THERMAL STRESSES
IN BUILDING ELEMENTS

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PRINCIPAL NOTATION

\(x, y, z\) = Rectangular co-ordinates.

\(\Phi\) = Airy stress function.

\(\sigma_x, \sigma_y, \sigma_z\) = Normal components of stress parallel to \(x, y\) and \(z\) axes.

\(\sigma_{xy}, \sigma_{yz}, \sigma_{zx}\) = Shear stress components.

\(\varepsilon_x, \varepsilon_y, \varepsilon_z\) = Strains in the \(x, y\) and \(z\)-directions.

\(U, V, W\) = Displacement components in the \(x, y\) and \(z\)-directions, respectively.

\(\sigma_{xy}, \sigma_{yz}, \sigma_{zx}\) = Shear strains.

\(E\) = Young's Modulus of elasticity.

\(\mu\) = Poisson's ratio.

\(\alpha\) = Linear coefficient of thermal expansion.

\(M\) = Bending moment.

\(M_x, M_y\) = Bending moments per unit length perpendicular to \(x\) and \(y\) axes, respectively.

\(M_{xy}\) = Twisting moment per unit length perpendicular to \(x\) axis.

\(N_x, N_y\) = Normal forces per unit length perpendicular to \(x\) and \(y\) directions, respectively.

\(N_{xy}\) = Shear force per unit length perpendicular to \(x\) axis.

\(D\) = Flexural rigidity of a plate.

\(U_0\) = G.E. = Complementary Energy.

\(\Delta T\) = Temperature difference.

\(\psi\) = Displacement function.
fn

eqn

L.H.S.

R.H.S.

l

h

\( A_R, E_R, \Delta T_R \)

\( A_W, E_W, \Delta T_W \)

k

Q

G

Q

H

T_s

T_a

T_{sa}

T_{IA}

\( \varepsilon \)

h_i

h_o

a

I

w

\( M_{11}, M_{12}, M_{21}, M_{22} \)

\( N_{11}, N_{12}, N_{21}, N_{22} \)

\( N_{11}^1, N_{12}^1, N_{21}^1, N_{22}^1 \)

n

= function.

= equation.

= Left hand side.

= Right hand side.

= length of wall.

= height of wall

= Values for roof.

= Values for wall.

= Thermal conductivity.

= Density of body.

= Specific heat.

= Rate of heat flow.

= Heat transfer coefficient of surface.

= Absolute temperature of body.

= Absolute temperature of outside air.

= Sol-air temperature.

= Inside-air temperature.

= Emissivity of surface.

= Air-film conductance for internal surface.

= Air-film conductance for external surface.

= Absorptivity of outside surface for solar radiation.

= Total incidence of solar radiation.

= Thermal frequency. = \( \frac{2\pi}{24 \text{ (hrs)}} \) rad/hr.

= Transfer coefficients for body.

= Transfer coefficients for external air.

= Transfer coefficients for internal air.

= Harmonic order.
SYNOPSIS

In the first part of this thesis, the analysis of temperature distribution in roof slabs from solar radiation is carried out employing Analogue computers as well as digital computer programs. The results obtained by both these methods compared favourably. The simplicity and speed with which solutions can be obtained by the Analogue computer method make it preferable.

In the second part, from the temperature distributions computed for roof slabs, the resulting thermal stresses are determined for different support conditions. Strength of materials solutions and minimum complementary energy methods are used.

In the third part, theoretical and experimental analyses of thermal stresses in roofs and walls due to temperature differentials between roof and walls are performed. Two models are treated - one with a reinforced concrete roof slab resting on brickwork walls and the other with an asbestos cement roof resting on asbestos cement walls. Heating was applied by infra red heaters.

In the final part, thermal crack formations in buildings are studied. Analytical solutions are presented for the spacing and location of expansion joints to accommodate the resulting thermal movements. Charts are drawn from which the spacing of expansion joints can be determined directly, depending on the geometric properties of sections, the tensile strengths of the materials employed and the magnitude of the maximum tensile thermal stresses to which the elements are subjected.
CHAPTER I
INTRODUCTION

1.1 GENERAL

Two of the most notable technological advances in this century are the development of nuclear energy sources and rocket-powered supersonic speed flights. Both of these achievements have involved high temperatures as well as in some applications severe temperature gradients.

Thermal stresses resulting from these temperatures are among the most important factors which determine the life of the materials employed in these applications and thus more work in this field is essential.

In building elements where temperature variations within the structure are of such a magnitude as to create a problem, determination of thermal stresses becomes important.

In tropical countries, thermal stresses are of considerable magnitude and it is well known for concrete structures to crack even before the application of any live load. The cracking is mostly caused by shrinkage and thermal stresses.

These temperature stresses caused by daily and seasonal fluctuations of atmospheric temperature are not accurately estimated in nuclear structural elements subjected to it. So more study is needed to develop accurate solutions for the thermal stresses involved.

1.2 HISTORY

The theory of elasticity was first presented in 1829 when Poisson published his famous memoir.
In 1835 Duhamel modified the theory of elasticity to take into account temperature change. At the same time Neumann formulated the thermoelastic equations in a form identical to that of Duhamel. Finally Barchardt (1873), Hopkinson (1879) and Alibandi (1900) worked towards the formulations of the thermoelastic equations used today.

**THERMO-ELASTIC STRESS-STRAIN RELATIONS**

\[ e_x = \frac{1}{E}(\sigma_x - \mu(\sigma_y + \sigma_z)) + \alpha T \]

\[ e_y = \frac{1}{E}(\sigma_y - \mu(\sigma_x + \sigma_z)) + \alpha T \]

\[ e_z = \frac{1}{E}(\sigma_z - \mu(\sigma_x + \sigma_y)) + \alpha T \]

\[ e_{xy} = \frac{1}{2} \gamma_{xy} = \frac{1}{2G} \sigma_{xy} \]

\[ e_{yz} = \frac{1}{2} \gamma_{yz} = \frac{1}{2G} \sigma_{yz} \]

\[ e_{zx} = \frac{1}{2} \gamma_{zx} = \frac{1}{2G} \sigma_{zx} \]

\[ G = \frac{E}{2(1 + \mu)} \]

\[ e_x = \frac{\partial U}{\partial x} \]

\[ e_y = \frac{\partial V}{\partial y} \]

\[ e_z = \frac{\partial W}{\partial z} \]

\[ e_{xy} = \frac{1}{2} \gamma_{xy} = \frac{1}{2} \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) \]

\[ e_{yz} = \frac{1}{2} \gamma_{yz} = \frac{1}{2} \left( \frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right) \]

\[ e_{zx} = \frac{1}{2} \gamma_{zx} = \frac{1}{2} \left( \frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} \right) \]
1.3 REVIEW OF PREVIOUS WORK

**ALLAN**

Calculation of thermal stresses in a heated restrained reinforced concrete slab has been done.

The stresses were calculated from:

\[ f = \frac{N}{I}(d - n) \]

where

- \( d \) = slab thickness
- \( n \) = distance from neutral axis to slab top

and

\[ M = \frac{\alpha(\ell E I)}{d} \]

where

- \( I \) = moment of inertia
- \( M \) = bending moment

The problem of the choice of moment of inertia was studied by taking a cracked section as well as an uncracked one. Final recommendations were to choose the moment of inertia of an uncracked section in calculations of thermal stresses.

**ALLECK**

Analysis of thermal stresses induced in a thin rectangular plate constrained at one edge by a uniform temperature change has been demonstrated.

The problem was first resolved into one of boundary tractions. Then assuming

\[ \sigma_x = f_1(x) + y f_2(x) + y^2 f_3(x), \]

the equilibrium equations and boundary conditions were satisfied. The stresses were expressed in terms of \( y \) and derivatives of \( f(x) \).

Using the principle of minimum strain energy, the problem was resolved into the solution of a system of simultaneous differential equations for \( f(x) \).
BOLEY AND WEINER

An intensive discussion of the fundamental theory of thermal stresses was presented with numerous illustrative examples.

Thermal stress problems as met in practice were analysed chiefly using strength-of-materials solutions - in this respect plates, beams and various structures were analysed.

The development of inelasticity theory to include temperature effects was also presented.

HANNANT

Thermal stresses in reinforced concrete sections have been fully investigated. Analysis has been worked out for the prediction of the residual stresses in the reinforcement due to shrinkage and creep. Reinforced concrete sections subjected to thermal gradients have been investigated and stresses in the tensile reinforcement and factors affecting these stresses, like creep, shrinkage, cracking and reduction in the coefficient of thermal expansion have been studied.

Investigations of the inelastic phenomenon have been carried out at high temperature using reinforced and unreinforced cylinders. The contribution of the reinforcement in restricting the formation of wide cracks has been noted. Recommendations on the amount of reinforcement have been concluded.

HENDRY AND PAGE

Elaborated on the different factors affecting the temperature distribution in reinforced concrete roof slabs as well as the effect of variations in these factors. Factors being discussed were:
1. Slab thickness.
2. Surface colour.
3. Wind speed.

They also outlined methods for determination of surface reflectivity.

Using the temperature distribution data as studied, they applied them to a number of illustrative examples in evaluating the thermal stresses in slabs, beams and walls.

For the calculations of the thermal stresses, they used a procedure based on Jasper's method as described by Borg and Genaro(4).

In conclusions they drew attention to the great need for comprehensive studies on thermal stresses from solar radiation especially in constructions involving ribbed slabs and composite beams. They also emphasized the necessity for expansion joints.

HELDENFELS AND ROBERTS(9)

They employed the principle of minimum complementary energy to determine the stresses in a heated flat plate, applying the theory of elasticity and using the approximate variational method.

Airy's stress function for a heated plate is given by:

\[ \nabla^4 \varphi = -E \varepsilon \nabla T \]

\[ \nabla^4 \varphi = -E \varepsilon \nabla T \]

Let \( \varphi = f \cdot g \)

where \( f = f(x) \); and \( g = g(y) \)

\[ \sigma_x = f'g'' \]

\[ \sigma_y = g'f'' \]
\[ \sigma_y = f''g \]
\[ \sigma_{xy} = -f'g' \]

The problem was solved by selecting an appropriate function \( g \) and using the principle of minimum complementary energy to determine a function \( f \) that gave the best approximation to the exact solution of equation (1).

A suitable selection of \( g \) was a function proportional to the stress function of an infinitely long plate subject to temperature distribution

\[ T = Y \]

The solution for the function \( f \) was obtained after satisfying the necessary boundary conditions.

Having determined \( g \) and \( f \), the stress condition in the plate was subsequently analysed. Equation (3) gave the stresses at any point.

This theoretical solution was checked by experimental work and good agreement was reported.

IYENGAR, CHANDRASHENKHARA AND ALWAR (10,11,12)

Thermal stresses in flat rectangular isotropic plates of constant thickness with arbitrary temperature distribution in the plane of the plate have been determined.

Fourier series and integrals have been assumed for the stresses satisfying the boundary conditions and the differential equations

\[ \nabla^4 \phi = -\alpha E \nabla^2 T \]

Numerical results have been produced.

Solutions for free rectangular plates, restrained rectangular plates/
plates and long rectangular plates constrained at one of the shorter edges have been given.

HOYLE (13)

Presented illustrations to the thermal stress and strain problem as well as definition of the elastic thermal stress. Solutions of thermal stresses in turbines were also given. Analysis of thermal stresses in cylinders (representing plane strain) and plates (representing plane stress) was also carried out.

JAMES (14)

Thermal stresses in an elastic continuum were calculated using an iterative numerical method for solving the resulting set of simultaneous equations. The method has been extended to cover axially-symmetrical thermoelastic problems - application was mainly done to an autoclave.

NEAL (15)

The general principles of thermal stress analysis for trusses and framed structures, employing the principle of virtual work were outlined.

MORT (16)

Thermal stresses in rectangular plates restrained along an edge have been analysed making the assumption that the rectangular plate was connected to a semi-infinite elastic plate and the temperature varied along plate surface only.
Method of variation has been used and the principle of minimum potential energy employed.

OGUNLESI (17)

A stress function called the Complex Biharmonic Eigenfunction was derived from considerations of complex variable theory. It was then applied to an arbitrarily loaded free plate.

The solution to the plate problem is of the form:

$$[A] = [\lambda]^{-1} [F]$$

where $[A]$ is a $(2n \times 1)$ column matrix whose elements are the arbitrary constants to be determined.

$[F]$ is a $(2n \times 1)$ column matrix whose elements were called "Load integrals".

$[\lambda]$ is an $(n \times n)$ Hessian Matrix referred to as "the characteristic matrix".

The stress function was then applied to thermal stress problems. It was first applied to a free rectangular plate subjected to a step function and polynomial temperature distributions. The solution was also applied to clamped plates as well as to laterally loaded thin rectangular plates.

Experimental investigations were carried out to verify the application of the theory. Photoelastic analysis was employed together with direct measurement of thermal stresses in a steel plate.

ROSENHAUPT, KOFMAN AND ROSENTHAL (18)

Thermal stresses/
Thermal stresses in unseparated walls, due to thermal expansion of roof resulting from temperature differential between roof and wall were computed.

Methods for reducing these thermal stresses by a separation of a horizontal sliding joint were studied.

Experiments were carried out to determine the coefficients of friction for different separating materials, and variable loads and age after casting.

SAMELSON AND TOR

Stresses in walls of underground reinforced concrete cylindrical tanks containing hot liquid of temperature between 50-500°F have been studied. Stresses for steady state temperature gradients as well as for transient state gradients have been evaluated.

TIMOSHENKO AND WOINOWSKY-KRIEGER

Thermal stresses in a clamped plate subjected to non-uniform heating were calculated on the assumption that temperature variation across thickness of the plate is linear and there was no temperature variation in planes parallel to the surface of the plate. The case of the heated plate was treated in a manner similar to that of a plate in pure bending.

Thermal stresses in simply supported rectangular plates were also determined.

THOMLINSON

A rational theory was developed by which the thermal stresses resulting/
resulting in any mass of concrete exposed to the daily and seasonal fluctuations of atmospheric temperature were evaluated. The theory was mainly intended for the determination of thermal stresses in concrete road slabs, but could be extended for concrete roof slabs, retaining walls, etc.

Zienkiewicz (22,23,24)

Used Airy's stress function formulations in computing shrinkage and thermal stresses in massive structures (22).

For plane strain \( \nabla^4 \phi = -\frac{E}{1-\mu} \nabla^2 (\alpha T) \)

For plane stress \( \nabla^4 \phi = -E \nabla^2 (\alpha T) \)

Using finite difference approximations, the solution was then resolved into a system of simultaneous equations. These simultaneous equations were then solved using relaxation techniques.

The problems of stresses and deformations resulting from imposed loads, displacements and temperature stresses in a concrete structure were also formulated on the assumption of a visco-elastic behaviour of concrete (23).

Zienkiewicz and Cruz (24) applied the principles of the slab analogy in determining thermal stresses in the general case of variable elastic or thermal properties.

Zuk (25)

The solution of thermal and shrinkage stresses in composite beams has been demonstrated.
The solution was reached by investigating the interface forces and couples between the slab and beam and satisfying the horizontal as well as the vertical compatibility conditions.

1.4 DISCUSSION OF PREVIOUS WORK

For the determination of thermal stresses an accurate solution of the temperature distribution should be done first. The methods developed so far are laborious. Thus fast methods for the solution of the existing rigorous solution by Gorcum (38) are considered as well as the development of more simple and accurate methods using finite difference approximations and employing Analogue Computers.

The existing methods of thermal stress analysis could be divided into three groups:

1. Methods using Strength of Materials Solution: Main contributors in this are Allan (1), Hendry and Page (8), Samelson and Tor (19), Timoshenko and Woinowsky-Krieger (20), and Zuk (25). These methods give an estimate of the magnitude of thermal stresses pending more experimental and theoretical verifications.

2. Energy Methods: Alleck (2), Heldenfels and Roberts (9), Mori (16) and Neal (15) are among the most eminent investigators. These methods, although reliable, are lengthy involving the assumption of functions and many integrations. Much work is thus needed before we arrive at the stresses in question.

3. Numerical Methods: Iyengar - Chandrashenkhara - Alwar (10, 11, 12), James (14), Rosenhaupt - Kofman - Rosenthal (18) and Zienkiwicz (24) developed solutions using these methods.

The solutions/
The solutions produced by Iyengar, Chandrashenkhara and Alwar involve Fourier series and integrals giving the thermal stresses in plates subjected to arbitrary temperature distributions. The solution of these Fourier series and integrals is not a fast process, accuracy depending on the number of terms taken.

However the solutions by Zienkiewicz and Rosenhaupt - Kofnan - Rosenthal are simple and give more exact solutions, accuracy depending on the size of meshes chosen.

CONCLUSION

All the previous analysis was mainly theoretical and was confined to plates. Very little work was done concerning the behaviour of building elements subjected to temperature fluctuations. Thus more experimental and theoretical work is needed in this field.

The endeavour of Rosenhaupt - Kofnan - Rosenthal was towards this direction, but the analysis was limited to the determination of thermal stresses resulting from the temperature differential between wall and roof assuming mean temperatures for wall and roof. No account was taken of the temperature variation through the wall surface and roof thickness.

More precise and rigorous solution is to be considered, taking into account the temperature variations along wall surface.

Very little work has been done regarding the location and spacing of movement joints.

Thus more investigation in this respect is needed.

1.5 SCOPE OF PRESENT INVESTIGATIONS/
1.5 SCOPE OF PRESENT INVESTIGATIONS

The main objectives of the present work could be summarised in the following manner:

1. The development of a quick and accurate method for the analysis of the temperature distribution in roof slabs from solar radiation.

2. Calculation of the thermal stresses in roof slabs subjected to temperature distribution obtained from the above analysis. Stress distribution in slabs for different support conditions is to be considered.

3. The analysis of thermal stresses in roofs and walls resulting from temperature differentials between roof and walls. Development of a rational theory for the calculations as well as experimental investigations were sought.

4. Study of thermal crack formations in buildings and methods for eliminating or minimizing these cracks by use of expansion joints.

   Analysis was mainly conducted for the location as well as the spacing of these joints.
CHAPTER II

TEMPERATURE DISTRIBUTION IN ROOF SLABS DUE TO SOLAR RADIATION

2.1 INTRODUCTION

2.1.1 Solar Radiation

The solar radiation consists of three parts:

a) direct solar radiation.
b) diffuse sky radiation
c) radiation reflected upon the surface from surrounding surfaces.

Thus total radiation = intensity of direct radiation +
intensity of diffuse sky radiation +
intensity of solar radiation reflected upon surface from surrounding surfaces.

The direct solar radiation is a function of the altitude angle, azimuth angle, hour angle, latitude, turbidity and water vapour of atmosphere.\(^{(26)}\)

The most recent value of the solar constant which is defined as the incident solar radiation on a surface normal to the sun's and located at outer limit of earth's atmosphere,\(^{\text{r.y.s.}}\) when the earth is at its mean distance from the sun, has been found by Stair and Johnson in 1956 as 453 ± 22 Btu/hr.ft\(^2\).\(^{(28)}\)

The solar radiation incident upon a surface may be absorbed, reflected or transmitted through the material:

\[ \varphi + a + \tau = 1 \]

where \( \varphi \) = reflectivity of surface
\( a \) = absorptivity of surface
\[ \tau = \text{transmissivity of surface} \]

The emissivity and absorptivity of surfaces are governed by Kirchoff's laws which state that emissivity of a surface is equal to the absorptivity provided that the incident radiation has the same wavelengths as those being emitted by the surface.

Direct solar radiation incident on a horizontal surface is given by:

\[ I_H = I \sin \beta \]

where \( \beta = \text{solar altitude} \) and \( I = \text{total incidence of solar radiation} \).

Component of direct solar radiation incident on a vertical surface is given by:

\[ I_V = I \cos \beta \cos \delta \]

where \( \delta = \text{surface solar azimuth} \)

### 2.1.2 Heat Transfer

In general there are three forms of heat transfer:

a) CONDUCTION: Fourier heat conduction equation:

\[ \frac{\partial}{\partial x}(k \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k \frac{\partial T}{\partial z}) + Q = \rho c \frac{\partial T}{\partial t} \]

b) CONVECTION: \( Q = H(T - T_0)^{5/4} \)

c) RADIATION: \( Q_r = \sigma(T_s^4 - T_a^4) \) - Stefan-Boltzmann's formula

\( \sigma = \text{Stefan-Boltzmann's constant} = 5.67 \times 10^{-8} \text{ Btu/ft}^2/\text{hr.} \cdot (\text{°K})^4 \)

### 2.1.3 Heat Transmission and Solar Radiation in Buildings

If a temperature differential exists between an internal enclosure and the surrounding environment, heat transfer takes place.

The fluctuating nature of the outdoor air temperature and solar radiation makes it a non-steady state for the external environment.
(a) Steady-state Heat Transfer

The rate of heat transfer through the surface shown in fig. is given by (26):

\[ q = U(T_{i,\alpha} - T_{o,\alpha}) \]

where \( U \) = overall heat transmission coefficient

\[ U = \frac{1}{\frac{1}{h_i} + \frac{x}{k} + \frac{1}{h_o}} \]

The temperature of the outside surface is raised by the sun's rays above that of the shade air temperature. The rate of temperature rise depends on (B):
1. Intensity of short-wave radiation.
2. Rate of outside air temperature rise.
3. Wind velocity.
4. Equivalent sky temperature.
5. Absorptivity and emissivity of outside surface.
6. Thermal properties of body, size, shape and thickness.
7. Internal thermal environment.

For a composite wall, overall heat transmission coefficient is:

\[ U = \frac{1}{\frac{1}{h_i} + \sum x_k + \frac{1}{C} + \frac{1}{h_o}} \]

\[ C = \text{conductance of air space} - \text{Btu/hr/ft}^2/\degree F \]
b) Periodic Heat Transfer\(^{(31,32,34,35)}\)

The rate of heat transfer from the external thermal environment to the outside surface of a roof or wall is\(^{(31)}\):

\[
q_o = aI - h_o(T_o - T_{oa}) - h_r(T_o - T_R) = -K \frac{dT_o}{dx} \tag{1}
\]

where \(h_c\) = convective heat transfer coefficient-Btu/ft\(^2\)/hr/\(^\circ\)F.

\(h_r\) = radiative heat transfer coefficient-Btu/ft\(^2\)/hr/\(^\circ\)F.

\(T_R\) = equivalent radiant temperature of sky and ground-\(^\circ\)F

Equation (1) can be expressed as:

\[
q_o = h_o(T_{oa} - T_o) + aI - h_r(T_{oa} - T_R) \tag{2}
\]

Using sol-air temperature concept, equation (2) can be expressed as:

\[
q_o = h_o(T_{sa} - T_o) \tag{3}
\]

where \(T_{sa} = T_{oa} + \frac{aI}{h_c + h_r} - \frac{h_r(T_{oa} - T_R)}{h_c + h_r}

= sol-air temperature

and \(h_o = h_c + h_r\)

**Sol-Air Temperature\(^{(33,34)}\)**

It is a hypothetical temperature of the outdoor air in contact with a wall or roof which would give the same temperature distribution through the roof or wall and rate of heat transfer as exists with the actual outdoor air temperature and incident total solar radiation.

According to Brown\(^{(36)}\):

\[
T_R = 1.2 T_{oa} - 14 \quad \text{For a horizontal surface}
\]

and \(T_R = 1.1 T_{oa} - 5 \quad \text{For a vertical surface}\)

The sol-air temperature expression has got three terms, the third of which varies with the sky conditions whether it is cloudy or clear and with day and night.
Thus for clear conditions (26):

\[ T_{sa} = T_{0,a} + \frac{a I}{h_o} - \frac{h_r}{h_o}(14 - 0.2 T_{0,a}) \]

Using the values of \( \frac{h_r}{h_o} = 0.4 \) and 0.3 for night and day respectively for a horizontal surface, and for sky represented by m oktas:  

\[ m \text{ oktas is a measure of the cloudiness of the sky, ranging from 0-8.} \]

For day-time \( T_{sa} = T_{0,a} + \frac{a I}{h_o} - \left(\frac{9 - m}{9}\right)(4.2 - 0.06 T_{0,a}) \)

For night-time \( T_{sa} = T_{0,a} - \left(\frac{9 - m}{9}\right)(4.2 - 0.06 T_{0,a}) \)

Since the thermal environment is non-steady, the outdoor air temperature and intensity of solar radiation together with the sky temperature can be represented by a Fourier series of the form (26, 37, 32, 33, 35):

\[ T = T_m + A_1 \cos w_1 t + B_1 \sin w_1 t + A_2 \cos w_2 t + B_2 \sin w_2 t + \ldots + A_n \cos w_n t + B_n \sin w_n t \]

where \( t = \) number of hours measured from midnight solar time

\[ T_m = \frac{1}{24} \int_0^{24} T dt \]

\[ A_n = \frac{1}{12} \int_0^{24} T \cos w_n t dt \]

\[ B_n = \frac{1}{12} \int_0^{24} T \sin w_n t dt \]

\[ w_n = n w_1, \quad w_1 = \frac{2\pi}{24} \text{ rad/hr.} \]
At any thickness $x$ of the wall, \[
\frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \frac{\partial T}{\partial t}
\]

At inside surface, $q_i = -k \left( \frac{\partial T}{\partial x} \right)_{x=L} = h_i (T_i - T_{i,a})$

At outside surface, $q_o = -k \left( \frac{\partial T}{\partial x} \right)_{x=0} = h_o (T_{e,a} - T_o)$

2.2 TEMPERATURE DISTRIBUTION IN ROOF SLABS

The temperature inside a r.c. slab was computed. The slab was assumed with the following properties:

- **Thickness** = 5 in.
- **Conductivity** = $k = 10$ Btu/ft$^2$/hr/$^\circ$F/in.
- **Density** = $\rho = 150$ lb/ft$^3$
- **Specific heat** = $c = 0.24$ Btu/lb/$^\circ$F
- **Diffusivity** = $a = \frac{k}{\rho c} = 0.023$ ft$^2$/hr
- **Emissivity** = $\varepsilon = 0.95$; solar absorptivity = $\alpha = 0.95$

Air film conductance for external surface

$= h_o = 3.5$ Btu/ft$^2$/hr/$^\circ$F (see W.S. Billington, "Thermal Properties of Buildings")

Air film conductance for internal surface

$= h_i = 1.65$ Btu/ft$^2$/hr/$^\circ$F

The solution was performed for two cases.
1. Assuming a sinusoidal variation of the sol-air and inside air temperatures of the form:

\[ T_{sa} = 95 + 15 \sin wt \]
\[ T_{1A} = 90 + 5 \sin wt \]

where \( w \) = thermal frequency = \( \frac{2\pi}{24} \) rad/hr

2. 24 hours record of observed outdoor air temperature and sky temperature on a horizontal roof. (Data from the summer design day for Khartoum, May 24th, 1962).

Assume inside air temperature \( T_{1A} = 80^\circ F \)

2.2.1 Sinusoidal Sol-air and Inside Air Temperatures

2.2.1.1 Solution by Transfer Matrices Method (Rigorous Solution)

This solution has been presented by Van Gorcum(38) in 1949 and elaborated in 1964 by Ogunlesi(17).

From this solution the amplitude of temperature at any distance \( x \) from the upper surface of the slab can be found from:

\[ T_x = \frac{1}{pq + rs} (CT_{1A} - r T_{sa}) \]

assuming a one-dimensional temperature distribution.

where 
\[ p = (M_{11})_x - x - R^1(M_{21})_x - x \]
\[ q = R(M_{11})_x - (M_{12})_x \]
\[ r = (M_{12})_x - x - R^1(M_{22})_x - x \]
\[ s = R(M_{21})_x - (M_{22})_x \]
\[ (M_{11})_x = \cos(\delta x) = (M_{22})_x \]
\[ (M_{12})_x = \frac{\sin(\delta x)}{K_\gamma} ; (M_{21})_x = \gamma K \sin(\delta x) \]
\[ \gamma^2 = -\frac{C h w}{K} ; w = \frac{2\pi n}{24} \]

The theory presented here is the basis of part of the first objective (see p. 13) of this work. This theory is used for the calculation, results are shown in table (2.1).
\( n = \text{harmonic order} \)

\[
R = \frac{1}{h_0} = \frac{1}{3.5} = 0.286
\]

\[
R^1 = \frac{1}{h_1} = \frac{1}{1.65} = 0.606
\]

For any particulars concerning this method of solution, see Appendix (A).

**ASSUMPTIONS**

1. Temperature varies through the thickness of the slab only.
2. Thermal properties of slab are constant; they do not change with either temperature or time.
3. Air (outside and inside) has no heat capacity

From the assumption (3),

\[
N_{11} = N_{22} = N_{11}^1 = N_{22}^1 = \cos 0 = 1
\]

\[
N_{21} = N_{21}^1 = \sin 0 = 0
\]

\[
N_{12} = -\frac{1}{k} = -R = -\frac{1}{h_0} = -\frac{1}{3.5} = -0.286
\]

\[
N_{12}^1 = -\frac{1}{k^1} = -R^1 = -\frac{1}{h_1} = -\frac{1}{1.65} = -0.606
\]

\[
\gamma^2 = -\frac{C_{w,i}}{k} = -\frac{150 \times 0.24}{10/12} \times \frac{2\pi, i}{24}
\]

\[
\gamma = 2.375 (-1 + i)
\]

\[
k\gamma = 1.98 (-1 + i)
\]

For the solution of the temperature equation \( (T_x) \), a computer program has been developed by which the temperature at any depth \( x \) and at any time of the day was easily obtained (Appendix B).

See table (2.1) for the results: \( T_0, T_1, T_2, T_3, T_4 \) and \( \gamma \) refer to points on the surface, 1” from surface, 2”, 3”, 4” from surface out on the inside surface respectively.
<table>
<thead>
<tr>
<th>TIME</th>
<th>TEMPERATURE - °F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T₀</td>
</tr>
<tr>
<td>1 a.m.</td>
<td>92.73</td>
</tr>
<tr>
<td>2</td>
<td>95.49</td>
</tr>
<tr>
<td>3</td>
<td>98.15</td>
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<td>8</td>
<td>104.50</td>
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<tr>
<td>9</td>
<td>103.80</td>
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<tr>
<td>10</td>
<td>102.40</td>
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<td>11</td>
<td>100.50</td>
</tr>
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<td>12 noon</td>
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<tr>
<td>13</td>
<td>95.42</td>
</tr>
<tr>
<td>14</td>
<td>92.67</td>
</tr>
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<td>90.01</td>
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<tr>
<td>23</td>
<td>87.68</td>
</tr>
<tr>
<td>24 midnight</td>
<td>90.07</td>
</tr>
</tbody>
</table>

**COMPUTED**

**TABLE 2.1:** TEMPERATURE DISTRIBUTION IN A R. CONCRETE ROOF SLAB.

"SINUSOIDAL SOL-AIR AND INSIDE AIR TEMPERATURES".
However the solution of the temperature equation involved multiplication of complex numbers which was performed in a similar manner to that presented by Muncey and Spencer (39).

2.2.1.2 Solution by General Purpose Analogue Computer

Fourier's equation for one-dimensional heat flow is given by:

\[
\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}
\]

where \( a \) = diffusivity

The method of the Analogue Computer solution is based on replacing this partial differential equation with a finite difference approximation. The Analogue Computer is then employed to solve the resulting set of finite difference differential equations.

In an analogue computer, the mathematical operations and relationships between the problem variables are obeyed by the computer voltages.

These operations are achieved by potentiometers, amplifiers (summers or integrators), multipliers, sign changers and function generators (12, 13). For more information see Appendix (C).

The author adopted a similar solution to that presented by Christie (42), for the solution of consolidation problems, since the equations governing heat conduction and consolidation of soils are of the same nature.

Terzaghi's equation for one-dimensional consolidation (40) is:

\[
\frac{d u}{d t} = C_v \frac{d u}{d z}
\]

where \( u \) = excess pore water pressure at depth \( z \) and time \( t \).

For any mesh point \( i \) at any time \( t \), and \( C_v \) = coefficient of consolidation, (see fig. below)

Fourier's equation can be approximated by:

\[
\frac{\partial T_i}{\partial t} = \frac{a}{(\Delta x)^2} \left( T_{i+1} + T_{i-1} - 2T_i \right)
\]

TIME SCALING:

Let \( \tau = \frac{a}{(\Delta x)^2} \frac{t}{2} \)

where \( \tau \) = computer time

\( t \) = actual time
The temperature of the external surface of the slab is given by (43, 44):

\[ T_0 = M_0 T_{sa} + (1 - M_0) T_1 \]

where \( M_0 = \frac{1}{1 + \left(\frac{k}{h_0 \Delta x}\right)} \)

\[ = 0.259 \]

The expression for \( T_0 \) is derived from consideration of heat flow, namely:

\[ \frac{k}{\Delta x}(T_0 - T_1) = h_0 (T_{sa} - T_0) \]

\[ \frac{k}{\Delta x} + h_0 T_0 = h_0 T_{sa} + \frac{k}{\Delta x} T_1 \]

\[ T_0 = \left(\frac{h_0}{\Delta x + h_0}\right) T_{sa} + \left(\frac{k}{\Delta x + h_0}\right) T_1 \]

if \( M_0 = \frac{1}{1 + \frac{k}{h_0 \Delta x}} \), then

\[ T_0 = M_0 T_{sa} + (1 - M_0) T_1 \]

The temperature of the internal surface of the slab is similarly given by

\[ T_1 = M_1 T_{1A} + (1 - M_1) T_4 \]

where \( M_1 = \frac{1}{1 + \frac{k}{h_1 \Delta x}} \)

For the first mesh point, \( \frac{dT}{dt} = T_2 + T_0 - 2T_1 \)

For the second/
For the second mesh point, \( \frac{dT_2}{dT} = T_3 + T_1 - 2T_2 \)

For the third mesh point, \( \frac{dT_3}{dT} = T_4 + T_2 - 2T_3 \)

For the fourth mesh point, \( \frac{dT_4}{dT} = T_1 + T_3 - 2T_4 \)

The expressions for the sol-air and inside air temperatures are given by:

\[ T_{sa} = 95 + 15 \sin \omega t \]
\[ T_{la} = 90 + 5 \sin \omega t \]

To transform these into computer variables, we have to scale the time to its corresponding computer value.

**TIME SCALING:**

\[ T = \frac{a \cdot t}{(\Delta x)^2} \]

For \( a = 0.023 \) and \( \Delta x = 1 \), the reader is referred to the note for the value selected for \( \Delta x \).

\[ T = (0.023 \times 144)t \]
\[ t = 0.3017 \cdot T \]

and \( \omega = \frac{2\pi}{24} \) for one day period and 1st harmonic

\[ T_{sa} = 95 + 15 \sin \left( 0.0791t \right) \]
\[ T_{la} = 90 + 5 \sin \left( 0.0791t \right) \]

Thus for \( t = 24 \) hours, computer time \( T = 79.55 \) sec.

If all integrator gains are multiplied by 10, then \( T \) corresponding to 24 hours = 7.955 sec.

Thus connect S.1 to 24V for integrators in main circuit (i.e. \( E_1, E_2, E_3, E_4 \)) and multiply potentiometer readings of \( (B_{11} \text{ and } B_{12}) \) by 10.

Reading of potentiometer \( B_{11} = 0.791 \)

Reading of potentiometer \( B_{12} = 0.791 \)
See fig. (2.1) for the computer network.

This network was wired in a patch panel which was then connected to the SC30 Solartron Analogue Computer.

However, for the integrators used in the main circuit (i.e. $E_1, E_2, \ldots$) the following initial conditions were fed:

At $t = 0$,

\[
\begin{align*}
T_0/4 &= 22.52^\circ F \\
T_1/4 &= 22.11 \\
T_2/4 &= 21.86 \\
T_3/4 &= 21.74 \\
T_4/4 &= 21.72 \\
T_1/4 &= 21.79
\end{align*}
\]

These initial conditions were obtained from the rigorous solution (2.2.1.1).

However if these initial conditions were not known, an iteration procedure similar to that presented by Christie and Soane (45) could be followed.

The output of the computer could be read direct from a digital voltmeter which was incorporated in the computer (see plate 2) by adjusting the switch selector to the required position and moving the timer to the desired time interval. However, to get a continuous record of the temperature variations during the whole day, we connected the output terminals to an X-Y plotter (see plate 1) and a continuous record for the temperature distribution throughout the whole slab section and for the whole day was obtained. Figs. show (2.2, 2.3, and 2.4) the temperature distribution for one day, three days and seven days respectively.
Fig. (2.1): ANALOGUE COMPUTER NETWORK
"SINUSOIDAL SOL AIR AND INSIDE AIR TEMPERATURES."
(b)

(c)

(d)

COMPUTER SUB-CIRCUITS
PLATE (1) : SOLARTRON SC-30 ANALOGUE COMPUTER
SET-UP WITH X-Y CO-ORDINATE PLOTTING TABLE.
PLATE (2) : SOLARTRON SC-30 ANALOGUE COMPUTER
PLATE (3) : X-Y CO-ORDINATE PLOTTING TABLE
Fig.(2.5):  COMPARISON OF ANALOGUE COMPUTER SOLUTION AND RIGOROUS SOLUTION.
"Sinusoidal Sol Air and inside Air Temperatures."
Since the sol-air and inside-air temperatures are sinusoidal, a sine function has to be generated and connected to the computer network at its appropriate place.

For the generation of sine and cosine functions, see Appendix (D).

2.2.1.3 CONCLUSIONS

1. Good agreement was reached between Analogue Computer solution and rigorous solution (by transfer matrices) as shown by fig. (2.5).

2. The temperature distribution inside the slab followed a sinusoidal variation similar to the assumed sol-air and inside air temperatures with the amplitude decreased and phase displaced as sections went deeper in the slab.

3. Repetitive identical curves were obtained for successive days if the same external and internal thermal environments were assumed for the successive days.

4. The Analogue Computer solution is thus very satisfactory. In about eight seconds the whole picture for the temperature distribution inside the slab was seen.

Thus quick solutions are obtained and one can manipulate the different parameters involved and detect - on the spot - the effect on the distribution.

2.2.2 24 Hours Record of Outdoor Air Temperature, Sky Temperature, Ground Temperature and Intensity of Solar Radiation on a Horizontal Surface

Summer design day data for Khartoum was obtained as shown in table (2.2). The hottest day in the hottest month was chosen. The inside air temperature was assumed equal to 80°F. (Tropical environment)
FIG. (2.2)  ANALOGUE COMPUTER RESULTS FOR TEMPERATURE DISTRIBUTION IN A CONCRETE ROOF SLAB (SINUSOIDAL SOL.-AIR TEMPERATURE)
FIG(2,3) ANALOGUE RESULTS FOR TEMPERATURE DISTRIBUTION IN A CONCRETE ROOF SLAB (SINUSOIDAL SOL-AIR TEMPERATURE)
FIG. (24)  ANALOGUE COMPUTER RESULTS FOR TEMPERATURE DISTRIBUTION IN A CONCRETE ROOF SLAB [SINUSOIDAL SOL-AIR TEMPERATURE]
The sol-air temperature was calculated from:

\[ T_{sa} = T_{0,a} + \frac{a I}{h_c + h_r} - \frac{h_r (T_{sa} - T_R)}{h_c + h_r} \]

Assuming the heat transfer coefficient \((h_c + h_r) = 3.5\) as before, the radiation transfer coefficient \(h_r\) was calculated as follows:

Radiation loss from surface is given by Stefan-Boltzman's formula:

\[ Q_r = \sigma \varepsilon (T_s^4 - T_a^4) \]

This can be put in the form:

\[ Q_r = h_r (T_s - T_a) \]

where \(h_r = \sigma \varepsilon \left( \frac{(T_s - T_a)^4}{(T_s - T_a)} \right) = \sigma \varepsilon \left( \frac{(T_s - T_a)(T_s + T_a)(T_s^2 + T_a^2)}{T_a - T_a} \right) \]

\[ = (T_s + T_a)(T_s^2 + T_a^2) \]

assuming \(T_s = T_a = T_{a,m}\) = mean air temperature

**Calculation Error in the assumption of \(T_s = T_a\),**

\[ h_r = \sigma \varepsilon \left( \frac{4T_{a,m}^3}{T_a - T_a} \right) \]

**Calculation Error in the assumption of \(T_s = T_a\),**

From table (2.2), \(T_{a,m} = (96.5 + 460) = 556.5^\circ \text{F}\)

\[ h_r = 0.174 \times 10^{-8} \times 0.95 \times 4(556.5^3) \]

\[ = 1.1395 \]

Having determined \(h_r\), the sol-air temperature was calculated as shown by table (2.2).

2.2.2.1 Rigorous Solution

For calculation purposes the sol-air temperature as given in table (2.2) was approximated by a Fourier series, namely (26,37,32,33)

\[ T_{sa} = T_{sa,m} + \sum_{n=1}^{12} A_n \cos (wnt) + \sum_{n=1}^{12} B_n \sin (wnt) \]
This equation can be put in the form:

\[ T_{sa} = T_{sa,m} + \sum_{n=1}^{12} C_n \sin (wnt - \alpha_n) \]

where \( C_n = \sqrt{\frac{2}{\pi} \cdot \frac{B_n}{A_n}} \)

\( \alpha_n = \tan^{-1} \left( -\frac{A_n}{B_n} \right) \)

\[ T_{sa} = \sum_{t=1}^{24} T_{sa} \cos(wnt) \]

\[ A_n = \frac{t = 24}{12} \sum_{t=1}^{24} T_{sa} \sin(wnt) \]

\[ B_n = \frac{t = 24}{12} \sum_{t=1}^{24} T_{sa} \]

\[ T_{sa,m} = \frac{1}{12} \sum_{t=1}^{24} T_{sa} = \text{mean sol-air temperature} \]

The equations governing the temperature distribution through the slab are:

\[ T_x = \frac{1}{pq + rs} (qT_{L,A} - r T_{sa}) \]

\[ T_{sa} = T_{sa,m} + \sum_{n=1}^{12} C_n \sin (wnt - \alpha_n) \]

\[ T_{L,A} = 80^\circ F \]

The solution/
<table>
<thead>
<tr>
<th>Local Civil Time</th>
<th>Air Temperature °F</th>
<th>Sky Temperature °F</th>
<th>Total Solar Radiation on Horizontal Surface Btu/hr.ft.²</th>
<th>Sol-Air Temperature °F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 am.</td>
<td>87</td>
<td>43</td>
<td>-</td>
<td>72.61</td>
</tr>
<tr>
<td>2</td>
<td>85</td>
<td>42</td>
<td>-</td>
<td>70.94</td>
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<td>-</td>
<td>69.94</td>
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<td>68.94</td>
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<td>11</td>
<td>105</td>
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<td>171.92</td>
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<tr>
<td>12 noon</td>
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<td>68</td>
<td>326</td>
<td>178.70</td>
</tr>
<tr>
<td>13</td>
<td>109</td>
<td>69</td>
<td>312</td>
<td>176.10</td>
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**TABLE 2.2** - Calculation of Sol-Air Temperature from Air Temperature, Sky Temperature and Total Solar Radiation Incident on Horizontal Surfaces - Data for the Summer Design Day for Khartoum. (Obtained from private source.)
The solution of these equations gives the temperature at any section $x$ and at any time $t$ as:

$$T(x,t) = \frac{1}{pq + rs} \left[ q x 80 - r \left( T_{sa,m} + \sum_{n=1}^{12} C_n \sin(wnt - \phi_n) \right) \right]$$

For the solution of $T(x,t)$, a computer program was developed as shown in Appendix (F). See table (2.3)

2.2.2.2 General Purpose Analogue Computer Solution

A graph of the sol-air temperature $T_{sa}$ versus time was plotted (fig. 2.6).

To feed this function into the computer, the following procedure was adopted:

1. The curve was approximated by a series of straight line segments as shown in fig. (2.6).
2. Then for every value of $\delta x$ along the time axis, the value of $\delta y$ along the temperature axis was determined.
3. Since the DIFG simulates a fn as a number of linear segments, the $\delta x$ and $\delta y$ segments were fed successively starting with $\delta x$. The proper sign for $\delta y$ should be put.
4. After setting all the segments, the monitor switch was put to level and the initial value of $T_{sa}$ was set. Fig. (2.6) shows the segments fed.
5. Finally the switch was set to 'IN USE'.

INITIAL CONDITIONS

The temperatures at zero time were found from the rigorous solution as:
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<tr>
<td>24 midnight</td>
<td>83.44</td>
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**TABLE 2.3 : TEMPERATURE DISTRIBUTION IN A R.CONCRETE ROOF SLAB.**

"24 HOURS VALUES OF SOL-AIR TEMPERATURE AND CONSTANT INSIDE AIR TEMPERATURE".
SOL-AIR TEMPERATURE APPROXIMATION

FIG. 2.6

SOL-AIR TEMPERATURE (Tsl/4) - °F

TIME (HOURS)

0 2 4 6 8 10 12 14 16 18 20 22

0 2 4 6 8 10 12 14 16 18 20 22

segment

δx 1922 10.6 5.7 4.24 5.31 4.64 10.6 1922
δy -1.5 18.5 7.0 2.3 1.7 4.2 16.0 4.5
\[ T_{0/4} = 20.86^\circ F; \quad T_{1/4} = 21.53^\circ F \]
\[ T_{2/4} = 21.97; \quad T_{3/4} = 22.17 \]
\[ T_{4/4} = 22.16; \quad T_{1/4} = 21.93 \]

These initial conditions were set in their corresponding potentiometers fig- (2.7).

**X-Y PLOTTING TABLE**

**X-axis**

let 7.96 secs. (in computer) = 24 hours

and 12 in. on graph-paper = 24 hours

.  volts needed range from 0 - 12 volts. Use sensitivity of 1 volt/in.

. Reading of potentiometer B511 = \( \frac{12}{7.96} \times 0.1 = 0.151 \) volts

See fig (2.7)

**Y-axis**

Temperature ranged from (20 - 40)

. volts needed range from (20 - 40) volts. Use sensitivity = 5 volts/in. Fig. (2.8) shows the outputs obtained.

**2.2.2.3 Conclusions**

1. Fig. (2.9) shows a satisfactory agreement between the Analogue Computer Solution and the rigorous solution.

2. However, the variations that exist could be attributed to the following causes:

a) Replacing of the partial differential equation by finite difference approximation.

b) Inaccuracy in the potentiometers and amplifiers used.

c) Inaccuracy in the X-Y plotting table equipment.

d) Drift error in the Diode Function Generator caused by long time usage.
3. The temperature variation in all sections of the slab followed
a similar pattern to that of the sol-air temperature curve
with amplitudes decreasing and phase angle increasing through
the section of the slab.

This was typical of what was expected in an actual roof
slab in the tropics where the inside reaches its maximum tempera-
ture some hours after the outside surface.

4. From the two solutions obtained by the Analogue Computer, one

can satisfactorily conclude that the Analogue Computer could be
successfully employed in the solutions of temperature distribution
problems and similar problems.

5) From fig. (2.9), the curve given by the Analogue is not perfectly
sinusoidal at the end which may be due to inaccuracy in the Diode Function
Generator and X-Y plotting table.
Fig. (2.7) ANALOGUE COMPUTER NETWORK.

"24 HOURS VALUES OF SOLAR AIR TEMPERATURE AND CONSTANT INSIDE AIR TEMPERATURE."
Fig. (2.7)
FIG. (2.8) ANALOGUE COMPUTER RESULTS FOR TEMPERATURE DISTRIBUTION IN A CONCRETE ROOF SLAB
(24 HOURS RECORD OF SOL-AIR TEMPERATURE AND CONSTANT INSIDE AIR TEMPERATURE.)
FIG. (2.9) COMPARISON OF ANALOGUE COMPUTER RESULT WITH RIGOROUS RESULT
(recorded values of sol-air temperature)
CHAPTER III
THERMAL STRESS DISTRIBUTION IN SLABS

3.1 INTRODUCTION

Thermal stresses are computed in slabs with different boundary conditions and subjected to: 1) temperature variations through the thickness of the slab only and 2) temperature variations throughout the surface of the slab only.

Computer programs were developed for the solutions.

3.2 GENERAL SOLUTIONS

3.2.1. Assumptions

1. One-dimensional temperature distribution. Temperature variation through the thickness of the slab only.

2. Thermal properties of the material do not vary with either temperature or time. (Conductivity, specific heat, diffusivity)


4. Sections plane before heating remain so after heating (Bernoulli's assumptions).

5. Slab completely free of surface traction.

Using the coordinate axes as shown in fig. (3.1),

where d = slab thickness

L = slab length

and from the assumptions mentioned above, the
Fig. (3.1): COORDINATE AXES

Fig. (3.2): SLAB SECTIONS
temperature variation can be expressed by:

\[ T = T(x) \] ..........(1)

Under these conditions it is reasonable to assume the stresses in the form:

\[ \sigma_{zz} = \sigma_{yy} = f(x) \] ..........(2)

\[ \sigma_{xx} = \sigma_{yy} = \sigma_{xz} = \sigma_{yz} = 0 \] ..........(3)

Then the equilibrium equations, compatibility conditions and boundary conditions are applied as given by Boley and Weiner(3).

3.2.2 Equations of Equilibrium:

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + x = 0
\]

\[
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + y = 0
\] ..........(4)

\[
\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + z = \theta
\]

\(X, Y\) and \(Z\) are the components of body forces in the \(x, y, \) and \(z\) axes respectively.

3.2.3 Compatibility equations(3):

\[
(1 + \mu) \nabla^2 \sigma_{xx} + \frac{\partial^2 \theta}{\partial x^2} + \alpha E \left( \frac{1 + \mu}{1 - \mu} \nabla^2 T + \frac{\partial^2 T}{\partial x^2} \right) = 0 \quad .... (a)
\]

\[
(1 + \mu) \nabla^2 \sigma_{yy} + \frac{\partial^2 \theta}{\partial y^2} - \alpha E \left( \frac{1 + \mu}{1 - \mu} \nabla^2 T + \frac{\partial^2 T}{\partial y^2} \right) = 0 \quad .... (b)
\]

\[
(1 + \mu) \nabla^2 \sigma_{zz} + \frac{\partial^2 \theta}{\partial z^2} + \alpha E \left( \frac{1 + \mu}{1 - \mu} \nabla^2 T + \frac{\partial^2 T}{\partial z^2} \right) = 0 \quad .... (c)
\]

\[(1 + \mu)/\]
\[ (1 + \mu)\nabla^2 \sigma_{xz} + \frac{\partial^2 \Theta}{\partial x \partial z} + \alpha E \frac{\partial^2 T}{\partial x \partial z} = 0 \]  
\[ \text{(d)} \]

\[ (1 + \mu)\nabla^2 \sigma_{yx} + \frac{\partial^2 \Theta}{\partial x \partial y} + \alpha E \frac{\partial^2 T}{\partial x \partial y} = 0 \]  
\[ \text{(e)} \]

\[ (1 + \mu)\nabla^2 \sigma_{yz} + \frac{\partial^2 \Theta}{\partial y \partial z} + \alpha E \frac{\partial^2 T}{\partial y \partial z} = 0 \]  
\[ \text{(f)} \]

where \( \Theta = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \)

\[ \alpha = \text{coefficient of thermal expansion} \]

\[ E = \text{Young's modulus}; \ u = \text{Poisson's ratio} \]

The equilibrium equations and the compatibility equations are satisfied by the previous assumptions of the state of stress if

\[ \frac{\partial^2}{\partial x^2} (f(x) + \frac{\alpha E}{1 - \mu} T) = 0 \]  
\[ \text{(6)} \]

\[ \therefore f(x) = -\frac{\alpha E}{1 - \mu} T + Ax + B \]  
\[ \text{(7)} \]

The constants \( A \) and \( B \) are determined from the boundary conditions.

\( A \) represents the restraining moment function.

\( B \) represents the restraining force function.

3.2.4 Boundary Conditions

For fixed edges there is a restraining force as well as a restraining moment. \( \therefore A = B = 0 \)

and \( \sigma_{zz} = \sigma_{yy} = f(x) = -\frac{\alpha E}{1 - \mu} T \)  
\[ \text{(8)} \]

For no-end restraints:

\[ \sigma_{zz} = \sigma_{yy} = f(x) = \frac{\alpha E}{1 - \mu} \left[ T + \frac{\partial}{\partial x} \int \frac{a/2}{b} bTdx + x \int \frac{a/2}{b} b_x Tdx \right] \]  
\[ -\frac{\partial}{\partial x} \int \frac{a/2}{b} bdx + x \int \frac{a/2}{b} b_x^2 dx \]  
\[ \text{(9)} \]
If there is a restraining force only:

\[
\sigma_{zz} = \sigma_{yy} = f(x) = \frac{\alpha E}{1 - \mu} \left( -T + x \int_{-\frac{d}{2}}^{\frac{d}{2}} b \cdot x \cdot T dx \right)
\]

3.3 SLABS WITH FIXED EDGES

i) From equation (8)

\[
\sigma_{zz} = \sigma_{yy} = -\frac{\alpha E}{1 - \mu} T
\]

A computer program was developed for the solution of stresses \( \sigma_{zz} \), \( \sigma_{yy} \) for the temperature distribution in the concrete slab subjected to the environmental thermal data for Khartoum on the hottest month. See Appendix (G). Figure (3.3) shows the stress distribution.

ii) Dividing the slab into horizontal sections, every section representing a plate (Figure 3.2). Then applying Timoshenko's solution for a plate with clamped edges to each section.

Assuming the middle plane of the plate is free to expand but the edges are clamped.

The non-uniform heating produces bending moments uniformly distributed along the edges given by (20):

\[
M = \frac{2 \alpha \Delta T D (1 + \mu)}{h}
\]

\[
= \frac{2 \alpha \Delta T E h^3 (1 + \mu)}{12(1 - \mu^2)h}
\]

\[
= \frac{2 \alpha E \Delta T h^2}{12(1 - \mu)}
\]
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**TABLE 3.1 - TEMPERATURE DISTRIBUTION IN A R.CONECTE ROOF SLAB**
### TIME THERMAL STRESSES - lb/in²

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</tr>
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**TABLE 3.2 - THE STRESSES IN A SLAB WITH RESTRAINED EDGES**
FIG. (3.3) TEMPERATURE AND STRESS DISTRIBUTION IN A SLAB WITH FIXED EDGES
where \( D = \frac{Eh^3}{12(1 - \mu^2)} \)

= bending rigidity of plate

\( h \) = plate thickness , \( \mu \) = Poisson's ratio, 

\( E \) = Young's modulus of elasticity

Equation (11) is obtained from:

\[ e_z = \frac{X}{r_z}; \quad e_y = \frac{X}{r_y} \]

\[ M_z = D \left( \frac{1}{r_z} + \frac{h}{r_y} \right) \]

where \( e_z \) = strain in the z-direction, \( r_z \) and \( r_y \) are radii of curvature corresponding to the z and y axes.

\[ e_z = \alpha \Delta T \]

\[ \frac{1}{r_z} = \frac{\alpha \Delta T}{h/2} = \frac{2\alpha \Delta T}{h} \]

\[ M_z = D \left( \frac{2\alpha \Delta T}{h} + \mu \cdot 2\alpha \Delta T \right) \]

\[ M_z = D(1 + \mu) \cdot \frac{2\alpha \Delta T}{h} \]

Stresses \( \sigma_{zz}, \sigma_{yy} \) are obtained from:

\[ \sigma_{zz} = \sigma_{yy} = \frac{M_x}{l} = \frac{2\alpha E \Delta T}{(1 - \mu)} \cdot \frac{x}{h} \quad \ldots \ldots \ldots (12) \]

Fibre stresses (max.) = \( \sigma_{zz} \text{ max.} = \sigma_{yy} \text{ max.} = \frac{2\alpha E \Delta T}{(1 - \mu)} \cdot \frac{h/2}{h} \]

\[ = \frac{\alpha E \Delta T}{1 - \mu} \quad \ldots \ldots \ldots (13) \]

Thus the expression for the stresses obtained is the same as that given by equation (8).

3.4 SIMPLY SUPPORTED SLABS/
3.4 SIMPLY SUPPORTED SLABS

Strength of Materials Solution:-

The expression for the stresses in a simply supported slab is given by equation (9) as:

\[
\sigma_{zz} = \sigma_{yy} = \frac{\alpha E}{1 - \mu} \left[ \frac{1}{-d/2} - \frac{1}{-d/2} \int \frac{\alpha E T}{d} \frac{d^2 T}{d^2 x} \right]
\]

Using the notation:

\[
N_T = \alpha E \int_{-d/2}^{d/2} T dx
\]

\[
M_T = \alpha E \int_{-d/2}^{d/2} T x dx
\]

Equation (14) can be written as:

\[
\sigma_{zz} = \sigma_{yy} = \frac{1}{1 - \mu} \left[ -\alpha E T + \frac{N_T}{d} + \frac{12x}{d^3} M_T \right]
\]

\[
\sigma_{xx} = \sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0
\]

A computer program was employed for the solution of equation (15) for a concrete roof slab subjected to the environmental thermal data for Khartoum on the hottest month.

Appendix (H) contains the computer program used.

Figure (3.4) shows the stress distribution obtained.

3.5 SLABS WITH TEMPERATURE VARIATION ALONG THE SURFACE ONLY

Use co-ordinate axes as shown in fig. (3.5.a). Now since temperature \( T \) varies along slab surface,

\[
T = T(x, y)
\]
<table>
<thead>
<tr>
<th>Time</th>
<th>H</th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
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**Table 3.3 - The Stress Distribution in a Simply Supported Slab**
FIG. (3.4) TEMPERATURE AND STRESS DISTRIBUTION IN A SLAB WITH SIMPLY SUPPORTED EDGES
Fig. (3.5)
The stresses in the slab are given by (3):

\[
\sigma_{xx} = \frac{1}{1 - \mu} \left\{ -\alpha E T + \frac{1}{d} \left[ (1 - \mu) N_x + N_T \right] + \frac{122}{d^3} \left[ (1 - \mu) M_x + M_T \right] \right\} 
\]

\[
\sigma_{yy} = \frac{1}{1 - \mu} \left\{ -\alpha E T + \frac{1}{d} \left[ (1 - \mu) N_y + N_T \right] + \frac{122}{d^3} \left[ (1 - \mu) M_y + M_T \right] \right\} 
\]

\[
\sigma_{xy} = \frac{1}{1 - \mu} N_{xy} - \frac{122}{d^3} M_{xy} 
\]

where \( N_x, N_y, N_{xy} \) are forces per unit length, obtained from:

\[
N_x = \frac{\partial^2 F}{\partial y^2} ; \quad N_y = \frac{\partial^2 F}{\partial x^2} ; \quad N_{xy} = -\frac{\partial^2 F}{\partial x \partial y} ;
\]

where \( F(x,y) \) is a stress function given by:

\[
\nabla^4 F = -\nabla^2 N_T 
\]

where \( N_T = \alpha E \int_{-d/2}^{d/2} T dz \)

\( M_x, M_y, M_{xy} \) are obtained from:

\[
M_x = -D \left( \frac{\partial^2 W}{\partial x^2} + \mu \frac{\partial^2 W}{\partial y^2} \right) - \frac{M_T}{1 - \mu}
\]

\[
M_y = -D \left( \frac{\partial^2 W}{\partial y^2} + \mu \frac{\partial^2 W}{\partial x^2} \right) - \frac{M_T}{1 - \mu}
\]

\[
M_{xy} = (1 - \mu) D \frac{\partial^2 W}{\partial x \partial y}
\]

\[
D \triangle^4 W = \frac{1}{1 - \mu} \sqrt{M_T} ; \quad M_T = \alpha E \int_{-d/2}^{d/2} T \cdot z \cdot dz 
\]
where \( W \) = displacement in the z-direction

\[ D = \text{bending rigidity of plate per unit length} = \frac{Ed^3}{12(1-\mu^2)} \]

The stresses expressed by equations (17, 18, 19), are determined by solution of equations (20 and 22).

\[ i.e. \nabla^4 F = -\nabla^2 N_T \]

\[ \text{and } D\nabla^4 W = - \frac{1}{1-\mu} \nabla^2 M_T \]

3.5.1 MINIMUM STATIONARY COMPLEMENTARY ENERGY METHOD

(I) \( \nabla^4 F = -\nabla^2 N_T \) \hspace{1cm} (I.1)

Using complementary energy method for the solution of the biharmonic equation (3.9),

let \( F = f.g \); where \( f = f(x) \); \( g = g(y) \)

\[ Nx = f.g'' \]

\[ Ny = f''g \]

\[ Nxy = -f'g' \]

\[ N_T = X + N_{T_0} \]

where \( N_{T_0} = \text{value of } N_T \text{ at } x = 0 \)

The biharmonic equation is solved by choosing a function \( f \) (proportional to the stress \( \lambda \) of an infinitely long slab - \( l \to \infty \)) subject to \( N_T = X \), then the principle of minimum stationary complementary energy is used to determine \( g \).

Thus \( f = F \) from eqn (I.1), \( F'''' = -N_T'' = -X'' \)

\[ f''' = -X'' \]

\[ f'' = -X + \lambda_1 x + \lambda_2 \]

From eqn (3.5.6), \( N_T(x) = \alpha E \int (100-\xi) dx = 420 \alpha E \)

\[ N_T = 420 \alpha E - 5 \alpha E x \]

Where \( \alpha = \text{unknown value, see } \text{eqn } 3.5.6 \).
BOUNDARY CONDITIONS

At $x = 0, l$, displacement $U = V = 0$ for a slab with fixed edges:

\[
Nx = \frac{Ed}{1 - \mu} \left( \frac{\partial U}{\partial x} + \mu \frac{\partial V}{\partial y} \right) - \frac{NT}{1 - \mu} = \frac{NT}{1 - \mu} = f
\]

\[\text{...............(I.5)}\]

\[
N_{xy} = \frac{Ed}{2(1 + \mu)} \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) = 0 = -f' \cdot \delta'
\]

\[\text{...............(I.6)}\]

\[
f(0) = -\frac{NT(0)}{1 - \mu}
\]

\[f(1) = -\frac{NT(1)}{1 - \mu}
\]

\[f'(0) = 0 \]

\[f'(1) = 0 \]

At $y = \pm 1/2$, $U = V = 0$

\[
N_{y} = -\frac{NT}{1 - \mu} + \frac{Ed}{1 - \mu} \left( \frac{\partial V}{\partial y} + \mu \frac{\partial U}{\partial x} \right)
\]

\[\text{...............(I.7)}\]

\[
N_{y} \left( \pm \frac{1}{2} \right) = \frac{NT \left( \pm \frac{1}{2} \right)}{1 - \mu}
\]

\[\text{...............(I.8)}\]

\[N_{xy} \left( \pm \frac{1}{2} \right) = 0 \]

From equation (I.3), "Equation" (I.3) is... relation. How does (I.3) follow from it?

\[
f = \frac{x^3}{6} + \frac{Ax^3}{6} + A_2 \frac{x^2}{2} + A_3 x + A_4 \]

\[\text{...............(I.9)}\]
C.E./unit length of slab is

\[ U_0 = \frac{1}{2E} \int_{-1/2}^{1/2} \left[ N x^2 + N y^2 - 2\mu N x N y + 2(1 + \mu) N x y^2 + 2E\alpha N_T (N x + N y) \right] dxdy \]  \[ \ldots \ldots \ldots \ldots \ldots \ldots \ldots (I.10) \]

Substituting from (I.2) into (I.10) we get

\[ U_0 = \frac{1}{2E} \int_{-1/2}^{1/2} \left[ B_1 \varepsilon''^2 + B_2 \varepsilon''^2 - 2\mu B_3 \varepsilon'' + 2(1 + \mu) \varepsilon''^2 \right] dxdy \]

\[ + 2E\alpha \left\{ (B_5 + B_6) \varepsilon + (B_7 + B_8) \varepsilon'' \right\} dy \]  \[ \ldots \ldots \ldots \ldots \ldots \ldots \ldots (I.11) \]

where

\[ B_1 = \int_0^1 f'' dx; \quad B_2 = \int_0^1 f''^2 dx; \quad B_3 = \int_0^1 f f'' dx = \left[ f' \right]_0^1 - B_4 \]

\[ B_4 = \int_0^1 f''dx; \quad B_5 = NT_0 \int_0^1 f'' dx = NT_0 \left[ f' \right]_0^1 \]

\[ B_6 = \int_0^1 f f'' dx; \quad B_7 = NT_0 \int_0^1 f dx; \quad B_8 = \int_0^1 X f dx \]

Principle of stationery C.E.: a stationary value of \( U_0 \) distinguishes the correct stress distribution among all distributions which satisfy the conditions of equilibrium and of prescribed surface tractions (3).

Thus minimizing the C.E. by setting the 1st variation of \( U_0 \) equal to zero:

\[ \delta U_0 = \frac{1}{E} \int_{-1/2}^{1/2} \left[ B_1 \varepsilon''^2 + B_2 \varepsilon''^2 - 2 \left\{ B_4 + \mu (B_3 + B_4) \right\} \varepsilon'' + E\alpha (B_5 + B_6) \right] dxdy \]

\[ - \frac{1}{E} \left\{ \left( \mu B_3 + 2(1 + \mu) B_4 \right) \varepsilon' - B_1 \varepsilon''' \right\} \delta \varepsilon \int_{-1/2}^{1/2} dy \]

\[ \frac{1}{E} \left[ B_1 \varepsilon''^2 - \mu B_3 \varepsilon'' + E\alpha (B_7 + B_8) \right] \delta \varepsilon' \int_{-1/2}^{1/2} \ldots (I.12) \]
Euler's differential equation for the fn g is:

\[ B_1 \cdot e^{iv} + B_2 g - 2 \left[ B_4 + \mu (B_3 + B_4) \right] g'' + B_5 g + B_6 = 0 \]

................(I.12)

The remaining terms set equal to zero are the boundary conditions.

...from boundary conditions \( B_1, B_2, B_3, B_4, B_5, B_6 \) can be found

Differential equation for g is of the form:

\[ a \cdot e^{iv} + b g'' + c g + d = 0 \]

................(I.13)

Solution of (I.13) is in the form:

\[ g = - \frac{d}{c} + (A_2 \cosh \alpha y \cdot \cos \beta y + A_3 \cosh \alpha y \cdot \sin \beta y + \]

\[ A_4 \sinh \alpha y \cdot \cos \beta y + A_5 \sinh \alpha y \cdot \sin \beta y) \]

The constants are obtained from boundary conditions.

Having determined \( f \) and \( g \), stress fn \( F \) and forces/unit length \( N_x, N_y \) and \( N_{xy} \) are determined from equation (I.2).

3.5.2 Finite Difference Approximations Method

(II) \( D \frac{h}{h} W = - \frac{1}{1 - \mu} \frac{2}{h^2} \frac{\partial^2 M_T}{\partial x^2} \)

................(II.1)

The above equation is approximated by finite difference methods and the resulting set of equations solved by a computer program.

Boundary Conditions

\[ W = \frac{\partial W}{\partial n} = 0 \quad \text{at} \ x = 0, 1 \quad \text{.........(II.2)} \]

and \( y = \pm \frac{1}{2} \)

From finite difference approximations, (see \( \epsilon \), 3.6.6)

\[ (\frac{h^4}{h^4}) W = \frac{1}{h^4} \left[ 8 \sum_{i=1}^{4} W_i - 2 \sum_{i=5}^{8} W_5 - \sum_{i=9}^{12} W_9 - 20 W_0 \right] \]

................(II.3)

\[ \frac{\partial^2 M_T}{\partial x^2} = M_{T1} + M_{T2} + M_{T3} + M_{T4} - 4M_T \]

But \( M_T = M_T(x) \); \[ \frac{\partial^2 M_T}{\partial x^2} = \frac{d^2 M_T}{dx^2} = \frac{(M_{T1} - 2M_{T0} + M_{T3})}{a^2} \]

................(II.4)
From (II.1, II.3 and II.4)

\[-8 \sum_{1}^{4} W_{1} + 2 \sum_{5}^{8} W_{5} + \sum_{9}^{12} W_{9} + 20W_{0} = -\frac{d^{2}}{D(1 - \mu)} x\]

\[(MT_{1} - 2MT_{0} + MT_{3}) \] \[\text{................(II.5)}\]

This relationship was satisfied at all mesh points (fig. 3.6.a.)

Computer program (Sparse Matrix Routines)

The deflection \(W\) is obtained by using a computer program (Appendix J). Having determined \(W, Mx, My\) and \(Mxy\) are obtained from equation (21). Thus knowing \(Nx, Ny, Nxy\) and \(Mx, My, Mxy\) the stresses are determined from equations (17, 18 and 19).
Fig. (3.G) MESH POINTS
CHAPTER IV

THERMAL STRESSES IN WALLS

4.1 INTRODUCTION

The temperature differential between roofs and walls leads to different degrees of expansion of roofs and walls resulting in the development of thermal strains in the roof slabs, in the roof-wall boundaries and in the wall sections.

When these thermal stresses exceed the allowable tensile strengths of the roof and wall materials, thermal cracks develop.

For the determination of the stress condition that exists in an actual building in the tropics subjected to the incidence of solar radiation, two models were studied.

The first model consisted of reinforced concrete roof slab and brickwork walls. The second model consisted of asbestos cement roof slab and walls.

The Finite Difference Method was used for the approximation of the Biharmonic equation governing the stress distribution as well as for the differential equations governing the boundary conditions. The resulting set of simultaneous equations was solved by matrix methods employing a computer program for that purpose.
4.2 THERMAL STRESSES IN WALLS RESULTING FROM THE TEMPERATURE DIFFERENTIAL BETWEEN THE ROOF AND WALL

ASSUMPTIONS

1) The slab is at a mean temperature higher than that of the wall.
2) Temperature varies through the slab thickness and wall height only.
3) Thermal properties as well as elastic properties of slab and wall do not change with temperature.
4) Sections plane before heating remain so after heating.
5) Plane stress condition.
6) Homogeneous wall.

Along the roof-wall boundary, compatibility condition as well as equilibrium conditions are satisfied.

COMPATIBILITY EQUATION

By the assumption that plane sections remain plane after heating:

\[ e_R - e_W = 0 \] ......(4.1)

where \( e_R \) = strain at the bearing plane of roof
\( e_W \) = strain at the bearing plane of wall

Since the roof slab is at a higher temperature \( T_R \) than the temperature of the wall \( T_W \), the roof exerts a tensile force \( F_W \) on the wall and the wall reacts by an equal and opposite compressive force \( F_R \) \((18,25)\) (fig. 4.1.b)

EQUILIBRIUM EQUATION

\[ F_W + F_R = 0 \] ......(4.2)

roof strain is given by:
FIG. (4.1)  ANALYSIS OF THERMAL STRESSES IN WALLS.
where $\Delta T_R$ = change in mean temperature of roof
$\Delta T_W$ = change in mean temperature of wall

wall strain is given by:

$$
e_W = \frac{\sigma_W}{E_W} \quad \ldots \ldots (4.4)$$

where $\sigma_W$ = stress at the top of the wall.

Substituting from (4.3) and (4.4) into equation (4.1), we get:

$$\alpha_R \Delta T - \frac{\sigma_W}{E_W} - \frac{\sigma_R}{E_R} = 0 \quad \ldots \ldots (4.5)$$

where $\Delta T = \Delta T_R - \Delta T_W$

Equation (4.2) can be expressed as:

$$\tau \cdot S \cdot L/2 + \sigma_R \cdot A_R = 0 \quad \ldots \ldots (4.2)'$$

where $\tau =$ shear stress at boundary, $S =$ wall thickness, $L =$ wall length

From fig. (4.1.c,d,e,f), the stress in any section at distance $y$ from the top is given by:

$$\sigma_W = \frac{F}{S \cdot h} + \frac{F \cdot h/2}{S \cdot h^3/12} (h/2 - y)$$

$$= \frac{F}{S \cdot h} \left[ \frac{h}{2} (h - y) + 1 \right] \quad \ldots \ldots (4.6)$$

$$F = - \sigma_R \cdot A_R \quad \ldots \ldots (4.7)$$

From equation (4.5),

$$\sigma_R = E_R \cdot \alpha_R \cdot \Delta T - \frac{\sigma_W}{E_W} \cdot E_R$$

Substituting in equation (4.7), we get:
Substituting for $F$ from equation (4.8) into equation (4.6) we get:

$$F = -E_R A_R \left( -\frac{\sigma_{WO}}{E_W} + \alpha_R \Delta T \right) \quad \ldots \ldots (4.8)$$

Substituting for $\sigma_{WO}$ from equation (4.8) into equation (4.6) we get:

$$\sigma_{WO} = -\frac{E_R A_R}{S \cdot h} \left[ -\frac{\sigma_{WO}}{E_W} + \alpha_R \Delta T \right] \left[ -\frac{6Y}{h} + 4 \right] \quad \ldots \ldots (4.9)$$
4.3 THERMAL STRESSES IN WALLS DUE TO TEMPERATURE GRADIENT ALONG WALL HEIGHT

Dividing the wall into horizontal sections (fig. 4.2.a) and each section assumed to represent a beam, then the solution for a rectangular beam restrained at its edges and with temperature variations through the thickness is applied to each section:

Thus $\sigma_{xx} = -\alpha E T(z)$ \hspace{1cm} \ldots \ldots \ldots (4.10)

Additional stresses are:

$\delta \sigma_{W1} = -\alpha E (T_1 - T_w)$
$\delta \sigma_{W2} = -\alpha E (T_2 - T_w)$
$\delta \sigma_{W3} = -\alpha E (T_3 - T_w)$
$\delta \sigma_{W4} = -\alpha E (T_4 - T_w)$
$\delta \sigma_{W5} = -\alpha E (T_5 - T_w)$
$\delta \sigma_{W6} = -\alpha E (T_6 - T_w)$ \hspace{1cm} \ldots \ldots \ldots (4.11)

where $T_w = \text{mean temperature of wall}$

Final stresses in the wall are thus obtained by superimposing those determined by equation (4.11) on those given by equation (4.6).

In equation (4.6) the slab has been assumed of a constant mean temperature $T_R$.

But if the temperature varies with the depth, the following adjustment of equation (4.6) is suggested:

Divide the slab into horizontal sections as shown in fig. (4.2.d).

Force in section (1) is $F_1 = -\alpha E R (T_1 - T_0) A_1$

Force in section (2) is $F_2 = -\alpha E R (T_2 - T_0) A_2$

Force in section (3) is $F_3 = -\alpha E R (T_3 - T_0) A_3$
WALL SECTIONS

(a)

TEMPERATURE DISTRIBUTION

(b)

CO-ORDINATE AXES

(c)

ROOF AND WALL SECTIONS

(d)

FIG. (4.2)
Horizontal force transmitted from roof to wall \( = \sum F \) .........(4.12)

Equation (4.6) should be accordingly changed into:

\[
\sigma_w = \frac{\sum F}{3.4h} \left[ \frac{6}{\gamma h} \left( \frac{h}{2} - y \right) + 1 \right] \quad .........(4.13)
\]
4.4 RIGOROUS ANALYTICAL SOLUTION OF THERMAL STRESSES IN WALLS

Assume co-ordinate axes as shown in fig. (4.3), and assume temperature variation along wall width and height, 

\[ T_m = T(x, y) \]

i.e. \( T_m = T(x, y) \) from meteorological data, obtained from calculating the outside and inside surface temperatures (Chap. II). Airy's stress function governing the stress condition along wall surface is given by (3):

\[
\nabla^4 \Phi = -\alpha E \nabla^2 T \\
\sigma_x = \frac{\partial^2 \Phi}{\partial y^2} ; \quad \sigma_y = \frac{\partial^2 \Phi}{\partial x^2} ; \quad \sigma_{xy} = -\frac{\partial^2 \Phi}{\partial x\partial y}
\]

Displacements are given by (4.8):

\[
U = -\frac{1 + \mu}{E} \frac{\partial \Phi}{\partial x} + \frac{1 + \nu}{E} \frac{\partial \Phi}{\partial y} - \alpha T x \\
V = -\frac{1 + \mu}{E} \frac{\partial \Phi}{\partial y} + \frac{1 + \nu}{E} \frac{\partial \Phi}{\partial x} + \alpha T y
\]

where \( \frac{\partial^2 \Phi}{\partial x\partial y} = \nabla^2 \Phi \) and \( \nabla^2 \psi = 0 \)

BOUNDARY CONDITIONS

i) At \( y = 0, \phi = 0 \), \( \sigma_{w,0} = \frac{\partial^2 \Phi}{\partial y^2} = \frac{\alpha E R \Delta T}{A_R} \frac{\partial \Phi}{\partial y} + \frac{S_E W}{A_R E_R} \frac{\partial \Phi}{\partial y} \)

The expression for \( \sigma_{w,0} \) is derived from considerations of compatibility and equilibrium conditions along roof-wall boundary.

From equation (4.3),

\[
e_R = \alpha_R \Delta T - \frac{\sigma_R}{E_R}
\]

But from equation (4.5),

\[
\sigma_{w,0} = E_R \alpha_R \Delta T - \frac{E_R}{E_W} \sigma_R
\]
FIG. (4.3)  CO-ORDINATE AXES FOR WALL.
From equation (4.2)

\[ \sigma_R = \frac{F}{A_R} = \frac{1}{A_R} \int - \frac{Q}{\partial x \partial y} \text{d}x \]

\[ = - \frac{S}{A_R} \frac{\partial \phi}{\partial y} \]

\[ \therefore \sigma_{Wo} = \frac{E}{W} \Delta T + \frac{E}{E} \frac{S}{A_R} \frac{\partial \phi}{\partial y} \]

ii) At \( x = \pm \frac{1}{2} \), \( \phi = 0 \), \( \frac{\partial \phi}{\partial x} = 0 \)

\[ \ldots \ldots (4.18b) \]

iii) At \( y = h \), \( U = V = 0 \);

\[ \frac{\partial \psi}{\partial x} = 0 ; \quad \frac{\partial^2 \psi}{\partial x \partial y} = \nabla^2 \phi \]

\[ \ldots \ldots (4.18c) \]

Since the wall surface is symmetrical about the y-axis, only one half of the wall is treated.

Using the finite difference method for the solution of the biharmonic equation governing the stress condition through wall surface as well as the differential equations involved in the boundary conditions, we divide over wall-surface into a grid of square meshes of \( d \) in. size. Fig(4.4)

Then the solution rests on satisfying the biharmonic differential equation (4.15) at all mesh points as well as satisfying the boundary condition as given by (4.18).

Fictitious points have to be assumed on the boundaries to supply the necessary number of functions that formulate the solution.

The finite difference approximations comprise the following relationships:

\[ - \Delta \phi = +\alpha \Delta T \quad \text{where} \quad T = \text{temperature change} \]
\[ \frac{1}{4} (8 \sum_{i=1}^{4} \Phi_i - 2 \sum_{j=5}^{8} \Phi_j - 2 \sum_{k=9}^{12} \Phi_k - 20 \Phi_0) \quad (\text{see fig. below}) \]

\[ = \alpha \cdot E \frac{(T_1 + T_2 + T_3 + T_4 - 4T_0)}{d^2} \quad \cdots \cdots (4.19a) \]

\[ \sigma_x = \frac{\partial \phi}{\partial y^2} = \frac{\Phi_2 + \Phi_4 - 2 \Phi_6}{d^2} \quad \cdots \cdots (4.19b) \]

\[ \sigma_y = \frac{\partial \phi}{\partial x^2} = \frac{\Phi_1 + \Phi_3 - 2 \Phi_7}{d^2} \quad \cdots \cdots (4.19c) \]

\[ \sigma_{xy} = -\frac{\partial \phi}{\partial x \partial y} = \frac{\Phi_5 - \Phi_6 + \Phi_7 - \Phi_8}{4d^2} \quad \cdots \cdots (4.19d) \]

\[ \frac{\partial \phi}{\partial y} = \frac{\Phi_2 - \Phi_4}{2d} \quad \cdots \cdots (4.19e) \]

\[ \frac{\partial \phi}{\partial x} = \frac{\Phi_1 - \Phi_3}{2d} \quad \cdots \cdots (4.19f) \]

\[ \frac{\partial^2 \psi}{\partial x^2} = \frac{\psi_1 - \psi_3}{2d} ; \frac{\partial^2 \psi}{\partial y^2} = \frac{-\psi_2 - \psi_4}{2a} \quad \cdots \cdots (4.19g) \]

\[ \nabla^2 \phi = \frac{\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 - 4\Phi_0}{d^2} \quad \cdots \cdots (4.19h) \]

\[ \frac{\partial^2 \psi}{\partial x \partial y} = \frac{-\psi_5 + \psi_6 + \psi_7 - \psi_8}{4d^2} \quad \cdots \cdots (4.19i) \]
FIG. (4.4) | GRID FOR FINITE DIFFERENCE FORMULATIONS

N.B. Numbers in parenthesis refer to displacement functions $\psi$. 
Satisfying the biharmonic equation (4.15) at all mesh points, we get the following set of equations:

\[ \begin{align*} 
\text{PT}(1): & \quad 8\Phi_1 - \Phi_3 + 16\Phi_7 - 4\Phi_8 - 2\Phi_3 - \Phi_1 - 20\Phi \nonumber \\
& \quad = + \alpha E_d^2(T(0,0) + 2T + T_{-2} - T_1) \\
\text{PT}(2): & \quad 16\Phi_8 + 8\Phi_4 + 8\Phi_5 - 4\Phi_7 - 4\Phi_9 - 2\Phi_4 - \Phi_5 - 20\Phi \nonumber \\
& \quad = + \alpha E_d^2(T_1 + 2T_8 + T_3 - 4T_2) \\
\text{PT}(3): & \quad 16\Phi_9 + 8\Phi_2 + 8\Phi_4 - 4\Phi_8 - 4\Phi_{10} - 2\Phi_5 - \Phi_1 - \Phi_5 - 20\Phi \nonumber \\
& \quad = + \alpha E_d^2(T_2 + 2T_9 + T_4 - 4T_3) \\
\text{PT}(4): & \quad 16\Phi_{10} + 8\Phi_3 + 8\Phi_5 - 4\Phi_9 - 4\Phi_{11} - 2\Phi_6 - \Phi_2 - \Phi_6 - 20\Phi \nonumber \\
& \quad = + \alpha E_d^2(T_3 + 2T_0 + T_5 - 4T_4) \\
\text{PT}(5): & \quad 16\Phi_4 + 8\Phi_6 + 8\Phi_9 - 4\Phi_{10} - 4\Phi_{12} - 2\Phi_7 - \Phi_3 - \Phi_4 - 20\Phi \nonumber \\
& \quad = + \alpha E_d^2(T_4 + 2T_{11} + T_6 - 4T_5) \\
\text{PT}(6): & \quad 16\Phi_{12} + 8\Phi_5 + 8\Phi_{41} - 4\Phi_{11} - 4\Phi_{40} - 2\Phi_{13} - \Phi_4 - \Phi_4 - 20\Phi \nonumber \\
& \quad = + \alpha E_d^2(T_5 + 2T_{12} + T_{53} + 4T_6) \\
\text{PT}(7): & \quad 3\Phi_1 - 2\Phi_2 + 8\Phi_8 - \Phi_9 + 8\Phi_{13} - 2\Phi_{14} - \Phi_9 - \Phi_3 - 20\Phi \nonumber \\
& \quad = + \alpha E_d^2(T(6,0) + T_1 + T_6 - 4T_7) \\
\text{PT}(8): & 
\end{align*} \]
\[ \Phi_{14} + 8 \Phi_7 + 8 \Phi_2 + 8 \Phi_9 - 2 \Phi_3 - 2 \Phi_1 - 2 \Phi_2 - 2 \Phi_4 - 2 \Phi_15 - \Phi_20 - \Phi_10 = 0 \]

\[ - 21 \Phi_8 = + \alpha E d^2 (T_7 + T_2 + T_9 + T_{14} - 4T_8) \quad \ldots \ldots \quad (8) \]

\[ \Phi_{15} + 8 \Phi_3 + 8 \Phi_9 - 2 \Phi_14 - 2 \Phi_2 - 2 \Phi_4 - 2 \Phi_16 - \Phi_21 - \Phi_7 = \]

\[ - \Phi_{11} - 21 \Phi_9 = + \alpha E d^2 (T_8 + T_3 + T_{10} + T_{15} - 4T_9) \quad \ldots \ldots \quad (9) \]

\[ \Phi_{16} + 8 \Phi_3 + 8 \Phi_4 + 8 \Phi_{11} - 2 \Phi_5 - 2 \Phi_3 - 2 \Phi_4 - 2 \Phi_7 - \Phi_{22} - \Phi_8 = \]

\[ - \Phi_{12} - 21 \Phi_{10} = + \alpha E d^2 (T_9 + T_4 + T_{11} + T_{16} - 4T_{10}) \quad \ldots \ldots \quad (10) \]

\[ \Phi_{17} + 8 \Phi_9 + 8 \Phi_2 - 2 \Phi_16 - 2 \Phi_4 - 2 \Phi_6 - 2 \Phi_8 - \Phi_{23} - \Phi_9 = \]

\[ - \Phi_{40} - 21 \Phi_{11} = + \alpha E d^2 (T_{10} + T_5 + T_{12} + T_{17} - 4T_{11}) \quad \ldots \ldots \quad (11) \]

\[ \Phi_{18} + 8 \Phi_9 + 8 \Phi_2 + 8 \Phi_{49} - 2 \Phi_{17} - 2 \Phi_5 - 2 \Phi_4 - 2 \Phi_3 - \Phi_{24} - \Phi_{10} = \]

\[ - \Phi_{43} - 21 \Phi_{12} = + \alpha E d^2 (T_{11} + T_6 + T_{52} + T_{18} - 4T_{12}) \quad \ldots \ldots \quad (12) \]

\[ \Phi_{1} + 8 \Phi_7 - 2 \Phi_8 + 8 \Phi_{14} - \Phi_5 + 8 \Phi_9 - 2 \Phi_{20} - \Phi_{25} - \Phi_{33} = \]

\[ - 20 \Phi_{13} = + \alpha E d^2 (T_{12,0} + T_7 + T_{14} + T_{19} - 4T_{13}) \quad \ldots \ldots \quad (13) \]

\[ \Phi_{20} + 8 \Phi_3 + 8 \Phi_8 + 8 \Phi_{15} - 2 \Phi_19 - 2 \Phi_7 - 2 \Phi_9 - 2 \Phi_21 - \Phi_{26} - \Phi_2 = \]

\[ - \Phi_6 - 20 \Phi_{14} = + \alpha E d^2 (T_{13} + T_8 + T_{15} + T_{20} - 4T_{14}) \quad \ldots \ldots \quad (14) \]
PT (15): \[ 8 \Phi_2 + 8 \Phi_4 + 8 \Phi_6 - 2 \Phi_{10} - 2 \Phi_{22} - \Phi_{27} - \Phi_{13} - \Phi_3 - \Phi_{17} - 20 \Phi_{15} = + \alpha E \cdot d^2 (T_{14} + T_9 + T_{16} + T_{21} - 4T_{15}) \]

\[ \cdots \cdots \cdots (15) \]

PT (16): \[ 8 \Phi_2 + 8 \Phi_4 + 8 \Phi_7 - 2 \Phi_1 - 2 \Phi_9 - 2 \Phi_{11} - 2 \Phi_{12} - \Phi_{28} - \Phi_{14} - \Phi_4 - \Phi_{18} - 20 \Phi_{16} = + \alpha E \cdot d^2 (T_{15} + T_{10} + T_{17} + T_{22} - 4T_{16}) \]

\[ \cdots \cdots \cdots (16) \]

PT (17): \[ 8 \Phi_3 + 8 \Phi_6 + 8 \Phi_8 - 2 \Phi_{12} - 2 \Phi_{10} - 2 \Phi_{22} - 2 \Phi_{24} - \Phi_{29} - \Phi_{15} - \Phi_5 - \Phi_{39} - 20 \Phi_{17} = + \alpha E \cdot d^2 (T_{16} + T_{11} + T_{18} + T_{23} - 4T_{17}) \]

\[ \cdots \cdots \cdots (17) \]

PT (18): \[ 8 \Phi_4 + 8 \Phi_7 + 8 \Phi_9 - 2 \Phi_{39} - 2 \Phi_{23} - 2 \Phi_{11} - 2 \Phi_{38} - \Phi_{30} - \Phi_{16} - \Phi_6 - \Phi_{44} - 20 \Phi_{18} = + \alpha E \cdot d^2 (T_{17} + T_{12} + T_{24} + T_{25} - 4T_{18}) \]

\[ \cdots \cdots \cdots (18) \]

PT (19): \[ 8 \Phi_5 + 8 \Phi_3 + 8 \Phi_20 - 2 \Phi_{14} - 2 \Phi_{26} - \Phi_{34} - \Phi_{7} - \Phi_{21} - 20 \Phi_{19} = + \alpha E \cdot d^2 (T_{18,0} + T_{13} + T_{20} + T_{25} - 4T_{19}) \]

\[ \cdots \cdots \cdots (19) \]

PT (20): \[ 8 \Phi_6 + 8 \Phi_1 + 8 \Phi_6 - 2 \Phi_{25} - 2 \Phi_{13} - 2 \Phi_{25} - 2 \Phi_{27} - \Phi_{8} - \Phi_{22} - 20 \Phi_{20} = + \alpha E \cdot d^2 (T_{19} + T_{14} + T_{21} + T_{26} - 4T_{20}) \]

\[ \cdots \cdots \cdots (20) \]
PT(21): $8\Phi_7 + 8\Phi_20 + 8\Phi_5 + 8\Phi_22 - 2\Phi_6 - 2\Phi_4 - 2\Phi_6 - 2\Phi_8 - \Phi_9$

- $\Phi_9 - \Phi_23 - 20\Phi_21 = +\alpha E_\mu d^2(T_{20} + T_{15} + T_{22} + T_{27})$

- $4T_{21}$

.............(21)

PT(22): $8\Phi_28 + 8\Phi_1 + 8\Phi_16 + 8\Phi_23 - 2\Phi_27 - 2\Phi_5 - 2\Phi_7 - 2\Phi_29 - \Phi_20$

- $\Phi_10 - \Phi_24 - 20\Phi_22 = +\alpha E_\mu d^2(T_{21} + T_{16} + T_{23} + T_{28} - 4T_{22})$

.............(22)

PT(23): $8\Phi_9 + 8\Phi_22 + 8\Phi_7 + 8\Phi_24 - 2\Phi_28 - 2\Phi_16 - 2\Phi_21 - 2\Phi_30 - \Phi_21$

- $\Phi_11 - \Phi_38 - 20\Phi_23 = +\alpha E_\mu d^2(T_{22} + T_{17} + T_{24} + T_{29} - 4T_{23})$

.............(23)

PT(24): $8\Phi_3 + 8\Phi_23 + 8\Phi_38 + 8\Phi_29 - 2\Phi_27 - 2\Phi_17 - 2\Phi_39 - 2\Phi_37 - \Phi_22$

- $\Phi_12 - \Phi_45 - 20\Phi_24 = +\alpha E_\mu d^2(T_{23} + T_{18} + T_{50} + T_{30} - 4T_{24})$

.............(24)

PT(25): $8\Phi_1 + 8\Phi_25 - 2\Phi_20 - \Phi_35 - \Phi_13 - \Phi_27 - 21\Phi_25 = +\alpha E_\mu d^2$

$(T_{24,0}) + T_{19} + T_{26} + T(30,6) - 4T_{25})$ .............(25)

PT(26): $8\Phi_25 + 8\Phi_20 + 8\Phi_27 - 2\Phi_19 - 2\Phi_21 - \Phi_14 - \Phi_28 - 21\Phi_26 = +\alpha E_\mu d^2(T_{25} + T_{20} + T_{27} + T(30,12) - 4T_{26})$.............(26)

PT(27): $8\Phi_2 + 8\Phi_21 + 8\Phi_28 - 2\Phi_20 - 2\Phi_22 - \Phi_25 - \Phi_15 - \Phi_29 - 21\Phi_27$

- $+\alpha E_\mu d^2(T_{26} + T_{21} + T_{28} + T(30,18) - 4T_{27})$.............(27)
\[
\text{PT(28)}: 8 \Phi_27 + 8 \Phi_22 + 8 \Phi_29 - 2 \Phi_21 - 2 \Phi_23 - 2 \Phi_26 - \Phi_16 - \Phi_30 - 2 \Phi_28
= \alpha E \cdot a^2 (T_{27} + T_{22} + T_{29} + T(30,24) - 4T_{28}) \cdots (28)
\]

\[
\text{PT(29)}: 8 \Phi_28 + 8 \Phi_23 + 8 \Phi_30 - 2 \Phi_22 - 2 \Phi_24 - \Phi_17 - \Phi_37 - 2 \Phi_29
= \alpha E \cdot a^2 (T_{28} + T_{23} + T_{30} + T(30,30) - 4T_{29}) \cdots (29)
\]

\[
\text{PT(30)}: 8 \Phi_29 + 8 \Phi_24 + 8 \Phi_37 - 2 \Phi_23 - 2 \Phi_38 - \Phi_28 - \Phi_46 - 2 \Phi_30
= \alpha E \cdot a^2 (T_{29} + T_{24} + T_{49} + T(30,36) - 4T_{30}) \cdots (30)
\]
Satisfying the boundary conditions given by equation (4.18.a) we have

\[ \frac{\partial^2 \Phi}{\partial y^2} = \alpha \frac{E}{R_W} \Delta T + \frac{S_{E}}{A_R B_R} \cdot \frac{\partial \Phi}{\partial y} = A + B \frac{\partial \Phi}{\partial y} \]

\[ \therefore \Phi_{31} - \frac{2x_0 + \Phi_1}{d^2} = A + B \left( \frac{\Phi_1 - \Phi_{31}}{2d} \right) \]

\[ \therefore \Phi_{31} + \Phi_1 = d^2 A + \frac{d}{2} B (\Phi_1 - \Phi_{31}) \]

\[ \Phi_{32} + \Phi_7 = d^2 A + \frac{d}{2} B (\Phi_7 - \Phi_{32}) \]

\[ \Phi_{33} + \Phi_9 = d^2 A + \frac{d}{2} B (\Phi_9 - \Phi_{33}) \]

\[ \Phi_{34} + \Phi_{19} = d^2 A + \frac{d}{2} B (\Phi_{19} - \Phi_{34}) \]

\[ \Phi_{35} + \Phi_{25} = d^2 A + \frac{d}{2} B (\Phi_{25} - \Phi_{35}) \]

\[ \Phi_{36} + 0 = d^2 A + \frac{d}{2} B (0 - \Phi_{36}) \]

Assuming linear variations of the function \( \Phi \) in the region of the following mesh points:

30-37-46; 24-38-45; 18-39-44; 12-40-43; 6-41-42

\[ \therefore \Phi_{30} + \Phi_{46} - 2 \Phi_{37} = 0 \]

\[ \Phi_{24} + \Phi_{45} - 2 \Phi_{38} = 0 \]

\[ \Phi_{18} + \Phi_{44} - 2 \Phi_{39} = 0 \]

\[ \Phi_{12} + \Phi_{43} - 2 \Phi_{40} = 0 \]

\[ \Phi_6 + \Phi_{42} - 2 \Phi_{41} = 0 \]
Satisfying the boundary conditions given by equation (4.18.c), we get the following set of equations:

\[ FT(6): \quad U_6 = 0 = \frac{1}{E} \frac{\partial \phi}{\partial x} + \frac{1}{E} \frac{\partial \psi}{\partial y} + \alpha T_x \]

\[ \therefore \quad 0 = \frac{1}{E} \phi_2 - \frac{\phi_1}{2d} + \frac{1}{E} \frac{\psi_1 - \psi_5}{2d} + \alpha T(0) \]

\[ \psi_{41} - \psi_5 = 0 \] 

\[ V_6 = \frac{1}{E} \frac{\partial \phi}{\partial y} + \frac{1}{E} \frac{\partial \psi}{\partial x} + \alpha T_y = 0 \]

\[ \therefore \quad - \frac{1}{E} \left( \frac{\phi_{41} - \phi_5}{2d} \right) + \frac{1}{E} \frac{\psi_{12} - \psi_{12}}{2d} + \alpha T(h) = 0 \]

\[ \therefore \quad - (1 + \mu) \left( \phi_{41} - \phi_5 \right) = - \alpha E T 2d h \]

\[ \therefore \quad \frac{\partial^2 \psi}{\partial x \partial y} = \nabla^2 \phi \]

\[ \therefore \quad \frac{\psi_{11} - \psi_{11} + \psi_{40} - \psi_{40}}{4d^2} = \frac{2 \phi_{12} + \phi_5 + \phi_{41} - 4 \phi_6}{d^2} \]

\[ \therefore \quad 2 \phi_{12} + \phi_5 + \phi_{41} - 4 \phi_6 = 0 \]

\[ FT(12): \quad U_{12} = 0 \quad \therefore \quad - (1 + \mu) \left( \phi_6 - \phi_6 \right) + \left( \psi_{40} - \psi_{11} \right) \]

\[ = \alpha E 2T d^2 \]

\[ V_{12} = 0 \quad \therefore \quad - (1 + \mu) \left( \phi_{40} - \phi_{11} \right) + \left( \psi_{12} - \psi_6 \right) \]

\[ = - \alpha E 2T d h \]
\[ \psi_0 = 0 \therefore (\psi_8 - \psi_1) + (\psi_{11} - \psi_{12}) + (\psi_6 - \psi_{12}) \]
\[ + (\psi_{40} - \psi_{12}) = 0 \]

\[ \Phi = \frac{\partial^2 \psi}{\partial x \partial y} \therefore - (\psi_{17} - \psi_{39}) + (\psi_{11} - \psi_{5}) - 4\Phi_6 - 4\Phi_{40} \]
\[ - 4\Phi_{11} + 16\Phi_{12} - 4\Phi_{18} = 0 \]

\[ \text{PT(18): } 4(1 + \mu)(\Phi_{12} - \Phi_{24}) + (\psi_{39} - \psi_{17}) = -\alpha E.T.4.d^2 \]

\[ \text{PT(24): } -(1 + \mu)(\Phi_{30} - \Phi_{38}) + (\psi_{24} - \psi_{13}) = -\alpha E.T.6.d^2 \]

\[ -(1 + \mu)(\Phi_{38} - \Phi_{23}) + (\psi_{30} - \psi_{18}) = -\alpha E.T.2.d.h \]

\[ -(\psi_{24} - \psi_{18}) + (\psi_{17} - \psi_{18}) + (\psi_{12} - \psi_{18}) + (\psi_{39} - \psi_{18}) = 0 \]

\[ -(\psi_{23} - \psi_{38}) + (\psi_{40} - \psi_{11}) - 4\Phi_{24} - 4\Phi_{17} - 4\Phi_{12} - 4\Phi_{39} \]
\[ + 16\Phi_{18} = 0 \]

\[ \text{PT(24): } -(1 + \mu)(\Phi_{30} - \Phi_{38}) + (\psi_{30} - \psi_{23}) = -\alpha E.T.6.d^2 \]

\[ -(1 + \mu)(\Phi_{38} - \Phi_{23}) + (\psi_{30} - \psi_{18}) = -\alpha E.T.2.d.h \]

\[ -(\psi_{30} - \psi_{24}) + (\psi_{23} - \psi_{24}) + (\psi_{18} - \psi_{24}) + (\psi_{38} - \psi_{24}) = 0 \]

\[ -(\psi_{29} - \psi_{37}) + (\psi_{39} - \psi_{17}) - 4\Phi_{30} - 4\Phi_{23} - 4\Phi_{18} - 4\Phi_{38} \]
\[ + 16\Phi_{24} = 0 \]

\[ \text{PT(30): } (1 + \mu)\Phi_{24} + (\psi_{37} - \psi_{29}) = -\alpha E.T.8.d^2 \]

\[ \text{PT(30): } (1 + \mu)\Phi_{24} + (\psi_{37} - \psi_{29}) = -\alpha E.T.8.d^2 \]
\[-(1 + \mu)(\Phi_{37} - \Phi_{29}) + (\psi_{49} - \psi_{24}) = -\alpha.E.T.2d.h \ldots (59)\]

\[(\psi_{49} - \psi_{30}) + (\psi_{29} - \psi_{30}) + (\psi_{24} - \psi_{30}) + (\psi_{37} - \psi_{30}) = 0\]

\[\ldots \ldots . . . . . (60)\]

\[-(\psi_{48} - \psi_{52}) + (\psi_{38} - \psi_{23}) - 4\psi_{29} - 4\psi_{24} - 4\psi_{37} + 16\psi_{30} = 0\]

\[\ldots \ldots \ldots \ldots \ldots \ldots (61)\]

\[PT(49): \quad \psi_{52} - \psi_{48} = -\alpha.E.T.10d^2 \quad \ldots \ldots \ldots \ldots (62)\]

\[\psi_{50} - \psi_{30} = -\alpha.E.T.2d.h \quad \ldots \ldots \ldots \ldots (63)\]

\[\quad (\psi_{50} - \psi_{49}) + (\psi_{48} - \psi_{49}) + (\psi_{30} - \psi_{49}) + (\psi_{52} - \psi_{49}) = 0\]

\[\ldots \ldots \ldots \ldots \ldots \ldots (64)\]

\[-(\psi_{47} - \psi_{51}) + (\psi_{37} - \psi_{29}) - 8\psi_{30} = 0\]

\[\ldots \ldots \ldots \ldots \ldots \ldots (65)\]

Assume linear variation of the function \(\psi\) in the region through mesh-points \((47-50-51; 48-49-52)\):

\[\therefore \quad (\psi_{47} - \psi_{50}) + (\psi_{51} - \psi_{50}) = 0 \quad \ldots \ldots \ldots \ldots (66)\]

\[\quad (\psi_{48} - \psi_{49}) + (\psi_{52} - \psi_{49}) = 0 \quad \ldots \ldots \ldots \ldots (67)\]

**SOLUTION**

The set of simultaneous equations \((1 - 67)\) can be put in matrix form \([A][X] = [B]\) where \([A]\) is the coefficient matrix, \([B]\) functions contains the temperature \(A\) and \([X]\) contains the unknown functions.

For the solution of this matrix a computer program \((47)\) was employed using "Comp Fact Routines". Appendix \((K)\) shows the computer program adopted.
After some manipulation with the computer results, it was found that more correct results were obtained if we took the differences of the functions $\psi$ since their magnitudes were much greater than those of the functions $\Phi$—order of magnitudes of $\psi$ being $10^{14}$ and those of $\Phi$ being $10^4$.

Thus the functions in the L.H.S. of equations (1-67) should be changed according to the following notation:

\[
\begin{align*}
\Phi_1 & \to \Phi_46 = \Theta_{46} \\
\psi_1 - \psi_5 & = \Theta_{47} \\
\psi_5 - \psi_6 & = \Theta_{48} \\
\psi_41 - \psi_6 & = \Theta_{49} \\
\psi_40 - \psi_{11} & = \Theta_{50} \\
\psi_48 - \psi_6 & = \Theta_{51} \\
\psi_{11} - \psi_{12} & = \Theta_{52} \\
\psi_48 - \psi_{12} & = \Theta_{53} \\
\psi_{24} - \psi_{12} & = \Theta_{54} \\
\psi_{39} - \psi_{17} & = \Theta_{55} \\
\psi_{37} - \psi_{18} & = \Theta_{56} \\
\psi_{38} - \psi_{23} & = \Theta_{57} \\
\psi_{30} - \psi_{18} & = \Theta_{58} \\
\psi_{25} - \psi_{24} & = \Theta_{59} \\
\psi_{37} - \psi_{29} & = \Theta_{60} \\
\psi_{35} - \psi_{24} & = \Theta_{61} \\
\psi_{26} - \psi_{30} & = \Theta_{62} \\
\psi_{52} - \psi_{48} & = \Theta_{63} \\
\psi_{50} - \psi_{30} & = \Theta_{64} \\
\psi_{48} - \psi_{49} & = \Theta_{65} \\
\psi_{51} - \psi_{47} & = \Theta_{66} \\
\psi_{47} - \psi_{50} & = \Theta_{67}
\end{align*}
\]
4.5 APPLICATION OF RIGOROUS SOLUTION TO REINFORCED CONCRETE -

BRICKWORK MODEL (A)

Model (A) is a 1/3rd scale model with reinforced concrete roof slab and brickwork walls. The slab is 5ft. square and 2in. thick. The walls are 5 ft. long by 3 ft. high.

For details of model see fig.(5.6).

The rigorous solution as outlined in section (4.4) was applied to model (A) with the temperature distribution through the slab and wall being obtained from experimental investigations. Fig(4.5) shows the experimental temperature distribution.

Applying the solution to this model, and assuming that the temperature varies with the wall height only, we get the following:

L.H.S. of equations (1-30) is not changed but the R.H.S. is changed after substituting the corresponding temperatures of the points considered as follows:

Equations (1,7,13,19,25) = + 346.32 α.E
Equations (2,8,14,20,26) = + 104.76 α.E
Equations (3,9,15,21,27) = + 46.08 α.E
Equations (4,10,16,22,28) = + 25.56 α.E
Equations (5,11,17,23,29) = + 5.40 α.E
Equations (6,12,18,24,30) = + 7.20 α.E

Equations (31-36) become:

\[ 1.02715 \Phi_{31} + 0.97285 \Phi_1 = + 944.07 \alpha.E \] ................................(31)
\[ 1.02715 \Phi_{32} + 0.97285 \Phi_7 = + 944.07 \alpha.E \] ................................(32)
\[ 1.02715 \Phi_{33} + 0.97285 \Phi_{13} = + 944.07 \alpha.E \] ................................(33)
\[ 1.02715 \Phi_{34} + 0.97285 \Phi_{19} = + 944.07 \alpha.E \] ................................(34)
\[ 1.02715 \Phi_{35} + 0.97285 \Phi_{25} = + 944.07 \alpha.E \] ................................(35)
Fig. (4.5) TEMPERATURE INCREASE THROUGH ROOF AND WALL OF MODEL(A).
\[ 1.02715 \Phi_{36} = 944.07 \alpha E \quad \ldots \ldots \ldots (36) \]

For the remaining equations (37-67), the L.H.S. is not changed except \((1 + \mu)\) is replaced by \((1, 1)\), but the R.H.S. is changed as follows:

Equations (37 to 42) = 0

Equations (43, 47, 51, 55, 59, 63) = -3434.4 \alpha E

Equations (44, 45, 48, 49, 52, 56, 57, 60, 61, 64, 65, 66, 67) = 0

Equation (46) = -572.4 \alpha E

Equation (50) = -144.8 \alpha E

Equation (54) = -1717.2 \alpha E

Equation (58) = -2289.6 \alpha E

Equation (62) = -2862 \alpha E
4.6 APPLICATION OF RIGOROUS SOLUTION TO ASBESTOS CEMENT MODEL (B)

Model (B) is a 1/6th scale model of asbestos cement - 30 in. x 30 in. x 0.9 in. slab resting on 30 in. x 18 in. x 0.625 in. walls with a 12 in. square opening in one of the walls as shown in fig. (5.7).

Taking the same number of mesh points as in model (A), we have mesh size, \( d = 3 \) in.

Thus for the temperature distribution in fig. (4.6) as obtained from experiment, equations (1-67) become:

L.H.S. of equations (1-30) is not changed, but the R.H.S. of these equations is changed in the following manner:

Equations (1, 7, 13, 19, 25) = 41.4 \( \alpha E \)
Equations (2, 8, 14, 20, 26) = -4.5 \( \alpha E \)
Equations (3, 9, 15, 21, 27) = 4.5 \( \alpha E \)
Equations (4, 10, 16, 22, 28) = -4.5 \( \alpha E \)
Equations (5, 11, 17, 23, 29) = 1.8 \( \alpha E \)
Equations (6, 12, 18, 24, 30) = 0.9 \( \alpha E \)

Equations (31-36) acquire the following forms:

\[
\Phi_{31} + \Phi_4 = d^2 \lambda + \frac{d}{2} B(\Phi_4 - \Phi_{31})
\]
\[A = \alpha E \; \Delta T = 26.96 \; \alpha E; \; B = \frac{S}{h} = \frac{0.625}{0.9 \times 15} = 0.0463\]

\[
1.0694 \Phi_{31} + 0.9306 \Phi_4 = 242.64 \; \alpha E \quad \ldots (31)
\]
\[
1.0694 \Phi_{32} + 0.9306 \Phi_7 = 242.64 \; \alpha E \quad \ldots (32)
\]
\[
1.0694 \Phi_{33} + 0.9306 \Phi_1 = 242.64 \; \alpha E \quad \ldots (33)
\]
\[
1.0694 \Phi_{34} + 0.9306 \Phi_{19} = 242.64 \; \alpha E \quad \ldots (34)
\]
\[
1.0694 \Phi_{35} + 0.9306 \Phi_{25} = 242.64 \; \alpha E \quad \ldots (35)
\]
\[
1.0694 \Phi_{36} = 242.64 \; \alpha E \quad \ldots (36)
\]
Fig. (4.6): TEMPERATURE INCREASE THROUGH ROOF AND WALL OF MODEL (B).
There are no changes in the L.H.S. of equations (37-67) except that \((1 + \mu)\) is replaced by \((1.17)\), but the R.H.S. of these equations undergo the following changes:

Equations (37-42) = 0

Equations \((43,47,51,55,59,63)\) = \(-1026\alpha E\)

Equations \((44,45,48,49,52,53,56,57,60,61,64,65,66,67)\) = 0

Equation (46) = \(-171\alpha E\)

Equation (50) = \(-342\alpha E\)

Equation (54) = \(-513\alpha E\)

Equation (58) = \(-684\alpha E\)

Equation (62) = \(-855\alpha E\)
CHAPTER V
EXPERIMENTAL INVESTIGATIONS OF THERMAL STRESSES

5.1 RESUME'

Investigations covered two models - a 1/3rd scale model (A) with a 5 ft. square by 2 in. thick reinforced concrete roof slab resting on 5 ft. x 3 ft. brick walls, and a 1/6th scale model (B) of asbestos cement roof and walls - the roof slab is 30 in. square by 0.9 in. thick and the walls are 30 in. x 18 in. x 5/8 in.

The two models were subjected to heat radiation from the top using Dimplex infra-red heaters with silicon contained elements. The inside air was kept in continuous circulation employing a simple ventilation system which consisted of a fan and two Perspex and cardboard ducts as inlets and outlets for air.

Temperature was measured by Cu-Constantan thermocouples and the thermal strains were measured by electrical resistance strain gauges.

5.2 HEATING SYSTEM

5.2.1 Description

1 K watt Dimplex infra-red heaters were used. The heater contains a heating spiral, a silica sleeve containing the spiral, a convex reflector which can be angled 10° up from the horizontal and 50° down allowing the beamed heat to be directed where required, a wall-plate and a wire guard.

The heater is switched on and off by a pull-cord.

5.2.2/
5.2.2 Characteristics of Infra-Red Heaters

A Dexion frame was assembled for the installation of the heaters as shown in plate (9).

A heater was installed 3 ft. from the surface of a 6 ft. square wooden board and the radiation characteristics of the heater as received by the surface were studied.

A grid 5 ft. square containing 25 small squares was drawn on a sheet of paper which was then fixed to the board. The grid represented the slab surface to which these radiations were to be applied later.

The illumination as received at the nodal points were measured using a Minilux Portable Illumination Meter (manufactured by Salford Electrical Instruments Ltd.)

The Minilux is a portable illumination meter for the measurement of different illumination levels. It consists of a moving coil indicator and a separate photo-electric cell unit. The cell unit is connected to the indicator by a flexible lead.

The instrument has four ranges adjustable by a switch on the front panel of the indicator. It is handy and easy to operate.

The instrument complies with the requirements of B.S.667

After measuring the illuminations at all mesh points, lines of equal illumination were drawn connecting points of equal illuminations. Fig. (5.1) shows the isothermals representing the radiation patterns of the heater. Belsey and Benseman (50) obtained similar radiation patterns for infra-red heaters.
5.2.3 Superposition of Heaters

Different arrangements of the heaters were studied by superimposition of the isothermals of the heaters to produce an even distribution of radiations throughout the 5 ft. square surface.

The arrangement shown in fig (5.2) was found to give a reasonably even distribution.

The heaters were then installed as shown in plate (9) and fig. (5.6). A probe of 2 in. x 2 in. x ½ in. cork painted black on the top side and covered with aluminium foil on the other sides was used for measuring temperature distribution from the heaters on the wooden board. A thermocouple was fixed to the top surface of the cork.

The temperature was measured at all mesh points and at the centre-lines of the board. Fig. (5.2) shows the readings obtained which gave a confirmation to the even distribution of temperature throughout the surface of the slab for model (A).

For model (B) of 30" square surface of roof slab, the arrangement of heaters as indicated by fig. (5.3.a) was found to give satisfactorily even radiations throughout the surface. Fig. (5.3.b) shows the readings obtained after superimposition of three heaters, giving the required distribution.

Thus for model (A), seven heaters arranged in parallel were used.

For model (B), three heaters were used. The heaters were assembled in parallel at a spacing of 15 in. centre-to-centre and installed at the centre and edges of the slab at a height of 3 ft. from the slab surface.
Fig.(5.2) : LAYOUT OF HEATERS FOR MODEL (A).
HEAT FLUX
(lux)

TEMPERATURE
(mV)

Fig.(5.2)
LAYOUT OF HEATERS FOR MODEL (B).

HEAT FLUX (lux)

Fig. (53)
5.3 STRAIN - MEASURING GAUGES AND RECORDING DEVICES

5.3.1 Electrical Resistance Strain Gauges

The gauges used in this study were manufactured by the Japanese Tokyo Sokki Kenkyujo Co. Ltd., and supplied by (Electro-Mechanisms Ltd.)

The gauges possessed the following properties:

Type PRS-5 and PRS-5-41

Gauge Length: 5 mm
Gauge Resistance: 120 ± 0.3 ohms
Gauge Factor: 2.03

The gauge was attached to the test specimen following the normal techniques. (51, 52)

Temperature Compensation

Since the dummy gauges were attached to specimens kept at room temperature and the active gauges were attached to the heated elements, a temperature differential existed between the active and dummy gauges. The strains obtained might be corrected making use of the Apparent Strain versus Temperature - curve supplied with the gauges.

The temperature corrections could be performed according to the relationships. (53)

\[
\text{Apparent Strain} = \frac{1}{k} \frac{\Delta R}{R} \Delta T = (\alpha_g - \alpha_b) \Delta T + \frac{1}{k} \gamma \Delta T
\]

where \( k \) = gauge factor

\( \alpha_g \) = thermal coefficient of expansion of gauge material

\( \alpha_b \) = thermal coefficient of expansion of base material (mild steel)
\[ \gamma = \text{temperature coefficient of resistivity of gauge material} \]
\[ \Delta T = \text{temperature change} \]

- if base material were mild steel,
\[ (\text{Apparent Strain) steel base} = \frac{\gamma}{k} \Delta T + (\alpha_{st} - \alpha_{g}) \Delta T \ldots (1) \]

- if base material were concrete,
\[ (\text{Apparent Strain) concrete base} = \frac{\gamma}{k} \Delta T + (\alpha_{\text{concrete}} - \alpha_{g}) \Delta T \ldots (2) \]

- From equations (1) and (2):
\[ (\text{Apparent Strain) concrete base} = (\text{Apparent Strain) steel base} + (\alpha_{\text{concrete}} - \alpha_{\text{steel}}) \Delta T \ldots (3) \]

Actual Strain = Measured Strain - Apparent Strain

However, a test was carried out to determine the behaviour of the gauges used under temperature effect only.

A PRS-5 gauge was mounted on a brickwork pier which was placed inside an oven. A dummy gauge was mounted on an identical specimen and kept at room temperature.

Then the oven was switched on. Readings of apparent strains and their corresponding temperatures were recorded. From the readings obtained, curves were plotted of apparent strains versus temperature differentials (Fig. 54.a).

The same procedure was repeated for a PRS-5-11 gauge mounted on an asbestos cement sheet. Results were plotted in fig. (54.b).

From the curves obtained, it was noticed that the Apparent Strain for PRS-5 gauges mounted on brickwork followed a straight line, while the Apparent Strain for PRS-5-11 gauge mounted on asbestos cement followed a curve. This might be due to the material properties of the asbestos.
TEMPERATURE DIFFERENTIAL—°C
(between active and dummy gauges.)

GAGE TYPE = PRS 5
TEST OBJECT = BRICKWORK.

(a)

APPARENT STRAIN — in./ln.x10^6

-100
-200
-300
-400

(b)

GAGE TYPE = PRS 5 II
TEST OBJECT = ASBESTOS CEMENT

APPARENT STRAIN — in./ln.x10^6

0
-20
-40
-60
-80
-100
-120
-140

FIG. (5.4): TEMPERATURE CORRECTIONS FOR ELECTRICAL RESISTANCE STRAIN GAUGES.
5.3.2 "Savage and Parsons" 50-way Strain Recorder

All strain measurements on the models were recorded on a "Savage and Parsons" 50-way Strain Recorder (Plate 11). The recorder voltage is 5 volts.

All connections from the active and dummy gauges to the recorder are done in a Multiple Strain Gauge Bridge Circuit having a gauge selector switch.

After the recorder is switched on, the light spot is first set to zero. Then the initial balance of the bridge is done by an 'Apex Resistance' potentiometer. This is performed by setting the gauge selector switch to indicate the required gauge and then by the "Apex Resistance" potentiometer, the light spot is set to zero.

Any change in active gauge resistance unbalances the bridge and the out-of-balance voltage is measured on the recorder. This out-of-balance voltage is a measure of the strain according to the relation:

\[
\phi = n \times 100 \times \frac{\Delta R}{R}
\]

where \( \phi \) = measured quantity

\( n \) = sensitivity factor (10, 2 or 1)

\( \Delta R \) = change in gauge resistance

\( R \) = active gauge resistance.

\[
\text{Strain} = \frac{1}{k} \frac{\Delta R}{R} = \frac{1}{k} \frac{\phi}{n \times 100}
\]

\( k \) = gauge factor.

Accuracy of the order of \( 5 \times 10^{-6} \text{in/in} \).
5.4 TEMPERATURE - MEASURING AND RECORDING EQUIPMENT

5.4.1 Copper-Constantan Thermocouples

Copper-Constantan thermocouples were employed for the measurement of temperature throughout this study.

5.4.1.1 Calibration

Soldering of all the junctions was carried out. The thermocouples were calibrated by comparison with a mercury-in-glass thermometer in a stirred water bath. The e.m.f.s. as indicated by the Honeywell recorder were recorded against their corresponding temperature as measured by the mercury-in-glass thermometer.

Then the recorded e.m.f.s. were plotted against their corresponding temperatures Fig. (5.5.b) is a representative sample of the calibration curves obtained for some of the thermocouples. The curves obtained were in good agreement with what is normally obtained for Cu-Constantan thermocouples (49) as shown in fig. (5.5.a).

5.4.1.2 Measuring and Reference Junctions

The measuring junctions after being soldered were embedded in 1/8 in. grooves made in the test specimens and then fixed in position with cement mortar.

For the reference junction, each thermocouple was inserted into a pyrex tube 1/8 in. diameter and all the tubes were placed in a thermos flask filled with ice cubes. The flask was firmly closed.

5.4.2 The Honeywell Recorder

The recorder used was an Electronik 15 Strip Chart Multipoint Recorder manufactured by Honeywell Controls Ltd. (Plate 12).

This recorder is a continuous balance potentiometer which measures and/
Fig. (5-5): **Calibration of Copper Constantan Thermocouples.**
which measures and records the magnitudes of up to 24 variables. Each variable is printed by a print wheel mechanism which identifies each variable by a specific number and symbol or colour and symbol.

These prints are made on a standard strip chart 122 ft. long.

After checking all connections, power is switched on to the instrument allowing it to warm up for 30 minutes or more. Then power is switched on to the chart driving system when recording is required.

Calibrated accuracy of output voltage under reference conditions of 85°F and 120 V (a.c) is ± 0.04%.

5.5 CALCULATION OF THERMAL STRESSES

After measuring the strains on the recorder and carrying out the necessary temperature corrections, the actual strains in the element are calculated according to:

Actual Strain = Measured Strain - Apparent Strain

Having calculated the strains in the three directions of the rosette, the stresses are calculated as follows:

\[
\sigma_x = \frac{E}{1 - \nu^2} (\varepsilon_a + \mu \varepsilon_a - (1 + \mu)\alpha \Delta T)
\]

\[
\sigma_y = \frac{E}{1 - \nu^2} (\varepsilon_c + \mu \varepsilon_a - (1 + \mu)\alpha \Delta T)
\]

\[
\sigma_{xy} = -\left(\frac{2eb - e_a - e_c}{e_a - e_c}\right) (\sigma_x - \sigma_y)
\]

\[
\sigma_{xy} = \frac{e_c}{2} (\sigma_x - \sigma_y)
\]
5.6 EXPERIMENT ON MODELS

5.6.1 Description of Models

Model (A) was one-third scale with 5 ft. square by 2 in. thick reinforced concrete roof slab and 5 ft. x 3 ft. x 1½ in. brick walls, with a 2 ft. square central opening at one of the walls. The walls were built on steel channels for complete fixity (fig. 5.6).

The concrete mix used was a (1:2:4) Rapid Hardening Portland Cement, sand, coarse aggregate mix with 0.55 water/cement ratio.

The steel reinforcement consisted of a mesh of 3/16 in. diameter bars at 6 in. centre-to-centre.

The bricks were one-third scale - 3 in. x 1.42 in. x 1 in.

The mortar joints were Rapid Hardening Portland Cement and sand in the ratio of 1:3.

Model (B) was 1/6th scale, made of asbestos cement roof slab and walls. The roof slab was 30 in. x 31 in. x 0.9 in. thick. The walls were 30 in. long by 18 in. high and 5/8 in. thick. An opening 1 ft. square was made in one of the walls. (fig. 5.7)

The walls and slab were cemented together by araldite.

The walls were clamped to steel channels as shown in plate (14).

To keep away air draughts, the models were built inside an 8 ft. square by 8 ft. high hardboard shelter.
Fig.(5.6): **ASSEMBLY OF HEATERS AND POSITIONS OF STRAIN GAUGES AND THERMOCOUPLES.**
Fig(5.7): POSITIONS OF STRAIN ROSETTES AND THERMOCOUPLES. MODEL(B).
5.6.2 Control Tests

5.6.2.1 Concrete

Three 4 in. cubes, three 4 in. diameter by 6 in. height cylinders, and three beams 4 in. x 4 in. x 20 in. were cast with the slab.

After 28 days, tests on the control specimens were carried out according to B.S.1881, 1952(54), and the following properties were noted:

- Mean cube compressive strength = 5745 lb/in²
- Mean cylinder crushing strength = 4845 lb/in²
- Mean modulus of rupture = 755 lb/in²

The cubes and cylinders were crushed in a Denison Testing machine with a maximum range of 250 tons.

The beams were tested in an Avery Testing machine using a rate of loading of 400 lb./minute.

5.6.2.2 Bricks

Ten bricks were crushed in an Avery Testing Machine in accordance with the requirements of B.S.1257,1945(55).

The mean crushing strength was found to be equal to 2890 lb/in².

5.6.2.3 Asbestos Cement

Two beams 3 ft. long and 3 in. x ½ in. cross-section were tested. Loads were applied by two pans suspended by wire ropes at the ends of the beams.

Mean modulus of rupture = 1545 lb/in²
5.6.3 Determination of Young's Moduli and Poisson's Ratios for Concrete, Brickwork and Asbestos Cement

5.6.3.1 Concrete

Three cylinders 4 in. diameter x 6 in. height were tested in a Lausenhausenwerk testing machine using Lamb's Extensometers (manufactured by A. Macklaw-Smith Ltd.) for measuring the strains.

The two extensometers (roller and lateral) were mounted onto the cylinder.

The elastic range governing the loading was calculated in the following manner:

Mean cube compressive strength = $C = \frac{5750}{20} \text{ lb/in}^2$

20 lb/in$^2$ stress would require a load of $20 \times \frac{\pi \times 4^2}{4} = 0.4 \text{ tons}$

$(1/3C + 20) \text{ lb/in}^2$ stress would require a load of

$(\frac{5750}{3} + 20) \frac{\pi \times 4^2}{4} \approx 10 \text{ tons}$

The loading range was chosen between 1 ton and 10 tons.

The test was then carried out by loading and unloading the specimens. Three cycles were performed. The difference between the maximum and the minimum strains was calculated for the second and third cycles and if the difference in the third cycle differed by more than 5% from the difference in the second cycle, more cycles were required.

From the obtained readings (table 5.1), curves were plotted of stress versus vertical strains, the slope of which gave Young's Modulus of elasticity (Fig. 5.8.a).

Three other curves of lateral strains versus vertical strains were drawn, and Poisson's ratios were determined from the slopes of the curves (fig. 5.8.b).

...
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<tr>
<th>STRESS (lb/in²)</th>
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<th>CYLINDER NO. (2)</th>
<th>CYLINDER NO. (3)</th>
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</thead>
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<tr>
<td></td>
<td>Longitudinal strain (in/in x 10⁻⁵)</td>
<td>Lateral strain (in/in x 10⁻⁵)</td>
<td>Longitudinal strain (in/in x 10⁻⁵)</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>178.24</td>
<td>4.92</td>
<td>0.69</td>
<td>4.92</td>
</tr>
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<td>10.82</td>
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**TABLE (5.1): STRESS-STRAIN RESULTS FOR CONCRETE**
Fig. (5.8): Young's Modulus and Poisson's Ratio for Concrete.
Fig. (5.8)
Fig (5.8)
5.6.3.2 Brickwork

Three brickwork beams 15 in. long and 1.42 in. x 0.5 in. X-section were built.

Electrical resistance strain gauges - type HPL-10 were fixed to the top surfaces of the beams at mid-spans in the longitudinal and lateral directions. Dummy gauges were fixed to a separate brickwork wall.

The beams were supported on roller and knife-edge supports. Loads were then applied to the beams by adding weights to two pans suspended by wire ropes 4 inches from the supports.

Loads were increased and the corresponding strains were measured on a "Savage and Parson's" 50-way strain recorder (Plate 4). Stresses were calculated from the relation:

\[
\text{Stress} = \frac{M}{I} = \frac{w \times l^4}{I} \frac{y}{I}
\]

From the readings obtained (table 5.2), the longitudinal strains were plotted against the stresses - the slopes of the curves giving Young's Modulus of Elasticity. Fig. (5.9.a).

Three other curves of lateral strains against longitudinal strains were plotted and Poisson's ratio determined from their slopes (fig. 5.9.b).

5.6.3.3 Asbestos Cement

Two asbestos cement beams 3 ft. long and 3 in. x ½ in. cross-section were tested following the same procedure adopted for brickwork beams (section 5.6.3.2).
PLATE (4) : BEAM TEST FOR THE DETERMINATION OF YOUNG'S MODULUS OF ELASTICITY AND POISSON'S RATIO FOR BRICKWORK.
<table>
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<tr>
<th>LOAD -lbs.</th>
<th>BEAM NO. (1)</th>
<th>STRESS -lb/in²</th>
<th>LONGITUDINAL STRAIN -in/ in. x 10⁻⁶</th>
<th>LATERAL STRAIN -in/ in. x 10⁻⁶</th>
<th>BEAM NO. (2)</th>
<th>STRESS -lb/in²</th>
<th>LONGITUDINAL STRAIN -in/ in. x 10⁻⁶</th>
<th>LATERAL STRAIN -in/ in. x 10⁻⁶</th>
<th>BEAM NO. (3)</th>
<th>STRESS -lb/in²</th>
<th>LONGITUDINAL STRAIN -in/ in. x 10⁻⁶</th>
<th>LATERAL STRAIN -in/ in. x 10⁻⁶</th>
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**TABLE (5.2): STRESS-STRAIN RESULTS FOR BRICKWORK BEAMS**
Fig (5.9): Young's modulus and Poisson's ratio for brickwork.
Fig (5.9)
Fig. (5.9)
The stresses at the top fibres were calculated from:

\[ f = \frac{M_y}{I} = \frac{M}{bd^2/6} = \frac{w \times 10^{10}}{bd^2/6} \]

The strains were recorded on a "Savage and Parson's" 50-way strain recorder.

From the readings obtained (table 5.3), curves were drawn of longitudinal strains versus stresses - the slope of which gave Young's Modulus (fig. 5.10.a).

Two curves of transverse strains versus longitudinal strains were also drawn and Poisson's Ratio determined from the slopes (fig. 5.10.b).

The mean values of Young's Modulus and Poisson's Ratio were calculated.
PLATE (5) : BEAM TEST FOR THE DETERMINATION OF YOUNG'S MODULUS OF ELASTICITY AND POISSON'S RATIO FOR ASBESTOS CEMENT
<table>
<thead>
<tr>
<th>LOAD (lbs)</th>
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</tr>
</thead>
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<td>STRESS (lb/in^2)</td>
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**TABLE (5.3): STRESS-STRAIN RESULTS FOR ASBESTOS CEMENT BEAMS**
Fig (5.10): YOUNG'S MODULUS AND POISSON'S RATIO FOR ASBESTOS CEMENT.
Fig. (5.10)
5.6.4 Determination of the Linear Coefficients of Thermal Expansion for Reinforced Concrete, Brickwork and Asbestos Cement.

Test Specimens

The test specimens consist of reinforced concrete prisms 2" x 1 ½" x 13.44", brickwork piers 3" x 1" x 13" and asbestos cement sheets (Plate 7).

Cu-Constantan thermocouples were embedded in the centres of the specimens as well as on their surfaces.

Test Apparatus (Plate 8)

The apparatus consists of a 7" x 5" x 1" mild steel base and two mild steel rods 1" in diameter x 14.5" long. The rods support two invar rods ½" diameter and 8" long. These two invar rods protrude to the surface of a Griffin oven with a 2" diameter hole in the top. A coil of an inductance transducer is to be rigidly attached to the protruding part outside the oven. The needle of the transducer is to rest on the top of a third invar rod 3/8" diameter by 8" long rigidly fixed to the specimen through a hole in the top of the specimen.

The principle of this test is the measurement of the differential thermal displacements of the mild steel rods and the specimen under test.

\[
\text{Differential displacement} = \text{Displacement in specimen} - \text{(measured by transducer meter)} \cdot \text{Displacement in steel rods.}
\]

Test Procedure

The coil of the inductance transducer was energised by an oscillator (5 volts, 1Kc/sec).
PLATE (6) : EXPERIMENTAL SET-UP FOR THE DETERMINATION OF THE LINEAR COEFFICIENTS OF THERMAL EXPANSION
PLATE (7) : SPECIMENS USED FOR THE DETERMINATION
OF THE LINEAR COEFFICIENTS OF THERMAL
EXPANSION
PLATE (8) : APPARATUS FOR THE DETERMINATION OF
THE LINEAR COEFFICIENTS OF THERMAL
EXPANSION
The relative movement of the needle and core altered the inductance. This signal was demodulated and displayed on a direct reading meter type C.51, in terms of displacements. Accuracy of these transducers was of the order of 0.00004".

Before starting the test, the calibrations of the transducer were checked against a micrometer. The micrometer shaft was firmly fixed to two steel rods with the shaft pointing upwards. The needle of the transducer was placed on the top of the shaft. Then for certain readings of the transducer meter, the corresponding readings of the micrometer were noted (Fig. 5.11).

The thermocouples were connected to a Honeywell recorder (Plate 6). The recorder was then switched on.

The transducer meter was set to zero.

Finally the oven was switched on and time was allowed until the specimen and the steel rods attained steady temperatures. At this condition the displacement as recorded by the transducer meter was noted. The procedure was repeated for different temperatures. The results obtained were shown in tables (5.4, 5.5 and 5.6). Graphs were then plotted of temperature changes versus strains – from the slopes of which the linear coefficients of thermal expansion were calculated (see fig. 5.12).
Fig. (5.11): CALIBRATION OF TRANSUDER METER.

CALIBRATION FACTOR = $25 \times 10^{-5}$ in./division

Range = 2.5
Scale = 100
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<tr>
<th>TEST NO.</th>
<th>TEMPERATURE RISE IN CONCRETE $^\circ$F</th>
<th>TEMPERATURE RISE IN STEEL $^\circ$F</th>
<th>DIFFERENTIAL DISPLACEMENT in. x 10$^{-3}$</th>
<th>CONCRETE DISPLACEMENT in. x 10$^{-3}$</th>
<th>STRAIN in./in. x 10$^{-6}$</th>
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**TABLE (5.4):** LINEAR COEFFICIENT OF THERMAL EXPANSION OF REINFORCED CONCRETE
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<th>TEST NO.</th>
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<th>TEMPERATURE RISE IN STEEL °F</th>
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**TABLE (5.5): LINEAR COEFFICIENT OF THERMAL EXPANSION OF BRICKWORK**
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<th>TEMPERATURE RISE IN STEEL °F</th>
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<th>ASBESTOS CEMENT DISPLACEMENT in. x 10^-3</th>
<th>ASBESTOS CEMENT STRAIN in./in. x 10^-6</th>
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**TABLE (5.6):** LINEAR COEFFICIENT OF THERMAL EXPANSION OF ASBESTOS CEMENT
Fig. (5.12.a): Linear coefficient of thermal expansion of reinforced concrete.
Fig. (5.12,b): **LINEAR COEFFICIENT OF THERMAL EXPANSION OF BRICKWORK.**

1. **Graph 1:**
   - **Output:** $\alpha = 6.75 \times 10^{-6}$ in/in/°F
   - **Temperature Increase (°F):** 0, 20, 40, 60, 80
   - **Strain (in/in \times 10^{-6}):** 0, 100, 200, 300, 400, 500, 600

2. **Graph 2:**
   - **Output:** $\alpha = 6.8 \times 10^{-6}$ in/in/°F
   - **Temperature Increase (°F):** 0, 20, 30, 40
   - **Strain (in/in \times 10^{-6}):** 0, 100, 200, 300

Fig. (5.12): Linear coefficient of thermal expansion of asbestos cement.
5.6.5 **Ventilation System**

A simple ventilation system was adopted. The system comprised a fan and two ducts (inlet and outlet).

The fan was positioned at the centre of the models. The inlet duct was located just under the lintel and at about three inches from the centre of the fan. The outlet duct was placed at the bottom of the opening (See plates. 9, 14).

5.6.6 **Experimental Procedure**

Aluminium sheets were placed in the spaces between the walls and the hardboard shelter at the level of the slab top, to reflect back any heat radiations from the hardboard to ensure that the walls did not receive any radiations from the hardboard shelter. (Plates 9, 13).

After fixing the gauges and thermocouples in the required positions, leads were connected to the "Savage and Parson's" Strain Recorder and the thermocouples connected to the Honeywell Recorder.

After checking all the connections, the Honeywell recorder was switched on and thirty minutes later, the "Savage and Parson's" 50-way strain recorder was switched on and all of the strain gauge circuits balanced.

Then the time was marked on the slip chart of the Honeywell Recorder.

Lastly the heaters and fan were switched on.

Readings of strain functions were then taken and the times at which these readings were taken were recorded.

Readings were taken hourly for about five hours, at the end of which the temperature of the slab top surface reached about 120°F.
PLATE (9) : EXPERIMENTAL SET-UP FOR MODEL (A)
PLATE (10) : RECORDING EQUIPMENT AND DUMMY GAUGES FOR MODEL (A)
PLATE (11) : SAVAGE AND PARSON'S 50-WAY STRAIN RECORDER
PLATE (13) : EXPERIMENTAL SET-UP FOR MODEL (B)
PLATE (14) : MODEL (B)
5.6.7 Experimental Results

From the tests carried out to determine the elastic properties of the materials of the models, the following results as shown in table were found:

<table>
<thead>
<tr>
<th>Material</th>
<th>Concrete</th>
<th>Brickwork</th>
<th>Asbestos Cement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus - (E) - lb/in²</td>
<td>2.9 x 10⁶</td>
<td>1.11 x 10⁶</td>
<td>0.52 x 10⁶</td>
</tr>
<tr>
<td>Poisson's Ratio (ν)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.17</td>
</tr>
<tr>
<td>Coefficient of Expansion (α)</td>
<td>7.85</td>
<td>6.78</td>
<td>7.825</td>
</tr>
<tr>
<td>in/in x 10⁻⁶/°F</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the measured thermal strains, the actual strains in the model elements were determined in the manner indicated in section (5.4.1).

From the actual strains obtained, the stresses were calculated as outlined in section (5.5).

Table (5.7) showed the maximum stresses obtained at the top of the wall, mid-height and bottom for model (A).

In table (5.8), the stresses for model (B) were shown.
Distance measured from wall top alone (inches) in/in in/in in/in g in/in x °F 2 lb/in 2 lb/in 2 lb/in

<table>
<thead>
<tr>
<th>Wall Section</th>
<th>Distance measured from wall top along wall height (inches)</th>
<th>$e_a$ in/in x $10^{-6}$</th>
<th>$e_c$ in/in x $10^{-6}$</th>
<th>$e_b$ in/in x $10^{-6}$</th>
<th>$\Delta T$ °F</th>
<th>$\sigma_x$ lb/in$^2$</th>
<th>$\sigma_y$ lb/in$^2$</th>
<th>$\sigma_{xy}$ lb/in$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre - Line of Wall</td>
<td>0</td>
<td>4365.78</td>
<td>188.44</td>
<td>150.0</td>
<td>34.32</td>
<td>142.16</td>
<td>-34.62</td>
<td>-12643</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>4179.6</td>
<td>51.52</td>
<td>110.63</td>
<td>12.05</td>
<td>104.98</td>
<td>-22.58</td>
<td>-4.89</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>42.83</td>
<td>50.23</td>
<td>60.07</td>
<td>7.95</td>
<td>-12.79</td>
<td>-5.34</td>
<td>-13.77</td>
</tr>
<tr>
<td>6&quot; from wall edge</td>
<td>18</td>
<td>4114.48</td>
<td>18.22</td>
<td>72.61</td>
<td>10.91</td>
<td>38.61</td>
<td>-5742</td>
<td>-6.25</td>
</tr>
</tbody>
</table>

**TABLE (5.7): EXPERIMENTAL STRESSES IN MODEL (A)**
<table>
<thead>
<tr>
<th>Wall Section</th>
<th>Distance measured from wall top along wall height (inches)</th>
<th>$e_a$ in/in x $10^{-6}$</th>
<th>$e_c$ in/in x $10^{-6}$</th>
<th>$e_b$ in/in x $10^{-6}$</th>
<th>$\Delta T_{oF}$</th>
<th>$\sigma_x$ lb/in$^2$</th>
<th>$\sigma_y$ lb/in$^2$</th>
<th>$\sigma_{xy}$ lb/in$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre</td>
<td>0</td>
<td>+131.0</td>
<td>+160.5</td>
<td>118.65</td>
<td>16.6</td>
<td>+4.07</td>
<td>+16.9</td>
<td>-12.65</td>
</tr>
<tr>
<td>Line of wall</td>
<td>9</td>
<td>+252.5</td>
<td>+159.6</td>
<td>+172.7</td>
<td>10.9</td>
<td>+93.6</td>
<td>+53.4</td>
<td>-13.6</td>
</tr>
<tr>
<td>3&quot; from wall edge</td>
<td>16.5</td>
<td>+113.11</td>
<td>+73.70</td>
<td>+71.24</td>
<td>9.75</td>
<td>+19.9</td>
<td>+2.4</td>
<td>+9.8</td>
</tr>
</tbody>
</table>

**TABLE (5.6): EXPERIMENTAL STRESSES FOR MODEL (B)**
### Table (5.9): Theoretical Stresses for Model (A)

<table>
<thead>
<tr>
<th>MESH POINT</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>232.9</td>
<td>1094.6</td>
<td>2364.4</td>
<td>3989.0</td>
<td>5909.2</td>
<td>7844.8</td>
<td>264.1</td>
<td>1082.4</td>
<td>2195.2</td>
<td>3684.4</td>
<td>5615.0</td>
<td>6219.4</td>
<td>340.8</td>
<td>927.0</td>
<td>1671.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MESH POINT</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>2674.9</td>
<td>3655.0</td>
<td>4387.4</td>
<td>214.4</td>
<td>714.7</td>
<td>986.2</td>
<td>1153.3</td>
<td>986.3</td>
<td>-76.5</td>
<td>151.6</td>
<td>601.1</td>
<td>449.7</td>
<td>263.2</td>
<td>-223.2</td>
<td>-1647.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MESH POINT</th>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>698.6</td>
</tr>
<tr>
<td>32</td>
<td>669.0</td>
</tr>
<tr>
<td>33</td>
<td>596.3</td>
</tr>
<tr>
<td>34</td>
<td>716.1</td>
</tr>
<tr>
<td>35</td>
<td>775.6</td>
</tr>
<tr>
<td>36</td>
<td>919.1</td>
</tr>
<tr>
<td>37</td>
<td>-4754.7</td>
</tr>
<tr>
<td>38</td>
<td>-2602.9</td>
</tr>
<tr>
<td>39</td>
<td>6759.6</td>
</tr>
<tr>
<td>40</td>
<td>11882.4</td>
</tr>
<tr>
<td>41</td>
<td>9031.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CENTRE-LINE OF WALL</th>
<th>DISTANCE FROM WALL TOP ALONG HEIGHT (INCHES)</th>
<th>( \sigma_x / dE )</th>
<th>( \sigma_y / dE )</th>
<th>( \sigma_{xy} / dE )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25.67</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>17.38</td>
<td>1.73</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>11.51</td>
<td>-2.18</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>9.77</td>
<td>-4.70</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>8.20</td>
<td>-16.92</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.43</td>
<td>-16.43</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>-20.81</td>
<td>20.81</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6(^a) FROM WALL EDGE</th>
<th>DISTANCE FROM WALL TOP ALONG HEIGHT (INCHES)</th>
<th>( \sigma_x / dE )</th>
<th>( \sigma_y / dE )</th>
<th>( \sigma_{xy} / dE )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25.75</td>
<td>0</td>
<td>1.90</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>8.28</td>
<td>-4.93</td>
<td>4.95</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-16.70</td>
<td>-15.54</td>
<td>5.36</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>-0.92</td>
<td>2.47</td>
<td>3.05</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>-8.36</td>
<td>17.41</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>-26.05</td>
<td>39.80</td>
<td>-9.65</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>-46.76</td>
<td>89.34</td>
<td>-24.92</td>
<td></td>
</tr>
</tbody>
</table>
### Table (5.10): Theoretical Stresses for Model (B)

#### Centre-Line of Wall

<table>
<thead>
<tr>
<th>Mesh Point</th>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>47.4</td>
</tr>
<tr>
<td>32</td>
<td>46.4</td>
</tr>
<tr>
<td>33</td>
<td>29.4</td>
</tr>
<tr>
<td>34</td>
<td>116.7</td>
</tr>
<tr>
<td>35</td>
<td>182.2</td>
</tr>
<tr>
<td>36</td>
<td>228.9</td>
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<tr>
<td>37</td>
<td>-495.6</td>
</tr>
<tr>
<td>38</td>
<td>125.4</td>
</tr>
<tr>
<td>39</td>
<td>1849.8</td>
</tr>
<tr>
<td>40</td>
<td>3358.1</td>
</tr>
<tr>
<td>41</td>
<td>2626.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distance from Wall Top Along Height (Inches)</th>
<th>( \sigma_x / dE )</th>
<th>( \sigma_y / dE )</th>
<th>( \sigma_{xy} / dE )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28.19</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>9.69</td>
<td>1.73</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>5.33</td>
<td>-4.54</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>7.67</td>
<td>-13.00</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>8.91</td>
<td>-20.66</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>3.20</td>
<td>-19.78</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>-18.42</td>
<td>18.42</td>
<td>0</td>
</tr>
</tbody>
</table>

#### 3" from Wall Edge

<table>
<thead>
<tr>
<th>Distance from Wall Top Along Height (Inches)</th>
<th>( \sigma_x / dE )</th>
<th>( \sigma_y / dE )</th>
<th>( \sigma_{xy} / dE )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25.95</td>
<td>0</td>
<td>6.58</td>
</tr>
<tr>
<td>3</td>
<td>5.15</td>
<td>2.65</td>
<td>7.48</td>
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<tr>
<td>6</td>
<td>-12.12</td>
<td>-3.21</td>
<td>6.37</td>
</tr>
<tr>
<td>9</td>
<td>-0.29</td>
<td>8.90</td>
<td>4.51</td>
</tr>
<tr>
<td>12</td>
<td>-4.02</td>
<td>20.46</td>
<td>3.24</td>
</tr>
<tr>
<td>15</td>
<td>-13.59</td>
<td>36.17</td>
<td>-0.69</td>
</tr>
<tr>
<td>18</td>
<td>-24.93</td>
<td>66.38</td>
<td>-9.65</td>
</tr>
</tbody>
</table>
Fig. (5.13): STRESS DISTRIBUTION AT CENTRE LINE OF WALL — MODEL (A).
Fig (5.13)

STRESS DISTRIBUTION AT
6° FROM WALL EDGE

MODEL (A)
Fig. (5.14): STRESS DISTRIBUTION AT CENTRE-LINE OF WALL.
MODEL (B).
Fig. (5.14): STRESS DISTRIBUTION AT 3° FROM WALL EDGE — MODEL (B).
5.7 CORRELATION OF EXPERIMENTAL AND THEORETICAL RESULTS

In table (5.9) the stress function and the derived stresses for model (A) are shown. The stresses in the centre-line of the wall and at 6 inches from the edge were plotted in fig. (5.13).

Similarly for model (B) the stresses are shown in table (5.10). Fig. (5.14) shows the stresses at the wall centre-line and at 3 inches from the wall edge.

From figures (5.13, 5.14), a reasonable correlation between measured stresses and calculated ones is observed. However this correlation is better in case of asbestos cement model (B) than in the reinforced-concrete and brickwork model (A). This is to be expected because asbestos cement is more homogeneous than brickwork.

However the differences that exist between measured and calculated stresses might be attributed mainly to errors introduced from temperature compensation (order of magnitude of temperature compensation of $200 \times 10^{-6}$ in/in. for a temperature differential of $20^\circ$C between active and dummy gauges (fig.5.4).
5.8 CONCLUSIONS

From the two models studied, the following conclusions can be drawn:

1. At the centre-line of the wall, the stress distribution was as outlined below:
   a) The longitudinal stress ($\sigma_x$) was tensile for the top 5/6ths of the wall height, having its maximum value at the top of the wall, and compressive for the bottom 1/6th.
   b) The axial stress ($\sigma_y$) was tensile through the top fourth of the wall, and compressive throughout, the remaining height except at the bottom where it is tensile.
   c) The sheer stress ($\sigma_{xy}$) was negligible throughout the height.

2. At 1/10th of wall length from the wall edge, the following was noticed:
   a) The longitudinal stress ($\sigma_x$) was tensile in the top third and compressive for the remainder of the height; the maximum tensile value occurring at the top and the maximum compressive value at the bottom.

   The maximum tensile values were $194$ lb/in$^2$ for brickwork and $106$ lb/in$^2$ for asbestos cement.

   b) The axial stress ($\sigma_y$) was negligible for the top 1/6th of the height, compressive for the next 1/6th and tensile for the remaining height.

   The highest compressive values were observed at 1/3rd of wall height. The highest tensile values were observed at the bottom.
c) The shear stress (σxy) was tensile for the top 5/6ths of wall height and compressive for the bottom 1/6th.

The highest tensile values were (40.5 lb/in²) for brickwork and (30.5 lb/in²) for asbestos cement.

The highest compressive values were (-187.5 lb/in²) for brickwork and (-39.3 lb/in²) for asbestos cement.

3) From fig. (5.13 and 5.14), some variations exist between measured and calculated stresses. Errors are mainly from temperature compensation.
CHAPTER VI
EXPANSION AND CONTRACTION JOINTS

6.1 INTRODUCTION

6.1.1 General

Cracks develop in building elements due to one of the following causes:

1) Applied loads
2) Differential settlement
3) Volume changes resulting from changes in temperature, shrinkage and moisture movements.

Buildings subjected to fluctuating temperatures tend to develop thermal stresses whose magnitude might be such as to cause cracks whenever the tensile strengths of the materials involved are exceeded.

To minimize or eliminate these cracks, three approaches are possible:

1) To minimize the temperature differential that exists between the interacting parts. This is achieved by use of insulating materials on the outside of the roof.

2) To decrease or eliminate the forces in the interacting parts resulting from the temperature differential. This is done by the provision of movement joints.

3) To increase the tensile strength of the wall by suitable reinforcement.

6.1.2 Review and Discussion of Past Studies

Diversity of opinions and practice exists on the most effective means of crack control in buildings - some advocate the use of reinforcement/
reinforcement; others advocate the provision of movement joints at the weakest sections of the building.

Those who support the use of reinforcement (Rensaa, Emel'yanov and Joint Committee on Standard Specifications for Concrete and Reinforced Concrete) maintain that steel reinforcement reduces the effect of the volume changes by imparting a restraining force on the element and thus reducing the width of cracks although the cracks might be greater in number than if the element is unreinforced. The reinforcement should be placed above and below openings (e.g. doors, windows, etc.). Rensaa recommends the use of two bars of 1 in. diameter in openings of outside walls and more reinforcement is needed for long walls whose thicknesses are greater than 8 inches. (58)

The Joint Committee on Joints claim that expansion joints are expensive and might be difficult to maintain. So they recommend the use of additional reinforcement and the avoidance of joints in walls but not in roofs.

Those supporting the provision of joints advocate that although the additional reinforcement might have prevented the occurrence of cracks, their experience indicated that it might not. (60)

Tippy regards the reinforcement as a supplementary aid in the prevention of shrinkage cracking. (61)

The provision of control joints finds the support of Merill, Marchant and Wild, Young, Tippy, and Copeland. (62) (59) (60) (63)

Many views exist on the use of expansion joints for different types of constructions:

1) For concrete constructions:

Those supporting the omission of expansion joints in such constructions/
constructions claim that the expansion caused by $100^\circ F$
temperature increase is less than the initial shrinkage of
the concrete. Thus the concrete will not be subjected to any
compressive stresses because any temperature increase will
tend to close up the shrinkage cracks only.

2) For concrete-masonry construction:

Views differ regarding the provision of joints in these construc-
tions. Emel'yanov and Tippy do not recommend the provision
of joints. Menzel suggests for the elimination of cracks the
improvement of the tensile strength of the block, of the mortar
bond and of the concrete construction.

6.2. TYPES OF MOVEMENT JOINTS

There are three main types of movement joints:

1) Expansion joints
2) Contraction joints
3) Construction joints.

6.2.1 Expansion Joints

These joints are intended to eliminate compressive stresses in the
member by accommodating expansions resulting from thermal and
moisture content changes.

The joint is formed by making a gap ($\frac{3}{8}$ in. - 1 in. wide) between
adjacent parts; this is filled with a compressible elastic
material to allow for the movements and provide a protection
against the entry of water and other matter. Jointing materials
used are joint fillers, sealers and water stops.

6.2.2 Contraction Joints/
6.2.2 Contraction Joints

Reductions in temperature and shrinkage cause tensile stresses in restrained concrete members. Contraction joints are thus provided at certain pre-determined locations to allow for contractions in order to release these tensile stresses.

There are two types of contraction joints:

a) True contraction joint which is formed by a complete gap between two adjacent sections.

b) Dummy (control) Contraction Joint:

No complete gap is provided between the adjacent sections, but only weakening the section at a specific location to allow for crack formation at that location only.

6.2.3 Construction Joints

These joints are merely separations formed by placing fresh concrete against the surface of hardened concrete (interrupted concreting). There are three main types: straight joints, tongue-and-groove joints and segmental joints.
6.3 SPACING OF JOINTS

6.3.1 Factors Influencing Spacing of Joints

1. Amount of reinforcement in the concrete. For large amounts of reinforcement, the spacing is increased and vice-versa.

2. Type of construction. In clay-brick construction, spacing depends on the age of the clay-brick as well as on the type of the mortar bond. In masonry construction, the wall pattern affects the resistance of the wall to cracks. For walls made of full-height blocks, the area of blocks resisting cracks was found to be equal to 46% of the total area of blocks, but for walls made of half-height blocks, the area was 50%.

3. Shape of building and length of outside walls.

4. External thermal environment and internal thermal conditions. In localities suffering from large temperature fluctuations, the structures subjected to the most adverse conditions (e.g., uninsulated walls and unheated buildings), the Joint Committee suggests a maximum spacing of 200 ft.

5. The presence of openings in walls. Merill recommends a spacing not more than 25 ft. for control joints in solid walls, and not more than 20 ft. for walls with openings.

6. The concreting operation. In floor construction Hunter proposes an alternate bay construction allowing two days between the concreting of adjacent panels.
6.3.2 Spacing of Expansion Joints

The existing practice of expansion joint spacing is summarised in the following table:

<table>
<thead>
<tr>
<th>TYPE OF CONSTRUCTION</th>
<th>EXPANSION JOINT SPACING (ft.)</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced Concrete</td>
<td>80 100 200</td>
<td>Hunter(65)</td>
</tr>
<tr>
<td>Plain Concrete Slabs up to 9 in. thick</td>
<td>100 60</td>
<td>National Building Code of Canada, (69)</td>
</tr>
<tr>
<td>Reinforced Concrete Roof Slabs with Insulation</td>
<td>65 30-50 40 60</td>
<td>Report of Joint Committee on Standard Specifications for Concrete and Reinforced Concrete(58)</td>
</tr>
<tr>
<td>(on masonry)</td>
<td></td>
<td>Joints in Concrete Construction, Published by Expandite Ltd., London.</td>
</tr>
<tr>
<td>(on concrete)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reinforced concrete roof slabs without Insulation</td>
<td>30 20 20 30</td>
<td>Polish Norm Pn-56/B-03260(66)</td>
</tr>
<tr>
<td>(on masonry)</td>
<td></td>
<td>E. Eichler(66)</td>
</tr>
<tr>
<td>(on concrete)</td>
<td></td>
<td>E. Eichler(66)</td>
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<td>Report of Joint Committee Polish Norm Pn-56/B-03260(66)</td>
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<td></td>
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<td>with openings (heated, non-insulated)</td>
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Thermal cracks usually tend to develop in:

1) The roof - if the roof temperature is lower than that of the walls.
2) The bearing plane of the roof.
3) The uppermost layers of the wall.

To determine the spacing of expansion joints which minimizes the thermal cracks or eliminates them, the following procedure is suggested:

a) Thermal stresses in the roof and wall are determined for the most severe temperature distributions. From these, the maximum tensile stresses in the roof and wall are evaluated and their locations noted.

b) Fissure lengths are then calculated for the three cases mentioned above.

c) The spacing of joints is then calculated from:

\[ \text{Spacing } L = 2x \]

where \( x \) = fissure length
6.3.2.1 Determination of Spacing of Expansion Joints in the Roof

From fig. (6.1), the frictional force set up between roof and wall is given by

\[ F = f \cdot w \cdot x \]  
(6.1)

where \( f \) = friction coefficient
\( w \) = roof load in lb/ft. run
\( x \) = fissure length in ft.

FORMATION OF FISSURES IN THE ROOF

Fissures form in the roof (tie beam) if its temperature is lower than that of the wall.

Stresses in the tie beam resulting from the frictional force \( F \) are given by:

\[ \sigma_T(z) = -\frac{f \cdot w \cdot x}{A_R} + \frac{(f \cdot w \cdot x \cdot d/2)}{I_R} (z - d/2) \]

\[ = f \cdot w \cdot x \left( -\frac{1}{A_R} + \frac{d/2}{I_R} (z - d/2) \right) \]  
(6.2)

assuming the beam to have symmetrical top and bottom reinforcement.

where \( d \) = beam thickness
\( Z \) = distance from bottom of beam to section where the stress is required.
\( I_R \) = moment of inertia of beam.
\( A_R \) = cross-sectional area of beam

If maximum stress in the beam occurs at Zo from beam bottom, then in the limiting state,

\[ \sigma_t(Zo) + \sigma_T(Zo) \leq \sigma_t \]  
(6.3)

where \( \sigma_t(Zo) \) = maximum tensile thermal stress in the beam
\( \sigma_T(Zo) \) = stress in the beam at Zo from bottom, due to frictional force \( F \)
Fig. (6.1): FRICTIONAL FORCE SET UP BETWEEN ROOF AND WALL.
\[ \sigma_t = \text{tensile strength of beam} \]

From equation (6.2),
\[ \sigma_f(z) = f.w.x\left(-\frac{1}{bd} + \frac{6}{bd^2}(z - d/2)\right), \quad b = \text{beam width} \]

If maximum thermal stress occurs at \( z = 0 \),
then \( \sigma_f(0) = f.w.x\left(-\frac{1}{bd} - \frac{3}{bd}\right) \)
\[ = x\left(\frac{f.w}{bd}\right)(-4) \]

From equation (6.3),
\[ -4x\left(\frac{f.w}{bd}\right) = \sigma_t - \sigma_f(\infty) \]
\[ = \sigma_t\left(1 - \frac{\sigma_f(\infty)}{\sigma_t}\right) \]
\[ = \sigma_t(1 - r) \quad \text{where} \quad r = \frac{\sigma_f(\infty)}{\sigma_t} \]

\[ \therefore \quad x = -\left(\frac{bd}{f.w}\right)\frac{2}{4}(1 - r)\sigma_t \]

If \( \sigma_f(z_0) \) occurs at \( z_0 = d/4 \),
then \( \sigma_f(d/4) = f.w.x\left(-\frac{1}{bd} + \frac{6}{bd^2}\left(d/4 - d/2\right)\right) = f.w.x\left(-\frac{1}{bd} - \frac{3}{2bd}\right) \)
\[ = f.w.x\left(-\frac{5}{2bd}\right) \]

\[ \therefore \quad x = -\left(\frac{bd}{f.w}\right)\frac{2}{5}(1 - r)\sigma_t \]

If \( z_0 = d/2 \), \quad \( x = -\left(\frac{bd}{f.w}\right)\frac{1}{1}(1 - r)\sigma_t \)

If \( z_0 = 3d/4 \), \quad \( x = +\left(\frac{bd}{f.w}\right)\frac{2}{2}(1 - r)\sigma_t \)

If \( z_0 = d \), \quad \( x = \left(\frac{bd}{f.w}\right)\frac{1}{2}(1 - r)\sigma_t \)

From above, minimum fissure length \( x = \frac{1}{2}\left(\frac{bd}{f.w}\right)(1 - r)\sigma_t \)
\[ \therefore \quad (6.4) \]
Critical spacing of expansion joints = 2x = \left(\frac{bd}{f_{w}w}\right)(1 - r)\sigma_{t}^{c} \tag*{(6.5)}

Fig. (6.2) gives the maximum spacing of expansion joints in tie beams for different beam sizes and tensile strengths as well as different ratios of maximum tensile thermal stresses to tensile strengths.

6.3.2.2 Determination of Spacing of Expansion Joints in the Bearing Plane of Beam

Damage in the bearing plane of beam can arise if the shearing strength of the wall is greater than that of the bearing plane (e.g. when roof rests on reinforced concrete walls). If roof rests on masonry walls, fissures go deeper - at the wall joint under the uppermost brick course.

According to Kucynski, if the resultant of the frictional forces between the roof and wall is F, then

\[ F = f_{w}w \cdot x \]

For a bearing plane of length x, the change in length due to F is

\[ \Delta x_{F} = \int_{0}^{x} \frac{F \cdot dx}{A_{R}E_{R}} = \int_{0}^{x} f_{w}w \cdot x \cdot dx = \frac{f_{w}w \cdot x^2}{2A_{R}E_{R}} \] \tag*{(6.6)}

For a bearing plane of length x, the change in length due to a temperature rise in the roof of \( \Delta T_{R} \) is given by:

\[ \Delta x_{T} = \alpha_{R}A_{R} \cdot x \] \tag*{(6.7)}

The resultant change of the length x will be zero if

\[ \alpha_{R}A_{R}x - \frac{f_{w}w \cdot x^2}{2A_{R}E_{R}} = 0 \]

i.e. if/
Fig.(6.2): SPACING OF EXPANSION JOINTS IN ROOFS.
Fig (6.2)
Equation (6.8) gives the critical length for no wall-heating.

If the temperature of the wall has also increased by $\Delta T_w$, then the critical length is found from:

$$x = \frac{2 \alpha_w E_w A_w \Delta T_w}{f_w} \quad \alpha_w = \text{sectional area of wall} \quad (6.9)$$

For a temperature rise $\Delta T_R$ in beam and $\Delta T_w$ in wall, the critical length is given by:

$$x_{Cr.} = \frac{2}{f_w} \left[ \alpha_R E_R A_R \Delta T_R - \alpha_w E_w A_w \Delta T_w \right] \quad (6.10)$$

Thus spacing of expansion joints in the bearing plane of the beam is given by equation (6.10) namely

$$L = x_{Cr.} = \frac{2}{f_w} \left[ \alpha_R E_R A_R \Delta T_R - \alpha_w E_w A_w \Delta T_w \right]$$
6.3.2.3 Determination of Spacing of Expansion Joints in Walls

The frictional force $F$ as given by equation (6.1) causes a stress $\sigma_f'(y)$ in the wall at a distance of $y$ from the wall-top, represented by:

$$\sigma_f'(y) = \frac{F}{A_w} + \frac{F h/2}{I_w} (h/2 - y)$$

where:
- $A_w$: cross-sectional area of wall
- $I_w$: moment of inertia of wall
- $h$: wall height

$$= f_{w,x} \frac{f_{w,x} h/2}{I_w} (h/2 - y) \quad \ldots \ldots (6.1)$$

If the maximum thermal stress in the wall occurs at $y_o$, then the limiting condition is:

$$\sigma_x(y_o) + \sigma_f'(y_o) \leq \sigma_{wt} \quad \ldots \ldots (6.12)$$

where:
- $\sigma_x(y_o)$ = maximum tensile thermal stress in the wall at $y_o$
- $\sigma_f'(y_o)$ = stress in the wall at $y_o$ caused by frictional force $F$
- $\sigma_{wt}$ = tensile strength of wall

From analytical and experimental analysis of thermal stresses in walls (Ch. IV and Ch. V), it was found that maximum tensile stresses occur at the top of the wall.

Thus from equation (6.11):

$$\sigma_f'(0) = f_{w,x} \left[ \frac{1}{A_w} + \frac{h/2}{I_w} (h/2) \right]$$

$$= f_{w,x} \left[ \frac{1}{S \cdot h} + \frac{h^2/4}{S \cdot h^3/12} \right]$$

$$= f_{w,x} \cdot \frac{1}{S \cdot h} \left[ 1 + \frac{2}{3} \right]$$

Substituting in equation (6.12) we get:
Curves in fig. (6.3) give the spacing for different wall properties and different tensile strengths as well as different ratios of maximum tensile thermal stresses to permissible tensile strength.
Fig. (6.3): SPACING OF EXPANSION JOINTS IN WALLS.
6.3.3 Spacing of Contraction Joints

Copeland (63) has determined the maximum permissible distance between control joints in blank walls. In his solution, Copeland (63) has adopted two methods; in the first method he neglected the effect of the variations in shrinkage, thermal contraction and extensibility of the masonry, but in the second method the effect of these variations was taken into account.

First Method

Assume a wall of length $L$ and height $H$, restrained at top and bottom edges. The tensile stress diagram at the centre-line of wall, as well as the shear diagram at top and bottom edges are shown in fig (6.4.a).

Tensile resistance of wall = $T = f_t A_t H$ \hspace{1cm} (6.15)

where $f_t = \text{average tensile strength of wall}$

$A_t = \text{effective wall area resisting tension per ft. of wall height}$.

Total shear force at top and bottom edges = $V = f_v \frac{L}{2} A_v$ \hspace{1cm} (6.16)

where $f_v = \text{maximum shearing strength of the mortar joint}$

$A_v = \text{effective joint area resisting shear per ft. of wall length}$.

For equilibrium $T = V$

\[ f_t A_t H = f_v \frac{L}{2} A_v \]

\[ L = 2 \left( \frac{f_t}{f_v} \right) \left( \frac{A_t}{A_v} \right) H \] \hspace{1cm} (6.17)

Thus the maximum wall length is defined by equation (6.17).
Second Method

Curves were plotted relating the average stress at centre-line of wall as a percentage of maximum stress \( \frac{f_{av}}{f_{max}} \) against ratio of wall length to wall height \( (L/H) \) as shown in fig. (6.4.b).

Assuming similarity of strain and stress distribution, occurrence of cracks when average stress or strain - and not the maximum - exceeds the ultimate values, and shrinkage and temperature stresses or strains are additive, Copeland suggested the following condition for crack elimination:

\[
P_{m} \leq \frac{e_{u}}{R(e_{d} + e_{t})}
\]

\[\ldots \ldots (6.18)\]

where \( P_{m} \) = ratio of average stress to maximum stress
\( e_{u} \) = ultimate tensile strain
\( e_{d} \) = drying shrinkage (between limits)
\( e_{t} \) = temperature contraction (between limits)
\( R \) = degree of restraint

The joint spacing is then determined from:

\[S = C \sqrt{H}\]

\[\ldots \ldots (6.19)\]

where \( C \) is a function of \( \frac{e_{u}}{R(e_{d} + e_{t})} \)

C should give a joint spacing at selected wall height which obeys the curves in fig. (6.4.b).

Curves were drawn of \( S \) against \( H \) for different values of \( C \) as shown in fig. (6.4.c), from which the spacing for any wall height and properties can easily be obtained.
FIG. (6.4): CONTRACTION JOINT SPACING.
6.4 LOCATION OF JOINTS

Thermal cracks are mostly observed in upper sections of outside walls where severe temperature stresses exist, in the tie beam, in the bearing plane of the tie beam and in cornices and long canopies. Cracks are also observed in weakened sections, namely openings for doors and windows.

Location of joints is thus accomplished by taking into account the areas of crack formations, the maximum permissible distance between joints and the requirement that the joints should completely separate the sections so that each section can function as an independent unit.

Joint locations are recommended at the following places \( (66,67) \):  
1) at the ends of long walls and at suitable distances along the wall length as specified by the maximum permissible spacing between joints.
2) at weakened sections e.g. openings and changes of cross-section.
3) at installations of vibrating machines.
4) where the concreting is interrupted.
5) at offsets and junctions of walls e.g. in L-, T-, H-, and U-shaped structures.

In section \( (6.3) \), the spacing for expansion and contraction joints has been determined.

Having determined the joint spacing, its location is found bearing in mind the above mentioned recommendations. In walls it is preferable to locate expansion and contraction joints at the same place.
For a wall 500 ft. long and interrupted by another cross-wall at 200 ft. from edge, the joint locations are as follows:

If maximum permissible expansion joint spacing = 150 ft.
and maximum permissible contraction joint spacing = 50 ft.
then expansion joints required are located as shown in fig. (a):

\[
\text{exp.} \quad \text{exp.} \quad \text{exp.} \quad \text{exp.} \quad \text{exp.} \quad \text{exp.}
\]
\[
\text{200 ft.} \quad \text{300 ft.}
\]
\[
\text{exp.} \equiv \text{expansion joint}
\]

Contraction joints are located as shown in (b):

\[
C \quad C \quad C \quad C \quad C \quad C \quad C \quad C \quad C \quad C \quad C \quad C
\]
\[
(b)
\]
\[
C \equiv \text{contraction joint}
\]

From diagrams (a) and (b) the final location of joints is shown in diagram (c):

\[
\text{exp.} \quad C \quad \text{exp.} \quad C \quad \text{exp.} \quad \text{exp.} \quad C \quad C \quad \text{exp.} \quad C \quad C \quad \text{exp.}
\]
\[
(c)
\]

However construction joints in beams and slabs are made in sections of minimum shear.
6.5 CONCLUSIONS

1. The spacing of expansion joints in the roof tie beam can be determined from equation (6.5)

\[ L = \left( \frac{bd}{f_w} \right) (1 - r) \sigma_t \]

Thus for any tie beam size and load, and for different tensile strengths and different ratios of maximum tensile thermal stresses to tensile strengths, the joint spacing can be found from curves in fig. (6.2).

Example: For a tie beam of area \( bd = 24.0 \text{ in}^2 \), \( f = 0.75 \), \( w = 735 \text{ lb/ft.} \), \( \sigma_t = 200 \text{ lb/in}^2 \) and \( \sigma_x(z_0) = 40 \text{ lb/in}^2 \), the maximum joint spacing is given by

\[ L = \left( \frac{24.0}{0.75 \times 735} \right) (1 - \frac{40}{200}) 200 \]

\[ = 70 \text{ ft.} \]

From the table summarising the existing practice of expansion joint spacing, it is shown that for reinforced concrete roof slabs with insulation, Hartmann suggests a spacing between 30-50 ft., while Eichler recommends a spacing of 40 ft. when roof rests on masonry walls and 60 ft. when it rests on concrete walls; the Polish Norm recommends a spacing of 60 ft.

For non-insulated reinforced concrete roof slabs, the Polish Norm recommends a spacing of 30 ft.; Hartmann supports 20 ft. while Eichler suggests 20 ft., when roof rests on masonry and 30 ft. when it rests on concrete.
2. The spacing of expansion joints in the walls is given by equation (6.14) namely

\[ L_0 = 2\left(\frac{h}{f_w}\right)\left(\frac{1}{4}\right)(1 - p)\sigma_{wt} \]

For different parameters, curves in fig. (6.3) give the required spacing.

E.g. Assume a wall of thickness \( S = 9 \text{ in.} \); \( f = 0.75 \); height \( h = 144 \text{ in.} \); \( w = 735 \text{ lb/ft.} \); \( \sigma_{wt} = 150 \text{ lb/in}^2 \); Length \( L = 500 \text{ ft.} \); max. tensile thermal stress \( \sigma_x(z_0) = 75 \text{ lb/in}^2 \).

\[ L_0 = 2\left(\frac{9}{0.75 \times 735}\right)\left(\frac{1}{4}\right)(1 - \frac{75}{150}) \times 150 \]

\[ = 90 \text{ ft.} \]

3. All the above solutions are theoretical pending more investigations and experimental verifications.
CHAPTER VII

CONCLUSIONS AND POSSIBLE FUTURE RESEARCH

The first object of this study was to develop a fast and accurate method for the solution of temperature distributions inside roof slabs, walls, etc. subjected to tropical conditions.

Analogue computers were found to give very satisfactory solutions of the temperature distribution problems. The solutions were quickly obtained and the accuracy obtained was very reasonable as compared with rigorous solutions using Digital Computers.

Thus it is the author's belief that Analogue Computers should be employed in the solution of heat conduction problems in slabs, walls and other similar problems.

The temperature distribution pattern in roof slabs subjected to sinusoidal outside air temperature as well as sinusoidal inside air temperature was found to follow a sinusoidal pattern displaced in phase and damped in amplitude.

For roof slabs subjected to actual weather data for a Tropical environment (Khartoum), in its hottest periods, the temperature distribution inside the slab was found to follow the same pattern as the external environment but with reduced amplitudes and displaced phases. This is in agreement with what is expected.

The second object of this study was to determine the thermal stress distribution in roof slabs subjected to the temperature distribution experienced in Tropical environment.

In general the stresses in roof slabs were found to vary according to the edge condition in the following manner: (see fig. 3.3)
a) For slabs restrained at the edges, the thermal stresses trace the same curves as the temperature.

The maximum compressive thermal stress obtained was about 800 lb/in^2 occurring at the outermost layer at about 2 p.m.; while the maximum tensile thermal stress was 200 lb/in^2 occurring at 6 a.m.

b) For simply supported slabs, the stress distribution is as shown in fig. (34).

The maximum compressive thermal stress was about 88 lb/in^2 occurring at the outermost layer at about 10.30 a.m., while the maximum tensile thermal stress was about 64 lb/in^2 occurring at the middle layer at about 10.30 a.m.

The third object was the theoretical and experimental analysis of thermal stresses in roofs and walls resulting from temperature differentials between them. An analytic solution was developed for the determination of thermal stresses in walls employing Finite Difference - Computer methods.

When the solution was applied to the two models, reasonable agreement was observed between experimental and theoretical stresses at the points where measurements were taken. There were however, odd points where there were appreciable differences between experimental and theoretical stresses. This might be attributed to errors introduced from temperature compensation (see section 5.7).

From the results obtained, it was noticed that the highest thermal stresses occurred at the wall top. These stresses were most severe in the centre-line of the wall. Thus the centre-line is a location for an expansion joint depending on the conditions outlined in Chapter (VI).
The final object of this research was the analytical study of thermal crack formations in buildings and the determination of the maximum permissible spacing between expansion joints as well as the location of these joints.

Study of fissure formations in roofs (tie beams), in bearing plane of beam, and in walls was carried out. From this study and after determining the maximum tensile thermal stresses in roofs and walls, the expansion joint spacing in these elements was evaluated.

Charts were drawn from which the maximum permissible spacing of joints could easily be determined, depending on the properties of the elements, the maximum tensile thermal stresses and the tensile strengths of the elements.
FURTHER RESEARCH

i) For the analysis of thermal stresses in roofs and walls resulting from temperature differentials between them, uniform temperature distribution on the roof surface as well as similar temperature variations along all the walls have been assumed in this study. Further work is needed to determine the stress condition for non-uniform temperature distribution.

ii) With regard to the problem of expansion joints, experimental study is needed to verify the theoretical solutions put forward in this study concerning the spacing and location of these joints.

There are, however, some problems involved in attempting to study crack formations in buildings in order to determine the spacing of expansion joints.

The most complicated problem is to distinguish thermal cracks from shrinkage cracks. Thus measurements of the magnitudes of thermal stresses in these elements is very essential.
REFERENCES


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42. |


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Appendix (A)

Temperature Distribution in Roof Slabs

SOLUTION BY TRANSFER MATRICES METHOD

This solution has been presented by Van Gorcum (38) in 1949 and elaborated in 1964 by Ogunlesi (37).

Van Gorcum has shown that a homogeneous plane material can be treated as a passive plate. For an infinite homogeneous slab, the amplitudes of temperature and heat flow at any plane surface can be calculated from those of another surface by transfer coefficients.

Thus for one-dimensional temperature distribution,

\[ \frac{\partial^2 T}{\partial x^2} = \frac{1}{a} \frac{\partial T}{\partial x} \quad \ldots \ldots \ldots (1) \]

Equation of conduction of heat is given by:

\[ Q = -k \frac{\partial T}{\partial t} \quad \ldots \ldots \ldots (2) \]

Equation of storage of heat is given by:

\[ \frac{\partial Q}{\partial x} = -C \frac{\partial T}{\partial t} \quad \ldots \ldots \ldots (3) \]

Equations (2) and (3) give

\[ \frac{\partial^2 Q}{\partial x^2} = \frac{C}{k} \frac{\partial Q}{\partial t} \quad \ldots \ldots \ldots (4) \]

Solution of equation (4) yields the following relationships:

\[ T_1 = M_{11} T_0 + M_{12} Q_0 \quad \ldots \ldots \ldots (5) \]

\[ Q_1 = M_{21} T_0 + M_{22} Q_0 \quad \ldots \ldots \ldots (6) \]

where

\[ T_1 = \text{Amplitude of temperature of inside surface of slab.} \]

\[ Q_1 = \text{Amplitude of heat flow of inside surface of slab.} \]

\[ T_0 = \text{Amplitude of temperature of outside surface of slab.} \]

\[ Q_0 = \text{Amplitude of heat flow of outside surface of slab.} \]

\[ M_{11} = \cos \theta \quad M_{22} = M \]
\[ M_{12} = -\frac{\sin\gamma 1}{k \gamma}, \quad M_{21} = \gamma k \sin\gamma 1 \]

\[ \gamma^2 = \frac{\sigma C}{k} \omega, \quad \omega = \frac{2\pi n}{24} \]

\[ n = \text{harmonic order} \]

For a building element consisting of more than one homogeneous slab in series, the matrix coefficients are found by multiplication of the matrices for each slab.

Applying this method to the concrete roof slab, the heat flow paths can be represented by the following schematic diagram.

The amplitudes of temperature and heat flows at any distance \( x \) from the upper surface of the slab can be expressed by:

\[
\begin{bmatrix}
T_x \\
Q_x
\end{bmatrix} =
\begin{bmatrix}
(M_{11})_x & (M_{12})_x \\
(M_{21})_x & (M_{22})_x
\end{bmatrix}
\begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix}
\begin{bmatrix}
T_{sa} \\
Q_{sa}
\end{bmatrix}
\]

The amplitudes of temperature and heat flow for the inside air can be found from:
From equations (7) and (8) the amplitudes of temperature and heat flow at any distance $x$ from the upper surface can be found from:

$$T_x = \frac{1}{pq + rs} (qT_{1A} - r T_{sa}) \quad \ldots \ldots (9)$$

$$Q_x = \frac{1}{pq + rs} (s T_{1A} + q T_{sa}) \quad \ldots \ldots (10)$$

where

$$p = (M_{11}^1)_1 - x - R(M_{21}^1)_1 - x$$

$$q = R(M_{11}^1)_x - (M_{12}^1)_x$$

$$r = (M_{12}^1)_1 - x - R(M_{22}^1)_1 - x$$

$$s = R(M_{21}^1)_x - (M_{22}^1)_x$$

**ASSUMPTIONS**

1. Temperature varies through the thickness of the slab only.
2. Thermal properties of slab are constant. They do not change with either temperature or time.
3. Air has no heat capacity.

From the above assumptions,

$$N_{11} = N_{22} = N_{11}^1 = N_{22}^1 = \cos \theta = 1$$

$$N_{21} = N_{21}^1 = \sin \theta = 0$$

$$N_{12} = -\frac{1}{k} = -R = -\frac{1}{3.5} = -0.286$$

$$N_{12}^1 = -\frac{1}{k^1} = -R^1 = -\frac{1}{1.65} = -0.606$$
\[ \gamma^2 = -\frac{C}{k} iw = -\frac{150 \times 0.24 \times 2\pi}{10 \times 24} \]

\[ \gamma = 2.375(-1 + i) \]

\[ k\gamma = 1.98(-1 + i) \]

For the solution of equation (9) a computer program has been developed by the author. See Appendix (b) for the computer program by which the temperature at any depth from the slab surface and at any time of the day was easily obtained.

However, the solution of equation (9) involved multiplication of complex numbers which was performed in a similar way to that presented by Muncey and Spencer (39).
APPENDIX (B)

COMPUTER PROGRAM FOR THE SOLUTION OF
TEMPERATURE DISTRIBUTION IN ROOF SLABS
(Sinusoidal Sol-Air and Inside Air Temperatures)

***A

JOB
CIE 023 /0000 0002/ EL-NOURANI, TEMPERATURE DISTRIBUTION
COMPUTING 2000 INSTRUCTIONS
OUTPUT
0 LINE PRINTER 1000 LINES
COMPILER AA

begin
integer t,n,y
real array MR11,MI11,MR12,MI12,MR21,MI21,MR22,MI22,pR,pI,qR,qI,rR,rI,sR,c
aI,pqR,pqI,rSR,rsI,BrB,BrC,DrD,DrI,BrFrFrG,FrI,h,x,k,z (0:5,0:3)

real a,cs,sn,ch,sh,T,Txt,x

cycle y=0,1,5
x=y/12

cycle n=0,1,1
if n=0 then ->1

->2
1:MR11(y,n)=1 ; MI11(y,n)=0 ; MR22(y,n)=1 ; MI22(y,n)=0
MR12(y,n)=-1.2X; MI12(y,n)=0 ; MR21(y,n)=0 ; MI21(y,n)=0

->3
2:a=2.375*x*sqrt(n) ; ch=0.5(exp(a)+exp(-a))) ; sh=0.5(exp(a)-exp(-a)))
cs=cos(a) ; sn=sin(a)

MR11(y,n)=ch*cs
MI11(y,n)=sh*sn
MR12(y,n)=-(ch*sn+sh*cs)/(3.96*sqrt(n)))
MI12(y,n)=-(ch*sn-sh*cs)/(3.96*sqrt(n)))
MR21(y,n)=(1.98*sqrt(n))*(ch*sn-sh*cs)
MI21(y,n)=-(1.98*sqrt(n))*(ch*sn+sh*cs)
MR22(y,n)=MR11(y,n)
MI22(y,n)=MI11(y,n)

3:repeat
repeat
cycle $y=0,1,5$

cycle $n=0,1,1$

$pR(y,n)=MR11(5-y,n)-.606MR21(5-y,n)$
$qR(y,n)=.25MR11(y,n)-MR12(y,n)$
$qI(y,n)=.25MI11(y,n)-MI12(y,n)$
$pR(y,n)=MR11(5-y,n)-.606MR21(5-y,n)$
$qR(y,n)=.25MR11(y,n)-MR12(y,n)$
$qI(y,n)=.25MI11(y,n)-MI12(y,n)$
$pR(y,n)=MR11(5-y,n)-.606MR21(5-y,n)$
$qR(y,n)=.25MR11(y,n)-MR12(y,n)$
$qI(y,n)=.25MI11(y,n)-MI12(y,n)$
$pR(y,n)=MR11(5-y,n)-.606MR21(5-y,n)$
$qR(y,n)=.25MR11(y,n)-MR12(y,n)$
$qI(y,n)=.25MI11(y,n)-MI12(y,n)$

$pqR(y,n)=pR(y,n)*qR(y,n)-pI(y,n)*qI(y,n)$
$pqI(y,n)=pR(y,n)*qI(y,n)+pI(y,n)*qR(y,n)$
$rSR(y,n)=rR(y,n)*sR(y,n)-rI(y,n)*sI(y,n)$
$rSI(y,n)=rR(y,n)*sI(y,n)+rI(y,n)*sR(y,n)$

$BR(y,n)=pqR(y,n)+rSR(y,n)$
$BI(y,n)=pqI(y,n)+rSI(y,n)$

$C(y,n)=(BR(y,n)^2+BI(y,n)^2)$
$DR(y,n)=qR(y,n)*BR(y,n)+qI(y,n)*BI(y,n)$
$DI(y,n)=qI(y,n)*BR(y,n)-qR(y,n)*BI(y,n)$

$BR(y,n)=DR(y,n)/C(y,n)$
$BI(y,n)=DI(y,n)/C(y,n)$

$k(y,n)=radius(ER(y,n),RI(y,n))$
$Z(y,n)=arctan(ER(y,n),EI(y,n))$
$FR(y,n)=-rR(y,n)*BR(y,n)-rI(y,n)*BI(y,n)$
$FI(y,n)=rR(y,n)*BI(y,n)-rI(y,n)*BR(y,n)$

$gR(y,n)=FR(y,n)/C(y,n)$
$gI(y,n)=FI(y,n)/C(y,n)$

$h(y,n)=radius(gR(y,n),gI(y,n))$
$X(y,n)=arctan(gR(y,n),gI(y,n))$

repeat
repeat

newlines(3)
\begin{verbatim}
caption \#\#\#RESULTS newlines(2)

cycle t=0,1,24
print (t,2,0)
spaces(4)
cycle y=0,1,5

sn=sin((2\pi/24)*t+X(y,1))
sc=sin((2\pi/24)*t+Z(y,1))
Ttxt=90k(y,0)+5k(y,1)*sc+95h(y,0)+15h(y,1)*sn

print f1 (Ttxt,3)
spaces(4)

repeat

newline

repeat

end of program
\end{verbatim}
In an Analogue computer, the mathematical operations and relationships are achieved by potentiometers, amplifiers (summers or integrators), multipliers, sign changers and function generators.

**POTENTIOMETERS:** used for multiplying by a constant which is less than unity.

**SIGN CHANGERS:** for reversing the sign of the signal.

**SUMMERS:** used for adding voltages. The output is equal in magnitude to the algebraic sum of the input voltages but of opposite sign.

**INTEGRATORS:** used for producing an output equal in magnitude but opposite in sign to the integral of the input voltage with respect to computer time.

**MULTIPLIER:** used for multiplication of one variable by another variable.

**FUNCTION GENERATOR:** used for producing an output voltage which is a function of the input voltage.

**PROGRAMMING AN ANALOGUE COMPUTER**

The general procedure could be summarised in the following steps:

1. Determination of method of solution by study of the model.
2. Construction of a scaled computer program including initial conditions.
Potentiometer

Integrator

\[ e_o = \frac{1}{C} \int (e_1(t) + e_2(t)) \, dt + e_0(0) \]

Summer

\[ e_o = -(e_1 + e_2 + e_3) \]

Multiplier

Diode Function Generator

Symbols used for analogue computer components.
3. Connection of the computer components into a patch panel.
4. Setting to correct values all potentiometers being used.
5. Connection of outputs to recorder inputs.

GENERAL CHARACTERISTICS OF THE SC-30 ANALOGUE COMPUTER

The computer used in this study was a Solartron SC-30 Analogue Computer (manufactured by Solartron Electronic Group Ltd.) and was available in the Department of Electrical Engineering, University of Edinburgh.

This computer is a general-purpose machine with reference voltage \( \pm 100 \) volts and incorporating the following equipment:

a). 60 coefficient potentiometers.
b) 30 operational amplifier units.
c) 4 servo multiplier units.
d) 2 general-purpose Diode Function Generators.
e) 4 symmetric Diode Limiter Bridges.
f) 4 Relay Amplifiers.

Besides the above mentioned equipment, the computer has a removable patch panel (into which the circuits are plugged), a central digital voltmeter (for recording outputs), a timer (for setting computer time corresponding to the time for the physical problem), comprehensive control, monitoring and checking facilities.

OPERATING MODES

1. POT SET: The coefficient potentiometers and any other potentiometers are set to the desired values.
2. PROBLEM CHECK: The initial conditions of the problem variables are set.
3. **COMPUTE:** The problem solution is started at this stage and integrators, function generators etc. start operating.

4. **HOLD:** This terminates the computing mode.

**X-Y CO-ORDINATES PLOTTING TABLE**

The plotting table used was manufactured by Bryans Aeroquipment Ltd.

This device consists of two position control servomechanisms which places a pen at a point on a sheet of paper corresponding to two variables in rectangular co-ordinates represented by voltages X and Y.

The maximum size of paper it can accommodate is 10 x 15 inch graph paper. The graph paper is normally held flat on the table by a vacuum but we used sellotape for holding it.

The plotting table contains three units:

a) **X-axis time base with sensitivity ranging from 0.25 - 10 volts/in**

b) **X-axis with sensitivity ranging from 0.1 - 5 volts/in**

c) **Y-axis with sensitivity ranging from 0.1 - 5 volts/in.**

**X-AXIS**

X-axis was connected to monitor hole (18) in the patch panel.

For one day in the problem, time in the computer is given by:

\[ t = 7.955 \text{ sec.} \]

On graph paper we require 12 in. to represent one day.

Thus using a sensitivity in the X-axis of 1 volt/in., volts needed range from 0 - 12 volts.

In fig. (2.1b), reading of potentiometer B51 should be equal to

\[ \frac{12}{7.955} \times 0.01 = 0.1512. \]
To adjust the X-scale, the amplifier selector was set to B4. The pen was then adjusted to be on the zero of the X-axis. Then, the computer was set to "COMPUTE" stage. With the fine screw, the pen was then adjusted to coincide with the 12 in.-line.

**Y-AXIS**

The Y-axis was connected to monitor hole (T) through a brown lead.

With the pen on zero of the Y-axis, (To) was connected to (A3) (fig. 24b). The amplifier selector was then set to (B2).

When Problem Check Stage was pressed, the pen recorded the initial value of To.

When COMPUTE was pressed, the pen followed the To-curve. This was repeated for all outputs.

Max. $T/4 = 30^\circ\text{F}$; Min. $T/4 = 20^\circ\text{F}$

Volts needed range from 20-30 volts.

Thus a sensitivity of 1 volt/in. was assumed.
GENERATION OF SINE AND COSINE FUNCTIONS IN ANALOGUE COMPUTERS

Consider the differential equations

\[ \frac{d^2 y}{dz^2} + w^2 y = 0 \]  

\[ \frac{dy}{dz} = 0 \]  

at \( z = 0 \)

solution \( y = A \cos wz \)

\[ |y|_{\text{max.}} = A \]

\[ \frac{dy}{dz} = -Aw \sin wz \]

\[ \frac{|dy|}{dz}_{\text{max.}} = Aw \]

\( y = \frac{A}{100} \bar{y} \); where \( \bar{y} \) = computer value

\[ \frac{dy}{dz} = \frac{Aw}{100} \frac{dy}{dz} \]

From equation (1): \[ \frac{dy}{dz} = -w^2 \int y \, dz \]  

Substituting from (4) and (5) into (a), we get:

\[ \frac{Aw}{100} \frac{dy}{dz} = -w^2 \int \frac{A}{100} \bar{y} \, dz \]

\[ \therefore \frac{dy}{dz} = -w \int \bar{y} \, dz \]  

From equation (a) \( y = \int \frac{dy}{dz} \, dz \)

Substituting from (4) and (5) in (b), we get:

\[ \frac{A}{100} \bar{y} = \int \frac{Aw}{100} \frac{dy}{dz} \, dz \]
Equations (i), (ii) and (iii) are the computer equations.

Thus setting a -100 volt initial condition on integenator generating $-\bar{y}$, we obtain the desired solution,

$$\bar{y} = 100 \cos \omega \tau$$

From equation (i) and (I),

$$\frac{d\bar{y}}{d\tau} = -\omega \int \bar{y} \, d\tau$$

$$= -\omega \int 100 \cos \omega \tau \, d\tau$$

$$= -\omega \cdot 100 \frac{\sin \omega \tau}{\omega}$$

$$\therefore \frac{d\bar{y}}{d\tau} = -100 \sin \omega \tau$$

$$w = \beta_1 k_1 = \beta_2 k_2; \quad y = 100 \cos \omega \tau$$

From equation (i) and (I),

$$\frac{d\bar{y}}{d\tau} = -\omega \int \bar{y} \, d\tau$$

$$= -\omega \int 100 \cos \omega \tau \, d\tau$$

$$= -\omega \cdot 100 \frac{\sin \omega \tau}{\omega}$$

$$\therefore \frac{d\bar{y}}{d\tau} = -100 \sin \omega \tau$$

$$\ldots \ldots (II)$$
APPENDIX(R)
COMPUTER PROGRAM FOR THE HARMONIC ANALYSIS
OF SOL-AIR TEMPERATURE

***A
JOB
CIE 023/0000 0003/BL-NOURANI /SOL-AIR TEMP.
OUTPUT
0 LINE PRINTER 1000 LINES
COMPILER AA
begin
integer t,n
real x,sn,cs,y,T,SUM
real array a,b (1:24,1:12),f,d,R,S,A,B,C,Z (1:12)

SUM=0
cycle t=1,1,24
read(y)

-1 if y=0

SUM=SUM+y
cycle n=1,1,12
x=2n*t/24
sn=sin(x)
cs=cos(x)
a(t,n)=y*sn
b(t,n)=y*cs
repeat

repeat
1: cycle n=1,1,12

R(n)=0
S(n)=0
cycle t=1,1,24
R(n)=R(n)+a(t,n)
S(n)=S(n)+b(t,n)
\[ A(n) = S(n)/12 \]
\[ B(n) = R(n)/12 \]
\[ Z(n) = \arctan(B(n), -A(n)) \]
\[ C(n) = \text{radius}(A(n), B(n)) \]

```
repeat
  repeat
    newlines(3)
    caption   THE ppp RESULTS
    newlines(2)
    print f1 (SUM/24,3)
    newlines (3)
    cycle n=1,1,12
    print(n,2,0)
    spaces(4)
    print(Z(n),1,3)
    spaces(4)
    print f1 (C(n),3)
    newline

repeat
  newlines(3)
  cycle t=0,1,24
  print(t,2,0)
  spaces(4)
  cycle n=1,1,12

f(n)=2\pi*n*t/24-Z(n)
sn=sin(f(n))
d(n)=C(n)*sn
```

repeat
\[ T = \frac{\text{SUM}}{24} + d(1) + d(2) + d(3) + d(4) + d(5) + d(6) + d(7) + d(8) + d(9) + d(10) + d(11) + d(12) \]

print fl (T,3)

newline

repeat

endofprogram

72.61 70.94 69.94 68.94 68.27 73.17 94.89 120.53
142.08 159.83 171.92 178.70 176.10 169.55 156.44
139.35 119.08 97.55 89.92 86.92 83.92 80.27
76.94 74.61 0

***Z
APPENDIX (F)

COMPUTER PROGRAM FOR THE SOLUTION OF TEMPERATURE DISTRIBUTION IN ROOF SLABS

(24 Hours Record Of Observed Outdoor Air Temperature, Incident Solar Radiation, and Sky Temperature on a Roof Slab)

```
begin
integer t, n, y
real array M11, M11, MR12, M12, MR21, M21, MR22, M22, pR, pI, qR, qI, rR, rI, sR, c
   sI, pR, pqI, rsR, rsI, BR, BI, C, DR, DI, ER, EI, FR, FI, gR, gI, h, x (0:5, 0:12), z, b (0:12)
real a, cs, sn, ch, sh, T, Txt, x, d

cycle y=0, 1, 5
   x=y/12

cycle n=0, 1, 12
if n=0 then ->1

->2
1: MR11 (y, n)=1 ; M11 (y, n)=0 ; MR22 (y, n)=1 ; M22 (y, n)=0
MR12 (y, n)= -1.2x ; M12 (y, n)=0 ; MR21 (y, n)=0 ; M21 (y, n)=0
->3
2: a=2.375x*sqrt(n) ; oh=(exp(a)+exp(-a)) ; sh=(exp(a)-exp(-a))
   cs=cos(a)
   sn=sin(a)

MR11 (y, n)=ch*cs
M11 (y, n)=sh*sn
MR12 (y, n)= ((ch*sn+sh*cs)/(3.96*sqrt(n)))
M12 (y, n)= ((ch*sn-sh*cs)/(3.96*sqrt(n)))
MR21 (y, n)= (1.98*sqrt(n))*(ch*sn-sh*cs)
M21 (y, n)= -((1.98*sqrt(n))*(ch*sn+sh*cs)
MR22 (y, n)=MR11 (y, n)
M22 (y, n)=M11 (y, n)

3: repeat
repeat

cycle y=0, 1, 5

cycle n=0, 1, 12

pR (y, n)=MR11 (5-y, n)-.606MR21 (5-y, n)
pI (y, n)=M11 (5-y, n)-.606M21 (5-y, n)
qR (y, n)=.2857MR11 (y, n)-MR12 (y, n)
qI (y, n)=.2857M11 (y, n)-M12 (y, n)
```
\[
\begin{align*}
rR(y,n) = & \text{MR12}(5-y,n) - 606 \text{MR22}(5-y,n) \\
rI(y,n) = & \text{MI12}(5-y,n) - 606 \text{MI22}(5-y,n) \\
sR(y,n) = & 2857 \text{MR21}(y,n) - \text{MR22}(y,n) \\
sI(y,n) = & 2857 \text{MI21}(y,n) - \text{MI22}(y,n)
\end{align*}
\]

\[
\begin{align*}
pqR(y,n) = & pR(y,n) * qR(y,n) - pI(y,n) * qI(y,n) \\
pql(y,n) = & pR(y,n) * qI(y,n) + pI(y,n) * qR(y,n) \\
rsl(y,n) = & rR(y,n) * sI(y,n) + rI(y,n) * sR(y,n)
\end{align*}
\]

\[
\begin{align*}
BR(y,n) = & pqR(y,n) + rsR(y,n) \\
BI(y,n) = & pql(y,n) + rsl(y,n)
\end{align*}
\]

\[
\begin{align*}
C(y,n) = & (BR(y,n) + 2) + (BI(y,n) + 2) \\
DR(y,n) = & qR(y,n) * BR(y,n) + qI(y,n) * BI(y,n) \\
DI(y,n) = & qI(y,n) * BR(y,n) - qR(y,n) * BI(y,n)
\end{align*}
\]

\[
\begin{align*}
FR(y,n) = & -rR(y,n) * BR(y,n) - rI(y,n) * BI(y,n) \\
FI(y,n) = & rR(y,n) * BI(y,n) - rI(y,n) * BR(y,n)
\end{align*}
\]

\[
\begin{align*}
gR(y,n) = & FR(y,n) / C(y,n) \\
gI(y,n) = & FI(y,n) / C(y,n)
\end{align*}
\]

\[
\begin{align*}
h(y,n) = & \text{radius}(gR(y,n), gI(y,n)) \\
X(y,n) = & \text{arctan}(gR(y,n), gI(y,n))
\end{align*}
\]

\[
\begin{align*}
& \text{repeat} \\
& \text{repeat}
\end{align*}
\]

\[
\begin{align*}
& \text{newlines(3)} \\
& \text{caption} \quad \text{RESULTS}
\end{align*}
\]

\[
\begin{align*}
& \text{newlines(2)} \\
& \text{caption} \quad \text{RESULTS}
\end{align*}
\]

\[
\begin{align*}
& \text{newline} \\
& \text{cycle} \quad n=0,1,12 \\
& \text{read} \quad (b(n), z(n)) \\
& \text{repeat} \\
& \text{cycle} \quad t=0,1,24
\end{align*}
\]
print (t,2,o)
spaces(4)
cycle y=0,1,5
T=0
cycle n=0,1,12
sn= sin (2r*n*t/24-z(n) +X(y,n) )
d=b(n)*h(y,n)*sn
T=T +d
repeat
Txt= 80 ER(y,0) +T
print f1 (Txt,3)
spaces(4)
repeat
newline
repeat
end of program

110.10  -1.5708  52.73  1.781  18.64  -1.559  0.9801  -0.113
3.478    1.636  0.4327  1.983  1.488  -1.458  0.1918  -0.526
0.5749  1.856  0.1490  0.959  0.5356  -1.491  0.1752  1.933
0.1458  1.571
***Z
APPENDIX (G)

COMPUTER PROGRAM FOR THE DETERMINATION OF THERMAL
STRESSES IN SLABS WITH RESTRAINED EDGES

This program is a continuation of the program in Appendix (F) for
the determination of the temperature distribution in roof slabs.

Thus the program starts after the printing instructions for
temperature (i.e. after line 53)

<table>
<thead>
<tr>
<th>caption</th>
<th>THESS SS STRESSES</th>
</tr>
</thead>
<tbody>
<tr>
<td>newline</td>
<td></td>
</tr>
</tbody>
</table>

| cycle   | t=0,1,24         |
| print   | (t,2,0)          |
| cycle   | y=1,2,11         |

DT=0

DT=TP(y,t) -TP(y,0)

f=15 DT

1 TP temperature of point
1 DT is temperature difference for a plane w.r.t. initial temperature
1 f is thermal stress in mid-plane of section
1 15 is αE/(1-μ)
1 α coefficient of thermal expansion =0.000004
1 E modulus of elasticity = 3000 000 psi
1 μ Poisson's ratio = 0.2

spaces (4)
print (f,4,2)
spaces (4)
repeat
newline
repeat
end of program
APPENDIX (H)

COMPUTER PROGRAM FOR THE CALCULATION OF

THERMAL STRESSES IN SIMPLY SUPPORTED SLABS

"Temperature variation through slab thickness only".

Similar to Appendix (G), this program is a continuation of that in Appendix (F):

Thus after line (53) of the program in Appendix (F), the program runs as follows:

```
caption  THERMAL STRESSES

newlines (3)

caption  F1 F2 F3 F4 F5 F6

newline

cycle  t=0,1,24

DT=0

NT(t) = 0

DM=0

MT(t) = 0

cycle  y=1,2,11

DT = TP(y,t) - TP(y,0)

NT(t)= NT(t) + DT

DM = (6-y) * DT/2

MT(t) = MT(t) + DM

repeat/
```
\begin{verbatim}
repeat
  cycle t=0,1,24
  print (t,2,0)
  cycle y=1,2,11
  f = 15 ( -(TP(y,t) - TP(y,0)) + \frac{1}{6} NT(t) + \frac{1}{36} (6-y) * NT(t) )
  TP temperature of point
  DT is temperature difference for a plane w.r.t. initial temperature
  f is thermal stress in mid-plane of section
  15 is \alpha E / (1-\mu)
  \alpha coefficient of thermal expansion = .00004
  E modulus of elasticity = 3000 000 psi
  \mu Poisson's ratio = 0.2
  spaces (4)
  print (f,4,2)
  spaces (4)
  repeat
  newline
  repeat
end of program
\end{verbatim}
APPENDIX (J)

COMPUTER PROGRAM FOR THE CALCULATION OF

THERMAL STRESSES IN SIMPLY SUPPORTED SLAPS

(With temperature variation along slab surface only)

In this program, use has been made of "Sparse Matrix Routines" (46).

\begin{verbatim}
integer m,n,wm
real p
integerarray ws(0:50)
routine spec wread (arrayname w, integerarrayname ws, integer c)
routine spec wprint (arrayname w, integerarrayname ws, integer a,d,e,m,n)
routine spec wadd (arrayname w, integerarrayname ws, integer a,b,c,
 x,y, real f)
routine spec wdiv (arrayname w, integerarrayname ws, integer a)
read (wm)
begin
array w(0:wm)
read (m,n,p)
wread (w,ws,1)
caption \& MATRIX \& A
wprint (w,ws,1,0,0,3,3)
wread (w,ws,2)
caption \& MATRIX \& B
wprint (w,ws,2,0,0,3,1)
wadd/
\end{verbatim}
wadd (w,ws,1,2,3,0,81,1)
caption A & A &\& B & A &\& compounded & by & A columns
wprint (w,ws,3,0,81,6,2)
wdx (w,ws,3)
caption A & Matrix & A & B
wprint (w,ws,3,0,0,6,2)
end

routine wread (arrayname w, integerarrayname ws, integer c)

.....................
.....................
.....................
end

routine wprint (arrayname w, integerarrayname ws, integer a,d,e,m,n)

.....................
.....................

routine wadd (arrayname w, integerarrayname ws, integer a,b,c,x,
y, real f)

.....................
.....................
end

routine wdiv (arrayname w, integerarrayname ws, integer a)

.....................
.....................
end

end of program
APPENDIX(K)

COMPUTER PROGRAM FOR THE ANALYSIS OF
THERMAL STRESSES IN WALLS

(47)

(Using COMP FACT Routines)

***A

JOB

CIE 03/0000 0008 /EL-NOURANI /THERMAL STRESSES /WALLS

OUTPUT

O LINE PRINTER 1000 LINES

COMPILER AA

begin

realarray  A(1:67, 1:67) , B,X( 1:67 ), D1(1:67), D2(1:67)
integerarray P(1:2,1:66)

integer i,j

routine comp fact(array name A,B,X,D1,D2, integer array name P, integer n,Q)

integer i,j,C,r,s
real max,E

->0 if n>0

caption comp#fact#DATA#FAULT,n<0

stop

6:-10if n>1

X(1)=B(i)/A(1,1)

->11

10:-2ifQ<1

cycle i=1,1,n

max=0

cycle j=1,1,n

if max < |A(i,j)| then max=|A(i,j)|

repeat

D1(i)=max

cycle j = 1,1,n

A(i,j)=A(i,j)/max

repeat

repeat

cycle i=1,1,n

max=0

cycle j=1,1,n

if max < |A(j,i)| then max=|A(j,i)|

repeat

D2(i)=max
cycle $j=1,1,n$
$A(j,i)=A(j,i)/\max$
repeat
repeat

\begin{itemize}
\item cycle $C=1,1,n-1$
\item max=0
\item cycle $i=C,1,n$
\item cycle $j=C,1,n$
\item if max > \|A(i,j)\| then \rightarrow 5
\item max=\|A(i,j)\|; r=i; s=j
\end{itemize}

5: repeat
repeat
P(1,C)=r; P(2,C)=s
cycle $i=C,1,n$
E=A(C,i)
A(C,i)=A(r,i)
A(r,i)=E
repeat
cycle $i=1,1,n$
E=A(i,C)
A(i,C)=A(i,s)
A(i,s)=E
repeat
E=A(C,C)
\item cycle $i=C+1,1,n$
\item $\Lambda(i,C)=A(i,C)/\max$
\item repeat
\item cycle $i=C+1,1,n$
\item E=A(i,C)
\item cycle $j=C+1,1,n$
\item $\Lambda(i,j)=A(i,j)-A(C,j) \times \max$
\item repeat
\item repeat
\item repeat
\item repeat

2: cycle $i=1,1,n$
X(i)=E(i)/E(i)
repeat
\item cycle $i=1,1,n-1$
E=X(P(1,i))
X(P(1,i))=X(i)
X(i)=E
\item cycle $j=i+1,1,n$
X(j)=X(j)-\max \times A(j,i)$
repeat
repeat

\rightarrow 99 \quad \text{if} \quad A(n,n) < 10^{-10}$

X(n)=X(n)/A(n,n)
\[ X(n) = 10000 \times D2(n) \]

\[
\begin{align*}
\text{cycle } i &= n-1, -1, 1 \\
E &= X(i) \\
\text{cycle } j &= i+1, 1, n \\
E &= E - X(j) \times A(i, j) \\
&\text{repeat} \\
X(i) &= E/A(i, i) \\
&\text{repeat} \\
\text{cycle } i &= n-1, -1, 1 \\
E &= X(i) \\
X(i) &= X(P(2, i)) \\
X(P(2, i)) &= E \\
&\text{repeat} \\
\text{cycle } i &= 1, 1, n \\
X(i) &= X(i)/D2(i) \\
&\text{repeat} \\
\text{end}
\end{align*}
\]

\[
\begin{align*}
\text{cycle } i &= 1, 1, 57 \\
\text{cycle } j &= 1, 1, 57 \\
A(i, j) &= 0 \\
&\text{read } (A(i, j)) \\
&\text{repeat} \\
&\text{repeat} \\
\text{cycle } i &= 1, 1, 57 \\
B(i) &= 0 \\
&\text{read } (B(i)) \\
&\text{repeat} \\
&\text{comp fact } (A, B, X, D1, D2, P, 57, 1) \\
\text{caption } \mathbf{M} \text{ ELEMENT} \\
&\text{newlines}(2) \\
\text{cycle } i &= 1, 1, 57 \\
\text{print } (i, 2, 0) \text{ ; spaces } (4) \\
\text{print } (X(i), 6, 3) \\
&\text{newline} \\
&\text{repeat} \\
&\text{endof program}
\end{align*}
\]
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