A description is given of the wind-tunnel and its associated equipment, and of their use under the control of a P.D.P.8 computer. An automatic windspeed measuring instrument utilising the shedding of vertices from a cylinder has been designed and tested.

Linear perturbation theory had been extended by including terms of higher order. Numerical results computed from this theory have been compared with experimental measurements made in the laminar boundary layer. The 2nd Harmonic Component has been shown to behave in a manner generally consistent with the predictions of theory. Other 2nd order effects, as predicted, were too small to be detected.

The possibility that at least some of the behaviour characterising the initial stages of transition might be explained by the theory has been explored. Although the available numerical results were not complete, discrepancies, already observed, make the extension of this theory into the transition region, appear a doubtful exercise.

A suggested explanation for the observations made in the early part of the transition region, based upon the appearance and growth of a disturbance not found in the laminar region, has been given. Results obtained in the later part of the transition region seemed to be consistent with the appearance and growth of turbulent spots.
COMPUTER AIDED INVESTIGATION OF FINITE DISTURBANCES IN THE

BOUNDARY LAYER ON A FLAT PLATE

Thesis submitted by

THOMAS ROBERTSON, B.Sc. (Edinburgh)

For the degree of

Doctor of Philosophy

The Fluid Dynamics Unit,
Department of Natural Philosophy,
University of Edinburgh.
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PREFACE

The research described in this thesis, was conducted in the Fluid Dynamics Unit of the Department of Natural Philosophy under the supervision of the director, Dr. M.A.S. Ross, and Dr. J.G. Burns.
SYMBOL TABLE

The undernoted symbols describe the quantities indicated in this list, unless otherwise stated in the text. Less used symbols are defined as they occur in the text.

\( X, x \)  distance from the leading edge of the flat plate.

\( Y, y \)  distance along the plate, perpendicular to the centre line.

\( Z, z \)  distance from the surface of the flat plate.

\( U_0 \)  free stream velocity.

\( U \)  mean velocity at a point in the boundary layer.

\( u' \)  root mean square value of the \( X \) component of fluctuation velocity.

\( v' \)  root mean square value of the \( Y \) component of fluctuation velocity.

\( w' \)  root mean square value of the \( Z \) component of fluctuation velocity.

\( u, v, w \)  instantaneous values of the components of fluctuation velocity.

\( f \)  oscillation frequency; the frequency of the fundamental component.

\( \alpha_r = \frac{2\pi}{\lambda} \)  wave number.

\( \alpha_i \)  complex part of the eigenvalue.

\( \rho \)  density of air.

\( \nu \)  kinematic viscosity

\( \delta \)  boundary layer thickness

\( \delta_1 \)  boundary layer displacement thickness, \( = 1.7208 \left( \frac{v X}{U_0} \right)^{1/4} \) for the Blasius distribution.

\( F \)  non-dimensional frequency \( = \frac{2\pi \nu}{U_0 \delta} \)

\( R_\delta \)  boundary layer Reynolds number.

\( R_W \)  the resistance of a hot-wire anemometer at temperature \( T_W \).
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<td>$R_a$</td>
<td>the cold resistance of a hot-wire anemometer at temperature $T_a$.</td>
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<td>$i$</td>
<td>the current through the hot-wire.</td>
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<tr>
<td>$e'$</td>
<td>the r.m.s. perturbation voltage recorded by a hot-wire anemometer.</td>
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<td>$N$</td>
<td>the frequency of vortex shedding from a cylinder.</td>
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<tr>
<td>$S$</td>
<td>the Strouhal number, $S = \frac{Nd}{U_0}$.</td>
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<tr>
<td>$d$</td>
<td>the diameter of a cylinder.</td>
</tr>
<tr>
<td>$R_D$</td>
<td>the diameter based Reynolds number, $R_D = \frac{U_0d}{v}$.</td>
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CHAPTER 1

HISTORICAL REVIEW

1.1 The Boundary Layer

In 1827 Navier derived the equations later to be known as the Navier-Stokes equations. These equations were to a great extent ignored during the remainder of the 19th century, attention being concentrated on the Euler equations. Viscous forces were known to be small for the fluids most commonly encountered, water and air, and it was hoped that the Euler equations would yield solutions in agreement with experimental results. The difficulty in obtaining exact solutions to the Navier-Stokes equations also encouraged concentration on the simpler Euler equations.

The result of this concentration on non-viscous theory, was seen in the large discrepancies between the predictions of theory, and experimental results. The best known discrepancy was the D'Alembert Paradox - non-viscous theory predicted that the drag on a body moving through a fluid would be zero - a result which was obviously at odds with reality.

No progress had been made towards understanding the contradictions by the end of the 19th century.

The answer to the problem was contained in Stokes' paper of 1851. Stokes treated the problem of a pendulum oscillating in a viscous fluid, by imposing a no slip condition at the surface of the bob of the pendulum. Other assumptions, made to simplify the Navier-Stokes equations, limited the solutions to situations where the velocities were small.
For the next 50 years, it was assumed that the limitation to small velocities, which applied to Stokes' results, applied also to the no slip condition.

In 1904 Prandtl introduced the concept of the boundary layer, a small region of fluid surrounding a body, when the body and fluid are in relative motion. Within this region the velocity changes, from the velocity of the body, to the free stream velocity. Large changes in velocity occur over a short distance, consequently the velocity gradients are large, and viscosity is important in this region. The approach to the free stream velocity at the outer surface is asymptotic. Outside the boundary layer, viscous effects are not important, and non-viscous theory can be applied.

The boundary layer assumption made possible the simplification of the Navier-Stokes equations. The problem of two dimensional flow over a semi-infinite flat plate was solved by Blasius in 1908 in terms of an infinite power series. This solution, which provided the distribution of velocity through the boundary layer, has since been improved.

Boundary layer theory for the flat plate was first verified experimentally by Burgers (1924) and by van der Hegge Zijnen (1925). Better agreement has been obtained by Dryden (1936) and Nikuradse (1942).

1.2 Hydrodynamic Stability

O. Reynolds (1883) investigated the flow of water through cylindrical pipes by introducing coloured liquid into the flow. He recognised two distinct types of flow:

a) Smooth, streamlined flow - Laminar flow
b) Disordered, eddying flow - Turbulent flow.

Reynolds investigated the transition between the two types of flow, and discovered a similarity law. He used pipes of different diameter $D$, different liquids, and different rates of flow $Q$. On the basis of his observations he defined a non-dimensional parameter, since called the Reynolds' Number $R$. 

$$R = \frac{UD}{v} \quad \text{where} \quad U = \frac{Q}{\pi D^2}$$

Reynolds discovered that transition occurred always at almost the same value of $R$. This value, called the critical Reynolds Number $R_{\text{crit}}$, was approximately 2,300. Subsequently it has been shown that $R_{\text{crit}}$ can be greatly increased by reducing disturbances in the fluid, in the laminar region.

There does exist a lower value of $R_{\text{crit}}$ below which the flow will be laminar, whatever disturbances are present.

The discovery of transition predated the work of Prandtl. The presence of the two kinds of flow in the boundary layer of a sphere was demonstrated by G. Eiffel, and by Prandtl (1914). Prandtl accounted for a variation in the drag coefficient of the sphere, as being due to transition.

Rayleigh investigated transition theoretically in a series of papers published between 1880 and 1915. He established the method of small disturbances, by examining the behaviour of small perturbations of a flow. He concluded that for a flow to be stable, any such perturbation must be damped.

Rayleigh considered the eigenvalue problem of the linearised equations of motion for a perturbation. He showed that for a
perfect fluid, the existence of a point of inflexion in the mean flow profile was a necessary condition for instability. Tollmien has since shown this condition to be sufficient.

As an alternative to the eigenvalue problem, attention was given to the energy balance, of the perturbation, and the total flow. Energy is transferred to the perturbation from the mean flow by the action of the Reynolds stresses. At the same time viscous damping has the opposite effect. The balance between these two processes determines whether or not the flow is stable.

Lorentz (1898) and Orr (1907) used this method to find a critical Reynolds number for Couette flow. The values which they obtained were respectively 288 and 177, which compared very poorly with the measured value of 1940. The reason for the poor agreement was that perturbations, which were not physically possible, were taken into account by this method. Only by satisfying the Navier-Stokes equations can such spurious perturbations be eliminated.

Future development was in the direction of the eigenvalue problem, and the energy balance method has been in general ignored.

1.3 Stability of Boundary Layers

Prandtl and Tietjens extended the work of Rayleigh by allowing viscosity to influence the perturbation.

Tietjens (1925) approximated the Blasius profile to a series of straight lines. With this profile he accounted for the viscous layer at the wall. Because the curvature of the velocity profile vanished throughout the boundary layer, the method did not allow consideration of the singularity at the critical layer. The critical
layer occurs where the perturbation velocity and the local velocity of the flow are the same.

Both Prandtl and Tietjens found that consideration of viscosity, resulted in the amplification of disturbances which nonviscous theory had suggested, would be damped. This paradoxical result made inevitable their failure to find a critical Reynolds number.

Heisenberg (1924) was the first to study the stability of a flow with a continuously curved profile. He showed that Poisseuille flow was unstable at large enough Reynolds numbers, but again failed to find a critical Reynolds number.

Tollmien (1929) adapted Heisenberg's method to study boundary layers, approximating the Blasius profile to a straight line and a parabola, thereby ensuring a finite curvature at the critical layer. Tollmien was able to consider viscosity in the critical layer and remove the singularity. Tollmien evaluated a critical Reynolds number for the flow. He also showed that disturbances in a certain frequency range would be amplified, and that all other disturbances would be damped. These results are normally expressed by a 'neutral stability diagram' on the \((\Phi, R_0)\) plane. The two branches of the curve are made up of points at which disturbances neither amplify nor damp. Inside the curve, all disturbances amplify, outside the curve all disturbances damp. Such a curve is shown in Fig. 1.1.

Schlichting recalculated and extended the results of Tollmien. In 1933 he calculated curves of constant amplification for the unstable oscillations contained within the neutral stability curve. Schlichting (1935) found the distribution of the eigenfunction
Fig. 4.1: The Neutral Stability Curve.
through the boundary layer for neutral oscillations on each side of the neutral stability curve.

The small disturbances are known as Tollmien-Schlichting waves.

The work which has been described, was concerned only with two-dimensional disturbances. Squire (1933) showed that for flow between parallel planes, a three-dimensional perturbation was equivalent to a two-dimensional perturbation at a lower Reynolds number. This result implied that to find the critical Reynolds number, only the two-dimensional disturbance need be considered. It must be emphasized that Squire's theorem applies only to small, time-dependent perturbations.

While the developments of small disturbance theory, which have been described, were being made, an alternative theory of transition was put forward by Taylor (1936).

Taylor suggested that a laminar boundary layer was inherently stable to small disturbances. His explanation for breakdown was based on the likelihood of free stream fluctuations entering the boundary layer. When this happened the pressure gradients associated with a fluctuation were expected to cause flow reversal and separation which would in turn result in breakdown.

The boundary layer was supposed to become more likely to take up a fluctuation, as it became thicker with increasing Reynolds number. Consequently a given level of disturbance in the free stream would only cause transition above a certain minimum Reynolds number. The analysis, based on the Karman-Polhausen theory of the velocity distribution in a boundary layer, related the free stream turbulence to the transition Reynolds number.
At this stage, with two apparently contradictory explanations for the transition process, experiments were performed to try to see whether either explanation was in accord with transition behaviour.

1.4 Boundary Layer Stability - Experiment

Dryden (1934, 1936, 1939) made a thorough investigation of boundary layer transition. Using spheres of different sizes in flows with different turbulence levels, he obtained results which seemed to agree with Taylor's predictions. Dryden suggested that both explanations of transition could be correct, and that the cause of transition could depend on the level of free stream turbulence. He attempted to find evidence of the presence of Tollmien-Schlichting waves, but was unable to do so.

Not satisfied by this failure, Dryden instigated in 1940, the investigation which was to lead to the publication in 1947 of Schubauer and Skramstad's famous paper.

Schubauer and Skramstad made a particular effort to keep the level of free stream turbulence low. They were able to prove the existence of disturbances of the Tollmien-Schlichting type. They concluded from their observations, that small disturbances present in the boundary layer were selectively amplified as they travelled downstream. A disturbance of a particular frequency reached its maximum amplitude at a Reynolds number which corresponded to the position of branch II of the neutral stability curve.

An electrically excited vibrating ribbon on the surface of the
plate was used to introduce controlled, small disturbances into the boundary layer. The behaviour of the controlled disturbances was shown to be the same as that of the natural disturbances. The controlled disturbances were then used to compare experimental results with the theoretical predictions of Tollmien and Schlichting.

A neutral stability curve was found in reasonable agreement with the theoretical curve of Tollmien. The distribution of the disturbance, across the boundary layer at branches I and II of the neutral stability curve, compared well with the results of Schlichting (1935).

Bennett (1953) investigated the natural transition of a boundary layer on a flat plate, for a range of free stream turbulence levels from 0.1% to 0.5%. He confirmed that the amplification of Tollmien-Schlichting waves could be regarded as the first stage of the transition process for all the levels of free stream turbulence used. He acknowledged the possibility that Taylor's theory might control the transition process at higher levels of free stream turbulence, but pointed out that such levels must be higher than had been thought.

Before the experimental investigation by Schubauer and Skramstad, the existence of Tollmien-Schlichting waves was doubted and Taylor's theory seemed the most likely explanation for transition. This situation was completely reversed by the 1947 paper.

The predictions of Tollmien and Schlichting were in agreement with the initial stages of the growth of a disturbance in a boundary layer. It soon became obvious that their theory would not suffice to explain the processes leading to transition. Theoretical work since 1947 has been concerned with the attempt to extend linear
theory, (Tollmien and Schlichting's theory) in such a way as to explain the behaviour of the boundary layer, up to and during the transition process. So far success has been limited.

1.5 Non-linear Theory

In general, extensions made to linear theory have been made for plane Poiseuille flow, and not boundary layer flow. There are a number of reasons for this;

1) The mean flow profile for Poiseuille flow is parabolic but for the Blasius boundary layer, the profile must be approximated to, by a numerical solution.

2) The Reynolds numbers are of the order of ten times greater in Poiseuille flow. This provides additional justification for asymptotic solutions.

3) In Poiseuille flow, the Reynolds number depends on the separation of the planes, and is constant at all downstream positions, in the absence of a pressure gradient. In boundary layer flow the local Reynolds number depends on the thickness of the boundary layer, which in turn depends on downstream position.

Consideration of Poiseuille flow allows a simplification of the mathematical problem, particularly desirable when dealing with extra terms of the non-linear equations. Conclusions drawn from consideration of Poiseuille flow were expected to be relevant to boundary layer flow.

The first predictions on the effect of the non-linear terms were made by Landau (1944), who produced a theory of successive instabilities. He obtained an equation which described the time
amplification of a disturbance. This equation contained non-linear terms.

Meksyn and Stuart (1951) produced an approximate method for the solution of the non-linear equation. They recognised three non-linear processes:
1) Distortion of the mean flow.
2) Distortion of the distribution of the fundamental.
3) Generation of harmonics.
They decided that the generation of harmonics was not likely to have important effects in the transition region, and considered the first two processes.

They found a threshold value for the disturbance amplitude in a region stable for infinitesimal disturbances. Above this threshold a disturbance would be amplified, below, it would be damped. As the disturbance amplitude was increased, the critical Reynolds number decreased until it reached a minimum value. Further increase in disturbance amplitude produced an increase in the critical Reynolds number.

Stuart (1958) considered only the non-linear terms corresponding to mean flow distortion. He showed that a stable disturbance of finite size could exist in a region unstable to infinitesimal disturbances.

Stuart (1958 and 1960) derived Landau's equation from energy principles. He indicated the significance of the three non-linear terms in the solution. These terms corresponded to the three processes dealt with by Meksyn and Stuart.

Lin (1958) made an order of magnitude analysis of the non-linear terms. In the critical layer his analysis led to the conclusion
that the generation of harmonics would become important when the fundamental was still too small to produce appreciable mean flow distortion. In the rest of the boundary layer, mean flow distortion was predicted to be the most important second order effect. The results of this work are rather vague, mainly because Lin did not define important.

Stuart (1960) distinguished between his own previous work, and the work of Lin. He studied Lin's analysis and concluded that the disturbance amplitudes which he had considered, were smaller than those discussed by Lin. He did not think that Lin's analysis would apply to smaller disturbances.

Stuart's analysis gives the relative size of the harmonics. If the fundamental component is of small order $A$, the amplitude of the 2nd harmonic, and the distortion of the mean flow will be of order $A^2$, and in general, the amplitude of the nth harmonic will be of order $A^n$.

All the theoretical work discussed, has dealt with time amplified disturbances. Bradshaw, Stuart, and Watson (1960) showed how the equations could be adapted to deal with spatially dependent perturbations. The importance of this extension is in the fact that experiments normally involve the study of spatially dependent perturbations. Gaster (1962) provided further evidence of the equivalence of the two types of perturbation.

Experimental results which will be discussed in the next section provided strong evidence of 3-dimensionality in the transition region. Much of recent theoretical work has attempted to produce agreement with these experimental results.

Benney and Lin (1960) studied the interaction of a two
dimensional perturbation with a three dimensional perturbation. The interaction produced four secondary disturbances. Two of the disturbances, which produced strictly two dimensional distortions, were equivalent to the non-linear effects discussed by Meksyn and Stuart (1951).

Benney (1961) discussed the other two secondary disturbances which were intrinsically three dimensional and non-periodic in time. He showed that these disturbances were systems of longitudinal vortices, which re-inforced each other at some spanwise positions, and cancelled at others. These vortices produced the spanwise periodicity of the flow.

The two systems of longitudinal vortices had different spanwise periodicity. The system which dominated when the three-dimensional perturbation was small, had twice the period of that which dominated when the three-dimensional perturbation was large.

Benney (1964) extended the previous work on Pois/seauille flow, to deal with an approximate boundary layer profile given by

\[
0 < z < 1 \quad u = z U_0 \\
1 < z \quad u = U_0, \quad U_0 = 1.
\]

He was again able to consider the four secondary flows, and noticed a tendency towards inflexions in the boundary layer profiles, outside the critical layer, at spanwise positions corresponding to re-inforcement of the vortices.

Stuart (1960) criticised Benney's results, because of Benney's assumption that the two interacting perturbations would have the same frequency and amplification factor. However Benney (1964) justified his results on the grounds of their agreement with experimental results.
Meksyn (1964) has shown that when perturbations of finite size are considered, the three-dimensional perturbations will be less stable than the corresponding two-dimensional perturbations.

1.6 Experimental Work on Finite Disturbances

Since the confirmation of small perturbation theory by Schubauer and Skramstad (1947), experimental interest has switched to the transition region. The first investigations focused attention on the last stages of the transition process.

Emmons (1951) used a water table to investigate transition in a boundary layer. He observed that turbulence appeared first in small spots, and that the appearance of these spots was random in position, and time. The turbulent spots were carried downstream by the flow, growing in the streamwise and spanwise directions. Spots appeared at different spanwise positions, grew towards each other and eventually merged. Eventually at some downstream position, the whole boundary layer became turbulent.

Emmons accounted for these results with a statistical theory. An intermittency factor (\( \gamma \)) was defined at every position, as the ratio of the time for which the flow was turbulent, to the total time.

Schubauer and Klebanoff (1955) confirmed the important part played by turbulent spots in the transition process. They devised a method of creating turbulent spots, using an electrical discharge. They satisfied themselves that the artificial spots behaved in the same way as naturally occurring spots, and then made a thorough investigation of the structure, and properties of turbulent spots.

Since 1955, a lot of work has been done in an attempt to explain
the origin of turbulent spots. The early stages of transition, before the first appearance of turbulent spots, have been investigated by a number of workers. Two techniques have been used - flow visualisation and hot-wire anemometry.

Fales (1955) was the first to perfect a flow visualisation method. The boundary layer on a flat plate, moving at uniform speed through water was studied by injecting dye from a tube into the water. Fales found that by placing a trip wire parallel to the leading edge of the plate, the initially uniform distribution of dye was concentrated into discrete lines. Downstream movement resulted in an increase in the velocity of the lines, and in the appearance of spanwise irregularities. Fales regarded the concentration of dye, as being caused by vortices shed by the trip-wire, and identified the dye lines as vortex lines.

Hama, Long and Hegarty (1957) used Fales' technique. They studied the three-dimensional nature of the vortices produced by the trip wire. As observed by Fales, the vortex lines became warped if the vortices were sufficiently strong. The warping resulted eventually in the formation of discrete vortex loops at different spanwise positions.

The vortex loops were stretched continuously in the downstream direction. This could be explained with reference to a theory developed by Lord Kelvin. This theory suggested that the front of the loop would be rotated into a region of higher velocity - further out in the boundary layer, inevitably the rear of the loop rotated towards the plate would move with a lower velocity. Hence the stretching was produced by the different velocities of the front and back of the vortex loops.
Bursts of turbulence appeared at spanwise positions corresponding to the shoulders of the vortices.

Hama (1960) repeated the work, using a vibrating ribbon to produce the disturbance in the boundary layer. This was in response to Schubauer’s suggestion (1958) that the behaviour of the disturbances introduced by a ribbon, might differ from that of the vortices introduced by the trip wire. Hama confirmed his previous results by showing that the sinusoidal induced vorticity rolled up into discrete vortices. Hence for large disturbances the method of introduction of the perturbation did not affect the results.

Discrepancies between his results, and those of Klebanoff and Tidstrom (1959), led Hama to examine his flow visualisation technique (1962). He conceded that the concentration of dye into discrete vortices could have been caused by properties of the streaklines and not by a rolling up of the disturbance. This result casts some doubt on the main conclusion of the work, that the formation of discrete vortices in the spanwise direction is a necessary prerequisite to transition.

A hot-wire anemometer study of the transition process, with special reference to the stages before the appearance of turbulent spots, has been made in the low turbulence wind tunnel at the National Bureau of Standards in Washington.

Klebanoff and Tidstrom (1959) investigated the behaviour of large disturbances in the boundary layer. They discovered a large spanwise variation in the amplification of such disturbances. The spanwise distribution of the disturbances was characterised by 'peaks' and 'valleys'. Evidence was found of energy transfer from the 'valleys' to the 'peaks'. Turbulent spots were observed only
at spanwise positions corresponding to 'peaks' of the disturbance amplitude.

It was concluded that turbulent spots were formed at the 'peaks' and then spread into the neighbouring 'valleys'. Variations in the boundary layer thickness were suggested, as an explanation for the periodic spanwise variations in disturbance amplitude.

Turbulent spots appeared to be formed before there was any sign of discrete spanwise vortices - a result which contradicted the findings of Hama, Long and Hegarty.

Klebanoff, Tidström and Sargent (1962) continued the investigation of the transition region. A change of smoothing screens reduced the spanwise variations of the mean flow to less than the measurement error. The disturbance was still three-dimensional in the transition region.

A controlled three-dimensional disturbance was produced, by placing strips of sellotape under the ribbon. This modification produced no detectable, spanwise effects on the mean velocity. Thus the boundary layer was assumed to be two-dimensional.

Counter rotating vortices were shown to exist on either side of each 'peak' in the amplitude distribution. The periodicity of the spanwise distribution of vortices seemed to be halved, as transition proceeded. These results agreed with the predictions of Lin and Benney.

Other phenomena, and theories were investigated from the standpoint of their influence on transition. The generation of harmonics, and the distortion of mean flow did not seem to be capable of producing large enough effects. A theoretical investigation by Gortler and Whitting (1958) was shown to predict results
which disagreed with the experimental results.

Investigations were also made into the conditions leading to the generation of turbulent spots. These spots seemed to be associated with hairpin eddies, and inflected mean flow profiles.

Tani (1960) and Tani and Kamoda (1962) investigated the effects of introducing a two-dimensional perturbation into a three-dimensional boundary layer. They obtained results which were broadly similar to those obtained by Klebanoff et al.

Kovasznay (1960) investigated the appearance and growth of turbulent spots in the boundary layer using multi-channel hot-wire units. His results were in accord with the results of Schubauer and Klebanoff (1955), and in particular, suggested strongly that turbulent spots would originate in the outer regions of the boundary layer.

1.7 Reviews

Boundary layer instability has been reviewed by Dryden (1955), Schlichting (1960), Stuart (1960) and Tani (1966).

Books by Lin (1955), Schlichting (1955), and the book edited by Rosenhead (1963) are also relevant.

1.8 The Edinburgh Work

The availability of high speed digital computers suggested the possibility of obtaining a numerical solution to the equations of motion for a perturbation. This work has now been in progress for some years. Concurrently an experimental investigation of the
behaviour of boundary layer disturbances up to the transition region has been in progress.

It was decided to begin the investigation by considering the linearised equations of motion. Jordinson (1970) used a numerical method, devised by Osborne, to derive eigenvalues and corresponding eigenfunctions for the linear equations of motion of a spatially dependent perturbation. Ross, Barnes, Burns and Ross (1970) and Ross (1969) compared the numerical results derived by Jordinson with the behaviour of the fundamental component of a small disturbance, measured experimentally.

Ross et al. found that the predictions of theory were in excellent agreement with experiment, as far as the distribution of the disturbance through the boundary layer, and the wavelength of the disturbance, were concerned. They found discrepancies in the downstream amplification which suggested that the assumption of separability made in the course of the derivation of the linear equations was not fully justified. In addition Ross et al. found a neutral stability curve in reasonable agreement with theoretical predictions, though they obtained a critical Reynolds number considerably lower than predicted theoretically.

In attempt to account for these discrepancies, Barry and Ross (1970) took account of the thickening of the boundary layer. The effects produced were small, but they did result in a new neutral stability curve which lay outside that obtained by ignoring boundary layer thickening. A value of the critical Reynolds number closer to the experimental value was obtained, but the discrepancy was still quite large.

Some observations of the 2nd harmonic components of boundary
layer disturbances were made by Kersley (1965) and Barnes (1966). The next stage in the computational process was the inclusion of higher order terms in the equations of motion for the boundary layer perturbation. This was done by Barry (1970) and resulted in predictions about mean flow distortion, the second harmonic component and distortion of the fundamental component.

The comparison of these results with experimental results was undertaken by the author of this thesis.

The second order theory, dealt with by Barry, allowed the magnitude of the disturbance to affect its behaviour. Consequently the effect of increasing the size of the disturbance - both experimentally and theoretically was of great interest.

Transition was recognised to involve the appearance of three-dimensionality at some stage. Two dimensional perturbations were considered during the derivation of the numerical results. The question arose as to how far into the transition region, the numerical solutions could be expected to extend. It was hoped that the present work would provide an answer to this question.

Results obtained throughout the transition region are compared with previous results both theoretical and experimental, and an attempt made to clarify some of the problems arising.
CHAPTER 2
EQUIPMENT

1(a) The Tunnel

The closed circuit wind tunnel was designed specifically for low turbulence work. The contraction area ratio of the settling chamber to the working section was 15:1. Three smoothing screens were fixed in the rapid expansion section in front of the settling chamber. Two moveable screens were placed 2 feet apart in the settling chamber.

The smoothing screens were made from 38 mesh 36 s.w.g. phosphor-bronze wire with a blockage coefficient of 0.506. The screens were cleaned at regular intervals.

The total turbulence level, above 8 c.p.s has been measured by Barnes (1966). Values at two representative speeds are

\[
\frac{1}{3}(u^2 + v^2 + w^2)^{1/2}/U_0 \times 100^\circ/\circ = 0.027^\circ/\circ \text{ at 65 ft./sec.} \\
= 0.006^\circ/\circ \text{ at 30 ft./sec.}
\]

An airline diagram of the tunnel is shown in Fig. 2.1

1(b) Drive and Control

The tunnel was powered by a hydrostatic servo-system based on an N.E.L. design which provided a complete hydraulic analogue of the Ward-Leonard system. A 35 H.P. squirrel cage induction motor drove the pump at a constant speed, and this in turn drove the hydrostatic motor coupled to the fan. The fan was designed to absorb 30 H.P.
at 750 R.P.M. to give the tunnel's top speed.

Two speed controls were used.

The coarse control altered the angle of the swash plate in the pump, thereby changing the rate of flow of oil through the pump, and consequently the rate at which the fan was driven. A 230 v. motor drove the swash plate, and its operation was controlled by either of two pairs of relays. One pair of relays was activated by control panel switches, the other pair by the P.D.P.8 computer. A control panel switch allowed one of those relay pairs to control the swash plate motor.

Fine control was achieved by regulating the amount of oil bypassing the hydrostatic motor, using an electrically controlled, variable aperture, moog valve between the high pressure delivery, and low pressure return, pipes of the pump. The current supplied to this valve was controlled by either of two variable 2000 ohm resistors, one of which was selected by a control panel switch. One resistor was varied through a 50:1 right angled gear box by a 12 volt stepper motor, controlled by the P.D.P.8 computer. The other resistor was varied manually with a panel control. The fine control allowed the speed of revolution of the fan to be varied by ± 0.1 R.P.M.

Control of the speed of the wind tunnel involves the switching of relays or driving a stepper motor. It was possible for this control to be either manual or by the P.D.P.8 computer.

The tunnel had built in safety devices. A temperature controlled switch resulted in power to the motor being cut off, if the tunnel was driven too fast. Other such devices are described by Barnes (1966).
Fig. 2.1 Airline Diagram of the Wind Tunnel
1(c) The Working Section

The working section was 10 feet long and of basic 4 feet square section. Along its length, perspex fillets were set diagonally into each corner, such that the actual cross section of the working section was octagonal. Each fillet could be flexed a distance of ±1 inch by means of 12 screws along its length. This provided a ±2% variation in the area of the working section. The mean area of the working section was 13 square feet.

Access to the working section was given by two doors at its ends. Breather holes at the back of the working section equalised the pressure in the working section and the control room. The pressure in the tunnel was equalised to atmospheric pressure between the third and fourth corners, where the velocity was lower than in the working section. Consequently the pressure in the control room was lower than atmospheric pressure, and the room was sealed.

The maximum speed attainable in the working section, fixed by its dimensions, and the maximum speed of rotation of the fan, was 140 feet/second.

The design, testing, and calibration of the wind-tunnel is described by Barnes (1966).

2. The Flat Plate

The flat plate was a sheet of perspex 9 feet long, 4 feet high and $\frac{1}{2}$ inch thick. The leading edge was symmetrically tapered over the front 6 inches to a $\frac{1}{32}$ inch diameter circular tip. The trailing edge was tapered over the back $7\frac{1}{2}$ inches to a $\frac{1}{64}$ inch diameter circular tip.

The plate was suspended from a heavy L-shaped duralumin bracket
bolted to the roof and off-set from the centre line of the working section by $\frac{1}{4}$ inch. Along its bottom edge the plate was wedged by small aluminium brackets screwed to the floor. The leading edge of the plate was 5½ inches from the front of the working section.

3. **The Traversing Mechanism**

a) **The carriages**

Hot wire probes were carried at the end of a boom attached to a carriage. Two carriages were used for the work described in this thesis.

The carriage used in the early part of the work has been described by Barnes (1966). The carriage used in the later stages was designed to be used in association with the P.D.P.8 computer. Only the later model will be described, the other model was basically similar. Both carriages were required to position a hot wire probe with an accuracy of 0.05 inches in the $x$ direction, 0.02 inches in the $y$ direction and 0.001 inches in the $z$ direction - the directions of the axes are shown in Fig. 2.2.

The carriage had a shaped leading edge, and a shaped trailing edge, to reduce its influence on the flow. The boom was of sufficient length to ensure that the hot-wire probe was uninfluenced by the carriage, the probe was a minimum of 16 inches from the leading edge of the carriage. The carriage was 4½ inches high, 1.75 inches wide and, including the streamlined leading and trailing edges, 20.5 inches long. See Fig. 2.3. This carriage has also been described by Ross (1969).
Fig. 2.2 The Co-ordinate Axes
Fig. 2.3 The Carriage
b)  The x movement

The carriage moved on ball races along a 1 inch diameter steel rod fixed to the floor of the tunnel, 4 inches from the flat plate. The steel rod and the bottom of the flat plate were parallel to an accuracy of 0.005 inches. A loop capstan drive allowed the carriage to be driven in the x direction by a SLO-SYN type 3825/1011 24 volt stepping motor via a 12:1 reduction gear.

c)  The z movement

The boom was mounted as shown in Fig. 2.4. The end of the boom was pivoted on the drive nut which was driven by the rotation of a 2 inch long micrometer screw of pitch 0.05 inch. The screw was rotated by a Muirhead 24 v. stepping motor type 11 M 3004, acting through a 100:1 right angled worm reduction gear box.

As the drive nut moved along the screw, the distance between the end of the boom, and its fixed pivot changed. To allow for this change, the boom was made in two parts; the end section attached to the drive nut could slide inside the other part which was pivoted and carried the hot-wire probe. The change in total length of the boom was small, and a linear relationship existed between the distance moved by the drive nut, and the distance moved by the hot-wire probe.

The boom on the older carriage was not attached to the drive nut, but was pushed by it. This removed the necessity that the boom should be able to change in length, but the coupling between the drive nut and the hot-wire probe was not so satisfactory.
Fig. 2.4 The Z Movement - Drive Screw and Boom
d) **The y movement**

The unit, which carried the z traversing mechanism, ran between vertical rails which were \( \frac{1}{2} \) inch in diameter. A \( \frac{1}{20} \) inch diameter steel cable connected the top of the unit to a 1 inch diameter threaded drum. The drum was rotated by a 24 volt Muirhead stepping motor type 11 M 3004 acting through an 80:1 Vactric Reduction Gearhead and a 12:1 worm reduction gearbox. The rotation of the drum resulted in winding or unwinding of the steel cable, thereby producing movement of the probe in the y direction.

Sliding coverplates attached to the unit maintained the streamlining of the carriage, during y movement. The coverplates limited the range of movement to 3 inches above or below the centre line.

e) **The carriage control unit**

The control unit has been described in detail by Ross (1969). The control unit allowed the three 24 volt stepping motors, which positioned the hot-wire probe, to be operated either manually or by the P.D.P.8 computer.

The stepping motors were driven by 24 volt pulses supplied to their 4 terminals in a given order. The manual drive circuit incorporated two pairs of transistors which functioned as bistable multivibrators. Each multivibrator could supply a pulse to either of the two stepping motor terminals associated with each stator of the motor. These multivibrators were triggered by 15 volt pulses from a free running multivibrator. The frequency of the triggering pulses controlled the speed at which the stepping motors were driven.
The order in which pulses were supplied to the 4 terminals of a stepping motor, controlled the direction of drive.

The supply of pulses from the computer, to drive stepping motors is discussed in a later chapter (4).

f) Measurement of position

The x co-ordinate was measured with a potentiometer wire running parallel to the rail on which the carriage moved. The wire was rigidly held, to prevent kinking, between two supports, each \( \frac{1}{4} \) inch from the rail. A D.C. voltage of around 2 volts was maintained across the wire. A contact attached to the carriage base slid along the wire.

The length of the wire could be measured with a metre rule. The linearity of the wire was checked at various positions. Small changes in the position of the carriage were measured using a travelling microscope, and compared with the changes in the voltage drop across the wire. The graph obtained by plotting voltage \( v \) distance was a straight line, and knowledge of its gradient, and the total voltage drop along the wire, made possible a second estimate of the length of the wire.

A typical graph is shown in Fig. 2.5.

Values obtained by the two methods, for the length of a wire were 99.25 inches and 99.3 inches respectively.

A small brass plug was set into the plate, at a distance of 1 foot from the leading edge. The centre of this plug served as the reference position. To calibrate the x measurement, the hot-wire probe was moved to coincide with this plug, and readings of
Fig. 2.5 Calibration Graph - X Potentiometer Wire
the fractional voltage $V_{XR}$ and the total voltage $V_{TR}$ noted.

The position of the carriage, knowing the fractional voltage $V_X$ and the total voltage $V_T$, could be calculated using the formula

$$X = \frac{V_X}{V_T} \times L - \frac{V_{XR}}{V_{TR}} \times L + 12 \text{ inches}.$$ 

$L$ was the total length of the potentiometer wire in inches. The $X$ coordinate could be measured with an accuracy of $\pm 0.05$ inches.

The $Y$ coordinate was measured using a potentiometer wire held vertically inside the carriage. The reference position was the centre line of the plate, which was found using the centre point of the brass plug. The procedure adopted for $Y$ measurement was the same as that for $X$ measurement.

The relevant formula was

$$Y = \frac{V_{XR}}{V_{TR}} \times L' - \frac{V_Y}{V_T} \times L'. $$

$L'$ was the total length of this potentiometer wire in inches. The $Y$ coordinate could be measured with an accuracy of $\pm 0.05$ inches.

The $Z$ coordinate was the distance from the hot-wire probe to the plate. This coordinate was not measured directly because of the difficulty experienced in trying to find the zero position. The obvious choice of zero position was the surface of the plate, but the hot-wire probe broke whenever it touched this surface.

The method used to fix the zero position, and provide the $Z$ coordinate depended on the characteristics of the Blasius boundary layer, and is described in a later chapter (4). The method required knowledge of a parameter, proportional to the actual distance,
between different hot-wire probe positions. Provided that the drive mechanism could be shown to behave linearly, a convenient parameter was the number of pulses supplied to the \(Z\) stepping motor. These pulses were automatically counted when the stepping motor was driven by computer, but a counter had to be used when the motors were driven manually.

The linearity of the drive mechanism was checked using a travelling microscope. Results of such a check are shown in Fig. 2.6. The supply of a pulse to the \(Z\) stepping motor resulted in a movement by the hot-wire probe of \(0.22 \times 10^{-3}\) inch.

With the old carriage, measurements from a clock gauge fixed to bear against the driven end of the boom, served as the required parameter for the measurement of \(Z\) position.

The problem of direct measurement of \(Z\) position has been given some thought. It was thought that a zero position might be obtained by bringing a pressure transducer fixed to the boom into contact with the plate.

Barnes showed that the waviness of the plate could result in variations of up to 0.006 inches, in the position of the surface, while moving a distance of 1 foot. Consequently to ensure that the pressure transducer accurately reflected the position of the hot-wire probe relative to the plate, it would have to be close enough to the probe, to affect the flow round it. Thus this method was rejected.

4. The Boundary Layer Perturbation

The vibrating ribbon technique was used to introduce disturbances of controlled frequency and amplitude into the boundary layer.
Fig. 2.6 The Linearity of the Z Movement
The technique was pioneered by Schubauer and Skramstad (1947) and since then has been used by many workers.

a) The Ribbon System

The arrangement is shown in Fig. 2.7.

The ribbon was a phosphor bronze strip 0.001 inch thick and 0.1 inch wide. It was clamped at the top of the tunnel and lay across the working side of the plate at right angles to the flow. The other end of the ribbon passed through a slit in the floor of the tunnel; a rod in this slit held the ribbon close to the surface of the plate. To this end of the ribbon was soldered a metal hook, from which weights could be hung to apply tension to the ribbon.

The central portion of the ribbon was held off the surface of the plate by two glass bridges, 0.0065 inch in diameter; one bridge was fixed 4 inches above the centre line, the other 4 inches below. Above the top bridge the ribbon was sellotaped to the plate, but below the lower bridge a strip of paper 0.2 inch wide was placed over the ribbon and sellotaped to the plate. The arrangement below the lower bridge allowed tension to be applied to the central portion of the ribbon, but did not allow sideways or outward movement.

Three permanent magnets, $\frac{1}{8}$ inches wide, of horseshoe type, were mounted on a perspex former, screwed to the reverse side of the plate in a position which resulted in the magnets being symmetrical about the centre of the ribbon. The ribbon oscillated in the plane perpendicular to the plate, when an A.C. current passed through it. The frequency of oscillation was the frequency of the current. Kersley (1965) showed the independence of the amplitude of oscillation from its frequency, provided that this forcing frequency
Fig. 2.7 The Ribbon
was greater than $\frac{1}{3}$ of the natural frequency of oscillation of the ribbon, and less than $\frac{5}{6}$ of that frequency. The amplitude was then strictly dependent on the amplitude of the current through the ribbon.

The ribbon was 12 inches from the leading edge of the plate for all the work described in this thesis. The brass plug, mentioned earlier, fitted into a hole in the plate, opposite the centre of the ribbon. The front surface of the plug lay flush to the surface of the plate. From the rear of the plug, a wire was taken to an oscilloscope. If the current through the ribbon was sufficiently large, the ribbon would strike the brass plug, producing a pulse on the screen of the oscilloscope.

By varying the frequency and amplitude of the current through the ribbon, the smallest current which could make the ribbon strike the plug was found. The frequency corresponding to this current was the resonance frequency. The resonance frequency could be altered by changing the tension applied to the ribbon. The resonance frequency was always adjusted so that both the fundamental and 2nd harmonic frequencies complied with Kersley's conditions. Under these conditions, it was not possible to make the ribbon hit the plug with the set-up available, when it was oscillating at the fundamental frequency. Consequently absolute measurement of the ribbon amplitude was impossible.

The oscillating part of the ribbon was 8 inches long. The presence of the ribbon was expected to result in an upstream movement of the position of natural transition. However Ross (1969) showed that with an 8 inch ribbon, natural transition did not occur on the plate, at speeds less than 35 feet per second. The work described in this thesis involved speeds below this limit, thus natural transition, speeded by the ribbon, imposed no limitation on
the working length of the plate.

b) **The Ribbon Drive Circuit**

This circuit, illustrated by the block diagram in Fig. 2.8 supplied an alternating current to the ribbon.

The signal was produced by a Hewlett Packard oscillator type 207. The frequency of the signal could be varied in the range 1 cps to 100 Kcps. The frequency could be adjusted to a required value with an accuracy of \( \pm 0.2 \) cps with the help of a Venner Frequency Meter type T.S.A. 3336/2. A Mullard 10 watt power amplifier allowed the current to be increased to a maximum of around 3 amps.

When the tunnel was run manually, control of the amplitude of the current supplied to the ribbon was achieved, using 2 devices. Variations in steps of 5 dB were obtained with a decade attenuator. The amplitude control of the oscillator allowed finer adjustments to be made.

For computer controlled running, it was necessary to be able to keep the ribbon current constant automatically, for a considerable time. This was achieved, using a computer controlled fine control.

The fine control consisted of a 470 K ohm resistor in series with a 20 K ohm potentiometer driven through a 50:1 right angled gear box by a 12 volt stepping motor. The device allowed a variable fraction of the voltage generated by the oscillator to be tapped off, and used after further amplification to drive the current through the ribbon. The variation made possible by this device was \( \pm 4\% \). The fine control is shown in Fig. 2.9.

A standard resistor was connected in series with the ribbon.
Fig. 2.8 Block Diagram - The Ribbon Circuit
20 Kilohms

4.70 Kilohms

Fig. 2.9 The Ribbon Current Fine Control
Measurement of the voltage across the 1 ohm resistor, gave the current passing through the ribbon. The voltage was read directly on a Solartron true R.M.S. Voltmeter, type VM 1484, during manual operation. The R.M.S. voltmeter also supplied a D.C. voltage proportional to the R.M.S. value, and this D.C. voltage could be read by a Solartron Digital Voltmeter, type LM 1420. Since this device was computer controlled, the ribbon current could be measured when the tunnel was being run automatically.

In fact during automatic running, all oscillating quantities were measured using the true R.M.S. voltmeter and the Digital voltmeter. To make this possible a switching network was required, which shall be described later in this chapter.

The supply of current to the ribbon could be cut off by a switch, controlling the power amplifier. This switch was operated manually, or by a computer controlled relay.

5) Hot-Wire Anemometry

The hot-wire anemometer is an electrically heated wire of small diameter which cools convectively when placed in an airflow.

Work was done by King (1914) which showed that the rate of loss of heat (H) from a cylinder placed normal to a flow was related to the velocity of the flow (U) by the equation:

\[ H = A + BU^n \]

A and B are constants depending on physical properties of the cylinder.

King gave the indice \( n \) the value 0.5 and with this value the above equation is known as King's Law.
When using such an anemometer to measure velocity it is normal to keep either the current through the wire, or the resistance of the wire, at a constant value. These anemometers are referred to as constant current, and constant temperature instruments, respectively. The anemometers used in the present work are constant current instruments. For such an anemometer King's equation can be written in the form:

\[ \rho = \rho_0 + m u^n \]

\[ \rho = \frac{R_w}{R_w - R_A} \]

\( R_w \) is the resistance of the wire under operating conditions.
\( R_A \) is the resistance of the unheated wire.
\( \rho_0 \) and \( m \) are constants for a given anemometer which must be determined by some calibration process.

Collis and Williams (1959) concluded that, when the velocity of the flow, and the diameter of the cylinder result in a Reynolds number based on their having a value in the range \( 0.02 < R_e < 44 \), King's Law should be modified slightly, and the above equation re-written.

\[ \rho = \rho_0 + m u^{0.45} \]

This equation was used throughout the work of this thesis.

6. The Hot-Wire Anemometer

a) The hot-wire probe

The hot-wire was carried by a head which screwed into a co-axial plug at the end of the boom attached to the carriage. Two prongs, one hard soldered to the stainless steel case of the head,
the other to an insulated central pin, carried the hot-wire. The prongs were nickel plated number 6 sewing needles. A streamlined perspex cap, fitting tightly into the steel case, completed the assembly, the whole being araldited together. The head is shown in Fig. 2.10.

An 0.002 inch diameter piece of Wollaston wire was soft soldered under slight tension to the tips of the prongs. The silver coating was removed by electrolytic etching in a fine jet of 10% nitric acid. The etched portion of the wire, 0.0002 inch in diameter and normally about 0.1 inch long, was the hot-wire anemometer. A typical working current, through the hot-wire was 50 mA, and the resistance was of the order of 10 ohms.

b) The Control Unit

The hot-wire control unit was designed by Kersley (1965). The unit had two channels and required a 12 volt D.C. power supply. It was designed to supply a current of between 1 mA and 100 mA to a hot-wire whose resistance was between 2 ohms and 100 ohms. It was stabilised such that the ratio of voltage changes to current changes was of the order of 300:1.

The voltage, across the hot-wire and some cables leading to it, was measured directly by the Solartron Digital Voltmeter LM 1420. The current through the hot-wire was found by measuring the voltage across a 10 ohm standard resistor in series with the hot-wire. This voltage was measured on a channel of the digital voltmeter.

Control of the current in the hot-wire was achieved with a coarse control, and a fine control. Both controls were potentiometers in series with the hot-wire, the former a 10 k ohm
Fig. 2.10 The Hot-Wire Probe
potentiometer, the latter a 250 ohm potentiometer. During on-line runs it was essential that the hot-wire current be kept constant. A switch on the panel, allowed either of two identical potentiometers to be used for fine control. One potentiometer was controlled manually, the other was driven by a 12 volt stepping motor acting through a 50:1 gear box. For a typical hot-wire current of 50 mA, the variation possible under computer control was ± 2 mA which was normally sufficient to allow the computer to maintain a constant current throughout an experiment.

A switch allowed a very large resistance, 200 K ohms to be placed in series with the hot-wire. This reduced the current through the hot-wire to a very small quantity, and allowed the resistance of the unheated wire to be measured.

7. Measurement and Recording Equipment

a) D.C. voltage

By measuring the D.C. voltage across the hot-wire, the mean velocity of the flow could be determined, using the formula given in Section 2.5. D.C. voltage measurements were performed by the Solartron Data Logging System which incorporated

- a Digital Voltmeter type LM 1420
- a Command Range Unit type EC 1475
- a Scanning Unit type LU 1461 and
- a Punch Encoder type LU 1467.

The system could be operated by the computer or by panel controls. Results were presented on a display.

The digital voltmeter was accurate to ± 0.05% of the maximum reading on any range selected. Ranges allowing the measurement of
D.C. voltages between 10 μ volts and 200 volts were available. The scanning unit allowed the voltage on any one of 20 channels to be measured. The command unit made possible continuous reading of a single channel, or else reading of a sequence of channels. The punch encoder allowed the results displayed on the screen to be punched on a decimal punch, or transferred to the interface of the P.D.P.8 computer.

b) Fluctuating voltages

The fluctuating velocity components were obtained from fluctuating voltages, using the relationship

\[
\frac{u'}{U_0} = \frac{u}{U_0} (\rho - 1)^2 \frac{e'}{0.45 \text{ } i \text{ } R_A(\rho - \rho_0)}
\]

where \( u' = \sqrt{u^2} \) the r.m.s. of the fluctuating velocity and \( e' \) is the r.m.s. of the fluctuating voltage. The above formula was derived by differentiating the modified version of King's equation, the derivation was given by Kersley (1965). The relationship assumes a linear relationship between fluctuating voltages and velocities - an assumption which will be reconsidered in a later chapter (7).

The output from the hot-wire, the voltage across it, was passed into a Tektronix type FH122 low level pre-amplifier with a nominal gain of either 100 or 1000. The amplifier was choke-capacitance coupled, thus only the fluctuating component could enter the actual amplifier.

When the tunnel was under manual control the output from the pre-amplifier was passed into a Brüel and Kjaer type 2107 frequency
analyser. This instrument was used as a filter to remove noise, and allow frequencies of interest to be studied—normally those frequencies were the fundamental and 2nd harmonic frequencies resulting from the ribbon input. The output from the filter could then be fed into a Brüel and Kjaer type 2305 pen recorder which gave a permanent record on a linear 10-110 millivolt scale. The frequency analyser was operated on its narrowest band-width of 45 dB octave selectivity. The permitted frequency range was between 20 cps and 20,000 cps and the output could be proportional to the RMS, peak, or average of the input—an amplification in steps of 10 dB was also possible.

The frequency analyser and pen recorder could also be used to give a frequency distribution of any voltage.

With the tunnel under computer control, a different procedure was used to record the fluctuating components. To allow two frequencies to be studied, two filters were used. The voltage output from the pre-amplifier was fed to the Brüel and Kjaer frequency analyser and the Krohn-Hite type 330 NR bandpass filter, the two filters being in parallel as shown by the block diagram—Fig. 2.11.

The filtered outputs were measured by the Solartron true RMS voltmeter whose D.C. output was measured in turn by the Digital Voltmeter.

Since three kinds of fluctuating signal had to be measured in this way, a switch was supplied to allow any one of the fluctuating voltages to be measured by the true R.M.S. voltmeter as required.

The variations which could occur in the R.M.S. values of the fluctuating voltages in the course of an experiment were of the
Fig. 2.11 Block Diagram - Automatic Recording of Data
order of $10^3$. Thus the provision of an automatic attenuator was a priority.

**The Switch**

This device was operated by two computer controlled relays. Any one of 4 input terminals could be directly connected to the output, depending on the states of the two relays. The system is shown in Fig. 2.12.

The inputs did not have a common earth to avert the possibility of earth loops producing interactions between the inputs.

**The Attenuator**

This device consisted of a chain of resistors accurate to 0.5%, whose values were chosen to agree with a decibel scale. The input signal was applied across the whole chain of resistors, attenuation was produced by tapping the output from a segment of the chain - the input and output having a common earth.

The point from which the output was taken, was controlled by the S.T.C. Magnetic Switching Device type ZM 53, which made a circuit between the output terminal and one of the possible tapping points.

Application of a pulse to one of the two input terminals of the magnetic switch, resulted in the attenuator being zeroed, and application of a pulse to the other input terminal resulted in the range of the attenuator being increased by one. The pulses were supplied by relays associated with each input terminal. The sequence; relay on - delay - relay off - ; resulted in the supply
Fig. 2.12 Automatic Switch
of a pulse to the terminal concerned, and a change in the range of the attenuator. 7 possible ranges gave a variation in amplification of -0 dB to -60 dB in steps of approximately 10 dB - the actual ranges are shown in Table 2.1. This range ensured that any fluctuation voltage, measured anywhere in the boundary layer, could be read on a single range of the Solartron true R.M.S. voltmeter. The resistor chain is shown in Fig. 2.13 and the magnetic switch in Fig. 2.14.

The program control of this device is discussed in Chapter 4.

8. Conclusions

The apparatus described in this chapter, was designed to allow experiments to be carried out under some degree of computer control. Computer control seemed to involve three processes:

1) Driving stepping motors
2) Switching relays
3) Selecting a channel of the digital voltmeter, and reading the voltage on it.

No consideration has as yet been given to the measurement of windspeed, this is the topic of the next chapter.
Fig. 2.13 Automatic Attenuator - Chain of Resistors
Fig. 2.14 Automatic Attenuator - Magnetic Switch
<table>
<thead>
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<th>Attenuation</th>
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</thead>
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</tr>
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</tr>
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</tr>
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<td>6</td>
<td>1014.6</td>
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</tbody>
</table>

Table 2.1 Attenuator Ranges
3.1 Measurement

A pitot tube was stationed 5 feet downstream of the beginning of the working section, on the working side of the plate, at a distance of 8 inches from the wall, and 2 feet 9 inches from the floor. The dynamic pressure in the free stream, recorded by the pitot tube, was measured with a Chattock Gauge.

The temperature in the working section was measured by a thermometer, on the floor. Measurement of atmospheric pressure then allowed the calculation of the density, and the kinematic viscosity, of the air in the wind-tunnel.

From the calibration constants of the Chattock Gauge, and the physical formulae, an empirical relationship was derived to give the free stream velocity,

\[ U_0 = 1.778 \frac{a(T + 273)}{P} \]

\[ U_0 \] = Free stream velocity in ft sec.
\[ P \] = Atmospheric pressure in mm. Hg.
\[ T \] = Air temperature in °C.
\[ a \] = The Chattock gauge reading in units of \( \frac{1}{2830} \) inch.

The Chattock Gauge could be read with an error of \( \pm 2 \) basic divisions. A windspeed of 12 feet per second gave a reading of 120 divisions. Since the velocity was proportional to the square root of this reading, the error in velocity was of the order of \( \pm 1\% \). In the same way a windspeed of 30 feet per second corresponded to a
Chattock gauge reading of 750 divisions, and the windspeed could be measured with accuracy of $\pm 0.2\%$.

3.2 **Computer Control**

As indicated in the previous chapter, the velocity of air in the tunnel could be altered by the computer. For complete computer control of the windspeed, an automatic measuring device was required.

Normally the velocity was calculated from the dynamic pressure, obtained from a pitot tube, using a pressure gauge. Computerisation of such a gauge would require the computer to measure the length of a column of liquid.

In general the measurement of windspeed, using the computer must involve the conversion of windspeed to some parameter directly measurable by the computer through its interface. Such parameters were in this case, voltage and frequency. No simple method of converting windspeed into a voltage presented itself. The possibility of converting windspeed into a frequency seemed more promising.

Vortices are shed from the back of a cylinder held perpendicular to a flow. If a hot-wire anemometer is placed close behind such a cylinder, the oscillating velocity field, produced by the passage of vortices, gives rise to oscillations in the voltage output from the anemometer. The transfer of these pulses to the computer, after any necessary filtering or shaping, would allow their frequency to be measured. If a simple relationship between the velocity of flow past a cylinder, and the frequency of vortex shedding from the back of the cylinder, could be found, a windspeed measuring device utilising the computer would be possible.
3.3 Previous Work on Vortex Shedding

The phenomenon of vortex shedding from the rear of bluff bodies, or rather the associated phenomenon of Aeolian Tones had been known since antiquity. The first experimental investigation of vortex shedding from a cylinder was carried out by Strouhal in 1878. Early in the present century it was shown that, at least at low velocities, the pattern of the shed vortices approximated closely to a Karman vortex street. A considerable amount of work has been done on the problem since 1945. Before looking at this work it is convenient to define two parameters which will assist in the description.

A Reynolds number $R_D$ based on the diameter of the cylinder can be defined $R_D = \frac{U_0 d}{\nu}$. The Strouhal number which is a non-dimensional frequency parameter can be defined, $S = \frac{N d}{U_0}$. $U_0$ is the velocity of the flow past the cylinder. $d$ is the diameter of the cylinder. $\nu$ is the kinematic viscosity. $N$ is the frequency of vortex shedding.

With a given cylinder, if the velocity of the flow increases from zero, a number of changes in the behaviour of the flow at the rear of the cylinder have been observed. It would seem possible to define ranges of $R_D$ in which the behaviour is unchanged.

In the range $0 < R_D < 1$, the equations for Stokes flow describe the flow, to a good approximation.

In the range $1 < R_D < 4$, vortices form behind the cylinder; normally one vortex forms before the other.

In the range $4 < R_D < 40$, which was defined by Kovaszny (1949), two stationary vortices are attached to the rear of the cylinder.

In the range $40 < R_D < 150$, vortices are shed from the rear of the cylinder at a regular frequency, forming a Karman vortex
street. This vortex street persists for a distance of many cylinder diameters downstream, eventually decaying by diffusion. Roshko (1953) was able to define a linear relationship between the Strouhal number $S$ and $R_D$.

In the range $150 < R_D < 300$ vortex shedding does not take place at a regular frequency, and the irregular bursts which are observed seem to correspond to a transition region. Transition, which takes place after the formation of vortices, results in quite rapid decay to fully developed turbulence.

In the range $300 < R_D < 3 \cdot 10^5$ defined by Roshko (1961) and named the subcritical region, the vortices are made up of turbulent fluid. Transition takes place in the region of formation of the vortices, which are again shed at a regular frequency. As the value of $R_D$ increases through this range, the transition point moves forward through the formation region.

This range was subdivided into 3 by Bloor (1964). When $R_D < 1.3 \cdot 10^3$ transition is regarded as being caused by 3-dimensional effects. For $R_D > 1.3 \cdot 10^3$ transition is regarded as being caused by Tollmein-Schlichting waves which were detected by Bloor. When $R_D < 8 \cdot 10^3$ transition is supposed to occur in the region where the fluid curls into the vortices. When $R_D > 8 \cdot 10^3$ the transition takes place in the separated layers before the vortices start to form. Hence the 3 subregions postulated by Bloor. These subregions do not seem to be reflected by changes in $S$; the results of Roshko (1953) suggest that $S$ remains virtually constant through the whole of the sub-critical range.

In the range $3 \cdot 10^5 < R_D < 10^6$ named by Roshko the supercritical region, vortex shedding again becomes irregular. In addition, the
cylinder wake narrows. It has been postulated that in this region laminar separation of the cylinder boundary layer occurs, then the separated layers become turbulent and re-attach to the cylinder. Finally turbulent separation gives rise to the narrow wake. The separated bubbles are very unstable, sometimes only one such bubble is present. This instability is responsible for the lack of a regular vortex shedding frequency. Pressure measurements made by Achenbach (1968) identified transition and separation points and confirmed the above explanation first put forward by Roshko in 1961.

In the range $R_D > 10^6$ named by Roshko the transcritical region, transition to turbulence occurs in the boundary layer of the cylinder before separation. As $R_D$ increases transition occurs sooner, but separation is not affected. The wake is wider in this region than in the supercritical region, and a regular frequency of vortex shedding reappears.

The mechanism which controls vortex shedding is not understood. Some of the features of the phenomenon are explained by Gerrard (1966).

### 3.4 Preliminary Findings

The results discussed in the previous section were examined with a view to designing a windspeed control. Only in a region of vortex shedding at a regular frequency was a relationship between that frequency, and the velocity of the flow past the cylinder, likely to exist. There appeared to be three such regions, separated by transition regions in which vortex shedding was irregular.
Since $S$ appeared to be constant through the sub-critical region, the vortex shedding frequency was expected to be directly proportional to the flow velocity. In this region, the vortices, being composed of turbulent fluid, were likely to be less stable than those observed for $40 < R_D < 150$.

Cylinders, several inches in diameter would have been required to make $R_D > 10^6$ for tunnel velocities in the range 5/feet/second to 100 feet/second. For this reason, the transcritical region was eliminated, since cylinders several inches in diameter would have been large enough to pose blockage problems.

An experimental investigation of vortex shedding behaviour, in the sub-critical range of $R_D^*$ seemed to be called for. The presence of a regular shedding frequency would have to be confirmed, and the extent of the $R_D$ range discovered. When this had been done, the variations of $S$ through the range could be investigated. Finally, if these results were satisfactory, the pulses obtained from the anemometer, used to detect the vortices, could be shaped or amplified as required.

3.5 The Experimental Arrangement

A cylinder was mounted on a panel which could replace one of the wind-tunnel windows. The cylinder, which was 11 inches long, was held at right angles to the direction of flow, at a distance of 10 inches from the wall of the wind tunnel. The hot-wire anemometer was mounted slightly downstream of the cylinder, with the axis of the probe in the direction of flow.

It is convenient to choose axes such that, the $x$ direction is
the direction of flow, and the z axis lies along the cylinder. It follows that the y axis will be perpendicular to the plane of the cylinder and the direction of flow. The z position of the probe was fixed opposite the midpoint of the cylinder, but the probe could be moved in the other two directions. The x position could be measured with accuracy 0.05 inch, while the y position could be measured with accuracy 0.001 inch.

The previously described control unit maintained a constant current through the hot-wire anemometer. The voltage output was amplified by the Tectronix amplifier before being passed to the Bruel and Kjaer frequency analyser. This device was employed normally as a filter, and its output was passed to the Venner frequency counter to allow the frequency of vortex shedding to be measured. The velocity was measured as described in the first section of this chapter.

The frequency analyser and its associated pen recorder were used to analyse the spectrum of the anemometer output, particularly in the first stage of the investigation.

3.6 The Probe Position

Working conditions were chosen which ensured that \( R_D \) had a value in the subcritical range as defined by Roshko. The cylinder was of diameter 0.093 inch and the windspeed was around 35 feet/second, giving \( R_D \) a value of the order of 1800.

The vortices were detected as pulses in the anemometer output. By altering the position of the anemometer both the magnitude of the pulses, and their ratio to the size of the background noise, could be changed. The aim was to make this ratio a maximum, and it was
hoped that under these conditions, the anemometer output would not require filtering.

At a given downstream position, the anemometer was tracked across the plane of the central axis of the cylinder, while the anemometer output was watched on an oscilloscope screen.

It was found that no probe position existed which would allow the anemometer pulses to be counted directly, without filtering. This result was confirmed for a larger cylinder of diameter 0.249 inch. This made the finding of the optimum position rather less critical.

Very close to the cylinder — less than one diameter from it, the anemometer output was very noisy, and it was difficult to detect the vortex shedding frequency. Outside this region movement in the x direction produced only a gradual fall in the size of the pulses. The output was a maximum on the centre plane of the cylinder, but it was also very noisy, with a particularly large component at double the shedding frequency, due to the probe picking up the effects of vortices above and below the centre plane. Movement away from this plane reduces the size of the pulses, but increases the ratio of the pulse amplitude to the noise. A balance must be struck between these two effects.

The work described in the next section was performed with the anemometer probe 2 diameters downstream of the cylinder and 1 diameter above the central plane.

3.7 Preliminary Investigation

A stainless steel cylinder of length 11 inches and diameter 0.093 inches was used for the first measurements. The frequency
of vortex shedding was calculated from 10 pulse counts each lasting for 10 seconds. The velocity was varied between 5 feet/second and 60 feet/second and graphs were plotted of $S$ against $R_D$. Two such graphs are shown in Figs. 3.1 and 3.2.

The results agreed in general form with the results of Roshko (1953) but there were discrepancies particularly in the Reynolds number ranges of the different regions. When $R_D$ was less than 400, $S$ increased systematically with $R_D$.

When $400 < R_D < 1,000$ $S$ varied but not systematically.

When $R_D$ was greater than 1,000, $S$ remained very nearly constant as $R_D$ varied. In both the cases illustrated, the standard deviation in the constant region is less than 0.20%, i.e. in Fig. 3.1 the best average value of $S$ is 0.2097 and the standard deviation is 0.0004.

Small systematic variations were observed in the best average values of $S$ obtained with the same cylinder from day to day. These variations lay within a range of 0.5% of the value of $S$.

These preliminary results confirmed that a windspeed control, based upon the proportionality of the frequency of vortex shedding, and the flow velocity past a cylinder, was feasible and suggested that the accuracy obtainable might be better than 1%. The lower velocity limit of the device would seem to correspond to a Reynolds number $R_D$ of 1,000. Accordingly an increase in the diameter of the cylinder, should have resulted in a lowering of the lower velocity limit.
Fig. 3.1 $S \times R$, Cylinder Diameter = 0.093 inches
Fig. 3.2 $S \times R_b$ Cylinder Diameter $= 0.093$ inches
3.8 Cylinders of Different Diameter

Two stainless steel cylinders of diameters 0.249 and 0.497 inch were used in an effort to extend the range of velocities for which $S$ was constant, in the direction of the lower limit. Graphs were obtained which could be compared with those produced with the smaller cylinder. The graphs for the larger cylinders are shown in Figs. 3.3 and 3.4.

The form of the graphs was unchanged, but the range of $R_D$, for which $S$ could be regarded as constant, varied considerably with the diameter of the cylinder. In particular the lower limit of $R_D$ for constant $S$, increased as the cylinder diameter increased, an effect which seemed likely to defeat the aims of this part of the investigation.

For the cylinder of diameter 0.249 inch, the region of constant $S$ began at $R_D = 1600$ which corresponded to a velocity of 12 feet/second.

For the cylinder of diameter 0.497 inch, the region of constant $S$ began at $R_D = 3600$ which corresponded to a velocity of 14 feet/second.

The best average values for $S$ differ for the different cylinders, but there does not seem to be any correlation between the diameters of the cylinders and the value of $S$.

- 0.093 inch cylinder $S = 0.2095$ with standard deviation 0.0004
- 0.249 inch cylinder $S = 0.2023$ with standard deviation 0.0005
- 0.497 inch cylinder $S = 0.2047$ with standard deviation 0.0011

The increased standard deviation for the largest cylinder, is at least in part explained by a systematic decrease in $S$ as $R_D$
Fig. 3.3 $S v R_b$ Cylinder Diameter = 0.249 inches
increases. The effect is small but obvious in Fig. 3.4.

From these results it would seem that to regard $S$ as a function of $R_D$ only is an oversimplification. The extent of the region of unsteady values of $S$ does seem to increase as the diameter of the cylinder increases, with a resulting increase in the lower limit of $R_D$ for constant $S$. The distinction between the unsteady and steady $S$ regions looks real enough on the graphs, but it is important to remember that all the graphs are very much 'stretched' along the $S$ axis. There was no evidence of a shift in the range of $R_D$, for which $S$ increased systematically with $R_D$, as the cylinder diameter was varied.

These results set a lower limit of 12 feet/second on the velocities which could be measured using this device.

3.9 The Suspension of the Hot-wire Anemometer

Kovasznay (1949) suggested that if the probe, carrying the detecting anemometer, lay along the wake of the cylinder, wake interference would result. Kovasznay worked at the low Reynolds numbers of the stable region. The possibility that the mode of suspension of the anemometer might affect the frequency shedding behaviour in the sub-critical region, had to be considered.

Accordingly the probe was mounted at right angles to its direction in the previous investigations, although the position of the hot-wire anemometer was unchanged. Using the two smaller cylinders, diameters 0.093 inch, and 0.249 inch, the results shown in Figs. 3.5 and 3.6 were obtained. Interest was concentrated on
the region for which \( R_D \) was less than 2,000 because it was thought possible that the changed mode of suspension might affect the lower limit of the region of constant \( S \).

The graphs, Figs. 3.5 and 3.6, are different from those obtained earlier. For neither cylinder was there a region of constant \( S \) at values of \( R_D \) below 2,000. The values of \( S \) were greater than recorded with the same cylinder previously - \( S = 0.23 \) compared with \( S = 0.21 \). For the larger cylinder, a sharp fall in \( S \) from 0.23 to 0.224 was noticed at \( R_D = 2,900 \).

The graphs make it obvious that this mode of suspension of the anemometer was useless for a windspeed measuring device. The graphs differ not only from those obtained previously in the present investigation, but also from those obtained by other workers using various methods of measurement (see, for example, Tsuchiya et al. (1970)). It must be concluded that mounting the probe at right angles to the wake of the cylinder has destroyed the normal mechanism of vortex shedding. This result contradicts that of Kovasznay (1949) though it should be remembered that while the present work refers to the sub-critical region, his work referred to the stable region.

3.10 Further Investigations

Groves et al. (1964) found that the value of \( R_D \) at which vortex shedding started was dependent on the diameter to length ratio. This ratio was much larger in the investigations of Groves et al., 0.25, than in any of the work described here. For this reason it was thought unlikely that the length of the cylinder
Fig. 3.5 S vs H. Cylinder Diameter = 0.093 inches
Fig. 3.6 $S \times R_c$, Cylinder Diameter = 0.249 inches
would be a relevant parameter. For the largest cylinder of diameter 0.497 inch, the diameter to length ratio was 0.045.

The cylinder of diameter 0.497 inch had its length halved to 5⅛ inches. Fig. 3.7 shows the result of comparing two cylinders of the same diameter, but different lengths. As expected there was no apparent difference in behaviour.

The surface of the cylinder could exert an influence upon transition, and consequently on the vortex shedding phenomenon. Since changes in surface texture of a cylinder used in a windspeed control over a long time were almost inevitable, an investigation of the effects of a large change in the surface texture was made.

A cylinder of diameter 0.497 inch had its surface very much roughened with coarse sandpaper. Results obtained with this cylinder are shown in Fig. 3.7 also. As can be seen, the changes in the surface did not seem to affect the vortex shedding phenomenon in the range of constant S.

The existence of a frequency of vortex shedding implies that a certain period will be associated with the shedding of each vortex. The stability of the process can be expressed by finding the standard deviation of this period. Table 3.1 shows this standard deviation expressed in terms of frequency, at a considerable number of velocities. Table 3.1 also shows the reduction in standard deviation obtained by considering the period associated with the shedding of 100 vortices. These latter results allowed the conclusion that where the frequency of vortex shedding was measured by counting the pulses produced in 1 second, the standard deviation in frequency was certainly not going to be larger than that in S.
Fig. 3.7 $S v R_0$. Cylinder Diameter = 0.497 inches
<table>
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<th>Frequency</th>
<th>Deviation</th>
<th>%/oage</th>
<th>Frequency</th>
<th>Deviation</th>
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</table>

Table 3.1 The Stability of the Vortex Shedding Period
Thus there was no instability in the vortex shedding process likely to reduce the attainable accuracy of the windspeed measurement.

3.11 Conclusions from the Experimental Investigation

The results described seemed to indicate that a windspeed measuring device accurate to $\frac{1}{2}^\circ$ at worst could be designed for use in the velocity range, 12 feet/second to 100 feet/second. The output from the anemometer would require to be filtered and amplified before being passed to the computer.

Results to which attention might be drawn include the lack of simple Reynolds number dependence and the striking effect of the positioning of the probe carrying the hot-wire. Obviously both these effects could be investigated in more detail, but as far as the design of a windspeed measurement device was concerned they were not strictly relevant.

On the basis of the results a permanent windspeed measurement device was constructed.

3.12 Windspeed Measurement Device

A cylinder of diameter 0.125 inch and of length $1\frac{1}{4}$ inches was fixed rigidly parallel to the tunnel wall, a distance of $6\frac{3}{4}$ inches from it. The device was placed on the non-working side of the perspex plate.

The anemometer was held by a probe, whose axis was in the direction of the flow, a distance of two diameters downstream from the cylinder and one diameter from the central plane of the
cylinder. The suspension could be slid back to allow replacement of the hot-wire anemometer when required. The anemometer could always be restored to the same position by means of a pin and hole which fixed the position of the suspension. The arrangement is shown in Fig. 3.8.

The anemometer was supplied by a simple control unit which allowed a stabilised current to be passed through the hot-wire. The output from the hot-wire was amplified nominally 1,000 times by a Tektronix type FM 127 low level pre-amplifier. Further amplification and also filtering of the output was required, and during the testing process, this was provided by the Bruel and Kjaer frequency analyser. This instrument was required for other purposes, described in the previous chapter.

A variable filter was constructed. The filter was based on an amplifier, CA 3010, together with a variable load. In fact the load consisted of one of a possible ten tuned circuits, each of which was used over a frequency range of 200 Hz. The switching between the tuned circuits could be accomplished either manually or, in the case of the first eight, automatically. A network of three relays allowed any one of those eight tuned circuits to be selected.

The amplification of the filter could be altered up to a maximum of 160X. Increase in the amplification resulted in a narrowing of the band of the filter, consequently to ensure that the ranges provided by the different tuned circuits overlapped, the amplification was normally of the order of 80X. The circuit of the filter is shown in Fig. 3.9 and the relay network in Fig. 3.10.
Fig. 3.8 Windspeed Measurement Device - Cylinder and Anemometer
Fig. 3.9 Filter Circuit
Fig. 3.10 Relay Network - Selection of Filter Ranges
The output from the filter was fed into the computer, where the pulses produced were counted. To provide a time interval for pulse counting, 1 second pulses were taken from the Venner frequency counter type TSA 3336/2 and used to gate the input of pulses to the computer.

The system which allowed windspeed measurement to be performed automatically is illustrated by the block diagram of Fig. 3.11.

The computer only counted pulses whose peak to peak size was greater than a threshold value, 12 volts. Thus the use of the wrong filter range, or too small a current in the hot-wire, could result in pulses being missed. Before using the device, the output from the filter was checked on the oscilloscope.

3.13 Calibration and Use of the Windspeed Measurement Device

The instrument was calibrated against velocity measurements made with the Chattock Gauge. A calibration curve is shown in Fig. 3.12. The ranges of the filter are presented in Table 3.2; the limits of each range are approximate, because the ranges must overlap.

The calibration constant connecting the flow velocity, and the frequency of vortex shedding was \( \frac{d}{S} \). \( d \), the diameter of the cylinder, was measured with a micrometer gauge. \( S \) was obtained from a calibration run. The value of \( \frac{d}{S} \) which was used throughout the first year of operation was 0.0478. It should be noticed that since \( S = \frac{Na}{U} \), the calibration constant for a cylinder is independent of \( d \).
Fig. 3.11 Block Diagram - Windspeed Measurement Circuit
<table>
<thead>
<tr>
<th>Range</th>
<th>Frequency</th>
<th>Velocity(ft/s)</th>
<th>Frequency</th>
<th>Velocity(ft/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>225</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>225</td>
<td>11</td>
<td>410</td>
<td>19.5</td>
</tr>
<tr>
<td>3</td>
<td>410</td>
<td>19.5</td>
<td>630</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>630</td>
<td>30</td>
<td>860</td>
<td>41</td>
</tr>
<tr>
<td>5</td>
<td>860</td>
<td>41</td>
<td>1050</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>1050</td>
<td>50</td>
<td>1320</td>
<td>63</td>
</tr>
<tr>
<td>7</td>
<td>1320</td>
<td>63</td>
<td>1550</td>
<td>74</td>
</tr>
<tr>
<td>8</td>
<td>1530</td>
<td>73</td>
<td>1720</td>
<td>82</td>
</tr>
<tr>
<td>9</td>
<td>1720</td>
<td>82</td>
<td>1800</td>
<td>86</td>
</tr>
<tr>
<td>10</td>
<td>1800</td>
<td>86</td>
<td>1950</td>
<td>93</td>
</tr>
</tbody>
</table>

Table 3.2 The Ranges of the Filter
Windspeed can be measured with an accuracy dependent on the accuracy of frequency measurement and on the stability of the calibration constant, as the flow velocity is varied.

Frequency measurements were obtained from 10 pulse counts, each lasting for 1 second. The minimum frequency measured was 200 cycles per second; thus the reading error was smaller than 0.10/o, for all frequency measurements.

The stability of the frequency of vortex shedding was considered in Section 3.10. The results presented in Table 3.1 show that any error from this source will be less than 0.20/o.

It was assumed during the calibration process that no systematic errors in the Chattock readings were present. Any other errors would have tended to increase the standard deviation of $S$, from its average value. This standard deviation was of the order of 0.30/o.

The calibration, shown in Fig. 3.12, gave a value for $S = 0.2200$ with the standard deviation, 0.0006. Checks performed at intervals showed that day to day variations did not result in changes in $S$, greater than the standard deviation.

The effect of the errors, dealt with, could be calculated by adding the percentage errors, since the factors concerned were multiplied to give the velocity. The conclusion that velocity should be measureable to an accuracy of 0.50/o resulted.

The full specifications of the device were:

<table>
<thead>
<tr>
<th>Range of measurement</th>
<th>fully automatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 feet/second to 82 feet/second</td>
<td></td>
</tr>
<tr>
<td>partially automatic</td>
<td>0.50/o</td>
</tr>
<tr>
<td>12 feet/second to 93 feet/second</td>
<td></td>
</tr>
<tr>
<td>$U_o$</td>
<td>$N$</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>11.03</td>
<td>228</td>
</tr>
<tr>
<td>11.71</td>
<td>242.4</td>
</tr>
<tr>
<td>12.40</td>
<td>258.6</td>
</tr>
<tr>
<td>13.10</td>
<td>277</td>
</tr>
<tr>
<td>13.28</td>
<td>280</td>
</tr>
<tr>
<td>13.90</td>
<td>292.6</td>
</tr>
<tr>
<td>14.65</td>
<td>309.2</td>
</tr>
<tr>
<td>15.41</td>
<td>327.2</td>
</tr>
<tr>
<td>17.27</td>
<td>364.2</td>
</tr>
<tr>
<td>18.04</td>
<td>380</td>
</tr>
<tr>
<td>18.82</td>
<td>396.2</td>
</tr>
<tr>
<td>19.75</td>
<td>416.8</td>
</tr>
<tr>
<td>20.72</td>
<td>438.6</td>
</tr>
<tr>
<td>21.84</td>
<td>462.4</td>
</tr>
<tr>
<td>22.72</td>
<td>481.8</td>
</tr>
<tr>
<td>23.55</td>
<td>498.2</td>
</tr>
<tr>
<td>24.64</td>
<td>522</td>
</tr>
<tr>
<td>25.77</td>
<td>546.4</td>
</tr>
<tr>
<td>27.69</td>
<td>585</td>
</tr>
<tr>
<td>28.58</td>
<td>602.8</td>
</tr>
<tr>
<td>29.36</td>
<td>618</td>
</tr>
<tr>
<td>30.11</td>
<td>635</td>
</tr>
<tr>
<td>30.93</td>
<td>653.8</td>
</tr>
<tr>
<td>32.10</td>
<td>679</td>
</tr>
<tr>
<td>33.12</td>
<td>700.6</td>
</tr>
<tr>
<td>33.99</td>
<td>719.6</td>
</tr>
</tbody>
</table>

Table 3.3 Calibration Data
The upper limit of measurement was specified by the filter ranges.

3.14 Conclusions

The windspeed measuring device has now operated satisfactorily for over a year.

It has been much used in keeping the windspeed constant during automatic experiments lasting for as long as three hours.

During an experiment lasting for some hours, the pressure and temperature will inevitably change. These changes alter the viscosity and thus affect both the Reynolds number, and another important experimental parameter, the F number. However because the calibration constant, \( \frac{d}{f} \), is independent of Reynolds number, the relationship between the frequency of vortex shedding, and the flow velocity is unaffected by temperature and pressure changes. Thus if the relevant frequency is held constant, the flow velocity will not change.

The measuring device has also been used to set the velocity of the tunnel to a required value before starting an experiment.

Hot-wire anemometers were normally calibrated manually against the velocity measurements made with the Chattock Gauge. Automatic calibration using the vortex shedding device was possible.

All these applications are discussed in the next chapter, as is the interface which allows the instrument to work.
CHAPTER 4

ON-LINE EXPERIMENTS

4.1 On Line Running

As a result of adaptations, and additions to the equipment, which have been described in the two preceding chapters, on-line running of wind tunnel experiments involved the demand that the computer should be able to perform four basic operations, in some required order, controlled by program.

These operations were:

a) Switching Relays
b) Driving Stepper Motors
c) Operating the data-logging system
d) Counting pulses (from the wind-speed control).

This chapter will describe briefly, the basic operations performed by the computer, and the interface which made these operations possible. The later parts of this chapter will be concerned with the programs which supplied the order in which the basic operations were performed.

4.2 Basic Operations

The interface was built by the late Mr. J. Whittaker.

(i) Switching of Relays.

The interface allowed the selection of one, of 24 relays, and permitted the selected relay to be activated, or de-activated.

Associated with each relay was a bistable circuit whose output controlled the state of the relay - a voltage level of 0v
switching the relay off while a voltage level of -3v switched the relay on. The output could be changed by supplying one of two input terminals with a positive going pulse and an enabling voltage level.

Two computer instructions supplied the pulses, each instruction being associated with a given input terminal, and thus with one of the two possible output voltage levels. In fact the instructions supplied pulses to the corresponding terminals of every bistable circuit.

Selection of a single relay was achieved by providing only one of the bistable circuits with the enabling voltage level. The circuits were arranged in three banks of 8, associated with each bank was an octal to binary converter. Selection of a relay involved placing a two digit octal number in a location whose contents controlled the supply of pulses to the converters. The smaller digit produced an input for each converter, which resulted in the enabling of a corresponding bistable circuit in each bank by the outputs from the converter. The larger digit controlled the selection of the banks, by enabling one of the converters.

The interface is shown in Fig. 4.1 which shows 4 bistable circuits - half a bank.

Switching of a relay involved placing a number corresponding to the relay in a store, and the supply of the required computer instruction.

(ii) Driving Stepping Motors

The interface made possible the selection of one of 8 stepping motors, and allowed the selected motor to be driven a given number
Fig. 4.1 The Relay Interface
of steps in a required direction.

The selection of a stepping motor was accomplished in the same way as the selection of a relay.

The pulses which drove the motors were provided by a multivibrator. This circuit was initially triggered by a pulse, supplied by the computer instruction, to drive a stepping motor. The output from a bistable circuit controlled the initial setting of the multivibrator, and thus the direction, in which it switched.

The pulses supplied to the motors were counted by a counter made up of eleven bistable circuits, each save the first, being triggered by the preceding circuit. The contents of the counter were initially set to the complement of the number of steps desired, by placing that number in a location, whose outputs controlled the setting of the counter. A sign digit in this location controlled the setting of the single bistable circuit, which determined the direction of switching.

When the pulses, supplied to the motors, had restored the contents of the counter to zero, a pulse, sent from the counter, stopped the production of pulses.

The counter is shown in Fig. 4.2 and the multivibrator in Fig. 4.3.

(iii) Operating the Data-Logging System

The interface made possible the selection of one of 20 channels; the voltage on that channel was measured by a digital voltmeter, and the reading could be transferred to the computer.

The selection of a channel has been performed in two ways. The system incorporated a selection device which allowed the channels
Fig. 4.2 The Pulse Counter - Stepping-Motor Interface
Fig. 4.3 Multivibrator - Supplies Pulses to Drive Stepping-Motors
to be referenced in order. Pulses supplied by computer instructions either increased the channel number by one, or returned that number to zero.

The advantage of this method was that by using existing equipment, the interface was made relatively simple. However in order to read the contents of any two channels, the contents of every intervening channel had to be read. Careful choice, of the order of referencing channels, and of the quantities measured on each channel, could reduce the number of useless readings, but to eliminate such readings was not possible. It was obvious that considerable savings of time, and computer space would be possible, if channels could be selected in any order.

Accordingly the selection device incorporated in the system was bypassed, and selection of channels was performed, using a network of 5 relays. These relays were arranged, such that each setting would provide a path for the contents of one channel, to the digital volt-meter. The relays were controlled by the computer.

The saving in time and computer space achieved is illustrated by comparing IMP programs to read successively channels 5 and 3 of the system, before and after the new interface was adopted.

(i) \%
Readdvm zero (x) (ii) \% Selectdvm (5)
Readdvm next (x) \% Readdvm (Store 1)
Readdvm next (x) \% Selectdvm (3)
Readdvm next (Store 1) \% Readdvm (Store 2)
Readdvm next (x)
Readdvm zero (x)
Readdvm next (Store 2)

The single operations above each occupy a comparable time. Normally 10 channels of the data-logging system were used in the course of an experiment.
The Solartron punch encoder type LU 1467 allowed the contents of a selected channel to be read, into the computer. The voltage on a selected channel was measured by the digital voltmeter, producing a four digit decimal result, which was displayed. Each digit was converted by the punch encoder to a binary number, and transferred to the computer. The conversion process was started by a data-logger pulse, but thereafter the conversion, and transfer, of each digit was triggered by a pulse, from the computer. When the 4 digits had been transferred, a data logger pulse allowed the start of the next conversion. The 4 binary numbers transferred to the computer were re-assembled by program, to give an octal number, which could be stored in a single computer location.

Some difficulties arose in the transfer, but these were overcome by programming, and will be discussed later in this chapter.

(iv) Counting Windspeed Pulses

The interface allowed the number of pulses, produced by the windspeed measurement device in a known time interval, to be counted.

The counter was that described earlier, as a part of the stepping motor interface. Pulses from the windspeed control could be counted only if a voltage level, which enabled a gate to the counter, was present. This level was produced by 1 second pulses provided by the Venner frequency analyser.

The counter was initially set to zero, and at the end of a count lasting 1 second, its contents were transferred automatically to an associated computer store.

Before describing the programs, which allow experiments to be
performed, a short account of the computer system is given.

4.3 The Computer

The wind tunnel experiments could be run by a PDP-8. Computer, which also played an important part in a number of other experiments, being performed in the Physics Department, at the same time. A time sharing, supervisor program was produced by Dr. J.G. Burns and Mr. S.T. Hayes. This program allowed different experiments to be run concurrently, by the computer.

The supervisor program established a queue of programs, such that at any time, the next operation to be performed in each experiment, would be controlled by a program in the queue. Associated with each interfaced peripheral, such as the data-logger, described in the previous section, was a bistable circuit called a flag. Depending on the voltage level, on one of the outputs of this circuit, the flag could be said to be raised, or lowered. Whenever a program attempted to use a peripheral, the relevant flag was raised.

The supervisor program scanned all the flags in order, until it came to one, which was raised. It would then service the associated experiment, by carrying out the instructions in the program running the experiment, until it encountered another peripheral. Supervisor then leaves the experiment, and scans through the flags as before. In this way it was ensured that no one experiment could monopolise the computer.

Some experiments were given a higher priority which allowed them to interrupt the normal sequence of operations, to receive immediate attention. Since the time spent waiting in the queue was
normally of the order of a milli-second, there was no requirement of high priority for the wind tunnel.

The supervisor program occupied a certain region of the core-space of the computer, and each user was assigned his own area of core. It was very important that no user should write programs which strayed either into supervisor core-space, or into the core-space of other users.

To provide a safeguard against the overlapping of programs in the computer core, and also to make programming easier, Mr. S.T. Hayes wrote a compiler which translated programs written in a high level language, IMP (a version of Atlas Autocode) into the lower level language of the P.D.P.8, PAL 3. This program ran on the machine at the Regional Computer Centre, first on a K.D.F. 9 computer, then on an I.B.M. 360/50.

The programs described later in this chapter, were written in IMP language. The compiler allowed the PAL 3 version to be produced, and simple corrections could be made at this stage, if required.

4.4 **Experiments**

Most experiments involve three stages.

i) General preparations and the setting of fixed parameters to the required values.

ii) The variation of parameters, and the collection of data.

iii) The processing of data.

Experiments were performed, with the aid of the computer, to make a comparison with numerical results derived from theory. This led naturally to other experiments where no numerical results
were available for comparison.

The three stages will be considered in some detail, with particular reference to the programs, which allowed computer control of parts of the experiment. Only the second stage was performed wholly on-line, but important parts of the other two stages could be automatically controlled.

(i) General preparations and the setting of fixed parameters.
   This stage could itself be subdivided into:
   (a) Switching on and adjustment of equipment.
   (b) Calibration of the Hot-wire anemometer.
   (c) Setting of the fixed parameters.

(la) This part of the experiment is self-explanatory.

(1b) Calibration of the Hot-wire Anemometer.

The anemometer was calibrated against the pitot-tube measurements made by the Chattock Gauge. The current carried by the anemometer was held constant while measurements of hot-wire voltage were obtained, at a number of windspeeds. The resistance of the anemometer, when cold, was found from a measurement of the voltage, when a current too small to heat the wire was used. Readings of temperature, and pressure, allowed the calculation of the windspeed from the Chattock readings.

The calibration constants were the gradient and intercept of the best straight line through data points calculated from the above measurements. The relevant formulae were:
\[ \phi = \frac{R_w}{R_w - R_a} \]

\[ \phi = \phi_0 + mU^{0.45} \]

The graph of \( \phi \) v. \( U^{0.45} \) yielded calibration constants \( \phi_0 \) and \( m \).

The calibration process was always performed manually. Automatic calibration was possible, using the available equipment. A program was written to calibrate the anemometer, against the vortex count from the automatic windspeed device.

The windspeed was set to some highish value, say 50 feet/second. The program set the current in the anemometer to its constant value, and read the voltage. A count of the frequency of vortex shedding, immediately yielded the tunnel velocity. The windspeed was then reduced by a predetermined amount, by switching on the tunnel decrease relay for a known time. After adjusting the range of the automatic filter, the previous procedure was repeated, to obtain readings of windspeed, and anemometer voltage. A set of at least 6 such readings, made possible the calibration of the hot wire anemometer, as described previously.

This procedure was not adopted because, while the experiments were proceeding, the windspeed control was to some extent under test. Consequently the additional velocity measurements obtained with the Chattock, should have made it easier to spot any inconsistencies in the behaviour of the windspeed control. The fact that no such inconsistencies have been observed, would suggest that calibration against the vortex counting device, automatically,
should be valid, and the accuracy of the device is such that the calibration should not be much less accurate. The relevant program is illustrated in Fig. 4.4.

10) Setting of fixed parameters.

In experiments involving the comparison between numerical and experimental results, the fixed parameters must be matched as closely as possible. The fixed parameters for each experiment were the Reynolds number $R_8$ and the non-dimensional frequency $F$. In fact the same value of $F$ was used throughout, in all the experiments, and numerical calculations.

The quantities $F$ and $R_8$ are not independent, consequently the choice of a value for $F$ will influence the range of choice for $R_8$. Certain obvious factors limit the range of values attainable. The length of the working region of the plate, and the limits of tunnel velocity provide limits for $R_8$, while the resonance frequency of the ribbon limits the $F$ number.

A suitable value for the frequency, $f$, of the perturbation introduced into the boundary layer, by the ribbon had to be found. Kersley (1965) defined the range in which the amplitude of oscillation of the ribbon was independent of frequency as $\frac{1}{3} f_{res} < f < \frac{5}{6} f_{res}$. To avoid the introduction of a perturbation of frequency $2f$ it was desirable that $2f$ should not be near to $f_{res}$. This additional limitation was demanded by the interest in the 2nd harmonic disturbance. The resonance frequency could be altered by changing the weights suspended from the free end of the ribbon, thereby changing the tension of the ribbon, but obviously there were limits to this procedure.
START

Set the Current

Read the H/W Voltage

Set the range of the Windspeed Filter

Read the Velocity

Output the Results

Have N Sets of Results been obtained?

Yes

No

Reduce the Velocity by a fixed Amount

STOP

Fig. 4.14. Flow Diagram - Hot-Wire Calibration
The anemometer output of fundamental frequency, and its harmonics, had to be separated from noise, at the same frequencies. Thus the spectrum of noise, picked up by the anemometer, was studied, with the anemometer at various positions in the boundary layer. Particular care had to be taken that neither the fundamental, nor even more important, the 2nd harmonic frequency, coincided with noise peaks. Such peaks included those at 50 cps, 100 cps, etc., due to the mains supply, and its harmonics; peaks at 25 cps due to vibration of the boom carrying the anemometer; and a peak at around 200 cps due to the rotation of the fan. Only the last mentioned peak was influenced by the speed of the tunnel.

On the strength of the above information one suitable value for $f$ was 45 cps, giving $2f = 90$ cps. Another value used in a number of the experiments was $f = 60$ cps, with $2f = 120$ cps. The resonance frequency was 132 cps in the first case, changing the weights suspended from the ribbon resulted in its increasing to 155 cps for the second case.

$$F = \frac{2\pi f v}{U_0^2};$$  whence if $f$ is known and $F$ chosen, $U_0$, the free stream velocity is determined, provided that the physical conditions which fix the viscosity are known. Knowledge of $U_0$ immediately limits the possible range of

$$R_\delta = \frac{U_0 \delta}{v} \quad \text{where} \quad \delta = \left(\frac{v x}{2U_0}\right)^{1/2},$$

obtainable along the plate.

It was thought desirable that both the damping regions, and the amplifying region, as described by linear theory, should be present along the length of the plate. Reference to the neutral
stability curves, showed that a choice of $F = 30.10^{-6}$ was suitable. The numerical calculations were all performed with $F = 80.10^{-6}$ and $f = 45$ cps.

A program was used to set up the fixed values of $F$ and $R_0$, see Figs. 4.6 and 4.7. The program calculated the coefficient of kinematic viscosity $\nu$ from the values of temperature and atmospheric pressure which were given to it. Since $f$ was known, the program was able to calculate the value of $U_0$ which gave the required value of $F$, normally $80.10^{-6}$.

The velocity of the tunnel was then adjusted by the program to the required value. The velocity had been brought manually to the correct range, using the coarse control. The program calculated the vortex count corresponding to the required velocity. This count was compared with the average of 10 vortex counts, each over a period of 1 second. The difference between the required and the actual values, was applied as a correction factor, to the stepping motor, controlling the fine windspeed control of the tunnel, thereby producing a correction proportional to the initial error. By repeating this process the windspeed was adjusted to the required value.

Knowing the velocity $U_0$ and the viscosity $\nu$, the required value of $R_0$ could be set up by choosing the correct $X$ position. The required $X$ position was obtained by comparing the required and actual values of the potentiometer voltage, and applying a proportional correction to the $X$ stepping motor. Repetition of this process allowed the required $X$ position to be attained.

Other initial values had to be set up, but these depended on
START

1. Read in T, P, & Fixed Parameters
2. Calculate Viscosity & Required Velocity
3. Set Velocity
4. Calculate the Required X Position
5. Measure the actual X Position
6. Find the Difference

- Yes: Is the Difference Zero?
  - Yes: Output the Starting Conditions
  - No: Apply a Correction proportional to the Difference

STOP

Fig. 4.6 Flow Diagram - Setting of Initial Conditions
the type of experiment, and consequently will be considered later, with the experiment concerned. Errors arising in the choice of the fixed parameters, will be considered in a later chapter (6), and also in Appendix 3.

(ii) The variation of parameters and the collection of data.

This stage of the experiment could be performed on-line. The relevant programs were made up of two parts; routines which performed single operations, and the body of the program which called these routines in the required order. The same routines were used in different programs, each routine being associated with a piece of equipment rather than a given experiment.

The routines had three functions:

a) keeping parameters constant,
b) varying parameters,
c) collecting data.

These types of routine will now be considered in more detail.

a) Keeping the parameters constant.

The parameters in question were, the windspeed, the current through the ribbon, the current through the anemometer, and sometimes the voltage across the anemometer.

In all cases the required constant value was stored, and the actual value of the parameter compared with it. A correction was then applied to the relevant stepping motor. The comparison and correction was repeated, until the required value was obtained.

The same control program was used for the ribbon current,
and the anemometer current. The correction applied was proportional to the initial discrepancy between the required and actual values. If the discrepancy was too large, greater than 40 units, no correction was made, and the actual value of the current was checked. Oscillation about the correct value was prevented, by providing that the correction should be halved, if the discrepancy does not continually fall in size.

The windspeed control program followed the same pattern, but oscillations, which could be caused by the long reaction time, of the windspeed, to a change in the angular velocity of the driving fan, had to be prevented. Accordingly the control routine involved only the acquisition of a single windspeed measurement. When ten such measurements had been accumulated, the average was found, and compared in the usual way with the required value.

Keeping the anemometer voltage constant implied moving on a streamline. Again the pattern was similar, but in this case, the corrections which were applied to the Z stepping motor, were small, and constant, to eliminate the possibility of oscillation. In addition, the anemometer current had to be checked throughout the process, to ensure that voltage changes were not due to current drift.

b) Varying parameters.

The position of the hot-wire anemometer probe was the variable parameter. Variation was produced by supplying a known number of steps, to the correct stepping motor. When the Z stepping motor was used, the number of steps taken was noted, to
give an indication of position.

When either of the other stepping motors was used, a reading from the relevant potentiometer, served the same function.

c) Collection of Data.

Data was collected by transferring data-logger readings to the computer. The data was stored in an array. When this array was filled, its contents were transferred to a single block on magnetic tape, by a routine, called automatically when the last location in the array was filled. Each block, which contained 128 12 bit binary numbers, was accompanied on magnetic tape by an identifier, which allowed the block to be referenced, whenever required.

In a previous chapter, the measurement of fluctuating voltages, using the digital voltmeter, was described. The fluctuating voltages were converted to D.C. voltages proportional to the r.m.s. values, by the Solartron true r.m.s. voltmeter. Because of the great variations in amplitude of the fluctuating components, through the boundary layer, an automatic attenuator, which scaled the voltages to some range of the true r.m.s. voltmeter, normally the 30 mV range, was constructed. The range of this attenuator could either be increased by 10 dB, or the attenuation could be reduced to zero.

With the true r.m.s. voltmeter on the 30 mV range, the attenuator was set to give an output between 7.5 mV and 27 mV. If the output was greater than 27 mV, the attenuation was increased by 10 dB; with an output below 7.5 mV, the attenuation was reduced
to zero, and successively increased in steps of 10 dB, to a value, 10 dB down on its previous value. This latter process was rather slow, and an attempt was made to reduce the number of times it happened.

In the experiments, a number of different fluctuating components were measured, at a large number of boundary layer positions. The order of measurement of the components, often 4 in number, was the same at every position. The attenuator range associated with each component, when it had been read previously was stored in an array, which had a location corresponding to each component. An intermediate step, which involved the setting of the attenuator, to the range stored in this array, was incorporated. If this range was still suitable, readings could be taken immediately, if not the range was adjusted, and the new range stored in the correct location. The associated range was always transferred to the computer, with the amplitude measurements.

The use of the range array saved a lot of time, since often the switching of ranges required was negligible, whereas without some such device, the range would have had to be zeroed, before every reading, or at best, the actual output, through many more ranges, would have had to be looked at, and compared with the limiting values, by the computer. This procedure is best illustrated by the flow diagram, Fig. 4.8.

When successive readings were taken through the same channel of the data-logging system, there was a tendency for spurious readings, in the form of large negative values, to appear. The D.C. voltages, produced from fluctuating components, were normally
Fig. 4-8. Flow Diagram - Control of the Automatic Attenuator
read about 20 times, and an average found. Consequently the above effect was disturbing.

It was found that a program delay between successive readings, eliminated the effect. This delay was incorporated in the relevant routine, as was a check, which allowed incorrect readings of the above type to be discarded.

The above described routines are illustrated in Figs. 4.9, 4.10 and 4.11.

**The programs**

(i) Z traverse.

At a previously selected number of equally spaced positions, through the boundary layer, measurements of the mean velocity and of the r.m.s. amplitudes of the fundamental and 2nd harmonic frequency fluctuating velocity components were collected. With the ribbon current off, the data yielded the Blasius mean flow profile, and the noise levels, at the two relevant frequencies. When the ribbon current was switched on, the measurements gave any existing mean flow distortion, and the magnitudes of the fundamental, and 2nd harmonic, components.

The program made use of the routines described in this chapter, and is illustrated in Fig. 4.12.

This program was started with the hot-wire at its closest to the flat plate. Positioning of the probe was achieved either by computer, or manually; the same procedure was used in either case. As the wire was moved towards the plate, the velocity of the air flowing past it decreased, consequently the anemometer voltage
START

Store Required Value

Read Actual Value

Find the Difference

Is the Difference Zero?

Yes

STOP

No

Yes

Is the difference Smaller

No

Reduce the Correction Factor

Correct by driving a stepping motor

Fig. 4.9 Flow Diagram - Control of a Fixed Parameter
Select D.V.M. Channel

Set Attenuator Range

Read the Contents of the DVM Channel

Is the Reading False?

Yes

Delay

No

Store the Reading

Yes

Have N Readings been taken?

No

Find the Average

Find an Estimate of the Error

Transfer Results to a Computer Array

Is the Array full?

No

Yes

Transfer the Results to Dectape

Zero the Array

STOP

Fig. 4.10 Flow Diagram - Collection of Data
START

Input Constant Parameters

Set $A = 0$

Switch off the Ribbon

Set the Current

Read the H.W. Voltage

No $\quad$ Does $A = 0$?

Yes

Set the Ribbon

Select Filter 1

Read Signal

Select Filter 2

Read Signal

No $\quad$ Does $A = 0$?

Yes

No $\quad$ Have N Sets of Readings been taken?

Yes $\quad$ Switch on the Ribbon

Set the Ribbon

Set $A = 1$

STOP

Move Outwards

Fig. 4.12 Flow Diagram - The Z Traverse
increased. A sudden drop in the voltage across the wire, close to the plate, indicated its entry into the buoyancy region, where the forced convection laws, operative at higher velocities, had broken down. Monitoring the hot-wire voltage, made it possible to drive the hot-wire into this region, where it was stopped, and then backtracked slightly to remove backlash.

A program to perform this operation is shown in Fig. 4.13.

(ii) X and Y traverse.

Similar data was obtained from these traverses, to that produced by a Z traverse. The position of the anemometer was varied along a streamline. The probe position was calculated from potentiometer readings.

X and Y traverses were performed by the same program, Fig. 4.14; numbers in a certain store determined which was carried out.

The results collected using the programs described in this section, had to be processed. This stage of the experiments is now considered.

c) Processing of Data

Data was stored on magnetic tape during the previous stage of the experiment. The data was in the form of voltages, obtained from the data-logging system, and in some cases, of associated attenuator ranges. The processing involved the conversion of voltages, into velocities, and, when the results were obtained from a
I Set the Current
Read the Voltage
Move the H. Wire in Z Steps
Set the Current
Read the Voltage
Has the Voltage decreased?
No
Is the increase greater than 1 unit?
No
Yes
Set Z = Z/2
Yes
Move the H. Wire out 5 Steps
STOP
Fig. 4.13 Flow Diagram - Initial Placing of the H.W. for a Z Traverse
Set the H.W. on the required Streamline

Obtain Data as in a Z Traverse

Record the variable Position Parameter (X or Y)

Have sufficient readings been taken?

No

Move the H.W. away from the plate

Change the variable Position Parameter (X or Y)

Yes

STOP

Fig. 4.14. Flow Diagram - X or Y Traverse
Z traverse, the location of the results exactly in the boundary layer. Two programs were used - one on-line, the other off-line.

The on-line program, run under supervisor control, prepared the previously stored data, for treatment by the off-line program. The recorded voltage levels were converted, when necessary, to true voltage levels, using the stored information about attenuator ranges, and the input information about other amplifier ranges, which were noted at the outset of the whole experiment. Noise corrections to the r.m.s. voltage levels were made by finding the difference of squares of the fluctuating signal voltage and the noise voltage at the same frequency, and then calculating the square root. The results emerged from this program, in the form of a paper tape, which could, if required, serve as part of the input, for the off-line program.

The off-line program performed several operations in order.

The calibration data was used to calculate the constants of the hot-wire, the gradient and intercept of the best straight-line through the data points. The constants were stored for later use.

The mean and fluctuating velocities corresponding to any voltage level could be calculated using the formulae given in the earlier section on anemometers, together with the aforementioned constants. Where the results had been obtained from X or Y traverse, this was all the information required, to plot distribution, and growth curves, for the fundamental, and second harmonic, components. Knowledge of the mean velocity, allowed the stream-line along which these results were obtained, to be located in the boundary layer, with reference to the Blasius Profile.
Input Instrument Ranges

Transfer Data obtained at a single B.L. position, from Magnetic Tape, into the Computer

Calculate the RMS values of Noise & Disturbance plus Noise

Eliminate the Noise

Output a position Parameter, the Mean Voltage, & the Disturbance Levels, in the Input Format for the off-line Program

Has each B.L. Position been considered?

No

Yes

STOP
Where the results were obtained from Z traverse the program served an additional function. The distance between successive Z positions was known in computer steps, but the actual distance, and the location of all the positions relative to the plate, were unknown. It was assumed, that the ten points nearest to the plate would obey the Blasius distribution, which was equivalent to assuming them to lie on a straight line, of known gradient. A facility was provided, which allowed the elimination of any points, in the buoyancy region. The gradient of the experimental best straight line was found, and compared with the Blasius gradient. The ratio of the gradients gives the relationship between computer steps and \( \frac{Z}{f} \) displacements in the boundary layer.

A graph of \( \frac{U}{U_0} \) against \( \frac{Z}{f} \) could now be plotted through the boundary layer, with the experimental results. The graph was compared with the Blasius distribution. If the discrepancies were within the range of error, it could be concluded that the boundary layer was laminar, and that the experimental points had been correctly located in the boundary layer. Distributions of any existing mean flow distortion, and of fluctuating components of fundamental, and second harmonic, frequency could now be plotted through the boundary layer.

Any change in the characteristics of the anemometer due to ageing, or being struck by dust particles, could result in a profile, which deviated from the Blasius form. Such instances were isolated, and any persistent discrepancy, would have to be treated with the utmost seriousness, since it could reflect either
Fig. 4.16 The Distribution of Mean Velocity close to the Plate
a fault in the traversing mechanism, or even more seriously faulty flow in the tunnel, due to separation or some such phenomena.

Knowledge of the actual distance moved by the probe for each computer step, made possible the calculation of \( \delta \), since \( \frac{z}{\delta} \), corresponding to a computer step was known. This allowed the direct calculation of \( R_S \), which could be compared with the indirectly calculated value, set up by the 'setting-up' program. The directly calculated value is valid in the case of any discrepancy, but again the reason for any discrepancy must be considered seriously as it could reflect a flow irregularity.

4.5 Conclusion

This chapter has dealt with the methods used to obtain results, with particular reference to the use of the computer. The results, which are described in later chapters, were obtained while the operation of the tunnel was being transformed, from a wholly manual process, to the present largely computerised process.

The system, described, has proved convenient for measuring mean velocities, and the r.m.s. values of fluctuating quantities. The use of an A to D converter, to take instantaneous readings of fluctuating quantities, should allow the application of correlation methods, which may yield information about phase, and allow better elimination of noise.

Whatever future developments of apparatus, and methods, take place, the routines, written to perform basic operations, should retain their usefulness, and the procedure contained in the programs should be adhered to, in order to work systematically, through the boundary layer.
'SET-UP' PROGRAM

```
0/o BEGIN
0/o INTEGER FACTOR, NU, TP, PRES, FREQ, WIND, WINDAV, DIF
0/o INTEGER VXR, VTR, RVX, VT, VXREQ, VX, DIFX, J, Z3
0/o INTEGER RREQ, F, U0, COUNTREQ
0/o DOUBLE TEMP1, TEMP2, TEMP3, B, Z1
0/o INTEGERFNSPEC SQRT (0/o DOUBLE B)
0/o INTEGERFNSPEC ABS (0/o INTEGER Y)

FACTOR = 204
0/o OPENHSR
0/o READOCTAL (TP)
0/o READOCTAL (PRES)
0/o READOCTAL (FREQ)
0/o READOCTAL (RREQ)
0/o READOCTAL (F)
0/o READOCTAL (VXR)
0/o READOCTAL (VTR)
0/o CLOSEHSR

TEMP1 = (343 * TP)/1000 + 935
TEMP2 = 452 + (248 * TP)/1000
TEMP3 = ((390 + TP/10) * PRES)/100
NU = 0/o LOW(((TEMP1 * TEMP2)/TEMP3)

0/o OPERATOR 0/o MESSAGE 'VISCOSITY = ' NU
TEMP1 = (628 * F)/FREQ
TEMP2 = TEMP1 * NU
U0 = SQRT (TEMP2)
```

Input of Data

Calculation of viscosity

Calculation of Required Velocity
'SET-UP' PROGRAM (Contd.)

°/° OPERATOR °/° MESSAGE 'VELOCITY = ' UO

TEMP3 = UO × FACTOR

COUNTREQ = °/° LOW(TEMP3/100)

1: TEMP1 = 0

°/° CYCLE J = -10, 1, -1

°/° READWINDSPEED (WIND)

TEMP1: TEMP1 + WIND

°/° REPEAT

WINDAV = °/° LOW (TEMP1/10)

DIF = COUNTREQ - WINDAV

°/° OPERATOR °/° MESSAGE 'REQUIRED COUNT' = COUNTREQ

→ 2 °/° IF ABS(DIF) < 1

°/° STEPPINGMOTORSELECT (5)

°/° STEPPINGMOTORDRIVE (-2× DIF)

°/° OPERATOR °/° MESSAGE 'COUNT DIF = ' DIF

→ 1

2: TEMP1 = (RREQ × RREQ)/(296 10)

TEMP2 = (NU × TEMP1)/UO

X = °/° LOW (TEMP2)

°/° OPERATOR °/° MESSAGE 'X POSITION IS: ' X

TEMP1 = (1000·VXR)/VTR

TEMP2 = (1000 × (X-100))/820

RVX = °/° LOW(TEMP1 + TEMP2)

3: °/° SELECTDVM (1)

°/° READDVM (VT)

TEMP3 = RVX × VT

VXREQ = °/° LOW (TEMP3/1000)

Setting of velocity to the required value

Calculation of required X position
'SET-UP' PROGRAM (Contd.)

0/o SELECTDVM (0)
0/o READDVM (vx)
0/o OPERATOR 0/o MESSAGE 'X VOLTAGE = ' vx
0/o OPERATOR 0/o MESSAGE 'TOTAL VOLTAGE = ' vt

DIFX = -(VXREQ - VX)
0/o IF DIFX = 0 0/o THEN → 4

0/o STEPPINGMOTORSELECT (1)
0/o STEPPINGMOTORDRIVE (DIFX)

→ 3

4: 0/o OPERATOR 0/o MESSAGE 'INITIAL CONDITIONS SET UP'

0/o STOP

0/o INTEGERFN SQRT (0/o DOUBLE B)

0/o SUSPEND

0/o STOP 0/o IF 0/o HIGH (B) < 0
0/o STOP 0/o IF 0/o HIGH (B) > 2047

Z3 = 2047

5: Z1 = Z3

Z3 = 0/o LOW ((B/Z1 + Z1)/2)

→ 6 0/o IF Z3 = 0

→ 5 0/o UNLESS Z3 < 0/o LOW(Z1)

6: 0/o RESULT = Z3

0/o END

0/o INTEGERFN ABS (0/o INTEGER Y)

0/o IF Y < 0 0/o THEN Y = -Y

0/o RESULT = Y

0/o END

0/o END 0/o OF 0/o PROGRAM
This program uses only integer and double integer arithmetic, and sets up the initial conditions with accuracy around 1\%.
THE ATTENUATOR

0% ROUTINE SETRANGE (0% INTEGER RN)

0% INTEGER RX, STORE

RX = RN - RO

0% IF RX = 0 0% THEN → 65

0% IF RX < 0 0% THEN → 64

CHANGERANGE (RX)

→ 65

64: ZERORANGE

CHANGERANGE (RN)

65: RO = RN

66: 0% READDVM (STORE)

DELAY (AAA)

0% READDVM (STORE)

0% IF STORE < 300 0% THEN → 67

0% IF STORE > 900 0% THEN → 68

→ 69

67: 0% IF RO = 0 0% THEN → 69

RO = RO - 1

ZERORANGE

CHANGERANGE (RO)

→ 66

68: 0% IF RO = 6 0% THEN → 69

CHANGE RANGE (1)

RO = RO + 1

→ 66

69: 0% END
% ROUTINE ZERORANGE
  % SWITCHONRELAY (3)
  DELAY (BAA)
  % SWITCHOFFRELAY (3)
  DELAY (ABB)
% END

% ROUTINE CHANGERANGE (% INTEGER RR)
% INTEGER RQ
  RQ = 0
  56: → 55 % UNLESS RQ < RR
  % SWITCHONRELAY (2)
  DELAY (ABA)
  % SWITCHOFFRELAY (2)
  DELAY (ABB)
  RQ = RQ + 1
  → 56
  55: % END

% ROUTINE DELAY (% INTEGER DD)
% STEPPINGMOTORSELECT (0)
% STEPPINGMOTORDRIVE (DD)
% END
CONTROL ROUTINES

a) H.W. Current and Ribbon Current

\*\* ROUTINE CONTROL (\*\* INTEGER CONST, ERROR, FF, NNN)

\*\* INTEGER DIFTOT, DIF, TEMP, M, DIFREQ

1: DIFTOT = 0

\*\* CYCLE M = - NNN, 1, -1

11: \*\* READDVM (TEMP)

DIF = TEMP - CONST

\*\* IF ABS (DIF) < 40

DELAY (AAA)

72: DIFTOT = DIFTOT + DIF

\*\* REPEAT

2: \*\* IF ABS (DIFTOT) < ERROR

3: \*\* IF DIFTOT > DIFREQ

4: \*\* STEPPINGMOTORSELECT (FF)

\*\* STEPPINGMOTORDRIVE (-4 \* DIFTOT/5)

DIFREQ = DIFTOT

1

3: DIFTOT = DIFTOT/2

4

2: \*\* END
b) H.W. Voltage for traversing along a streamline

```
% ROUTINE STREAMLINE (% INTEGER ERRO, CON)
% INTEGER VOLT, CORRECT
101: % SELECTDVM (8)
    CONTROL (CURRQ, ERRO, 6, CON)
% SELECTDVM (9)
% READDVM (VOLT)
    VOLT = VOLT - VOLTREQ
% RETURN % IF VOLT = 0
    CORRECT = 2
    CORRECT = - CORRECT % IF VOLT < 0
% STEPPINGMOTORSELECT (2)
% STEPPINGMOTOR DRIVE (CORRECT)
-> 101
% END
```
c) WINDSPEED

```
% ROUTINE WINDSPEED
% INTEGER WINDTEMP, DIFSPED
    Z = Z + 1
% READWINDSPEED (WINDTEMP)
    DIFSPED = WINDTEMP - SPEEDREQ
    WINDSTORE = WINDSTORE + DIFSPED
% IF Z<10 THEN
    IF ABS(WINDSTORE) < 10
        STEPPINGMOTORSELECT (5)
        STEPPINGMOTORDRIVE (WINDSTORE/5)
    6: Z = 0
    WINDSTORE = 0
  8: % END
```

The above programs make use of the integer function ABS

```
% INTEGERFN ABS (% INTEGER X)
% IF X<0 THEN X = -X
% RESULT = X
% END
```
RECORDING ROUTINES

\% ROUTINE AVERAGE (\% INTEGER MAX, NN)
\% INTEGER L, LLL, P, AV, POISS
\% INTEGER ARRAY C(1 : 24)
\% REAL TOTAL

LLL = 1
\% READDVM (C(LLL))
DELAY (AAA)
TOTAL = 0
\% CYCLE L = - NN, 1, -1
81: \% READDVM (C(LLL))
→ 82 \% IF C(LLL) < MAX
83: DELAY (AAA)
→ 81
82: → 83 \% IF C(LLL) < 0

DELAY (ABA)
TOTAL = TOTAL + C(LLL)
LLL = LLL + 1
\% REPEAT

AV = \% INTPT (TOTAL/NN)
OUTPUT (AV)
TOTAL = 0
\% CYCLE P = - NN, 1, -1
LLL = P + NN + 1
TEMP = C(LLL) - AV
TEMP = ABS (TEMP)
TOTAL = TOTAL + TEMP
RECORDING Routines (Contd.)

\%\% REPEAT

TOTAL = TOTAL/NN
POISS = \%\% INTPT (TOTAL)
OUTPUT (POISS)

\%\% END
TRANSFER OF DATA TO THE COMPUTER

```
% ROUTINE OUTPUT (% INTEGER JJ)
A(POINT) = JJ
POINT = POINT + 1
% IF POINT = 129 % THEN TRANSFER

% END
```

TRANSFER OF DATA FROM THE COMPUTER MEMORY TO DECTAPE

```
% ROUTINE TRANSFER
% RETURN % IF POINT = 1
A(1) = POINT
% WRITEDECTAPE (A(0), 1544, 5, 1)
POINT = 2

% END
```
CHAPTER 5

THE NUMERICAL RESULTS

5.1 Introduction

A brief account of the method used to obtain numerical results for comparison with experimental measurements is given in this chapter. The derivation of the numerical results involved two stages:

a) The setting up of the equations which described the flow.

b) The finding of numerical solutions for these equations.

The behaviour of a Newtonian fluid is fully described by the Navier-Stokes equations of motion. These equations do not in general have an analytic solution. In particular cases, an attempt is normally made to simplify the equations, by assessing the effects of the different terms in the equations, and eliminating those of least significance. Sometimes this procedure produces an equation with an analytic solution, more commonly the simplified equations must be solved numerically.

A numerical solution must be found for the boundary layer equations.

5.2 The Boundary Layer on a Flat Plate

Typically, the thickness of the boundary layer at some point, is much smaller than the distance from the leading edge of the plate. The rate of change of any quantity across the boundary layer is assumed to be much greater than the rate of change of the same quantity along the plate. The boundary layer is assumed to be two-dimensional.
These assumptions allow the simplification of the Navier-Stokes equations to give the boundary layer equations. These equations were first obtained by Prandtl in 1904 and by Blasius in 1908.

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial z^2} \tag{1}
\]

\[
0 = -\frac{\partial p}{\partial z}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]

Blasius solved the equations for the case of steady flow at zero angle of incidence to a semi-infinite flat plate. The limitless plate ensured that there were no downstream variations of the free stream parameters.

A stream function can be defined from the continuity equation. The Blasius method of solution assigns a value to the stream function:

\[
\psi = (2\nu U_0 x)^{\frac{1}{2}} f(\gamma) \quad \text{where} \quad \gamma = \left(\frac{U_0}{2\nu x}\right)^{\frac{1}{2}} z .
\]

These values can be substituted in the boundary layer equations to yield the equation:

\[
f'''' + ff'' = 0 \tag{2}
\]

Boundary conditions are supplied by the no-slip condition at the surface of the plate:
$u = w = 0 \quad \text{where} \quad z = 0$

and by the matching of the outer layers of the boundary layer to the free stream:

$$u \rightarrow U_0 \quad \text{as} \quad z \rightarrow \infty.$$ 

These boundary conditions can be expressed in terms of the function $f(\gamma)$:

$f(0) = f'(0) = 0$ and $f'(\infty) = 1$.

The boundary conditions can be used as starting values for a numerical integration of the equation for $f(\gamma)$. Tabulated values, which give the mean flow profile of the laminar boundary layer, are obtained.

5.3 The Perturbed Boundary Layer

The laminar boundary layer can be perturbed by a sinusoidal disturbance of frequency $\beta$. The Navier-Stokes equations describe the behaviour of the perturbed boundary layer. Elimination of the least significant terms from these equations is again possible, and the resulting equations are called the perturbation equations.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \Delta u$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \Delta w$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0.$$
The boundary layer is assumed to be two dimensional and the boundary conditions are the same as in the previous section.

The pressure terms can be removed from the first two equations by differentiating each partially, and subtracting the results. A stream function $\psi$ can be defined from the third equation, and substituted into the equation formed from the first two equations.

The resulting equation, expressed in terms of vorticity $h$, is:

$$\frac{\partial h}{\partial t} + \frac{\partial \psi}{\partial z} \cdot \frac{\partial h}{\partial x} - \frac{\partial \psi}{\partial x} \cdot \frac{\partial h}{\partial z} = \nu \Delta h \quad (4)$$

If a suitable expression for $\psi$ is chosen, the solution of this equation should be possible. Remembering that the perturbation was sinusoidal and of frequency $\beta$, Barry chose a double ended, Fourier series expansion of the harmonics of $\beta$:

$$\psi = \psi_0(x,z)e^0 + C \left[ \psi_1(x,z)e^{-i\beta t} + \psi_1^*(x,z)e^{i\beta t} \right]$$
$$+ C^2 \left[ \psi_2(x,z)e^{-2i\beta t} + \psi_2^*(x,z)e^{2i\beta t} \right] + \ldots \ldots \quad (5)$$

The order of magnitude of the components of frequency, $\beta$, is controlled by the scaling factor $C$. This factor is always a small quantity. The important assumption has been made, that the magnitude of the component of frequency $2\beta$ is controlled by $C^2$. This assumption, made by Stuart in his work on non-linear theory, is now built into the equations, and must affect the solutions obtained.

The expression (5) selected for $\psi$, can be substituted into the equation (4), to give an equation, which contains terms associated
with each harmonic, and which can be split into three equations, each associated with a single harmonic. These equations are given:

The mean flow, \(0^{th}\) harmonic

\[
\begin{align*}
\frac{\partial \psi_0}{\partial z} \cdot \frac{\partial}{\partial x} (\Delta \psi_0) - \frac{\partial \psi_0}{\partial x} \cdot \frac{\partial}{\partial z} (\Delta \psi_0) + c^2 \left[ \frac{\partial \psi_1}{\partial z} \cdot \frac{\partial}{\partial x} (\Delta \psi_1) - \frac{\partial \psi_1}{\partial x} \cdot \frac{\partial}{\partial z} (\Delta \psi_1) \right] + o(c^4) \\
= \nu \Delta \Delta \psi_0
\end{align*}
\]

The Fundamental, \(1^{st}\) harmonic

\[
\begin{align*}
c \left[ -i \beta \Delta \psi_1 - \frac{\partial \psi_0}{\partial z} \cdot \frac{\partial}{\partial x} (\Delta \psi_1) + \frac{\partial \psi_1}{\partial z} \cdot \frac{\partial}{\partial x} (\Delta \psi_0) - \frac{\partial \psi_0}{\partial x} \cdot \frac{\partial}{\partial z} (\Delta \psi_1) \right] \\
- \frac{\partial \psi_1}{\partial x} \cdot \frac{\partial}{\partial z} (\Delta \psi_0) + c^3 \left[ \frac{\partial \psi_1}{\partial z} \cdot \frac{\partial}{\partial x} (\Delta \psi_2) + \frac{\partial \psi_2}{\partial z} \cdot \frac{\partial}{\partial x} (\Delta \psi_1) \right] + o(c^5) \\
= c^2 \nu \Delta \Delta \psi_1
\end{align*}
\]

The \(2^{nd}\) harmonic

\[
\begin{align*}
c^2 \left[ -2i \beta \Delta \psi_2 + \frac{\partial \psi_0}{\partial z} \cdot \frac{\partial}{\partial x} (\Delta \psi_2) + \frac{\partial \psi_2}{\partial z} \cdot \frac{\partial}{\partial x} (\Delta \psi_0) - \frac{\partial \psi_0}{\partial x} \cdot \frac{\partial}{\partial z} (\Delta \psi_2) \right] \\
- \frac{\partial \psi_2}{\partial x} \cdot \frac{\partial}{\partial z} (\Delta \psi_0) + \frac{\partial \psi_1}{\partial z} \cdot \frac{\partial}{\partial x} (\Delta \psi_1) - \frac{\partial \psi_1}{\partial x} \cdot \frac{\partial}{\partial z} (\Delta \psi_1) \right] + o(c^4) \\
= c^2 \nu \Delta \Delta \psi_2
\end{align*}
\]
These equations apply to finite perturbations, for which \( c \) is finite, but small.

If infinitesimal perturbations are considered, \( C \) is infinitesimal and all terms of order \( C^2 \) or higher can be eliminated. The mean flow equation (6a) is reduced to the Blasius boundary layer equation. The fundamental equation (6b) reduces to the purely linear, homogeneous form. The second harmonic equation (6c) collapses to a form similar to the mean flow equation.

Even for finite perturbations \( C \) is small, and the influence of terms of order \( C^2 \) is not expected to be great. However such terms in equation (6a) produce distortion of the mean flow. Terms of order \( C^2 \) result in the prediction of a 2\(^{nd}\) harmonic component by equation (6c). Finally terms of order \( C^3 \) are expected to change the distribution of the fundamental component.

5.4 Separability

The perturbation is periodic in \( t \). If it is also assumed to be periodic in \( x \), the first order term of the expansion can be written

\[
\psi_1(x, z)e^{-i\beta t} = \psi_1^+(x, z)e^{i(ax-\beta t)}.
\]

The second order term can be expressed similarly.

If \( \alpha \) is complex, \( \alpha = \alpha_r + i\alpha_i \), there is a spatial amplification term \( e^{-\alpha_i x} \) in addition to the periodic term \( e^{-\alpha_r x} \). A perturbation for which \( \beta \) is wholly real, and \( \alpha \) complex, is a spatially dependent perturbation.

At this stage a separable solution is postulated. It is
assumed that the exponential, spatial amplification term carries all the \( x \) dependence of the perturbation. The first order term may now be written

\[ \psi_1(x, z) e^{-\beta t} = \phi_1(z) e^{i(\alpha x - \beta t)} \]

similarly

\[ \psi_2(x, z) e^{-2\beta t} = \phi_2(z) e^{i2(\alpha x - \beta t)} \]

Substitution of these expressions into the equations (6) given earlier, simplifies these equations. A homogeneous equation is obtained for \( \phi_1 \), which has an eigenvalue solution. Values of \( \beta \) and \( R \) are known, and solution of the equation involves finding complex eigenfunctions \( \phi_1(z) \) which correspond to the complex eigenvalues \( \alpha \).

The equation for \( \phi_2 \) is not homogeneous, consequently the solutions can be obtained directly. In particular the solutions for \( \phi_2(z) \) are not eigenfunctions with respect to eigenvalues \( 2\alpha \).

5.5 Solution of the Equations

The eigenvalues \( \alpha \) and corresponding eigenfunctions \( \phi_1(z) \) are obtained using a numerical method developed by Osborne. The application of this method to the present problem has been fully discussed by Jordinson (1968). The numerical solution was compared with the experimentally measured fundamental component by Ross (1969).

The numerical solution, produced by considering an infinitesimal perturbation, is in no way dependent on the magnitude of the perturbation. A linear relationship is predicted between the distributions...
of the fundamental produced by perturbations of different sizes.

The values for $\alpha$ and $\phi_1(z)$ can be substituted in equation (6o) and corresponding solutions for $\phi_2(z)$ evaluated. These solutions are compared with the experimental measurements of the 2nd harmonic component in the next chapter. The other 2nd order quantity, mean flow distortion can be evaluated in the same way from equation (6a).

If the size of the fundamental component is known, the size of the associated 2nd order effects is predicted by theory. The 2nd order quantities increase faster than the fundamental component when the perturbation is increased in size. The relationship between the distributions of a 2nd order quantity produced by perturbations of different sizes is predicted to be quadratic.

5.6 The Iteration Process

The eigenfunctions $\phi_1(z)$ corresponding to eigenvalues $\alpha$ are calculated for a Blasius mean velocity distribution. The perturbation produces distortion of the mean flow, which can be evaluated by substituting the values of $\phi_1(z)$ and $\alpha$ into the 2nd order equations (6a). New values of $\phi_1(z)$ and $\alpha$ may be calculated for the distorted mean velocity distribution. This is the first step in an iteration process.

Recalculation of the 2nd order terms produces a new value for the mean flow distortion. The next step of the iteration process is a recalculation of $\phi_1(z)$ and $\alpha$. Successive values of the same quantity, produced by the iteration process must converge, otherwise the method has broken down. Normally iteration is
repeated until two such successive values, differ by less than some predetermined value - see Fig. 5.1.

Whenever an experimental value of the fundamental component has been determined, a first value for the mean flow distortion can be found. This allows the iteration to be started. For fundamental disturbance levels of less than 1%, the changes produced by iteration were too small to be detected experimentally. Thus the numerical solutions obtained for comparison with the experimental results, discussed in the next chapter, had not gone through the iteration process.

The iteration method allows the magnitude of a perturbation to affect the form of the solution. The mechanism is the interaction of first and second order effects. The method used to obtain the result of this interaction is successive approximation, and no attempt is made to relate this mathematical method to the way in which the boundary layer adapts to the presence of a perturbation.

It was possible to explore the effects of increasing the magnitude of the fundamental component. Experimentally this results in the appearance of the first signs of transition to turbulence, at a disturbance level of 1%. Theoretically the only effect is a breakdown of the iteration method occurring at disturbance levels between 4% and 8% at realistic Reynolds Numbers. The breakdown takes the form of an oscillation of the successive values of the same quantity, and occurs at lower disturbance levels at higher Reynolds Numbers. Any attempt to relate the breakdown in a mathematical process to the physical phenomenon - transition to turbulence seems to be dangerous.
Fig. 5.1 The Iteration Process
5.7 Thickening Boundary Layer

Barry and Ross (1970) included in their equations for the perturbation, the main terms representing the growth of boundary-layer thickness. The equations given earlier in this chapter, include these terms. Numerical results were derived from these equations and compared with results derived from equations used in earlier work. Such a comparison is shown in Figs. 5.2 and 5.3.

The effects of this modification are small. The fundamental component is reduced by a maximum of 2%. The 2nd harmonic component is reduced by a maximum of 4%. The inclusion of the extra terms does result in a small shift in the position of the neutral stability curve. Branch 1 moves in the direction of lower Reynolds Numbers, thereby improving the agreement between theory and experiment (Ross, Barnes, Burns, Ross, 1970).

5.8 2-Dimensionality

The numerical results predict the behaviour of a 2-dimensional perturbation in a 2-dimensional boundary layer. Should span-wise variation be found in the experimental boundary layer the applicability of the numerical results becomes questionable.

Many experiments, e.g. Klebanoff, Tidstrom and Sargent (1962) have presented strong evidence that 3-dimensionality appears during the transition process. This question is returned to in a later chapter, for the moment it is sufficient to state that such results must reduce the possibility of explaining transition with 2-dimensional theory.
Fig. 5.3 The B.L. Thickening Terms - 2nd Harmonic Component
5.9 **Discrete and Separable Solutions**

Experimentally a perturbation is introduced with a vibrating ribbon, and travels downstream, amplifying or damping according to its frequency and the downstream position. A theoretical solution can be obtained for any Reynolds number corresponding to a downstream position, and this solution will be entirely independent of solutions obtained at Reynolds numbers corresponding to either upstream or perhaps less surprisingly, downstream positions. One implication of agreement between theory and experiment is that cumulative effects need not be considered to explain the conditions at a given position. Such agreement has been demonstrated to a considerable extent by Ross, Barnes, Burns, Ross, (1970), for linear theory. Second order theory makes predictions about the second harmonic component and mean flow distortion which are compared with experiment in the next chapter. Agreement will again imply the absence of cumulative effects.

When an attempt is made to predict theoretically the change in behaviour between two different Reynolds Numbers, that is between two X positions, difficulties are encountered. The assumption of a separable solution implies that all the X variation is carried in the term $e^{iax}$. It is known that $\phi(z)$ varies with $R\phi$ and thus with $X$, consequently it would appear that a separable solution exists only if $X$ is constant. A solution of the unseparated form $\psi(x, z)$ should be found, but unfortunately such a solution requires more boundary conditions than are available.

The good agreement obtained between theory and experiment by Ross, Barnes, Burns, Ross (1970) suggests that $\phi(z)$ must approximate
closely to $\psi(x, z)$, when $X$ is constant.

5.10 Conclusions

Linear theory has been extended by the inclusion of terms of higher order. The resulting equations have solutions which are compared with experimentally measured quantities such as the 2nd harmonic component. It is hoped that the extended theory might provide information about transition.

In favour of this idea is the dependence of the form of the solution on the size of the perturbation. The existence of a limit in the size of perturbation is also of interest in this connection, though the form this limit takes, makes any attempt to relate it to experiment rather dangerous. Against the possibility of second order theory of the type described, predicting transition must be ranged the prominent three dimensionality observed in the transition region, and the lack of any theoretical mechanism to allow cumulative effects.
6.1 Introduction

In their paper dealing with 3-dimensional effects Klebanoff, Tidström and Sargent (1962) made some observations of the 2nd harmonic component. These observations were made with a view to deciding whether the generation of harmonics could play an important part in the transition process. They came to a negative conclusion, and consequently, did not attempt a detailed investigation of the harmonic components.

Kersley (1965) and Barnes (1967) investigated the behaviour of the 2nd harmonic component. They pointed out differences between the distributions of the 2nd harmonic component, and the fundamental component, through the boundary layer. The present work is a continuation of the investigation, begun by them.

Ross et al. (1970) obtained good agreement between numerical results derived from linear theory, and the experimentally measured fundamental component. A natural extension of this work, was to include higher order terms in the theoretical equations, following Meksyn and Stuart (1951), and to see whether the agreement between theory and experiment could be maintained. Barry (1970) produced numerical results from the equations which had been extended by the inclusion of second order terms. These results predicted the behaviour of the 2nd harmonic frequency component, and also predicted distortion of the mean flow, and of the fundamental component, as compared with values derived from linear theory.
The work to be described in this chapter compares the numerical results obtained by Barry, with experimental results. In order to make this comparison, certain fixed parameters had to be matched, as closely as possible. For experimental reasons described in the previous chapter (L), an F number of 80 was regarded as convenient. Accordingly all the numerical results were obtained with $F = 80$, and experimental conditions were adjusted to approximate to this value as closely as possible. The choice of a fixed value for $F$ limited the quantity of data produced. The results of Ross et al. suggested that there would be no loss of generality arising out of this limitation.

Two important assumptions were made about the boundary layer in the course of the derivation of the equations from which Barry produced his numerical results:

1. The pressure gradient was assumed to be zero in the streamwise direction.

2. Strict two-dimensionality was assumed.

The extent to which the experimental arrangement satisfied these conditions, has to be examined.

6.2 The Pressure Distribution along the Flat Plate

The pressure distribution was measured by Barnes (1966) and his investigations were repeated in rather more detail by Ross (1969) as a preliminary to his measurements of the fundamental frequency component.

The results of these investigations are summarised in this section.
The static pressure at any position along the plate was found to be independent of the position of the carriage, provided that the trailing edge of the carriage was not less than 6 inches in front of the trailing edge of the flat plate.

The pressure gradient was zero within the range of experimental accuracy between 1 foot and 4 feet downstream of the leading edge of the plate. Between the 4 feet and 6\(\frac{1}{2}\) feet positions there was a small positive pressure gradient. With the probe downstream of 5\(\frac{1}{2}\) feet, the carriage was less than 6 inches from the trailing edge of the plate. This specified a downstream working limit of 5\(\frac{1}{2}\) feet.

Within the working region the assumption of zero pressure gradient appeared tenable as the Polhausen Shape Factor \(\Lambda\) never exceeded - 0.30. The static pressure is shown in Fig. 6.1.

Ross concluded that the movement of the carriage produced changes in blockage after the downstream limit had been passed and attributed these changes to an interaction between the wakes of the carriage and the plate.

6.3 Two Dimensionality

A necessary condition for strict two dimensionality would be that both the ribbon, and the plate, were infinite in the span-wise direction. In fact the span of the plate was 4 feet 6 inches, and the ribbon was only 8 inches long. Some variation in the span-wise direction was inevitable, but it was hoped that this variation would be small enough near the centre line of the plate to be ignored.

The spanwise distributions of the mean and fluctuating components of velocity were measured at a number of downstream positions.
Fig. 6.1 The Pressure Gradient in the Working Section
Fig. 6.2 shows results obtained at a Reynolds number of 1200.

The mean velocity was constant across the span of the plate, provided that the input signal to the ribbon conformed to rules discussed in the next section. The fluctuating components showed a considerable variation across the span of the plate, having an amplitude maximum close to the centre-line, and decreasing in amplitude as the distance from the centre-line increased. The variation of the 2nd harmonic frequency component followed closely the variation of the fundamental component. The relative variation of the 2nd harmonic was much greater than that of the fundamental.

Neither component had a symmetrical distribution about the centre-line. In the region below the centre-line, out to a distance of 0.8 inches, both fluctuating components showed very little variation with spanwise position. It appeared that in this region the local conditions should approximate very closely to the two dimensional assumption. For this reason, the results discussed in this chapter were obtained at a spanwise position within this region. Before each experimental run, the spanwise variation of the more sensitive 2nd harmonic component was examined close to the working position. The absence of spanwise variations around the working position led to the conclusion that two-dimensional theory could be applied.

This conclusion was supported by the good agreement obtained between experimental results and two-dimensional linear theory (Ross et al., 1970). However there was one region of disagreement which will be considered here.

Ross et al. compared the theoretically predicted growth in the
downstream direction, of the fundamental component with its experimental counterpart. Possible theoretical objections to this procedure have been discussed in the preceding chapter. Ross et al. noticed that towards the end of the amplifying region the experimentally measured fundamental component was always smaller than predicted. This discrepancy increased in the downstream direction (see Fig. 6.3).

The measurements of Ross et al. were obtained near the centre-line of the plate, where the fundamental component had its maximum amplitude. It is suggested that the spanwise distribution of amplitude shown in Fig. 6.2 will lead to the transfer of energy outwards from the central peaks. This transfer of energy will reduce amplitudes measured near the centre-line to below their predicted values. The reduction of amplitude will increase in the downstream direction because the transfer of energy must be expected to continue as long as the spanwise variations persist.

Thus the discrepancies observed by Ross et al. are suggested to be, at least in part, due to the imperfect experimental approximation to the two dimensional ideal.

Because of theoretical and experimental objections, the experimentally measured growth of the 2nd harmonic component in the downstream direction has not been compared directly with its theoretical counterpart.

The theoretical objections, indicated in Chapter 5, result from the assumption of a solution which is separable in \( x \) and \( z \).

The experimental objections, discussed in this section, are suggested to result from the three-dimensionality of the fluctuating components.
Fig. 6.2 Spanwise Distributions at $R_e = 1200$
Fig. 6.3 Downstream Growth for $F = 81 \times 10^{-6}$ Ross (1969)
The spanwise distribution of the fluctuating components probably reflected the amplitude variation along the length of the vibrating ribbon. If this assumption is correct, the spanwise variations in the central region can be reduced by using a longer ribbon. There are difficulties associated with the use of a longer ribbon.

Ross (1969) has shown that at a fixed velocity, the position of 'natural' transition moves upstream as the length of the ribbon is increased, possibly reducing the working length of the plate. A longer ribbon is more likely to twist, introducing undesirable perturbations.

A reduction in the spanwise variations might be obtained by shaping the magnetic field which drives the ribbon. A field whose strength increases with distance from the centre-line of the plate could have the desired effect. For the best results, careful adjustment of the field would be required, and an electromagnet could probably be used, replacing the permanent magnets used to produce a uniform field, at present.

6.4 Working Conditions

The 2nd harmonic component was small enough to be comparable with the noise component of the same frequency. An increased input perturbation produced a proportionately larger increase in the size of the 2nd harmonic component.

Two undesirable effects were likely to occur as the input increased in size. Higher order effects, unaccounted for by 2nd order theory, might have appeared and produced discrepancies between
the experimental and theoretical results. Any such effects would be predicted to be at least an order of magnitude down on the 2nd harmonic. The other undesirable effect, at this stage of the work, was the appearance of traces of the transition process. Whether these two effects can be regarded as in any way equivalent is a question which will be returned to, in the next chapter.

The criterion adopted for the size of the input perturbation was that it would produce at the relevant Reynolds Number, a fluctuating fundamental component of magnitude \( \frac{u'}{U_0} \) less than 0.7%. Under such conditions none of the behaviour associated with transition, to be described in the next chapter, was observed. Since the 2nd harmonic under those conditions was comparable with noise, the possibility of distinguishing higher order effects from noise can be ignored.

The results described in this chapter, conformed to the above condition.

6.5 The Numerical Data

The eigenfunctions and their derivatives were evaluated at 40 equally spaced positions across the boundary layer for values of \( Re \) in the range 900 - 1500. The F No. was always 80.

The relationship between the derivatives of the eigenfunctions and the experimental r.m.s. values have been given by Jordinson (1969)
\[
\begin{align*}
\mathbf{u}_{1\text{rms}} &= \sqrt{\mathbf{U}_{1}^2} = \frac{1}{\sqrt{2}} \mathbf{C} e^{-(a_1 \delta_1)x} \sqrt{\left(\beta_{1r}'^2 + \beta_{11}'^2\right)} \\
\mathbf{u}_{2\text{rms}} &= \sqrt{\mathbf{U}_{2}^2} = \frac{1}{\sqrt{2}} \mathbf{C}^2 e^{-2(a_1 \delta_1)x} \sqrt{\left(\beta_{2r}'^2 + \beta_{21}'^2\right)}.
\end{align*}
\]

For a given value of \( R_5 \), \( \frac{1}{\sqrt{2}} \mathbf{C} e^{-(a_1 \delta_1)x} \) and \( \frac{1}{\sqrt{2}} \mathbf{C}^2 e^{-2(a_1 \delta_1)x} \) are constants relating theoretical and experimental quantities. To evaluate these constants for a given \( R_5 \), the area contained by the experimental quantity between the plate and the phase change point was equated to the corresponding area bounded by the theoretical quantity.

The distribution of r.m.s. velocities produced by experiment normally consisted of up to 50 data points scattered through the boundary layer, not necessarily at equal intervals. A simple program produced values of the r.m.s. velocity at equal intervals through the boundary layer, using linear interpolation. The trapezoidal rule was applied to the equally spaced data points to evaluate the area of interest. The trapezoidal rule was then applied to the equally spaced values of \( |\delta'| \) to obtain the relevant theoretical area. The ratio of these areas supplied a multiplier which operated on all the theoretical values. A curve was drawn through the resulting points on a graph, and by plotting the experimental points on the same graph, comparison between the theoretical distributions, and the experimental distributions, through the boundary layer could be made.

As a result of applying this procedure to the fundamental \( x \) is non-dimensional.
and 2nd harmonic components at Reynolds numbers between 900 and 1500, the results illustrated in Figs. 6.4, 6.5, 6.6, 6.7, 6.8, 6.9 and 6.10 were obtained.

6.6 Distributions Through the Boundary Layer

The agreement between the experimental and theoretical distributions of the fundamental component was excellent. Small discrepancies appeared near the peaks of the distributions, particularly at higher Reynolds numbers. Such effects were also observed by Ross et al. (1970) so it is unlikely that they can be attributed to the fact that larger input perturbations were employed in this investigation.

It might have been expected that the fundamental component would have shown 2nd order effects because of its relatively large amplitude. Such effects, were predicted to alter the size of the distribution peak and would not show up at this stage.

It will be remembered that the distributions were located in the boundary layer by fitting the mean velocities to the Blasius' profile. The excellent agreement between theory and experiment in the location of such features of the fundamental distribution as the peak of amplitude, and the phase change or minimum point, which is evident at each downstream position, provided confirmation of the reliability of the method of location.

The comparison between the experimental and theoretical distributions through the boundary layer, of the 2nd harmonic components is also shown in Figs. 6.4 - 6.10. The agreement was generally remarkably good. That this agreement does not look
Fig. 6.4 Theoretical and Experimental Distributions at $R_\theta = 900$
Fig. 6.6 Theoretical and Experimental Distributions at $R = 1100$
Fig. 6.7 Theoretical and Experimental Distributions at $R_8 = 1200$
Fig. 6.8 Theoretical and Experimental Distributions at $R_e = 1300$
Fig. 6.9 Theoretical and Experimental Distributions at $R_s = 14.00$
Fig. 6.10 Theoretical and Experimental Distributions at $R_s = 1500$
as impressive as that for the fundamental component is not surprising when the fact that the 2nd harmonic was an order of magnitude smaller is remembered. Attention is drawn to the behaviour of the three obvious features of the 2nd harmonic distributions, the peak, the minimum and the point of inflexion.

The peak of the experimental distribution moved inwards as $R_\delta$ increased. At $R_\delta = 900$ the maximum was at $\frac{Z}{\delta} = 0.105$ but at $R_\delta = 1500$, it was at $\frac{Z}{\delta} = 0.055$. This compared with a theoretically predicted shift in the position of the maximum from $\frac{Z}{\delta} = 0.100$ to $\frac{Z}{\delta} = 0.0625$.

The minimum of the experimental distribution could not be located accurately, because the 2nd harmonic was too small in its region to be separated from the noise with any certainty. The results which were obtained were compatible with the theoretical location of the minimum points.

The change in gradient of the distribution, between the peak and the minimum, was present at every position except $R_\delta = 900$. As with the other features, as $R_\delta$ increased, the change in gradient moved towards the origin of the $\frac{Z}{\delta}$ graph.

Exact matching of the fixed parameters was not possible, the actual values of $\delta F$ and $R_\delta$ are shown in Table 6.1.

6.7 The Relative Sizes of the Fundamental and 2nd Harmonic Components

The theoretical expressions for the r.m.s. values of the fundamental and 2nd harmonic components are:
<table>
<thead>
<tr>
<th>$R_s$ Initial</th>
<th>$R_s$ Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>908</td>
</tr>
<tr>
<td>1000</td>
<td>985</td>
</tr>
<tr>
<td>1100</td>
<td>1103</td>
</tr>
<tr>
<td>1200</td>
<td>1219</td>
</tr>
<tr>
<td>1300</td>
<td>1327</td>
</tr>
<tr>
<td>1400</td>
<td>1407</td>
</tr>
<tr>
<td>1500</td>
<td>1491</td>
</tr>
</tbody>
</table>

$R_s$ Initial Set by program with accuracy 1%

$R_s$ Calculated Derived from $\delta = 0.341 \delta$ with accuracy 3%
The results discussed in the previous section were obtained by relating both fluctuation components to the corresponding theoretical quantity. This allowed the form of the distributions to be compared.

However once the relationship between the fundamental component and the first order eigenfunction had been computed, the 2nd harmonic component could be determined in magnitude and form without further reference to the experimental results.

Writing \( u_{1\text{ rms}} = A |\phi_1'| \) when \( A \) has been determined, the 2nd harmonic component can be found from \( u_{2\text{ rms}} = \sqrt{2} A^2 |\phi_2'| \).

In order to test this assumption reference was made to the results shown in Table 6.2. In this table are displayed the values of the constant relating the 2nd harmonic component and the 2nd order eigenfunction, obtained in two ways. In column 2, the constant is calculated from the corresponding constant for the fundamental, displayed in column 1. In column 3, the constant is calculated directly from the relationship between the 2nd harmonic and 2nd order eigenfunction as in the previous section.

The results show that the 2nd harmonic predicted from the magnitude of the fundamental was always of the same order of magnitude as the experimentally measured 2nd harmonic. Except for the region of Reynolds number 1100 - 1300, this experimentally measured value was greater than the predicted value. An explanation for this effect at the lower range of Reynolds number is offered in the next
<table>
<thead>
<tr>
<th>$R_e$</th>
<th>$A$</th>
<th>$\sqrt{2} A^2$</th>
<th>$2^{nd} H./2^{nd} O. EFn.$</th>
<th>$% age$</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>0.00332</td>
<td>0.0000156</td>
<td>0.0000281</td>
<td>180</td>
</tr>
<tr>
<td>1000</td>
<td>0.00441</td>
<td>0.0000276</td>
<td>0.0000387</td>
<td>140</td>
</tr>
<tr>
<td>1100</td>
<td>0.00583</td>
<td>0.0000481</td>
<td>0.0000469</td>
<td>97.5</td>
</tr>
<tr>
<td>1200</td>
<td>0.00629</td>
<td>0.0000560</td>
<td>0.0000477</td>
<td>85</td>
</tr>
<tr>
<td>1300</td>
<td>0.00759</td>
<td>0.0000815</td>
<td>0.0000915</td>
<td>112</td>
</tr>
<tr>
<td>1400</td>
<td>0.00169</td>
<td>0.0000041</td>
<td>0.0000079</td>
<td>196.5</td>
</tr>
<tr>
<td>1500</td>
<td>0.00244</td>
<td>0.0000084</td>
<td>0.0000129</td>
<td>154.5</td>
</tr>
</tbody>
</table>

Table 6.2 Actual and Predicted Magnitudes of the 2nd Harmonic Component
section. An explanation for the results obtained at Reynolds numbers of 1400 and 1500 was harder to find. It was possible that the effects of some cumulative process were being observed but no evidence is supplied in support of this.

The next two sections deal with some of the factors which could tend to obscure the results.

6.8 The Production of Spurious 2nd Harmonic

The possibility that a disturbance of the 2nd harmonic frequency would be introduced by the ribbon into the boundary layer has to be considered. There were three possible sources of such a disturbance.

Harmonics of the fundamental frequency were likely to be present in the driving current supplied to the ribbon by the oscillator. The response of the ribbon to the interaction of the current with the magnetic field was not likely to be purely linear. The production of the boundary layer perturbation by the vibrating ribbon is a little understood process, and an assumption of linearity could not be justified.

It is suggested that the boundary layer disturbance included two unrelated components of the 2nd harmonic frequency, the true 2nd harmonic component, and the spurious component whose generation has been explained. The behaviour of the spurious component could be predicted using linear theory, and inevitably its damping and amplifying regions would not coincide with those of the components of interest.

For the results described in the preceding section, the relevant
F number for the spurious component was 160. Reference to a neutral stability curve showed that the spurious component could be expected to reach a maximum value at $R_\infty = 860$, and thereafter to damp. It seemed possible that traces of this component might appear at positions, $R_\infty = 900$ and $R_\infty = 1,000$, but that further downstream the spurious component would be damped sufficiently to be ignored. The distributions at $R_\infty = 900$ and $R_\infty = 1,000$ were examined.

At the upstream position the distribution at the 2nd harmonic frequency did not show the expected change of gradient at $x/\theta = 0.4$, and was almost twice as large as predicted by theory. The distribution of a fundamental component at the frequency of the 2nd harmonic, at $R_\infty = 900$, is shown in Fig. 6.11a. The two distributions are very similar. At $R_\infty = 1,000$ the shape of the experimentally measured distribution, at the 2nd harmonic frequency, was similar to its theoretical counterpart but the magnitude of the component was still considerably larger than predicted. These results seemed to be consistent with the presence of a spurious component of 2nd harmonic frequency which was large enough to dominate at $R_\infty = 900$, and still significant at $R_\infty = 1,000$.

Fig. 6.12 shows a distribution in the downstream direction of the 2nd harmonic component. There is a small peak in the distribution at $R_\infty = 860$. Such a peak was always observed in downstream distributions. The position of this peak is as would be expected if it were caused by a double frequency fundamental component. The downstream distributions seemed to confirm the importance of the spurious 2nd harmonic component at Reynolds numbers below 1,000.

The suggested explanation for the discrepancies between the experimentally measured and the predicted, amplitudes of the
Fig. 6.11 Sources of Error

a) Spurious Fundamental

b) Spurious 2nd Harmonic generated by the H.W. R=1000
R=900
F=160
Fig. 6.12 Downstream Behaviour of the 2nd Harmonic Component
2nd harmonic component at low Reynolds numbers was in no way relevant to the discrepancies observed at higher Reynolds numbers. There appeared to be an effective lower limit, to the useful working region, caused by the generation of spurious 2nd harmonic, in this case at $R_8 = 1,000$. This restriction on working in the upstream region of the amplifying zone is present whatever the frequency of the perturbation.

Improvements could be sought, by reducing the spurious component perhaps by improving the oscillator; but given the lack of knowledge of the mechanism of introducing a perturbation into the boundary layer, such improvements may not be significant. The application of correlation methods to the measurements of the 2nd harmonic component might allow the two components to be separated.

6.9 Other Sources of Error

In the previous section the effect of the generation of harmonics in the input mechanism was considered. Harmonics may be generated in the measuring system.

The hot-wire anemometer which was used to sense the disturbances is not a linear instrument. The output voltage supplied by the anemometer depended upon the velocity of the flow over its surface. If this velocity consisted of a mean component together with a single frequency perturbation, the resulting output voltage would contain harmonics.

A program was written which Fourier analysed the output from the anemometer. The 2nd harmonic generated by the anemometer was a maximum with the hot-wire close to the plate, but was never significant.
for the range of disturbance sizes dealt with in this chapter.

Fig. 6.11b shows the distribution of 2nd harmonic generated, close to the plate.

The generation of harmonics is a characteristic of any amplifier. It was not possible to obtain an accurate estimate of the generation of harmonics in the measuring circuit by the various amplifying components. The passage of a sine wave through the circuit was followed, and showed that the quantity of 2nd harmonic generated was certainly less than 1% of the magnitude of the sine wave, but how much of the 2nd harmonic observed was due to straight amplification of an existing component, and how much was due to generation was impossible to estimate. No correction was made, but in the presence of any systematic variation from expected results an effort would have to be made. No such variation appeared in the results considered previously.

Noise presented the chief problem in the attempt to measure the 2nd harmonic. The noise level associated with the 2nd harmonic frequency, was less than that associated with the fundamental frequency, but this difference was much less than that between the actual components. This explained the difficulty in measuring the 2nd harmonic component. In addition, although the noise level dropped with the distance from the plate, the drop was far less rapid than for the 2nd harmonic component. This explained the extreme difficulty experienced in obtaining readings further out in the boundary layer, in fact in some cases it was impossible. The noise-levels associated with the two relevant frequencies had a distribution through the boundary layer which is illustrated in Fig. 6.13.

Correlation measurements could make it possible to eliminate
Fig. 6.13 Variation of Noise Levels with B.L. Position
this problem. However since the reduction in noise level achieved is dependent on the number of measurements made, the size of the noise level compared with the 2nd harmonic could result in a prohibitive number of readings being required.

6.10 Conclusions - 2nd Harmonic

The results discussed in this chapter seem to offer confirmation of the predictions of 2nd order theory, especially when the difficulties associated with the measurements are considered. The one serious unexplained discrepancy is in the magnitude of the 2nd harmonic component at Reynolds Numbers of 1400, and 1500. These errors are of the order of a factor of two.

Later in this chapter, observations of the growth downstream of the fluctuating components are presented, and although no direct comparison is made with numerical results derived from theory, additional confirmation of theory does emerge.

The position of the peak of the distribution of the fundamental component appears to depend on the product $F \times R^\circ$, the peak moving in towards the plate as the product increases. This result must apply to any spurious double frequency component. However the results discussed in a previous section suggest that the peaks of the spurious, and genuine 2nd harmonic distributions coincide. If the $F$ number for the 2nd Harmonic is supposed to be twice the $F$ number for the fundamental, this result can be explained by assuming that the peak of the 2nd harmonic distribution obeys the same rules as the peak of the fundamental distribution.

The observed fact that the peak of the 2nd harmonic distribution
always lies well inside the peak of the associated fundamental distribution, follows automatically.

6.11 Phase Distribution across the Boundary Layer

Barry's results supplied the distribution of the phase of the 2nd harmonic through the boundary layer.

It was hoped to use a photographic method to obtain results for comparison with theory. The input to the vibrating ribbon was projected on the screen of an oscilloscope together with the 2nd harmonic component. Measurement of the separation in the peak positions of the traces were made, with the anemometer at different positions in the boundary layer. The variation should have yielded the phase variation of the 2nd harmonic through the boundary layer.

The results were very disappointing, the comparison between a typical set and theory is shown in Fig. 6.14. Useful phase measurement must await the application of correlation measurements which might allow the influence of noise to be removed, and reduce the errors to manageable proportions.

6.12 Mean Flow Distortion

The mean flow distortion predicted by theory was a second order effect, and consequently very small. For a disturbance of amplitude 1% of the free stream velocity, the mean flow distortion will never exceed 0.2% of the free stream velocity.

In the inner regions of the boundary layer, the sensitivity of the hot-wire anemometer is a maximum and it may be possible to detect
Fig. 6.14. B.L. Distribution of 2nd Harmonic Phase at $R_b = 1300$
mean flow variations of 0.5% of the free stream velocity. However the sensitivity decreases as the distance from the plate increases.

It is not surprising that no traces of mean flow distortion were observed, while the results described earlier in this chapter were being collected. The comparison between the measured mean velocity, and the Blasius mean flow profile at the extreme measurement positions is shown in Figs. 6.15 and 6.16.

6.13  **Downstream Growth of the Disturbance**

a)  **Various input perturbations**

The growth of the fluctuating components measured along a streamline in the downstream direction is shown in Figs. 6.17 and 6.18. Each graph illustrates the behaviour of the fundamental and 2nd harmonic components, produced by two different input perturbations at the same $z/9$ position. Measurements were made near to $z/9 = 0.1$ because the ratio of the 2nd harmonic to the noise is most favourable in this region.

One of the 2nd order effects predicted by theory was a reduction in the local amplification factor, $a_1$, of the fundamental component. Although the change in $a_1$ was very small, it was thought that the effect on the fundamental component might be cumulative, resulting in a decrease in the ratio of the two fundamental components, produced by different input perturbations, in the downstream direction. Tables 6.3 and 6.4 showed no signs of this effect.

Reference to Section 6.7 shows that if the input perturbation
Fig. 6.15  B.L. Distribution of Mean Velocity at $R_e = 900$
Fig. 6.16 B.L. Distribution of Mean Velocity at $R_e = 1500$
is altered in such a way as to multiply the amplitude of the fundamental component at some position by \( B \), then the amplitude of the 2nd harmonic at the same position should be multiplied by \( B^2 \). Tables 6.3 and 6.4 show agreement to within 5\%, with this prediction of 2nd order theory.

b) Different positions in the boundary layer

Figs. 6.19, 6.20, 6.21 show the downstream growth and decay of disturbances at 3 different \( \frac{z}{\delta} \) positions, respectively \( \frac{z}{\delta} = 0.1 \), \( \frac{z}{\delta} = 0.25 \) and \( \frac{z}{\delta} = 0.4 \).

As was observed by Ross (1969) the position of maximum amplitude for the fundamental component moved upstream as the distance from the plate was increased. The graphs show that the position of maximum amplitude for the 2nd harmonic component moved downstream as the distance from the plate was increased. These results are summarised in Table 6.5.

At \( \frac{z}{\delta} = 0.1 \) the 2nd harmonic component began to decay where the fundamental component was still growing and at \( \frac{z}{\delta} = 0.4 \) the converse occurred. Remembering the very close links between the two fluctuating components, which have been obvious throughout the work, these results seem surprising.

Theory can only predict such behaviour on the basis of variations in the \( \phi \) functions. The downstream variations of \( |\phi_1'| \) and \( |\phi_2'| \) at the relevant positions are shown in Figs. 6.22 and 6.23. Comparison of these graphs with the experimental results of Figs. 6.19, 6.20, 6.21 suggests that at least part of the 'surprising' behaviour can be predicted from variations in the eigenfunctions.
<table>
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The average value of $(\text{Fund. Ratio})^2$ between $R_s = 1156$ & $R_s = 1435$ is $2.168$ with standard deviation $0.059$.

The average value of 2nd H. Ratio between $R_s = 1156$ & $R_s = 1435$ is $2.206$ with standard deviation $0.136$.

*Table 6.3 The Relationship between the Fundamental and 2nd Harmonic Components*
Fig. 6.17 Downstream Behaviour of Fluctuating Components at $\frac{z}{L} = 0.097$
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The Average Value of (Fund Ratio)² between R = 966 & R = 1391 is 1.585 with Standard Deviation 0.105.

The Average Value of (2nd H Ratio) between R = 966 & R = 1391 is 1.604 with Standard Deviation 0.057.

Table 6.4 The Relationship between the Fundamental and 2nd Harmonic Components
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Table 6.5a  Downstream Behaviour of the Fundamental Component
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\text{z/\delta} = 0.098 & \text{z/\delta} = 0.241 & \text{z/\delta} = 0.41 \\
R_\delta & u''/U_\infty & R_\delta & u''/U_\infty & R_\delta & u''/U_\infty \\
832 & 0.0010 & 819 & 0.00041 & 819 & 0.00095 \\
874 & 0.0016 & 866 & 0.00071 & 865 & 0.00078 \\
922 & 0.0013 & 907 & - & 908 & - \\
966 & 0.0013 & 949 & - & 950 & - \\
1008 & 0.0027 & 992 & 0.00082 & 992 & 0.00135 \\
1048 & 0.0041 & 1031 & 0.00108 & 1029 & 0.00187 \\
1089 & 0.0054 & 1068 & 0.00117 & 1067 & 0.0022 \\
1125 & 0.0077 & 1106 & 0.00155 & 1103 & 0.0026 \\
1161 & 0.0103 & 1141 & 0.00178 & 1139 & 0.0028 \\
1198 & 0.0130 & 1175 & 0.00232 & 1173 & 0.0030 \\
1233 & 0.0164 & 1210 & 0.0029 & 1202 & 0.0045 \\
1266 & 0.0215 & 1241 & 0.0032 & 1236 & 0.0057 \\
1297 & 0.0244 & 1274 & 0.0045 & 1265 & 0.0067 \\
1328 & 0.0268 & 1301 & 0.0051 & 1295 & 0.0079 \\
1360 & 0.0267 & 1331 & 0.0059 & 1326 & 0.0084 \\
1391 & 0.0230 & 1361 & 0.0066 & 1354 & 0.0090 \\
1420 & 0.0187 & 1390 & 0.0068 & 1384 & 0.0098 \\
1449 & 0.0146 & 1417 & 0.0058 & 1410 & 0.0090 \\
1477 & 0.0095 & 1444 & 0.0049 & 1437 & 0.0093 \\
1505 & 0.0062 & 1473 & 0.0031 & 1464 & 0.0067 \\
1532 & 0.0044 & & & 1490 & 0.0045 \\
& & & & 1516 & 0.0022 \\
\end{array}
\]

Table 6.5b Downstream Behaviour of the 2nd Harmonic Component
Fig. 6.19  Downstream Behaviour of Fluctuating Components at $\frac{\eta}{\delta} = 0.098$
Fig. 6.20  Downstream Behaviour of Fluctuating Components at $\frac{z}{h} = 0.241$
Fig. 6.21 Downstream Behaviour of Fluctuating Components at $\frac{z}{\delta} = 0.41$
Fig. 6.22 Downstream Behaviour of the Normalised Fundamental Eigenfunction, Derivative
Fig. 6.23 Downstream Behaviour of the 2nd Harmonic Eigenfunction Derivative
Conclusions

The predictions made by 2nd order theory have been compared with experimental measurements. The comparison was inhibited by the small size of the second order effects.

The distribution of the second harmonic component through the boundary layer has been shown to agree well with the predictions of theory. A relationship between the amplitudes of the fundamental and 2nd harmonic components was assumed during the derivation of the equations describing the behaviour of a disturbance in the boundary layer. This relationship has been justified experimentally.

An attempt was made to compare the distribution of the phase of the 2nd harmonic component through the boundary layer with the predictions of theory. Failure probably resulted from the fact that the 2nd harmonic component, and the noise were the same size.

Second order theory made predictions about distortion of the mean flow, and about changes in the fundamental component. These predicted effects were so small that the failure to observe them could be regarded as negative confirmation of theory.

Accepting the limitations imposed by the sensitivity of the measuring equipment, the agreement between the experimental measurements and the theoretical predictions must be regarded as satisfactory. Agreement in the laminar region suggested the attempt to extend the predictions by increasing the size of the perturbations. It was hoped that some of the characteristics of the transition region might be explained in this way.
CHAPTER 7
THE TRANSITION REGION

7.1 Introduction

The investigations described in this chapter were concerned with transition to turbulence. The results of these investigations are presented in the early sections of this chapter. Later, these results are considered in the light of other investigations, both theoretical and experimental, and an attempt is made to produce a consistent explanation for the observed behaviour.

The previously described automatic system could be used to obtain the results, the only modification being an increase in the number of readings taken to obtain the average of a fluctuating quantity. In addition, the anemometer output was displayed on the screen of an oscilloscope since some of the changes occurring in the transition region were most easily observed in this way.

The importance of 3-dimensional effects in the transition region had been emphasized by previous work, e.g. Klebanoff, Tidstrom and Sargent (1962), Tani (1960). Accordingly, the span-wise distribution of mean and fluctuating components in the transition region was looked at.

7.2 Span-wise Variation

The input perturbation was increased in size, beyond the limit specified by the criterion of the previous chapter. The span-wise distributions of the mean and fluctuating components of velocity were recorded at different downstream positions.

The downstream development of the span-wise distribution of
mean flow is shown in Fig. 7.1. The transition region was characterized by the appearance and growth of spanwise irregularities. The lack of symmetry about the centre line was very noticeable; the irregularities were much larger above, than below the centre line.

At $R = 1300$, the furthest upstream position, the mean velocity varied slightly, but there was no real evidence of the existence of maxima and minima.

At $R = 1400$, mean velocity maxima developed, particularly above the centre line. Peaks of mean velocity appeared 1.75 inches above the centre line, and 0.6 inches above the centre line. A much smaller peak of mean velocity appeared 1.75 inches below the centre line. The mean velocity variations were as great as 10% of the mean velocity of the undisturbed boundary layer.

At $R = 1500$ striking changes had taken place in the mean velocity distribution. The peak of mean velocity 0.6 inches above the centre line had been replaced by two peaks, a small peak 0.3 inches above the centre line, and a large peak 1.0 inches above the centre line. In fact a minimum of velocity between these peaks, was found 0.6 inches above the centre line. The mean velocity peak, 1.75 inches above the centre line remained, though there was some evidence that it was starting to divide in two. The slight inward shift to 1.7 inches above the centre line, and the possible traces of a subsidiary peak 2.1 inches above the centre line, were noticed. This downstream position also saw the appearance of a peak of mean velocity 0.7 inches below the centre line. The variations in the mean flow were as great as 140% of the mean velocity of the undisturbed boundary layer, at this downstream position.
Fig. 7.1 The Spanwise Distribution of Mean Velocity
The span-wise distributions of the fundamental component are shown in Figs. 7.2, 7.3 and 7.4.

At $R_\theta = 1300$ there was a spanwise distribution of the same shape as had been found in the laminar region.

At $R_\theta = 1400$ the spanwise distribution was characterised by maxima and minima. These maxima were 0.4 inches, 0.95 inches and 2.0 inches above the centre-line and 0.8 inches and 1.6 inches below the centre line. The maxima of the fundamental component did not coincide with the associated maxima of the mean velocity. In this region the fundamental component maxima coincided with mean velocity deficits. However the maxima of the fundamental components seemed to be close to the maxima of mean velocity for $R_\theta = 1500$.

At $R_\theta = 1500$, the distribution of the fundamental component was the same as that at $R_\theta = 1400$, except for a great increase in amplitudes, and a shift of the peak which had been 2.0 inches above the centre-line to a position 1.75 inches above the centre-line. In this region the maxima of the fundamental component and of the mean velocity coincided.

The spanwise distribution of the 2nd harmonic component followed that of the fundamental component closely, except for a deficit in the 2nd harmonic component which appeared at $R_\theta = 1300$, shown in Fig. 7.2.

The spanwise distributions were obtained close to the plate, at about $z_\theta = 0.1$ because the sensitivity of the hot-wire is a maximum in this region. Results obtained later showed that changes associated with transition occurred first, further out in the boundary layer. There were no indications of basic differences in the pattern of transition, at these outer positions, which would result
Fig. 7.2 Spanwise Distribution of Fluctuating Components at $R_s = 1300$
Fig. 7.3 Spanwise Distribution of the Fundamental Component at $Re = 1400$
Fig. 7.4  Spanwise Distribution of the Fundamental Component at $R_e = 1500$
in different spanwise behaviour.

7.3 The Distribution of the Mean and Fluctuating Components of Velocity Through the Boundary Layer in the Transition Region

a) The Influence of the size of the input perturbation.

Typical results of increasing the size of the input perturbation are shown in Figs. 7.5, 7.6, 7.7, 7.8, 7.9, 7.10. The results obtained at the same downstream position $R_g = 1330$ were produced by input perturbations in the ratio 1:2:3:6.

The development of the mean flow could be divided into three stages. With the smallest input, the mean velocity profile was indistinguishable from the Blasius profile, Fig. 7.7. The second stage, shown by Figs. 7.8 and 7.9, involved the appearance and growth of mean flow distortion, in the form of a velocity deficit. The effect was centred at about $Z/9 = 0.3$ and increased with the input perturbation. The third stage involved a change in the form of the mean flow distortion. The mean flow profile was now characterised by a point of inflexion at $Z/9 = 0.35$. Inside this point the velocity was greater than the Blasius value, outside, the converse was true, Fig. 7.10.

The peak of the distribution of the fundamental component moved outwards as the perturbation was increased in size. As the peak shifted to $Z/9 = 0.3$ the minimum point also shifted outwards to $Z/9 = 0.675$, Fig. 7.6a. The largest input perturbation produced a distribution with a peak amplitude at $Z/9 = 0.4$, but without any trace of a minimum point. In this case the amplitude fell quite gradually towards the outside of the boundary layer, Fig. 7.6b.
Fig. 7.5  B.L. Distribution of Fluctuating Components at \( R_b = 1330 \)

a) Input = A  
b) Input = 2A
Fig. 7.6  B.L. Distribution of Fluctuating Components at \( R = 1330 \)

a) Input = 3A  

b) Input = 6A
Fig. 7.7 B.L. Distribution of Mean Velocity at $R_e = 1330$, with an Input = $A$
Fig. 7.8  B.L. Distribution of Mean Velocity at $R_e = 1330$, with an Input = 2A
Fig. 7.9 B.L. Distribution of Mean Velocity at $R_e = 1330$, with an Input = 3A
Fig. 7.10 B.L. Distribution of Mean Velocity at $Re = 1330$, with an Input = 6A
The first sign that transition would occur, appeared in the distributions of the 2nd harmonic component. The smallest perturbation produced no distortion of either the fundamental component, or the mean flow from their predicted values. This perturbation produced a small peak in the distribution of the 2nd harmonic component, centred at $\frac{z}{c} = 0.25$, in addition to the predicted peak inside $\frac{z}{c} = 0.1$, Fig. 7.5a. Increase in the size of the input perturbation resulted in the growth of the second peak at a greater rate than that of the inner peak. The second peak moved outwards as the perturbation was increased in size, reaching $\frac{z}{c} = 0.34$ for the second largest perturbation, Figs. 7.5, 7.6a. The largest input perturbation produced a much flattened distribution of the 2nd harmonic component, without trace of either inner peak or minimum point, Fig. 7.6b. The similarity of the distributions of the fundamental and 2nd harmonic components, in the later stages was noticeable, Figs. 7.6.

b) The influence of the Reynolds Number $R_8$.

The results of following the downstream development of the transition process are shown in Figs. 7.11 to 7.15. Distributions of the mean and fluctuating velocities were produced at different downstream positions by the same input perturbation. In the transition region, the downstream development appeared to resemble closely the succession of changes occurring at a single position, as the input perturbation was increased in size.

At the three positions chosen; $R_8 = 1300, 1400$ and $1500$, the mean flow displayed the 3 types of behaviour previously observed in
Fig. 7.11 B.L. Distribution of Fluctuating Components, with an Input = B

a) At $R = 1300$

b) At $R = 1400$
Fig. 7.12 B.L. Distribution of Fluctuating Components at \( R_s = 1500 \), with an Input = B
Fig. 7.13  B.L. Distribution of Mean Velocity at $R = 1300$ , with an Input = B
Fig. 7.14 B.L. Distribution of Mean Velocity at $Re = 1400$, with an Input = B
Fig. 7.15  B.L. Distribution of Mean Velocity at $\theta = 1500$, with an Input = B
the transition region. The downstream growth of the fluctuating components also took the form described in the previous section. At \( R_\delta = 1300 \), the distributions resembled those obtained previously with the smallest input. At \( R_\delta = 1400 \) the development seemed further advanced than that obtained with the 2nd largest input in the previous section. At \( R_\delta = 1500 \) turbulence looked to be more fully developed than was the case for any of the inputs previously considered.

These results would seem to indicate the equivalence of downstream movement, and increase in the size of the perturbation, within the transition region.

7.4 Downstream Development of the Perturbation

The previous distributions had located certain positions of interest in the boundary layer. Streamlines corresponding to such positions were selected, and the downstream development along these streamlines towards turbulence was observed. Figs. 7.16, 7.17 and 7.18 show the results obtained at three such positions.

Fig. 7.16 corresponds to \( \frac{z}{\delta} = 0.1 \) where signs of turbulence appeared, last.

Fig. 7.17 corresponds to \( \frac{z}{\delta} = 0.25 \) where the first signs of transition were observed.

Fig. 7.18 corresponds to \( \frac{z}{\delta} = 0.4 \) where the largest fluctuating components were observed.

The mean velocity was also observed, and in the latter two cases a drop in its value was found to coincide with the first steep increase in the fluctuating components. In all three cases,
Fig. 7.16 Downstream Behaviour of Fluctuating Components at $\frac{x}{b} = 0.1$, with an Input = C
Fig. 7.17 Downstream Behaviour of Fluctuating Components at $\frac{z}{h} = 0.25$, with an Input = C
Fig. 7.18 Downstream Behaviour of Fluctuating Components at $\frac{R}{8} = 0.4$, with an Input = C
the change to an increased mean velocity was found to coincide closely with the peak of the fluctuating distributions.

The three sets of results at different positions, were produced by the same input perturbation. As expected the rise in fluctuating components associated with transition occurred first at $\frac{z}{\delta} = 0.25$ and then at $\frac{z}{\delta} = 0.4$. Coinciding with this rise, was a drop in the fluctuating components at $\frac{z}{\delta} = 0.1$. The fall was of fairly short extent, $\Delta R_\delta = 40$, and was followed by a rapid rise in the fluctuating components.

As expected the maximum values of the fluctuating components were obtained at $\frac{z}{\delta} = 0.4$ where the fundamental component reached an r.m.s. value of 7.5% in the case shown.

7.5 **Measurement Errors**

The fluctuating components were larger in the transition region than in the laminar region which preceded it. The problems due to noise were much reduced in the transition region. In particular the second harmonic component was sufficiently large compared with the noise to be observed throughout the boundary layer.

The spurious component of 2nd harmonic frequency did not seem to affect the results, because the transition region was always sufficiently far downstream to ensure that the spurious component would be damped out. Peaks in the growth curves for the second harmonic discussed in Section 7.4, probably showed the presence of the spurious components at lower Reynolds numbers.

Two errors arising from the response of the hot-wire anemometer to velocity changes must be considered. Both errors result from the
non-linearity of the response of the instrument.

The generation of 2nd harmonic by the hot-wire anemometer due to the non-linearity of the relationship between velocity and the hot-wire voltage was discussed in the previous chapter. The effect which depended upon the magnitude of the fundamental component was likely to be more important in the transition region. Direct correction of results has not been carried out, but Table 7.1 gives a good idea of the size of the error.

The other error arose from the assumption of a linear relationship between changes in voltage, and changes in velocity at any boundary layer position. The linear assumption became less acceptable as the changes in velocity caused by the fluctuations increased in size. The assumption resulted in overestimation of the amplitudes of fluctuating components in the transition region. The problem was considered by Schubauer and Klebanoff (1945). Ross (1969) followed their analysis, assuming the fluctuation to be sinusoidal, and obtained the table reproduced here, Table 7.2. This table gives some idea of the size of the effect. In order to remove this error, the linear relationship between voltage and velocity changes would have to be discarded, and a more accurate formula found. Possibly a quadratic relationship would suffice, though this would introduce great complications in the calculations, i.e. replace

\[
\frac{u'}{U_0} = \frac{(\rho - 1) U'}{0.45 I R_n (\rho - \rho_0)} \frac{U}{U_0}
\]

\[
= F \left( \frac{U}{U_0} \right) e'
\]
### Table 7.1 2nd Harmonic Generated by the Hot-Wire

<table>
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<tr>
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<th>( u'/U_0 = 2% )</th>
<th>( u'/U_0 = 4% )</th>
<th>( u'/U_0 = 8% )</th>
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<td>0.1</td>
<td>2.82</td>
<td>6.06</td>
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<tr>
<td>0.3</td>
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<tr>
<td>0.4</td>
<td>0.75</td>
<td>1.54</td>
<td>3.19</td>
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</tr>
<tr>
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<td>1.23</td>
<td>2.55</td>
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<td>1.04</td>
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<tr>
<td>0.7</td>
<td>0.44</td>
<td>0.89</td>
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Table 7.2  Error caused by the Non-Linearity of the Hot-Wire Response

<table>
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<th>$u'/U%$</th>
<th>$u'/U%$</th>
<th>Estimated Error $%$</th>
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<td>10</td>
<td>41</td>
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</tr>
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<td>33</td>
<td>19</td>
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<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
by \( \frac{u^t}{u_o} = G(\frac{U}{u_o})e^{2} + H(\frac{U}{u_o})e^{t} \)

where \( G(\frac{U}{u_o}) \) and \( H(\frac{U}{u_o}) \) would be obtained by considering the higher order terms of the differential. (See appendix).

The results presented in this chapter were not compared with numerical results. It is important that attention should be drawn to the errors which arose, but direct corrections were not at this stage necessary, as the conclusions to be drawn were qualitative.

7.6 Comparison with Theory

Theoretical work has attempted to extend two dimensional linear theory to allow the prediction of transition.

In a series of papers ending in 1968, Stuart working sometimes in collaboration, extended linear theory retaining the two-dimensional form, by considering higher order terms. Lin worked along the same lines, obtaining results which did not agree with those of Stuart.

Extension of theory to three dimensions was stimulated by the experimental work performed at the American National Bureau of Standards which culminated in the 1962 paper of Klebanoff, Tidstrom and Sargent. Working at the same time, Benney, in collaboration with others notably Lin, produced a three-dimensional theory which predicted some of the experimental results.

The results described in this chapter will be compared with those produced previously, particularly the results of the work mentioned above. The two possible methods of approach, two-dimensional, and three dimensional, are considered.
7.7 **2-Dimensional Work**

a) **Introduction.**

The results presented in Section 7.2 show strong evidence of 3-dimensional behaviour in the transition region. However it is possible that some of the other effects observed in the transition region might be 2-dimensional. It is even possible that such 2-dimensional effects might be more important in explaining the transition process than the obvious 3-dimensional effects.

It was with these possibilities in mind that the attempt was made to extend the region of agreement, between the predictions of second order theory, and experimental measurements, into the transition region.

Barry obtained numerical results for disturbances, much larger than those considered in the previous chapter. Using the iteration method discussed in Chapter 5, he took into account the mean flow distortion caused by the disturbance, and the effect of this distortion on the fundamental component. As yet the effect of distortion of the mean flow on the 2nd harmonic component has not been dealt with.

Barry's results can be compared in general with the results presented earlier in this chapter. Since Barry produced his results from equations corresponding to those of Stuart, this comparison can be regarded as a test of Stuart's conclusions.

b) **Stability.**

Table 7.3 shows the effect of the amplitude of the disturbance on the amplification factor $a_l$. The initial value which applies to
<table>
<thead>
<tr>
<th>$R_a$</th>
<th>%age Perturbation</th>
<th>$\lambda_{\text{initial}}$</th>
<th>$\lambda_{\text{final}}$</th>
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</thead>
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<td>0.0167345</td>
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<tr>
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<td>1</td>
<td>0.0006934</td>
<td>0.0006758</td>
</tr>
<tr>
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<td>0.0006258</td>
</tr>
<tr>
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<td>0.0066026</td>
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<tr>
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</tr>
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<tr>
<td>1750</td>
<td>2</td>
<td>0.0263312</td>
<td>Oscillates</td>
</tr>
</tbody>
</table>

Table 7.3 The Effect of Disturbance Amplitude on the Eigenvalue $\lambda$
small disturbances, was calculated assuming a Blasius mean flow profile. A distorted mean flow profile, produced by the finite amplitude disturbance was assumed during the calculation of the final value.

The amplifying (or damping) term is of the form \( e^{-\alpha_1 x} \) and a reduction in \( \alpha_1 \) produces an increase in the rate of amplification. Table 7.3 shows that an increase in the size of the disturbance in the amplifying zone, results in a decreased rate of amplification, and thus an increase in stability. Since it was hoped to predict the large rates of amplification in the transition region, illustrated by Figs. 7.16, 7.17 and 7.18, this result was rather disappointing.

Theory does predict a larger amplification zone for large disturbances, as indicated by the result at \( R_\theta = 1500 \) in Table 7.3. It also in general predicts a reduction in stability with disturbance size in the damping zone. As indicated in an earlier chapter (5), the iteration method breaks down if the disturbance level exceeds a certain value, which decreases with Reynolds number.

c) Fundamental component.

Barry obtained modified distributions of the fundamental component through the boundary layer, two such distributions are shown in Figs. 7.19 and 7.20.

The modifications took the form of a small redistribution of amplitude towards the outside of the boundary layer. The position of the amplitude peak was unaffected, but the minimum point moved outwards by at most 0.018.

Reference to the experimental distributions discussed in section
Figs. 7.19, 7.20  Theoretical B.L. Distributions of the Fundamental Component
7.3, shown in Figs. 7.5, 7.6 and 7.11, shows that the fundamental component varies much more than predicted in the transition region. The position of the amplitude peak shifts outwards as the disturbance increases in size, and there is also an outwards shift of the minimum point. This latter shift becomes much larger than that predicted by theory; of the order of $0.07\%$.

d) Mean flow distortion.

Associated with the modified distributions of the fundamental component, Barry predicted mean flow distortion whose distribution through the boundary layer is illustrated by Figs. 7.21 and 7.22. The maximum predicted distortion produced by disturbances of the order of $3\%$, is $\frac{1}{2}\%$ of the free stream velocity.

Comparison with experimental distributions of mean flow distortion, shown in Figs. 7.23 and 7.24, plotted from Figs. 7.9 and 7.14 indicates that the shape of the distortion is close to the predictions of theory. However the magnitude of the distortion is much greater than predicted.

e) The 2nd harmonic.

Unfortunately the effect of mean flow distortion on the 2nd harmonic component has not yet been computed.

All the theoretical distributions of the 2nd harmonic component, computed for a Blasius mean flow profile, show evidence of a change in gradient of the distribution between the peak and the minimum. At extreme positions, $R_\infty = 500$ and $R_\infty = 1750$, the change in
Figs. 7.21, 7.22 Theoretical B.L. Distributions of Mean Flow Distortion
Figs. 7.23, 7.24  B.L. Distributions of Mean Flow Distortion
Fig. 7.25 Theoretical B.L. Distributions of the 2nd Harmonic Component
gradient is replaced by a second peak (see Fig. 7.25). It is of some interest to see whether a distorted mean flow profile might result in the appearance of this peak at intermediate Reynolds numbers.

It will be remembered that the first apparent sign of transition was the appearance of a second peak in the 2nd harmonic distribution, see Fig. 7.5a.

f) Conclusions.

Although the evidence obtained so far is not complete, it seems likely that the changes taking place in the transition region, are too great to be explained by second order theory. Indeed the predicted increase in stability with the size of the disturbance is directly opposed to the experimental findings.

It is to be hoped that the effect of the mean flow distortion on the 2nd harmonic component will soon be computed. In addition it would be interesting to introduce mean flow distortion, of the order encountered experimentally, at some stage of the iteration process. Assuming that the iteration process did not break down, comparison of the fundamental and 2nd harmonic distributions obtained, with the corresponding experimental distributions would provide a satisfying and almost conclusive test of theory.

For the remainder of this chapter it is assumed that the two dimensional theory ceases to explain the behaviour of the boundary layer, when the first signs of transition appear. The attempt is made to understand the phenomena associated with transition from different standpoints.
The 2-Dimensional Work of Lin

Meksyn and Stuart (1951) concluded that mean flow distortion would be a larger effect than the generation of harmonics.

Lin (1955) made a careful order of magnitude analysis of the terms of the perturbation equation. His analysis showed that within the critical layer the generation of harmonics would be more important than distortion of the mean flow. Lin also showed that within the critical layer each harmonic would be of the same order of magnitude as the fundamental component. Outside the critical layer, Lin's analysis agreed with the work of Stuart and Meksyn.

Stuart (1960) suggested that Lin's analysis applied to large disturbances, and was unlikely to be relevant to the smaller disturbances discussed by Stuart.

This work applied to Poiseuille flow but was expected to be relevant to boundary layers.

The first sign of transition observed in the present work was an additional peak of 2nd harmonic at \( \frac{z}{\delta} = 0.25 \) as illustrated by Fig. 7.5a. This peak moved outwards as it grew (Figs. 7.5, 7.6). Hence the possibility that this component of 2nd harmonic frequency, found only for large disturbances, originated in the critical layer at \( \frac{z}{\delta} = 0.2 \), had to be admitted. This result would tend to support Lin's theory.

The conclusions reached about behaviour outside the critical layer were not supported by experiment. The second harmonic component was often observed where there was no sign of mean flow distortion. However this was probably due only to the fact that the measuring
equipment was much more sensitive to the presence of harmonics than to the presence of mean flow distortion.

Too much weight could not be placed on any agreement with Lin's work because of the vagueness of the paper. In particular the word 'important' is used without any attempt to define its meaning, in the context.

7.9 *Three-Dimensionality*

Two-dimensional theory failed to explain the effects which characterised the transition region. Only a three-dimensional theory can be expected to explain the spanwise variations described in Section 7.2.

A three-dimensional perturbation propagates at an angle to the direction of flow, and is periodic in both the spanwise and streamwise directions. The interaction of such a perturbation with a two-dimensional perturbation has been described in a number of papers produced by Benney, sometimes in collaboration, notably with Lin. The interaction has been considered for a number of mean velocity profiles, including a simple approximation to a boundary layer profile, consisting of two straight lines. The two perturbations were always in phase, and propagated downstream with the same velocities. The interaction of the two perturbations resulted in important secondary effects which could be either two-dimensional or three-dimensional.

The two-dimensional secondary effects were similar to the nonlinear effects produced by considering higher order terms in the equation of motion for a single perturbation. These have been discussed by Benney and Lin (1960) and Stuart (1960).
The three-dimensional secondary effects resulted in the appearance of vortices at periodic spanwise positions. These longitudinal vortices produced variations in boundary layer thickness across the span, when the simplified boundary layer profile was considered. The vortex separation was controlled by the choice of parameters for the perturbations, e.g., spanwise wavelength. Parameters could be chosen such that the vortex separation accorded with that produced experimentally by Klebanoff et al. (1962).

If the boundary layer conditions were such that the rate of growth of the three-dimensional disturbance was greater than that of the two-dimensional disturbance, the relative size of the former would increase. This increase was shown by Benney to result in the appearance of weak vortices, between the original vortices. If the increase continued, all the vortices would eventually become equal in strength, and there would at this stage be two vortices, with one on each side of the original vortex position, where before there was only one vortex. This process should be noticed as a halving of the spanwise wavelength of boundary layer thickness. The effect was observed by Klebanoff et al., and traces of it can be seen in Fig. 7.1, at \( R_\theta = 1500 \) in the present work.

There was good agreement between the predictions of Benney's theory and the spanwise effects observed by Klebanoff et al. The spanwise periodicity could in fact be selected to agree with these results, but no information was given as to whether any spanwise period was likely to be preferred, under given boundary layer conditions. The application of Benney's theory to the somewhat aperiodic
spanwise variations evident in this experiment seemed rather obscure, although the traces of doubling of the mean velocity peaks suggested at least some connection. The conditions that the two perturbations should be in phase, and travel with the same velocity were very restrictive and possibly could not be satisfied as was pointed out by Stuart (1960).

7.10 Explanation of Results

In the previous sections two possible approaches to the problem of explaining the results obtained in the transition region were considered.

An attempt was made to extend two-dimensional theory into the transition region. The inability of the theory to predict the large changes evident in this region, have suggested that this attempt will not succeed, though the results obtained so far are not complete.

The attempt made by Benney to explain spanwise phenomena, on the basis of the interaction between a two-dimensional perturbation and a three-dimensional perturbation has been discussed, and some of the limitations of his results indicated. It seems possible that the results obtained in this investigation might be explained by the appearance and growth of some disturbance, in the transition region. This disturbance could be three-dimensional.

Accordingly the results obtained in the transition region, before the appearance of an inflected mean velocity profile are interpreted in this way.
The Early Stages of Transition

Reference is made first to the results presented in Section 7.3 and illustrated by Figs. 7.5, 7.6, 7.7, 7.8, 7.9, 7.11, 7.13, and 7.14—distributions of mean and fluctuating components through the boundary layer.

It is suggested that the appearance of a second peak in the distribution of the 2nd harmonic, signals the appearance of the disturbance whose growth occupies the early stages of transition, (Fig. 7.5a). This disturbance is referred to in future as the transition disturbance. The distributions of the 2nd harmonic, taken in order, show the growth of the transition disturbance in amplitude and extent, (Figs. 7.5, 7.6, 7.11). The corresponding distributions obtained at the fundamental frequency, reflect similar behaviour of the transition disturbance, fundamental component. The mean flow distributions (Figs. 7.8, 7.9, 7.14) show the appearance and growth of a mean flow deficit whose boundary layer distribution resembles that of the fast, growing fluctuating components of the transition disturbance (Fig. 7.23, 24).

At each stage of the transition process, the characteristic disturbance dominates a certain region of the boundary layer. The disturbance amplitude drops rapidly on each side of this region, and the disturbance characteristic of a laminar boundary layer dominates the behaviour in the inner and outer regions of the boundary layer. This explains the persistence of the inner peak in the 2nd harmonic distribution, typical of the laminar boundary layer, well into the transition region (Figs. 7.6, 7.11). The outward movement of the minimum point in the distributions of the fluctuating components can be explained by the same effect. This
minimum is the boundary between the region dominated by the
transition disturbance, and the region in which the laminar dis-
turbance is still paramount. As transition proceeds, the region
dominated by the transition disturbance increases in size, hence
the outward movement of the boundary between the regions. Obvious-
ly this effect only becomes apparent when the transition distur-
bance has moved out into the region beyond the minimum point of
the laminar distribution.

It is suggested that in the transition region, a measurement
of a fluctuating component gives the result of the superposition
of two disturbances, one typical of the laminar region, the other
typical of the transition region. It is possible that instantaneous
variations in the values of fluctuating components are much larger
in the transition region, because of phase differences between
these disturbances. The superposition can result in an instan-
taneous addition or subtraction of the disturbances. It is of
course also possible that the variations are a characteristic of
the transition disturbance.

The later distributions (Figs. 7.6, 7.11) show that the peak
amplitude positions for the transition disturbance are the same
for both the fundamental and the 2nd harmonic components. In the
laminar boundary layer the amplitude peak of the 2nd harmonic com-
ponent is well inside that of the fundamental component. Both
types of disturbance exist in the transition region. The two peaks
which might appear in the distribution of the 2nd harmonic are
some distance apart ($\frac{2\alpha}{\delta} = 0.2$) and appear separately. The two
peaks which might appear in the fundamental distributions are very
close ($\frac{2\alpha}{\delta} < 0.1$) and usually appear as one peak. The same
explanation can be given for the fact that the first signs of transition appear in the 2nd harmonic distribution - any traces in the fundamental distribution are hidden by the peak of the laminar disturbance, (Figs. 7.5a, 7.11).

The similarity between the distributions of mean flow distortion, and distributions of the fluctuating components is strong evidence that the fast growing transition disturbance is fed locally with energy from the mean flow, (Figs. 7.23, 7.24).

Downstream distributions at $\frac{z}{\delta} = 0.25$ and $\frac{z}{\delta} = 0.41$ show the extremely rapid growth of the transition disturbance. The coincident drop in mean velocity can also be seen.

The downstream distribution at $\frac{z}{\delta} = 0.09$ is of greater interest, because coinciding with the rapid rise in fluctuating components further out in the boundary layer, at this position there is a drop in amplitude of these components. The possibility of a transfer of energy outwards through the boundary layer must be admitted. The fact that the drop in amplitude is a transition effect is well illustrated by Fig. 7.26. The drop is of short duration, and is followed by a steep rise in amplitude.

Reference to the spanwise distributions, particularly Figs. 7.1 and 7.3, shows that again a rise in the fluctuating amplitude is accompanied locally by a drop in mean velocity.

Having accounted for as many of the observations as possible on the basis of the existence of a disturbance typical of the transition region, it is useful to summarise the properties of this disturbance.

Some characteristics of the disturbance are:
Fig 7.26 Downstream Behaviour of the Fundamental Component at $\frac{z}{h} = 0.1$
1) The disturbance grows very rapidly, the rate of growth is much greater than ever encountered in the laminar region.

2) The distributions through the boundary layer of components of different frequency appear to be the same. This is well illustrated by the later distributions of the fundamental and second harmonic frequency components (Figs. 7.6, 7.11).

3) These distributions appear to depend on the size of the disturbances. An outward movement of the peak amplitude position occurs when either the input perturbation is increased in size or when the Reynolds number is increased. The distributions do not seem to depend directly on Reynolds number.

4) There is no evidence of a phase change in these distributions.

5) The similarity of the distributions of mean flow distortion and of fluctuating components suggests strongly that the disturbance gains energy from the mean flow locally.

6) The frequencies of the disturbance fundamental, and its harmonics were always the same as those of the dominant disturbance in the laminar region; the phase relationship is uncertain.

7) The appearance of the disturbance seems to depend upon the disturbance amplitude in the laminar region attaining a threshold value.

8) The growth of the disturbance seems to be irreversible and leads inevitably to turbulence.

According to Benney's arguments, the spanwise behaviour should be explainable as being due to the interaction between two disturbances, one of which must be three-dimensional. It seems possible that small,
probably unobservable, spanwise variations in mean flow might control the spanwise properties of such a three-dimensional disturbance. In the transition region the interactions of this disturbance might result in the reinforcement of the irregularities in the mean flow so that they can be observed. The possibility that the transition disturbance is three-dimensional is acknowledged.

7.12 The Later Stages of Transition

The growth of the transition region disturbance continued until random high frequency fluctuations appeared in the outer regions of the boundary layer. Coinciding with this event, other changes were observed in the boundary layer. The part of the transition region which sees the development from the first appearance of random fluctuations to a fully turbulent boundary layer is the subject of this section.

Measurements made in this region suggested the existence of two regimes of fluid flow. One regime was the continuation of that found in the earlier stages of transition, typified by a fast growing disturbance and a defect in the mean velocity. The other regime was a turbulent regime typified by random disturbances and an increased mean velocity. It is suggested that alternation between these two regimes explains many of the results observed in this region (Figs. 7.6, 7.10, 7.12 and 7.15).

Instantaneous values of the mean velocity seemed to alternate between values typical of the two regimes (Table 7.4). The measured average value gives in this region, an indication of the time spent by the boundary layer at the position of interest, under each of the two regimes. Increase in the Reynolds number, or the input
**H. W. Voltages**

<table>
<thead>
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<td>86</td>
<td>80</td>
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</tbody>
</table>

Voltage on the Streamline = 1190

An increase in Voltage implies a reduction in Velocity & conversely.

Table 7.4 Mean Voltage Fluctuations at the Later Stages of Transition
perturbation increased the time spent by the boundary layer in the turbulent state, with a consequent increase in the average value of the mean velocity. Eventually the mean velocity attains a steady value which corresponds to turbulent flow, the boundary layer spends all the time under the turbulent regime. These results are illustrated by Fig. 27.

In a turbulent boundary layer there are no preferred frequencies at which energy is concentrated; energy is distributed evenly over the frequency spectrum. In a laminar boundary layer, and during the early stages of transition energy is concentrated at preferred frequencies. During the later stages of transition the amplitudes of the components at the fundamental and 2nd harmonic frequencies, drop because of the increasing importance of the turbulent regime, in which these frequencies have no special significance. This drop does not coincide exactly with the first appearance of random fluctuations, because at first the increased amplitude during the time spent under the early transition regime can more than compensate for the smaller amplitude during the short time spent under the turbulent regime. Inevitably the stage is reached when the amplitudes of all the harmonic frequency components, and of all other frequency components, are the same - this is fully developed turbulence. These changes are illustrated by the downstream distributions of Figs. 7.16, 7.17 and 7.18.

The boundary layer profiles obtained in this region presumably represent the weighted mean of the profiles typical of the two regimes which characterise this region. The weighting is supplied by the time spent by the boundary layer under each regime. Fig. 7.28 illustrates this behaviour. The boundary layer distributions
Fig. 7.27 Downstream Behaviour of the Mean Velocity

- $z/h = 0.41$
- $z/h = 0.25$
- $z/h = 0.10$

$U/U_*$ vs. $R_y$ graph showing various downstream behaviour points.
Fig. 7.25 Mean Flow Profiles encountered in the Transition Region - Schematic
obtained in this region can also be regarded as resulting from a weighted mean. The relevant mean flow and fluctuating distributions are those shown in Figs. 7.6, 7.10, 7.12, 7.15.

The results discussed in this section, and the conclusions drawn are wholly consistent with an explanation of the later stages of transition, based upon the appearance and growth of turbulent spots as discussed by Emmons (1951).

7.13 **Conclusions**

The results of observations made in the transition region were presented in the early sections of this chapter.

Finite disturbance theory had been shown to agree satisfactorily with many of the results obtained in the laminar region. This theory made predictions about the behaviour of disturbances of the same order of magnitude (1 - 4%) as those observed in the transition region. A comparison of these predictions with observed results, although incomplete, seemed to show that the changes occurring in the transition region were much larger than predicted.

Spanwise variations had been observed in the transition region and attention was drawn to a theory, primarily due to Benney, which could explain spanwise variations on the basis of an interaction between two and three-dimensional perturbations.

On the basis of its behaviour, the transition region was divided into two sub-regions, and an attempt was made to supply a qualitative explanation for the observations made in each sub-region.

It was suggested that the appearance and growth of a disturbance not found in the laminar region could explain the observations made
of the early stages of transition. The properties of such a disturbance have been listed. It was thought that the later stages of transition were consistent with the appearance, growth and multiplication of turbulent spots.
CHAPTER 8

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

1. Comparison with Existing Theory

a) The laminar boundary layer.

Within the laminar region, second order theory has successfully predicted the distribution of the second harmonic component through the boundary layer, and the relationship between the amplitude of the 2nd harmonic component and the amplitude of the fundamental component. Some peculiarities of the downstream distribution of the 2nd harmonic component have been shown to be at least partially compatible with the predictions of 2nd order theory. Other 2nd order effects, the distortion of the mean flow, and variations in the fundamental component, have not been observed, but this is not surprising given the accuracy of the experimental measurements, and the predicted size of the effects.

No successful comparison was made between the experimental and theoretically predicted phase distributions through the boundary layer of the 2nd harmonic component. The availability of the computer, together with an A to D Converter, should make it possible to apply correlation methods to instantaneous measurements of the fluctuating voltages. Information about the phase distributions of both the fundamental, and the 2nd harmonic components should become obtainable.

In this way it should be possible to extend considerably the area of agreement between theory and experiment, and confirm the
conclusion of the present work that, within the laminar region, the behaviour of small finite perturbations is adequately described at a single downstream position, by linear theory and its extensions to higher order.

b) The transition region.

It was hoped that the application of 2nd order theory to large perturbations \( \left( \frac{u'}{U_0} \right) \) greater than 1% might result in the prediction of some of the effects observed in the transition region. Although the theoretical work has not been completed, it appears unlikely that the extension of two-dimensional theory will prove successful.

Agreement between some experimental results obtained in the transition region, and the predictions of a theory due to Lin was noted.

The spanwise distribution of the mean and fluctuating velocity components in the transition region, was investigated. Traces were observed of a doubling of the spanwise peaks, an effect whose prediction was possibly the main achievement of the theory due primarily to Benney.

2. **Extension of Results Obtained in the Transition Region**

It has been suggested in this thesis, that the observed behaviour in the early stages of transition might be explained, as the result of the appearance and growth of a disturbance with properties, differing from those of typical disturbances found
in the laminar region. At the same time, this disturbance characteristic of the transition region appeared to be linked by frequency with the disturbance in the laminar region preceding transition. The two disturbances appeared to co-exist in the transition region, though the faster growing transition region disturbance soon swamped the disturbance characteristic of the laminar region.

The properties of the disturbance characteristic of the transition region have been detailed as far as possible from the experimental evidence.

The application of correlation methods to results obtained in this region of the boundary layer, might make it possible to decide whether two disturbances are present, since components of different phase, but with the same frequency, can be separated in this way.

The results obtained in the later stages of the transition region seemed to be compatible with the appearance, growth and multiplication of turbulent spots. A detailed study of the behaviour of such spots, should assist in the understanding of the later stages of transition, and such a study is intended in Edinburgh.

It is to be hoped that an extension of theory might accompany the accumulation of experimental observations in the transition region. The extension of theory to three dimensions, as attempted by Benney, is a logical next step in view of the spanwise variations observed experimentally. In addition, the possibility that resonance effects might play an important part should perhaps be considered carefully.
APPENDIX I

Boundary layer perturbations were induced by a ribbon caused to vibrate near the surface of the flat plate. This technique, pioneered by Schubauer and Skramstad (1947), has been used in many succeeding experimental investigations of boundary layer behaviour. However it appears that little is known of the mechanism whereby the ribbon perturbs the boundary layer, and the relationship, between the controlled parameters, the frequency and amplitude of the ribbon driving current, and the characteristics of the induced boundary layer perturbation, is not obvious.

A simple theoretical treatment is presented here in an effort to clarify the problem. The behaviour of the vibrating ribbon is considered, and then a relationship between these vibrations and the perturbations is derived.

a) The vibrating ribbon.

The ribbon of length, 2L, carries an alternating current of amplitude, $i_0$, and frequency, $\omega$. The force acting on the ribbon in a uniform magnetic field, $B$, is:

$$ F = B 2L i_0 \sin(\omega t + \beta) . $$

The tension in the ribbon will generate a restoring force

$$ F_R = T \sin \theta $$

(see Fig. A.1.1)

$$ = T \theta $$

(\theta \ 10^{-3} \text{ radians})

$$ = T \frac{Z}{L} . $$

There will also be a damping force acting on the ribbon, as
Fig. A.1.1 The Ribbon
it moves through the air. This damping force might be proportional to the velocity of the ribbon, at any instant:

\[ F_D = 2Mk \ddot{z} \]  

in standard notation.

\( M \) is the mass of the ribbon.

Equating the forces acting, to the acceleration of the ribbon:

\[
MZ = F - F_R - F_0
\]

\[
= B.2L.i_o \sin(wt + \beta) - 2Mk\ddot{z} - T. \cdot \frac{Z}{L}
\]

\[
\therefore \text{ } \ddot{z} + 2k\dot{z} + n^2z = \frac{F_0}{M} \sin(wt + \beta) \tag{A.1.1}
\]

where \( F_0 = B.2L.i_o \)

\[ r^2 = \frac{T}{ML} \]

The equation (A.1.1) is a standard equation, whose solution as the sum of a particular integral, and a complementary function, is given in Jaeger (1951).

The solution is:

\[
z = A e^{-kt} \sin(n't + D) + \frac{F_0}{M \left(n^2 - \omega^2 \right)^2 + 4k^2\omega^2/\pi} \sin(wt + \beta - \phi) \tag{A.1.2}
\]

where \( n' = /\left(n^2 - k^2 \right) \).

\[ \text{A and D are constants, determined by the initial conditions} \]

\[ \phi = \text{arg} \left(n^2 - \omega^2 \right) + 2i k \omega \]

The first term of the solution dies away exponentially with time.

The second term is the steady state solution or forced
oscillation, and is the term of interest when dealing with the behaviour of a continuously vibrating ribbon.

\[ Z = \frac{F_0}{\lambda (n^2 - \omega^2)^2 + 4k^2 \omega^2 \frac{n}{2}} \sin(\omega t + \beta - \phi) \quad \text{(A.1.3)} \]

The ribbon oscillates with the frequency of the current passed through it, \( \frac{\omega}{2\pi} \).

The amplitude of the ribbon oscillation is proportional to \( i_0 \), the amplitude of the driving current, since \( F_0 = B.2L.i_0 \).

The displacement is out of phase, by \(-\phi\), with the driving current.

The amplitude of the ribbon oscillation clearly depends on the forcing frequency.

The amplitude has a maximum value when \( \frac{\omega}{2n} = \frac{n}{2\pi} \), the natural frequency of oscillation of the ribbon. This is a resonance effect, whose size depends on the importance of the damping term \( k \). As \( k \) increases, the resonance effect becomes smaller, and for \( k \) comparable with \( n \), vanishes altogether.

Kersley (1965) investigated experimentally the behaviour of an oscillating ribbon, and found a frequency range in which the amplitude of ribbon oscillation was independent of the frequency of the ribbon driving current. This frequency range was:

\[ \frac{1}{3}n < \omega < \frac{5}{6}n \]

One implication of this result is that the amplitude of oscillation for \( \omega = \frac{1}{3}n \), should be similar to the amplitude, for \( \omega = \frac{5}{6}n \). This demand imposes conditions on the size of the damping term \( k \) as can be seen in Table A.1.1. The observed peak
<table>
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<th>$\omega$</th>
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<th>$5/6,n$</th>
<th>$n$</th>
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<td>1.091</td>
<td>1.58</td>
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Table A.1.1 The Effect of Velocity Damping on the Amplitude of Ribbon Oscillation
of resonance imposes an upper limit on the value of $k$.

It is difficult to reconcile the conditions imposed on $k$, and the assumption of damping forces, proportional to velocity should probably be reconsidered.

b) The boundary layer perturbation.

It is suggested that the perturbation is induced by the ribbon as a result of the drag force exerted on the moving fluid. This force depends on the fluid velocity in the neighbourhood of the ribbon. As the ribbon oscillates, its position in the boundary layer varies and consequently the fluid velocity in its neighbourhood varies.

The Reynolds number based upon the thickness of the ribbon takes the value of 1 for a typical velocity in the neighbourhood of the ribbon. Thus there will be no vertex shedding from the trailing edge of the ribbon, and only viscous drag need be considered.

The force acting on unit length of ribbon of breadth $x$ is given by:

$$D = 1.32824 \phi u^{3/2} x^{1/2} v^{-1/2}$$  \hspace{1cm} (A.1.2)

(Goldstein 1938.)

In the Blasius boundary layer, close to the plate

$U = AZ$ where $A$ is constant, to a very good approximation.

It is probably upon this linearity of the relationship between velocity, and distance from the plate, in the Blasius
boundary layer, that the usefulness of the vibrating ribbon technique depends.

In A.1.2,

\[ D = (1.32824 \rho A^{3/2} v^{-1/2} x^{1/2} Z^{3/2} \]

\[ = E x^{1/2} Z^{3/2} \]

Now \( Z = Z' + Z_0 \cos \omega t \) where \( Z' \) is the equilibrium position of the ribbon referred to the plate and \( Z_0 \) the amplitude of oscillation,

\[ D = E x^{1/2} (Z' + Z_0 \cos \omega t)^{3/2} \]

\[ = E x^{1/2} Z^{3/2} \left( 1 + \frac{Z_0}{Z} \cos \omega t \right)^{3/2} \]

If \( M \) is the mass of fluid in the neighbourhood of the ribbon, on which the drag force acts, then

\[ D = M \dot{u} = M \frac{du}{dx} \]

To find the reduction in velocity produced by the ribbon as a function of time, integrate across the breadth of the ribbon.

\[ \int_0^U \dot{u} dU = F(1 + \frac{Z_0}{Z} \cos \omega t)^{3/2} \int_0^{x^{1/2}} dx \]

\[ \frac{1}{2} u^2 = \frac{2}{3} F B^{3/2} (1 + \frac{Z_0}{Z} \cos \omega t)^{3/2} \]

where \( F = E \frac{Z_0^{13/2}}{M} \)

This gives the diminution in velocity produced by the vibrating ribbon.

\[ ^* M \text{ is not constant} \]
\[ u = k \left( 1 + \frac{Z_0}{Z} \cos \omega t \right)^{3/4} \]

expanding, using the binomial method.

\[ u = k \left[ 1 + \frac{3}{4} \frac{Z_0}{Z} \cos \omega t - \frac{3}{64} \left( \frac{Z_0}{Z} \right)^2 \cos^2 \omega t + \ldots \right] \]

where \( k = \left[ \frac{4}{3} \cdot 1.32824 \cdot \frac{\rho}{\mu^2} \cdot (AB)^2 \right]^{1/3} \).

Higher order terms are neglected in the expansion because \( \frac{Z_0}{Z} < \frac{1}{3} \).

The component of fundamental frequency, \( \omega \), is proportional to \( Z_0 \), the amplitude of the ribbon oscillation, and thus to \( i_0 \), the amplitude of the ribbon driving current.

It was suggested in Section 6.3 that the perturbation induced by the vibrating ribbon might include a component of 2nd harmonic frequency. This suggestion is confirmed by equation A.1.3. The ratio of the amplitudes of the 2nd harmonic and fundamental components is \( \frac{L}{16} \left( \frac{Z_0}{Z} \right)^3 \).

Normally \( \left( \frac{Z_0}{Z} \right) < \frac{1}{3} \).

Consequently the ratio of the 2nd harmonic component to the fundamental component is less than 2%.

However, it should be remembered that the corresponding ratio for the true 2nd harmonic generated by the boundary layer is normally less than 5%. It seems probable, that in regions of the boundary layer where this component is damped, and the spurious component, introduced by the ribbon, is amplified, the latter will be dominant.
APPENDIX II

The generation of 2nd harmonic by the anemometer.

The relationship between the measured voltage across the hot-wire anemometer, and the velocity in its neighbourhood is not linear. Consequently a 2nd harmonic voltage component can be generated by a single frequency velocity disturbance.

Suppose that the total velocity is \( u + u' \cos \omega t \).

Using the notation and formulae given in Section 2.5

\[
\phi = \phi_o + m(u + u' \cos \omega t)0.45 \quad (A.II.1)
\]

where \( \phi = \frac{R_w}{R_w - R_a} = \frac{1}{1 - \frac{R_a}{R_w}} \)

i.e. \( \frac{1}{\phi} = 1 - \frac{R_a}{R_w} \)

and \( \frac{R_w}{R_a} = \frac{1}{1 - \frac{1}{\phi}} \).

Hence \( e = i R_w = i R_a \cdot \frac{\phi}{\phi - 1} \quad (A.II.2) \)

where \( e \) is the total voltage across the hot-wire anemometer.

Substituting A.II.2 in A.II.1 gives:

\[
e = i R_a \frac{\phi_o + m(u + u' \cos \omega t)0.45}{\phi_o + m(u + u' \cos \omega t)0.45 - 1} \quad (A.II.3)
\]

This voltage \( e \) will have a steady component corresponding to the mean velocity \( u \), and components of different frequencies \( \omega, 2\omega, 3\omega \) etc.

In order to separate these components, knowing \( u, u', \phi_o, m, i \)
A type of Fourier analysis is used.

Let \( e = e_0 + e_1 \cos \omega t + e'_1 \sin \omega t + e_2 \cos 2\omega t + e'_2 \sin 2\omega t \)

then substitute in both sides of equation A.II.3 values of \( \omega t \):

\[
0, \quad \frac{\pi}{4}, \quad \frac{\pi}{2}, \quad \frac{3\pi}{4}, \\
\pi, \quad \frac{5\pi}{4}, \quad \frac{3\pi}{2}, \quad \frac{7\pi}{4}.
\]

The 8 equations which result, allow values for \( e_0, e_1, e'_1, e_2, e'_2 \) to be calculated.

The ratio \( \frac{e_2^2 + e'_2^2}{e_1^2 + e'_1^2} \) gives the amount of 2nd harmonic generated by the wire, compared with the fundamental component.

This calculation was carried out by means of the program.
APPENDIX III

Sources of Error

a) The duration of experiments.

The results discussed in this thesis were collected during experiments lasting up to 4 hours. During such periods of time, considerable changes in the ambient temperature and pressure could take place. Since the kinematic viscosity depends on both temperature and pressure, it follows that viscosity variations took place during the experiments.

Two fixed parameters, $R_8$, and $F$, were specified for experiments and both these quantities depend on the kinematic viscosity. Consequently variations in the fixed parameters were inevitable. Table AIII.1 gives an indication of the size of these variations, noting that a temperature change of $3^\circ C$ and a pressure change of 3 mm. would be exceptional.

b) The Stability of the R.M.S. values of fluctuating velocities.

Table AIII.2 gives examples of the stability of fluctuating velocities, and noise components measured with the automatic system. The behaviour of the noise components well illustrates the difficulty experienced in measuring the r.m.s. value of a fluctuating velocity when the noise level was comparable in size to it.

The stability of the fluctuating components was much greater in the laminar region, than in the transition region; a possible explanation for this was suggested in a relevant section of Chapter 7.
\[
P = 760 \text{ mm Hg}
\]

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\[
T = 10 \text{°C}
\]

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**Table A.3.1** Variation of F and R with Temperature and Pressure
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<td>15.1</td>
<td>15.0</td>
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<td>72.6</td>
<td>18.8</td>
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<tr>
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<td>3.79</td>
<td>15.5</td>
<td>16.2</td>
<td>20.0</td>
<td>100.0</td>
<td>14.9</td>
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<tr>
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<td>5.12</td>
<td>39.9</td>
<td>9.3</td>
<td>18.4</td>
<td>100.0</td>
<td>16.6</td>
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<tr>
<td>0.719</td>
<td>5.64</td>
<td>93.7</td>
<td>7.6</td>
<td>—</td>
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<tr>
<td>0.818</td>
<td>9.31</td>
<td>66.3</td>
<td>12.5</td>
<td>—</td>
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S. D. Standard Deviation of the Disturbance + Noise
N.S. D. Standard Deviation of the Noise
Di/N Ratio of the Noise to the Disturbance + Noise

Table A.3.2 Instantaneous Variations in R.M.S. Fluctuating Voltages
REFERENCES


Bloor, M.S. 1964 J. Fluid Mech. 12, 290.


Collis, D.C. and Williams, M.J. 1959 J. Fluid Mech. 6, 357.


1936 N.A.C.A. T.M. 562.

1939 Turbulence and the Boundary Layer, J. Aero Sci. 6, 85 and 101.


Fales, E.N. 1955 J. Franklin Inst., 259, 491.


REFERENCES (Contd.)


See N.A.C.A. T.M. 1291.

Jaeger 1951 'An Introduction to Applied Mathematics', O.U.P.


Nikuradse, J. 1942 Monograph.


REFERENCES (Contd.)

1887 I11, 17.
1892 I11, 575.
1895 IV, 203.
1913 VI, 917.
1914 VI, 266.
1915 VI, 341.

Reynolds, 0. 1883 Phil. Trans. 174, 935.

Rosenhead, L. (Ed.) 1963 'Laminar Boundary Layers', O.U.P.


1955


REFERENCES (Contd.)

603, 1931.
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