K⁻d INTERACTIONS AT 1.45 AND 1.65 GeV/c AND
A REGGE POLE ANALYSIS OF THE REACTION K⁻n + Λπ⁻

Thesis

Submitted by

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ABSTRACT

This thesis gives details of a $K^-$ scattering experiment in liquid deuterium at incident kaon momenta of 1.45 and 1.65 GeV/c. As a member of one of the research groups collaborating in this experiment, the author describes data collection and total channel cross section calculations, in which he was involved.

The identification of resonance particles and the determination of their properties (mass, width, elasticity, spin, parity, isospin, baryon number and strangeness) is important in high energy physics. The method often employed for this is Partial Wave Analysis. Details of this technique are given as well as a review of analyses of the reaction $K^-N \rightarrow \Lambda \pi$ in the energy region about 2 GeV. Although this method is successful, it suffers from a degree of arbitrariness in the parameterisation of the non-resonant background.

In an attempt to overcome this, collaboration data, already successfully Partial Wave Analysed, are analysed using a Regge Pole formalism to predict the background. However this prescription is found to be too restricted to accurately predict the data. Nevertheless conclusions are drawn, which may be useful in the development of this method.
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CHAPTER 1

1.1 Introduction

For the past fifteen years, high statistic scattering experiments have been a major interest in the field of Particle Physics. Such experiments have resulted in the discovery of all but about thirty of the three hundred particles now established. But this could only have been achieved by the advent of three technical developments, namely the particle accelerator, the bubble chamber and the computer.

By the early fifties, synchrotrons had been developed sufficiently in energy and flux to provide a source of particles useful for scattering experiments. Before then the only source had been cosmic rays, whose occurrence was uncontrollable. Accelerators in contrast could provide a pulse of particles every two seconds, approximately. Further, a particular type of particle with a particular momentum could be selected.

The bubble chamber, developed during the fifties, provided an ideal detector for these experiments. The recycling time of a few milliseconds, with even shorter sensitive time, resulted in an almost complete absence of background tracks, while enabling it to cope easily with incoming pulses every two seconds. The tracks in the bubble chamber were virtually distortion-free and the liquid of the chamber itself, provided a high density of target particles for the scattering of the incoming particles.
Such experiments were first used to probe the nature of the strong interaction - the fundamental force responsible for nuclear stability. In these experiments particles, occasionally observed in cosmic radiation, could be produced in quantity and properties, such as spin and parity, which control particle interactions, could be determined.

Then structure in the scattering amplitude was discovered, which could be attributed to particles of lifetime $10^{-23}$ secs (resonance particles). Many more of these resonance particles were discovered, but only from statistical analysis of a large number of possible events. Spin and parity determination required even greater numbers.

The vast data-producing potential of accelerator and bubble chamber could produce sufficient statistics for resonance investigation in a few weeks. Even with the aid of computers the normal time for data-analysis is about three years and without them such investigations would be impossible.

Identification of particles and determination of their properties form an important step in the development of a unified theory of the strong interaction. While such a theory has not yet been formulated, new concepts have evolved and new laws defined. This chapter will deal, approximately chronologically, with those developments which are particularly relevant to this thesis.

1.2 Spin

The failure of the Schroedinger theory to predict the observed fine structure splitting of the hydrogen atom and other experimental evidence led Uhlenbeck and Goudsmit to propose that the electron possessed half a quantum of angular
momentum. To achieve a more theoretical derivation of this property, called spin, attempts were made to define a relativistic Hamiltonian operator for substitution into Schrödinger's equation.

The relativistic Hamiltonian was:

\[ H = \sqrt{G - e\mathcal{A}}^2 + m^2 + e\mathcal{\Phi} \]

but the operator would then be:

\[ H = \sqrt{(-iv^2 - e\mathcal{A})^2 + m^2} + e\mathcal{\Phi} \]

which would give a term in \( \sqrt{\gamma} \) which could not be interpreted.

The Klein-Gordon treatment was to remove the square root term by squaring.

\[ (H - e\mathcal{\Phi})^2 = (\sqrt{G - e\mathcal{A}}^2 + m^2)^2 \]

The Schrödinger form, \( H\psi = E\psi \), could not be used and the Klein-Gordon equation, in operator form was:

\[ \left[ \left( i \frac{\partial}{\partial t} - e\mathcal{\Phi} \right)^2 - (-iv - e\mathcal{A})^2 \right] \psi = m^2 \psi \]

But because of the squaring the value of \( E \) for any state of a free particle could be both positive and negative. This meant that \( \psi \) was not positive-definite (then held to be a necessary property of a wavefunction) and there was no physical interpretation for the negative \( E \) states. Finally, it gave the wrong predictions for the hydrogen atom and so was discarded, though later it was found to apply to spinless particles.

Dirac eliminated the square root by suggesting the term square rooted was in fact a perfect square.
\[
\sqrt{(|\mathbf{p}| - e\mathbf{A})^2 + m^2} = \mathbf{a} \cdot (|\mathbf{p}| - e\mathbf{A}) + \beta m
\]

Hence \(a_x^2 = a_y^2 = a_z^2 = \beta^2 = 1\)

\[\{a_i, a_j\} = 0 \quad \text{for } i \neq j\]

\[\{a_i, \beta\} = 0\]

\(a_1, a_2, a_3\) and \(\beta\) must therefore be operators and are normally taken to be \(4 \times 4\) matrix operators.

\[
a_x = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}, \quad a_z = \begin{pmatrix}
0 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}
\]

\[
a_y = \begin{pmatrix}
0 & 0 & 0 & -i \\
0 & 0 & 1 & 0 \\
0 & -i & 0 & 0 \\
i & 0 & 0 & 0
\end{pmatrix}, \quad \beta = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

Dirac's equation is therefore:

\[
(\mathbf{a} \cdot (-i\mathbf{v} - e\mathbf{A}) + \beta m + e\mathbf{A})\psi = i \frac{\partial}{\partial t} \psi
\]

where the wavefunction must be a \(4\) component column matrix

\[
\psi = \begin{pmatrix}
\psi_1 \\
\psi_2 \\
\psi_3 \\
\psi_4
\end{pmatrix}
\]

To investigate the properties of this equation consider a free particle \((\mathbf{A} = 0, \mathbf{v} = 0)\) with wavefunction of the form

\[
\psi = e^{i(\mathbf{p} \cdot \mathbf{r} - Et)} \begin{pmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4
\end{pmatrix}
\]

Dirac's equation becomes:

\[
\begin{align*}
\psi_1 - \psi_3 &= 0 \\
\psi_2 + \psi_4 &= 0
\end{align*}
\]
\[
\begin{pmatrix}
-E+m & 0 & p_z & p_- \\
0 & -E+m & p_+ & -p_z \\
p_z & p_- & -E-m & 0 \\
p_+ & -p_z & 0 & -E-m
\end{pmatrix}
\begin{pmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix}
\]

where \( p_\pm = p_x \pm ip_y \).

For a non-zero solution the determinant of the \(4\times4\) matrix must be zero and this reduces to:

\[
E = \pm (p^2 + m^2)^{1/2}
\]

Like the Klein Gordon equation, negative energy states are obtained but unlike it, the wave function is positive-definite.

In all, there are 4 linearly independent solutions of the Dirac equation for a free particle

\[
\psi_1 = A_1 e^{i(\vec{p} \cdot \vec{r} - Et)} \begin{pmatrix} p_z \\ p_+ \\ p_- \\ E-m \end{pmatrix}
\]

\[
\psi_2 = A_2 e^{i(\vec{p} \cdot \vec{r} - Et)} \begin{pmatrix} p_- \\ -p_z \\ 0 \\ E+im \end{pmatrix}
\]

for \( E = \pm (p^2 + m^2)^{1/2} \)

To interpret these solutions, first consider angular momentum \( \vec{L} = \vec{p} \times \vec{p} \). For \( \vec{L} \) to be a constant of the motion of the system, its components must commute with the Hamiltonian.
\[ [L_z, H] = -i(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x})(-i \bar{a}.v + \beta m) \]
\[ + i(-i\bar{a}.v + \beta m)(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}) \]
\[ = \alpha_x \frac{\partial}{\partial y} - \alpha_y \frac{\partial}{\partial x} \neq 0. \]

Define \( \sigma_z = i\alpha_x \alpha_y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \)

\[ [\sigma_z, H] = 2(\sigma_y \frac{\partial}{\partial x} - \sigma_x \frac{\partial}{\partial y}) \]

So \( [L_z + \frac{1}{2} \sigma_z, H] = 0. \)

Similarly if \( \sigma_x = i\alpha_y \alpha_z \) and \( \sigma_y = i\alpha_z \alpha_x \)
then \( [L_x + \frac{1}{2} \sigma_x, H] = 0 \) and \( [L_y + \frac{1}{2} \sigma_y, H] = 0. \)

Therefore \( [\bar{L} + \frac{1}{2} \bar{\sigma}, H] = 0. \)

So \( \frac{1}{2} \bar{\sigma} \) represents half a quantum of angular momentum - the intrinsic spin of the particle (spin \( \bar{S} = \frac{1}{2} \bar{\sigma} \)). The total angular momentum \( \bar{J} = \bar{L} + \bar{S} \) is therefore a constant of motion of the system. (In fact as the components of \( \bar{J} \) do not commute with each other only \( J_z \) and \( |\bar{J}|^2 \) can be constants of motion.)

Now we define a new operator, Helicity, as the orientation of spin with respect to the momentum vector \( \bar{p} \).

Helicity \( X = \frac{\bar{\sigma} \cdot \bar{p}}{|ar{p}|} \).

For simplicity take \( \bar{p} = (0, 0, p) \)

Then \( X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \).
so \( X\psi_1 = + \psi_1 \) (spin orientated parallel to momentum)

and \( X\psi_2 = - \psi_2 \) (spin orientated antiparallel to momentum).

So half integer spin with its two possible orientations is a natural consequence of the Dirac form of the relativistic Hamiltonian.

As for the negative energy solutions, consider the electron in an electromagnetic field (either or both of \( \phi \) and \( A \) non-zero). If \( e \) is taken as positive, then the negative energy states become positive. (Obviously in the absence of a field the nature of the charge cannot be determined.) So the 4 states may be identified as positively and negatively charged electrons each in two spin orientations.

Other properties of the positive electron (positron) were deduced from Dirac's Theory and indeed such a particle was identified by Anderson in a cosmic ray shower in 1932. The positron was also called the "antiparticle" of the electron.

Hyperfine splitting was rightly identified by Pauli as an interaction between the nuclear magnetic moment and the total electronic magnetic moment.

The splitting would therefore depend on the orientation of the nuclear magnetic moment. If, for example, a nucleus had spin \( s = 1 \), it could be orientated in three directions \( m_s = -1, 0, +1 \), resulting in 3-fold hyperfine structure splitting.

Investigation of hyperfine structure of atoms indicated that the nuclei could be divided into two general categories. Particles which had integer spin \( (s = 0, 1, 2, \ldots) \) were called
Bosons as they obeyed the Bose-Einstein formalism of statistical mechanics. Particles which had half integer spin \( s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots \) were called Fermions as they obeyed Fermi-Dirac statistics.

All particles, as yet discovered have been found to possess unique spin - in fact determination of a unique spin is as fundamental a test in the identification of a particle as finding its mass and charge - and its significance goes beyond statistical mechanics classification.

As angular momentum is conserved, (as yet there has been no evidence against this), then in decays or interactions of particles orbital angular momentum must account for any imbalance in the spins of the participating particles. This orbital angular momentum manifests itself as structure in the angular distribution of outgoing particles. In fact spins of particles are normally deduced from study of angular distributions (differential cross sections) using a process called Partial Wave Analysis (see Chapter 4).

Beyond this, spin has a very important role in classification of particles - symmetry schemes such as SU(3) which attempt to link up properties of different particles.

1.3 The Strong Interaction

Modern nuclear physics could be said to date from the discovery of the neutron in 1932. The neutron was the other constituent of the nucleus, with mass only slightly larger than the proton's mass but with zero charge. To account for nuclear stability there must exist a strong force (at that time only electromagnetic and gravitational forces were known) within the nucleus.
The mass defect of nuclei suggested that the binding energy per nucleon was approximately 6 MeV throughout the periodic table. The strong force must therefore satisfy a saturation condition. Stable nuclei showed a preference for equal numbers of protons and neutrons, though, of course, as the number of protons increased their Coulomb repulsion became significant, resulting in excess neutrons.

To account for these two properties, Heisenberg suggested that the strong force was an exchange force between proton and neutron. The exchange quantum must have positive and negative charge states.

The exchange processes would therefore be

\[ p \leftrightarrow n + Y^+ \]
\[ n \leftrightarrow p + Y^- . \]

Although the nature of \( Y^+ \) was not defined, the \( \beta \)-decay process was obviously an influencing factor.

Any strong proton-proton and neutron-neutron force would have to be equal, as the Coulomb repulsion was sufficient to account for the observed excess of neutrons in stable nuclei.

Yukawa proposed that the strong force could be described by the virtual exchange of a massive particle in much the same way as the Coulomb force between two charged particles could be attributed to the virtual exchange of a photon.

\[ e^- \rightarrow \gamma \rightarrow e^+ \]
\[ e^- \rightarrow \gamma \rightarrow e^- \]
The electromagnetic scalar potential satisfies the wave equation \( (V^2 - \frac{\partial^2}{\partial t^2})U = 0 \) with \( U \propto \frac{1}{r} \).

For a massive particle the wave equation becomes
\[
(V^2 - \frac{\partial^2}{\partial t^2} - m^2)U = 0 \quad \text{with} \quad U \propto \frac{e^{-mr}}{r}
\]

\( Y \) is Yukawa’s exchange particle of mass \( m \). The resulting potential therefore falls off exponentially. The approximate range of the force \( (r_0) \) will therefore be given by \( mr_0 \sim 1 \).

Hence if \( r_0 \sim 1.5 \text{ fm} \),
\[ m \sim 125 \text{ MeV/c}^2. \]

A similar result is obtained for the mass of Yukawa’s particle using Heisenberg’s Uncertainty Principle.

Considerations of spin would make Yukawa’s particle a boson with spin 0 or 1
\[
p \leftrightarrow n Y^+
\]
with spin \( \frac{1}{2} \leftrightarrow \frac{1}{2} + 0 \) or \( \frac{1}{2} \leftrightarrow \frac{1}{2} + 1 \).

Experimental evidence suggested that there was indeed a proton-proton force, hence neutron-neutron force by charge symmetry, and these forces were of the same magnitude as the proton-neutron force. This meant that the strong interaction was charge independent.
1.4 Isospin

The charge independence of the strong nuclear force gave rise to the suggestion that the proton and neutron were two charge states of the same particle. The equality of their spin, one half, and the approximate equality of their masses (proton mass \( \sim 938.3 \text{ MeV/c}^2 \) and neutron mass \( \sim 939.6 \text{ MeV/c}^2 \)) added weight to this.

The new particle was called a nucleon and could be described by five quantities: 3 spatial coordinates, its spin orientation coordinate \( m_s \), and a new coordinate to distinguish the proton and neutron state. By analogy with spin, where a spin \( \frac{1}{2} \) particle had two states \( (m_s = \pm \frac{1}{2}) \), this new coordinate was ascribed the same formalism as spin and called isospin. The nucleon, therefore, has isospin \( I = \frac{1}{2} \) and its states are therefore the orientation of the isospin vector along the z-direction of an imaginary isospin space. The orientation quantum number, \( I_3 \), therefore could have two values \( +\frac{1}{2} \) (proton) and \( -\frac{1}{2} \) (neutron).

Charge invariance suggests the absolute conservation of isospin in strong interactions. The conservation of \( I_3 \) is equivalent to charge conservation as

\[
\text{charge } Q = I_3 + \frac{1}{2}.
\]

Charge invariance also led to a development of Yukawa's Theory. To produce proton-proton scattering or neutron-neutron scattering two of Yukawa's particles must be exchanged.

![Diagram of proton-proton and neutron-neutron scattering involving Yukawa's particles](attachment:image.png)
This double exchange process would not result in a scattering amplitude equal to that of the single exchange proton-neutron scattering. So the strong interaction would not be charge invariant.

This problem was resolved by Kemmer who postulated a neutral particle of mass equal to that of Yukawa's particle. Hence

\[
\begin{array}{c}
p \quad p \\
\downarrow \quad \uparrow \\
Y^0 \\
p \quad p \\
\end{array}
\quad \quad \quad
\begin{array}{c}
n \quad n \\
\downarrow \quad \uparrow \\
Y^0 \\
n \quad n \\
\end{array}
\]

described proton-proton and neutron-neutron scattering.

Neutron-proton scattering could be described by two processes:

\[
\begin{array}{c}
p \quad n \\
\downarrow \quad \uparrow \\
Y^+ \\
n \quad p \\
\end{array}
\quad \quad \quad
\begin{array}{c}
p \quad p \\
\downarrow \quad \uparrow \\
Y^0 \\
n \quad n \\
\end{array}
\]

Bhabha gave the Yukawa particle the name meson, though later it was called pi-meson or pion. The process \( N \leftrightarrow N\pi \) (\( N \), nucleon; \( \pi \), pion) has these three fundamental interactions

1) 
\[
\begin{array}{c}
p \quad n \\
\downarrow \quad \uparrow \\
\pi^+ \\
p \quad \pi^+ \\
\end{array}
\]

2) 
\[
\begin{array}{c}
p \quad p \\
\downarrow \quad \uparrow \\
\pi^0 \\
p \quad \pi^0 \\
\end{array}
\]

3) 
\[
\begin{array}{c}
n \quad n \\
\downarrow \quad \uparrow \\
\pi^0 \\
n \quad \pi^0 \\
\end{array}
\]
If the coupling constants of interactions 1), 2) and 3) are a, b and c respectively, then the scattering amplitude for

i) proton-proton scattering is $b^2$.

ii) Neutron-neutron scattering is $c^2$.

iii) Proton-neutron scattering is $a^2 + bc$.

For charge invariance $bc + a^2 = b^2 = c^2$, which has two solutions

1) $b = c$ ; $a = 0$.

That is where only the $\pi^0$ exist, but this does not allow the charge exchange scattering which was experimentally observable.

2) $b = -c$ ; $a = \pm (2b^2)^{1/2}$

the sign of a is merely an arbitrary phase factor and by convention

$$a : b : c = \sqrt{2} : -1 : 1$$

This ratio can also be obtained by considering a spin $1/2$ particle decaying into a spin $1/2$ particle and a spin 1 particle.

Then

$$|^{1/2},^{1/2}\rangle \rightarrow |^{1/2},^{1/2};1,1\rangle$$

$$|^{1/2},^{1/2}\rangle \rightarrow |^{1/2},^{1/2};1,0\rangle$$

$$|^{1/2},^{-1/2}\rangle \rightarrow |^{1/2},^{-1/2};1,0\rangle$$

have relative amplitudes $\sqrt{2} : -1 : 1$ where $|S, m_s\rangle$ represents the decaying particle and $|S^F, m_s^F; S^B, m_s^B\rangle$ represents the decay products.

Using isospin instead of spin, the $N \rightarrow N\pi$ amplitudes are obtained.

The pion therefore has isospin $I = 1$ and there states $I^3 = 1$ ($\pi^+$), $I^3 = 0$ ($\pi^0$) and $I^3 = -1$ ($\pi^-$). Here charge $Q = I^3$. 
Strongly interacting particles can be divided into two groups: baryons, like the nucleon, which are fermions, and mesons, like the pion, which are bosons. If a new quantum number, Baryon number \( B \) is introduced where

\[
B = \begin{cases} 
1 & \text{for baryons} \\
-1 & \text{for antibaryons, the antiparticles of baryons.} \\
0 & \text{for mesons or their antiparticles.}
\end{cases}
\]

Then \( I_3 = \frac{2Q - B}{2} \) for both mesons and baryons. Thus conservation of \( I_3 \) implies conservation of baryon number as well as charge conservation in strong interactions.

1.5 Strangeness

The next development came from the search for Yukawa's particle. In the 1930's the only possible source was cosmic radiation and the detector was the Wilson cloud chamber. In 1936 a particle of mass about 200 times the electron mass and in two charge states (positive and negative) was detected\(^{(1)}\) but later it was found to have nuclear cross-section about three orders of magnitude too small to be a pion.

Photographic emulsions were developed as detectors and in 1947 the pion was observed in an emulsion\(^{(2)}\). But also that year Rochester and Butler observed two "strange" events in a cloud chamber\(^{(3)}\). The first was V-shaped and was interpreted as the spontaneous decay of a neutral particle. The other, a kinked track without any sign of a recoil, was interpreted as a spontaneous decay of a charged particle.

There then followed a series of 'strange' decays of particles produced by cosmic ray showers and detected in either cloud chamber or emulsion:
All these particles decay by weak processes (lifetimes \(10^{-10} - 10^{-8}\) secs) but were created in strong interactions. Pais suggested that these particles were created in pairs (Associated production) and postulated an extension of the concept of isospin(4).

Gell-Mann realised that suitable assignment of particles to isospin multiplets would, by conservation of \(I_3\), provide selection rules for strong interactions(5).

\[
\begin{align*}
\pi^- p & \rightarrow \Lambda^0 K^0 \\
I_3: & \ -1 + \frac{1}{2} \rightarrow 0 - \frac{1}{2} \quad \text{(allowed)} \\
\pi^- p & \rightarrow \Sigma^+ K^- \\
I_3: & \ -1 + \frac{1}{2} \rightarrow 1 - \frac{1}{2} \quad \text{(not allowed)}.
\end{align*}
\]

The \(K^+\) and \(K^0\) were assigned to an isospin doublet and their antiparticles \(K^-\) and \(\bar{K}^0\) also form a doublet. The \(\Lambda^0\) was identified as the neutral element of the \(\Sigma\) triplet but this does not invalidate the argument. The decay modes do not conserve isospin therefore cannot proceed by a strong interaction.

Gell-Mann and Nishijima developed this idea into a so-called strangeness theory(6). A new quantum number, strangeness (8), was defined. For mesons, pions were taken...
### Table 1.1  Meson Isospin Multiplets and Strangenesses.

<table>
<thead>
<tr>
<th>Multiplet</th>
<th>Charge</th>
<th>Strangeness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>Pion</td>
<td>$\pi^+$</td>
<td>$\pi^0$</td>
</tr>
<tr>
<td>Kaon</td>
<td>$K^+$</td>
<td>$K^0$</td>
</tr>
<tr>
<td>Antikaon</td>
<td>$\bar{K}^0$</td>
<td>$\bar{K}^-$</td>
</tr>
<tr>
<td>Eta</td>
<td>$\eta$</td>
<td></td>
</tr>
</tbody>
</table>

Meson charge centre
Table 1.2  Baryon Isospin Multiplets and Strangenesses.

<table>
<thead>
<tr>
<th>Multiplet</th>
<th>Charge</th>
<th>Strangeness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>Nucleon</td>
<td>p</td>
<td>n</td>
</tr>
<tr>
<td>Lambda</td>
<td>(\Lambda^0)</td>
<td>(-1)</td>
</tr>
<tr>
<td>Sigma</td>
<td>(\Sigma^+)</td>
<td>(\Sigma^0)</td>
</tr>
<tr>
<td>Xi</td>
<td>(\Xi^0)</td>
<td>(\Xi^-)</td>
</tr>
<tr>
<td>Omega</td>
<td>(\Omega^-)</td>
<td>-3</td>
</tr>
</tbody>
</table>

Nucleon charge centre
Table 1.3  Antibaryon Isospin Multiplets and Strangenesses.

<table>
<thead>
<tr>
<th>Multiplet</th>
<th>Charge</th>
<th>Strangeness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>Antinucleon</td>
<td>( \bar{n} )</td>
<td>( \bar{p} )</td>
</tr>
<tr>
<td>Antilambda</td>
<td>( \bar{\Lambda}^0 )</td>
<td>+( \frac{1}{2} )</td>
</tr>
<tr>
<td>Antisigma</td>
<td>( \bar{\Sigma}^- )</td>
<td>( \Sigma^0 )</td>
</tr>
<tr>
<td>Antixi</td>
<td>( \bar{\Xi}^- )</td>
<td>( \Xi^0 )</td>
</tr>
<tr>
<td>Antomega</td>
<td>( \bar{\Omega}^- )</td>
<td>+1( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

Anti-nucleon charge centre
to be non-strange \((S = 0)\) and the strangeness of any other meson was defined as twice the difference between the charge centre (average charge) of the isospin multiplet and the charge centre of the pions. This is shown in Table 1.1 for all the mesons discovered stable under strong decay.

For baryons (antibaryons) the non-strange multiplet is the nucleons (anti-nucleons) and strangenesses are calculated relative to them (Tables 1.2 and 1.3). The \(\Xi^0\) and \(\Omega^-\) were not discovered until 1959 and 1964 respectively.

Strangeness was held to be conserved in both strong and electromagnetic interactions. This results in there being no energetically possible strong channel, which also conserved strangeness, available for the decay of these particles so the only channels open are electromagnetic (e.g. \(\Sigma^0 \rightarrow \Lambda^0 \gamma\)) conserving strangeness or weak (e.g. \(\Sigma^+ \rightarrow n \pi^+\)) not conserving strangeness.

A further selection applies in weak decay, namely that strangeness can only change by one unit in any decay. This produces cascade decays

\[
\begin{align*}
\Omega^- & \rightarrow \Xi^- \pi^0 \\
\Lambda^0 \pi^- & \rightarrow p \pi^- 
\end{align*}
\]

1.6 Parity

The parity operator, \(P\), has the effect of reversing each of the coordinate axes

\[
\begin{align*}
x & \rightarrow -x \\
y & \rightarrow -y \\
z & \rightarrow -z 
\end{align*}
\]

If it is operated on a wavefunction, there are two possible eigenstates with eigenvalue \(\xi = \pm 1\). This is because two
operations of \( P \) return the wavefunction to its original form
\[ P^2 \psi = \psi. \]

The two eigenstates are:

\[ P\psi(xyz) = \psi(-x-y-z) = + \psi(xyz) \quad \text{even parity} \]
\[ P\psi(xyz) = \psi(-x-y-z) = - \psi(xyz) \quad \text{odd parity}. \]

Note that the probability function \( |\psi|^2 \) is invariant under parity transformation.

For orbital angular momentum states the spatial distribution is described by Legendre Polynomials.

\[ \ell = 1 \quad \psi \propto P_1(\cos \theta) = \cos \theta \]
\[ \therefore \quad P\psi \propto -\cos \theta. \]
\[ \ell = 2 \quad \psi \propto P_2(\cos \theta) = (3 \cos^2 \theta - 1) \]
\[ \therefore \quad P\psi \propto (3 \cos^2 \theta - 1). \]

In general, for two particles in angular momentum state \( \ell \) the parity eigenvalue of the state is \((-1)^\ell\).

Investigation of X-ray spectra produced empirical selection rules, whereby some transitions were allowed but others not. For example, for a one-electron atom there was an observed selection rule \( \Delta \ell = \pm 1 \). The emitting atom had to change its orbital angular momentum by one quantum, hence its parity had also to change.

As the radiation was characteristic of an electric dipole (spin=1; parity odd), this selection rule could be put in the form of a law of conservation of parity, where parity of atom before = (parity of atom after) \( \times \) (intrinsic parity of photon).

Further selection rules for X-ray emission from multi-electron atoms and also \( \gamma \)-ray emission provided strong evidence
for conservation of parity in both strong and electromagnetic interactions.

Selection rules in particle interactions were also interpreted by this law. For example

\[ \pi^- + d \rightarrow n + n \]

The \( \pi^- \) is captured in the s state \((\ell = 0)\). The total angular momentum of the initial state is therefore \( j = 1 \) (the deuteron spin being the only contribution).

The neutrons must be in an asymmetric state by the exclusion principle and total angular momentum \( j = 1 \). The only available state is \(^3P_1\) \((\ell = 1)\).

Hence, if parity is conserved,

\[
P(\text{pion}) \times P(\text{deuteron}) \times P(\text{orbital}) = P(\text{neutron})
\]

\[
\times P(\text{neutron}) \times P(\text{orbital})
\]

\[
\therefore P(\text{pion}) \times (+1) \times (-1)^0 = (\pm 1) \times (\pm 1) \times (-1)^1.
\]

Though the parity of the neutron is unknown, we get the result \( P(\text{pion}) = -1 \). The reaction \( \pi^- + d \rightarrow n + n + \pi^0 \) is forbidden by parity conservation (assuming \( \pi^0 \) has odd parity also) and this is confirmed by experiment.

Thus it appears that in both strong and electromagnetic interactions a state of definite total parity transforms into a state with the same total parity. The total parity of a state is made up of orbital contributions and the intrinsic parities of the individual particles.

The intrinsic parities of bosons can be determined by methods similar to that for the pions. Fermions, on the other hand, cannot be created except from or with another fermion, otherwise angular momentum could not balance. So at most we can tell whether the fermions have the same or different
<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass (MeV)</th>
<th>Lifetime (secs.)</th>
<th>Charge</th>
<th>Spin</th>
<th>Parity</th>
<th>Baryon No.</th>
<th>Isospin</th>
<th>$I_3$</th>
<th>Strangeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+$</td>
<td>139.6</td>
<td>$2.6 \times 10^{-8}$</td>
<td>+1</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>+1</td>
<td>0</td>
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<tr>
<td>$\pi^0$</td>
<td>135.0</td>
<td>$8.4 \times 10^{-17}$</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>139.6</td>
<td>$2.6 \times 10^{-8}$</td>
<td>-1</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$K^+$</td>
<td>493.8</td>
<td>$1.2 \times 10^{-8}$</td>
<td>+1</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>+$\frac{1}{2}$</td>
<td>+1</td>
</tr>
<tr>
<td>$K^0$</td>
<td>497.8</td>
<td>($8.6 \times 10^{-11}$ $K_\Sigma^0$)</td>
<td>0</td>
<td>0</td>
<td>-</td>
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<td>+1</td>
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<tr>
<td>$\overline{K^0}$</td>
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<td>($5.2 \times 10^{-8}$ $K_L^0$)</td>
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<td>-</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>+$\frac{1}{2}$</td>
<td>-1</td>
</tr>
<tr>
<td>$K^-$</td>
<td>493.8</td>
<td>$1.2 \times 10^{-8}$</td>
<td>-1</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
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<td>-1</td>
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<tr>
<td>$\eta$</td>
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<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p$</td>
<td>938.4</td>
<td>Stable</td>
<td>+1</td>
<td>$\frac{1}{2}$</td>
<td>+</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
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</tr>
<tr>
<td>$n$</td>
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<td>93</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>+</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>-$\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1115.6</td>
<td>$2.5 \times 10^{-10}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>+</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
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<tr>
<td>$\Sigma^+$</td>
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<td>$\frac{1}{2}$</td>
<td>+</td>
<td>1</td>
<td>1</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
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<td>0</td>
<td>$\frac{1}{2}$</td>
<td>+</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>1197.4</td>
<td>$1.5 \times 10^{-10}$</td>
<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>+</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>1314.7</td>
<td>$3.0 \times 10^{-10}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>+</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>+$\frac{1}{2}$</td>
<td>-2</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>1321.3</td>
<td>$1.7 \times 10^{-10}$</td>
<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>+</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>-$\frac{1}{2}$</td>
<td>-2</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>1672.5</td>
<td>$1.3 \times 10^{-10}$</td>
<td>-1</td>
<td>$\frac{3}{2}$</td>
<td>+</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-3</td>
</tr>
</tbody>
</table>
Table 1.4 Properties of Stable Hadrons (Continued)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass (MeV)</th>
<th>Lifetime (secs.)</th>
<th>Charge</th>
<th>Spin</th>
<th>Parity</th>
<th>Baryon No.</th>
<th>Isospin</th>
<th>$I_s$</th>
<th>Strangeness</th>
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<tbody>
<tr>
<td>$\bar{n}$</td>
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<td>93 secs.</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>-</td>
<td>-1</td>
<td>$\frac{1}{2}$</td>
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<td>0</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>938.4</td>
<td>Stable</td>
<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>-</td>
<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>- $\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>$\Lambda^0$</td>
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<td>0</td>
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<td>1</td>
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<td>+1</td>
</tr>
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<td>1</td>
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<tr>
<td>$\Sigma^+$</td>
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<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>-</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>1321.4</td>
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<td>+1</td>
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<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>+2</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>1314.7</td>
<td>$3.0 \times 10^{-10}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>-</td>
<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>- $\frac{1}{2}$</td>
<td>+2</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>1672.5</td>
<td>$1.3 \times 10^{-10}$</td>
<td>+1</td>
<td>$\frac{3}{2}$</td>
<td>-</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>+3</td>
</tr>
</tbody>
</table>

Data from Review of Particle Properties (ref. (11)).
parities. By convention the proton is taken to have even parity and all other fermion parities determined relative to the proton.

In the section on spin it was stated that conservation of angular momentum places restrictions on angular distributions of interactions. Similarly, conservation of parity places a further restriction on an interaction, so, again, it can be determined by Partial Wave Analysis (see Chapter 4).

Hence intrinsic parity, as well as spin, is used in the symmetry schemes mentioned in the section on spin.

The properties of all strongly interacting particles (Hadrons) which are stable under strong decay are given in Table 1.4

1.7 Resonances

By the early fifties accelerators had developed sufficiently to allow pion nucleon scattering experiments to investigate the strong force. The accelerated particles (protons) were directed into a heavy target. From the particles thus produced, pions of a specific momentum could be selected by use of electric and magnetic fields and collimators.

In 1954 the total cross-sections for $\pi^+p$ and $\pi^-p$ scattering had been measured up to pion laboratory energy $1500$ MeV(7). These showed a peak in cross section at about $160$ MeV pion lab. energy. The peak was considerably more pronounced in the $\pi^+p$ cross-section. When the cross-sections for the pure isospin states ($\pi^+p$ is pure isospin $\frac{3}{2}$, whereas $\pi^-p$ is a mixture of isospin $\frac{3}{2}$ and isospin $\frac{1}{2}$) were unfolded, the peak was observed in the isospin $\frac{3}{2}$ cross-section only.
The peak was analogous to the slow neutron capture cross-section of nuclei. There were peaks in the neutron capture cross-section at certain neutron kinetic energies—called resonant energies by analogy with absorption of electromagnetic radiation. These resonant energies were widely enough spaced so as not to interfere with one another. The cross section in the region of these resonances was parametrized by Breit and Wigner

\[ \sigma_c = S \frac{\lambda^2}{\pi} \frac{\Gamma \Gamma_n}{(E - E_0)^2 + \frac{\Gamma^2}{4}} \]

- \( S \) is a statistical factor
- \( \lambda \) is the de Broglie wavelength if the neutron in the centre of mass frame
- \( E_0 \) is the resonant energy
- \( E \) is the kinetic energy of the neutron
- \( \Gamma \) is the total width of the excited state
- \( \Gamma_n \) is the partial neutron decay width.

The width was related to the lifetime of the state \( (t) \) by the Uncertainty Principle

\[ \Gamma_t \sim \hbar \]

The pion nucleon peak was parametrized in similar fashion

\[ \sigma = 2\pi \lambda^2 \frac{\Gamma^2}{(E - E_0)^2 + \frac{\Gamma^2}{4}} \]

- \( \lambda \) is the pion centre of mass wavelength divided by \( 2\pi \).

This was fitted with \( E_0 = 159 \text{ MeV} \) and full width at half height approximately 100 MeV. The lifetime of such a resonant state would therefore be \( 10^{-23} \text{ secs.} \) approximately.
Consideration of angular distributions suggested that this resonance state had angular momentum eigenvalues $J = \frac{3}{2}, \ell = 1$. The orbital angular momentum eigenvalue gave the state positive parity.

$$P = P(p)P(\pi)(-1)^\ell$$

$$= +1 -1 -1 = +1$$

This resonant state had all the properties of a particle, definite spin, parity and isospin $(J^P I = \frac{3}{2}^+ \frac{3}{2})$. Its baryon number would be 1 and its strangeness 0. Therefore it was considered to be an excited state of the nucleon. As such, its mass would be about 1300 MeV and its lifetime of approximately $10^{-23}$ secs is consistent with a strong decay mode.

By 1960 meson-nucleon scattering experiments had produced sufficient data in three body final states for more resonant states to be detected. Alston et al.\(^9\) found a hyperon excited state in the reaction $K^- p \rightarrow \Lambda^0 \pi^+ \pi^-$ at 1.15 GeV/c by considering the reaction as a two stage process $K^- p \rightarrow Y^* \pi^+ \pi^- \text{ } \Lambda^0 \pi^+ \pi^- \text{.}$

Plotting the $Y^*$ effective mass showed a peak above phase space (the distribution obtained assuming no kinematic structure). This peak had mass 1380 MeV and width 64 MeV. Isospin, $I = 1$, Baryon number, $B = 1$, and strangeness, $S = -1$, would make it an excited state of the $\Sigma$ hyperon. Spin and parity assignment could not be made.

The same group also observed a strange meson resonance state\(^10\) in the reaction $K^- p \rightarrow K^{*-} p \rightarrow K^0 \pi^- \pi^+ \pi^- \text{.}$

The mass was approximately 885 MeV and width 16 MeV. Since then, a multitude of such resonant states have been observed\(^11\).
and it is convenient to consider them as particles - resonance particles - differing only from those listed in Table 1.4 in their instability under strong decay.

1.8 The Experiment

The experiment, whose data were used in the analysis presented in this thesis, was proposed in 1965, principally as a high statistic $K^-$ neutron scattering experiment to study the resonance particle, $\Sigma(2030)$, which was not well established at that time$^{(12)}$. To date it has resulted in four published papers.

(A) A partial wave analysis of the $K^- n \rightarrow \pi^- \Lambda$ channel, which found resonance parameters of the $\Sigma(2030)$ and the $\Sigma(1910)$. A resonance at 2080 MeV was also suggested$^{(13)}$.

(B) A study of production of hyperon resonances $\Sigma(1385)$, $\Lambda(1405)$ and $\Lambda(1520)$. Production cross sections, angular distributions and branching fractions of the decay modes were given$^{(14)}$.

(C) An investigation of a peak in the $\Lambda p$ mass spectrum in the reaction $K^- d \rightarrow \Lambda p \pi^-$, possibly due to a dibaryon bound state$^{(15)}$.

(D) A determination of the elasticity of the $\Sigma(2030)$ in the reaction $K^- n \rightarrow K^- n$ $^{(16)}$.

Each of the participating laboratories (at Birmingham, Edinburgh and Glasgow Universities and Imperial College, London) undertook a section of film analysis and data collection (see Chapter 2). The data from each laboratory had to be
compatible so that they could be merged. Therefore the rules for data extraction had to be standardised, as far as possible.

The data collection at Edinburgh was a combined effort of a number of researchers, whereas total cross section calculations which were necessary before data could be merged, were undertaken principally by the author (see Chapter 3). However it must be understood that both these sections of work were within the framework of the collaboration and methods used were those agreed at collaboration meetings.

1.9 The Λπ⁻ Channel

Where resonance formation occurs in high energy scattering, certain of the quantum numbers are fixed. For the strong process

\[ K^- n \rightarrow X \rightarrow \text{observed final states.} \]

Isospin, baryon number and strangeness are conserved. Hence for

\[ K^- n \rightarrow X \]

\[ I \ \frac{1}{2} \ rac{1}{2} \rightarrow l \text{ or } 0 \]

\[ I_3 \frac{1}{2} \frac{1}{2} \rightarrow -1 \ (\text{So } I \neq 0) \]

\[ B \ 0 \ 1 \rightarrow 1 \]

\[ S \ -1 \ 0 \rightarrow -1 \]

So in \( K^- \) neutron scattering the only resonance particles formed will have the quantum numbers characteristic of the \( \Sigma \) hyperon. Further properties (mass, width, spin and parity) are normally determined by partial wave analysis. In this the scattering amplitude is resolved into a sum of amplitudes which are eigenstates of spin and parity (partial wave amplitudes). An interaction is assumed to go by either of two
processes, \( K^- n \rightarrow \) observed final state or \( K^- n \rightarrow X \rightarrow \) observed final state.

No assumptions are made about the first process except that its partial wave amplitudes are slowly varying functions of energy (or, more usually, incoming centre of mass momentum) and therefore can be parameterized by the first few terms of a power series in centre of mass momentum.

For the second process, the resonance amplitude is inserted in the appropriate partial wave and its energy dependence can be explicitly calculated as a function of its mass and width. The total partial wave scattering amplitudes are obtained by adding resonant and non-resonant amplitudes,

\[
T = T_{\text{bg}} + T_{\text{res}}
\]

- \( T \) - Total partial wave amplitude
- \( T_{\text{bg}} \) - Background (non-resonant) partial wave
- \( T_{\text{res}} \) - Resonant partial wave amplitude.

A more detailed account of the formalism of partial wave analysis and techniques of fitting experimental data are given in Chapter 4.

The most energetically possible decay modes of the \( \Sigma(2030) \) were:

1. \( \Lambda \pi \)
2. \( \Sigma \pi \)
3. \( \bar{N}K \)
4. \( \Sigma(1385)\pi \)

Mode (d) was discarded as its analysis would be too complicated. The partial widths of the other three channels
(proportional to their relative probabilities) can be theoretically calculated using SU(3). This gave partial widths of 24.2 for (a), 13.7 for (b) and 16.4 for (c)\(^{17}\).

The relatively high non-resonant background in the NK channel would make resonance identification more difficult. Also polarisation determinations in the \(\Sigma\pi\) channel are complicated. Therefore the channel \(K^-n \to \Lambda \pi^-\) was selected for analysis because it was expected to produce the greatest number of \(\Sigma(2030)\) and also it was relatively simple to analyse. The results of this analysis are given in paper (A)\(^{13}\). The availability of these well-fitted data provided an opportunity to test an alternative form of background parameterization which did not suffer from the arbitrariness of that detailed above. The background parameterization tested was that predicted by the Generalised Interference Model.

1.10 The Generalised Interference Model

At low energies (incident beam momentum up to 2 GeV/c) scattering is dominated by direct channel resonances:

\[
\begin{array}{c}
K^- \\
\hline n \\
\hline \Sigma^* \\
\hline \\
\hline \pi^- \\
\hline \Lambda
\end{array}
\]

Therefore, as stated in the previous section, the scattering amplitude comprises resonance contributions plus non-resonant background. But at high energies (incident beam momentum over 5 GeV/c) scattering is described by cross channel exchanges:
Fig. 1.1 A typical total cross section curve (for K⁻p scattering). The dashed line represents the approximate shape of a Regge extrapolation into the lower energy region.

Data for this figure Refs. (11), (22), and (23).
The formalism of these processes, Regge Theory, is described in Chapter 5.

A typical cross section curve is shown in Fig. 1.1. In the low energy region resonances appear as bumps upon a smoothly varying background curve. At high energy the curve is smooth, asymptotically approaching a constant value. The intermediate region has characteristics of both high and low energy regions.

There have been two attempts to link resonance and Regge formalisms for the interpretation of data in the intermediate region. Both involve the extrapolation of Regge predictions which can then be extended further into the low energy region.

Such an extrapolation gave the approximate shape of the cross section curve but without any of the lower energy structure (Fig. 1.1). The Generalised Interference Model approach was to identify the Regge extrapolation with the non-resonant background\(18\). So an appropriate linear combination of the partial wave amplitudes in equation (1.1) could be written:

\[
    f = f_{\text{regge}} + f_{\text{res}} \tag{1.2}
\]

- \(f\) - scattering amplitude at intermediate or low energy
- \(f_{\text{regge}}\) - extrapolated Regge amplitude
- \(f_{\text{res}}\) - resonant amplitude.
Using Finite Energy Sum Rules, Dolen, Horn and Schmid (19) concluded that:

\[ f = f_{\text{regge}} + f_{\text{res}} - \langle f_{\text{res}} \rangle \]  

where \( \langle f_{\text{res}} \rangle \) is the locally averaged resonant amplitude.

If \( \langle f_{\text{res}} \rangle \sim 0 \) due to resonances entering with alternating sign and comparable strength then equations (1.2) and (1.3) would be equivalent. Whereas, if \( \langle f_{\text{res}} \rangle \sim f_{\text{res}} \) then:

\[ f \sim f_{\text{regge}} \]  

So any structure, which could be interpreted as resonances, would have to be included, in some form, in the Regge amplitude. Hence, Regge background subtraction, required by the Generalised Interference Model, would be equivalent to subtracting off resonant contributions as well.

Schmid went further, producing resonance properties by analysing a purely Regge amplitude into partial waves, in the intermediate region (20). Because of this, he suggested an equivalence between t-channel Regge poles and s-channel resonances. This equivalence was called "Duality" and, if true, would confirm equation (1.4). However there has been some scepticism of Schmid's technique. The resonant behaviour resulted from the phase of the Regge signature factor and the resonance parameters depended crucially on what assumptions were made about the Regge residue terms and other arbitrary assumptions.

Despite this, duality has become an established, though rather limited, concept. Whether or not the Generalised Interference Model involves serious "double counting" has not yet been resolved, most of the controversy resulting from
The latitude available in dividing an amplitude into resonant
and background parts.

However Reeder and Sarma\(^{21}\) successfully used the
Generalised Interference Model to fit the hypercharge
exchange reactions:

a) \(\pi^- p \rightarrow K^0 \Lambda\) down to beam momentum 2.6 GeV/c

b) \(\pi^+ p \rightarrow K^+ \Sigma^+\) at 3.23 GeV/c

c) \(K^- p \rightarrow \pi^0 \Lambda\) down to 4.1 GeV/c.

Their method has been used in attempting to fit the
\(\Lambda\pi^-\) channel data (see Chapter 6), and this should provide
a reasonably rigorous test of the theory, as these data have
already been well fitted and the parameters of the resonances
determined. By extending the method well into the low
energy region (beam momenta 1.45 and 1.65 GeV/c) there was
a substantial possibility of it failing, but, if successful,
then the background would be described by a few Regge para-
eters, some of which were well known, rather than a large
collection of arbitrary parameters. Further, the fewer
background parameters there were the more sensitive the
final fit should be.
REFERENCES FOR CHAPTER 1

CHAPTER 2

2.1 Introduction

The experiment was run at the Rutherford High Energy Laboratory (RHEL). Approximately 720,000 pictures, each comprising three photographs of the same expansion taken from different angles, were taken in the Saclay 80 cm Bubble Chamber filled with liquid deuterium. The incident $K^-$ mesons were of two momenta, 1.45 GeV/c and 1.65 GeV/c.

Due to the Fermi motion of the nucleons in the deuteron, the $K^-$ centre of mass energy spectrum ran from 1850 MeV to 2150 MeV (Fig. 2.1). The beam used was the $K^-$ separated beam, designed by A.M. Segar, which gave a momentum resolution of $\pm 1^\circ/c$.

The first and second runs were carried out in 1966. 240,000 pictures were taken at 1.65 GeV/c beam momentum and the film equally divided between the laboratories at Glasgow University, Birmingham University and Imperial College, London.

The third run, in 1967, comprised 105,000 pictures at 1.65 GeV/c and 375,000 pictures at 1.45 GeV/c, which were divided between the three laboratories named above and also Edinburgh University.

Each laboratory scanned and processed its own film but final data were available to all. A flow diagram for the Edinburgh scheme of data preparation is shown in Fig. 2.2.
Fig. 2.1  A typical centre of mass energy distribution of events.
Fig. 2.2  Flow diagram of data preparation.
Fig. 2.3 Scanning rectangle superimposed on the bubble chamber as seen on view 2 (the central view).

Fig. 2.4 Scanning regions defined within the scanning rectangle.
2.2 Scanning

The 1.65 GeV/c film was scanned twice for all topologies except one-prong or two-prong events with no associated decay.

To define a scanning region the rectangle, whose dimensions were the length and breadth of the back window of the chamber, as seen on view 2, the central view (Fig. 2.3), was divided by a grid, 10 units x 10 units. A rectangle was defined, within which the interaction vertex had to lie to be recorded, and any decay had to be within a larger rectangle (Fig. 2.4). The whole volume of the chamber was not used as some regions did not appear on all three views and also illumination was poor near the edges of the chamber.

The scanners recorded on sheets the following information for each scanned event.

a) The number of the roll of film.
b) The frame number of the picture.
c) The event number.
d) The topology of the event.

Events occurring on the same frame were numbered 1, 2, 3 etc. as they appeared from the bottom to top of the frame.

Using the CERN convention, the topology was described by the three digits, ABC, where A was the number of charged tracks (prongs) leaving the interaction, B was the number of charged particle decays from the interaction (these appeared as a kink on a track, as the neutral decay product would not leave a track) and C the number of uncharged particles decaying into two charged particles. This type of decay was called a $V_0$, as it appeared V-shaped on the film.

If a positive track could be identified as a spectator proton (the interaction involving the neutron only and leaving the proton with its Fermi energy) then it was not counted as a prong, instead the topology number was prefixed by the
letter 'S'. The spectator track had to be unbroken, projected length less than 15 cm and stopping within the chamber.

To be recorded an event had to occur on a beam track. For a $V_0$ to be recorded, the V-shape had to point back to within 2 cm of an interaction. So a $V_0$ could be associated with more than one interaction. To identify the cases where this occurred the letters 'CVO' (Common $V_0$) were noted in the comments column.

Electrons were readily identified as they tended to spiral in the chamber's magnetic field. Any track which turned through $180^\circ$ in the course of its flight in the chamber was deemed to be an electron and therefore not recorded.

In the interaction, electrons occurred as Dalitz Pairs (an electron and positron appearing to leave the interaction vertex) from the decay of the zero charged pion

$$\pi^0 \rightarrow \gamma e^+ e^-$$

or even

$$\pi^0 \rightarrow e^+ e^- e^+ e^- .$$

Charged pions often decayed in a chain reaction

$$\pi^- \rightarrow \mu^- \rightarrow e^- \rightarrow e^- \rightarrow \mu^- \rightarrow e^- \rightarrow \gamma e^+ e^-$$

For the pion (lifetime $2.8 \times 10^{-8}$ sec) to decay within the chamber, it must have virtually stopped. The $\pi^-$ will almost certainly have interacted before it could decay, so only the positive decay will be observed. The intermediate
\(\mu\)-track had to be small (approximately 1 cm) as only 30 MeV/c momentum was imparted to it from the pion decay, which occurred, as stated above, when the pion was effectively at rest.

This criterion was sufficient to distinguish \(\pi - \mu - e\) from the \(K - \mu\) decay, which was being recorded, for two reasons. Firstly only the \(K^-\) would occur commonly (the \(K^+\) could only be produced with one or more other strange particles) and secondly the 236 MeV/c momentum imparted to the muon would normally take it outside the chamber (certainly much further than 1 cm) before it decayed.

Finally \(\gamma\) conversion to a positron-electron pair occurred, resulting in a \(V_0\) with zero degree opening angle.

In all these cases the electron tracks were ignored, as was the \(\pi - \mu\) decay, though their presence was noted in the comments column.

Also scanned for were tau decays of the \(K^-\) mesons in the beam.

\[
K^- \rightarrow \pi^- \pi^- \pi^+ .
\]

These resulted in a 300 topology with all 3 tracks of minimum bubble density and maximum opening angle of 20°. Such events were recorded as 300 topologies but 'TAU' was inserted in the comments column.

e) The approximate coordinates of the interaction vertex.

This was recorded by a letter (displacement along the chamber) and a number (displacement across the chamber). (Fig. 2.4).

f) The approximate coordinates of kinks.

g) The approximate coordinates of \(V_0\)'s.

h) Comments.
Fig. 2.5  A typical Scan Card.
Already 'CVO', 'PME', 'Dalitz Pair', 'Electron pair' and 'TAU' have been mentioned. Also inserted were comments to help a third scanner or measurer and warnings of a scatter (an outgoing track striking another deuteron).

The 1.45 GeV/c was scanned in similar fashion but for neutron events only. By conservation of charge a neutron event had to have an odd number of prongs with possibly a spectator proton (criterion of identification unchanged). This saved considerable processing time with the loss of very few genuine neutron events. The 15 cm cut-off on the spectator length would correspond to an initial Fermi momentum of at least 300 MeV/c. This resulted in very few genuine neutron events being excluded (see Chapter 3, Section 6).

Initial scans were done independently by two teams of two paid scanners. The results were compared and events recorded in both scans were accepted. The film was scanned a third time by a physicist to settle differences.

The selected events were recorded on scan cards (Fig. 2.5). This information was logged on a magnetic tape (called a Scan Master) using an UPDATE program. The input tape for the measuring sequence was prepared from this tape.

2.3 Measuring

The measuring was carried out at Glasgow University using the SMP/7044 installation. This stage was done entirely by operators in Glasgow, who extracted the scan information from the Scan Master, measured the film on S.M.P's, using the IBM 7044 computer on-line for data collection, and ran
a pre-geometry program producing geometry input tapes.

2.4 Geometric and Kinematic Fitting

The events were processed using the RHEL geometrical reconstruction program, GEOM, and the RHEL kinematic fitting program (deuterium version) DTKIN. All events which failed to pass GEOM, for any reason, were remeasured but there was no kinematic remeasure.

The Geometry program produced a library tape of geometric information which served as an input for kinematics. Also produced was a listing which served to identify events for remeasure.

The kinematics program tested the successfully measured events against 93 hypotheses. These hypotheses included all the most common neutron events for the topologies scanned for, some of the most common proton events, a large number of scatter hypotheses to eliminate spurious kinks and three hypotheses with pion beam track.

As well as producing a library tape of fitting information for both successful and unsuccessful fits, the program also produced a detailed listing of the fitting.

A separate DTKIN run was done on all 300 topologies to test for tau decays.

2.5 Choosing

A fourth (Bubble Density) scan was done, again by physicists, on all kinematically fitted events. For each track in each fitted hypothesis the quantity \( \frac{1}{\beta^2 \cos \phi} \) was
calculated by the kinematics program where $\phi$ was the dip angle of the track, the angle between the track and the front window of the chamber. As $\frac{1}{\beta^2}$ is directly proportional to the number of bubbles produced per unit length along the track, then this quantity $\frac{1}{\beta^2 \cos \phi}$ would be a measure of bubble density as it appeared on the scanning table.

Although the scale ran from 1 to 40, only 1 to 5 could be distinguished, thereafter the track appeared as a solid black line. At best visual resolution of bubble densities was about $30^\circ/0$, requiring about 20 cm of track which was well illuminated and reasonably free of crossing tracks. Normally the resolution was nearer $50^\circ/0$ though this was sufficient to distinguish protons from pions except at momenta over 1200 MeV/c. On the other hand, pions and kaons could only be distinguished up to about 600 MeV/c. Still this did provide a useful criterion for selecting a fit or fits from a number of possible alternatives.

Therefore for a fit to be accepted the bubble densities of all the tracks had to be consistent with the calculated bubble densities of the fit.

In general when a fit was made momentum balance in three orthogonal directions and also energy balance was checked. This gave a so-called four constraint (4C) fit. If a track was so short that its curvature (hence magnitude of momentum) could not be measured then one of the constraint equations would be required to calculate this quantity, leaving a three constraint check on the fit - a 3C fit.

If there was a neutral particle which did not decay into a $V_0$, then three constraints would be used to calculate its
momentum leaving only energy balance as a test of the fit — a 1C fit.

The conditions for a fit to be accepted were as follows.

(a) Bubble densities consistent (see above).

(b) Confidence level
    >50% for a 1C interaction vertex fit
    >100% for a 4C or 3C interaction vertex fit.

The kinematics program already included a 10% confidence level cut-off for the overall fit.

(c) 4C or 3C fits were accepted in preference to 1C fits except for the so-called \( \Lambda - \Sigma \) ambiguity.

The two cases of this were:

\[
K^- n \rightarrow \Lambda \pi^- \rightarrow p\pi^- \quad \text{a 4C fit}
\]

ambiguous with

\[
K^- n \rightarrow \Sigma^0 \pi^- \rightarrow \gamma \Lambda \rightarrow p\pi^- \quad \text{a 1C fit}
\]

and

\[
K^- n \rightarrow \Lambda \pi^+ \pi^- \pi^- \rightarrow p\pi^- \quad \text{a 4C fit}
\]

ambiguous with

\[
K^- n \rightarrow \Sigma^0 \pi^- \pi^- \pi^+ \rightarrow \gamma \Lambda \rightarrow p\pi^- \quad \text{a 1C fit}
\]

where these ambiguities occurred and there was no other way of distinguishing them both were accepted.

d) 0C fits were discarded.

e) \( \pi \)-fits were discarded.

f) If the same hypothesis was fitted more than once, the most likely fit was selected.
Fig. 2.6 A typical Choice Card.
For each event selected a choice card was punched containing details of the selected fit or fits (Fig. 2.6). These cards together with the Kinematics Library tape were used as input for the 1NCO program which extracted information about the selected fit, summarised it and wrote it up on a Data Summary Tape (D.S.T.).

D.S.T.'s were prepared for 35 of the most common hypotheses, for all rare fits (fits which included production of more than one strange particle) and for tau decays.
3.1 Introduction

From analysis of the Edinburgh film, Data Summary Tapes were prepared containing over 5000 events fitting twenty seven $K$-neutron hypotheses at 1.45 GeV/c beam momentum, about 3500 events fitting $K$-neutron or $K$-proton hypotheses at 1.65 GeV/c (Table 3.1) and 79 events at either beam momentum fitting rare hypotheses (Table 3.2). To convert the raw numbers of events to cross sections various calculations and corrections had to be made. Roughly grouped, they were:-

a) Correction for random loss of events in processing.
b) Correction for systematic losses of events.
c) Estimation of pion beam contamination and allowance for its effect.
d) Assignment of ambiguously fitted events.
e) Corrections for effects due to there being two nucleons in the deuteron.
f) Estimation of the total track length of the beam.

3.2 Processing losses

It was assumed that events lost in scanning, measuring or fitting stages were lost randomly, due to essentially human factors (for example, events not being noticed at scanning or events being faultily or imprecisely measured).

a) Scanning

The two independent scans together with a third scan to assess discrepancies gave three numbers for each topology.

$N_1$ - No. of accepted events seen by Scan 1 only.
$N_2$ - No. of accepted events seen by Scan 2 only.
$N_{12}$ - No of accepted events seen by both scans.
<table>
<thead>
<tr>
<th>Channel</th>
<th>Number of Events</th>
<th>1.45 GeV/c Film</th>
<th>1.65 GeV/c Film</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^-n \rightarrow \pi^- n K^0 \rightarrow \pi^+ n$</td>
<td>464</td>
<td>271</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda^0 \rightarrow \rho n$</td>
<td>709</td>
<td>255</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \pi^- n^0 \Lambda^0 \rightarrow \rho n$</td>
<td>1075</td>
<td>448</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Sigma^0 \rightarrow \gamma \Lambda^0 \rightarrow \rho n$</td>
<td>516</td>
<td>179</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- p \bar{K}^0 \rightarrow \pi^- n$</td>
<td>3</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- p \bar{p} \Lambda^0 \rightarrow \rho n$</td>
<td>111</td>
<td>93</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- p \bar{p} \Lambda^0 \rightarrow \rho n$</td>
<td>11</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- p \bar{p} \Sigma^0 \rightarrow \gamma \Lambda^0 \rightarrow \rho n$</td>
<td>45</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow n K^- \rightarrow \pi^- n^0$</td>
<td>146</td>
<td>91</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow n K^- \rightarrow \mu^- \nu$</td>
<td>97</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \pi^\circ \Sigma^- \rightarrow \pi^- n$</td>
<td>202</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \rho \pi^- K^- \rightarrow \mu^- \nu$</td>
<td>28</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \rho \pi^- K^- \rightarrow \nu^- n^0$</td>
<td>14</td>
<td>13</td>
<td></td>
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<td>$\rightarrow \rho \pi^- K^- \rightarrow \mu^- \nu$</td>
<td>38</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \rho \pi^- K^- \rightarrow \nu^- n^0$</td>
<td>62</td>
<td>42</td>
<td></td>
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<tr>
<td>$\rightarrow \rho \pi^- \Sigma^- \rightarrow \pi^- n$</td>
<td>281</td>
<td>127</td>
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<tr>
<td>$\rightarrow \rho \pi^- \Sigma^- \rightarrow \pi^- n$</td>
<td>87</td>
<td>61</td>
<td></td>
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<td>$\rightarrow \rho \pi^- \Sigma^- \rightarrow \rho n^0$</td>
<td>89</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \rho \pi^- \Sigma^- \rightarrow \rho n^0$</td>
<td>138</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \rho \pi^- \Sigma^- \rightarrow \rho n^0$</td>
<td>10</td>
<td>10</td>
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</tr>
<tr>
<td>$\rightarrow \rho \pi^- \Sigma^- \rightarrow \rho n^0$</td>
<td>25</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Channel</td>
<td>1.45 GeV/c Film</td>
<td>1.65 GeV/c Film</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-----------------</td>
<td>-----------------</td>
<td></td>
</tr>
<tr>
<td>$\bar{K}n \rightarrow \rho \bar{K}n$</td>
<td>669</td>
<td>291</td>
<td></td>
</tr>
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<td>$\rightarrow \rho \bar{K}n$</td>
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<td>50</td>
<td></td>
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<td>$\rightarrow \rho \bar{K}n$</td>
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<td>22</td>
<td></td>
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<td>141</td>
<td></td>
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<td>$\rightarrow \rho \bar{K}n$</td>
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<td>151</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \rho \bar{K}n$</td>
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<td>79</td>
<td></td>
</tr>
<tr>
<td>$K^\pi$</td>
<td>752</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \rho \bar{K}n$ (Tau Decay)</td>
<td>307</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{K}p$</td>
<td>$\rightarrow \rho \bar{K}n$</td>
<td>4</td>
<td></td>
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<tr>
<td>$\rightarrow \rho \bar{K}n$</td>
<td>83</td>
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<td></td>
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<tr>
<td>$\rightarrow \rho \bar{K}n$</td>
<td>49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \rho \bar{K}n$</td>
<td>85</td>
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<td></td>
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<tr>
<td>$\rightarrow \rho \bar{K}n$</td>
<td>12</td>
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<tr>
<td>$\rightarrow \rho \bar{K}n$</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \rho \bar{K}n$</td>
<td>13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## TABLE 3.2

Numbers of events fitting rare hypotheses

<table>
<thead>
<tr>
<th>Channel</th>
<th>Number of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^- n \rightarrow K^0 \pi^+ \rightarrow \pi^- \Lambda^0$</td>
<td>16</td>
</tr>
<tr>
<td>$\rightarrow K^0 \pi^+ \rightarrow 2\pi^- \Lambda^0$</td>
<td>16</td>
</tr>
<tr>
<td>$\rightarrow K^0 \pi^+ \rightarrow \Lambda^0 \rightarrow \rho \pi^-$</td>
<td>24</td>
</tr>
<tr>
<td>$\rightarrow K^0 \eta^0 \pi^+ \rightarrow \pi^- \Lambda^0$</td>
<td>2</td>
</tr>
<tr>
<td>$\rightarrow K^0 \pi^+ \pi^- \rightarrow \Lambda^0 \rightarrow \rho \pi^-$</td>
<td>8</td>
</tr>
<tr>
<td>$\rightarrow K^0 \eta^0 \pi^+ \rightarrow \pi^- \Lambda^0$</td>
<td>1</td>
</tr>
<tr>
<td>$\rightarrow K^0 \pi^+ \pi^- \rightarrow \Lambda^0 \rightarrow \rho \pi^-$</td>
<td>2</td>
</tr>
<tr>
<td>$K^- \rho \rightarrow K^0 \pi^+ \pi^- \rightarrow \pi^- \Lambda^0$</td>
<td>10</td>
</tr>
</tbody>
</table>
The possibility of misidentification of topology was assumed to be very slight, as its nature had to be agreed upon by two pairs of scanners independently or else, by one pair of scanners and the third scanner, the latter making a selection aware of other interpretations.

If $N$ was the actual number of events of one topology present then:

Efficiency of Scan 1 = $E_1 = \frac{N_1 + N_{12}}{N}$

Efficiency of Scan 2 = $E_2 = \frac{N_2 + N_{12}}{N}$

Overall efficiency = $E = \frac{N_1 + N_2 + N_{12}}{N}$.

Also if loss of events is random:

$E_1 \times E_2 = \frac{N_{12}}{N}$.

So

$E_1 = \frac{N_{12}}{N_2 + N_{12}} \times 100^\circ/\circ$

$E_2 = \frac{N_{12}}{N_1 + N_{12}} \times 100^\circ/\circ$

$E = \frac{N_1 + N_2 + N_{12}}{N_1 + N_2 + N_{12} + N_3} \times 100^\circ/\circ$

where $N_3 = \frac{N_1 N_2}{N_{12}}$.

At scanning stage, the very short spectator tracks were often missed (e.g. an S101 topology being recorded as a 101 topology). Also with long spectator tracks there was the possibility of it being recorded as another prong (e.g. an S101 topology being recorded as a 201 topology). However, no distinction was made at kinematic fitting stage and the same hypotheses were tested, no matter whether the event was recorded, for example, as a 101,
TABLE 3.3
Scanning Efficiencies for 1.45 GeV/c Film

<table>
<thead>
<tr>
<th>Topology Group</th>
<th>$N_{12}$</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$E_1(^{\circ}/o)$</th>
<th>$E_2(^{\circ}/o)$</th>
<th>$E(^{\circ}/o)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>101, S101</td>
<td>2062</td>
<td>260</td>
<td>252</td>
<td>89.1</td>
<td>88.8</td>
<td>98.8</td>
</tr>
<tr>
<td>110, S110</td>
<td>1239</td>
<td>238</td>
<td>230</td>
<td>84.3</td>
<td>83.9</td>
<td>97.5</td>
</tr>
<tr>
<td>300, S300</td>
<td>909</td>
<td>132</td>
<td>127</td>
<td>87.7</td>
<td>87.3</td>
<td>98.4</td>
</tr>
<tr>
<td>301, S301</td>
<td>81</td>
<td>19</td>
<td>23</td>
<td>77.9</td>
<td>81.0</td>
<td>95.8</td>
</tr>
<tr>
<td>310, S310</td>
<td>544</td>
<td>100</td>
<td>116</td>
<td>82.7</td>
<td>84.7</td>
<td>97.3</td>
</tr>
</tbody>
</table>

N.B. Only every third roll of film was used to produce the data for this table.

TABLE 3.4
Scanning Efficiencies for 1.65 GeV/c Film

<table>
<thead>
<tr>
<th>Topology Group</th>
<th>$N_{12}$</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$E_1(^{\circ}/o)$</th>
<th>$E_2(^{\circ}/o)$</th>
<th>$E(^{\circ}/o)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>101, S101, 201</td>
<td>2818</td>
<td>508</td>
<td>204</td>
<td>93.3</td>
<td>84.7</td>
<td>99.0</td>
</tr>
<tr>
<td>110, S110, 210</td>
<td>1087</td>
<td>288</td>
<td>110</td>
<td>90.8</td>
<td>79.1</td>
<td>98.1</td>
</tr>
<tr>
<td>300, S300, 400</td>
<td>1234</td>
<td>174</td>
<td>83</td>
<td>93.6</td>
<td>87.6</td>
<td>99.2</td>
</tr>
<tr>
<td>301, S301, 401</td>
<td>180</td>
<td>43</td>
<td>10</td>
<td>94.7</td>
<td>80.7</td>
<td>99.0</td>
</tr>
<tr>
<td>310, S310, 410</td>
<td>622</td>
<td>180</td>
<td>91</td>
<td>87.2</td>
<td>77.6</td>
<td>97.1</td>
</tr>
</tbody>
</table>

$N_{12}$ - Number of events recorded by both scans.
$N_1$ - Number of events recorded by Scan 1 only.
$N_2$ - Number of events recorded by Scan 2 only.
$E_1$ - Efficiency of Scan 1.
$E_2$ - Efficiency of Scan 2.
$E$ - Overall Scanning Efficiency.
TABLE 3.5
Errors in numbers of events corrected for scan efficiencies and errors in calculated scan efficiencies for 1.115 GeV/c film.

<table>
<thead>
<tr>
<th>Topology Group</th>
<th>N</th>
<th>ΔN</th>
<th>E (°/o)</th>
<th>ΔE (°/o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101, S101</td>
<td>2606</td>
<td>6.3</td>
<td>98.8</td>
<td>0.25</td>
</tr>
<tr>
<td>110, S110</td>
<td>1751</td>
<td>7.6</td>
<td>97.5</td>
<td>0.4</td>
</tr>
<tr>
<td>300, S300</td>
<td>1186</td>
<td>4.6</td>
<td>98.4</td>
<td>0.4</td>
</tr>
<tr>
<td>301, S301</td>
<td>128</td>
<td>2.9</td>
<td>95.8</td>
<td>2.2</td>
</tr>
<tr>
<td>310, S310</td>
<td>781</td>
<td>5.5</td>
<td>97.3</td>
<td>0.7</td>
</tr>
</tbody>
</table>

TABLE 3.6
Errors in the numbers of events corrected for scan efficiencies and errors in calculated scan efficiencies for 1.65 GeV/c film

<table>
<thead>
<tr>
<th>Topology Group</th>
<th>N</th>
<th>ΔN</th>
<th>E (°/o)</th>
<th>ΔE (°/o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101, S101, 201</td>
<td>3567</td>
<td>6.8</td>
<td>99.0</td>
<td>0.2</td>
</tr>
<tr>
<td>110, S110, 210</td>
<td>1514</td>
<td>6.4</td>
<td>98.1</td>
<td>0.4</td>
</tr>
<tr>
<td>300, S300, 400</td>
<td>1503</td>
<td>3.8</td>
<td>99.2</td>
<td>0.25</td>
</tr>
<tr>
<td>301, S301, 401</td>
<td>235</td>
<td>1.8</td>
<td>99.0</td>
<td>0.75</td>
</tr>
<tr>
<td>310, S310, 410</td>
<td>919</td>
<td>6.2</td>
<td>97.1</td>
<td>0.65</td>
</tr>
</tbody>
</table>

N - Number of events corrected for scan efficiency.
ΔN - Error in N.
E - Overall Scanning Efficiency.
ΔE - Error in E.
S101 or 201 topology. So in calculating scan efficiencies no distinction was made between these different topologies. The calculated efficiencies for these topology groups are shown in Tables 3.3 and 3.4.

A more rigorous derivation of the above efficiency formulae is given in Appendix 1. This treatment results in an expression for the uncertainty, $\Delta N$, in the calculated total number, $N$.

$$E = \frac{N_1 + N_2 + N_{12}}{N}$$

$$|\Delta E| = \frac{N_1 + N_2 + N_{12}}{N^2} |\Delta N|$$

for $\Delta N$ small compared with $N$.

These errors are given in Tables 3.5 and 3.6.

b) Geometry

In calculating the geometry efficiency, it was assumed that there was a certain probability that any event would be faultily measured resulting in the failure of geometrical reconstruction. There could be many reasons for this (not enough fiducials measured, tracks measured in the wrong order on one of the three views, interruptions etc.). All, it was hoped, random occurrences. It was also possible that certain configurations (short decays, small angle decays etc.) posed particular problems for the measurer resulting in a greater failure rate, but, as these losses would be systematic, allowance can be made for them (see next section). Finally it was assumed that there were some "bad" events which would
### TABLE 3.7

Geometrical Efficiencies for 1.45 GeV/c Film

<table>
<thead>
<tr>
<th>Topology Group</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
<th>$N_4$</th>
<th>$E_1$ ($^\circ$/o)</th>
<th>$E_2$ ($^\circ$/o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101, S101</td>
<td>3540</td>
<td>637</td>
<td>225</td>
<td>139</td>
<td>82.1</td>
<td>96.8</td>
</tr>
<tr>
<td>110, 3110</td>
<td>887</td>
<td>168</td>
<td>65</td>
<td>39</td>
<td>81.2</td>
<td>96.4</td>
</tr>
<tr>
<td>300, S300</td>
<td>2462</td>
<td>502</td>
<td>236</td>
<td>129</td>
<td>79.6</td>
<td>95.8</td>
</tr>
<tr>
<td>301, S301</td>
<td>237</td>
<td>58</td>
<td>35</td>
<td>19</td>
<td>75.4</td>
<td>94.0</td>
</tr>
<tr>
<td>310, S310</td>
<td>1317</td>
<td>250</td>
<td>114</td>
<td>59</td>
<td>80.4</td>
<td>96.4</td>
</tr>
</tbody>
</table>

### TABLE 3.8

Geometrical Efficiencies for 1.65 GeV/c Film

<table>
<thead>
<tr>
<th>Topology Group</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$N_3$</th>
<th>$N_4$</th>
<th>$E_1$ ($^\circ$/o)</th>
<th>$E$ ($^\circ$/o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101, S101, 201</td>
<td>3035</td>
<td>403</td>
<td>149</td>
<td>62</td>
<td>86.7</td>
<td>98.2</td>
</tr>
<tr>
<td>110, 3110, 210</td>
<td>1373</td>
<td>198</td>
<td>66</td>
<td>33</td>
<td>85.7</td>
<td>97.8</td>
</tr>
<tr>
<td>300, S300, 400</td>
<td>1296</td>
<td>195</td>
<td>111</td>
<td>35</td>
<td>84.8</td>
<td>97.7</td>
</tr>
<tr>
<td>301, S301, 401</td>
<td>222</td>
<td>39</td>
<td>15</td>
<td>8</td>
<td>83.0</td>
<td>96.9</td>
</tr>
<tr>
<td>310, S310, 410</td>
<td>772</td>
<td>144</td>
<td>73</td>
<td>33</td>
<td>81.4</td>
<td>96.5</td>
</tr>
</tbody>
</table>

$N_1$ - Number of events passing Geometry on first measure.

$N_2$ - Number of events passing Geometry on second measure.

$N_3$ - Number of events failing Geometry on both measures.

$N_4$ - Number of events failing Geometry on both measures due to random causes.

$E_1$ - Single measure efficiency.

$E$ - Overall Geometrical Efficiency.
### TABLE 3.9

Errors in numbers of events corrected for geometry efficiencies and errors in calculated geometry efficiencies for 1.45 GeV/c film

<table>
<thead>
<tr>
<th>Topology Group</th>
<th>$N$</th>
<th>$\Delta N$</th>
<th>$E$ (°/o)</th>
<th>$\Delta E$ (°/o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101, S101</td>
<td>4316</td>
<td>17.2</td>
<td>96.8</td>
<td>0.4</td>
</tr>
<tr>
<td>110, S110</td>
<td>1094</td>
<td>9.4</td>
<td>96.4</td>
<td>0.8</td>
</tr>
<tr>
<td>300, S300</td>
<td>3093</td>
<td>17.6</td>
<td>95.8</td>
<td>0.55</td>
</tr>
<tr>
<td>301, S301</td>
<td>314</td>
<td>7.4</td>
<td>94.0</td>
<td>2.2</td>
</tr>
<tr>
<td>310, S310</td>
<td>1626</td>
<td>11.6</td>
<td>96.4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

### TABLE 3.10

Errors in numbers of events corrected for geometry efficiencies and errors in calculated geometry efficiencies for 1.65 GeV/c film

<table>
<thead>
<tr>
<th>Topology Group</th>
<th>$N$</th>
<th>$\Delta N$</th>
<th>$E$ (°/o)</th>
<th>$\Delta E$ (°/o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101, S101, 201</td>
<td>3500</td>
<td>10.4</td>
<td>98.2</td>
<td>0.3</td>
</tr>
<tr>
<td>110, S110, 210</td>
<td>1604</td>
<td>8.0</td>
<td>97.8</td>
<td>0.5</td>
</tr>
<tr>
<td>300, S300, 400</td>
<td>1524</td>
<td>8.1</td>
<td>97.7</td>
<td>0.5</td>
</tr>
<tr>
<td>301, S301, 401</td>
<td>269</td>
<td>4.1</td>
<td>96.9</td>
<td>1.5</td>
</tr>
<tr>
<td>310, S310, 410</td>
<td>949</td>
<td>8.5</td>
<td>96.5</td>
<td>0.9</td>
</tr>
</tbody>
</table>

$N$ - Number of events, corrected for Geometrical Efficiency.

$\Delta N$ - Error in $N$.

$E$ - Overall Geometrical Efficiency.

$\Delta E$ - Error in $E$. 
fail no matter how often or how carefully they were measured.

Again there were three numbers for each topology.

$N_1$ - No. of events passing on first measure.

$N_2$ - No. of events passing on second measure.

$N_3$ - No. of events failing both measures.

$N_3$ will comprise events which have failed both measures due to random losses ($N_{4i}$), systematic losses which will be allowed for later and "bad" events.

If $E_1$ is the efficiency of a single measure then

$$E_1 = \frac{N_1}{N_1 + N_2 + N_4} \times 100\%$$

and $E_2 = \frac{N_2}{N_2 + N_4} \times 100\%$

Therefore $N_4 = \frac{N_2^2}{N_1 - N_2}$.

Overall efficiency $E = \frac{N_1 + N_2}{N_1 + N_2 + N_4} = (1 - \frac{N_2^2}{N_1}) \times 100\%$.

The calculated efficiencies for the topology groups are shown in Tables 3.7 and 3.8.

A more rigorous derivation is given in Appendix 2 and again the uncertainty in the corrected number was obtained.

So if $E = \frac{N_1 + N_2}{N}$

then $|\Delta E| = \frac{N_1 + N_2}{N^2} |\Delta N|$

for $\Delta N$ small compared with $N$.

These errors are given in Tables 3.9 and 3.10.
Table 3.11 Kinematic Fitting Efficiencies and Processing Correction Factors
For 1.45 GeV/c Film.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Kin. Fitting Efficiency</th>
<th>Processing Correction Factor</th>
<th>Channel</th>
<th>Kin. Fitting Efficiency</th>
<th>Processing Correction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{K}n \rightarrow \pi^- n$ $\bar{K} \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\bar{K}n \rightarrow \pi^- n$ $\bar{K} \rightarrow \pi^+ \pi^0$</td>
<td>72</td>
<td>1.48</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda$ $\rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^+ \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.55</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^0 \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Sigma^0 \rightarrow \pi^- \pi^+$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Sigma^0 \rightarrow \pi^- \pi^+$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow \pi^+ \pi^-$</td>
<td>71</td>
<td>1.48</td>
<td>$\rightarrow \pi^- \pi^- \Xi^- \rightarrow \pi^- \pi^0$</td>
<td>68</td>
<td>1.57</td>
</tr>
</tbody>
</table>
Table 3.12  Kinematic Fitting Efficiencies and Processing Correction Factors for 1.65 GeV/c Film.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Kin. Fitting Efficiency (%)</th>
<th>Processing Correction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^- n \rightarrow \pi^- n K^0 \rightarrow \pi^+ n^- \rightarrow \pi^- n^- $</td>
<td>71</td>
<td>1.45</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Lambda \rightarrow p \pi^- $</td>
<td>71</td>
<td>1.45</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^0 \Lambda \rightarrow p \pi^- $</td>
<td>71</td>
<td>1.45</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \Sigma^0 \rightarrow \gamma \Lambda \rightarrow p \pi^- $</td>
<td>71</td>
<td>1.45</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Lambda \rightarrow K^0 \rightarrow \pi^+ n^- $</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Lambda \rightarrow p \pi^- $</td>
<td>73</td>
<td>1.56</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Lambda \rightarrow \pi^0 \Lambda \rightarrow p \pi^- $</td>
<td>67</td>
<td>1.56</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Lambda \rightarrow \Sigma^0 \rightarrow \gamma \Lambda \rightarrow p \pi^- $</td>
<td>67</td>
<td>1.56</td>
</tr>
<tr>
<td>$\rightarrow n K^- \rightarrow \pi^- n^0 $</td>
<td>73</td>
<td>1.43</td>
</tr>
<tr>
<td>$\rightarrow n K^- \rightarrow \mu^- \nu $</td>
<td>73</td>
<td>1.43</td>
</tr>
<tr>
<td>$\rightarrow \pi^0 \Sigma^- \rightarrow \pi^- n $</td>
<td>73</td>
<td>1.43</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- K^- \rightarrow \mu^- \nu $</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- K^- \rightarrow \pi^- n^0 $</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- n K^- \rightarrow \mu^- \nu $</td>
<td>72</td>
<td>1.48</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- n K^- \rightarrow \pi^- n^0 $</td>
<td>72</td>
<td>1.48</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \Sigma^- \rightarrow \pi^- n $</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \Sigma^- \rightarrow \pi^- n $</td>
<td>72</td>
<td>1.48</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Sigma^+ \rightarrow p \pi^- $</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Sigma^+ \rightarrow \pi^- n $</td>
<td>68</td>
<td>1.57</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Sigma^+ \rightarrow \pi^- n $</td>
<td>72</td>
<td>1.48</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \pi^- \Sigma^+ \rightarrow \pi^- n $</td>
<td>72</td>
<td>1.48</td>
</tr>
<tr>
<td>Channel</td>
<td>Kin. Fitting Efficiency</td>
<td>Processing Correction Factor</td>
</tr>
<tr>
<td>---------</td>
<td>-------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>$K^0 \rightarrow \rho K^- \pi^+$</td>
<td>73</td>
<td>1.41</td>
</tr>
<tr>
<td>$\rightarrow \rho K^- \pi^+ \pi^0$</td>
<td>70</td>
<td>1.47</td>
</tr>
<tr>
<td>$\rightarrow \rho \pi^- K^0$</td>
<td>70</td>
<td>1.47</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ K^- \pi^- n$</td>
<td>70</td>
<td>1.47</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \pi^0 \Lambda$</td>
<td>70</td>
<td>1.47</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \pi^0 \Sigma$</td>
<td>70</td>
<td>1.47</td>
</tr>
<tr>
<td>$K^- \rightarrow \pi^+ \pi^- \pi^- \pi^0$ (Tau Decay)</td>
<td>76</td>
<td>1.36</td>
</tr>
<tr>
<td>$K^- \rho \rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^+ \rho^- \rho$</td>
<td>71</td>
<td>1.45</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^+ \rho^- \rho$</td>
<td>67</td>
<td>1.56</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^+ \rho^- \rho$</td>
<td>67</td>
<td>1.56</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^+ \rho^- \rho$</td>
<td>67</td>
<td>1.56</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^+ \rho^- \rho$</td>
<td>67</td>
<td>1.56</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^+ \rho^- \rho$</td>
<td>67</td>
<td>1.56</td>
</tr>
<tr>
<td>$\rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^+ \rho^- \rho$</td>
<td>67</td>
<td>1.56</td>
</tr>
</tbody>
</table>
c) Kinematics

As stated in Chapter 2 there was no kinematic remeasure (i.e. remeasure of events which were successfully reconstructed by the Geometry program but failed to be fitted to any hypothesis by the Kinematics program.) Therefore no direct estimate could be made of kinematic fitting efficiency. So the efficiencies calculated by the Glasgow group were adopted. This was not at all unreasonable as the Edinburgh film was measured and fitted on the same machines, by the same operators, using the same programs and at the same time as the Glasgow film. These efficiencies and the overall processing corrections are shown in Tables 3.11 and 3.12.

3.3 Systematic losses

Events were lost due to the finite volume of the bubble chamber or, more precisely, the restricted scanning region used. For both charged and uncharged particle decays there was the possibility that the decay would occur outside the scanning region. Also, for a decay occurring close to the interaction vertex, there was a possibility that it would not be noticed. Uncharged particles could decay by a neutral decay mode (e.g. $K^0 \rightarrow \pi^0\pi^0$), so the decay would not be seen. Charged particle decays could be in configurations which made identification or measurement difficult. Finally events were lost due to the probability cut on selected fits.

a) Fiducial Region

It can be seen from the distribution of interactions along the length of the chamber (Fig. 3.1), that the restricted scanning region results in approximately uniform occurrence of
x displacement of interaction vertex (cm.)

Fig. 3.1 A typical distribution of interaction vertices along the chamber.

y displacement of interaction vertex (cm.)

Fig. 3.2 A typical distribution of interaction vertices across the chamber.

z displacement of interaction vertex (cm.)

Fig. 3.3 A typical distribution of interaction vertices vertically in the chamber.

The Fiducial Limits are shown by the dashed lines.
**TABLE 3.13**

**Fiducial Limits**

<table>
<thead>
<tr>
<th></th>
<th>Interaction Fiducial Volume</th>
<th>Decay Fiducial Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X Coordinate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Limit</td>
<td>20 cm</td>
<td>31 cm</td>
</tr>
<tr>
<td>Lower Limit</td>
<td>- 20 cm</td>
<td>- 21 cm</td>
</tr>
<tr>
<td><strong>Y Coordinate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Limit</td>
<td>12 cm</td>
<td>17 cm</td>
</tr>
<tr>
<td>Lower Limit</td>
<td>- 12 cm</td>
<td>- 17 cm</td>
</tr>
<tr>
<td><strong>Z Coordinate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper Limit</td>
<td>32 cm</td>
<td>37 cm</td>
</tr>
<tr>
<td>Lower Limit</td>
<td>20 cm</td>
<td>15 cm</td>
</tr>
</tbody>
</table>
events along the chamber, tailing off sharply at either end. A fiducial region was chosen to include as much of the plateau as possible.

The lateral and vertical limits on the fiducial region were chosen sufficiently wide to include most of the interactions, (Figs. 3.2 and 3.3). These limits cannot be the dimensions of the chamber as it was necessary to have the decay fiducial volume larger than the interaction fiducial volume (see later in this section). The fiducial limits are shown in Table 3.13.

b) Uncharged particle decays

(i) Neutral decay modes

The $\Lambda$ has two main decay modes $^{8}$.  

$\Lambda \rightarrow p\pi^{-}$ \hspace{1cm} 64.0\% \pm 0.7 (seen)  

$\Lambda \rightarrow n\pi^{0}$ \hspace{1cm} 36.0\% \pm 0.7 (unseen).

Thus for every seen $\Lambda$ decay there must be a correction of $1.56 \pm 0.02$ to account for the unseen decay mode.

The $K_s^0$ also has two main decay modes $^{8}$.  

$K_s^0 \rightarrow \pi^+\pi^-$ \hspace{1cm} 68.7\% \pm 0.5 (seen)  

$K_s^0 \rightarrow \pi^0\pi^0$ \hspace{1cm} 31.3\% \pm 0.5 (unseen).

So a correction of $1.46 \pm 0.01$ should be applied, but as only 50\% of $\bar{K}^0$ particles decay via the $K_s^0$ channel, the correction factor should be doubled. None of the $K_L^0$ decay modes are tested for, as its decay length is about twenty times the chamber length at the momentum at which they are normally produced.

Final correction factors are:-
Fig. 3.4 Lifetime Distribution of $K^0_s$.
The straight line represents the ideal distribution.

Fig. 3.5 Lifetime Distribution of $\Lambda$.
The straight line represents the ideal distribution.
\[ \Lambda = 1.56 \pm 0.02 \]
\[ \bar{K}^0 = 2.92 \pm 0.02. \]

(ii) Lifetime weighting.

The lifetime distributions of observed \( \bar{K}^0 \) and \( \Lambda \) decays are shown in Figs. 3.4 and 3.5. As a logarithmic scale is used the graph should theoretically be linear, but there are obvious losses both for very long and very short lived particles.

The long lived losses can be attributed to the decay being outside the decay fiducial volume (Table 3.13). The short lived decays would occur very close to the interaction so that some 101 topologies would appear to be three-prong events and 301 topologies five-prong events.

If an interaction occurred in which the neutral decay particle went off with momentum \( p \) in the laboratory frame in such a direction that it would have to travel a distance \( \ell \) to reach the decay fiducial boundary, then the probability of its decaying outside the fiducial region would be

\[ \exp\left(\frac{mt}{pt}\right) \]

where \( m \) is the mass of the neutral particle and \( t \) its lifetime.

If \( c \) (the short decay cut-off) is taken to be the decay length (as projected onto the plane of the chamber windows) below which the detection efficiency of decays drops, then the probability of a particle decaying within this length \( c \) would be

\[ 1 - \exp\left(-\frac{mc}{pt \cos \phi}\right). \]

As \( c \) is the projected track length then for a track
Fig. 3.6 Weighted total of $\Lambda$ events for various values of the short decay cut-off.

Fig. 3.7 Weighted totals of $\bar{K}^0$ events for various values of the short decay cut-off.
**TABLE 3.14**

Lifetime Weightings for Uncharged Particle Decays

<table>
<thead>
<tr>
<th></th>
<th>1.45 GeV/c Film</th>
<th>1.65 GeV/c Film</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{K}^0$ Weighting</td>
<td>1.06</td>
<td>1.05</td>
</tr>
<tr>
<td>$\Lambda$ Weighting</td>
<td>1.08</td>
<td>1.08</td>
</tr>
</tbody>
</table>

**TABLE 3.15**

Lifetime Weightings for Charged Particle Decays

<table>
<thead>
<tr>
<th></th>
<th>1.45 GeV/c Film</th>
<th>1.65 GeV/c Film</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma^-$ Weighting</td>
<td>1.09</td>
<td>1.07</td>
</tr>
<tr>
<td>$\Sigma^+_p$ Weighting</td>
<td>1.38</td>
<td>1.33</td>
</tr>
<tr>
<td>$\Sigma^+_n$ Weighting</td>
<td>1.14</td>
<td>1.13</td>
</tr>
</tbody>
</table>
dipping at angle $\phi$ to the plane of the chamber window the actual length would be $c / \cos \phi$.

Thus the probability of a "seen" decay would be

$$\exp(-mc/pt \cos \phi) - \exp(-ml/pt).$$

So events were weighted by the factor:

$$W = \left[ \exp(-mc/pt \cos \phi) - \exp(-ml/pt) \right]^{-1}$$

and $W = 0$ for events with decay lengths less than $c/\cos \phi$.

It is worth noting that the larger decay fiducial volume ensures that $t$ was at least 10 cm in most cases, so that, in general, individual weights would be small (in the region 1.0 to 1.5).

To find the optimum value of $c$ (fewest events discarded, but correct prediction of events lost) its value was increased in steps until the weighted total reached a constant value (Figs. 3.6 and 3.7). For both $K^0$ and $\Lambda$ decays ($\Sigma^0$ decays were included in the $\Lambda$ total as the probability of losing a $\Lambda$ from a $\Sigma^0$ decay was assumed to be the same as the probability of losing a $\Lambda$ from an interaction) a cut-off of 0.3 cm was suggested. The weights obtained using this value of $c$ are shown in Table 3.11.

c) Charged particle decays

Only the kinks due to $\Sigma$ decays were considered. These were

$$\Sigma^- \to \pi^- n$$
$$\Sigma^+ \to p \pi^0 \quad (\Sigma^+_p)$$
$$\Sigma^+ \to n \pi^+ \quad (\Sigma^+_n).$$

The $K^-$ decays, $K^- \to \pi^- \pi^0$ and $K^- \to \mu^- \nu$, have a decay length which is large with respect to the chamber dimensions, so most decays occurred outside the chamber. Where
decays were detected, then events were simply added to those in which the decays were unseen.

(i) Lifetime weighting

The procedure described in the previous section was adopted. The cut-off values were found to be 0.3 cm for $\Sigma^-$ and $\Sigma^+_n$ and 0.8 cm for $\Sigma^+_p$. The correction factors are shown in Table 3.15.

(ii) Small projected angle

Charged particle decays were normally detected by observing kinks in tracks. However it was possible for there to be little apparent change of direction on the film, between the decaying particle's track and the track of the charged decay product. Although such decays could be detected (change in bubble density and change in curvature) it was found that the detection efficiency and possibly the measuring efficiency was reduced. There were two reasons for these small angle kinks; firstly the decay angle itself could be small, secondly the decay could occur in a plane normal to the film plane.

The small angle decays could be accounted for by use of the empirical momentum dependent weight:

$$W = \frac{1}{1 - \alpha p}$$

$\alpha$ is a constant $= 0.02$ for $\Sigma^-$ and $\Sigma^+_n$

$= 0.19$ for $\Sigma^+_p$

$p$ is the lab. momentum of the $\Sigma$.

In $\Sigma$ decay most of the momentum will be transferred to the nucleon, as it is much heavier than the pion, so the
Fig. 3.8. The decay rotation angle ($\psi$) of the $\Sigma^- \rightarrow \pi^- n$ decay.
Fig. 3.9 Decay Rotation Angle distribution (folded about 90°) of $\Sigma^-$ decay.

Fig. 3.10 Decay Rotation Angle distribution (folded about 90°) of $\Sigma^+p$ decay.

Fig. 3.11 Decay Rotation Angle distribution (folded about 90°) of $\Sigma^+n$ decay.
Fig. 3.12. Correction factor for decay rotation angle losses for $\Sigma^-$ decay against number of bins used to calculate the correction factor and confidence levels of each.

Fig. 3.13. Correction factor for decay rotation angle losses for $\Sigma^+p$ decay against number of bins used to calculate the correction factor and the confidence levels of each.

Fig. 3.14. Correction factor for decay rotation angle losses for $\Sigma^+n$ decay against number of bins used to calculate the correction factor and confidence levels of each.
nucleon will tend more toward the original \( \Sigma \) direction than the pion. Therefore many more small angle kinks occur for \( \Sigma^+ p \) than for \( \Sigma^- \) or \( \Sigma^+_n \) hence the two values of \( a \).

As there was no significant difference between the two beam momenta, both sets of events were added to give better statistics. This gave average weights of approximately 1.02 for \( \Sigma^- \) and \( \Sigma^+_n \) and 1.2 for \( \Sigma^+ p \).

The orientation of the decay about the direction of the \( \Sigma \) should be isotropic. If \( \psi \) is the angle between the decay plane and the plane of the camera and \( \Sigma \) (Fig. 3.8), then the distribution of \( \psi \) should be flat (Figs. 3.9, 3.10 and 3.11). These graphs are folded about 90° and show obvious losses near 0° and 180°.

To calculate these losses, nine bins (from 90° downward) were fitted to a horizontal line. The resulting correction factor and confidence level of the fit were calculated. This procedure was repeated for ten up to all eighteen bins. These correction factors and corresponding confidence levels are shown in Figs. 3.12, 3.13 and 3.14. The most likely corrected factors were 1.08 for \( \Sigma^- \), 1.25 for \( \Sigma^+ p \) and 1.06 for \( \Sigma^+_n \).

The much larger \( \Sigma^+ p \) correction factor was due to a considerable number of events being lost even at a value of \( \psi \) well away from 0° or 180°. This was attributed to the larger number of small angle decays for the \( \Sigma^+ p \) decay mode. Thus only the second correction was adopted as it included the first.

d) Probability cut.

For genuine fits the probability (confidence level obtained from the chi-squared of the fit) distribution should be flat. Therefore the probability cuts applied at choosing
would result in 1°/o of good 4c fits and 5°/o of good 1c fits being discarded. Correction factors 1.01 and 1.05 respectively were applied.

3.4 Pion Contamination

At choosing, fits with \( \pi \) beam track were noted but not accepted for the D.S.T. For events, which were fitted by both \( \pi \) and \( K \) beam track hypotheses, only the \( K \)-fit was accepted. This resulted in a few genuine \( \pi \)-fits being recorded as \( K \)-fits.

The channels affected were:

\[
\begin{align*}
K^- n & \rightarrow p \pi^- K^- & (1) \\
& \rightarrow p \pi^- K^- \pi^0 & (2) \\
& \rightarrow \pi^+ \pi^- K^- n & (3)
\end{align*}
\]

which could be ambiguous with

\[
\begin{align*}
\pi^- n & \rightarrow p \pi^- \pi^- & (1) \\
& \rightarrow p \pi^- \pi^- \pi^0 & (2) \\
& \rightarrow \pi^+ \pi^- \pi^- n , & (3)
\end{align*}
\]

respectively.

There was no other pion channel which could readily be ambiguous with a kaon channel, so if pion beam contamination was small, the effect on other channels would be negligible.

Only case (1) above was analysed to estimate pion contamination, as it was well populated. The correction factor calculated was assumed to apply to the other cases.

The ambiguous events were divided in the ratio of the uniques (see Section 5 of this chapter).

Thus

\[
N_m = N_{ka} \times \frac{N_\pi}{N_\pi + N_{ku}}
\]
### TABLE 3.16

Pion Beam Contamination

<table>
<thead>
<tr>
<th></th>
<th>1.45 GeV/c Film</th>
<th>1.65 GeV/c Film</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of K fits</td>
<td>669</td>
<td>291</td>
</tr>
<tr>
<td>$N_{Ku}$</td>
<td>500</td>
<td>222</td>
</tr>
<tr>
<td>$N_{Ka}$</td>
<td>169</td>
<td>69</td>
</tr>
<tr>
<td>Number of unique π-fits ($N_{π}$)</td>
<td>29</td>
<td>9</td>
</tr>
<tr>
<td>Number of misidentified K-fits ($N_{m}$)</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Correction factor (%/o)</td>
<td>98.6 ± 0.4</td>
<td>99.0 ± 0.5</td>
</tr>
<tr>
<td>Corrected Number of K-fits ($N'_{K}$)</td>
<td>660</td>
<td>288</td>
</tr>
<tr>
<td>Corrected Number of π-fits ($N'_{π}$)</td>
<td>38</td>
<td>12</td>
</tr>
<tr>
<td>Pion Beam Contamination (%/o)</td>
<td>2.8 ± 0.8</td>
<td>2.0 ± 1.0</td>
</tr>
</tbody>
</table>

$N_{Ku}$ - Number of K-fits which are not ambiguous with π-fits.

$N_{Ka}$ - Number of K-fits which are ambiguous with π-fits.
\[ N_m \quad \text{No. of misidentified K-fits.} \]
\[ N_{Ka} \quad \text{No. of K-fits which are also ambiguous with a \( \pi \)-fit.} \]
\[ N_{Ku} \quad \text{No. of K-fits which are not ambiguous with a \( \pi \)-fit.} \]
\[ N_\pi \quad \text{No. of unique \( \pi \)-fits.} \]

So correction factor = \( \frac{(N_{Ku} + N_{Ka}) - N_m}{(N_{Ku} + N_{Ka})} \times 100^\circ /o \).

These quantities and results are given in Table 3.16. The errors quoted are statistical and are dominated by the error in the number of unique \( \pi \)-fits.

As the pion cross section is roughly twice the kaon cross section, then the pion beam contamination will be:

\[ \frac{N'_{\pi}}{2N'_k + N'_\pi} \times 100^\circ /o \]

where
- \( N'_{\pi} \) corrected number of pion events.
- \( N'_k \) corrected number of kaon events.

This gave beam contaminations of \( 2.8^\circ /o \pm 0.8 \) for the 1.45 GeV/c run and \( 2.0^\circ /o \pm 1.0 \) for the 1.65 GeV/c run. These results were consistent with the estimates of the collaborating laboratories, some of which used \( \delta \)-ray counts to estimate pion contamination.

3.5 Ambiguously fitted events

On the D.S.T. there were 895 events which were fitted to more than one hypotheses. Of these 513 were \( \Lambda - \Sigma \) ambiguities. Because of the large numbers of events involved, it was possible to resolve this ambiguity statistically. For
Fig. 3.15 Distribution of cosine of $\gamma$ decay angle in the $\Sigma^0$ decay frame for $\Lambda-\Sigma$ ambiguous events.

Fig. 3.16. Calculated percentage of $\Lambda$ events in $\Lambda-\Sigma$ ambiguous events against number of bins used in the calculation and the confidence levels of each.
### TABLE 3.17

**Allocation of $\Lambda - \Sigma$ Ambiguities**

<table>
<thead>
<tr>
<th>Channel</th>
<th>1.45 GeV/c Film</th>
<th>1.65 GeV/c Film</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ambiguos Total</td>
<td>Allocated Total</td>
</tr>
<tr>
<td>$\Lambda , \pi^- $</td>
<td>359</td>
<td>276</td>
</tr>
<tr>
<td>$\Sigma^o , \pi^- $</td>
<td>83</td>
<td>111</td>
</tr>
<tr>
<td>$\Lambda , \pi^- , \pi^- , \pi^+ $</td>
<td>23</td>
<td>18</td>
</tr>
<tr>
<td>$\Sigma^o , \pi^- , \pi^- , \pi^+ $</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>
genuine $\Sigma^0$ events, the decay distribution of the $\gamma$ in the $\Sigma^0$ rest-frame should be isotropic but when the distribution of $\cos \theta$ ($\theta$ being the $\gamma$ decay angle defined as angle between the direction of motion of the $\Sigma^0$ in the interaction centre of mass frame and the direction of the $\gamma$ in the $\Sigma^0$ rest-frame) was plotted for these ambiguous events there was a large backward peak (Fig. 3.15).

As this decay distribution for genuine $\Sigma^0$ events should be flat, the backward peak was attributed to genuine $\Lambda$ fits. To find the ratio of peak to flat background, a technique was used, similar to that for determining the charged decay notation angle losses. Ten bins from +1 downward were fitted to a straight line. This line was projected to -1 to define the $\Sigma^0$ background. The percentage peak ($\Lambda$-fits) was calculated as was the confidence level of the fit. The process was repeated for eleven up to sixteen bins and the results shown in Fig. 3.16. From these graphs it can be seen that the ambiguous fits should be divided in the ratio $77^0/0$ $\Lambda$ fits to $23^0/0$ $\Sigma^0$ fits. Table 3.17 shows the ambiguous events allocated in this ratio.

The other ambiguously fitted events were dealt with in a simpler manner by allocating the events to the competing hypotheses in the ratio of events uniquely fitting these hypotheses.

For example, there were seven on the 1.45 GeV/c film which fitted both the hypotheses $K^- n \rightarrow K^- n \rightarrow \mu^- \nu$ and $K^- n \rightarrow \Sigma^- \pi^0 \rightarrow \pi^- n$. 
Table 3.18  Allocation of Ambiguous Events in 1.45 GeV/c Film

<table>
<thead>
<tr>
<th>Channel</th>
<th>No. of Ambiguities</th>
<th>Channel</th>
<th>No. of Ambiguities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^-$</td>
<td>$\pi^- n K^0 \rightarrow \pi^- n$</td>
<td>Total</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \pi^- \Lambda \rightarrow \rho n$</td>
<td>361</td>
<td>278</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \pi^- n^0 \Lambda \rightarrow \rho n$</td>
<td>31</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \pi^- \Sigma \rightarrow \gamma \Lambda \rightarrow \rho n$</td>
<td>395</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \pi^- n^0 \rho K^0 \rightarrow \pi^- n$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \pi^- n^+ \Lambda \rightarrow \rho n$</td>
<td>23</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \pi^- n^+ n^0 \Lambda \rightarrow \rho n$</td>
<td>3</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \pi^- n^+ \Sigma = \gamma \Lambda \rightarrow \rho n$</td>
<td>27</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow n K^- \rightarrow \pi^- n^0$</td>
<td>47</td>
<td>265</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow n K^- \rightarrow \rho \gamma$</td>
<td>40</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \eta^0 \Xi^- \rightarrow \pi^- n$</td>
<td>20</td>
<td>135</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \rho n^- K^- \rightarrow \rho \gamma$</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \rho n^- K^- \rightarrow \pi^- n^0$</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow \pi^+ n^- n K^- \rightarrow \rho \gamma$</td>
<td>23</td>
<td>9</td>
</tr>
</tbody>
</table>
### Table 3.19 Allocation of Ambiguous Events in 1.65 GeV/c

<table>
<thead>
<tr>
<th>Channel</th>
<th>No. of Ambiguities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>$K^-n \rightarrow \pi^- n \rightarrow \pi^+ \pi^-$</td>
<td>0</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>112</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>19</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>123</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>20</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>0</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>26</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>22</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>16</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>15</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>4</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>5</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>17</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>29</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>0</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>27</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>1</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>1</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>8</td>
</tr>
<tr>
<td>$\rightarrow \pi^- \rightarrow \rho \pi^-\bar{\nu}$</td>
<td>3</td>
</tr>
</tbody>
</table>
### Table 3.19 (Continued)

<table>
<thead>
<tr>
<th>Channel</th>
<th>No. of Ambiguities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>(K^+ \rightarrow pK^+\pi^-)</td>
<td>2</td>
</tr>
<tr>
<td>(K^+ \rightarrow pK^+\pi^0)</td>
<td>3</td>
</tr>
<tr>
<td>(K^+ \rightarrow p\pi^-\pi^0)</td>
<td>1</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+K^-\pi^-)</td>
<td>23</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+\pi^-\pi^+)</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>34</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+\rho^-)</td>
<td>8</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+\pi^-)</td>
<td>0</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+\pi^-\pi^+)</td>
<td>2</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+\pi^-\pi^0)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+\pi^-\pi^0)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
The first hypothesis had 73 unique fits and the second 165.

Therefore number of events assigned to first hypothesis

\[ \frac{73}{73 + 165} \times 7 \sim 2 \]

and number of events assigned to second hypothesis

\[ \frac{165}{73 + 165} \times 7 \sim 5 \]

Ambiguously fitted events were assigned, the contributions to each channel totalled and the resultant allocations are shown in Tables 3.18 and 3.19.

3.6 The Deuteron

As stated earlier deuterium was used in the bubble chamber to provide neutron targets so that kaon-nucleon scattering could be studied in the pure Isospin 1 channel. The problems inherent in the use of deuterium will be discussed in this section.

a) Hulthen Distribution

If neutron events only are required then ideally the interaction should not affect the proton, leaving it merely as a "spectator". After such an interaction the spectator would have the momentum it possessed in the deuteron due to its Fermi motion. This assumption is called the impulse approximation.

The proton momentum distribution can be calculated from the wave function of the deuteron, first formulated by Hulthen\(^{(1)}\) and named after him.

Hulthen took a neutron-proton potential function
as an approximation to the Yukawa potential

\[ V(r) = - \frac{V_o e^{-(\beta-a)r}}{1 - e^{-(\beta-a)r}} \]  

(3.1)

where \( \mu \) is the reduced mass of the proton and neutron. This gave an exact analytic solution of the Schroedinger equation. Hulthen wave function:

\[ \psi(r) = k \frac{e^{-\alpha r} - e^{-\beta r}}{r} \]  

(3.3)

Chew\(^{(2)}\) showed that this function agreed with the solution, obtained by numerically integrating Schroedinger's equation using Yukawa's potential, to within 3\%o. Chew also found:

\[ K = \frac{a \beta}{2\pi} \left( \frac{a+\beta}{(a-\beta)^2} \right)^{1/2} \]  

(3.4)

\[ a = (\mu B)^{1/2} \]  

\( B \) is deuteron binding energy.

\[ \beta = 5.476a. \]

The momentum wave function is obtained by taking the Fourier Transform of the spatial wave function:

\[ \phi(p) = \int_{-\infty}^{\infty} e^{i \vec{p} \cdot \vec{r}} \psi(r) \, dr \]  

(3.5)

The momentum probability will therefore be:

\[ P(p) = 4\pi p^2 \phi^{*}(\vec{p}) \phi(\vec{p}) \]

\[ = K' \frac{p^2}{[(p^2+a^2)(p^2+\beta^2)]^2} \]  

(3.6)

with \( p = |\vec{p}| \).
Fig. 3.17 The shape of the Hulthen momentum distribution function.
Fig. 3.18 A spectator momentum distribution for $^4$C fits, compared with the Hulthen distribution.

Fig. 3.19 A spectator momentum distribution for $^1$C fits, compared with the Hulthen distribution.
Moravcsik found that the Hulthen wave function was best fitted for $\alpha = 45.7$ MeV and $\beta = 238$ MeV$^3$. The shape of the distribution thus obtained is shown in Fig. 3.17. Two examples of actual spectator momentum distributions are shown in Figs. 3.18 and 3.19. The first is for $4c$ fits and the second for $1c$ fits. There is a very obvious low momentum discrepancy in the $1c$ fit distribution and a less obvious high energy enhancement in both (not shown in the distributions).

b) Non-Impulsive events

From the Hulthen distribution approximately 1.6% of spectator protons should have momentum greater than 280 MeV/c. In fact, about 9% of events have spectator momentum greater than 280 MeV/c. The impulsive approximation does not hold absolutely.

The enhancement must be due to some secondary interaction with the proton. As would be expected, the percentage of non-impulsive events is channel dependent, so the value cited above is merely averaged over the channels.

If the secondary interaction is elastic then there will be no effect on the total channel cross section though such events will not be useful for producing angular distributions for outgoing particles. Inelastic secondary interactions would pose a problem but as they are likely to be a small part of the non-impulsive events, their effect should be negligible.

The spectator cut on the 1.45 GeV/c film will have an effect. Because of it, a proportion of non-impulsive events will be lost, which, although useless for partial wave analysis, are necessary for total cross section calculations. For the
1.65 GeV/c film, the average non-impulsive percentage was 12°/o but only 8°/o for the 1.45 GeV/c film. Therefore a 4°/o correction must be applied to all 1.45 GeV/c channels to account for this loss.

c) **Inserted spectator fits**

The low energy discrepancy is due to inserted spectator fits. Below 80 MeV/c the range of the spectator is less than 1 mm and so effectively unseen. According to the Hulthen distribution approximately five-eighths of events should have spectator momentum less than 80 MeV/c. The effect of the spectator being unseen would be to render an otherwise fittable \( l_c \) fit unfittable and reduce a \( l_c \) fit to a \( l_c \) fit. To overcome this, a routine PXYPYZ was used in kinematic fitting. In this, a spectator proton, if unseen, was inserted with momentum

\[ P_x = P_y = P_z = 0 \quad \text{and} \quad \Delta P_x = \Delta P_y = \frac{3}{4} \Delta P_z = 30 \text{ MeV/c}. \]

For \( l_c \) fits the inserted fits distribution follows the shape of the Hulthen distribution but for \( l_c \) fits there is not sufficient pull to take the momentum away from its starting value of 0 MeV/c. As the spectator momentum distribution determines the shape of the \( K^-n \) centre of mass energy distribution, \( l_c \) inserted spectator fits were not suitable for cross sections involving energy dependence.

d) **Glauber "shadowing" Correction**

Another deuterium effect is that named after Glauber, where the neutron cross section is reduced by the proton "shadowing" the neutron. Similarly the neutron can shadow the proton. In its original form the Glauber correction is defined thus:
### TABLE 3.20

Glauber Correction

<table>
<thead>
<tr>
<th></th>
<th>1.45 GeV/c Film</th>
<th>1.65 GeV/c Film</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deuterium cross section ( (\sigma_a) )</td>
<td>58.3 mb</td>
<td>58.0 mb</td>
</tr>
<tr>
<td>Proton cross section ( (\sigma_p) )</td>
<td>32.2 mb</td>
<td>34.3 mb</td>
</tr>
<tr>
<td>Neutron cross section ( (\sigma_n) )</td>
<td>28.4 mb</td>
<td>26.0 mb</td>
</tr>
<tr>
<td>Glauber correction ( (\sigma_g) )</td>
<td>2.3 mb</td>
<td>2.3 mb</td>
</tr>
</tbody>
</table>

### TABLE 3.21

Number of TAU decays and correction factors

<table>
<thead>
<tr>
<th></th>
<th>1.45 GeV/c Film</th>
<th>1.65 GeV/c Film</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of fitted TAU decays</td>
<td>554</td>
<td>182</td>
</tr>
<tr>
<td>Processing correction</td>
<td>1.50</td>
<td>1.36</td>
</tr>
<tr>
<td>Probability cut correction</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Corrected number of TAU decays</td>
<td>( 839 \pm 4^\circ/0 )</td>
<td>( 250 \pm 7^\circ/0 )</td>
</tr>
</tbody>
</table>
\[ \sigma_d = \sigma_n + \sigma_p - \sigma_g \]

where \[ \sigma_g = \frac{1}{4\pi} \left\langle r^{-2} \right\rangle \sigma_n \sigma_p \]

\( \sigma_d \) - deuterium cross section
\( \sigma_n \) - neutron cross section
\( \sigma_p \) - proton cross section
\( \sigma_g \) - Glauber correction
\( r \) - separation of the nucleons in the deuteron.

The value of \( \left\langle r^{-2} \right\rangle \) provides a problem. The solution of the Hulthen wave function gives the value 0.034 \( \text{mb}^{-1} \).

However Cool et al. took its value to be 0.0423 \( \text{mb} \), whereas Bugg et al. used the value 0.029 \( \text{mb} \). An approximate average of these was taken by Lynch:

\[ \left\langle r^{-2} \right\rangle \approx \frac{1}{400 \text{mb}} \]

and this value has been adopted.

\( \sigma_n \) and \( \sigma_g \) were calculated at 1.45 \( \text{GeV/c} \) and 1.65 \( \text{GeV/c} \) using Lynch's values for \( \sigma_d \) and \( \sigma_p \). (Table 3.20). In both cases the Glauber correction was 2.3 \( \text{mb} \) giving a 40\% reduction in cross sections due to shadowing.

e) Choice of Spectator

There was some ambiguity about the assignment of spectator where two nucleons occurred in the final \( K^- d \) state (e.g. \( K^- \pi^- pp \) and \( K^0 \pi^- np \)). In the two proton final state the lower energy proton was automatically chosen as spectator while this selection may have been wrong there could be no doubt that the final state was produced by a neutron interaction.
assuming, as before, that there was no inelastic secondary scattering. Therefore there will be no effect on the channel cross sections.

For neutron proton final states, the proton was required to have smaller momentum than the neutron. 72°/o of fitted hypotheses at 1.45 GeV/c and 69°/o at 1.65 GeV/c satisfied this condition. While it was likely that there were some genuine neutron interactions excluded by this criterion, these should be balanced by contamination of genuine proton interactions.

3.7 Total Track Length

Within the fiducial volume there were 554 fitted TAU decays at 1.45 GeV/c beam momentum and 182 at 1.65 GeV/c. When multiplied by the processing correction factor and probability cut corrections (the only correction factors applicable to the TAU decay) these totals become 839 decays at 1.45 GeV/c and 250 at 1.65 GeV/c (Table 3.21).

Not all the correction factor errors have been calculated but where they have, they were approximately equal to the statistical error in the number of events added by the correction. If this was generally true the resultant error would merely be the statistical error in the final corrected number. As this cannot be established the error taken was the statistical error on the raw numbers.

The lifetime of the K⁻ is 1.2371 x 10⁻⁸ secs and 5.58°/o of its decays are by the TAU channel(8). A K⁻ with momentum 1.45 GeV/c has velocity of 2.84 x 10⁸ ms⁻¹ and lifetime of 3.84 x 10⁻⁸ secs in the laboratory frame.
So to produce 839 TAU decays there must have been a total \( K^- \) track length

\[
L = \frac{839 \times 2.84 \times 10^8 \times 3.84 \times 10^{-8}}{0.0558} = 164,000 \text{ m} \pm 4\%.
\]

Similarly at 1.65 GeV/c the \( K^- \) velocity was \( 2.87 \times 10^8 \text{ m/s} \) and lifetime was \( 4.29 \times 10^{-8} \) secs.

So
\[
L = \frac{250 \times 2.87 \times 10^8 \times 4.29 \times 10^{-8}}{0.0558} = 55,800 \text{ m} \pm 7\%.
\]

### 3.8 Total Channel Cross Sections

The mass of an atom of deuterium is \( 3.33 \times 10^{-24} \text{ g} \) and, in the bubble chamber, the density of deuterium was \( 0.165 \text{ g cm}^{-3} \). Therefore one atom of deuterium occupied a volume

\[
= \frac{3.33 \times 10^{-24}}{0.165} = 2.02 \times 10^{-23} \text{ cm}^3.
\]

For the 1.45 GeV/c film the cross section per event was therefore

\[
\frac{2.02 \times 10^{-23}}{1.64 \times 10^7} \text{ cm}^2 = 1.23 \text{ \( \mu \)b} \pm 4\%.
\]

And similarly for the 1.65 GeV/c film the cross section per event was

\[
\frac{2.02 \times 10^{-23}}{5.58 \times 10^6} \text{ cm}^2 = 3.63 \text{ \( \mu \)b} \pm 7\%.
\]

Tables 3.22, 3.23 give the numbers of events in each channel, the corrections to be applied, the corrected numbers and the total channel cross sections.
The errors quoted on the corrected numbers are the statistical errors on the raw numbers plus assigned ambiguous events. The errors on the cross sections are the errors in the corrected numbers and the error in the microbarn equivalent added in quadrature.

The values for the total channel cross sections shown in Tables 3.22 and 3.23 were approximately consistent with those values calculated by the other three collaborating laboratories, though the spread of results for any hypothesis was appreciable. A fairly typical set of values is given in Table 3.24.

To date, the only published $K^-n$ partial cross sections at this energy have been from the B.E.G.I. collaboration(9).
<table>
<thead>
<tr>
<th>Channel</th>
<th>No. of Events</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Corrected Number</th>
<th>Total Channel Cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>π⁻νK⁰</td>
<td>431</td>
<td>-</td>
<td>1.48</td>
<td>2.92</td>
<td>1.06</td>
<td>-</td>
<td>-</td>
<td>0.72</td>
<td>1.04</td>
<td>1.04</td>
<td>1.05</td>
<td>1610 ±5%°/o</td>
<td>1.97mb ±7%°/o</td>
</tr>
<tr>
<td>π⁻Λ⁰</td>
<td>276</td>
<td>2.01</td>
<td>1.48</td>
<td>1.56</td>
<td>1.08</td>
<td>-</td>
<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.01</td>
<td>152 ±4%°/o</td>
<td>1.86mb ±6%°/o</td>
<td></td>
</tr>
<tr>
<td>π⁻π⁻Λ⁰</td>
<td>951</td>
<td>1.03</td>
<td>1.48</td>
<td>1.56</td>
<td>1.08</td>
<td>-</td>
<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.05</td>
<td>2770 ±3%°/o</td>
<td>3.40mb ±5%°/o</td>
<td></td>
</tr>
<tr>
<td>π⁻Σ⁻</td>
<td>84</td>
<td>2.17</td>
<td>1.48</td>
<td>1.56</td>
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<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.05</td>
<td>515 ±7%°/o</td>
<td>0.64mb ±8%°/o</td>
<td></td>
</tr>
<tr>
<td>π⁻π⁻K⁻⁻</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay unseen</td>
<td>17</td>
<td>-</td>
<td>1.52</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.05</td>
<td>34 ±22%°/o</td>
<td>42µb ±22%°/o</td>
<td></td>
</tr>
<tr>
<td>Decay seen</td>
<td>3</td>
<td>-</td>
<td>1.52</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.01</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π⁺π⁺Λ⁺</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay unseen</td>
<td>140</td>
<td>1.27</td>
<td>1.52</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>1.04</td>
<td>1.05</td>
<td>307</td>
<td>0.58mb ±9%°/o</td>
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</tr>
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<td>Decay seen</td>
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<td>1.23</td>
<td>1.52</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.01</td>
<td>163</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π⁺π⁺Σ⁺</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay unseen</td>
<td>30</td>
<td>1.27</td>
<td>1.52</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
<td>63</td>
<td>0.13mb ±14%°/o</td>
<td></td>
</tr>
<tr>
<td>Decay seen</td>
<td>13</td>
<td>1.64</td>
<td>1.66</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.05</td>
<td>39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π⁻pK⁻</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay unseen</td>
<td>621</td>
<td>-</td>
<td>1.46</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.99</td>
<td>1.04</td>
<td>1.04</td>
<td>1.01</td>
<td>985</td>
<td>1.27mb ±6%°/o</td>
<td></td>
</tr>
<tr>
<td>μ decay</td>
<td>14</td>
<td>1.45</td>
<td>1.57</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.01</td>
<td>34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π decay</td>
<td>4</td>
<td>1.50</td>
<td>1.57</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.04</td>
<td>1.04</td>
<td>1.01</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 3.22  Numbers of Events, Correction Factors, and Total Cross Sections for 1.45 GeV/c Film (Contd.)

| Channel | No. of Events | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | Corrected Total Channel 
|---------|---------------|----|----|----|----|----|----|----|----|----|----| Number Cross section |
| \( \pi^- p^0 K^- \) | | | | | | | | | | | | |
| Decay unseen | 42 | 1.02 | 1.52 | - | - | - | - | 0.99 | - | 1.04 | 1.04 | 1.05 | 73±10\(^{\circ}\)/0 | 90\(\mu\)b±16\(^{\circ}\)/0 |
| \( \mu \) decay | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( \pi \) decay | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( \pi^- p^+ n \) \( \pi^- n \) | | | | | | | | | | | | |
| Decay unseen | 67 | 1.09 | 1.52 | - | - | - | - | 0.99 | 0.72 | 1.04 | 1.04 | 1.05 | 153±10\(^{\circ}\)/0 | 0.19\(\mu\)b±10\(^{\circ}\)/0 |
| \( \mu \) decay | 13 | 1.69 | 1.48 | - | - | - | - | - | 0.72 | 1.04 | 1.04 | 1.05 | 27 |
| \( \pi \) decay | 13 | 2.23 | 1.48 | - | - | - | - | - | 0.72 | 1.04 | 1.04 | 1.05 | 35 |
| \( \pi^0 \Sigma^- \) | | | | | | | | | | | | |
| 165 | 1.08 | 1.46 | - | - | 1.09 | 1.08 | - | - | 1.04 | 1.04 | 1.05 | 347±10\(^{\circ}\)/0 | 0.43\(\mu\)b±9\(^{\circ}\)/0 |
| \( \pi^- \pi^+ \Sigma^- \) | | | | | | | | | | | | |
| 256 | 1.01 | 1.57 | - | - | 1.09 | 1.08 | - | - | 1.04 | 1.04 | 1.05 | 520±10\(^{\circ}\)/0 | 0.64\(\mu\)b±7\(^{\circ}\)/0 |
| \( \pi^- \pi^0 \Sigma^- \) | | | | | | | | | | | | |
| 43 | 1.53 | 1.48 | - | - | 1.09 | 1.08 | - | - | 1.04 | 1.04 | 1.05 | 136±12\(^{\circ}\)/0 | 0.17\(\mu\)b±13\(^{\circ}\)/0 |
| \( \pi^- \pi^- \Sigma^- \) | | | | | | | | | | | | |
| proton decay | 77 | 1.05 | 1.57 | - | - | 1.38 | 1.25 | - | - | 1.04 | 1.04 | 1.01 | 238 |
| neutron decay | 124 | - | 1.57 | - | - | 1.14 | 1.06 | - | - | 1.04 | 1.04 | 1.01 | 257 |
| \( \pi^- \pi^- \pi^0 \) \( \pi^- \pi^- \pi^0 \) | | | | | | | | | | | | |
| proton decay | 7 | 1.43 | 1.48 | - | - | 1.38 | 1.25 | - | - | 1.04 | 1.04 | 1.05 | 29 |
| neutron decay | 22 | 1.05 | 1.48 | - | - | 1.14 | 1.06 | - | - | 1.04 | 1.04 | 1.05 | 44 |

The correction factors are:
(1) for ambiguous events, (2) for processing losses, (3) for unseen decay modes,
(4) for short or long lived decay losses, (5) for decay notation angle losses,
(6) for pion beam contamination, (7) for spectator identification,
(8) for non-impulsive spectator losses, (9) for Glauber shadowing,
(10) for probability cut.
<table>
<thead>
<tr>
<th>Channel</th>
<th>No. of Events</th>
<th>Corrections</th>
<th>Corrected Number</th>
<th>Total Channel Cross Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^- p K^0 )</td>
<td>254</td>
<td>1.45 2.82 1.05</td>
<td>0.69 1.04 1.05</td>
<td>850 ( \pm 6^o/0 ) 3.09mb ( \pm 9^o/0 )</td>
</tr>
<tr>
<td>( \pi^- \Lambda^0 )</td>
<td>128</td>
<td>1.68 1.45 1.56 1.08</td>
<td>- 1.04 1.01</td>
<td>550 ( \pm 7^o/0 ) 2.0 mb ( \pm 10^o/0 )</td>
</tr>
<tr>
<td>( \pi^- \pi^0 \Lambda^0 )</td>
<td>388</td>
<td>1.04 1.45 1.56 1.08</td>
<td>- 1.04 1.05</td>
<td>1080 ( \pm 5^o/0 ) 3.92mb ( \pm 9^o/0 )</td>
</tr>
<tr>
<td>( \pi^- \Sigma^0 )</td>
<td>41</td>
<td>1.66 1.45 1.56 1.08</td>
<td>- 1.04 1.05</td>
<td>182 ( \pm 12^o/0 ) 0.66mb ( \pm 14^o/0 )</td>
</tr>
<tr>
<td>( \pi^- \pi^+ K^0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay unseen</td>
<td>19</td>
<td>1.02 1.47 - - -</td>
<td>1.04 1.05</td>
<td>40 ( \pm 20^o/0 ) 0.15mb ( \pm 21^o/0 )</td>
</tr>
<tr>
<td>Decay seen</td>
<td>6</td>
<td>1.43 - - - - -</td>
<td>1.04 1.01</td>
<td>9</td>
</tr>
<tr>
<td>( \pi^- \pi^+ \Lambda^0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay unseen</td>
<td>91</td>
<td>1.35 1.47 - - -</td>
<td>1.04 1.05</td>
<td>324 ( \pm 7^o/0 ) 1.17mb ( \pm 10^o/0 )</td>
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<tr>
<td>Decay seen</td>
<td>69</td>
<td>1.22 1.43 - - -</td>
<td>1.04 1.01</td>
<td>198</td>
</tr>
<tr>
<td>( \pi^- \pi^+ \Sigma^0 )</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay unseen</td>
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<td>1.25 1.47 - - -</td>
<td>1.04 1.05</td>
<td>100 ( \pm 13^o/0 ) 0.35mb ( \pm 15^o/0 )</td>
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<td>Decay seen</td>
<td>7</td>
<td>2.57 1.54 - - -</td>
<td>1.04 1.05</td>
<td>68</td>
</tr>
<tr>
<td>( \pi^- p K^- )</td>
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<td></td>
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<td></td>
</tr>
<tr>
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<td>274</td>
<td>1.01 1.41 - - -</td>
<td>0.99 1.01 1.01</td>
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<td>( \mu ) decay</td>
<td>7</td>
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<td>16</td>
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<tr>
<td>( \pi ) decay</td>
<td>4</td>
<td>1.63 1.57 - - -</td>
<td>1.04 1.01</td>
<td>11</td>
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<tr>
<td>( \pi^- \pi^0 p K^- )</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Decay unseen</td>
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<td>0.99 1.04 1.05</td>
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<tr>
<td>( \mu ) decay</td>
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<td>0</td>
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<tr>
<td>( \pi ) decay</td>
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<td></td>
<td>0</td>
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<td>No. of Events</td>
<td>Corrections</td>
<td>Corrected Numbers</td>
<td>Total Channel Cross sections</td>
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<td>---------------</td>
<td>-------------</td>
<td>-------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>$\pi^- + \pi^- K^-$</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Decay unseen</td>
<td>114</td>
<td>1.09 1.47</td>
<td>0.99 0.69 1.04 1.05</td>
<td>163 $^{+8^\circ/o} - 0.59^{+11^\circ/o}$</td>
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<tr>
<td>$\mu$ decay</td>
<td>13</td>
<td>1.50 1.48</td>
<td>0.69 1.04 1.05</td>
<td></td>
</tr>
<tr>
<td>$\pi$ decay</td>
<td>11</td>
<td>1.82 1.48</td>
<td>0.69 1.04 1.05</td>
<td></td>
</tr>
<tr>
<td>$\pi^0 \Sigma^-$</td>
<td>105</td>
<td>1.10 1.43</td>
<td>1.07 1.08</td>
<td>208 $^{+8^\circ/o} - 0.75mb^{+11^\circ/o}$</td>
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<tr>
<td>$\pi^+ \Sigma^-$</td>
<td>121</td>
<td>- 1.57</td>
<td>1.07 1.08</td>
<td>230 $^{+9^\circ/o} - 0.83mb^{+11^\circ/o}$</td>
</tr>
<tr>
<td>$\pi^- \pi^0 \Sigma^-$</td>
<td>32</td>
<td>1.64 1.48</td>
<td>1.07 1.08</td>
<td>97 $^{+14^\circ/o} - 0.35mb^{+16^\circ/o}$</td>
</tr>
<tr>
<td>$\pi^- \pi^0 \Sigma^+$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>Proton Decay</td>
<td>31</td>
<td>1.02 1.57</td>
<td>1.33 1.25</td>
<td>168 $^{+12^\circ/o} - 0.61mb^{+14^\circ/o}$</td>
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<tr>
<td>Neutron Decay</td>
<td>41</td>
<td>1.01 1.57</td>
<td>1.13 1.06</td>
<td></td>
</tr>
<tr>
<td>$\pi^- \pi^0 \Sigma^+$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Proton Decay</td>
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<td>- 1.48</td>
<td>1.33 1.25</td>
<td>39 $^{+24^\circ/o} - 0.14mb^{+25^\circ/o}$</td>
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<td>Neutron Decay</td>
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<td>1.09 1.48</td>
<td>1.13 1.06</td>
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</tr>
<tr>
<td>$\pi^+ \pi^- \Lambda^0$</td>
<td>277</td>
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<td>1.56 1.08</td>
<td>720 $^{+6^\circ/o} - 2.62mb^{+9^\circ/o}$</td>
</tr>
<tr>
<td>$\pi^+ \pi^- \pi^- \Lambda^0$</td>
<td>4</td>
<td>- 1.56 1.56</td>
<td>1.08</td>
<td>11 $^{+50^\circ/o} - 40\mu b^{+50^\circ/o}$</td>
</tr>
<tr>
<td>$\pi^+ \Sigma^-$</td>
<td>79</td>
<td>1.02 1.49</td>
<td>1.07 1.08</td>
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<tr>
<td>$\pi^- \Sigma^+$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>Proton Decay</td>
<td>43</td>
<td>- 1.49</td>
<td>1.33 1.25</td>
<td>264 $^{+9^\circ/o} - 0.95mb^{+11^\circ/o}$</td>
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<td>1.13 1.06</td>
<td></td>
</tr>
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<td>Corrections</td>
<td>Correction Number</td>
<td>Total Channel Cross Section</td>
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<tr>
<td>--------------------------</td>
<td>---------------</td>
<td>-------------</td>
<td>-------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>$\pi^+ \pi^+ \pi^- \Sigma^-$</td>
<td>11</td>
<td>1.51 1.07 1.08</td>
<td>1.04 1.01</td>
<td>76(\mu)b $\pm$ 31(^\circ)b</td>
</tr>
<tr>
<td>$\pi^+ \pi^- \Sigma^+$</td>
<td>6</td>
<td>1.57 1.33 1.25</td>
<td>1.04 1.01</td>
<td>0.15(\mu)b $\pm$ 25(^\circ)b</td>
</tr>
<tr>
<td>Proton Decay</td>
<td>6</td>
<td>1.57 1.33 1.25</td>
<td>1.04 1.01</td>
<td>16</td>
</tr>
<tr>
<td>Neutron Decay</td>
<td>11</td>
<td>1.09 1.13 1.06</td>
<td>1.04 1.01</td>
<td>24</td>
</tr>
</tbody>
</table>

N.B. 1) The correction factors are as for Table 3.22.

2) The last 6 channels are $K^-p$ interactions.
TABLE 3.24

Comparison of some total channel cross sections within the collaboration

<table>
<thead>
<tr>
<th>Channel</th>
<th>C (mb)</th>
<th>E (mb)</th>
<th>G (mb)</th>
<th>B (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda \pi^-\pi^0 )</td>
<td>3.56</td>
<td>3.40</td>
<td>5.2</td>
<td>4.04</td>
</tr>
<tr>
<td>( \Sigma^-\pi^+\pi^- )</td>
<td>0.76</td>
<td>0.64</td>
<td>0.9</td>
<td>0.88</td>
</tr>
<tr>
<td>( \Sigma^+\pi^-\pi^- )</td>
<td>1.03</td>
<td>0.61</td>
<td>1.09</td>
<td>1.04</td>
</tr>
<tr>
<td>( \bar{K}\pi^-\bar{n} )</td>
<td>2.86</td>
<td>1.97</td>
<td>4.0</td>
<td>3.17</td>
</tr>
<tr>
<td>( K^-\pi^-p )</td>
<td>1.50</td>
<td>1.27</td>
<td>1.7</td>
<td>1.08</td>
</tr>
<tr>
<td>( \Lambda\pi^-\pi^+\pi^- )</td>
<td>0.57</td>
<td>0.56</td>
<td>0.64</td>
<td>0.64</td>
</tr>
</tbody>
</table>

C - Collaboration total channel cross section (Ref. 9)
E - Edinburgh total channel cross section
G - Glasgow total channel cross section (preliminary)
B - Birmingham total channel cross section

Imperial College figures not available.

Beam momentum : 1.45 GeV/c.
REFERENCES FOR CHAPTER 3

(1) L. Hulthen Arkiv f"ur Fysik 28A (1942).
(2) G.F. Chew Phys. Rev. 74, 809 (1948).
    L.R.L. Preprint UCRL 19844.
CHAPTER 4

4.1 World Data

To date there have been three $\bar{K}N$ experiments which analyse the channel $\bar{K}N \rightarrow \Lambda \pi$ in the energy region 1.9 to 2.1 GeV.

One was run at the Lawrence Radiation Laboratory (L.R.L.) and analysed by groups at L.R.L., University of California (Los Angeles) and University of Illinois and two were run at the Rutherford High Energy Laboratory. One of these was analysed by College de France, RHEL and Saclay (CRS) and the other was the experiment described in this thesis.

The American experiment used the 72 inch hydrogen bubble chamber. Eight kaon beam momenta from 1.22 to 1.95 GeV/c were used. The bulk of the data collection was done at L.R.L. by Wohl et al., who did a partial wave analysis of the $\Lambda \pi^0$ channel\(^{(1)}\). The other collaborators, Trower et al. (Illinois) and Dauber et al. (U.C.L.A.) did not study this channel in depth. However all these data were added to data from a $K^-$ deuterium experiment of energy range 1.66 to 1.90 GeV, which had already been analysed by Smart et al.\(^{(2)}\), and the $K^-N \rightarrow \Lambda \pi$ channel analysed between 1.66 and 2.215 GeV by Smart\(^{(3)}\).

The C.R.S. experiment used the Saclay bubble chamber filled with hydrogen. Thirteen beam momenta from 1.263 to 1.845 GeV/c were used. A partial wave analysis of the $K^-p \rightarrow \Lambda \pi^0$ channel in the region 1.915 to 2.165 GeV...
Fig. 4.1 Energy distribution of deuterium data analysed by Smart.

Fig. 4.2 Energy distribution of hydrogen data analysed by Smart.

T - Data collected by Trower et al.
D - Data collected by Dauber et al.
All other data collected by Wohl et al.
Fig. 4.3 Energy distribution of hydrogen data analysed by the C.R.S. Collaboration.

Fig. 4.4 Energy distribution of deuterium data analysed by the B.E.G.I. Collaboration.
was performed by Berthon et al. \(^{(4)}\).

Finally the B.E.G.I. collaboration published results of a partial wave analysis of \(K^- n \rightarrow \Lambda \pi^-\) in the energy region 1.9 to 2.1 GeV\(^{(5)}\).

A summary of data used in each of these analyses is shown in Figs. 4.1, 4.2, 4.3 and 4.4. The techniques of analysis and results are described in Sections 3, 4 and 5.

Two further studies have been done using all or part of the above data. These were by Litchfield \(^{(6)}\) and Barbaro-Galtieri \(^{(7)}\). While they are more comprehensive than any of the three analyses described, their techniques are essentially similar.

4.2 Partial Wave Analysis

In this section the reaction considered will be meson (spinless) and baryon (spin \(\frac{1}{2}\)) scattering to meson (spinless) and baryon (spin \(\frac{1}{2}\)). This process can be represented in the centre of mass frame as

\[
\begin{align*}
\text{M}_1 & \quad \vec{K}_1 \\
\text{B}_1 & \quad \vec{K}_f \\
\text{M}_2 & \quad \theta \\
\text{B}_2 & \quad \vec{K}_i
\end{align*}
\]

\(\theta\) Scattering angle

\(k_i\) initial C of M momentum

\(k_f\) final C of M momentum.

A transition matrix \(M\) can be defined, which operates on the initial baryon spin state \(\chi_i\) to give the final wave
function $\psi_f$.

$$\psi_f = M \chi_1$$  \hspace{1cm} (4.1)

For conservation of parity $M$ must be scalar and has most general form

$$M = f(\theta) + ig(\theta) \sigma . \hat{n}$$  \hspace{1cm} (4.2)

$\sigma$ is the Pauli spin operators
$n$ is a unit vector normal to the production plane.

$$\hat{n} = \frac{\vec{k}_1 \times \vec{k}_f}{|\vec{k}_1 \times \vec{k}_f|}$$  \hspace{1cm} (4.3)

$f$ and $g$ are the spin non-flip and spin-flip amplitudes.

It can be shown that the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \text{Tr} M^+ M = |f|^2 + |g|^2$$  \hspace{1cm} (4.4)

and polarisation times differential cross section is

$$\bar{P} \frac{d\sigma}{d\Omega} = \frac{1}{2} \text{Tr} M^+ \sigma M = -2\text{Im}(f^* g) \hat{n} .$$  \hspace{1cm} (4.5)

$f(\theta)$ and $g(\theta)$ can be expanded in terms of the partial wave amplitudes $T^\pm_l$, where $l$ is the orbital angular momentum of the final state and the total angular momentum $J = l + \frac{1}{2}$.

$$f(\theta) = \sum_l [(l+1)T^+_l + lT^-_l] P^l_0(\cos \theta)$$  \hspace{1cm} (4.6)

$$g(\theta) = \sum_l [T^+_l - T^-_l] P^l_1(\cos \theta) .$$  \hspace{1cm} (4.7)

$P^l_0$ is the $l$-th order Legendre polynomial.

$P^l_1$ is the $l$-th order first associated Legendre polynomial.

$\kappa$ is the incident C. of M. wavelength divided by $2\pi$. 
An intermediate state, with $J = \frac{3}{2}$, can appear in the partial wave amplitudes $T^+_3$ or $T^-_4$. For a final state $\Lambda \pi$ the parity of the intermediate state is equal to the product of the intrinsic parities of the final state particles and the orbital parity of the final state.

$$\text{Parity} = P(\Lambda) P(\pi) P(\text{orbital})$$

$$= (+1)(-1)(-1)^\ell = -(-1)^\ell.$$  

So an intermediate state, $J^P = \frac{3}{2}^+$, appears in the $T^+_3$ amplitude whereas the state, $J^P = \frac{1}{2}^-$, appears in the $T^-_4$ amplitude.

It can be shown that $\frac{d\sigma}{dn}$ is invariant under parity transformation $T^+_l \leftrightarrow T^-_{l+1}$ so no distinction can be made between the two parity states but $P \frac{d\sigma}{dn}$ changes sign on parity transformation so this ambiguity - the Minami ambiguity - can be resolved.

Also both $\frac{d\sigma}{dn}$ and $P \frac{d\sigma}{dn}$ are invariant under the transformation $T^+_l \rightarrow e^{i\phi} T^+_l$ so in general it is necessary to define the phase of one of the partial wave amplitudes.

It is customary to expand the experimental distributions $\frac{d\sigma}{dn}$ and $P \frac{d\sigma}{dn}$ in a series of Legendre Polynomials.

$$\frac{d\sigma}{dn} = \kappa^2 \sum_{m=0} A_m P_m (\cos \theta)$$  

$$P \frac{d\sigma}{dn} = \hat{n} \kappa^2 \sum_{n=1} B_n P'_n (\cos \theta)$$  

$\frac{d\sigma}{dn}$ is derived directly from the angular distribution of events in an energy bin. The polarisation for each angular distribution bin was calculated from the observed $\Lambda$ decay asymmetry relative to the production normal.
\[
\bar{F} = \frac{3}{a_\Lambda} \langle \cos \xi \rangle \cdot \hat{n} \tag{4.10}
\]

where \( \cos \xi = \hat{p} \cdot \hat{n} \)

\( \hat{p} \) is the unit vector parallel to the momentum of the proton in the \( \Lambda \) decay frame

\( a_\Lambda = 0.65^{(12)} \).

Expressions for the A's and B's in terms of the partial wave amplitudes are normally obtained from tables \(^{(8)}\). For example

\[
A_2 = 4 \Re(T_0 T_2^- + T_1 T_1^-) + 6 \Re(T_0 T_2^+) \\
+ 2 |T_1^+| + 2 |T_2^-| + \frac{12}{7} \Re(T_2^- T_2^+) + \frac{24}{7} |T_2^+|
\]

expanded up to \( T_2^+ \).

The final stage of the analysis is to parameterise the partial wave amplitudes in terms of non-resonant background and Breit-Wigner resonances to best fit the experimental values of A's and B's.

The standard form of a Breit-Wigner amplitude is:

\[
T = \frac{\delta e^{-i\phi} (\Gamma_e \Gamma_r)^{1/2}}{E_R - E - i\Gamma/2} \tag{4.11}
\]

where \( \phi \) is the phase.

\( \Gamma_e \) is the partial width of the incident (elastic) channel.

\( \Gamma_r \) is the partial width of the final (reaction) channel.

\( E_R \) is the energy of the resonance.

\( \Gamma \) is the width of the resonance (the summation of the partial width over all the decay channel).
Where both resonant and non-resonant amplitudes occur in the one partial wave these amplitudes are normally added.

In general any number of parameters can be employed to fit the non-resonant background, though usually no more than eight are used in any one partial wave. For each resonance there are four parameters to be determined ($J^P$ must be assumed to allow an assignment to a partial wave) namely mass ($E_R$), width ($\Gamma$), phase ($\phi$) and an elasticity term $(x_e x_r)^{1/2}$

where $x_e (= \frac{\Gamma_e}{\Gamma})$ is the elasticity of the elastic channel and $x_r (= \frac{\Gamma_r}{\Gamma})$ is the elasticity of the reaction channel.

But as stated above the value of $\phi$ for one resonance must be fixed.

A fuller discussion of the theory of partial wave analysis is given in numerous articles and textbooks (9), (10).

4.3 The American Analysis

Smart's analysis was carried out on 11000 $K^- N \rightarrow \Lambda \pi$ events which were grouped into 18 energy bins between 1.66 and 2.215 GeV (Fig. 4.1 and 4.2). The angular distributions were required to have at least ten events in any one bin and forty events were required in each polarisation bin.

Expansion to the ninth partial wave required the determination of $A_0$ to $A_8$ and $B_1$ to $B_8$ from the observed $\frac{d\sigma}{d\Omega}$ and $\bar{P}\frac{d\sigma}{d\Omega}$ distributions.

The partial wave amplitudes, where non-resonant, were parameterised by an arbitrary complex function which could be
constant, linear or quadratic in \( k \) (the incoming momentum)

\[
T = A + Bk + Ck^2 \exp\left[i(D + Ek + Fk^2)\right]
\]

for \( A + Bk + Ck^2 > 0 \) \hspace{1cm} (4.12)

\[
T = 0 \hspace{1cm} \text{for} \ A + Bk + Ck^2 < 0
\]

The inserted resonances were of Breit Wigner form

\[
T = \frac{\text{Re} \left( \frac{1}{\sqrt{x} e^{ix}} \right)}{e^{\sqrt{x} e^{ix}}}.
\] \hspace{1cm} (4.13)

The main structure in the total cross section was at 1770 MeV and 2030 MeV. These were fitted by the resonances \( \Sigma(1770), \ J^P = \frac{3}{2}^- \) and \( \Sigma(2030), \ J^P = \frac{7}{2}^+ \). A resonance at 1910 MeV was tried in several partial waves but was best fitted in the \( T_3^- \) wave giving a \( J^P \) assignment \( \frac{5}{2}^- \). A resonance at 1880 MeV was suggested with \( J^P = \frac{1}{2}^+ \) but there was no convincing evidence for the \( \Sigma(2250), \ J^P = \frac{9}{2}^- \) reported by Cool et al. (11).

The Resonance parameters from Smart's best fit are shown in Table 4.1. The phase of the \( \Sigma(1770) \) was assigned \( \phi = 0 \) and all other phases were calculated relative to this.

In addition, thirty-five parameters were required to describe the non-resonant background.

4.4 The C.R.S. Analysis

The C.R.S. data, some 6000 \( K^+ p \rightarrow \Lambda \pi^0 \) events (Fig. 4.3), gave differential cross sections for 20 angular bins at 13 different energies. Polarisation bins were somewhat fewer
### TABLE 4.1
Resonance Parameters obtained from the Partial Wave Analysis of Smart.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Spin $J^P$</th>
<th>Parity</th>
<th>Mass $E_R$ (MeV)</th>
<th>Width $\Gamma$ (MeV)</th>
<th>Elasticity $\sqrt{\frac{x}{x_R}}$</th>
<th>Phase $\phi$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma$ (1770)</td>
<td>$\frac{5}{2}^-$</td>
<td>-</td>
<td>1775$^{+7}_{-14}$</td>
<td>146$^{+9}_{-9}$</td>
<td>0.27$^{+0.02}_{-0.03}$</td>
<td>0 (fixed)</td>
</tr>
<tr>
<td>$\Sigma$ (1880)</td>
<td>$\frac{1}{2}^+$</td>
<td>+</td>
<td>1882$^{+10}_{-40}$</td>
<td>222$^{+150}_{-162}$</td>
<td>0.11$^{+0.03}_{-0.02}$</td>
<td>-27$^{+36}_{-27}$</td>
</tr>
<tr>
<td>$\Sigma$ (1910)</td>
<td>$\frac{7}{2}^+$</td>
<td>+</td>
<td>1902$^{+11}_{-12}$</td>
<td>52$^{+25}_{-25}$</td>
<td>0.08$^{+0.02}_{-0.02}$</td>
<td>34$^{+21}_{-21}$</td>
</tr>
<tr>
<td>$\Sigma$ (2030)</td>
<td>$\frac{7}{2}^+$</td>
<td>+</td>
<td>2032$^{+6}_{-6}$</td>
<td>160$^{+16}_{-16}$</td>
<td>0.21$^{+0.01}_{-0.02}$</td>
<td>174$^{+8}_{-8}$</td>
</tr>
</tbody>
</table>

### TABLE 4.2
Resonance Parameters obtained from the Partial Wave Analysis of the C.R.S. Collaboration

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Spin $J^P$</th>
<th>Parity</th>
<th>Mass $E_R$ (MeV)</th>
<th>Width $\Gamma$ (MeV)</th>
<th>Elasticity $\sqrt{\frac{x}{x_R}}$</th>
<th>Phase $\phi$ (fixed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma$ (1910)</td>
<td>$\frac{3}{2}^+$</td>
<td>+</td>
<td>1910$^{+20}_{-20}$</td>
<td>60$^{+20}_{-20}$</td>
<td>0.1$^{+0.02}_{-0.02}$</td>
<td>0</td>
</tr>
<tr>
<td>$\Sigma$ (2030)</td>
<td>$\frac{7}{2}^+$</td>
<td>+</td>
<td>2030$^{+10}_{-10}$</td>
<td>165$^{+30}_{-15}$</td>
<td>0.2$^{+0.02}_{-0.02}$</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>
as approximately thirty events were required per bin.

Although A and B coefficients were calculated for comparison with other data, fitting was made directly to the angular distributions and polarisations. Beyond $A_7$ and $B_7$ the coefficients were compatible with zero, so only the partial wave amplitudes up to $T_{4^-}$ were present.

The partial wave non-resonant backgrounds were parameterised by a linear combination of Legendre Polynomials

$$T = \sum_{n=0}^{N} (A_n + i B_n) P_n(k')$$

where $k'$ was a linear function of the incident kaon momentum running from -1 to +1 across the region of the experiment. Legendre polynomials were used because inclusion of higher polynomials would not affect the values of A and B already calculated. In addition, the $\Xi(1770)$ was inserted with fixed parameters and phase, $\phi = 0$, in the $T_2^+$ partial wave.

The first fits, with all non-resonant backgrounds linear in $k$ ($N = 1$), established the positive parity of the $\Sigma(2030)$. This fit required 28 background parameters (no non-resonant background was inserted in the $T_3^+$ wave) but was about 4 standard deviations out.

An extension to cubic backgrounds ($N = 3$) resulted in a confidence level of about 40% but this improvement was also achieved by making only the $T_3^-$ and $T_1^+$ amplitudes cubic and leaving the rest linear (36 background parameters instead of 56).

The $T_3^-$ structure could be accounted for by a resonance,
Fig. 4.5. Distribution of cosine of γ decay angle in the Σ° decay frame for \( \Lambda - \Sigma \) ambiguous events with seen spectators.

Fig. 4.6. Distribution of cosine of γ decay angle in the Σ° decay frame for \( \Lambda - \Sigma \) ambiguous events with unseen spectators.
\[ J^P = \frac{1}{2}^+ \] at the low energy end of the region. Some data (at beam momenta 1.134, 1.153, 1.173 and 1.183 GeV/c) were added from a previous experiment (13) and the \( \Xi(1910) \) was established. The best fitted parameters for the \( \Xi(2030) \) and \( \Xi(1910) \) are given in Table 4.2.

If a resonance, \( J^P = \frac{3}{2}^+ \), were postulated to account for the \( T_1^+ \) structure it would have mass approximately 2060 MeV and width approximately 180 MeV, though this was not pursued.

4.5 The B.E.G.I. Analysis

Two separate analyses were done on the B.E.G.I. collaboration data. One performed on a sample of 1500 events having seen spectators only, spectator momentum between 80 and 280 MeV/c (see Chapters 2 and 3). This gave a particularly clean sample of \( \Lambda \) events for three reasons. First, inserted spectator fits, which might not be as kinematically sound as those with seen spectators, were excluded. Second, for \( \Lambda - \Sigma \) ambiguities most \( \Lambda \)'s could be selected by accepting as \( \Lambda \) fits those events which had the cosine of the \( \gamma \) decay angle in the \( \Sigma^0 \) fit between \(-1.0\) and \(-0.8\) (see Fig. 3.15). But when this distribution is separated for seen and unseen spectators (Figs. 4.5 and 4.6) the selection includes virtually all the \( \Lambda \) events for seen spectators but many fewer for unseen spectators. So the \( \Sigma^0 \) contamination is a minimum for seen spectator events. Lastly the non-impulsive events which will have obvious kinematic biases are excluded.
The second analysis, including the unseen spectator events, but not the non-impulsive events, gave a great increase in statistics (3700 events). However, the fits obtained tended to have lower confidence levels than those for the first analysis, despite more elaborate background parameterisation being possible. Perhaps this was a reflection of the relative quality of the events used.

For both analyses the $A$ and $B$ coefficients were calculated by method of moments

$$\frac{A_n}{A_0} = \frac{(2n+1)}{2(2n+1)} \sum W \frac{P_n(\theta)}{\sum W}$$  \hspace{1cm} (4.15)

$$\frac{B_n}{A_0} = \frac{3(2n+1)}{(n+1)\sigma^2} \sum W \frac{P_n(\theta) \cos \xi}{\sum W}$$  \hspace{1cm} (4.16)

The summations extend over all the events in the energy bin (8 bins for the first analysis and 11 bins for the second). $W$ is the individual event weight for short and long-lived decay, and $\xi$ is the angle between the proton direction in the decay frame of the $\Lambda$ and the production normal. $A_0$ was calculated from total cross section

$$A_0 = \sigma / 4\pi \times 2$$  \hspace{1cm} (4.17)

These coefficients were calculated up to $A_8$ and $B_8$ and fitted by partial wave amplitudes up to $T_4^+$. The background parameterisation in each partial wave was at most cubic.

$$T = A P_0(u) + B P_1(u) + C P_2(u) + i[D P_0(u) + E P_1(u) + G P_2(u) + H P_3(u)]$$  \hspace{1cm} (4.18)

where $P_n(u)$ are Legendre polynomials

and $u = (k - 0.68)/0.0735$. 
### TABLE 4.3
Resonance Parameters obtained from the 'Seen Spectator' Partial Wave Analysis of the B.E.G.I. Collaboration

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Spin $J$</th>
<th>Parity $P$</th>
<th>Mass $E_R$ (MeV)</th>
<th>Width $\Gamma$ (MeV)</th>
<th>Elasticity $\sqrt{X_e X_R}$</th>
<th>Phase $\phi$ (fixed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma(1910)$</td>
<td>$3/2^-$</td>
<td>$+$</td>
<td>$1898^{+8}_{-8}$</td>
<td>$56^{+23}_{-23}$</td>
<td>$0.13^{+0.02}_{-0.02}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\Sigma(2030)$</td>
<td>$5/2^-$</td>
<td>$+$</td>
<td>$2023^{+8}_{-6}$</td>
<td>$123^{+14}_{-14}$</td>
<td>$0.19^{+0.01}_{-0.01}$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$\Sigma(2080)$</td>
<td>$3/2^-$</td>
<td>$+$</td>
<td>$2129^{+12}_{-12}$</td>
<td>$94^{+38}_{-38}$</td>
<td>$0.24^{+0.05}_{-0.05}$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

### TABLE 4.4
Resonance Parameters obtained from 'All Spectator' Partial Wave Analysis of the B.E.G.I. Collaboration

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Spin $J$</th>
<th>Parity $P$</th>
<th>Mass $E_R$ (MeV)</th>
<th>Width $\Gamma$ (MeV)</th>
<th>Elasticity $\sqrt{X_e X_R}$</th>
<th>Phase $\phi$ (fixed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma(1910)$</td>
<td>$3/2^-$</td>
<td>$+$</td>
<td>$1903^{+10}_{-10}$</td>
<td>$77^{+27}_{-27}$</td>
<td>$0.09^{+0.02}_{-0.02}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\Sigma(2030)$</td>
<td>$5/2^-$</td>
<td>$+$</td>
<td>$2027^{+6}_{-6}$</td>
<td>$158^{+16}_{-16}$</td>
<td>$0.19^{+0.01}_{-0.01}$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$\Sigma(2080)$</td>
<td>$3/2^-$</td>
<td>$+$</td>
<td>$2082^{+4}_{-4}$</td>
<td>$87^{+20}_{-20}$</td>
<td>$0.16^{+0.03}_{-0.03}$</td>
<td>$0$</td>
</tr>
</tbody>
</table>
These were added to Breit-Wigner resonances (where present) except of the $T_3^+$ wave which was assumed to have no non-resonant background.

For the first analysis (seen spectator events only) all background parameterisation was constant ($T = A + iE$). The $\Sigma(1770)$ was inserted, with fixed parameters (12) and phase $\phi = 0$, in the $T_2^+$ partial wave and the $\Sigma(2030)$ was inserted in the $T_3^+$ partial wave. The fit was improved if a resonance was inserted in either $T_3^-$ or $T_0^+$ or $T_1^+$ partial waves. The $T_0^+$ "resonance" had fitted mass 2100 MeV and elasticity, $\sqrt{\frac{\Sigma}{\Xi}} = 0.36 \pm 0.12$. It seemed highly improbable that such a pronounced resonance would have gone unnoticed, however the structure could adequately be parameterised by the background. The $T_3^-$ and $T_1^+$ resonances had masses approximately 1900 MeV and 2130 MeV respectively. Therefore the best fit was for these two resonances and the $\Sigma(2030)$ (Table 4.3). This fit required 16 background parameters and had a confidence level of 53%/0.

With the addition of unseen spectator events, the increased statistics permitted more elaborate background parameterisations, which made the insertion of the $\Sigma(1770)$ unnecessary. This analysis approximately repeated the above method. The resonance parameters for the best fit are shown in Table 4.4. 31 background parameters were used and the confidence level of the fit was 0.27%/0.

4.6 Conclusions

All three analyses agree upon the presence of the resonances, $\Sigma(2030), J^P = \frac{7}{2}^+$, and $\Sigma(1910), J^P = \frac{5}{2}^+$. However
the $\Sigma(2080)$, $J^P = \frac{3}{2}^+$, was only observed in the C.R.S. and B.E.G.I. analyses, though only the latter was definite about its resonant nature. Of the two further studies only Litchfield (6) observed this resonance (mass ~ 2070 MeV) but both Litchfield and Barbaro-Galtieri (7) found Smart's $\Sigma(1880)$ but at 1920 MeV and 1950 MeV respectively. In addition, both these analyses found evidence for a $\Sigma(1940)$ resonance, $J^P = \frac{3}{2}^-$. 

These results demonstrate the strength and weakness of this method of partial wave analysis. The background criterion, partial wave amplitudes slowly varying but arbitrary functions of energy, allows sufficient freedom (normally a maximum of eight parameters per partial wave) to ensure a reasonably definite fit, provided the dominant resonances are inserted in their appropriate partial wave. However the background can mask less pronounced resonance structure. In the B.E.G.I. analysis, the $\Sigma(1770)$, a well-established resonance lying outside the energy region, was included, both with fixed parameters on top of a constant background and also within a quadratic background. In the C.R.S. analysis, structure near the edges of the energy region (at 1910 MeV and 2060 MeV) was fitted alternatively as resonance or background. More data was included to establish the $\Sigma(1910)$ resonance. However, the structure at 2060 MeV remained within a cubic background. Even when resonant nature is established the background can affect the resonance parameters (e.g. the different estimates of the masses of the $\Sigma(1880)$ and $\Sigma(2080)$). So the determination of whether structure is resonant (and if so, determination of its parameters) or not, is made difficult by the freedom in background
parameterisation.

The Generalised Interference Model, introduced in Chapter 1, will certainly provide a far more restricted background parameterisation. Hypercharge exchange scattering has been successfully described by this model, in the intermediate energy region \(^{(14)}\), using a background which was predicted by two Regge trajectories. Each trajectory possessed eight parameters, two of which were well known and four of which were known to an order of magnitude. Extending this method to the low energy region may result in loss of definition which the restricted background parameterisation cannot remove but, if accurate, this parameterisation should overcome the difficulty of identifying less pronounced resonant structure.
REFERENCES FOR CHAPTER 4

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(10) A.D. Martin & T.D. Spearman "Elementary Particle Theory"
5.1 Potential Scattering

For any physical scattering process the momenta of the particles involved have to be real and any interaction must occur in definite angular momentum states \((l = 0, 1, 2...\)). Useful information can be derived by extending the definition of the scattering amplitude to unphysical regions of complex momentum and angular momentum. The process to achieve this is called analytic continuation and will first be applied to the relatively simple case of non-relativistic potential scattering of spinless particles.

The wave function for potential scattering can be obtained by solving the Schrödinger equation, which, if the potential function is central, reduces to the radial form:

\[
\frac{d^2 U(r)}{dr^2} + \frac{l(l+1)}{r^2} U(r) - 2M(V(r) - E) U(r) = 0 \quad (5.1)
\]

\(M\) is the reduced mass of the two particles.

\(2ME = k^2\) where \(k\) is the centre of mass momentum.

The interaction potential is taken to be a linear combination of Yukawa potentials, since this best describes the strong interaction of particles.

\[
V(r) = \sum_n c_n \exp(-\mu_n r)/r \quad (5.2)
\]

As \(r \to \infty\) the Schrödinger equation tends to the form:-
\[
\frac{d^2 U(r)}{dr^2} + k^2 U(r) = 0 \tag{5.3}
\]

which has solutions \( U(r) = e^{\pm ikr} \).

A general solution for \( U(r) \) will therefore have asymptotic form:

\[
U(r) = A \exp[ikr - \frac{i\pi}{2}(\ell+1)] + B \exp[-ikr + \frac{i\pi}{2}(\ell+1)] \tag{5.4}
\]

The terms \( \frac{i\pi}{2}(\ell+1) \) are necessary for the definition of the constants \( A \) and \( B \).

The first term in equation (5.4) represents a purely outgoing spherical wave of amplitude \( A \) and the second term a purely incoming wave of amplitude \( B \). The ratio of these two amplitudes is by definition the S-matrix

\[
S = \frac{A}{B} \tag{5.5}
\]

In fact \( A \) and \( B \) are not constants but functions of \( k \) defined for fixed value of \( \ell \). Therefore \( S \) will also be a function of \( k \) for fixed \( \ell \). To emphasise this equation (5.5) is written in the form

\[
S_\ell(k) = \frac{A_\ell(k)}{B_\ell(k)} \tag{5.6}
\]

By unitarity \( |S_\ell(k)| = 1 \) so \( S_\ell(k) \) can also be written in terms of a phase shift in the \( \ell \)-th partial wave \( \delta_\ell(k) \)

\[
S_\ell(k) = \exp[2i \delta_\ell(k)] \tag{5.7}
\]

Equation (5.4) would serve to define \( A_\ell(k) \) and \( B_\ell(k) \) over the whole complex \( k \)-plane provided both functions are
Fig. 5.1 The cut k-plane over which $\Phi_\ell(k)$ and $\Phi_\ell(k)$ are analytic.

Fig. 5.2 Position of singularities (crosses) of $S_\ell(k)$ in the cut k-plane.

Fig. 5.3 Position of singularities (crosses of $S_\ell(k)$ in the cut E-plane.)
analytic in the whole region. In fact the functions are
analytic over a cut complex $k$-plane. The cuts are
$\text{Im}(k) = \mu \to \infty$ and $\text{Im}(k) = -\mu \to -\infty$ where $\mu$ is
the minimum $\mu_n$ from equation (5.2). (Fig. 5.1).

$S_\ell(k)$ will also be analytic over this region except
for poles which occur when $B_\ell(k) = 0$. It can be shown
from the causal and time reversal properties of the
Schrödinger equation that these poles occur on the positive
Im($k$) axis or symmetrically about the negative Im($k$) axis
(Fig. 5.2).

As $E = \frac{k^2}{2\hbar}$, $S_\ell(k)$ will be analytic except for
poles on the cut $E$-plane (Fig. 5.3).

For a pole $B_\ell(k) = 0$,

$$U(r) = A_\ell e^{ikr} .$$

(5.8)

For $E$ real and negative

$$k = iK \quad (K \text{ real and greater than zero}),$$

$$U(r) = A_\ell(k)e^{-Kr} .$$

(5.9)

The wavefunction in equation (5.9) is characteristic of a
bound state - the negative energy corresponding to the
binding energy.

For one of the poles in the $R_\ell(E)$ region

$$E_p = E_1 - iE_2 .$$

(5.10)

Near this pole $B_\ell(k)$ can be written
\[ B(k) = C(E - E_p) \]  \hspace{1cm} (5.11)

Also

\[ B_k^*(k) = C^*(E^* - E_p^*) \] \hspace{1cm} (5.12)

As

\[ B_k^*(k) = A_k(k) \] \hspace{1cm} (5.13)

Then equation (5.12) becomes

\[ A(k^*) = C^*(E^* - E_p^*) \] \hspace{1cm} (5.14)

For real \( k \), \( E \) is also real, \( k = k^*; \ E = E^* \)

\[ S_\ell(k) = \frac{C^*(E - E_p^*)}{C(E - E_p)} \] \hspace{1cm} (5.15)

\[ S_\ell(k) = \frac{C^*}{C} \frac{E - E_1 - iE_2}{E - E_1 + iE_2} \] \hspace{1cm} (5.16)

(provided \( E_2 \) is small).

This is the Breit-Wigner formula for a resonance with mass \( E_1 \) and width \( 2E_2 \). So these poles give rise to Breit-Wigner peaks in cross section in the physical region. The poles therefore can be identified with resonances.

Extending the \( S \)-matrix to complex values of angular momentum \( A \) and \( B \) will still be defined by equation (5.14) but will now be functions of the variables \( \ell \) and \( E \) (chosen in preference to \( k \)). Equation (5.15) becomes

\[ S(\ell,E) = \frac{A(\ell,E)}{B(\ell,E)} \] \hspace{1cm} (5.17)

\( A(\ell,E) \) and \( B(\ell,E) \) are analytic in the region \( \text{Re}(\ell) > -\frac{1}{2} \) and in the cut \( E \)-plane already defined (Fig. 5.3). \( S(\ell,E) \) can therefore be defined for this region and is analytic in it except for singularities. These singularities
Fig. 5.4 A typical Regge trajectory for a Yukawa potential projected onto the complex $t$-plane.

Fig. 5.5 A typical Regge trajectory for a Yukawa potential, shown on a Chew-Frautschi Plot. The square corresponds to a bound state and the dots to resonances.
are called Regge poles. In general the position of a Regge pole will be defined by a function:

\[ \phi(t,E) = 0 \quad (5.18) \]

This will define a two-dimensional surface in the four-dimensional space of complex \( t \) and \( E \). In general two two-dimensional surfaces in four-dimensional space will intersect at a point. For example

\[ t = n \quad (i.e., \quad \text{Re}(t) = n; \quad \text{Im}(t) = 0) \]

where \( n \) is a positive integer) gives the plane shown in Fig. 5.3, the intersection with \( \phi(t,E) = 0 \) being one of the poles.

The plane \( \text{Re}(E) = \text{const.}, \quad \text{Im}(E) = 0 \) is shown in Fig. 5.4. As \( \text{Re}(E) \) changes its value the pole traces out a trajectory on the complex \( t \)-plane.

The following features of the trajectory are consequences of the Schroedinger equation

1) For \( E < 0 \) the trajectory lies on the \( \text{Re}(t) \) axis.

2) At \( E = 0 \) (the scattering threshold) the trajectory moves into the complex \( t \)-plane with \( \text{Im}(t) > 0 \).

3) For \( E > 0 \) the trajectory curves backward into the region \( \text{Re}(t) < -\frac{3}{2} \).

The shape of the trajectory drawn in Fig. 5.4 is characteristic of a Yukawa potential.

Projecting \( \phi(t,E) = 0 \) onto the \( \text{Re}(E), \quad \text{Re}(t) \) plane produces a Chew-Frautschi Plot (Fig. 5.5). The physical states correspond to integer values of \( t \). For \( E < 0 \) these correspond to bound states and for \( E > 0 \) resonances. Hence
the Chew-Frautschi plot links resonances and bound states with different masses and spins. In general a potential will give rise to several of these trajectories.

In potential scattering the properties of Regge poles follow from the Schrödinger equation. Scattering of elementary particles, relativistic scattering, has no equivalent of the Schrödinger equation, so the properties of Regge poles derived from potential scattering are merely modified to preserve such relativistic properties as Lorentz invariance, unitarity and crossing symmetry.

5.2 Mandelstam Representation

When relativistic scattering is considered it is normal to use the Mandelstam Representation. In this, the basic interaction involves an initial state of particle 1 and particle 2, which scatter producing particle 3 and particle 4 in the final state. This is written: \( 1 + 2 \rightarrow 3 + 4 \)

\[
\begin{align*}
& P_1 & \rightarrow & -P_3 \\
& P_2 & \rightarrow & -P_4 \\
\end{align*}
\]

The outgoing momenta \(-P_3\) and \(-P_4\) are so close to make the Mandelstam variables, which will be defined below, symmetric in the labels 1, 2, 3, 4.

For simplicity all four particles are spinless and of mass \(m\). The amplitude for the total reaction is a function of two variables normally taken to be the total energy \(W\) and the cosine of the centre of mass scattering angle \(\theta\).
It is convenient to define three new variables, the Mandelstam variables:

\[ s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \]
\[ t = (p_1 + p_3)^2 = (p_2 + p_4)^2 \]  \hspace{1cm} (5.19)
\[ u = (p_1 + p_4)^2 = (p_2 + p_3)^2 \]

These are not independent as

\[ s + t + u = 4m^2. \]  \hspace{1cm} (5.20)

As all the masses are equal then the magnitudes of all the momenta in the centre of mass frame will be equal. This variable \( k_s \) together with \( z_s \) can be used as independent variables describing the scattering, so \( s, t \) and \( u \) can be expressed in terms of them.

\[ s = W_s^2 = 4(k_s^2 + m^2) \]
\[ t = -2k_s^2 (1 - z_s) \]  \hspace{1cm} (5.21)
\[ u = -2k_s^2 (1 + z_s) \]

For physical scattering \( k_s \) must be real and 
\(-1 < z_s < 1\) which gives the conditions

\[ s > 4m^2 \]
\[ t < 0 \]
\[ u < 0 \]  \hspace{1cm} (5.22)

For the process \( 1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \) (the bar denoting antiparticle) the scattering amplitude will be a function of
Fig. 5.6 Mandelstam Plot showing the region of physical scattering (shaded) in $s$, $t$, and $u$ channels for equal mass particles.

Fig. 5.7 Mandelstam Plot showing regions of physical scattering for the reaction $K^- n \rightarrow \pi^- \Lambda$. 
\( W_t \) (or \( k_t \)) and \( z_t \). So \( s, t \) and \( u \) become:

\[
s = -2k_t^2(1 - z_t) \\
\]

\[
t = W_t^2 = 4(k_t^2 + m^2) \\
\]

\[
u = -2k_t^2(1 + z_t) \\
\]

and for physical scattering

\[
s < 0 \\
\]

\[
t > 4m^2 \\
\]

\[
u < 0 \\
\]

Similarly for the reaction \( 1 + 4 \to 2 + 3 \)

\[
s = -2k_u^2(1 + z_u) \\
\]

\[
t = -2k_u^2(1 - z_u) \\
\]

\[
u = W_u^2 = 4(k_u^2 + m^2) \\
\]

and for physical scattering

\[
s < 0 \\
\]

\[
t < 0 \\
\]

\[
u > 4m^2 \\
\]

The reaction \( 1 + 2 \to 3 + 4 \) is called an \( s \)-channel reaction, \( 1 + 3 \to 2 + 4 \) a \( t \)-channel reaction and \( 1 + 4 \to 2 + 3 \) a \( u \)-channel reaction, the name merely denoting which of \( s, t \) or \( u \) represents the total energy.

As \( s + t + u = \text{const.} \), then these variables can be represented on a triangular plot (Mandelstam plot) (Fig. 5.6).

If the masses of the particles are not all equal the physical regions for the three scattering processes are not as defined in equations (5.22), (5.24) and (5.26). For the
scattering process \( K^- p \rightarrow \pi^- \Lambda \), the physical regions are shown in Fig. 5.7. Notice that as \( s \) or \( t \) or \( u \) tends to infinity the physical regions asymptotically approach the "equal masses" physical region.

5.3 Scattering Amplitude

For a physical scattering process the incoming particles travel in the same direction, so should be represented by a plane wave and not a spherical wave. The wavefunction for such a process will therefore be of the asymptotic form

\[
\psi(r) = e^{-ikz} + f(z,k) \frac{e^{ikr}}{r} \quad (5.27)
\]

where the incoming beam is along the \( z \)-axis. \( f(z,k) \) is called the scattering amplitude.

The wavefunction can also be expressed in a partial wave expansion in terms of \( U_\ell(r) \) defined by equation (5.1).

\[
\psi(r) = \sum_\ell (2\ell+1) \frac{U_\ell(r)}{r} P_\ell(z) \quad (5.28)
\]

\( U_\ell(r) \) has asymptotic form given by equation (5.4), therefore \( \psi(r) \) has asymptotic form

\[
\psi(r) = \frac{1}{r} \sum_\ell (2\ell+1) A_\ell(k) \exp\left[ikr - \frac{1}{2}(\ell+1)\pi\right] P_\ell(z) \\
+ \frac{1}{r} \sum_\ell (2\ell+1) B_\ell(k) \exp\left[-ikr + \frac{1}{2}(\ell+1)\pi\right] P_\ell(z) \quad (5.29)
\]

Equation (5.27) can be expanded in partial waves

\[
\psi(r) = \frac{1}{2kr} \sum (2\ell+1) \left[(-1)^\ell e^{-ikr} - e^{ikr}\right] P_\ell(z) \\
+ \frac{1}{r} \sum (2\ell+1) f_\ell(k) e^{ikr} P_\ell(z) \quad (5.30)
\]
Fig. 5.8  The position of singularities (crosses) on the cut $t$-plane.
Equating coefficients of $e^{ikr}$ and $e^{-ikr}$ in equations (5.29) and (5.30) gives:

$$e^{-\frac{1}{2}(l+1)\pi} A_\ell(k) = \frac{1}{21k} + f_\ell(k)$$

(5.31)

$$e^{\frac{1}{2}(l+1)\pi} B_\ell(k) = \frac{(-1)^l}{21k}$$

(5.32)

\[ \therefore S_\ell(k) = \frac{A_\ell(k)}{B_\ell(k)} = 1 + 21k f_\ell(k) \]

(5.33)

or

$$f_\ell(k) = \frac{S_\ell(k) - 1}{21k}$$

(5.34)

From now on the scattering amplitude will be considered instead of the S-matrix. Nevertheless the analyticity properties of $f(\ell,E)$ will be identical to those of $S(\ell,E)$.

5.4 Crossing Symmetry

For a t-channel process it is convenient to consider the scattering amplitude as a function of the complex variable $t$ and $\ell$. The amplitude $f^t(\ell,t)$ will be defined in the region $\Re \ell > -\frac{1}{2}$ and in the cut $t$-plane shown in Fig. 5.8.

$f^t(\ell,t)$ will be analytic in this region except for poles also shown in Fig. 5.8. These poles correspond to bound states if $\Re(t) < 4m^2$ or resonances if $\Re(t) > 4m^2$ ($\Re(t) = 4m^2$ now being the scattering threshold). At these points $\Re(t)$ will be the mass squared of the bound state or resonance.

Strong interaction processes are controlled by a number of conservation laws — baryon number, Isospin, charge, strangeness and C-parity are all conserved. A reaction will
therefore be characterised by a set of conserved quantum numbers. As all reaction amplitudes with the same set of quantum numbers will be coupled, by unitarity, a singularity in one amplitude implies a singularity in all. Therefore all such reactions will have the same Regge poles or Regge-trajectories could be considered as having characteristic sets of quantum numbers, and can be coupled with all channels with the same set of quantum numbers.

The existence of exchange forces in relativistic scattering can also be used. In t-channel scattering there are two possible exchanges

where again it is assumed that $m_1$ and $m_2$ are spinless particles.

Near the s-pole

$$f_s^t(t) \sim \frac{g_1}{s - m_1^2} = \frac{g_1}{-2k_t^2(1 - z_t) - m_1^2} \quad (5.35)$$

and near the u-pole

$$f_u^t(t) \sim \frac{g_1}{s - m_1^2} = \frac{g_2}{-2k_t^2(1 - z_t) - m_1^2} \quad (5.36)$$

Equation (5.35) is the Fourier transform of the Yukawa potential $-g_1 \exp(-\mu r)/r$ and as
Fig. 5.9  A Regge Trajectory for \( \Delta \) particles (\( B=1, S=0, I=\frac{3}{2}, P=+1 \)). The signature of this trajectory is \(-\).

Fig. 5.10  Regge Trajectories for \( \Delta \) particles (\( B=1, S=-1, I=0 \)). The squares represent positive parity particles (negative signature) and the cross, negative parity particles (positive signature). The two trajectories coincide and are therefore exchange degenerate.

Fig. 5.11  Regge Trajectories for \( N \) particles (\( B=1, S=0, I=\frac{1}{2} \)). The squares represent positive parity particles (negative signature) and the crosses, negative parity particles (positive signature).
\[
\int_{-1}^{1} \frac{dz_t \, P_\ell(z_t)}{-2k_t^2(1+z_t)-m_2^2} = (-1)^\ell \int_{-1}^{1} \frac{dz_t \, P_\ell(z_t)}{-2k_t^2(1-z_t)-m_2^2}
\]

then equation (5.36) is the Fourier transform of the potential
\[- (-1)^\ell g_2 \exp(-\mu_2 r)/r.
\]

The total contribution of these two exchanges to the interaction potential of the \(\ell\)-th partial wave is
\[
V_\ell(r) = - \left[ g_1 \exp(-\mu_1 r)/r + (-1)^\ell g_2 \exp(-\mu_2 r)/r \right].
\]

Therefore the force will be different in odd and even \(\ell\) states. So two different amplitudes can be defined
\[
\begin{align*}
\ell^+ \quad & \text{for} \quad \ell = 0, 2, 4 \ldots . \\
\ell^- \quad & \text{for} \quad \ell = 1, 3, 5 \ldots .
\end{align*}
\]

These amplitudes are separately continued into the complex \(\ell\)-region giving rise to two Regge trajectories which are in general unrelated. To distinguish them a signature factor (the plus or minus of the original definition of the amplitudes) is assigned to the trajectories.

As the parity of a bound state or resonance
\[ P = P_A P_B (-1)^\ell, \]
where \(P_A\) and \(P_B\) are the intrinsic parities of the external particles, then the parities of all bound states and resonances on a trajectory must be the same as \(\ell\) varies in steps of 2 along the trajectory.

Where the external particles are no longer spinless it is found that the total angular momentum, \(J\), still varies in steps of 2. Some examples of assignment of resonances to Regge trajectories are shown on Chew-Frautschi plots in Figs. 5.9, 5.10 and 5.11.
5.5 Sommerfeld-Watson Transformation

The total amplitude will therefore be

\[ f^t(t, z_t) = \sum_{\text{even } \ell} (2\ell+1) f^t_\ell(t) P_\ell(z_t) \]
\[ + \sum_{\text{odd } \ell} (2\ell+1) f^t_\ell(t) P_\ell(z_t) \quad (5.39) \]

If two new amplitudes are defined for either signature

\[ f^\pm(t, z_t) = \sum_{\text{all } \ell} (2\ell+1) f^\pm_\ell(t) P_\ell(z_t) \quad (5.40) \]

then

\[ f^t(t,z_t) = \frac{1}{2} [f^+(t,z_t) + f_+^+(t, -z_t)] \]
\[ + \frac{1}{2} [f^-_t(t,z_t) - f^-_t(t, -z_t)]. \quad (5.41) \]

The amplitudes \( f^\pm(t,z_t) \) can be extended into the complex \( \ell \)-region using the Sommerfeld-Watson transformation.

For a function \( F(x) \) analytic in the \( x \)-plane except for poles, then over a closed integral \( \Gamma \)

\[ \oint_{\Gamma} F(x) dx = 2\pi i \sum_n \text{Res } F(x_n) \quad (5.42) \]

where \( \text{Res } F(x_n) \) are the residues of the \( n \) poles enclosed by the contour.

\[ \text{Res } F(x_n) = \lim_{x \to x_n} (x - x_n) F(x) \quad . \quad (5.43) \]

Consider the function

\[ F^+(\ell) = \frac{(2\ell+1) f^+_\ell(t,z_t) P_\ell(-z_t)}{2i \sin \pi \ell} \quad (5.44) \]
Fig. 5.12 The complex \( \ell \)-plane showing the two contours of integration \( \Gamma \) and \( \Gamma' \).
which is analytic only for \( \text{Re}(t) > -\frac{1}{2} \) and has poles where \( t \) is equal to a real integer. Then integrating over the contour shown in Fig. 5.12 gives

\[
\oint_{\Gamma} F^\pm(t) dt = 2\pi i \sum_n \text{Res} F(t=n)
\]

\[
= 2\pi i \sum_n \lim_{t \to n} (2n+1) f^\pm(t) \frac{(t-t)P_n(-z_t)(t-n)}{2i \sin \pi t}
\]

\[
= \sum_n (2n+1) f^\pm(n, t) P_n(z_t) \tag{5.45}
\]

where \( n = 0, 1, 2, \ldots \).

The right hand side of equation (5.45) is identical to \( f^\pm(t, z_t) \) as defined in equation (5.40).

Integrating \( F^\pm(t) \) over the new contour \( \Gamma' \) (Fig. 5.12) gives

\[
\oint_{\Gamma'} F^\pm(t) dt = \int_{-i\infty}^{i\infty} F^\pm(t) dt + \int_{\text{semi-circle}} F^\pm(t) dt \tag{5.46}
\]

For a Yukawa type potential the second integral vanishes as the radius of the semi-circle approaches infinity.

The contour \( \Gamma' \) encloses the poles along the \( \text{Re}(t) \) axis as well as all the singularities (Regge poles) of \( f^\pm(t, t) \).

\[
\oint_{\Gamma'} F^\pm(t) dt = \int_{-i\infty}^{i\infty} F^\pm(t) dt
\]

\[
= 2\pi i \sum_{t=n} \text{Res} F^\pm(t) + 2\pi i \sum_{\text{Regge poles}} \text{Res} F^\pm(t) \tag{5.47}
\]

The first summation in equation (5.47) is simply the
amplitude \( f^\pm (t, z_t) \). If the \( i \)-th Regge pole has position on the \( \ell \)-plane \( \alpha_i(t) \) (hereafter called \( \alpha \) for simplicity) the residue of \( f^\pm (t, z_t) \) at this point is \( R_i^\pm (t) \) then:

\[
\text{Res } F^\pm (a) = \frac{(2a+1) R_i^\pm (t) \, P_a(-z_t)}{2i \sin \pi a} .
\]  

(5.48)

As \( -z_t \to \infty \)

\[ \begin{align*}
\text{a}) & \quad P_a(-z_t) \to \frac{\Gamma(a+\frac{1}{2}) (-2z_t)^a}{\Gamma(a+1) \Gamma(\frac{1}{2})} \\
\text{b}) & \quad 2\pi i \text{ Res } F^+(a) \to \frac{2\pi R_1^+(t)}{\Gamma(\frac{1}{2})} \frac{\Gamma(a+\frac{3}{2})}{\Gamma(a+1)} (-2z_t)^a \\
\text{c}) & \quad \int_{-i\infty}^{i\infty} F^+(\ell) d\ell \to 0 .
\end{align*} \]  

(5.49)  

(5.50)  

(5.51)

Therefore equation (5.47) becomes

\[
f^\pm (t, z_t) \to \sum_{i^\pm} \beta_i^\pm (t) \frac{\Gamma(a+\frac{3}{2})}{\sin \pi a} (-2z_t)^a
\]  

(5.52)

where the \( \beta_i^\pm (t) \) contain the remaining constants and \( t \)-dependent terms. Substitution into equation (5.41) gives

\[
f^\pm (t, z_t) \to \sum_{i=1}^{\infty} \beta_i(t) \frac{1 + \xi_i e^{-i\pi a}}{\sin \pi a} \frac{\Gamma(a+\frac{3}{2})}{\Gamma(a+1)} (-2z_t)^a
\]  

(5.53)

where the sum runs over all Regge poles and \( \xi_i \), the signature factor, equals \( +1 \) \((-1)\) if \( \alpha \) has even (odd) signature.

Equation (5.53) can be extended to unphysical values of \( t \) provided \(|t| \) is not too large, otherwise the background integral would dominate the amplitude. For fixed \( t \), small
and less than zero, as \(-z_t \to \infty\), \(s \to \infty\) and \(u \to -\infty\), which is within the physical \(s\)-channel region.

So \(f^t(t, z_t) = f^s(s, z_s)\). \(\quad (5.54)\)

From equations (5.23) it can be shown that

\[-2z_t = \frac{s-u}{2(-\frac{t}{4} + m^2)} \quad (5.55)\]

which can be written

\[-2z_t = \frac{s-u}{2s_0} \quad (5.56)\]

where \(s_0\) is an energy scale factor. So the asymptotic form of the \(s\)-channel scattering amplitude is:

\[f^s(s, z_s) = \sum_i \beta_i(t) \frac{1+\xi_4 e^{-i\kappa}}{\sin \pi \alpha} \frac{\Gamma(\alpha + \frac{3}{2})}{\Gamma(\alpha+1)} \left(\frac{s-u}{2s_0}\right) \quad (5.57)\]

Similarly for Regge poles in the \(u\)-channel, the asymptotic form of the scattering amplitude as \(z_u \to \infty\) and for \(u\) small and negative would be:

\[f^u(s, z_s) = \sum_j \beta_j(u) \frac{1+\xi_4 e^{-i\kappa}}{\sin \pi \alpha} \frac{(\alpha+3)}{(\alpha+1)} \left(\frac{s-u}{2s_0}\right) \quad (5.58)\]

The Regge amplitude in equation (5.7) will be used in the next chapter as a parameterisation of the non-resonant background in the analysis of the reaction \(K^- n \to \pi^- \Lambda\) at 1.45 and 1.65 GeV/c. More rigorous derivations of this equation can be found in many textbooks, (for example "Phenomenological Theories of High Energy Scattering", V.D. Barger and D.B. Cline, published by W.A. Benjamin, Inc. (1970), and "Elementary Particle Physics and Scattering Theory" edited by M. Chretien and S.S. Schweber, published by Gordon and Breach (1967)).
CHAPTER 6

6.1 Introduction

The reaction $K^- n \rightarrow \pi^- \Lambda$ is of the type pseudoscalar meson + spin $\frac{1}{2}$ baryon going to pseudoscalar meson + spin $\frac{1}{2}$ baryon. The presence of spin results in a second scattering amplitude being required. The amplitude, $f$, is identified with the spin nonflip amplitude and $g$, the spin flip amplitude, is introduced. The interaction matrix is therefore:

$$M = f + ig \sigma \cdot \hat{n}$$

(6.1)

where $\sigma$ are the Pauli spin matrices

and $\hat{n}$ is the unit vector normal to the interaction plane.

However, neither $f$ nor $g$ possess the analyticity properties required by Regge Theory. Instead, the Mandelstam invariant amplitudes, $A$ and $B$, are used.

$$A = \frac{\mu E}{L} \left\{ (\sqrt{s} + M)f + \frac{E}{\sqrt{1 - z_s^2}} \left[ (\sqrt{s} + M)z_s + \frac{(\sqrt{s} + M)E^2}{p_1 p_2} \right] \right\}$$

(6.2)

$$B = \frac{\mu L}{E} \left\{ f + \frac{E}{\sqrt{1 - z_s^2}} \left[ z_s - \frac{E}{p_1 p_2} \right] \right\}$$

(6.3)

where $E = \left[ (E_1 + m_1)(E_2 + m_2) \right]^{\frac{1}{2}}$

(6.4)

and $M = \frac{1}{2} (m_1 + m_2)$.

(6.5)

The incoming (outgoing) baryon has mass $m_1$ ($m_2$), centre of mass energy $E_1$ ($E_2$) and centre of mass momentum $p_1$ ($p_2$).
\( \mu_1 (\mu_2) \) is the mass of the incoming (outgoing) meson, 
\( s, t, u \) are the Mandelstam variables and \( z_s \) is the cosine of the s-channel scattering angle.

Another set of amplitudes, useful because of their ease of physical interpretation, are the helicity amplitudes, \( F_{ab} \), where \( b = + (-) \) if the incoming baryon has helicity \( +\frac{1}{2} (-\frac{1}{2}) \) and \( a = + (-) \) if the outgoing baryon has helicity \( +\frac{1}{2} (-\frac{1}{2}) \). Of these four amplitudes, only two are independent.

\[
F_{--} = F_{++} \\
F_{-+} = -F_{+-}
\]

by parity conservation.

A statement of the Generalised Interference Model is therefore:

\[
F_{+\pm} = F_{+\mp} + F_{+\pm} 
\]

(6.7)

6.2 *Regge Amplitude*

The \( t \)-channel helicity amplitudes can be expressed in terms of \( A \) and \( B \).

\[
F_{++} = A \left[ t - (m_1 + m_2)^2 \right]^{\frac{1}{2}} + \frac{BM}{[t - (m_1 + m_2)^2]^{\frac{1}{2}}} \left[ s - u + \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \left( \frac{\mu_1^2 - \mu_2^2}{m_1 - m_2} \right) \right] 
\]

(6.8)

\[
F_{+-} = \frac{Bd^{\frac{1}{2}}}{[t - (m_1 + m_2)^2]^{\frac{1}{2}}}
\]

(6.9)

where \( d = stu - t(\mu_1^2 - m_1^2)(\mu_2^2 - m_2^2) - s(m_1^2 - m_2^2)(\mu_1^2 - \mu_2^2) \)

\[-(\mu_1m_2^2 - \mu_2m_1^2)(\mu_1^2 + m_2^2 - \mu_2^2 - m_1^2)\]
However, in these definitions singularities have been introduced at \( t = (m_1 + m_2)^2 \). So to retain the analyticity properties of the Mandelstam invariant amplitudes, two new amplitudes are defined.

\[
G_+ = - \left[ t - (m_1 + m_2)^2 \right]^{\frac{1}{2}} F^t_+ \tag{6.11}
\]

\[
G_- = - \left[ t - (m_1 + m_2)^2 \right]^{\frac{1}{2}} d^{-\frac{1}{2}} F^t_- . \tag{6.12}
\]

Applying a Sommerfeld-Watson transformation to these amplitudes gives:

\[
G_+ \sim \sum \beta_+ \frac{1 + i e^{-i\alpha}}{\sin \pi \alpha} \frac{\Gamma(a + \frac{3}{2})}{\Gamma(a + 1)} \left(\frac{s-u}{s+} \right)^a \tag{6.13}
\]

\[
G_- \sim \sum \beta_- \frac{1 + i e^{-i\alpha}}{\sin \pi \alpha} \frac{\Gamma(a + \frac{3}{2})}{\Gamma(a + 1)} \left(\frac{s-u}{s-} \right)^{a-1} \tag{6.14}
\]

Equation (6.13) is identical to the equation derived in Chapter 5 (equation 5.57) and equation (6.14) can be derived by a similar method. If \( G_+ \) are defined over the whole complex \( t \)-plane, the term \( (\sin \pi \alpha)^{-1} \) will also produce singularities in the region \( \text{Re}(a) < \frac{-1}{2} \). These poles will be "killed" by the term \( \Gamma(a+1) \), so \( (\sin \pi \alpha \Gamma(a+1))^{-1} \) will be analytic over the whole region \( \text{Re}(a) < \frac{-1}{2} \). As only the first of the poles in \( (\sin \pi \alpha)^{-1} \), \( a = -1 \), will have much effect in the region \( \text{Re}(a) > \frac{-1}{2} \) then \( (\sin \pi \alpha (a+1))^{-1} \) can be approximated to \( (a+1)/\sin \pi \alpha \).

The functions \( \left[ \beta_+ \Gamma(a + \frac{3}{2}) \right] \) do not have poles at \( a = -\frac{3}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots \), by Mandelstam symmetry, so \( \Gamma(a + \frac{3}{2}) \) can be incorporated into the \( \beta_+ \).
The pole at \( \alpha = 0 \) corresponds to a 'ghost' state which, in the t-channel, appears to represent a particle with negative mass. For odd signature this singularity is eliminated, but for even signature the "ghost-killing" factor, \( \alpha \), has to be included.

So equations (6.13) and (6.14) become:

\[
G_+ = \beta^+_+ \frac{1 + e^{-i\pi\alpha}}{\sin \pi\alpha} \alpha(\alpha+1) \left( \frac{s-u}{2s^+_+} \right) ^\alpha \\
G_- = \beta^-_- \frac{1 + e^{-i\pi\alpha}}{\sin \pi\alpha} \alpha(\alpha+1) \left( \frac{s-u}{2s^-_-} \right) ^{-1} 
\]  

for even signature,

and

\[
G_+ = \beta^+_+ \frac{1 - e^{-i\pi\alpha}}{\sin \pi\alpha} (\alpha+1) \left( \frac{s-u}{2s^+_+} \right) ^\alpha \\
G_- = \beta^-_- \frac{1 - e^{-i\pi\alpha}}{\sin \pi\alpha} (\alpha+1) \left( \frac{s-u}{2s^-_-} \right) ^{-1} 
\]  

for odd signature.

The terms \( \beta^i \) contain the remaining \( \alpha \) and \( t \) dependent terms. Arbab and Chiu\(^{(1)}\) found good experimental agreement by assuming that \( \beta^-_- \) was proportional to \( \alpha \). Reeder and Sarma\(^{(2)}\) assumed that all four amplitudes vanished at the point, \( \alpha = -1 \). The term \( (\alpha+1) \) fulfilled this requirement for the even signature amplitudes but for the odd signature amplitudes, \( \beta^-_- \) were assumed to be proportional to \( (\alpha+1) \).

Therefore if these \( \alpha \) dependent terms are removed from the \( \beta^i \), the remaining functions, \( \gamma^+ \), should be \( t \)-dependent only.

For even signature
\[ G_+ = \sum \gamma_+ \frac{1 + e^{-i\alpha}}{\sin \pi a} \alpha(a+1) \left( \frac{s-u}{2s^+} \right)^a \]  
(6.19)

\[ G_- = \sum \gamma_- \frac{1 + e^{-i\alpha}}{\sin \pi a} \alpha^2(a+1) \left( \frac{s-u}{2s^-} \right)^{a-1} \]  
(6.20)

For odd signature

\[ G_+ = \sum \gamma_+ \frac{1 + e^{-i\alpha}}{\sin \pi a} (a+1)^2 \left( \frac{s-u}{2s^+} \right)^a \]  
(6.21)

\[ G_- = \sum \gamma_- \frac{1 - e^{-i\alpha}}{\sin \pi a} \alpha(a+1)^2 \left( \frac{s-u}{2s^-} \right)^{a-1} \]  
(6.22)

where the functions \( \gamma_+ \) are assumed to be constant or at most linear functions of \( t \).

For the reaction \( K^- n \rightarrow \pi^- \Lambda \), the exchange particle must have \( I = \frac{1}{2}, S = 1, B = 0 \), i.e. a \( K \) meson trajectory.

The trajectory must have natural parity, by conservation of spin and parity at the meson vertex. There are only two such Kaon resonances, the \( K^\pi \) (mass = 890 MeV; \( J^P = 1^- \)) and the \( K^{*\pi} \) (mass = 1420 MeV; \( J^P = 2^+ \)). The \( K^* \) lies on a trajectory with odd signature and the \( K^{*\pi} \) on a trajectory with even signature. To the total Regge contribution is:-

\[ G_\pi = G_A^\pi + G_B^\pi \]  
(6.23)
where the superscript \( A(B) \) refers to the \( K^+ (K^-) \) trajectory.

\[
G_+^A = D_+^A (a_A + 1)^2 \left( 1 + \tan \frac{\pi a_A}{2} \right) R_+^A \quad (6.24)
\]

\[
G_-^A = D_-^A (a_A + 1)^2 \left( 1 + \tan \frac{\pi a_A}{2} \right) R_-^A \quad (6.25)
\]

\[
G_+^B = D_+^B (1 + t/t_B) a_B (a_B + 1) \left( 1 - \cot \frac{\pi a_B}{2} \right) R_+^B \quad (6.26)
\]

\[
G_-^B = D_-^B a_B^2 (a_B + 1) \left( 1 - \cot \frac{\pi a_B}{2} \right) R_-^B \quad (6.27)
\]

where \( R_+^A = \left( \frac{s-u}{2(m_1+m_2)E^A_+} \right) ^{\mp 1/2} (a_A^{\mp \frac{1}{2}}) \quad (6.28) \)

and \( R_+^B = \left( \frac{s-u}{2(m_1+m_2)E^B_+} \right) ^{\mp 1/2} (a_B^{\mp \frac{1}{2}}) \quad (6.29) \)

As the \( t \)-dependences of the terms \( \gamma \) are unknown, then the simplest solution is to assume that they are all constant. However Reeder and Sarma found it necessary to make one of the terms linear in \( t \). Polarisation data\(^{(3)}\) on the reaction \( \pi^+ p \to K^+ \Sigma^+ \) (which is related to the \( K^- n \to \pi^- \Lambda \) reaction by crossing and \( SU(3) \) symmetries) showed polarisation changing sign at \( t \sim 0.5 \text{ GeV}^2 \) and becoming very large as \( t \) decreases. A change in sign of polarisation can be produced near a change in sign of \( a_A \) or \( a_B \). But whichever one is selected, the effect of the other has to be suppressed. The simplest method of achieving this is to make the \( G_+ \) amplitude of the chosen trajectory linear in \( t \). Consideration of differential cross section suggested that it is better to choose the \( K^{*+} \) trajectory. The \( t \)-dependence of \( \gamma_B \) is extracted as the term \( (1 + t/t_B) \), where \( t_B \) is of the order of \( 0.5 \text{ GeV}^2 \), and the terms \( D \) are unknown constants.
6.3 Resonant Amplitude

The partial wave expansions for the spin nonflip and spin flip amplitudes are:

\[ f = \kappa \sum_\ell [(\ell+1) T^\ell_+ + \ell T^\ell_-] P^\ell(z_s) \]  
\[ g = \kappa \sum_\ell [T^\ell_+ - T^\ell_-] P^\ell(z_s) \]

(Chapter 4, equations (4.6) and (4.7)).

If only Breit-Wigner resonances are present, these expansions can be written:

\[ f = \sum_{\text{Res}} (J + \frac{1}{2}) P^J_\ell(z_s) \left( \frac{1}{p_1 p_2} \right)^{\frac{1}{2}} \frac{\sqrt{x_e x_r}}{\varepsilon - 1} \]  
\[ g = \sum_{\text{Res}} (-1)^{J-\ell-\frac{1}{2}} P^J_\ell(z_s) \left( \frac{1}{p_1 p_2} \right)^{\frac{1}{2}} \frac{\sqrt{x_e x_r}}{\varepsilon - 1}. \]

The summations extend over all the resonances present.

The elasticity \( \sqrt{x_e x_r} \), can take positive or negative value depending on whether the resonance has phase 0 or \( \pi \).

\[ \varepsilon = \frac{s_R - s}{\sqrt{s_R}} \]

where \( s_R \) is the resonance mass squared.

Substitutions into equations (6.2) and (6.3) then into (6.8) and (6.9) give the helicity amplitudes for the resonant contribution.

In Chapter 4, section 2, it was stated that there was an overall phase ambiguity in defining scattering amplitudes. This is overcome by taking the phase of one resonance to be 0 or \( \pi \), and calculating the phases of all other resonances.
relative to it. The background is sufficiently free to enter in the appropriate phase, whatever the initial choice of reference phase. However, in this analysis, the phase of the Regge amplitude may not be so flexible, so that, although the fixing of the phase of one resonance will define the phases of all other resonances (either 0 or $\pi$), the background amplitude may not be compatible with this.

So an overall relative phase ($\delta$) is allowed between resonant and Regge amplitudes. Equation (6.7) therefore becomes:

$$F_{++} = F_{++}^{\text{Regge}} + e^{i\delta} F_{++}^{\text{Res}} \quad (6.35)$$

Lastly the differential cross section can be written:

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s p_1^2} (|F_{++}|^2 + |F_{+-}|^2) \quad (6.36)$$

and polarisation times differential cross section can be written:

$$-\mathbf{p} \cdot \frac{d\sigma}{dt} = \frac{1-z_s^2}{16\pi \sqrt{s} \Delta} \cdot \frac{p_1}{p_2} \text{Im}(F_{++}^* F_{+-}) \quad (6.37)$$

The differential cross section with respect to $z_s$ is easily calculated:

$$\frac{d\sigma}{dz_s} = \frac{d\sigma}{dt} \cdot \frac{dt}{dz_s} = 2 \, p_1 p_2 \frac{d\sigma}{dt} \quad (6.38)$$
TABLE 6.1

Energy Bin Limits and Mean Bin Energies.

<table>
<thead>
<tr>
<th>Bin Number</th>
<th>Lower Limit (MeV)</th>
<th>Upper Limit (MeV)</th>
<th>Mean Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1870</td>
<td>1930</td>
<td>1905</td>
</tr>
<tr>
<td>2</td>
<td>1930</td>
<td>1965</td>
<td>1950</td>
</tr>
<tr>
<td>3</td>
<td>1965</td>
<td>1990</td>
<td>1979</td>
</tr>
<tr>
<td>4</td>
<td>1990</td>
<td>2010</td>
<td>2000</td>
</tr>
<tr>
<td>5</td>
<td>2010</td>
<td>2030</td>
<td>2020</td>
</tr>
<tr>
<td>6</td>
<td>2030</td>
<td>2050</td>
<td>2039</td>
</tr>
<tr>
<td>7</td>
<td>2050</td>
<td>2085</td>
<td>2065</td>
</tr>
<tr>
<td>8</td>
<td>2085</td>
<td>2150</td>
<td>2113</td>
</tr>
</tbody>
</table>
6.4 The Data

The purpose of this analysis was to take data already fitted by the B.E.G.I. collaboration\(^{(5)}\) and fit it using the parameterisation given above. The seen spectator data was chosen for this trial because it was better fitted in the original analysis. So \(K^{-}n \to \pi^{-}\Lambda\) events were selected on the following criteria:

a) Spectator momentum between 80 and 280 MeV/c

b) Where an event was ambiguous with a \(\Sigma^{0}\)-fit, events with the cosine of the decay angle of \(\gamma\) in the \(\Sigma^{0}\) frame greater than -0.8 were discarded.

c) Events with effective mass of the \(\Lambda p\) combination between 2120 and 2160 MeV and also spectator momentum greater than 200 MeV/c were discarded as this region corresponded to a possible di-baryon bound state\(^{(6)}\).

The events thus selected were divided into 8 centre of mass energy bins (Table 6.1) and the angular distributions plotted for events weighted for short and long lifetime losses (Fig. 6.1). The errors plotted are statistical

\[
\Delta (\Sigma \omega) = \sqrt{\Sigma \omega^2}, \quad \text{(see Appendix 3)},
\]

where \(\omega\) are the individual event weights and the summations are over all the events in the angular bin.

Polarisations were calculated from the decay asymmetry of the proton in the \(\Lambda\) decay frame (Fig. 6.2).

\[
P_0 = \frac{3}{a_{\Lambda}} \cos \frac{1}{3} \quad \text{(see Chapter 4, eq. 4.10)}
\]

\[
\Delta P_0 = \left[ \frac{3 - (a_{\Lambda} P_0)}{n a_{\Lambda}} \right]^\frac{1}{2} \quad \text{(see Appendix 3)},
\]

where \(n\) is the total number of events in the polarisation bin.
Fig. 6.1 Angular Distribution Data and best fitted curves; for limits of the Energy Bins, see Table 6.1.
Fig. 6.2  (Continued)
Fig. 6.2  Polarisation data and best fitted curves; for limits of Energy Bins, see Table 6.1.
Fig. 6.2 (Continued).
The theoretical differential cross section and polarization were calculated as continuous functions of the centre of mass energy, \( E \), and the cosine of the centre of mass scattering angle, \( z \). So a double integration is required to predict the expected occupancy of any angular bin (limits \( E_1 \), \( E_2 \), \( z_1 \) and \( z_2 \)).

\[
N(E_1 \text{ to } E_2; z_1 \text{ to } z_2) = \frac{1}{E_2 - E_1} \int_{E_1}^{E_2} Q(E) dE \int_{z_1}^{z_2} \frac{d\sigma}{dz} dz
\]  

(6.39)

\( Q(E) \) is the dynamical centre of mass energy distribution normalised to the experimental microbarn equivalent and depends on the following factors:

1) The incoming beam momentum spread (B).
2) The energy spread due to the Fermi motion of the neutron (\( H \)).
3) A virtual neutron flux factor (F).

The beam spread was assumed to be Gaussian of full width 30 MeV/c for both beam momenta.

\[
B(p) = \frac{W_1}{\sqrt{2\pi} \sigma} \exp \left[ -\left( p - \bar{p}_1 \right)^2 / \sigma^2 \right] + \frac{W_2}{\sqrt{2\pi} \sigma} \exp \left[ -\left( p - \bar{p}_2 \right)^2 / \sigma^2 \right] \]

(6.40)

where \( p \) is the kaon beam momentum

\[
\bar{p}_1 = 1.45 \text{ GeV/c} \\
\bar{p}_2 = 1.65 \text{ GeV/c} \\
\sigma = 0.015 / \sqrt{2ln2} 
\]

The factors \( W_1 \) and \( W_2 \) are beam normalisations. The microbarn equivalent for the 1.45 GeV/c film was 3.0 events/\( \mu \)b and the total correction factor for processing, unseen decays,
shadowing (see Chapter 3), was 1.93. So \( \frac{w_1}{1.93} = 1.55 \) events seen/\( \mu b \).

Similarly for the 1.65 GeV/c film

\( w_2 = \frac{1.6}{2.09} = 0.77 \) events seen/\( \mu b \).

In the laboratory system a \( K^-n \) interaction appears thus:-

BEFORE

\[ \overrightarrow{K^-} (E_K, \overrightarrow{p}) \] (stationary)

AFTER

\[ (K^-n) \]

\[ \text{Spectator proton} \ (E_S, \overrightarrow{p}_S). \]

The \( K^-n \) centre of mass energy is:

\[
E = \sqrt{(E_K + m_d - E_S)^2 - (\overrightarrow{p}_K - \overrightarrow{p}_S)^2}
\]

(6.41)

If the interaction is impulsive then the distribution of \( p_S \) is given by the Hulthen momentum wave function.

\[
\phi_H \propto \frac{1}{(p_S^2 + \alpha^2)(p_S^2 + \beta^2)} \quad \alpha = 45.7 \text{ MeV} \]

\[
\beta = 238 \text{ MeV} \]

(see Chapter 3, Section 6).

The resulting \( E \) distribution is:

\[
H(E) = K \frac{E}{p} \int \frac{p_S \, dp_S}{[(p_S^2 + \alpha^2)(p_S^2 + \beta^2)]^{3/2}}
\]

(6.42)

where \( K \) is a normalisation factor.

Finally the virtual neutron flux factor, which is required to convert the theoretic neutron cross section to the physical neutron-in-the-deuteron cross section, has the
Fig. 6.3  Centre of mass energy distribution of events used in this analysis. The dynamically predicted distribution is also shown.
form (7):-

\[
F(E) = \frac{E p_1}{p E_s}
\]

where \( p_1 \) is the incoming centre of mass momentum. The overall normalisation factor, \( K' \), is obtained from the equation

\[
\int_{E_{\text{min}}}^{\infty} H(E) F(E) \, dE = 1 \quad (6.44)
\]

Therefore

\[
Q(E) = K' E p_1 \int \frac{B(p) dp}{p^2} \int \frac{p_s \, dp_s}{(p_s^2 + \alpha^2)(p_s^2 + \beta^2) E_s^2}.
\]

(6.45)

For seen spectators, the limits of integration of \( p_s \) are 80 to 280 MeV/c. \( Q(E) \), normalised to the total number of events, is plotted in Fig. 6.3 together with the experimental centre of mass energy distribution.

As the theoretical functions do not vary rapidly with energy they are evaluated at the mean energy of each bin. Comparison with a numerical integration showed that this simplification resulted in, at most, a 4% error. Therefore equation (6.39) becomes:

\[
N(E_1 \text{ to } E_2; z_1 \text{ to } z_2) = Q(E) \int_{z_1}^{z_2} (\frac{d\sigma}{dz})_E \, dz
\]

(6.45)

and

\[
P(E_1 \text{ to } E_2; z_1 \text{ to } z_2) = \hat{n} \int_{z_1}^{z_2} \left( \frac{d\sigma}{dz} \right)_E \, dz
\]

\[
\int_{z_1}^{z_2} \left( \frac{d\sigma}{dz} \right)_E \, dz
\]

(6.46)

A numerical integration in steps of 0.5 in \( z \) was found to be sufficient for evaluating \( N \) and \( P \).
6.5 Fitting

The experimental data, $\sum_\omega$ and $P_0$, were fitted to the theoretically derived numbers, $N$ and $P$, using the CERN minimisation program, MINUIT (long version). The function minimised was chi squared, whose derivation is given in Appendix 3.

$$\chi^2 = \sum_{\text{Ang Bins}} \frac{(\sum_\omega - N)^2}{\sum_\omega^2} + \sum_{\text{Pol. Bins}} \frac{a_\alpha n(P - P_0)^2}{(3 - a_\lambda P)} \quad (6.47)$$

$a_\alpha$ was taken to be quadratic in $t$ ($a_\alpha = 0.35 + 0.96t + 0.16t^2$, and $a_B$ linear in $t$ ($a_B = 0.24 + 0.69t$). The expression for $a_\alpha(a_B)$ was calculated by Reeder and Sarma from the $\rho(A_2)$ trajectory using SU(3) and is only approximately constrained to pass through the $K^*(K^{*\mp})$ pole in the positive $t$ region. Of the remaining background parameters, the energy scale factors, $E$, should be of the order of 1 GeV and the crossover term, $r_B$, should be approximately 0.5 GeV$^2$.

The initial fit used Reeder and Sarma's best fitted values (Table 6.2 column 1). The $\Sigma(2030)$ was inserted with fixed parameters (mass 2030 MeV, width 130 MeV and elasticity 0.2), and preliminary testing suggested a starting value of $\delta$, the phase between Regge and resonant amplitudes (equation (6.35)), of $\frac{\pi}{2}$. Using these parameters a chi squared of 540 was calculated for 180 data points (10 angular and polarisation bins in each of the 8 energy bins).

Using the TAUROS minimising routine of MINUIT the chi squared was reduced to 331 in 200 iterations, allowing the $E$ and $D$ factors, the crossover term and the phase to vary.
TABLE 6.2

Background Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^+_A$</td>
<td>25</td>
<td>25</td>
<td>24</td>
</tr>
<tr>
<td>$D^-_A$</td>
<td>0.44</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>$E^+_A$</td>
<td>2.12</td>
<td>0.53</td>
<td>0.59</td>
</tr>
<tr>
<td>$E^-_A$</td>
<td>1.81</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>$D^+_B$</td>
<td>60</td>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td>$D^-_B$</td>
<td>975</td>
<td>300</td>
<td>200</td>
</tr>
<tr>
<td>$E^+_B$</td>
<td>0.58</td>
<td>0.15</td>
<td>0.1</td>
</tr>
<tr>
<td>$E^-_B$</td>
<td>0.15</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>$t_B$</td>
<td>0.526</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>1.57</td>
<td>1.48</td>
<td>1.8</td>
</tr>
</tbody>
</table>


### TABLE 6.3

**Resonance Parameters**

<table>
<thead>
<tr>
<th>Values used in Fitting</th>
<th>Previously Determined Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) (2) (3) (4)</td>
<td>S  C  B  L  G</td>
</tr>
<tr>
<td><strong>Σ(2030)</strong></td>
<td></td>
</tr>
<tr>
<td>Mass (MeV)</td>
<td>2030  2037  2040  2039</td>
</tr>
<tr>
<td>Width (MeV)</td>
<td>130   200   184   185</td>
</tr>
<tr>
<td>Elasticity</td>
<td>0.2   0.203  0.19  0.19</td>
</tr>
<tr>
<td><strong>Σ(2080)</strong></td>
<td></td>
</tr>
<tr>
<td>Mass (MeV)</td>
<td>2085  2070  2078  2071</td>
</tr>
<tr>
<td>Width (MeV)</td>
<td>75    150   150   144</td>
</tr>
<tr>
<td>Elasticity</td>
<td>0.14  0.13  0.11  0.11</td>
</tr>
<tr>
<td><strong>Σ(1910)</strong></td>
<td></td>
</tr>
<tr>
<td>Mass (MeV)</td>
<td>1910  1896  1897  1890</td>
</tr>
<tr>
<td>Width (MeV)</td>
<td>70    33    30    39</td>
</tr>
<tr>
<td>Elasticity</td>
<td>0.1   0.07  0.05  0.048</td>
</tr>
</tbody>
</table>

| S  - Smart's Analysis  |
| C  - C.R.S. Analysis   |
| B  - B.E.G.I. Analysis |
| L  - Litchfield's Analysis |
| G  - Barbaro-Galtieri's Analysis |

*Summary of previously determined data from reference (8).*
No convergence was achieved, in fact convergence was never achieved throughout the analysis. Instead TAUROS terminated when either the time limit or the limit of calls to the routine which calculated chi squared was exceeded.

Inserting the \( \Sigma (2080) \) and \( \Sigma (1910) \) with fixed parameters (Table 6.3, column 1) and both with fixed phase of \( \pi \) relative to the \( \Sigma (2030) \), reduced chi squared to 304 on minimisation. Allowing the resonance parameters to vary further reduced chi squared to 269 (Table 6.3, column 2). During these minimisations the contribution to chi squared from the polarisation data remained constant, approximately 150. To make these data more significant the number of polarisation bins was reduced, with at least twenty events being required in each bin (Fig. 6.2).

The energy scale factors had consistently dropped during minimisation and so were arbitrarily set at a quarter of Reeder and Sarma's values. The \( D \) terms had also dropped, so were set at considerably reduced values. These starting values (Table 6.2, column 2) together with the original resonance starting values (Table 6.3, column 1) gave a chi squared of 320 for 134 data points (80 angular bins and 54 polarisation bins).

Further minimisations on various combinations of the parameters \( E, D, t_B \) and \( \phi \) produced only minimal improvement in chi squared (the total reduction was 25). But allowing resonance parameters to vary brought the chi squared down to 247 (Table 6.3, column 3).

The parameters of \( \alpha_A \) and \( \alpha_B \) were floated, producing a small improvement without greatly affecting \( \alpha_A \) and \( \alpha_B \).
Further minimisations allowing resonance parameters and the 
\( E_+ \) and \( D_+ \) terms (the \( E_- \) and \( D_- \) could vary markedly 
with little effect on chi squared) to vary reduced the chi 
squared to 235 (Table 6.3, column 4 and Table 6.2, column 3).

Finally, the other two resonances, which had been observed in the energy region though not by the B.E.G.I. collaboration, namely the \( \Xi(1880) \), \( J^P = \frac{3}{2}^+ \), and the \( \Xi(1940) \) 
\( J^P = \frac{3}{2}^- \), were inserted. The phases of these two resonances 
and of the \( \Xi(2080) \) were allowed to float with respect to the 
\( \Xi(2030) \) phase. The chi squared was reduced to 203. The 
resulting \( \Xi(2080) \) and \( \Xi(1910) \) phases, 3.54 and 3.59 respectively, were consistent with \( \pi \). However the \( \Xi(1880) \) 
and \( \Xi(1940) \) phases were 1.19 and 1.87 respectively, so the 
resonance interpretation was very doubtful. Also the reduction in chi squared was merely due to the introduction of new 
parameters which decrease the number of degrees of freedom of 
the fit.

The best fit was therefore given by the background listed 
in Table 6.2, column 3 and resonance parameters in Table 6.3, 
column 4. This fit is shown in Figs. 6.1 and 6.2.

6.6 Conclusions

The fitting terminated without convergence, with chi 
squared indicating a confidence level of approximately \( 10^{-9} \) 
for the best fit. Any further reduction in chi squared was 
only achieved by introducing more parameters, thereby re-
ducing the number of degrees of freedom, with no improvement
in confidence level.

The polarisation fits (for example Fig. 6.2) never really followed the trend of the data. However the relatively large errors ensured that the contribution to chi squared was not too large. But this resulted in a lack of sensitivity. During minimisation the major reduction in chi squared was achieved by improving the angular fit. A much more accurate polarisation fit would be required to resolve the Minami ambiguity (see Chapter 4, Section 2) to determine the intrinsic parity of a resonance.

However, there were four positive results from the analysis. Firstly, the margin of failure of the method was only very slight. The initial fit (chi squared of 540 on 160 data points using fixed parameters) was surprisingly good and indicated considerable physical validity of the method. But the background amplitude was too inflexible or too unsubtle to fit the experimental data accurately. This may have resulted from the assumptions and simplifications made in its derivation. For example, the $\alpha$-dependences of the terms $\beta'$ were assessed at higher energy and these approximations may not remain valid in the low energy region. Also the terms $\gamma$ were assumed to have minimum $t$-dependence. This was very reasonable because, if the correct $\alpha$-dependences had been removed ($\alpha$'s being functions of $t$), the remaining terms $\gamma$ should be at most very slowly varying functions of $t$.

Improved fitting would obviously be possible by allowing the $\gamma$'s to be linear, quadratic or even cubic in $t$, but in doing so, the arbitrariness, which this analysis was supposed
to avoid, would be introduced

Secondly, the innovation of introducing a phase between resonant and Regge amplitudes was justified as, on all minimisations, $\delta$ remained in the region of $\frac{\pi}{2}$.

Thirdly, fairly definite conclusions can be made concerning presence of resonances. The $\Sigma(2030)$ was indispensable in fitting and considerable improvement was achieved by inserting the $\Sigma(2080)$ and $\Sigma(1910)$. In all fits where the parameters were allowed to float, they consistently tended to the same values which were also consistent with previously determined values (Table 6.3). Of course, as minimisation never converged nor confidence level became significant, none of the sets of fitted resonance parameters could be considered as a definite determination. However there were two strong pieces of evidence against the $\Sigma(1880)$ and $\Sigma(1940)$, namely, that there was negligible improvement in confidence level on their insertion and that they were best fitted with unphysical values of phase (in general resonant amplitudes should have relative phases of 0 or $\pi$ - see Chapter 4 and this chapter). So this analysis agreed with the original B.E.G.I. analysis, in as much as the only resonant contributions were those of the $\Sigma(2030)$, $\Sigma(2080)$ and $\Sigma(1910)$.

Finally, there was no evidence of "double counting". This problem was discussed in Chapter 1, Section 10. If the Regge amplitude contained all resonant amplitudes, as duality suggested, then the insertion of Breit-Wigner amplitudes would be unnecessary. Thus, during fitting these would be eliminated
by reduction of their elasticities. However, the most pronounced resonance in the region, the $\Sigma(2030)$, showed no significant reduction in elasticity throughout the analysis and the slight reduction in the elasticities of the $\Sigma(2080)$ and the $\Sigma(1910)$ were still within the limits of previously obtained values (Table 6.3).

REFERENCES FOR CHAPTER 6

7. Reference (5), Appendix.
ACKNOWLEDGEMENTS

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Lastly I would like to thank Professor N. Feather, F.R.S., for allowing me use of the facilities of the Department of Physics, University of Edinburgh and also the University of Edinburgh for the award of a studentship.
APPENDIX 1

Scanning Efficiency

Let \( p, q, r \) and \( s \) be the probabilities of an event being recognised by neither scanner, first scanner only, second scanner only and both scanners respectively

\[ p + q + r + s = 1. \]  

If we assume no correlation between the scanning of the two scanners then we have

\[ \begin{align*}
    p &= (1 - E_1)(1 - E_2) \\
    q &= E_1(1 - E_2) \\
    r &= E_2(1 - E_1) \\
    s &= E_1E_2
\end{align*} \] 

where \( E_1 \) and \( E_2 \) are the scanners' efficiencies.

By eliminating \( E_1 \) and \( E_2 \) in equations (2) we get equation (1) and

\[ ps = qr. \] 

If we have a sample of \( N \) events (\( N \) is of course unknown) of which \( a, b, \) and \( c \) events are detected by first scanner, second scanner and both scanners respectively. The probability for this happening for a particular sample is

\[ P_N = \frac{N! P^{N-a-b-c} q^a r^b s^c}{a!b!c! (N-a-b-c)!} \] 

The expected value of \( a \) is

\[ \bar{a} = \sum_a \sum_b \sum_c a P_N(a) \]

\[ = \frac{Nq}{(a-1)!b!c! (N-a-b-c)!} \]

\[ = Nq (p+q+r+s)^{N-1} \] by the multinomial theorem

\[ = Nq. \]
Similarly \( \bar{b} = N r \) \hspace{1cm} (6)
and \( \bar{c} = N s \) \hspace{1cm} (7)

We obtain the "most likely" values of the unknowns \( N, E_1 \) and \( E_2 \) by equating \( \bar{a} \bar{b} \) and \( \bar{c} \) to the observed values of \( a \ b \) and \( c \).

So \( qr = s(1 - q - r - s) \). From (1) and (3)

\[
\bar{a} \bar{b} = \frac{c(N - \bar{a} - \bar{b} - \bar{c})}{N} = \frac{a + \bar{b} + \bar{c} + \frac{\bar{a} \bar{b}}{c}}{a + \bar{b} + \bar{c} - \frac{\bar{a} \bar{b}}{c}} \hspace{1cm} (8)
\]

The overall efficiency has the "most likely" value

\[
E = \frac{\bar{a} + \bar{b} + \bar{c}}{N} = \frac{a + \bar{b} + \bar{c} - \frac{\bar{a} \bar{b}}{c}}{a + \bar{b} + \bar{c} - \frac{\bar{a} \bar{b}}{c}} \hspace{1cm} (9)
\]

We now require the "standard error". For this we require the error matrix for \( a \ b \) and \( c \).

\[
(\Delta a)^2 = \frac{(a - \bar{a})^2}{a(a - 1)} = \frac{a^2 - a^2}{a(a - 1)} \hspace{1cm} (10)
\]

Now \( a(a - 1) = \sum \sum \sum a(a - 1) P_N(a, b, c) \)

\[
= q^2 N(N - 1) \sum \sum \sum (N - 2)! p^{N-a-b-c} q^{a-2} r^b s^c (a-2)! b! c! (N-a-b-c)! \]

\[
= q^2 N(N - 1)(p + q + r + s)^{N-2} \hspace{1cm} (11)
\]

\[
(a - a)^2 = q^2 N(N - 1) + Nq - (qN)^2 \]

\[
= q(1 - q)N = (\Delta a)^2 \hspace{1cm} (12)
\]

Similarly

\[
\frac{(b - \bar{b})^2}{(b - \bar{b})^2} = r(1 - r)N = (\Delta b)^2 \hspace{1cm} (13)
\]

and \( (c - \bar{c})^2 = s(1 - s)N = (\Delta c)^2 \hspace{1cm} (14) \)
\[(a - \bar{a})(b - \bar{b}) = \bar{a}b - a\bar{b}\]  \hspace{1cm} (15)

\[\bar{a}b = \sum_a \sum_b \sum_c abP_N(a,b,c) = N(N-1)qr\]  \hspace{1cm} (16)

So \[(a - \bar{a})(b - \bar{b}) = N(N-1)qr - Nq Nr\]

\[= - Nqr = (\Delta a \Delta b)\]  \hspace{1cm} (17)

Similarly \[(a - \bar{a})(c - \bar{c}) = - Nqs = (\Delta a \Delta c)\]  \hspace{1cm} (18)

and \[(b - \bar{b})(c - \bar{c}) = - Nrs = (\Delta b \Delta c)\].  \hspace{1cm} (19)

The standard error of any function is

\[\left(\Delta f\right)^2 = \left(\frac{\partial f}{\partial a}\right)^2 (\Delta a)^2 + \left(\frac{\partial f}{\partial b}\right)^2 (\Delta b)^2 + \left(\frac{\partial f}{\partial c}\right)^2 (\Delta c)^2\]

\[+ 2 \frac{\partial f}{\partial a} \frac{\partial f}{\partial b} (\Delta a \Delta b) + 2 \frac{\partial f}{\partial a} \frac{\partial f}{\partial c} (\Delta a \Delta c)\]

\[+ 2 \frac{\partial f}{\partial b} \frac{\partial f}{\partial c} (\Delta b \Delta c)\]  \hspace{1cm} (20)

So \[\frac{\left(\Delta f\right)^2}{N} = q\left(\frac{\partial f}{\partial a}\right)^2 + r\left(\frac{\partial f}{\partial b}\right)^2 + s\left(\frac{\partial f}{\partial c}\right)^2\]

\[- (q \frac{\partial f}{\partial a} + r \frac{\partial f}{\partial b} + s \frac{\partial f}{\partial c})^2\]  \hspace{1cm} (21)

If the function is \[N = a + b + c + \frac{ab}{c}\]

\[\frac{\partial t}{\partial a} = 1 + \frac{r}{s}\]

\[\frac{\partial t}{\partial b} = 1 + \frac{q}{s}\]

\[\frac{\partial t}{\partial c} = 1 - \frac{qr}{s^2}\]  \hspace{1cm} (22)
\[ \frac{(\Delta N)^2}{N} = q(1 + \frac{r}{s})^2 + r(1 + \frac{q}{s})^2 + s(1 - \frac{qr}{s^2}) \]

\[ = (q + r + s + \frac{qr}{s})^2 \]

\[ = \frac{qr}{s^2}(q + r + s + \frac{qr}{s}) \quad (23) \]

\[ \therefore \text{ Standard deviation } \]

\[ \Delta N = \frac{1 - \frac{E}{c}}{N} \quad (24) \]
APPENDIX 2

Geometry Efficiency

If \( p \), \( q \), and \( r \) are the probabilities of an event failing geometry both times, passing on the second measure and passing on the first measure respectively then

\[
p + q + r = 1 \quad (1)
\]

If \( E_1 \) is the efficiency of 1 measure

\[
p = (1 - E_1)^2 \\
q = (1 - E_1)E_1 \\
r = E_1 .
\]

Eliminating \( E_1 \) from (2) we get equation (1) and

\[
b = (1 - r)^2 \\
q = (1 - r)r .
\]

If \( N \) is the unknown number of events in the sample and \( a \) and \( b \) the numbers of events passing on first and second measure respectively then by a similar calculation to that of Appendix I, we get

\[
\bar{a} = Nq \quad (5) \\
\bar{b} = Nr . \quad (6)
\]

Taking the "most likely" values of \( a \), \( c \), \( b \) as the observed values, from (4) we get

\[
\bar{a} = (1 - \frac{\bar{b}}{N}) \bar{b} \quad (7)
\]

\[
N = \frac{\bar{b}^2}{\bar{b} - \bar{a}} \quad (8)
\]

and the Efficiency
\[
E = \frac{a + b}{N} = 1 - \frac{a^2}{b^2} \quad . \tag{9}
\]

For the error calculation we will only involve a 2x2 error matrix.

Again \((\Delta a)^2 = q(1 - q)N\)
\[(\Delta b)^2 = r(1 - r)N \quad \tag{10}\]
\[(\Delta a\Delta b) = qrN \quad . \quad \text{See Appendix 1}\]

Taking \(N = \frac{b^2}{b - a}\)
\[
\frac{\partial N}{\partial a} = \frac{b^2}{(b-a)^2} = \frac{r^2}{(r-q)^2} = \frac{1}{r^2} \quad \tag{11}\]
\[
\frac{\partial N}{\partial b} = \frac{b(b-2a)}{(b-a)^2} = \frac{r(r-2a)}{(r-q)^2} = \frac{2r-1}{r^2} \quad \tag{12}\]

\[
\left(\frac{\Delta N}{N}\right)^2 = q\left(\frac{\partial N}{\partial a}\right)^2 + r\left(\frac{\partial N}{\partial b}\right)^2 - (q \frac{\partial N}{\partial a} + r \frac{\partial N}{\partial b})^2 \quad \tag{13}\]
\[
= \frac{r(1-r) + r(2r-1)^2}{r^4} - 1
\]
\[
= \frac{(1-r)^2(2r-r^2)}{r^4}
\]
\[
= \frac{E(1 - E)}{E_1}\]

\[
\therefore \Delta N = \frac{\sqrt{E(1 - E)N}}{E_1} \quad \tag{14}\]
APPENDIX 3

Chi Squared

The data were presented binned in both centre of mass energy \( (E) \) and cosine of the centre of mass scattering angle \( (z) \). If \( k \) is the total number of bins and \( N \) the total number of events, the following experimental numbers were available:

- \( n_i \) number of events in the \( i \)-th bin
- \( c_i \) average value of \( \cos \xi \) in the \( i \)-th bin, where \( \xi \) is the angle between the direction of the proton in the \( \Lambda \) decay frame and the production normal.

Then the probability of an event with energy in the range \( E \) to \( E + dE \) and cosine of scattering angle in the range \( z \) to \( z + dz \) and cosine of proton direction angle in the range \( x \) to \( x + dx \) (where \( x = \cos \xi \)) is:

\[
P(E,z) \left[ 1 + x R(E,z) \right] \, dE \, dz \, \frac{dx}{2}.
\]

With normalisation

\[
\int \int P(E,z) \, dE \, dz = 1.
\]

\( P \) and \( R \) are calculated theoretically from Regge and resonance parameters. \( P_i \) and \( R_i \) are the integrals over the \( i \)-th bin of \( P \) and \( R \).

So the probability of getting the observed occupancy is

\[
L_1 = \frac{N!}{n_1! \ldots n_k!} \left( P_1 \right)^{n_1} \ldots \left( P_k \right)^{n_k}
\]

(1)
\[
\ln L_1 \sim N \ln N - N - \frac{1}{2} \ln(2\pi N) + \sum_{i=1}^{k} \left( n_1 \ln P_i - n_1 \ln n_1 - \frac{1}{2} \ln(2\pi n_1) \right) + \frac{1}{2} \ln(2\pi N). 
\]

This approximation will be valid provided not more than \( \frac{1}{5} \) of the bins have occupancy less than 5.

So
\[
\ln L_1 \sim \sum_{i=1}^{k} \left( n_1 \ln \frac{NP_i}{n_1} - \frac{1}{2} \ln(2\pi n_1) \right) + \frac{1}{2} \ln(2\pi N). 
\]

The maximum likelihood value is given by:
\[
\frac{\partial}{\partial P_i} \ln L_1 = 0 \quad \text{subject to} \quad \sum_{i=1}^{k} P_i = 1 
\]
i.e.
\[
\sum_{i=1}^{k} \frac{n_i}{P_i} dP_i = 0 \quad \text{subject to} \quad \sum_{i=1}^{k} dP_i = 0.
\]

So
\[
\frac{n_i}{P_i} = \lambda \quad \text{where} \quad \lambda \quad \text{is independent of} \quad i,
\]
or
\[
n_i = \lambda P_i.
\]

Summing both sides gives \( \lambda = N \).

Therefore
\[
P_i = \frac{n_i}{N}.
\]

For small deviations
\[
P_i = \frac{n_i}{N} + \varepsilon_i
\]
where \( \sum_{i=1}^{k} \varepsilon_i = 0 \).

So
\[
\frac{NP_i}{n_i} = 1 + \frac{N\varepsilon_i}{n_i}.
\]
\[
\ln L_1 = \sum_{i=1}^{k} n_i \left[ \ln \left( 1 + \frac{N_{e_i}}{n_i} \right) - \frac{1}{2} \ln (2\pi n_i) \right] + \frac{1}{2} \ln (2\pi N) 
\]

(9)

\[
\ln L_1 = \sum_{i=1}^{k} n_i \left[ \frac{N_{e_i}}{n_i} - \frac{1}{2} \left( \frac{N_{e_i}}{n_i} \right)^2 + O(\varepsilon_i^3) \ldots \right] 
\]

(10)

+ other terms independent of \( \varepsilon_i \).

If \( \frac{N_{e_i}}{n_i} \) is very much smaller than 1, the terms in the log series of order \( \varepsilon_i^3 \) and greater can be ignored.

Therefore

\[
\ln L_1 \sim -\frac{N^2}{2} \sum_{i=1}^{k} \frac{\varepsilon_i^2}{n_i} + \text{other terms}, 
\]

(11)

so

\[
\chi^2 = N^2 \sum_{i=1}^{k} \frac{\varepsilon_i^2}{n_i} = \sum_{i=1}^{k} \frac{(N_{P_i} - n_i)^2}{n_i} 
\]

(12)

where \( L_1 \sim \text{const. } \exp (-\chi^2/2) \).

The theoretically predicted number \( N_{P_i} \) is in fact the predicted number \( N_i \) divided by some theoretical weighting factor \( W_i \), which accounts for the systematic loss of events due to short or long lived decays.

So

\[
\chi^2 = \sum_{i=1}^{k} \frac{(N_i/W_i - n_i)^2}{n_i} 
\]

\[
= \sum_{i=1}^{k} \frac{(N_i - W_i n_i)^2}{W_i^2 n_i} 
\]

(13)

No such theoretical weighting factor has been calculated.

Instead, individual events in a bin have been ascribed a weight, \( \omega \) (see Chapter 3, Section 3). So the term \( W_i n_i \) is identified with the sum of the individual weights of the events in the \( i \)-th bin and \( W_i^2 n_i \) with the sum of the squares of the individual weights.
The other observed quantity, polarisation, is given by

\[ \frac{3}{a} \cdot c_1 \]

The likelihood of getting the \( c_1 \) is given by:

\[
L_2 = \prod_{i=1}^{k} \left[ \int \cdots \int \delta(n_1 c_1 - \sum_{j=1}^{n_1} x_j) \right]
\]

\[ \times \left( 1 - R_x x_j \right) \frac{dx_j}{2} \]

For \( n_1 \) large, then by central limits theorem

\[
L_2 = \prod_{i=1}^{k} \left[ (2\pi n_1 \sigma_1^2)^{-\frac{1}{2}} \exp \left[ -\frac{(n_1 c_1 - n_1 \bar{c}_1)^2}{2n_1 \sigma_1^2} \right] \right]
\]

\[ + o(\frac{1}{n_1}) \]

where \( \bar{c}_1 = \int_{-1}^{+1} x(1 + R_1 x) \frac{dx}{2} \)

and \( \sigma_1^2 = \int_{-1}^{+1} (x - \bar{c}_1)^2 (1 + R_1 x) \frac{dx}{2} \)

where \( \bar{c}_1 \) and \( \sigma_1^2 \) are the mean and variance of \( x \) in the i-th bin.

This gives \( \bar{c}_1 = \frac{R_1}{3} \) and \( \sigma_1^2 = \frac{3 - R_1^2}{9} \).
So \( L_2 = \text{const.} \exp\left[-\sum_{i=1}^{k} \frac{n_i(c_i - \frac{R_i}{3})^2}{2\left(\frac{1}{3} - \frac{R_i}{9}\right)}\right] \) \hspace{1cm} (17)

and \( \chi^2 = \sum_{i=1}^{k} \frac{n_i(R_i - 3c_i)^2}{(3 - R_i^2)} \) \hspace{1cm} (18)

The observed polarisations \( P_{o1} = \frac{3c_i}{a_\Lambda} \) and the theoretical polarisations \( P_i = \frac{R_i}{a_\Lambda} \).

So \( \chi^2 = \sum_{i=1}^{k} \frac{a_\Lambda n_i(P_i - P_{o1})^2}{(3 - P_i a_\Lambda)} \) \hspace{1cm} (19)

N.B. The polarisation bins need not be identical to the angular bins. For \( \ell \) polarisation bins

\[ \chi^2 = \sum_{j=1}^{\ell} \frac{a_\Lambda n_j(P_j - P_{o1})^2}{(3 - P_j a_\Lambda)} \] \hspace{1cm} (20)

The total likelihood function for both angular distributions and polarisations:-

\[ L = L_1 \cdot L_2 \] \hspace{1cm} (21)

So \( \chi^2 = \sum_{\text{Ang. bins}} \frac{(n_i - \bar{n}_i)^2}{\bar{n}_i w^2} + \sum_{\text{Pol. bins}} \frac{a_\Lambda n_i(P_i - P_{o1})^2}{(3 - P_j a_\Lambda)} \) \hspace{1cm} (22)