A Study of the CP Violation Parameter $\frac{\epsilon'}{\epsilon}$

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Abstract

NA48 is a fixed target experiment situated in the North Area High Intensity Facility at the CERN SPS. Its primary purpose is to study direct CP violation in the neutral kaon system, in particular $\frac{\epsilon'}{\epsilon}$, the ratio of direct to indirect CP violation. This thesis presents a study of $\frac{\epsilon'}{\epsilon}$ using data taken by NA48 in a six week run in September-October 1997. A method for calculating $\frac{\epsilon'}{\epsilon}$ and possible systematic effects are discussed.

The value of $\frac{\epsilon'}{\epsilon}$ obtained is $(1.58 \pm 0.45_{stat} \pm 0.60_{sys} \pm 1.3_{energy}) \times 10^{-3}$. 
Declaration

This work represents the effort of many members of the NA48 Collaboration. However, the final analysis is entirely my own and does not necessarily represent the views of the collaboration.
Acknowledgements

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1. INTRODUCTION.

Symmetries, conservation laws and their violations are an important topic in physics. In the 17th century Newton unified the motion of falling objects on Earth and the motion of the heavenly bodies, producing his theory of gravity and the idea of conservation of potential/kinetic energy. Today a widely favoured theory beyond the Standard Model is that of Super-symmetry, which unifies the interactions of bosons and fermions.

Until 1957, all particle interactions were thought to be invariant under three discrete, quantum mechanical, symmetries:

- **C** Charge conjugation, *i.e.* exchanging the roles of particles and anti-particles in an interaction. To be more precise charge conjugation exchanges the creation and annihilation operators, for example a vector field $V_\mu$ becomes $-V^{\mu \dagger}$. $V_\mu$ will annihilate a vector particle with its anti-particle, since $V_\mu^{\dagger}$ does the opposite this is equivalent to exchanging particles and anti-particles.

- **P** Parity, *i.e* swapping a left-handed coordinate system for a right-handed one. This reverses the direction of all momenta, although not the spin of a particle.

- **T** Time reversal *i.e* interchanging the initial and final states of an interaction, combined with complex conjugation of the wave-functions.
By convention, C and P symmetries are chosen to be unitary operators e.g. $PP^\dagger = P^\dagger P = 1$ and T is an anti unitary operator $TT^\dagger = T^\dagger T = -1$

Lee and Yang noted in 1956 [1] that there was no experimental evidence for the invariance of weak interactions under any of these operations. The next year Wu used aligned Cobalt atoms to measure the angular distribution of electrons emitted during the $\beta$ decay of the Cobalt nuclei [2]. She found an excess of electrons at a given $\theta$ compared with the angle $180 - \theta$, indicating parity violation.\(^1\) Parity is maximally violated, and as a consequence of this the anti-neutrino emitted in $\beta$ decay is right-handed i.e. the spin of the anti-neutrino is oriented parallel to the direction of motion. From Wu's result Lee et al [3] proved that symmetry under charge conjugation must also be violated. The most obvious example of this comes again from beta decay, where a right-handed anti-neutrino is produced. Applying charge conjugation the $\bar{\nu}_R$ becomes a right-handed neutrino $\nu_R$, which has never been seen.

Today it is understood that the violations of C and P symmetries, under the weak force, are due to the coupling of the W bosons to only the left-handed fermion doublets $(e_L, \nu_L)$, the $Z^0$ couples equally to left and right handed fermions. In the Standard Model only massless, left handed neutrinos exist. Results from the atmospheric neutrino experiment Super Kamiokande [4] suggest that neutrinos oscillate, implying that they are massive. This is the first evidence for physics beyond the Standard Model.

Although parity was seen to be violated, Landau's "combined inversion" [5], or CP, was still considered to be a good symmetry. Consider again the right-handed anti-neutrino $\bar{\nu}_R$ emitted in the decay of the Cobalt atom. Operating on this with\(^1\) If one considers the electron emitted at angle $\theta$ and applies a parity operation one obtains a electron emitted at a angle of $180 - \theta$. 

---

\(^1\) If one considers the electron emitted at angle $\theta$ and applies a parity operation one obtains a electron emitted at a angle of $180 - \theta$. 

---
the joint CP operator one obtains a left handed neutrino $\nu_L$. This is the observed state. CP appears to be conserved.

In 1964 Christenson, Cronin, Fitch and Turlay discovered the kaon CP eigenstate $K_2$ ($CP=-1$) decaying into two pions ($CP=+1$), albeit with a small branching ratio [6]. It was for this, the discovery of CP violation, that Fitch and Cronin won the Nobel prize in 1981.

Since then CP violation has been found in the decays $K_{\mu 3}$ ($K_L \rightarrow \pi \mu\nu_\mu$) [7] and $K_{e 3}$ ($K_L \rightarrow \pi e\nu_e$) [8]. Recently a CP violating, T odd asymmetry has been seen the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ [9]. Large CP violating effects are expected in the very rare ($10^{-12}$) decays $K_L \rightarrow \pi^0e^+e^-$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$. Observations of these decays would either constrain the flavour section of Standard Model or give hints of new physics. There are theoretical predictions of large CP violating effects in the $B$ system, experiments such as BaBar in SLAC, Belle at KEKB and Hera-B at DESY are being built to study CP violation in $B^0$ decays.

The only symmetry that is thought to be unbroken out of the C, P and T combinations is the overall CPT symmetry. But if CP is violated and CPT inviolate then T symmetry, which is implicitly assumed in both quantum and classical mechanics, must be violated. The consequences of this and CP symmetry breaking are not well understood, but CP violation is a necessary condition to generate the baryon asymmetry (matter anti-matter imbalance) present in the universe today.

The Sakharov conditions for the generation of a matter dominated universe from an initial symmetric state are:

- Baryon number violation (e.g. the decay of the proton.)
- CP violation.
- A period of localised thermal dis-equilibrium (known popularly as the infla-
1. Introduction.

Since the discovery of CP violation 35 years ago much effort has been put into understanding its origins and the size of the effects it produces. The Standard Model provides a framework for CP violation, but contributions from other models have not yet been ruled out. The relative size of the two major mechanisms, direct and indirect CP violation (denoted in the kaon system by $\varepsilon'$ and $\varepsilon$ respectively) is not well measured and precision experiments are underway at both CERN and Fermilab to measure this.

My own work is with the NA48 collaboration at CERN studying CP violation in $K_S$ and $K_L$ to two pion decays. This experiment is a follow up to NA31 and aims to measure the CP violating parameter $\frac{\varepsilon'}{\varepsilon}$ to an accuracy of better than one part in ten thousand. If a non zero value of $\frac{\varepsilon'}{\varepsilon}$ is measured this indicates the presence of direct CP violation, something expressly forbidden in the Superweak Model [10] of CP violation.
2. THE CP PHENOMENOLOGY OF THE KAON SYSTEM

2.1 The Kaon System

Under the strong interaction the neutral kaon system can be thought of as consisting of two distinct particles, the $K^0$ made up of $\bar{s}$ and $d$ quarks and the corresponding anti-particle the $\bar{K}^0$. The kaon is the lightest of the strange mesons and therefore can only decay via the weak interaction. Under the weak interaction, which does not conserve strangeness, the two mass eigenstates become non-degenerate and the weak interaction (without CP violation) instead recognises two linear combinations, the $K_1$ and $K_2$, of the strong eigenstates $K^0$ and $\bar{K}^0$:

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \quad |K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$  (2.1)

The $K_1$ state has CP=+1, the $K_2$ has CP=−1 and neutral pions have CP=−1. CP is a multiplicative quantum number, therefore a two pion state with angular momentum 0 has: $(CP= -1) \times (CP =-1) \equiv CP + 1$ and the three pion state, again with angular momentum = 0, has CP = -1. Thus the CP conserving decay of $K_1$ is, $K_1 \rightarrow \pi\pi$ and similarly $K_2 \rightarrow \pi\pi\pi$. The three pion decay is less likely, because of the smaller phase space, so the $K_2$ state should form the long lived kaon, $K_L$ with a lifetime of $\tau_L = (5.17 \pm 0.04) \times 10^{-8}$ and the $K_1$ the short lifetime state $K_S$, $\tau_S = (8.934 \pm 0.0008) \times 10^{-10}$ [11].

This is consistent, until one includes the effects of CP violation. Consider again
the long and short lived kaon states, $|K_L\rangle$ and $|K_S\rangle$, respectively. With CP non-conservation they become superposition states of $K_1$ and $K_2$. These states can be written as: [12]

$$
|K_S\rangle = |K_1\rangle + (\bar{\epsilon} + \delta)|K_2\rangle \quad |K_L\rangle = |K_2\rangle + (\bar{\epsilon} - \delta)|K_1\rangle
$$

(2.2)

The $K_L$ and $K_S$ states are no longer orthogonal and their mixing can be given by:

$$
\langle K_L | K_S \rangle = 2Re(\bar{\epsilon}) - 2iIm(\delta)
$$

(2.3)

If $T$ is conserved and CPT violated then $\bar{\epsilon}$ is zero and $\delta$ non-zero. Whereas if $T$ is violated and CPT conserved then $\bar{\epsilon}$ is non-zero and $\delta$ is zero. For CP conservation both $\bar{\epsilon}$ and $\delta$ are zero and $K_L$ and $K_S$ are the eigenstates $K_2$ and $K_1$ respectively.

### 2.2 CPT or T violation?

Having established experimentally that CP violation occurs [6] [7] [8], the next task is to find out whether either of the remaining symmetries, CPT or T is conserved. One therefore considers the evolution of a neutral kaon as a function of time:

The general time evolution of any decaying particle ($P$) can be written: [13]

$$
\frac{d|P\rangle}{dt} = (-iM - \Gamma/2) |P\rangle
$$

(2.4)

Where $M$ is the mass of the particle and $\Gamma$ is the decay width. The decay of the particle is balanced by the appearance of the final state(s) $F$:

$$
-\frac{d}{dt} |\langle P | P \rangle|^2 = \sum_F |\langle F | H | P \rangle|^2
$$

(2.5)
For a kaon state which is a mixture of \(|K_S\) and \(|K_L\) this gives two direct decay/production terms and a cross term from the \(|K_S\), \(|K_L\) mixing:

\[
\begin{align*}
\Gamma_S &= \sum_F |\langle F|H|K_S\rangle|^2 \\
\Gamma_L &= \sum_F |\langle F|H|K_L\rangle|^2
\end{align*}
\] (2.6) (2.7)

\[
\left(-i\Delta m + \frac{\Gamma_L + \Gamma_S}{2}\right) \langle K_L|K_S\rangle = \sum_F \langle F|H|K_L\rangle^* \langle F|H|K_S\rangle
\] (2.8)

The cross term, equation 2.8, is always CP violating. The \(\Delta m\) term \((M_L - M_S)\) if non zero, implies mixing between the kaon states as a function of time.

Now it is possible to define the CP violating amplitudes:

\[
\eta_+ = \frac{\langle F|H|K_L\rangle}{\langle F|H|K_S\rangle} \quad \text{final state CP} = +1
\] (2.9)

\[
\eta_- = \left[ \frac{\langle F|H|K_L\rangle}{\langle F|H|K_S\rangle} \right]^* \quad \text{final state CP} = -1
\] (2.10)

Which implies (using equation 2.8) that:

\[
\left(-i\Delta m + \frac{\Gamma_L + \Gamma_S}{2}\right) \langle K_L|K_S\rangle = \sum_F \eta_+ \Gamma_S
\] (2.11)

\[
\left(-i\Delta m + \frac{\Gamma_L + \Gamma_S}{2}\right) \langle K_L|K_S\rangle = \sum_F \eta_-^* \Gamma_L
\] (2.12)

Here we are using the partial widths for a kaon decaying to the relevant CP state.

The right-hand side of equation 2.8 is dominated by the most common, two pion, decays and therefore we can neglect equation 2.12. Re-writing equation 2.11 in terms of the two pion final state and neglecting terms in \(\Gamma_L\) gives [12]:

\[
\left(-i\Delta m + \frac{1}{2}\Gamma_S\right) \langle K_L|K_S\rangle = \sum_{00,+} \eta_{\pi\pi}^* \Gamma_{\pi\pi}
\] (2.13)

Recall equation 2.3 which defined the \(\langle K_L|K_S\rangle\) mixing in terms of T and CPT violation. It is now possible to equate this with equation 2.13 and define \(\phi\) (the
phase of the two pion decay) in terms of the decay width and $K_L K_S$ mass difference:

If $\textbf{CPT}$ is conserved ($\bar{\epsilon} \neq 0, \delta = 0$) Then:

$$\left(\frac{1}{\Gamma_S} + \frac{i}{2\Delta m}\right) 2Re(\bar{\epsilon}) = \eta_{\pi\pi}$$

$$\Rightarrow \phi_{\pi\pi} = \tan^{-1}\left(\frac{2\Delta m}{\Gamma_S}\right)$$  \hfill (2.14)

Similarly for $\textbf{T}$ conservation ($\bar{\epsilon} = 0, \delta \neq 0$)

$$\left(\frac{-i}{\Gamma_S} + \frac{1}{2\Delta m}\right) 2Im(\delta) = \eta_{\pi\pi}$$

$$\Rightarrow \phi_{\pi\pi} = \tan^{-1}\left(\frac{-\Gamma_S}{2\Delta m}\right)$$  \hfill (2.15)

Substituting in for the measured values of $\Delta m = (0.5301 \pm 0.0014) \times 10^{-10} \text{hs}^{-1}$ and $\Gamma_S = 1.1804 \times 10^{10} \text{s}^{-1}$ gives a value for the phase of the $\pi\pi$ decays of $43.7 \pm 0.03^\circ$ for the $\textbf{CPT}$ conserving case and $138.7 \pm 0.03^\circ$ for the $\textbf{T}$ conserving case.

Experimentally the phase of $\eta_{+-}$ is $43.5 \pm 0.6^\circ$ and $\eta_{00}$ is $43.4 \pm 1.0^\circ$ [11]. This implies that $\textbf{T}$ is clearly violated, whilst $\textbf{CPT}$ appears to be conserved. For the rest of this thesis $\delta$ will be taken as zero and the magnitude of the mixing between $\langle K_L | K_S \rangle$ as being $2Re(\bar{\epsilon})$.

In 1998 the CERN experiment CPLEAR measured the parameter $Re(\delta)$ directly using semi-leptonic kaon decays [14] they found a value of $Re(\delta) = (3.0 \pm 3.3_{\text{stat}} \pm 0.6_{\text{sys}}) \times 10^{-4}$, which is also consistent with zero.
2.3 Theoretical Models of CP Violation.

There are two main theoretical models for CP violation. The Standard Model with three quark families will allow CP violation via a phase factor in the CKM matrix (which parameterises the mixing between different quark types). Whereas Wolfenstein's Super Weak Model allows $K^0$ to $\bar{K}^0$ mixing via $\Delta S = 2$ transitions [10].

2.3.1 CP Violation in the Standard Model.

In 1973, Kobayashi and Maskawa [15] introduced a third generation of quarks into the Standard Model and noted that this provided a theoretical means of CP violation. In the two generation model there is only one mixing angle, the Cabbibo angle ($\lambda$), between the quark generations. However when the number of generations is increased to three, another two mixing angles and one phase factor are added to produce the full CKM Matrix:

$$
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
= 
\begin{pmatrix}
1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3[\rho - i\eta] \\
-\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\
A\lambda^3[1 - \rho - i\eta] & -A\lambda^2 & 1
\end{pmatrix}
(2.16)
$$

Here, the Wolfenstein parametrisation of the CKM matrix is used. $\eta$ represents the phase which, if non zero, produces CP violation. This parametrisation is correct up to terms of $O(\lambda^3)$. From the unitarity of the matrix one can formulate nine unitarity relations, the relevant one for the kaon system is:

$$
V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0
(2.17)
$$
Fig. 2.1: $\epsilon_K$ and the Constraints on $\rho$ and $\eta$. The solid lines are experimental calculations of various parameters, the dotted line shows the upper limit of $\Delta M_s/\Delta M_d$ and the contours represent a fit to the values of $\bar{\rho} = \rho(1-\frac{\lambda^2}{2})$ and $\bar{\eta} = \eta(1-\frac{\lambda^2}{2})$ from [16].

And for the $B_d^0$ system:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

(2.18)

These unitarity relationships are often shown in the form of a triangle (especially in the case of CP violation in $B_d^0$ decays). The area of each triangle is given by $J/2$ where $J$ is:

$$J = Im(V_{ud}V_{ts}V_{us}^*V_{td}^*) = Im(V_{ud}V_{cb}V_{ub}^*V_{cd}^*) \propto A^6 \lambda^6 \eta$$

(2.19)

A consequence of this relationship, is that area of the triangles must be non-zero to allow CP violation.

By measuring the sides and angles of the unitarity triangles its is possible to constrain the CKM parameters. In the kaon system it is difficult to access the elements $V_{cd}, V_{cs}, V_{td}$ and $V_{ts}$ directly, apart from in rare kaon decays such as $K_L \to \pi^0 \nu \bar{\nu}$ which would provide a very good measurement of $V_{td}$. This difficulty
and the fact that the side $V_{td}V_{ts}^*$ is so short means that is is difficult to constrain the CKM parameters using measurements from the kaon system. However the measurement of $\epsilon_K$ the asymmetry between the decays $K_L \to \pi l\nu$ and $K_L \to \pi l\bar{\nu}$ produces a constraint on the $\rho \eta$ plane (see Figure 2.1.)

**Direct and Indirect CP violation**

In the Standard Model CP violation in the kaon system occurs via two distinct mechanisms. The first, indirect CP violation, occurs via the transition of a $K^0$ to a $\bar{K}^0$ or vice versa (Fig 2.2). This CP violation process conserves the magnitude of isospin and is denoted by the parameter $\epsilon$.

![Diagram](image)

**Fig. 2.2: The Box Diagram, Indirect CP Violation**

Direct CP violation occurs in the actual decay of the kaon and is parameterised by the number $\epsilon'$. It proceeds via the penguin decay(s) shown in Fig 2.3. The penguin decay has both strong (gluonic) and electro-weak (via photon or $Z^0$ exchange) parts. Although the electro-weak decay has a much smaller amplitude it can proceed using the $\Delta I = \frac{3}{2}$ channel whereas the gluon interaction is purely $\Delta I = \frac{1}{2}$, it is the interference between the $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ channels that induces direct CP violation. Since the strong and electro-weak decays have opposite signs the photon channel becomes important. With the top quark mass fixed at $\approx 174$
GeV/c² the two diagrams come very close to cancelling each other out [17] and \( \epsilon' \) becomes much less than \( \epsilon \).

![Diagram](image)

Fig. 2.3: The Penguin Diagram, Direct CP Violation

2.3.2 The Superweak Model.

Wolfenstein’s Superweak Model [10] assumes the existence of \( \Delta S = 2 \) transitions analogous to the \( K^0\bar{K}^0 \) mixing given by diagram 2.2. However there is no analogous component to \( \epsilon' \), which is a \( \Delta S = 1 \) transition. This means that the ratio \( \frac{\xi'}{\xi} \) would, according to the Superweak Model, be exactly zero. A non-zero value of \( \frac{\xi'}{\xi} \) would exclude the Superweak Model of CP violation.

2.4 Isospin Analysis of the Two Pion Decay

We now wish to relate the physical observables \( \eta_{\pi\pi} \) (section 2.2) to the ratio of direct and indirect CP Violation, \( \frac{\xi'}{\xi} \) (section 2.3.)

The two final states (\( \pi^+\pi^- \) and \( \pi^0\pi^0 \)) can have isospin I=0 and I=2 (the I=1 state is antisymmetric and therefore forbidden by Bose-Einstein statistics.) We can therefore use Clebsch Gordon co-efficients to write the \( \pi^+\pi^- \) and \( \pi^0\pi^0 \) states
2. The CP Phenomenology of the Kaon System

in terms of their isospin components:

\[ |+\rangle = \sqrt{\frac{2}{3}} |I = 0\rangle + \sqrt{\frac{1}{3}} |I = 2\rangle \]  
\[ |00\rangle = -\sqrt{\frac{1}{3}} |I = 0\rangle + \sqrt{\frac{2}{3}} |I = 2\rangle \]  

The \( K^0 \) and the \( \bar{K}^0 \) both have \(|I| = \frac{1}{2}\) and the scattering amplitude for the \( \Delta I = \frac{1}{2} \) transition can be defined as [12]:

\[ \langle I = 0 | H | K^0 \rangle = A_0 e^{i\delta_0} \]  
\[ \langle I = 0 | H | \bar{K}^0 \rangle = A_0^* e^{i\delta_0} \]  

Similarly for the \( I=2, \Delta I = \frac{3}{2} \) decay.

\[ \langle I = 2 | H | K^0 \rangle = A_2 e^{i\delta_2} \]  
\[ \langle I = 2 | H | \bar{K}^0 \rangle = A_2^* e^{i\delta_2} \]  

\( \varepsilon \) and \( \varepsilon' \) can now be defined\(^1\):

\[ \varepsilon = \frac{\langle I = 0 | H | K_L \rangle}{\langle I = 0 | H | K_S \rangle} = \frac{ImA_0}{ReA_0} + \bar{\varepsilon} \]  
\[ \varepsilon' = \frac{1}{\sqrt{2}} \left[ \frac{\langle I = 2 | H | K_L \rangle}{\langle I = 0 | H | K_S \rangle} - \frac{ImA_0}{ReA_0} \right] \]  

\[ = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \left( \frac{ImA_2}{ReA_0} - \frac{ImA_0 ReA_2}{ReA_0 ReA_0} \right) \]  

The parameter \( \frac{\langle I = 2 | H | K_S \rangle}{\langle I = 0 | H | K_S \rangle} \) is also known as \( \omega \), the \( \Delta I = \frac{1}{2} \) rule means that this is of the order of \( \approx 0.05 \).

\(^1\) Remembering that the \( \Delta I = \frac{3}{2} \) transition is suppressed we neglect terms smaller than \( \frac{ReA_2}{ReA_0} \).
Since is only possible to measure the relative phase of the decay modes we can set \( A_0 \) to be real (this is the Wu-Yang phase convention [18]) and obtain:

\[
\epsilon = \tilde{\epsilon} \quad \epsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \left( \frac{Im A_2}{Re A_0} \right) \tag{2.28}
\]

Using this information and the isospin decomposition equations (2.20, 2.21) the \( K^0(\bar{K}^0) \to \pi\pi \) decay amplitudes can be written as:

\[
\langle + - | H | K^0 \rangle = \sqrt{\frac{2}{3}} A_0 + \sqrt{\frac{1}{3}} A_2 e^{i(\delta_0 - \delta_2)} \tag{2.29}
\]

\[
\langle 00 | H | K^0 \rangle = \sqrt{\frac{1}{3}} A_0 - \sqrt{\frac{2}{3}} A_2 e^{i(\delta_0 - \delta_2)} \tag{2.30}
\]

\[
\langle + - | H | \bar{K}^0 \rangle = \sqrt{\frac{2}{3}} A_0 + \sqrt{\frac{1}{3}} A_2^* e^{i(\delta_0 - \delta_2)} \tag{2.31}
\]

\[
\langle 00 | H | \bar{K}^0 \rangle = \sqrt{\frac{2}{3}} A_0 - \sqrt{\frac{2}{3}} A_2^* e^{i(\delta_0 - \delta_2)} \tag{2.32}
\]

Substituting for \( K_L \) and \( K_S \), one can now express \( \eta_{00} \) and \( \eta_{+-} \) in terms of the \( CP \) violating parameters.

\[
\eta_{00} = \frac{\langle 00 | T | K_L \rangle}{\langle 00 | T | K_S \rangle} = \epsilon + \epsilon' \tag{2.33}
\]

\[
\eta_{+-} = \frac{\langle + - | T | K_L \rangle}{\langle + - | T | K_S \rangle} = \epsilon - 2\epsilon' \tag{2.34}
\]

Approximating again, using the fact that \( \epsilon \) and \( \epsilon' \) are both close to zero one obtains:

\[
\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} \times \frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^+\pi^-)} = \left[ \frac{\eta_{00}}{\eta_{+-}} \right] = 1 - 6 Re \left( \frac{\epsilon'}{\epsilon} \right) \tag{2.35}
\]

It is the parameter \( Re \left( \frac{\epsilon'}{\epsilon} \right) \) i.e. the double ratio of \( CP \) violating \( K_L \) to \( CP \) conserving \( K_S \) decays in the charged and neutral modes, that NA48 will measure.
2.5 The Status of $\xi'$. 

During the 1980 two high statistics experiments were run to measure $\xi'$. NA31's result was $\xi' = (23 \pm 6.5) \times 10^{-4}$ [19], while the E731 experiment at Fermilab obtained a value of $\xi' = (7.4 \pm 5.9) \times 10^{-4}$ [20]. Although the numbers are consistent, different conclusions were obtained. The world average for $\xi'$ is $(15 \pm 8) \times 10^{-4}$ [11].

A recent result from KTeV [22] gives a measurement of $\xi' = (28.0 \pm 3.0 \text{(stat)} \pm 2.6 \text{(syst)} \pm 1.(MC\text{stat})) \times 10^{-4}$ confirming the NA31 discovery of direct CP violation. This would give a new world average of $(21.8 \pm 3.0) \times 10^{-4}$. Given the complexity of the measurement and the importance of the value of $\xi'$ it is essential that another high statistics measurement be performed.

Theoretical calculations of $\xi'$ are exceptionally difficult. First one uses operator product expansion and QCD perturbation theory to expand the weak Hamiltonian for the $K^0 \rightarrow \pi\pi$ transitions in terms of the local four fermion interactions (Feynman diagrams) each one multiplied by a Wilson Coefficient. The Wilson coefficients are calculated using renormalisable perturbation theory. The Wilson Coefficients and four fermion operators both depend on the selected renormalisation scale $\mu$ but the effective Hamiltonian must be independent of this. However as one tends to low renormalisation (mass) scales $M_b > \mu > M_c$ the amplitudes of the local operators and final state hadronisation become substantially modified by strong interactions and perturbation theory becomes unrealistic. For $K^0 \rightarrow \pi\pi$ decays the final state interactions cannot be neglected and cause a large uncertainty on $\xi'$.

There are eleven four fermion operators which contribute to $\xi'$. The most impor-

\[^2\] This includes an earlier result from E731 of $\xi' = (32 \pm 30) \times 10^{-4}$ [21].
tant of these are $B_6$, which corresponds to the gluonic penguin and $B_7$, $B_8$ which are the colour matched contributions from the two electroweak penguins. The interference between the operators $B_6$ and $B_7$, $B_8$ generates a non-zero value of $\epsilon'$. The calculations of these $B$ parameters uses a technique called lattice QCD whereby the interaction is simulated in a virtual lattice inside a computer (each point on the lattice is a point in space time where interactions can happen). The errors on the size of each of the B operators and the cancellations of the gluonic and electroweak penguins contribute the greatest uncertainty to theoretical calculations of $\frac{\epsilon'}{\epsilon}$.

Uncertainties on the strange quark mass also contribute to the error on $\frac{\epsilon'}{\epsilon}$. The lattice calculation of the strange quark mass is used as an estimate of the accuracy of the lattice QCD result. Lattices with a low output value of the strange quark mass (between 70 - 90 MeV) have a high value of $\frac{\epsilon'}{\epsilon}$ of the order of $10^{-3}$.

A review, before the announcement of the KTeV results, gave an estimate of $\frac{\epsilon'}{\epsilon} = (4.6 \pm 3.4) \times 10^{-4}$ [17]. This review also stated that due to uncertainties on the mass of the strange quark and the size of the relative contributions from the two penguin diagrams, the value of $\frac{\epsilon'}{\epsilon}$ can be of the order $10^{-3}$. Many theories beyond the Standard Model (including the Minimal Super Symmetric Model) propose other mechanisms for CP violation.

It is therefore necessary to measure $\frac{\epsilon'}{\epsilon}$ to a high degree of experimental accuracy in an effort to understand the origins of CP violation.
3. THE NA48 EXPERIMENT

This chapter details the NA48 experimental setup, as used in the 1997 run. It contains a brief overview of the experiment, followed by more detailed descriptions of each sub-system. Chapter 4 will describe the trigger system, dataflow and the two main parts of the reconstruction program.

Two nearly collinear beams of $K_S$ and $K_L$ are used concurrently. Both charged and neutral data are taken at once, so the relative intensity of the four decay modes cancels in the double ratio. The two beams coincide in space at the detector. Thus time dependent detection efficiencies and accidental effects should cancel in the evaluation of $\epsilon'.

3.1 A Brief Description of the NA48 Experiment

Figure 3.1 shows the NA48 experimental beamline and detector.

The co-ordinate system is defined such that, the $Z$ axis is along the $K_L$ beamline, $X$ horizontally towards the Jura mountains (left along the beam direction) and $Y$ vertically upwards. The co-ordinate origin is; $(X : 0, Y : 0)$ on the $K_L$ beamline and $Z = 0$ at the centre of the $K_S$ target. All distances are measured in cm and energies in GeV.

A burst\textsuperscript{1} of approximately $10^{12}$ protons, of energy 450 GeV, from the SPS ring

\textsuperscript{1} A burst is 2.5 seconds in duration, occurring every 14.4 seconds.
Fig. 3.1: The NA48 Experiment.
is directed down the K12 beamline. This primary proton beam interacts with a beryllium target, forming the $K_L$ beam. A residual beam (protons from the primary beam which were not absorbed by the $K_L$ target) of $3 \times 10^7$ protons per burst, is then partially deflected onto another beryllium target, 112m downstream, to form the $K_S$ beam. This deflection is caused by a special bent crystal. Between the bent crystal and the $K_S$ target a proton tagging detector is located. Any protons incident on the $K_S$ target must first pass through this detector, thus allowing the determination of whether a decay originated from the $K_L$ beam (untagged events) or $K_S$ beam (tagged events).

Both neutral kaon beams are cleared of charged particles, collimated and continue on to the decay (fiducial) region, the start of which is defined by an anti-counter at the end of the $K_S$ collimator. The kaons which decay in the fiducial region generate daughter particles which are detected in the sub-detectors. Figure 3.2 shows a GEANT generated drawing of the NA48 detector.

The sub-detectors are:

- A spectrometer consisting of four multi-wire Drift Chambers and a magnet for measurement of the momentum of charged particles.

- The $K_S$ and $K_L$ anti counters, $AKS$ and $AKL$ detect events containing particles which have X or Y positions outside the fiducial region. This helps define the acceptance of the detector and veto decays in which one of the particles would be un-detected.

- The Charged Hodoscope provides part of the level one trigger, indicating that a charged decay has occurred. Its high time-resolution means that it provides the event time for charged decays.

- The Liquid Krypton Calorimeter (LKR) is the NA48 electro-magnetic calorimeter
Fig. 3.2: A Schematic of the NA48 Detector.
and is used to measure the position, energy and time of photons from a $2\pi^0$ decay. It provides additional information about the energy of charged particles and is used for charged particle identification.

- The Hadron Calorimeter (HAC) is used to measure the energy of showers generated by charged pions and to observe single-strip hits produced by muons. This helps differentiate between these particles.

- The Muon Veto counts muons so that the $K_L \rightarrow \pi \mu \nu\mu$ decay and accidental muons in the beamline can be vetoed.

3.2 The K12 Beamline

3.2.1 The $K_L$ Target Area

A schematic diagram of the beamline is shown in figure 3.3, page 24.

On the diagram, B indicates the main proton bending magnets. The TAx is used as a retractable beam dump when only the $K_S$ or $K_L$ beam is required. Throughout the detector, except around the bent crystal and proton tagger, the beam is transported in a evacuated beam pipe, 16 cm in diameter.

The proton beam from the SPS is incident on the $K_L$ target at an angle of 2.4 milliradians (see figure 3.4). Both $K_L$ and $K_S$ targets are composed of four 10 cm long by 2 mm diameter beryllium rods placed one after each other. The protons undergo strong interactions with the quarks in the target to form a variety of hadronic particles and photons. Of these the photons are absorbed by the collimator, or pass straight down the beampipe, and charged hadrons are removed by the sweeping magnets. Of the remaining neutral hadrons the $K_L$ is the only one, apart from neutrons, which has a long enough lifetime to reach the fiducial
region before it decays.

Some of the remaining protons, which do not interact with the $K_L$ target, are then bent by the Bent Crystal onto the $K_S$ beamline.

3.2.2 The Bent Crystal

The application of the bent crystal to deflect the proton beam is a novel idea. It has several advantages over the traditional method of using magnets and collimators [23]:

- Stray muons in the beam are not deflected because their momentum is too high. Thus the beam is kept clean.

- Only a small fraction of protons is needed, which would require a very tight collimator. The crystal only deflects $5 \times 10^{-5}$ of the incident protons.

- The high energy of the protons (450 GeV) means that a large magnet of about 5m in length would be required to obtain the deflection angle of 9.6 milliradians. The crystal is only 56mm long.

The bent crystal is a single grain of high purity silicon aligned such that the proton beam passes through the crystal at a tangent to the bend. It is held in a mobile mount, which allows precise 3D alignment and applies the pressure needed to bend the crystal (figure 3.5).

The crystal is aligned such that the proton beam passes close to one of the major axes of the silicon lattice, the resulting electric field deflects a small fraction of the protons. The remaining undeflected protons continue on to a tungsten beam dump.
3. The NA48 Experiment

Fig. 3.3: The K12 Beamline

Fig. 3.4: Kaon Production at the Targets
BENT CRYSTAL:

Si cut // to (110) planes
[60 mm x 18 mm x 1.5 mm]

\[ \Phi = \pm 28.7^\circ \]
\[ \theta_0 = 18.7 \text{ mrad} \]

Deflection: \( \theta = 9.6 \text{ mrad} \)

Projection perpendicular to beam direction, z:

\[ y' = \frac{dy}{dz} \]
\[ dy/dx = \frac{1}{R \tan \Phi} \]
\[ = 0.61 \text{ mrad/mm} \]

Fig. 3.5: The Bent Crystal
3.2.3 The Proton Tagger

The proton tagger must be able to cope with a high rate of incident protons (> 1 MHz), high radiation dose environment (1 Gy per burst) and have a very good time resolution (≤ 500 ps) [24].

Any dead-time or misalignment will cause a $K_S$ decay to be mis-identified as $K_L$ decay ($K_S$ inefficiency) whereas accidental hits will mean that a $K_L$ decay is identified wrongly as coming from the $K_S$ beam ($K_L$ dilution). These inefficiencies should be decay mode independent, although they lead to a correction of the double ratio they cannot generate an artificial non-zero value of $\xi$. The tagging method is discussed in detail in section 7.1, page 95.

The proton tagging detector is made up of interleaved vertical and horizontal ladders of scintillating foil (figure 3.6) such that the rate in each counter is less than 1 MHz. The foils are 4mm deep parallel to the beam and range from a width of 200 $\mu$m at the centre of the beam to 3000 $\mu$m at the edge, such that the whole beam profile is covered with equal occupancy and efficiency. Anti-counters at the ends of the tagger remove halo particles while two trigger counters are used for efficiency studies. The whole tagger is mounted on a carbon fibre support structure parallel to the beam axis.

Each tagger foil can be moved vertically, horizontally or by rotation to ensure optimal alignment with the proton beam. This alignment ensures low hit multiplicity per incident proton, no geometrical inefficiencies, and good time resolution.

To be able to obtain the required pulse and overall time resolution the tagger is read out with a Flash Analogue to Digital Converter which has been built specially for NA48. The resolution of the system is 50 ps.
Fig. 3.6: The Tagging Detector
3.2.4 The $K_S$ Target and Associated Detectors.

As with the $K_L$ target (section 3.2.1) the $K_S$ target is made up of 40 cm of beryllium rods, the production angle in this case is 4.2 milliradians. The target is followed by a 6m collimator to define the beam.

The precise arrangement of the two targets, their production angles and collimators ensures that the decay spectra of the two beams is similar over the momentum range 70 - 170 GeV and $K_S$ lifetime range 0 - 4 $\tau_s$.

The Anti $K_S$ veto (AKS)

The AKS is positioned directly behind the $K_S$ collimator. It is composed of two radiation lengths of a tungsten/iridium crystal providing a photon converter for a three scintillator array. Un-decayed $K_S$ pass through the centre of this apparatus, which defines the beam size, while decayed kaons produce either gammas, which undergo pair production in the converter, or charged pions which are detected directly by the scintillators. Kaons also undergo scattering in the collimator, creating a fuzzy edge to the beam profile (known as the halo). The AKS provides a hardware definition of the beginning of the fiducial region.

For charged decays one can observe the amount of scattering in the collimator by looking at the size and shape of the halo composed of decays with a $XY$ vertex out-with the beam profile. However, for neutral decays, only the $Z$ vertex is known. It is therefore difficult to determine if scattering has occurred in a neutral decay. For this reason it was decided to use the tungsten/iridium crystal. There is a strong enhancement of the pair production cross-section along the crystal axis, so a thinner amount of converter can be used. The radiation length for the aligned crystal compared to that of an amorphous converter is between 1.5 to 5
times smaller, depending on the angle between the beam and the crystal axes. This minimises the effect of scattering, whilst maintaining the overall efficiency of the sub-detector [25].

The AKS also defines the neutral energy scale of the experiment. To find the Z vertex of a neutral decay the energies of the showers in the electro-magnetic calorimeter LKR are used (see section 5.1, page 59). Since all $K_S$ decays should occur downstream of the AKS, fine-tuning of the LKR calibration can be done by adjusting the energy scales such that all decays have a $Z$ vertex position consistent with the known position of the AKS.

3.2.5 The Anti $K_L$ Counters (AKL)

The main background to $K_L \rightarrow \pi^0\pi^0$ and $K_L \rightarrow \pi^+\pi^-$ comes from three-body decays of the $K_L$ meson. Three-body decays are more likely to contain a least one particle outside the active detector area. Thus to lower the rate of these decays into the Level Two Trigger and to define the $K_L$ beam acceptance the AKL can
be used in veto as part of the Level One Charged Trigger. In 1997 the decay rate in the detector was sufficiently low that it was decided not to use the AKL as part of the trigger.

The AKL consists of seven scintillator/iron pockets each defining a solid angle cone. The positions of the anti counters are shown in figure 3.8. The first four pockets contain 24 counters, each counter consisting of 35 mm of iron preceding 10 mm of scintillator, and the later three pockets contain 16 counters. The counters form an octagon shape outside the vacuum chamber or helium tank, see figure 3.7.
3. The NA48 Experiment

The end of the fiducial region is marked by the change from a vacuum tank to a helium filled environment. The two tanks are separated by a 2.4 mm (0.003 radiation lengths) thick kevlar window. Here the magnetic spectrometer is located, it consists of four proportional multi-wire drift chambers DCH 1 to 4 with a dipole magnet between DCH2 and DCH3. The spectrometer is used for fast and accurate reconstruction of charged particle momentum and vertex. The resolution of the spectrometer is characterised by:

\[ \frac{\sigma_p}{p} = 0.48\% \oplus 0.009p\% \quad (P \text{ in GeV}/c) \quad (3.1) \]

3.3 The Magnetic Spectrometer

3.3.1 The Drift Chambers.

Drift chambers 1, 2 and 4 contain 8 planes of 256 sense wires while drift chamber three contains only four such planes. The planes have four different orientations \(x(0^\circ), y(90^\circ), u(45^\circ), v(-45^\circ)\) with each orientation containing two staggered views \(xx', yy', uu', vv'\) to resolve left right ambiguities. DCH1, DCH2 and DCH4 contain \(xx', yy', uu', vv'\) orientations while DCH3 contains only \(xx'\) and \(yy'\) planes.

Each drift cell is made of four potential wires, held at a steady potential of 1250
volts, surrounding one sense wire (figure 3.9). The chambers are filled with a 60:40 Argon:Ethane mix.

The $X - Y$ resolution of a single eight plane drift chamber is 110 $\mu$m.

### 3.3.2 The Magnet.

The NA48 magnet is a modification of a previously used CERN magnet. It was designed to fulfil the following requirements.

- An aperture of $245 \text{ cm} \times 240 \text{ cm}$ to match the acceptance of the LKR.

- A field integral between DCH2 and DCH3 of 0.83 Tm, corresponding to a change in transverse momentum of 250 MeV/c (the maximum transverse momentum of a pion from a $K^0 \rightarrow \pi^+\pi^-$ decay.)

- The uniformity of the field in the active region better than 10% to allow fast momentum calculation by the Level Two Charged Trigger (see section 4.2.2).

- A fringe field at DCH2 and DCH3 smaller than 200 Gauss. This condition is required to avoid front to back perturbations in the behaviour of the chambers and to ensure a small deflection of particles between DCH1 and DCH2 and similarly between DCH3 and DCH4 [26].

### 3.4 The Charged Hodoscope

The Charged Hodoscope is used to provide the charged event time, and to generate a highly accurate and fast charged trigger as part of the Level One Trigger (see section 4.2.1, page 43.)
The Hodoscope is composed of two planes, of 64 scintillator strips, each 2 cm thick, (figure 3.10) the edge of the Hodoscope is at least 121 cm from the centre of the beam line. The strips are arranged horizontally in the first plane and vertically in the second. Each plane is divided into four quadrants of 16 strips. Each strip extending from either $X = 0$ or $Y = 0$ to the edge of the fiducial annulus where they are read out by photo-multipliers. The distance between the two planes is the maximum available (80 cm) to allow separation between prompt signals and signals due to back-scattering from the calorimeter.

![Fig. 3.10: The Charged Hodoscope](image)

The Hodoscope $\ell'_{e}$ trigger signal is defined as two hits in opposite extended quadrants. An extended quadrant is the 16 scintillator strips in that quadrant plus the nearest strip in the next quadrant. This has high efficiency for two-body decays,
as the particles have equal and opposite transverse momentum, while veto-ing some of the three-body decays that do not fulfil this condition.

The time resolution of the Hodoscope must be high as it provides one side of the time window for tagging charged decays. The poorer the time resolution, the larger the time window required and thus the probability of mis-identification of $K_L$ as $K_S$. As the tagging uses different detectors in charged and neutral modes this could produce a systematic bias in $\frac{\mathcal{L}}{\tau}$. This effect is discussed in more detail in section 7.1.

3.5 The Electro-magnetic Calorimeter (LKR).

The NA48 Electro-magnetic calorimeter uses Liquid Krypton as a homogeneous active detection medium, with a sophisticated tower readout structure, to provide precise shower energy and time measurement for neutral decays. The detector requirements for the $\frac{\mathcal{L}}{\tau}$ measurement are:[27]

- Stable detector response. A homogeneous medium has low stochastic fluctuations and a high signal to noise ratio.

- Detector geometry accurate to ±0.1 mm over distances of 1 m.

- A uniform detector response within the calibrated energy range (3-100 GeV). The energy resolution is parameterised by:

$$\frac{\sigma(E)}{E} = \frac{3.5\%}{\sqrt{E}} \oplus \frac{0.11 GeV}{E} \oplus 0.6\% \quad \text{Energy in GeV} \quad (3.2)$$

- A time resolution on single photon showers of 0.5 ns.
Fig. 3.11: A Cross Section Through the LKR.
3.5.1 The Detector Structure

The detector presents an octagonal face to the beam, the edge of the LKR is 128 cm from the \( K_L \) beam axis. It is composed of 13 248 cells lying parallel to the beam direction (figure 3.11.) The square face of each cell is 2 cm by 2 cm at the front plate and increases linearly to 2.022 cm at the rear of the calorimeter, thus producing the correct solid angle projectivity for the given fiducial volume. The LKR is 27 electro-magnetic radiation lengths in depth (\( \approx 125 \) cm) which is equivalent to two nuclear interaction lengths.

Each cell (figure 3.12) is composed of two electrodes, (alternating cathode and anode) parallel to the beam direction. The electrodes are made of a Cu-Be alloy, 40 \( \mu \)m thick and 18 mm in height, strung between the front and back plates of the calorimeter. Five spacer plates along the ribbon length maintain the proper spacing and create an accordion structure, the ribbons zig-zag with a angle of ±48 mrad. This structure minimises the dead area presented by the electrodes to a photon shower. The longitudinal arrangement of electrodes gives better position and energy resolution. At the same time this means that the position of each ribbon must be known to 50 \( \mu \)m to avoid non-linearities in the energy scale due to the increase in ionisation close to the anodes.

The Neutral Hodoscope

Attached to the second spacer plate within the LKR are the thirty two bundles of scintillating fibres which compose the Neutral Hodoscope (the LKR is split into four quadrants each with eight bundles). The Neutral Hodoscope provides a second neutral detector and is used to measure the efficiency and resolution of the Neutral Trigger and the calorimeter.
3.5.2 The LKR Readout Electronics.

A schematic representation of the LKR readout electronics is shown as part A of figure 4.2, page 47.

The first stage cold readout electronics are silicon pre-amplifiers, located on the rear plane of the calorimeter. This minimises electronic noise and allows quick extraction of the signal. The output from these pre-amplifiers is fed to the second stage warm amplification transceviers and thence to the Calorimeter Pipeline Digitiser (CPD) system.

The CPDs are 40 MHz FADCs with a ten-bit range which is swapped between four different gains. (the gain range is also recorded in bit format.) Each CPD reads out 16 daughter cards, one daughter card per cell. The CPDs contain a pipeline FIFO memory which retains all information recorded in the previous 204.8 μs. The output of the CPDs is fed simultaneously to the Neutral Trigger (section 4.2.3, page...
Due to the high trigger rate and large data volume produced by the CPDs a significant amount of data reduction is required before data is sent from the LKR to the Data Merger. The Data Concentrator provides a fast shower finding algorithm so that the data volume can be reduced by 80% whilst all relevant information is conserved.

The Data Concentrator algorithm has two stages. Firstly, seed cells above a certain threshold (100 MeV for charged and 25 MeV for neutral triggers) are found. Then expansion algorithms are performed around the seed cell. The expansion algorithm takes one of the cells already selected and performs either the “diamond” or “cross” expansions (see figure 3.13) starting with the cross expansion algorithm and alternating between the two. The cross algorithm consists of switching on any cell which is next to a cell which is already on (horizontally or vertically) whilst the diamond algorithm is for filling the corners in a square, a cell comes on if at least one of it’s two neighbours horizontally AND one of it’s two neighbours vertically has already been selected. The expansion continues in a particular direction unless the newly selected cells are below a threshold of 1 MeV or until the halo expansion.
reaches its maximum extent of $128 \times 128$ cells. Figure 3.14 shows some possible halo expansions.

The resultant selected cells are then read out to the Data Merger (the Data Merger is discussed in section 4.3.2, page 50.)

3.6 The Hadron Calorimeter (HAC)

The HAC is a secondary calorimeter (figure 3.15.) It is used for distinguishing between charged pions and muons for additional background removal. The HAC consists of an iron/scintillator sandwich, 1.2 m (6.7 nuclear interaction lengths) in depth. It is divided into back and front modules, where each module is composed of 24 planes of alternate horizontal and vertical strips. The strips extend across a half plane (from either $X = 0$ or $Y = 0$) to the outer edge and the HAC itself presents at 270 cm $\times$ 270 cm square face to the beams.

The HAC readout uses photo-multipliers connected to another CPD system (see section 3.5.2) the output of which is again fed to the Data Merger.
3.7 The Muon Veto

The Muon Veto is the last detector in the NA48 beamline and is used as part of the trigger logic to veto $K_{\mu 3}$ decays\(^2\) and to form a positive trigger for rare decays such as $K_L \rightarrow \mu^+ \mu^- \gamma$.

The Muon Veto consists of three scintillator planes each positioned behind an

---

\(^2\) During the 1997 run the Muon Veto was not used as part of the $\zeta^+_\ell$ trigger.
iron wall to prevent accidental coincidences. The first two planes have eleven scintillator strips each (horizontal strips in plane one and vertical in plane two.) Plane three has six horizontal strips (figure 3.16.) Each strip is read out via a photo-multiplier at each end, apart from the centre strips in each plane which are split in half by the beam pipe. All planes cover an active detector area of 270 cm x 270 cm square.

Planes one and two form the basis of the $1\mu$ trigger which is defined as, at least one strip hit in each of planes one and two. Plane three in coincidence with the HAC is used to monitor the efficiency of the Muon Veto.
4. TRIGGERS AND DATAFLOW

4.1 Overview of the NA48 Dataflow/Trigger System

To be able to measure the direct CP violation parameter \( \xi^0 / \xi \) to an accuracy of \( 10^{-4} \), NA48's experimental set up is designed to cope with a high trigger rate, to be almost deadtime-less and to have minimal systematic effects.

The NA48 trigger system is a three level system, with event building and reconstruction taking place between levels two and three. The Level One and Level Two triggers are implemented in hardware whilst the Level Three Trigger is a software based algorithm.

Event building is performed in hardware at the experimental site, data integrity checks are made and the data sent via fast FDDI link to the CERN CS2 machine. Here the Level Three and Real-time Reconstruction processes operate.

To obtain the high level of statistics needed for the \( \xi^0 / \xi \) measurement the trigger system must be able to cope with an input rate to the Level One of 100 kHz. Both Level One and Level Two triggers act to reduce this to the level of 12 000 events per burst (4.8 kHz). These events are then sent to the Data Merger for event building. This means the amount of background events rejected must be 60 % of input whilst maintaining an unbiased sample. Thus a highly efficient multi-level trigger system and de-coupled, pipelined read-out is required.
### Subsystem Signal Definition

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Signal</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon Veto</td>
<td>1 $\mu$</td>
<td>At least one strip in each plane</td>
</tr>
<tr>
<td>Muon Veto</td>
<td>2 $\mu$</td>
<td>$\geq 2$ strips in one plane and $\geq 1$ in the other</td>
</tr>
<tr>
<td>Charged Hodoscope</td>
<td>$Q_x$ (CHOD)</td>
<td>And of Hodoscope opposite quadrants</td>
</tr>
<tr>
<td>Neutral Hodoscope</td>
<td>$Q_x$ (NHOD)</td>
<td>As above (used for efficiency studies)</td>
</tr>
<tr>
<td>Neutral Trigger</td>
<td>$E_{TOT}$</td>
<td>Energy Sum of LKR and HAC $&gt; 30 GeV$</td>
</tr>
<tr>
<td>AKL</td>
<td>$AKL_{or}$</td>
<td>At least one strip hit, or of all pockets</td>
</tr>
<tr>
<td>AKL</td>
<td>$AKL_{and}$</td>
<td>At least one strip hit, and of all pockets</td>
</tr>
</tbody>
</table>

**Tab. 4.1: Level One Trigger Inputs**

A resume of the Dataflow/Trigger system is given schematically in figure 4.1. More detailed discussions of specific parts can be found below.

## 4.2 The Trigger System

### 4.2.1 The Level One Trigger

The Level One trigger is a fast "hit" trigger built in VME hardware. It combines the trigger outputs from the; Muon Veto (planes one and two only), AKL, Neutral Trigger and both Hodoscopes. Table 4.1 details the major trigger inputs. All inputs are asynchronous and are read in to the Level One Trigger Supervisor every clock cycle (25ns).\(^1\) Thus to ensure a good coincidence the OR of the present and previous clock cycles is used to perform the trigger logic.

---

\(^1\) The clock signal runs at 40 MHz and is distributed to every sub-detector.
Fig. 4.1: The NA48 Trigger and Dataflow System

Level One Trigger Logic

- Level One Trigger Supervisor
- Etot Signal
- Neutral Trigger
- Initiate Readout

Level Two Charged Trigger

- DCH data
- MBX yes/no
- Level Two Trigger Supervisor

Level Two Trigger Supervisor

DataFlow

- Det 1
- Det 2
- Det 15
- Optical links

Data Merger

- FEWS 1
- FEWS 2
- FEWS 3
- FEWS 4

L3 and RTR (CERN CS2 computer)
When the signals satisfy a particular trigger condition the Level One will issue a strobe to the Level Two Charged Trigger (section 4.2.2) and the Level Two trigger supervisor. The $\frac{e^+}{e^-}$ charged trigger is $Q_{e^+/e^-} / 2 \times E_{TOT}$. The neutral trigger remains autonomous from this chain and makes its own decision on whether the decay is an $\frac{e^+}{e^-}$ candidate. The only other trigger of interest for this analysis is the random trigger which is used for accidental studies. These studies are discussed in section 7.3.

The Level One Trigger Supervisor also provides a global time-stamp, corresponding to the clock cycle, for each event. This time-stamp is distributed to each sub-detector and is used for consistency checks in the Front End Workstations (section 4.3.3).

### 4.2.2 The Level Two Charged Trigger

The Level Two Charged Trigger uses information from the read-out of the drift chambers to calculate the invariant mass and decay vertex of any charged tracks. It makes a decision based on the hypothesis that a decay was $K^0 \rightarrow \pi^+\pi^-$ and relays this decision to the Level Two Trigger Supervisor.

The information from each drift chamber is held in a pipeline $200\mu s$ in depth, regardless of whether a trigger has been issued or not. On issue of strobe from the Level One Charged trigger the Level Two will access these buffers and extract data for the last $800$ ns. The Drift Chamber data contains both the event time-stamp (bined in units of $25$ ns) and a fine-time signal bined units in $25/16$ ns.

The information for each view in a chamber is processed by "Coordinate Builders". Using a look up table the Coordinate Builder transforms the hit wires and drift times into hit coordinates for that view\(^2\) and performs integrity checks on the

---

\(^2\) The number and orientation of the views in each chamber is discussed in section 3.3.1
data. This information is then passed to the main component of the Level Two Charged trigger, the Massbox.

The Massbox

The operating sequence of the Massbox is as follows;

1) The output of the Co-ordinate Builders is collected by the Event Dispatcher, ordered and sent to an Event Worker.

2) The Event Worker performs an association algorithm on the $x\ y\ u\ v$ co-ordinates to produce space-points.

3) The Combinatorial Dispatcher collects this information and converts it into a set of possible tracks. The track information is then sent to one of the “Two Track Workers”.

4) A Two Track Worker computes the associated vertex, kaon mass (for an assumed $\pi^+\pi^-$ decay.) and forwards it to the trigger collector.

5) A decision is taken as to whether the events is good, and the result forwarded to the Trigger supervisor. If the Massbox could not complete its decision making process with the 128 $\mu s$ time limit a “Massbox don’t know” signal is sent to the trigger supervisor and the event is kept.

During 1997 a good Massbox $\pi^+\pi^-$ event had to satisfy the follow criteria; lifetime (measured from the AKS) $\leq 4.0\tau_s$, $Z$ vertex $\geq 1.48$ m downstream from the $K_S$ target, closest distance of approach of the two tracks $\leq 5$cm, opening angle $\leq 15$ mrad and invariant mass greater than $0.95 \times M_{K^0}$.
4. Triggers and Dataflow

4.2.3 The Neutral Trigger (NUT)

The Neutral Trigger monitors information from the LKR and analyses this to find possible $2\pi^0$ decay candidates. It operates autonomously from the charged trigger, sampling information from the LKR every 25ns, performing its algorithm and returning the decision to the Level One and Two triggers. A schematic diagram is shown in figure 4.2 parts B, C and D.

4.2.4 Summation and Filtering of the LKR Signals

The calorimeter signals are analogue summed in $8 \times 2$ and $2 \times 8$ “super-cells”, in $X$ and $Y$ views respectively, on the CPD cards. The output of these super-cells is digitised by 10bit 40MHz FADCs and filtered.

The digital filter selects those channels which are higher than a given programmable threshold for at least two time slices ($2 \times 25$ns). Pedestal and undershoot correc-
tions are made and the highest time slice selected. Finally the output of the FADC is summed along $X$ or $Y$ projections to produce an array of 64 $X$ and 64 $Y$ strips and the results sent to "Peak-sum chips".

The "Peak-sum" Circuitry

The peak-sum and adder ASIC chips analyse the data in $X$ and $Y$ projections, each chip performing calculations on four adjacent strips of cells. Each strip $i$ has a weight assigned to it ($x_i$), corresponding to the $X$ (or $Y$) distance of the row from the centre of the calorimeter, this information is programmed into the chip software.

The circuitry computes the following;

- The total energy sum $\sum E_i$.
- The first moment $\sum E_i x_i$.
- The second moment $\sum E_i x_i^2$.
- The number of peaks, where a peak is defined to be, a row above a given threshold and higher than its neighbouring rows, both in space and time.\(^3\)

The Look-up Tables

The results from the Peak-sum finders are sent to a set of look-up tables, each implemented in a Xilinx programmable logic chip. This chip contains a set of boolean functions which are used to make a yes/no decision for this event.

The Look-up tables implement the following functions:

\(^3\) A programmable bit takes into account any effect from dead cells within rows.
- Combine $X$ and $Y$ views.

- Find a precise event time from the energy.

- Compute the following:

  Energy $\Sigma E_i$.

  Centre of gravity $C = \sqrt{(\Sigma E_i x_i)^2 + (\Sigma E_i y_i)^2 / \Sigma E_i}$

  $Z$ position of the decay vertex.

  Proper lifetime $\tau = (Z_{\text{decay}} - Z_{\text{aks}})/(\beta \gamma \tau_S)$.

During the 1997 run a good $2\pi^0$ decay was defined as having; $\tau < 5.5\tau_S$ and centre of gravity < 17 cm.

### 4.3 The Dataflow

This section will describe Dataflow requirements and the Data Merger. To describe the action of the Dataflow system as a whole a typical burst read-out sequence will then be described.

#### 4.3.1 The Dataflow Requirements

The number of events to be processed following the Level Two Trigger is, for a typical burst about 3.5 kHz from the $K_L$ beam and around 0.5 kHz from the $K_S$ beam for $\xi$ candidate events. Rare decay, calibration and random triggers make up a further 0.8 kHz (many events satisfy more than one trigger.) The total trigger rate from the Level Two Trigger is $\approx 4.8$ kHz (about 12k events/burst).

The event length is dominated by the data from the LKR calorimeter, for a $2\pi^0$ event the typical LKR sub-event size is 11.4 Kbytes. The other detectors generate
modest amounts of data for each trigger.

The event length is thus likely to be between 7 Kbytes and 20 Kbytes for a typical event, with calibration events up to 200 Kbytes. A typical event length is taken as being 15 Kbytes.

The Dataflow must therefore be able to process data at 72 Mbytes/sec (180 Mbytes/burst). The Data Merger was originally designed to handle a data rate of 75 Mbytes/burst but is able to cope with 2.5 times this level. A slight (5 %) amount of dead time within the burst has been observed but this should not bias the measurement of $\frac{t}{T}$.

4.3.2 The Data Merger

The Data Merger consists of a 9U VME crate containing a Single Board Computer (on which the Data Merger Control software runs), One Input Buffer (IB) per sub-system and an Output Formatter (FOF). The sub-systems are the outputs from the following parts of the detector; Tagger, Anti-counters, Drift Chambers, Charged Hodoscope, LKR (which makes up eight separate sub-systems,) Neutral Trigger, HAC and Muon Veto.

Data arrives as 40 bit words from each sub-system,\(^4\) over fast (10MHz) optical links. The data is written into memory on the IBs until required for read-out. If at any point during the cycle an IB’s memory becomes 97.5 % full an XOFF flag is raised and the optical link will stop transmitting data.

The read-out is controlled from the FOF by means of a token passing scheme. The FOF passes the token to the first IB and reads the data out via a special high-speed

---

\(^4\) Bits 0-31 contain the data, then there are five parity bits, two empty bits and the end of event bit.
bus, running at 25MHz, known as the R-path. The data packets are collected and stored by the FOF. When this sub-event has been read out the token is passed to the next IB in the daisy-chain. The sub-system packets are thus combined in an order defined by the token passing.

The completed event is then outputed via the front panel of the FOF onto a HiPPI link where it is fed via a HiPPI switch to the Front End Workstations (FEWS) for format checking and writing to disk.

Control of the IBs is provided by a Xilinx programmable gate array, all logic and intelligence functions are contained within it. It can be re-programmed whenever needed thus allowing flexibility of design.

4.3.3 The Burst Sequence

A typical read-out sequence is as follows;

1. One second before the SPS burst begins the signal WWE (warning of warning of ejection) is sent. This switches all sub-systems and the Data Merger into a mode ready for data taking.

2. On issue of a readout command from the Level Two Trigger Supervisor the sub-systems read-out begins.

3. All sub-systems sends their sub-events to the Data Merger in parallel. Although the sub-event can be of any length the first three data words must be:

   1) The sub-system identifier concatenated with the local event number from the sub-system.

   2) The global event number, coming from the Trigger Supervisor.
3) The event time-stamp. The local and global event number are used for format checking of the data.

4. The sub-event ends. The last word must be the local word count and have the end of event bit set TRUE. The IB also adds on its own word count to the end of the event. Now the R-path signal RP-NO-DATA goes low, indicating that at least one IB has a whole sub-event.

5. Now the whole of the first event is in the Data Merger. The signal RP-HAVE-DATA goes high, indicating that an event is ready to be read-out. The FOF sends out the token and data from the IBs is read-in and buffered on the FOF.

6. The token returns to the FOF. The FOF adds on its identifier word, padding words and a word count for the whole event. The event is then sent to the FEWS.

7. The read-in to the Data Merger, the read-out from IBs to the FOF and the output to the FEWS all occur asynchronously. This means there is no dead time in the dataflow mechanism unless one of the processes becomes overloaded. When the dataflow system becomes overloaded the following procedure is followed:

(a) One of the IBs has reached 97.5% full. XOFF is sent to the sub-system which is the data source for this IB.

(b) Since this sub-system can no longer send any data it too becomes full and propagates the XOFF to the Level One Trigger supervisor. The Trigger Supervisor will now stop issuing triggers until the XOFF disappears.
4. Triggers and Dataflow

(c) Read-out of the IBs is still continuing, though there is no data entering the IB with the XOFF condition. When this IB's memory drops below 90% full XOFF is de-asserted, read-in from that sub-system starts again.

(d) The XOFF removal propagates down the chain to the L1TS and normal data taking is resumed. The amount of lost events due to XOFFs from the Data Merger is \( \approx 1000 \) in a burst of 12000 good events. Thus the dataflow introduces a deadtime fraction of 5%.

8. At the end of the SPS cycle the End of Burst signal is received. The Trigger Supervisor stops sending triggers. The FOF also notes the EOB signal.

9. The FOF continues reading out events as normal. When RP-NO-DATA goes low this indicates that there are no more events or partial events left in the IBs. This condition \((\text{EOB} == \text{TRUE} \&\& \text{RP-NO-DATA} == \text{TRUE})\) is taken by the FOF as the end of the data taking for this burst.

10. The FOF adds the burst trailer word and burst word count to the end of the last event and passes it to the FEWS. It then breaks its connection to the FEWS, the IBs are cleared and readied for the next burst.

11. As soon as the burst is over, data on the FEWS is checked, formatted and written to a local disk buffer connected to the FEW. While writing to the disk, data is sent to the CS-2 via dedicated FDDI links which are multiplexed through a Serial HIPPI connection.

4.4 Reconstruction and the Level Three Trigger

This section will detail the two main reconstruction programmes; the spectrometer reconstruction and the LKR reconstruction. The cuts used by the Level Three \( \zeta \)}
4. Triggers and Dataflow

The design and implementation of the Level Three Trigger (L3) and Real Time Reconstruction (RTR) is also discussed.

4.4.1 The Charged (DCH) Reconstruction

The aim of the charged reconstruction is to find tracks using only hit wire information i.e. without using drift time information. These tracks are then refined using drift time information to increase the resolution and resolve left-right ambiguities. The momentum, production vertex and time of the tracks is then found.

Firstly, hit wires are used to build clusters in each view \((x, y, u, v)\) of each chamber. A cluster can have either one wire out of the \((x, x')\) planes in a view hit, one wire hit in each plane, or two hit wires in one plane and one in the other.

Clusters from chambers one and two are then combined into segments. The four segments for each chamber are aligned to form mini-tracks in chambers one and two.

The mini-tracks from the forward arm of the spectrometer are projected through to chamber four and associated with clusters in chamber four. The deviation due to the magnetic field has to be less than a given limit. The limits are: 45 mrad in the \(x\) view, 5 mrad in the \(y\) view and 35 mrad in the \(u\) and \(v\) views. Space-points are now formed in each chamber from the clusters and a track defined.

Having defined tracks using only hit wire information the definitions can now be refined using drift times. The track momentum is calculated using the four space-points and the known magnetic field. The vertex position for each track pair is calculated by minimising the closest distance of approach of the two tracks. [28]
4.4.2 The Neutral (LKR) Reconstruction

The neutral reconstruction measures the energy in the LKR cells. It groups hit cells into clusters and accurately calculates the energy, X Y position, rms width and time of each cluster. The program is designed to take into account the possibility of overlapping clusters and dead cells in the calorimeter.

The starting point of the neutral reconstruction is the times and energies of every cell read out from the LKR. Candidate cells with energy above > 0.2 GeV are designated as seed cells for clusters. If the seed cell is higher in energy than any of the surrounding eight cells then a cluster is designated as "found".

Once clusters have been found the next task is to compute which cells contribute energy to a particular cluster. Cells within \((R_{\text{cell}} - R_{\text{cluster}}) \leq 12\ \text{cm}\) \((12 \times 12\ \text{cell shower box})\) can contribute energy to a particular cluster. If a cell can belong to two or more clusters then the energy of that cell is apportioned between each cluster according to the expected shower profile. The shower profile comes from GEANT simulations of showers in the LKR calorimeter. The contribution of the cluster \(i\) to cell \(j\) is computed as the weight \(W_{i}^{j}\) and the energy is added to the cluster such that:

\[
E_{\text{added}} = \frac{W_{i}^{j}}{\sum_{k=1}^{n} W_{k}^{j}}
\]

(4.1)

Corrections are applied for dead cells and loss of energy due to the beam pipe at the centre of the calorimeter.

The position of the cluster \(i\) is then estimated using the energy of the seed cell and its eight surrounding neighbours. The width of the cluster is found using a five cells by five cells square box, centred on the seed cell. The cluster time and its error are estimated in a similar manner. [29]
4.4.3 The Level Three Trigger

The L3/RTR is a software program running in the CERN CS-2 machine. It is a program for data decoding, reconstruction and splitting of events into various output streams. Steering of the reconstruction is done via a set of ASCII configuration files, which can be updated at any time.

The Level Three trigger is the first part of the program. It looks at the trigger word of the events and according to the bit pattern, various actions are started. An action comprises of decoding, reconstruction and filtering routines. If the event passes the cut for each action then it is written to an output stream.

If any decoding, reconstruction or computation module returns an error, the error is internally counted and stored. The list of actions associated to the event can therefore be modified dynamically during the analysis of the event itself. The action for the $\ell_\ell$ trigger bits is; decode and reconstruct the DCH and LKR, if the event passes all cuts the event is written to the Golden stream.[30]

For events with the charged trigger bit set the cuts are:\(^5\)

- Lifetime as measured from the AKS $< 5.0\tau_s$.
- Invariant mass $0.4776$ GeV/$c^2 \leq M \leq 0.5117$ GeV/$c^2$.
- $E_{LKR}/P_{track} \leq 0.95$.
- Centre of gravity $\leq 12$ cm (Centre of Gravity is defined in section 5.2, page 64.)

For events with the neutral trigger bit the L3 trigger calculates an event time which is the mean of the cluster times in the LKR. The clusters in the events are then defined as variables and how they are calculated are found in Chapters 5, 6 and 7.

\(^5\) definitions of variables and how they are calculated are found in Chapters 5, 6 and 7.
formed into "four photon combinations", If one of the four photon combinations fulfils the following cuts then the event is kept. The cuts are:

- Energy of all clusters $\geq 2$ GeV.
- Cluster time - event time $\leq 12.5$ ns.
- Centre of gravity $\leq 15$ cm.
- $E_{LKR} \geq 60$ GeV.
- For one of the three possible combinations of four clusters the $\pi^0\pi^0$ mass combinations have to have masses between $0.12 \text{ GeV}/c^2 \leq M_\pi \leq 0.15 \text{ GeV}/c^2$.

Events in the Golden Stream are fully reconstructed. The output of the reconstruction is stored in a data format called COMPACT. The COMPACT data is then written to disk where the events can be analysed later.
5. DEFINING THE FIDUCIAL VOLUME AND

ACCEPTANCE STUDIES

To measure $\frac{\epsilon'}{\epsilon}$ one takes the double ratio of CP violating $K_L$ to two pion decays over CP conserving $K_S$ to two pion decay:

$$R = \frac{\Gamma(K_L \rightarrow \pi^0\pi^0)}{\Gamma(K_S \rightarrow \pi^0\pi^0)} \times \frac{\Gamma(K_S \rightarrow \pi^+\pi^-)}{\Gamma(K_L \rightarrow \pi^+\pi^-)} = 1 - 6 \text{Re} \left( \frac{\epsilon'}{\epsilon} \right) \quad (5.1)$$

The double ratio ($R$) is evaluated independently, via a log likelihood fit, for different kaon energies $E_{kaon}$, (see section 8.2, page 115.)

To find the true number of decays in each mode one takes the measured number of decays found and applies certain corrections. The major corrections are the following:

- **Acceptance $A(E_{kaon}, \tau)$**. Acceptance is the function relating the number of decays that occurred, for a given energy bin, to the number of decays that should have hit the detector and caused a signal. If one could build a detector covering $4\pi$ steradians it would have an acceptance of 100%. The acceptance studies are detailed in section 5.4, page 65.

- **Backgrounds, $B(E_{kaon})$**. The backgrounds are different for each of the four decay modes and do not cancel in the double ratio. Evaluation of backgrounds must be at the $10^{-3}$ level to avoid biases on $\frac{\epsilon'}{\epsilon}$, this is discussed in chapter 6 page 77.
5. Defining the Fiducial Volume and Acceptance Studies

- **Efficiency, \( E \).** This is the trigger and reconstruction efficiency for each of the four modes. Detection efficiencies should cancel to first order, if \( \xi' / \xi \) is independently measured at different kaon momenta. See sections 7.5, page 109 and 7.6, page 112.

- **Tagging.** The tagging has two effects on the number of events measured, depending upon whether the decay is \( K_L \) or \( K_S \). For true \( K_S \) decays any inefficiency (denoted by \( \xi \)) of the tagger will cause them to be identified as \( K_L \), whereas accidental hits in the tagger in-time with a \( K_L \) decay will cause a \( K_L \) decay to be misidentified as a \( K_S \) (an effect known as \( K_L - K_S \) dilution, :math: \( \alpha \)). Tagging effects are discussed in section 7.1 page 95.

To isolate each of the four decay modes cuts are made. These cuts reduce background and detector effects as well as defining each of the four modes. Where possible these cuts are made symmetrically, either the same for \( K_L \) and \( K_S \) decays or the same for charged and neutral modes. This minimises possible bias these cuts may make on \( \xi' / \xi \).

This chapter will detail the methods used for finding the decay vertex and energy of each decay mode and the acceptance studies. Isolation of each of the decay modes and removal of physics backgrounds are the subject of chapter 6. Tagging effects, efficiency studies, the energy scale and the effect of accidental particles will be dealt with in chapter 7.

### 5.1 Energy and Z Vertex Selection.

For neutral decays, finding the kaon energy is trivial, being the summed energies of the reconstructed photons (clusters) in the LKR. For charged decays the momentum of the kaon can be found by adding the four momenta of the charged
tracks, but a more accurate measurement of the kaon energy can be made by using the kaon mass as a constraint.

The track momenta are taken directly from the reconstruction program, the decay is assumed to be \( K^0 \rightarrow \pi^+\pi^- \).

The four momentum of the parent kaon is found using:

\[
E_{\text{kaon}} = \frac{1}{\theta} \sqrt{(M_K^2 - R M_{\pi}^2) R} \quad \text{where,} \quad R = 2 + \frac{E_1}{E_2} + \frac{E_2}{E_1} \tag{5.2}
\]

where \( \theta \) is the opening angle between the positive and negative tracks and \( E_1, E_2 \) their energies. A detailed proof of this formula can be found in appendix A.

For this analysis only kaons with energies between \( 70 \leq E_{\text{kaon}} \leq 170 \) GeV were selected.

For charged events tracks are reconstructed and used to calculate the \( X, Y \) and \( Z \) position of the kaon decay. For neutral decays tracks cannot be found, however it is possible to calculate the \( Z \) position of the decay vertex using the kaon mass as a constraint:

\[
Z_{\text{decay}} = Z_{\text{LKR}} - \left[ \frac{1}{M_K^2} \sum_{i,j(i<j)}^{N_{\text{clus}}} E_i E_j R_{ij} \right]^{\frac{1}{2}} \tag{5.3}
\]

\( R_{ij} \) is the radial distance between the \( i \)th and \( j \)th clusters and \( E_i, E_j \) their respective energies. A derivation of this is given in appendix A.

Once the energy and \( Z \) decay vertex of the kaon are known the lifetime can be calculated:

\[
\tau = \frac{(Z_{\text{decay}} - Z_{\text{aks}}) M_{\text{kaon}}}{E_{\text{kaon}} \tau_s} \tag{5.4}
\]

In this equation the lifetime is measured in units of the \( K_S \) lifetime \( \tau_s = 0.08927 \) ns and the kaon mass taken from the particle data booklet \( (M_{\text{kaon}} = 497.62 \pm 0.031 \)
5. Defining the Fiducial Volume and Acceptance Studies

$\tau = 0$ is defined to be the AKS position, \textit{i.e.} the beginning of the fiducial region. The AKS position is 609.4 cm for charged decays and 607.3 cm for neutral decays. Kaon decays are selected for analysis if they have $\tau_s \leq 3.5$.

5.2 Using the AKS to Define the Fiducial Region

The $Z$ vertex resolution for both charged and neutral decays is finite and non-negligible (for charged decays the resolution is about 30 cm and for neutral decays it is about 70 cm.) This means that certain decays with a true lifetime of greater than zero will have a reconstructed lifetime of less than zero and vice versa (recall that $\tau = 0$ is defined at the AKS and not the $K_S$ production target). If a software lifetime cut is applied at $\tau = 0$ then a fraction of events will be lost, when $Z_{true} \geq 0$ and $Z_{rec} \leq 0$ and gained when $Z_{true} \leq 0$ and $Z_{rec} \geq 0$.

This is not a problem for $KL$ decays. The lifetime of the $KL$ is so long that the $Z$ vertex distribution is flat over this range and therefore the amount of losses and gains will be equivalent, provided that the acceptance does not vary rapidly with $Z$ in this region. Figure 5.1 shows the acceptance as a function of $\tau$ for each of the four decay modes. The acceptance for $KL \rightarrow \pi^+\pi^-$ decays varies by 1.5\% in the region $\tau = 0.0 - 1.0 \tau_s$.

For $KS$ decays, the $Z$ vertex distribution is an exponential decay with a high decay rate. Due to the shape of this curve the amount of decays gained will be greater than the amount of decays lost, pushing the peak of the $Z$/lifetime distribution away from $\tau = 0$ towards higher lifetimes.

If the $Z$ vertex resolution was the same for the charged and neutral modes then this effect would not affect the value of $\xi/\xi$. This is because the decrease in the overall number of decays accepted would cancel between charged and neutral
Fig. 5.1: The Acceptance as a Function of Lifetime (Monte Carlo Simulation).
modes in the double ratio. The poorer $Z$ vertex resolution for neutral decays than for charged decays means that the $Z$ vertex distribution for $K_S \rightarrow \pi^0\pi^0$ will be shifted downstream with respect to the $Z$ vertex distribution for $K_S \rightarrow \pi^+\pi^-$ decays. This would have a noticeable effect on the value of $\xi$. The solution to this problem is provided by the AKS veto counter. For $K_S$ decays the AKS provides a hardware definition of the fiducial region. If a decay occurs before the AKS the decay products will hit this veto counter and the event will be rejected. Since this is a physical veto, rather than a software cut, its effect is symmetrical between charged and neutral decays. Any $K_S$ decays which have a reconstructed negative lifetime are assumed to be decays with a true lifetime $\geq 0.0$. These decays are kept.

For charged decays the pions are detected directly by the scintillating counters whilst for neutral decays the photon converts in the tungsten/iridium crystal and the resultant shower is detected by the scintillator. The position of the AKS for neutral and charged decays is therefore different by two centimetres, measured as 607.3 cm and 609.4 cm respectively.

To be effective the AKS must be highly efficient. The AKS inefficiency was measured by C. Biino to be $(3.5 \pm 0.6) \times 10^{-3}$ for neutral decays and $(2.7 \pm 0.6) \times 10^{-3}$ for charged decays [31]. With 6% of neutral and 4.5% of charged $K_S$ decays occurring before the AKS the effect on the $K_S \rightarrow \pi^+\pi^-$ decay mode is $(1.2 \pm 0.3) \times 10^{-4}$ and $(2.1 \pm 0.4) \times 10^{-4}$ for the $K_S \rightarrow \pi^0\pi^0$ mode. This gives an overall correction on the double ratio of $(1.1 \pm 0.5) \times 10^{-4}$ and can safely be taken as negligible.

For $K_L$ decays the beginning of the fiducial region is defined by a software cut at $\tau = 0.0\tau_s$ so that the same fiducial volume is used in all four decay modes.
There is a small (of the order of $10^{-3}$) amount of scattered kaons from the collimators. Because the $K_S$ collimators have to be close to the beginning of the fiducial region it is possible for these scattered kaons to decay and for their daughter particles to pass through the detector. To remove such events a cut on the centre of gravity of the events is used. The centre of gravity (COG) is defined as:

$$\text{COG} = \sqrt{\left(\frac{\sum_i E_i X_i}{\sum_i E_i}\right)^2 + \left(\frac{\sum_i E_i Y_i}{\sum_i E_i}\right)^2}$$

(5.5)

Where the $X$ and $Y$ positions are taken at $Z = Z_{LKR}$. Centre of gravity measures how close the event is to the $Z$ axis. Events with centre of gravity greater than 10 cm are rejected for all four decay modes. This cut also helps to remove background from three body decays with missing momentum.

5.3 The Weighting Procedure

The $K_S$ and $K_L$ decays have different $Z$ vertex distributions because of their different lifetimes. For a $K_S$ decay with an energy of 100 GeV (this is close to the mean energy for a kaon decay in the NA48 experiment) the average distance travelled is $\approx 5$ m. For a $K_L$ decay the average distance travelled by the kaon is 600 times this (3000 m). The total size of the fiducial region is about 90m, whilst for the measurement of $\xi$ only the first 40 m is used.

The acceptance varies as a function of $Z$ and is different for the neutral and charged modes. Therefore to reduce the uncertainty on the double ratio coming from the different $Z$ acceptances one weights the $K_L$ lifetime distribution to make it the same as the $K_S$. 

The $K_S$ Halo

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The acceptance varies as a function of $Z$ and is different for the neutral and charged modes. Therefore to reduce the uncertainty on the double ratio coming from the different $Z$ acceptances one weights the $K_L$ lifetime distribution to make it the same as the $K_S$. 

Each $K_L$ event is given a weight according to:

$$W = \exp \left( \frac{- (Z - Z_{aks})}{\lambda(p)} \right) \quad \text{where} \quad \lambda(p) = \left( \frac{p}{M_{K^0}} \right) \left( \frac{c\tau_S}{1 - \tau_S/\tau_L} \right)$$  

(5.6)

Although this increases the statistical error on $\frac{\xi}{\epsilon}$, it minimises any systematic error from the acceptance and therefore reduces the dependence on the amount of Monte Carlo statistics. Without the weighting procedure there is a 20% variation of the ratio of acceptances in the energy range 70 - 170 GeV. This leads an overall acceptance correction on $\frac{\xi}{\epsilon}$ of 1.3% (compared with 0.06% for weighted events).

### 5.4 Acceptance Studies of the Decay Modes

The number of decays that produce a hit in the detector is a function of the kaon energy and of the selected fiducial volume. The ratio of the number of events generated in a given energy and $Z$ region to the amount of events collected in the detector (for the same energy and $Z$ region) is called the acceptance. The energy band is defined as $70 \text{ GeV} \leq E_{\text{kaon}} \leq 170 \text{ GeV}$ and the fiducial volume by the lifetime (and therefore $Z$) cuts as described above (section 5.1, page 59). For neutral decays the $XY$ acceptance cuts are made at the LKR only. Events are accepted if the kaon decay products are at least 11 cm away from the outer edge of the calorimeter and at least 15 cm away from the centre of the beam pipe. (see figure 5.2, page 69, upper left plot). Photons outside this area will have poorly measured energies so the kaons producing such photons must be removed. This $XY$ cut is also made on charged decays to reduce background containing electrons (see section 6.6.1, page 88.)

During the 1997 run a column of the LKR had no high voltage and was unable to be used. Events with the $X$ position of a particle within $\pm 4$ cm of the centre of the dead column were rejected. This effect is also included in the evaluation of
For charged decays radial cuts are made at drift chambers 1, 2, 4 and the muon veto to ensure that tracks pass though the whole detector. A cut on the asymmetry of the decay is also used:

\[
\text{Asymmetry} = \frac{|E_{\pi_1} - E_{\pi_2}|}{E_{\text{kaon}}} \propto \cos \theta^*
\]

This is a direct cut on the centre of mass frame, \(\theta^*\) is the angle between the direction of the kaon and pion in the centre of mass. The maximum value of the asymmetry is 0.62 and the cut is parameterised as:

\[
|E_{\pi_1} - E_{\pi_2}| / E_{\text{kaon}} \leq 0.62
\]

\[
|E_{\pi_1} - E_{\pi_2}| / E_{\text{kaon}} < 1.08 - 0.0052E_{\text{kaon}}
\]

This is an empirical formula, coming from Monte Carlo acceptance studies.

Using this cut minimises the acceptance difference between \(K_S\) and \(K_L\). This difference in acceptance is due to the different illuminations of the beams on the detector, particularly on the magnetic spectrometer [32]. The asymmetry cut has the added advantage of removing background to \(K_S \rightarrow \pi^+\pi^-\) decays from the decay \(\Lambda \rightarrow p\pi\) (section 6.4, page 86). There is a slight loss in statistics, 2.05\% for the asymmetry cut compared with no cut. This cut also imposes a minimum track momentum of 26 GeV/c.

The neutral decays are far more asymmetric, due to the extra degrees of freedom from the \(\pi^0 \rightarrow \gamma\gamma\) decays. Using the asymmetry cut instead of the \(R_{LKR} \leq 15\) cm cut would lead to a 12\% loss in statistics in the neutral modes. Since the \(K_L \rightarrow \pi^0\pi^0\) decay is the decay mode with the least statistics it defines the statistical error on \(\xi\). It was therefore decided not to use the asymmetry cut for neutral decays. The fact that the photon is massless and the extra degrees of
freedom from the $\pi^0$ decay mean that the acceptance difference between $K_L$ and $K_S$ is much smaller in the neutral mode than in the charged mode.

To measure the acceptance a Monte Carlo simulation is used.

5.4.1 The Monte Carlo Simulation, NMC

NMC is the NA48 fast Monte Carlo [33]. According to a title file, defined by the user, it will generate the required kaon decay mode within a selected $E_{kaon}$ and $Z$ region. As each kaon decays its daughter particles are tracked through the detector and any possible interactions simulated. These simulated signals are then processed by the reconstruction package and the results written out into COMPACT format.

The kaon production spectrum is based on studies of $K^+, K^- \text{ data}$ [34] with an empirical correction devised by B. Hay[35]. The parametrisation is:

$$\frac{d^2N}{dpd\Omega} = \frac{\eta p^2}{4p_o} \left[1.30 e^{-(8.5 p/p_o + 3.0 p^2 \theta^2)} + 4.35 e^{-(13 p/p_o + 3.5 p^2 \theta^2)} \right]$$

$$\times \left[ e^{-\left(4.2 + 0.053 p \right)} + 0.016 \right] \quad (5.9)$$

Where $p_o$ is the proton momentum, $\theta$ is the angle between the incoming proton and the produced kaon and $p$ the momentum of the kaon. To obtain the decay spectrum of the kaon this is convoluted with lifetime distribution of the $K_S$ or $K_L$:

$$d(z) = \frac{1}{\lambda} e^{-z/\lambda} \quad \text{with} \quad \lambda = \beta \gamma c \tau \quad (5.10)$$

The decay $\Lambda^0 \rightarrow p\pi$ can also be selected. The program will allow; $\pi^0 \rightarrow \gamma\gamma$ decay, the Dalitz decay of the $\pi^0$ ($\pi^0 \rightarrow e^+e^-\gamma$) and $\pi \rightarrow \mu\nu$ decay.

The response of each sub-detector is simulated according to parametrisation from either data or from a GEANT Monte Carlo. To simulate the interactions in
the LKR and HAC the program uses a database of "showers". These showers are generated by a full GEANT simulation of the calorimeter using the energy range 2 - 100 GeV for photons and electrons and 2 - 200 GeV for pions. NMC reads this database and selects an appropriate shower according the type of showering particle and its energy. In addition multiple scattering of charged particles (in the chambers, beam pipe and other subdetectors) and gamma conversions are also simulated.

5.4.2 Comparisons of Data and Monte Carlo

To assess how accurately NMC simulates the detector one compares data and Monte Carlo distributions for various quantities. Since the primary use of the Monte Carlo is to measure the acceptance the most important distributions are those which affect the size of the acceptance correction.

The distributions to be compared are; X Y or radial positions at various detectors, lifetime distributions and momentum distributions.

Figure 5.2 shows comparisons of the cluster positions in the LKR for $K_L \rightarrow \pi^0 \pi^0$ decays. The plot in the upper left corner shows the hit positions of data photons in the calorimeter (the cut around the edge of the calorimeter and the dead column can be clearly seen). The other two plots show comparisons between data (points) and Monte Carlo (dotted line) for the lowest energy cluster. The agreement between photon energies is within statistical limits above 7 GeV below which the two distributions differ due to resolution tails in the data. The radial position shows good agreement across the whole distribution. Similar agreement is seen in the other three decay modes.

A comparison of the four lifetime distributions is shown as figure 5.3. While the $K_L$
Fig. 5.2: Comparisons of Data and Monte Carlo for $K_L \rightarrow \pi^0\pi^0$. The upper left plot shows the hit positions of clusters in the LKR (from data.) The other two plots compare data (red points) and Monte Carlo (dashed black line.)
Fig. 5.3: Comparisons of Lifetime Distributions.

Distributions show good agreement of the whole lifetime range the $K_S$ distributions show disagreement at low lifetimes. This again is due to the resolution of the detector being not well simulated in the Monte Carlo. Since no low lifetime cut is made for $K_S$ decays and because the calculation of $\xi/\epsilon$ is performed as an integral over lifetime the resolution does not matter.

Figure 5.4 show plots of the momentum spectra for data divided by that of Monte Carlo, for each of the decay modes. All four modes show a relative variation of
Fig. 5.4: Data / NMC for the Four Momentum Spectra. The plots have been normalised such that the number of events is the same data and Monte Carlo,
data/MC of 5-10%. This indicates that the Monte Carlo spectrum requires more tuning.

Since there are differences between the data and Monte Carlo spectra in each of the four modes one must fit the double ratio on a bin by bin basis. This minimises the acceptance correction and the bin size defines the accuracy required in the Monte Carlo. To determine the level of accuracy required in the Monte Carlo one considers the differences between the two expectation values for the acceptance, (one from data and one from Monte Carlo) given that there is a variation (\( \alpha \)) of the acceptance and of the spectrum (\( \beta \)) over the bin. Here is is assumed that any other difference between data and Monte Carlo can be neglected.

Define the momentum distributions of events to be \( N_G(p) \) for the Monte Carlo generated events and \( N_T(p) \) for the true number of events in the data. The expectation values for the acceptances (\( A_{mc} \) Monte Carlo, \( A_D \) data) can then be defined as:

\[
\langle A_{mc} \rangle = \frac{\int a(p)N_G(p)dp}{\int N_G(p)dp} \\
\langle A_D \rangle = \frac{\int a(p)N_T(p)dp}{\int N_T(p)dp} \\
\Delta A = \langle A_{mc} \rangle - \langle A_D \rangle
\]

The data and Monte Carlo spectra are related by:

\[
N_G(p) = b_0(1 + \beta(p - P_C))N_T(p)
\]

Here \( b_0 \) is the difference between the data and Monte Carlo spectra at some arbitrary point in the bin (for convenience we take the centre of the bin \( P_C = (P_2 + P_1)/2 \)). \( \beta \) is the variation of the difference spectra over the bin, this is assumed to be a linear function.
The variation in the acceptance across a bin between $P_1$ and $P_2$ is:

$$a(p) = a_0(1 + \alpha(p - P_C))$$  \hspace{1cm} (5.13)

If $a_0$ is the acceptance for momentum $P_C = (P_2 + P_1)/2$ (at the centre of the bin).

The fractional change in the acceptance is given by:

$$\frac{\Delta A}{a_0} = \alpha \beta \left(\frac{P_2 - P_1}{12}\right)^2$$  \hspace{1cm} (5.14)

The factor $b_0$ due to the overall difference in the spectra cancels in the evaluation of the expectation values. A detailed proof of equation 5.14 can be found in appendix B.

Now it remains to select a bin size $P_2 - P_1$ such that, the change in the acceptance (due to the differences between data and Monte Carlo spectrum $\beta$ and differences in the acceptance $\alpha$) is small i.e. $\frac{\Delta A}{a_0} \leq 1 \times 10^{-3}$.

A bin size of 5 GeV was selected. The maximum value of $\beta$ that can be obtained by reading off the gradients of the plots in figure 5.4 is in the range 150 - 155 GeV for the $K_S \rightarrow \pi^+\pi^-$ decay modes. The rate of change of the distribution is $\approx 2\%$ over 5 GeV i.e $\beta = 0.004(\text{GeV})^{-1}$ The value of $\alpha$ for this bin is (from figure 5.5) about 0.013 $(\text{GeV})^{-1}$. Thus the value of $\frac{\Delta A}{a_0}$ is $9.3 \times 10^{-6}$.

Now repeating this exercise for the maximum variation of acceptance:

Figure 5.5 shows the acceptances as a function of energy for the four decay modes. The biggest variation of acceptance of the bin comes in the low energy end of the $K_L \rightarrow \pi^0\pi^0$ when the variation is about 30% across the bin i.e $\alpha$ is about 0.06 $(\text{GeV})^{-1}$ and $\beta \approx 0.003(\text{GeV})^{-1}$ which implies $\frac{\Delta A}{a_0}$ is $3.12 \times 10^{-5}$.

Thus it has been demonstrated that the difference in the spectra between data and Monte Carlo causes an effect of $\leq \times 10^{-3}$ in any of the decay modes. For
Fig. 5.5: Acceptance of the Four Decay Modes.
the measurement of \( \xi \), the acceptance functions and weighted momentum spectra are similar in the charged and neutral modes (see figure 5.5) and thus the effect on the double ratio coming from differences in the data and Monte Carlo energy spectra cancels in the double ratio and can therefore be safely neglected.

### 5.4.3 Measuring the Acceptance

The experiment is designed such that the acceptance correction should be small and thus not heavily dependent on the accuracy of the Monte Carlo. The two beams \( K_S \) and \( K_L \) are produced is such a way that they have the same production spectra. The weighting procedure (section 5.3, page 64) minimises the effect of the different \( K_S \) and \( K_L \) lifetimes. These two effects should, in principle, cancel in the double ratio.

The acceptance must also take into account the difference between the \( \pi^0\pi^0 \) and \( \pi^+\pi^- \) decay modes and the fact the the two beam are not completely co-linear.

50 000 000 events of each of the four \( \xi \) modes were generated using the NMC Monte Carlo. The generated energy and \( Z \) decay vertex ranges were; \( 50 \leq E_{\text{kaon}} \leq 190 \) GeV and \( 200 \leq Z \leq 4800 \) cm. For this amount of generated events the input double ratio is one \( (\xi = \text{exactly } 0) \). The Monte Carlo is fully simulated and reconstructed, the output is then passed through the analysis program and \( \xi \) fitting routine. The output value of \( \xi \) is thus the size of the acceptance correction.

This method has the advantage of treating the Monte Carlo as if it were real data. Thus \( \pi \rightarrow \mu\nu \) decay, Dalitz decays of the \( \pi^0 \) and conversions are treated exactly as in the data. The only cut on true (as distinct from reconstructed) quantities is to require that all \( K_S \) decay after the \( AKS \), which is equivalent to requiring a fully efficient \( AKS \) veto.
The percentage acceptance for each mode was: $K_L \rightarrow \pi^0\pi^0$ 21.7%, $K_S \rightarrow \pi^0\pi^0$ 21.4%, $K_L \rightarrow \pi^+\pi^-$ 42.8% and $K_S \rightarrow \pi^+\pi^-$ 41.9%.\(^1\)

The size of the acceptance double ratio was measured to be $0.9963 \pm 0.0012$, which implies a correction to $\xi$ of $(0.62 \pm 0.2) \times 10^{-3}$. Figure 5.6 shows the acceptance double ratio correction as a function of momentum. The acceptance correction was applied on a bin by bin basis.

\(^1\) The numbers presented here are for un-weighted events.
6. EVENT SELECTION AND EVALUATION OF BACKGROUNDS

The main physics backgrounds to each of the four decay modes are given below (Table 6.1.) This chapter will explain the various cuts made to minimise backgrounds from each of the decay modes and the techniques used for evaluation of the background distributions.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Backgrounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_S \to \pi^+\pi^-$</td>
<td>$\Lambda \to p\pi$</td>
</tr>
<tr>
<td>$K_L \to \pi^+\pi^-$</td>
<td>$K_L \to \pi\mu\nu_\mu$</td>
</tr>
<tr>
<td></td>
<td>$K_L \to \pi e\nu_e$</td>
</tr>
<tr>
<td></td>
<td>$K_L \to \pi^+\pi^-\pi^0$</td>
</tr>
<tr>
<td>$K_S \to \pi^0\pi^0$</td>
<td>No significant background</td>
</tr>
<tr>
<td>$K_L \to \pi^0\pi^0$</td>
<td>$K_L \to \pi^0\pi^0\pi^0$</td>
</tr>
</tbody>
</table>

*Tab. 6.1: The Main Backgrounds to the Four $e^e$ Modes*
6. Event Selection and Evaluation of Backgrounds

6.1 Neutral Event Selection

To select good neutral events from data a preliminary event selection was made. All events were required to have the neutral trigger $\xi_e$ bit set. Since a good $2\pi^0$ event may have accidental activity associated with it "four photon combinations" are formed in each event. For example an event with five clusters would have $^5C_4 = 5$ possible four photon combinations. A maximum of twelve good clusters are allowed in an event. For a cluster to be considered good it must fulfil the following requirements:

1. Each cluster must have an energy greater than 3 GeV and less than 100 GeV. 3 GeV is the lower limit for which the LKR calibration is valid, below this energy the detection of clusters may not be fully efficient. Above 100 GeV saturation effects in cells may occur.

2. The centres of the clusters must hit the active area of the LKR. Radial position of the cluster $\geq 15$ cm, at least 4 cm away from the centre of the dead column and at least 11 cm from the edge of the active detector area.

3. A list of additional dead cells in the calorimeter was prepared on a run-by-run basis. The cluster position must not be within 2 cm of any of these dead cells. The number of dead cells in the calorimeter was $\approx 40$ out of a total number of $\approx 13300$.

Once good clusters are found they can be formed into four photon combinations. A good four photon combination must have:

1. The energy and lifetime of the kaons within the accepted range $70 \leq E_{\text{kaon}} \leq 170$, $\tau \leq 3.5 \tau_s$ and $\tau \geq 0.0\tau_s$ for $K_L$ decays only.
2. The event must not be in-time with a hit in the AKS.\textsuperscript{1} This cut is to remove \(K_S\) decays before the start of the fiducial region. Its importance is explained fully in section 5.2.

3. The centre of each cluster must be separated from all other cluster centres in the four photon combination by at least 10 cm to avoid reconstruction problems. The main problem is mis-measurement of the cluster energies. This is due to poorly apportioning the energy in a cell between two, or more, clusters.

4. The centre of gravity of the 4 photons must be less than 10 cm (see section 5.2, page 64.) This removes events that scrape the collimators and helps reduce background from \(K_L \rightarrow \pi^0\pi^0\pi^0\) decays.

5. To reject accidentals all of the photons must be within 5 ns of the average time. Any additional photons within this time window must have energy \(\leq 1.5\) GeV. Events with overflows in the drift chamber readout are also rejected.

### 6.2 Background in the Neutral Modes.

The \(K_S \rightarrow \pi^0\pi^0\) decay modes is the cleanest of the four \(\ell^+\ell^-\) decay modes, having no physics background, although decays from \(K_L \rightarrow \pi^0\pi^0\) which are mis-tagged as \(K_S\) decays do produce a noticeable effect, this is discussed in section 7.1.2, page 100.

For the \(K_L \rightarrow \pi^0\pi^0\) mode the background comes from \(K_L \rightarrow \pi^0\pi^0\pi^0\) with two lost photons. There are two ways in which a decay can lose two photons. One is when both photons from a \(\pi^0\) are outwith the active detector area. These decays are

\textsuperscript{1} The event time is the average of the four cluster times.
likely to have low momentum and high $Z$ vertex and are removed by the lifetime cut. A $K_L \rightarrow \pi^0\pi^0\pi^0$ decay can also lose two photons from different $\pi^0$s. To remove these events a test, similar to a chi-squared test, is made as to whether the four remaining photons appear to come from a $\pi^0\pi^0$ pair.

Recall the equation used for calculating the $Z$ vertex of a neutral decay (equation 5.3):

$$Z_{\text{Decay}} = Z_{LKR} - \left[ \frac{1}{M_K^2} \sum_{i,j(i<j)}^4 E_i E_j R_{ij}^2 \right]^{\frac{1}{2}} \tag{6.1}$$

The lifetime of the neutral pions is so short ($(8.4 \pm 0.6) \times 10^{-17}$ s) [11] that the decay vertex of the kaons is equivalent to that of the pions. Thus using the kaon mass constraint one can invert equation 5.3 and calculate the mass of the pion candidate:

$$M_\pi = \left[ \frac{E_{\gamma_1} E_{\gamma_2} R_{12}^2}{Z_{LKR} - Z_{\text{Decay}}} \right]^{\frac{1}{2}} \tag{6.2}$$

Where $\gamma_1$ and $\gamma_2$ are the two photons for this pion candidate. For a set of four clusters in the calorimeter two pion masses have to be calculated, therefore there are three possible combinations of photons to form pions. The two pion masses are correlated, due to the kaon mass constraint, therefore selection of the best combination is not done by using the calculated pion mass directly. Instead a pseudo chi-squared variable ($R_{\text{Ellipse}}$), on the sum and difference of the two pion masses, is constructed.

$$R_{\text{Ellipse}} = \left( \frac{M_{12} + M_{34} - 2M_\pi}{3\sigma^+(E_{M_{\pi^0}})} \right)^2 + \left( \frac{M_{12} - M_{34}}{3\sigma^-(E_{M_{\pi^0}})} \right)^2 \tag{6.3}$$

Here $\sigma^+$ and $\sigma^-$ are the resolution functions of the masses added ($M_{12} + M_{34}$) and masses subtracted ($M_{12} - M_{34}$) respectively. Since the resolution of the calorimeter is energy dependent both $\sigma^+$ and $\sigma^-$ are functions of the energy of the lowest
6. Event Selection and Evaluation of Backgrounds

Fig. 6.1: The Masses of the Pion Candidates.

energy cluster. The approximate values for $\sigma^+$ and $\sigma^-$ are 0.9 MeV and 2.1 MeV respectively (integrated over the whole energy range.)

The best pion combination is that with the smallest $R_{\text{Ellipse}}$. Events with the smallest $R_{\text{Ellipse}} \geq 1.5$ are rejected. This cut removes background from the $K_L \rightarrow \pi^0 \pi^0 \pi^0$ decays. Cutting on $R_{\text{Ellipse}}$ is equivalent to cutting out an ellipse on the $M_{12} M_{34}$ surface, with the centre of the ellipse at $M_{\pi^0} M_{\pi^0}$. This is shown graphically as Figure 6.1. The cut level of $R_{\text{Ellipse}} \leq 1.5$ is selected to keep the loss of $2\pi^0$ events to $\leq 5\%$ whilst maintaining a low level of background. The $R_{\text{Ellipse}}$ variable will be used to make an estimate of the background from $K_L \rightarrow \pi^0 \pi^0 \pi^0$ decays. The probability that the combination with the lowest $R_{\text{Ellipse}}$ value is not the correct combination was found, by using Monte Carlo, to be less than $10^{-4}$. 
Fig. 6.2: $R_{\text{Ellipse}}$ Distributions. $K_S$ distribution shown as crosses. $K_L$ as solid line (from data).

6.2.1 A Measurement of the Background from $K_L \rightarrow \pi^0\pi^0\pi^0$ Under $K_L \rightarrow \pi^0\pi^0$ decays.

The $3\pi^0$ background was measured by subtracting normalised $K_S \rightarrow \pi^0\pi^0$ from $K_L \rightarrow \pi^0\pi^0$ to expose the background $R_{\text{Ellipse}}$ distribution. The $R_{\text{Ellipse}}$ distributions for $K_L$ and $K_S$ decays are shown as Figure 6.2.

There are large tails seen in the $R_{\text{Ellipse}}$ distribution, both for $K_S$ and $K_L$. Tails with an $R_{\text{Ellipse}}$ value above 5, are understood to be due to gamma conversions and Dalitz decays before the magnet. The tails between $R_{\text{Ellipse}} = 2 - 5$ were studied using a GEANT based Monte Carlo and found to come from hadronic shower
fluctuations[36]. These tails are the same for $K_S \rightarrow \pi^0\pi^0$ and $K_L \rightarrow \pi^0\pi^0$, any difference between the two modes will be taken into account via the background subtraction procedure.

The background subtraction is performed in the following manner:

1. The data set is passed through the neutral event selection (see above), apart from the cut on $R_{\text{Ellipse}}$ which is released.

2. The decision of whether the decay is $K_S$ or $K_L$ is made on the basis of information from the tagger and LKR. If the difference between the tagger and LKR times is less than $\pm 2$ ns the decay is identified as a $K_S$ decay. Decays outwith this time window are declared to be $K_L$ decays.

3. The sample of $K_S \rightarrow \pi^0\pi^0$ decays contains a certain number of mis-tagged $K_L \rightarrow \pi^0\pi^0$ decays, due to accidental protons in the tagger. This fraction, known as “$K_L - K_S$ dilution”, can be measured (see section 7.1 page 95) and is independent of $R_{\text{Ellipse}}$. The number of $K_L$ decays in the $K_S$ distribution is subtracted as a function of $R_{\text{Ellipse}}$ to produce a “pure $K_S$” distribution.

4. The two distributions are normalised such that the number of events is the same in the first bin ($R_{\text{Ellipse}} 0.0 - 0.25$). This is shown as Figure 6.2. The two distributions are then subtracted to give the form of the background distribution. The background distribution is given as figure 6.3.

5. The number of events in the control region $445 R_{\text{Ellipse}}$ is counted. This is then extrapolated into the signal region to give the total amount of background. The ratio between the number of $K_L \rightarrow \pi^0\pi^0\pi^0$ events in the signal and control region is found from the Monte Carlo to be $= 1.22 \pm 0.21$.

This method of evaluating the size of the background makes several assumptions:
Fig. 6.3: The Background from $K_L \rightarrow \pi^0\pi^0\pi^0$ decays as a function of $R_{\text{Ellipse}}$. This is found by subtracting the normalised $K_S\ R_{\text{Ellipse}}$ distribution from the $K_L$ distribution (both $R_{\text{Ellipse}}$ distributions from data.)

1) That there is no background in the $K_S \rightarrow \pi^0\pi^0$ sample. 2) The tails in the reconstruction of $2\pi^0$ decays are the same $K_S$ and $K_L$. 3) That the difference between the two distributions is correctly assigned as $K_L \rightarrow \pi^0\pi^0\pi^0$.

The weighting procedure also has an effect on the number of background events. $K_L \rightarrow \pi^0\pi^0\pi^0$ decays are likely to have a high $Z$ vertex and thus a lower weight in the $K_L$ distribution. The background evaluation is therefore done on a weighted sample. The background was measured to be $(9\pm2) \times 10^{-4}$. The majority of the error comes from the low Monte Carlo statistics. This level of error is acceptable for the 97 data set but improved statistics is needed for the 98 analysis.

The background subtraction is done as a function of momentum. The momentum spectrum of the $K_L \rightarrow \pi^0\pi^0\pi^0$ is taken from events that passed the Monte Carlo acceptance cuts. This momentum spectrum is the same as that of the NMC $K_L \rightarrow \pi^0\pi^0\pi^0$ passing all cuts, but has approximately 100 times more statistics.
6.3 Preliminary Selection of Charged Events

Events must have the Massbox $\mathcal{E}$ trigger and no in-time muons in the Muon Veto. This is the definition of a "charged candidate event". Since events with more than two tracks are allowed, events can have multiple vertices - one for each pair of positive and negative tracks. A good vertex must have the tracks within 6 ns of each other and a closest distance of approach of less than 2 cm, to reduce the effect of accidental tracks. For each good vertex the following cuts are applied:

- Tracks must pass through the active area of the following: DCH1, DCH 2, DCH4 and the muon veto.
- Tracks must hit the active area of the LKR, as defined in Section 6.1. This is to lower the amount of background coming from $K_{e3}$ decays.
- The tracks must pass the asymmetry cut:

\[
\frac{|E_1 - E_2|}{E_{\text{kaon}}} \leq 0.62
\]
\[
\frac{|E_1 - E_2|}{E_{\text{kaon}}} < 1.08 - 0.0052E_{\text{kaon}}
\]

- The energy and lifetime of the kaons must be within the accepted range i.e. $70 \leq E_{\text{kaon}} \leq 150$, $\tau \leq 3.5\tau_s$ and $\tau \geq 0.0\tau_s$ for $K_L$ decays only.
- The vertex must not be in-time with a hit in the AKS.
- The Centre of Gravity of the vertex must be less than 10 cm.
- The invariant mass of the assumed $K^0 \rightarrow \pi^+\pi^-$ decay must be within $3\sigma$ (7.5 MeV/c^2) of the kaon mass ($M_{K^0} = 497.6$ MeV/c^2). This cut removes background from the three body decays of $K_L$. 
6. Event Selection and Evaluation of Backgrounds

6.4 Background to $K_S \rightarrow \pi^+\pi^-$ Decays

The physics background for the $K_S \rightarrow \pi^+\pi^-$ decay comes from the decay of a lambda or anti-lambda into a proton and a pion. The lifetime of the $\Lambda$ is approximately three times longer than that of the $K_S$, therefore lambdas form a background in $K_S$ but not in $K_L$ decays. In a decay where the proton and lambda are co-linear and the proton mis-identified as a pion the invariant mass of the two tracks falls within the kaon mass window.

It is possible to identify these decays as lambda decays using a different method. A proton co-linear with its parent lambda will carry off most of the lambda's
momentum. Thus the momentum of the two charged particles, proton and pion, will be very different, and the decay will have a large asymmetry. Figure 6.4 shows this graphically. The band corresponding to the $K_S \rightarrow \pi^+\pi^-$ decay can be clearly seen as well as the curve of the lambda decays. The asymmetry cut has a maximum value of 0.62 and therefore removes the majority of lambdas decays (see also section 5.4, page 65.)

The size of any remaining background was estimated using the lambda mass. By assuming the highest momentum track is a proton and the other track is a pion any remaining background from $\Lambda \rightarrow p\pi$ should form a peak at the lambda mass. Out of 5,926,748 charged $K_S$ and $K_L$ decays none were found to be within $5\sigma$ of the lambda mass. Therefore the background from $\Lambda \rightarrow p\pi$ decays is negligible.

6.5 $K_L \rightarrow \pi^+\pi^-\pi^0$ Decays.

$K_L \rightarrow \pi^+\pi^-\pi^0$ decays are treated as charged decays that have missing mass, due to the energy taken by the $\pi^0$. These decays should therefore be removed by the mass cut. A sample of 1 million $\pi^+\pi^-\pi^0$ in the kinematic range; $50 \text{ GeV} \leq E_{\text{kaon}} \leq 250 \text{ GeV}$ and $0 \text{ cm} \leq Z \leq 5600 \text{ cm}$ was generated using the NMC Monte Carlo. None of these events passed the mass cut, the closest event to the mass window was 30 resolution widths from the kaon mass (Figure 6.5.) The conclusion is that background from $K_L \rightarrow \pi^+\pi^-\pi^0$ decays is negligible for this analysis.

6.6 The Semi-leptonic Decays of $K_L$

These are the decays $K_L \rightarrow \pi\nu_e$ (known as $K_{e3}$) and $K_L \rightarrow \pi\mu\nu_\mu$, called $K_{\mu3}$. The decays have “missing momentum” due to the neutrino and therefore have a
Fig. 6.5: The Mass Distribution for $K_L \rightarrow \pi^+\pi^-\pi^0$ (Monte Carlo Simulation).

A wide mass distribution, part of which is within the kaon mass window. There are two way of reducing this background: 1) To identify either the muon or electron in the detector and thus veto the decay. 2) To make kinematic cuts on the missing momentum of the decay.

### 6.6.1 Muon and Electron Identification

Muon rejection is done at the event selection stage where all events with an in-time muon are rejected. Tracks are projected forward into the muon veto and associated with hits in planes 1, 2 and 3. A positive association is when the hit
position of the muon is within the maximum scattering angle for the track and the
time of the muon hit is within 4 ns of the event time from the charged Hodoscope.
All these events are then rejected [37]. This cut also rejects $K_S, K_L \rightarrow \pi^+\pi^-$
decays where the pion has decayed into a muon. This effect is dealt with as part
of the acceptance studies.

The liquid krypton calorimeter is used to identify electrons. Electrons will interact
in the same way as photons, depositing all of their energy within the 27 radiation
lengths of the LKR. The depth of the LKR is only two hadronic interaction lengths,
therefore pions will be less likely to shower within the calorimeter. Pion showers
are wider and more fragmented, thus a showering pion is more likely to generate
lower energy clusters compared with an electron of the same momentum.

Tracks are associated with the nearest cluster within a 6 cm radius around the
track impact point. If a track has an associated cluster and the ratio, $E_{\text{cluster}}/P_{\text{track}}$
is greater than 0.8 the track is identified as an electron and the event rejected.
This rejection method relies on the cluster energy in the LKR being well measured,
therefore it is more difficult to reject events close to dead regions of the calorimeter.

This cut is applied to both $K_L$ and $K_S$ decays to ensure, in principle, that there is
no correction on the double ratio due to pion rejection. The cut around the dead
column removes 15% of good $K_S \rightarrow \pi^+\pi^-$ decays.

6.6.2 Kinematic Rejection of Semi-leptonic Decays

To reject semi-leptonic decays kinematically a variable that measures the differ-
ence between the line of flight of the kaon and sum of the visible momentum is
used. If the decay is two body, the direction of the visible momentum will be
parallel with the line between the relevant target and the kaon decay vertex. The
KL and KS beams have different beam axes and divergences so to avoid biasing the background rejection this must taken into account.

Several background rejection variables were investigated and $P_{\text{vtx}}^2$ (a pseudo transverse momentum) chosen as being the best. The $P_{\text{vtx}}^2$ distribution is the same for KS and KL decays and has a high level of background rejection.

$P_{\text{vtx}}^2$ is defined as (figure 6.6):

$$P_{\text{vtx}}^2 = (P_1 + P_2)^2 \Theta^2$$

Where $\Theta$ is the angle between the sum of the momenta of tracks one and two, $(P_1 + P_2)$, and the line of flight of the kaon from the centre of target to a given $Z$ plane, in this case DCH1.

To calculate $P_{\text{vtx}}^2$ one must define whether the decay is KL or KS and select the
appropriate target position. Using the tagger to define whether the decay is $K_S$ or $K_L$ would mean that a certain percentage of true $K_L$ events would be misidentified as $K_S$ decays with a high $P^2_{vtx}$. Cutting on $P^2_{vtx}$ would therefore alter the $K_L - K_S$ dilution for the charged mode, which one wishes to avoid.

The $Y$ vertex position is used instead to define the decay type. Figure 6.7 shows the $XY$ and $YZ$ vertex distributions for $K_S$ and $K_L$ the separation of the two beams can be clearly seen. A $K_S$ decay has $Y > 3$ cm. The excellent collimation of the beams ensures a very high purity for this selection method, in a pure $K_S$ run only 1 out of 44460 ($2 \times 10^{-5}$) decays failed this cut. The amount of $K_L$ decays with $Y$ vertex $\geq 3$ cm was estimated to be $3 \times 10^{-5}$.

![Fig. 6.7: The Vertex Distributions for Charged Decays](image-url)
The $P_{\text{utx}}^2$ distributions are very similar for $K_S \to \pi^+\pi^-$ and $K_L \to \pi^+\pi^-$, so one can cut on $P_{\text{utx}}^2$ and then make the $K_L - K_S$ decision using the tagger. This method does not effect the size of the dilution factor. The $P_{\text{utx}}^2$ cut is $P_{\text{utx}}^2 \leq 200 \text{ (MeV/c)}^2$. This value is selected as it rejects only 2 per mill of $K_S \to \pi^+\pi^-$ events.

**Extrapolation of $K_{\mu3}$ and $K_{e3}$ background**

To obtain samples of $K_{\mu3}$ and $K_{e3}$ a data split was produced where the events satisfied the $K_L \to \pi^+\pi^-$ cuts, apart from the $P_{\text{utx}}^2$ and mass cuts. $K_{e3}$ events were selected by requiring that one track has $E_{\text{cluster}}/P_{\text{track}} \geq 0.95$ (a definite electron) and the other $E_{\text{cluster}}/P_{\text{track}} \leq 0.8$ (a definite pion). $K_{\mu3}$ events where selected with one and only one muon in time with a track.

A MINUIT program calling HMCLL [38] was used to extrapolate the semi-leptonic background under the $P_{\text{utx}}^2$ vs $M_K$ mass distribution. HMCLL uses a log likelihood fit [39] to fit N known distributions to a composite. In this case the known distributions are the backgrounds $K_{\mu3}$, $K_{e3}$ and the signal (given by $K_S \to \pi^+\pi^-$). The composite distribution to be fitted is $K_L \to \pi^+\pi^-$, without the $P_{\text{utx}}^2$ or $M_K$ cuts. The signal region was selected as; $P_{\text{utx}}^2 \leq 200 \text{ MeV}^2/\text{c}^2$ and $489.5 \leq M_K \leq 505.5 \text{ MeV}/\text{c}^2$, the amount of $K_L \to \pi^+\pi^-$ in the fitted region was fixed according to the measured amount of $K_S \to \pi^+\pi^-$ in this region. The amounts of each type of background were allowed to float freely.

A small fraction of $K_L \to \pi^+\pi^-$ (approximately 600 events in the whole data set) were found to have the correct mass and high $P_{\text{utx}}^2$, in excess of the background. This excess is seen only in $K_L$ decays and not in $K_S \to \pi^+\pi^-$. Neils Doble studied both neutron regeneration in the detector and scraping on the $K_L$ collimators were considered as possible sources of the background, they were found to be an order of magnitude too small to explain the mass excess[40]. The high $P_{\text{utx}}^2$ events did
not appear to come from any particular region of the detector, as would be the case in regeneration or scattering [41]. To avoid including these events in the fit, and thus over estimating the background level, the control region was selected to contain only the mass sidebands (from 478 to 489.5 and 505.5 to 518 MeV/c²).

The number and type of backgrounds for different control regions are shown in table 6.2. Figure 6.8 shows a typical fit result.

<table>
<thead>
<tr>
<th>Pt (MeV/c)²</th>
<th>$K_{e3} (\times 10^{-3})$</th>
<th>$K_{\mu3} (\times 10^{-3})$</th>
<th>Total (\times 10^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>400 - 1000</td>
<td>1.99 ± 0.13</td>
<td>0.59 ± 0.29</td>
<td>2.58 ± 0.4</td>
</tr>
<tr>
<td>400 - 1200</td>
<td>2.09 ± 0.13</td>
<td>0.45 ± 0.26</td>
<td>2.55 ± 0.3</td>
</tr>
<tr>
<td>400 - 1400</td>
<td>2.15 ± 0.15</td>
<td>0.37 ± 0.29</td>
<td>2.52 ± 0.4</td>
</tr>
<tr>
<td>400 - 1600</td>
<td>2.20 ± 0.14</td>
<td>0.29 ± 0.28</td>
<td>2.49 ± 0.3</td>
</tr>
<tr>
<td>500 - 1400</td>
<td>2.19 ± 0.17</td>
<td>0.26 ± 0.27</td>
<td>2.46 ± 0.4</td>
</tr>
<tr>
<td>600 - 1400</td>
<td>2.20 ± 0.20</td>
<td>0.23 ± 0.53</td>
<td>2.42 ± 0.6</td>
</tr>
</tbody>
</table>

*Tab. 6.2: The Charged Background*

The amount of background under the signal region was taken to be 2.5 × 10⁻³, made up of 2.1 × 10⁻³ $K_{e3}$ and 0.4 × 10⁻³ $K_{\mu3}$. An upper bound on the systematic error from this fit comes from the spread of the background values. This is 0.16 × 10⁻³, which is an acceptable systematic error for the purposes of this analysis.
Fig. 6.8: $P_{vtx}^2$ Projection of Charged Background Fit. All the distributions shown come from data samples.
7. REMAINING EFFECTS ON THE DOUBLE RATIO

This chapter will deal with effects specific to the NA48 experiment. These are; the tagging method, the energy scale, charged and neutral trigger efficiencies and the effect of accidental particles.

7.1 Tagging

Tagging is used to distinguish between decays originating in the $K_S$ target and those coming from the $K_L$ beam. Only protons in the $K_S$ beamline cause hits in the tagger. The time of these hits is then combined to form time(s) for the proton(s) associated with this event.

To determine whether the decay is in-time with a proton, the event time from the detector is used. For charged decays the event time comes from the Charged Hodoscope and for neutral decays it comes from the LKR. The time of the nearest proton to the event time is calculated, taking into account an offset for the time of flight between the tagger and the detector. If the time difference is within $\pm 2\text{ns}$ the event is tagged as $K_S$, otherwise the event is assumed to be $K_L$. Figure 7.1 shows the tagger time distribution for charged decays, together with the tagger time for $K_S$ and $K_L$ decays.

The number of protons passing through the tagger per burst is $\approx 3 \times 10^7$ and the rate of kaons exciting the final collimators in the two beamlines are $\approx 1.5 \times 10^7$.
Fig. 7.1: Tagging Time Distributions (from data).
in the $K_L$ beamline and $\approx 3 \times 10^2$ in the $K_S$ beamline [42]. Thus the rate of protons in the tagger is $\approx 10^5$ times greater than the number of $K_S$ decays in the fiducial volume and roughly twice that of the number of $K_L$ decays. Therefore the likelihood of a $K_L$ decaying in-time with a proton and being mis-tagged as a $K_S$ decay is high (roughly 11.2%, see section 7.1.2). This effect is called the $K_L - K_S$ dilution and is denoted by $\alpha$ on Figure 7.2. The tagger also has a slight (of order $10^{-4}$, see section 7.1.1) inefficiency denoted by $\xi$. These effects must be measured both in the charged and neutral modes. In principal, $\alpha$ and $\xi$ should be the same for charged and neutral modes, but since the event time is measured by different subdetectors there will be subtle differences between the two modes.

It is possible to relate the number of true $K_S$ and $K_L$ decays ($T_S, T_L$) to the
number of tagged decays \((N_S, N_L)\) using the two formulae below:

\[
N_S(\text{unweighted}) = (1 - \xi)T_S(\text{unweighted}) + \alpha T_L(\text{unweighted}) \quad (7.1)
\]

\[
N_L(\text{weighted}) = (1 - \alpha)T_L(\text{weighted}) + \xi T_S(\text{weighted}) \quad (7.2)
\]

### 7.1.1 The Tagging Efficiency.

#### Charged Decays

Measuring the tagging efficiency in the charged mode is trivial. One selects good \(K_S \to \pi^+\pi^-\) decays \((Y \text{ vertex} \geq 3.0 \text{ cm})\) and counts the number of these decays which are tagged as \(K_L\). Out of 2,289,578 \(K_S\) decays 332 were not tagged. This gives a tagging inefficiency of \((1.45 \pm 0.08) \times 10^{-4}\).

#### Neutral Decays

The tagging efficiency for neutral decays is more difficult to measure because there is no \(X Y\) vertex information. It is possible to measure the tagging efficiency using a \(K_S\) only beam, but this does not mimic the conditions of the mixed beam runs, especially accidental effects. The timing information from the \(LKR\) in charged \(\pi^+\pi^-\) events cannot be used, due to the large differences in shower development between photons and pions, which could induce differences in the \(LKR\) timing information.

Using the Dalitz decay \((\pi^0 \to e^+e^\gamma)\) of the \(K_S \to \pi^0\pi^0\) a measurement of the neutral tagging efficiency can be made throughout the run. The production of the electron - positron pair provides vertex information, whilst the shower shapes of electrons and photons are sufficiently similar to allow the \(LKR\) timing information to be used. The branching ratio \((BR = 1.198 \pm 0.032\%)\)\(^\text{[11]}\) and acceptance
(\( A = 4.65 \pm 0.3\% \))[43] for the Dalitz decay are both small, so this method is limited by statistics.

The neutral tagging efficiency was measured by D. Vattalo [44] using the method detailed above. She found 1 mis-tagged event out of 6823 events in the Dalitz sample. Taking into account the low statistics and possibility of backgrounds in the Dalitz samples, this gives a tagging inefficiency of \((1.44 \pm 0.6) \times 10^{-4}\).
7.1.2 The Dilution Factor

The dilution factor is the fraction of $K_L$ decays which are mis-tagged as $K_S$ decays, due to an accidental proton in the $\pm 2$ ns tagging coincidence window. The dilution factor must be applied as a correction to the double ratio. If the true value of $\xi^c$ is zero an artificial non-zero value of $\xi^c$ can only be generated if the measured dilutions are different charged and neutral.

Measuring the dilution factor for charged decays is simple, one counts the percentage of decays which have a $Y$ vertex positions compatible with a $K_L$ decay ($Y \leq 3$cm) but are also tagged as $K_S$. The charged dilution was measured to be $\alpha_{+-} = 11.21 \pm 0.03\%$.

In the neutral mode one does not have access to the $Y$ position of the kaon decay, so the neutral dilution cannot be measured directly. Instead the rate of protons on the tagger is compared for good neutral and charged $K_L$ events.

An event tagged as $K_L$ may contain protons outwith the $\pm 2$ ns tagging window. This probability of an event being tagged as $K_L$ and having an accidental proton in a different 4 ns time window can be measured independently for both charged and neutral decays. Figure 7.3 shows the probability of finding an accidental proton as a function of time away from the tagging window. The alternating structure of this distribution is due to the changing number of protons supplied by the SPS machine, which has a 200 MHz cycle.

This “accidental proton probability” (also denoted as APP) is different for charged and neutral due to the inefficiency of the charged trigger (in effect there is a difference in intensity between the charged and neutral modes). The difference between the two probabilities is directly proportional to the difference in the charged and neutral dilutions. The charged dilution can be measured directly,
this together with the difference in APP distributions, means that the neutral dilution can be calculated.

The two time windows either side of the tagging window have a low accidental proton probability, they are affected by the requirement that there be no proton within the tagging window. Time windows with $T_{\text{event}} - T_{\text{tag}} < -30\text{ns}$ and $T_{\text{event}} - T_{\text{tag}} > 15\text{ns}$ are too close to the edges of the event readout window. So the accidental proton probability was measured using the four time windows between $-25 < T_{\text{event}} - T_{\text{tag}} < -10\text{ns}$.

The accidental proton probability was found to be $10.68 \pm 0.02\%$ for charged and
10.75 ± 0.03% for neutral decays. The difference is therefore \((7 \pm 3.6) \times 10^{-4}\).

The charged dilution was measured to be 11.21 ± 0.03% and therefore the neutral dilution is calculated to be; Neutral = Charged + Difference = 11.21% + 0.07% = 11.28 ± 0.05%. The dilution effect is the largest correction to the double ratio, applying the dilution correction (with \(\alpha_{+-} = \alpha_{00} = 11.21\%\)) changes the double ratio by \(+60.5 \times 10^{-4}\). The difference between the two dilutions give an additional correction of \(+5.4 \times 10^{-4}\).

The \(K_S\) mode must have the background from \(K_L\) mis-tags removed as a function of \(p\) and \(\tau\). Therefore the dilution factor must either be proved to be independent of momentum and tau or must be applied as a function of \(p, \tau\). Figure 7.4 shows the accidental proton probability as a function of these two variables.

### 7.2 The Energy Scale

To measure \(\frac{\xi}{\epsilon}\) one must ensure that the correct number of decays fall within a given window \((p, \tau)\) as defined by cuts. To do this the energy scales for charged and neutral decays must be the same. *i.e.* for a given true energy \(E_{\text{TRUE}}\), the energy reconstructed by the spectrometer \((E_{\text{MEAS}\,+-})\) and the LKR \((E_{\text{MEAS}\,00})\) must be equivalent. Global energy scale corrections are applied according to the following formula.

\[
E_{\text{TRUE}} = E_{\text{MEAS}\,+-} C_{+-} = E_{\text{MEAS}\,00} C_{00} \tag{7.3}
\]

Where \(C_{+-}\) and \(C_{00}\) are the global energy scale corrections.

To measure, and correct if necessary, any global difference between the two energy scales one measures the \(Z\) position of the \(\text{AKS}\). The result of the measurement
can be related to the energy scale via the following formula:

\[
\text{energy scale} = \frac{Z_{lkr} - Z_{meas}}{Z_{lkr} - Z_{aks}} \quad (7.4)
\]

Here \(Z_{lkr}\) is the \(Z\) position of the LKR (12110.0 cm) \(Z_{meas}\) is the measured position of the AKS as obtained from a fitting procedure as detailed below and \(Z_{aks}\) is the real AKS position. The AKS detector is described in detail in section 3.2.4, page 28.

![Fig. 7.5: Z Vertex Distribution for K_S → π^0 π^0 Around the AKS.](image)

Fitting the \(Z\) vertex distribution of \(K_S\) decays will give the AKS position which must necessarily be different by 2.1 cm between charged and neutral decays. The
fit is made up of three components; an exponential decay of the $K_S$, a Gaussian (to simulate the resolution of the reconstruction around $Z = Z_{aks}$) and a flat part which is the contribution from the mis-tagged $K_L$ decays.

Figure 7.5 shows the $Z$ vertex distribution and fit result for neutral decays. The parameter $P_3$ is the difference, in cm, between the true AKS position and the result of the fit, $P_4$ is the neutral $Z$ vertex resolution (assuming the resolution function is a Gaussian). The AKS fit for charged decays is not shown.

The global energy scale corrections are; $1.00048 \pm 0.00002$ (negligible) for charged decays and $1.0022 \pm 0.0006$ for neutral decays. To correct the neutral data the $Z$ vertex and energy of the LKR clusters in neutral decays were multiplied by this correction factor, resulting in a correct $Z$ vertex measurement.

A second energy scale effect is non-linearities where the correction factors $C_{00}$ are different for different kaon energies. For example, suppose one added 100 MeV energy to a kaon of 70 GeV and kaon of 170 GeV. The resulting change in energy is one per mill at 70 GeV and 0.5 per mill at 170 GeV.

The effect of possible non-linearities was studied by I. Mikulec and G. Unal [45] additional energy corrections were applied to neutral events such that the result of the fit of the AKS position was flat in the energy range $70-170$ GeV. The change to double ratio was found to be a maximum of $1.5 \times 10^{-3}$. This variation of AKS position was found to be strongly correlated with the position of photons in the calorimeter. Additional “eta” calibration factors (from the analysis of $\eta \to \gamma \gamma$ and $\eta \to \pi^0 \pi^0 \pi^0$ decays) were applied to compensate for this. Thus the change on $R$ of $1.5 \times 10^{-3}$ is an upper limit on this systematic effect. Figure 7.6 shows the variation of the AKS position as a function of energy, after the global energy scale correction has been applied, but before applying the eta calibration factors. The large non-linearities at high energy are due to an non uniformity of the individual
Fig. 7.6: The Difference Between the Fitted (data) and Known AKS Position as a Function of Energy. The plot shows the difference in cm between the fitted ($Z_{FIT}$) and known ($Z_{AKS}$) positions, after the global energy scale correction has been applied.
cells over the calorimeter surface and a worse percentage error on the positions low radii (high energy) photons [46].

The etas were produced in a special run where a polyethene target 3.5 cm long and 5 cm in diameter was placed at the $Z$ position of the AKS, on the $K_L$ beamline. A beam of $\pi^-$ was directed onto this target producing etas which decay at the $Z$ position of the $CH_2$ target. With the $Z$ vertex of the decay and mass of the $\eta$ known precisely these constraints can be used to find the energies of the photons in the $LKR$ calorimeter. The energy scale of each calorimeter cell can then be corrected specifically.

7.3 Accidental effects.

At any given time the detector may contain random accidental activity as well as a “good” kaon decay. Such activity may include; noise in the detectors, scattering of particles from the collimators and activity in the beams.

During the 1997 run a “random” trigger was in operation. The rate of this trigger was proportional to the instantaneous intensity of the $K_S$ and $K_L$ beams, with the trigger fired randomly during the burst. This provides a un-biased sample of the activity in the detector. These events were overlayed on top of 10% of the normal data to assess what effect accidental activity would have on good $\xi^\prime$ events.

The accidentals cause two effects in the data:

1. Gains and losses of events that may move inside/outside of cuts. An example of this is when an accidental photon causes the $\pi^0$ masses to be mis-measured.

2. The additional noise and/or energy associated with these events will change
the measured energy of events and affect the weighting of \( K_L \) events. Overlaid events changing bins are strongly anti correlated and this must be taken into account when considering the effect of the accidentals.

The accidental correction was measured independently by G. Unal and B. Gorini\cite{36}, both found an accidental effect of a few times \( 10^{-4} \) which can be neglected in this analysis. However due to the low number of overlaid events the statistical error on this is \( 16 \times 10^{-4} \) and contributes a significant systematic to the double ratio.

### 7.3.1 Accidental Muons

Activity in the Muon Veto comes from one of three sources:

1. \( K_{\mu3} \) decay. The “no muon” condition is required to veto this decay.

2. \( \pi \rightarrow \mu \nu_\mu \) decay, where the charged pion has come from \( K_S \) or \( K_L \rightarrow \pi^+ \pi^- \) decay. The probability of charged pion decay is (once the \( K_L \) events have been weighted) independent of the beam type. Therefore the amount of \( K_S \) and \( K_L \rightarrow \pi^+ \pi^- \) decays which are rejected due to the pion(s) decaying should depend only of the acceptance of the muons in the muon veto.

3. Accidental muons is the beamline. The amount of accidental muons in the beamline was assessed by looking at \( K_S \), \( K_L \rightarrow \pi^0 \pi^0 \) decays and counting the number of muons\(^1\) in a 250ns readout window. The amount of \( K_L \rightarrow \pi^0 \pi^0 \) decays with an accidental muon was found to be \( 4.71 \pm 0.11\% \) for \( K_L \rightarrow \pi^0 \pi^0 \) decays and \( 4.78 \pm 0.06\% \) for \( K_S \rightarrow \pi^0 \pi^0 \) decays.

Since the effect of \( \pi \rightarrow \mu \nu_\mu \) decay is taken into account via the acceptance and the effect of accidental muons on the double ratio is symmetrical \( K_L, K_S \) to order

\(^1\) The reconstruction for this study did not require any tracks.
$10^{-4}$. The muon veto can be applied without causing any bias on $\frac{\epsilon}{\epsilon'}$.

### 7.4 DCH overflows

Particles may interact with material upstream of the drift chambers e.g. the beam pipe. These interactions can produce showers in the drift chambers hitting a large number of sense wires.

Drift Chamber overflows are caused when there are more than eight sense wires hit in a plane. The TDC card in the drift chamber is unable to cope with this amount of information and overwrites its buffers. At this point an overflow bit is set for this particular plane.

The redundancy in the charged reconstruction means that events with up to one overflow per view can still be reconstructed. However, the charged trigger and reconstruction efficiencies are affected by the drift chamber overflows (Section 7.5, page 109). The illumination of the drift chambers differs between $K_S$ and $K_L$ beams, thus the effect of the overflows will not cancel in the charged mode. To make sure that the deadtime introduced and that the effect of the overflows is symmetrical in all four modes, the overflow condition must be required in all four decay modes.

The overflow condition was defined as no overflow within $\pm 300\text{ns}$ of the event time. The percentage loss of events for each of the four decay modes is shown

---

2 The design of the drift chambers was discussed in Section 3.3, page 31
below;

\begin{align*}
K_S \rightarrow \pi^+ \pi^- & \quad 10.53 \pm 0.2\% \\
K_L \rightarrow \pi^+ \pi^- & \quad 11.10 \pm 0.3\% \\
K_S \rightarrow \pi^0 \pi^0 & \quad 20.00 \pm 0.4\% \\
K_L \rightarrow \pi^0 \pi^0 & \quad 20.93 \pm 0.6\%
\end{align*}

The percentage of overflows is far less in charged modes compared with neutral modes as good charged events must have the Massbox trigger bit set. The difference between $K_S$ and $K_L$ is thought to be due to the different illuminations of the two beams at the drift chambers and backgrounds in the $K_L$ beams. The amount of drift chamber overflows does not vary as a function of lifetime or energy in any of the four decay modes.

### 7.5 The Charged Trigger Efficiency

The charged trigger is made up of two parts: The level one trigger which is the coincidence of $Q_x/2$ and $E_{TOT}$. (see Section 4.2.1) and the level two trigger the Massbox, (Section 4.2.2).

Both parts of the level one trigger were found to have very small inefficiencies, the $Q_x$ inefficiencies are $K_S = (4.2 \pm 0.64) \times 10^{-3}$ and $K_L = (4.44 \pm 0.85) \times 10^{-3}$ [47] and the $E_{TOT}$ inefficiencies are $K_S = (0.22 \pm 0.08) \times 10^{-3}$ and $K_L = (0.84 \pm 0.23) \times 10^{-3}$ [48]. The major part of the charged trigger inefficiency comes from the Massbox.

To measure the Massbox efficiency events with the level one charged trigger bit set are selected. These event were then passed through the standard charged selection, and the number of events with the Massbox trigger bit counted. Events occurring directly after a drift chamber overflow have very high inefficiency, up to
80 %, due to missing hits in the drift chamber readout.

Additional sources of inefficiency which are not due to the Massbox itself, were identified. These inefficiencies were included in the efficiency calculation. They are \[49\];

- **Fine time edge effect.** The fine time signal (coming from the drift chambers) and the \(Q_{x}/2\) signal from the Charged Hodoscope cannot be perfectly synchronised. When the event time is near the edge of a clock pulse the fine time and \(Q_{x}/2\) signals maybe placed in different clock slices. This gives wrong timing information to the Massbox and causes an inefficiency.

- **No strobe.** This occurs when the Level One Trigger bit is set but no request is sent to the Massbox.

- **Watchdogs.** These are events where the Massbox response is too late for the Level Two Trigger Supervisor.

The main sources of inefficiency in the actual Massbox comes from two sources.

- **Chamber inefficiency.** The Massbox requires more information than the reconstruction program to make its decision. If a chamber is slightly inefficient the reconstruction may be able to recover an event which the Massbox cannot.

- **The Massbox algorithm.** The algorithm is limited, particularly by the number of hits in the chambers. If too many hits occur the Massbox will not be able to perform its computations and rejects the event.

The overall efficiency of the Massbox increases during the six week period of data taken in 1997, as problems were analysed and solved. However the \(K_{S} - K_{L}\)
difference is stable. A weighted average of the efficiency with time was calculated. No significant variation of efficiency with lifetime was found. Figure 7.7 shows the Massbox efficiency as a function of energy for $K_L$ and $K_S$. There is a possible energy dependence of the efficiency as a function of energy especially for $K_L$ decays. Because of this and the relatively low Massbox efficiency a correction for the Massbox efficiency is applied on a bin by bin basis.

![Fig. 7.7: The Massbox Efficiency](image_url)
The average efficiencies are;

\[
\begin{align*}
K_S \quad E_S &= 91.07 \pm 0.09\% \\
K_L \quad E_L &= 91.13 \pm 0.18\%
\end{align*}
\] (7.9) (7.10)

Applying this massbox efficiency correction changes the double ratio by \(15.6 \times 10^{-4}\).

### 7.6 The Neutral Trigger Efficiency

The Neutral Trigger efficiency was measured by G. Fischer [50]. He took Neutral Hodoscope minimum bias triggers and selected good \(2\pi^0\) candidates. The trigger efficiency was found to be:

\[
\begin{align*}
K_S \quad E_S &= 99.88 \pm 0.04\% \\
K_L \quad E_L &= 99.87 \pm 0.05\%
\end{align*}
\] (7.11) (7.12)

No variation with lifetime, momentum or run number was found, in the relevant kinematic range.
8. THE DOUBLE RATIO.

During the 1997 run NA48 collected; 459 061 $K_L \rightarrow \pi^0\pi^0$ events (after weighting 126 634 events), 1 097 260 $K_S \rightarrow \pi^0\pi^0$ decays, 985 681 $K_L \rightarrow \pi^+\pi^-$ events (237 253 after weighting) and 2 289 578 $K_S \rightarrow \pi^+\pi^-$ events. A summary of the cuts used to select these events is given in table 8.1 below.

To be able to fit the double ratio as a function of energy the distributions of reconstructed events must be corrected, by removal of backgrounds and correcting for $K_L - K_S$ dilution, to find the true numbers of events. Once the "pure" samples of the four decay modes are obtained, and the acceptance and massbox efficiency corrections applied, the double ratio can be fitted.

8.1 Background and Dilution Subtraction

Background Subtraction

To purify the $K_L$ decay modes one must subtract the relevant background(s), this is done using the formula given below:

$$K_{L_{True}}(p) = K_{L_{Rec}}(p) \left(1 - Bkg(p)\right) \quad (8.1)$$

Here $Bkg$ is the fractional background. The background(s) are subtracted as a function of momentum, the momentum spectra come from data in the cases of $K_{e3}$ and $K_{\mu3}$ decays and from Monte Carlo in the case of $K_L \rightarrow \pi^0\pi^0\pi^0$. The integrated
<table>
<thead>
<tr>
<th>Cut</th>
<th>value</th>
<th>reason</th>
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<tr>
<td><strong>Acceptance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{Kaon}$</td>
<td>70 - 170 GeV</td>
<td>Energy range</td>
</tr>
<tr>
<td>Ctau</td>
<td>$&gt; 0.0 \tau_s \text{ (from AKS)}$</td>
<td>Start of decay volume ($K_L$)</td>
</tr>
<tr>
<td>AKS no hit</td>
<td>$\pm 3 \text{ ns}$</td>
<td>Start of decay volume ($K_S$)</td>
</tr>
<tr>
<td>LKR hit</td>
<td>$&lt; 11 \text{ cm From outer edge}$</td>
<td>Outer acceptance cut</td>
</tr>
<tr>
<td>LKR hit</td>
<td>$&gt; 15 \text{ cm}$</td>
<td>Inner acceptance cut</td>
</tr>
<tr>
<td>LKR hit</td>
<td>$\pm 4 \text{ cm from dead strip}$</td>
<td>Reduces background</td>
</tr>
<tr>
<td>LKR hit</td>
<td>$&gt; 2 \text{ cm single dead cells}$</td>
<td>Reduces collimator scraping</td>
</tr>
<tr>
<td>COG</td>
<td>$&lt; 10 \text{ cm}$</td>
<td>Reduces collimator scraping</td>
</tr>
<tr>
<td><strong>Neutral</strong></td>
<td></td>
<td></td>
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<tr>
<td>NUT</td>
<td>$\xi'/\xi$ trigger</td>
<td>Defines data set</td>
</tr>
<tr>
<td>$\gamma$ separation</td>
<td>$&gt; 10 \text{ cm}$</td>
<td>Avoid energy mis-measurement</td>
</tr>
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<td>$\gamma$ energy</td>
<td>3-100 GeV</td>
<td>Energy within calibrated range</td>
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<td>$T_{\gamma} - T_{\text{event}}$</td>
<td>$&lt;5 \text{ ns}$</td>
<td>To remove out of time photons</td>
</tr>
<tr>
<td>extra in-time $\gamma$s</td>
<td>$&lt; 1.5 \text{ GeV Energy}$</td>
<td>Remove accidental effects</td>
</tr>
<tr>
<td>$R_{\text{Ellipse}}$</td>
<td>$&lt; 1.5$</td>
<td>Reduces $K_L \rightarrow \pi^0\pi^0\pi^0$</td>
</tr>
<tr>
<td><strong>Charged</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Massbox</td>
<td>$\xi'/\xi$ trigger</td>
<td>Defines data set</td>
</tr>
<tr>
<td>no Muons</td>
<td>within $\pm 6 \text{ ns}$</td>
<td>Reduces $K_{\mu3}$</td>
</tr>
<tr>
<td>$R_{DCH}$</td>
<td>$\geq 12 \text{ cm}$</td>
<td>Acceptance</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>$&lt; 0.62 \text{ or } &lt; 1.08 - 0.0052E_{kaom}$</td>
<td>Symmetrising acceptance</td>
</tr>
<tr>
<td>$T_{\text{track}} - T_{\text{event}}$</td>
<td>$&lt; 6 \text{ ns}$</td>
<td>Remove accidentals</td>
</tr>
<tr>
<td>$M_{\pi\pi}$</td>
<td>$489.5 - 505.7 \text{MeV}/c^2$</td>
<td>Reduces $K_L$ three body decays.</td>
</tr>
<tr>
<td>$E_{\text{cluster}}/P_{\text{track}}$</td>
<td>$&lt; 0.8$</td>
<td>Reduces $K_{e3}$</td>
</tr>
<tr>
<td>$P^2_{\text{vtx}}$</td>
<td>$&lt; 200(\text{MeV}/c)^2$</td>
<td>Reduces $K_{\mu3}$ and $K_{e3}$</td>
</tr>
<tr>
<td>Tagging</td>
<td>$\pm 2 \text{ ns}$</td>
<td>$K_S$ within cut, else $K_L$</td>
</tr>
<tr>
<td>DCH overflows</td>
<td>none in $\pm 300 \text{ ns}$</td>
<td>Symmetric deadtime</td>
</tr>
</tbody>
</table>

Tab. 8.1: Cuts Used to Select the Four Decay Modes.
backgrounds are (from sections 6.2, page 79 and 6.6, page 87) $2.1 \times 10^{-3} K_{e3}$, $0.4 \times 10^{-3} K_{\mu3}$ and $0.9 \times 10^{-3} K_L \rightarrow \pi^0\pi^0\pi^0$.

Dilution Effects

The tagging effects are less trivial recall equations 7.1, page 98:

$$N_S(\text{unweighted}) = (1 - \xi)T_S(\text{unweighted}) + \alpha T_L(\text{unweighted}) \tag{8.2}$$

$$N_L(\text{weighted}) = (1 - \alpha)T_L(\text{weighted}) + \xi T_S(\text{weighted}) \tag{8.3}$$

Here the number of true $K_S$ and $K_L$ decays is given by $(T_S, T_L)$ the number of tagged decays by $(N_S, N_L)$. The tagger inefficiencies $\xi$ are $1.4 \times 10^{-4}$ for the charged mode and $0.8 \times 10^{-4}$ for the neutral. Similarly the tagging dilutions $\alpha$ are 11.21% and 11.28%.

Rearranging to find the number of true decays for each mode gives:

$$T_S(\text{uw}) = \frac{1 - \alpha}{1 - \xi - \alpha} \left[ N_S(\text{uw}) - \frac{\alpha}{1 - \alpha} N_L(\text{uw}) \right] \tag{8.4}$$

$$T_L(\text{w}) = \left[ \frac{1 - \xi}{1 - \xi - \alpha} N_L(\text{w}) \right] + \left[ \frac{\xi}{1 - \xi - \alpha} N_S(\text{w}) \right] \tag{8.5}$$

The initials $w$ refers to weighted events and $uw$ to unweighted, $\xi$ is small and cancels, within statistical errors, between the charged and neutral modes, so neglecting terms in $\xi$ gives:

$$T_S(\text{uw}) = N_S(\text{uw}) - \frac{\alpha}{1 - \alpha} N_L(\text{uw}) \tag{8.6}$$

$$T_L(\text{w}) = (1 - \alpha)N_L(\text{w}) \tag{8.7}$$

8.2 Fitting the Double Ratio

To fit the double ratio the data is divided into twenty bins of equal energy width. The energy range is $70 \leq E_K \leq 170$ GeV. A log likelihood fit is performed to
find an estimate of the double ratio $R$. The procedure used to minimise the log likelihood function is given below.

All four distributions are taken to be Poisson distributions with unknown means, $\mu_{L00}(i), \mu_{L+}(i), \mu_{S00}(i), \mu_{S+}(i)$. The observed numbers of events are: $N_{L00}(i), N_{L+}(i), N_{S00}(i), N_{S+}(i)$. Here $L$ and $S$ refer to $K_L$ weighted events and $K_S$ unweighted events, $+-$ and $00$ refer to charged and neutral modes and $i$ is the bin number.

The Poisson distribution $P(n, \mu, i)$ for $n(i)$ events seen and a mean of $\mu(i)$ is:

$$P(n, \mu, i) = \frac{e^{-\mu(i)} \mu(i)^{n(i)}}{n(i)!} \quad (8.8)$$

and the log likelihood function is (omitting the factorial):

$$\sum_{i=1}^{\text{bins}} L = n(i) \log \mu(i) - \mu(i) \quad (8.9)$$

Combining the log likelihood for all four means gives [51]:

$$LL = \sum_{i} N_{L00}(i) \log \mu_{L00}(i) + N_{L+}(i) \log \mu_{L+}(i) + N_{S00}(i) \log \mu_{S00}(i) + N_{S+}(i) \log \mu_{S+}(i) - \mu_{L00}(i) - \mu_{L+}(i) - \mu_{S00}(i) - \mu_{S+}(i) \quad (8.10)$$

This is the function which has to be maximised, subject a constraint given by the double ratio (the double ratio is taken as being a constant over the energy range):

$$R = \frac{\mu_{L00}(i)}{\mu_{L+}(i)} \times \frac{\mu_{S+}(i)}{\mu_{S00}(i)} \quad (8.11)$$

$$\Rightarrow \mu_{L00}(i) \mu_{S+}(i) - R \mu_{L+}(i) \mu_{S00}(i) = 0 \quad (8.12)$$

The method of Lagrange multipliers states that for a given function, $f(x, y, z)$ subjected to the constraint $\phi(x, y, z) = 0$ it is possible to maximise $f$ by requiring
that:[52]

\[
\frac{\partial f}{\partial x} + \lambda \frac{\partial \phi}{\partial x} = 0 \tag{8.13}
\]
\[
\frac{\partial f}{\partial y} + \lambda \frac{\partial \phi}{\partial y} = 0 \tag{8.13b}
\]
\[
\frac{\partial f}{\partial z} + \lambda \frac{\partial \phi}{\partial z} = 0 \tag{8.13c}
\]

where \( \lambda \) is the Lagrange multiplier. Applying this to equations 8.11 and 8.10 means one minimises:

\[
f = -LL + \sum_i \lambda(i) \left[ \mu L_{00}(i) \mu S_{+-}(i) - R \mu L_{+-}(i) \mu S_{00}(i) \right] \tag{8.14}
\]

Using partial differentiation, with respect to the four \( \mu(i) \), gives:

\[
\frac{\partial f}{\partial \mu L_{00}(i)} = \left( \frac{NL_{00}(i)}{\mu L_{00}(i)} \right) - 1 - \lambda(i) \mu S_{+-}(i) = 0
\]
\[
\frac{\partial f}{\partial \mu L_{+-}(i)} = \left( \frac{NL_{+-}(i)}{\mu L_{+-}(i)} \right) - 1 + \lambda(i) [R \mu S_{00}(i)] = 0 \tag{8.15}
\]
\[
\frac{\partial f}{\partial \mu S_{00}(i)} = \left( \frac{NS_{00}(i)}{\mu S_{00}(i)} \right) - 1 + \lambda(i) [R \mu L_{+-}(i)] = 0
\]
\[
\frac{\partial f}{\partial \mu S_{+-}(i)} = \left( \frac{NS_{+-}(i)}{\mu S_{+-}(i)} \right) - 1 - \lambda(i) \mu L_{00}(i) = 0
\]

These equations can be simplified by defining a new Lagrange multiplier \( e(i) \) such that; \( e(i) = \lambda(i) \mu L_{00}(i) \mu S_{+-}(i) = \lambda(i) R \mu L_{+-}(i) \mu S_{00}(i) \).

\[
\mu L_{00}(i) = NL_{00}(i) - e(i) \tag{8.16}
\]
\[
\mu L_{+-}(i) = NL_{+-}(i) + e(i) \tag{8.16b}
\]
\[
\mu S_{00}(i) = NS_{00}(i) + e(i) \tag{8.16c}
\]
\[
\mu S_{+-}(i) = NS_{+-}(i) - e(i) \tag{8.16d}
\]

And the constraint equation becomes:

\[
[NL_{00}(i) - e(i)] [NS_{+-}(i) - e(i)] - R [NL_{+-}(i) + e(i)] [NS_{00}(i) + e(i)] = 0 \tag{8.17}
\]
The smallest root of this quadratic equation is the value of $e(i)$. If the estimator of $R$ is to be consistent then the sum over $e(i)$ tends to zero. This occurs when the log-likelihood function has been successfully minimised. I. Mikeluc compared the result of this fitting procedure with the value of the double ratio obtained by using an unbiased $\ln(R)$ fit. The bias between the two method was found to be $\leq 5 \times 10^{-5}$ on $R$ [53].

In practice values of $R$ are tested in a DO loop ($0.9 \leq R \leq 1.1$, step size $5 \times 10^{-4}$.) $R$ is then modified by the acceptance correction and the charged trigger efficiency to produce an estimator $R(i)$ for this bin. Equation 8.17 is solved and the values of $e(i)$ summed over the twenty momentum bins. The value of $R$ for which the $\sum_i e_i$ is closest to zero is taken as being the estimate of the double ratio. Figure 8.1 shows the log likelihood equation (equation 8.10), after all corrections, and the sum of $e_i$ over the twenty momentum bins as a function of $R$.

8.2.1 The Statistical Error on $R$

It is possible to find the error on the log likelihood fit using the following prescription: When the present value of the log likelihood fit is equal to the minimum value of the fit plus 0.5 then, the value of $R$ at this point is one standard deviation away from the "true" $R$ value.

$$LL_{NOW} = LL_{MIN} + 0.5 \quad \text{when} \quad R = R_{TRUE} + \sigma_R$$

(8.18)

From figure 8.1 the error on $R$ as given by the log likelihood fit is 0.0033. However this estimate of the error does not take into account the fact that the $K_L$ events are weighted and the weights are known to a high precession. By using this information a better estimate of the statistical error can be obtained.

For a Poisson distribution with $N$ total events the error is $\sqrt{N}$, e.g the error on the
Fig. 8.1: The Log Likelihood Function. The log likelihood function is the parabola. The straight line is the sum over the 20 energy bins of $e_i$; this is then divided by fifty to give an appropriate scale.
number of $K_S \rightarrow \pi^+\pi^-$ events is $\sigma_{S^+} = \sqrt{\Sigma[NS_{S^+}(i)]}$, where the sum denotes that this is the total number of pure $K_S \rightarrow \pi^+\pi^-$ decays, summed over all the bins.

For weighted events e.g. $K_L \rightarrow \pi^0\pi^0$ (taking the error on the weight as negligible) the error on the total number of weighted events ($\sigma_W$) is the individual weights ($w$) added in quadrature.

$$\sigma_W = \sqrt{\Sigma_{\text{events}} w^2} \quad (8.19)$$

The error on the double ratio ($\sigma_R$) is related to the errors on each of the four decay modes:

$$\sigma_R = R \sqrt{\left(\frac{\sigma_{L00}}{\Sigma NL_{00}(i)}\right)^2 + \left(\frac{\sigma_{L+0}}{\Sigma NL_{+}(i)}\right)^2 + \left(\frac{\sigma_{S00}}{\Sigma NS_{00}(i)}\right)^2 + \left(\frac{\sigma_{S^+}}{\Sigma NS_{S^+}(i)}\right)^2} \quad (8.20)$$

Substituting for the error on each of the decay modes we find that the error on $R$ is:

$$\sigma_R = R \sqrt{\frac{\Sigma[NL_{00}(i)^2]}{[\Sigma NL_{00}(i)]^2} + \frac{\Sigma[NL_{+}(i)^2]}{[\Sigma NL_{+}(i)]^2} + \frac{1}{\Sigma NS_{00}(i)} + \frac{1}{\Sigma NS_{S^+}(i)}} \quad (8.21)$$

### 8.2.2 The Fit Result

After weighting and splitting the data into 20 momentum bins the result of the log likelihood fit, before background, dilution and other corrections, gives a raw double ratio of $0.9770 \pm 0.0027$.

To this the following corrections (detailed in table 8.2) are applied in sequence. Each correction is applied on a bin by bin basis. The corrections due to Massbox efficiency and backgrounds vary as a function of energy. Effects such as accidentals and the neutral trigger efficiency produce such a small effect compared to the error on the measurements so they are not taken into consideration.
8. The Double Ratio.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Change to $R\ (10^{-4})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dilution $\alpha_{+-} = \alpha_{00}$</td>
<td>+ 60.5</td>
</tr>
<tr>
<td>Dilution Difference</td>
<td>+ 5.4</td>
</tr>
<tr>
<td>Massbox efficiency</td>
<td>+15.6</td>
</tr>
<tr>
<td>Neutral background</td>
<td>-8.0</td>
</tr>
<tr>
<td>Charged background</td>
<td>+ 25.0</td>
</tr>
<tr>
<td>Acceptance</td>
<td>+ 40.0</td>
</tr>
</tbody>
</table>

Tab. 8.2: Corrections to the Double Ratio. The corrections shown represent the resulting change to the fitted value of $R$ as the corrections are applied in sequence.

The resultant double ratio is $0.9905 \pm 0.0027$ corresponding to $\frac{e}{\bar{e}} = (1.58 \pm 0.45) \times 10^{-3}$.

8.3 Systematic Errors

Errors from acceptance ($A$), trigger efficiencies ($E$), backgrounds ($Bkg$), tagger inefficiency ($\xi$) and tagger dilution ($\alpha$) all produce systematic errors on the double ratio. The size of the tagging window, the $R_{\text{Ellipse}}$ cut, the $p_{vtz}$ cut and $M_{\pi^{+}\pi^{-}}$ cut were varied. The backgrounds, dilution, and tagging efficiency were re-calculated and a new double ratio produced in each case. The maximum effect on $R$ came from reducing the size of the tagging window from $\pm 2$ ns to $\pm 1.5$ ns, the effect was less than $1 \times 10^{-4}$. The double ratio is therefore insensitive to such changes.

The errors on the efficiencies, acceptance and backgrounds are all assumed to be independent (this is true to first order). The propagation of these errors is simple and all proceed in the same manner, the error propagation for the background subtraction is given below as an example.
For a function $F$ which depends on a set of independent variables $x, y, z,...$ the error on $F$ ($\sigma_F$) is given by:

$$\sigma_F^2 = \left( \frac{\partial F}{\partial x} \right)^2 \sigma_x^2 + \left( \frac{\partial F}{\partial y} \right)^2 \sigma_y^2 + \left( \frac{\partial F}{\partial z} \right)^2 \sigma_z^2$$ \hspace{1cm} (8.22)

The double ratio is calculated using:

$$R = A \left[ \frac{E_{L00}}{E_{L+0}} \times \frac{E_{S00}}{E_{S+0}} \right] \left[ \frac{K_L \rightarrow \pi^0\pi^0(\text{true})}{K_S \rightarrow \pi^0\pi^0(\text{true})} \times \frac{K_S \rightarrow \pi^+\pi^-(\text{true})}{K_L \rightarrow \pi^+\pi^-(\text{true})} \right]$$ \hspace{1cm} (8.23)

Here the true distributions are those obtained after correcting for backgrounds and dilution (see section 8.1, page 113.) $E_{L00}$ etc. represent the total trigger efficiency for the relevant decay channel.

Considering only the effect of the $K_L \rightarrow \pi^0\pi^0$ background (equation 8.1) the rest of the $R$ calculation equation can be taken as being a constant $C$.

$$R(K_L \rightarrow \pi^0\pi^0 \ Bkg) = C \times [N_L(p) (1 - Bkg(p))]$$ \hspace{1cm} (8.24)

Differentiating with respect to the background ($Bkg$) gives.

$$\frac{\partial R}{\partial Bkg} = -CN_L$$ \hspace{1cm} (8.25)

so the error on $R$ is simply:

$$\sigma_R^2 = (-C \cdot N_L)^2 \cdot \sigma_{Bkg}^2$$ \hspace{1cm} (8.26)

Since the background is very small we can approximate the number of true decays to the number of measured decays, with a negligible effect on the error of $R$.\(^1\) Thus the $C \cdot N_L$ in equation 8.26 becomes $R$ and the error on $R$ due to the background is trivially given by:

$$\sigma_R^2 = R^2 \sigma_{Bkg}^2$$ \hspace{1cm} (8.27)

\(^1\) E.g. for the $K_L \rightarrow \pi^0\pi^0$ decay mode this would change the relative error on $R$ by $9 \times 10^{-4}$.\)
The above method can also be used on the efficiencies and acceptance effects. The combined error is simply the addition in quadrature of all these effects:

$$\sigma_R^2 = R^2 \left[ \sigma_A^2 + \sigma_{Bkg L00}^2 + \sigma_{Bkg L+}^2 + \sigma_{Bkg L-}^2 + \sigma_{Eff L00}^2 \ldots \right]$$  \hspace{1cm} (8.28)

The Errors due to Tagging Inefficiency and Dilution

The effect of tagging errors on the double ratio is quite complex. However they produce similar effects in charged and neutral modes. It is therefore useful to think of the double ratio in terms of its components \( \eta_{+-} \) and \( \eta_{00} \).

The error on \( R \) with respect to the ratios \( \eta_{+-} \) and \( \eta_{00} \) is:

$$\sigma_R = R \sqrt{ \left( \frac{\sigma_{\eta_{00}}}{\eta_{00}} \right)^2 + \left( \frac{\sigma_{\eta_{+-}}}{\eta_{+-}} \right)^2 }$$

$$= R \sqrt{ \left( \frac{\partial \eta_{00}}{\partial (\xi_{00})} \right)^2 \sigma_{\xi_{00}}^2 + \left( \frac{\partial \eta_{00}}{\partial \alpha_{00}} \right)^2 \sigma_{\alpha_{00}}^2 + \left( \frac{\partial \eta_{+-}}{\partial (\xi_{+-})} \right)^2 \sigma_{\xi_{+-}}^2 + \left( \frac{\partial \eta_{+-}}{\partial \alpha_{+-}} \right)^2 \sigma_{\alpha_{+-}}^2 }$$  \hspace{1cm} (8.29)

\( \alpha \) is the dilution for the relevant mode and \( \xi \) the tagging inefficiency.

\( \eta \) is the number of true \( K_L \) decays divided by the number of true \( K_S \) decays. From equations 8.4, page 115 this is (for the neutral mode):

$$\eta = \frac{\frac{1-\xi}{1-\xi-\alpha} N_L(w) + \frac{\xi}{1-\xi-\alpha} N_S(w)}{\left( \frac{1-\alpha}{1-\xi-\alpha} \right) [N_S - \frac{\alpha}{1-\alpha} N_L(uw)]}$$  \hspace{1cm} (8.30)

$$= \frac{(1-\xi) (N_L(w) + N_S(w)) - N_S(w)}{-\alpha (N_S(uw) + N_L(uw)) + N_S(uw)}$$  \hspace{1cm} (8.31)

The partial differential of \( \eta \) with respect to \( \xi \) is:

$$\frac{\partial \eta}{\partial (1-\xi)} = \eta \left[ \frac{N_L(w) + N_S(w)}{(1-\xi) [N_L(w) + N_S(w)] - N_S(w)} \right]$$  \hspace{1cm} (8.32)
and similarly with respect to $\alpha$:

$$
\frac{\partial \eta}{\partial \alpha} = \eta \left[ \frac{N_S(uw) + N_L(uw)}{-\alpha [N_S(uw) + N_L(uw)] + N_S(uw)} \right] 
$$

(8.33)

Substituting into equation 8.29 gives the systematic error on $R$ due to tagging effects.

### 8.3.1 Main Systematic Errors

Table 8.3 lists all the systematics and the size of the resultant systematic error on the double ratio. The overall systematic is the addition in quadrature of all the effects given in table 8.3.

Those systematics marked with a $\dagger$ are statistical in nature and thus should diminish with the increase in data volume. Effects such as the error on the charged background fit need more understanding but do not necessarily limit the precision of the measurement.

These major systematics are, in order of size:

- **Massbox efficiency.** The error on measuring the Massbox efficiency comes from the number of Level One $\pi^+\pi^-$ triggers taken during the 1997 run. The amount of triggers taken was too few to allow the efficiency to be measured to greater than 2 per mill accuracy. Increasing the amount of Level One $\pi^+\pi^-$ triggers taken during the 1998 run will reduce the statistical error on the Massbox efficiency and allow any underlying systematics to be studied.

- **Accidental effects.** This systematic is again statistical in nature. Not enough data events were overlayed with randomly triggered events to allow a high accuracy measurement of the accidental correction. This could be reduced by overlaying more of the 1997 data set, however this is time and CPU
### Tab. 8.3: Systematic Effects on the Double Ratio

<table>
<thead>
<tr>
<th>Effect</th>
<th>Systematic ((\times 10^{-4}))</th>
<th>See Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error on (Q_X K_L)</td>
<td>8.4 †</td>
<td>7.5</td>
</tr>
<tr>
<td>Error on (Q_X K_S)</td>
<td>6.3 †</td>
<td>7.5</td>
</tr>
<tr>
<td>Error on (E_{TOT} K_L)</td>
<td>2.3 †</td>
<td>7.5</td>
</tr>
<tr>
<td>Error on (E_{TOT} K_S)</td>
<td>0.9 †</td>
<td>7.5</td>
</tr>
<tr>
<td>Error on MBX (K_L)</td>
<td>17.8 †</td>
<td>7.5</td>
</tr>
<tr>
<td>Error on MBX (K_S)</td>
<td>8.9 †</td>
<td>7.5</td>
</tr>
<tr>
<td>Charged BKg fit</td>
<td>1.5</td>
<td>6.6</td>
</tr>
<tr>
<td>Tagging Charged</td>
<td>2.0 †</td>
<td>7.1</td>
</tr>
<tr>
<td>Error on NUT (K_L)</td>
<td>4.0 †</td>
<td>7.6</td>
</tr>
<tr>
<td>Error on NUT (K_S)</td>
<td>5.0 †</td>
<td>7.6</td>
</tr>
<tr>
<td>Neutral Bkg</td>
<td>2.0 †</td>
<td>6.2</td>
</tr>
<tr>
<td>Tagging Neutral</td>
<td>11.1 †</td>
<td>7.1</td>
</tr>
<tr>
<td>Acceptance</td>
<td>12 †</td>
<td>5.4</td>
</tr>
<tr>
<td>Non linearities</td>
<td>(\leq 15)</td>
<td>7.2</td>
</tr>
<tr>
<td>AKS inefficiency</td>
<td>0.5</td>
<td>5.2</td>
</tr>
<tr>
<td>Accidentals</td>
<td>16 †</td>
<td>7.3</td>
</tr>
<tr>
<td><strong>Overall systematic</strong></td>
<td>36.2</td>
<td></td>
</tr>
</tbody>
</table>

-consuming. The percentage of overlayed events was increased from 10% in the 1997 data set to 15% in the 1998 data set to increase the precision of the measurement.

- Non linearities on the energy scale. The quoted value of the non-linearities was measured before the eta calibration factors were applied, these correc-
tions should decrease the size of the systematic. However, to be conservative the measured value of the non-linearities has been used as the systematic error. For NA48 to reach its ultimate goal of measuring \( \frac{\xi}{\epsilon} \) to an accuracy of \( 2 \times 10^{-4} \) it is essential that non-linearities in the energy scale be less than the value given above. A possible way to do this would be to reduce the energy range which is used to measure \( \frac{\xi}{\epsilon} \).

- Acceptance. This is a purely statistical error, coming from the lack of Monte Carlo statistics, producing and storing large quantities of Monte Carlo events is time and disk space consuming. It is planned to produce 100 times the amount of Monte Carlo available at present to decrease this systematic error to an acceptable level for the final \( \frac{\xi}{\epsilon} \) analysis.

- Tagging effects in neutral decays. This systematic comes from the small number of Dalitz events found in the 1997 data sample, meaning that the neutral tagging inefficiency is poorly measured. A new method of measuring the tagging inefficiency is being studied. This new method relates the timing distributions of electrons in the Charged Hodoscope and LKR and convolutes this with the known charged tagging inefficiency to obtain the neutral tagging inefficiency.

The value for \( \frac{\xi}{\epsilon} \) obtained using the method detailed in this thesis is:

\[
(1.58 \pm 0.45_{\text{stat}} \pm 0.60_{\text{sys}}) \times 10^{-3} \quad (8.34)
\]

\[
= (1.58 \pm 0.75) \times 10^{-3} \quad (8.35)
\]

8.4 The Energy Dependence of the Double Ratio

Figure 8.2 shows the double ratio as a function of energy. The ratio shows a distinct energy dependence. The double ratio as measured between 70-120 GeV
is $0.9835 \pm 0.0034_{\text{stat}}$ whilst in the energy range 120 -170 GeV an R value of $1.0010 \pm 0.0044_{\text{stat}}$ is obtained. These results are different by $17.5 \times 10^{-3}$. When the value of $R$ is fitted to a constant over the range 70-170 GeV the $\chi^2/ndf$ is $25.0/19 = 1.31$ [54]. This indicates that there maybe an energy dependence of the double ratio, although statistical fluctuations cannot be ruled out.

The strength of the energy dependence varies throughout the run. The variation of Massbox efficiency may account for some of the energy slope, although it is debatable to what level. The energy of the kaon is strongly correlated with the radial position of pions (or photons) hitting the detector. Thus by varying the size of radial cuts, especially in the charged sector it is possible to vary the size
of the energy slope (the Monte Carlo compensates partially for change in radial cut). Several checks on possible systematics have been done. These include:

- **Neutral Energy Scale.** For a global energy scale effect to cause this energy dependence of $R$ the distance between the AKS and the AKS would have to be wrong by 30-40 cm, this is far too large a shift to go unnoticed. Nonlinearity effects are too small to give this slope. [55].

- **DCH inefficiencies at low radius.** Large inefficiencies (up to 5% at $r=10$ cm) were simulated in a GEANT Monte Carlo: The change in the ratio of the acceptances for $K_S \rightarrow \pi^+\pi^-$ and $K_L \rightarrow \pi^+\pi^-$ (as compared with the ratio of acceptances for no inefficiency) was found to be $8 \times 10^{-4}$. [56]. This changes the energy dependence slightly but not enough to remove the observed slope.

- **The $K_L \rightarrow \pi^+\pi^-$ at high $p_{t_{\pi}}$.** were studied. A maximum number of 600 events in the signal region were found. This number of events cannot account for a large fraction of the energy dependence.

As of writing (9th April 1999) nothing has been found that could cause such an energy dependence, although studies are still in progress. It is planned to widen the energy range over which $R$ is measured to see if the possible energy dependence continues beyond 170 GeV.

Taking the standard deviation between the $R_{70-120}$ and $R_{120-170}$ values to be an additional systematic one arrives at a final value of $\frac{\epsilon'}{\epsilon}$ of:

$$\frac{\epsilon'}{\epsilon} = (1.58 \pm 0.45_{\text{stat}} \pm 0.60_{\text{sys}} \pm 1.3_{\text{energy}}) \times 10^{-3}$$ (8.36)

$$= (1.58 \pm 1.5) \times 10^{-3}$$ (8.37)
9. CONCLUSION

During the 1997 data taking the NA48 experiment collected 450 000 $K_L \rightarrow \pi^0\pi^0$ decays after all cuts. This thesis has presented an analysis of $\xi' / \xi$ using this data. The resulting value of $\xi' / \xi$ is $(1.58 \pm 0.85) \times 10^{-3}$, with a possible energy dependence that may increase the error to $1.5 \times 10^{-3}$. This is consistent with the four previous high statistics measurements of E731 (paper by Woods et al in 1988,)[21] E731 (paper by Gibbons et al in 1993) [20], NA31 [19] and KTeV [22]. It is consistent with $\xi' / \xi = 0$ as predicted by the Superweak model, however when taken in conjunction with the measurements listed above it points to a non-zero value of $\xi' / \xi$ and direct CP violation at the $10^{-3}$ level.

The two previous high statistics experiments who measured $\xi' / \xi$ are NA31 and E731. NA31 took $K_S$ and $K_L$ data in separate running periods with a moving $K_S$ train to minimise the acceptance difference between $K_L$ and $K_S$. E731 used a dual beam and a regenerator, allowing them to collect $K_S$ and $K_L$ data simultaneously. The regenerator was moved between the left and right beams each spill to avoid systematics due to the regenerator beam (partially $K_S$ partially $K_L$) and the vacuum beam (all $K_L$) hitting different areas of the detector.

A large source of systematic error for both experiments comes from uncertainties in the energy scale [19], [20]. The two subsequent experiments NA48 and KTeV
both use state-of-the-art electromagnetic calorimeters, the quoted resolutions are:

\[
\frac{\sigma(E)}{E} = 3.5\% \frac{0.110\text{GeV}}{E} \oplus 0.6\% \quad \text{NA48's Liquid Krypton calorimeter (9.1)}
\]

\[
\frac{2.0\%}{\sqrt{E}} \oplus 4.5\% \quad \text{KTeV CsI calorimeter (9.2)}
\]

Neutron interactions in the regenerator, another large source of systematic error for E731, have been considerably reduced in KTeV by the inclusion of a beryllium absorber directly after the regenerator target.

The largest source of systematics for NA31 was due to accidental effects which did not cancel between K_S and K_L running periods. NA48 detector takes K_S and K_L data simultaneously while using a proton tagger to distinguish the two beams, thus reducing the effect of accidental particles. KTeV was designed to collect all four modes simultaneously, however due to problems with their drift chamber readout and Level Three trigger their recently published result [22] uses neutral data taken during 1997 and charged data taken during 1996.

NA48 uses a weighting procedure to minimise any systematic coming from the different Z vertex distribution of the K_L and K_S beams whilst KTeV relies on a highly detailed Monte Carlo and "blind" analysis.

The recent result from KTeV gives a value of \( \xi \xi = (28.0 \pm 4.1) \times 10^{-3} \), confirming the discovery of direct CP violation by NA31. However due to the complexity of the measurement is is essential that another high statistics measurement of \( \xi \xi \) be made.

The weighted average for \( \xi \xi \) is \( (21.8 \pm 2.9) \times 10^{-4} \) from the four measurements of E731 (Woods 88)[21], E731 (Gibbons 93) [20], NA31 [19] and KTeV [22]. However due to the large (3.5\( \sigma \)) difference between the E731 and KTeV results the \( \chi^2/ndf \) of the results is 2.84. The prescription in the particle data book [11] for measurements (such as \( \xi \xi \)) which have a large \( \chi^2/ndf \) is to rescale the errors by a
Fig. 9.1: The Measurements of $\xi$. The yellow band shows the new world average is the energy dependence is ignored. The hatched band shows the world average if the energy dependence is included. The final point shows the result obtained in this thesis, solid line without energy dependence dotted line with energy dependence.

significance factor ($S = \sqrt{\chi^2/ndf}$). Applying this to the measurements of $\xi$ gives an average of $(21.8 \pm 4.9) \times 10^{-4}$. At present no explanation has been found to account for the difference between the E731 and KTeV result. If the E731 (Gibbons 93) result is rejected from the averaging procedure the world average becomes $(26.6 \pm 3.4) \times 10^{-4}$.

If the measurement presented in this thesis is included, the world average (without rescaling of errors) becomes $(21.2 \pm 2.8) \times 10^{-4}$ with a $\chi^2/ndf$ 2.24. Rescaling the
errors gives a new world average of \((21.1 \pm 4.1) \times 10^{-4}\).\(^1\) The new one sigma limit is shown as the yellow band on figure 9.1. This value for the size of direct CP violation is in agreement with the Standard Model, although on the high side of theoretical predictions. This may hint at new mechanisms of CP violation and physics beyond the Standard Model [57].

Including the possible energy dependence of this result as an additional source of error one can re-calculate a second weighted average of the five \(\xi\) results. This gives a weighted average of \((21.7 \pm 2.9) \times 10^{-4}\) with a \(\chi^2/\text{ndf}\) 2.17. Rescaling the errors gives a new world average of \((21.7 \pm 4.2) \times 10^{-4}\), this is shown as the hatched band on figure 9.1.

In the future KTeV expects to have 80% more statistics than at present from 1997 and to double their statistics in its 1999 run. The NA48 experiment will run into the year 2000 and expects to measure \(\xi\) with an accuracy of \(2 \times 10^{-4}\). The KLOE experiment at DAΦNE, using \(\Phi \rightarrow K_LK_S\), is also expected to publish a preliminary result of \(\xi\) soon.

\(^1\) If the E731(Gibbons 93) result is rejected from the averaging procedure the world average becomes \((25.2 \pm 3.1) \times 10^{-4}\) and does not require rescaling.
APPENDICES
A. CALCULATION OF KINEMATIC QUANTITIES

A.1 Calculation of Parent Momentum From Two Charged Tracks

The four momentum of the parent particle (assumed to be a kaon) is found using:

$$E_{kaon} = \frac{1}{\theta} \sqrt{(M_k^2 - R M^2) R} \quad \text{where, } R = 2 + \frac{E_1}{E_2} + \frac{E_2}{E_1} \quad (A.1)$$

To prove this consider the decay of a particle, mass \(M\) into two daughters each of mass \(m\), and four momenta \(P (P_1 = px_1, py_1, pz_1, E_1)\). The mass of the parent is given by:

$$M^2 = (P_1 + P_2)^2 = P_1^2 + P_2^2 + 2P_1P_2$$
$$= 2m^2 + 2E_1E_2 - |p_1||p_2|\cos\theta$$

(A.2)

If one assumes that the daughter particles are travelling close to the speed of light, i.e. \(m/E \ll 1\), and that the opening angle \(\theta\) is small then equation A.2 can be expanded:

$$M^2 = 2E_1E_2 \left[ 1 - \left( 1 - \frac{m^2}{E_1^2} \right)^{\frac{1}{2}} \left( 1 - \frac{m^2}{E_2^2} \right)^{\frac{1}{2}} \cos\theta \right] + 2m^2$$
$$= 2E_1E_2 \left[ 1 - \left( 1 - \frac{m^2}{2E_1^2} - \frac{m^2}{2E_2^2} \right) \left( 1 - \frac{\theta^2}{2} \right) \right] + 2m^2$$

(A.3)

$$= \theta^2 E_1E_2 + m^2 \left( 2 + \frac{E_1}{E_2} + \frac{E_2}{E_1} \right)$$
A. Calculation of Kinematic Quantities

Assuming the decay is a kaon going two pions and rearranging:

\[ \theta^2 E_1 E_2 = M_k^2 - R M_{\pi}^2 \]  

(A.4)

and from the kinematics of the decay one has:

\[ E_{\text{kaon}}^2 = (E_1 + E_2)^2 = E_1^2 + E_2^2 + 2E_1 E_2 \]

\[ \Rightarrow \frac{E_{\text{kaon}}^2}{E_1 E_2} = \frac{E_1}{E_2} + \frac{E_2}{E_1} + 2 \]  

(A.5)

\[ \Rightarrow E_1 E_2 = \frac{E_{\text{kaon}}^2}{R} \]

Substituting for \(E_1 E_2\) into equation A.4 gives the required formula, equation A.1, above.

A.2 Calculation of The Neutral Z Vertex

It is possible to calculate the \(Z\) position of the decay vertex using the kaon mass as a constraint (equation A.6 below):

\[ Z_{\text{decay}} = Z_{\text{lr}} - \left[ \frac{1}{M_K^2} \sum_{i<j}^{N_{\text{clus}}} E_i E_j R_{ij} \right]^{\frac{1}{2}} \]  

(A.6)

One can prove this by considering the decay of a kaon into two photons (the four photon case is the generalisation of this.) The kinematics of a two photon decay give the equation:

\[ M_K^2 = E_{\text{kaon}}^2 - p_k^2 \]

\[ = (E_1 + E_2)^2 - (p_1 + p_2)^2 \]

\[ = E_1^2 + E_2^2 + 2E_1 E_2 - (E_1^2 + E_2^2 + 2E_1 E_2(\cos \theta)) \]  

(A.7)

\[ = 2E_1 E_2(1 - \cos \theta) \]

\[ = E_1 E_2 \theta^2 \quad \text{(for small } \theta) \]
θ is the opening angle between the two photons, and can be approximated to

\[ \theta = \frac{R_{12}}{Z_{\text{decay}} - Z_{\text{kr}}} \]

where \( R_{12} \) is the radial separation of the two photons. From

the calorimeter reconstruction we obtain the \( X \) and \( Y \) positions and the energy of each cluster. Using this information we can substitute in for \( \theta \) in equation A.7:

\[ M_{K}^{2} = E_{1}E_{2} \left( \frac{R_{12}}{Z_{\text{decay}} - Z_{\text{kr}}} \right)^{2} \]  \hspace{1cm} (A.8)

Re-arranging gives:

\[ Z_{\text{decay}} - Z_{\text{kr}} = \left[ \frac{1}{M_{K}^{2}} E_{1}E_{2}R_{12}^{2} \right]^\frac{1}{2} \]  \hspace{1cm} (A.9)

\[ Z_{\text{decay}} = Z_{\text{kr}} - \left[ \frac{1}{M_{K}^{2}} E_{1}E_{2}R_{12}^{2} \right]^\frac{1}{2} \]  \hspace{1cm} (A.10)

For the four photon case equation A.7 becomes:

\[ M_{K}^{2} = (E_{1} + E_{2} + E_{3} + E_{4})^{2} - (p_{1} + p_{2} + p_{3} + p_{4})^{2} \]

\[ = E_{1}E_{2}\theta_{12} + E_{1}E_{3}\theta_{13} + E_{1}E_{4}\theta_{14} \]

\[ + E_{2}E_{3}\theta_{23} + ...... \]  \hspace{1cm} (A.11)

The proof then continues in the same manner as detailed above, to arrive at the
general case for \( N \) clusters (photons) of:

\[ Z = Z_{\text{kr}} - \left[ \frac{1}{M_{K}^{2}} \sum_{i,j(i<j)}^{N_{\text{clus}}} E_{i}E_{j}R_{ij}^{2} \right]^\frac{1}{2} \]  \hspace{1cm} (A.12)
B. HOW THE DIFFERENCES IN THE SPECTRA FEED INTO THE ACCEPTANCE

Recall equations 5.11 where the expectation values of the acceptance ($A_{mc}$ Monte Carlo, $A_D$ data) were defined:

\[
< A_{mc} > = \frac{\int a(p)N_G(p)dp}{\int N_G(p)dp} \quad (5.11a)
\]
\[
< A_D > = \frac{\int a(p)N_T(p)dp}{\int N_T(p)dp} \quad (5.11b)
\]
\[
\Delta A = < A_{mc} > - < A_D > \quad (5.11c)
\]

Where the momentum distributions of events are $N_G(p)$ for the Monte Carlo generated events and $N_T(p)$ for the true number of events in the data. To determine the level of accuracy required in the Monte Carlo one considers the differences between the two expectation values for the acceptance, given that there is a variation ($\alpha$) of the acceptance and of the spectrum ($\beta$) over the bin.

Consider a given bin between $P_1$ and $P_2$ and assume the momentum spectrum varies linearly across this bin. The data and Monte Carlo spectra are related by:

\[
N_C(p) = b_0(1 + \beta(p - P_C))N_T(p) \quad (B.1)
\]

Where $b_0$ is the difference is the overall difference spectra at momentum $P_C = (P_2 + P_1)/2$ (at the centre of the bin) and the acceptance at this point is given by
$a_0$. The difference in acceptance across the bin is therefore given by:

$$a(p) = a_0(1 + \alpha(p - P_C))$$  \hspace{1cm} \text{(B.2)}

To find $\Delta A$ one must now calculate each of the four integrals in equations 5.11. Firstly we define the total number of events in data $N_0$ to be:

$$\int_{P_1}^{P_2} N_T(p) dp = N_0 \Rightarrow N_T(p) = \frac{N_0}{P_2 - P_1}$$  \hspace{1cm} \text{(B.3)}

Thus the number of Monte Carlo events becomes:

$$\int_{P_1}^{P_2} N_C(p) dp = \int_{P_1}^{P_2} b_0(1 + \beta(p - P_C))N_T(p) dp$$

$$= \frac{N_0}{P_2 - P_1} \int_{P_1}^{P_2} (1 + \beta(p - P_C)) dp$$

$$= \frac{N_0}{P_2 - P_1} \left[ P_2 - P_1 + \beta \frac{P_2^2 - P_1^2}{2} - \beta P_C(P_2 - P_1) \right]$$

$$= N_0b_0$$  \hspace{1cm} \text{(B.4)}

Now the two more complex terms involving the acceptance. For the data:

$$\int_{P_1}^{P_2} a(p)N_T(p) dp = \frac{N_0}{P_2 - P_1} \int_{P_1}^{P_2} a_0(1 + \alpha(p - P_C))$$

$$= \frac{N_0a_0}{P_2 - P_1} \left[ P_2 - P_1 + \alpha \frac{P_2^2 - P_1^2}{2} - \alpha P_C(P_2 - P_1) \right]$$  \hspace{1cm} \text{(B.5)}

And for the Monte Carlo:

$$\int_{P_1}^{P_2} a(p)N_C(p) dp = \int_{P_1}^{P_2} a_0(1 + \alpha(p - P_C))b_0(1 + \beta(p - P_C))N_T(p) dp$$

$$= \frac{N_0a_0b_0}{P_2 - P_1} \int_{P_1}^{P_2} 1 + (\alpha + \beta)(p - P_C) + \alpha\beta(p - P_C)^2 dp$$  \hspace{1cm} \text{(B.6)}

$$= \frac{N_0a_0b_0}{P_2 - P_1} \left[ 1 + \alpha\beta\frac{(P_2 - P_1)^2}{12} \right]$$

Substituting these four terms into equation 5.11 we obtain

$$\Delta A = < A_{mc} > - < A_D > = a_0 \left[ 1 + \alpha \beta \frac{(P_2 - P_1)^2}{12} \right] - a_0$$  \hspace{1cm} \text{(B.7)}
Thus the fractional change in the acceptance is given by:

\[
\frac{\Delta A}{a_0} = \alpha \beta \frac{(P_2 - P_1)^2}{12}
\]

(B.8)
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