UNIVERSITY OF EDINBURGH
DEPARTMENT OF ELECTRICAL ENGINEERING

DYNAMOMETER FIELD CONTROL
TO SIMULATE REAL LOADS UNDER
DYNAMIC CONDITIONS

being a Thesis submitted by
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The trend of the work covered in this Thesis, is towards the application of an electric control system to the field current of a d.c. dynamometer to modify the speed torque curves so that a variety of real loads can be simulated such as constant, viscous and fan loads.

The proposed control system is a programmed closed loop control system with the controlled variable being the transmitted torque to the dynamometer.

The engine-dynamometer system, without the dynamometer field time constant, has two independent inputs, throttle opening and dynamometer field current, and two dependent outputs, engine speed and transmitted torque. Analytical investigation of the system indicated that a sudden jump exists in the response of transmitted torque for step variations in field current at constant values of throttle. Due to the sudden jump, the speed of response of transmitted torque is faster than that of engine speed. Because it is required to simulate loads that vary with engine speed, this result made the proposed method of control feasible even though
the system has not yet been controlled.

State variables of the controlled object, when the field time constant was considered, are defined and the system has two independent inputs, field voltage and throttle, and two dependent outputs, engine speed and transmitted torque. The system is non-linear and analogue computation was necessary. Computer investigation indicated that the field time constant, being the main contributor to dynamic lags in the system, affects the responses of transmitted torque and engine speed.

An estimate of the dynamic region of operation was made and the region was used to give the possible range of dynamic loads that can be simulated.

A bang-bang controller, being a time-optimal system, with silicon controlled rectifiers is to be used in controlling the field current where maximum and zero field voltages are switched across the field. A filter is to be used in conjunction with an ASEA ring torductor to filter out the expected engine noise in transmitted torque, the engine being the main contributor to noise in the system.

Analogue computation indicated that bang-
bang control is feasible and introduces limit cycle oscillations. The response of transmitted torque, in the transmitted torque control system, is faster than that of engine speed by a factor of 20 and hence bang-bang control is suitable.

The performance of the programmed transmitted torque control system was investigated on the computer. The programmed system simulates the desired loads, within ±2% error, under a maximum rate of change in throttle of 60 ft. lb. sec⁻¹. For this rate of change, three seconds are required to cover the range of the throttle. The main factor that limits the performance of the programmed system is the field time constant.


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1.1 Statement of Problem

In the past few years much has been done to develop the means for vehicle testing in test cells. Engine testing under steady state conditions gives satisfactory results for most automobile power train testing. However, it has become necessary to develop electric dynamometers to provide dynamic loads similar to actual conditions on the road.

The problem is to develop a control system to an electric dynamometer so as to simulate a variety of real dynamic loads similar to those encountered by engines while accelerating or decelerating. The control system provides a means to test engines under the following variety of loads:

1 - Constant loads or motor vehicle climbing a hill.
2 - Viscous loads or rolling resistance.
3 - Fan loads or wind resistance on a motor vehicle.

The system consists of an Austin commercial petrol engine mechanically coupled to an electric
field controlled dynamometer. It is required to control the dynamometer field current so as to make the transmitted torque, to the dynamometer, follow any of the above loads.

A programme unit is necessary to produce a reference signal directly proportional to the desired load. Because the loads to be simulated are functions of engine speed, a signal from a tachometer generator is fed to the input of the programmer.

A negative feedback control system suggests itself in that the controlled variable is the transmitted torque. Hence, the output signal from the programmer is compared with a feedback signal from a torque transducer to produce an error signal. The error signal actuates a controller that controls the dynamometer field current. The schematic diagram of the proposed programmed transmitted torque control system is shown in Fig. 1.1.1.

The proposed system poses an interesting problem in which automatic control will increase the usefulness of already existing equipment and may enable suitable alterations in engine settings and design to be made.

1.2 Literature Review

The recent development of programming methods
for automatic control have served to increase the usefulness of the dynamometer to provide a variety of engine tests under controlled conditions.

The earliest attempt to control the dynamometer so as to simulate loads similar to those encountered under accelerating conditions on the road was due to Bowers\(^{(1)}\) in 1950. An electronic control system was designed and built mainly for fuel rating work. A full throttle acceleration curve was obtained for a vehicle on a level highway. Duplication of this curve in the laboratory was attempted by proper design of the velocity, acceleration and error channels in the dynamometer control. The system is a closed servo system in which the output of a speed generator on the engine shaft is fed to the three channel control circuit. The d.c. signals from the control unit drive an amplifier which supplies the field current for the dynamometer to load the test engine. The system provides a means to simulate only an acceleration curve necessary for a particular type of engine test.

Knudsen, R.\(^{(12)}\) (1957), discussed two methods of dynamometer programming that may be used to simulate road conditions, namely cycling control and feedback signal programming. Cycling control is an open loop type of control without feedback signals to
engine speed and torque to assure duplication of desired load conditions. The feedback signal programming method is a closed loop system. Test runs are made on the road and the variations of engine speed and torque are recorded simultaneously on a tape recorder. The tape is then used to programme the dynamometer and engine controls in the laboratory. This method presents a problem in that, for different road conditions a different programme should be recorded on the tape. Knudsen also explained the system used by the General Motors Research Staff which consists of a typical Ward-Leonard control system used on d.c. dynamometers. Amplifying generator control is used on the generator and dynamometer fields to control speed or dynamometer torque. The torque control of a dynamometer can be modified by auxiliary controls to simulate various road conditions such as those mentioned in section 1.1.

Gallant(7) (1958) discussed briefly a dynamometer control system called the Electronic Proving Grounds which was designed and built by the Instrumentation Group of the Chevrolet Engineering Laboratory. The programmer, being the heart of the system, consists of a master timer that operates a mirco-switch which in turn operates the control contacts. A test schedule,
from information obtained on actual road test, is set up on the programmer. The programmer then directly controls the throttle and the dynamometer load which, in turn, indirectly control the speed and the dynamometer torque.

In 1961, Knudsen, F(11) reviewed in his paper the test cell dynamometer facilities, the techniques for using the test cell as a road simulator and described three different methods for programming a specific vehicle test. The description for the dynamometer control was made in terms of a Ward-Leonard control system used on a d.c. dynamometer.

The Ward-Leonard control system is the only system that can be used to simulate, separately the different types of loads mentioned in section 1.1.

1.3 Discussion of Proposed Control System.

The programmed transmitted torque control system, Fig. 1.1.1, is intended to simulate different types of real loads by means of controlling the dynamometer field current. In the treatment of the problem, a controller is to be used which will cause an increase or a decrease in the dynamometer field current for positive or negative error signals. This configuration is
necessary to make the transmitted torque follow the desired type of load that is to be simulated.

The transmitted torque, being the controlled variable, can be measured accurately by means of precision torque transducers that require no slip rings. Because this type of transducer was not available until recently, the methods discussed by Knudsen, R. and Gallant, section 1.2, have the dynamometer torque as the controlled variable instead of the transmitted torque. The proposed control system presents an economical solution to the problem as compared, say, to Ward-Leonard control system.

The programmed transmitted torque control system is investigated in four parts, Fig. 1.3.1.

Part (a): The engine-dynamometer system in which the dynamometer field time constant is not included.

Part (b): The controlled object which includes the engine-dynamometer system and the dynamometer field transfer function.

Part (c): The transmitted torque control system, being a closed loop system, which includes the controller, controlled object and a feedback signal proportional.
Part (d): The programmed transmitted torque control system, being the proposed control system.

The proposed method of control requires that the transmitted torque should respond to input reference variations, in the transmitted torque control system, more quickly than does the engine speed. The responses of transmitted torque and engine speed are investigated for the engine-dynamometer system, controlled object and transmitted torque control system.
CHAPTER 2

DIFFERENTIAL EQUATIONS AND SOLUTION

2.1 Introduction

In this chapter, the characteristics of the dynamometer and of the engine are first considered separately and the dynamic equations of the engine-dynamometer system are derived. The dynamic equations are then solved for step variations in field current at constant values of throttle opening. From the analytical solution the responses of engine speed and transmitted torque are investigated. A study was then made to investigate whether the transmitted torque response is faster than that of engine speed.

The controlled object is discussed and state variables are defined.

2.2 Dynamometer Characteristics

The swinging field type dynamometer is a separately excited d.c. generator. The armature is coupled to the shaft of the engine and the frame carrying the field winding is mounted on bearings in such a manner that unless constrained it can rotate about the engine shaft as centre. The frame carries a torque arm to balance the engine torque. The engine power generates
electric power in the armature circuit which is
dissipated in a load resistance. The dynamometer
circuit and mechanical lay-out diagrams are
shown in Fig. 2.2.1a and 2.2.1b respectively.

The dynamometer torque is a function of
engine speed and field current. The torque
equation is derived as follows:

Let \( T_d \) = dynamometer torque (ft. lb.)
\( \Omega_e \) = engine speed \( \text{(rad. sec.}^{-1}\text{)} \)
\( I_f \) = dynamometer field current (amps)
\( I_a \) = armature current (amps)
\( E \) = induced e.m.f. in the armature (volts)
\( R_L \) = load resistance (ohms)

The torque developed in the dynamometer is
known to be proportional to the product of the
field and armature currents i.e.

\[ T_d = k_1 I_f I_a \] ... ... ... ... ... ... 2.2.1

The equations of the armature current and
induced e.m.f. are respectively

\[ I_a = \frac{E}{R_L} \] and \( E = k_2 I_f \Omega_e \)

Eliminating \( (I_a) \) from equation 2.2.1, the
dynamometer torque equation becomes

\[ T_d = k_1 I_f^2 \Omega_e \] ... ... ... ... ... ... 2.2.2

where \((k)\) is a constant (ft. lb. sec. amp.\(^{-2}\)).

There is a linear relationship between \((T_d)\)
and \((\Omega_e)\) at constant values of \((I_f)\), while the
relation between \((T_d)\) and \((I_f)\) at constant values of \((y)\) is parabolic.

Patten(20) and Roberts(22) used the same steady state testing procedure on the engine-dynamometer system. Dynamometer torques were measured at constant values of dynamometer field currents and variable values of throttle openings. The measured values in both works have reasonable agreement. The dynamometer torque speed characteristic curves are reproduced in Fig. 2.2.2. It is noticed from these curves that the relation between dynamometer torque and engine speed is almost linear at constant values of \((I_f)\). The discrepancy from linear variation is mainly due to friction, windage and armature reaction. At constant values of engine speed, curves were drawn relating dynamometer torque and field current as shown in Fig. 2.2.3. The relation between them is almost parabolic.

There is thus a reasonable agreement between experimental and predicted results. If the slight effects of friction, windage and armature reaction are neglected, equation 2.2.2 can be used to define the dynamometer torque for different values of engine speed and field current. The experimental dynamometer torque speed characteristic curves were approximated by straight
lines and the value of the constant \( k \) was estimated to be 0.24 ft. lb. sec. \( \text{amp}^{-2} \).

### 2.3 Engine Characteristics

The torque of the engine is a function of throttle opening and engine speed, Roberts (22).

If

\[
\begin{align*}
T_m &\quad \text{engine torque (ft. lb.)} \\
\theta &\quad \text{throttle opening (degrees)} \\
y_i &\quad \text{engine speed (rad. sec.)}
\end{align*}
\]

then

\[ T_m = f(\theta, y_i) \] \hspace{1cm} \text{................. 2.3.1}

Because at steady state condition the engine and dynamometer torques are equal, Fig. 2.2.2 can be used to determine the variation of engine torque with engine speed and throttle opening. The engine torque speed characteristic curves are shown in Fig. 2.2.2 where each curve was drawn at a different constant value of throttle opening. These curves can be approximated by parallel straight lines and hence equation 2.3.1 can be written as

\[ T_m = -h y_i + z \] \hspace{1cm} \text{............... 2.3.2}

Where \( h \) and \( z \) are positive constants, the value of \( z \) depending on throttle opening. The estimated value of \( h \) is 0.5 ft. lb. sec. and the estimated range of \( z \) is \( 30 \leq z \leq 200 \) ft. lb.

### 2.4 Dynamic Equations

The torque relationships in the engine—
dynamometer system i.e. engine torque \((T_m)\), transmitted torque \((y_1)\) and dynamometer torque \((T_d)\), are represented in Fig. 2.4.1.

Let 
\[
\begin{align*}
\dot{y}_1 &= \text{engine acceleration (rad. sec.}^{-2}) \\
J_1 &= \text{engine moment of inertia (ft. lb. sec.}^2) \\
J_2 &= \text{dynamometer moment of inertia (ft. lb. sec}^2)
\end{align*}
\]

then, using the notation in the figure, the following dynamic equations can be written:

Transmitted Torque = \(y_2 = T_m - J_1 \ddot{y}_1\) \(\ldots\) 2.4.1

and \(y_2 = T_d + J_2 \dot{y}_1\) \(\ldots\) 2.4.2

These equations can be rearranged in the following form:

\[
\begin{align*}
y_2 &= T_m + T_d \frac{J_2}{J_1+J_2} + T_d \frac{J_1}{J_1+J_2} \ldots \ldots \ldots 2.4.3 \\
\dot{y}_1 &= T_m - T_d \frac{J_1}{J_1+J_2} \ldots \ldots \ldots 2.4.4
\end{align*}
\]

Substituting the values of \((T_d)\) and \((T_m)\), as defined in equations 2.2.2 and 2.3.2 respectively, in equations 2.4.3 and 2.4.4 gives

\[
\begin{align*}
y_2 &= (-h y_1 + \frac{Z}{J_1+J_2}) + \frac{J_2}{J_1+J_2} \left( k \frac{I_f^2}{J_1+J_2} y_1 \right) \ldots \ldots 2.4.5 \\
\dot{y}_1 &= -h y_1 + \frac{Z - k \frac{I_f^2}{J_1+J_2} y_1}{J_1+J_2} \ldots \ldots 2.4.6
\end{align*}
\]

In order to simplify the algebra of the dynamic equations, write

\[
\begin{align*}
\frac{k \frac{I_f^2}{J_1+J_2}}{J_1+J_2} &= k \frac{I_f^2}{J_1+J_2} = \chi_1 \quad (\text{sec}^{-1}) \\
\frac{Z}{J_1+J_2} &= \chi_2 \quad (\text{sec}^{-2}) \\
\frac{h}{J_1+J_2} &= \alpha \quad (\text{sec}^{-1})
\end{align*}
\]
Then, the dynamic equations of the system become

\[ y_2 = y_1 (x_1 J_1 - a J_2) + J_2 \dot{x}_2 \quad \cdots \quad 2.4.7 \]

and

\[ \dot{y}_1 + y_1 (x_1 + a) = \dot{x}_2 \quad \cdots \quad 2.4.8 \]

The engine-dynamometer system can be represented by a block diagram with two independent input variables, \((x_1)\) and \((x_2)\), and two dependent variables, \((y_1)\) and \((y_2)\), as shown in Fig. 2.4.2. Variations in the throttle opening or dynamometer field current produce changes in engine speed and transmitted torque.

2.5 **Solution of Dynamic Equations Under Certain Conditions**

The solution of equation 2.4.8 leads to the solution of equation 2.4.7 by direct substitution. Equation 2.4.7 is a non-linear, non-homogenous, first order differential equation with variable coefficients. Because the transmitted torque control problem is mainly concerned with the dynamometer field current, \((x_2)\) may be varied with respect to time at constant values of throttle opening.

The responses of engine speed \((y_1)\) and transmitted torque \((y_2)\), for positive step variations in \((x_1)\) at constant values of \((x_2)\), are investigated as follows:

The solution of equation 2.4.8 under the following conditions
\[ \chi_i = \chi_{i0} \quad \text{for } t \leq 0 \]
\[ \chi_i = \chi_i \quad \text{for } t > 0 \quad \text{where } \chi_i > \chi_{i0} \]
\[ \chi_i = \chi_i \quad \text{constant} \]
\[ y_i = 0 \quad \text{for } t = 0 \]

is \( y_i = \text{P.I} + \text{C.F.} \)

\[ y_i = \frac{X_2}{a + \chi_i} + A \exp. - (a + \chi_i) t \]

\[ y_i = \frac{X_2}{a + \chi_i} + \left( \frac{X_2}{a + \chi_{i0}} - \frac{X_2}{a + \chi_i} \right) \exp. - (a + \chi_i) t \]

At \( t = 0 \quad (y_i)_{t=0} = X_2 / (a + \chi_{i0}) \)

and at \( t = \infty \quad (y_i)_{t=\infty} = X_2 / (a + \chi_i) \)

where \( (y_i)_{t=\infty} < (y_i)_{t=0} \) and vice versa for a negative step in \( \chi_i \).

The response of \( y_i \) here is similar to that of a low-pass filter with a time constant of \( (a + \chi_i) \).

Thus, the response of engine speed to an increase in field current \( \chi_i \) is sketched in Fig. 2.5.1.

Substituting equation 2.5.1 into equation 2.4.7 gives the solution for the transmitted torque equation i.e.

\[ y_2 = J_2 X_2 + (J_1\chi_1 - aJ_2) \left[ \frac{X_2}{a + \chi_i} + \left( \frac{X_2}{a + \chi_{i0}} - \frac{X_2}{a + \chi_i} \right) \exp. - (a + \chi_i) t \right] \]

\[ \text{At } t = 0, \]

\[ (y_2)_{t=0} = \frac{(J_1 + J_2) \chi_{i0} X_2}{a + \chi_{i0}} \]

\[ \text{at } t = 0^+, \text{ the instant when the step is applied to } \chi_i, \text{ the term } (J_1\chi_1 - aJ_2) \text{ becomes } (J_1\chi_1 - aJ_2) \]

and hence,

\[ (y_2)_{t=0} = \frac{J_2 X_2 \chi_{i0} + J_1 X_1 X_2}{a + \chi_{i0}} \]
and at $t = \infty$

$$(y_2)_{t=\infty} = \frac{(J_z + J_1) X_z X_1}{a + X_1} \quad \cdots \quad 2.5.5$$

where $(y_2)_{t=0^+} > (y_2)_{t=0}$ and $(y_2)_{t=\infty} > (y_2)_{t=0}$ and vice versa for a negative step in $(x_i)$.

To sketch the response of $(y_2)$, it is important to investigate the condition that determines the value of $(y_2)_{t=0^+}$ as the value might be less, equal or greater than that of $(y_2)_{t=\infty}$.

Equating equations 2.5.4 and 2.5.5 to determine the condition for which $(y_2)_{t=0^+} = (y_2)_{t=\infty}$ gives,

$$\frac{J_z X_z X_{10} + J_1 X_1 X_z}{a + X_{10}} = \frac{X_2 X_1 (J_z + J_1)}{a + X_1}$$

which gives

$$J_1 X_1^2 - X_1 (J_z a + J_1 X_{10}) + J_z a X_{10} = 0$$

One of the roots of this equation, being $X_{10} = X_1$, is not accepted while the other root is

$$X_1 = \frac{a J_2}{J_1}$$

Hence, the condition for $(y_2)_{t=0^+} = (y_2)_{t=\infty}$ is when $X_1 J_1 = a J_2$ and thus there is only one value of $(x_i)$ that gives instantaneous response.

Similarly, it is possible to show that if

$$X_1 J_1 < a J_2 \quad , \quad (y_2)_{t=0^+} < (y_2)_{t=\infty}$$

and if

$$X_1 J_1 > a J_2 \quad , \quad (y_2)_{t=0^+} > (y_2)_{t=\infty}$$

Thus, the transmitted torque can have three
different types of response depending upon the value of \((X_1)\). For the conditions \(J_1X_1 < aJ_2\), \(J_1X_1 = aJ_2\) and \(J_1X_1 > aJ_2\), the responses for a positive increase in \((X_1)\) are sketched in Figures 2.5.2 a, b and c respectively. The sudden jump in the transmitted torque response, from \((y_2)_{t=0}^+\) to \((y_2)_{t=0}^-\), leaves transient amplitudes, from \((y_2)_{t=0}^+\) to \((y_2)_{t=0}^-\), which are covered exponentially with a time constant of \((\alpha + X_1)\).

The proposed transmitted torque control system requires faster torque responses to that of engine speed. To investigate this requirement using the engine-dynamometer system, it is of importance to study the magnitude of the ratio

\[ R = \frac{(y_2)_{t=0}^- - (y_2)_{t=0}^+}{(y_2)_{t=0}^- - (y_2)_{t=0}^+}. \]

If \(|R|\) is less than unity, the amplitude of the exponential variations of \((y_2)\) is reduced, due to the sudden jump, and at any instant of time the transmitted torque is nearer to its final value than that of engine speed. For example, when \(J_1X_1 = aJ_2\) the response of \((y_2)\) is instantaneous. The equation of the ratio \((R)\) is derived by substituting the values of \((y_2)_{t=0}^+\), \((y_2)_{t=0}^-\), \((y_2)_{t=0}^-\), and \((y_2)_{t=0}^-\) from equations 2.5.3, 4 and 5 which gives,

\[ R = \frac{aJ_2 - J_1X_1}{(J_1 + J_2)\alpha} \quad \cdots \quad 2.5.6 \]
Substituting the value of the constants, for the particular engine-dynamometer system, from Appendix I i.e.

\[ J_1 = J_2 = 0.4 \quad \text{(ft. lb. sec.}^2) \]
\[ a = 0.6 \quad \text{(sec.}^{-1}) \]
\[ 0 \leq x_1 \leq 1.5 \quad \text{(sec.}^{-1}) \]
gives,

\[ R = \frac{1}{2} \left( 1 - \frac{1}{0.6} x_1 \right) \]

A plot of this equation as \( x_1 \) varies from zero to \( 1.5 \) is shown in Fig. 2.5.3. The positive values of \( R \), zero values and negative values correspond to the conditions \( J_1 x_1 < a J_2 \), \( J_1 x_1 = a J_2 \) and \( J_1 x_1 > a J_2 \) respectively. The ratio \( R \) is always less than unity and the transmitted torque response is faster than that of engine speed.

Thus, even though the engine-dynamometer system has not yet been controlled, the condition that the transmitted torque should be faster than that of engine speed is satisfied.

2.6 Controlled Object

In section 2.4, the engine-dynamometer system was represented by a block diagram with two independent inputs and two dependent outputs. In fact, this block represents a part of the system to be controlled because the requirement is to
control the dynamometer field and not the field current squared \((\chi_i)\). The field current is to be controlled by varying the field voltage (as discussed in section 4.6) so that the time constant of the dynamometer field introduces an additional lag in the controlled object.

The block diagram of the controlled object is shown in Fig. 2.6.1 and it consists of three parts:

1. The transfer function of the dynamometer field. Let,

\[
\begin{align*}
T_f & = \text{field time constant (sec.)} \\
R_f & = \text{field resistance (ohms)} \\
X_3 & = \text{field voltage (volts)} \\
I_f & = \text{field current (amps)}
\end{align*}
\]

then the transfer function is

\[
\frac{I_f(s)}{X_3(s)} = \frac{1}{1 + \frac{R_f}{T_f s}} \quad \cdots \quad 2.6.1
\]

and

\[
I_f + T_f I_f = \frac{X_3}{R_f} \quad \cdots \quad 2.6.2
\]

2. A multiplier, where its output \((\chi_i)\) is proportional to field current squared. From the definition of variables for equations 2.4.5 and 2.4.6

\[
\chi_i = k I_f^2 \quad \cdots \quad 2.6.3
\]

3. The engine-dynamometer system block diagram defined by equation 2.4.7 and 2.4.8 i.e.
\[ y_2 = y_1 (\chi_1, J_1 - \alpha J_2) + J_2 \chi_2 \quad \ldots \ldots \quad 2.6.4 \]
\[ \dot{y}_1 + y_1 (\chi_1 + \alpha) = \chi_2 \quad \ldots \ldots \quad 2.6.5 \]

There are two independent inputs to the controlled object, field voltage \((\chi_3)\) and throttle opening \((\chi_2)\), and two dependent outputs, transmitted torque \((y_2)\) and engine speed \((y_1)\). In order to investigate the effect of the time lag on the responses of torque and speed, equations \((2.6.2 - 2.6.5)\) must be solved simultaneously. Because of the non-linearity of the equations, the response even to step variations in field voltage at constant throttle openings could not be determined analytically and was investigated on an analogue computer, chapter 5.

An attempt was made to develop approximate linear analysis for small perturbations, but the results were unsatisfactory.

2.7 State Variables of Controlled Object

The state variables of the controlled object are derived as follows:

Equation 2.6.2 can be rearranged as

\[ I_f' = \frac{\chi_3}{R_f T_f} - \frac{I_f}{T_f} \quad \ldots \ldots \quad 2.7.1 \]

Substituting equation 2.6.3 in equation 2.6.5 gives

\[ \dot{y}_1 = -y_1 (k I_f^2 + \alpha) + \chi_2 \quad \ldots \ldots \quad 2.7.2 \]

Equations \((2.7.1, 2.7.2)\) show that \((I_f)\) and \((y_1)\)
form a pair of state variables because \((\dot{I}_f)\) and 
\((\dot{y}_i)\) are functions of their instantaneous values
and of the inputs to the system only.

It is of interest, as discussed in chapter 5, to show that 
\((y_z)\) and \((y_i)\) form another pair of
state variables. Substituting equations 2.6.3
and \(J_1 = J_2\) in equation 2.6.4 gives,

\[ y_z = y_i (k I_f^2 - \alpha) J_1 + J_1 \chi_z \]  \[ \cdots \cdots 2.7.3 \]

Differentiating gives

\[ \dot{y}_z = \dot{y}_i (k I_f^2 - \alpha) J_1 + 2k J_1 y_i I_f \dot{I}_f + J_1 \dot{\chi}_z \]

substituting the values of \((\dot{I}_f)\) from equation

\[ \dot{y}_z = (\frac{y_z + y_i a J_1 - J_1 \chi_z}{y_i}) \dot{y}_i - a J_1 \dot{y}_i + J_1 \dot{\chi}_z - \]

\[ \frac{2}{I_f} \left( y_z + y_i a J_1 - J_1 \chi_z \right) + \frac{2k J_1 y_i \chi_3}{R_f I_f} \left( y_z + y_i a J_1 - J_1 \chi_z \right) \]

Hence

\[ \dot{y}_z = f(y_z, y_i, \chi_z, \chi_3) \quad \text{and} \quad (\dot{y}_i) \quad (\dot{y}_i) \]

form a pair of state variables.

2.8 Conclusion

Theoretical considerations show that the
response of transmitted torque, in the engine-
dynamometer system, is always faster than that of
engine speed for step variations in field current
at constant throttle openings.

State variables of the second order non-
linear controlled object are defined. The
system is to be investigated on an analogue
computer.
3.1 Introduction

Under steady-state conditions, zero acceleration, the transmitted torque is equal to both the engine and dynamometer torques. The proposed programmed transmitted torque control system is intended to make it possible to test engines under dynamic conditions, when there is non-zero acceleration. Because steady state data was used in determining the engine and dynamometer torque equations, only an estimate of the dynamic region of operation can be realised under non-steady state conditions.

The dynamic equations of the engine-dynamometer system, 2.4.5 and 2.4.6, together with the limitations on the physical variables are used to estimate the range of transmitted torques under dynamic conditions. The dynamic region is then used to give an estimate of the range of dynamic loads that might be simulated by the programmed transmitted torque control system.

3.2 The Steady State Region

The steady state region of operation, Fig. 2.2.2, is limited by the minimum and maximum
allowed values of engine speed, by throttle opening and by dynamometer field current.

The maximum engine speed is specified by the dynamometer designers. The minimum engine speed, when the engine stalls, occurs at certain values of dynamometer field currents and throttle openings. The maximum engine torque is obtained at maximum throttle opening and is limited by the engine design. The minimum values of throttle openings depend upon the values of the electrical load just before the engine stalls. The above limitations are a consequence of the engine-dynamometer design and hence it is not possible to extend their range on the existing equipment. The maximum value of the field current is limited by the dynamometer design. The minimum value of the dynamometer field current can be zero.

An approximation to the steady state region of operation is shown in Fig. 3.2.1, where the approximate dynamometer torque speed characteristic curves are defined by equation 2.2.2, \( T_d = 0.24 I_f^2 \), and the approximate engine torque speed characteristic curves are defined by equation 2.3.2, \( T_m = -0.5 J_1 + Z \). The limitations on the steady state region are represented by contour ABCDE in Fig. 3.2.2. The estimated minimum value
of engine speed, just before the engine stalls, is line AB. Line BC is the maximum field current
dynamometer torque speed characteristic curve and line CD is the maximum throttle opening engine
torque speed characteristic curve. The maximum engine speed is line DE and line AB represents
the no load dynamometer torque speed characteristic
curve at zero field current.

3.3 The Dynamic Region

In order to investigate the dynamic region
of operation, it is necessary to assume that the
engine and dynamometer torques, under accelerating
conditions, depend on engine speed, throttle
opening and field current in the same way as they
do under steady state conditions. This assumption
is made only as an approximation in the absence of
any other means of estimating the dynamic region.

The dynamic performance of the system is
constrained by the equation of motion, 2.4.6,
\[ \dot{y}_1 = -\frac{h+kI^2_f}{J_1+J_2} y_1 + \frac{Z}{J_1+J_2} \quad \ldots \quad 3.3.1 \]
by the speed limitations \(60 \leq y_1 \leq 160\) (rad. sec.\(^{-1}\))
by the dynamometer field
current rating \(0 \leq I_f \leq 2.2\) (amps)
and by the throttle opening \(30 \leq Z \leq 200\) (ft. lb.)

The dynamic region is plotted in a three
dimensional space with the transmitted torque \(y_2\),
given by equation 2.4.5, engine speed \( y_i \) and engine acceleration \( \dot{y}_i \) as the three axes. Planes of constant field currents and throttle openings can be defined within the region. Hence the parameters of the dynamic equations, 2.4.5 and 2.4.6, namely transmitted torque, engine speed, engine acceleration, field current, and throttle opening can be defined at any point in the space.

At steady state conditions, \( \dot{y}_i = 0 \), the relation between transmitted torque and engine speed is the steady state region of operation Fig. 3.2.2. This region, within which lines of constant dynamometer field currents and throttle openings are defined, represents a plane ABCDE in the \( y_z, y_i, \dot{y}_i \) space, at \( \dot{y}_i = 0 \), as shown in Fig. 3.3.1.

The three dimensional region of operation is determined by considering the relation between transmitted torque \( y_z \) and acceleration \( \dot{y}_i \), using equations 2.4.5 and 2.4.6, for constant engine speed \( y_i \).

Substituting the values of \( h = 0.5 \), \( k = 0.24 \) and \( J_1 = J_2 = 0.4 \) in equations 2.4.5 and 2.4.6 gives

\[
y_z = (0.12I_f^2 - 0.25)y_i + 0.5Z \quad \cdots \cdots \cdots \cdots \cdots \cdots 3.3.2
\]

\[
\dot{y}_i = 1.25Z - (0.3I_f^2 + 0.625)y_i \quad \cdots \cdots \cdots \cdots \cdots \cdots 3.3.3
\]
Eliminating \((Z)\) gives
\[
y_2 = 0.4 \dot{y}_1 + 0.24 I_f^2 y_1 \quad \ldots \quad 3.3.4
\]
Equation 3.3.4 shows that for constant speed \((y_1)\) and constant field current \((I_f)\), the relation between \((y_2)\) and \((\dot{y}_1)\) for different values of throttle openings \((Z)\) is a straight line.

Eliminating \((I_f^2)\) gives
\[
y_2 = -0.4 \ddot{y}_1 - 0.5 y_1 + Z \quad \ldots \quad 3.3.5
\]
Equation 3.3.5 shows that for constant speed and constant throttle opening, the relation between \((y_2)\) and \((\dot{y}_1)\) for different values of field current \((I_f)\) is a straight line.

To determine the region of operation at the lowest speed, 60 rad. sec\(^{-1}\), equations 3.3.4 and 3.3.5 are used simultaneously.

For minimum field current \(I_f=0\), \(y_2=0.4 \dot{y}_1\) \ldots 3.3.4a
For maximum field current \(I_f=22\), \(y_2=0.4 \dot{y}_1+70\) \ldots 3.3.4b
For minimum throttle opening \(Z=30\), \(y_2=-0.4 \ddot{y}_1\) \ldots 3.3.5a
For maximum throttle opening \(Z=200\), \(y_2=-0.4 \ddot{y}_1+170\) \ldots 3.3.5b

The limit of the region of operation, determined by equations 3.3.4 and 3.3.5 at 60 rad. sec\(^{-1}\), is shown in Fig. 3.3.1 as region AGKBH.

Consider now the highest permitted speed, 160 rad. sec\(^{-1}\).

For \(I_f=0\), \(y_2=0.4 \dot{y}_1\)
For \(I_f=22\), \(y_2=0.4 \dot{y}_1+186\)
For $Z = 30$, \( y_2 = -0.4 \dot{y}_1 - 50 \)
For $Z = 200$, \( y_2 = -0.4 \dot{y}_1 + 120 \)

The limit of the region of operation, determined by equations 3.3.4 and 3.3.5 at 160 rad sec\(^{-1}\), is shown in Fig. 3.3.1 as region NEMDPFN.

The procedure can be repeated for intermediate speeds and the estimated three dimensional dynamic region of operation is drawn in Fig. 3.3.2. Planes NMGGA and PJKV represent constant values of dynamometer field currents namely zero and maximum values respectively. Planes MGKV and NAJP represent constant values of throttle openings, maximum and minimum values respectively. The region of negative torque AFNEA is neglected because only real positive loads are to be simulated. The projections of the dynamic region on the $y_2$, $\dot{y}_1$ plane, $y_1$, $\dot{y}_1$ plane and $y_1$, $\dot{y}_1$ plane is shown in Figures 3.3.3a, b and c respectively.

The approximate range of transmitted torque is

\[ 0 \leq y_2 \leq 150 \text{ ft. lb.} \]

and the estimated maximum values of engine acceleration and deceleration are approximately 200 rad. sec\(^{-2}\) and -300 rad. sec\(^{-2}\) respectively.

3.4 Limitations on Loads that can be Simulated

The different types of controlled loads that are to be simulated under dynamic conditions are,
constant loads, \( y_2 = u \)
viscous loads, \( y_2 = v \cdot y_1 \)
fan loads, \( y_2 = \omega \cdot y_1^2 \)

These various loads represent surfaces in the \( y_z, y_1, \dot{y}_1 \) space and their intersection with the edge of the dynamic region give the boundary of the possible region of operation within which simulation will be possible. The position of the surfaces in the \( y_z, y_1, \dot{y}_1 \) space depend upon the values of \((u), (v), \) and \((\omega)\). The range of \((u), (v), \) and \((\omega)\) is limited by the dynamic region of operation.

The simulated load \( y_2 = u \) represents a plane in the \( y_z, y_1, \dot{y}_1 \) space parallel to the \( y_1, \dot{y}_1 \) plane. The region of operation in the \( y_1, \dot{y}_1 \) plane for different values of \((u)\) was derived graphically and is shown in Fig. 3.4.1a. The simulated load \( y_2 = v \cdot y_1 \) represents a plane in the \( y_z, y_1, \dot{y}_1 \) space which intersects the \( y_1, \dot{y}_1 \) plane in the \( \dot{y}_1 \) axis. The region of operation in the \( y_1, \dot{y}_1 \) plane for different values of \((v)\) was derived graphically and is shown in Fig. 3.4.1b. Similarly, the simulated load \( y_2 = \omega \cdot y_1^2 \) represents a parabolic surface in the \( y_z, y_1, \dot{y}_1 \) space which intersects the \( y_1, \dot{y}_1 \) plane in the \( \dot{y}_1 \) axis. The region of operation in the \( y_1, \dot{y}_1 \) plane for different values of \((\omega)\) was derived graphically and is shown
in Fig. 3.4.1c.

The maximum range of \((u), (v)\) and \((w)\) is

- \(0 \leq u \leq 150\) ft. lb.
- \(0 \leq v \leq 2.0\) ft. lb. sec.
- \(0 \leq w \leq 0.03\) ft. lb. sec^2

Fig. 3.4.2 represents the projection of the following torque surfaces on the steady state region of operation

1) \(y_z = 0; y_z = 100\) ft. lb.
2) \(y_z = 0.5y_1; y_z = 0.5y_1; y_z = 1.2y_1\) ft. lb.
3) \(y_z = 0.5y_1; y_z = 0.002y_1^2; y_z = 0.018y_1^2\) ft. lb.

This figure reveals the relation between transmitted torque and engine speed that it is desired to realise when the different types of loads are simulated under dynamic conditions.

3.5 Conclusion

The dynamic region of operation gives an estimate of the range of dynamic loads that can be simulated by the programmed transmitted torque control system.
CHAPTER 4

PRACTICAL CONSIDERATIONS AND DESCRIPTION OF TORQUE TRANSDUCER, PROGRAMMER AND CONTROLLER

4.1 Introduction

The performance of the proposed programmed transmitted torque control system will be limited by two factors, dynamic lags and noise. The principal contributors to these factors in the controlled object, which are of interest for further work, are discussed in this chapter. The principal dynamic lag is due to the dynamometer field time constant, and the irregular nature of the engine torque is the principal source of noise.

Additional contributions to dynamic lags and noise may be introduced by the components that are to be used in the proposed control system, Fig. 1.1.1, namely torque meter, programmer and controller. These components are discussed and their contribution to the above factors considered.

4.2 Field Time Constant

The field transfer function, being a part of the controlled object, was introduced in section 2.6. The value of the field time constant \( T_f \) was determined experimentally, as
discussed in Appendix II, and found to be approximately equal to one second.

4.3 **Engine Noise Frequency Range**

The engine is expected to be the main source of noise in the controlled object. The engine is a six cylinder four-stroke and hence there are three firing strokes in each revolution. The engine speed ranges from 60 to 160 rad. sec\(^{-1}\) or approximately from 10 to 25 revolutions per second, thus the expected noise frequency range in transmitted torque is

\[30 \leq f_n \leq 75\] cycles per second

4.4 **Transmitted Torque Transducer**

Transmitted torque can be measured by different means such as electrical strain gauges, accelerometer-dynamometer method and ASEA ring torductors.

Electrical strain gauges were not considered suitable because they usually require slip rings which introduce electrical noise and often cause unsatisfactory operation. Strain gauge torque meters that require no slip rings have recently been developed\(^{19, 24}\) but they are not available yet.

The accelerometer-dynamometer method for measuring the transmitted torque is based on
equation 2.4.2 i.e.

\[ y_2 = T_d + j_2 j_1 \]

A signal proportional to transmitted torque \((y_z)\) can be obtained by adding two signals one representing the dynamometer torque \((T_d)\) and the other represents the engine acceleration \((j_1)\).

The dynamometer torque can be measured easily by means of strain cells, F.M. capacitor gauges and F.M inductor gauges. Angular acceleration may be measured by the use of a drag-cup induction generator or a tachometer generator and a differentiator. These methods of measuring acceleration are not suitable because they are not sensitive enough and hence noise is a major problem under low acceleration.

A suitable method for measuring transmitted torque is by means of an ASEA ring torductor\(^{18}\). Such a torque unit enables accurate measurement of transmitted torque under steady state and dynamic conditions and require no slip rings. The dynamic response of the torductor is limited to 350 c.p.s. Because of the fast response, an additional filter should be used in conjunction with the torductor to filter out the expected engine noise in transmitted torque, section 4.3.

A specially designed shaft is supplied with the torductor unit. Shaft oscillations may be
another source of noise and they depend upon the design of the shaft. The range of the torductor unit that is to be used is (0-150 ft. lb.).

4.5 **Programmer Unit**

The programmer is a device used as a means to simulate different types of real loads that the engine may encounter while accelerating or decelerating. In practice, the unit consists of three potentiometers and a multiplier.

One of the potentiometers is to be supplied with a constant d.c. voltage from a battery to simulate constant loads. The second potentiometer is to be supplied with a signal proportional to engine speed from a tachometer generator so as to simulate viscous loads. Two signals proportional to engine speed are to be multiplied in a multiplier and its output is to be applied across the third potentiometer to simulate fan loads. A d.c. amplifier may be used as a summer so that an output signal proportional to any combination of the above loads may be obtained.

The programmer is represented by a computer circuit diagram, Fig. 4.5.1, because the programmed transmitted torque control system is to be investigated on an analogue computer, chapter 6.
4.6 **Controller**

The dynamometer field current might be controlled by means of linear controller, electronic or magnetic amplifiers, or non-linear bang-bang controller. In each case, the fastest field current response should be obtained and this is limited by the field time constant and the maximum permissible field voltage.

Because of the high power levels in the dynamometer field and large field inductance, electronic power amplifiers were not considered suitable. Magnetic amplifiers present many disadvantages which prevent them from being a good control element\(^{(8)}\).

A bang-bang controller using silicon controlled rectifiers is considered to be suitable for the following reasons:

1. It is known that bang-bang control gives the fastest step response i.e. it is a time-optimal system\(^{(6,9,13)}\). Because the field current response is limited by the field time constant and the maximum field voltage, the bang-bang controller will give the fastest possible field current response when maximum and zero voltages are switched across the field.
2. Silicon controlled rectifiers can have high current ratings to operate within the dynamometer range. They have very small turn-on and turn-off times in the order of $\mu$ seconds.

3. The SCR'S are cheap and hence this method of control is economical, compared to electronic amplifiers.

The preliminary design of a suitable controller is due to Lumsden (15) and is reproduced in Fig. 4.6.1. The controller is a full wave rectifier bridge in which the silicon controlled rectifiers are the controlled elements. The error signal of the programmed transmitted torque control system actuates a triggering circuit which acts as a switch for the silicon rectifiers. The SCR'S are switched on (or off) depending upon the sign of the error signal. If the error signal is positive, torque is less than required, the SCR'S are switched on and the bridge conducts to give a full wave rectified output voltage. The average value of the output voltage is equal to the maximum dynamometer field voltage. For negative error signals, torque is more than required, the SCR'S are switched off and hence they cause the bridge to
become in the non-conducting state with an output voltage of zero. Because a small specific value of positive or negative error signal voltage, depending upon the design, is required to operate the SCR's triggering circuit, the input output relationship of the controller may be approximated, for simplicity, to that of a relay with small hysteresis.

A bang-bang controller usually introduces limit cycle oscillations in controlled systems and hysteresis affects the amplitude of the limit cycle. The magnitude of the hysteresis should be as small as possible, in the triggering circuit, to produce minimum limit cycle amplitudes.

4.7 Conclusion

An additional filter is necessary to filter out the expected noise, in transmitted torque, produced by the engine. The dynamometer field current is to be controlled by a bang-bang controller where the maximum and zero field voltages are switched across the field. This type of control gives the fastest possible field current response and is limited by the field time constant and the maximum permitted field voltage. The bang-bang controller introduces limit cycle oscillations and they may be considered as an unavoidable noise in the programmed transmitted torque control system.
CHAPTER 5

SIMULATION AND DESIGN OF TRANSMITTED TORQUE CONTROL SYSTEM

5.1 Introduction

In this chapter, analogue computation was used to investigate two major factors namely, the feasibility of bang-bang control and the speed of response of transmitted torque with respect to engine speed for step input variations in the transmitted torque control system.

The controlled object was simulated on an analogue computer to investigate the transmitted torque and engine speed responses for step variations in the dynamometer field voltage as this was not possible analytically, section 2.6. Because the system is to be controlled by a controller which was approximated by bang-bang in section 4.6, trajectories for maximum and zero field voltages are drawn in the transmitted torque ($y_2$), engine speed ($y_1$) phase plane. The trajectories are necessary to investigate the feasibility of bang-bang control when the trajectories of the loads that are to be simulated, $y_2 = f(y_1)$, are considered as switching curves in the $y_2, y_1$ phase plane.
The transmitted torque filter, in the negative feedback loop, and the hysteresis of the controller are discussed. The transmitted torque control system was then simulated on the computer. Limit cycle oscillations are obtained and discussed. Finally, the transient response curves for transmitted torque and engine speed are obtained and compared with each other.

The investigation in this chapter is done under constant values of throttle openings.

5.2 Response of Controlled Object

The controlled object was discussed in section 2.6. The computer circuit diagram of the controlled object is shown in Fig. 5.2.1 and parts 1, 2 and 3 correspond to those in Fig. 2.6.1. The transient responses of transmitted torque and engine speed, for step variations in field voltage at different values of steady state values operating conditions, Table I d,f, were recorded from the computer and they are shown in Fig. 5.2.2 d and f. For any other steady state operating conditions, the transmitted torque and engine speed response curves have the same shape as those in Fig. 5.2.2 d and f. The rise time (defined as the time required for the transient response to reach 0.9 of its final value) of transmitted torque
and engine speed were estimated from the curves and it was found that the speed of response of torque is approximately twice as fast as that of engine speed for both Figures, 5.2.2 d and f.

Because the field transfer function was not included in the engine-dynamometer system, a sudden jump appeared in the response of the transmitted torque, Fig. 2.5.2. The response of the transmitted torque, in the controlled object, has no sudden jumps due to the time delay caused by the field time constant.

5.3 Switching Curves and Phase Plane Trajectories

The loads that are to be simulated by the programmed transmitted torque control system represent surfaces in the transmitted torque \((y_2)\), engine speed \((y_1)\), engine acceleration \((\dot{y}_1)\) space, as was discussed in section 3.4. The projection of the surfaces on the \(y_2, y_1\) plane, shown in Fig. 3.4.2, are curves representing the required load, \(y_2 = f(y_1)\), that it is desired to realise under dynamic conditions. Because \((y_2)\) and \((y_1)\) form a pair of state variables, section 2.7, the required \(y_2 = f(y_1)\) load can be used as a switching curve, for the programmed transmitted torque control system, in the \(y_2, y_1\) phase plane. Hence, to investigate the feasibility of bang-
Bang control, trajectories for maximum and zero field voltages are considered in the $y_2, y_1$ phase plane.

Using the controlled object computer circuit diagram in Fig. 5.2.1, trajectories (a) Fig. 5.3.1 were drawn in the $y_2, y_1$ phase plane when the maximum field voltage was switched on from different steady values of $(y_2)$ and $(y_1)$. Similarly when the zero field voltage was switched on from different steady state values of $(y_2)$ and $(y_1)$, trajectories (b) are shown in Fig. 5.3.1. Trajectories (a) and (b) were obtained under a constant value of throttle opening $Z = 100$ ft. lb. For any other constant value of throttle opening, trajectories for maximum and zero field voltages have the same shape as those in Fig. 5.3.1. The shape of the trajectories can be explained as follows:

The response of transmitted torque is faster than that of engine speed for step variations in field voltage, section 5.2. Thus, when the maximum or zero field voltage is switched on, the rate of change in transmitted torque would be greater than the rate of change in engine speed in the $y_2, y_1$ phase plane which explains the shape of the trajectories.

The different types of loads that are to be
simulated, \( y_2 = f(y_1) \), are
\[
\begin{align*}
y_2 &= u \\
y_2 &= v y_1 \\
y_2 &= \omega y_1^2
\end{align*}
\]
where the range of \((u)\), \((v)\) and \((\omega)\)/defined in section 3.4. The trajectories of these loads in the \(y_2, y_1\) phase plane can be considered as switching curves when superimposed on the trajectories for maximum and zero field voltage in Fig. 5.3.1. Under such configuration, it is possible to show that the path of trajectories \((a)\) and \((b)\) converge towards the switching curve \(y_2 = f(y_1)\) for all the range of \((u)\), \((v)\) and \((\omega)\). When trajectories \((a)\) intersect the \(y_2 = f(y_1)\) switching curve, the controller switches to trajectories \((b)\) and they converge towards the \(y_2 = f(y_1)\) switching curve. Hence bang-bang control is feasible for the different types of loads that are of interest.

5.4 Controller Hysteresis and Filter Time Constant

A filter is to be used in conjunction with the ring torductor to filter out the expected engine noise in transmitted torque as was discussed in section 4.4. The block diagram of the transmitted torque control system is shown in Fig. 5.4.1 where \((Y_2)\) is the input to the system and \((y_2)\) is
the output of the system. The computer circuit diagram of the system is similar to that in Fig. 5.2.1 with two additional major components, a relay in the forward loop and an R-C filter in the negative feedback loop.

The bang-bang controller introduces limit cycle oscillations in the system and they depend upon the hysteresis of the relay, the time constant of the filter and operating conditions.

Computer investigation indicated that the amplitude of limit cycle oscillations increases with increasing values of relay hysteresis at a constant value of filter time constant and steady state operating conditions. For two magnitudes of hysteresis, ± 2 volts and ± 4 volts, the transmitted torque limit cycle oscillations were recorded from the computer and they are shown in Fig. 5.4.2. The amplitude of the limit cycle oscillations increases by a factor of approximately 1.7 for an increase, by a factor of 2, in the magnitude of the hysteresis. Because it is desired to have minimum limit cycle amplitudes in the transmitted torque control system, the hysteresis in the controller, discussed in section 4.6, should be as small as possible.

An increase in the filter time constant causes an increase in the amplitude of the limit
cycle oscillations at a constant value of relay hysteresis and steady state operating conditions. For two values of filter time constant, 0.05 and 0.1 seconds, the transmitted torque limit cycle oscillations were recorded from the computer and they are shown in Fig. 5.4.3. Thus, an increase in the filter time constant by a factor of 2 will cause an increase in the amplitude of the limit cycle oscillations by a factor of approximately 1.4. The filter should filter out the expected noise and introduce minimum limit cycle oscillations. In order to design the filter, the following items must be considered:

1. The hysteresis of the controller.
2. The percentage of error introduced by noise while measuring the transmitted torque by means of the ring torductor.

These two items can only be determined experimentally. The design of the filter must make a compromise between item (2) and the amplitude of the limit cycle oscillation and hence requires experimental investigation. Because of the cost of the controller and the torductor unit, no attempt was made to evaluate experimentally the above two items. The values of the controller hysteresis and filter time constant were assumed to be
controller hysteresis \( \pm 0.01 \) volts

Filter time constant \((\tau_0)\) 0.01 second

The value of the filter time constant was assumed to be 0.01 seconds because for such a value and at the minimum expected noise frequency, 30 c.p.s., section 4.3, the gain of the R-C filter is approximately \(-6\) dB.

5.5 Investigation of Transmitted Torque Control System

Limit cycle oscillations and transient responses of engine speed and transmitted torque are investigated. The computer circuit diagram was discussed in section 5.4 where the time constant of the filter is 0.01 and the simulated relay has a hysteresis of \( \pm 0.01 \) volts.

5.5.1 Limit Cycles

The amplitude and frequency of the limit cycle oscillations depend upon steady state conditions. Transmitted torque limit cycles were recorded, at different steady state operating conditions Table I, and they are in Fig. 5.5.1 a, b, c, d, e and f.

The limit cycles have different shapes, symmetrical and unsymmetrical variations, depending upon the steady state operating point. If the operating point is in the middle of the steady
state region of operation, the transmitted torque has a value near the middle of its range, symmetrical limit cycles are obtained as shown in Fig. 5.5.1 (d, f). This happens because at such an operating point the dynamometer field current has a value near the middle of its range and as the maximum and zero voltages are switched on, symmetrical variations occur in the field current and consequently in transmitted torque. On the other hand if the transmitted torque has values near the edge of its range, unsymmetrical limit cycles occur as shown in Fig. 5.5.1 (a, b, c, e).

The minimum and maximum limit cycles amplitudes introduce an error in transmitted torque of approximately ± 0.1% and ± 0.7% respectively. The minimum and maximum limit cycle frequencies are approximately 8 c.p.s. and 16 c.p.s. respectively. These figures depend upon the assumed values for the filter time constant and controller hysteresis.

5.5.2 Transient Responses

The responses of transmitted torque and engine speed to step input variations, in the transmitted torque control system, are obtained and compared. The comparison is necessary because the proposed programmed transmitted torque
control system requires much faster transmitted torque responses than that of engine speed.

At different steady state operating conditions, Table I, two different magnitudes of input steps were applied and the respective transmitted torque and engine speed transient responses are shown in Fig. 5.5.2 a, b, c, d, e and f. Because Figures 5.5.2 (b, c) were obtained when the initial value of the dynamometer field current was \( I_f = 2.2 \) Amps, the maximum rated value, the transient response for positive steps were not considered. Figure 5.5.2 (a, e) were obtained when the initial value of \( (I_f) \) was \( 0.7 \) Amps and the transient response for negative steps were not considered.

Limit cycle oscillations are not visible in the engine speed response curves due to the slow speed of response of the engine speed. Thus, the effect of the limit cycles on the engine speed is negligible compared to their effect on the transmitted torque.

The rise time of the transmitted torque is almost the same for all the transient response curves and has an estimated value of approximately 0.1 seconds. Also the rise time of engine speed, for all the transient response curves, is almost the same and has an estimated value of
approximately 2 seconds. The response of transmitted torque for step input command is then 20 times faster than that of engine speed in the computer simulation. Hence feedback control improved the results obtained in section 5.2 by a factor of 10 and bang-bang control is suitable for the system.

5.6 Conclusion

Bang-Bang control is feasible when the load that is to be simulated, \( y_2 = f(y_1) \), is considered as the switching curve. The torque response of the transmitted torque control system is 20 times faster than that of engine speed and hence bang-bang control is suitable.

Limit cycle oscillations occur in the transmitted torque control system and they depend upon the hysteresis of the controller, the time constant of the filter and operating conditions. Due to the cost of the controller and the torductor unit, no attempt was made to obtain precise experimental values of the controller hysteresis and the filter time constant.
6.1 Introduction

In this chapter, the programmed transmitted torque control system was simulated and investigated on an analogue computer. The system is intended to simulate a variety of real loads that the engine might encounter under dynamic conditions, non-zero acceleration. In order to accelerate the engine so as to simulate the dynamic loads, the throttle opening should be varied.

The effect of varying the throttle on the performance of the programmed transmitted torque control system is investigated. Because there are no restrictions on how the throttle should be varied, step and ramp variations were considered. Trajectories of the programmed transmitted torque control system, being the switching curves, were simulated in the transmitted torque, engine speed phase plane. These are compared with the trajectories for the load that is to be simulated.

6.2 Performance of Programmed System

The block diagram of the programmed transmitted torque control system is shown in Fig. 6.2.1
The input to the system \( (Y_2) \) is the type of load that is to be simulated under dynamic conditions, constant load, \( Y_2 = u \) ft. lb. viscous load, \( Y_2 = v \cdot Y_1 \) ft. lb. fan load, \( Y_2 = \omega \cdot Y_1^2 \) ft. lb. and \( (Y_2) \) is the output of the system.

The performance of the programmed system should satisfy the following required condition under variations in the throttle opening, the output of the system \( (Y_2) \) should follow the required input \( (Y_2) \) within a small percentage of error, say \( \pm 2\% \).

The performance of the system is investigated in the following subsections when step and ramp variations were applied to the throttle so as to simulate constant, viscous and fan loads in the \( Y_2, Y_1 \) phase plane.

6.2.1 Performance for Step Variations in Throttle

For each of the following programmed loads

a) \( Y_2 = u = 60 \) ft. lb.
b) \( Y_2 = 0.5 \cdot Y_1 \) ft. lb.
c) \( Y_2 = 0.0034 \cdot Y_1^2 \) ft. lb.

three different steps (10, 20 and 40 ft. lb.) were applied to the throttle and the corresponding variation between the controlled transmitted torque \( (Y_2) \) and the engine speed \( (Y_1) \) is shown
in Figures 6.2.2 a, b, and c. The controlled torque \( y_3 \) does not follow the required simulated load \( (\gamma_3) \) for a certain range of the engine speed, depending upon the size of the step. These regions can be explained as follows:

Equations 2.7.1 and 2.7.3 are

\[
I_f = \frac{X_3}{R_f I_f} - \frac{I_f}{T_f} \quad \ldots \ldots \quad 6.2.1
\]

and

\[
y_2 = y_1 (k I_f^2 - \alpha) \frac{J_t}{J} + J \times 3 \quad \ldots \ldots \quad 6.2.2
\]

Where \( (X_3) \) is the field voltage, \( (I_f) \) is the field current and \( (X_2) \) is directly proportional to the throttle. The throttle opening, in equation 6.2.2, is directly proportional to the controlled transmitted torque \( (y_2) \). Thus, when a step is applied to \( (X_2) \), \( (y_2) \) would experience at that instant the same variation and this is shown in Figures 6.2.2. a, b and c. Because of the field time constant, a time delay will occur before \( (y_2) \) is controlled to reduce the error \( (\gamma_2 - y_2) \) to zero during which the engine speed would have been changed. When the field time constant was reduced by a factor of 2, two different steps (20 and 40 ft. lb.) were applied to the throttle under constant load simulation, \( Y_2 = u = 60 \) ft. lb. The variation between \( (y_2) \) and \( (y_1) \) under these conditions is shown in Fig. 6.2.3 a. A comparison between this figure and Fig. 6.2.2.a indicates that the region in which the controlled transmitted
torque does not follow the required load ($Y_2$) has been reduced by a factor of approximately 2.

Figures 6.2.2 a, b and c show that the transient response of the programmed transmitted torque control system in the $Y_2, Y_1$ phase plane is almost independent of the type of load that was simulated. The percentage of error between the required load that is to be simulated ($Y_2$) and the controlled transmitted torque ($Y_2$) depends upon the magnitude of the step that was applied to the throttle. Fig. 6.2.4a represents the variation between the estimated percentage of error and the magnitude of the step variations in throttle. This figure indicates that the programmed transmitted torque control system would perform as desired under small step variations in throttle with a maximum value of 5 ft. lb. If the field time constant was half its present value, the system would perform as desired for a step of 10 ft. lb., this value was estimated from Fig. 6.2.3 a. It is worth mentioning that the performance of the system would slightly improve under an increase in the rated field voltage. This is shown in Fig. 6.2.3 b which was obtained when the rated field voltage was increased by a factor of 4 under constant load simulation, $Y_2 = u = 60$ ft. lb.
6.2.2 Performance for Ramp Variations in Throttle

For each of the following programmed loads

a) \[ Y_2 = u = 60 \text{ ft. lb.} \]

b) \[ Y_2 = 0.5 Y_1 \text{ ft. lb.} \]

c) \[ Y_2 = 0.0034 Y_1^2 \text{ ft. lb.} \]

A ramp with different slopes (12, 60, 100 and 200 ft. lb. sec\(^{-1}\)) was applied to the throttle opening, where the slope \((s)\) of the ramp is the rate of change in throttle \((Z)\) with respect to time.

The variation between the controlled transmitted torque \((y_2)\) and the engine speed \((y_1)\) for each of the above programmed loads is shown in Fig. 6.2.5a, b and c. The performance of the programmed system under ramp variations in throttle is almost independent of the type of load that was simulated.

The percentage of error between \((y_2)\) and \((Y_2)\) increases with increasing values of \((s)\).

Fig. 6.2.4b represents the variation between the estimated percentage of error and the rate of change in throttle. Thus, the programmed transmitted torque control system would perform as desired under a maximum rate of change in throttle of approximately 60 ft. lb. sec\(^{-1}\). For such a rate of change, the range of the throttle, being \(30 \leq Z \leq 200\) ft. lb., can be covered in three seconds while simulating constant viscous and fan loads.
The programmed system would perform as desired under a higher rate of change in throttle if the value of the field time constant was less than one second. For example, when the field time constant was reduced by a factor of 2, the same procedure was repeated and it was found that the system performs as desired under a maximum rate of change in throttle of approximately 100 ft. lb. sec\(^{-1}\).

The programmed transmitted torque control system simulates the desired dynamic loads under ramp variations in throttle and the performance of the system under step variations is unsatisfactory.

6.3 **Simulation of Dynamic Loads**

In this section, constant, viscous and fan loads are simulated in the \(y_z, y_1\) phase plane for different values of \((u), (\nu)\) and \((\omega)\). The different types of loads are simulated separately when a ramp \((r)\) with a slope of 8 ft. lb. sec\(^{-1}\) was applied to throttle. For such a slow rate of change in the throttle, the trajectories of the programmed transmitted torque control system, being the switching curves, in the \(y_z, y_1\) phase plane may be considered as being simulated under steady state conditions.
6.3.1 Constant Load Simulation

For several values of \( u \), \( \gamma_2 = u = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110 \) and \( 120 \) ft. lb., the ramp \( r \) was applied to the throttle and the corresponding variation between the controlled transmitted torque \( (\gamma_2) \) and engine speed \( (\gamma_3) \) were obtained in the \( y_2, y_3 \) phase plane and they are shown in Fig. 6.3.1. The variation between \( y_2 \) and \( y_3 \), for the different values of \( u \), is a family of parallel straight lines to the \( (y_3) \) axes. Limit cycle oscillations are shown on each of the parallel lines and their amplitudes increase with increasing values of \( u \).

6.3.2 Viscous Load Simulation

The ramp \( r \) was applied to the throttle opening when the following values of \( r \) were considered,

\[
0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0 \text{ and } 1.2 \text{ ft. lb. sec.}
\]

The respective variation between \( y_2 \) and \( y_3 \) is shown in Fig. 6.3.2. The family of the straight lines pass through the origin, \( y_2 = y_3 = 0 \).

6.3.3 Fan Load Simulation

For each of the following values of \( \omega \)

\[
0.000667, 0.00134, 0.002, 0.00268, 0.00334, 0.005, 0.00667, 0.008, 0.00935, 0.0107 \text{ and } 0.012 \text{ ft. lb. sec.}^2.
\]
the corresponding variation between \((y_2)\) and \((y_1)\), when ramp \((r)\) was applied to the throttle, is shown in Fig. 6.3.3. The family of the parabolic curves converge towards the origin.

The maximum discrepancy between the required simulated load \(y_2 = f(y_1)\) and the controlled transmitted torque \((y_2)\) has a maximum value of approximately 0.7%.

6.4 Conclusion

The programmed transmitted torque control system does not work for step changes in throttle. The programmed system simulates the desired dynamic loads for a maximum rate of change in throttle, ramp variations, of 60 ft. lb. sec\(^{-1}\). Under such conditions, the engine experiences at each instant of time the desired simulated load within \(\pm 2\%\) error.

The performance of the programmed system would have been better if the value of the field time constant was less than one second.
CHAPTER 7

CONCLUSIONS

The programmed transmitted torque control system was investigated in different parts. The system is highly non-linear and analogue computation was necessary, analytical solution was only possible for the engine-dynamometer system. The performance of the programmed system is limited by dynamic lags and by noise. The main contributor to dynamic lags is the dynamometer field time constant and the engine is the main contributor to noise. The programmed transmitted torque control system does not work for step variations in throttle. For ramp variations with a maximum rate of change of 60 ft. lb. sec\(^{-1}\), the programmed system simulates the desired loads with a percentage of error being less than \(\pm 2\%\).

A detailed investigation of an approximate model of the programmed transmitted torque control system was given in the text. The main approximations and assumptions that were made are:

1. The engine and dynamometer equations were approximated from steady state data.
2. The controller characteristics were
approximated to that of a relay with small hysteresis.

3. The values of the controller hysteresis and filter time constant were assumed. Because of these factors, the performance of the real system would differ by an estimated value of approximately 5% from the performance of the approximate model in the computer simulation. The main difference would be in the amplitude of the limit cycle oscillations because they depend upon the hysteresis of the controller and the filter time constant.

The designed programmed transmitted torque control system is useful in test cells. The system can be used to simulate constant, viscous and fan loads on test engines and hence provides a means to test engines under accelerating or decelerating conditions caused by ramp variations in the throttle. Ramp variations are in fact suitable for engine testing under dynamic conditions, Knudsen, F. (11)

The designed programmed transmitted torque control system, when built, would enable suitable alterations in engine settings and design to be made.

In order to build the system shown in Fig. 1.1, the following components may be used:
1. Torque Transducer
   (ASEA Ring Torductor)
   Range (0 - 150 ft. lb.)
   Cost = 300 £

2. Tachometer Generator
   (Evershed Permanent Magnet
    Tacho-Generator)
   Type FF1/20
   Cost = 20 £

3. Programmer
   (Solartron standard analogue
    computing equipment) would be suitable
   Estimated cost = 400 £

4. Controller
   A refined design of the Lumsden
   controller\(^{(15)}\) is yet to be made.
   Estimated cost = 100 £.

When these components are assembled, the time
constant of the transmitted torque filter should
be adjusted experimentally to give a compromise
between the amplitude of the limit cycle
oscillations and the percentage of error in
transmitted torque due to the engine noise.

The estimated cost of the programmed
transmitted torque control system is approximately
850 £.
The usefulness of the designed system could be increased by the use of a dynamometer with a small field time constant, less than one second. If the value of the field time constant was half its present value, analogue computation indicates that the system would perform as desired under a maximum estimated rate of change in throttle of approximately 100 ft. lb. sec$^{-1}$. 
ACKNOWLEDGEMENTS

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APPENDIX I

Engine-Dynamometer Specifications -A-

I.1 Moment of Inertia

The moment of inertia of the engine \( (J_1) \) and the moment of inertia of the dynamometer \( (J_2) \) were taken from Roberts\(^{(22)}\) and the designers\(^{(16)}\) to be approximately

\[
J_1 = J_2 = 0.4 \text{ ft. lb. sec}^2.
\]

I.2 Dynamometer Field Specifications

- Rated d-c field voltage \( V_3 = 255 \) volts
- Rated d-c field current \( I_f = 2.2 \) amps

From Appendix II

- Field time constant \( T_f = 1.0 \) second
- Field resistance \( R_f = 116 \) ohms

I.3 Range of the Parameters of Dynamic Equations

The dynamic equations of the engine-dynamometer system are equation 2.4.7 and 2.4.8 where,

a) \( \chi_1 = 0.24 \frac{I_f^2}{J_1 + J_2} = k I_f^2 \) and its range is limited by \( (I_f) \),

\[
0 \leq \chi_1 \leq 1.5 \text{ sec}^{-1}
\]

b) \( \chi_2 = \frac{Z}{J_1 + J_2} \) (proportional to throttle opening) with a range limited by \( Z \)

\[
(30 \leq Z \leq 200),
\]

\[
40 \leq \chi_2 \leq 250 \text{ sec}^{-2}.
\]
c) \( y_z \) (transmitted torque)
\[ 0 \leq y_z \leq 150 \text{ ft. lb. wt.} \]
d) \( y_1 \) (engine speed)
\[ 60 \leq y_1 \leq 160 \text{ rad. sec}^{-1} \]
e) \( \dot{y}_1 \) (engine acceleration)
\[ -300 \leq \dot{y}_1 \leq 200 \text{ rad. sec}^{-2} \]
f) \[ a = \frac{0.5}{J_1 + J_2} \quad \text{(constant)} \]
\[ a = 0.6 \text{ sec}^{-1} \]
APPENDIX II

Measurement of Dynamometer Field Time Constant.

The field winding was demagnetized. At room temperature, the response of the dynamometer field current was recorded for step variations in field voltage, using a square wave generator and a power amplifier, and is shown in Fig. II. 1.

Assuming the field winding can be represented by a lumped parameter circuit, the transfer function of the field, equation 2.6.1, is

\[
\frac{I_F(s)}{X_3(s)} = \frac{1/R_F}{1 + T_F s}
\]

where \( T_F = L_F / R_F \)

The solution for field current after a step change in voltage is,

\[
i_f = (I_f)_{s.s.} \left[ 1 - e^{-t/T_f} \right]
\]

When \( t = T_f \), \( i_f = 0.632 (I_f)_{s.s.} \) and from Fig. II.1 the value of the field time constant was estimated to be 1.2 seconds.

The field resistance at room temperature was measured and is 95 Ohms. Hence \( L_F = T_F R_F = 114 \) Henries.

The interest lies in the value of the field time constant at actual operating conditions. Under such conditions, the field resistance has a greater value than that at room temperature due to heating effects. A steady state value of \( R_F \) was
reached after operating the system for about 30 minutes and is found to be 116 Ohms. Hence at operating conditions

\[ T_f = \frac{114}{116} \approx 1.0 \text{ second.} \]
APPENDIX III

Magnitude Scaling of Controlled Object
Equations For Computer Simulation

The three different parts of the controlled object, discussed in section 2.6, are scaled using the information about the range of the parameters given in Appendix I.

III.1 Magnitude Scaling of Field Transfer Function

The transfer function of the dynamometer field, equation 2.6.1, is

\[ \frac{I_f}{\chi_3} = \frac{1/R_f}{1 + T_f S} \quad \ldots \ldots \text{III.1} \]

The following variables are defined

\[ 2.2 \left( \frac{I_f}{2.2} \right) \text{ and } 255 \left( \frac{\chi_3}{255} \right) \]

Because \( R_f = 116 \) Ohms and \( T_f = 1 \) second, equation III.1 becomes

\[ \left( \frac{I_f}{2.2} \right) = \left( \frac{\chi_3}{255} \right) \quad \ldots \ldots \text{III.1a} \]

III.2 Magnitude Scaling of Multiplier Equation

The input-output relationship of the multiplier is defined by equation 2.6.3, i.e.

\[ \chi_s = k I_f^2 \quad \ldots \ldots \text{III.2} \]

Define the following variable, \( 1.5 \left( \frac{\chi_1}{1.5} \right) \).

Because \( k = \frac{0.24}{J_1 + J_2} \text{ (sec.}^{-1} \text{amp.}^{-2}) \), where \( J_1 = J_2 = 0.4 \text{ ft.} \text{ lb. sec.}^2 \), and \( I_f \) has been scaled
in section III.1 to be $22\left(\frac{I_2}{2.2}\right)$, equation III.2 becomes

$$\left(\frac{X_1}{1.5}\right) = \left(\frac{I_2}{2.2}\right)^2 \quad \ldots \quad \text{III.2.a}$$

III.3 **Magnitude Scaling of Engine-Dynamometer Equations**

The dynamic equations of the engine-dynamometer system, 2.6.4 and 2.6.5 are

$$y_2 = y_1 (\chi_1, J_1 - \alpha J_2) + J_2 \dot{\chi}_2 \quad \ldots \quad \text{III.3}$$

$$\ddot{y}_1 + y_1 (\chi_1 + \alpha) = \chi_2 \quad \ldots \quad \text{III.4}$$

Define the following variables,

$$150\left(\frac{y_2}{150}\right), 150\left(\frac{y_1}{150}\right), 250\left(\frac{\dot{J}_1}{250}\right) \text{ and } 250\left(\frac{\chi_2}{250}\right).$$

Because $\alpha = 0.6$ sec$^{-1}$, $J_1 = J_2 = 0.4$ ft. lb. sec$^2$ and $(\chi_1)$ has been scaled in section III.2 to be $1.5\left(\frac{\chi_1}{1.5}\right)$, equations III.3 and III.4 become respectively,

$$\left(\frac{y_2}{150}\right) = 0.8\left(\frac{\chi_1}{150}\right)\left[0.75\left(\frac{\chi_1}{1.5}\right) - 0.3\right] + 0.67\left(\frac{\chi_2}{250}\right) \quad \ldots \quad \text{III.3.a}$$

and

$$\left(\frac{\dot{J}_1}{250}\right) + \left(\frac{y_1}{150}\right)\left[0.9\left(\frac{\chi_1}{1.5}\right) + 0.3\right] = \left(\frac{\chi_2}{250}\right) \quad \ldots \quad \text{III.4.a}$$

Equations III.1.a, III.2.a, III.3.a and III.4.a are used to simulate the controlled object, in real time, on a 247 Solartron Analogue Computer. The components of the computer have an accuracy of 0.2%.

The controlled object computer circuit diagram is shown in Fig. 5.2.1.
APPENDIX IV

Engine-Dynamometer Specifications - B -

III.1 Engine

Makers: - Austin Motor Car Co., Ltd.,
Birmingham.

Type: - 6 cylinder, 4 stroke, commercial.
Cylinder dimensions: - 3.35" bore x 4" stroke.
Carburettor: - Zenith type 30 VM
Fuel: Commercial petrol.

III.2 Dynamometer

Makers: - The Macfarlane Engineering Co., Ltd.
Glasgow.

Armature ratings: Rev. - 1500 r.p.m.;
Volt - 285; Amp - 63; H.P. - 25.

The engine and dynamometer are shown in
Fig. IV.1.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>INITIAL VALUES</th>
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<tr>
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</tr>
<tr>
<td>amps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>throttle</td>
<td>$Z$</td>
<td>37</td>
</tr>
<tr>
<td>ft. lb.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIG. 111 PROGRAMMED TRANSMITTED TORQUE CONTROL SYSTEM
FIG. 2.2.1a. DYNAMOMETER CIRCUIT DIAGRAM

FIG. 2.2.1b. DYNAMOMETER LAYOUT TO MEASURE TORQUES
dynamometer torque curves

engine torque curves

\[ I_f = 2.2 \]
\[ \theta = 25^\circ \]
\[ \theta = 20^\circ \]
\[ \theta = 15^\circ \]
\[ I_f = 0.95 \]

\[ \theta = \text{throttle opening [degrees]} \]
\[ I_f = \text{dynamometer field current [amps]} \]

min. speed, \( I_f = 0 \) [no load], max. speed

engine speed - rad/sec

torque - ft. lb. w.t.
FIG 2.2.3  DYNAMOMETER FIELD CURRENT V.S. DYNAMOMETER TORQUE

\[ y_i = \text{engine speed [rad.sec]} \]
FIG. 2.4.1. TORQUES RELATIONSHIP

FIG. 2.4.2. BLOCK DIAGRAM OF ENGINE - DYNAMOMETER SYSTEM
FIG. 2.5.1. RESPONSE OF ENGINE SPEED TO STEP IN $x_i(t)$

(a) $J_x, J_z \neq a J_z$

(b) $J_x = a J_z$

(c) $J_x > a J_z$

FIG. 2.5.2. POSSIBLE RESPONSES OF TRANSMITTED TORQUE FOR DIFFERENT STEPS IN $x_i(t)$
FIG. 2.6.1 CONTROLLED OBJECT BLOCK DIAGRAM
FIG. 3.2.2 LIMITATIONS ON THE EXTENDED STEADY STATE REGION.
FIG. 3.3.1 PLANES IN THE $\gamma$, $x$, $\dot{y}$ SPACE
FIG. 3.3.2  DYNAMIC REGION OF OPERATION
FIG. 3.3a PROJECTION OF DYNAMIC REGION ON $y_x$, $y_z$ PLANE

FIG. 3.3b PROJECTION OF DYNAMIC REGION ON $y_x$, $y_z$ PLANE
FIG. 3.3.3c PROJECTION OF DYNAMIC REGION ON $y$, $\dot{y}$ PLANE

FIG. 3.4.1a REGION OF OPERATION IN $y$, $\dot{y}$ PLANE FOR CONSTANT VALUES OF $u^2$ ft-lb
FIG. 3.4.1b REGION OF OPERATION IN $\chi, \dot{\chi}$ PLANE FOR CONSTANT VALUES OF $^\prime V^\prime$ - FT. LB. SEC.

FIG. 3.4.1c REGION OF OPERATION IN $\chi, \dot{\chi}$ PLANE FOR CONSTANT VALUES OF $^\prime w^\prime$ - FT. LB. SEC.
FIG. 3.1.2 - PROJECTION OF DIFFERENT TORQUE PLANS ON THE EXTENDED STEADY STATE DESIGN.

- Constant torque: $y_2 = v = 100 \text{ ft.lb.}$
- Viscous load: $y_2 = 0.5y_1 \text{ ft.lb.}$
- Fan load: $y_2 = 0.002y_1^2 \text{ ft.lb.}$

Transmitted torque - ft.lb.wt.
Engine speed - rad.sec$^{-1}$
FIG. 4.5.1. PROGRAMMER COMPUTER CIRCUIT DIAGRAM
D = diode
s.l. = surge limiter

FIG. 4.6.1. CONTROLLER BRIDGE CIRCUIT [REF 15.]
FIG. 5.21  CONTROLLED OBJECT COMPUTER CIRCUIT DIAGRAM
Table: | Transmitted Torque | Engine Speed |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph of transmitted torque and engine speed responses for a step variation in field voltage (120 volts).](image)

**FIG. 5.2.2** CONTROLLED OBJECT TORQUE AND SPEED RESPONSES FOR A STEP VARIATION IN FIELD VOLTAGE (120 volts)

**FIG. 5.4.2** EFFECT OF HYSTERESIS ON LIMIT CYCLES AMPLITUDE

<table>
<thead>
<tr>
<th>Hysteresis of Simulated Relay</th>
<th>Transmitted Torque Limit Cycles ($y_k$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pm 2$ volts</td>
<td><img src="image" alt="Waveform" /></td>
</tr>
<tr>
<td>$\pm 4$ volts</td>
<td><img src="image" alt="Waveform" /></td>
</tr>
</tbody>
</table>
FIG. 5.4.1  BLOCK DIAGRAM OF TRANSMITTED TORQUE CONTROL SYSTEM

- **transmitted torque** \( y_2 \)
- **engine speed** \( y_1 \)
- **throttle** \( x_2 \)
- **field voltage** \( x_3 \)
- **field current** \( I_f \)
- **noise**

The diagram shows the transmitted torque control system with the following components:

1. Comparator for transmitted torque
2. Filter
3. Engine-dynamometer system
4. Comparator for throttle
5. Field current
6. Field voltage
7. Zero voltage input

Mathematical relations:

- \[ \frac{1}{R_f} \]
- \[ \frac{1}{1 + T_f S} \]
<table>
<thead>
<tr>
<th>Input torque (ft-lb)</th>
<th>Transmitted torque (ft-lb)</th>
<th>Engine speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>30</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

Note: The table above represents the relationship between input torque, transmitted torque, and engine speed. The values are illustrative and not actual data.
FIG. 5.5.2  TORQUE AND SPEED RESPONSE CURVES FOR STEP INPUT VARIATIONS IN THE TRANSMITTED TORQUE CONTROL SYSTEM
FIG. 6.2.1 BLOCK DIAGRAM OF PROGRAMMED TRANSMITTED TORQUE CONTROL SYSTEM
FIG. E2.3a CONSTANT LOAD SIMULATION [FIELD TIME CONSTANT=0.5 SEC.]

\[ y_2 \text{ ft.lb.} \]

\[ y_4 \text{ ft.lb.} \]

\[ z = 40 \quad z = 20 \]

\[ U = 60 \text{ ft.lb.} \]

\[ z \text{ step in throttle ft.lb.} \]
FIG 6.32 VISCOS LOAD SIMULATION

transmitted torque \( y \) ft.1b.

\[ V = \text{constant} \]

\[ V = 1.2 \text{ ft-lb. sec.} \]

\[ V = 1.0 \]
\[ V = 0.9 \]
\[ V = 0.8 \]
\[ V = 0.7 \]
\[ V = 0.6 \]
\[ V = 0.5 \]
\[ V = 0.4 \]
\[ V = 0.3 \]
\[ V = 0.2 \]
\[ V = 0.1 \]
FIG. 6.3.3  FAN LOAD SIMULATION

Transmitted torque $= W$, ft-lb.

Speed $\alpha$ rad. sec$^{-1}$
FIG. 11.1 FIELD CURRENT RESPONSE FOR A STEP IN FIELD VOLTAGE
FIG. IV.1 ENGINE AND DYNAMOMETER
REFERENCES

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