vacuum tank be evacuated to a pressure of $< 10^{-3}$ mbar. The downstream end of the tank is terminated by a thin, concave Kevlar window of 0.8 mm thickness. After the Kevlar window is the helium tank of the magnetic spectrometer followed by the subdetectors of the central detector. Outside of the vacuum tank and the helium tank are placed seven octagonal anti-counters (AKL).

3.2.2 The Anti-K$_L$ (AKL) Detectors

The AKL detectors are used to veto events in which the decay products' trajectories are outwith the active volume acceptance (defined by the electromagnetic calorimeter). When measuring $Re(\varepsilon'/\varepsilon)$ they are primarily used to remove $3\pi^0$ decays which occur at a rate which is 200 times that of the desired $K_L \rightarrow 2\pi^0$ mode.

The counters are arranged in seven octagonal rings, at suitable longitudinal positions, such that they cover a large solid angle outside of the detector. The inner radius of the rings increases in size going down the vacuum tank in the direction of the central detectors while the outer radius stays almost fixed. The radial acceptance and the $z$ coordinate, measured from the $K_S$ target, of each pocket are reported in Table 3.2. $R_{in}$ gives the radius of the circle inscribed in the octagon and $R_{ext}$ the maximum outer radius fully covered by the scintillator plates.

A schematic of the AKL counters (pockets) is shown in Figure 3.8. Each pocket contains two layers of NE110 scintillator, of 10 mm thickness, each of which is preceded by 35 mm of iron. Pockets 1–4 contain 24 counters and pockets 5–7 contain 16. Each of these counters are read out at both ends using EMI9814 photomultipliers. Custom-made electronics discriminate the signals, using a constant-fraction discriminator (CFD), then the signals from each end are put in coincidence using a “mean-timer”. Finally the logical ORs of the signals from each anti-counter ring are recorded in a Pipelined Memory Board (PMB) clocked at 40 MHz.
3. The Experiment

3.3 The Magnetic Spectrometer

The momenta of charged particles are measured in a magnetic spectrometer consisting of a central dipole magnet and two sets of drift chambers on each side (as
3. The Experiment

<table>
<thead>
<tr>
<th>pocket no.</th>
<th>$R_{in}$ (mm)</th>
<th>$R_{ext}$ (mm)</th>
<th>$z$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>480.</td>
<td>960.</td>
<td>28.887</td>
</tr>
<tr>
<td>2</td>
<td>600.</td>
<td>960.</td>
<td>41.227</td>
</tr>
<tr>
<td>3</td>
<td>720.</td>
<td>1200.</td>
<td>53.538</td>
</tr>
<tr>
<td>4</td>
<td>820.</td>
<td>1200.</td>
<td>65.762</td>
</tr>
<tr>
<td>5</td>
<td>920.</td>
<td>1200.</td>
<td>77.987</td>
</tr>
<tr>
<td>6</td>
<td>1100.</td>
<td>1390.</td>
<td>99.495</td>
</tr>
<tr>
<td>7</td>
<td>1200.</td>
<td>1390.</td>
<td>112.095</td>
</tr>
</tbody>
</table>

Tab. 3.2: AKL Pocket coverage and position.

Fig. 3.8: The AKL counters.

shown in Figure 3.9). The field integral is equivalent to a change in transverse momentum, $p_T$, of 250 MeV/c. The spaces between the chambers are filled with helium. The gas is required to equalize the pressure between the interior of the drift chambers and the fiducial region while minimising the multiple scattering of charged particles.

The magnetic spectrometer is used for fast, online reconstruction of the decay
vertex and the two-particle invariant mass. This allows the rejection of decays outwith the fiducial region and also to remove 3-body decays, such as $K_{e3}$ ($K_{e3} \equiv K_L \rightarrow \pi^+ e^+ \nu_e$) and $K_{\pi3}$ ($K_{\pi3} \equiv K_L \rightarrow \pi^+ \pi^- \pi^0$), when measuring $Re(\varepsilon'/\varepsilon)$.

![Diagram of the Magnetic Spectrometer](image)

**Fig. 3.9: The Magnetic Spectrometer.**

### 3.3.1 The Magnet

The central dipole magnet (MNP33) is shown in Figure 3.10. The magnet has an aperture of 2.40 m and an overall height of 4.00 m. The field integral of the main field component ($B_y$) for particles passing through the centre of the magnet is $\int B \, dl = 0.83 \, \text{Tm}$ which is equivalent to the $p_T$ "kick" of 250 MeV/c mentioned previously. The fringe field at the location of the inner drift chambers, at $z = \pm 2.70$ m, is less than 0.02 T and the non-uniformity of the field integral within the fiducial region is better than 5% with the exception of the areas close to the coils [29].

### 3.3.2 The Drift Chambers

There are four drift chambers in the NA48 spectrometer, with 8 planes of 256 wires in the first, second and fourth chambers, and only four planes in the third chamber. The three similar chambers have planes with wires in the orientation
Fig. 3.10: The Central Dipole Magnet (MNP33).

"xx'y'y'uu'vv'" and the remaining plane "xx'yy". The eight planes of grounded sense wires are oriented in four different directions orthogonal to the beam axis: $0^\circ (x, x'), 90^\circ (y, y'), -45^\circ (u, u')$ and $+45^\circ (v, v')$. Each view contains two staggered planes of wires to resolve left-right ambiguities [30].

A schematic of one of the drift cells is shown in Figure 3.11.

The readout electronics link the amplifiers at the drift chamber on one end to the Data Merger on the other end. The function of the electronics is to measure and record the arrival time of signals, to store the data for sufficient time to build a
trigger, to release and format the data on request by the trigger and to detect and report errors. The readout electronics is identical for each chamber and is housed in a single VME crate next to each chamber. There is, in addition, a “Master” crate in which data from each individual chamber are collected, signals are broadcast and commands are received from the run control program.

Each plane is equipped with two electronics modules called the TDC card and the Ring card. The TDC card measures the time of hits in the signal wires relative to the 40 MHz experimental clock in bins of 1.6 ns (1/16 of a clock cycle). Addresses and times of the channels which were hit are stored in an on-chip FIFO. The content of the TDC registers is transferred to an output FIFO from where it is sent to the Ring card [31].

Note: The third drift chamber (DCH3 in Figure 3.9) was not in place during the 1995 run but it is not strictly necessary as a vertex can be calculated from chambers 1 and 2 and with a knowledge of the field map and track segments reconstructed in chamber 4 the momentum of particles can be calculated. The design specification for the full spectrometer with four chambers was to obtain a kaon mass resolution of 3 MeV/c² and with three chambers a mass resolution of 2.5 MeV/c² has already been achieved. The principal use of the third chamber is to resolve geometrical ambiguities and to improve the efficiency.

3.4 The Central Detector

This section describes the elements of the central detector. These are the subdetectors which are used to record the decay products of the kaons. Each subdetector
has a specific task and it is by combining the information from all subdetectors that particle types and energies can be obtained.

3.4.1 The Charged Hodoscope

The charged hodoscope is used to define the event time, $t_0$, of each charged event. It is very important that the event time, produced by a trigger from this subdetector, be defined with sub-nanosecond accuracy. This is necessary as the event time is used as a "gate" to check for protons passing through the tagger. The lower the resolution of $t_0$ the larger the gate has to be in order to compensate for fluctuations. Increasing the size of the gate increases the probability of identifying a $K_L$ decay as a $K_S$ due to accidental coincidences within this time window. This will have a systematic effect in the measurement of $Re(\varepsilon'/\varepsilon)$ as, unlike the case of accidentals in the tagger, a distinction is made between charged and neutral modes and so the "mis-tagged" events will not cancel in the double ratio (Equation 2.43 on page 14).

The charged hodoscope is shown in Figure 3.12. It consists of two planes of scintillator counters: 64 horizontal counters in the first plane and 64 vertical counters in the second plane. Each plane is divided into four quadrants of 16 counters. The scintillation light from each counter is collected from one side via a plexiglass light guide, with a fish-tail shape, and recorded by a photomultiplier. The opposite end of the counter has been roughened and painted with an absorbing black coat to avoid reflected light from hitting the photomultiplier.

The distance between the two planes is the maximum available $^3$, and not less than 50 cm. This allows the separation of prompt signals from those due to back-scattering from the calorimeter.

The signals available from the hodoscope trigger logics are shown in Table 3.3. Each of these trigger signals have an online resolution of the order of 1 ns. The $Q_x$ condition is defined to be a hit in opposite quadrants of the hodoscope. It is important for the measurement of $Re(\varepsilon'/\varepsilon)$ as the desired charged decays ($\pi^+\pi^-$) are two-body and so the $p_T$ must sum to zero. This will filter out some of the

---

3 There is an 80 cm separation in the 1995 run, with the LKr calorimeter not in place, and a 70 cm separation in the 1996 run, with the LKr calorimeter in place.
3. The Experiment

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Fig. 3.12: The Charged Hodoscope.

<table>
<thead>
<tr>
<th>Charged Hodoscope Trigger Signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORH</td>
</tr>
<tr>
<td>ORV</td>
</tr>
<tr>
<td>TOTAL OR</td>
</tr>
<tr>
<td>COINCIDENCES</td>
</tr>
<tr>
<td>(Q_{13}^x)</td>
</tr>
<tr>
<td>(Q_{24}^x)</td>
</tr>
<tr>
<td>(Q_x)</td>
</tr>
</tbody>
</table>

Tab. 3.3: Charged Hodoscope Trigger Signals.

three-body decays, in which this condition is not necessarily met\[32\].
3.4.2 The Electromagnetic Calorimeter

The energy and position of decay products from neutral decays (\(K_{L/S} \rightarrow 2\pi^0 \rightarrow 4\gamma\)) are recorded in a large, homogeneous, liquid krypton (LKr) calorimeter which is designed for very high particle rates and excellent energy, space and time resolution, i.e.:

\[
\begin{align*}
\text{Energy} & : 4.1\%/\sqrt{E[GeV]}, \\
\text{Space} & : 4.5mm/\sqrt{E[GeV]}, \\
\text{Time} & : 0.5ns \text{ for } E > 10GeV.
\end{align*}
\]

Two showers should be resolved if separated by more than 4 cm.

To achieve these design goals a LKr calorimeter with a tower read-out structure has been developed. This structure is essential to guarantee good time resolution, to minimise the effect of overlapping clusters from pile up events and to reduce the ambiguity in pairing photons associated with each \(\pi^0\). A tower readout also permits low capacitance detector channels allowing fast ionization charge readout essential to cope with the \(\sim 1\) MHz single rate. The calorimeter structure is shown schematically in Figure 3.13.

The calorimeter is equipped with \(\sim 13,500\) cells formed by thin Cu-Be electrodes stretched longitudinally. The cell size is 2 cm \(\times\) 2 cm. Each cell is formed by two drift gaps sharing the collection electrode and read out by the same pre-amplifier (see Figure 3.14). The ribbons are passed through five slotted spacers made from glass-fibre reinforced epoxy resin. To minimise non-uniformities in response due to the position of the impact point of the particle relative to the collection electrode the electrodes are positioned so that they form a “zig-zag”. There are five such changes in direction for each electrode from front-plate to back-plate of the calorimeter: one at each of the spacers. At each spacer the deviation of the electrode is 50 mrad. Each drift gap is 1 cm wide, corresponding to a drift time of \(\sim 2.8\)\,\(\mu s\) for a bias of 5 kV. The active krypton length is 1.25 m corresponding to \(\sim 27X_0\).

During data acquisition the signals from each channel of the LKr calorimeter are shaped by a \([(CR)^2 - (RC)^4]\) circuit with differential and integration time constants of 20 nsec and read out using a “Calorimeter Pipeline Digitizer” (CPD).
Each CPD channel is a gain-switching, 10-bit FADC with 200 \( \mu \text{s} \) pipeline storage. There are four gains, chosen according to the pulse height of the incoming pulse, in order to increase the dynamic range of the system. The "effective" number of bits is 14.

As the LKr calorimeter has the potential to overwhelm the available bandwidth of the readout path (described in the next chapter) it is heavily zero-suppressed using a module known as the "data-concentrator".

**Note:** This subdetector was not in place during the 1995 run and will be commissioned during 1996 and 1997.

### 3.4.3 The Neutral Hodoscope

The neutral hodoscope determines the event time for neutral kaon decays (e.g. \( \Lambda \to \pi^0\pi^0 \to 4\gamma \)) and is installed inside the LKr calorimeter. Electromagnetic showers are registered by bundles of scintillating fibres housed inside Stesalit tubes of 7 mm, outer diameter. The tubes are stacked, in a vertical orientation, between the ribbons of the calorimeter such that they form a plane across the
active region of the calorimeter at a depth of 9.5 $X_0$. Several fibre bundles are coupled to one photomultiplier tube (10 per photomultiplier in the central region and 22 per photomultiplier in the peripheral part of the calorimeter). The signals from the bundles can be OR’ed logically to produce an online neutral event time independent of the offline reconstruction of event time in the LKr calorimeter.

Note: This subdetector was not in place during the 1995 run and will be commissioned during 1996 and 1997.

3.4.4 The Hadronic Calorimeter

The hadronic calorimeter (HAC) is used to record the energies and positions of particles showering in the material of this subdetector. In the final configuration this will mainly be charged pions, which will, in general, only deposit a small amount of energy in the LKr calorimeter. During the 1995 run in which there
was no electromagnetic calorimeter the HAC also had to record electromagnetic showers from electrons and γ’s.

The hadron calorimeter consists of an iron-scintillator sandwich of 1.2 m total iron thickness (6.7 nuclear interaction lengths). It is divided longitudinally into two separate modules (front: HACF, back: HACB), each consisting of 24 steel plates, 25 mm thick, of dimensions $2.7 \times 2.7 \text{ m}^2$ as shown in Figure 3.15.

Each scintillator plane, inserted between the steel plates, consists of 44 separate strips. Each strip spans only half of the calorimeter so that each plane is comprised of two half-planes. Around the beam hole the two central strips of each half-plane are 108 mm wide and 1242 mm long. In HACB the two outer strips in each half-plane are 1150 mm long and 108 mm wide. All other strips are 1300 mm long and 119 mm wide. The thickness of each scintillator is 4.5 mm. There are 24 planes in HACF and 25 in HACB. In consecutive planes the strips are alternately aligned in the horizontal and vertical directions. In each module the strips with an identical alignment (i.e. a stack of strips 24 (25) deep for HACF (HACB)) are coupled to the same photomultiplier using a plexiglass light-guide.

The high-voltage of the photomultipliers is adjusted so that there is a homogeneous response from all channels. This is done using muons recorded by a dedicated monitoring system. The absolute energy scale was determined, for electrons, during 1995 by a fixed energy electron beam which was scanned across the entire active region of the detector. The energy scale for pions may be determined by either a pion calibration beam and/or comparison with spectrometer data.

During 1995 the HAC was read out using a temporary Fast-Encoding-Readout-ADC (FERA) system (LeCroy module 4300B). These modules were read out over VME using the front-panel FERA bus to transfer the channel contents to a high-speed memory (HSM). From the HSM the data was transferred, using a “block-mover”, to the optical link boards where it was fed into the experiment dataflow. In 1996 this system will be replaced entirely by a CPD readout similar to that of the LKr calorimeter. In order to make the fast scintillator pulses of the HAC similar to slow pulses from electron drift in liquid krypton the pulses are shaped prior to input into the CPD modules.
The primary task of the muon veto is to eliminate $K_{\mu 3}$ ($K_{\mu 3} \equiv K_L \rightarrow \pi^\pm \mu^\mp \nu_\mu$) decays. It is also forseen that it could form a positive component of the trigger in studies of rare kaon decays.

The muon veto consists of three planes of NE110 plastic scintillator. The first two are 10 mm thick and the third is 6 mm thick. These planes are separated from each other by 0.8 m of iron which is sufficient to stop all particles, except for muons, from penetrating and triggering the detector\textsuperscript{4}. The iron walls between the planes prevent coincidences in the planes due to radioactive decay in the iron and the wall behind the third plane is required to absorb low-angle back-scattering from the beam-dump. The first two planes are used to produce the online muon trigger. The first plane consists of eleven horizontal strips (the central one having

\textsuperscript{4} A small number of pions penetrate the first iron wall and trigger plane 1. This is known as "punch-through"
3. The Experiment

Table 3.4: Muon Veto Trigger Signals.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1\mu$</td>
<td>single particle trigger (coincidence of planes 1 and 2)</td>
</tr>
<tr>
<td>$2\mu$</td>
<td>dimuon trigger (2 counters in each plane)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>OR of plane 1 counters</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>OR of plane 2 counters</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>OR of mean-timed plane 3 counters</td>
</tr>
</tbody>
</table>

Online the muon veto provides five trigger conditions, shown in Table 3.4.
In addition to the online triggers signals from each counter are read out into the dataflow every event. There is no pulse-height information only one bit to indicate whether the photomultiplier pulse was above threshold or not. During the 1995 run these pulses were read out using a Fast-Encoding-Readout-ADC (FERA) system which was in effect used as a pattern unit. From 1996 each channel will be read out using a 1 GHz 1-bit Flash-ADC (FADC) (called “TDC”) which will also provide timing information with 1 ns accuracy. Using the information from the counters in the two orthogonal planes the position of the muon can be found. Ambiguities due to multiple muons can be resolved using either the timing information from the TDC readout or using tracks extrapolated from the spectrometer to the muon veto planes.

During the test runs of NA48 in 1993 and 1994 I designed and implemented a monitoring and slow control system for the muon veto subdetector using conventional CAMAC electronics read out by a Macintosh IIfx computer. This system was designed to monitor the performance of the muon veto and provide slow control for the high-voltage (HV) supplied to the photomultiplier tubes. During 1995 this system was merged with a similar system for the hadron calorimeter. This monitoring system is described in detail in Appendix A.
4. THE NA48 TRIGGER AND DATAFLOW

In this chapter the trigger and dataflow schemes implemented in NA48 are described. The requirements, in terms of number of events needed to produce an acceptable statistical error, are outlined and the implications for the trigger system and the dataflow are described in more detail.

The trigger used during 1995 is also described.

4.1 Overview

In order for NA48 to measure $Re(\varepsilon'/\varepsilon)$ with an accuracy of $2 \times 10^{-4}$ the combined statistical and systematic errors, compared with NA31, have to be reduced by a factor of three. The dominant error sources in NA31 were:

1. $K_L \rightarrow \pi^0\pi^0$ event statistics,
2. possible differences in $\pi^0\pi^0$ and $\pi^+\pi^-$ energy scales,
3. background to $K_L \rightarrow \pi^0\pi^0$ by $K_L \rightarrow \pi^0\pi^0\pi^0$ decays with only four detected photons,
4. background to $K_L \rightarrow \pi^+\pi^-$ decays from residual three-body decays, especially $K_L \rightarrow \pi e\nu$ and $K_L \rightarrow \pi^+\pi^-\pi^0$,
5. differential effects of accidental activity in the detector between $K_S$ and $K_L$.

Systematics effects due to the difference in energy scales can be reduced by the choice of production spectra and by applying a weighting scheme to the events which depends on the initial state ($K_L$ or $K_S$), the decay mode, the $z$-vertex of the decay and the momentum of the kaon. Weights are applied to the $K_L$ decays such
that the resultant $z$-vertex distribution is identical to that of the K$_s$. Background in the charged and neutral decays are reduced by kinematic cuts imposed by sophisticated charged and neutral triggers described in this chapter. Differential effects due to accidental activity are reduced substantially by recording all four decay modes concurrently.

However, the major source of error comes from event statistics. In order to reduce the statistical error by a factor of three NA48 is designed to handle a tenfold increase in the beam intensity compared to NA31. It is important to note that this also decreases the systematic errors which are mainly limited themselves by statistical limitations on the test samples used to study the effects.

The design criteria for the trigger system are:

- high rate capability (up to 10 kHz),
- large and unbiased background rejection,
- minimal deadtime.

These criteria can only be met by a multi-level trigger used in conjunction with a high-speed, pipelined readout for each subdetector.

### 4.2 The Trigger System

The trigger design consists of three levels. Each stage is increasingly complex and implements more sophisticated cuts to reject background than the previous stage. The Level 1 and Level 2 triggers are implemented in hardware while it is intended that the Level 3 – software filtering of the data – will be implemented by a farm of processors which will reconstruct each event in real-time. Figure 4.1 shows the position of the levels in relation to the dataflow scheme.

#### 4.2.1 The Level 1 Trigger Logic

The Level 1 is a fast two particle trigger generated using the hodoscope, muon veto, anti-counters, chamber 1 multiplicity, energy sums from both calorimeters
and number of peaks in the LKr calorimeter. Each subdetector is driven by a 40 MHz clock which is generated centrally and distributed to each subdetector and the trigger supervisors. Information from each subdetector is clocked into the trigger supervisors continuously in 25 ns bins.

For charged decays the coincidence of hits in the charged hodoscope on opposite
sides of the beam-pipe \((Q_x)\) acts as the pre-trigger. During 1995 an alternative pre-trigger called \(Q2\) was used for the minimum bias trigger which was defined as a majority of two subcoincidences in the hodoscope planes. Information from the hodoscope, muon veto, anti-counters and drift chambers is passed on to the Level 1 Trigger Supervisor (L1TS).

For neutral decays there may be the possibility to use the neutral hodoscope as the pre-trigger (see Section 3.4.3). The Neutral Trigger itself combines parts of Level 1 and Level 2. Some information, such as energy above a certain threshold, are passed on to the L1TS while more complex information is passed directly to the Level 2 Trigger Supervisor (L2TS).

*The 1995 Trigger Setup*

During the 1995 run a simple trigger setup was used. Several trigger options were implemented by hardware in the pre-trigger and the functionality of Levels 1 and 2 were bypassed such that their only function was the transmission of the trigger word to the sub-detector front-end readout controllers. The front-end controllers would then send the part of their memory buffers corresponding to the event time to the “data-merger” (see Section 4.3).

The trigger bits implemented were:

- **1\(\mu\)**: single muon trigger from the muon-veto
- **2\(\mu\)**: dimuon trigger (majority of two hits in planes 1 and 2)
- **\(Q_2\)**: majority of two hits in the charged hodoscope
- **\(Q_x\)**: two hits in opposite quadrants of the charged hodoscope
- **\(EHL\)**: energy in hadron calorimeter greater than 6 GeV
- **\(EHH\)**: energy in hadron calorimeter greater than 40 GeV

*Note:* The values of the energy thresholds in the calorimeters were defined using a muon calibration and were found later to be underestimated – e.g. the \(EHL\) threshold is found to be 14 GeV from analysis of random events (see Section 7.1.1 on page 95).

The \(Q_2\) trigger is implemented using “sub-quadrants” as shown in Figure 4.2. A coincidence of any two of the sixteen sub-quadrants causes the \(Q_2\) trigger to fire.
The trigger conditions, during which the data analysed in this thesis was recorded, were:

\[ T = \mu\mu\gamma + (\pi\pi)D1 + (\text{min.bias})D2 + \text{random} \]

Where:

- \( \mu\mu\gamma \equiv Q_2.EHL.2\mu \) used for a rare decay study of \( K_L \rightarrow \mu\mu\gamma \)
- \( \pi\pi \equiv Q_x.EHH.1\mu \) used to record CP-violating decays of \( K_L \rightarrow \pi^+\pi^- \)
- \( \text{min.bias} \equiv Q_2.EHL \) “minimum-bias” trigger used to record 3-body decays of \( K_L \)

and D1 and D2 are downscaling factors of 1/128 and 1/256 respectively. These trigger bits were recorded in pipelined-memories and were used offline to select minimum-bias events only.

### 4.2.2 The Level 1 Trigger Supervisor (L1TS)

The purpose of the L1TS is to identify likely two-body charged decays so as to limit the rate of mass calculations in the second level charged trigger (L2C). It receives
4. The NA48 Trigger and Dataflow

information from the hodoscope, calorimeters, muon veto and the drift chambers and from this information determines whether a L2C calculation is required. If the L1TS decides to launch a mass calculation it inserts a request in the L2C queue along with the time of the event. A schematic of the L1TS is shown in Figure 4.3.

The L1TS consists of four parts. The first part aligns the information from the
various subdetectors with the global clock. The second stage compensates for the 
"edge effect" in which the inputs from different subdetectors are in different 25 ns 
bins. It is possible for this to occur when events are near the transition from 
one bin to the next. This compensation is achieved by making a logical OR of 
each time bin with the previous and subsequent time bins. On the basis of the 
hodoscope fine time, the output of a particular OR gate is selected. The third 
stage is a lookup table comprised of a 64K x 4 bit static RAM. Trigger conditions 
are selected by loading the memories accordingly. The 16 bit address is specified 
by the output of the second stage and the resulting data word is then the control 
word sent to the L2C. The fourth stage of the L1TS is the output stage. Output 
is sent to two places: the L2C queue and the L2TS. The information sent to the 
L2C queue is the trigger code, the time stamp corresponding to the bin of interest 
and a debug bit. For each time bin the L1TS also sends a complete copy of the 
information received and the trigger code generated to the L2TS.

4.2.3 The Neutral Trigger

The Neutral Trigger Dataflow

The neutral trigger monitors the signals coming from the LKr calorimeter and 
signals to the readout electronics whenever a \( \pi^0 \pi^0 \) candidate event has occurred. 
In the present design there is no pre-trigger.\(^1\) The calorimeter signals are con-
tinuously monitored and a new trigger decision is computed every 25 ns. This 
computation is fully pipelined and the result occurs at a fixed time delay after 
the incoming calorimeter pulses. The neutral trigger must notify the L2TS of its 
decision within 100 \( \mu s \) of the decay (see Section 4.2.5), however, with the pipeline 
design the decision will be available after about 5 \( \mu s \).

The LKr calorimeter readout and the Neutral Trigger chain is shown in Figure 4.4.
The 13340 cells of the calorimeter are summed up in (2 x 8) cells in both x and 
y views. This is done in an analogue unit. Each calorimeter channel features in 
the sum of one FADC in the x projection and one FADC in the y projection. The 
summed groups of channels are then flash digitised by 8-bit 40 MHz FADCs and

\(^1\) There may be the possibility to use the neutral hodoscope in an analogous manner to the 
charged hodoscope in the L1 charged trigger.
Fig. 4.4: The Neutral Trigger and the LKr Readout Path

passed on to a conditioning module which corrects for undershoots and removes pedestals. The groups of 16 are then summed digitally to produce energies in the whole of one row/column. These 1-D projections are then presented to the “peak-sum” chips. These chips are ASICs which are designed to calculate the number of peaks in x and y, $n_x$ and $n_y$, sum up the total energy in the x and y projections, $m_{0x}$ and $m_{0y}$, and calculate the first and second moments of the energy, $m_{1x}$, $m_{1y}$ and $m_{2x}$, $m_{2y}$ respectively (see also the section on “Neutral Trigger Algorithms”). These quantities are then passed on to 3 stages of lookup tables which combine the x and y peaks and sums the total electromagnetic energy with that of the hadronic calorimeter. The lookup tables also calculate the centre-of-gravity, the vertex position of the decay and the proper lifetime of the kaon. The output of the final stage is sent to the L2TS to be recorded and correlated with signals from other triggers.

The Neutral Trigger Algorithms

There are five main quantities which are computed for each time bin and which are used to make the trigger decision:

- The number of 1-dimensional peaks in x and y views. Good events are required to have $\leq 4$ peaks in each view in order to remove $K_L \rightarrow \pi^0\pi^0\pi^0$ background. This cut could be relaxed to $\leq 5$ peaks to allow for accidental activity in the detector.

- The total energy $E$. This must be above a given threshold for the event to be considered good. The hardware must interpolate between the time bins
of the FADC system to find the true maximum. The interpolation procedure also produces a precise event time to a fraction of a time bin.

- The centre-of-gravity of the FADC system to find the true maximum. The interpolation procedure also produces a precise event time to a fraction of a time bin.

  \[ C = \frac{\sum x_i E_i + \sum y_j E_j}{\sum E_i + \sum E_j} \]

  where the summations are over \( i \) (strip number in the x projection) and \( j \) (strip number in the y projection) respectively. The \( x_i \) and \( y_j \) are the coordinates of the center of each strip and the \( E_i \) and \( E_j \) are the sums of the energies contained within the cells in a horizontal or vertical strip in the calorimeter. \( C \) is required to be close to the centre of the beam axis, i.e. close to zero. Since the \( K_S \) and \( K_L \) beams have different divergences (angular spreads) this cut is required to be rather weak. It is required to remove single tracks.

- The vertex position of the decay \( z = z_c - d \) where \( d \) is the distance between the vertex position and the calorimeter and \( z_c \) is the (constant) distance between the collimator and calorimeter. \( d = \frac{\sqrt{(E(m_{2x} + m_{2y}) - E^2 C^2)/m_K}}{E_{2x} E_{2y}} \)

  where \( m_{2x} = \sum x_i^2 E_i \) and \( m_{2y} = \sum y_i^2 E_i \). The quantity \( d \) is required to be above some threshold.

- The proper lifetime of the kaon \( \tau = z/\sqrt{\gamma c \tau_s} \). This cut is equivalent to a cut in the \( E, d \) plane.

The \( K_L \) backgrounds are as follows. \( \pi^\pm e^\mp \nu \) (branching ratio 39\%), \( \pi^\pm \mu^\mp \nu \) (27\%) and \( \pi^+\pi^-\pi^0 \) (12\%) events are killed by the proper lifetime cut. The missing mass (either because it is carried by a massive neutral pion, or is missed in the neutrino) means that kinematically the lifetime will never be computed to be a low enough value to cause a trigger. However these events may fire the charged trigger. \( K_L \rightarrow \pi^0\pi^0\pi^0 \) events in which one or more reasonably energetic photons miss the calorimeter will also not pass the lifetime cut. \( K_L \rightarrow \pi^0\pi^0\pi^0 \) events in which all of the photons hit the calorimeter comprise the majority of the remaining background after the level 2 trigger.

4.2.4 The Level 2 Charged Trigger (L2C)

The L2C is designed to calculate the invariant mass and the decay vertex of charged kaon decays using information supplied by the drift chamber readout.
The Drift Chamber Dataflow

As in all other sub-detectors in NA48 the readout of the drift chambers is pipelined with no requirement for a pre-trigger or start signal. The chamber wires are continuously read out by modules, known as TDCs, which are driven by the 40 MHz clock. Each TDC chip is connected to 16 wires. As the occupancy of the chambers is low (2 out of 256 wires for a $\pi^+\pi^-$ event) the TDC modules incorporate zero-suppression of the data. In each 25 ns time bin the TDC provides a time-stamp, a fine time (4 bits, binned in 1/16th clock period increments of 1.6 ns @ 40 MHz) and the wire number (encoded in 4 bits). Successive hits are pushed into an internal 128 words deep FIFO memory. For each plane all the FIFO memories are merged, through a collection circuit, towards a single storage location where the time history of this plane is built. The serial storage rate capability of this merging tree is in excess of 10 MHz. The final collection FIFO in the tree is then read by a controller circuit which stores the resulting data in a time-stamp addressed circular buffer called a “ring”. At any time, the ring contains the wire hits that were observed in the last 200 $\mu$s time interval. The rings of each drift chamber are addressed by the L2C to extract the data from the required time.

The Implementation of the L2C

A schematic of the components of the L2C charged trigger is shown in Figure 4.5. The L2C is initiated upon a request from the L1TS. This request is broadcast to the ring controller of each drift chamber in the form of a time-stamp plus a fine-time derived from the hodoscope $Q_x$. The ring controllers then extract data from an 800 ns time interval and form structured event blocks which are sent to the next stage staggered plane coincidence and coordinate computation are performed.

Each view in a chamber is processed by a “Coordinate Builder” card. The wire hits in the two staggered A and B planes are observed to check vicinity in space and time. The drift values are used to calculate the associated coordinates. A look-up table contains individual time offset values for each group of 16 wires to take into account the small differences in cable length and also the time-of-flight between the chambers. The coordinate building is implemented in a pipelined circuit which
detects pairs of A and B hits in the same chamber cell and observes their drift times to decide if they are compatible with the event time. The coordinates are then passed on to the next stage of the L2C: the "Massbox".

The "Massbox" is comprised of an "Event Dispatcher" card plus four "Event Workers" which are implemented using a farm of VME-based DSP processors. The Event Dispatcher is a routing card which collects the coordinate packets coming from the Coordinate Builders and dispatches them to the Event Workers. The function of the Event Workers is to compute spacepoints, tracks, vertex and mass for the event. This process is split into several stages:

- The Event Dispatcher collects the data from the Coordinate Builders, serializes it and sends it to a free Event Worker.
- Using one or more processor the Event Worker associates projective coordinates $X, Y, U, V^2$ to produce space points.

\footnote{\textit{X}, \textit{Y}, \textit{U} and \textit{V} correspond to the 4 pairs of planes, $xx'$, $yy'$, $uu'$ and $vv'$ as described on}
• One worker, called the “Combinatorial Dispatcher” receives the spacepoints, generates all possible pairs of tracks and distributes them to a farm of DSPs called the “2-Track Workers”.

• Each 2-Track Worker computes the vertex and mass for this combination of tracks and sends its results to a “Collector DSP”.

• The Collector gathers the results of all the 2-Track Workers and sends them to a central interface which serializes the answers before transmitting them to the trigger supervisor.

If the Massbox cannot complete all its computations before the 100 $\mu$s deadline imposed by the L2TS it will stop all calculation and send a “don’t know” answer to the L2TS.

4.2.5 The Level 2 Trigger Supervisor

The L2TS is the destination for trigger decisions from the L1TS, containing information from the charged pre-trigger, the L2C and the “neutral trigger”. It correlates the trigger information from the local trigger sources, makes a trigger decision and dispatches it to the front-end systems in order to initiate a readout. Some other tasks that it has to perform are trigger counting, trigger downscaling, recording of trigger conditions, dead-time control and trigger monitoring. A block diagram of the L2TS is shown in Figure 4.6.

The L2TS can receive both synchronous and asynchronous trigger information. Each trigger source can provide between 8 and 24 bits of data and a data strobe to validate them, at a maximum continuous rate of 40 MHz. The asynchronous trigger systems (e.g. the L2C) also provide a 27-bit time-stamp labelling each individual 25 ns time-slice within a 2.5 s beam spill, while for the synchronous trigger sources the time information can be provided either by the source or be reconstructed within the L2TS by dedicated 40 MHz counters.

Trigger data from different sources is re-synchronized before a decision can be taken. This is performed by storing the incoming data in dual-ported 4K fast
static RAMs, one RAM per trigger source, at an address which is derived, in the case of asynchronous systems, from the 12 low-order bits of the time-stamp coming with the data. For synchronous systems the address may be generated internally by the L2TS. The chosen memory size allows for a maximum of 102.4 μs latency of trigger data, and this is therefore the time deadline for trigger data to arrive at the L2TS.

Simultaneously with asynchronous writing the synchronization memories are also
read out in a sequential way by the second port at a 40 MHz rate, collecting all
the stored trigger information after a fixed time delay of 100 \( \mu s \) since the event
time. The extracted trigger data bits are also presented in parallel on an output
port to be continuously read and stored by Pipeline Memory Boards (PMBs). The
information in these PMBs will be available in the dataflow as part of the charged
hodoscope readout.

The local trigger information extracted from the synchronization memories is then
passed to a set of cascaded look-up tables which are also implemented in fast static
SRAMs. These SRAMs are pre-loaded to give the required response when a valid
trigger is seen. A decision from the third level of the look-up table is then stored
in the Trigger Queue Buffer (TQB) along with the 27-bit time-stamp of the “event
time”. The main purpose of the TQB is to de-randomize triggers in such a way
that a fixed minimum time interval between them is guaranteed. Whenever trigger
information is present in the TQB and the minimum time from the last trigger
dispatched has elapsed the data is extracted from the TQB and passed to the
transmission stage.

The transmission stage receives a 59-bit trigger packet consisting of the 16-bit
event number, the 16-bit trigger word and the 27-bit time-stamp. It has now to
send the trigger packet to each subdetector readout system on fast serial links. At
the receiving end the packet is de-serialized and checked for transmission errors.
No acknowledge protocol is implemented. The trigger word is decoded and the
time-stamp used to extract data from the front-end memories.

During the 1995 run a prototype L2TS system (pTS) was used. The prototype
has the same functionality as the complete system but accommodates fewer trigger
sources and provides signals for fewer trigger destinations. This did not pose
a problem due to the absence of the LKr calorimeter and the combining of the
hadron calorimeter and muon veto readout systems for this run.

4.2.6 The Level 2B Neutral Trigger (L2B)

The primary goal of the Level 2B trigger is to complement the selection power of
the main pipelined neutral trigger. It is used to apply more sophisticated cuts in
the cases of neutral events which look like background but could be good events
with accidental activity superimposed. The L2B is software based and so can be easily adapted to deal with poor gain uniformity, poor online time resolution or dead channels in the calorimeter. It is also able to consider charged events. The L2B can be used to look for energy clusters in events, correlate them to momentum measurements from the L2C and supply a rejection of electrons of $K_e^0$ background by applying a cut on the cluster energy, $E$, divided by the track momentum, $p$. For electrons $E/p$ tends to be large compared to pions, which may not interact until they enter the hadron calorimeter and, even in the case that they do interact in the krypton, the longitudinal development of hadronic showers is such that some energy may "leak" from the back of the LKr calorimeter and be deposited in the hadron calorimeter. These effects contribute to a low $E/p$ value for pions.

The L2B is highly integrated with the LKr calorimeter readout, and performs processing in parallel with the zero suppression of the calorimeter data. When a trigger is received by the LKr readout data is fetched from the front-end memories and wait for an available processing slot in the readout hardware. As the data is read out a summary is also created and sent to the L2B computer. The L2B performs it’s cuts and if it decides that the event should be rejected it instructs the LKr readout to suppress the data and send an "empty event". The other subdetectors will send full events. The event will finally be killed completely by the Level 3 trigger.

The input data passes to an peak finding program which reconstructs energy, position and relative arrival time of each $\gamma$ found. The algorithm proposed is relevant only for events which have 5 or 6 $\gamma$’s from the peak finding program. If the event has 0 to 4 or more than 6 peaks it is immediately selected and further rejection will only be done offline.

If the individual photon times are not consistent with a single event then the event is accepted as it may be an interesting event with an accidental.

For the remaining events the excess photons are each put to one side and each 4 photon combination is tested against the hypothesis of a $2\pi^0$ event. The pion masses are reconstructed and required to be consistent with the known pion mass. For 5 photon events 15 combinations are tested and for 6 photon events 45 combinations are tested. If any combination reveals a plausible $2\pi^0$ event then the event is kept, otherwise the event is rejected.
4.2.7 The Level 3 Trigger

During the 1995 run there was no Level 3 trigger (L3) implemented. In future it is intended that every event should be reconstructed in “real-time” by the offline reconstruction program and further cuts applied to the data before it is written out to disk or tape. Essentially two implementations have been proposed so far:

- reconstruction in the front-end workstations (FEWS) as data arrives from the dataflow,
- reconstruction by the Meiko CS-2 at the CERN central data recording facility.

The FEWS and the CS-2 are described further in Section 4.3.

Without reconstruction of the tagger or LKr calorimeter the reconstruction time for one event is about 100 ms (Sun HyperSPARC processor on CS-2). The final trigger rate is expected to be about 3 kHz which implies 7500 triggers per 2.6 s burst. A burst occurs every 14.4 s so the “DC” load is about 500 Hz. This implies that a L3 implemented on the CS-2 would require a minimum of 25 nodes (2 processors per node). With LKr and tagging this may increase by 50-100% requiring up to 50 nodes of the CS-2 which will operate with 64 nodes in 1996.

4.3 The Dataflow

4.3.1 Overview

The expected rate from the L2TS during data taking for the $e'/\epsilon$ measurement will be approximately 1.5 kHz from $K_L$ and 0.15 kHz from $K_S$. Interleaved with these triggers may be a number of “rare decay” triggers, a heavily downscaled fraction of “low bias” events to check trigger stability (of the order of 1 kHz rate) plus calibration events, to check pedestals and random activity in the detectors, at a rate of approximately 0.2 kHz. Thus the overall rate from the level 2 is expected to be of the order of 3 kHz. For a 2.5 s spill this implies 7.5 k events/burst.
The data volume is dominated by the liquid krypton calorimeter readout but is very heavily dependent on the efficiency of the zero-suppression and data-compression algorithms. For a $K^0 \rightarrow 2\pi^0 \rightarrow 4\gamma$ four showers will be generated in the LKr calorimeter each of which will be contained in a 15 x 15 shower box. Thus the total number of cells to be read out, after zero-suppression, is of the order 900. For each cell 8-10 time slices from the 14-bit FADC system will be read out: 3 before the pulse to measure the pedestal effects (essential in an DC-coupled device where the pile-up of events causes the pedestal to drift) plus 5-7 around the peak to measure the amplitude and time of the pulse. This leads to an uncompressed, zero-suppressed data volume of 14-18 kbytes/event. The combined data volume from the drift chambers, hodoscope, anti-counters, muon veto, tagger and triggers will be of the order 2-3 kbytes/event. Therefore the total data volume per event is expected to be of the order 7-21 kbytes (the larger events being the neutral events). The typical event length is taken to be 10 kbytes/event.

Assuming the above estimates for the trigger rates and event length the dataflow has to be able to read out at a rate of 30 Mbytes/s. Allowing for the fact that the trigger is not as good as expected, or that zero-suppression or data-compression is not as efficient as hoped for, the readout has been designed for a 10 kHz trigger rate from the level 2 and an overall data rate of 100 Mbytes/s.

During the run in 1995 the trigger rate was limited to 1.5 kHz due to problems in the zero-suppression of the hodoscope readout system plus dead-time introduced due to the temporary FERA readout of the HAC. The HAC readout will be replaced in 1996 with a CPD readout similar to that of the LKr calorimeter (see Section 3.4.4). The event size was typically 2-3 kbytes and thus well below the design limit of the dataflow system.

To achieve such a high bandwidth the readout is implemented mainly as simple point-to-point links between buffers and FIFO memories. Only vertical transfers occur between the elements and there is no horizontal linkage between the data sources until the "data merging". A very simplified schematic of the readout is shown in Figure 4.7.

Flow control for the readout is implemented using an XON-XOFF protocol. Each source sends data at its maximum rate until inhibited by an XOFF from the destination. The condition of an XOFF propagating back to the front-end pipelined
memories of the subdetectors' readout is treated as an error condition and should not occur. The L2TS is responsible for ensuring that the rate at which triggers are issued does not exceed the capacity of the system.

In order to distribute the data-transfer load from the "AC"-like characteristics of one SPS 2.6 s burst every 14.4 s to a "DC", load large memory buffers are used to store the data. These are implemented as 320 Mbyte fast memories in DEC Alpha 3000/500 workstations.

### 4.3.2 The Hardware Implementation

Physically the dataflow is regarded as that part of the experiment from the readout controllers of each subdetector until the data is archived to hard-disk or magnetic tape. This data path is comprised of several interconnected subsystems. A schematic of the hardware implementation of the dataflow is shown in Figure 4.8.
The individual hardware subsystems are described as follows:

- **Subdetector**: A VME or FastBus crate containing a card with a fibre-optic link source module. A subdetector generates sub-events which are sent over the optical link to the data merger.

- **Data Merger (DM)**: A 9U VME crate containing up to 8 cards called Input Buffers (IB). Each IB has two channels, each of which is equipped with an optical link destination module. The crate contains standard VME
backplanes in J1 and J2 and a custom-developed high-speed bus, called R-path in J3. A module called the FOF, which contains a high-speed FIFO and a HiPPI source module, assembles sub-events read out from each IB channel over the R-path. The IBs are read out sequentially using a "Token Ring" protocol. The FOF sends the assembled events over a HiPPI link to one of the front-end workstations via a HiPPI switch.

- **HiPPI Switch**: A commercial HiPPI cross-bar switch equipped with 4 input and 4 output ports (know as a $4 \times 4$ configuration). This may be extended to $8 \times 8$ at a later date. A word in the data packet sent from the FOF, known as the I-field, tells the switch which Front-end Workstation should receive the event. The I-field is written to a FIFO in the FOF by the Data Merger Controller (DMC) before the beginning of each spill so that all events from one spill are received by the same Front-end Workstation.

- **Front-end Workstation (FEWS)**: The Front-end Workstations are TURBOchannel based DEC Alpha workstations each of which is equipped with one custom-designed HiPPI to TURBOchannel interface, an FDDI interface and a large, fast 320 Mbyte memory buffer. The HiPPI to TURBOchannel interface allows the workstation to accept HiPPI packets sent from the FOF via the HiPPI switch. An entire burst (75–250 Mbytes) is stored in the memory buffer of one front-end workstation. The FEWS perform some elementary data integrity checks during a spill before making the data available to the CS-2 via dedicated FDDI links. The FEWS also make available some of the data to the monitoring workstations via Ethernet.

- **Meiko CS-2**: The Meiko CS-2 is a parallel computing surface based on SUN HyperSPARC processors. These processors communicate via a high-speed internal network implemented using $8 \times 8$ cross-point switches. These links are capable of data transfers of up to 50 Mbyte/s between processors. The current system has 32 nodes each of which is comprised of two CPUs plus connections to the network switches (in 1996 the number of nodes will be increased to 64). The machine is equipped with 4 FDDI interfaces, 1 UltraNet interface and 1 HiPPI interface. The CS-2 is part of the CERN Central Data Recording (CDR) system. During data taking the CS-2 receives data files, containing data gathered during an SPS spill, from the FEWS. The data is
transferred via an FDDI concentrator over a single FDDI link between the control room and the CS-2 at the CERN computing centre. The CS-2 can perform full physics analysis of the data after archiving it to hard-disk or magnetic tape.

The hardware subsystems of NA48 are connected using the following links:

- **Optical Links**: The task of the optical link is to deliver, asynchronously, data from the ReadOut Controller (ROC) of a subdetector to the Input Buffers of the Data Merger. The links transfer data at 10 Mbytes/s over multi-mode optical fibres of around 200 m length.

- **HiPPI Links**: The HiPPI links between the Data Merger and the FEWS are uni-directional, point-to-point links. The HiPPI link is 32-bit wide and is capable of sustained transfer rates of up to 90 Mbyte/s. Links up to 25 M in length are implemented using copper twisted-pair cable. Longer links are implemented using Serial HiPPI which is transferred over single-mode optical fibres.

- **FDDI Links**: The FDDI links between the FEWS and the CS-2 are standard FDDI links over a single multi-mode optical fibre. Each FDDI link is capable of sustained data transfer rates of 6–7 Mbytes/s.

- **EtherNet**: The EtherNet linking the FEWS with the Monitoring Workstations is a standard EtherNet over 50Ω coaxial cable. This connection is capable of a few 100 kbytes/s.

### 4.3.3 The Data Transmission Mechanism

Each time a trigger is issued to a subdetector it produces a data packet (a *subevent*). The format of the data packet is defined by the ROC of each subdetector and is in general different between subsystems. The task of the Data Merger is to collect the subevents and to concatenate them to form an *event*. This is then sent to a workstation farm for processing.

Each subdetector is equipped with a ROC which interfaces the subdetector electronics to an optical link. The optical link transfers the subevent from the ex-
peridental hall to the control room where it enters an IB in the Data Merger crate.

Events are stored in the IB until required for readout. There is one IB channel per optical link. The readout is controlled by a token-passing scheme. When an IB receives a token it transfers its oldest event onto the R-path backplane before passing on the token to the next IB.

Events transferred along the R-path are received by the FIFO Output Formatter (FOF). The FOF initiates the readout of the IBs by issuing a token to the first IB channel. It then reads all the data on the R-path into a FIFO until it receives the token back from the last IB and thus builds a global event. The FOF is equipped with a HiPPI interface which it uses to transfer global events to a workstation farm via a HiPPI link.

The I-field added to the data packet by the FOF instructs the HiPPI switch to send the data to a given workstation in the farm. Each FEW receives one burst of data which it stores in a large memory buffer. The data in the memory buffer is processed while the subsequent bursts are transferred to other workstations. The number of workstations is determined by the amount of time needed to process one burst. To provide time for more processing it is merely necessary to add more workstations. ³

When one of the FEWS has successfully received data from the Data Merger it starts writing the data into a file on the local disk space. At the same time it sends a message to the CS-2 CDR-Daemon (CDR stands for Central Data Recording). This daemon starts a transfer process to copy the data to the parallel filesystem on the CS-2. Several processes are involved in the central data recording: "stagein" which uses several nodes of the CS-2 for the transfer of data from the FEWS to the parallel filesystem, "stageout" which is responsible for copying older data from the parallel filesystem to magnetic tape when the available disk space falls below a defined threshold. In addition several nodes can run the reconstruction software to perform analyses of the data - a Level 3 trigger could be implemented in this

³ What is defined by "processing" is (as of 1996) still not fixed. It could be imagined that some fast filtering of data, such as removal of events rejected by the L2B, could be done or that data compression could be applied to reduce the data volume. During 1995 online monitoring of a subset of events, as a check of detector performance, plus an event display were implemented.
way.

As a backup solution, in case the CS-2 is unavailable, each FEW has its own DLT tape drive (Digital Linear Tape). If the connection to the computer centre is interrupted data is written to local tapes.

A simplified dataflow diagram of the CDR system is shown in Figure 4.9.

Fig. 4.9: Dataflow and CDR configuration.
5. DATA REDUCTION

5.1 Introduction

The data used in this analysis was recorded in a 5 day period in September 1995 using a $K_L$ beam with the protons for the $K_S$ dumped in a TAX assembly. During this run there were several triggers driving the readout with different downscaling factors applied to each input bit. The "mixed-trigger" has already been described in Section 4.2.1. The $\mu\mu\gamma$ triggers dominate the data volume, accounting for $2/3$ of the events in each burst. In the first instance it was decided to remove all of these events in order to work with a more manageable data sample.

Due to the high-bandwidth constraints in the dataflow minimal processing is performed on the data and it is archived to storage media (disk or DLT tape) as raw data in binary format. During data recording a process in the FEWS, called the "check-formatter", reads the data to verify the format and check for data corruption. As this process does not perform any physics analyses but only makes simple checks, such as word counts, it can be made very fast. A modified version of this program, called "split_raw", was used to decode the trigger bits stored in the data and then rewrite the raw data with corresponding corrections to word counts, etc. in the headers. In this way a sample of raw data was obtained containing only "minimum bias", $\pi \pi$ and "random" events. The advantage of rewriting raw data over reconstructed quantities was that, at this time, new calibrations for the hadron calorimeter and the drift chambers were being released with very high frequency. Nevertheless a reduction by a factor of 3 in the time required to reconstruct the entire data sample was obtained. The volume of the data on disk was reduced further by compressing the data using "gnuzip". Using compression the data volume could be reduced by another factor of 2, giving an overall factor of 6.
Further event selection was performed using a full reconstruction of each event.

5.2 Event Reconstruction

Calibration of raw data and reconstruction of events at NA48 is performed by the program RAW. RAW is a hybrid of C and FORTRAN77; using the former to decode the raw data and the latter for the calibration and reconstruction. RAW uses the ZEBRA FZ[33] package to write an event output, although this was not used due to the large file size generated by this event output format. An alternative data format using Column Wise Ntuples (CWN)[34] with bit-packing and variable length arrays was defined and used in all subsequent analyses.

The compression of data required some small changes to the reconstruction program. The modified program first checks if the raw data is compressed and if so uncompress it into a temporary file. The data in the temporary file is then read by the reconstruction program. After reconstruction the temporary file is deleted.

After an event has been read the raw data information is decoded. At this point an event will be rejected in the case of an error in the data format. The C decoding will signal the reconstruction code not to attempt any reconstruction for this event and will then pass to the next event. Upon receipt of the decoded data the reconstruction code firstly applies calibration constants to the data. Pedestals and conversion constants for the hadron calorimeter plus time-origins (known as $t_0$'s) for each wire in the drift-chambers are fetched from a database implemented using the HEPDB[35] package. Using this information strip energies in the HAC and drift-times in the chambers are calculated.

Calculation of strip energies in the hadron calorimeter

For each strip in the HAC there is a pedestal value, $P_i$, and a calibration constant, $C_i$, defined for each run. The pedestal is defined to be the average noise in the strip and is determined from the analysis of random events taken during normal running.

The raw ADC signal for a strip, $A_i$, has the pedestal subtracted and is then mul-
tiplied by the calibration constant which converts the ADC counts to its energy equivalent in GeV. The calibration of the strips in the HAC front module is determined by using electrons from $K_{e3}$ decays in which the momentum is known from the charged spectrometer. Only tracks in which the electron impacts at the centre of a strip are used to avoid leakage of energy into adjacent cells. The typical strip sensitivity for the front module is $\sim 0.2$ GeV/ADC count.

As electrons deposit all their energy in the front module the strips in the back module are intercalibrated with those in the front using muons. A clean sample of tracks identified as muons using the muon veto reconstruction is obtained. As the muons are minimum ionising they should, on average, deposit the same energy in the back and front modules. Using the calibrated front energies constants are applied to the back energies such that this condition is met. For strips in the back module, where the scintillator is older than in the front, the typical strip sensitivity is $\sim 0.05$ GeV/ADC count.

At this point zero-suppression is applied to the data e.g. HAC cells with energies less than the pedestal value plus $\sigma_{ped}$ are removed.

The final strip energy $E_i$ is:

$$E_i = C_i(A_i - P_i)$$  \hspace{1cm} (5.1)

After calibration of the data a charged reconstruction is attempted. If the charged reconstruction is successful a neutral reconstruction is attempted to find any remaining photons in the HAC. Due to the absence of the LKr calorimeter in 1995 no independent neutral reconstruction is performed. However, for the analysis of $K_{\mu3}$, $K_{e3}$ and $K_{\pi3}$ the charged reconstruction is essential whereas the neutral reconstruction is not.

5.2.1 Charged Reconstruction

The charged reconstruction begins with the drift chamber information. Wire address information plus drift-times are used to build clusters in each view of each drift chamber. For each track passing through a chamber there is, in general, a signal on one wire in each view. In the drift chambers used in NA48 there are
4 views: x, y, u and v, as shown in Figure 5.1.

For each view there are two staggered planes called A and B so that each track produces a signal on a total of 8 wires per chamber. Associated with each hit is a drift time e.g. $t_\tau$ in the figure. Knowing the drift time plus the position of the wire there is a left-right ambiguity on the position of the hit relative to the wire. This ambiguity can be resolved using the staggered planes.

The charged reconstruction routine makes two passes to find tracks in the spectrometer. The first pass finds tracks without taking into account the drift time measurement and the second defines a new set of tracks using the drift times.

Firstly 1-dimensional objects called "clusters" are defined for each view of each of the three chambers. A cluster is defined by combining hits from the A and B planes in each view. A cluster can be made of one hit, in either the A or the B plane (called "singlets"), two hits, one in each of the A and B planes (called "doublets"), or three hits, one in A and two in B, or vice versa, (called "triplets"). The triplets arise from particle tracks which pass very close to a sense wire e.g. if a track passes within $\sim 90 \mu m$ of a B wire then both the A wires on either side may also register a hit.

Fig. 5.1: The distribution of signals for a single track in a drift chamber.
Clusters are then combined to form 2-dimensional objects called “segments”. A segment is defined by the combination of clusters in the same view in chambers 1 and 2. The slope of a segment must be less than 20 mrad. Four segments belonging to four different views are combined to make a track in the forward arm of the spectrometer.

After building tracks in the forward arm of the spectrometer space points are built in the third chamber (chamber 4 in 1995) using all combinations of clusters found in this chamber.

Using the tracks in chambers 1 and 2 and the space points in chamber 4 tracks through the entire spectrometer are found by asking that the deviation through the magnet is in a given range: 45 mrad for the x view, 5 mrad for the y view and 35 mrad in the u and v views.

Having found these tracks without using the drift time information they are used in the definition of new tracks which do make use of the drift times. In a given chamber, for most of the cases, a set of four clusters defines one space point. If a set of four clusters contains many hit wires more than one space point can be generated. Thus several tracks can be generated from the original track. The “best” track is defined to be the one with the lowest $\chi^2$ by fitting a straight line through the space points generated in chambers 1 and 2.

To calculate the momentum of each track they are parametrised by

$$
x = x(0) + \frac{dx}{dz}(z - z(0)) + \frac{C_x}{p} \tag{5.2}
$$

$$
y = y(0) + \frac{dy}{dz}(z - z(0)) + \frac{C_y}{p} \tag{5.3}
$$

where $C_x$ and $C_y$ are respectively the double integral of the field $B_x$ and $B_y$ between $z_0$ and $z$, $x$ and $y$ are coordinates at a given $z$ and $p$ is the momentum. The best values of the five parameters: $x$, $y$, $dx/dz$, $dy/dz$ and $p$, are calculated by minimising

$$
\chi^2 = \sum \frac{(x - x_i)^2 + (y - y_i)^2}{\sigma^2} \tag{5.4}
$$

where $x$ and $y$ are calculated coordinates and $x_i$ and $y_i$ are measured coordinates.
Using the momentum and the coordinates a vertex and closest distance of approach (cda) are calculated for each 2-track combination.

The tracks are then extrapolated into the hadron calorimeter and a reconstruction of showers performed. A “shower box” is formed around each track’s impact point and the energies summed within this box. Using the first moments of the energy within each shower box a “centre-of-gravity”, $(x, y)$, is calculated for each shower and using the second moments an $rms$ width is also calculated as shown in Equations 5.5–5.8.

\[
x = \frac{\sum_i E_i x_i}{\sum_i E_i} \tag{5.5}
\]

\[
y = \frac{\sum_j E_j y_j}{\sum_j E_j} \tag{5.6}
\]

\[
\sigma_x = \sqrt{\frac{\sum_i E_i x_i^2}{\sum_i E_i} - \left(\frac{\sum_i E_i x_i}{\sum_i E_i}\right)^2} \tag{5.7}
\]

\[
\sigma_y = \sqrt{\frac{\sum_j E_j y_j^2}{\sum_j E_j} - \left(\frac{\sum_j E_j y_j}{\sum_j E_j}\right)^2} \tag{5.8}
\]

The quantities used in the calculation are the positions of the centre of the HAC strips plus the energies in each strip as in Figure 5.2. Finally the energy is corrected for attenuation loss in the scintillator using a linear approximation $E'_i = E_i(1 + bd_i)$, where $b$ is a constant of the order of $8 \times 10^{-4}$ cm$^{-1}$ and $d_i$ is the distance of the energy deposition from the end of the scintillator strip measured in cm. As the strips are 120 cm long this corresponds to correction of $\leq 10\%$.

5.2.2 Neutral Reconstruction

In the absence of the electromagnetic calorimeter during 1995 a reconstruction of photons was attempted in the HAC albeit with a much poorer energy resolution.

1 As the hadron calorimeter is made up of scintillator strips (as described in Section 3.4.4) the readout of a shower’s energy is via two one-dimensional projections of the shower. Thus a “box” is actually a “cross”. See also Figure 5.2.
After the charged reconstruction has been performed in the HAC a neutral reconstruction routine is run to find any remaining clusters arising from photon showers. Energy which has been attributed to a charged track is firstly removed from the data banks. The method then used is to look for strips with energy above a given threshold. Looking along the strips in the projection of each quadrant a 1-D neutral cluster is begun when the first strip above the energy threshold is found. The end of the 1-D cluster is defined to be the strip at which the energy falls below the threshold value. Having formed any 1-D neutral clusters it is attempted to combine them into 2-D clusters. In the case of only one 1-D cluster in each projection of a quadrant the solution is totally unambiguous. If there are two separated 1-D clusters in each view then the 1-D clusters with the most similar energies are combined. When there are two 1-D neutral clusters in one projection and one in the other the energy of the strips containing the overlap is shared using the energies in the separated 1-D clusters as weights. If the energies of the separated 1-D projections are $E_{y1x}$ and $E_{y2x}$ and the energy of the overlapping 1-D projections is $E_{y12y}$, as shown in Figure 5.2, then the energies allocated to $\gamma_1$ and $\gamma_2$ from $E_{y12y}$ are:
5. Data Reduction

\[ E_{\gamma 1y} = \frac{E_{\gamma 12y}E_{\gamma 1z}}{E_{\gamma 1x} + E_{\gamma 2z}} \quad (5.9) \]

\[ E_{\gamma 2y} = \frac{E_{\gamma 12y}E_{\gamma 2z}}{E_{\gamma 1x} + E_{\gamma 2z}} \quad (5.10) \]

More complex topologies of overlaps are not resolved.

5.3 Selecting an Event Sample

Due to the simplicity of the trigger during the 1995 run there were many triggers issued when there was not a “good” two track event in the detector. Many triggers appeared to result from low energy showers passing through the detector. These could be observed in the high multiplicity of the charged hodoscope.

Initial event selection involved very simple criteria to remove these high multiplicity events and leave candidates for kaon decay. For high multiplicity events the charged reconstruction will fail as the drift chamber electronics have a “clear” mechanism which removes all data in the ring-buffer of any view which has greater than 15 hits in a 200 ns window. Events were selected in which there were two or more tracks resolved by the charged reconstruction. For these events reconstructed information from the spectrometer, hadron calorimeter, muon veto plus bits recorded in the digital PMBs were written into a CWN. In addition the reconstructed quantities of random events were also written into the ntuple. This data sample was used as the basis for all further analyses.

Included in this sample are:

- All random events,
- \( K_L \to \pi e \nu \),
- \( K_L \to \pi \mu \nu \),
- \( K_L \to \pi^+ \pi^- \pi^0 \),
- \( K_L \to \pi^+ \pi^- \),
• Other charged decay modes with branching ratios less than $10^{-3}$.

Neutral decay modes, such as $K_L \rightarrow \pi^0 \pi^0 \pi^0$ and $K_L \rightarrow \pi^0 \pi^0$, are not found in the sample due to the presence of the multiplicity trigger from the charged hodoscope, $Q_2$. 
6. MONTE CARLO SIMULATION

The number of events which decay in the fiducial region of the apparatus is not equal to the number detected. The geometrical acceptance, trigger efficiency and reconstruction efficiency determine the total number of each type of decay which is observed. To make any meaningful measurement it is necessary to calculate these effects and apply the resulting corrections to the data.

The geometrical acceptance is defined to be the number of decays observed by the detector within a given fiducial region divided by the total number of decays within this region. In principle the acceptance is just an integral over the solid angle subtended by the detector taking into account the kaon momentum spectrum, n-body phase space, the matrix element of the decay (form-factors) and the decay distribution. Due to the complexity of the problem it is impractical to evaluate this integral analytically so a numerical integration is performed using a Monte Carlo simulation. The Monte Carlo program generates decays with kinematic variables sampled from known distributions e.g. the momentum spectrum of the kaons produced at the target.

The acceptance is a different function of momentum and decay vertex for each of the decays:

\[
\begin{align*}
KL & \rightarrow \pi e\nu \quad (6.1) \\
KL & \rightarrow \pi \mu\nu \quad (6.2) \\
KL & \rightarrow \pi^+\pi^-\pi^0 \quad (6.3) \\
KL & \rightarrow \pi^+\pi^- \quad (6.4)
\end{align*}
\]

To calculate branching ratios of these decays it is necessary to evaluate the acceptances for each of these modes individually. To obtain a reliable result requires an accurate knowledge of the geometry of the detector, the matrix elements of the decays plus the decay spectrum.
The trigger decisions are based on analogue quantities and may not be easily reproduced offline. To remove the sensitivity to the trigger geometrical and kinematic cuts are applied to the event sample. These ensure that no part of the event sample is situated near to regions where distributions, such as the efficiency of the calorimeter threshold, are varying rapidly.

The generation of kaon decays and the simulation of the detector response to the decay products are described in the next section.

6.1 The New Monte Carlo Program (NMC)

Two Monte Carlo programs exist for the NA48 experiment. The first, NMC, is a fast Monte Carlo designed to produce high statistics results and the second, NASIM, is a GEANT [36] based Monte Carlo designed for studies of detector response.

For the calculation of the acceptances of the above four decay modes NMC was used. A brief description of the processes involved in the generation of an event are shown below.

6.1.1 Generation of a $K_L/K_S$

A kaon is generated at either the $K_S$ or $K_L$ target. The four momentum is sampled from the following spectrum

$$\frac{d^3N}{dpd\Omega} = \frac{\eta p^2}{4p_0} \left[ 1.30 e^{-\left( \frac{8.5p/p_0 + 3.0p^2\theta^2}{p^2} \right)} + 4.35 e^{-\left( \frac{13p/p_0 + 3.5p^2\theta^2}{p^2} \right)} \right]$$

(6.5)

where $\theta$, the production angle, is 4.2 mrad for $K_S$ and 2.4 mrad for $K_L$ and $p_0$ is the momentum of the incident proton beam (450 GeV/c). This spectrum is obtained from precision measurements of $K^+$ and $K^-$ production spectra made by Atherton et al[37]. Using parton model arguments the $K_L^0$ and $K_S^0$ spectra can be shown to be approximated by

$$\sigma(K_L^0) = \sigma(K_S^0) = \frac{1}{2} \left( \sigma(K^0) + \sigma(\bar{K}^0) \right) \simeq \frac{1}{4} \left( \sigma(K^+) + 3\sigma(K^-) \right)$$

(6.6)
To demonstrate this we begin by writing the proton initial state in terms of its constituent valence and sea quark distributions

\[ p = (F_v^u + F_s^u)u + (F_v^d + F_s^d)d + F_s^u\bar{u} + F_s^d\bar{d} + F_s^s s + F_s^\bar{s} \bar{s} \]  

(6.7)

where \( F_v^q \) is the fraction of valence quark \( q \), and \( F_s^q \) is the fraction of sea quark \( q \), in the proton wavefunction. The possibility of heavier flavour quark pairs, such as \( \langle c\bar{c} \rangle_s \), is neglected. Then, assuming that the kaon is produced by a single hard scatter followed by hadronization (as in deep inelastic scattering) we can write the cross sections for the production of \( K^0, \bar{K}^0, K^+, \) and \( K^- \) in terms of the probabilities of scattering from single free partons within the proton’s sea and valence distributions. For example, to obtain a \( K^0 \) there are two processes:

1. an \( \bar{s} \) quark, from the proton sea distribution, is knocked out of the proton and then combines with a \( d \) quark in the hadronization process,

2. a \( d \) quark, from the proton sea or valence distributions, is knocked out by the scattering process which then combines with an \( \bar{s} \) sea quark.

The second process is suppressed, relative to the first, by a factor \( \gamma_s \), which reflects the relative abundances of \( u\bar{u} : d\bar{d} : s\bar{s} \) in the quark sea. In PYTHIA[38], which is an event generator used to generate hadron-hadron interactions, this ratio is approximately 1 : 1 : 0.3 so in the following cross sections \( \gamma_s = 0.3 \) – although the result is generally true irrespective of the value of \( \gamma_s \). Using similar arguments to that for \( K^0 \) above we obtain the cross sections:

\[ K^0(\bar{s}d) \rightarrow \sigma(K^0) = \kappa \left[ F_s^\bar{s} + \gamma_s (F_v^d + F_s^d) \right] \]  

(6.8)

\[ \bar{K}^0(s\bar{d}) \rightarrow \sigma(\bar{K}^0) = \kappa \left[ F_s^s + \gamma_s F_s^d \right] \]  

(6.9)

\[ K^+(\bar{s}u) \rightarrow \sigma(K^+) = \kappa \left[ F_s^\bar{s} + \gamma_s (F_v^u + F_s^u) \right] \]  

(6.10)

\[ K^-(s\bar{u}) \rightarrow \sigma(K^-) = \kappa \left[ F_s^s + \gamma_s F_s^\bar{u} \right] \]  

(6.11)

The factor \( \kappa \) is a normalisation factor which, due to the near degeneracy in their masses, will be the same for each of the production cross sections.

Substituting (6.8) and (6.9) in the left side of (6.6) and using the relations \( F_s^q = F_s^\bar{q} \) and \( F_s^u \sim F_s^d \), for the sea quarks, and \( F_v^u = 2F_v^d \), for the valence quarks, we can
show that
\[ \frac{1}{2}(\sigma(K^0) + \sigma(\bar{K}^0)) = \frac{\kappa}{2} \left( F_s^d + \gamma_s(F_v^d + F_s^d) + F_s^s + \gamma_s F_s^d \right) \]
\[ = \frac{\kappa}{2} \left( 2F_s^s + \gamma_s(F_v^d + 2F_s^d) \right) \quad (6.12) \]

Similarly, if we substitute (6.10) and (6.11) in the right side of (6.6) we obtain
\[ \frac{1}{4}(K^+ + 3K^-) = \frac{\kappa}{4} \left( F_s^d + \gamma_s(F_v^u + F_s^d) + 3(F_s^s + \gamma_s F_s^u) \right) \]
\[ = \frac{\kappa}{4} \left( 4F_s^s + \gamma_s(F_v^u + 4F_s^u) \right) \]
\[ \approx \frac{\kappa}{2} \left( 2F_s^s + \gamma_s(F_v^d + 2F_s^d) \right) \quad (6.13) \]

which is identical to (6.12). Thus we see that the relationship (6.6) holds within the parton model.

The K+ and K− production spectra have been measured between 60 and 300 GeV/c. Below 60 GeV/c the spectrum in NMC is simply an extrapolation from the higher momentum data.

The decay spectrum is obtained by weighting the production spectrum with the decay distribution
\[ \frac{dN}{dz} = \frac{1}{\gamma \beta cT} e^{-z/\gamma \beta cT} \quad (6.14) \]
and integrating over the fiducial volume in which the decays are generated, \( z_{\text{min}} \) to \( z_{\text{max}} \). Thus, if we denote the production spectrum as \( F(p) \) and the decay spectrum as \( F'(p) \),
\[ F'(p) = F(p)e^{(z_{\text{target}}-z_{\text{min}})/\gamma \beta cT} (1 - e^{(z_{\text{min}}-z_{\text{max}})/\gamma \beta cT}) \quad (6.15) \]

The four momentum of the kaon plus the coordinates of its production and decay vertices are stored in the PART bank of a linked list, implemented using MZ ZEBRA[33], as shown in Figure 6.1.

6.1.2 Generation of Kaon Decays

In NMC the decay products are generated in the kaon rest frame and then Lorentz-boosted into the frame of reference of the detector.

The decay \( K_L \rightarrow \pi^+\pi^- \) is completely constrained by conservation of energy and momentum. The magnitude of the momentum in the centre-of-mass, \( p^*_\pi \) is given
The direction of one of the charged pions is generated with uniform probability in $\cos \theta$ and $\phi$, where $\theta$ is defined to be the azimuthal angle with respect to the direction of flight of the kaon and $\phi$ is the angle of rotation around this axis. By momentum conservation the other pion has the opposite direction.

For the 3-body decays, $K_{\mu3}$, $K_{e3}$ and $\pi^+\pi^-\pi^0$ the directions and momenta of the decay products are chosen according to 3-body phase space modified by form-factors (as described in Section 2.4 on page 16). The energies of the two charged products are generated uniformly within the kinematic limits and the energy of the neutral particle is then determined by conservation of energy. If it is not possible for this configuration to satisfy conservation of momentum a new configuration is

$$p^+_{\pi^+} = p^-_{\pi^-} = \sqrt{\frac{m_{K^0}^2}{4} - m_{\pi^\pm}^2} \quad (6.16)$$
generated.

In the NMC $K_{\mu 3}$ decay generator the slope of the linear dependence of the form-factor $f_+$ on $t$ is set to be $\lambda_+^{\mu} = 0.034$. This corresponds to the PDG evaluation $\lambda_+^{\mu} = 0.034 \pm 0.005$[20]. The value for $\xi(0)$ is taken from the same source and is set to the central value of $\xi(0) = -0.11 \pm 0.09$.

For $K_{e3}$ only one form factor, $f_+$, is involved. The slope parameter used in the decay generator is $\lambda_+^{e} = 0.0300 \pm 0.0016$, which is again taken from the PDG compilation[20].

The Dalitz plot of $\pi^+\pi^-\pi^0$ is parametrised by a series expansion introduced by Weinberg[39]

$$|M|^2 \propto 1 + g \frac{(s_3 - s_0)}{m_{\pi^+}^2} + h \left[ \frac{(s_2 - s_1)}{m_{\pi^+}^2} \right]^2 + j \left[ \frac{(s_2 - s_1)}{m_{\pi^+}^2} \right]^2 + \ldots$$

(6.17)

where $m_{\pi^+}^2$ is used to make the coefficients $g, h, j$ and $k$ dimensionless and

$$s_3 = (P_K - P_i)^2 = (m_{K^0} - m_i)^2 - 2m_{K^0}T_i, \ i = 1, 2, 3,$$

$$s_0 = \frac{1}{3} \sum_{i=1}^{3} s_i = \frac{1}{3} \left( m_{K^0}^2 + m_1^2 + m_2^2 + m_3^2 \right).$$

(6.18)

(6.19)

$P_K$ and $P_i$ are the four-momenta of the kaon and the pions and $m_i$ and $T_i$ are the mass and kinetic energy of the $i^{th}$ pion. The index 3 is used for the odd pion ($\pi^0$).

The coefficient, $g$, is a measure of the slope of the parameter $s_3$ (or $E_{\pi^0}$) while $h$ is a measure of the quadratic dependence on $s_3$. The parameters $j$ and $k$ measure, respectively, the linear and quadratic dependence on $(s_2 - s_1)$. The linear coefficient, $j$, in $(s_2 - s_1)$ must be zero if $CP$ invariance holds.

In the NMC $\pi^+\pi^-\pi^0$ generator only the linear coefficient in $(s_3 - s_0)$ is included. The value of $g$ used is 0.67 which corresponds to the PDG average, $g = 0.670 \pm 0.014$[20]. The coefficients $h, j$ and $k$ have a much smaller effect on the Dalitz plot and are neglected.

After the generation of the decay products in the CMS all the particles are boosted into the laboratory frame along the kaon direction of flight. The four-momenta, production vertex and decay vertex of each of the decay products are stored in the PART bank of the ZEBRA bank.

Some of the particles produced by the event generators are themselves unstable
and may decay within the fiducial volume. The decay of charged and neutral pions is also simulated in NMC.

The decay of neutral pions

\[
\begin{align*}
\pi^0 &\rightarrow \gamma\gamma \quad 98.8\% \\
\pi^0 &\rightarrow e^+e^-\gamma \quad 1.2\%
\end{align*}
\]

is generated at the kaon decay vertex. As the \(\pi^0\) lifetime is \((8.4 \pm 0.6) \times 10^{-17}\) s and \(\gamma = E_{\pi^0}/m_{\pi^0}\) is certainly less than 1000 the range of the \(\pi^0\) is less than 25 \(\mu\)m. Charged pion decay, \(\pi \rightarrow \mu\nu\), is generated along the line of flight of the pion. If the decay vertex is generated to be upstream of the electromagnetic calorimeter the pion is flagged as decayed and decay products are produced and stored in the PART bank else the pion is flagged as having not decayed within the fiducial region and no decay products are produced.

6.1.3 Simulation of the Detector

After generation of all the decay products the Lorentz-boosted particles are tracked through the detector. The initial momenta and particle directions are extracted from the PART bank and each particle is extrapolated to several planes corresponding to the positions of the various subdetectors. The positions and dimensions of the detector elements are shown in Table 6.1.

Charged particles, such as electrons, muons and charged pions undergo multiple scattering within the kevlar window, drift chambers and other subdetectors. After extrapolating a particle to a plane the scattering angle is generated by sampling from a Gaussian distribution of width\[40\]

\[
\sigma_{ms} = \frac{14.1\text{MeV}}{E} \sqrt{\lambda \left(1 + \frac{1}{9}\log_{10}(\lambda)\right)}
\]

where \(\lambda\) is the thickness of the material in radiation lengths. Gammas do not undergo multiple scattering and are projected in straight lines to the calorimeter.

The coordinates of the particles and their directions at each of the detectors are stored in a TRAK bank as shown in Figure 6.1
<table>
<thead>
<tr>
<th></th>
<th>x (cm)</th>
<th>y (cm)</th>
<th>z (cm)</th>
<th>Inner radius (cm)</th>
<th>Outer radius (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_L target</td>
<td>0.</td>
<td>0.</td>
<td>-12000.</td>
<td>0.</td>
<td>0.1</td>
</tr>
<tr>
<td>K_S target</td>
<td>0.</td>
<td>7.2</td>
<td>0.</td>
<td>0.</td>
<td>0.1</td>
</tr>
<tr>
<td>AKS</td>
<td>0.</td>
<td>6.8</td>
<td>607.</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>AKL 1</td>
<td>0.</td>
<td>0.</td>
<td>3489.</td>
<td>48.</td>
<td>96.</td>
</tr>
<tr>
<td>AKL 2</td>
<td>0.</td>
<td>0.</td>
<td>4723.</td>
<td>60.</td>
<td>96.</td>
</tr>
<tr>
<td>AKL 3</td>
<td>0.</td>
<td>0.</td>
<td>5954.</td>
<td>72.</td>
<td>120.</td>
</tr>
<tr>
<td>AKL 4</td>
<td>0.</td>
<td>0.</td>
<td>7176.</td>
<td>82.</td>
<td>120.</td>
</tr>
<tr>
<td>AKL 5</td>
<td>0.</td>
<td>0.</td>
<td>8399.</td>
<td>92.</td>
<td>120.</td>
</tr>
<tr>
<td>Kevlar window</td>
<td>0.</td>
<td>0.6</td>
<td>9531.</td>
<td>8.1</td>
<td>118.</td>
</tr>
<tr>
<td>Chamber 1</td>
<td>0.</td>
<td>0.6</td>
<td>9708.</td>
<td>9.6</td>
<td>135.</td>
</tr>
<tr>
<td>AKL 6</td>
<td>0.</td>
<td>0.</td>
<td>10548.</td>
<td>110.</td>
<td>140.</td>
</tr>
<tr>
<td>Chamber 2</td>
<td>0.</td>
<td>0.3</td>
<td>10628.</td>
<td>9.6</td>
<td>135.</td>
</tr>
<tr>
<td>AKL 7</td>
<td>0.</td>
<td>0.</td>
<td>11808.</td>
<td>120.</td>
<td>140.</td>
</tr>
<tr>
<td>Magnet</td>
<td>0.</td>
<td>0.</td>
<td>10898.</td>
<td>0.</td>
<td>240.</td>
</tr>
<tr>
<td>Chamber 3</td>
<td>0.</td>
<td>0.2</td>
<td>11168.</td>
<td>9.6</td>
<td>135.</td>
</tr>
<tr>
<td>Chamber 4</td>
<td>0.</td>
<td>0.</td>
<td>11888.</td>
<td>9.6</td>
<td>135.</td>
</tr>
<tr>
<td>Hodoscope</td>
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<td>0.</td>
<td>12060.</td>
<td>9.6</td>
<td>121.</td>
</tr>
<tr>
<td>LKr calorimeter</td>
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<td>0.</td>
<td>12235.</td>
<td>8.6</td>
<td>130.</td>
</tr>
<tr>
<td>Hadron calorimeter</td>
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<td>0.</td>
<td>12510.</td>
<td>9.6</td>
<td>130.</td>
</tr>
<tr>
<td>Muon veto 1</td>
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<td>0.</td>
<td>12719.</td>
<td>10.6</td>
<td>135.</td>
</tr>
<tr>
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<td>0.</td>
<td>12884.</td>
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<td>135.</td>
</tr>
<tr>
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<td>0.</td>
<td>0.</td>
<td>12997.</td>
<td>10.6</td>
<td>135.</td>
</tr>
</tbody>
</table>

Tab. 6.1: Geometry of the NA48 Beam and Subdetectors.
The projected impact points of charged tracks in the chambers, after multiple scattering, are smeared by a Gaussian of width 100 µm to simulate the intrinsic resolution of the chambers. Then, from the position of the track, hits are generated in the wires closest to the passage of the particle. From the distance between the track and the wire a drift time is generated. If the track passes within 90 µm of a potential wire then drift times are generated for the sense wires on both sides of the potential wire.

Showers in the electromagnetic and hadronic calorimeters are obtained from a "shower library" which is generated using a full GEANT simulation of the calorimeters. In the shower library there are showers from gammas and electrons generated between the ranges of 2 GeV and 100 GeV and between 2 GeV and 190 GeV for pions. These energy ranges are divided into 20 energy bins. During run-time NMC extracts a shower from the library according to the particle type and energy bin which corresponds to the momentum of the primary particle. Each shower in the library contains a number of cells (scintillator strips in the case of the HAC). For each cell there is stored the actual energy deposited within the cell by the shower. Using the cell energies from the shower library and the impact point from the tracking a shower is generated within the calorimeter.

The energy deposition of muons in the hadron calorimeter is calculated using a Molial function as an approximation to a Landau distribution. The energy loss is sampled from the distribution[40]

\[
N(E) = \exp \left[ \left( \frac{-E + E_{\text{HAC}}}{2\sigma} \right) - \exp \left( \frac{-E + E_{\text{HAC}}}{\sigma} \right) \right] \tag{6.22}
\]

where \( E_{\text{HAC}} = 1.4 \text{ GeV} \) is the mean energy loss of a muon in the HAC with the standard deviation \( \sigma = 0.09 \text{ GeV} \). The distribution is shown in Figure 6.2.

If selected, gamma conversion can be simulated in the beam-pipe and the kevlar window upstream of the helium tank. Gammas are tracked to the Kevlar window and the various tubes and flanges which constitute the beam-pipe. The probability of the production of an \( e^+e^- \) pair is then computed using[40, 41]

\[
P(x) = 1 - e^{-(\frac{x}{\sigma})} \quad \text{with} \quad \sigma = \frac{7}{9} \frac{A}{X_o \cdot N_a} \tag{6.23}
\]

where \( x : \) path length in the material [cm]
Fig. 6.2: The Molière Distribution for Muon Energy Deposition in the HAC.

\[
\sigma : \text{pair production cross section [cm}^2]\n\]
\[
\rho_i = \frac{N_A}{A} \cdot \rho : \text{density of interaction centres [cm}^{-3}]\n\]
\[
X_o : \text{radiation length of material [g cm}^{-2}]\n\]
\[
A : \text{atomic number of material}\n\]
\[
N_A : \text{Avogadro number [mol}^{-1}]\n\]
\[
\rho : \text{density of material [g cm}^{-3}].\n\]

The distribution of the energy sharing between the \(e^+\) and \(e^-\) is calculated according to [40]

\[
d\sigma(E_\gamma, E_{e^+}) \propto \frac{dE_{e^+}}{E_\gamma^3} \cdot (E_{e^+}^2 + E_{e^-}^2 + \frac{2}{3} E_{e^+} E_{e^-}) \cdot \left(\ln \left(\frac{2 E_{e^+} E_{e^-}}{m_e E_\gamma} \right) - \frac{1}{2}\right) \quad (6.24)
\]

where

- \(E_{e^+}, E_{e^-}\) : energies of the \(e^+ e^-\)
- \(E_\gamma\) : energy of gamma
- \(m_e\) : electron mass

The kinematics of the electrons produced are stored in the PART bank and the particles are tracked through the detector as for any other particle.

The quantities generated by the detector simulation are written into a ZEBRA bank structure which has the same structure as that in the reconstruction program. Calibrated data, such as drift times and cell energies, are written into the...
PRE bank which contains "pre-processed data" (PDS). The PDS is the meeting point between data and Monte Carlo (see Figure 6.3). The event output produced by NMC is written to a file using FZ ZEBRA (FZ is a format for writing ZEBRA banks in sequential files). This file is read by a program, "NMCREC", which recreates the ZEBRA structure in memory and then calls the reconstruction algorithms which are used in the reconstruction program. Finally a REC bank is produced which contains the reconstructed quantities from the subdetectors - such as cluster energies, track momenta, hits in hodoscope and muon veto, etc. Ntuples containing Monte Carlo reconstructed data plus Monte Carlo "truth" (the values from the PART and TRAK banks before the simulation of the detector response) were written with the same format as the data ntuples described in Section 5.3 on page 77. The data in these ntuples was then subject to identical cuts as those applied to the data.

The acceptances for $K_{\mu 3}$, $K_{\pi 3}$, $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-$, before cuts on the fiducial region, are shown in Figures 6.4–6.7. Additional cuts made on the data are described in Chapters 7 and 8.

In the analysis chapters Monte Carlo distributions are compared against those produced by data. This comparison is vital to ascertain that the Monte Carlo
program simulates the experiment correctly. For example, the Monte Carlo should produce vertex distributions and centre-of-mass quantities, such as invariant mass, which agree with data. However, while a disagreement between Monte Carlo and data generated distributions indicates a problem with the Monte Carlo, agreement does not exclude any error which removes or adds events uniformly across the distribution. Differences between the geometry of the experiment and the Monte Carlo and in reconstruction efficiencies of data and Monte Carlo data both have a systematic effect on the acceptances calculated and cannot be ruled out.
Fig. 6.5: Acceptance for $K_L \rightarrow \pi e\nu$. 
Fig. 6.6: Acceptance for $K_L \rightarrow \pi^+\pi^-\pi^0$. 
Fig. 6.7: Acceptance for $K_L \rightarrow \pi^+ \pi^-$. 
7. THE MEASUREMENT OF $\Gamma(K_L \to \pi\nu)/\Gamma(K_L \to \pi\nu)$

The first analysis in this thesis; a measurement of the relative branching ratios $\Gamma(K_L \to \pi\nu)/\Gamma(K_L \to \pi\nu)$, is presented in this chapter. The selection of events and identification of $K_{\mu3}$ and $K_{e3}$ is described. Sources of background are evaluated and removed from the data sample. The ratio is calculated in 10 bins in the $z$-vertex position and each bin is then corrected individually for losses and inefficiencies. The statistical error is calculated and an estimation is made of the systematic errors inherent to the measurement.

7.1 Event Selection

The initial event selection of two-track events has already been outlined in Chapter 5 on page 77. The resulting data sample contains $K_L \to \pi\nu$, $\pi\mu\nu$, $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-$ decays.

The next selection criteria concern the coordinates of tracks extrapolated to the hadron calorimeter and muon veto subdetectors. In order to identify the end-state particles it is necessary that the particles are not only accepted by the drift chambers in the spectrometer but also pass through the hadron calorimeter and the muon veto. To ensure that this is the case, a radius cut is made at 20 cm at the hadron calorimeter (inner radius 9.6 cm) and at 15 cm at the muon veto (inner radius 10.6 cm). Both the hadron calorimeter and muon veto are square in section so a further cut is made to ensure that the tracks do not fall within 5 cm of the edge of the subdetector.

To avoid the problems of overlapping showers, events are chosen in which the topology of the tracks minimises the possibility of identifying energy with the wrong track. The selection procedure is as follows:
1. events in which the tracks point to opposite quadrants of the calorimeter are accepted,

2. events in which the tracks are in adjacent quadrants are accepted if they are separated by at least 36 cm (3 strip widths) i.e. if both tracks are in the top two or bottom two quadrants they must be separated in the $x$ coordinate - if they are both in the left two or right two quadrants they must be separated in the $y$ coordinate.

### 7.1.1 Trigger Efficiency

As mentioned in Section 4.2.1 on page 50 the minimum bias trigger was defined to be a concidence between a majority of two sub-quadrants in the charged hodoscope and an energy threshold, $E_{HL}$, in the hadronic calorimeter. The sub-quadrants condition can be easily simulated in the Monte Carlo while the efficiency of $E_{HL}$ as a function of energy deposited in the calorimeter is measured from the data.

Most of the triggers ($\mu\mu\gamma$, $\pi\pi$ and "min. bias") have $E_{HL}$ present in coincidence with other logic signals from the scintillator hodoscopes and so cannot be used to measure the efficiency. The random triggers, however, do not require the $E_{HL}$ trigger to fire and therefore can be used to calculate the efficiency of $E_{HL}$ as a function of energy. For every random event the total energy in the hadron calorimeter is calculated and histogrammed. If the $E_{HL}$ bit is also set for this event the total energy is filled into another histogram. Both of these histograms are shown in Figure 7.1. Dividing the two distributions; energy in hadron calorimeter with $E_{HL}$ bit set divided by energy in hadron calorimeter for all random events, gives the efficiency of $E_{HL}$, $\varepsilon_{E_{HL}}$, as a function of the energy deposited in the calorimeter (Figure 7.2).

The threshold set on the calorimeter should correspond to a step function as the digitised energy sum from the FERA readout is derived from the same signals as are input to the discriminator which produces the $E_{HL}$ bit (as shown in Figure 7.3). The data clearly shows Gaussian smearing around the threshold point. As the signals to both the FERA readout and the discriminator should be identical this smearing may correspond to a resolution effect introduced by the electronics or to a difference $|\Sigma E_{\text{offline}} - \Sigma E_{\text{online}}|$. A major difference between the online and
Fig. 7.1: Energy in Hadronic Calorimeter for (a) Random Events; (b) Random Event with $EHL$ set.

offline energy sums is in the treatment of pedestals: the energy sum to the $EHL$ discriminator is not pedestal subtracted whereas the offline energy sum is. This corresponds to

$$\Sigma E_{\text{offline}} = \Sigma E_{\text{online}} - \Sigma \text{pedestal}$$

(7.1)

However, assuming that the pedestals are stable\(^1\), this will only move the online threshold by a constant value.

\(^1\) The pedestal mean was found to be stable to within 1 GeV over the period in which data used in this analysis was recorded.
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There may also be timing effects if the relative times of the $EHL$ bit and the charge collection gate for the FERA’s are different for minimum bias and random events. For example, for minimum bias events the time between the charge collection gate produced by the L1 trigger is fixed with respect to $EHL$ (the minimum bias trigger is defined to be $EHL.Q_2$). For random events the time difference between the charge collection gate and the $EHL$ bit, if it is set, is not fixed. Any mis-timing of the charge collection gate relative to $EHL$ will reduce the energy, calculated from the integrated charge, by an amount depending on the fraction of the pulse which falls within the charge collection gate (see Figure 7.4).
Another difference arises from the different methods in which energy is defined in the offline energy sum and the EHL trigger bit. If we make the approximation that the pulse formed by the analogue energy sum, which is used as the input to the EHL discriminator, is triangular then the collected charge is proportional to the peak voltage of the pulse \( q = \int i(t) dt = \int V(t)/R dt \approx V_{max} \Delta t/2R \); where \( R \) is the impedance of the input stage of the FERA and \( \Delta t \) is the baseline width.
of the pulse). If the photomultiplier system is linear then the charge, and thus the voltage, is also proportional to the energy. At this point, given no resolution effects in the electronics and assuming all of the charge falls within the collection gate, the efficiency distribution would correspond to a step function. Now, if a random background is added which consists of pulses with a peak voltage less than that of the EHL threshold this will have no effect on the EHL bit unless it is coincident with any other pulse such that the threshold voltage is achieved. However, the random background will always contribute to the energy calculated from the integrated charge and so the offline energy may be larger than the energy sum which fired the EHL bit. This effect is illustrated in Figure 7.5. As can be seen in Figure 7.1 the mean energy in a random event is of the order of 4 GeV and the sigma (assuming a Gaussian distribution) is of the order of 2 GeV. Thus we might expect some Gaussian smearing around the trigger threshold with a sigma of ~2 GeV plus an offset of, at most, 4 GeV between the energy from the integrated charge and the energy “seen” by the EHL discriminator.

To measure the trigger threshold a binary sigmoid (or “Fermi” function) is fitted to the efficiency curve:

\[
\varepsilon_{EHL}(E) = \frac{\varepsilon_{EHL}^{\text{max}}}{1 + e^{-\sigma(E-E_{\text{thr}})}}
\]

(7.2)

where \(\varepsilon_{EHL}^{\text{max}}\) is the maximum efficiency of the EHL bit, \(\sigma\) is a steepness parameter – as \(\sigma \to \infty\) the binary sigmoid becomes a step function, and \(E_{\text{thr}}\) is the energy
corresponding to the discriminator threshold. In Figure 7.2 (page 97) P1–P3 correspond to the fit parameters $\varepsilon_{EHL}^{mag}$, $\sigma$ and $E_{thr}$ respectively.

Each of the three parameters are allowed to vary in the fit, which is performed using a $\chi^2$ minimisation with MINUIT\cite{42}. The fit takes into account the error in the calculated efficiency, which is given by $\Delta \varepsilon = \sqrt{\varepsilon(1 - \varepsilon)/N}$ where $N$ is the number of random events in a given energy bin in Figure 7.1(a). The fit is shown, superimposed on the data, in Figure 7.2. The first derivative of the sigmoid is also shown (with an arbitrary normalisation) to illustrate the position of the threshold.

The mean threshold corresponds to 14.4 GeV. By fitting a Gaussian to the derivative the width of the threshold is estimated to be $\sim 4$ GeV.

That the maximum efficiency above the threshold is measured to be 77.6% may be due to the variable timing between the $EHL$ bit and the L1 trigger for random events. It is not inconceivable that there is some offset between the charge collection gate and the interval recorded by the digital PMB's. This could be inconsequential for minimum bias events, where the timing relative to the L1 is fixed, but for some subset of random events the $EHL$ bit may fall before or after the PMB recording window even though all of the charge is collected by the FERA gate.

The efficiency of the trigger for the various decay modes is dependent on the end-state particles. The end state particles in the pertinent decays are as shown below:
\[ K_L \rightarrow \pi^\pm e^{\mp}\nu \]
\[ \pi \rightarrow \mu^{\pm}\nu \ (\sim 3\%) \]

\[ K_L \rightarrow \pi^\pm \mu^{\mp}\nu \]
\[ \pi^\pm \rightarrow \mu^{\pm}\nu \ (\sim 3\%) \]

\[ K_L \rightarrow \pi^+\pi^-\pi^0 \]
\[ \pi^\pm \rightarrow \mu^{\pm}\nu \ (\sim 6\%) \]
\[ \pi^0 \rightarrow \gamma\gamma \ (\sim 98.8\%) \]
\[ \pi^0 \rightarrow e^+e^-\gamma \ (\sim 1.2\%) \]

\[ K_L \rightarrow \pi^+\pi^- \]
\[ \pi \rightarrow \mu^{\pm}\nu \ (\sim 6\%) \]

For \( K_{e3} \) decays the electron always deposits energy in the HAC, as does the pion in the case that it does not decay in flight. For \( K_{\mu3} \) there is only the single pion which deposits energy in the calorimeter. The energy deposited by a muon is approximately independent of energy and averages at 1.4 GeV.

The hadron calorimeter is calibrated in electromagnetic energy (primarily for the \( \mu\mu\gamma \) rare decay studies) so the ratio of energy to track momentum, \( E_{clus}/p_{track} \), averages to one. For pions there is a lower reconstructed cluster energy compared to an electron of similar energy due to the larger fluctuations in hadronic showers. Electromagnetic showers propagate via a cascade of pair production and bremsstrahlung until the photon energy drops beneath the threshold for pair production, 1.02 MeV. Both the photons and the electrons produce molecular excitation in the scintillator which is observed by the photomultipliers. Hadronic showers produce correspondingly less energy in the scintillator, depending on the number of \( \pi^0 \)'s in the early stages of the shower, and more energy is thermalised in the iron due to nuclear interactions. The electromagnetic/hadronic energy ratio is measured to be 1.4 (see Figure 7.6). Two Gaussians are fitted to the \( E_{clus}/p_{track} \) distribution in the range 0.4–1.4. Parameters P1–P3 are, respectively, the height, mean and standard deviation of the first Gaussian. P4–P6 are the same but for the second Gaussian. Thus, for pions, the 14.43 GeV threshold corresponds to an equivalent energy threshold of 20.2 GeV.
Therefore, to produce a data sample which is independent of the \( EHL \) trigger a momentum cut is made at 20 GeV on each track. A cut is made on momentum rather than on energy in the calorimeter as the kinematic quantities calculated with the spectrometer are known with greater accuracy and can be simulated in the Monte Carlo with more confidence than shower development in the hadronic calorimeter. By making a cut which is above the energy threshold for either a single electron or a single pion no bias is introduced between \( K_{e3} \) and \( K_{\mu 3} \).

### 7.1.2 Removal of \( K_L \to \pi^+\pi^0\pi^0 \)

To select preferentially the semileptonic decays a cut is first made on the kinematic variable \((p_0')^2\)[43, 44]. This variable is defined to be

\[
(p_0')^2 = \frac{(m_{K^0}^2 - m_{\pi^0}^2 - m_c^2)^2 - 4(m_{\pi^0}^2 m_c^2 + p_T^2 m_{K^0}^2)}{4 (p_T^2 + m_c^2)^2} \tag{7.3}
\]

where \( m_{K^0} \) is the kaon mass, \( m_{\pi^0} \) the neutral pion mass, \( m_c \) is the effective mass of the two charged tracks interpreted as \( \pi^+\pi^- \) and \( p_T \) equals the transverse momentum of the charged pair. In a true \( K_L \to \pi^+\pi^0 \) decay \( p_0' \) is the momentum of the \( K_L \) in the Lorentz frame in which the longitudinal momentum of the charged...
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The pion pair is zero. Thus it is equal in magnitude to the longitudinal momentum of the $\pi^0$ and so $(p'_0)^2$ must be positive definite. As $p'_0$ is calculated from Lorentz Invariant quantities it is also Lorentz Invariant. In the case of $K_{e3}$ and $K_{\mu3}$ the wrong mass assignment is made for the lepton. This results in an incorrect invariant mass and a $(p'_0)^2$ spectrum which is dominantly negative. The distribution of $(p'_0)^2$ is shown in Figure 7.7.

![Figure 7.7: The $(p'_0)^2$ Distribution for $K_L$ Data.](image)

The Monte Carlo was used to generate distributions of $(p'_0)^2$ for $K_{\mu3}$, $K_{e3}$ and $K_{\pi3}$. The resulting distributions are shown in Figures 7.8-7.10.

The effect of pion decay on the distributions is also shown. The small $Q$-value in pion decay $(m_{\pi\pm} - m_\mu = 33.9 \text{ MeV})$ compared to 252.5 MeV for $K_{\mu3}$ and 357.6 MeV for $K_{e3}$ means that the shape of the $(p'_0)^2$ is altered very little due to pion decay - apart from a slight broadening. For $K_{\pi3}$ the $Q$-value is somewhat smaller (83.5 MeV) as there are 3 massive particles in the end-state. There is, therefore, a correspondingly larger effect on the $(p'_0)^2$ distribution.

By applying a cut at $(p'_0)^2 < -0.004[\text{GeV}/c]^2$ 97.5% of the $K_{\pi3}$ is removed while keeping 96.9% of $K_{\mu3}$ and 98.6% of $K_{e3}$. The efficiency of this cut is re-evaluated after the fiducial region cut is imposed as this has an effect on the pion decay probability.
Fig. 7.8: The $(p_0')^2$ Distribution for $K_{\mu 3}$ Monte Carlo Data.

### 7.1.3 Separation of $K_{\mu 3}$ and $K_{e3}$

After removal of the $K_{\pi 3}$ the remaining sample contains $K_{\mu 3}$, $K_{e3}$ and a small background of the CP violating decay $K_L \rightarrow \pi^+\pi^-$. Assuming, for the moment, that the contamination from $\pi^+\pi^-$ is negligible the requirement is to separate the $K_{\mu 3}$ and $K_{e3}$ events.

As can be seen from Figure 7.6 electrons and pions cannot be easily distinguished in the hadron calorimeter due to the large overlap in the distribution of $E_{clus}/p_{track}$.
for these particles. Using the variable $E_{\text{max}}/p_{\text{track}}$, where $E_{\text{max}}$ is the sum of the energies from the cells in the shower maximum in the $x$ and $y$ projections, gives an improved separation (see Figure 7.11) as electromagnetic showers are predominantly contained in one cell of the calorimeter whereas hadronic showers are contained in 3–5 cells. The parameters P1–P6 are those obtained from a fit of two Gaussians to the distribution. P1 is the height at the maximum of the fit to the “pion” $E_{\text{max}}/p_{\text{track}}$ distribution, P2 is the mean and P3 is the standard deviation of the Gaussian. P4–P6 are the same variables but for the “electron” part fo the distribution.
However the separation is still not ideal and to separate the electrons and pions in the $K_{\mu 3}$ and $K_{e3}$ sample would require the evaluation of cut efficiencies using clean samples. As any variables used to select a "clean" sample, such as shower width, are correlated this could introduce a systematic bias in the result. Instead the strategy adopted is to identify all events with a muon in the end-state as $K_{\mu 3}$. It is then assumed that the Monte Carlo can correctly predict the loss of $K_{e3}$ due to pion decay in flight.

Fig. 7.10: The $(p_0')^2$ Distribution for $\pi^+\pi^-\pi^0$ Monte Carlo Data.
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Fig. 7.11: $E_{\text{max}}/p_{\text{track}}$ for Electrons and Pions: the Gaussian fits are to guide the eye only.

**Evaluation of the Muon Veto Efficiency**

The signals available from the muon veto are described in Section 3.4.5. Logical OR's of each plane are recorded for each event (8 x 25 ns time slices) in PMB's and the pattern of discriminator signals from each strip in planes 1 and 2 are recorded in FERA ADC's within a 200 ns gate provided by the level 1 trigger.

To calculate the efficiency of the muon veto planes requires that the muon can be identified in subdetectors other than the one whose efficiency is under study. Well separated tracks from the spectrometer were identified as muons from their signature in the hadron calorimeter. The separation condition in the calorimeter was that the tracks should extrapolate to opposite quadrants of the calorimeter.
and was required to avoid overlapping showers. Then for measuring the efficiency of each of the three planes a muon signal was required in two of the planes and the third plane checked for efficiency. Thus the efficiency of each plane is defined to be

\[
\epsilon_{\mu_1} = \frac{\langle \mu_{HAC} \rangle \cdot \mu_1 \cdot \mu_2 \cdot \mu_3}{\langle \mu_{HAC} \rangle \cdot \mu_2 \cdot \mu_3} \\
\epsilon_{\mu_2} = \frac{\langle \mu_{HAC} \rangle \cdot \mu_1 \cdot \mu_2 \cdot \mu_3}{\langle \mu_{HAC} \rangle \cdot \mu_1 \cdot \mu_3} \\
\epsilon_{\mu_3} = \frac{\langle \mu_{HAC} \rangle \cdot \mu_1 \cdot \mu_2 \cdot \mu_3}{\langle \mu_{HAC} \rangle \cdot \mu_1 \cdot \mu_2}
\]

Firstly the condition \(\mu_{HAC}\) has to be defined. A sample of separated tracks associated with a reconstructed hit in each of the first two planes of the muon veto is selected. For each of these tracks a search is made in a 3 x 3 shower box in the hadron calorimeter for the strip with the maximum energy. The maximum strip energies for the x and y projections are then summed. This is done separately for both the front and back modules of the calorimeter. The muon energy distribution for the front module is shown in Figure 7.12. As the muon is minimum ionising it will deposit the same energy in the front and back modules. The \(\mu_{HAC}\) condition is defined to be \(E_{\text{front}} < 2 \text{ GeV}\) and \(E_{\text{back}} < 2 \text{ GeV}\). The momentum cut at 20 GeV means that this is very unlikely to be satisfied by electrons or pions.

Having defined \(\mu_{HAC}\) the efficiency of the muon planes can be calculated as a function of position using tracks extrapolated from the spectrometer. Any background of electrons or pions in the \(\mu_{HAC}\) sample should be removed by the requirement of a muon signal from two out of three of the muon veto planes.

The efficiency of the muon planes is calculated to be

\[
\epsilon_{\mu_1} = 99.59 \pm 0.04\% \\
\epsilon_{\mu_2} = 97.49 \pm 0.09\% \\
\epsilon_{\mu_3} = 92.2 \pm 0.3\%
\]

where the quoted error is statistical only and is calculated using the identity

\[
\Delta \epsilon = \sqrt{\frac{\epsilon(1-\epsilon)}{N}}
\]
where $\varepsilon$ is the efficiency calculated and $N$ is the total number of muons in the sample used for the measurement.

Thus defining a muon as $\mu_1, \mu_2$ has an efficiency of $97.1 \pm 0.1\%$ and a majority of two out of three of the planes ("maj. 2") has an efficiency of $99.8 \pm 0.1\%$. The majority signal, "maj. 2", is used to define a muon. By studying the rates in time bins adjacent to the signal region it is estimated that the rate from incoherent coincidences was less than 0.5% for the "maj. 2" signal.

The estimation of these combined efficiencies, $\mu_1, \mu_2$ and "maj. 2", has been calculated using a signal from plane 3 to flag that a muon has indeed passed through planes 1 and 2 of the muon veto. At low momentum this does not give a correct result for the muon efficiency as muons "range out" and stop in the iron walls. Thus, at low momentum, $\varepsilon_{\mu_1} > \varepsilon_{\mu_2} > \varepsilon_{\mu_3}$. To check if this effects the measurement of the efficiencies tracks were selected which were identified as a muon using the $\mu_{HAC}$ condition plus a reconstructed hit in plane 1 of the muon veto from the FERA data. For each of these tracks the momentum of the track,
from the charged reconstruction, was histogrammed. Two further histograms of the track momentum were filled if the $\mu_2$ or $\mu_3$ PMB bits were set. By dividing the momentum histograms with the plane 2 or plane 3 PMB bits set by the histogram obtained by only requiring the $\mu_{HAAC}$ condition plus a hit in plane 1 associated with the track the efficiency of planes 2 and 3 as a function of momentum is obtained.

The efficiencies are shown in Figure 7.13. The hashed areas of the two histograms correspond to regions below the minimum track momentum cut that is imposed in the $K_{\mu3}/K_{e3}$ analysis. It can be seen that the efficiency of the $\mu_2$ and $\mu_3$ bits vary very little above the 20 GeV/c track momentum cut and so it is reasonable to multiply the efficiencies of various planes to obtain efficiencies of combinations of the planes.

![Figure 7.13: Plane 2 and 3 efficiency as a function of momentum.](image)

**Effect of Accidental Muons**

In addition to random coincidences in the counter there is also the possibility of an accidental muon, either "in-time" (produced by the same proton as the kaon) or "out-of-time". To try to evaluate the magnitude of this effect events are selected which have the "maj. 2" signal present. Hard cuts on shower width
and \( E_{clus}/p_{track} \) are used to positively identify the pion. Then if \( E_{max}/p_{track} \) is histogrammed for the “non-pion” this should correspond to a muon. The left-hand plot in Figure 7.14 shows the \( E_{max}/p_{track} \) for the muon candidate. There is a clear peak close to one which corresponds to electrons from \( K_{e3} \).

![Fig. 7.14: \( E_{max}/p_{track} \) for Muon Candidate and “Clean” Muon Samples.](image)

Comparing this distribution with a sample of muons selected by correlating spectrometer tracks with reconstructed muon information from the FERA data there is an excess in the first sample. Making a cut on \( E_{max}/p_{track} \) at 0.5, 2.88% of the muon candidate sample is above this cut compared with 0.24% of the clean muon sample.

The conclusion is that 2.6% of two-track events have an accidental muon associated with the event. To remove this effect an additional criterion is added to the \( K_{\mu3} \) selection: for each event with the “maj. 2” signal present one track must have an \( E_{max}/p_{track} < 0.5 \). This should result in a loss of \((0.24 \pm 0.03)\%\) of \( K_{\mu3} \) (the error quoted is purely statistical).

Having separated the \( K_{\mu3} \) and \( K_{e3} \) samples using the muon identification a background arising from \( K_L \to \pi^+\pi^- \) can be observed in the \( K_{e3} \) sample.
7.1.4 Removal of $K_L \to \pi^+\pi^-$ from $K_{e3}$ Sample

Although the branching ratio of $K_L \to \pi^+\pi^-$ is only $\sim 2 \times 10^{-3}$ it has a larger acceptance than the semileptonic decays and so constitutes a non-negligible background to the $K_{e3}$.

The signal can be seen in Figure 7.15, where the transverse momentum squared is plotted against the invariant mass, $m_\pi$, assuming that both particles are charged pions. The effect of the $(p_\pi^T)^2$ cut can be seen in the lower left corner of the plot.

As the CP violating decay is two-body the transverse momentum of the charged pions must cancel. Taking an event sample with $p_\pi^T < 4 \times 10^{-3} (\text{GeV}/c)^2$ illustrates the signal more clearly (shown in Figure 7.16).

![Figure 7.15: Transverse Momentum Squared vs. Invariant $\pi^+\pi^-$ Mass.](image)

To remove the $\pi^+\pi^-$ signal events are discarded with
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Fig. 7.16: Invariant $\pi\pi$ Mass for Low $p_T$ Events.

(0.491 < $m_c$ < 0.503 GeV ; $p_T^2 < 4 \times 10^{-3}(\text{GeV}/c)^2$)

Using the Monte Carlo it is found that these cuts remove 95.4% of $K_L \to \pi^+\pi^-$ decays while removing only 0.5% of $K_{e3}$ decays.

7.2 Comparison with Monte Carlo

To gain some confidence that the Monte Carlo is simulating the experiment correctly a comparison is made between Lorentz invariant quantities and kinematic quantities measured in the laboratory frame. The Lorentz invariant quantities compared are $p_T^2$, $m_c$ and $(p_0)^2$. Figures 7.17 and 7.18 shows these quantities for data and Monte Carlo and also the ratio of the two distributions: (data)/(Monte Carlo)
The Measurement of $\Gamma(K_L \rightarrow \pi \mu \nu)/\Gamma(K_L \rightarrow \pi e \nu)$

Carlo), to the right of each plot.

![Graphs of Lorentz invariant quantities](image)

**Fig. 7.17:** Comparison Between Data and Monte Carlo Lorentz Invariant Quantities for $K_{\mu 3}$ (hashed line is Monte Carlo). (data/MC) shown on right.

The Lorentz invariant quantities show reasonable agreement between the data and the Monte Carlo, apart from resolution effects at the boundaries of the distributions.

The momentum spectrum of the kaons has an effect on the acceptances calculated using the Monte Carlo. To verify that this is the same in the Monte Carlo and in the data the vertex distribution, radial distribution of tracks at the drift chambers
and the track separation at the first chamber are compared for data and Monte Carlo. The results of the comparison are shown in Figure 7.19, for $K_{\mu 3}$, and Figure 7.20, for $K_{e 3}$. As before the ratio of the two distributions, $(\text{data})/(\text{Monte Carlo})$, is shown to the right of each plot.

It can be seen that the average track distance from the beam-pipe at drift chamber 1 is smaller for data. This indicates that the momentum spectrum for the Monte Carlo is harder than for data. A method was devised to reconstruct the kaon
momentum spectrum of the semileptonic decays so that it could be checked against the Monte Carlo spectrum.

**Kaon Momentum Spectrum Reconstruction for Semileptonic Decays**

For the semileptonic decays it is not possible to reconstruct the kaon momentum due to the loss of kinematic information from the neutrino. However, there are sufficient constraints to allow us to calculate two possible kaon momenta, $p_L$ and $p_H$. 

Fig. 7.19: Comparison Between Data and Monte Carlo Laboratory Quantities for $K_{\mu 3}$ (hashed line is Monte Carlo). (data/MC) shown on right.
The kinematics of a $K_{l3}$ decay (i.e. $K_{e3}$ or $K_{\mu3}$) is shown in Figure 7.21.

The subscripts $\pi$, $l$, $\nu$ and $K$ refer to the pion, the lepton, the neutrino and the kaon respectively and the subscripts $\perp$ and $\parallel$ refer to transverse and longitudinal components of the 3-momentum in the $CMS$ and lab frames.

Defining the invariant mass, $m_{l\pi}$, to be the mass of the combined pion and lepton system we obtain

$$P_{l\pi} = P_l + P_\pi$$

(7.11)
Fig. 7.21: Kinematics of a Semileptonic Decay in Centre-of-mass and Laboratory Frames.

trivially from 4-momentum conservation.

The two following identities can be derived for CMS quantities:

\[ p^{\star}_{\nu} = \frac{m_K^2 - m_{\pi}^2}{2m_K} \]  \hspace{1cm} (7.12)

\[ E^{\star}_{\nu} = \frac{m_K^2 + m_{\pi}^2}{2m_K} \]  \hspace{1cm} (7.13)

Since \( p^{\star}_{\nu}^2 = p^{\star}_\nu \) the total neutrino momentum is known. By conservation of 3-momentum the transverse momentum \( p^{\star}_{\nu \perp} = p^{\star}_{\nu \perp} \). The transverse components of the momentum are not affected by the Lorentz boost from the CMS to lab frames and so the measured transverse momentum of the lepton/pion system in the lab gives the transverse momentum of the neutrino in the CMS. Thus the only ambiguity in the system of kinematic equations is the forward or backward direction of the neutrino in the CMS (\( p^{\star}_{\nu \parallel} = \pm \sqrt{p^{\star}_{\nu}^2 - p_{\nu \perp}^2} \)).

From the Lorentz transformation between the CMS and laboratory frames (transformation between a frame \( \Sigma \), in which the kaon is at rest, and a frame \( \Sigma' \) moving at velocity \(-\beta\) along the direction of flight of the kaon):
7. The Measurement of $\Gamma(K_L \rightarrow \pi\mu\nu)/\Gamma(K_L \rightarrow \pi\nu\nu)$

\[
\begin{pmatrix}
    p_{l\pi\perp} \\
p_{l\pi\parallel} \\
E_{l\pi}
\end{pmatrix} =
\begin{pmatrix}
    1 & 0 & 0 & 0 \\
    0 & 0 & \gamma & -(-\beta)\gamma \\
    0 & 0 & -(-\beta)\gamma & \gamma
\end{pmatrix}
\begin{pmatrix}
p_{l\pi\perp}^* \\
p_{l\pi\parallel}^* \\
E_{l\pi}^*
\end{pmatrix}
\]  

(7.14)

we obtain

\[
p_{l\pi\perp} = p_{l\pi\perp}^* 
\]  

(7.15)

\[
p_{l\pi\parallel} = \gamma p_{l\pi\parallel}^* - (-\beta)\gamma E_{l\pi}^* 
\]  

(7.16)

\[
E_{l\pi} = -(-\beta)\gamma p_{l\pi\parallel}^* + \gamma E_{l\pi}^* 
\]  

(7.17)

from which (using $\gamma = E_K/m_K$ and $\gamma\beta = p_K/m_K$) we can derive

\[
p_{l\pi\parallel} = \left(\frac{E_K}{m_K}\right) p_{l\pi\parallel}^* + \left(\frac{p_K}{m_K}\right) E_{l\pi}^* 
\]  

(7.18)

Equation 7.18 can be solved for $p_{L,H}$,

\[
p_{L,H} = m_K \frac{p_{l\pi\parallel} E_{l\pi}^* - p_{l\pi\parallel}^* E_{l\pi}}{(E_{l\pi})^2 - (p_{l\pi\parallel})^2} 
\]  

(7.19)

where $p_L$ and $p_H$ correspond to the low and high solutions for $p_K$ obtained from the possible choices of sign in the relation

\[
p_{l\pi\parallel}^* = \pm \sqrt{(p_{l\pi}^*)^2 - (p_{l\pi\perp})^2} 
\]  

(7.20)

Ignoring acceptance effects there is a 50% chance that $p_L$ (or conversely $p_H$) is the correct solution for each decay. This is due to the fact that the kaon is a pseudoscalar and so the orientation of the decay products is isotropic in the CMS. For fixed $p_K$ a plot of $p_L$ or $p_H$ is as shown in Figure 7.22.

For each event $p_L$ and $p_H$ is calculated. To obtain the original momentum spectrum, $p_K$, a transformation is obtained from the Monte Carlo which allows one to reconstruct the momentum spectrum from the $p_{L,H}$ spectrum.

The transformation is obtained as follows:

1.) $K_{e3}$ or $K_{\mu3}$ decays are generated by the Monte Carlo and tracked through the subdetectors. For decays which are accepted the two kaon momentum solutions $p_L$ and $p_H$ are calculated.
2.) For each event the number of entries in a 2-d histogram bin are incremented by 1 for each of the coordinates \((p_L,p_K)\) and \((p_H,p_K)\). The histogram contains 20 x 10 GeV/c bins in \(p_K\) between 0–200 GeV/c and 80 x 5 GeV/c bins in \(p_L,H\) between 0–400 GeV/c. In this way a matrix relating \(p_{L,H}\) to \(p_K\) is formed as shown in Figure 7.23.

3.) Each row in the histogram is then normalised to 1. (corresponding to a flat \(p_K\) spectrum). The entries in each bin then correspond to the probability of obtaining a \(p_{L,H}\) value for any given \(p_K\).

4.) For the data \(p_L\) and \(p_H\) are calculated for each event and filled into a 1-d
histogram as shown in Figure 7.24.

\[ N_i = \sum_{j=1}^{80} P_{ij} \phi_j \]  \hspace{1cm} (7.21)

where

- \( N_i \) = the measured flux in \( p_{L,H} \) in the interval \((5(i - 1): 5i)\) GeV/c
- \( P_{ij} \) = the probability of obtaining a \( p_{L,H} \) value in the interval \((5(i - 1): 5i)\) GeV/c from a \( p_K \) in the interval \((10(j - 1): 10j)\) GeV/c
- \( \phi_j \) = the real flux in \( p_K \) in the interval \((10(j - 1): 10j)\) GeV/c

Thus we have a system of 80 simultaneous equations relating the known measured flux in \( p_{L,H} \) \( (N_i) \) to the real flux in \( p_K \) \( (\phi_j) \) via the Monte Carlo generated probabilities \( (P_{ij}) \). This overdetermined system is solved for the \( \phi \)'s using a least-squares method (CERNLIB TLS routine[45]).

The result for \( K_e3 \) is shown in Figure 7.25. It can be seen that the measured spectrum has many more low momentum kaons than predicted by the Monte Carlo.
Carlo. The discrepancy at high momentum has far less effect due to the reduced statistics.

Fig. 7.25: The Momentum Spectrum Calculated from K_{e3} Decays (solid lines – data, hashed lines – Monte Carlo). The ratio (data/MC) is shown to the right.

To correct the momentum spectrum, $F(p)$, in the Monte Carlo the transformation

$$ F(p) \rightarrow F(p)(e^{-(A+BpK)} + C) \quad (7.22) $$

was applied to the production spectrum. The values chosen for the constants, $A$, $B$ and $C$ were 4.2, 0.053 and 0.0016 respectively. These values were obtained by fitting a function of the form $e^{-(A+BpK)} + C$ to the ratio $F'(p : data)/F'(p : MC)$, where $F'(p)$ is the decay spectrum (after acceptance corrections). To obtain the decay spectrum from the production spectrum one has to weight the decay spectrum by the decay probability (as described in Chapter 6 on page 82) and then correct for the acceptance. As the decay probability and the acceptance are both single-valued functions of the kaon momentum any correction derived from the ratios of the decay spectra (data/MC) can be applied to the production spectrum in the Monte Carlo. In Figure 7.26 the distributions $p_L$ and $p_H$, for K_{e3}, are compared with Monte Carlo generated distributions. The left-hand side shows the Monte Carlo distributions with the original spectrum and the right-hand side with the altered spectrum.

A further check was made using $K_L \rightarrow \pi^+\pi^-$ events where the kaon momentum
can be easily reconstructed for each event. The $\pi^+\pi^-$ events were selected by requiring no hit in any of the planes of the muon veto, applying strict kinematic cuts ($0.493 < m_{\pi\pi} < 0.503$ GeV, $p_T < 0.02$ GeV/c) and removing any events with a track which had a value of $E_{\text{max}}/p_{\text{track}} > 0.7$. Almost all the $K_{\mu3}$ events are removed by the muon veto condition (multiplying the inefficiencies of the three planes gives a global inefficiency of $8 \times 10^{-6}$) and the background from $K_{\mu3}$ is reduced by the $E_{\text{max}}/p_{\text{track}}$ cut. In the final sample 885 $\pi^+\pi^-$ events were
selected. By extrapolating the background from adjacent regions in $m_{\pi\pi}$ it is estimated that the background from $K_{e3}$ is less than 5%. The ratio of the resulting $p_K$ spectrum divided by that predicted by the Monte Carlo (with the modified spectrum obtained from the $K_{i3}$ analysis incorporated into the kaon generation routine) is shown in Figure 7.27. The spectra agree to within 10% between 30-170 GeV/c (the hatched area encloses ±10% of the central value obtained by fitting a constant to the ratio of the decay spectra). Due to the low statistics, within each bin in $p_K$, from $K_L \to \pi^+\pi^-$ in the data this test of the Monte Carlo is dominated by the statistical error. However it results in a verification of the transformation applied to the Monte Carlo spectrum (Equation 7.22) independently of the method used for the three-body decays.

<table>
<thead>
<tr>
<th>$\chi^2$/ndf</th>
<th>34.04 / 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_0$</td>
<td>0.1081</td>
</tr>
</tbody>
</table>

Fig. 7.27: The ratio of Decay Spectra for $K_L \to \pi^+\pi^-$ events: (data)/(Monte Carlo).

The distributions shown in Figures 7.17-7.20 were generated using the unaltered kaon spectrum. Similar distributions generated using the altered spectrum showed no improvement in the agreement of data and Monte Carlo for invariant quantities while the radial distributions of the tracks at various detectors showed a marked improvement. In Figures 7.17 and 7.18, which compare data and Monte Carlo for the Lorentz invariant quantities $p_T^2$, $M_\pi$ and $(p_0')^2$, the data distributions are consistently wider than the corresponding Monte Carlo distributions. This could be attributed to a resolution effect in the simulation of the charged spectrometer. The spectrometer resolution is limited by two effects:
1. **Multiple scattering.** Interactions of the charged particle with material in the kevlar window and chambers results in deviations of the particles direction.

2. **Sagittal error.** The position of the space-points reconstructed within each chamber have an uncertainty of 100-200 $\mu$m arising from the drift-time measurements.

These physical processes are included in the NMC Monte Carlo. The finite resolution results in a Gaussian smearing of the true values of the momentum. Thus, any quantities calculated from the track momenta will also be effected by the resolution. The data distributions are wider and so we conclude that the spectrometer resolution in the Monte Carlo is over-optimistic. If the non-agreement is simply due to this effect, alteration of the production spectrum will make no improvement to comparisons between data and Monte Carlo. This is what is observed in Figures 7.17 and 7.18.

**Additional Cuts Applied**

A cut on the z-vertex is applied at 10 m in both the data and the Monte Carlo. This region (0–10 m) lies immediately after the $K_L$ collimator and so any regeneration of $K_S$ from kaons interacting in the collimator would be observed at this point. From Figure 7.25 it can be seen that the mean kaon momentum is $\sim$80 GeV/c. This corresponds to a mean range of $\gamma \beta c \tau_{K_S} = 4.3$ m. Thus the cut at 10 m corresponds to a cut at 2.3 $K_S$ lifetimes and so $\sim 90\%$ of any regenerated $K_S$ will have decayed before this point. A further cut is applied at 60 m as the acceptance becomes very small beyond this point. Thus the fiducial region considered is the region (10–60 m).

In Figures 7.19 and 7.20 it can be seen that in the ratio of data:Monte Carlo for the radial distribution at the first drift chamber, there are far fewer tracks in close proximity to the beam-pipe in the data. A radius cut is made at 15 cm to remove the region around the beam-pipe. A cut is then applied in the separation of the tracks at chamber 1. Only events in which the tracks are separated by more than 30 cm are accepted.

During the "$\mu\mu\gamma$" run approximately $3.4 \times 10^7$ triggers were recorded. Of this
7. The Measurement of $\Gamma(K_L \to \pi\nu)/\Gamma(K_L \to \pi\nu)$

<table>
<thead>
<tr>
<th>Cut</th>
<th>Remaining Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. bias PMB bit set</td>
<td>649 598</td>
</tr>
<tr>
<td>Two tracks</td>
<td>647 395</td>
</tr>
<tr>
<td>One pos. plus one neg. track</td>
<td>645 553</td>
</tr>
<tr>
<td>$p_{\text{track}} &gt; 20 \text{ GeV}/c$</td>
<td>180 798</td>
</tr>
<tr>
<td>Z-vertex</td>
<td>140 192</td>
</tr>
<tr>
<td>Track radius at chamber 1</td>
<td>97 030</td>
</tr>
<tr>
<td>Track separation at chamber 1</td>
<td>94 785</td>
</tr>
<tr>
<td>Muon veto/HAC acceptance</td>
<td>84 475</td>
</tr>
<tr>
<td>Separation of tracks in HAC</td>
<td>83 632</td>
</tr>
<tr>
<td>$(p'_0)^2 &lt; -0.004 \text{ (GeV}/c)^2$</td>
<td>76 023</td>
</tr>
</tbody>
</table>

Tab. 7.1: Summary of number of events surviving the various cuts in the data reduction.

number less than 1/5 correspond to the downscaled minimum bias triggers. After selecting events in which there are at least two reconstructed tracks in the spectrometer and the PMB readout of the trigger bits is not corrupted 649 598 events remain. A summary of the statistics remaining, after each of the selection cuts are applied, is shown in Table 7.1.

7.3 Calculation of $\Gamma(K_L \to \pi\nu)/\Gamma(K_L \to \pi\nu)$

Using the selection criteria mentioned previously $K_{\mu3}$ and $K_{e3}$ decays are extracted from the data sample. In total 76023 events are selected. The branching ratio $\Gamma(K_L \to \pi\nu)/\Gamma(K_L \to \pi\nu)$ is calculated in $10 \times 6$ m bins in the $z$-vertex distribution between 10–60 m from the $K_S$ target. In each bin the branching ratio is calculated from

$2$ This means that in 90% of the minimum bias events either the charged reconstruction failed to find tracks or the PMB readout produced incorrect information. In this run many events were observed with very high multiplicities in the hodoscope and hadron calorimeter. These events, possibly arising from photon interactions in the beam-pipe, will fire the simple minimum bias trigger and could perhaps explain the low reconstruction efficiency.
\[
\frac{\Gamma(K_L \rightarrow \pi \mu \nu)}{\Gamma(K_L \rightarrow \pi \nu \nu)} = \frac{N_{\pi \mu \nu} \cdot A_{\pi \nu \nu} \cdot C_{\pi \mu \nu}}{N_{\pi \nu \nu} \cdot A_{\pi \nu \nu} \cdot C_{\pi \nu \nu}} \quad (7.23)
\]

where \(N_{\pi \nu \nu} (l = \mu \text{ or } e)\) is the number of decays of each type in this bin in \(z\), \(A_{\pi \nu \nu}\) is the acceptance and \(C_{\pi \nu \nu}\) are any correction factors applied. All the geometrical and kinematic cuts are folded into the acceptance so the main corrections come from the effect of pion decay in flight plus inefficiencies in the identification of muons. For \(K_{e3}\) the effect of pion decay is (neglecting inefficiencies in the muon veto) to move the event from the \(K_{e3}\) sample to the \(K_{\mu3}\) sample. In practice the muon identification is \((99.6 \pm 0.1)\%\) efficient: \((99.8 \pm 0.1)\%\), from the efficiency of the "maj. 2" signal, combined with \((99.76 \pm 0.03)\%\), from the \(E_{\text{max}}/p_{\text{track}}\) cut on muons in the hadron calorimeter. In the case of \(K_{\mu3}\), pion decay results in the loss of the event as the energy deposited by two muons is below the threshold \(EHL\). A fraction of \(K_{\mu3}\) will be identified as \(K_{e3}\) due to the imperfect efficiency of the muon identification. A system of simultaneous equations can be written in terms of \(K_{\mu3}\) and \(K_{e3}\), the samples initially attributed to \(K_{\mu3}\) and \(K_{e3}\), \(P_{\pi}\), the probability for pion decay in flight for \(Re_{3}\) decays and \(e_{\mu}\), the efficiency of muon identification:

\[
K_{\mu3} = \epsilon_{\mu} K_{\mu3} + P_{\pi} \epsilon_{\mu} K_{e3} \quad (7.24)
\]

\[
K_{e3} = (1 - P_{\pi} \epsilon_{\mu}) K_{e3} + (1 - \epsilon_{\mu}) K_{\mu3} \quad (7.25)
\]

The solutions for this system in terms of \(K_{e3}\) and \(K_{\mu3}\) are:

\[
K_{e3} = \frac{K_{e3} \ast -K_{\mu3} \ast (1 - \epsilon_{\mu})/\epsilon_{\mu}}{1 - P_{\pi}} \quad (7.26)
\]

\[
K_{\mu3} = \frac{K_{\mu3} \ast (1 - P_{\pi} + (1 - \epsilon_{\mu})/\epsilon_{\mu}) - K_{e3} \ast P_{\pi}}{1 - P_{\pi}} \quad (7.27)
\]

The resulting branching ratio as a function of \(z\)-vertex is shown in Figure 7.28. There does not appear to be any significant variation in \(\Gamma(K_L \rightarrow \pi \mu \nu)/\Gamma(K_L \rightarrow \pi \nu \nu)\) as a function of \(z\).

Finally a correction is applied to allow for the remaining background from \(K_L \rightarrow \pi^+ \pi^- \pi^0\) decays which are not removed by the \((p_0')^2\) cut. Generating Monte Carlo
7. The Measurement of $\frac{\Gamma(K_L \rightarrow \pi\mu\nu)}{\Gamma(K_L \rightarrow \pi\nu\nu)}$ as a function of $z$-vertex.

Fig. 7.28: $\frac{\Gamma(K_L \rightarrow \pi\mu\nu)}{\Gamma(K_L \rightarrow \pi\nu\nu)}$ as a function of $z$-vertex.

samples of $K_{\mu3}$, $K_{e3}$ and $K_{\pi3}$, each with $10^6$ events, and applying all the fiducial region cuts plus the cut on $(p_0')^2$ only 422 of the $K_{\pi3}$ survived compared to 38459 of the $K_{\mu3}$ and 36682 of the $K_{e3}$. Out of the 422 $K_{\pi3}$ events 415 of them have a muon in the end state produced by pion decay – the long tail at negative values of $(p_0')^2$ for $K_{\pi3}$ can be seen in Figure 7.10 on page 106. Using the values of the branching ratios, for these channels, taken from the “Review of Particle Properties” it can be shown that the $K_{\pi3}$ background results in a fractional correction to the $K_{\mu3}$ of $(−4.9 \pm 0.2) \times 10^{-3}$ and to the $K_{e3}$ of $(−6 \pm 2) \times 10^{-5}$. These corrections are, in fact, an upper bound as the acceptance for $K_{\pi3}$ is most likely to be overestimated by NMC. The overestimation arises from photon interactions with matter in the detector. In NMC gamma conversions are only simulated in the beam-pipe. In a study made by H. Dibon comparing gamma conversions in NMC and NASIM[46, 47], albeit with different generation limits (70–170 GeV/c), it was found that for $K_L \rightarrow \pi^+\pi^-\pi^0 \rightarrow \pi^+\pi^-\gamma\gamma$ 3.58% of the events included a gamma conversion in the beam-pipe. Including all detectors in the simulation predicted a gamma conversion in 32.7% of the events. Depending on the position of the gamma conversion, and the distribution of any shower products, these events may be lost. In the case that an electromagnetic shower passes through a chamber the large number of wires hit will prevent reconstruction of the tracks from the charged pions. This is particularly important if the shower occurs upstream of the magnet.
as the magnetic field will have the effect of spreading the electrons and positrons out before they hit the final chamber.

If no $K_{\pi 3}$ events were lost due to gamma conversion the effect of the estimation and subsequent subtraction of the $K_{\pi 3}$ background would be to reduce $\Gamma(K_{\mu 3})/\Gamma(K_{e3})$ from 0.704 to 0.700.

### 7.3.1 Systematic Errors in the Calculation of $\Gamma(K_L \rightarrow \pi\mu\nu)/\Gamma(K_L \rightarrow \pi\nu\nu)$

The main systematic errors arise from the acceptance calculation plus the determination of the residual background from $K_{\pi 3}$ decays.

The acceptance of a particular decay channel is a function of the kaon momentum, for any given fiducial region (this is shown for $K_{\mu 3}$, etc. in Figures 6.4–6.7 in Chapter 6). As the kaon momentum is not known on an event-by-event basis for the semi-leptonic decays the relative branching ratio, $\Gamma(K_{\mu 3})/\Gamma(K_{e3})$, is determined by integrating over the entire kaon momentum range. If it was possible to compare branching ratios in bins of $p_K$ any discrepancy in the spectra would cancel in the Monte Carlo acceptance calculation. Thus it is important that the kaon production spectrum in the Monte Carlo agrees well with that of the experiment. A comparison between the data and NMC kaon decay spectrum has been made using semi-leptonic decays and a small sample of $CP$-violating $K_L \rightarrow \pi^+\pi^-$ decays. After a correction was applied to the production spectrum in NMC the $p_K$ spectra in the data and NMC agree to within 10% over the entire momentum range. To estimate the effect of the uncertainty in the production spectrum two different Monte Carlo data sets were produced, each of which contained $10^6 K_{e3}$ and $10^6 K_{\mu 3}$ events. In the first set the spectrum in Equation 7.22 was weighted linearly such that there was a 10% excess at 20 GeV/c and a 10% deficit at 200 GeV/c. The second set is weighted in a similar manner but with the excess at 200 GeV/c and the deficit at 20 GeV/c.

For the first Monte Carlo data set the $K_{\mu 3}$ acceptance is decreased by 2.29% and the $K_{e3}$ is decreased by 2.24%. However, the ratio of acceptances changes by only 0.06%. In the second data set the $K_{\mu 3}$ acceptance increases by 2.57% while the $K_{e3}$ acceptance increases by 3.70%. This results in a systematic shift.
in the ratio of the acceptances by 1.1%. Thus the uncertainty in the production spectrum of the KL beam contributes significantly to the systematic error of \( \Gamma(K_L \to \pi\mu\nu)/\Gamma(K_L \to \pi e\nu) \). This will certainly improve in future years as increased \( K_L \to \pi^\pm\pi^- \) statistics will allow the decay spectrum to be measured from data. This in turn will result in a more accurate production spectrum in the Monte Carlo. In addition, the NA52/SPY\(^3\) collaboration at CERN have taken data in April 1996 to measure the fluxes of charged pions and kaons below 60 GeV/c, produced by 450 GeV/c protons on beryllium targets\[^48\]. This will provide information on the kaon production spectrum at lower momenta than the Atherton measurement\[^37\]. The SPY measurements were made principally to determine the neutrino fluxes for future neutrino experiments at the CERN West Area Neutrino Facility but will also provide useful data for comparison with the NA48 particle spectra. The results of the SPY collaboration have yet to be published.

To check the variation of the acceptance due to uncertainties in the form-factors in the \( K_{\mu3} \) and \( K_{e3} \) NMC generators several Monte Carlo data sets were generated with different values for the parameters \( \lambda_3 \) and \( \xi(0) \), for \( K_{\mu3} \), and \( \lambda_+ \) only for \( K_{e3} \). The values were varied between \( \pm 1\sigma \) of the central value quoted in the 1996 PDG “Review of Particle Properties”. The maximum variation in the \( K_{\mu3} \) case is 0.53% from the acceptance calculated at the central values of \( \lambda_+ \) and \( \xi(0) \). For \( K_{e3} \) the maximum variation from the central value is 0.61%. However, the \( \lambda_+ \) parameter is common to both form-factors and so there is a correlation between these e.g. when \( \lambda_+ \) is increased the acceptance increases for both \( K_{\mu3} \) and \( K_{e3} \). Taking this into account the variation is estimated to be the maximum difference between the acceptances for \( K_{\mu3} \) and \( K_{e3} \) when \( \lambda_+ \) is varied coherently, between \( \pm 1\sigma \) of the central value, in the two form-factors. This gives a systematic uncertainty of 0.1%. In the \( K_{\mu3} \) there is also the parameter \( \xi(0) \). Variation of this parameter within the one \( \sigma \) bounds changes the \( K_{\mu3} \) acceptance by 0.3%.

Figure 7.29 shows the \( K_{\mu3}/K_{e3} \) branching ratio, uncorrected for background from \( K_{e3} \), as a function of the minimum momentum cut, \( p_{\text{min}} \). Below the nominal cut \( (p_{\text{min}} = 20 \text{ GeV/c}) \) the branching ratio appears to decrease. This can reasonably be attributed to the \( EHL \) threshold in the HAC where the single pion in \( K_{\mu3} \) has insufficient energy to fire the \( EHL \) discriminator. Between the \( p_{\text{min}} = 20 \text{ GeV/c} \)

\[^3\text{Secondary Particle Yields}\]
result and the $p_{\text{min}} = 30 \text{ GeV/c}$ result there is an increase of 0.43% in the $K_{\mu 3}/K_{e3}$ ratio. This variation is used as the estimate of the systematic uncertainty arising from the minimum momentum cut.

A similar exercise was performed for the cut on the kinematic variable $(p_0')^2$. The $K_{\mu 3}/K_{e3}$ branching ratio was determined for the values $(p_0')^2 < -0.002, -0.004$ and $-0.006 \text{ (GeV/c)}^2$. The variation in the $K_{\mu 3}/K_{e3}$ ratio is less than 0.2%. As the $(p_0')^2$ distributions for the semileptonic decays are reasonably flat around this region we would expect that the cut would be fairly insensitive to discrepancies between the resolution of the spectrometer in the data and the Monte Carlo.

Uncertainty in the $K_L \to \pi^+\pi^-\pi^0$ correction arises mainly from the uncertainty in the acceptance due to gamma conversions. The result quoted below assumes no events are lost due to gamma conversions. This gives rise to a one-sided systematic due to the possibility that the final subtraction of $\pi^+\pi^-\pi^0$ events from the $K_{\mu 3}$ is overestimated.

Table 7.2 gives a summary of all the systematic errors evaluated. These errors are regarded as independent and are added in quadrature. The result for $K_{\mu 3}/K_{e3}$ is therefore:

$$\frac{\Gamma(K_L \to \pi\mu\nu)}{\Gamma(K_L \to \pi e\nu)} = 0.700 \pm 0.007_{-0.009}^{+0.010}$$

(7.28)

Fig. 7.29: $\Gamma(K_L \to \pi\mu\nu)/\Gamma(K_L \to \pi e\nu)$ as a function of $p_{\text{min}}$. 

\begin{center}
\includegraphics[width=\textwidth]{figure7.29.png}
\end{center}
7. The Measurement of \( \frac{\Gamma(K_L \to \pi \mu \nu)}{\Gamma(K_L \to \pi e \nu)} \)

<table>
<thead>
<tr>
<th></th>
<th>Syst. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum spectrum</td>
<td>±1.1%</td>
</tr>
<tr>
<td>( K_{13} ) form-factors</td>
<td>±0.3%</td>
</tr>
<tr>
<td>Cut on ( p_{\text{min}} )</td>
<td>±0.43%</td>
</tr>
<tr>
<td>Cut on ( (p'_0)^2 )</td>
<td>±0.2%</td>
</tr>
<tr>
<td>( K_{\pi3} ) background</td>
<td>+0.2%</td>
</tr>
</tbody>
</table>

Tab. 7.2: Systematic Errors in \( K_{\mu3}/K_{e3} \) Measurement.

where the first error is statistical and the second is systematic.

7.4 Interpretation of Result

A comparison of the present result for \( \frac{\Gamma(K_{\mu3})}{\Gamma(K_{e3})} \) with several previous measurements is shown in Figure 7.30. The world average is denoted by the dotted line with the one sigma errors as solid lines.

Note: the present result is not used in generating the ideogram or in the calculation of the world average.

The measured value is in very good agreement with both the current world average for previous experiments, \( \frac{K_{\mu3}}{K_{e3}} = 0.697 \pm 0.010^{[20]} \), and the previous best measurement by Cho et al \( \frac{K_{\mu3}}{K_{e3}} = 0.702 \pm 0.011^{[22]} \).

Comparing the measured value with that predicted by Equation 2.82 on page 21 we find that

\[
\frac{K_{\mu3}/K_{e3}^{(\text{measured})}}{K_{\mu3}/K_{e3}^{(\text{theoretical})}} = \frac{0.700 \pm 0.012}{0.677 \pm 0.009} = 1.03 \pm 0.02 \quad (7.29)
\]

so the relative branching ratios of the semileptonic decays of \( K^0 \) are consistent with the current algebra model described in Section 2.4.

The phenomenology of the semileptonic decays of charged kaons is identical to that of the neutral kaons. The \(|\Delta I| = 1/2\) rule predicts that

\[
\langle \pi^0 |V^\mu|K^+ \rangle = \frac{1}{\sqrt{2}} \langle \pi^- |V^\mu|K^0 \rangle \quad (7.30)
\]

\footnote{This ideogram is defined as described in the PDG “Review of Particle Properties”\[49\].}
7. The Measurement of $\Gamma(K_L \to \pi \mu \nu)/\Gamma(K_L \to \pi e \nu)$

The Measurement of $\Gamma(K_L \to \pi \mu \nu)/\Gamma(K_L \to \pi e \nu)$

WEIGHTED AVERAGE 0.697±0.010

Fig. 7.30: Comparison of present result with previous measurements of $K_{\mu 3}/K_{e 3}$.

Thus, if one neglects radiative corrections, the value of $R_{\mu e}^0 = \Gamma(K_{\mu 3}^0)/\Gamma(K_{e 3}^0)$ should be identical to $R_{\mu e}^+ = \Gamma(K_{\mu 3}^+)/\Gamma(K_{e 3}^+)$.

If we compare the measured result for $K_{\mu 3}/K_{e 3}$ with the world average taken from the PDG[20] we obtain:

$$\frac{R_{\mu e}^0}{R_{\mu e}^+} = \frac{0.700 \pm 0.012}{0.680 \pm 0.013} = 1.03 \pm 0.03 \quad (7.31)$$

which is consistent with the $|\Delta I| = 1/2$ rule.

A measurement of $K_{\mu 3}/K_{e 3}$ does not allow a determination of either of the form factors. However it does fix a relationship between either $\lambda_+$ and $\xi(0)$ or $\lambda_+$ and $\lambda_0$. Figures 7.31 and 7.32 depict this relationship (obtained from the linear approximation to Equation 2.81 on page 21) in terms of the two sets of parameters, $\lambda_+$ and $\xi(0)$ and $\lambda_+$ and $\lambda_0$, respectively. As the value of $\lambda_+$ is known rather accurately from $K_{e 3}$ decays, where there is no contribution from $\lambda_0$, the value
obtained for $K_{\mu 3}/K_{e 3}$ favours a larger value for $\lambda_0$. Taking the PDG value for $\lambda_+$ and the measured value of $K_{\mu 3}/K_{e 3}$ we obtain

$$\lambda_0 = 0.04 \pm 0.01$$  \hspace{1cm} (7.32)

where the majority of the error arises from the measurement of $K_{\mu 3}/K_{e 3}$. Varying the value of $\lambda_+$ within one sigma error bars of the present world average contributes 0.1% to the systematic error. As the $K_{\mu 3}/K_{e 3}$ result is dominated by the systematic error in the kaon momentum spectrum, followed by the statistical error, the present measurement is insensitive to the $\lambda_+$ parameter.

In the current PDG compilation the experiments measuring the $K_{\mu 3}/K_{e 3}$ ratio and/or the form factors can be divided into two groups: bubble chamber experiments and spectrometer experiments. Both groups have certain advantages and
disadvantages. Bubble chamber experiments, such as Cho[22], tend to suffer from low statistical precision, due to the dead-time in the expansion cycle and the low beam rate which must be maintained in order to distinguish the tracks, and also rather poor identification of the end state particles. The particle identification problems can be alleviated by using a large volume hydrogen chamber or a heavy liquid bubble chamber in which the particles can be identified via interactions, decays or stops. The great advantage of bubble chamber experiments is their uniform acceptance across the Dalitz plot. In the case of spectrometer experiments identification of the particles, from their signature in the calorimeters and muon counters, is usually much better than in the case of bubble chambers. These detectors can also tolerate a much higher rate of decay particles passing through the various sub-detectors and so can collect very high statistics in a relatively
short period of time. However, the acceptance of the spectrometer experiments is non-uniform across the Dalitz plot and so any measurements of the form factors (e.g. Birulev et al.[21]) are heavily dependent on Monte Carlo simulation of the efficiency of the apparatus as a function of position within the Dalitz plot. Much more care is needed to avoid generating false values for \( \lambda_+ \) and \( \lambda_0 \) arising from incorrect simulation of the apparatus.

The Birulev method eliminates the problems arising from the two-fold ambiguity in the \( K_L \) momentum. Events are selected in which the two solutions for \( t \), the square of the four-momentum transferred to the leptons, are within \( 10^2 \) (MeV)\(^2 \) of each other. These "diagonal" events correspond to events where the neutrino direction vector in the centre-of-mass system is nearly perpendicular to the kaon direction of flight. For \( K_{e3} \) decays, for example, the Dalitz plot is parametrised by

\[
\rho(E_\pi, E_\nu) \propto A f_+^2(t) = A f_+^2(0) \left( 1 + \frac{2\lambda_+ t}{m_\pi^2} + \frac{\lambda_+^2 t^2}{m_\pi^4} \right) \quad (7.33)
\]

By plotting the pion spectrum, as a function of \( t/m_\pi^2 \), for the data and then dividing by a similar distribution generated by the Monte Carlo, assuming no form factor alteration to the pure phase space prediction, a quadratic relationship is obtained from the ratio of data/(Monte Carlo). The parameter \( \lambda_+ \) can be extracted from this quadratic relationship. Alternatively both solutions for \( t \) can be placed on the 3-dimensional Dalitz plot \( t_1 \times t_2 \times E_\nu \). The value of \( \lambda_+ \) can then be determined as a free parameter by fitting this distribution to Monte Carlo simulated distributions. For \( K_{\mu3} \) events a similar analysis can be made using the more complex Dalitz plot parametrisation of Equation 2.51 on page 16.

With the high-precision spectrometer and good particle identification in the electromagnetic calorimeter and muon veto this analysis could be easily repeated by NA48 - especially as several collaborators in the Birulev paper are now members of the Dubna NA48 group\(^5 \). The statistics of the Birulev measurement were limited to 74 000 \( K_{e3} \) and 150 000 \( K_{\mu3} \). This could be achieved with a single day of running in NA48.

\(^5\) During the NA48 collaboration meeting in Dubna, at the end of February 1997, G. Tatishvili, of the Dubna group presented an analysis of the \( K_{e3} \) Dalitz plot. Using the analysis method described in the Birulev paper he obtained a preliminary value of \( \lambda_+ = 0.028 \pm 0.008 \).
8. A STUDY OF $\Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \text{all charged})$

The second analysis, a study of the branching ratio $\Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \text{all charged})$, is described in this chapter. In Chapter 7 it was possible to produce a clean sample of the semileptonic decays by imposing a cut on $(p)^2$. However there is no such cut which will produce a sample of $K_{\tau3}$ uncontaminated by backgrounds from $K_{e3}$ and $K_{\mu3}$. An alternative solution to the problem is obtained using maximum likelihood fits to estimate the proportions of $K_{f3}$, the sum of the $K_{e3}$ and $K_{\mu3}$, and $K_{\tau3}$ in the data sample.

8.1 Event Selection

Events are selected with identical cuts to the $K_{\mu3}/K_{e3}$ analysis except that the cut on $(p')^2$ is removed so that $K_{\tau3}$ are not removed from the sample. The resulting distributions for $m_{\tau}$, the effective $\pi\pi$ mass, and $(p')^2$ are shown in Figure 8.1. The $(p')^2$ distribution has the best separation between the $K_{\tau3}$ events and the $K_{f3}$ events and so is used to estimate the relative proportions of each type in the sample.

8.2 Method of Measurement

As shown in Figure 8.1, we can obtain distributions of kinematic variables for $K_{\mu3}$, $K_{e3}$ and $K_{\tau3}$ samples in which we aim to calculate the proportions, $P_j$, of each of these decay modes.

As there is no analytic form for the distributions of $m_{\tau}$ and $(p')^2$ for the three decay types, $K_{\mu3}$, $K_{e3}$ and $K_{\tau3}$, the Monte Carlo is used to generate these distributions for each of these decays. Then the predicted frequency distribution, $f_i$,
is approximated by histogramming the generated values of \( m_c \) or \((p_\theta)^2\). If there are \( m \) sources each contributing to the frequency distribution, \( f_i \), then the total number of events in each bin, given by the proportions \( P_j \) and the number of Monte Carlo events \( a_{ji} \) from source \( j \) in bin \( i \) is

\[
    f_i = N_D \sum_{j=1}^{m} P_j a_{ji} / N_j
\]  

where \( N_D \) is the number of entries in the data sample and \( N_j \) the number of entries in the Monte Carlo sample for source \( j \). Then, if \( d_i \) are the number of events from real data that fall within bin \( i \) and there are \( n \) bins in the histogram,

\[
    N_D = \sum_{i=1}^{n} d_i \quad \text{and} \quad N_j = \sum_{i=1}^{n} a_{ji}.
\]  

The \( P_j \) are the proportions of each data type and \( \sum_{j=1}^{m} P_j = 1 \). Incorporating the normalisation factors into the proportions, \( P_j \), such that we define

\[
    p_j = \frac{N_D P_j}{N_j}
\]  

then equation 8.1 can be rewritten as
\[ f_i = \sum_{j=1}^{m} p_j a_{ji}. \]  

(8.4)

One method to solve this for the \( p_j \) could be to adjust their values such as to minimise

\[ \chi^2 = \sum_{i=1}^{n} \frac{(d_i - f_i)^2}{d_i} \]  

(8.5)

which assumes that the \( d_i \) are distributed according to a Gaussian distribution whereas they are actually distributed according to Poisson statistics. Assuming that the true value for the frequency distribution in a given bin is \( f_i \) then the probability of observing a particular \( d_i \) in the data distribution is

\[ e^{-f_i} \frac{f_i^{d_i}}{d_i!}. \]  

(8.6)

Then the best estimated values for the \( p_j \) are given by maximising the total likelihood

\[ \mathcal{L} = \prod_{i=1}^{n} e^{-f_i} \frac{f_i^{d_i}}{d_i!} \]  

(8.7)

or, for convenience, its logarithm (and omitting the constant factorials)

\[ \ln \mathcal{L} = \sum_{i=1}^{n} (d_i \ln f_i - f_i). \]  

(8.8)

This method takes account of non-Gaussian fluctuations in the occupancy of the histogram bins in the data sample but does not make any allowance for the fact that there are also non-Gaussian fluctuations in the \( a_{ji} \) due to the finite Monte Carlo statistics. Disagreements between a particular \( d_i \) and \( f_i \) arise from incorrect \( p_j \), from fluctuations in \( d_i \) and also from fluctuations in \( f_i \). From equation 8.1 it can be seen that the fluctuations in \( f_i \) are damped by a factor \( N_D/N_j \) so producing a Monte Carlo sample which has many times the statistics of the data sample reduces the fluctuations in the \( a_{ij} \).
To take account of the fluctuations in the Monte Carlo distributions correctly, equation 8.4 is rewritten as

$$f_i = \sum_{j=1}^{m} p_j A_{ij}$$

(8.9)

where $A_{ij}$ are the expected number of events. For each $A_{ij}$ the corresponding $a_{ij}$ is generated assuming that the distribution of the $a_{ij}$ is Poisson. (In the limit $\lim_{n \to \infty} a_{ij} \to A_{ij}$)

Then the correct $p_j$ are obtained by maximising the combined probability of the observed $d_i$ and $a_{ji}$

$$\ln \mathcal{L} = \sum_{i=1}^{n} \left( d_i \ln f_i - f_i \right) + \sum_{i=1}^{n} \sum_{j=1}^{m} \left( a_{ij} \ln A_{ji} - A_{ji} \right).$$

(8.10)

The solution to the maximisation problem is given by differentiating equation 8.10 with respect to $p_j$

$$\sum_{i=1}^{n} d_i A_{ji} \frac{A_{ji}}{f_i} - A_{ji} = 0 \quad \forall j$$

(8.11)

and with respect to $A_{ji}$

$$\frac{d_i p_j}{f_i} - p_j + \frac{a_{ji}}{A_{ji}} - 1 = 0 \quad \forall i, j.$$  

(8.12)

These equations can be solved numerically using the HBOOK routine HMCMLL written by R. Barlow [34]. This routine uses MINUIT [42] to maximise the likelihood $\mathcal{L}$.

In our problem the proportions $P_j$ are the relative proportions of the summed semileptonic distributions and the $K_{e3}$ distributions for the variables $(p'_0)^2$ and $m_c$. Using the result of the $K_{\mu3}/K_{e3}$ branching ratios and their relative acceptances, calculated using the Monte Carlo, the semileptonic Monte Carlo distributions are produced according to the equation

$$K_{l3}^{\text{MC}} = K_{\mu3}^{\text{MC}} \frac{\Gamma(K_{\mu3})}{\Gamma(K_{e3})} + K_{e3}^{\text{MC}}$$

(8.13)
where \( K_{\mu 3}^{MC} \) and \( K_{e 3}^{MC} \) are Monte Carlo generated distributions of either \((p'_0)^2\) or \(m_c\) with the relative acceptances already taken into account. Similar distributions are generated for \( K_{\pi 3} \). An additional constraint is placed on the \( K_{\pi 3} \) Monte Carlo events in the calculation of the acceptance: any event in which a photon from the decay of the \( \pi^0 \) hits the central beam-pipe within the spectrometer is removed. This is done as a photon impinging on the steel of the beam-pipe will initiate an electromagnetic shower producing many \( e^+e^- \) pairs. If the shower enters one of the drift chambers the event will almost certainly be lost as the multiplicity check in each view will overflow, causing the ring-buffers in the front-end readout to be cleared.

The Monte Carlo generated distributions are shown in Figure 8.2, for the \( K_{\mu 3}^{MC} \) distributions, and Figure 8.3, for the \( K_{e 3}^{MC} \) distributions.

\[
\begin{align*}
\text{Fig. 8.2: } & m_c \text{ and } (p'_0)^2 \text{ Distributions from } K_{\mu 3}^{MC} \text{ Sample.} \\
8.3 \text{ Calculation of } & \Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \text{all charged}) \nonumber \\
\end{align*}
\]

Taking the proportions, \( P_{\pi l\nu} \) and \( P_{\pi^+\pi^0} \), from the maximum likelihood analysis the branching ratio \( \Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi\mu\nu + K_L \to \pi\nu\nu) \) can be
calculated from
\[ \frac{\Gamma(\pi^+\pi^-\pi^0)}{\Gamma(\pi\mu\nu + \pi\nu\nu)} = \frac{P_{\pi^+\pi^-\pi^0} A_{\pi\nu\nu} C_{\pi^+\pi^-\pi^0}}{P_{\pi\nu\nu} A_{\pi^+\pi^-\pi^0} C_{\pi\nu\nu}} \]  
(8.14)

where \( A_{\pi^+\pi^-\pi^0} \) and \( A_{\pi\nu\nu} \) are the calculated acceptances for \( \pi^+\pi^-\pi^0 \) events and semileptonic events respectively. \( C_{\pi^+\pi^-\pi^0} \) and \( C_{\pi\nu\nu} \) are introduced as correction factors which may be used to allow for background corrections or detection efficiencies which are not taken into account in the acceptance calculation. The acceptance for the combined \( K_{e3} \) and \( K_{\mu3} \) events is given by
\[ A_{\pi\nu\nu} = \frac{A_{\pi\mu\nu} B_{\pi\mu\nu} + A_{\pi\nu\nu} B_{\pi\nu\nu}}{B_{\pi\mu\nu} + B_{\pi\nu\nu}} \]
\[ = \frac{A_{\pi\mu\nu} B_{\pi\mu\nu} / B_{\pi\nu\nu} + A_{\pi\nu\nu}}{B_{\pi\mu\nu} / B_{\pi\nu\nu} + 1} \]  
(8.15)

where \( B_{\pi\mu\nu} \) and \( B_{\pi\nu\nu} \) are the branching ratios \( \Gamma(K_L \rightarrow \pi\mu\nu) \) and \( \Gamma(K_L \rightarrow \pi\nu\nu) \) respectively. The value of \( B_{\pi\mu\nu} / B_{\pi\nu\nu} \) has been measured in Chapter 7.

As the CP violating \( K_L \rightarrow \pi^+\pi^- \) signal has been removed there are no corrections due to any background (no other charged decay modes exist with branching ratios greater than \( 10^{-4} \)). A correction has to be applied to the \( \pi^+\pi^-\pi^0 \) decays due to the decay \( \pi^0 \rightarrow e^+e^-\gamma \). These events will be lost due to the requirement of only

Fig. 8.3: \( m_c \) and \( (p_\theta)^2 \) Distributions from \( K_{\pi3}^{MC} \) Sample.
two tracks in the spectrometer. This will then result in a loss of $1.198 \pm 0.032\%$ of $\pi^+\pi^-\pi^0$ decays.

The results of the log-likelihood fit of $K_{l3}^{MC}$ and $K_{\pi3}^{MC}$ distributions to the data distribution is shown in Figure 8.4. From HMCMLL, $P_{\pi l\nu} = 0.934 \pm 0.009$ and

![Log-likelihood fits superimposed on $(p_0)^2$ data distribution; ratio (data)/(MC fit).](image)

$P_{\pi^+\pi^-\pi^0} = 0.067 \pm 0.002$ (where the one sigma errors are estimated using the MINOS package of MINUIT). Using Equation 8.14, while tentatively setting the corrections, $C_{\pi^+\pi^-\pi^0}$ and $C_{\pi l\nu}$, to 1.0 (any corrections from backgrounds are ex-
expected to be less than 2%), one obtains

\[
\frac{\Gamma(\pi^+\pi^-\pi^0)}{\Gamma(\pi\mu\nu + \pi e\nu)} = \frac{0.067 \, 37412}{0.934 \, 16980} = 0.159. \quad (8.16)
\]

Using PDG 1996 values one would expect to find a value of 0.190. A second test was made to calculate the ratio \(\Gamma(\pi^+\pi^-\pi^0)/\Gamma(\pi e\nu)\). Identical cuts were applied except that any events with a muon in the muon veto were removed. This data was also analysed using the same log-likelihood fit method and the results are shown in Figure 8.5. Again assuming no corrections the resulting branching ratio is (recalculating acceptances to allow for pion decay)

\[
\frac{\Gamma(\pi^+\pi^-\pi^0)}{\Gamma(\pi e\nu)} = \frac{0.107 \, 35351}{0.893 \, 15383} = 0.274. \quad (8.17)
\]

Comparing this with result with an NA31 measurement, \(\Gamma(\pi^+\pi^-\pi^0)/\Gamma(\pi e\nu) = 0.336 \pm 0.003 \pm 0.007 \) [50], it can be seen that both measurements made via the log-likelihood method are \(\sim 15\%\) lower than expected.

As mentioned on page 128 in Section 7.3, some of the \(K_{\pi3}\) events will not be observed due to the effects of electromagnetic showers “flooding” one or more of the chambers of the spectrometer. Either of the two gammas from the decay of the \(\pi^0\) may undergo pair production while traversing part of the detector. This is induced by the strong electric field that surrounds the nuclei. The nucleus in the vicinity absorbs part of the momentum of the photon but, due to its large mass, its energy remains approximately constant and the sum of the energies of the \(e^+e^-\) pair is very near to that of the original photon. The total probability for pair production per radiation length, \(\mu_0\), has been calculated by Rossi[51]: \(\mu_0 \sim 7/9\). The radiation length for iron is 1.76 cm. Thus any gammas hitting the beam-pipe or any of the seven anti-counters (which are constructed of an iron-scintillator-iron sandwich containing a 10.5 cm depth of iron in the z-direction) will undergo pair-production. Depending on the energies and directions of the resulting \(e^+e^-\) pair an electromagnetic shower may be initiated from direct pair production by the charged particles. Direct pair production is initiated as a bremsstrahlung process where the bremsstrahlung photon pair produces in the vicinity of a nucleus (as shown in Figure 8.6). While the photons have more than \(2m_e\) energy, where \(m_e\) is the mass of the electron, further pair production can continue.
8. A Study of $\Gamma(K_L \rightarrow \pi^+\pi^-\pi^0)/\Gamma(K_L \rightarrow \text{all charged})$

Fig. 8.5: Log-likelihood fits superimposed on $(p_0')^2$ data distribution $(\bar{\mu})$; ratio $(\text{data})/(\text{MC fit})$.

The NA48 spectrometer and calorimeters are shown in Figure 8.7. It can be seen that anti-counters 6 and 7 are directly in front of chambers 2 and 4 respectively. Thus any gammas which convert in these anti-counters will flood the chamber with electrons and positrons and the chamber readout will be cleared due to the high multiplicity – even if the readout continued it is doubtful whether a charged reconstruction could be performed due to the large number of space points generated in the chamber. While this is unfortunate for $K_{\pi3}$ it is vital for the neutral decay modes ($2\pi^0$, $3\pi^0$) so that the angular acceptance for photons at the
A preliminary study has been made of the distribution of the gammas from $K_{e3}$ using NMC015 in which gamma conversion is simulated in the beam-pipe. Out of 100K events generated 9058 events passed all the kinematic cuts. For each of these events a check was made for a gamma conversion and, if one occurred, the position of the conversion was noted. In total 677 gamma conversions occur in the beam-pipe which accounts for $\sim 7\%$ of all events. The distribution of gamma conversions is shown in Figure 8.8. The rectangles depict the elements which make up the beam-pipe – this is represented in NMC as a steel tube surrounded by cylindrical flanges used for connecting the pipe to the Kevlar window, drift chambers, etc. The Kevlar window is represented by the vertical line at 9531 cm. The positions of the drift chambers and the anti-counters are also shown.

About 50\% of the conversions in the beam-pipe occur in a steel flange, of 8.1 cm inner radius and 1.5 cm thickness, at the beginning of the beam-pipe (shown in Figure 8.9). The remainder of the conversions occur along the length of the beam-pipe from low angle gammas grazing the inner edge of the pipe.

In addition, for each of the events passing the cuts, the gamma was tracked to the
z-position of each of the seven anti-counters which surround the fiducial region (shown in Figure 8.7) and the \((x, y)\) coordinates were calculated to ascertain if the gamma collided with the anti-counter ring. The occupancy of the anti-counters, for the same event sample, is shown in Figure 8.10. The rate in anti-counter six is quite high and may cause problems in chamber two. If we assume that any interaction in an anti-counter will result in one of the chambers being flooded then a further 33% of the \(K_{\pi3}\) events will be lost. This is, no doubt, a gross overestimate but within these limits there is the possibility of up to 40% of \(K_{\pi3}\) events with soft electromagnetic showers produced by gamma conversions.
By reducing the $p_{\text{min}}$ cut, on the minimum momentum per track, from 20 GeV/c to 15 GeV/c the discrepancy between the measured and expected result increases to 25%. If we apply the same cuts to the Monte Carlo sample 19189 events pass the kinematic cuts. Of these events 1222 events have one or more gamma conversion in the beam-pipe and 7909 events have one or more gamma hitting an anti-counter. Looking at the ratio (num. events with a gamma conversion in the beam-pipe)/(number events passing kinematic cuts) for the $p_{\text{min}} = 20$ GeV/c and $p_{\text{min}} = 15$ GeV/c samples it can seen that the fraction of events in which a gamma collides with the beam-pipe decreases from $\sim 7\%$ to $\sim 6\%$. This is expected as the
lower $p_{\text{min}}$ cut includes more low energy kaons where the opening angles between the gammas are larger. The larger opening angles are also reflected in the fraction of events with a gamma in one of the anti-counters which increases from 34% to
41%

From this qualitative study it can be seen that there is ample scope for gamma conversions to account for the low measurement of the ratio $\Gamma(K_L \rightarrow \pi^+\pi^-\pi^0)/\Gamma(K_L \rightarrow \text{all charged})$.

For a future, more quantitative analysis, a full GEANT analysis using NASIM could be used, taking into account all the physical processes for particle decay, bremsstrahlung, etc. in the detector medium. It is also possible that using the GEANT analysis this behaviour could be sufficiently well parametrised so that it could be simulated in the fast Monte Carlo - NMC. However, this study is outwith the scope of this thesis.
9. SUMMARY OF RESULTS

This thesis presents first results from the NA48 experiment at the CERN SPS. This experiment is designed to measure the direct CP-violating parameter \( \epsilon'/\epsilon \) via the "double-ratio" method described in the introduction (page 4) and Chapter 2. To measure this parameter requires very high precision in the statistical and systematic errors. To obtain the high statistics required the detector has been designed to be read out at a very high rate (see Chapter 4). This high rate capability allows large statistics of the major branching ratios of \( K_L \) and \( K_S \) to be collected quickly. The 5 day running period in 1995 which provided the data for this analysis was primarily a rare decay search for \( K_L \rightarrow \mu \gamma \). The downscaling in the mixed trigger used to drive the readout (page 51) implies that the "minimum bias" sub-set of the data corresponds to approximately one day of running with pure "minimum bias" triggers. From Chapter 7 it can be seen that a competitive result can be obtained within a very short period of running time.

The measured value for \( \Gamma(K_L \rightarrow \pi \mu \nu)/\Gamma(K_L \rightarrow \pi e \nu) = 0.700 \pm 0.012 \) is in good agreement with both the current world average for previous measurements of this branching ratio, \( \Gamma(K_L \rightarrow \pi \mu \nu)/\Gamma(K_L \rightarrow \pi e \nu) = 0.697 \pm 0.010[20] \), and the previous best measurement, \( \Gamma(K_L \rightarrow \pi \mu \nu)/\Gamma(K_L \rightarrow \pi e \nu) = 0.702 \pm 0.011[22] \). The error is dominated by systematic effects. The large error arising from the uncertainty in the \( K_L \) production spectrum will, in future, decrease as more \( K_L \rightarrow \pi^+ \pi^- \), \( 2\pi^0 \) are recorded. This will allow a precise determination of the decay spectra in both the charged and neutral decay modes of \( K_L \). The lack of an electromagnetic calorimeter in 1995 created some problems with the subtraction of remaining \( K_L \rightarrow \pi^+ \pi^- \pi^0 \) in the \( K_{\mu 3} \) sample. With the electromagnetic calorimeter in place this background could be further suppressed by requiring that semi-leptonic candidates are not accompanied by a neutral cluster in the calorimeter. At the level of this measurement we are not sensitive to uncertainties in the form factor \( \lambda_+ \). Using the semi-leptonic branching ratio measurement plus the PDG average
for $\lambda_+$ a value of $\lambda_0 = 0.04 \pm 0.01$ is obtained. A more precise determination of $K_{\mu 3}/K_{e 3}$ would require that the form factors in the decay matrix element be re-evaluated. This could be done using the method of Birulev et al [21], as discussed in Section 7.4. All other systematic errors are internal to the experiment, such as effects arising from differences between the Monte Carlo and data in spectrometer resolution, vertex reconstruction, etc. This situation will improve as feedback from the data contributes to a more accurate Monte Carlo simulation.

The measurement has been shown to be in good agreement with the theoretical expectation predicted by the $V - A$ model described in Section 2.4. No significant deviation from $\mu - e$ universality is observed in the semileptonic branching ratio. The near equivalence of the measured value of $R_{\mu e}^0$ with $R_{\mu e}^+$ (from the PDG compilation[20]) is consistent with the predictions of the $|\Delta I| = 1/2$ rule.

As a preliminary measurement from an entirely new apparatus the agreement between the measured result and the previous best measurement should give some confidence in the detector and in the Monte Carlo and reconstruction software.

A preliminary study of the ratio $\Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \text{all charged})$ has been made looking at the ratios $\Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi\nu\nu)$ and $\Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi e\nu)$. A method to calculate these branching ratios using log-likelihood fits to the kinematic variable $(p_0^2)$ has been described and first results are presented. The results are found to be consistently low. It is conjectured that this comes about from interactions of the gammas in the beam-pipe and the anti-counters which surround the fiducial region. This effect would not be observed if it was also required that both gammas are detected in the calorimeter. Due to the absence of the electromagnetic calorimeter and the poor resolution of the hadronic calorimeter for gammas no attempt was made to identify the gammas in the end state with the result that the gammas could end up in the anti-counters or hitting the beam-pipe. After 1996 this analysis should be repeated – reconstructing both gammas in the LKr calorimeter to obtain the $\pi^0$ energy or simply using the calorimeter to define the acceptance and performing a log-likelihood analysis as described in Chapter 8.
A. THE MUON VETO MONITORING SYSTEM

This appendix describes the “Muon Veto Monitoring System”. During test runs in 1993 and 1994 I was responsible for the design and implementation of the hardware and software for this system and during 1995 for the hardware only.

The purpose of the muon veto monitoring system is to provide online information on the status of the detector channels so that a shift crew can check if the detector is working correctly. Data is also written to disk so that further analyses can be made offline.

A.1 Monitored Quantities

The quantities which are recorded during a run are:

1. pulse-height distributions of all 56 PM tubes,
2. timing information from each channel,
3. trigger bit information from muon veto and HAC,
4. integrated rates of individual counters plus trigger bits.

From these quantities several other quantities, relevant to the correct functioning of the detector, can be calculated:

Attenuation Length of Scintillator Counters

An important quantity determining the quality of the scintillator is the attenuation length ($\lambda$). The attenuation length is defined to be the distance a scintillation pulse travels along a counter before the light intensity is reduced by a factor of $e^{-1}$. Thus the light intensity as a function of path length in the scintillator is:
The loss of light occurs in two basic ways: escape through the scintillator boundaries and absorption by the scintillator material. To alleviate the first effect the scintillators are wrapped on aluminium foil so that light incident at an angle greater than the Brewster angle, $\theta_B$, is reflected back into the scintillator. The Brewster angle is defined to be:

$$\theta_B = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{scint}}}/\lambda\right) \quad (A.2)$$

with $n_{\text{scint}}$ and $n_{\text{air}}$ being the refractive indices of the scintillator and air respectively. For the NE110 scintillator of the muon veto, which has a refractive index of 1.58, the Brewster angle is $39^\circ$.

By taking the pulse-height distribution from each PM and splitting it into several distributions corresponding to different positions along the counter the attenuation length can be calculated by fitting a function of the form (A.1) to the means, $\Delta_m$, or modes, $\Delta_p$, of the Landau distribution plotted as a function of path length ($x$). In practice it is easier to fit to the peak, $\Delta_p$, as the means are often biased due to the truncation of the Landau distribution, an effect caused by the dynamic range of the ADCs used to record the analogue pulses.

A low attenuation length could be indicative of ageing in the scintillator and can be due to the formation of colour centres in the plastic or “crazing” on the surface of the scintillator. “Crazing” is the formation of cracks on the surface which occurs on most plastic scintillators with time.

**Efficiencies of Trigger Bits**

Several signals derived from the muon veto may be used in the Level 1 trigger or simply recorded on an event-by-event basis. A list of the bits produced is shown in Table 3.4 on page 45. It is necessary to verify that the signals are both efficient and produce the least noise possible. The most important bits are those produced using planes 1 and 2 of the muon veto. A local trigger is constructed, using the hadron calorimeter and plane 3 of the muon veto, which detects muons independently of the other two scintillator planes.
By triggering on a “muon trigger” defined by a coincidence of a signal in the hadron calorimeter and plane 3 and recording the presence, or absence, of the trigger bits derived from planes 1 and 2 the following efficiencies can be calculated:

\[ \varepsilon_{1\mu} = \frac{HAC1.1\mu.\mu3}{HAC1.\mu3} \quad (A.3) \]
\[ \varepsilon_{\mu1} = \frac{HAC1.\mu1.\mu3}{HAC1.\mu3} \quad (A.4) \]
\[ \varepsilon_{\mu2} = \frac{HAC1.\mu2.\mu3}{HAC1.\mu3} \quad (A.5) \]

where \(1\mu\), etc. are as defined in Table 3.4 and HAC1 is defined to be a single particle trigger formed using the hadron calorimeter.

The numerator is defined to be the number of events in which all three trigger conditions are met and the denominator the events without requiring either of \(1\mu\), \(\mu1\) or \(\mu2\). As a “good” muon is defined by HAC1.\(\mu3\) this then gives the absolute efficiency of either of the three trigger bits derived from planes 1 and 2.

A.2 Hardware Implementation

The monitoring system differed in hardware and software implementation between the test runs in 1993 and 1994 and also during the physics run in 1995. The changes between 1994 and 1995 were due to the merger of the monitoring systems of the muon veto and the hadron calorimeter.

A.2.1 Setup in 1993

The initial tasks were to “boot-strap” the detector into a working status. This required the counters to be “plateaued” and then the signals had to be timed in to the electronics. Plateauing consisted of ramping up the High Voltage (HV) supplied to each photomultiplier base and measuring the efficiency of all the counters at each voltage and at several discriminator settings. Having plateaued the
counters the analogue signals from the PM’s were timed in such that they were synchronous when they reached the discriminators of the trigger electronics.

**Plateauing the Counters**

Figure A.1 shows a schematic of the electronics used during the plateauing. One half-plane, i.e. one side of a plane, was plateaued at a time. The trigger consisted of a coincidence of a PM in one half-plane with a transverse strip in the other plane, picked so that it was on the edge next to the trigger PMs. The trigger signals generated were recorded in a CAMAC input register (LeCroy 2341), so that the strip through which the muon passed was known, and also logically ORed to produce a “gate” for the input register and a “start” signal for some TDCs. The “stop” signals for the TDCs were produced by discriminated signals from the PMs being plateaued. Events were only read out when the multiplicity in the input register was one. Hence, it can be seen from the diagram that if the bit is set in the input register for one of the left-hand channels there should be a stop present on the corresponding right-hand channel unless there is an inefficiency. In the case of an inefficiency no signal is present and there is an “overflow” recorded in the TDC. Thus the efficiency of each counter is the number of TDC stops divided by the number of triggers recorded for this counter in the input register. When the efficiency of the PMs facing the trigger PMs is measured, a lower bound should be obtained for this PM/scintillator. A lower bound is obtained as light from the muon hits closer to the PM being studied will suffer less attenuation. Hence the number of photoelectrons produced by the photocathode should be greater.

An example of one of the plateau curves produced, as a function of voltage, is shown in Figure A.2.

**Timing in Signals to the Trigger**

The trigger used to plateau the counters had very loose timing. For the $1\mu$ trigger the required properties are:

- timing resolution less than 1 ns,
- low accidental rate.
The accidental rate comes from two sources. The first is from real muons passing through the detector. Near “in-time” accidentals may come from interactions in the target of the same proton which produced the kaon. There is also a high flux
of "out-of-time" accidentals which can be seen in \(\sim 4\%\) of all kaon decays. The second source comes from incoherent coincidences between counters in the two planes.

To achieve the desired timing resolution, and also to be able to use short gates to reduce the number of accidentals, it is necessary to time in the signal from each counter so that they are synchronous at the trigger electronics.

A simple trigger setup was used as shown in Figure A.3. By triggering on one of the half-strips a trigger was produced which should produce coincident signals at each end of the strips transverse to it. The signals are not, however, coincident when they arrive at the electronics due to the variation in the transit times of the PMs (dependent on voltage supplied) and different cable delays. The timing of all the PMs was completed using a setup in which the trigger was used to start a LeCroy 2228A TDC and the PMs under study were used to provide the stops. The counters in each plane were then aligned in time using programmable delay units so that hits in the center were coincident for each PM in the entire plane. The timing between strips was done by assuming the propagation velocity of the scintillation pulse in the trigger counter.
An example of the timing data is shown in Figure A.4. What is shown in the histogram is the plot of the difference, in time, of the arrival of signals from PMs L0 and R0 at a TDC stop. As the trigger selects events from the centre of the strip the time difference should be zero. The mean value of 3.12 ns, obtained for this example, indicates that L0 arrives later than R0 by this amount.

A.2.2 Setup in 1994

Between the 1993 and 1994 test beam periods a more generalised monitoring system was designed and, in addition, a laser calibration system was added so that the channels could be tested and timed without the use of beam.

The Laser Calibration System

A schematic of the laser calibration system is shown in Figure A.5. The laser system uses quartz fibres to distribute ultraviolet (UV) light to each of the scintillator counters. UV light (337 nm) is transported from a nitrogen laser, situated at the proton tagger, along a 215 m long primary fibre (600 μm diameter) to a shutter box which is situated at plane 1 of the muon veto. From the shutter box the UV illuminates a bundle of 85 secondary quartz fibres (200 μm diameter). The opposite ends of the secondaries are inserted into perspex prisms. For the strips with PMs at both ends there are two prisms positioned at the centre of each
counter while for the half strips terminating at the beam-pipe two prisms were situated 5 cm from the beam-pipe. The UV light produces local molecular excitation next to the prism which de-excites with the emission of scintillation light. Thus, the laser system mimics the passage of a muon through the scintillator. As the positions of the prisms are known this allows one to time in the counters using laser pulses.

The secondary fibres were cut to different lengths according to the plane to which they were connected. Using the known refractive index\(^1\) the path lengths were

\(^1\)Refractive index of 1.48 as quoted by the manufacturer: CeramOptec GmBH, Bonn, Germany.
arranged such that a laser pulse arrived at each plane with the same timing as a muon travelling at the speed of light\(^2\).

**Readout Electronics**

The readout system used during 1994 is depicted in Figure A.6.

The 44 analogue signals from the horizontal and vertical planes (HR, HL, VT and VB) were amplified using LeCroy 612AM photomultiplier amplifiers. These modules also act as a fan-out for the signals. One signal from the fan-out was dis-

\(^2\) As the typical energy of a muon in NA48 is \(\sim 30\) GeV, and corresponds to \(\beta = 0.9999\), this is a reasonable approximation.
Fig. A.6: The Monitor Electronics in 1994.
A. The Muon Veto Monitoring System

criminated to produce a logic pulse and another signal was delayed using "Lemo" cable before being input to LeCroy 2249A ADCs. The 44 logic pulses were used for 3 purposes:

1. to generate a logical OR of all 44 channels called \((H+V)\),

2. to produce TDC "stop" signals for timing information,

3. to provide rate information recorded on LeCroy 4432 scalers.

The analogue pulses from plane 3 were split using a passive resistor network and, as in planes 1 and 2, one half of the signal was used to produce a logic pulse and the other fed to the ADCs. The strips of plane 3 were discriminated and then "mean-timed", using a LeCroy 624, so that the time of the signal derived from plane 3 is independent of the position of the hit on the plane. The 6 mean-timed signals were then fanned in to produce a signal called \(3MT\). The logic pulses from the mean-timer were also counted using LeCroy 2251 NIM input scalers.

The trigger used to gate the ADCs and provide "start" signals for the TDCs was:

\[(H+V).3MT\]

where the timing was arranged such that the H or V signals arrived before the 3MT signals so that the trigger time was defined by plane 3 and so independent of the position of the hit on the detector. The analogue signals for the ADCs are delayed such that the PM pulse falls within the gate generated by the trigger.

The trigger was self-blocking, using a dual-timer as a flip-flop. A trigger bit was set in an input/output (i/o) register which was polled by the readout software for the presence of a trigger. The ADCs and TDCs were then read out using a Macintosh IIfx. The Macintosh accessed the CAMAC modules using a "MICRON" card resident on the computer bus which was interfaced to a "Mac-CC" CAMAC crate controller. This allowed readout of the ADCs, TDCs, etc. by mapping the slots and channels of the various modules to memory locations in the Macintosh[52, 53]. After all modules had been read out the software caused a reset pulse to be generated by the i/o register which freed the trigger.

In addition to the particle triggers several hundred pedestal triggers were read out during each burst. A random trigger was produced by a discriminated signal from
a photomultiplier tube with scintillator/Americium source assembly attached. A bit in the i/o register was set so that this event could be identified as a pedestal event.

During the 1994 test beam 127 data runs were recorded with this configuration. Each run consisted of 100–500 SPS bursts with ~2000 muons recorded per burst.

Typical pulse-height and timing distributions, taken during an early run, are shown in Figure A.7.

![Pulse-height and timing distributions](image)

Fig. A.7: Pulse-height and timing distributions.

In the upper histogram the muon signal produces a Landau distribution. This corresponds to the right-hand portion of the distribution. The left-hand part of the distribution corresponds to the pedestal i.e. the charge obtained when there is no muon present in the scintillator. An undesirable feature observed in the pedestal is the secondary peak situated in the minimum between the muon signal
A test for coherent fluctuations in the pedestal distribution, which could arise from a floating ground, was made by calculating the mean of all 56 pedestal distributions \( \langle \text{ped} \rangle \) and then calculating:

\[
\text{Pedestal fluctuation} = \frac{1}{56} \sum_{i=1}^{56} \text{sign}(\text{pulseheight}_i - \langle \text{ped} \rangle)
\]

(A.6)

where \( \text{sign}(\text{pulseheight}_i - \langle \text{ped} \rangle) \) is defined to be the sign of \( (\text{pulseheight}_i - \langle \text{ped} \rangle) \) i.e. \( \pm 1 \). The value of the pedestal fluctuation was histogrammed for all events in one run. If there were coherent noise fluctuations in the pedestals the distribution of the pedestal fluctuations from the above equation should show peaks at \( \pm 1 \). This was not found to be the case as shown in Figure A.8. So coherent noise fluctuations were ruled out and the noise appeared to occur randomly in the separate channels. Improving the grounding of the crates did not have any effect and this problem was not resolved in 1994.

In later runs, in which I played a lesser contribution to the cabling and general setup of the detector, it was intended that tests should be made on the effect of “choking” the ground of the high-voltage cables by wrapping them around ferrite toroids. This would decouple the photomultipliers from any fluctuations in the ground of the LeCroy HV system.

Nevertheless, during 1994 a large amount of data was recorded, the results of which are summarised in Section A.5.

\[ \text{A.2.3 Setup in 1995} \]

After the 1994 run the ADCs, TDCs and scalers in the readout electronics were kept in the same configuration. However, the readout and the trigger were merged with the monitoring system of the hadron calorimeter. This allowed a muon trigger to be formed without using planes 1 or 2 of the muon veto. This was deemed better for efficiency calculations. Unfortunately, after the merger was in progress, it was discovered that the HAC signals were too late to provide a gate for the muon veto ADCs with the delay cables in use at that time. To get round this problem a rather complicated “master-slave” trigger was built as shown in Figure A.9.
The trigger was initiated by the 3MT signal from the muon veto. The muon veto ADCs and TDCs were sent a trigger immediately and would begin to digitise. The 3MT signal was also delayed and put into coincidence with the signal particle trigger generated by the HAC (HAC1). If there was a coincidence between these two pulses a signal called README was produced which was detected by the Macintosh, via the i/o register, and readout of the ADCs and TDCs was initiated. If there was no coincidence a signal called CLEARME was produced. This signal had the effect of clearing all the ADCs and TDCs and then freeing the trigger after the 5 μs settling time required for these modules.

The single particle trigger from the HAC is obtained by forming the energy sums of the four HAC quadrants (Q1, Q2, Q3 and Q4 – see Figure A.10). Each quadrant is discriminated to produce a logic pulse and the four combinations

\[ L = (Q1 + Q4), \quad R = (Q2 + Q3), \quad T = (Q1 + Q2), \quad B = (Q3 + Q4). \]  \hspace{1cm} (A.7)

are made using a LeCroy 4564 Logic unit. Then the single particle trigger is defined to be:

\[ \text{HAC1} \equiv (L + R)(T + B) \]  \hspace{1cm} (A.8)

The muons from the monitoring system are used by the HAC for energy inter-
calibration between the strips in the front and back modules (as described in Section 5.2 on page 71). To ensure that the energy deposited in the HAC comes from the muon only, and that there are no contributions from overlapping pion or
electron showers, events with energy greater than 5 GeV are vetoed by the signal \textbf{EVETO}, as shown in Figure A.9. This signal is formed by summing the energies from the back and front modules and then discriminating the analogue signal. The threshold is set such that it is not triggered by a minimum ionizing particle. Events with hadronic or electromagnetic showers in the HAC are strongly suppressed.

In addition the $1\mu$ signal from the muon veto could be used to provide a trigger which was independent of the HAC. It was necessary to use this trigger for making high-voltage adjustments to the photomultipliers in the HAC.

\subsection*{A.2.4 Setup in 1996}

To reduce the complexity of the 1995 trigger additional delays were added for the ADC and TDC inputs (RG-58 coaxial cable for the analogue signals and twisted-pair flat-cable for the digital signals). This meant that a simple coincidence:

\texttt{HAC1.3MT}

could be implemented with far fewer modules in the trigger. The $1\mu$ signal could also be used as the monitor trigger. The trigger driving the readout was selectable using output channels of the i/o register. In both cases the trigger was vetoed by the \textbf{EVETO} signal (this does not apply to pedestal triggers – although the \textbf{EVETO} should not be set in this case anyway). A schematic of the trigger is shown in Figure A.11.
A. The Muon Veto Monitoring System

A.3 Software Implementation

The readout of the monitor system has used 1–3 Macintoshes, in various configurations, for recording, histogramming and presenting data to the user. All of the modules in the monitor (ADCs, TDCs, scalers and FERAs – in the HAC part of the readout) are implemented in the CAMAC standard and are read out over the crate backplane by a crate controller. The interface between the Macintosh computer and the CAMAC modules comprises of a MICRON card [52], which is resident on the NuBus inside the Macintosh, and a Mac-CC crate controller [53] which occupies slots 24 and 25 in the CAMAC crate. These modules are connected by twisted-pair flat-cables (34 pairs). There is one cable for transmission of data and one for addressing information.

The readout of the CAMAC modules is realised using “memory mapping”. Crates and modules within these crates are mapped to various locations within the address space of the Macintosh. Then reading or writing to these modules corresponds to reading and writing to memory locations which can be easily accomplished in the C language by using pointers. When an address within the MICRON
A. The Muon Veto Monitoring System

card is accessed a request is set up between the MICRON card and the Mac-CC. The protocol used is similar to the VME standard and requires a data acknowledge after each transfer. After receipt of the request the Mac-CC is responsible for generating the addressing and strobe signals required for a CAMAC cycle. In the case of a write cycle the word written to the memory location in the MICRON is transferred to the register of the CAMAC module. For a read cycle the data transmitted to the CAMAC backplane, by the ADC/TDC/..., is buffered by the Mac-CC before being transferred to the MICRON card where it is available to a program in much the same way as standard memory.

The programs used during 1993 for plateauing and initial timing in of the signals to the electronics were written solely for that purpose and were not used during later development of the monitor software. An exception to this was the C library "lrs1445.c" which contained subroutines written to allow the Macintosh to interact with the high-voltage crate (LeCroy 1440 crate with 1445A crate controller) via an RS-232 connection.

The software models adopted for the 1994 run and for 1995, and subsequent runs, are shown in Figure A.12. The monitor electronics are situated in the experimental hall approximately 100 m from the control room. The requirement for data acquisition in the hall and control of the readout and presentation of data in the control room meant that a distributed Macintosh system had to be used. In 1994 the muon veto monitor software was based on the “MultiDAQ for Macintosh” package written by H. Blümer and U. Koch of the Mainz ETAP group. The main components of the system were two programs – one of which ran in the control room, and was called the “shell”, and another which ran in the experimental hall, called the “kernel”. Both of these programs were implemented as state-machines. A diagram of the state model implemented is shown in Figure A.13. Changes of state made by the operator in the control room are propagated to the experimental hall. Thus both programs should be in the same state. The exception is when the user requests a change of state during an SPS burst. At the beginning of each burst the kernel informs the shell that a burst has begun. At this point the shell starts to queue any events, such as mouse clicks which, would interfere with the data acquisition of the kernel. At the end of each burst the kernel tells the shell that the burst is over and any events in the queue are then transferred. This is different for the post-1994 software (“MIDAS”) where the “Analyzer” queues the
In addition to the state model around which each process is modelled there are, loosely defined, 5 other software packages:

1. GUI: the graphical user interface. This allows the user to interact with the monitor. It is based on the Apple GUI and is designed to look and act like normal Macintosh applications. Dialog boxes are used to request information from the user and to report warnings and errors. A menu bar allows the user to send commands to the kernel (such as "BeginRun") and to access histograms.

2. MHIST: histogramming is implemented using the "MHIST" library, written by H. Blümer, which contains many of the common HBOOK commands and also routines for displaying histograms. I was responsible for implementing the subroutines for displaying 2-D histograms in the HBOOK styles.
"BOX" and "TEXT".

3. COMMS: the communications between the processes use AppleTalk Hi-Level events. Hi-Level events allow the format of the data buffer sent with the event (if any) to be defined by the user. Thus a flexible communications system can be designed which allows transfer of different data types, e.g. commands, messages, histograms, ..., using the same subroutines. Another advantage of Hi-Level events is that if a local process id (PID) does not correspond to the id in the header of the event the event will be posted across the network. Therefore the software used to transfer information between processes is independent of whether the process exists on the same CPU, Macintosh bus (NuBus) or AppleTalk Zone.

4. CAMAC: a collection of subroutines were written to provide easy access to the CAMAC modules. Routines to read out all the common ADCs, TDCs, FERAs, scalers and to set thresholds in discriminators were implemented.
5. HV: up until the 1995 run the high-voltage for the muon veto was controlled and monitored using the Macintosh in the experimental hall. Routines were written to set up the communications between the serial communications controller (SCC) chip in the Macintosh and the LeCroy 1445A HV crate controller and to read and write voltages supplied to the photomultipliers. HV monitoring could be selected by an option in a dialog box at “Begin Run” time. Requested and supplied voltages could be displayed on the Macintosh in the control room. For the 1995 run I had written a GUI for the HV control and monitoring of the muon veto and hadron calorimeter. This software was not integrated into the monitoring system as these subdetectors were integrated into the global NA48 slow control system during 1995 and 1996.

The kernel does most of the work in MultiDAQ. During a burst it polls the i/o register for a trigger and if one is present reads out all the ADCs and TDCs. Data from each channel, such as digitizations of time or charge, are written sequentially to a large buffer in memory. The data of each module is preceded in the buffer by a header containing a number which is the unique identifier for this module. The number of channels per module is also written in the header. Until the End-of-Burst (EOB) signal from the SPS the buffer fills up with events. When EOB is received the shell firstly writes the data buffer to binary file on a local disk then starts reading the buffer and histogramming ADC and TDC data. If the user in the control room requests a histogram the bin contents of the histogram are packed into a High-Level event and sent to the shell. The purpose of the shell is simply to provide a user interface and to display histograms of data: such as timing distributions, pulse-height distributions, hit maps, ....

Between 1994 and 1996 the software was altered considerably. A third CPU was added to the system and slow control of the HV was removed. The third CPU (a “Radius Rocket” card situated on the NuBus of the Macintosh in the experimental hall) was installed to provide more analysis power online. The purpose of the Macintosh in the control room (now called the “Presenter”) stayed the same while the data acquisition and histogramming tasks were split up between the “Analyzer” and the “Reader”. Finally the software name was changed from “MultiDAQ” to “MIDAS”.

The merging of the HAC and muon veto monitoring systems meant that the event
size had become much larger and the performance of the histogramming between bursts fell. To improve the situation the histogramming was divided between the Analyzer and the Reader processes. It was also intended that more elaborate analyses should be performed using the Analyzer to provide more sophisticated results, such as attenuation lengths, online.

A.4 Online Results

Some “screen-snaps” illustrating the GUI of the monitoring system are shown in Figures A.14–A.16. The user is presented with some simple options on startup and end of the monitoring program: in general the default options, which are stored in the resource fork of the Macintosh program, will suffice so that is is easy for a non-expert to operate.

![File Edit Run Hist Options Plot Special]

Fig. A.14: Begin Run Dialogue Box.
A. The Muon Veto Monitoring System

Figure A.15 shows how histograms are accessed via the Hist menu. A selection of the most useful histograms can be accessed using this menu. Other histograms are accessed via the histogram manager by typing in the required histogram id in a dialogue box. Also shown in Figure A.15 are histograms of the distribution of muon hits in the muon veto and hadron calorimeter. The ratio of these two histograms gives the efficiency of the muon veto (also shown although partially obscured by the histogram menu).

![Plot Menu with muon distribution 2-D histograms.](image)

Fig. A.15: Plot Menu with muon distribution 2-D histograms.

A.5 Offline Results

In this section some of the results derived from the data recorded in 1994 are presented.

The data written to disk by “MultiDAQ” could be read back and analysed offline
A. The Muon Veto Monitoring System

Fig. A.16: End Run Dialogue Box.

using a program, “ReadData”, written by S. Luitz. This program could run on both Unix or VMS workstations and allowed the user access to the usual CERN-LIB tools: HBOOK, PAW, .... Definitive ADC distributions were produced, for all 56, PM's, using the following selection criteria for the data:

1. raw data i.e. events over a whole strip,

2. side data i.e. events over the area from PM to mid-strip,

3. box data i.e. events in coincidence boxes defined by 11 x 11 strips.

The second criterion is of interest for the $1\mu$ trigger as the signal from the two ends of each counter is linearly added, using LeCroy 428F's, before being discriminated and so this corresponds to the distribution 'seen' by these discriminators. The third criterion is used to make a measurement of the attenuation length of the
scintillator (as described in section A.5.1). Examples of the distributions, using the PM’s VT0, VB0, HL0 and HR0, are shown in Figure A.17.

Fig. A.17: ADC Distributions from VT0, VB0, HL0 and HR0.
A.5.1 Attenuation Length of the Scintillator

Using the pulse-height distributions from the coincidence boxes, i.e. correlated with position along the scintillator strip, the attenuation length of the scintillator was calculated as follows:

1) the peak of the Landau distribution was estimated, for each of the 11 coincidence boxes in each strip, by iteratively fitting Gaussians around successively smaller ranges about the peak,

2) the mid-point of each coincidence box is accurately measured and so the attenuation length can be calculated by fitting the data points, \((x, \Delta p(x))\), to the equation:

\[
\Delta p(x) = \Delta p(0) \exp \left( -\frac{x}{\lambda} \right) \tag{A.9}
\]

where \(\Delta p(x)\) is the peak of the ADC distribution as a function of position in the scintillator strip and \(\lambda\) is the attenuation length of the scintillator. Examples of the attenuation length fits are shown in Figure A.18.

The scintillator used in the muon veto is NE110 and was previously used in the NA31 experiment. The attenuation length is quoted by the manufacturers (Nuclear Enterprises Technology Ltd., Edinburgh EH11 4BY, Scotland) to be 4 m. In practice the attenuation length will, almost certainly, be smaller than this as it is dependent on the geometry of the scintillator counters. If we imagine that the light produced by the scintillation process is isotropic then only a small solid angle is subtended by the ends of the counter. All light outwith this solid angle will undergo reflections before it reaches the light-guide. If the angle between the incident light and the perpendicular to the surface of the scintillator is greater than the Brewster angle there will be no internal reflection. While some of this will be reflected back in to the counter by the foil wrapping it will be heavily attenuated. The results obtained using the above method summarised in Table A.1.
A. The Muon Veto Monitoring System

Fig. A.18: Attenuation Fits from VT0, VB0, HL0 and HR0.

<table>
<thead>
<tr>
<th>Plane 1 Summary</th>
<th>Plane 2 Summary</th>
<th>Plane 3 Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\lambda}$ = 1.3 m</td>
<td>$\bar{\lambda}$ = 1.5 m</td>
<td>$\bar{\lambda}$ = 1.8 m</td>
</tr>
<tr>
<td>$\sigma$ = 0.2 m</td>
<td>$\sigma$ = 0.1 m</td>
<td>$\sigma$ = 0.5 m</td>
</tr>
</tbody>
</table>

Tab. A.1: Summary of Attenuation Lengths for Planes 1, 2 and 3

A.5.2 Efficiency of 1$\mu$ Trigger

By using the trigger **HAC1.3MT** it was possible to calculate the efficiency of the muon veto:
with position information coming from the HAC electronics. It was found that the efficiency was nearly 100% everywhere except along some sides where it dropped to about 50% (see Figures A.19–A.20). This was probably due to bad timing in the electronics and was fixed for the run in 1995.

\[ \varepsilon_{1\mu} = \frac{\text{HAC1.}(1\mu).3\text{MT}}{\text{HAC1.3MT}} \quad (A.10) \]

A.5.3 Multiplicities of Events

For each ‘normal’ event, i.e. the trigger bit was set but not the pedestal bit nor the laser bit, the multiplicity of the event in each of the principal planes was calculated. The multiplicity was calculated by counting strips with a hit in them for each plane. A ‘hit’ in a strip was defined to be either of the two PM’s attached to the strip above a set ADC cut. A summary of the multiplicities in planes 1 and 2 is shown in Table A.2. Histograms of the multiplicities of normal events in the muon veto are shown in Figure A.21.

The events with multiplicity = 0 correspond to events in which a hit was registered in plane 3 and only one of either plane 1 or plane 2. These events certainly do not

Fig. A.19: Efficiency of the Muon Veto (box)
correspond to inefficiency for muons as their large number contradicts the high efficiency of $\ell$ as measured previously but are in fact an artefact of the ADC cut. In the scintillator strips of planes 1 and 2 there is an overlap of $\sim$5%. This should show up as 5% of all events having multiplicity = 2. In both of the principal planes there are over twice the expected number of these events.
This appendix describes the work I did on the NA48 muon veto as part of the commissioning of the detector. The development phases of both the hardware and software have been described and some examples of the results which can be derived from the data have been shown.

The combination of the Macintosh/Mac-CC readout has allowed between 1500–2500 events to be read out during each SPS burst (all of which was available offline). Using the MultiDAQ software approximately 15% of the raw data could be histogrammed for ADC and TDC distributions before the arrival of the next burst. Using MIDAS this increased to nearer 50%. The remaining work still to
be done is the implementation of an increasing amount of the "offline" analysis to the online system e.g. online attenuation length plots.


[46] H. Dibon. Comparison of $\gamma$ Conversion in NMC and NASIM for the Decays $K_L^0 \rightarrow 2\pi^0, 3\pi^0, \pi^+\pi^-\pi^0$. *NA48 Note*, 95-34, March 1995.

[47] H. Dibon. Comparison of $\gamma$ Conversion in NMC and NASIM for the Decays $K_L^0 \rightarrow 2\pi^0, 3\pi^0, \pi^+\pi^-\pi^0$. *NA48 Note*, 95-7, March 1995.


[50] Kreutz et al. Determination of the branching ratios $\Gamma(K_L \rightarrow 3\pi^0)/\Gamma(K_L \rightarrow \pi^+\pi^-\pi^0)$ and $\Gamma(K_L \rightarrow 3\pi^0)/\Gamma(K_L \rightarrow \pi e\nu)$. *Zeitschrift fur Physik C*, 65:67–74, 1995.
