Capability of MLS Instruments in the Observations of Atmospheric Gravity Waves

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Declaration

I declare that this thesis was written by myself, and that the work it describes is my own except where otherwise stated.
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I am very grateful to friends and colleges in the department for their help and warm hospitality given to me during my period of the study.

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Abstract

We presented in this thesis model analysis on the capability of MLS instruments in the observations of atmospheric gravity waves in the middle atmosphere. This capability is considered in terms of the temperature (or amplitude) response and variance response of the instruments which carry the observations to an individually observed wave. The response results for each wave are reported principally as a function of the horizontal and vertical wavelength components and its propagation direction on the horizontal plane.

We found that the temperature response functions of both UARS MLS and EOS MLS instruments for waves with typical vertical wavelength of 1-20 km are sharply peaked at certain horizontal wavelengths but their amplitudes are always less than ~60% of the original amplitude in case of UARS MLS and 80% for EOS MLS. The responses at vertical wavelength of 10-20 km, in particular, are considerably high to waves with horizontal wavelength of scales ~200-500 km which are propagating away from the satellite position with respect to the line-of-sight direction. These are waves that should be most detectable with both of the MLS instruments in the real observations in terms of the amplitude response.

The variance response functions in the 32-measurement limb-tracking mode of UARS MLS and 150-measurement limb-tracking mode of EOS MLS have also peaked at some certain horizontal wavelengths for waves of vertical wavelength 1-20 km but their variance amplitudes never exceeds ~20% of the original value for UARS MLS and 30% for EOS MLS. Like the temperature response, the variance responses at vertical wavelengths of 10-20 km are maximum to waves with horizontal wavelength of scales ~200-500 km which are propagating away from the satellite position with respect to the LOS. In the contrary, gravity waves with vertical wavelength of few kilometers are unlikely to be much visible to the MLS instruments due to the very low response regardless of their horizontal wavelength scales or directions of propagation.

The comparison of the variance responses from the observations in the limb-tracking and limb-scanning modes of UARS MLS suggested that the scanned MLS variances might be unreliable for the study due to the vertical variations in scan data associated with the varying of instrument's viewing angle. These effects are less prominent in case of EOS MLS but the average variance responses in limb-scanning mode should be significantly lower than that in limb-tracking mode and, therefore, less effective in the observations of gravity waves in the real atmosphere compared to that carried in limb-tracking mode.
# Contents

Abstract

1 Introduction

1.1 Background and Aims of the Thesis

1.2 Microwave Limb Sounding Technique

1.2.1 Geometry of Limb Sounding Observations

1.2.2 Advantages of Limb Experiments

1.3 The Significance of Atmospheric Gravity Waves

1.3.1 The Significance to the Middle Atmosphere

1.3.2 Observations of Gravity Wave Characteristics

1.4 The UARS MLS Experiment

1.4.1 Mission Background

1.4.2 The Significance of the Experiment

1.4.3 MLS Instrument Overview

1.5 Gravity Waves Observations with UARS MLS

1.5.1 MLS 63-GHz Radiance Measurements

1.5.2 Temperature Weighting Functions

1.5.3 Temperature Variance Analysis

1.6 Thesis Outline

2 The Forward Model

2.1 Introduction

2.2 Microwave Radiative Transfer

2.2.1 Radiative Transfer Theory

2.2.2 Microwave Radiative Transfer Equation

2.3 Oxygen Absorption and Emission Model

2.3.1 The Formulation of Absorption Coefficient

2.3.2 The JPL Catalogue Data

2.3.3 Spectral Line Shapes

2.3.4 Oxygen Absorption Coefficient Analysis
Chapter 1

Introduction

1.1 Background and Aims of the Thesis

We present in this thesis model analysis on the capability of the Microwave Limb Sounder (MLS) instruments in the observations of atmospheric gravity waves in the middle atmosphere in both cases of UARS MLS and EOS MLS. This capability is described in terms of temperature (or amplitude) response and variance response of the instrument to an individually observed wave with different amplitudes, wavelength scales, and orientations.

The UARS MLS is one of ten instruments on board NASA’s Upper Atmosphere Research Satellite (UARS) which is in operation since 12 September 1991. It is a passive microwave radiometer designed primarily to measure profiles of molecular abundances (mainly O₃, ClO, and H₂O), temperature and pressure in middle atmosphere (stratosphere and mesosphere) using emission features near 63, 183, and 205 GHz [Waters 1993; Barath et al. 1993]. The instrument is the first implementation of atmospheric limb sounding from space operating in the microwave region and it has been proved very successfully so far in measuring profiles of many important gases especially those involved in the study of ozone (O₃) destruction in the stratosphere (see Section 1.4 for more details). The NASA’s Earth Observing System (EOS) Microwave Limb Sounder is a greatly enhanced version of the UARS MLS which is scheduled to be launched in June 2003. Its purposes and technical details are very similar to that of UARS MLS and will be described later in Chapter 6.

Although not designed for this purpose at first, Wu and Waters at Jet Propulsion Laboratory (JPL) have found that UARS MLS 63-GHz radiometer could observe small-scale temperature fluctuations at 30-80km altitudes [Wu and Waters 1996a, b, 1997]. Their initial analyses suggested that these fluctuations might be due to propagating gravity waves in the middle atmosphere as first proposed by Hines [1960]. As a result, they produced the first global maps of atmospheric temperature variances at ~100km horizontal scales associated to gravity wave activity in stratosphere and mesosphere found in their studies. With a good global coverage and long sequence of measurements made by UARS MLS, it was hoped that these variance data could provide useful information for studying geographical distributions...
and spectral distributions of gravity wave in the middle atmosphere at global scale, which no other previous method has ever achieved. Recent reports on the analysis of UARS MLS temperature variance are given by McLandress et al [2000] and Jiang and Wu [2001].

Though, the UARS MLS seems to be capable enough in detecting small-scale gravity waves in the middle atmosphere as reported so far, however, the interpretation of the MLS radiance variances in term of gravity wave model is still rather difficult in principle. This because, due to the geometry of limb sounding and pattern of MLS observation, it is suggested that the observed magnitude of the wave-induced UARS MLS radiance variances should be a result of convolutions of wave spectra, wave propagating directions, instrument weighting functions, and sampling patterns. All these factors could greatly complicate the problem and make the quantitative study of gravity wave information retained in the UARS MLS variance data is, to some extent, still quite limited. In addition, the gravity wave spectra can vary largely time, height, and place, which adds complexity to interpretation of the radiance fluctuations observed with the instrument.

The most effective way to understand the capability of gravity wave observations with the MLS instruments is to perform a model simulation of the experiment for a model atmosphere with gravity waves included where all these factors are taken into account. The analysis of this kind is important, because it could provide better knowledge of the kinds of waves that the MLS instruments should be able to observe in the real observation, but it is still not thoroughly examined in all works cited above. It is hoped that the results reported in this thesis could lead us to better understanding on the capability of the MLS instruments (both UARS MLS and EOS MLS) in detecting the gravity waves in the middle atmosphere with different amplitude, wavelength scale, and orientation in the real operations. In addition, this knowledge may allow us to improve diagnosis of gravity waves in both cases of UARS MLS study and, maybe, of EOS MLS in the near future.

The remainder of this chapter consists of five main sections. In Section 1.2, the basic concept and advantages of microwave limb sounding technique are given. In Section 1.3, the importance and observational techniques for the study of atmospheric gravity waves in the middle atmosphere are presented. Then, in Section 1.4, a brief review on missions and achievements of the UARS MLS experiment are addressed. Details of gravity wave study with UARS MLS reported by Wu and Waters are summarised and discussed in Section 1.5. Examples of MLS global variance maps of small-scale temperature variations are also shown therein. Finally, an outline of the following chapters is given in Section 1.6.
1.2 Microwave Limb Sounding Technique

We present in this section basic concepts and advantages of microwave limb-sounding technique which has been applied in the MLS experiments. However, only brief detail of the technique is given here for being a background of the further study. More information on the observing geometry of the MLS instruments and related microwave radiative transfer theory will be given in Chapter 2 and Appendix A.

1.2.1 Geometry of Limb Sounding Observations

Normally, there are two distinct kinds of viewing geometry that the satellite radiometers use in their measurements of atmospheric radiation: (1) the near-nadir view in which radiation is observed leaving the atmosphere in directions near to the local vertical, (2) the limb view in which radiation leaving the atmosphere nearly tangentially is observed. Figure 1.1 shows the geometry of limb-sounding from a satellite radiometer, which is in operation at an orbital altitude $H$ above the Earth's surface and viewing the atmospheric limb at tangent height $h_T$.

![Figure 1.1: Geometry of limb sounding observations illustrating the tangent height $h_T$ and the projected thickness of an arbitrary layer at height $z(x)$ along the ray path.](image-url)
From the arrangement shown in Figure 1.1, the radiometer receives the radiation emitted principally by the atmosphere along a ray path (x-axis). In this observing co-ordinate system, the point closest to the surface (where \( x = 0 \)) is called tangent point and its height is called tangent height denoted by \( h_T \). In the experiment, the instrument may scan atmosphere by sweeping its viewing direction along the vertical direction (z-axis) while the satellite is moving along horizontal direction. For example, the UARS MLS measures atmospheric thermal emission in microwave region by looking sideways (at 90° from the orbital path) and step-scanning the atmosphere through the limb at altitudes \(-0-100\) km while the satellite is orbiting the Earth at \( H = 600\) km along y-direction (out of page). In case of EOS MLS, the satellite orbits the Earth at altitude = 700 km along x-direction, while the instrument viewing ahead (along the orbital path) and step-scanning the atmosphere at attitudes \(-0-65\) km.

As described in Chapter 2, the observed radiance at the satellite radiometer in case of limb sounding can be expressed as a sum of two contributions. The first one is called the background term, which accounts for the radiation due to the background surface (the cold cosmic space) reduced by the attenuation due to the propagation through the entire atmosphere. The second term is the so-called atmospheric term, which accounts for the emission arising from the limb atmosphere along the ray path itself. Typically, the principle emitters in the Earth's atmosphere at microwave frequencies 1-300 GHz are water vapour (H\(_2\)O) and oxygen (O\(_2\)). Water vapour emissions are essentially associated with a single line at 22 GHz and 183 GHz while oxygen has absorption band between 50-70 GHz (known as the 60-GHz oxygen complex) and an additional line at 118.75 GHz. Ozone (O\(_3\)) also has some spectral lines at frequencies > 100 GHz but they are typically weak compared to those of H\(_2\)O or O\(_2\). Microwave emission in the middle atmosphere, in particular, is dominated by O\(_2\) molecules as H\(_2\)O molecules are usually confined to the lower atmosphere. Generally, radiometric measurements in 50-70 GHz portion of the oxygen band are used to derive temperature profiles both in clear and cloudy atmosphere. The influence of water vapour is small and can often be neglected when analysing the measurements within the oxygen band due to the large frequency separation between the water vapour lines and the oxygen complex. Measurements around the 22-GHz water vapour line are used to obtain column water abundance and the measurements around the 183-GHz water vapour line are normally used to obtain humidity. More detailed explanation of the radiance observation with MLS instruments are given in Chapter 2 for UARS MLS, and Chapter 6 for EOS MLS.
1.2.2 Advantages of Limb Experiment

The geometry of limb sounding technique shown previously in Figure 1.1 provides a number of advantages for remote sensing application over downward-looking experiments. The following summary is excerpted from Gille and House [1971].

1. High inherent vertical resolution. The combination of the spherical geometry and the exponential decrease of gas density with height normally provides data which are heavily weighted around the tangent point and gives high vertical resolution. Most of the emitted energy in this case comes from a layer of only 2- to 3-kilometres above the tangent point. This is equivalent to saying that limb-sounding observations are characterised by very narrow vertical weighting functions. The instrument’s narrow field of view is also assumed in this situation. (Note that, this assertion is generally valid only to the observation in non-saturation case as discussed in Chapter 2).

2. Cold (and constant) background. With the cold dark sky (the cosmic space with temperature = 2.7 K) serving as background, emission and reflection by the Earth’s surface are no longer of concern. Thus the limb-sounding technique avoids the ambiguity that occur in nadir viewing in which the background (the Earth’s surface) is hot and variable.

3. Long sensitivity. For a given tangent height $h_T$, the horizontal path contains up to 60 times more emitter than a corresponding vertical path. This means that temperatures can be measured to higher altitudes than with vertical sounders, and low concentration gases, such as NO, NO$_2$, and ClO, can be better detected.

4. Large surface coverage. Since atmospheric emission is being observed, the measurements can be made day or night and in any view direction relative to the satellite motion. This makes the continuous observations in global scale possible.

There are, of course, disadvantages associated with these features. The long ray paths mean that, even for rather transparent spectral regions, measurements in the lower atmosphere may suffer from a cloud along the path that could act as a body of infinite opacity. This might cause a considerable alteration in the observed radiation with varying seriousness depended on the observed wavelength. Another disadvantage is that horizontal resolution of limb sounding is considerably poor as the sharp vertical weighting function is associated with a horizontal region that may stretch 200-300 km or more along the ray path. This makes the MLS instruments tend to underestimate the small-scale temperature variation along the ray path occurred over this observing distance as we shall see later in this thesis.
1.3 The Significance of Atmospheric Gravity Waves

Atmospheric gravity wave is an important phenomenon that is currently believed to play a crucial role in controlling the large-scale circulation of the Earth's middle atmosphere through the saturation mechanism. We shall discuss in this section of their significance in the middle atmosphere and the difficulty to observe them with the conventional techniques in use so far. More background theory of these waves will be given in details in Chapter 3.

1.3.1 The Significance to the Middle Atmosphere

The theory of atmospheric gravity waves was initially proposed by Hines [1960] to explain for the observations of small fluctuating of ionospheric wind in the upper atmosphere. These waves are believed to be generated from the sources in the lower atmosphere (troposphere and lower stratosphere) by a variety of different mechanisms, e.g., topography, synoptic disturbances, or convective activity, and they cover a wide range of temporal and spatial scales. They then propagate upward to higher atmosphere both in the vertical and horizontal directions with amplitude growing exponentially with height. Hines also anticipated at the beginning some of the important effects of these waves on the dynamics of middle atmosphere. These included the transport of energy, the generation of turbulence, the likelihood of a non-linear cascade of energy to smaller-scale waves as the result of large gravity wave amplitudes, and the possible modulation of the middle atmosphere response due to variable characteristics and energies of the waves. These suggestions by Hines were reiterated later by Houghton [1978] to explain for the drag mechanism needed to balance the thermal and momentum budgets of the middle atmosphere.

Houghton [1978] suggested that gravity waves are the most likely candidate for balancing the thermal and momentum budgets of the middle atmosphere because of their known source in the lower atmosphere, their ability to transport momentum vertically, and their convergence of momentum flux in regions of (turbulent) dissipation. However, it was Lindzen [1981] who first proposed a simple scheme by which the principal effects of gravity waves in the middle atmosphere could be calculated. In his well-known study, Lindzen considered monochromatic wave motions and assumed that the exponential growth of wave amplitudes with height would be limited by the formation of convective instabilities that cause the wave to break beyond some certain point. Further vertical amplitude growth will then cease (become saturated) by dissipation through the irreversible extraction of wave energy.
energy into the production of turbulent diffusion just a level required to maintain saturation
amplitudes. Lindzen [1981] also generalised the expression for turbulent diffusion obtained
by Hines [1970] to apply to arbitrary mean flows, including critical levels, and obtained a
corresponding expression for mean flow accelerations induced by wave dissipation and
momentum flux convergence. The latter is essential in understanding the role of gravity
wave drag in balancing the thermal and momentum budgets of the middle atmosphere.

Because the mean flow acceleration accompanying wave momentum flux
convergence drives the mean flow toward the phase speed of the wave, the saturation of
gravity waves with non-zero phase velocities also offers an explanation for the mean zonal
wind reversals observed in the upper mesosphere and lower thermosphere. Following the
study by Lindzen [1981] there has been a resurgence of interest in gravity wave saturation
processes and effects. Lindzen's parameterisation scheme was then modified and included in
various numerical models to simulate of large-scale thermal dynamics, and constituent
structure of the middle atmosphere [e.g., Dunkerton 1982; Holton 1982, 1983; Schoeberl et
al. 1983; Garcia and Solomon 1985]. A number of theories have been developed in recent
years to give more understanding in the processes of gravity wave propagation and saturation
as well as their possible effects on the middle atmosphere. Details of these theoretical and
observational studies are reviewed in several sources, for examples, Fritts [1984, 1989],
Fritts and Rastogi [1985], Dunkerton [1989], and Hines [1991a, b, c]. For a historical review
of the gravity wave theory see, for examples, Hines [1972, 1989] and Fritts [1984].

There has been much subsequent progress in the understanding momentum transport
by middle-atmosphere gravity waves recently. Radar observations of gravity wave
momentum flux divergence have shown that these waves are capable of imposing a drag on
the background winds as large as 50-100 m s\(^{-1}\)/day in the middle and upper mesosphere
[Vincent and Reid 1983; Fritts and Vincent 1987]. The vigorous residual circulations
induced by this wave driving are sufficient to explain observed high-latitude mesospheric
temperatures, which depart from radiative equilibrium by as much as 50 K [von Zahn and
Meyer 1989; Hitchman et al. 1989]. The simple parameterisation of gravity wave momentum
flux divergence by Lindzen [1981] also permitted reasonably realistic modelling of the
observed thermal structure of the mesosphere [Lindzen 1981; Holton 1982, 1983; Schoeberl
et al. 1983; Garcia and Solomon 1985] and upper stratosphere [Hitchman et al. 1989]. These
studies do significantly emphasise the importance of gravity wave in the middle atmosphere.
In addition, it has been shown by a number of previous studies that wave-induced temperature variances is dominated by the longer horizontal wavelengths, which by the dispersion relation, are of low frequency. A scale analysis of frequently-observed gravity wave spectra leads Fritts [1984] to the conclusion that only about 1/3 of the momentum is transported by waves with periods longer than 2 h. Fritts and Vincent [1987] corroborate this with observations that approximately 70% of the mesospheric momentum flux is transported by gravity waves with periods less than 2 h. Using the dispersion relation and typical observed vertical wavelengths of 10 km, and typical Brunt-Väisälä periods of 5 mins, these momentum transporting waves should have horizontal wavelengths of roughly several hundred kilometres.

1.3.2 Observations of Gravity Wave Characteristics

The basic characteristics of gravity waves are commonly identified by the observed small-scale fluctuations in profiles of wind velocity, density, and temperature of the background atmosphere. A wide variety of measurement techniques have been applied to study these fluctuations throughout the lower and middle atmosphere. These measurements provide us enormously the information of gravity waves characteristics and activities in the middle atmosphere (for reviews, see Fritts [1984, 1989] and Tsuda et al. [1994]). Brief details of the conventional observational techniques in use so far are given below along with their shortcomings, which might be overcome in the MLS experiments.

Gravity Wave Observations in the Middle Atmosphere

A number of recent observational techniques have been employed to study small-scale atmospheric fluctuations (density, temperature, and background wind profiles in particular) which are usually associated to propagating gravity waves in the middle atmosphere. Before the UARS MLS instrument was in the operation, most of the information on gravity waves in the middle atmosphere have been provided mostly by radar, lidar, rocket sounding, balloon-borne, and satellite-based (in infrared region) measurements (see Figure 1.2 for examples of wave-induced temperature fluctuation profile observed with lidar experiments). However, all these techniques still have some shortcomings that might limit their ability in monitoring gravity wave variability and activities in global scale like the UARS MLS, or EOS MLS, does.
Figure 1.2: Examples of the average atmospheric temperature and their corresponding fractional temperature perturbation profiles in the lidar experiments. The dotted line in the upper figure corresponds to an adiabatic lapse rate of $-9.8 \text{K/km}$ and labels on the temperature scale apply only to the first profile, while subsequent profiles are shifted by 20 K (from Whiteway and Carswell [1995]).

Figure 1.2 illustrates examples of the vertical structure and temporal variability of temperature fluctuations and their corresponding fractional temperature perturbation profiles in the 30- to 60km region from the lidar experiments for the night of March 19 and 20, 1992 reported by Whiteway and Carswell [1995]. A distinct dominant vertical wavelength was often observed in these experiments but there was substantial variability within an observational period of several hours. However, the case of March 19 and 20 shown above was much more typical with a time evolution of the dominant vertical wavelength. The increase of wave amplitude with height is also apparent but not quite remarkably as the theory predicted. The adiabatic lapse (dotted lines) is shown in order to identify regions of convective instability as, in principal, it is impossible for wave amplitude to grow beyond the point of inducing marginal convection instability (the saturation point). Note that, the magnitude of the wave-induced temperature perturbation shown here is always less than $\approx 2$-3 % which is in accordance with the fluctuation of amplitude less than $\approx 5$-8 K if the typical background temperature of 250 K of the middle atmosphere is taken into account.
The observations of gravity wave structure, distribution, and variability, with conventional methods mentioned earlier reveal a wide range of spatial and temporal scales of the wave that exist in the middle atmosphere. These observed gravity-wave scales include horizontal wavelengths of ~10-1000 km, vertical wavelengths of 1-20 km, horizontal phase speed up to 100 m s\(^{-1}\), and intrinsic frequencies from inertial frequency, \(f \leq 2\pi/(12 \text{ hours})\) to the Brunt-Väisälä frequency, \(N \sim 2\pi/(5 \text{ minutes})\) [Fritts 1984]. Recent observations suggest vertical wavelength scales of 10-15 km and horizontal wavelengths longer than several hundred kilometres are typical for middle atmosphere gravity waves while the vertical wavelengths of less than 10 km may dominate the wave in the lower atmosphere [e.g. Meek et al. 1985; Smith et al. 1987; Fritts et al. 1989]. The vertical wavelength of a gravity wave motion, in particular, is an important quantity. This because, as explained in Chapter 3, it is seen to yield a direct measure of the intrinsic phase speed of the wave motion provided that the local buoyancy frequency \(N\) is known and horizontal wavelength is relatively much longer than the vertical wavelength (see Eq. 3.20). These are waves that most observational methods except satellite limb measurement mostly observe. Thus a distribution of vertical wavelengths in the middle atmosphere implies some knowledge of the range of gravity wave phase speeds and the potential for wave mean-flow interaction in regions of wind shear.

**Disadvantages of the Conventional Techniques**

We now discuss the shortcomings of some conventional observational methods, i.e., radar, lidar, rocket sounding, and radiosounde. Normally, the horizontal and vertical wind fluctuations could be observed by radars both in the troposphere and lower stratosphere [Balsley and Garello 1985; Fritts et al. 1988] and in the upper mesosphere [Vincent 1984; Meek et al. 1985; Vincent and Fritts 1987] but radars are generally blind in most part of the middle atmosphere (at about 20-60 km altitudes). This is because this altitude range is either too high or low to have sufficient back-scattered signals so they provide little information about the middle atmosphere in this region. In the contrary, a large number of data from rocket soundings have allowed statistically the study of gravity wave characteristics from wind and temperature fluctuations in the 20-60km altitude [Hirota 1984; Hirota and Niki 1985; Eckermann et al. 1994], however, these data are still obtained in a sporadic manner. Balloon-borne radiosonde instruments are good at giving access to the fine structure of the fluctuation fields [Barat 1982; Cot and Barat 1986; Tsuda et. al. 1991] but the altitude range is usually limited up to \(\approx 30\) km only.
Rayleight lidar offers unique feature of high resolution routine measurements of the density and temperature fluctuations, in the 30-70 km altitude range where radars and in situ measurements are not possible and rocket soundings sporadic [Chanin and Hauchecorne 1981; Hauchecorne et al. 1987; Wilson et al. 1991a, b; Duck et al. 1998]. Nevertheless, all but the most powerful one operable only on cloud-free nights. Satellite remote-sensing (in infrared region) has previously provided global measurements of gravity wave variances with large (100's km) horizontal scale [Fetzer and Gille 1994; Picard et al. 1998], but not of much smaller-scale features because of spatial averaging and sparse sampling. In addition, recent observations of the cloud-free atmosphere by the SPIRIT 3 radiometer on the MSX satellite have provided, for the first time, high resolution mid-wave infrared imagery of wave-like structure at stratospheric altitudes near 40 km which is believed to be originated from internal a monochromatic gravity wave [Picard et al. 1998; Dewan et al. 1998].

It should be noted that because all these techniques (except the satellite observations) are essentially based on single-station observations to obtain the physical quantity of gravity waves, they yield good temporal and vertical resolutions usually at one geographic station. The observable parameters at each station are vertical wavelengths and Doppler-shifted frequency, in addition, to the mean field. Horizontal wavelength and intrinsic frequency, for example, have to be mostly estimated directly with the aid of theoretical considerations (e.g., the dispersion relation) by assuming the dominance of monochromatic waves.

As described later, the use of MLS instruments in studying the gravity wave in the 20- to 80-km altitude does offer the better opportunity than most conventional methods in the long-term observation of activity of the gravity waves in the middle atmosphere (with horizontal wavelength of ≈ 100-500 km). The very high sampling rates over most of the Earth and high signal-to-noise ratio in the measurements ensure high statistical significance. The MLS observations have sufficient spatial resolution to monitor the dominant gravity wave scales, by several thousand daily soundings, and by near-global coverage. For these reasons, they are an important supplement to more conventional observations mentioned above. The major drawback to the studying of small-scale features in the MLS data is its insufficient temporal resolution to infer frequency, or insufficient spatial resolution to detect the variation of all three components of the wave propagation vector at once. Therefore, some fundamental properties of gravity waves, especially their dispersion relation, cannot be directly tested with the experiments.
1.4 The UARS MLS Experiment

1.4.1 Mission Background

The Upper Atmosphere Research Satellite (UARS) launched by the Space Shuttle Discovery on 12 September 1991, is a NASA mission dedicated to the comprehensive and integrated study of the Earth’s middle atmosphere [Reber 1993]. The Microwave Limb Sounder (MLS) is one of ten instruments on board the UARS. It is designed primarily to measure profiles of molecular abundances (ClO, O$_3$, H$_2$O) using thermal emission features in microwave spectral region near 63 GHz (for pressure and temperature measurements), 183 GHz (for H$_2$O and O$_3$ measurements) and 205 GHz (for ClO and O$_3$ measurements) [Waters 1993].

The altitude range of measurements for chlorine monoxide (ClO) is from ~15-45 km, ozone (O$_3$) from ~15-80 km, water vapour (H$_2$O) from ~15-85 km, and temperature and pressure from ~30-60 km (see Figure 1.3). The pressure measurements provide the vertical reference for composition measurements in the data analysis.

The UARS is a near circular orbit satellite at altitude ~600-km height with an inclination of 57° to the equator, and the orbit plane processes by 360° every 72 days. MLS samples the Earth’s limb 90° from the orbital velocity on the shaded side of the satellite along a minor circle (tangent track) where the tangent point of the observation path (where the signals mostly originated) is approximately 23° away from the sub-orbital track of the satellite. The 57° inclination of the URS orbit thus allows MLS to perform measurements over a latitude range from 34° in one hemisphere to 80° in the other (see Figure 1.6), and the hemisphere receiving maximum coverage reverses approximately every 36 days when UARS executes a 180° yaw manoeuvre. There are approximately 15 orbits per day, and the orbit precesses slowly with respect to Sun-Earth direction so that the local solar times at measurement locations do not vary appreciably with longitude in a given day but can vary greatly with altitude. The UARS orbit plane processes so that all local times are covered at all latitudes during each ~36-day period. All UARS MLS measurements are performed continuously, day and night. Limitations on spatial resolution due to radiative transfer through the limb are typically ~2-3 km in the vertical and ~300 km in the horizontal direction along the line-of-sight (LOS).
Figure 1.3: Vertical range of published measurements obtained from UARS MLS experiment where $T$ is temperature and $P$ is pressure. Solid lines indicate useful individual profiles. Dotted lines indicate zonal (or other) means, and the dashed lines for ClO at lower altitudes indicate individual measurements when ClO is enhanced in the polar winter vortices (from Waters et al. [1999]).

Since the measurements are made every 2 seconds, the horizontal distance between adjacent points is approximately 15 km (i.e., the distance travelled by the spacecraft in this time interval). In normal limb-scanning mode, a vertical scan in discrete steps over the altitude range ~0-100 km is performed every minute with approximately 32 measurements included. It is important to note that the measured radiance in each measurement does not correspond to an exact point in space but represents an average value which has been integrated over a three-dimensional volume of atmosphere (see Section 3.4 for more detail). An important feature of MLS is that its measurements are not degraded by ice clouds (such as the polar stratospheric clouds on which heterogeneous chemistry detrimental to ozone can occur) or volcanic aerosols (such as those from the Pinatubo volcano which erupted a few months before the UARS launch). The composition measurements are fairly insensitive to variations or uncertainties in atmospheric temperature. Overall calibration accuracy is ~3%, and radiometric calibration is done on each limb scan by observing an ambient blackbody target and cold space [Barath et al. 1993].
1.4.2 The Significance of the Experiment

The UARS MLS experiment has proved very successful so far in their original missions especially in the study of ozone depletion in the polar stratosphere. It has long been recognised that chlorine can catalytically destroy stratospheric ozone [Molina et al. 1974] which is of great concern as industrial chlorofluorocarbons (CFC’s), the major source of atmospheric chlorine, have led to a pronounced increased in stratospheric chlorine content [WMO 1990]. The first discovery of the Antarctic ozone hole [Farman et al. 1985] and subsequent measurements of this phenomena have revealed the important role of stratospheric heterogeneous chemistry [Molina et al. 1987] which is taking place in polar stratospheric clouds [Solomon 1990] and could lead to greatly enhanced chlorine and subsequent destruction of chlorine.

The global measurements of ClO by the MLS is very useful in monitoring the destruction of ozone (O₃) in the stratosphere. This because ClO is the rate-limiting molecule in the chlorine destruction of O₃ and its abundance gives a measure of the rate at which chlorine destroys ozone. UARS MLS is the first instrument to measure ClO on a global scale, and coupled with its measurements of stratospheric O₃, is providing an important contribution to the understanding and monitoring of ozone destruction by chlorine [Waters et al. 1993; Manney et al. 1994; Waters et al. 1996].

The UARS MLS water vapour measurements are also giving valuable information on the distribution and variability of stratospheric water vapour [Harwood et al. 1993; Carr et al. 1995]. Water vapour plays an important role in radiative and photochemical processes in middle atmosphere, while its distribution may help explaining on the dynamical processes operating in the stratosphere and mesosphere. Stratospheric water vapour is an important greenhouse gas through its absorption and emission of infrared radiation and so accurate knowledge of its distribution is required for global climate modelling. Water vapour is also one of the primarily molecules involved in the production of hydrogen-oxygen (HOₓ) compounds. These highly reactive HOₓ compounds are of particular importance to stratospheric chemistry, especially OH, which plays a significant role in the photochemistry of stratospheric O₃ [Shimazaki 1985]. Simultaneous MLS measurements of H₂O and O₃ by 183-GHz radiometer to higher altitudes (up to ~80 km), higher than previously explored on a global basis, have significantly provided additional information on middle-atmosphere ozone chemistry.
Secondary MLS measurement goals, beyond that for which the instrument was primarily designed, include volcanic injections of SO\textsubscript{2} injected into the stratosphere by the Pinatubo volcano [Read et al. 1993], upper-tropospheric H\textsubscript{2}O (e.g. Read et al. 1995), lower-stratospheric HNO\textsubscript{3} (Santee et al. 1995; Santee et al. 1997; Santee et al.1998), temperature variances associated with gravity waves in the stratosphere and mesosphere [Wu and Waters 1996a,b, 1997; McLandress et al. 2000]. The vertical range of measurements published to date from UARS MLS experiment is as shown in Figure 1.3. Recent work has extended the temperature measurement upward to \~85 km altitude.

The UARS MLS experiment is led by Jet Propulsion Laboratory (JPL) in the United States with collaboration from Heriot-Watt and Edinburgh Universities, and Rutherford Appleton Laboratory in the United Kingdom. Validation of the MLS primary data products, and their accuracies and precisions, are described in a special issue of the Journal of Geophysical Research (Vol.101, No. D6, 30 April 1996) on UARS data evaluation: temperature and pressure by Fishbein et al. [1996]; O\textsubscript{3} by Froidevaux et al. [1996], Cunnold et al. [1996a, 1996b], and Ricaud et al. [1996]; H\textsubscript{2}O by Lahoz et al. [1996]; and ClO by Waters et al. [1996]. A comprehensive review on the achievement of the UARS MLS experiment is given by Waters et al. [1999] and relevant references therein.

1.4.3 MLS Instrument Overview

The typical microwave radiometer of UARS MLS uses the so-called heterodyne (or superheterodyne) principle, where both the technique and the terminology date from the early days of radio. A heterodyne receiver is one in which the received signal, called the radio-frequency, or RF, signal, is translated to a different and usually lower frequency (the intermediate-frequency, or IF, signal) before it is detected.

UARS MLS has three radiometers, each based on a superheterodyne receiver (see Barath et al. [1993] for a description). For the purpose of the experiment, the instrument can be divided into three subsystems, the antenna, radiometers, and filter banks. Figure 1.4 shows signal flow block diagram of the UARS MLS instrument. The limb is scanned by the three-mirror antenna, which focuses limb emission into three horns of radiometers. The switching mirror accepts radiation either from the antenna, from an internal target, or from space. The space and target views are used during ground data processing for radiometric calibration of the instrument by optimised algorithms [Peckham 1989].
A dichroic plate following a switching mirror separates a signal to the 63-GHz radiometer (for 62.998- and 63.569-GHz O₂ lines). A polarisation grid then separates signals to the 183-GHz radiometer (for 183.310-GHz H₂O and 184.378-GHz O₃) and to the 205-GHz radiometer (for 204.352-GHz ClO, 206.132-GHz O₃, 204.575-GHz H₂O₂, and several weak lines of HNO₃). The RF signal measured by each radiometer is combined with a constant-frequency signal generated by a local oscillator (LO) by a double-sideband mixer. Local oscillators are at frequencies of 63.283, 184.777, and 203.267 GHz. From this process, the superimposed signals are down-converted to intermediate frequency (IF) bands in the range of 0-3 GHz. The radiometers have approximately equal response at IF frequencies above and below the local oscillator frequency. The IF signals, after amplification, are further frequency-converted to six spectral bands, each centred at 400 MHz with approximately 500-MHz instantaneous spectral bandwidth. These bands are input to six filter banks that split the signal into 15 separated spectral channels, simultaneously measure the power in each of these channels, and digitise the result for transmission to ground for data processing via the Command & Data Handling microprocessor. Table 1.1 shows spectral detail and the primary retrievals from each of the six spectral MLS measurement bands.
Table 1.1: Microwave Limb Sounder (MLS) Radiometers, Spectral Bands, and Primary Measurements

<table>
<thead>
<tr>
<th>Radiometer</th>
<th>LO Frequency (GHz)</th>
<th>Wavelength (mm)</th>
<th>Band Designation</th>
<th>IF Range (MHz)</th>
<th>Primary Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63.283</td>
<td>4.74</td>
<td>B1</td>
<td>90-540</td>
<td>pressure, temperature</td>
</tr>
<tr>
<td>2</td>
<td>203.267</td>
<td>1.48</td>
<td>B2</td>
<td>830-1340</td>
<td>ClO</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B3</td>
<td>1053-1563</td>
<td>H$_2$O$_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B4</td>
<td>2610-3120</td>
<td>O$_3$</td>
</tr>
<tr>
<td>3</td>
<td>184.778</td>
<td>1.62</td>
<td>B5</td>
<td>1213-1723</td>
<td>H$_2$O</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B6</td>
<td>145-655</td>
<td>O$_3$</td>
</tr>
</tbody>
</table>

Individual filter bandwidth varies from 128 MHz on band edge to 2 MHz at band centre. The 15 collective 63-GHz channels are refereed to as “band 1” and are numbered 1-15. Channels have similar characteristics on each side of band centre at channel 8 (e.g., channels 2 and 14) and are narrower toward the centre. The signal to noise at each channel is roughly proportional to the square root of the channel width. Table 1.2 gives the nominal positions (offset from line centre) and widths of these 15 spectral channels of MLS radiometers. The instrument integration time for each minor frame is ~2 seconds and a vertical scan in discrete steps over the altitude range ~0-100 km is performed each minute.

It should be noted here that the MLS optics are diffraction-limited by the aperture of the primary mirror shown in Figure 1.4 whose dimension is 1.6 meters in the vertical and 0.8 meters in the horizontal. The half-power beamwidth of MLS FOV could be approximated by $\Theta_{HP} \equiv 1.28\lambda/D$, where $D$ is the antenna diameter, therefore, it will have $\Theta_{HP} \equiv 0.8\lambda$ radians in vertical and $1.6\lambda$ radians in horizontal direction, or $\equiv 45.84\lambda$ degrees and $91.67\lambda$ degrees, respectively. For example, half-power beamwidth for the 63-GHz measurements is ~0.2° in vertical and ~0.4° in horizontal directions for UARS MLS [Jarnot et al. 1996]. The knowledge of half-power beamwidth of MLS antenna is necessary in the construction of three-dimensional temperature weighting function for the use in model simulation of gravity wave observation with the MLS instruments as described in principle in Chapter 3.
Table 1.2: Positions and Widths of MLS Spectral Channels

<table>
<thead>
<tr>
<th>Channel</th>
<th>Frequency Channel offset (MHz)</th>
<th>Channel bandwidth (MHz)</th>
<th>Channel</th>
<th>Frequency Channel offset (MHz)</th>
<th>Channel bandwidth (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-191</td>
<td>128</td>
<td>9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-95</td>
<td>64</td>
<td>10</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>-47</td>
<td>32</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>-23</td>
<td>16</td>
<td>12</td>
<td>23</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>-11</td>
<td>8</td>
<td>13</td>
<td>47</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>-5</td>
<td>4</td>
<td>14</td>
<td>95</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>-2</td>
<td>2</td>
<td>15</td>
<td>191</td>
<td>128</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

UARS was designed for an 18-month duration mission, but until recently the MLS (along with most other UARS instruments) continues to operate after ~7 years in orbit with no degradation in its 63- and 205-GHz measurements except for time-sharing of these measurements with those of other UARS instruments due to decrease in power available from the spacecraft. The MLS 183-GHz radiometers failed in April 1993 after 18 months of excellent data had been obtained. The 63-GHz radiometer was turned off 13 July 1997 to reduce MLS power consumption due to failure of a UARS battery. More recent information on UARS MLS experiment could be found from its website at http://mls.jpl.nasa.gov.

1.5 Gravity Wave Observations with UARS MLS

In this section we present a brief review on the gravity wave study in the middle atmosphere as reported by Wu and Waters. There are 3 parts in this section. Section 1.5.1 explains in detail the radiance measurements with MLS 63-GHz radiometer. Then, the temperature weighting functions of the observations for the saturated MLS channels are shown in Section 1.5.2. In Sections 1.5.3, the method to derive wave-associated variances from the small temperature fluctuations appeared in saturated MLS radiance profiles is described and examples of global variance maps produced according to this method are also given.
1.5.1 MLS 63-GHz Radiance Measurements

Now, we shall consider the radiance measurements with UARS MLS 63-GHz radiometer which measures the radiation near 63 GHz originated mainly from the emission by molecular oxygen (O₂) along the ray path. There are two general scanning mechanisms that have been implemented in the MLS operation for atmospheric sampling which are called limb-scanning and limb-tracking modes. Details of these two operating modes are as follows.

**MLS Limb-Scanning Mode**

In the routine limb-scanning mode, the instrument step-scans the atmospheric limb in ~65 seconds at 2 seconds/step from ~100 km to the surface producing increments of ~5 km in the mesosphere and 1-3 km in the stratosphere. Figure 1.5 shows an example of observed radiance profiles, described in terms of brightness temperatures, from a single scan of the MLS 63-GHz radiometer, which resolves an O₂ emission line into 15 spectral channels for retrieving atmospheric tangent pressure and temperature. (Note that the radiance profiles of channels 9-15 are not shown here but they are similar to those of channels 7-1 respectively.)

![Figure 1.5: An example of MLS radiance profiles near 63 GHz as the instrument step-scans from the mesosphere to the surface (~0-100 km altitudes). The tangent height of 18-km for the limb-tracking mode is also marked as a reference (from Wu and Waters 1997).](image-url)
From these profiles, we see that as the instrument scans down into the mesosphere region, the observed radiance at line centre (channel 8) starts saturating (having constant value regardless of the tangent height) at tangent height ~80 km. Radiances near the line centre saturate at higher altitudes than those near line wings due to stronger line absorption. However, all radiances are saturated when the instrument views tangent heights below ~18 km. The saturated radiance is a sign that the observed atmosphere is optically thick and the absorption along the ray path is very strong. In this situation, the observed radiances depend little on tangent height of observation, which is quite contrary to the unsaturated radiances observed at high tangent altitudes in which the dependence of the radiances on tangent height (or tangent pressure) is very strong. The saturated radiance is a good measure of atmospheric temperature at various altitude layers in the middle atmosphere and it is also very sensitive to small changes in the unperturbed atmospheric temperature at that layer (see Figure 2.14a for example). Thus, as the satellite moves along, the variations in MLS saturated radiances could reflect atmospheric temperature variations along the observing path. It is the saturation of the MLS radiances that is necessary for the use in the study gravity wave perturbation with the MLS instrument described later in Chapter 3.

**MLS Limb-Tracking Mode**

Radiance measurements in the non-standard limb-tracking mode has been implemented periodically since December 1994 and was hoped to provide a more useful information for studying small-scale variability in the atmosphere than the normal limb-scanning mode.

In this operating mode, the instrument views the limb at approximately constant tangent height of 18 km and performs its usual 2-second sampling frequency as in limb-scanning mode (equivalent to ~15km horizontal resolution along the suborbital track). Since all 63-GHz channel radiances are mostly saturated in the 18-km-limb-tracking observations (as seen in Figure 1.5), the MLS instrument in this operating mode could then measure atmospheric temperature continuously with a good horizontal resolution and long sequence of measurements. We shall prove in Chapter 5 for UARS MLS and Chapter 6 for EOS MLS that such a data set of radiances from these measurements is more reliable than that obtained in the limb-scanning mode for the use in the study of gravity waves with the MLS instruments. And, from this reason, we will report mostly on the possibility of gravity wave observation with the instruments operating in limb-tracking mode in this thesis.
Figure 1.6 shows the sampling tracks for the limb-tracking mode on 28 December 1994 with a high sampling rate along the observing track. As illustrated in the inset, the locations of individual measurement are separated by horizontal displacements of ~15km with a larger gap when the instrument takes 6 seconds for calibration. The limb-tracking mode was used nearly continuously during 23-30 Dec. 1994, 1 Feb.-20 Mar. and 7-15 Apr. 1995, and schedules for every-third day operation since while MLS is on.

![Sampling tracks](image)

**Figure 1.6**: UARS MLS sampling tracks on 28 December 1994 marked by the first measurement of each major frame (~65 seconds). The inset details the set of individual measurements in a single major frame with the short lines indicating the orientation of the temperature weighting functions. On this day MLS was preferentially observing the Northern Atmosphere. The ascending portion of orbit 1 is highlighted with solid lines (from Wu and Waters 1997).

The radiance brightness temperature observed at 18-km tangent height is a strong function of latitude due to atmospheric temperature climatology. As shown in Figure 1.7a for 28 December 1994, the radiance brightness temperature from channels 3-5 (at 38-48 km altitudes) have large gradients near the edge of the polar vortex (~40-50°N) that dominate all other variations. The weaker latitudinal gradients in channels 6-8 radiances suggest that the vortex is weakening at altitude above 50 km. Coherent variations can be seen in the radiance fluctuations at different channels (or altitudes) with scales from hundreds to thousands of kilometres. Coherent variations seen in channels 7 and 8 at northern high-latitudes extending over thousands of kilometres are expected due to strong wintertime planetary waves. Small-scale variations are also evident in many places.
Figure 1.7a: Channels 3-8 radiance measurements in limb-tracking mode at 18-km tangent height from the ascending part of orbit 1 on 28 December 1994 (from Wu and Waters 1997).

Figure 1.7b: Radiance fluctuations derived from Figure 1.7a with large-scale (>1000km) variations removed. Radiance fluctuations of each channel are displaced by 5 K with the channel number indicated at the left of each measurement series (from Wu and Waters 1997).
Although the small-scale temperature fluctuations are well displayed in the MLS radiance profiles, the interpretation of large-scale radiance variations from channels 7, 8, and 9 (not shown in Figure 1.7a), should be treated with caution. This because these channels are close to the O\(_2\) line centre and can vary by up to 3-8 K due to Zeeman effect associated with Earth's magnetic field. These variations generally change slowly along the orbital track except above 70°N and below 25°S (during this observing day) where the viewing angle changes rapidly with respect to the magnetic field lines.

The small-scale fluctuations can be seen more readily in Figure 1.7b where large-scale (> 1000 km) variations are removed from the radiances in Figure 1.7a. The filtered data in Figure 1.7b are obtained by differencing the raw and smoothed data (i.e., average over ~1000 km). The radiance fluctuations from the first ascending orbit show a magnitude of 0.5-3K at 10°-30°S over South America where deep convection is known to be strong during this period. The oscillations with an amplitude of ~1-2 K are apparent at mid- and high-latitudes of the Northern Hemisphere, but are not as strong as in the Southern Hemisphere. This is likely due to the unfavourable viewing angle from the ascending orbit observing these propagating waves. As shown later in the variance maps (Figure 1.10), the observed radiance fluctuations are actually strong at these latitudes if one combines both ascending and descending measurements with viewing angles. The important role of background wind speed is also evident in these observations, as has been emphasised in the recent numerical simulations [Alexander 1996]. Phase coherence and amplitude growth with height are clearly seen at some latitudes, suggesting the presence of vertically propagating waves. The 0°-30°N region is relatively quiet where the fluctuations are mainly due to instrument noise. Radiance measurements from channels 1, 2 and 14, 15 are not shown because these channels are not fully saturated at high altitudes with the 18-km-limb-tracking mode and pointing variations may contaminate the radiance variances.

The observed phase coherence and amplitude growth with height of the temperature variations indicate the interference of gravity waves in MLS measurements in the middle atmosphere. However, to extract the gravity wave data from these observed fluctuations in MLS radiance profiles as shown in Figure 1.7b is still rather difficult due to the reason given earlier. More accurate interpretation on the relations between MLS radiance variations and the parameters (wavelength scale and direction in particular) of wave that produce them requires further study using model analysis of MLS radiance measurements with gravity waves included. And this is the main objective of this thesis.
1.5.2 Temperature Weighting Functions

As mentioned earlier, the instrument’s weighting function and sampling patterns are keys to determine scales of waves that are visible in MLS experiments. We will briefly explain here the roles of the MLS temperature weighting functions in the observation of gravity waves in the middle atmosphere with MLS instruments. The theoretical derivation of the weighting functions presented here is discussed in detail in Section 2.4.

Figure 1.8: Temperature weighting functions of saturated channels 1-15 of the MLS 63-GHz radiometer when the instrument viewing the limb at 18-km tangent height.

Figure 1.8 shows examples of the MLS temperature weighting functions for the observations at 18km-tangent-height calculated by Eq. 2.40 in Chapter 2 showing eight altitude layers (with ~10-15km thickness) range from ~30 to 80km where the temperature is measured by the saturated radiances of different channels. As the weighting function represents the contribution from each atmospheric later to the observed radiance at each channel, it is indicated from this figure that the measured radiance in this case, originate from a localised layer at a given height with a resolution of about 10-15km. There is no contribution from the tangent height layer (~18km altitude) to the observed radiances in this case due to the very strong absorption of the atmosphere along the ray path. Table 1.3 summarises the key parameters of weighting functions and noise for each of MLS channel.
Table 1.3: The 63-GHz Channel Parameters for 18km-Tangent-Height Observation

<table>
<thead>
<tr>
<th>Channel</th>
<th>Approximate Height (km)</th>
<th>Layer Thickness (km)</th>
<th>Noise (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 15</td>
<td>30</td>
<td>10</td>
<td>0.07</td>
</tr>
<tr>
<td>2, 14</td>
<td>35</td>
<td>10</td>
<td>0.08</td>
</tr>
<tr>
<td>3, 13</td>
<td>40</td>
<td>10</td>
<td>0.12</td>
</tr>
<tr>
<td>4, 12</td>
<td>44</td>
<td>10</td>
<td>0.18</td>
</tr>
<tr>
<td>5, 11</td>
<td>48</td>
<td>10</td>
<td>0.26</td>
</tr>
<tr>
<td>6, 10</td>
<td>53</td>
<td>10</td>
<td>0.37</td>
</tr>
<tr>
<td>7, 9</td>
<td>62</td>
<td>10</td>
<td>0.49</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>15</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The saturated channels in limb-tracking mode are very sensitive to the small changes in background temperature at the layer defined by the weighting function in Figure 1.8. As a consequence, the multichannel observations with MLS 63-GHz radiometer in this mode should therefore map simultaneously natural variation of background temperature at different heights with a reasonable height resolution and this would give a great advantage to the gravity wave study in the middle atmosphere. It should be noted that, because the UARS MLS line-of-sight (LOS) direction is perpendicular to the orbit velocity, horizontal averaging in each measurement are then ~100-300 km cross-track (perpendicular to suborbit path) due to radiative transfer through the limb path, and ~30 km along-track (parallel to suborbit path) due to the antenna FOV smearing. These vertical and horizontal averagings can substantially reduce the magnitudes observed from actual atmospheric temperature fluctuations, but they are still detectable due to low radiometer noise (varying from 0.07K for channels 1 and 15, to 0.45K for channel 8) as shown in Table 1.3.

In the next section, we shall outline the variance analysis methods that have been used in the derivation of UARS MLS gravity wave variances reported so far. These methods are different for the usable data from the observations in limb-tracking and limb-scanning modes as described below.
1.5.3 Temperature Variance Analysis

The data used in the MLS variance analysis are optically thick (or saturated) radiances from both limb-scanning and limb-tracking modes. In limb-scanning mode, where the atmospheric limb is scanned from ~100 km to the surface, only the radiances from 6 measurements at the bottom portion of each 32-measurement scan are saturated and thus usable for the analysis. In limb-tracking mode, where the limb is viewed at a fixed height of ~18 km, all 32 measurements in a single scan are saturated and usable in the study.

To calculate the temperature variance in the 6-measurement limb-scanning mode, a linear variation is firstly removed from the 6 radiancé measurements, which remove effects of the weak tangent pressure dependence and large-scale wave modulations. Subsequently, the estimated total radiance variance for a given channel, \( \langle \sigma^2 \rangle \), is then defined as

\[
\langle \sigma^2 \rangle = \frac{1}{4} \sum_{i=1}^{6} (y_i - a - bz_i)^2 , i=1,2,\ldots,6
\]

where \( y_i \) and \( z_i \) denote, respectively, measured radiance temperature of the channel and tangent heights. Parameters \( a \) and \( b \) are determined from the linear least squares fit to the 6 measurements, and \( 4 \) is the degree of freedom. The total radiance variance, \( \sigma^2 \), is mainly due to atmospheric temperature fluctuations, instrument noise and non-linear terms of the pressure variation, namely,

\[
\sigma^2 = \sigma_{gw}^2 + \sigma_N^2 + \sigma_{NL}^2
\]

where \( \sigma_N^2 \) is the variance due to instrument noise for the given channel and is known from instrument calibrations in each limb scan. The non-linear pressure contribution, \( \sigma_{NL}^2 \), is small and only important for channel 1,15 and 2,14, and can be reasonably estimated from radiance models (when these channels are used in the analysis).

As a result, the atmospheric fluctuation, \( \sigma_{gw}^2 \), can be derived by subtracting \( \sigma_N^2 \) from the estimated total radiance variance \( \langle \sigma^2 \rangle \). We could now interpret the atmospheric temperature fluctuations contributing to the radiance variance as a manifestation of upward propagating gravity waves as stated earlier. Other fluctuation sources, such as the antenna pointing, are either insignificant or very occasional and, therefore, neglected in their study. This method effectively retains only the fluctuations of horizontal scales less than ~100 km to be taken into account.
For the variance analysis in limb-tracking mode, all 32 measurements in a single scan can be used in the calculation but the data are Fourier-filtered first along the orbit track to retain only horizontal scales of less than ~ 500 km (the approximate horizontal distance covered by the 32 measurements). The variance is then computed from these 32 filtered radiances by the method as described in Chapter 3. A typical day of scan or track radiance data yields approximately 1400 variance measurements per channel. Since the channels are symmetrically located about the line centre, the average variances from opposite pairs of channels are typically reported. The uncertainty in the estimated total radiance variances is the fundamental limit for detecting weak gravity wave signals, and this depends on the number of data points averaged and the instrument noise. To reduce this uncertainty, Wu and Waters generally average variance data from the measurements over a period of time for each latitude-longitude grid.

Global Map of Gravity Wave Variances

UARS MLS sampling covers latitude from 34° in one hemisphere to 80° in the other and UARS make 10 yaw manoeuvres per year allowing alternating views of high altitudes in the two hemispheres. Figures 1.9a and b shows examples of the 40-day-averaged global variance map in 32-measurement limb-tracking mode for two periods near solstices which are (a) January (20 December 1992-29 January 1993) and (b) July (18 June to 28 July 1993), centred on UARS yaw days [Wu and Waters 1997]. Both figures show the resulting maps at seven altitudes in the middle atmosphere, and the striking features in these maps are large amplitudes associated with the stratospheric polar vortex in the winter hemisphere and subtropical land masses in summer hemisphere. These features evolve with height and change remarkably above the stratopause.

Background winds are expected to play a major role in determining the gravity-wave variance amplitudes observed with MLS. Regions with strong winds are where gravity waves are most observable partly due to the Doppler shift to longer vertical wavelength of the observed waves. Theoretical studies [Schoeberl and Strobel 1984; Miyahara et al.1987] show that a strong background wind is a favoured condition for gravity waves to propagate vertically because of the large intrinsic phase speed (i.e., difference between horizontal wave phase speed and the background wind) that prevents the wave from breaking. The background wind in the lower atmosphere then acts like a filter in selecting the waves that could propagate to the higher atmosphere (see for detailed explanation in Chapter 3).
Figure 1.9a: Gravity wave variance maps for January (20 December 1992-29 January 1993) showing variances at seven altitudes in the middle atmosphere. Latitude and longitude bins are respectively 5 and 10 with more than 40 measurements in each grid point. The variances are in a unit of K^2 and coloured in a logarithmic scale, i.e., log_{10}(\sigma_{GW}^2), shown in the vertical panel on the right-hand side. Displayed wind data (up to ~1 hPa) are averaged over the same periods (from Wu and Waters [1997]).
Figure 1.9b: As in Fig. 1.9a but for July.
The enhanced variance associated with the stratospheric polar jet could then be interpreted by the selective filtering effect of the jet stream that acts to reshape the wave spectrum by allowing upstream propagating waves to grow more efficiently with height than others. This is the likely cause of the variance enhancement observed in the jet stream. As another result of background wind filtering, the variances in the subtropical hemispheres show larger amplitudes at the latitude 10°S-30°S in January where winds are stronger.

![Figure 1.10](image-url): Variance growth with height at different altitudes during “January” and “July” compared with the exponential growth expected for non-breaking waves. The variances are normalised by the squared mean brightness temperature [from Wu and Waters 1996a].

The height variation of normalised variance seen in Figure 1.10 reveals some important aspects of the propagating nature of the observed perturbations. Despite very different amplitudes in the lower stratosphere, these variances exhibit approximately the same growth rate with height throughout the stratosphere, which is consistent with the theoretical exponential growth for non-breaking gravity waves and rocket observations [Hirota and Niki 1985]. This property in variance growth further supports the gravity wave interpretation of the MLS radiance variance observations. Saturation of the normalised variances is observed in the mesosphere, implying wave breakdown and saturation and momentum drag at these altitudes [Fritts 1984]. The results from the global variance maps derived from UARS MLS radiance data are thus very useful in the study of gravity wave activity in the atmosphere but the better understanding of the capacity of instrument in the observations of gravity wave is still required the model simulation as done in this thesis.
1.6 Thesis Outline

The main objective of this thesis is to present the analysis on the capability of the MLS instruments in both cases of UARS MLS and EOS MLS in the observations of atmospheric gravity waves considered in terms of the amplitude response and variance response. The study is relied principally on the forward model developed in Chapter 2 to simulate the MLS radiance measurements in the model atmosphere where the gravity waves with different wavelength scales and orientations are included. It is hoped that the model results reported in this thesis could provide us better understanding on the capability of both the MLS instruments in the observations of the waves in the real atmosphere. This knowledge might lead to better diagnostic of these wave observed with both UARS MLS at present and with the EOS MLS in the near future. Contents for the rest of the thesis could be outlined as follows.

Chapter 2: The development of forward model for the use in the simulation of radiance measurements with MLS 63-GHz radiometer is described. Theory of microwave radiative transfer for oxygen emission and the applications of the model to our study are two main subjects to be considered.

Chapter 3: Background theory of atmospheric gravity waves and the way to include them in the forward model from Chapter 2 are described. The method to derive for the temperature response and variance response of the instrument to each individual wave is finally presented.

Chapter 4: Model results of the temperature response obtained from method explained in Chapter 3 are presented and discussed. The response results are expressed as a function of wavelength components and orientation of the waves that produce them.

Chapter 5: Model results of the variance response of the instrument to the observed gravity waves are presented and interpreted. The comparison of the responses obtained in both limb-tracking and limb-scanning mode of operation is also addressed.

Chapter 6: The possibility of gravity wave observations with the EOS MLS instrument is investigated. The response results are shown and compared to those of UARS MLS and some conclusions are derived.

Chapter 7: A summary of the main findings of the thesis is presented and brief discussion of the gravity-wave filter for different observational methods is given.
Chapter 2
The Forward Model

2.1 Introduction

As mentioned in Chapter 1, the main objective of this thesis is to study the capability of the MLS instruments in the observations of atmospheric gravity waves with different horizontal and vertical scales and orientation. The results of the study are obtained from the model simulation of the MLS radiance measurements in both limb-scanning and limb-tracking modes as detailed in the previous chapter but with gravity waves included. We present in this chapter the theoretical development of the so-called “forward model” which is the essential tool for the use in the simulation of MLS gravity wave observations performed in the study. The forward model described here is an analytical-numerical model developed to simulate the measurements of radiance profiles (expressed in terms of brightness temperature) made with the UARS MLS 63-GHz radiometer and EOS MLS 118-GHz radiometer. These observed radiances are assumed to originate purely from the thermal emission by molecular oxygen (O\(_2\)) at altitudes of \(= 0\)-120km in the model atmosphere which is assumed to be stationary, non-scattering, and under local thermodynamic equilibrium.

The rest of this chapter is divided into 5 main sections. In Section 2.2, the theory of microwave radiative transfer is described and the brightness temperature equation is derived as a result. Section 2.3 provides the description on the absorption and emission mechanism of microwave radiation by molecular oxygen (O\(_2\)) in the middle atmosphere. The oxygen absorption coefficient is then computed as a function of atmospheric temperature and spectral frequency. In Section 2.4, the applications of the model to simulate MLS radiance measurements are considered. Model results of the temperature weighting functions and simulated radiance are also displayed. In Section 2.5, the sensitivity of MLS instrument and its corresponding weighting function to the small change in background temperature is investigated and the perturbation form of the microwave radiative transfer equation in the saturation case is derived. Finally, a summary of the main work achieved in this chapter is presented in Section 2.6.
2.2 Microwave Radiative Transfer

The starting point on the development of the forward model used in satellite observations is the radiative transfer equation that describes the flow of radiant energy along the ray path to be measured by the radiometer onboard. The scalar form of this equation is remarkably simple in the Rayleigh-Jeans limit, and is sufficient to treat the large majority of microwave applications. We briefly outline here the formulation of this equation in microwave case. More general discussions of the radiative transfer theory could be found in several sources elsewhere [e.g. Chandrasekhar 1960; Liou 1980; Ulaby et. al. 1981; Tsang et. al. 1985; Goody and Yung 1989; Waters 1993, Salby 1996].

2.2.1 Radiative Transfer Theory

The classical form of the radiative transfer theory was developed by Chandrasekhar [1960]. The theory describes the intensity of electromagnetic radiation propagating in a general class of media that absorb, emit, and scatter the radiation. It is ideally suited for the study of radiative transfer in media such as atmosphere in which the flow of energy plays the central role. The main objective of the theory is to describe the variation of the radiation field in terms of the specific radiance intensity $I(v, s)$ along the ray path where $v$ is the radiance frequency and $s$ is any point along the path.

As illustrated in Figure 2.1, the variation of the radiance intensity at a point $s$ along a line in the direction of propagation is obtained by considering the sources and sinks of the radiation in a volume element along that line. This leads us to a differential form of the transfer equation:

$$\frac{dl(v, s)}{ds} = -\alpha(v, s)I(v, s) + J(s) \tag{2.1}$$

where $\alpha(v, s)$ is the extinction coefficient due to all involved atmospheric gases at frequency $v$ and position $s$, and $J(s)$ is a source term at that point. In the general theory, scattering into and from other directions can lead to both losses and gains to the intensity along a given direction and can be taken into account in the terms $J$ and $\alpha$. In this case, the first term on the right-hand side of Eq. 2.1 represents the loss of radiation by both absorption and scattering, and the second represents the total gained by thermal emission and scattering into the direction of propagation, respectively. However, effects of the scattering in microwave case
are practically small due to long microwave wavelengths compared to average radius of the scattering particles (e.g., cloud or aerosols). Moreover, the concentration of these particles in the middle atmosphere is rather low as most of them are confined to the lower atmospheric region. Therefore, we assume in our model simulation that the total power lost from the path of propagation due to scattering is relatively small and is ignored in the radiance calculation.

\[
\begin{align*}
    s &= s, \\
    I &= I(s) \\
    ds &= ds \\
    dI &= dI(s) \\
    RAY PATH & \\
    s &= s, \\
    I &= I(s) \\
    ds &= ds \\
    dI &= dI(s) \\
    dl &= dl(v, s) = -\alpha(v, s)I(v)ds + Jds
\end{align*}
\]

**Figure 2.1:** Geometry of the radiative transfer equation 2.1.

With the neglect of scattering in the calculation and the assumption that the observed atmosphere is homogeneous and in local thermodynamic equilibrium (LTE), the source function \( J \) in Eq. 2.1 then represents only contribution from the local thermal emission which is given by Kirchhoff’s law as

\[
J(s) = \alpha(v, s)B[v, T(s)]
\]  \hspace{1cm} (2.2)

where \( B[v, T(s)] \) is the Planck function,

\[
B[v, T(s)] = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}
\]  \hspace{1cm} (2.3)

where:

- \( \nu \) = frequency of radiation (in Hz),
- \( c \) = speed of light in vacuum, \( 2.998 \times 10^8 \) m/s,
- \( h \) = Planck constant, \( 6.626 \times 10^{-34} \) J s
- \( k \) = Boltzman constant, \( 1.381 \times 10^{-23} \) J/K,
- \( T \) = absolute temperature in degrees Kelvin (K).
By substituting for $J$ from Eq. 2.2 into Eq. 2.1, we obtain the radiative transfer equation in the new form

$$\frac{dI(\nu, s)}{ds} = \alpha(\nu, s)[B(\nu, T) - I(\nu, s)].$$  

(2.4)

This expression is known as Schwarzschild's equation and $\alpha(\nu, s)$ is now represented the absorption coefficient along the ray path. Hence, the equation of transfer (2.1) becomes a standard differential equation (2.4) for which the complete solution is readily obtained as

$$I(\nu, s_2) = I(\nu, s_1)e^{-\phi(s_1, s_2)} + \int_{s_1}^{s_2} \alpha(\nu, s)B[\nu, T(s)]e^{-\phi(s, s_2)}ds$$  

(2.5)

where the integration extends through the atmosphere along the ray path from some starting point $s = s_1$, to the point $s = s_2$ where the radiometer is located (see Figure 2.1).

Equation 2.5 is an integral form of the transfer equation 2.1 and is usually referred to as the Chandrasekhar's equation. It is indicated from Eq. 2.5 that in MLS radiance measurements, the observed radiance at the satellite could be expressed as a sum of two major contributions. The first one is called the background term, which accounts for the radiation due to the background surface (e.g. cold cosmic space for a limb sounding, and the Earth’s surface for a nadir sounding) reduced by the attenuation due to the propagation through the entire atmosphere. The second one is the atmospheric term, which accounts for the emission arising from the atmosphere along the ray path itself.

The dimensionless parameter $\phi(s_A, s_B)$ introduced in Eq. 2.5 is called the optical thickness between points $s_A$ and $s_B$ along the ray path defined as

$$\phi(s_A, s_B) = \int_{s_A}^{s_B} \alpha(\nu, s)ds.$$  

(2.6)

By the above definition, the optical thickness $\phi(s_A, s_B)$ is a measure of the cumulative depletion that the beam of radiation has experienced as a result of its passage through the layer between point $s_A$ and $s_B$.

\[\text{In this thesis we use } \phi \text{ as a symbol for optical thickness defined by Eq. 2.6. Other authors may use different symbols for this parameter.}\]
It should be noted that the absorption coefficient \( \alpha(v,s) \) shown in Eqs. 2.4 and 2.5 is the \textit{volume} absorption coefficient, which differs from the defined \textit{mass} absorption coefficient in Chandrasekhar's expression by a factor of \( \rho^{-1} \), where \( \rho \) is the density of the absorbing substance. The volume absorption coefficient used throughout this thesis has dimensions of reciprocal metres \( (m^{-1}) \), which could be related to another frequently-used values in decibels per kilometre by \( 1 \ m^{-1} = 4.34 \times 10^3 \ dB/km \) [Waters 1976]. The absorption coefficient is the important parameter that represents the intensity of the absorption (or emission) of electromagnetic radiation occurred in the transition process and its accurate form is rather complex. The formulation of oxygen absorption coefficient is to be given in Section 2.3.

\subsection*{2.2.2 Microwave Radiative Transfer Equation}

The radiative transfer equation 2.5 is valid for the radiation in all spectral frequency ranges. However, in microwave case, this equation could be presented in a simpler form called the \textit{brightness temperature equation}, which is commonly used in the microwave remote-sensing applications. The derivation of this equation is the subject to be done in this section.

\textbf{Definition of Brightness Temperature}

We introduce the microwave case by considering the low frequency range of microwave region where \( v \sim 1-300 \ GHz \) which gives \( h v < kT \) at typical atmospheric temperatures \( (T \sim 200-300 \ K) \). This condition is known as Rayleigh-Jeans limit which allows the Planck function \( B(v,T) \) in Eq. 2.3 to be approximated by

\[
B(v,T) = \frac{2v^2kT}{c^2} = \frac{2kT}{\lambda^2}
\]

in which \( \lambda (= c/v) \) is the wavelength of the radiation. The significant feature of this approximation is the linear relationship of the Planck function with physical temperature \( T \).

This naturally suggests us a scaling of the radiant intensity as

\[
T_b(v) = \frac{\lambda^2}{2k} I(v)
\]

This is the definition of the \textit{brightness temperature}, \( T_b \), a quantity that will play the role of radiant intensity in the microwave radiative transfer but that has been dimensionally scaled.
to give units of degrees Kelvin. It should be noted that this definition is exact and not an approximation in itself. Therefore, it could be applied as appropriate to the study of electromagnetic radiation without referring to the Rayleigh-Jeans approximation.

However, the definition of brightness temperature is not unique. The definition given in Eq. 2.8 is usually referred to as the Rayleigh-Jeans equivalent brightness temperature, and it must be distinguished from another frequently-used definition called thermodynamic brightness temperature. The latter is obtained by inverting the Planck function in Eq. 2.3, giving the temperature \( T \) of a blackbody that would produce the given radiance at that frequency. The Rayleigh-Jeans equivalent brightness temperature is particularly appropriate in a radiative transfer context because it is simply a scaling of intensity, and radiative transfer integrals are sums over intensities. The thermodynamic definition can lead to confusion in this context, although it has the advantage that the connection to physical temperature is strictly maintained. However, in microwave case, these two definitions are theoretically equivalent as far as the Rayleigh-Jeans approximation is concerned.

**Derivation of Brightness Temperature Equation**

By substituting the definition of \( T_b \) given by Eq. 2.8 in Eq. 2.5, we then have

\[
T_b(v,s_2) = T_{bo}e^{-\phi(v,s_2)} + \int_{s_1}^{s_2} \frac{T(s)}{r(v,T)} \alpha(v,s)e^{-\phi(v,s)} ds
\]

(2.9)

where the background brightness temperature \( T_{bo} \) is the is derived from the general boundary condition as

\[
T_{bo} = \frac{\lambda^2}{2k} I(v, s_1) = \frac{\lambda^2}{2k} B(v, T_c)
\]

(2.10)

where \( T_c = 2.74 \) K is the cosmic background temperature. The factor \( r(v,T) \) is defined as

\[
r[v, T(s)] = \frac{2kT}{\lambda^2} \frac{1}{B(v, T)} = \frac{kT}{h\nu} \left( e^{\frac{h\nu}{kT}} - 1 \right)
\]

(2.11)

which is equivalent to the ratio of the physical temperature \( T \) of a blackbody emitter to its defined brightness temperature \( T_b \), \( r(v,T) = T/T_b \). By expanding function \( r(v,T) \) in terms of \( h\nu/kT \), we have
Chapter 2

The Forward Model

\[ r(\nu, T) = 1 + \frac{1}{2!} \left( \frac{h \nu}{kT} \right) + \frac{1}{3!} \left( \frac{h \nu}{kT} \right)^2 + \ldots \]

where we see that \( r(\nu, T) \) is always greater than unity, and approaches unity in the Rayleigh-Jeans limit where \( h \nu kT << 1 \) and the two definitions of brightness temperature mentioned above are equivalent.

Equation 2.9 is the general form of the so-called brightness temperature equation. This equation is exact as far as the Planck law is concerned and is suitable to be used for most applications in microwave remote sensing without concerned for the errors introduced by the Rayleigh-Jeans approximation. The Rayleigh-Jeans approximation could be applied by setting \( r(\nu, T) = 1 \) in Eq. 2.9, giving

\[ T_h(\nu, s_2) = T_0 e^{-\phi(s_1, s_2)} + \int_{s_1}^{s_2} T(s) \alpha(\nu, s) e^{-\phi(s, s_2)} ds \]  \hspace{1cm} (2.12)

This is the simplified form of the brightness temperature equation commonly used in microwave remote-sensing applications for observed frequencies up to 300 GHz. At typical atmospheric temperatures, the errors in Planck function caused by the Rayleigh-Jeans approximation are typically only 3-4% therefore they are usually ignored in the microwave case. However, for the generality, we will retain the full formula of brightness temperature equation given by Eq. 2.9 for the use in our forward model. This makes the corrections to account for the Rayleigh-Jeans approximation not necessary in the model results of the MLS radiances obtained in this thesis.

2.3 Oxygen Emission and Absorption Model

To find a solution for radiance brightness temperature expressed by Eq. 2.9 for the MLS measurements, the calculation of the absorption coefficient \( \alpha(\nu, s) \) is needed. This section gives description of the background theory of microwave emission and absorption model by molecular oxygen \((O_2)\) that leads to the derivation of its absorption coefficient as required. Most of the material presented here is summarised from Waters [1976, 1990, 1993] and Rosenkranz [1993].
2.3.1 The Formulation of Absorption Coefficient

As mentioned in Chapter 1, a number of atmospheric molecules have spectral absorption (and emission) lines at microwave frequencies 1-300 GHz but the most prominent ones are water vapour (H₂O) and oxygen (O₂). Water vapour has dominant spectral lines at 22.235 and 183.31 GHz, and oxygen has absorption band between 50-70 GHz (known as the 60-GHz oxygen complex) and a single line at 118.75 GHz. The spectral line absorption occurs when a quantised system such as a molecule, interact with an electromagnetic radiation field and makes a transition between two quantum states of the system. In this process, the resonant frequency, \( \nu_{ul} \), at which the transition can occur is given by the Bohr formula:

\[
\nu_{ul} = \frac{|E_u - E_l|}{h}
\]

where \( E_u \) and \( E_l \) are the internal energies of the higher and lower states which are involved in the transition respectively, and \( h \) is Planck constant. Energy is absorbed if the transition is from a lower to a higher energy level, and emitted if the transition is from a higher to a lower energy level.

The absorption coefficient \( \alpha \) is an important parameter that represents the strength of the absorption (or emission according to Kirchoff's law) occurred in the transition process described above. It is found from quantum theory that \( \alpha \) depends principally upon at least five factors: (1) resonant frequency, (2) intrinsic line strength, (3) square of the dipole moment strength, (4) population of molecules that can participate in a transition, and (5) line-shape factor. A rapid insight can be achieved by considering the general mathematical model used to express for the absorption coefficient \( \alpha_{ul}(\nu) \) between quantum states \( l \) and \( u \) in the microwave case given by Waters [1976] as:

\[
\alpha_{ul}(\nu) = \left( \frac{1}{4\pi\varepsilon_0} \right) \left( \frac{8\pi^3\nu}{3hc} \right) \left[ n_l |\mu_{lu}|^2 - n_u |\mu_{ul}|^2 \right] F(\nu, \nu_{ul})
\]

---

1 Equation 2.14 assumes use of the SI system. The first factor in parentheses, \( 1/4\pi\varepsilon_0 \), is not present when Gaussian units are used.
where:
\[ a_{ll}(v) = \text{contribution to the total absorption at resonant frequency given by the} \]
molecular transition from energy state \( l \) to energy state \( u \),
\[ v_{ul} = \text{frequency of resonant transition defined by Eq. 2.13}, \]
\[ F(v, v_{ul}) = \text{line-shape factor (to be discussed in Section 2.3.3)} \]
\[ n_l \text{ and } n_u = \text{populations of the energy levels } l \text{ and } u, \]
\[ |\mu_{ul}|^2 \text{ and } |\mu_{ul}|^2 = \text{squares of the dipole matrix elements derived from quantum mechanical} \]
analysis of the transition.

**General Form of Absorption Coefficient**

For computational purposes, it is generally useful to rewrite Eq. 2.14 in the form,
\[ a_{ll}(v) = n\Psi_{ul}(v, T)F(v, v_{ul}) \quad (2.15) \]
where \( \Psi_{ul}(v, T) \) is called the “line intensity” function at temperature \( T \) of a single line for a
single molecule, and \( F(v, v_{ul}) \) is a spectral line-shape function (see details in Section 2.3.3).
Equation 2.15 is the general form of the absorption coefficient which shows the dependence
of the absorption coefficient upon the three principal functions; the total number density \( n \),
the line intensity function \( \Psi_{ul}(v, T) \), and the spectral line shape \( F(v, v_{ul}) \).

The general form of \( \Psi_{ul} \) for the dipole transition between states \( u \) and \( l \) in case of
microwave emission in thermodynamic equilibrium, is given by Waters [1993] as
\[ \Psi_{ul}(v, T) = \left( \frac{1}{4\pi\varepsilon_0} \right) \left( \frac{8\pi}{3hc} \right) \left( \frac{v |\mu_{ul}|^2 q_L^2}{Q(T) g_u} \right) e^{-|\epsilon_l - \epsilon_u| kT} \quad (2.16) \]
where \( Q(T) \) is the partition function. Generally, equation 2.16 is sufficient to be adopted in
the forward model for both water vapour and oxygen. But in the model calculation, it is not
necessary to determine each of the physical quantities in the equation individually.
Considerable effort was saved by the use of a spectroscopic database in the JPL Catalogue
described below.
Chapter 2

The Forward Model

2.3.2 The JPL Catalogue Data

The JPL Spectral Line Catalogue [described in Pickett et al. 1998] contains the physical parameters which describe the rotational transitions of all the significant gases in the Earth’s atmosphere. It includes all the sub-millimetre, millimetre, and microwave spectral lines in the frequency range between 0 and 10000 GHz (i.e., wavelengths longer than 30 μm). The catalogue is intended to be used as a guide in the planning of spectral line observations and as a reference that can facilitate identification and analysis of observed spectral lines. The catalogue data are available on-line via the world-wide-web at http://spec.jpl.nasa.gov.

In the catalogue a value of the so-called “line intensity”, $S_{ul}$, is listed for each microwave transition. This parameter incorporates many of the terms involved in the definition of the ‘intensity’ function given by Eq. 2.16. The line intensity $S_{ul}$ given in the catalogue is obtained from

$$S_{ul}(v) = 10^{12} \left( \frac{1}{4\pi \epsilon_0} \frac{8\pi^3}{3hc} \left( \frac{\nu_{ul}}{Q_{rs}(T)} \right)^2 \right) e^{-E_l/kT} - e^{-E_u/kT}$$

where $\nu_{ul}$ is the resonant line frequency, $S_{ul}$ is the line strength parameter, $\mu$ is the dipole moment along the molecular axis $x$, $E_l$ and $E_u$ are the lower and upper state energies, respectively, and $Q_{rs}$ is the rotation-spin partition function, and other symbols are as previously defined. In this definition, $S_{ul}$ is given in units of m$^2$ Hz and evaluated at temperature $T = 300$ K. The values of $S_{ul}$ for 39 oxygen lines between 50-70 GHz and a single line at 118.75 GHz are listed in Table 2.1. The expression of $S_{ul}$ in Eq. 2.17 is slightly different from that given by Eq. 2.16 but Waters [1993] shows that $\mu^2 S_{ul}^2 g^2 = S_{ul}(\mu)^2$ in case of rotational transition for oxygen molecule so there is no need to introduce these new terms here. Although the line intensity defined in Eq. 2.17 is derived for an electric dipole transition, the values listed in the catalogue include those for molecules, which do not have

---

1 The constant $10^{12}$ arises in Eq. 2.17 which is different from the definition of $S_{ul}$ given by Eq.1 in Pickett et al. [1998] because their equation is for $\nu_{ul}$ in MHz (catalogue unit), whereas Eq. 2.17 (and Eq. 2.19) is for $\nu_{ul}$ in Hz.
an electric dipole moment. In these instance the line intensity is defined with an equivalent electric dipole vector magnitude $|\mu_u|$, which gives the same results as the electric quadrupole or magnetic dipole it represents.

Values of $S_{ul}$ at other temperatures can derived from Eq. 2.17 once the dependence of $Q_{rs}$ on $T$ is known. For linear molecule, like $O_2$, $Q_{rs}$ is approximately proportional to $T$ in the limit where the energy spacing are small compared with $kT$ (Rayleigh-Jeans limit):

$$\frac{Q_{rs}(T)}{Q_{rs}(T_0)} = \frac{T}{T_0}$$

(2.18a)

where $T_0 = 300$ K. For asymmetric molecule, like $H_2O$ and $O_3$, $Q_{rs}(T)$ is proportional to $T^{3/2}$ in the same limit:

$$\frac{Q_{rs}(T)}{Q_{rs}(T_0)} = \left(\frac{T}{T_0}\right)^{3/2}$$

(2.18b)

Explicitly, $S_{ul}$ at other temperatures, or $S_{ul}(T)$, can be written as

$$S_{ul}(T) = S_{ul}(T_0)\left(\frac{Q_{rs}(T)}{Q_{rs}(T_0)}\right) \left(\frac{e^{-E_1/kT} - e^{-E_u/kT}}{e^{-E_1/kT_0} - e^{-E_u/kT_0}}\right)$$

$$\equiv S_{ul}(T_0)\left(\frac{T_0}{T}\right)^{d+1} e^{-\frac{1}{T} \left(\frac{1}{T_0} E_u/kT_0\right)}$$

(2.19)

where $d = 1$ for a linear molecule and $3/2$ for a non-linear molecule. Equation 2.19 requires that $|E_u - E_1|$ is small compared with $kT$ and $kT_0$.

By substituting the line intensity $S_{ul}(T)$ given by Eq. 2.19 above into the absorption coefficient defined in Eq.2.15, we obtain the absorption coefficient equation that is to be used in the forward model itself as

$$\alpha_{ul}(\nu) = (10^{-12})n Allen(\nu)S_{ul}(T)F(\nu, \nu_{ul}) \text{ m}^{-1}$$

(2.20)

where $S_{ul}(T)$ is calculated from Eq. 2.19 as the lower state energies $E_1$ are also listed for each transition in the JPL catalogue in units of wavenumbers$^1$.

---

$^1$ The inverse of the wavelength (in cm) of a photon having that amount of energy.
Table 2.1. Spectroscopic parameters of major $^{16}\text{O}_2$ lines in natural air. Line intensity is expressed in base 10 logarithm of the integrated intensity in units of nm$^2$ MHz and at temperature 300K.

<table>
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<tr>
<th>$N_\pm$</th>
<th>Line Centre</th>
<th>Line Intensity</th>
<th>$N_\pm$</th>
<th>Line Centre</th>
<th>Line Intensity</th>
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<td></td>
<td>(GHz)</td>
<td>(nm$^2$ MHz)</td>
<td></td>
<td>(GHz)</td>
<td>(nm$^2$ MHz)</td>
</tr>
<tr>
<td>39-</td>
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<td>5-</td>
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2.3.3 Spectral Line Shapes

In nature, there are various processes of line broadening which give an emission or absorption feature a finite frequency width. In this case, emission or absorption is not concentrated at a single frequency and this deviation from monochromatism is important in atmospheric observations. Naturally, line broadening in microwave case results mainly from two factors: pressure-induced collisions (pressure broadening) and thermal motion of gaseous molecules (Doppler broadening) whose details are given as follows.

**Pressure Broadening**

The mechanism that dominates the microwave line broadening in the lower atmosphere is pressure broadening (or collisional broadening) due to pressure-induced collisions and mutual interaction between molecules of absorbing gas. The thermal agitation of the molecules in the gas means that they continually collide with one another, and such collisions disturb the interaction between radiation and the dynamical states of the molecule (i.e., its vibration and rotation). This can stimulate a transition and thus shorten the lifetime of such a state, and consequently the line is broadened.

The line-shape function for pressure broadening may take various forms according to the theoretical model used to describe the nature of the collisions. For the middle atmosphere (stratosphere and mesosphere) where the atmospheric pressure is rather low and there is no overlap of spectral lines from the same molecules, then the line-shape function in Eq. 2.15 can be approximated by the Lorentz shape factor for an isolated line near resonance given by [Liebe et al. 1977]:

\[ F_p(v, \nu_{ul}) = \frac{1}{\pi} \frac{\Delta v_p}{(\nu_{ul} - v)^2 + \Delta v_p^2} \]  

(2.21)

where \( \Delta v_p \) is the line half-width defined by

\[ \Delta v_p = \frac{1}{2\pi \Delta t_c} \]  

(2.22)

where \( \Delta t_c \) is the mean time between collisions which is approximately \( 10^{-10} \) s for a typical atmospheric gas at sea level. Normally, \( \Delta v_p \) is a function of both temperature and pressure.
and it is quite difficult to be determined for each spectral line exactly. However, its empirical expression is conventionally given in the form

$$\Delta \nu_p = \Delta \nu_0 \left( \frac{p}{p_0} \right) \left( \frac{T}{T_0} \right)^{-x}$$

(2.23)

where $x$ is constant exponent usually between 0.7 and 1.0, and $\Delta \nu_0$ is the value of $\Delta \nu_p$ at reference pressure $p = p_0$ and temperature $T = T_0$. Values of $\Delta \nu_0$ and of $x$ may be different for collisions involving different types of molecules, because different types of intermolecular force may be involved. Eq. 2.23 allows $x$ and $\Delta \nu_0$ to vary from line to line. The values of the coefficients $\Delta \nu_0$ and $x$ for oxygen lines used in our model are given by Rosenkranz [1993] for the reference temperature $T_0 = 300$ K and $p_0 = 1$ bar. Practically, the Lorentz line-shape function described by Eq. 2.21 gives a good approximation to the shape of pressure-broadened lines only when $\nu$ is not very far from line centre (where $|\nu_{ul} - \nu| \ll \Delta \nu_p$). Departures from the Lorentz shape in the line wings are very difficult to measure and are a major source of uncertainty in atmospheric transfer. In our model, we chose to use mainly the Lorentz profile in the calculation of oxygen absorption coefficient due to its simple form and its ability to convolve with the Doppler line-broadening function to form a well-known Voigt line-shape function to be discussed later in this section.

**Doppler Broadening**

The second source of line broadening comes from the thermal motion of the molecules of a gas during emission or absorption of radiation. This process could give rise to a Doppler shift of the radiation frequency and result in producing the Doppler line broadening. The Doppler line-shape function is derived directly from the velocity distribution of the molecules, which is Gaussian distribution. Therefore, the Doppler line shape is also Gaussian and given by the formula [Salby 1996]:

$$F_D(\nu, \nu_{ul}) = \frac{1}{\alpha_D \sqrt{\pi}} \exp \left[ -\left( \frac{\nu_{ul} - \nu}{\alpha_D} \right)^2 \right]$$

(2.24)

where $\alpha_D$ is the line-width factor related to the Doppler half-width $\Delta \nu_D$ by,
in which \( m \) is the mass of the molecule (in gram), and \( M \) is its molecular weight (in gram/mole), e.g. \( M = 18 \) for \( \text{H}_2\text{O} \) and \( 32 \) for \( \text{O}_2 \), and \( \Delta \nu_D \) and \( \nu_{ul} \) have the same units. Unlike the pressure-induced half-width, \( \Delta \nu_p \), the Doppler half-width, \( \Delta \nu_D \), depends primarily on atmospheric temperature \( T \) and does not depend on atmospheric pressure \( p \).

The Doppler broadening of typical atmospheric lines is \( \sim 10^{-6} \) times the line frequency, and for microwave lines, it is surpassed by pressure-induced broadening in the troposphere and lower-stratosphere (see Figure 2.3). The Doppler broadening becomes more significant in the mesosphere and thermosphere where the atmospheric pressure is rather low and pressure-broadening function is small. In this case effects of Doppler broadening must be taken into account in the calculation of the absorption coefficient. This convolution between Doppler and Lorentz line-shape functions results in the so-called Voigt line shape as detailed below.

**Voigt Line Shape**

At high altitude in the mesosphere when both pressure-induced and Doppler broadening are compatible, the suitable line-shape factor is given by the convolution over frequency of the Doppler line with whichever collisional line shape being used. If the Lorentz line is used, the resulting Doppler-Lorentz convolved function is called the *Voigt line-shape function*, which is given by Armstrong and Nicholls [1972, p.218] as

\[
F_V(\nu, \nu_{ul}) = \frac{1}{\alpha_D \sqrt{\pi}} K(x, y) \tag{2.26}
\]

where \( K(x, y) \) is the Voigt function, defined by

\[
K(x, y) = \frac{y}{\pi} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{y^2 + (x-t)^2} \, dt \tag{2.27}
\]

where
where the functions $\Delta \nu_p$ and $\alpha_D$ are given by Eqs. 2.23 and 2.25 respectively. Spectral half-width of Voigt function could be approximately given by [Olivero and Longbothum 1977]

$$\Delta \nu_V = 0.5346 \Delta \nu_p + (0.2166 \Delta \nu_p^2 + 0.6931 \alpha_D^2)^{1/2} \quad (2.28)$$

Figure 2.3 plots line half-widths, $\Delta \nu_p$, $\Delta \nu_D$, and $\Delta \nu_V$, versus altitude in the terrestrial atmosphere up to 120 km for the 63.0-GHz oxygen line and for the atmospheric temperature profile shown later in Figure 2.5. The Voigt profile ($\Delta \nu_p$) is a result from the convolution between the pressure and Doppler line broadening and it shows the domination of each broadening mechanism at different height in the atmosphere. We see that the Doppler width in this case is relatively small (~0.07 MHz), and it is overcome by pressure-induced broadening in the troposphere and lower mesosphere. However, the Doppler broadening becomes more significant in the upper mesosphere and thermosphere where the atmospheric pressure is low and the pressure-induced broadening is significantly reduced in magnitude. Generally, effects of Doppler broadening could be completely neglected compared to the pressure broadening at altitudes up to ~40 km and comparatively small up to ~60 km. Between altitudes 70-80 km, $\Delta \nu_D$ and $\Delta \nu_p$ are comparable in magnitudes, however, above ~80 km the Doppler broadening becomes the principal mechanism of line broadening.

Practically, the Voigt function is only required when $\Delta \nu_D \geq \Delta \nu_p$ and, therefore, it is used in our model only for the simulation of radiance measurements at high altitudes (above ~70 km) by MLS channel 8 (at line centre) whose weighting function is peaked at that level (~80-km altitude). For the measurements made by other channels at lower attitudes the Lorentz line shape will be applied through out. The Voigt function needs to be calculated numerically and this could be done by using the VOIGT function in the IDL program. The algorithm of IDL's VOIGT function is based on the formulation made by Armstrong [1967]. It requires the terms $x$ and $y$ given above as input parameters and returns as a result function $F_V(n, n_{ul})$ in Eq. 2.26.
2.3.4 Oxygen Absorption Coefficient Analysis

We summarise in this section, the method to derive oxygen absorption coefficient used in the forward model based on the background emission and absorption theory described so far. First, we note that, from Eq. 2.15, the absorption coefficient due to a single transition is considered. In reality a quantum system usually has many allowed transitions, and Eq. 2.15 must be extended to account for this. If there is no interaction between the various transitions, and the radiation from different transitions is incoherent, then the total absorption coefficient at any given frequency is simply the summation from all individual line directly:

\[
\alpha(v) = \sum \alpha_{al}(v) \quad \text{(all transitions)}
\]

\[
= n \sum \Psi_{al}(v,T) F(v,v_{al})
\]

or, according to Eq. 2.20,

\[
\alpha(v) = (10^{-12}) n \sum \frac{v}{v_{al}} \Sigma_{al}(T) F(v,v_{al}) \quad \text{m}^{-1}.
\]

Figure 2.3: Line-width parameters, \(\Delta \nu_p\), \(\Delta \nu_D\), and \(\Delta \nu_v\), plotted versus altitude from ground surface up to 120 km for the 63.0-GHz oxygen line.
Equation 2.30 is usually valid for the measurements in the middle atmosphere where the atmospheric pressure is relatively low and all the oxygen lines involved in absorption and emission process are completely resolved [Meeks and Lilley 1963; Reber 1972].

In the lower atmosphere where the pressure is high, individual spin-rotation lines of oxygen have significant overlapping due to the strong pressure-induced broadening. In this case, the summation from all individual line directly in Eq. 2.30 is no longer justified. This has the effect of increasing absorption near the centre of the band but reducing it in the wings. In order to calculate the total absorption coefficient, in this case, we need a collisional transfer matrix for which the calculation is a very complicated and it is beyond the scope of our interest here. More details on this subject can be found in Rosenkranz [1975, 1988] who has applied the theory of bands composed of overlapping lines to the 60-GHz oxygen complex and employed reasonable approximations to reduce the complexity of the computation for practical use in remote sensing.

Though, oxygen has major 39 spectral lines in spectral region between 50-70 GHz (see Table 2.1) but they all are well resolved in the middle atmosphere and within the spectral range of UARS MLS band 1 (~62.5-64.0 GHz), only two O$_2$ lines at 62.998 and 63.569 GHz are significant in the observations [Fishbein et al. 1996]. These two resonant frequencies are nearly independent to each other and are designed to be centre of the lower and upper sideband of the MLS 63-GHz radiometer as described in Chapter 1. In our model simulation, we will consider only the atmospheric radiation from 62.998-GHz line observed in the lower sideband of the instrument. The computation includes all 15 channel frequencies where the spectroscopic line strengths are obtained from the JPL Catalogue described earlier and channel spectral line frequencies are as listed in Table 1.2. In the calculation, the number density $n$ of oxygen molecule is given by (using ideal gas law)

$$n = n_{air} = aP_{air} / kT$$

$$= (0.724 \times 10^{26})aP_{air \text{, mbar}} / T$$

(2.31)

where $a$ is the volume mixing ratio of gas, $n_{air}$ is the total density of air molecules, $P_{air}$ is the total air pressure, and the gas law $P_{air} = n_{air}kT$ has been used; $[P_{air}]_{\text{mbar}}$ is the total air pressure in millibars, or,
\[ n = (0.152 \times 10^{26}) \frac{[P]_{\text{mbar}}}{T} \]  

(2.32)

where the concentration of oxygen \( a = 0.21 \) is used. The line-strength factor \( S_{ul}(T) \) is given by Eq.2.19:

\[ S_{ul}(T) = S_{ul}(T_0) \left( \frac{T_0}{T} \right)^2 e^{-\left( \frac{1}{T_0} \right) E_{1/k}} \]  

(2.33)

where the constant \( d = 1 \) for \( \text{O}_2 \) is used. The value of \( S_{ul}(T_0) \) for \( T_0 = 300 \text{ K} \) are as listed in Table 2.1.

The line-shape function \( F(v, v_{ul}) \) to be used for the simulated radiance measurements with channels 1-7 and 9-15 is the Lorentz function given by Eq. 2.21:

\[ F_P(v, v_{ul}) = \frac{1}{\pi} \frac{\Delta v_p}{(v_{ul} - v)^2 + \Delta v_p^2} \]  

(2.34)

where \( \Delta v_p \) is defined by Eq. 2.23 as

\[ \Delta v_p = (1.139 \times 10^{-3}) \left( \frac{\frac{p}{p_0}}{\frac{T}{T_0}} \right)^{-0.8} \text{ GHz} \]  

(2.35)

Values of the coefficients \( \Delta v_0 = 1.139 \text{ GHz/bar} \) and \( x = 0.8 \) for 62.998 GHz \( \text{O}_2 \) line are from Rosenkranz [1993] for the reference temperature \( T_0 = 300 \text{ K} \) and \( p_0 = 1000 \text{ mbar} \). Only the radiance measurements with channel 8 that the Voigt function is employed and its values are calculated by the IDL’s VOIGT function mentioned earlier.

Following the above procedure, examples of the absorption coefficient profile for 63-GHz oxygen line are computed and plotted in Figure 2.4. This is for \( \Delta v = \nu_{ul} \nu \leq 500 \text{ MHz} \) and at temperatures \( T = 200, 250, 300 \text{ K} \), and pressure \( p = 1 \text{ mbar} \) (~ 50-km altitude). As we could expect, the absorption coefficient is maximum at the line centre and decreased in value rapidly for frequencies further away towards line wings. At the same frequency, the absorption coefficient decreases when the temperature increases from 200K to 300K but these differences are less noticeable at frequencies very close to line centre. At the same temperature, the absorption coefficient decreases rapidly with frequencies offset from line centre and form a bell-curve centred at resonant frequency.
It should be also noted here that the other effect that is neglected in our model calculation is the Zeeman effect associated with the Earth's magnetic field, which is important in the mesosphere. The Zeeman effect occurred from the Earth's magnetic field which will remove the $2J+1$ degeneracy of $O_2$, and spread the energy levels over a frequency range of $\sim 1$MHz. Only at pressures below approximately 3 mbar (\textasciitilde\textasciitilde 40km altitudes), where the pressure broadening is of the order of, or smaller than, $\sim 1$MHz, is the Zeemann splitting noticeable. The absorption and emission by the Zeemann components is both polarised and isotropic, and the scalar radiative transfer equation given by Eq. 2.9 is not an adequate description of the absorption and emission process. The Zeemann effect could result in the variation by $\sim 3-8$ K in MLS observations by channels 7 and 8 which sensed the high altitude regions [Wu and Waters 1996b], therefore it does not have much effect in the measurements with the MLS instrument in general and, as a result, it will not be included in the forward model employed in this thesis.
2.4 Model Applications to MLS Radiance Measurements

We have explained in the previous section the formulation and algorithm for the computation of the oxygen absorption coefficient required in the model simulation of MLS limb radiance measurements. In this section, we will apply the model in the derivation of the observed radiances described in terms of the brightness temperature given by Eq. 2.9 for UARS MLS experiment. The simulated radiances for EOS MLS experiment will be shown in Chapter 6.

2.4.1 Solutions to the Radiative Transfer Equation

In the model simulation, the solution of radiance brightness temperature \( T_b(\nu, x_s) \) observed with the sensor onboard the satellite is found by integrating along the instrument line-of-sight (x-axis in Figure 1.1) from \( x = -\infty \) (cold background atmosphere) to \( x = x_s \) (the assumed satellite position). This process could be described according to Eq. 2.9 as

\[
T_b(\nu, x_s) = T_{b0} e^{-\phi(1, x_s)} + \int_{-\infty}^{x_s} \alpha(\nu, x) T_r(\nu, x) e^{-\phi(x, x_s)} dx
\]

where \( T_r = T(x)/r(\nu, x) \) and other parameters are defined as usual. This equation is usually better expressed in terms of temperature weighting function \( W(\nu, x) \) as:

\[
T_b(\nu, x_s) = T_{b0} e^{-\phi(1, x_s)} + \int_{-\infty}^{x_s} T_r(\nu, x) W(\nu, x) dx
\]

where \( W(\nu, x) \) is defined as

\[
W(\nu, x) = \alpha(\nu, x) e^{-\phi(x, x_s)}
\]

Written in this form, the weighting function \( W(\nu, x) \) in Eq. 2.38 represents the contribution of each atmospheric layer to the total radiance received at the radiometer on board.

It is clearly seen that Eq. 2.37 is relied heavily on the integration along the ray path of \( T_r \) and \( W \) to find the solution for \( T_b \). However, as most of the atmospheric data involved in the integrands, especially background temperature \( T \), are usually tabled with altitude \( z \) from the Earth’s surface, it might be more convenient to transform the horizontal integration in Eq. 2.37 to be its equivalent height integration instead. We show in Appendix A details of such a transformation from which enables us to rewrite Eq. 2.37 as
\[ T_b(\nu, h_T) = T_{b0}e^{-2\Phi(h_r, \infty)} + \int_0^T W(\nu, h_T, z(x))dz. \quad (2.39) \]

where \( z(x) \) is the altitude at point \( x \) along the ray path derived from Eq. A.10 and the weighting function \( W(\nu, h_T, z) \) is given by Eq. A.17 as

\[
W(\nu, h_T, z) = \alpha(\nu, z)g(z, h_T)[e^{-\Phi(z, z)} + e^{-\Phi(z, z)}] \quad \text{for } z > h_T
\]

\[
W(\nu, h_T, z) = 0 \quad \text{for } z < h_T \quad (2.40)
\]

in which \( g(z, h) \) is the geometrical term defined in Eq. A.11. It should be noted that these weighting functions still do not take the FOV smearing effects of the instruments into account. These effects are rather important in the radiance measurements with the MLS instruments, especially in the saturation case, and they will be discussed in more detail later in Section 2.4.3.

The solution to the radiative transfer equation 2.39 requires a knowledge of, at least, (1) the atmospheric temperature \( T \) as a function of altitude \( z \), (2) the absorption coefficient \( \alpha \) as a function of frequency and altitude, and (3) the tangent height \( h_T \) of the observation. The general procedure used to solve this equation in our model is to ascribe a temperature to each atmospheric layer at height \( z \) in the model atmosphere which is assumed to be homogeneous, and in hydrostatic equilibrium. Once defined, an absorption coefficient is calculated for each layer from Eq. 2.31. This can lead to the derivation of optical thickness from Eq. A.13 and weighting function from Eq. A.17 (or Eq. 2.40). The radiance brightness temperature is then calculated by simple numerical integration within the altitude ranges defined by Eq. 2.39 and the results are described as a function of observed frequency \( \nu \) and tangent height \( h_T \) of the observation, \( T_b = T_{b0}(\nu, h_T) \).

Figure 2.5 shows examples of simulated brightness temperature \( T_{b0}(\nu, h_T) \) calculated from Eq. 2.39 at tangent pressures 0.1 mbar for background temperature \( T = 240 \) K, 1 mbar for \( T = 260 \) K, and 10 mbar for \( T = 280 \) K. These given pressure levels are equivalent to tangent altitudes of ~65, 48, and 17 km respectively. The channel frequencies are not shown here but they are as listed in Table 1.2 and the spectral positions \( \Delta \nu \) are offset from line centre at 62.998 GHz.
We can see that the dependence of the radiances on frequency and pressure of each channel is quite strong whenever the pressure is low (at high altitudes) and the channels are not fully saturated. For example, at tangent pressure 0.1 mbar (~65-km), the observed radiances are considerably high in the middle channels (Ch.6-10) and all the unsaturated radiances are approximately symmetric around the line centre (Ch.8). For channels at line wings with |Δν| > 10 MHz, their observed radiances are rather low and they are dominated by the cold cosmic background radiation. At tangent pressure 1 mbar (~48-km), nearly all middle channels with |Δν| < 15 MHz (Ch.5-11) are saturated. There are only channels on the line wings (Chs.1-4 and 12-15) that still unsaturated and their radiances are determined primarily by the channel’s frequency (from line centre). However, for the observations in lower atmosphere, it appears that all MLS channels saturated above pressure level 100 mbar (meaning at lower altitudes than ~17 km). The saturated radiance is a sign showing that the atmosphere under the observations is becoming optically thick and the absorption along the ray path becomes very strong. The cold background term in this case is negligible due to this strong absorption along the ray path and could be neglected in the model calculation. The saturated MLS radiance is independent from channel’s frequency and tangent height of observation and being a good measure of the observed background atmospheric temperature.

Figure 2.5: Examples of thermal emission of O\textsubscript{2} at tangent pressure 0.1 mbar, 1 mbar, and 100 mbar for isothermal atmosphere at temperatures 240, 260, and 280 K respectively.
2.4.2 The Model Atmosphere

The simulated radiance profiles shown in Figure 2.5 are for the observations in the isothermal atmospheres, which are the idealistic cases in our study. In the further study, we will consider the radiance measurements in more realistic model atmosphere. The model atmosphere we chose for our study is the 1976 US standard atmosphere, which provides the zonal-mean temperature, profiles in the terrestrial atmosphere up to 120 km at different latitudes. The selected temperature profile for the used in the radiance simulation is derived from a monthly zonal-mean profile in July at 65°S as shown in Figure 2.6. The equivalent pressure for altitude $z = H \ln(p_0/p)$ where the typical atmospheric scale height $H = 7$ km in the middle atmosphere and $p_0 = 1013$ mbar are applied, is also given as a reference in the vertical scale on the right-hand side. Data of the profile are averaged and indexed vertically every 1 km at altitude ranges between 0-120 km and they show the typical temperature decrease with altitude in the troposphere and mesosphere, and increase in the stratosphere. This profile is also represented the unperturbed background temperature required in the model simulation for the gravity wave study described later in Chapter 3.

![Zonal Mean Temperature Profile at 65°S 1976 US Standard Atmosphere](image)

**Figure 2.6:** The averaged monthly zonal-mean temperature profile at 65°S from the 1976 US standard atmosphere for July at altitudes between 0-120 km.
2.4.3 FOV Smearing Effects

The radiance calculation performed so far is for a single ray and does not take effects of the FOV smearing into account. In practice, a real instrument will have a field-of-view with a finite width in the vertical and horizontal directions. However, it is the vertical width that is more significant in the MLS measurements and the total measured radiance is normally given by the integration of \( T_b(v, h_T) \) over the field of view profile \( F(z) \):

\[
T^*_b(v, h_T) = \int_{\Delta z} T_b(v, z) F(z) \, dz
\]

(2.41)

where the FOV function \( F(z) \) is defined in terms of the angle between the half-power point of the antenna response. In case of a vertical sounder it is generally possible to assume that the intensity is uniform over the field of view. This is not usually the case with a limb sounder, because of the typically rapid variation of radiance with the tangent altitude. As a result, the FOV smearing effects must be taken into account in limb sounding to gain more accuracy in the radiance measurements whenever the dependence of \( T_b \) on tangent altitude is strong. For the MLS experiments, the FOV factor, \( F(z) \), could be approximated by the normalised form of the so-called antenna function \( A(\theta) \) having the Gaussian form

\[
A(\theta) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{\theta^2}{2\sigma^2} \right)
\]

(2.42)

where \( \theta \) is the angle from the pointing direction, and \( \sigma \) is width of the Gaussian width. It is related to \( \gamma \), the full-width at half-maximum (FWHM) of the FOV beam, by \( \gamma = 2\sigma (2\ln 2)^{1/2} \).

By replacing function \( F(z) \) in Eq. 2.41 by function \( A(\theta) \) in Eq. 2.42, we get

\[
T^*_b(v, h_T) = \frac{\int_{\Delta z} T_b(v, z) A(z) \, dz}{\int_{\Delta z} A(z) \, dz}
\]

(2.43)

in which the relation between parameters \( z \) and \( \theta \) (in degrees) is given by (Eq. A.7 in Appendix A)

\[
z^* = 2\theta \sqrt{(H - h_T)} \text{ km}
\]

(2.44)

where \( z^* = z - h_T \) is the vertical distance from tangent point and \( H = 600 \text{ km} \) is the orbital altitude and \( h_T \) is the tangent height of observations.
For the limb observation with the 1.6-m vertical diameter of the MLS antenna, it gives the 63-GHz and 118-GHz radiometers having FOV widths of ~0.21° and 0.11° [Jarnot et al. 1996]. For the observation at 18-km tangent height, these are corresponding to approximately 10-km and 5-km vertical width centred at tangent point respectively. Figure 2.7 plots FOV function $F(z^*)$ for the MLS observations at $h_T = 18$ km of both UARS MLS 63-GHz radiometer and EOS MLS 118-GHz radiometer. Effects of FOV convolution are generally included in the model calculation of MLS radiance brightness temperatures performed in this thesis in both cases of the MLS instruments. These inclusions of FOV factor will be described in more detail in Chapter 3 when the three-dimensional temperature weighting functions of the observations are constructed.

![Normalised FOV Functions for 18km—Tangent—Height Observation](image)

**Figure 2.7:** The normalised vertical FOV functions for MLS 63-GHz and 118-GHz radiance measurements at 18-km tangent height.

Figures 2.8a and b show examples of FOV smearing effects in UARS MLS radiance measurements for a single limb scan made with channel 4. In Figure 2.8a, the computed radiance profiles for a single ray both (with and without FOV smearing included) are presented and the differences in values of these two profiles are then highlighted in Figure 2.8b. We see that the incorporation of FOV factor in the calculation has resulted in the smoother radiance curve and the significant changes in values of non-saturation radiances (of
Figure 2.8a: Synthetic radiance profiles for a single-ray and FOV-convolved observations made with channel 4 of the UARS MLS 63-GHz radiometer.

Figure 2.8b: The differences between the single-ray and FOV-convolved radiance profiles shown in (a).
up to 20 K) at tangent height of ~40-70 km. In the contrary, the FOV factor does not affect much on values of the radiances at very high altitude (e.g. with $h_T > 70$ km) which is dominated by cosmic background radiation and at low altitude (e.g. with $h_T < 30$ km) where most channel radiances are fully saturated. The great variation of the non-saturation radiance at tangent height of ~40-70 km is due to the fact that the observed radiance in this case originates mostly from the emission at the tangent-height layer which makes it very sensitive to the change in tangent height and also the inclusion of FOV factor.

For the MLS measurements in optically-thin atmosphere, the dependence of limb radiance on the tangent height over the FOV might accounts for approximately half the temperature sensitivity of unsaturated 63-GHz radiances observed in the middle atmosphere [Fishbein 1996]. In the contrary, the FOV factor is not important for the measurements in optically-thick atmosphere where the radiation comes mostly from the later far above the tangent point. However, as explained in Chapter 3, the FOV functions are important in the observations of gravity waves with the MLS instruments and must be included in our study.

2.4.4 Simulated MLS Radiance Measurements

Figure 2.9 shows examples of the radiance profiles for a singe limb-scan made by UARS MLS 63-GHz radiometer where the instrument is assumed to step-scan the atmospheric limb between altitude range 0-120 km with 1-km increment at each step. The radiance profile for each channel is calculated directly from Eq. 2.43 and for the background atmospheric temperature profile shown in Figure 2.6. The profiles are plotted separately for each channel from 1 to 8. Data for channel 9-15 are not shown here but they are equivalent to those of channel 7-1 respectively. The 18-km tangent height for the observation in limb-tracking mode is also marked as a reference.

We find that the patterns of the computed radiance profiles generated by our model are, in general, similar to those obtained from real instrument reported by Wu and Waters (see Figure 1.4 for example) especially on the saturation altitudes for each channel. This similarity significantly confirms the practicability of the model in simulating radiance measured with the real MLS instrument under the similar atmospheric condition. Many of the characteristics of the radiance profiles shown in Figure 2.9 have already been discussed in the previous section for the radiance results shown in Figure 2.5. These include sources of the radiance contribution and the dependence of observed radiance profiles on tangent height for the observations at different tangent altitudes.
Figure 2.9: Synthetic radiance profiles for channels 1-8 of UARS MLS 63-GHz radiometer in a single scan from altitude 120 km to Earth's surface. The 18-km tangent height is marked by dashed line as a reference.

We see that, at the very high altitude (e.g. at $h_T > 80$ km), the observed radiances at most channels (except chs.7 and 8), are very low and they originate mainly from the cosmic background radiation (with amplitude of $\sim 2.7$ K). For the observations in the mesosphere (at altitudes $\sim 50-80$ km), the radiances observed with channel 8 (at line centre) and two other middle channels (chs.6 and 7) start to saturate (having constant value regardless of the tangent height of observation). These channels saturate at higher altitudes than those near line wing because of the stronger in line absorption along the ray path as mentioned earlier. At this level all channels apart from those three middle channels are still unsaturated. For the non-saturation case, most of the radiances originate from the atmospheric layer of a few kilometres close to the tangent point. This makes it very sensitive to the change in tangent amplitude during the measurements and the FOV factor is rather crucial for the radiance measurements made at these altitudes. In the contrary, the observed radiance in the saturation case comes mostly from the emission layer far above the tangent-height layer (see Figure 2.12 for details) and, therefore, should not be affected much if the FOV factor is not included in the calculation (for the unperturbed atmosphere).
The saturated MLS channels are very sensitive to the small changes in background temperature as to be seen in section 2.4.4 and their radiance data is suitable for the use in the study of gravity wave perturbations with the MLS instruments. This is quite contrary to the other measurements made with the MLS instrument (e.g. for H₂O or O₃) which prefer using unsaturated radiance data obtained from the observation at optically-thin atmosphere instead due to the requirement in the employed retrieval theory for the data analysis.

**Contributions to Observed MLS Radiance**

We now consider in more detail about sources of the observed MLS radiances at different tangent altitudes as shown in Figure 2.9. Normally, there are three main sources along the ray path in limb sounding that give rise to the radiance received with the instrument onboard. As described in Appendix A, these include the contributions from (1) the cold cosmic background \((T_{\text{cos}})\), (2) the emission at the atmospheric layer section close to the sensor, or nearer than the tangent point, \((T_{b1})\), and (3) the emission at the atmospheric layer section away from the sensor, or beyond tangent point, \((T_{b2})\). These three sources have dominated the observed radiance \(T_b\) with different proportions at different altitudes.

Figure 2.10a illustrates the percent contribution from these sources to the simulated radiance profile of MLS channel 4 as a function of tangent height. From Figure 2.10b we see that the total optical thickness of the observed atmosphere along the ray path at a given frequency does depend strongly on the tangent height of the observations. It is clear that at the very high tangent altitude (above ~80 km), most of the contribution comes from the cold background surface \((T_{\text{cos}})\) as the atmosphere is very thin at this level. At altitudes ~50-80 km where the optical thickness is still not very high, the observed radiances are provided from all three sources but in different proportions. At tangent height 60-80 km, in particular, the contribution from \(T_{b1}\) and \(T_{b2}\) are very compatible (with \(T_{b1} \equiv T_{b2}\)). However, the contributions from \(T_{\text{cos}}\) and \(T_{b1}\) are dramatically reduced at tangent height less than ~60 km and virtually vanished at tangent height lower than ~45 km. The observed radiances at this stage are saturated and their amplitude are determined solely by \(T_{b1}\). This because the observed atmosphere in this case is very optically thick (with optical thickness >> 1) and the absorption along the ray path is high. Figure 2.10b shows the total optical thickness along the ray path of the observed atmosphere at each tangent height to accompany the results shown in Figure 2.10a.
Chapter 2  \hfill The Forward Model

Contribution Ratio to Simulated MLS Channel 4 Radiance Measurements

Figure 2.10a: Contribution proportion from three main sources ($T_{\text{tot}}$, $T_b$, $T_{b2}$) along the ray path to the observed radiance $T_b$ made with MLS channel 4 at different tangent height.

Figure 2.10b: Total optical thickness along the ray path for the radiance observations with UARS MLS channel 4 whose contribution ratio are shown in (a).
2.4.5 Temperature Weighting Functions

As suggested by Eq. 2.39, the temperature weighting function in MLS measurements represents the contribution from each atmospheric layer to the observed radiance at the satellite, therefore, it is the key to understand the origin of the radiance (and its variation) along the ray path. Figure 2.11 shows examples of the weighting function calculated from Eq. 2.40 for the observation in non-saturation case of channel 4 when the instrument viewing the atmospheric limb at 60-km tangent height both with and without the FOV function included. Similarly, in Figure 2.12, the weighting functions of all channels (with FOV-function convolved) for a saturation case made at 18-km tangent height are presented.

Typically, in non-saturation case where the atmosphere is still not optically thick, most of the observed radiances in this case originate from the emission within the thin atmospheric layer centred at tangent point. For the observation with channel 4, this layer has a half-width of ~10 km which are larger than the layer half-width for a single-ray case due to the broadening effect cause by the FOV smearing effects. As a result, the value of unsaturated radiance depends strongly on the tangent altitude as we can see in Figure 2.9 and contains mostly the information at the tangent-height layer.

![Temperature Weighting Functions at Tangent Height of 60.0 km](image)

**Figure 2.11:** The FOV-convolved temperature weighting function in a non-saturation case of the measurement with channel 4 when the instrument views the atmosphere at tangent height of 60 km. The function is plotted with height above the tangent point and the function for a single-ray observation (dashed line) is also given for a comparison.
Figure 2.12: Examples of the FOV-convolved temperature weighting function for the saturation case of MLS channels 1-15 when the instrument views the atmosphere at 18-km tangent height (dashed line).

It is indicated from Figure 2.12 that all the radiance contributions in the saturation case made at 18-km tangent height are mostly from narrow atmospheric layer located far above the tangent point. This is in the contrary to the observation in non-saturation case where most radiance originates from the tangent-height layer (at ~60-km altitude). For example, most of the contribution to the observed radiance with channel 8 (at line centre) are from the emission at 80km-altitude layer with full-width at half-maximum of ~15 km and as the channel frequency moves away from line centre, the main contribution comes from deeper and deeper layers. In general, we found that the saturated radiance at each channel originates mainly from a localised layer centred at different heights in the middle atmosphere (range from ~30 to 80 km) with a vertical resolution of ~10 km for all channels except ~15 km for channel 8. The contribution from very deep layers (e.g. below 20-km altitude) is vanished because the absorption along the ray path in this case is very strong (see Figure 2.10b) and all radiation from these layers is completely absorbed along the path. It should be noted that the weighting function of channel 8 has a considerable greater half-width than of other channels due to the broadening effect caused by the Voigt function used in the computation of the function for this channel.
Table 2.2: Weighting Function Parameters for MLS 18-km-Tangent-Height Observation

<table>
<thead>
<tr>
<th>Channel</th>
<th>Frequency Offset (MHz)</th>
<th>Approximate Height (km)</th>
<th>Layer Thickness (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,15</td>
<td>191</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td>2,14</td>
<td>95</td>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>3,13</td>
<td>47</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>4,12</td>
<td>23</td>
<td>44</td>
<td>10</td>
</tr>
<tr>
<td>5,11</td>
<td>11</td>
<td>48</td>
<td>10</td>
</tr>
<tr>
<td>6,10</td>
<td>5</td>
<td>53</td>
<td>10</td>
</tr>
<tr>
<td>7,9</td>
<td>2</td>
<td>62</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>80</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 2.2 summarises some key parameters of the weighting functions shown in Figure 2.12 where line frequencies are offset from the line centre at ~63 GHz. The MLS weighting functions shown here are the same to those shown in Figure 1.8 which are somewhat different from that reported by Wu and Waters [1997] especially on heights of the saturated layer for channels on the far wing of the line (e.g. chs. 1,2 and 14,15) in which ours are slightly higher (~1-2 km). These differences might be due to the effects of line-coupling in the 60-GHz oxygen band in the lower atmosphere which are not included in our model. Principally, the line-overlapping effects will lower absorption on the line wings and raise it in the middle of the band [Rosenkranz 1988]. This makes the saturation altitudes of the wing channels of MLS instruments lower if these effects are taken into account. Though, the MLS weighting functions have rather narrow vertical width from geometry of the limb sounding, they do, however, have pretty broad corresponding horizontal widths along the ray path. Figure 2.13 shows examples of the temperature weighting functions for channels 4 and 8 illustrated in Figure 2.12 plotted as a function of distance x from the tangent point along the ray path towards the satellite. Note that only the weighting function for the layer close to the satellite that is significant in the measurements in saturation cases as seen in Figure 2.10a. The broad horizontal widths (of about 300-400 km) of both functions are quite apparent and this is a typical character of weighting function for all saturated MLS channels.
Figure 2.13: Temperature weighting functions of saturated MLS channels 4 and 8 plotted with distance $x$ from the tangent point along the ray path towards the satellite.

The bell-shape of the weighting functions along the ray path in the saturation case could be qualitatively described as follows. Because of strong line absorption along the ray path this means that the emission from very far from the satellite (especially from the cold background space or at distances beyond the tangent point) is totally absorbed on the way to the instrument and could be neglected in the calculation (see Figure 2.10a for details). Because of competition between increasing emission and decreasing transmission along the ray path near to the satellite, the temperature weighting functions increase smoothly with $x$ from a zero value to a single maximum and then smoothly decay to small values. Due to broad horizontal width of the weighting function along the ray path, this could make the temperature variation whose scale is small or comparable to width of weighting function difficult to be detected as its amplitude might be dramatically attenuated on the way to the satellite. Practically, amount of the reduction depends mainly on the wavelength scale of the variation (both in horizontal and vertical directions) compared to those of weighting function and also on the direction upon which the variation occurred. Effects of the averaging by weighting function along the ray path on amplitude of gravity waves observed with the MLS instruments are to be studied in detail in Chapters 4 for UARS MLS and 6 for EOS MLS.
2.5 MLS Observations of Small Temperature Variations

We shall consider in more detail in this section on the sensitivity of the MLS instruments to the small variations occurred in background temperature. This is to support the suggestion that the MLS radiance data in the saturation are suitable for the use in the study of gravity wave perturbations made with the instruments.

2.5.1 Sensitivity of MLS to Small Temperature Variation

The sensitivity of the MLS instruments to the small temperature variation in background atmosphere could be verified simply by comparing the variation in observed radiance profiles when the background temperature $T_0$ (in Figure 2.5) is shifted by some small amounts, i.e. by 5 or 10 K. This makes the input temperature profile $T(z)$ in Eq. 2.39 becomes $T = T_0 + \Delta T$ where $\Delta T = 5$ and 10 K. The corresponding radiance profiles of these new temperatures are derived and the results are shown in Figure 2.14a for a comparison. We find that the displacing of the background temperature profile by up to 5 or 10 K is hardly noticeable when the radiances are not fully saturated but these changes have become more clearly visible when the channel saturates at altitudes $z < 40$ km. These results indicate that the MLS instruments should be most sensitive to the small variations in background temperature when the observed radiances are saturated but they are less sensitive when the radiances are still unsaturated. Figure 2.14b shows the proportion of the changes in background temperature that are detected with the UARS MLS instrument at each tangent height. It is suggested from this figure that over 90% of the changes could be detected with the instrument at tangent height lower than ~ 40 km where observed radiances with channel 4 are already saturated.

It should be noted here that the MLS temperature weighting function also depends on background temperature through the definition of absorption coefficient but this dependence is typically weak in microwave case. As a result, effects of the variation in weighting function with temperature are normally ignored in the microwave remote sensing. Therefore, only the variation in temperature function $T_r$ in Eq. 2.9 with temperature that should contribute most to the variations of the radiance profiles shown in Figures 2.14. In the further study the small changes in the background atmosphere will be assumed to have a wave-like form and the results will be more complicated than those shown in both figures.
Figure 2.14a: Simulated radiance profiles for a single scan of MLS radiance measurements with channel 4 for the perturbation case \((T = T_0 + \Delta T)\) and the unperturbed case \((T = T_0)\) where \(\Delta T = 5.0\) and 10.0 K respectively.

Figure 2.14b: Ratio of the small changes in background temperature given in Figure 2.14a that is detected with the instrument.
2.5.2 Perturbation Form of Radiative Transfer Equation

Finally, we shall derive here the so-called ‘linearised radiative transfer equation’ for the radiance observation in optically-thick atmosphere which is very useful in the analysis of gravity wave perturbations in MLS measurements. We start from the microwave radiative transfer equation 2.39 could be reduced to have a simple form as

\[ T_b(\nu, h_T) = \int_{h_T}^{T(z)} W(\nu, h_T, z) dz. \]  

(2.45)

where the radiant contribution from could cosmic background is neglected here and the weighting function \( W(\nu, h_T, z) \) is given by Eq. 2.40 as

\[ W(\nu, h_T, z) = a(\nu, z)g(z, h_T)e^{-\phi(\nu, z)} \]  

(2.46)

in which all parameters are defined as usual. As stated earlier, the FOV factor is not shown in Eq. 2.46 but it is also included in the calculation. We could readily apply Eq. 2.46 to the observations of small change in background temperature \( T_0 \) with the instrument by replacing temperature \( T \) by \( T_0 + \Delta T \) in Eq. 2.45, this leads to the relation

\[ T_b(\nu, h_T) = T_{b0} + \Delta T_b \]  

(2.47)

where

\[ T_{b0}(\nu, h_T) = \int_{h_T}^{T_0(z)} W(\nu, h_T, z) dz \]  

(2.48a)

and

\[ \Delta T_b(\nu, h_T) = \int_{h_T}^{\Delta T(z)} W(\nu, h_T, z) dz. \]  

(2.48b)

Equation 2.48b is the ‘perturbation’, or ‘linearised’ form of the radiative transfer equation 2.39 which gives us the approximate changes in radiance \( T_b \) when the perturbation term \( \Delta T(z) \) is identified. It is this equation that is normally applied in our study. The linearisation theory of microwave radiative transfer equation in more general cases could be found in many sources, e.g., Westwater et al. [1990,1993] and Canavero [1990].
2.6 Chapter Summary

We have described in this chapter the formulation of the forward model to be used in the simulation of MLS radiance measurements made with UARS MLS 63-GHz radiometer and EOS MLS 118-GHz radiometer at altitude ranges 0-120 km. The observed radiance is assumed to be purely from the emission by molecular oxygen in the middle atmosphere.

We first presented in Section 2.2, background theory of microwave radiative transfer, which is the necessary for the development of the forward model used in this thesis. From this theory, we have derived for radiative transfer equation which is suitable for the use in the model calculation of radiances observed with the MLS instruments. It is indicated from this equation that the contributions to the observed radiances in this case come from the two important sources, which are the cosmic background radiation and the emission along the ray path itself. We later showed that the first contribution dominates radiances observed when the instrument views the atmosphere at very high altitudes (e.g. with $h_T > 80$ km) and where the absorption along the ray path is still very weak. The contribution from the second source is more significant when the viewing altitudes lowered into the mesosphere and it completely dominates the radiances at low tangent altitudes (e.g. with $h_T < 50$ km for UARS MLS channel 4) where the observed atmosphere along the ray path is optically thick.

In Section 2.3, we described in details on the formulation of oxygen absorption coefficient, which is required in the model calculation of MLS radiances. We showed that the absorption coefficient is a strong function of both the altitudes of the observation, the background temperature, and the observed frequency relative to line centre. This makes the absorption of the MLS channels close to line centre significantly higher than those on the line wings for the observations at some constant altitude and background temperature. We later showed in Section 2.4, examples of the calculated radiance profiles in a single limb scan of UARS MLS where the radiances for all channels saturated at tangent height below ~18 km. The saturated radiance is a sign showing that the atmosphere under the observations is optically thick and the absorption along the ray path becomes very strong. The temperature weighting function of each saturated channel indicated that the observed radiance at this stage originate mostly from the emission at atmospheric of ~10 km thickness for all channels except ~15 km for channel 8 (at line centre). As the saturated channels are proved to be very sensitive to small changes in background temperature, the radiance data obtained in this case are, therefore, useful in the gravity wave study with the MLS instruments.
The sensitivity of the saturated channels to the small changes in background temperature occurred in their observed layer was studied in more details in Section 2.5. It was shown that the amplitude of the small variations $\Delta T$ in background temperature could be detected by more than 90% of the original value with saturated MLS channels for $\Delta T \leq 10$ K. In the contrary these variations are barely visible if the channels are not fully saturated. These results support our believe on the capability of the MLS instruments in the observation of wave-induced temperature variation along the observing path which always have amplitude less than ~10 K. However, in realistic cases, the proportion of the wave amplitude to be observed with the instruments are not as high as shown here due to the finite scales of the wavelength and orientation of the waves that involved in the observations as discussed in detail later in Chapter 3.
Chapter 3
Atmospheric Gravity Waves

We present in this chapter background theory of atmospheric gravity waves and how to incorporate them into the forward model developed in the previous chapter for further study. In this thesis, we will focus our attention mostly on the simplest kind of the gravity waves which is produced by the buoyancy force in the atmosphere called "internal gravity wave", whose wavelength scales are believed to be most detectable with the MLS instruments.

This chapter is divided into five sections. Section 3.1 provides background theory of atmospheric gravity waves where the internal waves are considered in particular. In Section 3.2, some basic characteristics and properties of the waves are described in detail to provide more understanding on nature of their propagation in the middle atmosphere. We then present in Section 3.3 the description of MLS wave-observing technique implemented in the model simulation. This includes details of the instrument's observing geometry, wave model hypothesis, and the temperature weighting functions in three-dimension. In Section 3.4, the principles of gravity wave interference on MLS radiance measurements is described in terms of instrument's temperature response and variance response. Finally, the important contents presented in this chapter are summarised in Section 3.5.

3.1 Background Theory

Atmospheric gravity waves are important phenomena in the middle atmosphere because they are currently believed to play a major role in balancing the momentum and thermal budgets through their dissipation process. The simplest kind of gravity waves is called the "internal gravity wave", whose restoring mechanism is purely the atmospheric buoyancy force due to gravity. These waves have relatively short horizontal wavelength in global scale (~10-1000 km) while the typically observed vertical wavelengths are in scales of 1-20 km. The periods of propagation of these waves are normally found to be a few minutes to an hour or so with horizontal phase speeds of up to 100 m/s. The other kind of gravity waves commonly found in the middle atmosphere is called the "inertio-gravity wave", whose restoring force is resulted from the combination of the buoyancy force and the Coriolis force due to Earth's
rotation. The inertio-gravity waves have common structure like the internal waves except in considerably larger space and time scales. The horizontal wavelengths of these waves are normally greater than 1000 km with periods of up to several hours. Recent observations with lidar techniques and some in situ measurements suggest the typical middle-atmosphere gravity waves have dominant vertical wavelengths of 10-15 km and horizontal wavelengths longer than several hundred kilometres [Smith et al. 1987; Fritts et al. 1989]. In addition, gravity waves are also found to be common in the lower atmosphere with recent aircraft and radiosonde measurements (e.g. Allen and Vincent [1995]; Bian et al. [1996]).

The mathematical theory of atmospheric gravity waves is quite well established and its details can be found in several sources, for examples, Hine [1960], Beer [1972], Gossard and Hooke [1974], Lighthill [1978], Gill [1982], Andrews et al. [1987], Lindzen [1990], and Holton [1992]. In the remaining of this section, we will address only the standard form of wave solutions derived from the theory and some explanations on their basic characteristics that are usually found in nature.

3.1.1 Nature of Gravity Wave Propagation

As mentioned in Chapter 1, the theory of atmospheric gravity wave was first introduced by Hines [1960] to explain for the wave-like characteristics of irregular winds usually observed in the lower thermosphere. Attempts to account for their properties theoretically were biased initially toward a turbulence interpretation but Hines has argued convincingly that these fluctuations could be attributed to the internal gravity waves at that level. These waves were believed to be propagating to middle and upper atmosphere from their sources at lower levels (usually in the troposphere and lower stratosphere) with amplitude growing exponentially with height for a non-saturated wave. Since that time this theory has become almost universally accepted and has provided a firm theoretical foundation for subsequent studies (for a historical review see Hines [1972, 1989] and Fritts [1984]).

The popular model of gravity wave accepted at present is that of a wave propagating upward (both in vertical and horizontal direction) from the lower atmosphere region with its amplitude growing exponentially (in response to decreasing background air density) until at some height their amplitude will grow too large and the waves become unstable. Further vertical amplitude growth will then cease (the wave becomes saturated) by dissipation through the irreversible extraction of wave energy into the production of turbulence. This will result in mixing of heat and constituents while a convergence of the vertical flux of
horizontal momentum, associated with the waves, will be deposited into the background mean flow and induced the acceleration (or deceleration) of the mean flow. Theoretically, individual wave groups are considered to propagate and saturate independently, aside from the effects of superposition. However, there is still no unanimous concept of the detailed physical mechanisms involved in the entire process from the generation of the waves to their saturation. Excellent reviews on theories of gravity wave saturation and its effects in the middle atmosphere are given by Fritts [1984,1989].

3.1.2 Standard Form of Wave Function

From the gravity wave theory, the standard form of a monochromatic wave propagating in the Earth’s atmosphere, which is assumed to be stationary, frictionless, and non-dissipation, could be given by [Gill 1982]:

$$\Psi(x, y, z, t) = A_0 e^{z/2H} \exp[i(kx + ly + mz - \omega t)]$$  \hspace{1cm} (3.1)

This is a simple plane-wave form whose origins are assumed to be on Earth’s surface (z = 0) where $$\Psi(x,y,z,t)$$ is a wave function at any point (x,y,z) in space at time t, $$A_0$$ is the initial amplitude at source origin, the exponential term $$e^{z/2H}$$ represents the amplitude growth with height where H is the atmospheric scale height (~7 km in the middle atmosphere). The wavenumbers (k, l, m) are components of wave vector k in three-dimension given by,

$$k = (k, l, m) = \left(\frac{2\pi}{\lambda_x}, \frac{2\pi}{\lambda_y}, \frac{2\pi}{\lambda_z}\right)$$  \hspace{1cm} (3.2)

where $$\lambda_i$$ is the wavelength in i direction. In this co-ordinate system, x and y represent the two orthogonal directions on horizontal plane and z represents the vertical direction. The wave vector k is directed perpendicular to lines of constant phase, and in the direction of phase increase (propagation direction), and $$\omega$$ is called the intrinsic, or Doppler-shifted, frequency measured in the reference frame of background mean wind. For propagating wave, the intrinsic frequency $$\omega$$ is used to define period of wave motion, $$T = 2\pi/\omega$$
Theoretically, for the dispersive waves, the intrinsic frequency $\omega$ could be related to the wavenumbers $(k, l, m)$ by the formula called *dispersion relation* which is different for different type of waves. In a simple case of a monochromatic gravity wave travelling in two-dimensional $xz$-plane, this relation is given by

$$\omega^2 = \frac{N^2 k^2 + f^2 (m^2 + \alpha^2)}{k^2 + m^2 + \alpha^2}$$

(3.3)

where $f$ is the Coriolis parameter due to Earth’s rotation, $N$ is the buoyancy, or *Brunt-Väisälä* frequency, and constant $\alpha = 1/(2H)$. Equation 3.3 is valid for both the internal and inertio-gravity waves but in case of internal waves the effects of rotation could be neglected ($f = 0$) and Eq. 3.3 is reduced to

$$\omega^2 = \frac{N^2 k^2}{k^2 + m^2 + \alpha^2}$$

(3.4)

Equation 3.4 implies that the intrinsic frequency $\omega$ of the internal waves is always smaller in magnitude than the buoyancy frequency $N$. For average conditions in middle atmosphere $N \sim 0.02 \text{ s}^{-1}$, therefore, the period of motion of internal waves is typically greater than $2\pi/0.02$ seconds $\sim$ 5 minutes in this region.

It should be noted here that the intrinsic frequency $\omega$ is wave frequency measured in the reference frame of the background wind which is normally different from the ground-based observed frequency $\omega_0$. However, these two frequencies can be related to each other through the relation

$$\omega(z) = \omega_0 - k_H U_0(z)$$

$$= \omega_0 - |k_H| U_0(z) \cos \alpha.$$  

(3.5)

where $k_H$ is horizontal wave vector, $k_H \equiv (k, l), U_0(z)$ the mean horizontal background wind speed at height $z$, $U_0(z) \equiv [U_{0x}(z), U_{0y}(z)]$, and $\alpha$ is the angle of wave propagation direction relative to the background wind. As a result from Eq. 3.5, the determination of intrinsic frequency $\omega$ in most ground-based wave observing systems is generally incomplete without simultaneous background wind profiles and knowledge of wave propagation direction.
3.2 Gravity Wave Properties and Characterisation

We discuss in this section some important characteristics and properties of propagating gravity wave which include the wave energy and activity, wave phase and group velocity, and wave saturation process. In the atmospheric observation, gravity waves are usually identified from the small-scale wave-like fluctuations (both in space and time) appeared in the observed profiles of the background wind \((U)\), atmospheric density \((\rho)\), pressure \((p)\), or temperature \((T)\). To analyse these effects qualitatively, the perturbation theory is usually employed. In this theory, the perturbed atmospheric profile is presented as a combination of the background (unperturbed) term and the wave-induced oscillating component:

\[
\Phi(x, y, z, t) = \Phi_0(z) + \Phi'(x, y, z, t)
\]  

where \(\Phi\) represents any generic variable \((U, \rho, p, T)\). The subscript 0 and prime indicate the background and perturbation component of the profile respectively. In this case, it is necessary to assume that the unperturbed term for each variable is much larger than its respective perturbation term except for \(U\) as the solutions are still valid if horizontal mean wind does not exist. The background term is assumed to depend only on height \(z\) from Earth’s surface while the perturbation term could vary both with position and time.

In our study, the gravity wave function to be included in the model is defined by the real part of wave solution \(\Psi\) given in Eq. 3.1,

\[
\Phi'(x, y, z, t) = A_0 e^{z/2H} \cos(kx + ly + mz - \omega t)
\]  

where all parameters are defined as usual. The growth factor \(e^{z/2H}\) for wave amplitude is required theoretically just to ensure the wave energy per unit volume is conserved during the propagation in non-dissipation atmosphere (see Section 3.2 for more explanation). Figure 3.1 shows example of vertical structure of wave function \(\Phi'\) describe by Eq. 3.7 for waves with vertical wavelength 10 and 20 km. The sources of these waves are assumed to be on the Earth’s surface (at \(z = 0\)). The amplitude growing exponentially with height is apparent and this is the most recognisable aspect of propagating gravity waves that is usually found in both ground-based and in-situ measurements discussed in Chapter 1. In this figure, the inertial amplitude \(A_0\) has been adjusted so that the average amplitudes of the waves are between 1-5 K in the middle atmosphere (at \(z = 20-60\) km).
Chapter 3  

Atmospheric Gravity Waves

Figure 3.1: Vertical structure of atmospheric gravity waves with vertical wavelength 10 and 20 km whose origins are assumed to be on the Earth’s surface (at \( z = 0 \)). The growth factors \( e^{z/2H} \) and \( -e^{z/2H} \) (dashed-dot lines) are also shown as a reference.

3.2.1 Wave Energy and Activity

One of the most important properties of gravity waves is their capability to carry energy and momentum generated from the source at lower atmosphere to the higher level in the middle atmosphere or thermosphere. Generally, the energy of a travelling wave is defined in terms of the energy density \( E \) which is the sum of average kinetic and potential perturbation energy per unit volume [Gills 1982],

\[
E = \langle E_k \rangle + \langle E_p \rangle \\
= \frac{1}{2} \rho_0(z) \langle \mathbf{U}' \cdot \mathbf{U}' \rangle + \frac{1}{2} \rho_0(z) N^2 \langle \xi^2 \rangle
\]  

(3.8)

where \( \rho_0 \) is the unperturbed background density, \( \mathbf{U}' \) the wave-induced wind speed, \( N \) the buoyancy frequency, and \( \xi \) for a small vertical displacement relative to an equilibrium position of the oscillation. The brackets indicate a spatial or temporal average. In the absence
of dissipation, the energy density $E$ of the waves must be conserved. Since atmospheric density is decreasing with height as $\exp (z/H)$, to retain the constancy of energy density in Eq. 3.8 requires that the perturbation components $U'$ and those related to $<\xi^2>$ increase in proportion to $\exp (z/2H)$. This is the origin of growth factor introduced in wave function described by Eq. 3.1.

The knowledge on the possessed energy of the waves leads to the definition of wave activity described in terms of the average potential energy per unit mass,

$$E_{pm} = \frac{1}{2} N^2 <\xi^2>.$$ (3.9)

By considering that the density or temperature fluctuations are also associated with vertical displacement of the oscillating atmospheric fluid and therefore related to the available potential energy of the fluctuations, we can rewrite Eq. 3.9 as

$$E_{pm} = \frac{1}{2} (g/N)^2 <(\rho'/\rho_0)^2> = \frac{1}{2} (g/N)^2 <(T'/T_0)^2>$$ (3.10)

where $g$ is acceleration due to gravity, $\rho_0$ and $T_0$ are unperturbed background atmospheric density and temperature respectively. Eq. 3.10 gives the definition of wave activity which is commonly used in the atmospheric studies where the wave-induced density or temperature fluctuations are made available (e.g. in Wilson et al. [1991a,b]; Whiteway and Carswell [1994, 1995]; and Duck et al. [1998]). Written in this form, we can see how the observed fluctuations in atmospheric density or temperature profiles are crucial in the identification of wave activity in the atmosphere through their corresponding variance terms, $<(\rho'/\rho_0)^2>$ and $<(T'/T_0)^2>$ respectively. Most ground-based measurements obtain these density or temperature variance functions from the observed variation with time of these background properties at some specific locations. The MLS experiments, on the contrary, derive the temperature variances from the spatial fluctuations of background temperature seen along the observing track. This makes the MLS variances become more suitable in the study of gravity wave activity in global scale as discussed in Chapter 1.
Chapter 3

Atmospheric Gravity Waves

3.2.2 Phase Velocity and Group Velocity

We now discuss briefly on the definitions of wave phase velocity and group velocity and their relations with wavenumbers \((k, l, m)\) and intrinsic frequency \(\omega\). In theory, the group velocity represents the direction of energy flow, whereas the phase velocity is the observed movement of the wave peaks and troughs. The components of intrinsic phase velocity (measured with relative to background wind) are defined by

\[
(V_{px}, V_{py}, V_{pz}) = (\omega k, \omega l, \omega m)
\]  

(3.11a)

and the group velocity (relative to the fixed frame of reference) by

\[
(V_{gx}, V_{gy}, V_{gz}) = \left( \frac{\partial \omega_0}{\partial k}, \frac{\partial \omega_0}{\partial l}, \frac{\partial \omega_0}{\partial m} \right)
\]  

(3.11b)

To express these component of wave velocity in terms of wavenumbers \((k, l, m)\), we must use the dispersion relation given by Eq. 3.4 and the relation between \(\omega\) and \(\omega_0\) in Eq. 3.5.

We consider here a simple two-dimensional case of a wave travelling in the \(xz\)-plane in which amplitude of horizontal background wind speed becomes \(|U_0(z)| = U_0(z)\), where we have from Eq. 3.4,

\[
\omega = \frac{\pm Nk}{(k^2 + m^2 + \alpha^2)^{1/2}}
\]  

(3.12)

and from Eq. 3.5,

\[
\omega_0 = kU_0 + \frac{Nk}{(k^2 + m^2 + \alpha^2)^{1/2}}.
\]  

(3.13)

The plus (minus) sign is taken for convenience as for eastward (westward) phase propagation relative to the background wind respectively. Eq. 3.12 implies that the gravity-wave intrinsic frequency \(\omega\) will always less in value than the buoyancy frequency \(N\), or \(\omega \leq N\). With these relations, horizontal and vertical phase velocity are then given by

\[
(V_{px}, V_{pz}) = \frac{\pm N}{(k^2 + m^2 + \alpha^2)^{1/2}} (1, k/m)
\]  

(3.14a)

and the group velocity by
\[
(V_{ux}, V_{uz}) = \left( U_{0x} \pm \frac{N(m^2 + \alpha^2)}{(k^2 + m^2 + \alpha^2)^{3/2}}, \pm \frac{-Nkm}{(k^2 + m^2 + \alpha^2)^{3/2}} \right)
\] (3.14b)

Therefore, the horizontal components of group velocity and phase velocity are of the same sign, while vertical components are of opposite sign. This assertion is true for both the positive and negative root of Eq. 3.12. As wave energy is transported at the group velocity, for an internal gravity wave, the downward phase propagation implies upward energy propagation, and vice versa. In the atmosphere, gravity waves that are generated in the troposphere may propagate energy upward many scale heights into the middle and upper atmospheres, even through individual fluid parcel oscillations may be confined to vertical distances much less than a kilometre.

In addition, the horizontal intrinsic phase velocity \(V_{px}\) (relative to background wind) could be related to the observed ground-based phase velocity \(c = \omega_\ell / k\) by (from Eq. 3.5)

\[
V_{px} = c - U_{0x}
\] (3.15)

Therefore, the intrinsic characteristics of the internal gravity waves (both frequency and phase velocity) depend directly on the background wind velocity (and direction) while determinations of apparent phase velocity \(c\) depend primarily on the source of the waves. Effects of background mean wind on gravity wave observation in the middle atmosphere will be discussed in more detail in Section 3.2.4

### 3.2.3 Linear Wave Saturation Theory

It was recognised before that wind and temperature perturbation associated with an upward propagating wave would at some height grow too large and the wave would become unstable. The amplitude growth would then cease or “saturated” above this “breaking” level by the dissipation process. This could result in the transfer of energy and deposition of momentum from the waves into the surrounding atmosphere. A number of theories have been developed so far to explain for the mechanism of wave saturation process, however, there is still no universal concept to explain the whole process of wave saturation. In this section, we will consider only the well-known traditional “linear saturation theory” in which the convective instability of individual wave is taken to be a major factor in limiting the growing of its amplitude.
The linear saturation theory was first used to address gravity wave instability and turbulence production by Hodges [1967, 1969] assuming a vertical convective instability of the wave field. The basic idea of linear theory is that the amplitude of a monochromatic gravity wave propagating upwards will grow to the point where part of the wave would become statically unstable. Here the convective instability criterion is used to determine this wave breaking level, and the wave amplitude above that height is maintained at or below this instability limit. The instability criterion is achieved where the wave perturbation causes the total temperature lapse rate to become superadiabatic, i.e., where

\[ \frac{\partial \theta}{\partial z} < 0 \]  

(3.16)

where \( \theta \) is the atmospheric potential temperature. This is equivalent to the condition on horizontal perturbation wind speed [Fritts 1984]

\[ U' > |c - U_{ox}| \]  

(3.17)

where \( U' \) and \( U_{ox} \) are the horizontal induced-wind velocity and the background mean flow in the direction of wave motion, respectively, and \( c \) is the horizontal ground-related phase speed of the wave. The condition given in Eq. 3.17 leads to the definition of an amplitude threshold of induced wind speed for wave instability given by

\[ U' = |c - U_{ox}|. \]  

(3.18)

Equation 3.18 implies that saturation amplitude of a single wave depends on the amplitude of the background mean wind and its direction relative to phase speed \( c \). If they are in the same direction, the amplitude of \( U' \) is likely to be reduced, while their being in the opposite direction tends to increase value of \( U' \) and also the breaking level altitude of the waves.

From the relation given in Eq. 3.15, we can replace the amplitude threshold \( U' \) by the intrinsic phase velocity \( |V_{px}| \) where we have

\[ U' = |V_{px}| = \frac{\omega}{k} = \frac{N}{m[1 + (k/m)^2 + (\alpha/m)^2]^{1/2}}. \]  

(3.19)

Equation 3.19 indicates that waves with high intrinsic frequency and relatively long horizontal wavelength (low \( k \)) should have large saturation amplitude and, therefore, should
saturate at higher altitude. For waves whose horizontal wavelength is considerably long compared to vertical wavelength (with $k \ll m$) and $m \gg 1/2H$, their saturation amplitudes should be proportional to the vertical wavelengths ($\lambda_z = 2\pi/m$). This could be written in a simple form as

$$|V_{\text{sat}}| = \frac{N}{m} = \frac{N\lambda_z}{2\pi}.$$  

As to be shown in Chapters 4 and 5, MLS is most sensitive only to the waves with relatively long vertical wavelength (e.g. with $\lambda_z \gg 10$ km). These waves are likely to have large saturation amplitude and could travel high into the middle atmosphere before they reach their saturation level. The saturation conditions place no constraints on small-amplitude wave motions. However, wave motions with fluctuation amplitude that exceed saturation values are subjected to turbulent dissipation that acts to prevent the further growth of the wave amplitude.

### 3.2.4 Effects of Background Winds in MLS Wave Observations

We discuss here briefly the concept of the wave-filtering process due to the presence of background mean wind in MLS experiments. As discussed earlier, the background mean wind is expected to play a significant role in determining the wave intrinsic frequency and phase speed. This might affects the breaking level and saturation amplitude of the waves that depend on these two parameters. Generally, there are two conditions on the intrinsic frequency that define the maximum height to which gravity waves can propagate from their source in the lower atmosphere:

1. Where the wave ground-based phase speed $c$ matches the background wind speed $U_{0x}$ in both magnitude and direction which makes the intrinsic frequency and phase speed become zero or, $\omega$ and $V_{p\alpha} \to 0$ (from Eq. 3.15), and the wave will be absorbed there. This level is traditionally referred to as the **critical level**.

2. Where the intrinsic frequency $\omega \to N$, this makes the vertical number $m \to 0$ for waves with relatively short vertical wavelength where $m \gg 1/2H$ (from Eq. 3.12). So the solutions above this level have $m$ imaginary, and so the wave undergoes total internal reflection and such levels are normally referred to as **turning points**.
In principle, the intrinsic frequency and phase speed of the wave will vary with height due to the variation of background mean wind at different altitude in the atmosphere, as a result, the vertically-travelling gravity waves may find a critical level or turning point somewhere that could prevent them from moving further upwards. Wave reflection tends to occur in regions where the background wind shear causes growth in the intrinsic wave frequency and phase speed while wave absorbing tends to occur where the background shear causes the intrinsic frequency and phase speed to shrink in magnitude. The latter condition could also be applied to the breaking of the waves as small results in small amplitude threshold \( U'_x \) according to Eq. 3.19 that could, in turn, indicate the low breaking-level altitude of the waves. The background mean wind thus act like a filter in selecting the wave transmitted to the higher atmosphere from wave sources.

Under the strong background mean flow, the gravity waves propagating upstream, against the wind, will be Doppler shifted or refracted to relatively large intrinsic phase speed and long vertical wavelength (low \( m \)). Hence, this condition favours gravity waves to travel vertically to the higher atmosphere without breaking. On the contrary, if the background wind shear change sign, these upstream propagating waves are likely to be Doppler shifted toward smaller intrinsic phase speed and vertical wavelength which support the breaking of the waves at lower altitudes. For gravity wave motion under relatively weak background wind speed compared to their ground-based phase speed \( c \), the intrinsic frequency and phase speed are then mostly determined by the magnitude of \( c \). Waves with large \( c_l \) also tend to have relatively large intrinsic phase speed and their breaking levels should be much higher than those with small \( c_l \). Hence, the selective filtering by background mean wind may cause a prevailing direction of propagation of the waves against the mean flow. Assuming the strong background wind conditions are in favour of upstream propagation against the wind of gravity waves (without breaking), there should be a positive correlation between wind intensity and wave activity. This correlation was found in several observations using different kinds of technique, e.g., radar [Vincent and Fritts 1987], rocket sounding [Ekermann and Vincent 1989], lidar (Wilson et al.1991b; Whiteway and Carswell 1994), aircraft (Hartmann et al. 1989) and the UARS MLS instrument [Wu and Waters 1996a, b; 1997].
Chapter 3

Atmospheric Gravity Waves

3.3 MLS Gravity Wave Observation Model

We have learned from Chapter 1 that the UARS MLS 63-GHz radiometer (in its saturation mode) could efficiently detect small variations in the observed background temperature which might be due to gravity wave perturbation and the model analysis performed in Chapter 2 did strongly confirm of this capability. In this chapter, we shall describe in details how to use the MLS instruments as a tool in the observation of small-scale gravity waves in the middle atmosphere as mentioned earlier. These include the description of viewing geometry for gravity wave observations, the construction of three-dimensional weighting functions, and the definition of temperature (or amplitude) response and variance response of the observations.

3.3.1 Geometry of MLS Wave Observations

Figures 3.2 shows example of UARS MLS viewing geometry for the observations of atmospheric gravity waves on both horizontal (xy) and vertical (xz) plane. The new coordinate system is introduced here where the origin has shifted from the tangent point as shown in Figure 1.1 to be \( z = z_c \) where \( z_c \) is the peak position of weighting function for the observing channel (e.g. at 44 km for channel 4). The directions x, y, and z, are slightly modified in this new system due to the change in the origin of the coordinate system.

We see that, now, the origin of the system is located along the “data track” of the observing channel which is the path traced out by the saturated radiance measurements (not the “tangent track” as done in Chapter 2). The “across-track” direction (x) is perpendicular to the data track and is positive in the direction towards the satellite while the “along-track” direction (y) is parallel to the data track and is positive in the direction of the satellite motion. In this configuration, the x-direction is nearly aligned with the instrument’s LOS direction because the elevation angle \( \beta \) of the LOS with respect to the horizontal plane is always smaller than \( \sim 7^\circ \) in both UARS MLS and EOS MLS experiments. The geometry shown in Figures 3.2 are for UARS MLS channel 4 which has \( z_c = 44 \) km and the tangent height of observation of 18 km and satellite’s orbital altitude of 600 km are used in this case. The observations with other channels have the similar configuration to those shown in Figures 3.2, only the value of \( z_c \) of each channel that is significantly different. This new observing geometry will be used as a basis on the analysis of the gravity wave observations with the MLS instruments done in this thesis.
Figure 3.2: (a) Horizontal and (b) vertical viewing geometry of UARS MLS for gravity wave observations. The thick arrows indicate the across-track (x), along-track (y), and vertical (z) co-ordinate system, whose origin is at the point where the instrument LOS (dashed-dot) intersects the height where the channel saturates ($z_c$). The heights of the tangent point and satellite are given by $h_T$ and $H$ respectively. The solid lines on either side of the LOS indicate the instrument beam width and $\alpha_T$ and $\beta$ represent the data track angle and FOV elevation angle of the observation respectively.
From Figure 3.2b, we find that the elevation angle $\beta$ of the LOS with respect to the horizontal plane is relatively small in both UARS MLS and EOS MLS experiments where the value of $\beta$ at each tangent height $h_T$ is given by

$$\cos \beta = \frac{R_0 + h_T}{R_0 + z_c}$$  \hspace{1cm} (3.20)

where $R_0 = 6378$ km is the Earth's mean radius. For the observations in limb-tracking mode, the angle $\beta$ is fixed at $= 5.3^\circ$ in (for $h_T = 18$ km) but it could slightly vary in the saturated limb-scanning mode form $5.3^\circ$ (for $h_T = 18$ km) to $6.6^\circ$ (for $h_T = 0$ km). Whereas, the data track angle $\alpha_T$ shown in Figure 3.2a is positive counter clockwise from the east.

### 3.3.2 Wave Model Hypothesis

In accord with the new observing geometry that shown in Figures 3.2, we now assume in the model simulation that the wave-induced temperature fluctuations at any point $(x,y,z)$ and time $t$ in the new system have a simple plane-wave form as defined by Eq. 3.7,

$$T'(x,y,z,t) = A_0 e^{\pi/2H} \cos(kx + ly + mz - \omega t)$$  \hspace{1cm} (3.21)

where $z^* = z + z_c$ is the height from Earth's surface. Now, $z$ is represented the height from the co-ordinate origin, where all other parameters are defined as usual. Equation 3.21 is the typical form of the three-dimensional wave functions due to propagating gravity waves with horizontal wavelengths $\lambda_x$ and $\lambda_y$, vertical wavelength $\lambda_z$, and intrinsic frequency $\omega$ where directions of $x$, $y$, and $z$ axes are as given in Figures 3.2.

Figure 3.3 shows examples of wave functions $T'(x,y,z,t)$ of waves with wavelength $\lambda_x = 100$ and 500 km and $\lambda_z = 100$ km given by $T'(x,x,z,t) = e^{\pi/2H} \cos[2\pi(x/100 + z/100)]$ and $T'(x,y,z,t) = e^{\pi/2H} \cos[2\pi(x/500 + z/100)]$ respectively. These functions are plotted with distance $L$ (in km) from the origin along the LOS where $|L| = (x^2 + z^2)^{1/2}$ and $z = x \tan \beta$ (from Figure 3.2a). The weighting function along LOS for channel 4 is also displayed as a reference (dot line). Effects of growth factor, $e^{\pi/2H}$, are shown as the increase in amplitude of the wave peak (unity at the origin) with increasing scale of $L$ towards the satellite position.
Chapter 3  Atmospheric Gravity Waves

Figure 3.3: Structure of waves with wave functions $T'(x,y,z,t) = e^{i2H} \cos[2\pi(x/100+z/100)]$ (dashed-dot line) and $T'(x,y,z,t) = e^{i2H} \cos[2\pi(x/500+z/100)]$ (solid line) along the LOS. The weighting function MLS channel 4 (multiplied by 50) is also shown as a reference (dot line).

As shown later in Figure 3.6a, the visibility in amplitude of these waves to the observing MLS instrument depends directly on scales of their apparent wavelength $\lambda_L$ along LOS direction which, in this case, could be approximated by the relation

$$\lambda_L = \frac{\lambda_x}{\cos \beta + \left( \frac{\lambda_x}{\lambda_z} \right) \sin \beta} = \frac{\lambda_z}{\sin \beta + \left( \frac{\lambda_x}{\lambda_z} \right) \cos \beta}.$$  \hspace{1cm} (3.22)

For the observations with $\beta = 5.3^\circ$, this gives $\sin \beta = 0.092$ and $\cos \beta = 0.996$, hence, if the horizontal wavelength $\lambda_x$ is smaller, or comparable, to vertical wavelength $\lambda_z$ (e.g. with the ratio $\lambda_x/\lambda_z \leq 1$), we have $\lambda_L = \lambda_x$. But if $\lambda_x$ is much longer than $\lambda_z$ (e.g. with $\lambda_x/\lambda_z >> 1$), then value of $\lambda_L$ will depend strongly on both scales of $\lambda_x$ and $\lambda_z$ as described above. For example, wave with $\lambda_x = \lambda_z = 100$ km (dashed-dot line in Figure 3.2) has $\lambda_L = 92$ km while wave with $\lambda_x = 500$ km and $\lambda_z = 100$ km (solid line), has $\lambda_L = 343$ km. Note that $\lambda_x$ could have both positive and negative sign in Eq. 3.22 depended on sign of wavenumber $k$. 

86
It is important to note here that, in the model simulation, we have tacitly assumed that the wave structure described by Eq. 3.21 is “frozen-in” and does not change during the short period of the observation in a single set of 32 measurements made in both limb-scanning and limb-tracking mode of MLS. This means all wave parameters, especially the phase factor $\alpha_\zeta$, are assumed to be constant throughout this period. This is true in case of UARS MLS (and also with EOS MLS discussed in Chapter 5), where the each period of observation takes ~ 65 seconds to be completed while the periods of propagating gravity waves in the atmosphere are normally found to be much longer (up to an hour or more).

### 3.3.3 Wave Perturbation on Radiance Measurements

Effects of the gravity wave perturbation in the radiance measurements with MLS instruments could be identified directly from the variation in the observed brightness temperature of the instruments when the wave-induced perturbation term is included in the model. We start by considering the brightness temperature equation for the observations in optically-thick atmosphere which is given by Eq. 2.45 as

\[
T_b(\nu, h_T) = \int_{\eta_T}^\infty T_\zeta(z)W(\nu; h_T, z)dz
\]  

(2.45)

where $z^* = z + z_c$ as defined earlier in the new co-ordinate system and the FOV factor and cosmic background atmosphere are not included in the calculation at this stage. Written in this form, however, only wave-induced temperature fluctuations along LOS that are taken into account in the computation of $T_b$. As in reality gravity waves could propagate horizontally as well as vertically, to understand what kinds of waves that could be observed well with the MLS instruments, we have to consider the three-dimensional nature of the gravity wave perturbation on background atmosphere. This process is required the use of three-dimensional temperature weighting functions to accommodate the integration described in Eq. 2.45 and details of their constructions are as described below.

### Three-Dimensional Radiative Transfer Equation

To study effects of wave-induced perturbation in three-dimension in the new co-ordinate system, we have to consider the three-dimensional form of the radiative transfer equation 2.45 which is given as
\[ T_b(v, h_r) = \int_{x_0}^{x_1} \int_{y_0}^{y_1} \int_{z_0}^{z_1} T_r(x, y, z, t) W_x(x, z) W_y(y) W_z(v, h_r, z) \, dx \, dy \, dz. \tag{3.23} \]

where the temperature-related term \( T_r(x, y, z, t) \) is now defined by

\[ T_r(x, y, z, t) = T(x, y, z, t) / r(T). \tag{3.24a} \]

Function \( T(x, y, z, t) \) in this case is the combination of the background-temperature term and wave-induced perturbation term given by

\[ T(x, y, z, t) = T_0(z^*) + A_0 e^{-z^*/2h} \cos(kx + ly + mz - \alpha t) \tag{3.24b} \]

where function \( r(T) \) is defined by Eq. 2.11,

\[ r(T) = \frac{kT}{h} \left( e^{h\nu/kT} - 1 \right) \tag{2.11} \]

and all other parameters are defined as usual. Scales of the integration performed in each direction \( (x, y, z) \) in Eq. 3.23 depend on the widths of the weighting functions described along that direction which are \( \sim 400 \) km in \( x \)-direction, 40 km in \( y \)-direction, and 30 km in \( z \)-direction (see Figures 3.4a and 3.4b below).

Three-Dimensional Temperature Weighting Function

The weighting function involved in the calculation of MLS radiances performed with our model from Eq. 3.23 is now composed of three components. These are \( W_{xz} \), which is the normalised FOV function on \( xz \)-plane, \( W_y \) the normalised FOV function along \( y \)-direction (data track of UARS MLS), and \( W_z \) the vertical weighting function of channel 4 as shown in Figure 2.12. From the geometry of the observation shown in Figure 3.2 for small elevation angle \( \beta \), the normalised FOV functions \( W_{xz} \) and \( W_y \) could be approximated by the relations

\[ W_{xz} = \frac{1}{a} \exp \left[ -\left( \frac{z - x \tan \beta}{\delta_{xz}} \right)^2 \right] \tag{3.25a} \]

and
The constants $\delta_{xz}$ and $\delta_y$ are the Gaussian widths of the functions which are related to the FOV half-power beam widths $\gamma_{xz}$ and $\gamma_y$ at the origin by $\gamma = 2\delta(\ln 2)^{1/2}$. As the vertical half-power beam width of UARS MLS is \( \sim 0.21^\circ \) and horizontal width \( \sim 0.41^\circ \) [Jarnot et al. 1996] these approximately give $\delta_{xz} \sim 5.9$ km and $\delta_y \sim 11.6$ km (from Eq. 2.44 with $h_T = 44$ km) while $a$ and $b$ are the normalised factors of $W_{xz}$ and $W_y$ respectively. In these approximations, changes in the beam width along the LOS have been ignored and are replaced by a constant value at the origin instead. This is reasonable as we can find from Eq. 2.44 that the vertical beam widths at $h_T = 40$ and 60 km (saturated layer of channel 4) are changed by only \( \sim 0.4\% \) and \( 1.4\% \) from the width at $h_T = 44$ km respectively.

Figures 3.4 shows the projection of the 3-dimensional weighting function described in Eq. 3.25 on both vertical $xz$-plane, which is defined by $aW_{xz}W_z$, and on horizontal $xy$-plane, defined by $abW_{xz}W_y$ (for $z = 0$), for the saturated MLS channel 4. We can see that the weighting function has an asymmetric shape in the vertical plane, showing slight tilting from the LOS direction towards the horizontal plane. The horizontal projection of the weighting function is also asymmetric around the origin with a width of \( \sim 20 \) km and length \( \sim 150 \) km. Because of the spatial asymmetry of the temperature weighting functions, the magnitude of radiance fluctuations observed depends on the angle between the LOS direction and wave vector. Data from figure 3.4a indicate that only waves with wavefronts more aligned with the LOS direction that should be most detectable with the instruments in terms of observed amplitude. In the contrary, waves whose wavefronts perpendicular to the LOS should be strongly attenuated due to the severe averaging in amplitude of waves crests and troughs along the ray path. The asymmetry of horizontal weighting function shown in Figure 3.4b also indicates that only waves with horizontal wavefronts more aligned with the $x$-direction that should be most visible to the instruments while those with wavefronts perpendicular to $x$-direction should be least visible. It must be emphasised here that the latter conclusion is valid only for the two-dimensional waves travelling on horizontal $xy$-plane and it could not be applied for the observations of three-dimensional waves in general. Detailed analysis of the observations for these two-dimensional waves will be investigated in particular in Section 3.4.1 while geometry of these observations is shown in Figure 3.5.
Figure 3.4: Projection of idealised three-dimensional temperature weighting function on (a) vertical xz-plane and (b) horizontal xy-plane. These weighting functions are constructed for the observations of gravity waves with saturated MLS channel 4.
Figure 3.5: Configuration of MLS wave observation on horizontal plane. The footprints of the weighting function (small ellipse) move across wavefronts (shaded) with an angle $\theta$ along the observing track (y-direction) where $\lambda_H$ is the horizontal wavelength of the observed wave.

Figure 3.5 shows the geometry of MLS wave observations on horizontal $xy$-plane where the observing track is running along $y$-direction and $\theta$ is the angle of the wavefronts on the horizontal plane (shaded) of the wave relative to $x$-direction. Wave propagation direction is defined by the direction of wave vector $k_H$, which is perpendicular to the constant phase lines (or wavefronts) in the above figure in direction of phase increase. This gives the horizontal wave-vector components $(k, l)$ in this case defined by

$$ (k, l) = (k_H \sin \theta, k_H \cos \theta) = \frac{2\pi}{\lambda_H} (\sin \theta, \cos \theta). $$

where $\lambda_H$ is the horizontal wavelength of the wave. We could understand more clearly from this figure why waves with wavefronts more aligned with $x$-direction (with $\theta \approx 0^\circ$) should be most visible to the satellite. In the contrary, those with wavefronts more perpendicular to $x$-direction (with $\theta \approx \pm90^\circ$) will encounter strong averaging in amplitude of wave crests and troughs that makes them less visible to the instrument in terms of observed amplitude. Variation of instrument’s amplitude response with angle $\theta$ is shown later in Figure 3.6b.
3.4 Temperature Response and Variance Analysis

We present in this section the method to identify effects of gravity wave interference in MLS radiance measurements in terms of the defined temperature response and temperature variance. The temperature response and variance analysis method given here is similar in concept to the method that Wu and Waters have used in their study in limb-tacking mode but without any instrument noise or other possible error sources taken into account.

3.4.1 Definition of Temperature Response

Generally, the modified radiative transfer equation 3.23 is sufficient for the analysis on effects of gravity wave interference in MLS experiment that is performed with our model. However, these effects could be identified more clearly if we replace the temperature-related function \( T_r(x, y, z, t) \) as a combination between the background term \( T_0(z) \) and the perturbation term \( T'(x, y, z, t) \) as suggested by Eq. 3.24b into Eq. 3.23. This leads us to write the outcome radiance brightness temperature \( T_b \) as a combination of background-related term \( T_{b0} \) and the perturbation component \( \Delta T_b \), or,

\[
T_b(v, h_T) = T_{b0} + \Delta T_b
\]

where

\[
T_{b0} = \int \int \frac{T_0(z)}{r(T_0)} W_x(\nu; h_T, z) W(x, z) dx dz
\]

and

\[
\Delta T_b = \int \int \frac{T'(x, y, z, t)}{r(T)} W_x(x, z) W_y(y) W_\tau(\nu; h_T, z) dx dy dz.
\]

To achieve this, we have assumed that the weighting function \( W_x \) and function \( r(T) \) are not sensitive much on the small changes in background temperature which is generally true in saturation cases of MLS measurements as shown in Chapter 2.

The expression of \( T_b \) as the sum of \( T_{b0} \) and \( \Delta T_b \) enables us to clearly identified scales of wave perturbation effect by the observed magnitude of the perturbation terms \( \Delta T_b \) which is nearly independent from the background-related term \( T_{b0} \). We can now express \( \Delta T_b \) from Eq. 3.28b in terms of wave function \( T'(x, y, z, t) \) as
\[
\Delta T_b = \int_y \int_x A_0 \frac{e^{z^*/2H}}{r(T)} \cos(kx + ly + mz - \omega t) W_x(x, z) W_y(y) W_z(\nu; h_T, z) dx dy dz. \tag{3.29}
\]

where \(z^* = z + z_c\) is the height from Earth's surface defined earlier. By replacing \(z^* = z + z_c\) into Eq. 3.29 and assuming that the initial amplitude \(A_0\) is constant throughout the observed saturated layer, we can rewrite Eq. 3.29 as

\[
R(k, l, m, \omega) = \int_y \int_x e^{z^*/2H} \cos(kx + ly + mz - \omega t) W_x(x, z) W_y(y) W_z(\nu; h_T, z) dx dy dz \tag{3.30}
\]

where

\[
R = \frac{\Delta T_b}{A_0 e^{z^*/2H}} \tag{3.31}
\]

is called the ‘temperature response’ of the instrument for waves with wavenumbers \((k,l,m)\) and intrinsic frequency \(\omega\). By definition in Eq. 3.31, the temperature response represents the percentage of the maximum amplitude of the wave at the origin that is detected by the instrument at any time \(t\). Therefore, it might also be called the ‘amplitude response’ for the wave at time \(t\) in this sense. The temperature response is a good measure on the sensitivity of MLS to the wave-induced small-scale temperature fluctuations in background temperature and its values depend primarily on wavelengths, intrinsic phase, and propagation direction of the observed waves with respect to LOS direction by definition.

From Eq. 3.21, if we assume that the initial amplitude \(A_0\) and the response function \(R\) is nearly constant for individual wave observed by saturated MLS channels at different altitude level, we could suggest the observed wave-induced radiance fluctuation \(\Delta T_b\) at each channel should be in phase with average amplitude growing exponentially with height. These are the usual characteristics that we have found in the small-scale MLS radiance fluctuations shown in Figure 1.7b. However, as we can see from that figure that the average amplitude of the fluctuations of MLS is found to be rather low in general (only \(\sim 1-3\) K) and, normally, the maximum temperature response for each wave is only \(\sim 1.0\), this makes the amplitude term \(A_0 e^{z_c/2H}\) in Eq. 3.31 to be rather small. For example, if we assume \(R = 0.5\) for all waves, we will have values of \(A_0 e^{z_c/2H} \sim 2.6\) K for \(\Delta T_b = 1-3\) K.
Figure 3.6 gives examples of temperature response $R$ calculated from Eq. 3.30 for (a) waves moving along LOS with wavelengths $\lambda_L$ between 1-1000 km (like the ones shown in Figure 3.3) and (b) propagating waves with angle $\theta$ between $-\pi$ and $\pi$ (see Figure 3.5). Here, we assume the first group of waves is simply represented by wave function

$$T'(x, y, z, t) = e^{\pi^2/2\lambda_L} \cos\left(\frac{2\pi L}{\lambda_L}\right)$$  \hspace{1cm} (3.32a)

where $L$ is the distance from the origin along LOS and the second group by

$$T'(x, y, z, t) = e^{\pi^2/2\lambda_H} \cos\left(\frac{2\pi}{\lambda_H} (x \sin \theta + y \cos \theta)\right)$$  \hspace{1cm} (3.32b)

where $\lambda_H$ is the horizontal wavelength given by $\lambda_H = \lambda_x \sin \theta = \lambda_y \cos \theta = 100, 200, \text{ and } 500 \text{ km}$ in this calculation. Note that in the first case we imply that the waves have horizontal wavelength $\lambda_y = \infty$ (for wave propagating only on $xz$-plane), and in second case, they are just waves moving only on horizontal $xy$-plane with unity amplitude at the origin.

It is clearly shown in Fig. 3.6a that the temperature response to waves moving along LOS increases monotonically with wavelength scales of the waves. Waves with relatively short wavelength compared to width of weighting function along LOS (e.g. with $\lambda_L < 100$ km) are barely visible to MLS due to the FOV smearing and severe averaging in their amplitude along the path. Waves with wavelength scales of $\sim 200$-$600$ km are more visible to the instrument with the response amplitude increases rapidly from $\sim 0.3$-$1.0$. For waves with relatively long wavelength along LOS (e.g. with $\lambda_L > 600$ km), they are in the group of waves that most detectable in amplitude with response of up to more than 100%. This could happen due to the growth factor of wave amplitude that is involved in the calculation. The very long $\lambda_L$ of the observed wave appeared when they have wavefronts that are more aligned with the LOS direction. These results support our prior suggestion that only under this situation that the propagating gravity waves should be most visible to the MLS instruments in the real observations. The variation of amplitude response with wavelength $\lambda_L$ shown here is very useful to our study as we could estimate the response of the instruments to any wave under the observation directly once the scale of its apparent wavelength along LOS is given (i.e. from the relation in Eq. 3.22).
Figure 3.6a: Examples of the MLS temperature response plotted with wavelength along the LOS direction.

Figure 3.6b: The MLS temperature response plotted with angle $\theta$ for waves propagating on horizontal plane with horizontal wavelength $\lambda_H = 100$, 200, 500 km.
Chapter 3

Atmospheric Gravity Waves

The results shown in Figure 3.6b are for waves moving on horizontal xy-plane which are most familiar in everyday life but not quite realistic in representing the gravity waves that are normally propagating in three-dimension with vertical wavelengths of few kilometres. However, these results could be regard as for the idealistic case (at $\lambda_z = \infty$) which could be compared with results from more realistic cases (for $\lambda_z = 1\text{-}20 \text{ km}$) shown later in the next chapter. For these two-dimensional waves, they are most visible at angle $\theta = 0, \pm \pi$ which means their propagation direction is approximately perpendicular to the x-direction with their wavefronts more aligned with the LOS and encounter less attenuation. Waves with $\theta = \pm \pi/2$ are least visible (especially those with short $\lambda_H$) because they have wavefronts that are more perpendicular to the LOS direction and undergo severe attenuation along the ray path. The response to waves with very long $\lambda_H$ (e.g. with $\lambda_H > 500 \text{ km}$) is less sensitive to the direction of wave propagation as expected. The amplitude response for the three-dimensional waves with $\lambda_z = 1\text{-}20 \text{ km}$ are quite different from the idealistic case shown here as described later in Chapter 4. However, the response results in Figure 3.6b are more applicable if the observed vertical wavelengths are relatively large (e.g with $\lambda_z >> 20 \text{ km}$).

3.4.2 Variance Analysis Method

From the knowledge of wave-induced brightness temperature fluctuations (described in terms of temperature response), we can derive the temperature variances associated to these fluctuations using the variance analysis method described as follows. First, we consider the standard definition of the temperature variance, $\sigma^2$, which is given by

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\Delta T_{b,i} - \bar{\Delta T}_b)^2, i=1,2,\ldots,N \quad (3.33)$$

where $N$ is the total number of measurements used in each calculation and $\bar{\Delta T}_b$ is the average radiance fluctuation from these $N$ measurements given by

$$\bar{\Delta T}_b = \frac{1}{N} \sum_{i=1}^{N} \Delta T_{b,i}, i=1,2,\ldots,N \quad (3.34)$$

The term $\Delta T_{b,i}$ represents individual radiance perturbation obtained at the $i$-th measurement given by Eq. 3.29. As we have assumed earlier that the satellite observing track is along the y-direction (see Figure 3.5), therefore, the only term that changes explicitly between each
adjacent measurements along the track is \( ly = (2\pi/\lambda_y)y \). For convenience in the calculation, we assume that the first measurement is made at \( y = 0 \) and, therefore, the radiance response for the \( i \)-th measurement along the data track (in terms of temperature response) is given in accordance with Eq. 3.30 by

\[
R_i = \int_{y}^{\frac{\pi}{2\lambda_y}} \int_{t}^{r(T)} \cos\{kx + l[(i-1)\Delta y - y] + mz + \phi\} W_x W_y W_z dx dy dz 
\]

where \( \Delta y \) is the gap between each individual measurements and \( \phi = -\alpha t \) represents constant phase during the measurements at time \( t \) and \( i = 1, 2, ..., N \). This means we could suggest that only the temperature variation occurred along the observing track (\( y \)-direction) that could distribute most to the temperature variances derive from MLS radiance data.

By substituting \( \Delta T_{b,i} \) in Eq. 3.33 by function \( R_i \) from Eq. 3.35, we can express the temperature variance \( \sigma^2 \) in terms of \( R \) as

\[
\sigma^2 = \frac{1}{N-1} \sum_i (R_i - \bar{R})^2, i = 1, 2, ..., N
\]

where \( R_i \) is given by Eq. 3.35. The term \( \sigma_R^2 \) is called the 'variance response', which is defined by

\[
\sigma_R^2 = \frac{\sigma^2}{A_0 e^{-y/1H}}
\]

By this definition, apart from the temperature variation along \( y \)-axis, the variance response also depends significantly on the number of measurements \( N \) which is different between the variance analysis in MLS limb-tracking and limb-scanning mode. This because, in limb-scanning mode, only the bottom 6 measurements of each 32-measurement set in one single scan which meet the saturation requirement are used while in limb-tracking mode, all 32 measurements could be used in the analysis. Since the MLS measurements are made every 2 seconds, the horizontal distance, \( \Delta y \), between adjacent points along data track is then approximately 15 km (satellite velocity is \( \sim 7.5 \) km/s). Therefore, the variances obtained from data in limb-tracking mode will cover longer horizontal distance (\( \sim 465 \) km) than from limb-scanning mode (\( \sim 75 \) km). In practice, the distance \( \Delta y \) is hardly change significantly in MLS operation therefore, in this thesis, we are going to keep \( \Delta y \) constant at 15 km but vary the value of \( N \) as appropriate to see its effects on the MLS variance analysis with our model.
The definition of temperature variance given by Eq. 3.36 is just for a single set of measurements for wave with wave numbers \((k, l, m)\) and phase \(\phi\) and it is different from the reported variances, which are normally described in terms of the average value calculated over some period of time. It is possible that each of these average variances might be associated to waves with the same wavelength but it is unlikely for those waves to have the same phase \(\phi\) that varies with time. From this reason, we therefore decided to express the temperature variances produced a single wave with wave numbers \((k,l,m)\) by the average value of variance response, \(<\sigma_R^2>\), calculated for this wave with 100 random phases \(\phi_j\), or

\[
<\sigma_R^2> = \frac{1}{100} \sum_j \sigma_{R,j}^2, \quad j = 1, 2, ..., 100
\]

where \(\sigma_{R,j}^2\) is the variances for each chosen phase \(\phi_j\) of the wave. All variance responses reported later in this thesis will be referred to this average value described above unless stated otherwise. It should be noted that the average variance response \(<\sigma_R^2>\) depends only on the wavelength components \((\lambda_x, \lambda_y, \lambda_z)\) of the waves but not on intrinsic phase \(\phi\).

### 3.5 Chapter Summary

We presented in this chapter the background theory of atmospheric gravity waves and their incorporation into the forward model used for the simulation of MLS radiance measurement. In Sections 3.1 and 3.2, we described nature of the propagation of the waves in three-dimension and their basic properties and characteristics that are usually found in the real observations. These included energy and activity, phase and group velocities, and the saturation process. The mathematical form of gravity wave function for the use in the model was then defined. The gravity wave hypothesis employed here is that of a non-breaking monochromatic wave propagating upwards, both in vertical and horizontal direction, from the source situated on Earth's surface, with amplitude grows exponentially with height.

In Section 3.3, the construction of the three-dimensional MLS weighting functions was considered. We showed that the weighting function in this case has a asymmetric shape in the vertical plane, showing slight tilting from the LOS direction towards the horizontal plane. The horizontal projection of the function is also asymmetric around the origin with a width of \(~20\) km and length \(~150\) km. Because of the spatial asymmetry of the weighting functions, it is expected that the amplitude response to each wave.
In Section 3.4, we described in detail of the analysis methods used for the derivation of temperature (or amplitude) response and variance response in our study. We then showed, for examples, the results of temperature response to waves propagating in two-dimension which indicate that the instrument should be most sensitive to waves whose phase lines are more aligned with the LOS direction due to less attenuation in their amplitude along the ray path. Waves with phase lines perpendicular to the LOS are less likely to be detected well due to severe averaging along the path unless they have very long horizontal wavelength. However, we emphasised that this explanation is only valid for the two-dimensional waves moving on horizontal plane but not for the three-dimensional waves which have vertical wavelength scales of few kilometres found in real atmosphere. The response results for the latter case will be presented in the next chapter.
Chapter 4
Temperature Response Analysis

We have already described in detail in Chapter 3 the analysis method used to derive the temperature (or amplitude) response and variance response of the MLS instruments in the observations of atmospheric gravity waves studied in this thesis. We shall now consider in this chapter the application of the method to find the temperature responses of the UARS MLS instrument to the observed gravity waves with different scales of wavelength components and orientations. The results will be reported first as a function of wavelength components in three-dimension and, later, as a function of horizontal and vertical wavelengths and wave propagation direction on the horizontal plane.

In this study, we shall consider in particular the response to the gravity waves with horizontal wavelength scale of 1-1000 km and vertical wavelength scale 1-20 km, which are typically found in the middle atmosphere. As mentioned in Chapter 3 that, in the model simulation, we have tacitly assumed that the wave structure and all waves parameters are “frozen-in” and do not change during short period of a single scan made by the instrument. This assumption is valid in both cases of UARS MLS and EOS MLS operations in which they take approximately 65 and 25 seconds respectively to complete a single scan (with ~32 measurements for UARS MLS and 150 measurements for EOS MLS). These times are substantially smaller than the periods of propagating gravity waves in the atmosphere, which are normally found to be up to an hour or more.

This chapter is divided into 5 major sections. First, in Section 4.1, the variation of temperature response on intrinsic phase and effects of the growth factor on the derivation of the response are considered. In Section 4.2, the variation of the response with horizontal and vertical wavelengths is determined where some results and the interpretation are given. In Section 4.3, we investigate the variation of response with wave propagation direction on horizontal plane in more realistic cases. Section 4.4 discusses effects of some additional factors on the derivation of temperature response of the instrument. These include small changes of the viewing direction (or tangent height) occurred in limb-scanning mode, and the difference in vertical widths of weighting function among saturated channels. Finally, summary of the work and its results achieved in this chapter are presented in Section 4.5.
4.1 Dependence on Wave Phase and Growth Factor

We first consider in this section the dependence of the temperature response \( R(k,l,m) \) defined in Chapter 3 on the intrinsic phase \( \phi \) of the observed wave and effects of the growth factor in the calculation of the instrument’s temperature response.

4.1.1 Variation with Intrinsic Wave Phase

To study the importance of intrinsic phase, \( \phi = -\alpha x \), in the derivation of temperature response to a single wave, for convenience, we first assume the wave has a simple form described by

\[
T'(x, y, z, t) = e^{z/\lambda_x^2} \cos(kx + mz + \phi) + 01
\]

which represents a monochromatic gravity wave travelling on vertical \( xz \)-plane with amplitude growing with height where the wavenumber \( l = 2\pi\lambda_y \) could be neglected in this case. From Eq. 3.30, the temperature response to this wave is then given by

\[
R_{xz}(k,m,\phi) = \int \int_{v} \frac{e^{z/\lambda_x^2}}{r(T)} \cos(kx + mz + \phi) W_x(x,z)W_z(v;h_T,z)dx dz
\]  

where response function \( R_{xz} \) represents the dependence of \( R(k,l,m) \) on wavelengths \( \lambda_x \) and \( \lambda_z \) and phase \( \phi \), but not on \( \lambda_y \). In principle, the dependence of \( R \) on wavelength \( \lambda_y \) could be treated separately as shown in the next section.

Figure 4.1 show examples of the response function \( R_{xz} \) calculated from Eq.4.2 for waves with wavelengths \( \lambda_x = 300, 500, 1000 \) km, \( \lambda_z = 10 \) km, and phase \( \phi \) between \(-\pi\) and \( \pi \). It is immediately seen that the temperature response to all waves varies periodically with \( \phi \) with the same period of \( 2\pi \) but different in amplitude. However, as the average response for all waves shown here is very low (always less than \(-1\%)\), it indicates that these waves might not be visible much to the instrument. We will show later in this chapter that the responses to waves with \( \lambda_z = 1-20 \) km increase rapidly with increasing scale of \( \lambda_z \) and only those moving away from the satellite with respect to \( x \)-direction (with \( k < 0 \)) that are most visible.
Temperature Response with Intrinsic Phase $\phi$ (for $\lambda_z = 10$ km)

<table>
<thead>
<tr>
<th>$\lambda_x$ (km)</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.000</td>
</tr>
<tr>
<td>500</td>
<td>0.010</td>
</tr>
<tr>
<td>1000</td>
<td>0.020</td>
</tr>
</tbody>
</table>

**Figure 4.1:** Variations of temperature response $R_{xz}$ with intrinsic phase $\phi$ for gravity waves with wavelengths $\lambda_x = 300, 500, 1000$ km, and $\lambda_z = 10$ km ($\lambda_y = \infty$ in this case).

We notice from Figure 4.1 that the maximum magnitude of function $R$ for each wave occurred at position $\phi_{\text{max}} = -0.3\pi$ and its values increase steadily with increasing horizontal wavelength $\lambda_x$. For examples, for $\lambda_x = 300$ km we have $R_{\text{max}} = 0.001$ while for $\lambda_x = 1000$ km we have $R_{\text{max}} = 0.013$. The explanation for the variation of these response functions with $\phi$ seen in Figures 4.1 lies in the definition of the response given by Eq. 4.2 itself. As proved in Appendix B, in principle, the response function $R_{xz}$ for each wave at any given phase $\phi$ could be written as a linear combination of the response at phase $\phi = 0$ and $\pi/2$, or,

$$R_{xz}(k, m, \phi) = a_1 \cos \phi + a_2 \sin \phi \quad (4.3)$$

where the constants $a_1$ and $a_2$ are defined by

$$a_1 = R_{xz}(k, m, \phi = 0) = \int \int e^{t/2H} \cos(kx + mz) W_x(x, z) W_z(v; h_T, z) dx dz \quad (4.4a)$$

and
\[ a_2 = R_{xz}(k, m, \phi = \frac{\pi}{2}) = -\int_{x} e^{\frac{z^2}{2h}} \sin(kx + mz) W_x(x, z) W_z(v; h_T, z) \, dx \, dz. \] (4.4b)

Therefore, the value of temperature response at each phase \( \phi \) could be derived directly from Eq. 4.3 if values of the constants \( a_1 \) and \( a_2 \) are known.

From Eq. 4.3, we see the \( R_{xz} \) in this case is a sinusoidal function of period \( 2\pi \) and the maximum response will occur at \( \phi = \phi_{\text{max}} \) given by

\[ \phi_{\text{max}} = \tan^{-1}(a_2 / a_1) \] (4.5)

For example, for \( \lambda_x = 500 \, \text{km} \), it has \( a_1 = 0.0032 \) and \( a_2 = -0.0042 \), therefore, its response function could be written in the form

\[ R_{xz}(\phi) = 0.0032 \cos \phi - 0.0042 \sin \phi. \]

Therefore, the maximum response should occur at \( \phi_{\text{max}} = \tan^{-1}(-0.0042/0.0032) = -0.3\pi \) and \( 0.7\pi \) as shown in Figure 4.1.

From this knowledge, we can conclude that the temperature response to an individual wave will vary periodically with phase \( \phi \) where the maximum response will occur at phase \( \phi_{\text{max}} \) defined by Eq. 4.5. This maximum form of the response function \( R_{xz} \) in this study could be written as

\[ R_{xz}(k, m) = \int_{x} \int_{z} e^{\frac{z^2}{2h}} \cos(kx + mz + \phi_{\text{max}}) W_x W_z \, dx \, dz. \] (4.6)

This is the maximum response that a wave with wavelength \( \lambda_x \) and \( \lambda_z \) could obtain under the observation with MLS instrument. Note that the increase in the horizontal wavelength \( \lambda_x \) not only affects the maximum value of response to each wave, but also makes the response more symmetrical about \( \phi = 0 \) as we could expect from the relations shown in Eqs. 4.3 and 4.4 given above. It is the temperature response function in the maximum form as shown in Eq. 4.6 that will be employed later throughout this thesis.

103
4.1.2 Effect of Amplitude Growth Factor

From the definition of response function $R_{xz}$ described by Eq. 4.6, now we will consider briefly effect of the growth factor, $e^{z/2H}$, in the derivation of temperature response for UARS MLS. This factor was initially introduced in Eq. 3.30 to explain for the amplitude growing with height from the origin suggested by the gravity wave theory.

<table>
<thead>
<tr>
<th>Horizontal Wavelength $\lambda_z$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0</td>
</tr>
<tr>
<td>62.5</td>
</tr>
<tr>
<td>83.3</td>
</tr>
<tr>
<td>125.0</td>
</tr>
<tr>
<td>inf</td>
</tr>
<tr>
<td>250.0</td>
</tr>
<tr>
<td>125.0</td>
</tr>
</tbody>
</table>

Figure 4.2: Temperature response $R_{xz}$ plotted as a function of wavenumber $k$ in unit of cycles/50 km for gravity waves with $\lambda_z = 10$ km. The response results are shown for both the calculation with growth factor (dotted line) and without growth factor (dashed line) included. The corresponding wavelengths in kilometres are also marked along the opposite axes.

Figure 4.2 shows examples of the response function $R_{xz}(k,m)$ described as a function of horizontal wavenumber $k$ for waves with wavelength $\lambda_z = 10$ km. We find from this figure that effect of the growth factor is very small and could be negligible for waves with $k > 0$ regardless of the scales of $\lambda_x$. However, this effect is more profound for waves with $k < 0$, especially those with $\lambda_x$ close to the optimal value of $\approx 200$ km where the average response is increased by over 10% at this point. Therefore, in terms of accuracy, the inclusion of the
term $e^{z/2H}$ is necessary in the calculation of instrument's response to these waves. However, if this term is neglected, the exponential growth of wave amplitude with height is still pronounced by the term $e^{z/2H}$ remain in the definition of response function in Eq. 3.31. From this knowledge, we could expect the observed amplitude of the non-breaking wave with MLS instrument to increase rapidly with height of the saturation altitude $z_c$ as seen in Figure 1.7b. The observed MLS temperature variance in this case, by definition, should also be increased in proportion to $e^{z/2H}$. The reports of UARS MLS variances by Wu and Waters illustrated in Figure 1.10 do support the idea of exponential growth of wave amplitude with height as expected from the theory.

From geometry of the observations shown in Figure 3.2b, the positive value of $k$ indicates waves which are propagating towards the satellite with respect to $x$-direction (or LOS direction) and the negative $k$ means they are moving away from the satellite relative to that direction. As we have assumed at first that the gravity waves always propagate vertically upwards, this means the vertical wavenumber $m$ of the waves must have only the positive value ($m > 0$). But, as the waves could move freely in any direction on horizontal plane, the horizontal wavenumbers $k$ and $l$ could have both positive and negative value. However, as the weighting function $W_y$ is symmetric about the origin, the change in sign of $l$, therefore, does not affect the calculation of response to the observed waves much like it does in the change in sign of $k$ as seen in Figure 4.2.

The change in sign of wavenumber $k$ not only affects the propagation direction but, more importantly, also the structure of the wave along LOS direction as shown, for example, in Figure 4.3 for waves with wave functions \( T'(x,y,z,t) = e^{z/2H}\cos[2\pi(x/200 + z/10)] \) and \( T'(x,y,z,t) = e^{z/2H}\cos[2\pi(-x/200 + z/10)] \). These are gravity waves with wavelength $\lambda_x = 200$ km and $\lambda_z = 10$ km propagating along vertical $xz$-plane with unity amplitude at the origin. It is clear that the change in sign of $k$ from positive to negative has substantially increased scale of the apparent wavelength $\lambda_L$ of the wave along LOS (from $\lambda_L = 70$ km to $\approx 240$ km). This means waves with $k > 0$ tend to have phase lines that are more perpendicular to the LOS and so are strongly attenuated while those with $k < 0$, however, have phase lines that are more aligned with the LOS and so undergo less attenuation. The asymmetry of temperature response about $k = 0$ has profound effects in the gravity wave observations with the UARS MLS instrument as to be shown more clearly in the next section.
4.2 Dependence on Horizontal and Vertical Wavelengths

In this section, we will consider the dependence of the response function \( R(k,l,m) \) with wavelength components \((\lambda_x, \lambda_y, \lambda_z)\) of the observed wave in greater detail. The dependence of the function on wavelength \( \lambda_x \) will be treated separately from that on wavelengths \( \lambda_y \) and \( \lambda_z \).

In this thesis, the former dependence is represented by the function \( R_y(l) \) defined below and the latter by the function \( R_{xz}(k,m) \) already given by Eq. 4.6.

### 4.2.1 Variation with y-Horizontal Wavelength

As stated earlier in this chapter, we will represent the response function \( R(k,l,m) \) of the instrument to an individual wave with wavelength components \((\lambda_x, \lambda_y, \lambda_z)\) by the maximum form given by

\[
T'(x,y,z,t) = e^{x/\lambda_x}\cos[2\pi(x/\lambda_x + z/\lambda_z)]
\]

\[
T'(x,y,z,t) = e^{x/\lambda_y}\cos[2\pi(x/\lambda_y + z/\lambda_z)]
\]

---

**Figure 4.3:** Structure along LOS direction of gravity waves whose wave functions are given by

\[
T'(x,y,z,t) = e^{x/\lambda_x}\cos[2\pi(x/\lambda_x + z/\lambda_z)]
\]

and

\[
T'(x,y,z,t) = e^{x/\lambda_y}\cos[2\pi(x/\lambda_y + z/\lambda_z)]
\]
\[ R(k, l, m) = \int \int \int \frac{e^{ij2H}}{r(T)} \cos(kx + ly + mz + \phi_{\text{max}}) W_x W_y W_z \, dx \, dy \, dz. \] (4.7)

where \( \phi_{\text{max}} \) is given by Eq. 4.5. This is a numerical expression of the dependence of the temperature response function on wavelengths \((\lambda_x, \lambda_y, \lambda_z)\) to be considered in our study. Examples of the results obtained from this equation showing the variation of \( R \) with \( \lambda_y \) at some chosen values of \( \lambda_x \) and \( \lambda_z \) are shown in Figure 4.4.

![Temperature Response with Y-Horizontal Wavelength](image)

**Figure 4.4:** Variation of temperature response with wavelength \( \lambda_y \) to waves with \( \lambda_x = 200, 500, 1000 \) km (with \( k < 0 \), and \( \lambda_z = 10 \) km

Data from Figure 4.4 indicate that the response is very low (i.e., less than 5%) to waves with \( \lambda_y < 20 \) km regardless of the scales of wavelength \( \lambda_x \) due to the FOV smearing effects. However, the average response increases rapidly with increasing scales of \( \lambda_y \) from 20 to 100 km, as the FOV smearing is less significant at this stage. However, the response to all waves become saturated when \( \lambda_y \) is approximately greater than 200 km. Value of the response to each \( \lambda_x \) in this case is independent of \( \lambda_y \) and is determined only by scales of \( \lambda_x \). For example, the response at \( \lambda_x = 200 \) km is now = 0.19 and at \( \lambda_x = 1000 \) km is = 0.06.
Figure 4.5: The response function \( R_y(l) \) and its squared amplitude \([R_y]^2\) as a function of horizontal wavelength.

Model results of the response function shown in Figure 4.4 could be explained from the knowledge that the response function \( R(k,l,m) \) could be expressed mathematically as a product of two associated functions \( R_y(l) \) and \( R_{xz}(k,m) \), or, we can write

\[
R(k,l,m) = R_y(l)R_{xz}(k,m)
\]  

(4.8)

as proved in Appendix B where \( R_{xz}(k,m) \) is as defined in Eq. 4.6 and \( R_y(l) \) defined by

\[
R_y(l) = \int_y \cos(ly) W_y(y) dy.
\]  

(4.9)

Function \( R_y(l) \) depends only on wavelength \( \lambda_y \) along and its value for \( \lambda_y = 1-1000 \) km (and its square) is shown in Figure 4.5. The squared amplitude of \( R_y \) is for the use in the analysis of the variance response shown in Chapter 5. It is suggested from this figure that the response function \( R_y(l) \) is significant in the calculation of temperature response only when scales of \( \lambda_y \) approximately less than 200 km. If \( \lambda_y \) are greater than 200 km, the response to each wave will be determined only by function \( R_{xz}(k,m) \) as we have \( R_y(l) = 1 \) at this stage.
4.2.2 Variation with x-Horizontal Wavelength

We shall consider here the dependence of the temperature response on wavelengths $\lambda_x$ and $\lambda_z$ which is described through the response function $R_{xz}(k,m)$ given by Eq. 4.6. We first consider the variation of the response with wavelengths $\lambda_x$ in this section and with $\lambda_z$ in the following section.

![Graph showing the response function $R_{xz}$ as a function of wavenumber $k$ for waves with wavelengths $\lambda_z = 10, 15, 20, and 50$ km. The corresponding wavelengths in kilometres are marked along the opposite axes as a reference.]

Figure 4.6: Response function $R_{xz}$ displayed as a function of wavenumber $k$ for waves with wavelength $\lambda_z = 10, 15, 20, and 50$ km. The asymmetry of the response to each $\lambda_z$ is now clearly pronounced with the peak amplitudes substantially shifted to the left (in negative-$k$ region), especially those with short $\lambda_z$. From these results, they are primarily suggested that the average response to the typically-found waves with $\lambda_z = 10-20$ km will be only $0.2-0.7$ and only those moving away from the satellite relative to LOS that are most visible in terms of observed amplitude.
As seen in Figure 4.6, the response to horizontal wavenumber $k$ in the $x$-direction is cut-off at high positive and negative number due to the FOV smearing and broad width of weighting function along LOS (~200 km). As discussed earlier, at each $\lambda_z$, waves with $k > 0$ have phase lines that are more perpendicular with LOS direction so encounter the strong attenuation and receive rather small response. This response exhibits a monotonic increase with increasing $\lambda_x$ (especially at $\lambda_x = 200$-1000 km) indicating only the longest wavelengths are best observed. On the contrary, waves with $k < 0$, however, have phase lines that more aligned with the LOS and so undergo less attenuation. Moreover, the responses are no longer monotonically increasing but they sharply peak as some value of $\lambda_x$. For example, a wave with $\lambda_z = 10$ km has the largest response of $= 0.2$ at $\lambda_x = 200$ km and wave with $\lambda_z = 20$ km has the maximal response of $= 0.7$ at $\lambda_x = 500$ km. Note that, the response rises dramatically and becomes more symmetric about $k = 0$ as vertical wavelength increases from 10-50 km.

### 4.2.3 Variation with Vertical Wavelength

We have presented in the previous section the variation of the response function $R_{xz}$ with wavelengths $\lambda_x$ at some values of $\lambda_z$. In this section, we are going to consider the variation of this function with vertical wavelength $\lambda_z$ in more detail.

Figure 4.7 shows examples of profile of the response function $R_{xz}$ with vertical wavenumber $l$ for waves with wavelength $\lambda_x = 100, 200, 500, \text{ and } 1000$ km. The responses to both positive and negative values of wavenumber $k$ (for each $\lambda_x$) are expressed. Like in case of the variation with $\lambda_x$ shown in Figure 4.6, the response to waves with $k > 0$ increases continuously with increasing scales of $\lambda_z$ that means only the longest vertical wavelengths that are most visible and response to waves with $k < 0$ shapely peaked at some certain value of $\lambda_z$ for each $\lambda_x$. For example, the maximum response to wave with $\lambda_x = 200$ km is $= 0.4$ at $\lambda_z = 20$ km and to wave with $\lambda_x = 500$ km is $= 0.9$ at $\lambda_z = 50$ km. The low response to small vertical wavelengths is the direct effects of the FOV smearing and broad vertical weighting function of UARS MLS. The average response to each $\lambda_x$ increases rapidly with increasing scales of $\lambda_x$, especially for $\lambda_x = 100$-500 km, and becomes more symmetric about $l = 0$. Note that, the maximum response at $\lambda_x = 1000$ km is slightly greater than unity due to the inclusion of growth factor in the calculation as discussed earlier.
Chapter 4

Temperature Response Analysis

Figure 4.7: Variation of temperature response $R_{xz}$ with vertical wavelength to waves with wavelength $\lambda_x = 100, 200, 500$ and $1000$ km.

Data from figures 4.6 and 4.7 have provided us details of the amplitude response of UARS MLS as a function of wavelengths $\lambda_x$ and $\lambda_z$ of the wave under the observation. To summarise the results, we present in Figure 4.8 the temperature response of the instrument as a function of both wavenumbers $k$ and $l$ for $R_{xz} = 0.01, 0.1, 0.3, 0.6$, and $1.0$ (or the observed amplitude of $1\%, 10\%, 30\%, 60\%$, and $100\%$). As discussed earlier, waves with $k > 0$ have phase lines that are more perpendicular to the LOS direction and so encounter stronger attenuation along the ray path due to the severe averaging in amplitudes of wave crests and troughs. The response to these waves exhibits a monotonic decrease with increasing $k$ and $m$, indicating that only waves with the longest wavelengths $\lambda_x$ and $\lambda_z$ that are most visible.

Waves with $k < 0$, in the contrary, have phase lines that are more aligned with the LOS and so undergo less attenuation and their response is no longer monotonically decreasing but will sharply peak at some particular value of $\lambda_z$ which could be approximately estimated from data in this figure directly if contours of the amplitudes of $R_{xz}$ are sufficiently given. For example, the maximum response to waves with $\lambda_x = 200$ km is $\approx 0.4$ at $\lambda_z \approx 20$ km as shown earlier in Figure 4.7.
Figure 4.8: Temperature response as a function of horizontal wavenumber \( k \) and vertical wavenumber \( l \). The contours are for response amplitudes of 0.01, 0.1, 0.3, 0.5, and 1.0. The equivalent wavelengths in km are marked on the opposite axes.

Figure 4.8 not only provides us the approximate response at some certain values of \( \lambda_x \) and \( \lambda_z \) but also a good chance to identify both horizontal and vertical scales of the gravity waves that should be observed with the instrument at some certain response amplitude. For examples, only waves with scales of the wavelengths \( \lambda_x \) approximately > 125 km and \( \lambda_z > 8 \) km that have the amplitude detected more than 10 % with the UARS MLS and no waves with \( \lambda_z < 25 \) km could have response amplitude greater than = 0.8. However, to define the approximate minimum scales of wavelengths that the instrument could detect in reality, it depends not only the instrument’s response to that waves but also the amplitude of the observed waves themselves. Commonly, the acceptable fluctuation radiance in the observation should have amplitude, at least, comparable or greater than the average noise signal and the uncertainty in the calibration during the measurement. For UARS MLS band 1 this value is normally less than = 0.5 K and in case EOS MLS it is less than = 0.1 K. This means only waves that produce the radiance fluctuations with amplitude greater than = 0.5 K that should be taken into account in the gravity wave study with the UARS MLS instrument and = 0.1 K for EOS MLS instrument.
As a result, if we know the average amplitude of gravity waves in the observed atmospheric layer of each channel, we might be able to estimate minimum scales of the gravity waves that could be well detected with the instrument. From the reports by many recent experiments (e.g. Fetzer and Gille [1993] and Eckermann et al. [1995]), the observed amplitude of the waves in the middle atmosphere are typically between 1-5 K (see Figure 1.2 for example). This except at the very high-latitudes in the Northern Hemisphere and low-latitudes in the Southern Hemisphere where the average wave amplitudes could be as high as = 5-10 K. From this knowledge, we could expect that only waves with scales of $\lambda_x$ greater than $\sim 200$ km and $\lambda_z \sim 10$ km be mostly seen with UARS MLS in terms of the amplitude response as they could produce the radiance fluctuations with amplitude significantly greater than 0.5 K in the observation. The UARS MLS instrument is thus most sensitive only to gravity waves with relatively long vertical wavelength (e.g. with $\lambda_z > 10$ km) that are not typically included in the rocket, or balloon-based, experiments, therefore, they are unique to the observations with the satellite-based limb sounding only.

In addition, if we assume the average amplitudes of 1-5 K to waves observed with UARS MLS, therefore, at typical wavelength of $\lambda_z = 10-15$ km that dominate the middle atmosphere, the observed radiance fluctuation should be in magnitude of = 0.2-3 K as the average responses are = 0.2-0.6 for these waves. This expectation is in good agreement with the observed results shown in Figure 1.7b where the observed amplitudes of the fluctuations are always less than $\sim 3$-4 K. The average amplitudes found in the UARS MLS radiance fluctuations are then depended principally on the initial amplitude and amount of waves with rather large scale of vertical wavelength during observation. In the area where these large scale waves are allowed to propagate higher from their sources into the middle atmosphere without breaking (due to the support of strong background wind), we could expect the amplitude of the observed fluctuations in MLS radiance be significantly higher than that in the area where only waves with short-scale vertical wavelength (e.g. with $\lambda_z < 10$ km) are dominated. As the amplitude of the non-breaking waves is significantly growing with height this could enhance the perturbation effect of these large-scale waves on background atmosphere and make it more visible with the instrument as seen in Figure 1.7b at low-latitudes of the Southern Hemisphere and middle- and high-latitudes of the Northern Hemisphere as mentioned above.
Chapter 4  

Temperature Response Analysis

4.3 Dependence on Wave Propagation Direction

We have already described in Section 4.2 the temperature response of the UARS MLS instrument on wavelength components \((\lambda_x, \lambda_y, \lambda_z)\) of the observed wave in general cases. In this section, we shall describe this response in terms of vertical and horizontal wavelengths and propagation direction on horizontal plane of the observed waves, which might be more close to the situations we actually found in the real measurements. As described in Chapter 3, The propagation direction in our study is defined through the angle \(\theta\) which describes the alignment of wavefronts with respect to positive x-axis as seen in Figure 3.5.

4.3.1 Variation with Propagation Direction

If we consider the projection of the observed waves on horizontal plane as shown in Figure 3.5, we found from Eq. 3.26 that the relations between two horizontal wavenumbers \(k\) and \(l\) of these waves are simply given by

\[
(k, l) = (k_H \sin \theta, k_H \cos \theta) = \frac{2\pi}{\lambda_H} \sin \theta, \cos \theta
\]  

(4.10)

and these give the relation of wavelengths \(\lambda_x\) and \(\lambda_y\) described by

\[
(\lambda_x, \lambda_y) = \left[ \frac{\lambda_H}{|\sin \theta|}, \frac{\lambda_H}{|\cos \theta|} \right]
\]  

(4.11)

where \(\lambda_H\) is the horizontal wavelength of the observed wave and \(\theta\) is the angle of wavefronts relative to x-direction. Written in this form, the horizontal wavelengths \(\lambda_x\) and \(\lambda_y\) are now no longer independent from each other but related by terms of \(\lambda_H\) and \(\theta\), and this enables us to write for the perturbation function \(T'(x,y,z,t)\) given by Eq. 3.21 in the new form as

\[
T'(x, y, z) = e^{2\pi i z} \cos \left( \frac{2\pi}{\lambda_H} (x \sin \theta + y \cos \theta) + \frac{2\pi}{\lambda_z} z + \phi_{\text{max}} \right)
\]  

(4.12)

where the phase term \(\phi_{\text{max}}\) is described by Eq. 4.5 and \(z^*\) is replaced by \(z\) in this expression. This is the form of wave-induced fluctuation with unity amplitude at origin at \(t = 0\) as we have normally used before.
The temperature response $R(k,l,m)$ could now be calculated using Eq. 4.7 and the relations shown in Eq. 4.10. Figures 4.9a and b show response results from these calculations for waves with $\lambda_H = 100, 200, 500,$ and $1000$ km and $\lambda_z = 10$ km in (a) and 20 km in (b). It appears that, at $\lambda_z = 10$ km, only waves with $\theta < 0$ that are most visible in the observations as the response for all waves at $\theta > 0$ is always less than ~ 2%. Waves with relative long $\lambda_H$ (e.g. with $\lambda_H > 500$ km) are also less visible due to the very low response to them (always less than 10 %). There are only waves with $\lambda_H = 100$-500 km that could be most observable to the UARS MLS (with average response > 10%). However, the average response to each wavelength $\lambda_H$ in Figure 4.9a does increase substantially at $\lambda_z = 20$ km especially those with long $\lambda_H$ (e.g. with $\lambda_H > 500$ km). The maximum responses are now = 0.6-0.7 to waves with $\lambda_H = 100$-1000 km. These values are much higher than that in Figure 4.9a which are always less than 0.2. It is indicated from results in Figure 4.9 a that, the response to waves with short scales of $\lambda_H$ (e.g. with $\lambda_H < 200$ km) will be sharply peaked at two distinct values of $\theta$ close to 0 and $-\pi$ and the responses to waves with $\lambda_H > 200$ km peaked at $\theta = 0.5\pi$.

It should be noticed that the response results to each $\lambda_H$ shown here for waves with $\lambda_z = 10$-20 km are significantly different from that for $\lambda_z = \infty$ shown in Figure 3.6b in terms of amplitude and pattern of variation with $\theta$. For ideal waves with very large vertical wavelength scales (i.e., with $\lambda_z >> 20$ km), the dependence of the response on $\lambda_z$ is less important and the pattern of variation of the response with $\theta$ could be approximated by that shown in Figure 3.6b. The response in this case could be as high as 100% to waves with wavefronts aligned with the x-direction and they should be less visible if their wavefronts are perpendicular to that direction (as if the LOS is along x-direction). But, for typical waves with $\lambda_z = 10$-20 km which are more likely to be found in the real atmosphere, their structure on the vertical plane is as important as that on the horizontal plane. This makes pattern of the response to each $\lambda_H$ dramatically change if $\lambda_z$ increases from 10 to 20 km. In this case, waves with wavefronts aligned with the x-direction (with $\theta = 0, \pm \pi$) are no longer have the maximum response but those with wavefronts more perpendicular with the x-direction (with $\theta = \pm \pi$) that are most detectable (for $\lambda_H > 200$ km). The response to each $\lambda_H$ is also not symmetric about $\theta = 0$ as in case of large $\lambda_z$ but it is symmetric about $\theta = \pm 0.5\pi$ instead.
Figure 4.9: Variation of the temperature response with angle $\theta$ for waves with horizontal wavelength $\lambda_H = 100, 200, 500, 1000$ km, where vertical wavelength $\lambda_Z = 10$ km in (a) and $20$ km in (b).
4.3.2 Results Interpretation

The data of the temperature response shown in Figures 4.9a and b are very useful in helping us understand the relation of propagation direction in horizontal plane with the response to gravity wave observed with the UARS MLS instrument in the real atmosphere. However, the pattern of these response results could be explained more clearly by considering the response to waves with wavelengths $\lambda_x$ and $\lambda_y$ that are related to $\lambda_H$ and $\theta$ through Eq. 4.11 at $\lambda_z = 10$ and 20 km, respectively. First, as we have assumed $\lambda_H$ always $\geq 100$ km here, hence, their corresponding $\lambda_x$ and $\lambda_y$ must be always $\geq 100$ km by definition. But as we learned from Figure 4.5 that the total response $R(k,l,m)$ is nearly independent from $\lambda_y$ for $\lambda_y > 100$ km, this makes the response shown in both Figures 4.9a and b be determined mostly by the response function $R_{xz}$ given by Eq. 4.6, or, we have

$$R(k,l,m) = R_{xz}(k,m) \quad \text{for } \lambda_H > 100 \text{ km.}$$

Because the function $R_{xz}$ have already been examined thoroughly in the previous sections we then could use that results to explain for the variation of the response to the waves with $\lambda_H$ and $\theta$ found in Figures 4.9.

To achieve this, we first divide angle $\theta$ into four sections as described below and then calculate for the corresponding wavenumber $k$ and wavelength $\lambda_x$ for each section of $\theta$ from Eqs. 4.10 and 4.11 respectively. This gives us the following results:

$$\theta_1 = [-\pi, -0.5\pi], \quad k = \left[0, \frac{-2\pi}{\lambda_H}\right], \quad \lambda_x = [\infty, \lambda_H]$$

$$\theta_2 = [-0.5\pi, 0], \quad k = \left[-\frac{2\pi}{\lambda_H}, 0\right], \quad \lambda_x = [\lambda_H, \infty]$$

$$\theta_3 = [0, 0.5\pi], \quad k = \left[0, \frac{2\pi}{\lambda_H}\right], \quad \lambda_x = [\infty, \lambda_H]$$

$$\theta_4 = [0.5\pi, \pi], \quad k = \left[\frac{2\pi}{\lambda_H}, 0\right], \quad \lambda_x = [\lambda_H, \infty]$$

In the first quarter where $\theta = [-\pi, -0.5\pi]$, all waves have $k < 0$ with wavelength $\lambda_x = [\infty, \lambda_H]$. From Figure 4.6, we see that waves with $\lambda_z = 10$ km have maximum response $R_{max} = 0.2$ at
\(\lambda_\chi = 200\) km and waves with \(\lambda_z = 20\) km have \(R_{max} = 0.7\) at \(\lambda_\chi = 500\) km in this region. We can then suggest the response to the waves with \(\lambda_h \geq \lambda_\chi(\text{max})\) in both cases of \(\lambda_z\) to increase continuously with decreasing \(\theta\) from \(\theta = -\pi\) (where \(\lambda_\chi = \infty\)) to \(\theta = -0.5\pi\) (where \(\lambda_\chi = \lambda_h\)). But in case of \(\lambda_h < \lambda_\chi(\text{max})\), however, their response will increase with decreasing \(\theta\) until it reaches the peak value at \(\lambda_\chi = \lambda_\chi(\text{max})\), or, equivalently from Eq. 4.11, at optimal angle \(\theta_{\text{max}} = -\sin^{-1}[\lambda_h/\lambda_\chi(\text{max})]\), after this point the response will drop continuously until it gets to \(\theta = -\pi/2\). As a result, we could expect the optimal angle \(\theta_{\text{max}}\) for waves with \(\lambda_h = 100\) km and \(\lambda_z = 10\) km to be at \(\theta = -\sin^{-1}[100/200] = -0.177\pi\) and \(-0.83\pi\) as shown in Figure 4.9a.

In the second quarter where \(\theta = [-0.5\pi, 0]\), we are still in the negative portion of \(k\) with wavelength \(\lambda_\chi = [\lambda_h, \infty]\). The response to each wave in this case just varies in the reverse order (with respect to \(\lambda_\chi\)) of that in the first quarter. This makes these responses look like the reflection of the responses shown in the first quarter. All response functions in the negative-\(k\) portion of the waves are then essentially symmetric about \(\theta = -0.5\pi\).

In the third quarter where \(\theta = [0, 0.5\pi]\), we are now in the positive-\(k\) portion of the waves with \(\lambda_\chi = [\infty, \lambda_h]\). From data for \(k > 0\) in Figure 4.6, we can expect the response for each wave to decrease continuously with increasing \(\theta\) from \(\theta = 0\) (where \(\lambda_\chi = \infty\)) to \(\theta = 0.5\pi\) (where \(\lambda_\chi = \lambda_h\)). For waves with \(\lambda_z = 10\) km in particular, their response in this region is extremely small (< 1%) regardless of the scale of \(\lambda_\chi\). For waves with \(\lambda_z = 20\) km, their response will decrease with increasing \(\theta\) until it reaches the lowest value at \(\theta = 0.5\pi\) (where \(\lambda_\chi = \lambda_h\)). Finally, in the fourth quarter where \(\theta = [0.5\pi, \pi]\), we are still in the positive-\(k\) portion of Figure 4.6 with \(\lambda_\chi = [\lambda_h, \infty]\). Hence, pattern of the response to each wavelength \(\lambda_h\) in this case is no more than a reflection of the response shown in the third quarter.

We now summarise in Figures 4.10 the variation of UARS MLS temperature response with horizontal wavelengths \(\lambda_h\) angle \(\theta\) to waves with wavelength \(\lambda_z = 10\) km in (a) and 20 km in (b). We clearly see from these figures that the maximum responses to waves with \(\lambda_z = 10\) km are \(= 0.15-0.2\) occurred at \(\lambda_h = 100-300\) km while the maximum responses to waves with \(\lambda_z = 20\) km are \(= 0.6-0.7\) occurred at \(\lambda_h = 200-1000\) km. And only waves with \(\theta < 0\) that are most detectable in both cases.
Chapter 4  

Temperature Response Analysis

(a) Temperature Response with Wave Propagation Direction  
(for $\lambda_z = 10$ km)

![Graph](image)

**Figure 4.10a:** Variation of temperature response with $\theta$ and $\lambda_H$ for waves with $\lambda_z = 10$ km. 
The contours are for amplitudes 0.001, 0.01, 0.05, 0.10, 0.15, and 0.18.

(b) Temperature Response with Wave Propagation Direction  
(for $\lambda_z = 20$ km)

![Graph](image)

**Figure 4.10b:** As in Fig. 4.10a but for $\lambda_z = 20$ km and amplitudes 0.05, 0.10, 0.30, 0.60, and 0.70.
Chapter 4

4.4 Additional Remarks

We have discussed so far the simulation of UARS MLS temperature response to the wavelength components and propagation direction of the waves that are most likely to be found in the middle atmosphere. In this section, however, we will consider some additional functions in MLS operation that might affect the calculation of the response with our model. These include the small variation of instrument’s viewing angle in limb-scanning mode and the difference in vertical width of the channel’s vertical weighting function.

4.4.1 Effect of the Variation in Viewing Angle

In the analysis so far, we have assumed the elevation angle $\beta$ of the instrument to be constant at $5.3^\circ$. This assumption is not true if the measurements are performed in saturated limb-scanning mode which is involved in the small changes of viewing angle varied from $\beta = 5.3^\circ$ (at $h_T = 18$ km) to $= 6.6^\circ$ (at $h_T = 0$ km) for most channels. The variation of $\beta$ with tangent height might significantly affect the value of temperature response to the observed waves that we have studied earlier and this effect is to be considered in this section.

Figures 4.11 shows the variation of the temperature response with wavenumber $k$ to waves with $\lambda_z = 10$ km in (a) and 20 km in (b) at viewing angle $\beta = 5.3^\circ$ and $6.6^\circ$ respectively. We can immediately see that the small change in $\beta$ might cause a substantial change in the response to all waves especially those with $k < 0$. Though, the maximum response is still conserved in the increase of $\beta$ in this case but the sensitivity of the instrument now considerably shifted towards smaller-scale waves (e.g. with $\lambda_x < 200$ km). This makes the response of the instrument to these waves increase significantly, for example, for $\lambda_x = 100$ km and $\lambda_z = 10$ km, the response increases from 0.02 to be 0.08, or about 400%. However, the differences tend to be small for waves with $k > 0$ in both cases of wavelength $\lambda_z$ especially at $\lambda_z = 10$ km. Hence, we can primarily conclude here that the variation of viewing angle form $\beta = 5.3^\circ$ to $6.6^\circ$ in limb-scanning mode does significantly increase the possibility of waves with $\lambda_x = 100$-200 km and $k < 0$ to be observed with the UARS MLS instrument. This might also affect the variance response to these waves as to shown later in the next chapter.
Figure 4.11a: Variation of the temperature response with wavenumber $k$ for gravity waves with $\lambda_z = 10$ km in at two different viewing angles.

Figure 4.11b: As in Fig. 4.11a but for $\lambda_z = 20$ km.
4.4.2 Effect of the Variation in Weighting Function Width

In the UARS MLS experiment, all saturated channels have weighting function with vertical width $\approx 10$ km except for channel 8 (line centre) which is $\approx 15$ km (see Figure 2.12). This difference in vertical width can cause a substantial change in the response to the observed waves with these channels as illustrated below.

![Diagram showing the variation of temperature response of saturated UARS MLS channels 4 and 8 with vertical wavenumber $m$ for waves with wavelength $\lambda_x = 500$ km.]

The difference in vertical width can cause a substantial change in the response to the observed waves with these channels as illustrated below.

**Figure 4.12**: Variation of temperature response of saturated UARS MLS channels 4 and 8 with vertical wavenumber $m$ for waves with wavelength $\lambda_x = 500$ km.

Figure 4.16 shows the variation of temperature response of saturated UARS MLS channels 4 and 8 for waves with wavelengths $\lambda_x = 500$ km. We can see that the average response of channel 8 to all waves are considerably lower than that of channel 4 due to the broader width of its weighting function especially to waves with $\lambda_x < 20$ km. If we take the response 0.1 as a cut-off amplitude of the study, this means the cut-off wavelength $\lambda_z$ of channel 4 should be $\approx 10$ km and channel 8 be $\approx 15$ km at $\lambda_x = 500$ km and $k < 0$. The broader width of weighting function of channel 8 should, therefore, greatly reduce the response of the instrument to the waves with typical wavelength $\lambda_z = 10-20$ km found in the middle atmosphere. Hence, we must take this effect into account if the radiance observations with UARS MLS channel 8 (or EOS MLS channel 13) are involved in the study.
Chapter 4

Temperature Response Analysis

4.5 Chapter Summary

We have presented in this chapter model analysis of the temperature response for the observations of atmospheric gravity waves with saturated channel 4 of UARS MLS 63-GHz radiometer in limb-tracking mode where the instrument views the atmosphere at 18 km tangent height. The response to each observed wave is described both as a function of its wavelength components in three-dimension \((\lambda_x, \lambda_y, \lambda_z)\), and a function of the horizontal and vertical wavelengths and propagation direction defined by the orientation of the wavefronts on horizontal plane with \(x\)-direction.

We first showed in Section 4.1 that the temperature response to an individual wave varies periodically with the intrinsic phase \(\phi\) of the observed wave. We then proved that the response to at any phase \(\phi\) could be written as a linear combination of response at \(\phi = 0\) and \(\phi = \pi/2\) and the value \(\phi_{\max}\) could also be defined from this relation. To avoid effect of varying \(\phi\) in the calculation of the response to each wave, we assumed this response to be computed at \(\phi_{\max}\) which is slightly different for each individual wave. We also showed in this section that the inclusion of the amplitude growth factor in the input wavefunction is necessary in making our study be more realistic and the response results more credible (especially to waves with typical scale of \(\lambda_z = 1-20\) km).

We then considered in Section 4.2 the variation of the response with wavelength components \((\lambda_x, \lambda_y, \lambda_z)\) of an observed wave. The wavelength scales of interest for vertical wavelength are \(= 1-20\) km which are typically found in the atmosphere. We also supposed all these three components to be independent to each other in the analysis. We found in Section 4.2.1 that the dependence of the response function \(R(k,l,m)\) defined by Eq. 4.7 on wavelength \(\lambda_y\) is unique and independent from both wavelengths \(\lambda_x\) and \(\lambda_z\), and phase \(\phi\) of the waves. This dependence is defined by function \(R_y(l)\) given by Eq. 4.9 and plotted in Figure 4.4 for \(\lambda_y = 1-1000\) km. We concluded from the model results that in the observation of gravity waves, the UARS MLS is mostly sensitive to waves with relative long wavelength \(\lambda_y\) (e.g. with \(\lambda_y > 100\) km) and the response is no longer depended on \(\lambda_y\) for waves with \(\lambda_y > 200\) km. At this stage, the response to each wave is defined only by scales of wavelengths \(\lambda_x\) and \(\lambda_z\) through the response function \(R_{xz}(k,m)\) given by Eq. 4.6.
In Sections 4.2.2 and 4.2.3, we proceeded to examine the dependence of response function $R(k,l,m)$ on horizontal wavelengths $\lambda_x$ and vertical wavelength $\lambda_z$ which is defined in terms of the response function $R_{xz}(k,m)$ mentioned above. We showed in section 4.2.2 that the variation of function $R_{xz}$ with $\lambda_x$ depends strongly on both the scale of $\lambda_x$ itself and the sign of wavenumber $k$. By definition, the positive $k$ represents waves that are propagating towards the satellite with respect to $x$-direction and negative $k$ for those moving away from the satellite along that direction. We found that waves with $k > 0$ have phase lines that are more perpendicular with the LOS direction so encounter strong attenuation and have rather small response. The response to these waves exhibits a monotonic increase with increasing scales of $\lambda_x$ (especially for $\lambda_x = 200-1000$ km) indicating only the longest wavelengths are best observed. On the contrary, waves with $k < 0$, however, have phase lines that more aligned with the LOS and so undergo less attenuation. Moreover, their responses are no longer monotonically increasing but sharply peaked as some certain value of $\lambda_x$. We found that, in case of UAS MLS, the response to waves with $\lambda_z = 10$ km is generally low (always less than $\sim 20\%$) but this response does increase dramatically at $\lambda_z = 20$ km (up to 70%). For typical wavelength $\lambda_z$ of 10-20 km, we found that only waves with $\lambda_x > 200$ km that should be most visible to the satellite (with response greater than 20%).

The summary on the variation of response function $R_{xz}$ with wavenumbers $k$ and $m$ is presented in Figure 4.9. It shows that the response to waves with $k > 0$ exhibit a monotonic decrease with increasing $k$ and $m$, indicating that the largest scales of wavelengths $\lambda_x$ and $\lambda_z$ are most visible. The response to waves with $k < 0$ at each $\lambda_z$, however, is no longer monotonically decreasing but sharply peaked at some certain value of $\lambda_x$. The maximum value of the response $R_{xz}$ to a wave with wavelengths $\lambda_x$ and $\lambda_z$ could also be estimated from this figure. In addition, as scales of the vertical wavelength increase, the response function becomes more symmetric about $k = 0$.

In Section 4.3.3, the dependence of the response on the horizontal and vertical wavelengths and propagation direction (defined by angel $\theta$ of the wavefronts on horizontal plane with respect to positive $x$-direction) was analysed. The results are shown, for examples, only at $\lambda_z = 10$ and $20$ km. We found that only waves with $\theta < 0$ that are most detectable with the instrument in both cases as they all have $k < 0$ in these directions. The response to waves with $\lambda_z = 10$ km are generally low (always less than 20%) regardless of
their horizontal wavelength $\lambda_H$ or propagation direction $\theta$ and only those with $\lambda_H = 100-500$ km that are most visible. On the contrary, the response to all waves at $\lambda_Z = 20$ km is substantially increased from that at $\lambda_Z = 10$ km, especially those with very large $\lambda_H$ (e.g. with $\lambda_H > 500$ km). In this case, the maximum response of $\approx 0.7$ could be achieved at $\theta = -0.5\pi$ and with $\lambda_H = 400-500$ km.

In Section 4.4, we considered effects of some additional factors in the derivation of temperature response for UARS MLS observations. These include the small changes in instrument’s viewing direction occurred in limb-scanning mode and the difference in the vertical width of weighting function of channels 4 and 8. We found that the increase of viewing angle from $\beta = 5.3^\circ$ (at $z_t = 18$ km) to $\approx 6.6^\circ$ (at $z_t = 0$ km) does significantly increase the response to waves with relatively short $\lambda_x$ (e.g., with $\lambda_x < 200$ km) and $k < 0$, but it does not affect much on wave with $k > 0$ regardless of the scale of $\lambda_x$. On the contrary, the increase of vertical width of weighting function from $\approx 10$ km for most channels to $\approx 15$ km for channel 8 has substantially reduced the response to most waves, especially those with $\lambda_Z < 25$ km and $k < 0$, and all waves with $k > 0$ due to the stronger averaging in amplitude along the ray path. From these results, we suggested that effects of these factors should also be taken into account in the study of gravity waves the MLS instruments as appropriate.
Chapter 5

Variance Response Analysis

We have presented in the previous chapter model results of the amplitude response to gravity waves observed with channel 4 of UARS MLS 63-GHz radiometer both in limb-tracking mode at 18-km tangent height. In this chapter, we shall consider variance response of the instrument to the observed waves associated to the waves using the variance analysis method described in Section 3.3.4.

By the definition of variance response in Eq. 3.36, the variance response to an individual wave in MLS observations in limb-tracking mode is determined principally from the fluctuations of background temperature it produced along the data track (y-direction in Figure 3.2a for UARS MLS and x-direction for EOS MLS). Each calculated variance represents the wave-induced fluctuation over a certain distance along the observing path defined by the number of measurements employed and the gap between the adjacent measurements. Like the temperature response, model results of the instrument's variance response will be reported in two different ways, first as a function of wavelength components \((\lambda_x, \lambda_y, \lambda_z)\) and later as a function of horizontal and vertical wavelengths \(\lambda_H\) and \(\lambda_Z\) and angle \(\theta\) of wavefronts with respect to x-axis.

This chapter is divided in to five major sections. In Section 5.1, we consider first the dependence of the variance response on intrinsic phase and the number of measurements used in the calculation of each response. In Section 5.2, variation of the response to wavelength components \((\lambda_x, \lambda_y, \lambda_z)\) of the observed wave is analysed to investigate the dependence of the instrument on these parameters. In Section 5.3, the dependence of the variance response on horizontal and vertical wavelengths and wave propagation direction is considered where some selected results are then shown and interpreted. In Section 5.4, we provide some additional considerations on effects of some other factors in the derivation of the variance response with the model. These include the small variations of FOV direction occurred in limb scanning mode and the difference in vertical width of the channel's weighting functions. Finally, the summary and discussion of the work achieved in this chapter are presented in Section 5.5.
5.1 Dependence on Phase and Number of Measurements

We first consider the dependence of the variance response on intrinsic phase $\phi = -\alpha x$ of the wave and the number of measurements $N$ used in the calculation of each response. The results could provide us better understanding of the significant of these two parameters in the observations of gravity waves with the MLS instruments.

5.1.1 Variation with Intrinsic Wave Phase

We have already shown in Chapter 4 that the temperature response to an individual wave is strongly dependent on its intrinsic phase $\phi$ (see Figure 4.1 for example) and we expect the variance response, $\sigma_r^2$, to have the same characteristic according to its definition given by Eq. 3.36. We present in Figure 5.1, for example, the variation of this defined variance response with phase $\phi$ for waves with wavelengths $\lambda_x = 200$ km ($k < 0$), $\lambda_y = 100, 200, 500,$ and $1000$ km, and $\lambda_z = 10$ km. These response results are calculated directly from Eq. 3.36 with $\Delta y = 15$ km and $N = 32$ and each calculated response covers the fluctuation along $y$-direction over a distance of $\sim 500$ km. This value of $N$ and $\Delta y$ are chosen in accordance with the real observations made with the UARS MLS instrument in limb-tracking mode as described in Chapter 1.

We see quite clearly from Figure 5.1 that, in the case of $N = 32$, the variance response to waves with $\lambda_y < 500$ km is rather independent from phase $\phi$ but the dependence becomes significantly stronger to waves with $\lambda_y >> 500$ km. However, the variation of the response with $\phi$ at each wavelength $\lambda_y$ is still in a periodic function like that of the temperature response but with the oscillating period varies with value of $\lambda_y$. As the response function to each wavelength $\lambda_y$ fluctuates about some mean value, therefore, it is possible to take this value as being the approximated response of the instrument to each $\lambda_y$, especially for those with scales less than 500 km. For examples, the approximated response to waves with $\lambda_y = 100$ km is $= 0.015$ and for $\lambda_y = 500$ km is $= 0.019$ from this figure. These results suggest that, only the response to wave with scale of the along-track wavelength $\lambda_y >> 500$ km that depends much on wave phase $\phi$ in the 32-measurement limb-scanning mode of the UARS MLS.
Chapter 5

Temperature Variance Analysis

**Figure 5.1:** Variation of the variance response with intrinsic phase $\phi$ for waves with wavelengths $\lambda_x = 200$ km ($k < 0$), $\lambda_y = 100, 200, 500, \text{ and } 1000$ km, and $\lambda_z = 10$ km.

**Figure 5.2:** Comparison of the average response (solid line) and the response at phase $\phi = 0$ (dotted line) to waves with $\lambda_x = 200$ km ($k < 0$), $\lambda_y = 1-1000$ km, and $\lambda_z = 10$ km.
Figure 5.2 gives example of the average response calculated from the response to 100 different values of phases $\phi$ chosen randomly between $-\pi$ and $\pi$ and the response calculated at $\phi = 0$. The comparison shown here indicates that the average response and the response at $\phi = 0$ are comparable for waves with $\lambda_y$ less than $\approx 500$ km and the dependence of the response on phase $\phi$ is rather insignificant at this stage. The difference between these two responses are shown more clearly to waves with wavelength $\lambda_y > 500$ km where the dependence of the variance response to each wave on phase $\phi$ is rather strong as seen in Figure 5.1. As all reported UARS MLS variances are the average value collected from the measurements over some period of time, it is therefore reasonable to report the variance response obtained in our study in the same manner. We can then conclude from the result in Figure 5.2 that, in the 32-measurement limb-scanning mode, the UARS MLS instrument should be sensitive most (in terms of the variance response) to waves with $\lambda_y = 200-500$ km. Henceforth, the term 'variance response' given to an individual wave will be refereed to the 'average' value calculated from the random phase $\phi$ as described above.

5.1.2 Variation with Number of Measurements

The number of measurements $N$ is important in determining the distance along observing track covered in the calculation of each response. For example, at $N = 10$, the covered distance along $y$-direction in each observation should be $\approx 150$ km (with $\Delta y = 15$ km) and for $N = 20$ km, this should be $\approx 300$ km. The difference in the sampling distances could affect values of the instrument's response to each observed wave as shown in Figure 5.3.

Figure 5.3 shows the variance response plotted with $\lambda_y$ to waves with wavelengths $\lambda_x = 200$ km (and $k < 0$) and $\lambda_z = 10$ km with number of measurements $N = 6, 10, 20, 32,$ and 40. The response results given in this figure provide knowledge on scales of the along-track wavelength that should be most detectable to the UARS MLS in the observations. For examples, at $N = 6$, only waves with wavelength $\lambda_y = 100$ km that should be most visible to the instrument while at $N = 32$, only those with $\lambda_y = 100-500$ km that are most visible as suggested earlier. These results indicate that the response to waves with scales of $\lambda_y > 200$ km increase significantly with increasing value of $N$ and this should greatly enhance the opportunity of these waves to be observed with the instrument in the real observation. Note that at $N \leq 10$, the response will sharply peaked at some certain values of $\lambda_y < 200$ km only.
Figure 5.3: Variation of the variance response with wavelength $\lambda_y$ at different number of measurements $N$ for waves with wavelengths $\lambda_x = 200$ km, $\lambda_y = 1\text{--}1000$ km, and $\lambda_z = 10$ km.

The response results found in Figure 5.3 suggest that the temperature variances derived from data in the 32-measurement limb-tracking mode should contain mostly the contribution from the waves with wavelength scales of $= 100\text{--}500$ km along data track. The variances obtained from the 6-measurement observation in limb-scanning mode, on the contrary, should contain mostly the contribution from waves with $\lambda_y < 200$ km. Note that the maximum response possible to each wave depends only on their wavelengths $\lambda_x$ and $\lambda_z$ but not on $\lambda_y$ or number of measurements $N$. This because, from the mathematical analysis shown in Appendix B, the variance response to a single wave with wavelengths $\lambda_x$, $\lambda_y$, and $\lambda_z$ of the MLS instrument could be approximated by the relation (from Eq. B.10)

$$<\sigma^2_y> \equiv (a_1^2 + a_2^2)R^2_y <\text{var}[\cos \phi_y]>$$ (5.1)

where the constants $a_1$ and $a_2$ are as defined in Eqs. 4.4a and 4.4b, $R_y(l)$ is the response function given by Eq. 4.9, and $<\text{var}[\cos \phi_y]>$ is the average value of variance of $\cos[\phi_y]$. 

130
where $\phi_y$ is defined by Eq. B.9. In this expression, the dependence of the variance response to each wave on wavelengths $\lambda_x$ and $\lambda_z$ is defined through terms of $a_1$ and $a_2$, on wavelength $\lambda_y$ through the terms $R_y(l)$ and $\langle\text{var}[\cos \phi_y]\rangle$, and on number of measurements $N$ through the term $\langle\text{var}[\cos \phi_y]\rangle$. We see from Eq. 5.1 that if wavelengths $\lambda_x$ and $\lambda_z$ are set to be constant, as to waves shown in Figure 5.3, value of the variance response to each wave will depend only on $\lambda_y$ and $N$ defined by the product term $(R_y)^2 \langle\text{var}[\cos \phi_y]\rangle$. This term is shown, for example, in Figure 5.4 for $\lambda_y = 1$-1000 km and $N = 6, 10, 20, 32, \text{ and } 40$.

![Figure 5.4: Variation of the product term $(R_y)^2 \langle\text{var}[\cos \phi_y]\rangle$ with along-track wavelength $\lambda_y$ and number of measurements $N$.](image)

By the comparison of Figure 5.4 to Figure 5.3, we can conclude that it is this product term that controls the pattern of variation of the response shown in Figure 5.3. Equation 5.1 is very useful to the analysis of the UARS MLS variance response for the real observations as it explains explicitly sources of the amplitude of the response (through functions $a_1, a_2,$ and $R_y$) and role of the variation along data track in defining the total value of the response to each wave found in our study (through function $\langle\text{var}[\cos \phi_y]\rangle$). We notice that for the
Chapter 5  

Temperature Variance Analysis

UARS MLS observations with \( N > 10 \), the maximum value of the term \( (R_y)^2 \var{\cos \phi_y} \) will be \( \approx 0.5 \) at wavelength \( \lambda_y = 150-15N \), we could then expect the maximum response possible to an individual wave observed with the instrument to be given by

\[
< \sigma_R^2 >_{\text{max}} \equiv 0.5(a_1^2 + a_2^2).
\]  

(5.2)

This means the maximum variance response of the MLS instruments to each observed wave is determined only by functions \( a_1 \) and \( a_2 \) of the temperature response defined in Eqs. 4.4a and b. As a result, the maximum variance response possible to an observed wave in MLS observations should depend only on scales of \( \lambda_x \) and \( \lambda_z \) but not on scale of \( \lambda_y \) or number of measurements \( N \) as seen in Figure 5.3.

For the further study in this chapter we will consider mostly the variance response in the 32-measurement limb-tracking mode of UARS MLS calculated directly from Eq. 3.38. The response results for the 6-measurement limb-scanning mode will also be given later in this chapter to compare with those from limb-tracking mode.

5.2 Dependence on Horizontal and Vertical Wavelength

We consider in this section the dependence of the variance response on the wavelength components \( (\lambda_x, \lambda_y, \lambda_z) \) of the observed wave for the observations with \( N = 32 \). Scales of the horizontal wavelength in this study is assumed to be \( \lambda_x = 1-1000 \) km and vertical wavelength of \( \lambda_y = 1-20 \) km. Model results and their interpretation are as shown below.

5.2.1 Variation with y-Horizontal Wavelength

We now examine the dependence of the variance response on along-track wavelength \( \lambda_y \) in some more detail. Figure 5.5 show the variation of the response with \( \lambda_y \) for waves with wavelengths \( \lambda_x = 300 \) km (\( k < 0 \)) and \( \lambda_z = 10, 12, 15, \) and \( 20 \) km.

It is appeared that the fluctuations along data track of wavelength scales < 10 km should not be visible to the satellite due to the very low variance response and only those with scales of \( \approx 200-500 \) km that are most detectable. The response to each \( \lambda_z \) increses rapidly with \( \lambda_y \) for \( \lambda_y = 10-200 \) km and it steadily decreases if \( \lambda_y > 500 \) km due to the value of \( N = 32 \) used in the calculation of each variance as we could expect from Figure 5.4. As the
Chapter 5

Temperature Variance Analysis

Figure 5.5: Variation of variance response with wavelength \( \lambda_y \) for waves with wavelengths \( \lambda_x = 300 \) km \((k < 0)\), and \( \lambda_z \) = 10, 12, 15, 20 km. The calculation is for the observations in 32-measurement limb-tracking mode of UARS MLS.

The maximum response at each \( \lambda_z \) between 10-20 km depicted in this figure increases substantially from \( \lambda_z = 10 \) km to \( \lambda_z = 20 \) km (from \(-0.02\) to 0.2, or, \(-2\%\) to 20 \% of the squared amplitude at origin are detected). This means wave with longer \( \lambda_z \) should have better opportunity of being detected with the UARS MLS instrument than the shorter one in average. The variation of the variance response on vertical wavelength \( \lambda_z \) will be described in more detail in the next section. The results in Figure 5.5 confirm the suggestion obtained from Figure 5.4 that the dependence of the variance response on \( \lambda_y \) is strong only when scales of \( \lambda_y \) are not between \(-200-500 \) km. At scales of \( \lambda_y < 200 \) km, the variance response is controlled by \( \lambda_y \) mainly through the term \( R_y(l) \) as function \( <\text{var}\cos \phi_y> \) is constant at these scales \((= 0.5)\) and, at \( \lambda_y > 500 \) km, it is controlled by \( \lambda_y \) mostly through the function \( <\text{var}\cos \phi_y> \) as function \( R_y(l) \) is constant at these scales (at amplitude = 1.0 in Figure 4.5).
5.2.2 Variation with x-Horizontal Wavelength

We now consider in this section, the dependence of the variance response on the across-track wavelength $\lambda_x$ of the observed wave. We will show here only the response results for waves with vertical wavelength $\lambda_z \approx 10-20$ km and $\lambda_y = 500$ km. This scale of $\lambda_y$ is chosen to give the maximum response possible to each wave in the calculation according to Figure 5.4.

![Graph showing variation of variance response with horizontal wavenumber $k$](image)

**Figure 5.6**: Variation of the variance response to waves with vertical wavelength $\lambda_z = 10-20$ km with horizontal wavenumber $k$. The equivalent wavelengths $\lambda_x$ in kilometres are marked along the opposite axe as a reference.

Figure 5.6 show variance response as a function of horizontal wavenumber $k$ to waves with vertical wavelength $\lambda_z = 10-20$ which are likely to be visible most with UARS MLS in the real observations (from the knowledge of the temperature response presented in Chapter 4). We see that the variance responses of the instrument at these scales of $\lambda_z$ are always less than 25% of the squared amplitude of the wave at origin regardless of the scale of $\lambda_x$ which are rather low compared to that obtained in the ground-based or in situ methods. It is obvious that the response of UARS MLS to waves with $\lambda_z < 10$ km should be always
less than $\leq 2\%$ and, therefore, the gravity waves of these vertical wavelength scales should be hardly visible to the instrument. Like the temperature response, the response function to each $\lambda_z$ is not symmetric about $k = 0$ but shifted towards the negative-$k$ region with amplitude peaked at some certain values of $\lambda_x$. The optimal scales of $\lambda_x$ vary from $\approx 200$ km for $\lambda_z = 10$ km to $\approx 500$ km for $\lambda_z = 20$ km. These results indicate that, in the observations of gravity waves with $\lambda_z = 10\text{–}20$ km, UARS MLS should be most sensitive to those with scales of wavelength $\lambda_x$ in scales of few hundred kilometres which are propagating away from the satellite with respect to $x$-direction. Waves with $k > 0$, in this case, are less likely to be seen with the instrument as the response to them are always less than 10% in the observations.

5.2.3 Variation with Vertical Wavelength

Now, we shall consider the dependence of the variance response with vertical wavelength $\lambda_z$ in greater detail from that shown in Figure 5.6. Figure 5.7 shows examples of the response results as a function of vertical wavenumber $m$ to waves with wavelength $\lambda_x = 200, 300, 500,$ and $1000$ km.

We see in this figure that for waves with $k > 0$, the variance response to each $\lambda_x$ increases monotonically with increasing $\lambda_z$ that means the longest vertical wavelengths are most visible. But for those with $k < 0$, which have wavefronts more aligned with the LOS direction, the response is no longer monotonically increased and is sharply peaked at some certain value of $\lambda_z$ like the temperature response (Figure 4.7). For examples, the maximum response to waves with $\lambda_x = 200$ km is $\approx 0.1$ at $\lambda_z = 15$ km and for waves with $\lambda_x = 500$ km is $\approx 0.4$ at $\lambda_z = 50$ km. If we consider only for the response to wavelength $\lambda_z = 10\text{–}20$ km which are most likely to be found in the middle atmosphere, we see that the average response to all $\lambda_x$ is rather low (always $< 0.2$) regardless of the scales of $\lambda_x$ as already seen in Figure 5.6. These results suggest that, at some certain value of $\lambda_z$, the average variance response of UARS MLS should increase significantly with increasing scales of $\lambda_x$ and only waves $k < 0$ that are most visible to the satellite (in terms of the variance response). The cut-off wavelength in the observations where the response is $= 0$ I Figure 5.7 is at $\lambda_z = 10$ km for waves with $k > 0$ and at $\lambda_z \approx 8$ km for those with $k < 0$. 

135
Chapter 5  

Temperature Variance Analysis

Vertical Wavelength $\lambda_z$ (km)  

$\begin{array}{cccccc}
6.25 & 8.33 & 12.5 & 25.0 & \infty & 25.0 \\
6.25 & 8.33 & 12.5 & 25.0 & \infty & 25.0 \\
6.25 & 8.33 & 12.5 & 25.0 & \infty & 25.0 \\
6.25 & 8.33 & 12.5 & 25.0 & \infty & 25.0 \\
\end{array}$

Vertical Wavenumber $m$ (cycles/Skm)  

U, C

\[0.60 \quad 0.50 \quad 0.40 \quad 0.30 \quad 0.20 \quad 0.10 \quad 0.00\]

\[\text{NEGATIVE } K \quad \text{POSITIVE } K\]

$\lambda_x = 200 \text{ km}$  
$\lambda_x = 300 \text{ km}$  
$\lambda_x = 500 \text{ km}$  
$\lambda_x = 1000 \text{ km}$

Figure 5.7: Variation of the variance response with vertical wavenumber $m$ to waves with wavelengths $\lambda_x = 200, 300, 500,$ and $1000$ km and $\lambda_y = 500$ km.

From the results of the variance response shown in both Figures 5.6 and 5.7, we can primarily conclude that UARS MLS in its 32-measurement limb-tracking mode should be most sensitive to waves with relatively long $\lambda_z$ (e.g. with $\lambda_z \gg 10$ km) and medium scale of along-track wavelength $\lambda_y$ (e.g. with $\lambda_y \approx 200$-500 km). The scale of the across-track wavelength $\lambda_x$ to which the instrument is most sensitive to depended on the scale $\lambda_z$ as seen in Figure 5.6 but should be between $\approx 100$-500 km for $\lambda_z = 10$-20 km. Also the response at these scales of $\lambda_z$ to all waves should be less than $\approx 20$-25 % of the regardless of the scales of $\lambda_x$ as seen in Figure 5.6. Waves with relatively short scales of $\lambda_x$ and $\lambda_z$ (e.g. with $\lambda_x < 100$ km and $\lambda_z < 10$ km) are unlikely to be much detectable with the instrument due to the very low response caused by the FOV smearing and severe averaging in amplitudes of wave crests and troughs along the path. Finally, we summarise in Figure 5.8, the UARS MLS variance response as a function of horizontal wavenumber $k$ and vertical wavenumber $l$ for the calculation with $\lambda_y = 500$ km. The contours are for response amplitude of 0.0001, 0.001, 0.01, 0.1, 0.3, and 0.5, respectively.
Chapter 5

Temperature Variance Analysis

Figure 5.8: The UARS MLS variance response plotted as a function wavenumbers $k$ and $m$ of the observed waves where the corresponding wavelengths $\lambda_x$ and $\lambda_z$ are marked along the opposite axes. The calculation is for waves with wavelength $\lambda_y = 500$ km and $N = 32$.

We see in Figure 5.8 that the variance response to $\lambda_z$ does increase dramatically for $\lambda_z = 6-25$ km (from $= 0.0001$ to 0.3). This indicates the rapid improvement in the visibility of these waves to the UARS MLS instrument. In practice, we do not expect the waves with response amplitude as low as 0.001 (0.1%) or 0.01 (1%) to be much observed with the instrument due to the contamination of the instrument noise and the uncertainty in the measurements. Typically, the noise variance of the UARS MLS channel 4 is $= 0.01$ K$^2$ and any observable wave-induced variances should have their amplitudes (at least) comparable or considerably greater than this. If we take the variance of 0.01 K$^2$ as being the cut-off value for the study and assume the average wave variances of 1-10 K$^2$ at origin, the waves with response in order of $10^{-3}$ or $10^{-2}$ should still be visible to the instrument. This is corresponding to waves with approximate wavelengths $\lambda_x > 100$ km and $\lambda_z > 7$ km shown in Figure 5.8 and we expect the observed UARS MLS variance to waves with $\lambda_z < 20$ km to be always less than $= 2$ K$^2$, or less than $= 1$ K$^2$ for $\lambda_z < 15$ km. The statistical analysis of the measurements of weak gravity wave signals will be discussed in more detail in Appendix C.
This conclusion is in broad agreement with the frequent observed variances of UARS MLS in the middle atmosphere are normally much less than 1.0 K² (see Figure 1.9 for example). These values are relatively low compared to those typically obtained with radiosonde experiments (= 1-5 K²) and rocket observations (= 1-10 K²) as mentioned earlier. However, due to the low noise variance of UARS MLS, it is expected that any waves with variance response as low as 1 % might still be well visible to the instrument provided that the original variance is not too low (at least should be greater than = 1 K).

5.3 Dependence on Wave Propagation Direction

We have described in the previous section the variation of the variance response of the UARS MLS instrument with wavelength components \( \lambda_x, \lambda_y \) and \( \lambda_z \). In this section, we shall consider the variance response described in terms of the horizontal and vertical wavelengths (\( \lambda_H \) and \( \lambda_z \)) and angle \( \theta \) of the wavefronts on horizontal plane with respect to the x-direction.

5.3.1 Variation with Propagation Direction

As discussed in Chapter 3, the propagation direction on horizontal plane of the observed waves in our study is defined through angle \( \theta \) and we shall consider here the dependence of the variance response on horizontal wavelength \( \lambda_H \) and angle \( \theta \) of each wave using their relations with horizontal wavelengths \( \lambda_x, \lambda_y \) described by Eq. 4.11 in Chapter 4. Figures 5.9a-c demonstrate the response results to waves with \( \theta = [\pi, \pi] \), \( \lambda_H = 50, 100, 200, 500, \) and 1000 km where \( \lambda_z = 10 \text{ km} \) in (a), 15 km in (b), and 20 km in (c).

We can see in Figure 5.9a that the average responses to all waves with \( \lambda_z = 10 \text{ km} \) are rather low (always less than 0.02) as we have found earlier and only those with \( \theta < 0 \) that should be visible to the instrument. The response function to each \( \lambda_H \) has two peaks with equal amplitudes between \( \theta = 0 \) and \( -\pi \) where the minimum value = 0 occurred at \( \theta = -0.5\pi \) for all waves. At this angle, the waves have wavefronts that are perpendicular to x-axis and should exhibit no variation during the observation along y-direction of UARS MLS. Like the temperature response, at \( \lambda_z = 10 \text{ km} \), the average variance response is considerably higher to waves with scales of \( \lambda_H = 100-200 \text{ km} \) which maximum amplitudes of about 0.015-0.02. Waves with \( \lambda_H < 100 \text{ km} \) might be still well detectable but only at some certain directions.
(a) Variance Response with Direction of Wave Propagation
(for $\lambda_z = 10$ km and $N = 32$)

Figures 5.9a: The variation of variance response with angle $\theta$ for propagating waves with horizontal wavelength $\lambda_H = 50, 100, 200, 500, 1000$ km and vertical wavelength $\lambda_z = 10$ km.

(b) Variance Response with Direction of Wave Propagation
(for $\lambda_z = 15$ km and $N = 32$)

Figures 5.9b: As in Fig. 5.9a but for $\lambda_z = 15$ km.
Figures 5.9c: As in Fig. 5.9a but for $\lambda_z = 20$ km.

The average response to each $\lambda_H$ shown in Figure 5.9a does increase dramatically for waves with $\lambda_z = 15$ km shown in Figure 5.9b and 20 km in Figure 5.9c where the maximum response possible becomes $\approx 0.12$ and $0.22$ respectively. These values are in good agreement with the response results to each $\lambda_z$ shown in Figure 5.6. Data in these figures again indicate the significant increase in average with increasing $\lambda_z$ especially to those with long horizontal wavelength $\lambda_H$ (e.g. with $\lambda_H > 500$ km). This means wave at these wavelength scales should become more visible in the variance observation with the UARS MLS instrument when their vertical wavelengths are longer (e.g. with $\lambda_z >> 10$ km). However, for the observations at wavelength $\lambda_z = 10$-20 km presented here, the most visible waves to the instrument are still those with horizontal wavelength $\lambda_H = 200$-500 km propagating on horizontal plane at some certain directions as shown in Figures 5.9 with $\theta < 0$. Notice that, the variance response to each $\lambda_H$ is not symmetric about $\theta = 0$ as the real waves are propagating in three-dimension not only on horizontal plane and the LOS direction is also not totally on the horizontal plane. However, the response will become more symmetric about $\theta = 0$ if the vertical wavelength is getting longer as seen in Figures 5.9.
Chapter 5  

Temperature Variance Analysis

The expression of response to each observed wave in terms of horizontal wavelength $\lambda_H$, vertical wavelength $\lambda_Z$, and angle $\theta$ presented in this section might more convenient for the use in the real observation. This because it is the scales of wavelength $\lambda_H$ and angle $\theta$ of wave motion on horizontal plane that are more convenient to identify in the background atmosphere not the wavelength components $\lambda_x$ and $\lambda_y$. If these two parameters are known and we assume the dominate vertical wavelength $\lambda_Z$ in the middle atmosphere to be $= 10-20$ km, the above three figures showing response at $\lambda_Z = 10$, 15, and 20 km should be sufficient to give us some idea of the capability of UARS MLS in the real wave observations. However, if the descriptions of the observed waves are given in terms of wavelength components $(\lambda_x, \lambda_y, \lambda_Z)$, the analysis method done in Section 5.2 should be more appropriate in this circumstance.

5.3.2 Variation with Horizontal Wavelength

The response results given in Figures 5.9 are just for some selected values of $\lambda_H$ and we shall now consider these responses in more general cases of $\lambda_H$. Figures 5.10a-c show the response results plotted with $\lambda_H$ and $\theta$ for waves with $\lambda_H = 1-2000$ km, and $\lambda_Z = 10$ km in (a), 15 km in (b), and 20 km in (c). Data from these figures provide us more convenient way to approximate the variance response at each pair of parameters $\lambda_H$ and $\theta$, or identify the waves that could produce some certain response value at each $\lambda_Z$ if sufficient contours of the response amplitude are given (there are only 5 selected contours shown here in each figure). For example, if we want to know scales of wave that could produce the response amplitude of 0.1 in the observations, then we find from data in these figures that no waves with $\lambda_Z = 10$ km should be the candidate while at $\lambda_Z = 15$ km only waves with $\lambda_H = 100-400$ km should be capable of, and at $\lambda_Z = 20$ km, these scales have been extended to $= 100-800$ km. However, these wavelength scales must be accompanied by the preferable directions defined by angle $\theta$ as seen in Figures 5.9 to produce such value of response. Note that the response to waves with $\theta < 0$ (or with $k > 0$) is considerably lower than that to waves with $\theta > 0$ mostly due to the stronger attenuation of their amplitude along the ray path as described by the temperature response results shown in Chapter 4.

141
Figure 5.10a: Variance response to waves with $\lambda_z = 10$ km as a function of $\lambda_H$ and $\theta$ and for the response amplitudes of 0.001, 0.005, 0.01, 0.015, and 0.017 respectively.

Figure 5.10b: As in Fig. 5.10a but for $\lambda_z = 15$ km and the response amplitudes of 0.005, 0.01, 0.015, 0.05, and 0.1 respectively.
5.4 Additional Remarks

After having investigated thoroughly the dependence of UARS MLS variance response on the wavelength components ($\lambda_x, \lambda_y, \lambda_z$) and wave propagation direction in the previous two sections, we now consider some factors in the UARS MLS operation that might cause an additional variance in the observations. These include the small variations of viewing angle occurred in limb-scanning mode and the difference in vertical width of the temperature weighting function of saturated channels.

5.4.1 Variance Response in Limb-Scanning Mode

In Chapter 4, we have shown that the small changes in FOV viewing angle of the UARS MLS instrument from $= 5.3^\circ$ (at $h_T = 18$ km) to $= 6.6^\circ$ (at $h_T = 0$ km) in limb-scanning mode does significantly affect the temperature response calculated for each wave, especially those with short $\lambda_x$ and $k < 0$. We then suggested that this might also be true for the derivation of
variance response from data in limb-scanning mode compared to those obtained in limb-tracking mode with the same value of $N$. Regarding to this suggestion, we consider here the comparison of variance responses derived for the observation in both limb-scanning mode with $N = 6$ and in limb-tracking mode with $N = 6$ and 32. Figures 5.11a-c show the comparison of the response to waves with $\lambda_x = 100$ km and $\lambda_z = 10$ km in (a), $\lambda_x = 150$ km and $\lambda_z = 15$ km in (b), and $\lambda_x = 200$ km and $\lambda_z = 20$ km in (c). These values of $\lambda_x$ for each $\lambda_z$ are for $k < 0$ and chosen based on the results of temperature response for limb-scanning mode as shown in Figures 4.11. These are wavelengths to which the variance responses are expected to be affected most from the changes in viewing angle in limb-scanning mode.

We first consider the case of $\lambda_x = 100$ km and $\lambda_z = 10$ km in Figure 5.11a, it is quite clear in this case that the important source of the variance obtained in this 6-measurement method performed in limb-scanning mode is due to the variation in viewing angle not the fluctuation of wave amplitude itself. Without the variation in viewing angle (as in limb-tracking mode), the response to each wave is dramatically decreased and it is comparable to the response in the 32-measurement method at $\lambda_y < 100$ km. However, the average responses in both operating modes are still very low at these scales of $\lambda_x$ and $\lambda_z$ (always less than 0.001 or 0.1%). These waves are, therefore, unlikely to be detected with the instrument in the real observation otherwise it might cause a great problem in the interpretation of the UARS MLS variances obtained in the 6-measurement limb-scanning mode reported so far.

The responses to waves with $\lambda_x = 150$ km and $\lambda_z = 15$ km shown in Figure 5.11b indicate that effect of the variation in viewing angle is less severe in this case. However, the average variance response in limb-scanning mode is still significantly higher than those obtained in limb-tracking mode especially to waves with $\lambda_x < 300$ km. As the response amplitude in order of 1% might still be detectable with UARS MLS due to its low noise variance as discussed earlier, this effects then could not be ignored in the analysis of MLS variances obtained in limb-scanning mode. Notice that if we consider only the response to waves with $\lambda_y = 100$-500 km, which are mostly visible in the 32-measurement limb-tracking mode, we could expect from Figure 5.11b that the average response of the instrument in this mode should be comparable to that to these waves in the $N = 6$ limb-scanning mode at amplitude $= 0.02$ while the average response in $N = 6$ limb-tracking mode should be $= 0.005$ or four times less than that of the other two methods mentioned above.
Chapter 5  

Temperature Variance Analysis

Figure 5.11a: The comparison of variance response in different measuring modes of UARS MLS to waves with $\lambda_x = 100$ km, $\lambda_y = 10$-1000 km and $\lambda_z = 10$ km.

Figure 5.11b: As in Fig. 5.11a but for $\lambda_x = 150$ km and $\lambda_z = 15$ km.
Temperature Variance Analysis

Figure 5.11c: As in Fig. 5.11a but for $\lambda_x = 200$ km and $\lambda_z = 20$ km.

In the third case where $\lambda_x = 200$ km and $\lambda_z = 20$ km shown in Figure 5.11c, we see that effects of the vertical variation in viewing angle are much less significant in this case as they could increase the response to each wave by only up to $\approx 10\text{-}20\%$ from the normal value (in $N = 6$ limb-tracking mode). Now, only waves with small $\lambda_y$ (e.g. with $\lambda_y = 50\text{-}300$ km) that should be most visible in the $N = 6$ measuring method in both operation modes. It is clear from the results shown in Figures 5.11 that, though, the impact of additional variance produced in the 6-measurement limb-scanning mode is significantly reduced when $\lambda_z$ is getting longer but it still can not be ignored in the observations of waves with $\lambda_z = 10\text{-}20$ km in general. Therefore, results of the variances obtained from data in the 6-measurement limb-scanning mode should be less credible than those obtained from the 32-limb-scanning mode under the same atmospheric condition. The calculation of UARS MLS variances reported by McLandress et al. [2000] does confirm the importance of the effects of the variation in viewing angles in producing additional variances in limb-scanning mode. They clearly showed that the zonal-average variances at 38 km obtained from the 6-measurement limb-scanning mode of UARS MLS are comparable to those obtained in the 32-measuremenent limb-tracking mode while the variances in the 6-measurement limb-scanning mode are significantly lower than those in the first two methods mentioned above.
5.4.3 Variance Comparison for Channels 4 and 8

We have shown in Section 4.4.3 that the broader width of weighting function for saturated UARS MLS channel 8 could substantially reduced the temperature response of the waves with $\lambda_z = 10-20$ km on both sides of $k$. We shall prove in this section that the derivation of the variance response to these waves is also affected by this broadening.

![Image of graph showing variance comparison](image)

**Figure 5.12:** The comparison of variance response (as a function of vertical wavenumber $m$) of saturated UARS MLS channels 4 and 8 calculated for waves with wavelengths $\lambda_x = 500$ km and $\lambda_y = 500$ km.

Figure 5.12 shows the variance responses (as a function of vertical wavenumber $m$) of saturated MLS channel 4 compared with those of channel 8 to waves with $\lambda_x = 500$ km and $\lambda_y = 500$ km. We see that the variance response of channel 8 in average is less than that of channel 4 by up to $\approx 15-20\%$ at $\lambda_z < 25$ km with $k < 0$ and at all scales of $\lambda_z$ for $k > 0$. Only at wavelengths $\lambda_z \approx 25-100$ km that the response of channel 8 slightly exceeds that of channel 4 but as these scales of are unlikely to be found in nature, we could then safely say that, in general, the response to a wave of channel 8 will always less than that of channel 4 by up to 10-20 % depended on scale of $\lambda_z$ and direction of wave propagation defined.
5.5 Chapter Summary

We have presented in this chapter model analysis of the variance response to gravity waves observed with saturated channel 4 of UARS MLS in the 32-measurement limb-tracking mode. The results are reported first as a function of wavelength components (λ_x, λ_y, λ_z) of the observed wave and later as a function of horizontal and vertical wavelengths λ_H and λ_Z and angle θ of constant phase line with respect to positive x-axis.

We first demonstrated in Section 5.1 that the variance response of the UARS MLS instrument to an individual wave depended rather weakly on its intrinsic phase if the along-track wavelength λ_y is less than ≈15N km where N is the number of measurements made in a single scan. For the 32-measurement method used in limb-tracking mode, this conclusion is generally valid for waves with λ_y less than ≈500 km. It was also suggested by the results found in this section that in the N-measurement method where N ≥ 6, only waves with λ_y = 100-15N km that should be most visible to the instrument during the observation. The increase in value of N, in particular, could greatly enhance the possibility of waves with longer λ_y to be observed with the instrument in terms of the variance response.

In Section 5.2, we presented the model results of the variance response to an individually observed wave as a function of wavelength components (λ_x, λ_y, λ_z). We found that for the 32-measurement method used here only waves with λ_y = 200-500 km that are most visible in the observations where waves with λ_y < 20 km are unlikely to be seen due to the FOV smearing effects. It was also shown that the response to waves with λ_y > 200 km is virtually independent from λ_y and is defined only by scales of λ_x and λ_z at this stage. We found that, at some certain value of λ_y, the response waves with k > 0 exhibit a monotonic decrease with increasing value of k and m indicating that only the longest wavelengths λ_x and λ_z that are most visible in this situation. Waves with k < 0, in the contrary, have phase lines that are more aligned with the LOS and so undergo less attenuation and their response is no longer monotonically decreasing but will sharply peaked at some certain value of λ_Z. We also found that the maximum response to waves with vertical wavelength scales of ≈10-20 km which are thought to dominate the middle atmosphere are only ≈10-20 % of the amplitude of the original variance.
In Section 5.3, the variation of the variance response with horizontal and vertical wavelengths $\lambda_H$ and $\lambda_z$ and angle $\theta$ of horizontal wavefronts with respect to x-direction are given. We found that at $\lambda_z = 10-20$ km, only waves with $\theta < 0$ that are most visible to the instrument especially at $\lambda_z = 10$ km and the average variance to each wave substantially increased with increasing $\lambda_z$ especially to those with relatively long $\lambda_H$ (e.g. with $\lambda_H > 500$ km). All waves are invisible at $\theta = \pm 0.5\pi$ as we have $\lambda_y = \infty$ in this case and no variations along the observing track are expected to be seen. Waves with short $\lambda_H$ (e.g. with $\lambda_H < 100$ km) could still be well visible but only at some specific directions. In general, at $\lambda_z = 10-20$ km, we can conclude that only waves with wavelength $\lambda_H = 100-500$ km that could be most visible to the instrument but the preferred directions of the propagation on the horizontal plane for the best observation to each wave are varied. However, the total response possible in all waves in this case (for $\lambda_z = 10-20$ km) is still always less than ~20% but this will increase greatly with increasing $\lambda_z$ (e.g. with $\lambda_z >> 20$ km).

Finally, we examined in Section 5.4 effects of the small variations in viewing angles of the instrument in the derivation of variance response in the 6-measurement limb-scanning mode. We found that these effects should be very significant for waves with $\lambda_z = 10$ km but are less important for waves with larger scale of $\lambda_z$. However, at $\lambda_z = 10-20$ km, these effects are still considerably strong and could not be ignored in the analysis of UARS MLS wave variances obtained from radiance data measured in limb-scanning mode. We also showed in this section that the broader width of weighting function of saturated channel 8 should substantially reduce the instrument’s response to waves with $\lambda_z = 10-20$ km. This reduction in response amplitude could be as high as 15-20% from that obtained with saturated channel 4 under the same situation.
Chapter 6
Possibility of Gravity Wave Observations with EOS MLS

6.1 Introduction

We have presented in the previous two chapters model analysis on capability of the UARS MLS instrument in the observations of atmospheric gravity waves described in terms of the temperature response and variance response. In this chapter, we shall consider the possibility of gravity wave observations with NASA’s Earth Observing System (EOS) Microwave Limb Sounder (MLS), which is scheduled to be launched in June 2003, using the analysis method that has been applied to the UARS MLS observations described in Chapter 3.

There is still no plan to implement the EOS MLS instrument for the observation of gravity wave at present but if this could be done it might benefit to the study of gravity wave activity in the global scale. This because the EOS MLS observes in the orbital plane (look-forward pattern) thus providing latitude coverage between 82°S and 82°N on each orbit which is greater than that obtained with UARS MLS instrument in general. In addition, we will show later that EOS MLS should be better than UARS MLS in detecting waves with scales of vertical wavelength less than 20 km which are typically found in nature.

The rest of this chapter is divided into four main sections. In Section 6.2, the initial purposes of the EOS MLS mission and details of its 118-GHz radiometer are addressed. The simulated results of EOS MLS radiance profiles and the corresponding weighting functions of the saturated channels are then presented. In Section 6.3, the dependences of EOS MLS temperature response on wavelengths $\lambda_x$, $\lambda_y$, and $\lambda_z$, and propagation direction of the observed waves are investigated. In Section 6.4, the variance response for the EOS MLS observations is derived as a function of wavelength components and propagation direction of the observed wave. Finally, Section 6.5 provides the summary of important results obtained in this chapter.
6.2 The EOS MLS Experiment

We present in this section brief detail of EOS MLS mission and experiment along with the overview of the 118-GHz radiometer, which is designed for the temperature and pressure measurements like the UARS MLS 63-GHz radiometer. Examples of radiance profiles from the simulated radiance measurements with the instrument and the corresponding temperature weighting functions for the saturated channels are also given subsequently.

6.2.1 The EOS MLS Mission

The NASA's Earth Observing System (EOS) Microwave Limb Sounder (MLS) is a follow-on MLS experiment which is due to be launched on the EOS Aura satellite in June 2003. The EOS MLS instrument is designed to improve on the UARS MLS in several aspects especially in providing more and better measurements of the important trace gases involved in the process of ozone (O₃) depletion; i.e. HNO₃, OH, ClO, HCl, and H₂O [Waters et al. 1999]. Figure 6.1 summarizes primary measurement objectives of the EOS MLS currently being studied. These objectives include source gases, radicals, and reservoirs in all the chemical cycles thought to significantly affect stratospheric ozone. Measurements are made with EOS MLS radiometers which observe in spectral bands centered near 118 GHz (for temperature and pressure), 190 GHz (for several molecules including H₂O, O₃, N₂O, and HNO₃), 240 GHz (mainly for O₃ and CO), 640 GHz (for several molecules including O₃, N₂O, ClO, and HCl), and 2500 GHz (primarily for OH).

The simultaneous and commonly calibrated MLS measurements of ClO, HNO₃, HCl, N₂O, O₃, and temperature in the stratosphere are hoped to provide a powerful suite to improve understanding of key processes that could lead to greater ozone loss in the Arctic and to provide diagnostics of observed ozone loss. Among these, ClO abundances allow estimates of the amount of ozone loss due to chlorine chemistry. Abundances of HNO₃ and H₂O, and temperature, critically affects the microphysics leading to formation of surfaces upon which heterogeneous chemistry can occur and convert chlorine from reservoir to reactive forms. Abundances of HNO₃ also affect the rate at which reactive chlorine is converted back to reservoir forms. More information on the EOS MLS mission can be found on the EOS website at http://eos-aura.gsfc.nasa.gov.
Pattern of Observations

EOS MLS has pattern of observations that is rather different from the UARS MLS. The EOS MLS has a near polar orbit (98° inclination, sun synchronous) at orbital altitude ~700 km which allows nearly pole-to-pole observation coverage on each orbit, whereas the UARS orbit (57° inclination) and its precession pattern forces its MLS to switch between northern and southern high latitude measurements on the approximately monthly basis. Another key difference is that EOS MLS observes in the orbital plane (looking forward) that provides latitude coverage between 82°S and 82°N on each orbit. The UARS MLS, in the contrary, looking to the side in a direction 90° to the satellite velocity vector and the tangent track extends over a latitude range from 34° in one hemisphere to 80° in the other.

Each EOS MLS limb scan, under current nominal operation, will be performed approximately every 1.5° great circle along the orbital track (about 165 km distance and 25 s in time). The nominal scan range is 15-62.5 km for the THz radiometer, and 2.5 to 62.5 km.
scan spends more time in the lower stratosphere and for the GHz radiometers, in the upper troposphere, to emphasize these atmospheric regions which are currently of great scientific interest. The scan will be performed continuously (i.e. non-stepped) with an individual measurement integral time of ~0.17 s (for a single minor frame along the data path). This provides radiance measurements every ~0.3 km in the vertical in the upper troposphere and lower stratosphere and every ~ 1 km in the middle and upper stratosphere. Alternative scan programs will be used occasionally to provide measurements at higher altitudes in the mesosphere and (for some measurements) in the lower thermosphere.

### 6.2.2 Instrument Overview

We consider in this section some technical details of EOS MLS instrument. The EOS MLS instrument has radiometers in five spectral regions as stated earlier. The 118-GHz radiometer, covering the strong 118-GHz O₂ line, is chosen to measure atmospheric temperature and pressure needed for the designed measurements. The 190-GHz radiometer is chosen to measure the 183-GHz H₂O line, as done by UARS MLS, and to measure a strong band of HNO₃ lines. The 240 GHz radiometer is chosen to cover very strong O₃ lines in a spectral region where upper tropospheric absorption (mainly by water vapour continuum) is sufficiently small to allow measurements of upper tropospheric O₃. The 640 GHz radiometer is chosen to measure the lowest-frequency line of HCl, the strongest rotational line of ClO, and a strong line of N₂O. The 2.5 THz is chosen for OH mainly because of the relatively clean spectral region around the pair of very strong OH lines at 2.510 and 2.514 THz.

Figure 6.2 shows a signal flow block diagram for EOS MLS. As mentioned earlier, EOS MLS has radiometers in spectral bands centred near 118, 190, 240, 640, and 2500 GHz. A three-reflector antenna system, which is mechanically scanned in the vertical, received microwave radiation from the atmospheric limb coming along the ray path. Note that the antenna design is very similar to that of UARS MLS, with a primary reflector dimension of 1.6 m projected in the vertical direction at the limb tangent point. A switching mirror accepts radiation either from the antenna or, for calibration, from an internal target or a space view. An optical multiplexer following the switching mirror separates a signal into separated beams for each of the MLS radiometers. Each radiation beam is then combined with the signal generated by local oscillator to down-convert the signal to intermediate-frequency (IF).
band in the range of 0-3 GHz. The EOS MLS radiometers, except that operating near 118 GHz, are double-sideband (having approximately equal responses at IF frequencies above and below the local oscillator frequency). Each IF spectral band, after amplifying, then passes into the spectrometers, where it is split into 25 contiguous fixed-bandwidth filters, or channels. The power of output signal from each channel is finally measured and the results are digitised for transmission to ground for data processing via the Command & Data Handling microprocessor.

Table 6.1 gives the nominal positions (offset from line centre) and widths of these 25 spectral channels of the EOS MLS radiometers. The channel widths of the EOS MLS filter bank are symmetrically arranged about a central channel (Ch.13), making a total bandwidth of 1246 MHz per sideband (compared to 512 MHz for UARS MLS). The FOV’s vertical half-power beamwidth of EOS MLS is \(~0.11^\circ\) and horizontal width is \(~0.21^\circ\) which are equivalent to the distance \(~6\) and \(12\) km from tangent point respectively (from Eq. 2.44 with \(H = 700\) km and \(h_r = 18\) km). This means EOS MLS should have much better resolution in the observations at tangent point than that of UARS MLS discussed in Chapter 3.
### Table 6.1: Positions and Widths of EOS MLS Spectral Channels

<table>
<thead>
<tr>
<th>Channel</th>
<th>Frequency Channel offset (MHz)</th>
<th>Frequency Channel bandwidth (MHz)</th>
<th>Channel</th>
<th>Frequency Channel offset (MHz)</th>
<th>Frequency Channel bandwidth (MHz)</th>
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#### 6.2.3 Model Radiance Synthesis

We now consider the simulated radiance profiles for EOS MLS observations where the instrument is assumed to step-scan the atmosphere at altitudes between 0-120 km and for the background temperature shown in Figure 2.6. The forward model employed in the radiance calculation is as described in Chapter 2 but for the oxygen 118.75 GHz emission line and at frequencies (offset from line centre) shown in Table 6.1. However, we consider first in Figure 6.3 which shows an example of calculated spectra (in terms of brightness temperature) for emission from some isothermal atmospheres at frequencies ±200 MHz from line centre (covered the observed spectral range of Chs.6-20). The emission is from ray trajectories with tangent pressure at 0.01 mbar for background temperature $T_0 = 240$ K, 1 mbar for $T_0 = 260$ K, and 100 mbar for $T_0 = 280$ K. The antenna field-of-view (FOV) smearing is not included in the calculation in this case.
Figure 6.3: Synthetic radiance profiles for $\text{O}_2$ 118.75 GHz emission for the EOS MLS observations at tangent pressure 0.01 mbar (~81 km) for background temperature $T_0 = 240$ K, 1 mbar (~48 km) for $T_0 = 260$ K, and 100 mbar (~18 km) for $T_0 = 280$ K of. The spectral positions are offset from line centre at 118.75 GHz. Channel frequencies are not marked here but they are as detailed in Table 6.1.

Figure 6.3 shows model results of the synthetic radiance profile for the EOS MLS observations at various observed frequencies from line centre. Similar to the UARS MLS radiance profiles shown in Figure 2.5, The EOS profiles found here indicate that the observed radiances with each channel are strongly dependent on tangent height and observed frequencies, especially when the pressure is low (at high altitudes). Only when the instrument views the limb at low altitudes (e.g. less than 18 km) that the observing channel’s radiances are fully saturated and independent from the tangent height of the observations. As the saturated radiance data are required in the study of gravity waves with the MLS instrument, therefore, we can expect from Figure 6.3 that all EOS MLS channels should be available for the use in the study with our model if the tangent height of viewing are ~17 km or lower. In this thesis, we are going to analyse in detail of the temperature response and variance response for saturated EOS MLS channel 9 whose observing altitudes are at ~35-50 km along the ray path (like UARS MLS channel 4).
Synthesis Limb Radiance Profiles

We now consider model results of the radiance profiles from the EOS MLS observations in normal limb-scanning mode where the instrument is assumed to step-scan the atmospheric limb at tangent altitudes between 0-120 km with increment of 1km/step.

Figure 6.4 shows examples of the calculated radiance profile from a single scan of the instrument using background temperature shown in Figure 2.6. These profiles are plotted separately for each channel from 1 to 13. (Profiles of channels 14-25 are not shown here but they are similar to those of channels 13-1 respectively). The FOV function for the 118-GHz radiometer similar to that shown in Figure 2.7 is also included in the calculation. Note that radiance profiles for channels 1-4 are not labelled due to lack of space but they are implicitly identified by the order of the lines from the bottom up and the 18-km tangent height is also marked as a reference.

We immediately see that the observed radiances made with the EOS MLS instrument at the very high tangent altitudes (e.g. at $h_T > 80$ km) are primarily dominated by the contribution from cold background atmosphere like those of UARS MLS which are very low (always < 3 K). However, the observed radiances at each channel increase rapidly with decreasing tangent height for $h_T < 80$ km before they saturate at some certain point in the lower atmosphere. Radiances near line-centre saturate at higher altitudes than those near line-wing due to the stronger line absorption and most channels are fully saturated when tangent height of observation less than 18 km. These simulated radiance results of EOS MLS shown in Figure 6.4 suggest that the instrument should be capable of observing the gravity wave in the middle atmosphere using the saturated channels as we did with the UARS MLS but with better resolution.

6.2.4 Temperature Weighting Functions

This capability is more pronounced when we consider for the temperature weighting function of each saturated channel when the instrument’s viewing altitude is at 18 km shown in Figure 6.5. In this figure, only the weighting functions of channels 5-21 which are fully saturated for the observations made at 18-km tangent height (see Figure 6.4) are displayed. The weighting functions for channels 1-4 and 22-25 are not shown here but they are similar to that of channels 5 and 21 in shape but with slightly lower altitude at peak (~1 km apart for each channel).
Chapter 6  
Possibility of Gravity Wave Observations with EOS MLS

Figure 6.4: Synthesis radiance profiles for a single scan of EOS MLS 118-GHz radiometer from altitudes 120 km to Earth’s surface. The profiles are plotted separately for each channel from channel 1 (at line-wing) to channel 13 (at line-centre).

Figure 6.5: Temperature weighting functions for saturated channels 5-21 of the EOS MLS 118-GHz radiometer when the instrument is viewing the limb at 18-km altitude.
The weighting functions shown in Figure 6.5 represent the radiance contribution from different atmospheric layers to the saturated radiances of saturated channels 5-21 for the observations made at 18-km tangent height. Details of these functions suggest that the emission observed at each channel originated mainly from a localised layer at a specific height in the middle atmosphere (at ~20-80 km altitudes) with a resolution of about 10 km for all channels except ~15 km for channel 13 (line centre). These are very similar to those of the UARS MLS (see Figure 2.12) except for the peak altitudes of each weighting function due to the difference in channel’s frequency of both instruments. Table 6.2 summarises some key parameters of EOS MLS weighting functions shown in Figure 6.5. Line frequencies are offset from the line centre at 118.75 GHz.

Table 6.2: Weighting Function Parameters for 18-km-Tangent-Height Observation with EOS MLS 118-GHz Radiometer.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Frequency Offset (MHz)</th>
<th>Approximate Height (km)</th>
<th>Layer Thickness (km)</th>
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</table>

Data from the synthetic radiance profiles shown in Figure 6.4 and details of the weighting functions for saturated channels depicted in Figure 6.5 indicate that the multichannel observations of EOS MLS 118-GHz radiometer should also be able for the use in gravity wave study like the UARS MLS 63-GHz radiometer. To gain more understanding on the possibility of wave observations with the EOS MLS instrument, we shall consider in the next two sections the temperature response and variance response of the instruments to the assumed waves under the observations in 18-km limb-tracking mode.
6.3 Temperature Response Analysis

We first consider the temperature response of the EOS MLS instrument to the wave-induced temperature variation in the atmosphere whose functions are described by Eq. 3.18 as

\[ T'(x, y, z, t) = A_0 e^{-z/2H} \cos(kx + ly + mz - \omega t) \]  \hspace{1cm} (6.1)

where all parameters are defined as usual. The calculation is for the measurements at 18-km tangent height of saturated channel 9 whose observed atmospheric layer along the ray path is situated at altitude heights \(-35-50\) km (see Figure 6.5) with peak altitude at \(z_c \sim 41\) km.

**Figure 6.6:** Vertical geometry of gravity wave observation with EOS MLS channel 9 where \(H = 700\) km, \(z_c = 41\) km, \(h_r = 18\) km, and \(\beta = 5.3^\circ\) and the observing track is along \(x\)-axis.

The vertical geometry of gravity wave observations with EOS MLS is shown in Figure 6.6, which is similar to that of UARS MLS shown in Figure 3.2b apart from the fact that now the observing track of the instrument is now on the \(x\)-direction not \(y\)-direction like UARS MLS. The difference in the observing directions significantly affects the response of the instruments to horizontal wavelength components \(\lambda_x, \lambda_y\) in the derivation of variance response but has no effects, in principle, on the analysis of the temperature response to each
wave. In this section, we shall present the model results of the temperature response of EOS MLS instrument, first, as a function of wavelength components \((\lambda_x, \lambda_y, \lambda_z)\) and, second, as a function of horizontal and vertical wavelengths and the propagation direction of the observed wave. Concepts of the temperature response analysis used here are as described in Section 3.4.1 for the UARS MLS observations apart from the slightly difference in values of related weighting functions shown in Eq. 3.23. The vertical and horizontal FOV beamwidths of EOS MLS are approximately \(-5\,\text{km}\) and \(10\,\text{km}\) at \(z_c = 41\,\text{km}\) and this gives the constants \(\delta_{xz}\) and \(\delta_y\) in Eq. 3.35 to be \(-3.0\,\text{km}\) and \(6.0\,\text{km}\) respectively. The viewing angle \(\theta\) seen in Figure 6.6 is also \(5.3^\circ\) for limb-tracking mode with \(h_T = 18\,\text{km}\) and \(z_c = 41\,\text{km}\) of saturated EOS MLS channel 9.

### 6.3.1 Dependence on Horizontal Wavelengths

We first consider the dependence of the total response function \(R(k,l,m)\) on the horizontal wavelength components \(\lambda_x, \lambda_y, \lambda_z\). From the definition of \(R(k,l,m)\) given in Eq. 4.8,

\[
R(k,l,m) = R_y(l) R_{xz}(k,m),
\]

we can then examine the dependence of function \(R\) on wavelengths \(\lambda_x\) and \(\lambda_y\) separately where it depends on \(\lambda_x\) (and \(\lambda_z\)) through function \(R_{xz}(k,m)\) and on \(\lambda_y\) through \(R_y(l)\).

#### Dependence on Horizontal y-Wavelength

Figure 6.7 shows the response function \(R_y(l)\) and its squared-amplitude \([R_y(l)]^2\) calculated for waves with wavelength \(\lambda_y = 1\text{-}1000\,\text{km}\). We see that the function \(R_y(l)\) for EOS MLS is compatible to that of UARS MLS shown in Figure 4.5 but with better resolution at short \(\lambda_y\) (i.e. for \(\lambda_y < 20\,\text{km}\)) due to the smaller FOV width along this direction. For example, the wavelength cut-off of EOS MLS is \(-8\,\text{km}\) (compare to \(-16\,\text{km}\) of the UARS MLS) which indicates that waves with relatively short \(\lambda_y\) should be more visible to EOS MLS. The response \(R_y(l)\) increases rapidly with increasing \(\lambda_y\) for \(\lambda_y = 10\text{-}100\,\text{km}\) and saturates with unity amplitude at \(\lambda_y > 200\,\text{km}\) which means the response to each wave is determined only by the response function \(R_{xz}(k,m)\) according to Eq. 4.8 shown above.
Chapter 6  

Possibility of Gravity Wave Observations with EOS MLS

Figure 6.7: The response function $R_x(l)$ and its squared amplitude $[R_x]^2$ of saturated EOS MLS channel 9 at wavelength $\lambda_x = 1\text{-}1000 \text{ km}$.

**Dependence on Horizontal x-Wavelength**

The dependence of EOS MLS temperature response on wavelength $\lambda_x$ is defined through function $R_{xz}(k,m)$ described in Eq. 4.6. We show in Figures 6.8 examples of function $R_{xz}$ for waves with vertical wavelength $\lambda_z$ between 5-20 km.

We find from data shown in this figure that the EOS MLS should have significantly higher sensitivity to waves with relatively small scales of both wavelengths $\lambda_x$ and $\lambda_z$ (e.g. with $\lambda_x < 200 \text{ km}$ and $\lambda_z < 20 \text{ km}$) than that of UARS MLS shown in Figure 4.6. This difference is clearly seen for waves with relatively short $\lambda_z$ (e.g. at $\lambda_z \leq 10 \text{ km}$) with $k < 0$ which could attain the response amplitude as high as $\approx 0.4 \text{ (40\%)}$ with EOS MLS but always less than 20% with UARS MLS. This means more chances of these short vertical-scale waves to be detected with EOS MLS due to the better resolution of the instrument. The response to each $\lambda_z$ in this figure is asymmetric about $k = 0$ where peak position is shifted towards $k < 0$ like that of UARS MLS. This makes waves with $k < 0$ that are most visible to the instrument while waves $k > 0$ could be observed only at very large wavelength scales.
Figure 6.8: Temperature response $R_{xz}(k,l)$ plotted as a function of wavenumber $k$ to waves with vertical wavelength $\lambda_z = 5, 7, 10, 15$, and $20$ km.

6.3.2 Dependence on Vertical Wavelength

As mentioned earlier, the dependence of temperature response on vertical wavelength $\lambda_z$ is also defined through the response function $R_{xz}(k,m)$. As a result, we present in Figure 6.9 examples of function $R_{xz}$ plotted with wavenumber $m$ for waves with $\lambda_x$ between $100, 125, 200, 500$, and $1000$ km where both signs of $k$ (for each $\lambda_x$) are shown.

The results found in this figure suggest that in the observation of gravity waves, the EOS MLS instrument should be most sensitive to waves with long vertical wavelength (e.g. with $\lambda_z \gg 20$ km) where the response amplitude is very high ($= 1.0$ or 100%). However, at typical scales of $\lambda_z$ between 1-20 km the maximum response of $\approx 0.8$ could be achieved which is still considerably high compared to $\approx 0.6$ of UARS MLS. It is obvious that waves with rather short $\lambda_x$ (e.g. with $\lambda_x \leq 100$ km) should be more visible to the EOS MLS than to the UARS MLS due to the significant increase in response amplitude to these waves. (The response to them always < 0.03 with UARS MLS but could be as high as $\approx 0.3$ with EOS MLS). The response to each $\lambda_x$ is more symmetric about $m = 0$ if its scale is getting longer.
Figure 6.9: Temperature response $R_{xz}(k,l)$ plotted as a function of vertical wavenumber $m$ for waves with wavelength $\lambda_x = 100, 125, 200, 500,$ and $1000$ km.

Figure 6.10: Variance response of EOS MLS as a function of horizontal wavenumber $k$ and vertical wavenumber $m$ at amplitude $0.01, 0.1, 0.3, 0.6,$ and $1.0$. 
We summarise in Figure 6.10 the variation of temperature response $R_{xz}(k,l)$ of the EOS MLS instrument with scales of $\lambda_x$ and $\lambda_z$ at some selected amplitudes. The maximum response to waves with typical vertical wavelength of $\approx 10$-$20$ km in the middle atmosphere are between $= 0.5$-$0.8$ at $\lambda_x = 100$-$500$ km (with $k < 0$) which are considerably higher than those of UARS MLS where they are only $= 0.2$-$0.6$ (see Figure 4.9). This means the average amplitude of the observed wave-induced fluctuations in EOS MLS radiance profiles should be significantly higher than that of UARS MLS. For examples, if we assume normal wave amplitudes of $1$-$5$ K into account, we could then expect the fluctuation of amplitude $= 2$-$4$ K to be typically-found in EOS MLS radiance profiles while for UARS MLS it is $= 0.5$-$3$ K. The low noise signal of EOS MLS, which is typically less than $0.05$ K in most channels, also helps to increase the possibility of waves with short wavelengths to be observed. For the background wave amplitude of $1$ K, it is expected that all waves with response greater than $0.1$ should be well visible to the instrument, in principle. This includes waves with approximate wavelengths $\lambda_x \geq 70$ km and $\lambda_z \geq 6$ km from data in Figure 6.10.

### 6.3.3 Dependence on Wave Propagation Direction

We have already reported in the previous sections the EOS MLS temperature response as a function of wavelength components $(\lambda_x, \lambda_y, \lambda_z)$ of the observed wave. Now, we shall consider this response as a function of horizontal and vertical wavelengths ($\lambda_H$ and $\lambda_z$) and wave propagation direction on horizontal plane defined in terms of angle $\theta$ of wavefronts related to $x$-direction. The response to each wave is calculated using the relation of $\lambda_H$ and $\theta$ with wavelengths $\lambda_x$ and $\lambda_y$ given by

$$ (\lambda_x, \lambda_y) = \left[ \frac{\lambda_H}{\sin \theta}, \frac{\lambda_H}{\cos \theta} \right]. $$  

The total response $R(k,l,m)$ to an individual wave could be described now as a function of horizontal wavelength $\lambda_H$, vertical wavelength $\lambda_z$, and angle $\theta$. Figures 6.11a-c show examples of this function plotted with $\theta$ for waves with $\lambda_H = 50$, $100$, $200$, $500$, and $1000$ km. This response is calculated from the wavefunctions described by Eq. 4.12 with $\lambda_z = 10$ km in (a), $15$ km in (b), and $20$ km in (c) respectively.
Figure 6.11a: Temperature response as a function of horizontal wavelength $\lambda_H$ and angle $\theta$ for waves with $\lambda_H = 50, 100, 200, 500, 1000$ km, and $\lambda_Z = 10$ km.

Figure 6.11b: As in Fig. 6.11a but for $\lambda_Z = 15$ km.
Figure 6.11c: As in Fig. 6.11a but for $\lambda_z = 20$ km.

We see that the average amplitude of the response to each wave increase steadily with scale of vertical wavelength $\lambda_z$. At $\lambda_z = 10$ km, the most observable waves are those with $\lambda_H = 100-200$ km with maximum responses of $\approx 0.3-0.4$ at $\theta$ around $-0.5\pi$. Waves with $\lambda_H = 50$ km could still be visible but at limited directions close to $\theta = 0$ and $-\pi$. Waves with very large horizontal scale (e.g. with $\lambda_H > 500$ km), in this case, are unlikely to be much visible to the instrument due to very low response (always less than 0.1) as well as those with $k > 0$. At $\lambda_z = 15$ km, the most observable waves are those with $\lambda_H = 200-500$ km with maximum responses of $\approx 0.4-0.7$ at $\theta$ around $-0.5\pi$. Waves with $\lambda_H = 50$ and 100 km could still be well visible but at limited directions. The response to waves with $\lambda_H > 500$ km, in particular, has substantially increased at this value of $\lambda_z$. Finally, at $\lambda_z = 20$ km, the most observable waves are those with $\lambda_H = 200-1000$ km with maximum responses of $\approx 0.5-0.8$. It should be noticed that the response to waves with $k > 0$ has increases significantly to those with long $\lambda_H$ (e.g. with $\lambda_H > 500$ km) while waves with $\lambda_H < 200$ km are still very much invisible to the instrument in this portion of $k$. 

167
We can conclude from the results shown in Figures 5.11 that the amplitude response of EOS MLS to waves with \( \lambda_z = 10-20 \) km of our interest depends strongly on both scales of \( \lambda_H \) and \( \lambda_z \), and direction of the horizontal propagation defined from \( \theta \). Waves with small scale of wavelength \( \lambda_H \) (e.g. with \( \lambda_H < 100 \) km) are still well visible to the instrument but only at some limited directions. Waves with \( \lambda_H = 200-500 \) km are those that the EOS MLS should be most sensitive to especially ones with greater scale of \( \lambda_z \) and have \( \theta \) close \(-0.5\pi\). Waves with \( \lambda_H > 500 \) km could be best visible only at large scale of \( \lambda_z \) (e.g. at \( \lambda_z \geq 15 \) km) while at \( \lambda_z < 10 \) km, they are unlikely to be detected much with the instrument. In addition, the response to all waves with \( k > 0 \) (means they are moving toward the satellite at the time of the observation) is relatively low compared to those with \( k < 0 \) and they should be very much overlooked by EOS MLS in the real observation. It should be noted that the patterns of response variation shown in Figures 6.11a-c for UARS MLS are very similar to those of UARS MLS shown in Figures 4.9a-c but with significantly higher amplitude at each selected scale of wavelengths \( \lambda_H \) and \( \lambda_z \).

We summarize in Figures 6.12a-c the response function \( R_{xz}(k,l) \) of EOS MLS in more general cases of horizontal wavelength \( \lambda_H \) where the vertical wavelength \( \lambda_z = 10 \) km in (a), 15 km in (b) and 20 km in (c). This kind of data is useful not only in the estimation of the response at some values of \( \lambda_H \) and \( \theta \) but also in the prediction of scales and directions of the waves (in terms of \( \lambda_H \) and \( \theta \)) that could produce some certain response amplitude if the amplitude contours are sufficiently given. For example, at \( \lambda_z = 10 \) km, no waves should have the response greater than \( = 0.4 \) regardless of their scales of \( \lambda_H \) or the horizontal propagation direction but for \( \lambda_z = 20 \) km, that limit should be \( = 0.8 \). If we know the cut-off response that we have to impose upon the analysis due to the instrument noise or the uncertainty in the measurement, we could then find from these figures the scales of and \( \lambda_H \) and \( \theta \) that are within the scope of acceptance in the study. As the values of horizontal wavelength and direction of propagation of the waves are more convenient to identify in the real observation than the wavelength components in three-dimension, it is believed that the describing of the response function \( R_{xz}(k,l) \) with wavelengths \( \lambda_H, \lambda_z, \) and angle \( \theta \) shown here should be better for the use in the study of atmospheric gravity waves that the MLS instruments in both cases of UARS MLS and EOS MLS.
Chapter 6  Possibility of Gravity Wave Observations with EOS MLS

Figure 6.12a: Temperature response $R_{x_0}(k,l)$ as a function of horizontal wavelength $\lambda_H$ and angle $\theta$ for waves with $\lambda_z = 10$ km. The contours are for response amplitudes of 0.05, 0.1, 0.3, and 0.4 respectively.

Figure 6.12b: As in Fig. 6.12a but for $\lambda_z = 15$ km and response amplitudes of 0.01, 0.1, 0.3, 0.4, and 0.5 respectively.
6.3.4 Temperature Response in Limb-Scanning Mode

We now consider the temperature response of the EOS MLS instrument obtained from the observation in limb-scanning mode with viewing angle varies from $\beta = 5.3^\circ$ (at $h_T = 18$ km) to $6.6^\circ$ (at $h_T = 0$ km). There are approximately 50 measurements within this vertical range but we will consider only the response at the beginning and the end of the observation.

Figures 6.12a and b show the temperature response as a function of wavenumber $k$ to waves with $\lambda_z = 10$ km in (a) and 20 km in (b). We see that the response profiles in both cases of $\beta$ are rather compatible for $k > 0$, but are significantly difference at $k < 0$, especially to those waves with relative short $\lambda_x$ (e.g. with $\lambda_x < 200$ km). This means the small changes of viewing angle in limb-scanning mode could greatly affect value of the response to each individual wave as shown in these figures. However, the variation in EOS MLS temperature response with viewing angles seen here should not produce much profound effect on the derivation of the variance response from data in limb-scanning mode as it does in case of UARS MLS as we shall demonstrate in Section 6.4.4.
Figure 6.13a: Temperature response $R_{xz}(k,l)$ as a function of horizontal wavenumber $k$ to waves with $\lambda_z = 10$ km at two different viewing angles.

Figure 6.13b: As in Fig. 6.13a but for $\lambda_z = 20$ km.
6.4 Variance Response Analysis

We have already shown in Section 6.3 the dependence of EOS MLS temperature response on wavelength components and propagation direction of an individual wave. In this section, we will consider for the analysis of variance response for EOS MLS wave observation, which is slightly different from that of UARS MLS due to the change in direction of observing track (along x-direction for EOS MLS and y-direction for UARS MLS). For a comparison with UARS MLS study, we will also concentrate here on deriving the variance response for the observation in limb-tracking mode when the instrument views limb atmosphere at 18-km tangent height where most of the EOS MLS channels are already fully saturated.

6.4.1 Dependence on Number of Measurements

In principal, the variance analysis method described in Chapter 3 for the UARS MLS study could be applied directly to the analysis of the EOS MLS variance response performed here with appropriate changes in some parameters that are different between the two instruments. These parameters include the number of measurements $N$ performed in a single continuous scan and the distance $\Delta x$ between the individual measurements along the observing path, and also the vertical and horizontal widths of weighting functions involved in the radiance calculation. We first consider in this section the definition of EOS MLS variance response and its dependence on number of measurements $N$ of the operation.

Definition of the Variance Response

In the model calculation, we assume that the EOS MLS 118-GHz radiometer is in limb-tracking mode of operation where it views the limb atmosphere at constant tangent height of about 18 km. The instrument has the integral time of $\sim$0.17 s for the measurement in each minor frame, and it takes $\sim$ 25 seconds to complete a single set of continuous measurements in one major frame. This means, there are approximately 150 measurements performed in each 25-second period with the distance $\Delta x$ between each individual measurement of $\sim$1.3 km (satellite velocity is $\sim$7.5 km/s along the orbital path). Therefore, the individual variance obtained from the model computation will cover the temperature variation of distance $\sim$200 km along the observing track (compare to $\sim$500 km in the 32-measurement method of the UARS MLS). In the model calculation, we shall assume the values of $N = 150$ and $\Delta x = 1.3$ km for the EOS MLS measurements in limb-tracking mode in most cases.
For convenience in the calculation, we assume that the first measurement is made at the origin where \( x = 0 \) and, therefore, the temperature response for the \( i \)-th measurement along the data track (x-direction) is given in accordance with Eq. 3.30 by

\[
R_i = \int_{\omega_y} \int_{\omega_z} \frac{e^{z/2H}}{r(T)} \cos\{k[(i-1)\Delta x - x] + ly + mz + \phi\} W_x W_y W_z dxdydz
\]

where \( \Delta x = 1.3 \text{ km} \) is the gap between each adjacent measurements along the data track and phase \( \phi = -ax \) represents intrinsic phase during the measurements at time \( t \) and \( i = 1, 2, ..., N \).

The variance response to an individual wave from this \( N \)-measurement method is then defined from Eq. 3.36 by the relation

\[
\sigma^2_R = \frac{1}{N-1} \sum_{i=1}^{N} (R_i - \bar{R})^2, i = 1, 2, ..., N
\]

where \( R_i \) is given by Eq. 6.3. As discussed in Chapter 3, it would be more appropriate to report the variance response to each wave in our study in term of its average value, \( <\sigma^2_R> \), calculated from 100 random phases, or,

\[
<\sigma^2_R> = \frac{1}{100} \sum_{j=1}^{100} \sigma^2_{R,j}, \quad j = 1, 2, ..., 100
\]

where \( \sigma^2_{R,j} \) is the variances for each chosen phase \( \phi_j \) of the wave. Henceforth, the term 'variance response' reported in this chapter will be refereed to the 'average response' defined by Eq. 6.5 above unless stated clearly otherwise.

**Dependence on Number of Measurements**

The dependence of variance response with number of measurements \( N \) is significant in wave observation with MLS instruments as discussed in Chapter 5 for UARS MLS. This is also true in case of EOS MLS study. Figure 6.14a shows examples of the variance response as a function of wavenumber \( k \) to waves with \( \lambda_y = 200 \text{ km}, \lambda_z = 15 \text{ km}, \text{ and } N = 30, 50, 100, 150, \text{ and } 200 \) respectively. It is clear that the average response increases rapidly with increasing value of \( N \) for \( N < 100 \) but the rate of the increase has gradually slowed down for \( N > 100 \) and nearly negligible for \( N = 150-200 \). Therefore, it is clear that the response to each wave in the observations is strongly dependent on the number of measurements selected.
Figure 6.14a: EOS MLS variance response plotted with wavenumber \( k \) for gravity waves with \( \lambda_y = 200 \text{ km}, \lambda_z = 15 \text{ km}, \) and \( N = 30, 50, 100, 150, \) and 200.

Figure 6.14b: Values of function \( \langle \text{var}[\cos \phi_x] \rangle \) for \( \lambda_x = 1-1000 \text{ km} \) and \( N = 30, 50, 100, 150, \) and 200.
The reason of this dependence is that, due to the definition variance response given by Eq. 6.5, we could estimate the variance response for an individual wave generated from our model from the relation (as proved in Appendix B)

\[
<\sigma^2_R> \equiv (a_1^2 + a_2^2)R_y^2 <\text{var}[\cos\phi_x]>
\]  

(6.6)

where \(\phi_x\) is as defined in Eq. B.10. Equation 6.6 indicates that the variance response of the instrument depend on wavelength \(\lambda_x\) through both functions \(a_1\) and \(a_2\) defined in Chapter 4 and \(<\text{var}[\cos\phi_x]\)>, on wavelength \(\lambda_y\) through function \(R_y(t)\), and on vertical wavelength \(\lambda_z\) through functions \(a_1\) and \(a_2\). Function \(<\text{var}[\cos\phi_x]\) also depends on \(N\) by definition and we show for example in Figure 6.14b the variation of this function with \(\lambda_x\) for \(N = 30, 50, 100, 150,\) and 200. We see that the average value of function \(<\text{var}[\cos\phi_x]\) is nearly constant at \(= 0.5\) to wavelength \(\lambda_x\) less than \(= 1.3N\) km and it will drop continuously if scale of \(\lambda_x\) greater than this value. This means that only the response to wave with \(\lambda_x\) greater than \(= 1.3N\) km that should be affected most from the presence of term \(<\text{var}[\cos\phi_x]\> in the N-measurement method. For example, at \(N = 150\), only the response to waves with \(\lambda_x > 200\) km should be affected most from the decrease in value of \(<\text{var}[\cos\phi_x]\) at this stage (see Figure 6.14b).

This knowledge could help explaining pattern the variance response of waves with \(\lambda_z = 15\) km shown in Figure 6.14a. As we know that amplitude of the response function \(R_y\) for \(\lambda_y = 200\) km is \(\approx 1\) (see Figure 4.5), therefore, the response to each wave in this figure is approximately given only by the product term \((R_{xz})^2<\text{var}[\cos\phi_x]\) from Eq. 6.6 where \(R_{xz}\) at \(\lambda_z = 15\) km is as shown in Figure 6.8 and the function \(<\text{var}[\cos\phi_x]\) in Figure 6.14a. As the peak value of \(R_{xz}\) in this case is \(\approx 0.7\) at \(\lambda_x = 200\) km (with \(k < 0\)), we could then expect the maximum response at \(N \geq 150\) to be \(\approx 0.25\) at \(\lambda_x = 200\) km. We can clearly see in Figure 6.14a that the increasing in value of \(N\) from 30 to 150 (or 200) in this case does help waves with \(\lambda_x = 100-500\) km to be more visible to the instrument in terms of the variance response. For the purpose of the study, the case of \(N = 150\) in limb-tracking mode is going to be considered thoroughly in Sections 6.4.2-6.4.4 while the case of \(N = 50\) in both limb-tracking and limb-scanning modes will be considered in particular for a comparison in Section 6.4.5.
6.4.2 Dependence on Horizontal Wavelengths

Now, we will consider the dependence of EOS MLS variance response on the horizontal wavelengths $\lambda_x$ and $\lambda_y$ for the simulated observations in limb-tracking mode with $N = 150$, $\Delta x = 1.3$ km, and $h_T = 18$ km and the observing track of the instrument is along $x$-direction (the satellite's orbital path) as stated earlier.

**Dependence on Horizontal $y$-Wavelength**

We first consider the dependence of the response on across-track wavelength $\lambda_y$. Figure 6.15 shows the variance response plotted as a function of $\lambda_y$ to waves with wavelengths $\lambda_x = 100$, 200, 300, and 400 km (with $k < 0$), and $\lambda_x = 10$ km. Though, the response amplitude to all waves shown here are considerably low (always less than 0.06 or 6%) but the pattern of the variation is still well presented and could be explained as follows.

The response to waves with short scale of $\lambda_y$ (e.g. with $\lambda_y \leq 10$ km) is very small due to the FOV smearing effect along $y$-direction. As wavelength $\lambda_y$ getting longer, this effect is less and less important and the response to each $\lambda_x$ increases rapidly with increasing $\lambda_y$ for $\lambda_y = 10-200$ km. The response to all the waves becomes saturated at $\lambda_y > 200$ km due to the saturation of the response function $(R_y)^2$ at unity in this stage (see Figure 6.7). The saturation amplitude of the response to each wave could be approximated by the relation (From Eq. 6.6)

$$< \sigma_R^2 > \equiv (a_1^2 + a_2^2) < \text{var}[\cos \phi_x] > \quad (6.7)$$

where all functions are as defined earlier. This relation gives, for examples, amplitude of the saturated response $= 0.05$ for $\lambda_x = 100$ km, 0.06 for $\lambda_x = 200$ km, and 0.005 for $\lambda_x = 400$ km in good agreement with the values shown in Figure 6.15. As a result, the dependence of the variance response on $\lambda_y$ is no longer of the concern for the observation of waves with scale of $\lambda_y$ (across-track wavelength in case of EOS MLS) greater than $= 200$ km and the variance response to each observed wave in this situation could be approximately given by Eq. 6.7 described above.

176
Chapter 6  

Possibility of Gravity Wave Observations with EOS MLS

### Figure 6.15: Variation of variance response with wavelength $\lambda_y$ for $\lambda_y = 1$-1000 km and for wavelengths $\lambda_x = 100, 200, 300, \text{ and } 400 \text{ km (all for } k < 0), \text{ and } \lambda_z = 10 \text{ km.}$

**Dependence on Horizontal x-Wavelength**

We now consider the dependence of the variance response on the along track wavelength $\lambda_x$ of the EOS MLS observations. Figure 6.16 shows the response as a function of wavenumber $k$ to waves with wavelength $\lambda_y = 200 \text{ km and } \lambda_z = 7, 10, 12, 15, \text{ and } 20 \text{ km.}$

We see that the response to each $\lambda_z$ is not symmetric about $k = 0$ and its amplitude is peaked at some certain value of $\lambda_x$ in the negative-$k$ portion. For examples, the optimal wavelength $\lambda_x$ is at $= 100 \text{ km for } \lambda_z = 7 \text{ km and } = 250 \text{ km for } \lambda_z = 20 \text{ km with the amplitude of } = 0.05$ and 0.35 respectively. The response to all waves with $k > 0$ is extremely small and could be negligible in the study and only those with $k < 0$ that are most visible. The average response to each wave significantly increases with increasing $\lambda_z$ as we could expect and the maximum responses at typical scales of $\lambda_z = 10$-20 km are $= 0.1$-0.35 which is considerably higher than those of UARS MLS which are only $= 0.02$-0.22 (see Figure 5.6). This means the EOS MLS in limb-scanning mode should be much better than UARS MLS in the observations of gravity waves in the middle atmosphere especially those with short scales of $\lambda_z$ (e.g. with $\lambda_z < 10 \text{ km}$).
6.4.3 Dependence on Vertical Wavelength

Now, we consider the dependence of the response on vertical wavelength $\lambda_z$ in more detail. Figure 6.17 shows examples of the response as a function of wavenumber $m$ to waves with $\lambda_y = 200$ km where $\lambda_x$ are chosen between 100-1000 km for both $k < 0$ and $k > 0$.

Results in Figure 6.17 indicate that only waves with $\lambda_x = 200$-500 km that should be most visible to the instrument at typical wavelengths $\lambda_z = 10$-20 km. The response to waves with very long $\lambda_x$ (e.g. with $\lambda_x > 1000$ km) are very low in this case and could be neglected in the real observations (no fluctuations of these wavelength scales should be observed). If we consider only waves with $\lambda_z = 10$-20 km, the response of amplitude between $\approx 0.1$-0.3 should be attainable as seen earlier in Figure 6.16. It is clearly shown again here that waves with $k > 0$ in this vertical range are extremely low (except at the very large scales of $\lambda_z$) and they are unlikely to be seen much with the EOS MLS in the real observations. The variance response to each $\lambda_x$ found in this figure is sharply peaked at some certain value of $\lambda_z$ in the negative-$k$ portion (e.g. at $\lambda_z = 9$ km for $\lambda_x = 100$ km).
Figure 6.17: Variance response as a function of wavenumber $m$ to waves with $\lambda_y = 200$ km and $\lambda_x = 100, 125, 200, 500, \text{ and } 1000$ km.

Figure 6.18: Variance response of EOS MLS instrument plotted as a function of horizontal wavenumber $k$ and vertical wavenumber $m$. The contours are for amplitudes 0.001, 0.001, 0.01, 0.1 and 0.25 respectively.
Figure 6.18 gives a summary of the variance response of EOS MLS reported so far in this section plotted as a function of horizontal wavenumber \( k \) and vertical wavenumber \( m \) at some selected amplitudes. This knowledge is very useful if we want to know wavelength scales of the waves that could be well visible to the instrument in practice where the noise variance is getting involved. For example, as the noise variance is expected to be less than 0.005 K\(^2\) in most channels of EOS MLS, all waves that could produce the variance of amplitude comparable or higher than this value should be detectable to the instrument. If we assume the wave variance amplitude of 1-5 K\(^2\) for the background atmosphere, it is possible that waves with response as low as 0.01 could still be well visible to EOS MLS in terms of the variance response. These are waves with vertical wavelength \( \lambda_z \) as short as \( \approx 6-7 \) km which are virtually invisible to UARS MLS due to its much lower response to these waves (see Figure 5.8 for a comparison). For waves with \( \lambda_z = 10-15 \) km in particular, their average maximum responses are between 0.1-0.3 which means they should be able to produce the variance of amplitude up to \( \approx 0.1-1.5 \) K\(^2\) in the observation with EOS MLS. This is much better than in case of UARS MLS where we expect the maximum response at these scales of \( \lambda_z \) to be \( 0.02-0.6 \) K\(^2\) only.

### 6.4.4 Dependence on Wave Propagation Direction

We consider in this section the variance response as a function of the vertical and horizontal wavelength (\( \lambda_H \) and \( \lambda_z \)) and propagation direction on horizontal plane defined by angle \( \theta \). Figures 6.19a-c show the variation of the response with \( \theta \) for waves with \( \lambda_H = 50-1000 \) km and \( \lambda_z = 10, 15, \) and 20 km in (a), (b), and (c), respectively.

We see that, like UARS MLS, only waves with \( \theta < 0 \) that are most visible to the instrument at \( \lambda_z = 10-20 \) km while waves with \( k > 0 \) are unlikely to be much detectable. The latter are waves with wavefronts more perpendicular with LOS direction and, therefore, encounter stronger attenuation in amplitude along the LOS as described earlier in Chapter 4. The response to waves with \( \lambda_z = 10 \) km shown in Figure 6.19a is relative high at wavelength \( \lambda_H = 100-200 \) km with average amplitude of \( \approx 0.1 \) while waves with \( \lambda_H > 300 \) km are unlikely to be seen much with EOS MLS due to the very low instrument's response to them. Waves with short \( \lambda_H \) (e.g. with \( \lambda_H = 50 \) km) could still be well visible to the instrument but
at only some particular directions as seen in the figure. At $\lambda_z = 15$ km shown in Figure 6.19b, the response amplitude to each $\lambda_H$ has substantially increased from that at shown in (a), especially, those with $\lambda_H = 100$-200 km. For example, the maximum amplitude of response to waves with $\lambda_H = 200$ km in (a) is $= 0.06$ while in (b) is $= 0.25$. However, the response to waves with $\lambda_H > 500$ km is still relatively low and they are still unlikely to be seen much at this value of $\lambda_z$. Finally, the response to waves with $\lambda_z = 20$ km shown in Figure 6.19c is slightly improved from that at $\lambda_z = 15$ km and the most visible waves are still at wavelength scales $\lambda_H = 100$-200 km with average amplitude of $= 0.3$. The response to waves with wavelength $\lambda_H > 500$ km has significantly increased in this case and they are likely to be seen better if the value of $\lambda_z$ is getting higher.

![Figure 6.19a](image)

**Figure 6.19a:** Variance response as a function of angle $\theta$ to waves with $\lambda_z = 10$ km at some selected values of horizontal wavelength $\lambda_H$. 

181
Figure 6.19b: As in Fig. 6.19a but with $\lambda_z = 15 \text{ km}$.

Figure 6.19c: As in Fig. 6.19a but with $\lambda_z = 20 \text{ km}$.
The patterns of variance response shown in Figures 6.19a-c could be well explained by using the results of the response at each set of wavelength components \((\lambda_x, \lambda_y, \lambda_z)\) derived in the previous section as the relation of \(\lambda_H\) and \(\theta\) with \(\lambda_x\) and \(\lambda_y\) was already given by Eq. 4.11 in Chapter 4 as

\[
(\lambda_x, \lambda_y) = \left[ \frac{\lambda_H}{|\sin \theta|} \right] \left[ \frac{\lambda_H}{|\cos \theta|} \right].
\]  

(4.11)

For example, as we know that the variance response of EOS MLS is independent from \(\lambda_y\) if scales of \(\lambda_y\) are greater than \(= 200\) km, therefore, the response to all waves with \(\lambda_H \geq 200\) km should depend only on the value of \(\lambda_x\) defined above. The response to wavelength \(\lambda_x\) shown in Figure 6.16 indicates that at \(\lambda_z = 10-15\) km, waves with \(\lambda_H \geq 200\) km should have the maximum response at \(\lambda_x = \lambda_H\), or at \(\theta = -0.5\pi\) and their response will be symmetric about this point as seen in Figures 6.19a and b. Figures 6.20a-c summarise variation the EOS MLS variance response with angle \(\theta\) and horizontal wavelength \(\lambda_H\) at \(\lambda_z = 10-20\) km in more general cases of \(\lambda_H\).

(a) Variance Response with Propagation Direction
(for \(\lambda_z = 10\) km and \(N = 150\))

Figure 6.20a: Variance response as a function of angle \(\theta\) and horizontal wavelength \(\lambda_H\) to waves with \(\lambda_z = 10\) km at amplitude of 0.001, 0.01, 0.05, and 0.08.
Figure 6.20b: As in Fig. 6.20a but for \( \lambda_z = 15 \text{ km} \) and amplitude 0.005, 0.01, 0.05, 0.1 and 0.25.

Figure 6.20c: As in Fig. 6.20a but for \( \lambda_z = 20 \text{ km} \) and amplitude 0.01, 0.05, 0.1 and 0.3.
6.4.5 Variance Response in Limb-Scanning Mode

As it might be more convenient in conducting the gravity wave observation with EOS MLS in normal limb-scanning mode than in the special limb-tracking mode in practice, we shall consider in this section the variance response obtained in both cases. The approximated number of measurements, \( N = 50 \), made at tangent height less than 18 km is to be used in observation in limb-scanning mode while both values of \( N = 50 \) and 150 will be used in the calculation for limb-tracking mode.

Figures 6.21a-c show the variance response as a function of horizontal wavenumber \( k \) for waves with \( \lambda_y = 200 \) km, and \( \lambda_z = 10 \) km in (a), 15 km in (b), and 20 km in (c). These results are for the radiance measurements with \( N = 50 \) in limb-scanning mode and \( N = 50 \) and 150 in limb-tracking mode. It appears that at \( \lambda_z = 10-20 \) km, the average responses in the 50-measurement method in both cases are rather comparable in magnitude. Only at observed wavelength \( \lambda_x \) approximately less than 150 km that effects of the variation in viewing angle are clearly seen but they are not as strong as in case of the UARS MLS observations (see Figure 5.11). In all cases of \( \lambda_z \), the average response in the 150-measurement limb-tracking mode is substantially higher than that in both 50-measurements methods especially when scale of \( \lambda_z \) is getting longer. This is unlike the case of UARS MLS where the response in the 6-measurement limb-scanning mode could be as high as that in the 32-measurement limb-tracking mode in some situations.

These results indicate that though the gravity wave study with the 50-measurement limb-scanning mode could be carried in the EOS MLS experiment but the efficiency of this method might not be as good as that in the 150-measurement limb-tracking mode as the expected response to all waves with \( \lambda_z = 10-20 \) km in this case is always less than 0.1. However, due to the low variance noise of EOS MLS (always less than 0.005 K²), it is believed that the study of gravity waves with the 50-measurement limb-scanning mode is still worth considering as a primary aim of the mission. The successful of the gravity wave study with UARS MLS 32-measurement method where the variance response to waves with \( \lambda_z \leq 20 \) km is never greater than \( = 0.2 \) (see Figure 5.9c for example) could guarantee the benefit of using EOS MLS (even in limb-scanning mode) in observing waves with vertical wavelength = 10-20 km in the middle atmosphere that the other conventional methods are still not able to provide.
Chapter 6  
Possibility of Gravity Wave Observations with EOS MLS

Figure 6.21a: Comparison of the variance response of EOS MLS obtained in limb-scanning mode with $N = 50$ and in limb-tracking mode with $N = 50$ and 150. The results are for waves with $\lambda_y = 200$ km and $\lambda_z = 10$ km.

Figure 6.21b: As in Fig. 6.21a but for $\lambda_z = 15$ km.
Figure 6.21c: As in Fig. 6.21a but for $\lambda_z = 20 \text{ km}$.

6.5 Chapter Summary

We have presented in this chapter model analysis on the possibility of gravity-wave observations with EOS MLS 118-GHz radiometer described in terms of the temperature response and variance response of the instrument to each observed wave in both the 150-measurement limb-tracking method and 50-measurement limb-scanning method.

First, we found from the results of the simulated radiance profiles obtained in limb-scanning mode that most EOS MLS channels are also fully saturated at tangent height less than $18 \text{ km}$ like in the case of UARS MLS. This knowledge primarily indicates that the EOS MLS instrument should be used as a tool in the observation gravity waves in the middle atmosphere as the UARS MLS instrument has been successfully done before. To examine this ability of the instrument in greater detail, we chose the saturated EOS MLS channel 9 whose observing atmospheric layer is situated at altitudes $= 35-50 \text{ km}$ along the ray path to be an example for the further study. The analysis methods used for the derivation of temperature response and variance response of the instruments are as described in Chapter 3 but for the observations in the assumed 150-measurement limb-tracking mode of EOS MLS.
We then showed in Section 6.3 the temperature response of the instrument both as a function of wavelength components in three dimensions \((\lambda_x, \lambda_y, \lambda_z)\) of the observed waves and as a function of horizontal and vertical wavelengths \((\lambda_H, \lambda_z)\) and propagation direction in terms of angle \(\theta\) of the wavefronts with respect to the \(x\)-axis. We found that, like UARS MLS, only waves with \(k < 0\) that are most visible to the instrument and the average response increases with increasing values of both \(\lambda_x\) and \(\lambda_z\). The maximum response amplitude of \(\approx 0.4-0.8\) could be obtained to waves with \(\lambda_z = 10-20\) km which are believed to dominate the middle atmosphere. These values are considerably higher than that of UARS MLS which are \(\approx 0.2-0.6\). This means EOS MLS should be significantly better than UARS MLS in detecting amplitude of the gravity waves in real observations especially to those with relative short wavelength scales in both vertical and horizontal direction. In terms of \(\lambda_H\) and \(\theta\) for waves with vertical wavelength \(\lambda_z = 10-20\) km, EOS MLS should be most sensitive to waves with \(\lambda_H = 200-500\) km and with \(\theta\) close to \(-0.5\pi\). The average response to waves with relatively long \(\lambda_H\) (e.g. with \(\lambda_H > 500\) km), especially those with \(\theta > 0\), are significantly increased with increasing scale of \(\lambda_H\).

In Section 6.4, the analysis of variance response in the EOS MLS wave observations with the 150-measurement limb-tracking mode was presented. It was found from the model results that, at \(\lambda_z = 1-20\) km, EOS MLS should be most sensitive only to waves with \(k < 0\) as the response to those with \(k > 0\) is relatively low and should be neglected in the real observation. The maximum responses to waves with \(\lambda_z = 10-20\) km are found to be \(\approx 0.1-0.3\) (or \(\approx 10-30\%\) of original amplitude) which is relatively high compared to that of UARS MLS which are \(\approx 0.02-0.2\). At these scales of vertical wavelength \(\lambda_z\), only waves with horizontal wavelength \(\lambda_H = 200-500\) km that are most visible to the satellite, especially to those with wavefronts are aligned more along with the LOS direction (with \(\theta > 0\)). Waves with short \(\lambda_H\) (e.g. with \(\lambda_H < 100\) km) could still be clearly seen with the instrument but only at some particular directions. Like the temperature response, in terms of variance response, EOS MLS should be substantially better than UARS MLS in detecting waves with vertical scale less than \(\approx 20\) km which are most likely to be found in nature, especially to those with shorter scales of \(\lambda_z\) (e.g. with \(\lambda_z < 10\) km) which are virtually invisible to UARS MLS.
We also investigated in this section the response to waves with $\lambda_z = 10-20$ km in the assumed $N = 50$ limb-scanning method compared to that obtained in $N = 50$ and 150 limb-tracking method. We found that in general, the average responses in both 50-measurement methods are comparable though that of limb-scanning mode is slightly higher to waves with $\lambda_x < 150$ km. However, these responses are still significantly lower than that obtained in the 150-measurement method, especially when $\lambda_z$ is getting longer. In general, the maximum responses in both of the 50-measurement methods are always less than 0.1 while that of the 150-measurement method in limb-tracking mode could be as high as $\approx 0.3$ for $\lambda_z = 15-20$ km. However, as the 50-measurement method in limb-scanning mode is more convenient, technically, to be used in the EOS MLS observation of the gravity waves, we suggest from the results of our study that it is worth considering to do so. This because the maximum response of $= 0.1$ achieved in this method should still be sufficient for the observation of waves at vertical wavelength $= 5-15$ km compared to the successful of UARS MLS observations in which the response to all waves at these wavelength scales is also less than 0.1 in general. The EOS MLS instrument, therefore, could provide a better opportunity to the observation of gravity waves at these wavelength scales in the middle atmosphere that is still lack in most conventional methods operated so far.
Chapter 7
Summary and Discussion

We have presented in this thesis model analysis of the capability of MLS instruments in the observations of atmospheric gravity waves described in terms of the temperature response and variance response of the instruments to an observed wave. The study is for both the observations with UARS MLS 63-GHz radiometer which is currently in operation and EOS MLS 1 18-GHz radiometer which is due to be launched in June 2003. The work is focused on finding the responses to waves with horizontal wavelength of scale ≈ 1-1000 km and vertical wavelength of ≈ 1-20 km which are typically found in the atmosphere.

7.1 Thesis Summary

We summarise here the achievement of the work done in this thesis, which has been presented in Chapters 2-6. The main content and the important results found in each chapter could be briefly described as follows:

In Chapter 2, we described in detail of the forward model developed to simulate the radiance measurements made with the UARS MLS 63-GHz radiometer in the realistic background atmosphere at altitudes = 0-120 km. The observed radiances in this case mostly originate from the emission of molecular oxygen at frequency around 63 GHz along the ray path. We found that the radiance profiles generated from our model for the a single limb scan are generally in good agreement with the results reported from the real observations of the UARS MLS instrument, especially, on the saturation amplitude of each channel. These results primarily confirm the validity and practicability of the model in imitating the radiance measurements in the UARS MLS operation for the purpose of our study. By analysing the temperature weighting function of all the saturated channels, we found that the observed radiances from each individual channel at this stage are contributed mainly from a localised layer with a resolution of about 10-15 km whose centre spanned at eight different attitudes in the stratosphere and mesosphere. These saturated channels were proved to be very sensitive to the small variations of background temperature occurred in their observing layer which
might be due to the interference by propagating gravity waves. This knowledge did support our believe in the capability of UARS MLS as a new tool for the observation of the gravity waves at various altitudes in the middle atmosphere as demonstrated in some reports earlier.

In Chapter 3, the mathematical theory and some important properties of atmospheric gravity waves were given as a background for the understanding of the wave functions used in our study. The assumed gravity wave function for the use in the model simulation is that of a plane wave moving upward from the lower atmosphere into the middle atmosphere in both vertical and horizontal directions with its amplitude growing exponentially with height. However, the possibility of this wave to be observed with the MLS instrument depends on many factors, both the intrinsic properties of the wave itself and the technical details of the instrument involved in the observation. We explained in this chapter how to incorporate these factors into the calculation of the so-called ‘temperature response’, or ‘amplitude response’, and ‘variance response’ of the instrument to each observed wave. The amplitude of these responses represents the sensitivity of the instrument to the wave under observation, which depends both on its wavelength scales and the propagation direction. We chose to report in this thesis only results of the temperature and variance responses from the simulated observations in limb-tracking mode of saturated UARS MLS channel 4 when the instrument views the atmospheric limb at fixed tangent height of 18 km. Results from the other channels (except channel 8) are expected to be similar to those shown here regarding to the resemblance in shape and amplitudes of their weighting functions.

In Chapter 4, model results of temperature response of the UARS MLS instrument in the observations of the gravity waves were presented. The response results obtained here are reported in two different ways; first, as a function of wavelength components in three-dimension ($\lambda_x$, $\lambda_y$, $\lambda_z$) of an individual wave and, second, as a function of horizontal and vertical wavelengths and the propagation direction of the wave on the horizontal plane. It was found that, waves with relatively short $\lambda_z$ (e.g. with $\lambda_z \leq 10$ km) should not be visible much to the instrument as their amplitude responses are rather low (always less than $\approx 20\%$ of the original amplitude). However, the average response increases rapidly with increasing scale of $\lambda_z$ and at $\lambda_z = 10$-20 km the response of amplitude $\approx 0.2$-0.6 (or 20-60%) could be attained by the instrument to these waves. Generally, waves with wavenumber $k < 0$ are most visible as they have phase lines that are aligned more with the LOS direction and encounter less attenuation along the ray path. Waves with $k > 0$ are less detectable with the instrument as they have phase lines that more perpendicular to the LOS and undergo severe averaging in
amplitude of wave crests and troughs along the path. Typically, only waves with scales of horizontal wavelength $\lambda_H = 200-500$ km that are most visible to the UARS MLS instrument. However, waves with longer horizontal wavelength (e.g. with $\lambda_H > 500$ km), especially those with $k > 0$, should be more detectable if the vertical wavelength $\lambda_Z$ is getting longer.

In Chapter 5, model results of the variance response derived for the 32-measurement limb-tracking mode of UARS MLS were presented. It was firstly shown that, apart from the wavelength scales and propagation direction, amplitude of the variance response to each observed wave also depends principally on the number of measurements used in each calculation. For the 32-measurement method of UARS MLS, only waves with scales of along-track wavelength ($\lambda_p$) of $= 200-500$ km that are most visible in the observations. Waves with relatively short vertical wavelength $\lambda_Z$ (e.g. with $\lambda_Z \leq 10$ km) are unlikely to be much observable to the instrument, in this case, due to very low response to them (always less than $= 2\%$) caused by the FOV smearing effects. However, the average response increases substantially at $\lambda_Z = 10-20$ km where the maximum response of $= 0.2$ (20%) could be achieved at $\lambda_Z = 20$ km. At these scales of $\lambda_Z$, they are waves with $\lambda_H = 200-500$ km that still be most detectable with the instrument while waves with $\lambda_H > 500$ km are less likely to be observed especially those with $k > 0$. Waves with $\lambda_H < 100$ km might still be well visible but only at some specific directions of propagation. It was finally shown that, the variance response found in the 6-measurement limb-scanning mode of UARS MLS could be greatly contributed from the variation in viewing angle occurred during the operation apart from the variation of the wave amplitude along the observing path itself. This indicates that the UARS MLS variances obtained from this method are not quite suitable in the study of gravity waves with the instrument like those in the 32-measurement limb-tracking method describes above.

In Chapter 6, detailed analysis of the temperature response and variance response of the gravity wave observations with the EOS MLS 118-GHz radiometer was presented. In terms of the temperature response, the obtained model results indicated that, at vertical wavelength $= 1-20$ km, EOS MLS should be significantly better than UARS MLS in the detecting of atmospheric gravity waves, especially those with small scales on both horizontal and vertical directions. For example, the maximum amplitudes of the temperature response to waves with $\lambda_Z = 10-20$ km are between $= 0.4-0.8$ for EOS MLS while they are only $= 0.2-0.6$ for UARS MLS. However, the scales of horizontal wavelength $\lambda_H$ that both instruments
most sensitive to are rather compatible at wavelength $\lambda_H = 200-500$ km for waves with \( k < 0 \).

In the calculation of variance response of the instrument, we considered in both the response results from the observations in 150-measurement limb-tracking mode and 50-measurement limb-scanning mode made by saturated EOS MLS channel 9. In the comparison of the model results between these two operating methods, we found that the average responses in the 50-measurement limb-scanning method are significantly lower than those obtained in the 150-measurement limb-tracking mode. (The response amplitude is always less than $= 0.1$ in case of the 50-measurement method and $\approx 0.3$ in case of the 150-measurement method).

However, we expect that though the response to each observed wave in the 50-measurement limb-scanning mode is rather small but this should be sufficient for the observation of gravity waves with typical wavelength scales (e.g. with $\lambda_H = 50$-500 km and $\lambda_z = 5$-15 km) with the instrument.

This conclusion is based the successful observations of these waves with UARS MLS where the response to them is also never greater than 0.1 and the lower amplitude of noise variance of EOS MLS involved in the operation. In comparison, the 150-measurement limb-tracking mode of EOS MLS should provide better opportunity in detecting waves with typical vertical scales of 1-20 km than the 32-measurement method of UARS MLS. For example, the maximum amplitudes of the variance response to waves with $\lambda_z = 10$-20 km are $\approx 0.2$-$0.3$ for EOS MLS but they are only $\approx 0.02$-$0.2$ for UARS MLS. Though, the operation in limb-scanning mode of EOS MLS is still not in the current plan (as well as the study of gravity waves with the instrument), however, model results of the variance response obtained in this thesis strongly suggested that this mission might be worth considering of even only the 50-measurement limb-scanning method could be implemented in reality.

It should be noted here that, the analysis method of temperature response and variance response described in this thesis is for general purposes and it can be applied for the radiance measurements made by UARS MLS 183-GHz, or EOS MLS 190 GHz, channels as well. Similar to the 63-GHz (or 118-GHz) radiometer, the 183-GHz (or 190-GHz) radiances will saturate to the atmospheric temperature of various altitude layers. The differences, however, are their temperature weighting functions and a narrower beamwidth for the 183-GHz (or 190-GHz) channels which could provide better resolution for the observations of smaller-scales waves. It is possible that, these differences could allow us to compare the variances calculated from the two radiometers so as to gain more knowledge about the height variations of temperature fluctuations and the vertical wave structure as a consequence.
7.2 Gravity Wave Filter Functions

The results of the temperature and variance responses of the MLS instruments reported in this thesis have provided us better understanding of the wavelength scales and propagation direction of gravity waves that the instruments should be most sensitive to in the real observations. If we consider only the scale of vertical wavelength, which is most convenient to identify in with ground-based and in situ observational methods, we found that the vertical scales of the waves that both MLS instruments are most sensitive to are rather incompatible to those of conventional methods that analyse vertical profiles from radiosondes, rocket soundings, and lidar. That means each of the observational techniques has uniquely different portion of the wavelength spectrum that they are most sensitive to. Thus, the total wave variance derived for the entire spectrum must be filtered by an appropriate filter function unique to each observational method. Examples of filter functions for vertical wavelength of rocket sounding [Eckermann et al. 1995] and radiosonde [Allen and Vincent 1995] analyses are shown in Figure 7.1 compared to that of both UARS MLS and EOS MLS derived in this thesis. The first two methods have bandpass filters that allow only waves with relatively short vertical wavelength (e.g. with $\lambda_z < 10$ km) be taken into account. The MLS instruments, on the contrary, have no definite maximal cut-off values of $\lambda_z$ explicitly identified. The filter function for UARS MLS is taken from the maximum variance response of the instrument to each $\lambda_z$ as shown in Figure 5.8 and Figure 6.17 for the EOS MLS. Both MLS response filters are multiplied by some appropriate constants (1.8 in both cases) to make the final response at $\lambda_z = 100$ km become unity for the convenience in the comparison with other two methods.

Both radiosondes and rocket sounding methods have higher vertical resolution than that of MLS but cover a smaller range of altitudes. We see that both MLS filter functions are essentially being a low-pass filter in vertical wavenumber (as they are sensitive only to the relatively long vertical wavelength waves), so they provide rather different measures of gravity wave activity from the radiosonde and rocket sounding. All together, these four observation techniques could cover a very broad range of vertical wavelength. It must be emphasised here that the response functions for the MLS instruments also depend strongly on the wave propagation direction as well as the wavelength components and this factor must be taken into account to make an appropriate filter for the use in the variance analysis for the MLS instruments in the real observations.
Figure 7.1: Observational filter functions showing fractional variance response amplitude observed as a function of vertical wavelength for rocket sounding (solid), radiosonde (dotted), UARS MLS (dashed), and EOS MLS (dashed-dot) comparisons. Rocket and radiosonde filters are both modelled as simple band-pass filters with ranges of 2-10 km and 0.125-7 km, respectively, while both MLS filters are taken from the variance response results derived in Chapters 5 and 6 as detailed in text.

As seen in Figure 7.1, there is little overlap in the vertical wavelength domain between the MLS and the sounding filters. These two types of observations then are looking at two unique portions of the spectrum, so the waves that each technique observes have unique propagation properties and fundamentally different interactions with the atmosphere. This underscores what is perhaps the obvious fact that no single observation technique can see the whole spectrum of waves likely to be present. This also suggests something less obvious; that extrapolating the results of a single set of observations to the effects of the full spectrum of waves can lead to erroneous conclusions. Note that, the vertical profile of the temperature variance observed by the MLS instruments may be an important piece of information for gravity wave parameterisations. These profiles describe the energy growth in the low portion of vertical wavenumber $m$ (with relatively long $\lambda_2$) of gravity wave energy spectrum and might provide a constraint the parameterisation in this region, which is still difficult to obtain from most other methods.
Appendix A

Detailed Analysis of MLS Observational Method

We present here more detailed analysis of MLS observational method to give more understanding in the technical details of the operation and theoretical background of the observed radiance made with the MLS instruments.

A.1 Geometry of Limb Sounding Technique

We first consider the geometry of microwave limb sounding technique shown in Figure A.1. As mentioned earlier in Chapter 1, Microwave limb sounding measures atmospheric thermal emission at millimetre and submillimetre wavelengths as the instrument field of view (FOV) is scanned through the limb from above.

Figure A.1: Geometry of limb sounding observations illustrating the tangent height $h_T$ and the projected thickness of an arbitrary layer at height $z(x)$ along the ray path.
From Figure A.1, the radiometer is on a platform at an orbital altitude $H$ above the Earth’s surface. It receives the radiation emitted by the atmosphere along a ray path (dashed-dot line) that may be identified by the height $h_T$ of its tangent point. The atmosphere may be scanned by sweeping the viewing direction along the vertical direction (z-axis) while the satellite is moving along horizontal. The solid lines on either side of the ray path indicate the instrument beam width. Here we assign the observing co-ordinate system to have x-direction along the instrument’s line of sight (LOS), y-direction along the observing track (perpendicular to the paper plane), and z-direction along the vertical direction from Earth’s surface. In this arrangement, the distance between the satellite location and tangent point of observation is given by

$$L = \sqrt{(R_0 + H)^2 - (R_0 + h_T)^2}$$

$$= \sqrt{(2R_0 + H)(H - h_T)}$$

(A.1)

and angle $\theta$ of the FOV from the pointing direction (LOS) by

$$\tan \theta = \frac{z - h_T}{L} = \frac{z - h_T}{\sqrt{(2R_0 + H)(H - h_T)}}$$

(A.2)

where $R_0 = 6370$ km is the mean Earth’s radius and the approximation $h_T << R_0$ has been used. This condition is always true in case of MLS observations in which the tangent height $h_T$ is always between 0-100 km. The angle $\theta_H$ in Figure A.1, which a straight-line ray path makes with the local horizontal at the satellite position, is given by

$$\theta_H = \frac{R_0 + h_T}{R_0 + H}$$

(A.3)

An important geometric quantity which involved the FOV width is rate of change of $z$ with $\theta$ (for small $\theta$ and at constant $h_T$), which is just a distance $L$, or

$$\frac{dz}{d\theta} = \frac{1}{L} \frac{1}{\sqrt{(2R_0 + H)(H - h_T)}}$$

(A.4)
For situations in which $H$ is also much less than $R_0$, which included low-orbit satellite and balloons, Eqs. A.1 and A.4 can be approximated by

\[ L \equiv 113\sqrt{H - h_T} \text{ km} \quad (A.5) \]

and

\[ \frac{dz}{d\theta} \equiv 113\sqrt{H - h_T} \text{ km/radian} \quad (A.6a) \]

\[ \equiv 2\sqrt{H - h_T} \text{ km/degree} \quad (A.6b) \]

where $R_0 = 6370$ km has been used, and $H$ and $h_T$ have units of km. By integrating Eq. A.6b we have

\[ z^* \equiv 2\theta\sqrt{H - h_T} \text{ km} \quad (A.7) \]

where $z^* = z - h_T$ is the vertical distance from tangent point.

As an example, the orbit height of UARS is $H = 600$ km, and for $h_T = 18$ km, we have $S = 2780$ km, $\theta_H = 23.5^\circ$, and $dz/d\theta = 46$ km/degree. An FOV width of $\equiv 0.06^\circ$ is thus required for UARS instruments to provide the $\equiv 3$-km vertical resolution needed for profile measurements at tangent point. For the limb observation with the 1.6-m vertical diameter of the MLS antenna, it gives the 63-GHz and 118-GHz radiometers having FOV widths of $\sim 0.21^\circ$ and 0.11$^\circ$ [Jarnot et al. 1996]. For the observation at 18-km tangent height, these are corresponding to approximately 10-km and 5-km vertical width centred at tangent point respectively. In addition, the limb-path distance $\Delta L(h_T, z)$ from tangent point to a point at height $z$ along LOS is just given by Eq. A.5 with $H = z$:

\[ \Delta L(h_T, z) = 113\sqrt{z - h_T} \text{ km} \quad (A.8) \]

where $z + h_T << R_0$ is applied. This makes a limb path extending to heights 5 km above the tangent point on both sides has a length of $\equiv 500$ km. This long observational path is always encountered in the radiance measurements with the MLS instruments both in the saturation and non-saturation case. This makes them tend to underestimate the amplitude of short-scale temperature fluctuations along the ray path as shown in Chapter 4 for UARS MLS and in Chapter 6 for EOS MLS.
A.2 Formulation of Height Integration

As we learned in Eq. 2.36, the integration along the instrument line-of-sight (x-axis in Figure A.1 for both UARS MLS and EOS MLS) from \( x = -\infty \) (cosmic background) to \( x = x_s \) (the assumed satellite position) is required in the model calculation of radiance observed with the MLS instruments. However, as most atmospheric data (i.e., temperature, pressure) are tabulated with altitude, therefore, the integration with height should be more convenient for the use in practice.

The height integration for a limb-viewing instrument could be derived similarly to its horizontal counterpart except some geometrical term must be taken into account. Assuming a spherical geometry, the distance \( x \) along the ray path is related to the altitude \( z \) by

\[
(R_0 + h_T)^2 + x^2 = (R_0 + z)^2,
\]

where \( R_0 \) is the Earth’s mean radius (~6370 km). If we consider a spherical layer (shell) of thickness \( dz \), this is related to the horizontal distance \( dx \) by

\[
dx = \frac{z + R_0}{\left[(z + R_0)^2 - (h_T + R_0)^2\right]^{1/2}} dz = g(z, h_T)dz
\]

If we take into account that \( R_0 \) is usually much larger than \( z \) or \( h_T \), then function \( g(z, h_T) \) can be approximated by

\[
g(z, h_T) \approx \frac{R_0}{\sqrt{z - h_T}}
\]

Through this relation, Eq. 2.36 can be transformed into an integral over \( z \) by

\[
T_b(v, h_T) = T_{b0}e^{-\phi(h_T, \infty)} + \int_{h_T}^{\infty} \alpha(v, z)T_r(z)g(z, h_T)\left[e^{-\phi(v, z)} + e^{-\phi_1(v, z)}\right]dz
\]

where

\[
\phi(h_T, \infty) = \int_{h_T}^{\infty} \alpha(v, z')g(z', h_T)dz'
\]

and

\[
\phi_1(v, z) = \int_{z}^{\infty} \alpha(v, z')g(z', h_T)dz'
\]
\[ \phi_z(v, z) = \int_{h_r}^{z} \alpha(v, z') g(z', h_r) dz' + \int_{h_r}^{\infty} \alpha(v, z') g(z', h_r) dz' \quad (A.13c) \]

The presence of two terms in the integral component of Eq. A.12 is the result of the fact that \( x \) is a double-valued function of \( z \) (Eq. A.9). In this case, the first term corresponds to the emission from the layer section nearer to the sensor while the second term for the emission from layer section away from the sensor (see Figure A.1), which is seen through a much thicker atmosphere.

The total spectral radiance from Eq. A.12, therefore, can be written as the sum of contributions from three sources; the background surface \( T_{\text{cos}} \), the layer section close to satellite \( T_{b1} \), the layer section away from the satellite \( T_{b2} \), or

\[ T_h(v, h_r) = T_{\text{cos}}(v, h_r) + T_{b1}(v, h_r) + T_{b2}(v, h_r) \quad (A.14) \]

where

\[ T_{\text{cos}} = T_e e^{-2\phi_{(h_r, \infty)}} \quad (A.15a) \]

\[ T_{b1} = \int_{h_r}^{\infty} \alpha(v, z) T_r(z) g(z, h_r) e^{-\phi_z(v, z)} dz \quad (A.15b) \]

\[ T_{b2} = \int_{h_r}^{\infty} \alpha(v, z) T_r(z) g(z, h_r) e^{-\phi_z(v, z)} dz \quad (A.15c) \]

Equation A.12 can be rewritten in terms of weighting function as

\[ T_h(v, h_r) = T_e e^{-2\phi_{(h_r, \infty)}} + \int_{0}^{h_r} T_r(v, z) W(v, h_r, z) dz \quad (A.16) \]

where the weighting function \( W(v, h_r, z) \) is given by

\[ W(v, h_r, z) = \alpha(v, z) g(z, h) [e^{-\phi_z(v, z)} + e^{-\phi_z(v, z)}] \text{ for } z > h_r \]

\[ W(v, h_r, z) = 0 \text{ for } z < h_r \quad (A.17) \]

Examples of the weighting functions convolved with the FOV function in case of UARS MLS observations are shown in Figure 2.12 and in case of EOS MLS in Figure 6.5.
Appendix B
Mathematical Detail in the Response Analyses

Though, the model results of the temperature response and variance response in both cases of UARS MLS and EOS MLS have been derived numerically from the relevant equations given in Chapters 3 and 4. However, these response results could be better interpreted by using the mathematical analysis shown below.

Temperature Response Analysis

We first consider the definition of temperature response given by Eq. 3.30,

\[ R(k, l, m, \phi) = \int_{\tau} \int_{z} \int_{x} \frac{e^{z/2H}}{r(T)} \cos(kx + ly + mz + \phi) W_{xz}(x, z)W_{x}(y)W_{z}(v; h_T, z) \, dx \, dy \, dz \]  

(B.1)

where \( \phi = -\alpha x \). By considering the cosine term in Eq. B.1 as a real part of the complex function \( \exp[i(kx + ly + mz + \phi)] \), this leads to the relation

\[ R(k, l, m, \phi) = \text{Re} \left[ \int_{\tau} \int_{z} \int_{x} \frac{e^{z/2H}}{r(T)} e^{i(kx + ly + mz + \phi)} W_{xz}(x, z)W_{x}(y)W_{z}(v; h_T, z) \, dx \, dy \, dz \right] \]  

(B.2)

As the weighting functions \( W_{xz} \) and \( W_{x} \) given by Eq. 3.25 also have the exponential form, this enable us to bring terms involved parameter \( y \) to be integrated separately, or, we can write from Eq. B.2,

\[ R(k, l, m, \phi) = \text{Re} \left[ \int_{z} \int_{x} e^{z/2H} \frac{1}{r(T)} e^{i(kx + ly + mz + \phi)} W_{xz}(x, z)W_{x}(y)W_{z}(v; h_T, z) \, dx \, dz \right] \]  

(B.3)
Appendix B
Mathematical Detail in the Response Analyses

By replacing the exponential term $\exp[i(ly)]$ in terms of the combination $\cos(ly) + i\sin(ly)$ in the first integration with respect to $y$, we have

$$R(k,l,m,\phi) = \int_y W_y(y) \cos(ly)dy \cdot \text{Re}\left[ e^{\frac{z^2}{2H}} r(T) e^{i(\kappa z + m\phi)} W_{xz}(x,z)W_z(v,h_T,z)dx dz \right]$$

(B.4)

Equation B.4 indicates that the dependence of response function $R$ on wavelength $\lambda_y$ could be expressed explicitly by the first integration involved functions $W_y$ and $\cos(ly)$. This integration term is the origin of response function $R_y(l)$ given previously in Eq. 4.9.

The dependence of function $R$ on wave phase $\phi$ could be examined similarly to that of $\lambda_y$ and this gives us the relation

$$R(k,l,m,\phi) = \int_y W_y(y) \cos(ly)dy \cdot \text{Re}\left[ e^{\phi} e^{\frac{z^2}{2H}} r(T) \cos(kx + mz) W_{xz}(x,z)W_z(v,h_T,z)dx dz \right]$$

(B.5)

which could be expressed in the form:

$$R(k,l,m,\phi) = R_y(l) \cdot [a_1 \cos \phi + a_2 \sin \phi]$$

(B.6)

where the constants $a_1$ and $a_2$ are defined by

$$a_1 = R(k,l,m,\phi = 0) = \int_y e^{\frac{z^2}{2H}} \cos(kx + mz) W_{xz}(x,z)W_z(v,h_T,z)dx dz$$

(B.7a)

and

$$a_2 = R(k,l,m,\phi = \frac{\pi}{2}) = -\int_y e^{\frac{z^2}{2H}} \sin(kx + mz) W_{xz}(x,z)W_z(v,h_T,z)dx dz$$

(B.7b)

These are also the definitions of constants $a_1$ and $a_2$ given by Eqs. 4.4a and b respectively.

Equation B.6 describes the dependence of temperature response $R$ on wave intrinsic phase $\phi$ for a single wave with wavelength components $(\lambda_x, \lambda_y, \lambda_z)$. As the amplitude response of the MLS instruments to an individual wave will depend strongly on the intrinsic phase of the wave that is changed continuously with time, in the model calculation of the response to each wave, therefore, the specific phase $\phi$ must be clearly identified.
Appendix B

Mathematical Detail in the Response Analyses

Variance Response Analysis

The expression of temperature response function in terms of wavenumber components $(k,l,m)$ and intrinsic phase $\phi$ in Eq. B.6 leads us to write for the variance response $\sigma_R^2$ defined in Eq. 3.36 for UARS MLS observation as

$$\sigma_R^2 = [R_y(l)]^2 \cdot \{ (a_1)^2 \operatorname{var}[\cos\phi_y] + (a_2)^2 \operatorname{var}[\sin\phi_y] \} \quad (B.8)$$

where $\phi_y$ is the wave phase for the $i$-th measurement defined by

$$\phi_{y,i} = \phi + \Delta\phi_{y,i} = \phi + (i - 1)\frac{2\pi}{\lambda_y}, \quad i = 1,2,\ldots, N \quad (B.9)$$

where $\Delta y$ is the gap between each individual measurement which is 15 km for UARS MLS and $N$ is the total number of measurements used in each calculation of the response. From definition of a single variance response $\sigma_R^2$ given in Eq. B.8, we can then write for the averaged response $<\sigma_R^2>$ for 100 different values of random phase $\phi$ defined by Eq. 3.38 as

$$<\sigma_R^2> = (a_1^2 + a_2^2)R_y^2 <\operatorname{var}[\cos\phi_y]> \quad (B.10)$$

where the fact that $<\operatorname{var}[\cos\phi_y]>$ and $<\operatorname{var}[\sin\phi_y]>$ are nearly equivalent in this case is employed. Equation B.10 describes the dependence of the average variance response $<\sigma_R^2>$ reported in our thesis on wavelengths $\lambda_x$ and $\lambda_z$ through the terms $a_1$ and $a_2$ defined by Eq. B.7 and on $\lambda_y$ through functions $R_y$ and $<\operatorname{var}[\cos\phi_y]>$. In case of EOS MLS where the observing path is along $x$-direction, Eqs. B.9-B.11 are still applicable but the parameter $\Delta y$ and $\lambda_y$ in Eq. B.9 must be replaced by $\Delta x = 1.3$ km and $\lambda_z$ respectively.
Appendix C

Statistical Analysis on the Measurements of Weak Gravity Wave Signals

As mentioned earlier in Chapter 5, the uncertainty in the observed total radiance variance of the MLS instruments is the fundamental limit for the detecting gravity waves with weak variance signal, and this depends on the number of data points averaged and the instrument noise as described below.

This effect could be seen more clearly if we write the observed MLS temperature variance, $\sigma^2$, as the sum of variances contributed from wave signal ($\sigma_{GW}^2$), the instrument noise ($\sigma_N^2$), and other possible error sources such as non-linear terms in the pressure dependence ($\sigma_{NL}^2$) as described by Eq. 1.2 as,

$$\sigma^2 = \sigma_{GW}^2 + \sigma_N^2 + \sigma_{NL}^2$$  \hspace{1cm} (C.1)

Generally, the non-linear pressure contribution, $\sigma_{NL}^2$, is small and only important for channel 1, 15 and 2, 14, and can be reasonably neglected in the variance analysis of the other channels [Wu and Waters 1996a]. As a result, the atmospheric fluctuation, $\sigma_{GW}^2$, can be derived by subtracting $\sigma_N^2$ from the observed total radiance variance $\sigma^2$. However, in this thesis, we have reported the variance results as the average value from 100 measurements for each set of wavenumber components ($\lambda_x, \lambda_y, \lambda_z$), therefore, we can write for the average observed variance, $<\sigma^2>$, from Eq. C.1 as

$$<\sigma^2> \equiv <\sigma_{GW}^2> + <\sigma_N^2>$$ \hspace{1cm} (C.2)

where $<\sigma_N^2>$ is the average value of noise variances. Typically, for the measurement of strong wave signal compared to average noise signal, the expected noise variance involved
Appendix C  Statistical Analysis on the Measurements of Weak Gravity Wave Signals

in each measurement could be replaced by the average value, which are \(-0.01\) \(K^2\) for UARS MLS channel 4 and \(-0.005\) \(K^2\) for EOS MLS channel 9.

In reality, the noise signal is not constant for each set of measurements but has some uncertainty around the mean value given above and it is this uncertainty that determines the fundamental limit for the observations of weak gravity wave signal with the instruments. Statistically, this kind of uncertainty depends principally on the number of data point averaged of the instrument noise and average value of the noise variance involved. The noise variance, \(\sigma_N^2\), shown in Eq. C.1 as calculated from \(n\) radiance measurements is chi-squared distributed with mean \(<\sigma_N^2>\) and standard error \(s\). This distribution may be approximated by the normal distribution for large values of \(n\) (Tamhane and Dunlop 2000),

\[
s = \sqrt{\frac{2}{n} <\sigma_N^2>}. \quad (C.3)
\]

From the knowledge of the uncertainty in noise variances of the instrument, we could then reasonably identify waves that are statistically significant, or observable, in the observations with the MLS instruments to be those with variance observed at the satellite comparable or greater than the uncertainty in noise variance, or, which in this case given by

\[
<\sigma_{GW}^2> \geq \sigma_{limit}^2 \quad (C.4)
\]

where \(\sigma_{limit}^2\) represents the uncertainty of noise variance during the measurements at some level of the one-sided confidence in the statistical analysis. For example, in case of the 95% confidential level, the amount of this uncertainty could be approximated by 1.65\(s\) which gives values of \(\sigma_{limit}^2 \approx 2.33 \times 10^{-3}\) \(K^2\) for UARS MLS and \(\approx 1.16 \times 10^{-3}\) \(K^2\) for EOS MLS respectively. In this case, values of \(<\sigma_N^2> = 0.01\) \(K^2\) for UARS MLS and \(= 0.005\) \(K^2\) for EOS MLS, and \(n = 100\) are used in the calculation of \(s\) from Eq. C.3. These results indicate that the average wave variances in order of \(10^{-3}\) \(K^2\) should be statistically significant in the observations of atmospheric gravity waves with both instruments.
From the definition of variance response in Eq. 3.37, we could replace the observed variance term $<\sigma_{GW}^2>$ in Eq. C.4 by the product of the wave variance at the measurement altitude (where $z = z_c$) given by $\sigma_0^2 = A^2$, where $A$ is the amplitude of the wave at that level, and the average variance response $<\sigma_R^2>$, which depends on both wavelength scales and direction of wave propagation, or, we can write from Eq. C.4

$$<\sigma_{GW}^2> \geq A^2 <\sigma_R^2>.$$  \hspace{1cm} (C.5)

From Eqs. C.4 and C.5 we could reasonably define the cut-off wavelengths at some fixed amplitude $A$, or the cut-off amplitude at some give wavelength scales defined through the knowledge of variance response $<\sigma_R^2>$, by using the relation

$$<\sigma_R^2>_{cut-off} = \frac{\sigma_{limit}^2}{A^2}$$ \hspace{1cm} (C.6a)

or,

$$A_{cut-off} = \left[ \frac{\sigma_{limit}^2}{<\sigma_R^2>} \right]^{1/2}.$$ \hspace{1cm} (C.6b)

Figure C.1 shows examples of the variation of the cut-off amplitude $A$ with wavelengths $\lambda_x$ and $\lambda_z$ in accordance with the relation described by Eq. C.6b for $A = 0.1, 0.2, 0.5, 1, 2, 3,$ and $5$ K where results in Fig. C.1a are for UARS MLS and in Fig. C.1b for EOS MLS. The values of $<\sigma_R^2>$ used in the calculation are taken from Fig. 5.8 for UARS MLS and Fig. 6.18 for EOS MLS. This makes the results shown in Fig. C.1a are for the observations of waves with wavelength $\lambda_y = 100$-500 km which are most observable with UARS MLS and results in Fig. C.1b are for those with $\lambda_y$ greater than $\sim 100$ km that are most visible to EOS MLS. From the knowledge of $A$ as a function of $\lambda_x$ and $\lambda_z$ shown in both figures, we could find the cut-off wavelengths at some fixed value of $A$, or vice versa. For example, at $A = 1.0$ K, only waves with scales of $\lambda_z$ approximately greater than 8 km that are likely to be detectable with UARS MLS and greater than $\sim 5$ km in case of EOS MLS. This gives a chance for waves with low amplitude (e.g. with $A < 1.0$ K) to be observed more with EOS MLS, especially those with relatively short wavelength scales in both vertical and horizontal directions.
Appendix C  
Statistical Analysis on the Measurements of Weak Gravity Wave Signals

Figure C.1a: The variation of cut-off wave amplitude $A$ with wavelengths $\lambda_x$ and $\lambda_z$ as given by Eq. C.6b with $A = 0.1, 0.2, 0.5, 1.0, 2.0, \text{ and } 5 \text{ K}$. These results are for the measurements with $\sigma_{\text{limit}}^2 = 2.5 \times 10^{-3} \text{ K}^2$ of UARS MLS and for $\lambda_y = 100-500 \text{ km}$.

Figure C.1b: As in Fig. C.1a but with $\sigma_{\text{limit}}^2 = 1.2 \times 10^{-3} \text{ K}^2$ of EOS MLS and for $\lambda_y$ greater than $\sim 100 \text{ km}$.
Appendix C  Statistical Analysis on the Measurements of Weak Gravity Wave Signals

In addition, we could also derive from Eq. C.6 the cut-off amplitude of the waves observed with both instruments as a function of horizontal wavelength $\lambda_H$ and propagation direction defined through angle $\theta$ of wave vector on horizontal plane with respect to the positive x-direction (see Fig. 3.5). We show for examples in Figs. C.2 and C.3 the variation of some cut-off amplitudes with $\lambda_H$ and $\theta$ for waves with $\lambda_z = 10$ km and 15 km. Results shown in Fig. C.2 are for UARS MLS measurements using values of the variance response $\langle \sigma_k^2 \rangle$ taken from Fig. 5.10 and results in Fig. C.3 for EOS MLS measurements using values of $\langle \sigma_k^2 \rangle$ from Fig. 6.20 at the same value of $\lambda_z$.

It is quite apparent from the comparison at each $\lambda_z$ of Figs. C.2 and C.3 that EOS MLS are much better than UARS MLS in detecting gravity waves with small amplitude, or with relatively short horizontal scale. For example, at $\lambda_z = 10$ km, the minimum amplitudes required for the observable waves with UARS MLS at scales of $\lambda_H \sim 100$-500 km are in the range of $\sim 0.5$-1.0 K while at these scales of $\lambda_H$ the required wave amplitudes for the observations with EOS MLS are $\sim 0.2$-0.8 K only. We see that these cut-off amplitudes at each $\lambda_H$ are reduced dramatically in both cases with the increasing in values of $\lambda_z$ from 10 to 15 km. That means waves with low amplitude (e.g with $A < 1$ K) should be more visible to the MLS instruments if they have longer vertical wavelength. For example, waves with horizontal wavelength $\lambda_H \sim 50$-200 km and $\lambda_z = 10$ km should be well visible to UARS MLS with the amplitude of $\sim 0.5$ K while at $\lambda_z = 15$ km they need only $A \sim 0.1$-0.2 K in order to be comfortably detectable with the instrument. This conclusion is also true for the gravity wave observations with EOS MLS where at $\lambda_z = 15$ km, the amplitudes of less than 0.1 K are mostly needed in order to make waves with $\lambda_H \sim 50$-300 km visible to the instrument. However, it is worth emphasising that, in terms of the variance response discussed here, the different viewing geometries make UARS MLS and EOS MLS most sensitive to different wave directions during the observations. The UARS MLS is good at detecting waves with wavefronts aligned more with the LOS direction while the EOS MLS observes best to waves with wavefronts aligned more perpendicular to the LOS direction (waves propagating along N-S direction in particular).
Appendix C

Statistical Analysis on the Measurements of Weak Gravity Wave Signals

(a) Cut-off Amplitude with Wave Propagation Direction
(for $\lambda_z = 10$ km and $N = 32$)

Figure C.2a: The contours of cut-off amplitude $A = 0.5, 1.0, 1.5,$ and $2.0$ K plotted as a function of $\lambda_H$ and $\theta$ for waves with $\lambda_z = 10$ km observed with UARS MLS.

(b) Cut-off Amplitude with Wave Propagation Direction
(for $\lambda_z = 15$ km and $N = 32$)

Figure C.2b: As in Fig. C.2a but for $A = 0.2, 0.3, 0.4, 0.5,$ and $0.6$ K and $\lambda_z = 15$ km.
Appendix C  Statistical Analysis on the Measurements of Weak Gravity Wave Signals

(a) Cut-off Amplitude with Propagation Direction
(for $\lambda_z = 10$ km and $N = 150$)

Figure C.3a: The contours of cut-off amplitude $A = 0.2, 0.3, 0.5, 0.7, 1.0,$ and $1.5$ K plotted as a function of $\lambda_H$ and $\theta$ for waves with $\lambda_z = 10$ km observed with EOS MLS.

(b) Cut-off Amplitude with Propagation Direction
(for $\lambda_z = 15$ km and $N = 150$)

Figure C.3b: As in Fig. C.3a but for $A = 0.1, 0.2, 0.3, 0.4,$ and $0.5$ K and $\lambda_z = 15$ km.

210
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