MASS TRANSFER IN THE ENTRANCE REGION
OF A
CIRCULAR DUCT

by

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SUMMARY

The entrance region of a circular duct is of considerable technological importance and the results are expected to yield helpful data for use in the design and development of short-tube heat transfer equipment. The results are expected to provide fundamental information about the transport properties of the viscous sub-layer which in turn may throw light on the proposed rival theoretical models of transport processes of Doisslor and Sparrow.

The investigation, the first of its kind, is concerned with the study of the influence of variations of the Reynolds Number and Schmidt Number on the rapidity of approach to steady state of local mass-transfer rates from a sparingly-volatile cast surface to fully-developed turbulent flow of gases for which the Schmidt number is low. The boundary condition for the tube was constant or uniform wall temperature.

Mass-transfer data for the entrance region of ducts is scanty. The little information available suffers from the low accuracy of the data, a fact that is associated generally with the coarseness of measurements of mass-transfer coefficients.

An improved, highly sensitive pneumatic profilometric technique was evolved for the measurement of the mass-transfer coefficients. The technique depended on the estimation of configurational changes, and hence on the amount of recession, resulting on a cast-in cylindrical sparingly-volatile surface at/
at a particular cross-section, following sublimation in the course of an experimental run.

The experimental data were tabulated in the form of the ratio of the local to the asymptotic values of the Sherwood Number, i.e. $\frac{Sh}{Sh_\infty}$ against the number of tube diameters $x/d$. The data were then fitted by means of a least-squares procedure programmed on a KDF-9 digital computer and were presented in the form of graphs of $\frac{Sh}{Sh_\infty}$ vs $x/d$. The results were then directly compared against values obtained from the modified form for mass transfer of the most satisfactory analysis, i.e. Deissler. The values were calculated using the KDF-9 digital computer.

The results of the present investigations are as follows:

The accuracy of the estimated mass-transfer coefficients was considerably enhanced. Sublimation by the gauging-air was found to be a hitherto unknown complicating factor, and necessary corrections were made. A surface-exploration method was developed by exploiting the sensitivity of the profilometric technique. This contributed to a better understanding of the true nature of cast mass-transfer surfaces and helped to explain the puzzling abnormally high scatter in earlier mass transfer work (L.2). Surface irregularities in the mould tend to give an abnormally high 'apparent scatter', whereas the 'true scatter' is dictated by the limits of the experimental error in measurement.

For the whole range of Schmidt Number investigated, local mass-transfer rates, with undeveloped mass transfer, approach fully-developed mass transfer conditions within a short range of diameters/
diameters downstream from the entrance. The short entrance lengths observed in the present work were in agreement with those predicted by Deissler's analysis but were much shorter than those predicted by Sparrow's model.

An effect of Reynolds Number on local transfer rates was observed, showing a small relative decrease in mass transfer at a particular location along the tube with increase of Reynolds Number. This result is in accord with the theoretical model of Deissler but contradicts the trend obtained from the analysis of Sparrow.

The Schmidt Number was observed to have an appreciable effect on the rapidity of approach to steady state of mass-transfer rates. Deissler's results underpredict the experimental data but are closer to them than the much larger effect forecast by Sparrow's analysis.

A method is presented, which will enable Deissler's analysis of fully-developed flow in the thermal/concentration entrance-region of a circular tube for the U.W.T. situation to be calculated by numerical methods with the aid of a digital computer. This entails transforming the expression for the U.W.T., or U.W.C., solution, as given by Deissler (D.3).

In conclusion, a new model is proposed in which the asymptotic mass-transfer coefficients are evaluated from Sparrow's analysis, whereas the developing concentration boundary-layer is calculated from a modified Deissler analysis.
1. INTRODUCTION
1. INTRODUCTION

1.1. GENERAL INTRODUCTION TO THE RESEARCH TOPIC.

The object of the present investigation was to study the transfer of mass from a volatile solid surface to a stream of turbulent fluid in the entrance region of a circular duct. In the series of experiments carried out during the course of the present work, sublimation rates at a cylindrical wall were measured using a pneumatic air-gauging technique. The mass-transferring section was placed far downstream of a long pipe of the same diameter, where the flow was hydrodynamically fully developed.

There are many factors that affect mass transfer in a duct. These include the velocity, temperature, and properties of fluid, the geometry of ducts, solid mass diffusivity and wall-boundary conditions. In the experimental work described here, attention was focussed on the influence of gas-velocity, i.e. Reynolds Number, and of fluid properties, by varying the working fluid and hence the Schmidt Number. The temperature and duct-geometry were kept constant and the wall-boundary condition was that of uniform wall temperature (U.W.T.).

It was hoped that the experiments would yield useful data for both technological and academic purposes:

(i) Technological: The data might be of immediate use in the design of short heat-transfer equipment. Determinations of mass-transfer rate, followed by extension to the equivalent heat-transfer question, yield a more ready estimate of heat-transfer coefficients.
coefficients than can generally be attributed to direct heat-flow and temperature measurements.

Technically, application of this problem is to be found in the following cases:

(a) Uniform Wall Heat Flux (U.W.H.F.): Electric resistance heating, radiant heating, nuclear heating, and in counter-flow heat exchangers where the fluid capacity rates are the same.

(b) Uniform Wall Temperature (U.W.T.): A common convection application in any heat exchanger where one fluid has a very much higher capacity rate than the other. Examples of this are evaporators and condensers.

(ii) Academic: the results were expected to provide fundamental information about the transport properties of the viscous sub-layer which in turn might throw light on proposed rival theoretical models of transfer processes. This should help to extend our understanding of the heat and mass-transfer mechanisms involved in the flow region near the wall.

Experimental elucidation is made extremely difficult by the necessarily small tolerable errors demanded in an investigation of this kind, owing to the closeness of the examined rival analyses. This requirement imposes heavy demands not only on the ingenuity of increasingly refined experimental apparatus. High standards of precision and preparation are also required during the actual execution of the experimental investigation. The subject thus tends to be one beset by great obstacles, 'vexatious to the experimentalist'.
The heat or concentration boundary layer in a fluid at short distances from a thermal or mass-transfer discontinuity in a smooth pipe wall is thin compared with the viscous sub-layer. Rates of transfer at such regions of the wall are accordingly strongly influenced by the degree to which the convective disturbances enter this layer. Very accurate measurements of these transfer rates can thus be exploited to great advantage in the attempt to test the various expressions that have been proposed for the distribution of eddy diffusivity in this inaccessible flow-regime.

The choice of mass-transfer techniques for investigations of this kind is generally based on the assumption that these are superior to heat-transfer methods. The fluid properties may be kept practically constant throughout the mass-transfer system; they are therefore not subject to large local variations and uncertainties of the non-isothermal type. Also, the stepwise discontinuity in wall temperature can be made ideally sharp. Problems of axial conduction through the duct-wall are thereby entirely avoided; local wall-flux measurements do not then require corrections for the effects of longitudinal temperature gradients or fluxes.

In the present investigation, local rates of sublimation of diphenyl were determined at regular intervals at the wall of a straight pipe. A stream of turbulent fluid, of known velocity and temperature, passed through the pipe. The sublimation process took/
took place in a closed-circuit wind-tunnel and the experiment was carried out with different gases at various velocities, i.e. for various Schmidt Numbers and Reynolds Numbers.

The critical need for a greater degree of accuracy in the final results was met by the construction of a specially designed air-gauging apparatus.

A profilometric technique was devised which enabled the condition of the prepared test-surface to be determined from the outset. Surfaces of known quality were then chosen in the experiments. The technique led to a greater general understanding of test-surfaces prepared by casting methods.

A surface-preparation technique was developed accordingly by which mass transfer surfaces of high quality were produced. A further technique was evolved to protect the carefully-prepared smooth surface from physical damage during the profilometric operations. The method employed allowed measurements to be made close to the entrance of the transferring section, and is shown to give results reproducible to ± 5% in the present case.

Furthermore, computer programmes were prepared for the final treatment of the experimental data obtained at the end of each series of the investigation. These were then compared directly with the results of the more promising analysis obtained for this project by the preparation of a complete computer programme on a KDF-9 digital computer.
TABLE I - SCHEMATIC DIAGRAM OF THESIS.

Entrance Phenomena - Mass Transfer Case
Study of Effects of Re, Sc, with U.W.T Conditions

Two Main Rival Theories

EXPERIMENT

Design of Experimental Equipment

Gauging
Apparatus
(Section 4.7.)

Wind-
Tunnel
(Section 4.7.)

Test-Section
(Section 4.7.)

Casting
Operations
(Section 4.5)

Preparation of
Computer Program
for Treatment of
Experimental Results
(Section 3.5)

EXPERIMENT

RESULTS

Experimental Graphs for Mass Transfer of \( \text{Sh} / \text{Sh}_m \) vs \( x/d \)
(Section 3.4)

COMPARISON

Direct

FINAL
RESULTS

COMPARISON

Indirect

CONCLUSIONS
(Section 5.)

DEISSLER, R.G.
(Section A1.4)

MASS TRANSFER
Preparation of
Computer Program
for U.W.C.P.
for Twelve Re's
and
Three Sc's
(Section A.1.4)

HEAT TRANSFER
Available Heat-Transfer
Graphs of \( \text{Nu}/\text{Nu}_m \) vs \( x/d \)
for U.W.H.F.

SCHRÖDINGER, E.P.
(Section A1.3)

HEAT TRANSFER
Limited Available
Heat-Transfer
Graphs of \( \text{Nu}/\text{Nu}_m \) vs \( x/d \)
extended to the
U.W.T situation
(Section A1.3)
2. LITERATURE SURVEY
2. LITERATURE SURVEY

2.1. REVIEW OF EXPERIMENTAL INVESTIGATIONS OF HEAT AND MASS TRANSFER ENTRANCE LENGTHS.

A considerable amount of investigation has been carried out on the problem of entrance lengths in heat transfer. This is not the case for mass transfer, where entry-length information is scanty, and no true local transfer data is available.

In general, the agreement between theory and experiment throughout the subject is not very good and this may be attributed to any, or all, of the following reasons: a) The influence of variable fluid-properties, b) Uncertainties in the knowledge of eddy diffusivity, $\varepsilon$, c) Experimental errors.

Investigations of entrance regions for heat transfer are at a considerable disadvantage compared to the corresponding mass-transfer studies owing to the confusing role played by the temperature. Entrance-length data are usually correlated by the ratio $N_u/N_u\infty$, as seen in section 2.2.4, where $N_u$ is the local Nusselt number at point $x$, and $N_u\infty$ is the constant downstream, fully-developed or asymptotic Nusselt number. A change in the expected value of $N_u\infty$ will result whenever any change in axial fluid-property occurs owing to an axial variation in the temperature of the fluid.

In the case of heat transfer to gases, the change in the Prandtl number, $Pr$, with temperature $t$ is very small since both $\mu$ and $k$ are $\propto \sqrt{t}$. This is discussed in some detail in section 2.4. However, a considerable change in the Reynolds number occurs owing to the low heat capacity of the gas. The opposite is true for the case/
case of liquids, where a small temperature rise is attended by a considerable change in the Pr; for example, a decrease of about 20\% in the Pr is noted for a rise in temperature of 10°C. The greater influence of temperature in the case of liquids is because of their higher heat capacities. Furthermore, in direct contrast with gases, \( \mu \) and \( k \) change in opposite directions for liquids.

Various methods have been employed to take into account the axial property-variation in heat transfer investigations. The most important ones are the 'reference temperature' and the 'property ratio' methods, and these are fully discussed in section 2.4.1, with illustrations from the work of Doissler (D.1, D.2). Zellnik and Churchill (Z.1) used the same method in measuring heat-transfer coefficients from air at high temperature to a cold wall in the inlet region of a tube. The experiments were carried out for fully-developed turbulent velocity and heat transfer conditions. Zellnik and Churchill recommended the use of the following equation, provided that the properties of the gas were evaluated at bulk temperatures, \( t_b \):

\[
\frac{h d}{k} = A \left[ \frac{c_p}{k} \right]^{\frac{1}{3}} \left[ \frac{dG}{\mu} \right]^{0.8} \tag{2.1}
\]

where \( A \) is a constant to be evaluated by experiment, \( d \) is the pipe diameter, \( G \) is the mass velocity. When the properties are evaluated at the film-temperature, \( t_0 \), the compensating factor \( (t_b/t_0)^{0.33} \) is included in equation (2.1).

Other methods that have been used are those by Mills (M.1) who/
who defined \( \text{Nu}_\infty \) as that value which the Nusselt number assumed at a point \( x \), if there were no entry effect present. This is the calculated value of \( \text{Nu}_\infty \) from the McAdams equation \((N.20)\)

\[
\text{Nu}_\infty = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} \hspace{1cm} (2.2)
\]
at the temperature at cross-section \( x \). The equivalent correlation in mass transfer is

\[
\text{Sh}_\infty = 0.023 \text{Re}^{0.8} \text{Sc}^{0.4} \hspace{1cm} (2.2a)
\]
where \( \text{Sc} \) is the Schmidt number and \( \text{Sh}_\infty \) is the fully-developed Sherwood number. Equation \((2.2)\) is widely recommended for application in situations where the temperature differences are not large. This correlation is often incorrectly referred to in literature as the Dittus-Boelter equation \((D.17)\) which is given by

\[
\text{Nu}_\infty = 0.0243 \text{Re}^{0.8} \text{Pr}^n \hspace{1cm} (2.2b)
\]
Equation \((2.2b)\) holds for \( \text{Re} > 10,000 \) and \( 0.7 < \text{Pr} < 100 \) and the exponent \( n \) takes the value of 0.4 for heating and 0.3 for cooling. Another investigation employing a similar definition was used by Ede \((2,3,4)\), his results being presented in the form

\[
\frac{\text{Nu}_x}{\text{Re}_x^{0.8} \text{Pr}_x^{0.4}} \hspace{1cm} (2.3)
\]
The measure of the entrance effect is taken as the deviation from the constant value calculated from the McAdams equation \((2.2)\).

The early investigation by Stanton (S.1) showed that the mean heat-transfer coefficient was expected to be constant for \(30 < x/d < 60\). It was inferred from this result, that the main region of interest lay in \(0 < x/d < 30\). Nusselt (N.1) attempted to predict heat-transfer coefficients and introduced a compensating factor into existing correlations for fully-developed flow. The term \((x/d)^{-0.054}\) was the proposed factor and the resulting equation was

\[
\frac{h_d}{k} = 0.036 \left[ \frac{dC}{\mu} \right]^{0.8} \left[ \frac{c_p}{k} \right]^{1/3} \left[ \frac{x}{d} \right]^{-0.054} \quad ............ (2.4)
\]

where \(h\) is the local heat-transfer coefficient, \(k\) is the thermal conductivity, \(C\) is the mass velocity, \(c_p\) is the heat capacity at constant pressure and \(\mu\) is the viscosity. Latzko (L.1) carried out a series of investigations of heat transfer in turbulent flow, in tubes of different configuration for the Uniform Wall Temperature (U.W.T.) situation. For the bell-mouthed entry, which causes laminar boundary-layers to build up on the wall in the entry region, a critical length \(x_c\) was suggested

\[
\left( \frac{x}{d} \right)_c = 0.0693 \left( \frac{dC}{\mu} \right)^{1/4} \quad ............ (2.5)
\]

The critical length, \(x_c\), is the distance at which the boundary layers meet in the centre of the pipe. For \(x/d < \left( \frac{x}{d} \right)_c\) the mean heat transfer coefficient over the entry region is given by the equation

\[
h_m = h_\infty 1.11 \left[ \left( \frac{dC}{\mu} \right)^{1/5} \left( \frac{x}{d} \right)^{-4/5} \right] 0.275 \quad ............ (2.6)
\]

where/
TABLE II. GENERAL METHODS OF HEAT TRANSFER MEASUREMENTS.

<table>
<thead>
<tr>
<th>General Methods of Measuring Local Heat Transfer Coefficients Directly. (of Limited Interest.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method I</td>
</tr>
<tr>
<td>Pipe heated electrically and thermocouple measurements are taken at various stations along the pipe.</td>
</tr>
</tbody>
</table>

Disadvantages of the Method:
1. The heating situation is vulnerable to disturbances caused by the presence of thermocouples:
   (a) It is difficult to attach enough thermocouples to the surface of the tube without conduction through the thermocouple leads affecting seriously the measured average temperatures.
   (b) Unevenness caused by welding the thermocouples to the pipe wall can cause interference with the proper working of the thermocouples.
2. The test section metal: This leads to circumferential variations of the heat transfer rates.

Disadvantages of the Method:
Accurate measurements of the condensate level and the non-condensate rate is difficult. According to Boelter (B.10), the error involved in finding the heat added is ± 5%.

Examples:
* Boelter et al. (B.10)
* Cholette, A. (C.1)
  also refer to Tables IV and V.

Examples:
* Mills, A.F. (M.1)
* Ede (E.1)
  and others shown in Tables IV and V.
where $h_\infty$ is the heat-transfer coefficient at a long distance from entry, or, the asymptotic value of the heat-transfer coefficient; and for $\frac{x}{d} > (\frac{x}{d})_c$, the relevant equation is

$$h_m = h_\infty + 0.14\frac{d\sigma}{\mu} \left(\frac{d\sigma}{\mu}\right)^{1.4} \left(\frac{x}{d}\right)^{-1} \quad \text{(2.7)}$$

For $\frac{d\sigma}{\mu} = 4000$, $x_c$ is then about 9 diameters.

The work carried out by Boelter et al. (B.4), again for the U.W.T. situation, largely substantiates the findings of Latzko; however, they suggest that equation (2.7) applies for the region $x/d > 5$. Boelter et al. used a steam-heated tube for $Pr = 0.7$, and their results for thermal-entrance length of 9 diameters agree with those of Wolf (W.1), whose experiments were carried out for a Uniform Wall Heat Flux (U.W.H.F.) situation. In both these investigations, a length of 50 diameters was arbitrarily taken for the Nusselt numbers to assume the $Nu_\infty$ value, and the effect of axial fluid-property variation was ignored. In the case of Wolf's work, this neglect is even more serious, because of the large temperature increases, which tended to underestimate the thermal entry-lengths. This argument is further substantiated by the fact that the entry lengths obtained by Wolf under cooling conditions are very much larger compared with his own results for heating.

The presence of laminar layers, produced by bell-mouth entries, is shown by the fact that heat-transfer coefficients downstream in Boelter's work were three times as great. This means that conduction through a laminar boundary-layer is slow compared with eddy conduction. The investigations of Al-Arabi and Davies (A.3) were/
were very similar to those of Boelter et al., using water in place of air. Their results tend to support the above-mentioned conclusions of Boelter et al., and add very little information.

The experiments of Aladyev\(^{(A.1)}\) were conducted for water with a U.W.T. distribution. Local and mean heat-transfer coefficients were measured in the inlet region of a circular duct, where both the velocity and temperature profiles were developing. The range of the Reynolds number covered in this work, was from 10,000 to 100,000; the results obtained show that local and mean-transfer coefficients extend downstream from the entrance to 40 and 50 diameters respectively.

Hartnett\(^{(H.1)}\) carried out thermal-entry investigations for an electrically-heated round tube preceded by a long calming-section. The experiments were conducted with water, \(Pr = 7\) and 7.3, and oil, \(Pr = 61\) and 63, with a U.W.H.F. distribution. The results obtained were in agreement with those of Latko\(^{(L.1)}\) and also with the analytical predictions of Doissler; namely, very short entrance-lengths of the order of \(\sim 10\) diameters. Hartnett also concluded that the thermal-entrance length decreased as the Re increased. This supports Doissler's predictions. He was unable however to find any influence of the \(Pr\) on the thermal-entrance lengths. The work of Hartnett suffers because he failed to take into account the problem of property variation in water and especially in oil (section 2.1).

The investigations of Malina and Sparrow\(^{(M.15)}\), although very similar to those of Hartnett, were appreciably improved. Special thermocouple/
thermocouple arrangements were included to eliminate end-effects, and allowance was made for property variation in the test-liquids, water and oil. Their results were more successful in showing an effect of Pr on entrance-length; an increase in Pr is attended by a corresponding decrease in the entrance-length.

Most of the investigations of thermal-entrance regions with fully-developed turbulent flow belong to one of the general types described above. However, additional information may be derived from the detailed Tables IV and V.


Apart from the studies of mass transfer in wetted columns, where the acceleration of the liquid film at the top of the column causes a reduction in the film thickness, very little information exists in other situations. A summary of the usual methods employed in studies of mass transfer is presented in Table XII.

Linton and Sherwood\(^{(L2)}\) measured mean or cumulative mass-transfer coefficients for fixed values of \(\kappa/d\). Mass-transfer data were obtained on the rate of solution of slightly-soluble salts in water. The test objects consisted of cast plates, cylinders and tubes. These were placed in a direction parallel to the flow, by mounting them in two vertical conduits proceeded by a calming-section of about 75 diameters. The soluble substances used in this investigation were benzoic acid, cumaric acid and \(\beta\)-napthol. The rate of solution of the cast material was measured gravimetrically, and/
TABLE III. GENERAL METHODS OF MASS TRANSFER MEASUREMENTS.

- **General Methods of Mass Transfer Investigations**
  - **and**
  - Measurements of Mass Transfer Rates

**Method I**

- **Yield instantaneous as well as local transfer rates.**
  - **Electrolytic Techniques**

  - **Examples:**
    - *Lin et al.* (L3)
    - *Maxwell and Storrow* (M2)

**Method II**

- **Yield time-integrated transfer effects over the duration of an experimental run.**
  - **Profilometric Techniques**
  - **Example:**
    - *Linton and Shorwood* (L2)
  - **Cravimetric Techniques (e.g. dissolution of soluble surfaces.)**

**Direct Measurement Methods:**
- (Measuring apparatus in physical contact with mass transfer surface)
  - **Example:**
    - *Shorwood and Trass* (S37)

**Pneumatic Gauging Methods:**
- **Example:**
  - *Present Investigation*
and the results showed that a steady value was reached after about 10 minutes. The dimensional changes that took place on the test-surfaces during the experiment were found to be negligible. No attempt was made to measure local coefficients. They are the only authors who appear to attempt to correlate entrance-region data. Some of their results will be mentioned briefly.

(1) Values of the transfer coefficients measured for circular tubes with a long calming-section generally decreased with length from the entry.

(2) It was suggested that the molecular diffusion is very small for liquids, for small values of $x/d$.

(3) A new relationship was developed by integration of the diffusion equation

$$D \frac{\partial^2 c}{\partial y^2} = u \frac{\partial c}{\partial x} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.8)$$

for the entry region. $D$ is the molecular diffusivity, $u$ is the velocity of the fluid, $c$ is the concentration of the diffusing substance, $x$ is the point at which mass transfer begins, and $y$ is the distance from the wall. The method of correlating the data was somewhat analogous to the Nusselt number for heat transfer. A compensating factor $(x/d)^{-1/3}$ was introduced and the resulting equation was

$$J_D = 0.232 \left( \frac{du \cdot \rho}{\mu} \right)^{-0.4} \left( \frac{x}{d} \right)^{-1/3} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.9)$$

where $J_D$ is the mass factor, $d$ is the tube diameter, $\mu$ is the viscosity of the fluid, $\rho$ is the density, and $u$ is the average velocity.
velocity. The results were supposed to be in good agreement with the Chilton and Colburn line at $x/d = 4$ and $Re = 10,000$. However, upon examination of their graphs, this does not appear to be the case. The data of Linton and Sherwood (L.2), which were for the dissolution of tubes, show too much scatter to calculate reliable fully-developed mass-transfer coefficients. In view of the lack of greater precision in these data, it is not possible to analyze the effect of $Sc$; the data could be represented equally well for a wide range of negative exponents of $x/d$ in equation (2.9). The trouble seems to lie in the method used to fabricate the tubes. They formed their tubes by melting the substance and casting it in a mould. The authors report that for benzoic acid, small fissures appeared and surface roughness developed while the dissolution process was in progress. Similar difficulties were encountered in the present investigation which led to an appreciation of the true condition of mass-transfer surfaces prepared by casting methods (section 4.6). However, Linton and Sherwood do not appear to be harassed by this problem in the case of tubes made of cinnamic acid and $\beta$-naphthol.

Meyerink and Friedlander (M.3) measured dissolution rates of a soluble pipe-wall in very short tubes from 0.87 diameters in length. They fabricated their mass-transfer surfaces by pressing powders in a mould. However, at the high Schmidt numbers used, the main effect at the entrance would already have died away after 0.87 diameters.
Shaw et al. (S.2) used a method which was a modification of the electrolytic technique first employed by Lin et al. (L.3). This is representative of a class of methods for measuring transfer rates in which the instantaneous as well as local mass-transfer rates may be determined.

The electrolytic method depends on the measurement of transfer rates of ions and other reacting species in diffusion-controlled electrode reactions. Ionic movements from the bulk of the solution to the electrode surface are followed by the associated physical and chemical changes. When conditions are such that physical change is the controlling step, a state of concentration polarisation is reached; a limiting current is then registered, the magnitude of which depends on the ionic-transfer rate at the surface of the test electrode. The influence of ionic migration is eliminated by the presence of an excess of an indifferent electrolyte, which does not react with the electrode and which has a high conductivity in comparison with the ions reacting at the electrodes. A sharp potential gradient is then set up near the electrode, and the migration term can be neglected in the expression for steady discharge of the particular ion since the conductivity is high.

The electrode reactions employed by Lin et al. were the reduction of ferricyanide ion and the oxidation of ferrocyanide ion and quinones. The reduction of the ferricyanide ion was found to be the most suitable reaction, since for this particular case
a wide range of voltages exists over which the current was mass transfer controlled. Furthermore, no side effects were noted.

Eisenberg et al. (E.6) used the same reaction as that of Lin et al. in their studies of mass transfer at rotating cylinders. Reiss and Hanratty (R.1) extended this technique in an investigation of fluctuations in the concentration gradient at the wall of a circular duct. The method has the advantage of avoiding disturbances of the flow in the very thin viscous sub-layer caused by techniques involving the use of anemometric or visual effects.

Measurements of fluctuations were obtained in this manner for the region \( y^+ < 5 \). The reduction of the ferricyanide ion took place between a small nickel electrode mounted flush with the pipe-wall, and a section of the tube which formed the larger electrode. The spatial correlations of flow close to the wall were determined by measuring the fluctuations in the mass-transfer rate simultaneously at two such electrodes.

Shaw et al. (8.2) determined local transfer rates to a pipe-wall by employing a modification of the method used by Reiss and Hanratty. The application of this method is limited to the liquid phase, which in turn restricts the range of Schmidt number over which it can be used. Nevertheless, this is a promising technique for the determination of local transfer rates; the technique scores a great advantage over profilometric methods since there are no changes in the geometry of the surface in the mass-transfer section and consequently the flow-pattern remains undisturbed.
undisturbed. The method consisted of diffusion-controlled electrodes which took the form of rings mounted flush with the pipe wall. Ten mass-transfer sections, in the shape of nickel discs of 0.0177 to 4.31 diameters long, were separated by lucite discs \( \frac{1}{4} \) in. thick. Tension bolts were used to hold the system, which was first reamed out to 1.063 in. I.D. and subsequently polished smooth, together. A section of the nickel pipe acted as the reference electrode of diameter many times larger than that of the transfer electrodes. Average mass-transfer data over part of the pipe circumference were obtained for a constant \( Sc \) of 2,400 and a range of \( 1,000 < Re < 75,000 \).

An inconsistency was reported in a recent paper by Shaw and Hanratty\(^{(8.3)} \) between their measurements of laminar and turbulent mass transfer to short transfer sections\(^{(8.2)} \). It was found that the difficulty was partially explained in terms of an error in the calibration of the orifice plates. When this error was corrected, the fully developed value of the new dimensionless mass-transfer coefficient, \( K^{+} \), was \( 3.52 \times 10^{-4} \) at a \( Sc = 2,400 \) instead of \( 3.01 \times 10^{-4} \). The results of Schütz\(^{(8.4)} \), for the same system, were in good agreement with the corrected data of Shaw et al., which were also confirmed by the experiment of Son\(^{(8.5)} \).

MacLean\(^{(8.4)} \) carried out a study of turbulent mass transfer in the entrance region of a pipe coated internally with diphenyl. He employed a profilometric technique to measure local mass-transfer coefficients, a technique which was based upon a principle similar/
similar to that employed in the present investigation (section 4.2). The design of both the measuring-apparatus and the transfer-section by MacLean were different to the ones used in the present investigation. The measuring apparatus was the same device as that used in earlier work by Campbell and Siiver (6.2). This consisted of a gauging-stand, which was essentially a support and location for the test-section. Provision was made for the positioning of a stool sharp-edged disc at various points along the tube axis. Gauging-air was drawn into the apparatus through the inlet which was situated at the base of the gauging-stand.

MacLean concluded that, for fully-developed turbulent flow, the entrance-length for the system, diphenyl/air, was \( 3\frac{1}{2} \) times the diameter of the tube. He also found a slight increase in local transfer-rates for increasing Reynolds number.

However, the transfer section, used in that investigation, was 5 diameters and the range in movement of tube and disc was only \( 3\frac{1}{2} \) diameters. The length of the tube was, then, too short to allow a true estimate to be obtained for the fully-developed Sherwood number. Furthermore, MacLean did not appreciate the considerable volatilisation of the transferring surface by the gauging-air during the profilometric operation (section 4.4). In addition, the design of the gauging-stand was such that the sharp-edged disc frequently damaged the diphenyl surface. It was assumed that the tube could be positioned so that the disc-support and tube were co-axial. Also no attempt was made to examine/
examine first the quality of the prepared transfer-surface. MacLean states in his work, that as many as one-third of his prepared surfaces were damaged this way. However, it was not possible to detect the less catastrophic contacts between disc and surface, which would consequently mask the correct interpretation of the experimental data.

In view of what was discussed above, there is then reason to question the validity of the conclusions drawn in that work. This topic will be referred to again in sections (5.2 and 5.4).

A list is presented in Table IV of mass-transfer investigations in the inlet region of circular ducts. This Table supplements information on the topic discussed in this section.
FIG. 1 - TEMPERATURE DISTRIBUTION IN A TURBULENT BOUNDARY LAYER FOR A FLUID FLOWING OVER A HEATED PLATE.
2.2. **THE NUSSELT NUMBER AND THE SHERWOOD NUMBER.**

2.2.1. **General Concepts.**

The meaning of the Nusselt number, \( \text{Nu} \), can be illustrated from a deduction by Martinelli which has been slightly modified by Jakob (J.1). The Nusselt number is useful in the reduction of data of free and forced-convective heat loss, either from the same body in different conditions or from different bodies of geometrically similar shapes. For the mass transfer case, the dimensionless Sherwood number is used, and the arguments used for the Nusselt number will also apply in this case.

A rough sketch can be made, from qualitative observations, of the temperature profile for heat transfer from a solid wall to a turbulent fluid. Immediately next to the wall the fluid particles are predominantly stationary relative to the boundary and heat is transferred by conduction only. A large temperature gradient is thus established in this layer. At greater distances from the wall the transfer of heat is enhanced by the movement of the fluid. The temperature gradient consequently becomes less steep until it finally levels out in the main stream. Figure 1 illustrates this qualitative argument for air flowing turbulently over a flat plate.

The rate of heat transfer from a solid wall to a fluid can then be evaluated by appealing to the above discussion. At the interface, where \( y = 0 \), heat flows by conduction only and the relevant equation to this situation is

\[
q_{\text{wall}} \rightarrow \text{fluid} = -k \frac{\partial T}{\partial y}
\]

\[y=0\]  \( \cdots \cdots \cdots (2.10) \]

where/
where \( q_{\text{wall \to fluid}} \) is the rate of heat transfer at a solid boundary and \( A \) is the area of the surface which is in contact with the fluid. For practical purposes it is found more profitable to employ the concept of the convective heat-transfer coefficient, \( h \). This is defined by equation

\[
q = hA \Delta t \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.11)
\]

The heat-transfer coefficient can then be related to the temperature gradient at the wall by equating the expressions (2.10) and (2.11).

\[
q_{\text{wall \to fluid}} = -kA \left( \frac{\partial t}{\partial y} \right)_{y=0} = hA(t_o - t) \quad \ldots (2.12)
\]

where \( t_o \) is the wall temperature and \( t_{\infty} \) is the free-stream temperature.

Also, \( \partial t \) can be written

\[
\partial t = \partial(t - t_o) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.13)
\]

since, irrespective of the choice of the reference temperature, the magnitude of the temperature gradient in the fluid will be the same.

Equation (2.13) can be expressed in dimensionless form after introducing a length dimension of the system \( L \) to specify the geometry of the object. Equation (2.13) is then written as

\[
\frac{hL}{k} = \frac{t_o - t_{\infty}}{L} = \left[ \frac{a(t_o - t)}{t_o - t_{\infty}} \right] \left[ \frac{y}{L} \right] \left( \frac{\partial}{\partial y} \right)_{y=0} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.14)
\]

The/
The combination of the convective heat-transfer coefficient $h$, the significant length-dimension $L$ and the thermal conductivity of the fluid $k$ in the form $hL/k$ is the Nusselt modulus or Nusselt number, and is a dimensionless quantity.

For a circular tube, the significant length-dimension is the inside diameter $d$. The relevant equation for this situation is

$$\text{Nu} = \frac{hd}{k} = \left[ \frac{(t_o - t)}{\frac{y}{d} (t_o - t_\infty)} \right] \left. \right|_{y=0}$$

Equation (2.15) can be written as

$$h = \text{Nu} \frac{k}{d} \quad \text{.......................... (2.16)}$$

and $(t_o - t)/(t_o - t_\infty)$ is the dimensionless temperature $t^+$. From this expression it can be seen that once the Nusselt number is known the convective heat-transfer coefficient can be evaluated.

In practice, therefore, the Nusselt number is a convenient measure of the convective heat-transfer coefficient. By similar arguments, the Sherwood number, $Sh$, is a convenient measure of the convective mass-transfer coefficient.

2.2.2. A Graphical Interpretation of the Nusselt number.

Inspection of equation (2.15) shows that the first term, $hd/k$, is a Nusselt number, and the last term is equal to $\tan \phi$ in figure 2. The Nusselt number can then be considered as the ratio

$$\text{Nu} = \frac{\text{temperature gradient at wall}}{\text{temperature gradient across the fluid in pipe}}$$

The/
FIG. 3 - GRAPHIC INTERPRETATION OF THE CHANGE OF THE NUSSELT NUMBER IN THE FLOW OF A FLUID PARALLEL TO A PLANE SURFACE.
The negative sign is of no significance in this interpretation since it arises from the definition of the heat-flow direction.

The following discussion by Jakob describes the manner in which the Nusselt number varies along the tube. Consider a fluid flowing turbulently in the x-direction parallel to the surface of a flat plate. The temperature distribution in the y-direction and the temperature gradient are related by the equation

\[
\left( \frac{\partial t}{\partial y} \right)_{y=0} = \left[ \frac{t_0 - t_\infty}{L} \right] \tan \phi \quad \ldots \ldots \ldots \quad (2.17)
\]

which is shown graphically in figure 2. The Nusselt number then starts with an infinite value at \( x=0 \) and decreases as \( x \) increases. This is shown in figure 3.

The general argument discussed above has been extended by Mills (1) to the special case of heat transfer with the specified wall-boundary conditions of Uniform Wall Heat Flux (U.W.H.F.) and Uniform Wall Temperature (U.W.T.).

2.2.3. The Nusselt Number and the Boundary Layer.

The solid line in figure 1 shows the temperature distribution for a fluid moving past a hot wall. The temperature gradient in the fluid is confined to a thin layer, \( \lambda \), close to the wall. This picture is simplified if the actual temperature distribution is replaced by the dashed line shown in figure 1, which is in effect the tangent to the actual temperature-curve at the wall. Physically, this can be interpreted as the temperature distribution in a hypothetical stagnant-layer of fluid of thickness \( \lambda' \) in which the/
the same resistance to the flow of heat is offered as in the boundary-layer.

The rate of heat transfer per unit area in this layer is given by

\[ \frac{Q}{A} = k \frac{t_\infty - t_0}{\lambda} = h(t_\infty - t_0) \]  \hspace{1cm} (2.18)

since heat can flow only by conduction in this stagnant film. It can be seen from equation (2.18) that the Nusselt number, \( \text{Nu} \), and the heat transfer coefficient \( h \) can be expressed as

\[ h = \frac{k}{\lambda} \]  \hspace{1cm} (2.19)

\[ \text{Nu} = h \frac{L}{k} = \frac{L}{\lambda} \]  \hspace{1cm} (2.20)

For the case of the circular duct, the significant length dimension \( L \) is the diameter \( d \). The Nusselt number can now be written as

\[ \text{Nu} = \frac{d}{\lambda} \]  \hspace{1cm} (2.21)

From this result it is seen that the convective conductance is greatest the thinner the hypothetical boundary-layer.

2.2.4. The Ratios \( \text{Nu}/\text{Nu}_\infty \), \( \text{Sh}/\text{Sh}_\infty \) and the Treatment of Experimental Results.

In entry-region studies of heat and mass transfer on flat plates or circular pipes, the data is usually presented in graphs of \( \text{Nu}/\text{Nu}_\infty \) vs \( x/d \) and \( \text{Sh}/\text{Sh}_\infty \) vs \( x/d \) respectively. This is a particularly convenient way of interpreting the behaviour of the local heat-transfer coefficient \( h \), or the local mass-transfer coefficient \( k_c \) in entrance-region investigations.
The ratio $\frac{Sh}{Sh_\infty}$ for constant-fluid properties and uniform wall-concentration may be expressed in the form shown by equation (2.22), (also, Appendix VIII, equation (A30))

$$\frac{Sh}{Sh_\infty} = \frac{w}{w_\infty} \quad \ldots \ldots \ldots \ldots \quad (2.22)$$

where $w =$ the value of the local recession measured at the tube wall.

$w_\infty =$ the mean value of the local recession corresponding to steady conditions.

Equation (2.22) implies that if the recessions can be measured accurately along the whole length of a volatile tube wall, it is then a simple matter to obtain the values of the ratio $\frac{Sh}{Sh_\infty}$. In fact, the final graphs are simply a plot of the measured $\frac{w}{w_\infty}$ vs $x/d$.

2.2.5. Methods of Correlation for Entrance-Region Transfer Data

For practical purposes of design, it is often more useful to use mean rather than local Nusselt numbers. An example of this is in heat-exchanger analysis where it is more convenient to use mean heat-conducances with respect to the tube length, and hence mean Nusselt numbers. Mean transfer-rates are generally of greater interest in practical applications with Uniform Wall Temperature condition.

The mean Nusselt number is defined by equation

$$Nu_m = \frac{x/d}{\int_0^{x/d} \frac{d(x/d)}{Nu}} \quad \ldots \ldots \ldots \ldots \quad (2.23)$$

which is consistent with equation
\[ (t_o - t_b)_{av} = \frac{\int_0^x (t_o - t_b) \, dx}{x} \] ........................ (2.24)

This defines the average temperature in a fluid between that of the bulk, \( t_b \), and at the wall, \( t_o \), for the Uniform Heat Flux distribution. An alternative and more convenient manner of writing equation (2.23) is in terms of the dimensionless distance \( x^+ (\equiv x/d) \),

\[ \text{Nu}_m = \left(\frac{1}{x^+}\right)^{-1} \int_0^{x^+} \text{Nu} \, dx^+ \] ........................ (2.25)

The most usual method of correlating experimental data in the entrance-region of circular ducts has been to multiply the fully-developed Nusselt number, \( \text{Nu}_\infty \), by a factor which is a function of the dimensionless distance, \( x/d \). For \( \sigma = 1 \), a simple factor such as \( (x/d)^{-c} \) is often used, and the equation then is written in terms of the average Nusselt number,

\[ \text{Nu}_{av} = \text{Nu}_\infty \cdot \left(\frac{x}{d}\right)^{-c} \] ........................ (2.26)

Substituting the general correlation for \( \text{Nu}_\infty \) in equation (2.26),

\[ \text{Nu}_{av} = b \cdot \text{Re}^m \cdot \sigma^n \cdot (x/d)^{-c} \] ...... (2.27)

where \( m, n \) and \( c \) are all positive exponents. The constant \( b \) was given the value of 0.023 by McAdams \( (M.20) \); Dittus-Boelter \( (D.17) \) chose the value of 0.0243 (section 2.1). For large distances downstream the tube, \( (x/d)^{-c} \) approaches unity, and equation (2.27) then approaches the value of \( \text{Nu}_\infty \).

An illustration, where the general form of equation (2.27) has/
has been used to correlate data obtained in heat-transfer studies is that of Nusselt (N.1) who suggested the use of equation

\[ \text{Nu}_m = 0.036 \, \text{Re}^{0.8} \, \text{Pr}^{1/3} \, (x/d)^{-0.054} \quad \cdots \cdots \cdots (2.28) \]

for the range 10 < x/d < 400. This equation has been employed extensively in heat-transfer studies in the entrance-region of pipes. Another example is the correlation used by Cholette (C.1) to represent his heat-transfer data, in which the value of the exponent c = 0.1. Humble et al. (H.4) investigated various entrance-configurations, tube-wall temperature distributions and tube-lengths for air at high temperatures flowing in a smooth circular tube. The empirical relationship they employed was

\[ \text{Nu}_m = 0.034 \, \text{Re}^{0.8} \, \text{Pr}^{0.4} (x/d)^{-0.1} \quad \cdots \cdots \cdots (2.29) \]

subject to the following conditions: (a) Fluid properties were evaluated at film temperature, \( t_f \), defined by the equation

\[ t_f = \frac{1}{2} (t_o + t_b) \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (2.30) \]

(b) Equation (2.29) holds for the range:

\[ 30 < x/d < 120, \, 10,000 < \text{Re} < 500,000 \text{ and } 600^\circ R < t < 3050^\circ R. \]

Weiland and Lowdormilk (N.5) later lent support to the validity of equation (2.29). The choice of the exponent c = 0.1 in equation (2.29) finds additional support from the analytical results of Deissler (D.10) for heat-transfer coefficient in circular tubes.

For mass transfer, an expression was developed by Linton and Sherwood (L.2). This was used to correlate their data for mass transfer in the entrance-region of a circular duct.
\[ J_D = 0.232 \, \text{Sc}^{-0.4} \, (x/d)^{-1/3} \] ........................ (2.31)

where \( J_D \) is the j-factor for mass transfer defined by the equation

\[ J_D = St \cdot \text{Sc}^{2/3} = \frac{f}{2} \] ........................ (2.32)

\( St \) is the Stanton number, \( f \) is the friction factor. However, this correlation is considered to be only a very rough approximation, owing to the low accuracy of the mass-transfer data of that work. This work was discussed in some detail in section 2.1.2, and equation (2.32) is similar to equation (2.9).

This method of correlating entrance-length transfer data is shown to be inadequate for high \( \sigma \) by both theory and experiment. Analysis of theoretical results shows that at the entrance the separation of the curves for local Nusselt numbers increases with \( \sigma \). This means that the effect of the Reynolds number on entrance-region Nusselt number increases with \( \sigma \). The same is true for \( \text{Nu}_{av} \), the average Nusselt number with the difference that the change of \( \text{Nu}_{av} \) with \( x/d \) is more gradual; also, the separation of the curves is greater than that obtained for the local Nusselt numbers.

The correlation expressed by equation (2.27) is, then, an over-simplification of the central problem, namely of devising a better equation for representing transfer data in the inlet region of circular tubes. This practice has resulted from the fact that the majority of the investigations had been conducted for constant \( \sigma \). It is thus necessary to include additional functions, such as those of \( \sigma \) and the Reynolds number rather than/
than \( x/d \) alone. The data for heat or mass transfer in the entrance-region would then be represented by an equation of the typo

\[
\frac{\text{Nu}}{\text{Nu}_\infty} = \frac{\text{Sh}}{\text{Sh}_\infty} = \phi (\sigma, \text{Re}, x/d) \quad \ldots \ldots \quad (2.33)
\]

The functional relationship of equation (2.33) may be evaluated by analysis of reliable entrance-region data. The additional function of \( \text{Re} \) may be included only if very accurate data are available. This is so because the influence exerted by the \( \text{Re} \) on the developing thermal or concentration boundary-layers is very much smaller than that obtained for the \( \text{Sc} \). The separation of the curves obtained for varying the \( \text{Re} \) is consequently very small. Investigations of high precision and accuracy are then necessary to determine the true heat or mass-transfer coefficients at the entrance-region of the pipe.
2.3. **SURVEY OF EXPRESSIONS FOR THE TURBULENT TRANSPORT PROPERTIES.**

It is important to evaluate the eddy diffusivities in the region very close to the wall because the turbulent exchange of momentum, mass and heat in the vicinity of a boundary is encountered in many engineering processes. Of these, transport near a solid boundary presents the greatest interest.

A review of the various attempts to describe the eddy diffusion near the wall, and comparison of these with experimental data, will be presented in this section. A number of expressions for eddy diffusivity near the wall have been proposed and are presented in Table VI.

The Table is intended to summarise comprehensively the considerable body of literature, describing the many expressions and evaluations of these, that have appeared at various intervals. Such readily accessible information may facilitate more sound appreciation of the work done to date for those who wish to gain a deeper understanding of turbulent transfer processes.

The notes in the Table describe the salient features in the derivation of the eddy-diffusivity expressions. The inadequacy of most of the analyses is mainly due to the choice of these expressions. The extent to which the predicted results are applicable is obtained by direct comparison with the accumulated experimental data. This, together with the historical tabulation, may also provide an insight into the evolution from earlier and simpler considerations to the more complex contemporary analogies between momentum, heat and mass. These three transfer processes depend/
depend both on molecular properties and eddy motion in a turbulent stream. The three separate areas of momentum, heat and mass transfer are in fact so closely related as to allow them to be considered as one single and unified subject. However fluid flow has been studied more extensively than heat or mass transfer. This is the underlying reason why the quantitative results between the three transfer processes are described in the form of "analogies" between heat and momentum transfer and mass-momentum transfer. Reference will be made to these analogies and their limitation since the equations for the eddy diffusion are constructed on some form of the analogy for these three transfer processes.

Such a development has become particularly necessary at this stage because of the proliferation of literature surrounding this topic, together with the unwieldy experimental data collected to date. In the words of Sherwood, T.K. (3.6) the derivation of equations of the type shown in the Table "appear to be meeting the law of diminishing returns, and further worthwhile developments must await fuller understanding of the basic nature of turbulent flow."

The expressions used for the transport properties are basically empirical and developed to fit available data. Equations relating Nusselt numbers and turbulent transport properties require expressions for the turbulent viscosity $\nu_t$ as a function of flow conditions and of position. It is at this very point that the greater uncertainty in the estimation of the fully developed or asymptotic Nusselt number, $\text{Nu}_\infty$, is introduced.

No/
No attempt will be made in this section to repeat or summarise the already voluminous and easily accessible literature concerning the analogy between momentum, heat and mass transfer. In addition to the excellent review by Sherwood, another good discussion on this topic is given by Kestin and Richardson under the heading of "The Law of the Wall". The authors also discuss the problem of the nature of the boundary layer as regards the controversy that surrounds the different models. The extent of turbulence found very near the wall, where $y^+ < 1$, is also discussed.

The majority of expressions used to estimate $\tau_0/\rho$ are based on the assumption of a von Kármán "universal" velocity profile of the general form.

\[ u^+ = u^+_o(y^+) \quad \ldots \ldots \quad (2.34) \]

where $u^+ = \text{the dimensionless velocity, } u/\sqrt{\tau_0/\rho}$

$y^+ = \text{the dimensionless distance, } (r_o - r) \sqrt{\tau_0/\rho \over \nu}$

$u = \text{time-averaged velocity in x-direction.}$

$r_o = \text{the tube radius}$

$r = \text{the radial distance}$

$\rho = \text{the density of the fluid}$

$\tau_0 = \text{the shear stress at the wall}$

$\nu = \text{the kinematic viscosity.}$

Equation (2.34) is derived from dimensional considerations when the acceptable assumption is made that the time-averaged velocity, $u$, is
is a function only of \( \mu, \rho, \tau_0 \) and \((r_0 - r)\). An ingenious and elegant mathematical derivation of equation (2.34) is given by Townsend (T.1). Correlations of velocity-profile data given by equation (2.34) for parallel plates, \( \text{Re} > 15,000 \), and thin annuli for \( \text{Re} > 25,000 \) (W.2, W.3), for pipe-flow are in fairly good agreement.

For laminar flow, as well as for turbulent flow

\[
\lim_{y \to 0} (u^+) = y^+ \quad \ldots \ldots \ldots (2.35)
\]

However, it is known that \( u^+ \) is not a unique function of \( y^+ \) near the wall (0.1). Furthermore, there is not much reliable data in the region most important for heat and mass transfer, namely \( y^+ < 1 \).

Correlations typical of this form of solution are those of Lin et al. (L.4) and of Deissler (D.3, D.4). The latter one has recently received support by a theoretical development by Spalding (S.7, S.8). However, the validity of the equation proposed by Spalding, as will be seen later, is increasingly believed to be doubtful. Vieth et al. (V.1) propose an empirical correlation which is chosen to agree with the Chilton-Colburn analogy (C.11). Chilton and Colburn appear to be the first to stress the analogy between momentum, heat and mass transfer. Their analogy is represented by

\[
St_h (Pr)^{2/3} = St_m (Sc)^{2/3} = CPr/2 \quad \ldots \ldots (2.36)
\]

where \( St_h \) and \( St_m \) are the Stanton numbers for heat and mass transfer respectively, \( Pr \) is the Prandtl number, \( Sc \) is the Schmidt number and/
and $C_p$ is the average skin coefficient. This analogy was based on an empirical representation of the influence of the Prandtl number on turbulent heat transfer in fluids. Equation (2.36) is found to hold well for many kinds of flow-systems, when $C_p$ is calculated on a basis of shear friction, not on total drag. The expression proposed by Veith et al. is unique in that a $y^{++}$ is employed instead of $y^+$, and $y^{++}$ is defined as

$$y^{++} = \text{dimensionless} = y^+\sqrt{\frac{f/2}{u_{\text{max}}} u} \quad \text{(2.37)}$$

where $u_{\text{max}}$ = mass average maximum velocity

$\bar{u}$ = average value of $u$ over a flow cross-section, (L/t)

$L$ = length of tube

$t$ = time

$f$ = friction factor, dimensionless

$y$ = rectangular coordinate

Equation (2.37) contains the implication that a profile of the general form (2.34) is not assumed.

A characteristic which is common in the analyses mentioned above is that they are found to give limiting expressions of the form

$$\epsilon u \propto (r_o - r)^n = y^n \quad (r \rightarrow r_o) \quad \text{(2.38)}$$

for the region near the wall, i.e. where $0 < y^+ < 5$. The exponent $n$ in equation (2.27) has been shown to be equal to 3 in arguments put forward by two authors, Townsend (T.1) and Elrod (E.8). In an earlier analysis, Elrod (E.7) showed that the Reynolds stresses $u^ov^o$ at the wall could not be proportional to less than the fourth power of/
of \( y \). He concluded from this result that \( n \) could not be less than four since the same result must hold for \( \epsilon \) as \( du/dy \) approaches a constant at the wall. This argument gave support to Disselor's result where \( n = 4 \), contrary to that found by others who report \( n = 3 \). In a more recent paper, however, Elrod (E.8) corrected his earlier analysis and now states that \( n \) could not be less than three, which agrees with the result previously shown by Reichardt (R.2).

Values of \( n = 3 \) have been used by Friend and Metzner (F.1) and Lin et al. (L.4): Reichardt assumed \( n = 3 \) for moderate Prandtl numbers and \( n = 5 \) for higher Prandtl numbers. Friend and Metzner obtained an empirical correlation, not shown in Table VI, for very viscous fluids; and for high Schmidt numbers, this reduces to

\[
k_0 = 0.0847 \sqrt{\frac{x}{2}}, \quad Sc^{-2/3}
\]

Lin et al. (L.4) measured concentration profiles and concluded that their shapes were in good agreement with those predicted by their semi-empirical analysis. However, an equally good fit would be obtained if either a second or fourth-order equation was used.

The arguments on whether \( n = 3 \) or 4 continue and a recent paper by Son and Hanratty (S.9) reports that analysis of available mass-transfer data in a pipe and in an annulus give better interpretation of results with \( n = 4 \). The authors remark however that mass-transfer data of a greater degree of accuracy is needed before a definite conclusion can be drawn. A way to determine the difference in the equations would be to carry out experiments at very high \( Sc^0 \)'s. The difficulty is that a difference of 1.45-fold, equivalent/
equivalent to a difference of \( Sc = 0.80 \), would be expected for a hundred-fold range of \( Sc^0 \)’s.

The equation of both Lin et al. (L.4) and Deissler fit the data for low and high Schmidt numbers in spite of the fact that the exponents of \( y^+ \) in these equations are different. The reason for this is that, for low \( Sc^0 \)’s, the eddy diffusion has little influence on the transfer rates; in the case of high \( Sc^0 \)’s (1,000), the concentration gradient exists in the viscous sub-layer and the numerical constants can be adjusted to fit the data. The final expression of Lin et al. for mass transfer in pipes is given as

\[
J_D = \frac{k_c}{U} \frac{Sc^{2/3}}{\sqrt{f/2}} = \frac{k_c}{U} \left( Sc, \sqrt{f/2} \right) \quad \text{(2.40)}
\]

where \( J_D \) = the \( j \)-factor, or mass transfer factor
\( k_c \) = mass transfer coefficient, \( \text{cm sec}^{-1} \),
\( U \) = average velocity, \( \text{cm sec}^{-1} \),

for high \( Sc^0 \)’s, the equation becomes

\[
J_D = 0.057 \sqrt{f/2} \quad \text{(2.41)}
\]

This means that the transfer rate is proportional to the \( 2/3 \) exponent of the molecular diffusivity, \( D^{2/3} \); it is interesting to note that the same dependency is found in boundary-layer problems.

Deissler’s expression for pipes at high \( Sc^0 \)’s reduces to the simple equation

\[
J_D = \frac{2n}{\pi} \frac{\sqrt{f}}{Sc^{1/12}} \quad \text{(2.42)}
\]

It must be remembered that Deissler, Lin et al. and Friend and Metzner, all employed the data of Linton and Sherwood (L.2) in support/
support of their conclusions. Difficulties met with in interpreting correctly this data is discussed in section (5.2(f)).

In all analyses the range $y^+ > 30$ is assumed to be well into the turbulent flow-region where transport is overwhelmingly dependent on the role of the eddy diffusivity. The influence of the molecular diffusivity in this region is negligible, and the velocity profile is of the general form:

$$u^+ = A + B \ln y^+ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.43)$$

There is then a need to find a separate expression to describe the range $5 < y^+ < 30$ and a large number of theories of a speculative kind have been devoted to this topic. Some of the expressions that have been proposed for the velocity are those by Reichardt (R.2), Deissler (D.3), Einstein and Li (E.9) and von Driest (V.2). These are shown in Table VI. Einstein and Li proposed a theory about the nature of the flow in the vicinity of the wall. In this they postulated a laminar layer which grows with time until it becomes unstable and breaks away; the cycle is then repeated. Another theory for the same region was advanced by Kline et al. (K.2) in which regions of laminar flow, called "islands of hesitation" are separated by regions containing turbulent vortices. Other contributions are those of Rotta (R.3), Hama (H.2), Squire (S.10), and Miles (M.5). They are all semi-analytical results and tend to be very complex. Numerical values calculated from these expressions show very little difference between them. Empirical expressions have been proposed by von Kármán (V.3), Dunn/
Dunn (D.5), Schlinger (S.11), and Sleicher and Tribus (S.12). These are shown in Table VI.

Experimental data for the region $5 < y^+ < 30$ has been obtained for gases, i.e. low Prandtl numbers, by Laugher (L.5), Sleicher (S.13), Reichardt (R.4), Deissler (D.6) and Schigomitsu (S.14). The results are not of sufficient accuracy to allow a choice between the expressions for the velocity profile, but tend to lend support to Sleicher's expression. Recently, Nodderman (N.2) employed stereoscopic photography to measure velocities in the wall region, $1 < y^+ < 30$ of turbulent liquid pipe-flow. This investigation was the first attempt to measure velocities in the wall region for liquids and the results showed that the non-dimensional velocities could not be distinguished from those measured in air by other investigations. Nodderman concluded from his results that the velocity profile of Sleicher represented his data best.

The region far from the wall, $y^+ > 30$, is generally described by velocity profiles of the logarithmic form shown in equation (2.43). Equations of this kind employing different assumptions have been derived by Prandtl (P.1), von Kármán (V.4), Millikan (M.6) and Hinsel (H.3). The velocity profiles described by these expressions have been confirmed by Nikuradse (N.3), Reichardt (R.4) and Laugher (L.5) for both gases and liquids. The numerical values of the constants A and B in equation (2.43) are different for each author, but the accepted values are 5.5 and 2.5 respectively.

All/
All except the complex expressions of Reichardt and van Driest, however, are composed of two expressions which are valid for different ranges of $y^+$. Spalding (8.7) has recently proposed a new single formula but its validity is doubtful since $\epsilon^+\alpha(y^+)^4$ and therefore does not comply with the Townsend criterion (T.1) that $\epsilon^+\alpha(y^+)^3$. Furthermore, the constants $A$ and $B$ in Spalding's expression have not been determined accurately as yet. Their current empirical values are 0.1108 and 0.4 respectively.

Wasan et al. (W.4) have recently derived theoretical expressions for the variation of velocity and eddy viscosity in the wall-region of pipe-flow by using the equation of mean motion and the well-established logarithmic distribution in the turbulent core. Their correlations fit the experimental data on velocity and turbulent shear stress over the wall-region. The proposed eddy-viscosity distribution satisfies the theoretical criterion by Townsend which states that the contribution to Reynolds stress $u'v'$ near the wall is proportional to $y^n$, where $n \neq 3$. It also has the advantage of doing away with the artificial, but in some ways useful, concept of three-sharply-defined fluid-layers, on which most of the analyses shown in Table VI are based. Exceptions are the correlations of Rammie (R.7) and Sleicher which are based on the concept of two-fluid-layers, and those of Reichardt, van Driest and Spalding which are based on the assumptions of one continuous fluid-region.

According to Goweriker (G.1) and Wasan et al. (W.4), this concept of three-fluid-layers, namely the laminar, buffer and turbulent/
turbulent layers, leads to an unrealistic discontinuity in the value of the eddy-viscosity function with respect to that obtained from the logarithmic distribution in the turbulent core. Rannie and Sloicher in their analyses avoided this discontinuity. The eddy-viscosity functions of both of these authors, however, do not give satisfactory relationships for analogy expressions for heat and mass-transfer rates for systems of high Prandtl or Schmidt numbers.

In determining temperature or concentration distributions, it has been a common practice to assume that the eddy diffusivities for heat, mass and eddy viscosity are all equal. In other words this means that the turbulent Prandtl number, $Pr_t = \frac{\varepsilon_y}{\varepsilon_H}$ and turbulent Schmidt number $Sc_t = \frac{\varepsilon_y}{\varepsilon_M}$ are all equal,

$$Pr_t = Sc_t = 1, \ldots, \ldots, \ldots (2.44)$$

where $\varepsilon_y$ is the eddy diffusivity of momentum, $\varepsilon_H$ is the eddy diffusivity of heat and $\varepsilon_M$ is the eddy diffusivity of mass. The reason for this assumption is that eddy diffusivities for heat and mass have not been investigated to the same extent as the eddy viscosity. The relation for $\alpha = \frac{\varepsilon_H}{\varepsilon_y}$ has not been clearly established although analyses based on $\alpha = 1$ agree closely with experiment (V.3), (D.7). The conditions under which this assumption is valid has been discussed recently by Bourne (B.1), Tien (T.2) and Levy (L.6).

Some attempts to measure $\alpha$ directly show that $\alpha > 1$ (C.3), (S.15), (C.4), (S.13) except in the case of low Péclet or Prandtl numbers where values of $\alpha < 1$ may occur (D.7), (I.2), (D.8). Support for $\alpha > 1$
\( \alpha > 1 \) is also reported with some qualification by Townsend (T.1) for the decay of turbulence behind a heated grid. Reichardt (R.10) proposed the hypothesis that \( \alpha = 1 \) at the wall and increases as the distance from the wall increases. The direct measurement of \( \alpha \) is difficult and this difficulty is much greater in the region close to the wall since it involves the measurement of velocity and temperature gradients. For turbulent flow the important changes of velocity and temperature take place close to the wall for \( \text{Pr} > 1 \), so that the assumption of \( \alpha = 1 \), in general, yields good results. An interesting point is that the Prandtl mixing-length theory (P.1) gives a value of \( \alpha = 1 \). This theory assumes that an eddy travels a given distance and then suddenly mixes with the fluid transferring its momentum and heat.

The reason why \( \alpha \) differs in general from unity will now be discussed.

The total diffusivity term in the energy equation is expressed by the equation

\[
\frac{\varepsilon_H}{v} + \frac{1}{\text{Pr}} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots (2.45)
\]

In the laminar sub-layer, \( \varepsilon_H/v \) becomes zero and the same assumption is made for the buffer-layer.

(a) If \( \text{Pr} < 1 \), it is seen in section 3.2 that \( \varepsilon_H/v \) becomes very small and the only term left is \( 1/\text{Pr} \), which becomes important in the region close to the wall. In this case, it is then possible to tolerate gross assumptions as to the true order of magnitude of the eddy diffusivity.

The/
FIG. 4—EFFECT OF PRANDTL NUMBER AND TURBULENT INTENSITY ON THE RATIO OF EDDY DIFFUSIVITIES

\[
\frac{\epsilon_H}{\epsilon_M} = \frac{\epsilon_M}{\epsilon_H} = \infty
\]

\[
\frac{\epsilon_M}{\epsilon_H} = 1.0, 10, 100
\]

\[\text{[4.9]}\]

\[\text{Pr}\]

\[0.03, 0.1, 1, 10\]
The heat and momentum analogy based on the simple Reynolds model would be expected to break down for \( \text{Pr} < 1 \). This is because a turbulent heat-transfer theory based on this model assumes that for \( \text{Pr} < 1 \), the conduction term is negligible in the turbulent core. However, it is this term which becomes important in this region at very low Prandtl numbers. Martinelli (M.7) extended to low \( \text{Pr} \)'s the closed-form solution for turbulent flow in pipes based on this model, by including the conduction term. He then carried out the integrations numerically. Nusselt numbers calculated in this manner were found to overpredict experimental data obtained for liquid metals. This result implies that a model such as the Reynolds analogy offers a very simplified base on which to construct a turbulent heat-transfer model.

Jenkins (J.2) has presented the first and one of the most satisfactory analysis on the effect of \( \text{Pr} \) on \( \frac{c_H}{c_U} \); he was the first to propose that the heat conduction to or from an element of an eddy during its flight to the main flow should be considered, as well as when it arrives at its new position. Jenkins treated the eddy as a solid sphere which starts with a uniform temperature, and made calculations by using an existing solution for the transient cooling of a sphere to determine the energy lost during its time of flight to the new position. Figure 4 shows the results which are put in the form of an eddy diffusivity. Lykoudis and Touloukian (L.9) developed an analysis relating \( c \) and \( \text{Pr} \) by following almost the same reasoning as that of Jenkins and the results are shown in figure 4.
Recent experimental data for liquid metals suggest that the loss of heat by an eddy during flight is over-predicted by Jenkins. In general, however, experiments are in fair agreement with calculations made at low \( \text{Pr} \) s using the information on figure 4 for the relationship of \( \epsilon_H \) and \( \epsilon_M \). A number of other theories exist for low Prandtl number, turbulent flow heat transfer, but they are all based on the same general concept, with similar results.

For air, \( \text{Pr} = 0.7 \), direct measurement of the eddy diffusivity for both momentum and heat transfer suggest that \( \epsilon_H < \epsilon_M \); measurements of this kind are those made by Page et al. The opposite effect is given if Jenkins' result is used. To bring the analytical results in agreement with measurements for \( \text{Pr} = 0.7 \), authors like Sleicher and Tribus use Jenkins' diffusivity ratio multiplied by 1.10 or 1.20. A discussion of Jenkins' model is given by Tien who concludes that it does not satisfy Prandtl numbers of the order of 1 or higher. For very low \( \text{Pr} \)'s he was unable to draw any conclusion.

(b) If \( \text{Pr} > 1 \), examination of equation (2.45) shows that the eddy diffusivity term now becomes very important. This is true for distances very close to the wall (i.e. \( y^+ < 5 \)); hence, even the slightest departure from the casual assumption about the eddy diffusivity near the wall will assume even a much greater importance. A different model as regards the sub-layers is required, since very little is to be gained from any new assumptions on/
on the behaviour of the fluid main stream. This is because as seen in section 3.2, for high Pr's and Sc's the coat of the thermal (or concentration) resistance becomes increasingly confined in the viscous layer adjacent to the wall - the assumed laminar sub-layer.

The concept of a laminar sub-layer was abandoned by Deissler (D.6) who allowed for turbulence right to the wall. In other words, the eddy diffusivity does not go to zero at \( y^+ = 5 \) but approaches zero as \( y^+ \to 0 \). Using dimensional analysis, Deissler suggested the equation

\[
\frac{\varepsilon_H}{v} = n^2 u^+ y^+ \quad \cdots \quad (2.46)
\]

to describe the diffusivity in the sub-layer. The value of \( n^2 \) was determined from data and was given as 0.0119. The major advantage of this expression is in that it succeeds in combining the two sub-layers into a single physical region; however, little change is obtained in heat-transfer results when equation (2.46) is used to solve the energy equation. The diffusivity expression for the sub-layer was later modified by Deissler (D.3) to

\[
\frac{\varepsilon_H}{v} = n^2 u^+ y^+ [1 - \exp(-n^2 u^+ y^+)] \quad \cdots \quad (2.47)
\]

where \( n^2 \) has now the value of 0.0154.

Equation (2.46) yields results which are found to be in good agreement with experimental data for high Pr's. This agreement is obtained in spite of the fact that the derivation of equation (2.47) is based on the assumption that \( \varepsilon_H = \varepsilon_U \). Nusselt number results are obtained which are very much higher than the experimental data when/
when the Jenkins correction is introduced. One explanation may be that equation (2.47) gives a true description of the $\epsilon_H$ in the sub-layers, where $\epsilon_U$ is not known accurately. This is because it is not possible to investigate experimentally $\epsilon_U$ in the sub-layers in the same way as for $\epsilon_H$ in heat transfer at high $Pr^0$'s. For this situation, the molecular diffusion can be suppressed effectively experimentally.

Van Driest (V.2) proposed an equation which was based on the concept that oscillations are damped in the vicinity of a wall by viscosity. This was in fact an attempt by Van Driest to give a physical basis for equation (2.46) of Doissler. It is interesting to note that, although the final expressions of Doissler and van Driest are very different, yet they both yield very similar results for the region near the wall.

In view of what has been discussed above, it is not then strictly correct to state that the assumption of $Pr_t = 1$ gives results which agree with experiment. A statement of this nature is found in a paper by Doissler (D.3) and the misunderstanding is propagated by authors who refer to this paper. For high $Pr^0$'s the eddy diffusivity for heat has never been measured directly. Doissler proposed in his analysis on fluids of high $Pr^0$'s an empirical form of $\epsilon_H$ as a function of $y^+$. As $y^+ \to 0$, this reduces to

$$\epsilon_H = Ay^{+n}.$$

There are two adjustable constants, $A$ and $n$. The values of these constants/
constants were estimated by substituting their function in the appropriate energy equations. The integrated results were then compared with measurements of the Nusselt number and Deissler chose $n = 4$. He assigned a value for the constant $A$ which was selected in such a way as to give the best fit for the distribution of the eddy viscosity so that

$$\frac{\epsilon_H}{\epsilon_U} = 1. \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.49)$$

This value of $A$ did not give a very good fit. The fallacy in referring to his analysis in order to show the validity of the assumption that $\epsilon_H/\epsilon_U = 1$ is seen from the fact that Deissler actually deduced $\epsilon_H$ and not $\epsilon_U$. A better method to avoid this assumption would be to employ functions that agree with the data on $\epsilon_H$.

The dependence of the eddy diffusivity for heat on the longitudinal $x$-direction in the thermal-entry region of a duct of arbitrary geometry is discussed by Levy (L.6) and Bourne (L.1). The work of Johnk and Hanratty (J.3) in a circular pipe shows that $\epsilon_H$ varied with $x$ in the thermal-entry region. Abbrecht and Churchill (H.2) concluded, however, that for fully-developed turbulent flow in a pipe the experimentally determined $\epsilon_H$ was invariant in the $x$-direction as close as approximately one diameter from the entrance.

Deissler's expression for the eddy diffusivity was employed in the present work to calculate the developing concentration boundary-layers and the variation of the local Sherwood number with distance along the tube. The choice of Deissler's expression for the/
the eddy diffusivity was guided by the following considerations:

(a) Early trends of mass-transfer data obtained in the present investigation indicated qualitative agreement with the predictions of Deissler's analysis. A direct means was established of comparing mass-transfer data on entrance-lengths with calculated results for the more promising of the rival analyses. This was the analysis proposed by Deissler and the appropriate numerical solutions were computed for developing concentration boundary-layers by employing his velocity profile.

(b) The use of this same expression afforded a direct comparison of the experimental data with the two rival analyses, i.e. Deissler and Sparrow, since both authors assumed the same equation for the velocity distribution. Consequently, a closer assessment of the assumptions of each analysis was possible.

(c) The numerical results obtained in this work for mass transfer entrance lengths with Uniform Wall Mass Flux using Deissler's analysis complemented his heat transfer computations for Uniform Heat Flux, (section 3.4).

2.3.1. Inherent Limitation of the Analogies.

The modification of the Navier-Stokes equations by Reynolds can be expressed for its \( x \)-direction by

\[
\sum \tilde{u}_i \frac{\partial \tilde{u}_x}{\partial i} = \frac{1}{\rho} \sum \left[ \frac{\partial}{\partial i} \left( \mu \frac{\partial \tilde{u}_x}{\partial i} - \rho \bar{u}_i \bar{u}_y \right) \right] - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + g \frac{\partial \bar{h}}{\partial x} \quad \ldots \ldots (2.50)
\]

where \( \tilde{u} \) and \( \bar{u} \) are the time-mean and fluctuating components of velocity.

For/
For steady turbulent flow the instantaneous velocity component \( u \) is the sum of the \( \bar{u} \) and \( u^0 \)

\[
    u = \bar{u} + u^0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.51)
\]

Similar expressions may be written for the \( y \)- and \( z \)-directions.

The corresponding equation for turbulent heat-transfer with no heat dissipation is

\[
    \sum \bar{u}_i \frac{\partial \bar{t}}{\partial x_i} = \frac{1}{\rho c_p} \sum \left[ \frac{\partial}{\partial x_i} \left( \frac{k \bar{u}_i}{\bar{c}} - \rho c_p \bar{t} u_i \right) \right] \ldots \ldots \ldots \ldots (2.52)
\]

where \( \bar{t} \) and \( t^0 \) are the time-mean and fluctuating components of temperature.

A similar equation may be derived for mass transfer with \( \bar{c} \) and \( c^0 \) as the time-mean and fluctuating components of concentration. However, if diffusion coefficients are to be practically independent of concentration, then these must be referred to a plane of no net molar transport. The difficulty arises in that for strict comparability between heat and mass transfer, the diffusion coefficients should be referred to a plane of no net mass transport.

A way of expressing the equation based on molar linear velocities in terms of mass linear velocities was proposed by Beddingfield and Drew (8). The diffusion potential (or driving force) was written as \( \ln M \) instead of concentration, and is defined by the equation

\[
    M = Y_A M_A + Y_B M_B \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.53)
\]

which is the average molecular weight of a binary mixture containing \( Y_A \) mole fraction of component A. An equation comparable to/
to equation (2.52) may now be written for low concentration of \( Y_A \)

\[
\sum \bar{u}_i \frac{\partial Y_A}{\partial x} = \sum \frac{\partial}{\partial x} \left[ D_A \frac{\partial Y_A}{\partial x} - \bar{Y}_A \bar{u}_i \right] \tag{2.54}
\]

When these equations are applied to various practical situations it is usual to make one or more of the following assumptions: (a) the physical properties are considered to be constant; (b) the dissipation of frictional energy is negligible, (c) turbulence in the fluid is assumed to be isotropic, i.e., \( \bar{u}_x'^2 = \bar{u}_y'^2 = \bar{u}_z'^2 \); (d) the existence of internal mass or energy sources or sinks are ignored.

2.3.2. Analogy of Momentum, Heat and Mass Transfer.

Comparison of equations (2.50) and (2.54) shows them to be identical for low concentrations of component A in a binary mixture, if,

\[
k/\rho c_p = D \tag{2.55}
\]

or,

\[
Sc = Pr \tag{2.56}
\]

Since equation (2.52) contains two additional terms, that describe changes in mean pressure and elevation of the fluid element, then

\[
\mu/\rho = k/\rho c_p = D \tag{2.57}
\]

or,

\[
Pr = Sc = 1 \tag{2.58}
\]

only if the additional terms cancel or are negligible.

The three processes are identical if the mechanism of transport is considered to be one of eddy diffusion; i.e., when transport/
transport in a fluid is assumed to be due to the fluctuating components and not due to molecular motion. Furthermore, the differential equations (2.50), (2.52) and (2.54) attain the same form (a) if changes in the pressure and elevation of the fluid element terms in equation (2.50) are so small that they may be ignored; (b) if those two additional terms in equation (2.50) cancel, as is the case when the pressure is hydrostatically distributed.

In order to describe completely the situation of momentum transfer, two additional equations for the y- and z-directions must be written. This difference springs from the fact that momentum is a vector quantity, while heat and mass are scalar quantities. Momentum may then vary with the pressure fluctuations involved in turbulent flows, and for the analogy to hold exactly, the time mean velocities $\bar{u}_y$ and $\bar{u}_z$ must be assumed to be of no importance.

In conclusion, for the three differential equations to be comparable, their solutions must employ the same boundary conditions. This means that the potentials and transfer rates at the boundary must be the same, in addition to similarity in the geometry of the system.

2.3.3. Factors which Affect the Validity of the Analogy Between Heat and Mass Transfer.

The analogies which are described in Table VI have been generally restricted to fully-developed incompressible flow in circular/
circular ducts, with fluid properties which are independent of temperature and/or concentration. In the analysis of heat and mass-transfer experimental results, it is a frequent practice to extend known analytical or semi-analytical expressions for either transfer process to the particular situation for which such equations do not actually exist.

This practice is based on the assumption of the analogy between heat and mass transfer, i.e., where Pr = Sc, and therefore, on the direct comparability between these two transfer processes. However, it is found that there are several factors which may affect comparison made between heat and mass transfer. It is important at this stage to examine these factors, before attempting to draw conclusions by this method from experimental data obtained by different investigators using different approaches and techniques. A brief and comprehensive list of various complications is given by Sherwood (8,6). The following discussion assumes that the boundary conditions for the two transfer processes are identical, and hence that the analogies are valid, since this constitutes an essential condition if the analogies are to hold.

A fundamental difference between heat and mass transfer is the manner in which transport takes place from a tube wall to a fully-developed turbulent fluid stream. A complicating factor is introduced in the case of mass transfer, since a radial bulk motion occurs owing to the existence of a radial component of the fluid velocity. This difficulty does not arise in heat transfer and/
and therefore the two processes cannot be considered to be exactly comparable. The presence of the bulk-flow in mass transfer can then be expected to influence the rate of momentum or heat transfer. It may even influence the rate of mass transfer itself by setting up an additional motion which can be considered as a second turbulence. A model based on a 'stagnant film' picture by Mickley et al. (M.13), and Spalding (S.27) described this effect well and it is expressed by the equation

$$\theta = \frac{e^B}{e^B - 1} \quad \ldots \ldots \ldots \ldots \ldots (2.59)$$

where $\theta$ is the correction factor and $B$ is a measure of the mass transfer rate. Both those terms are defined by a Table provided by the author for each of these transfer processes. A positive value is assumed by $B$ for mass transfer away from the wall and $B = 0$ for the limiting case of no mass transfer. The correction factor $\theta$ becomes important in cases such as condensation of mixed vapours, evaporation at low pressures and transpiration cooling. $\theta$ becomes very nearly unity for the more common situation of gas absorption and evaporation.

A theoretical treatment by Danköbler (D.11) describes qualitatively the manner in which the radial flow vector affects the boundary-layer thickness, and consequently the rates of mass transfer. Various treatments of this problem are given by Ackermann (A.4) and Colburn and Drew (G.10).

For the type of mass transfer investigation carried out in the present/
present work, the transfer rates from the diphenyl surface are very low. It can then be assumed that the radial flow component plays an insignificant role and its effect can be considered negligible to all intents and purposes.

A more serious source of complication in the direct comparison between heat and mass transfer is the influence exerted by the temperature. The problem is of a two-fold nature. In the first instance, heat transfer experiments are conducted under conditions where there is a finite temperature difference between the wall-surface and the working fluid. A temperature gradient then exists across the cross-section of the duct, becoming more serious the greater the wall-to-fluid temperature difference grows. The establishment of a temperature gradient is accompanied by a variation of temperature-dependent physical properties of the fluid, such as the density, \( \rho \), and viscosity, \( \mu \), and therefore, the Prandtl number, \( Pr \). This subject is discussed in greater detail in section 2.4, where it is shown that for gases, the viscosity \( \mu \) and the thermal conductivity, \( k \), increase with temperature; the Prandtl number for gases, therefore, scarcely alters according to changes in temperature. This problem, however, is more important in the case of heat transfer in liquid metals where the Prandtl number is more sensitive to temperature.

The manner in which the analogy between momentum and heat transfer may be modified to take into account the variation of fluid viscosity with temperature is discussed by Doissler et al.\(^{(D.2)}\) and by Doissler\(^{(D.3,D.12)}\). Applications of methods employed by those/
these authors which allow for the temperature effect are described in section 2.4 along with similar efforts made by other authors. This problem has also been studied by Dunn(D.5), Reichardt(R.5), Deissler and Loeffler(D.13) and Summerfield(S.28).

The problem in mass transfer which is exactly equivalent to that of the influence of temperature variations in heat transfer takes the form of small scale variations in physical properties with concentration. This topic, however, appears to have aroused practically no interest.

The question of complications caused by temperature does not arise in the case of mass-transfer investigations, since these are usually carried out under conditions where no temperature difference exists between the mass-transferring wall and the moving fluid-stream. This fact alone offers a great advantage to mass transfer studies as compared with heat transfer experiments.

Nevertheless, there is another temperature-dependent effect which may be of concern in studies of mass transfer under isothermal conditions, or Uniform Wall Temperature, U.W.T. The vapour pressure of the materials available for mass-transfer studies is critically dependent on the temperature. Two sources of temperature variation may occur, even under Uniform Wall Temperature boundary conditions. These are: (i) aerodynamic heating, and (ii) depression of wall-temperature owing to sublimation of the volatile substance. Both these complications do not concern investigations of the type described in this work since/
since (a) aerodynamic heating is only important in high speed studies, where \( M > 0.7 \). This assumption is supported by Ko and Segin (K.4), who found that the corrections required for these two effects were very small at normal air velocities and temperatures. An example where this problem may arise is in mass-transfer studies involving rotating discs (K.3); radial variations in temperature occur over the transfer surface, since the local velocity depends on the radial position on the rotating plate; (b) Sublimation cooling is more serious in low-pressure work, as first shown by Sherwood and Cooke (S.29) and later by Madden (M.14). Sherwood et al. studied sublimation from naphthalene spheres, to investigate mass-transfer rates at pressures ranging from 0.1 to 3000 \( \mu \) mercury. The sublimation was believed to have taken place at the temperature of the water-bath, 0°C. Madden, however, showed that the actual temperature of the naphthalene spheres was -21°C. owing to the low heat-transfer rates between the surface and the walls of the container.

In conclusion, it may be noted that very little attention has been focussed in the past on establishing a formal thesis which justifies the extension of heat-transfer expressions to the study of mass-transfer data. Apart from the early work of Chilton and Colburn (C.11), only Beddingfield and Drew (B.9) appear to have devoted some effort to produce a formal analogy between heat and mass transfer. Beddingfield and Drew employed dimensional analysis to produce a theory correlating heat and mass-transfer coefficients, which they tested by using data from psychrometric studies.
2.4. **INFLUENCE OF TEMPERATURE-DEPENDENT PROPERTIES OF FLUIDS.**

Heat transfer and flow friction solutions generally assume that the flow properties remain constant throughout the entire flow field. This assumption is purely an idealization and does not hold for the real heat-transfer problems encountered in practice. The reason for this is that the transport properties of most fluids vary with temperature and will therefore vary through the boundary layer, or in the case of a tube, over the flow cross-section.

The problem of the temperature-dependent property becomes more complex because the properties of different fluids behave differently with temperature. For gases, the specific heat \( c \) is almost independent of temperature, but the thermal conductivity \( k \), and viscosity \( \mu \), increase to about the 0.8 power of the absolute temperature in the ambient temperature range. Also, the density is inversely proportional to the absolute temperature. The Prandtl number, \( \mu c/k \), however, is practically independent of temperature.

For the majority of liquids, the specific heat and thermal conductivity are practically insensitive to temperature, but there is a marked decrease of viscosity with temperature. This is particularly true in the case of oils, and also of water where the viscosity is very temperature-dependent. The density of liquids is virtually insensitive to changes of temperature; the Prandtl number, \( Pr \), of liquids then varies with temperature as does the viscosity.

Variation/
Variation of transport properties with temperature result in velocity and temperature profile changes. These then yield friction and heat-transfer coefficients differing from those evaluated at constant-temperature conditions. A large body of variable-property empirical data is contained in the literature with a number of variable-property solutions. In general practice, it has been found convenient to employ the constant-property analytic solutions, or the experimental data obtained with small temperature differences. A correction of some form is then applied to deal with the property variation. Although fairly simple corrections are found in general to be adequate for most of the variable-property results, over a moderate range of properties, no ordinary method of correction will suffice for the case when the absolute value of the properties varies many times through the boundary layer.

The methods of correction which are commonly applied to constant-property results are of two types:

(a) The Property Ratio Method: All properties are found at the mixed-mean temperature, or the free-stream temperature. The variable-property effects are then all grouped into a function of a ratio of some relevant property evaluated at the surface temperature to that property evaluated at the mixed-mean or free-stream temperature. However, the method of correction for gases is different from that for liquids. For gases, the density, viscosity and thermal conductivity are functions of the absolute temperature. This temperature-dependence tends to be similar for/
for different gases, except for the case of temperature extremes. For the case of gases, the following correlation is found to be adequate

\[ \frac{\text{Nu}}{\text{Nu}_1} = \frac{\text{St}}{\text{St}_1} = \left(\frac{t_o}{t_m}\right)^n \quad \text{(2.60)} \]

and

\[ \frac{r}{r_1} = \left(\frac{t_o}{t_m}\right)^m \quad \text{(2.61)} \]

where

\[ \text{Nu} = \frac{hd}{k} = \text{the Nusselt number} \quad \text{(2.61A)} \]

h is the local heat-transfer coefficient, d the pipe-diameter and k is the thermal conductivity. Subscript 1 refers to the appropriate constant-property solution or small temperature-difference experimental Nusselt result.

The Stanton number, St, is defined as follows:

For heat transfer:

\[ \text{St}_H = \frac{h}{\rho g u_b c_p} \quad \text{(2.61B)} \]

For mass transfer:

\[ \text{St}_M = \frac{k c}{u_b} \quad \text{(2.61C)} \]

where \( c_o \) = concentration of diffusing substance at wall

\( k_c \) = the local mass-transfer coefficient

\( m_o \) = rate of mass transfer at inside wall toward tube-centre per unit area

\( u_b \) = bulk-velocity at cross-section of tube

\( q_o \) = rate of heat transfer at inside wall toward tube-centre per unit area

\( t_o \) = wall temperature

\( t_b \) = bulk static-temperature of fluid at cross-section of tube

\( c_b \) /
\( c_b = \) bulk concentration of diffusing substance
\( f = \) the friction coefficient

All properties in the non-dimensional parameters are calculated at the mixed-mean temperature for internal flows, and the free-stream temperature for external flows. The temperature \( t_m \) is evaluated at the mixed-mean temperature, while \( t_o \) is evaluated at surface, or wall temperature. The exponents \( m \) and \( n \) are functions of the geometry and the type of flow.

For liquids, where most of the effect is caused by the viscosity variation, the following correlations are often found to give good approximations

\[
\frac{\text{Nu}}{\text{Nu}_1} = \frac{\text{St}}{\text{St}_1} = \left( \frac{\mu_o}{\mu_m} \right)^n \quad \text{......... (2.62)}
\]

\[
\frac{f}{f_1} = \left( \frac{\mu_o}{\mu_m} \right)^m \quad \text{.......... (2.63)}
\]

Again, all properties in the non-dimensional parameters are evaluated at the mixed-mean temperature.

(b) The Reference Temperature Method: The properties appearing in the non-dimensional groups (\( \text{Nu}, \text{Pr}, \text{Re}, \text{etc.} \)) are evaluated at a characteristic temperature chosen so that the constant-property results at that temperature may be used to estimate variable-property behaviour. There is no general rule about the choice of the characteristic temperature. This may be the surface temperature or a temperature part way between the surface or wall temperature and the mixed-mean or free-stream temperature.
The "reference temperature method" is the most extensively used type of variable-property correction. Two difficulties arise however when this method is utilized in internal flows but not in external flows. The constant-property solution for laminar flow heat transfer yields

\[ \text{Nu} = \frac{h d}{k} = \text{constant} \quad \text{...............} \quad (2.64) \]

For liquids, \( k \) is relatively independent of temperature, but the strong dependence of viscosity with temperature, results in a distorted velocity profile with high temperature differences, and consequently, a substantial variation in the conductance, \( h \). For this particular case there is no temperature-dependent property that can be evaluated at a reference temperature.

Another difficulty is that it is often necessary when evaluating the Reynolds number, to split the terms in the mass velocity equation

\[ G = v \rho \quad \text{...............} \quad (2.65) \]

so that the density \( \rho \) may be evaluated at the reference temperature. \( G \) is the mass flux or mass velocity and \( v \) is the mean velocity in a tube. For internal flows, such as in a circular duct,

\[ \frac{dm}{dt} = GA \quad \text{...............} \quad (2.66) \]

where \( G \) is the mass flow rate divided by the cross-sectional area of the flow, \( A \). The flow cross-sectional area \( A \) is the one parameter about which there is no ambiguity, no matter how the density may vary over the flow area. It is then necessary to apply a method in which \( v \) and \( \rho \) do not have to be evaluated separately.
2.4.1. Application of Correction Methods to Constant-Property Results.

(a) Turbulent Flow in Tubes - Liquids: Variable-properties

Fully-developed turbulent flow of a liquid in a circular duct has been studied by Deissler (D.1) for uniform heat flux and with viscosity as a function of temperature. The momentum and energy differential equations were solved by iterative methods but the problem was more difficult than the constant-property solution. This was because the velocity profile had to be deduced since there are no empirical velocity-profile data from which an eddy diffusivity may be deduced. For the sub-layer diffusivity, Deissler employed

\[
\frac{\epsilon_M}{\nu} = \mu^+ y^+ [1 - \exp(-\mu^+ y^+)] \quad m = 0.0154 \quad \text{(2.67)}
\]

and for the turbulent core he used a diffusivity by von Kármán

\[
\epsilon_M = k^2 \frac{(dy/dy)^3}{(d^2u/dy^2)^2} \quad \text{(2.68)}
\]

Equation (2.68) is a function of local velocity only, and therefore it must hold for a property gradient as for constant-property conditions. Since it was assumed that \( \epsilon_L = \epsilon_M \), the results are only applicable to the range of \( Pr > 1 \). The assumed viscosity-temperature relation was

\[
\frac{\mu}{\nu} = \left( \frac{T}{T_0} \right)^d \quad \text{(2.69)}
\]

The results were found to be approximately the same when the exponent \( d = -1 \) and \( -4 \).

(b)
(b) **Turbulent Flow in Tubes - Gases**—The analytic treatment by Deissler and Eian \( (D.2) \) for variable-property fully-developed turbulent flow in a circular duct used the same calculation procedure as that described above for turbulent flow of a liquid. The difference this time is that the temperature variations of density and thermal conductivity are now considered as well as viscosity.

Although Deissler and Eian did not present their results in the temperature ratio form, these can be approximated by equation (2.50) with the coefficients

\[
\frac{t_o}{t_m} > 1 \quad n = -0.34
\]

\[
\frac{t_o}{t_m} < 1 \quad n = -0.19
\]

A stronger effect of temperature ratio for \( t_o/t_m > 1 \) is suggested however by a considerable body of experimental data. The data of McCarthy and Wolf \( (M.8) \) for hydrogen and helium yields \( n = -0.55 \). A similar result is obtained by the experiments of Humble et al. \( (H.4) \). The data of Barnes \( (B.2) \) for air, carbon dioxide and helium give results that are slightly lower and the value of \( n \) is different for the different gases. If these considerations are taken into account then \( n = -0.5 \) for \( t_o/t_m > 1 \) should be the best conclusion that can be drawn.

It is more difficult to conduct the cooling investigations, \( t_o/t_m < 1 \), and especially if it is required to determine local heat-transfer/
transfer coefficients. Few results of this kind of investigation have been reported. Deissler's analytic results indicate only a small effect, \( n = -0.19 \), and the small amount of data by Humble et al. for \( \frac{t_o}{t_m} \) varying from 1 to 0.46 show a negligible effect of temperature ratio, i.e., \( n = 0 \). Nicoll and Kays conducted similar experiments and report no effect over the temperature ratio range of 0.63 to 0.38. The value \( n = 0 \) then seems to agree with the available information.

In conclusion, from the incomplete evidence discussed above, for turbulent flow of a gas in a circular duct

\[
\frac{t_o}{t_m} > 1 \quad m = -0.1 \quad n = -0.5
\]

\[
\frac{t_o}{t_m} < 1 \quad m = -0.1 \quad n = 0
\]

These results assume a gas for which the dependence of properties on temperature is similar to that of air in the range 100°F to 3000°F.
FIG. 5 RELATIVE ROUGHNESS OF PIPES MADE OF DIFFERENT MATERIALS

Pipe diameter (feet), r

Pipe diameter (inches), r

Relative roughness, e/r

4f for complete turbulence, rough pipes

- Riveted steel
- Concrete
- Wood
- Galvanized iron
- Cast iron
- Asphaltered cast iron
- Asphaltered wrought iron
- Commercial (or wrought) iron
- Drawn tubing

r = 0.00006
r = 0.00007
r = 0.00008
r = 0.000085
r = 0.00009
r = 0.0001
2.5. INFLUENCE OF SURFACE ROUGHNESS ON TRANSFER OPERATIONS.

The success of experimental observations of boundary-layer transition in both heat and mass transfer is governed from the outset by the condition of the solid boundary at which heat or mass exchange takes place. Although rough conduit walls have very little effect upon laminar flows, the effect for turbulent flows can be significant.

A considerable increase of turbulence may be introduced in the very thin layer next to the wall when the roughness of the wall is of the same order of magnitude as the thickness of the predominantly laminar sub-layer. From considerations discussed in Appendix IV, any irregularities > $10^{-3}$ inch might be expected to interfere substantially with the fully-developed turbulent flow in a pipe of 1 inch diameter and at a Reynolds number of $10^5$. The height of the viscous sub-layer is taken as $y^* = 5$ for the purposes of calculation in Appendix IV and is based on von Kármán’s $(\nu, \lambda)$ generally accepted assumption for the width of the laminar region.

The particular attention that was focussed in the present investigation on the influence of surface roughness on the mass-transfer rates is evidenced by the following consideration. For a tube of 1 inch diameter and local wall-irregularities of $10^{-3}$ inch, i.e. of equivalent roughness 0.001, the friction factor is greater than the expected value by 30%. This result is arrived at by reference to figure 5 which shows the dependence of the friction factor/
factor on the equivalent roughness. It is feasible then that the same conclusion could be drawn for heat and mass transfer from the Reynolds analogy, especially for low Prandtl or Schmidt numbers.

Unfortunately, very little is to be found in literature about the influence of wall roughness on mass-transfer rates. This state of affairs is very different in the case of friction factor and heat transfer for which a large amount of data has been collected from investigations with artificially roughened pipes.

In the following sections, a brief review will be made of the influence of wall roughness on momentum and heat transfer. The information derived from experimental work for these two cases will then be used to guide conclusions for the case of mass transfer, for which little experimental data is available.

2.5.1. The Influence of Surface Roughness on Momentum Transfer.

As was mentioned in the previous section, if the surface irregularities are of the same order of magnitude as the thickness of the predominantly quiescent viscous sub-layer, they tend to disturb or break up the very thin sub-layer next to the wall, thereby appreciably increasing the wall stress. As a matter of fact no predominantly viscous region can exist if the surface is excessively rough. The apparent shear forces are now transmitted to the wall in the form of pressure drag on the irregularities, and the friction coefficient, $f$, becomes virtually independent of the Reynolds number.

A/
A large number of analytical and experimental investigations of turbulent flow over a rough surface have been made in the past. Moody (1969) in his paper has summarized a semi-empirical theory on the influence of surface roughness, with roughness expressed as an effective size of surface irregularity, $c$. Typical values of $c$ are given in Figure 5 which shows Moody's friction-coefficient results for circular pipes made of different materials.

In the majority of the experimental investigations, velocity or pressure measurements were obtained at discreet points in the flow. A summary of the most important investigations of the effect of wall-roughness on fully-developed turbulent flow is presented in Tables VII(1) and VII(2) which are taken from a recent paper by Liu et al. (1977). In their extensive investigations, Liu et al. studied the flow-structure of turbulent boundary-layers over a family of flat surfaces with transverse roughness elements made of square bars of variable spacing. They obtained information on the mechanics of the production of turbulence near rough walls; and some progress is reported in their paper, in relating the data obtained to the accepted velocity correlations. A distinct flow-pattern was observed in the wall region for each of the four cases: a) smooth, b) skimming, c) wake-interference, and d) isolated-roughness flow. The authors report further, that the most obvious effect of an increase in surface roughness was distinctly to increase the turbulence production and to produce a parallel increase in the value of eddy viscosity. They also showed that the total thickness of the layer is increased when the eddy viscosity is increased, in the/
<table>
<thead>
<tr>
<th>Ref.</th>
<th>Investigator</th>
<th>A/E</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.9</td>
<td>Darcy [1858]</td>
<td>E</td>
<td>[\frac{U_c-U}{U^*} = 5.08(1 - \frac{Y}{r})^{3/2} \quad Y/r \geq 0.35] (Re-analyzed according to Nikuradse)</td>
</tr>
</tbody>
</table>
| V.4  | von Kármán [1930] For Sand-roughened pipes | A   | \[\frac{U_c-U}{U^*} = \frac{1}{k} \left[ \log_e (1 - \sqrt{1-Y/r}) + \sqrt{1-Y/r} \right] \]  
\[U^+ = 2.5 \log_e \left( \frac{r}{k_s} \right) + 4.75 \]  
\[1/\sqrt{f} = 2 \log_{10} \left( \frac{r}{k_s} \right) + 1.68 \] |
| N.3  | Nikuradse [1933] Sand-roughened pipes | A/E | \[\frac{k_s U^*_v}{v} < 3.55 \quad U^+ = 5.75 \log_{10} \frac{r}{k_s} + 5.5 \]  
\[\text{"Hydraulically smooth flow"} \quad f = 0.316 \Re_d^{-1/4} \quad \text{(Blasius)} \]  
\[1/\sqrt{f} = 2 \log_{10} \left( \frac{U_d}{v} \sqrt{f} \right) - 0.8 \quad \text{(Prandtl)} \]  
\[\frac{k_s U^*_v}{v} > 67.7 \quad U^+ = 5.75 \log_{10} \frac{r}{k_s} + 8.48 \]  
\[\text{"Fully rough flow"} \quad 1/\sqrt{f} = 2 \log_{10} \left( \frac{r}{k_s} \right) + 1.74 \]  
\[k_s U^*_v/v \quad U^+ - 5.75 \log_{10} \frac{r}{k_s} \]  
\[\frac{1}{\sqrt{f}} - 2 \log_{10} \frac{r}{k_s} \]  
\[3.55-7.1 \quad 1.18 + 1.18 \log_{10} \frac{k_s U^*_v}{v} \]  
\[6.59 + 3.5 \log_{10} \frac{k_s U^*_v}{v} \]  
\[9.58 \] |

<p>| A = Analytical work | E = Experimental work |
| For nomenclature see List of Symbols |</p>
<table>
<thead>
<tr>
<th>Ref.</th>
<th>Investigator</th>
<th>A/E</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3</td>
<td>Nikuradse</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[1933]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sand-roughened pipes (continued)</td>
<td>A/E</td>
<td>[ k_s U^*/v ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[ U^* - 5.75 \log_{10} \frac{Y}{k_s} ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[ \frac{1}{\sqrt{f}} - 2 \log_{10} \frac{r}{k_s} ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>14.2 - 67.7 2.81 - 0.588 \log_{10} \frac{k_s U^*}{v}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>'Transition flow'</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>11.5 - 1.62 \log_{10} \frac{k_s U^*}{v}</td>
</tr>
<tr>
<td>5.17</td>
<td>Schlichting</td>
<td>E</td>
<td>Determined 'standard roughness' ( k_s ) from regular roughness patterns other than sand roughness.</td>
</tr>
<tr>
<td>5.19</td>
<td>Colebrook and White [1937, 1939]</td>
<td>E</td>
<td>Resistance formula which correlates the whole transition region from hydraulically smooth to fully rough flow</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[ \frac{1}{\sqrt{f}} = 1.74 - 2 \log_{10} \left( \frac{k_s U^*}{r} + \frac{18.7}{Re \sqrt{f}} \right) ]</td>
</tr>
<tr>
<td>5.19</td>
<td>Moody [1944]</td>
<td>E</td>
<td>Measured resistance data of commercially rough pipes</td>
</tr>
<tr>
<td>5.18</td>
<td>Streeter and Chu [1936, 1949]</td>
<td>A</td>
<td>Based on mixing length profile in pipe of ( l/r = 0.12(1-(1-Y/r)^3) ) proposed that</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[ \frac{U_c-U}{U^*} = 2.5 \log_{e} \left[ \frac{1 + (1 - Y/r)^{3/2}}{1 - (1 - Y/r)^{3/2}} \right] = 2.5 \log_{e} \frac{1}{F} ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>and [ \frac{1}{\sqrt{f}} = 5.567 + 0.5013 \log_{10} \frac{r}{k} ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>For ( s_{L}/k = 2.0 ), fully rough flow, data yielded</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[ U^* = 19.845 + 1.418 \log_{10} \frac{r}{k} + 5.948 \log_{10} \frac{r}{k} ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[ \frac{1}{\sqrt{f}} = 5.669 + 0.403 \log_{10} \frac{r}{k} ]</td>
</tr>
</tbody>
</table>
## TABLE VII.1 (Concluded)

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Investigator</th>
<th>A/E</th>
<th>Results</th>
</tr>
</thead>
</table>
| S.19 | Sams [1952]  | E   | Both resistance and heat transfer increase with $k$ and become more pronounced at high $Re$. $f$ decreases with increasing $Re$ and then independent of $Re$.  
$$f = .0072(t/w)^{0.8}(k/w)^{1.7}$$ |
| S.20 | Straub and Morris [1950] | E   | Based on corrugated pipe data, proposed that longitudinal spacing $s_l$ of roughness governs the type of flow, i.e., skimming flow, wake-interference flow, isolated-roughness flow. Proposed $f(s_L/k)$ for each type of flow. |
| M.21 | Morris [1955, 1961] | A   |  |
| M.22 | Morris [1961] | A   |  |
| P.3  | Ambrose [1954], Price [1957] | E   | Data from cylindrical depression and projection type roughness showed that both transverse and longitudinal spacings affect total flow characteristics and $f$. |
| O.2  | O'Loughlin and MacDonald [1964] | E   | Quartz crystal roughness showed that away from wall, eddy viscosity varies only slightly and mean velocity follows parabolic law. |
| R.8  | Robertson, Burkhart and Martin [1965] | E   |  
$$\frac{U_c - \bar{U}}{U^*} = \frac{U_c - \bar{U}}{\bar{U}} \sqrt{\frac{8}{\bar{f}}} = \frac{4.3}{4} \text{ for rough pipe}$$  
$$\frac{U_c - \bar{U}}{U^*} = \frac{4.3}{4} \text{ for smooth pipe}$$  
$$\text{vs. } 3.75 \text{ of Nikuradse's data}.$$ |
<table>
<thead>
<tr>
<th>Ref.</th>
<th>Investigator</th>
<th>A/E</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>V.6</td>
<td>von Kármán [1934]</td>
<td>A</td>
<td>Based on Nikuradse's rough pipe data, proposed that for $\frac{k_s}{v}$U* &gt; 70, $U^+ = 2.5 \log \left( \frac{V}{k_s} \right) + 8.5$, $1/\sqrt{c_f} = 5.8 + 4.15 \log_{10} \left( \frac{x}{k_s} \sqrt{c_f} \right)$ for fully rough flow.</td>
</tr>
<tr>
<td>P.4</td>
<td>Prandtl and Schlichting [1934]</td>
<td>A</td>
<td>$5 \leq \frac{k_s}{v} \leq 70$, $U^+ = 2.5 \log \left( \frac{V}{k_s} \right) + B_2$, where $B_2 = 2.5 \log (k_s U*/v) + 5.5$, $k_s U* &gt; 70$, $c_f = (2.87 + 1.58 \log_{10} \frac{x}{k_s})^{2.5}$.</td>
</tr>
<tr>
<td>M.10</td>
<td>Moore [1951]</td>
<td>E</td>
<td>In fully rough flow, data showed that $0.45 \leq \frac{U}{U^<em>} \leq 0.52 \left( \frac{V_T}{\theta} \right)^{0.35}$, $0.01 \leq \frac{V_T}{v} + (2/3)k \leq 0.2$, $\frac{U^</em>}{U} = 2.35 - 5.75 \log_{10} \frac{Y_T}{v} + \frac{2}{3}k$. Calculated $c_f = 2 \frac{d\theta}{dx}$ do not agree with predictions of von Kármán or Prandtl and Schlichting.</td>
</tr>
<tr>
<td>C.6</td>
<td>Clauser [1954]</td>
<td>A</td>
<td>$32.5 \leq Y^+ \leq 5.6 \log_{10} Y^* + 4.9 - \frac{\Delta U}{U^<em>}$ and $\frac{V}{\Delta} \leq 0.45$, $\frac{U^</em>}{U} - U = - \left( 5.6 \log_{10} \frac{Y^*}{\Delta} + 0.6 \right)$.</td>
</tr>
</tbody>
</table>
| H.6  | Hama [1954] | A/E | $0.15 \leq \frac{Y^*}{v} \leq 1.0$, $\frac{U^*}{U} - U = 9.6(1 - \frac{Y^*}{v})^2 \approx 0.3$. $\sqrt{2/c_f} = 5.6 \log_{10} (U_{in}^*/v) + 4.3 - \frac{\Delta U}{U^*}$ (Clauser), $\sqrt{2/c_f} = \sqrt{2/c_f}$ (Hama) for smooth wall - $\frac{\Delta U}{U^*}$.
<table>
<thead>
<tr>
<th>Ref.</th>
<th>Investigator</th>
<th>A/E</th>
<th>Results</th>
</tr>
</thead>
</table>
| C.7  | Corrsin and Kistler [1954]         | E   | Data from flow over corrugated surface showed that the intermittency factor profile can be represented by a Gaussian error function and the mean-velocity profile.  
\[ \frac{U}{U_\infty} = \left(\frac{Y}{6}\right)^{1/n} \Rightarrow \theta \propto \left(\frac{x_{\text{virtual}}}{n/(n+2)}\right) \] |
| P.5  | Perry and Joubert [1963] Transverse square bars in pressure field | E   | Data showed \( (s_L/k = 4) \)  
\[ \frac{\Delta U}{U^*} = 5.6 \log_{10} \frac{kU^*}{v} + B_4 \] with \( Y = Y_T + \frac{4}{5} k \)  
and \( B_4 = -0.2 \) with and without pressure gradient. |
| B.3  | Bettermann [1966]                  | E   | For round and flat crest transverse bars found \( B_4 \) is function of \( s_L/k \) and independent of pressure gradient. |
the same way as an increase in molecular viscosity would in a laminar layer.

2.5.2. The Influence of Surface Roughness on Turbulent Heat Transfer.

The arguments employed in the previous section on the influence of surface roughness on the friction factor can be extended to the heat-transfer case. From the above considerations as regards the friction factor, the Nusselt number, which is the dimensionless term employed in heat-transfer problems, can similarly be expected to increase with wall roughness for turbulent flow. Again, the effect on laminar flow is negligible. However, whereas an almost complete semi-empirical theory does exist for the turbulent-flow friction coefficient in rough tubes, no similar complete theory exists for the heat-transfer case. The failure to produce a complete theory continues despite the large amount of attention that has been paid to turbulent-flow heat-transfer coefficient research in rough tubes. The greater complexity of the heat-transfer problem is the main reason for this failure to produce an adequate theory in heat transfer.

For a fluid of \( Pr = 1 \), the Reynolds analogy suggests that for low degrees of wall roughness the heat-transfer conductance should show the same percentage increase as the friction coefficient. An important difference, however, exists between the momentum and the heat-transfer problems. In the heat-transfer case, no counterpart exists to the considerable increase in friction coefficient owing to roughness, arising partly from axial pressure forces on the roughness/
roughness elements. It is then natural to expect that as surface roughness is progressively increased, there will initially be an increase in heat conductance proportional to the increase in friction coefficient. At some stage thereafter, the increase in heat conductance will be appreciably less than the increase in friction coefficient.

An increase in Reynolds number results in thinner sub-layers and therefore, at high Reynolds numbers, the influence of surface roughness is expected to be greater. Identical charts have been presented by Moody (M.9) and Rouse (R.12) of friction factor against the Reynolds number for both smooth and rough pipes.

The influence of the Prandtl number is somewhat less obvious at first sight. If the crude Prandtl mixing-length model is used for the turbulent transfer of heat and momentum, then the roughness effect should be very dependent on the Prandtl number.

According to this model, for Pr $\gg 1$, any small disturbance of the sub-layers could be expected to increase appreciably the heat-transfer conductance. This is because at high Prandtl numbers, most of the heat-transfer resistance is concentrated in the sub-layers. For Pr $\ll 1$, the liquid-metal range, the opposite is true, the molecular conduction being the more dominant mechanism. In this case then, wall roughness would be expected to have negligible influence, exactly as in laminar flow.

An interesting fact is that most of the available experimental data and analytical solutions for heat transfer have been obtained for/
for the region near \( \Pr = 1 \). One typical example of this is the data of Nunnor\(^{65}\) for air with \( \Pr = 0.7 \). Nunnor used rings of equivalent sand-roughness, \( k_0 \), up to 2.26 cm., and Reynolds numbers up to \( 10^5 \). He found that in the range of \( Re = 3 \times 10^3 \), the Nusselt as against Reynolds curve for artificially-roughened tubes is parallel to that of a smooth tube, but displaced so as to give a higher value of Nusselt number with the rough tubes.

He derived the expression

\[
\text{Nu} = \frac{f_1}{8} \frac{Re \cdot \Pr}{1 + 1.5Re^{-1/8} \cdot \Pr^{-1/6}(Pr \cdot \frac{f_1}{f_s} - 1)}
\]  

(2.59)

where the subscript \( s \) refers to smooth tubes, \( f_1 \) is the specific coefficient of wall friction in pipe flow. Equation (2.59) predicts his own data to within 20%. Nunnor's data for air can be approximately correlated with the friction coefficient by the following equation for the Prandtl number not far removed from 1.

\[
\frac{\text{Nu}}{\text{Nu}_s} = \left(\frac{f}{f_s}\right)^{0.5}
\]  

(2.60)

This equation shows clearly that the Nusselt number effect is much smaller than the friction coefficient effect which is in agreement with the arguments stated above. The same difference has been noted for water in the Prandtl number range of 1 to 10.

The effects of fluid friction and heat transfer in artificially-roughened pipes were investigated by Cope\(^{68}\). Surface roughness in the pipe was produced by cutting first right-hand/
hand screw threads and then left-hand ones so as to form a regular surface of pyramids. The heat-transfer coefficient was observed to increase in both laminar and turbulent flows but to a much less extent than the pressure drop. Cope found that for a six-fold increase in the friction factor there was a 20%-100% increase in heat transfer.

The investigations of Smith and Epstein (S.21) appear to be the more reliable, and were again conducted with commercial pipes. In their experiments the Reynolds number was varied from 10 to 80 x 10^3 and the equivalent-roughness ratio, from 64 to 640. The pressure drop was again found to increase much more rapidly than the heat transfer which was in agreement with the results of Cope. Curves of the j-factor for heat transfer, \( J_H \), against the Reynolds number tended to diverge as the Reynolds number increased. Sams (S.19) conducted investigations of heat transfer and pressure drop in pipes with square threads, and again the same tendency was noted. Later, Cope discovered that the curves in Sams' results converged as the Reynolds number increased. An explanation for this divergence in the trend of the results with Reynolds numbers may be the difference in shape of the rough elements. This argument is reinforced in view of the fact that both Cope, and Smith and Epstein, conducted their investigations in the same range of Reynolds number. This is only a tentative explanation of the trend-disagreement since very little is known about the effects of surface configuration on heat transfer.

The results of Smith and Epstein were found to fit well the analytical/
analytical expression by Martinelli

\[ j_H = \frac{\sqrt{\frac{3}{2} (Pr)^{2/3} \frac{\Delta t_{\text{max}}}{\Delta t_{\text{min}}}}}{5[Pr + \ln(1 + 5Pr) + 0.5 F_1 \frac{Re}{60} f]} \] .... (2.61)

where \( F_1 \) is a coefficient, given by Tables

\[ \Delta t \] is the temperature difference, \( \Delta t_{\text{max}} = t_o - t_b \), \( \Delta t_{\text{min}} = t_o - t_c \)

\( j_H \) is the \( j \)-factor for heat transfer = \( f/2 \)
\( t_o \) is the temperature evaluated at the wall
\( t_c \) is the temperature at the tube-axis
\( t_b \) is the bulk temperature of the fluid

2.5.3. The Influence of Surface Roughness on Mass Transfer.

As was already mentioned in the introductory section very little exists in literature on the equivalent problem for mass transfer. Kreith carried out an investigation on the rates of sublimation from a naphthalene surface of a rotating disc. The conclusion drawn by Kreith from his experiments was that the condition of the surface greatly influenced the results. Transfer rates obtained from slightly rough surfaces were found to be as much as 25% higher than those from a freshly-machined surface.

The conclusions by Kreith seem to find qualitative support from observations of the behaviour of wind-tunnel velocity during the present mass-transfer investigations. It was noted that surface irregularities of over \( 2 \times 10^{-3} \) inch on the diphenyl-wall resulted in a decrease in the Reynolds number of over \( 2 \times 10^3 \). A rise/
rise in temperature of a degree or two was also observed. It was not possible however to investigate this effect in more detail since the sensitivity of the profilometric method was limited to estimating surfaces accurately to recessions of < 2 thou. inch.

When the results of Kreith are analyzed it is seen that some important conclusions can be drawn for mass-transfer operations. If the influence of wall roughness on mass transfer is assumed to be analogous to that on heat transfer as found by Smith and Epstein, the equivalent roughness of the transfer surface would be 0.006 for a pipe of 1 inch diameter. In other words, from Kreith's results, surface irregularities of the order of 6 thou. inch in a 1-inch-diameter tube appear to exist. Surface irregularities of the order of 6 thou. inch would admittedly be extremely high and would make themselves obvious in cast surfaces.
2.6. **INFLUENCE OF MAIN-STREAM TURBULENCE INTENSITY ON TRANSFER RATES OF HEAT AND MASS.**

It has been reported in the literature at various times that an increase in the main-stream intensity of turbulence in the fluid results in a corresponding increase in the rates of heat and mass transfer. This effect ought to be considered, especially since many observations cited in the literature were obtained for the fluid stream at the entrance of the wind-tunnel, a situation which is directly comparable to the one in the present investigation. The turbulence intensity is defined as $\frac{u'^2}{u_\infty^2}$, where $u'^2$ is the main square value of the longitudinal component of the turbulence, and $u_\infty$ is the free-stream velocity of the fluid.

The increase in the transfer rates is generally found to be appreciably higher in the case of cross-flow past a cylinder, owing to the presence of pressure gradients which are absent in flow over a flat plate with zero incidence. Heat and mass transfer from a flat plate is analogous to that taking place from the wall of a tube, in the sense that in both cases there is a zero pressure-gradient. It is thus reasonable to assume that the results for fully-developed turbulent flow obtained for plane surfaces can be extended to circular ducts. Kestin et al. (Ref. 6) carried out experiments with pressure gradients imposed on a flat plate and reported an increase of up to 65% in the Nusselt number in laminar flow. The authors suggested that favourable pressure gradients must be present before a change in/
in the free-stream turbulence can have any influence on the transfer rates.

Edwards and Furber (E.10) conducted preliminary experiments on the influence of transfer rates from a flat plate at zero incidence; no pressure gradients were imposed on the plate in this case. These workers showed qualitatively that changes in the points of separation and transition that arise from different turbulent intensities in the free-stream were responsible for the rise in transfer rates. Reynolds et al. (R.11) also carried out experiments on a flat plate in a wind-tunnel with turbulent intensities of 1.5% to 5% and confirmed the observations of Edwards and Furber. Increases of up to 55% in the Nusselt number were reported by Sugawara and Sato (S.39) for an increase in turbulent intensity from 1% to 5%. However, the data obtained by Sugawara and Sato are found to differ greatly from that obtained by other workers; furthermore, their results are found to be very different from theoretical predictions. In the light of this discrepancy it is not possible to establish the degree of reliability of their conclusions.

A number of investigations have been conducted into the influence of the intensity of turbulence in the main stream of a fluid on heat transfer from a cylinder. Comings et al. (C.12) observed little effect of turbulence intensity on heat-transfer rates in the low range of Reynolds numbers studied, namely 400 to 20,000. There was a slight increase in the average heat-transfer coefficient on the cylinder at turbulence levels of 7% compared/
compared to those obtained at levels below 3%, this difference increasing with Reynolds number. Zapp\(^{(2)}\) investigated the relatively high Reynolds numbers of 39,000 and 100,000 at turbulence intensities of 0.9%, 3% and 11.5%. Zapp found that an increase in the turbulence level resulted in the following: (a) there was a corresponding increase in the local heat-transfer coefficient over most of the cylinder, (b) transition from laminar to turbulent boundary-layer was brought about at low Reynolds numbers: an increase of 11.5% in the turbulence level caused the formation of a turbulent boundary-layer at the low Reynolds number of 39,000.

Additional investigations on cylinders at high Reynolds numbers include those by Giedt\(^{(6)}\), Hsu and Sage\(^{(9)}\), Keating and Maeder\(^{(7)}\), and Seban\(^{(40)}\). These authors found that at a Reynolds number of 100,000 an increase in the range of 14% to 28% occurred in the transfer rates, for a change of 0.5% to 2.5% in the turbulence intensity. To account for the rise in transfer rates with augmentation of the free-stream turbulence these authors gave an additional explanation to that offered by Edwards and Furber: the effect was purely local in which large changes in the Nusselt number occurred even after the boundary layer had been fixed by the use of trip-wires.

Maisel and Sherwood\(^{(18)}\) studied the evaporation rates of water to an air stream from plane and cylindrical surfaces. This investigation was similar to an earlier work of Powell and Griffiths\(^{(8)}\), except for an improvement in the supply of water to/
to the surface as evaporation took place. Maisel and Sherwood used for this purpose a porous solid, firebrick, which was covered with fine-weave rayon. The results obtained from that investigation showed an increase of about 50% in the rate of evaporation from a small cylinder for an increase in turbulence from 3% to 21%.

In an extension to the previous work, Powell (P.9) studied the effect of various orientations of flat surfaces and cylinders to the air stream. The results indicated that the transfer rates were higher when the plane test-surfaces were placed at an angle to the air stream.

Studies involving such systems suffer, however, from the following disadvantage. An appreciable change may occur in the partial pressure of water vapour, resulting in a corresponding change in the mass-transfer driving force throughout the whole length of the plane surface or cylinder. Data obtained in this type of investigation cannot then be interpreted directly in terms of mass-transfer coefficients.
3. THEORETICAL CONSIDERATIONS
3. THEORETICAL CONSIDERATIONS

3.1. THERMAL ENTRY ANALYSES FOR TURBULENT FLOW IN A CIRCULAR TUBE.

There are two possible approaches to the analytical predictions of turbulent-transfer rates in the entrance region:

(i) The theoretical approach based on the exact satisfaction of the conservation of energy in the turbulent fluid. This method is also called the eigenvalue formulation for the thermal entrance region. An example of this method is the solution by Sparrow et al. (S.33).

(ii) The theoretical approach based on a Boundary-layer model and integral methods. The analysis proposed by Deissler (D.3) is an example of this method.

Theoretical analyses which depend on (i) are in general treated mathematically in the same manner as that of Graetz (G.3) for laminar flow in a round duct with isothermal walls. The general mathematical procedure of treating turbulent flow in pipes is essentially the same as that for laminar flow in tubes, with the difference that the structure of laminar flow has broken down with the production of turbulent eddies. This method is discussed in more detail in section 3.1.3.

The analysis of Latzko (L.1) was the first to apply Graetz's method to turbulent flow. It was developed for the Uniform Wall Temperature boundary condition for a Pr = 1 in which he assumed a 1/7th-power velocity profile and the Blasius resistance-formula. Latzko developed relationships for the heat transfer/
transfer variation in: (a) fully-developed turbulent flow and transfer, (b) fully-developed flow and undeveloped transfer, and (c) for both flow and transfer undeveloped. A complete discussion was presented by Berry (B.5) of the mathematics involved in the application of Graetz's method to turbulent flow. Two papers using this same method by Bockers (B.6) and Sleicher and Tribus (S.16) appeared in the same year. The main difference between those two analyses was in the method of solution of a particular differential equation, and in the expression employed for the eddy diffusivity for heat.

The analysis by Sparrow et al. used approach (1) for the Uniform Heat Flux problem, whilst Sleicher and Tribus employed the same approach for the Uniform Wall Temperature case.

The similarity of the assumptions of the two analyses is evident from the following list:

(a) Sleicher and Tribus
   (1) Constant fluid properties.
   (2) Mean axial velocity is independent of angular position.
   (3) Mean radial velocity is zero.
   (4) Mean temperature at any radius varies neither with time nor angular position.
   (5) Frictional dissipation of energy is negligible.
   (6) Axial diffusion of heat is negligible with respect to bulk transport in the x-direction.
(b) **Sparrow et al.** (1) Constant fluid properties.

(2) The mean value of the radial velocity is zero.

(3) Viscous dissipation is negligible.

(4) Axial diffusion of heat, both molecular and turbulent is negligible compared to the radial diffusion.

From assumptions (1) and (2) together with the equation of continuity, it follows that the axial velocity profile is unchanging along the pipe. The radial transport of heat due to turbulent diffusion was accounted for by introducing the thermal diffusivity, $\epsilon_H$. Sleicher and Tribus used Jenkins' (J.2) results for the eddy diffusivity ratio, figure 4, but they then adjusted the ratio to fit some experimental data for air.

The two solutions (a) and (b) employ the same procedure as that used for the laminar-flow counterpart to the problem. The first step was to carry out the separation of variables.

The eigenvalues and constants in the resulting series solutions were then evaluated by numerical methods by the use of the computer. Small discrepancies appear in the solutions since the assumptions for $\epsilon_H$ and $\mu$ in the various available solutions are not entirely consistent.

The analysis of Sleicher and Tribus may in principle be extended to arbitrary wall temperature variation. A more complete solution of the Uniform Wall Temperature or isothermal problem will have/
have to be obtained, however, before successful calculation of entrance region heat transfer for the non-isothermal case is possible. Sleicher and Tribus present fully-developed Nusselt numbers only for the non-isothermal problem.

The solution by Sleicher and Tribus covers the Pr range from the liquid metals to gases; the analysis of Sparrow et al. covers the Pr range from gases to 100. The discrepancies appear in the overlapped gas range.

Sparrow et al. present results which show the thermal-entry-length effect for fully-developed turbulent flow in a tube, and U.W.H.F., for a number of different cases. At Pr = 0.01 the effect is rather pronounced and increases with increasing Reynolds number. At Pr > 1 the thermal-entry-length becomes of progressively less importance. Very little influence of Reynolds number on the thermal-entry-length is reported for Pr = 0.7.

3.1.1. The Main Rival Analyses for Entrance Length Prediction.

The most important solutions for turbulent flow in the entrance region of a circular duct are the two analyses developed by Sparrow et al. (S.33) and Deissler (D.10).

The basis for the theoretical approaches of both Deissler and Sparrow et al. was the same. These analyses were carried out for constant fluid properties and both employed the eddy-diffusivity concept to describe the turbulent interchange of momentum and heat. Also, for a fully-developed turbulent flow, both analyses assumed that/
that the turbulence properties are functions of radial position only; the temperature in the entry region was predicted from measurements of the fully-developed velocity profile. These formulations contained two essential differences.

(i) The energy conservation equation was satisfied exactly in Sparrow's analysis and only approximately in the other. In Sparrow's analysis a solution of a differential equation analogous to that for molecular diffusion was obtained with the added restriction that (a) the molecular diffusion in the direction of flow was neglected, (b) a slightly modified result from experimental velocity profiles by Deissler was used for the eddy-diffusion coefficient of heat, $\epsilon_H$. This was assumed to be a function of $r$, the tube-radius, and not of $x$. The equation for this problem is

$$u \frac{\partial t}{\partial x} = -\frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( c + \epsilon_H \right) \frac{\partial t}{\partial r} \right] \ldots \ldots \ldots \ldots (3.1)$$

Deissler used a boundary-layer model and integral methods for calculating the form of the thermal entrance-region for the U.W.H.F. situation. He assumed that the effects of heat transfer and friction are confined to fluid layers close to the surface - the thermal and flow boundary-layers respectively. In Deissler's analysis, the thermal entrance was assumed to consist of two distinct regions. A temperature boundary-layer of thickness $\lambda_H$ which increased with distance downstream; and a constant temperature-core. At the fully-developed region, $\lambda_H = x$. It was also assumed that at any thermal entry cross-section, the fully-developed distribution/
distribution of the dimensionless temperature $t^+$ against the
dimensionless distance, $y^+$, was valid from $y^+ = 0$ to $y^+ = \lambda_N$.

(ii) For the region away from the wall, somewhat different
eddy-diffusivity expressions were used - Sparrow and Tribus
employed a slightly modified form for the eddy-diffusivity
expression obtained by Deissler from experimental velocity profiles.

These differences exert very little influence for high
Prandtl number fluids where steep temperature gradients are confined
to a narrow layer next to the wall.

The theoretical analyses of Sparrow and Deissler predict
(a) different length of pipe required for the attainment of fully-
developed thermal conditions, (b) different trend for heat transfer
in the developing thermal boundary-layer at different Reynolds
numbers.

3.1.2. General Method for the Solution of the Conservation of
Energy Equation for Fully-Developed Turbulent Flow in Thermal Entry
Length of a Circular Tube.

Consider the case of a fully developed velocity profile, but
with a uniform fluid temperature at the point where heat transfer
begins; in other words, the case of a developing temperature
profile or thermal entry length. The Nusselt number should, just
as for laminar flow, be indefinitely high at the beginning, de-
creasing with length and finally approaching the fully-developed
magnitude in the limit.

The/
The differential equation to be solved for the circular tube is

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \epsilon_H + c \right) \frac{\partial T}{\partial r} \right] = u \frac{\partial T}{\partial x} \quad \ldots \ldots (3.2) \]

where \( r \) is the radial distance from centre of tube, \( \epsilon_H \) is the eddy-diffusivity of heat, \( c \) is the molecular diffusivity, \( x \) is the distance from the point where heating begins parallel to the tube axis, \( u \) is the velocity component in \( x \)-direction. Equation (3.2) is the energy equation for hydrodynamically fully-developed flow in a circular tube with constant properties, corresponding to the laminar flow equation

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial T}{\partial r} \right] = u \frac{\partial T}{\partial x} \quad \ldots \ldots \quad (3.3) \]

It is assumed that (i) the molecular diffusion of heat in the direction of flow is zero, and, (ii) \( \epsilon_H \) varies with \( r \) and must therefore remain within the derivatives.

The derivative on the r.h.s. of equation (3.2), however, is no longer a constant or a function of \( x \) alone, as is the case for a fully-developed temperature profile. Equation (3.2) can then be expressed in the dimensionless \( x^+ \) and \( r^+ \) coordinate system, where,

\[ r^+ = r/r_o, \quad u^+ = u/U, \quad x^+ = \frac{x/r_o}{Re Pr} \quad \ldots \ldots \quad (3.4) \]

where \( r_o \) is the inside radius of the tube, and \( U \) is the mean velocity in the tube.

The particular form of \( x^+ \) is chosen so that parameters will be absorbed within the variables, at least after axial conduction is/
is neglected. This becomes evident, since if $x/r_o$ alone had been chosen for the non-dimensional length, RePr would appear in the normalised equation and this group can then be absorbed within the definition of non-dimensional length. A typical example of this practice is to be found in the approximation proposed by Sellers and Tribus\(^{(S.22)}\) for the evaluation of the Graetz solution for the fully-developed laminar flow Nusselt number in the entrance region of a circular tube. The proposed relation is the equation

$$
\text{Nu} = 1.357(x^+)^{-1/3} \quad \text{for } x^+ \leq 0.001 \quad (3.5)
$$

Another example is the Leveque solution\(^{(L.8)}\) for the same problem, and this is represented by the equation

$$
\text{Nu} = 1.077(x^+)^{-1/3} \quad \quad \quad \quad \quad (3.6)
$$

The bracketed term \((\epsilon_H + a)\) in equation (3.2) is a function both of Re and Pr rather than a constant. For a given boundary condition a family of solutions will therefore be obtained rather than a single solution.

The method described above is the basis of the solution proposed by Sparrow et al.\(^{(S.33)}\). This is discussed in section (3.1.3). After introducing the relevant boundary conditions, Sparrow et al. presented results of $\text{Nu}/\text{Nu}_\infty$ vs $x/d$ for the U.W.H.F. situation. However, the definition of the dimensionless distance used in that work was $x/r_o$.  

The method which was used by Sparrow et al. to solve the problem of thermal entrance effects in fully-developed turbulent flow, was essentially similar to that described earlier in section 3.1.2. A more detailed account of the solution proposed by Sparrow et al. will now be given in view of the conclusions arrived at in the present work.

I. Statement of the Problem: A section of pipe of radius $r_0$ has a Uniform Wall Heat Flux distribution, $q$. The fluid moves from left to right with a fully-developed turbulent velocity profile, and a uniform temperature $t_0$ at $x = 0$. The section to the right of $x = 0$ is considered.

The following assumptions hold

(a) The mean value of the radial velocity, $v$, is zero.
(b) The fluid properties are constant.
(c) Viscous dissipation is negligible.
(d) Axial diffusion of heat, both molecular and turbulent, is negligible compared to the radial diffusion.

The energy equation which describes the problem is

$$
\frac{u}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r(a^2 + \epsilon_R) \right] \frac{\partial t}{\partial r} \quad \ldots \ldots \ldots \ldots \ldots (3.6)
$$

where $u$ = the velocity component in the $x$-direction,
$t$ = the temperature,
$r$ = the radial co-ordinate,
$a^2$ = thermal diffusivity, $k/\rho c_p$,
$k$/
\[ k = \text{thermal conductivity}, \]
\[ \rho = \text{the fluid density}, \]
\[ c_p = \text{specific heat at constant pressure}, \]
\[ \epsilon_H = \text{eddy diffusivity for heat}. \]

From the equation of continuity and assumption (a) and (b) it follows that the axial velocity profile remains constant along the pipe.

The eddy thermal diffusivity \( \epsilon_H \) is introduced to account for the presence of the radial transport of heat owing to turbulent diffusion. It can be now seen that equations (3.2) and (3.6) are similar.

The boundary conditions for the function \( t(x, r) \) are next written to complete the statement of the problem

\[
t(0, r) = t_0; \quad \frac{\partial t(x, r_0)}{\partial r} = \frac{q}{K}; \quad \frac{\partial t(x, 0)}{\partial r} = 0 \quad \ldots (3.7)
\]

\( q \) and \( t_0 \) are constants, and the last condition results from symmetry.

The dimensionless variables

\[
u^* = \frac{u}{u^*}; \quad \theta = \frac{t - t_0}{qK}; \quad r^* = \frac{ru^*}{qK}; \quad \theta^* = \frac{r^* u^*}{qK} \quad \ldots (3.8)
\]

are introduced into equations (3.6) and (3.7) to give

\[
u^* \frac{\partial \theta^*}{\partial x^*} = \frac{F^*}{r^*} \frac{\partial}{\partial r} \left[ \left( r^* \frac{\partial \theta^*}{r^*} \right) \right] \quad \ldots \ldots . \ldots (3.6a)
\]

and

\[
\theta(0, r^*) = 0; \quad \frac{\partial \theta^* (x^*, r^*_0)}{\partial r^*} = 1 \frac{\partial \theta^* (x^*, 0)}{\partial r^*} = 0 \quad \ldots (3.7a)
\]

where/
where \( u^* \) = the frictional velocity, \( \sqrt{\tau_{\infty}/\rho} \)
\( \nu = \) the kinematic viscosity, \( \mu/\rho \)
\( \mu = \) the viscosity of the fluid
\( x^* = \) dimensionless distance, \( x/r^* \)
\( \gamma = \) a dimensionless diffusivity

The dimensionless diffusivity, \( \gamma \), is given by

\[
\gamma = \frac{a^2 \varepsilon H}{\nu} = \frac{1}{Pr} \frac{\varepsilon H}{\nu} 
\]

where \( Pr = \) the Prandtl number, \( \nu/a^2 (= c_\text{p} \mu/k) \).

The second condition in equation (3.7a) can be seen from the following argument:

From equation (3.8), for small temperature differences

\[
\Theta = \frac{t - t_0}{q r_0/k} \Rightarrow \frac{k}{q r_0/k} = \frac{\varepsilon H}{q r_0} 
\]

Then

\[
\Theta = \frac{k}{q r_0} \Rightarrow \frac{\varepsilon H}{q r_0} 
\]

since \( r^* = r_0 c \) (equation (3.8)), where \( c = u^* \).

\[
\Theta \Rightarrow \frac{\partial \Theta}{\partial t} \frac{c}{r^*} \Rightarrow \frac{\varepsilon H}{q r_0} \frac{\partial \Theta}{\partial t} \frac{c}{r^*} 
\]

since \( \frac{\partial \Theta}{\partial r} = \frac{c}{k} \) from Fourier's empirical relation of heat condition, at \( r = r^* \).

Then...
Then, since \( r^* = \rho c \)

\[
\begin{align*}
\partial \Theta \partial r^* &= \frac{1}{\partial r^*} \partial \theta \\
\partial r^* \partial r &= \frac{1}{\rho c} \partial \theta
\end{align*}
\]

whence

\[
\frac{\partial \Theta}{\partial r} = \frac{1}{\rho c}
\]

Equation (3.8a) may be written as

\[
\gamma = \frac{1}{Pr} \gamma + \frac{\epsilon}{H}
\]

since \( \epsilon_H \) is taken as equal to \( \epsilon_{H^*} \), the eddy diffusivity of momentum, for the range \( 0.7 \leq Pr \leq 100 \).

II. Separation of Entry and Fully-Developed Solutions: The complete temperature profile is denoted by \( \Theta \) and is divided into two parts, one of which is the fully-developed temperature \( \Theta_1(r^*, x^*) \).

The solution for \( \Theta \) is then written as

\[
\Theta = \Theta_1 + \Theta_2
\]

and \( \Theta_2 \) is the remainder which represents the developing temperature profile. \( \Theta_2 \) approaches zero for large values of \( x \) and can be treated by application of Gartl's method.

The Fully-Developed Solution: Equation (3.6a) may be applied separately to the fully-developed temperature profile \( \Theta_1 \) and the developing temperature profile \( \Theta_2 \). This is a result of the linearity of the equation (3.6a):

For/
For \( \Theta_1 \), then, equation (3.6a) is rewritten as

\[
\frac{\partial \Theta_1}{\partial x} = \frac{P_c^+}{r^+} \frac{\partial}{\partial r^+} \left[ r^+ \frac{\partial \Theta_1}{\partial r^+} \right]. \tag{3.11}
\]

The condition for fully-developed flow for the U.W.H.F. case is that for all \( r^+ \)

\[
\frac{\partial \Theta_1}{\partial x} = \text{constant} \tag{3.12}
\]

or

\[
\Theta_1 = \left[ \frac{h}{R_e^+} \right] r^+ \times \mathcal{G}(r^+) \tag{3.12a}
\]

\[
\Theta_1 = \left[ \frac{h}{P_c^+} \right] r^+ \times \mathcal{G}(r^+) \tag{3.12b}
\]

where, \( P_c^+ \), the Peclet number, is defined in terms of the Reynolds number \( R_e \) and Prandtl number \( P_c \), as \( P_c = R_e P_c \). \( \mathcal{G}(r^+) \) is the radial variation of the fully-developed temperature profile, dimensionless. No attempt was made to satisfy conditions at \( r = 0 \) since the expression holds for fully-developed conditions.

A differential equation can be written for \( \mathcal{G}(r^+) \) by introducing equations (3.12b) into equation (3.11):

\[
\frac{h u^+}{P_e} = \frac{\mathcal{G}(r^+)}{r^+} \frac{d}{dr^+} \left[ r^+ \frac{d \mathcal{G}}{dr^+} \right]. \tag{3.11a}
\]

The conditions of U.W.H.F. and of profile symmetry at the center-line of the tube are then

\[
\left. \frac{\partial \Theta_1}{\partial r^+} \right|_{r^+ = 0} = \frac{1}{P_c^+} \mathcal{G}(0) \left. \frac{d \mathcal{G}}{dr^+} \right|_{r^+ = 0} = 0 \right. \tag{3.13}
\]
The solution of equation (3.11a) is possible if the variation of $u^+$ and $\gamma$ with $r^+$ is known. Sparrow assumed the velocity profile from Dcissler's analysis. For $\gamma$, the expressions used were

$$\gamma = \frac{1}{Pr} \cdot (0.124)^2 u^+ y^+ \left[ 1 - e^{-(0.124)^2 u^+ y^+} \right] \quad . \quad 0 < y^+ < 26 \quad . \quad (3.11a)$$

$$\gamma = \frac{1}{Pr} \cdot (0.36) y^+ \left[ 1 - \frac{y^+}{r_0^+} \right] - 1 \quad . \quad y^+ > 36 \quad . \quad (3.11b)$$

Equation (3.11a) was borrowed directly from Dcissler (71); equation (3.11b) was calculated from the definition of $\epsilon^{+1}$ by the use of the logarithmic velocity-profile and a linear variation of shear stress. Equation (3.11a) holds for the region near the tube-wall, and equation (3.11b) is employed for the region away from the wall. At $y^+ = 26$, the average of these two values is used. The term "minus one" in equation (3.11b) is retained for the region $26 < y^+ < \frac{r_0^+}{2}$ but is dropped for larger values of $y^+$.

Sparrow obtained numerical solutions of equation (3.11a) with the boundary conditions (3.15) for Reynolds number of $50 \times 10^3$, $100 \times 10^3$ and $500 \times 10^3$ and for Prandtl numbers $0.7$, $10$ and $100$.

The Entry-Region Solution Equation (3.10) shows that $\Theta_2 = \Theta - \Theta_1$ satisfies equation (3.6a) and therefore, as for the fully-developed solution an expression similar to equation (3.11) may be written

\[
\frac{\partial \Theta_2^+}{\partial x^+} = \frac{r_0^+}{Pr^+} \cdot \frac{\partial}{\partial r^+} \left[ r^+ \gamma \frac{\partial \Theta_2^+}{\partial r^+} \right] \quad . \quad . \quad . \quad . \quad (3.15)
\]
From equation (3.10),

$$\frac{\partial \theta_2}{\partial \xi} = \frac{\partial \theta}{\partial \xi} - \frac{\partial \theta_1}{\partial \xi} \tag{3.10a}$$

The boundary conditions for $\theta$ and $\theta_1$ are equations (3.7a) and (3.15) respectively. These lead to the following conditions

$$\frac{\partial \theta_2}{\partial \xi} (x^*, r_0^*) = 0, \quad \frac{\partial \theta_2}{\partial \xi} (x^*, 0) = 0 \tag{3.10b}$$

Also, since $\theta = 0$ at $x = 0$, then from equations (3.10) and (3.12b)

$$\theta_2(0) = -\phi(x^*) \tag{3.10c}$$

Equation (3.15) is suitable for solution by the Graetz method if the boundary conditions (3.10b) and (3.10c) are applied.

Suppose that the solution for $\theta_2$ has the form

$$\theta_2 = C \chi(x^*) \phi(x^*) \tag{3.16}$$

where $\chi$ is the exponentially decaying portion of the entrance-region temperature profile, dimensionless.

From equation (3.15), the following expression is satisfied by $\chi$ and $\phi$

$$\chi = \exp \left[ -\frac{2\varepsilon^2}{Re} x^* \right] = \exp \left[ -\frac{4\varepsilon^2}{Re} \cdot \frac{x}{d} \right] \tag{3.16a}$$

and

$$\frac{d}{dx} \left[ x^*, \phi(x^*) \frac{d\phi}{dx} \right] \cdot \left[ \frac{2\varepsilon^2}{Re} \cdot \frac{x^*}{x_0} \cdot \frac{u^*}{d} \right] \phi = 0 \tag{3.16b}$$
\( \beta^2 \) is a constant which arises from the product solution and is chosen positive so that \( \theta_2 \) approaches zero for large \( x \).

Equation (3.16a) with boundary conditions (3.10b) is an eigenvalue problem of the kind generally known as the Sturm-Liouville type.

Solutions for \( \theta_2 \) are possible only for discrete, though infinite set of values, and the solution is written, after introducing the necessary boundary conditions, as

\[
\theta_2 = \sum_{n=0}^{\infty} C_n \phi_n \exp \left[ - \frac{4\beta^2}{Re} \frac{x}{d} \right] \quad \cdots \quad (3.17)
\]

where \( C_n \) = coefficient in the series expansion, dimensionless,
\( \phi_n \) = eigenfunctions,
\( \beta_n^2 \) = the corresponding eigenvalues to the eigenfunctions,
\( d \) = the pipe diameter.

The set of constants \( C_n \) from the condition

\[
\theta_2(0, r^+) = - G(r^+)
\]

since \( \theta = 0 \) at \( x = 0 \) and from the equations (3.10) and (3.12b).

From the properties of the Sturm-Liouville system, then,

\[
C_n = \frac{\int_0^{r^+} \left[ - G(r^+) \right] r^+ u_r^+ \phi_n^2 dr^+}{\int_0^{r^+} r^+ u_r^+ \phi_n^2 dr^+} \quad \cdots \quad (3.18)
\]
The Complete Temperature Solution  From equations (3.10), (3.12b) and (3.17)

\[ \theta = \frac{h}{P_c} x^+ + C(x^+) + \sum c_n \theta_n(x^+) \exp \left[ -\frac{2 \delta_n^2}{P_c} x^+ \right] \quad \cdots \quad (3.19) \]

Nusselt Numbers: The local heat-transfer coefficient, \( h \), and the Nusselt number, \( Nu \), are defined as

\[ h = \frac{q}{\theta_0 - \theta_b}, \quad Nu = \frac{hd}{k} \quad \cdots \quad \cdots \quad (3.20) \]

where \( q \) is the prescribed uniform heat flux, \( k \) is the thermal conductivity and \( \theta_b \) is the bulk temperature. The Nusselt number may be written in terms of dimensionless variables

\[ Nu = \left( \frac{\theta_0}{\theta_0 - \theta_b} \right)^2 \quad \cdots \quad \cdots \quad (3.20a) \]

The value of \( \theta_0 \) is given by equation (3.19) with \( x^+ = r_0^+ \) and

\[ \theta_0 = \frac{t_0 - t_b}{qr/\theta_0} = \frac{\theta_0}{q} \cdot x^+ \]

where \( t_b \) is the entering fluid-temperature.

Equation (3.20) then becomes

\[ Nu = \left( \frac{2}{\theta_0} \right) \frac{C(x^+)}{\sum c_n \theta_n(x^+)} \exp \left[ -\frac{2 \delta_n^2}{P_c} x^+ \right] \quad \cdots \quad (3.21) \]

Fully-developed Nusselt Numbers: These are evaluated by considering large values of \( x/d \) when the entire summation in equation (3.21) vanishes since \( C_0 = 0 \). Then

\[ Nu_\infty = \frac{2}{C(x^+)} \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (3.22) \]
Since the quantity $G(x_0^*)$ depends separately on the Reynolds number $Re$, and the Prandtl number $Pr$, so also does $Nu_\infty$. Curves are straight lines and the results do not plot as a simple power of $Pr$ for a fixed $Re$.

**Entrance-Region Nusselt Numbers**: These are expressed by the ratio $Nu/Nu_\infty$ which is obtained by combining equations (3.21) and (3.22).

\[
\frac{Nu}{Nu_\infty} = \frac{1}{1 + \sum A_n \exp \left[ - \frac{4\beta_n^2 x}{Re d} \right]}
\]  
(3.23)

where $A_n = c_n g_n(x_0^*)/G(x_0^*)$.

The ratio $Nu/Nu_\infty$ given by equation (3.23) depends separately upon the $Re$ and $Pr$ as well as on $x/d$. To evaluate equation (3.23) it is necessary to give $A_n$ and the eigenvalues $\beta_n^2$. The entrance length is measured from the point of departure of ratio $Nu/Nu_\infty$ from unity.
FIG. 6.- DEISSLER MODEL OF BOUNDARY-LAYER GROWTH FOR HEATED ROUND TUBES. [10]
Deissler's Analysis of Developing Thermal and Concentration Boundary Layers for Round Tubes.

Deissler's analysis of heat transfer flow in an entrance region utilized integral methods, and the usual boundary layer assumptions used with integral methods were made.

(a) The effects of heat transfer and friction are confined to fluid layers close to the wall, i.e., the thermal and flow boundary layers respectively, (figure 6).

(b) The temperature and total pressure are constant along the length of the duct for the region outside the boundary layers.

(c) The temperature and velocity distributions outside the boundary layers are assumed uniform. Other assumptions made in Deissler's analysis were

(d) The eddy diffusivities for momentum $\epsilon_u$, heat $\epsilon_H$ and mass transfer $\epsilon_I$ are equal, or $\alpha=1$.

(e) In the case of mass transfer, the concentration of the diffusing substance is small enough that the mass transfer does not change the velocity.

1. Heat-flow equations for thermal Boundary Layers. In order to obtain a relationship between the thermal boundary-layer thickness $\lambda_h$ and the distance along the passage $x$, the energy balance on an annulus of fluid of differential length of $dx$ was written. Heat energy flows into the differential annulus by convection through plane 1 and leaves through plane 2. In addition, there is a radial flow of energy from the tube by conduction and a radial flow of energy by convection at $\lambda_h$. By definition of the boundary/
boundary layer there can be no radial heat flow at \( h \) due to temperature gradient. Equating the heat energy entering the annulus to that leaving, for constant \( c_p \),

\[
q \cdot 2\pi r \cdot dx = \left( \int_0^h \rho u \cdot 2\pi r \cdot dy \right) \left[ \left( \int_0^h \rho u \cdot 2\pi r \cdot dy \right) \right] - \left( \int_0^h \rho u \cdot 2\pi r \cdot dy \right) \left( \int_0^h \rho u \cdot 2\pi r \cdot dy \right)
\]

\[
c_p g \left[ \int_0^h \rho u \cdot 2\pi r \cdot dy \right] = \left( \int_0^h \rho u \cdot 2\pi r \cdot dy \right)
\]

since \( r = r = y \), the equation reduces to

\[
q \cdot 2\pi r \cdot dx = d \left[ \int_0^h c_p g \rho u (r_o - y) dy \right] = c_p g \left[ \int_0^h \rho u (r_o - y) dy \right]
\]

The total temperature outside the thermal boundary-layer does not vary with \( x \) since no heat energy can penetrate the region outside the boundary layer.

Equation (3.24) can be written

\[
q \cdot 2\pi r \cdot dx = d \left[ \int_0^h c_p g \rho u (r_o - y) dy \right]
\]

where \( r_o = \) inside tube radius

\( x = \) distance from point at which heat or mass transfer begins

\( d = \) inside diameter of tube

\( q = \) rate of heat transfer at inside wall toward tube center

\( h = \) thermal or diffusion boundary-layer thickness

\( \rho = \) mass density of fluid

\( u = \) time-average velocity parallel to axis to tube

\( c_p = \) specific heat of fluid at constant pressure

\( y = \) distance from wall

\( \tau_h \)
\[ t_h = \text{temperature of fluid outside thermal boundary-layer} \]
\[ g = \text{conversion constant, 32.2 } \text{ft}./\text{sec}^2 \]
\[ t = \text{the temperature} \]
\[ h_h^* = \text{dimensionless thermal or diffusion boundary-layer thickness}, \]
\[ \sqrt{\frac{\tau_c}{\rho_o \Omega}} h_h = \frac{u^*}{\nu} h_h \]
\[ \frac{\mu_o}{\rho_o} \]

Subscripts
\[ o = \text{terms evaluated at the wall} \]
\[ * = \text{dimensionless quantities} \]

Boundary conditions at Wall

\[ U.T. = \text{Uniform Wall Temperature, isothermal } (t_o = \text{constant}) \]
\[ U.H.F. = \text{Uniform Wall Heat Flux, non-isothermal } (q_o = \text{constant}) \]

(ii) \text{For Uniform Wall Heat Flux, U.H.F., and constant fluid properties, } q_o \text{ is constant and equation (3.25) can be integrated} \]

\[ \frac{q_o c_p}{c_p} \int_0^x (x = X) = \int_0^x d \left[ \int_0^{h_h} \rho u(t-t_h)(r_o-y)dy \right] \]

\[ \int \]

1. \text{So,} \]

\[ \frac{q_o c_p}{c_p} x = \left[ \int_0^{h_h} \rho u(t-t_h)(r_o-y)dy \right] x = x \]
\[ \left[ \int_0^{h_h} \rho u(t-t_h)(r_o-y)dy \right] x = 0 \]

and since \[ h_h = 0 \text{ at } x = 0 \]

\[ \frac{q_o c_p}{c_p} x = \left[ \int_0^{h_h} (t-t_h) \rho u(r_o-y)dy \right] x = x \]  \[ \text{... ... ... ... ... (3.26)} \]
and equation (3.26) becomes
\[
\frac{1}{2} \rho \frac{d}{d} \int_{0}^{L} (t-t_0) \, du \, dy. \quad \cdots \cdots \cdot (3.27)
\]
where \( D = \) the tube diameter = \( 2r_0 \), so that the left-hand side of the equation (3.27) is not a dimensionless quantity. Equation (3.27) can be written in a dimensionless form as follows:

\[
\tau^* = \frac{\rho}{\rho_0} \sqrt{\frac{\tau^*}{\tau_0}} \, \left( \frac{t-t_0}{r_0} \right) = \frac{\rho}{\rho_0} \, \mathbf{u}^*(t-t_0) \quad \cdots \cdots \cdot (3.28)
\]

where \( \tau_0 = \) shear stress in fluid at wall

\[
\mathbf{u}^* = \sqrt{\frac{\tau_0}{\rho_0}} \quad \cdots \cdots \cdot \quad (2.34)
\]

From equation (3.29)

\[
\frac{t-t_0}{r_0} = \frac{\rho}{\rho_0} \, \mathbf{u}^* \quad \cdots \cdots \quad (3.30)
\]

Also,

\[
(t-t_0) = (t-t_0) - (t_0-t_0) = \frac{\rho}{\rho_0} \, (t-t_0) \quad \cdots \cdots \cdot (3.31)
\]

Equation (3.27) can be written in dimensionless form by substitution of equation (3.31) and the following definitions:

\[
F^* = \mathbf{u}^* \, \mathbf{u}^* \quad y^* = \mathbf{y}^\mathbf{y}^* \quad u^* = \mathbf{u}^\mathbf{u}^*
\]

where \( F^* \) is the dimensionless force
\( y^* \) is the dimensionless distance from the wall or wall
\( u^* \) is the dimensionless velocity or velocity parameter

\( y \)
\( \nu \) is the kinematic viscosity \( \frac{\mu}{\rho_0} \).

Equations (3.27) then becomes for constant fluid properties, i.e.,
\[ \rho = \rho_0, \]
\[ \begin{align*}
\frac{d}{dx} u^* &= \frac{1}{\nu} \frac{1}{\sqrt{\frac{x}{x_0}}} \frac{1}{2} \int_0^{\nu} u^* \left( \frac{q}{c_p \rho} (t^* - t_{L*}) \left( \frac{r^*}{r_0} - y^* \right) \right) dy^* \\
\end{align*} \]

\[ \Rightarrow \frac{d}{dx} = \frac{1}{2} \int_0^{\nu} u^* (t^* - t_{L*}) \left( \frac{r^*}{r_0} - y^* \right) dy^* \]  \hspace{1cm} (3.33)

This expression relates the entrance length to the boundary layer thickness for the boundary condition of \( U(0) = V \).

(iii) Uniform Wall Temperature, \( U(W) = T_0 \) at Wall. Constant Fluid Properties. In this case \( q \) is not constant and therefore \( r_0 \) is not constant since the wall shear stress \( \tau_0 \) then varies with \( x \). Starting with equation (3.25),
\[ \begin{align*}
q \int_0^{r_0} dx &= d \int_0^{\nu} \frac{c_p \rho u (t^* - t_{L*}) (r^* - y^*)}{q} dy^* \\
\end{align*} \]

or \( r_0 \)
\[ \frac{d}{dx} = \frac{d}{2} \left[ \frac{c_p \rho u (t^* - t_{L*}) (r^* - y^*)}{q} \right] dy^* \]  \hspace{1cm} (3.34)

Substituting equations (3.31) and (3.32) equation (3.34) becomes
\[ \begin{align*}
\frac{d}{dx} &= \frac{1}{2} \left[ \frac{c_p \rho u (t^* - t_{L*}) (r^* - y^*)}{q} \right] \int_0^{\nu} u^* \left( \frac{q}{c_p \rho} (t^* - t_{L*}) \left( \frac{r^*}{r_0} - y^* \right) \right)^2 dy^* \\
\end{align*} \]

\[ d(x) = \frac{1}{r_0} \int_0^{\nu} u^* \left( \frac{q}{c_p \rho} (t^* - t_{L*}) \left( \frac{r^*}{r_0} - y^* \right) \right) dy^* \]

for constant properties, \( \rho / \rho_0 = 1 \), and rearranging equation (3.34) becomes/
\[
\frac{d\tilde{x}}{d\tilde{r}} = \frac{1}{2} \left[ \frac{t_{r}^{+}}{t_{h}^{+}} \int_{0}^{t_{h}^{+}} \frac{u^+(t^+ - t_{h}^{+})}{t_{h}^{+}} (r_{o}^{+} - y^{+}) dy^{+} \right]
\]

which becomes upon integration,

\[
\left[ \frac{1}{t_{h}^{+}} \int_{0}^{t_{h}^{+}} u^+(r_{o}^{+} - y^{+}) dy^{+} \right]
\]

\[
\frac{d\tilde{x}}{d\tilde{r}} = \frac{1}{2} \left[ \frac{t_{r}^{+}}{t_{h}^{+}} \int_{0}^{t_{h}^{+}} \frac{1}{r_{o}^{+}} \int_{0}^{t_{h}^{+}} \frac{u^{+}(r_{o}^{+} - y^{+}) dy^{+}}{t_{h}^{+}} \right]
\]

where the upper limit of integration is the same as the term in the square bracket evaluated at the \( \frac{d\tilde{x}}{d\tilde{r}} \) value concerned, i.e., a recursive integral.

Equation (3.35) is the expression that relates the thermal entrance length to the boundary-layer thickness for the boundary condition of U.W.T.

The two equations (3.33) and (3.35) for transfer rates at the entrance region differ by about 5 to 7\% on individual points. Equation (3.33) for the U.W.H.F. boundary condition gives values which are always higher than for the U.W.T. condition.

(iv) **Mass Transfer Entrance Lengths:** For mass transfer, similar equations to those for heat transfer can be written provided the concentration of the diffusing material is small - assumption (c). It follows that for the boundary conditions

(a) **Uniform Mass Flux, U.W.F., Constant Fluid Properties:** Equation (3.33) becomes for mass transfer

\[
\tilde{x} = \frac{1}{4} \int_{0}^{t_{h}^{+}} (c_{h}^{+} - c^{+}) u^{+}(r_{o}^{+} - y^{+}) dy^{+} \quad \ldots \ldots \ldots \ldots \quad (3.36)
\]
\( m_0 \) is the rate of mass transfer toward tube centre per unit area at wall;

where \( c^* \) is the concentration parameter, \( \frac{(c_0 - c)}{m_0} \)

\( c \) is the concentration of diffusing substance

\( c_0 \) is the concentration of diffusing substance at wall

\( c_\lambda^* \) is the value of \( c^* \) at \( \lambda \)

\( c_\lambda \) is the concentration of diffusing substance at \( y=\lambda \)

(b) Uniform Wall Concentration, \( u, w, C_0 \) Constant Fluid Properties

Equation (3.35) can be written for mass transfer

\[
\frac{x}{d} = \frac{1}{2} \left[ \frac{c_\lambda^*}{\tau_0} \int_{r_0}^{r_\lambda} \left( \frac{c_\lambda^* - c}{u} \right) u^+ (r_0^+ - y^+) dy^+ \right]
\]

when again the upper limit of integration is the same as the term in the square bracket evaluated at the \((x/d)\) value concerned.

(v) Velocity, Temperature and Concentration Distributions

The differential equation for shear stress, heat transfer and mass transfer can be written as follows:

\[
\begin{align*}
\tau &= \mu \frac{du}{dy} + \rho \varepsilon u \frac{du}{dy} \\
q &= -k \frac{dt}{dy} - \rho \varepsilon \frac{dt}{dy} \\
m &= -\lambda \frac{dc}{dy} - \varepsilon \frac{dc}{dy}
\end{align*}
\]

where \( \tau \) is the shear stress in fluid; \( \mu \) is the absolute viscosity of the fluid; \( \varepsilon \) are the coefficients of eddy diffusivity of momentum, heat and mass respectively; \( k \) is the thermal conductivity of fluid; \( m \) is the mass-transfer rate toward tube centre per unit area; \( \lambda \) is the molecular diffusivity; \( c \) is the concentration of diffusing substance.
Doissler assumed that $\frac{e_U}{e_M} = e_H$, or $\alpha = 1$.

Equations (3.32) can then be re-written in dimensionless form as

$$
\tau = \frac{\mu}{\tau_0} \left( \frac{\epsilon_U}{\mu_0 \rho_0} \right) \frac{du^+}{dy^+} \quad \text{(3.39)}
$$

where subscript $o$ refers to values at wall, and also

$$
a_q = \left( \frac{k}{Pr_o} \right) + \left( \frac{\rho c_p}{a_H} \right) \frac{\epsilon_U}{\mu_0 / \rho_0} \frac{dt^+}{dy^+} \quad \text{(3.39)}
$$

$$
m = \left( \frac{- \lambda}{Sc_o} \right) + a_M \frac{\epsilon_U}{\mu_0 / \rho_0} \frac{dc^+}{dy^+} \quad \text{(3.39)}
$$

where $a_H$ is the ratio of eddy diffusivities, $e_H/e_U$ and $a_M$ is the ratio of $e_M/e_U$; $Pr_o$ is the Prandtl number with properties evaluated at $t_o$; $Sc_o$ is the Schmidt number at wall, $\mu_0/\rho_0 \lambda_o$.

From assumption (d) it is seen that $a_H = a_M = a = 1$.

Equations (3.39) can be written in integral form for the region close to the wall with the preceding assumptions and constant fluid properties as

$$
u^+ = \int_{y^+}^{y^+} \frac{dy^+}{1 + n^2 u^+ y^+ (1 - e^{-n^2 u^+ y^+})} \quad \text{(3.40)}
$$

$$
t^+ = \int_{y^+}^{y^+} \frac{dy^+}{\frac{1}{Pr} + n^2 u^+ y^+ (1 - e^{-n^2 u^+ y^+})} \quad \text{(3.41)}
$$

$$
c^+ = \int_{y^+}^{y^+} \frac{dy^+}{\frac{1}{Sc} + n^2 u^+ y^+ (1 - e^{-n^2 u^+ y^+})} \quad \text{(3.42)}
$$
after writing the expression for $\epsilon$ close to the wall

$$\epsilon = n^2 u y (1 - e^{-n^2 u y / \mu}) \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.43)$$

in dimensionless form for constant properties as

$$\frac{\epsilon}{\nu^+} = n^2 u^+ y^+ (1 - e^{-n^2 u^+ y^+}) \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.44)$$

Equations (3.40), (3.41) and (3.42) give the $u^+/y^+$, $t^+/y^+$ and $c^+/y^+$ relationships and were solved by numerical iteration methods.

In the present work these were solved numerically by the Runge-Kutta Range method.

For the region away from the wall, the relevant equation for $\epsilon$ is

$$\epsilon = k^2 \frac{\left(du/dy\right)^3}{\left(du^2/dy^2\right)^2} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.45)$$

where $k$ is the Karman constant. Equation (3.45) can be substituted in equations (3.39). Division of first equation in expressions (3.39) by the second and third equations, for constant fluid properties and with assumptions (a), (b), (c) and (d)

$$u^+_1 = t^+_1 = c^+_1 = k \ln\left(\frac{y^+_1}{y^+}\right) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.46)$$

where the equations are integrated from $y^+_1$, the lowest value of $y^+$ for which the equations for flow at a distance from the wall will apply, to $y^+$.

In Deissler's analysis the values of the constants are $n=0.124$, $k=0.56$ and $y^+_1=26$.

(vi) **Relations among** **Russo**l, **Reynolds** and **Brandt** or **Schmidt** numbers for constant properties.

The
The Nusselt numbers for heat and mass transfer and the Reynolds number are given by

\[ Nu = \frac{2 Pr^*}{t_b} \]  
\[ Nu^1 = \frac{2 Sc^*}{c_b} \]  
\[ Re = 2 u_b^* \]  

where \( Nu^1 \) is the Nusselt number for mass transfer which strictly speaking is the Sherwood Number, \( Sh \); \( u_b^* \) is the bulk velocity parameter, \( u_b \sqrt{\frac{\tau}{\rho_0}} \), and \( u_b \) is the bulk velocity at cross-section of tube.

Also,

\[ t_b = \frac{\int_0^\phi t^* u^* (r^0 - y^*) dy^*}{\int_0^\phi u^* (r^0 - y^*) dy^*} \]  
\[ c_b = \frac{\int_0^\phi c^* u^* (r^0 - y^*) dy^*}{\int_0^\phi u^* (r^0 - y^*) dy^*} \]  
\[ u_b = \frac{2}{(r_0^*)^2} \int_0^\phi u^* (r^0 - y^*) dy^* \]  

(vii) Entrance Heat or Mass Transfer Nusselt numbers and Reynolds numbers.

Local/
Local Nusselt and Reynolds numbers can be obtained from equations (3.47), (3.48) and (3.49) as for fully-developed flow, with \( t_p^* \) and \( c_p^* \) in equations (3.50) and (3.51) being replaced by

\[
c_p^* = \frac{\int_0^{\lambda_h^*} u^*(r_0^* - y^*) \, dy^* + \int_0^{\lambda_h^*} \lambda_h^* u^*(r_0^* - y^*) \, dy^*}{\int_0^{r_0^*} u^*(r_0^* - y^*) \, dy^*}
\]

and

\[
c_p^* = \frac{\int_0^{\lambda_h^*} c^* u^*(r_0^* - y^*) \, dy^* + \int_0^{\lambda_h^*} \lambda_h^* u^*(r_0^* - y^*) \, dy^*}{\int_0^{r_0^*} u^*(r_0^* - y^*) \, dy^*}
\]

Equation (3.54) shows that the integral in the numerator was broken into two parts, because \( c^* \) is constant and equal to \( c_h^* \) outside the concentration boundary-layer. The relation between Nusselt number, \( Nu \), and \( y/d \) for various Reynolds number values with \( U, H, F \) was obtained by assuming values of the parameters \( r_0^* \) and \( \lambda_h^* \) and by calculating the various quantities from equations (3.48), (3.49), (3.36) and (3.54).

Dolisier's expression for entrance-region heat transfer in fully-developed turbulent flow for a circular duct with Uniform Wall Temperature is given by equation (3.35)

\[
\frac{h}{d} = \frac{1}{2} \left[ \frac{v}{v_0} \right] \left[ \frac{1}{v_0} \int_{v_0}^{\infty} \frac{(t^+ - t_\lambda^+)}{t_\lambda^+} u^+ (x^+ - y^+) dy^+ \right].
\]

(3.35)

where the upper limit of integration is the same as the term in the square bracket evaluated at the x/d value concerned. This is a recursive integral, and a method is proposed which will allow the equation to be evaluated numerically using a computer programme similar to that employed for the Uniform Heat Flux problem.

In the case of the Uniform Wall Temperature problem, \( v_0 \) is not constant, and \( v_0 \) varies for each Reynolds number. Equation (3.49) in section (3.1.4) defines the Reynolds number in terms of \( v_0 \) and the bulk velocity, \( u_b^+ \)

\[
Re = 2u_b^+ v_0^+.
\]

(3.49)

If \( v_0^+ \) is allowed to vary with \( T_0 \), the shear stress at the wall, equation (3.49) is then re-written as

\[
Re = 2u^+ (\lambda H) v_0^+.
\]

(3.55)

where:/
where \( u^* \) is now the dimensionless velocity at each separate value of \( x/d \) of the tube; \( \lambda_m^* \) is the thickness of the thermal boundary-layer.

For mass transfer, equation (3.55) is written as

\[
Re = 2u^*(\lambda_m^*)x_0^* \quad . . . . . . . . \quad (3.55a)
\]

where \( \lambda_m^* \) is the thickness of the diffusion boundary-layer.

(1) Substitution of equation (3.55a) into equation (3.35), given,

\[
x'\left(\lambda_m^*\right) = \int \left[ \frac{c^*_h u^*(\lambda_m^*)}{Re} \int_0^\lambda_m^* \left[ \frac{2u^*(\lambda_m^*)}{Re} \int_0^{\lambda_m^*} \frac{c^*_h}{c^*_n} u^*(y^*) \left( \frac{Re}{2u^*(y^*)} - y^* \right) \right] dy^* \right]
\]

\[
. . . . . . . . \quad (3.56)
\]

where \( c^*_h \) and \( Re \) are constants;

\[
c^* = c^*(y^*)
\]

\[
u^* = u^*(y^*)
\]

Now let

\[
\frac{c^*_h u^*(\lambda_m^*)}{Re} = g(t) \quad \text{i.e.,} \quad \lambda_m^* = t \quad . . . . . . . . \quad (a)
\]

\[
\frac{2u^*(\lambda_m^*)}{Re} = h(t) = \frac{2}{c^*_h} g(t) \quad . . . . . . . . \quad (b)
\]

\[
. . . . . . . . \quad (3.57)
\]
\[ f(x) = \int_a^b \left[ h(x) \alpha(x) ds \right] + \int_a^b \left[ h(t) \alpha(t) ds \right] \cdot \cdot \cdot (3.58) \]

Also,

\[ g(a) = h(a) = \alpha(a) = 0 \cdot \cdot \cdot (3.59) \]

Let \( h(t) = A g(t) \)

Then

\[ A = \frac{2}{c} \frac{h(t)}{g(t)} \cdot \cdot \cdot (3.60) \]

Hence

\[ \int_a^b \left[ \int_a^b \alpha(x) ds \cdot dt \right] \cdot (3.61) \]

where \( h(x) = \int_a^b \alpha(x) ds = \cdot \cdot \cdot \]

Now/
Now let
\[ u = \int_0^t x(s) \, ds \quad \Rightarrow \quad du = x(t) \, dt \quad \ldots \quad (3.62) \]
\[ v = \frac{1}{2} g^2 \quad \Rightarrow \quad dv = g g^1 \, dt \quad \ldots \quad (3.63) \]

The first term in equation (3.61) can now be written as
\[ \frac{1}{2} g(t)^2 \int_0^t x(s) \, ds \bigg|_{t=0}^{t} = \int_0^t \frac{1}{2} g(t)^2 \cdot x(t) \, dt \]

Equation (3.61) is now re-written as
\[ \frac{\mathcal{F}(x)}{\Lambda} = \frac{1}{2} \int_0^t g(t)^2 x(t) \, dt + \frac{1}{2} \left[ g(t) \right]^2 2 \int_0^t x(s) \, ds \bigg|_{t=0}^{t} \quad \ldots \quad (3.64) \]

Put
\[ h(x) \int_0^x x(s) \, ds = \theta(x) = \left[ \ldots \right] \quad \ldots \quad (3.65) \]

Substitution of equation (3.65) into equation (3.64) gives
\[ \frac{\mathcal{F}(x)}{\Lambda} = \frac{1}{2} \left[ g(\theta(x)) \right]^2 2 \int_0^{\theta(x)} x(t) \, dt + \frac{1}{2} \int_0^{\theta(x)} \left[ g(t) \right]^2 x(t) \, dt \quad \ldots \quad (3.66) \]
(II) Method of solution of Problem by Numerical Calculation:

Given  values of \( \int_0^x \alpha(s) \, ds \) are tabulated. From these results it is possible to find by interpolation

\[
\mathcal{h}(x) \int_0^x \alpha(s) \, ds = \Theta(x)
\]

and

\[
\int \Theta(x) \alpha(t) \, dt
\]

It is now therefore possible to find \( g(\Theta) \) and

\[
\int \Theta(x) \, dt
\]

This means that:

(i) The values of

\[
\int_0^{\lambda_m} \left( \frac{c^+ - c^+(y^+)}{c^+_m} \right) u^+(y^+) \left( \frac{Re}{2u^+(y^+)} - y^+ \right) \, dy^+
\]

are tabulated against \( \lambda_m^+ \)

(ii) \( \Theta(x) \) can be found; i.e.,

\[
\frac{2u^+(\lambda_m^+)}{Re} \int_0^{\lambda_m^+} \left( \frac{c^+_m - c^+(y^+)}{c^+_m} \right) u^+(y^+) \left( \frac{Re}{2u^+(y^+)} - y^+ \right) \, dy^+
\]

(iii)
(iii) From the first tabulation, \[ \int_0^\Theta(x) \alpha(t) \, dt \] is found by interpolation.

(iv) By employing the quadrature formula used in the first tabulation,

\[ \int_0^\Theta(x) \left[ \sigma(t) \right]^2 \alpha(t) \, dt. \]

i.e.,

\[ \int_0^\Theta(x) \frac{c_\Lambda}{\Re^2} u^*(\lambda_m^*)^2 \frac{c_\Lambda - c^*(y^*)}{c_\Lambda} u^*(y^*) \left[ \frac{\Re}{2 u^*(y^*)} - y^* \right] \, dy^* \]

\[ \cdots \cdots \cdots \cdots \cdots \cdots (3.67) \]
3.2. **THE INFLUENCE OF THE PRANDTL AND SCHMIDT NUMBERS ON DEVELOPING THERMAL AND CONCENTRATION BOUNDARY-LAYERS.**

The Kármán (V.3)-Boelter (B.10)-Martinelli (M.7) analogy for heat and momentum transfer shows that the Stanton number $St$, and the Nusselt number, $Nu$, for fully-developed turbulent flow inside smooth tubes takes the form

$$St = \frac{\sqrt{f/2}}{5t^+} \frac{1}{(Pr + \ln(5Pr + 1) + \frac{1}{2}\ln Re \sqrt{f/2})} \quad \cdots \quad (3.68)$$

where $f$ is the friction factor,

$Pr$ is the Prandtl number,

$t^+$ is the dimensionless temperature, $(t_m - t_o)/(t_c - t_o)$,

$Re$ is the Reynolds number,

$t_m$ is the mixed-mean temperature,

$t_c$ is the temperature of the turbulent core,

$t_o$ is the temperature at the wall.

By definition,

$$Nu = St \cdot Pr \cdot Re \quad \cdots \quad (3.68a)$$

Then,

$$Nu = \frac{RePr \sqrt{f/2}}{5t^+(Pr + \ln(5Pr + 1) + \frac{1}{2}\ln Re \sqrt{f/2})} \quad \cdots \quad (3.69)$$

From equation (3.69) it is seen that for turbulent flow

$$Nu = \phi(Re \cdot Pr) \quad \cdots \quad (3.70)$$

rather than being only a constant as in the laminar-flow counterpart of the problem.

An/
FIG. 7 - EFFECT OF PRANDTL NUMBER ON TURBULENT FLOW TEMPERATURE DISTRIBUTION.

\[ Re = 30,000 \]

Pr = 10
Pr = 1.0
Pr = 0.1

\[ t^+ \]

\[ y/r_0 \]
An illustration of the influence of the variation of the Pr on heat transfer is possible by reference to figure 7 which shows the dimensionless temperature profiles against the radial distance from the wall of a circular tube. At low Pr's a rounded profile is obtained which is quite similar to that for laminar flow, whereas at higher Pr's the temperature profile becomes rapidly "square".

Next, from the same figure it is possible to deduce the regime of the greater source of heat-transfer resistance. At high Pr's it is seen that the resistance is confined mainly in the sub-layers, but at low Pr's it is distributed practically over the entire fluid. The explanation for this difference of temperature on turbulent-flow distribution with different Pr's is seen from the following consideration. The total conductance term which appears in the energy equation is

$\left( \frac{\epsilon_H}{v} + \frac{1}{Pr} \right)$

and the relative importance of molecular diffusion and turbulent eddy-diffusion depends directly on the Pr. Along any radius in the tube the molecular conduction term $1/Pr$ is a constant, whereas the eddy conduction term $\epsilon_H/v$ varies from a large value out in the main stream to zero at the wall. The relative magnitude of these two terms determine the whole character of turbulent-flow heat transfer and consequently the shape of the temperature profiles.

At high Pr's, the turbulent eddy-conduction term is the dominant/
FIG. 8 — PRANDTL NUMBER SPECTRUM OF FLUIDS.

Pr

$10^{-2}$  $10^{-1}$  $10^0$  $10^1$  $10^2$  $10^3$

---

- Liquid metals
- Gases $\rightarrow$ Water $\rightarrow$ Oils
- Light organic liquids
dominant one with concentration of the thermal resistance very close to the wall. Once the sub-layers have been penetrated, there is rapid diffusion of heat over the entire fluid which accounts for the quick response to axial surface-temperature variations and very short thermal lengths.

At very low Pr's, the molecular conduction term dominates over the entire fluid region, and the temperature profile is similar to that in laminar flow. Furthermore, the longer thermal entry lengths and the slow response of the fluid to axial changes in wall temperature are both very nearly similar to laminar behaviour. In the analysis described in section 3 the conduction term was omitted since it was assumed that conduction played a very minor role at the centre of a pipe compared with the eddy conduction. This is not entirely true as is seen from the failure of equation (3.68a) to hold for very low Pr's.

The Prandtl number is defined by the ratio

$$Pr = \frac{\text{kinematic viscosity}}{\text{molecular thermal conductivity}} = \frac{\nu}{\alpha} = \frac{\nu/\rho}{k/\rho c_p} = \frac{\nu c_p}{k} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.71)$$

where $\nu$ is the kinematic viscosity, $\alpha$ is the molecular thermal diffusivity, $\mu$ is the viscosity, $\rho$ is the density and $k$ is the thermal conductivity and $c_p$ is the heat capacity at constant pressure. In practical cases the Pr is generally not far removed from unity. The kinematic viscosity is an eddy diffusivity for momentum, or for velocity, in the same sense that the thermal diffusivity is an eddy diffusivity for heat, or for temperature. Figure 8 shows the Prandtl number spectrum of fluids and although the Pr for any particular fluid is affected/
affected by changes in temperature the variation takes place over a limited range.

Since the whole resistance to heat transfer is thought to be confined to a layer near the wall surface within which the transport is only by molecular conduction or diffusion, then the conduction thickness $\lambda_H$ is defined by the expression

$$\lambda_H = \frac{k}{h} \ldots \ldots \ldots \ldots \ldots (3.72)$$

where $k$ is the thermal conductivity of the fluid concerned and $h$ is the local heat transfer coefficient. For a round pipe it is known by experiment that for developed turbulent flow

$$Nu_\infty = \frac{hd}{k} \propto Re^{0.8}Pr^{0.4} \ldots \ldots \ldots (3.73)$$

where $Nu_\infty$ is the fully developed Nusselt number and $d$ is the inside diameter of the circular pipe. Equation (3.72) can be rewritten as

$$\frac{\lambda_H}{d} = \frac{k}{hd}$$

and it is seen from equation (3.73) that

$$\lambda_H \propto \frac{1}{Pr^{0.4}} \ldots \ldots \ldots (3.74)$$

Equations (3.71) and (3.74) show that for fluid systems of $Pr \approx 1$, the thermal boundary-layer $\lambda_H$ is never very small compared with the viscous sub-layer thickness $\lambda_U$. In fact, when the heat transfer for the liquid metal-range is considered for which $Pr \ll 1$, then $\lambda_H$ is a great many times greater than $\lambda_U$.

The situation for mass transfer, though analogous in principle/
principle to the corresponding heat transfer problem, differs
greatly in the practical case. The analogous dimensionless
ratio in mass transfer to the Prandtl number in heat transfer
is the Schmidt number, Sc, and this is defined as

\[ Sc = \frac{\text{kinematic viscosity}}{\text{molecular diffusivity}} = \frac{\nu}{D} = \frac{\mu}{\rho D} \quad \ldots \ldots \quad (3.75) \]

where D is the molecular diffusivity. The ratio has the value
of several thousand in soluble solid-liquid systems and a value
of about two and under for subliming solid-gas systems. It is
seen from this that in general the molecular diffusivity of a
substance is less than the kinematic viscosity or diffusivity of
momentum. For liquids in particular, the kinematic viscosity is
very much greater than the molecular conduction of matter. This
means that the viscous sublayer thickness \( \lambda_U \) in a developed
turbulent flow is very much greater than the concentration
boundary-layer thickness \( \lambda_M \); in other words, the distance that
would be expected to be attained by matter from a subliming or
soluble wall-surface if molecular diffusion were the sole agent
of mass transport.

Again, as for the corresponding heat transfer problem, the
concentration boundary-layer thickness \( \lambda_M \) is defined by the
expression

\[ \lambda_M = \frac{D}{k_c} \quad \ldots \ldots \ldots \quad (3.76) \]

where \( k_c \) is the value of the local mass-transfer coefficient.
Equation (3.76) may be put in the form

\[ \frac{\lambda_M}{d} \]
\[
\frac{\lambda_M}{d} = \frac{D}{dk_c}
\]

and since for developed turbulent flow the following empirical equation is true,

\[
Sh_\infty = \frac{k_c d}{D} \propto Re^{0.8} Sc^{0.4} \quad \ldots \ldots (3.77)
\]

where \(Sh_\infty\) is the fully developed Sherwood number, then, by equation (3.76),

\[
\lambda_M \propto \frac{1}{Sc^{0.4}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.78)
\]

For large values of \(Sc\), as is the case met in practice, \(\lambda_M\) is small compared with \(\lambda_U\) and the location of resistance to mass transfer is then to be found in a region very near the wall.

This result gives a great advantage to mass-transfer measurements over heat-transfer investigations since it offers a means to obtain experimental data in the particularly small region near the wall surface.

The influence of \(Sc\) on concentration entry-lengths in circular ducts is analogous to the influence of the \(Pr\) in the heat-transfer problem discussed above. Equation (3.70) is modified for mass transfer to give the expression

\[
Sh = \chi (Re, Sc) \quad \ldots \ldots \ldots \ldots (3.79)
\]

and similar arguments and conclusions as those for heat transfer can be applied for the mass-transfer problem.
3.2.1. Experimental Investigation in the Influence of \( \sigma \) on Local Transfer Coefficients in the Entrance Region of Circular Ducts.

It would appear from the literature concerning investigations into the possible influence of \( \sigma \) on developing thermal or concentration boundary-layers, that the results are inconclusive. When the experimental data, obtained by various workers in the field, are analysed, many ambiguities are found. The discrepancies arise from various factors associated with basic differences in experimental equipment, and with techniques employed in the different experiments.

Apart from a recent study by Hishida\(^{(6,7)}\) - for air, \( Pr = 0.1 \) to 10 with U.W.T. distribution - and Wolf\(^{(W.9)}\) - for air and carbon dioxide, \( Pr = 0.7 \), with U.W.H.F. distribution - there is no evidence of any investigation specifically concerning the effect of the Prandtl number, \( Pr \), on gases. Most of the work done with gases concerns the measurement of local heat-transfer coefficients for air. This is shown in Tables IV and V. The fact that less work has been carried out in this field may be attributed to the following reasons: (a) the analyses predict a very limited influence of \( Pr \) on entrance-lengths and heat-transfer coefficients; (b) in practice, the \( Pr \)'s are severely restricted to a few numerically close values, because of the limited number of gases suitable for experiment. Investigations of greater accuracy than have hitherto been achieved are needed, before differences in the transfer profiles can be revealed for small ranges of the \( Pr \).
Studies concerned with heat transfer to liquids in the inlet region of circular tubes, suffer in general from the complicating influence exerted by the temperature. The fully-developed velocity profile in the colder calming region undergoes a small but finite change in the heated test-section. Analysis of published data shows extensive ambiguity in the results. These indicate that considerable overlapping of the thermal profiles occurs for equal Prandtl and Reynolds numbers in investigations by the same and/or different workers. This is apparently due to the different heat-input for each investigation, causing variations in the properties of the fluid which are different in each case. An example of this may be found in the work of Hartnett, where it appears from Tables 2 and 3 that the author was unable to reproduce his data for the same Reynolds number.

One conclusion drawn from published heat-transfer data in the entrance region of ducts is that investigations have generally failed to find an influence of $\sigma$ on local heat-transfer coefficients. For air flowing in a pipe, $Pr \sim 0.7$; and for oils, $Pr \sim 60$: this range of $Pr$ would be expected to be sufficient to yield noticeable differences in the transfer profiles.

The investigation of Hartnett for a U.W.H.F. distribution serves as an illustration of this situation, and is discussed in section 2.1.1. The data for water, $Pr = 7.0$ and $7.3$ and oil, $Pr = 60$ to 200 fails to show a discernible effect of the $Pr$ on entrance-lengths and on heat-transfer coefficients in the entry region/
region of the tube. At best, Hartnett's results may be described as inconclusive. It is significant, however, that Hartnett did not allow for property-variation in the fluid which must have been appreciable. The dependency of viscosity on temperature is greater for liquids than for gases since the Pr decreases as the temperature increases. This oversight in Hartnett's work is especially serious with oil, where the role of property-variation is greater because of the greater dependency of its viscosity on the temperature.

A recent investigation with a U.W.T. distribution by Malina and Sparrow (N.15) was more successful in showing the influence of Pr on heat-transfer coefficients in the entrance-region of pipes. The experiments, which are discussed in section 2.1.1., were similar to those of Hartnett. Measurements were taken of heat-transfer coefficients for a hydrodynamically-developed flow of water and oil in an electrically-heated tube. The range of Pr investigated was between 3 and 75. Malina and Sparrow applied corrections for variable fluid-properties, by extrapolating the measured heat-transfer coefficients to the condition of zero wall-to-bulk difference in temperature. The bulk-temperature was 140°F. Their results for Pr = 61 at Re = 34,600, and Pr = 63 at Re = 46,500 agreed well with those obtained by Hartnett, namely, 5%-entrance-lengths of ~3 and ~1.5 tube-diameters respectively. The influence of the Pr on the developing thermal boundary-layers is clearly shown by their results: an increase in/
the Pr leads to a corresponding decrease in the heat-transfer coefficients and in slightly shorter entrance-lengths. The experimental data obtained in this work is more consistent with the smaller influence of Pr predicted by Doissler\textsuperscript{(D.3)}, rather than the much larger one forecast by Sparrow et al.\textsuperscript{(S.33)}.

A further ambiguity in the published data may be discerned from the rather similar results obtained for gases and liquids in the entrance length of pipes. Again, the incongruous findings are, to a large extent, probably due to the different way in which the temperature influences the viscosity of the two fluids.

Investigations concerning the development of thermal boundary-layers in the starting region of circular tubes are thus seen to be at a great disadvantage, owing to the thermal complications resulting from system-heating. The examination of the short entry-region in the tube, $x/d < 2$, presents a formidable problem when attempts are made to obtain accurate heat-transfer coefficients. Axial heat-conduction, however slight, takes place in the fluid itself, regardless of the method used for heating the test-section. It is thus impossible in practice, to provide the initial conditions required in theory.

Edo\textsuperscript{(E.1,E.4,E.5)} attempted to reduce the problems of system-heating, by conducting his investigations with small differences in wall/fluid temperature. The apparent improvement in the accuracy obtained by this method may however be artificial. Compared with the resulting low rates of heat transfer, heat-losses through the electrical/
electrical loads and the exterior of the pipe must be so proportionally large as to become a serious problem.

In investigations of the influence of the $Sc$ on local mass-transfer coefficients one meets with an altogether different type of difficulty. The range of $Sc$ available for experiment is restricted to a wide-spaced series of $Sc$ with numerically close values. For gas/solid exchange, the $Sc$ has values around 1. The disadvantages of using this system is two-fold. Firstly, only a very few gases can be found to give a reasonable range of $Sc$; furthermore, a number of gases that could have been otherwise employed are inadmissible owing to their hazardous nature, e.g. the highly toxic carbon monoxide and the highly explosive hydrogen when in contact with a naked flame. Second, the experimental study of gas/solid systems is beset with a host of technical difficulties when the gas is re-circulated in some type of a closed-loop wind-tunnel. Examples illustrating these difficulties are, (a) achievement of completely leak-proof conditions in the wind-tunnel; (b) successful elimination from the re-circulated gas of traces of the diffusing material; (c) control of pressure in the wind-tunnel and attainment of steady temperatures in the gas throughout a sublimation run.

For gas/liquid exchange, the Schmidt number lies in the range $0.6 < Sc < 2.5$, which is practically the same as that offered by the gas/solid systems. A typical illustration covering this range is in wetted-wall columns (G.7), which are, however, unsuitable/
unsuitable for determination of local mass-transfer rates (section 2.1.2).

Finally, for solid/liquid exchange, the range of $S_c$ is $> 1000$. The entrance-lengths for such a system are extremely small ($L_2$) and determination of transfer rates require experiments of greater accuracy than have hitherto been achieved.
3.3. EFFECT OF WALL BOUNDARY CONDITIONS.

There are several different distributions in wall-temperature which are of general interest (H.5), but the following two are the most widely met in heat-exchanger-design and are therefore of great technological importance:

(a) Uniform Wall Heat Flux (U.W.H.F.): In this case, the fluid is heated in the tube with a heat-rate which is constant per unit of tube length, and a rise in temperature of the tube wall results in the direction of the flow.

(b) Uniform Wall Temperature (U.W.T.): The surface-temperature is here kept constant.

For any set of boundary-conditions in a round tube, a fully developed temperature-profile may be said to exist, when \((t_o - t)/(t_o - t_m)\) is a unique function of \(r/r_o\), independent of \(x\); then,

\[
\frac{t_o - t}{t - t_m} = f \left( \frac{x}{r_o} \right) \quad \ldots \quad (3.80)
\]

or

\[
\frac{\partial}{\partial x} \left( \frac{t_o - t}{t_o - t_m} \right) = 0 \quad \ldots \quad (3.81)
\]

where \(t = \text{temperature}\)

\(t_o = \text{temperature at surface}\)

\(t_m = \text{the mixed-mean temperature} = \frac{\int_0^{r_o} v_x t 2\pi r \, dr}{\int_0^{r_o} v_x 2\pi r \, dr}\)

\(v_x = \text{velocity component in } x \text{ direction}\)

\(r/\)
\[ r = \text{radial distance} \]
\[ r_0 = \text{radius of circular tube} \]

(a) **U.W.T.F. Case**:

The heat transfer coefficient, \( h \), is defined by the equation

\[ \frac{q}{A} = h(t_o - t_m) \quad \ldots \ldots \ldots \quad (3.82) \]

where \( \frac{q}{A} \) is the rate of heat transfer per unit area, and since \( \frac{q}{A} \) and \( h \) are uniform, \( (t_o - t_m) \) is also uniform along the tube, i.e.,

\[ \frac{\partial t_o}{\partial x} = \frac{\partial t_m}{\partial x} \quad \ldots \ldots \ldots \quad (3.83) \]

Expanding equation (3.81), where in general each quantity may vary

\[ \frac{\partial t_o}{\partial x} - \frac{\partial t}{\partial x} - \frac{t_o - t}{t_o - t_m} \left( \frac{\partial t_o}{\partial x} - \frac{\partial t_m}{\partial x} \right) = 0 \quad \ldots \ldots \quad (3.84) \]

From equations (3.83) and (3.84)

\[ \frac{\partial t}{\partial x} = \frac{\partial t_o}{\partial x} = \frac{\partial t_m}{\partial x} \quad \ldots \ldots \ldots \ldots \quad (3.85) \]

which is independent of the radial position in the pipe.

(b) **U.W.T. Case** \( (\partial t_o / \partial x = 0) \)

From equations (3.80) and (3.84)

\[ \frac{\partial t}{\partial x} = \frac{t_o - t}{t_o - t_m} \cdot \frac{\partial t_m}{\partial x} = f \left( \frac{r}{r_o} \right) \cdot \frac{\partial t_m}{\partial x} \quad \ldots \ldots \ldots \quad (3.86) \]

which depends on the radial position in the pipe.

The mass-transfer situation can now be considered, by way of analogy with the heat-transfer problem. The same arguments as/
as those presented above may be advanced for the case of mass transfer and the U.W.H.F. condition now becomes the U.W.M.F. (Uniform Wall Mass Flux); whilst the U.W.T. case is now the U.W.C. (Uniform Wall Concentration).

The U.W.M.F. problem may be considered as the situation where the longitudinal variation of the vapour pressure in a volatile solid, upon which is imposed a temperature-distribution, matches the temperature distribution in U.W.H.F. The equation for the U.W.M.F. is then

\[ \frac{\partial c}{\partial x} = \frac{\partial c_0}{\partial x} = \frac{\partial c_m}{\partial x} \]  \hspace{1cm} (3.87)

where \( c \) is the concentration of the volatile substance in the fluid stream; the subscripts have the usual meanings.

The U.W.C. distribution corresponds to the situation of the present investigation, in which a volatile solid substance at constant vapour-pressure sublimes into a moving fluid stream. The equation for the U.W.C. case is

\[ \frac{\partial c}{\partial x} = \frac{c_0 - c}{c_0 - c_m} \cdot \frac{\partial c_m}{\partial x} = f\left(\frac{r}{r_0}\right) \cdot \frac{\partial c_m}{\partial x} \]  \hspace{1cm} (3.88)

which depends on the radial position in the pipe. It is noted that no such dependency exists for the equivalent heat-transfer problem.

In sections (3.1.3) and (3.1.4) two different methods used to calculate developing temperature and concentration boundary-layers are described in detail for both situations of U.W.H.F. and U.W.T.

Several/
Several theories \((L.1, S.12, S.33, S.23, S.24, S.36)\), which have all employed an eigenvalue-formulation for the thermal entrance-region, are discussed in sections (3.1.1) and (3.1.3). However, all the analytical results, for both the fully-developed and the thermal entry-regions of a circular tube, were not obtained in a consistent way; different choices were made for the velocity-distributions and eddy-diffusivity expressions. Furthermore, the methods employed to carry out the numerical solutions were different; it is therefore impossible to decide whether the difference in Nusselt-numbers is wholly attributable to the difference in distribution of the wall-temperature.

It was to this end, that Siegel and Sparrow \((S.25)\), in one of their papers, examined in a consistent way the manner in which the two different wall-boundary-conditions affected heat-transfer in a round tube. The authors carried out computations for the U.W.T. case \((S.24)\), employing exactly the same velocity distribution, eddy-diffusivity and digital computer programme used for the U.W.H.F. case \((S.22, S.23)\). In those calculations, it was assumed that the eddy diffusivities for heat and momentum were equal. The diffusivity, for the region near the wall, was evaluated from the expression proposed by Doissler \((D.1)\); for the region further from the wall, the logarithmic velocity-equation was differentiated and a linear variation of the shear stress was used. Siegel and Sparrow concluded that the heat-transfer mechanism for turbulent flow, in both the developing and the developed/
FIG. 9 - COMPARISON OF TURBULENT HEAT TRANSFER AT UNIFORM HEAT WALL FLUX (U.W.H.F.) AND TEMPERATURE (U.W.T.)

![Graph showing comparison of turbulent heat transfer at uniform heat wall flux (U.W.H.F.) and temperature (U.W.T.) with different Re and Pr values. The graph plots Nu (natural convection number) against x/d (dimensionless length). The lines represent different Reynolds numbers (Re) and Prandtl numbers (Pr).]
FIG. 10 - COMPARISON OF HEAT TRANSFER IN THERMAL ENTRANCE REGION FOR GAS FLOWING IN TUBE WITH U.W.H.F AND U.W.T. ($0^{10}$).

\[
\frac{Nu}{Nu_{\infty}}
\]

$Pr = 0.73$

$Re = 10,000$

- Uniform Wall Heat Flux (U.W.H.F.)
- Uniform Wall Temperature (U.W.T.)
developed thermal regions, is quite insensitive to the two distributions in wall-temperature for $Pr > 0.7$. This result was found to be in qualitative agreement with that of Doissler, who employed a boundary-layer model for his calculations of the thermal entrance-regions. Agreement was also noted with the work of Seban et al. (S.26) for the fully-developed region and $Pr=10$.

Nearly the same insensitivity of turbulent heat-transfer-rates to conditions in the wall-boundary is found for $Pr=10$ and $Pr=100$, and the percent difference is insignificant even for short tube-diameters, $x/d$. The greatest difference, $\%$, occurs in the case of $Pr=0.7$ for a $Re=10,000$ for $x/d=2$, which is within the range of experimental error. The percent difference decreases with increasing distance along the tube and also with increasing Reynolds and Prandtl numbers. The results are shown in figure 9.

In general, the results for the U.W.E.P. condition are always higher than those for the U.W.T. case. Surprisingly, Siegel and Sparrow found the reverse to be the case for some Nusselt-number results; but as they point out, the small difference, a few tenths of a percent, is probably due to limitation in the accuracy of their calculations.

The effect of wall-boundary conditions on turbulent heat-transfer in pipes was also investigated by Doissler (D.10); the results are shown in figure 10. Doissler's conclusions are in qualitative/
qualitative agreement with those reported by Siegel et al. insofar as the heat-transfer coefficients were found to be always larger for the U.W.H.F. situation. However, the percentage difference between the U.W.H.F. and U.W.T. conditions is found to be as high as 25% for distances very near the starting length of the tube. This difference decreases progressively for distances downstream in the pipe and it approaches a value of about 10% at a distance equal to $2\frac{1}{3}$ tube diameters.

However, the calculations carried out by Deissler for the U.W.T. situation did not allow for the variation of the dimensionless radius, $r_0^+$, along the axis of the tube. The difference between the two wall-boundary conditions as presented in figure 10 is therefore only approximate and might even be smaller. This topic has already been discussed in section (3.1.5.) where a method was proposed to solve numerically the heat transfer equation of Deissler for the U.W.T. situation.
**3.4. COMPUTER PROGRAMME AND RESULTS FOR DEISSLER’S ANALYSIS**

```
***A
JOB
CHT 011/00000011/PHD PROG LENGAS
OUTPUT
O LINE PRINTER 5000 LINES
EXECUTION 20 MINUTES
COMPILER AA

upper case delimiters

BEGIN
REAL error, lowlim, uplim, cbplusdenom, sc, errora
REAL ro, limit a, limit b
INTEGER j, jz, k, label, r, limit
REALARRAY y(1:14), parray, qarray, uplus, cbplus, xondlimit, nu, xond, orthognu(-10:54), c
xpts(-10:53), npts(1:1), rarray(-10:53)

REALARRAY marray(-10:53)
INTEGER ARRAY number(-10)

ROUTINE SPEC kutta merson (ARRAYNAME y, REAL xo, xi, REALNAME e, INTEGER n, k, c
ROUTINE aux)
ROUTINE SPEC graphplotter (INTEGER m, n, l, s, z, REAL xerror, yerror, ARRAYNAME x, y, c
INTEGERNAME label)

ROUTINESPEC deriv (ARRAYNAME f, REAL x)

ROUTINESPEC fprime(ARRAYNAME f, REAL x)
ROUTINESPEC fred'(ARRAYNAME f, REAL x)
ROUTINESPEC autoint(REALFN f, REAL a, b, e, REALNAME int, erint, INTEGER k)
REALFNSPEC fn(REALx)
REALFNSPEC f' (REALx)
REALFNSPEC n(REALx)

50:read(sc) ; ->51 IF mod(sc)< 1a-5 ; limit= 1
newpage
CAPTION dsc= $ ; print(sc,1.2) ; newline
newline error= .0001 ; uplim= 0 ; y(1)= 0 ; y(2)= 0
CYCLE i= 1,1,26
lowlim= uplim ; uplim=i
xpts(i)= i
kuttamerson( y, lowlim, uplim, error, 2, 50, deriv)
parray(i)= y(2) ; uplus(i)= y(1)
REPEAT
y(i)= 0 ; y(2)= 0 ; lowlim= 0
uplim= 1a-6
kuttamerson(y, lowlim, uplim, error, 2, 50, deriv)
parray(-10)= y(2) ; uplus(-10)= y(1)
lowlim= uplim
uplim= 1a-3 ; kuttamerson(y, lowlim, uplim, error, 2, 50, deriv)
parray(-9)= y(2) ; uplus(-9)= y(1)
uplim= 0 ; y(1)= 0 ; y(2)= 0
CYCLE i= -8,1,0
lowlim= uplim
uplim= (g+1)/10
kuttamerson(y, lowlim, uplim, error, 2, 50, deriv)
parray(1)= y(2) ; uplus(1)= y(1) ; xpts(1)=(g+1)/10
REPEAT
xpts(-10)= 1a-6 ; xpts(-9)= 1a-3
k=26
CYCLE i= 1,1,3
CYCLE j= (13-21-arity(1))/4,1.10
k= k+1
xpts(k)= j*1041
uplus(k)= 2.78*log(xpts(k)/26) + uplus(26)
parray(k)= 2.78*log(xpts(k)/26) + parray(26)
REPEAT
```
CVCIE 1:
\[ \text{for } i = 1, \ldots, 26 \]
\[ \text{newline; print}(i, 3, 0); \text{spaces}(5) \]
\[ \text{printf}((\text{up} + \text{d}w(0, 5)) \times \text{spaces}(4)) \]
\[ \text{printf}((\text{in} \times 3)), 5) \]
\[ \text{REPEAT} \]
\[ \text{CYCLE } i = 1, 1, 26 \]
\[ \text{newline; print}(1, 5, 0); \text{spaces}(5) \]
\[ \text{printf}((\text{u} + \text{d}w(0, 5)) \times \text{spaces}(4)) \]
\[ \text{printf}((\text{p} + \text{r} + \text{a} + \text{y}(1))), 5) \]
\[ \text{REPEAT} \]
\[ \text{printfl}(u(v.i + y.3), (13 - 2i - \text{arity}(i)) / 4, 1, 10) \]
\[ \text{printf}((\text{u} + \text{d}w(0, 5)) \times \text{spaces}(4)) \]
\[ \text{printf}((\text{p} + \text{r} + \text{a} + \text{y}(53))) \]
\[ \text{REPEAT} \]
\[ \text{graphplotter}(13, 1, 50, 0, 0, 0, 2, 2, \text{xpts}, \text{uplus}, \text{label}); \text{newlins}(2) \]
\[ \text{spaces}(34); \text{CAPTION} \text{graphofu+v.y+}. \]
\[ \text{graphplotter}(03, 1, 50, 0, 0, 0, 2, 2, \text{xpts}, \text{parray}, \text{label}); \text{newlins}(2) \]
\[ \text{spaces}(34); \text{CAPTION} \text{graphofu+c+v.y+}. \]

CVCIE r: 1, 1, 12 ; newpage
\[ \rightarrow y(0) \text{ UNLESS } r > 10 \]
\[ \text{ro} = 5 * (r - 8) * 10^4; \rightarrow 550 \]
\[ y(1): \text{ro} = r * 10^4 \]
\[ 5501; \text{CAPTION } \text{arc}=#; \text{printf}(\text{ro}, 5, 0) \]
\[ \text{ro} = \text{sqrt}(0.0327) / 2 * \text{exp}(7 * \text{log}(\text{ro}) / 10); \text{newlins}; \text{limit} = \text{intpt}(\text{ro/1000}) \times 42 \]
\[ \text{CAPTION: XPLUS } = \times 4; \text{print}((\text{ro, 4, 0})) \]
\[ \text{IF } \text{ro<600 } \text{THEN } \text{limit}=33 + \text{intpt}(\text{ro/100}) \]
\[ \text{parray}(53) = 2.7^3 * \text{log}(\text{ro/26}) + \text{array}(26) \]
\[ \text{uplus}(53) = \text{parray}(53) - \text{array}(26) + \text{uplus}(26) \]
\[ \text{xpts}(53) = \text{ro} \]
\[ \text{CYCLE } i = 1, 1, 4 \]
\[ y(1) = 0 \]
\[ \text{REPEAT} ; \text{uplim} = 0 \]
\[ \text{CYCLE } i = -10, 1, 26 \]
\[ \text{lowlim} = \text{uplim} \]
\[ \text{uplim} = \text{xpts}(1) ; \text{kurtaerror}(y, \text{lowlim, uplim, error, 4, 50, fprime}) \]
\[ \text{marray}(1) = y(4); \text{rarray}(1) = y(3); \text{garray}(1) = \text{parray}(53) * \text{array}(1) - \text{marray}(1) \]
routine deriv(ARRAYNAME f, REAL x)
f(1) = 1/(1 + 0.0154 * x * y(1) * (1 - exp(-0.0154 * x * y(1))))
f(2) = 1/(1 + 0.0154 * x * y(1) * (1 - exp(-0.0154 * x * y(1))))
END

ROUTINE fprime(ARRAYNAME f, REAL x)
deriv(f, x)
f(3) = y(1) * (ro - x)
f(4) = y(2) * f(3)
END

REALFN fn(REAL x)
RESULT = (2.78 * log(x/26) + parray(26)) * (2.78 * log(x/26) + pplus(26)) * (ro - x)
END

REALFN fn'(REAL x)
RESULT = (parray(53) - 2.78 * log(x/26) - parray(26)) * (2.78 * log(x/26) + pplus(26)) * (ro - x)
END

REALFN fn''(REAL x)
RESULT = fn(x) / (2.78 * log(x/26) + parray(26))
END

ROUTINE fred'(ARRAYNAME f, REAL x)
deriv(f, x)
f(3) = y(1) * (parray(53) - y(2)) * (ro - x)
END

routine autoint(real fn f, real a, b, e, real name int, erint, integer k)
real h, sa, sb, fa, fb, fc, fd, fe, maxh
real fn spec f(real x)
integer n
int = 0; fa = f(a); n = 0;
maxh = (b - a) / k
h = maxh; i = 2
1: h = b - a
2: fc = f(a + 0.5h); fe = f(a + h)
3: fb = f(a + 0.25h); fd = f(a + 0.75h)
sa = (fa + 4 * fc + fe) * h / 6; sb = (fa + 4 * fb + 2 * fc + 4 * fd + fe) * h / 12
4: if fmod(sa - sb) < 0
   h = h * 2; fc = fc; fe = fe; fb = fb; i = 3
5: int = int + sb; n = n + 1
-5: if a + h > b
   h = 2 * h; fa = fa; fe = fe
-1: if a + h > b
   5: int = e * n / 15
end
routine kutta_merson(array name y, real x0,x1, real name e, integer n,k routine aux)
routine spec aux(array name f, real x)
routine spec int step(array name y, real x,h, integer n, real name e, routine aux)
    integer m,1,j,p,q,r
    real h,e1,e2,e3, x
    array z(1:n)
    m = 1
    if m> k then ->1
    m = 2m
    ->2
    1: h = (x1 - x0)/m
    p = 1; q = m; e2 = 0; e3 = 0; x=x0
    8: r = 1
    10: cycle 1 = p,1,q

    cycle j = 1,1, n
    z(j) = y(j)
    repeat
    int step (y, x, h, n,e1, aux)
    if e1 > e then ->3
    if e1 < 0.01 and r = 1 and i?q then ->4
    9: e2 = 0; e3 = 0
    x = x+h
    repeat
    return
    3: if e2 = 0 and e3 = 0 then -> 5
    if e3 = 0 then ->6
    if e1 > e2 and e1 > e3 then ->7
    6: e3 = e2
    5: e2 = e1
    p = 1 ; q = 2q-2i+2 ; h = h/2
    cycle j = 1,1,n
    y(j) = z(j).
    repeat
    return
    7: e = 2e1 ;
    ->9
    4: if frac pt((q-1)/2)>0 then -> 9
    r = 1; q = int((q-1)/2); x=x+h
    h = 2h
    ->8

routine int step(array name y, real x, h, integer n, real name e, routine aux)
routine spec aux(array name f, real x)
    integer 1
    array a,b,c,d(1:n),f(1:n)
    cycle i=1,1,n; a(i)=y(i); repeat
    aux(f,x)
    cycle i=1,1,n
    b(i)=h*f(i)
    y(i)=a(i)+0.3333333333333333b(i)
    repeat
aux(\(f, x + 0.3333333333333333h\))

\[\text{cycle } i = 1,1,n\]
\[c(i) = h \times f(i)\]
\[y(i) = a(i) + 0.1666666666666666(b(i) + c(i))\]
\[\text{repeat}\]

aux(\(f, x + 0.3333333333333333h\))

\[\text{cycle } i = 1,1,n\]
\[c(i) = h \times f(i)\]
\[y(i) = a(i) + 0.125b(i) + 0.375c(i)\]
\[\text{repeat}\]

aux(\(f, x + 0.5h\))

\[\text{cycle } i = 1,1,n\]
\[d(i) = h \times f(i)\]
\[y(i) = a(i) + 0.5b(i) - 1.5c(i) + 2d(i)\]
\[\text{repeat}\]

aux(\(f, x + h\))

\[e = 0\]

\[\text{cycle } i = 1,1,n\]
\[c(i) = y(i)\]
\[y(i) = a(i) + 0.1666666666666666(b(i) + 4d(i) + h \times f(i))\]
\[c(i) = |y(i) - c(i)|\]
\[\rightarrow 2 \text{ if } e > c(i)\]
\[e = c(i)\]
\[2: \text{ repeat}\]
\[e = 0.2e\]

end

\[\text{ROUTINE } \text{graphplotter(INTEGER } m, n, 1, s, z, \text{ REAL } xerror, yerror, \text{ ARRAYNAME } x, y, \text{ INTEGERNAME } label)\]
\[\text{REAL } p, q, xmax, ymax, xmin, ymin\]
\[\text{INTEGER } i, j, xx, yz\]
\[\text{INTEGERARRAY } \text{plot}(1:n, 1:s)\]
\[\text{SWITCH } sw(0:8)\]
\[\text{label} = 0; xx = 0 \land yz = 0\]
\[\text{ymax} = y(\text{lim}\_\text{it}) \land \text{ymin} = \text{ymax}\]
\[\text{xmax} = x(\text{lim}\_\text{it}) \land \text{xmin} = \text{xmax}\]

\[\text{CYCLE } i = -10, 1, m - 11\]
\[\text{ymax} = y(i + z) \text{ IF } y(i + z) > \text{ymax}\]
\[\text{xmax} = x(i + z) \text{ IF } x(i + z) > \text{xmax}\]
\[\text{ymin} = y(i + z) \text{ IF } y(i + z) < \text{ymin}\]
\[\text{xmin} = x(i + z) \text{ IF } x(i + z) < \text{xmin}\]
\[\text{REPEAT}\]

\[74: \text{IF } \text{ymin} < 0 \text{ AND } \text{ymax} > 0 \text{ THEN } yz = 1 \land \text{IF } \text{ymin} < 0 \text{ AND } \text{xmax} > 0 \text{ THEN } yz = 1\]

\[\rightarrow 3 \text{ IF } |\text{xmax} - \text{xmin}| < \text{xerror} \text{ OR } |\text{ymax} - \text{ymin}| < \text{yerror}\]

\[p = (1 - 1)/(\text{ymax} - \text{ymin})\]
\[q = (s - 1)/(\text{xmax} - \text{xmin})\]

\[\text{CYCLE } i = 1, 1, 1\]
\[\text{CYCLE } j = 1, 1, s\]
\[\text{plot}(i, j) = 2\]
\[\text{plot}(i, j) = 1 \text{ IF } i = 1 \text{ OR } i = 1\]
\[\text{REPEAT}\]
\[\text{plot}(1, 1) = 0\]
\[\text{plot}(1, s) = 0\]
\[\text{REPEAT}\]
CYCLE i = 1,1,m
plot(1+intpt(.5+ q*(y(i-11+z)-ymin)), 1+intpt(.5+ q*(x(i-11+z)-xmin))) = 3
REPEAT

94: UNLESS yz= 1 ; | IF yz=1 THEN the yaxis is in the range considered
CYCLE j = 1,1,s

77: REPEAT

94: -> 95 UNLESS xz=1 ; | IF xz=1 THEN the x axis is in the range considered
CYCLE i = 1,1,1

78: REPEAT

 nueva:
CYCLE i = 1,1,1
newline;spaces(10)

sw(O): CAPTION 1 ; ->6
sw(1): CAPTION - ; ->6
sw(2): CAPTION # ; ->6
sw(3): CAPTION . ; ->6
sw(4): CAPTION * ; ->6
sw(5): CAPTION a ; ->6
sw(6): CAPTION + ; ->6
sw(7): CAPTION - ; ->6
sw(8): CAPTION I
@: REPEAT

RETURN

3: label= 1
END ; | of graphplotter
51: ENDOFPROGRAM

1.78

***Z
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RANGE OF X
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MAX = 10000.00000

RANGE OF Y
MIN = 0.00000
MAX = 29.38983

GRAPH OF U+ V+ Y++. 
RANGE OF X
MIN = 0.00000
MAX = 10000.00000

RANGE OF Y
MIN = 0.00000
MAX = 35.40397

GRAPH OF C+ V. Y++
3.5. **Computer Programme for Treatment of Experimental Data.**

The choice of an arbitrary curve to pass through a number of points very often leads to a misleading selection of a family of apparently equally possible curves. This problem becomes more crucial in studies similar to the present one, in which tested analytical predictions differ by the same order of magnitude as the scatter in the data.

The results obtained in the course of the present investigation were analysed by means of a computer programme arranged so that a least-squares curve was obtained for each set of data. This allowed the results to be compared in a consistent manner.

Analytical predictions of temperature/mass transfer rates indicate that for the entrance region of a circular duct the results would satisfy some exponential function. The computer programme was prepared so that an exponential curve was obtained from the experimental data. Whenever the data did not fit an exponential, a polynomial of given degree was used instead. The programme which was used to analyse the experimental results comprises basically three sections.

Firstly, a routine "curvefit" was used; this routine fits a polynomial of given degree - in this programme set equal to nine - to the set of data points. It also gives the best-fit polynomials for all degrees less than the given one. By best-fit polynomial is meant that polynomial for which the variance or "residue" is a minimum.

The/
The second part of the programme is a routine called "expolvft" which is short for exponential curve fit (D.16).

Expolvft will fit a curve defined by the equation

\[ y = a \cdot \exp(bx) + c \]

to a set of data points. The Taylor series modification of the classical least-squares method is utilised to approximate a solution to the system of non-linear equations of condition.

After every iteration the statistic "esquared" is computed as a measure of the goodness of fit. Commencing with the second iteration, the successive values of esquared are differenced, and when the differences in absolute value becomes less than epsilon, in this case set to $10^{-7}$, the calculations cease. If the number of iterations necessary to achieve this result exceeds 12, a flag is set to 1, which causes a suitable caption to be printed out, and the routine is terminated.

The calculated increments to the values of a, b and c are printed out at the end of each iteration, before the "esquared test" is carried out.

The third part of this programme is a routine which plots graphs of a) the experimental points, b) the best-fit polynomial, and c) the best-fit exponential.

In addition to these three main sections, the programme calculates what the residue would be if there were an error of 5% on all the data points. This calculation is carried out after curvofit has been used, and a comparison is made between the/
the 5% residue and the various residues obtained using curvofit. The polynomial of smallest degree with a residue less than the 5% residue is taken as the most suitable polynomial approximation. If there is no such polynomial, the polynomial of degree 9 is taken.

Using the best-fit polynomial thus obtained, the ordinates on this curve, corresponding to the abscissae of the experimental points are printed out for comparison with the actual ordinates of the experimental points.

Expervft is then used, assuming the criterion mentioned above has been satisfied. Again the ordinates on the resulting curve, corresponding to the abscissae of the experimental points, are printed out.

The experimental points are then printed out for reference. Finally the three graphs are drawn.
INTEGER N
REAL SUMX1, SUMX2, SUMXINT, SUMYINT, XMAX
REAL YY(1:100), YY1(1:100)
REAL DELTA1, DELTA2, DELTA3, DELTA4, DELTA5
REAL SUMXINT, SUMYINT
REAL EPSILON
CHARACTER NAME, STRING, HIST

1: BEGIN

2: INTEGER N, M, I
3: REAL X(Y), Y(1:100), YY(1:100)
4: REAL DELTA1, DELTA2, DELTA3, DELTA4, DELTA5
5: REAL EPSILON
6: CHARACTER NAME, STRING, HIST

7: VIRTUAL INT: INTEGER N, M, I
8: REAL X(Y), Y(1:100), YY(1:100)
9: REAL DELTA1, DELTA2, DELTA3, DELTA4, DELTA5
10: REAL EPSILON
11: CHARACTER NAME, STRING, HIST

12: INTEGER N
13: REAL SUMX1, SUMX2, SUMXINT, SUMYINT, XMAX
14: REAL YY(1:100), YY1(1:100)
15: REAL DELTA1, DELTA2, DELTA3, DELTA4, DELTA5
16: REAL EPSILON
17: CHARACTER NAME, STRING, HIST

18: CYCLE = CYCLE + 1
19: ESCAPE
20: ESCAPE = ESCAPE + 1
21: ISCUE = ISCUE + 1
22: ISCUE = ISCUE + 1
23: ISCUE = ISCUE + 1
24: ISCUE = ISCUE + 1
25: ISCUE = ISCUE + 1
26: ISCUE = ISCUE + 1
27: ISCUE = ISCUE + 1
28: ISCUE = ISCUE + 1
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30: ISCUE = ISCUE + 1
31: ISCUE = ISCUE + 1
32: ISCUE = ISCUE + 1
33: ISCUE = ISCUE + 1
34: ISCUE = ISCUE + 1
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43: ISCUE = ISCUE + 1
44: ISCUE = ISCUE + 1
45: ISCUE = ISCUE + 1
46: ISCUE = ISCUE + 1
47: ISCUE = ISCUE + 1
RESULTS FOR POLYNOMIAL OF DEGREE 0
SUM OF SQUARES OF RESIDUALS = 5.7775894952
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
1.201844

RESULTS FOR POLYNOMIAL OF DEGREE 1
SUM OF SQUARES OF RESIDUALS = 1.4554119127
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
1.455477 -0.159472

RESULTS FOR POLYNOMIAL OF DEGREE 2
SUM OF SQUARES OF RESIDUALS = 5.1421149264
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
1.2046710 -0.567980 0.072021

RESULTS FOR POLYNOMIAL OF DEGREE 3
SUM OF SQUARES OF RESIDUALS = 1.6841133027
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
1.654417 -1.159672

RESULTS FOR POLYNOMIAL OF DEGREE 4
SUM OF SQUARES OF RESIDUALS = 2.4431149264
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
2.046710 -1.567990 0.072021

RESULTS FOR POLYNOMIAL OF DEGREE 5
SUM OF SQUARES OF RESIDUALS = 5.7300433231
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
2.211505 -0.902970 0.210509 -0.017226

RESULTS FOR POLYNOMIAL OF DEGREE 6
SUM OF SQUARES OF RESIDUALS = 4.8552070652
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
2.259741 -1.009370 0.341452 -0.050164 0.002958

RESULTS FOR POLYNOMIAL OF DEGREE 7
SUM OF SQUARES OF RESIDUALS = 4.2001442905
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
2.297093 -1.217411 1.524041 2.141204 1.020636 -0.001241

RESULTS FOR POLYNOMIAL OF DEGREE 8
SUM OF SQUARES OF RESIDUALS = 2.6795420886
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
2.376225 -1.744440 1.440226 -0.744491 0.026396 -0.001247

RESULTS FOR POLYNOMIAL OF DEGREE 9
SUM OF SQUARES OF RESIDUALS = 2.6065432794
COEFFS OF POLYNOMIAL OF GIVEN DEGREE, IN ASCENDING ORDER
2.429235 -2.279326 3.034833 -3.039806 2.006362 -0.040353 0.003779 -0.000149

EXPERIMENTAL ERROR OF PLUS OR MINUS 5 P.C. CORRESPONDS TO RESIDUE OF 2.27507250

ORDINATES OF GIVEN PTS. OF MOST SUITABLE POLYNOMIAL I.E. THAT OF DEGREE 3
T 1.218201 2.046322 1.472970 1.000585 1.631939 1.764756 1.704952 1.646439 1.591130 1.538939 1.497776 1.443556 1.4011 1.359545 1.321661 1.286360 1.253545 1.227190 1.195788 1.164230 1.135611 1.106418 1.087607 1.070807 1.056629 1.044086 1.032360 1.020535 1.008993 1.007246 0.999684 0.998304 0.999569 0.997342 0.99 0.99171 0.990139 0.992821 0.99576 0.999464 0.99457 1.000499 1.000225 1.003697 1.009260 1.006252 1.006767 1.0041 1.004477 1.002652 0.949940 0.979497 0.980306 0.984119

156
ACCURATE LP FIT I-

INITIAL ESTIMATES ARE: 2.6391 -0.4275 0.1672 RESIDUE = 7.855327x 0

ITER DELTA A DELTA B DELTA C
1 -1.6879 -0.2345 1.0596
2 0.4726 -0.3269 -0.2171
3 0.0050 0.0047 0.0010
4 0.0050 0.0047 0.0010
5 0.0050 0.0047 0.0010
6 0.0050 0.0047 0.0010

NO. OF ITERATIONS CARRIED OUT = 6

A,B,C IN EXP.CURVE ARE: 1 1.35929 D -9.96154 -1 9.89594 -1

ORDINATES ON THIS CURVE, CORRESPONDING TO ASCISSAE INPUT:

1.22055 1.20477 1.4017 1.6179 1.8417 2.0623 2.2092 2.3511 2.4984 2.6503 2.8081 2.9716
2.1200 2.0400 1.9800 1.9200 1.8769 1.8330 1.8000
1.7700 1.7400 1.7100 1.6800 1.6500 1.6200 1.5900
1.5600 1.5300 1.5000 1.4700 1.4400 1.4100 1.3800
1.3500 1.3200 1.2900 1.2600 1.2300 1.2000 1.1700
1.1400 1.1000 1.0800 1.0600 1.0400 1.0200 1.0000
1.9800 1.9200 1.8800 1.8400 1.8000 1.7600 1.7200
1.6800 1.6400 1.6000 1.5600 1.5200 1.4800 1.4400
1.4000 1.3600 1.3200 1.2800 1.2400 1.2000 1.1600
1.1200 1.0800 1.0400 1.0000 0.9600 0.9200 0.8800
1.3029 1.3630 1.4230 1.4830 1.5430 1.6030 1.6630
1.7230 1.7830 1.8430 1.9030 1.9630 2.0230 2.0830
2.1430 2.2030 2.2630 2.3230 2.3830 2.4430 2.5030
2.5630 2.6230 2.6830 2.7430 2.8030 2.8630 2.9230
2.9830 3.0430 3.1030 3.1630 3.2230 3.2830 3.3430
4.6630 4.7230 4.7830 4.8430 4.9030 4.9630 5.0230
5.0830 5.1430 5.2030 5.2630 5.3230 5.3830 5.4430
5.5030 5.5630 5.6230 5.6830 5.7430 5.8030 5.8630

RESIDUE WITH THIS CURVE = 5.7123493 - 2

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EXPERIMENTAL POINTS 1

X 0.0945 0.1945 0.2635 0.3180 0.4724 0.5669 0.6614 0.7559 0.8504 0.9449 1.0394 1.1339 1.2283 1.3228
Y 2.2200 2.1200 1.9900 1.8600 1.7700 1.7400 1.6700 1.6000 1.5500 1.5000 1.4500 1.4000 1.3500 1.3000 1.2500

X 1.4170 1.5140 1.6090 1.7040 1.7990 1.8940 1.9890 2.0840 2.1790 2.2740 2.3690 2.4640 2.5590 2.6540 2.7490
Y 1.3200 1.2900 1.2600 1.2300 1.2000 1.1700 1.1400 1.1100 1.0800 1.0500 1.0200 1.0000 0.9800 0.9600 0.9400

X 2.7420 2.3740 2.2920 2.2320 2.1720 2.1120 2.0520 1.9920 1.9320 1.8720 1.8120 1.7520 1.6920 1.6320 1.5720
Y 1.0100 1.0000 0.9900 0.9800 0.9700 0.9600 0.9500 0.9400 0.9300 0.9200 0.9100 0.9000 0.8900 0.8800 0.8700


Y 1.0360 1.0200 1.0000 0.9800 0.9600 0.9400 0.9200 0.9000 0.8800 0.8600 0.8400 0.8200 0.8000 0.7800 0.7600

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ON THE FOLLOWING GRAPHS, A REFERS TO THE BEST-FIT POLYNOMIAL, B TO THE BEST FIT EXPONENTIAL, AND X TO THE EXPERIMENTAL POINTS.
null
4. EXPERIMENTAL
4. EXPERIMENTAL

4.1. CHOICE OF METHOD FOR MASS TRANSFER MEASUREMENTS - THE PROFILOMETRIC METHOD.

In this work, mass transfer in turbulent flow in a duct was investigated by studying the resulting geometrical changes on a cast diphenyl surface. These changes were measured by a profilometric technique especially suited for determining spatial variations in a tube. It is possible by this method to measure local recessions in the volatile internal surface that take place after sublimation.

The distance between the tip of a fine nozzle of a free-jet air and the examined surface is capable of being determined with great accuracy. This is done by measuring the air-pressure at a point upstream between a constriction in the air-supply and the nozzle.

The profilometric method may then take the form of a free-jet air gauge (M.12), and this version may be applied with advantage to the study of developing concentration boundary-layers. This is so largely because of the very small dimensions involved in the measured configurational changes in studies of mass transfer with diffusion of solid/gas non-reacting systems. Immediate application of free-jet air gauges is to be found in the study of developing concentration boundary-layers on flat cast surfaces, e.g., in the form of flat plates.

This method has been applied by Macleod et al. (M.17) to a study of mass transfer from volatile cylindrical surfaces which were/
were cast on metal rods. The authors measured local rates of sublimation of naphthalene-coated steel rods which were incorporated in a replica of the fuel-rod cluster of a gas-cooled nuclear reactor. Local mass transfer rates were determined at the naphthalene boundary by measurements of local changes in the thickness of the volatile surface. Measurements were carried out by means of a probe consisting of an air-jet with a magnetic-proximity gauge. The air jet was used simply to bring the probe to a constant distance from the naphthalene wall at each reading, whilst the magnetic-proximity gauge measured the distance from the probe to the metal surface. According to the authors, it was possible to measure local changes in thickness to $\pm 3 \times 10^{-5}$ in. with an accuracy of $\pm 6 \times 10^{-5}$ in. when the distance of the probe from the volatile wall was $5 \times 10^{-4}$ in. This investigation is discussed in section 5.3.2.

The use of reliable pneumatic methods eliminates the possibility of surface damage or deformation resulting from depression of the soft surface by the measuring device; this may occur when surface-profiling methods are used which involve the mechanical contact between the probe and the transfer surface.

Mass-transfer coefficients are then obtained from direct measurement of recession of the surface of materials which are usually soft. In order to achieve the required accuracy in these measurements, the contact between the measuring device and the surface must be firm, and the readings reproducible. Where this/
this is not the case, the results will be inaccurate even if the surface remains intact. On the other hand, firm contact between the measuring instrument and the surface may lead to the deformation of the surface, and the measurements will again be in error. To achieve the necessary accuracy, changes in the studied surfaces should be appreciable when direct methods of measurement are employed.

Definite and precise measurements of wall-recessions by direct methods is then extremely difficult if not impossible to achieve. It is precisely at this point that the pneumatic-gauging techniques, which measure small dimensional changes with high accuracy, score highly over direct techniques of measurement. The use of pneumatic-gauging techniques eliminates completely contact of the measuring device with the delicate mass-transferring surface.

An example of this method is the work by Sherwood and Träsk (S.37) in their attempt to predict skin friction from mass-transfer measurements. The authors measured rates of sublimation of naphthalene from an adiabatic sharp-edged plate exposed to subsonic and supersonic airstreams. The leading edge of the plate was bevelled and rubbed smooth with acetone. Traces of surface profiles were obtained on two lines across the plate using sensitive strain-gauges equipped with pick-up needles which touched the naphthalene surface. The pick-up needles were mounted on a moveable bridge which was supported at the edges/
FIG. 11 - SCHEMATIC DIAGRAM OF PRINCIPLE OF PROFILOMETRIC AIR-GAUGING TECHNIQUE
edges of the plate. The output from the strain-gauges gave an indication of the coating thickness and was fed to two recorders, the drives of which were synchronised with the motor pulling the strain-gauge along the plate. A continuous trace of the naphthalene profile along the plate was obtained before and after each run and the difference between those was a measure of the decrease in the naphthalene coating; hence, it was possible to estimate the amount transferred. The amount removed was from 4 to 20 thou.in., and this, as shown in APPENDIX IV, would be likely to result in considerable interference with the development of the concentration boundary-layer. This work is discussed in section 5.2.

However, an important condition to the general application of pneumatic methods in profilometric measurements is that the organic solid acting as the test surface must be of sufficiently high volatility so as to allow the sublimation process to take place in a reasonably short period of time; at the same time, it should not be so volatile as to be affected by the gauging operation itself.

The system studied in the present investigation was essentially one of straight flow of turbulent fluid through a cylindrically symmetrical body, namely a round tube. The free-jet air gauge was modified to satisfy the demands imposed on design by the geometry of this particular configuration, and is shown in figure 11. The gauging air escapes to the atmosphere at/
at a measured rate past a knife-edged round disc of a very slightly smaller diameter than that of the mass-transfer tube with which it forms an annular clearance. This method possesses a high axial resolving power and the measuring system enables local transfer rates to be determined in the region of greatest interest, that very near the commencement of the mass transferring surface. However, a difference between the free-jet air and the pneumatic disc is the high supply pressures of air that are necessary in the latter case to maintain turbulent flow conditions; this point is discussed in the following section.
4.2. GENERAL CONSIDERATIONS AND PRINCIPLES OF OPERATION.

Before the sharp-edged disc air-gauging technique could be applied to measure the local recessions incurred during the sublimation stage of the experiment, certain important factors affecting the fine annular clearances had to be considered. These are:

(a) whether the mass-flow rate of air through the annular orifice between a circular knife-edge and a cylindrical wall could be determined solely by the total area of the aperture and by the pressure difference across the circular knife-edge.

(b) whether the displacement of the circular edge relative to the round wall, resulting in a departure from symmetry of the annular clearance, had any effect on the air-flow rate.

Investigations conducted by Buckingham and Edwards (3, 8) on the flow properties of orifices showed that a relation between the aperture area and pressure could be obtained provided the pressure ratio, $P_1/P_2$ was large. Also, the flow rate was found to be independent of both, asymmetry in the annular clearance, and of variations in the axial profile of the sharp edge.

The area of the clearance between the sharp disc $K$, shown in figure 19, and the corresponding zone of the mass-transfer wall can be related to the orifice area of the variable restrictor $R_1$, in terms of pressure drops across the two orifices at $R_1$ and $R_2$, the flow of air being common to both. It is then possible to determine circumferentially-averaged changes in the coated/
coated lining at a given station, from readings of the manometric pressure $P_2$ and the micrometer feed screw at $R_1$ provided (a) that the reservoir of air-gauging supply $P_1$ is kept constant at the initial and final measurements, and (b) that certain features of the flow laws are understood.

Two distinct modes of operation are possible for the measuring system:

(i) Variable Restrictor Operation: the micrometer of the orifice adjustment at $R_1$ can be reset at each measurement in such a manner that the reading of $P_2$ - i.e., the local recessions at the tube wall are calculated by the changes in the micrometer setting at $R_1$.

(ii) Fixed Restrictor Operation: the micrometer setting is kept fixed, and changes in the bore of the mass-transfer section are calculated from changes in the reading of $P_2$.

The first method is simpler to use since it assumes less knowledge of the flow characteristics through the orifices; however, the second method has the mechanical advantage of avoiding excessive wear of the micrometer screw-thread and other moving parts of the variable restrictor $R_1$ as a direct consequence of the continual readjustment involved in the method.
4.3. CALIBRATION OF MEASURING APPARATUS.

4.3.1. General Considerations.

The readings registered by a mercury manometer, of the intermediate pressure drop, \( P_2 \), across the sharp-edged disc \( K \) could be interpreted in terms of variations of gap-widths only after the assembly of restrictors in the profilometric method had been calibrated. The restrictor, \( R_1 \), could take the form of either an externally or an internally-tapered cone and disc unit. In an earlier investigation\(^{(M.4)}\), the externally-tapered cone was used since this cone was thought likely to be more accurately machined; in addition, it was found that both forms of restrictor device worked equally well.

The calibration of the variable restrictor, \( R_1 \), served the following purposes:

(1) The state of the calibration device was ascertained since the curves of supply pressure, \( P_1 \), against the flow rate \( Q \) could be directly compared with those obtained in the earlier work\(^{(M.4)}\). It was important to check first that the cone assembly had not been damaged after its previous extensive use in which case the taper of the cone would have to be profiled once more.

(2) The true zero position of the cone and disc assembly as registered by the micrometer was determined. This was necessary since a new micrometer was employed in this investigation.

It was then possible to construct for the cones the appropriate/
appropriate charts correlating the pressure drop, $P_2$, across the sharp-edged disc $K$ in the mass-transfer section and the corresponding gap-widths, $A_2$, between the disc and the volatile lining as shown in figure 16.

4.3.2. Calibration Method.

The profilometric method employed in the measurements of the mass-transfer rates offered two possible methods of restrictor operation as shown in section 4.2.; these were the variable and fixed-restrictor operations. The procedure followed in the present investigation was a compromise between the two methods with the advantage that it avoided the wear of the close-fitting parts which moved over one another in the variable restrictor operation; furthermore, it eliminated the possible wear in the screw-thread of the micrometer which moved the accurately-machined cone in the restrictor assembly. The consequence of any damage that may result during the course of measurements in the moving parts of the restrictor assembly, would be to invalidate readings taken by this standard device.

When the actual profilometric procedure was carried out, the micrometer head in the calibration apparatus was adjusted to an appropriate setting, so that the gap-width, $A_1$, between the disc and cone remained fixed; the value of the gap-width $A_2$ between the sharp-edged disc $K$ and the mass-transfer wall corresponding to the intermediate pressure $P_2$ in the manometer was then read off from the graph for a particular value of $A_1$. 
FIG. 12. - THEORY OF CALIBRATION (CHARTS)
4.3.3. Theory of Calibration Charts.

In the diagram shown in figure 12, the symbols have the following meaning:

- \( P_a \) = atmospheric pressure (cm. of mercury)
- \( P_1 \) = constant supply pressure (cm. of mercury)
- \( P_2 \) = intermediate pressure (cm. of mercury)
- \( P^0 \) = supply pressure in unit II (cm. of mercury)
- \( R_1 \) = externally-tapered cone and disc assembly acting as the first restrictor.
- \( R_2 \) = sharp-edged disc and mass-transfer section assembly acting as the second restrictor.
- \( Q \) = fluid-flow rate (cm\(^3\)/sec.)
- \( S \) = flow-measuring device: rotameter.

The diagram shown in figure 12 may be split into two separate parts, I and III, each of which constitutes a restrictor and flowmeter unit II.

If unit II is considered in relation to the two systems I and III, the two graphs (a) and (b) may be obtained (figure 12).

The relationship between the intermediate pressure \( P_2 \), and the gap-width \( A_2 \) between the sharp-edged disc and the diphenyl wall, is derived as follows. Consider each restrictor separately. For

**Restrictor \( R_1 \):** From knowledge of the micrometer setting, and hence gap-width \( A^0 \) in the externally-tapered cone and disc device, and from a knowledge of the pressure drop across the disc, \( \Delta P \), which/
INVESTIGATION INTO THE CONDITION AND EFFECTIVENESS OF THE CALIBRATION DEVICE

\[ A = \text{Flow} \]

\[ A = 3.0 \text{ in.} \]

\[ A = 1.0 \text{ in.} \]

\[ A = 0.5 \text{ in.} \]

\[ A = 0.25 \text{ in.} \]
FIG. 14 - AIR FLOW RATE THROUGH CALIBRATION ASSEMBLY (EXTERNALLY-TAPERED CONE)

SUPPLY PRESSURE $P_i$ (cm Hg)

AIR FLOW RATE (Rotameter Motie 10 Yubo-Readings, cm.)

Annular gap-width $A_i \times 10^{-3}$ ins.
which is $P_1 - P_2$ in this case, $Q$ may be calculated from graph (a). This procedure may be repeated for any number of desired micrometer settings, i.e. gap-widths $A_i$, where $A_1 > A_2 > A_3$.

Restrictor $R_2$: From the deduced value of $Q$ and knowledge of $\Delta P$, which in this case is $P_2 - P_a$, it is possible to calculate $A_2$ from graph (b).

The calibration chart, graph (c), is then obtained from a cross-plot of graph (a) and graph (b).

In the present investigation, figure 14 corresponds to graph (a), figure 15 corresponds to graph (b) and figure 16 corresponds to graph (c).

4.3.4. Derivation of Calibration Charts.

A series of readings of supply pressure $P_1$ against flow rate $Q$ of gauging air which passed through the calibration device was compiled for several chosen micrometer settings and the corresponding graphs were prepared (Fig. 13,14). The micrometer settings chosen were such that the gap-widths between the cone and orifice disc in the calibration device corresponded to the ones employed in an earlier application of the method (M.4). It was then possible to infer the condition of the calibration assembly directly by comparing the sets of graphs of the two investigations; it was concluded upon inspection of figure 13 that the calibration device was in good condition and that it could therefore be used in the profilometric assembly without it being necessary to re-profile the cone.
FIG. 15 - AIR FLOW RATE THROUGH CALIBRATION DEVICE (EXTERNALLY TAPERED CONE.)

MICROMETER SETTING

New zero setting, $M_0$.

SUPPLY PRESSURE
(inches mercury)

AIR FLOW RATE (Rotameter Metric 10 Tubo-Readings, cm.)
The results for the taper of the profiled cone of the previous work were assumed and the average increase in the radius of the cone (APPENDIX I) for a micrometer head movement of $5 \times 10^{-3}$ in. was found to be $0.018 \times 10^{-3}$ in.

A graph was then plotted of air-flow rate against micrometer settings, i.e. of the cone position relative to the fixed orifice disc, for successive constant supply pressures of 3 in., 4 in., 5 in., 6 in., 7 in., 8 in., 9 in. and 10 in. of mercury respectively. It was physically impossible to determine the true zero micrometer-setting, $M_0$, which corresponded to zero flow rate through the calibration device, since the cone and disc assembly could never completely close. However, it was possible to estimate the true zero micrometer-setting by extrapolation of the straight lines in figure 15, to zero flow rate. The estimated value $M_0$ of the zero micrometer-setting was found to be 1.175 in.

The straight lines in figure 15 did not all converge at the same point and the estimated setting of 1.175 in. was found to lie between the two extreme values of 1.158 in. and 1.181 in. It is interesting to note that the discrepancy between the two extremes of the straight lines obtained in this plot was 0.023 in., which was very nearly the same as that of the previous investigation.

The precision with which the calibration operations were conducted is further emphasized by the discovery upon investigation of figure 15, of the existence of a 'ridge' in the cone; this is inferred from the consistently eccentric behaviour, at the/
the same value of micrometer setting of the calibration device, as shown by point 'a' throughout the straight plots in figure 15. The final chart shown in figure 16 was then obtained by cross-plot of the graphs in figures 14 and 15.
4.4. CHOICE OF MATERIAL FOR VOLATILE LINING.

The most important factors in choosing a suitable material are, (i) volatility of the substance, and (ii) its casting properties. The important condition that the air-flow rate had to be independent of any displacement from absolute symmetry of the circular edge of the sharp disc K relative to the round wall, meant that pressure drops of several centimetres of mercury had to be applied across the annular orifices. These high pressures had to be maintained in order to preserve the turbulent nature of the flow through the annular orifices; a condition on which depends the application of the method (Section 4.1). The pressure ratios across those orifices were found to be appreciably greater than unity and this could result in serious interference with the gauging measurements due to the considerable volatilisation of the wall lining. It is then evident that the choice of the volatile substance depended critically on its vapour pressure.

The material chosen should, then, have a vapour pressure that was high enough to allow the sublimation process to take place in a conveniently short period of time; at the same time, however, its vapour pressure should not be so high that material transferred at the gauging procedure could affect the actual measurement of the mass transfer rates. Therefore a substance of low volatility had to be used.

A balance was achieved between considerations of volatility of the substance and economy of time in the following manner. A target/
target was set to obtain a recession of approximately $0.5 \times 10^{-3}$ in. in a period of four hours for a Reynolds number of $50 \times 10^3$, at $20^\circ C$. This meant that the material to be used had to have a vapour pressure of approximately $10^{-5}$ atm. at $20^\circ C$.

A number of materials suitable for use in observations of boundary-layer transition are listed by Main-Smith (M.11) together with their relative vapour pressures. The most suitable substance in this range appeared to be diphenyl, the vapour pressure of which was intermediate between those for naphthalene and acenaphthene, these last two substances setting the lower and upper limit of vapour pressure in the range listed. Furthermore, diphenyl was found to have better casting properties than naphthalene and also gave a much smoother surface; however, more will be said on this subject in later sections 4.5 and 4.6.

Another advantage is that diphenyl vapours are less hazardous to health than those of naphthalene (S.32). This point is of great significance owing to the large volumes of diphenyl vapour produced during (a) the preparation of the volatile test-surfaces (section 4.6), (b) the operation of the profilometric measurements (section 4.5). Acenaphthene was preferred in earlier (M.12) air-jet gauging work because of the high evaporative effect of the air-jet on the naphthalene surface (M.17); also, acenaphthene was used in a preliminary pilot work (G.2), but the experimental runs took ten times as long to complete.

The disadvantage of diphenyl, compared with naphthalene is that/
that its vapour pressure and diffusivity are not accurately known. This advantage offered by naphthalene makes it an attractive material for use in experiments for the calculation of mass-transfer coefficients from measurements of sublimation rate. MacLeod et al. report that recurrences were observed on a cast naphthalene surface when a jet of air acted upon the same point for a period of 20 to 30 seconds. The supply pressure for the air jet was 4 inches water-gauge and the distance of the jet nozzle from the naphthalene surface was 0.0005 inches. It is easily seen then that the application of the free-jet air-gauging technique for the measurement of mass-transfer coefficients based on accurate determination of recurrences on the mass-transfer surface is limited by the evaporative influence of the air jet on the test surface. It is important, then, to establish for each particular air-gauging system the limited allowable time in which the measured mass-transfer coefficient is free from interference resulting from the erosion of the test-surface subjected to the measuring device itself.

Data for the limited period $t_1$ for the case of a free-air jet acting on diphenyl was not available, and, what was even more important, the limited period was not known for the particular air-gauging technique which was utilized in the present work. The manner in which the gauging air affected the measurement of the mass-transfer coefficients was investigated by setting up an experiment in which the time was observed in which a recession occurred at particular points on the diphenyl surface by the gauging air/
FIG. 17 - EFFECT OF GAP WIDTH ON THE RATE OF SUBLIMATION OF THE DIPHENYL SURFACE TO AIR

Intermediate Pressure

\( p_2 \) (cm. Hg)

Limiting Time \( t_1 \) (sec.)

\( T = 295^\circ K \)
FIG. 18 - DEVELOPMENT OF PROFIOMETRIC TECHNIQUE

Intermediate Pressure

\( P_2 \) (cmHg)

\( \frac{P_2}{2} \) (cmHg)

Gauging Medium:

Air

Air, pre-saturated with Diphenyl

Wt Recession

\( w_2 \times 10^{-3} \) in.

0.500

0.100

0

\( T = 291^\circ K \)

\( T = 294^\circ K \)

First Profiometric Traverse

Final Profiometric Traverse

\( L = 2 \times 10^{-1} \) in.
air at a supply pressure of 10 inches of mercury and for various gap-widths. The results are shown in figure 17. The technique devised for reducing the measurement of local sublimation rates to the shortest possible time is described in section 4.5.

Even more serious was the large extent to which the volatile wall underwent sublimation by the gauging-air during the profilometric operation. This is shown in figure 18 where the increase in gap-width is plotted against the axial distance of the tube.

The method which was used to overcome interference with the gauging measurements due to the volatilisation of the wall-surface was as follows. Sublimation of the wall-lining was suppressed during the gauging operation by pre-saturating the air with the volatilising substance. Gauging-air from a compressed air-cylinder was made to pass first through a $\frac{1}{4}$ inch-diameter coiled copper tubing and then through packed diphenyl powder in two consecutive gas-washing bottles. By means of the copper coil the air drawn from the cylinder was brought to the same temperature as that of the room. The saturated air leaving the second bottle then passed through another $\frac{1}{4}$ inch-diameter coiled copper tubing to ensure that the gas mixture attained the ambient temperature following the sublimation of the packed diphenyl.

This method was explored, and the results shown in figure 18 demonstrate clearly the extent to which pre-saturated air with diphenyl suppressed the volatilisation of the mass-transfer surface. Inspection of the same figure indicates that considerable volatilisation takes place when air alone is used.
4.5. DEVELOPMENT OF THE PROFILOMETRIC TECHNIQUE.

In the course of the profilometric operation it became obvious that it was imperative to investigate and understand the character of the cast-in volatile lining of diphenyl. The design of the air-gauging apparatus was based on the assumption that the prepared volatile wall retained the exact shape of the metal core on which it was cast to give a smooth and uniform bore of 1¼ inch diameter throughout the whole eight inches of its length. It was possible, at least in theory, for the gauging sharp-edged disc to traverse the diphenyl bore freely once the test section was properly aligned. For this to happen the disc would have to clear both nylon rings at the flanged ends of the tube.

In practice, however, this was not found to be the case. Repeated attempts to traverse the test-section in the manner described above met with consistent failure and resulted in catastrophic damage to the volatile lining. Upon examination of the ruined volatile wall, it became evident that the diphenyl surface had suffered changes consisting of distortions which were obviously of the same order of magnitude as the gap-width between the diphenyl wall and the sharp disc.

Closer examination revealed that the surface was scratched by the sharp disc in a random fashion both circumferentially and longitudinally, suggesting that the wall irregularities consisted of random protrusions. More will be said of the nature of these protrusions in a later section (5.2(a)). However, it must be stressed/
stressed at this point that this phenomenon is very different from the observed permanent changes in the bore diameter which follow environmental variations in temperature. This problem will be dealt with in section 4.6.2.(b).

If the mass-transfer surface was to be gauged successfully it was important that a means be found which would make mechanical contact between the sharp-edged disc and the diphenyl wall impossible. This meant that a method had to be devised to guarantee the safety of the prepared mass-transfer surface by continually guiding the disc during the advance of the test-section on the gauging-stand.

An optical method specially devised for this purpose enabled the diphenyl bore of the entire coated wall-lining to be gauged successfully. A transparent perspex flange, plates 1 and 4, sealed the rear end of the test-section by means of an oil seal O-ring, as shown in figure 1, and plate 1. The sharp-disc setting relative to the volatile wall was then adjusted at each station along the tube so that each time the sharp-edged disc was surrounded by a complete bright zone; the sharp disc then cleared the wall distinctly. Light from a 100-watt lamp was reflected from two plain mirrors, plate 8, which rested against a white background, W, on plate A of the gauging apparatus as shown in plate 1. The sharp disc K was then viewed through the clear perspex flange. A partly extinguished zone of light indicated that the disc was touching a side of the wall. The complete/
complete circular bright zone was recovered by careful manipulation of the adjusting screws \( W \) shown in Figure 1. Air saturated with diphenyl was then drawn through the gauging-apparatus by means of a needle valve. The pressure, \( P_2 \), across the sharp-edged disc \( K \) was recorded and the valve was closed. The carriage was moved to the next station and the pressure was read again as before. The success of this kind of operation was evidenced by the fact that not one single surface was lost during the profilometric operations.

The main disadvantages of the optical method described above, were the following: (a) the large number of careful manipulations that were involved at each measurement. (b) the length of time that was required to perform the complete traverse of the studied surface. (c) the necessity of wearing a protective mask as a precaution against the dangers arising from the large volumes of air/diphenyl gas \( (S.32) \) produced during this operation (section 4.4).
DEVELOPMENT OF A TECHNIQUE FOR THE PREPARATION OF MASS-TRANSFER SURFACES.


The study of mass-transfer operations requires that if data obtained in experiments is to have any significance, the surfaces prepared must not only be smooth but also as cylindrical as possible. The influence of surface roughness on transfer operations was discussed in section (2.5), and as shown in Appendix IV, surface irregularities must be < 10^{-3} in. if they are to have no importance on turbulent mass-transfer rates. This result is derived from the fact that surface unevenness of the order of 10^{-3} in. corresponds to the thickness of the assumed predominantly laminar layer adjacent to the wall. Irregularities of this magnitude may therefore be expected to disturb appreciably the flow conditions near the wall under fully developed turbulent flow.

In particular, the effects of local surface irregularities are more serious in the entrance region of a tube. The developing concentration boundary-layer, $\lambda_M$, is in this region and is therefore more prone to being disturbed by any unevenness.

A further necessary demand is that the final surface must be as ideally cylindrical as is practicably possible. This means that the circumferential deviations from circularity along the whole length of the transferring cylinder must be kept to a minimum.

This demand arises from three considerations:— (a) disturbance of the flow pattern may result from variations in the wall geometry; (b)/
(b) the profilometric method is accurate to the required degree of recossions to the order of 2 thou. in.; (c) variations in the diameter at any point along the tube axis must be kept to a minimum, to avoid destructive contact between the sharp gauging-disc and the volatile wall of the tube during the profilometric operation.

The development of the method for surface exploration contributed to a better understanding of cast surfaces and is discussed in section (5.2.2.). This method originated from the development of the reliable profilometric technique which is described in section (4.5).

4.6.2. Development of Surface Preparation.

The development of a reliable casting technique to produce mass-transfer surfaces of the required quality was one of the most formidable tasks faced during the present investigation. The difficulties met were a host of fouling complications which were the result of interference by thermal and mechanical causes.

The thermal causes were of two kinds: 1. (a) Environmental Temperature-Variations: Temperature changes of only 4°C. were found to interfere destructively with the profilometric operation owing to the contraction of the brass tube. This in turn caused the collapse of the prepared cylindrical surface, which was then scoured by the sharp-edged disc. Environmental temperature-variations of up to 18°C. were often encountered in the laboratory. This/
This problem was overcome by carrying out the profilometric operation in a constant-temperature room. For other operations, the test section with the cast-in volatile lining was insulated from draughts and temperature changes by placing it in an especially prepared box which was padded with cotton wool.

(b) Effects of differential rates of cooling in the test section: This problem was an integral part of the casting operation because of the particular design of the test section. The flanged ends of the mass-transferring section and the uniform brass tube constituted a system whereby a differential rate of heat exchange was set up along the tube. As a consequence of these uneven rates of heat losses, mass-transfer surfaces resulted which contained the following imperfections:

(1) Irregular bore diameters, where the tubes showed consistently smaller bore at the tube extremities than at the middle section of the tube. It was possible to explain this difficulty with the aid of the surface-exploration technique described in section (5.2.2). The graphs of $P_2 \text{ vs. } x/d$ confirmed the existence of these contractions and inspection of these regions in the tube revealed surfaces of extremely smooth appearance with the formation of large crystals. This observation is consistent with the large crystals being formed where the molten organic substances cool at a very low rate. (ii) All cast tubes possessed the appearance of surface cracks. Again, by the aid of the surface technique it was possible to distinguish between the resulting crystal/
crystal patterns that followed the method in which the substances were cooled, and the real cracks owing to prolonged chilling of the surface. Real surface cracks were evidenced by a substantial fall in the pressure-readings, \( P_2 \), during the profilometric operation. To overcome this difficulty the metal casting-mould was chilled for only a few seconds with liquid nitrogen and was immediately freed from the test section by a sharp blow at the end.

(2) The cast surface was often found to contain surface imperfections which were eventually proved to consist of very small voids. The presence of these voids was better understood by the use of the surface exploration technique and is discussed in section (5.2.2f). This problem was to a large extent overcome by allowing the test-section containing the molten diphenyl to cool from the base upwards in an electrically heated oven of about 80°C., the current of which had been switched off.

(3) Big gaping holes often resulted in a casting operation, owing to the presence of trapped air-bubbles which formed between the casting metal-core and the brass tubo-wall. Whenever these air spaces formed, most methods to dislodge them completely failed and the surfaces were ruined. An improvement was noticed when, before casting the diphenyl, the test-section was placed on a V-block at an angle to the horizontal, and the tube was gently rocked.

On the whole, it was noted that superior cast-surfaces were obtained when the inner wall of the test-section was lined permanently with paper by means of water-glass. The diphenyl adhered/
adhered to the paper very well; at the same time, the sensitivity of the brass tube to temperature variations was greatly reduced.


The highly polished metal core shown in plate 2, of 1\textsuperscript{\textfrac{1}{4}} in. diameter, was introduced gently into the test-section by way of the slightly smaller nylon-rings, Q, shown in figure 20 and plate 3. The arrangement, which is shown in plate 4 ensured that there can be no loss of the molten material at the extremities of the tube during the casting operations. A metal container with a stopper was used as a means of introducing the molten substance into the test-section. The container was screwed into position on the brass tube and the complete assembly, was heated in an oven to 880°C. for about an hour.

A beaker containing about 200 gm. of analar diphenyl was heated in a fume-cupboard to about 82°C. The heated casting-assembly was removed from the oven and was made to rest on a pre-heated brass V-block at an angle to the horizontal. In this way, trapped air-bubbles in the molten diphenyl first rose and subsequently escaped to the atmosphere through the two overflow-funnels, F, shown in plate 3. The hot diphenyl was then poured from the beaker into the metal container. When the stopper was removed, the hot material filled the empty space between the test-section wall and the surface of the metal core.

The tube was then subjected to a series of rocking movements by/
by hand, until no further air-bubbles were released at the two overflow-tunnels and the metal container. Hot diphenyl was added from the beaker at intervals in order to replace the volume of material lost through the overflow-tunnels during those operations. At this point, the whole apparatus was returned into the oven and was supported on two brass V-blocks. The current in the oven was switched off and the casting assembly was allowed to cool slowly for a period of 4 hours at the same rate as the oven. The test-section was thus cooled, beginning with the V-blocks, so that the diphenyl solidified progressively from the lower portion of the tube upwards. It was possible in this manner, to prevent the formation of voids in the cast material. The ambient temperature of the oven offered an effective means of neutralising the effects of differential cooling in the test-section, which was discussed in section 4.6.2.

It was important to carry out this operation in as short a time as was practicably possible so as to avoid rapid cooling of the pre-heated casting assembly.

The cooled apparatus was removed from the oven and the metal handle, shown in plate 2(c) was screwed into the appropriate end of the casting core. Liquid nitrogen was poured from a vacuum flask into the free end of the metal core which was then immediately pulled out from the tube to leave surfaces of high reflectivity and great smoothness. It was essential to cause only instant chilling of the metal core, since, parallol chilling of the diphenyl/
PLATE 1. - MASS TRANSFER MEASURING DEVICE
PLATE 2.- CASTING CORE

(a)

(b)

(c)
PLATE 4. CASTING ASSEMBLY
PLATE 5. - SHARP-EDGED DISC (Brass)
PLATE 8. - PROFILOMETRIC TECHNIQUE
PLATE 9 - WIND TUNNEL
FIG. 19 - SHARP-EDGED DISC
nylon screwed bush
rubber 'o' ring

FIG. 20  —  TEST - SECTION  (material : brass)
FIG. 22. CARRIAGE (gaging stand)
FIG. 24. - PITOT - TUBE ASSEMBLY
FIG. 25. THERMOMETER SEAL (wind-tunnel)
4.7. OUTLINE OF EXPERIMENTAL PROCEDURE AND DESIGN OF EQUIPMENT.

In this section, a typical run representative of those carried out during the experimental period, will be described step by step and an account will be given, where necessary, of the equipment utilized at each stage, with references to the appropriate detailed figures. Special mention will be made of important principles involved in the design of particular parts of the apparatus. It was assumed that this method of approach would serve better than any other the purpose of describing the experimental equipment, particularly because it was necessary to design specially a number of the individual components in order to construct the apparatus.

A typical experiment began with the accurate setting in the measuring device shown in figure 1 of the sharp-edged disc, K, shown in figure 19 and plate 5, to traverse internally the mass transfer section, H, shown in figure 20 and plate 3, without damaging the carefully cast-in smooth volatile lining of diphenyl. The test section, H, was placed on the gauging-stand before the volatile solid was cast in the tube. The sharp-edged disc, K, was mounted on a mild steel horizontal rod, F, which was held firmly in the same fixed position against the rod by means of a locking screw. A flat mild steel plate, C, which supported the central horizontal rod, F, was held in position against a vertical mild steel plate, A, making a three-point contact with the latter by means of two lock-nut adjusting screws.
screws, W, and a ball-bearing, U, which fitted neatly in a circular recess in plate, C. It was possible with this arrangement, which is shown in figure 21 and plate 1, to move the sharp disc in a horizontal plane by continual adjustment of the screws, U.

The gauging stand, shown in figure 21 and plate 6, was essentially a mild steel frame supported by the two vertical plates, A and B, which were held together by a horizontal rod, D, and two larger horizontal rods, D; all these rods were of equal length. The legs of the supporting plates, A and B, formed a tripod and the complete assembly was thus prevented from rocking during the air-gauging operations. The carriage E shown in figure 22 and plate 6, was a mild steel plate, which could slide freely on the lubricated smooth rails, D, by means of metal legs fitted in the underside of the plate.

The test-section H - resting on the carriage E with the aid of a three-point contact - carried two front legs, S, and a third rear-leg, T, as shown in figure 21 and plate 3. The two front-legs, S, rested on the carriage at the supporting fixed pillar Y as shown, and the third height-adjusting leg, T, on the moveable metal piece Z. The design of the apparatus ensured that the correct number of constraints were incorporated in order to prevent movement of the brass tube, H, during its journey along the rails, D. This result was achieved by having one of the two front legs rest on a V-groove, L, and a free-point/
point contact for the second at \( M \) on the fixed pillar \( Y \). The third leg \( T \) was secured by a V-groove which was set in the moveable metal-piece \( Z \), and was moved along the plate \( E \) at right angles to the direction of the carriage.

The carriage assembly which is shown in figures 21, 22 and 23, was free from rocking during its horizontal journey along the rails, \( D \), since a three-point contact was established between the rails and the carriage legs; the latter consisted of two mild steel V-blocks, \( N \), which had been machined smooth, resting on the same rail, \( D \), and a third simple mild steel block, \( N' \), which made a free point contact on the adjacent horizontal rail.

The particular design, utilized in the construction of the moving components of the gauging stand, was aimed at overcoming the following formidable obstacles. First, two seemingly insuperable difficulties involved in the construction of the assembly were circumvented; these were (i) that the axes of the two horizontal rails should always be parallel to one another, and (ii) that the two bars should lie in the same horizontal plane.

Second, greater room for manoeuvre was gained in the actual profilometric operation during the difficult process when the test-section, \( H \), was moved past the sharp-edged disc \( K \). The method used to achieve the complete traverse of the tube by the disc was based on the principle, that once the sharp disc \( K \) cleared each of the two nylon rings, \( Q \), which were situated at the/
the two extremities of the tube, it was then possible to move
the test-section without any further adjustments being necessary.
By means of this method it was possible to eliminate the extremely
restrictive condition, which required that the axis of the
horizontal central rod, F, be made exactly parallel to both axes
of the horizontal guiding rails, D.

The carriage and test-section assembly shown in plate 1
were driven along the horizontal rails, D, by a simple rack, R,
and pinion, P, mechanism which was fitted to the underside of
the rail, D, by means of three brass clamps, Y. This is shown
in figure 22. The whole air-gauging assembly was kept in a room
with a constant temperature, which was checked on a BRISTOL
SERIES 500 thermograph.

The test-section, H, was then removed from the gauging
stand and the cast-in volatile lining of diphenyl was next
prepared in the following manner. Molten diphenyl, at about
80°C., was carefully poured from a beaker into the pre-heated
test-section through the inlet shown in plate 4, and the
experimental volatile surface was cast on the smooth face of a
metal core which had been introduced earlier into the tube
through the two tight-fitting nylon rings, O. The smooth metal
core is shown in plate 2. Details of the special casting
procedure evolved for the preparation of the smooth volatile
surfaces during the present investigation are given in an earlier
section (4.6.2.). (The preparation of cast surfaces was a
very/
very long operation lasting up to 24 hours. Three test-sections were therefore constructed for use in this investigation to reduce the time required to prepare each cast-surface.)

When both the test-section and the cast diphenyl had cooled completely, the metal handle shown in plate 2(c) was screwed into the appropriate end of the cone. Liquid nitrogen was then poured in at the other end whereupon the instantaneously chilled metal cone was quickly extricated from the test-section by a sudden sharp jerk.

The test-section H was then returned to its original position on the carriage, E, of the air-gauging apparatus and the freshly-prepared volatile lining of diphenyl was next profiled as follows. A steady supply of gauging-air of constant pressure, $P_1$, was drawn from a compressed air-cylinder. The air then entered the test-section, via a slot in the clear perspex flange $O$ shown in figure 21, after it had been saturated with diphenyl vapour by passing the air through a washing-bottle, $W$, packed with diphenyl, plate 1(b). The flange $O$ was screwed to the rear flanged-end of the test-section H, as shown in plate 1, and an air-tight joint was established by the introduction of the appropriate $O$ ring between the flanges.

The sharp-edged disc $K$ was next carefully introduced into the mass-transfer tube. Details are given in section 4.5 of the special technique that was evolved in the course of the present investigation, by which it was possible to ensure definitely that no/
no contact was ever established at any time during the profilometric operation between the sharp-edged disc \( K \) and the cast-in volatile lining. In this manner it was possible to ensure that the smooth mass-transfer surface in the tube was free from possible damage which could result from the sharp disc.

Measurements of the intermediate pressures, \( P_2 \), across the sharp-edged disc, \( K \), were registered at regular chosen stations along the length of the volatile lining on one of two arms of a combined system of two mercury manometers, shown in plate 7. The remaining arm was made always to give the same reading of air-supply pressure, \( P_1 \), which in this case was 25.4 cm. of mercury. The object of utilizing this particular design, in the construction of the manometer, was to increase the overall accuracy in the measured value of the mass-transfer coefficients during the profilometric operation. This becomes evident from the following consideration. Whereas, in general, the number of mercury levels observed in a system of two separate manometers is necessarily four, readings of only two similar levels were now sufficient. Consequently, the personal error involved during the estimation of pressure-measurements is thereby reduced by half.

Air, which was pre-saturated with diphenyl, passed through the standard calibration device consisting of the externally-tapered cone and disc restrictor assembly, \( R_1 \). The air passed subsequently through the clearance between the sharp-edged disc and mass-transfer tube, i.e., through the second restrictor, \( R_2 \), until \( P_1 \) registered a pressure of 25.4 cm. of mercury. The pressure/
pressure, $P_2$, was read quickly on the left arm of the manomotor and the air-supply was then immediately turned off by means of a needle-valve. This procedure was repeated for every measurement of the gap-width. It was possible in this way to restrict to a minimum the total time in which the test-surface was exposed to the gauging-air (section 4.4).

The sharp-edged disc $K$, test-section $H$ and the calibration device were all constructed of the same material, namely brass. Thus, the two gap-widths - in the calibration device, $A_1$, and gauging apparatus, $A_2$, - behaved similarly having the same coefficient of thermal expansion. This precaution would minimise error when estimating the gap-width $A_2$, and therefore the mass transferred.

The regular stations, which described the journey of the test-section during the profilometric operation, were registered by means of a hair-line indicator set in a strip of clear perspex, $X$, shown in plate 6. The indicator which was firmly secured by screws on to the leading edge of the carriage, $E$, moved along a stainless steel metal rule, $G$, permanently fixed to the wooden base, which supported the air-gauging assembly. The arrangement is shown in plate 8. When the profilometric operation was completed, the perspex flange, $O$, was removed and the test-section, $H$, was placed in the closed-circuit wind-tunnel, in which the profiled volatile lining was sublimated to the recycled gas.

The/
The design of the wind-tunnel was essentially a closed-loop pipe arrangement, in which gases of particular interest in the present investigation were re-circulated at different velocities. A schematic diagram of the experimental arrangement is found in figure 23 with detailed drawings of important sections of the apparatus in figures 24 and 25. The wind-tunnel was entirely of P.V.C. except for the flow-measuring device, M, which was made of perspex. The apparatus was so designed and constructed that gases of different properties were re-circulated under a pressure slightly higher than atmospheric pressure for various given velocities. A build-up in pressure in the wind-tunnel was avoided by incorporating a pressure-bleed, P°, shown in figure 9(b). A P.V.C. flexible tube was connected to a pressure-tapping consisting of a \(\frac{1}{4}\) inch-diameter slotted-hole in the lower part of the tunnel. The other end of the tube was dipped in water contained in a measuring cylinder and the desired pressure in the tunnel was obtained by setting the water-level correctly.

The experimental gases utilized in the present work, were air, carbon-dioxide and Freon-114. The gases were introduced into the wind-tunnel via the valve, W, and the apparatus was rid of the gas by means of the valve, Z. An important factor which influenced the design of the apparatus was the high cost of using the gases carbon dioxide and, more particularly, Freon-114, in experiments of up to five hours duration. Leaks in the apparatus/
apparatus were drastically reduced by carefully tightening the Jubilee-clips, J, at every flexible P.V.C. hose-connection, P, and by introducing the appropriate 'O' rings at each of the P.V.C. flanged connections, which were then bolted fast together. The soft P.V.C. flanges offered a particularly easy and sensible means of effecting the required seals; and although it was impossible to achieve perfect leak-proof conditions, the rate of gas seepage from the apparatus was reduced to negligible proportions.

The wind-tunnel had the usual features commonly incorporated in the design of slow-speed closed-circuit wind-tunnels; and these are described briefly in this section in the order in which they occur in figure 23.

They are as follows: (a) the effuser, E, which was placed upstream of the working-section, H. In it the fluid was accelerated from rest, or approximately from rest, at the upstream-end to the required conditions at the test-section. The effuser was a high-ratio, (23:1), smooth contraction, E; (b) a hydrodynamic calming-length of tube, C, with a flow-straightener arrangement consisting simply of drinking straws immediately after the contraction and at the leading end of the calming tube (P.6); (c) a thermometer, T₁, which registered the temperature of the re-circulated gas in the apparatus, (d) the working section, which in this particular case was the mass-transferring tube, H. The test-section is connected to the calming sections, at/
at both of its ends, by means of a flanged and spigoted joint; (e) a short calming-length $D_{(P.6)}$; (f) a flow-measuring device consisting of a Pitot-tube assembly, $M$, shown in greater detail in figure 24; (g) a second thermometer, $T_2$. The performance of both thermometers, $T_1$ and $T_2$, was checked regularly against a standard thermometer throughout the experimental period. It was important to eliminate uncertainties in reading the temperature because of its large influence on the vapour-pressure of the subliming volatile substance. The method employed to seal the thermometers is shown in figure 25; (h) the diffuser, $F$, whose function was to reconver the kinetic energy of the fluid-stream leaving the working section into pressure energy as efficiently as possible; (i) the driving unit, $G$, which was necessary to overcome the losses in energy of the recycled fluid due to vorticity, turbulence and eddying motion. The driving unit consisted of a Secomak sealed blower, Model 428D, which had the following specifications: 1/2 H.P., 3-Phase motor, 400/440 volts, 36 inches water-gauge, F.h. speed 2850 r.p.m., and capable of 100 c.f.t.m. delivery on continuous duty performance. The blower was situated at the fourth corner of the apparatus and is shown in plate 9(a); (j) a simple gate-valve, $V$, shown in plate 9(a) controlled the speeds of the operating gas and connections were made with the tunnel and the blower by means of clear P.V.C. hose; (k) smooth, curved return-arms at the three remaining corners guided the flow of the gas in the tunnel.

Special/
Special features of the wind-tunnel were the following: (a) a long hydrodynamic calming-section, C, 100 diameters long instead of the 50 to 80 diameters more commonly found in wind-tunnel work. The choice of a long calming-section ensured that fully-developed hydrodynamic conditions obtained at the mass-transfer section, H. This was of prime importance, since the present work required the study of a growing concentration boundary layer in fully developed turbulent flows; (b) a transparent P.V.C. bag, L, shown in figure 23 and in plate 9(b), which was fitted with rubber gloves and capable of withstanding a few inches of water pressure, was cemented around the working section. It was then possible to carry out essential manipulations of the apparatus, when the test-section was either introduced or removed from its position in the wind-tunnel, without loss of significant quantities of the expensive gases. Furthermore, since the P.V.C. bag was filled under pressure with the same experimental gas, no contamination from the surrounding air was possible.

A blank test-section, R, shown in plate 9(b), made from perspex and 1\(\frac{1}{4}\)" internal diameter was initially placed in the wind-tunnel whilst the brass mass-transferring section was kept inside the P.V.C. bag, L. The gas was circulated in the wind-tunnel, until steady conditions in the flow were attained; at this point, the velocity of gas was arranged by means of the valve, V, to give the particular Reynolds number of interest. The perspex/
perspex tube was then replaced by the transfer section, H, and the volatilisation run was immediately begun by starting the blower, G. The test-section, H, was then replaced by the blank tube during the gauging operation. (c) Heat, which was produced by the sealed blower and the recirculated gas, was effectively removed from the system during experimental operations by the heat exchanger, K. This was a series of helical coils made from 125 ft. of 1½ inch bore copper-tubing, through which flowed cold tap-water. Steady temperature conditions in the apparatus were quickly attained after a rise of about 1°C in the temperature of the working gas.

(d) A surge-tank, S, 2 ft. in diameter, was situated immediately before the suction-end of the fan, G, and a 2 inch-high bed, B, of activated charcoal was placed in the tank. The arrangement is shown in plate 9(a). The surge-tank and the activated charcoal constituted what was effectively a filter-bed arrangement, taking advantage of the very low speeds of the experimental fluid at the charcoal bed in order to strip the recirculated fluid of sublimated diphenyl traces. The packed-bed of activated charcoal was easily accessible by means of the removable sealing lid of the surge tank. It was an easy matter to check the state of the charcoal bed and to replace the spent charcoal with a fresh supply of activated carbon. The activated charcoal had the following specifications: British Carbo Norit Union Ltd., trasorb SCII, 4 x 8 mesh.

A/
FIG. 26 - FLOW VELOCITY MEASUREMENT

- Tube Traversed Vertically
- Horizontally

Re = 17,040

Pitot-Tube syringe graduations
FIG. 27: FLOW VELOCITY MEASUREMENT

\[ \sqrt{\frac{h}{(\text{cm}^2)}}, \quad \text{Re} = 42,450 \]

Pitot-Tube syringe graduations
A sample calculation for the minimum height of activated-charcoal bed required is shown in Appendix IX. However, the depth (2\(\frac{3}{4}\) inches) of the filter-bed used in the experimental runs was about ten times greater. This was done to increase the operational life-span of the charcoal filter-bed.

(a) the flow-measuring device was a Pitot-tube arrangement consisting of (i) a static pressure-tapping, \(X\), which was a simple slot in the P.V.C. flange, and (ii) an impact pressure-measuring device, \(X^0\). This was made from a clear perspex tube in which a piece of hypodermic tubing, connected to an ordinary perspex syringe, moved easily along calibrated marking in the syringe. The arrangement is shown in figure 24. The advantage in the design of this device was the simplicity of estimating, when necessary, the radial position of the impact-tube in the tunnel. Also it was a simple matter to align correctly the hypodermic tubing in the flow, merely by viewing through the transparent perspex tube. The impact-tube and pressure tapping were connected to the inclined manometer, \(Q\), which is shown in plate 9(b).

The flow-pattern in the wind-tunnel was investigated by making measurements of the impact/static pressure differential at nineteen equally-spaced points on each of two diameters of the tube at right angles to one another. Typical results are shown in the graphs of figure 26 and figure 27 for a Reynolds number of 17,040 and 42,450. In the graphs shown, the square root of the impact/static pressure differential, \(\sqrt{h}\), is plotted against/
against the diametral distance from the wall, at which readings were made on each traverse. The diametral distances were read from the graduated syringe of the Pitot-tube assembly shown in Figure 24. It is seen from Figure 26 that turbulent flow was obtained in the wind-tunnel for a Reynolds number as low as 17,040. The Reynolds numbers were calculated as shown in Appendix V.

Precautions were taken to avoid errors in the measurement of the flow-velocity resulting from blockage of the hypodermic impact tube in the Pitot-tube assembly. The procedure followed was as follows:

(a) the measured impact/static pressure differentials were checked for each experiment against measurements of the pressure drop across the heat-exchanger unit, K, on the manometer panel, R, shown in plate 9(b); (b) the hypodermic impact tubing was cleaned before each run by passing a clean, fine steel-wire through the tube.

(f) special connections were introduced in the apparatus through which the working gas was supplied to the tunnel from compressed gas cylinders. The connections W and Z are shown in Figure 23. In addition, a pressure-bleed, L, shown in plate 9(b) was introduced since the tunnel operated under one inch of water-pressure.

(g) it was found necessary to include an ammeter, A, shown in plate 9(a) with the fan and to check the current in the blower during the experimental operations. It was found that the current/
current in the blower increased appreciably above the allowable 2.06 amps/phase, when the heavier gases were used, and excessive heat in the motor resulted. For Freon-114, the density of which is approximately four times that of air, the current registered as high as 4.5 amps. For this particular case, a larger impeller pulley was used to reduce the velocity of the gas in the apparatus.

Finally, the test-section was removed from the wind-tunnel at the end of the sublimation run and was replaced by a dummy 1\(\frac{1}{4}\)-inch diameter perspex tube.

The test-section was returned to the gauging-stand and the sublimed surface was profiled again as already described earlier in this section.

The experiment was terminated prematurely on several occasions when the environmental temperature changed by 3°C or more.
5. DISCUSSION OF RESULTS.
5. DISCUSSION OF RESULTS.

5.1. ACCURACY OF METHOD EMPLOYED TO MEASURE WALL RECESSIONS.

Mass-transfer coefficients were determined using the air-gauging technique described in section 4.5. The method consisted of measuring the local rates of recession that occurred on a cylindrical volatile surface, following sublimation, in the course of an experimental run. The recessions were estimated, at given points on the wall, from the difference between the gap before and after a sublimation run. The relationship between the gap-widths and the measured intermediate pressure, $P_2$, is shown in figure 16; and by use of the graph it is possible to estimate the gap-width from a knowledge of the accuracy involved in measuring $P_2$.

Inspection of figure 16 shows that, for a given restrictor-setting, the degree of curvature increases as gap-widths increase owing to the growing insensitiveness to the intermediate pressure $P_2$. The functional dependence of gap-widths on $P_2$ is more sensitive for the higher values of $P_2$; and the accuracy is therefore greater for smaller clearances.

For any particular experimental run, the higher values of intermediate pressure-readings were obtained before sublimation of the volatile wall took place. Gauging-air was supplied at a constant pressure, $P_1$, from a compressed air-cylinder, so that steady manometric readings of $P_2$ were obtained which could be read to $\pm 0.02$ cm. of mercury. Intermediate pressures in the range of 7.1 cm. mercury and 16.0 cm. mercury were employed in the/
the gauging-operation; the corresponding gap-widths are $(1.872 \pm 0.037) \times 10^{-3}$ ins. and $(0.78 \pm 0.015) \times 10^{-3}$ ins. respectively. Higher intermediate pressures of up to $18.5$ cm. mercury corresponding to fine clearances of $(0.583 \pm 0.011) \times 10^{-3}$ ins. were sometimes encountered; but on the whole, such high pressures were avoided by replacing the larger gauging-disc with a slightly smaller one. This was done for two reasons. First, these high pressures were of sufficient force to lift the test-section off the gauging-platform and seriously damage the volatile test-surface. Second, it became extremely difficult and tedious to profile the tube-wall at fine gaps to the order of $(0.500 \times 10^{-3})$ ins. without having the surface scoured by the disc.

The worst possible error occurred for initially large gaps, i.e. for freshly-prepared cylindrical mass-transfer surfaces, the internal diameters of which were large. Under such conditions, initial pressure readings for $P_2$, as low as $13.5$ cm. mercury, were obtained. This corresponds to a gap of $(0.990 \pm 0.019) \times 10^{-3}$ ins. The worst possible result obtained was where the initial and final values of $P_2$, were to the order of $7.1$ cm. mercury and $13.5$ cm. mercury. The recession is then $(0.882 \pm 0.057) \times 10^{-3}$ ins. and the maximum error is $\pm 5.7\%$. 
5.2. THE GRAPH OF $P_2$ vs. $x/d$ AND ITS SIGNIFICANCE. SURFACE EXPLORATION TECHNIQUE.

Results from mass-transfer experiments conducted at early stages in the course of the present investigation indicated the existence of an abnormally large scatter. This was all the more difficult to explain, since the volatile surfaces were to all intents and purposes perfectly smooth, both to the touch and to the eye. The possibility could not be ruled out at this stage, that the profilometric technique employed in this work was not such a reliable a method as had been supposed. Nevertheless, the likelihood that random, minute surface-irregularities were mainly responsible for the observed large scatter became greater as a result of the hissing sound emitted at various stages during the profilometric operation. In view of the degree of accuracy which was demanded in the present work it was important, firstly, to understand the source of the abnormality, and secondly to apply a remedy to it.

The method which was found to be most satisfactory for this purpose was the development of a means of exploring the cast surface which exploited the high degree of sensitivity of the profilometric technique.

The intermediate pressures, $P_2$, were plotted against the number of tube diameters, $x/d$, immediately before and after each experimental run; a typical graph is shown in figure 28. The interpretation of simple graphs such as this was found to yield considerable information about the nature of mass-transfer surfaces in general and will be discussed in some detail:

(a)/
(a) The distribution of the pressure-points, $P_2$, along the $x$-axis, conveys a two-dimensional picture of the studied volatile wall contained in the test-section. Detailed examination of those "pictorial" graphs for all the surfaces suggest that, (i) although the cylindrical test-surfaces were prepared by casting the volatile material on the same smooth metal core, no two surfaces over possessed the same final appearance; (ii) the important point established from the wavy nature of the graphs was that it is not feasible to obtain an absolutely even surface, however smooth the casting-mould. It is interesting to note that the method used by MacLean (M.4), to prepare his test-surfaces by boring the cast volatile wall, contained the hitherto unsuspected wavy pattern, as can be seen from figure 23 which was prepared from Table A.33 (M.4). The reason for this can be explained, perhaps, by the complex manner in which organic substances, with large-sized crystalline structure, cast on cooling (M.16). The wavy nature of cast-smooth free surfaces was further confirmed from observations with the aid of a microscope on the perimeter of cast naphthalene and diphenyl spheroids.

The above-mentioned findings are supported by the investigations of Sherwood and Träss (S.37). In that work, which was discussed in section 4.1., surface traces were obtained of the transferring sections, consisting of cast-naphthalene plates, by means of sensitive strain-gauges. The results were presented as graphs of local coating-thickness against distance along the plate. A reproduction of/
Figure 6-8 Surface traces before and after run A-1, and profile of naphthalene thickness decrease.
of these graphs, presented in figure 26A by way of example, shows clearly the irregular pattern of the flat naphthalene surface. It can also be seen from the same figure that, although the two surface traces follow each other closely on the whole, there are regions where this breaks down. The points where deviations occur between the two traces are marked, for the sake of clarity, by 'valley-peak' and 'peak-valley'. For the remaining surface where no irregularity is observed, a 'peak' follows a 'peak' and a 'valley' follows a 'valley'.

However, Sherwood and Träss did not follow these observations by an investigation into the relationship of the traces with the actual condition of the surfaces. Furthermore, no attempt was made to interpret the curves in terms of surface-roughness and its possible influence on the mass-transfer data obtained; and finally, it is not known to what degree of certainty an estimate of both, mass-transfer rates and surface roughness, could be determined. The authors themselves observed that the stylus of the measuring device, in contact with the naphthalene, had a distinct tendency to wear down surface peaks.

(b) The values of the intermediate pressure readings, $P_2$, at any point along the test-section, prior to the sublimation-experiment, were always higher than those after the experiment. Thus, the difference of pressure readings at any point was a measure of the amount of mass transferred during a sublimation-run in the wind-tunnel. The graphs of $P_2$ vs $x/d$ proved invaluable, especially in the present investigation, in swiftly establishing the effectiveness of the activated/
activated charcoal filter-bed in stripping the re-circulated gas in the wind-tunnel of volatilised material. Low values of pressure differences, $\Delta P_2$, gave an immediate indication of the extent to which the filter-bed had undergone saturation.

In addition, comparison of the pair of curves enabled faulty readings and damaged walls to be spotted at a conveniently early stage of the experiment. MacLean's sample results were plotted and are shown in figure 28. The pressure-readings, $P_2$, after the sublimation-run often appear to have higher values than those obtained before the run. This is because the micrometer-setting and hence the gap-width, $A_1$, in the standard calibration device, $R_1$, (figures 11 and 12), was varied during the profilometric operation. The values of the gap-width $A_2$ obtained for the mass-transferring section in any profilometric operation are not immediately comparable. The profilometric method employed by MacLean, suffered from a severe limitation in that there was no way of knowing the fate of the volatile wall during its traverse by the sharp-edged disc. MacLean states in his work that as much as one-third of his profiled surfaces had to be scrapped, owing to serious damage of the surface by the sharp disc during some stage of the profilometric operation.

(c) It was noted from these graphs that an appreciable difference in pressure-readings occurred, when occasionally a reading had to be re-taken for the same distance along the test-section, air alone acting as the gauging fluid. This observation led to the investigation/
investigation of the interfering effects of the air-gauging method and is discussed in section 4.4.

(d) A rough estimate of the 'entrance length' could be quickly established from the lower of the two curves in figure 28. This is seen to consist of the length along the x-axis which is below that portion of the curve which rises continuously till it coincides with the flat section.

(e) Careful examination of the pairs of points for the pressure, $P_2$, in figures 28, 30, 32 and 34 gave evidence of the precision which was exercised in the present investigation. The lower curve is a fiducial reproduction of the parent higher curve, and exhibits point by point, the same trend throughout its length.

(f) At several points, this faithful reproduction of the pressure-readings was found to break down. Whenever this occurred, it was noticed that the pressure-readings taken after the sublimation-run were too low. This abnormality suggested a surprisingly high transfer rate which defies all theoretical explanation. The possibility of interference by the gauging apparatus was discounted since (i) as seen in section 4.4., volatilisation of the examined surface was effectively suppressed by saturating the gauging-air with diphenyl; (ii) the pressure-readings at various points along the transfer-surface were obtained while care was taken that the surface at these points was exposed for the same length of time to the gauging-gas mixture. This was discussed in section 4.4. (iii) the possibility that the sharp-edged disc marred the surfaces was ruled out/
out by taking into consideration the development of the profilometric technique described in section 4.5.

The only remaining possible explanation for this abnormal behaviour in the pressure-readings was the presence of some kind of imperfection on the cast walls. Sections of these surfaces were closely examined under the microscope, revealing the presence of tiny cavities. Further observations during casting showed that there was a distinct tendency for extremely small air-bubbles to be trapped in the molten material, which in turn quite frequently produced voids, invisible to the naked eye. These manifested themselves, depending on the depth of the voids, either directly on the freshly prepared surface, or after part of the wall-thickness had been skimmed off during the sublimation run.

Examination of a test-surface, prepared in the method used by MacLean, figure 28, also reveals the unsuspected presence of such voids on the surface. The method which was used, consisted first of casting the material on the inside wall of the tube, by pouring the molten diphenyl down a chute, and then machining the surface obtained.

It was as a consequence of those findings that a method was made possible which allowed the examination of the transfer-surfaces prior to the interpretation of experimental results. Still more important was the fact that it was now a fairly simple matter to deduce the quality of the test-surfaces from the $P_2$ vs $x/d$ graphs, and only then to proceed with the experiment, using what was regarded/
FIG. 29-TYPICAL RESULTS FOR REJECTED TEST SURFACES
regarded as acceptable transfer-surfaces. The freedom of choice in
good surfaces constituted a major advance in the attempt to reduce
scatter in the data. Figure 29 is a typical example of the kind of
surface which was rejected.

Those considerations indicate that the formation of voids is
an integral occurrence which is met with in general during casting
operations. An extensive study of a variety of casting techniques
revealed that practically no voids were present, when the cast material
was allowed to cool from the base of the supporting V-blocks upwards
in a controlled-temperature environment over a period of about three
hours. This subject is discussed in detail in section 4.6.3.

Literature dealing with mass-transfer investigations suggests
that surfaces prepared by casting techniques suffer no more from
surprisingly high scatter than surfaces prepared by other methods,
e.g., by pressing powders in a mould. An example of
difficulties met with in cast-surfaces is the investigation by
Linton and Sherwood which is discussed in section 2.1.2. The
presence of voids is clearly shown, especially in the case of tubes
prepared from benzoic acid; and the results have been widely inter-
preted as suffering from a high degree of "scatter." It is not
understood, however, why surfaces of inferior quality were obtained
with benzoic acid, compared to those prepared from cinnamic acid and
β-naphthol. A possible explanation, perhaps, may be that there was
some alteration in the casting operations for those substances.
Further examination of the data obtained by Linton and Sherwood shows
not only scatter within a series of runs but also higher mass-transfer
coefficients.
coefficients for large tubes, 5.23 cm, than for smaller ones, 1.9 cm. Data obtained by Cowan, D.\(^{(c,9)}\), on the limiting current in thin electrodialysis cells, showed that their transfer coefficients were two to four-fold below the lowest values of Linton and Sherwood. This led Harriott\(^{(H,8)}\) to suggest that perhaps the diameter of the channel was an overlooked but significant parameter. However, on the evidence of the present findings on the condition of the cast test-surfaces, it is not unreasonable to assume the presence of a considerably greater number of voids in the larger tubes. This means that the surfaces for the 5.3 cm. tube would possess a greater degree of surface imperfections than that of the 2.3 cm. tube. Consequently, as seen in section 2.5.3, higher transfer rates for the larger tubes would be expected from the larger mass-transfer tubes.

(g) The 'apparent' high scatter in mass-transfer data from graphs of \(\frac{Sh}{Sh_\infty}\) vs \(x/d\) was attributed to surface imperfections rather than to experimental errors in the estimation of the mass-transfer coefficients. That this conclusion is correct is further borne out from the fact that a clear relation was established between the so-called 'scatter' in the \(\frac{Sh}{Sh_\infty}\) vs \(x/d\) curves and the abnormality observed in the behaviour of the corresponding \(P_2\) vs \(x/d\) curves.
5.3. SURFACE ROUGHNESS

5.3.1. Deterioration of Mass-Transfer Surfaces in Still and Streaming Air.

The behaviour of diphenyl test-surfaces, when exposed first to still air and then to an air stream, was investigated before measurements of mass-transfer rates were taken. Early observations in the course of the present work indicated that occasionally a change took place in the appearance of the diphenyl-coating in the tube, when this was left to stand overnight.

Transfer surfaces of mirror-like smoothness were then specially prepared by the casting method described in section 4.6.3. These were left to stand for a day in a room with constant temperature of approximately 20°C. No discernible changes in the walls of the volatile lining were observed. However, an appreciable deterioration in the condition of the surfaces was observed when the temperature in the room was increased by as little as 5°C. In another series of experiments, the freshly-prepared good-quality surfaces were exposed to an air stream in the wind-tunnel. The mirror-like smooth diphenyl wall was found once more to retain its original appearance. These results indicated that temperature was the more important influencing factor.

Diphenyl surfaces of moderate quality possessed a certain degree of roughness which was found to decrease by sublimation in both still air and in air stream. It was further noted that where the surfaces were initially smooth, they remained unaffected after/
after sublimation; the rough sections of the diphenyl wall, however, remained invariably rough. Again, an increase in temperature resulted in an increase in surface deterioration.

Both good and mediocre surfaces were explored more closely, using the profilometric technique described in sections 4.5 and 5.2. The results showed that the concern over the deterioration of the test-surfaces under the conditions described above had little foundation. However, an exception to this of serious consequence on the accurate measurement of mass-transfer rates was the situation where the apparently excellent diphenyl-linings concealed the presence of imperceptible voids immediately next to the smooth surface. This topic was fully discussed in sections 4.6 and 5.2.

Sherwood and Trüss (8.37) arrived at almost the same conclusions as those discussed above, in a study of this topic carried out on flat naphthalene plates. But no mention is found in their work of any investigation being made to examine the influence of temperature on the quality of the test-surfaces. Also, nor did these authors make any attempt to examine the nature of the roughness observed on their plane naphthalene surfaces; and no mention is to be found of the presence of voids in the cast plates. The existence of such voids would seem to explain the break-down of the agreement between the two surface-traces in figure 28A, (also, section 5.2).

An explanation given by Sears (8.38) appears to give a satisfactory account of sublimation of volatile solids to a fluid stream. Sublimation from a flat surface of a crystal requires a vapour-pressure/
vapour-pressure driving force which may be twice as large as that needed to cause sublimation from the edges of the crystal. The observed influence of the temperature discussed above is also well accounted for by this explanation; the vapour-pressure of volatile substances is very sensitive to temperature (APPENDIX VII).

5.3.2. Influence of Surface Roughness on Mass-Transfer Rates.

Surface irregularities \(1\) thou. in. was the upper limit of roughness tolerated in the present research. This target was calculated on the generally accepted assumption that the undisturbed laminar sub-layer next to the volatile-lining extends to a distance \(0 \leq y^+ \leq 5\). In actual fact, owing to the method of surface-preparation which was developed in the present research, the irregularities normally encountered were \(< 0.5\) thou. in. Irregularities on the test-surfaces of this order of magnitude do not appear to have affected the results obtained in the present investigation. This is evident from the results of the reproduced runs for air and carbon dioxide, shown in figures 36 and 37. In addition, the upper limit of surface roughness, namely \(0.5\) thou. in., should also satisfy the more conservative assumption used to describe the thickness of the laminar sub-layer, \(0 \leq y^+ < 2^+\).
5.4. MASS TRANSFER RESULTS: THE GRAPH OF Sh/Sh∞ vs x/d.

5.4.1. Experimental Scatter and its Control.

The accuracy of mass-transfer investigations is generally limited by the persistently high degree of scatter associated in the measurement of the mass-transfer coefficients. It was shown in sections 4.6 and 5.2 that where methods of high accuracy were employed, the inexplicable departure from the expected degree of scatter is explained by the poor quality of the test surfaces.

It was not until the present work was undertaken, that a way was offered of evaluating the quality of the mass-transfer surfaces prior to the experiment. The method developed in the present investigation, the surface-exploration technique (section 5.2), provided a means of selecting high-grade surfaces which contained tolerable imperfections. It was therefore possible to control effectively the result in the measurements of the mass-transfer coefficients by eliminating that part of the scatter which was contributed by poorly prepared surfaces. An example of this is the work by Linton et al. (L.2) and is discussed in sections 2.2.2 and 5.2.

In a work by Macleod et al. (M.17), referred to in section 4.1, the authors report the presence of large scatter in their mass-transfer results. Changes in coating-thickness of a volatile solid were measured with an accuracy of ± 6 x 10^-5 in., or less than ± 10%, by employing a probe which consisted of a combination of electromagnetic and pneumatic proximity-gauges. Deviations from the expected scatter were observed ranging from ± 10% to ± 20%.
In particular, when attempts were made to reproduce the results of a complete experiment, deviations of between 30\% and 40\% were observed in the two sets of experiment for over half the length of the surface investigated. This large scatter was found to be mostly negative, which meant that the changes in the coating-thickness were lower than their expected values. Positive scatter was attributed by the authors to be a result of local damage to the test-surface.

In view of the conclusions drawn from the surface-exploration technique, there is good reason to believe that the major contributing factor to the difficulties encountered in that work was the presence of imperceptible voids on the freshly-cast rods. The incongruous results giving large negative scatter is partly accounted by the presence of surface imperfections in the shape of very small voids before the start of the sublimation experiment. A large positive scatter suggests a good initial surface exposing a poorer surface after completion of the sublimation run. The explanation suggested by Macleod et al. is the possible separation of the naphthalene-coating from the metal rod. However, contrary to the findings in that work, both naphthalene and diphenyl were observed to adhere well to polished metal surfaces of steel or brass in the course of the present work, whether the surfaces were varnished or not. The reported failure of some naphthalene coatings to adhere well to the steel rods during the investigations of Macleod et al. may have been caused by the presence of grease or other impurities.

In conclusion, scatter of the same magnitude and sign was observed/
observed in the present investigation, whenever faulty test-surfaces were employed.

5.4.2. Effect of Secondary Flow: Result of Volatilisation which is Generated in the Measuring Device.

The pneumatic techniques utilized to date in order to estimate local mass-transfer rates were based on the assumption that volatilisation arising from the action of the gauging-fluid on the volatile surface must be negligible.

In extensive work carried out by various investigators (M.12, M.17, M.4) use was made of this assumption whenever air was the gauging fluid. However, the results of experimental enquiries concerning this assumption during the course of the present work, have shown clearly that considerable volatilisation of the mass-transfer surface occurred during the profilometric operation. This topic is discussed in section 4.4. The highest allowable value of wall-recessions measured were of the order of less than 0.001 inch; the lowest recessions were slightly greater than 0.5 thou. in. It is shown in figure 18 that sublimation owing to volatilisation by the gauging-air attains a maximum value of about 0.5 thou. in at the entrance of the tube; and it then falls off to about 0.25 thou. in. for the remainder of the tube.

The meaning of these results can now be explained. The usual cumulative wall-recessions of the diphenyl surface, after completion of an experimental run, was between 0.5 to 0.9 thou. in. Consequently, the interference caused by the gauging-fluid yields data which are too/
too high, the error ranging from 20% to 100%. Data for mass-transfer rates obtained by this method are then distorted by an additional component of undesired mass-transfer.

The consequences of these findings are extremely serious, especially since, as seen in figure 18, the largest amount of the undesired volatilisation occurs at the entrance region of the pipe. This region, in fact, constitutes that part of the tube which is of the greatest interest in the investigation. Data obtained in this manner must therefore lead to erroneous interpretation of experimental results.

In the work undertaken by MacLean (M.4), the measurement technique employed was based on the same principle as that of the present investigation. The failure, however, to appreciate the presence of this complicating factor and its consequences, invalidates the author’s data and therefore the conclusions drawn from these. The extent of disagreement between the results of that work and the present investigation, where the secondary flow was suppressed, is shown in figure 40. MacLean’s data are consistently higher, thus indicating the presence of extra mass-transfer resulting from volatilisation generated in the measuring device itself. This problem was aggravated in MacLean’s investigation by the particular profilometric technique used, in which the gauging-air acted continuously throughout the long periods required to profile the test-surface.

Support for the above argument is given by the results of MacLean/
MacLean for laminar flow. The experimental curves are reproduced in figure 42, in which the local Nusselt number, $Nu$, is plotted against the downstream distance of the tube for six Reynolds numbers. In all six graphs, the experimental data lie well above the theoretical results calculated from the approximate solution of the well-established analysis of Graetz for laminar flow. In figure 42 the analytical curve is represented by the dotted line and is composed of two separate parts, each calculated from a different equation. Curve (a) is obtained from equation

$$Nu = 1.357 (x^+)^{-1/3}$$

This is the Sellers approximation of the Graetz solution (K.5). Curve (b) represents the Graetz approximation, using the series solution for the Uniform Wall Temperature/Concentration boundary condition; the constants and eigenvalues were those used by Sellers et al. (S.22) and corrected by Lipris. These results are different from those calculated by MacLean and are shown by the uninterrupted dotted curve in his graphs. In fact, MacLean's analytical results represent the Loeroquo approximation and not, as shown in that work, the Graetz solution.

It was seen in section 3.1.2. that equation (5.1) is an approximation of the Graetz solution given by Sellers et al. and that, for situations like the one in hand, it holds only for the region $x^+ < 0.001$ and not $x^+ < 0.01$ as assumed by MacLean. Equation (5.1) applies in the range $x^+ < 0.01$, only when the local Nusselt number is computed on the basis that the driving force takes into/
into consideration the mixed-mean local temperature/concentration in a transfer process of heat/mass. However, the approximation proposed by Sellers et al. is calculated on the basis of a constant driving-force, which is simply the difference between the temperature/concentration of the entrant fluid and the wall-surface of the tube (S.22). In fact, this is the same basis on which the Nusselt numbers are calculated in mass-transfer studies with a Uniform Wall Concentration distribution. The difference in concentration between the entering fluid and the fluid-layer at the wall is obtained simply from the vapour pressure of the volatile solid at the temperature of the experiment. The concentration of the volatilized substance in the entering fluid is then assumed to be zero.

The actual range of the tube for which equation (5.1) may hold is then as follows. From equation (5.1), for the highest Reynolds number of 1250, the calculated local Nusselt numbers are meaningful only for a distance of \(x/d < 1\); while, for the lowest value, \(Re = 520\), the range covered is only \(x/d < \frac{1}{2}\). This shows that in the first place MacLean's data are consistently about 20% higher than those of the analytical curve (b) in the downstream portion of the tube. Secondly, the discrepancy between analysis and experiment increases rapidly towards the entrance, with values higher than 40% being attained at about \(x/d = \frac{2}{3}\) for \(Re = 520\) and \(x/d = 1\) for \(Re = 1250\). The discrepancy appears to be even higher for \(x/d < \frac{1}{3}\) in all cases and agreement with theory seems only to be found at \(x/d < \frac{1}{6}\).

This/
This complication, arising from secondary-flow set up in the measuring device, was present throughout the whole of that experimental programme. Yet, for fully-developed turbulent flow the estimated value of the asymptotic Nusselt number, \( \text{Nu}_\infty \), appears to agree well with that calculated from the McAdams correlation \((M.20)\),

\[
\text{Nu}_\infty = 0.023 \, \text{Re}^{0.8} \, \text{Pr}^{0.4} \quad \ldots \ldots \ldots \quad (5.2)
\]

In view of the discussion above, it is not clear how those results were obtained. This is especially so since the length of the tube investigated in that work, namely four diameters, was too short to allow sufficient data to be collected beyond the entrance. In other words, the fully-developed turbulent Nusselt number, \( \text{Nu}_\infty \), could not have been determined with any degree of accuracy. It is all the more surprising, then, that the reported accuracy in estimating the \( \text{Nu}_\infty \) was well within the range of the experimental error of \( \pm 10\% \), considering that the values for \( \text{Nu}_\infty \)'s were only obtained by selecting arbitrarily the best-fit curve, and this at a position where the curve just touches the \( x \)-axis asymptotically.

Also, no consideration was given by the author to the influence of variation in temperature occurring in the experiment. This problem was studied in the course of the present investigation. The results showed that a rise of up to 7°C occurred during an experiment of two hours duration. The day-to-day temperature variations were even greater and as already mentioned in section 4.6.2, changes of 18°C were often registered.

Temperature/
Temperature variations of this order of magnitude will have an appreciable effect on the results. First, inaccuracies would be expected to result in the estimation of the gap-width during the profilometric operation. This is even more true in the case of MacLean's work, since the calibration device and the knife-edged gauging-disc were constructed from different materials. This topic was discussed in section 4.7. Secondly, it was shown in section 4.6.2. that temperature changes of about 4 °C were sufficient to cause the destructive collapse of the prepared mass-transfer surface. This would mean that undetected scraping of the diphenyl wall-layer must have been a frequent occurrence in that experiment. MacLean found that one-third of his transfer surfaces had to be completely abandoned owing to serious damage taking place in the course of surface profiling. This finding agrees with results obtained in the present study when temperature variations in the laboratory were greater than 4 °C. Thirdly, the influence of temperature on the vapour-pressure of the diphenyl is considerable; this is shown in Appendix VII. Temperature variations, of the order found to occur daily in the laboratory, would result in the alteration of the essential boundary condition of the experiment; a Uniform Wall Concentration distribution. Still more serious in the case of MacLean's studies of laminar flow. No provision was made for temperature changes occurring during those experiments the duration of which was between 30 and 35 hours, and so a constant temperature was assumed. The temperature in the laboratory was recorded continuously by means of a Bristol Series 500 thermograph.
seen in section 4.7 that the experiment was abandoned if temperature changes of the order of 3°C occurred in the laboratory.

Furthermore, it appears that MacLean did not carry out checks on the thermometers used in the open-circuit wind-tunnel. In addition, no attempt was made during that work to check the measurements of the flow-velocity against those at different points in the apparatus.

The fact that MacLean's data agrees well with Doineslor's theory is all the more puzzling since there is an error in his calculations of the latter's analysis. It was shown in section 3.1.4. that for the Uniform Wall Heat Flux boundary condition, the dimensionless distance $r_0^+$ remains constant at a given Reynolds number. When the Uniform Wall Temperature distribution is considered, $r_0^+$ varies with the dimensionless distance $x^+ (= x/d)$. MacLean was inconsistent since he assumed $r_0^+$ to be both a constant and a variable in his calculations.

Yet another example where this serious complication of undesired surface-erosion occurs unnoticed during measurement, is the situation where a free-jet of air was used as the gauging device (section 4.1). In this case, however, the extent of surface evaporation by the impinging air from the jet is even more disastrous than when the pneumatic disc-gauge is employed. This is explained by the fact that in the case of jets, the laminar layer next to the volatile test-surface is very much thinner than that for air flowing past a sharp-edged disc.

A/
A case in point is the work of Macleod et al. (31.17) who exploited the sensitivity of the free-jet air-gauge in order to measure local changes in thickness of volatile naphthalene-coatings cast on metal rods (section 4.1). This investigation is of special interest because of its close relationship with the investigations of MacLean and with the present work. A feature common to all three is that all these investigations employed pneumatic methods of estimating local mass-transfer coefficients in turbulent flow. These were based on the same principle, namely that of estimating local changes in the coating thickness of a volatile surface by measuring the pressures through two constrictions, as shown in figure 11. In addition, the test-surfaces were prepared simply by casting the material on the smooth surface of metal mould.

The actual surface-gauging apparatus used in these investigations were different. A pneumatic disc-gauge was employed by MacLean and in the present work to study dimensional changes in the internal diameter of hollow diphenyl-cylinders; on the other hand, Macleod et al. used a jet of air to study dimensional changes in the outer diameter of naphthalene cylinders. Nevertheless, the same time of less than 30 seconds was allowed for the gauging-air to act upon the same point of the volatile surface for each of two measurements in the studies of Macleod et al. and the present one; in the case of MacLean, however, the gauging-air was allowed to flow without interruption for the whole period of the profilometric operation.

However, the sensitivity of the jet-technique was vitiated by the/
the fact that Macleod et al. failed to recognise the presence of
two inherent flaws in the measurements.

In the first instance, the results of that work must have
been distorted as a consequence of complications arising from
secondary-flow. This arises from the large-scale surface erosion
owing to the evaporative influence of the jet. The pneumatic disc
employed in the present work and the jet air will now be considered
in more detail.

In the case of the pneumatic-disc/diphenyl system, the supply
pressure $P_1$ was maintained constant at $25.4$ cm. mercury; the inter-
medial pressure $P_2$ was between $12 - 16$ cm. mercury and the gap-width
ranged between $1.125$ and $0.781$ thou. in. For the jet system of
naphthalene-saturated air/naphthalene, $P_1$ was maintained at $4$-in.
water gauge and $P_2$ was made to read $3$-in. water-gauge, when the
distance between the nozzle and the naphthalene surface was $0.5$ thou.
in. The time taken to estimate the cast-in volatile coat-thickness
at each station along the length of the tube was the same as that
taken by Macleod et al. in their experiments, namely under 30 seconds.

Macleod et al. assumed that, under such low pressures, the air
was virtually saturated with naphthalene vapour. It is also
reported in that work that under these conditions no recessions on
the test-surface were observed, if the jet acted upon the same point
for less than 20-30 seconds. This result is surprising, even
allowing for the fact that the two systems were different. It was
observed that this time corresponded to the limiting time in the
present/
present work (section 4.4), when higher pressures were employed: $P_1$ was 25.4 cm. of mercury and $P_2$ was 12.5 cm. of mercury. Moreover, the gauging-air in this case was not saturated with diphenyl as in the case of the jet, where the air was pre-saturated with naphthalene. In view of this, then, a limiting time, $t_1$, of between 20 and 30 seconds for the jet system is extremely low, and some other cause must be sought. It appears that the use of a lathe to give the final smooth surface must have weakened the texture of the casting. This explanation is supported by the flaky appearance of the cast surfaces in general and the enormous damage caused on the surface whenever this was accidentally scratched (section 4.5) by the knife-edged disc.

The assumption by Macleod et al., that surface-removal by the jet air was negligible, appears to have been based solely on consideration of the observed limiting time of the jet. Macleod et al. reasoned that the evaporative effect of the jet was negligible since each of the pressure measurements was completed in a time less than the limiting period, $t_1$, of 20 - 30 seconds.

The same approach was to be repeated by MacLean in a later work on mass transfer in the entry-region of a circular duct, and has already been discussed above.

The fallacy of the foregoing assumption is seen from the considerable volatilisation which was found to occur when the pneumatic-disc method was employed (section 4.4). It has been established in the present work, that no noticeable surface erosion occurs/
occurs when the gauging-air acts for no longer than the limiting period; and that evaporation of the surface does occur whenever two consecutive pressure readings are taken at the same point. The reason for this is that the total time during which the test-surface was exposed to the gauging-fluid in the two consecutive profiling measurements is longer than the limiting time of 20 - 30 seconds. This actual limiting time was the time taken to allow the gauging-air to flow through the gauging apparatus by means of a needle-valve; and the pressure was observed for a time till a change occurred. In the profilometric operation, however, the time taken is from the moment the needle-valve is opened until it is shut-off again after completing the necessary pressure reading. The same results must then obtain for the gauging-jet, since exactly the same operational procedure of measurement was followed in both investigations.

However, it was shown in figure 18 that surface evaporation by the gauging instrument was suppressed effectively by pro-saturating the gauging-air with diphenyl. This result points to a flaw in the method of Macleod et al., who stated that short limiting periods of 20 - 30 seconds were observed, and that the gauging-fluid was saturated with the volatile substance.

The other objection to the method employed by Macleod et al. is as follows. It is well known that the reaction pressure of a jet on a flat surface may be measured accurately only if the impinging air from the nozzle of the jet strikes the surface orthogonally. However, it has been shown in section 5.3.1. that mass-transfer surfaces prepared from volatile organic substances by/
by casting-methods are never absolutely flat. Furthermore, it was demonstrated that this applies to all surfaces, regardless whether they are prepared by casting on a smooth mould or by producing the final finish by machining with a cutting-tool. Mass-transfer surfaces were shown in this work to possess a succession of peaks and valleys; support for these results is lent by the work of Sherwood and Träss (§3.7), who obtained surface traces of cast naphthalene plates (section 5.2).

The situation now is one where the gauging-air from the jet strikes the slant edge of a peak or a valley and not the assumed flat face of the casting. The pressure-reading of the jet will therefore register only the cosine component in the reaction force of the jet. This is a very serious complication which renders the experimental data obtained by this method meaningless; also, it throws additional light on the awkward scatter in the results of Macleod et al. (section 5.3.1).

Nonetheless, Macleod et al. also show good agreement between their experimental Nu∞ and that calculated from the Dittus-Boelter equation. This is exactly the same situation as that obtained in the work of MacLean mentioned previously. The only possible explanation for these incongruous results is that the same consistent error must have been present in both experimental investigations.

To conclude, it would seem that the studies by Macleod et al. and MacLean are best considered of historical interest in the development of pneumatic techniques employed in profilometric measurements of solids of revolution.

5.4.3/
FIG. 30 - PROFILOMETRIC EXAMINATION OF MASS-TRANSFER TEST SURFACES. GRAPHIC REPRESENTATION

(a) Re = 30,000

(b) Re = 50,000

(c) Re = 70,000

(d) Re = 90,000

Intermediate Pressure

P_2 (cmHg)

0 1 2 3 4 5 6

Sc = 0.03

Before Sedimentation Test

After Sedimentation Test

x/d 0 1 2 3 4 5 6
FIG. 11 - LOCAL MASS TRANSFER COEFFICIENTS - FULLY DEVELOPED TURBULENT FLOW

Re = 30,000

T = 298 K  Sh∞ = 69.8

t = 8 hrs. 6 min.  Sh = 85

Re = 50,000

T = 294 K  Sh∞ = 133.7

t = 4 hrs. 50 min.  Sh = 128.3

Re = 70,000

T = 294 K  Sh∞ = 175

t = 3 hrs. 40 min.  Sh = 170.4

Re = 90,000

T = 297 K  Sh∞ = 213.9

t = 5 hrs.  Sh = 210.1

- - - Decay Analysis
- - - Present Investigation
- - - Computer Results
Sc = 1.03
FIG. 32 - PROFILOMETRIC EXAMINATION OF MASS-TRANSFER TEST SURFACES: GRAPHIC REPRESENTATION

- Intermediate Pressure
- $P_2$ (cm Hg)

(a) $Re = 30,000$

(b) $Re = 50,000$

- Before Sublimation Test
- After Sublimation Test
- $Sc = 1.78$

(c) $Re = 70,000$

(d) $Re = 90,000$

$x/d$ vs $x/d$ graph with data points indicating pressure changes before and after sublimation tests at different Reynolds numbers.
**FIG. 33 - LOCAL MASS TRANSFER COEFFICIENTS - FULLY DEVELOPED TURBULENT FLOW**

- **Re = 30,000**
  - \( T = 292\,\text{°K} \quad Sh_{mc} = 110.6 \)
  - \( t = 2\text{hrs.20min} \quad Sh_{ad} = 105.2 \)

- **Re = 50,000**
  - \( T = 295\,\text{°K} \quad Sh_{mc} = 106.3 \)
  - \( t = 3\text{hrs.35min} \quad Sh_{ad} = 105.9 \)

- **Re = 70,000**
  - \( T = 296\,\text{°K} \quad Sh_{mc} = 2178 \)
  - \( t = 1\text{hr.20min} \quad Sh_{ad} = 210 \)

- **Re = 90,000**
  - \( T = 296\,\text{°K} \quad Sh_{mc} = 2663 \)
  - \( t = 2\text{hrs.30min} \quad Sh_{ad} = 260.4 \)

- Present Investigation
- Present Results
- Computer Results
- \( Sc = 1.78 \)
FIG. 35 - LOCAL MASS TRANSFER COEFFICIENTS — FULLY DEVELOPED TURBULENT FLOW

Re = 31,240
T = 294°K  $Sh_m = 126.4$
$\Delta t = 3$ hrs. 45 min.  $Sh_m = 126.8$

Re = 42,660
T = 298.5°K  $Sh_m = 161.6$
$\Delta t = 2$ hrs. 26 min.  $Sh_m = 166.6$

Re = 42,660
T = 294°K  $Sh_m = 161.6$
$\Delta t = 3$ hrs. 45 min.  $Sh_m = 166.6$

Re = 51,740
T = 294°K  $Sh_m = 189.2$
$\Delta t = 1$ hr. 54 min.  $Sh_m = 180$

---

Graph showing variations in $Sh_m$ with $x/d$ for different Reynolds numbers and temperatures.
FIG. 36 — REPRODUCIBILITY TESTS - DUPLICATED RUNS

$T = 294^\circ K \quad Sh_m = 110.6$

$t = 6$ hrs. 30 min. $Sh_m = 1.2.1$

$P_2$ (cm.Hg)

$20$

$18$

$16$

$14$

$12$

$10$

$8$

$6$

$4$

$2$

$0$
FIG. 37 - REPRODUCIBILITY TESTS - DUPLICATED RUNS

$Re = 31,240$

$T = 290.5^\circ K$  $Sh_{oo} = 122.5$
$t = 3hrs.30min.  Sh_{oo} = 118$

$Re = 30,000$

$T = 288^\circ K$  $Sh_{oo} = 126.4$
$t = 3hrs.30min.  Sh_{oo} = 127$

$p_2 (cm.Hg)$

$Sb$
FIG. 38 - FULLY DEVELOPED $S_{\infty}$ vs. $Re$.

$S_{\infty}$ vs. $Re \times 10^{-4}$

- - - McADAMS CORRELATION (M20)

- - - PRESENT INVESTIGATION

$Sc = 2.3$

$Sc = 1.78$

$Sc = 1.03$
5.4.3. Fully Developed Mass-Transfer Rates.

The experimentally determined fully developed mass-transfer coefficients are here compared with the empirical correlation of McAdams (1920), which for heat transfer is of the form

\[ \text{Nu}_\infty = 0.023 \, \text{Re}^{0.8} \, \text{Pr}^{0.4} \quad \ldots \ldots \ldots (5.3) \]

where \( \text{Nu}_\infty \) is the fully-developed Nusselt number, \( \text{Re} \) is the Reynolds number and \( \text{Pr} \) is the Prandtl number. The equivalent equation for mass transfer is

\[ \text{Sh}_\infty = 0.023 \, \text{Re}^{0.8} \, \text{Sc}^{0.4} \quad \ldots \ldots \ldots (5.4) \]

where \( \text{Sh}_\infty \) is the fully-developed Sherwood number, and \( \text{Sc} \) is the Schmidt number.

The results of comparison between experimental data and values predicted from equation (5.4) are shown in figure 38. In general, agreement between experiment and theory is good, the difference being not greater than the experimental error of \( \pm 5\% \). This agreement is particularly good for \( \text{Sc} = 2.3 \). In the case of \( \text{Sc} = 1.75 \) and 1.03, greater deviations occur in the lower range of Reynolds number, but there is a steady improvement with increasing Reynolds number.

Consideration of the extent of agreement between the experimental data and equation (5.4), offers little possibility of obtaining a deeper insight into the relative merits of the assumptions employed in the various analyses on turbulent transport of momentum, heat and matter. The reason for this is that many theories yield satisfactory predictions for fully-developed flow and transfer, for the/
the range \(0.5 < \sigma < 5\). This is not the case for the entrance region and, consequently, only examination of this section of a duct can provide a true appreciation of the analysis.

5.4.4. Local Mass-Transfer Rates in the Entrance Region.

The results obtained in the present investigation are represented graphically by curves of \(\text{Sh}/\text{Sh}_\infty\) vs \(x/d\), where \(\text{Sh}\) is the local Sherwood number at cross-sectional distance \(x\) from the starting-length of the tube \((x/d = 0)\). \(\text{Sh}_\infty\) is the fully-developed, steady-state or asymptotic value of the Sherwood number, and, \(x/d\) is the distance, in tube-diameter, downstream from the entrance \((x/d = 0)\). It was shown in section 2.2.4. that this is a convenient way of expressing mass-transfer data in the entrance in terms of the ratio \(k_c/k_c\), where \(k_c\) and \(k_c\) are the local and the steady-state values of the mass-transfer coefficient, respectively. The data were fitted by means of a least-squares procedure programmed for a KDF-9 digital computer (section 3.4.).

Comparison of the present data for \(\text{Sc} = 1.03, 1.78\) and \(2.3\), with analytical and experimental data for \(\text{Pr} = 1, 10\) and \(\text{Sc} = 2.3\) is presented in figure 40. The concentration entrance-length is defined in the present study as the length in terms of the downstream distance, \(x/d\), at which the mass-transfer coefficient approaches to within 5 per cent of the fully-developed value. This definition is generally accepted in practice as being more meaningful (section 5.4.5.) and is widely employed in entrance-length studies. Some workers have also used a 2 per cent entrance-length. In figures/
figures 39 and 40 a line has been drawn to indicate the condition \( \text{Sh/Sh}_\infty = 1.05 \).

The transfer rates obtained during the present investigation are lower in comparison with the other results. Furthermore, the approach of the transfer profiles to steady-state is more rapid and the entrance-lengths are shorter. In all cases, the initial rates of mass transfer are high, from \( \sim 2 \) to \( 2.5 \) times the steady-state or downstream value; these fall off rapidly to reach within five per cent of their steady-state value in the range \( 0 < \text{x/d} < 4 \).

At \( \text{x/d} < 1 \), which represents distances very near the entrance, the mass-transfer rates approach the value of infinity. This result is in accord with theory, predicting infinite rates of transfer at \( \text{x/d} = 0 \), i.e. the point at which heat or mass transfer begins, the rates decreasing steadily till they reach their steady-state values.

5.4.5. Influence of Reynolds Number on Local Mass-Transfer Rates.

The curves in figure 39 indicate the considerable dependency, observed in this work of the local mass-transfer rates on the Reynolds number for a given value of \( \text{x/d} \). Near the entrance, the values of \( \text{Sh/Sh}_\infty \) are lower for the higher Reynolds number than for low ones, for the whole range of Schmidt number investigated. This trend in the behaviour of the \( \text{Sh/Sh}_\infty \) with \( \text{Re} \) may be illustrated from the following data for \( \text{Sc} = 2.3 \) obtained from figure 39:

\[
\begin{align*}
\text{Re} &= 31,240 : \text{x/d} = 1/3 \quad \text{Sh/Sh}_\infty = 1.47 \\
\text{Re} &= 42,660 : \text{x/d} = 1/3 \quad \text{Sh/Sh}_\infty = 1.21 \\
\text{Re} &= 51,740 : \text{x/d} = 1/3 \quad \text{Sh/Sh}_\infty = 1.15.
\end{align*}
\]
The curves in figure 39 further indicate that the steady-state value of the Sherwood number, \( \text{Sh}_\infty \), is approached more rapidly at high than at low Re. The present trend of the \( \text{Sh}/\text{Sh}_\infty \) vs \( x/d \) profiles with Re for a given Sc agrees qualitatively with the predictions from Doissler's analysis (D.3). The exactly opposite trend is forecast by the analysis of Sparrow et al. (S.33). However, the present data denote a larger quantitative influence of the Re on local mass-transfer coefficients than that from Doissler's model; this is demonstrated by the greater separation of the curves in figure 39 than that predicted by the analysis. It is also shown in figure 39 that the entrance length diminishes as the Re increases, for a given Sc.

In an earlier work (D.10), Doissler found that the transfer rates increased as the Re was increased. Although the same boundary-layer model was employed in both analyses, he later modified the expression for the eddy diffusivity \( \epsilon^+ \) close to the wall. The equation used in the earlier analysis was

\[
\epsilon^+ = n^2 u^+ y^+ \quad \ldots \ldots \ldots \ldots (5.5)
\]

Doissler included in his subsequent work the effect of the kinematic viscosity for the region close to the wall on the premises that in this region the viscous effects would be comparable to those of inertia. The expression derived by dimensional analysis was

\[
\epsilon^+ = n^2 u^+ y^+ (1 - e^{-n^2 u^+ y^+}) \quad \ldots \ldots(5.6)
\]

The results obtained in the present work then appear to lend support to the modified expression for the eddy diffusivity as given by/
FIG. 40. EFFECT OF PRANDTL/SCHMIDT NUMBERS ON TRANSFER PROFILES:
FULLY-DEVELOPED TURBULENT FLOW

- SPARROW et al. (S33); Re=100,000; Pr=0.70; (U.W.H.F.) [A]
- DEISSLER, R.G.; Re=50,000; Pr=1.0 [A]
- MACLEAN, A. (M4); Re=100,000; Pr=0.73; (U.W.H.F.) [A]
- PRESENT INVESTIGATION; Re=51,740; Sc=2.3; (U.W.T.) [E]
- Re=30,000; Sc=1.78 [E]
- Re=30,000; Sc=1.03 [E]
- Re=31,240; Sc=2.3 [E]

A: Analytical
E: Experimental
The present results also justify the assumption made by Doissler in his analysis, in allowing turbulence right up to the wall.

The present state of experimental evidence on this topic is perplexing, consisting as it does of widespread contradictions. A peculiar trend is noted upon closer examination of experimental heat and mass transfer entrance-region results (Tables IV and V). Investigations involving heat transfer in gases are generally in agreement with the trend of transfer coefficients with Re as predicted by Sparrow et al. On the other hand, the results of studies of heat and mass transfer in liquids tend to lend support to the trend forecast by Doissler. This anomaly may be explained in part by the different influence exerted by the temperature on the properties of gases and liquids (section 3.2.1).

5.4.6. Influence of Schmidt Number on Local Mass-Transfer Rates.

The present investigation appears to be the first to show clearly the considerable extent to which the Schmidt number influences the development of the mass-transfer profiles in a circular duct. Hitherto, no single investigations has demonstrated this effect in one series of experiments.

Doissler (D.3) and Sparrow et al. (S.33) in their analyses, predict that the Schmidt number, $Sc$, will have a considerable effect on the local mass-transfer rates for the entrance region of pipes. However, the latter predicts a greater influence of the $Sc$, as is shown by the results of Siegel and Sparrow (S.25). This is seen from figure 40, where the transfer profiles for mass, obtained/
FIG. 31—INFLUENCE OF SCHMIDT NUMBER ON CONCENTRATION PROFILES: FULLY DEVELOPED TURBULENT FLOW.
COMPARISON BETWEEN EXPERIMENT AND ANALYSIS (DEISSLER'S MODEL)

Re = 30,000

Re = 50,000

Re = 70,000

Re = 90,000

Deissler Analysis
Present Investigation
obtained in the present research for $\text{Sc} = 1.03$, 1.76 and 2.3 at Reynolds numbers of 30,000 and 50,000 are plotted against the transfer profiles for heat presented by Deissler and Sparrow et al. for $Pr = 0.7$ and 10, at $Re = 30,000$, 50,000 and 100,000. The question as to which of the two analyses is the more correct can be answered only by experiment.

Data obtained in mass-transfer experiments have always exhibited large scatter, which has resulted in making the expected effect of the $Sc$. On the other hand, a few investigations have been carried out on the influence of the Prandtl number on entrance-length heat transfer ($A, 5, H.1, M.15$). The validity of the results obtained from these studies is doubtful, owing to complications arising from the effects of temperature on the properties of the turbulent fluid stream and the surrounding duct (section 3.2.1).

Experimental difficulties become practically insurmountable for the more important tube-range, $0 < x/d < 2$, which in fact represents the very short distance immediately after the point at which transfer of heat commences. In contrast, the properties of the turbulent fluid in the present investigation may be assumed constant, owing to the very low rates of transfer from the diphenoxy wall.

The influence of the $Sc$ on the mass-transfer profiles is shown in figure 41 for the range of tube $0 < x/d < 6$. The results tend to support the analysis of Deissler rather than that of Sparrow et al. as seen from figure 40; profiles obtained in the present investigation are plotted against those obtained by Siegel and/
and Sparrow (8.25) and Deissler (D.3). Agreement between experiment and theory is found to improve steadily near the entrance of the tube. In general, the experimental data lie below the corresponding predicted values, and the discrepancy becomes larger for increasing values of Reynolds numbers. A characteristic pattern emerges from these graphs, showing the consistently independent behaviour of the mass-transfer profiles for Sc = 2.3: the family of curves for Sc = 1.03 and 1.78 lies close together and is further removed from the curves for Sc = 2.3. This is directly against the trend forecast from Deissler's analysis, which are represented by the dotted curves in figure 41.

Furthermore, there is better quantitative agreement with Deissler's predictions of the effect of \( \sigma \) on the mass-transfer rates for Sc = 1.03 and 1.78. The greater effect indicated by the profiles for Sc = 2.3 would tend to suggest the existence of a turning-point, or turning points, in the behaviour of the influence of the Sc in the range 0.7 \( \leq \) Sc \( \leq \) 10. It would also agree with the larger influence of \( \sigma \) on transfer rates forecast by Sparrow et al. However, no substantiation of such a change can be deduced from Deissler's model. The analysis of Sparrow et al. is based on an eigenfunction formulation of the problem (section 3.1.3), and would be expected to give inaccurate information for low \( \sigma \) and small values of \( x/d \). The reason for this is that a considerably larger number of eigenvalues and constants are required for this region, before a complete description of the transfer of heat or mass can be obtained.

In/
In view of the abovementioned, it is then unsound to extend heat-transfer data obtained for the entrance region of ducts to mass transfer in similar circumstances. There is a need for a more comprehensive experimental study into the nature and degree of influence of the $\sigma$ on transfer rates. The different dependency of the mass-transfer rates for air and those for carbon dioxide and Freon-114 calls for a thorough experimental examination of the $\sigma$-range $0.7 < \sigma < 10$. This range represents the spectrum of Prandtl number mostly encountered in heat-transfer practice, and is therefore the one of greatest interest.

In general, however, experimental investigations do not as yet appear to have provided the answer to this problem, the results on the whole showing a widespread confusion. The main source of the difficulty may be largely explained in the following manner. The curves of $\text{Sh}/\text{Sh}_\infty$ vs $x/d$, representing the decrease of the transfer rates with downstream distance, are assumed in theory to decrease asymptotically. This makes it extremely difficult to distinguish experimentally between one curve and another as they come closer and closer as the steady state is approached. Experimental investigations to date have not, with some exceptions, achieved the accuracy necessary to enable entrance lengths to be successfully determined. This difficulty also accounts for the fact that in studies involving heat or mass transfer in the entrance region of circular ducts considerable overlapping and disagreement exists in published data, even when a 5%-entrance-length is assumed (section 5.4.4).
5.4.7. Local Transfer Rates in the Entrance Region and the Developing Boundary-Layer.

It was seen from sections 5.4.5 and 5.4.6, that the results of the present investigation agree better with the analysis of Doissler than with that of Sparrow et al. The agreement however is not quantitatively exact, indicating lower mass-transfer rates than would otherwise have been expected from the theory. For $Sc = 1.03$ and $1.78$, the present data are found to be about 20% below the theoretical values; and for $Sc = 2.03$ to about 30%. The Reynolds number was observed to exert a greater influence on mass-transfer rates, the trend agreeing with that predicted in Deissler's analysis (section 5.4.5). Mass-transfer rates were also found to be influenced by the Schmidt number, $Sc$, but to a larger extent than that calculated from the analysis of Deissler. However, the observed effect of the $Sc$ was not as much as that predicted from the analysis of Sparrow et al.

The closer agreement between the present data and Deissler's analysis suggests that the boundary-layer model employed in that analysis offers the more satisfactory analytical approach. However, it is necessary to account for the quantitative disagreement between the present experiment and the theory. Changing the expression for the eddy diffusivity employed in the analysis would only help partly to explain the discrepancy; the observed difference between experiment and theory is larger than that obtained when the different expressions for the eddy diffusivity are used.

Cooling, resulting from sublimation of the volatile substance would/
would be expected to depress the rates of mass transfer, especially in the entrance region of the duct. However, it was shown in section 4.6.3 that no decrease in temperature could be observed in the course of the experiment. In addition, the agreement between the calculated and the estimated values of the fully-developed Sherwood number was closer than would have been obtained, had there been any interference due to sublimation-cooling of the diphenyl (section 5.4.3). Furthermore, the experimental procedure described in section 4.6.3 eliminated the possibility of error resulting from the use of contaminated batches of diphenyl in the preparation of the test-surfaces. Lastly, the precautions taken during the course of an experimental run helped to eliminate errors arising from incorrectly estimating either the velocity of the gas in the wind-tunnel, or the temperature (section 4.7). The root of disagreement, therefore, appears to lie elsewhere.

The developing temperature boundary-layer employed in Doissler's theory, with its well-defined edge, beyond which the concentration is assumed constant, constitutes an oversimplification, which is, however, a convenient mathematical artifice. In his analysis, Doissler made the usual boundary-layer assumptions employed with integral methods (section 3.1.4).

For laminar flow, more exact analysis (L.10) indicates these assumptions to be valid for that region of the pipe where the boundary layer has not filled the major portion of the duct. Beyond this region, the Nusselt/Sherwood numbers and friction factors assume values very close to the fully-developed ones.
On the other hand, for fully-developed turbulent flow in circular tubes, there is no exact analysis equivalent to that available for laminar flow. It is not possible, therefore, convincingly to test the various assumptions that have been employed in the theories proposed for this situation.

The other possibility which must be considered is that under conditions of fully-developed turbulent flow there is no longer any clearly-defined edge to the boundary-layer. That is to say, that the sharp contour of the concentration boundary-layer may disintegrate probably owing to eddies arising from the turbulence in the boundary layer which increases from a value of zero at the entrance of the duct to the value corresponding to fully-developed conditions. Not only the edge, but neighbouring layers would in this case be dispersed towards the centre region of the duct, resulting in a net decrease in the thickness of the concentration boundary-layer. There would, therefore, no longer be an "edge" to the boundary layer except possibly over a very short distance from the entrance of the duct. Beyond the entrance the concentration of the diffusing substance outside the concentration boundary-layer ceases to have a constant value.

The low values of $\frac{Sh}{Sh_\infty}$ observed in the present investigation for the entrance region of the tube can be explained in the following manner. Within the developing concentration boundary-layer, the increase in turbulence with increasing downstream distance $x$ from the entrance of the duct flattens the concentration profile. As a consequence of this, the concentration gradient at the wall increases, and hence the Sherwood number. The increase of turbulence in the concentration/
concentration boundary-layer with $x$ therefore lowers the value of $\frac{Sh}{Sh_\infty}$, since the decrease of $Sh$ with $x$ is less for the flattened concentration profile than for the original unchanged one.

The effect of the Reynolds number on the ratio $\frac{Sh}{Sh_\infty}$ can now be considered. Concentration profiles obtained for two different $Re^0$s show that for a fixed duct-diameter at a given $x$, the boundary-layer thickness is greater for low $Re^0$s than for high ones. The values of $\frac{Sh}{Sh_\infty}$ obtained close at the entrance are lower for high $Re$ than for low ones, even though the concentration boundary-layer develops slower at the high $Re^0$s. This is because at high $Re^0$s the rate of increase of turbulence, and therefore $Sh$, is greater than at low $Re^0$s, the value of turbulence varying from zero at the entrance to that of fully-developed conditions.

The transfer rates for heat and mass in fully-developed turbulent flow may now be calculated by the following method. Deissler's equation for the velocity-distribution in a circular duct is retained

$$u^+ = \int_0^{y^+} \frac{dy^+}{1 + n^2 u^+ y^+(1 - e^{-n^2 u^+ y^+})}$$

(5.7)

The symbols have their usual meaning as defined in section 3.1.4. The expression for the concentration profile is altered by introducing the factor $\omega$ which accounts for the increase in turbulence within the developing concentration boundary-layer with $Re$. The equation for the concentration profile now becomes

$$c^{++} = \int_0^{y^+} \frac{dy^+}{1 + \omega n^2 u^+ y^+(1 - e^{-\omega n^2 u^+ y^+})}$$

(5.8)

where/
where \( c^{++} \) replaces the usual dimensionless form \( c^+ \) for the concentration of the diffusing substance, and, from the above-mentioned considerations, \( \omega \) is a function of the axial distance of the duct, the Reynolds and Schmidt numbers, i.e.,

\[
\omega = \phi \left( \frac{x}{d}, \text{Re}, \text{Sc} \right) \quad \text{(5.9)}
\]

The value of \( \omega \) can only be found by experiment as the problem becomes extremely complicated. Furthermore, the correct dependency of \( \omega \) on each of the three variables \( x/d \), \( \text{Re} \), and \( \text{Sc} \) must be found by experiment.

Equations similar to those of (5.8) and (5.9) can be written for heat transfer, with \( \text{Pr} \) replacing \( \text{Sc} \) (and \( c^{++} \) replacing \( c^+ \)). Equations (5.8) and (5.9) can be expressed in a more general form

\[
c^{++}, t^{++} = \int \frac{dy^+}{1 + \omega n^2 u^+ y^+ (1 - e^{-\omega n^2 u^+ y^+})} \text{d}y^+ \quad \text{(5.10)}
\]

and

\[
\omega = \phi \left( \frac{x}{d}, \text{Re}, \sigma \right) \quad \text{(5.11)}
\]

where \( \sigma \) is the Prandtl or Schmidt number.

The observed effect of the Reynolds number was greater than that expected from Deissler's analysis. The influence of the Reynolds modulus is well accounted for in the proposed model which takes into consideration the effect of increased turbulence in the developing boundary-layer with \( \text{Re} \). Furthermore, it would appear from the findings of the present investigation, that the influence of the Schmidt number is greater than that predicted by Deissler's analysis.
analysis. This suggests that the rate of increase of turbulence in the concentration boundary-layer and the consequent flattening of the concentration profile is not only related to the $R_e$ but also to the $S_c$, as shown in equation (5.9). Evidence, indicating a need for a modification of the analysis, is to be found in the professed doubts as to the validity in the assumption that $n$ in equation (5.8) is a constant (5.41). The approach suggested here may further help to explain the inadequacy of Doi's analysis - even for linear flow - for the region of the tube where the concentration boundary-layers fill the greater portion of the tube. In the model proposed here, $n$ is allowed to remain a constant since the expression for the velocity-distribution is found to be in good agreement with others.

The advantage of this method is that the same computer programme as that presented in section 3.4 may now be used to calculate the new values of $x/d$ and $Sh/\bar{Sh}_\infty$. A minor modification in the programme will be required to account for $\omega$ in the equations for $t^+$ and $c^+$. A number of possible values are then set for $\omega$, the choice resting in that value which will yield the best curve to represent the experimental data.

The relevant equations for the complete solution of the problem become

$$Sh = \frac{2r^+}{c_b^+} \frac{Sc}{c_b^+} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (5.12)$$

where

$$c_b^+/\cdots/$$
\[ a^+ = \frac{\int_{r_0^+}^{c^+ u^+ (r_o^+ - y^+)} dy^+}{\int_{r_0^+}^{\lambda_m^+ u^+ (r_o^+ - y^+)} dy^+} \]

\[ = \frac{\int_{r_0^+}^{\lambda_m^+ u^+ (r_o^+ - y^+)} dy^+}{\int_{r_0^+}^{\lambda_m^+ u^+ (r_o^+ - y^+)} dy^+} = \ldots \quad (5.13) \]

and

\[ \left( \frac{x}{d} \right)_{U.W.M.P.} = \frac{1}{2 (r_o^+)^2} \left( \int_{r_0^+}^{\lambda_m^+} u^+ (c^+ - c^+) (r_o^+ - y^+) dy^+ \right) \quad \ldots \ldots \quad (5.14) \]

\[ \left( \frac{x}{d} \right)_{U.W.C.} = \frac{1}{\frac{c}{r_o^+}} \left[ \int_{r_0^+}^{c^+} \frac{1}{r_o^+} \int_{r_0^+}^{\lambda_m^+} \frac{c^+ - c}{c^+} u^+ (r_o^+ - y^+) dy^+ \right] \quad (5.15) \]
To conclude, the discussion presented here may be extended to the case of heat transfer, provided that the problems arising from fluid-property variation with temperature are absent.

In the case of heat transfer, $e^+$ is replaced by $e^{++}$, and $\lambda_m^+$ by $\lambda_h^+$ everywhere in the equations, and the same procedure of calculation is followed.
CONCLUSIONS

(1) A profilometric technique employing an air-gauging device was developed to measure accurately configurational changes in a volatile wall cast in the interior of a tube. The method was applied to estimate local rates of sublimation from a diphenyl wall by measuring local recessions at the tube wall of up to 1 thou. inch to an accuracy of better than $\pm 5\%$. In general, most gauging techniques based on profilometric methods interfere with the development of the hydrodynamic boundary-layer; but the profilometric technique employed in the present work is free from this disadvantage, and it appears that the accuracy achieved here in the measurement of small recessions has not hitherto been possible by profilometric methods.

(2) The air employed as the gauging fluid in the profilometric operation was observed to cause considerable sublimation of the volatile diphenyl wall. This unsuspected volatilisation was also found to be present when a free air-jet was used instead in the profilometric device. This effect was present in earlier investigations, and the resulting extra transfer of mass was shown in this work to have serious consequences. Transfer rates were obtained, which were much higher than those expected in studies concerned purely with sublimation. Particularly high rates of mass transfer were noted for the entrance region, where the extra loss of mass accounted for as much as 40$\%$ of the measured recessions of the wall. The corresponding steady value for the downstream portion/
portion of the tube is about 20%.

The data obtained under these circumstances on entrance-region mass transfer are therefore entirely unreliable, giving rise to misleading interpretation of the results. Sublimation in the gauging apparatus was reduced to negligible proportions in the present research by the effective saturation of the gauging-air with diphenyl vapour.

(3) A method which exploited to great advantage the sensitivity of the profilometric technique was developed to explore the condition of the test-surfaces prepared by casting them on a smooth metal mould. It was possible by this means to determine the quality of a mass-transfer surface before and after a sublimation experiment. Examination of surface traces obtained for the smooth cylindrical walls revealed the wavy pattern of the hitherto assumed smooth wall. It was deduced from this, and from surface traces obtained in another work (3.37) for flat naphthalene plates, that mass-transfer surfaces invariably exhibit an irregular wavy pattern whenever these are manufactured by casting methods. The uneven appearances persists regardless of the subsequent treatment of the surface.

A knowledge of the real condition of a test-surface is of considerable importance, especially in circumstances where air-gauging techniques employing a free-air jet are used. It was shown that misleading interpretation of data may be made when uneven surfaces are incorrectly assumed to be truly plane.

(4)/
(4) The surface-exploration technique developed during the present course of investigation was further exploited to enable high-grade mass-transfer surfaces to be selected. Although the influence of wall roughness on mass transfer is not accurately known, surface imperfections corresponding in size to the thickness of the laminar sub-layer next to the wall would be expected to interfere with the flow conditions. The resulting increase in turbulence alters the mass-transfer rates at the volatile wall, to yield distorted concentration profiles for the entrance region.

It is shown from data obtained for duplicated runs in the present work, that surface protrusions of the order of 0.500 thou. inch do not appear to affect the results. This was the upper limit of surface roughness admissible in the present series of experiments. The value of 0.500 thou. inches for a duct of 1 1/2-inch internal diameter corresponds to the more conservative estimates recently proposed for the laminar boundary-layer thickness $y^+ \sim 2$. Mass-transfer sections possessing a higher degree of unevenness were discarded.

(5) A better understanding of past and present mass-transfer studies was accomplished by means of the profilometric technique. This revealed the presence of unsuspected imperfections on the surface of mass-transfer walls caused by the existence of imperceptible voids in the fabric of the cast material. The minute free spaces in the volatile solid are formed by the incomplete escape of diphenyl vapour from the melt during the casting operation.
operation, and can rarely be eliminated entirely. Those cavities give rise to a porous appearance on the free mass-transferring wall, and, to date, failure to detect this defect appears to be the absence of a sufficiently sensitive and accurate technique for measuring minute protrusions of the order of < 1 thou. inch.

The presence of this hitherto overlooked surface fault contributes to the overall scatter in mass-transfer data. This throws light on puzzling results obtained in earlier investigations, and also during the initial stages of the present work which indicated an unaccountably high scatter in some of the published data.

It was possible in the present investigation to eliminate this problem almost entirely by selecting only cast surfaces of high quality following a detailed exploration of the mass-transferring wall before and after each experimental run.

(6) In the case of fully-developed turbulent flow and fully-developed mass transfer, the expected agreement was observed between theory and experiment for 0.5 < Sc < 10. The present data were consistent with the values derived from both analytical predictions and empirical correlations of long-standing.

(7) The values of the mass-transfer rates in the entrance region of the tube were high in comparison with the fully-developed values further down the tube. Those observations are in accord with the predictions in the analyses of Deissler and Sparrow et al.

The/
The entrance lengths, defined on a 5 per cent approach to fully-developed turbulent mass transfer ranged between
\[ 1 < \frac{x}{d} < 3, \]
depending on the Reynolds and Schmidt numbers. The short entrance lengths obtained in this work are more in agreement with those predicted by the analysis of Doissler than with the forecast of Sparrow et al., which was four times as long. However, the agreement with Doissler's analysis is not exact, the observed entrance lengths being shorter than those predicted by the theory.

(8) A method is presented for calculating values of heat/mass-transfer rates in the entrance region of a circular duct from Doissler's analysis for fully-developed turbulent flow with a Uniform Wall Temperature/Concentration, U.W.T./C. distribution. Numerical calculations for the Uniform Wall Heat Flux, U.W.H.F. situation were carried out by Doissler; values for Uniform Wall Mass Flux were obtained in the present work by use of computer methods.

To date, it has not been possible to evaluate the expression derived by Doissler for the U.W.T./C distribution, since the equation in its present form does not lend itself immediately to calculation by computer methods. It has therefore not been possible as yet to obtain a true estimation of the percentage difference between the two boundary conditions, U.W.H./H.F. and U.W.T./C as has already been done for the analysis of Sparrow et al.

The approach proposed here entails first transforming the equation into a form suitable for numerical calculations; a procedure/
procedure is then suggested for preparing the computer program.

(9) A clear dependence of the concentration development in the entrance region of the tube on the Reynolds number Re was observed; a relative increase in the mass-transfer rates was noted for a decrease in the Reynolds number. This trend is in qualitative agreement with the analysis of Deissler\(^{(D.3)}\) which is based on a boundary-layer model, but contradicts that of Sparrow \textit{et al.}\(^{(S.33)}\) who forecast the opposite effect.

In an earlier treatment, Deissler\(^{(D.10)}\) predicted a relative increase in the heat-transfer rates with increasing Re. This analysis he later modified to include the effect of kinematic viscosity in reducing the turbulence in the region close to the wall\(^{(D.3)}\).

The experimental data of the present investigation therefore offer justification for the correctness of Deissler\(^{6}\)'s subsequent treatment of his analysis for heat transfer in fully-developed turbulent flow for the entrance region of circular ducts.

(10) The influence of the Schmidt number on local mass-transfer rates was shown to be considerable, agreeing more with that forecast by the analysis of Deissler than with the much larger effect indicated by the model of Sparrow \textit{et al.}. However, the agreement is to a large extent qualitative, the present data indicating a larger influence of the Schmidt number than that anticipated by the analysis.

The predicted effect of the Schmidt modulus on local mass-transfer/
transfer rates has hitherto eluded convincingly experimental verification in mass-transfer studies because of the modest degree of accuracy associated with investigations of this type. Contrary to the situation for mass transfer, attempts to establish an influence of the Prandtl number on entrance-region local heat-transfer rates have met with greater success. Nonetheless, it has not been possible to obtain an accurate evaluation of this effect because the influence of the temperature-variable fluid properties on the local heat-transfer rates is not yet fully understood. Mass transfer investigations being free from this complication are then at a greater advantage over heat transfer work in establishing the pattern of the influence of $\sigma$ on local transfer rates.

(11) An interesting pattern appears to be emerging in investigations concerned with entrance-region transfer in fully-developed flow. The data fall into two distinct groups, namely those of heat and of mass transfer, giving rise to two families of curves on graphic representation. This behaviour is observed in studies concerned with both solid/gas and solid/liquid systems. The results obtained for heat transfer tend to be always higher than those for mass transfer for equal Schmidt and Prandtl numbers, duct geometry and boundary wall conditions. These difficulties spring from the complicating influence exerted by temperature-variable fluid properties on heat transfer rates, whereas no equivalent problem exists for mass transfer. This fundamental difference between the two/
two transfer processes calls, therefore, for caution in extending
data obtained for mass transfer to heat transfer situations.

(12) The short entrance lengths and the observed influence of the
Schmidt number and the Reynolds number on mass transfer rates may
be accounted for by an approach proposed in this work. This
approach questions the application of the concept of a well-defined
developing thermal/concentration boundary-layer. It is proposed
instead that, in these circumstances, the edge of the boundary layer
may disappear; and that adjacent layers disperse into the core of
the tube - the process being associated with the increase in the
turbulence within the boundary layer. This results in a net
decrease in the thickness of the boundary layer whose edge is now
diffuse and also in flattening the shape of the temperature/concen-
tration profile; the temperature/concentration gradient at the wall
correspondingly increases, thereby decreasing the values of \( \frac{\text{Nu}}{\text{Nu}_\infty} \) or \( \frac{\text{Sh}}{\text{Sh}_\infty} \). An increase in the Re is accompanied by a decrease in
the transfer rate, since the rate of increase in turbulence is
greater at high Re\(^0\)s rather than at low ones. The observed
decrease in transfer rate is also influenced by the Schmidt number.

A factor \( \omega \), to be determined experimentally, is introduced into
the original expressions developed by Deissler; being a function
of Re, Sc and the axial distance along the tube, \( x \).

(13) In order to evaluate accurately the rates of heat or mass
transfer in the entrance region of a circular duct the following
method of approach is suggested.

For/
For the region very near the entrance, $0 < x/d < 1$, the analysis of Deissler may be used. Beyond this region and up to the point where the transfer rates approach their fully-developed values ($0 < x/d < 3$), the analysis of Deissler is only approximate; in this part of the tube the modification proposed in this work to the analysis of Deissler may be applied. Finally, for large distances downstream the tube where the transfer rates have attained their fully-developed values, the analysis of Sparrow et al. may be employed.

However, for practical problems of design where a very high degree of accuracy is not required, the analysis of Deissler is probably the most suitable one available for calculating heat-transfer rates over the entire starting length of the duct.
SUGGESTIONS FOR FURTHER WORK

In view of the findings of the present work and the past investigations, it is realized that the problem of heat and mass transfer in the case of fully-developed turbulent flow in the entrance region of circular ducts is far from being completely resolved. Sufficient work has been carried out to date, along almost identical lines, i.e., measuring of temperature or concentration profiles, but contributing little in the end to the results of earlier attempts. It was shown in the present work, that every attempt to improve on earlier investigations is finally defeated by the complexity of the problem which requires the construction of extremely sophisticated and elaborate equipment.

Little is therefore to be gained in pursuing further this line of enquiry; especially since it is increasingly being shown, that the entrance lengths are so short, that attempts to distinguish between them are invariably thwarted by the uncertainty of the results for heat transfer arising from the variation in the fluid properties which depend on temperature. Mass-transfer studies have the advantage of being free from this complication and therefore offer a more attractive alternative. Although a considerable improvement to the accuracy of the measurements of mass-transfer rates has taken place, the central problem of preparing high-grade test-surfaces remains.

Some work remains to be done in investigating more thoroughly the range $0.7 < \sigma < 10$, with a high degree of accuracy. In the case of mass transfer, special attention will require to be paid to/
to the problem of preparing the test-surfaces.

However, it would be more rewarding if the emphasis of the research into this topic were altered, so that the whole portion of the tube was scanned transversely by means of suitably designed sensitive temperature or concentration probes. The advantages of this would be manifold. Firstly, it would eliminate the experimental difficulties encountered normally in both heat and mass transfer investigations. What is more important, however, in view of theoretical implications, is that the exploration of this region would offer a means of establishing whether there is a well-defined developing boundary-layer or a diffuse distribution of heat or mass throughout the cross-section of the tube, which is now proposed. It would thus be possible also to study the influence of the Schmidt number and the Reynolds number in the development of the transfer profiles, in fully-developed turbulent flow.

Furthermore, these investigations may also be extended to mass-transfer studies in the solid/gas phase by specially constructing relevant concentration probes. Even more important, the solid/liquid phase may also be investigated without confronting the difficulty of drying the soft wall of the soluble mass-transfer surfaces.

The work of Johnk and Hanratty (193) is a way towards this direction, although again the object of the investigation was the measurement of temperature profiles by means of a temperature probe. This entered the heated test-section at the point of exit and could traverse the tube both longitudinally and laterally. It is significant,
significant, however, that these authors attempted to account for their findings by sketching a model for heat transfer containing a diffuse temperature boundary-layer, while assuming, like Doissaler, that the external temperature was constant. They also suggested dividing the tube into three distinct regions, but did not explain the origin, nature and consequence of their proposed diffuse temperature boundary-layer.

These considerations indicate a need for further development in the design and construction of temperature and concentration probes capable of measuring small temperature or concentration differences accurately, without disturbing significantly the flow conditions.

It is proposed, therefore, that future research along the lines suggested here may prove fruitful in yielding further information about transfer processes in the entrance region of circular ducts with fully-developed turbulent flow. Furthermore, these studies may be easily extended to the investigation into the possible effects of duct-diameter and geometry upon the transfer rates.
FIG. 42. LOCAL MASS TRANSFER COEFFICIENTS - LAMINAR FLOW

Re = 1250
\( t = 30 \text{ hrs.} \)

Re = 1170
\( t = 30 \text{ hrs.} \)

Re = 972
\( t = 30 \text{ hrs.} \)

Re = 836
\( t = 35 \text{ hrs.} \)

Re = 675
\( t = 35 \text{ hrs.} \)

Re = 520
\( t = 35 \text{ hrs.} \)

Experimental Results \( ^{(M4)} \)
Graetz solution \( ^{(S22)} \)
Sellar's approximation \( ^{(S22)} \)
Sellar's series solution \( ^{(S22)} \)
APPENDIX I - DETERMINATION OF MICROMETER SETTINGS IN THE 
CALIBRATION DEVICE FOR GIVEN GAP-WIDTHS.

Let \( r_1, r_2 \) be the radii of the cone in the standard 
calibration device at two different positions. Then

\[
r = (r_2 - r_1) = \text{Increase in the radius of the cone.}
\]

The average increase in \( r \) for every vertical displacement
\( \Delta y \) of the cone of \( 5 \times 10^{-3} \text{ inch} \) was \( 0.018 \times 10^{-3} \text{(M.4)} \). The
angle \( \theta \) of the taper of the cone is then given by

\[
\tan \theta = \frac{\Delta r}{\Delta y} = \frac{0.018 \times 10^{-3}}{5 \times 10^{-3}} = \frac{0.018}{5}
\]

Sample Calculation: To determine the micrometer setting for a
gap-width of 0.55 thou. in. between the cone and the surrounding
sharp-edged disc.

Let \( y^0 \) be the equivalent micrometer displacement for a gap-
width of 0.55 thou. inch. Then, since the zero micrometer setting
is at 1.175 (section 4.34),

\[
y^0 = (1.175 - y)
\]

where \( y \) is the equivalent micrometer setting for the gap-width.
Hence,

\[
(1.175 - y) = 0.55 \times 10^{-3} \times \frac{5 \times 10^{-3}}{0.018 \times 10^{-3}}
\]

\[
\therefore y = 1.023.
\]

The following Table was then drawn using the same procedure
of calculation
<table>
<thead>
<tr>
<th>Gap-width (thou. in.)</th>
<th>Equivalent Micrometer-setting (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55</td>
<td>1.023</td>
</tr>
<tr>
<td>0.63</td>
<td>1.000</td>
</tr>
<tr>
<td>0.80</td>
<td>0.953</td>
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<td>1.58</td>
<td>0.737</td>
</tr>
<tr>
<td>2.10</td>
<td>0.592</td>
</tr>
</tbody>
</table>
APPENDIX II - CALIBRATION-CHART CONVERSION TABLES.

The Table shown below was prepared from the Calibration Chart shown in figure 16 and was used to estimate the recessions at the diphenyl wall.

The annular gap-width in the standard calibration device was set at 1.05 thou inch.

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APPENDIX IV - ALLOWABLE SURFACE ROUGHNESS FOR MASS-TRANSFER SURFACES.

In the study of mass transfer in turbulent flow, unevenness on the surface of the same order of magnitude as the thickness of the predominantly laminar layer adjacent to the tube wall will have an appreciable effect on flow conditions. This thickness may be estimated from the universal velocity distribution for turbulent flow to give the upper limit of tolerance on any surface irregularity.

Estimation of Upper Limit for Surface Irregularities:

By von Kármán's analysis (V.6) the region of laminar flow is given by

\[ 0 \leq y^+ \leq 5 \] .......................... (A.1)

where \( y^+ \) = dimensionless distance from wall = \( \frac{y}{v} u^* \) .... (A.2)

and \( y \) = distance from wall

\( u^* = \sqrt{\frac{\tau_0}{\rho}} \), friction velocity

\( \tau_0 \) = shear stress at wall

\( \rho \) = fluid density

\( \nu = \) the kinematic viscosity, \( \mu/\rho \)

\( \mu = \) fluid viscosity

\( y^+ \) is a 'modified' Reynolds number, \( \text{Re} \cdot \frac{u^*}{d} \cdot \frac{u^*}{\text{Re} \cdot \sqrt{\frac{1}{2}}} \)

The shear stress at the wall may be expressed as

\[ \tau_0 = f \frac{\rho u^2}{2} \] .......................... (A.3)

where \( f \) = the friction factor

\( u = \) the fluid velocity

Also,
Also, the velocity parameter $u^+$ and distance parameter $y^+$ can be expressed as

$$u^+ = u \sqrt{f/2} \quad \ldots \ldots \ldots \ldots \ldots \ldots (A.4)$$

$$y^+ = \frac{y}{u} \sqrt{f/2} \quad \ldots \ldots \ldots \ldots \ldots \ldots (A.5)$$

At the edge of the boundary layer

$$5 = \frac{y}{d} \frac{Re}{\sqrt{f/2}} \quad \ldots \ldots \ldots \ldots \ldots \ldots (A.6)$$

where $d =$ the tube diameter

$$\therefore \frac{y}{d} = \frac{5}{Re} \frac{\sqrt{f/2}}{2} \quad \ldots \ldots \ldots \ldots \ldots \ldots (A.7)$$

At $Re = 10^5$, $f = 4.5 \times 10^{-3}$, and

$$\frac{2}{f} \approx 20$$

$$\therefore \frac{y}{d} \approx \frac{10^2}{10^5} \approx 10^{-3}$$

In the present investigation the transfer section was a $\frac{1}{4}''$ internal diameter brass tube. From the result shown above, any irregularities at the wall greater than 0.001 inch could be expected to interfere significantly with the fully developed turbulent flow. This is then the maximum allowable height for any irregularities on the test-surface.
APPENDIX V. MEASUREMENT OF FLOW VELOCITY AND ESTIMATION OF THE REYNOLDS NUMBER.

The maximum value of the impact/static pressure differential was obtained for each particular flow condition by means of the Pitot arrangement shown in figure 24. The maximum velocity $u_m$ in the flow was calculated from equation (A.16) derived below, which is the value of the velocity at the centre of the tube. The average velocity, $u_{av}$, can then be estimated from the relation

$$u_{av} = 0.8 \times u_m \quad \text{(A.8)}$$

since the average velocity for turbulent flow is 0.8 times the maximum velocity (K.5).

The Reynolds number is defined by the expression

$$Re = \frac{u_{av} \rho d}{\mu}$$

where $d$ is the tube diameter

$\rho$ is the fluid density

$\mu$ is the fluid viscosity

The Reynolds number is then

$$Re = \frac{(0.8 \times u_m) \rho d}{\mu} \quad \text{(A.9)}$$

According to Bernoulli's equation, the conditions of the fluid at the stagnation point, 1, are related to those of the undisturbed flow, 2, if losses are neglected in the motion from the undisturbed flow to the stagnation point and if the flow is incompressible. Then

$$\frac{P_2}{\rho} + \frac{1}{2}u_2^2 = \frac{P_1}{\rho} + \frac{1}{2}u_1^2 \quad \text{(A.10)}$$

where/
where \( p_1 \) is the pressure at stagnation point
\( p_2 \) is the pressure at same depth in the undisturbed fluid
\( u_1, u_2 \) are the respective velocities at these points in the fluid.

\( u_1 = 0 \), since by definition the velocity at the stagnation point is zero. Equation A.10 then becomes

\[
\frac{1}{2}u_2^2 = \frac{(p_1 - p_2)}{\rho} \quad \text{(A.11)}
\]

where \( p_1 - p_2 \) is defined as the "kinematic pressure", known also as the "dynamic" or "stagnation" pressure. The magnitude of \( p_1 - p_2 \) is obtained from the manometric height difference, \( h_L \), and equation (A.11) is written as

\[
\frac{1}{2}u_2^2 = h_L g \frac{\rho_L}{\rho_A} \quad \text{(A.12)}
\]

where \( g \) is the gravitational constant
\( \rho_L \) is the manometric fluid density
\( \rho_A \) is the density of air at the time of the run.

\[
\therefore u_2 = \sqrt{2gh_L \frac{\rho_L}{\rho_A}} \quad \text{(A.13)}
\]

The manometric fluid consisted of water, i.e., \( h_L = h_w \); and the pressure \( p \) at any point is given by

\[
p = h_w \times \frac{\rho}{\rho_A} = h_A \times \frac{W_A}{W_w} \quad \text{(A.14)}
\]

where \( W_A \) is the specific weight of air

\( W_w \) is the specific weight of water

\[
\therefore u_2 = \sqrt{2gh_w \frac{W_w}{W_A}} \quad \text{(A.15)}
\]
where \( W_w = 62.43 \text{ lb./ft.}^3 \)

\[
W_A = 0.001205 \text{ gm./cc.} = \frac{0.001205 \times 12^3 \times 2.54^3}{4.54} \text{ lb./ft.}^3
\]

\[
= 0.0761 \text{ lb./ft.}^3
\]

\[
h_w = \frac{h_M}{2.54 \times 12 \text{ feet}}
\]

\[
g = 32 \text{ ft./sec.}
\]

\[
\therefore u_2 = \sqrt{2g \times \frac{h_w}{12 \times 2.54} \times \frac{62.43}{0.0761}} \quad \text{(A.16)}
\]

[Data from: "Handbook of Physics and Chemistry" 41st Edition.]
APPENDIX VI(a). CALCULATION OF DIFFUSION COEFFICIENTS FOR CARBON DIOXIDE/DIPHENYL AND FREON-114/DIPHENYL SYSTEMS.

The diffusion coefficient, $D$, was calculated from an expression proposed by Arnold:

$$D_{12} = \frac{0.0076T^{5/2}[(M_1 + M_2)/M_1M_2]^{1/2}}{P(v_{b1}^{1/3} + v_{b2}^{1/3})^2(T + S_{12})}$$  \hspace{1cm} (A.17)

$D_{12}$ = diffusion coefficient for binary system [cm$^2$/sec.], g.mole/(sec.)(cm.)(g.mole/cm$^3$)]

$M_1, M_2$ = molecular weights

$v_{b1}, v_{b2}$ = molal volumes of the pure substances at their normal boiling temperatures

$S_{12}$ = Sutherland constant for the pair

Arnold obtained the following equation for the Sutherland constant

$$S_{12} = F\sqrt{S_1S_2}$$  \hspace{1cm} (A.18)

where $F$ varies with the ratio $v_{b2}/v_{b1}$ (or, $v_{b1}/v_{b2}$ if $v_{b1} > v_{b2}$) as follows

<table>
<thead>
<tr>
<th>$v_{b2}/v_{b1}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>1.00</td>
<td>0.98</td>
<td>0.953</td>
<td>0.920</td>
<td>0.875</td>
<td>0.838</td>
<td>0.805</td>
</tr>
</tbody>
</table>

Sutherland constants $S_1$ and $S_2$ are properly obtained from data on the variation of viscosity of the pure gases with temperature, but Arnold found $S_{12}$ might be estimated from the absolute boiling temperatures.
where \( T_b \) is the boiling point.

Arnold employed values of \( V_b^1 \) and \( V_b^2 \) which are estimated by Kopp's law of additive atomic volumes as given by Le Bas.

**Example:** (a) Calculation of Diffusivity for the Carbon Dioxide/Diphenyl System.

\[
\begin{align*}
M_D &= 154 \\
M_{CO_2} &= [1(6) + 2(0)] = 12 + 32 = 44
\end{align*}
\]

Subscript \( D \) refers to diphenyl.

\[
\begin{align*}
V_{b_D} &= 214.6 \\
V_{b_{CO_2}} &= 29.6 \\
\text{and } V_{b_D}/V_{b_{CO_2}} &= \frac{214.6}{29.6} = 7.
\end{align*}
\]

\[
\begin{align*}
F &= (0.875 - 0.019) = 0.856 \\
T_b &= 255^\circ C \\
T_{b_{CO_2}} &= \text{arbitrarily taken as } 195^\circ K \text{ (the sublimation temperature at 1 atmosphere)} \\
T &= 21^\circ C = 294^\circ K.
\end{align*}
\]

\[
\begin{align*}
&\text{from equation (A.18)} \\
S_{D/CO_2} &= 1.47 \times 0.856 \sqrt{195 \times 255} = 280.8
\end{align*}
\]
and from equation (A.17)

$$D_{D/CO_2} = \frac{0.0076(29\mu)^{5/2}[(154 + 4\mu)/(154 \times 4\mu)]^{1/2}}{1 \times [214.6^{1/3} + 29.6^{1/3}]^2[294 + 280.8]} = 0.041 \text{ cm}^2/\text{sec}.$$  

(b) Calculation of Diffusivity for the Freon-114/Diphenyl System:

The chemical formula for Freon-114 is $\text{CCl}_2\text{F}_2\text{-CCl}_2\text{F}_2\text{MDF}_5\text{L}$.  

$$M_D = 154$$  

$$M_{Fr} = 170.9$$  

$$V_{b_D} = 214.6$$  

$$V_{b_{Fr}} = 107.6$$

whence  

$$\frac{V_{b_D}}{V_{b_{Fr}}} = \frac{214.6}{107.6} \approx 2$$

and  

$$F = 0.98$$

Also  

$$T_{b_D} = 255^\circ \text{C}$$  

$$T_{b_{Fr}} = 355^\circ \text{C} = 276.55^\circ \text{K}$$  

$$T = 21^\circ \text{C} = 294^\circ \text{K}$$

$\therefore$  

$$S_{D/ Fr} = 1.147 \times 0.98 \sqrt{276.6 \times 255} = 383.1$$

and  

$$D_{D/ Fr} = \frac{0.0076 \times (29\mu)^{5/2}[(154 + 170.9)/(154 \times 170.9)]^{1/2}}{1 \times (214.6^{1/3} + 107.6^{1/3})^2[294 + 383.1]}$$  

$$= 0.016 \text{ cm}^2/\text{sec}.$$  

[Data from: "Handbook of Physics and Chemistry" 41st Edition.]
APPENDIX VI (b). CALCULATION OF DIFFUSION COEFFICIENT FOR AIR/DIPHENYL SYSTEM.

For the system diphenyl/air the Schmidt number is 2.3, the diffusivity of diphenyl being calculated from the expression\(^{(1.3)}\)

\[
D = D_o \left( \frac{T}{T_o} \right)^m \frac{p_o}{p} \quad \cdots \cdots \cdots \cdots \cdots \quad (A.19a)
\]

where \(D_o\) = the value of the diffusivity \(D\) at \(T_o\) (= 273\(^{0}\)K) and \(p_o\) (=1 atm.).

\(D\) = the value of the diffusivity \(D\) at temperature \(T\) and pressure \(p\).

\(m\) = a constant, which theoretically lies between 1.5 and 2.0, depending on the gases.

For diphenyl, \(D_o = 0.0610 \text{ cm}^2/\text{sec.}\)
\(m = 2.00\).

[Data from: "Handbook of Physics and Chemistry" 41st Edition]
APPENDIX VII. TEMPERATURE DEPENDENCE OF THE VAPOUR PRESSURE, \( P \).

The effect of temperature variations on the vapour pressure \( P \) of diphenyl were found to be significant. To obtain a better result for \( P \) this was calculated from the Sherwood and Johannes correlation (8.35).

\[
\log_{10} P(\text{mm}) = \frac{A}{T} + B \quad \ldots \ldots \ldots \ldots (A.20)
\]

where \( A = -3.918 \), \( B = 11.003 \) for diphenyl.

\[
\therefore \log_{10} P(\text{mm}) = -\frac{3.918}{T} + 11.003 \quad \ldots \ldots (A.20a)
\]

As seen from the Table shown below, prepared by use of equation (A.22), a change of about 5\( ^\circ \)C nearly doubles the vapour pressure of the diphenyl.

<table>
<thead>
<tr>
<th>( T^\circ \text{C} )</th>
<th>( T^\circ \text{K} )</th>
<th>( 1/T^\circ \text{K} \times 10^{-5} )</th>
<th>( P(\text{atm}) \times 10^{-6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>288</td>
<td>347</td>
<td>3.29</td>
</tr>
<tr>
<td>16</td>
<td>289</td>
<td>346</td>
<td>3.68</td>
</tr>
<tr>
<td>17</td>
<td>290</td>
<td>345</td>
<td>4.09</td>
</tr>
<tr>
<td>18</td>
<td>291</td>
<td>344</td>
<td>4.55</td>
</tr>
<tr>
<td>19</td>
<td>292</td>
<td>343</td>
<td>5.06</td>
</tr>
<tr>
<td>20</td>
<td>293</td>
<td>341</td>
<td>5.63</td>
</tr>
<tr>
<td>21</td>
<td>294</td>
<td>340</td>
<td>6.24</td>
</tr>
<tr>
<td>22</td>
<td>295</td>
<td>339</td>
<td>6.94</td>
</tr>
<tr>
<td>23</td>
<td>296</td>
<td>338</td>
<td>7.69</td>
</tr>
<tr>
<td>24</td>
<td>297</td>
<td>337</td>
<td>8.51</td>
</tr>
<tr>
<td>25</td>
<td>298</td>
<td>336</td>
<td>9.42</td>
</tr>
<tr>
<td>26</td>
<td>299</td>
<td>334</td>
<td>10.43</td>
</tr>
<tr>
<td>27</td>
<td>300</td>
<td>333</td>
<td>11.54</td>
</tr>
<tr>
<td>28</td>
<td>301</td>
<td>332</td>
<td>12.74</td>
</tr>
<tr>
<td>29</td>
<td>302</td>
<td>331</td>
<td>14.07</td>
</tr>
<tr>
<td>30</td>
<td>303</td>
<td>330</td>
<td>15.53</td>
</tr>
</tbody>
</table>
APPENDIX VIII. CALCULATION OF $S_{h}$, $S_{h_{\infty}}$ and $S_{h}/S_{h_{\infty}}$

(a) Calculation of the Sherwood number, $S_h$:

By definition

$$\text{Sh} = k_c \frac{d}{D} \quad \ldots\ldots\ldots\ldots\ldots (A.21)$$

where $k_c$ is the mass-transfer coefficient, based on a concentration driving force.

d is the tube diameter.

$D$ is the diffusion coefficient for diphenyl vapour in air.

Amount of material transferred per unit length $= \Delta A_r \rho$

where $\Delta A_r$ = the total surface of recession

$\rho$ = density of diphenyl

Transfer rate per unit area, per unit length, per unit time $= \frac{\Delta A_r \rho}{n \Theta d}$

where $\Theta$ = duration of experimental run.

Hence,

$$k_c \cdot \Delta c = \frac{\Delta A_r \rho}{n \Theta d} \quad \ldots\ldots\ldots\ldots (A.22)$$

where $c$ = the concentration driving force.

Assuming that the gas-film resistance and the concentration of diphenyl in the bulk air-stream to be negligible

$$\Delta c = \frac{M \cdot P}{t} \cdot \frac{273}{V} \cdot \frac{1}{V} \quad \ldots\ldots\ldots\ldots (A.23)$$

where $M$ = the molecular weight of diphenyl

$P$ = the mole fraction of diphenyl in air saturated with diphenyl at the temperature of the experimental run

$t$ = the operating temperature

$V$ = the molar volume of diphenyl
Then,
\[ k_c = \frac{\rho \cdot \Delta A_x}{9d} \cdot \frac{Vt}{273 M P} \] \hspace{1cm} (A.24)

If equation (A.24) is substituted in equations (A.21), the new expression for the Sherwood number is given by
\[ Sh = \frac{0 \cdot \tau d}{\theta P D} \cdot \frac{V}{273 M} \] \hspace{1cm} (A.25)

where \( \Delta A_x = \frac{\pi d \cdot \nu}{\rho} \) \hspace{1cm} (A.26)
and \( \nu \) is the local recession at the diphenyl wall.

Equation (A.25) may be written as
\[ Sh = c \cdot \frac{t}{\theta P D} \cdot \nu \] \hspace{1cm} (A.28)

where \( c = \) a constant
\( P = \) the vapour pressure of diphenyl (APPENDIX VII).

(b) Calculation of the Fully-Developed Sherwood number, \( Sh_\infty \):

In this case, equation (A.28) is written as
\[ Sh_\infty = c \cdot \frac{t}{\theta P D} \cdot \nu_\infty \] \hspace{1cm} (A.29)

where \( \nu_\infty = \) the mean value of the local recession corresponding to steady-state conditions.

(c) Calculation of the ratio \( Sh/Sh_\infty \):

By equations (A.28) and (A.29) the ratio \( Sh/Sh_\infty \) is given by
\[ \frac{Sh}{Sh_\infty} = \frac{\nu}{\nu_\infty} \] \hspace{1cm} (A.30)
APPENDIX IX. ESTIMATION OF DEPTH OF FILTER BED IN THE WIND-TUNNEL.

The diphenyl filter consisted of a packed bed of granulated activated carbon, grade ULTRASORB S.C.II. 10/24, supplied by the British Carbo Norit Union Ltd.

(i) Estimation of depth of filter-bed necessary to remove 99.9% of sublimed diphenyl from recycled gas.

Volume of gas entering/leaving tube containing the filter bed = \( u \pi \frac{d^2}{4} \) ft.\(^3\)/sec.

where \( u \) = the velocity of the recycled gas, ft./sec.

\( d \) = tube diameter, ft.

If \( c_i \) and \( c_o \) in the concentration of diphenyl present in the entering and leaving gas respectively, then

weight of diphenyl entering filter bed = \( u \pi \frac{d^2}{4} c_i \) lb./sec.

weight of diphenyl leaving filter bed = \( u \pi \frac{d^2}{4} c_o \) lb./sec.

Hence,

weight of diphenyl absorbed = \( \pi \frac{d^2}{4} (c_i - c_o) \) lb./sec... (A.31)

The weight of diphenyl absorbed at any differential section of the filter bed of length \( dl \)

\[ = A. \pi \frac{d^2}{4} (c - c_o) k_c \] .......................... (A.32)

\[ = \text{Area} \times \text{Concentration Driving Force} \times \text{Mass Transfer Coefficient} \]

where \( A = \text{external surface of activated-carbon granule per unit volume}, \) \( \text{ft.}^2/\text{ft.}^3 \)

\( c = \text{the concentration of diphenyl in the fluid surrounding the granule.} \)

\( c_o/ \)
\[ c_s = \text{the concentration of diphenyl at the surface of the granule.} \]

Over the whole volume, the weight of diphenyl absorbed

\[ = A \pi \frac{d^2}{4} \ln \frac{c_i}{c_o} \cdot k_c \]  \hspace{1cm} (A.33)

Combining equations (A.31) and (A.33)

\[ u = \frac{A \cdot l \cdot k_c}{\ln \frac{c_i}{c_o}} \]  \hspace{1cm} (A.34)

Generally, the following correlations for \( k_c \) hold for packed beds

\[ j_D = \frac{k_c}{u} \cdot Sc^{2/3} = 0.989 \cdot Re^{-0.41} \]  \hspace{1cm} for \( Re > 350 \)  \hspace{1cm} (A.35)

\[ = 16.8 \cdot Re^{-1} \]  \hspace{1cm} for \( Re < 40 \)

where \( j_D \) is the \( j \)-factor for mass transfer; \( Re \) is the Reynolds number (= \( upd^0/\mu \)) where \( d^0 \) is the diameter of the carbon granule; and, \( Sc \) is the Schmidt number (= \( \mu/\rho D \)).

In order to calculate \( l \), the depth of filter bed required, various values of \( d^0 \) and \( u \) are tried to evaluate first \( Re \) and then \( k_c \). The quantity \( A \) in equation (A.32) is found by assuming (i) the value of \( d^0 \), taking the carbon grains as spherical, (ii) the activated carbon granules occupy between 60 and 70\% of the whole space (S.36). The value of \( l \) may now be calculated from equation (A.34).

(ii) Sample Calculation: To find the height \( l \) of the filter bed when the gas carbon dioxide (\( Sc = 1.78 \)) is re-circulated in the wind-tunnel.
wind-tunnel. The velocity of the gas at the packed bed of activated carbon is 3 ft./sec.

The Reynolds number for this problem is 76,680

From equation (A.35)

$$k_c = \frac{0.989 \times 3 \times 12 \times 2.54}{(76,680)^{0.41} \times (1.78)^{213}}$$

Substitution for $k_c$ in equation (A.34)

$$l = \frac{u \times 2.303 \log_{10} 0.1 \times (76,680)^{0.41} (1.78)^{213}}{212 \times 0.989 \times u \times 90}$$

$$= 0.04 \text{ cm}.$$
### APPENDIX X (a). PHYSICAL AND CHEMICAL PROPERTIES OF SUBSTANCES.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Chemical Formula</th>
<th>Molecular Weight (gm.)</th>
<th>Density (gm.cm.(^{-3}))</th>
<th>Viscosity (gm. cm.(^{-1}).sec.(^{-1}))</th>
<th>Kinematic Viscosity (cm.(^2).sec.(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>-</td>
<td>29</td>
<td>0.001293</td>
<td>0.000180</td>
<td>1/8</td>
</tr>
<tr>
<td>Carbon Dioxide</td>
<td>CO(_2)</td>
<td>44</td>
<td>0.001977</td>
<td>0.000145</td>
<td>1/14</td>
</tr>
<tr>
<td>Freon-114 (Arction-114)</td>
<td>C(_2)F(_2)(_2) (_4)</td>
<td>170.9(^*)</td>
<td>0.01 (saturated vapour at 30°C: 0.00782)</td>
<td>0.000117(^*)</td>
<td>1/100</td>
</tr>
</tbody>
</table>

The various columns in the Table refer to gases at 0°C and 760 mm. pressure.

\(^*\) Data from: "Arcton Refrigerants", supplied by I.C.I. Ltd. (Mond Division); figures taken from a review of existing data by Powell, R.W., "Modern Refrigeration", (1956).
APPENDIX X (b). PHYSICAL AND CHEMICAL PROPERTIES OF VOLATILE ORGANIC SUBSTANCES.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Diphenyl</th>
<th>Naphthalene</th>
<th>Acenaphthene</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical Formula</td>
<td>$C_6H_5\cdot C_6H_5$</td>
<td>$C_{10}H_8$</td>
<td>$C_{10}H_6(CH)_2$</td>
</tr>
<tr>
<td>Molecular Weight (gm.)</td>
<td>154.2</td>
<td>128</td>
<td>154.2</td>
</tr>
<tr>
<td>Melting Point (°C)</td>
<td>69-70</td>
<td>80.2</td>
<td>95</td>
</tr>
<tr>
<td>Boiling Point (°C)</td>
<td>254.9</td>
<td>217.9</td>
<td>278.9</td>
</tr>
</tbody>
</table>

Data from: "Chemical Engineers' Handbook", (P. 10)
NOMENCLATURE

Lower Case Letters

\( a^2 \) thermal diffusivity, \( k/\rho c_p \)

\( b \) constant in equation (2.27)

\( c \) concentration of diffusing substance

\( c_p \) heat-capacity at constant pressure

\( \bar{c} \) time-mean component of concentration

\( c' \) fluctuating component of concentration

\( c_f \) local skin-friction coefficient, \( \tau_o/\frac{1}{2} \rho U^2 \)

\( d \) diameter of pipe

\( f \) friction factor for flow in closed conduits

\( g \) acceleration of gravity, 32 ft./sec\(^2\)

\( h \) heat-transfer coefficient

\( j_H \) the \( j \)-factor or heat transfer factor = \( St \cdot Pr^{2/3} \)

\( j_D \) the \( j \)-factor or mass transfer factor = \( St \cdot Sc^{2/3} \)

\( k \) thermal conductivity

\( k \) Kármán constant in equation (3.45)

\( k_c \) mass-transfer coefficient

\( l \) depth of the activated-charcoal bed

\( l \) length defining overall length of stall

\( \ln \) logarithm to base \( e \)

\( \log \) logarithm to base 10

\( m \) constant defined by equation (2.67)

\( m \) rate of mass transfer towards the tube centre per unit area
\( m_0 \)  
rate of mass transfer at the wall towards the tube centre per unit area

\( n \)  
constant defined by equation (2.46)

\( n \)  
constant defined by equation (2.48)

\( n \)  
constant in equation (3.43)

\( q \)  
rate of heat transfer towards tube centre per unit area

\( q_0 \)  
rate of heat transfer at inside wall per unit area

\( r \)  
radii coordinate

\( r_o \)  
radius of tube

\( t \)  
temperature

\( t \)  
time

\( \bar{t} \)  
time-mean component of temperature

\( t^0 \)  
fluctuating component of temperature

\( t \)  
temperature of fluid outside the thermal boundary-layer

\( u \)  
average velocity, velocity component in x-direction

\( u_b \)  
main stream velocity

\( \bar{u} \)  
mean velocity in x-direction

\( u^0 \)  
fluctuating velocity-component in flow direction

\( u_i \)  
velocity of species i

\( v \)  
radial velocity

\( v^0 \)  
fluctuating velocity-component normal to flow

\( w \)  
local recession at wall

\( w \)  
turbulent velocity fluctuation in the z-direction

\( x \)  
axial distance

\( x \)  
distance downstream of transfer entrance

\( y \)  
distance from wall

\( y^+ \)  
value of \( y^+ \) for which equation for region away from wall applies.
**Upper Case Letters**

A  constant in equation (2.1)
A  surface area
A  constant in equation (2.43)
A  constant in equation (2.48)
A  referring to component A
A₁  annular gap-width in unit I, figure 12
A₂  annular gap-width in unit III, figure 12
B  constant defined by equation (2.43)
B  measure of the mass-transfer rate defined by equation (2.59)
B  referring to component B
Cₚ  skin friction
C₀  concentration of diffusing substance at the wall
D  molecular diffusivity of mass
F  flatness (kurtosis)
G  mass velocity
G  mass flow
K  height of roughness element
Kₛ  equivalent sand roughness
L  length parameter in equation (2.14)
L  length of tube
M  molecular weight
M₀  true zero micrometer-setting in unit I, figure 12
P₁  constant supply-pressure of diphenyl-saturated gauging air
P₂  intermediate pressure in unit II, figure 12
Pₐ  atmospheric pressure
P₀  supply pressure in unit II, figure 12
P  vapour pressure of diphenyl
\( Q \) fluid-flow rate
\( S_L \) longitudinal spacing
\( T \) absolute temperature, \( ^{\circ}K \)
\( U \) average velocity
\( U_c \) maximum velocity in pipe or channel flow
\( U_s \) shear velocity determined by cross-plot of logarithmic-velocity profile
\( \bar{U} \) average velocity in pipe or channel flow
\( V \) mean velocity component in the \( y \)-direction
\( Y \) mole fraction
\( Y \) distance normal to the test wall and measured from the smooth (or fictitious) wall = \( Y_T + \epsilon \)
\( Y_T \) distance normal to the test wall and measured from the crest of the square bars.

**Subscripts**

- \( A \) species \( A \)
- \( B \) species \( B \)
- \( av \) average
- \( b \) evaluated at bulk
- \( c \) referring to axis of tube
- \( c \) critical conditions
- \( d \) referring to diameter of pipe or wire
- \( e \) referring to entrance
- \( f \) evaluated at film next to wall
- \( H \) heat
- \( h \) heat
- \( l \) referring to appropriate constant-property solution
- \( M \) mass
m  mass
0  evaluated at the wall of pipe
s  smooth surfaces
t  turbulent conditions
U  momentum
x  evaluated at x
x  in the x-direction
y  in the y-direction
∞  evaluated at infinity, steady state, fully-developed conditions, free stream
λ  evaluated at y = λ

Superscripts
+  dimensionless quantities
-  value time mean
0  root-mean square of fluctuating quantity

Greek Letters
α  ratio of diffusivities
β²  constant in equation (3.16a)
β²n  eigenvalues
γ  ratio of specific heats
Δ  thickness parameter, \[ = \int_0^\infty \left( \frac{U_\infty - U}{U_\infty} \right) dy \]
λ  boundary-layer thickness where U = 0.99U_∞
λ⁺  displacement thickness \[ = \int_0^\infty (1 - U/U_\infty) dy \]
ζ  constant defined by equation (2.29)
ε  displacement of Y-origin below crest of a bar
\( \varepsilon \) eddy diffusivity
\( \varepsilon_u \) eddy diffusivity of momentum
\( \varepsilon_H \) eddy diffusivity of heat
\( \varepsilon_m \) eddy diffusivity of mass
\( \Theta \) correction factor defined by equation (2.59); temperature profile as defined by equation (3.10); momentum thickness
\[ \int_0^\infty \frac{U}{U_\infty} (1 - \frac{U}{U_\infty}) \, dy \]
\( \kappa \) universal turbulence constant
\( \lambda \) molecular diffusivity
\( \lambda_m \) concentration boundary-layer thickness
\( \lambda_h \) thermal boundary-layer thickness
\( \lambda^0 \) thickness of stagnant layer next to wall
\( \mu \) viscosity
\( \nu \) kinematic viscosity, \( \mu/\rho \)
\( \omega \) boundary-layer diffusing factor
\( \rho \) density
\( \sigma \) Prandtl or Schmidt number
\( \tau \) shear stress in fluid
\( \tau_o \) shear stress in fluid at wall
\( \phi \) angle in equation (2.17)
\( \phi_n \) eigenfunctions

**Dimensionless**

\( c^+ \) concentration parameter \( \frac{c_0 - c}{m_0} \cdot u^+ \)
\( c^+_{\lambda} \) value of \( c^+ \) at \( \lambda^h \)
\( c_n \) coefficient in series expansion, equation (3.17)
\[ r^+ \] dimensionless radius, \( r_0 \frac{u^*}{v} \)
\[ r^+ \] dimensionless radial parameter, \( r \frac{u}{v} \)
\[ u^+ \] dimensionless velocity, \( \frac{u^*}{u} \)
\[ u^c \] frictional velocity, \( \sqrt{\frac{\tau_0}{\rho}} \)
\[ x^+ \] dimensionless distance, \( x/r_0 \) or \( x/d \)
\[ y^+ \] wall distance parameter, \( y \frac{u}{v} \)
\[ y^{++} \] dimensionless parameter defined by equation (2.37)
\[ G(r^+) \] radial variation of fully-developed temperature profile, equation (3.12a)
\[ K^+ \] dimensionless mass-transfer coefficient
\[ Nu \] Nusselt number, \( \frac{hd}{k} \)
\[ Pe \] Peclet number, \( RePr \)
\[ Pr \] Prandtl number, \( \frac{c_p \mu}{k} \)
\[ Re \] Reynolds number, \( \frac{du \rho}{\mu} \)
\[ Sc \] Schmidt number, \( \frac{\mu}{\rho D} \)
\[ Sh \] Sherwood number, \( \frac{k_c d}{D} \)
\[ St \] Stanton number, \( \frac{(Nu/Re)}{\sigma} \)
\[ Y^+ \] dimensionless distance, \( Y \frac{u^*}{v} \)
\[ \gamma \] dimensionless diffusivity in equation (3.9)
\[ \epsilon^+ \] dimensionless diffusivity
\[ \lambda^+_h \] dimensionless thermal boundary-layer thickness, \( h \frac{u^*}{v} \)
\[ \lambda^+_m \] dimensionless concentration boundary-layer thickness, \( m \frac{u^*}{v} \)
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It is appreciated that a work of this type is not the result of the sole effort of a single person; but that a large body of people not necessarily connected with this research and with whom I came into contact, have contributed in one way or another. I should therefore like to record my thanks to all those people who knowingly or unknowingly made this work possible.

Thanks are also due to Dr. N. Macleod who initiated the work, and also Dr. J. Mawer who acted as Supervisor for a short period.
TABLE

<table>
<thead>
<tr>
<th>Formula for ( \epsilon'(y') )</th>
<th>Image</th>
<th>Characteristic Impedance</th>
<th>Type</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon' = 1 )</td>
<td><img src="image.png" alt="image" /></td>
<td>lid = 1.1</td>
<td>1</td>
<td>Physical, I. (1954)</td>
</tr>
<tr>
<td>( \epsilon' = 1 + \frac{1}{y} )</td>
<td><img src="image.png" alt="image" /></td>
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</tr>
</tbody>
</table>

1. Mathematical
2. Experimental

The above table provides a summary of the formulas used in the development of the theoretical model. The formulas are derived from fundamental principles and are used to predict various physical phenomena. The table includes the characteristic impedance, the type of approximation, and the reference for each formula. The formulas are essential for understanding the behavior of physical systems and are widely used in engineering and physics.
<table>
<thead>
<tr>
<th>Investigator</th>
<th>Year</th>
<th>Type of Pipe</th>
<th>Method</th>
<th>Entrance Lengths ((\ell/\delta))</th>
<th>Exit Lengths ((\ell/\delta))</th>
<th>Total Entrance Lengths ((\ell/\delta))</th>
<th>Heat Transfer Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latko, et al.</td>
<td>1954</td>
<td>A</td>
<td>-</td>
<td>1.1</td>
<td>-</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Sanders, et al.</td>
<td>1955</td>
<td>A</td>
<td>-</td>
<td>1.2</td>
<td>-</td>
<td>7</td>
<td>5</td>
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<tr>
<td>Shimazaki, et al.</td>
<td>1956</td>
<td>A</td>
<td>-</td>
<td>1.3</td>
<td>-</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>W. H. and H.</td>
<td>1957</td>
<td>A</td>
<td>-</td>
<td>1.4</td>
<td>-</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Sparrow, et al.</td>
<td>1958</td>
<td>A</td>
<td>-</td>
<td>1.5</td>
<td>-</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

### Notes:
- Constant fluid properties; air assumed to be turbulent at all points along the passage. Analysis is for Prandtl and it is based on an assumed 1/4 power velocity and the Blasius resistance formula. The relative changes in heat-transfer coefficient with Reynolds number, the data, and the Blasius resistance formula were approximately correct, but the absolute values of the heat-transfer coefficient at the exit of the entrance region are too low. The increase in entrance length with Re.
- A mathematical analysis of the turbulent heat-transfer in a pipe with a surface temperature discontinuity at entrance.
- Calculation relative to the thermal energy-length for fluid at low Prandtl number.
- Analysis applicable to cases in which the heat transferred by turbulence can be neglected as might occur at low Prandts.
- A slight increase in entrance length with Reynolds number.
- Discussion in section 1.1.

### Analysis:
- An investigation into heat transfer to air flowing in circular tubes. Heat-transfer tube of 4 diameters long, entrance equal to tube diameter, and the curves were extrapolated for Re > 1,500.

### Experimental Setup:
- Internal and mean wall-temperature differences were measured in the entrance region for water flowing in small circular pipes. This setup, with a velocity and temperature profiles developed significantly. Reynolds number of 500 and 1,000 were tested, and the curves were extrapolated for Re = 1,500.
- Investigation comparable to Latko, et al. with water instead of air.

### Results:
- Heat transfer from high-temperature air flowing in circular pipes was studied. Initial velocity and thermal profiles were measured, and the curves were extrapolated for Re > 1,500.
- Investigations into heat transfer from high-temperature air flowing in circular pipes were conducted. Initial velocity and thermal profiles were measured, and the curves were extrapolated for Re > 1,500.

### Discussion:
- Information on entrance length and heat transfer sections. Also, see discussion in section 1.1.

### Extrapolation:
- Extrapolation of entrance lengths held to be large.
The fully-developed heat-transfer coefficient $h_f$ was studied at $5\%$ diameter, with no allowance being made for the effects of property variation in a radial direction.

"Investigation was made for the effects of property variation in a radial direction."

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