SOIL-STRUCTURE INTERACTION IN ARCH BRIDGES

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Abstract

European Community directives now insist upon the imposition of 11.5t axle weights for the assessment of highway bridges and structures. This need for heavier loads arises from the Community wide harmonisation of transport policy. Its successful implementation requires the urgent assessment of our bridge stock of some 75000 masonry arches.

The analysis of arch bridges has long lacked an accurate method of assessing the loads transmitted to the arch ring by the surrounding soil. This thesis proposes pressure distributions suitable for use in the analysis of arch bridges. It examines, by way of instrumented small scale and in-situ tests, the soil-structure interaction effects arising from the backfill material. Observations of zones of soil displacement around a loaded arch are made in order to better describe the interactive effects. A finite element analysis of the instrumented tests was done and a parametric study was used to assess the effects of various material properties upon the system's behaviour.

The inclusion of the interactive effects observed, and modelled, intends to lead to cost savings in the arch bridge assessment programme by reducing the conservatism inherent in the most common assessment methods. Design curves incorporating soil-structure interaction effects are presented where significant capacity increases can be seen compared with analyses ignoring the effects.
Declaration

This thesis is submitted to the University of Edinburgh for the Degree of Doctor of Philosophy. The work described in this thesis was carried out under the supervision of Dr. David A. Ponniah and Dr. Jin Yeam Ooi. The work was undertaken in the Department of Civil Engineering and Building Science at the University of Edinburgh. Industrial supervision of the research was carried out by Mr. John Page of the Transport Research Laboratory, Crowthorne.

In accordance with the Regulations of the University of Edinburgh governing the requirements for the Degree of Doctor of Philosophy, the candidate submits that the work presented in this thesis is original unless otherwise referenced within the text. In particular, the observation of the zones of fill displacement, the measurement of extrados shear stresses, the measurement of in-situ soil pressures, and the elastic finite element analysis are all claimed as original.

The following papers published in journals or conference proceedings were derived from the work in this thesis. A set of these papers is bound, where possible, with the thesis and may be found inside the back cover. Full permission from the relevant publisher or copyright holder has been obtained. The numbering sequence makes no attempt to follow that of the thesis proper, the pagination follows that of the parent journal or proceedings as appropriate.


Charles A. Fairfield, BEng, MIHT
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Finally, the author wishes to thank his mother, father, and sister, for their advice, encouragement, and support. To them, this thesis is dedicated.
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The definitions of the symbols used in this thesis are listed below. Roman characters are given first, followed by Greek characters. The symbols are defined where they first appear in the text. In the few cases where more than one definition has been assigned to a symbol, the meaning will be evident from the context in which it is used.

**Roman characters**

- $a_{ij}$: calibration constant matrix entry, $i^{th}$ row, $j^{th}$ column
- $A_p$: load platen area
- $b_p$: load platen breadth
- $C_A$: cell action factor
- $C_u$: uniformity coefficient
- $d_c$: cover to the crown
- $D_{10}$: effective grain size, diameter at which 10% of grains are finer
- $D_{60}$: diameter at which 60% of grains are finer
- $e$: end wall to springer distance
- $E$: reading from eccentricity channel (ST's)
- $E_a$: arch modulus
- $E_p$: pavement modulus
- $E_s$: soil modulus
- $F_j$: force delivered by one loading jack
- $I_p$: influence value for vertical stress increase
- $I_\sigma$: influence value for normal stress increase
- $I_t$: influence value for shear stress increase
- $K_0$: earth pressure coefficient at-rest
- $L$: span
- $L_r$: linear dimension scale factor
- $m$: modular ratio
- $m_p$: pavement modular ratio
- $N$: reading from normal stress channel (ST's)
- $q$: average applied stress
Q  applied load
r  radius of extrados
S  reading from shear stress channel (ST's)
t  arch ring thickness
W  collapse load
W_i  load causing formation of i\textsuperscript{th} hinge
W_p  collapse load of prototype
x  horizontal distance from the crown
(x/r)  dimensionless horizontal coordinate
(x/r)_i  dimensionless horizontal coordinate of i\textsuperscript{th} hinge
z  depth through the fill

Greek characters

\gamma  bulk unit weight of model arch ring
\gamma_p  bulk unit weight of prototype arch ring
\Delta \varepsilon  eccentricity of \Delta \sigma, live load value
\sigma  total normal stress on the extrados
\Delta \sigma  live load normal stress on the extrados
\sigma_1  major principal stress
\sigma_3  minor principal stress
\sigma_{DL}  dead load normal stress on the extrados
\sigma_{cell}  stress transducer reading
\sigma_{ff}  free field stress
\Delta \sigma_z  vertical stress increase
\tau  total shear stress on the extrados
\Delta \tau  live load shear stress on the extrados
\tau_{DL}  dead load shear stress on the extrados
\tau_{xy}  generalised shear stress
\phi  angle of shearing resistance
\phi_m  mobilised angle of shearing resistance
\omega  uniformly distributed load
Chapter 1  Introduction

1.1  General introduction to the arch bridge problem in the U.K.

Archaeological evidence places the dawn of the masonry arch at circa 3600 B.C. in the ancient kingdoms of Egypt and Mesopotamia. The Romans used the masonry arch form of construction to great effect. The remains of many substantially Roman arches are a testament to both the skills of their builders and the inherent durability of the arch. The importance of the arch bridge in Britain's infrastructure increased between the 17th and 19th Centuries. Over 40000 arches were built in this period and they contribute to an approximate 75000 masonry arches in our road, rail, and waterway networks today. These arches are required to bear ever increasing loads, well beyond those foreseen at the design and construction stage. Many will need strengthening or replacing by January 1999, when European Community directives make the 40t gross vehicle weight a statutory requirement for assessment purposes. The maximum axle weight is also set to increase from 10t to 11.5t. It is the axle weight, equal to some fraction of the gross vehicle weight, that causes the damage to roads and bridges. Some member states have suggested gross vehicle weight increases to 44t with an associated axle load increase. Already, in Britain, the 44t lorry is legal on routes from major railheads(1) and this is unlikely to be the last load increase, given the increased flow of goods by road within Europe.

To cater for these increased loads, the existing bridge stock has to be assessed. Based on existing assessment methods costs have been estimated at £1400 million for the upgrading and subsequent work(2). Such costs would be particularly onerous to the Department of Transport and Local Authorities, necessitating the diversion of funds away from other areas of infrastructure maintenance and development. Improvements in the methods of assessment should lead to cost savings. The costs, mainly borne by industry and passed on to the consumer, of being unable to use 40t lorries are estimated at a minimum of £100 million per annum(2). The bridge assessment programme is urgent and improvements in the procedures and methods used to analyse arch bridges are essential to permit a reduction in expenditure between now and 1999.
1.2 Features of a masonry arch bridge

Fig. 1.1 shows the salient parts of a typical masonry arch bridge and the terminology associated with this type of structure. The masonry components might be made of dressed masonry, brickwork, coursed masonry, or random coursed rubble. The fill material is often variable and may contain voids, substantial inclusions, or even coursed masonry backing. The road pavement is usually dressed macadam or asphaltic surface layer(s).

The load is transmitted through the road pavement, distributed through the fill and applied to the extrados. The high modulus masonry arch ring attracts stress and transmits the load to the abutments through the springers. The vertical loads are transmitted in the form of a thrustline around the profile to the foundations. The advantages of this form of construction are its inherent flexibility, large compressive strength, aesthetic appeal, and low whole life costs.

1.3 A brief history

The arch was first used by the ancients to support roof structures. Such arches would originally be made of reeds, woven and bound into bundles and formed into arcuate constructions. The marsh dwellers of Southern Iraq used this form of construction until the 1960's. Elsewhere, notably in ancient Egypt, the reeds were replaced by sun baked blocks and more durable arches were built. Arches are still used today throughout the Middle East but more for architectural and building purposes than as part of the infrastructure of the region. Several notable examples of arches survive where the walls and other elements of the buildings have long since disappeared. A vaulted roof from the mausoleum of Miriam Bidiyah, at Qalhat in the Sultanate of Oman, is shown in Fig. 1.2. This arch has survived from the mid-15th Century in a harsh desert environment.

The Greeks, for all their architectural talents, did not tend to use arched forms, preferring architecturally squalid and cumbersome beams. With the rise of the Roman empire the arch increased in popularity. It is a misconception stemming from modern times that the Romans used only semicircular arches. Smith points us to the wide range of arched bridges and viaducts with a variety of span to rise ratios. Roman use of the arch extended from sewers such as the Cloaca Maxima to
viaducts and bridges such as the Pont du Gard near Nîmes in France. They used the arch for roofing and vaulting on a grand scale throughout their empire. A typical Roman multispan road bridge is shown in Fig. 1.3. Substantial portions are of original Roman construction dating from the 2nd century but some evidence of modern repairs can be seen.

With the decline and fall of the Roman empire the skills and knowledge of that golden age were consigned to the black hole of history. Rules for aesthetically proportioning arches contrived by Vitruvius were soon lost. Few new arches were built and little maintenance was carried out on those still standing. It was only in mediaeval times that religious orders, known as "bridge brothers", were started to assist with the development of Europe's infrastructure. With renewed interest in arching came increased efforts to understand the workings of the arch and we find early work on arches published in the 17th Century by the Royal Society. The relevant literature on the subject shall be reviewed in Chapter 2 from Hooke onwards.

Bridge building in Europe continued apace as successive countries emerged from Reformation into Revolution and as the transport, trade and communication needs of societies developed. Some notable examples illustrating the range of profiles to be found in the arch bridge stock are given below for reference purposes.

Britain's longest span masonry arch bridge is the Grosvenor bridge, Chester built in 1833 with a 61m span and a 12.8m rise (span to rise ratio 4.77). Brunel's elliptical arch bridge carrying trains over the Thames at Maidenhead since the 19th Century is a 39m span with a 7.4m rise (span to rise 5.27). In British cathedral construction Gothic arches predominate but few of these pointed arches can be found in the transport network today. In France the Pèdrousse viaduct uses a pointed central arch to support the spans on the upper tier of the structure. This is ideal as pointed arches are well able to bear highly concentrated loads at their crowns. An examination of the experimental work reviewed in Chapter 2 shows many more examples of the diverse nature and extensive possibilities of the arch form of construction.

Once "modern" materials such as steel and prestressed concrete made their debut the arch bridge waned once again. As these modern materials arrived too late for the boom following the industrial revolution Britain was left with the legacy of a large
number of arch bridges in key positions of our transport networks. As imposed vehicle loads increased interest in the arch was renewed.

1.4 The methods and reasons for evaluating the capacity of an arch bridge

Originally the heaviest possible load needing to be borne by a road bridge was a horse pulling a fully laden cart. Rough calculations give a line load of approximately 10 to 25kN per axle, or 5 to 12.5kN per wheel, for such a vehicle. Modern axles are allowed to carry up to 11.5t (112.8kN): up to 25% of these vehicles may well be overloaded(6). Vehicle speeds have also risen rapidly from a limit of 4mph (6.4kph) to 70mph (112.7kph); this increases dynamic loads on a structure and damage incurred in any accident.

These load and speed increases have also occurred on Britain's railway bridges: as some of the problems of overloading were recognised and the state of the art knowledge was more advanced at the time of construction of the majority of our railway bridges there are fewer problems involving their analysis and assessment. The vast railway stock of arch bridges must still be maintained to keep the axle loads up to current standards and any new, heavier, vehicle must still be assessed for safe passage over any "marginal" bridges. The generally older, more vulnerable, road bridge stock poses more of a problem today for the assessment engineer.

1.4.1 Current methods of assessment

There are basically four methods of arch bridge assessment in Britain today: the MEXE method, the mechanism method, elastic analyses, and finite element analyses. These will be described and details will be given for individual programs based on these methods: ARCHIE for the mechanism method, CTAP for the elastic analysis, and MAFEA for the finite element analysis.

1.4.1.1 The MEXE method

Pippard et al.(7-13) derived the Military Engineering Experimental Establishment (MEXE) method for the rapid assessment of masonry and brickwork arches during
the 2nd World War. He used steel voussoir arches and tested them to failure with various load and arch geometries. By comparison with the behaviour of a continuous steel rib he concluded that the voussoir arch problem could be treated elastically.

From his tests a general formula relating arch span, thickness, and crown construction thickness to collapse load was derived as:

\[ W = \frac{740(d_e + t)}{L^{13}} \]

Eqn 1

The collapse load obtained is a provisional axle load and as such it must be modified by a variety of factors. These factors are: profile factor for span to rise ratios other than 4, material factors for fill, joints, and arch ring, joint factors for depth and thickness, condition factor for a judgement by the assessor of the overall condition of the structure, and an axle factor for lift-off or no lift-off cases. These factors are multiplied together and then multiplied by the provisional axle load to arrive at a modified axle load. This is then compared with the heaviest category of vehicle currently operational at, or below, that value and the bridge is limited to that gross vehicle weight or axle load, whichever restriction is the more onerous. As a rapid assessment method it is useful but the modifying factors cast doubt on the accuracy of the technique. Despite its limitations the method is recommended in Department of Transport Standards, most recently, BD21/93(14) and its associated advice note, BA16/93(15).

1.4.1.2 The mechanism method

Various workers(16-19) have assisted in the development of the mechanism method and their work will be reviewed fully in Chapter 2. The method considers the four hinged mechanism mode of failure illustrated in Fig. 1.4, with the following assumptions: no tension develops in the arch ring, infinite compressive strength of the voussoirs, infinite elastic modulus of the voussoirs, and sufficient friction present to prevent voussoir slip. The arch is divided into five segments if the hinges are allowed to move up from the springers or three segments if the hinges are restricted to the springers. The dimensions of these segments are known as is the load position. By taking moments of all forces acting about an axis through any of the hinge positions the equations for static equilibrium may be derived for three of
the hinges. These may be solved simultaneously and the horizontal and vertical reactions obtained as well as the load needed to cause such a mechanism. Iteration is used to arrive at the minimum value of applied load causing a mechanism to form. This load is considered to be the collapse load for the bridge.

1.4.1.3 Castigliano's elastic (no tension) analysis

Castigliano(20), in 1879, developed the theorems of minimum elastic strain energy and his approach to arch analysis was based on the assumption that the thrustline remained within the middle third of the arch ring. A thrustline consistent with the applied load set is calculated and areas of the rib in tension are discounted for the next loop in the iterative analysis. The new thrustline, passing through compression only zones, is drawn and the process continued until no tension exists anywhere in the rib. Compressive stresses are then calculated based on an available rib depth and can be compared with the permissible stress for the appropriate material or used to calculate deflections in the rib.

The method has been computer coded by workers at the University of Wales, Cardiff(21,22) as program CTAP. Castigliano's basic strain energy equation is differentiated to find the abutment forces at one side of the span. The basic principle underlying the program's code is that of ignoring tensile zones, which are deemed to have cracked at some lower load. These regions of tensile cracking define the hinge positions in the event of a mechanism type failure and after several iterations the ultimate limit state is reached and a collapse load range output. The collapse load cannot be stated exactly as it lies between the applied load values used in the last two load increments. The lower bound of this range is acceptably close given small increments of loading.

There are marked similarities in the structural engineering between the mechanism method and a Castigliano type analysis. Identical collapse loads would be achieved for a bare arch rib analysed by each method. Differences in the way each analyst has incorporated the fill pressures, both from dispersal of the surface load and from redistribution as the arch deforms under some applied load, account for the discrepancies between the two methods when applied to complete bridge structures.
1.4.1.4 Finite element analyses for arch bridge assessment

Different teams of researchers have devoted a considerable amount of time and effort to the use of powerful finite element analyses for the solution of the arch bridge problem\(^\text{(23-26).}\) Due to the complex and time consuming nature of finite element work this research is not yet used in routine bridge assessment. However, it has provided valuable insight into the two dimensional behaviour of the arch bridge which has significantly influenced present thinking and has even led to a usable finite element program developed by the University of Nottingham and British Rail Research\(^\text{(27-31).}\)

This package, MAFEA, is based on tapered beam elements to model the arch ring. These one dimensional elements are assigned a thickness at their end nodes which renders them effectively two dimensional. As nodal cracking occurs under load the elements thin, thus defining the hinge positions and the failure mode, giving a reduced depth of section through the arch ring. The cracked tensile zones are ignored for subsequent load increments and as the available cross section decreases the masonry begins to yield in compression, reducing the section depth further. The thinning, under both tensile and compressive stresses, is shown in Fig. 1.5.

The element equations are solved for global arch displacements at each load step. Convergence within any one load increment is based upon the attainment of a limiting incremental change in effective section depth. This limiting change is set as small as possible for greatest accuracy consistent with the rapid convergence of the iterations. The next load increment is then applied. Failure criteria are: limiting deflection, limiting deflection rate, or ring separation.

Some concessions to the presence of the fill material are made through load dispersal in the fill and the effects of lateral fill pressures acting on the extrados. Load spread angles of between 0° and 90° are specified by the user for the limits to the range of what may be compared to a simple Boussinesq distribution of the contact stress down to the extrados.

Lateral pressure redistribution is catered for by the use of fill elements which act horizontally, causing a stress increase as the arch deforms into them. Lateral earth pressures from the at-rest state to the full mobilisation of the passive state are possible. Both soil springs and 8-noded fill elements have been used in MAFEA's
finite element based core. For the soil springs the failure criterion was basically that of an elastic, perfectly plastic material whereas for the 8-noded fill elements a Mohr-Coulomb law was applied. The fill properties currently required for a MAFEA run are: elastic modulus, angle of shearing resistance or passive pressure coefficient, and bulk unit weight.

British Rail Research are assessing the efficacy of a soil-soil shear model based on imaginary columns of soil transferring the applied load along their vertical edges\(^{(32)}\). Such a model was tried, with little success, by Ponniah\(^{(33)}\) who found the load spread and distribution on the extrados to be heavily dependent upon the number of shear planes between these soil columns. The omission, thus far in its development by British Rail Research and in its entirety by Ponniah, of the complementary horizontal shear stresses accompanying the vertical shears renders the model invalid. As such it defies equilibrium and no possible Mohr's circle of stress could be drawn for the imagined force distribution on the imaginary edge areas of the soil columns.

1.5 Deficiencies in the evaluation of arch bridge capacity

This section of the chapter will cover the perceived shortcomings in the way in which arch bridges are assessed in Britain today. The shortcomings are not entirely the fault of the methods used in bridge assessment but are due to the paucity of information available about the soil-structure interaction. This has led to undue conservatism in the way in which some arches have been assessed.

Deficiencies identified in the evaluation of arch bridge capacity are: the omission of three dimensional effects, omission of soil-structure interaction effects, and the omission of the effect of repairs upon the assessed capacity. Three dimensional effects such as spandrel wall, wing wall, and parapet wall contributions to overall capacity are ignored because of the difficulties encountered when attempts have been made to model their stiffnesses and strengths. This omission leads to added conservatism in assessments of arch capacity.

The omission of soil structure interaction effects is possibly more critical to the assessment of arches. The soil and road pavement stiffnesses govern the spread of an axle's load down onto the extrados. The intimate contact between the fill and the
arch governs the load transfer both normal and tangential to the arch ring. It is these applied stresses on the extrados, governed by the stress field in the surrounding fill, that manifest themselves as the more familiar, easily observed, load-deformation behaviour and eventual failure mode of the arch ring.

To ignore the fill's contribution to the carrying capacity of a soil-arch system is obviously conservative as its presence and inherent stiffness imply that it must contribute to the overall capacity of the bridge. The difficulty to date has been the qualification and quantification of the interactive effects. This topic is the primary concern of this thesis.

The lack of research of a geotechnical nature in the field of arch bridges has led to the gross simplifications we see in our present day assessment methods. Attempts are made to model various effects such as load distribution through road and fill materials. These are, at best empirical, and at worst erroneous. Some attempts have been made at modelling the lateral pressure mobilisation behind a deforming arch ring: as with the simulation of the load dispersal, these attempts are often simplifications made in an attempt to rationalise and analyse, rather than arrive at a fundamental understanding of, the actual behaviour of the fill behind the arch.

The omission of the effects of repairs on arch capacity is another unknown factor in the assessment of bridges. Such effects are outwith the scope of this thesis but are briefly discussed here for the sake of completeness. Arches have long lives and as such they often pass from owner to owner. Each has had its own repairs carried out based on its requirements of the day. Excellent references to repair techniques useful for brickwork and masonry structures can be found in Sowden(34).

However; little information about the stiffness and strength of different types of repair is made available. The influence of each type of remedy is discussed in qualitative terms but no quantification is given of the updated capacity following repair. This topic is of interest where the extent of the repairs is major. Examples of "major" repairs would be: grouting of the fill, saddling the extrados, adding tie bars between spandrels, and lining the intrados. Some of the aforementioned techniques necessitate revaluation of the soil-structure interaction effects. A grouted fill has a different angle of shearing resistance, hence different dispersive ability, from a dry, unbonded fill. Mass concrete saddles change the interface friction effects and the applied stress distributions by virtue of their different stiffnesses.
1.5.1 Basis for soil-structure interaction in arch bridges

Having identified the above deficiencies in the assessment and analysis of arch bridges the author and his predecessors at the University of Edinburgh set about establishing a basis from which investigation of the effects the fill has upon an arch could be described and evaluated in detail\(^{33, 35-39}\). The aim of the project was to find sufficient information to assist structural engineers in the day to day problems of bridge assessment.

The logical sequence of the thought process was as follows: initially, a load is applied to the road pavement's surface, then through the fill below, and finally onto the arch itself. From the arch the load is transferred to the abutments by way of the springers. The obvious interaction occurs in the road pavement and fill immediately beneath the loaded area. The fill has sufficient stiffness to disperse the load over a wider area prior to its affecting the arch ring. Therefore the primary mode of soil-structure interaction was easily identified as being that of load dispersal. This is shown in Fig. 1.6.

Second to the load dispersal comes the lateral earth pressure mobilisation and redistribution as the arch deforms. This effect was derived in the following manner: the load dispersal onto the extrados tends to cause the arch ring to move away from the surrounding fill. The pressure state behind this portion of the extrados will then change, depending on the applied surcharge load and the arch deformation. As the arch deforms the extrados on the side of the arch remote from the load will tend to be pushed outwards and into the fill. This will also mobilise pressures substantially different from those acting in the at-rest state. This is shown in Fig. 1.7.

At higher loads the arch begins to develop a geometry associated with its ultimate limit state. Arches where soil-structure interaction effects dominate tend to be semicircular or steeper haunched profiles where there is a substantial fill mass surrounding the extrados with which it can interact. Semicircular and steeper haunched arches tend to develop four or five hinged failure mechanisms. Hinges involve the rotation of segments of the arch into and away from the surrounding fill mass. These rotations have their accompanying soil pressure changes in a similar, but more localised manner, to that demonstrated in Fig. 1.7. The localised pressure changes behind segments of a typical semicircular arch are shown in Fig. 1.8.
With the pressure changes caused by a live load on the road surface comes the mobilisation of substantial normal and shear stresses on the extrados. These pressures, caused by a combination of load dispersal and lateral pressure redistribution, were identified as key factors in the soil-structure interaction present in a backfilled arch bridge.

Following observations made during the load test to collapse on the bridge at Bargower\(^{40}\), arching effects were identified as contributors to the net interaction between the arch ring and the fill. Arching may be defined as the change in stress caused by the inclusion of materials of different moduli from the free-field stress that would exist were the inclusion not present. In this form the arching effects simply refer, in portmanteau fashion, to the changes in the stress state around the arch caused by the live load and the ensuing arch ring deformations. More particularly, arching action may be used to refer to the transferral of stress from one part of a structure to another. This happened at Bargower when, at a high load, some voussoirs dropped out of the arch ring. This left a gap in the barrel over which the thrustline arched by leaving the barrel and passing through the stiffer fill. The fact that the arch did not collapse immediately following the loss of these few voussoirs implied that arching over the gap had occurred. This was then identified as a further soil-structure interaction effect and is illustrated in Fig. 1.9.

The three factors mentioned above: load dispersal, pressure redistribution, and arching action, make up the basis of the soil-structure interactions in the soil-arch system. This thesis sets out to investigate, both qualitatively and quantitatively, the effects postulated, observed, and speculated on by various workers in the past.

### 1.6 Thesis outline

An attempt has been made to make each chapter of this thesis self-contained as far as is possible. Thus all the relevant literature is reviewed in Ch. 2. All the information pertaining to the small scale model tests is to be found in Ch. 3, and so on. This modular approach to the thesis, it is hoped, will make it more readable and allow readers to access relevant sections of the work more rapidly. The remaining chapters contained within this thesis are outlined below.
Chapter 2  A Review of the Relevant Literature

Pertinent research and practice is described in chronological order from Hooke\textsuperscript{(5)}, 1675, to date. The view is divided into two sections: theory and experiment.

Chapter 3  Small Scale Model Arch Tests

Tests on 0.700m span timber arches with dry sand backfill are presented. Zones of fill and arch displacement are identified and various interactive effects noted. Collapse loads are given for all tests.

Chapter 4  Large Scale Model Arch Tests

Instrumented tests on 2m span brickwork arches are discussed. Arch displacements, normal, tangential, and end wall contact stresses were measured. A final test to collapse is presented and compared with current methods of assessment and analysis.

Chapter 5  Full Scale Field Tests

The instrumentation of Kimbolton Butts bridge, Cambridgeshire, is described. Heavy axle loads of up to 30t were used during the tests. Analysis of the extrados contact normal stress, fill vertical stress distributions, and arch strains and displacements is described.

Chapter 6  Finite Element Analysis of the Soil-Arch System

An elastic finite element analysis is shown to provide excellent predictions of the stress field around a backfilled arch. Investigations of the effects of various material properties upon the interactive behaviour are provided. Road pavements are analysed. Comparisons with other tests are found to be favourable at lower stress levels.
Chapter 7  Conclusions

Although a discussion and conclusions may be found at the end of each chapter, a full summary is included here to round off the thesis and present the salient results as concluding statements all located in one place within the thesis.

Chapter 8  Recommendations for Future Research

Attention is given to possible ways in which the research might proceed after the completion of this project. This chapter follows the sequence of previous chapters in its recommendations which are given topic by topic.

Chapter 9  References

Ch. 9 contains, in sequence, numbered references making up the body of cited evidence necessary for the support of this thesis and the orientation of the reader.

Appendix  Published Work

Papers arising from the research which have been either published or presented.
Figure 1.1  The salient parts of an arch bridge

Figure 1.2  Mausoleum of Miriam Bidiyah, Qalhat, sultanate of Oman
Figure 1.3  Ponte Pietra, Verona, Italy

Figure 1.4  A typical failure mechanism
Figure 1.5  The thinning process used by MAFEA (After Choo\textsuperscript{(27)})
Figure 1.6  Load dispersal

Figure 1.7  Lateral pressures at low loads
Figure 1.8  Lateral pressures at high loads

Figure 1.9  Arching action behind displaced voussoirs
Chapter 2  
A Review of the Relevant Literature

2.1  Introduction

The voussoir arch has been a subject of research for 300 years. From the development of rules of thumb for pioneering arch bridge builders the first more detailed investigations into the nature of the thrust in arches were inspired. These investigations were largely practical, being concerned with issues such as: the thrust in an arch ring, the forces exerted on abutments, piers, and centring pieces, and the failure modes of arches of various configurations. Theories and models were developed, often in tandem, often in relative ignorance and often with incredible insight. Modern research has continued to use models and theories to provide understanding of the complexities of the soil-arch system and this has led to the analysis of soil-structure interaction effects in the 1990's. For the purpose of this literature review the different lines of progress have been identified as analysis and experiment. An introduction to the literature pertaining to soil-structure interaction theory and experiment is also included. When a worker, or research group, are involved in both theory and experiment, this will be indicated and details of the work will be presented in each of the appropriate sections of the review.

2.2  Theoretical work pre-20\textsuperscript{th} Century

The analysis of masonry arch bridges can be split into two distinct sections: that pre-dating the 20\textsuperscript{th} Century and that of the modern era. The work of the early investigators is described here, from Hooke in 1675 to Alexander and Thomson in 1902.

The earliest recorded research into the arch was that by Robert Hooke in 1675\textsuperscript{(5)}. In order to prevent plagiarism and to overcome scientific jealousy within the Royal Society he published his principal findings in the form of an anagram, the translation of which reads:

"As hangs the flexible line, so but inverted will stand the rigid arch."
Hooke's work looked at the catenary form for hanging chains and inverted the thought process to determine the correct mathematical and mechanical form for all manner of arches for building purposes. Force vector diagrams were derived to expound the concept of the thrustline in an arch ring.

In France Pierre La Hire worked on the arch problem and between 1695 and 1731 published two treatises on the subject\(^{(41,42)}\). His work may be seen as a precursor to the mechanism method of analysis, still used today. He concerned himself with the weight needed to ensure overall stability in an arch ring, assuming perfectly smooth, rigid voussoirs. Further study involved the derivation of the profile needed, for a given material bulk unit weight, to give rise to a catenary shaped thrustline following the geometrical centreline of the derived profile. His most important contribution involved the study of the effects of abutment movement and the ensuing hinge formation in a semicircular arch. La Hire correctly deduced the positions of the three hinges arising from inward or outward movement of the arch abutments or springers. Whilst La Hire was working in France, the holder of the Chair of Mathematics at the University of Edinburgh, one David Gregory, was following similar lines of thought. He suggested the theoretically correct shape for an arch centreline where the arch took the form of Hooke's inverted catenary. His published contribution, in 1697\(^{(43)}\), stated that an arch will stand only if a catenary could be wholly contained within the thickness of the arch ring. The precursors of modern tools such as mechanism and plastic analyses may be seen in the work of Gregory, although they were not recognised as a mechanism and a "safe theorem" at that time. Also evident in this early research is the birth of what we today would call the "middle third rule" for arch analysis which leads to the deduction of a geometrical factor of safety as used by Pippard and Heyman. In 1729, in France, Bélidor\(^{(44)}\) tried to advance the work of La Hire and Gregory. He assumed, incorrectly, that hinges formed in the arch at 45° around the ring, as measured from the springers. He, also incorrectly, made the thrustline tangential to the arch centreline at these hinge points. Using vector force diagrams he graphically produced failure mechanisms, albeit incorrect ones, for semicircular arches.

Simultaneously, and possibly as a result of collaboration with Bélidor, Couplet derived further ideas on thrustlines, collapse mechanisms, and stability. These he published as two "Mémoires" in 1729 and 1730\(^{(45)}\) in which one minor error must be noted. In his analysis of the self weight stability of semicircular arches he claims a limiting thickness to rise ratio of 0.101. This is based on erroneous assumptions...
regarding the hinge positions which were apparently used to give a simpler solution to the problem of determining the minimum thickness to rise ratio needed for stability. The error was to be corrected by Heyman in the 20th Century and this will be noted in the appropriate section of the review. Regardless of this minor error, Couplet made concise statements of his assumptions and produced what was, for the times, a seminal paper propounding a near complete, self-contained, analytical method for the solution of arch bridge collapse and stability problems.

In 1748, a rather brilliant young engineer officer called Poleni was called upon by the authorities of the day to analyse the hemispherical dome of St. Peter's(46). This marked the start of arch type analyses in what is now Italy. The dome was sliced into imaginary lunes, akin to the segments of an orange, and the stability of pairs of diametrically opposite lunes was considered. For each pair of segments the thrustline was shown to lie within the dome's thickness, thereby demonstrating the safety of the entire roof structure. Part of his work had previously been published by French and British workers, notably that pertaining to thrustlines and the "safe theorem". This could be seen as a measure of the ability of the Poleni team. Coulomb, in 1773(47), reiterated Gregory's conclusion that for an arch to stand the thrustline could not leave the arch ring. An addition to the bank of knowledge of the day came in the form of a method which allowed the intrados hinge(s) to occur where mechanics dictated. This improved Couplet's method which fixed the hinge positions prior to analysis.

Other work which Coulomb became famous for included the theory governing the thrust a wedge of soil exerts upon the back face of an earth retaining structure and his yield criterion for soils. Both these ideas form much of the basis of soil mechanics used to analyse soil-structure interaction problems. Coulomb however did not link these two strands of his considerable scientific repertoire to analyse an arch with the addition of fill material. This is due, in part, to no oversight of Coulomb's but because at that time there were few arch bridges backfilled to carry roads which needed assessing for the effects of increasing axle loads.

The early arch research efforts appear, on the basis of the evidence presented above, to have been directed towards the basic understanding of the arch alone. The work involving fill material and its added complexities was to come later.
Leading on from the early work there was a period of consolidation of existing knowledge. France and Great Britain played the leading roles with Gauthey (1809), Navier (1833), and Heather (1853) producing collections of available material for teaching and reference purposes. A University of Edinburgh Professor, one David Robison, occupant of the Chair of Natural Philosophy, wrote short articles for the Encyclopaedia Britannica (3rd edition, 1797 and supplement, 1801) on:

"...the construction of arches and centres for bridges."

Little new knowledge was added in these articles but they raised awareness of current practice as regards forces on centring pieces and arches during construction.

The most significant contribution any of the aforementioned three made to the field stemmed from the fact that, for the first time, most of the previous work on the subject was collated and made available to the students of the day. This served to overcome the lack of communication which was the hallmark of the previous centuries and the reason for the duplication of certain elements of people's work. This is not to say that nothing new developed during the first half of the 19th Century. Coinciding with the arch's heyday was the work of Moseley in 1835 who based his efforts on thrustline concepts. Barlow furthered this work in 1846 by stating, and demonstrating most effectively to the Institution of Civil Engineers, that the thrustline could occupy a variety of different positions within the arch ring. Possible configurations are shown in Fig. 2.1. It may be seen that, depending on which joint is removed, the thrustline can fluctuate wildly within the arch.

Across the Channel, Yvon Villarceau proposed a simple inverse design method for arch bridges in 1854. At this stage the funicular polygon concept was applied to the arch bridge problem. The French and the Swiss had been particularly active in the field of graphical statics in previous centuries with experts such as Coulomb and Culmann dominating the area. Fuller, in 1875, applied the funicular polygon solution to an arch to locate the thrustline: to this day the method known as "Fuller's construction" is still used. These methods furthered the use of the "middle third rule" in a variety of forms. The last significant contribution of the era came from Italy where Castigliano derived powerful elastic analysis equations for statically determinate and indeterminate structural forms. By now the arch bridge was on the wane and it was only because of the rapid expansion of the European
railway network that the theoretical interest in arches continued. Armed with the hard won knowledge of their predecessors, railway engineers could build arches with more economy and safety than those in the road networks of the day.

There were workers of great renown who contributed to the arch bridge field around the turn of the 20th Century. Their work marks the transition between the old and the new; the designers and the assessors. A brief summary of their work is given below.

Alexander and Thomson\(^{(56)}\) derived simple rules for the scientific design of masonry arches and published them over the years 1883 to 1902. They also considered soil thrust on retaining structures with substantial reference to the work of William McQuorn Rankine. Like Coulomb before them they did not relate soil thrust to arch behaviour, preferring to treat the problem from a purely structural viewpoint. Rankine\(^{(57)}\) gave a rule of thumb approach to arch design where quick calculations were used to assist in the proportioning of the crown depth, pier width, and span to rise ratio. Like others before he based his rules on an imaginary "middle third". Rankine, more famous for his geotechnical expertise, did not, surprisingly, analyse the effects of a superimposed fill load on the arch.

2.2.1 Theoretical work in the 20th Century

The following section describes, in more detail, the 20th Century work on arch bridges from a theoretical viewpoint. The work is predominantly assessment driven, rather than design driven but it is necessary to note the possible design uses, for new arch structures, of the most recent "assessment" methods.

2.2.1.1 Pippard et al.: peacetime interest, wartime necessity

Pippard, in conjunction with Baker, Chitty, Ashby, and Tranter worked on the arch bridge problem from 1936 to as late as 1968\(^{(7-13)}\). During this considerable time, including the 2nd World War, they carried out tests on model arches (section 2.3.1.1) of various compositions and configurations. The tests showed that the arch behaved as an elastic member at low loads and as a mechanism at loads above those generally causing the first hinge to form. In his theoretical deliberations he invoked
the "middle third" rule to limit tensile stresses in the arch ring. This conservatism was later relaxed to the more familiar "middle half" rule. The other limiting criterion used by Pippard was that based on an upper bound to the compressive stress in the arch ring. An analytical method of assessment was developed from these observations, test results and theoretical limiting criteria. The method was essentially a mechanism solution. It was developed for use in wartime situations where rapid assessments by untrained personnel were needed for abnormal loading from tanks, tank carriers and other heavy goods vehicles.

Taking his two limiting rules - compressive stress and "middle half" - he and his co-workers formulated equations relating: span, rise, thickness, and fill depth at the crown to vehicle type. In this way, a span could be checked for suitability against tables of axle loads for the vehicles of the day. This formed the basis of the nomogram, shown in Fig. 2.2, which has come to be known as the MEXE method, still in use today.

2.2.1.2 Selberg

Immediately after the 2nd World War, Selberg\(^58\) proposed a method of assessment based on a spandrel wall section which tapered into the fill until it met the opposite wall. Such a superstructure would be inordinately stiff and there appears to have been little use for the Selberg method. The analysis was based on the premise that no moment carrying capacity was present. A thin strip was analysed with forces applied to allow for spandrel interaction effects such as: bond with the arch ring, self weight, and fill-wall friction along the interface. These empirically derived forces were used to examine the arch thrustline. Selberg also recommends methods of increasing the capacity of his arch bridges such as the use of grout injection and concrete saddles over the extrados.

2.2.1.3 Heyman: the rise of the plastic method

Professor Jaques Heyman has produced extensive papers and publications on arch bridge analysis from 1966 to the present day\(^{19,59-64}\). All are based on mechanism failures allowing an ultimate load assessment to be made easily. Famous for his
research into the plastic methods of design and analysis it was natural that he should have applied these to the arch bridge problem.

Heyman's work is essentially the mechanism methods of Coulomb and the uplet with the addition of the fundamental theorems of plasticity. His method relies on the accurate choice of four hinge points on the arch ring. Moment equations are then used, which must include the unknown reaction forces at the springers, to solve the problem. The hinges are iteratively moved and the static equilibrium reanalysed until a minimum collapse load and its accompanying mechanism is discovered. The lower bound theorem of plasticity says that as long as there is one such thrustline and hinge mechanism, the arch will stand, it being at least as "intelligent" as the analyst.

For assessment purposes, Heyman proposed a geometrical factor of safety. This may be defined as the ratio of actual arch ring thickness to that needed to sustain a certain load set. If this factor is greater than one, the arch is safe, as far as this analysis goes. It must be remembered that at no stage does Heyman consider the interaction between the fill and the arch ring. The fill, of Heyman's methods, contributes dead load only with none of the benefits of interaction and strengthening of the soil-arch system. For arches without fill, such as cathedral arches, vaults in buildings, and open spandrel bridges, Heyman's methods provide reliable, tabular form, solutions which are often used today.

2.2.1.4 Walklate and Mann: Fuller's construction revisited

In 1983, the two authors published a practical method of assessment of load carrying capacity\(^{65}\). A thrustline is produced for a given applied load set and the load is varied to determine bounds on the load which just keep the thrustline within the middle third of the arch ring. This will give answers close to those predicted by pure mechanism methods even allowing for the simplifications inherent in the Walklate and Mann analysis such as: fixed outer hinge positions, purely vertical loading, and reactions based on an equivalent span linear beam element.

The analysis fails to account for any of the interactive effects, observed experimentally by this time, between the fill and the arch ring. The authors do not
even comment on the significance of their omission of lateral fill forces. However, the method is quick and lends itself to computer manipulation and solution.

2.2.1.5 Sawko et al.: a finite element approach

Sawko, Rouf, and Towler combined to propound one of the earliest finite element approaches to the modelling of the arch bridge problem(66-68). Linked non-linear beam elements were used for the arch ring which was given realistic stress-strain properties with a parabolic plot and a "falling branch" at large strains. They have, with some degree of success, modelled bridges tested to failure by a variety of workers. The load-deflection plots matched their experimental counterparts, even without restraining the thrustline and without the possibility of shear in the arch ring and without the effects of soil-structure interaction. In light of such omissions, agreement with experiment may have been entirely fortuitous. Sawko and Rouf later improved the program and incorporated shear in the masonry elements resulting in slightly improved correlation with experiment(69,70).

2.2.1.6 Harvey et al.: arch bridge ARCHIE

The work of Dundee University has resulted in the development of a mechanism method based computer program, arch bridge ARCHIE(16-18,71). Using Heyman's work as a starting point Harvey added the third theorem of plasticity: a masonry yield criterion. This supplants Heyman's assumption of infinite material strength and adds to the upper and lower bound criteria already in place in Heyman's method.

Most notable is the inclusion of load dispersal, through both road pavement materials and fill materials as well as the modelling of the effects of lateral fill forces. Harvey and co-workers were the first to attempt the modelling of the complexities of soil-structure interaction. A range of techniques have been tried by Harvey and Smith: Boussinesq type models, passive pressure factors, at-rest pressure distributions, and soil shear stiffness. The program has been marketed and has sold well to local authorities responsible for arch bridge assessment on a day to day basis.
Considerable effort has been devoted to the analysis and assessment of multi-span viaducts and arches. Investigations of pier thrust, limiting pier thickness and the behaviour of the wedge of fill, or backing, between successive spans have been carried out. The ARCHIE suit of programs can now analyse viaducts up to the ultimate limit state.

2.2.1.7 Vilnay and Cheung: confusion amidst contention

Vilnay published work on two, three, and four voussoir arches. The work concerned stability in relation to arch ring thickness to voussoir length\(^{(72)}\). Work and energy concepts were used with some confusion over the use of real or virtual deflections. The discrepancies, errors and confusions present in the reasoning were pointed out in discussions by Heyman and Harvey\(^{(73)}\). The parameters used, and their allowable ranges were criticised by Walklate and Mann\(^{(74)}\) in the same discussion.

Vilnay and Cheung incorrectly inverted the mechanism method principal that no tension should be allowed to develop at hinges. In doing so they effectively claimed that the collapse load was governed by the presence of tensile forces at joints and hinges. A final flaw in the work of Vilnay arose when he neglected the self weight of the voussoirs in his calculations, despite their size. The work of Vilnay and co-authors seemed to cease before the 1990’s with no contribution being made to the field beyond 1988.

2.2.1.8 Jennings: mechanism methods and soil pressures

A computer analysis\(^{(75)}\) was developed based on the mechanism method with the addition of zones of active or passive failure behind sagging or hogging hinges respectively. Little new knowledge or understanding arose from the research apart from recognition of the use of load safety factors rather than geometrical safety factors.

Jennings did include interaction between the arch ring and the fill but it is unlikely that the pressure distributions on the extrados implied by his work could exist in
practice. This is because the displacement required to mobilise full passive pressure is considerable and unlikely, even at the ultimate limit state.

2.2.1.9 Davies: the funicular polygon method and MARCH

Davies\(^{76,77}\) of the University of Edinburgh has produced software for the analysis and assessment of arch bridges. His MARCH program is based upon the funicular polygon method and other programs have been developed for implementation of Heyman's method with the addition of a loop to enable calculation of the collapse load by gradually increasing the applied load. Initially no soil pressures were included in the mechanism program resulting in some conservatism when steep haunched arches are analysed.

The MARCH package allows the user to apply soil pressures from a database of distributions which has, as one option, the facility to impose a user-defined distribution upon the extrados. The program calculates thrustline positions and collapse loads based on collapse occurring when the thrustline leaves the arch ring. The program adds no significant knowledge to the field but it is a good learning tool as it allows a user to move the thrustline around wildly and examine the ensuing effects upon the system.

2.2.1.10 Crisfield: mechanism and finite element analyses

Crisfield worked at the Transport Research Laboratory on the development of computer models for arch bridges. His mechanism method program\(^{78}\) used horizontal soil pressures, with the dangerous proviso that they could cause an overestimate of the collapse load, and virtual work equations rather than the more common moment equations. Various problems regarding hinge positions were encountered but these were explained away by considerations of load dispersal through the fill.

The first of the two finite element analyses\(^{23,24}\) adopted a continuum approach to the modelling of the arch ring. Allowance was made in the material strengths and stiffnesses for the fact that joints were present in reality but not in his model. These appear to have been made with no evidence, experimental or otherwise to support
them. The analysis was non-linear, both materially and geometrically and in some cases, most notably in the analysis of Bridgemill, able to achieve good correlation with the load-deflection curve or the collapse load but not both simultaneously.

His second model is more complex. It involves full two dimensional treatment of the arch and the fill. A Mohr-Coulomb yield criterion is used for the fill and plane stress conditions pertain throughout the system. Full Gaussian integration was used, accurate material properties, based on test results, were incorporated and the anisotropy of the arch ring was included as an effect. For all these additions the new model failed to provide a useful assessment or analysis tool. The correlation with it and full scale field tests appears to be worse than that achieved with the first, simpler approach.

2.2.1.11 Hughes: the elastic approach, Castigliano revived

Hughes, at Cardiff, has produced an assessment program based on Castigliano's strain energy analysis(21,22). This is known as CTAP, a commonly used package in the day to day assessment of arch bridges.

The arch is treated as an elastic fixed rib. Load is applied incrementally and areas subjected to tensile stresses are discounted from the arch ring thickness for the next loop of the analysis. In this way the arch thins until it just contains the thrustline within its reduced cross sectional area. The current applied load is the collapse load for the system. The incremental nature of the method allows load-deflection plots to be produced with the advantages of the geometric nonlinearity built into the method. The collapse load falls between two applied loads: that applied pre-collapse and that eventually causing collapse. If too large an increment is applied, convergence is difficult to reach. Soil effects are included by the imposition of horizontal forces on the extrados in a manner similar to that used by Crisfield(78).

2.2.1.12 Franciosi and Franciosi: the cells method

By treating the voussoirs as rigid blocks with elastic mortar between them, the two Franciosi’s have produced a new technique for arch analysis(79) known as: "the cells method". Movement of the voussoirs, or cells, causes movement in the elastic
jointing material. The stresses in the joint are calculated and a series of stress blocks through the arch ring is produced: they lead to what is essentially a Heyman type analysis of the thrustline location. Similar criteria are used to curtail the applied load and the collapse state is obtained with a load-deflection plot derived from the joint stresses being output. The technique has been computer coded but appears to be little used in practice.

2.2.1.13 Choo et al.: Nottingham and the MAFEA program

MAFEA has been developed between Nottingham University and British Rail Research after extensive work by Choo and his team (27-29) into the arch bridge assessment field. The core of the method lies in a powerful finite element analysis with a simple user interface wrapped around it. The program models the arch as a series of linked one dimensional elements whose properties render them more akin to a two dimensional voussoir with all its strength and stiffness grouped along its centreline. User defined load dispersal, of a Boussinesq type, is included. Mobilisation of earth pressure behind passive arch segments is included and is calculated from knowledge of the current deflections. Deflection causes a certain pressure change which can be seen as the program runs through its incremental analysis.

The modelling of defects is made possible by thinning the line elements where appropriate and proceeding with the analysis. A variety of stopping criteria can be applied which enable the accurate simulation of models where the deflections may be large due to the low arch ring stiffnesses used. For the analysis of real arches the stopping criteria can be tightened to allow only small total deflections, low rates of movement per unit applied load, or ring separation effects. Choo has obtained good agreement with the results of field tests in terms of load-deflection data and to a lesser extent, collapse loads.

2.2.1.14 Cooke: large deformation analysis

Cooke's work (80), from the University of Canterbury, New Zealand, is based upon the examination of the stability of a masonry arch system. The onset of instability is predicted by a large deformation analysis where the deflections are calculated by
considering the potential energy changes in the system. The arch alone is considered as a rigid bar failing by formation of a four hinged mechanism. The basic assumptions used by Heyman also apply here: infinite material strength is assumed, voussoir friction prevents slipping of the stones, abutment movement does not take place, the spandrels are filled with homogeneous rigid fill, and the arch is rigid. A failure mechanism is chosen and a collapse load is arrived at by plotting hinge loci and potential energy changes associated with the arch rotations. The solution is an upper bound on the collapse load and iteration using various mechanisms must be used to determine the "true" collapse load.

Crisfield pointed out, in a discussion on Cooke's work, that there was an increase in capacity arising from the use of a more distorted profile of arch due to the fact that the analysis used large displacements. Such an increase could not occur as the undeformed profile has the highest capacity. Crisfield queried the use of the principle of superposition in Cooke's method. No use has been made of the method in practical bridge assessment situations.

2.2.1.15 Oran: buckling of a continuous flexible rib

The last piece of theoretical work reviewed is that of Oran(81) from the University of Missouri. As such his methods have not been used or further referred to in this thesis but the work is reviewed for the sake of completeness. His approach treats the arch as a flexible, continuous rib. He then analyses the buckling behaviour under load as well as under a load plus some precompression of the arch ring. Various boundary conditions are modelled such as springs, yielding ends, fixed ends and pinned supports. Collapse loads are derived from elastic buckling criteria. For everyday arch assessment the method is of little use but it has possibilities for the serviceability limit state analysis of shallow arches failing in "snap-through" buckling. It is included here as it points the reader towards an excellent range of references pertinent to the elastic buckling treatment of arches and curved beams.

2.2.2 A summary of the theoretical work

Most of the modern methods of analysis, and the current ways of thinking about the arch bridge problem, stem directly from the earliest research as described in section
2.2. This is not to say that the new is merely a reworking of the old: early misconceptions have been elucidated and corrected, methods have been combined elegantly e.g. Heyman's use of the mechanism method and plastic analysis, Davies's use of the funicular polygon in tandem with the modern ideas of soil thrust and Harvey's addition of load dispersal and passive pressures to the simple mechanism method.

The newest methods are finite element analyses which have relied upon the power of the modern computer to both clarify, study parametrically and improve on old methods. With the finite element methods come a variety of problems: more complexity implies more time analysing structures. More variables and parameters imply more scope for error in assigning values to them and more laboratory testing. More sophistication requires a better analyst and finally, time is of the essence in our assessment of the bridge stock to new European Community standards. Any new method must be easy to use in practice for assessment purposes. If a structure warrants more complex treatment the newer methods can then be justified.

The gradual recognition, amongst the arch analysts, of the need to incorporate the effects of soil-structure interaction into their analyses may be traced through the history and details of the methods given in this chapter. They gradually highlight the need for more accurate qualification and quantification of the soil pressures acting on the extrados for a given applied load. The leading edge of the technology appears to have been reached and honed to a fine degree as far as the arch is concerned. The thrust of the research of the day lies firmly in the area of soil pressures and how they affect the load carrying capacity of a soil-arch system. A chronology showing the salient contributions to the theoretical aspect of the work on arches is given, for reference purposes, at the end of this chapter.

2.3 Experimental work - a brief history

The experimental work on arch bridges, both full and small scale, will be outlined. The work carried out before the 20th Century will be discussed briefly followed by a lengthier, more detailed review of that done in the modern era.

Since the first thinkers and early men of science turned their minds to arches, there have been tests examining the stability, the geometry, the stiffness and the strength
of arches. Romans, in all probability, built mock-up, scale models of their arched structures, knowing that stability was unaffected by scale. Only historical evidence from other fields lends credence to this supposition as no record remains of Roman model tests on arches. The labour, time, and cost involved in construction rendered full scale tests to destruction unfeasible.

The French led the way in the use of tests to look at the behaviour of arches. Gautier, in 1717, built small scale timber model half spans spanning between a wall and a level base. These he destroyed by removing lateral support from behind the springer and extrados until the arch collapsed. Ten different configurations were used with blocks of known mass to enable measurement of the abutment thrust and the reduction needed to cause collapse. Also in France, Danyzy tested small plaster voussoir arches to determine failure modes and minimum pier sizes for stability. His work matched the findings of Couplet, several years previously but was not published immediately. Coulomb did not seem to know of Danyzy's work when he developed his contribution to the field but the conclusions of both men are generally in agreement.

Boistard, again a Frenchman, built larger scale models to determine the collapse modes under various applied load geometries, the minimum abutment requirements and the forces exerted on the centring piece during construction. Boistard noted, correctly, that a semicircular arch will not sit on a centring piece under its self weight: the lower voussoirs will stand proud of it due to the thrust generated in the arch ring. He also confirmed Couplet's earlier findings on limits required for arch stability upon decentring. Heyman notes that Boistard's contribution did not include any calculations. The work was reported by Lesage in a volume of "Mêmoires" in 1810.

Barlow presented his models, shown in Fig. 2.1, to the Civils in 1846 as part of a paper presentation on arches and thrustlines. His voussoirs were timber and his joints were made up of thin sheets of board which could be inserted and withdrawn between the voussoirs. By the simple expedient of alternately inserting and extracting joint boards he shifted the hinge positions wildly, demonstrating the possible positions of the thrustline within the arch.

Other studies were carried out in Britain, France and Austria by engineers "on the job", whilst they built arches for the road, rail, and waterway networks. Using the
knowledge of the day they calculated for the purposes of safety, design and economy. As such these did not fall into the category of tests and are not described here. Heyman (1959) gives excellent references to such work and brief descriptions of what was involved. Suffice to say that little new knowledge was added but existing rules were confirmed or at least bolstered by more evidence.

2.3.1 Experimental work in the 20th Century

There were many masonry arch bridges in our transport infrastructure by the middle of the 20th Century. Many had never been subjected to the heavy loads we expect our bridges to bear today. Pre-2nd World War little was done as no new arches were built, loading was light and onerous assessment requirements were not required. During the war assessments were needed, quickly, of arch bridges and their load carrying capacity. This marked the start of the assessment led work of Pippard. Nowadays arch bridge experiments are still common as axle loads are increasing at a rate unforeseen only twenty years ago. The tests and experimental work of the 20th Century engineers and scientists will be described in the following section of the review.

2.3.1.1 Pippard et al.: the tests behind the MEXE method

Pippard's first tests (8,9) used smooth steel voussoirs and dry bed joints. He compared the collapse loads with a continuous steel rib and concluded that elastic methods were appropriate for arch analysis. All the tests used a span to rise ratio of 4 and they failed by the formation of four or five hinged mechanisms.

He then progressed to mass concrete voussoirs in 3m spans with a span to rise ratio of 4 again. Lime and cement mortar with either limestone or granite concrete was used to build these arches. The models were tested to collapse: the most important outcome was the relaxation of the "middle third" to a "middle half" rule. A selection of these tests was carried out minus the jointing material: these proved inconclusive, failure being determined by a mixture of voussoir slip, crushing, spalling and delamination. Dynamic loads were used but little decrease in capacity was noted.
2.3.1.2 Davey: Building Research Station work

Davey\(^{(86)}\) carried out tests to destruction on three bridges in England for what was then the Ministry of Transport during and before the 2nd World War. Proving tests were carried out on 18 other arches. The tests stopping short of destroying the bridges. The three collapse tests were interesting in that attempts were made to quantify the contribution of each element of the structure to its overall capacity. Various components were removed: firstly the parapets, secondly the road surfacing and the fill. These partially demolished structures were compared with an intact system to isolate the contribution of individual elements. Tests were also carried out, at a later date, to assess the effect of crown cover upon collapse load. Increasing fill depth resulted in increased capacity but the analysis extends no further.

2.3.1.3 Chettoe and Henderson: extending Davey's work

In the 1950's a broad based study of masonry arch bridges was done by Chettoe and Henderson\(^{(87)}\). Tests applying up to 90 tons load were carried out; deflection and abutment spread measurements were made on 13 bridges in Britain. The principal conclusion was that arches behave elastically and that at normal loads it was not necessary to consider composite action with the fill. They concluded that the 45° load spread through the fill was acceptable for assessment purposes. Substantial agreement with Pippard's work was reached and the findings, especially those from deflection measurements, are similar. As will be demonstrated in this thesis, interaction between arch ring and fill does occur, to the benefit of the load carrying capacity. The author is puzzled as to how, without pressure measurements in the fill and on the extrados, the sweeping statement of the acceptability of the 45° load spread could be made. Any attempt to discuss soil pressures is purely speculative without direct experimental evidence as support. Given the movements observed by Chettoe and Henderson some allowance must be made for the presence of the fill and its restraining effect upon arch displacement. Notwithstanding the above criticisms, their deflections and spreads provide valuable information about elastic arch behaviour. The additional information about inelastic unrecoverable displacements after passage of abnormal load(s) was novel and to this day we have little information about residual displacements.
Over a twenty five year period from the late 1950's the Highways Research Station, based in Madras, conducted tests on a wide range of arch bridges. Single and multispan bridges were tested to failure and to a proof load. Information was collected about the contribution each component made to the capacity in a manner similar to that used by Davey in Britain. The Indian study was more extensive as they had, at that time a relatively large number of redundant multispan arches. The piers between some of these spans were sufficiently squat for the arches to be considered as single spans. Tests with various parts removed were carried out quickly and with few of the logistics problems facing Davey. The arch ring's contribution was observed to be 46% and the rest of the structure, including the fill's contribution was therefore 54% of the overall load carrying capacity. The fill's contribution was reckoned to stem from the ability of a well compacted fill to disperse stress.

Further conclusions were drawn relating spread and deflection under a proof load to that which would be produced by a gross vehicle weight of 40 tons. In this way arches could be empirically assessed based on the passage of a light axle, provided the resulting movements were small.

Some model tests were carried out as part of the Indian research contribution to the field but these were of surprisingly poor quality given the care that went into the field tests. The models, largely because of design faults and scale effects, failed to replicate many of the facets of the arch behaviour observed in-situ.

2.3.1.5 Sawko et al.: tests to check finite element analyses

As noted under 2.2.1.5, Sawko and his co-workers tested 4m span arches with the aim of verifying their finite element model's predictions of the ultimate limit state. Good correlation was found to exist between model tests and computer predictions. The models did not use fill material but they simulated the effects of the fill's self weight by placing weights on steps cast up against the extrados.
Melbourne, based in Bolton, has carried out several tests\cite{92-94} on large scale models for the Transport Research Laboratory. Fill pressure measurements were undertaken with limited success. The fill tended to be a dry, stiff, crushed limestone conforming to the Department of Transport "Type 1" specification. Pressure measurements in such a fill will always be difficult but what information the cells yielded is useful in that it indicated mobilisation of earth pressures substantially different from those acting in the at-rest state. Considerable load dispersal was also observed, probably because of the fill's high stiffness. The collapse load was much higher than analytical predictions indicated it would be. This is, in the author's view, almost entirely due to the fill's contribution.

The recent work at Bolton has investigated the effect of ring separation upon the carrying capacity. Models with lubricated rings to prevent mortar bond formation have been tested and the reduction in capacity quantified in relation to the amount of debonding present.

2.3.1.7 Harvey et al.: Dundee's model tests

The team at Dundee have tested several models\cite{16,95} to destruction for information on field tests and for use in the verification of their program, ARCHIE. Fill pressure measurements have been tried with some success: good correlation with the assessment program ARCHIE was achieved in the qualitative, and to a lesser extent the quantitative, sense. Measurements of extrados stress were made with and without the road pavement materials to assess the differences in load dispersal caused by the pavement.

The mobilisation of active pressures behind the segment of the arch moving away from the fill was observed as was the partial mobilisation of the passive state behind the segment of the extrados remote from the load. The load test to failure showed a mechanism type collapse mode developing. Full collapse was not possible due to interference from the test rig at high loads but this seemed to have had little bearing on the results.
A series of 1.5m span models have also been tested at Dundee. No fill was used and simulation of the self weight of the fill was brought about by the use of weights on steps cast onto the extrados. Some defects were introduced to examine the strength reduction caused but little change was observed. Finally, Harvey and Smith have, for demonstration purposes, a timber model of a multispan viaduct. Load is applied to the extrados of the end span, for example, and the effects of this load can easily be seen in adjacent spans. This model is purely qualitative and it affords the user a rapid understanding of the behaviour of pier-arch systems under any load geometry.

2.3.1.8 Hendry et al.: the University of Edinburgh tests

The University of Edinburgh have tested two masonry arches to collapse. Bridgemill\(^{96}\) was a flat arch with a span to rise ratio of 6.4 tested in 1984. Acoustic emission monitoring and theodolite displacement measurements were carried out as the load was increased. Failure did not occur as the loading jacks reached the end of their travel. The collapse load was predicted from the load-deflection curve up to the end of the test. At the end of the test the bridge appeared to exhibit signs of an elastic snap-through buckling failure rather than a four hinged mechanism. This is possibly to be expected given the high span to rise ratio of Bridgemill. Acoustic emission monitoring showed how the onset of cracking could be detected well before visual inspection revealed cracks. No fill pressure measurements were included in this test.

Bargower\(^{40}\), a semicircular arch built on a 10° skew, was tested to collapse by line loading it at the third span. Fill pressure measurements were included with some success. Acoustic emission transducers and theodolite displacement measurements were undertaken. Considerable interaction with the fill was observed to take place; the large fill depth over the arch gave rise to considerable load dispersal. As the arch deformed pressures significantly higher than at-rest values were observed on the side of the arch remote from the load. Rock formations behind one springer added to the strength of this structure and also prevented arch rotation into this backing. Collapse occurred by local voussoir crushing and loss of masonry beneath the load point. Evidence of hinge formation elsewhere was found. The analysis of the results from this test led substantially to the development of the ideas about soil-structure interaction by Ponniah\(^{33,35,36}\) at the University of Edinburgh.
Hendry, Royles, Davies and Ponniah (1997) tested a range of models from Bridgemill to the model tests described in chapters 3 and 4 of this thesis. The early models, tested by Hendry were used to determine the contributions made to the capacity by various components of the structure, especially the spandrel walls. Due to the narrowness of the models in relation to their spans, shown in Fig. 2.3, the spandrels have influenced the model behaviour to a large extent. Fill pressure measurements were made on a range of these models by Ponniah.

Models (1997) by Hendry and Royles ignored the fill and simulated its dead load with weights on steps cast onto the extrados. A small demonstration model with third scale bricks as fill has been used to simulate interaction between backing and an arch ring. This small scale model provides a simple demonstration of the lateral forces the arch exerts upon the fill.

2.3.1.9 Other tests by the Transport Research Laboratory

Using others as subcontractors or their own staff, the Transport Research Laboratory have carried out collapse load tests on the following arches: Bridgemill (1996), Bargower (40), Preston on the Wealds Moor (1998), Prestwood (1998), see Fig. 2.4, Torksey (1999), Shinafoot (1999), see Fig. 2.5, Barlae (100), Strathmashie (100) and Kimbolton Butts (101) (see chapter 5). Loading has been applied as a line across the full available width of bridge at points between third and fifth span and increased until collapse except in the case of the new arch at Kimbolton Butts. A variety of failure modes was noted and some differences of opinion exist between the "official" version in the contractor's report and the interpretations of others, most notably those of Smith in his doctoral thesis (16).

2.3.2 A summary of the experimental work

The experiments can be divided into field and model tests. Model tests have provided valuable insight into the behaviour of soil-arch systems where the fill has been included in the model. Those models without fill are of no use beyond the demonstration of simple modes of arch ring behaviour under directly applied extrados loading.
The field tests provided valuable information but in many cases measurements were not sufficiently comprehensive. The innate variability of real arch bridge construction causes difficulty in isolating phenomena and comparing test results with both models of the tests themselves and other field tests. However, considerable understanding of arch behaviour has been gleaned from these field tests over the 20th Century. Increasing cost and decreasing availability of suitable arches will reduce the number, already small, of full scale tests in the future. It must be stated that any opportunity to instrument new or existing arches should include fill pressure measurement. To summarise the work, theoretical and experimental, on arch bridges a chronology of notable events is given below.

- c.100 Emperor Caesar Augustus, Pont du Gard, Nimes
- 537 Emperor Justinian, Ayasophia, Istanbul
- 1570 Palladio, rules for sizing arches
- 1570's Gil de Hontañon, rules for sizing arches
- 1640 Grumbold's Clare College bridge
- 1675 Hooke, catenary analogy
- 1695 La Hire, funicular polygon
- 1697 Gregory, catenary analogy
- 1712 La Hire, cracking patterns
- 1716 Ecole des Ponts et Chaussées founded
- 1717 Gautier, model tests
- 1729 Couplet, model tests
- Bé lidor, thrustline, fixed hinges incorrectly
- 1730 Couplet, corrections to hinge positions, pier design
- 1732 Danyzý, model tests
- Wade's bridge, Aberfeldy
- 1747 Labelye's Westminster bridge
- 1748 Poleni, method of slices
- 1768 Perronet's Pont de Neuilly
- 1769 Smeaton's bridge, Perth
- 1773 Coulomb, thrustline and mechanism approach
- 1779 Iron bridge, Coalbrookdale
- 1800 Boistard, model tests
- 1831 Rennie's London bridge
- 1833 Hartley's Grosvenor bridge, Chester
- Navier, collected lessons published

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2.4 A review of the relevant soil-structure interaction literature

When compared with the history of the arch, soil mechanics is a young science. The study of soil-structure interaction is almost as old as soil mechanics. The first workers were aware of the need to provide analytical solutions to engineering problems which nearly always involved a structure of some description. This section of the literature review covers the work used in the development of soil-structure interaction thinking in the arch bridge field. The arch may be seen as a retaining structure and so some ideas have been garnered from the seminal studies into wall-soil behaviour. Curved retaining structures, such as buried culverts and curved "containing" structures, such as pipelines have been subjected to soil-structure interaction analysis for design purposes. As such, they have been examined for ideas to apply to the arch-soil problem. The most general description of the causes and effects of soil-structure interaction are to be found in Thorburn\(^{(102)}\), where various cases are investigated, presented and summarised. He also provides a comprehensive collection of other references.
2.4.1 Luscher et al.: buckling of soil surrounded tubes

Luscher\textsuperscript{(103,104)}, based at Massachusetts Institute of Technology (MIT), used a Winkler soil spring model to analyse the buckling failure of buried culverts in an elastic soil mass. His work included theory and experiment. His principal conclusions were: that soil surrounding culverts significantly increases buckling resistance of the culvert ring and that as soil stiffness is increased the culvert becomes more likely to fail by local crushing rather than in a multi-wave buckling manner.

Davis\textsuperscript{(105,106)}, again based in the United States, worked on backfilled concrete arch culverts. Soil pressure measurements were used to investigate the effects of soil placement, compaction, crown cover and culvert displacement under load. The concept of soil arching was used to explain the load-deformation and load-culvert pressure behaviour. Soil pressures were converted to an equivalent sand density for purposes of normalisation. Davis cleverly used softer, more compressible backfills in certain zones to analyse the effects upon the stresses elsewhere around the extrados. The patterns of soil pressure change with applied load change were monitored and he found mobilisation of the active state behind inwards moving segments and a fraction of the passive state behind outwards moving segments.

Höeg\textsuperscript{(107)}, based at MIT, has studied the stress fields around, and the displacements of, underground structural cylinders. Large scale tests were used, with copious measurements of normal soil pressure on the ring, displacement of the ring and surface contact pressures beneath the load. Normalised compressibilities and flexibilities were used in his analysis of the tests. Theoretical pressure distributions were derived by assuming local soil yield around moving segments of the ring. In this way the effects of arching could be predicted and compared to his experimental results. He concluded that: interface friction, and hence shear stress along the interface, was important, and the actual pressure distributions lay between the analytical predictions for "full slip" and "full friction" on the interface. He also showed that arching contributes to the strength of the ring, and that finite element analysis would be an ideal way to proceed.

Moore\textsuperscript{(108-111)}, based in Australia, has provided theoretical input to the problem of the assessment of the stability of buried culverts for design purposes. His solutions are largely finite element based. Various parameters such as: shape of culvert, fill...
depth, fill to culvert stiffness ratio, fill Poisson's ratio and load geometry were investigated. Critical hoop thrust conditions were examined and criteria for determining the failure modes were drawn up. This gave a set of conditions which could be used to determine whether crushing or buckling would dominate. The studies are all numerical and no experimental evidence is provided beyond comparison with the tests of Luscher et al\cite{103,104}. The work and ideas of Moore represent the state of the art in culvert analysis and they have been applied successfully to a range of real structures both in the design stage and in the back analysis after failure stage. Possible stress distributions\cite{108-111} around buried circular structures are shown in Fig. 2.6 for a variety of surface frictions and flexibilities. This represents a good summary of the soil-structure interaction investigations undertaken in the field of buried pipes and culverts.

2.4.2 Retaining wall analyses

Various workers have examined the stability of walls and the soil behind them. The points of most relevance to the arch bridge problem are the correlation of deflection and rotation with mobilised pressure, the zones of soil movement, and the progression of yield in the soil. The work of Potts, Fourie, Jardine, and Burland at Imperial College\cite{112-114}, London, is of relevance. They have undertaken extensive parametric studies, using finite element methods, small scale models, and full scale field tests, to examine the behaviour of soil-wall systems. Zones of fill displacement were traced for various loads and geometries of wall using the models. Pressure distributions behind moving walls were derived from the finite element results and these have been used for design and analysis purposes. The influence of wall-soil stiffness ratio was also explored in their work. Correlations between wall displacement and observed, or modelled, stresses are shown in Fig. 2.7.

Measurements made during full scale field tests have been used to improve design and analysis methods. Such tests are numerous and an example is reviewed here for reference purposes. Sims and Jones have published results\cite{115} from an instrumented motorway retaining wall which show the pressure distributions in comparison with the theoretical distributions of Coulomb, Jaky, and Spangler. The changes in this pressure distribution with time were recorded along with wall movements. Finite element modelling was used to predict the experimental results with some success. The various theories were found to be unable to model the changes in pressure with
either time or wall deformation. Elastic analyses were found to give the best predictions of the stress state at low loads and as such they recommended the use of two dimensional elastic finite element analysis for wall design.

2.4.3 Application of soil-structure interaction to arch bridges

The ideas gleaned from the soil-structure interaction literature, a sample of which has been reviewed, were applied to the soil-arch system by Ponniah at the University of Edinburgh over the period 1987 to date\(^{(33,35,36)}\). Four modes of soil-structure interaction were postulated: load dispersal, lateral earth pressure mobilisation and redistribution as the arch ring deformed, mobilisation of circumferential shearing resistance, and arching behind displaced portions of the arch ring. The modes of interaction were based on observations and measurements made during the field tests at Bargower and subsequent model tests reviewed in section 2.3.1.8. Various arch tests were examined and the discrepancies between theory and practice were ascribed to the previously mentioned soil-structure interaction effects.

It is on Ponniah's work that the studies and tests described in this thesis have been developed. As the work was developed the latter two interactive modes have become largely similar to the second mode mentioned above. The presence of circumferential shear stresses on the extrados and the onset of arching are due, almost wholly to the deformation of the arch ring under load. As such they may be lumped under the broader category of the effects of the mobilisation and redistribution of the fill pressures around the extrados.

2.5 Concluding remarks

The review has covered the lengthy history of the arch bridge analysts and practitioners along with the development, in recent years, of soil-structure interaction theories as applicable to the arch bridge problem. As with any transportation/infrastructure type problem, the tracing of its history closely follows the history of the civilisations dependent upon that infrastructure. One is always aware of that fact when writing a general review of this nature and one is conscious of the need to stand on the shoulders of giants to undertake such a task. The giants
in this case being Heyman\(^{(19,59-64)}\), Harvey\(^{(17,18,71,116)}\), and Smith\(^{(16)}\) who have provided excellent, all-encompassing reviews of the salient points in the history of the arch bridge.

For all the efforts devoted to solving the arch bridge problem little appears to have been achieved in the way of agreement across the wide spectrum of researchers active in the field. Each partial solution has been, of necessity, tailored to fit the prevalent circumstances. This thesis sets out to provide answers in the area of arch-soil interaction which will be of widespread use, regardless of other underlying structural assumptions inherent in the reader’s choice of assessment or analysis method.
Figure 2.1  Possible thrustline configurations (After Barlow(53))

Figure 2.2  The MEXE method nomogram
Figure 2.3  Edinburgh model tests

Figure 2.4  Prestwood Bridge before collapse
Figure 2.5  Shinafoot Bridge before collapse

Figure 2.6  Stress distributions around buried pipes
Figure 2.7  Earth pressures with increasing wall displacements
Chapter 3  Small Scale Model Arch Tests

3.1  Introduction

As discussed in Chapters 1 and 2 of this thesis, there are many masonry arch culverts and bridges in the transport network of the UK which are currently in need of assessment. European Community directives seeking to harmonise transport policy with regard to heavy goods vehicles ask for larger loads to be carried, resulting in the need to assess a substantial percentage of the existing bridge stock.

Arch bridges are currently assessed conservatively, using simplified techniques such as the MEXE method\(^\text{(14)}\), as currently required by the Department of Transport standard, BD21/93, or mechanism methods\(^\text{(17)}\). Arch bridges are also analysed using Castigliano’s method in computerised form\(^\text{(22)}\), and finite element analyses are also available\(^\text{(23)}\). The nature of these analyses is expounded upon, at greater length in Ch. 2. Yet an undefined factor in these techniques is the contribution which the fill material or soil makes towards their strength. Studies to evaluate this contribution are currently being done\(^\text{(117,118)}\).

The effect of the fill may be investigated by a number of means, such as theoretical methods, field tests and model tests. While theoretical techniques can be used to investigate many variables quickly and cheaply; because of the idealisations required in such analyses they are limited in their usefulness. On the other hand while field tests are realistic they are enormously expensive and also relevant only to a limited number of parameters. Model tests incorporating much of the real structure can be used to investigate a wide range of parameters. This chapter describes a series of model tests used to investigate the effect of fill on buried arches.

The model did not purport to be a scale representation of any particular bridge. The following points must be made with regard to the scaling of collapse loads for comparison with prototype bridges: the model and prototype are rarely homologous, the stresses are very much lower in the model thus the effects of material failure in the prototype can not be modelled, the stability of the model does not differ from that of the prototype, and the scale factor to be used can vary according to whether material densities or material strengths are used. The other fundamental difference
between the model, described above, and prototype arches is that the model lacked:
a road pavement, spandrel, wing, and parapet walls, and jointing material between
the voussoirs. The omission of these structural components causes the scaled
collapse loads to be below that for a prototype arch bridge. Centrifuge modelling
may be undertaken to correctly model the fill stresses but then observation of the
model during testing and steady application of a point load on the fill's surface
becomes difficult. Comparisons are made in this chapter between the collapse loads
obtained using the models and those obtained in full scale bridge tests.

The aim of the investigation was to examine the interaction between the arch and
the surrounding fill, particularly; the dispersal of the surface load and the
mobilisation and redistribution of earth pressures acting on the arch. This was to be
achieved by observation of the: zones of fill displacement around the loaded arches,
collapse loads for various fill densities and fill depths, and the effect of varying the
span to rise ratio of the model.

The tests fell into two series:
(i) examination of the zones of fill displacement
(ii) a parametric study of both span to rise ratio of 2 (semicircular), and span to
rise ratio of 4 arches.

The parameters investigated included: end wall position, fill density, fill depth and
load position.

3.2 Model description

An arch span of 700mm was constructed in timber with 45 voussoirs in the
semicircular arch and 25 voussoirs in the span to rise ratio of 4 arch, shown in Figs
3.1 and 3.2. The loading system is shown in Fig. 3.3. To minimise the effects of
friction a gap was required between the side of the arch and the 4mm thick glass
side wall. Thus these walls were not structural, i.e. not spandrel walls, and would
not affect the collapse loads unduly. Polythene film was placed over the extrados in
strips, preventing the fill material escaping through the gaps between the side walls
and the arch.
Early tests using the model showed that the polythene film was adequate for the purposes of retaining the fill. Had fill escaped and fallen between the ends of the voussoirs and the side walls, the collapse loads measured would be unduly high because of sand being trapped thus preventing free movement. Friction between the fill and the side walls could not be totally eliminated, even with the use of polythene film and the gap left between the side walls and the voussoirs. As the arch and fill move under the influence of the applied load, frictional resistance is generated along the inside of the glass walls. This friction will increase the collapse load by what was found to be a negligible amount.

A ring thickness of 35mm was chosen, giving a thickness to rise ratio below 0.106 which renders the arch unstable without the fill\(^{(64)}\). Thus the arch was supported by a wooden former until backfilling was sufficiently advanced for lateral earth pressures to stabilise the arch.

The extrados, with its smooth timber voussoirs and polythene film, was more representative of brick than masonry. Examination of the effects of extrados roughness on the collapse load could be made the subject of future research into masonry arch bridges. Such model tests were beyond the scope of this study.

### 3.3 Fill properties

The fill was a medium, uniformly graded dry silica sand with rounded particles. The particle size distribution obtained by dry sieving is shown in Fig. 3.4. It was placed from a scoop from zero drop height in 50mm layers. Each layer was given 20 blows per 200mm longitudinally, with a steel tamping rod. A series of density box tests with 0.156m\(^3\) samples gave an average density of 1515 kgm\(^{-3}\). Shear box tests yielded an internal friction angle of 40° at this density. The variation of the angle of shearing resistance with density is shown in Fig. 3.5.

To check the efficacy of the manual laboratory compaction, a series of tests were carried out on the semicircular arch with a 100mm cover at the crown. A standard number of blows per quantity of fill were applied and the resulting spread of bulk densities over a set of ten tests was checked. The average bulk density was 1515kgm\(^{-3}\) with a range of 8.5kgm\(^{-3}\) or approximately ±0.3% of the average value. This indicated good repeatability of the bulk density and a sensible choice of
compaction method. The average value of $1515\text{kgm}^{-3}$ corresponds to the middle value used in the tests to assess the effects of bulk density upon the collapse load of the model. This was achieved in both series of tests by using 20 blows per 50mm fill depth per 200mm longitudinally.

The densities were measured by two methods: firstly the average mass of sand per scoop was estimated from tests on 100 scoops. The number of scoops needed to fill the model to a certain level, hence a certain fill volume was recorded. The bulk density was then calculated by dividing the mass of fill by its occupied volume. Alternatively, a more accurate measure of the mass of fill placed was obtained by weighing a bucket full of the fill and then placing sufficient fill to carry out the test. The remaining sand could then be weighed to give the mass used. The density was then calculated as before but based on a more accurate mass of fill used. This second method was used throughout after initial trials of the methods.

Such close correlation between independent test series, and such good repeatability need not be discussed further: suffice it to say that the method of fill pluviation, compaction, and density measurement are all acceptable.

The properties of the fill that were most important were those needed to classify and describe it, its angle of shearing resistance, its density - compactive effort relationship, its angle of shearing - density relationship, its strength, and its stiffness. The same fill type was to be used for the later, large scale tests described in Ch. 4 of this thesis. As these later tests were to be compared with finite element analyses, some accurate evaluation of the fill's elastic properties was needed. For the sake of economy of words and space, the fill tests and results have all been presented here.

The tests and descriptions contained herein conform to the relevant sections of BS812\textsuperscript{(119)}, BS1377\textsuperscript{(120)}, and BS5930\textsuperscript{(121)}. The fill was described as a light brown, medium sized, uniform, silica SAND. The particles were rounded with smooth surface texture. The fill contains only $5.8\%$ finer than $600\mu m$. This dropped to $2.9\%$ finer than $600\mu m$ after 2 months use in the test rig. This decrease may be explained by the loss, to atmosphere, of the dust and fine sand fraction caused by the bridge collapsing and the fill falling into the receiving tank beneath the test rig. Its uniformity coefficient was 1.59, making it a uniform material.
Approximately 5% by mass of sample was black, coal based particles, in the larger size range. No other organic material was present and testing with 5% HCl solution revealed the absence of limestone based particles.

The sand was tested for equilibrium moisture content. The following tests were carried out: fresh from the bag, after two weeks drying at 20°C and 60% relative humidity, and thereafter at six monthly intervals. The fresh moisture content, ex-bag, was 0.3% and this fell to 0.1% after two weeks drying in the laboratory. Thereafter, no test gave an equilibrium moisture content greater than 0.1%. From these tests it is apparent that the moisture content may effectively taken as zero, hence the bulk and dry densities being the same.

The results of density box tests on 0.156m³ samples were used to reveal the possible density variations for different blow counts with the hand held tamping rod. At each density, shear box tests were done to determine the variation of the angle of shearing resistance with bulk density. The results were shown earlier, in Fig. 3.5. The fill is observed to stiffen considerably under increased compactive effort as would intuitively be expected.

Triaxial tests on the dry sand were carried out to determine, more accurately, the shear strength parameters. Typical stress-strain plots, for various confining pressures, are shown in Fig. 3.6. The triaxial tests yielded the typical Mohr's circles of stress shown in Fig. 3.7.

From the stress-strain plots presented in Fig. 3.6, elastic moduli could be derived for the fill. In accordance with advice found in various soil mechanics textbooks(122,123), a secant modulus at half the peak stress was calculated. For the purposes of the finite element analyses discussed in Ch. 6 of this thesis, an elastic modulus of 10MPa was used. Theoretically it is possible to determine Poisson's ratio from the results of triaxial tests but the value obtained is widely variable depending on the stress range, or increment, over which it is calculated. For most sands the Poisson's ratio lies between 0.3 and 0.4. A value of 0.4 was chosen for future reference(124).

These results summarise the material used and the tests needed to determine certain material properties. The tests needed to clarify the behaviour of typical bridge fills may be found in published work by the author(3). The results presented, above, need
little discussion beyond statements to the effect that the fill is suitable, "well behaved", easily handled and controlled in a laboratory environment, and consistent with respect to the ease with which properties such as density and angle of shearing resistance can be controlled.

3.4 Construction and testing of the models

The construction sequence is shown in Figs 3.8 to 3.10. Fig. 3.8 shows the arch ring over its forming piece, prior to backfilling. Fig. 3.9 shows the partially filled haunches with the polythene film in place. Fig. 3.10 shows the model ready for testing with the platen in place.

The load was applied to the model using a counterbalanced loading lever above the fill surface, shown on Fig. 3.3. Holes were drilled in the lever providing ten load positions with the load added at a steady rate until failure.

3.4.1 Test parameters

A total of 148 tests were carried out with the following parameters;

a. A set of tests was repeated to check the repeatability of the collapse loads observed.

b. The first three tests were to establish that the end walls were sufficiently remote from the arch. The semicircular, steeper haunched, arch was used for these tests because the restraining effect was likely to be greater than for the flatter arch.

c. The next four tests on the semicircular arch were to determine the zones of fill displacement. A $d_c$ value of 45mm was used for these tests with four load points; $(x/r) = 0.0, -0.4, -0.75$ and -0.85.

d. The next 120 tests encompassed a parametric study on both the semicircular and span to rise ratio of four, arches. The variables used were fill depth at the crown $d_c$ and load position $(x/r)$. Six fill depths in conjunction with ten load positions
were tested for each arch profile, giving a total of 120 tests in the parametric study.

3.4.2 Methods of recording

Two methods of recording the movements of the arch and the fill were used throughout the tests. Still photography for both the arch/fill system and close-up details, at particular load levels, while a video camera recorded the complete test until collapse occurred.

The still photographs and the video recordings were used to deduce the zones of deformation of the soil-arch system. The former were processed as slides. These were alternated between two identical projectors placed adjacent to each other. Successive slides were aligned using the 12mm wire grid pattern on the glass side walls. Thus the loci of sand particles during the test could be traced and zones of fill and arch displacement identified. Slow motion running of the video was used to locate the hinge formation leading to collapse.

3.5 Results

This section will describe: the tests used to determine the repeatability of certain aspects of the models, the suitability of one critical dimension; namely the spacing between the springers and the end walls of the testing rig, the zones of fill and arch displacement, and the parametric study undertaken on the two profiles, one semicircular and one at a span to rise ratio of four. The effects of fill density and depth are also presented.

3.5.1 Repeatability of the collapse loads

The semicircular arch was filled to 6 different depths \(d_c\) and loaded at \((x/r) = -0.20\). Each test was carried out three times. The greatest spread of collapse loads occurred at \(d_c = 100\) mm and the values were found to have lie in the range, \(243N \pm 34N\), a spread of \(\pm 14\%\) about the mean using all 18 tests. This narrow range indicates an
acceptable degree of repeatability for the tests. As such these results need no further discussion.

3.5.2 The effect of end wall spacing

Three values of the distance, e, between end wall and springer, were tested. Plywood inserts were placed at distances of 100mm, 130mm and 160mm from the extrados at the springers. The semicircular arch was tested to collapse with the load at \((x/r)=-0.40\) and \(d_e=45\)mm. At the distances of 100mm, 130mm, and 160mm from the springer, the failure loads were 182N, 159N and 153N, respectively. The collapse load was stabilising around 150N, at just over 160mm from the springer. This distance of 160mm was adopted for all subsequent tests. The importance of this parameter will be discussed below.

The distance between the end walls and the springer did affect the collapse load of the arch. If the end walls were too close to the arch, an increased lateral pressure would have been exerted at the springings due to the greater degree of confinement in the fill. This increases the collapse load by preventing arch displacement.

In extreme cases the fill can become locked under the lateral stresses and can form "backing", which was added, at time of construction, as haunching to many arch bridges for extra strength. Such stiffer fill, or backing, can sustain the thrustline, enabling it to leave the arch ring. This also increases the collapse load. The study of this phenomenon was not an objective of this stage of the investigation, and so e was set to 160mm. The criterion for using this distance was finding a distance sufficiently large so that further increase in this distance had little effect on the collapse load.

3.5.3 Observation of the zones of fill and arch displacement

Zones of movement were obtained for each of the four load positions. The results are shown on a representation of the arch overlain with a 25mm grid in Figs 3.11 to 3.14. Squares within any contour show regions at which fill movement was observed at the corresponding stage, representing a percentage of the failure load for the particular test, given in Figs 3.11 to 3.14. As no geotechnical
instrumentation was built into the model tests, the pressures can not be stated with certainty. Speculation about the pressure changes is made possible by the judicious use of the observations of the arch's, and associated fill's, displacements.

3.5.3.1 Load at \((x/r)=0\); Fig. 3.11

Between \((x/r)=-1\), i.e. the springer, and hinge A no significant movement was observed. Fill pressures on this segment of the arch would be assumed to be the at-rest values. Above hinge A the fill was displaced radially outwards, corresponding to a rotation of segment AB about A, into the fill. The pressures here may be represented by some fraction of the full passive pressure. Between B and D the fill was displaced towards the arch. This movement was vertical beneath the load and became increasingly circumferential towards the hinges at B and D. This corresponds to movement of these segments about their instantaneous centres of rotation. The corresponding soil pressures behind these segments would tend towards the active pressure values, the displacements being sufficient to cause this pressure reduction. From D to E the displacement patterns are seen to be similar to those observed between A and B. From hinge E to \((x/r)=1\), at rest pressures would act because the arch displacement was negligible until \(0.8W\) had been applied (\(W\) being the collapse load for the system). As in the other load cases, the outer pair of extrados hinges did not form at the springers due to the confining effect of the fill. The fill pressure has the effect of reducing the effective span of the arch to the distance between the outer pair of hinges. This effect confirms observations made by Melbourne\(^{(125)}\), Smith\(^{(16)}\) and Crisfield\(^{(126)}\) in both analytical work and model tests.

3.5.3.2 Load at \((x/r)=-0.40\); Fig. 3.12

Here, as in the other load positions, the arch and the fill were not displaced between \((x/r)=-1\) and hinge A. Arch segment AB rotated about A, away from the fill which would tend to cause the fill pressures to fall towards the active state. Beneath the load point, fill displacements were predominantly vertical. These displacements became increasingly circumferential towards A and the crown of the arch at \((x/r)=0\). Segment BC's motion about its instantaneous centre of rotation was such that no fill or arch displacements were observed beyond the crown of the arch as far
as hinge C at \((x/r) = 0.10\). Segment CD rotated about D into the fill: this would tend to cause partial mobilisation of passive pressure. Fill displacement behind CD was not seen until a load of 0.46W had been applied, just before the hinge at C formed. The displacements increased substantially after this hinge formed. Below hinge D, little movement of the arch and fill is seen, and then only at high loads.

### 3.5.3.3 Load at \((x/r) = -0.75\); Fig. 3.13

The hinge pattern may be seen in Fig. 3.13 and the pattern of fill displacements was similar to that described above for a load at \((x/r) = -0.40\). The differences being that below the load point the displacements were larger due to the reduced confining effect of the arch, and the horizontal fill displacements towards the right hand edge of the segment BC were larger. This is due to the load being further from the arch. The fill displacements behind CD do not become significant until 0.51W has been applied. The fill displacements below the level of hinge D do not become significant until a larger fraction of W has been applied relative to that necessary to cause significant displacements in the test loaded at \((x/r) = -0.40\).

### 3.5.3.4 Load at \((x/r) = -0.85\); Fig. 3.14

The displacements for this load position were similar to those observed for a load at \((x/r) = -0.75\) with the same consequences of a reduction in confinement beneath the load being found. Significant displacement above the passive segment CD was not noted until 0.67W had been applied.

### 3.5.3.5 General remarks on the zones of fill and arch displacement

In general the fill behind the arch was not displaced horizontally due to the confining effect, and it would be inappropriate to assume, as in some analyses\(^76\), that the fill pressures act horizontally. The displacement contours show that the zones of movement around the arch do not significantly change shape during the test. However the displacements do increase in magnitude as the applied stress increases. The implication is that the zones of active and passive pressure assumed
for the collapse mode may also be applied to an elastic, or serviceability limit state, analysis.

As was expected, the span to rise ratio of four arch exhibited the same patterns of fill displacement as the semicircular arch. Due to the shallower fill depths required to cover the flatter arch the lateral earth pressures would be lower and the arch movements above the springers were consequently greater. The effect of this reduction in lateral restraint was observed in the hinge locations where in the flatter arch the outer hinges formed closer to the springers.

3.5.4 Parametric study

Each of the six fill depths and ten available load positions were tested to collapse for both the semicircular and flatter arches. In all tests bearing failure of the fill occurred before arch collapse. The downwards movement of the load platen into the fill was observed to be proportional to the fill depth between the load and the extrados. Figs 3.15 and 3.16 show the effects on the collapse load, W, of changing the load point and the fill depth beneath the load platen for both model arches.

The variation of collapse load with load position can be described as follows. The collapse load W peaked at the mid span and fell gradually until a minimum W occurred between \((x/r)=-0.30\) and \(-0.40\). This is consistent with the findings of other workers\(^{11,19,64}\). Beyond here, the value of W increased again as the load moved towards the end of the span. Had the load point been off the span itself it is supposed that this increase would have continued as the load moved further beyond the springer and a proportion of the applied stress was dispersed away from the arch.

The increases in collapse load with fill depth are shown for both arches and all load positions in Figs 3.15 and 3.16. The observed minimum collapse loads all occurred in the range \((x/r)=-0.30\) to \(-0.40\) but the curves were flatter than expected. This arose because of the behaviour of the fill material. The most critically loaded portion of the arch is not directly below the load point but slightly closer to the crown. There is some combination of depth and radial distance away from the centre of the load which gives a higher stress increase on the extrados than that
found at points directly below the load. This effect is more pronounced at load points away from \((x/r)=0\).

Further complications arise due to bearing failure beneath the platen which then moves into the fill, thus reducing the distance between the load point and the extrados. This has the effect of reducing the load dispersal through the fill thereby reducing the collapse load. This renders a test at \((x/r)=-0.75\), more similar to one at \((x/r)=-0.65\) in terms of the load distribution involved: therefore the observed minima are flatter than expected.

Evidence from full scale tests\(^{40,96}\) shows that bearing failure of the surface occurred before collapse. The load point then changes which must be accounted for in any analysis. Each full scale bridge exhibited the same trends throughout the tests but with different magnitudes involved.

### 3.5.4.1 Hinge positions

For each load position, irrespective of the fill depth used, the pattern of hinges formed at collapse was consistent to within one voussoir. Tables 1 and 2 give the hinge positions in relation to the load position for each arch.

<table>
<thead>
<tr>
<th>Load point, ((x/r))</th>
<th>(W_1)</th>
<th>((x/r)_1)</th>
<th>(W_2)</th>
<th>((x/r)_2)</th>
<th>(W_3)</th>
<th>((x/r)_3)</th>
<th>(W_4)</th>
<th>((x/r)_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>60</td>
<td>-0.73</td>
<td>89</td>
<td>-0.10</td>
<td>98</td>
<td>-0.94</td>
<td>100</td>
<td>0.36</td>
</tr>
<tr>
<td>-0.95</td>
<td>59</td>
<td>-0.69</td>
<td>76</td>
<td>-0.03</td>
<td>98</td>
<td>-0.93</td>
<td>100</td>
<td>0.36</td>
</tr>
<tr>
<td>-0.85</td>
<td>58</td>
<td>-0.64</td>
<td>78</td>
<td>0.03</td>
<td>98</td>
<td>-0.92</td>
<td>100</td>
<td>0.42</td>
</tr>
<tr>
<td>-0.75</td>
<td>45</td>
<td>-0.59</td>
<td>83</td>
<td>0.03</td>
<td>98</td>
<td>-0.84</td>
<td>100</td>
<td>0.48</td>
</tr>
<tr>
<td>-0.65</td>
<td>54</td>
<td>-0.53</td>
<td>83</td>
<td>0.03</td>
<td>99</td>
<td>-0.84</td>
<td>100</td>
<td>0.59</td>
</tr>
<tr>
<td>-0.55</td>
<td>53</td>
<td>-0.48</td>
<td>83</td>
<td>0.10</td>
<td>99</td>
<td>-0.73</td>
<td>100</td>
<td>0.64</td>
</tr>
<tr>
<td>-0.40</td>
<td>34</td>
<td>-0.30</td>
<td>78</td>
<td>0.10</td>
<td>99</td>
<td>-0.73</td>
<td>100</td>
<td>0.64</td>
</tr>
<tr>
<td>-0.30</td>
<td>50</td>
<td>-0.23</td>
<td>88</td>
<td>0.17</td>
<td>99</td>
<td>-0.69</td>
<td>100</td>
<td>0.69</td>
</tr>
<tr>
<td>-0.20</td>
<td>48</td>
<td>-0.10</td>
<td>80</td>
<td>0.17</td>
<td>99</td>
<td>-0.53</td>
<td>100</td>
<td>0.73</td>
</tr>
<tr>
<td>0.00</td>
<td>36</td>
<td>0.03</td>
<td>85</td>
<td>±0.20</td>
<td>100</td>
<td>±0.76</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>


The first hinge (B in Fig. 3.12) formed in the extrados beneath the load. The hinge formed almost underneath the load, but marginally closer to the crown. The second hinge (C in Fig. 3.12) formed in the intrados close to the crown. The third hinge (A in Fig. 3.12) formed in the intrados to the left of the load and the final hinge (D in Fig. 3.12) formed in the extrados on the other side of the mid span; this caused the collapse of the arch. The percentage of the collapse load $W$ at which each of the hinges formed was quantifiable and appeared to be repeatable to within $\pm 10\%$. This is also shown in Tables 1 and 2. For a load at $(x/r)=0$; a five hinged mechanism arose which may be seen in Fig. 3.11. This was predicted by Heyman(1959) and Pippard(11). The order of hinge formation, A, B, C, D, was exactly as predicted by Heyman and Pippard but the hinge positions did not correspond exactly to those predicted by them. The difference lay in the positioning of the pair of outer hinges; this facet of the failure mechanism can be explained in terms of the patterns of fill displacement and the corresponding pressures acting on the arch.

The arches failed by the formation of four hinged mechanisms with the exception of those loaded at $(x/r)=0$. The results showed that the first hinge formed, not directly beneath the load but marginally closer to the crown. This may be explained by considering the most critical combination of depth and radial distance away from the load point. The second hinge formed in the vicinity of the crown where the restraining soil pressure would tend to be minimised because this was where the minimum fill depth occurred. That the pressure restraining the arch is minimised...
explains why, regardless of load point, the second hinge formed here. The third and fourth hinges did not form at the springers but because of the tendency of the earth pressures to resist arch rotation they form at a level above them.

The percentages of $W$ at which the hinges formed was consistent for all tests. The first hinge formed at a small fraction of $W$ and the next hinge only formed at a much higher load. The remaining two hinges formed extremely rapidly at between 0.98$W$ and the collapse load. This is consistent with the location of the thrust line. The deviation of the thrust line required from the at-rest position to form two hinges is large but then the subsequent deviation needed to form the other two hinges is small as the thrust line already lies close to the extremities of the arch ring (16,17).

Heyman's collapse mechanism places the two outer hinges (A and D in Figs 3.12 to 3.14) at the springers. Due to the effects of the lateral earth pressures acting on the arch these two hinges did not form at the springers but closer to the crown in all cases. The lateral earth pressure will tend to its maximum value at the springers because this is where the fill depth is greatest. Where the fill depth is greatest, the vertical self-weight stress is maximised and so too is the lateral stress. As mentioned above it was this earth pressure that prevented hinge formation at the springers. Heyman's model uses only the fill's dead weight and it assumes no further interaction but the two inner hinges are similar in both Heyman's analysis and the model tests.

The position of the inner hinges is affected in the following manner: Heyman's load point hinge forms directly beneath the load but due to dispersal of the load through the fill the model tests have proved that, in practice, it forms marginally closer to the crown of the arch. The other inner hinge, C in Figs 3.12 to 3.14, was found to occur closer to the crown than predicted by Heyman's analysis. This is because the lateral fill pressures acting at greater depths are sufficiently large to prevent arch rotation and hinge formation. The findings of the model tests, with respect to hinge locations are in agreement with the work of Harvey and Smith (16,17).

3.5.5 The effect of fill density and depth upon the collapse load

The effect of fill density and fill depth have been combined into one section of results and discussion here. The two are inextricably linked; the single section
covering them is justified by the use of the common threads of load dispersal and self-weight stress increase, caused by either increased cover or increased density, and the ensuing contribution towards higher collapse loads.

The achievable density variations were found to be small given a uniformity coefficient for the fill material of 1.59. The uniformity coefficient is a standard term used in granulometrics to express the spread of particle sizes present in an aggregate sample. It is defined as the diameter at which 60% of the sample is finer divided by the diameter at which 10% is finer. The relevant diameters are obtained from the particle size distribution test results and they are the equivalent spherical diameters based on the British Standard test sieve aperture sizes. The larger the uniformity coefficient, the wider the range of particle sizes in the sample. By definition the minimum value of the uniformity coefficient is 1: a value of 1.59 indicates a uniform, or one-sized, sample.

The blow count was varied to change the density. Blow counts of 10, 20 and 40 per 50mm layer per 200mm longitudinally were used giving densities of 1495, 1515 and 1548 kg m\(^{-3}\) respectively. The load was applied at \((x/r) = -0.40\) with \(d_\text{c} = 45\text{mm}\). Collapse loads increased from 143N to 161N over this density range, an increase of 13\% for a 3\% density increase. The increase in collapse load, \(W\) is plotted against density in Fig. 3.17.

To separate the effects of increased dead load and increased live load dispersal, a series of tests was carried out with the load below the intrados. The load was applied using weights hung from the intrados at \((x/r) = -0.40\). The fill depths were increased and the increase in \(W\) was noted. This is shown in Fig. 3.18.

As no live load dispersal occurs for a load hung from the intrados, the difference between the increase in \(W\) for a surface load and an intrados load is due to increased load dispersal. The difference between the increase in \(W\) for a surface load (dependent on dead load and live load dispersal) and an intrados load (dependent on dead load) is due to load dispersal and not increased dead load arising from the increase in the \(d_\text{c}\) values; thus the effects of the two factors: dead load increase and load dispersal increase are separated. As \(d_\text{c}\) was increased from 10mm to 20mm the proportion of the ensuing increase in \(W\) due to the dead load was 40\%, giving a 60\% contribution from increased live load dispersal. As \(d_\text{c}\) was increased from 70mm to 100mm the dead load increase contributed 30\% of the increase in \(W\),
whilst the increase in live load dispersal contributed 70%. The increase in $W$ may be caused by increasing either $d_c$ or the fill density with the effects of increased dead load and increased live load dispersal being separated as indicated.

An increase in the fill density causes the collapse load to increase for two reasons: an increase in support stiffness and an increase in dead weight. The higher stiffness of the denser fill permits greater dispersal of the applied stress. The increased dead weight of the soil-arch system means that a larger live load is needed to deviate the thrust line sufficiently to form the collapse mechanism. The findings of this study bear these reasons out even for the small range of fill densities tested. The study in which the effects of increased dead load and increased live load dispersal were separated gave interesting results. As $d_c$ increased, the dead load increased but made only between 30% and 40% of the increase in collapse load. The live load dispersal made up between 60% and 70% of the increase in collapse load. This was as expected because the dead load increases linearly with $d_c$, whereas the live load dispersal is increased by a power generally greater than 1 as $d_c$ increases\(^{(123)}\).

Supporting evidence for this claim is presented as follows: for a constant cross sectional area of fill (as occurs over an arch backfilled to at least crown level) the volume of fill is directly proportional to the fill depth therefore the weight force exerted on the system by the fill is also directly proportional to the fill depth. A typical load dispersal formula is that of Boussinesq\(^{(123)}\):

$$\Delta \sigma_z = \left( \frac{Q I_p}{z^2} \right)$$

Eqn 2

Where:

- $\Delta \sigma_z$ is the vertical stress increase due to a point load on the surface,
- $Q$ is the applied point load on the surface,
- $I_p$ is the influence value for vertical stress increase and,
- $z$ is the depth through the fill to the point at which the vertical stress increase is required.

The influence value for points directly below the load is 0.478\(^{(123)}\) and assuming a unit point load on the fill surface, the vertical stress increase may be expressed as:

$$\Delta \sigma_z = \left( \frac{0.478}{z^2} \right)$$

Eqn 3
Because this represents the stress increase, the dispersal is obtained by subtracting this increase from the original applied load (unity in this case). The dispersal may then be expressed as:

\[ "Dispersal" = 1 - \left( \frac{0.478}{z^2} \right) \]  

Eqn 4

It can be seen that as the fill depth, \( z \) is increased the dispersal is increased by a power greater than one, i.e. faster than the increase in the dead load stress.

3.6 Comparisons with other methods of analysis

This section purports to demonstrate the differences between currently available analysis techniques and their attempts to model the small scale problems presented in this chapter. The methods used are: Heyman's plastic analysis\(^{(64)}\), Dundee University's ARCHIE program\(^{(16)}\), and MAFEA, the finite element package assembled by workers\(^{(27,28)}\) at the University of Nottingham and British Rail Research. Complete descriptions of these methods, with associated references, may be found in Ch. 2 of this thesis.

3.6.1 Heyman's plastic analysis

The tests were compared to a simple load dispersal, no soil support analysis given by Heyman\(^{(64)}\). This plastic analysis allows the fill to have dead weight but no inherent strength. The span to rise ratio of 4 arch was checked using this method and at \( d_e = 5 \)mm the analysis predicted \( W \) to be 38.4N compared with a test value of 151N. The discrepancy was exacerbated at larger \( d_e \) values. At \( d_e = 50 \)mm the analysis gave \( W \) as 69.3N compared with a test value of 300N. The span to rise ratio of four arch was used because Heyman's method is based upon the same geometry. It was therefore assumed that his analysis would return its best result for this configuration. By "best", the author implies that lying closest to the experimental collapse load.

These analytical collapse loads imply Heyman's plastic analysis is 75% lower than the true collapse load at \( d_e = 5 \)mm and 77% low when \( d_e = 50 \)mm. Such discrepancies are clearly unacceptable for the accurate analysis and assessment of
model arch tests. In the field where stiffer fills than the dry laboratory sand would be encountered, Heyman's analysis would, it is postulated, become even more conservative.

The discrepancies arise because of the omission of the effects of soil-structure interaction by Heyman in his method. No load dispersal is permitted and no lateral resistance to arch ring deformation is provided by the fill. Heyman's fill is therefore little more than a dense jelly which provides a self-weight contribution but no stiffness or strength. For a bare arch ring, Heyman's analysis is accurate; as is demonstrated in the assessment of the stone arches at Lincoln Cathedral\(^ {64}\).

3.6.2 The mechanism method: program ARCHIE

Dundee's mechanism analysis program was used to model the semicircular arch. A cover to the crown, \(d_c\) of 45mm was used with a line load at \((x/r)=-0.40\). The model tests indicated a collapse load of 153N for this configuration. ARCHIE produced a value for the collapse load of 105N, some 31% below the observed value. The passive pressure factor was set to 0.40 for the analysis. The remainder of the material properties were input from known laboratory test data.

The 31% difference between ARCHIE and the actual collapse load is acceptable for this type of problem. ARCHIE does not purport to be able to solve small scale model problems with all their associated material properties. During data input it was noted that arch properties were well outwith ranges stipulated in the program documentation. The values were typed in and the models analysed regardless. ARCHIE, as detailed elsewhere in this thesis, contains some of the effects of soil-structure interaction. Most importantly it models load dispersal and lateral pressure redistribution. The inclusion of these phenomena allied to the mechanism method has produced a solution closer to the experimental observations than a simple plastic analysis did.

A possible source of discrepancy between ARCHIE's collapse load and the observed value is the side wall friction in the model. This friction, between the fill and either the glass or the polythene film was not modelled in ARCHIE. Omission of this strengthening effect could, in part, lead to the 31% lower ARCHIE collapse load value.
3.6.3 A finite element analysis: program MAFEA

The MAFEA suite of programs was used to analyse the semicircular arch in the same configuration as described above for the ARCHIE analysis. MAFEA predicted a collapse load of between 80N and 90N. This equates to some 48% to 41% lower than the experimental value. Although worse than ARCHIE in its attempt to model this small scale problem, MAFEA was still a great improvement over Heyman's simpler plastic analysis. Again, like ARCHIE, MAFEA does not claim to be able to analyse small scale problems. The reasons for the discrepancy of between 48% and 41% are, given that MAFEA also incorporates its own model for load dispersal and lateral pressure redistribution, similar to those given for ARCHIE, above.

3.6.4 Comparisons with full scale tests

Two bridges\(^{40,96}\) tested by Edinburgh University for the Transport Research Laboratory were used for comparative purposes. Various scale factors were used to attempt to model the collapse loads found in these prototype arch bridges. The collapse load at Bridgemill was approximately 3100kN and that at Bargower was 5600kN. Using a scaling factor involving material densities and the linear dimension scale factor, collapse loads for the models (using the tests with the closest corresponding fill depths and load positions) were obtained. These were 722kN and 572kN for Bridgemill and Bargower respectively. The relationship between model and prototype collapse loads is given below:

\[
\left( \frac{W_p}{W} \right) = \left( \frac{\gamma_p\gamma}{\gamma_p} \right) L_r^2
\]

Eqn 5

Where: \( W_p \) and \( W \) are the prototype and model collapse loads respectively, \( \gamma_p \) and \( \gamma \) are the prototype and model arch densities respectively and, \( L_r \) is the linear dimension scale factor.

To allow for the omission of any pavement, wing, parapet and spandrel walls, and jointing material in the models tested, these scaled collapse loads were increased to 1581kN and 2677kN for Bridgemill and Bargower respectively. This increase is justified by reference to the work of Hendry \textit{et al.} on a variety of arch bridges\(^{40,96,127,128}\) which examined the individual contributions of each structural
component in the soil-arch system towards the overall load carrying capacity. These factored and scaled collapse loads represent \(0.51W_p\) and \(0.48W_p\) for Bridgemill and Bargower respectively. These are satisfactory results for models of this type which are primarily aimed at a study of the phenomena present in a soil-arch system rather than models of any particular bridge. These results, which draw together discussion of scale effects and comparisons with full scale tests, need not be discussed further.

3.7 Conclusions

1. Rapid, consistent testing of small scale model arches with sand backfill was achieved.

2. Soil-structure interaction effects contributed significantly to the load carrying capacity of such models.

3. The models were able to simulate two of the postulated soil-structure interaction effects: load dispersal and lateral earth pressure distribution.

4. The collapse load increased as the fill depth over the crown increased.

5. The increase in collapse load with increasing fill depth was made up of contributions from increased dead load and increased live load dispersal; the increase arising from the dispersal being predominant.

6. The collapse load increased as the fill density increased. This effect is linked with conclusion 5, as has been expanded upon in this chapter.

7. The minimum collapse loads were found to occur for load points at between \((x/r)=-0.30\) and \(-0.40\).

8. The hinge patterns leading to development of the ultimate limit state were consistent and repeatable both in terms of hinge locations and percentages of the collapse loads at which they formed.

9. Modelling of the tests was carried out using three other analyses: in order of increasing accuracy, with the associated percentage discrepancy in

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parenthesis, these were: Heyman's plastic analysis (75% to 77% low), MAFEA (41% to 48% low), and ARCHIE (31% low).

10. Scaling of the models to compare their collapse loads with those obtained in full scale field tests to destruction was carried out. Only the judicious use of scaling factors taking into account the omission of the various components of a complete arch bridge structure gave any semblance of sense to the resulting scaled collapse loads (48% and 51% of the observed failure loads at Bargower and Bridgemill respectively).

11. Many phenomena observed by other workers have been reproduced; the dissimilarities discovered have been accounted for in terms of the soil-structure interactions inherent in the soil-arch system.
Figure 3.1 Salient dimensions of test rig for semicircular arch

Figure 3.2 Salient dimensions of test rig for \((L/r) = 4\) arch
Figure 3.3  Loading system

Figure 3.4  Particle size distribution - model tests
Figure 3.5  Angle of shearing resistance *versus* dry unit weight

Figure 3.6  Stress - strain plots from triaxial compression tests
Figure 3.7  Mohr circles from triaxial compression tests

Figure 3.8  Model arch and formwork
Figure 3.9  Partially filled haunches

Figure 3.10  Complete model before testing
Figure 3.11  Fill displacements, load at \((x/r) = 0\)

Figure 3.12  Fill displacements, load at \((x/r) = -0.40\)
Figure 3.13  Fill displacements, load at $x/r = -0.75$

Figure 3.14  Fill displacements, load at $x/r = -0.85$
Figure 3.15  Collapse load versus fill depth for various load positions, \((L/r) = 2\)

Figure 3.16  Collapse load versus load position for various fill depths, \((L/r) = 4\)
Figure 3.17  Variation in collapse load with fill density

Figure 3.18  Variation in collapse load with fill depth
Chapter 4    Large Scale Model Arch Tests

4.1    Introduction

To verify and quantify the soil-structure interaction effects identified qualitatively by the author in section 1.5.1 and Ch. 3 of this thesis and by his co-workers at the University of Edinburgh\(^{(33,35,36)}\), a programme of instrumented tests on brickwork arches was developed. The scale was chosen to lie between the purely qualitative timber arch models and the full scale test *in-situ*. The results and principal findings from the tests may also be found in papers by the author\(^{(117,118)}\).

The objectives of this part of the investigation are: to measure geostatic stresses as the arch was being backfilled, to measure shear and normal stresses on the extrados under live load on the backfill surface at a variety of load points, and to measure shear and normal stresses on the extrados as the arch was taken to collapse with the load at the third span.

4.2    Model description

The arches were designed in-house for testing in the laboratory under controlled conditions. The span was chosen as a 2.00m semicircle because this profile was similar to that at Bargower and to those tested in the small scale tests of Ch. 3. Such a profile also gives rise to the greatest interactive effects. A thickness of half a brick was adopted for the arch ring. The salient dimensions of the soil-arch system can be seen in Fig. 4.1.

No spandrel, or wing walls were provided. This was to ensure the models were representative of a thin, two dimensional slice through the centreline of a bridge. Complications due to soil-spandrel interaction, and arch-spandrel interaction were thereby eliminated. The effects of the presence of spandrels upon the collapse load and behaviour of the models cannot be predicted, or separated as yet. It was felt necessary to reduce their effect in order to examine just the soil-arch interactions as would occur in the centre of a prototype arch bridge. Timber side walls were constructed to retain the fill and were independent of the arch. Light bracing with a
pair of diagonal rolled steel angle sections (60x60x5) was used for stability. To prevent loss of fill between the arch and the side walls, heavy duty polythene strips were lapped 0.1m over the extrados and 0.1m up the inside of the timber walls. These polythene strips were fixed to the timber side walls but not to the arch. This was to minimise edge effects and ensure that the movement of the arch and fill was independent of the walls of the testing tank. Concertina folding and ample lap lengths were provided, in the polythene strips, to allow movement whilst retaining the fill.

Timber end walls were set a sufficient distance back from the springers of the arch (Fig. 4.1) to avoid the fill becoming locked under lateral stress as the arch deformed under load. This distance was estimated by finite element analysis of the model arches, and with the end wall-springer distance tests on the small scale bridges described in Ch. 3 of this thesis. For stability the end walls were braced using a soldier and waler arrangement on their external faces.

The arch was built of grey Class B Engineering bricks set in a 1:1:6 cement: lime: sand mortar. Gauged joints were specified to avoid the need for rubbing bricks. A maximum joint thickness of 10mm was imposed on the intrados. The arch was set in English bond, half bricks being cut every second course for the facing of the arch ring. Crushing tests on the bricks revealed a characteristic strength, perpendicular to the likely line of thrust in the arch, of 42.2Nmm⁻². Cube crushing tests on the specified mortar mix revealed a cube strength of 3.6Nmm⁻² at 28 days. Fig. 1a of B.S. 5628(129) was used to infer a characteristic brickwork strength of 9.0Nmm⁻².

4.3 Fill properties

The fill material used was the same uniform, medium SAND, that was used for the tests described in Ch. 3 of this thesis. Density tests indicated a bulk density of 1515kgm⁻³ for the chosen placement method (see section 4.4). Shear box tests on the sand gave an angle of shearing resistance of 40° for this bulk density. These were backed up by large diameter triaxial tests which gave a marginally lower angle of shearing resistance for the same bulk density. The particle size distribution for the sand and further details are given in Ch. 3 of this thesis.
4.4 Construction of the model

Timber formwork was provided for the construction of the arch. Due to the light loads imposed upon the centring piece design focussed on accuracy of construction to a radius of 1.00m, rather than upon the structural design of the former. The formwork was struck after 28 days curing time had been allowed for the mortar in the voussoir joints to strengthen. This allowed the installation of instrumentation on the intrados prior to backfilling the arch. The dry silica sand fill was placed from zero drop height, by hand, and compacted with a hand held tamping rod, in 50mm layers. The photograph in Fig. 4.2 shows the test tank before instrumenting and backfilling.

4.5 Instrumentation

To enable quantative analysis to be made of the soil-structure interaction, instrumentation was incorporated into the tests. This will be discussed below, as follows: vibrating wire gauges (VWG's) on the end walls, stress transducers (ST's) on the extrados, and linear variable differential transducers (LVDT's) on the intrados. Finally the datalogging system and the instrumented loading system are included for reference purposes only. More details may be found in the appropriate manufacturer's literature which will be referenced accordingly.

4.5.1 End wall stresses: the VWG's

To confirm that the end walls were sufficiently far from the springers, VWG's were mounted on the wall's inner faces. It was expected that sufficient spacing of the walls from the springers would result in only small stress increases in the horizontal direction as the arch deformed under load. The VWG's were manufactured by Gage Technics. Their full stress range extended from zero to 500kPa in compression with excellent linearity of response over the working stress range, even at low stresses of only 5kPa.

The VWG's were calibrated using the apparatus shown in Fig. 4.3. The air/water cylinder supplied the pressure in the water bag to exert a uniform stress on the thin sand layer. The cell was exercised through many cycles of up to 250kPa to
eliminate hysteresis and any consequent non-linearity from the response. Once a linear, repeatable response was achieved, the cell was calibrated using the same datalogging system and connectors that were later to be used in the tests. A typical calibration chart for a VWG is shown in Fig. 4.4.

A simple correction compensating for the loss of stress due to dispersal through the thin sand layer and friction between the sand and the side walls of the pressure vessel was used to correct the applied stress at the active face of the cell for calibration purposes. All calibrations were carried out at 20°C±1°C eliminating the need for any temperature/drift correction in-situ because the tests were carried out under similar conditions.

The VWG's were mounted in cored holes through the end walls. The rear boss behind the cell projected out to allow cable runs to be easily accessed in the event of damage. The active face projected into the fill 15mm proud of the inner face of the end walls. The cell was packed by a stiff timber surround, details of which may be seen in Fig. 4.5.

Once installed, each VWG was connected to the datalogging system and subjected to a small stress to ensure electrical continuity and the absence of handling damage. Cell action factors were derived for the VWG's to calculate the percentage under or over registration. The method outlined by Hanna\(^{(130)}\) was used to derive a cell action factor, \(C_A\) of 1.19±0.01. This implies over registration by 19% ± 1% of the free field stress which would act if the cell were not present. The cell's estimate of the free field stress is then given by Eqn 6:

\[
\sigma_{ff} = \left( \frac{\sigma_{cell}}{C_A} \right)
\]  
Eqn 6

For the presentation of the test results all stresses output by the VWG's were corrected in accordance with Eqn 6.

4.5.2 Extrados stresses: the Cambridge ST's

To monitor the stresses on the extrados, Cambridge In-Situ's stress transducers\(^{(131)}\) were used. They were chosen because they could measure the stress normal to the
extrados, the stress tangential to the extrados and the eccentricity of the normal stress. They were extremely sensitive and at the same time robust enough to withstand the rigours of the collapse load test. As such they were ideally suited to the quantification of the soil-structure interaction effect in the soil-arch system. A full working stress range of zero to 500 kPa normal to the cell in compression and -250 kPa to +250 kPa in shear along the active face of the cell was sufficient to encompass the likely stress range in-situ. Both normal and shear stresses could be read to ±0.1 kPa. A schematic of an ST is shown in Fig. 4.6. Assembly and wiring of the ST's was carried out in-house, as was the fabrication of the cell housings and active faces.

Calibration of the cells was carried out by mounting each cell in the apparatus shown in Fig. 4.7. Normal stress was applied by adding slotted weights to the hanger shown, and shear stress was applied by loading the hanger attached to the wire running over the pulley at the end of the frame. Eccentricity of the normal load was achieved by moving the knife edge load set distances away from the centre of the cell's active face.

Each of the three output channels was exercised over its full working stress range to eliminate hysteresis. The ST's were checked, on all three output channels: normal, shear, and eccentricity, for cross-sensitivity and a matrix of calibration constants was derived. This matrix was a 3x3 array containing entries a_{11} to a_{33} inclusive. Designating channels 1, 2, and 3 as normal, shear, and eccentricity respectively, entry a_{11} represented the response on the normal stress channel to an applied normal stress, a_{22} the response on the shear stress channel to an applied shear, and a_{33} the response on the eccentricity channel to an applied eccentricity of load. Entry a_{ij} represented the response of channel i to external action j. The ST's were designed to render a_{ii}, 1 ≤ i ≤ 3, as large as possible whilst reducing all elements off the leading diagonal to almost zero. This was to reduce the cell's cross-sensitivity. The matrix of calibration constants was inverted and the stresses in-situ could be calculated as shown in Eqn 7:

\[
\begin{pmatrix}
\Delta \sigma \\
\Delta \tau \\
\Delta \varepsilon
\end{pmatrix} =
\begin{pmatrix}
\Delta N \\
\Delta S \\
\Delta E
\end{pmatrix}
\begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{pmatrix}
\]  

Eqn 7
Where: $\Delta N$, $\Delta S$, and $\Delta E$ were the changes in logged output readings from the normal, shear, and eccentricity channels respectively; $\Delta \sigma$, $\Delta \tau$, and $\Delta e$ are the calculated normal and shear stresses and the eccentricity in-situ.

Typical calibration charts, and the accompanying matrix $a_{ij}$, are shown in Fig. 4.8. for one of the ST's.

The ST's were mounted inside aluminium alloy housings surmounted by an active face machined from the same material. A 0.005" (0.127mm) feeler gauge was passed around each active face to ensure that it was free to deform without restraint from the cell housing. The active face was screwed onto the body of the ST and the upper surface was covered with a thin sheet of garnet sandpaper designed to roughen the surface to the consistency of the fill material. This was necessary to avoid under registration of shear stress due to the presence of the smooth machined alloy surface at the active face.

The complete cell, in its housing, was set into pre-cut pockets in the extrados. Cable connections were run through a 10mm diameter hole cut through the arch ring for ease of access. The surrounding gap between the housing and the brickwork was made good with dental plaster and sanded to a level flush with the surrounding extrados. Once the plaster had set a small stress was applied to the active face to ensure electrical continuity and the absence of handling damage.

No cell action factor, $C_A$ was necessary for the ST's as they sat flush with the extrados and did not form an inclusion in the free field which would alter the stress state. The active face deflections were also designed to be small thus causing little stress change away from the free field stress state. The stresses could be calculated, without correction, direct from Eqn 7.

For comparative purposes, a Kulite stress transducer was used to ensure that the stresses, as measured by two different types of instrument, were consistent within the error bounds and accuracies of the cells. This cell was chosen for its robust nature, its working stress range (zero to 700kPa in compression) and its accuracy ($\pm 0.2kPa$).

Calibration of the cell was undertaken in the same manner as that for the VWG's described in section 4.5.1. The cell was surrounded by a relatively stiff guard ring
that eliminated the need for additional housing. The cell was mounted in a pre-cut pocket on the extrados which was made good with dental plaster once the cell was in place.

Cell data and dimensions were used to derive a cell action factor, $C_A$ of 1.04 for the Kulite cell. This implied an under registration of 4% above the free field stress. The cell action factor was used (see Eqn 6) for the Kulite cell because, although it sat flush with the extrados, its active face was liable, under stress, to deflect substantially more than that of the ST's. Such deflections could cause stress changes away from the true free field stress state, thus necessitating the use of $C_A$, albeit low. Details of the mounting of the ST and the Kulite cell at the crown may be seen in Fig. 4.9.

### 4.5.3 Intrados displacements: the LVDT's

To measure arch deformations under load, ten pairs of LVDT's were mounted around the intrados. A mixture of long (100mm) and short (50mm) travel transducers were used. The transducers used a simple potentiometric circuit which varied the output voltage according to the displacement of the slider. They were linear over their working range with an accuracy of ±0.01mm.

The transducers were calibrated on a polished steel table, levelled by micrometer screws. Standard spacer blocks with protective slip covers were used to provide accurate displacements for which output voltages could be measured. The transducers were calibrated over their entire working ranges (50mm or 100mm) using the datalogging system to be used later in the arch tests. A typical set of calibration results is presented in Fig. 4.10.

The transducers were mounted on the intrados using wooden blocks with cold formed steel angle brackets to transmit the vertical and horizontal displacements to the sliders. Each pair of LVDT's was mounted on a scaffold frame which could be unclipped and withdrawn rapidly as collapse approached. This was necessary to avoid damaging the transducers as the arch failed but it meant that no displacements could be recorded during the final stages of the collapse load test.
4.5.4 Datalogging

The following datalogging systems were used for the calibration and testing of the instrumentation and throughout the test programme: a Gage Technics GT1169 D1 acoustic VWG reader for the VWG's on the end wall, a Microlink® 12 bit analogue to digital converter for the ST's and the Kulite cell on the extrados, and a Schlumberger Orion® datalogger for the LVDT's. The VWG readings were recorded manually as each gauge was scanned. The analogue to digital converter's output was written to diskette by the datalogging software developed in-house for the BBC Microcomputer. The output from the Orion system was sent direct to both a printer and a diskette. Care was taken to ensure that the same connectors, cables and equipment were used at all stages of the programme.

4.6 The bridge loading system

The arch models were constructed on the laboratory's strong floor to enable inclusion of the ground beams, jacks, and load platen. One ground beam, bolted to the strong floor, ran alongside each face of the arch, parallel to the span. Tension jacks were screwed onto vertical rods fixed at their lower ends to the underside of the top flanges of the ground beams. Two load cells were used, in series with each jack, to monitor the force applied by the jacks. At the top of each vertical rod two small diameter plates held a steel cross beam into position. The flexible strip footing, used as a platen to transmit a uniform stress to the fill's surface, sat beneath the cross beam. The loading system may be seen in Fig. 4.11.

The jacks were hand pumped from a single point to ensure equal force was applied by each one. As the tension in the jacks increased the cross beam was pulled down, reaction coming from the bolted ground beams. The compressive stress beneath the platen was thus increased. A digital voltmeter gave the force applied per jack when used in conjunction with its calibration chart.

The applied force per jack, $F_j$ was converted to an average applied stress, $q$ using Eqn 8:

$$q = \left( \frac{2F_j}{A_p} \right)$$  \hspace{1cm} \text{Eqn 8}
Where \( A_p \) is the surface area of the load platen: 0.180m by 1.72m in this case. Where needed for analysis of results or comparative purposes, the average applied stress, \( q \) was converted to an equivalent line load, \( \omega \) in kN/m using Eqn 9:

\[
\omega = qb_p
\]

Eqn 9

Where \( b_p \) is the breadth of the load platen (0.180m in this case). The load was applied at a steady rate such that \( q \) increased by 1kPa per minute.

By means of a summary, Fig. 4.12 shows the positions of the instruments and the sign conventions used throughout this Chapter.

4.7 Measurements made during backfilling

Knowing the dead load stress state in a model arch is useful for verification of later finite element analyses (see Ch. 6 of this thesis) and for verification of the GEOSIM analysis (see section 4.9.5.2) giving stresses on the extrados. Information of this nature could be easily incorporated into the source codes of analyses such as ARCHIE\(^{(16)}\), CTAP\(^{(21)}\), and MAFEA\(^{(29)}\), so that the initial stress state is both realistic and correct. This would remove some of the geotechnical simplifications, entirely justified thus far because of lack of quantitative geotechnical information, inherent in current methods of analysis and assessment.

There are two possible ways of examining live load stresses: they can be considered as a part of the total stress with the dead load stress, or as a change in stress from dead load values. These are shown in Eqns 10 and 11 respectively.

\[
\sigma = \sigma_{DL} + \Delta \sigma
\]

Eqn 10a

\[
\tau = \tau_{DL} + \Delta \tau
\]

Eqn 10b

Or:

\[
\Delta \sigma = \sigma_{LL}
\]

Eqn 11a

\[
\Delta \tau = \tau_{LL}
\]

Eqn 11b
Where: $\sigma$ is the total normal stress, composed of $\sigma_{DL}$, the dead load normal stress and $\Delta \sigma$, the live load normal stress read by the cell, and $\tau$ is the total shear stress, composed of $\tau_{DL}$, the dead load shear stress and $\Delta \tau$, the live load shear stress read by the cell. Whichever stress, total or live load only, is used will be clearly stated.

All instruments were scanned at regular intervals during the backfilling stage. Lifts of 50mm were used; "balanced" backfill depths were maintained on both left and right hand sides of the arch. No fill level was allowed to exceed another by more than 200mm. Table 3 details the backfilling process up to a crown cover of 150mm.

Table 3: Backfilling: fill levels (mm down from top of test tank)

<table>
<thead>
<tr>
<th>Stage</th>
<th>Left side/ mm</th>
<th>Right side/ mm</th>
<th>$d_{left}$/ mm</th>
<th>$d_{right}$/ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1292.5</td>
<td>-1292.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-1292.5</td>
<td>-800</td>
<td>0</td>
<td>492.5</td>
</tr>
<tr>
<td>3</td>
<td>-800</td>
<td>-800</td>
<td>492.5</td>
<td>492.5</td>
</tr>
<tr>
<td>4</td>
<td>-800</td>
<td>-700</td>
<td>492.5</td>
<td>592.5</td>
</tr>
<tr>
<td>5</td>
<td>-700</td>
<td>-700</td>
<td>592.5</td>
<td>592.5</td>
</tr>
<tr>
<td>6</td>
<td>-700</td>
<td>-600</td>
<td>592.5</td>
<td>692.5</td>
</tr>
<tr>
<td>7</td>
<td>-600</td>
<td>-600</td>
<td>692.5</td>
<td>692.5</td>
</tr>
<tr>
<td>8</td>
<td>-600</td>
<td>-400</td>
<td>692.5</td>
<td>892.5</td>
</tr>
<tr>
<td>9</td>
<td>-400</td>
<td>-400</td>
<td>892.5</td>
<td>892.5</td>
</tr>
<tr>
<td>10</td>
<td>-400</td>
<td>-200</td>
<td>892.5</td>
<td>1092.5</td>
</tr>
<tr>
<td>11</td>
<td>-200</td>
<td>-200</td>
<td>1092.5</td>
<td>1092.5</td>
</tr>
<tr>
<td>12</td>
<td>-150</td>
<td>-150</td>
<td>1142.5</td>
<td>1142.5</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>1292.5</td>
<td>1292.5</td>
</tr>
</tbody>
</table>

4.7.1 End wall stresses

All five VWG's held their zero readings until sufficient depth of fill had been placed above their centrelines to cause a change in stress. Generally the stress increases in the horizontal direction were linear with increasing fill depth above
each VWG. The results giving increases in horizontal stress versus fill depth may be seen in Fig. 4.13.

It is clear that the horizontal stress increase on the end walls is small and linear with depth. Where the stress increases are small the percentage error in any reading is relatively large as the absolute error remains the same over the entire working stress range of the VWG. The deviations from the general trend of linear increase in horizontal stress with depth should not be treated with suspicion. Suffice to say that the stress increases measured were sufficiently small to ensure the end effects were negligible.

Using the bulk density obtained in box tests (1515kgm$^{-3}$), approximate overburden pressures at the level of the centreline of each VWG were calculated. Earth pressure coefficients for the at-rest state, $K_0$, were derived by dividing the measured horizontal stress by the calculated vertical overburden stress. Values of $K_0$ lay in the range 0.18 to 0.58.

The $K_0$ values indicate some deviation from the ideal geostatic, plane strain situation. Comparison with Jáky's empirical value for $K_0$ given by Eqn 12 shows that the calculated values are of the correct order of magnitude.

$$K_0 = 1 - \sin \phi$$  \hspace{1cm} \text{Eqn 12}

For the fill used, $\phi=40^\circ$, giving $K_0$ values in the range 0.34 to 0.37. This range lies in the middle of that calculated from the estimated overburdens and the VWG readings.

### 4.7.2 Extrados stresses

All five ST's and the Kulite cell held their zero readings until fill over the active faces was in place. Once fill depths increased the normal and shear stresses on the extrados both began to increase. Initially the stresses were very low and some of the results are spurious because the relative error at low stress is large given a constant absolute error over the entire working stress range. Normal and shear stresses around the extrados are given in Fig. 4.14. The Kulite cell, measuring the normal stress at the crown, is represented by a single point on the graph.
The normal stress, as measured by the ST's is symmetrical about the crown at \((x/r) = 0\). The shear stress is rotationally symmetric about the origin because of the sign change for \(\tau\) from one side of the arch to the other in this dead load only case: as the shear stress, \(\tau\) changes direction, so it changes sign as shown on Fig. 4.12.

The normal stress at the crown, as measured by the Kulite cell, was within 5% of the corresponding ST's value for \(\sigma_{DL}\) when the fill depth over the crown reached 150mm. This demonstrates the accuracy and precision of the chosen instrumentation.

Finally the cells held their zero readings whilst the fill was being added, indicating that no "nipping" of the cells occurred. "Nipping" would imply that movement of the arch had squeezed the edges of the cells in their dental plaster pockets, giving an unknown initial prestress, dependent on the cell's cross-sensitivity, across the active faces of the cells.

### 4.7.3 Intrados displacements

As backfilling progressed small cyclical changes in the arch's profile were recorded by the intrados mounted LVDT's. When the arch was more deeply filled on one side, that side tended to move away from the fill towards the more lightly loaded side. Given the dry granular nature of the fill it is likely that the sand followed this movement, maintaining intimate contact with the extrados. When the fill was brought up to the same level on each side this sway was corrected. From the LVDT results this sway was less than \(\pm 0.5\)mm horizontally, as shown in Fig. 4.15.

No gross distortion of the arch profile was observed. The LVDT's on the springers gave displacements less than \(\pm 0.2\)mm horizontally and less than \(\pm 0.28\)mm vertically. By stage 13 of the backfilling the springers had settled by 0.15mm and 0.12mm vertically downwards on the left and right hand sides respectively due to bedding down and elastic compression. The horizontal movements of the springers by stage 13 indicated that the span had increased by 0.10mm over 2m, an increase of 0.01%.
The crown, by stage 13 had dropped by 0.42mm vertically downwards. It did not move horizontally beyond the aforementioned side sway which was always corrected as the fill was brought up to the same level on each side of the arch.

### 4.8 Live load tests

The testing programme then continued to examine the behaviour of the soil-arch system under live, or superimposed loading. The loading system detailed in section 4.6 was used for all the tests discussed here. Table 4 gives details of the testing programme.

<table>
<thead>
<tr>
<th>Test</th>
<th>Load point, (x/r)</th>
<th>No. of stages</th>
<th>Max. q/ kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.00</td>
<td>4</td>
<td>77.50</td>
</tr>
<tr>
<td>2</td>
<td>-0.75</td>
<td>4</td>
<td>75.56</td>
</tr>
<tr>
<td>3</td>
<td>-0.50</td>
<td>4</td>
<td>75.48</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>4</td>
<td>75.42</td>
</tr>
<tr>
<td>5</td>
<td>-0.33</td>
<td>9</td>
<td>115.2</td>
</tr>
</tbody>
</table>

Test 5 took the bridge to failure. Tests 1 to 4 are discussed here. Analysis of the observed stresses and displacements will be used to quantify and qualify the soil-structure interaction in backfilled arch bridges. Differences in the observed stress distributions and lateral earth pressure redistribution as the arch deforms under load will be analysed for each of the first four different load points.

#### 4.8.1 End wall stresses

Throughout each of tests 1 to 4 the VWG's registered small horizontal stress increases on the end walls. The most severe stress increases on the end walls were found to occur during test 1 where the load platen was centered on (x/r)=-1.00, *i.e.* over the left hand springer line. The largest horizontal stress increase was found to be 24.5kPa at VWG 5. This VWG position represents the most critical combination of depth and radial distance from the load platen, in terms of the imposed stress increase in the horizontal direction.
The most severe horizontal stress increase on the end wall remote from the loaded side of the span occurred during test 5, the test to destruction. This result will be discussed further in section 4.9.1. Whereas in tests 1 to 4 the most severe horizontal stress increase on the end wall remote from the loaded side of the span occurred during test 3, with a load centred over \((x/r)\)= -0.50. VWG 2 registered a horizontal stress increase of 9.8kPa. The side of the arch under the load tended to move away from the load whilst pushing the remote side of the span into the fill. This caused lateral earth pressure redistribution and the ensuing small horizontal stress increase at the end wall. This pattern of deformation was identical to that illustrated in Ch. 1 of this thesis which postulated the likely modes of soil-structure interaction in the soil-arch system.

The ST's results, discussed below in section 4.8.2, will be used to show that these small end wall stress increases are related to the changes observed in the stress state around the extrados of the arch. The LVDT results, discussed below in section 4.8.3, will also be used to prove that the stress changes may all be related to deformations of the arch under load.

No other VWG results warrant analysis because only small horizontal stress increases were measured. The two worst case results have been presented above: one for "direct" loading by the platen and the other for "indirect" loading as a result of arch deformation. The fact that the remainder of the VWG measured stresses were below these two maxima indicates that the end walls were far enough apart to prevent undue interaction between them and the arch.

### 4.8.2 Extrados stresses

The ST's were scanned at each stage for normal and shear stress readings in each test. The resulting total stresses on the extrados, both shear and normal, may be seen in Figs 4.16 to 4.31. Each of the four tests have four figures, giving the variation of normal and shear stress with load position and the average applied stress, \(q\), on the fill's surface. From these plots influence values for \(\Delta \sigma\) and \(\Delta \tau\) have been derived. These dimensionless values for \(\Delta \sigma\) and \(\Delta \tau\) were calculated by Eqn 13:

\[
I_\sigma = \left( \frac{\Delta \sigma}{q} \right) \tag{13a}
\]
The influence values are a means of determining the change in a quantity relative to the application of a unit amount of the same quantity elsewhere in the system. They may be used to derive normalised stress distributions around an arch of this geometry. Different geometries would obviously necessitate the derivation of different influence values. Provided the geometry of the soil-arch system remains unchanged, the influence values \( I_\sigma \) and \( I_\tau \), also remain unaffected by any change in scale.

The results of tests 1 to 4 will be presented and discussed test by test. Final discussion and analysis will draw them together for comparative purposes, before section 4.8.3 examines the arch deformations under these stresses. The results plotted are all live load values, \( \Delta \sigma \) and \( \Delta \tau \).

Fig. 4.16 shows the live load normal stress distribution for a load at the left hand springer; \((x/r) = -1.00\). As expected the stress increase at ST 1; \((x/r) = -0.62\) is the most severe as this point is closest to the load platen. The normal stress increases here (see Fig. 4.18) at a rate of 18kPa per 78kPa of applied stress. This would indicate an influence value for \( \sigma \) at \((x/r) = -0.62\) of 0.23.

The normal stress across the flatter, upper portion of the extrados remained constant throughout test 1. This indicates that little load was reaching the crown directly from the load platen due to dispersal effects in the fill. This will be discussed further in section 4.8.3.

Nowhere else is there a significant stress increase normal to the extrados until ST's 4 and 5, sited at \((x/r) = 0.35\) and 0.62 respectively, show increases of 2 and 3kPa respectively under an applied stress, \( q \) of 78kPa. This small stress increase is a result of the arch deforming outwards, away from its centre of curvature, into the fill. The resulting movement tended to mobilise partial passive pressures. The earth pressure coefficient did not deviate substantially from that found at-rest because the arch deflections were small. The deflections were small because at \((x/r) = -1.00\), the load was some considerable distance from the arch itself. For test 1 the dominant interaction was the dispersal of the surface applied stress, \( q \) through the fill.

\[
I_\tau = \left( \frac{\Delta \tau}{q} \right)
\]

Eqn 13b

\[ I_\tau = \left( \frac{\Delta \tau}{q} \right) \]
lateral earth pressure redistribution was minimal because of the small sidesway of the arch ring.

Examination of Fig. 4.17, showing the shear stress distribution around the extrados, indicates that the loaded side of the span underwent a decrease in shear stress whilst the side of the arch remote from the load underwent a similar small shear stress increase. These changes had, at their extremes, influence values of less than 0.10, on both sides of the arch. Geotechnics conventionally ignores stress changes outwith the "bulb of pressure"; the outer limit of the bulb of pressure is generally defined as the 0.10 influence value contour\(^{(123)}\). The explanation of the shear stress changes is as follows: on the loaded side of the arch the barrel is pushed downwards and sand fill slides over the extrados towards the left hand springer. This causes a shear in the right to left direction, hence the drop in the ST reading. On the opposite side of the arch the barrel is pushed outwards, into the fill. This mobilises some circumferential shearing resistance as the sand fill is pushed towards the right hand springer. Such displacement patterns were clearly identified in Ch. 3 of this thesis in tests to destruction on the small scale arch models. They also match the displaced shapes postulated in Ch. 1 of the thesis.

Fig. 4.19 clearly shows the effect these movements of the arch had upon the shear stresses around the extrados. Here the influence values are indicated by the slopes of the plots of stress versus applied stress, \(q\). The slopes are similar in magnitude but opposite in direction on opposite sides of the arch.

Figs 4.20 to 4.23 show the results from test 2 where the load was positioned over \((x/r) = -0.75\). The plots follow the same sequence as those from test 1. Fig. 4.20 shows the normal stress distribution around the extrados for the load platen over \((x/r) = -0.75\). As could be expected the ST located at \((x/r) = -0.62\) gave the largest normal stress increase. This arose because this ST was directly in the pressure bulb beneath the load platen. Fig. 4.22 gives a rate of normal stress increase of 33kPa per 76kPa of applied stress on the fill's surface. This corresponds to an influence value of 0.43. This represents an increase of 0.20 from the corresponding value from test 1. It must be noted that, as postulated in Ch. 1 and demonstrated qualitatively in Ch. 3, the most critical point on the extrados is not directly below the centreline of the load platen. It occurs at some other combination of depth and lateral distance along the extrados, closer to the crown.
The normal stress distribution has now encroached further onto the span compared with that found in test 1. The ST at the crown registered an increase in normal stress such that its influence value was 0.10: the least significant stress increase defining the limits of the pressure bulb beneath the load.

Although these results discussed so far for test 2 all come from the side of the arch under load no decrease in stress has been observed. A fall in stress, possibly towards the active state, may have been expected as the arch moved inwards and away from the fill. However, due to the imposition of the considerable surcharge, q, above the fill, no stress decrease was noted. The fill and the arch have therefore remained in intimate contact for the continued transfer of load across the interface. If such contact were broken, as may occur in a stiff cohesive fill, the extrados stresses would decrease.

On the side of the arch remote from the load, smaller changes in normal stress were noted. Where the arch ring rotated away from the fill the stress fell initially and thereafter remained constant, 8kPa below its at-rest value (see Fig. 4.20, ordinate for ST at (x/r)=0.35). Further round the extrados, at (x/r)=0.62 the normal stress rose by some 8.5kPa giving an influence value for the "indirect" normal stress increase of 0.11.

Figs 4.21 and 4.23 show changes in shear stress around the extrados during test 2. The trends and quantities involved are close to those observed for normal stress increase. The "direct" and "indirect" stress increase influence values were 0.40 and 0.10 respectively, marginally less than those found for the normal stress increases. The I of 0.40 represents an increase of greater than 0.30 from the corresponding peak in test 1. Over the crown the change in shear stress was small. This is because the shear stress at the crown is, in effect, a horizontal stress and also because the normal stress is small. Initially, the dead load shear is zero at the crown and substantial change to this value can only be caused by relative lateral motion at the crown. Small changes in the shear stress at the crown may be caused by vertical motion but these were not detected. The displacements causing the aforementioned stress changes in test 2 will be discussed in section 4.8.3.

The stress state around the extrados in test 3, with the load at (x/r)= -0.50, is presented in Figs 4.24 to 4.27. The normal stress distribution (Fig. 4.24) continues the trend of tests 1 and 2. It exhibits a peak normal stress of 38kPa at ST 2, situated
at \((x/r) = -0.35\). This occurs for an applied stress of 75kPa, giving an influence value for normal stress increase of 0.51. This is an increase in \(I_\sigma\) of 0.08 above the peak value found in test 2. The peak also occurs closer to the crown as the load point is shifted further onto the span. The most highly stressed point must also move correspondingly closer to the crown.

To the left of the load platen the stress falls rapidly: ST1, situated at \((x/r) = -0.62\) registered an \(I_\sigma\) of less than 0.10. This is because of the depth of fill above ST1. The pressure bulb moved, with the load platen, closer to the crown.

On the remote side of the arch the peak normal stress increase occurred at \((x/r) = 0.62\) giving an influence value for "indirect" stress increase of 0.10. Like test 2, this increase in normal stress took place almost immediately upon loading. Little further increase was observed thereafter, regardless of the applied stress.

Examination of the shear stress distributions around the extrados (see Fig. 4.25) shows that the peak shear stress increase took place at \((x/r) = -0.35\) on the loaded side of the span. The influence value here was 0.13. This was matched by a peak "indirect" influence value for the remote side of 0.12.

The \(I_\tau\) value under the load was less than that found for test 2 because the fill depth between the arch and the load platen decreases markedly as the load's centreline moves from \((x/r) = -0.75\) to -0.50. Classical stress distribution methods\(^{123}\) show that as the stress increase becomes predominantly vertical, there is less of an increase in other, non-vertical directions.

The recorded arch deformations can be used to explain the stress changes around the extrados. Relative movement of the interface between the soil fill and the arch mobilises circumferential shearing resistance. Correlation will be made between displacement and normal and shear stresses mobilised in section 4.8.3.

The stress state around the extrados in test 4, with the load at \((x/r) = 0.00\), the crown, is presented in Figs 4.28 to 4.31. The normal stress distribution is approximately symmetrical about the crown at \((x/r) = 0.00\). Any non-symmetry possibly arose from small errors in the initial placing and seating of the load platen over the crown and compaction induced anisotropy.
Peak influence values were noted as follows: beneath the platen, 0.85, and on either side; 0.12 to 0.14 at the haunches.

Figs 4.29 and 4.31 show shear stress increase influence values of 0.35 at the crown and less than 0.10 elsewhere. The shear stress distribution has rotational symmetry about the crown. The change in $\tau$ is negative on the right and positive on the left of the arch. The high $I_\tau$ value at the crown is perhaps surprising given that the load was directly above ST3. This will be analysed in section 4.8.3 when the deformations of the arch are discussed.

The experimental stress states around the extrados for a variety of load positions are presented in table 5. Peak influence values, $I_\sigma$ and $I_\tau$ are given for both the "direct" and "indirect" loading cases. $I_\sigma$ and $I_\tau$ values for "direct" loading arise from the proximity of the load platen whilst those for the "indirect" case stem from stresses mobilised as the arch deforms.

<table>
<thead>
<tr>
<th>Test</th>
<th>Load</th>
<th>$I_{\text{indirect}}$</th>
<th>$(x/r)$</th>
<th>$I_{\text{direct}}$</th>
<th>$(x/r)$</th>
<th>$I_{\text{indir}}$</th>
<th>$(x/r)$</th>
<th>$I_{\text{indir}}$</th>
<th>$(x/r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.00</td>
<td>0.23</td>
<td>0.62</td>
<td>&lt;0.1</td>
<td>0.62</td>
<td>&lt;0.1</td>
<td>0.62</td>
<td>&lt;0.1</td>
<td>0.62</td>
</tr>
<tr>
<td>2</td>
<td>-0.75</td>
<td>0.43</td>
<td>0.62</td>
<td>0.40</td>
<td>0.62</td>
<td>0.11</td>
<td>0.62</td>
<td>0.10</td>
<td>0.62</td>
</tr>
<tr>
<td>3</td>
<td>-0.50</td>
<td>0.51</td>
<td>0.35</td>
<td>0.13</td>
<td>0.35</td>
<td>0.10</td>
<td>0.35</td>
<td>0.12</td>
<td>0.62</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>0.85</td>
<td>0.00</td>
<td>0.39</td>
<td>0.00</td>
<td>0.14</td>
<td>±0.14</td>
<td>&lt;0.1</td>
<td>±0.62</td>
</tr>
</tbody>
</table>

The largest peak value for normal stress increase occurs when the load platen is at the crown. This is because, at the crown, where the depth of fill above the arch is minimised, the stress increase is greatest. This does not correspond to the weakest position for the arch, because of other geometrical factors(34) the collapse load would be lower in the vicinity of the third to quarter span ($x/r$=-0.33 to -0.50).

Table 5 gives the maximum contour, and limits of the pressure bulb as the load traverses the span. It is evident from all the above results that a considerable amount of dispersal occurs in the fill over the arch.

### 4.8.3 Intrados displacements

The arches all behaved as postulated in Ch. 1 and as observed in Ch. 3 of this thesis. The salient points from the LVDT results will be presented here test by test.
Fig. 4.32 gives the displaced shapes for each test. They are not drawn to scale and are all slightly exaggerated for clarity.

The displacements in test 1 are small, the lateral movement being less than 1.5mm at all LVDT positions. These small deformations explain the low stress changes observed. Where the displacement change is small the mobilised stresses are small, especially for a relatively weak dry granular fill material with a low soil modulus. Obviously, larger displacements cause larger stresses until a maximum friction angle is mobilised. On reaching this maximum friction angle, a dense granular soil shears and the particle interlock is overcome. Shearing then takes place at a lower stress than that needed to cause failure, but at a larger strain. Shearing then continues until the critical state is reached: here, the soil shears at constant volume and at constant mobilised friction angle(133).

Test 2 has been used to demonstrate the relationship between stress mobilised and the displacement causing the stress change. It was noted that, as the load was moved over the span for test 2, the displacements were larger than those from test 1. The displaced shape was basically similar but the outwards movements of the remote side of the arch were greater in test 2.

Fig. 4.33 correlates both normal and shear stresses with both horizontal and vertical displacements for test 2. A pair of LVDT's was mounted on the intrados opposite each ST mounted on the extrados. At \((x/r) = \pm 0.62\) and \(\pm 0.35\) the shear and normal stresses were dependent on each of the measured vertical and horizontal displacements. No one stress was dependent on purely one displacement. This is to be expected on a curved surface such as the extrados of an arch where normal stress occurs perpendicular, and shear stress tangential, to the interface. This interface, at the outer two pairs of ST's, is not orthogonal to the axes of Fig. 4.33.

At the crown, the shear stress could not be correlated with the vertical, or y-displacement. This is seen in the lower right hand portion of Fig. 4.33. The plot for ST3 shows no influence of displacement upon the shear stress at the crown. As discussed earlier, this could be expected as the shear at the crown, where the extrados is horizontal, would occur predominantly horizontally. Little correlation between vertical movement and horizontal stress was expected, or found.
Where the displacements were largest, close to the load at ST's 1 and 2 in this test, the mobilised stresses were largest. Where the stress changes were small, the displacements were also small. Such accurate correlation is useful for the quantification of the interactive nature of the arch analysis problem.

Knowing the relationship between displacement and stress helps the analyst assess the likely stress distribution currently acting behind an arch. With displacement monitoring, the stresses changes could be predicted and used for subsequent arch bridge capacity analyses.

For comparison with classical geotechnical engineering theory, Fig. 4.34 was produced. This shows the percentage of the classical Rankine earth pressure states \(^{(123)}\) mobilised for certain displacements. Where the normal stress on the extrados drops below the at-rest value, partial mobilisation of active pressure occurs. Where \(\sigma\) increases, the passive state is being approached. The calculations for Fig. 4.34 are based on displacements normal and tangential to the extrados, corresponding to the normal and shear stresses.

It may be seen that the active state is mobilised at small displacements. Full active pressure is mobilised for displacements of approximately twice \(D_{10}\), the effective grain size of the sand. If the initial stress, because of load, or compaction induced anisotropy, is above that required for Rankine's at-rest state, there is then more scope for stress decrease. If the stress were initially \(x\) kPa above the at-rest pressures there would then be scope for an extra \(x\) kPa stress decrease whilst the stress decrease from at-rest state required to mobilise the active state remains unchanged.

Sokolovskii's coefficients \(^{(134)}\) are available as an alternative means of determining active and passive states. However, Sokolovskii's method is open to criticism \(^{(63)}\) for its inability to handle mixed state problems where one zone of fill is in a state of plastic equilibrium and others retain their elastic equilibrium. Therefore it is not used here. Neither Coulomb's method \(^{(47)}\) for earth pressure on walls nor Culmann's graphical construction \(^{(63)}\) are used because these methods are relatively impractical.

The active state, once fully mobilised, continues to govern the soil fill's behaviour. The distinct peak at 150% of the Rankine value indicates the point at which particle interlock is overcome and shearing takes place beyond this peak at constant volume. This is indicated by the flat portion of the graph having an ordinate of
approximately 100% beyond resultant displacements of 1mm (approx. 10 times $D_{10}$).

On the passive side of the graph (the upper right hand quadrant) there is considerable stress mobilised for only small displacements of the arch. What is surprising is that 100% of the Rankine passive pressure coefficient is mobilised. It has long been held true that the passive state is never fully mobilised behind the arch\(^{(16)}\). This is true for the segment on the side of the arch remote from the load platen. It may be seen (Figs 4.32 and 4.33) that the displacements on the right hand side of the arch are smaller than those beneath the load platen. These points correspond to the lower displacements in the upper right quadrant of Fig. 4.34. Here no more than 40% of full passive pressure is mobilised at a displacement of 0.5mm. There is a dearth of points on the passive side of the plot until around 82% of the Rankine value. This, and the cluster of points around 100% of the Rankine value, derives from points on the left hand, loaded, side of the span. Here the arch is moving away from the load but because of the surcharge stress, q on the fill's surface the stress actually increases. This is not a classical, "wall being pushed into fill" passive state situation. This is purely the effect of stress dispersal through the fill. The 40% value represents an acceptable limit to the pressure mobilisation which should be allowed on the remote, "passive" side of an arch.

The remaining two tests need no further discussion beyond that required to explain the anomalous crown movement in test 4. Here the load was directly above the crown yet the crown appeared to rise after stage 3 of the loading process. Initially it dropped, as would be expected, under load: after stage 3 the crown pair of LVDT's indicated a small horizontal displacement and some recovery of the initial vertical movement. The reason for this is not known. The stress changes did not match this displacement in any way that could shed light on this behaviour. This anomaly did not, as far as can be ascertained, appear to affect the remainder of the test. This concludes the analysis of the results from tests 1 to 4.

4.9 Test to destruction

Test 5 of the sequence used a load at $(x/r)=-0.33$ to take the arch to failure. The test conditions of 1 to 4 apply to test 5. The same instrumentation was used with the exception of the LVDT's at high loads. The LVDT's were removed before collapse.
to avoid damaging them. There are therefore no displacement readings for the final two stages of test 5. This section examines the results from the VWG's, ST's, and LVDT's. It then discusses the observed failure mode and compares collapse load values with other current methods of assessment. Analyses giving extrados stresses are provided for comparative purposes.

4.9.1 End wall stresses

These warrant little discussion as they were everywhere smaller than the observed maxima from tests 1 to 4. Continuing the trend of previous tests, the horizontal stress increase was larger on the end wall closest to the load platen. It was smaller on the remote side of the span. Even at the much higher stresses used in this test (up to 115.2kPa compared to 76kPa previous to this) there were no significant increases in the VWG readings. This is because the arch displacements were correspondingly larger, thus causing stress relief at the end walls as the fill moved with the arch. Again the VWG readings prove that the end walls were far enough apart to prevent interaction.

4.9.2 Extrados stresses

Figs 4.35 to 4.38 show the stress state around the extrados. Table 6 gives the loading stages for test 5, the load rate was kept constant until gross deformation of the arch caused a drop in q after stage 6. A set of readings was taken at the lower applied stress before pumping the jacks again. This set of readings is included as stage 7.

It may be seen that the normal stress distribution is similar to that found for tests 1 to 4. There is a peak of normal stress closer to the crown than the load platen, a relatively flat distribution over the crown, and a smaller peak of normal stress as the arch gets pushed outwards into the fill on the side of the arch remote from the load.
ST2 gives the largest response (see Fig. 4.37) which is to be expected for a load at \((x/r)=-0.33\). However the influence value was not as high as that found during test 3. The peak normal stress increase for a load at \((x/r)=-0.33\) would occur closer to the crown than the load platen, as happened in tests 1 to 4. The absence of any other instrumentation on the extrados between \((x/r)=-0.35\) and the crown means that the pressures can only be speculated on, not stated with certainty. Examination of the finite element analysis used in Ch. 6 to derive stresses on the extrados reveals a peak influence factor for normal stress increase of 0.79 at \((x/r)=-0.23\). This is in accordance with the first four entries in Table 5 of this chapter. Table 5 has been modified to include the test 5 results for peak influence factors. This appears as Table 7, below.

The stress normal to the arch increases at higher stresses, \(q\) but the influence value decreases. The normal and shear stresses are no longer increasing at the same rate as the applied stress, \(q\). This lag effect is due to the arch's deformation pattern
under high load. This will be discussed further in section 4.9.3. As the arch deforms, stress relief occurs which is compensated for by the steady increase in q. The stress relief is caused by the arch moving inwards, away from the fill and this causes the influence value to decrease.

Significantly, on the side of the arch remote from the load platen, the influence values for both normal and shear stresses increase with applied stress, q. The reasoning is exactly the opposite to that applied above. Here, at $(x/r) > 0$, the arch is being pushed outwards into the fill and the pressures are increasing as the displacement increases. As this is a stress increase situation, rather than a stress relief situation with the associated lag effects described above, the influence values can keep up with the increase in q. They increase because of the increasing displacement of the arch into the fill. This will be discussed further in section 4.9.3.

Between stages 7 and 8 the applied stress dropped, either because of leakage in the hydraulic lines feeding the loading jacks, or because of movement of the arch, or a combination of these factors. The instruments were scanned and small stress decreases were noted before the load was pumped back to slightly above that acting prior to the decrease.

When stage 9 loading was reached the arch took no more stress and collapse occurred. A final scan of the ST's showed the stress dropping rapidly as the arch underwent gross deformations immediately before collapsing. The maximum reading on the loading jacks indicated that collapse occurred at an average applied stress, q of 115.2kPa. Little importance should be attached to the stress state on the extrados at stage 9 as collapse was imminent. The maximum stresses should perhaps be read from the plots for stage 8, these being the last set of results before gross deformations affected the stress field around the arch.

4.9.3 Intrados displacements

The displacements of the intrados up to stage 7 of test 5 are shown in Fig. 4.39. The results from the LVDT's show the typical sidesway movement of the arch under load. The left hand side moves inwards and down whereas the right hand side moves outwards and up. The movements were, in the later stages accompanied by hinging.
The crown was displaced to the right, away from the load by 1.5mm horizontally at the most. As shown in Fig. 4.38, ST3 at the crown, did not register any significant change in shear during the test. A small change in normal stress was observed at the crown (see ST3, Fig. 4.37) consistent with the 2.5mm vertically downwards displacement.

Where the arch has moved into the fill may clearly be seen in Fig. 4.39. These displacements correspond to the normal and shear stress increases seen in Figs 4.35 and 4.36 for ST's 4 and 5 on the right hand side of the arch. The displacements were eventually large enough to cause heave at the fill surface above the right hand side of the span. The heave on the fill's surface must have allowed either some stress relief or increased arch displacement. Were a stiff road pavement present this heave would not occur until a much higher stress had been applied. The fact that, on the "passive" side of the arch the stresses are increasing with q, as discussed previously, suggests that the displacements are being allowed to increase because of the absence of a road pavement over the fill rather than stress relief effects occurring for the same reason.

Errors will be present in all LVDT results for stages where the displacements were large. This is largely due to rotational effects. Where the arch hinges, translations in the horizontal and vertical directions are accompanied by substantial rotations. The LVDT's on their scaffold frame measure displacements in a fixed plane of reference: horizontally and vertically. They continue to do so independent of the rotation of the arch. As such an LVDT could fail to register any displacement if its point of contact with the mounting bracket on the intrados underwent purely rotational movement. The point of contact would slide relative to the probe on the LVDT without displacing the sliding potentiometer arm. Video recordings taken during the test have been used to confirm the accuracy of the displaced shapes presented in Fig. 4.39 on a qualitative basis only. Without extensive image processing the video record of the tests is not suitable for quantitative analysis of deformations.

4.9.4 Failure mode

The arch failed once a four hinged mechanism had developed. Initial loading causes elastic compressive strain in the arch ring. This strain shortens the arch enough to
allow slight rotation to occur. This rotation is accompanied by cracking of the mortar joint or by bond failure at the mortar/vousoir interface. This rotation and cracking is known as a hinge. Once four hinges have formed, alternating between the extrados and intrados, the arch becomes unstable and collapses. The hinge positions and the applied stresses, \( q \) causing their formation are presented in Table 8 below.

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Location</th>
<th>( q ) Kpa</th>
<th>( (q/ W) ) %</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.30</td>
<td>34.6</td>
<td>30</td>
<td>Extrados</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
<td>64.5</td>
<td>56</td>
<td>Intrados</td>
</tr>
<tr>
<td>3</td>
<td>-0.64</td>
<td>99.0</td>
<td>86</td>
<td>Intrados</td>
</tr>
<tr>
<td>4</td>
<td>0.69</td>
<td>115.2</td>
<td>100</td>
<td>Extrados</td>
</tr>
</tbody>
</table>

The directions of rotation of each segment between hinges are shown in Fig. 4.40. The measured displacements are seen to be consistent with the hinges in test 5. It should also be noted that the percentages of the collapse load, \( W \), at which the hinges formed is consistent with the percentages observed in tests on the small scale model arches (see Ch. 3 of this thesis). The hinge locations of test 5 are also consistent with those seen in tests to collapse on small scale arches.

4.9.5 Comparisons with other analyses

Comparisons will be made, using commercially available software where possible, between the actual collapse load and the assessed collapse load. Two hand calculated values of collapse load are obtained from the MEXE method and Heyman's plastic method of analysis. A method derived by the author for calculating the extrados pressures under both dead and live loading is presented. Comparisons are drawn between the measured and calculated extrados stresses.

4.9.5.1 Analyses giving collapse loads

The methods used are tabulated, in order of decreasing accuracy, in Table 9. All methods gave collapse loads below the experimental value.
Table 9 Collapse load comparisons, 2m span arch

<table>
<thead>
<tr>
<th>Method</th>
<th>$\omega$/kNm$^{-1}$</th>
<th>$(22.1-\omega)/$ kNm$^{-1}$</th>
<th>% error</th>
<th>$(\omega/22.1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEST 5</td>
<td>22.1</td>
<td>0</td>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>MAFEA</td>
<td>21.4</td>
<td>0.7</td>
<td>-3</td>
<td>0.97</td>
</tr>
<tr>
<td>ARCHIE</td>
<td>17.9</td>
<td>4.2</td>
<td>-19</td>
<td>0.81</td>
</tr>
<tr>
<td>MARCH</td>
<td>15.0</td>
<td>7.1</td>
<td>-32</td>
<td>0.68</td>
</tr>
<tr>
<td>Cascade</td>
<td>13.0</td>
<td>9.1</td>
<td>-41</td>
<td>0.59</td>
</tr>
<tr>
<td>Heyman</td>
<td>8.56</td>
<td>13.54</td>
<td>-61</td>
<td>0.39</td>
</tr>
<tr>
<td>MEXE</td>
<td>7.77</td>
<td>14.33</td>
<td>-65</td>
<td>0.35</td>
</tr>
</tbody>
</table>

The average applied stress, $q$, has been converted to the equivalent line load using Eqn 9. The difference between experiment and assessment is expressed in both absolute and relative terms in columns 3 and 5 respectively. The methods used are described in detail in Ch. 2 of this thesis with the exception of the Cascade software. Cascade is a mechanism method based analysis with little interactive effects incorporated beyond a simple soil stress distribution model.

British Rail and Nottingham University's MAFEA suite gave the closest prediction of $\omega$, their result being only 3% below the experimental value. This represents an excellent prediction from which safety factors could be applied economically. The second best answer was achieved by Dundee University's ARCHIE program. This gave a 17% discrepancy; again an excellent prediction of $\omega$ from which to apply the relevant safety factors. These two methods represent the only acceptable results, admittedly in the absence of CTAP, to which the author did not have access.

The MAFEA and ARCHIE methods incorporate the basic soil-structure interaction effects, as described in Ch. 2 of the thesis. They both cater, to a certain extent, for stress dispersal and lateral earth pressure redistribution. The remainder of the methods take little account of the interactive effects postulated by the author: the MEXE method ignores the presence of the fill and Heyman's plastic method gives it dead weight only. The latter would be equivalent to placing a dense jelly with no shear strength or stiffness over the arch.

For these reasons the remaining methods do not produce acceptable results for this arch model. Their suitability may be improved for the analysis of different arch geometries. Such a study is outwith the scope of this present thesis but a partial, and
confidential, study using MEXE, ARCHIE, an older version of MAFEA, and CTAP has been undertaken by the Department of Transport. This indicates the sensitivity of each method to certain parameter changes for a variety of tests to destruction.

It must be stated that the MEXE result is very subjective. A condition factor of 0.3 was used to modify the provisional axle load. This was to account for the absence of spandrels, wing walls, parapets, and road pavement strata. A condition factor of 0.9 would produce a collapse line load, $\omega$ of 23.3kNm$^{-1}$. Such an answer is clearly unsafe as it exceeds the actual collapse load by some 5.5%. Also unacceptable is the wide possible variation in collapse loads caused by permissible variations in only one MEXE modification factor. Given that the collapse load is to be assessed by "eye" and "experience" such a wide spread of collapse loads is perhaps to be expected from the MEXE method.

The thrust of this discussion leads to the conclusion that MAFEA and ARCHIE remain acceptable for the analysis of test 5 because of their inclusion of basic interactive effects. Less sophisticated methods were unacceptable, would have been uneconomical in-situ, and were no easier to apply than MAFEA or ARCHIE.

4.9.5.2 Analyses giving extrados stresses

This section presents the results of a method$^{(135)}$ derived by the author and his colleagues at the University of Edinburgh. The program, called GEOSIM, runs as a spreadsheet with added macro routines in Microsoft's Excel version 4.0®. The extrados is divided into 200 equal intervals covering the span. The stresses on each segment are then calculated. Mohr's circle of stress is used to calculate the normal and shear stress changes induced by the horizontal and vertical stress state. The program calculates influence values for stress increase by various methods: Boussinesq$^{(123)}$, Poulos$^{(136)}$, and BD21/93$^{(14)}$. These influence values are used to calculate the live load normal and shear stress state around the extrados at each of the 200 points. The results may be compared to the measured stresses.

The dead load stress state is derived as shown in Fig. 4.41. The dead load vertical stress is assumed to be geostatic and equal to depth multiplied by bulk unit weight. The dead load horizontal stress is assumed to be $(1-\sin \phi)$ multiplied by that acting
vertically. Mohr's circles are then used to calculate the stresses normal and tangential to the extrados. The results of a parametric study using GEOSIM to examine the dead load stresses are presented in Fig. 4.42.

The higher the fill’s bulk unit weight, the larger the extrados stresses. The fill depth variation produces the same effect. The variation of stress with $\phi$, the angle of shearing resistance of the fill is also shown. The higher the $\phi$ value the lower the normal stress and the higher the shear stress. A higher $\phi$ value indicates a fill better able to support itself through internal friction. Less load transfer then occurs normal to the extrados.

The limits to the curves are also of interest: the normal stress at the crown is simply the depth multiplied by bulk unit weight whilst that at the springers is the equivalent free field, geostatic horizontal stress. The plots of shear stress show no shear at the crown under dead load only and stresses tending to zero at the springers. These zero values are to be expected as the analysis assumes major and minor principal planes orthogonal, and mutually perpendicular to the coordinate axes. The shear stress on either principal plane is, by definition, zero.

The effects of introducing a linear density gradient with depth through the fill are also shown in Fig. 4.42. These effects are seen to be small and need not be discussed further. They were checked and processed to enable the elimination of the effects of non-homogeneity from the analysis.

The measured dead load stress state from the model arch tests is superimposed upon the GEOSIM stress state in Fig. 4.43. Fig. 4.14 has been used to provide the test results from stage 13 of the backfilling process. GEOSIM is seen to give an accurate prediction of the dead load stress state measured after completion of the backfilling process. The differences between theoretical and measured are small enough to assume the results in Fig. 4.42 are accurate. The use of GEOSIM is to be recommended for analysis of the dead load stress state around an arch. The differences are greatest at the haunches of the arch where the effects of the relatively rigid arch upon the stress field would be expected to be greatest.

The use of GEOSIM for the live load stresses is demonstrated in Fig. 4.44. Test 5 is analysed using Boussinesq, Poulos and BD21/93 influence values. The
experimental results at stages 1 and 4 are superimposed for comparative purposes upon the GEOSIM results at the same stages.

The normal stress in the vicinity of the load platen is modelled most accurately by the Boussinesq analysis. The BD21/93 codified stress distribution severely overestimates the stress applied to the extrados as does the Poulos method incorporating the effects of the rough rigid arch boundary. On the side of the arch remote from the load platen none of the GEOSIM calculations model the experimental stress state for stages 1 to 4.

The reasons for the similarities are perhaps fortuitous: that Boussinesq's method for isotropic, homogeneous, elastic, semi-infinite half masses should model the peak normal stress beneath the load platen is surprising. The presence of the arch itself violates the principal assumptions behind Boussinesq's method. However; the results indicate that at low load levels, Boussinesq can predict the peak applied stress for this geometry.

The Poulos distribution is essentially the same as Boussinesq's with the addition of a concentration factor giving higher stresses at the interface between fill and arch. The concentration factor generally increases the stress by a factor of 1.4. This increase is obviously too severe for the arch tested here. The arch is not perfectly rigid: it deforms away from the load allowing some stress relief (which will be offset by the steady increase in surcharge loading on the fill's surface). This is why Poulos overestimates the stress on the extrados. BD21/93 overestimates grossly as it is simplistic but necessarily conservative until information such as that above is translated into current practice.

None of the elastic methods described above can model the stress changes observed on the remote side of the span. The calculated distributions produce no direct stress increase at such large horizontal distances away from the load platen. They do not model the arch deformations therefore cannot recreate the partial passive pressure mobilisation observed in test 5. The Boussinesq and Poulos analyses are highly sensitive to Poisson's ratio for the fill and as such the derived horizontal stress distributions spread over an unrealistic horizontal distance. The distributions have been curtailed at an influence value of 0.10 to avoid this directional sensitivity. These stresses may be used as input to an ARCHIE type mechanism analysis or as the "standard" stress distribution option in the MAFEA suite. Inclusion of such
parameters is not possible without access to the relevant source codes but it is hoped that the inclusion of such information would result in more accurate and economical arch bridge assessment.

4.10 Conclusions

1. Large scale model tests on instrumented arches have been successfully completed.

2. The chosen instrumentation functioned adequately for the purposes and could be used again with ease.

3. The chosen datalogging and post-processing systems all functioned well throughout the test sequence.

4. Dead load stress states were measured around the extrados: these compared favourably with theoretical predictions based on assumed principal planes and geostatic stresses in the fill.

5. Live load stress distributions were identified: these were correlated with the measured displacements of the arch ring. Soil-structure interaction effects such as stress dispersal and lateral earth pressure redistribution were quantified. Peak stress normal to the arch occurred for live loading over the crown. Influence values of 0.85 were found for normal stress. Partial mobilisation of passive pressures occurred on the remote side of the span. No more than 40% of the Rankine passive pressure was mobilised. This was caused by the movement of the arch into the fill and the subsequent redistribution of the lateral earth pressures.

6. Significant shear stresses were measured around the extrados: these tended to resist arch movement. The shear stresses were correlated with the deflections of the arch ring to give typical "design" stress distributions under live loading at a variety of load points from springer line to crown.

7. A test to collapse was carried out with loading at \( (x/r) = 0.33 \). The arch failed in a four hinged mechanism at an average applied stress of 115.2kPa.
8. Assessment methods MAFEA and ARCHIE gave excellent predictions of the collapse load (3% and 17% low respectively). Methods ignoring or simplifying the interactive effects gave unacceptable results.

9. Theoretical analyses were carried out to compare with the experimental results: at lower loads, before the gross deformations associated with failure could occur, Boussinesq's method gave good predictions of the peak normal stress on the loaded side of the span. None of the methods used could model the partial passive pressure mobilisation on the side of the arch remote from the load platen.

10. Favourable comparisons were drawn between the failures observed using the small scale models of Ch. 3: hinge locations, critical load points, and interactive behaviour were all reproduced in these large scale tests.

11. The stress distributions derived or measured during the test programme may be incorporated into current assessment methods with a view to their improvement.

Figure 4.1  Salient dimensions, 2m span arch
Figure 4.2  Testing tank before instrumenting and backfilling

Figure 4.3  VWG calibration chamber
Figure 4.4  VWG calibration chart

Figure 4.5  VWG mounting detail
Figure 4.6  Schematic diagram of an ST

Figure 4.7  ST calibration apparatus
Figure 4.8  ST calibration chart and matrix

Figure 4.9  ST and Kulite cell mounting detail
Lvdt 4 'sticks' @ mid-range, probe filed down for test use. Calibration constant unaffected.

Figure 4.10  LVDT calibration chart

Figure 4.11  The bridge loading system
$\square$ = Stress Transducer
$\times$ = Displacement Transducer Pair
$\|_i$ = VWG

Figure 4.12  Instrument location and sign convention

Figure 4.13  Horizontal stress increase on end walls (backfill stage 13)
Figure 4.14  Stresses on the extrados (backfill stage 13)

Figure 4.15  Deformation of the arch during backfilling
Figure 4.16  Normal stress on the extrados, test 1, load at \((x/r)=-1.00\)

Figure 4.17  Shear stress on the extrados, test 1, load at \((x/r)=-1.00\)
Figure 4.18  Change in normal stress with applied stress, q: test 1

Figure 4.19  Change in shear stress with applied stress, q: test 1
Figure 4.20  Normal stress on the extrados, test 2, load at (x/r)=0.75

Figure 4.21  Shear stress on the extrados, test 2, load at (x/r)=0.75
Figure 4.22 Change in normal stress with applied stress, q: test 2

Figure 4.23 Change in shear stress with applied stress, q: test 2
Figure 4.24  Normal stress on the extrados, test 3, load at \((x/r)=-0.50\)

Figure 4.25  Shear stress on the extrados, test 3, load at \((x/r)=-0.50\)
Figure 4.26  Change in normal stress with applied stress, q: test 3

Figure 4.27  Change in shear stress with applied stress, q: test 3
Figure 4.28 Normal stress on the extrados, test 4, load at \((x/r) = -0.00\)

Figure 4.29 Shear stress on the extrados, test 4, load at \((x/r) = -0.00\)
Figure 4.30  Change in normal stress with applied stress, q: test 4

Figure 4.31  Change in shear stress with applied stress, q: test 4
Figure 4.32  Displaced shapes: tests 1 to 4

Figure 4.33  Correlation of stress and displacement, test 2
Figure 4.34  Active and passive pressure mobilisation, test 2

Figure 4.35  Normal stress on the extrados, test 5, load at (x/r) = -0.33
Figure 4.36  Shear stress on the extrados, test 5, load at \((x/r) = -0.33\)

Figure 4.37  Change in normal stress with applied stress, \(q\): test 5
Figure 4.38  Change in shear stress with applied stress, q: test 5

Figure 4.39  Displaced shapes: stages 0 to 7, test 5
Figure 4.40  Collapse mechanism, test 5

Figure 4.41  Dead load stress state: GEOSIM
The effect of introducing a denigrad & pligrad to m-moistest

Figure 4.42  Dead load stress state: GEOSIM, parametric study
Figure 4.43  Dead load stress state versus GEOSIM stress state
Stress on the extrodos, elastic analyses

**MMODTEST stage 1, q = 17kPa**

- Experimental
- BD 21/43
- Boussinesq
- Poulos

**MMODTEST stage 4, q = 65kPa**

- Experimental
- BD 21/43
- Boussinesq
- Poulos

Figure 4.44  Live load stress state: GEOSIM
Chapter 5  Full scale field tests

5.1  Introduction

This chapter describes the load tests on the newly constructed brickwork arch bridge over the river Kym at Kimbolton Butts, Cambridgeshire. The bridge was designed by Cambridgeshire County Council in 1992 to replace the old filler joist bridge carrying the B660 out of Kimbolton. It was decided that a new brickwork arch would provide the most aesthetically pleasing and economical solution through low maintenance costs and a long service life. It was also decided that valuable information on the behaviour of full scale arch bridges would be obtained if the structure were fully instrumented.

To this end Cambridgeshire County Council invited: the University of Edinburgh, the Transport Research Laboratory (TRL), and Ceram Building Technology to instrument the structure. Overall funding for the project was provided by the Department of Transport. The objectives of this study were: to provide results pertaining to the stress dispersal through a stiff road pavement and the well compacted fill, the quantification of earth pressure mobilisation as the arch deformed under load, and enhancement of the existing knowledge of the behaviour of arch bridges.

5.2  Bridge description

The bridge and its salient dimensions are shown in Figs 5.1 and 5.2. An elevation and cross section are shown in Fig. 5.2. It spans 8m at a span to rise ratio of 4. The arch ring is built in Accrington Nori Smooth Gold bricks with a compressive strength of 105Nmm⁻². These Engineering Class A bricks were set in gauged joints composed of a 1:1:6 cement: lime: sand mortar mix. A ring thickness of 0.440m was adopted with a crown cover of 0.450m. In plan the carriageway was wide enough for two 3m lanes with 1.5m verges to each side. The pavement comprises 200mm of Type 1 sub-base surmounted by 50mm hot rolled asphalt wearing course.
The arch ring was waterproofed with Stirling Lloyd's "Eliminator" membrane prior to placement of the drainage blanket and fill. The entire structure was founded on mass concrete pads supported by the underlying Oxford clay bed to a depth of 1.4m below which Kellaways sand was to be found.

Between the pavement and the extrados, Carrstone fill was used to make the formation level and to backfill the haunches of the arch. The fill was a brown silty ferruginous SAND with some gravel. Its particle size distribution may be seen in Fig. 5.3. The 12% silt content precludes the use of this potentially frost susceptible soil within 600mm of formation level. Due to the inclusion of an Enkadrain fabric drainage layer between the fill and the arch this requirement was overlooked. Triaxial tests on large samples gave an angle of shearing resistance of $35^\circ$ at a bulk unit weight of 21.8kNm$^{-3}$. This was equal to the in-situ bulk unit weight as measured by nuclear density meter during filling and compaction. Typical results from a triaxial tests may be seen in Fig. 5.4.

The fill particles could be described as honeycombed in the gravel fraction and rounded for all smaller visible sizes. The honeycombed gravel particles produced considerable interlock and internal friction, especially at such high relative densities. TRL carried out triaxial tests for their report on the exercise(137): their tests yielded an angle of shearing resistance of $28^\circ$ but no indication of load rate, sample density or moisture content was given.

The secant modulus at half peak stress was calculated from the graphs of deviator stress versus axial strain. This modulus was found to be dependent upon the confining pressure as given in Eqn 14.

$$E_s = 0.2 \sigma_3^{0.7}$$

Eqn 14

As such it is not needed here but it is included for reference for those wishing to proceed with a finite element analysis of the soil-arch system. The Poisson's ratio was estimated as 0.4 but this was highly dependent upon the stress range over which it was calculated. It must be borne in mind that these fill properties are sensitive to changes in moisture content, method of compaction, load rate, and to a lesser extent, stress history.
5.3 Instrumentation

With the above general description of the structure in mind the instrumentation was planned to yield information on the dispersal of an applied stress through the road pavement, the fill and onto the arch. Extrados stresses were to be measured to further quantify the passive pressure mobilisation on the side of the arch remote from the loaded axle. These would also give the final stress distribution results from axle through pavement and then fill onto the extrados.

To enable quantitative analysis to be made of the soil-structure interaction, instrumentation had to be incorporated into the structure. This will be discussed below, in the following order: pressure cells beneath the pavement and in the fill, vibrating wire gauges (VWG's) on the extrados, and type T thermocouples in free air, the fill, and the arch ring itself.

Additional instrumentation was carried out by TRL for the monitoring of strains and intrados displacements. Their results remain confidential and may be obtained upon direct application to the client: the Department of Transport in this case. They have not been presented in this thesis. Where, for comparative or illustrative purposes, it has been necessary to quote TRL's results, permission has been given by John Page, Project Manager, TRL.

The specification of all the instruments took into account the need for: robustness, sensitivity, accuracy, wide working range, low cost, fast response time, and long term stability. TRL envisage monitoring the structure for only one year: it is hoped that the instrumentation will survive for at least twenty years enabling genuinely long term readings, over several freeze/thaw cycles, to be carried out.

5.3.1 Vertical stress increase in the fill

For measurement of the vertical pressures in the fill, Soil Instruments Ltd.'s pressure gauges were used. The instrument consists of two circular active faces, 0.100m in diameter, with oil of a similar elastic modulus to the surrounding fill, between these faces. A small bore pipe connects the sealed oil chamber to a VWG transducer activated by the oil pressure which deflects a thin flexible diaphragm. One of these instruments is shown in Fig. 5.5, prior to being buried in the fill above.
the extrados. The cells have a working stress range of zero to 500kPa in compression; similar to the VWG's described below. Cabling consisted of 12mm diameter sheathed and armoured coaxial cable which could be run through the fill without ducting for protection.

5.3.2 Extrados stresses

For measurement of the stress normal to the extrados, Gage Technics Ltd.'s vibrating wire gauged (VWG) pressure cells were specified\(^\text{(139)}\). The instruments were previously used in the large scale laboratory tests described in Ch. 4 of this thesis. The VWG's have a circular active face 0.145m in diameter and a boss 0.120m long behind the cell to permit housing of the transducer body and cable connection points. One of these VWG's is shown on the extrados in Fig. 5.6, prior to burial. The cells have a working stress range of 0kPa to 500kPa and a sensitivity of ±1kPa over its full working stress range. Cabling consists of 4mm diameter coaxial cable which is not armoured but runs in 38mm diameter reinforced ducting through the fill to the cable termination manhole. The ducting may also be seen in Fig. 5.6.

All cables, plugs, jointing materials, and draw tools were supplied by RS components to a specification compatible with both the instruments and the datalogging systems used.

5.3.3 Temperature measurements

Due to the slight temperature sensitivity of the instruments specified, type-T thermocouples were buried in the arch ring and the fill to measure arch ring and fill temperatures respectively. For test purposes an extra thermocouple was placed on the parapet coping stone to monitor the air temperature. The thermocouples were of the copper-constantan type with insulated, unducted wires leading to the cable termination manhole. The sensitivity of the instrument is such that it can respond to a temperature change of 0.1°C in a matter of seconds.
5.3.4 Calibration

The Soil Instruments pressure cells were calibrated in the same apparatus as the VWG's described in Ch. 4 of this thesis but a slot had to be cut into the side of the pressure chamber to allow insertion of the cell midway up the vessel. Two vertically aligned fishplates were used to cover the slot with the exception of the hole required to allow passage of the small bore tube between the cell and the transducer. This allowed both faces of the cell to be covered by the graded Carrstone fill, with no loss of fill from the slot in the side of the chamber, thus replicating the in-situ conditions as closely as possible. The fill was graded through a 2.36mm BS test sieve in the calibration chamber: this was the same as that being used to protect and surround the cell in-situ. A typical calibration chart is shown in Fig. 5.7.

To calibrate the instruments for temperature sensitivity a cell was read under zero stress in a thermostatically controlled oven, open air - both indoors and outside - as well as in a thermostatically controlled fridge/freezer unit. In this way the change in zero reading could be found for a certain change in surrounding temperature. The variation in zero reading, typically 0.5% for a threefold Celsius temperature increase, for a range of temperatures is given in Fig. 5.8.

The Gage Technics. VWG's were also calibrated using the apparatus described in Ch. 4 of this thesis. The packing in the bottom of the steel pressure chamber was used to replicate the restraining effect of the arch ring upon the cell in-situ. The fill used in the steel pressure chamber was the Carrstone fill, graded through the 2.36mm BS test sieve.

The thermocouples, provided by the Transport Research Laboratory, were direct reading and needed no calibration before use.

All the pressure measuring instruments were checked, in the calibration apparatus, for: hysteresis, non-linearity of response, cross-sensitivity, temperature sensitivity, and response time. All calibrations were carried out with identical cables, plugs, and reading systems to ensure accurate replication of the test conditions.

No calculation of cell action factor, \( C_A \), was needed for the cells. The cells in the fill had two active faces and the cell's modulus was, by design, similar to that of the fill to eliminate arching effects across the cells. The small bore pipe was sufficiently
long to ensure that the transducer housing was far from the region in which stress measurements were being taken.

The VWG's on the extrados were, with the addition of the Enkadrain blanket, effectively flush with the surrounding brickwork. Thus they did not form any substantial inclusion in the fill which could cause stresses different from the free-field values to be measured.

The thermocouples in the arch and fill were placed a suitable distance away from the cells to prevent any unquantifiable interactions from taking place.

5.3.5 Installation

The Gage Technics VWG's were placed into counterbored holes in the extrados of the arch. A haunched surround of dental plaster was made for each VWG to hold it in position during backfilling over the bridge and subsequent testing. The 4mm cable was drawn up through the plaster surround and into 38mm diameter ducting. The ducts were then run into the cable termination manhole. The disposition of the VWG's on the extrados and the lines of the duct runs may be seen in Fig. 5.9.

Once the plaster surround had set, a 50mm covering of the specially graded, 2.36mm down, Carrstone fill was placed over the VWG's active face. This was to protect the cell from possible damage by mechanical plant and sharp point contact with coarser aggregate particles in the fill. This fill was compacted with a hand held tamping rod.

Placement of the Enkadrain blanket over the cell was followed by the cutting of holes through this geotextile membrane to expose the partially buried VWG. Placement of the remainder of the non-graded fill was then allowed to continue.

Once the fill over the arch had reached the required level, as given in Fig. 5.9, the ground was prepared for the installation of the Soil Instruments pressure cells. This entailed the hand excavation of a 100mm deep pocket in the fill which was half filled with the graded calibration soil as shown in Fig. 5.5. This was then compacted with a hand held tamping rod. The cell was then placed in the shallow pocket and covered with a further 50mm of the specially graded fill. The upper
layer of this was also compacted and once the cell was installed, placement of the next layers of ungraded fill could continue. The armoured cable was run through the fill to the cable termination manhole.

One Soil Instruments cell, wrapped in polythene bubble packing, was placed in the manhole and kept under nominally zero applied stress. The purpose of the extra cell in the manhole was to enable assessment of the change in zero reading due to the measured temperature changes during the tests without the added complication of having a stress change due to an axle load above the cell's active face. During the tests this cell, which had been calibrated for change in zero, unloaded, reading with temperature change (see Fig. 5.8), was laid horizontally on the kerb and used to assess the amount of correction needed on the readings from the other cells of this type.

The type-T thermocouples were installed in the arch and the fill. The arch thermocouple was built into the brickwork during construction. The fill thermocouple was hand driven into the fill in the middle of the vertical line of Soil Instrument pressure cells but offset some 100mm to the side of the array of cells. This was to prevent the presence of the thermocouple probe interfering with the readings from the pressure cells. This gauge was taken as being representative of the temperature throughout the zones of fill where pressure measurements were being taken. The unducted wire was also run back to the cable termination manhole.

The thermocouple in free-air was mounted on the parapet coping stone and fixed, by its connecting cable, onto the wall with adhesive tape. It was not permanently installed and was only laid on the parapet during tests on the bridge.

5.3.6 Datalogging

A terminating junction box for all the fill and extrados pressure and temperature measuring instruments was made. This was waterproofed and sealed to be kept in the manhole on a permanent basis. Connection to, and readings from, this box were made using a Gage Technics acoustic strain gauge meter with a channel selector and a digital display. The thermocouples were connected separately to their dedicated reader, a Comark thermocouple measuring unit.
Results were recorded on a laptop personal computer for post-processing. Hard copy of all results was produced simultaneously to prevent loss of data in the event of power failure or corruption. The datalogging system was such that the same equipment was used for both the calibration and the subsequent testing.

5.4 Test sequence

Table 10 gives the test schedule to be read in conjunction with Fig. 5.10 which shows the position of the load lines traversed by the HB trailer.

<table>
<thead>
<tr>
<th>Test</th>
<th>Line</th>
<th>Nominal load/ t</th>
<th>Wheel loads/ kg Wheel 1</th>
<th>Wheel loads/ kg Wheel 2</th>
<th>Wheel loads/ kg Wheel 3</th>
<th>Wheel loads/ kg Wheel 4</th>
<th>Total load/ kg</th>
</tr>
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<tbody>
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<td>6200</td>
<td>5450</td>
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<td>2</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(23900)</td>
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<td>4450</td>
<td>4700</td>
<td>4650</td>
<td>18750</td>
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</table>

Table 10 gives the wheel loads as measured with digital weigh pads before each test. The total mass for test 2 was assumed to be the same as that measured before test 1. Each mass was measured to an accuracy of ±50 kg.

For each wheel, an average applied stress was calculated for later use in the calculation of stress influence values. The average applied stress was derived from tyre contact area measurements immediately before test 1 commenced. Typically the contact area was approximated to by a rectangle 0.550 m long by 0.250 m wide. This gave an average contact area of 0.1375 m² which was then used to calculate the average applied stress on the road pavement's surface. These stresses are given in Table 11 with references to wheel numbers given in Fig. 5.10. The contact stresses for test 2, with the load at line 2, were assumed equal because of symmetry about the bridge's longitudinal centreline.
Table II  Average applied stresses

<table>
<thead>
<tr>
<th>Test</th>
<th>Line</th>
<th>Nominal load/ t</th>
<th>Contact stress/ kPa Wheel 1</th>
<th>Contact stress/ kPa Wheel 2</th>
<th>Contact stress/ kPa Wheel 3</th>
<th>Contact stress/ kPa Wheel 4</th>
</tr>
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<td>421</td>
<td>442</td>
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<td>335</td>
<td>332</td>
</tr>
</tbody>
</table>

The arch filling was completed on 27 Nov. 1992 and the bridge was paved and open to traffic by 16 December 1992. The load tests were done after completion of the structure during an overnight possession on 27/28 Feb. 1993. At each nominal axle load and load line, the HB trailer was pushed across the bridge from South to North. Eleven scans were taken as the trailer was parked at 1m intervals from "Load off", to 1m off the springer line, through 1m (one eighth of the span) intervals to the North springer and subsequent "Load off" reading on the North side of the span.

5.5 Measurements made during backfilling

The instruments were scanned individually as they were unwrapped on site to obtain an updated zero reading pertinent to the prevailing atmospheric conditions. The gauges were all scanned in their installed positions before the 50mm graded fill covering was applied. The gauges were all scanned upon completion of the filling process to a level some 50mm above the crown. TRL later scanned the cells on 16 Dec. 1992 upon completion of the road pavement. New zero readings were taken before the heavy axle load tests and these will be used to calculate the live load, stress change, values.

The changes in stress on the extrados from the initial, pre-installation zero readings, to those observed on 16 Dec. 1992 are small and follow trends similar to those seen above the upper portions of the semicircular arches tested and described in Ch. 4 of this thesis.
The vertical stresses measured in the fill were small and almost directly proportional to the fill depth above the centre of the instrument. Small deviations from this ideal geostatic stress state would have been caused by lateral stress changes induced by the placement and compaction of the fill around the cells. The cells were placed sufficiently far from the arch to prevent the presence of a relatively rigid inclusion affecting the stress readings. These readings need little further discussion: for the live load tests the stress changes from the new zero readings will be used, rendering these dead load readings insignificant. They have been included to demonstrate the efficacy of the specified instrumentation and datalogging systems. They also quantify the dead load stress state and show the similarities between this and other tests described elsewhere.

The air and fill temperatures were measured by a 76mm immersion, mercury in glass thermometer before the thermocouples were available. They were used to verify the accuracy and precision of the thermocouple readings once they were installed. The dummy cell in the cable termination manhole exhibited only small changes in zero, unloaded, reading during the filling process. This is consistent with the small (<4.0°C) temperature changes observed over the course of a typical winter's day. Therefore no stress corrections need be applied to the dead load results to compensate for temperature effects.

TRL's results over this period are presented to demonstrate the accuracy of the effects observed above. They are discussed, in brief qualitative terms, below.

Continual shrinkage and settling-in effects were observed from installation (13 to 15 Oct. 1992). Small movements were observed upon striking of the centring piece: these tended to increase the tension in the arch ring. Increasing compression in the arch ring then occurred in tandem with the increasing fill depth over the extrados. Upon completion of the filling a slight decrease in compression took place. This was offset by the subsequent increase in compressive strain induced by the addition of the dead load from the sub-base and road pavement.

These are consistent with the stress measurements: added dead load would be expected to put the arch further into compression by the transfer of the fill's dead weight across the interface between the soil and the arch.
5.6 Heavy axle load tests

The test sequence follows Table 10, with the load positions given by Fig. 5.10. All pressure cells were scanned thrice at each load point and the average reading used to calculate the stress change from the calibration charts (see Fig. 5.7). Temperature readings were taken at convenient intervals throughout the testing. This section presents and discusses the results from the 6 tests. TRL’s results are not presented here but are referred to on occasion for explanation or elucidation. The temperature results are presented first because the conclusions from them are relevant to the whole test programme.

5.6.1 Temperature readings: tests 1 to 6

The temperature readings in air, fill, and arch are shown in Fig. 5.11. The temperature in the fill and on the arch did not vary significantly during the night; hence obviating the need for any correction to be applied to the pressure readings. The Soil Instruments cell, left out to assess the change in unloaded cell reading with temperature, confirmed this by registering little significant change throughout the test sequence.

As expected the thermal capacity of the massive soil-arch system ensured that the arch and fill temperatures lagged the slowly falling air temperature. The fill, insulated by both pavement and brickwork, remained at a higher temperature than the exposed arch ring. Due to the fact that new zero readings were taken at 2315 on 27 Feb. 1993, prior to the load moving onto the span, any temperature induced stresses before this time could be discounted. Beyond these zero readings the temperature change remained too small to affect the readings.

5.6.2 Stresses in the fill: tests 1 to 6.

The vertical stress increases in the fill under heavy load were measured. The reciprocal theorem was invoked to extend these results by saying that if an axle produced a stress increase of 1kPa at a certain location, then when the axle was above that certain location the stress increase at the same depth below its former position would also be 1kPa. This means that the results from only five instruments
could be used to derive the stress bulb beneath the axle provided sufficient tests were completed to offset the relative lack of instruments.

The derived stress bulb, containing all results giving an influence value for vertical stress increase greater than 0.001, from tests 1 to 6 inclusive is shown in Fig. 5.12. Here the contours represent the influence values, multiplied by 1000 to give "sensible" values for plot clarity, for vertical stress increase at each of three levels. The uppermost plot represents the load spread 525mm below the axle: this is the dispersal arising from the 450mm road pavement make-up plus a 75mm layer of fill. The middle plot represents the dispersal occurring over a total depth of 950mm whilst the lowest plot gives the furthest extent of the pressure bulb at 1200mm depth.

The plots represent the stress increase occurring beneath one wheel out of the four present on the loaded axle. The plots above may be superimposed with the horizontal separation given by Fig. 5.10 to obtain the stress increase below the entire axle if so desired. A vertical section may be taken through the contour plots to show the stress dispersal effect of the pavement and fill. This is presented in Fig. 5.13.

The equivalent side slope may be seen to be 1 in 0.47 if the zone of influence is curtailed at an influence value of 0.01. This was derived simply by scale drawing. This represents a considerable spread of the applied stress through just 450mm of pavement structure and 75mm of dense fill.

The peak influence value observed in tests 1 to 6 was 0.097: this occurred when the load was run along line 2 until it was 2m South of the crown. As shown on Fig. 5.9 the vertical stress measurements were taken 2.15m South of the crown. This offset of 0.15m may mean that a slightly higher influence value may have been found if the axle had been centered exactly above the cells (2.15m from the crown). The resulting increase in vertical stress influence value is thought to be small: this may easily be borne out by simple elastic stress dispersal analysis. The 0.15m offset is insignificant relative to the width of the applied load; this gives further support to the claim that the maximum influence value was approximately 0.097.

The results taken from the array of cells immediately below the road pavement imply that the pavement plus a skim coating of fill are capable of dispersing
approximately 90% of the applied surface stress. The stresses are seen to be so low because of several reasons. The pavement is relatively stiff and disperses the applied stress widely. The fill is of relatively low elastic modulus: the lower the fill modulus, the lower the stress it attracts for a given input of externally applied work. The fill is undergoing predominantly vertical displacement which, when its relatively low modulus is taken into account, would not seem unreasonable. This last statement implies the fill mass is moving without being unduly stressed. This could occur if the arch were observed to displace by an amount sufficient to remove a certain amount of restraint from the fill. If this restraint were otherwise present the fill would be subjected to a greater stress for the same externally applied work input.

It is envisaged that the three factors listed above occur simultaneously. The dominant effects are thought to be those of the pavement’s dispersive capabilities and the fill’s relatively low modulus. TRL’s measured displacements[137] are not deemed substantial enough to have caused mass fill movement downwards with the arch. The findings of Ch. 3, where the fill depth was seen to influence the collapse load of the system bear out the conclusion concerning load dispersal. There it was shown that the increase in capacity was caused by a combination of increased dispersal and increased dead weight. The dispersal effect was identified as more important, it accounted for between 60% and 70% of the capacity increase arising from an increased cover at the crown. The finite element analysis discussed in Ch. 6 provides supporting evidence for the low stresses in the fill and a parametric study will be presented where the fill’s modulus is one of the fundamental variables.

The stress bulb presented here may be applied over the whole span of the arch. Curtailment of it will obviously occur at different depths as the bulb traverses the arch and the extrados is met at shallower depths but the relevant influence values may still be read off the plots wherever the extrados is encountered.

Comparisons between the experimentally deduced stress distribution and that given empirically in the codified method of assessment[14], BD21/93, are discussed with a view to improving the codified stress dispersal method for future bridge assessments.
5.6.2.1 Comparisons with other analyses

The results of the Boussinesq\(^{(123)}\) and Poulos\(^{(136)}\) elastic stress distribution analyses are shown in Fig. 5.14 for each of the soil pressure measuring cells. They are compared with the actual results obtained from in-situ observations.

The Boussineq method is seen to overestimate the fill stresses by a variable amount. The difference is considerable for those cells far from the fill surface. Where the fill surface is close to the cell the stress measured is close to the predicted elastic, Boussinesq values. For cells close to the load and the fill's surface the inhomogeneity introduced by the rough, relatively rigid arch does not have as much of an effect as the cells are above crown level. Boussinesq's theory predicts, reasonably accurately, the vertical stress increase at these points. Closer to the arch, the Boussinesq prediction fails to model the stress state because of the violation of most of the fundamental assumptions governing the use and application of Boussinesq's formulae.

This arch is shallower than the semicircular profiles tested for Ch.4 of this thesis. There is less fill mass, and in cross section, less fill area through which the assumptions of isotropy, homogeneity, and semi-infinity of the elastic half-space are to hold true. The arch accounts for a greater percentage of the cross section and structural stiffness where the span to rise ratio is large than it does in the semicircle case where \(L/r\) is only 2. The shallower arch, forming more of a foreign inclusion in the fill, effectively prevents the Boussinesq analysis from performing adequately.

The above forms the main reason for the limited use of the Boussinesq model in the case of the shallower arch at Kimbolton. The fact that Boussinesq can predict the stresses immediately beneath the road pavement is of lesser importance to the arch bridge assessment problem, and of greater relevance to serviceability requirements.

Poulos's elastic analysis overestimates the vertical stresses by a consistently greater amount than the Boussinesq method. It too is better at predicting the stresses close to the surface than at depth. The reasons for the failure to predict the stress state at depth are the same as those applied to the discussion of Boussinesq's results. Poulos's prediction gives a greater overestimate than Boussinesq because it applies influence factors incorporating the proximity of a rough, rigid boundary. These, at peak stress values, result in a factor of approximately 1.4 being applied to the basic
Boussinesq stresses. The overestimate gets larger as the extrados of the arch is approached and the effects of the boundary make themselves felt in the Poulos analysis.

The codified stress dispersal uses the "1 in 2" sideslope method of load spreading. This is the recommended BD21/93 method for calculation of the extrados stresses. The method has been applied to the calculation of the fill stresses. The results of this comparison may be seen in Fig. 5.15. This gives elevations of the uppermost plot from Fig. 5.12 with the BD21/93 version of the load spread superimposed. The codified influence value is a uniform 0.90 across this level (525mm below the road surface): this compares with an observed peak value of 0.097. This renders the BD21/93 method some 859% higher than the observed results would suggest was an accurate stress increase immediately beneath the road pavement. This error is continued for all depths, BD21/93 always grossly overestimates the increase in vertical stress in the fill.

More critically it also underestimates the load spread angle through pavement and fill. The fact that it fails to predict the influence value is excusable: the fill modulus could change by many orders of magnitude yet BD21/93 would continue to predict identical influence values at any given depth. Not spreading the load over a sufficiently wide area is less fortunate as will be seen when the extrados stresses are analysed. This has the effect of concentrating the load upon the extrados which, if used as input for a mechanism analysis in the form of an equivalent set of resolved point forces, would result in an underestimate of the arch capacity: as such its use is extremely uneconomical.

It must be pointed out, in defence of the codified method of stress dispersal, that it is flat topped and is being compared with "bell shaped" distributions resulting from experimental observations and theoretical distributions. The BD21/93 influence value is, at a given depth, constant irrespective of the lateral distance from the axle's centreline. At no time does it defy equilibrium, all the force applied at the surface of the fill gets applied to the extrados of the arch. This is also the case with the Boussinesq and Poulos methods.

The codified method is empirical and the points made in the preceding paragraph must be remembered. It still remains that the empiricism results in gross errors when it comes to the calculation of the stress state in the fill. The empiricism that
results in equal side slopes being applied to both sides of the load spread is unnecessarily conservative. The difference in load spread angle is clearly seen in Fig. 5.15 to be greater on the springer side of the axle due to the greater fill depth on that side of the load available for dispersing the contact stress. Recommended load spreads are given in Fig. 5.15 which may be used empirically if desired.

The final points to be made concerning the fill stresses involve the dispersive power of modern road pavement materials. The pavement causes a large reduction in stress whichever analysis or result is used. The reduction in stress is demonstrated by both Boussinesq and Poulos although neither method specifically includes a higher modulus layer. Scope exists\(^{(140)}\) to allow for multi-modulus strata in each case but beyond proving that the stress dispersal is increased for stiffer pavements, little use may be made of those results here. In the case of the 450mm of road pavement at Kimbolton, this reduction amounts to some 90% of the contact stress. A stiffer, or deeper, pavement would intuitively cause greater stress dispersal. It would also add a small amount to the dead load on the arch, thus giving even greater carrying capacity. The finite element analysis of Ch. 6 varies the pavement stiffness and depth to examine the reduction in fill and extrados stresses arising from increased pavement thicknesses or rigidities. This concludes the analysis of the stress state in the fill.

5.6.3 Stresses on the extrados: tests 1 to 6

The VWG's have been used to produce the results for this section of the chapter. Changes in stress, normal to the extrados from tests 1 to 6 are shown in Fig. 5.16. Channels 1 to 4 correspond to the instrument locations given in Fig. 5.9.

The VWG's on the extrados gave significant pressure changes during the passage of the heavy axle. The graphs of measured pressure, normal to the extrados, versus load position for each of tests 1 to 6 are shown in Fig. 5.16 to be significant. The peak stress was found to occur as the loaded axle passed above the transducer and it dropped away as the axle moved towards the crown. As the axle moved to the remote side of the bridge, opposite the VWG's, the registered pressure increased again as the arch began to be pushed back into the fill on the side remote from the loaded axle. This represented partial mobilisation of the passive pressure state.
pressures measured when the arch was being pushed into the fill were substantially lower than those found as the axle passed above the VWG's.

The results from tests 2, 3 and 6, where the loaded axle traversed the bridge along line 2, (see Fig. 5.10), are presented in Fig. 5.17. The increase in peak measured stress with applied nominal axle load is clearly linear within the error bound associated with each result.

Higher stresses were measured for axle loads moving along line 1 (see Fig. 5.10) because of the reduced distance between the load point and the VWG's all along line 1. The results from line 2 are plotted because all three different nominal axle loads were used along this line. Table 11 has been used to plot the actual force exerted because the graph exhibited a small degree of non-linearity when the nominal axle loads were plotted. This amounted to a linear best fit giving a correlation coefficient of 0.97 or 97%; whereas using the actual axle loads increased this correlation coefficient to 0.99 or 99%.

From the VWG readings, a bulb of pressure can be derived for the critical load position, close to the quarter span point or -2m from the crown of the arch. This bulb is shown in plan and elevation on Fig. 5.18 for a 30t axle along line 1 positioned at the quarter span.

The contour values are influence values for normal stress on the extrados, i.e. they are equal to the measured stress divided by the average applied stress on the road pavement surface. The stresses were expressed in this way to enable the results to be seen as typical stress distributions on the extrados of such an arch. In this way the results become more generally useful.

The peak influence value was found to be 0.548, this was the largest of all the results from tests carried out on the structure. The contour map has been curtailed at the 0.100 influence value contour as values below this were deemed insignificant. When viewed in elevation the same limits apply. The 0.55m wide loaded area was seen to have spread to 3.2m linearly over the extrados.

The peak influence value observed on the side of the arch remote from the axle load was only 0.10. This represents partial passive pressure mobilisation. The passive Rankine state would only be achieved upon the mobilisation of approximately five
times the observed stress. The maximum mobilisation of passive pressure here is lower than that found behind a semicircular arch for the reasons discussed in Chapters 3 and 4. During tests on the semicircular arch 40% of the full Rankine passive pressure was mobilised. Here, no more than approximately 18% of the full passive pressure has been mobilised. The measured displacements\(^{137}\) confirm this qualitatively because they are small and will not cause the mobilisation of large stresses.

The graphs of measured pressure versus load position (Fig 18) all show how the normal pressure on the extrados increased as the loaded axle approached the VWG’s positions. The registered pressures sharply increase as the axle moves far enough onto the span so that the axle’s zone of influence encompasses the VWG’s position. The pressure peaks as the axle passes over each VWG and drops as the axle’s zone of influence moves away from the VWG’s location. When the axle was on the side of the arch remote from the VWG’s positions, reversal in the direction of the normal pressure was observed. This is exactly as discussed previously with reference to the fill stress cells.

The above describes the basic trends seen in all six tests. Differences in the magnitude of the peak pressures observed in these tests may be accounted for in two ways:

1. The load line may be further away from the cells, thereby giving a lower peak pressure. The cells registered their peak pressures for an axle moving along load line 1; the lowest peak pressures measured were for the most remote load line, no. 3, in the right hand lane of the road. This may be confirmed by simple stress dispersal analyses such as Boussinesq, Newmark or Westergaard\(^{123}\).

2. The arch was subjected to biaxial bending stresses. These arose because the central portions of the arch barrel were considerably less stiff than the restrained outer edges and corners of the arch barrel. VWG’s 1 and 2, see Fig. 5.9, were positioned in the middle portion of the arch whereas VWG’s 3 and 4 were further out towards the edges of the arch. The pressures measured at VWG’s 1 and 2 were lower than those at 3 and 4 for loads on line 1 because of the lower stiffness of the central, more flexible portions of the arch barrel. Where the arch was stiffer, i.e. closer to the corbelled
corners or the torsion beam running the length of the span, the pressure was seen to be substantially higher, even for a lower strain along the length of the arch's span. This is borne out by the strain and displacement results obtained by TRL and presented here as Figs 5.19 and 5.20. Over the longitudinal centreline the displacements are larger, indicating movement of the arch away from the fill, hence the lower stresses on the central portion of the arch. The longitudinal strains in the central section, where the structure is generally more flexible, are larger, as are the lateral strains. This is shown in Figs 5.19 and 5.20. Such behaviour may imply that analysis of the arch barrel in three dimensions, using the load dispersal patterns presented, may be facilitated by treating it as a shell buckling problem with the appropriate edge restraint conditions.

The results shown in Fig. 5.17 need little discussion: they show clear, quasi-linear pressure increases with axle load increases. The linearity implies perfect elasticity in the soil-arch system. TRL's displacement results, which show less than 0.05mm unrecovered deformation in the worst case, confirm the elasticity of the structure. The VWG's at locations 1 and 2, in the more flexible central section of the arch, registered higher pressures because of the proximity of the load which was, along line 2, in the middle of the road. The overall impression gained from Fig. 5.17 is one in which the VWG's appeared to behave well, giving sensibly consistent pressure measurements of the right order of magnitude.

The pressure bulb on the extrados, derived from the VWG readings for a load at the quarter span, shown in Fig. 5.18, gives an idea of the extent of the load dispersal. The codified method causes a higher influence value to act as well as a narrower dispersal width compared to that measured by the VWG's. The implications for assessment or design of masonry arches are: lower stresses would be deemed to act on the extrados, therefore a higher axle limit could be applied, or in a design situation, a reduced thickness of arch could be used for the same axle load limits. The stress concentration would not be as severe as imagined under the codified method which could only be beneficial. The typical pressure bulb shown (Fig. 5.18) could be used in conjunction with a mechanism analysis, to correctly model the applied load set. This would probably increase the assessed capacity of the soil-arch system: firstly because of the lower peak stress and secondly because of the wider spread of the load. This incorporation into a mechanism method of analysis could take the form of a resolved set of point loads or an equivalent uniformly distributed
load over the limits of the pressure bulb (Fig. 5.16). These loads could then be used to iterate towards a collapse mechanism based on an improved starting point where the applied loads are better known.

No pressure measurements were made at the crown due to the proximity of the road pavement materials and the ensuing complexities of calibration of the VWG's to this situation. The stresses at the crown may well have been higher for a load above the crown but geometrical factors imply that the minimum collapse load does not necessarily occur at this point\(^{64}\). The reasons for the apparent stress concentration on the extrados may be summarised as follows: BD21/93 takes no account of the presence of the relatively stiff road pavement layer(s), it allows very little dispersal of stress below the level of the crown of the arch and it fails to apply any stress other than a purely vertical one to the arch. For all these reasons the VWG's were used to give a more accurate picture of the actual extrados pressure distribution.

5.6.3.1 Comparisons with other analyses

Boussinesq's analysis has been carried out for the stresses on the extrados. The vertical and horizontal live load induced stress changes were calculated. Mohr's circle of stress was used, in similar fashion to that described in Ch. 4 of this thesis, to derive the resulting normal stress on the extrados, \(\sigma\). The stresses calculated by Boussinesq's method were all irregular and no clear trends were established. On occasion the actual stress was greater than the predicted stress: on other occasions it was much less than the Boussinesq analysis prediction.

The discrepancies are probably due to a number of factors: the movement, albeit small, of the arch, the anisotropy and inhomogeneity of the fill-arch system, and the presence of the arch itself. The soil-structure interaction effects effectively prevent Boussinesq's method from predicting the stress state, even around a shallow arch where less scope for interactive behaviour exists.

Paradoxically, it is in the case of the semicircular profile (see Ch. 4), where maximum scope for interactive behaviour exists, that Boussinesq's method has been proved to predict the stresses accurately at low applied loads. This is because there is a greater area of fill, when the soil-arch system is viewed in elevation, within which Boussinesq's solution can provide more accurate results. The distribution of
stress becomes established in the fill, according to classical, elastic analyses, and only when points close to the arch are examined do the aforementioned discrepancies become more apparent.

Poulos's distribution also produces inadequate predictions of the stress state around the extrados for the span to rise ratio encountered at Kimbolton. Therefore, no graphical results are presented here and no further discussion of the two elastic methods will be entertained.

Stress dispersal by the codified, "1 in 2" side slope, method was undertaken for a 0.55m wide loaded area, akin to the 0.55m long tyre used on the HB trailer. A unit stress was applied and the corresponding influence values for vertical stress were derived. Mohr's circle of stress was then used to calculate the resulting normal stress on the extrados, having assumed a horizontal stress equal to $K_0$ multiplied by the vertical stress. $K_0$ is the assumed earth pressure coefficient for the at-rest state as described in Ch. 4 for the GEOSIM analysis.

The normal pressure distribution thus derived may be seen in Fig. 5.18, superimposed upon that actually measured. The peak influence value according to BD21/93 was 0.650, some 0.102 higher than that measured. The decrease of 0.102 from code to practice represents a drop of 16% of the code value. More significantly, the load distribution occurs over a much narrower width than was the case in practice. The BD21/93 load distribution extends over a mere 1.05m, compared to 3.2m observed in practice. As discussed in section 5.6.2.1 the failure to spread the load over a wide enough area is overly conservative for the economical assessment of arch bridges.

The principal practical consideration arising from any analysis of the dispersal, be it experimental or theoretical, is that stiff, well laid road pavement overlays may be used to increase the capacity of older bridges which would otherwise fail current assessment checks. The dispersive power of the pavement is sufficient to reduce the peak fill stress to 10% of its surface value. When this is continued through the fill onto the higher modulus arch material, the peak stress is only 54.8% of the applied contact pressure. Were it not for the action of the pavement and the fill this peak extrados pressure would be greater. It is the stress state around the extrados that governs the displacements and subsequent collapse load of the arch. The lower the applied stress on the extrados, the greater the capacity of the structure. This premise
assumes the presence of structurally sound spandrel and parapet walls and foundations. The use of overlays for the upgrading of arch bridges is the subject of papers by the author and his co-workers\(^{141,142}\).

The relationship between peak applied stress and assessed collapse load is explored further in Ch. 6 of this thesis where finite elements are used to provide the most detailed picture yet of the stress state behind an arch bridge. This comparative work, showing the relationship between stress on the extrados and capacity as assessed by ARCHIE and MEXE converts the work of this thesis into convenient, practical assessment and design charts aimed at Local Authority Engineers.

This concludes the analysis of the results from the heavy load tests on Kimbolton Butts bridge. The principal findings will be summarised below. The project is deemed complex enough to warrant summary conclusions split into appropriate sections rather than presented as a single list of points.

5.7 Conclusions

The following section summarises the principal conclusions relating to: the instrumentation, the methods of calibration, the installation and testing, the temperature measurements, the wheel contact stress measurements, the stresses on the extrados, and the stresses in the fill.

5.7.1 Conclusions: instrumentation

1. The instrument types specified all proved simple to use, both in the calibration stage and \textit{in-situ}. They all yielded useful results for reasonable cost and effort.

2. The types of instrument used were easily installed and tested by one person in reasonable time, hence reducing the possibility of delaying the main contractor.

3. The use of geotechnical instrumentation has provided valuable insight into the complex behaviour of a soil-arch system. 
4. The use of such instrumentation would be beneficial on all future arch tests as this is the area of assessment and analysis suffering most from a paucity of information.

5.7.2 Conclusions: wheel loads, contact areas and temperatures

1. The use of individual wheel loads is necessary for analysis of the influence values but average contact stresses may be used with little error in analysing such systems.

2. Temperature measurement by simple thermocouple and a "dummy" gauge on the kerbside proved that even overnight in winter, insufficient temperature change occurred to warrant application of a temperature correction to the instrument readings.

3. Thermocouples and "dummy" gauges should be incorporated into the instrumentation system design for future tests as little information pertaining to the effects of temperature change on the pressures measured is currently available.

5.7.3 Conclusions: stresses in the fill

1. A typical distribution of the vertical soil stress immediately beneath the road pavement was derived.

2. Peak influence values of only 0.097 were observed. The dispersive effect of the stiff road pavement materials is the cause of the low stresses measured in the fill.

3. The BD21/93 dispersal method gives higher stresses spread over a narrower area leading to its inherent conservatism.

4. The pressure distributions found in the fill, combined with those found on the extrados could be incorporated into a mechanism analysis in an attempt to improve the assessed capacity of a masonry arch bridge.
5.7.4 Conclusions: stresses on the extrados

1. A typical distribution of the pressure normal to the extrados was derived.

2. A peak influence value of 0.548 was observed: this was some 16% lower than the BD21/93 peak influence value.

3. The loaded axle's zone of influence was considerably larger than that allowed for in BD21/93.

4. Some passive resistance was observed as the arch was pushed into the fill on the half-span remote from the loaded axle. This was not more than approximately 18% of the classical Rankine value which compares favourably with the 40% mobilisation observed in tests on semicircular arches (see Ch. 4).

5. The pressure readings were affected by the varying flexibilities of different portions of the arch. Further complications were introduced by the identification of biaxial bending action in the arch barrel.
Figure 5.1  Kimbolton Butts Bridge

a) the completed bridge

b) during construction
Figure 5.2  Salient dimensions, Kimbolton Butts Bridge
Figure 5.3  Particle size distribution, Carrstone fill

Figure 5.4  Typical stress-strain plot from triaxial compression test
Figure 5.5  Installation of fill pressure cell

Figure 5.6  VWG on the extrados
\[ \Delta \sigma = 0.515 \times (10^{11} / \text{reading}^2) \]

Figure 5.7  Fill pressure cell calibration chart

Figure 5.8  Fill pressure cell, temperature sensitivity
### Gauge Locations

- **E1 - E4** gauges embedded in arch extrados
- **F1 - F5** gauges embedded in Fill

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Distance from longitudinal centreline (mm)</th>
<th>Horizontal distance from crown (mm)</th>
<th>Depth below extrados at crown (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>180</td>
<td>1310</td>
<td>-</td>
</tr>
<tr>
<td>E2</td>
<td>290</td>
<td>2220</td>
<td>-</td>
</tr>
<tr>
<td>E3</td>
<td>2480</td>
<td>1300</td>
<td>-</td>
</tr>
<tr>
<td>E4</td>
<td>2470</td>
<td>2220</td>
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<td>F1</td>
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<td>2650</td>
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<tr>
<td>F5</td>
<td>2650</td>
<td>3782</td>
<td>750</td>
</tr>
</tbody>
</table>

**Figure 5.9** Instrument locations
Figure 5.10  Loading lines and HB trailer
Figure 5.11  Temperature variation during the night 27/28 Feb. 1993
Influence values for vertical stress increase $\times 10^3$

Figure 5.12  Vertical stress increase beneath a loaded wheel
Figure 5.13  Vertical section through pressure bulb
Figure 5.14  Stress state in the fill: Boussinesq, Poulos, and experiment
Figure 5.15  Stress state below road pavement: BD21/93 and experiment
Figure 5.16  Normal stress on the extrados: tests 1 to 6
Figure 5.17  Normal stress versus axle load, tests 2, 3, and 6
Figure 5.18  Normal stress on the extrados
Figure 5.19  Crown displacements, test 2

Figure 5.20  Longitudinal strain at the crown, test 2
Chapter 6 Finite Element Analysis of the Soil-Arch System

6.1 Introduction

Many different methods have been used to predict and model the behaviour of arch bridges, with or without their backfill and road pavement strata. These include: plastic methods\(^{(64)}\), empirical methods\(^{(14)}\), elastic analyses\(^{(72)}\), mechanism methods\(^{(16)}\), and funicular polygon techniques\(^{(52)}\). Finite element analyses have also been used recently with some success. Discussion of examples of these methods and analyses is included in the review of the relevant literature comprising Ch. 2 of this thesis.

The purpose of this part of the research is to show that comparatively simple finite element techniques which include appropriate material properties can accurately model the soil-structure interaction in backfilled arch bridges. The stress fields before and after application of live load were both explored. The finite element results were compared with the experimental results presented in Ch. 4.

In this study both the fill and the arch were modelled as elastic homogeneous solids. It is entirely heuristic in character: it in no way suggests that the fill and arch remain elastic for all applied loads. It allows the importance of complex material modelling to be assessed and should form a basis for comparisons with more complicated non-linear analyses. An elasto-plastic analysis was also carried out. This forms only a minor part of the finite element study. Material non-linearity was used but geometric non-linearity, which could be used for the analysis of certain large displacement cases, was not deemed necessary at this stage.

As the assessment problems become more pressing, the use of finite elements to analyse arch bridges will increase. Most existing analyses using the finite element method do not give stresses around the soil-arch interface on the extrados. Many of these methods do not consider the action or influence of the fill upon the arch. MAFE\(^{(29)}\), the Nottingham University and British Rail Research finite element suite uses complex three dimensional coding to predict the arch deformations and stresses. However; its soil model\(^{(32)}\) is simplistic.
The MAFEA suite has been used in a detailed parametric study by British Rail Research. In many of the finite element studies carried out no attempt has been made to show that complex, non-linear material characterisation is needed. With the exception of MAFEA no other finite element analysis for arch bridge design or assessment has been subjected to a parametric study to identify variables of importance.

This chapter seeks to overcome the geotechnical omissions made in earlier finite element analyses. It provides predictions of the stress state around the extrados for both dead and live load conditions. Classical methods are used for comparative purposes. Given the satisfactory nature of these comparisons, the study proves that simple, elastic finite element methods may be safely used to analyse backfilled arches. The chapter concludes with a design chart incorporating the soil-structure interaction effects into practice. This chart will enable Local Authority assessors to calculate the capacity increase arising from the inclusion of the effects of soil-structure interaction. The methods of assessment currently available can be presented on this chart for assessment purposes. A method statement for the use of this chart is included. Examples of the capacity increase arising from the inclusion of backfill interaction effects are included.

The finite element analysis was performed using the AFENA program, developed by The Centre for Geotechnical Research, University of Sydney, New South Wales\(^{143}\). Mesh generation was performed by the GENTOP\(^{144}\) program. Checking of the meshes was carried out manually with subsequent graphical checks being done by the CHKFE\(^{145}\) program. Post-processing of the results was carried out using programs: MESHGUT\(^{146}\), (gut the mesh to form an outline only); AIDSC\(^{147}\), (Assemble Information for Displacement and Stress Contouring); and FELPA\(^{148}\), (Finite Element Plotting Algorithm); all developed at The Centre for Geotechnical Research as referenced above.

The development of AFENA and its allied post-processors is being undertaken at The University of Edinburgh by Rotter and Ooi's research team investigating silo-stored solid interaction problems. Extensive test runs and comparisons with classical analytical results have been carried out to prove the validity of this program. AFENA is commercially available worldwide. The author has conducted a series of tests examining the algorithm's ability to model: the elastic half-space stress distribution problem, the shear box problem, and the elastoplastic bearing capacity.
problem. The results from these were satisfactory and are not included here for economy of space. Suffice to say that they prove the ability of program AFENA to model common geotechnical situations. Documentation of some of the aforementioned test meshes is to be found in projects partly supervised by the author whilst at The University of Edinburgh\(^{(149,150)}\).

### 6.2 Finite element formulation

The arch, the fill, and the road pavement are all given linear elastic, homogeneous, isotropic properties. This assumption is almost certainly invalid for mixed material problems of this nature\(^{(151)}\). By establishing the nature of predictions arising from the simplest elastic theory, it is possible to determine the assumptions necessary for more complex modelling, and also to learn which facets of the soil-structure interaction and the derived stress distributions arise from more complex material properties. Elastic predictions of yield may be made which then allows an estimate of the serviceability limit state.

The arch presented in Ch. 4 is to be modelled. The salient dimensions are identical to those used for the analysis of the large scale tests. The finite element method\(^{(152)}\) is used for the discretisation of the soil-arch system. The elements used are 8-noded isoparametric serendipity elements. The 8-noded element was used in conjunction with a 3 by 3, nine point, Gaussian integration routine. The characteristic geometry and a typical mesh to represent a thin section through the soil-arch system are shown in Fig. 6.1. The mesh shown in Fig. 6.1 models the arch ring as a continuous curve.

The element stiffness matrix is formed, using the necessary assumptions from the theory of virtual work. The global stiffness matrix is then formed in the usual manner. The procedure from this point onwards follows that of many previous finite element algorithms. These are all well documented elsewhere\(^{(152-4)}\) and shall not be discussed further. Sufficient investigation of the program and its characteristics had been undertaken by the author\(^{(149,150)}\) under the guidance of Ooi, at The University of Edinburgh, to explore the package’s abilities.

The arch and fill materials were given isotropic properties. No variation of moduli or Poisson’s ratio with depth was used. The densities used were those found from...
laboratory testing of the fill and brickwork materials described in Ch. 4 of this thesis. For the elasto-plastic investigation the yielding of the elements was governed by the Mohr-Coulomb yield criterion. Subsequent plastic flow behaviour was governed by an non-associated flow rule.

The material properties are given in Table 12. AFENA's element type 27, a plane strain isoparametric element, was used to model the fill and the arch. Where a road pavement was included, it too was modelled by element type 27. For increased mesh refinement under the load point(s), 7-noded transition elements were used. These elements could be placed at the rate of two per eight noded element edge, as the transition element does not have a lower mid-side node. Only minor modifications are required to the shape functions and the finite element solution algorithm for such elements.

In Table 12 the bulk and dry unit weights are equal as the arch fill is assumed to be well drained and the phreatic surface is assumed to be below springer level.

<table>
<thead>
<tr>
<th>Property</th>
<th>Arch</th>
<th>Fill</th>
<th>Pavement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus/ kPa</td>
<td>1x10⁷</td>
<td>1x10⁴</td>
<td>5x10⁵</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.20</td>
<td>0.40</td>
<td>0.35</td>
</tr>
<tr>
<td>Bulk unit weight/ kNm⁻³</td>
<td>21.5</td>
<td>14.5</td>
<td>14.5</td>
</tr>
<tr>
<td>Dry unit weight/ kNm⁻³</td>
<td>21.5</td>
<td>14.5</td>
<td>14.5</td>
</tr>
<tr>
<td>Coeff. of earth pressure at-rest</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The properties given in Table 12 represent typical values used for the analyses. For the parametric study some of these have been varied whilst the others have been held constant. Where this has been done, a note to this effect will be made in the text of the thesis. For the purposes of the simple elastic analysis, initial runs were done with no road pavement materials added. Later runs included a pavement. Where applicable, any mesh or material changes will be indicated clearly in the text.

The elasto-plastic analysis used the material properties of Table 12, including the pavement material, with an apparent cohesion of 1kPa, and an angle of shearing resistance was of 40°. This reflects the behaviour of a dry SAND fill, with no silt.
or plastic fines content. The apparent cohesion was set to 1kPa to prevent unduly premature yielding of the fill under very low stresses. Given a Mohr-Coulomb failure criterion for the fill, yield could occur at the surface of the strata where for numerical reasons, the principal stress ratio was made large by division by a very small number, the minor principal stress in this case. A value of 1kPa was found to be sufficient to prevent premature yield.

No attempt was made to model the interface between the arch extrados and the fill. The dry SAND fill was deemed to be sufficiently free flowing to remain in intimate contact with the arch at all times. The stress measurements of Ch. 4 provide evidence in support of this statement. Test meshes were developed which incorporated a Goodman joint\(^{(155)}\), element type 28 in AFENA, between the fill and the arch. The 6-noded, zero thickness, contact element failed to model the stress state on a curved surface. It was thought to be due to the lack of continuity of slope, and curvature, \(x\) and \(y\) being global \((x,y)\)-coordinates for the mesh. The development of the interface element for curved boundaries is the subject of further research at the University of Edinburgh.

Having completed the presentation of the basic finite element formulation, and described the types of tests carried out, this chapter continues by examining the results from the analytical study. The elastic analysis of the dead load stress state is discussed, followed by presentation of the results from the live load study. The live load study begins without a road pavement and then explores the effects upon the stress field of the addition of a road pavement. The elasto-plastic analysis is discussed briefly and then the finite element method is compared with the results from the arch bridge tests of Ch. 4.

### 6.3 Elastic analysis of the dead load stress state

This section presents and discusses the results from the simple elastic finite element analysis of the stress state in a soil-arch system. The investigation is original and necessary as it provides an accurate starting point from which live load analyses may be undertaken. Elastic analysis is sufficient to model the at-rest stresses as no yield is expected to occur for this condition. The differences between the geostatic stress state and the actual at-rest pressures are examined and are found to be small over the upper portion of the arch but more significant around the haunches and
springers. Nowhere is the difference so significant, relative to the magnitude of the live load stresses, that the geostatic stress state would become unsuitable as a starting point for a live load analysis.

The at-rest pressures are presented for various sections through the fill and the arch. The effects of varying the ratio of the fill's modulus to that of the arch is examined with respect to the changes in the stress field in and around the arch. Finally, the vertical, end-wall, boundaries are moved outwards to assess the effects of the proximity of these walls upon the stress state.

The notation and sign convention for the analyses are presented in Fig. 6.2. Stresses in the fill follow the sign convention and notation used in classical soil mechanics. Extrados contact stresses are expressed as $\sigma$, normal to the arch and $\tau$, tangential to the arch. For the stresses on sections radially through the arch, the sign convention remains the same but as the orientation of the section now lies at right angles to the extrados the directions of $\sigma$ and $\tau$ are rotated by 90° accordingly. This is clearly shown in Fig. 6.2.

### 6.3.1 Geostatic stresses or total body forces?

The facility exists, within AFENA, to switch on geostatic stresses or to allow the self-weight element body forces to be applied at the relevant nodes to set up the stress state arising from these body forces. Both these options were explored to ascertain which stress state is more applicable to the analysis of arch bridges. The stress state for the geostatic option is given in Eqns 15 to 17.

$$\sigma_{vertical} = \sigma_1 = \gamma_b z \quad \text{Eqn 15}$$

$$\sigma_{horizontal} = \sigma_3 = K_0 \gamma_b z \quad \text{Eqn 16}$$

$$\tau_{xy} = 0 \quad \text{Eqn 17}$$

Where $\gamma_b$ is the fill's bulk unit weight and $z$ is the depth below the fill's surface. The parameter $K_0$ is the coefficient of earth pressure at-rest, assumed to be 0.5 here.
6.3.2 Stresses in the fill

The vertical and horizontal stresses in the fill are shown in Fig. 6.3. Here the geostatic stresses as well as those arising from the applied total body forces are presented. The plots represent the stresses on sections H1 to H10, taken horizontally through the fill at the depths indicated on Fig. 6.4.

Away from the arch the stresses are seen to be close to geostatic values. Over the haunches, springers, and close to the extrados the confining effect of the arch alters the stress state. Generally the vertical stresses increase with depth and with increasing proximity to the extrados. Above the level of the crown, where the analysis can not yet "sense" the presence of the arch, the stresses are close to geostatic values. Where the arch is steep the vertical stress increases as the extrados is approached. Where the arch flattens, at x≈-0.80m or (x/r)≈-0.80, the stress does not change significantly as the extrados is approached.

The horizontal stresses are seen to fall towards the extrados whereas the vertical stress generally increased in magnitude. The horizontal stress increases with depth but the stress ratio is not constant with either depth or position for the non-geostatic, total element body force case. The ratio is never sufficiently far enough from 0.5 to invalidate the analysis based on geostatic stresses. Given the magnitude of the live load stresses the initial stress state is insignificant. Obviously, for the geostatic case the stresses are in the ratio: 0.5 horizontal stress to 1 vertical stress, because the coefficient of earth pressure at-rest, K₀ was set at a constant value of 0.5.

There are significant non-zero shear stresses introduced where the stresses are allowed to attain their free-field, non-geostatic values. The geostatic stress state has no shear stresses on horizontal and vertical planes. As will be seen in section 6.3.3, these shear stresses, and the omission of them for the geostatic case do not alter the normal and shear stresses on the extrados by a significant amount. The shear stresses in the fill are shown in Fig. 6.5.

The shear stress above the crown is small and unaffected by the presence of the extrados. Below crown level the shear stress increases with depth. The vertical boundary is assumed to be free to displace vertically without shearing restraint. The shear stress increases towards the arch, but decreases again as the extrados is
approached. This reflects the low interface shear stresses which will be seen on the extrados and discussed in section 6.3.3.

The effect of the arch, acting as a stiff inclusion in the fill mass, causes differences in the stresses observed from those predicted assuming a geostatic distribution of stress. As will be demonstrated in section 6.4, these stresses, and the difference between them and the geostatic stress field, are insignificant given the large live load stresses.

6.3.3 Stresses on the extrados

The stresses on the fill are transferred to the arch ring across the extrados-fill interface. This transfer of stress governs the increase in the stresses in the arch: this in turn governs the collapse load of the arch by determining at what live load the failure mechanism will develop. The normal and shear stresses, $\sigma$ and $\tau$, on the extrados are shown in Figs 6.6 and 6.7. The equivalent geostatic values are also presented.

The geostatic values are close to those observed when total body forces were used in the analysis. Over the crown of the arch the differences between the elastic predictions and the geostatic stresses are small. This arises because the fill above the crown is more likely to be under geostatic conditions. Between $(x/r) = \pm 0.30$, the arch profile is relatively flat and thus it forms little in the way of a foreign inclusion in the fill. It is in this region that the normal stress on the extrados is quasi-geostatic. Outwith $(x/r) = \pm 0.30$ the normal stress is greater than the geostatic values by as much as 3kPa or 50%. This is because the arch, by now steep in profile, forms a significant stiff inclusion in the fill mass. This stiff body attracts considerable stress because of its relatively high elastic modulus. The in-situ conditions are far from geostatic. The larger than expected normal stress is beneficial to the load carrying capacity of the arch because it ensures the dead load thrust line is closer to the intrados profile than it would have been assumed to be if geostatic stresses were used.

The live load has to then displace the thrust line further to form the four hinged failure mechanism. Greater displacement of the thrust line requires greater externally applied work: hence the higher collapse load obtained from the inclusion
of soil-structure interaction effects. Program ARCHIE\(^{(16)}\) could easily be modified to include this accurate dead load stress state; it currently uses a geostatic model to predict a dead load thrust line position.

The apparent roughness in the total body force version of the normal stress state around the extrados may be accounted for by the fact that the geostatic assumption totally ignores the presence of the bottom boundary whereas the finite element analysis can model the complete geometry including the bottom boundary. The stress fluctuation near the springers is not expected to have any significant effect on the overall prediction. The presence of a fixed boundary also affects the stress state predicted by the finite element model, which could easily examine the effects of different boundary conditions upon the predicted stresses. This effect was seen for those sections plotted in Figs 6.3 and 6.5.

Comparisons between the geostatic shear stress and that predicted by the use of total body forces may be made as follows: the shear stresses are similar over the crown where the arch is shallower, the limiting values of zero at the crown, where the extrados is horizontal, and zero at the springers, as well as the maxima encountered at \((x/r) = ±0.90\) are reproduced by both methods. The differences in magnitude are small relative to the live load stresses.

The reasons for the similarities in stress state over the upper portion of the arch are as discussed above for the normal stress state. It must be noted that the shear stress predicted using total body forces is greater than the geostatic version: for the normal stress this order was reversed over the upper portions of the arch. Where the normal stress was higher by geostatic calculations the shear stress was lower and \textit{vice versa}.

Differentiating the GEOSIM equation for \(\tau\) on the extrados with respect to position, \((x/r)\) and solving for the \((x/r)\) which gave the maxima and minima produced maxima at \((x/r) = ±0.88\). This compares favourably with the total body force predicted maxima at \((x/r) = ±0.90\). The geostatic value of \(\tau\) at these points is lower by some 40\%. This is because of the presence of the arch as discussed above. The maxima occur at the same position because the geometry of the soil-arch system is the same for both methods. The expected minima at the crown and \((x/r) = ±1.00\) are reproduced.
Further examination of the stress state around the extrados may be undertaken by considering the stress ratio and friction angle mobilised as a result of the predicted stress state. The ratio of the major principal stress to the minor principal stress versus position is plotted in Fig. 6.8. The associated mobilised friction angle is also plotted.

The local maxima seen at \((x/r) = \pm 0.30\) mark the transition between the shallow and steep portions of the extrados. Here, both the normal and shear stresses start to increase rapidly as the arch starts to affect the stress distribution more markedly. The peak flow value of 3.92 coincides with the greatest difference in the principal stresses. This also represents the largest friction angle mobilised around the extrados. The same local maxima at \((x/r) = \pm 0.30\) occur for the mobilised friction angle. This angle is calculated from Eqn 11 below with \(\tau\) and \(\sigma\) as before.

\[
\phi_m = \tan^{-1}\left(\frac{\tau}{\sigma}\right)
\]

Eqn 18

Nowhere does \(\phi_m\) exceed 40°, that being the angle of shearing resistance specified in the Mohr-Coulomb yield criterion. The final point about all the plots for the dead load stress state is that they exhibit reflective symmetry about an axis through \((x/r) = 0.00\) or the centreline of the mesh. This is to be expected given the symmetry of the self-weight loading.

The stress state presented here could easily be incorporated into current analyses; improvements to assessed capacity being derived from the use of an accurate initial stress field in the manner described within this section of the thesis.

To conclude the discussion of the results from elastic analysis of the dead load stress state on the extrados it may be said that the difference between geostatic and total body force versions of the stresses is so small as to be insignificant when compared to the magnitude of the live load normal and shear stresses. It must be borne in mind that the failure of soils and soil-structure interfaces is governed by stress ratio, or even stress difference. Also soil is very stress history dependent. If part of the soil yields under gravity loading, the stress path under live load would be quite different.
6.3.4 Stresses in the arch ring

The stresses in the arch ring under dead load only are examined. Two curved sections are used to plot various quantities around the arch. These are sections XX and YY as shown in Fig. 6.4. The normal and shear stresses are plotted for XX and YY in Figs 6.9 and 6.10 respectively. These follow the sign convention shown in Fig. 6.2 for the stresses in the arch.

Fig. 6.11 shows principal stress rosettes in the upper portion of the arch. This, in combination with the stresses plotted in Figs 6.9 and 6.10, infers that there is a linear distribution of stress radially through the arch ring. This produces the familiar stress block also shown in Fig. 6.11. The linear stress distribution through the arch is what would be expected from a classical structural analysis. Under self-weight the stresses are compressive everywhere. The principal stress rosettes follow the arch profile: this relates to the well documented dead load thrust line profiles, all of which follow the arch's profile. The magnitude of the major principal stress, akin to the normal stress of Fig. 6.9, is greatest along the intrados. This too shows the location of the dead load thrust line.

Some principal stress rotation was seen at the springers where the fixed boundary of the mesh provided the reaction to the self-weight of the arch. This was not worthy of further comment. No discussion of the stress ratio, or mobilised friction angle is relevant here as the masonry is sufficiently stiff and strong to eliminate the radical stress changes associated with material failure by ensuring that it can not possibly yield. Justification for this is provided by Heyman, who assumed an infinitely strong material, resistant to crushing under the arch's compressive thrust. Intuitively, failure of the arch material would not be expected under the self-weight stress regime. As such the stresses in the arch ring need no further analysis.

6.3.5 The effects of the relative stiffnesses of the arch and the fill upon the predicted stress state

The relative stiffness of the arch and fill is defined by Eqn 19:

\[ m = \left( \frac{E_a}{E_s} \right) \]  

Eqn 19
The values examined were as follows: \( m = 1 \times 10^3 \) and \( 1 \times 10^5 \). Dead load stresses were calculated from the total body forces and not simple geostatic analysis. The stresses in the fill were unaffected by changes in modular ratio, \( m \) when it was varied by way of altering the modulus of \( \text{ii. arch, } E_a \). The graphs in Fig. 6.12 show how little the normal stress on the extrados was affected by an increase in \( E_a \). Similar small changes were observed for the shear stress on the interface. The stiffer arch caused marginally higher normal stresses on the extrados and marginally lower shear stresses. Over the upper portions of the arch the stresses were not affected by the modular ratio, \( m \).

When \( m \) was altered by changing \( E_a \) the stresses were affected to a greater extent but still not significantly, especially under dead load only conditions. Whether \( m \) was increased by altering \( E_a \) or \( E_s \), the stiffer the arch, the higher the normal stress on the extrados. Where the extrados was subjected to higher normal stresses, less load transfer occurred in shear tangential to the interface. The stress ratios around the extrados for different \( m \) values followed the trends of Fig. 6.8 with small decreases in \( N_b \) being observed close to the springers when the arch stiffness was increased. These differences were not significant and need not be discussed further until application of this analysis is applied to the live load cases in section 6.4.3.

6.3.6 The effects of boundary proximity

For this study the \( x \)-coordinates of the vertical end wall boundaries were changed from \( x = \pm 1.625 \text{m} \) to \( x = \pm 4.00 \text{m} \). The parameters investigated were those governing the stress transfer from fill to arch, namely: \( \sigma \) and \( \tau \) on the extrados. If the boundaries were too close to the arch at \( x = \pm 1.625 \text{m} \), then significant changes to \( \sigma \) and \( \tau \) would be found as the boundaries were moved further away to \( x = \pm 4.00 \text{m} \). An analysis using the total body forces was carried out and the resulting stress changes may be seen in Figs 6.13 and 6.14.

Fig. 6.13 shows only small changes in normal stress, \( \sigma \) for a large change in the boundary's position. Fig. 6.14 shows the same trend for shear stress, \( \tau \). For a detailed study of this nature, confirmation had to be sought as to whether the stress state changes would still be comparably insignificant for various degrees of mesh refinement and various material properties. Several analyses were done to ensure
that, under dead load, no significant changes were observed for the two chosen boundary positions. At no point could the stress changes be deemed significant.

These results are confirmed by the vibrating wire gauge readings taken on the end walls of the model described in Ch. 4 of this thesis. Evidence from the small scale tests of Ch. 3 adds support to this conclusion because if no significant stress changes were found by altering the boundary position, then the end wall separation was sufficient to prevent its influencing the results. This effectively concludes the examination of the dead load stress state; the thesis continues by using the mesh, boundary conditions, and boundary positions of Fig. 6.1 to analyse the effect of live load upon the system.

6.4 Analysis of the live load stress state

This section examines the effects of live load position and magnitude, relative stiffness (as governed by the modular ratio, m, Eqn 19), fill depth over the crown, the addition of stiffer road pavement strata, and the relative stiffnesses of the fill and pavement, upon the behaviour of the soil-arch system. Finally the preliminary elasto-plastic analysis of the stress state is presented. The mesh of Fig. 6.1 is used throughout, with the exception of the analyses carried out to investigate the effects of adding a stiffer road pavement. For these analyses, extra material properties and transition elements were added as specified in section 6.1 and Table 12.

6.4.1 Stresses in the fill

This section examines the stresses in the fill, on the extrados, and finally, in the arch ring. This follows the order in which the stresses are transmitted from the applied load to the arch. For the purposes of this section the mesh of Fig. 6.1 is loaded at \((x/r) = -0.33\). Loading was applied in stages. The magnitude of the applied stress at any stage was the same as that outlined in Table 6 of Ch. 4 for the loading on the 2m span semicircular arch. The stresses along the various horizontal sections shown on Fig. 6.4 are used to present the information about the stress state in the fill under live loading at \((x/r) = -0.33\). The stresses along H1 are shown in Fig. 6.15. Little change in any of the three stresses plotted was observed. This is to be
expected as the stress is dispersed through the fill above H1 to such an extent that the applied load has no significant effect.

Sect. \( \eta \) H2, at a depth of 1.01m, exhibited some signs of stress increase with live load increase. The stresses along H2 may be seen in Fig. 6.16. The vertical stress increase on the left hand, loaded side of the arch is greater than that seen on the side of the arch remote from the load. This is a function of simple stress dispersal effects rather than an indication of the change in mobilisation of lateral earth pressures in the fill. The effect of mobilisation of higher lateral earth pressures on the "passive" side of the arch may be seen in the plot of horizontal stress along H2. Here the horizontal stress is clearly greater on the remote side of the span.

The trends discussed above continued for sections H3 to H6. Fig. 6.17 shows the stresses on H7, immediately below crown level. The vertical stress on the loaded side of the arch is large because the available depth of fill for stress dispersal is now only 0.21m. On the unloaded side of the arch the vertical pressure exhibits no change under an increasing live load. The points on the right hand side of the mesh are now too far away from the load to be directly influenced by the load spread. It is interesting to note that the horizontal stresses on the remote side of the arch still increase as the arch deforms into the fill under load. Where these stresses have been proven, by considering the vertical stresses, not to have arisen as a consequence of the load spread, they may be said to have been a product of lateral pressure mobilisation.

Section H8, immediately above the crown is examined and the associated stresses are plotted in Fig. 6.18. The vertical stresses are tending towards the classical Boussinesq "bell shaped" load distribution. The limits of the pressure bulb may also be seen to be wider than those used in the MEXE analysis where a "1 in 2" side slope is assumed.

6.4.2 Stresses on the extrados

The total stresses, \( \sigma \) and \( \tau \), normal and tangential to the arch respectively are plotted in Fig. 6.19. These stresses result from a live load applied at \( (x/r) = -0.33 \). The normal stress is seen to peak at an influence value of 0.595 times the applied stress, \( q \). The peak values occurred at \( (x/r) = -0.25 \), closer to the crown than the load, for
the reasons discussed in Chs 3 and 4 of this thesis. The limits to the load spread were found to be \(-0.58 \leq (x/r) \leq -0.07\). This was considerably wider than MEXE's load spread. The normal stress on the remote side of the arch increased slightly with added live load; it did not increase by as much as would have been expected based on the evidence of the horizontal stresses in the fill.

The effect of scale on the graph should also be considered; the stress change on the remote side of the arch will be shown to be close, at low loads, to the experimental results when a more suitable scale is chosen. On the scale needed to show the peak stresses on the loaded side of the arch, the relatively small increases on the other side are dwarfed. More important for the purposes of this section is the live load dispersal effect and its quantification.

The shear stress on the extrados behaves as would be expected. The shear stress changes direction on the side of the arch remote from the load point. The shear stress over the crown increases from zero as the live load is increased. The increase is small: influence values of less than 0.1 are recorded at the crown. The peak shear stress is at \((x/r) = -0.35\), directly beneath the load point. This peak recorded an influence value of 0.217: less than the peak influence value for the normal stress. The shear stress also changed sign on the right hand side of the load point: indicating a change in direction of the shear stress. The influence value corresponding to the negative peak was -0.186.

The stress distributions described here should be used for the future analysis of arch bridges. Failure to use the correct stress state around the extrados can result in costly imposition of unduly conservative axle limits on bridges capable of bearing much more load.

Currently, engineers are asked to use a load spread of "1 in 2" to comply with the Department of Transport's standard\(^{(14)}\). Fig. 6.20 shows the load distribution through just the fill, with no road pavement added, for the soil-arch system analysed here. The normal stress increase occurs between \((x/r) = -0.58\) and -0.07. The shear stress increase occurs between \((x/r) = -0.90\) and \((x/r) = 0\). As is shown in Fig. 6.20, both normal and shear stresses occur over significantly wider areas giving much wider than "1 in 2" side slopes. The slopes are not symmetrical because the arch is curved. Therefore the load to extrados distance is constantly changing with \((x/r)\). There is more fill to the left of the load in which stress dispersal can take place. The
arch's confining effect is more severe as the load to extrados separation is reduced on the right of the load.

The MEXE method allows no shear to develop at the extrados. MEXE allows only vertical stress to develop at the extrados, this is clearly incorrect as it defies equilibrium. The estimate of vertical stress has been seen to be conservative, to the detriment of the current arch bridge assessment programme. The stresses have now been quantified and presented in Figs 6.19 and 6.20. As such they may be used to increase a bridge's assessed capacity by providing an accurate estimate of the load transmitted through the fill onto the arch. Further assessed capacity increases will arise from the addition of a relatively stiff road pavement layer.

6.4.3 Stresses in the arch

The stresses in the arch ring are plotted for curved sections XX and YY, in Fig. 6.21. Naturally, the magnitude of the stresses would increase if the arch modulus were to be increased. Under the load where the stress on YY, the outer section, is large, the corresponding value on XX is small. This reflects the stress block plotted in Fig. 6.11 for the dead load stress state. It is similar to that found for continuous curved beams and bars\(^{156}\). It reflects the structural behaviour of the arch ring. There is compression on the extrados face under the load but tension on the intrados face. This leads to the conditions for hinge formation. The stresses are also presented in Fig. 6.22 for the seven sections radially through the arch ring. These show the stress block to good effect. They are used to plot the thrustline and predict the hinge positions: this is also shown in Fig. 6.22. The experimentally observed hinge locations are superimposed.

6.4.4 The effect of load position upon the system's behaviour

This section examines the stresses in the fill, on the extrados, and finally, in the arch ring. This follows the order in which the stresses are transmitted from the load to the arch. The mesh of Fig. 6.1 is loaded at \((x/r) = 0.00, -0.25, -0.33, \text{ and } -0.50\). Loading was applied in stages. The magnitude of the applied stress at any stage was the same as that outlined in Table 6 for the loading on the 2m span semicircular arch.
6.4.4.1  Stresses in the fill

The vertical stress on section H8, see Fig. 6.4, is used to illustrate the different load dispersal patterns arising from the application of the load at different points. The vertical stresses are plotted in Fig. 6.23 for each load point. The greatest peak stress occurred at the crown for a load at \((x/r)=0.00\). This maximum arose because of the proximity and confining effect of the arch and also because of the minimal depth of fill available for stress dispersal. As the load is positioned further away from the crown the peak value decreases and the load spread widens. The stress to the left of the load points is seen to be less than that to the right. This is because the arch is closer to the load points on the right hand side than the left. Therefore less fill is available for stress dispersal and the confining effect is made more significant.

The relative effect of the load point may be seen when influence values for the vertical stress are considered. These are presented in Table 13.

<table>
<thead>
<tr>
<th>Load pos'n</th>
<th>Peak (\sigma_{vertical}/) kPa</th>
<th>Influence value</th>
<th>Relative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>114</td>
<td>0.990</td>
<td>1.000</td>
</tr>
<tr>
<td>-0.25</td>
<td>111</td>
<td>0.964</td>
<td>0.974</td>
</tr>
<tr>
<td>-0.33</td>
<td>101</td>
<td>0.877</td>
<td>0.886</td>
</tr>
<tr>
<td>-0.50</td>
<td>90</td>
<td>0.781</td>
<td>0.789</td>
</tr>
</tbody>
</table>

The influence values were calculated by dividing the predicted stress by the applied stress of 115.2kPa. The relative values arise from the division of the influence value by 0.990, that being the peak influence value found. The stresses in the fill are then transmitted to the extrados and these will be discussed in terms of the normal stress \(\sigma\) in the following section.

6.4.4.2  Stresses on the extrados

These stresses, \(\sigma\) and \(\tau\), on the extrados form the principal part of the study. Their importance arises from them being the stresses that govern the load transfer from the fill to the arch. Their variation is shown in Fig. 6.24 for each of the four load points.
For loads distant from the crown considerable dispersal occurs. Not only are the peak normal and shear stresses lower but the load spread is much wider when points away from the crown are considered. These load spreads and stress distributions may be scaled to any arch of these material properties and relative geometry for use in assessment methods or design codes. Normalization of the stresses may be achieved by the use of influence values as presented in Table 14.

<table>
<thead>
<tr>
<th>Load pos'n</th>
<th>Peak σ/ kPa</th>
<th>Influence value</th>
<th>Relative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>112</td>
<td>0.972</td>
<td>1.000</td>
</tr>
<tr>
<td>-0.25</td>
<td>86</td>
<td>0.747</td>
<td>0.769</td>
</tr>
<tr>
<td>-0.33</td>
<td>72</td>
<td>0.625</td>
<td>0.643</td>
</tr>
<tr>
<td>-0.50</td>
<td>53</td>
<td>0.460</td>
<td>0.473</td>
</tr>
</tbody>
</table>

It may be seen that the stresses on the extrados are below those acting vertically on section H8. This is to be expected due to the consideration of stress dispersal effects.

6.4.4.3 Stresses in the arch

The reduced extrados stresses arising from increased load dispersal cause proportionally lower arch ring stresses. The shapes of the stress distributions around XX and YY were reproduced with the relevant peak stresses shifted according to the applied load's position. The stress blocks were unchanged in shape, only in magnitude were they different from that discussed above. It must be remembered that any change in arch modulus would result in its attracting a higher stress for a given external work input. Suffice to say that the reduced extrados stresses arising from increased load dispersal for loads away from the crown are beneficial to the assessment of arch bridge capacity.
6.4.5 The effects of the relative stiffnesses of the arch and the fill upon the predicted stress state

The mesh of Fig. 6.1 is loaded, in stages, at \((x/r) = -0.33\) with the magnitude of the applied stress at any stage as outlined in Table 6 of Ch. 4. In this study the modular ratio was varied by altering \(E_s\). The following values of \(E_s\) were used in conjunction with an arch modulus, \(E_a\) of \(1 \times 10^7\) kPa: \(1 \times 10^4\) kPa, \(4 \times 10^4\) kPa, \(7 \times 10^4\) kPa, and \(10 \times 10^4\) kPa. These gave modular ratios as follows: 1000, 250, 143, and 100.

The normal and shear stresses on the extrados are used to illustrate the effects of \(m\) upon the system's behaviour. The results are shown in Fig. 6.25. Little difference in peak stress was noted for the range of \(m\) values analysed. The stiffer fills dispersed the load more and caused a reduction in the stress transmitted to the arch. The stress reduction caused by inclusion of a stiffer fill had less of an effect than would be expected.

The lack of effect of \(E_s\) upon the interactive behaviour of the soil-arch system is not surprising for the range of moduli chosen. Extreme values of \(m=1\), or infinity, could be analysed to determine the limits within which the stresses could possibly vary with the variation of \(E_s\). Such analyses provide little information of relevance to practical arch bridge assessment problems. To conclude this section it can be said that the soil's modulus, over the range of values analysed here, has little effect upon the capacity of the arch.

6.4.6 The effects of increased fill depth over the crown

This section leads into the study involving the addition of road pavement strata. The results are essentially the same: whether the stress dispersal is caused by an increase in fill depth, or whether it arises from the superposition of a stiff pavement layer is irrelevant. The dominant parameter under examination is still the stress state around the extrados: it is this stress state that governs the capacity of the soil-arch system to withstand external load. Increasing the fill depth caused stress reduction on the extrados, even when allowance was made for the dead load stress increase due to the extra surcharge load from the fill.
6.4.7 The effects of the addition of a road pavement

The mesh of Fig. 1, with a 75mm thick road pavement replacing the top 75mm of fill, is loaded at \((x/r) = -0.33\). Loading was applied in stages as outlined in Table 6. The pavement material properties are given in Table 12. These were chosen to suit the wide range of pavement material properties encountered in practice\(^{157}\). The selection of moduli and Poisson's ratios must be done with due care: the range of moduli encompasses that of subgrade soils of low stiffness to asphaltic cements of very high stiffness.

6.4.7.1 Stresses in the fill

Section H8 was used to show the stress decrease arising from the addition of a road pavement above the fill. Without a pavement, the peak vertical stress was 82kPa: addition of the pavement having 50 times the stiffness of the fill reduced this to 56kPa. This equates to a reduction in vertical stress of 32\% of the fill only peak value. Similar stress reductions were observed along the other sections but were not any more worthy of comment than those described on section H8. The vertical stress decrease will manifest itself as a reduced stress on the extrados.

6.4.7.2 Stresses on the extrados

The pavement caused increased stress dispersal at section H8 as described above. This caused lower stresses to be transmitted to the extrados. The total cover to the crown was the same whether or not a pavement was used. The pavement merely changed the properties of the upper layers of what were fill elements in the original, unaltered mesh. This ensured that the stress dispersal observed was solely that arising from the increase in stiffness of the upper surface of the mesh. The dual effects of stress dispersal from increased depth and that caused by increased stiffness were thus separated. The normal and shear stresses on the extrados are shown in Fig. 6.26 for a mesh having a pavement 50 times as stiff as the underlying fill.

The peak normal stress was seen to drop from 67.5kPa to 54.5kPa. This equated to a decrease of 19\% of the original fill only value. Such a decrease in applied stress on the extrados would result in a considerable capacity increase. The load spread
angle was increased with the addition of the pavement. This too would be beneficial
to an arch bridge's assessed capacity. The influence values for peak normal stress
were: 0.586 for the fill only case and 0.475 when the pavement was included.

The shear stresses on the extrados were also reduced. The reduction of both shear
and normal stresses was such that the mobilised friction angle, influenced by the
ratio of $\tau$ to $\sigma$, was not affected significantly: $\phi_m$ dropped slightly on all points
around the extrados. The implication of this was that the stresses were marginally
further from those required to cause yielding of the soil-arch interface.

The reduction in normal stress, 19% of the fill only value, was not as large as the
reduction of stress in the fill (32%) caused by the pavement. This discrepancy
occurred regardless of the fact that the extrados was at greater depth than any point
along section H8. This may be explained as follows: the depth to the extrados was
greater therefore the dispersal was greater and the stresses were lower at extrados
level giving less scope for percentage reduction in stress. Also, the extrados marks
the boundary between the weaker fill and the relatively rigid arch. Such a rigid
inclusion tends to cause stress concentration, this would have made the percentage
decrease in stress lower from fill to extrados than that observed from pavement to
fill.

6.4.7.3 Stresses in the arch

The addition of a pavement with 50 times the stiffness of the fill caused the
reduction of the stresses in the arch. The peak normal stress in the arch fell from
92kPa to 82kPa, a reduction of 11%. The peak shear stress fell from 39kPa to
32.5kPa, a reduction of 17%. The shear stress drops by more because of the change
in direction of the stresses in the arch: the shear in the arch is more akin to the
normal stress on the extrados in terms of direction of line of action.

The migration of the location of peak stress towards the crown, seen on the extrados
stress plots in Fig. 6.26, is due to stress dispersal considerations. Some combination
of depth and horizontal distance from the load point is more critical than a point
directly beneath the load's centreline.
6.4.7.4 The effects of the relative stiffnesses of the pavement and the fill upon the predicted stress state

The mesh plus pavement was analysed with a total of 0.150m crown cover. Various pavement moduli, \( E_p \) were used, the pavement modular ratio, \( m_p \) is given in Eqn 20 in terms of fill and pavement moduli.

\[
m_p = \left( \frac{E_p}{E_r} \right)
\]

Eqn 20

The following values of \( m_p \) were used: 1, 10, 50, and 100. The \( m_p \) value of 1 was the analysis done without the pavement, each element above the arch used only the fill's properties. The stress state on the extrados will be used to illustrate the effects of using different stiffnesses of pavement. The normal and shear stresses are plotted in Fig. 6.27 for each different pavement used. The peak stresses and associated influence values are presented in Table 15.

<table>
<thead>
<tr>
<th>( m_p )</th>
<th>Peak ( \sigma ), kPa</th>
<th>Influence value</th>
<th>Relative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>67.5</td>
<td>0.586</td>
<td>1.000</td>
</tr>
<tr>
<td>10</td>
<td>65.0</td>
<td>0.564</td>
<td>0.962</td>
</tr>
<tr>
<td>50</td>
<td>54.5</td>
<td>0.473</td>
<td>0.807</td>
</tr>
<tr>
<td>100</td>
<td>49.0</td>
<td>0.425</td>
<td>0.726</td>
</tr>
</tbody>
</table>

In the region between the crown and the load point, the normal stress peaked. This peak was reduced with successive increases in pavement stiffness. As no depth, or cover, changes were made to the mesh between analyses, the increase in dispersal arising from this pavement stiffness increase causes the stress decrease. The width of the load spread was also increased as the pavement stiffness increased.

Such stress distributions lead to the conclusion that the use of bituminous pavement overlays could be an economical way of increasing the assessed capacities of arch bridges. The experimental evidence of the bridge test at Kimbolton Butts\(^{101}\) shows a large amount of stress dispersal through the road pavement. The author's paper on the use of pavement overlays as a means of increasing assessed capacities\(^ {141} \) also shows how current, possibly conservative, assessment methods predict considerable...
axle limit increases with the addition of stiffer road pavements or increased crown cover.

6.5 Elasto-plastic analysis of the stress state on the extrados

This section looks at the effects of allowing fill material failure under live load application. It does this in two ways: firstly it examines the elastic prediction of yield, and secondly it uses the relevant elasto-plastic, non-infinite cohesion, material properties to allow yield to occur.

The elastic prediction of yield was carried out as follows: the yielded zones in the fill were plotted for different live load stages, and then the mobilised friction angle, \( \phi_m \), was plotted for a given applied load.

The elasto-plastic analysis used an apparent cohesion of 1kPa, just sufficient to prevent yield under geostatic stresses, to model the fill. The live load was incremented in 100 stages to 115.2kPa. The stress distribution on the extrados was used to compare with that observed in the elastic analysis(Fig. 6.19).

6.5.1 Elastic prediction of yield

Fig. 6.28 shows the elastic prediction of yield zones at the percentages of the 115.2kPa applied stress indicated on the contours. The fill directly beneath the load platen is prevented from yielding by the confining effect of the load itself. The yield zones begin on either side of the loaded area. There are two distinct yield zones. The area beneath and to the right of the load yields because of the radially inwards movement which tended to change the stress from the at-rest values. The yield zone on the side of the arch remote from the load point was caused by the tendency for the extrados to move radially outwards into the fill. This movement caused the stress to change from the at-rest values; stress increases indicating partial passive pressure mobilisation were observed. This was coincident with the location of the second yield zone. Good correlation between these yield zones and those seen behind the small scale models described in Ch. 3 of this thesis was noted.
Fig. 6.29 shows the development of the yield zones at different applied stresses. The mobilised friction angle, $\phi_m$ at 10% of the 115.2kPa applied live load stress is indicated by the contour values in Fig. 6.29. Again two distinct yield zones are seen. The displacements are small at only 10% of the total applied stress therefore only the crown of the arch suffers the effects of the load. At larger loads, the arch deforms more and it also deforms at points closer to the springings. This was indicative of the progressive onset of yield.

6.5.2 Elasto-plastic analysis of the soil-arch system

The normal and shear stresses on the extrados are used to illustrate the effects of including soil yield in the analysis. The stress state on the extrados remained the same as that derived from the simple elastic analysis until stage 4 loading. This applied a stress of 65kPa, or 56% of the final stage 9 loading to the fill’s surface. Beyond this applied load the stress redistribution associated with yielding caused a reduction in applied stress and a large increase in the deformations of both the fill and the arch. The stresses from the elastic and the elasto-plastic loading are compared at stage 6 loading in Fig. 6.30. The stress generally fell as the fill yielded.

The stress relief effect would result in increased assessed capacities. However little discussion of this is included in this thesis. It is felt that the elastic analysis is sufficient for the safe assessment of arches. The deformations in the elasto-plastic analysis are primarily dependent on the arch modulus. This thesis does not seek to examine the properties of the arch; in the main it is interested in the soil-structure interaction effects occurring around the arch.

A relatively weak arch would suffer gross deformations under load: the ensuing stress reduction in any elasto-plastic analysis would be correspondingly large. As the arch displacements are rarely known in practice, the quantification of the stress state on the extrados is impossible. Use of an elasto-plastic analysis could not therefore always be recommended as it could predict unduly low stresses on the extrados which would lead to an overestimate of the arch’s capacity.
6.6 Comparisons with experiment & assessment improvements

The stress state on the extrados will be compared with that observed experimentally in the 2m span arch described in Ch. 4. Secondly the influence values for $p_e \times k$ stress normal to the extrados will be used to suggest improvements to current assessment techniques by incorporating the interactive behaviour of the fill and the arch.

6.6.1 Comparison with 2m span tests

The stress state on the extrados will be used here to illustrate the efficacy of the elastic finite element analysis. The normal and shear stresses on the extrados are shown at stages 1 and 4 in Figs 6.31 and 6.32. The elastic finite element prediction of the stress state can be seen to be in excellent agreement with the experimental results. Comparisons have already been made in Ch. 4 between experimental results and those derived from Boussinesq$^{(123)}$ and BD21/93$^{(14)}$ analyses. These results show that the finite element method has provided an excellent prediction of the stress state up to stage 4 loading. It also provided the only means of modelling the normal stress increase on the side of the arch remote from the load platen.

Beyond stage 4 live loading the stress states diverged rapidly with increasing load. This represents the limit to the validity of the elastic analysis. This also coincided with the fact that the stresses were equal until stage 4 loading had been applied regardless of whether or not an elastic, or an elasto-plastic analysis was used. In the elasto-plastic analysis, beyond stage 4, the stresses differed significantly from those predicted by the elastic analysis.

The shear stress at stage 1 is predicted well by the finite element analysis; by stage 4 the analytical values differ from those observed experimentally. It must be remembered that the experimental results were subject to small error bounds which could render the comparisons marginally better, or worse, at any given stage. The elasto-plastic analysis proved no better at modelling the gross deformations observed in the tests of Ch. 4.

That the finite element analysis predicted almost exactly, those stresses acting at low loads was surprising given the material properties, the difficulties of modelling
interactive behaviour numerically, and the presence of a curved interface in the mesh. The main conclusion derived from the comparisons made here is that the normal stress predicted by the finite element analysis is in close agreement with experimental observations and the shear stress is slightly less well predicted in this case.

The other information gleaned from the finite element study may be said to be accurate because the extrados stresses have proved the efficacy of the analysis. The extrados stresses are the hardest to model numerically: evidence for this is provided by the number of finite element analysis papers dealing with interface modelling\(^{155,158-162}\). References 158 to 162 are given because they themselves provide excellent bibliographies for the reader interested in this particular area of numerical modelling.

6.6.2 Improvements to current assessment methods

The results of the finite element analysis, in particular those relating to the normal stresses on the extrados, have been used to suggest improvements to the MEXE method. A design chart is presented below showing the assessed capacity increases arising from use of the simple MEXE method with inclusion of the soil-structure interaction effects. This chart is plotted in Fig. 6.33.

The axes in Fig. 6.33 represent the fill depth at the crown and the capacity, as assessed by the MEXE method. Superimposed on the horizontal axis are two sets of influence values for the peak stress normal to the extrados, \(\sigma\). The lower set are derived from the MEXE method. The upper set are the result of the elastic finite element analysis based on third span loading. The influence values show how the applied load causes progressively less normal stress on the extrados as the cover to the crown is increased. The assessed capacity of the arch rises accordingly. The MEXE stress influence values were calculated using the "1 in 2" side slope for stress dispersal.

The finite element analysis was used to provide predictions of the influence values for this peak normal stress. These are also plotted on Fig. 6.33's horizontal axis. They represent the influence values, and hence the stress state, obtained when soil-structure interaction effects were included in the modelling of the system. For a
given crown cover the finite element analysis influence values were lower than those predicted by the MEXE method.

The procedure for the use of the chart in Fig. 6.33 is as follows:

1. Calculate the MEXE capacity in the usual manner.

2. Read off the corresponding influence value from the finite element method.

3. Move along the horizontal axis until the MEXE influence value equal to the finite element influence value, from 2, above is found.

4. Move vertically upwards until the MEXE capacity corresponding to this modified influence value is reached.

5. Read off the modified MEXE capacity which incorporates the soil-structure interaction effects inherent in the finite element analysis.

An example of this procedure is given in Fig. 6.33 with arrows and numbers showing the sequence of events associated with steps 1 to 5 above. This has been carried out for a range of crown covers and the capacity increase arising from the use of interactive effects has been shown in Fig. 6.34.

The capacity increase over the simple MEXE method is considerable. The method is perceived as being "safe": it merely increases the MEXE assessed capacity by including an improved representation of the stress state on the extrados. The finite element analysis influence values also include the effects of lateral earth pressure redistribution mobilised as a result of arch deformations.

Fig. 6.33 could easily be redrawn to show the ARCHIE(16), MARCH(77), or MAFEA(29) versions of the capacity increase with crown cover. The influence values would have to be amended for each different method depending on the stress dispersal model used by each respective method. The ensuing capacity increases with these more sophisticated methods would not be as large as those found when adding interactive effects to the MEXE method. The bridge of Ch. 4 has its assessed capacity increased from 7.77kNm⁻¹ to 13.2kNm⁻¹ with the inclusion of
interactive effects. This then renders the MEXE result only 40% lower than the experimentally observed collapse load for the 2m span arch.

The chart in Fig. 6.3 is presented as the simplest way of including the beneficial effects of the soil-structure interaction. As such it can easily be used by the assessing engineers without the time required for expensive finite element analyses, hand analysis of stress dispersal, or complex interactive analyses involving correlation of pressures with observed displacements. The chart provides a convenient means of assessing arch bridges with some allowance for the interactive effects imposed upon the system by the fill's presence.

6.7 Conclusions

1. Elastic finite element analysis was successfully used to model the dead load stresses in the soil-arch system.

2. Geostatic stresses provided an adequate starting point for an analysis. The use of stresses derived by considering total body forces was not deemed necessary.

3. The stress state in the fill and around the extrados was found to be significantly affected by the presence of the arch. These effects were attributed to, and quantified in terms of, postulated and observed interactive behaviour.

4. The arch's elastic modulus did not affect the predicted fill or extrados stresses significantly.

5. The finite element mesh was found to have adequate boundary conditions and end wall separations for the consistent modelling of the soil-arch system.

6. The elastic finite element analysis successfully predicted the experimentally observed stress state until a stress of 65kPa had been applied. This represents 56.4% of the collapse load observed in experiments.
7. The stress states on the extrados were presented in a format suitable for use in arch assessment routines, *i.e.* normal and shear stress distributions on the extrados.

8. The elastic finite element analysis proved that the addition of a road pavement overlay enhanced the assessed capacity by increasing the stress dispersal between the load point and the extrados.

9. The stresses on the extrados decreased significantly as the elastic modulus of the pavement was increased.

10. The elastic analysis gave satisfactory predictions of yield zones in the fill. These showed good correlation with those observed in small scale model tests.

11. The elasto-plastic finite element analysis provided satisfactory predictions of yield zones in the fill: it did not provide useful results for the stress state on and around the arch.

12. The analyses were used to produce a chart showing how the assessed capacity increased with the inclusion of soil-structure interaction effects.

13. The MEXE method was used to demonstrate the assessed capacity increase. Analytical values for the peak normal stress on the extrados were used to give the assessed capacity increase by assuming these stresses could be said to be acting in the MEXE method.

14. The capacity increase could easily be calculated, and presented in similar design chart form, for other more sophisticated arch assessment and analysis methods.
Figure 6.1  Typical finite element mesh

Figure 6.2  Sign convention and notation
Figure 6.3  Vertical and horizontal stresses in the fill

Figure 6.4  Location of sections for stress plots
Figure 6.5  Shear stresses in the fill, dead load only

Figure 6.6  Normal stress on the extrados, dead load only
Figure 6.7  Shear stress on the extrados, dead load only

Figure 6.8  Mobilised angle of shearing resistance and stress ratio on the extrados
Figure 6.9  Normal stress, sections XX & YY, dead load only

Figure 6.10  Shear stress, sections XX & YY, dead load only
Figure 6.11  Principal stress rosettes and dead load stress block

Figure 6.12  Normal stress on the arch for different $E_a$ values, dead load only
Figure 6.13  The effect of boundary proximity on $\sigma$, dead load only

Figure 6.14  The effect of boundary proximity on $\tau$, dead load only
Figure 6.15  Stresses on H1, load at (x/r)=-0.33

Figure 6.16  Stresses on H2, load at (x/r)=-0.33
Figure 6.17  Stresses on H7, load at \( (x/r) = -0.33 \)

Figure 6.18  Stresses on H8, load at \( (x/r) = -0.33 \)
Figure 6.19  Stresses on the extrados, load at \((x/r) = -0.33\)

Figure 6.20  Stress distributions on the extrados
Figure 6.21  Stresses in the arch

Figure 6.22  Stresses on radial sections through the arch
Figure 6.23  Vertical stress on H8, various load points

Figure 6.24  Stresses on the extrados, various load points
Figure 6.25 Normal stress on the extrados for different $E_s$ values

Figure 6.26 Stresses on the extrados with a road pavement added
Figure 6.27  Stresses on the extrados for various pavement moduli

Figure 6.28  Yield zones at % increments of the applied stress
Figure 6.29  Yield zones at 10% total applied stress, contours of $\phi_m$

Figure 6.30  Stress on the extrados: elastic and elasto-plastic analyses
Figure 6.31 Extrados stresses: FEA and experiment, stage 1

Figure 6.32 Extrados stresses: FEA and experiment, stage 4
Figure 6.33  Capacity increase based upon influence values for $\sigma$

Figure 6.34  Capacity increase arising from inclusion of interactive effects
Chapter 7  Conclusions

7.1 Introduction

This chapter presents the summary conclusions derived from the findings presented in the thesis. It represents an amalgamation of the conclusions written at the end of each chapter. It serves to eliminate duplication of the conclusions drawn thus far and to render each chapter's conclusions applicable to the entire thesis.

7.2 General Conclusions

The purpose of the project was to quantify the extent of the interactive effects present in a soil-arch system with a view to their use in improved assessment methods. To achieve this objective the work has been carried out in phases which have been described in this thesis and the appropriate conclusions drawn where necessary. The following general conclusions can be made:

1. Zones of fill displacement have been identified behind a loaded arch.

2. Typical normal and shear stress distributions on the arch extrados have been produced for possible use in bridge assessments.

3. The stress state around a loaded arch has been correlated with measured arch displacements.

4. Simple finite element analysis has been used to identify the importance of various parameters in the soil-arch system.

7.3 Specific conclusions

1. Soil-structure interaction effects contributed significantly to the load carrying capacity of the arch bridge models analysed in this thesis.
2. The models were best able to illustrate two of the postulated soil-structure interaction effects: load dispersal and lateral earth pressure distribution.

3. The collapse load increased as the fill depth over the crown increased. The increase in collapse load with increasing fill depth was made up of contributions from increased dead load and increased live load dispersal; the increase arising from the dispersal being predominant.

4. The collapse load increased as the fill density increased. This effect is linked with 3 above.

5. The minimum collapse loads were found to occur for load points between \((x/r)=-0.30\) and \(-0.40\).

6. Measured dead load stresses around the extrados compared favourably with theoretical predictions based on assumed principal planes and geostatic stresses in the fill.

7. Peak stress normal to the arch occurred for live loading over the crown. Influence values for normal stress of 0.85 times the applied live load stress were found. No more than 40% of Rankine's passive pressure was mobilised on the remote side of the span due to arch movements.

8. Significant shear stresses were measured around the extrados, which tended to resist arch movement.

9. Assessment methods MAFEA and ARCHIE gave excellent predictions of the collapse load (3% and 17% low respectively). Methods ignoring or simplifying the interactive effects gave unacceptable results.

10. Theoretical elastic analyses, \(i.e\). Boussinesq's method gave good predictions of the peak normal stress at low loads on the loaded side of the span. None of the methods used could model the partial passive pressure mobilisation on the remote side of the arch.
11. Temperature measurement proved that, even overnight in winter, insufficient temperature change occurred to warrant the application of a temperature correction to instrument readings on a full scale bridge.

12. Peak influence values for normal stress on the extrados of 0.548 were observed: this was 16% lower than the BD21/93 peak influence value. The loaded axle's measured zone of influence was considerably larger than that allowed for in BD21/93.

13. Some passive resistance was observed as the side of the full scale arch remote from the load was pushed into the fill. This was not more than 18% of the classical Rankine values.

14. In the field tests the pressure readings were affected by the relative flexibilities of different portions of the arch with complications introduced by biaxial bending action in the arch ring.

15. Peak influence values for vertical stress of only 0.097 were observed in the field tests. This was due to the dispersive power of the relatively stiff road pavement. The BD21/93 dispersal method gave higher fill stresses spread over a narrower area leading to its inherent conservatism in this case.

16. In the finite element analysis the arch's elastic modulus did not affect the predicted fill or extrados stresses significantly.

17. The elastic finite element analysis successfully modelled the experimentally observed stress state until 56% of the eventual collapse load had been applied.

18. The elastic finite element analysis proved that the addition of a road pavement overlay enhanced the assessed capacity by increasing the stress dispersal between the load point and the extrados. The stresses on the extrados were decreased significantly as the elastic modulus of the pavement was increased.
19. The elasto-plastic finite element analysis provided satisfactory predictions of yield zones in the fill but it did not provide useful results for the stress state on and around the arch.

20. Good comparisons were drawn between the small scale models, the 2m span brickwork model, and the field tests. The interactive behaviour of the soil-arch system in all its aspects was reproduced at each of these scales.
Chapter 8  Recommendations for Future Research

8.1  Introduction

This chapter sets out the possible future progress of the research project. The recommendations follow a logical progression through the work presented in the thesis. Each section of the project has been examined for future possibilities. Each chapter has answered the salient questions and points raised in the course of the study; simultaneously, further issues beyond the scope of this thesis have been raised. This chapter is to be read in conjunction with the review of the relevant literature presented in Ch. 2. In this way, any progression of the "state of the art" through future research may be better coordinated with the research of the present day and that of its forebears.

Further applications of the research described in this thesis to other areas of engineering science are briefly outlined at the end of the chapter. These cover a wide range of applications in both the civil and structural engineering fields.

8.2  Small scale model tests

The work done thus far on these small scale models may be found in Ch. 3 of the thesis. Future research should concentrate on the analysis of different arch ring profiles: a semicircle and a span to rise ratio of four have been investigated so far. Other possible profiles could be: one intermediate to the two tested already, for example a span to rise ratio of three arch, and a profile flatter than a span to rise ratio of four. The model of the flatter arch would have only a shallow covering of fill at the crown and little interaction between the fill and arch ring would occur. However, valuable insight into the behaviour of flatter arches could be gained from examining the relationships between: collapse load and cover depth, collapse load and load position, collapse load and load geometry, and finally, collapse load and arch ring thickness.

Further investigations into the effects of "backing" upon the behaviour of the single span model arches should be carried out. Timber backing pieces of various shapes
could be placed behind the extrados at the haunches of an arch model. These pieces could be cut to taper outwards from crown level or from the quarter span points, as is often the case in older arch structures. The models, with the addition of such backing pieces, could be tested to destruction in order to determine the changes in the failure mode and collapse load engendered by the backing material.

The final recommendation for future research on the simple, single span arch models is this: a study of the effects of different load geometries should be carried out. Tests done so far have used only one load platen. This simulates the effects of a line load over the span, rather than those of a double or triple axle bogie. Careful attention would need to be paid to the proportioning and scaling of such a load geometry.

An interesting development stemming from the single span models is the development of similar, small scale, multispans arch models. Initial proving trials on these models are currently underway at the University of Edinburgh in conjunction with Napier University, Edinburgh. The investigation could encompass the detection of zones of fill and arch displacement for a variety of load positions, geometries, and fill depths.

Different methods of transferring the horizontal thrust from the loaded to the unloaded span should be investigated. Some suggestions for configurations to be used in the zone between the two spans are: sand fill only, loose timber blocks to quarter span level surmounted by sand fill, loose timber blocks laid to crown level surmounted by a layer of sand fill simulating the cover over the crown, and one solid timber wedge replacing the lower voussoirs on opposite sides of the inter-span gap. By testing the arch at several load positions on one span the effect of these aforementioned types of "backing" upon the collapse load can be investigated.

A rigid wall could be inserted between the two spans, thereby making the model equivalent to a set of two, disparate, single arches. The collapse load of each single span, for a variety of fill depths and load positions, could then be compared to the multispans model's corresponding failure loads.
8.3 Large scale model tests

The number of parameters and variables involved in tests of this nature is large enough to provide scope for future tests. The obvious subjects of future investigation are: arch span, profile, thickness, crushing strength, and number of spans for the masonry, or brickwork, materials. For the fill the obvious targets for future research would be: fill type, strength, stiffness, and density.

A logical progression from the author's tests would be the addition of the other components of a typical arch bridge structure. These would be: spandrel walls, wing walls, parapets, and road pavement strata. Appropriate instrumentation should be included in an attempt to quantify the contribution made by each structural element to the load carrying capacity of the whole model.

Following recent thinking in the field of flat arches, notably by the Department of Transport, some large scale model tests should be carried out on this type of structure. The author envisages a study whereby the abutment blocks could be jacked together, or apart, to simulate the effects of varying the initial stress state in the arch. Abutment thrusts could be measured for various sets of live loads at various positions across the span. Tests to destruction could be carried out on a variety of profiles, strengths, stiffnesses, and arch thicknesses: the stress-strain, or load-deflection behaviour of the arch should be noted as well as its ultimate load.

Such arches would have only a skim coat of fill over the crown and the usual road pavement strata surmounting that. The soil-structure interaction problem shifts its emphasis away from the arch ring to the abutments, foundations, and the underlying soil. This makes the measurement of abutment thrusts and even their displacements or rotations essential. Such deflections could be correlated with the ground pressures behind the abutment and suitable geotechnical analyses could then be carried out on the abutment block.

8.4 Full scale field tests

Due to the wide range of structures found in-situ, specific recommendations can not be made about the types of test required. As and when such bridges become available for testing, the tests and their instrumentation are planned in a manner
similar to that described in Ch. 5. Obviously all future tests should be as intensively instrumented as possible, within the constraints of time and budget.

8.5 Finite element analysis

The author recommends the following future progression of the computer based analysis of the soil-structure interaction in arch bridges: advances in the modelling of the interface, elasto-plastic analysis with full interface modelling, and eventually, three dimensional analysis.

The principal effect requiring attention in the future is the interface behaviour between the fill and the extrados of the arch. Having successfully modelled the interface, elasto-plastic finite element analyses could be done. It is not thought necessary to incorporate the effects of geometrical non-linearity for the accurate modelling of such situations. Whether material non-linearity improves the predictions of arch behaviour remains to be seen.

Three dimensional finite element analysis, incorporating spandrels, parapets, and pavements should be carried out. It is recommended that careful comparisons with the results of various field and model tests are done at all stages, especially as any three dimensional modelling will be extremely complex. It is not envisaged that such a time consuming analysis would become standard practice for arch assessment.

8.6 Applications to other areas of engineering interest

The research leading to the presentation of this thesis has broad based applications within engineering. Conservation of our heritage, in the form of arches, extends beyond the transport infrastructure to cathedrals, monuments, and older listed buildings which often use arcuate construction as a structural form.

Given the shape of the soil-arch system, this research lends itself particularly well to the analysis of cut and cover tunnels, buried culverts and pipes of a range of shapes and sizes. The tests on 2m span models may also replicate the behaviour of a culvert as well as that of a masonry arch bridge. The finite element analyses carried out
may, with suitable dimensions, simulate the live loading of a buried pipe rather than an arch bridge. The small scale timber arch models may reproduce some of the effects associated with a segmental tunnel lining, given suitable depths of fill to represent the cover to the crown. As such, the research, because of its mixture of small, large, and full scale tests with classical and finite element predictions of soil and arch behaviour, lends itself to a wide range of other engineering applications.

8.7 Summary

Recommendations for future research have been provided for each distinct area of interest in this thesis. Some of these pieces of work have been started but are either at an exploratory or an incomplete stage. A final proviso must be made here: any future research into arch bridges must be assessment driven and carried out in conjunction with the people facing the burden of assessing the nation's bridge stock. Sufficient academic understanding of the problems appears to be available, it now remains to translate this into safe and economical design and assessment methods.
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Appendix  Published Work

The following papers published in journals or conference proceedings were derived from the work in this thesis. A set of these papers is bound, where possible, in this appendix. Full permission from the relevant publisher or copyright holder has been obtained. The numbering sequence makes no attempt to follow that of the thesis proper, the pagination follows that of the parent journal or proceedings as appropriate. Each entry in the list below is followed by a brief statement detailing whether or not the paper has been included.


5. FAIRFIELD, C. A. & PONNIAH, D. A., Earth pressure measurements at Kimbolton Butts Bridge, Cambridgeshire. TRRL Contractor Report, TRRL, Crowthorne, 1993. (Not included for copyright reasons)


13. FAIRFIELD, C. A. & PONNIAH, D. A., Heavy axle load tests on a new brickwork arch bridge. 5th Int. Conf. Struct. Mas. for Developing Countries, Santa Catarina, 1994. (Not included for copyright reasons)

THE INSTITUTION OF CIVIL ENGINEERS

INSTITUTION MEDAL AND PREMIUM (LOCAL ASSOCIATIONS) COMPETITION 1992

"SOIL - STRUCTURE INTERACTION IN A MASONRY ARCH BRIDGE TEST"

CHARLES ALEXANDER FAIRFIELD BEng
Graduate Member

Edinburgh and East of Scotland Association

East Anglian Association
Thursday 22nd October 1992 at 2.30 p.m.
SOIL-STRUCTURE INTERACTION IN A MASONRY ARCH BRIDGE TEST

C. A. Fairfield BEng

Abstract

An investigation into the soil-structure interaction in a soil-masonry arch system is described. The principal modes of interaction observed were surface load dispersal, lateral earth pressure redistribution and circumferential shear stress mobilisation. A laboratory test on an instrumented, 2m span semicircular brickwork arch was undertaken. Measurements of arch displacement and normal and tangential earth pressures on the extrados were carried out as the applied load was increased until collapse. Failure was by a four hinged mechanism at a collapse load higher than that predicted by current methods of assessment. Comparison is made between the measured pressures and those allowed for in the current U.K. assessment code for highway bridges and structures. In conclusion, the current methods of assessment are shown to be conservative due to their omission of the effects of soil-structure interaction.

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1.0 INTRODUCTION

Archaeological evidence places the dawn of the masonry arch at circa 3600 B.C. in the ancient kingdoms of Egypt and Mesopotamia\(^{(1)}\). Such corbelled arches still stand throughout the Old World. The principles of arching were known in non-analytical terms to Bronze Age societies but were never used on a grand scale until the Roman's extensive use of the semi-circular arch\(^{(2)}\). The Romans extended these principles to vaults and domes such as those in the magnificent basilica of St. Sophia, built in Istanbul under Justinian's authority in 527 A.D. The Roman's Pont du Gard aqueduct still stands near Nîmes in France, a testament to the skills of their engineers. On the waning of the golden ages of Greece and Rome such skills were lost until mediaeval times when monastic orders of "bridge brothers" sprang up over Europe. They aimed to provide bridges for travellers and it was M. Bénezet, the only engineer canonized for his efforts, who restored the masonry arch to its previous role in infrastructure development.

The importance of the masonry arch in Britain's infrastructure increased between the 17th and 19th centuries with over 40,000 being constructed. These still exist in our road and rail networks today under ever increasing axle loads, well beyond those predicted at time of construction. Many will require strengthening or replacing. The European Commission (EC) requires the Department of Transport (DTp) to raise the maximum allowable gross vehicle weight (GVW) from 38t to 40t by 1999. The maximum axle weight is to be increased to 11.5t. Some EC member states would like to see further increases in GVW to 44t by a similar date. To cater for these increased loads, current assessment methods require improvements in the bridge stock. These costs have been estimated at around £1400 million\(^{(3)}\). Such cost would be particularly onerous to the DTp., necessitating the diversion of funds away from other essential areas of infrastructure maintenance and development. The costs, mainly borne by industry and passed on to the consumer, of being unable to use 40t lorries have been estimated at a minimum of £100 million per annum\(^{(3)}\).

The bridge assessment programme is urgent, as current methods of assessment\(^{(4)}\) are conservative and often result in unnecessary repair work. Improvements on the codified method\(^{(4)}\) have been made by Heyman\(^{(5)}\), Harvey\(^{(6)}\) and Hughes\(^{(7)}\) . These involve plastic and mechanism type analyses with some inclusion of the effects of soil-structure interaction. Finite element analyses are used\(^{(8)}\) in the analysis of arch bridges and these can model the effects of the fill around the arch ring with varying degrees of complexity. The experimental work described here aims to provide information on the earth pressures acting on an arch as it is loaded to failure.
2.0 SOIL-STRUCTURE INTERACTION EFFECTS

There are four modes of interaction inherent in the soil-arch system. These are postulated by Ponnieh as follows:

1. Load dispersal through the fill onto the extrados
2. Lateral earth pressure redistribution as the arch deforms
3. Mobilisation of circumferential shear stresses
4. Arching action behind displaced voussoirs.

The first three of the above modes are discussed in this paper. The load dispersal effect is shown in Figure 1. The applied stress is considerably larger than the stress on the extrados due to dispersal in the fill. The codified assessment method allows an engineer to assume dispersal at a slope of 1 in 2 to no deeper than the level of the crown of the arch. As will be shown, in practice, a greater amount of the load is dispersed before reaching the extrados rendering the current assessment method conservative.

The deformation patterns at low and high loads are shown in Figures 2 and 3. Under load the portion of the arch beneath the load point moves away from the fill, causing the pressures to fall from the at rest values to the active state. On the side of the arch remote from the load, arch displacements are into the fill, causing partial mobilisation of passive pressures. It must be noted that a substantial portion of the arch ring above each springer is undeformed. At rest pressures still act at the springers, preventing arch deformation. This phenomenon prevents hinges forming at the springers as predicted by Heyman's analysis and it thereby reduces the effective span of the arch. This has the effect of increasing the collapse load above that found by an analysis without soil support in a horizontal direction.

As a consequence of the arch deformations, circumferential shear stresses are generated. These stresses are significant as will be shown by the experimental results. Existing analyses only provide for normal earth pressures. No provisions for pressures other than horizontal pressures are made and the omission of the effect of circumferential stresses is conservative to an as yet unknown degree.

3.0 EXPERIMENTAL INVESTIGATION

To further understand and quantify the effects described above, a test to collapse on an instrumented 2m span semicircular brickwork arch was carried out in the laboratory.
3.1 DESCRIPTION OF THE ARCH

The leading dimensions of the model are shown in Figure 4. Also shown is the coordinate system and the notation used to denote the stresses acting on the extrados. The dimensionless horizontal coordinate is expressed as a ratio of distance from the origin to the radius of the extrados. This coordinate then varies from -1 at the left hand springer to +1 at the right hand springer. The stresses, \( \sigma \) and \( \tau \), act normally and tangentially to the extrados respectively.

Mass concrete base slabs were provided beneath the springers: these were bolted to the laboratory's strong floor. The arch ring was constructed in engineering brick with a 1:1:6 mix mortar in the joints. The arch ring thickness was 102.5mm. Side walls of 18mm thick plywood were provided: these were to retain the fill but not act as spandrel walls to the arch. This ensured that the arch was effectively a 2-d slice without the complications of arch-soil-spandrel interaction. Heavy duty polythene was lapped 100mm up the side walls and nailed into place. Multiple folds and lap joints were incorporated into the polythene to allow movement of the fill relative to the walls to occur with only minimal frictional restraint. The polythene sheeting was lapped 100mm over the extrados of the arch but was not in any way fixed to it. End walls, at a distance shown on Figure 4, were built using plywood. A series of 30x30x5 RSA's were bolted to the outside of the plywood walls as additional stiffening. The end walls were stiffened by a pair of soldier beams and three walers.

3.2 FILL PROPERTIES

The fill material was a uniform, dried silica sand with an effective grain size, \( D_{10} \) of 0.6mm. It was placed from zero drop height and compacted in 50mm layers to a depth of fill over the crown of 150mm. From a series of laboratory tests in accordance with BS1377:1975(10) the specific gravity of the material was found to be 2.64. The initial moisture content of the sand was 0.3%, falling to <0.1% after one week in the laboratory. Density tests revealed an average bulk unit weight of 15.5kNm\(^{-3}\), for the compactive effort used in backfilling around the arch. No road pavement was used above the fill in this test as analysis of the load dispersal through multi-layered, multi-modulus materials was deemed too complex, at this stage.

3.3 INSTRUMENTATION

The instrumentation layout is shown in Figure 5. Under the arch, mounted on a scaffolding frame, were 18 linear variable differential transducers. These work on
the same principle as a simple potentiometric circuit where any displacement of the active parts of the instrument gives a change in output voltage relative to the input, excitation voltage. Calibration was by means of standard spacer blocks and a digital voltmeter. The transducers were mounted in pairs at equal intervals around the intrados; one transducer measuring vertical displacements, the other, horizontal displacements.

A vertical line of vibrating wire gauged pressure cells (VWG's) was provided on the centreline of each end wall to assess the horizontal pressures acting there. The VWG's were calibrated in a pressure chamber using the same fill material as in the model test. The pressure was provided by an air-water cylinder connected to a water bag within the pressure chamber. Output from the VWG's as the pressure varied was recorded by data logger.

Cambridge Instruments stress transducers (ST's) were set into pockets cut in the extrados of the arch prior to construction. Dental plaster was used to retain each transducer. The ST's measured both the normal stress, \( \sigma \) and the tangential stress, \( \tau \), on the arch. The directions of these stresses are shown in Figure 4. Calibration was carried out using a rig where weights could be hung from the transducer causing stress changes at the active faces of the ST's. A pulley system to apply horizontal load was used to calibrate the ST's under the action of shearing, or tangential stresses. All output was recorded on a datalogger. The outer surface of the active faces of the ST's was roughened by fixing pieces of garnet sandpaper to them.

Load cells were used in series with the hydraulic jacks applying the load to the arch. These were calibrated in tension in an Avery testing rig with output being monitored by a digital voltmeter.

All instrumentation systems were checked for hysteresis, non-linearity of response over the working stress or displacement ranges, cross-sensitivity, temperature sensitivity and response time. During the test all systems were read by a Microlink® datalogger connected to a micro-computer with dual disk drives.

### 3.4 Test Procedure

Load was applied by two hydraulic jacks anchored to the laboratory's strong floor and connected, at their upper ends, to a steel spreader beam surmounting a timber load platten. The spreader beam and the timber platten ran across the whole width of the model between the internal faces of the side walls, perpendicular to the span of the arch. The load was centred on \( (x/r) = 0.33 \) over a loaded width of 160mm. Nine increments of load were applied until collapse with readings taken from all instruments at each increment. The applied stress under the platten was increased at
a rate of 1kNm$^2$ per minute. To avoid damage to the displacement transducers the frame was withdrawn prior to collapse; as a result, no displacements are available immediately before collapse.

4.0 RESULTS

Collapse occurred at an applied stress of 115.2kNm$^2$ which corresponds to a line load of 22.1kNm$^1$. The failure was by formation of a four hinged mechanism with the hinge locations as shown in Figure 6. The order of hinge formation is also shown in Figure 6 and the loads at which the hinges formed is given below.

Table 1 Hinge positions.

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Position (x/r)</th>
<th>Load @ formation</th>
<th>% of collapse load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.30</td>
<td>34.6kNm$^2$</td>
<td>30.0</td>
</tr>
<tr>
<td>2</td>
<td>+0.03</td>
<td>64.5kNm$^2$</td>
<td>56.0</td>
</tr>
<tr>
<td>3</td>
<td>-0.64</td>
<td>99.0kNm$^2$</td>
<td>86.0</td>
</tr>
<tr>
<td>4</td>
<td>+0.69</td>
<td>115.2kNm$^2$</td>
<td>100</td>
</tr>
</tbody>
</table>

At the collapse load, the fill did not suffer bearing capacity failure. The vertical displacement of the platten was less than 5mm. The dotted line in Figure 6 shows the profile of the fill surface prior to collapse.

The displacement measurements from the transducers gave displaced shapes at each load increment apart from the final stages of the test. The deformed shape of the arch matched that shown in Figures 2 and 3. Due to these displacements the fill pressures around the extrados changed significantly during the test.

The observed stresses around the extrados at 50% of the collapse load are shown in Figure 7. The stresses from the arch test are plotted directly onto the graph of stress versus position. The comparison is made, on the figure, between the experimental data and the codified assessment method’s version of the stress state. The vertical stress increase according to the code(4) has been calculated using the 1 in 2 slope for load dispersal. The resulting normal and tangential stresses, $\sigma$ and $\tau$ respectively, were calculated by using Mohr’s circle of stress(13). Various analyses have been used to compare collapse load values: these are given in Table 2 below. All properties and values used in these analyses are taken directly from laboratory tests or measurements. The analyses were all carried out pre-test, as would be the case if a Local Authority was asked to carry out the assessment.

The VWG’s gave few meaningful results; only very small stress changes were measured with no clear trends being noticed.
Table 2 Comparison of collapse load values, (all values in kNm\(^{-1}\)).

<table>
<thead>
<tr>
<th>Test value</th>
<th>Codified method(4)</th>
<th>% error</th>
<th>Heyman’s analysis(5)</th>
<th>% error</th>
<th>ARCHIE analysis(16)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.1</td>
<td>7.77</td>
<td>64.8</td>
<td>8.56</td>
<td>61.3</td>
<td>17.9</td>
<td>19.0</td>
</tr>
</tbody>
</table>

5.0 DISCUSSION & ANALYSIS

The collapse mode was as expected; four hinges formed turning the arch into an unstable structure. The first hinge formed, not directly beneath the load, but slightly closer to the crown at \((x/r) = -0.30\). This was due to load dispersal through the fill. The most stressed point on the arch was not vertically below the load but at a point where the combination of depth and horizontal distance from load to extrados gave a larger stress increase. The second hinge formed close to the crown of the arch. Here the restraining soil pressure was minimal, thus allowing sufficient rotation of segment 1-2, (see Figure 6), for hinge formation. The third and fourth hinges did not form at the springers, but because of the large at rest earth pressures acting below the levels of hinges 3 and 4 (see Figure 6), which resist arch rotation, they form at a level well above the springers. These hinge locations do not correspond with those predicted by Heyman\(^{(5)}\) in his plastic analysis which uses only the fill’s dead weight and no further interactive properties. The effective span of the arch was the horizontal distance between hinges 3 and 4, \(i.e.\) 1.51m: this is substantially less than the actual span of 2m used in current assessment methods\(^{(4)}\). This effective decrease in span is not used in current analyses except Dundee’s ARCHIE analysis program\(^{(14)}\).

The percentages of ultimate load at which the hinges formed is worthy of comment. The first hinge forms at only 30% of ultimate load and only when a further 26% of the ultimate load is added does the second hinge form. The remaining two hinges form extremely rapidly at high percentages of the ultimate load. Consideration of the thrustline leads to an explanation as follows: the deviation of the thrustline from the at rest position required to form two hinges is large; the subsequent deviation needed to form the final two hinges is small as the thrustline already lies close to the extremities of the arch\(^{(6,14)}\).

The stresses shown in Figure 7 show the normal stress on the arch beneath the load to be some 25% lower than the values predicted by the code\(^{(6)}\). The actual load dispersal through the fill is greater in practice than the code allows. On the side of the arch remote from the load point the normal stress found experimentally is significantly higher than that allowed for in the code. The fact that the code ignores
this potentially beneficial partial mobilisation of passive fill pressure makes it
unduly conservative. The shear stresses measured are greater than those predicted
by current methods of assessment. Again, the code ignores the potential benefits of
allowing some stabilising shear stress to be mobilised. This is shown in Figure 7
where it may be seen that, according to the code’s analysis, only those pressures
due to the self weight of the fill are permitted on the side of the arch remote from
the load. No increased stress, normally or tangentially, is permitted.
Collapse load values by other analyses are all conservative; in the case of Heyman’s
analysis(9) and the codified MEXE analysis(9), unduly so. Dundee’s ARCHIE
analysis(14) underestimates the collapse load by only 19%, an better prediction for
this type of problem. Analyses ignoring the stabilising effects and overestimating
the destabilising effects are uneconomical as their use would lead to unnecessary
repair work leading to diversions and delays for commercial vehicles. Such
diversions and delays have far reaching implications: increased noise and
atmospheric pollution would result(15), increased prices for goods where
transportation is a major cost element such as: steel, bricks and beer(16) also result.

6.0 CONCLUSIONS

1. Improved arch bridge assessment methods are needed if economical
   implementation of EC directives is to be achieved.
2. The load carrying capacity of the test arch was increased due to soil-structure
   interaction effects.
3. Current methods of analysis underestimate the strength of the test arch.
4. The best prediction of the collapse load was given by Dundee’s ARCHIE
   analysis.
5. Further tests of this nature are needed to add to existing knowledge of the
   composite behaviour of the soil-arch system.

7.0 ACKNOWLEDGEMENTS

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continual support and guidance of the author’s supervisor, Dr. D. A. Ponniah, is
also acknowledged.
8.0 REFERENCES

FIG. 1. LOAD DISPERGAL
LOAD

Road level

Passive

Active

Rock

FIG. 2 DEFORMATION AT LOW LOADS
Fig. 3 Deformation at high loads
FIG. 4 DIMENSIONS OF MODEL ARCH
(GEOMETRY & NOTATION)
FIG. 5 INSTRUMENTATION LAYOUT

- = STRESS TRANSUDCER
× = DISPLACEMENT TRANSUDCER PAIR
| = VWG
**FIG. 7a SOIL Pressures (Normal)**

Normal pressure around the extrados

\[ \text{Normal pressure/ kPa} \]

- Experimental values
- Codified method \(^{(4)}\)

**FIG. 7b SOIL Pressures (Shear)**

Shear pressure around the extrados

\[ \text{Shear pressure/ kPa} \]

- Experimental values
- Codified method \(^{(4)}\)
Abstract
An element of an on-going investigation into the soil-structure interaction in a soil-masonry arch system is described. A laboratory test on an instrumented, 2m span semicircular brickwork arch was undertaken. Measurements of arch displacement and normal and tangential earth pressures on the extrados were carried out as the applied load was increased until collapse. Comparison is made between the measured pressures and those allowed for in the current UK assessment code for highway bridges and structures. Current methods of assessment are shown to be conservative due to their omission of the effects of soil-structure interaction.

1.0 INTRODUCTION
Archaeological evidence places the dawn of the masonry arch at circa 3600 B.C. in the kingdoms of Egypt and Mesopotamia. The principles of arching were used on a grand scale with the Romans' use of the semi-circular arch. The importance of the masonry arch in Britain's infrastructure increased between the 17th and 19th centuries with over 40,000 being constructed. These still stand today; under ever increasing axle loads, well beyond those predicted at time of construction. Many will require strengthening or replacing by 1999 due to new legislation on allowable axle weights. To cater for these increased loads, the existing bridge stock has to be reassessed. Costs have been estimated at around £1400 million for the upgrading and subsequent work. Such cost would be particularly onerous to the Department of Transport, necessitating the diversion of funds away from other areas of infrastructure maintenance and development. The costs, mainly borne by industry and passed on to the consumer, of being unable to use 40 t lorries are estimated at a minimum of £100 million per annum.

The bridge assessment programme in the UK is urgent. Current methods of assessment are conservative and often result in unnecessary repair work and thus improvements to the methods are needed. Improvements on the codified method are suggested by Heyman, Harvey and Hughes and involve plastic and mechanism type analyses with some considerations of the effects of soil-structure interaction. Finite element analyses are also used in the assessment of arch bridges. This paper presents results from a backfilled, 2m span brick arch tested to failure. The collapse load was higher than that predicted by current assessment methods. The paper's principal conclusion is that any future assessment technique must account for the soil-structure interaction observed in this, and other arch bridge tests.

The economics of the construction of small span arch bridges have been studied over the last few years and new arch bridges are built in the UK, albeit rarely. Suitable siting of the bridge, use of locally available materials, close liaison between consultant and contractor and further research into the arch's complex behaviour will all combine to give a cost-effective solution to bridging problems. Once the arch is completed, little maintenance is required when compared to typical concrete spans, thus making the life cost of the structure competitive. Aesthetically speaking the arch has few near rivals.

2.0 SOIL-STRUCTURE INTERACTION EFFECTS
Four modes of interaction postulated by Ponniah are:
1. Load dispersal through the fill
2. Lateral earth pressure redistribution as the arch deforms
3. Mobilisation of circumferential shear stresses
4. Arching action behind displaced voussoirs

The first three of the above modes are discussed in this paper. The load dispersal effect is shown in Fig. 1. The applied stress is considerably larger than the stress on the extrados due to dispersal in the fill. The codified assessment method allows an engineer to assume dispersal at a slope of 1 in 2 to no deeper than the level of the crown of the arch. As will be shown, a greater amount
of the load is dispersed before reaching the extrados rendering the current assessment method conservative.

The deformation patterns at low and high loads are shown (Figs 2 & 3). Under load the portion of the arch beneath the load point moves away from the fill, causing the pressures to fall from the at rest values to the active state. On the side of the arch remote from the load, the arch displacements are into the fill, causing mobilisation of passive pressures. At rest pressures still act at the springers; preventing arch deformation. This prevents hinges forming at the springers and it reduces the effective span of the arch. This has the effect of increasing the collapse load above that found by an analysis without horizontal soil support.

As a consequence of the arch deformations, circumferential shear stresses are mobilised. Existing analyses only provide for normal earth pressures and no provisions for pressures other than horizontal pressures are made. Thus the omission of the effect of circumferential stresses is conservative to an as yet unknown degree.

3.0 EXPERIMENTAL INVESTIGATION

The leading dimensions of the model, the coordinate system and the stress convention are shown (Fig. 4). The dimensionless horizontal coordinate is expressed as a ratio; distance from the origin, $x/r$. This coordinate then varies from -1 at the left hand springer to +1 at the right hand springer. The stresses, $s$ and $t$, act normally and tangentially to the extrados respectively.

Concrete base slabs, bolted to the laboratory's strong floor, were provided beneath the springers. The arch was constructed in engineering brick with a 1:1:6 mix mortar in the joints. The arch thickness was 102.5mm. Side walls of 18mm thick plywood were provided: these were to retain the fill but not act as spandrel walls. This ensured that the arch was effectively a 2-D slice without the complications of arch-soil-spandrel interaction. End walls, at a distance shown on Fig. 4, were built using plywood. 30x30x5 RSA’s were bolted to the outside of the plywood walls as additional stiffening. The end walls were stiffened by a pair of soldier beams and three walers.

3.1 FILL PROPERTIES

The fill material was a uniform, dried, silica sand with an effective grain size, $D_{10}$ of 0.61 mm. It was placed in 50mm layers to a depth over the crown of 150mm. A series of tests to BS1377:1975 gave the specific gravity as 2.64. The initial moisture content was 0.3%, falling to <0.1% after one week in the laboratory. Density tests revealed an average bulk unit weight of 15.5kNm$^3$. No road pavement was used as analysis of the load dispersal through multi-layered, multi-modulus materials was deemed too complex, at this stage.

3.2 INSTRUMENTATION

Fig. 5 shows the instrumentation layout. Under the arch, mounted on a scaffolding frame, were 18 linear variable differential displacement transducers. The transducers were mounted in pairs at equal intervals around the intrados; one transducer measuring vertical displacements, the other, horizontal displacements.

A vertical line of vibrating wire gauged pressure cells (VWG’s) was provided on the centreline of each end wall to assess the horizontal pressures acting at the end walls. The VWG’s were calibrated in a pressure chamber using the same fill material as the model test.

Cambridge Instruments stress transducers (ST’s) were set into pockets cut in the extrados of the arch before backfilling. Dental plaster was used to retain each transducer. The ST’s measured the normal stress, $s$ and the tangential stress, $t$, on the arch. Calibration was carried out using a rig where weights could be hung from the transducer causing stress changes at the active faces of the ST’s. A pulley system to apply horizontal load was used to calibrate the ST’s under the action of shearing, or tangential stresses. The outer surface of the active face of each ST was roughened by affixing pieces of garnet sandpaper. Load cells were used in series with the hydraulic jacks to apply the load to the soil surface above the arch.

All instrumentation systems were checked for hysteresis, non-linearity of response, cross-sensitivity, temperature sensitivity and response time. During the test all systems were read by a Microlink* datalogger connected to a micro-computer.
3.3 TEST PROCEDURE

Load was applied by two hydraulic jacks and a steel spreader beam surmounting a timber load platten 160mm wide. The spreader beam and the timber platten ran across the whole width of the model between the internal faces of the side walls, perpendicular to the span of the arch. The load was centred on (x/r) = -0.33. Nine increments of load were applied until collapse with readings taken from all instruments at each increment. The applied stress was increased at a rate of 1kNm\(^2\) per minute.

4.0 RESULTS

Collapse occurred at an applied stress of 115.2kNm\(^2\), corresponding to a line load of 22.1kN/m\(^1\). The failure was by formation of a four hinged mechanism with the hinge locations as shown (Fig. 6), the order of hinge formation is also shown. The loads at which the hinges formed is given below (Table 1).

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Position (x/r)</th>
<th>Load @ formation (kNm(^2))</th>
<th>% of collapse load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.30</td>
<td>34.6kNm(^2)</td>
<td>30.0</td>
</tr>
<tr>
<td>2</td>
<td>+0.03</td>
<td>64.5kNm(^2)</td>
<td>56.0</td>
</tr>
<tr>
<td>3</td>
<td>-0.64</td>
<td>99.0kNm(^2)</td>
<td>86.0</td>
</tr>
<tr>
<td>4</td>
<td>+0.69</td>
<td>115.2kNm(^2)</td>
<td>100</td>
</tr>
</tbody>
</table>

At collapse the fill did not suffer bearing capacity failure. The vertical displacement of the platten was less than 5mm. The dotted line (Fig. 6) shows the profile of the fill surface immediately before collapse.

The displacement measurements from the transducers gave displaced shapes at each load increment apart from the final stage of the test. The deformed shape of the arch matched that shown (Figs 2 & 3) and it may be compared with the hinge positions and rotations are shown (Fig. 6). Due to these displacements the fill pressures around the extrados changed significantly during the test.

The observed stresses on the extrados at a typical load level, 50% of the collapse load, are shown (Fig. 7). The comparison is made between the experimental data and the codified assessment method's version of the stress state. The vertical stress according to the code\(^6\) has been calculated using a 1 in 2 slope for load dispersal. The resulting normal and tangential stresses, \(s\) and \(t\) respectively, were calculated by using Mohr's circle of stress\(^6\).

Various analyses have been used to compare collapse load values, (Table 2). All properties and values used in these analyses are taken directly from laboratory tests or measurements. The analyses were all carried out pre-test, as would be the case if a Local Authority was asked to carry out the assessment.

The VWG's measured very small stress changes, less than 5kN/m\(^2\); implying that the end walls were suitably distant from the springers. Therefore little interaction between the arch and the end walls occurred.

Table 2 Comparison of collapse load values, (all values kNm\(^1\)).

<table>
<thead>
<tr>
<th>Test value</th>
<th>Codified method (4)</th>
<th>% difference</th>
<th>Heyman's analysis (5)</th>
<th>% difference</th>
<th>ARCHIE analysis (14)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.1</td>
<td>7.77</td>
<td>64.8</td>
<td>8.56</td>
<td>61.3</td>
<td>17.9</td>
<td>19.0</td>
</tr>
</tbody>
</table>

5.0 DISCUSSION & ANALYSIS

The failure mode was as expected; four hinges forming a collapse mechanism. The first hinge formed, not directly beneath the load, but slightly closer to the crown at (x/r) = -0.30. This was due to load dispersal through the fill. The most stressed point on the arch was not vertically below the load but at a point where the combination of depth and horizontal distance gave the largest stress increase. The second hinge formed close to the crown of the arch. Here the restraining soil pressure was minimal, allowing sufficient rotation of segment 1-2, (Fig. 6), for hinge formation. The third and fourth hinges did not form at the springers, but, because of the at rest earth pressures acting below the levels of hinges 3 and 4 (Fig. 6), which resisted arch rotation, they formed at a level above the springers. These hinge locations did not correspond with those predicted by Heyman\(^5\) in his plastic analysis which uses only the fill's dead weight. The effective span of the arch was the horizontal distance between hinges 3 and 4, \(i.e.\ 1.51\)m: substantially less than the actual span of 2m used currently\(^6\).

The percentages of ultimate load at which the hinges formed is worthy of comment. The first hinge formed at
only 30% of ultimate load and only when a further 26% of the ultimate load is added did the second hinge form. The remaining two hinges formed at high percentages of the ultimate load. Consideration of the thrustline leads to an explanation: the deviation of the thrustline from the at rest position required to form two hinges is large; the subsequent deviation needed to form the final two hinges was small as the thrustline already lay close to the extremities of the arch.

Fig. 7 shows the normal stress on the arch beneath the load to be 25% lower than that predicted by the code. The load dispersal through the fill was greater, in practice, than the code allows. On the side of the arch remote from the load the normal stress found experimentally was significantly higher than that allowed for in the code. The fact that the code ignores this potentially beneficial partial mobilisation of passive fill pressure makes it unduly conservative. The shear stresses measured are greater than those predicted by current methods of assessment. The code ignores the potential benefits of allowing some stabilising shear stress to be mobilised as shown in Fig. 7. According to the code’s analysis, only those pressures due to the self weight of the fill are permitted on the side of the arch remote from the load. No increased stress, normal or tangential, is permitted.

Estimated collapse loads by other analyses were conservative; in the case of Heyman’s analysis and the codified MEXE analysis, the code’s analysis underestimated the collapse load by only 19%, a better prediction for this type of problem. Analyses ignoring the stabilising effects and overestimating the destabilising effects are uneconomical as their use would lead to unnecessary repair work leading to diversions and delays for commercial vehicles. Such diversions and delays imply: increased noise and atmospheric pollution and increased prices for goods where transportation is a major cost element such as: steel, bricks and beer.

6.0 CONCLUSIONS

1. Improved arch bridge assessment methods are needed if economical implementation of EC directives is to be achieved and if new arch bridges are to be built cost-effectively.
2. The load carrying capacity of the test arch was increased due to soil-structure interaction effects.
3. Current methods of analysis underestimate the strength of the test arch.
4. The closest prediction of the collapse load was given by ARCHIE.
5. Soil pressures measured were up to 25% lower, in practice than allowed for in the codified method.
6. Further tests of this nature are needed to add to existing knowledge of the composite behaviour of the soil-arch system.

7.0 ACKNOWLEDGEMENTS

The assistance of the SERC and the Transport & Road Research Laboratory under CASE Award No. 90552123 is gratefully acknowledged. The author gratefully acknowledges the assistance of the Technical Staff in the department along with that of his student Mr R. Puri.

8.0 REFERENCES


Fig. 1 Load dispersal

Fig. 2 Deformation at low loads
Fig. 3 Deformation at high loads

Fig. 4 Dimensions of model arch (geometry & notation)
Fig. 5 Instrumentation layout

Fig. 6 Hinge locations
Fig. 7a Soil pressures (normal)

Fig. 7b Soil pressures (shear)
Influence values (x1000) for vertical stress under road pavement

FIG. 6 LOAD SPREAD THROUGH PAVEMENT KIMBOLTON BUTTS BRIDGE
Measured normal stress
Codified normal stress
Measured shear stress
Codified shear stress

FIG. 5 SHEAR AND NORMAL ARCH PRESSURES, 2M SPAN MODEL
FIG. 2 STRESS TRANSDUCER CELL CALIBRATION

FIG. 3 VHG READINGS, BARCOWER BRIDGE
the load spread 0.075m below the base of the road pavement. The influence values given showed that a unit stress on the surface of the 0.450m thick pavement was reduced to 10% of its surface value by the pavement. Current codified methods and analytical techniques allow some load dispersal but rarely as much as was seen to have occurred here.

CONCLUSIONS

1. The instruments used all functioned well and were easily calibrated and installed.

2. The geotechnical information yielded by the instrumentation has proved valuable in helping to understand the behaviour of a soil-arch system.

3. The results have helped to explain some of the conservatism inherent in arch bridge assessment techniques.

4. The results have justified the inclusion of geotechnical instrumentation in arch bridge tests to come.

ACKNOWLEDGEMENTS

The assistance of the SERC and the Transport Research Laboratory under CASE award No. 90552123 is gratefully acknowledged. The authors would also like to thank the technical staff in the Department of Civil Engineering & Building Science, and Messrs. K. Blackie, R. Mallinson and R. Puri for their unstinting efforts on the arch bridge research project.

REFERENCES


During the test, in which a line load at the third span was gradually applied to the road surface, the VWG's were read by a dedicated Gage Technique Ltd. acoustic gauge strain meter GT1169. Readings of soil pressure on the extrados for a variety of surface applied loads were recorded. A sample set of results from one channel is shown (Fig. 3). The results showed the feasibility of measuring interface pressures in such a situation. They demonstrated the considerable load dispersal which takes place above the arch because the measured pressures were substantially lower than the average applied contact stress on the road's surface. Correlation of measured pressures and arch displacements also showed the effects of lateral pressure redistribution as the arch deformed.

**Bargower Bridge Model Test**

A model test was carried out with a view to replicating the behaviour of the field test to destruction on the full scale bridge. Soil pressure measurements on the extrados beneath the load line were required, this being the most heavily loaded portion of the arch. Access to the extrados was simple as the instrumentation could be installed prior to backfilling. Kulite soil pressure cells were used for stress measurements. The cells were calibrated using the apparatus shown in Fig. 1. The Kulite transducer was developed specifically for soil stress measurement. Being fluid filled the active face exhibits very little deflection under load and the active to total area ratio is such that the insertion of the cell into the system being measured causes little arching around the cell itself. The basic transducer element is a solid state silicon pressure sensor working on the four arm strain gauge bridge principle. The data logger was a standard type Microlink connected to a microcomputer for data storage and analysis.

A sample set of results is shown (Fig. 4). These too show the considerable load dispersal occurring in the fill over the arch as shown in Fig. 3. The dashed lines on Fig. 4 are estimated plots for shallower fill depths covering the crown. At lower fill depths the dispersal is reduced and the measured pressure on the extrados forms a larger proportion of the applied surface pressure. The model successfully reproduced the failure mechanism observed in the field test and with judicious use of scaling factors between model and field test, some approximate failure load back analyses were possible.

**A 2m Span Brick Arch Model Test**

A semicircular 2m span half brick thick arch was tested to destruction in the laboratory to further quantify the soil pressure distributions acting on the arch. Evidence of substantial shearing stresses along the extrados led the authors to believe it necessary to measure both contact normal and shear stresses during the test. To this end Cambridge Instruments contact stress transducers were embedded in dental plaster filled pockets cut into the extrados. The cells, being sensitive to both normal, shear and eccentric loading, were calibrated using the equipment shown in Fig. 2. The Cambridge Instruments ST's consist of an active face, roughened by the addition of garnet sandpaper, surmounting sets of very thin strain gauged webs. The webs are thin enough to register an appreciable linear strain at small applied loads. Three separate circuits are wired into the body of the transducer; one each for normal, shear and eccentric load measurement. The entire cell is housed in a machined aluminium alloy casing with cables run out the base of the instrument. Each of the three circuits may be assigned to an individual channel for data logging purposes. The gauges are temperature compensated. Data logging during the test was carried out using Microlink equipment and a microcomputer.

A sample set of results from an intermediate stage of the test is given (Fig. 5). Considerable changes in pressure from the at rest state were observed as well as significant shearing stresses around the extrados. The dashed lines on Fig. 5 represent the codified version of the stress state on the extrados for the same applied surface load. It may be seen that the codified method overestimates the normal stress and underestimates the tangential, or shear, stress. This gives rise to some of the conservatism in current methods of arch assessment.

**Kimbolton Butts Bridge, Heavy Load Tests**

Cambridgeshire County Council's Butts Bridge at Kimbolton was completed in December 1992 and opened to traffic that month. It is an 8m span, 4m rise, circular segmental brickwork arch backfilled with nominally 1/4" (38mm) down Carrstone fill. Designed to the new EC axle limits and gross vehicle weights, the bridge represents a major turning point in the revival of the story of the arch bridge and its builders. The Transport Research Laboratory, with the help of Edinburgh University, load tested the bridge in February 1993. Geotechnical instrumentation was incorporated into the fill with a view to measuring the dispersal of heavy wheel loads through road pavement and fill. Contact normal stress measuring VWG's were also installed on the extrados but this section will concentrate on the gauges embedded wholly in the fill.

Soil Instruments Ltd's pressure gauges were used, having been calibrated in a manner similar to that shown in Fig. 1. The instrument consists of two circular active faces, 0.100m diameter, with oil of a similar elastic modulus to the surrounding soil between these faces. A small bore pipe connects the sealed oil chamber to a VWG transducer activated by the oil pressure deflecting a thin diaphragm. The VWG transducer cable is armoured and is able to lie in a narrow trench in the fill. The active faces of the instrument must be surrounded with the graded calibration fill, as was used in the steel pressure chamber of Fig. 1. Once the sensitive parts of the instrument are buried, placement of the rest of the ordinary bridge fill may continue. A wide range of standard VWG readers are available for this type of cell. During the tests at Kimbolton the results were recorded using the Gage Technique Ltd. reader described previously.

From a vertically aligned array of these Soil Instruments Ltd. gauges, the pressure bulb beneath a typical loaded wheel was obtained. The results are shown in Fig. 6 for
SOIL PRESSURE MEASUREMENT FOR ARCH BRIDGE ASSESSMENT

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Soil-structure interaction is a major contributor to the load carrying capacity of masonry arch bridges, of which there are many in the UK. A large number of these structures are threatened due to increased axle limits and the inherent conservatism of current assessment methods. Research is being carried out on the assessment methods and the paper describes soil pressure measuring instruments used in the research. Sample results showing the fill and interface pressures derived from the use of such instrumentation are presented. Simple geotechnical instrumentation is shown to be useful in furthering the understanding of the complexities of a soil-arch system.

INTRODUCTION

There are, in the UK, many masonry arch bridges which are being reassessed under European Community (EC) directives specifying higher axle loads and gross vehicle weights. Due to an unknown degree of conservatism inherent in current assessment techniques, research into the fill’s contribution to the load carrying capacity is being carried out. The fill’s contribution takes the form of: load dispersal, lateral pressure redistribution during deformation and arching action(1). To quantify the extent to which the fill surrounding the arch helps it carry load, soil pressure measurements are needed.

This paper will describe the specification and calibration of the instruments used. It will then outline their use during: the test to destruction of Bargower Bridge(2), the model test simulating Bargower Bridge(2), a 2m span brick arch model test(3) and the heavy load test on the newly constructed Kimbolton Butts Bridge in Cambridgeshire. Sample results from each type of instrument are then presented.

METHOD

SPECIFICATION

The specification of the instrumentation took into account the need for: robustness, sensitivity, accuracy, wide working stress range, low cost, fast response time and long term stability. All cabling had to be either flexible enough to permit drawing through ducts or armoured to prevent damage during installation. The specification for the data logging systems was such that the same system could be used for calibration and test purposes with easily repeatable readings being stored, for post-processing, or output directly to hard copy. The design of the calibration system was such that the in-situ stress state was replicated during calibration.

CALIBRATION

Due to the different types of instrument used, two calibration methods were needed. For the Bargower field and model tests and the Kimbolton Butts Bridge heavy load tests the apparatus shown in Fig. 1 was used. A known pressure was applied by an air/water cylinder to the water bag encased in a steel pressure chamber. The known pressure change at the water bag/fill interface caused a certain change in the instrument reading. By using a range of applied pressures the cell’s response could be calibrated.

For the 2m span brick arch model tests the arrangement shown in Fig. 2 was used. The stress transducer cells (ST’s) measured both normal and shear stresses so a pulley system was used to apply a horizontal load simulating shear stress on the active face. A simple hanger with slotted weights was used to simulate the application of a stress normal to the cell.

All instrumentation systems were checked for hysteresis, non-linearity of response, cross-sensitivity, temperature sensitivity and response time.

BARGOWER BRIDGE FIELD TEST

For this test, Gage Technique Ltd. vibrating wire gauges (VWG’s) were installed on the extrados of the arch. The VWG’s have a diameter of 0.145m across the active face with a 0.120m long boss behind to permit cable connection and mounting. As the arch was already backfilled, access to the extrados was obtained by rotary coring of the voussoirs. The VWG’s were then inserted into the cored hole to be surrounded with the calibration fill used in the steel pressure chamber (Fig. 1). Once the hole was filled, by VWG and calibration fill, the cell was held in place by four threaded bars and a rear template secured with Rawlbolts to the voussoirs. The bolts were then tightened until the cell just registered a small pressure change.
FIG. 2 PARTICLE SIZE DISTRIBUTIONS FOR TYPICAL BRIDGE FILLS
FIG. 1 MODES OF INTERACTION
ACKNOWLEDGEMENTS

The assistance of the SERC and the Transport Research Laboratory under CASE award No. 90552123 is gratefully acknowledged. The authors would also like to thank the technical staff in the Department of Civil Engineering & Building Science, and Messrs. S. Z. George and R. Puri and Ms. S. F. Fahey for their efforts on the arch bridge research project.

REFERENCES


Table 1 Summary of bridge fill properties.
choice between drained or undrained testing lies with the analyst - the soil type also predetermines, to a certain extent, the type of test carried out - and their chosen method of analysis. The shear box is useful for granular fills because sample preparation is simple but the extent of the information yielded by the test is more limited than in the triaxial test.

RESULTS

The results are presented "bridge by bridge" below. They represent the likely range of fill properties found in situ and in the model tests used to replicate the in situ behaviour. Table 1 summarises the salient properties of each bridge's fill material. Figure 1 shows the interactions occurring behind an arch which are affected by those soil properties measured. The detailed nature of the soil-structure interaction is reported elsewhere. Figure 2 shows the range of particle size distributions encountered.

BRIDGEMILL, OLD RIVER BRIDGE

The fill beneath the road pavement structure's subbase was stratified with two strata: one a gravelly SAND; the other, lower strata, a gravelly, sandy CLAY fill. The bulk of the fill was gravelly SAND, of a predominantly sandstone based nature. As the tests at Bridgemill were conducted early on in the project, little information about the fill over the arch was deemed necessary.

BARGOWER BRIDGE, FIELD TEST

The fill at Bargower Bridge was stratified below the road pavement as follows: 0.7m of very silty, gravelly SAND overlying 1.0m of silty, gravelly SAND below which lay sandstone BOULDERS with a little sand. The Boulder layer extended down to the level of the bridge abutments and beyond. Particle size distribution tests were carried out on the fill as well as: multistage triaxial, density and compaction tests.

BARGOWER, MODEL TEST

The stratification beneath the road pavement in the model tests simulating Bargower was as follows: coarse SAND overlying gravelly SAND below which lay crushed stone COBBLES with a little sand. The layer depths were appropriately scaled. Each layer in the model test corresponds, as closely as possible, to its equivalent layer in situ. Particle size distribution tests were not carried out due to the obvious problems associated with the scaling of individual particle sizes. Density and triaxial tests were carried out on the model fill materials.

A 2M SPAN BRICK ARCH MODEL TEST

At a later stage in the research, the authors wished to concentrate on accurate pressure measurement on the extrados. To simplify this a uniform dry silica, medium SAND was used throughout this series of model tests. It was of a similar angle of shearing resistance to those fills found in situ. The tests carried out on the fill were as follows: particle size distribution by both wet sieving and hydrometer methods, density, shear box and triaxial.

BALMOOR BRIDGE, INVERURIE

A large bulk sample of the fill material was collected from a trial pit dug over the span. The fill was described as a silty SAND with gravel and it predominated in the make up of the backfill over the arch. The following tests were carried out on the materials: particle size distribution by both wet sieving and hydrometer methods, density and compaction tests.

KIMBOLTON BUTTS BRIDGE, CAMBRIDGESHIRE

The fills described for the bridges above were predominantly sandy, often with appreciable silt content and some gravel. The model fills used matched the actual bridge fills with the exception of the siltier materials. The tests used to classify and assess the soil properties were typically those used in practice for granular materials. The authors take cognisance of the fact that with an appreciable silt content the behaviour can change markedly. The silt fraction causes unwanted water retention by reducing the permeability, capillarity by the nature of its particle size distribution and frost susceptibility. There are difficulties in modelling the behaviour of silty materials but where the percentage of non-plastic fines is small the model's sand fills proved sufficient for the narrow range of fill types encountered in the field.

A typical arch bridge fill could be described as: granular, frictional, mainly sandy soil with an angle of shearing resistance of between 35° and 45°. Some gravel is usually present as well as around 10% silt content. However, fine fractions of up to 25% are not unknown in the bridge fills in the UK.

The most common, and simplest, methods of sampling the fill behind arch bridges have been found to be trial pit excavation and rotary coring through the structure. The most useful tests for the purpose of bridge assessment have been found to be: particle size distribution by both wet sieving and hydrometer methods, light hammer or vibrating hammer compaction tests, density tests, shear box tests and triaxial tests.
ARCH BRIDGE BACKFILL PROPERTIES

C A Fairfield and D A Ponniah, University of Edinburgh, Scotland

As part of the nationwide bridge assessment programme, a variety of field and model tests have been carried out on masonry arch bridges. This paper describes the typical soils used as backfill in these structures, the tests best used to classify the soils and the soil properties deemed most important for analysis and assessment purposes. All sampling procedures described are in accordance with BS 5930, the British Standard Code of Practice for Site Investigation; all soil tests described are in accordance with BS1377, the British Standard Methods of Test for Soils for Civil Engineering Purposes. The most important tests were those for: particle size distribution, density and strength in either the shear box or triaxial apparatus.

INTRODUCTION

Under European Community directives, the maximum axle limit on the UK's highway bridges is to increase from 10t to 11.5t by 1999. Many of these structures are of masonry arch construction, backfilled to carry a road pavement. Much attention is currently focused upon the assessment of masonry arches; here the separate branches of structural and geotechnical engineering intertwine to analyse the soil-structure interaction inherent in a soil-arch system. Typical interactive modes identified are: load dispersal, earth pressure redistribution and arching action. These are shown in Fig.1 for a typical semicircular arch. Much information has been obtained pertaining to the materials used in the construction of the spandrels, wing walls, parapets and arch barrels. A greater degree of uncertainty lies in assessing the nature of what lies above the extrados of an arch.

This paper represents a distillation of practical experience gained on field and model tests since 1984. Typical sampling methods are described followed by test methods best suited to analyse the bridge and its backfill. Test results are presented from: Bridgemill's old river bridge(1), Bargower Bridge field test(2), Bargower Bridge model test(3), a laboratory test on a 2m span brick arch(3), Balmoor Bridge, Inverugie(4) and Kimbolton Butts Bridge, Cambridgeshire.

The sampling described is in accordance with BS5930(5) and the testing with BS1377(6). From the tests described the best picture of what lies behind the extrados of a typical arch bridge can be obtained rapidly and at little cost. The model test fills described give an idea of the range of properties used to simulate those found in situ.

METHOD

SAMPLING PROCEDURES

The most common methods of sampling the fill above extant arch bridges are: trial pits in the road pavement going through to the fill(2) and cored holes drilled through the arch's wingwalls, spandrel walls or barrel(4) into the soil. In the case of the model tests, or indeed for the construction of a new arch bridge, fill samples may be taken directly from the borrow pit or local quarry for laboratory testing. A variety of these methods were used for obtaining the soil samples described below.

TESTING PROCEDURES

Samples brought from the field are laboratory tested to BS1377(6) to determine their salient properties for use in the subsequent arch bridge analysis. For classification purposes, the stratigraphic record and associated engineering descriptions form a good starting point for the lab. tests. Classification may be most effectively achieved by wet sieving as outlined in Test 7a of BS1377. The combined clay and silt fraction can be calculated from this test and if further breakdown of this is required, Test 7d, a hydrometer method, may be carried out.

The bulk density of the fill behind an arch is of importance in determining the self-weight stress state in the structure. To determine the fill's bulk density from field samples Tests 12, 13 or 14 of BS1377 may be used. These tests also give the relationship between moisture content and dry density which would be useful in predicting the fill's compactive performance during placement behind either a new full scale bridge or a lab. model test.

The soil's angle of shearing resistance and elastic modulus are critical parameters in any interactive analysis of the stress field above an arch bridge. Elasto-plastic finite element analyses require the input of material moduli and "friction angles". Classical methods require the use of the interaction angle between the fill and the rough surface of the arch: this is a property derived directly from the angle of shearing resistance. Test 21, the triaxial test, has many variant forms with varying degrees of suitability for different soil types. Where samples are rare, multistage triaxial testing is recommended to maximise the amount of data obtained from the minimum number of samples. The
FIG. 3 INFLUENCE VALUES FOR VERTICAL STRESS VERSUS POSITION, \( \frac{x}{r} \)
FIG. 1: SALIENT DIMENSIONS OF THE SOIL–ARCH SYSTEM

FIG. 2: MEXE AND ARCHIE CAPACITIES VERSUS FILL DEPTH AT THE CROWN


<table>
<thead>
<tr>
<th>Material and geometric properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Span/ m</strong></td>
</tr>
<tr>
<td><strong>Rise/ m</strong></td>
</tr>
<tr>
<td><strong>Ring thickness/ m</strong></td>
</tr>
<tr>
<td><strong>Cover at crown/ m</strong></td>
</tr>
<tr>
<td><strong>Fill depth at crown/ m</strong></td>
</tr>
<tr>
<td><strong>Pavement depth at crown/ m</strong></td>
</tr>
<tr>
<td><strong>ϕf/ degrees</strong></td>
</tr>
<tr>
<td><strong>γf/ kNm^-3</strong></td>
</tr>
<tr>
<td><strong>γa/ kNm^-3</strong></td>
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<tr>
<td><strong>γp/ kNm^-3</strong></td>
</tr>
<tr>
<td><strong>Passive pressure factor</strong></td>
</tr>
</tbody>
</table>

Table 1 Material and geometric properties

<table>
<thead>
<tr>
<th>Modification factor</th>
<th>Cover=0.15m</th>
<th>Cover=0.25m</th>
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</thead>
<tbody>
<tr>
<td>Span to rise factor</td>
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<td>1.00</td>
</tr>
<tr>
<td>Profile factor</td>
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<td>0.69</td>
</tr>
<tr>
<td>Barrel factor</td>
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<td>1.00</td>
</tr>
<tr>
<td>Fill factor</td>
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<td>0.70</td>
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<tr>
<td>Material factor</td>
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<tr>
<td>Mortar factor</td>
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<td>0.90</td>
</tr>
<tr>
<td>Joint factor</td>
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<td>0.73</td>
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<tr>
<td>Axle factor</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 2 MEXE modification factors
The examples cited were intended to give the reader an idea of the costs of typical arch rehabilitation jobs. All the jobs were necessary, in some cases, urgently. Few of the arches had only one problem; spalling accompanied cracking and spandrel wall bulging accompanied foundation problems. Such is the complex nature of the soil-arch system. In all cases action was taken quickly and efficiently. The costs of the examples used ranged from £70000 to £1 25 million. The disturbance to traffic caused by the methods used ranges from the minimal at Balmoor Bridge and Spencer River Bridge to complete closure for a period of time in some cases. The authors propose that the use of pavement overlays could enhance the capacity of a large number of the nation’s arch bridges and culverts at very low cost. The disturbance to traffic is minimal if only half the carriageway is resurfaced at one time. On a capacity increase per unit cost basis the overlay method gives a cheap axle limit increase.

Obviously for the arch bridges discussed in the paper the repairs used were fully justified but for very little extra cost even greater axle limits could have been achieved by the judicious use of pavement overlays to give increased load dispersal. If the arch itself shows signs of deterioration, as would be indicated by a reduced MEXE condition factor after a routine inspection, then the other more costly methods of repair must be used. If the structure is, in other respects, safe, then capacity increases could be brought about cost effectively by the overlay method and other, more expensive methods need not be used.

The disadvantages of using pavement overlays may be summarised as follows: where the arch bridge is humpbacked the addition of 0.10m of asphalt could result in the new level of the road surface having to be maintained for a considerable distance off the span itself to keep to an acceptable vertical alignment. If the road level were raised then the kerb and footpath levels would also need raising, thus adding to the overall time and cost of the method. Finally, if the road level were raised, the parapet level may need raising to keep within safety guidelines. As many arch bridges are listed structures any work affecting the external appearance of the bridge, such as parapet raising, would have to be designed and executed with due regard for the aesthetics of the structure. This would add to the time and cost of the proposed overlay method.

Within the constraints discussed the overlay method, because of its beneficial increases in stress dispersal, could be used to achieve higher allowable axle limits for arch bridges at low cost. This would reduce the financial burden of meeting new EC directives thus allowing money to be saved or diverted to other areas of infrastructure development in the future.

CONCLUSIONS

1. As the cover above an arch is increased its capacity, when calculated by two current assessment methods, increases.

2. The extra cover causes a capacity increase by allowing increased stress dispersal as demonstrated by the codified method and Bousinesq's analysis.

3. For the arch analysed, capacity increases of up to 61% and vertical stress reductions of up to 21% were found.

4. The pavement overlays alone were shown to be a cost effective means of increasing allowable axle limits within certain constraints.

5. The use of pavement overlays in conjunction with other repair techniques, where needed, could give even greater increases in allowable axle limits at little extra cost compared to the prices of these other repair methods when used alone.

ACKNOWLEDGEMENTS

The assistance of the SERC and the Transport Research Laboratory under CASE Award No. 90552123 is gratefully acknowledged. The assistance of Drs. W. J. Harvey and F. W. Smith of the Department of Civil Engineering, University of Dundee is gratefully acknowledged.

REFERENCES


The cost of adding a pavement overlay to a typical arch bridge was based on information obtained relating to the cost per square metre of a 0.06m thick hot rolled asphalt (HRA) base course plus a 0.04m thick HRA wearing course. The combination of the two gave a typical 0.10m pavement overlay. Rates of £8 and £6 per square metre were quoted for the base and wearing courses respectively. Combining these costs gave a total cost of £14 per square metre. The quoted rates were high because of the small quantities involved. The total cost of the works was estimated to be no more than £150000 for a typical 10m span, 6m carriageway width, arch bridge. The time taken for the work, from mobilisation to completion, was estimated at one week, assuming fair weather conditions.

Other commonly used arch rehabilitation methods were costed using information obtained about jobs already completed. In 1988 the Bridge of Alford, Grampian Region, was restored at a cost of £900000(5). The work entailed replacement of some defective masonry and replacement of the fill with a pulverised fuel ash/cement mixture. In 1986 the three span Spencer River Bridge was widened and strengthened(16). A variety of options were considered and costed; the cheapest was the restraining of individual spans by tying between the springers using post tensioned steel bars, the most expensive was the strengthening of the substructure using small diameter bored piles. The estimated cost of the works ranged from £70000 to £210000. The option selected for the Spencer River Bridge was the lining of the existing arches with precast concrete segmental units. The estimated cost of this scheme was £1500000.

An alternative to lining the intrados is the placing of a concrete saddle over the extrados as was done at Chertsey, Surrey in 1991 by contractor Mowlem(18). The works here also included the addition of a reinforced concrete deck and replacement of the stone parapet walls over all six spans. The cost of the works to Surrey County Council was £1.25million.

Other typical cost results obtained were as follows: the use of a deck waterproofing scheme on the Lovat Bridge(17) cost £150000 in 1985, complete replacement, as was done at Bridge of Avon(5), Grampian Region, cost £503000 in 1990 and the use of steel tendons through the fill and spandrel walls at Balmoor Bridge, Grampian Region, cost £90000 in 1990-91.

**Discussion**

The capacity analyses are discussed together to enable comparisons to be made between the two methods used. Following this the stress dispersal analyses are discussed and the evidence supporting the capacity increases is produced. Finally the cost data are discussed as are some limitations to the usefulness of the pavement overlay method.

**Cost Data**

The capacity increased with the cover over the arch because of the increased stress dispersal arising from the extra cover. The extra cover causes a small dead load increase therefore the live load required to cause collapse will also increase because of the greater deviati of the thrust line needed to form the collapse mechanism. The MEXE analysis cannot distinguish between the addition of extra fill and the addition of extra cover by means of a stiffer road pavement overlay. The ARCHIE analysis, being more sophisticated, can make the distinction, hence the larger capacity increase for the same extra cover when ARCHIE's overlay facility is used. The ARCHIE capacity was larger than the MEXE capacity at any one fill depth for the arch analysed as ARCHIE is a more detailed analysis. The MEXE method takes no account of factors such as the earth pressures resisting arch deformation under load, hence its inherent conservatism in this case. It is obvious that different arch geometries and material properties will give rise to different analysed capacities and the MEXE method may not always be as conservative in some instances whereas in others it may be even more conservative. The configuration analysed here was chosen to demonstrate the effect of increasing the fill depth, or cover, over an arch. A full comparative study of the relative merits of current methods of analysis is beyond the scope of this paper. The following section discusses the stress dispersal analyses used in order to produce evidence supporting the capacity increases discussed above.

**Stress Dispersal Analyses**

The stress dispersal analyses carried out with the cover at 0.15m and 0.25m all showed the reduction in vertical stress arising from the extra cover provided. The extra cover places the load point further away from the extrados, hence the reduction in the influence values for the vertical stress. Due to the reduction of the stress on the arch the analysed capacity is increased when the fill depth, or cover, is increased. The codified analysis predicted higher stresses than the Boussinesq analysis because the code allows only limited dispersal as described above. The plot of influence value versus position (Fig. 3) looks unrealistic for the codified method of dispersal because of the unnatural limits imposed on its lateral extent by the code. The Boussinesq dispersal, allowed to continue until an influence value of 0.100 was encountered, appears more natural. The conservatism of the codified dispersal method is demonstrated experimentally by the authors elsewhere(12,13). It is obvious that different arch geometries will give rise to different analysed dispersal patterns and the codified method may not always be as conservative in some instances whereas in others it may be even more conservative. The configuration analysed here was chosen to demonstrate the effect of increasing the fill depth, or cover, over an arch. A full comparative study of the relative merits of available methods of stress dispersal is beyond the scope of this paper.
applied load to determine the ring thickness needed to carry that load. The program includes some allowance for the earth pressures acting on the extrados. The program can assess the effect of altering any of the variables given in Table 1. In this study the overlay facility was used to assess the effect of increasing the fill depth upon the capacity. The other variables were kept constant whilst overlays of 0.04m, 0.06m and 0.10m were added to the 0.15m of fill over the crown. A single axle load with no lift-off was applied to the arch. For each depth of overlay the load position was moved horizontally across the upper surface of the soil-arch system in 0.20m increments until the critical position was found. The load was then increased until the formation of a four hinged collapse mechanism.

**STRESS DISPERSAL ANALYSIS BY THE CODIFIED METHOD(7)**

The codified method assumes that the load exerted by a tyre on the road surface is distributed, at the level of the arch extrados, over the total width of the applied stress plus half the distance from the road surface to the extrados. A unit stress was applied to the upper surface of the soil-arch system with its centreline at $(x/r)=-0.33$ and edges at $(x/r)=-0.33\pm0.08$. Influence values for the vertical stress on the extrados were calculated. The influence values for vertical stress were calculated, by this method, with the fill depth over the crown at both 0.15m and 0.25m.

**STRESS DISPERSAL ANALYSIS BY BOUSSINESQ'S METHOD(11)**

The stresses within a semi-infinite, elastic, homogeneous, isotropic mass of soil due to a uniform stress on its surface were determined by Boussinesq(15). A unit stress was applied to the upper surface of the soil-arch system with its centreline at $(x/r)=-0.33$ and edges at $(x/r)=-0.33\pm0.08$. Influence values for the vertical stress on the extrados were calculated using the geometry of the arch. The assumption of a semi-infinite soil mass was violated by the presence of the arch and homogeneity would be unlikely in practice. The method was used as it was felt that it could offer the benefits of more realistic stress dispersal, regardless of the violation of two assumptions. The influence values for vertical stress were calculated, by this method, with the fill depth over the crown at both 0.15m and 0.25m.

**COST DATA**

The information about the cost of the proposed overlay method was obtained through a survey of published articles relating to arch bridge rehabilitation projects in Britain, and by private communications with a contractor and a regional council.

**RESULTS**

The results from each of the capacity and dispersal analyses are presented. Increases in arch capacity and decreases in the vertical stress on the extrados are shown to have occurred. Comparisons between the overlay and other capacity increasing methods are made.

**CAPACITY ANALYSIS BY THE MEXE METHOD**

The variation in capacity with fill depth for the MEXE method is shown (Fig. 2). At a fill depth of 0.15m the MEXE capacity was 57.5kNm$^{-1}$, increasing to 89.3kNm$^{-1}$ at a fill depth of 0.25m. This represented a capacity increase of 55% for an extra 0.10m of cover.

**CAPACITY ANALYSIS BY ARCHIE**

The variation in capacity with fill depth for the ARCHIE analysis is shown (Fig. 2). At a fill depth of 0.15m ARCHIE gave an arch capacity of 72kNm$^{-1}$, increasing to 116kNm$^{-1}$ at a fill depth of 0.25m. This represented a capacity increase of 61% for an overlay of 0.10m. The ARCHIE capacity was, for the arch shown (Fig. 1), always higher than the MEXE capacity. The ARCHIE capacity was increasing faster than the MEXE capacity as the cover increased.

**STRESS DISPERSAL ANALYSIS BY THE CODIFIED METHOD(7)**

The influence values for the vertical stress on the extrados are shown (Fig. 3). The codified method gave a peak vertical stress of 0.510 times the applied stress for a fill depth of 0.15m. This peak stress decreased to 0.401 times the applied stress for a fill depth of 0.25m. This represented a decrease in the vertical stress on the extrados of 18% for a 0.10m increase in fill depth. The stress distribution produced by the codified method was narrow with no stress being distributed onto the extrados outwith the range $-0.50<(x/r)<0.10$.

**STRESS DISPERSAL ANALYSIS BY BOUSSINESQ'S METHOD(11)**

The influence values for the vertical stress on the extrados are shown (Fig. 3). Boussinesq's method gave a peak vertical stress of 0.349 times the applied stress for a fill depth of 0.15m. This peak stress decreased to 0.287 times the applied stress for a fill depth of 0.25m. This represented a decrease in the vertical stress on the extrados of 18% for a 0.10m increase in fill depth. The stress distribution produced by the Boussinesq analysis was wider than that predicted by the codified method; stresses were distributed onto the extrados over the range $-1.00<(x/r)<0.20$. At $(x/r)=-1.00$ the springer of the arch was encountered so no further stress distribution occurred. At $(x/r)=0.20$ the influence values had fallen to below 0.100; this value being regarded as the edge of the bulb of pressure for most purposes, the stress distribution was curtailed at this point.
Bituminous road pavement overlays are proposed as a method of economically increasing the load carrying capacity of masonry arch bridges. A codified MEXE analysis and a computerised mechanism method confirm the increase in arch capacity arising from these proposals. Stress dispersal analyses are used to show the benefits of such overlays. A 0.10m overlay was shown to increase the arch capacity by 61% whilst decreasing the vertical stress on the arch by 21%. Cost data is presented for the typical pavement overlays analysed. The use of pavement overlays, where feasible, is shown to be economical whilst giving beneficial load carrying capacity increases. The paper is written with the purpose of obtaining feedback on an idea to reduce the cost of rehabilitating those arches currently failing routine assessments in the UK.

**INTRODUCTION**

Archaeological evidence places the dawn of the masonry arch at circa 3600 B.C. in the kingdoms of Egypt and Mesopotamia\(^1\). The importance of the masonry arch to Britain's infrastructure cannot be overemphasised. Approximately 40000 masonry arch bridges were built between the 17th and 19th centuries. These still stand today; under axle loads well beyond those predicted by their builders. The roads authorities in Britain bear the legal responsibility for the assessment and maintenance of the nation's bridge stock\(^2\). Under new European Commission (EC) directives, the Department of Transport (DTP) has to increase the maximum allowable gross vehicle weight (GVW) from 38t\(^3\) to 40t and the maximum axle weight from 10t\(^4\) to 11.5t. Some EC member states have suggested further increases in GVW to 44t. To cater for these increased loadings, the existing bridge stock has to be reassessed. The cost of the upgrading and rehabilitation work has been estimated at £1400million\(^4\) for the country as a hole and between £100million and £200million for Scotland's 8850 rural road structures alone\(^5\).

Many arches are subject to overloading due to inadequate cover. Lack of cover over the arch leads to a vertical stress concentration on the extrados\(^6\). The purpose of this study is to examine the effects of increasing the cover over the arch. The use of bituminous overlays is proposed as a cost effective means of increasing an arch bridge's allowable axle limit.

The paper presents results from two of the current assessment methods to demonstrate the benefits arising from an increase in the total crown depth of a soil-arch system. These assessment methods are the codified MEXE method\(^7\) and ARCHIE\(^8,9,10\), a widely used computerised mechanism analysis. Stress dispersal calculations are undertaken according to the codified method\(^7\) and Boussinesq's analysis\(^11\). The stress dispersal techniques show the reduction in vertical stress on the extrados caused by the addition of a pavement overlay. Further information on the stress dispersal in a soil-arch system may be found in the work of Fairfield and Ponniah\(^12,13\).

Costs are given for the addition of a typical 0.10m thick overlay and these are compared with the costs of other typical arch rehabilitation techniques. A 0.10m overlay is shown to give a 61% capacity increase and a 21% decrease in the peak vertical stress on the extrados. The paper's principal conclusion is that pavement overlays should be considered as an economical way of upgrading arch bridges.

**METHOD**

The capacity and stress dispersal analyses used are described. They have been applied to a typical soil-arch system (Fig. 1) with the geometric and material properties given (Table 1). The coordinate system for stress and position used throughout the study is also shown (Fig. 1). The method of collection of the cost data is then described.

**CAPACITY ANALYSIS BY THE MEXE METHOD**

The MEXE method assumes a limiting compressive stress in the masonry and this condition is used to permit calculations on different spans, rises, ring thicknesses and fill depths. The results were correlated, tabulated and presented in the nomographic form used today. A provisional axle load was calculated for the arch which was multiplied by the relevant modification factors (Table 2). The fill depth over the crown was varied in four stages from 0.10m to 0.25m and the capacity calculated at each stage.

**CAPACITY ANALYSIS BY ARCHIE**

Dundee University's ARCHIE program uses the mechanism method\(^14\) to analyse an arch under some
Fig. 7 shows the normal stress on the arch beneath the load to be 25% lower than that predicted by the code. The load dispersal through the fill was greater, in practice, than the code allows. On the side of the arch remote from the load the normal stress found experimentally was significantly higher than that allowed for in the code. The fact that the code ignores this potentially beneficial partial mobilisation of passive fill pressure makes it unduly conservative. The shear stresses measured are greater than those predicted by current methods of assessment. The code ignores the potential benefits of allowing some stabilising shear stress to be mobilised as shown in Fig. 7. According to the code's analysis, only those pressures due to the self weight of the fill are permitted. No increased stress, normal or tangential, is permitted. Estimated collapse loads by other analyses were conservative, in the case of Heyman's analysis and the codified method by 60%. ARCHIE underestimates the collapse load by only 19%, a better prediction for this type of problem. Analyses ignoring the stabilising effects and overestimating the destabilising effects are uneconomical as their use would lead to unnecessary repair work leading to diversions and delays for commercial vehicles. Such diversions and delays imply increased noise and atmospheric pollution and increased prices for goods which transportation is a major cost element such as: steel, bricks and beer.

Conclusions
1. Improved arch bridge assessment methods are needed if economical implementation of EC directives is to be achieved.
2. The load carrying capacity of the test arch was increased due to soil-structure interaction effects.
3. Current methods of analysis underestimate the strength of the test arch.
4. The closest prediction of the collapse load was given by ARCHIE.
5. Soil pressures measured were up to 25% lower in practice than allowed for in the codified method.
6. Further tests of this nature are needed to add to existing knowledge of the composite behaviour of the soil-arch system.

Acknowledgements
The assistance of the SERC and the Transport Research Laboratory under CASE Award No. 90552123 is gratefully acknowledged. The author gratefully acknowledges the assistance of the Technical Staff in the department along with that of his student Mr R. Puri.

References
Geotechnical considerations in arch bridge assessment

<table>
<thead>
<tr>
<th>Test value</th>
<th>Codified method</th>
<th>Collapse ratio (MEXE)</th>
<th>Heyman's Analysis</th>
<th>Collapse ratio (Heyman)</th>
<th>ARCHIE analysis</th>
<th>Collapse ratio (ARCHIE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.1</td>
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<td>8.56</td>
<td>2.58</td>
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Table 2 Comparison of collapse load values, (all values kNm⁻¹).

Discussion and analysis

The failure mode was as expected; four hinges forming a collapse mechanism. The first hinge formed, not directly beneath the load, but slightly closer to the crown at (x/r)=0.30. This was due to load dispersal through the fill. The most stressed point on the arch was not vertically below the load but at a point where the combination of depth and horizontal distance gave the largest stress increase. The second hinge formed close to the crown of the arch. Here the restraining soil pressure was minimal, allowing sufficient rotation of segment 1-2, (Fig 6), for hinge formation. The third and fourth hinges did not form at the springers, but, because of the at rest earth pressures acting below the levels of hinges 3 and 4 (Fig 6), which resisted arch rotation, they formed at a level above the springers. These hinge locations did not correspond with those predicted by Heyman in his plastic analysis which uses only the fill’s dead weight. The effective span of the arch was the horizontal distance between hinges 3 and 4, i.e. 1.51m: substantially less than the actual span of 2m used currently.

The percentages of ultimate load at which the hinges formed is worthy of comment. The first hinge formed at only 30% of ultimate load and only when a further 26% of the ultimate load is added did the second hinge form. The remaining two hinges formed at high percentages of the ultimate load. Consideration of the thrustline leads to an explanation: the deviation of the thrustline from the at rest position required to form two hinges was large; the subsequent deviation needed to form the final two hinges was small as the thrustline already lay close to the extremities of the arch.
The stresses, then, vary from -1 at the left hand radius of the extrados, r. This coordinate as a ratio; distance from the origin, x

The leading dimensions of the model, the coordinate system and the stress convention are shown (Fig. 4). The dimensionless horizontal coordinate is expressed as a ratio; distance from the origin, x: radius of the extrados, r. This coordinate then varies from -1 at the left hand springer to +1 at the right hand springer. The stresses, \( \sigma \) and \( \tau \), act normally and tangentially to the extradoses respectively.

The applied stress was increased at a rate of 115.2 kN/m², corresponding to a line load of 22.1 kN/m. The failure was by formation of a four hinged mechanism with the hinge locations as shown (Fig. 6), and the order of hinge formation is also shown. The loads at which the hinges formed is given below (Table I).

<table>
<thead>
<tr>
<th>Hinge</th>
<th>Position (x/r)</th>
<th>Load @ formation</th>
<th>% of collapse load</th>
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<tr>
<td>1</td>
<td>-0.30</td>
<td>34.6 kN/m²</td>
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<tr>
<td>2</td>
<td>+0.03</td>
<td>99.0 kN/m²</td>
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<tr>
<td>3</td>
<td>-0.64</td>
<td>115.2 kN/m²</td>
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</table>

Table 1 Hinge Positions.
Geotechnical considerations in arch bridge assessment

by C A Fairfield and D A Ponniah

An element of an on-going investigation into 'the soil-structure interaction in a soil-masonry arch system is described. A laboratory test on an instrumented, two metre span semicircular brickwork arch was undertaken. Measurements of arch displacement and normal and tangential earth pressures on the extrados were carried out as the applied load was increased until collapse. Comparison is made between the measured pressures and those allowed for in the current UK assessment code for highway bridges and structures. Current methods of assessment are shown to be conservative due to their omission of the effects of soil-structure interaction.

Archaeological evidence places the dawn of the masonry arch at circa 3600 BC in the kingdoms of Egypt and Mesopotamia. The principles of arching were used on a grand scale with the Romans' use of the semicircular arch. The importance of the masonry arch in Britain's infrastructure increased between the 17th and 19th centuries with over 40000 being constructed. These still stand today; under ever increasing axle loads, well beyond those predicted at time of construction. Many will require strengthening or replacing by 1999 due to new legislation. The European Commission (EC) requires the Department of Transport (DoT) to raise the maximum allowable gross vehicle weight (GVW) from 38t to 40t and the maximum axle weight to 11.5t. Some EC member states have suggested further increases in GVW to 45t. To cater for these increased loads, the existing bridge stock has to be reassessed. Costs have been estimated at around £1400m for the upgrading and subsequent work. Such costs would be particularly onerous to the DTP necessitating the diversion of funds away from other areas of infrastructure maintenance and development. The costs, mainly borne by industry and passed on to the consumer, of being unable to use 40t lorries are estimated at a minimum of £100m per annum. The bridge assessment programme is urgent. The load dispersal through the fill is suggested by current assessment methods. The codified assessment method allows an engineer to assume dispersal at a slope of 1 in 2 to no deeper than the level of the crown of the arch. As will be shown, a greater amount of the load is dispersed before reaching the extrados rendering the current assessment method conservative. The deformation patterns at low and high loads are shown (Figs 2 & 3). Under load the portion of the arch beneath the load point moves away from the fill, causing the pressures to fall from the at rest values to the active state. On the side of the arch remote from the load, the arch displacement are into the fill, causing mobilisation of passive pressures. At rest pressures still act at the springers; preventing arch deformation. This prevents hingess forming at the springers and it reduces the effective span of the arch. This has the effect of increasing the collapse load above that found by an analysis without horizontal soil support. As a consequence of the arch deformations, circumferential shear stresses are mobilised. Existing analyses only provide for normal earth pressures and no provision...