ASPECTS OF PARTON MODELS
OF DEEP INELASTIC SCATTERING

Thesis

Submitted by

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DECLARATION

The work in this thesis is entirely my own, except where otherwise indicated.
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ABSTRACT

This thesis considers several different (though not unrelated) aspects of parton models of deep inelastic scattering.

In the first chapter a parton model is set up. All the usual results are obtained, viz.: scaling, Callan-Gross relations. Sum-rules are derived for polarized and unpolarized electromagnetic and for unpolarized weak deep inelastic scattering.

In Chapter Two we motivate and construct a parton model in the target hadron rest frame, and compare with predictions of the usual infinite momentum frame formulation. Some extra predictions are presented, and subasymptotic behaviour examined. Comparison is made with experiment.

We discuss the need to include confinement within the parton model in Chapter Three. Consequences of this for the net jet charge are examined. This is followed by a general discussion of the effects of confinement on the parton model structure functions. These effects are found to be vanishingly small in the Bjorken limit. A particular (quantum mechanical) model is constructed, in which partons are localised. Consequences of this localization for polarized and unpolarized valence and sea partons are examined.

The Fourth Chapter attempts, with partial success, to explain the Melosh transformation in terms of relativistic kinematics. Also we show how confinement leads to a spin rotation which lowers the predicted value of the axial/vector coupling constant ratio from the SU6 value.

In the Fifth and final chapter we compare the QCD predictions for deep inelastic scattering with those of the subasymptotic parton model, and conclude that dynamical higher twist effects must be
small. A broader investigation into the relation between parton models and field theory follows, from which we develop a prescription for telling us what field theory underlies a given parton model.
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## CHAPTER 5 PARTON MODELS AND FIELD THEORY

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If by "scaling" or "scale invariance" we mean that an equation of physics is unaltered by an arbitrary change of length scale, then it is not difficult to visualize the circumstances under which scaling might be expected to hold (almost) in nature. Upon consideration of the various length scales that are known to exist we note that they are separated into well-defined regions; modestly altering the length scale of a system in one of these regions might be reasonably expected to leave the system unchanged, since the gap that must be crossed to reach another scale (thus providing a reference length) may be much larger than the "modest alteration".

From the uncertainty principle we may substitute "energy scale" for "length scale" in the above discussion.

\[
\begin{array}{c|c|c}
\text{eV} & \text{length} \\
10^{-28} & \text{galactic} & 10^{25} \\
10^{-3} & \text{atomic} & 10^{-8} \\
10^{7} & \text{nuclear} & 10^{-12} \\
10^{8} & \text{nucleon} & 10^{-13} \\
& \text{subnucleon} & \\
\end{array}
\]

(Data taken from Ford, 1963).

If only one energy/length scale existed in nature, the above would suggest that scaling is exact. However this is not the case. Hence the "almost" in the first sentence means that scaling may pertain within somewhat fuzzy boundaries surrounding the above regions; stepping beyond these boundaries incurs the penalty of "scale breaking".
A familiar example of scaling is the problem inherent in judging the distance of a faraway object, such as a star in space. The scale-breaking associated with this is the phenomenon of parallax.

The above motivation for scaling has been taken literally in one hadron model, that of Kogut and Susskind to be discussed in §29 below. Complementing this approach is that of dilation invariance (see e.g. Roy, 1975). A very simple demonstration of how this leads to scaling in deep inelastic scattering (DIS) is given in the next section.

DIS aims to uncover the structure of hadrons by probing them with very high energy (high resolution) leptons. At high energies we expect the hadron to appear less complex than at lower energies because in this region the time scale for DIS is so much shorter than any hadron time scale (e.g. the time scale for interactions between the hadron's constituents) that the hadron appears frozen. This is equivalent to saying that the probe energy is much greater than the hadron energy scales (e.g. constituent interaction energy) that the constituents appear to be free. This will be discussed more fully in Chapter 2. Later we shall also see that the above-mentioned property of scaling obtains in DIS experiments. The simplest DIS models reproducing this now-well-verified scaling (and scale breaking!) phenomenon are the so-called "parton" models. These provide an intuitively appealing framework for displaying the experimental results, as many experimental facts are built into parton models. An example of this is the Callan-Gross (CG) relation, as shown in §8.

This thesis will consider several aspects of these parton models of DIS. An incomplete though hopefully comprehensive background is provided in Chapter 1. A simple parton model is set up; the aim here is threefold: to touch upon and relate several areas discussed in
Chapters 2-5, to expose many well-known facets of DIS parton models, and to provide a bibliography.

Chapter 2 introduces a rest frame parton model and takes up the much-discussed scale breaking of DIS structure functions. Kinematics is discussed in detail.

In Chapter 3 the problem of confinement, normally neglected in parton models, is included in the simplest possible way. Firstly quantum number exchange, necessary for the DIS final state to exhibit integer quantum numbers, is investigated. Secondly in a quantum mechanical model we examine the effects of confinement upon the constituent momentum and spin distributions.

In Chapter 4 various spin rotations are discussed with a view to shedding light on the Melosh transformation. The now-popular "null plane" interpretation of partons is sketched.

The final chapter discusses the effects of an underlying field theory of hadrons on DIS parton models. We review and extend the Kogut and Susskind approach, and compare the naive parton model predictions with those of QCD.
§1 Scale Invariance in DIS

In the introduction we suggested that dilation invariance might be a symmetry of hadronic interactions (in particular). We take this idea literally here for those hadronic interactions under consideration i.e: for the DIS of leptons off hadrons. We shall show that any model which assumes that DIS is unaltered by a change in scale leads to "Bjorken scaling".

For most of this thesis we shall be considering naive parton models of DIS. These are characterized by the assumption that at extreme energies the lepton probe sees the hadron as an incoherent bundle of point-like constituents (see §2). The emphasis here is on point-like: this ensures that naive parton models respect dilation invariance. The result of this section then guarantees that naive parton models exhibit Bjorken scaling.

Later on, in §29, we shall present less-primitive parton models, which are not dilation invariant, and shall find as a consequence they display scaling violations.

The scale transformation shall be written as \( r \rightarrow r' = e^\varepsilon r, \varepsilon \) real, for spacetime coordinates \( r \). The effect of this on a spinor field is (Roy, 1975, pp. 67-8. All references here will be to Roy).

\[
\psi(r) \rightarrow \psi'(r) = e^{\frac{3\varepsilon}{2}} \psi(e^\varepsilon r) \quad (1.1)
\]

as is easily shown. We might thus expect the nucleon EM current to transform as

\[
J_\mu(r) \rightarrow J'_\mu(r) = e^{3\varepsilon} J_\mu(e^\varepsilon r) \quad (1.2)
\]
From (1.1), using \( |p\rangle = a^+(p)|0\rangle \) and expressing the creation operator in terms of the field, we get

\[
|p\rangle \rightarrow |p\rangle' = e^{-\varepsilon} e^{-\varepsilon p}\rangle
\] (1.3)

assuming that the vacuum is unchanged by a scale transformation (p. 75).

(Here the symbol inside the ket refers to the particle's momentum, so that \( p \rightarrow p' = e^{-\varepsilon} p \), as we would expect from the uncertainty principle).

The hadronic vertex of DIS is characterized by a structure tensor (see §2 or Roy, or Close, 1979)

\[
W_{\mu\nu}(p,q) = (2\pi)^2 \frac{p_0}{M} \int d^4r \, e^{iq \cdot r} \langle p | J_\mu(0) J_\nu(r) | p\rangle . \quad (1.4)
\]

We wish to dilate this but are faced immediately with a problem: how do we deal with the masses? These should not be altered by a scale transformation, and yet if this is the case then objects like \( p_0 \) do not behave simply under scale transformations. This exposes the main weakness of the dilation invariance idea. In fact dilation invariance can only be exact if hadron masses are continuous or zero (this is evident from \( p^2 = M^2 \)). The problem can be circumvented here by assuming that the hadron is moving with infinite momentum, so that \( p_0 \sim |p| \) and hence scales. Thus dilation invariance cannot possibly be a rest frame symmetry: it is at best an approximate symmetry for extreme relativistic hadrons.

With this in mind we apply eqns. (1.2) and (1.3) to the structure tensor (1.4) to find that the scale transformation \( r \rightarrow r' \) leads to

\[
W_{\mu\nu}(p,q) \rightarrow W'_{\mu\nu}(p,q) = W_{\mu\nu}(p',q')
\] (1.5)

where \( p' = e^{-\varepsilon} p \), etc.
W_{\mu\nu} can be expanded in terms of p_\mu, q_\nu and scalar "structure functions" W_{1,2}, as we shall see in §2. The result for EM DIS is

\[ W_{\mu\nu}(p,q) = (-g_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2}) W_1(\nu, Q^2) + \]

\[ + \frac{1}{M^2} (p_\mu + \frac{p^* q_\mu}{Q^2})(p_\nu + \frac{p^* q_\nu}{Q^2}) W_2(\nu, Q^2) \]

where \nu, Q^2 are Lorentz scalars formed from the available four vectors:

\[ \nu = \frac{p^* q}{M}, \quad Q^2 = -q^2. \]

Invariance of hadronic interactions under a change in scale means that W_{\mu\nu} is unaltered by the above scale transformation

\[ W'_{\mu\nu}(p,q) = W_{\mu\nu}(p,q) \quad (1.6) \]

(1.5) and (1.6) together mean that

\[ W_{\mu\nu}(p',q') = W_{\mu\nu}(p,q). \]

In terms of the EM structure functions this yields

\[
\begin{align*}
W_1(\nu', Q^2') & = W_1(\nu, Q^2) \\
W_2(\nu', Q^2') & = e^{2\epsilon} W_2(\nu, Q^2)
\end{align*}
\]

where \nu' = e^{-2\epsilon}\nu, Q^2' = e^{-2\epsilon}Q^2. We choose to express W_{1,2} as functions of (x,Q^2) instead of (\nu, Q^2), where \[ x = \frac{Q^2}{2M\nu} \] is called the "Bjorken scaling variable" (Bjorken and Paschos, 1969) for reasons that shall soon become apparent. Note that it is dilation invariant: x' = x.

From (1.7) we see that

\[
\begin{align*}
MW_1(x,Q^2) & \equiv F_1(x,Q^2) = F_1(x,e^{-2\epsilon}Q^2) \\
MW_2(x,Q^2) & \equiv F_2(x,Q^2) = F_2(x,e^{-2\epsilon}Q^2)
\end{align*}
\]

\(\epsilon\) is arbitrary and so we have the result that the EM structure functions
are unaltered by an arbitrary change in $Q^2$, i.e.:

$$F_{1,2}(x,Q^2) = F_{1,2}(x).$$

This is Bjorken scaling.

The above calculations were done for EM DIS. It is easy to extend them to weak DIS and to polarized EM DIS. The only change necessary is the replacement of the general expansion for $W_{\mu\nu}$ by the weak or polarized EM analogue (cf. eqns. (5.2), (7.2)). The result is that scaling obtains for

$$M_{W_1} \equiv F_1$$
$$\nu W_{2,3,4,5} \equiv F_{2,3,4,5}$$

for the weak case, and

$$\nu X_1 \equiv g_1$$
$$\frac{\nu^2}{M} X_2 \equiv g_2$$

for the polarized EM case.

Thus the above dimensional analysis arguments demand that Bjorken scaling obtains for weak and for polarised and unpolarized EM DIS in the limit of large momenta. For reasons discussed above we expect the same result to hold for naive parton models, to which we now turn.

§2 A Parton Model of EM DIS

DIS consists of scattering very high energy leptons off target hadrons (in practice, protons or neutrons) i.e: $eH \rightarrow e'X$ as indicated in Fig. (2.1). Leptons ($\mu, e, \nu, \bar{\nu}$) are convenient probes because they are known to be pointlike and so any structure that is revealed in DIS is sure to be hadron structure. The leptons are given
extreme energies because in this case we expect hadron structure to be simpler than at lower energies, as discussed in §0. In particular we expect the composite hadron to appear as a collection of quasi-free pointlike constituents in this high energy limit.

Assuming single-photon exchange dominates (Close, 1973) we can write for the DIS double differential cross-section (e.g.: Close, 1979; Roy, 1975)

\[
\frac{d^2\sigma}{d\Omega'dE'} = \frac{\alpha^2}{Q^4} \frac{E'}{E} L_{\mu\nu} W_{\mu\nu}
\]

where \(-Q^2 = q^2 < 0\) is the four-momentum transfer squared, and where \(E, E'\) are defined in Fig. (2.1). The lepton vertex tensor is

\[
L_{\mu\nu} = e_{\mu'} e'_{\nu} + e_{\mu'} e_{\nu} - g_{\mu\nu} \cdot q
\]  

(2.1)

while \(W_{\mu\nu}\) is the hadron vertex tensor (e.g.: Roy, 1975; Close, 1979)

\[
W_{\mu\nu} = (2\pi)^2 \frac{P_o}{M} \int d^4 r e^{-i q \cdot r} \langle p | J_\mu (0) J_\nu (r) | p \rangle
\]  

(2.2)

where \(P_o\) is the hadron energy, \(M\) its mass, and where \(J_\mu\) is the hadron current appropriate to the DIS process under consideration. We are able to write \(L_{\mu\nu}\) in the form (2.1) because leptons are pointlike, and we are able to evaluate the lepton current matrix elements. We are unable to do this for \(W_{\mu\nu}\); we cannot proceed beyond eqn. (2.2) without making some assumptions about the hadronic current matrix elements. To this end we assume Hermiticity, gauge invariance, \(P\) and \(T\) invariance, so that we may expand (2.2) in terms of available four vectors, as in §1:

\[
W_{\mu\nu}(p, q) = (-g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2}) W_1 (v, q^2)
\]

\[
+ \frac{1}{M^2} (p_{\mu} - \frac{p \cdot q}{q^2} q_{\mu}) (p_{\nu} - \frac{p \cdot q}{q^2} q_{\nu}) W_2 (v, q^2).
\]

(2.3)
Imposing the above symmetries has thus allowed us to extract the Lorentz structure, leaving two invariant structure functions $W_{1,2}$ to represent our ignorance of EM DIS. $v$ and $Q^2$ are measured experimentally (in the LAB frame) through the kinematic relations

$$
\begin{align*}
v &= E - E' \\
Q^2 &= 4EE' \sin^2 \frac{\theta'}{2}
\end{align*}
$$

We can proceed no further without making a dynamical assumption about hadron structure. The parton model dynamical assumption is that in DIS

(i) the hadron appears to the virtual photon ($\gamma^*$) probe to consist of independent pointlike spin $\frac{1}{2}$ constituents called partons. By "independent" we mean that $\gamma H$ scattering appears in the deep inelastic limit to be just the incoherent sum of elastic lepton-parton scattering. This notion is most believable when the momentum transfer (and hence $v, Q^2$) is very large, since in this case the hadron constituents will appear to be quasi-free to the $\gamma^*$ probe $^\dagger$.

We implement assumption (i) by introducing (complete) sets of multiparton states into $W_{\mu\nu}$. If we symbolically write the matrix element of eqn. (2.2) as $\langle p | J J | p \rangle$ then from (i)

$$
\begin{align*}
\langle p | J J | p \rangle &= \sum_{123} \langle p | 1 \rangle \langle 1 | j_2 \rangle \langle 2 | j_3 \rangle \langle 3 | p \rangle + \\
&\sum_{1234} \langle p | 1 \rangle \langle 1 | j_{23} \rangle \langle 23 | j_4 \rangle \langle 4 | p \rangle + \\
&\sum_{123456} \langle p | 12 \rangle \langle 12 | j_{34} \rangle \langle 34 | j_{56} \rangle \langle 56 | p \rangle + ..... \\
&\equiv (1,0) W_{\mu\nu} + (1,2) W_{\mu\nu} + (2,2) W_{\mu\nu} + ..... 
\end{align*}
$$

$^\dagger$ This was discussed in §0. More formally we may say that we believe the partons to be quasi-free because the underlying field theory, of which partons are the quanta, is soft. This will be discussed in detail in Chapter 5.
where the hadronic current $J$ is now just the free pointlike parton current $j$ and spectator partons are not indicated. Explicitly

$$
(1,1)_{\mu\nu} = (2\pi)^2 \frac{p_o}{M} \sum_{ijk} \int d^3k_1 \ d^3k_2 \ d^3k_3 \ \Sigma_{\sigma_1 \sigma_2 \sigma_3} \ \delta^3(x_{1,2,3}) \ <p|k_1 \gamma_\mu X_1|_1 \cdot (2\pi)^2 \left< k_1 \sigma_1; X_1 | j_\mu(0) | k_2 \sigma_2; X_2 \right>_j j < k_2 \sigma_2; X_2 | \gamma^\nu j_\nu(q) | k_3 \sigma_3; X_3 >_k \cdot < k_3 \sigma_3; X_3 | p>
$$

Here $\sum_X$ means integrating over the momenta and summing over the spins of $X$, where $X$ represents the "spectator" partons, i.e.: those not entering into the interaction with the EM probe. $k, \sigma$ represent the interacting partons' momentum, spin and $ijk$ indicate parton type or "flavour". $j(q)$ is the Fourier transform of the coordinate space free parton current

$$j_\nu = \sum_{k \ell} \bar{\psi}_k \gamma_\nu Q_{k \ell} \psi_\ell : \quad (2.6)$$

We calculate the current matrix element to be

$$j < k_2 \sigma_2; X_2 | \gamma^\nu j_\nu(q) | k_3 \sigma_3; X_3 >_k = (2\pi)^4 \delta^4(k_3 + X_3 + q - k_2 - X_2) X_2 | X_3 >$$

$$\cdot \frac{1}{(2\pi)^3} \frac{\mu}{\sqrt{k_2 k_3}} \left[ \bar{u}(k_2 \sigma_2) \gamma_\nu Q_{jk} u(k_3 \sigma_3) - \bar{v}(k_3 \sigma_3) \gamma_\nu Q_{kj} v(k_2 \sigma_2) \right]$$

and similarly for $< j_\mu(0) |$ but without the $\delta$ function. Here we have pulled the $X$'s through the current, since they are spectators.

Using the identity

$$\int \frac{d^3k_2}{k_2^0} \delta^4(k_1 + q - k_2) = \delta(k_1 \cdot q - \frac{1}{2}q^2)$$

we can, after some calculation, reduce (2.5) to
\[(1,1) \mathcal{W}_{\mu\nu} = \frac{P_0}{M} \sum_{ij} Q_{ij} Q_{ji} \int \frac{d^3 k_1}{\sigma_1} \delta(k_1 \cdot q - \frac{1}{2} Q^2) \sum_{X_1} \frac{P_{i}^i(k_1; X_1) \frac{\mu^2}{k_0^2}}{\sigma_1 \cdot X_1} \cdot \left[ u(k_1 \sigma_1) \gamma_\mu u(k_1 + q \sigma_2) \cdot \bar{u}(k_1 + q \sigma_2) \gamma_\nu u(k_1 \sigma_1) + \text{antiparton term} \right] \tag{2.7} \]

where \( P_{i}^i(k_1; X_1) = <p \mid k_1 \sigma_1; X_1 \rangle \cdot <k_1 \sigma_1; X_1 \mid p> \) is interpreted as the probability for finding the target hadron in state \( |k_1 \sigma_1; X_1 \rangle \), i.e. as a Dirac parton (with momentum \( k_1 \) and spin \( \sigma_1 \)) plus spectators \( X_1 \).

We write \( e_i^2 = \sum_{j} Q_{ij} Q_{ji} \) and assume that \( e_i^2 \) is independent of \( \sigma \) for unpolarized hadrons.

Thus \( P_{i}^i(k_1) = \frac{1}{2} \bar{P}_i(k) \) where \( P_{i}^i(k_1) = \int_{X_1} P_{i}^i(k_1; X_1) \) and so we have

\[(1,1) \mathcal{W}_{\mu\nu} = \sum_i e_i^2 \frac{P_0}{M} \int \frac{d^3 k}{\sigma_1} \delta(k \cdot q - \frac{1}{2} Q^2) \frac{1}{2} \bar{P}_i(k) \cdot \left[ 2 k_\mu k_\nu + k_\mu q_\nu + q_\mu k_\nu - g_{\mu\nu} k \cdot q \right] + \text{antiparton term} \tag{2.8} \]

for the simple-parton contribution to \( \mathcal{W}_{\mu\nu} \). Note that \( [ \hspace{1cm} ] \) in (2.8) is just the \( L_{\mu\nu} \) of eqn. (2.1) with parton momentum replacing lepton momentum. The reason for this, of course, is that we have assumed partons are free field quanta.

Let us temporarily assume that

\[(iii) \ k_\mu = x \ p_\mu \]

i.e. that the parton entering the EM vertex carries a momentum \( k \) that is just an overall fraction \( x \) of the hadron momentum \( p \). This assumption is reasonable in a frame in which the hadron possesses very large momentum (e.g. Bjorken and Paschos, 1969). Thus we expect \( 0 \leq x \leq 1 \).

Assumption (iii) requires us to alter eqn. (2.8) as follows:

\[ \int d^3 k \ P_{i}^i(k) \ ... \to \int_0^1 d\bar{x} \ f_{i}(\bar{x}) \ ... \]
with \( \int d^3k \, p_i(k) = 1 \) implying \( \int_0^1 dx \, f_i(x) = 1 \). Also

\[
\delta(k \cdot q - \frac{1}{2} Q^2) \rightarrow \frac{1}{p \cdot q} \delta(\frac{1}{x} - x)
\]

where \( x = \frac{Q^2}{2 M^2} \) is the famous Bjorken scaling variable, here interpreted as a momentum fraction. Thus (2.8) becomes

\[
(1,1) W_{\mu \nu} = \sum_i e_i^2 \frac{1}{2} f_i(x) \left( \frac{1}{p \cdot q} \left[ p \cdot q (-\varepsilon_{\mu \nu} + q_{\mu} q_{\nu}) + 2x(p_{\mu} - \frac{p \cdot q}{q^2} q_{\mu})(p_{\nu} - \frac{p \cdot q}{q^2} q_{\nu}) \right] + \text{antiparton term.} \right)
\]

Comparing with the general expansion (2.3) we see immediately that

\[
\begin{align*}
MW_1(v, Q^2) & \equiv F_1(x) = \frac{1}{2} \sum_i e_i^2 f_i(x) \\
vw_2(v, Q^2) & \equiv F_2(x) = x \sum_i e_i^2 f_i(x)
\end{align*}
\]

(2.9)

Eqn. (2.9) presents us with the most important predictions of the primitive parton model of EM DIS:

(i) The structure functions scale, i.e.: are functions of the ratio \( x \) only.

(ii) The CG (Callan, Cross, 1969) relation, relating \( F_{1,2} \).

(iii) The "master formula" (Close, 1979) for \( F_2(x) \).

Prediction (i) was not in disagreement with the early data (Close, 1973) as shown in Fig. (2.2), though subsequently some scaling violation has been observed (we will have a lot more to say about this later on).

Prediction (ii) is also realized in the data (Bodek et al., 1979): see Fig. (2.3). The third and most important prediction leads to many sumrules, as we shall see in §6. Most of these sumrules are in rather good agreement with experiment. Thus already the naive parton model
has a powerful body of support indicating that hadrons indeed appear to be composed of free pointlike Dirac particles, when probed at extreme energies.

§3 Form Factors

The matrix element for the elastic scattering of composite Dirac particles (see Fig. 3.1) is

$$\langle p's' | J^{EM}_\mu | ps \rangle = \frac{1}{(2\pi)^3} \frac{M}{\sqrt{p'_0 p_0}} \overline{u}(p's').$$

(3.1)

$$\cdot \{F_1(Q^2)\gamma_\mu + \frac{i\kappa}{2M} F_2(Q^2)\sigma_{\mu\nu} q^\nu\} u(ps)$$

whereas for pointlike Fermions we find (cf. §2)

$$\langle p's' | j^{EM}_\mu | ps \rangle = \frac{1}{(2\pi)^3} \frac{M}{\sqrt{p'_0 p_0}} \overline{u}(p's')\gamma_\mu u(ps)$$

and so to calculate with composite Fermions rather than pointlike ones we make the substitution $\gamma_\mu \rightarrow \Gamma_\mu = \{ \}$. The form of $\Gamma_\mu$ is obtained by expanding in terms of available four-vectors, subject to gauge invariance (e.g.: Jarlskog, 1974, Close, 1979). The functions $F_{1,2}$ are known as form factors, and represent our ignorance of elastic scattering of composite particles. For pointlike particles we see that $F_1(Q^2) = 1, F_2(Q^2) = 0$ for all $Q^2$. Thus form factors play an analogous role to structure functions; the former appear in the elastic scattering amplitude whereas the latter appear in the inelastic scattering cross-section.

$\kappa$ is the anomalous magnetic moment of the composite Fermion (i.e.: over and above the magnetic moment we would expect for a pointlike particle obeying the Dirac equation, vis. $\frac{e}{2M}$). The form factors satisfy $F_{1,2}(0) = 1$. This identification of $\kappa$ and the threshold
values of $F_1, F_2$ come from the non-relativistic reduction of the Dirac equation with EM interaction (see Gasiorowicz, 1966; Baym, 1974; Bjorken and Drell, 1964).

It is conventional to define

$$G_M \equiv F_1 + F_2 \kappa$$

$$G_E \equiv F_1 - \frac{Q^2}{4M^2} F_2 \kappa$$

known as magnetic, electric form factors since $G_M(0) = \mu = 1+\kappa$, $G_E(0) = 1$, for the proton. Experimentally

$$\frac{G_M(Q^2)}{G_M(0)} = \frac{1}{1 + \frac{Q^2}{0.71 \text{ GeV}^2}} \sim Q^4$$

for large $Q^2$ for the proton (e.g.: Close, 1979; Wilson, 1971).

Let us assume that partons are not in fact pointlike, but are composite. From the above discussion we see that to take this into account we substitute $\gamma_\mu \rightarrow F_\mu$. Repeating the calculations of §2 with this change we find that the structure functions are altered as follows:

$$F_1(x) \rightarrow \left[ F_1^2(Q^2) + \frac{Q^2 \kappa^2}{4\mu^2} F_2^2(Q^2) \right] F_1(x)$$

$$F_2(x) \rightarrow \left[ F_1^2(Q^2) + \frac{2Q^2 \kappa^2}{4\mu^2} F_2^2(Q^2) \right] F_2(x).$$

Thus if partons are composite then scaling is broken and the CG relation no longer holds. But as was discussed in §2 the CG relation is approximately satisfied (to within $\sim 20\%$, as we shall see later) so that $F_2(Q^2) \sim 0$ at present machine energies. Furthermore scaling was observed to hold in the early experiments (Bjorken and Paschos, 1969; Wilson, 1971) so that $F_1(Q^2) \sim 1$. Thus, at least in the early data, partons appear to be pointlike. This is the main conclusion of
We cannot yet close the door on parton structure, however. Whilst partons presented themselves as pointlike objects to the $\gamma^*$ probes of the early 1970's, the higher-energy $\gamma^*$ that will be produced by future generations of accelerators may resolve a parton structure. This is because higher energy $\gamma^*$ can probe smaller distances, by the uncertainty principle. In fact, as we shall discuss in detail below, scaling violations are being seen today and so perhaps partons are beginning to show some structure. On the other hand these violations can be explained kinematically within the framework of the naive (pointlike) parton model presented in §2 (see section §13 below). We shall leave the question of the "compositeness" of partons to future sections, for now we accept the evidence presented above and say that, from the early data, partons appear to be pointlike.

§4 Cats Ears, Z Graphs, and Bubbles

In §2 we calculated the one parton contribution $(1,1)W_{\mu\nu}$ to the DIS structure function $F_2$. We saw that this involved the probability density $P(k) = \int |<k;X|p>|^2$ and we represent this contribution by Fig. (4.1).

We devote this section to an examination of the multiparton terms in $W_{\mu\nu}$. These fall into four distinct groups, the first of which appears in $(2,2)W_{\mu\nu}$ and is the (i) two-parton handbag diagram as shown in Fig. (4.2). It is easy to see that this involves

$$\int d^3k_2 |<k_1 k_2;X_{12}|p>|^2,$$

which is just the same as the previous probability density, since the parton carrying momentum $k_2$ is unseen by the $\gamma^*$ probe, and acts like just another spectator.
The second contribution also appears in $W_{\mu\nu}^{(2,2)}$ and is the 

(ii) cats ears diagram

as illustrated in Fig. (4.3). After a calculation similar to that of §2 we see that the cats ears involve the overlap

$$\int d^3k_1 \int d^3k_2 \langle p|k_1+q,k_2; X_{12}\rangle \langle k_1,k_2+q; X_{12}|p\rangle ...$$

We expect this to vanish in the Bjorken limit because of Landshoff's (1974) argument, which requires that the amplitude for finding a parton with momentum of order $\sim q$ dies with increasing $Q^2$. In field theory language this requirement is equivalent to the statement that the underlying field theory of hadrons is soft, i.e.: super-renormalizable. We shall discuss more in Chapter 5.

(iii) Z graph

This appears in $W_{\mu\nu}^{(1,3)}$ and is shown in Fig. (4.4). Its contribution to the structure function is

$$p_0 x \int \frac{d^3k}{k_0} P(k) 2k\cdot q \delta(k,q + \frac{1}{4}Q^2)$$

$$\rightarrow 2x \int_0^1 d\tilde{x} \delta(\tilde{x} + x) = 0$$

where in the last line we have invoked assumption (iii) of §2. This is the result quoted by Roy that in the infinite momentum frame the Z graph vanishes. From the discussion of §9 below it is apparent that Z graphs should also vanish in the hadron rest frame.

(iv) Bubble diagram

This also appears in $W_{\mu\nu}^{(1,3)}$ and is shown in Fig. (4.5). We calculate its contribution to $F_2$ to be

$$p_0 x \delta^3(0) \int \frac{d^3k_1}{k_1^0} \int \frac{d^3k_2}{k_2^0} 2k_1\cdot q \delta^4(k_1 + k_2 - q) .$$
This diagram has nothing to do with DIS; we see that the $\gamma^*$ does not even probe the hadron. In fact the bubble will disappear in the QED renormalization procedure. Thus we may ignore it here.

With our assumption of free (pointlike) parton currents we see that there are only two vertices. Because of this, all the multiparton contributions to $W_{\mu\nu}$ (i.e.: to $F_2^\prime$) must be of the type (i) - (iv). Introducing more partons into the calculations will not add any new diagrams, but will just add spectators to the above diagrams.

The result of this section, then, is that the only contribution to the DIS structure function that survives the Bjorken limit is the handbag diagram, Fig. (4.1), which we have calculated in §2.

§5 Extending the Model to Weak DIS

Weak DIS (e.g.: $\nu H \rightarrow \mu^+ X$) seeks to uncover the weak structure of the hadron target by probing at high energies with neutrinos or antineutrinos. In formulating parton models of weak DIS we find that the main change is the way in which the probe couples to the partons. Anticipating the identification of partons with quarks, which will be discussed more fully in the next section, we note that the fractional weak charges are unity ($w^2 = 1$) since partons have the same weak charge (i.e.: isospin) as the lepton probe. Furthermore the weak interactions are known to violate parity maximally; this is reflected in the $V^\pm A$ nature of the weak current, which means that we must make the substitution $\gamma_\mu \rightarrow \gamma_\mu (1 \pm \gamma_5)$ for $\bar{\nu}/\nu$ in the current (2.6).

Straightforward calculation shows that the $\gamma_5$ term adds an extra $+i\varepsilon^{\mu\nu\alpha\beta} q_\alpha k_\beta$ to $(1,1) W_{\mu\nu}$ (cf. eqn. 2.8). Making assumption (iii) of §2 then leads to the following expression for the weak vertex tensor

\[ W_{\mu\nu} = \frac{G_F}{\sqrt{2}} \bar{\nu} (1 \pm \gamma_5) \gamma_\mu \nu \]

\[ + i G_F \bar{\nu} (1 \pm \gamma_5) \gamma_5 \gamma_\mu \nu \]

\[ = G_F \bar{\nu} (1 \pm \gamma_5) (\gamma_\mu \nu + i \gamma_5 \gamma_\mu \nu) \]
\[ (1,1)_{w}^{\mu \nu} = \frac{1}{M} \frac{1}{p \cdot q} \sum_{i} f_{i}(x) \left[ 2x p^{\mu} p^{\nu} + (p^{\mu} q^{\nu} + q^{\mu} p^{\nu}) - p \cdot q g^{\mu \nu} \pm i \epsilon^{\mu \nu \alpha \beta} q_{\alpha} p_{\beta} \right]. \quad (5.1) \]

The general expansion for \( w_{\mu \nu} \) in terms of available four-vectors, taking into account Hermiticity and T invariance, is (Roy, 1975; Wray, 1972)

\[ w_{\mu \nu}(p,q) = -g^{\mu \nu} w_{1}(\nu,Q^{2}) + \frac{p^{\mu} p^{\nu}}{M^{2}} w_{2}(\nu,Q^{2}) \]

\[ -\frac{1}{2M^{2}} \frac{i}{\epsilon^{\mu \nu \alpha \beta}} q_{\alpha} p_{\beta} w_{3}(\nu,Q^{2}) \]

\[ + \frac{q^{\mu} q^{\nu}}{M^{2}} w_{4}(\nu,Q^{2}) + \frac{1}{2M^{2}} (p^{\mu} q^{\nu} + q^{\mu} p^{\nu}) w_{5}(\nu,Q^{2}). \quad (5.2) \]

The last three terms were absent in the EM case. The \( w_{3} \) term violates parity, and \( w_{4,5} \) terms are easily shown to be proportional to the divergence of the axial current \( \frac{\mu}{\alpha} A^{\mu} \).

Comparing eqns. (5.1) and (5.2) we may extract the parton model weak structure functions

\[ Mw_{1}(\nu,Q^{2}) \equiv F_{1}(x) = \sum_{i} f_{i}(x) \]

\[ w_{2}(\nu,Q^{2}) \equiv F_{2}(x) = 2x \sum_{i} f_{i}(x). \quad (5.3) \]

Thus we have the weak CG relation \( 2xF_{1} = F_{2} \).

There is a change in sign in the \( w_{3} \) term depending upon whether the lepton probe is a \( \nu \) or an \( \bar{\nu} \). But \( \nu q \) scattering is the same as \( \bar{\nu} q \) scattering (i.e.: \( \bar{\nu} q \leftrightarrow \nu \bar{q} \)) so that, for \( \nu \)

\[ w_{3}(\nu,Q^{2}) \equiv F_{3}(x) = 2 \sum_{i} \left[ f_{i} q(x) - f_{i} \bar{q}(x) \right] \quad (5.4) \]

where \( q \) stands for (anti)quark. In this notation \( f_{i}(x) \) in (5.3) equals \( f_{i} q(x) + f_{i} \bar{q}(x) \).
The $W_{4,5}$ terms in $W^{\mu\nu}$ pick up coefficients proportional to the lepton mass squared when contracted with the lepton vertex tensor $L_{\mu\nu}$. Thus $W_{4,5}$ do not contribute significantly to DIS. Nevertheless for completeness we write down their parton model expressions:

$$W_4(v, Q^2) = 0$$

in this particular model, and

$$W_5(v, Q^2) \equiv F_5(x) = 2 \sum_i f_i(x)$$

so that $2F_5 = F_2$.

Again we find that scaling pertains. This is observed experimentally (Benvenuto et al., 1979) and so we are encouraged to consider further the naive (i.e.: pointlike) parton model of weak DIS.

§6 Unpolarized Sumrules, Without Charm

Here we shall show how parton model sumrules are constructed, and shall compare these with experiment. Firstly let us endow the partons that enter into DIS with the quantum numbers of quarks, and call them quark-partons, $QP$. These quarks are the three (spin $\frac{1}{2}$) elements of the fundamental representation of SU3, and their quantum numbers are given in Table (6.1).

To simplify matters we shall consider only SU3 $QP$, i.e.: we shall neglect charm, bottom, top ... For a discussion of sumrules with charm see, for example, Close (1979). Furthermore we shall confine ourselves to unpolarized DIS: sumrules for polarized DIS are considered by Bartelski (1980).

The master formula of the naive parton model is given in eqn. (2.9):

$$F_2(x) = \sum_i e_i^2 x f_i(x) \equiv \sum_i e_i^2 i(x)$$
where $i = u, d, s$ is the distribution in momentum fraction for up, down, and strange QP. With the usual charge assignments (Table 1) we find for EM DIS

\[ F_{2}^{\text{YP}} = \frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d}) + \frac{1}{9}(s + \bar{s}) \]

\[ F_{2}^{\text{YN}} = \frac{4}{9}(d + \bar{d}) + \frac{1}{9}(u + \bar{u}) + \frac{1}{9}(s + \bar{s}) \]

for (p)roton and (n)eutron. Here we have used isospin symmetry:

\[ u^{p}(x) = d^{n}(x). \]

Let us divide each distribution into a "valence" and "sea" portion $i = i_{V} + K$. Then we find

\[ F_{2}^{\text{YP}} = \frac{4}{9} u_{V} + \frac{1}{9} d_{V} + \frac{4}{9} K \]

\[ F_{2}^{\text{YN}} = \frac{1}{9} u_{V} + \frac{4}{9} d_{V} + \frac{4}{9} K . \]

We see that $\frac{1}{4} \leq \frac{F_{2}^{\text{YN}}}{F_{2}^{\text{YP}}} \leq 4$ is a strict bound imposed on the EM structure functions by the partronmodel master formula, with isospin symmetry. In the SU3 valence approximation we neglect the effects of the sea (set $K = 0, 2d = u$) and so obtain

\[ \frac{F_{2}^{\text{YN}}}{F_{2}^{\text{YP}}} = \frac{2}{3} = \frac{i^{n} e_{i}^{2}}{i^{p} e_{i}^{2}} . \]

Experimentally the situation is as in Fig. (6.1). The ratio is $\sim 1$ near $x = 0$, which indicates in our model that the sea must dominate here. For midrange $x$ we find the ratio is near the SU3 value of $\frac{2}{3}$ and so we conclude that this is the valence region ($K \sim 0$: the sea has dropped out). Finally, for $x \rightarrow 1$ the data seems to indicate that the ratio $\rightarrow \frac{1}{4}$, which suggests the "active" QP ($u$ for proton, $d$ for neutron) carries most of the hadron momentum in this region.

\[ ^{\dagger} \text{Valence QP are the three SU3 QP that determine the nature of the hadron.} \]

\[ \text{Sea refers to all other QP constituents.} \]
The weak probe couples to QP isospins: $\nu^d(u) + \mu^u(d)$, neglecting the Cabibbo angle (Jarlskog, 1974), so that from the weak master formula, eqn. (5.3)

$$
\begin{align*}
F_{2}^{vp} &= 2(d + \bar{u}) \\
F_{2}^{vn} &= 2(u + \bar{d})
\end{align*}
$$

(6.2)

The factor 2 comes from the V-A nature of weak interactions. From (6.1) and (6.2) we have

$$
\frac{F_{2}^{vn} + F_{2}^{vp}}{F_{2}^{vn} + F_{2}^{vp}} = \frac{\frac{5}{9}(u + \bar{u} + d + \bar{d}) + \frac{2}{9}(s + \bar{s})}{2(u + \bar{u} + d + \bar{d})} \approx \frac{5}{18}.
$$

Experimentally (e.g.: Close, 1979) this ratio is $\approx \frac{5}{18}$ for $x \gtrsim 0.2$, so we again conclude that sea effects are small beyond small $x$.

The first sumrule we shall construct is the momentum conservation sumrule (MCSR). Using the definition of $i(x)$ we see that

$$
\int_{0}^{1} dx(u + \bar{u} + d + \bar{d} + s + \bar{s}) = 1 - \varepsilon
$$

(6.3)

The integral represents the total momentum fraction carried by QP. $\varepsilon$ is the momentum fraction of non-quark partons (i.e.: those that do not couple to the DIS probe, such as the intermediate gauge "gluons" of QCD). From (6.1) and (6.2) we see that (6.3) can be written as

$$
\text{MCSR: } \int_{0}^{1} dx \left[\frac{9}{2}(F_{2}^{vp} + F_{2}^{vn}) - \frac{3}{4}(F_{2}^{vp} + F_{2}^{vn})\right] = 1 - \varepsilon
$$

(6.4)

substituting experimental values we find (Close, 1979) $\varepsilon \approx 0.46$.

Since by definition $\frac{1}{x}i(x)$ is the probability for finding QP $i$ with momentum fraction $x$ in the proton, we can construct a strangeness conservation sumrule. Both proton and neutron have zero strangeness, so that
\begin{align*}
0 &= \int_{0}^{1} \frac{dx}{x} (s - \bar{s}) \quad (6.5)
\end{align*}

Similarly, using the fact that proton(neutron) has EM charge \(1(0)\) we can express charge conservation as
\begin{align*}
1 &= \int_{0}^{1} \frac{dx}{x} \left[ \frac{2}{3}(u - \bar{u}) - \frac{1}{3}(d - \bar{d}) \right] \\
0 &= \int_{0}^{1} \frac{dx}{x} \left[ \frac{2}{3}(d - \bar{d}) - \frac{1}{3}(u - \bar{u}) \right] \\
\end{align*}

where we have made use of (6.5) and of Table (6.1).

From (6.6) we see that
\begin{align*}
2 &= \int_{0}^{1} \frac{dx}{x} (u - \bar{u}) \\
1 &= \int_{0}^{1} \frac{dx}{x} (d - \bar{d}) \\
\end{align*}

so the integrals in eqns. (6.5) and (6.7) indicate the number of \(s, u, d\) valence QP in the proton. From (6.1) and (6.6) we construct the charge conservation sumrule:
\begin{align*}
\text{CCSR} : \int_{0}^{1} \frac{dx}{x} (F_{2}^{P} - F_{2}^{N}) &= \frac{1}{3} \\
\end{align*}

where we have assumed \(i = i_{v} + \kappa\), as above. Experimentally the integral is \(0.28 \pm 0.06\) (Close, 1979), in agreement with the QP model.

From the flavour-counting sumrules (6.5) and (6.7) we construct the Adler sumrule:
\begin{align*}
\text{Adler sumrule:} \int_{0}^{1} \frac{dx}{x} (F_{2}^{\nu P} - F_{2}^{\nu P}) &= 2
\end{align*}

which is the weak analogue of the CCSR.

From eqn. (5.4) we see that
\begin{align*}
F_{3}(x) &= \frac{2}{x} \sum_{i} e_{i}^{2} (i - \bar{i})
\end{align*}
so that we have for weak DIS

\[ x F_{3}^{v_{p}} = x F_{3}^{v_{n}} = 2(d - u) \]
\[ x F_{3}^{v_{p}} = x F_{3}^{v_{n}} = 2(u - d) \]  (6.8)

from which follows (Landshoff, 1974) the

Landshoff sumrule: \[ x(F_{3}^{v_{p}} - F_{3}^{v_{n}}) = -6(F_{2}^{v_{p}} - F_{2}^{v_{n}}) \].

From (6.8) and the flavour-counting sumrules (6.5), (6.7), we have the

Gross Llewellyn-Smith sumrule:

\[ \int_{0}^{1} dx(F_{3}^{v_{p}} + F_{3}^{v_{n}}) = 6. \]

Experimentally the latter is satisfied to within \( \sim 20\% \) (Ellis, 1976).

From the form for the weak DIS double-differential cross-section

\[ \frac{d^2 \sigma}{d\Omega' dE'} = \frac{G^2}{(2\pi)^2} \frac{E'}{E} L_{\mu \nu} W_{\mu \nu} \]

where \( L_{\mu \nu} = L_{EM} \pm i \varepsilon_{\mu \nu \alpha \beta} k^\alpha q^\beta \)

(cf. eqn. (2.1)) and where \( W_{\mu \nu} \) is given by eqn. (5.2) we find (Close, 1979)

\[ \sigma_{\nu} = \frac{G^2 \pi}{\alpha} \int_{0}^{1} dx \left[ \frac{2}{3} F_{2}^{\nu}(x) + \frac{1}{3} x F_{3}^{\nu}(x) \right] \]

so that from eqns. (6.2), (6.8) we have, for an isoscalar target\(^{\dagger}\)

\[ R = \frac{\sigma_{\nu}}{\sigma_{\nu}} = \frac{1 + 2r}{3 - 2r} \]

where \( r = \frac{<q>}{<q + \bar{q}>} \), \( q = \sum_{i} i \).

\(^{\dagger}\) The situation is somewhat different for a non-isoscalar target.
Experimentally $R = 0.43$ (Close, 1979), $R = 0.47 \pm 0.02$ (Heagy et al., 1981) which indicates again the presence of a small but nonzero sea ($r = 0.1$). Equivalently we can write

$$B = \frac{<xF_3>}{<F_2>} = \frac{<q - \bar{q}>}{<q + q>} = 1 - 2r.$$

Experimentally $B = 0.8$ (Close, 1979), $B = 0.77 \pm 0.04$ for $<Q^2> = 5 \text{ GeV}^2$, $B = 0.67 \pm 0.05$ for $<Q^2> = 20 \text{ GeV}^2$ (Heagy et al., 1981), so that again $r = 0.1$.

From the $F_{2,3}$ data and the above discussion we see that $q, \bar{q}$ can be extracted from the experimental data, see Fig. (6.2). In fact we can extract $u, \bar{u}, d, \bar{d}$ etc. separately, as shown in Fig. (6.3). We shall not be discussing symmetry breaking, but show the data for the sake of completeness (Buras, 1980).

Now let us briefly summarize this section. We have seen that the naive QP model yields several sumrules that are supported by the data, and several that have yet to be tested. It also gives us a consistent picture: apart from the three valence QP, there exists in the proton and neutron a small ($\sim 10\%$) sea concentrated at low $x$ ($< 0.2$).

**TABLE (6.1)**

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$d$</th>
<th>$s$</th>
</tr>
</thead>
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<td>$\frac{2}{3}$</td>
<td>$-\frac{1}{3}$</td>
<td>$-\frac{1}{3}$</td>
</tr>
<tr>
<td>$I_3$</td>
<td>$\frac{1}{2}$</td>
<td>$-\frac{1}{2}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$s$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

§7 Extending the Model to Polarized EM DIS

The experimental data on polarized DIS is too poor to be useful (Alguard et al. 'See also Chapter 3) but this should not deter us from
extending our parton model into this domain. The predictions we obtain can be compared with those of other models, and can await future experiments for support, or otherwise.

Evidently the assumption (ii) of §2, namely $P^i_\sigma(k) = \frac{1}{2}P^i_\sigma(k)$, will have to be altered here. Instead we make the assumption

\[(ii)' \quad P^i_\sigma(k) = \delta^\tau_\sigma P^i_\tau(k) + \delta^-_\tau P^i_\tau(k)\]

i.e.: QP contribute to polarized DIS if their spins $\sigma$ are aligned/antialigned with the target hadron spin $\tau$. From (ii)' we see that

\[P^i(k) = \Sigma P^i_\sigma(k) = P^i_\uparrow(k) + P^i_\downarrow(k)\]

The structure tensor is changed from the unpolarized expression, eqn. (2.7), to

\[(1,1)^{\mu\nu}_w = \Sigma e_i^2 \frac{P_0}{M} \int \frac{d^3k}{k_o} \mu^2 \delta(k \cdot q - \frac{1}{2}Q^2) \Sigma_{\sigma,\tau} \]

\[
(\delta^\tau_\sigma P^i_\tau(k) + \delta^-_\tau P^i_\tau(k), (1 + \gamma_5 f)\bar{u}(k\sigma)\gamma_\mu \left(\frac{k + q + \mu}{2\mu}\right)u(k\sigma) + \text{antiparton term})
\]

\[= (1,1)^{\mu\nu}_w + (1,1)^{\mu\nu}_s\]

where

\[(1,1)^{\mu\nu}_s = \Sigma e_i^2 \frac{P_0}{2M} \int \frac{d^3k}{k_o} \delta(k \cdot q - \frac{1}{2}Q^2) (P^i_\uparrow(k) - P^i_\downarrow(k)) \gamma_\mu i \epsilon^{\nu\alpha\beta} q_\alpha \sigma_\beta \]

where $(1,1)^{\mu\nu}_w$ is the unpolarized tensor, given by eqn. (2.8), and where $\uparrow(\downarrow)$ refers to parton spin $\sigma$ being (anti)aligned with hadron spin $\tau$.

If we now make assumption (iii) of §2 that $^\uparrow k = x\mu$ then we

\[\uparrow \text{This assumption means that the parton spin } \sigma^\mu = \frac{k^\mu}{\mu} \text{ can be written}
\]

\[\sigma^\mu = \frac{x}{\mu} \frac{P^\mu}{\mu} = \frac{Mx}{\mu} \tau^\mu \text{ where } \tau^\mu \text{ is the hadron spin.}\]
find for the polarized contribution

\[(1,1)_{\mu\nu} = \frac{1}{2Mv} \sum \epsilon_i^2 \epsilon^{1\mu\nu\alpha\beta} q_\alpha \tau_\beta (f_i^+(x) - f_i^+(x)). \quad (7.1)\]

The general expansion for \(S^{\mu\nu}\) (Roy) is

\[S^{\mu\nu} = i\epsilon^{\mu\nu\alpha\beta} \frac{q_\alpha}{M} \tau_\beta X_1(v, Q^2) + i\epsilon^{\mu\nu\alpha\beta} \frac{q_\alpha P_\beta Q^\tau}{M^3} X_2(v, Q^2) \quad (7.2)\]

Note that in parton models we expect that the \(X_2\) term will be absent \((q \cdot \tau = 0\) by angular momentum conservation, as is easily shown). In fact this is the case: comparing (7.1) and (7.2) yields for the polarized structure functions \(X_1, 2\)

\[\nu X_1(v, Q^2) \equiv g_1(x) = \sum \epsilon_i^2 \frac{1}{2} (f_i^+(x) - f_i^+(x))\]

which is the usual result (Kuti and Weisskopf, 1971; Feynman, 1972; Kaur, 1977) and

\[X_2(v, Q^2) = 0.\]

Thus we predict that scaling obtains in polarized EM DIS for the structure function \(g_1\).

This is a convenient point to gather together the results of this introductory chapter.

In \$2 we constructed a parton model of EM DIS. The predictions of this model were entirely in accord with those of other parton models, as was indicated variously throughout the text, and was based upon the following three assumptions

(i) the observed DIS cross-section is the incoherent sum of elastic lepton-QP scattering cross-sections.

(ii) The probability of finding a parton in an unpolarized hadron is independent of the parton's spin.
(iii) The QP momentum is an overall fraction of the parent hadron momentum, $k^\mu = x p^\mu$.

The second of these is eminently reasonable. The third is reasonable in a frame in which the hadron has very large momentum. Intuitively the crucial first assumption is also most believable in this frame.

In §§3,4 we showed that the partons appeared to be pointlike in the early data and that multiparton contributions were negligible in the Bjorken limit. In §§5,7 the model was extended to weak and to polarized EM DIS.

The main predictions of this model are scaling, CG relations, and various sumrules. In §6 we showed how these sumrules are constructed from the "master formula" and compared with experiment. Those sumrules that have been tested show good agreement with the data. Also, experiment shows that scaling approximately holds and that the CG relations are satisfied. It is our belief that the above experimental evidence constitutes a powerful body of support for the naive parton model of DIS.

§8 Projectors and CG Relations

We insert this section parenthetically at this point in part because we shall soon need the projectors constructed here. The main aim, though, is to derive quite generally the conditions necessary for the CG relations to hold. We shall find that for the particular case of parton models they must be true.

From the general expression (2.3) for the EM hadron vertex tensor we construct the projection operators for structure functions $W_{1,2}$:
\[
F_{\mu \nu}^{1} = \frac{1}{4} \left( \frac{p_{\mu} p_{\nu}}{M^2 - \frac{(p \cdot q)^2}{q^2}} - g_{\mu \nu} \right)
\]

\[
F_{\mu \nu}^{2} = \frac{1}{4} \left( \frac{3p_{\mu} p_{\nu}}{M^2 - \frac{(p \cdot q)^2}{q^2}} - g_{\mu \nu} \right) \frac{M^2}{M^2 - \frac{(p \cdot q)^2}{q^2}}
\]

and so the projectors for \( F_{1,2} \) are, in the Bjorken limit

\[
M_{P_{1}}^{\mu \nu} = \frac{1}{4} M \left( \frac{Q^2}{v^2} \frac{p_{\mu} p_{\nu}}{M^2} - g_{\mu \nu} \right)
\]

\[
\nu P_{2}^{\mu \nu} = M x \left( \frac{3Q^2}{v^2} \frac{p_{\mu} p_{\nu}}{M^2} - g_{\mu \nu} \right)
\]

so that

\[
F_{1}(x,Q^2) = \frac{1}{4} M \left( \frac{Q^2}{v^2} \frac{B}{A} - A \right)
\]

\[
F_{2}(x,Q^2) = M x \left( \frac{3Q^2}{v^2} \frac{B}{A} - A \right)
\]

where

\[
A \equiv g_{\mu \nu} \mathcal{W}_{\mu \nu}
\]

\[
B \equiv \frac{p_{\mu} p_{\nu}}{M^2} \mathcal{W}_{\mu \nu}
\]

From (8.2) it is evident that any model for which

\[
\frac{Q^2}{v^2} \frac{B}{A} \rightarrow_{Bj} 0
\]

(Bj stands for Bjorken limit) will lead to the CG relation \( 2x F_{1} = F_{2} \).

It is now a simple matter to check that parton models necessarily predict CG relations. A parton model is one in which the DIS of a lepton off a hadron is seen as the incoherent sum of elastic scatterings off the constituent partons (cf. §2). Thus \( \mathcal{W}_{\mu \nu} = \sum_{i} \mathcal{W}_{\mu \nu}^{i} \).
for each parton \( i \). Naive partons are pointlike and so \( W^i_{\mu\nu} \) is like the lepton vertex tensor, eqn. (2.1). From this it is easy to see that the scalars \( A, B \) of eqn. (8.2) are both proportional to \( \nu \), so that (8.4) is satisfied. Hence \( 2x_F^1 = F_2 \) for all parton models (cf. eqn. (2.9)).

In general, given the \( W_{\mu\nu} \) of a model of hadron structure, eqns. (8.3), (8.4) will tell us whether or not the CG relations follow.

For the polarized EM structure functions there is no corresponding relation, since their projection operators are linearly independent, as is easily seen.

With a little more work analogous conditions for weak interaction CG relations can be obtained. In addition to eqns. (8.3) define

\[
C \equiv \frac{p^\nu q^\mu W^W_{\mu\nu}}{M^2} \tag{8.3}
\]

\[
D \equiv \frac{q^\nu q^\mu W^W_{\mu\nu}}{\nu^2}
\]

where now \( W_{\mu\nu}^W \) is given by eqn. (5.2). Note immediately that the parity violating \( W_3 \) term is the only one antisymmetric in \( \mu, \nu \) so that its projector will be unrelated to the others. Thus we cannot obtain any CG-type relation involving \( W_3^+ \). We obtain, in the target hadrons rest frame,

\[
A = -4W_1 + W_2 - \frac{Q^2}{M^2} W_4 + \frac{\nu}{M} W_5
\]

\[
B = -W_1 + W_2 + \frac{\nu^2}{M^2} W_4 + \frac{\nu}{M} W_5
\]

\[
C = -W_1 + W_2 - \frac{Q^2}{2M^2} W_4 + \frac{\nu}{2M} W_5
\]

\[
D = \frac{Q^2}{\nu^2} W_1 + W_2 - \frac{Q^4}{M^2\nu^2} W_4 - 2xW_5
\]

\[\dagger\]

It is possible to obtain such a relation within the context of parton models, vis. \( xF_3 = F_2 \). However this requires the additional assumption that no antipartons are present. See §5.
It is tedious but straightforward to invert these eqns. to obtain, in the Bjorken limit

\[ W_1 = -\frac{1}{4}A + C - \frac{1}{4}D \]
\[ W_2 = \frac{Q^2}{\nu^2}(-\frac{1}{4}A + 3C + \frac{\nu^2}{Q^2}D) \]
\[ W_4 = \frac{M^2}{\nu^2}(\frac{1}{4}A + B - 3C + \frac{3}{2}D) \]
\[ W_5 = \frac{M}{\nu}(-A + 4C - 3D) \]

so that for weak CG relations to hold we require

\[ \frac{\nu^2}{Q^2} \frac{D}{A} \rightarrow 0 \]
\[ \frac{C}{A} \rightarrow 0 \]

Again we choose the parton model for demonstration. Because in this case \( W_{\mu\nu}^W \) is of the same form as \( L_{\mu\nu}^W \) the terms containing \( q_{\mu,\nu} \) in \( W_{\mu\nu}^W \) contribute to the DIS cross-section with a factor proportional to the parton mass and become vanishingly small in the Bjorken limit (Wray 1972). Thus we cannot observe \( C, D \) in this limit and so set them equal to zero yielding once more \( 2xF_1 = F_2 \). In addition we find \( xF_5 = F_2 \) as in §5. \( W_4 \) is the only structure function involving \( B \) and so is not specified in terms of the others here.
§9 Justification and Kinematics

A key assumption of parton models is that partons are free inside the hadron; it is because of this that the first parton models (e.g.: Bjorken and Paschos, 1969) chose a reference frame in which the hadron moves with infinite momentum. In such a frame the interaction time $\tau$ of the parton with the $\gamma^*$ is much less than the parton lifetime $T$ (see Fig. (9.1)) so that in DIS the constituents appear to be more or less free. Furthermore $\tau$ is much less than any other time scale, such as that of spectator interactions (so the parton model incoherence assumption can be made) or that of final state interactions (so that confinement effects can be ignored).

The same motivation can be used for the hadron rest frame. Here $q_o = \frac{P \cdot q}{M} = v$ so that $\tau \sim \frac{1}{q_o} \rightarrow 0$. The parton lifetime $T$ ($\sim \frac{1}{|k|}$ in a naive collision time picture) remains finite. If the hadron size is $R$, then the time it takes for a parton to cross the hadron is $\sim R$, in natural units. If partons are to be regarded as free then we must have $\tau \ll R$ so that confinement effects can be ignored in this context. We shall consider aspects of confinement more fully in subsequent chapters.

The assumption that partons are free (or nearly so) inside hadrons means that partons are on mass shell (or nearly so) before and after being struck by the virtual $\gamma^*$:

$$k^2 = u^2$$  \(9.1\)

$$(k + q)^2 = m^2 .$$  \(9.2\)

Here we have allowed for the possibility that the final parton may
have a different mass to the initial one (strictly speaking the EM interactions of QP require $m = \mu$. Nevertheless we shall maintain the distinction for notational purposes.)

In the infinite momentum frame (IMF) it is natural to interpret the Bjorken scaling variable $x = \frac{Q^2}{2MV}$ as an overall momentum fraction (e.g.: Bjorken and Paschos, 1969; Roy, 1975; Feynman, 1977; Chapter 1) $k = xp$. Making this identification in the hadron rest frame $p = (M; 0)$ means interpreting $x$ as a mass fraction, $x = \frac{\mu}{M}$ and neglecting parton three-momentum. This means that partons are motionless inside the hadron and that either $x$ is constant or partons are off shell. The first of these at least is very naive and so we reject the overall momentum fraction interpretation for $x$ in the rest frame.

Is there another interpretation available? From (9.2)

$$z \equiv \frac{k_0}{M} = x + \frac{k^3}{M}. $$

Assuming that the parton three-momentum density $P(k)$ is spherically symmetric in the hadron rest frame, we can write

$$<z> \equiv \int \frac{d^3k}{Bj} P(k)z = \frac{1}{Bj} x$$

so that $x$ is the expected energy fraction of the initial QP, in this frame. Thus we can say $0 \leq x \leq 1$.

This interpretation allows us the following prediction: if $\#$ is the average number of QP in the hadron and $\varepsilon$ is the average number of non-quark partons (i.e.: gluons) then we have

$$\# \cdot <z> = 1 - \varepsilon$$

where $<z> \equiv \int_0^1 dx \frac{1}{\#} f(x) <z> = <x>$ so that

$$<x> = \frac{1 - \varepsilon}{\#}. $$
Parameterizing the parton x distribution as \( f(x) = (p+1)(1-x)^p \) gives us \( \langle x \rangle = \frac{1}{p+2} \) so that we have a "counting rule":

\[
P = \frac{\#}{1-\varepsilon} - 2.
\]

Experimentally \( \varepsilon = \frac{1}{3} \) (cf. §6) so that \( p = 2\# - 2 \). Present theoretical prejudice favours three QP, \( \# = 3 \), which yields \( p = 4 \). Data will tolerate \( p = 3-4 \) near \( x = 1 \) (Landshoff and Scott,1977). The Drell-Yan-West (1971) relation suggests \( p = 3 \) in which case the counting rule yields \( \varepsilon = \frac{2}{3} \).

Is there anything else that can be gleaned from rest-frame kinematics?

Combining eqns. (9.1) and (9.2) yields

\[
\langle x \rangle = \frac{1}{8j} \left[ x + \frac{1}{x} \left( \frac{<k^2(x)>_{QP} + \mu^2}{M^2} \right) \right].
\]

Substituting eqn. (9.3) gives us

\[
<k^2_{\perp}(x)>_{QP} = M^2x^2 - \mu^2.
\]

The \( x \)-dependence of QP transverse momentum has been discussed extensively in the literature (Landshoff and Scott 1977; Close et al. 1977; Davies and Squires 1977). We obtain here the result that \( k_{\perp} \) increases as \( x + 1 \), in agreement with Landshoff and Scott, Davies and Squires. We shall delay further comment until §13.

§10 The Franklin Model

Franklin (1977) and Bjorken and Paschos (1969) construct their parton models starting from the expression

\[
\hat{W}^i_2 = e^2_i \delta(v_i - \frac{Q^2}{2\mu}) = \frac{1}{v} F^i_2(k,x,Q^2)
\]

where \( v_i = \frac{k_i \cdot q}{\mu} \), for the structure function of the ith QP.

Applying the parton model incoherence assumption they wrote for the hadron structure function
\[ F_2(x,Q^2) = \sum_i \int d^3k \, P_i(k) F_2^i(k,x,Q^2) \]
\[ = \sum_i e_i^2 \mu \int d^3k \, P_i(k) \delta(k \cdot q - \frac{1}{2} Q^2) . \]

In the rest frame model of Franklin the QP momentum density is spherically symmetric, so

\[ F_2(x) = \sum_i e_i^2 \frac{2\pi \mu}{\kappa(x)} \int dk \, k \cdot P_i(k) \left( \frac{1}{2M} x - \frac{1}{x} \frac{u^2}{M^2} \right) . \]

This is Franklin's main result: a scaling structure function which can be obtained from the primordial parton momentum density. Many more predictions have since been calculated, and the model extended to include off shell initial and final QP (Denny, 1980).

One criticism of this model (Squires, private communication) is that the starting point, eqn. (10.1) does not properly take into account the lepton-hadron flux. For this reason we shall not consider the Franklin model further here, but shall instead construct a rest frame model which does deal correctly with this flux factor. The two models are very similar, for reasons that will soon become obvious. Therefore the Franklin model predictions are included as an appendix to this chapter so that comparison can readily be made with the model that is about to be presented.

\S 11 The Rest Frame Model

Consider lowest order Compton scattering of a lepton with momentum \& off QP \( i \) with momentum \( k \) (Fig. (11.1)). The cross-section for this process can be written (Bjorken and Drell, 1969; Close, 1979)
\[ d\sigma_i = \frac{1}{|v_{\ell} - v_p|} \frac{1}{2E} \frac{1}{2k_o} |A|^2 \frac{d^3 l'}{(2\pi)^3 2E'} \frac{d^3 k'}{(2\pi)^3 2k'_o} (2\pi)^4 \delta^4(k + l - k' - \ell) \]

where \[ |A|^2 = \frac{e^4}{Q^4} e_i^2 e_{i'}^2 L_{\mu\nu} W_{\ell 1}^{\mu\nu}, \]
\( e_i \) = fraction of electron charge carried by QP \( i \). The flux factor is \[ \frac{1}{|v_{\ell} - v_p|} \]
where \( v_p \) is the target hadron velocity (and not the QP velocity) since experimentally the cross-section that we measure is that for lepton-hadron scattering. In the hadron rest frame this flux factor is unity.

Integrating over the (undetected) final QP momentum \( k' \) yields
\[ \frac{d^2 \sigma_i}{d\Omega dE'} = \frac{\alpha^2}{Q^4} e_i^2 \frac{E'}{2k_o E} L_{\mu\nu} W_{\ell 1}^{\mu\nu} \delta(2k\cdot q - Q^2) \tag{11.1} \]

where \[ L_{\mu\nu} = 2 \left[ \ell_{\mu} \ell_{\nu}' + \ell_{\nu} \ell_{\mu}' - g_{\mu\nu} \ell\cdot \ell' \right] \]
and \( W_{\ell 1}^{\mu\nu} = 2 \left[ k_{\mu} k_{\nu}' + k_{\nu} k_{\mu}' - g_{\mu\nu} (k\cdot k' - \mu^2) \right] \).

The DIS cross-section is
\[ \frac{d^2 \sigma}{d\Omega dE'} = \frac{\alpha^2}{Q^4} \frac{E'}{E} L_{\mu\nu} W_{\ell 1}^{\mu\nu} \tag{11.2} \]

with \( L_{\mu\nu} \) as before and with \( W_{\ell 1}^{\mu\nu} \) the general expansion in terms of structure functions, eqn. (2.3).

We find
\[ L_{\mu\nu} W_{\ell 1}^{\mu\nu} = 8 \left[ 2(\ell\cdot k)^2 - Q^2 \ell\cdot k + \frac{1}{4} Q^4 - \frac{1}{4} Q^2 \mu^2 \right] \tag{11.3} \]

and, in the rest frame
\[ L_{\mu\nu} W_{\ell 1}^{\mu\nu} = 4E'E' \left[ 2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right] \tag{11.4} \]

From the parton model incoherence assumption we have
Putting together eqns. (11.1) - (11.5) we obtain the hadron structure functions in terms of QP momentum densities:

\[
E dE' \left[ 2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right] = \sum_i \int \frac{d^3k}{k_0} P_i(k) \delta(2k\cdot q - \frac{1}{2}Q^2) \cdot \left[ 2(k\cdot l)^2 - Q^2 l\cdot k + \frac{1}{4}Q^4 - \frac{1}{4}Q^2 \mu^2 \right]
\]

Performing the angular integrations we find, after some calculation,

\[
2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} = \frac{\pi}{\nu} (1 - \frac{Mx}{\nu}) \sum_i \int \frac{dk}{k_0} \frac{k}{k_0} P_i(k) \cdot (2M^2 x^2 + (Q^2 - 2\mu^2) \sin^2 \frac{\theta}{2} + 8M^2 x^2 \frac{(k_0 - Mx)}{\nu} \left[ \cos^2 \frac{\theta}{2} + (1 + \frac{\nu^2}{Q^2}) \sin^2 \frac{\theta}{2} \right] + \frac{2M}{\nu} x (2k_0 Mx - M^2 x^2 - \mu^2))
\]

By comparing coefficients of \( \sin^2 \frac{\theta}{2}, \cos^2 \frac{\theta}{2} \) we extract the structure functions

\[
F_1(x, Q^2) = \pi M^2 x \sum_i \frac{e_i^2}{4Mx(k_0 - Mx) - 2\mu^2} \int_{\kappa} dk \frac{k}{k_0} P_i(k) \{1 + \frac{4Mx(k_0 - Mx) - 2\mu^2}{Q^2} \}
\]

(11.6)

\[
F_2(x, Q^2) = 2\pi M^2 x^2 \sum_i \frac{e_i^2}{12Mx(k_0 - Mx) - 2\mu^2} \int_{\kappa} dk \frac{k}{k_0} P_i(k) \{1 + \frac{12Mx(k_0 - Mx) - 2\mu^2}{Q^2} \}
\]

Including subasymptotic corrections it is tedious but straightforward to show that

\[
\kappa = \frac{1}{M} \left| \frac{x}{M^2} \right| = \left| \frac{M^2}{Q^2} (x^2 - \frac{\mu^2}{M^4x^2}) \right|
\]

As an initial check on (11.6) we see that in the Bjorken limit scaling obtains and the CG relation \( 2xF_1 = F_2 \) holds.

Asymptotically (11.6) becomes
Go

\[ F_2(x) = \frac{2\pi M^2 x^2}{B_j} \sum_{i=1}^{e^2} \int_{k_0}^{\infty} \frac{dk}{k_0} P_i(k) \]  

(11.7)

which is similar to the Franklin form, eqn. (10.2), with the replacement \( \frac{M^2 x^2}{k_0} \rightarrow \mu \). Thus if the average QP momentum is not too large we expect eqns. (10.2), (11.7) to yield similar structure functions.

Finally as a check note that if we apply the projector \( P_{\mu\nu}^{\mu\nu} \) of §8 to the structure tensor \( W_{\mu\nu} \) of eqn. (2.8) we recover the structure function (11.7) in the rest frame. Thus from two different calculations we have arrived at the same expression for the rest frame structure function. Henceforth, then, we work with eqn. (11.7) and not with the Franklin form, eqn. (10.2).

§12 Asymptotic Predictions of the Model

[1] It is easy to show that the form (11.7) for \( F_2(x) \) is equivalent to

\[ F_2(x) = \sum_{i=1}^{e^2} \int \frac{d^3k}{k_0} M^2 x^2 \delta(k_0 - Mx) P_i(k), \]  

(12.1)

Now in the usual IMF models the structure function has the form

\[ F_2(x) = \sum_{i=1}^{e^2} x f_i(x) \]  

(12.2)

(cf. eqn. (2.9)) where \( f_i(x) \) is normalized to 1. It is important that the rest frame structure function can be written as in (12.2) because without this we cannot construct the experimentally successful parton model sumrules (§6). From (12.1) and (12.2) we have

\[ f_i(x) = M^2 x \int \frac{d^3k}{k_0} \delta(k_0 - Mx) P_i(k). \]  

(12.3)

From the above discussion we see that the usual sumrules can be constructed in this rest frame parton model if and only if eqn. (12.3)
is normalized to 1. It is easy to verify that this is the case, if the
typical parton momentum is not too large (i.e. $P_1(k) < 1$ for $k \gtrsim M$).
The same restriction holds for the Franklin model (cf. appendix). Hence
all the predictions of §6 follow from this rest frame QP model.

[2] We now discuss moments of the structure function, defined as

$$M_{n+2} = \int dx \, x^n \, F_2(x) \quad . \quad (12.4)$$

For simplicity we take the QP to be massless, $\mu = 0$. Then it is not
difficult to show from eqns. (12.1) and (12.4) that

$$M^\mu=0_n = \sum_i e_i^2 \frac{2^n \langle k^{n-1} \rangle}{n!} \quad . \quad (12.5)$$

e.g.: $M_1 = \sum_i e_i^2$, $M_2 = \frac{4}{3} \sum_i e_i^2 \langle k \rangle$ .

From (12.2), (12.5) we have

$$\langle x^n \rangle = \frac{2^{n+1} \langle k^n \rangle}{n! \langle x \rangle} \quad . \quad (12.6)$$

This equation is interesting because the expectation value on the
r.h.s. refers to all partons described by the momentum density $P(k)$,
whereas the l.h.s. refers only to those partons that couple to the
probe, the QP. For $n = 1$ we have (recall $\mu = 0$)

$$\langle k_0 \rangle = \frac{3}{4} M \langle x \rangle \quad . \quad$$

In §9 we showed that $M(x) = \langle k_0 \rangle$, the average QP energy. Thus

$$\langle k_0 \rangle = \frac{3}{4} \langle k_0 \rangle$$

i.e.: the average energy of all those partons with momentum density
$P(k)$ is $\frac{3}{4}$ of the average energy of the QP described by the same $P(k)$. 
This tells us that non-quark partons carry less energy/momentum on average than do QP.

From (12.2), (12.4) and the results of §9 we may write

\[ M^2 = \sum_i e_i \langle x \rangle \]

with \( \langle x \rangle = \frac{1-\varepsilon}{\#} \). Also \( \sum e_i^2 = \# \bar{e}^2 \), where recall \# is the number of QP and \( \bar{e} \) is their average charge. Thus \( M^2 = \# \bar{e}^2 \left(1 - \frac{\varepsilon}{\bar{e}^2}\right) \) or

\[ \varepsilon = 1 - \frac{M^2}{\bar{e}^2} . \]

For the proton \( \bar{e}^2 \) is between \( \frac{2}{3} \) and \( \frac{1}{3} \) depending on how much sea is present (Bjorken and Paschos 1969). Experimentally \( M^2 \approx 0.17 \) (Anderson 1978) so that \( \varepsilon = \frac{1}{4} - \frac{1}{2} \) (compare §6).

Finally it is perhaps worth pointing out that this moments discussion is very model-dependent; the Franklin model, for instance, yields different results (appendix).

§13 Subasymptotic Predictions

[1] It should be stressed at the outset that, though subasymptotic corrections to QP model scaling are calculable, they should be treated with caution. This is because the parton model incoherence assumption is strictly valid only in the Bjorken limit (§9), and because graphs other than the handbag contribute below this limit (§4). Thus we come to the conclusion of Petronzio et al. (1976) that subasymptotic corrections due to QP model kinematics are possibly not right and, even were they correct, perhaps not the whole story. Having said this, it is interesting to note their qualitative agreement with the data.

We will establish this here. Eventually in Chapter 5 we will discuss moments quantitatively, and there the relevance of QP model subasymptotic
corrections will become apparent.

We shall take \( u, d, s \) QP to be massless, for simplicity. Later on we shall introduce the charmed QP, which is given a mass. These considerations apart, the discussion will be entirely general (in particular, we shall not have to specify \( \mathbb{P}(k) \)).

From (11.6)
\[
F^\mu = 0(x,Q^2) = 2\pi M^2 x^2 \sum_i e_i^2 \int dk \mathbb{P}_i(k) \left[ 1 + \frac{12 M x(k - Mx)}{Q^2} \right]
\]
(13.1)
\[
\kappa = \frac{1}{2} M x (1 - \frac{M^2 x^2}{Q^2}) \equiv \frac{1}{2} M x'.
\]

We take the Bjorken limit and invert the eqn. to obtain \( \mathbb{P}(k) \) in terms of \( F_2(x) \):
\[
\sum_i e_i^2 \mathbb{P}_i(k) = -\frac{1}{\pi M^3} \left[ \frac{d}{dx} \left( \frac{F_2(x)}{x^2} \right) \right]_{x = \frac{2k}{M}}.
\]

Substituting back into (13.1) gives the subasymptotic structure function in terms of the asymptotic function:
\[
F_2(x,Q^2) = \frac{x^2}{x^2 - F_2(x')} - \frac{6 M^2 x^2}{Q^2} \left[ F_2(x) - x \int \frac{1}{y^2} F_2(y) \right]
\]
(13.2)

From (13.2) it is not difficult to show that, in limit \( x \to 0 \)
\[
F_2(x,Q^2) \sim F_2(x)
\]
so that QP model scaling violation is negligible in this region. We can also show that, in the limit \( x \to 1 \)
\[
F_2(x,Q^2) \sim \frac{Q^2}{Q^2 + 4 M^2 x^2} F_2(x)
\]
so that for finite \( Q^2 \) scaling violations may be substantial for large \( x \).

A more concrete picture is obtained by considering the sub-asymptotic moments
\[ M_{n+2}(Q^2) = \int_0^1 dx x^n F_2(x,Q^2) \ . \]

Substituting eqn. (13.2) we find after a short calculation

\[ M_{n+2}(Q^2) = M_{n+2} + \frac{M^2}{Q^2} (n - 1 + \frac{6}{n+4}) M_{n+4} \ . \]  

(13.3)

The asymptotic corrections to our rest frame QP model thus make two predictions about the \( Q^2 \) dependence of moments:

(i) The moments decrease with \( Q^2 \) to their asymptotic values.

(ii) They decrease faster for larger \( n \).

Both these are realized in the data, Fig. (13.1) (taken from Duke and Roberts 1980). Note that (13.2) reinforces our suggestion above that scaling violations are more important at large \( x \) than at small \( x \).

[2] Over the past few years it has become something of an industry to parametrize the QP model scaling violations via a subasymptotic scaling variable, such as \( x' \) above. At least half a dozen such variables have been proposed in the literature. Let us see if eqn. (11.6) can be parametrized in this fashion.

If we express \( P(k) \) in terms of the asymptotic structure function \( F_2(x) \) we obtain an expression which reduces, in the limit of zero QP mass, to eqn. (13.2) above. The "awkward" last term in (13.2) can be eliminated by differentiating both sides. The result is, after some calculation

\[
\begin{aligned}
\frac{d}{dx} \left( \frac{F_2(x,Q^2)}{x^3} \right) &= -\frac{3}{x^4} \left( 1 - \frac{2\mu^2}{Q^2} \right) + \frac{\mu^2}{Q^2} \left[ \frac{4}{x^2} - 6 \frac{\mu^2}{x^2} - \frac{12}{x^4} \frac{\mu^2}{M^2} \right] F_2(x) \\
&+ \left\{ \frac{1}{x^3} \left( 1 - \frac{2\mu^2}{Q^2} \right) + \frac{\mu^2}{Q^2} \left[ -\frac{8}{x} + \frac{4}{x^3} x^2 - \frac{\mu^2}{M^2} \right] + \frac{6}{x^3} \frac{\mu^2}{M^2} \right\} \frac{d}{dx} F_2(x) \\
&- \frac{1}{x^2} \left| x^2 - \frac{\mu^2}{M^2} \right| \frac{\mu^2}{Q^2} \frac{d}{dx^2} F_2(x) . \end{aligned}
\]  

(13.4)
Now let us parametrize the scaling violations as

\[ F_2(x,Q^2) = (1 + \frac{M^2}{Q^2} a)F_2(\zeta), \quad \zeta = x + \frac{M^2}{Q^2} b \]  

(13.5)

and substitute in (13.4). We will then obtain three eqns. (by equating the coefficients of \( F_2, F_2', F_2'' \)) constraining \( a, b \). If these equations are consistent then we can say that the subasymptotic corrections of this model can be described in terms of the scaling variable \( \zeta \).

Unfortunately the three equations are not in fact consistent, so this model does not admit to an exact scaling interpretation, unlike the Franklin model (see Franklin 1977 or the appendix). However we can obtain a scaling variable in the limit of large \( x \) by dropping the first of the three equations\(^\dagger\) (the coefficient of \( F_2(x) \)). The two remaining equations yield

\[ F_2(x,Q^2) = \frac{Q^2}{Q^2 + 2(2M^2x^2 - \mu^2)} F_2(\zeta) \]

\[ \zeta = x + x\frac{M^2}{Q^2}(\frac{\mu^2}{M^2} - x^2) \]  

(13.6)

This \( \zeta \) is just the scaling variable of the Franklin model.

Note that in the limit \( x \to 1 \) where \( \zeta \) is valid we have (for \( \mu = 0 \))

\[ \zeta \to x_{BG} = \frac{x}{1 + x\frac{M^2}{Q^2}} + O(\frac{1}{Q^4}) \]

where \( x_{BG} \) is the phenomenologically successful (at large \( x \)) Bloom-Gilman variable (Bloom and Gilman, 1971). So once again we see that the kinematic \( Q^2 \) corrections to exact scaling of this model reflect the data, at least qualitatively, for large \( x \).

\(^\dagger\) We expect \( F_2 \) to be small compared to \( F_2', F_2'' \) for large \( x \). This is so, for example, if we parametrize \( F_2(x) \sim (1 - x)^P \) for large \( x \).
The picture that emerges from subsections 1, 2 is one of small scaling violations at low \( x \) and large violations at high \( x \) (with \( F_2 \) decreasing as \( Q^2 \) increases). Experimentally this is only partially supported, as seen in Fig. (13.2): there is a significant rise near \( x = 0 \) as \( Q^2 \) increases. QP model kinematics cannot explain this. Can we find some dynamical explanation?

The increase in \( F_2(x,Q^2) \) with increasing \( Q^2 \) at small \( x \) is often associated with charm production (Scott, Close 1979). We shall see that this occurs naturally in our rest frame model.

There are no valence charm QP in the nucleon and so \( c\bar{c} \) pairs must be created. This requires the final state mass squared \( W^2 = (p+q)^2 \) to be greater than \( 4m^2_c \). This corresponds to a threshold value of \( x \) given by

\[
x_{\text{TH}} = \frac{Q^2}{Q^2 + \omega_c^2}
\]

where \( \omega_c^2 = 4m^2_c - M^2 \sim 8 \text{ GeV}^2 \). \( x \) must be less than \( x_{\text{TH}} \) before \( c\bar{c} \) pairs can be created. Thus \( x \) is restricted to low values unless \( Q^2 \) is very large. With this knowledge, and bearing in mind the large charm mass, it is not difficult to show that the charm contribution to \( F_2(x,Q^2) \), eqn. (13.6), is approximately

\[
F_2^c(x,Q^2) = \frac{Q^2}{Q^2 + 2m^2_c} F_2^c(x)
\]  

where \( F_2^c(x) \) is the asymptotic contribution. Thus \( F_2(x,Q^2) \) increases as \( Q^2 \) increases, for low \( x \). In fact the form (13.7) is very similar to that obtained by Scott (1977) using different arguments, and can explain the observed \( Q^2 \) behaviour of \( F_2(x,Q^2) \) quite well (Scott 1977; Close 1979).

The overall conclusion of this section is that this rest frame QP model can qualitatively explain the observed scaling violations at large
§14 QP vs. Gluons

Integrating eqn. (12.3) we can extract the following QP momentum distribution

\[ P_{QP}(k) = \frac{k}{k_0} P(k). \quad (14.1) \]

This is interpreted here as follows: if the primordial momentum of all hadron constituents is described by the density \( P(k) \), then the QP density seen by the \( \gamma^* \) probe is given by (14.1). In the limit \( \mu \to 0 \) this becomes \( P_{QP} = (1 - \cos \theta)P \), as illustrated in Fig. (14.1). Thus the backward direction is emphasized, which is to be expected because the probe should interact preferentially with these QP constituents that approach it. Thus we have correctly taken into account the flux factor mentioned earlier.

Again from eqn. (12.3) we can express primordial quantities by inverting to obtain \( \mu = 0 \)

\[ P(k) = \frac{1}{\pi M^3} \left[ \frac{d}{dx} \left( \frac{f(x)}{x} \right) \right] \quad x = \frac{2k}{M} \]

so that for example the average primordial energy of all hadron constituents

\[ <k_0>_{all} = 4\pi \int_0^\infty dk \, k^3 P(k) , \quad \mu = 0 \]

can be expressed as \( <k_0>_{all} = \frac{3}{4} M <x> \). This is just the result given earlier in §12.2. Similarly for transverse momentum we find (again with \( \mu = 0 \))

\[ <k_{\perp}^2>_{all} = \frac{1}{3} M^2 <x^2> \]
so that from eqn. (9.4) we have \( \langle k^2 \rangle_{\text{all}} = \frac{1}{3} \langle k^2 \rangle_{\text{QP}} \), i.e. QP have much larger average transverse momentum than do non-quark partons (gluons).

§15 Summary and Conclusions

To end this chapter we summarize the results obtained and state our conclusions.

Our aim has been to construct a naive parton model of DIS in the target hadron rest frame, at the same level of rigour as the conventional IMF formulation of, for example, Feynman (1972) or Roy (1975). To this end we discussed rest frame kinematics, from which we conclude that the crucial incoherence assumption can be made. We found that the Bjorken scaling variable \( x \) can be interpreted as an average energy fraction of QP constituents. A phenomenologically acceptable counting rule was constructed, and the QP average transverse momentum is predicted to rise with \( x \). This latter prediction is absent in IMF models.

We then constructed our rest frame model and obtained all the usual results (cf. Chapter 1), i.e.: scaling, CG relation, sumrules. In several ways we showed that, although the basis of their derivation is less sound, the subasymptotic predictions of the model are at least in qualitative agreement with experiment. These subasymptotic predictions arose from rest frame kinematics (high \( x \)) and charm production (low \( x \)).

Finally, because our model gives a relation between the primordial parton momentum density \( P(k) \) and the QP \( x \) distribution \( f(x) \), we were able to extract information about the non-quark constituents of hadrons.
Our overall conclusion is that this rest frame model is every bit as valid as the conventional IMF models, and is in some ways richer: for example, the relation between \( P(k) \) and \( f(x) \) is absent in the IMF formulation.
APPENDIX

[1] Here we shall present some of the predictions of the Franklin model, discussed in §10. We saw that the structure function $F_2$ is given by, in the Bjorken limit

$$F_2(x) = \sum_i e_i^2 2\pi u \int \frac{dk \cdot k}{\kappa(x)} P_i(k),$$

$$\kappa(x) = \frac{1}{M} \left| x - \frac{1}{x} \frac{\mu^2}{M^2} \right| \quad (A1)$$

and so we see immediately that the Franklin model does not allow zero mass QP to contribute. Also $F_2(x=0) = 0$, which is in disagreement with experiment.

If eqn. (A1) is to be compatible with the QP model "master formula", eqn. (2.9), then the QP $x$ distribution is

$$f_i(x) = \frac{2\pi u}{x} \int_0^\infty \frac{dk \cdot k}{\kappa(x)} P_i(k) \quad (A2)$$

which must be normalized to 1. If for illustration we choose a Gaussian form for the momentum density

$$P_i(k) = \frac{d^3 k}{\pi^{3/2}} e^{-k^2 d^2} \quad (A3)$$

then we find for the normalization

$$N \equiv \int_0^1 dx f_i(x)$$

$$= \left( \frac{2v}{\pi} \right)^{1/2} e^v K_o(v), \quad v = \frac{1}{2} \mu^2 d^2$$

where $K_o$ is a Bessel function of imaginary argument. This is shown in Fig. (A1): note that $N$ is nearly 1 for most values of $v$, so that the master formula is applicable to the Franklin model for most values of $v$. 
Note also that the expectation value of $x$ is given by

$$<x> = \frac{1}{N} \mu \frac{B}{M}$$  \hspace{1cm} (A4)$$

In rest frame models, recall, $<x>$ is just the average QP energy fraction, and so from (A4) we see that $\frac{1}{N}$ is just the Lorentz dilation factor.

[2] For the Gaussian momentum density (A3) we calculate the DIS moments

$$M_n = \int_0^1 dx x^{n-2} F_2(x)$$

to be

$$M_0 = \sum_i e_i^2 \left( \frac{\mu}{M} \right)^{-1}$$

$$M_2 = \sum_i e_i^2 \left( \frac{\mu}{M} \right)$$

$$M_{2n+2} = \frac{(4n-2)}{M^2d^2} M_{2n} + \frac{\mu^4}{M^4} M_{2n-2}$$  \hspace{1cm} (A5)$$

$M_{2,4,6}$ have been determined experimentally (for the proton, at $Q^2 = 40$ GeV$^2$). See Anderson 1978) to be $0.17 \pm 0.02$, $0.013 \pm 0.001$, $0.003 \pm 0.001$ respectively.

Franklin obtains a good fit to the proton structure function for

$$\frac{\mu}{M} = \frac{1}{7}$$  \hspace{1cm} (A6)$$

$$M^2d^2 = 20.$$  

Substituting these values for the parameters into (A5) we obtain $M_2 = 0.14$, $M_4 = 0.016$, $M_6 = 0.005$.

\[ This \ result \ for \ M_2 \ can \ in \ fact \ be \ obtained \ independent \ of \ our \ choice \ for \ P_1(k). \]
For the odd moments we find

\[ M_1 = \sum_i e_i^2 N \]  \hspace{1cm} \text{(A7)}

\[ M_{2n+3} = \sum_i e_i^2 \left( \frac{\mu^2}{M^2} \right)^{n+1} \left( \frac{2v}{\pi} \right) e^v K_{n+1}(v) . \]

Experimentally (Duke and Roberts 1980) \( M_3 \sim 0.04 \), \( M_5 \sim 0.007 \) at \( Q^2 \sim 20 \text{ GeV}^2 \), whereas eqns. (A6), (A7) yield \( M_3 = 0.042 \), \( M_5 = 0.009 \). Better agreement can be obtained by increasing \( M^2 d^2 \) to \( \sim 25 \).

We give two examples of sumrules that can be constructed in this model (cf. §6). We find without difficulty that the charge conservation sumrule takes the form

\[ M_1^p - M_1^n = \frac{1}{3}N . \]

Superscripts refer to (p)roton and (n)eutron. We see that, because \( N \) is less than 1, \( M_1^p - M_1^n < \frac{1}{3} \). In fact from (A6) and Fig. (A1) we have \( M_1^p - M_1^n = 0.25 \). The experimental figure normally quoted (Close 1979) is 0.28, with unknown error. This number is unreliable because of uncertainties in measuring the 1st moments, but it seems to be nearer the Franklin result than to the accepted value of \( \frac{1}{3} \).

The second sumrule, easily constructed, is

\[ M_2^n = \frac{2}{9} (1 - \epsilon)N \]

where \( \epsilon \) is the average gluon energy fraction. Substituting the experimental \( M_2^n \) (Anderson 1978) we find \( \epsilon = 0.43 \pm 0.07 \) at \( Q^2 = 10 \text{ GeV}^2 \) and \( \epsilon = 0.28 \pm 0.13 \) at \( Q^2 = 20 \text{ GeV}^2 \). This is to be compared with \( \epsilon \sim 0.46 \) in the IMF models (e.g.: Roy 1975, or Chapter 1).
[3] In the Franklin model we can take into account the effects of offshell QP. For the final QP (i.e.: that with onshell mass \( m \) in the notation of §9) this generalization makes no difference to the structure functions to \( O(\frac{1}{Q^2}) \) as we shall see; for initial QP (on-shell mass \( \mu \)) the structure functions are altered in leading order, but the effect is small.

We write the structure function in the form

\[
F_2(x,Q^2) = \sum_i e_i^2 2\mu \int d^3k \frac{P_i(k)}{k^2} \int dm^2 \rho_i(m^2) \delta((k+q)^2 - m^2)
\]

(A8)

The initial QP mass is \( \mu \), as before, but now the final QP effective mass is variable. The spectral function \( \rho(m^2) \) describes the mass-squared distribution, and is normalized to 1. For onshell final QP, \( \rho(m^2) \) is a \( \delta \) function and (A8) reduces to the old form, eqn. (A1).

Now, however, from (A8)

\[
F_2(x,Q^2) = \sum_i e_i^2 2\mu \int \frac{Q^2}{Q^2 + 2M^2x^2} \int dm^2 \rho_i(m^2) \int dk \frac{K(m^2,x,Q^2)}{k^2} P_i(k)
\]

where \( k = |k| \) and where

\[
k(m^2,x,Q^2) = \frac{M}{\mathcal{F}} \left( \frac{1}{x} \frac{\mu^2}{M^2} - x \right) + \frac{M^2}{Q^2} \left( x^2 + \frac{\mu^2 - m^2}{M^2} - \frac{\mu^2 m^2}{M^4 x^2} \right) + O(\frac{M^4}{Q^4}).
\]

Here we have assumed that \( \rho_i(m^2) \) has support only when \( m^2 \ll Q^2 \). Note that in the Bjorken limit \( k(m^2,x,Q^2) \) reduces to \( \kappa(x) \), so that \( F_2(x,Q^2) \) reduces to the previous expression (A1).

We define the subasymptotic scaling variable in the usual manner (Akama 1974, Franklin)

\[
\kappa(\xi) = \kappa(m^2,x,Q^2)
\]

so that the subasymptotic structure function can be obtained from the asymptotic expression via the replacement \( x \rightarrow \xi \). In fact we find
\[ \xi = x + x \frac{M^2}{Q^2} (\frac{m^2}{M^2} - x^2) \]

(cf. §13.2) and

\[ F_2(x,Q^2) = \frac{Q^2}{Q^2 + 2M^2 x^2} F_2(<\xi>) + o \left( \frac{M^2}{Q^2} \right) \quad (A9) \]

where

\[ <\xi> = x + x \frac{M^2}{Q^2} (\frac{m^2}{M^2} - x^2) \]

\[ <m^2> = \int dm^2 \rho(m^2) m^2 \]

In writing down the above expressions we have made the assumptions \( \mu^2 << N^2 \) and that either \( \mu^2 = <m^2> \) or else \( \mu^2 << m^2 \).

In (A9) we have a neat expression describing the subasymptotic behaviour of the Franklin model. We have seen that letting the final QP off shell makes no difference to \( F_2 \) up to \( O(\frac{1}{Q^2}) \).

[4] Now we allow the initial QP off shell. This is more difficult than the previous case because the initial QP mass is present in the Bjorken limit, and so a different expansion scheme is required to calculate offshell effects.

From (A.1) we find \(^\dagger\)

\[ <\mu> \langle F_2(x\mu^2) \rangle = \sum_i \epsilon_i^2 \frac{2\pi}{2} <\mu> \int d\mu^2 \rho(\mu^2) I(x\mu^2) \]

\[ I(x\mu^2) = \int dk k \rho(k) \kappa(x\mu^2) \]

where \( \kappa(x,\mu^2) \) is as before except now the initial QP four-momentum

\(^\dagger\)It is more convenient to evaluate the expression on the l.h.s. of (A10) than the more direct expectation value \( <F_2(x,\mu^2)> = \int d\mu^2 F_2(x\mu^2) \rho(\mu^2) \). We assume these two expressions are very similar so that we may take (A10) to be the observable structure function. Note from (11.7) that this problem does not arise in the rest frame model of §11.
squared \( k^2 = \mu^2 \) is allowed to vary.

Let us expand \( I(x, \mu^2) \) in a Taylor series about \( \langle \mu^2 \rangle \):

\[
I(x, \mu^2) = I(x, \langle \mu^2 \rangle) + (\mu^2 - \langle \mu^2 \rangle) \left[ \frac{d}{d\mu^2} I(x, \mu^2) \right]_{\mu^2 = \langle \mu^2 \rangle} + \frac{1}{2} (\mu^2 - \langle \mu^2 \rangle)^2 \left[ \frac{d^2}{d\mu^2} I(x, \mu^2) \right]_{\mu^2 = \langle \mu^2 \rangle} + \ldots
\]

We expect this series to converge quickly if \( \rho(\mu^2) \) is sharply peaked about \( \langle \mu^2 \rangle \). The zeroth order term of \( \langle F_2 \rangle \) is just the previous on-shell term, eqn. (A1)

\[
\langle > \rangle_0 = \sum_i e_i^2 \frac{2\pi}{\langle \mu \rangle} \int dk \frac{P(k)}{<\kappa>}
\]

\[
\langle \kappa \rangle = \frac{1}{M} \left| x - \frac{1}{x} \frac{\langle \mu^2 \rangle}{M^2} \right|
\]

The first order term is zero. The second order term is found to be

\[
\langle > \rangle_2 = -\sum_i e_i^2 \frac{2\pi}{\langle \mu \rangle} \frac{1}{4M^2x^2} \left[ P(\langle \kappa \rangle) + \langle \kappa \rangle \frac{dP(\kappa)}{d\kappa} \right]
\]

Eqn. (All) is the lowest order (in \( \frac{\Delta \mu^2}{M^2} \)) correction that contributes to the structure function. We plot this, and the onshell term, in Fig. (A2) for our Gaussian momentum density (A3). We have taken \( \Delta \mu^2 = \langle \mu^2 \rangle \). We see that the effect on the structure function of letting the initial QP offshell is negligible. Even for a much broader mass distribution, \( \Delta \mu^2 = 4\langle \mu^2 \rangle \), the offshell effect is very small for \( x > 0.2 \). (Landshoff and Scott (1977) have argued that the initial QP must be offshell at small \( x \). We see from Fig. (A2) that offshell effects are more pronounced in this region).
It is important to note that the above generalization to offshell QP is not unique to the Franklin model; our rest frame model of §11 can be similarly generalized. However the details are slightly different for the initial QP case, as can readily be seen by comparing the asymptotic structure functions, eqns. (A1) and (11.7).

[5] The subasymptotic behaviour of the Franklin model is readily seen by expanding eqn. (A9):

\[
F_2(x, Q^2) = F_2(x) - \frac{2M^2}{Q^2} x^2 F_2(x) - \frac{x^2}{M^2} \frac{dF_2}{dx} + O\left(\frac{x^4}{Q^4}\right) \quad (A12)
\]

where for simplicity we have assumed that both initial and final QP are onshell and are of equal mass.

The third term of (A12) should dominate the subasymptotic corrections for most values of \(x\) because experimentally (Anderson et al. 1977) \(\frac{dF_2}{dx} \gg F_2\). In fact \(\frac{dF_2}{dx} < 0\) for all \(x\) (Anderson et al. 1977; Gordon et al. 1979) and so from (A12) we conclude that

\[x > \frac{\mu}{M} \quad \text{implies} \quad F_2(x, Q^2) > \frac{\mu}{M} F_2(x)\]

(ignoring the small second term). Indeed this behaviour is reflected in the data, Fig. (13.2). Note in particular the prediction that there should (almost) be a crossover point at \(x = \frac{\mu}{M}\), i.e.

\[F_2\left(\frac{\mu}{M}, Q^2\right) = F_2\left(\frac{\mu}{M}\right)\].

This is also the case experimentally (Anderson 1977): we obtain \(\frac{\mu}{M} \sim 0.15 - 0.25\), in broad agreement with Franklin's value of \(\frac{1}{7}\).

From (A12) it is easy to show that the subasymptotic moments of \(F_2\) are given by

\[M_n(Q^2) = M_n + \frac{M^2}{Q^2} (n-1)(M_{n+2} - \frac{\mu^2}{M^2} M_n),\]
(compare eqn. (13.3)). We expect $M_{n+2} > \frac{\mu^2}{M^2} M_n$ and so conclude that $M_n(Q^2)$ decreases monotonically to its asymptotic value of $M_n$ as $Q^2$ increases to infinity, and decreases more rapidly as $n$ is increased. As was discussed in §13, both those predictions are supported experimentally.

\[\text{Fig. (A1)}\]

\[\text{Fig. (A2)}\]
CHAPTER 3

CONFINEMENT WITHIN THE PARTON MODEL

§16 Introduction

As we have seen in Chapters 1, 2 the QP models of DIS have enjoyed considerable phenomenological success. Scaling and CG relations have been observed, approximately; those scaling violations that do exist are describable, at least qualitatively, in terms of kinematic corrections to the asymptotic parton model.

However one aspect of parton models is in complete disagreement with experiment, and that is the basic requirement that QP be free. They should be detected in the debris resulting from the target hadron breaking up in DIS: $2H \rightarrow \ell'X$. Hence the well-known dilemma:

Experimentally verified predictions arise from the QP model assumption that QP are free, yet this assumption is itself in contradiction with experiment.

We are reluctant to throw away the parton models because of their successes, and also because they provide a simple appealing picture of hadrons at high energies. Therefore we do not say that partons are unphysical but instead conclude that they cannot be seen on their own, i.e.: some final state interactions, which do not otherwise interfere with the QP model predictions, come into effect to bind the partons into observable particles.

It can easily be seen why we might expect these final state interactions not to affect the remaining QP model predictions (Landshoff and Scott 1977). We can say that the length of time associated with these confinement phenomena will be $\sim R$ (the hadron size). In order for the parton notion to be independent of these phenomena we must
demand that the interaction time $\tau$ be much less than the confinement time, $\tau \ll R$. We expect this to be the case (we saw in Chapter 2 that $\tau \to 0$ in the Bjorken limit).

In light of the above discussion we believe it is necessary to introduce confinement into the QPM but that this will not alter the asymptotic predictions. However in the words of Close (1978): "While this may be a reasonable zeroth order picture, one should clearly worry about the interactions that bind the quarks and ask whether they do not indeed affect the scaling predictions of the naive quasi-free model."

This chapter is divided into two parts. In the first we examine the quantum number exchange which must take place so that the final state has observable (i.e.: integer) quantum numbers. This is done within the parton model framework but independently of any model of confinement. In the second part we look at a particular simple model which localizes partons within hadrons, and examine the consequences of this localization.

§17 Average Net Jet Charge

If the "handbag" picture is right then the final state particles fall into two distinct "jets", one from the struck QP and one from the (hadron minus parton), as shown in Fig. (17.1). They are distinct in that they are separated in momentum; $W_p$ is of order $p$ ($\sim M$ in LAB frame) whereas $W_q \sim q$. This two-jet structure is basic to all parton models. Because the jets are distinct we require their quantum numbers to be integral, but this is not the case in Fig. (17.1).

We see that an antiparton (or some other animal with antiparton quantum numbers) must be exchanged between the two jets, as in Fig. (17.2a).
If this is not to spoil the parton model incoherence assumption then we must require that the time scale governing the exchange be much larger than that of DIS (i.e.: the exchange takes place when the jets are widely separated - this will be discussed more fully later on). Thus by the uncertainty principle the energy of the exchanged antiparton should not be too large. This is what we expect in the parton model: the antiparton arises from the hadron sea, which is confined to small energy/momentum \( (m \sim M) \). Also in the spirit of the parton model we note that this antiparton will be onshell, since it is so long-lived.

We will discuss consequences of the above paragraph later on. Right now we are interested in the effects of the exchanged anti-parton on the net jet charge. Our basic assumption is that the struck parton \( (k') \) prefers to interact with a sea antiparton (not a sea parton) as in Fig. (17.2b) (cf. G. Ross 1980 in this regard).

We are now in a position to calculate the jet charges. The result is shown in Table (17.1). Here is a sample calculation:

If the hadron is a proton then the probability for the \( \gamma^* \) probe to scatter off a \( u \) QP is \( \frac{2}{3} \). Neglecting strangeness the probability for this struck \( u \) QP scattering off an \( \bar{Q}P \) with charge \( +\frac{1}{3}, -\frac{2}{3} \) is, from our assumption, \( \frac{1}{2} \). Thus a \( u \) QP produces a jet with charge \( +1, 0 \) with equal probability.

Table 1 shows the probability \( H_{PC}^{QP} \) that the QP jet emanating from hadron \( H \) has charge \( c \). D stands for deuteron. By charge conservation we see that the probability \( H_{PC} \) that the hadron jet has charge \( c \) is given by

\[
H_{PC} = H_{PC}^{QP}_{c_H - c}, \quad c_H = \text{hadron charge}.
\]

\[\text{Here we assume the probability for the } \gamma^* \text{ scattering of an } \bar{Q}P \text{ is negligible. The results we obtain are easily generalized to non-zero probability.}\]
From these probabilities we can calculate the average charge

\[ \langle c_{QP}^H \rangle = \sum_{i=-1}^{1} c_{i} P_{i} \] 

of the QP jet emanating from hadron H; these are given in Table (17.2). The numbers in brackets correspond to predictions for the usual QPM. We see that the effect of QP exchange is to lower the naive result by \( \frac{1}{6} \). This is seen in other models, e.g.: Farrar and Rosner (1973), Erickson et al. (1979).

If, in an obvious notation, we write the proton as \( p = (uud) + \kappa (uudd) \) and allow the probe to scatter off sea QP and \( \bar{Q} \) as well as valence QP, then after some calculation we find for the average net charge \( \langle c_{P}^p \rangle = \frac{1}{6 + 8\kappa} \), which reduces to the previous result in the limit \( \kappa \to 0 \). From the discussion in §6 we believe that \( \kappa \) is \( x \) dependent, being large near \( x = 0 \) and dropping off to zero for \( x \gtrsim 0.2 \). Thus the average net jet charge becomes \( x \) dependent, rising from some minimum value at \( x = 0 \) to a maximum of \( \frac{1}{6} \) for large \( x \). This is reflected in the data (Erickson et al.).

If we include strangeness, \( p = (uud) + \kappa (uudd\bar{s}\bar{q}) \), then we find \( \langle c_{P}^p \rangle = \frac{1}{3} \frac{1}{1+\kappa} \), which we expect to rise to the value \( \frac{1}{3} \) as \( x \) increases. Thus the effect of QP exchange on the net jet charge is diminished by the presence of strangeness.

**TABLE (17.1)**

<table>
<thead>
<tr>
<th>c</th>
<th>( P_{p} )</th>
<th>( n_{p} )</th>
<th>( d_{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>-1</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{4} )</td>
</tr>
</tbody>
</table>

**TABLE (17.2)**

<table>
<thead>
<tr>
<th>H</th>
<th>( \langle c_{H}^{QP} \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>( \frac{1}{6} ) (( \frac{1}{3} ))</td>
</tr>
<tr>
<td>n</td>
<td>( -\frac{1}{6} ) (0)</td>
</tr>
<tr>
<td>d</td>
<td>0 (( \frac{1}{6} ))</td>
</tr>
</tbody>
</table>
§18 Quantum Number Exchange Within the Parton Model

[1] In §17 we argued that the handbag diagram of the naive QP model must be replaced by the diagram of Fig. (17.2a) to incorporate quantum number exchange. We further argued that within the QP model this exchanged $\overline{Q}$P ought to be onshell, and have small four-momentum $m$. Here we shall discuss the consequences of this new diagram for the structure function $F_2$ of DIS. Note that if the QP model is self-consistent then the extra terms in $F_2$ generated by this new diagram must be negligible compared to the original parton model expression.

To calculate the new $F_2$ we must square Fig. (17.2a) yielding Fig. (18.1) for the contribution of the $i$'th QP. We write the structure tensor for this diagram as

$$\hat{W}_{\mu\nu}^i(k,q,m) = 2(k_{\mu}k'_{\alpha} + k'_{\mu}k_{\alpha} - k\cdot q g_{\mu\alpha}) \frac{\omega^{\alpha\beta}}{Q^2}$$

$$\cdot \frac{2(k_{\beta}k'_{\nu} + k'_{\beta}k_{\nu} - k\cdot q g_{\beta\nu})}{\omega^{\alpha\beta}}$$

where $\omega^{\alpha\beta} = \omega^{\alpha\beta}(m,k')$ contains the blob of Fig. (18.1) and can be written in general as

$$\omega^{\alpha\beta} = A g^{\alpha\beta} + B(k'm_{\alpha} + m_{\alpha}k')$$

$$+ C k'k'_{\alpha\beta} + D m_\alpha m_\beta$$

Equation (18.1) requires some explanation. The lines representing a parton of momentum $k$ and the $\gamma^*$ of momentum $q$ are connected to the new blob by a line representing an onshell parton of momentum $k'$ and so we expect the $kq$ vertex to take exactly the same form in eqn. (18.1) as in the original $W_{\mu\nu}^i$. For the same reason (cf. also §16) we expect the $kq$ vertex to be physically remote from the blob of Fig. (18.1) and so $\omega^{\alpha\beta}$ can depend on $k',m$ only.
It is a simple matter to show that the first term of $\omega_{ab}$, when
substituted into eqn. (18.1), yields this familiar gauge invariant
form (cf. §11). We automatically have $q_{\mu \nu} J^i_{\mu \nu} = 0$.

The scalars $A,B,C,D$ of eqn. (18.2) are functions of the
available invariants, viz. $k'^2, m^2, m'k'$. Let us initially make the
assumption that those QP entering the blob are massless (K. Ellis
et al. 1979), $k'^2 = m^2 = 0$, so that $A,B,C,D$ are functions of only
one variable, $m'k'$. In this case by dimensional analysis we must
have $A = c_p, B,C,D = \frac{c_{2,3,4}}{m'k'}$ with $c_{1,2,3,4}$ dimensionless constants.

Putting all this together we find for the new structure tensor

$$
\frac{i}{2}W^i_{\mu \nu} = -\frac{i}{2}c_1 (k_{\mu}k'_{\nu} + k'_{\mu}k_{\nu} - \frac{1}{4}Q^2 g_{\mu \nu}) + \\
+ \frac{c_4}{Q^2 m'k'} \left[ m'k (k_{\mu}k'_{\nu} + k'_{\mu}k_{\nu}) + (m'k')^2 k_{\mu}k_{\nu} \\
+ (m'k)^2 k'_{\mu}k'_{\nu} - \frac{1}{4}Q^2 m'k (m_{\mu}k'_{\nu} + k'_{\mu}m_{\nu}) \\
- \frac{1}{4}Q^2 m'k' (m_{\mu}k_{\nu} + k_{\mu}m_{\nu}) + \frac{1}{4}Q^4 m^2 m'm_{\mu}m_{\nu} \right].
$$

(18.3)

By construction the first term is (up to a constant) the old parton
structure tensor. We saw in Chapter 2 that in the target hadron rest
frame we may write the hadron structure tensor $W_{\mu \nu}$ in terms of the
parton structure tensor $W^i_{\mu \nu}$ as follows (cf. §11)

$$
W_{\mu \nu}(p,q) = \sum_i e_i^2 \int \frac{d^3k}{k_o} P_i(k) \delta(k'q - \frac{1}{4}Q^2) \frac{i}{2}W^i_{\mu \nu}(k,q)
$$

(18.4)

where $P_i(k)$ is the probability for QP $i$ (with momentum $k$) being
ejected from the hadron. For our new structure tensor this must be
generalized to
\[ W_{\mu \nu}(p,q) = \sum_{ij} e_i^2 \int \frac{d^3k}{k_0} P_i(k) \int d^3m \overline{P}_j(m) \delta(k \cdot q - \frac{1}{4}Q^2) \int d^3m \overline{W}^i_{\mu \nu}(k,q,m) \]  
(18.5)

If we substitute for \( W^i_{\mu \nu} \) into eqn. (18.5) we find, after some algebra

\[ F_2(x,Q^2) = \sum_{ij} e_i^2 \int \frac{d^3k}{k_0} P_i(k) \int d^3m \overline{P}_j(m) \delta(k \cdot q - \frac{1}{4}Q^2) \]

\[ \cdot M x Q^2 \left( -c_1 + \frac{m \cdot k}{Q^2} c_4 \right) \]  
(18.6)

in the hadron rest frame.

Because \( m, k \) are both \( \sim M \) we see that the quantum number exchange \( (c_4) \) term is negligible in comparison to the parton model \( (c_1) \) term, in the Bjorken limit. This is the desired result: we can include quantum number exchange within the QP model, and this does not spoil the original asymptotic QP model predictions.

Furthermore it is apparent from (18.6) that we can (up to an arbitrary constant) calculate the subasymptotic correction to \( F_2(x) \) due to quantum number exchange. We shall do this in subsection [4].

[2] It remains for us to show that the \( c_1 \) term of eqn. (18.6) can in fact be reduced to the old rest frame result (cf. Chapter 2).

\[ F_2(x) = \sum_{ij} e_i^2 M^2 x^2 \int \frac{d^3k}{k_0} P_i(k) \nu \delta(k \cdot q - \frac{1}{4}Q^2) \]  
(18.7)

For simplicity we shall assume no SU3 symmetry breaking, i.e.: \( P_i(k) \) is independent of \( i \), etc. Identifying (18.6) with (18.7) leads to the equality

\[ \int d^3m \overline{P}(m) (-c_1) = 1. \]  
(18.8)

Hence in this particular case of DIS the coefficient of the \( g_{\alpha \beta} \)
term of $\omega_{\alpha\beta}$ is identified as minus the inverse normalization of the QP sea. This identification guarantees that the $c_1$ term (the dominant term) of our new $F_2$, eqn. (18.6), is exactly the old QP model structure function, eqn. (18.7).

[3] Here we answer two questions which might be asked of subsection [1].

(i) What happens if we relax our assumption that QP are massless? In this case the scalars $A, B, C, D$ that describe the blob of Fig. (18.1) become functions of $k'^2, m^2$, as well as $m \cdot k'$. The end result of this is that $c_{1,4}$ are no longer constants, they are functions of the dimensionless ratios $\frac{k'^2}{m \cdot k'}$, $\frac{m^2}{m \cdot k'}$, which depend on $Q^2$. Thus we are unable to say that the usual parton model term dominates. In fact we cannot even guarantee that the structure function scales. This is why we have to assume that QP are massless.

Note that if we apply QCD corrections to the parton model quantum number exchange calculation we will introduce a mass scale $\Lambda^2$ into $\omega_{\alpha\beta}$, even with massless QP. However, as we shall see in Chapter 5 the QCD structure functions do not scale anyway.

(ii) Is our starting point, Fig. (17.2a), sufficiently general? We have claimed that a low energy onshell QP is exchanged between the two blobs. This is apparently in contradiction with the Feynman-Field (FF) model of final-state interactions, which has a flux tube of glue connecting the blobs. The FF model is formulated within the parton model framework and so if Fig. (17.2a) is the general parton model diagram for final state interactions, then it ought not to be in contradiction with this particular (FF) model.

We show that the two are compatible. FF regard the tube of glue as an elastic band which stretches as the two groups of QP separate.
The elastic snaps (i.e.: forms $\bar{q}q$ pairs) repeatedly until the last $\bar{Q}P$ has a momentum compatible with that of the hadron-minus-$Q\bar{P}$ blob. This is shown schematically in Fig. (18.3). Comparing with Fig. (17.2a) we see that FF is a particular model for the quark jet blob of Fig. (17.2a). Squaring this statement, we can say that Fig. (18.4) is the FF blob of Fig. (18.1).

[4] It is not difficult to show from eqn. (18.6) that the sub-asymptotic contribution to $F_2(x,Q^2)$ from the (E)xchange term is given by

$$F_2^E(x,Q^2) = 4\pi\Sigma \int \frac{J}{Mx} dk k P_1(k)$$

where $<m_0> = \int d^3m \bar{P}(m)m_0$. The ratio of this exchange term to the asymptotic structure function behaves like

$$\frac{F_2^E(x,Q^2)}{F_2(x)} \rightarrow 0, \quad x \rightarrow 0$$

$$\rightarrow c'_4 \frac{M^2}{Q^2}, \quad x \rightarrow 1$$

(18.9)

with $c'_4 = \frac{<m_0>}{M} c_4 = \text{constant}$. As we shall see, experiment suggests that $c'_4$ is small, so that from eqn. (18.9) the exchange term is unimportant.

It is not difficult to show that the n'th moment of $F_2^E(x,Q^2)$ is

$$M_n^E(Q^2) = c'_4 \frac{M^2}{Q^2} \left(\frac{n+2}{n+1}\right)^{n+1} M_{n+1}.$$  

This must be added to the subasymptotic moments discussed in Chapter 2. Thus we obtain

$$M_n(Q^2) = M_n + \frac{M^2}{Q^2} \left( c'_4 \left(\frac{n+2}{n+1}\right)^{n+1} M_{n+1} + (n-3 + \frac{6}{n+2}) M_{n+2} \right)$$
We note that the second term vanishes for \( n = 1 \), and so a clean test for \( c'_4 \) can be obtained by looking at the \( Q^2 \) dependence of \( M_1(Q^2) \). Unfortunately this moment is not well known, so we look at \( n = 2 \) (cf. Fig. (3.1)). This yields \(-0.1 \leq c'_4 \leq 0\).

Thus the exchange correction is small compared to the kinematical corrections of Chapter 2.

[5] The calculations of this section have so far been performed in the hadron rest frame. Here we shall rederive our main result in the infinite momentum frame (IMF). This will enable us to see which predictions are frame dependent and which are not.

We choose the Drell-Yan (1971) frame (see also Roy, 1975) in which

\[
\begin{align*}
 p^\mu &= (P + \frac{M^2}{2P}; 0, P) \\
 k^\mu &= (xP; 0, xP) \\
 q^\mu &= \left( \frac{2Mv-Q^2}{4P}; q_\perp, \frac{-(2Mv+Q^2)}{4P} \right).
\end{align*}
\]

Here the proton momentum \( P \) approaches infinity faster than any observable, such as \( v, Q^2 \), so that we may ignore \( O(\frac{1}{P^2}) \) terms. In the frame (18.10) we therefore have \( p^2 = M^2, k^2 = 0, Q^2 = q_\perp^2, p \cdot q = Mv \). In addition we take the \( QF \) momentum to be

\[
 m^\mu = (yP; 0, yP)
\]

so that \( m^2 = 0 \). The \( QP \) (\( QF \)) three-momentum is thus a fraction \( \frac{x}{y} \) of the parent hadron three momentum \( P \) in this frame.

If we redo the calculations of subsection [1] in this new frame we find for the analogue of eqn. (18.6):
\[ F_2(x) = \sum_{ij} \frac{e_i^2}{p_i^2} \int_0^1 \frac{dx f_i(x)}{x} \int_0^1 dy \, f_j(y) \theta(1-y-x) \frac{1}{M^2} \delta(x-x) \] 

which simplifies to

\[ F_2(x) = \sum_{ij} \frac{e_i^2}{p_i^2} x f_i(x) \int_0^1 dy \, f_j(y) \theta(1-x-y)(-c_1) \] .

If this is to reduce to the old IMF expression for \( F_2 \) (the "master formula")

\[ F_2(x) = \sum_{ij} \frac{e_i^2}{p_i^2} x f_i(x) \]

then we must have

\[ f_i(x) + \frac{\sim}{\sim} f_i(x) = (-c_1) f_i(x) \int_0^1 dy \, \bar{f}(y) \theta(1-x-y) \]

(18.11)

where we require both \( f_i(x), f_i(x) \) to be normalized to 1. If \( f_i(x), \bar{f}(y) \) are such that \( x+y < 1 \), then (18.11) reduces to the IMF analogue of (18.8):

\[ \int_0^1 dy \, \bar{f}(y)(-c_1) = 1 \]

This slight complication represented by the \( \theta \) function in (18.11) arises because here we have explicitly taken into account the constraint that QP momentum plus QP momentum must be less than or equal to hadron momentum.

Note that in this IMF we have \( m \cdot k = 0 \) so that the \( c_4 \) term is absent. Thus quantum number exchange effects are \( O(1/Q) \) here. This suggests that the subasymptotic corrections are frame-independent, though in both rest frame and IMF the parton model handbag term wins.

Note also that the mass-squared of the QP jet is \( W_q^2 = 2m \cdot k' \) and that in this frame \( 2m \cdot k' = \frac{x}{Q^2} \), so that in this case the average
QP jet mass squared is predicted to rise with $Q^2$.

So far in this chapter we have looked at the effects of final state interactions on parton model predictions. We now turn inward to the momentum and spin distributions of the primordial hadron constituents. In the particular model considered we shall see that these distributions can all be obtained in terms of a "confinement factor" which describes how the constituents are localized.

§19 A Localized Parton Model

Suppose a spin-$\frac{1}{2}$ constituent is at rest within the hadron. We may write for its spinor

\[ \psi(0, \mathbf{r}; \epsilon) = S(\mathbf{r}) \omega_0(\epsilon) \]  

at time $t = 0$, where $\omega_0(\epsilon)$ is a rest frame spinor and $\epsilon$ is a spin index. $S(\mathbf{r})$ is a spatial distribution function which localizes the constituent to within a volume of radius $R$, i.e.: $R \propto \langle r \rangle = \int d^3r \, r |S(\mathbf{r})|^2$, where $R$ is some length scale characteristic of hadrons. $S(\mathbf{r})$ is spherically symmetric and is normalized to 1.

What are the consequences of our confinement assumption? We write

\[ \psi(t, \mathbf{r}; \epsilon) = \left( \frac{d^3k}{(2\pi)^3} \right)^{1/2} \sqrt{\frac{\mu}{k_0}} \sum_{\sigma} \left[ b_\epsilon(k, \sigma) u(k, \sigma) e^{-ik \cdot r} + d_\epsilon^*(k, \sigma) v(k, \sigma) e^{ik \cdot r} \right] \]  

as the general solution to the free Dirac equation $\frac{i}{\hbar} \frac{\partial}{\partial t} \psi(t, \mathbf{r}) = (\beta \mu - i \alpha \cdot \mathbf{v}) \psi(t, \mathbf{r}) = H\psi(t, \mathbf{r})$ where $b, d^*$ are respectively the amplitudes for positive, negative energy solutions $u, v$ and are

\[
\text{† Thus we are constructing a first quantized model.}
\]
chosen such that (19.2) satisfies the initial conditions (19.1). We shall calculate $b, d^*$ below, but first let us discuss our interpretation of (19.1).

The solutions $u(k,\sigma)e^{-ikr}, v(k,\sigma)e^{ik\cdot r}$ are eigenstates of the Hamiltonian operator $H$ with eigenvalues $+k_0, -k_0$ respectively. The expectation value $<H> = \langle\psi(t,r)|H|\psi(t,r)\rangle$

$= \int d^3k \sum_{\sigma} [|b(k,\sigma)|^2 + |d(k,\sigma)|^2]k_0$ is independent of time. It is the sum of the expectation values of $H$ for eigensolutions $u, v$. Because it is time-independent we may calculate $<H>$ from eqn. (19.1) to obtain $<H> = \langle u |$ (for any $S(r)$). Thus in an obvious notation $<H> = \langle u | $ $<k_0> u + <k_0> v$. Similarly we find $<k> = 0 = <k> u + <k> v$.

Thus we regard the solution (19.1) as representing a spin-$\frac{1}{2}$ particle at rest, and as being composed of a linear combination of moving positive and negative energy solutions $u, v$.

So far the discussion is entirely general and holds for any point-like Fermion. In the present context we shall call the moving solutions "daughter partons" and the stationary constituent a "parent parton"$^\dagger$

We shall leave open the correspondence of these partons with the constituent quarks, current quarks, and QP seen in the literature, though it seems natural to identify parents with SU6 constituent quarks (cf. Chapter 4).

Thus the effect of confining a parent parton is to generate positive and negative energy daughters, with amplitudes $b, d^*$. We can in fact calculate the average number $#$ of daughters; it is not difficult to show that $#^{-1} = \frac{<u>}{k_0}$.

For the sake of completeness we briefly examine the time development of (19.1). The full time-dependent solution $\psi(t,r)$ will not in

$^\dagger$ Alternative names might be "partons" and "valons" (cf. Hwa et al. 1981).
general be localized except at \( t = 0 \) (there is no confining term in the Hamiltonian). This is not important in DIS, however. Because of the discussion of §16 we may assume that the DIS process takes place instantaneously at \( t = 0 \), so that the \( \gamma^* \) probe sees a snapshot of the target hadron\(^\dagger\).

Now we calculate \( b, d^* \). Fourier analyzing eqn. (19.2), at \( t = 0 \), and equating with (19.1) we find

\[
|b_\epsilon(k,\sigma)|^2 = \frac{1}{4}(1 + \frac{\mu}{k_0}) |\hat{S}(k)|^2 |U_\sigma U_\epsilon|^2
\]
\[
= B_\sigma^\epsilon(k) \tag{19.3}
\]

\[
|d_\epsilon(k,\sigma)|^2 = \frac{1}{4}(1 + \frac{\mu}{k_0}) |\hat{S}(k)|^2 |U_\sigma^+ \gamma\cdot k U_\epsilon|^2
\]
\[
= D_\sigma^\epsilon(k) .
\]

Here we have used

\[
\hat{S}(k) = \frac{k + \mu}{2\mu} U_\sigma
\]

for the daughter parton spinors, where \( \frac{k}{\mu} = \frac{k_0 + \mu}{k_0} \) and

\[
\omega_\epsilon(\sigma) = \begin{pmatrix} U_\epsilon \\ 0 \end{pmatrix}, \quad U_\epsilon = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \omega_\epsilon = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

for the parent spinors. \( \hat{S}(k) \) is the Fourier transform of \( S(r) \). \( B_\sigma^\epsilon(k), D_\sigma^\epsilon(k) \) are respectively interpreted as being the probability for finding a daughter, antidaughter of momentum \( k \) and spin \( \sigma \) in a parent of spin \( \epsilon \) at rest. Note that

\(^\dagger\) This last statement is in contradiction with the usual IMF parton model. We shall examine this more closely in §21.
P(k) = \sum_\sigma [B_\sigma^c(k) + D_\sigma^c(k)] = |\tilde{S}(k)|^2

is independent of \( \varepsilon \). Henceforth we drop the spin index whenever \( \sigma \) is summed.

The daughter and antidaughter contributions to the momentum density \( P(k) \) are

\[
B(k) = \frac{1}{2} (1 + \frac{\mu}{K_0}) P(k) \\
D(k) = \frac{1}{2} (1 - \frac{\mu}{K_0}) P(k)
\]

so that \( B > D \).

The situation thus far is as follows. We take account of confinement in a very simple way in eqn. (19.1). This localization gives rise to parton and antiparton momentum and spin densities which we can calculate given the confinement factor \( S(r) \). The Fourier transform squared of this spatial distribution, \( P(k) \), is the starting point of many parton models (Akama, 1974; Landshoff and Scott, 1977; Franklin 1977) and gives rise to the structure function \( F_2(x) \) via

\[
F_2(x) = \sum_{ Bj } e_i^2 \frac{2\pi M^2 x^2}{\kappa} \int \frac{dk}{k} \frac{k}{K_0} P(k), \quad \kappa = \frac{1}{4} M x - \frac{\mu^2}{M^2} \tag{19.5}
\]

in the hadron rest frame, eqn. (14.7). In writing down (19.5) we make the central parton model assumption that \( F_2(x) \) is the incoherent sum of individual QP contributions; it is a matter of personal preference at this point whether we choose \( i \) to label parent partons or daughter partons.

This model naturally gives rise to an antiparton distribution. Thus we may extract from \( F_2(x) \) a "sea" contribution by assuming that the daughter parton momentum distribution in the sea equals the antidaughter distribution, \( B^c_{\text{sea}}(k) = D(k) \). Hence from (19.4)
the "valence" distribution is \( \frac{1}{k_0} P(k) \) and the "sea" distribution is
\( (1 - \frac{1}{k_0})P(k) \). Substituting those in (19.5) gives the valence and sea
contributions to the structure function. Note though that the sea
contribution is only that of the "free sea", i.e.: arising from each
parent individually, rather than from interactions between parents.

In addition to giving parton and antiparton \( x \) distributions this
marriage of parton model and confinement yields information about parton
and antiparton spin distributions, which we shall now investigate.

§20 Including Spin Dependence

In this section we take \( \mu = 0 \) (this is only for ease of cal-
culation, it is not essential) so that for instance the \( x \) distri-
bution \( f(x) \) becomes, from (19.5'), \( f(x) = 2\pi M^2 x \int dk P(k) \) (cf.
eqn. (12.3)). Similarly we define \( b^E_\sigma(x), d^E_\sigma(x) \) but \( B^E_\sigma(k),
D^E_\sigma(k) \) replacing \( P(k) \). Obviously \( [b^E_\sigma(x) + d^E_\sigma(x)] = f(x) \),
which is normalized to 1.

Define the parent-daughter polarization asymmetry as

\[
a(x) \equiv \frac{b^E_\sigma(x) - d^E_\sigma(x)}{b^E_\sigma(x) + d^E_\sigma(x)}
\]

which is a measure of the correlation between the parent spin and that
of its constituent daughters. From (19.3) we see that \( a(x) = 1 \).

The hadron-parent asymmetry \( a^H_1(x) \) measures the correlation
between the parents' spin and that of the hadron they form. We assume
that there are three such parents (uud for the proton and ddu for the
neutron) and that the symmetry between them is rest frame SU6, in which
case the probability is \( \frac{1}{3} \frac{5}{9} \) for the \( u \) parent and \( \frac{2}{3} \frac{1}{9} \) for the
d parent quark having spin (anti-)parallel to the proton spin. \( u \leftrightarrow d \)
for the neutron. Thus we find \( a_1^H(x) = \frac{1}{3} \) (a result we could have obtained from simple statistical arguments).

Lastly the hadron-daughter asymmetry \( a_2^H(x) \) is obtained by combining the above. We can write: (probability for hadron spin being aligned with daughter spin) = (probability for hadron spin being aligned with parent spin) \( \times \) (probability for parent spin being aligned with daughter spin) + (aligned + antialigned). We find \( a_2^H(x) = \frac{1}{3} \).

Similarly we can find the antiparton asymmetries \( \bar{a} \), with \( d_\sigma^e \) replacing \( b_\sigma^e \). Of interest to us is the parent-antidughter asymmetry \( \bar{a}(x) \) which we calculate to be \( \mu = 0 \)

\[
\bar{a}(x) = \int \frac{dk}{M_x} \frac{P(k)}{M_x} \left( 1 - \frac{4Mx}{k} + \frac{2x^2}{k^2} \right)
\]

If we express \( P(k) \) in terms of \( f(x) \) this yields

\[
\bar{a}(x) = 1 + \frac{8x^2}{f(x)} \left[ \int_0^1 \frac{dy}{y^3} f(y) - 2x \int_0^1 \frac{dy}{y^4} f(y) \right]
\]

from which it is straightforward to show that the expectation value \( \langle \bar{a} \rangle = \int_0^1 dx \bar{a}(x) f(x) \) equals \( -\frac{1}{3} \), independent of \( f(x) \). Hence in this model, as in that of Close and Sivers (1977), the sea is polarized. In our case this polarization is a result of confinement.

As an example we choose the popular parametrization \( f(x) = 4(1-x)^3 \) and find

\[
\bar{a}(x) = \frac{8x^2(3+2x)}{(1-x)^3} \ln \frac{1}{x} - \frac{40x^2}{(1-x)^2} - \frac{4x}{1-x} - \frac{1}{3}
\]

which is shown in Fig. (20.1). This behaviour is similar to that obtained by Close and Sivers (1977). If the \( x \) distribution falls off faster, say as \( (1-x)^5 \) or \( 7 \) then because \( \langle \bar{a} \rangle = -\frac{1}{3} \) we see that the region over which \( \bar{a}(x) \) is negative must be squashed towards \( x = 0 \).
Of more immediate experimental interest is the electroproduction asymmetry
\[ \Lambda^{\gamma H}(x) = \frac{1}{2} \frac{3}{2} \sigma^\gamma - \sigma^\gamma + \]
where \( \frac{1}{2}, \frac{3}{2} \) refers to the total photon plus hadron spin. In the parton model we may write, following Close (1979)
\[ \sigma^{\gamma H} = \sum \sigma^{\gamma q} p^{q H} \]
where \( p^{q H} \) is the probability for quark spin being (anti)aligned with the hadron spin and where \( \sigma^{\gamma q} \) is the \( \gamma^* \)-quark cross section.

We have yet to decide whether \( q \) stands for parents or daughters. Parent quarks are at rest and so \( \sigma^{\gamma q^+} = 0 \), by angular momentum conservation. Thus, if \( q \) refers to parents, then it is easy to show that
\[ A^{\gamma P} = \frac{5}{9}, \quad A^{\gamma n} = 0 \]
which are the usual results (e.g.: Close 1979).

If, on the other hand, \( q \) refers to daughters we cannot say \( \sigma^{\gamma q^+} = 0 \), because daughters have a nonzero transverse momentum (Close, 1979). We write
\[ \sigma^{\gamma q^+} = \sigma \cdot \sin^2 \frac{1}{2} \bar{\eta} \]
\[ \sigma^{\gamma q^-} = \sigma \cdot \cos^2 \frac{1}{2} \bar{\eta} \]
in which case it is not difficult to show that
\[ A^{\gamma P}(x) = \frac{5}{9} \frac{1}{2} (a(x) + \bar{a}(x)) \cos \eta \] (20.1)

\[ + \] Note that this \( \cos \eta \) factor can be obtained, even for the case \( q = \) parent quark, if we rotate the quark 2-spinor \( U_{\sigma} = (\frac{1}{0}), (\frac{0}{1}) \) to
\[ U_{\sigma}(\eta) = \begin{pmatrix} \cos \frac{1}{2} \eta \\ -\sin \frac{1}{2} \eta \end{pmatrix} \begin{pmatrix} \sin \frac{1}{2} \eta \\ \cos \frac{1}{2} \eta \end{pmatrix}. \] This spin rotation will be discussed in detail in Chapter 4.
Even if we take $\cos \eta = 1$ then $A^{\gamma p}$ is still less than the SU6 result of $\frac{5}{9}$ for most of the $x$ range, because $\frac{1}{2}(a(x) + \bar{a}(x)) < 1$ except at $x = 1$. We plot $A^{\gamma p}(x)$ in Fig. (20.2) (for the $\bar{a}(x)$ given in Fig. (20.1)). The result is compatible with experiment (Alguard et al., 1978) and is similar to that of other, more sophisticated models of quark spin distribution (Close, 1974; Kaur, 1977; Hughes, 1977) as shown in Fig. (20.3) (taken from Alguard et al., 1978).

It is a straightforward exercise to show that allowing a non-zero quark mass means that $A^{\gamma p}$ lies somewhere between eqn. (20.1) and the constant SU6 value of $\frac{5}{9}$.

We define the polarized structure function $g_1^H(x)$ in the usual manner (e.g.: Kaur, 1977)

$$2 \times g_1^H(x) \equiv F_2(x) A^{\gamma H}(x)$$

and introduce the Bjorken sumrule (Bjorken, 1966; Close, 1979; Feynman, 1972)

$$\int_0^1 \! dx (g_1^p(x) - g_1^n(x)) = \frac{1}{6} \frac{|g_A|}{g_V}$$

(20.2)

where $g_{V,A}$ are the $\beta$-decay vector and axial coupling constants.

From (20.2) the SU6 (parent quark) value is $\left| \frac{g_A}{g_V} \right| = \frac{5}{3}$, which compares with the experimental value of $1.25 \pm 0.01$ (Alguard et al., 1978).

From eqn. (20.1) we calculate $\left| \frac{g_A}{g_V} \right| = \frac{5}{9}$. Introducing a nonzero daughter quark mass means that

$$\frac{5}{9} \leq \left| \frac{g_A}{g_V} \right| \leq \frac{5}{3}$$

(cf. eqn. (24.2) below). This range includes the experimental value.

Thus if we treat parent quarks as the QP of DIS then we obtain the old SU6 values for spin-dependent quantities. If, however, we allow
for confinement, i.e.: treat daughter partons as the QP of DIS, we obtain values lower than the SU6 predictions, and in general introduce an \( x \) dependence absent in the SU6 case.

§21 Do Daughter Partons Exist on the Light Cone?

It can be argued that, because of the finite extent of the hadron, the \( \gamma^* \) probe does not see a "snapshot" at \( t = 0 \) but instead sees a "light cone snapshot" at \( x_+ = 0 \), where \( x_+ = t + z \). (This \( x_+ = 0 \) formulation of the parton model in the hadron rest frame is equivalent to the usual IMF parton model. This will be discussed in more detail in Chapter 4). Squires (private communication) claims that in \( x_+ = 0 \) models there are no daughters generated by confinement, at least within the context of bag models. If this is also the case here, then the results of §§19, 20 are peculiar to \( t = 0 \) models, and have no meaning in the \( x_+ = 0 \) case.

The attitude we take is that the time scale involved in DIS is so much smaller than that of confinement that the hadron appears to be frozen; in terms of the confinement factor we say \( S(t=0) \sim S(t=R) \), where \( R \) represents the hadron size. Thus taking \( t=0 \) or \( x_+ \neq 0 \) should not make any difference.

For the sake of completeness, however, it is perhaps advisable to examine the localized parton model on the light cone. From eqn. (19.2) we see that the general solution is

\[
\psi(x_+ = 0) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{e^{i(k_3 x_3 + k \cdot x_1)}}{\sqrt{k_0}} \sum \left[ b(k\sigma)u(k\sigma)e^{i(k_3 x_3 + k \cdot x_1)} + d^*(k\sigma)v(k\sigma)e^{-i(k_3 x_3 + k \cdot x_1)} \right]
\]

(21.1)

where \( k_+ = k_0 + k_3 \). The total probability
\[ 1 = \int d^3x \psi^\dagger(x_+ = 0) \psi(x_+ = 0) \]

is changed from the \( t=0 \) result to

\[ = \int d^3k \sum_s \left[ |b(ks)|^2 + |d(ks)|^2 \right] \frac{k_0^2}{k_+} \]  

\[ (21.2) \]

i.e.: an extra factor \( \frac{k_0}{k_+} \).

Analogous to eqn. (19.1) we write

\[ \psi(x_+ = 0) = S(x_+ = 0) \omega(\epsilon) \]  

\[ (21.3) \]

where \( S \) localizes the light cone solution. The total probability can thus be written

\[ 1 = \int d^3x |S(x_+ = 0)|^2 \]  

\[ (21.4) \]

\[ \omega^\dagger(\epsilon) \omega(\epsilon) = 1. \]

If we now define the "light cone Fourier transform" of \( S \) as

\[ S_+(k) \equiv \frac{1}{(2\pi)^{3/2}} \int d^3x \ e^{i(k_+ x_3 + k_1 \cdot x_\perp)} S(x_+ = 0) \]  

\[ (21.5) \]

then (21.4) becomes

\[ 1 = \int d^3k |S_+(k)|^2 \frac{k_+}{k_0} \]  

\[ (21.6) \]

The integrand is interpreted as the probability density for finding a daughter parton with momentum \( k \) inside a light cone parent parton. From (21.2) and (21.6) we thus require

\[ \sum_s \left[ |b(ks)|^2 + |d(ks)|^2 \right] = |S_+(k)|^2 \left( \frac{k_+}{k_0} \right)^2. \]  

\[ (21.7) \]
Repeating the calculations of §19 we find

\[ b(ks) = \frac{k^+_0}{k_0} S^+_+(k) \sqrt{\frac{\mu}{k_0}} u^+(ks)\omega(\varepsilon) \]

\[ d^*(ks) = \frac{k^+_0}{k_0} S^+_+(k) \sqrt{\frac{\mu}{k_0}} v^+(-ks)\omega(\varepsilon) \]

If we choose the localized parent parton spinor to be on the light cone, \( \omega(\varepsilon) = \omega_+^o(\varepsilon) = \frac{1}{\sqrt{2}} \begin{pmatrix} U_\varepsilon \\ \sigma_3 U_\varepsilon \end{pmatrix} \) then

\[ \Sigma_s |b(ks)|^2 = \frac{1}{2} |S^+_+(k)|^2 \left( \frac{k^+_0}{k_0} \right)^3 \]

\[ \Sigma_s |d(ks)|^2 = \frac{1}{2} |S^+_+(-k)|^2 \left( \frac{k^-_0}{k_0} \right)^2 \left( \frac{k^+_0}{k_0} \right)^2 \]

where the daughter spinors \( u, v \) have been taken to be free, as in the \( t = 0 \) case. Equations (21.8) satisfy (21.7) if \( k_3 = k_0 \) (we expect this here), in which case the negative energy term goes to zero, as suggested by Squires.

If, on the other hand, we choose parents to be at rest, as in the \( t = 0 \) case, \( \omega(\varepsilon) = \omega_0^c(\varepsilon) = (0_\varepsilon^c) \) then we find

\[ \Sigma_s |b(ks)|^2 = |S^+_+(k)|^2 \left( \frac{k^+_0}{k_0} \right)^2 \left( \frac{k^+_0 + \mu}{2k_0} \right) \]

\[ \Sigma_s |d(ks)|^2 = |S^+_+(-k)|^2 \left( \frac{k^-_0}{k_0} \right)^2 \left( \frac{k^+_0 - \mu}{2k_0} \right) \]

which satisfies (21.7) if for example \( k_3 = 0^+ \). We expect \( S^+ \) to be cylindrically symmetric so that \( P(k_\perp) = |S^+_+(k_\perp)|^2 \) with

\[ \begin{pmatrix} B(k_\perp) \\ D(k_\perp) \end{pmatrix} = \frac{1}{2} \left( 1 + \frac{\mu}{k_0} \right) P(k_\perp) \]

i.e.: entirely analogous to the \( t = 0 \) case, eqn. (19.5), but with

\[ \dagger \text{The general requirement is } |k^+_0 S^+_+(k)|^2 = |k^-_0 S^+_+(-k)|^2. \]
Thus the effect of putting parent partons on the light cone \( x_+ = 0 \) depends upon the parent momentum. If \( \omega = \omega_+ \) then Squires' result for the bag model (see also Hughes, 1977, in this regard) also holds here: the negative energy components drop out. If, on the other hand \( \omega = \omega_0 \) then the resulting distributions are similar to the \( t = 0 \) case. \( \omega = \omega_0 \) may seem more natural here, since the parent partons' momenta automatically add up to the hadron momentum.

§22 Summary

This chapter has dealt with final state interaction effects within the parton model. In the first half we suggested that these effects should not be important in the Bjorken limit, and demonstrated this explicitly by taking quantum number exchange into account, within the context of parton models. The result is that this confinement contributes only in \( O(\frac{1}{Q^2}) \) and in a frame-dependent way. Even at this order the confinement contribution is small compared to kinematic contributions, as was shown by examining the DIS moments \( M_n(Q^2) \). We found that we can make no predictions, and cannot even guarantee scaling, unless QP are massless. Also, at least in the IMF, we have \( W_q^2 \propto Q^2 \)

In the second half of the chapter we found that, in a quantum mechanical model, confinement introduces a parton momentum due to the uncertainty principle. We found that sea partons arise naturally out of confinement, and that spin distributions can be calculated without introducing further parameters. The sea is polarized. Confinement introduces an \( x \)-dependence into spin-dependent quantities and depresses them below the SU6 predictions. Putting parent partons on the light cone need not drastically alter these conclusions.
CHAPTER 4

SPIN ROTATIONS

§23 Motivation

This chapter is in part a development of the previous discussions on polarized DIS. If in §20 we replace the two-spinor \( U_\sigma(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \) by \( U_\sigma(\eta) = \begin{pmatrix} \cos \frac{i}{2} \eta \\ -\sin \frac{i}{2} \eta \\ \sin \frac{i}{2} \eta \\ \cos \frac{i}{2} \eta \end{pmatrix} \) then it is not difficult to show that the electroproduction asymmetry \( A^H \) becomes \( A^H \cos \eta \) (cf. footnote, p. 73) and so \( \eta \) is equivalent to the "spin rotation angle" of Close (1974, 1979), with \( \cos \eta = \langle \sigma_3 \rangle \). Similarly we see that the polarised structure function \( g_1 \) becomes \( g_1 \cos \eta \) so that \( \eta \) is the "spin dilution factor" introduced by Kaur to describe the loss of quark spin to gluons (in a broken SU6 model of DIS). Both these authors discuss their angles in terms of the Melosh transformation (MT).

The MT is a unitary transformation relating constituent quarks (CQ) spinors with those of current quarks (CrQ) (Close, 1979; Hey, 1974; Bell, 1974; Ruegg, 1975) where CQ are the non-relativistic SU6 quark model building blocks, whilst CrQ can be interpreted as the extreme relativistic QP of DIS (e.g. Soper 1977). Both models are very successful in their own regimes, and yet CQ and CrQ are not the same animals, because the MT is not trivial.

The main object of this chapter is to shed some light on the origin and physical significance of the MT. It is hoped that the above paragraphs will provide sufficient justification for including this in a thesis about parton models of DIS. If not, note that one of the main phenomenological motivations for the original MT was a desire to reduce \( g_A/g_V \) from the SU6 value of \( \frac{5}{3} \), and that in Chapter 3 we showed how this reduction might be possible in a localized
rest frame parton model. This link between MT and QP model will be discussed first. Later on we will introduce the kinematic Wigner rotations and see how they relate to the MT and to rest frame parton models.

§24 Lorentz Spin Rotation

If we project the QP spin along some axis \( \hat{n} \)

\[
\langle \hat{\sigma}, \hat{n} \rangle = \frac{\phi^+(k,s) \sigma \cdot \hat{n} \phi(k,s)}{\phi^+(k,s) \phi(k,s)}
\]

(Bjorken and Drell, 1964) we find for the free particle spinors (cf. §19) that the expected value along the \( z \) direction is given by

\[
\langle \sigma_3 \rangle = \cos \theta = \frac{k_0 (k_0 + \mu) - \frac{\mu^2}{k_0}}{k_0 (k_0 + \mu)} \tag{24.1}
\]

Now in the nucleon rest frame we assume that parton momenta is distributed spherically symmetrically, so that averaging over \( k \) yields

\[
\langle \cos \theta \rangle = \frac{1}{3} + \frac{2}{3} \langle \frac{\mu}{k_0} \rangle. \tag{24.2}
\]

Thus we have \( \frac{1}{3} \leq \langle \cos \theta \rangle \leq 1 \). If \( |k| \) is nonzero the spin will be tilted away from the \( z \) direction. Recall that in Chapter 3 \(|k|\) became nonzero as a result of localization. This leads us to the following scenario:

Three parent partons at rest with their spins in the \( z \) direction are confined to form a hadron with its spin in the \( z \) direction. Confinement introduces nonzero three-momentum, which rotates the partons' spins away from the \( z \)-direction, as indicated in eqn. (24.2).
We can visualize this scenario in one of two ways:

(i) Confinement introduces pair creation (daughter partons). This increase in parton numbers\(^\dagger\) dilutes the average parton spin.

(ii) Confinement introduces a momentum parameter \(k\) that causes the (stationary) parent parton two-spinor to rotate:

\[ U_\alpha(0) + U_\alpha(\eta) \quad \text{with} \quad \eta = \eta(k) \quad \text{given in (24.1) above.} \]

The first is closer in spirit to the discussion of Chapter 3. Here \(k\) is the daughter parton momentum. In (ii) the physical interpretation of \(k\) is rather more obscure. This disadvantage is made up for by the fact that we deal directly with the parent parton two-spinors, rather than four-spinors or spin vectors; this will allow an easier comparison with the MT, to which we now turn.

§25 The Melosh Transformation

[1] Firstly a brief excursion onto the light cone. A particle is said to be on a light cone if its momentum satisfies \(|k_3| = k_0\), i.e.: it is moving in the \(\pm z\) direction at the speed of light. It is natural, then, to change variables from \(k_0, \delta\) to 

\[ k_\pm \equiv \frac{1}{\sqrt{2}}(k_0 \pm k_3) \]

so that the four-momentum \(k_\nu = (k_0; k_\perp, k_3)\) becomes \(k_\nu = (k_-; k_\perp, k_+)\). \(k_-\) corresponds to energy (Kogut and Soper, 1970). The spinor of a particle on the \(\pm\) light cone is denoted \(\phi_\pm\), so that

\[ 1/2(1 \pm a_3)\phi_\pm = \phi_\pm \]

\(^\dagger\) Recall from Chapter 3 that the average number \# of daughters is given by \(#^{-1} = \frac{<\eta>}{k_0}\) so that from (24.2) \(<\cos \eta> = \frac{1}{3} + \frac{2}{3\#}\) which decreases as \# increases.\]
where \( a \) is the spinor velocity operator (Bjorken and Drell, 1964). Thus \( \frac{1}{2}(1 \pm a_3) \) are light cone projection operators and so

\[
\frac{1}{2}(1 \pm a_3)\phi_+ = 0
\]

which means that the spinor is being projected onto the wrong side of the light cone. Our Lorentz boost convention is that of Bell (1974) (the "active" viewpoint – cf. Martin and Spearman, 1970); \( \phi_+ \) is the spinor of a particle boosted in the \(+z\) direction, so that \( k_+ = +k_3 \).

The CrQ of current algebra are light cone objects (e.g.: Bell, 1974; Adler and Dashen, 1968) as are the QP of the usual IMP parton model. It is for this reason that we make the identification CrQ = QP. The reasons for choosing the IMP have been elucidated above, for parton models, and elsewhere (Bell, Adler and Dashen) for current algebra. Indeed, according to Bell the light cone is the natural frame of reference to choose because it is "what we see" (Hey, 1974).

On the other hand, deep inelastic experiments are done in the hadron rest frame, and so it may seem desirable to formulate parton models of DIS in this frame. This has been the attitude taken in Chapters 2, 3: we look at the hadron "where its at" (Franklin, 1978).

The two formulations come together in the work of Soper (1977): "The collection of partons in a hadron is often described by giving the amplitude to find the partons in a given configuration at time \( t' = 0 \) in a reference frame in which the hadron is moving in the \( z \) direction with nearly the speed of light. As viewed from the rest frame of the hadron, this wave function tells the parton configuration as it would be determined by making local measurements on a space-time surface that is nearly the surface \( x^0 + x^3 = 0 \). Thus an
economical approach ... is to treat the coordinate \( x^+ = (x^0 + x^3)/\sqrt{2} \) as a 'time' coordinate and to describe the hadron by the amplitude for the partons to be in a given configuration at a fixed 'time' \( x^+ \)."

Thus from Soper's point of view the QP are those daughter partons of Chapter 3 that are on the light cone, so that the MT relates these light cone daughter partons to the parent partons (CQ).

We now introduce the MT and see how it is connected to the Lorentz spin rotation of §24.

[2] A free spinor for a particle with momentum \( p \) and spin in the \( z \) direction may be written as

\[
\phi(p, \sigma) = \frac{p_{\sigma} + \mu}{2\mu} \left( \begin{array}{c} U_{\sigma} \\ \sigma \cdot \vec{p} U_{\sigma} \end{array} \right)
\]  

(25.1)

where \( \vec{p} = \frac{p}{p_{\sigma} + \mu} \). The unitary matrix representing the transformation of Melosh is (Bell, 1974; Ruegg, 1975)

\[
V_{M}(k) = \left( (\mu + k_{o} + k_{3})^{2} + k_{\perp} \right)^{-\frac{1}{2}} \left( (\mu + k_{o} + k_{3}) + k_{\perp} \right).
\]

(25.2)

which affects the free-particle spinor (25.1) as follows

\[
\phi + \phi_{M} = V_{M} \phi
\]

\[
= \left( \frac{p_{\sigma} + \mu}{2\mu} \right) \left( \begin{array}{c} \left( \mu + k_{o} + k_{3} \right)^{2} + k_{\perp} \end{array} \right)^{-\frac{1}{2}} \left( \begin{array}{c} (\sigma \cdot k \cdot (p_{\sigma} + \mu)U_{\sigma} \\ -\sigma \cdot k \cdot U_{\sigma} + (\sigma \cdot \vec{p} U_{\sigma}) \end{array} \right)
\]

(\( \gamma \) matrix conventions of Bjorken and Drell, 1964).

If we write

\[
U_{\sigma}(\gamma) = \begin{bmatrix} \cos \frac{1}{2} \gamma \\ \sin \frac{1}{2} \gamma \end{bmatrix}, \begin{bmatrix} -\sin \frac{1}{2} \gamma \\ \cos \frac{1}{2} \gamma \end{bmatrix}
\]

\[
\phi_{M}^{\dagger} \phi = \frac{p_{0}}{\mu} \text{ requires } U_{\sigma}^{\dagger}(\gamma)U_{\sigma}(\gamma) = 1.
\]

\( ^{\dagger} \) We can always write two-spinors like this, since the normalization \( \phi_{M}^{\dagger} \phi = \frac{p_{0}}{\mu} \) requires \( U_{\sigma}^{\dagger}(\gamma)U_{\sigma}(\gamma) = 1 \).
for the large components of \( \Phi_M \), then we find

\[
\begin{align*}
\cos \frac{1}{2} \gamma & \rightarrow \frac{(\ )}{((\ )^2 + k_\perp^2)^{\frac{1}{2}}} \quad , \quad p_3 \rightarrow \infty \\
|\sin \frac{1}{2} \gamma| & \rightarrow \frac{k_\perp}{((\ )^2 + k_\perp^2)^{\frac{1}{2}}} \quad , \quad p_3 \rightarrow \infty 
\end{align*}
\]

(25.3)

so that the MT is just a spin rotation. Of course we require

\[
\cos^2 \frac{1}{2} \gamma + \sin^2 \frac{1}{2} \gamma = 1
\]

and we note that this is indeed the case above. However it does not hold away from the limit \( p_3 \rightarrow \infty \), as is easily seen. Also it is only in the limit \( p_3 \rightarrow \infty \), that the two-spinor of small components rotates in the same way, eqn. (25.3). This demonstrates that the MT is meaningful only in this limit (Ruegg, 1975).

Thus the MT rotates the light cone spinor

\[
\phi_+(\sigma) = \frac{p_o + \mu \frac{1}{2}}{2\mu} \left( \begin{array}{c} U_\sigma \\ \sigma_3 U_\sigma \end{array} \right)
\]

(25.4)

(compare eqn. (25.1)) to

\[
\phi_M = \frac{p_o + \mu \frac{1}{2}}{2\mu} \left( \begin{array}{c} U_\sigma(\gamma) \\ \sigma_3 U_\sigma(\gamma) \end{array} \right)
\]

The momentum parameter \( k \) has yet to be interpreted.

Note that if the third component of null-plane momentum goes to zero, \( k_3 = k_+ \rightarrow 0 \), then (25.3) reduces to

\[
\cos \frac{1}{2} \gamma + \frac{\mu}{(k_+^2 + \mu^2)^{\frac{1}{2}}} \quad , \quad k_+ \rightarrow 0
\]

Note also that if the third component of "ordinary" momentum goes to zero, \( k_3 \rightarrow 0 \), then the Lorentz spin rotation reduces to

\[
\cos \eta = \frac{\mu}{(k_+^2 + \mu^2)^{\frac{1}{2}}}
\]
from eqn. (23.1). We see that $\eta = \frac{1}{2} \gamma$ in the limit of third component of appropriate momentum going to zero. This relation is rather mysterious, since it relates a spinor rotation angle to a vector rotation angle in different limits. In order to clarify things we therefore consider the spinor rotation corresponding to the vector rotation of eqn. (23.1). This "Lorentz spinor rotation" is just half the vector rotation $^{\dagger} \cos^{\frac{1}{2}} \eta = \left(\frac{1}{2} (1 + \cos \eta)\right)^{\frac{1}{2}}$. Explicitly

$$\cos^{\frac{1}{2}} \eta = \left(\frac{2k_{o}(k_{o}+\mu) - k_{\perp}^{2}}{2k_{o}(k_{o}+\mu)}\right)^{\frac{1}{2}}.$$  \hspace{1cm} (25.5)

Comparing (25.5) with the spinor rotation induced by the MT, eqn. (25.3), we note that they are equal in the limit $k_{3} \to 0$, i.e.:

$$\eta = \gamma , \hspace{0.5cm} k_{3} = 0,$$

so that in this limit the MT is simply a light cone Lorentz spinor rotation.

The physical origin of this equality is easy to understand if we regard the MT as a Lorentz transformation. Below, (a) represents the Lorentz transformation that generates the spinor rotation of (25.5) in the limit $k_{3} \to 0$, while (b) represents the MT in this limit. They are separated only by an infinite boost in the $z$ direction, which does not alter the transverse components.

$^{\dagger}$ We note that if a two-spinor $U(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is rotated to $U(\eta) = \begin{pmatrix} \cos^{\frac{1}{2}} \eta \\ \sin \eta \end{pmatrix}$ then the corresponding spin vector is rotated from $s(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ to $s(\eta) = \begin{pmatrix} -\sin \eta \\ 0 \\ \cos \eta \end{pmatrix}$. This is true for a particle at rest (so we are utilizing interpretation (ii) here). We shall see later that this half-angle relation also holds on the light cone.
Thus it is tempting to consider the MT as a Lorentz transformation. This naturally leads us into a discussion of the Wigner rotation.

§26 The Wigner Rotation

[1] In this subsection (which is taken mostly from Weinberg, 1970) we shall derive the general expression for a Wigner rotation (WR).

The Lorentz transformation matrices $\Lambda^\mu_\nu (\Lambda^\mu_\nu \Lambda^\rho_\nu = \delta^\mu_\rho)$ are represented in spinor, vector ... space by unitary operators $U(\Lambda)$, with $U(\Lambda_1)U(\Lambda_2) = U(\Lambda_1 \Lambda_2)$. A single particle state of momentum $\mathbf{p}$ and $z$-component of spin $\sigma$ is obtained by boosting the corresponding state from rest through

$$|\mathbf{p},\sigma> = \sqrt{\frac{P_0}{\mu}} U(L(p)) |0,\sigma>$$

where $L(p)$ is the Lorentz boost and where the $\sqrt{\frac{P_0}{\mu}}$ factor gives our noncovariant (Bjorken and Drell, 1964) normalization. For spin $\frac{1}{2}$ particles $U(L(p))$ is just a Foldy-Wouthuysen (FW) transformation, when acting on rest states.

Operating on these states with a general Lorentz transformation $U(\Lambda)$ yields

$$U(\Lambda)|\mathbf{p},\sigma> = \sqrt{\frac{P_0}{\mu}} U(AL(p)) |0,\sigma>$$

$$= \sqrt{\frac{P_0}{\mu}} U(L(\Lambda p))U(L^{-1}(\Lambda p)AL(p)) |0,\sigma> .$$

The operator $R \equiv L^{-1}(\Lambda p)AL(p)$ gives the four-vector $\overline{p} = (\mu;O)$
first a three-momentum $p$, then $Ap$, and then $0$ again. It is thus a rotation, so that

$$U(A)U(L(p))|Q,\sigma> = U(L(p'))U(R)|Q,\sigma> \quad (26.1)$$

$$p' = Ap$$

Of interest to us will be the case where $A$ is simply a boost, $A = L(k)$. Thus from (26.1) we see that two Lorentz boosts can be written as a single boost and a spin rotation.

The picture is shown pictorially in Fig. (26.1) (Perl, 1974; Martin and Spearman, 1970). We now examine the spinor and vector WR, and their relation with the MT.

[2] A Lorentz boost may be written in the form (Perl, 1974; Martin and Spearman, 1970)

$$L(p) = R(0,\theta,0)L(p_3)R^{-1}(0,\theta,0) \quad (26.2)$$

with Euler angles $\alpha, \beta, \gamma = 0,\theta,0$. For simplicity we restrict the triangular sequence of boosts to the $xz$ plane, as in Fig. (26.1).

Before calculating the WR angle $\omega$ for spin $\frac{1}{2}$ particles let us note that we might expect the result to resemble the Melosh rotation angle $\gamma$ of §25.2; from (26.1) we see that a quark of momentum $p$ in a hadron at rest is given a momentum $p'$ and a spin rotation when the hadron is given a boost of momentum $k$. Choosing $k_3 = 0$ we anticipate a relation between MT and WR.

Rotation and boost of a spinor is accomplished via application of the $4 \times 4$ matrices

$$U(R(\theta)) = \begin{bmatrix} R(\frac{1}{2}\theta) & 0 \\ 0 & R(\frac{1}{2}\theta) \end{bmatrix}$$
\[ U(L(p_3)) = \begin{pmatrix} \cosh \rho & \sigma_3 \sinh \rho \\ \sigma_3 \sinh \rho & \cosh \rho \end{pmatrix} \]

respectively (see Perl, Martin and Spearman). Here

\[ R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \]

is the familiar \(2 \times 2\) rotation matrix \(d^{1\gamma}_{\sigma\sigma'}(\theta)\). Thus from (26.2)

\[ U(L(p)) = \begin{pmatrix} \cosh \rho & R(\theta) \sigma_3 \sinh \rho \\ R(\theta) \sigma_3 \sinh \rho & \cosh \rho \end{pmatrix} \]

in the \(xz\) plane. Here \(|p| = \mu \sinh \rho, \quad p_0 = \mu \cosh \rho\). Performing the calculation of eqn. (26.1) it is tedious but straightforward to derive the constraints, independent of \(WR\) angle \(\omega\),

\[ p'_\perp = p_\perp \quad (26.3) \]

\[ \mu p'_0 = k_0 p_0 + k \cdot p \]

which is just the result we obtain by boosting the four-vectors directly (see next subsection). For the rotation angle \(\omega\) we obtain

\[ \cos^2 \frac{1}{2} \omega = \frac{2(p_0 + \mu)(k_0 p_0 + k_3 p_3 + \mu^2) - (k_0 - \mu) p_\perp^2}{2(p_0 + \mu)(k_0 p_0 + k_3 p_3 + \mu^2)} \quad (26.4) \]

Of interest to us is the limit \(k_3 \to \infty\), which yields, after some algebra,

\[ \gamma = \omega \]

i.e: the Melosh rotation angle is just a \(WR\) angle of a quark on the light cone (a QP).
For the sake of completeness we look at the WR of a four-vector.

We must calculate the rotation angle \( \omega \) from

\[
L(k_3)L(p) = L(p')R(\omega)
\]

where

\[
L(k_3) = \begin{bmatrix}
\cosh \kappa & 0 & 0 & \sinh \kappa \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\sinh \kappa & 0 & 0 & \cosh \kappa
\end{bmatrix}
\]

\[
R(\omega) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \omega & 0 & -\sin \omega \\
0 & 0 & 1 & 0 \\
0 & \sin \omega & 0 & \cos \omega
\end{bmatrix}
\]

and where \( L(p) = R(\theta)L(p_3)R^{-1}(\theta) \). The result is

\[
\cos \omega = \frac{\left[(p_o+p_3)^2(p_o+\mu)^2 - p_{\perp}^2(p_o+p_3)^2\right]^{\frac{1}{2}}}{(p_p+p_3)(p_o+\mu)}
\]

in the limit \( k_3 \to \infty \). It is not difficult to show that this is consistent with the expression (26.4) for \( \cos \frac{1}{2} \omega \), i.e.: the two-spinor rotation angle is just \( \frac{1}{2} \) the four-vector rotation angle on the light cone (also at rest, as discussed earlier).

As a check note that in the limit \( p_3 \to 0 \) we have

\[
\cos \omega \rightarrow \frac{\mu}{(p_{\perp}^2+\mu^2)^{\frac{1}{2}}} , \quad p_3 \to 0
\]

as demanded by the discussion of subsections §§25.2, 26.2. Furthermore in the limit \( \mu \to 0 \) we obtain the well-known result that the WR corresponds to the rotation of the velocity vector (Gasiorowicz, 1966).
From the discussion of this chapter so far we might come to the conclusion that the MT is simply a kinematic spin rotation of moving quarks: localizing the constituents inside a hadron introduces a momentum, and boosting the hadron to infinite momentum leads to a Wigner rotated quark spin. Identifying the constituents in the hadron at rest as $CQ$ and those in the light cone hadron as $CrQ$ or $QP$ means that $WR = MT$.

The fact that $<\sigma>_3 < 1$ need not be associated with the MT. It may be regarded as a consequence of confinement (cf. §24).

A slight flaw develops if we take seriously the suggestion made earlier that the MT is simply a Lorentz transformation. Bell (1974) shows explicitly that the MT is "...essentially the old FW transformation specialized to good components", and we know that the FW transformation is a unitary Lorentz boost (when applied to spinors at rest). So what is the flaw?

If we treat the MT as a Lorentz transformation then we see from (26.1) that

$$V_M(k,\phi_+(\sigma)) = \sqrt{\frac{p_o}{p_o'}} \phi_W(p',\sigma)$$

(26.5)

$\phi_+$ is the light cone spinor of eqn. (26.3), here subjected to a MT. Substituting from eqn. (25.2) we find

$$\frac{1}{((u+k_0+k_3)^2 + k_2)^{\frac{1}{2}}} \begin{bmatrix} u+k_0+k_3 + k_3 U_G \\
\sigma_3 \end{bmatrix} = \begin{bmatrix} \frac{p_o'+u}{p_o'} \\\n\sigma \end{bmatrix} \begin{bmatrix} U_\sigma(\omega) \\
\sigma U_\sigma(\omega) \end{bmatrix}$$

so that $WR = MT$ only in the limit $p_3' \to \infty$. Thus the spinors on both

$\frac{1}{2}(1+\alpha_3)$. 

A good component is one that survives the light cone projection
sides of eqn. (26.5) are on the light cone, and because the MT relates CrQ to CQ we arrive at the conclusion:

The MT is a WR only in the limit of infinite momentum CQ. It seems intuitively undesirable to give CQ an infinite momentum, because of the success of rest frame SU(6) (Close, 1979; Hey, 1974) although perhaps we may want CQ to have nonzero momentum (in fact this is what initially led to SU6W, constituent. Hey discusses this in some detail).

§27 Conclusions

In Chapter 3 we found it desirable to reduce the average QP spin <σ₃> from 1. This was achieved here in eqn. (24.2) via a Lorentz spin rotation, i.e.: the rotation undergone by a spin vector due to a Lorentz boost. We attribute this boost momentum to confinement.

This Lorentz spin rotation was found to be a limiting case of the MT, which led us to consider the MT as a Lorentz transformation. Indeed we found that the Melosh rotation angle equals the WR angle on the light cone. However, the two transformations themselves can only be considered equivalent if CQ are on the light cone. If CQ are nonrelativistic (or at rest, as in Chapter 3) then there is more to the MT than relativistic kinematics.
In this chapter we shall progress from the naive intuitive parton models of Chapters 1-4 by looking at the consequences that an underlying quantum field theory of hadrons has for the structure functions of DIS. We shall briefly review the important work of Kogut and Susskind (1974), and present the philosophy that we adopt. Consequences of this philosophy will be investigated in later sections. Up to this point the discussion will have been very general, and will have shown that different underlying field theories alter the intuitive parton model results in different ways. Inevitably, though, we shall focus on a particular field theory: quantum chromodynamics. This will be examined in some detail. Finally we will compare the DIS predictions of QCD with those of our intuitive model of Chapter 2. The result is that the data slightly favours QCD, and that the so-called "dynamical-higher-twist" contribution to structure function moments is small, experimentally.

As a necessary prerequisite to a discussion of field theory in this context, we introduce the notion of an effective coupling constant.

§28 Effective Coupling Constant

This is a more accurate description than the popular "running coupling constant" which is self-contradictory. The essence is given by a classical analogue (Llewellyn-Smith, 1978): that of an interaction between two charges in a dielectric. The potential is

\[ V(r) = \frac{Q_1 Q_2}{4\pi \varepsilon r} \]

where \( \varepsilon > \varepsilon_0 \) due to polarization of the intervening dielectric.
molecules, which shield the charges. For $r$ much less than the molecular spacing $d$ there is no shielding, so that $\varepsilon \rightarrow \varepsilon_0$. Define the effective charge as

$$Q(\frac{r}{d}) \equiv Q \sqrt{\frac{\varepsilon_0}{\varepsilon(\frac{r}{d})}}$$

which tends to $Q$ for $r \ll d$ and tends to a constant (less than $Q$) as $r \rightarrow \infty$. In terms of $Q$ the potential is given by

$$V(r) = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$$

which has the same form as if there were no dielectric present.

The situation is similar in field theory. We can use either a fixed coupling constant defined at some given momentum transfer ($Q^2 = 0$ for QED) which will be shielded for different $Q^2$ by vacuum polarization, or we can calculate an effective coupling constant in terms of which the interactions are simpler.

Next consider the above interaction in QED. The first order amplitude for the Coulomb scattering of an electron (Fig. (28.1)) is (Bjorken and Drell, 1964)

$$A_1(e^2,Q^2) = -i \frac{e^2}{Q^2} \bar{u} \gamma_\mu u$$

The second order correction to this amplitude, due to vacuum polarization, is shown in Fig. (28.2). Calculating Fig. (28.2) (Bjorken and Drell) and adding to $A_1$ yields

$$A_2(e^2_R,Q^2) = -i \frac{e^2_R}{Q^2} \bar{u} \gamma_\mu u (1 + \frac{a}{3\pi} \ln \frac{Q^2}{m^2}), \quad Q^2 \gg m^2$$

where $e^2_R$ is the renormalized electron charge. In the limit $Q^2 \rightarrow 0$
this is
\[ e_R^2 = e^2 \left( 1 - \frac{\alpha}{3\pi} \ln \frac{\Lambda^2}{m^2} \right) + O(\alpha^2) \]
where \( \Lambda \) is the loop momentum cutoff.

We can define the QED effective coupling constant by absorbing the vacuum polarization into Fig. (28.1), i.e.:
\[ A_1(e^2(Q^2), Q^2) \equiv A_2(e^2_R, Q^2) \quad (28.1) \]
so that
\[ e^2(Q^2) = e_R^2 \left( 1 + \frac{\alpha}{3\pi} \ln \frac{Q^2}{m^2} \right) + O(\alpha^2) \quad (28.2) \]

The analogy with the classical case is made more clear if we look at the effect of vacuum polarization upon the electron charge distribution, Fig. (28.3). We see that the bare charge \( e \) is shielded so that a second electron at infinity feels a renormalized charge \( \int_0^\infty \rho(r) d^3r = e_R \), where \( \rho(r) \) is the electron charge density (Bjorken and Drell, 1964).

At some finite distance \( R \) this second electron feels a stronger effective charge \( \int_0^R \rho(r) d^3r = \bar{e}(R) \). Thus the effective charge increases as \( R \) decreases (as in the classical case) or as \( q \) increases† (eqn. (28.2)).

The above discussion leads us to make the following statement: a "bare" diagram (Fig. 28.1) becomes "dressed" (Fig. 28.2) because of higher order interactions. These interactions may be accounted for by replacing the fixed coupling constant of the original bare diagram with a calculable effective coupling constant, which depends upon momentum transfer.

† At truly astronomical values of momentum transfer, \( \bar{e} \) starts decreasing. From Fig. (28.3) we see that this occurs when \( Q^2 > \Lambda^2 \). This means that the \( O(\alpha) \) approximation of eqn. (28.2) breaks down. The next generation of theorists need not worry, however; eqn. (28.2) will hold up to \( Q^2 \approx 10^{56} m^2 \).
For the sake of completeness we note here that the effective coupling constant can be calculated for any renormalizable field theory by using the so-called renormalization group equation (Llewellyn-Smith, 1978, and references therein). This equation is derived from the requirement that any physical theory must be independent of the renormalization point (the value of $q$ at which the renormalized coupling constant is defined). See Nash (1978), Callan (1976). All the leading logarithm terms are summed (not just the first term, as in eqn. (28.2)) and yield

$$e^2(Q^2) = \frac{e_R^2}{1 - \frac{\alpha}{3\pi} \ln \frac{Q^2}{m^2}} + O(\alpha^2) \quad (28.3)$$

which agrees with (28.2) to $O(\alpha)$.

What are the physical consequences of eqn. (28.2) or (28.3)? A photon with momentum $q$ impinges upon an electron. The fact that the effective photon-electron coupling increases with the photon momentum means that a high momentum photon will see self-interactions of the electron that a low momentum photon is unable to resolve, as suggested by Fig. (28.4). The higher its momentum, the more interactions the photon will see. QED is thus a "hard" theory in the sense that there is no length or mass scale beyond which interactions can be ignored.

Now we shall briefly consider a "soft" theory. In super-renormalizable field theories such as $g^3$ the effective coupling constant $\bar{g}(Q^2)$ decreases as $Q^2$ increases, so that interactions get switched off at high momentum. We can understand this from the following example. In Fig. (28.5) we show the $g^3$ analogue of Coulomb scattering. Some $O(g^3)$ and $O(g^5)$ corrections to this process are shown in Fig. (28.6). Of these only the first of Fig. (28.6)
diverges (logarithmically). In fact this is the only source of divergence in the theory. Hence there are less renormalization constants in $g\phi^3$ than in "just" renormalizable theories such as QED. This is why $g\phi^3$ is termed "super-renormalizable".

The reason for super-renormalizability is easy to trace (e.g. Nash, 1978): the coupling constant $g$ must have units of mass. Because of this the $O(g^A)$ diagrams of $g\phi^3$ must have $A-B$ more powers of momentum (including external momentum $q$) in the denominator of the corresponding Feynman integral than do diagrams of $O(g^B)$, $B < A$. Thus as $q$ increases the higher order diagrams get suppressed, unlike QED. Changing over to the effective coupling constant point of view $g\phi^3$ Coulomb scattering, Fig. (28.7), we expect $\bar{g}(Q^2)$ to decrease with increasing $Q^2$. In fact the renormalization group equation tells us that $\bar{g}(Q^2) \sim Q^{-\beta}$, $\beta > 0$.

The physical consequence of this decreasing effective coupling constant is that a probe of ever-increasing momentum sees less and less structure until, at truly asymptotic momenta, there are no interactions at all and only free "bare" particles survive. The theory is "soft".

We might anticipate from the above paragraph that parton models correspond to soft or super-renormalisable field theories. This is in fact the case, as will be demonstrated later on.

Let us finish this section by mentioning fixed point and asymptotically free field theories. In the first of these the effective coupling constant tends to a constant $\bar{g}(Q^2) \rightarrow g^*$ as $Q^2 \rightarrow \infty$. Thus fixed point theories become scale invariant: the probe of momentum $q_1$ sees the same density of interactions as the probe of momentum $q_2$, $q_{1,2}$ large. This does not mean that fixed point theories exhibit Bjorken scaling in DIS, despite the discussion of §1, as will become apparent in later sections.
In asymptotically free theories the effective coupling constant behaves as (e.g.: Llewellyn-Smith, 1978) \( \bar{g}(Q^2) \sim \frac{c}{\ln Q^2} \) as \( Q^2 \to \infty \). Thus for asymptotic momentum transfer the interactions get switched off slowly. This does not mean that asymptotically free theories behave like super-renormalizable theories at large \( Q^2 \), however. We shall see that, because the switch-off is so slow, interactions can never be ignored. The significance of asymptotically free theories to DIS is, of course, the fact that today's major contender for True Theory of strong interactions is QCD, an asymptotically free gauge theory. We shall return to QCD in §34.

§29 Scale Invariant Parton Model

In the last section we saw how different field theories evolve as the length scale (or probe momentum) is changed. Here we shall see how this evolution affects our intuitive parton model DIS predictions. We proceed by reviewing the 1974 papers of Kogut and Susskind, quoted in the references.

Kogut and Susskind (KS) begin by assuming that nature is organized into discrete clusters, Fig. (29.1): nuclei, nucleons, quarks, subquarks, ..., labelled by an index \( N \). In the limit of large \( N \) the length scales to be associated with these clusters is hypothesized to be independent of \( N \): \( \frac{R_N}{R_{N+1}} \equiv \Lambda \gg 1 \). Given this assumption it is not difficult to motivate the following expression for the structure function \( F_2(x,N+1) \) of clusters of type \( N+1 \) having longitudinal momentum fraction \( x \) (in the IMP)

\[
F_2(x,N+1) = \int_x^1 \frac{dy}{y} f_{N+1,N} \left( \frac{x}{y} \right) F_2(y,N) \tag{29.1}
\]

where \( f_{N+1,N} \left( \frac{x}{y} \right) \) is the probability to find a cluster of type \( N+1 \)
and longitudinal fraction $x$ in a cluster of type $N$ and longitudinal fraction $y$.

Using Laplace transforms KS show that the moments $M_{\alpha+1}(N) = \int_0^1 \frac{dx}{x} x^\alpha F_2(x,N)$ satisfy $M_{\alpha}(N+1) = m_{\alpha} M_{\alpha}(N)$ where

$m_{\alpha+1} = \int_0^1 \frac{dy}{y} y^\alpha f_{N+1,N}(y)$. For fixed point (FP) theories the distributions $f_{N+1,N}(y)$ become independent of $N$ for large $N$, so that

$$M_{\alpha}(N) = (m_{\alpha})^N M_{\alpha}(0) \quad (29.2)$$

Now it is because of the discrete nature of the clusters that the parton idea may be applied. A $\gamma^*$ probing a hadron will have wavelength $\lambda \sim \frac{1}{Q}$, in IMF. For $\lambda \sim R_N$ the $\gamma^*$ will be able to resolve clusters of type $N$ into those of type $N+1$, but the latter will appear point-like; in other words the $\gamma^*$ will see the clusters as being made up of partons described by a probability distribution $f_{N+1,N}(x)$. Now

$$\frac{R_\alpha}{R_N} = \sqrt{\frac{Q^2}{Q_0^2}} \sim \Lambda^N \quad \text{so that} \quad N \sim \frac{\ln(Q^2/Q_0^2)}{\ln \Lambda^2}.$$ Thus the limit of large $N$ corresponds to the DIS limit $Q^2 \rightarrow \infty$. Substituting into (29.2) we see that

$$M_{\alpha}(Q^2) \sim \left(\frac{Q^2}{Q_0^2}\right)^{-d_{\alpha}} \quad (29.3)$$

where $d_{\alpha} = -\frac{\ln m_{\alpha}}{\ln \Lambda^2} > 0$, so that if the underlying field theory of hadrons is FP then the moments of the DIS structure functions should decrease as some power of $Q^2$. Furthermore because $f(y)$ is a probability we see that $m_2 = 1$ so that $M_2$ must be independent of $Q^2$, i.e.: the area under $F_2(x,Q^2)$ is constant. Putting these two facts together we see that the structure function must shift towards $x = 0$ as $Q^2 \rightarrow \infty$, in the manner suggested by, for instance, Fig. (32.1) below, and as reflected in the data, Fig. (13.2).

In their second paper KS show that the result (29.3) is modified
for asymptotically free (AF) theories. The $Q^2$ dependence of the moments is softened to $\ln Q^2$. (We might have anticipated this: recall that $g(Q^2)$ decreases as a power for FP and as a $\ln$ for AF). Also, from particular assumptions about $f_{N+1,N}(y)$ they find that
de $\alpha = \text{constant}$ as $\alpha \to \infty$ for nongauge field theories whereas
d $\alpha = \text{constant}.\ln \alpha$ for gauge theories.

Thus the KS approach has shown us the signatures that different underlying field theories have for the moments of DIS structure functions. This information, which will be used in future sections, is summarized in Table (29.1).

**TABLE (29.1)**

<table>
<thead>
<tr>
<th>Field theory</th>
<th>Behaviour of $M_\alpha(Q^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nongauge FP</td>
<td>$(Q^2)^{-c}$</td>
</tr>
<tr>
<td>Gauge FP</td>
<td>$(Q^2)^{-c.\ln \alpha}$</td>
</tr>
<tr>
<td>Nongauge AF</td>
<td>$(\ln Q^2)^{-c}$</td>
</tr>
<tr>
<td>Gauge AF</td>
<td>$(\ln Q^2)^{-c.\ln \alpha}$</td>
</tr>
</tbody>
</table>

The main objection to the KS approach is the initial assumption about discrete clusters: this is certainly untrue in QCD, for instance, or for any field theory of QP. Here there is a continuous resolution by the probe of a quark into quark plus gluon, etc. (Llewellyn-Smith, 1978; Ellis, 1976). In fact KS discuss this and write down the continuous version of eqn. (29.1):

$\dagger$ For completeness we shall simply quote a result obtained by Llewellyn-Smith (1977): scaling is broken by powers of $\ln Q^2$ for spin $\frac{1}{2}$ theories; these sum to powers of $\ln Q^2$ in AF theories and to powers of $Q^2$ in other theories. See also Drell and Yan, 1971, and references therein.
\[ \frac{\partial F_2(x, \tau)}{\partial \tau} = \int \frac{dy}{y} F_2(y, \tau) f\left(\frac{x}{y}\right), \quad \tau = \ln Q^2 \]

which in fact is very similar to the Altarelli-Parisi evolution equation obtained from QCD (see, for example, Close, 1979). KS feel that their "discrete" approach is not invalidated, though perhaps the application of parton ideas is more dubious in the continuous approach.

We will pursue this matter further in §§31-2 below. The attitude we adopt will be slightly different to that of KS presented here, and will be based upon our simple underlying philosophy, to which we now turn.

§30 Philosophy

The philosophy that we shall adopt over the next few sections for our investigation into the consequences for DIS structure functions of an underlying field theory of hadrons is that of Ellis (1976) and Llewellyn-Smith (1978):

(i) Assume that hadrons are made up of constituents that are governed by some quantum field theory.

(ii) The uncertainty principle implies that resolving power increases as momentum increases. Thus a \( \gamma^* \) probing a hadron will see a bare constituent at some momentum \( Q_0 \), with more and more structure being exposed as \( Q \) increases (Fig. (28.4)).

(iii) In the parton model framework we can say from (ii) that the constituent probability distribution decreases at large \( x \) and increases at small \( x \), as \( Q \) increases.

Thus if the underlying theory of hadrons is a quantum field theory, we expect the DIS structure functions to shift towards small \( x \) as \( Q^2 \) increases. Exactly how fast this shift occurs depends upon the particular
theory, as we shall see.

This is the intuitive approach we shall adopt, with the addition

(iv) From (iii) we see that $<x>$ is a decreasing function of $Q^2$.

It is therefore reasonable for us to assume that the average number $#$ of QP constituents increases with $Q^2$. (See Fig. (28.4)).

By assuming merely that there is a nontrivial underlying field theory, we are differing from the naive parton model point of view, Chapters 1-4, as should be apparent from (iv). We shall now look at consequences of our new viewpoint for different types of parton distribution.

§31 Composite Constituents

[1] The QP distributions are now considered to be functions of $Q^2$ as well as $x$, because of §30. We wish to examine some of the different forms that $f(x,Q^2)$ can take, and so in this section we shall initially assume that $f(x,Q^2)$ can be factorized:

$$f(x,Q^2) = g(x) h(Q^2) \quad (31.1)$$

This is in the spirit of a parton model with composite QP. The underlying field theory describes the QP constituents, whereas the QP themselves are free within the hadron. From the discussion of §3 we see that $h(Q^2)$ represents the QP form factors. Close (1979) also views the factorizable distribution in this light.

Our attitude here is reminiscent of the KS "cluster" approach (see also Hwa et al., 1981) but it is not quite the same; in fact we shall see that eqn. (31.1) is suitable only if the underlying theory is nongauge.

Let us further assume that
\[ g(x) \rightarrow \frac{c_0}{x^{1+a}}, \quad x \rightarrow 0. \]

Experimentally \( F_2(x, Q^2) \) is a constant, or is possibly an increasing function of \( x \) near \( x = 0 \) (c.f. Fig. (13.2)) and so we expect \( a \geq 0 \).

We will also assume \( g(x) \rightarrow c_1 (1-x)^b \) as \( x \rightarrow 1 \) though this is not important since the \( x \rightarrow 1 \) behaviour is not significant to this discussion, as will become clear.

The normalization condition is
\[ \int_{x_0}^{x_1} dx f(x, Q^2) = 1 \quad (31.2) \]

Drell and Yan (1971) have shown that the usual IMF parton model ideas are applicable to DIS only if \( x_0 \gg \frac{\Lambda^2}{Q^2} \), where \( \Lambda \) is some hadronic mass scale. Obviously we also require \( x_0 < < \langle x \rangle \), where \( \langle x \rangle \) is a decreasing function of \( Q^2 \), from §30. Thus we conclude that \( x_0 \) is a decreasing function of \( Q^2 \), but is not equal to zero at finite \( Q^2 \). This will have important consequences for the QP form factors.

We write \( x_1 = 1 - \gamma \) where from momentum conservation \( \gamma \) is easily shown to be \( \gamma = \# x_o = \frac{(1-c)}{<x>} x_o \).

Substituting (31.1) and differentiating we see that
\[ \frac{d}{dQ^2} \left( \frac{1}{h(Q^2)} \right) = \left( \frac{dx_1}{dQ^2} \right) c_1 (1-x_1)^b - \left( \frac{dx_0}{dQ^2} \right) \frac{c_o}{x_0^{1+a}}. \]

The \( c_o \) term dominates the right hand side. The equation is readily solved to yield for the QP form factors
\[ h(Q^2) = \frac{a}{c_o} x_o^{a}. \]

We will see below that \( a \gtrsim 12 \).
Performing the above trick of differentiating and then reintegrating on the equation

\[ \langle x^n \rangle = \int_{x_o}^{x_1} dx^n f(x, Q^2) \]

we find

\[ \langle x^n \rangle = \left[ K_n - c_1 I_{bn}(\gamma) - c_0 \frac{x^n_{o-a}}{a-n} \right] h(Q^2), \quad n \neq a \]

where \( K_n \) is the integration constant and where \( I_{bn} = \int d\gamma y^b(1-\gamma)^n \) is expected to be small. For \( n < a \) the \( c_0 \) term dominates (because \( x^n_{o-a} \) is an increasing function of \( Q^2 \)) and so we find for the moments of the structure function \( F_2 \)

\[ M_{n+1}(Q^2) = \sum_i e_i^2 \langle x^n \rangle \]

\[ = \frac{(1-\varepsilon)}{\langle x \rangle} e^2 \frac{e_{\langle x \rangle}}{a-n} x^n_o, \quad n < a. \quad (31.3a) \]

Here we have used \( \sum_i e_i^2 = \# e^2 \) where \( e^2 \) is the average QP charge squared. The average number \( \# \) of QP is related to \( \langle x \rangle \) through momentum conservation: \( \#<x> = (1-\varepsilon) \) (cf. §9).

For \( n > a \) we expect the \( K_n \) term to win, yielding

\[ M_{n+1}(Q^2) = \frac{(1-\varepsilon)}{\langle x \rangle} e^2 \frac{e_{\langle x \rangle}}{c_0} x^n_o K_n, \quad n > a. \quad (31.3b) \]

Firstly notice that in the limit \( n \to \infty \) the \( Q^2 \) behaviour of the moments is independent of \( n \). Recall from §29 that this is the signature of nongauge theories. Hence the field theory underlying the factorizable QP distribution must be nongauge.

From Chapter 2 we know that \( M_n(Q^2) \) decreases faster and faster with \( Q^2 \) as \( n \) gets larger, at least up to \( n = 12. \) This is compatible with eqn. (31.3b) only if \( a \lesssim 12. \)
We see that the structure function is given by

\[
F_2(x,Q^2) = \frac{(1 - \varepsilon)}{\langle x \rangle} \frac{e^2}{c_0} \frac{a}{x_o^a} x g(x)
\]

which decreases with \(Q^2\) except near \(x = x_o\) where it increases, as shown schematically in Fig. (31.1). This agrees qualitatively with the data, Fig. (13.2).

Note however that \(\# = \frac{1 - \varepsilon}{\langle x \rangle} \approx 3, \frac{e^2}{c_0} \approx \frac{2}{a}, a \approx 12\) implies \(F_2(x_o,Q^2) \approx 8\) which is not in agreement with experiment. This means that our factorisability assumption is excluded by experiment, at least presently available \(Q^2\), and so we must modify eqn. (31.1).

[2] In place of eqn. (31.1) we write

\[
f(x,Q^2) = g(x)h(Q^2) + f(x,Q^2)
\]

where \(f\) is nonfactorizable but otherwise arbitrary. Normalizing \(f(x,Q^2)\) as before yields

\[
h(Q^2) = \frac{a}{c_0} x_o^a (1 - f)
\]

where \(f = \int_{x_0}^{x_1} dx f(x,Q^2)\). If \(h(Q^2)\) represents QP form factors then \(h \geq 0\) so that \(0 \leq f \leq 1\). Proceeding as before we find for the moments

\[
M_{n+1}(Q^2) = \frac{(1 - \varepsilon)}{\langle x \rangle} \frac{e^2}{a-n} \left[ \frac{a}{a-n} (1 - f) x_o^n + \frac{\nabla}{\nabla} \right], \quad n < a
\]

\[
= \frac{(1 - \varepsilon)}{\langle x \rangle} \frac{e^2}{a-n} \left[ K \frac{a}{c_0} x_o^a (1 - f) + \frac{\nabla}{\nabla} \right], \quad n > a
\]

where \(\nabla_{x^n} = \int_{x_0}^{x_1} dx x^n f(x,Q^2)\). If the first term wins, the theory is nongauge, \(x_o\) as before. Otherwise it is unspecified until we say
something about $f(x,Q^2)$. The structure function itself is given by

$$F_2(x,Q^2) = \frac{(1 - e)}{<\alpha>} e^2 \left[ x g(x) \frac{a}{c_0} \chi^a (1 - \bar{f}) + f(x,Q^2) \right]$$

$$= \frac{(1 - e)}{<\alpha>} e^2 \left[ a(1 - f) + f(x_0,Q^2) \right], \quad x \to x_0.$$

There is no longer any disagreement with data, near $x = x_0$.

We have interpreted $g(x)$ as the probability for finding a composite QP with momentum fraction $x$ inside a hadron, and $h(Q^2)$ as the form factors (squared) for this QP. What interpretation, if any, can we place upon $f(x,Q^2)$? It seems natural to think of the extra term as arising from interactions (interactions generate extra QP which will contribute an extra piece to the probability distribution $f(x,Q^2)$). We might imagine that at some low momentum scale $Q^2_0$ the form factor term dominates, so that the hadron consists of just three free composite valence QP described by the distribution (31.1). (these are naturally interpreted as the "parent partons" of Chapter 3 or the "valons" of Hwa et al.). At some larger momentum scale $Q^2_1$ the form factors have died, leaving just the interaction term: $f(x,Q^2) = f(x,Q^2)$. Hence, in this framework, we have the normalization $f$: $0 \to 1$ as $Q^2:Q^2_0 \to Q^2_1$, i.e.: the QP probability distribution evolves in $Q^2$ from $f(x,Q^2_0) = g(x) h(Q^2_0)$ to $f(x,Q^2_1) = f(x,Q^2_1)$. We might now write for the interaction piece

$$f(x,Q^2) = \sim \sim \sim \sim \sim \sim$$

where the first term dominates at $Q^2 \sim Q^2_1$, the second term wins for $Q^2 \sim Q^2_2 \gg Q^2_1$ etc. $h(Q^2)$ represents the sub-QP form factors. In this way we uncover more and more structure as $Q^2$ increases.

This point of view again recalls the cluster model of KS. It is
also reminiscent of Chapter 3, where we had "composite" parent/valence partons giving rise to daughter/sea partons.

If the above interpretation of $\hat{\gamma}$ is taken, then we expect the nonsinglet moments of the structure function at momentum scale $Q^2_{0,1}$ ... to be given by eqns. (31.3), since the interaction (sea) term $\hat{\gamma}$ is absent for these moments. Hence in this interpretation the distributions (31.4), as well as (31.1), is appropriate for nongauge theories.

§32 Bjorken and Paschos Amended

[1] In the last section we chose a QP distribution and applied our philosophy to obtain information about the underlying field theory. Here we shall see how the assumptions of §30 affect a well-known intuitive parton model. In the Bjorken and Paschos (1969) model the structure function is

$$F_2(x) = \sum_{N=3, \text{odd}} P(N) \sum_{i=1}^{N} e_i^2 x f_N(x)$$

(32.1)

where $\Sigma$ is a sum over parton configurations; $P(N)$ is the probability of finding a configuration of $N$ partons in the target hadron, and is chosen to be $\frac{C}{N(N-1)}$, $C^{-1} = 1 - \ln 2$, so that $F_2 \rightarrow$ constant as $x \rightarrow 0$; $\Sigma e_i^2 = \frac{2}{3}N$ for the neutron and $\frac{2}{3}N + \frac{1}{3}$ for the proton; $f_N(x)$ is the probability for finding a parton with momentum fraction $x$ in a configuration of $N$ partons, and can be written

$$f_N(x) = \int_0^1 dx_2 \ldots dx_N \frac{F_N(x_1 \ldots x_N)}{\delta(1 - \sum_{i=1}^{N} x_i)}.$$

† i.e. nonsinglet in flavour, e.g.: $F_3$, $F_2^{p-n}$. These do not involve the $\bar{q}q$ sea.
For large \( N \) we expect that the distribution will not be greatly altered by adding an extra parton-antiparton pair, i.e. \( f_N \) (the probability for finding a configuration of \( N \) partons with momentum fractions \( x_1 \ldots x_N \)) becomes independent of \( N \) for large \( N \) (cf. KS) and so must be a constant. This yields \( f_N(x) = (N-1)(1-x)^{N-2} \).

Because of the simplicity of its derivation we shall refer to this as the "natural" behaviour of \( f_N(x) \).

Note that the Bjorken and Paschos model has no composite partons; all the constituents are free, pointlike QP. Their number is not fixed. If we now switch on the underlying field theory we expect their average number \( \langle N \rangle = \# \) to increase as \( Q^2 \) increases (point (iv) of §30). Say \( N_{\text{min}} \equiv n \) is an increasing function of \( Q^2 \).

Then the Bjorken and Paschos structure function (32.1) is generalized to

\[
F_2(x,n) = \sum_{N=n, \text{odd}}^{\infty} P(N,n) \frac{2}{2N} + \frac{1}{3} x f_N(x)
\]

for the proton, where \( P(N,n) = \frac{C(n)}{N(N-1)}, \quad \left[ C(n) \right]^{-1} = \sum_{N=n, \text{odd}}^{\infty} \frac{1}{N(N-1)} \).

Explicitly

\[
F_2^P(x,3) = \frac{1}{1-\ln 2} \left\{ \frac{2}{9} \left( \frac{1-x}{2-x} \right) + \frac{1}{6} \frac{x}{(1-x)^2} \left[ \ln \left( \frac{2-x}{x} \right) - 2(1-x) \right] \right\}
\]

\[
F_2^P(x,5) = \frac{1}{5-\ln 2} \left\{ \frac{2}{9} \left( \frac{1-x}{2-x} \right)^3 + \frac{1}{6} \frac{x}{(1-x)^2} \left[ \ln \left( \frac{2-x}{x} \right) - 2(1-x) - \frac{2}{3}(1-x)^3 \right] \right\}
\]

are plotted in Fig. (32.1). Note the expected shift to lower \( x \) as \( n \) (i.e.: \( Q^2 \)) increases. For neutrons the square brackets are zero.

It is easy to see that \( F_2^n + F_2^P \) as \( n \to \infty \).
[2] The moments \( M_{\alpha+2}(n) = \int_0^1 dx x^\alpha F_2(x,n) \) are of course constant in the original Bjorken and Paschos model. Now however they become functions of \( Q^2 \) through \( n \). We calculate

\[
M_{\alpha+2}(n) = \frac{2}{9} \frac{1}{2} C(n) \cdot B(\alpha+1,n-1) + O(\frac{1}{n})
\]

where \( B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \) is the Euler \( \beta \) function. We shall work in the limit of large \( n \) and so shall neglect the extra proton term in \( F_2 \). It is not difficult to show that, in this limit, \( C(n) \to 2n \). Then, with \( B(x,y) \to x^{-y}\Gamma(y) \) as \( x \to \infty \), we can show that

\[
M_{\alpha+2}(n) \to \frac{2}{9} n(\alpha+1)^{-(n-1)}\Gamma(n-1), \quad \alpha \to \infty.
\]  

(32.3)

In (32.3) we are thus considering the limit of large \( Q^2 \) (large \( n \)) and large \( \alpha \). Now if we define \( n-1 \equiv \ln f(Q^2) \) then we have

\[
M_{\alpha+2}(n) \to \frac{2}{9} \ln f(Q^2) \Gamma(\ln f(Q^2)) \left[ f(Q^2) \right]^{-\ln n} \alpha
\]

(32.4)

From §29 we recall that such an \( \alpha \) dependence is characteristic of gauge theories. Thus if the Bjorken and Paschos form (32.1) and the assumptions of §30 are both to be satisfied, then the underlying quantum field theory of hadrons is a gauge theory.

Also from §29, and equation (32.4), we note that if this underlying theory is to be FF then we require \( f(Q^2) \sim Q^2 \), whereas if it is AF then \( f(Q^2) \sim \ln Q^2 \).

If we keep to the limit of large \( n \) but now consider small \( \alpha \), we can replace eqn. (32.3) with the form

\[
M_{\alpha+2}(n) \to \frac{2}{9} \frac{\alpha!}{n} \alpha^n
\]

Thus the amended Bjorken and Paschos moments decrease with \( Q^2 \), and
decrease faster for larger $\alpha$. We have seen in Chapter 2 that this is reflected in the data. Note $M_2 = \frac{2}{9} = \text{constant à la KS.}$

The example of this section serves to show that, by postulating merely that there is an underlying field theory (§30), the structure function and moments are altered from the intuitive parton model result in a well-defined way.

§33 The Natural Distribution

In §32 we suggested that the "natural" behaviour for the QP distribution is

$$f(x, Q^2) = (N-1)(1-x)^{N-2}$$  \hspace{1cm} (33.1)

with $N = N(Q^2)$ an increasing function of $Q^2$. It is illuminating to examine (33.1) in more detail, independent of the Bjorken and Paschos model. For this distribution the expected momentum fraction is $<x> = \frac{1}{N}$. We shall see below that $N$ is related to the average number of QP, $\#$. (It need not have the same interpretation as in §32.)

From the parton model master formula of Chapter 1 we may write for the DIS structure function

$$F_2(x, Q^2) = \frac{(1-\varepsilon)}{<x>} e^2 x f(x, Q^2)$$ \hspace{1cm} (33.2)

(cf. also §§9, 31). This is the general expression for $F_2$ taking into account the underlying quantum field theory of hadrons within the parton model. With the distribution (33.1) we find for the moments

$$M_\alpha(Q^2) = (1 - \varepsilon) e^2 N(N-1)B(a, N-1) .$$

Now we have $M_2(Q^2) = (1 - \varepsilon) e^2$ and so is not a constant here, in contrast to the KS conclusion (§29) unless $\varepsilon = \text{constant}$. Thus in
the limit of very large $Q^2$, where kinematic effects (target mass, etc.) may be ignored, we suggest that any deviation of $M_2$ from a constant value is due to $Q^2$ dependence of the gluon average momentum fraction $\epsilon$.

We now follow the last section by writing $N-1 = \ln f(Q^2)$ and making use of the asymptotic form of $B(\alpha, N-1)$ to obtain

$$M_2(Q^2) \approx (1-\epsilon)e^{2(1+\ln f)}\ln f \Gamma(\ln f)f^{-\ln \alpha}$$

and so the natural distribution (33.1) leads to the same asymptotic behaviour as gauge theories (§29). Choosing $f(Q^2) = Q^2/Q_0^2$ gives us a FP gauge theory whereas $f(Q^2) = \ln(Q^2/Q_0^2)$ yields an AF gauge theory.

We may give a physical interpretation to $f(Q^2)$ by noting that, for the scale invariant FP theories, we expect $\epsilon$ to be a constant so that from $\langle x \rangle = 1-\epsilon$ we have $\langle x \rangle \propto 1/\#$. But $\langle x \rangle = 1/N$ so that $\# \propto \ln f$. Thus for FP gauge theories we have the following result for the number of QP resolved by a probe of momentum $Q$:

$$\# \propto \ln Q^2$$

whereas for AF gauge theories

$$\# \propto \ln \ln Q^2$$

Note that for super-renormalizable field theories, where interactions drop out at large $Q^2$ we expect $\epsilon \to 0$ and $\# \to$ constant, so that $\langle x \rangle = \text{constant}$. From the above discussion this implies that $F_2$ scales exactly, as is easily shown. This illustrates the comment made earlier that naive parton models correspond to an underlying theory of hadrons that is super-renormalizable.

To briefly summarize: the natural choice for QP distribution
function (33.1) has been shown to be appropriate for gauge theories. The asymptotic $Q^2$ behaviour of the number $\#$ of QP within the hadron has been derived for FP and AF gauge theories (and, trivially, for super-renormalizable theories).

So far in this chapter we have examined the relationship between the naive intuitive parton models and various underlying field theories. At this point we specialize to a particular theory: quantum chromodynamics. The phenomenological success of the constituent quark models (e.g.: Hey, 1975; Close 1979) and of QP models, at very different energy scales, can be reconciled if the underlying quantum theory of hadrons is AF. The reasons for supposing that it is a gauge theory, in particular a colour gauge theory with vector particles mediating the interactions, have been expounded in the literature (e.g. Ellis and Sachrajda, 1979) as will be discussed below. The theory that emerges uniquely from all these considerations is QCD. The experimental status of QCD predictions for DIS will be discussed in §36, where we shall regain contact with the intuitive parton model of Chapter 2 via a phenomenological comparison. This comparison will shed light on QCD and on the limitations of our intuitive model.

QCD is a very complicated theory, and because of this we must include two sections of review before discussing predictions. There is insufficient space to include all aspects of the theory, so we limit this review to essential topics. We hope the brevity will not be at the expense of coherence. A fuller picture can be obtained from the many references given.

§34 Gently Introducing QCD: $a_s(Q^2)$

Given the interaction Lagrangian of a particular quantum field theory we can calculate the corresponding DIS structure functions for
that theory. Drell and Yan (1971) do this by calculating directly from the definition of $W_{\mu\nu}$, eqn. (2.2) (dressing up the parton currents) whereas Llewellyn-Smith (1978) calculates the $\gamma^*$ cross-section:

$$\sigma = 4\pi \varepsilon^*_\mu \varepsilon_{\nu} W^{\mu\nu}$$

$$\Sigma \Sigma \int |A|^2 \cdot \text{(invariant phase space)}$$

where $A$ is the amplitude for the $\gamma^*$ (with polarization vector $\varepsilon_\mu$) scattering off field quanta. Both approaches work to lowest order in a $g\phi^3$ theory.

Unfortunately nature has most likely chosen a more complex theory for hadrons and so the actual computation is more involved. More importantly is the fact, explained below, that lowest order QCD perturbation theory cannot unambiguously be compared with experiment, so calculations must be taken to higher order.

As mentioned in §28, QCD is an AF theory. This can be shown by analogy with QED. As in §28 we can calculate the amplitude for some QCD process, such as $qq$ scattering (Field). Including higher order graphs in this calculation requires the introduction of a renormalized coupling constant $\alpha_s(\mu)$ defined at some arbitrary reference mass (renormalization point) $\mu$. (e.g. Lichtenberg, 1980; Field, 1978; Atwood, 1980; Ross, 1980; Stevenson, 1980, 1981; Bjorken, 1979; Llewellyn-Smith, 1977; Ellis and Sachrajda, 1979) so that the amplitude is written, for momentum transfer squared $Q^2$,

$$A(Q^2, \mu^2, \alpha_s(\mu^2)) = A(Q^2_{\mu^2}, \alpha_s(\mu^2))$$

where the right hand side follows from dimensional analysis. Requiring that $A$ is independent of $\mu$ (i.e.: "renormalization group invariant") leads to (compare eqn. (28.1))
\[ A(Q^2, \alpha_s(Q^2)) = A(\mu^2, \alpha_s(\mu^2)) \]

(cf. Field, 1978) i.e.: the 1-parameter ambiguity introduced by the necessity of renormalization (Stevenson, 1980) manifests itself in a momentum-dependent effective coupling constant \( \alpha_s(Q^2) \) given by, after summing leading graphs that contribute to \( A \),

\[
\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + B \alpha_s(\mu^2) \ln \frac{Q^2}{\mu^2}} \tag{34.1}
\]

where \( B \) is a positive constant (compare eqn. (28.3)).

As in the QED case, \( A \) is now the bare (lowest order) QCD amplitude with the bare coupling \( \alpha_s \) replaced by eqn. (34.1).

Field (1978) derived (34.1) by calculating the amplitude for \( q q \rightarrow q q \). Ross (1980) derived it for \( e^+ e^- \rightarrow X \) and went on to show the same \( \alpha_s(Q^2) \) results from applying the renormalization group eqn. to \( \sigma_{e^+ e^-} \rightarrow X \), i.e.: the renormalization group equation automatically sums the leading graphs. That the renormalization group method is equivalent to the above-presented derivation is verified by differentiating (34.1):

\[
\frac{d\alpha_s(Q^2)}{d\mu^2} = 0
\]

Let us now introduce \( \Lambda \), defined by

\[
\Lambda^2 = \mu^2 \exp \left[ -\frac{1}{B \alpha_s(\mu^2)} \right] \tag{34.2}
\]

It is easy to check that \( \frac{d\Lambda^2}{d\mu^2} = 0 \) so that \( \Lambda^2 \) is massive and is renormalization group invariant. It is thus the fundamental mass scale of perturbative QCD (analogous to \( m^2 \) in QED, cf. eqn. (28.3)).

Of course the same expression for \( \Lambda^2 \) can be obtained from the...
renormalization group: from dimensional arguments we have

\[ \Lambda^2 = \mu^2 f(\alpha_s) \]

for some \( f \). If \( \Lambda^2 \) is a fundamental mass scale then it must be independent of \( \mu^2 \):

\[
\left[ \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right] \Lambda^2 = 0
\]

which has the solution

\[
f(\alpha_s) = \exp \left[ - \int \frac{d\alpha'_s}{\beta(\alpha'_s)} \right].
\]

From its definition \( \beta(\alpha_s) \equiv \frac{\partial \alpha_s}{\partial \mu^2} \) the \( \beta \) function is found to be, in lowest non-trivial order, \( \beta(\alpha_s) = -B\alpha_s \) which yields

\[
f(\alpha_s) = \exp \left[ - \frac{1}{B\alpha_s} \right].
\]

Substituting above we recover eqn. (34.2) for \( \Lambda^2 \).

This expression for \( \Lambda^2 \) allows us to rewrite (34.1) as

\[
\alpha_s(Q^2) = \frac{1}{B \ln \frac{Q^2}{\Lambda^2}}
\]

(34.3)

This shows that QCD is an AF theory.

Calculating \( \beta(\alpha_s) \) to higher order modifies eqn. (34.3); consequences of this for the \( \Lambda \) parameter will be discussed below.

§35 Perturbative QCD

We wish to compare DIS predictions of QCD with those of the naive parton model of Chapter 2. Before we can do this, however, we must review QCD perturbation theory. Our concern will not be the details of QCD calculations; the references given in the following cursory discussion contain these.
Close (1979), Ellis and Sachrajda (1979), Llewellyn-Smith (1977), Ross (1980) all show that there is strong experimental evidence for coloured vector gluons mediating quark interactions. The quantum field theory of coloured vector gluons and quarks is QCD, which has vertices given in Fig. (35.1).

Note that the theory is non-Abelian. This is the essential feature which ensures that the QCD effective coupling becomes weaker at large $Q^2$, in contrast to QED (e.g.: Bjorken, 1979; Lichtenberg, 1980; Llewellyn-Smith, 1977). The constant $B$ of $\alpha_s(Q^2)$, eqn. (34.3) is given by

$$B = \frac{33 - 2f}{12\pi}$$

where $f$ is the number of quark flavours. This second term is negative and so contributes to screening à la QED. The first term dominates however; this "antiscreening" comes about because the gluons are charged (self-interacting) and hence will smear out the strong charge distribution at a given vertex. From Gauss' Law we might argue that the apparent charge decreases as momentum increases.

From the form of $\alpha_s(Q^2)$ we see that QCD perturbation theory breaks down at low $Q^2 \sim \Lambda^2$, so if we hope to do calculations we must go to large momentum so that $\alpha_s$ is small. But even here there are difficulties: there are no coloured quarks or gluons observed in nature (possibly QCD is confining at large distances) and so there are no asymptotic scattering states (Bjorken, 1979) and so how can we reliably construct an $S$ matrix? In Bjorken's words "it seems necessary to solve QCD in order to formulate it".

In order to circumvent these difficulties we turn to R.K. Ellis et al. (1979) who have proved that, in the region where QCD perturbation theory is feasible (i.e.: large $Q^2$-parton model country) the physical
cross-section can be factored into a "hard" piece and a "soft" non-perturbative piece, as shown for DIS in Fig. (35.2). The quarks leaving the soft blob have low momenta \( \sim \Lambda \) and are considered to be asymptotic states within the hadron, so that perturbative QCD can be applied to the hard blob. Thus we can hope to obtain QCD predictions by juggling with calculated observables so as to eliminate the uncalculable piece. (It is for this reason that QCD people must resort to moments – see §36.3).

We thus expect that QCD will modify rather than completely destroy the parton model predictions for DIS. This is the attitude taken by Ross (1980): add gluons to the parton model and calculate.\(^1\) In fact it has been shown (Llewellyn-Smith, 1978; Polkinghorne, 1977) that the hard blob of Fig. (35.2) becomes the ladder diagram of Fig. (35.3) in the limit of large \( Q^2 \). This ladder diagram is just the old parton handbag diagram with gluon rungs.

Before we present the perturbative QCD predictions for DIS and compare with experiment, there is one further theoretical aspect of the theory which must be mentioned, and that is the renormalization prescription dependence of \( \Lambda \).

From its definition we see that \( \Lambda \) depends on \( \alpha_s(u^2) \), which is prescription-dependent. Thus the value of \( \alpha_s(Q^2) \) at a given \( Q^2 \), and hence the rate of convergence of the perturbation expansion, depends upon prescription. To any finite order in perturbation theory the QCD predictions are therefore dependent upon the particular renormalization prescription used in the calculation. This is obviously an artifact of the expansion since observables cannot depend upon the details of renormalization, so the name of the game is to minimize this artificial

\(^1\) According to Bjorken (1979) our attitude should be: "... if the calculation is not obviously wrong, its right. But it is possible that this postulate, while not obviously wrong, may not be right."
dependence. There have recently been at least two methods proposed for doing this (Stevenson, 1980, 1981; Celmaster and Sivers, 1980. For phenomenological consequences see Monsay and Rosenzweig, 1981).

Not only is $\Lambda$ prescription-dependent, it is also completely arbitrary to $O(\alpha_s)$ and it is process-dependent, as is seen from the following equation (Stevenson, 1980; Field, 1978)

$$\frac{1}{\ln \frac{Q^2}{\Lambda^2} + C} = \frac{1}{\ln \frac{Q^2}{\Lambda^2} - \frac{C}{n^2}} + \ln \frac{Q^2}{\Lambda^2}, \text{ large } Q^2.$$ 

So to completely specify $\Lambda$ we must specify the renormalization prescription and the process, and must work to $O(\alpha_s^2)$.

### §36 Comparison with Experiment

[1] The QCD parameter $\Lambda$ must be determined experimentally before the theory has any predictive power. $\Lambda$ can be extracted from the structure function data via the "evolution equation", which describes the $Q^2$-dependence of the structure functions (cf. §29 or, for example, Abbott et al., 1980), or from moments. The analysis is complicated, for reasons discussed above, and different methods (and different experimenters) obtain different values for $\Lambda$. We shall merely quote the results here (in GeV).

Field (1978) obtains a value $\Lambda = 0.4 - 0.5$, Barnett (1979) finds $\Lambda = 0.3 - 0.4$. Buras (1981) quotes a value $\Lambda \sim 0.7$, whilst Donnachie and Landshoff (1980), in an extreme model which attempts to attribute most DIS scaling violations to higher twist terms (see later), obtain $\Lambda \sim 0.1$. Bollini et al. (1981) find the same value, and Coignet (1981)

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† Atwood refers to experiment as "... the fodder for all theoretical ruminations."
quotes $\Lambda = 0.1 \pm 0.1$. Gabathuler (1980) too obtains $\Lambda \sim 0.1$ from EMC data and quotes a result $\Lambda \sim 0.25$ from SLAC data. He claims these two are consistent if higher twist\(^\dagger\) effects are taken into account. Roberts (1981) claims that CDHS, SLAC, EMC data are all consistent with $\Lambda = 0.4$. D.P. Roy (1981) reconciles the different experimental values on the grounds of charm production.

The effect of higher twist on the value of $\Lambda$ is discussed by Abbott et al. (1980), who parametrize the structure function $F_2$ as follows

$$
F_2(x,Q^2) = F_2(x,Q^2) \left[ 1 + \frac{\mu^2}{(1-x)Q^2} \right]
$$

where $\mu^2$ is a higher twist mass scale. The result for DIS is that $\Lambda$ is a decreasing function of $\mu^2$ (see Fig. (36.1)) so that including this term ($\mu^2 \neq 0$) produces a lower value of $\Lambda$.

The above results are to leading order in $\alpha_s$. From the last section we know that $\Lambda$ is totally arbitrary to this order, so that the above $\Lambda$'s are "effective" values. Taking into account next to leading order Buras finds $\Lambda_{\overline{\text{MS}}} = 0.40$, $\Lambda_{\overline{\text{MS}}} = 0.50$ and $\Lambda_{\text{MOM}} = 0.85$. The subscripts refer to three popular subtraction schemes: minimal subtraction, modified (or mutilated) minimum subtraction, and momentum subtraction, respectively. Ross (1980) obtains $\Lambda_{\overline{\text{MS}}} = 0.610 \pm 0.035$ which reduces to $\Lambda_{\overline{\text{MS}}} = 0.20 \pm 0.15$ when higher twist is included. Duke and Roberts (1980) obtain $\Lambda_{\overline{\text{MS}}} = 0.42$. Coignet (1981) quotes $\Lambda_{\overline{\text{MS}}} \approx 0.1 \pm 0.1$.

The general conclusion we draw here is that the effective value of $\Lambda$ is in the region $\Lambda \sim 0.1 - 0.5$. Inclusion of higher twist terms produces a smaller value of $\Lambda$ (a similar conclusion is reached by

\(^\dagger\) Higher twist terms are non-leading terms that appear in the operator product expansion of the DIS currents. See Roy (1975) for details.
The ratio $R = \frac{\sigma_L}{\sigma_T}$ of longitudinal to transverse $\gamma^* \gamma$ cross-sections can in principle provide a test of QCD and of intuitive parton models. It is not difficult to show that in parton models with massless QP (e.g.: Feynman, 1972)

$$R_{QP} = \frac{4k_{\perp}^2}{Q^2}.$$  \hspace{1cm} (36.1)

Earlier we saw that $<k_{\perp}^2> = M^2x^2$ where $M$ is the hadron mass (Chapter 2). Averaging over $x$ we find

$$<R>_{QP} = \frac{0.14\text{GeV}^2}{Q^2}$$

since $<x^2> \sim M_3 \sim 0.04$ where $M_3$ is the third moment of the proton structure function. The QCD expression for $R$ contains this primordial parton model term plus a "perturbation" term proportional to $\alpha_s$ (Ross, 1980; Buras, 1979; Atwood, 1980).

$$R_{QCD} = R_{QP} + \frac{r(x)}{\ln \frac{Q^2}{\Lambda^2}}$$

which dominates at large $Q^2$. $r(x)$ is large at small $x$ and small at large $x$ (Field, 1978; Gabathuler, 1980). Thus the QCD prediction is markedly different to that of the intuitive QP model in both $x$ and $Q^2$ dependence, and so an experimental determination of $R$ is desirable.

$R$ is experimentally observed as the following combination of structure functions (e.g.: Close, 1978; Söding, 1981).

$$R(x,Q^2) = \frac{F_2 - 2xF_1}{2xF_1} + \frac{4M^2x^2}{Q^2} \frac{F_2}{2xF_1}.$$
(This reduces to eqn. (36.1) in the parton model of Chapter 2, as is easily seen). The data is summarized below (taken from Barish (1980). All experimental references can be found in Barish, Smadja (1980); Barnett (1979). For very recent reviews see Montgomery (1981), Sciulli (1980)).

\[
\langle R \rangle = 0.20 \pm 0.10 \quad \text{SLAC-MIT} \quad \text{ep} \quad Q^2 = 2 - 20 \text{ GeV}^2
\]

\[
\begin{align*}
0.44 \pm 0.25 \pm 0.19 & \quad \text{HIC} \quad \uparrow \quad 1 - 12.5 \\
0.15 \pm 0.10 \pm 0.04 & \quad \text{BEBC-GGM} \quad \nu N \quad 0.1 - 50 \\
0.03 \pm 0.05 \pm 0.10 & \quad \text{CDHS} \quad \nu N \quad 2 - 200 \\
0.18 \pm 0.06 \pm 0.04 & \quad \text{HPN} \quad \nu N \quad 2 - 200
\end{align*}
\]

N = nucleon.

The quoted errors are statistical and systematic. There is no obvious \( Q^2 \) trend. The \( x \)-dependence can be seen in Fig. (36.2), taken from Barnett (1979). There is no obvious \( x \)-dependence either. The solid curves show QCD with no higher twist, the dashed curve is QCD plus a diquark model of higher twist (Barnett).

The lack of any apparent \( Q^2 \) dependence is more difficult to reconcile with the QP model than with QCD, since the QP model predicts a stronger falloff. Both QCD without higher twist and the QP model fall below the experimental \( R \); again this is worse for the QP model, which has no "outs". QCD advocates can construct higher twist models that fit the data as in Fig. (36.2).†

† Diquark models can be formulated within the framework of the naive QP model (Donnachie and Landshoff, 1980) but the resulting picture is inconsistent with data (Close and Roberts, 1980).
Thus we conclude that the experimental value of $R$ slightly favours QCD over the intuitive QP model. The error bars are large, however, and in fact $R$ is never more than two standard deviations from zero, so perhaps it is premature to rule out the QP model or low-twist QCD just yet. This point is emphasized if we look at the neutrino data on its own, (Fig. (36.3), taken from Atwood, 1980) which yields $\langle R \rangle = -0.03 \pm 0.04$. Abramowicz et al. (1981) find $\langle R \rangle = 0.10 \pm 0.07$.

[3] From §35 we expect that many QCD predictions will differ from naive QP model predictions only in $O(\alpha_s)$. Thus for example the Gross-Llewellyn-Smith sumrule becomes

$$\int_0^1 dx \left[ F_3^{\text{VP}} + F_3^{\text{VN}} \right] = 6 \left( 1 - \frac{\alpha_s}{\pi} \right)$$

in QCD. Recall from Chapter 1 that in QP models the right hand side is just 6. The Bjorken sumrule too is altered but the Adler sumrule, which expresses charge conservation, remains the same. These differences between QP model and QCD are small at large $Q^2$, and present experiments cannot distinguish between the two predictions (for example the Gross-Llewellyn-Smith sumrule is satisfied only to within $\sim 20\%$ - see Chapter 1).

A theoretically much cleaner test is provided by the moments of structure functions. (For a recent review see Söding, 1981). Intuitive QP models scale and so their moments are constants whereas QCD moments are functions of $\ln Q^2$ (cf. §29). In fact, in leading order QCD (e.g.: Atwood, 1980)

$$M_{NS}(Q^2, n) = \frac{C}{\int_{\ln \frac{Q^2}{A}} d_n n}$$

(36.2)
where NS refers to nonsinglet. C is a constant which is not calculable perturbatively (cf. Fig. (35.2) and related discussion) and \( d_n \) is given by, for four quark flavours,

\[
d_n = \frac{4}{25} \left[ 1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^{n} \frac{1}{j} \right].
\]

Note that this tends to constant \( \ln n \) as \( n \to \infty \), in accordance with the prediction of KS (§29).

The unknown C can be eliminated by noting that

\[
\frac{d_n M_{NS}(Q^2,n)}{d_m M_{NS}(Q^2,m)} = \frac{d_n}{d_m}.
\]

Here then is a possible test: plot the logarithm of the nth moment against the logarithm of the mth moment. QCD says the result should be a straight line of calculable slope. The experimental results (Duke and Roberts, 1980) are shown in Fig. (36.4), and are consistent with QCD. Unfortunately this is not a good test (or, more accurately, not a definitive one) because in the words of J. Ellis et al. (1979):

"Claudia Cardinale could be fitted to a straight line on a log-log plot". Also because QP model moments are constant (at large \( Q^2 \)) their ratios are also straight lines.

A better test results by raising eqn. (36.2) to the power \( (-d_n)^{-1} \). This QCD predicts (in leading order, without higher twist)

\[
\left[ M_{NS}(Q^2,n) \right]^{-d_n^{-1}} = \ln \frac{Q^2}{\Lambda^2}.
\]  

(36.3)

This has been tested experimentally (Fig. (36.5)) and the agreement is good. Note that QCD cannot predict the constant of proportionality in (36.3), however from this equation an effective value for \( \Lambda \) can be obtained.
Can the naive QP model fit the data as well as does QCD? Recall that the rest frame QP model of Chapter 2 predicts (eqn. (13.3))

$$M_n(Q^2) = M_n \left[1 + \frac{M^2}{Q^2} (n - 3 + \frac{6}{n+2} \frac{M_{n+2}}{M_n})\right].$$  \hspace{1cm} (36.4)

Over a limited $Q^2$ range this $\frac{1}{Q^2}$ behaviour might look like the $\ln^{-1}Q^2$ behaviour of QCD (Berger, 1979). In fact Williams (1980) has found that the data can be parametrized as

$$M_n(Q^2) = \left[1 + \frac{0.447n^2}{Q^2}\right] M_n$$ \hspace{1cm} (36.5)

for $n = 2 - 10$ and $Q^2 > 2$ GeV$^2$. This fit is no worse than the above QCD fit for any $n$ and is actually slightly better for $n > 8$. Perkins (1981) has obtained a good fit with

$$M_n(Q^2) = \left[1 + (n - 1.5) \frac{1.28}{Q^2}\right] M_n$$ \hspace{1cm} (36.6)

for $n \leq 7$. Thus asking if eqn. (36.4) can fit the data reduces to asking if (36.4) is consistent with (36.5) or (36.6). The answer is no: consistency of (36.4) and (36.5) requires

$$\frac{M_{n+2}}{M_n} = 0.5n$$

which is impossible for $n > 2$. Consistency of (36.4) and (36.6) requires

$$\frac{M_{n+2}}{M_n} = 1.5$$

which is impossible. Thus although QP model scaling violations are certainly in the right direction, they are insufficient to explain all the scaling violations seen experimentally. So we now have a test which distinguishes between the intuitive QP model and QCD (in favour of the latter).
We do not give up on the subasymptotic QP model yet, however. Eqns. (36.5) and (36.6) have shown us that higher twist (i.e. $\frac{1}{Q^2}$ - see Roy, 1975) can by itself explain the data. Also we have seen that leading order QCD alone can do so. Thus a large range of combinations of the two will agree with experiment. Now in order to take higher twist effects into account within QCD, we can alter the QCD moments to (Field, 1978)

$$\hat{M}(Q^2,n) = M(Q^2,n) \left[ 1 + (n-2) \frac{\Delta}{Q^2} \right]$$

(36.7a)

or to (Ross, 1980)

$$\hat{M}(Q^2,n) = M(Q^2,n) \left[ 1 + n \frac{\tau^2}{Q^2} \right].$$

(36.7b)

Optimum fits are obtained for $\Delta \sim 0.1 - 0.2$ GeV$^2$ and $\tau^2 = 0.03 \pm 0.23$ , i.e. $\tau < 500$ MeV. Comparing these $\frac{1}{Q^2}$ terms with the corresponding QP model term in eqn. (36.4) yields

$$\frac{\Delta}{M^2}, \frac{\tau^2}{M^2} \sim \frac{M_{n+2}}{M_n}$$

which gives very reasonable values for $\frac{M_{n+2}}{M_n}$. Thus QP model $\frac{1}{Q^2}$ corrections are consistent with experimentally determined higher twist corrections to low-twist QCD predictions.

The significance of this is the following. We have found that the leading order QCD log plus parton model (i.e.: kinematic) corrections yield optimum fits to the moments data. This implies that dynamic (i.e.: non-kinematic) higher twist effects ought to be small. (This is in agreement with Duke and Roberts, 1980; Roberts, 1981; Ellis and Sachrajda, 1979; and with Ross, 1980. See also Gunion, 1980. It is not in agreement with Barnet et al., 1979).
Summary

In this chapter we have investigated the consequences an underlying hadron field theory has for the structure functions of DIS. Initially we adopted the simple intuitive philosophy of Ellis (1976), and by way of illustration applied this to the well-known Bjorken and Paschos (1969) naive parton model. The amended model exhibited scaling violations as expected, and in such a way as to indicate that the field theory underlying this amended model is a gauge theory. We also discussed two natural choices for the QP distribution:

\[ f(x, Q^2) = g(x) h(Q^2), \quad (N-1)(1-x)^{N-2}, \]

and found that these correspond to nongauge and gauge theories respectively. We saw that the number \( \# \) of QP constituents increases with momentum transfer as

\[ \# \sim \ln Q^2 \]

for FP gauge theories and

\[ \# \sim \ln \ln Q^2 \]

for AF gauge theories (and, of course, \( \# = \text{constant} \) for super-renormalizable theories).

The second moment \( M_2 \) was found to be constant only if the expected gluon energy/momentum fraction \( \epsilon \) is constant. In obtaining the above results we made no assumptions beyond those of KS.

The second half of the chapter reviewed QCD (the most likely candidate field theory at this time) and compared the DIS predictions of QCD with those of our intuitive rest frame QP model of Chapter 2, and with experiment. Our main conclusion is that dynamical higher twist may be only a small effect in the experimentally-observed structure functions.

Let us conclude this chapter with an appropriate quote from K. Ellis et al. (1979): "Even though many of the detailed predictions of the original parton model are changed by the inclusion of hard higher-order QCD interaction effects the physical picture underlying the model survives remarkably well. The detailed account of hard
interaction effects ..... makes the parton model an even more valuable tool for the analysis of hadron scattering."

CONCLUSIONS

We conclude this thesis by listing the results that we have obtained. Some of these results are well-known and so we cannot claim that they are new. In these cases, though, the derivation is original.

(i) We have derived the conditions under which the Callan-Gross relations are expected to hold, and have shown that for parton models these conditions are automatically satisfied.

(ii) We have shown in different ways (momentum conservation sumrule, counting rule) that there exist hadron constituents that do not couple to the deep inelastic probe (i.e.: gluons).

(iii) We have shown that it is sensible to construct parton models in the rest frame of the target hadron, and have presented such a model. The rest frame formulation produces all the familiar results of the old IMF models, plus a few extras. For example, we have seen that, in the rest frame, QP transverse momentum increases with $x$, and that offshell effects can be accounted for. Also, rest frame models give rise naturally to a sub-asymptotic scaling variable that can be interpreted as a QP average energy fraction. This scaling variable qualitatively explains the observed $Q^2$ dependence of the DIS structure functions in a model-independent way.

(iv) Comparison of the subasymptotic behaviour of our rest frame parton model with the predictions of QCD has demonstrated that dynamical (as opposed to kinematic) higher twist contributions to the moments of structure functions may be small.
(v) We have shown quite generally that confinement can (and should) be included within the parton model, and that the effects of this on the parton model structure functions is vanishingly small in the Bjorken limit. We have seen how confinement affects the net jet charge, and have constructed a quantum-mechanical model in which partons are localized within the hadron. In this particular model we have seen that confinement naturally gives rise to a sea distribution and to spin distributions that are not in contradiction with experiment, without introducing extra parameters.

(vi) We have found that QP spin is tilted because of confinement.

(vii) The phenomenologically-successful MT is related to the kinematical Lorentz spin rotation and WR in various momentum limits. There is more to the MT than relativistic kinematics, however, unless we are prepared to believe in infinite momentum CQ.

(viii) We have investigated the relationship between parton models of DIS and the underlying field theory of hadrons. This investigation, in conjunction with the work of KS, has enabled us to determine, for example, the type of field theory that underlies a given QP distribution. Several lesser results were also obtained, for example the number of QP in a hadron described by an AF gauge theory increases as $n \sim \ln \ln Q^2$.

We might have finished this thesis with the quotation given at the end of Chapter 5. Here, though, is another that equally well expresses my own attitude towards parton models (Girardi, 1980): "It is worthwhile to note that this very simple model allows us to make very interesting qualitative and quantitative predictions which are, by the
way, never far from what is observed. This is the reason why, even having quantum chromodynamics at hand, physicists always like to refer back to the parton picture." It is our belief that the results obtained in this thesis enhance this parton picture of DIS.
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FIGURE HEADINGS

FIG. (2.1): DIS $\ell p \rightarrow \ell' X$ in the one photon exchange approximation.

FIG. (2.2): Early data (cf. Close, 1973) demonstrating scaling, i.e.
the $Q^2$ (= $4E E' \sin^2 \theta$) independence of the structure
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Fig. (9.1)

Fig. (11.1)

Fig. (13.1)
\[ a > a_0 \]

Fig. (28.4)

Fig. (28.5)

Fig. (28.6)

Fig. (28.7)
Fig. (36.5)