STUDIES IN DIRECT BREAK UP REACTIONS

Thesis

Submitted by

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ABSTRACT

A semi-classical trajectory calculation has been undertaken in order to model light ion projectile break up for the reaction $^{208}_{\text{Pb}}(^7_{\text{Li}},\alpha t)$ at $E_{\text{Li}} = 70$ MeV. The projectile is considered to be clustered into two fragments throughout the calculation; initially the clusters are given the spatial possibilities within the classical limits. As the projectile approaches the target the fragments interact with the target and the resulting individual fragment trajectories are determined to high numerical accuracy. The calculation takes full account of all potential interactions between the three particles in the final channel. Inelastic interactions of the fragments with the target are considered by the use of complex interaction potentials. The calculation is capable of predicting the cross sections for several different reaction channels.

It is found that the results of the calculation predict correctly the magnitudes and trends of a substantial body of experimental data. Since there are no adjustable parameters in the calculation this finding is both remarkable and significant. This discovery is particularly valuable since there is no completely satisfactory quantum mechanical solution of this problem. Another valuable feature of this semi-classical calculation is that the simplicity of the mathematical formulation lends itself to a direct model picture of the essential dynamics. For particular kinematic situations several pictorial representations are discussed. Such representations should help the experimentalist to focus his experiments towards the most interesting physics.

As a complement to the semi-classical evaluations several quantum mechanical break up calculations have also been investigated. A
calculation based on the plane wave approximation for all interacting particles reproduces some of the trends seen in the experimental data. A calculation based on the quasi-free break-up model is also undertaken in a restricted method, but it is found that this method does not predict any results that are in reasonable agreement with the experimental data. Finally, the results of an initial investigation are given into the feasibility of performing a full calculation where the distortion of all the particle waves are correctly accounted for.
This thesis has been composed by myself, and the approach presented is entirely my own work.
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CHAPTER I

INTRODUCTION

I.1 General Introduction

Much information about nuclear structure and nuclear reaction mechanisms is obtained as a result of bombarding an energetic nuclear particle (projectile) on a nucleus (target). When two nuclei barely touch each other, either they keep their identities, or more intimate contact leads to a new nuclear system. The results of both aspects must be studied fully to find out the physical relations governing this microscopic world.

Experimentally, fast accelerators can produce highly energetic particles. The charged projectile will be able to overcome the mutual Coulomb barrier with the target giving the possibility to study a wide variety of different nuclear processes. Theoretically, bigger and faster computers increase the domain of theoretical investigations and analysis.

Fragmentation processes are a common phenomenon in nuclear reactions, in which the projectile may, together with the target, decompose to substructures of the originals. These fragmentation processes could be induced by light and heavy ions, at non-relativistic and relativistic energies. A prominent feature of very asymmetric heavy ion reactions at non-relativistic energies (8 - 20 MeV/nucleon) is the copious production of fragments with mean velocities close to that of the projectile.

The energy spectrum of an ejectile at a particular angle often shows the following characteristics:
At low ejectile energy the distribution is continuous and may have the Maxwellian shape due to evaporation of the ejectile from the compound nucleus, and may have added to it the lower energy component of the pre-equilibrium emission of the ejectile (Hau 52, Kik 68, Vog 68, Bla 75, Flo 79).

In the high energy part of the spectrum, sharp peaks result from reactions leaving the residual nucleus in a well defined state (Sha 63, Bro 67, Boh 69, Eis 76, Iac 79).

In between those limits, a continuous region exists, as a result of several processes in which one considers the ejectile to come from a high level density region of the compound nucleus with level spacing comparable or less than the natural width of the states, or to come from sharp states with poor resolution of the detection system. This continuum region may contain the high energy ejectiles out of the pre-equilibrium process or may also be due to containing special structure like isobaric analogue resonances, or multipole collective motions, e.g. giant resonances. The following references deal with the last two regions (McC 68, Aus 70, Jac 70, Hod 78, Hod 81, Sat 83).

During the last few years, much attention has been devoted to this continuum region, since it includes a large variety of different processes. The one which has growing interest is the projectile fragmentation or break up processes of light ions.

Break up is a peripheral process in which the projectile is fragmented into smaller parts in the Coulomb and/or nuclear field of the target nucleus.

The simplest type of the break up occurs when the projectile divides into two fragments (binary break up). It is found that the
the cross section increases rapidly with energy as soon as it exceeds the threshold of the break up.

The break up can take place, together with other competing processes in the continuum region, producing the same kind of particles so it is desirable to detect the two fragments in coincidence with clear identification of the particles. The main trend to date is to understand more of the characteristics of the break up, starting with light ion projectiles \( (z = 1, 2, 3) \) before studying heavier-ion break up.

The projectile break up was first considered some fifty years ago. It started with the simplest composite particle, the 'deuteron'. Oppenheimer and Phillips (Opp 35a, Opp 35b) suggested a model based on the target Coulomb field as the principal interaction for the break up of the projectile. Also Landau and Lifshitz (Lan 48) described the Coulomb break up. The first experiment reported on the deuteron was done by Helmholtz (Hel 47) and its earliest theoretical interpretation was given by Serber (Ser 47) based on his spectator model.

More sophisticated experiments on deuteron break up (Fin 69, Jar 73, Gem 76, Mat 82) shows that as the energy increases the nuclear interaction will become more important than the Coulomb interaction.

It was found for the deuteron \( (E_d = 5.6 - 200 \, \text{MeV}) \) that the break up cross section is \( 12 - 43\% \) of the total, cross section and increases as the target mass increases (Pam 78).

Inclusive measurements from \( (d,p), (d,n) \) showed a pronounced break up bump at forward angles, centred at beam velocity \( \sim \frac{1}{4} E_d \) and its full width at half maximum \( \text{FWHM} = \Delta E \) decreases with increasing target mass (Mat 80b) and has this relation (Ser 47):
\[ \Delta E = 1.5(E_d \epsilon)^{1/3} \]

where

- \( E_d \) = deuteron kinetic energy
- \( \epsilon \) = binding energy of the deuteron \( \sim 2.2 \text{ MeV} \).

A shift for the centroid position of the break up bump, as observed for heavy targets, was attributed to the target Coulomb field (Mat 80b).

The break up cross section was found to be proportional to \( A^{2/3} \), where \( A \) is the target mass number, at low energy \( E_d = 15.5 - 25.2 \text{ MeV} \) (Pam 78, Bis 80, Kle 81) and at high energy \( E_d = 56 - 200 \text{ MeV} \) (Hel 47, Wu 79, Bec 80, Mat 80b).

The elastic break up of the deuteron, in which the target is left in its ground state, measured by coincidence at forward angles, may account for up to 48\% of the total break up cross section (Jar 73, Mat 82). Here the angular correlation showed an enhancement for wide angle measurements, in which the two ejectiles are detected at opposite sides of the beam axis. Kleinfeller et al. (Kle 81) suggested that Coulomb break up by heavy targets may become important for an incident deuteron energy near or above the Coulomb barrier. On the other hand, Jarzyk et al. (Jar 73) suggested that Coulomb forces are not important at \( E_d = 12 \text{ MeV} \) on Au, Pb for elastic break up. So the role of Coulomb break up is still an open question.

For \(^3\text{He}\), inclusive measurements Matsuoka et al. (Mat 78) found that the deuteron spectrum, as a result of \((^3\text{He}, d)\) has a bump at about \( \frac{2}{3}E_{^3\text{He}} - \frac{1}{3}E_C \) where \( E_{^3\text{He}} \) is the incident kinetic energy of \(^3\text{He}\) and \( E_C \) is the Coulomb barrier. The cross section is again strongly forward peaked with FWHM.
\[ \Delta E = 1.26 \left( E^3_{\alpha} \cdot \epsilon \right)^{1/3} \]

\( \epsilon \) is the separation energy of the deuteron from \(^3\)He. The inclusive break up cross section is found to be proportional to \( A^{1/3} \). They also estimated the total break up cross section \( \sim 17\% \) of the geometrical cross section of heavy target \(^{209}\)Bi and up to \( 45\% \) for light target nuclei \(^{12}\)C, where the geometrical cross section is \( \pi R_A^2 = \pi \times 1.96 \text{A}^{2/3} \). Similar features are also obtained by (Dja 83) at \( E^3_{\alpha} = 130 \text{ MeV} \).

The elastic break up from coincidence data \(^3\)He(dp) also has a forward peak for elastic break up and can account for \( 50\% \) of the inclusive cross section at \( \theta_d = 15^\circ \) (Mat 80b). For \( \alpha \) break up at \( E_\alpha = 80 - 172 \text{ MeV} \) (Bud 78, Wu 78, Wu 79, Koo 79) inclusive measurements show a bump about \((m/m_\alpha)E_\alpha\) where \( m \) is the mass of the ejectile. The centroid and the width were found to be independent of the target mass. The total break up accounts for up to \( \sim 30\% \) of the total reaction cross section, and is proportional to \( A^{1/3} \). The coincidence measurements show the following:

i) sequential break up into \( p+t, d+d \).

ii) direct break up of \( \alpha \) into \( p+t, d+d, n+^3\)He.

iii) absorptive break up, characterized by fast spectator and an evaporated particle. This process was the dominant one (\( \sim 90\% \) of the inclusive).

iv) as the projectile energy increases from 65 MeV to 140 Mev \((\alpha, dd), (\alpha, pt)\) cross sections increase by one order of magnitude (Koo 79). They also found that at \( E_\alpha = 140 \text{ MeV} \), on different targets strong contributions are present from absorptive break up reaction via Li-particles.
(\(^5\text{Li}^* \rightarrow \alpha + p, \quad \text{\(^6\text{Li}^* \rightarrow \alpha + d, \quad \text{\(^7\text{Li}^* \rightarrow \alpha + t\))}.

(Bud 78) found that inelastic break up in \(^{58}\text{Ni}(\alpha, \text{tp})\) is four times the elastic break up and the difference was attributed to pre-equilibrium processes.

For heavier projectiles, the process becomes more complicated due to the possibility of exciting the individual fragments as well as the projectile itself in the early stages, and causing more complicated interaction and de-excitation mechanisms.

For general reviews of non-relativistic energies see (Wil 80, Gel 80, Bau 84) and for relativistic energies see (Gol 78, Nag 82).

\(^6,7\text{Li}\) break up has also been studied (see Neu 80, Neu 82, Cas 80, Cun 80, Sho 81, Tab 82, Pla 86). Planeta et al. (Pla 86) studied the non-elastic break up of \(^6\text{Li}\) (156 MeV) on \(^{40}\text{Ca}\). By measuring the coincidence events between \(\gamma\) emitted from the target and a projectile fragment at the beam velocity, it was found that the total break up cross section is 50% of the total reaction cross section and the non-elastic break up is about 62% of the total break up.

In a kinematically complete experiment, Shotter et al. (Sho 81) observed the direct and sequential break up components of 10 MeV/nucleon \(^7\text{Li}\) projectiles on \(^{12}\text{C}\) and \(^{208}\text{Pb}\) into the \(\alpha + t\) channel. Bice (Bic 80) investigated \(^{12}\text{C}\) break up into 3 \(\alpha\)'s and \(^8\text{Be}\) during his study of fast \(\alpha\)-particle components in \(\alpha\)-energy spectrum resulting from the reaction \(^{208}\text{Pb} + \text{^{12}C}\) (132, 187, 230 MeV). Bhowmik et al. (Bho 79) found from correlations between \(\alpha\)-particles and heavy ion ejectiles in the reaction \(^{14}\text{N}(148 \text{ MeV})\) on \(^{58}\text{Ni}\) that some \(\alpha\)-particles are emitted at an early stage prior to the formation of deep inelastic fragments and they suggested that the break up of \(^{14}\text{N}\) may take place, with one of the projectile fragments subsequently interacting with the
target leading to the emission of other particles. One of the possible interactions of this fragment with the target is the complete absorption, while the other fragment behaves like a spectator. Also, they reported that other fragmentation processes were observed but account for only a few per cent of the break up cross section.

Tabor et al. (Tab 81) also studied $^{14}$N at 62 MeV on $^{27}$Az and they obtained the energy distribution of C, B, Be, Li fragments. The first three particles have pronounced peaks at the beam velocity and they attributed that mainly to the break up of $^{14}$N, followed by the absorption of one part of the projectile and the detection of the remaining part (spectator). Also, the angular distributions were fitted with a simple approach based on the internal momentum distribution of the fragment of the projectile.

Fröhlich et al. (Frö 79) in a study of the $^{40}$Ca($^{20}$Ne, $^{16}$O) reaction at 10 MeV/nucleon, observed a beam velocity component in the spectrum of $^{16}$O at forward angles which could not be explained by the $\alpha$-transfer process and they attributed this component to the break up of $^{20}$Ne. Their theoretical analysis of $^{16}$O reproduces the energy spectra well (Uda, 79c).

The experimental studies of light ion induced projectile break up are mainly performed at the following laboratories:

<table>
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<tr>
<th>Projectile</th>
<th>E/A (MeV/nucleon)</th>
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<tr>
<td>Bonn</td>
<td>d</td>
<td>7.5-25</td>
</tr>
<tr>
<td>Jülich</td>
<td>d</td>
<td>40</td>
</tr>
<tr>
<td>Maryland</td>
<td>d, $\alpha$</td>
<td>40</td>
</tr>
<tr>
<td>Osaka</td>
<td>d, $^3$He</td>
<td>23-37</td>
</tr>
<tr>
<td>Groningen</td>
<td>$^3$He, $\alpha$</td>
<td>16-20</td>
</tr>
<tr>
<td>Daresbury</td>
<td>$^8$Li, $^7$Li, $^9$Be</td>
<td>10</td>
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1.2 Motivation for Studying the Break up and Fragmentation Reactions

The motivation for the study of break up and fragmentation can be summarised as a study of the following:

1. To find out the main characteristics of the break up reaction and its role on other competing processes (Bod 80, Aar 81).

2. The Fermi distribution of the spectator inside the projectile in the ground state or probably excited as well. However, there are other factors to hinder clean momentum distribution determination such as
   i) projectile target orbital angular momentum dispersion;
   ii) final state interaction effects on the ejectiles;
   iii) multistep processes.

   All these are expected to be minimized at high energies.

3. Production of fragments far from the stability line (Sym 79).

4. Absorption cross section and limiting angular momenta.

5. Projectile spectroscopic properties, e.g. studying single particle strength in continuum by choosing projectile energy for optimal kinematical conditions for the excitation of various angular momentum states "resonances". (Gal 82).

6. Studying clustering probability. It was proved (Bau 79) that the break up dominates the surface region so it must be taken into account in theories of the interaction of composite particles.

7. Creation of exotic states of nuclei in projectile fragments at very high energy and studying anomalies, e.g. short mean free path of secondary nuclei (Tin 83).

8. $\beta$-decay half life measurements of the fragments (Mur 82).

10. As the break up characteristics are identified, e.g. energy spectrum from the break up process, this can help to subtract background spectra in the high energy region, for cleaner study of nuclear structure, and also giant resonances (Nom 78).

11. Together with the strong absorption model spin information for α-decay states could be extracted (Rae 84).

I.3 Projectile Break up Reaction Processes

The break up reaction processes can be divided into two modes.

I.3.1 Direct process

The projectile breaks up in the first step, due to the interaction with the target nucleus, and at least one of the ejectiles should be a part of the projectile. The energy spectrum of this ejectile has a bell-shaped distribution centred near the projectile velocity and the spectrum decreases rapidly as the scattering angle increases. The interaction time is of the order of nuclear transit time. The cross section of this direct break up process increases as the projectile and/or target mass(es) increase(s).

This process could be subdivided into

i) Elastic: the target is left in the ground state, and ejectiles are the projectile fragments (sometimes called quasi-elastic)

\[ a + A \rightarrow b + x + A \ . \]

ii) Inelastic: the target is left in a definite excited state, and ejectiles are again the projectile fragments

\[ a + A \rightarrow b + x + A^* \ . \]
iii) **Transfer:** The projectile breaks up first, and one of its fragments exchange one or more nucleons with the target

\[ a + A \rightarrow b + x + A^{(*)} \]

\[ \rightarrow y + B^{(*)} . \]

iv) **Absorption:** The projectile breaks up first, and one of the fragments captured by the target. The new subsystem may disintegrate via evaporation of light particles. Such a process is known as break up fusion

\[ a + A \rightarrow b + x + A^{(*)} \]

\[ \rightarrow B^* + y + C^{(*)} . \]

The other fragment in the last two mechanisms can be considered as a spectator.

I.3.2 **Indirect process (sequential break up)**

For this reaction process the projectile interacts first with the target, exciting it to a particle unstable state, then subsequently decays into the particle channel. There is a distinct separation assumed between the production of this excited particle and its decay. The decay process is assumed to take place outside the interaction (mainly nuclear) region of the target nucleus, and therefore the fragments should have the properties of the unbound system. Now the question of time is related to the life-time of the resonant state of the projectile compared with the transit time of the projectile across the target. Consequently, the information about the energy exchange mechanism, (e.g. by Coulomb and/or nuclear, ...), leading to the excitation of the projectile may be lost due to relatively long life-time of such resonant state.

1) (*) means the possibility of excitation is included.
The sequential break up could be subdivided into:

i) **elastic**: in which the projectile excited, then decays, while the target is left in its ground state.

\[
a + A \rightarrow a^* + A \rightarrow b + x .
\]

ii) **inelastic**: both projectile and target become excited

\[
a + A \rightarrow a^* + A^* \rightarrow b + x .
\]

iii) **transfer**: one or more nucleons exchanged with the target, and a particle is produced which subsequently decays

\[
a + A \rightarrow c^* + B(*) \rightarrow b + y .
\]

Note that in the case of multifragment production

\[
x = \sum_{i=1}^{n} x_i,
\]

\[
y = \sum_{i=1}^{n} y_i.
\]

Such sequential processes were suggested first for relativistic energies where the nuclear continuum can be strongly populated. Here Coulomb and/or nuclear forces may cause the excitation to a resonant state for the target and may also for the projectile, e.g. via multipole collective motion. The particle decay of this resonance will contribute significantly to the fragmentation spectra (Win 79).
I.4 Model Description of the Break Up Phenomenon

In the continuum region several reaction processes occur. A wide variety of models have been proposed. Some of them describe the fragmentation of the projectile in the peripheral region. These can be divided into several groups.

I.4.1 Simple Models

These models are proposed to describe mainly inclusive measurements.

I.4.1.1 Serber model (Ser 47)

The basic idea is that the projectile consists of two substructures, the spectator (b) and the participant (x). For the peripheral reaction the participant x interacts with the target nucleus, while the spectator does not interact at all with the target. Naturally the spectator will pass into the forward direction. The velocity distribution has a bell shape, and it is peaked around the projectile velocity $v_p$, equivalent to energy $E_b$:

$$E_b = \frac{m_b}{m_p} E_p$$

where

$m_p$ is the mass of the projectile

$m_b$ is the mass of the spectator

$E_p$ is the kinetic energy of the projectile.

The width of the velocity distribution is governed by the Fermi motion of the spectator inside the projectile. The interactions of the participant with the target could be compound, pre-equilibrium, deep inelastic or a direct process.
The calculations of the cross section are based on the probability density distribution $|\psi(r)|^2$, $|\psi(p)|^2$ where $r$ is the relative distance between $x$ and $b$ and $p$ is the relative momentum. Many Serber type calculations (Mat 78, Wu 78, Bud 78, Shy 79, Kam 79, Tab 81) reproduce the inclusive data qualitatively, suggesting that the light ion break up process occurs within the peripheral region of the target nucleus. The model is a limiting case in the framework of multi-step direct reaction theories (Fes 80). The model has its limitations:

1. It predicts only inclusive spectra.
2. It neglects projectile distortion effects due to Coulomb or nuclear forces.
3. The model assumes total interaction of the participants and total transparency of the spectator.

Using the same idea of dividing the projectile into $b, x$, Hussein and McVay (Hus 79) have shown that for heavy ion induced reaction, the break up cross section is proportional to $|\phi(K)|^2$ where $\phi(K)$ is the momentum space wave function of the observed fragment in the projectile, $\hbar K$ its internal momentum inside the projectile. Such an approach accounts for the main features of the angular distribution for the reactions $^{208}\text{Pb}(^{16}\text{O},^{12}\text{C})$, $^{208}\text{Pb}(^{16}\text{O},^{15}\text{N})$ and $(\alpha, h)$ on various targets.

I.4.1.2 Abrasion ablation model (Wes 76)

As the projectile hits the target peripherally, their joint parts are swept out in a quasi-equilibrated high excitation and subsequently decay into fragments, whilst the rest of the projectile
and the target remain as spectators. The momentum distribution is found (to first order approximation) in the Gaussian form

$$\frac{d\sigma}{dp} \sim e^{-\frac{p^2}{2\sigma^2}};$$

its width $\sigma$ is obtained by applying the Fermi gas model as a description of the nucleus and can be written as (Fes 73, Gol 74) as

$$\sigma^2 = \sigma^2 \frac{m_F(m_p - m_F)}{m_p - 1} = \frac{<p^2>_F}{5} \frac{m_F(m_p - m_F)}{m_p - 1}$$

where

$m_p$ is the projectile mass.

$m_F$ is the fragment mass.

$P_F$ Fermi momentum of the nucleon.

with average momentum $<p^2> = \frac{3}{5} <p^2>_F$ .

Another approach is to consider the projectile itself to be heated up to a temperature $T$, then after thermal equilibrium it decays subsequently with the same kind of Gaussian momentum distribution, but the width depends on its temperature $T$, nucleon mass ($m$), and the projectile and fragment masses, $m_p, m_F$ (Gol 74)

$$\sigma^2 = \frac{m m_F(m_p - m_F)}{m_p} kT$$

$k$ is the Boltzmann constant. For inclusive experiments it is difficult to distinguish between these two descriptions.

I.4.1.3 Simple parametric model

Baur et al. (Bau 80b) show that the reaction amplitude is mainly composed of grazing partial waves and could be described by
where

\[ L_0 \] is the grazing angular momentum, at which break up is peaked.

\[ B_0 = \frac{L_0}{q_a} \] impact parameter at grazing conditions and almost independent of the incident energy.

\[ E_a \] incident energy,

\[ q_a \] projectile wave number.

\[ E_{B\cdot E} \] binding energy of the two fragments in the projectile.

\[ \beta(E_a, E_{B\cdot E}) \] describes the strength of the break up process. It saturates at high incident energies and vanishes at low energies.

\[ \Delta L \] diffuseness of the angular momentum (spread of number of waves contributing to the break up).

\[ \Delta R = \frac{\Delta L}{q_a} \]

The total break up probability of the reaction \((a, b)\)

\[ \sigma^b_T(a, b) = 2\pi \beta(E_a, E_{B\cdot E}) \int_{0}^{\infty} dB B e^{-\left(\frac{B - B_0}{R}\right)^2} \approx 2\pi^{3/2} \beta B_0 \Delta R . \]

All these models were based on inclusive experiments and it is difficult to distinguish between different break up processes which illustrates the necessity for coincidence experiments.
I.4.2 Distorted Wave Born Approximation (DWBA)

There are currently two different ways in which the transition amplitude, using distorted waves, can be calculated for break up.

I.4.2.1 Prior DWBA

This is an idea based on the inelastic excitation of the projectile into an unbound state (continuum) which rapidly decays into two fragments, so the resulting transition amplitude is the coherent sum of the products of spectroscopic and transition amplitudes over the transferred angular momenta (Uda 80, Uda 81, Uda 79).

The spectroscopic amplitude depends on several physical quantities that determine the possible break up process, e.g. the intrinsic kinetic energy \( E^*_K \) of the projectile and its binding energy.

The break up cross section depends on the transition amplitude and the prior form for binary break up is (applying ingoing boundary conditions)

\[
T_{\text{Prior}}(\vec{k},\vec{k}',\vec{K}) = \langle \phi^{(-)}_K(\vec{r}) \mid \phi^{(-)}_K(\vec{r}) \rangle |_{1A} \left( \vec{r}_1 \right) + \langle \phi^{(-)}_K(\vec{r}) \mid \phi^{(+)}_{g.s}(\vec{r}) \rangle U^+_K(\vec{R})
\]

\( \phi^{(-)}_K(\vec{r}) \) is the elastic scattering wave function of the first fragment and the second fragment.

\( \phi^{(+)}_{g.s}(\vec{r}) \) : ground state wave function of the projectile

\( U^+_K(\vec{R}) \) : outgoing relative wave function of the projectile on the target at momentum \( \vec{K} \).

\( U^-_{K'}(\vec{R}) \) : ingoing relative wave function between centre of mass of the fragmented system and the target.
\( V_{iA}(r_i), \, i=1,2 \) : interaction potential between target A and particle \( i \). It could be expanded into several orders, mainly first is the highest and higher orders are responsible for dynamical stretching (by coupled channel).

The advantage of this form is that it accounts fully for the final state interaction. Udagawa et al. (Uda79c) made a parametrised form of DWBA overlap integral (Frö79, Uda79a,b) to evaluate the triple differential cross section for \(^{20}\)Ne break up.

1.4.2.2 Post DWBA

In this approach the outgoing distorted wave is the fragments' elastic waves on the target with the interaction in the final channel. The transition amplitude takes the form (applying outgoing boundary conditions)

\[
T_{\text{Post}}(k_1, k_2; R) = \langle U_k(r_1) U_k(r_2) | V_{12}(r_{12}) | \phi_{gs}(R) U_R^+(R) \rangle
\]

\( U_k(r_i) \) is the elastic outgoing wave between the fragment \( i \) and the target according to the interaction potential \( V_{iA}(r_i) \),

\( V_{12}(r_{12}) \) is the interaction potential between the two fragments,

\( \phi_{gs}(r), U_R^+(R) \) have the same definitions as in the case of prior form.

This method was used to explain the Coulomb break up by Landau et al. (Lan47), then applied for deuteron break up by Baur et al. (Bau76,
Bau 79) successfully at low energy and extended to light ions in (Bau 84, and references therein) with some success. Also they showed the importance of the recoil effects and the qualitative agreement was fairly good.

I.4.3 3-body Models

The first, and simplest, theoretical approach to the exclusive measurements for binary break up is the 3-body model. Such a model considers the projectile as two separate entities in a bound state, coming into the entrance channel of the reaction.

The general form for the 3-body system in quantum mechanics, interacting via binary potentials, could be described in the Schrodinger equation.

\[
[H_{\text{rel}} + V_{12}(\vec{r}) + V_{1A}(\vec{r}_1) + V_{2A}(\vec{r}_2) - E] \psi(\vec{R}, \vec{r}) = 0,
\]

where

\( <T_r> \) = relative kinetic energy operator between particles composing the projectile.

\( <T_R> \) = relative kinetic energy operator between the centre-of-mass of the projectile and the target.

\( V_{12}(\vec{r}) \) potential between the two particles inside the projectile.

\( V_{1A}(\vec{r}_1) \) potential between ith particle and the target.

\( E \) total energy

Fig. I.1: Schematic representation of the 3-body coordinate system.
ψ(R, r) wave function describing the 3-body system in terms of relative coordinates R, r.

Next we shall give a brief account of the methods involved for solving this equation.

I.4.3.1 Faddeev Method

It is one of the most general methods to solve the 3-body system. The total wave function of the system ψ(R, r) could be written as

ψ(R, r) = ψ₁(r₁, r₂) + ψ₂(r₁, r₂) + ψ₃(R, r).

This helps to decompose the 3-body Schrodinger equation into

[<T_r> + <T_r> + V₁₂(r) - E] ψ₃(R, r) = -V₁₂(r)[ψ₁(r₁, r₂) + ψ₂(r₁, r₂)]

[<T_r> + <T_r> + V₁₁(r₁) - E] ψ₂(r₁, r₂) = -V₁₁(r₁)[ψ₃(R, r) + ψ₁(r₁, r₂)]

[<T_r> + <T_r> + V₁₂(r₂) - E] ψ₁(r₁, r₂) = -V₁₂(r₂)[ψ₂(r₁, r₂) + ψ₃(R, r)]

Each ψ₁ is decomposed and the two-body subsystem is again decomposed by a partial wave expansion into radial, angular, and spin parts. Practically the number of partial waves is limited to the significant ones. Carrying out an angular momentum analysis results in a set of coupled partial differential equations of two variables r, R which could be solved numerically to obtain the radial parts of the wave functions to be used in evaluating the transition matrix elements.
This shows the huge computation task of solving these equations. Also the integral equation approach results in a set of integral equations of two variables which need nonlocal-separable (or effective local separable) potentials to reduce the integral equations, so that the integration involves one variable only to save computing time.

The method has several other difficulties such as implying the full range Coulomb effect and antisymmetrisation to enforce the Pauli principle. Also it requires a proper set of two body potentials that can satisfy at the same time reversal essential properties of the subsystems involved, e.g. binding energies. In other words, those potentials which can describe on-shell scattering may not have the right off-shell behaviour.

Such difficulties put limits on the physical problems which could be solved. Examples of the problems that can be solved, are the triton ground state (Pay 80a), the three nucleon system (Fri 81, Pay 80b) and d-α scattering (Ban 83, Miy 85).

I.4.3.2 Coupled discretised - continuum channels (CDCC).

The basic idea is to expand the wave function of the system as a sum over bound and scattering states of the relative wave functions of the two clusters of the projectile \( \phi(r) \), multiplied by the relative wave function between projectile and target \( X_{JM}(\hat{P}, \hat{R}) \)

\[
\psi(\vec{R}, \vec{r}) = \sum_i \phi_i(\vec{r}) X_{JM}(\vec{P}, \vec{R}) + \sum_{\ell = 0}^{\ell_{\text{max}}} \sum_{|J - \ell| \leq J + \ell} \int_0^{k_{\text{max}}} \phi_{\ell}(k, \vec{r}) X(\vec{P}, \vec{R}) d\vec{R}
\]
\[ \phi_i^b(\vec{r}) \] represents the bound states
\[ \phi_j^s(\vec{k}, \vec{r}) \] represents the scattering wave function of the first cluster on the second inside the projectile at angular momentum \( \ell \) and momentum \( k \).

The range of \( k \) is divided into bins, each obtained by

\[ \phi_{i, \ell, m}(\vec{r}) = \frac{1}{\sqrt{\Delta k_i}} \int_{k_i}^{k_{i+1}} \phi_{\ell m}(\vec{k}, \vec{r}) dk, \]

with
\[
\langle \phi_i | \phi_j \rangle = \delta_{ij}, \\
\langle \phi_i^b | \phi_j \rangle = 0.
\]

and
\[
\epsilon_i = \frac{\hbar^2 k_i^2}{2\mu} = \frac{\hbar^2}{2\mu} \left[ \frac{k_i + k_{i-1} - 2}{2} + \frac{1}{12} (\Delta k_i)^2 \right]
\]

\[ \Delta k_i = k_{i+1} - k_i. \]

By direct substitution of this wave equation in the Schrödinger equation, we get a set of coupled differential equations for \( \chi \)'s which should be solved to get the wave functions. Applying boundary conditions on each channel we get the S-matrix and then the cross sections. (Raw 75, Yah 81) applied this method to the break up of \( ^2H_1 \) and they get an overall agreement.

The method has some disadvantages. It needs a big configuration space, comparable to the maximum that the computer can handle, i.e. it puts computational limitations on \( \ell_{\text{max}}, k_{\text{max}} \) rather than the physical one. It also assumes mass partition (clustering) rather than suggesting it. Also they ignore the possibility of the existence of resonance states and they treat all above the threshold as continuum.
1.4.3.3 Adiabatic Approximation

In this approximation, the internal parameters of the projectile and target are considered "frozen" during the interaction. It is appropriate for use with model spaces obtained from macroscopic or collective dynamical models, such as the cluster description of the nucleus or collective mode. So, one can assume the internal wave function is not affected during the interaction and replace the internal Hamiltonian by its energy eigenvalue. The approximation can not be extended to very highly excited states having life-times less than collision time. (Ros 60) demonstrate the similarity of adiabatic and distorted waves in weak coupling. There are two models for this approximation.

1.4.3.3.1 High energy adiabatic model

The basic idea is that the projectile excitations are neglected with respect to its incident energy, namely if the wave function is factorized to

\[ \psi(\vec{R}, \vec{r}) = \phi(\vec{r}) X(\vec{R}, \vec{r}) \]

The Schrödinger equation takes the form

\[ [<T_R> + V_{1A}(\vec{r}_1) + V_{2A}(\vec{r}_2) + \epsilon] \phi(\vec{r}) X(\vec{R}; \vec{r}) = E\phi(\vec{r})X(\vec{R}, \vec{r}) \]

and

\[ [<T_r> + V_{12}(\vec{r}) - \epsilon]\phi(\vec{r}) = 0 \]

\( \epsilon \) = binding energy of the two clusters in the projectile.
If one expands $\chi(\vec{R}, \vec{r}) = \sum_{\ell L J} \chi_{\ell L J}(R, r) [Y_\ell(\hat{r}), Y_L(\hat{R})]_J$

and similarly the potentials, one can get a coupled set of differential equations for $\chi_{\ell L J}(R, r)$ for fixed values of $r$ and $J$.

Then once these equations are solved (for a proper set of potential parameters which fit elastic scattering of the fragments on the target to get $V_{A1}', V_{A2}'$) one can obtain $r$-dependent $T_{\ell', L', \ell, L}(r)$ for each value of $r$ for all values of $\ell, L, \ell'$ and $L'$. Then it is possible to obtain the break up cross section for each $L$ in the input channel, and then the total break up cross section. For more explicit details of the method is obtained (see Ama 79, Ama 81).

1.4.3.2 Low energy adiabatic model

The basic assumption is that the projectile relaxes gradually during the collision. The whole projectile has the same velocity. So if

$$\psi(\vec{R}, \vec{r}) = U(\vec{R})\phi(\vec{r}; \vec{R})$$

then

$$<T_R>\psi(\vec{R}, \vec{r}) \approx [E - \varepsilon(\vec{R})] \psi(\vec{R}, \vec{r})$$

and

$$[<T_R> + V_{12}(\vec{r}) + V_{1A}(\vec{r}_1) + V_{2A}(\vec{r}_2) - \varepsilon(\vec{R})] \phi(\vec{r}; \vec{R}) = 0.$$ 

The last equation should be solved for each value of $R$ independently. The approach can use the 2-centre shell model (Gre 71) and consider the break up as a perturbation, with no velocity couplings, all break up is virtual and the projectile recovers, i.e. no final break up in the outgoing channel.
I.5 Current Studies

Several approaches tackled the break up phenomenon, more exclusively, Thompson et al. (Tho 83) applied the high energy adiabatic approximation to calculate the elastic break up of $^7$Li into $\alpha$ and triton (t) clusters, with two non adiabatic modifications. First, the break up energy is not fixed to the ground state value but varies with the channels considered. Second, the full adiabatic wave function is used at different physical values of the relative energies between the two clusters inside the T-matrix. They consider the distortions of incoming waves due to nuclear and/or long range Coulomb dipole potential. The best agreement in the angular distribution of the elastic break up was for angles greater than the grazing angle with nuclear force responsible for the break up and distortion by Coulomb and nuclear potential.

The use of the adiabatic approach with Coulomb forces is not consistent because the adiabicity assumes short transition period such that the internal excitations (status of the two clusters in the projectile) does not change and in $^7$Li we have two clusters with different charges.

Recently, the CDCC method is used by Sakuragi et al. (Sak 86, and references therein) first to investigate the elastic scattering of $^7$Li on $^{12}$C and $^{48}$Ca, $^{58}$Ni, $^{208}$Pb at $E_{lab} = 63 - 148$ MeV. They obtained a reasonable agreement with one free parameter "$N_1$" to adjust in each case. The method is remarkable in getting the best fit, but on account of the large numbers of channels this eventually needs large and fast computer (8 Mb processor).

By extending the method to investigate the elastic break up
(via non resonant states) and inelastic projectile break up (via resonance states) using the nuclear force only. The non resonance states were extended to 3 MeV above the break up threshold for $^7\text{Li}$ at 70 MeV on $^{120}\text{Sn}$. They obtain a good agreement for the elastic and close agreement for the resonance states but still with one free parameter $N_I$ to be adjusted from the elastic scattering data. For the $^{12}\text{C}$ target they got an agreement with relatively large oscillations, and predict direct elastic (non-resonance) which has not been seen experimentally so far. The authors also consider the Coulomb force of $^7\text{Li}$ on $^{120}\text{Sn}$ and $^{208}\text{Pb}$ at the same laboratory energy, but restrict the channel space to 4-states only of the projectile and neglecting the non resonant states to simplify the calculations. The agreement with the data for $^{120}\text{Sn}$ target lies within a factor of 3 and for $^{208}\text{Pb}$ target occurs within a factor of 4; they attribute these differences to the non-resonance states, not being considered in the calculation.

I.6 The Objective of the Thesis

The current research lines for break up reactions are based exclusively on quantum theory, and for heavy ion reaction the number of channel waves and states involved are eventually increasing with the creation of a big computational task. Also the solutions involved are stationary solutions (time independent). Our objective is to investigate a new semiclassical approach which depends on both classical and quantum theory to study heavy ion reactions and, in particular, the direct break up process, using the reaction model
The method is based on calculating the trajectory of the main particles under consideration with phenomenological potential and considering the absorption in a semiclassical way. The method accounts fully for the recoil of the target and the time factor is considered in the evolution of motion of the particles.

Another objective is to investigate the extension of the application of some recent quantum mechanical models for direct break up process to non S-state projectiles, e.g. \(^{7}\text{Li}\).
II.1 Introduction

II.1.1 General view

In heavy ion reactions, the quantum mechanical methods are not easily applicable to many dynamical problems such as fragmentation reactions. If one uses the asymptotic "stationary" solutions the increasing number of partial waves and the number of states involved makes the calculations more complicated and not simple to resolve the role of each partial wave and state. Nonstationary techniques such as the time dependent Hartree-Fock method make the way to the solution even more complicated. A classical model containing the essential physics is constructed, upon which other simple models are built and/or could be used by itself as a testing ground.

II.1.2 Basic characteristics of classical approach

At intermediate and high projectile energy heavy ion reactions (≥ 10 MeV/nucleon) are characterised by the small De Broglie wavelength \( \lambda \). The sum of the radii of the two interacting nuclei \( R = R_1 + R_2 \) is generally much bigger than the wavelength \( \lambda \). The wave packet describing the projectile motion (usually covering several \( \lambda \)'s) has small spatial width, which is less than the range of the strong interaction potential between the two nuclei. Also if the spread of the wave packet can be neglected over the time of interaction,
then one may assume that the wave packet acts as a particle moving under the influence of the interaction potential.

Under these conditions, it is possible to consider the dynamics of the reaction on a classical basis.

Below the Coulomb barrier, the Sommerfeld parameter is introduced to describe these conditions in atomic and nuclear problems. Its definition

$$\eta = \frac{Z_1 Z_2 e}{\hbar v}$$  \hspace{1cm} (II.1)

$Z_1, Z_2$ are the charge numbers of the interacting particles,
$v$ relative velocity between particles,
and the condition to use the classical approach is $\eta >> 1$.

Above the Coulomb barrier, another parameter is used, which is

$$\zeta = KR$$  \hspace{1cm} (II.2)

$K$ wave number of De Broglie wavelength of the projectile,
$R$ the sum of radii of the two interacting nuclei $R_1 + R_2$
and the condition for classical treatment is

$$\zeta >> 1$$

Another parameter $Ka$ is also considered, where $a$ is the diffuse
ness of the nuclear potential and the condition is $Ka > 1$.

Under this condition, the quantal effects are less enhanced and one can get a qualitative and may be quantitative picture of the re-
action without considering quantum structure effects.

It is possible under the laws of classical dynamics to determine simultaneously the coordinates and momenta as distinct points in phase
space, and obtain an unambiguous orbit which might help to provide us with a kinematical window for a specific process. Such treatment considers fully the dynamical changes in the system under investigation due to different forces at the same time, including the recoil effect of the target. Another characteristic feature of the classical approach is that it describes different reaction processes with the same kind of potential.

In general, classical and semi-classical methods are often useful in establishing the mechanism of the heavy ion reaction since the reactants may be internally excited rather than changing their original trajectory.

II.1.3 Classical Classification of Heavy Ion Reactions

The classical picture of heavy ion reactions at incident energy above the Coulomb barrier can be classified as follows:

1. At small impact parameter $b < R$ fusion and deep inelastic processes occur.
2. At impact parameter comparable to $R$ peripheral collisions (or grazing collisions) set in.
3. At larger impact parameter $b > R$ reactions are mostly dominated by elastic scattering but some other inelastic or quasi-elastic processes may take place.

Below the Coulomb barrier, classically the reaction is mainly Rutherford elastic scattering.
11.1.4 Historical Review

In the last two decades, there have been several nuclear reaction studies based on classical dynamics. They vary in their treatment from using clusters (Gre 83) to nucleons (Bod 77,80) and as a fluid (Pro 83). Also some methods consider the classical approach as part of their treatment (Bro 72, Kno 76). Other studies also establish the connections between classical and quantum mechanical treatment (Suz 83).

11.1.4.1 Classical Trajectory Technique without Dissipative Forces

11.1.4.1.1 Cluster Models

Boneh et al. (Bon 67) calculated the classical trajectory for long range α-particles emitted in spontaneous fission of $^{252}$Cf. They neglect the nuclear forces and consider only the Coulomb forces to determine the trajectories of the 3-body system composed of the α-particle and the two fission fragments. Their motion was restricted to the initial fixed plane containing them. This study produced a good agreement for the α-energy spectrum integrated over all angles for particular mass ratios of the fragments. This helped to establish the idea of moving the fission point toward the light fragment, which explains part of the dynamics of the fission process. Camp et al. (Gam 78) use the same trajectory method for the reaction $^{32}$S + $^{197}$Au at incident energy 373 MeV. The $^{32}$S moves in Coulomb trajectory to the distance of closest approach with $^{197}$Au, then they consider $^{32}$S as ($\alpha + ^{28}$Si) and calculate the trajectory of the
3-body system under Coulomb forces only on a fixed plane. The idea of α-particle emission taking place between the projectile and the target region during the interaction time is confirmed by matching the predicted and experimental angular distributions. Gregoire et al. (Gre 83) proposed a model for projectile energy (10-100 MeV/nucleon). Such a projectile is assumed to have an α-particle, and the rest of the projectile is assumed to fuse with the target, i.e. partial fusion process. Their study considers mainly the α-emission. They describe the α-particle link to the rest of the projectile using a non-stationary wave function (coherent state). This wave function provides the extension of the α-particle in the projectile. The total energy of the α-particle is the sum of two terms.

\[ E_\alpha = E_{c_\alpha} + \frac{1}{2} \hbar \omega. \] (II.3)

\[ E_{c_\alpha} = \] the classical energy of the α-particle associated with the motion of the centroid of the coherent state.

\[ \frac{1}{2} \hbar \omega = \] the zero point energy for the radial motion of α inside the projectile.

The α-particle will be emitted when the interaction of the projectile with the target lowers the effective barrier between α and the rest of the projectile, i.e. 3-body dynamics determine the α-emission. They predict the integrated α-energy spectrum with a broad characteristic bump at the beam velocity. The method provides a direct explanation of the α-emission.
Another class of classical microscopic calculation is based on calculating the trajectories of nucleons with two-body forces between all pairs of nucleons. For high energy (1 GeV) heavy ion collision Bodmer et al. (Bod 77) used attractive and repulsive Yukawa potentials, neglecting the Coulomb interaction, the possibility of meson production and the effect of the intrinsic spins of the nucleons. The classical trajectory of each nucleon is obtained in the projectile and in the target. When a set of nucleons come close together, forming a coalescence during the interaction, they calculate the averages of the densities, global velocities to get the realistic equation of state.

For dense cases they applied a hydrodynamical limit using the Euler-Navier Stokes equation, and for dilute systems, they applied the cascade calculations. For central and near central collisions none of these approaches was adequate, but later Bodmer et al. (Bod 79) included finite range interaction effects and discussed non-central collision with various parameters. A simpler approach was used by Calloway et al. (Cal 79) and later by Kitazoe et al. (Kit 81), where they considered an effective nucleon-nucleon potential. The conditions for a set of nucleons to form a fragment is that the distance between each other is less than the radius of the potential and, at the same time, form a known isotope structure. They obtained, in general, good agreement with the angular distribution.

A similar approach was made by Kiselev (Kis 85) for Ne + U, Ca + Ca, and Nb + Nb at 400 MeV/nucleon, using the same criteria for the fragment but a different method of analysis by constructing a kinetic energy flow tensor.
\[ F_{ij} = \sum_{\nu} \frac{p_i(\nu)p_j(\nu)}{2m_\nu} \]  

(II.4)

\( p_i(\nu) \)  the final momentum component \( i \) in the centre of mass of the reaction for a fragment \( \nu \),

\( m_\nu \)  the mass of the fragment \( \nu \).

This tensor is characterized by 6-numbers, considered as ellipsoidal parameters, which show where the kinetic energy accumulated. Considering the scattering angle \( \theta \) along the major axis of the ellipsoid, its main axes are obtained from the ratios of the eigenvalues of the tensor. There was similarity in comparing with other predictions of the flow angle \( \theta \) by Cugnon (Cug 80), and more important was the prediction of the angular distribution, impact parameter dependence for two reactions.

Aichelin et al. (Aic 85) used the Boltzman-Uehling-Uhlenbeck equation to describe the time evolution of the single particle phase space distribution function \( f(\vec{p}, \vec{r}) \) in a mean field potential which will be a specified function of the local density. The evolution was carried out by Newtonian mechanics.

II.1.4.3  **Classical Trajectory Technique with Dissipative Forces**

These techniques are characterised by including friction forces in the equation of motion. It is used basically to describe the deep inelastic and partial fusion and total fusion reaction, or more generally, damped collisions. Bondorf et al. (Bon 74) suggest a classical model which involves dissipative force to describe heavy
ion reactions and obtain a critical angular momentum for fusion and, more extensively, later Birkeland et al. (Bir 79) continued similar study and they also obtained good agreement with the fusion cross-section.

Deubler et al. (Deu 77) studied deep inelastic collision by two deformed ions with radial and tangential friction depending on the overlapping density distribution and also introduced damping vibrational forces. They restrict the intrinsic spins and relative angular momenta normal to the plane of the classical trajectory. They obtain the deflection function for Kr (525 MeV) + Bi, and fit the angular distribution for $\theta_{\text{c.m.}}$ for the reaction $^{40}\text{Ar} (388 \text{ MeV}) + ^{232}\text{Th}$.

Royer et al. (Roy 85) used the liquid drop model and proximity potential with a friction form factor to describe the fusion path of the heavy system and they predicted the dynamical double humped fusion barriers.

Harrison (Har 78) used the three body classical friction model to describe deep inelastic collisions. He obtained angular correlations and energies of $\alpha$ particles emitted from the reaction $^{16}\text{O} (100 \text{ MeV})$ on $^{27}\text{Al}$, $^{58,64}\text{Ni}$.

II.1.4.4 Semiclassical Techniques

There are also several studies which depend partially on classical dynamics, usually named as the semiclassical approach.

Their basic principle is to describe the relative motion between particles involved classically and preserve the quantum nature of each particle individually, see (Bro 81). There are other approaches
which have some link to classical dynamics. For example, Knoll et al. (Kno 76) suggest another alternative which can account for the diffraction and absorption effects, by obtaining the classical solution of the Hamilton-Jacobi equations with complex potential, then constructing waves through W.K.B. approximation in terms of a multiple reflection series inside the active region. Applying several mathematical criteria they select the complex trajectories which produce the radial wave function and the scattering amplitude and so the cross-section. They have been able to reproduce almost the elastic scattering data and account for Fraunhofer diffraction.

Bertesh et al. (Ber 77) studied the dynamics of heavy ion collisions by solving the time dependent Schrödinger equation for a potential well moving along the classical trajectory. They conclude that single particle dynamics can be characterised classically at early stages of the collision but this does not give accurate velocities or transfer probabilities at later stages of the reaction.

Sukumar and Brink (Suk 83) have studied inelastic scattering, using the path integral method, by solving the Schrödinger equation for the wave function of the internal coordinate $\xi$ evolving under the influence of a time dependent interaction potential $V(\bar{r}(t), \xi)$. The function $\bar{r}(t)$ is the classical orbit.

Guidry et al. (Gui 84) investigate the Coulomb nuclear interface with the classical limit S-matrix, in which the S-matrix is generated classically and manipulated by quantum mechanical rules. There was a considerable amount of backward angle scattering in which the deflection function is monotonic, and they were able to assign certain partial waves with each scattering angle, i.e. probing
the surface.

Carlitz et al. (Car 85) used the classical path to obtain the classical action $S$ and developed a method to extract energy levels and wave functions for a one-dimensional system with simple linear and quadratic potentials, and they were able to reproduce W.K.B. results.

II.1.4.5 Relation with Quantum Mechanics

There are also other studies which establish the connection between classical and quantum aspects of particle motion. Suzuki (Suz 83) established the relation between time dependent Hartree-Fock and their classical trajectories in phase space.

Hahn et al. (Hah 84) compares the classical trajectory calculation and wave packet scattering model results for fusion, deep inelastic and quasielastic components. They showed that quantum effects tend to widen the finite classical values for the critical angular momentum for fusion and maximum angular momentum for nuclear interaction.

II.2 The Model

II.2.1 Introduction

2.1.1 Basic Features

Any model is an abstraction of reality, projecting an interesting part of it. Our object is to study the classical model for a 3-body system of a fixed identity (mass, charge) under the influence of the
mutual forces between them, and then to compare the results of this
model to those obtained from experimental data for some 3-body re-
actions. Since there is no fundamental physical force to de-
scribe the 3-body force known classically, we shall restrict the
description of the forces to the mutual two-body forces, namely
the Coulomb and nuclear forces between each pair of the 3-body
system, excluding other types of forces such as friction which
could be considered in further developing stages.

The analytical description of the motion of the 3-body system
(Taf 85, Mar 68, Sze 67, Fin 58) has been limited to two special
cases:

1. The restricted problem, in which a small mass (planetoid)
   moves in the gravitational field of the other two finite
   masses.

2. Spatial stationary configuration of the 3-body system
   solved by Lagrange in 1772.

Neither of these are applicable for the general case so one
may resort to numerical methods to simulate the motion of the 3-
body system on a computer.

II.2.1.2 Computer Trajectory Program

The program basically divides the time between successive in-
teractions among different elements of the system. The evolution
of this system with time is a stochastic one, obeying the classical
laws of dynamics. The basic idea has been done before by others
(Har 78, Bic 80) but with poor numerical method and low statistics.
The method is improved technically by using more sophisticated methods and introducing a semiclassical factor which describes the absorption of the individual clusters by the target. This factor is shown to be very important.

The method of calculation has been particularly applied to the reaction $^{208}$Pb($^7$Li, αt)$^{208}$Pb in order to study the final products. The reasons we have chosen to study this 3-body reaction are:

1. Experimental data exists.
2. The binding energy of α,t clusters inside the $^7$Li nucleus is relatively small, ~ 2.47 MeV, so expectedly the break up is more likely than for other systems.
3. The α,t cluster structure of $^7$Li is well known (Dub 84) and has the following properties.
   i) α-t clusters are well separated since the size of each cluster is ~ 2 fm and the separation distance is about 3.5 fm (Wal 85).
   ii) Fractional parentage coefficient of α,t clusters in $^7$Li is relatively large.
4. For this reaction it is possible to apply classical mechanics because $KR \sim 47 > 1$, where $K$ is the wave number associated with the relative motion between $^7$Li and $^{208}$Pb, $R$ is the sum of their radii, and also the Sommerfeld parameter is 12.6 > 1.
5. Relativistic corrections are not necessary since $v/c \sim 0.1$, and considered to be negligible.
II.2.2 Assumptions of the Model

There are several assumptions to make.

1. We assume that the $^7\text{Li}$ projectile composed of the $\alpha+t$ cluster system, and $\alpha$, $t$, $\text{Pb}$ preserve their identities and internal energy states during the reaction.

2. The 3-body classical dynamics are governed by conservative potentials and no dissipative forces are involved.

3. Nuclear densities of the three bodies do not change during the collision.

The behaviour of the process is described by the Coulomb and Woods-Saxon form of the optical potential in an elementary sense, in which the real part is responsible for the trajectory, while the imaginary part describes the attenuation of the clusters. The potential takes the form:

$$V(r) = \frac{V_0}{1 + e^{(r-R_0)/a}} + i \frac{W_0}{1 + e^{(r-R'_0)/a'}}$$

The values of the potential parameters are taken from literature which describe the elastic scattering of each individual cluster at the relevant energy. The values of the parameters are stated in Table II.1

For $\alpha-t$ we took the charge radius as the sum of the charge radii = 3.55 fm. We define the nuclear interaction peripheral region by two distances. First, the strong interaction radius at which the nuclear force almost vanishes and, second, the critical fusion radius at which the two bodies fuse together if their separation distance is equal or less than this critical value. The values
<table>
<thead>
<tr>
<th></th>
<th>$V_0$ (MeV)</th>
<th>$R_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W_0$ (MeV)</th>
<th>$R'_0$ (fm)</th>
<th>$a'_0$ (fm)</th>
<th>$R^c$ (fm)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(40 MeV) α - Pb</td>
<td>-181.1</td>
<td>7.82</td>
<td>0.62</td>
<td>-15.13</td>
<td>8.00</td>
<td>0.85</td>
<td>7.41</td>
<td>Per 81</td>
</tr>
<tr>
<td>(20 MeV) t - Pb</td>
<td>-150.3</td>
<td>7.41</td>
<td>0.707</td>
<td>-13.9</td>
<td>8.41</td>
<td>0.816</td>
<td>7.41</td>
<td>Arm 69</td>
</tr>
<tr>
<td>α - t</td>
<td>-91.2</td>
<td>2.05</td>
<td>0.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3.55</td>
<td>Kub 72</td>
</tr>
</tbody>
</table>
are stated in Table II.2.

<table>
<thead>
<tr>
<th></th>
<th>α-Pb (fm)</th>
<th>t-Pb (fm)</th>
<th>Li-Pb (fm)</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong interaction radius</td>
<td>11.23</td>
<td>11.01</td>
<td>11.718</td>
<td>(Bir 76)</td>
</tr>
<tr>
<td>Fusion radius</td>
<td>7.51</td>
<td>7.36</td>
<td>7.838</td>
<td>(Gla 75)</td>
</tr>
</tbody>
</table>

II.2.3 Equations of Motion

The basic principle is the Newton Law of motion (Principia, 1686) which relates the force and the acceleration of the body by the mass constant, namely, the equation of motion used to describe the process is:

$$m_i \frac{d^2 \vec{r}_i}{dt^2} = \vec{F}_i(\vec{r}_{ij}, \vec{r}_{ik}), \quad i, j, k = 1, 2, 3, \ i \neq j \neq k$$

(II.5)

where,

- $m_i$ is the mass of the $i$-th particle.
- $\vec{r}_{ij} = \vec{r}_j - \vec{r}_i$ is the relative position vector of particle $j$ relative to particle $i$,
- $\vec{r}_i, \vec{r}_j$ are the position vectors of particle $i, j$, respectively,
\( \vec{F}_i \) is the total force acting on particle \( i \)

\[
\vec{F}_i = \sum_{k=1}^{2} \vec{F}_{ik} \quad \text{for} \quad k \neq i
\]  

\( \vec{F}_{ik} \) is the mutual force between particle \( i \), and particle \( k \)

\[
\vec{F}_{ik} = \vec{F}_{ik}^c + \vec{F}_{ik}^N
\]  

\( \vec{F}_{ik}^c \) is the Coulomb force and is defined by the relation

\[
\vec{F}_{ik}^c = \frac{Z_i Z_k e^2}{r_{ik}^3} (\vec{r}_i - \vec{r}_k), \quad r_{ik} \geq R_{ik}^c
\]

\[
\vec{F}_{ik}^c = \frac{Z_i Z_k e^2}{R_{ik}^c} (\vec{r}_i - \vec{r}_k), \quad r_{ik} < R_{ik}^c
\]

\( i, k = 1, 2, 3, i \neq k \)

where

\( Z_i, Z_k \) are the atomic numbers of the particles \( i, k \) respectively.

\( \vec{r}_i, \vec{r}_k \) are the position vectors of the particles \( i, k \) respectively.

\( r_{ik} = |\vec{r}_i - \vec{r}_k| \) is distance between particles \( i \) and \( k \),

\( R_{ik}^c \) is the charge radius between particles \( i, k \) and for cluster target case = \( r^c A_T^{1/3} \), and \( r^c = 1.25 \text{ fm} \).

\( \vec{F}_{ik}^N \) is the nuclear force and defined by the relation:

\[
\vec{F}_{ik}^N = \frac{V_{oi k}}{a_{ik}} \frac{x}{(1 + x)^2} \cdot \frac{1}{r_{ik}} \cdot (\vec{r}_i - \vec{r}_k)
\]

\( i, k = 1, 2, 3, i \neq k \)

\( x = \exp \left[ \frac{r_{ik} - R_{oi k}}{a_{ik}} \right] \)

\( R_{oi k} \) is the half nuclear density radius
$r_{ik}$ is the distance between the particles $i, k$

$a_{ik}$ is the diffuseness of the potential between $i, k$

$V_{0ik}$ is the depth of the potential between $i, k$.

This force is the first derivative of the Woods-Saxon potential which describes the overall nuclear potential of a nucleus (Hod 78).

The vector diagram used to describe the 3-body system is shown in Fig. II.1.

The typical values of the total potential force are shown in Fig. II.2 for $\alpha$-Pb and Fig. II.3 for $t$-Pb, Fig. II.4 for the $\alpha$-$t$ system and also the radial kinetic energy between the two clusters.
Fig. II.2 The total potential energy and the total force between the alpha and the lead target.
The distance between the triton and lead (fm).

Fig. II.3 The total potential energy and the total force between the triton and the lead target.
The potential

The force

The kinetic energy

The distance between alpha and triton (fm).

Fig. II.4 The total potential energy, force, and kinetic energy between alpha and triton in $^7\text{Li}$. 
II.2.4 The Transmission Factor

As each of the clusters passes the target, there are several processes which may take place, other than elastic scattering, such as inelastic or complete absorption of the cluster by the target. One way to account for the reduction in the elastic channel due to non-elastic processes is to use a complex potential with negative imaginary part \( W \). The relation between this imaginary potential and the reduction rate (Boh 69, Jac 70, Bro 81) is:

\[
\text{reduction rate per particle} = -\frac{2W}{\hbar} \quad (\text{II.10})
\]

The strength of the potential \( W \) is linked to the mean free path \( \lambda \) (Jac 70)

\[
W = -\frac{\hbar v}{2\lambda} \quad (\text{II.11})
\]

or

\[
\frac{1}{\lambda} = -\frac{2W}{\hbar v} \quad (\text{II.12})
\]

This reduction rate could be calculated locally for each time step along the classical trajectory. The probability that the cluster transmits across the target region is given by

\[
T = e^{-\int_{\text{classical path}} \frac{2W(r)}{\hbar v(r)} dS} = e^{-\sum_{n} \frac{W(r_n)}{\hbar v(r_n)} \Delta S_n} \quad (\text{II.13})
\]

where \( r_n \) is the distance between the cluster and the target at step \( n \).

\( W(r_n) \) is the imaginary potential at distance \( r_n \).

\( v(r_n) \) is the relative velocity between the cluster and the target.
\( \Delta S_n \) is a segment of the path at step \( n \).

Outside the strong interaction radius the transmission factor is almost unity.

II.3 Methods of Calculation

II.3.1 Numerical Methods

The 3-body trajectory is obtained from the solution of the set of coupled second order differential equations numerically. The first step is to change this set to another set of first order differential equations whose solutions are calculated using the following numerical procedure:

\[
\frac{\vec{r}_{i,n}}{r_{i,n}} = \frac{\vec{F}_{i,n}}{m_{i,n}}, \\
\vec{r}_{i,n+1} = \vec{r}_{i,n} + \vec{r}_{i,n} \Delta t_n \\
\vec{r}_{i,n+1} = \vec{r}_{i,n} + \vec{r}_{i,n} \Delta t_n
\]

where, \( i \) is the index for particle number, \( n \) is the index for the integration step number, \( \Delta t_n \) is the time step of the integration.

There are a wide range of numerical methods to apply, the principal task is to find the most efficient numerical integration procedure. It is not easy to prove which is the best analytically but if one takes a set of cases covering the physical range under interest and makes use
of a subset of known numerical methods, then it is possible to compare them with the following factors:

1. Accuracy and stability of the solutions obtained which depend mainly on:
   i) Truncation error due to truncated series used by the numerical method.
   ii) Round off error which depends on the machine precision.

2. The total time of the central processing units (CPU) of the computer which depends mainly on:
   i) Numerical formula used.
   ii) Ability to change the step size of the integration for certain levels of accuracy.

Since the nuclear force is the strongest in nature the program used must be very sensitive to account properly for such force.

   For the factor 1.ii) one should use the double precision mode during the calculations to reduce the round off error, and for the other factors, i), 2, a finite number of cases covering the physical range has been calculated for comparative purposes.

Three methods were investigated:

1. Euler method: simple one and used in previous reference (Bic 80).

2. Runge-Kutta fourth order (RK4): popular one.

3. Runge-Kutta-Fehlberg "RK7(8)" (Feh 68).

The numerical equations of each method are stated in Appendix I. All of them are single step methods and are chosen to make the control of the time interval in each individual step of integration easier than multistep methods.
II.3.2 Time Step Control and Scaling

One of the great sensitive issues in trajectory calculations in its integration process is to use a reasonable time step for a required accuracy of the solution. If the step size is made small to reduce the effect of truncation error, this would have implied a larger number of steps to perform the span of the integration and a large number of arithmetic operations to be performed; consequently this produces larger round off errors, and greater CPU time.

Several methods were suggested for the time step, other than being average constant time step (Bic 80) $0.725 \times 10^{-25}$ sec.

Boneh et al. (Bon 67) suggest for Coulomb trajectory an exponential parametric function, namely the time step

$$\Delta t_n = t_{n-1}(e^a - 1) = t_n - t_{n-1} \quad (II.15)$$

where

- $a = 0.1$,
- $t_n = t_o e^{na}$, total time until step $n$.
- $t_o = 10^{-22}$ second.

This suggestion may be reasonable for Coulomb force but not for the nuclear force, since it is stronger and varies much more rapidly.

Another approach is to assume that the local truncation error in each step dominates the total error of this step so it should be calculated (Cal 58, Col 60, Bri 69, Feh 68) and related inversely to the time step size. If one combines a physical factor which accounts for the physical quantities involved, such as velocity and acceleration also inversely with the time step, it becomes more sensitive. The best relation obtained among
formulae, which has the lowest CPU time, can relate the successive
time steps by the equation,

\[ \Delta t_n = \Delta t_{n-1} \left[ \frac{1}{(TE)^2 \left( \frac{3}{i=1} \dot{r}_i^2 + \frac{3}{i=1} \ddot{r}_i^2 \right)} \right]^{1/16} \]  (II.16)

where

- \( n \) is the step index
- \( t \) is the time interval,
- \( TE \) estimate of the truncation errors,
- \( \dot{r}_i, \ddot{r}_i \) velocity and acceleration of particle \( k \),
- \( \Delta t_0 = 10^{-24} \) second.

The exponent is a very sensitive factor and the most reasonable
one is found to be \( \frac{1}{16} = 2^4 \) and it will remain exactly constant
in the computer.

This formula II.16 produces the time step with lowest average
CPU time for 240 events covering the physical range of the trajectories involved in such a reaction.

A distribution of the time step repetition in one case is shown
in Fig. II.5.

If the derivatives involved, such as \( \dot{r}, \ddot{r} \) are different by
orders of magnitude for a certain system of units, e.g. velocity \( \sim 10^9 \) cm/sec, acceleration \( 10^{31} \) cm/sec\(^2\) in our case here, it would
be reasonable to change the system of units in the calculations to
another system such as

- unit of time = \( 10^{-23} \)
- unit of distance = \( 10^{-13} \) cm.
- unit of mass = 1 a.m.u = \( 1.661 \times 10^{-24} \) gm.
Fig. II.5 The distribution of the time step in a typical three body trajectory case.
This new system of units will bring the velocity and acceleration to the same order of magnitude numerically and makes the calculations away from overflows or underflow in the computers and gives a better estimate for the truncation error (TE) and the time step factor in Eq. (II.16).

II.3.3 Checks and Comparison of Different Numerical Methods

To check the stability of the solutions and the accuracy of the method we perform the calculations on a limited number of cases and evaluate after each few steps of integration the following quantities:

1. total energy of the system = \[ \sum_{i=1}^{3} \left( \frac{p_i^2}{2m} + v_i \right) \]

2. square of the linear momentum = \[ \sum_{i=1}^{3} p_i^2 \]

3. square of the angular momentum = \[ \sum_{i=1}^{3} L_i^2 \]

Since no external forces or torques are acting on the 3-body system, and we are using conservative forces, then these quantities should be constant through the whole integration procedure. The most sensitive test is found to be the total energy since it involves both the coordinates and the velocities.

Another check is made by calculating the trajectory and the speed of the centre of mass of the 3-body system. The speed is constant \( (1.432211 \times 10^8 \text{ cm/sec}) \) and the C.M. trajectory is a straight line.
A further check is done by reversing the trajectory back, and it is found that the error within 0.1 fm from the original position for the the percentage change in the total energy (percentage error) is 0.027%.

The accuracy could be controlled by the time step in the case of the Euler and the RK4 methods and a certain parameter called tolerance, "Tol", which limits the maximum value of the leading truncation error in RK7(8).

The comparison between 3-methods was carried out under the same initial conditions and integration limits, over several cases. It was found that the Li-scattering error has the largest CPU time and largest error. The results of one Li-scattering event are shown in Table II.3.

Table II.3

<table>
<thead>
<tr>
<th>Method</th>
<th>Time step</th>
<th>No. of Steps</th>
<th>CPU (sec)</th>
<th>Percentage error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler</td>
<td>$1 \times 10^{-25}$</td>
<td>17542</td>
<td>38</td>
<td>2% (1.4 MeV)</td>
</tr>
<tr>
<td></td>
<td>$7.5 \times 10^{-25}$</td>
<td>2263</td>
<td>25</td>
<td>10% (7 MeV)</td>
</tr>
<tr>
<td>RK4</td>
<td>$5 \times 10^{-25}$</td>
<td>3541</td>
<td>23</td>
<td>$1.4 \times 10^{-6}$% (1 eV)</td>
</tr>
<tr>
<td></td>
<td>$5 \times 10^{-24}$</td>
<td>355</td>
<td>5</td>
<td>0.028% (19 keV)</td>
</tr>
<tr>
<td>RK7(8)</td>
<td>variable: Tol = $10^{-2}$</td>
<td>813</td>
<td>20</td>
<td>$6.4 \times 10^{-6}$% (4 eV)</td>
</tr>
<tr>
<td></td>
<td>Tol = $10^{-4}$</td>
<td>197</td>
<td>4</td>
<td>0.03% (21 keV)</td>
</tr>
</tbody>
</table>
The Euler method is the simplest but takes a large number of steps and CPU time. The second method is RK4, which is popular because it involves a lesser number of steps and CPU. The third method proved to be the most efficient one (lowest CPU for certain percentage error). Although the calculation is larger per step, the ability to choose a reasonable value for the next time step reduces the number of steps. The reduction in this number is about 10-15% of the three-body trajectory and could be up to 95% for the two-body trajectory. Table II.4 shows the average CPU carried on (12) test cases on the same machine for different tolerance values for the RK7(8) method.

Table II.4

<table>
<thead>
<tr>
<th>Tolerance</th>
<th>Average CPU/event (sec)</th>
<th>Percentage Error in total energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^5$</td>
<td>3.3</td>
<td>0.2%</td>
</tr>
<tr>
<td>$10^3$</td>
<td>8.0</td>
<td>0.006%</td>
</tr>
<tr>
<td>10</td>
<td>10.4</td>
<td>0.001%</td>
</tr>
<tr>
<td>1</td>
<td>13.25</td>
<td>0.0004%</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>15.0</td>
<td>$1.4 \times 10^{-4}$%</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>20.9</td>
<td>$10^{-4}$%</td>
</tr>
</tbody>
</table>

II.3.4 Monte Carlo Method

The initial conditions of each trajectory may vary within certain physical limits in an infinite number of ways, and at the same time the trajectories obtained from a few cases is not enough to determine the
characteristics of the reaction under study, so a large number of cases must be simulated. An artificial sampling method is used to obtain these characteristics. In the current model the trajectory was calculated under certain criteria which are set for particular events to take place. The method is based on:

1. The randomness of the process, which is to be considered at the starting initial conditions, namely impact parameter $b$, the distance between the two clusters $R$, and the initial orientation of $\alpha$-particles in space, determined by the polar angles $\theta, \phi$ by mapping their distribution functions to the uniform distribution of a random variable $q$ which is limited to $0 \leq q < 1$. The computer can provide a pseudo random number, from which the random value required for each parameter is deduced to start the trajectory calculations. Appendix II shows the procedure to obtain the pseudo random number. As a necessary check on the physical values obtained Fig. 11.6 shows a histogram of the values obtained for the impact parameter with a little increase in number per cell at higher values of $b$, due to the distribution function imposed, Eq. 11.18.

Fig. II.7 shows the histogram of the values of $R$. It differs from the average number per cell by $\pm 2.5\%$. Similar fluctuations are obtained for $\theta, \phi$, with each cell size $5^\circ$.

2. The large number of trials of different initial conditions to obtain distributions of physical significance to the experimental data obtained and the computing time available.

3. The statistical analysis of results is done afterwards. The advantages of the method are:
Fig. II.6 The distribution of the number of events with the impact parameter for the total number simulated.
The relative distance (fm).

Fig. II.7 The distribution of events with the starting relative distance between clusters.
1) It simplifies the way to obtain the characteristics of the 3-body problem and is able to bring it to its fundamentals.

2) Modifications can be carried out to the model directly.

But the method is subject to limitations, namely:

1) Computing time limit and speed of calculations.

2) Accuracy of the calculation itself.

3) The large number of simulated events required to produce significant results.

4) The accuracy of the experimental data involved in the calculations.

It is necessary to check the distribution of the initial parameters and also the correlated distribution to make sure that a fair number of each parameter is carried out and cover the physical range required. To improve the statistics with limited computing time, instead of using the distribution assigned to the initial parameter, one can assume primarily that it has a uniform distribution and later in the analysis procedure it is possible to assign a weight to each event simulated according to the proposed distribution for each parameter. The value of the parameter here acts as a central value of a small cell and its frequency is proportional to the weight obtained from the distribution function. A large number of events simulated brings the sizes of these cells to small values and may not be equal, but close. This is called the sophisticated Monte Carlo method (Fox 62), which is used to improve the computing efficiency for rare events.
II.3.5 **Program Lay-out**

The program is mainly divided into three main parts. The first part started by assigning a certain impact parameter, \( b \), for the \(^7\text{Li} \) projectile at a large distance from the target, \( 10^4 \) \( \text{fm} \). This impact parameter may be taken step-wise, or via the pseudo random number \( q \) for a specific region \([b_{\text{min}}, b_{\text{max}}]\), in the later case as

\[
b = \left( q(b_{\text{max}}^2 - b_{\text{min}}^2) + b_{\text{min}}^2 \right)^{\frac{1}{2}} , \quad (II.17)
\]

\[0 < q < 1\]

since the frequency distribution of the number of projectiles as a function of the impact parameter \( b \) is

\[
N(b) = \frac{2b}{b_{\text{max}}^2 - b_{\text{min}}^2} . \quad (II.18)
\]

The projectile moves under the mutual Coulomb force between the projectile and target until at a distance 25 \( \text{fm} \) from the target. At this distance, or more, the nuclear force between the target and the two clusters inside the projectile are negligible.

No change in the trajectories was observed if we changed this point to, say, 100 \( \text{fm} \), so we moved it to 25 \( \text{fm} \) to save unnecessary computing time.

The second stage starts at this point by considering the projectile as two clusters in a bound state. The binding energy \( E_B = 2.47 \text{ MeV} \), and a separation distance \( R \) is assigned within the classical range (0.6 - 4.1 \( \text{fm} \)). The two clusters have a relative angular momentum \( l_\text{in} \), the initial orientation of the two clusters in space is defined by the polar angles \((\theta_1, \phi_1)\) of the \( \alpha \)-cluster.
where $\theta_1$ is the projection angle defined by the range $0 \leq \theta_1 < \pi$. $\phi_1$ is the azimuthal angle defined by the range $0 < \phi_1 < 2\pi$. The graph of this geometry and rotation matrices for coordinate and velocity transformations are shown in Appendix III.

The values of $\theta_1$, $\phi_1$ can be either increased in a step-wise or via pseudo random number $q$, uniformly distributed.

$$
\theta_1 = q \pi \quad 0 \leq q < 1 \quad (II.19)
$$

$$
\phi_1 = 2q \pi \quad 0 \leq q < 1 \quad . \quad (II.20)
$$

The initial coordinates are calculated for each cluster and the initial velocity of each cluster is evaluated by taking the vector sum of the intrinsic velocity of the cluster inside the projectile and the centre of mass velocity of the projectile itself. The target at this moment has a recoiling velocity due to the mutual Coulomb force (very small). The radial velocity between the two clusters is obtained from the relation

$$
V(R) = \frac{2}{\mu} \left( E_B - V_N(R) - V_C(R) - V_\kappa(R) \right) \quad (II.21)
$$

where

* $\mu$ is the reduced mass of the two clusters,
* $E_B$ binding energy of the two clusters in the projectile,
* $V_N$ nuclear potential (Woods-Saxon),
* $V_C$ Coulomb potentials
* $V_\kappa$ centrifugal potential

$$
= \frac{41.2}{\mu R^2} \quad \text{MeV}.
$$
Starting from these initial conditions of the 3 particles involved, the motion of the system is integrated under the mutual forces between them, defined by Eqs. II.8, II.9, until the distance between one of the clusters and target has increased from its minimum value to 50 fm. We extend the calculation to this distance in order to account for those cases where the fragments emerge at small angles between them.

Once the full trajectories have been calculated, tests are then made on the conditions of the clusters and target relative positions and we obtain the following possibilities:

1. If the 2 clusters remain at a distance less than 15 fm with the target the event was considered as total fusion. At this distance or larger it is impossible for the cluster to form a bound state with the target "classically".

2. If only one cluster is still inside this distance and the other escapes, the case is considered partial fusion.

3. If none of them is less than 15 fm, a further test was made in terms of the relative distance between the two clusters $r_{12}$
   i) if it is less than or equal to the maximum classical distance (4.1 fm) the event was considered as Li-scattering.
   ii) if it is bigger than 4.1 fm, it was considered as break-up. For this case another test is made later to make sure that the two clusters do not converge and recombine again to form Li nucleus.

In cases 2, 3 i), the trajectory calculation is converted to the two body case by using the centre of mass of the two adjacent
particles instead of both of them. This was introduced to reduce the amount of calculation. In the case of break-up, the 3 body trajectory, and in all cases the calculation, is proceeded to distance $10^4$ fm from the target.

Then the final energies and the directions of the ejectiles are obtained. During the motion of each cluster inside the strong interaction radius with the target several quantities have been monitored,

1. distance of closest approach to the centre of the target.
2. Time duration inside the strong radius.
3. Distance traversed inside the strong interaction radius.
4. Absorption of each cluster and its transmission probability, as described by Eq. II.13.
5. Average mean free path of each cluster on the surface of the target

$$\bar{\lambda} = \frac{\sum \lambda_i(r) \Delta S_i}{\sum \Delta S_i}$$

and $$\lambda_i(r) = - \frac{\hbar v(r)}{2W(r)} .$$

**II.4 Kinematical Considerations**

Ohlson (Ohl 65) studied the non-relativistic kinematics of reactions end with 3 body final products. Later Fuchs (Fuc 82) gave another more accurate proof for some of the relations obtained by Ohlson. One can use these relations to study the kinematical limits of the 3-body process, i.e. find the kinematical region in phase space.
of such processes. Also the distribution of the energy (or momentum) density of states for each particle; the standard form of this relation for particle 1,

\[ \rho(E_1) = \frac{m_1 m_2 m_3}{2m_2 + m_1 + m_3} \frac{p_1 p_2}{(p_1 - p) \cdot p_2} \cdot \frac{1}{h} \]  

(II.23)

where

- \( m_1, m_2 \) are the masses of the ejectiles,
- \( m_3 \) the mass of the residual nucleus,
- \( p_1, p_2 \) the laboratory momenta of the ejectiles,
- \( p \) the laboratory momenta of the projectile,
- \( E_1 \) the laboratory energy of particle 1 (under consideration).

If the residual nucleus mass \( m_3 \) is very large compared with the mass of the ejectiles \( m_1, m_2 \), i.e. \( m_3 \gg m_1, m_2 \) the density of the states reduces to the form

\[ \rho(E_1) \propto \frac{m_1 m_2}{h^6} \cdot \frac{p_1 p_2}{h^6} \]  

(II.24)

which is symmetric with respect to both ejectiles.

Such kinematical relations are basically independent of the reaction mechanism and the density of states is an essential factor in the evaluation of the transition probability rate and eventually the differential cross-section.

In the framework of the participant-spectator model, one can check the possibility of the direct break-up process, by comparing the maximum energy of the spectator (as one of the ejectiles) \( E_{1\text{lab}}^{\text{max}} \) at a particular angle for the ejectiles, with its kinetic energy
share $E_1$ in the projectile incident kinetic energy, i.e. energy of the spectator at the beam velocity

$$E_1 = \frac{m_1 E_p}{m_1 + m_2}$$  \hfill (II.25)

if $E_1 \leq E_1^{\text{max}}$ lab direct break-up (with particle 1 as spectator) is possible at these angles

if $E_1 > E_1^{\text{max}}$ lab direct break-up is not likely to take place, since the kinematical limit will be violated.

So the conditions for $E_1^{\text{max}}$ lab to be large make the direct break-up possible. $E_1^{\text{max}}$ could be obtained from the relation

$$E_1^{(\text{max})} = E_p - E_r^{(\text{min})} - E_B$$  \hfill (II.26)

where $E_1^{\text{max}}$ is the maximum energy of the ejectile.

$E_r$ is the recoil energy of the target and the participant.

$E_B$ is the binding energy of the spectator and the participant in the projectile.

i.e. $E_r$ is small when the mass of the target is basically very large, so conservation of momentum implies less recoil energy and emission in the forward direction for both ejectiles.

$E_B$ is small and this occurs for a loosely-bound system such as $\alpha + t$ in $^7\text{Li}$.

On the other hand if $E_1$ is small, this is possible if the masses of the spectator are smaller than the participant. For comparable masses of participant and spectator, both are likely to occur, which I believe makes direct break-up have maximum opportunity.
Fig. 11.8 shows the energy density distribution of the spectator (t) in the reaction $^7\text{Li}$ (70 MeV) + $^{208}\text{Pb}$ → αt + $^{208}\text{Pb}$, where α and t are at horizontal angle 18° from the beam direction and separation angle (vertical) $\Delta\lambda = 16.8^\circ$.

$E_1 \sim 30\text{ MeV}$

$E_{1\text{max}} \sim 67\text{ MeV}$

so from the kinematical point of view this direct break up is more likely to take place.

Finally, a program has been constructed based on the kinematics and of the 3-body system used to check the final results of the direct break up obtained from the classical trajectory calculations as a final stringent test for the data obtained before carrying out the analysis, and this will be discussed in the next chapter.
Fig. II.8 The distribution of the density of states of the triton for horizontal angle $\psi_0 = \psi_t = 18^\circ$ and vertical separation angle, $\Delta \lambda = 5.9^\circ$. 
CHAPTER III

RESULTS AND DISCUSSION OF THE CLASSICAL TRAJECTORY MODEL

III.1 Introduction

III.1.1 General

In this chapter we shall discuss the results of the simulation of the reaction $^7\text{Li}(70 \text{ MeV})$ on $^{208}\text{Pb}$. This simulation has been carried out in several stages. The first objective was to study the production dependence of the ejectiles, due to different reaction mechanisms, on the impact parameter of the projectile. Such a study helps to define the important range for each reaction mechanism and some of its characteristics. The second stage is to study the global energy distributions (integrated over all angles) and the angular distributions (integrated over all energies) for each reaction process at the peripheral region. The third stage results are concerned with exclusive conditions in which data obtained in the second stage, making use of the symmetries of the model, are folded and compared with the exclusive experimental results. Other interesting events like lithium stretching and pure Coulomb interaction with the target are also discussed. The role of absorption in the final results is considered and its importance is shown at the relevant points.

Although the numerical solutions have been tested for energy conservation, further kinematical checks for break up events have been made for the ejectile energies and their direction and compared with the 3-body kinematical solutions. It was found that almost all the break up results have less than 1% error in the expected energy of one ejectile; this expected energy is calculated in terms of the energies
of the other ejectile and the target (as a residual nucleus) and the directions of the ejectiles. Such an independent check provides another measure for the accuracy of the calculations carried out.

III.1.2 Method of Analysis

The results obtained from the trajectory program (Section II.2.1.2) are further processed with a set of programs designed to perform certain functions, such as averaging and folding for all kinds of reaction events and, in particular, to obtain the angular and energy distributions. The trajectory data obtained are weighted according to the following:

1 - Transmission factor

This factor is based on the transmission of each cluster across the target nucleus as defined in Section II.2.4. This therefore gives a measure for the probability of cluster survival. If $T_t$ is the transmission probability of the triton, and $T_\alpha$ is the transmission probability of $\alpha$, with the assumption that each transmission probability is a function of the trajectory of each cluster independently, then the weight (WT) of each event could be assigned as follows:

i) break up and Li-scattering events

$$WT = T_\alpha \cdot T_t$$  \hspace{1cm} (III.1)

ii) partial fusion has two possibilities, if $\alpha$ escaped and the triton fused into the target

$$WT_\alpha = T_\alpha \cdot (1 - T_t)$$  \hspace{1cm} (III.2)
or the triton escaped and α-fused into the target

\[ WT_t = T_t \cdot (1 - T_\alpha) \]  

(III.3)

2. - Intensity normalization

In the case of stepwise selection of the impact parameter, the number simulated for each impact parameter is constant, so in order to obtain a uniform intensity from all impact parameter values considered, one should introduce a normalization factor. This factor could be deduced as follows: the area of the annulus at impact parameter \( b = 2\pi b \Delta b \), \( \Delta b \) is the increment of \( b \). The number simulated at each impact parameter is \( n(b) \) and is constant for all the steps of \( b \). Then the total number

\[ N = \sum_{i=1}^{m} n(b) = m n(b) \]

where \( m \) is the number of steps. The intensity is equal to

\[ \frac{\text{total number}}{\text{total area}} = \frac{N}{\pi (b_{\text{max}}^2 - b_{\text{min}}^2)} \]

The normalized intensity at each impact parameter \( b = 2\pi b \Delta b \cdot f_b \) where \( f_b \) is the normalization factor which is then defined as

\[ f_b = \frac{N}{n(b)} \frac{2b}{b_{\text{max}}^2 - b_{\text{min}}^2} \Delta b \]

\[ = m \Delta b \frac{2b}{b_{\text{max}}^2 - b_{\text{min}}^2} \]  

(III.4)

The total weight of each event in this case must include this normalization factor and be equal to

\[ WT_s = f_b \cdot WT \]  

(III.5)

For comparative purposes, we set \( WT = 1 \) to clarify the role of
absorption. The idea of assigning a weight for the initial starting conditions of the projectile in terms of its internal parameters $R, \theta_1, \phi_1$ was rejected, since we specify the starting point for considering the 3-body system and in reality all possible values of $R, \theta_1, \phi_1$ are taking place at this starting point, considering a long enough period of time. However, if one assumes that the projectile itself may have certain internal momentum distribution, one can assign another weight which depends on the internal momentum. One method is to use the momentum wave function of the cluster inside the projectile. The method to obtain this wave function and the probability distribution are shown in Appendix IV. The weight in this case is

$$W_p = |\psi(p)|^2 p^2$$

(III.6)

where $p$ is the relative momentum between the two clusters inside the projectile, and this value of $p$ is set at the initial starting conditions of the 3-body trajectory calculation.

For the case of break up events, since we are using spherical potentials, each event is cylindrically symmetric about the Z-axis. So the results obtained from the trajectory calculations can be used to build up another break up event, which can take place at the same distance $r$ from the target, but with different orientations in space. This could be achieved simply by rotating the coordinate system around the direction of the incident particle along the Z-axis by angle $\Delta \phi$, where $\Delta \phi$ is defined within the range $0 \leq \Delta \phi < 360^\circ$, or equivalently by changing the azimuthal angles of the ejectiles $\phi'_a, \phi'_t$ by the same amount $\Delta \phi$ into new $\phi'_a, \phi'_t$, namely

$$\phi'_a = \phi_a + \Delta \phi$$

$$\phi'_t = \phi_t + \Delta \phi$$

(III.7)
This build up process can take place indefinitely, but was restricted to cover the $\Delta \phi$ range in a reasonable number of steps which depend on the geometry of the detection system of the experimental results under consideration.

Fig. III.1: Schematic representation of the cylindrical symmetry of the break up event.

This building up process is not necessary for the case of Li-scattering and partial fusion, because the final channel is a 2-body system and will be symmetric around the Z-axis.

III.2 Impact parameter dependence

As a starting point we needed to scan all physical possibilities of the results, the impact parameter (b) was taken between $b = 0$ to $b = 11$ fm, in steps of 1 fm; for each value of $b$, 360 cases have been simulated, covering the physical range of the initial projectile parameters $R, \theta_1, \phi_1$, stepwise. The reason for the upper limit of $b_{\text{max}} = 11$ fm is the strong interaction radius of $^7\text{Li}$ on $^{208}\text{Pb}$ which is 11.718 fm (Bri 76) and which corresponds to a grazing impact parameter
\( \approx 8 \text{ fm} \), measured from the centre of the projectile, assuming that one of the clusters may be at a distance up to 3 fm from the centre of the projectile, suggesting that the maximum value is \( b = 11 \text{ fm} \). To get a preliminary idea about the impact parameter region of each process, Fig. III.2 shows the probability for the break up events with the impact parameter, and it is found to exist mostly around \( b = 8-10 \text{ fm} \). The shape, in general, is in agreement with quantum mechanical calculations a similar reaction type by \( \beta_{\text{nucl}}(\beta_{\text{coll} \ 8\text{pb}}) \) and also with Utsunomija (Uts 83) on the break up fusion of \( ^{7}\text{Li} \) on \( ^{159}\text{Tb} \).

Partial fusion events have two possibilities, either \( \alpha \) escapes and the triton is captured by the target in the reaction \( ^{208}\text{Pb}(^{7}\text{Li}, \alpha) \), or the triton escapes and the \( \alpha \) is captured in the reaction \( ^{208}\text{Pb}(\text{Li}, t) \). Fig. III.3 shows that most of the partial fusion events occur between \( b = 7.0 \) to 9.0 fm at a range of impact parameter less than the break up range.

To check the imaginary potential used to describe the absorption of the clusters by the target, one defines the absorption probability in terms of the transmission factors at impact parameter \( b \).

\[
\frac{n(b)}{\sum_{i=1}^{\infty} [(1 - T_\alpha)(1 - T_\text{t})]_i} n(b)
\]

where \( n(b) \) is the number of events simulated at impact parameter \( b \).

This absorption probability varies with the impact parameter, as shown in Fig. III.4 for the classical model. It is possible to compare this absorption probability with other simple quantum mechanical methods using reasonable parameters. There are two methods:

1. Using the Coulomb waves to describe the relative motion between the target and the projectile \( ^{7}\text{Li} \) (composed of these two clusters,
Fig. III.2 Break up probability distribution with respect to the impact parameter.
The Impact parameter (fm).

$^{208}$Pb ($^7$Li (70 MeV), $\alpha$), $^{208}$Pb ($^7$Li (70 MeV), t).

Fig. III.3 Partial fusion probability distribution with respect to the impact parameter.
The classical model
Quantum mechanical (I)
Quantum mechanical (II)

The impact parameter (fm).

Fig. III. 4. The total absorption of the two clusters.
at incident energy 70 MeV and calculate the transmission probability of these waves into the target, represented by a spherical well of fixed depth \(- V_0\) and radius \(R_0\) (Mar 65, Pre 62). The basic assumption is that the transmitted part of the wave to the interior of this sphere will be completely absorbed. The value of the transmission probability is found to be very slowly dependent on the value of \(V_0\) but more sensitive on \(R_0\). The parameter values used \(V_0 = 200\) MeV and \(R_0 = 10.0\) fm, which are equal to the sum of two radii of the projectile and the target = \(1.27(A^{1/3}_{Li} + A^{1/3}_{Pb})\) fm.

2. Using the optical potential with Woods-Saxon form which produces the best fit to the angular distribution of elastically scattered \(^7\)Li on \(^{208}\)Pb at 68 MeV (Dav 84) and the absorption probability \([1 - |\eta(b)|^2]\) where \(\eta(b)\) is the reflection coefficient. The parameters of the optical potential obtained using Code (HERMES) (Coo 84) are

\[
V_0 = -101.73\text{ MeV}, \quad R_0 = 8.21\text{ fm}, \quad a_r = 0.7\text{ fm}
\]

\[
W_0 = -25.47\text{ MeV}, \quad R_0 = 1.24\text{ fm}, \quad a_i = 0.885\text{ fm},
\]

particularly at the larger impact parameter range which is the most important, together with the reasonable values of \(V_0, R_0\), giving a higher degree of confidence in using the imaginary potential parameters of the two clusters used to produce their transmission probabilities \(T_o, T_t\).

Also the curve showing the transmission coefficient is very similar to that obtained by Baur et al. (Bau 80b) in studying \(\alpha\)-particle break up by \(^{62}\)Ni isotopes.

As the one main interest is in the direct break up process, we restrict the maximum and minimum values of \(b\) to 8 and 10 fm, respectively. 798,000 different cases were simulated by Monte Carlo method. To study the impact parameter dependence on different physical and spatial properties of the products inside this region,
it was necessary to divide this impact parameter range to 20 equal cells, each having 0.1 fm width, and obtain histograms for the physical quantities under interest.

The data obtained from the simulation is weighted with WT and averaged over the number of events of the particular process in each cell. The average of any quantity \( Q \) is obtained by evaluating

\[
\bar{Q} = \frac{\sum_{i=1}^{n_j} (WT)_i \cdot Q_i}{\sum_{i=1}^{n(b)} (WT)_i}
\]

\((WT)_i\) - weight of the event (i) of a particular process \( j \) as described by equations III.1, III.2, III.3.

\(Q_i\) - physical quantity of event (i) of a particular process.

\(n_j\) - total number of events of a particular process \( j \) in a particular impact parameter cell.

From this definition the value of the average quantity inside each cell will be close to the highest weight which, in the case of the absorption considered, represents the most surviving cases. In some cases, we consider the averages but without absorption, i.e. \( WT = 1 \) for all events, and this is done to get an idea for the role of absorption inside this region of \( b = 8 - 10.0 \) fm.

### III.2.1 Projectile Scattering

The major contribution of events inside this region of impact parameter \( b = 8.0 - 10.0 \) fm comes from the projectile scattering, with 65% of the total number of events. Fig. III.5 shows the contribution from different competing processes, namely, Li elastic scattering,
The Impact parameter (fm).

Fig. III.5. The distribution of events with the impact parameter.
break up and partial fusion without considering the absorption.

The Li-scattering events are always increasing, while the break up events show systematic decrease. Partial fusion is generally decreasing except for a little bump between $b = 9.2 - 9.7$ fm, due to the Coulomb forces between $\alpha$ and the target-triton system, as will be explained later in Sec.III.2.2. For the same reason, the systematic decrease in break up events is almost sustained for $b = 9.6 - 9.9$ fm.

Considering the Li-scattering events only, the classical trajectory calculations can provide us with the duration of each cluster remaining inside the strong interaction radius, i.e. within the region where the mutual nuclear forces with the target are taking place.

Fig. III.6a shows the average time interval for $\alpha$ and for triton in Li-scattering events without absorption and Fig. III.6b is similar but with the absorption considered. Both show a systematic decrease in the time interval with a plateau. The time interval for both clusters decreased considerably with the increase of the impact parameter and this region ($b < 8.3$ fm) is dominated by the nuclear interaction. The plateau for $\alpha$ has the range $b = 8.6 - 9.0$ fm and for $t$ it starts at $b = 9$ fm, the plateau defines a new region where the nuclear and Coulomb forces are comparable with each other, making the cluster partially orbiting the target. Afterwards, in the case of $\alpha$, the time interval decreased monotonically for $b > 9.1$ fm.

The time interval of the triton is less than the time interval of the $\alpha$-cluster for $b < 9.6$ fm, suggesting that the elastic scattering for $b = 8 - 9.6$ fm is more likely to occur when the $\alpha$-cluster is nearest to the target, while for $b = 9.6 - 10.0$ fm the reverse holds true, suggesting that $\alpha$ is pushed away by Coulomb repulsion from the target, and the triton comes between the target and the $\alpha$-cluster for a longer period.
Fig. III. 6. The average time interval of each cluster inside the strong interaction radius of the target.
inside the target region while the triton itself is still in its plateau region.

The absorption, aside from its effect of overall reduction in the time interval, does not change the main features of the time interval histograms for the clusters in Li-scattering events, and this can establish the idea that the absorption has generally little effect on the mechanism of such a process at this range of the impact parameter.

### III.2.2 Partial Fusion

Such a process can take place in two different ways, as mentioned earlier in Section III.1.2. Fig. III.7 shows the probability of each process. The process in which $\alpha$ escapes and the triton fuses covers the entire region of $b = 8 - 10.0$ fm, but the process in which $t$ escapes and $\alpha$ fuses has smaller range $8 - 9.1$ fm and this can be explained in terms of the Coulomb distance of closest approach. For $b = 9.1$ fm the distance of closest approach due to Coulomb force only between projectile and the target is 12.1 fm, while the strong interaction radius between $\alpha$ and the target is 11.23 fm. If the $\alpha$ can move to $\sim 1.4$ fm from the centre of the projectile at distance 12.1 fm, then the $\alpha$ could be inside the strong interaction radius by just $\sim 0.5$ fm, in which the nuclear force is not enough to make the $\alpha$ fuse into the target since the repulsive Coulomb force will be the dominant one (see Fig. II.2). For the triton case the maximum distance from the centre of the projectile could be 2.7 fm, which may put the projectile at a distance $\sim 9.4$ fm from the target inside the attractive nuclear region (see Fig. II.3). This means that the triton may then fuse into the target.

To get another look at the process, the average distance travelled
The impact parameter (fm).

FigIII.7. The distribution of the ejected alphas and tritons in partial fusion processes.

The number of transmitted ejectiles in each cell.
by each escaped cluster without considering absorption is shown in Fig. III.8. The α-clusters, in general, travel larger distances than those escaped tritons in the second process at the same impact parameter and this explains why the probability of α-escapes is generally smaller for \( b = 8 - 8.9 \) fm, since their crossing distances are larger. This shows at \( b \leq 9.1 \) fm that the α is affected by the nuclear field, while at \( b \geq 9.1 \) fm the α is only affected by the Coulomb force. For the tritons there is an exceptionally high value for the crossing distance at \( b = 9.1 - 9.2 \), which indicates orbiting and this is absent in Fig. III.7 because their orbiting gives very low probability to escape. The actual number of events for triton escape process inside this cell is 3 events and the reason for this small number is the use of the Woods-Saxon potential which has a relatively narrow force region peaked at the target surface.

In considering the momentum weighting, the momentum probability distribution is shown in Fig. AIV.2. The average time interval of the outgoing α is represented in Fig. III.9, with and without momentum weighting (without considering absorption). It shows in the case of momentum weighting an overall increase in the average time interval which means that the highest momentum probability components stay, on average, for a longer period. These components have generally low relative momentum, as shown in Fig. AIV.2, i.e. low relative velocity components staying longer. Fig. III.10 shows the same trend for the triton with relatively smaller change between with and without momentum weighting and one can attribute this to its higher speed than that of the α which makes the change in time relatively smaller. This indicates that the partial fusion processes occur well inside the strong interaction radius.
Fig. III. 8. The average crossing distance of each outgoing cluster inside the strong interaction radius of the target.
$^{208}_{\text{Pb}}(^{7}_{\text{Li}}(70 \text{ MeV}),\alpha)$. 

Without momentum weighting

With momentum weighting

---

Fig. III.9 The crossing time distributions with and without momentum weighting for the ejected alpha.
$^{208}\text{Pb}(^7\text{Li}(70\text{ MeV}),t)$.

Without momentum weighting

With momentum weighting

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1110.png}
\caption{The crossing time distributions with and without momentum weighting for the ejected triton.}
\end{figure}
III.2.3 Direct Break up:

One of the advantages of this type of calculation is to get an idea of how the recoil energy of the target changes with the impact parameter. Fig. III.11 shows the average recoil energy for this kind of event, with and without absorption. Without absorption it is much higher, up to 1.9 MeV, and decreases sharply at $b = 9.5 - 9.7$ fm, while with absorption the average recoil energy is less than 0.2 MeV and its changes are very small over the entire region. This indicates that those break up events which survive exchange very little energy between the projectile and the target.

The transmitted number across the target in direct break up events has some distinct features. Although the number of $\alpha$'s is the same as the tritons', the final number transmitted is different due to the different dynamical behaviour of each inside the target, as shown in Fig. III.12a. $\alpha$ particles are transmitted much more than tritons, which shows again that the Coulomb force repels the $\alpha$ away from the target nucleus region consistently with the impact parameter, while the triton is not just decreasing sharply to $b = 8.5$ fm, but stays with low transmission probability up to $b = 9.5$ fm, due to orbiting around the target, and then starts to increase due to less orbiting path, and both $\alpha, t$ behave similarly, $b = 9.6 - 10.0$ fm. The probability of both to be transmitted is the product of both probabilities of transmission as defined in Eq. III.1 and is shown in Fig. III.12a and on smaller scale on Fig. III.12b. We find the original peak, first shown in Fig. III.2 splits into two peaks, one at about $b = 8.8$ fm and another one at $b = 9.7$ fm, 0.9 fm apart. One can explain this by looking
The impact parameter (fm).

Fig. III. 11. The average recoil energy of the target for breakup reaction.

$^{208}$Pb ($^7$Li (70 MeV), α).
With absorption
Without absorption

The average recoil energy (MeV).
Fig. III. 12. The transmision of alpha, triton, and both together out of 798000 events simulated.
at Figs. III.13a,b and Fig. III.14a,b which display the average time interval and average distances, with and without absorption for both clusters respectively. With absorption, the $\alpha$ does not touch the strong interaction region for $b \geq 9.2$ fm and very little for lesser values. The result of this indicates that in the first peak, $b = 8.8$ fm, one or both clusters may participate with the target, while in the second peak at $b = 9.7$ only the lowest charged cluster (triton) passes the target nuclear region and the $\alpha$ cluster acts as a spectator. This emphasizes the idea of the participant-spectator model but it shows also that such an idea is not the only mechanism for direct break up, nevertheless one can set as a general principle that the highest charge cluster acts as a spectator.

Fig. III.14a shows the average distance of an $\alpha$-cluster without considering the absorption for $b = 8.0-8.4$ fm, the $\alpha$-cluster moves inside the strong interaction radius and barely touches it for the range of $b = 8.5 - 9.2$ fm, while the average distance for the triton is relatively larger between $8 - 28$ fm.

Considering the absorption, Fig. III.13b and Fig. III.14b also show that the triton is partially orbiting for $b = 8.5 - 10.0$ for a longer average period and distance inside the nuclear region, i.e. the tendency of triton orbiting is possible for direct break up events but is not definitely the case for $\alpha$.

To get an idea about the average transmission coefficient for each cluster, Fig. III.15a shows the histogram for these values without weighted averages. The transmission for the $\alpha$ is almost unity for $b \geq 8.5$ fm, while the triton is attenuated most between $b = 8.5 - 9.3$ fm. Fig. 15b shows the average value of the transmission factor of the most surviving break up events (i.e. by
The impact parameter (fm).

**Fig. III.13.** The average time interval of each cluster inside the strong interaction radius of the target.
Fig. III. 14. The average distance of each cluster inside the strong interaction radius of the target.
Fig. III. 15. The average transmission coefficients for each cluster to cross the target.
considering the absorption) for $\alpha$ is around 1.0 fm for $b > 9.2$ fm, while for the triton the average is about 0.5 for $b > 8.5$ fm.

The average energy of each cluster corresponding to a zero absorption is shown in Fig. III.16a. At low values of $b = 8 - 9.3$ fm, the energy corresponding to the average is slightly above the beam velocity, while at $b = 9.3 - 9.9$ fm, it becomes less. On the contrary, for the average energy for the triton. Comparing with Fig. III.14a (and also Fig. III.13a) we can see that the average distance of the triton inside the target decreased significantly at $b = 9.6$ fm. This means it becomes on average closer to the $\alpha$, i.e. the attraction between the $\alpha$ and the triton becomes more effective in the presence of the target, and this is the reason why the $\alpha$ is affected by the nuclear field of the target via the triton. This causes the $\alpha$ to share more energy with the triton and the target, which shows a drop in the average energy of $\alpha$, and an increase in triton energy. On our case here, the target is so massive that such an effect can be seen. However, in increasing the impact parameter both the $\alpha$ and the triton act together in a way that causes little increase in the recoil energy of the target at $b > 9.7$ fm, as shown in Fig. III.11.

This idea did not change much for the case of most surviving break up events, as shown in Fig. III.16b.

Another facet on this idea could be seen in the average distance of closest approach of each cluster.

Fig. III.17a shows the average distance of closest approach without considering the absorption. For the triton, starting from $b = 9.4 - 9.7$ fm, the average distance increased significantly, from 4.7 fm to about 10 fm, which is around the surface of the target.

Fig. III.17b shows the weighted average distance with absorption,
Fig. III.16. The variation of the average energy of each cluster with the impact parameter.

The impact parameter (fm).

The average energy (MeV).

\( ^{208}\text{Pb} \left( ^{7}\text{Li}, 70 \text{ MeV}, \text{at} \right) \).

Without absorption.

With absorption.
Fig. III. 17. The average distance of closest approach for each cluster with respect to the target center.
showing a systematic increase for the $\alpha$ cluster, but for the triton between $b = 8.0 - 8.6$ fm, it was almost outside the target region at about 12.5 fm and touching the target at about 10 fm in the rest of the values of the impact parameter.

Both the Coulomb force and nuclear force contribute to peripheral collisions and for comparative purposes, the values of the grazing angle of scattering/pure Coulomb deflection angles are listed in Table III.1, at the strong interaction radii defined in Table II.2.

To see the relation between the average scattering angle and the impact parameter, Fig. III.18a shows that the $\alpha$ average scattering angle decreases systematically but for $b = 8 - 8.5$ fm, it decreases rapidly, starting at 64°. Considering Fig. III.13a, the $\alpha$ is mainly deflected by the nuclear field at angles $\geq$ grazing angle. This means that it is at the negative side of the beam direction, and afterwards at $b = 8.6 - 9.7$ fm decreasing smoothly as the nuclear force becomes less effective and the Coulomb force starts taking part. At $b = 9.8 - 10$ fm the faster decrease is due to the Coulomb scattering only.

Also, the average scattering angle ($\theta_t$) of the triton (without absorption) as shown in Fig. III.18a, varies dramatically, starting from 80° to the peak at 140° at $b = 9.1$ fm and then decreases very fast to about 13° at $b = 9.8$ fm. This wide change is due to the condition of orbiting of the triton in that region, but most important, both the $\alpha$ and $t$ have similar scattering angles around 20° at $b = 9.7 - 10.0$ fm.

Considering the absorption, Fig. III.18b shows great reduction in the values of the scattering angles below $b = 9.7$ fm and in
Fig. III.18. The average scattering angle (θ) of each cluster.
TABLE III.1

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Grazing angle (deg.)</th>
<th>Strong Interaction radius (fm)</th>
<th>Grazing impact parameter (fm)</th>
<th>Coulomb Scattering angle</th>
<th>Charge/Energy (10^x) MeV(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^4\text{He}(40,\text{MeV}) + ^{208}\text{Pb})</td>
<td>42.28°</td>
<td>11.32</td>
<td>7.64</td>
<td>45.78</td>
<td>30.00</td>
</tr>
<tr>
<td>(^3\text{He}(30,\text{MeV}) + ^{208}\text{Pb})</td>
<td>25.28°</td>
<td>11.01</td>
<td>8.77</td>
<td>31.4</td>
<td>20.31</td>
</tr>
<tr>
<td>(^7\text{Li}(70,\text{MeV}) + ^{208}\text{Pb})</td>
<td>32.42°</td>
<td>11.718</td>
<td>8.713</td>
<td>39.79</td>
<td>25.94</td>
</tr>
</tbody>
</table>
general for most surviving events deflections of \( \alpha \) are greater than those of the tritons at all impact parameters 8 - 10 fm, mainly due to its larger charge.

The average relative energy between the two ejectiles, as shown in Fig. III.19, it is reduced radically if one considers the absorption and is systematically decreasing with the impact parameter. Alternatively, the average distance of closest approach is systematically increasing as displayed in Fig. III.20.

The average scattering angle of the centre of mass of the \( \alpha \) and triton without absorption, is decreased from 40° to 27° at \( b = 8.5 \) fm, since both are affected by the nuclear field, then increased again due to the orbiting of the triton which decreased systematically to smaller angles at \( b = 9.7 \) fm. For most survival the average centre of mass angle increased from about 10° to about 15°, showing that those events \( \alpha, t \) can exist on both sides of the beam at \( b = 9.7 \), and together on the same side at \( b > 9.7 \) fm using Fig. III.18a, b.

In this peripheral region at \( b = 8.0 \) fm the average mean free path was found to be 0.1 fm for both \( \alpha, t \) and in partial fusion the maximum values obtained for escaped \( \alpha \) is 2 - 3 fm and for tritons is 1 - 2 fm, while for break up process the average mean free path for \( t \) is raised to 11 fm, indicating larger possibility of orbiting.

III.3 The Energy Distribution

In this section the energy distributions are integrated over all directions of emission of the ejectiles. The energy is divided into cells, each 1 MeV. Absorption is taken into account in all cases.
$^{208}\text{Pb}(^{7}\text{Li}(70\text{ MeV}),\alpha t)$.

With absorption

Without absorption

The average relative energy (MeV).

The impact parameter (fm).

Fig. III.19. The average relative energy between clusters after the break up reaction.
$^{208}$Pb($^7$Li (70 MeV), αt).

With absorption.

Without absorption

The average distance of closest approach (fm).

Fig. III.20. The average distance of closest approach for the center of mass.
$^{208}\text{Pb}({}^7\text{Li}(70 \text{ MeV}), \alpha_t)$.  

With absorption.  

Without absorption.  

Fig. 111. 21. the average scattering angle of the center of mass of the two clusters ejected.
III.3.1 Projectile Scattering

The energy distribution of the centre of mass energy of the two outgoing clusters is shown in Fig. III.22. It is noticed that the outgoing energy is not a monochromatic energy due to the following reasons:

1. The relative kinetic energy and the potential between the two ejectiles are not considered at the final evaluation of the energy.
2. Recoil energy of the target varies within 0.5 MeV for \( b = 8 - 10 \) fm.
3. Change in the Coulomb potential energy at the initial start of the 3-body trajectory, due to the difference in orientation of the two clusters, which is \( < 0.1 \) MeV.

These cause a spread in energy about 2-3 MeV on both sides of the mean value.

III.3.2 Partial Fusion

Fig. III.23 shows the energy of the outgoing \( \alpha \)-particle from the first process in which \( \alpha \) is the ejectile. The peak is shifted above the beam velocity (at 40 MeV) to higher values (at 49 MeV) due to the Coulomb repulsion by the triton-target system to the \( \alpha \)-particle causing the increase in its energy more than its average ingoing energy.

Similarly, Fig. III.24 shows that the triton energy distribution in the second process with the same features, with less peak shift than the average ingoing (\( \sim 30 \) MeV) to \( \sim 33 \) MeV. Most of these high energy components come from regions outside the target region in which they are not absorbed nor affected by the attraction of the nuclear forces.

To investigate this idea the impact parameter is extended to lower
$^{208}\text{Pb} (^{7}\text{Li} (70 \text{ MeV}), ^{7}\text{Li})$.

With absorption

Without absorption

Fig. III.22. The center of mass energy distribution for the $^{7}\text{Li}$ integrated over all angles.
Fig. III. 23. The energy distribution for the alpha particles integrated over all angles.
Fig. III.24. The energy distribution for the triton particles integrated over all angles.
values, $b = 6.0$ fm, for a limited number of (18,000) events it is found that (with the absorption considered) the energy of the ejectiles ($a$ or $t$) increase systematically with the decrease of the impact parameter without much change to the peak position. This suggests that as the fused cluster goes into the target while the other cluster stays almost away from the nuclear field and is affected by the Coulomb repulsion of the combined system, which leads to the increase of its average energy.

III.3.3 Direct Break up

The energy distributions of both ejectiles produce an asymmetric shape as shown in Figs. III.25 and III.26. The peak of the $\alpha$ cluster is about 36 MeV, while for the triton about 33 MeV, and the main cause of their shifts, other than their average beam energy, is that the total weight of a break up event is the product of both transmission coefficients, which will be the highest when both are away from the nuclear region, and those were located before at $b = 9.6 - 9.8$ fm (see Fig. III.15a and Fig. III.16b). At that region the triton exists on average between the $\alpha$ and the target (see Fig. III.17a,b) and act as an extension for the nuclear force of the target to affect the $\alpha$, which share much more momentum with this system, causing reduction in its kinetic energy and increase in the triton energy. These energy distributions produce an integrated picture from the usual experimental energy distributions which are usually taken at a particular direction.

The relative energy between the two ejectiles are shown without absorption in Fig. III.27a. Clearly, there are two main components, low relative energy component and high relative energy component which is the largest. The nuclear force of the target plays an
The energy of alpha particles (MeV).

\( ^{208}\text{Pb} \rightarrow ^{7}\text{Li} (70 \text{ MeV}, \alpha) \).

Fig. III. 25. The energy distribution for the alpha particles integrated over all angles.
The energy of triton particle (MeV).

Fig. III.26. The energy distribution for the triton particles integrated over all angles.
essential role for the production of the high energy component. The region between these two components has relatively low number and its range depends very much on the diffuseness of the Woods-Saxon potential used between the clusters and the target. If the diffuseness is small, the spread of the nuclear force at the surface of the target becomes small. The possibility for the nuclear force being involved or not in the break up process confined to small space, giving low possibility of such middle events to exist. To confirm the connection between high relative energy component and its creation deep into the target nuclear field, Fig. III.27b shows the relative energy distribution with the absorption considered. The high energy component almost completely disappears, since the clusters went deep inside the strong interaction radius and were absorbed by the target, and the low energy component is then dominant. It is noticed also that the number of events at small relative energy \( \leq 1 \) MeV shows a dip due to the Coulomb barrier between \( \alpha - t \) inside the projectile which in turn "after the break up" gives more kinetic energy for those of relative energies resulting in reduction in numbers of this cell. Another correlation is also obtained for the relative energy and the angle between ejectiles. Without considering absorption Fig. III.28a shows that the high relative energy component has a wide angle break up due to the nuclear field of the target, while if the absorption is considered, this component vanishes, as shown in Fig. III.28b and the low relative energy events have an angle between ejectiles less than 90°.
The relative energy between the two clusters (MeV).

Fig. III. 27. The relative energy distribution for the two fragments.
The relative energy between the two clusters (MeV).

Fig. III. 28. The correlation between the relative energy between clusters and the separation angle between them, the number of events is proportional to the area of each square.
III.4 The Angular Distributions

In this section, the angular distributions of the ejectiles integrated over all energies is displayed. The angle of scattering considered here is the angle between the ejectile and the projectile direction (Z-axis), irrespective of the left or right of the beam direction. There are no exact experimental data to these distributions basically because of the limitation on the impact parameter values between 8.0 fm and 10.0 fm, but comparison will be considered with the closest experimental conditions.

III.4.1 The Projectile Scattering

The angular distribution of the scattered Li-projectile is shown in Fig. III.29, with and without considering absorption. The particular shape of the distribution is due to the limits of the impact parameter around the grazing conditions. The maximum of the distribution occurs at the angle interval 27-30°, which is little less than the grazing angle \( \sim 32° \) due to the small effect from the nuclear force. As the absorption is considered, the shape does not change with respect to the angles, but there is a decrease in the magnitude of the cross section. At the angle \( \theta = 28.5° \) the average differential cross section without absorption is 2175.1 mb and with absorption 1705.4 mb, while the nearest experimental results at the projectile energy 68 MeV (Dav 84), the cross section changes rapidly from 3487 mb at \( \theta = 28° \) to 1909.2 mb at \( \theta = 30° \). Although the results are close, the differential cross section with the absorption considered is less than the experimental values, suggesting that the absorption may be little overestimated. However, the close agreement of the experiment and trajectory calculations give confidence in the correctness of the trajectory calculations.
$^{208}\text{Pb}(^{7}\text{Li}(70 \text{ MeV}),^{7}\text{Li})$.

With absorption  \quad \quad \text{Experiment}

Without absorption  \quad \quad \text{at 68 MeV.}

\textbf{Fig. III.29.} The angular distribution for the $^{7}\text{Li}$ particles integrated over all energies.
III.4.2 The Partial Fusion

Fig. III.30 shows the angular distribution of $\alpha$ and triton (t) ejected from two different processes of partial fusion considering the absorption. The two peaks are not far from each other. Table III.1 shows the Coulomb scattering angles at beam velocity covering the impact parameter range under study. The right side to the peak of the t-angular distribution lies within the Coulomb range of scattering, which means that some of the tritons ejected are more likely to be affected on average by the Coulomb force only, while for the $\alpha$-ejectiles the right side of the peak is shifted slightly to smaller angles due to little effect from nuclear forces. Also the triton angular distribution has larger width than the $\alpha$-angular distribution because it has a smaller mass and consequently its oscillations around the centre of mass of the projectile has larger amplitude, spreading its existence over a larger range of impact parameter and eventually those less than the grazing impact parameter are attracted by the nuclear force toward small angles, and those of larger impact parameter than the grazing follow the Coulomb trajectories toward small angles as well. Another factor, the triton can easily be drifted due to its small mass. The peak of the triton angular distribution exists at $18^\circ$-$21^\circ$ with an average value of the cross section of 107 mb/sr, while the experimental value of the inclusive cross section (Day 86) (including all processes) at $\theta = 22.5^\circ$ is $\sim 110 \pm 15$mb, which suggests a large contribution for this partial fusion among other processes producing triton ejectiles. And the peak of the $\alpha$ angular distribution, inside the range $24^\circ$-$27^\circ$, and the average value of the cross section 171.2 mb/sr while the inclusive cross section for $\alpha = 25.5^\circ$ is $370 \pm 60$ mb, suggesting 46% contribution from the partial fusion process. Both results show that the partial fusion is a major process near the
Fig. III.30. The angular distribution for the alpha and triton integrated over all energies (absorption considered).
III.4.3 The Direct Break up

Fig. III.31 shows the angular distributions of $\alpha$ and $t$ as ejectiles from break up events. They have similar features as the partial fusion but the two peaks of the distribution are separated more and the reason is that $\alpha$-particles are mostly affected by the Coulomb force, only as shown in Fig. III.13a,b and Figs. III.14a,b, while the tritons in most of the events have an effect from the nuclear forces of the target. Also the triton has a wider spread angular distribution for the same reasons as those discussed in Section III.4.2.

The two peaks have an overlap region which is of importance to coincidence experiments. The peak of this region exists about $17^\circ$ and one cannot expect maximum rate of coincidence at this angle, especially for narrow geometry, as will be shown in Section III.5.

III.5 Exclusive Distributions

The full kinematical experiments provide a more precise and accurate description of the break up process, so it is necessary to compare with this data to check the classical trajectory model.
Fig. III.31. The angular distribution for the alpha and triton, integrated over all energies.
III.5.1  Projected Energy Distribution

For elastic break up events of the same reaction under study, Shotter et al. (Sho 81) obtained the energy distribution of the ejected triton in coincidence with $\alpha$, and their summed energies are selected at a value corresponding to the target being left in its ground state. This projected energy distribution is obtained at vertical separation angle $16.8^\circ$ and horizontal angle $18^\circ$ ($\beta < 8^\circ$).

Fig. III.32 shows the projected energy distribution of triton for both experimental and simulation results at the same angles, normalised to the same number. The absorption is considered for the theoretical prediction, although it has little effect in these cases. The upper and lower limits of the triton energy are slightly less than the experimental distributions. The predicted central value was about 29 MeV. The shapes of the spectra are approximately similar. This agreement with the exclusive experimental data and the classical trajectory model again gives some confidence that the classical model even in these complicated situations is of great value in helping to understand the reaction mechanism.

III.5.2  Angular Distribution

The double differential cross section for both $\alpha$ and triton break up events is obtained under the experimental conditions. The two detectors in close geometry experiments (Sho 81) are set at the same horizontal angles and the vertical separation angle $\Delta \lambda = 5.9^\circ$. The angular size of each detector is $\sim 3^\circ$ in both the horizontal and vertical directions. The relation between this angular system
\[ ^{208}\text{Pb}(^{7}\text{Li} \ (70 \text{ MeV}), \alpha t) \].

The energy of the triton (MeV).

Fig. III. 32. The projected energy spectrum of triton for separation angle $\Delta \lambda = 16.9^\circ, \psi_s = \psi_t = 18^\circ$. 
of coordinates and spherical polar coordinates is shown in Appendix V. Within those limits the data obtained from the trajectory calculations are built up as mentioned in Section III.1.2 by rotating break up events around the Z-axis in steps of 3° to be comparable with the detector size. The double differential cross section obtained is shown in a histogram, Fig. III.33, together with the experimental data points.

The simulation results produced have a good quantitative agreement near the peak range which indicates that the break up process could be explained by this approach within this peak range. It emphasized that the agreement in absolute magnitude is achieved without any adjustable parameters. At smaller angles, where the experiment is limited by high elastic scattering background, the simulation predicts the existence of direct break up which is obtained from the lower part of the impact parameter range with the nuclear forces becoming more effective.

At higher deflection angle the simulation results decreased very rapidly compared with the experimental data of the direct break up and can be explained in terms of the following factors:

1. Some of the break up events could occur due to the wave nature of the cluster in which they are able to penetrate the classical barriers to produce further break up events.
2. The geometrical absorption parameters should be adjusted to give better account for those events by allowing the surviving clusters to go deeper inside the target region to feel the Coulomb repulsion more by reducing the absorption radius.

A noticeable gap exists in the double differential angular distribution between 6° - 12° and one can attribute its existence to
The classical model

The experimental data

\[ ^{208}\text{Pb}(^7\text{Li}, \alpha t) \]

Fig. III.33. The double differential cross section for alpha and triton, the separation angle \( \Delta \lambda = 5.9 \) deg.
the internal rotation motion of the clusters inside the projectile, if both are in the region where the Coulomb and nuclear forces are balanced. It is unlikely for the two clusters to break up and be emitted closely. So, according to this picture at small angles both clusters are affected by the nuclear field, while at larger angles both are affected by the Coulomb field.

One can try to look at the angular distribution differently by selecting those events coming from one side of the target and deflected by the nuclear field to the other side, conventionally known as negative deflection. Fig. III.34 shows a schematic representation for the simulation of the scattering process. The angular distribution obtained on the negative side has very low number and does not show a

![Diagram](image-url)
recognizable pattern so, by integrating both sides, it is found that the negative deflection is \( \sim 5.5\% \) of the total number received at this vertical separation \( \Delta \lambda = 5.9^\circ \) for all horizontal angles, which is a relatively low ratio.

In studying the impact parameter dependence at this close geometry, most of the double differential cross section was located between \( b = 9.5 \) to \( 10.0 \) fm, as shown in Table III.2.

<table>
<thead>
<tr>
<th>b</th>
<th>fm.</th>
<th>Percentage of the total break up cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0 -</td>
<td>8.49</td>
<td>-</td>
</tr>
<tr>
<td>8.5 -</td>
<td>8.49</td>
<td>15.9%</td>
</tr>
<tr>
<td>9.0 -</td>
<td>9.49</td>
<td>17.7%</td>
</tr>
<tr>
<td>9.5 -</td>
<td>10.0</td>
<td>66.4%</td>
</tr>
</tbody>
</table>

Recently a wide angle break up experiment took place for \(^7\)Li (70 MeV) on \(^{208}\)Pb which has been undertaken at Daresbury Laboratory by Yorkston et al. (Yor 86). The detectors were at the same horizontal plane and the triton was detected at angles 20°, 30° on the negative side of the beam and \( \alpha \) detectors on the positive side at angles 20° - 40°. Fig. III.35a, b show the experimental and the theoretical histograms. Although the values of the cross sections are of the same order of magnitude (less than 10 mb), the pattern of the distributions is somewhat random, and this is due to low statistics at these wide angles, which is about \( \sim 10\% \) of all break up data, as will be shown in Section III.7. Nevertheless the overall agreement is quite remarkable.
Fig. III.35 The double differential cross section for wide angle break up.
These results clarify more the characteristics of the direct break up process. However, two points should be made.

1. The assumption of the projectile to be divided uniquely into two clusters, α and triton in our case, is virtual but not far from reality [Wa85], but other substructures may exist. This means that the cross section calculated by this semi-classical approach needs a reduction factor which accounts for the proportion of this substructure among all possible substructures.

2. The classical range of the separation of the two clusters inside the Li-projectile \( R = 0.6 - 4.1 \) fm. covers only 75% of the quantum mechanical description of the projectile and the rest, which is 25%, is smearing out the classical existence of the two clusters to a wider range of \( R \) values. These wider separations could also be responsible for further break up events, due to its wave nature.

These two points, quantitatively, are opposite to each other.

III.6 The Effect of the Target Coulomb Force Only

To study this effect, 18,000 events were simulated for the impact parameter range \( b = 6.0 - 10.0 \) fm and only the Coulomb forces of the target are allowed to act on the projectile substructures. No break up or partial fusion events take place and the only kind of event is the projectile scattering. So from this classical treatment there is no break up or partial fusion can take place without the nuclear forces of the target. The differential Coulomb force acting on the two
Fig. III. 36. The angular distribution of $^7$Li particles, compared with Rutherford cross section.
clusters is not strong enough to cause the break up within the classical limits. The energy of the Li-scattered is between 68-70 MeV with the maximum at 70 MeV cell and the variation in the ejectile energy is due to different recoil energy taken by the target 1.18 MeV and change in the projectile status, as will be discussed further in Section III.8.

Fig. III.36 shows the angular distribution of the scattered Li particles under the Coulomb forces only. The width of the distribution is limited between 28° - 45° as calculated independently from the Rutherford trajectory at impact parameters b = 10.0 - 6.0 fm, respectively while the distribution obtained lies between 27° and 48°, thus causing the first and last cells to be out of the sequence. The maximum of the angular distribution exists at θ = 31.5° (grazing angle). The Rutherford cross section at lab. conditions is also compared with simulation, clearly in very good agreement with the simulation results, which again emphasize the accuracy of such calculations.

III.7 Out of Plane Dependence

In binary break up experiments the ejectiles emerge virtually in all directions but sometimes there is some correlation. One of these correlations is the relative direction of emission of both ejectiles. With cylindrical symmetry of the break up process mentioned in Section III.1.2, this correlation could be studied on the following basis. Define a plane by the incident projectile direction (θ = 0,
\( \phi = 0 \) and one of the ejectile directions \((\theta_1, \phi_1)\) (\(\alpha\)-particle). Define another plane by the incident projectile direction and the other ejectile (triton) direction \((\theta_2, \phi_2)\). The angle between these two planes is the difference between their azimuthal angles \(\phi_2 - \phi_1 = \Delta \phi\), as shown in Fig. III.38.

![Diagram showing the geometry of out-of-plane study.](image)

**Fig. III.37.** The geometry of out-of-plane study.

The correlation of interest is the yield as a function of the angle between the two planes \(\Delta \phi\) and the scattering angles of the ejectiles.

A simple trend is found. Fig. III.38a shows the \(\Delta \phi\) distribution
Fig. III.38. The distribution of events with the angle between ejectile planes for different sets of cluster deflections.
for the ranges of $\theta_1, \theta_2$ defined as follows: $5 \leq \theta_1 < 15,$ $5 \leq \theta_2 < 15.$ At these forward angles with 7.2% of the total number considered in this study, within this range the distribution is almost constant, suggesting no preference toward each other. Fig. III.38b shows the $\Delta\phi$ distribution inside the range $15 \leq \theta_1 < 25,$ $15 \leq \theta_2 < 25.$ It has 82.8% of the total number and a hump peaked at separation angle $\Delta\phi$ around 50° which means that the ejectile emerges at this range with some correlation in the form of attraction in between the substructures of the projectile, while they are in and out of the reaction. This suggests that this range is of importance for close geometry break up experiments. Fig. III.38c shows $\Delta\phi$-distribution for another range $25^\circ \leq \theta_1 < 35^\circ,$ $25^\circ \leq \theta_2 < 35^\circ,$ which has 10% of the total number. At this wide angle scattering $\Delta\phi$ is peaked around 180°, i.e. both ejectiles are close in the same plane but on different sides of the beam direction, which suggests the importance of this angular range for wide angle break up experiments.

III.8 Projectile Stretching

The energy distribution of the projectile scattering has a spread in its emerging energy due to the reasons mentioned in Sec. III.3.1. One major factor is that the projectile in some cases is affected by the fields of the target in a way which distorts its original status of internal motion. The projectile has originally a specific binding energy as a result of the kinetic energies of the substructures and the potential in between. If an external force acts differently on these substructures, their kinematic conditions will change, resulting
Fig. III.39. The projection of the classical trajectory of the 3-body system on the cartesian planes.
in a new form of orbiting each other with a new angular frequency and new maximum separation distance. A simulation was carried out on a high precision calculation (with 2-3 eV change of the total energy) and shows that the Li projectile is stretched to maximum distance \( \sim 6 \text{ fm} \) from each other and produces a new orbit of the individual clusters inside the projectile. Fig. III.39 a, b, c shows the trajectories in the Cartesian planes x-y, y-z, z-x, respectively.

There are two reasons which cause such ejectile cases to emerge in this way:

1. The projectile itself has high incident momentum and eventually its substructures keeping their direction forward.
2. The attractive force, even being very small between the two clusters, extends to that distance (6.1 fm) in between.

This makes the recombination possible even after 30 fm from the target. This leads us to try to find a relation between the ejectile energy of its mass centre and its maximum separation distance \( R_{\text{max}} \) in terms of a correlation between both. Since \( R_{\text{max}} \) could be a measure for the distortion of the ejectile, Fig. III.40 shows that for one energy value (69 MeV) the \( R_{\text{max}} \) could vary widely. The general trend with low number in each cell in which, as \( R_{\text{max}} \) increases, the value of the energy decreases. The ratio of those ejected with energy different from 69-70 MeV range was \( \sim 15\% \). This idea of projectile recovery seems in agreement with earlier explanations by Thompson et al. (Tho 83) to account for their over-estimated results of break up of \(^7\text{Li}\) using the DWBA calculations.
\[ ^{208}\text{Pb} (^7\text{Li} (70 \text{ MeV}), ^7\text{Li}) \]. Without absorption.

Fig. III. 40. The correlation between the energy of the scattered \(^7\text{Li}\) and the maximum separation distance between its two clusters.
CHAPTER IV

QUANTUM MECHANICAL MODELS

IV.1 Introduction

The exact quantum mechanical treatment for the 3-body stationary problem is well known formally in terms of the Faddeev equations, which have been introduced in Chapter I. Practically, these equations are very difficult to use for heavy ion reactions due to the large number of waves and states involved in the reaction. For direct break up, the problem becomes even more complicated due to the infinite number of ways the ejectiles emerge. Several authors proposed simpler approaches to the problem, such as Matsuoka et al. (Mat 80a), who used the plane Born approximation, and Aartz et al. (Aar 82), who used the quasi free model based on the spectator-participant model for the projectile, and the distorted wave Born approach by Baur et al. (Bau 76).

In this chapter we extend the application of some of these models to heavier non S-state projectiles such as $^7\text{Li}$, in which the alpha and the triton clusters have $\ell=1$(the relative angular momentum).

The triple differential cross section for elastic break up in the laboratory has the general form (Mei 85),

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = \frac{2\pi ma}{\hbar^2 k_{a}} |T_{f1}|^2 \rho(E_1)$$  (IV.1)
where \( m_a \) is the reduced mass of the projectile approximated to the mass of the projectile itself;

\( \hbar k_a \) is the relative momentum between the projectile and target;

\( T_{fi} \) is the transition probability from the initial to the final system;

\( \rho(E_i) \) is the number of possible states for the final system in phase space, in which particle 1 has energy between \( E_i \) and \( E_i + dE_i \) and is emitted in the direction within the solid angle \( d\Omega_1 \) and the other ejectile inside the solid angle \( d\Omega_2 \) at energy \( E_2 \) determined by the conservation laws of energy and linear momentum.

The transition amplitude carries the physics of the process (often considered on a stationary basis). There are two basic approaches (Sat 80) to calculate \( T_{fi} \) and these are considered below.

1. **Perturbative approach**

The initial and final states of the system are described by the unperturbed eigenfunctions, and the perturbation is selected to be the interaction potential for the process

\[
T^{\text{Post}}_{fi} = \langle x_f^- | W_f | x_i^+ \rangle \quad \text{(IV.2)}
\]

or

\[
T^{\text{prior}}_{fi} = \langle x_f^- | W_i | x_i^+ \rangle \quad \text{(IV.3)}
\]

\( W_i, W_f \) are the interaction potentials in the initial and final states, respectively.
2 Gellmann-Goldberger Approach (Gel 53)

This method is based on taking part of the full channel potential to distort the free particle wave function and the rest remains as interaction potential (Gre66). The transition matrix elements take the form (Bau 76)

$$\tau_{fi}^{\text{prior}} = \langle \psi_f^{-} | V_{Ax} + V_{Ab} | e^{i\vec{q}_i \cdot \vec{R}_a} \psi_a (r_b) \rangle \tag{IV.4}$$

while

$$\tau_{fi}^{\text{post}} = \langle e^{i\vec{q}_b \cdot \vec{r}_b} e^{i\vec{q}_i \cdot \vec{r}_x} | V_{Ax} + V_{Ab} + V_{bx} | \psi_i^+ \rangle \tag{IV.5}$$

where

$$\psi_i^+$$ is the eigenfunction of the initial system, selected with outgoing boundary conditions,

$$\psi_f^-$$ eigenfunction of the final system, selected with ingoing boundary conditions.

$$e^{i\vec{q}_i \cdot \vec{r}_i}$$ the wave function of the free particle i with respect to the target,

$$V_{ij}$$ is the potential between particle i and particle j,

$$\psi_a$$ is the wave function of the projectile.

The quantum approaches differ by the approximations imposed on the full eigen-functions of the initial and/or final systems, also the interaction potentials considered and the kind of distortion.

IV.2 Plane Wave Born Approximation (P.W.B.A)

IV.2.1 Introduction

The P.W.B.A. is a simple quantum mechanical calculation that is useful to study surface reactions, and it also gives some insight into the process under study. For the P.W.B.A. the role of
momentum and angular momentum is clearer than for the more complicated quantum methods since the transition matrix is easily factorized. This method is used by Matsuoka et al. (Mat. 80a) in studying the elastic break up of $^3$He (90 MeV) on light and medium targets $^{12}$C, $^{51}$V, $^{90}$Zr. They obtained an overall quantitative agreement for inclusive and exclusive data. Later Meijer et al. (Mei 85) applied the same model at lower projectile energy and a different target $^{28}$Si with similar success. So, our aim here is to extend the application of this model for heavier non-S state projectile such as $^7$Li. The main assumption of this approach is the relative motion between the clusters and the target in the outgoing channel and between the projectile and target are approximated to plane waves, assuming the Coulomb and nuclear distortions to be negligible. The assumption could be justified by considering the elastic break up process to take place just outside the target nucleus, i.e. at the tail of the nuclear potential where the total potential is small and the conditions of the plane Born approximation can be met. In fact, the results of the classical trajectory model show that the surviving break up events mainly take place around the strong interaction radii of the clusters with the target.

![Vector diagram of the 3-body system used in quantum mechanical calculations.](image)
To obtain the cross section for direct elastic break up, one should calculate the transition matrix element. The vector diagram for the 3-body system is shown in Fig. IV.1.

### IV.2.2 Formalism

Considering the perturbative approach in post form, the wave functions of the incident channel is

\[ |X_{i}^{+}⟩ = e^{i \mathbf{k}_{a} \cdot \mathbf{R}_{a}} \psi_{a}(\mathbf{r}_{bx}, \xi_{b}, \xi_{x}) \psi_{A}(\xi_{A}) \]  

where \( \mathbf{k}_{a} \) in the number of the projectile 
\( \mathbf{R}_{a} \) is the position vector of the projectile with respect to the target 
\( \psi_{a} \) is the wave function of the projectile system \( (a) \) and could be expanded as

\[ = (C^{2S})^{\frac{1}{2}} \psi_{a}(\mathbf{r}_{bx}) \psi_{b}(\xi_{b}) \psi_{x}(\xi_{x}) + \ldots \]

\( (C^{2S})^{\frac{1}{2}} \) is the spectroscopic amplitude for the substructures \( b, x \) in the projectile, 
\( \mathbf{r}_{bx} \) relative spatial vector between \( b, x \) particles,

\( \psi_{b}(\xi_{b}) \) intrinsic wave function of the cluster \( b \) with \( \xi_{b} \) representing all the internal coordinates of \( b \).

\( \psi_{x}(\xi_{x}) \) intrinsic wave function of the cluster \( x \) with \( \xi_{x} \) representing all the internal coordinates of \( x \).

\( \psi_{A}(\xi_{A}) \) intrinsic wave function of the target, with \( \xi_{A} \) representing all the internal coordinates of \( A \).

while the final channel wave function

\[ \langle X_{f}^{-} | = e^{-i \mathbf{k}_{b} \cdot \mathbf{R}_{b}} e^{-i \mathbf{k}_{x} \cdot \mathbf{R}_{x}} \psi_{b}(\xi_{b}) \psi_{x}(\xi_{x}) \psi_{A}(\xi_{A}) \]  

(IV.7)
where
\[ \vec{k}_b, \vec{k}_x \] are the wave numbers of the clusters \( b, x \) respectively,
\[ \vec{R}_b, \vec{R}_x \] are the vector position of the clusters \( b, x \) with respect to the c.m. of the (target-other cluster) respectively,
\[ \psi_b(\xi_b), \psi_x(\xi_x), \psi_A(\xi_A) \] are the same as the incident channel.

The interaction is taken to be the full potential between the two clusters \( b, x = V_{bx} \). The transition matrix element then has the form
\[ T_{\text{post}} = \int \int \int \int \int e^{ik_a \cdot R_a} e^{-i\vec{k}_b \cdot \vec{R}_b} e^{-i\vec{k}_x \cdot \vec{R}_x} \psi_b(\xi_b)\psi_x(\xi_x)\psi_A(\xi_A) \]
\[ V_{bx}(\vec{r}_{bx})\psi_a(\vec{r}_{bx}, \xi_b, \xi_x)\psi_A(\xi_A) d\vec{r}_a d\vec{r}_b d\vec{r}_x d\xi_b d\xi_x d\xi_A. \]

The description of the projectile wave function \( \psi_a \) is restricted to the first main cluster term. Therefore it is assumed that the other possibilities for the projectile substructures are not too important. In this way it is possible to reduce the complexity of the problem and the calculations. This assumption is equivalent to setting \( (C^A, S) = 1 \). Since the potential is independent of any of the internal coordinates of the 3-bodies under study, the integration over the intrinsic wave functions "by orthogonality" = 1.

For heavy targets \( \vec{R}_x \cong \vec{r}_{xA} \) and \( \vec{R}_b \cong \vec{r}_{bA} \), so the following transformations can be used
\[ \vec{r}_{xA} = \vec{R}_a - \frac{m_b}{m_a} \vec{r}_{bx} \]
\[ \vec{r}_{bA} = \vec{R}_a + \frac{m_x}{m_a} \vec{r}_{bx} \]

(IV.8) (IV.9) (IV.10)
The transition matrix element now reduces to the factorized form:

\[
T_{\text{post}}^{f_1} = \int d\vec{R}_a e^{i\vec{Q} \cdot \vec{R}_a} \int d\vec{r}_b V_{bx}(\vec{r}_b) \psi_a(\vec{r}_b)e^{-i\vec{q} \cdot \vec{r}_b} \quad (IV.11)
\]

= \bar{T}_1(\vec{Q}) \bar{T}_2(\vec{q})

where

\[
\vec{Q} \text{ is the momentum transferred to the target}
\]

\[
\vec{q} = \vec{k}_a - \vec{k}_b - \vec{k}_x \quad (IV.12)
\]

\[
\vec{q} \text{ is the relative momentum of the two clusters inside the projectile}
\]

\[
= \frac{m_x}{m_a} \vec{k}_b - \frac{m_b}{m_a} \vec{k}_x \quad . \quad (IV.13)
\]

If the 1st integral is carried out over all R-space it leads to \(\delta(\vec{Q})\), which is only finite for a value of \(\vec{Q} = 0\). Such a situation cannot apply for the break up process due to the energy required for dissociation. As shown in the classical trajectory calculation as well as quantum calculations, for the reaction under study (see Fig. III.4), projectiles with impact parameter \(b < 6.0\) fm are almost absorbed totally. This result may be used to set a criterion that \(R_a\) is limited to a value greater than the critical fusion radius defined in Table II.2. Accordingly the integration over \(R_a\) covers the exterior of a sphere of radius \(R_C = 7.83\) fm in this case. Similar criteria were used by Matsuoka (Mat 80a),

\[
\therefore \bar{T}_1(\vec{Q}) = \int_{R > R_C} e^{i\vec{Q} \cdot \vec{R}} d\vec{R} = -4\pi R_C^2 j_1(QR)/Q . \quad (IV.14)
\]
The second integral includes the ground state wave function of the two clusters $b, x$ in the projectile and is taken to be

$$\overline{r_{bx}} = \overline{r}$$

$$\psi_{a}(\overline{r}) = \frac{1}{\sqrt{3}} \sum_{m=-1}^{1} R_{1}(r) Y_{1,m}(\Omega_{r})$$  \hspace{1cm} (IV.15)

$R_{1}(r)$ is the radial wave function normalized to unity

$Y_{1,m}(\Omega)$ is the angular wave function "spherical harmonics".

In the expression for $T_{2}(q)$ (eqn. IV.11) $e^{i q \cdot \overline{r}}$ is expanded as:

$$e^{i q \cdot \overline{r}} = 4\pi \sum_{\ell'=-\infty}^{\infty} \sum_{m'=-\ell'}^{\ell'} (i)^{\ell'} j_{\ell'}(qr) Y_{\ell',m'}^{*} (\Omega_{r}) Y_{\ell,m} (\Omega_{q})$$

$$\ell'=0 \text{ m'}=-\ell'$$  \hspace{1cm} (IV.16)

where $j_{\ell'}(qr)$ is the spherical Bessel function.

If the Woods-Saxon potential between $a, t$ clusters is taken for $V_{bx}$ (with parameters defined in Table II.1), then $T_{2}(q)$ has the form:

$$T_{2}(q) = \frac{4\pi}{\sqrt{3}} \sum_{\ell'=0}^{\infty} \sum_{m'=-\ell'}^{\ell'} \frac{1}{m=-1} (i)^{\ell'} \int j_{\ell'}(qr) R_{1}(r) V(r) r^{2} dr$$

$$\times \int Y_{1,m}(\Omega_{r}) Y_{\ell',m'}^{*} (\Omega_{r}) d\Omega_{r} \cdot Y_{\ell,m}(\Omega_{q})$$

$$= \frac{\pi}{\sqrt{3}} \int j_{1}(qr) R_{1}(r) V(r) r^{2} dr \cdot i \sum_{m=-1}^{1} Y_{1,m}(\Omega_{q})$$

$$= \int j_{1}(qr) R_{1}(r) V(r) r^{2} dr \cdot \frac{4\pi i}{\sqrt{3}} \left[ \sqrt{\frac{3}{4\pi}} \cos q \right]$$

$$- 2i \sqrt{\frac{3}{2\pi}} \sin q \sin q$$  \hspace{1cm} (IV.17)
IV.2.3 Calculations and results

The integral is evaluated numerically between \( r = 0 \) to \( r = 15 \) fm to cover the entire significant range.

The triple differential cross section is given by:

\[
\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = \frac{2}{\hbar} \frac{m_e}{P_a} |T_{\text{post}}|^2 \rho(E_1). \tag{IV.18}
\]

\( \rho(E_1) \) is defined by eqn. (II.23).

To obtain the double differential cross section

\[
\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \int \frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} dE_1, \tag{IV.19}
\]

the integration is done numerically, covering all the kinematical range of \( E_1 \).

Fig. IV.2a shows the projected spectrum for the triton as predicted by the plane wave Born approximation and can be compared with the experimental data in Fig. IV.2b, obtained at \( \psi_t = \psi_\alpha = 18.0^\circ \) and \( \Delta \lambda = 5.9^\circ \) (Bic 80). Clearly, the two distributions are different; the theoretical curve has two widely separated peaks. The reason why the cross section falls to almost 0 when \( E_t \sim 30 \) MeV is that the direction of \( q \) becomes normal to the z-axis (\( \theta_q = 90^\circ, \phi_q = 0 \)) for this geometry of detection.

The experimental data, shown in Fig. IV.2b, has a similar feature to the calculated results in that it also has two peaks, but however the experimental widths of the peaks are much narrower than the calculated results. There are two reasons for such differences. First, the calculations use only plane waves for all the wave functions.
(a) using plane born approximation.

\[ { }^{208}\text{Pb}(^7\text{Li},\alpha t) \]

\[ \psi_a = \psi_c = 18.0^\circ \]

\[ \Delta \lambda = 5.9^\circ \]

(b) The experimental data

Fig. IV. 2. The projected energy distribution of the triton using the plane wave approximation.
However, for both ejectiles emerging near the beam \((v_a \sim v_t)\)
the Coulomb forces will distort the outgoing waves. This effect
has already been discussed in Sec. III.4 in connection with the
semi-classical model. The second reason that we should expect a
difference between the calculated and experimental results is that the later is
very strongly influenced by how the detector geometry efficiency
varies with triton energy. For \(E_t\) near the beam velocity this
efficiency is expected to be at a minimum due to the finite separa-
tion between the detectors. For values of \(E_t\) greater and less
than beam velocity, the efficiency will first increase, as the radius
of the kinematic break up sphere becomes greater than the minimum
detector separation distance, and then finally sharply decrease as
the radius of the kinematic sphere becomes larger than the
maximum detector separation distance. This efficiency effect, due
to the finite size of the experimental detector collimators, will
be different than that implicit in the calculated distribution
because the calculation assumed ideal point detectors.

Fig. IV.3 shows the P.W.B.A. predictions for the differential
break up cross section. The experimental results are also shown.
The calculation shows rapid diffraction oscillations which are
not seen in the data. However the calculation corresponds to
point counters, while the data have been averaged by the finite
detector collimators by about 3° in both the vertical and horizontal
directions. Therefore in order that a more valid comparison be made,
the calculated values are averaged over a bin size of 4°, centred
at the angles for the experimental date. These averaged calculation
points are also shown in Fig. IV.3. The agreement between the
Fig. IV.3. The double differential cross section for alpha and triton, the separation angle $\Delta \lambda = 5.9$ deg.
averaged calculated values and the experimental results for angles
greater than 20° is quite reasonable in absolute magnitude and
trend. It is worth bearing in mind that there are no adjustable
parameters in the calculation. For very forward angles (less than
∼ 20°) the Coulomb force is expected to play a large role in the
total break up process. It is known that the phase of this
break up amplitude is opposite to that of the nuclear component.
Therefore since no account is given of Coulomb component in the
P.W.B.A. calculation, a comparison between this calculation and
experiment below ∼ 20° will not be very meaningful.

For a wide angular separation between the break up fragments,
the P.W.B.A. break up calculation produces a bell shaped spectrum
for the projected triton spectrum, Fig. IV.4. The predicted peak
of the cross section has a value of 9000 mb/sr²MeV⁻¹. This is a
much larger value than that corresponding to the closer geometry
shown in Fig. IV.2a. There is some preliminary data for very wide
angular separation (∼ 40°) which suggest that the \( \frac{d^2\sigma}{d\Omega_1 d\Omega_2} \) cross section does indeed increase for modest angles but certainly
not so much as indicated by the 9000 mb/sr²MeV for separation 30°
compared to ∼ 3 mb/sr²MeV for 5.9°. However the wider the angle
between the fragment, the higher the relative momentum and energy
between them. This means that such events are probably associated
with those collisions that involve a large interaction between the
projectile and the target. If this is true then the fragments
will be strongly influenced by the nuclear force which will
obviously distort the outgoing waves, and so invalidates the plane
wave approximation for this situation. The P.W.B.A. calculation
$^{208}$Pb ($^7$Li (70 MeV), αt).

$\psi_t = -15.0^\circ$

$\psi_a = 15.0^\circ$

**Fig. IV. 4.** The projected energy distribution of the triton using the plane wave approximation, for wide angle.
calculation should therefore only be considered for that reaction of the data that corresponds to the "soft" interaction, i.e. where only a small internal energy has been implanted into the projectile.

The angular part of the projectile internal wave function \( l = l \hbar \) affects radically the scattering amplitude and eventually the projected energy spectrum. Fig. IV.5 shows the variation of the square of the angular part of the transition amplitude with the energy of the triton.

Clearly, the minimum at \( E_t = 29 \text{ MeV} \) coincide with the minimum at the projected energy spectrum in Fig. IV.2a.

One may suggest at this point to extend this study to use coupled channel plane Born approximation to know the contributions from higher angular momentum states in the projectile cluster system. The idea of a fully absorbing sharp disc edge with the critical fusion radius, without considering any reflection or refraction to the plane wave describing the relative motion, may be unrealistic, so one could resort to the other methods which include these factors and distort such plane waves.

IV.3 Quasi Free Model

IV.3.1 Introduction

As discussed in Sec. III.2 most direct elastic break up is obtained if the triton (lower charge) passes the target at its strong interaction radius, while the \( \alpha \) (higher charge) is outside the strong interaction radius. This idea is very close to the
$^{208}\text{Pb} (^7\text{Li (70 MeV), at}), \quad \psi_a = \psi_t = 18.0^\circ.$

$\Delta \lambda = 5.9^\circ.$

The energy of the triton (MeV)

Fig. IV.5. The square of the angular part of the scattering amplitude using the plane wave approximation.
quasi free model developed by Aartz (Aar 82). This suggests that the quasi free model non-S state projectile could be extended. The model is reviewed briefly here.

IV.3.2 Formalism

The transition matrix element for direct elastic break up in the prior form is

$$T_{\text{prior}} = \langle \chi_f^- | W_1 | \chi_i^+ \rangle \quad \text{(IV.20)}$$

where

$$| \chi_i^+ \rangle = | \phi_a \psi_a \psi_A \rangle$$

$$\phi_a(\vec{k}_a')$$ is the projectile wave which is approximated from Coulomb wave to plane wave with local Coulomb corrected momentum $\vec{k}'$ at the Coulomb distance of closest approach.

$$\psi_a, \psi_A$$ are the wave functions of the projectile and the target.

$$\langle \chi_f^- \rangle = \langle \phi_b \chi_x \psi_b \psi_x \psi_A \rangle .$$

$$\phi_b(\vec{k}_b')$$ is the Coulomb wave of the spectator "b" approximated to plane wave with local Coulomb corrected momentum $\vec{h}k_b'$.

$$\chi_x$$ is the elastic optically scattered wave of the participant.

$$\psi_x$$ is the intrinsic wave function of the participant.

If the interaction potential is taken as

$$W_1 = V_{\chi_A} (\vec{r}_{\chi_A}) - \frac{Z_x}{Z_a} V_{aA} (\vec{r}_{aA}) \quad \text{(IV.21)}$$
The transition matrix element becomes

\[ T_{fi}^{QF} = \int \, d\mathbf{x}_A \, x^*(-k_x) \left[ V_{xA}(r_{xA}) - \frac{Z_x}{Z_a} \psi_a(r_{xA}) \right] \times \exp[i(k'_{aA} - k'_{bA}) \cdot r_{xA}] \]

\[ \times \int \, d\mathbf{r}_b \, \psi_a(\mathbf{r}_b) \exp[-i [k'_{bB} - \frac{m_b}{m_a} k'_{aA}] \cdot \mathbf{r}_b] \]

\[ = T_1 \times T_2(q') \]

where

\[ q' = k'_{bA} - \frac{m_b}{m_a} k'_{aA} \]

\[ k'^2 = k^2 - 2u_{12} Z_1 Z_2 e^2/h^2 R_{CA} \]

\[ \frac{\mathbf{k} \cdot \mathbf{k}'}{kk'} = \cos(\tan^{-1}(Z_1 Z_2 e^2/2E_x b_{CA})) \]

\[ \mathbf{k}' \]

is the local momentum at the distance of closest approach \((R_{CA})\) and impact parameter \((b_{CA})\).

\(Z_1, Z_2, \mu_{12}\)

are the atomic numbers and reduced mass of particles 1, 2.

The first transition matrix \(T_1\) represents the scattering of the incoming local Coulomb corrected plane wave to elastic wave via the interaction \(V_{xA}(\mathbf{r}_{xA}) - \frac{Z_x}{Z_a} \psi_a(\mathbf{r}_{xA})\) where \(\mathbf{r}_{xA}\) is an approximation to the \(\mathbf{r}_{aA}\) in the second term. This interaction vanishes at large distances, i.e. there is no contribution to the scattering amplitude from partial wave with large angular momenta. At small distances the interaction \(V_{xA}\) is considered to be the
the dominant component and gives the scattering amplitude of elastically scattered triton on the target.

This situation imposes the condition of cut off for large values of angular momenta and this can be achieved by introducing a smooth cut off function \((1 - |\eta_\ell|^2)\) (see Sec. III.2) within the transition matrix

\[
T_1 = \frac{i\hbar^2}{T} \sum_\ell (2\ell+1) P_\ell(\cos\theta)(1 - |\eta_\ell|^2)[e^{2i\delta_\ell} - 1] \tag{IV.26}
\]

where

- \(\delta_\ell\) is the total (Coulomb + Nuclear) phase shift.
- \(\eta_\ell\) is the reflection coefficient.

The transition amplitude in a similar way to the plane Born approximation leads to the final form

\[
T_2(q') = \sqrt{4\pi} i \int j_\ell(q'r)\psi_a(r)r^2dr \left[\cos\theta_{q'} - \sqrt{2} i \sin\theta_{q'} \sin\phi_{q'}\right] \tag{IV.27}
\]

**IV.3.3 Calculations and Results**

The calculations with local Coulomb corrected momentum reduce the dynamical range of the ejectiles, since there are limits for the kinetic energy of one cluster to be able to reach the distance of closest approach from the target which has been specified from the trajectory calculations. The values of \(R_{CA}\) for the spectator is \(R = 13\) fm and for the participant \(R = 10.3\) fm.

The first integral of the transition amplitude is calculated from the phases obtained for the best fit to the elastic data of the
triton at mean energy 33 MeV as given in ref. (Coh 85). The second integral of the scattering amplitude is calculated numerically, and the cross section is finally obtained in eqn. (IV.18).

Fig. IV.6 shows the values of the cross section at the dynamically accepted range of energy of the participant. The comparison of this shape with the experimental distribution, Fig.IV.7 shows also the angular distribution for small-angle break up. Again the calculation and experiment do not agree. It may be that the lack of elastic data for the participant (t) over the dynamical range, that is relevant, is not adequate enough to describe the participant behaviour.

In summary, one of the main difficulties is determining the correct potentials for the various fragment energies. In the absence of specific experimental data it may be worthwhile to investigate this model using a global potential.

IV.4 Distorted Wave Born Approximation

IV.4.1 Introduction

The D.W.B.A. calculation has been applied to situations involving break up of projectiles of mass number A < 4 (Bau 76, Bau 84). These calculations have been reasonably successful but they normally involve the zero range approximation, i.e. \( V_{bx} = V_o \delta(\vec{r}_b - \vec{r}_x) \). This approximation might be justifiable if the clusters of the projectile are in a relative S-state (i.e. \( \ell = 0 \) for the relative orbital angular momentum of the clusters) but the approximation is unjustifiable for higher relative \( \ell \) states. For such situations (e.g. \(^7\text{Li}, \alpha\) and \(t\) cluster have \( \ell = 1 \)) the finite size of the cluster system and therefore the interaction should be considered.
\[ ^{208}_{\text{Pb}}(^{7}\text{Li}(70 \text{ MeV}), \alpha) \]

\[ \Delta \lambda = 5.9^\circ \quad \quad \psi_e = \psi_c = 18.0^\circ \]

Fig. IV. 6. The projected energy distribution of the triton using the quasi free model.
The quasi free.

The experiment

Fig. IV.7. The double differential cross section for alpha and triton, the separation angle $\Delta \lambda = 5.9$ deg.
IV.4.2 Formalism

Recently Shyam et al. (Shy 85) have discussed a method to extend the D.W.B.A. calculations to include the finite range of the interaction. They include the finite range effects of the interaction, by using a method similar to Braun-Munzinger and Harney (Bra 74) which depends on the local momentum approximation. Each wave function is then factorised in terms of a local coordinate (covering the interaction), and another covering the particle target coordinate. For this calculation the resulting differential cross section for elastic scattering has the form:

\[
\frac{d^3\sigma}{d\Omega_x d\Omega_b dE_x} = 2\pi(\mu_x \mu_b \mu_a / (2\pi\hbar)^6)(k_x k_b / k_a) \sum_{\ell m} |\beta_{\ell m}|^2
\]

where \(\mu_x, \mu_b, \mu_a\) are the reduced masses of the spectator, participant and

\(k_x, k_b, k_a\) are the wave numbers of the spectator, participant and the projectile, respectively,

\(\beta_{\ell m}\) is the elastic amplitude for angular momentum state and its projection \(m\) of the projectile. This amplitude could be expressed as

\[
\beta_{\ell m} = ((4\pi)^{5/2} / k_a k_b) \sum_{\ell_a} \sum_{\ell_b} \sum_{\ell_x} \sum_{\Gamma M} L_a - L_b - L_x + \ell_a + \ell_b - \ell_x \gamma_{\ell_a \ell_b \ell_x} (\Omega_{k_b}, \Omega_{k_x}) (2L_a + 1)(2L_b + 1)^2(2L_x + 1)^2 (2\ell_a + 1)^2(2\ell_b + 1)^2(2\ell_x + 1)^2 (2\Gamma + 1)^{-1}
\]
\[ \langle \ell_a M_r - m_z \ell m_z \mid \Gamma \ell \rangle \langle \ell_a 0 \ell - 0 \mid \Gamma 0 \rangle \]

\[ \langle \ell_b 0 \ell x 0 \mid \Gamma 0 \rangle \langle \ell_b 0 \ell x 0 \mid \Lambda a 0 \rangle \]

\[ \langle \ell x 0 \ell x 0 \mid \ell x 0 \rangle \langle \ell_b 0 \ell b 0 \mid \ell b 0 \rangle \]

\[ \langle \ell a 0 \ell a 0 \mid \ell a 0 \rangle \]

\[
\begin{bmatrix}
\ell_b & \ell_x & \ell_a \\
\ell_b & \ell_x & \Lambda a \\
\ell_b & \ell_x & \Gamma
\end{bmatrix}
\times 
\]

\[ \times G^b_{\ell_b \ell x \ell a} (K_b, K_x, K_a) \]

\[ \times R_{\ell_b \ell x \ell a} (k_b, k_x, k_a) \]

where \[ \mathcal{F}_{\ell_b \ell x} (\hat{k}_b, \hat{k}_x) \] is the double spherical harmonics and is given by

\[
= \sum_{\nu_x} \langle \ell_b - \nu_x \ell_x \nu_x \mid \ell a 0 \rangle \\
\times Y_{\ell_b, -\nu_x} (\hat{k}_b) Y_{\ell x, \nu_x} (\hat{k}_x) 
\]

Each chevron bracket represents Clebsch-Gordan coefficient, \[ Y_{\ell, m} \] is the spherical harmonics, the curly bracket contains the 9-j coefficient, all are defined in ref. (Bri 75). \[ x \] refers to the participant (triton), \[ b \] refers to the spectator (\[ \Psi \]),
is the finite range factor

\[ G_{\ell_a, \ell_x, \ell_b} (K_b, K_x, K_a) \]

\[ = \int r_1^2 \, dr_1 \, j_{\ell_b} (M_x/(M_A + M_x) K_b \, r_1) \, j_{\ell_x} (K_x \, r_1) \]

\[ V(r_1) \, U_{\ell_a} (r_1) \, j_{\ell_a} ((M_x/M_A) K_a \, r_1) \]

\[ j \]

is the spherical Bessel function

\[ \ell_a, \ell_x, \ell_b \] are the angular momentum indices

\[ V(r_1) \]

is the interaction potential between \( \alpha \) and \( \tau \) in the projectile \((a)\)

\[ U_{\ell_a} \]

is the bound state radial wave function between the two clusters \( \beta, x \) in the projectile \((a)\) in a state of angular momentum \( \ell \).

\( \hbar K_x, \hbar K_b, \hbar K_a \) are the local momenta at the closest distance of approach from the target, as defined in Sec. IV.3.2.

\( M_x, M_b, M_a, M_A \) are the masses of the participant, spectator, projectile and the target, respectively.

\[ R_{L_b, L_x, L_a} \]

is the radial integral which represents the overlap of the wave functions of the reactants and products with respect to the target.

\[ = \int dr \, \chi_{L_b} (k_b, (M_A/(M_x + M_A)) r) \, [\chi_{L_x} (k_x, r)/k_x r] \, \chi_{L_a} (k_a, r) \]

where \( \chi_{L_b}, \chi_{L_x}, \chi_{L_a} \) are the radial functions of the elastically scattered waves representing the participant, spectator and the projectile on the target.
L_x, L_b, L_a are the orbital angular momenta of the fragments and the projectile with the target.

\[ \mathcal{L}_x, \mathcal{L}_b, \mathcal{L}_a \]

are the coupled angular momenta,

\[
\begin{align*}
\mathcal{L}_x &= \bar{L}_x + \bar{\ell}_x \\
\mathcal{L}_b &= \bar{L}_b + \bar{\ell}_b \\
\mathcal{L}_a &= \bar{L}_a + \bar{\ell}_a
\end{align*}
\]

The angular momentum coefficients provide the necessary selection rules to reduce the amount of calculations.

To evaluate this expression for the differential cross section requires considerable C.P.U. time. The main reason for this is that for high projectile energies all the particles participating in the break up reaction will have large orbital angular momentum \( L_a \) values. The calculation therefore has to span a considerable portion of the 3-D \( L_x, L_a, L_b \) angular momentum phase space.

I have made some preliminary investigations as to the feasibility of evaluating the above cross section for the \( \text{Pb}(^7\text{Li} \rightarrow \alpha + t) \) break up reaction.

IV.4.3 Calculations and Results

The calculation of the finite range factor is relatively straightforward. The \( \alpha-t \) interaction potential is taken to be Woods-Saxon shape with parameters given in Table II.1. To give an idea as to the value of these integrals, Table IV.1 gives the values of the integrals for a somewhat arbitrary selection of \( \alpha \) values and energy for the outgoing triton (the \( \alpha \) energy is directly related to the triton energy).
To investigate the range of $\ell$ values that must be considered at each outgoing trion energy, the value of $\ell_a$ was first selected, and then the integrals were evaluated and inspected for a range of values for $\ell_b$ and $\ell_x$. From this inspection an upper limit for $\ell_b$ and $\ell_x$ could be defined in such a way that the integral value of $G$ would be less than $10^{-7}$ for any values of $\ell_b$ and $\ell_x$ above the limits. This process was repeated for $\ell_a = 0, 1, 2, 3, \ldots$.

From Table IV.2 it can be seen that the limits for $\ell_x, \ell_b$ are large. Typically to evaluate the cross section for a particular $E_x$ would require the evaluation of $\sim 200$ integrals.

For a particular value of $E_x$ the evaluation then proceeds by calculating the radial integrals $R_{\ell_x, \ell_b, \ell_a}$. The total number of these integrals needed depends upon the range of $\ell_x, \ell_b, \ell_a$ that are...
important in the reaction. Fortunately the break up reaction is confined closely to the peripheral surface interaction region between the two ions. This therefore naturally restricts the $L$ values. Even so, the spread in the significant $L$ values is such that the total number of integrals $R_{L_x,L_b,L_a}$ needed to be evaluated is of the order of 150,000. The evaluation of this number of integrals is clearly the difficulty to evaluation of the cross section. However to investigate how the total radial dependence depends on the partial wave of the projectile, the quantity $\sum_{L_x,L_b} |R_{L_x,L_b,L_a}|^2$ was evaluated as a function of $L_a$.

This quantity is shown in Fig. IV.8. For this projectile energy the grazing angular momentum is 50 $\hbar$. It is clearly seen that break up probability peaks at the grazing angular momentum but has a considerable spread about this value. For the results of this calculation to be compared with the experimental data a range of

<table>
<thead>
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<th>$L_a$</th>
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</tbody>
</table>

Table IV.2
Largest values of $L_a$, $L_b$, $L_x$ for $|G_{L_a,L_b,L_x}| < 10^{-7}$
The projectile orbital angular momentum ($h$).

Fig. IV.8. The sum of the square of the radial integrals dependence on the projectile orbital angular momentum.
values for $E_x$ must be considered. It is estimated that this would involve a huge amount of computing time, since for each $E_x$ all the functions would have to be re-evaluated. A further difficulty is that the functions $\chi_a, \chi_b, \chi_x$ vary with the energy $E_x$. It is therefore necessary to have accurate elastic scattering data to calculate the radial $\chi$ functions.

In summary, although the finite calculation is one of the best in principle, the practical evaluation involves enormous difficulties. The successful application of this calculation must wait for better universal elastic data and most important of all, bigger and faster computers.
In the field of heavy-ion reactions there is no one universal theory that takes account of all the different phenomena that can be inferred from the experimental data. Break up of the projectile is one type of phenomena that is at present attracting considerable interest. This is not only because the break up process is interesting in its own right but it is also now realised that the break up process can have a strong influence on other reaction channels. Most of the current theoretical research into break up reactions is based on a purely quantum mechanical theory. However to date there is no completely satisfactory quantum calculation because of the complexity of dealing with three fragments interacting through finite nuclear forces. This complexity mainly arises due to the large orbital angular momenta of the projectile and fragments with the target, which in turn necessitates the evaluation of a huge number of radial integrals - this presents a formidable computing task. This difficulty associated with the quantum mechanical approach encourages the search for other methods to calculate the reaction process. Now, the very reason why the quantum approach is difficult, i.e. large \( L \) values, means that a semi-classical approach to the problem may be justifiable. Indeed, if the Sommerfeld parameter for the projectile and target is much greater than 1, then the evolution of the central coordinates of the projectile quantum wavepacket will correspond to that predicted by the classical equation of motion. Therefore, in view
of the difficulties encountered by the quantum mechanical calculation, it seems well worth while to investigate the break up reaction with classical dynamics for the specific case where the Sommerfeld parameter is large. This I have done, and this work forms the main thrust of this thesis.

In Chapter I, a review is presented of the various experiments and theoretical calculations undertaken to study the break up reaction. A review of classical and semi-classical type calculations that have been used to parameterise heavy-ion reactions is given in the first part of Chapter II.

In the second part of Chapter II it is shown how the classical model may be constructed to study the direct break up phenomenon based on the classical trajectory of the clusters of a composite projectile. The motion of these cluster particles is governed by the real part of the elastic optical potential of each cluster with the target, and the Woods-Saxon potential for the two clusters inside the projectile. The absorption of the individual clusters has been considered semi-classically by considering the imaginary part of the same elastic potential with the target. This absorption is calculated from the transmission factor representing the probability of survival for the cluster to pass the nuclear region.

Several numerical methods to calculate the cluster trajectories have been tested. It was found necessary to evolve a more sophisticated technique to integrate the motion of the 3-body system than had previously been used. The integration time step is controlled in such a way as to reduce the number of steps required to cover
a fixed trajectory for a certain degree of accuracy.

The calculations were checked during the development and testing stages of the program by the following methods:

1. A search for any deviations from the conservation laws.
2. A check to see if the trajectory of the centre-of-mass of the 3-body system is a straight line, and the speed of the centre-of-mass is constant.
3. At the end of the trajectory, it is reversed back to its starting point by reversing the motion of the 3-body system. A check is made to determine if the original starting point is recovered.

These tests were performed for a range of kinematic conditions so as to ensure the accuracy of the numerical methods over the complete phase space associated with these reactions.

In Chapter III the results of the semi-classical calculations are presented in some detail. The following general conclusions were drawn. The direct break up and the partial fusion processes occur at certain ranges of the impact parameter. The partial fusion takes place at relatively lower values for the impact parameter than the break up process. This means the partial fusion takes place deeper inside the surface. The absorption of the clusters does not affect the shape of most of the distributions for projectile elastic scattering or partial fusion processes, but for break up the role of absorption is more essential. For break up, the process can be divided into two main components corresponding to low and high relative energy between the break
up ejectiles. The component corresponding to high relative energy almost vanishes due to the high probability of cluster absorption. This absorbed component becomes part of the break up fusion reaction in which the break up takes place first, and is then followed by the absorption of one cluster.

Low energy break up proceeds by two mechanisms. In the first mechanism the two clusters are interacting with the nuclear field of the target. This occurs at lowest impact parameter values $b = 8 - 8.5$ fm. For the second mechanism only one cluster interacts with the nuclear field. (This second mechanism is reminiscent of the spectator, participant model).

It is shown in Chapter III that the results from the semiclassical calculation are in reasonable agreement with a substantial body of experimental data. In particular the angular distribution for the reaction considered ($^7$Li + α + t at $E = 70$ MeV), and for where there is a small angle between the break up fragments, gives reasonable agreement with the experimental values. The most remarkable thing about this calculation is that it contains no free parameter and yet it gives an absolute cross section in agreement with the data. The projected energy spectra of the fragments are also in fair agreement with the experimental results. They peak at the same energy, but the calculation gives somewhat narrower distributions than measured experimentally.

Those break up events from the calculation corresponding to wide separation angle between the fragments, again were in agreement with the limited measured data. This agreement is particularly
significant because it gives some justification to the procedure followed for the transmission of the fragments through the interior region of the target. The calculation yields elastic scattering and partial fusion cross sections as well as break up cross sections. It is found that the magnitude of the calculated elastic cross section is in agreement with the experimental value at the grazing angle. Again it is emphasized that there are no adjustable parameters in the calculation. This means that both the break up and elastic scattering absolute cross sections are correctly reproduced by the same set of parameters for the interaction. The comparison of the partial fusion reaction channel between experiment and calculation, is difficult because the experimental partial fusion is inferred from inclusive measurements. These inclusive measurements involve all reaction processes, whereas the simulation results are concerned only with the partial fusion component. An indication that the calculation does not include all processes, is that the calculated partial fusion yield tends to be less than the experimental value.

For the specific break up reaction studied in this thesis, the semi-classical calculation yields results that are in reasonable agreement with the experimental measurements. This is a significant result because it means that such a calculation would be applied to other types of complex reactions, at least for those situations where the Sommerfeld parameter is much greater than unity. The value of this type of calculation is that in principle it is straightforward, and because of this, it is capable of giving real insight into complex reaction mechanisms since it is possible to
"see" what is happening on the trajectory for a particular final result, i.e. break up, elastic scattering, etc. This is almost impossible to do for a quantum mechanical type calculation where there are several particles in the final channel. For example, in the semi-classical calculation, if the trajectories are inspected for those events corresponding to break up, then it is found on average that the t cluster comes closer to the target than the a cluster. However, for elastic scattering the reverse holds. Such insight into the dynamics of the reaction mechanism is given for other examples in Chapter III.

Chapter IV deals with a selection of models for break up reactions based on the quantum theory. The first model considered was a plane wave Born approximation where all particles are considered to be plane waves. This type of calculation has been applied to light projectiles such as $^3\text{He}$, where the fragments have zero relative orbital angular momentum, i.e. $\lambda = 0$, so the formulation was extended to apply to this case. It is found that the projected energy spectrum is characterised by two peaks widely separated at 15, 45 MeV, while the experimental results show the separations to be at 25, 35 MeV. The double differential cross section for break up process agrees with the experimental results in a reasonable way for scattering angles $> 20^\circ$. However, the cross section predictions for wide angle break up give unreasonably large values. The main reason for this is probably due to the neglect in the calculation of the interaction of the fragments with the target which becomes more important for wide angle break up.
The model based on quasi free scattering gave cross sections in very poor agreement with the experimental results. One possible reason for this is the lack of experimental elastic scattering data from which the correct optical model potentials can be deduced.

The final quantum mechanical model considered was based on the D.W.B.A. approximation where the distortion of all particle waves is correctly accounted for. For this model it was only possible to evaluate the difficulties of performing a full calculation. It was found that the main difficulty in this type of calculation is the need to calculate a very large number of radial integrals and the sums associated with these integrals. It is concluded that further progress in this type of calculation would need the use of bigger and faster computers.

From the above results of Chapter IV it is concluded that at present it is very difficult to perform a satisfactory quantum mechanical calculation for the break up process. However, Chapter III shows that the semi-classical calculation can yield results in agreement with experiment. This type of calculation is straightforward and because of this, gives considerable insight into the dynamics of the reaction process.

The success of the semi-classical calculation for the fragmentation reaction should encourage other people to apply this type of calculation to other complex reaction situations. It would also be worthwhile investigating the relationship between the classical trajectory calculation and the evolution
of quantum wave packets. Such an investigation may lead to a reaction theory that retains the "picture" aspect of the classical but also takes more account of the wave nature of the reaction particles.
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**APPENDIX I**

**Numerical Equations for the Trajectory Calculations**

The numerical equations to solve the simultaneous set of second order ordinary differential equations for the three methods under test are given in this appendix.

The original differential equations have the form

\[ \vec{r}_i = \vec{F}_i (x_1, y_1, z_1, x_2, \ldots, z_3, t)/m_i \]  \hspace{1cm} (Al.1)

\( i = 1, 2, 3 \)

where:
- \( i \) is the index which refers to the particle number,
- \( \vec{r}_i \) is the position vector of particle \( i \),
- \( \vec{F}_i \) is the total force exerting on particle \( i \).

This differential equation could be decomposed into the Cartesian representation as follows:

\[ \ddot{x}_i = F_{x_i} (x_1, \ldots, z_3, t)/m_i \]  \hspace{1cm} (Al.2)

\[ \ddot{y}_i = F_{y_i} (x_1, \ldots, z_3, t)/m_i \]

\[ \ddot{z}_i = F_{z_i} (x_1, \ldots, z_3, t)/m_i \]

From now onwards the equations will be shown in terms of the \( x \)-component only, and other components will have the same form.

For the Euler method the numerical equation is

\[ x_{i,n+1} = x_{i,n} + \Delta t \cdot \dot{x}_{i,n} \]  \hspace{1cm} (Al.3)

\[ \dot{x}_{i,n+1} = \dot{x}_{i,n} + \Delta t \cdot \ddot{x}_{i,n} \]

\[ = \dot{x}_{i,n} + \Delta t \cdot F_{x_{i,n}} (x_{1n}, y_{1n}, \ldots, z_{3n}, t_n)/m_i \]
where

- $i$ is the index for particle number
- $n$ is the index for the integration step number.
- $\Delta t$ is the time step, and it is considered constant.

For the Runge Kutta fourth order method (Ald 66)

\[
\begin{align*}
\ddot{x}_{i,n,1} &= F_i(x_{i,n}, y_{i,n}, \ldots, t_n)/m_i \\
\ddot{x}_{i,n,2} &= \ddot{x}_{i,n,1} + \frac{\Delta t}{2} \cdot \ddot{x}_{i,n,1} \\
\ddot{x}_{i,n,2} &= F_i(x_{i,n,2}, y_{i,n,2}, \ldots, t_n + \frac{\Delta t}{2})/m_i \\
\ddot{x}_{i,n,3} &= \ddot{x}_{i,n,2} + \frac{\Delta t^2}{4} \ddot{x}_{i,n,1} \\
\ddot{x}_{i,n,4} &= F_i(x_{i,n,3}, y_{i,n,2}, \ldots, t_n + \frac{\Delta t}{2})/m_i \\
\ddot{x}_{i,n,4} &= \ddot{x}_{i,n,2} + \frac{\Delta t}{2} \ddot{x}_{i,n,1} + \frac{\Delta t^2}{2} \ddot{x}_{i,n,2} \\
\ddot{x}_{i,n,4} &= F_i(x_{i,n,4}, y_{i,n,4}, \ldots, t_n + \Delta t)/m_i
\end{align*}
\]

and finally

\[
\begin{align*}
\ddot{x}_{i,n+1,1} &= \ddot{x}_{i,n,1} + \frac{\Delta t^2}{6} \left( \dddot{x}_{i,n,1} + \dddot{x}_{i,n,2} + \dddot{x}_{i,n,3} \right) \\
&\quad + \Delta t \cdot \dddot{x}_{i,n}
\end{align*}
\]

For the Runge-Kutta-Fehlberg method (Feh 68) the numerical equations have the form:

\[
\begin{align*}
\ddot{x}_{i,n+1} &= \ddot{x}_{i,n} + \frac{\Delta t}{6} \left( \dddot{x}_{i,n,1} + 2\dddot{x}_{i,n,2} + 2\dddot{x}_{i,n,3} \\
&\quad + \dddot{x}_{i,n,4} \right)
\end{align*}
\]
where

\[
x_{i,n+1} = x_{i,n} + \Delta t \cdot \sum_{\mu=0}^{10} C_{\mu} x_{i,n,\mu}
\]

\[
x_{i,n+1} = x_{i,n} + \Delta t \cdot \sum_{\mu=0}^{10} C_{\mu} x_{i,n,\mu}
\]

\[
x_{i,n+1} = x_{i,n} + \Delta t \cdot \sum_{\mu=0}^{10} C_{\mu} x_{i,n,\mu}
\]

\[
\beta_{\mu} \dot{x}_{i,n,\mu}, \quad \mu=1, \ldots, \mu-1
\]

\[
\beta_{\mu} \dot{y}_{i,n,\mu}, \quad \mu=1, \ldots, \mu-1
\]

\[
\dot{x}_{i,n,\mu} = \dot{x}_{i,n,\mu} (t + \alpha_{\mu} \Delta t) =
\]

\[
\dot{x}_{i,n,\mu} = \dot{x}_{i,n,\mu} + \Delta t \alpha_{\mu} \dot{x}_{i,n,\mu}
\]

\[
\dot{x}_{i,n,\mu} = \dot{x}_{i,n,\mu} + \Delta t \alpha_{\mu} \dot{x}_{i,n,\mu}
\]

\[
C_{\mu}, \beta_{\mu}, \alpha_{\mu} \text{ are constants listed in Table A1.1. The leading truncation error is obtained by evaluating } x, \ddot{x} \text{ at two more values of } \alpha_{\mu}, \text{ using the 8th order expansion}
\]

\[
\hat{x}_{i,n,\mu} = \dot{x}_{i,n,\mu} + \Delta t \alpha_{\mu} \ddot{x}_{i,n,\mu}
\]

\[
\hat{x}_{i,n} = x_{i,n} + \Delta t \cdot \sum_{\mu=0}^{12} C_{\mu} \hat{x}_{i,n,\mu}
\]

The constants \( C_{\mu} \) also are listed in Table A1.1. The leading truncation error (LTE) for the velocity component \( \ddot{x} \) of particle \( i \) is

\[
(LTE)_{\ddot{x}_i} = \frac{41}{840} (x_{i,n,0} + x_{i,n,10} - x_{i,n,11} - x_{i,n,12}) \Delta t
\]

and similar one for the coordinate \( x \) of particle \( i \).
TABLE A1.1: FOR THE CONSTANTS OF THE RK7(8) METHOD.

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\[(\text{LTE})_{x_i} = \frac{41}{840} (\ddot{x}_{i,n,0} + \ddot{x}_{i,n,10} - \ddot{x}_{i,n,11} - \ddot{x}_{i,n,12}) \Delta t\] 

(Al.11)

The method is very good in controlling the accuracy level of the calculations, and at the same time the integration step size.
APPENDIX II

RANDOM NUMBER

The pseudo random numbers are produced by the computer and obtained through the evaluation of a periodic function with very long period. Most main frame computers as well as G.E.C. mini-computers with OS400 or OS600 operating systems are able to provide the pseudo random number directly via callable subroutines from the Fortran program. For example, on G.E.C. one should first call SETRAN followed by a call for the subroutine RANDOM to obtain the random number as an output. This procedure is independent of the processing time of the program (CPU time), but other machines like Vax 11/780 need to set seeds for the pseudo random generators. These seeds should be changed for each time of using the program, otherwise the results of calculations will be reproduced. To obtain different random numbers automatically for each run on the computer one can use the time subroutines, e.g. IDATE and SECNDS and convert their outputs to be used as variable seeds to the subroutine RANDU. This helps to prevent the production of repetitive data, since the sequence of pseudo random numbers produced depends on the value of these seeds and the processing time of the program.
APPENDIX III

Coupling of the Clusters and Projectile Motions

To determine the initial boundary conditions of the integration of motion of the 3-body system one should calculate the transformation of the clusters coordinates and velocities inside the projectile to the laboratory frame (xyz). The projectile coordinate system \((x''', y'''', z''')\) has its origin at the centre of mass of the projectile, \(x''' - y'''\) defined as the plane of rotation of the two clusters, i.e. the rotation around the \(z''\)-axis. The position of the centre of mass of the projectile is \((x_c', y_c', z_c')\) with respect to the lab. system. The set of coordinate axes \((x', y', z')\) are parallel to the lab. axes \((x, y, z)\) at the point \((x_c, y_c, z_c)\).

The plane of rotation \(x''' - y'''\) intersects the \(x' - y'\) plane at the \(z'''\)-axis. Assuming the \(a\)-cluster has the initial coordinates \((0, r_1, 0)\) in the \((x''', y''', z''')\) projectile system, and defined in the \((x', y', z')\) system by the position vector \(\vec{r}_1(r_1, \theta_1, \phi_1)\), the \(z'''\)-axis makes an angle \(\pi/2 - \theta_1\) with the \(z'\)-axis. To obtain the coordinates and velocity components of the clusters in the lab. system \((xyz)\) one should perform the following transformations:

1. Rotation by angle \(\left(\frac{\pi}{2} - \theta_1\right)\) about the \(x'''\)-axis, clockwise to the \((x'', y'', z'')\) system, using the transformation matrix

\[
T_1 = \begin{bmatrix}
1 & 0 & 0 \\
0 & \sin \theta_1 & -\cos \theta_1 \\
0 & \cos \theta_1 & \sin \theta_1
\end{bmatrix}
\] (AIII.1)
2. Rotation by $\phi - \frac{\pi}{2}$ about the $z''$-axis clockwise to $(x', y', z')$

system by the transformation matrix

$$T_2 = \begin{bmatrix} \sin \phi & \cos \phi & 0 \\ -\cos \phi & \sin \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(AIII.2)}$$

... total rotational transformation

$$T = T_2 T_1 = \begin{bmatrix} \sin \phi & \cos \phi \sin \theta & -\cos \phi \cos \theta \\ -\cos \phi & \sin \phi \sin \theta & -\sin \phi \cos \theta \\ 0 & \cos \phi & \sin \phi \end{bmatrix} \quad \text{(AIII.3)}$$

3. Transformation from the projectile centre of mass to the laboratory system (xyz)

Fig. AIII.1  Lab. and projectile coordinate system.
As the α-cluster is at position \((0, r_1, 0)\) in the \((x'', y'', z'')\) system, the lab. coordinates will be

\[
\begin{align*}
x_1 &= x_c + r_1 \cos \phi_1 \sin \theta_1 \\
y_1 &= y_c + r_1 \sin \phi_1 \sin \theta_1 \\
z_1 &= z_c + r_1 \cos \theta_1 \\
\end{align*}
\tag{AIII.4}
\]

With the triton at position \((0, r_2, 0)\) in the \((x'', y'', z'')\), the lab. coordinates are

\[
\begin{align*}
x_2 &= x_c - r_2 \cos \phi_1 \sin \theta_1 \\
y_2 &= y_c - r_2 \sin \phi_1 \sin \theta_1 \\
z_3 &= z_c - r_2 \cos \theta_1 \\
\end{align*}
\tag{AIII.5}
\]

Assuming the motion of the projectile parallel to the z-axis in the positive direction, and \(\alpha\) has the velocity components \((\omega r_1, v_1, 0)\) in the projectile system and the triton \((-\omega r_2, -v_2, 0)\), the lab. components are for \(\alpha\)

\[
\begin{align*}
v_{x_1} &= \omega r_1 \sin \phi_1 + v_1 \cos \phi_1 \sin \theta_1 \\
v_{y_1} &= -\omega r_1 \cos \phi_1 + v_1 \sin \phi_1 \cos \theta_1 \\
v_{z_1} &= V + v_1 \cos \theta_1 \\
\end{align*}
\tag{AIII.6}
\]

for the triton

\[
\begin{align*}
v_{x_2} &= -\omega r_2 \sin \phi_1 - v_2 \cos \phi_1 \sin \theta_1 \\
v_{y_2} &= \omega r_2 \cos \phi_1 - v_2 \sin \phi_1 \cos \theta_1 \\
v_{z_2} &= V - v_2 \cos \theta_1 \\
\end{align*}
\tag{AIII.7}
\]

where
\[ r_1 = \frac{m_2 R}{m_1 + m_2} \]
\[ r_2 = \frac{m_1 R}{m_1 + m_2} \]
\[ v_1 = \frac{m_2 v}{m_1 + m_2} \]
\[ v_2 = \frac{m_1 v}{m_1 + m_2} \]
\[ R = r_1 + r_2 = \text{initial distance between clusters} \]
\[ v = \text{initial relative velocity between clusters:} \]
\[ \text{it may have positive value for outward motion,} \]
\[ \text{and negative for inward motion.} \]

\( \omega \) is the angular velocity of the two clusters around their centre of mass.
APPENDIX IV

MOMENTUM DISTRIBUTION

To obtain the momentum distribution using a quantum mechanical method, the first step is to get the spatial wave function

\[ \psi(r) = \frac{1}{2l+1} \sum_{m=-l}^{l} R_{l}(r) Y_{l,m}(\Omega_{r}) \]  

(AIV.1)

which represents the α-\(t\) bound structure of \(^7\)Li in the ground state with binding energy 2.47 MeV with relative angular momentum \(l\), using Code BOUND (Sm169) to obtain the radial wave function which fits the binding energy. Fig. AIV.1 shows the square of the relative wave function between the two clusters and the probability distribution as functions of the relative distance between the α,\(t\) clusters. The classical range of this distance is limited between 0.6 fm and 4.1 fm which covers only 75% of the total probability. The total probability is checked by extending the integral limits of \(r\) between 0.0 and 15.0 fm and it was found to be \(1.00 \pm 10^{-6}\).

The second step is to obtain the momentum wave function by calculating the Fourier transform of the spatial wave function, carried as follows for a single component of the spherical harmonics.

\[ \mathcal{Y}(\vec{k}) = \frac{1}{\sqrt{3}} \sum_{m_{k}=-1}^{1} \psi(k) Y_{l,m_{k}}(\Omega_{k}) \]  

(AIV.2)

Substituting for the plane wave expansion (Mes 62)
\[ e^{i \mathbf{k} \cdot \mathbf{r}} = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (i)^{\ell} j_{\ell}(kr) Y_{\ell}^{m}(\Omega_{r}) Y_{\ell}^{-m}(\Omega_{r}) \quad \text{(AIV.3)} \]

One can get,

\[
\frac{1}{\sqrt{3}} \sum_{m_1=-1}^{1} \psi(k) Y_{1, m_1}(\Omega_{k})
\]

\[
= \frac{1}{\sqrt{3}} \frac{4}{(2\pi)^{3/2}} \int_{0}^{\infty} R(r) j_{1}(kr) r^2 dr \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} R(r) j_{1}(kr) r^2 dr \sum_{m=-1}^{1} Y_{1, m_1}(\Omega_{k})
\]

\[
\therefore \psi(k) = \frac{2i}{(2\pi)^{1/2}} \int_{0}^{\infty} R(r) j_{1}(kr) r^2 dr \quad \text{(AIV.4)}
\]

Fig. AIV.2 shows the modulus squared of the momentum wave function and the momentum probability distribution.

The classical range of the values of \( k \) from 0 to 2.03 fm\(^{-1}\) covers 87.41% of the total probability, then the momentum probability distribution is better than the spatial wave function in describing the closest stationary distribution to the classical limits of \( r \) values.
The distance between the two clusters (fm).

Fig. AIV.1 The square of the wave function and the probability distribution as functions of the relative distance between the alpha and the triton in $^7$Li nucleus.
The wave number $k$ (fm$^{-1}$).

Fig. AIV.2 The square of the momentum wave function and the momentum probability distribution for the alpha-triton clusters in $^7$Li nucleus.
APPENDIX V

ANGULAR TRANSFORMATIONS

In this thesis the angular coordinates considered are two types; the first is the spherical angular coordinates \((\theta, \phi)\). The second is created to adapt the experiment conditions in terms of a horizontal angle \(\psi\) measured from the z-axis in the y-z plane, and the vertical angle \(\lambda\) measured from the x-axis but on the vertical plane passing the line which makes the angle \(\psi\). Both systems are shown in Fig. AV.1.

![Schematic representation of the angular coordinate systems.](image)

The exact transformation between the two systems is given by the equations:
\[
\psi = \tan^{-1} (\tan \theta \sin \psi)
\]
\[
\lambda = \cos^{-1} (\sin \theta \cos \phi)
\]
\[
\theta = \cos^{-1} (\cos \psi \sin \lambda)
\]
\[
\phi = \tan^{-1} (\sin \psi \tan \lambda)
\]

The differential elements of these angles are transformed by the Jacobian, leading to the definition of the solid angle between solid angles in both systems.

\[
d\Omega = \sin \theta \, d\theta \, d\phi = \frac{\sin \lambda \sin^2 \psi + \sin \lambda \cos^2 \lambda \cos^2 \psi}{\cos^2 \lambda + \sin^2 \lambda \sin^2 \psi} \, d\lambda \, d\psi
\]

The transformation between laboratory and centre of mass scattering angles \( \theta_L \), \( \theta_C \) respectively are also stated.

\[
\tan \theta_L = \frac{\sin \theta}{m_1 + \cos \theta C}
\]

\[
\tan \theta_C = \frac{(\sin \theta_L / (\cos \theta_L - 1.0 / (\cos \theta_L - 1.0 / (\cos \theta_L + \sqrt{\cos^2 \theta_L + \frac{m_2^2}{m_1^2} - 1}))))}{(AV.4)}
\]

\[
\phi_L = \phi_C
\]

and the solid angles are related by the relation

\[
d\Omega_{\text{lab}} = \frac{m_2^2 (m_2 + m_1 \cos \theta_C)}{(m_1^2 + m_2^2 + 2m_1m_2 \cos \theta_C)^{3/2}} \, d\Omega_{\text{C.M.}}
\]