THE DEEP ELECTRICAL STRUCTURE OF THE 
GREAT GLEN FAULT, SCOTLAND

MAXWELL AZU MEJU

DOCTOR OF PHILOSOPHY
UNIVERSITY OF EDINBURGH
1988
DECLARATION

This thesis has been composed by me and has not been submitted for any other degree. Except where acknowledgement is made, the work is original.

Maxwell A. Meju
ABSTRACT

The deep structure of the Great Glen Fault (GGF) is an outstanding problem of Scottish geology. The magnetotelluric (MT) technique in the frequency range 0.016 - 640 Hz has been applied to the determination of the electrical resistivity structure across this fault. Data from 32 stations along 3 traverses in this region have been processed using standard tensorial techniques with attention being paid to the biasing of the impedance tensor. Application of digital filtering techniques enhanced further the quality of noise degraded data sets.

One-dimensional inversion algorithms which account for errors on the data and the non-unique nature of the solutions have been developed and applied to the processed data sets to yield an approximate geoelectric structure for the region. Subsequent two-dimensional (2-D) finite difference modelling has revealed some hitherto undiscovered features. To the east and west of the fault zone, the structure is characterized by a highly resistive (2000 - 100000 Ωm) upper crust which thickens from about 20Km at the traverse extremes to 30 - 42 Km near the fault and by a lower crust having a resistivity in the range 150 - 400 Ωm. The basic structure of the GGF is an outcropping low resistivity (150 - 200 Ωm) "dyke-like" body, about 1 Km wide and connected with the lower crust at a depth of about 30 Km. Similar concealed dykes exist to the east and west of the fault and are interpreted as steep shear zones.

The 2-D model explains the many geophysical and geological observations made in and around the fault and is therefore the first unequivocal model for the region; the Great Glen is interpreted as a fossil rift or a contact zone between accreted electrically distinct terrains. An integrated interpretation of MT and gravity data sets has indicated other zones of crustal thickening in the Scottish Caledonides. These are interpreted as ancient continental collision (or microplate suture) zones and shown to be consistent with the known regional geology. A tectonic model is proposed for the Scottish Caledonides and plausible explanations are offered for the low resistivity nature of the lower crust and the dyke-like bodies.
"But if any human being earnestly desire to push on to new discoveries instead of just retaining and using the old; to win victories over Nature as a worker rather than over hostile critics as a disputant; to attain, in fact, to clear and demonstrative knowledge instead of attractive and probable theory; we invite him as a true son of Science to join our ranks."

BACON (Novum Organum, Prefatio, 1620)
ACKNOWLEDGEMENTS

I am very grateful to Dr V.R.S. Hutton for suggesting and supervising the magnetotelluric project, and for letting me participate in numerous international field projects: the experience is invaluable. Her great kindness, generosity and hospitality are thankfully acknowledged. Dr R.A. Scrutton was my second supervisor and contributed in no small amount to the success of this project: thanks for everything. Special thanks to Dr Anna Thomas-Betts, my tutor at Imperial College, for introducing me to the magnetotelluric method and for her continued support.

I extend my warmest thanks and appreciation to all members of the Geophysics Department for their help and friendship. I benefitted from discussions with Drs R. Hipkin and B.A. Hobbs. Special thanks to G. Dawes for technical support and for developing the improved S.P.A.M. system – it's been "MT without tears". I thank my colleagues: E.R.G. Hill for helping me understand inverse theory; Sergio Luiz Fontes for introducing me to digital filtering and for field help and his friendship; T. Harinarayana, A. Tzanis and R. Parr for useful discussions; and D. Galanopoulos and Phil Jones for field assistance.

I am grateful to Drs Kathy Whaler and G. Stuart of Leeds University for help with my fieldwork. I thank NERC (and especially Mr. Valiant of the Geomagnetic Equipment Pool) for a loan of their AMT system and frequency analyser.

My thanks also go to the officials of the Locharber Forestry Commission (and especially Mr. Biggin, Torlundy office) and owners of various estates (in particular Loch Eil estates) for their help and cooperation.

I am indebted to the BGS (Edinburgh) and especially Mr. D.W. Redmayne for providing me with historical earthquake records of the Great Glen region; the illuminating discussions with Redmayne are gratefully acknowledged.

I express my profound gratitude to the Nigerian Federal Government for a scholarship and the CVCP (London) for an overseas research students award: this study would not have been undertaken without their financial support.

Finally, my deepest thanks go to my family for all their support and especially to 'Chi' Rosaline for her love, patience and understanding during the last four years.
## CONTENTS

<table>
<thead>
<tr>
<th>Abstract</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgements</td>
<td>5</td>
</tr>
<tr>
<td>Contents</td>
<td>6</td>
</tr>
<tr>
<td>List of figures</td>
<td>10</td>
</tr>
<tr>
<td>List of tables</td>
<td>12</td>
</tr>
</tbody>
</table>

### CHAPTER 1 INTRODUCTION

1.1 The quest for deep crustal information

1.2 Aspects of geology of northern Scotland

1.2.1 Tectonic and geologic setting

1.2.2 Geology of the Great Glen study region

1.3 History of investigation

1.4 Problems for research

1.5 Outline of research

### CHAPTER 2 THE MAGNETOTELLURIC METHOD

2.1 Source field

2.2 Maxwell's equations

2.3 One-dimensional (1-D) MT theory

2.3.1 Uniform half-space

2.3.2 n-layered half-space

2.4 Two- and Three-dimensional MT theory

2.4.1 Two-dimensional (2-D) structures

2.4.2 Three-dimensional (3-D) structures

2.5 Earth response functions

2.5.1 Indicators of structure

2.5.2 Indicators of structural dimension

2.5.2.1 Electrical strike direction and Tipper
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5.2.2</td>
<td>Impedance skew</td>
<td>40</td>
</tr>
<tr>
<td>2.5.2.3</td>
<td>Ellipticity and Eccentricity</td>
<td>41</td>
</tr>
<tr>
<td>2.6</td>
<td>Estimation of the impedance matrix</td>
<td>41</td>
</tr>
<tr>
<td>2.7</td>
<td>Coherence</td>
<td>43</td>
</tr>
<tr>
<td>2.6</td>
<td>Estimation of the impedance matrix</td>
<td>41</td>
</tr>
<tr>
<td>2.7</td>
<td>Coherence</td>
<td>43</td>
</tr>
<tr>
<td>3.1</td>
<td>Field equipment specification</td>
<td>44</td>
</tr>
<tr>
<td>3.2</td>
<td>Field instrumentation</td>
<td>45</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Magnetic and electric field sensors</td>
<td>45</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Data recording equipment</td>
<td>45</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Equipment Calibration</td>
<td>49</td>
</tr>
<tr>
<td>3.3</td>
<td>Field procedure and practical considerations</td>
<td>49</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Site selection</td>
<td>49</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Field set-up</td>
<td>53</td>
</tr>
<tr>
<td>3.4</td>
<td>Survey details</td>
<td>53</td>
</tr>
<tr>
<td>4.1</td>
<td>Data transfer</td>
<td>57</td>
</tr>
<tr>
<td>4.2</td>
<td>Data processing</td>
<td>57</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Standard analysis</td>
<td>57</td>
</tr>
<tr>
<td>4.2.1.1</td>
<td>General window analysis</td>
<td>57</td>
</tr>
<tr>
<td>4.2.1.2</td>
<td>Averaging of windows results</td>
<td>58</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Bias analysis</td>
<td>61</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Cultural noise analysis</td>
<td>61</td>
</tr>
<tr>
<td>4.2.3.1</td>
<td>Noise types found in the MT records</td>
<td>63</td>
</tr>
<tr>
<td>4.2.3.2</td>
<td>Digital filtering</td>
<td>63</td>
</tr>
<tr>
<td>4.2.3.3</td>
<td>Frequency domain window editing</td>
<td>69</td>
</tr>
<tr>
<td>4.3</td>
<td>Magnetotelluric field responses</td>
<td>71</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Glen Garry profile</td>
<td>71</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Loch Arkaig profile</td>
<td>72</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Glen Loy profile</td>
<td>72</td>
</tr>
<tr>
<td>5.1</td>
<td>Linear least squares inversion</td>
<td>75</td>
</tr>
<tr>
<td>5.2</td>
<td>Constrained linear least squares inversion</td>
<td>77</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Inversion with simplicity measures</td>
<td>78</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Inversion with prior information</td>
<td>79</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.3</td>
<td>Nonlinear least squares problems</td>
<td>80</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Iterative least squares inversion</td>
<td>80</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Ridge regression</td>
<td>83</td>
</tr>
<tr>
<td>5.3.2.1</td>
<td>Properties of ridge solution</td>
<td>84</td>
</tr>
<tr>
<td>5.3.2.2</td>
<td>Damping factor and the stability of the regression estimates</td>
<td>85</td>
</tr>
<tr>
<td>5.4</td>
<td>Solution appraisal</td>
<td>86</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Goodness-of-fit</td>
<td>86</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Parameter resolution matrix</td>
<td>86</td>
</tr>
<tr>
<td>5.5</td>
<td>Errors and extreme parameter sets</td>
<td>87</td>
</tr>
<tr>
<td>5.5.1</td>
<td>Parameter covariance matrix</td>
<td>87</td>
</tr>
<tr>
<td>5.5.2</td>
<td>Extreme parameter sets</td>
<td>87</td>
</tr>
<tr>
<td>5.6</td>
<td>Geometric interpretation of constrained inversion</td>
<td>89</td>
</tr>
<tr>
<td>CHAPTER 6</td>
<td>1-D MODELLING AND INVERSION OF MT RESPONSES</td>
<td>92</td>
</tr>
<tr>
<td>6.1</td>
<td>Forward modelling</td>
<td>92</td>
</tr>
<tr>
<td>6.2</td>
<td>Inversion of MT data</td>
<td>92</td>
</tr>
<tr>
<td>6.3</td>
<td>Model search</td>
<td>94</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Linearising parameterizations</td>
<td>94</td>
</tr>
<tr>
<td>6.3.2</td>
<td>The optimization scheme</td>
<td>95</td>
</tr>
<tr>
<td>6.3.3</td>
<td>Computational details</td>
<td>98</td>
</tr>
<tr>
<td>6.3.3.1</td>
<td>The singular value decomposition (SVD) of a matrix:</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>an overview</td>
<td></td>
</tr>
<tr>
<td>6.3.3.2</td>
<td>Application of SVD to inversion</td>
<td>99</td>
</tr>
<tr>
<td>6.3.3.3</td>
<td>Estimation of damping factors for ridge analysis</td>
<td>99</td>
</tr>
<tr>
<td>6.3.4</td>
<td>Convergence characteristics of the optimization algorithm</td>
<td>100</td>
</tr>
<tr>
<td>6.4</td>
<td>Model interpretation methods</td>
<td>108</td>
</tr>
<tr>
<td>6.4.1</td>
<td>Damped most squares method</td>
<td>108</td>
</tr>
<tr>
<td>6.4.2</td>
<td>The a priori information approach</td>
<td>113</td>
</tr>
<tr>
<td>6.4.3</td>
<td>Multi-station interpretation</td>
<td>114</td>
</tr>
<tr>
<td>6.4.3.1</td>
<td>Joint inversion</td>
<td>114</td>
</tr>
<tr>
<td>6.4.3.2</td>
<td>Simultaneous inversion</td>
<td>115</td>
</tr>
<tr>
<td>6.4.4</td>
<td>Occam's razor technique</td>
<td>116</td>
</tr>
<tr>
<td>6.4.5</td>
<td>Nibett-Bostick (N-B) transformation</td>
<td>117</td>
</tr>
<tr>
<td>6.5</td>
<td>One-dimensional models</td>
<td>118</td>
</tr>
<tr>
<td>6.5.1</td>
<td>1-D response functions and the existence of solutions</td>
<td>118</td>
</tr>
<tr>
<td>6.5.2</td>
<td>Comparison of methods</td>
<td>118</td>
</tr>
<tr>
<td>6.5.3</td>
<td>Interpretive geoelectric sections</td>
<td>130</td>
</tr>
</tbody>
</table>
6.5.4 Effective phase sections 134

CHAPTER 7 TWO-DIMENSIONAL NUMERICAL MODELLING STUDIES 136
7.1 Introduction 136
7.2 Model construction 136
7.2.1 Computer modelling technique 137
7.2.1 Programs and execution 137
7.2.1.2 Mesh design considerations 138
7.2.2 Prior information and initial models 139
7.3 2-D geoelectric models 146
7.4 Inverse calculations 167
7.4.1 Optimal resistivity distribution 167
7.4.2 Errors on the model parameters 167

CHAPTER 8 INTERPRETATIONS AND CONCLUSIONS 170
8.1 Geoelectric structural appraisal 171
8.1.1 Relationship with previous geophysical and geological observations 171
8.1.2 Integrated interpretation of MT and Gravity data 178
8.2 Deep geology of northern Scotland 181
8.2.1 Gross geological overview: problematic observations 182
8.2.2 Evidence of deep structure 184
8.2.3 Regional synthesis: an alternative tectonic view 194
8.2.4 Tectonic summary 199
8.3 Interpretation of high electrical earth conductivity 201
8.4 Discussions 203
8.5 Conclusions 204
8.6 Recommendations and plans for future research 206

REFERENCES 208

Appendix I Glen Garry field results 227
Appendix II Loch Arkaig field results 235
Appendix III Glen Loy field results 241
Appendix IV Individual site 1-D models 262
# LIST OF FIGURES

1.1 Seismic and electromagnetic survey map of the British Isles  
1.2 Map showing the tectonic units of northern Britain and NW Ireland  
1.3 Simplified geological map of the Great Glen region  
2.1 Typical amplitude spectrum of magnetic variations in the ELF range  
2.2 Homogeneous half-space model  
2.3 Skin depth as a function of period and ground resistivity  
2.4 n-layer earth  
2.5 Two-dimensional structure  
2.6 Rotation of axes by angle $\theta$  
3.1 Block diagram of complete infield MT system  
3.2 Simplified flowchart of the infield process  
3.3 Instruments response curves  
3.4 Map showing the locations of the MT observational sites  
4.1 Some results of conventional data processing  
4.2 Example of bias in response estimates assuming noise-free E or H channels  
4.3 Frequency response of a delay line filter  
4.4 Time series for two data windows before and after the application of a delay line filter  
4.5 Some results of data processing with a simple delay line filter  
4.6 Results of frequency domain noise analysis for some data windows.  
5.1 Ridge trace for typical geoelectric model parameters  
5.2 A two-dimensional simplification of the p-dimensioned interactions in the vector space.  
6.1 A simplified flowchart of the minimization algorithm  
6.2 Estimation of the damping factors in ridge analysis  
6.3 Ridge regression models for synthetic and actual field data  
6.4 Convergence characteristics of the algorithm
6.5 A simplified flowchart of the iterative most squares algorithm
6.6 COPROD models
6.7 Comparison of ridge regression and most squares models for site L18
6.8 Some models for site L1
6.9 Occam and most squares models for L19
6.10 Most squares and N-B results for L8 and L5
6.11 Interpretive geoelectric sections for Glen Garry, Loch Arkaig and Glen Loy profiles
6.12 Glen Loy invariant phase section
7.1 East–west profiles of apparent resistivity across the Great Glen fault for E– and H–polarisations and frequencies 8, 14, 20, 24 and 32 Hz
7.2a&b 2–D geoelectric models for the Glen Loy profile
7.3a–j Fit of the Glen Loy 2–D response curves to observed data
7.4 Fit of the Glen Loy 2–D model to the Loch Arkaig data
7.5 A 2–D geoelectric model for the Glen Garry profile
7.6 Fit of the Glen Garry 2–D response curves to observed data
7.7 Most squares resistivity model for the Glen Loy profile
8.1 Some geophysical results for northern Scotland
8.2 Location map of the Gravity and MT profiles
8.3 Integrated geophysical model for the Great Glen region
8.4 Integrated geophysical model for northern Scotland
8.5 A comparison of MT/Gravity and seismic structure across northern Scotland
8.6 Comparison of the MT/Gravity model and previous models for northern Scotland
8.7 The lid of the continental collision zone (orogen) and its frontal wedges model
8.8 A model for the deep geology of northern Scotland
8.9 Schematic model of fluid circulation systems in regional metamorphic belt
# LIST OF TABLES

**Table**

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Site specifications</td>
<td>56</td>
</tr>
<tr>
<td>6.1</td>
<td>Model resolution matrices for sites L11, L16 and G1</td>
<td>109</td>
</tr>
<tr>
<td>6.2</td>
<td>A demonstration of the simultaneous multi-site interpretation technique</td>
<td>129</td>
</tr>
<tr>
<td>7.1</td>
<td>Grid size used in 2-D modelling of Glen Loy data</td>
<td>142</td>
</tr>
</tbody>
</table>
The resistivity of a material is its ability to impede the flow of current through it. Rocks, the building materials of the earth, are aggregates of minerals. The electrical properties of a rock will therefore depend on the composition of the constituent minerals, the amount and interconnectivity of pore space between the grains, the presence of volatiles (in connected channels) and temperature. The resistivity of earth materials varies over a wide range (see Haak and Hutton 1986, Fig 1) especially when there are temperature variations or in the presence of specific minerals and volatiles (in connected pores). This feature enables important and in some respects unique information to be deduced about the earth's crust and mantle by determining earth resistivity. The geophysical techniques for earth resistivity determination (and the depth probed) fall into the following groups:

1. A.C. Resistivity methods (≈ 300m)
2. D.C. Resistivity methods (≈ 2km)
3. Magnetotelluric (MT) method (≈ 600km)
4. Geomagnetic depth sounding (GDS) and Horizontal gradient method (≈ 1000km) and
5. Spherical harmonic analysis of global magnetic time variation data (≈ 2000km)

They all have wide applications. Of particular importance is the determination of structural and compositional information about the earth from lateral resistivity variations.

In this chapter, a brief historical background of the magnetotelluric method is given and followed by discussions of the known geology and geophysics of the study area. The project outline and objectives are also given.

1.1 The quest for deep crustal information

Over the past decade, especially in northern Britain, there has been a major effort to examine the deep structure of the Lithosphere by seismic and electromagnetic profiling techniques. Some of the various experimental profiles - completed or proposed - in Britain (including the recent surveys in Ireland) are shown in fig 1.1. The importance of these investigations cannot be over-emphasised. These techniques provide good images of structural patterns in the subsurface. However, despite the vast geophysical observations in northern Britain the deep geology is still not well understood. The role of
Fig 1.1 Seismic and electromagnetic survey map of the British Isles.

Heavy lines are the British Institution's Reflection Profiling Syndicate's (BIRPS) seismic traverses (only those relevant to this study are named). MOIST (Moine and Outer Isles Seismic Traverse), WINCH (Western Isles-North Channel), DRUM (Deep Reflections from the Upper Mantle) and SHET (Shetland) are deep seismic reflection lines. The LISPB (Lithospheric Seismic Profiling in Britain) and the recent seismic refraction experiments in Ireland (Univ. College, Dublin) are shown by the dashed lines. Beaded lines are the electromagnetic profiles conducted in Britain by Edinburgh University and in Ireland by the Applied Geophysics Unit, Univ. College, Galway.
some of the major crustal dislocations (faults) in the structural evolution of the regions is at best, poorly understood. One of such faults is the Great Glen fault in northern Scotland. The present project is designed to probe the deep structure of this outstanding crustal feature using an electromagnetic profiling technique. Of the electromagnetic (EM) techniques currently available for probing the earth, the magnetotelluric (MT) method is the most commonly used.

Magnetotellurics is a geophysical exploration technique employing simultaneous measurements of natural transient electric and magnetic fields to infer the electrical conductivity (the reciprocal of resistivity) distribution within the earth beneath the site of the surface fields. The history of the MT prospecting method is comparatively recent, and was first proposed by Cagniard in 1953, although the Russian scientist Tikhonov (1950) and the Japanese scientists, Kato and Kikuchi (1950) and Rikitake (1950) had published on the subject of deep crustal electrical conductivity investigations without applying the results to practical geophysical exploration.

The method differs from other EM geophysical techniques that require artificial sources of EM energy to probe the earth. The naturally occurring interminable variations in the earth's magnetic field induce eddy currents, called telluric currents, in the conductive crust of the earth that are detectable as electric field variations at the surface (and within the earth). Cagniard (1953) showed that if the variation in the magnetic field is assumed to be derived from a plane EM wave propagating vertically into the earth, the EM impedance (i.e., the ratio of the horizontal electrical field in the ground to the orthogonal horizontal magnetic field), measured at a number of frequencies, gives earth resistivities as a function of frequency or period resulting in a form of depth sounding.

This is a wide-band depth sounding technique, the frequency ranging from <0.001 Hz to >1000 Hz, and is suited for shallow as well as deep structural investigations. The depth of sounding can be roughly related to frequency by the use of the skin depth defined as

$$\sigma \approx 0.5 \ (\text{Resistivity/Frequency})^{1/2} \ \text{Km}$$

where resistivity is in ohm–metres and frequency in Hertz.

The MT method finds its application in the studies of global structures such as crust, mantle, rift valleys etc. (Madden and Swift 1969; Hutton et al, 1981; Stanley 1984); in mineral exploration (Strangway et al, 1973; Strangway, 1983);
in geothermal resources exploration and evaluation (Hermance and Grillot, 1974; Hutton et al 1984; Devlin 1984); in basin evaluation for petroleum exploration (Tikhonov and Berdichevsky 1966; Vozoff, 1972; Alperovitch et al, 1982; Stanley et al, 1985); and in civil engineering, ground water and archaeological problems (Guineau, 1975). This method has been used in offshore conditions (eg Cagniard and Morat, 1967; Morat 1974; Hoehn and Warner, 1983) and for earthquake prediction (Honkura et al, 1977).

Of the conventional deep probing EM techniques, the MT method is the most advantageous in terms of resolution, sophistication of interpretation, and ease of logistics. Geomagnetic depth sounding interpretations suffer from a lack of resolution, and controlled-source studies (an artificial source variant of MT) are logistically cumbersome and currently simplistic (one-dimensional) in their interpretation.

The MT method has now developed from its initial reconnaissance applications into a powerful subsurface mapping tool second only to seismics in the depth and quality of information it provides. In fact it is now being argued that a coincident MT study should be made wherever a seismic reflection survey is undertaken and that the interpretation of reflection images be constrained by electrical conductivity information (Jones 1987).

Two difficulties beset MT data interpretation: depth resolution of earth structures and bias effects on the sounding curves. EM sounding experiments cannot resolve sharp boundaries or thin layers except with ideal observations (Langer 1933; Jones 1987); the diffusive nature of the energy propagation "smears out" the real earth structure. Bias effects are a consequence of noise corruption of the MT signals and produce frequency-independent dc-like ("static") shift or frequency dependent biasing of the apparent resistivity sounding curves which limits model resolution to a large extent. The phase of the impedance (not obtained by the D.C. methods) in such instances provides a useful constraint.

The seismic reflection technique has got its problems too: while it provides good images of horizontal and sub-horizontal structures, vertical boundaries are very poorly imaged and may only be inferred from zones in which sub-horizontal reflectors are absent, and then not with certainty (Hall, 1986). Areas of thick sedimentary or volcanic sequences are unfavourable to the seismic method.

MT is best suited to locating vertical boundaries and for studying deep sedimentary basins or areas hampered by surface volcanics. Jones (1987) gave
an interesting account of cases in which MT results aided the
gеоlосіс/tectonic interpretations of the seismic sections. Berkman et al
(1984) have successfully used the combined techniques to delineate the
structure of the South Clay basin, Utah. An integrated interpretation of MT,
gravity and aeromagnetic data sets has also been recommended by Prieto et
al. (1985) who showed that in basalt-covered areas, reasonable rock
compositions and regional structural information can be determined from the
combined data sets.

1.2 Aspects of geology of northern Scotland

1.2.1 Tectonic and Geologic setting

The Northern and Grampian Highlands of northern Scotland are separated
by the Great Glen fault zone and constitute the Metamorphic Caledonides of
Britain. The Metamorphic Caledonides are separated from the northwestern
Lewisian Foreland by the Moine Thrust zone and from the Midland Valley to the
south by the Highland Boundary fault (fig 1.2)

The tectonic style of the region is related to Caledonian crustal deformation
of about 500my ago. Several workers have tried to explain the apparent
complex geology in terms of plate tectonics (Dewey, 1971, 1974; Garson and
Plant, 1973; Phillips et al, 1976; Lambert and McKerrow, 1976) but the structural
framework appears to indicate that the Caledonian mountains were not built
according to the simple rules of modern plate tectonics (Brown 1979). It is
believed, however, that the Caledonian orogenic cycle resulted in welding of
previously separated European continents and the North Atlantic continent
(composed of what are now Greenland and North America), together with a
number of trapped smaller crustal units (Dewey, 1969). Scotland being derived
from the marginal portions of the North Atlantic continent (Watson, 1984). The
manner of assembly of the various crustal units (Massifs) and their accretion to
the large continental margin is largely unknown. The Lewisian foreland is
thought to represent a fragment of the stable North Atlantic shield (Watson
1984).

The geology of northern Scotland is very complex and the distribution of
the major crustal units is shown in fig 1.2. The northwestern part of the
Metamorphic Caledonides is certainly underlain by Lewisian continental crust
(Watson and Dunning 1979). The basement rocks are mainly highly deformed
gneisses and granulites metamorphosed at about 2700 Ma (Scourian) and 1750
Ma (Laxfordian) (see Watson and Dunning, 1979; Johnstone et al, 1979).
Fig 1.2 Map showing the tectonic units of northern Britain and NW Ireland. The inset (lower right) shows the distribution of the Moine and Dalradian rocks (from Watson, 1984).
Lewisian type gneisses are tectonically interleaved with metasediments of the Moine throughout the Northern Highlands and are inferred from similar basement outcrops in Islay and Colonsay to underlie much of the Grampian Highlands (Watson 1984). A basement cover of Torridonian sandstone and younger formations is found in the foreland region. These Foreland rocks are overridden by Moine rocks at the low-angle Moine Thrust.

The Metamorphic Caledonides are represented mostly by the Moine and Dalradian rocks, a complex sequence of metamorphosed sediments (psammites and pelites) which are thought to be of fluviatile or deltaic origin (Johnstone, 1975). Their respective distribution is shown in fig 1.2 (inset). Three structural/lithostratigraphic units are recognised in the Moines northwest of the GGF (Johnstone et al 1969) - from west to east, the Morar, Glenfinnan and Lock Eil divisions with the Sgurr Beag slide (Tanner et al, 1970; Tanner 1971) and the Loch Quoich line (Roberts and Harris 1983) as successive demarcations. An initial late Proterozoic (> 750 Ma) deformation and metamorphism affected these moine rocks which were already crystalline at the onset of the Caledonian cycle (Watson 1984). Immediately to the east of the GGF is a zone of mixed rocks consisting of Moinian ('young Moines' or Grampian Division; see Watson 1984) and Dalradian lithofacies. Further east is the Dalradian assemblage which extends to the Highland Boundary fault. The rocks to the east of the GGF have not suffered any metamorphism prior to the Grampian orogeny (Watson, 1984).

Although a great deal of the surface geology of northern Scotland has been uncovered, the deep structure across the orogen is still poorly understood. The major Caledonian tectonic boundaries trend NE-SW to ENE-WSW and there appears to be some geological evidence for appreciable motion associated with them but no unequivocal model has as yet been developed that is consistent with the vast geological and geophysical observations in this region. The Great Glen fault is one of the main tectonic boundaries and its deep structure and significance form the theme of this study.

1.2.2 Geology of the Great Glen Study region

The area of study lies between Fort William to the south and Fort Augustus to the north and extends for about 20 Km on either side of the GGF.

The surface geology of the study area is rather complex with a dominant NE-SW structural trend. Several structural discontinuities occur in the area of which the GGF is the most outstanding. The GGF is a major crustal feature
Fig 1.3. Simplified geological map of the Great Glen region (after IGS Sheet 62E and Geological map of the U.K. North).
which traverses some 160 Km of the Scottish mainland (Harris et al 1978). The fault zone, consisting of crushed and sheared rocks as well as indurated rocks, is about half a kilometre wide and is bordered on either side by extensively shattered strata. Caught up in the fault zone on the south-east side of Loch Lochy (fig 1.3), and largely bounded by individual fault planes, are narrow strips of Old Red Sandstone sediments representing remnants of an originally more extensive cover to the Caledonian metamorphic rocks. The country rocks are mostly metasediments but plutonic rocks are common. Moine rocks occur to the west of the fault and the zone of interfolded Moine and Dalradian rocks extends eastwards from the fault to the Fort William Slide – a surface along which at an early stage in the Caledonian orogeny Upper Dalradian units became detached from and slid bodily over the underlying rocks cutting out several Dalradian formations (see IGS Sheet 62E). The fault zone is flanked by outstanding igneous complexes eg Clunes, Glen Loy and Ben Nevis. Outcrops of granitic gneisses are also common. Dykes and sheets of Ultrabasic to acidic composition are ubiquitous; a suite of areally restricted (southwest of GGF) Camptonite dykes have been observed to cut the crush rock of the GGF but do not penetrate it to the south-east side (IGS Sheet 62E), a feature that perhaps suggests large-scale horizontal movement along the fault.

As the GGF has been the source of much discussion and speculation, some geological and geophysical observations on the fault and adjacent areas will be summarised in the next section.

1.3 History of Investigation

The GGF has been the subject of intensive studies and various workers (eg Kennedy, 1946; Marston, 1967; Holgate, 1969, Garson and Plant, 1972; Winchester, 1973; Storetvedt, 1974a,b; Chesher and Bacon, 1977; Harris et al 1978; Van der Voo and Scotese, 1981; Smith and Watson, 1983; Briden et al, 1984; Torsvik, 1984; Hall, 1986) have discussed its long history of movement and have reached conflicting conclusions as to the nature and extent of translations along it. The state of stress adjacent to the GGF has been studied by Parson (1979) and does not support any appreciable lateral motion along the fault. The course of the fault off the mainland of Scotland has also been studied. Its precise course to the SW beyond Loch Linhe is almost uncertain, partly because of probable splays. It has been interpreted as extending southwestwards beneath the sea for a distance of about 70 miles by Ahmad (1966). According to McQuilllin and Binns (1973) the main fault appears to pass north of the island of Colonsay whilst a southern branch has been inferred to
pass through Islay connecting with the Leannan Fault in Ireland (Pitcher et al., 1964; Dobson and Evans, 1974). From north of Colonsay the main fault was inferred to continue towards the west or southwest for at least 160 Km by Bailey et al (1975). Its submarine extension has been traced northeastwards to the Shetland Islands where it is represented by the Walls Boundary and Nesting Fault systems (Flinn, 1961,1969). Its total length thus probably exceeds 720 Km (Harris et al 1978). Correlation of the fault zone with similar faults in Newfoundland has been discussed by Wilson (1962), Kay (1967), Webb (1969) and Pitcher (1969) among others. A magnetic high was attributed to the fault by Avery et al (1968) and Ahmad (1966) which was confirmed by the ridge-like features observed along the fault on the regional aeromagnetic map of northern Britain (Hall and Dagley 1970). Hall and Dagley point out that some of the anomalous magnetic features appear to be offset by the fault and consider this as evidence of major sinistral strike-slip movement which is in accord with Kennedy's (1946) interpretation. Recorded seismic activity of the fault has been reported by Dollar (1949) and Ahmad (1966, 1967).

The Strontian and Foyers granite complexes (at the flanks of the fault) — interpreted by Kennedy (1946) as probably part of one rock mass based on outcrop pattern — have been intensively studied by various workers in an attempt to establish the nature and translation on the fault by geophysical means. Ahmad (1966, 1967) concludes from seismic, radiometric, magnetic and gravimetric observations that the Foyers and Strontian granites probably are not part of one rock mass, that the GGF is an active fault and supports a dip-slip movement on the fault since all known epicentres lie to the SE of the fault. The anomaly over the Foyers complex on the smoothed aeromagnetic map of northern Britain was interpreted (Hall and Dagley, 1970) as due to a deep-seated (15Km) magnetic body extending to a depth of about 26 Km which agrees with Ahmad's interpretation, whereas a shallow body extending from the surface to 0.4 Km was suggested for the Strontian granite.

Sparse heat flow measurements (Pugh 1977; Oxburgh et al 1980) are available for the region and point to the deep presence of radiogenic materials in the Great Glen area and especially near the Foyers Granite and southwestwards from Loch Tay. Dimitropoulos (1981) constructed a gravity model for the Grampian region requiring a granitic layer at 7 to 19 Km depths below the surface between the GGF and Loch Tay fault to the south to fit the observed gravity anomaly and this adds credence to the previous heat flow observations.
The 1974 LISPB deep seismic refraction investigation was carried out along a N–S traverse across Britain (fig 1.1) and the results have been analysed by various workers (Bamford et al, 1977, 1978; Bamford 1979; Faber and Bamford 1979, 1981; Hall 1980). Three crustal layers were identified beneath the Scottish Caledonides and the lower crustal layer was found to thicken southwards from the foreland region, but the top of the mid-crustal layer was not delineated with certainty between the Great Glen and Highland Boundary faults (Bamford 1979). Bamford and others also provide evidence for a step in the Moho north of the GGF. These features will be discussed further in chapter 8.

Geoelectromagnetic studies have also been carried out in this region. Hutton et al (1977, 1980, 1981) studied the crust and upper mantle in Scotland and show evidence of a deep conductive structure associated with the GGF and Kirkwood et al (1981) also found a deep-lying conductor at the fault zone. Mbipom (1980) and Mbipom and Hutton (1983) from a traverse parallel to the LISPB line showed the presence of a conductor in the GGF area and the lower crust was found to be distinctly conducting. However, these geoelectromagnetic studies were based on long period (20 -> 1000 sec) observations at stations over 15 Km apart and thus suffered from insufficient shallow depth information and inaccurate spatial resolution of small-scale features.

High resolution deep seismic reflection profiling in offshore Britain and especially in northern Scotland (Smythe et al, 1982; Brewer et al, 1983; Brewer and Smythe 1984; McGeary and Warner, 1985) show spectacular reflections from the Moho and from thrust zones within the Caledonian fold belt and foreland. In particular, Brewer et al (1983) and Hall and others (1984) confirmed the presence of lateral variations in the deep structure in the region of the GGF and the possible association of zones of high seismic reflectivity in the lower crust with electrically conducting layers is of considerable interest at the present time.

1.4 Problems for Research

The exact role of the Great Glen fault (GGF) in the structural evolution of the region is not well understood. Geological reconstructions suggest lateral movements along the fault whereas studies of the state of stress adjacent to the fault (Parson, 1979) in the Fort Augustus area do not indicate lateral displacements. Earth tremors related to the fault zone are recorded in historic time and it is still active.
The aeromagnetic data show that there is a marked contrast in crustal structure across the Great Glen. A NE-SW trend, consistent with the known geology, is dominant and the magnetic field is predominantly positive and of deep seated origin. Magnetic profiles across this positive magnetic zone are interpreted (Hall and Dagley, 1970) to indicate the presence of a thick (~10Km) normally magnetized (about 3 A/m) near-horizontal sheet at depths of 8-15 Km below the surface south of a nearly vertical plane coincident with the GGF; the apparent absence of any strong regional gravity anomaly associated with the GGF on land makes it difficult to accept this interpretation as such thick and magnetic bodies would be associated with density contrasts (Hipkin and Hussain, 1983). Faruquee (1972) found that the main feature of the gravity field associated with the possible extension of the GGF south of Mull and southwest of Colonsay is a broad gravity low. This does not provide a simple solution to the problem posed by the magnetics. However, one wonders if this magnetic sheet can be related to Dimitropoulos’ (1981) Grampian granite layer.

Previous geoelectromagnetic work (Hutton et al, 1981, Mbipom 1980) showed the presence of a 10 Km wide crustal zone of low resistivity in the Great Glen region which connects at depth with a conductive lower crust/upper Mantle layer. Such a large-scale lateral inhomogeneity would be associated with gravity contrasts. This apparent dilemma needs to be resolved. Also, Hutton et al (1977) showed that anomalous conductive zones exist along the GGF and in the north-west corner of Sutherland which correspond with Garson and Plant’s (1972) ancient plate boundaries in the region. This leads us to ask if the region is a plate tectonic synthesis.

Bamford and others (1978) show continuity of layered structure across the fault with a Moho offset to the north of the fault (Faber and Bamford 1981) while Brewer et al (1983) show the presence of lateral variations in the region of the fault. The MOIST reflection profile (Smythe et al 1982) which traverses the offshore extrapolation of the Caledonian Foreland and Orogenic belt (west of the GGF) shows lateral variations in the deep structure of the region as confirmed later by Brewer and others. One wonders if these offshore features have any land analogues.

Having summarised the various observations on the Great Glen fault and adjacent regions, we are now faced with the problems of reconciling geologically inferred lateral movements on the fault with its present day seismicity and inferences from known epicentre distribution (Ahmad 1966), correlation of offshore deep seismic structure with mainland conductivity
structures, the possible existence of other features hitherto undiscovered that may provide the missing link between gravimetric and magnetic data and the speculative possibility that the region is a plate tectonic synthesis. Most of these issues will be addressed in this study.

1.5 Outline of Research

The deep structure of the Great Glen fault is still unresolved and constitutes a major problem of Scottish geology. It was therefore deemed worthwhile to carry out a high resolution magnetotelluric study of this fault as it probably holds the key to the deep geology of this region.

The main objectives of the study are: (1) to determine the conductivity distribution in the neighbourhood of the Great Glen fault and (2) to correlate any observed conductivity structures with available geophysical and geological data.

In line with these objectives, a broadband (.016 - 640 Hz) tensorial MT technique was used to determine the crustal conductivity distribution across the Great Glen fault. Intensive field experiments were conducted in 1985 and 1986 using state of the art technology. Conversion of observed responses (experimental data) into earth conductivity structures in line with modern trends in geophysical data analysis was done by use of efficient computer programs developed by the author. To prevent accidental discoveries by the computer and "structural over-interpretation", the problem of resolution and non-uniqueness was addressed using an iterative most-squares (Jackson, 1976) technique - never before used in the Magnetotelluric situation.

The two-dimensional results were evaluated in the light of the available geophysical and geological data and found to be regionally consistent. An integrated two-dimensional interpretation of MT and gravity data was carried out and this appeared to resolve the apparent dilemma in the previous Great Glen gravity interpretations. Extension of this integrated approach to the adjacent regions indicated the presence of zones of deep crustal/mantle structures; these findings shed some light on the deep structure of the Scottish Caledonides. A speculative tectonic model was proposed for the Scottish highland region.
CHAPTER 2
THE MAGNETOTELLURIC METHOD

The general applicability of the magnetotelluric method can be easily assessed if the differences in spatial behaviour of the electric and magnetic field components, as well as their different behavioural patterns in two- and three-dimensional environments are understood. Following a brief description of the energy source used to probe the earth, the principles of the MT method are outlined in this chapter.

2.1 Source field

The MT source field is the transient portion of the earth’s magnetic field. The time-varying magnetic field induces current fluctuations in the earth which have a wide spectrum. A typical average amplitude spectrum of these magnetic variations shows a minimum at about 1 Hz (fig 2.1) and allows the source field to be classified into two types of activities, one above and the other below 1 Hz.

![Typical amplitude spectrum of magnetic variations in the Extra Low Frequency range](image)

Fig 2.1 Typical amplitude spectrum of magnetic variations in the Extra Low Frequency range (after Keller and Frischknecht, 1966).

The main source of MT fields of frequencies above 1 Hz is worldwide thunderstorm activity, which is extensively concentrated in the tropics and tends to peak in the early afternoon, local time. There are 3 storm centres (Brazil, Central Africa and Malaysia) with an average of 100 storm days per year; their geographic distribution is such that during any hour of the day there is perhaps a storm in progress in one of the centres.

Some of the thunderstorm energy is converted to EM fields which are
propagated with slight attenuation in the earth–ionosphere interspace as a
guided wave (Budden, 1961) and at large distances from the source this is a
plane wave of variable frequency. These MT fields penetrate the earth’s
surface to produce the telluric currents. Generally, these signals from the
lightning strokes (termed sferics) have higher amplitudes at lower latitudes and
the weak currents induced by these fields in the subsurface have amplitude
peaks at distinct frequencies (the Schumann resonances: 8, 14, 20, 25, 32 Hz)
which are used in MT applications. At frequencies of about 2 KHz there is a
strong absorption in the wave guide (Strangway et al, 1973) resulting in
reduction of signal intensity. The signal levels are generally higher in the
summer than in winter (Patra and Mallick, 1980).

Other minor sources of signals above 1 Hz are man–made power
distribution systems which are generally localised and restricted to 50Hz and
harmonics. Spurious signals may be generated in the magnetic field at very
low frequencies by wind vibration of the coil or ground vibration. Ground
discharge currents from the direct lightning strokes also generate spurious
signals in the electric field. However, some of these spurious signals are
uncorrelated in electric and magnetic field records and are readily detected.

The primary contributors to the source field between 1 and $10^{-3}$ Hz are
pulsations in the earth’s magnetic field (Troitskaya, 1967). The pulsations have
some correlation with ionospheric phenomena and are assumed to result
directly from the motion of charged particles above the base of the ionosphere;
current opinion is that they are the magnetic effects of hydrodynamic waves
trapped in the magnetosphere. So far no electrical processes have been
discovered within the earth’s crust which would generate activity of this
frequency range and subcrustal processes would have their waves severely
attenuated by the overlying crust. The reader is referred to the detailed
descriptions of pulsations given by Orr (1973) and Rokityansky (1982).

There are latitudinal variations in the amplitude of the signals but this will
be unimportant for small scale MT measurements. The work of Price (1962),
Orange and Bostick (1965) and Berdichevsky et al (1973) seems to suggest a
very widespread source for the low and middle latitudes. According to
Berdichevsky et al (1973), at mid–latitudes the external field changes
insignificantly over distances of 100 – 200 Km, along the meridian and 200 –
500 Km along the parallel. This determines somewhat the size of the region
within which the use of models with a plane wave as the external field is
justified.
Thus far MT investigators use only 5 of the quantities available at the surface for exploration: three perpendicular components of the magnetic field variations (Hx, Hy, Hz) and two of the electric field variations (Ex, Ey). The vertical electrical field (Ez) and currents (Jz) are not used.

2.2 Maxwell's equations

The magnetotelluric method of determining the electrical conductivity of the subsurface strata depends on the fact that natural electromagnetic waves penetrate into the earth's crust to depths dependent on their frequency and on rock conductivity. Application of Maxwell's equations enables us to determine conductivity as a function of depth from surface measurements of electric and magnetic fields.

Maxwell's field equations in a homogeneous isotropic medium, assuming a time dependence of the type $e^{i\omega t}$ and a charge-free space, are

\begin{align*}
\nabla \times E &= -\frac{\partial B}{\partial t} = -i\omega \mu H \\
\nabla \times H &= J + \frac{\partial D}{\partial t} = (\delta + i\omega \epsilon)E \\
\n\nabla \cdot B &= 0 \\
\n\nabla \cdot D &= 0
\end{align*}

Where $\epsilon$ (F/m) is the permittivity, $\delta$ (mho/m) is the conductivity and $\mu$ (H/m) the magnetic permeability of the medium in which the fields propagate, $B$ is the magnetic induction, $D$ is the displacement vector, $J$ is the current density vector, $E$ is the electric field vector, $H$ is the magnetic field vector and $\omega$ is the angular frequency of the source field.

2.3 One-dimensional (1-D) MT theory

In a 1-D situation, conductivity is a function of depth only. We shall adopt the cartesian coordinate system represented by the axes x(east), y(north) and z(vertically downwards), and assume that the conditions of a plane wave normally incident on the earth are satisfied.

2.3.1 Uniform half-space

Consider a homogeneous half-space model in which the solid earth conductivity, $\delta$ occupies the half-space $z \geq 0$, and $z < 0$ is free space (fig. 2.2). The current sheet induced by a time-varying magnetic field flows parallel to the earth's surface along $0x$. 
Fig 2.2 Homogeneous half space model.

Maxwell's equations combine to give the wave equation

\[ \nabla^2 E = \mu \delta \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2} \]  \hspace{1cm} 2.2

If we apply the vector Laplacian operator to the three rectangular components of \( E \) and note that \( E_y = E_z = 0 \) and \( \partial / \partial x = \partial / \partial y = 0 \), we obtain the expression

\[ \frac{\partial^2 E_x}{\partial z^2} = \mu \delta \frac{\partial E_x}{\partial t} + \varepsilon \frac{\partial^2 E_x}{\partial t^2} \]  \hspace{1cm} 2.3

The elementary solution of eq. 2.3 is:

\[ E_x = A e^{Yz} + B e^{-Yz} \]  \hspace{1cm} 2.4a

The MT field is known to be quasi-periodic in the frequency range used and for a harmonically time-varying field we have a general solution:

\[ E_x = A e^{i(\omega t + Yz)} + B e^{i(\omega t - Yz)} \]  \hspace{1cm} 2.4b

where \( Y = \pm (i\omega \mu \delta - \varepsilon \mu \omega^2)^{1/2} \) is the wave propagation constant referred to as the radian wave-length or wave number, and \( A \) and \( B \) are arbitrary constants which are evaluated by applying the boundary conditions, i.e., the continuity of the tangential electric and magnetic components across the interface. However, at the frequencies of field variations considered in MT work, resistivity is more dominant than the dielectric constant or the magnetic permeability and \( Y = (i\omega \mu \delta)^{1/2} \).

In the present problem, \( E_x \) vanishes as \( z \rightarrow \infty \) so that

\[ E_x = B e^{-Yz} \]  \hspace{1cm} 2.5

From Maxwell's equation (2.1a) we obtain

\[ \frac{\partial E_x}{\partial z} = -i\omega \mu H_y \]

Differentiating eq. 2.5 with respect to \( z \) and rearranging we obtain
\[ H_y = \frac{1}{i \omega \mu} \gamma B e^{-\gamma z} \]  
and

\[ Z = \frac{E_x}{H_y} = \left( i \frac{\omega \mu}{\delta} \right)^{1/2} \]  

where \( Z \) is the wave (or Cagniard) impedance of the homogeneous ground, defined by the ratio of orthogonal components of \( E \) and \( H \).

If however, we consider the magnetic field \( H_x \) along \( OX \) (fig. 2.2) instead of the electric field and solve the equation

\[ \nabla^2 H = \mu \delta \frac{\partial H}{\partial t} + \mu c^2 \frac{\partial^2 H}{\partial t^2} \]

we obtain

\[ Z = \frac{E_y}{H_x} \]

From eq. 2.7 we can define Cagniard's apparent resistivity

\[ \rho = \frac{1}{\omega \mu} |Z|^2 \]

with a constant phase of 45° between \( E \) and \( H \).

The propagation constant can be separated into real and imaginary parts

\[ \gamma = (\omega \mu / 2 \rho)^{1/2} + i(\omega \mu / 2 \rho)^{1/2} \]

(indicating an exponential decay of amplitude along the travel path and an oscillatory nature of the wave amplitude), and can be used to define the skin depth (the depth at which the amplitude of the field has been attenuated by 1/e of its value at the surface) given by

\[ d = 1 / \text{real} (\gamma) = (2 \rho / \omega \mu)^{1/2} \]

We therefore can calculate the penetration depth of EM waves in a homogeneous earth for different resistivities using eq. 2.9b as shown in fig 2.3.
Fig. 2.3 Skin depth as a function of period and ground resistivity.
2.3.2 n-layered half-space

Let us now consider a model in which the earth is represented by \( n \) horizontal layers where the resistivities of the layers are \( \rho_1, \rho_2, \ldots, \rho_n \) respectively and the thicknesses of the top \( n-1 \) layers are \( h_1, h_2, \ldots, h_{n-1} \) as shown in fig 2.4.

\[
\begin{array}{c}
| \hline
| h_1 & \rho_1 \\
| h_2 & \rho_2 \\
| \vdots \\
| h_{n-1} & \rho_{n-1} \\
\hline
\end{array}
\]

Fig 2.4 n-layer earth

The ratio of orthogonal components of \( E \) and \( H \) yields the impedance in the general medium

\[
Z = -\frac{i\omega \mu}{\gamma} \coth(\gamma z + \ln A/B)
\]

2.10

The constants \( A \) and \( B \) can be eliminated by considering boundary conditions or by considering only the ratios of wave impedances at two different points. If we utilize the ratio of wave impedances and evaluate the impedance \( Z_2 \) at depth \( z_2 \) with reference to \( Z_1 \) at \( z_1 \) in the same medium and solve eq. 2.10 for \( \ln \sqrt{A/B} \) we find that

\[
Z_2 = -\frac{i\omega \mu}{\gamma}[\coth(\gamma(z_2 - z_1) - \coth^{-1}(Z_1\gamma/i\omega \mu))]
\]

2.11

Eq. 2.11 holds for \( z_1 \) and \( z_2 \) in the same medium. For a homogeneous semi-infinite medium, with \( z_2 = 0 \), and \( (z_2 - z_1) = h_1 \), the thickness of the top layer, then \( Z_2 \) at the ground surface \( (z = 0) \) becomes

\[
Z_2 \text{ (surface)} = \frac{i\omega \mu}{\gamma}\coth(\gamma h_1 + \coth^{-1}(\gamma Z_1(z=h_1)/i\omega \mu))
\]

2.12

In a layered earth, the continuity conditions which must hold at each boundary permit us to express the wave impedance observed at the surface in terms of the wave impedances at each of the lower layers. The general expression for the impedance at the surface of an \( n \)-layered half-space is

\[
Z_0 = \frac{i\omega \mu}{\gamma_1}\coth(\gamma_1 h_1 + \coth^{-1}(\gamma_1 h_1 + \coth^{-1}(\gamma_2 h_2 + \coth^{-1}(\gamma_2 h_2 + \ldots + \coth^{-1}(\gamma_{n-2} h_{n-2} + \coth^{-1}(\gamma_{n-1} h_{n-1} + \coth^{-1}(\gamma_{n-1} h_{n-1} + \ldots))))))
\]

2.13

Equation 2.13 is used in the 1-D MT forward problem (discussed in chapter 6) to calculate the apparent resistivity and phase of a layered earth structure at
any given frequency.

2.4 Two- and Three-dimensional MT theory

When the ground conductivity varies in the horizontal direction and with depth, $\delta(x,y,z)$, the relation between horizontal electric and magnetic field components cannot be expressed by eq. 2.7 but by a tensor impedance to accommodate lateral inhomogeneities and anisotropic effects (Cantwell, 1960): that is, the scalar impedance relationship $E = ZH$ becomes

$$
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix} =
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
H_x \\
H_y
\end{bmatrix}
$$

where $Z_{xy}$, $Z_{yx}$ are the principal impedances and $Z_{xx}$, $Z_{yy}$ are the subsidiary (or additional) impedances due to contributions from parallel components of the magnetic fields. In this situation, the E fields are dependent on both parallel and orthogonal H fields and the impedance matrices vary as a function of measurement position (with respect to geoelectric contrasts) and frequency. If a structure extends over a great distance only in one direction, this direction is called the strike. In the MT situation, the criterion for 2-dimensionality is that extension along strike is great compared to the skin depth of the incident EM fields.

2.4.1 Two-dimensional (2-D) structures

In a 2-D environment, conductivity is a function of depth and one horizontal direction, i.e. $\delta = \delta(y,z)$, as shown in figure 2.5.

![Fig 2.5 Two-dimensional structure](image)

Assuming that the source field assumption still holds and an invariance with respect to the $x$-axis (fig 2.5), the total EM field splits into two independent modes, E- and H-polarisations with the E and H fields respectively polarised in the strike direction. The impedances due to E- and H-polarisations
are different (O'Brien and Morrison, 1967) depending on the frequency and the location of measurements with respect to the resistivity discontinuity.

Starting from Maxwell's equations (2.1 a and b) and neglecting displacement currents as before, we formulate the appropriate field equations in two dimensions as follows:

(a) for the H-polarisation case (H=Hx(y,z) and E=(Ey,Ez))

\[
\begin{align*}
\delta Ez/\delta y - \delta Ey/\delta z &= -i\omega \mu Hx \quad 2.15a \\
\delta Ey &= \partial Hx/\partial y \quad 2.15b \\
\delta Ez &= \partial Hx/\partial z \quad 2.15c
\end{align*}
\]

and the Hx function satisfies the equation

\[
\text{div} \left( 1/\delta \text{ grad } Hx \right) - \gamma^2/\delta Hx = 0
\]

(b) for the E-polarisation case (E=Ex(y,z), H=(Hy,Hz)) we have

\[
\begin{align*}
\delta Ex &= (\delta Hz/\delta y - \delta Hy/\delta z) \quad 2.16a \\
\delta Ex/\delta y &= iw\mu Hz \quad 2.16b \\
\delta Ex/\delta z &= -i\omega \mu Hy \quad 2.16c
\end{align*}
\]

and the Ex function satisfies the equation

\[
\text{div} \left( 1/\mu \text{ grad } Ex \right) - \gamma^2/\mu Ex = 0
\]

That is, equations 2.15 and 2.16 combine to give the Helmholtz equation of the form

\[
\left( \partial^2 F/\partial y^2 + \partial^2 F/\partial z^2 \right) - \gamma^2 F = 0 \quad 2.17
\]

and satisfy the conditions of the continuity of F and \( \partial F/\partial n \) at the relevant boundaries, where F=Hx or Ex, and \( \gamma^2 = i\omega \mu \delta \), and n is the normal to the layer boundary if we consider horizontally inhomogeneous media (Patra and Mallick, 1980).

It is not always possible to evaluate the response of 2-D structures analytically. Analytical solutions exist only for simple structures (e.g. Rankin, 1962; d'Erceville and Kunetz, 1962; Weaver, 1963; Hobbs, 1975). In most cases numerical methods are used to calculate the responses of arbitrary 2-D resistivity structures. There are four main methods in current use: the transmission line analogy (Madden and Thompson, 1965; Wright 1969; Swift 1967,1971; Ku et al, 1973), the integral equation (Patra and Mallick, 1980), the finite element (Coggon 1971; Silvester and Haslam, 1972; Reddy and Rankin, 1973; Wannamaker et al 1985, 1986) and the finite difference (Patrick and
Bostick 1969; Jones and Price 1970; Jones and Pascoe, 1971,1972; Williamson et al 1974; Brewitt-Taylor and Weaver 1976; Weaver and Brewitt-Taylor, 1978) methods. In all these methods the region to be modelled is divided into a mesh of elements, with the edges set far enough from any lateral discontinuities so that the boundary conditions are satisfied.

The finite difference method (which was used in this study) is centred on solving the Helmholtz EM field equation (2.17) for both E- and H-polarizations. The main procedure involves setting up a rectangular grid, representing the conductivity distribution, with variable grid spacings to suit the requirements near conductivity boundaries and near the surface of the model where the normal derivative must be estimated for calculating the field components. The equations for EM fields are represented by a set of finite difference equations for each point on the grid. Derivatives are replaced with difference quotients thus reducing the differential equations to a set of linear equations for the field values at the grid points. Boundary conditions are also reduced to difference conditions and added to the system of algebraic equations that are solved by the Gauss-Seidel iterative method.

The finite difference scheme is conceptually the simplest and probably the most widely used method. From its early days of fixed grid spacings and sharp transitions in conductivity across boundaries it has developed into a greatly improved (in terms of accuracy of solutions and ease of operation) modelling tool. Williamson and others corrected the original formulation of Jones and Pascoe to allow for variable grid spacings, Brewitt-Taylor and Weaver's smooth transitions in conductivity helped overcome the early problem of fitting field values across sharp conductivity boundaries and the work of Weaver and Brewitt-Taylor led to improved boundary conditions.

It should be borne in mind that the various numerical techniques essentially approximate otherwise continuous functions by values at discrete points within a mesh of finite dimension and assume some (commonly linear) functional relation for the field variations between the grid points. The calculated responses are therefore prone to errors if the discretization is not good enough and the solutions become unstable when the overall grid size becomes too large. It is imperative therefore that attention be paid to the way and manner in which a problem is discretized.
2.4.2 Three-dimensional (3-D) structures

In many MT problems, the conductivity distribution is actually 3-D. In a 3-D structure, the conductivity is a function of all coordinates, i.e. $\sigma = \sigma(x,y,z)$. Analytical solutions to Maxwell's equations in 3-dimensions are difficult and intractable and such problems at best are solved by the approximate techniques. The most successful methods of solution include the differential equation methods (e.g. Jones and Pascoe, 1972; Lines and Jones, 1973 a,b; Jones and Vozoff, 1978; Pridmore et al, 1981), the integral equation techniques (Raiche, 1974; Weidelt 1975a; Ting and Hohman, 1981; Das and Verma, 1982; Wannamaker et al, 1984), thin sheet approximations (Dawson and Weaver 1979; Ranganayaki and Madden, 1980; Madden, 1980; Madden and Park 1982; Park et al, 1983; Park 1985) and hybrid techniques (Lee et al, 1981). Analogue scale modelling experiments have also been used to study 3-D structures (e.g. Rankin et al, 1965; Dosso, 1966, 1973; Dosso et al, 1980; Nienaber et al, 1981).

2.5 Earth response functions

The MT data recorded on the ground surface provide valuable information about the earth. Any function that can be derived from such a record is an earth response function (Rokitvansky 1982). Such a function characterises the conductivity structure of the earth and could be the impedance tensor, the apparent resistivity or the phase. The conductivity strike and the dimensionality characteristics of the earth are auxiliary parameters which illuminate the nature of the conductivity structures.

2.5.1 Indicators of structure

Eq. 2.14 defines the relationship between the horizontal components of the electric and magnetic fields in the frequency domain with the time variations of the E fields given by the convolution of the earth response function $Z_a$, $Z_b$, $Z_c$, $Z_d$ say, with the H fields. Put simply,

$$E = Z.H$$

where the elements of $Z$ are the Fourier transforms of $Z_a$, $Z_b$, $Z_c$ and $Z_d$. or

$$E_x = Z_{xx}H_x + Z_{xy}H_y$$

$$E_y = Z_{yx}H_x + Z_{yy}H_y$$

In a 1-D environment, satisfying Cagniard's (1953) tabularity conditions,

$$Z_{xx} = Z_{yy} = 0$$

$$Z_{xy} = -Z_{yx}$$

so that the scalar impedance relationship (2.7) holds good. Cagniard (1953)
defined an apparent resistivity (eq. 2.8) which in practical units is given by
\[ \rho_a = 0.2T |Z_{xy}(yx)|^2 \]  
and phase
\[ \phi = \arg(Z_{xy}(yx)) \]
where \( T \) is the period in seconds, and \( E \) and \( H \) fields are measured in mV/Km and gammas (or nanoteslas) respectively and \( \rho_a \) is in ohm-metres.

In a 2-D environment, \( Z_{xx} \) and \( Z_{yy} \) are non-zero and \( Z_{xy} \neq -Z_{yx} \) in general. The elements of \( Z \) vary as the coordinate axes are rotated. When the measurement axes are aligned with the structural strike \( Z_{xx} = Z_{yy} = 0 \), so that
\[ Z_{xy} = Ex/Hy = E_{\parallel} \]
and
\[ Z_{yx} = Ey/Hx = E_{\perp} \]

However, strictly 2-D structures are rare in nature and MT measurements are made in two orthogonal directions which may not be aligned with the (unknown) strike.

Suppose our measurement axes \((x,y)\) form an angle \( \theta \) (measured clockwise from the \( x \)-axis) with the true strike, we want to determine our field components in the (preferred) principal anisotropy axes \((x',y')\). Let \( R \) be the coordinate rotation matrix for a vector in the \((x,y)\) plane given by
\[ R = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \]

On rotation of the tensor elements from \((x,y)\) to \((x',y')\) by an angle \( \theta \) about \( Z \) (fig 2.6), the rotated impedance tensor becomes
\[ Z' = RZR^{-1} \]
so that the impedance tensor relationship in the rotated frame is

\[ E' = RZR^{-1}H^{-1} = Z'H' \]  

2.23

Expanding eq. 2.22 we obtain (for the general case) the elements of \( Z' \) in terms of the elements of \( Z \), viz:

\[ Z'_{xx}(\theta) = \frac{(Z_{xx} + Z_{yy})}{2} - \frac{Z_{0}(\theta + \pi)}{4} \]  

2.24a

\[ Z'_{yy}(\theta) = \frac{(Z_{yy} + Z_{xx})}{2} + \frac{Z_{0}(\theta + \pi)}{4} \]  

2.24b

\[ Z'_{xy}(\theta) = \frac{(Z_{xy} - Z_{yx})}{2} + \frac{Z_{0}(\theta)}{2} \]  

2.24c

\[ Z'_{yx}(\theta) = \frac{(Z_{yx} - Z_{xy})}{2} + \frac{Z_{0}(\theta)}{2} \]  

2.24d

where

\[ Z_{0} = \frac{(Z_{xx} + Z_{yx}) \cos 2\theta - (Z_{xx} - Z_{yy}) \sin 2\theta}{2} \]

If the real and imaginary parts of \( Z_{0} \) are plotted on an Argand diagram for varying \( \theta \), it is obvious that a 1-D structure will give a single point locus, whereas a 2-D structure will generate a straight line and a 3-D structure will generate an ellipse (Sims, 1969).

In practise, on rotation through 180 degrees, two minima for the diagonal elements are obtained and the corresponding axes are termed the principal conductivity axes. The apparent resistivities and phases in these directions are called major and minor apparent resistivities and phases. These are expressed as

\[ \rho_{maj} = 0.2T |Z'_{xy}|^2, \phi_{maj} = \text{arg}(Z'_{xy}) \]  

2.25a
\[
\rho_{\text{min}} = 0.2T \left| Z'yx \right|^2, \quad \phi_{\text{min}} = \arg(Z'yx)
\]

It is now relatively common to define an effective response function for a medium which is rotationally invariant (Tikhonov and Berdichevsky, 1966; Ranganayaki, 1984). The effective impedance, which is related to the determinant of the system of equations 2.14 is given by

\[
Z_{\text{eff}} = (ZxxZyy - ZxyZyx)^{1/2}
\]

It has the physical sense of mean impedance for the medium. The corresponding apparent resistivity and phase are given by

\[
\rho_{\text{eff}} = \frac{1}{\mu \omega} \left| ZxxZyy - ZxyZyx \right|
\]

and

\[
\phi_{\text{eff}} = \text{phase of } (ZxxZyy - ZxyZyx)
\]

In a strictly 2-D environment, \(\rho_{\text{eff}} = (\rho_{xy} \rho_{yx})^{1/2}\) and \(\phi_{\text{eff}} = \phi_{xy} + \phi_{yx}\). These two quantities \((\rho_{\text{eff}}, \phi_{\text{eff}})\) are the geometric mean of parallel and perpendicular resistivities and phases and have been interpreted by Ranganayaki (1984) as scalar averages for the medium by analogy with mixtures, where the geometric mean gives an accurate estimate of the physical properties (Madden, 1976).

2.5.2 Indicators of structural dimension

The earth is complex and in order to obtain a good picture of the subsurface structural patterns, information from the parameters discussed in section 2.5.1 are interpreted together with the constraints provided by the dimensionality indicators. They point to the degree of anisotropy of the earth and are extracted from the rotated impedance tensor \(Z'\).

2.5.2.1 Electrical strike direction and Tipper

If a well defined conductivity strike can be obtained, then the structure is at least 2-D. Two principal directions can be found (section 2.5.1) by determining the angle \(\theta_{o}\) and \((\theta_{o} + 90\ degrees)\) at which \(Z'xy(\theta)\) is maximized and \(Z'yx(\theta)\) is minimized. One of these directions is the strike for a 2-D structure. The common practice is to maximize some suitable functions of \(Zxy\) and \(Zyx\) during coordinate axes rotation. Everett and Hyndman (1967a) maximize \(\left| Z'xy \right|\); and Swift (1967) maximizes \(\left| Z'xy \right|^2 + \left| Z'yx \right|^2\) analytically using the formula

\[
\theta_{o} = 1/4 \arctan \frac{(Zxx - Zyy)(Zxy + Zyx)^* + (Zxx - Zyy)^*(Zxy + Zyx)}{\left| Zxx - Zyy \right|^2 - \left| Zxy + Zyx \right|^2}
\]

where * denotes complex conjugate.
Ideally, $Z'_{xx}$ and $Z'_{yy}$ should be zero at this position but they rarely vanish. However, Jones and Vozoff (1978) have indicated that even for a 3-D variation, eq. 2.28 will yield a general strike direction.

Generally, the observed vertical magnetic field $H_z$ is used to help determine which of the 2 principal directions is the strike direction. In the 2-D case, the horizontal direction in which the magnetic field is most highly coherent with $H_z$ will be perpendicular to the electrical strike direction (Vozoff, 1972).

The relationship between the measured $H_z$ and the horizontal components at a single site can be expressed (Everett and Hyndman 1967b, Madden and Swift, 1969) in a simple form

$$H_z = A H_x + B H_y$$

where $A$ and $B$ are the geomagnetic single station transfer functions calculated from cross-spectral estimates. These functions can be visualized as operating on the horizontal magnetic field and tipping part of it into the vertical. Using the coefficients $A$ and $B$, Vozoff (1972) defined a quantity, tipper or vertical field transfer function given by

$$T = \sqrt{A^2 + B^2}$$

and * denotes complex conjugate and $<>$ an average over a frequency band.

In a 1-D structure the vertical transfer function is zero. $A$ is zero for an x-trending 2-D structure and $T$ can thus be used to assess the 2-D character of the MT data. The differences in the strike directions determined by impedance tensor rotation and the vertical-horizontal magnetic field relationships are thought to be a measure of 3-dimensionality (Vozoff, 1972).

2.5.2.2 Impedance Skew

The impedance skew (Swift, 1967) is the amplitude of the ratio of off-diagonal to diagonal elements of the impedance tensor and is expressed as

$$Skew = \left| \frac{Z_{xx} + Z_{yy}}{Z_{xy} - Z_{yx}} \right|$$

It is a measure of the EM coupling between the measured $E$ and $H$ fields along coincident directions (Vozoff, 1972). There will be no coupling in a 1-D
situation or if our measurement axes coincide with the principal axes of a 2-D structure or if our measurement site is located over a point of radial symmetry in a 3-D structure (Word et al, 1970; Kaufman and Keller 1981). It is rotationally invariant; the parameter combinations \((Z_{xx} + Z_{yy})\) and \((Z_{xy} - Z_{yx})\) are independent of \(\theta\) and can be used to define two invariant quantities

\[
I_1 = Z_{xx} + Z_{yy}
\]

\[
I_2 = (Z_{xy} - Z_{yx})/2
\]

(see Berdichevsky and Dmitriev, 1976) which have found use in 1-D MT modelling studies. Skew is zero for ideal 1-D and 2-D cases and is non-zero for general 3-D structures. However, in practical field situations, skew is non-zero and there is no consensus as to what value it should take in a 2- or 3-D structure (see Kao and Orr, 1982). However, from 3-D modelling studies Reddy et al (1977a), Ting and Hohman (1981), and Park et al (1983) suggested upper limits of 0.4, 0.12 and 0.5 respectively for the onset of 3-D behaviour. A value of \(< 0.4\) is generally used in routine studies as indicative of 2-dimensionality.

2.5.2.3 Ellipticity and eccentricity

Ellipticity (Word et al 1970) is a 3-D parameter related to the principal radii of the Z rotational ellipse, defined as

\[
\beta_0 = \frac{|Z_{xx}(\theta_0) - Z_{yy}(\theta_0)|}{|Z_{xy}(\theta_0) + Z_{yx}(\theta_0)|}
\]

\(\beta_0\) is zero for 1-D and 2-D cases. It is used as a semi-quantitative measure of 3-dimensionality of the structure and the degree of coupling between individual 3-D features (Word et al, 1970).

Eccentricity is an indicator of a 3-D structure. Word et al, (1970) defined eccentricity of the rotation ellipse as

\[
\beta(\theta) = (Z_{xx} - Z_{yy})/(Z_{xy} + Z_{yx})
\]

It is dependent on the rotation angle and is zero for a 2-D structure when evaluated at the strike direction. Thus \(\beta(\theta_0) = \beta_0\).

2.6 Estimation of the impedance matrix

The estimation of the impedance tensor elements is usually done in the frequency domain. Sims and Bostick (1969) and Sims et al (1971) have discussed a classical least squares spectral analysis procedure for optimizing the estimates of the tensor elements from a large number of independent record sets.
The natural MT signals are generally contaminated with noise which add onto the spectral estimates of the complex amplitudes of the E and H field variations at a given frequency degrading the relationships

\[ \begin{align*} 
E_x &= Z_{xx}H_x + Z_{xy}H_y \\
E_y &= Z_{yx}H_x + Z_{yy}H_y 
\end{align*} \]

The method of Sims and Bostick (1969) and Sims et al (1971) uses the complex conjugate of each component and minimizes the noise power on the magnetic or electric channels to yield four cross-spectral equations (for each of the two matrix equations). Any two of the four equations (6 possible combinations) may be solved simultaneously for the relevant tensor element so that six distinct equations emerge for each of the tensor elements. For example, six different estimates of the element \( Z_{xy} \) may be computed using the equations

\[ \begin{align*} 
\langle Z_{xy} \rangle &= \langle H_x \bar{E}_x \rangle \langle \bar{E}_x E_x \rangle - \langle H_x \bar{E}_y \rangle \langle \bar{E}_y E_x \rangle \quad 2.34a \\
\langle Z_{xy} \rangle &= \langle H_x \bar{E}_x \rangle \langle \bar{E}_x E_y \rangle - \langle H_x \bar{E}_y \rangle \langle \bar{E}_y E_y \rangle \quad 2.34b \\
\langle Z_{xy} \rangle &= \langle H_x \bar{E}_y \rangle \langle \bar{E}_x H_x \rangle - \langle H_x \bar{E}_x \rangle \langle \bar{E}_x H_y \rangle \quad 2.34c \\
\langle Z_{xy} \rangle &= \langle H_x \bar{E}_y \rangle \langle \bar{E}_y H_x \rangle - \langle H_x \bar{E}_x \rangle \langle \bar{E}_x H_y \rangle \quad 2.34d \\
\langle Z_{xy} \rangle &= \langle H_x \bar{E}_y \rangle \langle \bar{E}_x H_y \rangle - \langle H_x \bar{E}_x \rangle \langle \bar{E}_x H_y \rangle \quad 2.34e \\
\langle Z_{xy} \rangle &= \langle H_x \bar{E}_y \rangle \langle \bar{E}_y H_y \rangle - \langle H_x \bar{E}_x \rangle \langle \bar{E}_x H_y \rangle \quad 2.34f 
\end{align*} \]

where * denotes complex conjugate and \( \langle Z_{xy} \rangle \) is one measured estimate of \( Z_{xy} \).

The \( Z_{ij} \) are usually assumed to be slowly varying functions of frequency (although this may not always be true as in the case near vertical conductivity boundaries) so that \( \langle A_i B_j \rangle \) may be regarded as averages over some finite bandwidth or the cross-power spectrum between \( A_i \) and \( B_j \) at some centre frequency. Two of the equations (2.34c and d) are relatively unstable in the 1-D case where the fields are unpolarised since \( \langle E_x \bar{E}_y \rangle, \langle E_x \bar{H}_y \rangle, \langle E_y \bar{H}_x \rangle \) and \( \langle H_x \bar{H}_y \rangle \) tend to zero. The other four equations are stable in the absence of highly polarised incident fields (Sims et al, 1971). Eqs. 2.34e and f are
biased downward by random noise on H (and not by noise on E) and eqs. 2.34a and b give estimates that are biased up by random noise on E (and not by noise on H). Eq. 2.34f is most commonly used in MT data analysis where it is assumed that the H field is less contaminated with noise than the E field. The remote reference technique (Gamble et al., 1979) uses eq. 2.34f but with H' and Hx obtained from a remote station with (assumed) independent local noise contributions.

2.7 Coherence

Let S(t) and X(t) be two time series and S(f) and X(f) be their respective Fourier transforms. The coherence between the two signals is given by

$$C_{sx}^2 = \frac{\langle S^*(f) X(f) \rangle \langle S(f) X^*(f) \rangle}{\langle S(f) S^*(f) \rangle \langle X(f) X^*(f) \rangle} \quad 2.35$$

where $0 \leq C_{sx}^2 \leq 1$ for all frequencies (Bendat and Piersol, 1971). $C_{sx}^2$ will be unity if the two signals are perfectly correlated and will be zero if they are uncorrelated.

Real signals contain additive noise. Now, let us consider S(t) as a linear combination of two signals U(t) and V(t) so that

$$S(f) = U(f) + V(f) \quad 2.36$$

with an expected value $\bar{S}(f)$ given by

$$\bar{S}(f) = \langle U(f) \rangle + \langle V(f) \rangle \quad 2.37$$

where $\langle U(f) \rangle$ and $\langle V(f) \rangle$ are averaged values of the transfer functions in equation 2.36.

The coherence between a signal S(f) and its predicted value $\bar{S}(f)$ is known as the Predicted Coherence defined as

$$C_{ss}^2 = \left| \langle U(f) \rangle \langle V(f) \rangle \right|^2 / \left| \langle U(f) \rangle \right|^2 \left| \langle V(f) \rangle \right|^2 \left( \left| \langle U(f) \rangle \right|^2 + \left| \langle V(f) \rangle \right|^2 \right) \quad 2.38$$

where $P_{AA} = \langle A(f) A^*(f) \rangle$ and $P_{AB} = \langle A^*(f) B(f) \rangle$.

Equation 2.36 is very similar to the MT matrix equations 2.18b. If we substitute the values of the impedance estimates (section 2.6) into eq. 2.18b we obtain the predicted values of the two electric fields Ex and Ey. The coherences between the pairs (Ex, Ex) and (Ey, Ey) are known as their predictabilities (Swift, 1967) and for Ex, say is expressed as

$$Coh(Ex, Ex) = \frac{1}{\left[ \left( \langle Ex Ex^* \rangle \langle Ex Ex^* \rangle \langle Ex Ex^* \rangle \right)^{1/2} \right]} \quad 2.39$$

In MT work, the predicted coherence is preferred to the ordinary coherence.
CHAPTER 3
FIELD STUDIES

A description of the audiofrequency magnetotelluric (AMT) data acquisition system, its calibration and field installation are now given, followed by information about the survey sites.

3.1 Field equipment specification

The most remarkable practical interest of the MT method comes from the fact that the depth of penetration is linked to the measurement frequencies, and if use is made of a sufficiently large range of frequencies one can explore electrical, crystalline basement or sedimentary subsoil of great thickness. The aim of this project is to obtain depth and resistivity distributions across the Great Glen fault. The depth of interest is from about 100m to 30–60Km. Previous studies in the region have shown that upper crustal resistivities are in the range 1000 -> 10000 Ohm·m and that the lower crust/upper mantle is characterised by resistivities in the range 50–500 Ohm·m.

Using the above information, the potential maximum and minimum measurement frequencies may be estimated through the skin depth equation (2.9b). Selecting conservative resistivity values of 100–3000 Ohm·m as possible crustal averages for the region and a frequency range 600–0.01 Hertz, skin depths are approximately 204m–1.1Km at 600Hz and 50Km–274Km at 0.01Hz. However, since the skin depth is practically several times greater than the actual maximum depth from which information might be derived and considering an overestimation by a factor of about 5 (Vozoff et al, 1982), we expect information from about 40–220m to 10–55Km. It was thus considered satisfactory to employ the range 0.01–600Hz in this study.

The nature of the MT signals was also considered when specifying the type of equipment to be used. The main characteristics of the signals are:

1) a practically continuous spectrum of frequency; only signals with frequencies of interest need be retained, and

2) an amplitude that is frequency dependent and also varies as a function of time (or season). Values less than 10 mV/Km (electric signal) and a few milligamma (magnetic signal) are not uncommon and we therefore need to be able to detect (with precision) and amplify the signals.

Moreover, the high frequencies of interest dictate that the signals be
received and stored at a suitable rate. For all these reasons, the desired MT measurement system must possess an excellent signal-to-noise ratio, strong amplification, ability to process digitally and record data for a broad range of frequencies.

3.2 Field instrumentation
The basic MT instrumentation consists of the signal detecting system (magnetic and electric sensors) and the data recording equipment.

3.2.1 Magnetic and electric field sensors
The magnetic sensors used in this study consisted of two sets (CM16 and CM11E) of three induction coils (sensitivity 50 mV/γ) supplied by Societie ECA - France (the sensors were developed by Dr G.Clerc, Garchy, France). There is a flat response between 3 and 800 Hz for the CM16 coils and 0.012 and 100 Hz for the CM11E coils. Each coil consists of turns of fine copper wire wound on a core of high magnetic permeability, and a preamplifier with low-noise connections and components, all enclosed in a waterproof casing. The three orthogonal magnetic fields were measured with separate coils. For the electric field measurements, polarizable electrodes (copper rods) were mostly used. The copper electrodes were about 30cm long and 2cm in diameter; they were tapered at one end for easy insertion into the ground and each had a (clip) cable-connection at the other end. Petiau and Dupis (1980) have shown that non-polarizable electrodes have the lowest noise characteristics and are superior to the polarizable types at frequencies below 10Hz but that both types produce similar results at frequencies above 10Hz. However, in this study "Dupis" dyke electrodes were used only at a few sites for measurement below 10Hz. Each of the "Dupis" electrodes consisted of a lead tube in lead chloride solution housed in a plastic tube with a porous base. The dimensions are about the same as for the copper rods. The electric field in the earth was measured with 2 pairs of electrodes placed about 50–100m apart.

3.2.2 Data recording equipment
The Short Period Automatic Magnetotelluric (S.P.A.M.) MKII real-time data acquisition and analysis system developed at Edinburgh University by Dawes (1984) was used to record data digitally on cartridge tapes over the frequency range 0.016–640Hz. The S.P.A.M. MKII is a portable battery operated, digital 7-channel system and incorporates full tensorial analyses, a remote reference facility and one-dimensional inversion to resistivity as a function of depth. A detailed description of the S.P.A.M. MKII system can be found in Hutton et al,
(1984) and only the essential features shall be recounted here. A block diagram of the complete in-field system is given in fig 3.1.

Two versions of S.P.A.M. were used in this study. In one version (belonging to Edinburgh University) the frequency sounding bandwidth was from 640 to 0.09Hz, while the other version (on loan from N.E.R.C.) the frequency bandwidth was 128-0.016Hz. The total S.P.A.M. frequency range is divided into 4 overlapping bands to prevent the risk of large amplitude signals saturating any of the channels or of signals of weak amplitude being obliterated by the background noise. The gain can therefore be optimized. The main units of the system are (i) a microcomputer (LSI 11 with a 64 Kilobyte RAM), a programmable amplifier/filter bank and a power regulator; (ii) twin cartridge (programs and data) storage deck; (iii) alphanumeric pocket VDU and keyboard; (iv) 40 column miniature electrosensitive printer for listing and plotting results; (v) sensor distribution box and (vi) a power distribution box.

The field programs are read off the program tape by the computer and any useful results written onto the other tape. These 4–track TU58 tapes can each hold sufficient data for all the 4 bands recorded at each field location. The main program facilities are: software band selection; instrument response corrections; automatic gain adjustment and signal selection, with manual override; alteration of signal selection criteria, and continuous or switchable display of results (which can be listed on the printer or stored on tape).

The sensor distribution box contains the telluric preamplifiers and connections to the induction coils. The signal from each electrode pair is connected to a preamplifier with a gain of 40. The signals from the induction coils are brought to the sensor distribution box by 3m long connecting cables. A 50m individually shielded multiconductor cable powers the box (the telluric preamplifiers and the coil amplifiers in the coil housings) and takes the electric and magnetic signals back to the amplifier/filter bank. The maximum allowable (but changeable) input range of the analog to digital converter (ADC) is ±5V and all incoming signals are tested to see that they did not exceed this range. All the 7 channels have switchable 50 and 150Hz notch filters to eliminate noise from power transmission lines. Each amplifier/filter unit has automatic (or manual) gain adjustment with hardware band selection. From the amplifier/filter bank the signals are sent to the multiplexer – ADC unit where the channels are sequentially sampled and digitized. The sampling frequency is different for each frequency band. Each data set is called an ‘event’ or ‘window’ and consists of 256 samples for each of the 7 channels.
Fig 3.1 Block diagram of complete in-field MT System (after Dawes, 1984)
Fig 3.2 Simplified flowchart of the in-field system (after Dawes, 1984)
The data window is transferred in real-time into computer memory for infield analysis and is stored if it satisfies certain preset conditions. The computer then initiates the acquisition of the next window. Depending on the quality of the data, about 24–30 data windows are considered adequate for a particular frequency band. A simplified infield flowchart is shown in fig 3.2.

Infield analysis enables a very quick appraisal of the data being collected and basically involves converting the data into apparent resistivities, phases and other vital information as functions of frequency. Essentially, by the Fast Fourier transformation of the data, the intensity of each signal is obtained over a range of frequencies and then corrected for the instrument response to yield the earth response functions desired. Data analysis will be discussed in detail in chapter 4.

3.2.3 Equipment Calibration

The MT instruments described above required calibration before and after each field campaign to obtain their frequency response characteristics for accurate determination of the earth response functions. Facilities for precise calibration of the induction coils are lacking at Edinburgh University but this task was undertaken at Centre de Recherches Geophysiques (C.R.G.) in Garchy, France. The calibration data from Garchy were used in the present study and are plotted in fig 3.3a.

The telluric pre-amplifiers and the S.P.A.M. filter/amplifier units were calibrated by the author in Edinburgh using a frequency analyser. This analyser has an internal signal generator and outputs the instrument response functions (amplitude and phase) for a range of frequencies. The telluric pre-amplifiers frequency response curves are shown in fig. 3.3b. The S.P.A.M. amplifier/filter response curves for one of the 7 identical channels are shown in fig 3.3c for the 4 overlapping frequency bands.

3.3 Field procedure and practical considerations

3.3.1 Site Selection

Ideally MT soundings are conducted in open flat-lying areas. Since the region of study is mountainous with very difficult terrain conditions, measurements were restricted to the more accessible wooded forests and farmlands inbetween impressive topographic features. The open ground except on farmlands is mostly boggy (or water logged). However, care was taken to locate the sites as far away as possible from topographic features. A chosen site would preferably be an open or clear area for 60–100 metres square.
Fig 3.3. Instrument response curves.

a. Calibration curves for the induction coils

b. Calibration curves for the Edinburgh University and NERC telluric preamplifiers.

c. The S.P.A.M. amplifier/filter response curves of one of the seven identical channels for four overlapping frequency bands. The frequency bands for the Edinburgh University and NERC S.P.A.M. systems are respectively 640–66 Hz and 128–16 Hz for band 0; 66–8 Hz and 16–2 Hz for band 1; 8–1 Hz and 2–0.25 Hz for band 2; and 1–0.09 Hz and 0.25–0.016 Hz for band 3.
Amplitude in dB vs. Frequency in octaves

BAND 3
BAND 2
BAND 1
BAND 0
away from trees, steep ground, fences and telephone lines, and with the nearest power transmission line at about 400m to 1Km distance. At some locations however, measurements were made in forest clearings of about 50 square metres.

3.3.2 Field set-up

The sequence of operations in setting up an MT site is described below.

(1) The centre for the sensor systems is located. The sensor box is left at this position. A prismatic compass is used to lay out 25–50m electric cables in four orthogonal directions (preferably magnetic north, south, west and east). The electrodes are planted in the ground at the end of these lines and the cables connected with the clips at the top. The contact resistance between an electrode pair (N–S, or E–W) is measured. A typical resistance is about 2–5kΩ. Values greater than 10kΩ do not give good results and if necessary the electrodes are checked to see that they are firmly planted in the ground and that the cables are properly connected.

(2) Shallow trenches are dug for the two horizontal induction coils (since they are very sensitive to wind vibrations) preferably in the E–W and N–S directions. A deep hole is dug with the aid of an auger for the vertical coil. The horizontal coils are aligned and all are levelled with a spirit level after ensuring that they are adequately supported. As a precaution, the same coil is used for the same magnetic element throughout the field campaign. The coils are connected to the sensor box with 3m long cables. Care must be taken that the cables do not vibrate in the wind and so clods of dirt or pieces of rock are placed along their lengths.

(3) the multiconductor cable is connected to the sensor box at one end and to the S.P.A.M. system (placed in the back of a Landrover) at the other end about 50–100m away.

(4) The whole system is earthed and then tested (initial system Calibration) before the measurements commence.

Only a crew of 2 is needed to install the various systems and typical set-up time is 30–40 minutes. Measurements can be made at one or two sites in a day depending on the signal activity and required data quality. In this study however, each site was occupied for at least one day.

3.4 Survey details

In two fieldwork campaigns in 1985, MT measurements in the frequency range 0.09–640Hz were undertaken at 22 sites by the author. In 1986, together with D.Galanopoulos and Drs K. Whaler and G. Stuart of Leeds University, additional soundings were performed at 12 sites in the frequency range
Fig 3.4. Map showing the locations of the MT observational sites.
The MT surveys consisted of 7 tensor soundings along the Glen Garry (GG') profile, 6 along the Loch Arkaig (AA') profile and 19 soundings along the Glen Loy (LL') traverse. The observational sites are shown in fig 3.4. The average station spacing was about 2Km (1Km along LL' in 1986) which was considered adequate for accurate spatial resolution of possible short wavelength anomalies. The S.P.A.M. MKII 7-channel system was used with ECA induction coils for all the soundings. Remote reference techniques (Gamble et al, 1979) were not used in this study. Only data which satisfy certain preset conditions (predicted coherencies, signal-to-noise ratio and frequency content) were accepted. The minimum coherency criterion used was 0.8. The accepted data sets were initially analysed in real-time using standard impedance tensor techniques and the raw data and computed result output to a cartridge recorder and to a small Brother HP printer enabling immediate scrutiny of the results. Thus the data quality was controlled, and the success of the sounding assessed before leaving any field location; it was also possible to ascertain if the station spacing was satisfactory or whether a greater station density was required.

The Glen Garry traverse runs approximately east-west with the extreme sites at about 20Km and 1Km to the west and east of the GGF respectively. The Loch Arkaig profile extends westwards from the GGF for about 20Km and all but one of the sites lie within 10Km distance from the fault and on the north-side of Loch Arkaig. The Glen Loy traverse runs across the fault in an approximately northwest-southeast direction. It presented the only easy access to the region east of the fault and together with the westernmost AA' site 607 (projected) forms the most extensive (38Km long) and necessarily representative traverse across the Great Glen study region. It has the closest network of stations (see fig 3.4) around the fault. The British national grid references of the sites are given in table 3.1 (sites with less than two band recordings neglected).
<table>
<thead>
<tr>
<th>Site Number</th>
<th>Profile Code</th>
<th>Grid Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>553</td>
<td>G1</td>
<td>NN 117 998</td>
</tr>
<tr>
<td>574</td>
<td>G2</td>
<td>NN 173 986</td>
</tr>
<tr>
<td>550</td>
<td>G3</td>
<td>NH 203 006</td>
</tr>
<tr>
<td>551</td>
<td>G4</td>
<td>NH 231 002</td>
</tr>
<tr>
<td>552</td>
<td>G5</td>
<td>NH 281 009</td>
</tr>
<tr>
<td>555</td>
<td>G6</td>
<td>NH 337 051</td>
</tr>
<tr>
<td>556</td>
<td>G7</td>
<td>NN 318 976</td>
</tr>
<tr>
<td>573</td>
<td>A1</td>
<td>NN 122 913</td>
</tr>
<tr>
<td>572</td>
<td>A2</td>
<td>NN 150 896</td>
</tr>
<tr>
<td>571</td>
<td>A3</td>
<td>NN 174 887</td>
</tr>
<tr>
<td>570</td>
<td>A4</td>
<td>NN 194 887</td>
</tr>
<tr>
<td>557</td>
<td>A5</td>
<td>NN 182 901</td>
</tr>
<tr>
<td>607</td>
<td>L1</td>
<td>NM 148 615</td>
</tr>
<tr>
<td>560</td>
<td>L2</td>
<td>NN 089 849</td>
</tr>
<tr>
<td>606</td>
<td>L3</td>
<td>NN 097 847</td>
</tr>
<tr>
<td>605</td>
<td>L4</td>
<td>NN 107 845</td>
</tr>
<tr>
<td>603</td>
<td>L5</td>
<td>NN 111 840</td>
</tr>
<tr>
<td>601</td>
<td>L6</td>
<td>NN 122 828</td>
</tr>
<tr>
<td>559</td>
<td>L7</td>
<td>NN 133 833</td>
</tr>
<tr>
<td>602</td>
<td>L8</td>
<td>NN 136 823</td>
</tr>
<tr>
<td>558A</td>
<td>L9A</td>
<td>NN 146 827</td>
</tr>
<tr>
<td>558B</td>
<td>L9B</td>
<td>NN 146 825</td>
</tr>
<tr>
<td>561</td>
<td>L10</td>
<td>NN 160 831</td>
</tr>
<tr>
<td>609</td>
<td>L11</td>
<td>NN 141 797</td>
</tr>
<tr>
<td>562</td>
<td>L12</td>
<td>NN 165 821</td>
</tr>
<tr>
<td>611</td>
<td>L13</td>
<td>NN 155 789</td>
</tr>
<tr>
<td>563</td>
<td>L14</td>
<td>NN 173 804</td>
</tr>
<tr>
<td>565</td>
<td>L15</td>
<td>NN 222 823</td>
</tr>
<tr>
<td>608</td>
<td>L16</td>
<td>NN 233 798</td>
</tr>
<tr>
<td>604</td>
<td>L17</td>
<td>NN 253 795</td>
</tr>
<tr>
<td>566</td>
<td>L18</td>
<td>NN 285 821</td>
</tr>
<tr>
<td>610</td>
<td>L19</td>
<td>NN 350 797</td>
</tr>
</tbody>
</table>

Table 3.1. Site specifications
CHAPTER 4
DATA ANALYSIS AND RESULTS

This chapter describes the procedure adopted in processing the MT field records to yield the response functions needed for qualitative and quantitative (computer modelling and inversion) studies. For many purposes the infield standard automatic data processing is sufficient. However, at the base laboratory in Edinburgh the various "raw" data sets were reprocessed on the mainframe computer (EMAS) using additional or different data selection criteria so as to obtain more reliable estimates. Inspection of these results showed that certain sections of the data were contaminated with noise. The impedance tensor elements were then averaged (Sims et al, 1971) to minimize bias effects and digital filtering techniques were applied to enhance further the quality of the data.

4.1 Data transfer

On return to Edinburgh, the "raw" field data that were initially stored on cartridge tapes (TU58) were transferred onto floppy discs and then to a 1/2" magnetic tape. The data were finally transferred from the magnetic tape onto the local network for a very rigorous analysis.

4.2 Data Processing

Basically, MT data processing is the conversion of the recorded time series for the 5 field components to earth response functions. Hermance (1973) has reviewed the different methods for MT data analysis and details of the techniques are described in Vozoff (1972), Sims and Bostick (1969) and Sims et al (1971). The various formulations will not be reproduced here. The classical, and by now standard, tensor impedance analysis techniques (see section 2.6) were used in this study.

4.2.1 Standard analysis

4.2.1.1 General window analysis

Time series analysis was done using the data analysis program originally written and described by Rooney (1976) and re-written for computational efficiency by G.Dawes. Analysis was done in the frequency domain using equation 2.34f. The following operations are executed for each of the windows of the 4 frequency bands by the program.

(i) The time series are first scaled using the recording gains. The field
components are optionally rotated into desired coordinates. Instrument correction tables are then set up.

(ii) The data are reduced to zero trend and normalized to unit standard deviation (Bendat and Piersol, 1971) using the least squares method.

(iii) A cosine bell window (Harris, 1978) taper is applied to the first and last 10% of the data set.

(iv) The data set is augmented with zeros to a power of 2 to satisfy the requirement of the Fast Fourier transform (FFT) algorithm (Cooley and Tukey, 1965). The complex Fourier coefficients of each component are then calculated using a general FFT routine.

(v) The Fourier transform coefficients are corrected for the instruments (telluric preamplifier and CM16 & CM11E coils) responses.

(vi) Auto- and cross-spectral estimates are then calculated and band-averaged over 8 neighbouring frequencies so as to reduce the variance of the estimates.

(vii) The initial results of the analysis are then calculated using the smoothed spectral estimates. These are the impedance tensor elements using eq. 234f, the Cagniard resistivities and phases (i.e. in the measurement directions), polarisation, predicted Hz, Ex and Ey coherencies, Hz transfer functions (tipper coefficients); these are saved in a file.

These procedures are executed for each recorded window in turn.

4.2.1.2 Averaging of window results

A comparison was made (Sule, 1985) of the three conventional methods of averaging the individual data windows (auto- and cross-spectral averaging assuming a normal distribution, impedance tensor averaging assuming a lognormal distribution and apparent resistivity averaging assuming a lognormal distribution) and it was found that impedance tensor averaging is practically the most convenient. The impedance tensor averaging technique was thus adopted in this study.

The unrotated tensor elements obtained from general window analysis are subjected to further statistical analysis and winnowing leaving only reliable estimates for averaging - this minimizes the variance of the resultant earth response functions.

The averaging program of Rooney (1976) was adapted for short period MT data by G. Dawes. Only estimates with minimum predicted coherencies of 0.8 are accepted subject to other conditions. For the pairs Zxx, Zxy and Zyx, Zyy and the tipper coefficients, the predicted Ex, Ey and Hz coherencies are used.
respectively. Those with small signal to noise ratio are rejected and it is also required that each frequency set possesses a preferred sign of the off-diagonal elements of the impedance tensor. Outliers (all estimates lying outside ±2.2 standard deviations from the mean value) are rejected. A new mean and standard deviation are then calculated and the procedure repeated. The impedance tensor elements derived from windows which satisfy the coherency, power, sign and outlier rejection criteria are then averaged over all the 4 frequency bands assuming a lognormal distribution (Bentley, 1973) and the tipper coefficients averaged assuming a normal distribution. These are then subdivided to give equispaced estimates on a logarithmic scale (usually 8–10 per frequency decade/band).

Several parameters are then computed (as functions of frequency) from these mean estimates. They include: the apparent resistivities and phases (unrotated and rotated), the azimuth of the principal impedance axis, and the impedance skew values. In this study, apparent resistivities and phases were also obtained with axes parallel and perpendicular to N49E (magnetic), the direction of the GGF, thus providing the E- and H-polarization field response curves required for 2-D modelling. The effective MT responses for 1-D modelling were also obtained. Vertical magnetic field information is both scanty and noisy and was deleted from the interpretation process.

The data from 32 stations were analysed using the methods discussed above. Some examples of the apparent resistivity and phase results from standard data analysis are shown in figure 4.1. The error bars assigned to the parameter estimates are ±1 standard deviation about the mean assuming a normal distribution (lognormal for those on log scale) (Bentley, 1973). In this figure are shown the apparent resistivity and phase information obtained at various frequencies. Note that there is no overlap between the first segment (band 0) and the rest of the curve. This discrepancy can only be explained in terms of noise corruption of the data. This feature was found to be common to the high frequency (band 0) data at some sites and at others there is a large scatter in the values of the responses at high frequencies. The lower frequency data at some sites were also degraded in quality by noise contamination (see fig 4.1). This seems to suggest that the data window acceptance criteria used in routine analysis are not effective in eliminating noisy records since any coherent noise with good power characteristics would be accepted as a good signal by the program. A different way of noise detection and elimination seems imperative.
Fig 4.1 Some results of conventional data processing
4.2.2 Bias analysis

To assess the bias effects on the response curves it was found necessary to reprocess all the data sets using the 6 different methods of estimating the impedance tensor elements (i.e. eqs. 2.34 a–f). The effect of noise on the data will be to elevate or decrease the apparent resistivity estimates. If the noise is mainly associated with the telluric measurements the impedance estimator 2.34a would produce the maximum values for the apparent resistivitites. On the other hand if the noise is in the magnetic channels then eq. 2.34f would produce curves that are biased downwards. The range between these extremes is taken to characterise the bias in the data for each site. For each site the 6 different response curves were inspected for bias effects. While the curves derived with eqs. 2.34a,b were generally shifted upward and those from eqs. 2.34d, f, shifted downward with respect to the others, the bias range was small (less than half a decade) especially at high frequencies as shown by the example from one site in fig 4.2.

It was envisaged that an approach to the noise problem would be to process the “raw” data using an acceptance criterion that is based on the scatter between the 4 different stable equations (2.34 a,b,e and f), the stability criterion of Sims et al (1971) defined as

$$S = \frac{|Z(a)| \cdot |Z(b)|}{|Z(e)| \cdot |Z(f)|}$$

where Z(a), Z(b) etc refer to the estimates from eqs. 2.34a,b etc. S will be unity if the 4 estimates are identical and will increase as the estimates diverge. A value of $0.7 < S < 1.3$ was used but it did not improve the mismatch between the high and moderate frequency segments of the response curves. This time consuming approach was therefore abandoned. However, to provide unbiased response curves (i.e. with minimized frequency independent shifts), the 4 stable solutions were averaged (Tikhonov and Berdichevsky, 1966; Sims et al, 1971) and only these mean response functions have been used in subsequent studies. The frequency-dependent noise problem still remains.

4.2.3 Cultural noise analysis

It is by now obvious that the automatic standard or bias analysis procedures are not of much help in the presence of certain types of noise. It was therefore decided that each data window of a given set be investigated and analysed as appropriate.
Fig 4.2 Example of bias in response function estimates assuming noise-free E or H channels.
4.2.3.1 Noise types found in the MT records

Examination of some data windows revealed the presence of two main types of noise:

(i) Impulsive signals which may be correlated or uncorrelated in the corresponding components. Such noise affecting the telluric components may be caused by electric sources (leakage currents, electric fences, etc) and in the magnetic channels such noise may be due to movement of vehicles or heavy machines (e.g. agricultural and mining). It should be remarked that extensive quarrying activities and road construction work were going on at the Glen Garry Forest at the time of the 1985 investigations and that the worst noise was observed in the 1985 data sets.

(ii) Strong coherent near-stationary perturbations in all components. The commonest of these signals are 50Hz and harmonics affecting the bands 0 and 1 data. The source of these is the electric mains. Other less stable perturbations (5–6Hz, 35–36Hz and 106–110Hz) are also present in some of the 1985 data and may have been caused by motors working at or close to 300, 2100 or 6480 rpm respectively. It was thus obvious that the infield (S.P.A.M.’s) 50Hz and 150Hz analog rejection (notch) filters did not eliminate completely the electric mains noise at some locations.

4.2.3.2 Digital filtering

The remarkable time-invariance of some of the high frequency noise suggested that a simple comb (delay-line) or notch filter could be used to remove them. The impulsive noise (spikes) could also be eliminated using a suitable de-spiking filter. These filters are routinely used in geophysical data analysis and the reader is referred to the excellent text on this matter by Kanasewich (1981).

Using the Z transform method (i.e. letting $e^{-j\omega} = Z$) we can design a delay line filter to remove unwanted near-stationary signals. The Z transform of the impulse response of a simple subtractive delay-line filter is given by

$$W(z) = 1 - Z^n$$

where $n$ is the number of digital time units by which an input signal is delayed.

If in the z plane the input data $X(z)$ is convolved with the impulse response of the filter $W(z)$ we obtain an output whose z transform is given by

$$Y(z) = W(z) \cdot X(z)$$
or

\[ Y(z) = (1 - Z^n) \cdot X(z) \]  

The equation for use in digital filtering on a computer is

\[ Y_i = X_i - X_{i-n} \]

where \( X_i \) is the \( i \)th sample of the 5 time series of a data window, \( Y_i \) is the recovered signal sample for the 5 components and \( n \) is the delay unit given by

\[ n = \text{[Sampling frequency/perturbation frequency]} \]

The operation represented by equation 4.4 effectively cancels 50Hz (and other perturbations) and harmonics in synthetic data, but the simple comb filter (eq. 4.2) is not a very good filter in that it distorts the spectrum at all other frequencies between zero and \( F_{nq} \) where there is a gain of two. \( F_{nq} \) is given by

\[ F_{nq} = RF \frac{(2K + 1)}{2}, \quad K = 0,1,2... \]

where \( RF \) is the rejection frequency. This behaviour is illustrated in fig 4.3. However, we are only interested in the ratios between corresponding field components and this problem is therefore trivial.
Fig 4.3 Frequency response of a delay line filter
Fig 4.4 Time series for two data windows (a) before and (b) after the application of a delay line filter.
Digital notch filters (see Kanasewich (1981) for a treatment of design considerations) were also tested on synthetic data and found to be satisfactory. Digital filters were then built into the existing MT data analysis program and the various data sets reprocessed. A remarkable improvement in the quality and degree of overlap between adjacent data bands was achieved by application of the delay-line filter. In figs 4.4a & b are shown respectively the time series for 2 windows of a high frequency band before and after the application of the delay-line filter. Unwanted single frequencies were removed by notch filtering. In figure 4.5 are shown the results of analysis for some sites after the application of a simple subtractive delay-line filter (compare with the results from conventional analysis shown for the same sites in fig 4.1). Note the considerable improvement achieved for the band 0 data using the delay-line filter.

The low frequency data are generally noisy (a consequence of the polarizable electrodes used for the field recordings). However, the subtractive delay-line filter appears to be ineffective at low frequencies and Fischer (1982) suggested an additive delay-line filter (not used in this study) given by

\[ Y_i = X_i + X_{i+n} \]  

4.7

It was observed that no amount of digital filtering was helpful if the data were contaminated with irregular noise. A different strategy had to be adopted for such cases.
Fig 4.5 Some results of data processing with a simple delay line filter.
4.2.3.3 Frequency domain window editing

An additional program (written by G. Dawes) was used to inspect the different stages of analysis, window by window, in order to separate the distinctly corrupted and the uncorrupted data. This manual window editing was begun by inspecting each window in the frequency domain for a given frequency band. From the inspection of the cross-spectral amplitudes for the electric and magnetic components and the Cagniard resistivities along measurement axes one can establish which of the signals is significantly contaminated with noise and may either delete the affected windows from the analysis process or employ the appropriate impedance estimator. In fig 4.6 are shown the results of analysis of nine windows of a noisy frequency band. Each window in fig 4.6 has four sets of analysis against frequency: the Cagniard resistivity from Ex and Hy (scaled to 0.1 to 10Ωm); the averaged cross-spectral amplitudes for Hy and Ex, with E normalised to H; the cross-spectral amplitudes for Hx and Ey and the Cagniard resistivity from Ey and Hx. Windows which have much greater power than normal in some or all of the 4 field components and with steeply rising (or decreasing) resistivity curves are rejected. In this case it can be seen that the noise is strong in Ey and Hx and that the associated curves rise too steeply; the whole frequency band was rejected. For other sites, it was possible to distinguish between the heavily contaminated and the less disturbed windows.

The existing data analysis program was therefore modified by the author to allow for data-dependent processing, viz: (1) manual intervention so that only selected windows (after prior visual inspection) can be analysed, (2) any of the 6 (or an average of 4) methods of impedance tensor estimation can be used to determine the response functions and (3) digital filtering techniques can be applied repeatedly to noise-degraded data.

The mean results obtained using this modified analysis procedures will be discussed in the next section.
Fig 4.6 Results of frequency domain noise analysis for some data windows. For each of the windows (labelled 19 - 25) are shown the Cagniard apparent resistivity (scaled to .1 to 10Ωm) for the N-S and E-W directions, and the cross spectral amplitudes of orthogonal E and H fields as functions of frequency.
4.3 Magnetotelluric field responses

Data from 32 sites have been processed with attention being paid to the biasing of the impedance tensor. The mean tensor response functions (presented in appendices I, II and III) are qualitatively assessed in this section. For each site, plots are given of the frequency dependence of the major and minor apparent resistivity and phase, azimuth of major impedance and skew factor with an indication of coherence and the number of averaged (mean) estimates. The azimuths refer to magnetic coordinates and 0, 90, −90 degrees denote respectively magnetic north (N), east (E), west (W). The azimuths indicate the preferred direction of the electric field and have a 90 degrees ambiguity. For a simple 2-D structure, the major principal direction will be parallel to strike if on the conductive side of the lateral inhomogeneity and perpendicular to strike on the resistive side of the contrast (Reddy and Rankin, 1972; Reddy et al, 1977b). Site names and profile code are as given in table 3.1.

In general, the regional geoelectric profile is characterised by at least 3 geoelectric units: an upper unit of moderate resistivity, a highly resistive intermediate unit (resistor) and a lower unit of low resistivity (conductor). The results for each survey profile are presented in an appendix. For each profile, the individual site results are presented starting from the west through to the east. The low frequency (<1Hz) data are generally noisy because the signal strength is notoriously low in the range .1 to 1Hz (the so called "dead band") and also due to additive noise from the polarizable electrodes used in this study. The results for the various profiles are now discussed.

4.3.1 Glen Garry profile

In appendix I are shown the results for sites 553 (G1), 574 (G2), 550 (G3), 551 (G4), 552 (G5), 555 (G6) and 556 (G7). Site 555 has been projected onto this traverse from about 5Km to the north (see fig 3.4). The quality of the data based upon scatter, error bars, and data point continuity is generally good at most sites in the frequency range 612 to 0.5Hz. Lower frequency data are of good quality only at sites 551 and 574. Apparent resistivities at most sites are only moderately anisotropic i.e. the values in the principal directions differed by less than half a decade but the phase information seems to suggest a 2- or 3-D environment. Lateral variations occur between G2 and G3 and to the east of G5 and there appears to be an indication of a decrease in depth to the basal conductor at G3. The skew factors are generally low in value and appear to increase from 0.1−.2 at those sites far away from the GGF (and at G7) to about
0.24–0.4 near the fault.

The azimuth is about 45 degrees E of N (magnetic) at the western margin of the profile and changes to about 45 degrees W of N near the fault. This is the expected behaviour from MT theory assuming that the Great Glen fault zone is of lower resistivity than the surrounding crust. This is thus the first signature of the fault on the MT records. Note the rotation of azimuth at site 556 from about 40 degrees W of N between 612 and 13 Hz to about 10–20 degrees E of N at about 1 Hz.

4.3.2 Loch Arkaig profile

The results for the Loch Arkaig stations 573 (A1), 572 (A2), 557 (A5), 571 (A3) and 570 (A4) are shown in that order in appendix II. The results are generally of poor (although usable) quality. The high frequency data were corrupted by 5–6 Hz, 32–36 Hz and about 100 Hz noise. In addition, most of the apparent resistivity curves appear to have been shifted along the vertical axis between 100 Hz and 10 Hz. This might be due to near-surface inhomogeneities - the so called static distortion (Berdichevsky and Dmitriev 1976; Hermance, 1982; Park et al, 1983). Near-surface resistivity variations of limited extent typically cause vertical displacement of both the major and minor resistivity curves without changing the phase information. In this situation, the major resistivity or the effective resistivity curve provides some useful information (Berdichevsky and Dmitriev, 1976). It is obvious from these plots that the apparent resistivities are generally anisotropic.

At site 573 (located in an area of E–W camptonite dykes) the azimuth is 0 or 90 degrees between 612 Hz and 1 Hz and rotates to the regionally consistent strike of 45 degrees E of N at lower frequencies. The telluric field appears polarised in the NE–SW direction (45 degrees E of N) at site 572 and may suggest the presence at depth of a low resistivity linear structure such as a fault. East of this station, the azimuth is 45 degrees W of N, i.e. perpendicular to the Great Glen fault axis. This is as expected from 2–D MT theory. These sites therefore require at least 2–D interpretation.

4.3.3 Glen Loy profile

The Glen Loy traverse is the most interesting in that it transects different structural units and with sites distributed in a manner that permits accurate spatial resolution of short wavelength features in the vicinity of the GGF. In appendix III are shown the results for sites 607 (L1), 560 (L2), 606 (L3), 605 (L4), 603 (L5), 601 (L6), 559 (L7), 602 (L8), 558A, 558B (L9), 561 (L10), 609 (L11), 562
(L12), 611 (L13), 563 (L14), 565 (L15), 608 (L16), 604 (L17), 566 (L18) and 610 (L19) in that order. In general the data imply a strong 2-D, anisotropic, electrical earth with 3 to 4 distinct layers.

The quality of the data is good at some sites but noise-degraded sections of the sounding curves are a common feature (the data acquired in the 1985 fieldwork using the Edinburgh University S.P.A.M. MK II system appear contaminated by 5-6Hz and 32-36Hz noise). The difficult terrain conditions limited the experiments to accessible areas which are mostly inhabited or farmed.

Site 607 is located within the Glenfinnan Division (Johnstone et al, 1969) of the Moines and is the least anisotropic of all the observations with skew values which decrease from 0.4 at around 100Hz to about 0.2 at frequencies below 5Hz. The azimuthal values are scattered and this feature together with the somewhat isotropic resistivities suggest that 1-D interpretation of the results could provide a reasonable indication of the actual resistivities at this site.

Sites 560 and 606 are respectively located to the west of and at the periphery of the Glen Loy Gabbro Complex. Their azimuthal values vary from 30 degrees E of N at high frequencies to a northerly (or westerly) trend at mid-frequencies rotating back to 30-40 degrees E of N at frequencies below 1Hz. At 560 the skew values are generally low (about 0.2) at frequencies below 100Hz and the resistivity anisotropy is less than half a decade at high frequencies. The L3 (606) curves suggest the presence of near-surface inhomogeneities. The skew values are about 0.1 and 0.3 above and below 1Hz respectively. The anomaly is suspected to be 3-D here.

The sites to the east of L3 and west of L12 (562) are situated within the Glen Loy intrusive complex and exhibit anomalous features. The major apparent resistivity at site 605 shows only little frequency dependence (a feature seen at 560 and 606), fluctuating around 10000 Ωm above 0.1Hz. The azimuthal direction is persistently N60°E. A geoelectric contrast is inferred in the neighbourhood of this site. Sites 603 and 601 are slightly anisotropic with low skew values (0.1 - 0.2) and with the same azimuthal behaviour as at 605. The resistivity variations at the other sites to the west of L12 are similar and the azimuth varies from N30°W to almost E-W. Sites 558A and 558B are only tens of metres apart and were occupied at different field seasons. Only site 558B, the repeat sounding, will be interpreted in this study. The nature of the anomaly in the Glen Loy complex appears to be 3-D.

Sites 609 and 562 are situated in the Great Glen fault zone. The apparent
resistivities are strongly anisotropic and the telluric field is strongly polarised in the direction of the fault axis, N45°E as expected from MT theory assuming that this zone of thoroughly comminuted and indurated rock is conductive. Skew factors are about 0.2. The "dead band" data are noisy. The structure is strongly 2-D here.

The dead band is noisy at all sites east of the fault zone except at 566 which was measured in the early hours of the morning (about 0200 hrs) but the high frequency data are of good quality. Site 611 is located close to a zone of shattered rocks bordering the fault zone and the apparent resistivity is very anisotropic with the electric field polarised in the NE/SW direction. Site 563 shows anisotropic character and the azimuthal directions vary from about N55°W above 5Hz with skew values of about 0.2 to E-W below 5Hz with skew values greater than 0.4. The azimuthal direction shows a rotation from N40°W above 1Hz to about 10°–30° E of N below 1Hz at site 565 with a comparable frequency dependance of the skew factors as at 563. The level of cultural noise at site 565 precluded bands 0 and 3 measurements.

Site 608 is located within the Fort William Slide zone and site 604 just outside it. The data are at least 2-D in character (large skew values, strong anisotropy, very different phase data for the two principal directions). Sites 566 and 610 are situated to the east of the surface position of the Fort William Slide. The apparent resistivities are anisotropic but the skew values are generally only about 0.2. The azimuth shows abrupt 90 degrees rotation from N40°W to about N45°E at 1Hz suggesting a possible different trend in the basement rocks or the presence of a complex structure at depth. Lateral variations in apparent resistivities occur in the vicinity of the Fort William Slide and there appears to be an indication of a decrease in depth to the lower geoelectric conductor in the region of the slide. Two-dimensional interpretation (at least) is requiried for these sites.
CHAPTER 5
ASPECTS OF DISCRETE INVERSE THEORY

Inverse theory is an organised set of mathematical techniques for reducing experimental data to useful information about the physical world. It provides a formalism in which many questions fundamental to geophysical data analysis may be addressed. For instance, consider the problem of determining the resistivity distribution of the subsurface from MT data. The resistivity distribution within the earth is a continuous function of depth which can only be uniquely determined if the MT response functions are known for all frequencies in the range \([0, \infty]\), a problem in continuous inverse theory. However, geophysical data with associated uncertainties, are acquired over a limited bandwidth, and we seek the minimum set of parameters that describe the earth. The problem is therefore discretized. Discrete inverse theory is concerned with the approximations of otherwise continuous functions with a finite number of discrete parameters.

The underlying concepts of some of the procedures used for deriving useful information from a finite set of observations are outlined in this chapter. In this short account, the least squares formalism is adopted because of its mathematical robustness when the experimental data are, in the words of Jackson (1972), “inaccurate, insufficient, and inconsistent”. A complete survey of the art of geophysical inversion can be found in the book by Menke (1984); and Jackson (1972, 1976, 1979), Aki and Richards (1980) among others have published excellent treatments of the underlying mathematical concepts while Lines and Treitel (1984) give an easily digestible review of least squares inversion in geophysics. A comprehensive treatment of the inverse methods in remote sensing is given in Twomey (1977) on which some of the material in this chapter draws heavily.

Many physical inverse problems are nonlinear. To explain how to tackle nonlinear problems, it is instructive to consider first the simpler and better-understood linear inverse problem. The same ideas are then extended to the nonlinear case.

5.1 Linear least squares inversion

Consider a physical system (such as the earth) that can be characterised by \(P\) unknown model parameters which represent members of a vector

\[
m = \text{col} (m_1, m_2, \ldots, m_P)
\]  

5.1
Suppose that \( N \) measurements are performed on this system in a particular experiment, we can represent the observed data by the vector of random variables

\[
d = \text{col} (d_1, d_2, \ldots, d_n)
\]

Assuming that a theory exists for predicting the outcome of the experiment, let \( F \) be the set of \( n \) (usually nonlinear) functions of the model parameters that describes the theoretical predictions. We also assume that the errors in the observations are additive, Gaussian, statistically independent, with zero mean and of unit variance. In general, the observations are related to the physical model by

\[
d = F(m) + e
\]

where \( e \) is the vector of additive noise (errors) expressed as

\[
e = \text{col} (e_1, e_2, \ldots, e_n)
\]

Our goal is to discover the \( p \) variates, \( m \) that describe our physical system (i.e. the solution of equation 5.3). If the functions \( F(m) \) are linear in \( m \), we may write the problem in matrix form

\[
d = Gm + e
\]

where the \( n \times p \) matrix \( G \) is a linear functional of the parameters and is sometimes called the design matrix.

As usual in physical sciences, the number of data points exceeds that of the model parameters (an overdetermined system) and the resulting system of equations is solved in the least squares sense.

We may rewrite eq. 5.5 as

\[
e = d - Gm
\]

The method of least squares estimates the solution of an inverse problem of this kind (5.6) by finding the model parameters that minimizes a particular measure of the length of the estimated data, namely, its euclidean distance from the observations.

We define a quadratic function \( Q \) by the formula

\[
Q = e^T e = (d - Gm)^T (d - Gm)
\]
(here and in future formulations $a^T$ signifies the transpose of $a$) and further define the least squares estimator, $\hat{m}$, to be the estimator that minimizes $Q$, the sum of squares of the errors. Minimization is effected by setting to zero the derivatives of $Q$ with respect to each of the $p$ parameters $m_j$ and solving the resulting equations, that is,

$$\begin{align*}
\frac{\partial Q}{\partial m_j} &= d^T d - 2m^T G^T d + m^T G^T G m_j = 0 \\
\frac{\partial Q}{\partial m} &= -2G^T d + 2G^T G \hat{m} = 0
\end{align*}$$

This yields the matrix equation

$$G^T G \hat{m} = G^T d$$

the so-called "normal equations" for the parameters $m$. This may be solved for $\hat{m}$ providing that $(G^T G)^{-1}$ exists:

$$\hat{m} = (G^T G)^{-1} G^T d$$

Eq. 5.10 is known as the unconstrained least squares (or Gauss-Newton) solution of eq. 5.6. A solution of eq. 5.5 in the above absence of errors and when $n = p$ (i.e. $G$ is a square matrix) is

$$m = G^{-1} d$$

The least squares method uses the Lanczos inverse (Lanczos, 1961) instead of the direct inverse of $G$ and is given by

$$G_L^{-1} = (G^T G)^{-1} G^T$$

$G_L^{-1}$is also known as the natural generalized inverse of $G$.

5.2 Constrained linear least squares inversion

In many physical problems it is possible to generate a set of different solutions that adequately explain some experimental data especially in the presence of measurement errors. Ultimately one solution has to be selected and interpreted. To do this we often have to add to the problem some information not contained in the equation $d = G m + e$. This extra information is referred to as a priori information (Jackson, 1979) and can take many forms. It is noted here that the matrix $G^T G$ may be singular or near-singular. This situation produces some undesirable effects but the difficulty can be obviated by using a priori data.
5.2.1 Inversion with simplicity measures

A very effective way of inverting a finite collection of inexact data is to impose the constraint that the desired solution be smooth. If it is desired that the model parameters vary slowly with position, say, then we may choose to minimize the difference between physically adjacent parameters \((m_1 - m_2), (m_2 - m_3), \ldots (m_{p-1} - m_p)\). These first differences can be written in the form

\[
Dm = \begin{bmatrix}
1 & -1 \\
1 & -1 \\
\vdots & \vdots \\
1 & -1 \\
\end{bmatrix}
\begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_{p-1} \\
m_p \\
\end{bmatrix}
\]

where \(D\) is the difference operator known here as the smoothness matrix and \(Dm\) is the smoothness of the vector \(m\).

We may quantify the roughness of the solution by considering instead the second differences so that \(D\) is of the form

\[
D_2 = \begin{bmatrix}
0 & 1 & -1 \\
-1 & 2 & -1 \\
-1 & 2 & -1 \\
\end{bmatrix}
\]

To gauge the smoothness of our least squares solution we use a quadratic measure \(q(m)\) given by

\[
q(m) = m^T D^T D m
\]

Let us define a symmetric matrix, \(H\), given by

\[
H = D^T D
\]

We may state the constrained problem as: minimize \(q = m^T H m\) under the condition \(|d - Gm|^2 \leq Q_1\) (or specifically \(|d - Gm|^2 = Q_1\)) where \(Q_1\) is the maximum tolerable sum of squared residuals.

Using the method of Lagrange multipliers we find an unconstrained extremum of the function

\[
\phi = (d - Gm)^T (d - Gm) + \beta m^T H m
\]

where \(0 \leq \beta \leq \infty\) is an undetermined multiplier.

This requires that for all \(j\)

\[
\frac{\partial}{\partial m_j} (d^T d - m^T G^T d - d^T G m + m^T G^T G m - Q_1 + \beta m^T H m) = 0
\]

78
so that

\[(G^T G + \beta H)m = G^T d\]

from which we obtain the solution

\[m = (G^T G + \beta H)^{-1}G^T d\]

Equivalent we can minimize \(|d - Gm|^2\), holding \(q\) constant. Proceeding as before, we find an absolute extremum of the Lagrange function

\[\phi = m^T Hm + \beta^{-1}(d - Gm)^T(d - Gm)\]

If we assign a value \(Q_2\) to \(q\) it follows that

\[\frac{\partial}{\partial m_j} [m^T Hm - Q_2 + 1/\beta(d^T d - 2m^T G^T d + m^T G^T Gm)] = 0\]

so that

\[(G^T G + \beta H)m = G^T d\] as before, \(\beta\) being undetermined.

When \(H\) is an identity matrix we obtain a special constrained inversion formula. If in this case the constraint equations are arranged to form an expression equal to zero (Marquardt, 1970; Twomey, 1977, pp.134–135) such as

\[Dm = \begin{bmatrix} 1 & & & \bar{m} \\ & 1 & & \\ & & \ddots & \ddots \\ & & & 1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ \bar{m} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\]

and a weight \(\beta\) is assigned to each of the constraint equations and a weight of unity to the actual \(n\) data equations, we can solve the augmented problem in a least squares sense. Since the right-hand side of the constraint equations is zero, they do not contribute to the right-hand side of the final equations. The special least squares solution in this circumstance is

\[m = (G^T G + \beta I)^{-1}G^T d\]

This important formula will be used later when dealing with nonlinear problems.

5.2.2 Inversion with prior information

We can incorporate previously obtained information about the model parameters in our inversion process. This could be in the form of results from previous experiments or quantified expectations dictated by the physics of the problem. Generally, these external data help single out a unique solution among all the equivalent ones. The procedure is an extension of the solution
simplicity technique. We employ linear equality constraints that are to be satisfied exactly and of the form

\[ Dm = h \]  \hspace{1cm} 5.24

where \( D \) is an identity matrix as in eq. 5.22 and \( h \) is a \((p \times 1)\) vector of a priori values of \( m \). We wish to bias \( m \) towards \( h \). We simply minimize

\[ S = (d - Gm)^T(d - Gm) + \beta (m - h)^T(m - h) \]  \hspace{1cm} 5.25

Setting the derivatives of \( S \) with respect to the model parameters to zero we find that

\[ GTGm - GTd + \beta m - \beta h = 0 \]

or

\[ (GTG + \beta I)m = (GTd + \beta h) \]  \hspace{1cm} 5.26

from which we obtain the biased solution

\[ m_b = (GTG + \beta I)^{-1}(GTd + \beta h) \]  \hspace{1cm} 5.27

However this procedure should only be used when it is reasonably justified for it produces undesirable effects when \( h \) is unrealistic (see Twomey, 1977, p.138).

In the cases discussed so far, the constraints are implemented by arranging the constraint equations as rows in \( Gm = d \). The parameter \( \beta \) is chosen by trial and error, and we seek a \( \beta \) value that produces a predicted residual of the same order of magnitude as the actual misfit of observed and calculated data. It is also possible to solve the constrained equation directly for \( \beta \) and the constrained solution \( \hat{m}_c \) (see Meyer, 1975, p.398).

5.3 Nonlinear least squares problems

In nonlinear problems the experimental data are related to the model parameters via a nonlinear forward functional \( F \) and the general problem is expressed as

\[ d = F(m) + e \]

where the various quantities have been previously defined.

5.3.1 Iterative least squares inversion

In solving nonlinear least squares problems, a Taylor series approximation is often used to linearize the problem so that the simple least squares method can be used to solve the linearized set of equations. We wish to minimize

\[ Q_o = (d - F(m))^T(d - F(m)) \]  \hspace{1cm} 5.28
Differentiation as before yields a system of nonlinear equations that are difficult to solve. The general procedure is to start with a first approximation to the actual model parameters and then successively improve the values of the estimates until $Q_0$ is minimized.

We may assume that the model response function $F(x)$ is approximately linear around some starting model $m_j^0$ ($j=1, 2, \ldots, p$) such that perturbation of the model response about $m^0$ can be represented by the first-order Taylor series expansion:

$$F(m) = F(m^0) + \sum_{j=1}^{p} \left. \frac{\partial F}{\partial m_j} \right|_{m^0} (m_j - m_j^0)$$

5.29

Define an $n \times p$ matrix of partial derivatives or Jacobian, $A$, computed by the forward theory and with elements

$$A_{ij} = \left. \frac{\partial F_i}{\partial m_j} \right|_{m^0}$$

5.30a

and a ($p \times 1$) parameter change vector $x$ with elements

$$x_j = m_j - m_j^0$$

5.30b

Eq. 5.29 can now be re-expressed as

$$F(m) = F(m^0) + Ax$$

5.31a

Let us define further an ($n \times 1$) discrepancy vector, $y$, containing the differences between the initial model response $F(m^0)$ and the observed data

$$y = d - F(m^0)$$

5.31b

Substituting eqs. 5.31a,b in eq. 5.28 we find that

$$Q_0 = e^T e = (y - Ax)^T (y - Ax)$$

5.32

We need to solve for the $x$ that satisfies

$$y = Ax$$

5.33

Using the least squares method it follows that

$$\frac{\partial Q_0}{\partial x} = \frac{\partial}{\partial x} (y^T y - x^T A^T y - y^T A x + x^T A^T A x) = 0$$

5.34

which is equivalent to the vector equation

$$A^T A x - A^T y = 0$$

or

\[ \text{81} \]
\[ A^T A x = A^T y \] 5.35

Assuming that \([A^T A]^{-1}\) exists, we have from eq. 5.35

\[ x = [A^T A]^{-1} A^T y \] 5.36

which is the parameter correction to be applied to our initial model, \(m^0\). The successive application of this procedure minimizes eqs. 5.28 or 5.32. Let \(x^0 = m^0\) and \(x^1 = x^0 + (A^T A)^{-1} A^T y\). The iterative formula can be written as

\[ x^{k+1} = x^k + [A^T A]^{-1} A^T y \] 5.37

where the Jacobian \(A\) is evaluated at \(x^k\).

The main drawbacks of this technique are that a good approximation to the best estimate is required for the procedure to converge and that the matrix \(A^T A\) may be singular or near-singular producing some undesirable effects. Even if \(A^T A\) is non-singular, the solution may diverge or converge very slowly. Fortunately, techniques have been developed to overcome some of these problems. It is remarked here that the \((n \times p)\) matrix, \(A\), can be factored (Lanczos, 1961) into a product of three other matrices

\[ A = U \Lambda V^T \]

where \(U(n \times p)\) and \(V(p \times p)\) are the data space and parameter space eigenvectors respectively and \(\Lambda\) is a \((p \times p)\) diagonal matrix containing at most \(r\) non-zero eigenvalues of \(A\), with \(r \leq p\). The eigenvalues of \(A^T A\) are explicitly related to those of \(A\) and \(A^T A\) is said to be ill-conditioned if the eigenvalues are small.

To prevent unbounded solution growth when \(A^T A\) is ill-conditioned, Levenberg (1944) suggested a method of "damped least squares" to damp the absolute values of the increments during the successive applications of Taylor approximations; arbitrary positive weights are added to the main diagonal of \(A^T A\). Levenberg also showed that the directional derivative of the residual sum of squares has a minimum when the weights are equal. The idea of adding a biasing constant to the diagonal of \(A^T A\) was later used by Marquardt (1963, 1970) and Hoerl and Kennard (1970a) to propose very useful nonlinear least squares algorithms. This technique is commonly known as the Marquardt–Levenberg or ridge regression method. This approach is just one of a class of damped approximate inverses (Jupp and Vozoff, 1975).
5.3.2 Ridge regression

In this method, we minimize some combination \( \phi \) of the prediction error and the solution length. We form the function

\[
\phi = Q_1 + \beta Q_2 = \mathbf{e}^T \mathbf{e} + \beta (x^T x - L_0^2)
\]

where we have placed a bound, \( L_0^2 \), on the energy of the parameter increments and \( \beta \) is an undetermined Lagrange multiplier which determines the relative importance that is given to \( Q_1 \) and \( Q_2 \). Here \( \beta \) is referred to as the damping factor. Minimization of \( \phi \) in a manner exactly analogous to the least squares derivation (section 5.3.1) yields the normal equations

\[
(A^T A + \beta I)x = A^T y
\]

from which we obtain the ridge estimator of the increments,

\[
x_R = (A^T A + \beta I)^{-1} A^T y
\]  

This is similar to the noniterative constrained linear inversion formula for the case \( H=1 \) (eq. 5.23) and may be regarded as its nonlinear, iterative analogue. The process can be iterated until a solution to the general inverse problem is obtained. If our starting model is \( x^0 \) such that the first linear approximate estimate of the parameters is \( x^1 = x^0 + x_R \), nonlinearity is dealt with using the iterative formula

\[
x^{k+1} = x^k + (A^T A + \beta I)^{-1} A^T y
\]

Note however that we can base our measures on solution length on the estimate \( x^{k+1} \) instead of simply limiting the perturbation length \( x_R \). To do this, we treat the estimates \( x^k \) as the a priori data in the nonlinear equivalent of the formulations described in sub-section 5.2.2.

Examination of eqs. 5.36 and 5.40a shows that the latter is an effective way of dealing with singularities or near-singularities in \( A^T A \). Ridge regression is in effect a hybrid technique in the sense that it combines the steepest descent and the least squares methods. The steepest descent method dominates when the starting model is far from the solution while the least squares method becomes effective as the solution is approached. Marquardt (1970) showed that ridge regression is similar in character to generalized inversion, and that \( (A^T A + \beta I)^{-1} A^T \) approaches the generalized inverse of Penrose (1955) as \( \beta \to \infty \). Unlike Hoerl and Kennard (1970a), Marquardt also interpreted the matrix \( \beta I \) as the covariance matrix of a set of a priori data. The properties of the ridge solution have been discussed in great detail by Marquardt (1970a) and
Hoerl and Kennard (1970a) and will be discussed only briefly here.

5.3.2.1 Properties of ridge solution

Since the ridge solution \( \hat{x}_R \) is a biased estimate of the least squares solution \( \hat{x} \), its mean squared error is defined by application of the expectation operator as

\[
\text{MSE}(x_R) = E[(x_R - \hat{x})^T(x_R - \hat{x})] = \text{trace} \{ \text{Var}(x_R) + (E(x_R) - \hat{x})^T(E(x_R) - \hat{x}) \} = \text{variance} + (\text{bias})^2
\]

Let \( \lambda_1, \lambda_2, ..., \lambda_p \) be the eigenvalues of the matrix \( A^T A \). We can express eq.5.41a in terms of the eigenvalues of \( A^T A \) and \( \beta \). The variance of \( x_R \) is

\[
\text{Var}(x_R) = \delta^2 (A^T A + \beta I)^{-1} A^T A (A^T A + \beta I)^{-1} = \delta^2 V \Lambda V^T
\]

where \( (\Lambda, j) = \lambda_j / (\lambda_j + \beta)^2 \) and the data are assumed to be uncorrelated and of equal variance \( \delta^2 \).

We have that \( \text{trVar}(x_R) = \delta^2 \sum_{j=1}^{r} \lambda_j / (\lambda_j + \beta)^2 \)

Where \( \text{tr} \) denotes the trace and there are \( r \) non-zero eigenvalues of \( A^T A \).

Since \( E(x_R) = (A^T A + \beta I)^{-1} A^T A \hat{x} = Z \hat{x} \), we obtain from eq 5.41a

\[
(\text{Bias } (x_R))^2 = \hat{x}^T (Z - I)^T (Z - I) \hat{x} = \beta^2 \hat{x}^T V \Lambda \hat{x} V^T \]

where \( (\Lambda, j) = (\lambda_j + \beta) \). Substituting \( Y^T = (Y_1, Y_2, ..., Y_r) = \hat{x}^T V \), we find that

\[
\text{MSE } (x_R) = \delta^2 \sum_{j=1}^{r} \lambda_j (\lambda_j + \beta)^{-2} + \sum_{j=1}^{r} \gamma_j^2 \beta^2 (\lambda_j + \beta)^{-2}
\]

It can therefore be demonstrated that the variance of \( x_R \) decreases as \( \beta \) increases while the squared bias increases with \( \beta \). However, Hoerl and Kennard (1970a), Marquardt (1970) and Marquardt and Snee (1975) showed that there exists a value of \( \beta > 0 \) such that the mean square error (misfit) of the ridge estimator is less than for the least squares estimator. While this provides a justification for using the ridge regression method in this study, it is widely known in geophysical modelling that the best fitting solution is not necessarily unique.
5.3.2.2 Damping factor and the stability of the regression estimates

Hoerl and Kennard (1970a) suggested a subjective graphical technique for selecting the damping factor, $\beta$. This involves the use of a ridge trace (plots of the components of $x_R$ against $\beta$). The ridge trace is examined and a value of $\beta$ chosen in the region where the estimate have stabilised. An example of a ridge plot is shown in fig 5.1.

![Ridge Trace](image)

**Fig 5.1 Ridge trace for typical geoelectric model parameters**

The ridge trace can also be used in the alternative solution stabilization technique. This involves deleting those parameters with small eigenvalues (see Wiggins, 1972) - the singular value truncation method. From the ridge plots, those parameters whose estimates $x_{Rj}$ are unstable with either sign changes or that rapidly decrease to zero are deleted from the inverse problem. We retain only those $x_{Rj}$ that hold their predicting power, or become important as a small bias is added to the problem.

However, for automatic inversion the common practise is to set $\beta$ first to a large positive value thus taking advantage of the good initial convergence properties of the steepest descent method and thereafter $\beta$ is multiplied by a
factor less than unity after each iteration so that the linear least squares method predominates near the solution. A variant of this procedure (Johansen, 1977) assumes as $\beta$ the smallest eigenvalue of $A^TA$ matrix; if divergence occurs it is substituted by the next largest eigenvalue until the solution is obtained.

5.4 Solution Appraisal

One question that is fundamental to physical data analysis is, how representative of the real physical system is our reconstructed least squares model? Inverse theory, in addition to the model parameters, provides us with a plethora of related information which can be used to gauge the "goodness" of the least squares solution to the inverse problem. Two of such "auxiliary parameters" are discussed below.

5.4.1 Goodness-of-fit

Assuming that our data $d_i$ (or $y_i$) are normally distributed about their expected values and with known uncertainties $\delta_i$, we can assess the fit between the observed and calculated data by calculating the statistical parameter, $Q$, defined as

$$Q = \frac{\sum_{i=1}^{N} (d_i - F(m))^2}{\delta^2}$$

For $N$ independent observations and $p$ independent parameters, $Q$ is distributed as $\chi^2$ with $(n - p)$ degrees of freedom. For an unconstrained solution we reject the fit at the $Q_T$ level of significance (see Meyer, 1975 p.397) if

$$\Pr\{\chi^2(n - p) \geq Q_T\} \leq Q_T$$

For a constrained solution with $L$ independent constraint equations the number of degrees of freedom is $n - p + L$, and we reject or accept the fit based on $Q_T$ as given by (Meyer 1975, p.397)

$$Q_T = P_r\{\chi^2(n - p + L) \geq Q\}$$

by comparing the calculated value with $Q(\chi^2;n - p+L)$ from $\chi^2$ tables.

5.4.2 Parameter resolution matrix

For a linear system, we can assess the quality of the model derived from a given data set by examining the parameter resolution matrix (Jackson, 1972) given by

$$R = HA = \{V\Lambda^{-1}U^T\}U\Lambda V^T = VV^T$$

where $U$ and $V$ have been previously defined and the matrix $H$ is the inverse
used. $R$ is of dimension $p \times r$ where $r$, the number of non-zero eigenvalues, is the degree of freedom of the problem. $R$ is evaluated at an acceptable model. If $R = I$, i.e., $p = r$, then each model parameter is uniquely determined. The deviation of the rows of $R$ from those of the identity matrix, $I$, measures the lack of resolution for the corresponding parameters.

For a nonlinear system we can obtain the resolution of a linear problem that is in some sense close to the nonlinear one. For the constrained inversion process, the parameter resolution matrix is given by

$$R_c = VV^T + \frac{V\lambda^2V^T}{\beta I} = \frac{V\lambda^2V^T}{\lambda^2 + \beta I}$$

where $\beta$ is as previously defined.

As usual in the physical sciences, we may want to estimate the range of values which the model parameters might have as a result of the uncertainties associated with the experimental observations.

### 5.5 Errors and extreme parameter sets

An important aspect of data analysis is the determination of bounds on the various model parameters that are consistent with the data and the associated errors. For the sake of clarity we shall adopt the notations of section 5.3.

#### 5.5.1 Parameter covariance matrix

The simplest form of error estimation is the determination of the limits of the parameters from the covariance matrix, $C$. The parameter covariance matrix depends on the covariance of the data and the way in which we map the data errors into parameter errors (Menke, 1984, p.65). If the data are uncorrelated and of equal variance $\delta^2$, then

$$C = [(A^TA)^{-1}A] \delta^2 [(A^TA)^{-1}A^T] = \delta^2 [A^TA]^{-1}$$

The square roots of the diagonal components of this matrix are generally referred to as standard deviations of the least squares estimates and may be used to estimate the bounds of the model parameters. The off-diagonal elements, the covariances, of $C$ indicate the correlations between the parameters.

#### 5.5.2 Extreme parameter sets

We may elect to determine a solution with the maximum tolerable sum of square residuals. One method of extremal inversion is the Most Squares technique of Jackson (1976) in which a value is determined for each parameter which is maximum (or minimum) under the constraint that the misfit of the
observed and calculated data is equal to some desired value. We extremize the linear objective function $x^T b$ under the constraints

$$|y - Ax|^2 = Q_T$$

where the functional $b$ is a vector of zeros with the kth element (to be maximized) equal to 1, i.e. $b^T = (0, ..., 0, 1^k, 0, 0)$ and $x$ is the vector of parameters.

Introducing the Lagrange multiplier $\mu$, we have that

$$\frac{\partial}{\partial x} [x^T b + 1/2 \mu (x^T A^T A x - 2 x^T A^T y + y^T y - Q_T)] = 0 \quad 5.49$$

or

$$A^T A x = A^T y - \mu b$$

from which we obtain the Most squares solution

$$x_m = [A^T A]^{-1} [A^T y - \mu b] \quad 5.50$$

Thus the value of $\mu = 0$ corresponds to the least squares solution. However, eq. 5.50 must satisfy eq. 5.48, therefore

$$Q_T = \mu^2 b^T [A^T A]^{-1} b + \mu b^T [A^T A]^{-1} A^T y + y^T y - y^T A [A^T A]^{-1} A^T y$$

so that

$$\mu = \left( \frac{Q_T - y^T y + y^T A [A^T A]^{-1} A^T y}{b^T [A^T A]^{-1} b} \right)^{1/2} = \pm \sqrt{Q_T - Q_{LS}} 5.52$$

where $Q_{LS}$ is the sum of the square residuals of the optimal least squares solution.

Whenever $Q_T > Q_{LS}$ there will be $2P$ solutions (for $P$ parameters) since there are two solutions for $\mu$ for each parameter. If the errors on the data are assumed univariant and uncorrelated, it is expected that $Q_T$ will have a value equal or close to the number of the data, i.e. $Q_T \approx n$.

For cases in which the matrix $A^T A$ is ill-conditioned Jackson (1976) suggested a modification of (5.50) using instead $H$, the generalized inverse of $A$. $H$ and $A$ satisfy the Penrose (1955) conditions. Thus from

$$A H A = A = U \Lambda V^T (V \Lambda^{-1} U^T) U \Lambda V^T$$

we find that

$$H = V \Lambda^{-1} U^T$$

so that
\[ x_{ms} = H (y - \mu H^T b) \quad 5.53 \]

and
\[ \mu = \pm [(Q_T - Q_{LS})/b^T H H^T b]^{1/2} \quad 5.54 \]

Thus the Most squares method, in effect, provides us with an envelope of solutions that is maximally consistent with the observed data where the ± refers to the upper and lower envelopes.

We can compare directly the most squares and the least squares solutions. Eq. 5.50 can be expressed as
\[ x_{ms} = (A^T A)^{-1} A^T y - \mu (A^T A)^{-1} b \]
\[ = x_{LS} \pm ((Q_T - Q_{LS})/b^T (A^T A)^{-1} b)^{1/2} (A^T A)^{-1} b \]

where \( x_{LS} \) is the least squares solution. The most squares solution envelope may thus be interpreted as the confidence limits of the least squares solution.

5.6 Geometric interpretation of constrained inversion

Recall that constrained inversion requires minimization of some combined function
\[ \phi_c = \| y - Ax \|^2 + \beta x^T H x = Q_1 + \beta Q_2 \quad 5.55a \]

or
\[ \phi_c = \beta \| y - Ax \|^2 + x^T H x = \beta Q_1 + Q_2 \quad 5.55b \]

Let \( H = I \) so that \( Q_2 = \sum x_i^2 \) with an absolute minimum only when \( x_1 = x_2 = \ldots = x_p = 0 \). In the constrained solution process we first determine a solution for a particular value of the Lagrange multiplier \( \beta \). \( Q_1 \) and \( Q_2 \) are then calculated. The process is repeated for several values of \( \beta \). From these calculations we can examine the path of the solution in function space. Such a solution trajectory is illustrated in fig 5.2. In this figure are shown the \( Q_1 \) contours (solid lines) and the \( Q_2 \) contours (broken lines) which in the general situation would represent the \((p-1)\)-dimensioned hypersurfaces of constant \( Q_1 \) or \( Q_2 \). \( Q_1 \) attains a minimum at A and \( Q_2 \) at B.
Fig 5.2 A two-dimensional simplification of the p–dimensioned interactions in the vector space (after Twomey, 1977).

If eq. (5.55a) is minimized, the solution travels from A through C to B as $\beta$ is varied from zero to an infinitely large value and the direction of travel is reversed when eq. (5.55b) is minimized. If, however, we assign a fixed value $\alpha$ to $Q_1$ and minimize (5.55a) or equivalently minimize (5.55b) holding $Q_2$ at a constant value of $\gamma$, a solution C is obtained (the point at which the $Q_1 = \alpha$ and $Q_2 = \gamma$ contours touch in fig 5.2) and it solves both problems. Note that the solution $C_1$ also satisfies the condition $Q_2 = \gamma$ even though it may be a false or undesired solution, a situation that may arise if our initial guess model is very far from the true solution.

However, most practical problems are formulated as the search for the smoothest solution among all possible solutions with $Q_1 \leq Q_T$, where $Q_T$ is the maximum permissible error. Here again $Q_T$ may be such that an undesirable solution may result. To illustrate this point let us minimize $Q_2$ under the condition $Q_1 \leq (Q_T = \alpha")$ and with reference to fig 5.2. The surface $\alpha"$ encloses the point B where $Q_2$ is smoothest and therefore solves the problem; $C_1$ is also a possible solution for the problem. These may not be the desired solutions to the above problem.

It cannot be over-emphasised that a suitable choice of $Q_1$, not so large as to enclose the point B, and a realistic step length for the undetermined multiplier $\beta$ during successive (ie. iterative) application are essential ingredients in the constrained interpretation process. If we use very coarse steps in the $\beta$
factors we may overshoot the true solution \( C \) say, and obtain \( C^1 \) as the best solution to our problem (see also Twomey 1977, p.148).

In geophysical data modelling, our search is often confined to the neighbourhood of \( Q_1 = A \) especially when we begin our iterative search with a good approximation to the true solution and we seek the solution with the smallest misfit between actual and calculated data. The applications (with necessary modifications) of the various concepts outlined in this chapter to practical magnetotelluric problems will be discussed in chapter 6. It is remarked here that most of the current research in nonlinear inversion falls into three main classes: (i) approximate methods of solution, (ii) simpler and more effective numerical procedures for solving the normal equations and (iii) methods for estimating errors or bounds on the parameters of the nonlinear problem. These trends in conventional data analysis will be reflected in the work presented in chapters 6 and 7.
CHAPTER 6
1-D MODELLING AND INVERSION OF MT RESPONSES

Theoretical modelling techniques are used as tools to improve the understanding of the relationship between the MT response functions and the various subsurface resistivity discontinuities that have generated them. The basic theoretical background is contained in chapter 5, and the present chapter elaborates both those aspects of it which were utilised in the present study and those which have been modified and/or extended by the author. The eigenstate analysis of Lanczos (1961), also known as singular value decomposition (SVD), was used extensively in this study.

6.1 Forward modelling.

Traditionally, 1-D interpretation of MT field data involves comparison with theoretical master curves (Cagniard, 1953; Yungul, 1961; Wait 1962b; Srivastava, 1967). These curves are generated by application of equation 2.13 assuming a horizontally homogeneous model with 1-D resistivity distribution. This constitutes the forward approach: given some information on the values of the set of parameters (here, the number of layers and their resistivities and thicknesses) a theoretical relationship is used to derive the values of some measurable quantities (here, apparent resistivities and phases).

Forward modelling by interactive computing is a more versatile extension of the original curve-matching technique. The theoretical curves generated for an initial model are displayed together with the field curves on an interactive terminal. The model parameters are adjusted and the operations repeated until an acceptable visual fit is obtained between the field and theoretical curves.

6.2 Inversion of MT data

An important procedure which has been commonly adopted in recent years for MT data interpretation is the inverse problem in which a conductivity structure is directly retrieved from the data: given some information on the values of some measured quantities (field data) we use a theoretical relationship to derive the values of the set of parameters that explains our field observations as discussed in chapter 5.

The inversion of MT data for subsurface resistivity distribution is a non-unique and nonlinear process, and this poses an interpretation problem. The non-uniqueness stems from the fact that an infinite number of completely different resistivity structures exist that satisfy a finite collection of MT data.
The inversion of actual field data is complicated by additional sources of non-uniqueness: errors on the data, the limited bandwidth of observations, bias effects due to lateral variations in the earth's resistivity structure and cultural noise, and the fact that electromagnetic (EM) sounding data cannot resolve sharp boundaries or thin layers (since the real earth structure is "smeared out" as the propagating energy becomes diffused).

A variety of methods exist for inverting MT data. The commonest technique is the layered earth (parametric) inversion in which a succession of layer resistivities and thicknesses which reproduce the observations is sought (eg. Wu, 1968; Nabetani and Rankin, 1969; Laird and Bostick 1970; Jupp and Vozoff, 1975; Jones and Hutton, 1979b; Larsen 1981). This method requires a reasonable initial guess to the resistivity structure to enable the algorithm to converge rapidly. Direct or exact schemes that do not require a first guess model have been developed by Parker (1980), Parker and Whaler (1981) and Fischer et al (1981). Other workers (eg Parker, 1970, 1977; Oldenburg, 1979; Hobbs, 1982) have developed inversion schemes that produce a resistivity structure that is isotropic and a continuous function of depth. Along similar lines, Niblett and Sayn-Wittgenstein (1960), Becher and Sharpe (1969) and Bostick (1977) have developed techniques to obtain a continuous resistivity function which approximately reproduces the data. Most of the methods referred to above seek a model that either fits the data best or to within an expected tolerance.

Considering the nature of the MT data, it is not expected that any single model will satisfy the recorded data uniquely in the absence of a priori information. Partial remedies to this problem are provided by the random search (Monte Carlo) and linear (Backus-Gilbert) methods. However, the Monte Carlo method (eg Jones and Hutton 1979b), in which a large number of randomly generated models are tested against the data, is too consumptive of computing time. Moreover, such a random search can never be exhaustive. A deficiency of the Backus-Gilbert (Backus and Gilbert 1968) linearized appraisal is that the unique averages refer only to those models that are linearly close to the constructed model and thus it cannot illuminate the true variability of model space (Oldenburg, 1983) and its 'averaged parameters' may not have physical significance (eg. see Jackson 1979, p.144). Oldenburg (1983) has developed techniques for appraising the non-uniqueness inherent in the inversion of MT data for a subsurface resistivity which is a continuous function of depth, and Oldenburg et al (1984) favour the use of a variety of
interpretation algorithms to explore the range of acceptable models as a means of appraising the non-uniqueness. As a way of accounting for non-uniqueness, Constable et al (1987) have developed an algorithm for generating smooth models which are devoid of the sharp discontinuities that typify conventional least squares inversion. It is clear from the above discussion that there is no unequivocal method as yet for resolving non-uniqueness in MT inversion.

In the subsequent sections of this chapter, the well developed tools of generalized matrix inversion (Lanczos, 1961; Jackson, 1972; Wiggins, 1972) are used to invert MT data from the Great Glen region. A technique, similar to Jackson's (1976), is developed to quantify the non-uniqueness inherent in the inversion process. In the use of alternative approaches we incorporate a priori information (Jackson, 1979) to produce acceptable models. The various algorithms are characterised by numerical stability and rapid convergence. The full inversion process is discussed under the two headings: Model Search and Model Interpretation methods.

6.3 Model Search

The solution to a linear inverse problem can be obtained in one step. For the non-linear MT problem, we need to linearize and iterate towards a solution. Since the inherent nonlinearity might be too strong for the linearized technique to be applicable, a variety of stabilization techniques are employed in the solution process.

6.3.1 Linearising parameterizations

Parameterization is the process of selecting variables to represent data and model parameters. In determining subsurface resistivity distributions the model parameters are the layer resistivities and thicknesses or alternatively resistivity-thickness products and the data are the field observations and the model responses calculated from the forward theory. The apparent resistivity is well known to display a lognormal distribution rather than a normal one. It can be demonstrated that the nonlinearity is reduced when the logarithms of the apparent resistivity data are considered instead of the apparent resistivity data themselves. In fact such transformed variables are conventionally used in MT work; the logarithm of model parameters are used in the present inversion process, i.e. \( m = \{ \log \rho_1, \log \rho_2, ..., \log \rho_c, \log T_1, \log T_2, ..., \log T_{c-1} \} \) where \( 2C-1 = p \), the number of model parameters and \( \rho \); and \( T_i \) are the \( i \)th layer's resistivity and thickness respectively.
6.3.2 The optimization scheme

Model search involves finding a solution that minimizes a suitable cost function. It is desired that this solution be both numerically and statistically stable. Statistical stability is important here because of the differing standard errors associated with each observation.

Assuming the data errors $\delta_i$ to be statistically independent, let us define a $(n \times n)$ diagonal matrix, $W$

$$W = (1/\delta_1, 1/\delta_2, 1/\delta_3,...,1/\delta_n)$$

For the sake of clarity we shall keep the same notations as in section 5.3. Let us define further the quantity $Q$, that measures the goodness of our estimated solution as a possible subsurface resistivity distribution

$$Q = e^T e = \sum_{i=1}^{n} [Wy_i - WAx_j]^2 \quad j=1,...,p$$  \hspace{1cm} 6.1

$Q$ is known to be distributed as $\chi^2$.

We state the constrained optimization problems as: Given a maximum admissible misfit $Q_T$, find in the set of possible solutions (on account of observation errors and model errors) with $Q < Q_T$, an acceptable solution which is the smoothest as judged by the measure $x^THx$.

The optimization scheme is simple. We minimize

$$Q_1 = (Wy - WAx)^T(Wy - WAx)$$

Subject to the constraint

$$Q_2 = x^THx$$

where the smoothness measure $Q_2$ represents the squared solution step-length when $H = I$. Note that each datum is scaled by its associated error to prevent undue weight being given to poorly estimated data points. The cost function to be minimized is

$$\phi = \{(Wy - WAx)^T(Wy - Ax) + \beta(x^THx - Q_2)\}$$  \hspace{1cm} 6.2

Recall that $H = D^T D$. Rearranging the constraint equations to form an expression equal to zero and proceeding in the manner described in section 5.3. We obtain the scaled least squares normal equations

$$[(WA)^T\beta^{1/2}D^T][WA\beta^{1/2}D^T]x = [(WA)^T\beta^{1/2}D^T][Wy\begin{bmatrix} \beta^{1/2}D^T \\ 0 \end{bmatrix}$$  \hspace{1cm} 6.3

which may be solved for the constrained solution.
When H = I, we have a member of the family of ridge estimators (Hoerl and Kennard, 1970a; Marquardt, 1970). As in section 5.3 nonlinearity is dealt with using an iterative procedure of the form

\[ x^{k+1} = x^k + [(WA)^T(WA) + BH^{-1}]^{-1} (WA)^T(Wy) \] (6.5)

If the matrix D is a first difference operator (eq. 5.13) we in effect obtain a smooth model (Constable et al., 1987). There are two ways of implementing this algorithm: we either augment the actual data equations with extraneous data (the constraint equations) as shown in equation 6.3 and with \( \beta \) chosen by trial and error or add a constant value of \( \beta \) to the main diagonal of \( A^T A \) (directly augmented singular values) with automatic selection of \( \beta \); these two procedures are used respectively by the optimization routines SMTHF and RIDGE in the inversion program MTINV which has been written by the author.

A flowchart of this optimization scheme is shown in figure 6.1. To speed further the convergence of (6.5) we employ a technique for scaling the Jacobian matrix. Each column of the matrix A is scaled by the root mean sum of squares value of the coefficients (Marquardt, 1963). The scaled matrix \( A^s \) is

\[ A_{ij}^s = A_{ij}/S_j = D^{-1}A_{ij} \]

where

\[ S_j^2 = 1/N \sum_{i=1}^{N} (A_{ij})^2 \]

and the diagonal matrix D is of the form

\[ D = \begin{bmatrix} S_1 & 0 & \ldots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & S_n \end{bmatrix} \]

Let Z = WA, where W is the matrix of observation errors. The least squares estimate of the scaled parameter change vector \( x_s \) is

\[ x_s = [(D^{-1}Z)^T (D^{-1}Z) + BH^{-1}]^{-1} (D^{-1}Z)^T (Wy) \]

\[ = D[Z^T Z + BH^{-1}]^{-1} Z^T (Wy) \]

\[ = D[(WA)^T(WA) + BH^{-1}]^{-1} (WA)^T(Wy) = Dx_R \]

To obtain the original parameter perturbation we then rescale the resultant solution in the form

\[ x_R = D^{-1}x_s \]
Fig 6.1 A simplified flowchart of the minimization algorithm. 1 and 2 refer to optional minimization techniques.
However, scaling is done only when the coefficients in one row are widely different from those in another row. It is avoided when all values are about the same order of magnitude since the additional round-off error incurred during the scaling operation itself may adversely affect the accuracy. As a pre-scaling check, the largest and smallest elements of each column are initially output by the scaling routine SCALEJ in the program MTINV thus permitting user intervention.

Additional stabilizing side constraints come into play during the solution process. For physical reasons bounds are placed on the size of the perturbations using the "smoothness criterion" of Jackson (1973) defined as

$$S = \left\{ \frac{1}{p} \sum_{j=1}^{p} (x_j)^2 \right\}^{1/2} \leq 1$$

At each iteration any perturbation $x_j$ greater than 1 is regarded as unsuccessful or multiplied by a factor ($< 1$) decreasing the length without changing the direction. While this operation prevents the solution from wildly "over-shooting" the linear range, it does slow the convergence. Another side constraint is that the model parameters are non-negative scalar functions.

6.3.3 Computational details

6.3.3.1 The Singular value decomposition (SVD) of a matrix: an overview

Lanczos (1961) discussed the extension of eigenvector analysis to a real $n \times p$ matrix. For such a matrix, $A$, there is an $n \times n$ orthogonal matrix $U$ and a $p \times p$ orthogonal matrix $V$ such that $U^TAV$ assumes one of the two following forms:

(i) $U^TAV = [\Lambda] \quad$ if $n \geq p$

(ii) $U^TAV = [\Lambda \ 0] \quad$ if $n \leq p$

This decomposition is known as the Singular value (or spectral) decomposition (Lanczos, 1961) of $A$. The columns of $U$ and $V$ are the left and right singular vectors of $A$ respectively (ie. pairwise orthogonal unit vectors). $\Lambda$ is an $r \times r$ diagonal matrix, its diagonal entries $\lambda_1, \ldots, \lambda_r$ are the positive singular values of $A$. The number $r$ is the rank of $A$ (or the potential number of degrees of freedom in our data).

$$r \leq \min (p,n) \text{ and } \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_r \geq \lambda_{r+1} = \ldots = \lambda_p = 0$$

However, when $n >> p$, as in most geophysical problems, it is impractical to
store all the matrix $U$. $U$ is often partitioned in the form

$$U = \begin{bmatrix} U_1 & U_0 \end{bmatrix}$$

where $U_1$ is $n \times p$ so that we have the factorisation

$$A = U_1 \Lambda V^T$$

For simplicity we shall refer to $U_1$ as $U$.

The SVD method has been shown to be more robust numerically than most other methods of solving the normal equations for $x$ (see Stewart, 1973; Lawson and Hanson, 1974; Lines and Treitel, 1984). The SVD algorithm used in this study is a variant of the one given by Golub and Reinsch (1970) and was translated from ALGOL by R.L. Parker.

6.3.3.2 Application of SVD to inversion

The least squares solution given by eq 5.36 can be written in terms of the SVD of $A$ as

$$x = [A^T A]^{-1} A^T y = [V \Lambda^{-2} V^T] \Lambda \Lambda^{-1} U^T y = V \Lambda^{-2} U^T y$$

where we have assumed that $r = p$ and the inverse of $A^T A$ (i.e. $V \Lambda^{-2} V^T$) exists.

In ridge regression (the present application), we replace the $1/\lambda_j$ in the $\Lambda^{-1}$ matrix by the element

$$\lambda_{Rj}^{-1} = \lambda_j / (\lambda_j^2 + \beta)$$

If we drop the weighting term $W$ in equation 6.4 and consider the case where $H = I$, the ridge estimator in terms of SVD is

$$x_R = (A^T A + \beta I)^{-1} A^T y = V \Lambda_{R}^{-1} U^T y$$

where $\Lambda_{R}^{-1}$ is related to $\Lambda^{-1}$ by the transform (6.7).

The advantages of using these transformed variables are obvious: when $\lambda_j$ is much larger than $\beta$, $\lambda_j + \beta$ differs very little from $\lambda_j$ and when $\lambda_j$ is close to zero, $A^T A$ will not be so close to singularity.

6.3.3.3 Estimation of damping factors for ridge analysis

The optimum damping factors are determined automatically in ridge regression. The largest and smallest singular values of $A^T A$ are respectively multiplied by 10 and 0.1, and the range divided into 10 values, $Q_k$. This procedure is illustrated graphically in fig 6.2.
Fig 6.2. Estimation of the damping factors in ridge regression. \( Q_s = \lambda_s/10 \) and \( Q_L = 10\lambda_L \) where \( \lambda_s \) and \( \lambda_L \) are respectively the smallest and largest singular values of the problem.

The formula for the Kth \( Q \) is

\[
Q_K = \frac{(100 \; Q_s - Q_L + (Q_L - Q_s)K^2)/99 \; K = 1,...,10}{99}
\]

The values of \( Q \) are squared to give the damping factors. It is clear from fig 6.2 that this technique involves coarse steps in \( \beta \) in the region of \( \beta_L \). These become finer towards \( \beta_s \). \( \beta \) is initially set to the largest value and then the successively smaller values are used until divergence occurs (ie. solution becomes unstable). If no divergence occurs a value of zero is assigned to \( \beta \) (yielding the unbiased estimate). This "down range" search for solutions with minimum residuals is performed by the routine RIDGE and the solution before divergence is used as the next iterate in the main (calling) routine MTINV. The extra computation time is minimal; once the eigenvectors and singular values of the \( A^T A \) matrix have been obtained by the main program, ridge regression estimates can easily be calculated for any value of \( \beta \).

As noted in section 5.6 coarse steps in \( \beta \) may lead us past the true solution. On the other hand, very fine steps in \( \beta \) involving multiplication by a constant value <1 will slow down the convergence. The approach adopted here can be interpreted as intermediate between these two extremes and this feature together with the side stabilizing constraints ensures that the algorithm is characterised by rapid convergence and numerical stability.

6.3.4 Convergence characteristics of the optimization algorithm

To assess the applicability of MTINV in geoelectric interpretation it is instructive to apply the inversion methods first to synthetic data generated using equation 2.13. We generate the responses, at discrete frequencies, of
Fig 6.3. Ridge regression models for synthetic and actual field data.

(a)&(b) 3-layer models for synthetic data
(c) 4-layer models for synthetic data
(d) 3-layer model for actual field data
(e) 4-layer model for actual field data

S = starting model, F = final model. The parameter CHISQ represents the minimum achievable misfit in models (a) & (b) and the expected tolerance in the other models. DF is the damping factor used in (c). Ri and Di are respectively the bounds on the model resistivities (in $\Omega$m) and depths(m) to interfaces. The Niblett-Bostick transformation are also shown with error crosses. Details of the synthetic models are given on page 107.
1D RIDGE REGRESSION MODEL FOR SITE RAID TEST

CHISQ=0.01  NFREQ=38
3 LAYERS  AMP+PHASE FIT

RESISTIVITY OHM.M

FREQUENCY IN HZ

PHASE DEGREES

APPARENT RESISTIVITY OHM.M
1D RIDGE REGRESSION MODEL FOR SITE RR1D TEST

CHISQ=7.24  DF=0.0003815  NFREQ=38
4 LAYERS

RESISTIVITY OHM.M

FREQUENCY IN HZ

DEPTH M.
1D RIDGE REGRESSION MODEL FOR SITE L10 INVARIANT

CHISQ=15.83  NFREQ=29
3 LAYERS  AMP+PHASE FIT
RESISTIVITY OHM.M

PHASE DEGREES
0. 15. 30. 45. 60. 75. 90. 10
FREQUENCY IN HZ

DEPTH M.

S  F  S

R1 1322 1322
01 1453 145
R2 7891 7891
02 17862 178
R3 1178 1178
1D RIDGE REGRESSION MODEL FOR SITE L5

CHISQ=43.99  NFAEQ=34
4 LAYERS  AMP-PHASE FIT

EFFECTIVE

RESISTIVITY OHM.M

S

F

S

APPARENT RESISTIVITY OHM.M

PHASE DEGREES

15° - 90° - 85° - 82.1° - 10° - 100000

FREQUENCY IN HZ
two models that typify the situation in northern Scotland (as determined by previous studies), A and B shown below

**Model A (3 layers)**

- 1000 Ohm-m (1 Km)
- 10000 Ohm-m (10 Km)
- 1000 Ohm-m (∞)

**Model B (4 layers)**

- 1000 Ohm-m (1 Km)
- 10000 Ohm-m (10 Km)
- 200 Ohm-m (30 Km)
- 2000 Ohm-m (∞)

We add 10% noise to the calculated responses to reflect possible field conditions. The minimization routine is applied to these data sets using widely different starting models. Two 3-layer models (derived from Model A data) with the minimum achievable error are shown in figs 6.3 a&b. In fig 6.3c are shown two recovered 4-layer models (from Model B data) with a prescribed error: in this case an optimum model is first sought and then the model parameters are changed in opposite direction until the error condition is satisfied. These results are satisfactory. The Bostick (1977) transformation of the data are also shown for comparison. The results are compatible. Two applications to actual field data are shown in figs 6.3 d&e.

The convergence information is represented in fig 6.4. The error for 10 iterations are shown in order that we can appreciate the convergence rate; the sum of squared residuals decreased by a factor of about 13 in only two iterations. Notice that the decrease in error between iterations 4 and 10 is barely perceptible.

![Fig 6.4 Convergence characteristics of the algorithm](image-url)
Comparison of the ridge regression method as proposed by Hoerl and Kennard (1970a) and the generically related procedure of augmenting the actual data with extraneous data showed that the latter on account of computational cutbacks (damping factor determined post facto) is about 40% less consumptive of computing time but the former is preferred when the starting model is very far from the true solution.

6.4 Model interpretation methods

It is obvious from the previous section that we have at our disposal efficient routines for minimizing a functional subject to some constraints. However, the optimal solutions may not be mathematically unique. We may examine the parameter resolution matrix of any acceptable model to see what features of the model are important. The exact damped resolution matrix is given by equation 5.46.

The model resolution matrices for optimal models derived from actual field data are shown in tables 6.1a–c. The rows of the matrices show the extent to which each model parameter can be resolved. It is obvious that the main diagonals of the matrices are delta-like i.e. R = I. However, the MT problem typically admits an infinite number of solutions. In this situation, the favoured approach is to incorporate a priori information in the interpretation scheme to yield solutions that are coherent with the geological background and other geophysical data. In this study the matrix $A^TA$ is always of full rank and $R = I$.

6.4.1 Damped most squares method

This is an iterative extremal inversion technique never before applied in the MT situation and is used here to quantify the non-uniqueness inherent in the inversion problem. We formulate the constrained extremal inversion problem as follows: given an optimal solution to an inverse problem, $x_0$ with error $Q_0$, and a specified maximum tolerable residual error $Q_T$, search around this solution for other solutions which fit the data to the expected tolerance; or equivalently, extremize the linear parametric function $x^Tb$ under the constraint

$$(Wy - WAx)^T(Wy - WAx) + Kx^THx = Q_T$$  \hspace{1cm} 6.10$$

where the quantity $Kx^THx$ is a penalization criterion that measures the solution’s departure from smoothness and helps stabilize the iterative solution process.
### Table 6.1: Model resolution matrices for sites (a) L11, (b) L16, and (c) G1. The first 3 members of the rows and columns of the matrices correspond to the resistivities and the others to the thicknesses of the model layers.
The functional \( b \) consists of zeros except at the \( i \)th position since we wish to extremize the \( j \)th component. The inner product \( x^T b \) thus represents a projection of the unknown vector \( x \) along a known vector \( b \). For the sake of clarity, we shall drop the error term \( W \) in equation 6.10 but keep in mind that the equations have been scaled by their associated standard errors.

We proceed in the same manner as in subsection 5.5.2 and find the constrained solution at the extremum. The minimization is straightforward. Introducing the Lagrange multiplier \( 1/2 \mu \), we have that

\[
\exists \mu \left[ \frac{1}{2} \mu \left( x^T A x - x^T A y - y^T A x + y^T y + K x^T H x - Q_T \right) + x^T b \right] = 0 \quad 6.11
\]

\[
0 = A^T A x + x^T A^T A + K H x + K x^T H - 2 A^T y + b
\]
or

\[
[A^T A + K H] x = [A^T y + \mu b]
\]

from which we obtain the damped least squares solution

\[
x_0 = [A^T A + K H]^{-1} (A^T y - \mu b) \quad 6.12
\]

Here we will consider the case where \( H = I \) so that

\[
x_0 = [A^T A + K I]^{-1} (A^T y - \mu b) \quad 6.13
\]

where \( K \) is the biasing constant added to the main diagonal of \( A^T A \) during the search for the optimal solution.

When the quadratic constraint (eq. 6.10) is satisfied, we have that

\[
Q_T = x^T [A^T A + K I] x - 2 x^T A^T y + y^T y
\]

\[
= (y^T A - \mu b^T) (A^T A + K I)^{-1} (A^T y - \mu b) + y^T y - 2 [(y^T A - \mu b^T) (A^T A + K I)^{-1} A^T y]
\]

After some algebra we find that

\[
Q_T = y^T y + \mu^2 b^T (A^T A + K I)^{-1} b - y^T A (A^T A + K I)^{-1} A^T y
\]

from which we obtain

\[
\mu = ((Q_T - y^T y + y^T A (A^T A + K I)^{-1} A^T y) / b^T (A^T A + K I)^{-1} b)^{1/2}
\]

\[
= \pm ((Q_T - Q_0) / b^T (A^T A + K I)^{-1} b)^{1/2} \quad 6.14
\]

Provided that \( Q_T > Q_0 \), there exist two solutions for \( \mu \) yielding 2p solutions for the \( p \)-parameters which are maximally consistent with the data. The expected value of \( Q_T \) is close to \( N \), the number of observations.

Using the singular value decomposition of \( A \), we re-write eq. 6.13 as
\[ x_D = V\Lambda_D^{-1}U^T y - \mu V\Lambda_D^{-2}V^T b \]  \hspace{1cm} 6.15

where

\[ \Lambda_D = \text{diag} \left( (\lambda_i^2 + K)/\lambda_i, \quad j=1,...,p \right) \]

The initial iteration of equation 6.13 gives the first linear approximation to the extreme model \( x_m \)

\[ x_m^1 = x_0 + x_0 \]

The minimization at iteration \( K+1 \) is

\[ x_m^{K+1} = x_m^K + (A^T A + K I)^{-1} (A^T y - \mu b) \]  \hspace{1cm} 6.16

where the Jacobian, \( A \) and \( \mu \) are evaluated at \( x_m^K \). We may re-express eq. 6.15 as

\[ x_D = V\Lambda_D^{-1}U^T y \pm |\mu| V\Lambda_D^{-2}V^T b \]  \hspace{1cm} 6.17

where the first term on the right hand side is the 'ridge' solution and \(|\mu|\) denotes the absolute magnitude of \( \mu \). So that \( x_D \) may be interpreted as providing the confidence limits of our optimal solution.

The damping of the singular values corresponds to the a priori assumption that the parameter change vectors are small. The damping ('filtering') procedure therefore confines the parameter change vector to a region where the linearization yields meaningful approximations to the nonlinear function (Marquardt, 1963). This approach was adopted since Jackson's (1976) prescription for dealing with singularities in \( A^T A \) was found to be inadequate for practical MT data. The use of the formula (eq. 6.16) ensures that the solutions do not oscillate wildly and that the condition given by eq. 6.10 is satisfied. A flowchart of this interpretation process is shown in fig 6.5. One pitfall in most squares interpretation is that a negative model parameter may be reached before the stipulated condition is satisfied. This situation arises when the error parametric surface is open in certain directions so that the parameter can be changed without any perceptible change in the sum of the squared residuals. To arrest this likely situation, an upper bound is fixed for the parameter changes (33% of the actual value of the parameter) in the algorithm, leading to a trade-off between convergence rate and stability. This side constraint also ensures that the sought quantities do not change their sign too soon in pathological situations; the search is terminated in any one direction if the parameter assumes a negative value. This scheme is executed by the routine MOSTSQ in the interpretation program MTINV.
START

OPTIMAL MODEL($X_o$),
$\beta$, ERROR($Q_o$)

INITIALIZE VARIABLES
$Q_T = Q_o \times \text{FACTOR}$, $J = 0$

BEGIN ITERATIVE SEARCH
$J = J + 1$, $m_j = X_o$, $Q_T = Q_o$

FORM PROJECTION VECTOR $b_j$

CALCULATE JACOBIAN, $A$

CALCULATE SVD OF $A$

1

GET $\lambda + \beta$

2

CALCULATE $\mu$, $X_d$

CONTROL RATE OF CHANGE OF $m_j$

NEW ESTIMATE
$m_j = m_j + X_d$

CALCULATE NEW ERROR
$Q_T = \sum |y - Ax|^2$

FOR ALL PARAMETERS?

YES

SOLUTIONS OBTAINED?

YES

STOP

NO

CHANGE SIGN OF $\mu$, $m_j = X_o$, $Q_T = Q_o$

SEARCH

OBTAIN NEXT ITERATE

OUTPUT

Fig 6.5 A simplified flowchart of the iterative most squares algorithm. 1 and 2 refer to optional optimization procedures.
The main advantage of the damped most squares interpretation method is that the errors on the data and the non-unique nature of the solution are accounted for using the class of solutions which are maximally consistent with the observations. In this way the interpreter can decide, at a glance, what features of the model are important. It is hoped that this procedure will prevent the over-interpretation of sparse data. The method also forms a compromise between the time-consumptive Monte-Carlo and the faster single-model interpretation methods.

6.4.2 The a priori information approach

A priori knowledge about the values of some parameters can be incorporated right from the start in the optimization scheme to yield a unique solution (Jackson, 1979). For instance, we may obtain values for certain layer resistivities or thicknesses from borehole logs or other geophysical or geological observations and wish to retain these values in our final result. In the same vein, for an MT site A with a sparse or biased data set we may presume that the resistivity distribution at a neighbouring site B (with good quality data) is similar and incorporate its resistivity-depth information in the interpretation of site A responses.

Let the parameter vector \( m = (m_1, m_2, \ldots, m_p) \) characterise the resistivity-depth distribution under the site whose data we wish to invert. Let the vector \( c = (c_1, c_2, \ldots, c_p) \) be the known results from prior geophysical observations etc. This interpretation problem is the same as posed by equation 5.25. The differences between the vectors \( m \) and \( c \) in the present problem are represented by the vector \( h \) in equation 5.25. Here we minimize

\[
(Wy - WAx)^T(Wy - WAx) + \beta (x - (m - c))^T(x - (m - c))
\]

6.18

The solution for the parameter increment is

\[
x_a = [(WA)^T(WA) + \beta I]^{-1} (WA)^T(Wy) + \beta h
\]

6.19

where \( h_j = m_j - c_j \).

Equation 6.19 is applied iteratively to minimize eq. 6.18. In the first iteration, however, \( h \) is a null vector since our initial best guess model \( m_0 \) is the a priori model \( c \). The minimization at iteration \( k+1 \) is

\[
m^{k+1} = m^k + [(WA)^T(WA) + \beta I]^{-1} [(WA)^T(Wy) + \beta (m^k - c)]
\]

6.20

In the implementation of this algorithm we write the a priori data in the form given by equation 5.24. The augmented data equations lead to the alternative
least squares normal equations of the form given by eq. 6.3 (where h is a null vector). It follows from this that the ridge regression method uses a different a priori model at each iteration. This procedure is optionally executed by the routine SMTHF in the interpretation program MTINV. A value of unity is assigned to B in practice.

It is normal practice to invert the amplitude and phase data simultaneously. Often we find that apparent resistivity curves have been shifted statically. The inversion of the phase data only with a priori parameters can therefore yield a much more realistic geoelectric structure than would be obtained from incorporation of the apparent resistivity data.

6.4.3 Multi-station interpretation

The interpretation of data sets from sites along a traverse poses a further problem since we often find zones in which the electrical properties are in total contrast to those of the adjacent regions (i.e. anomalous). Such anomalies may be artifacts of the inversion algorithm used or may have resulted from over-interpretation on the part of the interpreter when faced with a catalogue of equivalent models. The non-subjective interpretation method adopted in this study involves inversion of the various data sets either jointly or simultaneously.

6.4.3.1 Joint inversion

The independent data sets are combined and inverted to yield a representative structure. If this structure appears unacceptable then the data sets do not characterise the same geoelectric structure. For the acceptance criterion we use the joint misfit defined as

$$\chi^2 = \left\| W_y - W y_0 \right\|^2$$

where

$$y_0 = \begin{bmatrix} y_1 \\ \vdots \\ y_K \end{bmatrix}, \quad A_0 = \begin{bmatrix} A_1 \\ \vdots \\ A_K \end{bmatrix}$$

K is the total number of sites and $\left\| . \right\|$ denotes the usual Euclidean norm. This procedure is an optional facility in the interpretation program MTINV.
6.4.3.2 Simultaneous inversion

We are interested in obtaining a laterally smooth profile. We start off with a smooth profile (i.e., the same model for all sites) and invert all the various data sets simultaneously with the constraint that the differences in layer parameters between physically adjacent sites be minimal and that the solutions be statistically stable.

This technique is now demonstrated using a two-site example. Extension to higher dimensions is straightforward. The data and constraint equations are partitioned in the form

\[
\begin{bmatrix}
A_1 & 0 \\
-1 & 0 \\
0 & A_2 \\
1 & -1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= 
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
\]

where \( A_* \) is an \((n+n'+p)x(p+p)\) matrix containing the partial derivatives \( A_1, A_2 \) plus the \( px2p \) smoothness matrix for the two sites, \( x_* \) contains the solution vectors \( x_1 \) and \( x_2 \) each of dimension \( px1 \) and the vector \( y_* \) contains the discrepancy vectors \( y_1 (nx1) \) and \( y_2 (n'x1) \) plus the augmenting data \( h_1, \ldots, h_p \) which are the desired differences in the values of the layer parameters. In this example \( h \) is a null vector.

We state the optimization problem as the search among all possible solutions with \( |y_* - A_* x_*|^2 \leq Q_T \) (the maximum tolerable error), for the smoothest solution as judged by \( |x_1 - x_2|^2 \).

We form the Lagrange function

\[
\phi (x_1, x_2) = \sum_{j=1}^{p} (x_{j1} - x_{j2})^T (x_{j1} - x_{j2}) + \mu^{-1} \left( \sum_{i=1}^{n} (y_{i1} - \sum_{j=1}^{p} A_{ij1} x_{j1})^2 + \sum_{k=1}^{n'} (y_{k2} - \sum_{j=1}^{p} A_{kj2} x_{j2})^2 - Q_T \right)
\]

6.21
where \( \mu \) is an undetermined multiplier and the numbers 1 and 2 indicate the site contributing the data; these data have been standardized but the W term is neglected for clarity. Setting its derivatives with respect to the model parameters to zero we obtain for the first linear approximation

\[
\frac{\partial \phi}{\partial x_1}(x_1, x_2) = x_1 - x_2 + \mu^{-1}(A_1^TA_1x_1 - A_1^TY_1) = 0
\]

giving \((A_1^TA_1 + \mu I)x_1 = A_1^TY_1 + \mu x_2\)

or

\[
\frac{\partial \phi}{\partial x_2}(x_1, x_2) = x_2 - x_1 + \mu^{-1}(A_2^TA_2x_2 - A_2^TY_2) = 0
\]

giving \((A_2^TA_2 + \mu I)x_2 = A_2^TY_1 + \mu x_1\)

so that

\[
x_1 = (A_1^TA_1 + \mu I)^{-1}(A_1^TY_1 + \mu x_2) \tag{6.22a}
\]

and

\[
x_2 = (A_2^TA_2 + \mu I)^{-1}(A_2^TY_2 + \mu x_1) \tag{6.22b}
\]

The above equations (6.22a&b) must be solved simultaneously for the solutions and may be applied successively to recover the desired profiles. It is clear from equations 6.22a&b that the solutions are coupled via \( \mu \). If the constraint equations are weighted more heavily than the data equations then the differences between the values of the parameters are minimized at the expense of increasing the prediction error of the other equations. A very small value of \( \mu \) would produce the usual rough models. \( \mu \) is determined by trial and error until an acceptable total sum of squared error is obtained. This technique may be interpreted in simple terms as moving a resolving kernel across the surface instead of down the geoelectric section as is customary in geophysical modelling. With appropriate constraints one might deduce a less complicated resistivity structure for the traverse than would be obtained using conventional interpretation methods. The interpretation program MT4INV handles a maximum of 4 sites; modification to interpret \( > 4 \) sites is trivial.

6.4.4 Occam's razor technique

This is an interpretation technique that produces a very smooth layered structure. The mathematical formulation is as given in section 5.2 and applied in subsection 6.3.2. The constraint equations are of the form given by eq.5.13.
The method is conceptually similar to the multi-site lateral smoothing technique the difference being that the smoothness measures are applied down a geoelectric section here. Unlike conventional simple layered inversion we allow for a large number of layers (dependent on the number of field responses) with the constraint that the differences between the values of physically adjacent parameters be minimal (ie. "shaved off by Occam's razor"). The desired misfit is fixed at or near the number of field data. Operationally, a starting model is constructed by the routine PREP (called by MTINV) using a simple data transformation and the step at each iteration is optimized by the routine SMTHF until a model that satisfies the prescribed misfit condition is found. Only a few iterations (<5) are required in general and computational time is minimal (<6 sec. CPU).

The advantage of this technique is that the solution does not require or depend on a user-supplied preferred (or guess) model but on the data. Constable et al, (1987) applied a similar technique to practical EM sounding data and termed it Occam's inversion.

6.4.5 Niblett-Bostick (N-B) transformation

The N-B transformation (Niblett and Sayn-Wittgenstein, 1960; Bostick 1977; Jones 1983) is a simple semi-continuous mapping of MT data into resistivity-depth information using an algorithm based upon the asymptotic response of the impedance data in a horizontally layered model with infinitely conducting or infinitely resistive substrata. Parameters for a single layer are provided, by this technique, for each resistivity and phase point, so that a continuous resistivity-depth curve may be obtained for the resistivity and phase independently. The relevant expressions are

\[ h = \left[ \frac{\rho_a}{(\mu_0 \omega)} \right]^{1/2} \]

and

\[ \rho_\alpha(h) = \rho_a \left[ (\pi/2\phi) - 1 \right] \]

where \( \omega \) is the angular frequency, \( h \) is the penetration depth in a half space medium of resistivity equal to the observed value \( \rho_a \) at the particular frequency and \( \rho_\alpha \) is an alternative expression that incorporates phase (\( \phi \), in radians) information for the N-B resistivity at depth \( h \) (Weidelt et al, 1980).
6.5 One-dimensional models

6.5.1 1-D response functions and the existence of solutions

Weidelt (1972, 1986), Parker (1977, 1980, 1983) and Parker and Whaler (1981) provide us with conditions that a given MT data set must satisfy for a 1-D solution to exist. While the Great Glen region is at least 2-D in character, it is now well-known from 2-D and 3-D modelling studies that we can recover representative 1-D profiles in regions of complex structures (Tikhonov and Berdichevsky, 1966; Berdichevsky and Dmitriev, 1976; Ranganayaki 1984). The effective response functions given by equations 2.27a&b have been shown by Ranganayaki (1984) to be very useful in 1-D modelling studies and were considered invertible in this study.

6.5.2 Comparison of methods

It is almost customary to interpret the 1979 COPROD data (Jones, 1979) as a check on any 1-D inversion scheme. Parker (1982) shows that any model for the COPROD data can only be constrained to a maximum depth of about 300Km. The model obtained using the Occam's razor technique is shown in fig 6.6a. It is noteworthy that the Occam model shows no structure above 5Km and below 300Km where there are apparently no constraining data. The Occam model is superposed on the 'smoothest' model derived by Constable et al (1987) for the COPROD data in fig 6.6b. The Monte Carlo model of Jones and Hutton (1979b) for the same data is also shown in this figure for comparison. In fig 6.6c the 'smoothest' and Monte Carlo models are superposed on the most squares models for the COPROD data for comparison. Notice the high degree of concordance between the various models. Also, it can be noted that as might be expected the most squares models cover a zone within which all the other structures lie.

For most of the Great Glen observation sites the effective data sets were inverted using the multi-purpose user-friendly interpretation program MTINV. A typical value for the time for a detailed interpretation (ie. model search and most squares analysis) for a 4-layer problem is 45 seconds (CPU). For a smooth (Occam) model with <20 layers the run time is <10 seconds (CPU). The resultant models for five selected sites are presented in figs 6.7-6.10 and table 6.2 to illustrate various aspects of the different procedures.

A comparison of the results obtained using the ridge regression method, the most squares technique and the N-B transformation of the data is shown in figs 6.7a&b. The same error condition has been applied in the two former
methods. The most squares models clearly cover a range of possible models and thus provide a measure of the non-uniqueness. The result of the joint interpretation of E- and H-polarisation data for site L1 is shown in fig 6.8a. The low frequency data (< 1Hz) are typically noisy but 1-D interpretation could provide a realistic estimate of the resistivity distribution under this location as also suggested by the most squares models for the joint data set (fig 6.8b). The Occam model (amplitude fit only) with superposed N-B results for L1 is shown in fig 6.8c. Note the general agreement between the Occam and N-B results. However, the Occam model does not fit the phase data satisfactorily at the low and very high frequencies. An independent interpretation of the phase data alone produced an unacceptable model as shown in figure 6.8d. This suggests that simultaneous inversion of phase and amplitude data is required for site L1.

The adequacy of interpreting the amplitude data only for site L19 is suggested by the Occam and most squares models shown in figs 6.9 a&b. Notice the excellent fit of both the amplitude and phase data for this site. The simultaneous multi-site interpretation technique is demonstrated for two sites L8 (S1, 33 frequencies) and L5 (S2, 34 frequencies) in the computer printout from MT4INV (table 6.2). The individual station (L8 and L5) interpretations (using MTINV) are presented in figs 6.10 a&b for comparison.
fig 6.6. COPROD models

(a) Occam model.

(b) Comparison of Occam, Monte Carlo (Jones and Hutton 1979b) and 'smoothest' (Constable, Parker and Constable 1987) models.

(c) Comparison of most squares, Monte Carlo and 'Smoothest' models.

The type of data inverted, apparent resistivity (AMP) and/or phase are indicated. NFREQ is the number of data points. An acceptable model will have a CHISQ value comparable to NFREQ.
OCCAM MODEL FOR SITE 1979 COPROD

COPROD Model, rms 1.0

RESISTIVITY OHM.M

LOG$_{10}$ resistivity ($\Omega$·m)

CHISQ=14.75  NFREQ=15
15 LAYERS  AMP FIT

LOG$_{10}$ resistivity ($\Omega$·m)

Depth (m)

COPROD Model, rms 1.0

C&P\&C

SMOOTHEST MODEL

(1987)

JONES \& HUTTON

(1979) MODEL "J"

Period (s)

Phase (degrees)

10$^2$  10$^3$

2.1

2.2

2.3

2.4

2.5

80

60

40

20

0

10$^3$
MOST-SQUARES SM MODELS FOR SITE 1979 COPROD

CHISQ=33.66  NFREQ=15
4 LAYERS  AMP+PHASE FIT

RESISTIVITY OHM.M

DEPTH M.

FREQUENCY IN HZ
Fig 6.7. Comparison of (a) Ridge regression and (b) Most Squares models for site L18.
Fig 6.8. Some models for site L1.

(a) Joint interpretive regression model
(b) Joint most squares models
(c) Occam and N-B results
(d) Independent interpretation of phase data

The data types used in the inversion are apparent resistivity (AMP) and/or phase angle are indicated. NFREQ is the number of data points. An acceptable model will have a CHISO value comparable to NFREQ. Model (d) satisfies the CHISO criterion but is rejected as it does not fit the apparent resistivity data.
JOINT INTERPRETIVE MODEL FOR SITE L1

CHISQ=845.20  NFREQ=84
3 LAYERS  AMP FIT

RESISTIVITY OHM.M

MOST-SQUARES SM MODELS FOR SITE L1  JOINT POL

CHISQ=718.26  NFREQ=64
3 LAYERS  AMP FIT

RESISTIVITY OHM.M
OCCAM MODEL FOR SITE L1 EFFECTIVE

CHISQ=31.89  NFREQ=32
16 LAYERS  AMP FIT

RESISTIVITY OHM.M

DEPTH M.

OCCAM MODEL FOR SITE L1 EFFECTIVE

CHISQ=22.74  NFREQ=32
16 LAYERS  PHASE FIT

RESISTIVITY OHM.M

DEPTH M.
Fig 6.9. (a) Occam and (b) Most squares models for L19. The N-B results are also shown for comparison.
Fig 6.10. Most squares and N-B results for (a) L8 and (b) L5.
Fig 6.10. Most squares and N-B results for (a) L8 and (b) L5.
<table>
<thead>
<tr>
<th>RESISTIVITY</th>
<th>DEPTH(S1)</th>
<th>RESISTIVITY</th>
<th>DEPTH(S2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000.00</td>
<td>1500.00</td>
<td>2000.00</td>
<td>1500.00</td>
</tr>
<tr>
<td>8000.00</td>
<td>30000.00</td>
<td>8000.00</td>
<td>30000.00</td>
</tr>
<tr>
<td>300.00</td>
<td></td>
<td>300.00</td>
<td></td>
</tr>
</tbody>
</table>

**EXACT SUM OF SQUARED MISFIT(SSQ), FIELD/MODEL DATA**

1497.216

---

<table>
<thead>
<tr>
<th>RESISTIVITY</th>
<th>DEPTH(S1)</th>
<th>RESISTIVITY</th>
<th>DEPTH(S2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975.35</td>
<td>1457.78</td>
<td>2628.73</td>
<td>987.47</td>
</tr>
<tr>
<td>5072.25</td>
<td>27922.00</td>
<td>6860.70</td>
<td>36013.31</td>
</tr>
<tr>
<td>1098.23</td>
<td></td>
<td>696.14</td>
<td></td>
</tr>
</tbody>
</table>

**PREDICTED SUM SQUARED DEVIATIONS**

75.67929

**EXACT SUM OF SQUARED MISFIT(SSQ), FIELD/MODEL DATA**

151.8023

---

<table>
<thead>
<tr>
<th>RESISTIVITY</th>
<th>DEPTH(S1)</th>
<th>RESISTIVITY</th>
<th>DEPTH(S2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1872.60</td>
<td>1299.61</td>
<td>2178.69</td>
<td>599.54</td>
</tr>
<tr>
<td>5146.33</td>
<td>29633.41</td>
<td>7499.34</td>
<td>39653.50</td>
</tr>
<tr>
<td>906.58</td>
<td></td>
<td>565.53</td>
<td></td>
</tr>
</tbody>
</table>

**PREDICTED SUM SQUARED DEVIATIONS**

75.02228

**EXACT SUM OF SQUARED MISFIT(SSQ), FIELD/MODEL DATA**

76.09239

**PREDICTED SUM SQUARED DEVIATIONS**

75.31596

---

**Table 6.2** A demonstration of the simultaneous multi-site interpretation technique. Model resistivities are in $\Omega$m and depths in metres. SSQ is the actual squared misfit between field and calculated model data and the predicted value is the squared residuals $|y - Ax|^2$.
The individual station results for the three traverses are presented in appendix IV. The interpretive composite sections derived from the 1-D results are now presented and discussed.

6.5.3 Interpretive geoelectric sections

For each traverse the 1-D most squares results are ‘pieced together’ into a coherent structure that can be interpreted as characterising the resistivity distribution of the region and by implication its geotectonic structure. These are called the geoelectric sections. Only the top 40Km is shown except for the Glen Loy traverse.

The interpretive section for the Glen Garry traverse is shown in fig 6.11a. A 1–2Km thick superficial layer (probably consisting of weathered igneous and metamorphic rocks) with resistivity in the range 1500–3000Ωm is underlain by a highly resistive (4000–11000Ωm) crystalline upper crustal unit that is thickest near the Great Glen fault (20–30Km) and under G1 (interpreted here as due to a plutonic body which outcrops in the area). Between G2 and G4 this unit is only 6–9 Km thick. A lower resistivity (300–1500Ωm) unit underlies the resistive upper crust. The depth to this low resistivity unit appears to be shallow (~6 Km) in the neighbourhood of G3. However, 2-D modelling is required to ascertain the presence of any lateral inhomogeneity. Recall that this feature was also inferred from qualitative studies of the data (chapter 4).

The Loch Arkaig geoelectric section (fig 6.11b) shows a less complicated structure than the Garry section. Three crustal layers are distinguished. A supercrustal layer, 1–2 Km thick and of 1000–3200Ωm resistivity can be traced continuously from L1 to A4 west of the GGF. This layer probably consists of weathered Caledonian metamorphics and is underlain by a highly resistive (2500–7000Ωm) layer, about 13–16 Km thick. A lower crust of 200–500Ωm resistivity forms the basal unit of the structure in this area.

The much longer Glen Loy geoelectric section is shown in fig 6.11c. Each geoelectric unit exhibits a marked horizontal inhomogeneity with irregular bounding surfaces that possibly reflect the complex tectonics of the region. The upper crust is in general very resistive (2000–22000Ωm) and appears to thicken from 10–15 Km at the traverse extremes to about 35 Km over a 12 Km wide zone that is asymmetric about the GGF. A deep trough-like structure of low resistivity (100–1000Ωm) occurs at the position of GGF. The lower crust/upper mantle layer is of low resistivity (50–400Ωm) but its thickness is not well-determined.
Fig 6.11. Interpretive geoelectric sections for (a) Glen Garry (b) Loch Arkaig and (c) Glen Loy profiles. The Vertical bars in fig 6.11c represent the most squares limits of the 1-D models.
An intermediate layer, about 10–15 Km thick, with resistivity in the range 900–1500Ωm appears to be present from about 8 Km east of the GGF. This layer is separated from the zone of thickened crust by a low resistivity dyke–like body. The surface layer is thickest in the fault zone and its immediate surroundings and is shallower than 2.5 Km elsewhere. This structure will be compared with the regional 2-D model and interpreted tectonically in chapter 8.

6.5.4 Effective phase sections

Ranganayaki (1984) demonstrated the usefulness of pseudosections of the phase of the effective impedance for structural interpretation in 2-D environments. Traditionally, pseudosections provide a general picture of resistivity or phase variations with frequency; only qualitative inferences can be made from such sections. In this study however, the phase data are mapped onto their appropriate depths via the Niblett–Bostick transformation to give a more realistic structural pattern. Only the Glen Loy section (shown in fig 6.12) will be considered here. In a homogeneous medium the effective phase is 90 degrees, and it is higher or lower than this value in conductive or resistive strata respectively. It is obvious in fig 6.12 that the top 10 Km (about 30–40 Km near the GGF) of the crust is resistive and is underlain by a conductive layer. The phase contours show lateral variations in structure especially to the east of the GGF. Note the indication of a buried vertical contact in the neighbourhood of sites L16 and L17. Thus this new method of presentation of the phase data provides an equivalent interpretive cross-section to that obtained from the more rigorous data inversion (fig 6.11c). Additional structural information are also provided by the phase section. Notice the indication of structural discontinuity at the near-surface in the vicinity of L17. This probably corresponds to the Fort William Slide. The underlying structures appear severely contorted. The western part of the region appears to be relatively less deformed. The Glen Loy Gabbro may be funnel-shaped with a bottom depth of about 25 Km; and appears to be fault–bounded at the near-surface in the vicinity of L4 and L5. The lack of observations between L2 and L1 militates against any detailed interpretation west of L2. A general eastward dip of the deep structures in the vicinity of the GGF is apparent.
Fig 6.12 Glen Loy invariant phase section. Contour interval is 5 degrees. The pecked lines correspond to approximate boundaries between resistive ($\Phi_{\text{eff}} < 90^\circ$) and conductive ($\Phi_{\text{eff}} > 90^\circ$) areas.
CHAPTER 7
TWO-DIMENSIONAL NUMERICAL MODELLING STUDIES

The structural complexity of the Great Glen region requires the use of 2- or 3-dimensional modelling procedures for realistic quantitative interpretations. Fortunately the main structural elements are, to a fair approximation, two-dimensional with a dominant N49°E (magnetic) trend. The relevant MT responses have been obtained with axes parallel and perpendicular to this direction (see Chapter 4). The determination of 2-D models along with a limited search for the range of possible models that fit the measured responses for the various traverses will be discussed and the models presented following an overview of the conventional techniques for deriving 2-D geoelectric information.

7.1 Introduction

The search for a 2-D model proceeds either through forward calculations in which the MT response of a prior resistivity model is obtained and compared with measured data or by inverse calculations in which a model is discovered from the field data. These techniques are plagued by the non-uniqueness problem.

Direct inversion of 2-D MT data is difficult and highly non-linear. The commonest way to solve the inverse problems at the present time is to linearize them and apply the tools of generalized linear inversion (e.g. Weidelt, 1975; Jupp and Vozoff, 1977; Oristaglio and Worthington, 1980; Hill, 1987). The greatest disadvantage with the linear inverse methods is that a realistic first guess is required and even with a good initial model the iterative computations do not always converge.

The forward solution process is cheaper and most widely used in MT work. This process allows the modeller to have a feel for the structure and to incorporate a priori information so that the ultimate model will be (at least to some extent) consistent with the known geology and any available geophysical data.

7.2 Model Construction

Two-dimensional model construction is a difficult task and the search for acceptable models can never be exhaustive for the same reasons as enumerated in the previous chapter. The strategy adopted in this study was to explore a limited range of models that are consistent with the field
observations. For this purpose, a finite difference scheme was adopted with the Glen Loy geoelectric section as the starting model. The resultant models were analysed for structural similarities with the available geophysical models for the region and the known geology. Inverse calculations were then performed on the most appealing models to determine the bounds on the parameters.

7.2.1 Computer modelling technique

7.2.1.1 Programs and execution

The finite difference computer modelling program based on the method of Brewitt-Taylor and Weaver (1976) and written by Brewitt-Taylor was used in this study. A good account of the finite difference method and its application to the 2-D EM induction problems has been given by Brewitt-Taylor and Weaver (1976) and the underlying theory will not be reproduced here. Rather, the essential features of the numerical modelling technique will be recounted.

In the main, the region to be modelled is divided into a mesh of rectangular elements (or cells) of variable sizes (the intersections of vertical and horizontal grid lines form the grid points or nodes). Conductivity values are allocated to the centres of these rectangular cells allowing for a smooth conductivity variation between neighbouring cell centres. Using central difference formulae, the finite difference representation of the Helmholtz equation (2.17) is obtained for smoothly varying conductivity functions in which the value at each grid point is given by a weighted mean of the specified values of the four ambient cells. The applicable boundary conditions are as follows:

1. The conductivity is only a function of depth at infinitely large horizontal distances (i.e. tabularity conditions prevail far away from the region of interest).
2. A constant magnetic field is assumed at the air-earth interface in the H-polarization case and for the E-polarization case it is assumed that there is an intervening thick air layer at the top of which the magnetic field is constant.
3. The fields vanish at great depths. A zero field value is assigned to each of the nodes at the lower boundary and the 1-D finite difference solution obtained; the field values so derived are assigned to the side boundaries.
4. The tangential components of the electric and magnetic fields as well as the normal components of H are continuous across internal conductivity boundaries.

The reader is referred to the excellent paper on the improved boundary conditions for EM induction problems by Weaver and Brewitt-Taylor (1978). For
each polarization and with the relevant boundary conditions, the finite difference formulations are derived for the interior points of the grid and the field solutions are obtained. Once a solution ($E_x$ or $H_x$ for the $E$- or $H$-polarization respectively) has been obtained, the other necessary field solutions can be computed via eqs. 2.16 b & c or 2.15 b & c as appropriate. From the field values the surface responses (apparent resistivity and phase) may be obtained.

The 2-D modelling process consists of two main stages:

1. The data preparation stage. The problem is defined by the grid dimension, geometry and properties (relative permeability and permittivity, and the conductivity of the subregions), the unit of distance, the frequency (or period), the field components for evaluation, and the boundary conditions. The information is read by the 2-D EM data preparation program PREP and assembled in a form suitable for the next stage.

2. The problem solution stage. The main arithmetic program (ARITH) utilizes the information provided by PREP to solve the 2-D induction problem. The field values at the grid points are written onto files.

The form of presentation of the results depends on the user. The method adopted in this study was to display the surface responses together with the actual field data on the same plot for visual comparison using a computer program written by the author. The above procedures are repeated for each desired frequency and for each different model. In this study 9–11 equispaced (on a logarithmic scale) frequencies were used. ARITH was modified by the author to calculate the solutions at more than one frequency going through the first stage only once. A reworked (more accurate and computationally efficient) version of ARITH (Hill, 1987) was used in the final stages of the modelling studies.

7.2.1.2 Mesh design considerations

The accuracy of the numerical solutions to the 2-D problem depends strongly on the level of discretization used as shown by Muller and Losecke (1975). However, not much has been published in the literature on grid design but the following general rules (see Wannamaker et al, 1985) were applied in the grid design.

1. The ratio of the size of adjacent mesh elements should not exceed 2 to 4.

2. Near a boundary the element dimensions should be less or equal to a quarter skin depth in the medium where the element resides; at least 4 such
elements are necessary on either side of the contrast.

3. Far away from vertical contrasts (2 or 3 skin depths) the size of the elements may be increased to about one skin depth of the medium of residence.

4. At least 3 elements in the horizontal direction and 2 elements in the vertical direction are needed to define a resistivity subregion.

5. Vertical grid spacings may be increased logarithmically with depth since the fields decay exponentially. The same applies above the ground surface but only a few elements (8-9) are needed because of the long wavelength of the field in the air. Within the earth however, the maximum vertical spacings are kept to 1 or 2 skin depths in the appropriate medium. Near the bottom, the dimension of the elements is kept uniform.

7.2.2 Prior information and initial models

From the surface geology it is known that the GGF is a major crustal dislocation and that the zone of crushed rocks is about half a kilometre wide (see fig 1.3 and IGS Sheet 62E). The dominant structural trend as defined by geology and aeromagnetics (Hall and Dagley, 1970) is NE-SW. The GGF is probably a deep feature as suggested by past geoelectromagnetic work (Mbipom 1980; Mbipom and Hutton 1983) and deep seismic reflection profiling (Brewer et al, 1983) and its total length possibly exceeds 720Km (Harris et al, 1978). A structure with such characteristics may be assumed to be 2-dimensional.

MT studies of dyke and fault structures have been carried out by d'Erceville and Kunetz (1962) and Rankin (1962) among others and their theoretical responses are known. If the Great Glen fault zone is likened to a dyke and assuming that a resistivity contrast exists across its borders, then the recorded MT data along a profile across it would exhibit the known MT behaviour across a dyke.

The actual apparent resistivities obtained at frequencies of 8,14,20,24 and 32 Hz for the E- and H-polarization cases are plotted in fig 7.1 against the station distance from the fault axis for the Glen Garry and Glen Loy profiles. As expected from MT theory, there is greater discontinuity in the apparent resistivity values for the H-polarization than for the E-polarization data at the fault zone. This provides a justification for the 2-D interpretation of the field data and a first quantitative indication of the basic structure of the fault – an outcropping "dyke-like" body. The spatial resistivity plots for the Glen Loy traverse also show that the resistivity (200-400 Ohm-m) of the fault zone is an
Fig. 7.1 East-west profiles of apparent resistivity across the Great Glen fault for E- and H-polarisations and frequencies 8, 14, 20, 24 and 32 Hz. The direction of the electric field in each case is indicated on the top right-hand corner of the plot.
order of magnitude less than that of the surrounding crust (2000 → 10000 Ohm-m) in the E-polarization case (Meju and Hutton, 1987). Also, the low resistivity zone appears to be about 2Km wide instead of about 0.5Km as defined by surface geology. Notice that somewhat similar structures are apparent at about 12Km west of the GGF on the Glen Gary profile and at about 6–7Km east of the GGF on the Glen Loy traverse.

The geoelectric section for the Glen Loy traverse (fig. 6.11c, chp. 6) also showed features that are common with the above observations and was considered suitable as a 2-D starting model. The structure was extended over great distances to the left and right of the traverse extremes. The problem was well discretized with a grid size of 60 x 62 elements (see table 7.1). The Glen Loy data sets were considered first and the fault zone was modelled as a 4Km deep trough-like conductive structure. The fit of the calculated response to the observed data was only reasonable for the H-polarization (H-pol) data. The conductive structure was made to extend to 10Km depth and about 2Km wide. This provided a better fit to the H-pol data but the E-polarization (E-pol) data were still poorly fitted especially at frequencies below 10Hz. The 10Km thick resistive upper crust was made thicker (20Km) and this proved a better model than previous ones (as judged by the E-pol fit). The conductive dyke representing the GGF was later found to connect with the lower crust and this basic structure was retained in all the subsequent models. The dyke was modelled as an inclined or a vertical structure and as splayed structures, and a single vertical structure extending from the surface to at least 20Km depth was favoured. Other features were added onto this basic framework with necessary adjustments until a reasonable fit was obtained for both polarizations. Another conductive dyke was found to exist at about 6–7Km east of the GGF and buried under a 4–7Km thick resistive cover. It should be mentioned here that all these modifications to the preliminary models were done keeping in mind the trade-off between thicknesses and resistivities.

Having obtained a satisfactory model for Glen Loy, the Glen Garry profile was modelled next. A satisfactory model here means that the model response curves lie within the zone defined by the error bars of the E-pol and H-pol data for most of the sites. A comforting feature was that the same general
Table 7.1

Grid size used in the 2-D modelling: Glen Loy profile.

Number of Y-grids=60 and Z-grids=62

Y-Grid positions (km)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-500</td>
<td>13</td>
<td>-10.1</td>
<td>-0.30</td>
<td>1.45</td>
<td>49</td>
<td>10.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-350</td>
<td>14</td>
<td>-9.60</td>
<td>-0.20</td>
<td>1.55</td>
<td>50</td>
<td>13.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-100</td>
<td>15</td>
<td>-8.40</td>
<td>-0.10</td>
<td>1.90</td>
<td>51</td>
<td>14.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-44.0</td>
<td>16</td>
<td>-7.20</td>
<td>0.00</td>
<td>3.10</td>
<td>52</td>
<td>14.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-20.0</td>
<td>17</td>
<td>-6.80</td>
<td>0.20</td>
<td>5.50</td>
<td>53</td>
<td>15.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-14.4</td>
<td>18</td>
<td>-6.40</td>
<td>0.40</td>
<td>6.30</td>
<td>54</td>
<td>15.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-12.6</td>
<td>19</td>
<td>-6.00</td>
<td>0.60</td>
<td>6.70</td>
<td>55</td>
<td>16.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-12.0</td>
<td>20</td>
<td>-5.40</td>
<td>0.80</td>
<td>6.80</td>
<td>56</td>
<td>20.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-11.4</td>
<td>21</td>
<td>-4.80</td>
<td>0.95</td>
<td>6.90</td>
<td>57</td>
<td>50.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-11.0</td>
<td>22</td>
<td>-4.40</td>
<td>1.10</td>
<td>7.00</td>
<td>58</td>
<td>120.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-10.7</td>
<td>23</td>
<td>-4.00</td>
<td>1.25</td>
<td>7.40</td>
<td>59</td>
<td>350.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-10.4</td>
<td>24</td>
<td>-3.60</td>
<td>1.35</td>
<td>8.60</td>
<td>60</td>
<td>500.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Z-Grid positions (km)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-145.0</td>
<td>14</td>
<td>0.70</td>
<td>27</td>
<td>6.31</td>
<td>40</td>
<td>33.90</td>
<td>53</td>
<td>62.40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-46.0</td>
<td>15</td>
<td>1.60</td>
<td>28</td>
<td>10.90</td>
<td>41</td>
<td>38.60</td>
<td>54</td>
<td>62.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-16.0</td>
<td>16</td>
<td>1.90</td>
<td>29</td>
<td>20.00</td>
<td>42</td>
<td>40.16</td>
<td>55</td>
<td>62.60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-6.50</td>
<td>17</td>
<td>2.00</td>
<td>30</td>
<td>23.00</td>
<td>43</td>
<td>40.66</td>
<td>56</td>
<td>62.80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-3.20</td>
<td>18</td>
<td>2.10</td>
<td>31</td>
<td>24.00</td>
<td>44</td>
<td>40.83</td>
<td>57</td>
<td>63.40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-1.40</td>
<td>19</td>
<td>2.22</td>
<td>32</td>
<td>24.30</td>
<td>45</td>
<td>41.00</td>
<td>58</td>
<td>65.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.44</td>
<td>20</td>
<td>2.50</td>
<td>33</td>
<td>24.40</td>
<td>46</td>
<td>41.60</td>
<td>59</td>
<td>67.40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.00</td>
<td>21</td>
<td>3.40</td>
<td>34</td>
<td>24.50</td>
<td>47</td>
<td>44.00</td>
<td>60</td>
<td>85.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.10</td>
<td>22</td>
<td>3.70</td>
<td>35</td>
<td>24.60</td>
<td>48</td>
<td>51.00</td>
<td>61</td>
<td>250.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.20</td>
<td>23</td>
<td>4.00</td>
<td>36</td>
<td>24.90</td>
<td>49</td>
<td>58.00</td>
<td>62</td>
<td>500.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.30</td>
<td>24</td>
<td>4.30</td>
<td>37</td>
<td>25.80</td>
<td>50</td>
<td>61.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.40</td>
<td>25</td>
<td>4.60</td>
<td>38</td>
<td>28.50</td>
<td>51</td>
<td>62.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.50</td>
<td>26</td>
<td>5.00</td>
<td>39</td>
<td>31.20</td>
<td>52</td>
<td>62.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The model (apart from minor supracrustal differences and the presence of a concealed dyke at about 10–11Km to the west of the GGF) produced a satisfactory fit to the various data sets and this attests further to the 2-dimensionality of the region. On the basis of structural similarities between the two profiles, a concealed (4–7Km depth) conductive dyke was introduced in the Glen Loy model at about 10–11 Km west of the GGF and this improved the fit of the observed and calculated responses.

The noise-degraded Loch Arkaig data were plotted against the Glen Loy model and a satisfactory fit was observed. No separate computations were done for this profile. The Glen Loy model was then accepted as typical of the region and other equally acceptable models were calculated for the region.
Fig 7.2 a & b. 2-D geoelectric models for the Glen Loy profile.
2-D GEOELECTRIC MODEL: GLEN LOY TRAVERSE

Model resistivities in ohm-meters. 

GGF

1000-2500

80000

50

200

300

8000

3002

2000

10km

20km

30km

40km

50km

60km

0km

<2Km>
2-D GEOELECTRIC MODEL: GLEN LOY TRAVERSE

Model resistivities in ohm-meters
7.3 2-D geoelectric models

Two acceptable and equivalent 2-D models for the Glen Loy traverse are shown in figs 7.2 a & b. Only the relevant portion of each model is presented even though the actual dimension used in the 2-D calculations was much larger. An outcropping low resistivity (150–200 Ohm-m) “dyke-like” body, about 1Km wide, occurs at the position of the GGF and is bordered on either side by thickened (30–45Km) resistive crust. This low resistivity zone connects with the lower crust at depth. Two other similar dyke structures are concealed (about 4–7Km depth) at 6–7Km east and 10–12Km west of the GGF. A superficial layer of variable thickness (1–2Km) and resistivity (1000–3000 Ohm-m) overlies a very resistive upper crust. The upper crust has a resistivity of 8000–80,000 Ohm-m and thickness which varies laterally from about 20Km at the traverse extremes to about 40Km near the fault zone. The lower crust is characterised by low resistivities (50–300 Ohm-m) and the whole structure is underlain by a semi-infinite layer of 2000 Ohm-m at about 63Km depth which is in accord with the depth derived from previous EM induction studies in the region (Mbipom, 1980). The fit of the model curves to the observed E- and H-polarization data for all the sites was good and is shown in figs 7.3 a–j. Note the good fit in both amplitude and phase for the E- and H-polarizations at most of the sites. However, the E-polarization data for those sites situated over the Glen Loy Gabbro, an obviously 3-D mass, appear to have been elevated statically as suggested by the near parallelism of the computed and observed response curves at all frequencies. Three-dimensional modelling (beyond the scope of the present study) would be appropriate in studying such anomalies. Sites L16, L17 and L19 also appear to show 3-D features affecting mostly the E-polarization data.

In figs 7.4a & b are shown the fit of the Glen Loy model responses and the Loch Arkaig data. The degree of fit is satisfactory.

A model for the Glen Garry traverse is shown in fig 7.5 to illustrate the differences in superficial crustal structures between this area and the Glen Loy region. The superficial layer exhibits a marked horizontal inhomogeneity. It is of variable thickness (400m–2Km) and has resistivities in the range 1000–4000 Ohm-m. It appears to deepen towards the fault from the traverse extremes. The fit of the model responses to observational data was good and is shown in figs 7.6 a – d.
Fig 7.3 a-j. Fit of the Glen Loy 2-D response curves to observed data. The apparent resistivity and phase information are presented for different polarisations (E & H) at each station. The station position with respect to the fault axis is indicated at the top of each set of plots.
STATION DIST. = -6.6KM

STATION DIST. = -5.6KM
STATION DIST. = -5.0KM

STATION DIST. = -3.5KM
STATION DIST. = -2.9KM

STATION DIST. = -2.2KM
STATION DIST. = 4.4KM

STATION DIST. = 6.9KM
STATION DIST. = 15.5KM
Fig 7.4 a & b. Fit of the Glen Loy 2-D model curves to Loch Arkaig data. The apparent resistivity and phase information are presented for different polarisations (E & H) at each station. The station distance with respect to the fault axis is indicated at the top of each set of plots.
STATION DIST. = -9.0KM

STATION DIST. = -5.7KM
STATION DIST. = -3.4KM

STATION DIST. = -1.8KM
Fig. 7.5 A 2-D geoelectric model for the Glen Garry profile.
Fig 7.6 a–d Fit of the Glen Garry 2-D response curves to observed data. The apparent resistivity and phase information are presented for different polarisations (E & H) at each station. The station position with respect to the fault axis is indicated at the top of each set of plots.
7.4 Inverse calculations

7.4.1 Optimal resistivity distribution

Using a 2-D linearized inversion program written by Hill (1987) an optimal resistivity distribution was derived for the region. The underlying theory of the inversion algorithm is similar to that discussed in chapters 5 and 6 and can also be found in Hill (1987). The main feature of the program is that inversion is by ridge regression or singular value truncation (Jupp and Vozoff 1975, 1977) techniques. The input models were the two Glen Loy models shown in figs 7.2a & b. Basically each resistivity unit of an input model is divided into a number of smaller blocks of the same resistivity and the problem is solved in the least squares sense. The partial derivatives are computed by determining numerically the effect due to a finite change in the resistivity of each of the resistivity blocks. While this approach is less accurate than analytic differentiation, it was considered adequate for the present study. The result of this operation is a model with a minimum achievable error.

7.4.2 Errors on the model parameters

Assuming that a model with the minimum achievable error had been found, we need to estimate the range of values which the parameters may have. The errors are either determined from the parameter covariance matrix or by a non-iterative most squares technique (Jackson, 1976). In the least squares approach the model parameters may be correlated with each other so that the diagonal components of the covariance matrix may not provide correct estimates of the individual parameters. Using Hill's program, the most squares method was applied to the two resultant models. The errors associated with the model parameters were estimated by setting the threshold residual sum of squares error (see sec. 5.5.2) at

\[ Q_T = Q_{LS} \times 1.1 \]

where \( Q_{LS} \) is the sum of squared residuals of the optimal solution.

The Jacobian obtained for the final iteration (i.e. evaluated at the optimal model) was used in the calculations. No iterations were conducted for the error estimations which, therefore, may have been incorrectly estimated. Unresolved adjacent parameters (blocks of similar values) were amalgamated and the corresponding columns of the Jacobian were summed. A new set of parameters was then calculated and the individual errors re-estimated.
Fig 7.7 Most squares resistivity model for the Glen Loy profile
"Plate tectonics has dominated our thinking for many years and continues to do so while we struggle to fit our geology to the theory and its variations. I wonder what the ruling theory will be in 30 years or more, and how much of present tectonic models will form part of it."

W.H. Poole, 1983.
INTERPRETATIONS AND CONCLUSIONS

The interpretive geoelectric models are appraised in the light of the available geological and geophysical data and 2-D gravity models are constructed for the Great Glen region and the Scottish Caledonides. An integrated interpretation of the various data sets was carried out within the framework of plate-tectonics to unravel the deep structure of the region. The possible causes of the observed crustal conductive structures are discussed based largely on the concept of large-scale convective cells in capped overpressured systems (Etheridge et al, 1983). Suggestions are made for further work in this region and for the development of new MT data interpretation techniques.

8.1 Geoelectric structural appraisal

Much of the discussion in this section will draw upon the MT results from the Glen Loy traverse since it cuts across most of the geological features of interest in the area of study. The 1-D and 2-D models show common features of perhaps regional interest. An important feature of these models is the presence at depth (4-7 Km) of a conductive "dyke-like" body that connects with the lower crustal layer at about 6-7 Km east of the GGF. Its surface position corresponds to a major geological feature in the area - the Fort William Slide, a surface along which at an early stage in the Caledonian deformation upper crustal rocks became detached from and slid bodily over the underlying rocks. An outcropping conductive dyke-like body occurs at the position of the GGF. These two structures do not appear to be connected together. The zone of thickened crust is asymmetric about the GGF in all the models but is about 12 Km wide in the 1-D case and about 18 Km wide in the 2-D model. The thicknesses of the upper crustal blocks are underestimated by the 1-D models by about 20%. A deep-lying conductive dyke-like body appears to be present at about 10-12 Km west of the GGF along the profile (this body was better constrained by the Glen Garry data) and is connected with the lower crustal conductor. The lower crust appears to be of the same electrical character on either side of the fault.

8.1.1 Relationship with previous geophysical and geological observations

In many respects the 2-D geoelectric model (redrawn in fig 8.1a) is in accord with the results obtained from previous geological and geophysical studies in the region. Hutton et al (1977) from magnetometer array studies
Fig 8.1 Some geophysical results for northern Scotland.

(a) A simplified 2-D geoelectric model for the Great Glen region.

(b) An electrical model for northern Scotland (from Mbipom 1980).

(c) Magnetic anomalies of the Great Glen region (from Smoothed Aeromagnetic map of Great Britain and Northern Ireland, IGS 1970).

(d) Seismic structure along the LISPB line (after Bamford et al 1978). The numbers in the figure give P-wave velocity in km/s.

(e) Part of the WINCH profile in the sea of Hebrides (after Hall 1986). The 2-D MT model is superimposed for comparison.

(f) The SHET deep seismic section across the Walls Boundary Fault (from McGeary 1987).
showed that an anomalously conducting zone exists along the Great Glen (the conductive zone being slightly offset to the east of the fault). Mbipom (1980), Mbipom and Hutton (1983) and Hutton et al (1981) showed the presence of a conductor in the Great Glen area and the lower crust was found to be distinctly conducting while the upper 10–15 Km of the crust in this region was found to be resistive (5000–10000 Ωm) with a 10 Km gap at the GGF where an upper conducting layer connects with the conductive lower crust/upper mantle (fig 8.1b). The 10 Km gap may at first sight appear over-estimated but when one considers their station spacing (10–15 Km) and the frequency sounding bandwidth (5x10^-2 – 3x10^-4 Hz) it seems probable that they observed the combined signatures of the conductive dykes of the Great Glen system. Kirkwood et al (1981) required models ranging from a line current at 80 Km depth to a uniform current sheet, 60 Km wide, at 10 Km depth to fit long period magnetovariational observations near the GGF and favour an interpretation in terms of a conducting zone in the 20–80 Km depth range for the fault. However, the structure of the fault was not resolved by these previous studies that lacked adequate control of the upper crust. The present study thus provides new information on the resistivity structure across the Great Glen area.

According to Ahmad (1966,1967) the Great Glen fault is active and the Strontian and Foyers granite complexes extend to 22Km or greater depth, more or less as has been shown in the present 2-D geoelectric model. Handa and Sumitomo (1985) and Meju and Hutton (1987) from studies in Japan and Scotland respectively have suggested that upper crustal dyke-like zones of low resistivity may be hallmarks of active fault systems. Dimitropoulos’s (1981) Grampian granitic body with a top at 7Km and bottom at 19Km between the GGF and Loch Tay fault to the south, – an interpretation rejected by Hipkin and Hussain (1983) – and the inferences from heat flow measurements (Pugh 1977, Oxburgh et al 1980) are consistent with the thick (20–30Km) highly resistive upper crust south-east of the GGF presented here.

The magnetic ridge over the GGF as seen on the Northern Sheet of the Aeromagnetic anomaly map of Great Britain (fig 8.1c) coincides with the position of the inferred zone of thickened electrical crust. From three profiles across this anomaly Hall and Dagley (1970) suggested the presence of a thick horizontal sheet with a top at 8–15Km and bottom at 15–26Km (i.e. from the Loch Ness–Moray Firth area to the Foyers granite complex) and bounded in the northwest by a nearly vertical plane coincident with the GGF. However,
another profile across this anomaly in the Loch Lochy area was interpreted as due to a normally magnetized unit bounded by the GGF to the south and extending from a shallow depth to about 15Km or greater. In either case, the bottom of this magnetic sheet agrees somewhat with the top of the lower crustal conductor in the 2-D model and the GGF is of a vertical disposition.

The seismic refraction model of Bamford et al (1978) is shown in fig 8.1d. Note that the 20Km refractor may be correlated within the bounds of the geoelectric model with the top of the lower crustal conductor.

A section of the WINCH deep seismic reflection profile across the offshore extension of the GGF is shown in fig 8.1e with the Glen Loy 2-D model superimposed for comparison. Note the correspondence between the base of the resistive upper crust and the zone of prominent reflectors in the lower crust. BIRPS' deep seismic reflection profiling in the north of the Shetlands – the SHET survey (see fig. 1.1) – crosses the Walls Boundary Fault (WBF), a suggested northern extension of the GGF. The seismic section is shown in fig 8.1f. The results show the WBF as a near-vertical structure which transects the entire crustal thickness and offsets the continental Moho; the Moho reflection time was found to be significantly different on either side of the fault zone implying a small wavelength structure on the Moho directly beneath the surface position of the WBF (McGeary, 1987). Assuming structural continuity between the GGF and the WBF, crustal thickening as detected by geoelectrics may be invoked to explain the local Moho topography in travel-time. Note the presence of sub-Moho reflectors in the SHET and WINCH (Fig. 8.1e) profiles. It is now known that beneath the essentially vertical frictional slip horizon of strike-slip faults is a zone of quasi-plastic shearing (Sibson 1983) which in some cases may be sub-horizontal decoupling shear zones (Lachenbruch and Sass, 1980; Prescott and Nur 1981). This may be the case at the typical Moho depth beneath the Great Glen and the Walls Boundary faults. However, caution must be exercised when comparing mainland and offshore structures in Scotland since the exposed metamorphic Caledonides narrow rapidly on the mainland from north-east to south-west. More so, Hall (1985,1986) suggests that the Dalradian terrain of SW Scotland occupies an area geophysically distinct from that of the Grampian Highlands and separated from it by the Cruachan lineament.

The 2-D geoelectric model can also be shown to accord with the known geology of the region. The highly resistive upper layer is consistent with a metamorphosed, highly granitized upper crust. Outcrops of granitic and other
crystalline rocks abound in the area. The outcropping conductive dyke at the position of the GGF may be related to the crushed rocks of the fault zone; these rocks probably derived in part from the initial sedimentary cover of the type now represented by the narrow strips of sediments (Old Red Sandstone) that occur in the fault zone on the south-east side of Loch Lochy (fig. 1.3). The concealed conductive dykes and the thickened resistive crust re-affirm the long held geological views of a deeply sheared basement (Watson, 1977) that is perhaps extensively block-faulted (Johnstone et al, 1969; Tanner et al, 1970; Watson and Dunning, 1979) thus forming basins within which were deposited the pre-Caledonian cover-units whose thickness suggest active subsidence (Johnstone, 1975). The emplacement of the Fort William Slide and the Loch Quoich Line (Clifford 1957; Roberts and Harris, 1983), two major tectonic boundaries to the east and west of the GGF, may have been controlled by these buried shear zones. The GGF is inferred to form splays off the west coast of Scotland and this feature may possibly be related to the multiple conductive dyke systems of the geoelectric model. Although the GGF is about 2Km wide at the near surface, it narrows to about 800m at >4Km depth which agrees with the 500m suggested from surface geology (see IGS Sheet 62E).

8.1.2 Integrated interpretation of MT and Gravity data

Although a large coherent magnetic anomaly is associated with the GGF, the gravity expression across the faults on land is relatively featureless (broad negative low) and presents an interpretation problem (Hipkin and Hussain, 1983). Computation of pseudogravity anomalies (Jarvis 1980) failed to resolve the apparent conflict between gravity and magnetic anomalies over this fault and Hipkin and Hussain suggest that a plane truncation in the lower crust by the Curie isotherm may resolve the dilemma.

Since the 2-D MT model appears to be consistent with much of the geophysical and geological observations on the fault, an integrated interpretation of MT and gravity data sets was considered a worthwhile approach to help clarify the nature of the Great Glen gravity anomaly.

The integrated interpretation problem is posed as follows: Assuming that the structural geometry is reasonably well determined by the MT method and that there is a regional vertical gravity gradient that does not generate any gravity anomaly, we want to estimate the departures from this background field which adequately reproduce the observed gravity data. As in the MT case 2-dimensionality is assumed. The gravity data (described by Hipkin and Hussain 1983) were obtained (with a point spacing of 2Km) from the land
Fig 8.2 Location map of the gravity and MT profiles
Fig 8.3. An integrated geophysical model for the Great Glen region (a) Bouguer gravity anomaly of profile GG'.

(b) MT/gravity model for profile GG'
gravity data bank for northern Britain (Hipkin and Hussain, 1983) for a profile approximately parallel to the MT profiles and as clear as possible of known granites (about 11Km north of Glen Garry). The gravity profile (labelled GG' in fig 8.2) is coincident with that of Jarvis (1980) and runs from the Moine Thrust to Loch Erich fault (194E843N–266E778N, British national grid coordinates in Km). The Bouguer gravity anomaly of the profile (fig 8.3a) has been described by Hipkin and Hussain (1983). The data were fitted by the author by 2-D forward modelling with a Taiwani (Taiwani et al. 1959) algorithm (the computer programs were provided by R. Hipkin and T. Genc).

The resultant 2D MT/Gravity model is shown in fig 8.3b. This model was obtained only after a few forward calculations. Note the good fit between the observed gravity data and the calculated responses of the MT/Gravity model (fig 8.3a). The regional MT model incorporates recent geoelectric results (Hill, 1987) from the Moine Thrust area (described in subsection 8.2.2). The zones of thickened electrical crust coincide with the broad gravity lows. These zones are characterised by smaller density contrasts (≅−0.01g/cc) than the surrounding regions (≅+0.025 g/cc). Thus an interesting observation in the Great Glen area is that the magnetic field values (>150nT) are high where the gravity field values are low and the electrically resistive crust is thickest.

It cannot be emphasized enough that the 2-D geoelectric model appears to be consistent with the Great Glen gravity data and that crustal thickening (as detected by geoelectrics) can be invoked to explain some of the major gravity anomalies in the Scottish Highlands. The author has no doubt that the 2D geoelectric model shows significant (hitherto undiscovered) features compatible with much of the geophysical and geological observations in the Great Glen region. On these grounds, it is reasonable to propose a close relation between the complex crustal electrical structure and regional tectonics.

8.2 Deep geology of northern Scotland

As the Great Glen forms a natural boundary between the Northern and Grampian Highlands it is also appropriate that the results of this study be interpreted in terms of the deep structure of the Scottish Highlands as a whole. Previous geological and geophysical results from the areas adjacent to the Great Glen area are now considered and reconciled with the present geoelectric model into a feasible tectonic framework. Firstly, however, a synopsis is presented of some outstanding geological problems in the region which any tectonic model should try to explain.
8.2.1 Gross geological overview: problematic observations

Current hypotheses suggest that most of the Moine rocks north of the GGF as well as some areas immediately to the southeast of the Great Glen, have suffered pre-Caledonian - Grenville (1000 Ma) and Morarian (720 Ma) - deformation and metamorphism and have merely been recrystallised during the Caledonian orogeny. The areal extent of these orogenies remains unclear (Johnson et al 1979) and their influence on the metamorphic zonation in these areas is still uncertain (Fettes 1983). These problems are compounded further by the discovery of only Caledonian deformations in the Moines of Sutherland by Soper and Wilkinson (1975).

Detached slices and wedges of Lewisian rocks are interleaved with Moine rocks towards the boundary between the Morar and Glenfinnan Divisions forming a laterally extensive (250Km) NNE trending zone (Tanner et al 1970). Tanner and coworkers suggested that these may have been derived from a basement rise dividing the Moinian depositional basin into two troughs. Johnstone et al (1979), adopting the assumptions of Tanner and coworkers, raised the possibility that whereas the western trough was filled largely by fluvialite or deltaic sediments of westerly provenance, the easterly trough received detritus from different sources. The location and areal extent of these depocentres are at best conjectural. Also, Watson (1984) argues that about 25-35 Km of crystalline material was eroded from the Highlands forming sediments 1.5 times this volume and that most of this volume should have found its way into Ordovician and Silurian basins in or at the margins of the North Atlantic continent. Such basins have not yet been identified.

Garson and Plant (1973) suggested a subduction related Morarian event with a suture coinciding with the GGF but Johnson (1975) postulates that the suture is hidden by the Caledonian (Glenelg and Ross-shire) nappes (see also Stewart 1982, p.418) and argued for a continental collision tectonics for the Morarian episode, with the Moines deposited on one continent (the Baltic Shield or microcontinent) and the Stoer and Torridon groups on another (the Laurentian). These hypotheses need considerable testing as microplate structure of this part of the Caledonides has not been identified.

Four regional deformation episodes (D1-D4) are recognized in the Glenfinnan and Loch Eil divisions of the Moines and Roberts and Harris (1983) suggested that the D1-D2 structures are of pre-Caledonian age. Structural studies in the Glen Garry area (Holdsworth and Roberts 1984) showed a N-S transport direction during D2 which is markedly different from the Caledonian
WNW-directed movements. The significance of this N-S trend is not known. The relationship between these northern Moines and those to the south of the GGF is still debatable (see Johnson et al 1979; Piasecki 1980; Piasecki et al 1981; Baird 1982; Roberts and Harris 1983).

The origin and nature of the GGF as well as the translations on it is also still controversial. Considerable strike-slip movements have been suggested from regional studies by several workers (see section 1.3). However, evidence for some brittle or ductile deformation of the wall rocks indicative of large-scale strike-slip movement is missing; near the GGF, Parson (1979) detected small-scale structures indicating NW-SE compression and no structures indicative of lateral NE-SW displacement were observed.

The enormously thick Dalradian rocks of the Grampian Highlands are traditionally believed to have accumulated in a Dalradian basin whose exact position is a matter of conjecture. The nature of the basin (whether continental or oceanic) is not fully resolved (e.g. Phillips et al 1976; Graham and Bradbury 1981). The high pressure to which much of the Dalradian fold-mass was subjected is thought to be indicative of extreme thickening and shortening of the weak basin fill while the anomalous low pressure Buchan region of NE Scotland (which is permeated by large volumes of basic magma) represents an area of least thickening (Watson 1984). However, no satisfactory mechanism has been found as yet to achieve the required crustal thickening (see Watson 1984, p.198). The southwest Dalradian basaltic activity was also thought to have occurred along a NE-SW trending (Caledonoid) rift related to the presumed opening of the Iapetus Ocean to the southeast (e.g. Graham 1976; Graham and Bradbury 1981; Plant et al 1984) but Fettes and Plant (1985), Fettes et al (1986) and Graham (1986) present radically different interpretations based largely on trans-Caledonoid deep structures. However, no direct, and by all means compelling, geophysical evidence for the deep structure of the region has so far been provided.

Along the southern margin of the Dalradian and largely in faulted contact are a series of exotic rocks (black phyllites, cherts, grits, amphibolites, spilitic lavas and serpentinites) known as the Highland Border series. Henderson and Robertson (1982) interpreted these rocks as tectonically emplaced ophiolites (see also Garson and Plant 1973) derived from an ocean basin to the north of the Highland Boundary (in the Central Highlands or farther north) but there remains the chief difficulty of identifying where the respective ocean basin developed. According to Henderson and Robertson, the position of the
postulated basin bears strongly on the problem of 'root-zone' to the Tay nappe and related structures which are outstanding geological puzzles.

Although the isotope systematics of some Caledonian granites indicate a dominance of crustal provenance (Hamilton et al 1980) and a change in the age of the basement across the Grampian Highlands (Halliday 1984), the Caledonian granites are not easily related to active subduction and there is no obvious mechanism for their generation (Halliday 1984).

Multi-disciplinary evidence of the deep structure across the Scottish Highlands will now be presented that will test some of these geological hypotheses and hopefully facilitate a clearer assessment of the above problems.

8.2.2 Evidence of deep structure

A recent short period (0.1 – 800 Hz) MT study of the Moine Thrust region (Hill, 1987) (profile CC' in fig 8.2) revealed an anomalous resistivity distribution to the west of the Lairg area. Although the data are sparse with poor control on the precise location of lateral variations (5Km average station spacing), most of the measurements were made between Loch Oykel and Lairg, that is in the region of the inferred Moho offset of Faber and Bamford (1981) and the effective orogenic front of Watson and Dunning (1979). Hill derived a 2-D model which showed the presence of a 10–20 Km wide zone of thickened (>40Km) resistive (>80000 Ωm) crust at the centre of the profile, bordered by a normal 16–20Km thick resistive (8000 Ωm) upper crust. The lower crust was found to be more conductive (200 – 1000 Ωm) everywhere. Mbipom (1980) indicated a thickening of the resistive upper crust and a thinning of the underlying conductive layer in the region of the Dalradian outcrop (see fig 8.1b).

On the basis of the interesting results obtained from the Great Glen MT/Gravity modelling (fig 8.3a&b), it seemed worthwhile to fit the gravity data along an extended profile (HH' in fig 8.2) running from the Lewisian Foreland (164E870N) to the Midland Valley (335E720N) and parallel to the WINCH seismic reflection profile (Brewer et al 1983). The objective was to shed some light on the deep structure of the Scottish Highlands and to see if some of the features seen offshore could be detected on the mainland using techniques other than seismics. The Bouguer gravity anomaly for this profile is as described by Dimitropoulos (1981). A good agreement between the calculated and observed regional gravity data was obtained by the author as shown in fig 8.4a for the model of fig 8.4b. An interesting feature of this resistivity-density structural
Fig 8.4. An integrated geophysical model for northern Scotland.

(a) Bouguer gravity anomaly along profile HH'

(B) MT/gravity model for profile HH'
Fig 8.5. A comparison of MT/Gravity and seismic (after Brewer et al, 1983) structure across the Scottish Caledonides
model is the need for another zone of thickened crust (~42Km) in the region of the Dalradian outcrop. The model thus confirm and refine Mbipom's model for the Dalradian region. This alternative interpretation may possibly shed some light on some of the on-land problematical geological phenomena and the unexplained features on the WINCH seismic section (assuming lateral continuity of structure offshore).

The regional MT/Gravity model of fig 8.4 is shown again in fig 8.5 together with the WINCH seismic section (on the same horizontal scale). Note the correspondence between the zones of thickened (~42Km) crust in the MT/Gravity model and the regions of truncated seismic reflections that extend from the near-surface to sub-moho depths (~42Km). It can hardly be coincidental that almost identical structures are revealed across the Caledonian orogen by the three geophysical methods since each measures a different rock property. Because each of these methods is most effective when the contrast in the observed parameter is greatest and the contrasts may occur at different depths for each method, this structural concordance is seen as evidence of strong lateral (NW-SE) differences in the deep structure of the Scottish Highlands. No such correspondence has been reported elsewhere to the author's knowledge. It is perhaps noteworthy that Bamford et al (1978) ascribed the cause of the uncertainty in LISPB interpretation in the Grampian Highlands (see fig 8.1d) to rapid deepening to the south of the GGF and north of the Highland Boundary Fault. This inference is consistent with the present interpretation and the MT/Gravity model perhaps explains the seismically detected Moho offset (Bamford et al 1978; Faber and Bamford 1981).

The approximate locations of the inferred zones of thickened crust are shown in fig 8.6a. Note the southward increase in the size of these crustal anomalies.

Further geophysical evidence of the deep structure of the region comes from the magnetometer arrays study of Hutton et al (1977) the results of which are shown for a period of 700 sec in fig 8.6b. In this figure, notice that the hypothetical-event Z (vertical magnetic field component) zero contours (labelled B and C) seem to correlate with the Great Glen and Dalradian anomalous zones respectively. Hutton et al (1977) drew attention to the relation between their data and the results of Garson and Plant (1973) and suggested that these anomalous magnetic variation features may also be related to ancient plate boundaries. It is also noteworthy that the areas of recent deep-focus (22Km) earthquakes (BGS data), Kintail (10-08-74), Fort William (13-11-72) and Oban...
Fig 8.6. Comparison of the MT/Gravity model and previous models for northern Scotland.

(a) Location map of the MT/gravity inferred zones of crustal thickening.

(b) Magnetic variation anomaly map of Scotland (after Hutton et al 1977). The Z contour interval is 0.2 nT and the zero Z contours are represented by the dotted lines.

(c) Distribution of belts of alpine type ultramafic rocks in Scotland (from Garson and Plant 1973).

(d) Map showing the distribution of wedges and slices of Lewisian rocks in the Moine rocks of Scotland (from Johnstone 1975).

(e) Distribution of late Caledonian granites and associated rocks in Scotland (from Watson 1984).

(f) Distribution of late Caledonian granites and the location of the Mid-Grampian Line (after Halliday 1984). The MT/gravity profile HH' and the position of the Dalradian crustal anomaly are indicated.
Zone of discontinuous outcrops of alpine-type ultramatic rocks
Site of ancient oceanic trench and direction of lithospheric subduction

Scale: 5 km
lie within the inferred zones of crustal thickening.

Much geological evidence can be shown to support the above geophysical inferences. The NNE trending belts of alpine type ultramafic rocks in the region discussed by Garson and Plant are presented in fig 8.6c. They agree in geographical location not only with the magnetic variation anomalies but also with the three anomalously thick crustal zones discussed in this thesis especially in the Great Glen and Dalradian areas. The northernmost zone correlates with the laterally extensive NNE trending belt of Lewisian rocks in the northern Moines (fig 8.6d). Also, a remarkably good agreement is seen in the northwest corner of Sutherland and in the Southwest Highlands (Oban area) between the Z zero contours (8.6b) and the anomalous trans-Caledonoid zones marked by gravity gradients and appinite pipes (fig 8.6e) namely the Loch Shin line (Watson, 1984) and the Cruachan lineament (Watson 1984; Plant et al 1984; Fettes et al 1986; Graham 1986; Hall 1986). The appinite suite of rocks is regarded as being derived from mantle sources and the Cruachan lineament has been shown by the above workers to be a deep basement feature separating crustal blocks of contrasting physicochemical characteristics. The MT/Gravity model also shows some correspondence with the thick-skinned or autochthonous model for the Moine thrust area suggested by Watson and Dunning (1979) and Stewart (1982).

Isotopic evidence can also be shown to support the above geophysical inferences. From the distribution of inherited zircons (≤1000 Myr old) Pidgeon and Aftalion (1978) proposed a major Proterozoic crustal component in granites north of the Highland Boundary fault, and isotope systematics (Hamilton et al 1980) of some of the Caledonian granites (such as Strichen, Ben Vuirich, Ben Nevis, Etive and Foyers intrusions and Strontian biotite granite) also indicate a dominance of crustal provenance. Recent isotopic data (Halliday, 1984) confirmed and refined these previous models of the basement under northern Britain. It was shown (Halliday 1984) that although a major mantle component is important, the lower and middle continental crust have affected the isotopic compositions of the late Caledonian granites to the extent that a change in the age of the basement is recognizable across the Grampian Highlands. Halliday placed the basement boundary through the middle of the Grampian Highlands and called it the Mid-Grampian Line (fig 8.6f). It is significant that the Mid-Grampian Line correlates with the northern margin of the inferred zone of thickened crust and Hutton et al's (1977) magnetic variation anomaly in the Dalradian region.
As mooted previously, the appearance of linear features in the same geographical location plus quantitative estimation of source depth in different data sets suggests the presence of strong lateral differences in structure across the region and a deep-seated origin of the structures. Thus the available geophysical and geological data form a coherent body of evidence for the presence of deep Caledonoid and non-Caledonoid structures in the region. The association of alpine type belts with zones of thickened crust is significant as it seems to suggest that alpine type tectonic processes may have been operative in this region.

The origin and emplacement of alpine type ultramafic rocks have been discussed by Gresens (1970) and Chidester and Chady (1972) among others. They are believed to originate in rifts in ocean-floors or in the mantle underneath continental crusts. In continental environments they are emplaced tectonically through the crust into the overlying thick pile of basin sediments. However, Fyfe (1967, p.47) suggested that dunites and peridotites can also result from in situ differentiation of mafic or ultramafic magma. Garson and Plant (1973) have interpreted the suites of ultramafic rocks in Scotland as ophiolites implying oceanic conditions and subduction.

Areas of soda metasomatism characterised by crocidolite, probably of early Devonian age, occur in the Inverness region in association with breccia pipes and the chemical changes have been tentatively related by Deans, Garson and Coats (1971) to saline diapiric intrusions or to deep-seated carbonatite intruded in the plane of the GGF. Evaporite deposits often form, given certain limitations, in the early rifting stage of the fragmentation of continents and of the formation of proto-oceans (Windley 1977, p.215), and carbonatites, which result sometimes from partial fusion of mantle peridotites (Wylie and Huang 1975), are also critical components of rift magmatism (Windley 1977). Whichever interpretation is preferred, however, it is evident that the attendant processes are deep-seated and probably involve megafractures. It can be no accident that the aegirine granulites of Glen Lui (with blue amphiboles), situated near the Loch Tay fault, resemble some of the rocks from the Great Glen (Deans et al 1971). The question thus arises, 'can we reconcile the above observations into one feasible structural framework?' This is attempted in the next section.
8.2.3 Regional synthesis: an alternative tectonic view

Although it is generally believed that the Northern and much of the Grampian Highlands are underlain by Lewisian-type material (Bamford et al 1978; Watson and Dunning 1979; Watson 1984), the MT/Gravity model presented here shows discontinuity in structure across the Scottish Caledonides. Also, a comparison (Hamilton et al 1980) of the Sm-Nd systematics of the Lewisian gneisses with those of some early Caledonian granites shows that these granites are not dominated by a Lewisian component. Thus the simplest interpretation of the MT/Gravity model (fig 8.4b) is that the zones of thickened upper crust demarcate different terranes. These terranes were probably continental fragments with attached crust and upper mantle as is the case in the accreted terranes in the northern part of the Phillipines (Karig, 1983). They may have been separated by ephemeral basins some of which were probably oceanic or quasi-oceanic and individually filled; these basins could have formed in rift systems in the underlying crust and may have developed as part of the early Palaeozoic continental shelf (or shelves) as envisaged by Graham (1976), Graham and Bradbury (1981) and Plant et al (1984) for the Dalradian. These basins were closed in space and time by convergence leading to continental collision and expulsion of the basin-fill sediments as major nappes. Oblique movements as envisaged by Badham (1982) for the Hercynides were probably important in the development of the present crustal architecture. It follows from this terrane accretion interpretation that the geophysically inferred areas of thickened crust are suture zones. This hypothesis needs considerable testing.

This model is consistent with Hutton et al’s (1977) plate-boundary interpretation for the Great Glen and Dalradian regions. It is also consistent with Halliday’s (1984) proposal – rejected by Watson et al (1984) – that a crust, younger than the basement to the north of the Mid-Grampian Line, was thrust under the south side of the Grampian Highlands. This would seem to suggest that the suture (concealed) in the Dalradian region lies near the Mid-Grampian Line and Hutton et al’s magnetic variation anomaly. A suture in the Kintail-Oykel region would support Johnson’s (1975) and Stewart’s (1982) proposal of a concealed Morarian suture in NW Scotland. Garson and Plant (1973) suggested the Great Glen as a Morarian suture and the Dalradian belt of alpine type ultramafics was interpreted as another suture. This is in remarkable agreement with the geophysical evidence presented here. However, considering the effective phase information (fig 6.12), and Hutton et al’s (1977)
magnetic variation anomaly, it is also possible that the Great Glen suture is probably hidden by the Caledonian Fort William slide. It probably corresponds to the concealed conductive dyke observed in this study to the east of the GGF. Another plausible explanation is offered below for the Great Glen fault.

Areas of fenite-type soda metasomatism occur in the Great Glen region (from south of Foyers to north of Inverness) in association with breccia bodies (Deans et al 1971). These workers ascribed the chemical changes of the country rocks to a deep-seated carbonatite intruded in the plane of the GGF. The close association between continental rifting and alkaline magmatism is well known and rift magmatism may be associated with abnormal enrichment of volatiles (and alkalis) which may lead to highly explosive and fragmental volcanism (see Windley 1977, p.220). Also, peralkaline (soda-rich) subvolcanic ring complexes (whose rocks include carbonatites and fenites formed by in situ alkali metasomatism of the country rocks) are associated with rift valleys (Windley 1977, p.222). Thus the Great Glen is probably a fossil rift system. The rifting is probably of late Silurian or early Devonian age if it can be dated by the soda metasomatic event (Deans et al 1971). The breccia bodies in the Great Glen probably suggest explosive events and may be fossil earthquakes, affected by later strike-slip movements. The fenite-type rocks probably represent exhumed subvolcanic ring complexes. If these contentions are correct, then we can speculate that the apparent absence of fenite-type rocks around Strontian and between Strontian and Foyers (Deans et al 1971) is suggestive of a southwestward deepening magma chamber or may be a reflection of the deeper erosion in the northern Great Glen region.

The above hypothesis may be used to shed some light on the aeromagnetic anomaly of the Great Glen region. It is possible that the linear magnetic anomaly in this region arises from magmatic rocks associated with the proposed rifting. From three profiles across the magnetic ridge over the GGF, Hall and Dagley (1970) suggested the presence of a thick horizontal sheet with a top at 8 to 15 Km and bottom at 15 to 26 Km from the Loch Ness – Moray Firth area to the Foyers granite complex and bounded in the northwest by the GGF. This implies a southward dip for the magnetic sheet and accords with the author's suggestion that subvolcanic complexes probably lie deep further south of Foyers. However, another profile across this magnetic anomaly in the Loch Lochy area was interpreted by Hall and Dagley as due to a normally magnetized unit bounded by the GGF to the south and extending from a shallow depth to about 15Km or greater. This would imply a restricted
occurrence of this unit and an initial southeastward underthrusting followed by sinistral strike-slip movement along the fault axis. However, dykes and sheets of ultrabasic to acidic composition occur to the west and southwest of Loch Lochy and the magnetic unit is probably related to these intrusive bodies.
Fig 8.7. The lid of the continental collision zone (orogen) and its frontal wedges model (after Laubscher 1983). Essential points are (a) two subducted slabs with intervening funnel of detached middle and lower crustal material that may be detached (as slices and wedges) and transformed into ophiolite and (b) the surficial masses are detached and transformed into recumbent folds and nappes. Lateral displacement between the two slabs may be accommodated along a fault (F).
The structural model presented above appears to the author to form a basis for other speculations. In the Northern Highlands a complex geological history has been suggested from geological evidence, and isotopic evidence (see Powell et al 1983, for a bibliography) supports two or more cycles of orogenic activity. However, no internal unconformities that might relate the polyphasal nature of both the deformation and regional metamorphism to the isotopic data (see Powell et al 1983, p.813) have yet been recognised within the Moine rocks in this region. It appears that all the major lithological divisions of at least the southwestern Moine share the same deformational and metamorphic history (Baird 1982; Roberts and Harris 1983). The Kintail-Oykel crustal anomaly may therefore provide the important missing link in the complex history of NW Scotland. The complex structural pattern of the Scottish Caledonides may be explained by the hypothesis of episodic and local collision and expulsion of basin-fill sediments as major nappes. Collision tectonics as they relate to nappe and recumbent fold formation can easily be demonstrated in the light of the "orogenic lid and its frontal wedges", model developed by Laubscher (1983) for the central Alps. The basic concepts are illustrated in fig 8.7.

Northwest of the GGF the structural vergence is in general NW but structures with eastward vergence occur in the Morarian alpine-type belt of Johnson (1975). The root zone of the Morarian antiforms is exposed in the Glenelg region where the western part of the zone has been truncated by the Moine thrust, and in Sutherland it is possible that the root zone lies wholly or in part beneath the Moine thrust (Johnson 1975). The Morarian deformation obviously pre-dates the formation of the Moine thrust and the suggested root zone seems to correlate with the Kintail-Oykel anomaly. The Great Glen region appears to be the root zone of the later Moine thrust. This view is supported by Coward (1980) and by the recent estimates of the displacement in the Moine thrust zone by Butler (in Coward 1983) which suggest that the Cambrian strata can be restored to some 60Km east of the present trace of the Moine thrust. The upright to NW verging Sgurr Beag slide may also be rooted in the Great Glen region. This is supported by Coward (1983), who considered the slide to have a sigmoidal form branching off a floor thrust in the lower crust and estimated the footwall ramp to be some 60Km east of the Moine thrust outcrop and east of the outcrop of the Sgurr Beag slide.

Southeast of the GGF the structures are dominated by large fold nappes which face the northwest near the Great Glen but face southeast near the Highland Boundary fault. According to Thomas (1979) the structures (F1
nappes) appear to diverge from an upward facing axial zone in the Central Highlands but Shackleton (1979) pointed out that it is hard to see how such enormous volumes of nappes could be ejected, fountain-wise, from this steep belt. The author agrees with the interpretation of Thomas (1979) in the light of Laubscher’s (1983) model but suggests that the ‘root zone’ of these nappes probably lies in the region of the proposed Dalradian suture.

8.2.4 Tectonic summary

The tectonic setting for the Scottish Caledonides involves compressive as well as extensional movements.

The geophysical models presented and discussed here evoke a picture of a basement that is divided into blocks. The orogen is viewed therefore as a mosaic of accreted terranes or microplates. It is noteworthy that Coward (1983, p.808) pointed out that the Scottish Caledonides have many similarities to the North American Cordillerra (see Coney et al 1980) and may be a mixture of allochthonous or suspect terranes, and relics of an autochthonous foreland. It is remarked here that Peddy and Keen (1987) suggested that the Flannan reflector seen on the DRUM seismic section (McGeary and Warner 1985) may be a fossil subduction zone.

The foregoing analysis of the Caledonian orogen is crude but draws on the increasingly popular concept of suspect terranes of orogenic belts (Coney et al 1980; Williams and Hatcher 1983), the strike-slip orogen hypothesis of Badham (1982), the transpression model of Saleeby (1981) involving a combination of strike-slip and convergence and Laubscher’s (1983) model for alpine-type collision. The structural model presented here provides a basis for informed speculations by other authors as well as a fresh approach to Caledonian syntheses. An interpretation of the deep geology of northern Scotland in the light of geophysical data is given in fig 8.8. This model shows some similarities with that proposed by Yardley et al (1982) for the southern part of the Scottish Caledonides and with Garson and Plant’s (1973) model for the Scottish Highlands.
Fig. 3.2 A model for the deep geology of northern Scotland.
8.3 Interpretation of high electrical earth conductivities

The causes of high rock electrical conductivities have been discussed extensively in the literature (e.g. Brace 1971; Shankland and Ander 1983; Haak and Hutton 1986). For physical reasons conductivity is expected to increase generally with depth. It depends on porosity, pressure, temperature and other parameters. As regards the lower crust/upper mantle layer in northern Scotland, it can only be said that an impressive array of factors may be responsible for its conductive nature, e.g. petrology, presence of fluids in interconnected spaces (electrolytic conduction), pressure, solid conduction through the bulk silicate material itself due to increased temperature, and partial melt. Partial melting seems unlikely and solid conduction through hydrated silicate minerals may be possible but is unattractive. Petrological factors may be called upon for geological reasons but this can be misleading as the presence of water affects a variety of petrological materials (gabbro, tonalite, granite) in similar ways (Lambert and Wylie, 1970).

When crustal blocks collide, they bore into each other forming “wedge into split-apart” (Laubscher, 1983) structures since the respective originally continuous sequences are forced apart (as during sill emplacement), with the uplifted splits being bounded by basal detachment surfaces. The basement and cover responses to deformation will be different resulting in a contrast in style and fabrics. This mechanism may well account for the complex wedge-type structures seen on the MOIST, WINCH and SHET profiles. This implies that some of the seismic reflective zones may not be zones of strong lithological contrast but are merely structurally developed zones within the basement. The sheared nature of the deep crust as suggested from geological studies (Tanner et al 1970; Watson 1977; Coward 1983; Soper and Anderton 1984) and confirmed by recent seismic reflection profiling (Smythe et al 1982; Brewer et al 1983; Brewer and Smythe 1984) and the present study make the presence of fluids in connected cracks at deep crustal levels an attractive possibility. Moreso, the presence of stress-aligned liquid-filled microcracks pervading at least the top 10 to 20Km of the earth's crust has been recognized by Crampin (1985) among others from shear-wave splitting in various geotectonic environments and may be a plausible explanation for the steep conductive zones found in the Great Glen region. Significantly, the Great Glen fault is still active and frictional heating with perhaps subterranean contributions may be another factor contributing to the high conductivity of this structure. However, it may be unsafe to ascribe the high conductivity of this structure to an exotic
geothermal reservoir such as a deep-lying congealed carbonatite magma in the plane of the GGF (Deans et al 1971): a more practical hypothesis is preferred.

It has been demonstrated by Etheridge and Cooper (1981) among others that metamorphic fluids may be focused or channelled through high permeability pathways that are structurally or lithologically determined. It is also known from petroleum geology that the presence of an impermeable cap, say a shale layer, within a thick sequence of water-saturated sediments can lead to the development of anomalous pressures underneath the cap rock. Etheridge et al (1983) used the concept of large-scale convective cells in capped over-pressured systems to demonstrate that large-scale fluid circulation is possible in regional metamorphic belts. The basic concept is illustrated in fig 8.9. In this figure, we see that fluids are focused by deep-rooted shear zones. There is limited circulation for anatectic regions but circulation becomes increasingly active at upper parts of the metamorphic pile. A relatively impermeable cap separates the overpressured metamorphic system from upper crustal hydrostatic systems preventing exchange between the two hydro-systems. Thus the whole pile is deformed with the upper crustal open system losing its fluid and electrolyte conduction capabilities while large amounts of fluids are trapped at its base (Etheridge et al 1983).

The above concept may be used to explain the observed conductivity distribution in northern Scotland. It also allows a great deal of flexibility in the petrological or physical characterization of crustal conductive structures. The necessary conditions for its operation are rife and the preponderance of deep structural discontinuities in this region ensures that the basal fluid system is in circulation. For a steep shear, large-scale fluid circulation is possible along the higher permeability crushed zones. This probably explains the conductive nature of the system of dykes in the Great Glen area. The original cap rocks have become severely eroded or lost so that the Great Glen fault zone has become open to the meteoric system. This concept can also be used to throw light on the distribution of metamorphic zones in the region. For a normal (or thrust) fault, the emplacement of a slab of hot, deeper lying material over cool, water-saturated crust may initiate a convective circulation in the footwall block, with some fluid penetration into, and retrogression of, the hanging wall (Etheridge et al 1983). In other words, thrusting of rocks undergoing metamorphism over cold foreland rocks would result in retrogression in the overlying slab and a prograde metamorphism of the footwall rocks. If these contentions are correct, then we expect a significant amount of fluids at the
base of the resistive upper crust or in the lower crust in general and especially along the numerous detachment surfaces. This fluid system, together with the contributions from the connected steep shears, may have influenced the petrological, electrical and seismic character of the lower crust to a large extent.

Fig 8.9 Schematic model of fluid circulation systems in regional metamorphic belt (after Etheridge et al 1983).

8.4 Discussions

As shown in the discussion which follows the main objectives of this project have been realized. The deep electrical structure of the Great Glen Fault has been clarified and structural problems encountered by seismics, gravimetrics and geology have been overcome by this geoelectric study. All available useful geophysical and geological data have been integrated into a coherent structural framework to illuminate the previously elusive deep structure of the Scottish Caledonides. Some light has also been shed on the possible nature of movements on the GGF.

While the MT technique has been successfully used in this study, it must be noted that the data acquisition were hampered by numerous factors of which cultural noise and terrain conditions are the most problematical; resistive
mountainous terrain propagates electrical noise over great distances and the noise degraded low frequency data limited model resolution to some extent. The use of non-polarizing electrodes for low frequency measurements and of a simple delay-line filter is strongly recommended.

The effect of the surrounding seas (‘coast effect’) has been discounted in this study on the grounds of the numerical modelling studies by Mbipom (1980). At the geomagnetic latitude of Scotland, source fields associated with polar electrojet or substorm activity have complicated morphologies such that the recognition of internal anomalies may be difficult. However, the work of Hutton et al (1977) showed that source field effects are not very significant at periods shorter than 1000 sec. Recent geological studies in the Dalradian region (Fettes et al 1986) show considerable along-strike variations in structures, therefore the 2-D geoelectric model should be regarded as the best approximation to the more realistic 3-D earth structure.

This study shows that the MT technique is suited to the detection or reconstruction of tectonic processes in orogenic belts. However, it should be emphasized that this was made possible only by use of a dense network of observational sites and appropriate sounding frequencies; short-wavelength anomalies were thus more accurately resolved. This study also shows that the effective 1-D resistivity and phase sections can provide very useful structural information in areas that show strong 2-D features and also stresses the need to use a priori information to resolve non-uniqueness in MT inversions. But the author argues that there is no case for forcing geoelectric models into conformity with say, seismic models unless it is justified by the MT data: the MT technique is as useful and independent as any other technique for structural mapping and its value in resolving structural disputes is evident in the above multi-disciplinary synthesis. Each geophysical technique is beset with particular problems and so efforts should be directed towards improving current geoelectric interpretation methods as attempted in this study. The author wonders why attention is directed so strongly to use of seismics when as shown in this study MT and the simpler Gravity techniques can help solve major crustal problems and are substantially less expensive.

8.5 Conclusions

Deep magnetotelluric profiling in the Great Glen region, together with information derived from the adjacent regions, has provided new insights into the architecture of the crust beneath the deformed Scottish mountain chains. Not only has it been shown that anomalous Caledonoid (NE–SW) trending
zones of thickened crust exist in the alpine-type belts (see Johnson 1975; Garson and Plant 1973) of Scotland but also that continental collision tectonics has been a common ingredient in the evolution of the Scottish mountain belts. Light has been thrown on the possible microplate or accreted terrane structure of the Scottish Caledonides.

The following main conclusions are drawn from this study:

1. The invariant 1-D MT sections are useful structural mapping tools but the depths to interfaces are underestimated in this area of complex geology.
2. The Great Glen study area is shown to be 2-dimensional. The 2-D model derived from the study is consistent with the results of numerous geophysical and geological studies of the GGF. It is thus the first unequivocal model for the Great Glen region.
3. The upper crust in this region is very resistive but with different values on either side of the fault. Its thickness varies laterally from about 20Km at the traverse extremes to about 40Km near the fault zone. The zone of thickened upper crust is about 18Km wide and is asymmetric about the main fault axis. The lower crust is of low resistivity (150–400 Ωm) which agrees with past observations in the region.
4. The basic structure of the GGF is an outcropping low resistivity (150–200 Ωm) dyke-like body, about 1Km wide, and transects the whole of the upper crust in a vertical fashion but may be sub-horizontal at lower crustal/upper mantle depths. Comparable structures are concealed underneath the surface position of the Fort William slide and east of the Loch Quoich Line at about 4–7Km depth; these are interpreted as shear zones and may have developed in response to compression during microplate collision. The Great Glen is interpreted as a fossil rift system or contact zone (suture) between two accreted geoelectrically different terranes.
5. The interpretation of MT data sets from 3 profiles with dense networks of observational sites has ensured good resolution of possible short-wavelength anomalies in the Great Glen region and an integrated interpretation of MT and gravity data sets has clarified the previously problematical gravity anomaly.
6. The crustal architecture of the Scottish Caledonides is attributed to rifting and collision tectonics. The coherent body of geological and geophysical evidence presented above goes some way in solving some of the problems of the deep structure of the Caledonides. Furthermore, the
The proposed model provides us with a reasonably detailed picture to account for the observed deformation (alpine type structures and strike-slip faults) and metamorphism (considerable amount of fluid transport is envisaged in the conductive structures). However some of these hypotheses need considerable testing.

7. The value of magnetotellurics in resolving geological and other geophysical disputes is evident. Each of the inferred collision zones apparently has a distinctive geoelectric structure by which it may be identified. Gravity provides a useful constraint. It is suggested that the Great Glen model be used as a ‘pathfinder’ for any other undiscovered Scottish microcontinental collision zone(s). A corollary that follows, but needs further testing is that the electrical anomaly across the Southern Uplands Fault (Sule 1985) is suggestive of a collision zone in that region. If this proves to be true, then this combined MT/Gravity approach may be of equal value in resolving structural problems in other mountain belts. MT has summarily been shown to be a very powerful and indispensable geological mapping tool in the deformed (?Caledonian) mountain chains of northern Scotland.

8. Above all, this study has shown that, in the words of Hatcher Jr., Williams and Zietz (1983, p1), "capability of integrating the multitude of data from geologic and geophysical investigations in orogenic belts is essential to arriving at solutions to some of the great problems related to the development of mountain chains".

8.6 Recommendations and plans for future research

1. The history and deep structure of the crust in Scotland can only be well understood if the nature and extent of translations along the major crustal dislocations are fully ascertained. While this study has thrown some light on the nature of the Great Glen Fault, the extent of translations along it is still uncertain. The nature and extent of movement along the various geotectonic boundaries in Scotland should be regarded as research problems of high priority.

2. The proposed zone of crustal thickening north of the Highland Boundary Fault should be investigated by the Magnetotelluric method. The determination of other zones of crustal thickening will improve our understanding of the past history of the region.

3. Development of improved MT interpretation methods should also be
regarded as projects of high research priority in view of the depth of information that can be obtained from MT studies under ideal conditions. New, accurate and cost-effective 2-D and 3-D interpretation methods that can cope with complex earth problems need to be developed. Along this line, it is planned that the applicability of the method of summary representation (Polozhii decomposition) (Polozhii 1965; Tarlowski et al 1984) in EM modelling should be investigated. A simple 2-D data transformation (see Coen et al 1983) is feasible and would cut down on computing costs. Also, the amount of unambiguous information produced by 2-D geoelectric modelling can be greatly increased by employing suitable a priori data as shown in the 1-D case. As a potential improvement of 1-D geoelectric interpretation method, the possibility of joint inversion of MT and Transient electromagnetic and/or other geoelectric data will be examined in the near future.

4. The importance of improved MT observational techniques on land cannot be over-emphasized. Efforts should be geared towards improving the data acquisition method in order to reduce the ambiguity which arises during data interpretation.

5. For crustal structural studies an integrated interpretation of various geophysical and geological data sets is highly recommended. MT and Gravity (or MT and seismics) should be regarded as essential combinations.

6. A joint inversion technique for gravity, magnetics and/or MT data is currently under study. An empirical relationship between gravity and MT anomalies in Scotland is also being sought as this will aid MT/gravity interpretation. It is hoped that the integrated interpretation approach will be extended to the southern Caledonides of Britain in the near future.


Brewer J.A. and Smythe D.K., 1984. MOIST and the continuity of crustal reflector geometry along the Caledonian-Appalachian orogen, J. Geol. Soc. Lond., 141, 105-120.


Jones A.G. and Hutton V.R.S., 1979b. A multi-station magnetotelluric study in


Roberts A.M. and Harris A.L., 1983. The Loch Quoich Line - a limit of early Palaeozoic crustal reworking in the Moine of the Northern Highlands of


APPENDIX I

Glen Garry field results

The MT results obtained at 7 sites along the Glen Garry profile (G1–7) are presented here.

For each site, the results shown are: the major and minor apparent resistivity and phase, coherence, the number of estimates averaged, skew and the azimuths of the major impedance.
SITE: 553A TELLURIC

<table>
<thead>
<tr>
<th>CARTRIDGE</th>
<th>553A</th>
<th>5531A</th>
<th>5532A</th>
<th>5533A</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>SAMPLES/WINDOW</td>
<td>256</td>
<td>256</td>
<td>256</td>
<td>256</td>
</tr>
<tr>
<td>NUMBER WINDOWS</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>WINDOW RATE HZ</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>PLOT HPF</td>
<td>55.00</td>
<td>0.00</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>PLOT LPF</td>
<td>550.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>FREQUENCIES/DECade</td>
<td>180</td>
<td>15.86</td>
<td>6.85</td>
<td>0.05</td>
</tr>
<tr>
<td>FREQUENCIES/BAND</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>MIN COHERENCY</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>REJECTION LOOPS</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

MINOR

<table>
<thead>
<tr>
<th>FREQUENCY IN HZ</th>
<th>0.00</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
<th>0.35</th>
<th>0.40</th>
<th>0.45</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>REJECTION LOOPS</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Component</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CARTRIDGE</td>
<td>579</td>
<td>579</td>
<td>579</td>
<td>579</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sampling/Window</td>
<td>29</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number Windows</td>
<td>65</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sampling Rate Hz</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plot LPF</td>
<td>775</td>
<td>775</td>
<td>775</td>
<td>775</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freq/Decade</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freq/Band</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RIN Coherency</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rejection Loops</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SITE: 555A TELLURIC

<table>
<thead>
<tr>
<th>Component</th>
<th>5550A</th>
<th>5551A</th>
<th>5552A</th>
<th>5553A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Samples/WINDOW</td>
<td>256</td>
<td>256</td>
<td>256</td>
<td>256</td>
</tr>
<tr>
<td>Number Windows</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>23</td>
</tr>
<tr>
<td>Sample Rate Hz</td>
<td>2018</td>
<td>259</td>
<td>32</td>
<td>4</td>
</tr>
<tr>
<td>PLOT HPF</td>
<td>64.00</td>
<td>6.00</td>
<td>0.50</td>
<td>0.05</td>
</tr>
<tr>
<td>PLOT LPF</td>
<td>500.00</td>
<td>64.00</td>
<td>8.00</td>
<td>1.00</td>
</tr>
<tr>
<td>FREQ/BAND</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>MIN COHERENCY</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>REJECTION LOOPS</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

---

**MAJOR**

**MINOR**

**NUMBER OF ESTIMATES**

**SHEEN**

**REJECTION IN DEGREES**
SITE: 556A TELLURIC

<table>
<thead>
<tr>
<th>CARTRIDGE</th>
<th>556A</th>
<th>5561A</th>
<th>5562A</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>SAMPLES/WINDOW</td>
<td>256</td>
<td>256</td>
<td>256</td>
</tr>
<tr>
<td>NUMBER WINDOWS</td>
<td>40</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>SAMPLE RATE Hz</td>
<td>2016</td>
<td>256</td>
<td>92</td>
</tr>
<tr>
<td>PLOT HPF</td>
<td>61.55</td>
<td>6.90</td>
<td>0.59</td>
</tr>
<tr>
<td>PLOT LPF</td>
<td>525.50</td>
<td>68.00</td>
<td>16.00</td>
</tr>
<tr>
<td>FREQ/DECADE</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FREQ/BAND</td>
<td>6</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>MIN COHERENCY</td>
<td>0.70</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>REJECTION LOOPS</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

FREQUENCY IN HZ

PHASE DEGREES

NUMBER OF ESTIMATES

RVMUTH IN DEGREES

SHEW

FREQUENCY IN HZ

PHASE DEGREES

0 10 20 30 40 50 60 70 80 90 100

FREQUENCY IN HZ

234
APPENDIX II

Loch Arkaig field results

The MT results obtained at 5 sites along the Loch Arkaig profile (A1–5) are presented here.

For each site, the results shown are: the major and minor apparent resistivity and phase, coherence, the number of estimates averaged, skew and the azimuths of the major impedance.
SITE : 573A TELLURIC

<table>
<thead>
<tr>
<th>CARTIDGE</th>
<th>5733A</th>
<th>5731A</th>
<th>5732A</th>
<th>5733A</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>SAMPLES/MIN</td>
<td>25A</td>
<td>25B</td>
<td>25B</td>
<td>25A</td>
</tr>
<tr>
<td>NUMBER WINDOW</td>
<td>57</td>
<td>58</td>
<td>69</td>
<td>41</td>
</tr>
<tr>
<td>SAMPLE RATE Hz</td>
<td>2519</td>
<td>258</td>
<td>32</td>
<td>4</td>
</tr>
<tr>
<td>PLOT HFF</td>
<td>51.05</td>
<td>6.00</td>
<td>0.55</td>
<td>0.05</td>
</tr>
<tr>
<td>PLOT LPF</td>
<td>555.25</td>
<td>69.00</td>
<td>15.05</td>
<td>0.60</td>
</tr>
<tr>
<td>FREQUENCY/DEC</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FREQUENCY/BAND</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FILTERS</td>
<td>5.05</td>
<td>6.00</td>
<td>6.00</td>
<td>0.60</td>
</tr>
<tr>
<td>REJECTION LOOPS</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FREQUENCY IN HZ</th>
<th>10.00</th>
<th>100.00</th>
<th>1000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.10</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>
SITE: 571A TELLURIC

CARTRIDGE: 571A 571R 5712A 5713A
BAND: 1 2 3 4
COMPONENTS: 4 5 6 7
SAMPLES/WINDOW: 256 256 256 256
NUMBER WINDOWS: 10 10 10 10
SAMPLE RATE HZ: 2212 2212 2212 2212
PLOT HPF: 0.00 0.00 0.00 0.00
PLOT LPF: 0.00 0.00 0.00 0.00
FREQS/DECADE: 2 0 0 0
FREQS/BAND: 2 2 2 2
MIN COHERENCY: 0.80 0.80 0.80 0.80
REJECTION LOOPS: 2 2 2 2
SITE: 557A TELLURIC

<table>
<thead>
<tr>
<th>CARTRIDGE</th>
<th>5579A</th>
<th>5571A</th>
<th>5572A</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>SAMPLES/WINDOW</td>
<td>25A</td>
<td>25B</td>
<td>25C</td>
</tr>
<tr>
<td>SAMPLE RATE</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
</tr>
<tr>
<td>PLOT HPF</td>
<td>64.00</td>
<td>64.00</td>
<td>64.00</td>
</tr>
<tr>
<td>PLOT LPF</td>
<td>625.00</td>
<td>625.00</td>
<td>625.00</td>
</tr>
<tr>
<td>FREQ/DECADE</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>FREQ/BAND</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>COHERENCY</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>REJECTION LOOPS</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

FREQUENCY IN HZ:

1000.00 1000.00 1.00 0.01

NUMBER OF ESTIMATES:

1.01 0.1 0.01

AZIMUTH IN DEGREES:

0.0 0.4 0.8 0.0 0.4 0.8

FREQUENCY IN HZ:

1000.00 1000.00 1.00 0.01
APPENDIX III

Glen Loy field results
The MT results obtained at 19 sites along the Glen Loy profile (L1-19) are presented here.

For each site, the results shown are: the major and minor apparent resistivity and phase, coherence, the number of estimates averaged, skew and azimuths of the major impedance.
SITE : 560A TELLURIC

CARTRIDGE   = 5605A  5601A  5602A  5603A
BRAND   = 1  2  3  4
COMPONENTS = 5  5  5  5
SAMPLES/WINDOW = 258  258  258  258
NUMBER WINDOWS = 40  40  40  24
SAMPLE RATE Hz = 2048  256  256  256
PLGT HPF = 64.00  8.00  0.50  0.05
PLGT LPF = 500.00  64.00  0.05  1.00
FREQ/DECADE = 8  8  8  8
FREQ/BAND = 8  7  10  11
MIN COHERENCY = 0.85  0.85  0.80  0.80
REJECTION LOOPS = 2  2  2  2
SITE: 60GA TELLURIC

<table>
<thead>
<tr>
<th>CARTRIDGE</th>
<th>06569A</th>
<th>05901A</th>
<th>0692A</th>
<th>05539A</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>SAMPLES/METON</td>
<td>256</td>
<td>256</td>
<td>256</td>
<td>256</td>
</tr>
<tr>
<td>NUMBER WINDOWS</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>SAMPLE RATE HZ</td>
<td>60</td>
<td>84</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>PLOT HPF</td>
<td>12.00</td>
<td>8.00</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>FREQUENCY/DEC/DECRE</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>FREQUENCY/HPF</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>MIN COHERENCY</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>REJECTION LOOPS</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

FREQUENCY IN HZ
0.01 0.10 1.00 10.00 100.00

PHONE DEGREES
0 10 20 30 40 50 60 70 80 90 100

APPARENT RESISTIVITY CM-H 1000 10000 100000

MAJOR

MINOR

PLOT LPF

NUMBER OF ESTIMATES
0.0 0.5 1.0 1.5 2.0

10 100 1000

AZIMUTH IN DEGREES
-90 -45 0 45 90

FREQUENCY IN HZ
0.01 0.10 1.00 10.00 100.00

PLOT LPF

NUMBER OF ESTIMATES
0.0 0.5 1.0 1.5 2.0

10 100 1000

AZIMUTH IN DEGREES
-90 -45 0 45 90

FREQUENCY IN HZ
0.01 0.10 1.00 10.00 100.00

244
<table>
<thead>
<tr>
<th>Component</th>
<th>11</th>
<th>11</th>
<th>11</th>
<th>11</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site</td>
<td>1080.00</td>
<td>1080.00</td>
<td>1080.00</td>
<td>1080.00</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1.00</td>
<td>0.10</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Site</td>
<td>1050.00</td>
<td>1050.00</td>
<td>1050.00</td>
<td>1050.00</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1.00</td>
<td>0.10</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Site</td>
<td>1020.00</td>
<td>1020.00</td>
<td>1020.00</td>
<td>1020.00</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1.00</td>
<td>0.10</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Site</td>
<td>1000.00</td>
<td>1000.00</td>
<td>1000.00</td>
<td>1000.00</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1.00</td>
<td>0.10</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**SITE:** 558B TELLURIC

<table>
<thead>
<tr>
<th>Component</th>
<th>11</th>
<th>11</th>
<th>11</th>
<th>11</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site</td>
<td>1080.00</td>
<td>1080.00</td>
<td>1080.00</td>
<td>1080.00</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1.00</td>
<td>0.10</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Site</td>
<td>1050.00</td>
<td>1050.00</td>
<td>1050.00</td>
<td>1050.00</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1.00</td>
<td>0.10</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Site</td>
<td>1020.00</td>
<td>1020.00</td>
<td>1020.00</td>
<td>1020.00</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1.00</td>
<td>0.10</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Site</td>
<td>1000.00</td>
<td>1000.00</td>
<td>1000.00</td>
<td>1000.00</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1.00</td>
<td>0.10</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
SITE: 562A TELLURIC

<table>
<thead>
<tr>
<th>CARTRIDGE</th>
<th>5020A</th>
<th>5621A</th>
<th>5622A</th>
<th>5623A</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>SAMPLES/MIN</td>
<td>256</td>
<td>256</td>
<td>256</td>
<td>256</td>
</tr>
<tr>
<td>SAMPLE RATE</td>
<td>90</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>PLOT HPP</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>PLOT LPF</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>FREQU/DECAD</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>FREQU/BAND</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>KIN COHERENCY</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>REJECTION LOOPS</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
### SITE: 611A TELLURIC

<table>
<thead>
<tr>
<th>CARTRIDGE</th>
<th>6110A</th>
<th>6111A</th>
<th>6112A</th>
<th>6113A</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>SAMPLES/WINDOW</td>
<td>256</td>
<td>256</td>
<td>256</td>
<td>256</td>
</tr>
<tr>
<td>NUMBER WINDOWS</td>
<td>30</td>
<td>30</td>
<td>45</td>
<td>32</td>
</tr>
<tr>
<td>SAMPLE RATE Hz</td>
<td>612</td>
<td>640</td>
<td>640</td>
<td>640</td>
</tr>
<tr>
<td>PLOT HPF</td>
<td>0.00</td>
<td>2.00</td>
<td>0.10</td>
<td>5.02</td>
</tr>
<tr>
<td>PLOT LPF</td>
<td>120.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.10</td>
</tr>
<tr>
<td>FREQ/DECAD</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>FREQ/BAND</td>
<td>10</td>
<td>6</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>MIN COHERENCY</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>REJECTION LOOPS</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

### FREQUENCY IN HZ

<table>
<thead>
<tr>
<th>FREQUENCY IN HZ</th>
<th>100000</th>
<th>10000</th>
<th>1000</th>
<th>100</th>
<th>10</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREQUENCY IN HZ</td>
<td>512</td>
<td>512</td>
<td>512</td>
<td>512</td>
<td>512</td>
<td>512</td>
</tr>
</tbody>
</table>

### NUMBER OF ESTIMATES

<table>
<thead>
<tr>
<th>NUMBER OF ESTIMATES</th>
<th>100</th>
<th>10</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUMBER OF ESTIMATES</td>
<td>100</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

### AZIMUTH IN DEGREES

<table>
<thead>
<tr>
<th>AZIMUTH IN DEGREES</th>
<th>90</th>
<th>60</th>
<th>30</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>AZIMUTH IN DEGREES</td>
<td>90</td>
<td>60</td>
<td>30</td>
<td>0</td>
</tr>
</tbody>
</table>

### SKEN

<table>
<thead>
<tr>
<th>SKEN</th>
<th>60</th>
<th>30</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>SKEN</td>
<td>60</td>
<td>30</td>
<td>0</td>
</tr>
</tbody>
</table>

### PHASE DEGREES

<table>
<thead>
<tr>
<th>PHASE DEGREES</th>
<th>180</th>
<th>90</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>PHASE DEGREES</td>
<td>180</td>
<td>90</td>
<td>0</td>
</tr>
</tbody>
</table>

### APPARENT RESISTIVITY OHM.M

<table>
<thead>
<tr>
<th>APPARENT RESISTIVITY OHM.M</th>
<th>10000</th>
<th>1000</th>
<th>100</th>
<th>10</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPARENT RESISTIVITY OHM.M</td>
<td>10000</td>
<td>1000</td>
<td>100</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

### FREQUENCY IN HZ

<table>
<thead>
<tr>
<th>FREQUENCY IN HZ</th>
<th>100000</th>
<th>10000</th>
<th>1000</th>
<th>100</th>
<th>10</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREQUENCY IN HZ</td>
<td>100000</td>
<td>10000</td>
<td>1000</td>
<td>100</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
SITE : 563A TELLURIC

<table>
<thead>
<tr>
<th>CARTRIDGE</th>
<th>563AR</th>
<th>563AR</th>
<th>563AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>SAMPLES/MIN</td>
<td>258</td>
<td>258</td>
<td>258</td>
</tr>
<tr>
<td>NUMBER WINDOWS</td>
<td>48</td>
<td>48</td>
<td>33</td>
</tr>
<tr>
<td>SAMPLE RATE HZ</td>
<td>2580</td>
<td>2580</td>
<td>35</td>
</tr>
<tr>
<td>PLOT HPF</td>
<td>64.00</td>
<td>8.00</td>
<td>0.50</td>
</tr>
<tr>
<td>PLOT LPF</td>
<td>500.00</td>
<td>64.00</td>
<td>8.00</td>
</tr>
<tr>
<td>FREQS/DECIDE</td>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>FREQS/BAND</td>
<td>6</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>MIN COHERENCY</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>REJECTION LOOP</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

![Graphs showing apparent resistivity and phase degrees vs. frequency in Hz for major and minor orientations.](image-url)
SITE: 608A TELLURIC

<table>
<thead>
<tr>
<th>CARTRIDGE</th>
<th>608A</th>
<th>608A</th>
<th>608A</th>
<th>608A</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>COMPONENTS</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>SAMPLES/MIN</td>
<td>256</td>
<td>256</td>
<td>256</td>
<td>256</td>
</tr>
<tr>
<td>SAMPLE RATE</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>SAMPLES/NODS</td>
<td>512</td>
<td>64</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>PLOT HPF</td>
<td>8.00</td>
<td>2.00</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>PLOT LPF</td>
<td>120.00</td>
<td>8.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>PLOT DEC</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>FREQUENCIES</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>REJECTION LOOP</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

MAJOR

MINOR

NUMBER OF ESTIMATES

SHAPE

AZIMUTH IN DEGREES

FREQUENCY IN HZ

PHASE DEGREES D TO 90°

APPARENT RESISTIVITY OHM.M

FREQUENCY IN HZ

1000.00 100.00 10.00 1.00 0.10 0.01
SITE : 566A TELLURIC

| CARTRIDGE | 566A 566A 566A 566A |
| BAND   | 1 2 3 4 |
| COMPONENTS | 5 5 4 4 |
| SAMPLES/HINDOW | 250 250 250 250 |
| NUMBER WINDOW | 40 40 65 65 |
| SAMPLE RATE HZ | 204.6 204.6 92 4 |
| PLOT HPF | 0.00 0.00 0.05 0.05 |
| PLOT LPF | 0.00 0.00 0.00 1.00 |
| FREQUENCIES/DECIDE | 0 0 0 0 |
| FREQUENCIES/BAND | 0 7 10 11 |
| MIN COHERENCY | 0.00 0.00 0.00 0.00 |
| REJECTION LOOPS | 2 2 2 2 |

**FREQUENCY IN HZ**

**1.00 0.10 0.01**

**PHASE DEGREES**

**1.00 0.10 0.01**

**APPARENT RESISTIVITY OMN.M**

**1.00 0.10 0.01**

**PLOT MINOR**

**FREQUENCY IN HZ**

**1.00 0.10 0.01**

**PHASE DEGREES**

**1.00 0.10 0.01**

**APPARENT RESISTIVITY OMN.M**

**1.00 0.10 0.01**

**PLOT MAJOR**

**FREQUENCY IN HZ**

**1.00 0.10 0.01**

**PHASE DEGREES**

**1.00 0.10 0.01**

**APPARENT RESISTIVITY OMN.M**

**1.00 0.10 0.01**

**PLOT MAJOR**
Individual site 1-D models

The individual station 1-D inversion results are presented here, going from the west to the east, for the Glen Garry (G1-G7), Loch Arkaig (A1-A4) and Glen Loy (L1-L19) profiles. Only the most squares and in some cases the Occam and Niblett-Bostick models are shown. Irreducible bias effects limited model resolution to a large extent for the Loch Arkaig stations. Consequently, station 557 (A5) has been deleted from the interpretation process. Notice in the Glen Loy models that the sites to the east of the surface position of the Fort William slide (L17, L18, L19) show a somewhat different deep structural behaviour from the rest of the profile.
1D MOST-SQUARES SM MODELS FOR SITE G1 EFFECTIVE

CHIQ=22.80  NFREQ=25
3 LAYERS  AMP+PHASE FIT
RESISTIVITY OHM.M

10 MOST-SQUARES SM MODELS FOR SITE G2 EFFECTIVE

CHIQ=27.36  NFREQ=30
3 LAYERS  AMP+PHASE FIT
RESISTIVITY OHM.M
10 MOST-SQUARES SM MODELS FOR SITE G3 EFFECTIVE

OCCAM MODEL FOR SITE S50A EFFECTIVE
1D MOST-SQUARES SM MODELS FOR SITE G4 EFFECTIVE

CHISQ=47.09  NFREQ=33
3 LAYERS  AMP+PHASE FIT
RESISTIVITY OHM.M

OCCAM MODEL FOR SITE 551A EFFECTIVE

CHISQ=17.08  NFREQ=33
16 LAYERS  AMP FIT
RESISTIVITY OHM.M
1D MOST-SQUARES SM MODELS FOR SITE 65 EFFECTIVE

CHISQ=29.18  NFREQ=32
3 LAYERS  AMP+PHASE FIT
RESISTIVITY OHM.M

10 MOST-SQUARES SM MODELS FOR SITE 66 EFFECTIVE

CHISQ=24.62  NFREQ=27
3 LAYERS  AMP+PHASE FIT
RESISTIVITY OHM.M

266
1D MOST-SQUARES SM MODELS FOR SITE A1 EFFECTIVE

$\chi^2=73.10$  \text{ NFREQ=34}
3 LAYERS  \text{ AMP+PHASE FIT}

RESISTIVITY OHM.M

1D MOST-SQUARES SM MODELS FOR SITE A2 EFFECTIVE

$\chi^2=34.10$  \text{ NFREQ=31}
3 LAYERS  \text{ AMP+PHASE FIT}

RESISTIVITY OHM.M
1D MOST-SQUARES SM MODELS FOR SITE A3 EFFECTIVE

CHISQ=40.71  NFREQ=20
3 LAYERS  AMP+PHASE FIT
RESISTIVITY OHM.M

1D MOST-SQUARES SM MODELS FOR SITE A4 EFFECTIVE

CHISQ=47.29  NFREQ=32
3 LAYERS  AMP+PHASE FIT
RESISTIVITY OHM.M
1D MOST-SQUARES SM MODEL FOR SITE L9 EFFECTIVE

CHISQ=23.71  NFREQ=26
3 LAYERS  AMP+PHASE FIT

RESISTIVITY OHM.M

1D MOST-SQUARES SM MODEL FOR SITE 5588 EFFECTIVE L9

CHISQ=5.83  NFREQ=26
3 LAYERS  PHASE FIT

RESISTIVITY OHM.M
1D MOST-SQUARES SM MODELS FOR SITE L16 EFFECTIVE

CHISQ=48.33  NFREQ=29
3 LAYERS  AMP-PHASE FIT
RESISTIVITY OHM.M

1D MOST-SQUARES SM MODEL FOR SITE L17 EFFECTIVE

CHISQ=73.60  NFREQ=33
4 LAYERS  AMP-PHASE FIT
RESISTIVITY OHM.M