PARTICLE MOTION AND HEAT TRANSFER

IN ROTARY DRUMS

by

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Presented for the Degree of Doctor of Philosophy
Edinburgh University
1988
to
Mum and Dad
For everything
The work described in this thesis is the original work of the author except where specific reference is made to other sources. It has not been submitted in whole or in part for any degree at any other University.
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En hvatki es missagt es i fraedum pessum, 
pa es skylt at hafa pat heldr, es sannara reynisk

Islendingabok
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ABSTRACT

The aim of the present work was to study the heat transfer to the bed of solids in an externally heated rotating drum by conduction from the hot wall. Examination of the existing work in this field identified both the difficulties involved in direct heat transfer measurements and the importance of the mixing behaviour of the solids to the heat transfer. Accordingly bed motion was investigated experimentally, rather than heat transfer, although heat transfer experiments with temperature sensitive liquid crystal ink were considered and preliminary tests carried out.

Residence time distribution is also dependent on the mixing patterns within the bed so that it was logical to include this aspect of the operation of rotary drums in this work.

The mixing pattern in the bed was studied by following the path of a single particle in a two dimensional bed, using a technique by which video recordings of bed motion could be analysed by a computer. The effects of rotational speed, drum size, filling and inter-particle friction were studied.

Heat transfer and residence time distribution models were developed. These were used with both experimental and simulated idealized data to investigate the effects of different mixing patterns on the heat transfer in the bed and the residence time of the solids in a drum.

The importance of a relatively stagnant "core" area in the bed was identified in both models. This area of the bed has been shown to be of particular importance in residence time modelling.
CHAPTER 1
INTRODUCTION

Rotating drums are widely used in industry where their purpose is often to heat solid materials. Heat may be transferred either from inside the drum, from a flame or a flow of hot gas through the drum, or from outside, for example by hot gases flowing through an external jacket. Heat transfer is in general a combination of convection, radiation and conduction. The aim of the present work is to investigate the heat transfer to solids in externally heated drums where the predominant mechanism is conduction from the hot wall to the solid bed.

Although this work is concerned primarily with externally heated drums many of the features of heat transfer in such drums are common to internally heated drums. For this reason a study of the available literature on all aspects of heat transfer in rotating drums was undertaken. This study showed that most convective and radiative heat transfer paths are well documented and satisfactory models had been developed for these heat flows. In contrast very little work has been carried out on the conduction transfer between the wall and the bed, and has produced no adequate model of this heat transfer. Both experimental and theoretical work is needed in this area to elucidate the mechanism heat transfer. The present work is largely theoretical, in conjunction with experiments on mixing behaviour which affects heat transfer.

Direct study of heat transfer in rotary drums has proved difficult in the past. Temperature measurement in the interior of a moving particulate bed is complicated by the fact that introducing a measuring device into the bed must disturb the pattern of mixing of the solids. Non-intrusive temperature measurement using temperature sensitive liquid crystal ink coated particles was considered but experimentation showed that the method was impractical (see appendix IX). From previous work it appeared that the mixing motion of the bed affected the conductive heat transfer, while the radiative and convective paths were largely unaffected by the mixing pattern of the solids, and that the lack of information on this mixing behaviour could be a major factor in the difficulties encountered in modelling heat transfer from the wall. Hence experiments were performed to investigate this mixing, rather than to study the heat transfer directly. Investigation of previous work on the residence time
distribution in rotating drums showed that that knowledge of the mixing
behaviour of the bed could also provide a new insight into this aspect of the
operation of rotating drums.

Several types of bed motion in rotating drums have been identified:
slipping, slumping, rolling, cascading, cataracting and centrifuging (see figure
1.1). The type of motion depends on the operating conditions of the drum and
the properties of the solids; for example rotational speed, bed height, wall to
particle friction and interparticle friction. The relationship between these factors
and the motion of the bed is not well established.

In a slumping bed large segments of the bed periodically detach from the
upper half of the bed surface and slump towards the lower half of the bed in a
avalanche type of motion. Between slumps the bed travels round with the wall
of the drum. A slump occurs when the solids reach an upper angle of repose
(\(\alpha\)) (see figure 1.1).

At higher speeds slumping is replaced by rolling; here there is continuous
movement of solids down the bed surface. In the rolling mode the upper bed
surface is flat.

Further increase of rotational speed produces cascading behaviour; here
the motion is similar to rolling except that the bed surface is kidney shaped.

Cataracting and ultimately centrifuging occur at yet higher rotational
speeds. Centrifuging occurs at speeds over the 'critical speed' (\((g/R_d)^{0.5}\)).
Normally cataracting and centrifuging do not generally occur in rotary kilns and
reactors.

The most important types of motion are the rolling and cascading motion.
These are the types of motion usually found in industrial kilns. In the present
work the bed will be assumed to be showing either rolling or cascading
behaviour throughout.

Three types of slipping motion occur. In the first type the solids move as
one body with the wall until they reach some inclination, lower than their angle
of repose, at which point they slide back down the wall, and the process
repeats itself. In the second type of slipping motion the bed moves upwards
with the drum wall, but at a lower rotational speed. Solids tumble down the
Figure 1.2 Path of material through drum

Figure 1.3 Cross-section of bed
surface so that this motion is similar to rolling except that there is slip between the wall and the solid bed. The final slipping mechanism is where the bed remains stationary with the solids continuously slipping against the wall.

The particulate solids follow a spiralling path as they pass through the drum, as shown in figure 1.2. There are two distinct phases to this spiralling motion. In the first particles in the bulk of the bed move round fixed relative to the drum wall until they reach the upper surface of the bed. They then tumble down this inclined surface, advancing axially as they do so. The particles then reenter the bed on the lower half of the bed to begin another circular path. If a cross-section of the bed is considered (figure 1.3) a typical particle might commence its circular path at point A. It then moves round to B, at the same angular speed as the drum wall. Once it has reached point B on the bed surface it tumbles down the bed surface, beginning a new circular path when it re-enters the bulk of the bed at C. The point C would be at a different axial position from A and B because of the particle's axial advance as it tumbled down the surface. This pattern of motion allows two distinct parts of the bed to be identified.

1. The 'Fixed Bed'— In this part of the bed the particles remain fixed relative to each other and to the drum wall until they reach the bed surface.

2. The 'Cascade Layer'— This is the layer formed on the bed surface by the solids tumbling down it. All mixing of solids takes place on this layer.

A few workers [R4,12] have identified a third distinct region of the bed cross-section; the 'core'. This is shown as the shaded area in figure 1.3. This part of the bed is usually described as a stagnant area. There is little mixing between this part of the bed and the fixed bed and cascade layer surrounding it. This is because particles from the core cannot easily pass into the cascade layer and cannot therefore mix with the particles leaving and re-entering the fixed bed. There is circulation and mixing of the solids within this core. This core area is of great interest in the present work.

All mixing between particles originating from different radii in the fixed bed takes place on the cascade layer. This mixing is termed transverse or radial mixing, as opposed to axial mixing which is the mixing of solids from different axial positions along the drum; which also takes place on the cascade layer.
Slipping

Slumping

Rolling

Cascading

Cataracting

Centrifuging

Figure 1.1 Types of Bed Motion
There are several terms which are commonly used in studies of rotating drums. Referring to figures 1.2 and 1.3, the length \((l)\) and radius \((R_d)\) of the drum are self-explanatory. The kiln inclination is the angle between the longitudinal drum axis and the horizontal. The angle between the bed surface and the axis of the drum \((\beta)\) is important in consideration of particle motion on the surface of the bed.

In figure 1.3, \(\phi\) is called the filling angle of the bed. This is used in calculating bed height and filling fraction. The bed height is the maximum thickness of the bed:

\[
H_b = R_d(1 - \cos\phi)
\]

The filling fraction is the fraction of the drum which is filled with solids. The filling fraction may be the overall filling \((f_c)\); simply the total volume of solids divided by the total drum volume, or it may be the filling at a particular axial position \((f_c(x))\) where

\[
f_c(x) = \frac{\text{area of solid bed}}{\text{cross-sectional area of drum}}
\]

\(x\) is the distance from the feed end of the drum, along the axis.

The objective of the experimental work was to study the effect of varying operating conditions on the mixing pattern in the bed. The parameters identified as likely to influence the mixing were: rotational speed, degree of filling of the drum; surface characteristics of the solids; particle size and size distribution. A technique was developed by which the path of a single particle could be followed in a 2-dimensional drum. The technique was based on a system of image acquisition and analysis using a video recorder, video digitizer and microcomputer. The overall mixing behaviour of the bed was inferred from the path of the single particle. The mixing pattern was described by means of a matrix of the rates at which material passed between different parts of the bed.

The experiments were carried out with varying rotational speeds, fillings, particle surfaces and ratios of particle to drum diameter. The effect of particle size distribution was not investigated. Analysis of the mixing in a bed with
more than one size of particle would be very time consuming, but is clearly an important topic for future work.

The experiments clearly showed the existence of the 'core' area in the bed which proved to be important in modelling both heat transfer and residence time distribution. The presence of this distinct bed region had been little reported previously.

The present work is concerned with kilns with no internal fittings. The presence of flights, chains or any other internals would be expected to affect the mixing of the solids but these are not considered here.

Models of heat transfer and residence time distribution based on the mixing information provided by the experiments were developed. The importance of the core is highlighted by the results produced by these models. The effect of the presence of the core on previously developed models of mean residence time, which had not allowed for the core, was also considered.

The review of previous work on both residence time distribution and heat transfer forms chapter 2. Chapter 3 contains a description of the experimental apparatus and method and of the experiments carried out, with a discussion of the results produced. The development of the residence time distribution model, along with simulations produced using the model and consideration of previous models is contained in Chapter 4. Heat transfer modelling is described in Chapter 5. Finally the conclusions drawn from this work and suggestions for further possible work are presented in Chapter 6.
CHAPTER 2
LITERATURE REVIEW

2.1. Introduction

The main functions of rotary drums are to provide the retention time and heat required for the process taking place in them. The two main aspects of rotary drum operation which have been studied therefore are the residence time and heat transfer in these drums. A significant amount of work has been done in these fields and this is discussed below.

The published work on residence time distribution may be divided into three classes:

- Experimental work on full size equipment, using radioactive tracers
- Small scale experimentation, investigating the effect on the residence time distribution of varying operation conditions
- Modelling of mean residence time and residence time distribution

The field of heat transfer in drums has attracted more attention than residence time, perhaps because there is such a wide range of heat transfer conditions in drums, depending on the heating method and the temperature in the drum. The heat transfer work can be divided into two types of study:

- Overall simulation of the complete operation of a drum
- Investigation of a specific heat transfer mechanism in drums

The development of the areas of interest outlined above is discussed below, residence time is considered first.

2.2. Residence Time Distribution
2.2.1. introduction

In any vessel with material passing through it under some flow regime other than true plug flow, different elements of material will require different lengths of time to pass through the vessel, this is true of solids in rotating drums. Analysis of the distribution of the times taken by material to pass through vessels is useful and methods of measuring and analysing these distribution are well established.

The exit age of an element of material is the length of time it has spent in the vessel. The distribution of these times for a vessel is known as the ‘exit age distribution’ E, or the residence time distribution (RTD). This is conveniently represented as a normalized distribution; the ‘E curve’, see figure 2.1. The distribution is normalized, so that the total area under the E curve is equal to 1, ie:

\[ \int_{0}^{\infty} E \, dt = 1 \]  \hspace{1cm} (2.1)

the fraction of the exit stream between \( t \) and \( t + dt \) will be

\[ \int_{t}^{t+dt} E \, dt \]  \hspace{1cm} (2.2)

The fraction younger than age \( t_1 \) will be

\[ \int_{0}^{t_1} E \, dt \]  \hspace{1cm} (2.3)

And the fraction older than \( t_1 \) will be

\[ 1 - \int_{0}^{t_1} E \, dt \]

A 'closed vessel' is defined as one in which material enters and leaves solely by plug flow, ie there is no axial diffusion or dispersion over the entrance and exit so that all material moves into and out of the vessel by bulk flow alone. Varying velocities, back diffusion, swirls and eddies are not permitted at the exit and entrance of a closed vessel. A rotating drum may
reasonably be assumed to be a closed vessel.

For steady flow the holding time or mean residence time, $t$, is given by:

\[ \bar{t} = \frac{V}{\bar{V}} \]

$V$ = vessel volume
\(\bar{V}\) = volumetric flowrate

(2.4)

It can be shown that the mean of the E curve for a closed vessel must be equal to this mean residence time, $\bar{t}$.

Experimental information on residence time distribution is often presented in the form of a 'C curve'. This represents the normalized response to an instantaneous pulse of tracer in the stream entering a vessel (see figure 2.2). Here the measured concentration, $c$, is divided by the total area under the concentration-time graph for the experiment to give the C curve, where $C$, as defined below, is plotted against time.

\[ \int_{0}^{\infty} C \, dt = \int_{0}^{\infty} \frac{c}{Q} \, dt = 1 \quad \text{where} \quad Q = \int_{0}^{\infty} c \, dt \]

For a closed vessel the C curve is identical to the E curve, allowing the residence time distribution to be obtained directly from experimental data.

Residence time distribution is frequently characterized by its mean and variance. The mean for a concentration-time curve is

\[ \bar{t} = \frac{\int_{0}^{\infty} t \, c \, dt}{\int_{0}^{\infty} c \, dt} \]

for discrete time intervals $t_i$

\[ \bar{t} = \frac{\sum t_i c_i \, \Delta t_i}{\sum c_i \Delta t_i} \]

(2.5)

The variance $\sigma^2$ is defined as
Figure 2.1 'E' Curve

Figure 2.2 'C' Curve
\[
\sigma^2 = \int \frac{(t - \bar{t})^2}{\int c \, dt} \, c \, dt = \int \frac{t^2}{\int c \, dt} \, c \, dt - t^2
\]

in discrete form

\[
s^2 = \frac{\sum(t_i - \bar{t}^2) \, c_i \, \Delta t_i}{\sum c_i \, \Delta t_i} = \frac{\sum t_i^2 \, c_i \, \Delta t_i}{\sum c_i \, \Delta t_i} - t^2
\]

For normalized distributions in closed vessels these expressions (in discrete form) are simplified to:

\[
\bar{t} = \sum t_i \, E_i \, \Delta t
\]

\[
s^2 = \sum t_i^2 \, E_i \, \Delta t - \bar{t}^2
\]

### 2.2.2. Modelling Methods

Several techniques have been used to model the residence time distribution of solids in rotary drums. Most commonly used is the axial dispersion model [A1,A2,A3,A6,H9,K2,R41. Other types which have been used are a finite stage transport model; made up of stages representing the mixing in individual bed turns [M4,M5] and a Monte Carlo simulation [R2]. These models are described below.

#### 2.2.2.1. The Axial Dispersion Model

A dispersion model for non-ideal flow uses an analogy between the mixing in the flow and a diffusional process. The dispersed plug flow model envisages the non-ideal flow as being a perfect plug flow with a degree of back- or inter-mixing superimposed upon it. No allowance is made for bypassing or stagnant areas in the vessel in this type of model.

The contributions to backmixing of the material may be described by the expression:
\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - u_x \frac{\partial C}{\partial x} \tag{2.9}
\]

which is analogous to Fick's Law for molecular diffusion. The parameter \( D \) is known as the longitudinal or axial dispersion coefficient. In dimensionless form equation (2.9) becomes

\[
\frac{1}{\theta} \frac{\partial C}{\partial \theta} = \left( \frac{D}{uL} \right) \frac{\partial^2 C}{\partial z^2} - \frac{\partial C}{\partial z} \tag{2.10}
\]

where \( \theta = \frac{t}{L} \), \( z = \frac{x}{L} \)

\( D/uL \) is the 'vessel dispersion number'. The Peclet or Bodenstein number, defined as \( uL/D \), the inverse of the vessel dispersion number is often used.

The variance of the \( C \) curve can be derived and is found to be [L4,L5]

in dimensionless form:

\[
\sigma^2 = \frac{\sigma^2}{t^2} = \left( \frac{D}{uL} \right)^2 - 2 \left( \frac{D}{uL} \right)^2 (1 - \exp(-Pe)) \tag{2.11}
\]

For this model to be applied to experimental data from a continuous rotary drum with a particulate tracer several assumptions must be made:

1. The steady state in the drum is not perturbed by the introduction of the tracer
2. The bed height is constant along the whole length of the drum, leading to a constant mean axial velocity, \( v \).
3. Tracer concentration is radially uniform.
4. The Peclet number (or axial dispersion coefficient) is constant for fixed operating conditions.

The initial and boundary conditions for a closed vessel may then be applied:

\[
\text{initial condition} \quad C(0,z) = 0 \\
\text{(there is no tracer material in the drum initially)}
\]
inlet boundary conditions \((z=0)\)
\[ C(0,0) - \frac{1}{Pe} \frac{\partial C(\theta,0)}{\partial z} = \delta(\theta) \]

at the outlet
\[ \frac{\partial C(\theta,1)}{\partial z} = 0 \]

that there is no dispersion at the entrance and exit.

A solution for \(C(\theta,1)\) and hence \(E\) has been derived [A3,Y3] viz:

\[ C(\theta) = 4 \sum_{n=1}^{\infty} \lambda_n \left( \frac{2\lambda_n \cos \lambda_n \theta + Pe \sin \lambda_n \theta}{4\lambda_n^2 + 4Pe + Pe^2} \right) \cdot \exp \left( \frac{Pe - \frac{Pe^2 + 4\lambda_n^2}{4Pe^2}}{2} \right) \]  

(2.12)

where \(\lambda_n\) is the \(n^{th}\) positive root of the transcendental equation

\[ \tan \lambda = \frac{4\lambda Pe}{4\lambda^2 - Pe^2} \]

Equation (2.12) forms the basis of the axial dispersion model for dimensionless residence time distribution.

Brenner [B7] devised a modified solution of equation (2.12) for large Peclet number or small \(\theta\). The resulting solution is:


\[ C(\theta) = \left\{ \frac{1}{4} \frac{1}{(Pe)}^{\frac{1}{2}} \frac{\theta}{\theta + 1} - \left( \frac{1}{Pe} \right)^{\frac{1}{2}} \left( \frac{\theta}{\theta + 1} \right)^{\frac{1}{2}} \right\} \]

\[ \left[ \Psi \left( \frac{\theta}{\theta + 1} \right) \left\{ \frac{1}{2} - \frac{1}{\theta + 1} + \frac{Pe(1 - \theta)}{4\theta^2} \right\} + \chi \left( \frac{\theta}{\theta + 1} \right) \right] \}

\[ \exp \left[ \frac{-Pe(1 - \theta)^2}{4\theta} \right] \]

(2.13)

where

\[ \Psi(\zeta) = \sum_{k=0}^{\infty} (-1)^k \left[ \frac{2\zeta(1 - \zeta)}{Pe} \right]^k \phi_k(\zeta) \]

with

\[ \phi_k(\zeta) = 1.3... \ (2k+1) \left[ \frac{1}{2k+1} - \frac{\delta \zeta}{2} + 4(k + 1)\zeta^2 \right] \]

and

\[ \chi(\zeta) = \sum_{k=0}^{\infty} (-1)^k \left[ \frac{2\zeta(1 - \zeta)(k-1)}{Pe} \cdot \frac{2(1 - 2\zeta)\phi_k(\zeta)}{Pe} \right] + \left[ \frac{2\zeta(1 - \zeta)}{Pe} \right]^k \Omega_k(\zeta) \]

where

\[ \Omega_k(\zeta) = 1.3... (2k+1)[-6 + 8(k + 1)\zeta] \]

The Peclet number, required for equation (2.13) can be calculated from the experimentally determined dimensionless variance (see equation (2.11)). For a large Pe the approximation

\[ \sigma^2 \theta = \frac{2}{Pe} \]

is acceptable.

Equation (2.13) provides a good estimate of the exact solution of (2.12) for large Pe or small \( \theta \), for which (2.12) itself is not useful. A simplified solution, based on (2.13), has been proposed [A3] which gave satisfactory results:
\[
C(\theta) = \frac{Pe \exp\left\{\frac{2(1-\theta)^2Pe}{4}\right\}}{4\pi} \tag{2.14}
\]

Equations (2.13) and (2.14) have been used to predict residence time distributions in drums from experimentally derived Peclet numbers [A3,B7,A3,Y3].

### 2.2.2. Finite Stage Transport Model

Mu and Perlmutter [M4,M5] used a model of this type to simulate the residence time distribution in a rotary drum. This model simulated the bed motion by means of a series of the model blocks shown in figure 2.3. The different regions and connecting flows in the block represented the sections of the bed and the flow of solids in an axial slice of the bed. The plug flow region of the block represented the fixed bed, while the cascade layer was represented by the well mixed flow. Backmixing was represented by the recycle stream. Using numbers of these blocks in series it was possible to derive an expression for the residence time distribution (see appendix I).

Methods were proposed for calculating \(Q\), the recycle ratio; \(P\), the volume fraction of the cascade layer and \(N\) the number of stages for given drum speed, dimensions and material properties. The bypass fraction, \(a\), could only be found by fitting the model to an experimentally determined residence time distribution.

### 2.2.2.3. Monte Carlo Simulation

A third modelling method was used by Rogers and Gardner [R2], who predicted the path of individual particles through the bed using a Monte Carlo method.

A Monte Carlo simulation can be regarded as a mathematical experiment by which the expected outcome of a stochastic process is estimated by random sampling from the probability distributions which govern the events making up the process.

The particle motion model developed by Saeman [S1] and Vahl and Kingma [V1] was used to determine the time spent and axial advance for one turn through the bed as functions of the radial position of the particle in the bed.
Figure 2.3 Model Block Used by Mu & Perlmutter
These models are discussed further in chapter 4.

In order to predict the path of a particle through a rotating drum Rogers and Gardner used a Monte Carlo method to select the point on the bed cascade layer at which a particle tumbling down the surface would reenter the fixed bed. They assumed that the bed was well mixed so that the reentry point was independent of the exit point. The probability of a particle reentering at any radius was then a function only of the reentry radius, the drum radius and the drum filling. In fact the cumulative probability that a particle would reenter within any radius was given by

\[ F(r) = \frac{\text{volume of bed with radius } < r}{\text{total volume of bed}} \]

for \( r \) (see appendix II). The result was a random value of \( r \) chosen with frequency \( f(r) \), produced from a uniform random distribution of values of \( y \), where \( f(r) \) was the probability that a particle would enter the bed at a point between \( r \) and \( r + dr \).

This selection of reentry point introduced a small amount of dispersion to the residence by introducing a distribution of times and axial advances for each bed turn. This effect was very small however and was not of the correct magnitude to simulate real drums. A further source of dispersion was introduced by assuming the axial advance of particles on the surface to be normally distributed about the mean value as predicted by the particle motion model used. Incorporating this effect into the Monte Carlo simulation gave residence time distributions more in agreement with experiment.

This model required an experimentally determined dispersion coefficient (see appendix) which was used to define the normal distribution of the axial advance on the surface. Two cases were simulated:

1. A horizontal batch cylinder with no bulk flow of solids, in this case the bed height was constant over the whole length of the drum, and the effect being modelled was purely the dispersion due to the distribution of the axial movement of the particles. The dispersion coefficient was determined experimentally and used to calculate the standard deviation of the distribution of the particle path used in the simulation (see appendix). The simulation procedure generated the path of the particle over a set time. The final
positions of 1000 such particles were used as a particle number distribution. This distribution was fitted to the theoretical distribution derived from the one-dimensional dispersion equation to find a value of the dispersion coefficient, D, for comparison with the original value found experimentally.

2. Horizontal cylinders with flow of solids; here the simulation was used to account for both dispersion and bulk flow. The paths of 1000 particles were generated and their exit ages used to form the residence time distribution. Experimental data on the holdup of material in the drums and the bed profile was necessary for the simulation, as well as the experimental residence time distribution. It was also necessary to have some information on the variation of the standard deviation of the particle axial paths with fractional filling, in order to allow for the change in bed height along the drum. It was assumed that the angle between the bed surface and the drum axis was constant for the full length of the drum. The equations

\[ D = b_0 f_c^m \quad \text{and} \quad D = b_0 f_c^m(x) \]  

were used to relate D, the dispersion coefficient and the fractional filling \( f_c \). Equation (2.16) was used for flow situations where the filling was a function of \( x \), the axial position. \( b_0 \) and \( m \) were constants determined experimentally in batch drums.

These three models have been tested as a means of simulating the behaviour of solids in rotary drums. It is an obvious feature of all these models that a large amount of experimental data was needed before the models could be applied. The dispersion models required the Peclet number \( P_e \) to be experimentally determined. The finite stage model required the bypass fraction on the cascade to be found by fitting the model to experimental results. The Monte Carlo method required the dispersion coefficient as derived from batch experiments as well as the experimental constants \( b_0 \) and \( m \), above. The first two models cannot therefore be used to simulate the residence time distribution of a drum without an experimental residence time distribution. The Monte Carlo model did not require a residence time distribution but did require data from batch experiments.
All the types of model described here have been shown to simulate small scale experimental results well [A1, A2, A3, A6, H9, K2, R4, I3, M6].

The finite stage model and the axial dispersion models produced simulated residence time distributions which were very similar to the experimental data on which they were based. An interesting finding in the finite stage model was that the cascade bypass fraction was high in both the examples used [M5]. This might suggest that the method used to calculate the speed of particles on the cascade layer was underestimating the true speed on the bed surface.

The Monte Carlo model [R2] produced residence time distributions which could be made to fit the experimental data, but not as closely as the other models. This could have been because of the assumption of constant bed angle, which is not good in horizontal drums, or because of the simple relationship used to relate filling fraction and dispersion coefficient.

2.2.3. Experimental Work

There have been several studies with radioactive tracers in full sized kilns and driers [A4, C3, G1, G7, H6, L1, M7, O1, R3, S6, Z2]. However none of these studies gave enough data for comparison with the models described above. The conditions were often extremely unrepresentative of drums in general, either because of the materials; eg chopped seaweed [G1] which would be unlikely to behave as a granular material; or because of the test method. For example Shevstov et al. [S6] used a brick capsule to contain the radioactive source. This would almost certainly not have followed a path typical of the particulate solids in the kiln. Physical property changes, for example in cement kilns [C3, G7, H6, M7, O1, R3, S6, Z2] also make residence time distributions more difficult to compare with theoretical models.

Much work has been concerned with the technique of injection and detection of the tracer substances rather than with the analysis of the resulting residence time distributions.

There is little scope in full scale plant for investigation of the effect of varying operating conditions on the residence time distribution. This has only been possible in small scale drums.
The most extensive experimentation has been carried out by Abouzied [A1,A2,A3] who studied mainly horizontal drums with constricted discharge. He considered the effect of rotational speed, material feedrate and particle size and the effect of drum inclination.

A wide range of rotational speed was used in these experiments, from 10rpm to 120rpm (6.7% to 80% of the critical speed). This would have included a range of bed motion from rolling at the lower speeds and cataracting at the highest speeds, with cascading motion in an intermediate range. With only five experimental points in this wide range of behaviour it is not easy to draw conclusions from these results.

The residence time distributions were characterized by their variances. At the lower rotational speeds the variance decreased slightly with increasing speed. At higher speeds, however, the variance increased with speed. This suggests that the transition from rolling to cascading motion might have a significant effect on the residence time distribution (see figure 2.4).

In the other experiments (varying feedrate, particle size and inclination) a drum speed of 28% of critical (42rpm) was used. This would have corresponded to either rolling or cascading bed behaviour.

The RTD variance decreased with increasing feedrate (figure 2.5). Because these experiments were carried out at constant rotational speed the bed profile would have been dependent on the feedrate (holdup increases with feedrate at constant drum speed).

Comparing dimensionless variances it was found that above a certain feedrate the dimensionless variance ($\sigma^2/\bar{\sigma}$) was virtually constant. Below this rate the dimensionless variance decreased rapidly with increasing feedrate. Only one experimental point lay below the critical feedrate so that it could not be definitely identified.

Particle size was found to have no effect on the residence time distribution.

Even small deviations from the horizontal (0.5°) had a dramatic effect on the residence time distribution, both the mean and the variance dropped substantially when the drum was inclined slightly. The decrease in the mean
was due to the decreased holdup in the inclined drums. Both the mean and the variance varied linearly with the angle of inclination for inclined drums, and both showed a marked discontinuity between values for horizontal and inclined drums (figure 2.6).

In a later paper [A2] Abouzied derived a 'scale up variance' which was a function only of a characteristic time and the material properties and which was valid only for horizontal drums (see appendix III).

Abouzied used this scale up variance with data from one experimental drum to predict scale up variances for drums with different diameter to length ratios. Comparison of predicted and experimental values of scale up variance showed a mean deviation of 15% between the two values. The maximum deviation was 40%.

Residence time distributions predicted from these scale up variances show the peak of the residence time distribution displaced in time and the predicted residence times were less skewed than the experimental distributions. Never the less this method does present a way of estimating a residence time distribution for horizontal drums where there is no experimental residence time data.

Rutgers [R4] investigated the effect of fractional filling, feedrate, speed of rotation and drum inclination on residence time distribution. He expressed his findings in terms of the effects on the dispersion coefficient, rather than the RTD variance.

It was found that increasing filling, by altering the discharge weir height and changing the solids feedrate gave dispersion coefficients approximately proportional to $f_c^{0.5}$. To arrive at this conclusion average holdups and dispersion coefficients for sets of experiments carried out with the same discharge diameter were used for comparison. Since the experiments within any of these sets were carried out at different holdups this averaging process seems invalid because it obscured the effect of the variation of holdup within the sets. For experiments carried out with a constant discharge weir radius and varying feedrate the dispersion coefficient was not proportional to $f_c^{0.5}$.

The dispersion coefficient increased with decreasing feedrate, this was in agreement with Abouzied's findings.
Figure 2.4 Variation of RTD variance with Rotational Speed

Figure 2.5 Variation of RTD variance with Feedrate

Figure 2.6 Variation of RTD variance with Drum Inclination
The dispersion coefficient, and variances estimated from it using (2.13), increased with increasing rotational speed. The decreasing relationship found by Abouzied at low speeds was not evident because only one experiment in the appropriate speed range was carried out. There were only four data points to cover a range of 7.5% to 92% of the critical speed so that the relationship between RTD and rotational speed was not well defined.

The above experiments were all in horizontal drums. One experiment in an inclined drum was also carried out but unfortunately it was not at conditions which could be compared to any of the other results because it had been performed with a fractional filling and feedrate different from all the other runs.

Karra and Fuerstenau [K2] studied the effect of discharge plate geometry on the residence time distribution in horizontal drums. With constant feedrate and rotational speed (27% of critical) they found that increasing the holdup (by changing the discharge diameter) initially produced a decrease in variance, but that further increases in holdup then produced an increase in the variance. Therefore a discharge diameter existed which would produce a minimum RTD variance.

Hehl et al [H9] found that, at constant feedrate, in a horizontal drum, increasing rotational speed produced a decrease in the variance of the RTD. The speeds used were between 1.2% and 11.8% of the critical speed. These results then confirmed Abouzied's conclusion that variance of residence time at low rotational speeds decreases with increasing speed.

2.2.4. Mean Retention Time Modelling

A further body of work is concerned with the prediction of mean residence time from drum operating conditions and material properties [M5,S7, H5,S1,V1]. This type of model is based on the calculation of the rate at which solids emerge from the fixed bed onto the cascade layer and the rate of their axial movement down the drum. This type of model is reasonably successful in predicting mean residence time, although some of the parameters needed are difficult to obtain, in particular angles of repose and the coefficient of friction on the cascade layer [S7]. In these models no allowance has been made for the presence of the core area of the bed and the bed has been assumed to have no transverse mixing. These models are discussed in more detail in Chapter 4.
2.2.5. Summary of Residence Time Literature

Previous work on residence time in rotating drums can be divided into four groups: modelling of the residence time distribution, experimentation on industrial drums, small scale experimentation and modelling of the mean retention time.

Of the three techniques which have been used to model the residence time distribution in this type of equipment the most widely used have been the axial dispersion models. The other two types of model; the Monte Carlo simulation and the finite stage model have found only limited use. All three techniques require experimental data in order to simulate the residence time distributions in drums.

Residence time distributions in industrial kilns provide limited data because of the relatively crude experimental methods reported and are of course specific to the conditions in each individual drum.

Experimentation in small scale drums has identified the effect on residence time distribution of the main variables in drum operation; Rotational speed, solids feedrate, particle size and drum inclination. The main findings of the small scale experimentation are:

- Variance of residence time distribution decreases with increasing rotational speed at relatively low speeds and increases with speed at higher rotational speeds.

- RTD variance decreases with increasing solids feedrate, at constant speed.

- Particle size has no effect on the variance or mean of the residence time distribution.

- There is a marked decrease in variance between horizontal and inclined drums. For inclined drums the variance decreases with increasing drum inclination.

Mean residence time can be estimated using models based on the motion of particles in the bed. In these models it is generally assumed that there is no transverse mixing of the solids. This is in contrast to the Monte Carlo method of residence time distribution simulation where it is assumed that there is complete transverse mixing. These different assumptions concerning the degree of transverse mixing on the cascade layer is discussed further in chapter 4, and
is an important consideration in the present work.

2.3. Heat Transfer

2.3.1. Introduction

As stated previously the rotating drum is widely used in industrial processes where it is necessary to heat or cool a particulate solid. There is a wide range of operating conditions in such equipment but there are three main classes of drum:

- The direct fired kiln, in which the solids are heated by a flame within the drum.
- The direct heater or cooler where the solids are heated or cooled by a co- or counter-current flow of gas within the drum.
- The indirectly heated kiln which is heated by hot gases in an external jacket.

The basic heat transfer routes are the same for all three classes of drum; there are five fundamental heat flows:

- Gas-solid charge
- Gas-wall
- Wall-solids
- Conduction through the wall
- Outside wall-surroundings

The mechanism of exchange may be conduction, convection, radiation or a combination of these. The relative importance of the individual mechanisms and the relative magnitudes of the heat flows will depend on the operating conditions. For example, radiation would obviously be important in a direct fired kiln, while one might expect convection to dominate in a direct cooler.

There are two main bodies of work on heat transfer in rotating kilns; the
first is concerned with simulating the conditions in process kilns, taking into consideration all the heat transfer paths as well as mass transfer and reaction; the second type of work examines a specific mechanism or path of heat transfer with the object of developing a method of determining the appropriate heat transfer coefficient. Even in the cases where a specific heat transfer route is studied the model produced is strongly influenced by the process considered and the assumptions made about the various mechanisms. Some workers have considered only radiative exchange [G4], for example, while others have neglected radiation entirely [K3,L3,T2,W1]. An assumption which was almost universally applied although not always explicitly stated in both types of work was that the solid bed was radially well mixed and therefore had a uniform radial temperature distribution at any axial position. Imber and Pashkis [11] provided evidence that this assumption was more satisfactory than a radially unmixed or segregated path model.

2.3.2. Heat Transfer Models

It is useful to look at the existing experimental and theoretical work in terms of the different heat transfer routes and mechanisms investigated first and then to examine how this work has been used in the simulations of complete kiln behaviour.

2.3.3. Gas–Solid Exchange

2.3.3.1. Convection

Three empirical correlations have been used for the convective heat transfer between the bed surface and the gas stream:

\[
\begin{align*}
\hat{h}_{gs(c)} &= 17.8(k_gD)^{0.2}(U_g/C_{pg})^{0.8} \\
\hat{h}_{gs(c)} &= 4.17(G_g/A)^{0.67} \\
\hat{h}_{gs(c)} &= 0.023(k_g/D)Pr^{0.4}Re^{0.8}
\end{align*}
\]

Equation (2.19) is simply the normal heat transfer expression for fully developed turbulent flow in a long tube. Equation (2.18) is given in Perry [P3] for the heat transfer coefficient from the gas stream to the kiln wall. It has been assumed
by some workers that this is equal to the heat transfer coefficient between gas and solid charge [M1,R1,S4,V3,W9] but this has been shown in experiments not to be the case [T2,W6]. In fact the gas-solid coefficient has been shown to be approximately 10 times the gas-wall coefficient [T2]. The first expression was given by Khodorov [K6] who derived it from data on several cement kilns.

Watkinson and Brimacombe [W6] experimented with a direct fired pilot rotary kiln. They found that at low rotational speeds the gas-solids heat transfer rate was governed by mixing within the bed, while at higher speeds the heat flow was limited by the gas side heat transfer. This change corresponded to the slumping-rolling bed motion transition identified by Henein [H3]. It was discovered that convective heat transfer accounted for over 70% of the total gas to solids heat flow when under gas-side control. This is much higher than would have been predicted by the empirical relationships. The measured coefficients were around 150W/m²K while those calculated from equation (2.18) were around 10W/m²K.

There are two factors which could account for this higher than predicted coefficient; the large effective surface area of the particulate bed and the increase in Reynolds number due to the motion of the particles cascading through the gas.

Tscheng and Watkinson [T2] identified the unsuitability of equations (2.18) and (2.19) for heat transfer calculations between gas and solids, pointing out that these equations had been derived for completely different conditions. They systematically investigated the effects of gas temperature, flowrate and bed filling on the heat transfer coefficient.

They used the same kiln as Watkinson and Brimacombe, but used direct heating with preheated air rather than direct firing. In these experiments the radiative coefficients between gas and solids and gas and wall were calculated to be 35 to 70 times lower than the corresponding convective coefficients. In order to eliminate variations of bed surface area along the kiln the solids flow and angle of inclination of the kiln were adjusted to give uniform filling along the length of the bed.

The heat transfer coefficients were determined from heat balance calculations using solid and gas temperature measurements taken along the length of the kiln. Uniform radial solid and gas temperatures were assumed,
implying that the bed was well mixed.

It was found that the gas-solid convective coefficient increased with increased gas flowrate. The rotational speed had only a slight positive effect on the coefficient. All the experiments were carried out with rolling bed motion. $h_{gs(c)}$ appeared to decrease slightly with increasing degree of fill. It was suggested that this was due to the change in the ratio of the bed surface area to volume which accompanies a change in filling. Bed surface area increases with filling ($f_c$) as

$$A_s \propto f_c^{0.27}$$

for $0.04 < f_c < 0.30$

transfer to the bulk of the bed was by the mixing of the heated surface layer of particles into the bulk, so if the ratio of surface to bulk particles is decreased overall heat transfer would be affected. However, Tscheng and Watkinson's data concerning this affect showed considerable scatter and the relationship is less than certain.

Particle size and type of material were found to have no detectable effect on convective heat transfer coefficients.

In conclusion Tscheng and Watkinson presented a dimensionless expression for the convective heat transfer between a rolling bed and the gas stream:

$$Nu_{gs} = 0.46Re^{0.555}Re_w^{0.104}f_c^{-0.341}$$  \hspace{1cm} (2.20)

where $Re_w$=Rotational Reynold's number=$D_e^2\omega/\nu$

Gorog et al [G5] fitted a simpler relationship to the data of Tscheng and Watkinson:

$$h_{gs} = 0.4G_g^{0.62}$$  \hspace{1cm} (2.21)
This eliminates the dubious filling term and is very similar to equation (2.18) in form.

2.3.3.2. Radiation

The degree of complexity of the radiative exchange in a kiln depends on the nature of the media involved. The gas may be transparent, in which case it plays no part in radiative exchange, or it may emit and absorb radiation either in distinct bands or as a 'grey' gas. A grey gas is one for which the emissivity is constant for all wavelengths of radiation. The solids and kiln wall may be assumed to be either black or grey although this is probably not strictly true. The actual emissivities of the refractories and the solids in kilns is generally not known, values given in various sources are in the range 0.5 to 1.0.

Cross and Young [C5] assumed that the combustion gases in the direct fired iron ore kiln which they studied acted as a grey gas and that the solids were radiatively grey, all emissivities being independent of temperature. Since emissivity increases only slightly with increasing temperature [B5] this approximation should not have introduced significant errors in comparison with the uncertainty in the values of the emissivities themselves.

The radiative heat flow was given as:

\[ Q_{gs} = 2R \sin \phi \phi \cdot J_g (T_g^4 - T_s^4) \]  

(2.22)

\( 2R \sin \phi \) is the chord length of the bed, where \( \phi \) is the filling angle of the bed, \( \alpha \) is the Stefan-Boltzmann constant and \( J_g \) is the 'radiation interchange factor' between the gas and the charge surface. This factor \( J_g \) was used to account for the view factor, \( F_{sg} \), between the gas and the solid and for the emissivities of the gas and the solids (\( \varepsilon_g \) and \( \varepsilon_s \) respectively). The view factor from the gas to the charge is the ratio of the bed surface area to the sum of the bed surface area and the exposed wall surface area, this was given approximately by:
Of the radiation from the gas which reaches the bed surface some is absorbed and some is reflected to the wall, a portion of that being reflected is reabsorbed by the gases. Radiation reaching the wall is then absorbed or re-reflected, either being absorbed by the gas or reaching the bed or another area of the wall. Taking into account these multiple reflections Cross and Young [C5] arrived at the following expression:

\[ J_g = \frac{(EB - 1)}{(\varepsilon_s + 1 - \varepsilon_B - 1)} \]  

(2.24)

where \( \varepsilon_s \) was the bed emissivity and \( \varepsilon_B \) was the effective emissivity from the wall to the bed surface, given by:

\[ \varepsilon_B = \frac{\varepsilon_s F_{sw} \left[ 1 + \frac{F_g (1 - \varepsilon_s) (1 - \varepsilon_B)}{(F_{sw} + 1) \left[ 1 - (1 - \varepsilon_s) (1 - \varepsilon_B) (1 - \varepsilon_w) \right]} \right]}{(F_{sw} + 1)} \]  

(2.25)

\( F_{sw} \) the view factor of the bed surface from the wall and \( \varepsilon_w \) the wall emissivity.

Gorog, Brimacombe and Adams [G4] made a detailed study of the purely radiative heat transfer in rotary kilns. A large part of the work was devoted to consideration of the behaviour of the gas, which was treated as a real gas rather than a grey or transparent gas. The effect of a temperature gradient along the kiln on the radiative behaviour of the gas was also considered.

It was concluded that the grey gas approximation, if applied to gases which actually absorb radiation in distinct bands is not satisfactory. The reason for this is that the transmissivity of a real gas for its own radiation is much lower than for a grey gas (\(<0.2\) as opposed to \(0.8\)). Thus the grey gas approximation could lead to errors as great as 20% if the emissivity is less than 0.8. However if the emissivities of the wall and solids are high (\(>0.8\)) the grey gas assumption may be acceptable, since there will be little re-reflection.

It was found that 86% of the energy received by the solids came from the gas within an axial slice 0.3 kiln diameters either side of the point. Again this was explained by the low transmissivity of the gas for its own radiation. There
was no long-range radiative exchange and therefore temperature gradients had little effect on the transfer between gas and solids. The error introduced by ignoring temperature gradients in the gas was less than 4%.

In calculating the overall radiative heat exchange between the gas and solids the effect of reflection to and from the walls and absorbance by the gas was taken into account. A reflection method was used to describe the net radiant loss of the three media. After two reflections the majority of the energy emitted by the gas had been absorbed so that after two reflections the remaining energy was distributed between the wall and the solids as if they were black. After two reflections the gas was treated as transparent since all energy within the banded regions would have been absorbed previously. Thus figure 2.8 was produced, this shows the final distribution of energy originating from the gas. From this diagram the overall gas–solids radiative exchange may be determined. Figures 2.7 and 2.9 show the distribution of radiant energy originating from the solids and walls. The net gain of energy of the wall, gas and solids may be determined by summing the terms represented on these diagrams.

Examination of the equations accompanying figures 2.7, 2.8 and 2.9 shows that if any of the common assumptions; black solids or wall or transparent gas, are made the calculation of net transfer is much simplified.

Generally radiative exchange is well described by theory and radiative transfer in experimental work is allowed for rather than investigated.

2.3.3.3. Regenerative Transfer

A secondary mechanism of heat transfer between the gas and the solids is via the regenerative effect of the wall. Here the uncovered wall is heated by the gas and then transfers some of its accumulated heat to the underside of the solids when it becomes covered by the bed. This mechanism has been shown to be important in the relatively cold sections of direct fired kilns [G5] as well as in unflighted direct coolers and heaters [K3].

The heat transfer coefficients used in the determination of this heat flux have been those described in sections 2.2.1 and 2.3.1 below for gas–wall convection and wall–solids conduction respectively. The calculations are somewhat more complicated for the wall–solids element of the transfer than in
Figure 2.7 Radiative exchange from Solids

Figure 2.8 Radiative exchange from Gas
Figure 2.9 Radiative exchange from Wall

Equations corresponding to numbered heat transfer paths on figures 2.7, 2.8 and 2.9. (Lm is the mean path length)

1. \( q_s = \varepsilon_s A_s E_s \)
2. \(-\alpha_s \tau_g (2L_m) F_{ws} F_{sw} \rho_w \varepsilon_s A_s E_s \)
3. \(-\alpha_s \tau_g (3L_m) F_{ws} F_{ww} F_{sw} \rho_w^2 \varepsilon_s A_s E_s \)
4. \(- (F)_{ws} \tau_g (3L_m) F_{ww}^2 F_{sw}^2 \rho_w^3 \varepsilon_s A_s E_s \)
5. \(- (F)_{ws} \tau_g (3L_m) F_{ws} F_{sw}^2 \rho_s \rho_w^2 \varepsilon_s A_s E_s \)
6. \(-\alpha_s \tau_g (L_m) F_{ws} \varepsilon_w A_w E_w \)
7. \(-\alpha_s \tau_g (2L_m) F_{ws} F_{ww} \rho_w \varepsilon_w A_w E_w \)
8. \(-\alpha_s \tau_g (3L_m) F_{ws} F_{ww}^2 \rho_w^2 \varepsilon_w A_w E_w \)
9. \(-\alpha_s \tau_g (3L_m) F_{ws}^2 F_{sw} \rho_s \rho_w \varepsilon_w A_w E_w \)
10. \(-F_{ws} \tau_g (3L_m) F_{ww}^3 F_{sw}^3 \varepsilon_w A_w E_w \)
11. \(-2F_{ws} \tau_g (3L_m) F_{ws} F_{ww} F_{sw} \rho_s \rho_w^2 \varepsilon_w A_w E_w \)
12. \(-\alpha_s \varepsilon (L_m) A_s E_g \)
13. \(-\alpha_s F_{ws} \rho_w \tau_g' (L_m) \varepsilon_g (L_m) A_w E_g \)
(14) \[ q_w = \varepsilon_w A_w E_w \]
(15) \[-a_w \tau_g (L_m) F_{sw} \varepsilon_s A_s E_s \]
(16) \[-a_w \tau_g (2 L_m) F_{ww} F_{sw} \rho_w \varepsilon_s A_s E_s \]
(17) \[-a_w \tau_g (3 L_m) F_{ww}^2 F_{sw} \rho_w^2 \varepsilon_s A_s E_s \]
(18) \[-a_w \tau_g (3 L_m) F_{ws} F_{sw}^2 \rho_s \rho_w \varepsilon_s A_s E_s \]
(19) \[-F_{ww} \tau_g (3 L_m) F_{ww}^2 F_{sw} \rho_w^3 \varepsilon_s A_s E_s \]
(20) \[-2F_{ww} \tau_g (3 L_m) F_{ws} F_{sw}^2 \rho_s \rho_w^2 \varepsilon_s A_s E_s \]
(21) \[-a_w \tau_g (L_m) F_{ww} \varepsilon_w A_w E_w \]
(22) \[-a_w \tau_g (2 L_m) F_{ww}^2 \rho_w \varepsilon_w A_w E_w \]
(23) \[-a_w \tau_g (2 L_m) F_{ws} F_{sw} \rho_s \varepsilon_w A_w E_w \]
(24) \[-a_w \tau_g (3 L_m) F_{ww}^3 \rho_w^2 \varepsilon_w A_w E_w \]
(25) \[-2a_w \tau_g (3 L_m) F_{ws} F_{ww} F_{sw} \rho_s \rho_w \varepsilon_u A_w E_w \]
(26) \[-F_{ww} \tau_g (3 L_m) F_{ww}^3 \rho_w^3 \varepsilon_u A_w E_w \]
(27) \[-2F_{ww} \tau_g (3 L_m) F_{ws} F_{ww} F_{sw} \rho_s \rho_w^2 \varepsilon_w A_w E_w \]
(28) \[-F_{sw} \tau_g (3 L_m) F_{ww}^2 F_{sw} \rho_s \rho_w^2 \varepsilon_w A_w E_w \]
(29) \[-a_w \varepsilon_g (L_m) A_w E_g \]
(30) \[-a_w \varepsilon_{ww} \rho_w \tau_g' (L_m) \varepsilon_g (L_m) A_w E_g \]
(31) \[-a_w \varepsilon_{sw} \rho_s \tau_g' (L_m) \varepsilon_g (L_m) A_s E_g \]

(33) \[ q_g = \varepsilon_g (L_m) A_g E_g \]
(34) \[-[1 - \tau_g (L_m)] F_{sw} \varepsilon_s A_s E_s \]
(35) \[-[\tau_g (L_m - \tau_g (2 L_m))] F_{sw} \rho_w \varepsilon_s A_s E_s \]
(36) \[-[\tau_g (2 L_m) - \tau_g (3 L_m)] F_{ww} F_{sw} \rho_w^2 \varepsilon_s A_s E_s \]
(37) \[-[\tau_g (2 L_m) - \tau_g (3 L_m)] F_{ws} F_{sw}^2 \rho_s \rho_w \varepsilon_s A_s E_s \]
(38) \[-[1 - \tau_g (L_m)] \varepsilon_w A_w E_w \]
(39) \[-[\tau_g (L_m - \tau_g (2 L_m))] F_{ww} \rho_w \varepsilon_w A_w E_w \]
(40) \[-[\tau_g (2 L_m) - \tau_g (3 L_m)] F_{ww}^2 \rho_w^2 \varepsilon_w A_w E_w \]
(41) \[-[\tau_g (2 L_m) - \tau_g (3 L_m)] F_{ws} F_{sw} \rho_s \rho_w \varepsilon_w A_w E_w \]
(42) \[-[\tau_g (2 L_m) - \tau_g (3 L_m)] F_{ws} F_{ww} F_{sw} \rho_s \rho_w \varepsilon_w A_w E_w \]
(43) \[-[\tau_g (L_m) - \tau_g (2 L_m)] F_{sw} F_{sw} \rho_s \varepsilon_w A_w E_w \]
(44) \[-a_g \rho_w \varepsilon_g (L_m) A_w E_g \]
(45) \[-[\rho_w F_{ww} + \rho_s F_{ws}] \rho_w \tau_g' (L_m) \varepsilon_g (L_m) A_w E_g \]
(46) \[-[a_g' + \rho_w \tau_g' (L_m)] F_{sw} \rho_s \varepsilon_g (L_m) A_s E_g \]
2.3.4. Gas–Wall Exchange

2.3.4.1. Convection

The conduction–convection heat transfer from the gas to the wall (or vice versa) is simpler than the gas–solids exchange because the surface area and Reynolds number are unambiguous. The following correlations have been used to predict the convective heat transfer coefficient between the flowing gas and the kiln wall:

\[ h_{gw(c)} = 0.023 \frac{k_g}{D} \Pr^{0.4} \Re^{0.8} \]  
\[ (2.26) \]

\[ h_{gw(c)} = 4.17 \left( \frac{G}{A} \right)^{0.67} \]  
\[ (2.27) \]

\[ h_{gw(c)} = 17.8 \left( \frac{k_g}{D} \right)^{0.2} \left( \frac{U_g}{Cp_g} \right)^{0.8} \]  
\[ (2.28) \]

\[ h_{gw(c)} = 0.036 \frac{k_g}{D} \Re^{0.8} \Pr^{0.33} \left( \frac{D}{L} \right)^{0.055} \]  
\[ (2.29) \]

\[ h_{gw(c)} = 1.26 \left( \frac{k_g}{D} \right) \left[ \left( \frac{\Re \Pr D}{2(L-x)} \right) \right]^{0.4334} \]  
\[ (2.30) \]

Equation (2.26) is the usual equation for heat transfer for fully developed turbulent flow in a non-rotating cylinder. Equation (2.29), which was used by Gorog, Adams and Brimacombe [G5] is a correlation for non-fully developed turbulent flow in a cylinder with \( 10 < L/D < 400 \), as is equation (2.30) used by Watkinson and Brimacombe [W6]. This is quite typical of a rotary kiln where the length to diameter ratio is normally less than 50:1. The heat transfer coefficients calculated from equation (2.29) are typically in the range of 10 to 30 W/m²K; those calculated from equation (2.26) are some 25% lower.

Equation (2.27) is given in Perry [P3] and presumably results from empirical work on kilns, although no source is given. Equation (2.28) was taken from Khodorov's experimental work on kilns [K6] and was used by Manitus et al [M1].

The effect of rotational speed on the gas–wall convective transfer has
been investigated. Watkinson and Brimacombe [W6] experimented on an empty pilot kiln and found that varying the speed of rotation between 0.25 and 1.0 rpm had no effect. However Tscheng and Watkinson [T2] did find a relationship between heat transfer coefficient and rotational speed; that \( h_{gw} \) varied with \( \omega^{-0.297} \). However the data from which this relationship was derived does not justify this degree of accuracy and it is in fact questionable that the any relationship between the two quantities was demonstrated. It was suggested that the decrease in heat transfer coefficient with rotational speed was due to a stabilization of the laminar flow at the wall due to the motion of the wall. This has been shown to be the case only for small diameter pipes rotating at high speeds, it seems unlikely, therefore, that this would occur in large, slowly rotating kilns.

Tscheng and Watkinson also found that solid throughput, bed inclination and degree of filling had no effect on the gas-wall heat exchange but they did find that the coefficient varied positively with the mass flowrate of the gas. The correlation they proposed for the gas-wall convective coefficient was:

\[
\text{Nu}_{gw} = 1.54 \text{ Re}^{0.575} \text{ Re}_{w}^{-0.292}
\]  

(2.31)

where \( \text{Re}_{w} \) is the rotational Reynolds number as in section 2.1.1. If the rotational effect is neglected the heat transfer coefficient becomes a function of the gas flowrate only. In terms of \( h_{gw} \) this becomes:

\[
h_{gw} = 1.54 (k_g/\text{D}) \text{ Re}^{0.575}
\]  

(2.32)

2.3.4.2. Radiation

Much of the previous discussion of radiative exchange between the gas and the solid charge applies to the exchange between the gas and the wall.

Cross and Young gave the expression:

\[
Q_{gw(r)} = \text{Ro} J_w (T_G^4 - T_w^4)
\]  

(2.33)

\[
J_w = (1/\varepsilon_i + 1/\varepsilon_w - 1)
\]  

(2.34)
\( \varepsilon_w \) is the wall surface emissivity and \( \varepsilon_s \) is the effective emissivity from the gas flow to the wall, determined by summing the reflections between wall and solids (see section 2.1.2). The wall and the gas were assumed to be grey.

\[
\varepsilon_i = \frac{\varepsilon_g \left\{1 + F_{sw}(1-\varepsilon_g)(1-\varepsilon_s)\right\}}{(F_{sw} + 1)}
\]

(2.35)

\( \varepsilon_s \) is the solids emissivity.

\[
F_{sw} = \frac{\sin\phi}{(\pi - \sin\phi)}
\]

(2.36)

The discussion of Gorog and Brimacombe's work for radiation between gas and wall is much the same as for that between gas and solids. The net exchange may again be found by summing the terms on figures 2.7, 2.8 and 2.9.

2.3.5. Wall-Solids Transfer

2.3.5.1. Conduction

The heat transfer between the underside of the solid bed and the covered wall can be considered to be by conduction, although heat transfer within a packed bed always involves some convection and radiation in the voids (see appendix VIII). This mechanism of transfer of heat into the bed is of particular importance in an indirectly heated kiln. It should be noted that this is the only case where the majority of the heat is initially transferred to the underside of the solids, rather than to the cascade layer. In this case only a small part of the total solids burden is heated initially, whereas if the cascade layer is heated all particles in the bed are exposed to the heat source. The bulk of the bed in the indirectly heated drum is heated when the thin wall layer of hot particles mix into the bulk after tumbling down the bed surface. This mechanism is also important where the regenerative heating effect of the wall is significant.

In some cases this transfer route may be neglected without incurring large errors. Brimaconbe and Watkinson [B4,W6] found that the wall and solid temperatures in their direct fired pilot kiln were very similar and so neglected
the heat transfer between them.

It has been common to approximate wall-solids heat transfer coefficient as a multiple of the wall-gas coefficient, \([L2,R1,S4]\), usually 5 times the gas-wall convection coefficient. This is an empirical estimate based on plant data and which has no basis in the physical processes involved in the transfer.

Heat transfer between the covered wall and the bed has been considered by Wachters and Kremers [W1] and by Lehmberg et al. [L3]. In both studies the wall-solids heat transfer coefficient \((h_{ws})\) was measured experimentally in the same basic way. Sand was heated in a small scale copper drum while the drum was rotated to ensure a uniform initial temperature by mixing the solids. The outside of the drum was then exposed to rapid cooling by jets of cold water. The solids temperature was recorded during the cooling period and these temperatures used to calculate the heat transfer coefficients.

Bed temperatures in these experiments were measured using thermocouples. Wachters and Kremers used thermocouples projecting into the bed while Lehmberg et al. used a thermocouple dipped into the cascade layer of the bed. Wachters and Kremers used thermocouples at different radii to identify any radial temperature gradient present. No such variation was found but this may have been because the thermocouples disturbed the mixing pattern in the bed. In Lehmberg's experiments no attempt was made to investigate radial gradients. It is very likely that the thermocouple in the cascade layer disturbed the mixing pattern on the cascade layer. There must also be some doubt as to whether the temperatures measured by the thermocouples were those of the gas phase or of the solid phase. This is always a problem in temperature measurement in particulate beds under unsteady state conditions, where the solid temperature would lag behind the gas temperature.

Lehmberg et al attempted to improve on the experimental method by use of a device to reduce the heat losses across the upper surface of the bed. To do this they introduced a surface, held parallel to the bed surface, which had its temperature held at the bed surface temperature. This was intended to reduce any radiative loss from the bed. However it is unlikely that the radiative losses were too important in Wachters' experiments where the maximum temperature used was 90°C. Lehmberg used a maximum temperature of 200°C.
Both workers initially compared their experimental results with heat transfer coefficients predicted by penetration theory. The average heat transfer coefficient may be derived as shown below, for a semi-infinite solid, initially at temperature $T_o$ in contact at $y=0$ with a surface with temperature $T_w$. The one-dimensional Fourier equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2}$$

(2.37)

where $\alpha$ is the thermal diffusivity of the solid, yields the solution:

$$\frac{T_w - T}{T_w - T_o} = \text{erfc} \left( \frac{y}{\sqrt{2(\alpha t_c)^{0.5}}} \right)$$

(2.38)

The wall flux is given by:

$$q_w = q \bigg|_{y=0} = -k \left. \frac{\partial T}{\partial y} \right|_{y=0}$$

$$q_w = \frac{k}{(\pi \alpha t_c)^{0.5}} (T_w - T_o)$$

(2.39)

The total heat transferred over the contact time $t_c$ is then:

$$Q_w = \int_0^{t_c} q_w \, dt = 2k \left( \frac{t_c}{\pi \alpha} \right)^{0.5} (T_w - T_o)$$

(2.40)

The average heat transfer coefficient over the contact time is then:

$$h_{ws} = 2k \left( \frac{t_c}{\pi \alpha} \right)^{0.5}$$

(2.41)

The experimental heat transfer coefficients were calculated from the heat balance equation:
\[ h_{ws}A(T_b - T_w) = -\rho C_p V \frac{dT_b}{dt} \] (2.42)

where \( T_b \) is the bulk solids temperature, \( T_w \) is the wall temperature, \( A \) is the area of contact between the solids and the wall, and \( V \) is the total volume of solids.

On integration this gives:

\[ \frac{T_b(t) - T_w}{T_b(0) - T_w} = \exp\left(-\frac{t}{\tau}\right) \] (2.43)

\[ \tau = \frac{\rho C_p V}{h_{ws}A} = \frac{\rho C_p R \left( \frac{\phi}{2} \sin \frac{\phi}{2} \right)}{h_{ws} \left( \frac{2\phi}{2} \right)} \]

Having found that this model did not satisfy their experimental results both Wachters and Kremers and Lehmberg et al proposed other models to explain the discrepancy.

Wachters and Kremers identified a layer of particles in contact with the wall of the drum which preferentially returned to the wall after tumbling down the bed surface. They cited experiments with coloured particles to support this theory but unfortunately did not present any evidence of the results of these experiments. They noted that the segregation of the wall layer was especially evident at high rotational speeds. This layer of particles which did not mix with the bulk formed the basis for Wachters and Kremers heat transfer model. Lehmberg et al found no evidence of the existence of this layer in their coloured particle experiments.

In the derivation of the heat transfer model Wachters and Kremers assumed that the wall layer particles were well mixed amongst themselves, and that the remaining bulk of the bed was also well mixed. Figure 2.10 shows the distribution within the bed at the beginning and end of the contact time.

It was assumed that:
Figure 2.10 Temperature Distribution in Bed at beginning and end of contact time
- temperature of the bulk far from the wall remained constant over the contact time.
- The penetration depth was small compared to the radius of the drum, so that the curvature could be neglected.
- tangential heat transfer could be neglected.

\[
\frac{\alpha}{\partial t} = \kappa \frac{\partial^2 \theta}{\partial y^2}
\]
\[
\text{where } \theta = T_w - T
\]

with boundary and initial conditions:

\[
\theta(0, 0 < y < d) = \theta_i \quad \theta(t, 0) = 0
\]
\[
\theta(0, y > d) = \theta_b \quad \theta(t, \infty) = \theta_b
\]

The solution, found by superposition of plane sources is:

\[
\theta = \theta_i \text{erf} \left( \frac{y}{2(\alpha t)^{0.5}} \right) + \theta_b - \theta_i + \text{erf} \left( \frac{y + d}{2(\alpha t)^{0.5}} \right) + \text{erf} \left( \frac{y - d}{2(\alpha t)^{0.5}} \right)
\]

It was also assumed that the average temperature of the layer at the wall (i.e. 0 < y < d) does not change over the contact time, that is that the temperature change is small enough to be neglected. The average heat transfer coefficient was then found from:

\[
h_{ws} = \frac{1}{t_c \theta_b} \int_0^{t_c} k \theta(t, 0) \, dt
\]

\[
\sigma = \frac{t}{t_c}, \quad \eta = \frac{y}{2(\alpha t_c)^{0.5}}, \quad \delta = \frac{d}{2(\alpha t_c)^{0.5}}, \quad \Theta = \frac{\theta}{\theta_b}
\]

\[
\Theta_i = \frac{\Theta_i}{\Theta_b}, \quad \Psi = \frac{2k}{h_{ws} (\pi \alpha t_c)^{0.5}}
\]
\[ \psi^{-1} = \frac{h_{ws}(\pi \sigma t_c)^{0.5}}{2k} = 1 - \frac{\phi^2(\delta)}{2\phi(\delta) - 0.5\phi(2\delta)} \] (2.47)

\[ \phi(\delta) = \delta(\pi)^{0.5}\text{erf} \delta - \exp(-\delta^2) + 1 \]

Three special cases were identified:

1. Where there was no wall layer formed, i.e., \( \delta = 0 \) and \( \Psi = 0 \), this gave

\[ h_{ws} = \frac{2k}{(\pi \sigma t_c)^{0.5}} \] (2.48)

This is identical to equation (2.37).

2. Where \( d > 4(\sigma t_c) \) i.e., \( \delta > 2 \) and \( \Psi = 3 \), this is the case where the wall layer thickness is greater than twice the penetration depth, here the heat transfer coefficient is given by:

\[ h_{ws} = \frac{2k}{3(\pi \sigma t_c)^{0.5}} \] (2.49)

which is three times smaller than the case where there is no wall layer.

3. An intermediate case where \( \delta < 0.6 \) and \( \Psi(\delta) = 1 + 2\delta \) where

\[ h_{ws} = \frac{2k}{d\pi^{0.5} + (\pi \sigma t_c)^{0.5}} \] (2.50)

Wachters and Kremers plotted \( \tau F(\Phi) \) against \( \omega^{-0.5} \), where

\[ F(\Phi) = \Phi^{0.5}(\Phi \sin \Phi)^{-1} \] (2.51)

and \( \tau \) is as in equation (2.39) for case 1.
\[ \tau F(\Phi) = \frac{R}{4}(\pi - \omega)^{0.5} \]  

(2.52)

for case 2

\[ \tau F(\Phi) = \frac{3R}{4}(\pi - \omega)^{0.5} \]  

(2.53)

and for case 3

\[ \tau F(\Phi) = \frac{R}{4}(\pi - \omega)^{0.5} + \frac{R}{4}(\pi - \omega)^{0.5} \cdot \frac{d}{\Phi^{0.5}} \]  

(2.54)

Plots of \( \tau F(\Phi) \) against \( \omega^{-0.5} \) for equations (2.52) and (2.53) straight lines passing through the origin with the gradient of the second case being three times that of the first. Equation (2.54) also produced a straight line with a gradient identical to that of (2.52) but with the intercept on the \( \tau F(\Phi) \) axis at the point

\[ \tau F(\Phi) = \frac{R}{4}(\pi - \omega)^{0.5} \cdot \frac{d}{\Phi^{0.5}} \]  

(2.55)

Clearly if \( d/\Phi^{0.5} \) is not known, as is the case for all practical situations, equation (2.54) cannot be evaluated. Wachters and Kremers used a value of \( 1.12 \times 10^{-5} \)m in their calculations, which gave the best fit for their experimental data.

It was found from the plot of \( \tau F(\Phi) \) against \( \omega^{-1} \) for the experimental data that case 1, simple penetration theory, never applied. Case 2, where the thickness of the wall layer was greater than twice the penetration depth fitted the data well at relatively high rotational speeds ( >1 rad/s or 12% of the critical speed). The results indicated that at lower speeds the wall layer thickness, \( d \), decreased. Increasing contact times, corresponding to decreasing rotational speed, led to increasing interference between the temperature profiles at the two boundaries of the wall layer. Wachters and Kremers observed a change on bed flow pattern at 1 rad/s, probably from rolling to
cascading behaviour.

This model successfully accounted for the experimental data presented.

Lehmberg et al. also proposed a layer at the wall which restricted the heat transfer. In this case the layer was of gas rather than being a special part of the solid bed. The basis on which this assumption was made was the existence of a similar layer between the solids and wall in gas fluidized beds [B6,D2].

The similarity between the situation in a fluidized bed and in a rotary drum is not obvious. This thin layer of gas between the wall and the solids would cause a temperature jump at the wall (i.e. at \( y = 0 \)). The heat flux across this thin film at \( y = 0 \) is given by

\[
q_0 = h_g (T_w - T_{g0})
\]  

(2.56)

where \( T_{g0} \) is the temperature of the gas at the wall and

\[
q_0 = -k_g \frac{\partial T}{\partial y}_{y=0}
\]  

(2.57)

(by definition of \( k_g \), the thermal conductivity of the gas.)

Since there is no storage of heat in the gas layer

\[
\left( \frac{\partial T}{\partial y} \right)_{y=0} = - \frac{h_g}{k_g} (T_w - T_g)
\]  

(2.58)

Equation (2.57) is the boundary condition for this model. The initial condition is:

\( T = T_0 = \text{const at } t=0 \text{ for all } y \)

The solution is
\[ T = T_w \left[ \text{erfc} \left( \frac{y}{2 \sqrt{\alpha t_c}} \right) - \exp \left( \frac{h_g y + \left( \frac{h_g}{k_g} \right)^2 \sqrt{\alpha t_c}}{k_g} \right) \right] \]

\[ \cdot \text{erfc} \left( \frac{y + \frac{h_g}{k_g} \sqrt{\alpha t_c}}{2 \sqrt{\alpha t_c}} \right) \quad \text{for } 0 < y < \infty \]  

(2.59)

This leads to the solution for \( h_{ws} \):

\[ h_{ws} = b_e \left( \frac{2}{\pi} - \frac{k_g}{h_g \sqrt{\alpha t_c}} + \frac{k_g}{h_g \sqrt{\alpha t_c}} \right) \cdot \exp \left( \frac{h_g \sqrt{\alpha t_c}}{k_g} \right) \text{erfc} \left( \frac{h_g \sqrt{\alpha t_c}}{k_g} \right) \]  

(2.60)

where \( b_e \) is the penetration depth.

If the dimensions of equation (2.60) are considered the heat transfer coefficient appears to have units \( \text{ms}^{-0.5} \) which is clearly not correct. The source of this anomaly is the definition of penetration depth used. The penetration depth for heat transfer as normally defined in heat transfer textbooks (e.g., Myers [M6]) is the distance from a heat source at which the change in temperature over a given time is equal to a set fraction of the temperature change at the surface in contact with the heat source. The fraction used varies, but the dimension of the penetration depth, as defined in this way, is always length.

Lehmberg et al. defined the penetration depth as:

\[ b_e = \sqrt{k C_p \rho} \]  

(2.61)

This produces units for the 'penetration depth' of \( J/m^2 \text{Ks}^{-0.5} \), clearly not length. If these unusual units are used in equation (2.60) the dimensions of the heat transfer coefficient are correct. The equation is therefore consistent, it is the definition of its variables which is unconventional. The confusion is not lessened by the absence of any units or dimensions in the nomenclature of this paper, apart from for the contact time and the thermal properties of the materials used.
An inconsistency in equation (2.60) which cannot be explained by the definition of the penetration depth lies in part of the third term of the equation;

\[ \text{erfc}\left(\frac{h_g c t_c}{k_g}\right) \]

The expression within the brackets here has dimension length. In order to have a meaningful error function value the expression here should have been dimensionless. There are other minor examples of what are presumably typographical errors in the mathematics of this work.

Lehmberg et al proceeded by plotting experimental results in the form of \( h_{WS} \) against the inverse of the contact time. They then fitted equation (2.58) to these graphs to produce values for the ratio of \( h_g \) to \( k_g \), showing that with increasing particle diameter the gas film thickness increased. No other factors were discussed which might have affected the ratio of heat transfer coefficient to thermal conductivity. It is possible that some property of the solids other that the particle size may have affected this ratio since experiments with sand and sodium carbonate with similar diameters produced a large difference in the value of \( h_g/k_g \). If these results are compared to those for different sizes of sand it appears clear that some other effect was present.

<table>
<thead>
<tr>
<th>particle size</th>
<th>sand</th>
<th>soda</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{WS} )</td>
<td>157</td>
<td>794</td>
<td>( \mu m )</td>
</tr>
<tr>
<td>( k_g )</td>
<td>3607</td>
<td>1330</td>
<td>( m^{-1} )</td>
</tr>
</tbody>
</table>

When equation (2.60) was fitted to the experimental data the behaviour of \( h_{WS} \) was well described by the model.

Direct comparison of the two sets of data from the two papers is difficult because there are several parameters for which values are not given in the two studies. Wachters and Kremers did not give the density, thermal conductivity or heat capacity of the sand they used in their experiments. This made it impossible to extract the values of the heat transfer coefficients from their data without guessing these quantities. Lehmberg did not report the degree of filling used in his experiments, so that the data from those experiments cannot be plotted in the form used by Wachters.
Lehmberg stated that the heat transfer coefficients in Wachters experiments were double those in his own work. It is not obvious, given the problems in calculating coefficients from Wachters' data, how this statement was arrived at.

It is possible to fit equation (2.50) to Lehmberg's data and the fit appeared to be as good as that for equation (2.59). Smaller values of \( d \), the thickness of the wall layer were needed than for Wachters' own data. It was also necessary to use more than one value of \( d \) for each set of results to cover the full range of contact times used. This is not unreasonable since Wachters and Kremers observed that the wall thickness depended on the rotational speed of the drum.

Both models describe the experimental data well, but both depend on parameters which can only be found by fitting equations to experimental data. The actual reason for the discrepancy between penetration theory and the experimental heat transfer has not been identified by the work done to date.

2.3.5.2. Radiation

Again Gorog and Brimacombe provide the most definitive account of the radiative exchange between these two media. Figures 2.7, 2.8 and 2.9 provide the basis for calculating the net transfer of radiant energy between the wall and the solids.

2.3.6. Conduction Through the Wall and Losses to Surroundings

The two remaining heat transfer routes; through the wall and from the outer wall to the surroundings, depend only on the properties of the wall and the conditions outside the kiln once the wall temperature has been established. These are of less fundamental interest than the internal heat flows since the heat transfer coefficients which can be derived for these processes are not directly related to the behaviour of the bed. These heat fluxes are however of great importance in the development of overall heat balances in kilns, where the heat losses through the wall may be substantial.

2.4. Overall Kiln Simulation

The first model of a rotary kiln which took into account the heat transfer within the kiln was by Gilbert [G3] who made a detailed study of the operation of a cement kiln. He was concerned principally with the overall heat balance
over the kiln and used plant data rather than calculated heat transfer coefficients to formulate his model. The detailed data presented in this work has been of use to other workers in testing their models [S4,11]

Lyons et al. [L2] also modelled a cement kiln, using an analogue computer. They incorporated heat transfer, chemical reaction and drying considerations in their simulation, which consisted of 11 differential equations and 11 algebraic equations covering the heat and mass balances, reaction rate and calculation of parameters such as heat transfer coefficients. The source of the heat transfer coefficients used was not given, it is possible they were derived from plant data. The convective coefficient for transfer between gas and wall \( h_{gw} \) used was larger than the gas-solid coefficient \( h_{gs} \), which has been shown to be incorrect [T2,W6]. The wall-solid coefficient was some 8 times larger than that for the gas-solid exchange, which is unusually high.

Sass [S4] presented a simplified model of kiln heat transfer which did not take into account chemical reaction or mass transfer. The model was intended to be general so that it could be applied to any kiln used purely for heating solids. The heat transfer coefficients were determined from correlations available in the literature at that time (1964), which was before much of the experimental work described above appeared. The gas-wall and gas-solid coefficients were assumed to be equal and were determined from equation (2.18) above with the addition of a term to account for radiation. This radiation term included a correction factor, the value of which was determined by fitting the model to existing plant data. This correction term accounted for absorption by the gas as well as the view factors. The wall-solid coefficient was taken to be five times the gas-solid coefficient, Sass acknowledged that this was at best a rough approximation.

An alumina kiln was simulated by Riffaud et al. [R1] in a model which took heat transfer and chemical reaction rates into account. The heat transfer coefficients for gas-wall and gas solid convection were calculated from (2.18), as in Sass [S4]. The value of the coefficient for the wall-solid transfer was taken from Lyons [L2].

Wingfield et al. [W9] modelled the heat and mass transfer and reaction in an iron ore reducing kiln. Again heat transfer coefficients between gas and wall and gas and solids were assumed equal. In this case equation (2.17) above was
used to calculate these coefficients. No value for the wall to solids coefficient was given, neither was its method of calculation. The gas was assumed to be transparent, and the solids and wall black to simplify the model.

Dust carry-over, chemical reaction and drying as well as heat transfer were considered by Manitus et al. [M1] in a model of an alumina kiln. Allowance was made for the variation of the thermal properties of the materials with temperature, which previous workers had neglected. The convective coefficients \( h_{gs} \) and \( h_{gw} \) were calculated from (2.17). The wall–solid heat transfer was calculated from empirical work by Khodorov [K6].

Venkateswaran and Brimacombe [V3] modelled the complete process in an iron ore kiln. Again heat transfer coefficients for the convective transfer were assumed to be equal and calculated from (2.18), with the wall–solids conduction coefficient assumed to be five times greater.

All these overall simulations were developed before the bulk of the detailed experimental work was carried out, which accounts for the simple correlations used to calculate the heat transfer coefficients.

There seems to have been no recent work on kiln simulation, perhaps because the method is well established and the main refinements which can be made to the existing models lie in the use of the improved heat transfer correlations now available.

2.4.1. Summary of Heat Transfer Literature

Of the five heat transfer routes identified at the beginning of this section three are of particular interest; gas–solid exchange, gas–wall exchange and wall–solid exchange.

Heat transfer between the gas and the wall and between the gas and the solid bed are well documented. Both convective and radiative transfer have been studied for these routes, and correlations which correspond well to experimental data are available.

The conduction from the wall to the solid bed is less well explored and the two studies which have been made of this mechanism produced conflicting conclusions. The heat transfer rate from the wall to the solids is lower than that predicted by simple penetration theory, assuming that the bed is radially
well mixed. The proposed explanations for this discrepancy are that there is a layer of either gas [L3] or relatively hot solids [W1] between the bulk of the bed and the wall, which has an insulating effect.

The radial mixing behaviour of the bed is important in indirectly heated kilns where the mixing of the hot wall layer into the bulk controls the heat transfer into the bulk, but is less important where initial heat transfer is to the cascade layer.

Overall heat transfer simulation of drums is a well established technique, although there has been little work reported recently.

2.5. Overall Summary

The two main groups of previous work on rotary drums; residence time and heat transfer experimentation and modelling have been outlined in this review. In general heat transfer is the more developed area, with correlations available for the calculation of most of the heat fluxes within a drum from basic operating parameters of the drum. The only aspect of heat transfer which remains to be adequately explained is the transfer of heat from the wall to the solids burden by conduction.

Residence time distribution models developed to date depend heavily on extensive experimental data for the particular drum being modelled. Only one method [A2] has been developed by which data on the residence time distribution in one drum may be used to predict the behaviour of the solids in a different drum, this method is only applicable to horizontal drums. Methods of predicting the mean residence time by modelling the motion of particles in the bed have been developed which give good results over a limited range of conditions.

An important factor which has been identified is the relationship between the radial mixing of the solids and the heat transfer from the wall. The assumption that the bed is well mixed does not give good agreement with experimental results in this case. Radial mixing is also important in residence time modelling. The particle motion models leading to estimates of the mean retention time in drums are based on the assumption that the bed is unmixed.

The present work aims to investigate the radial mixing of a bed under a
range of experimental conditions and to use the information thus collected in modelling heat transfer from the wall and residence time distribution in solid beds in rotating drums.
3.1. Introduction

Examination of previous work on heat transfer and residence time in rotating drums has identified transverse mixing as an important factor in both areas. Transverse or radial mixing occurs when particles leave the fixed part of the bed at one radius and re-enter it at another, after tumbling down the cascade layer.

Previously it has been assumed that the bed is either a) 'well mixed', i.e. that the re-entry point of particles is random, or b) that the bed is segregated; each particle always returning to the radial path from which it emerged onto the cascade layer. In general heat transfer work has been based on the assumption the bed is well mixed, so that it could be assumed that there are no radial temperature gradients in the bulk of the bed. However in work concerned with the prediction of bed profile and mean residence time in drums the segregated type of bed motion has been widely assumed as the basis of models of the solids motion [S7]. The core has never been included in modelling either heat transfer or residence time.

The fact that both the well mixed model and the segregated model of bed motion have been used with some success in models describing different aspects of beds in rotating drums could suggest that the true mixing pattern lies between these two extremes. It is also true that in the majority of the heat transfer mechanisms, where heat is initially transferred to the cascade layer, the actual extent of radial mixing is not very important because all the fixed bed particles pass down the cascade layer and are heated. In the case of heating from the wall of the drum [L3,W1] it has been found that the well mixed assumption does not produce a model which is in agreement with experimental measurements, again suggesting that the actual motion of the bed is not the well mixed motion proposed.

Previous investigations of the radial mixing in rotating drums have been undertaken using coloured particles to follow the mixing pattern. Both Wachters
and Kremers [W1] and Lehmbreg et al [L3] carried out experiments of this type.

Lehmbreg et al published photographic records of their experiments. The method used was to replace a section of a stationary bed with a block of coloured particles having the same physical properties. The bed was then rotated and the distribution of the coloured particles in the bed recorded photographically. The distribution of the coloured particles through the bed was used as a qualitative measure of the radial mixing in the bed. However it is uncertain how conclusive such experiments are, since it is impossible to discern mixing between particles originating at different radii from mixing amongst particles from the same radius in the bed.

Wachters and Kremers gave no detailed account of the experimental method they used in their investigation of mixing. It is possible that their method differed from that of Lehmbreg et al. The major finding reported from Wachters and Kremers' experiments was that particles originally adjacent to the wall tended to return to the wall preferentially after tumbling down the cascade layer. This wall layer of particles which did not mix with the bulk formed the basis of the heat transfer model produced by Wachters and Kremers. No information about the behaviour of the inner bulk of the bed was published.

Lehmbreg did not agree that such a wall layer existed, though it is not clear how the type of experiment they used could prove or disprove the existence of such behaviour.

At best the experiments described above could provide only crude qualitative information about the bed motion. A more quantitative study of the mixing behaviour of the bed, and the factors influencing it would be useful in modelling both heat transfer and residence time.

Radial mixing depends on the relationship between the radii of the paths of particles in consecutive passes through the fixed bed. If the particles always re-enter the fixed bed from the cascade layer at the same radius from which they left it the bed is completely unmixed or segregated. If the radius at which particles re-enter the fixed bed is independent of their previous radius the bed is well mixed. Between these two extremes the path radii for consecutive cycles, conveniently represented by the points at which particles emerge onto the cascade layer and re-enter the fixed bed on the lower half of the bed, are linked by some function giving the probability that a particle leaving from one
radius will re-enter at another.

In the core area particles do not behave in the same way as those moving between the fixed bed and the cascade layer. Solids in the core tend to remain there for a relatively long period of time. There is a limited amount of mixing between the upper surface of the core and the lower surface of the cascade layer which allows some interchange of solids from the core with those from the fixed bed. However in considering the two extremes of bed motion, the well mixed and segregated models, the core will be assumed not to mix with the cascade layer at all, since this represents the idealized extreme behaviour of this part of the bed.

The passage of solids from the fixed bed into the core from the cascade layer can be represented by a probabilistic model, in the same way as transitions between different radii in the fixed bed can. For a particle passing out of the core into the cascade there are two stages to consider. Firstly there is the probability that a given particle will escape the core in a given period of time. Secondly there is the probability distribution governing the point at which a particle, having escaped from the core, will enter the fixed bed.

If the fixed bed is divided into a number of discrete concentric layers of equal depth the probabilities linking the exit and re-entry points in the fixed bed and the core may be combined with the rates at which solids leave each layer of the bed to form a 'mixing matrix' which describes the transverse motion of the bed.

The mixing matrix $M$ consists of elements $m_{ij}$ where $m_{ij}$ is the rate at which material leaves layer $i$ and re-enters the bed at layer $j$. The matrix is square, with order equal to the number of layers in the fixed bed plus one. The extra row and column of the matrix represent the behaviour of the core.

The element representing transitions from the core to the core (the bottom right hand corner of the matrix) is always zero. Unlike the fixed layers there is no fixed rate at which particles must emerge from the core. Only transitions between the core and a fixed layer are considered to be significant. The case where particle escapes briefly from the core on to the cascade to return to the core is equivalent to the particle simply remaining in the core since the boundary of the core and cascade layer is not well defined. The probability of a particle escaping the core to enter the fixed bed can be calculated from the volume of the core and the rate at which particles leave and enter it.

The sum of the elements of any row or column corresponding to a layer
of the fixed bed is equal to the overall rate at which material enters and leaves that layer of the bed. This can be calculated from the radius of the layer, the thickness of the layer and the drum speed:

$$\sum_j m_{ij} = \omega r_i \delta$$  \hspace{1cm} (3.1)

where $\omega$ is the drum speed, $r_i$ is the mean radius of layer $i$ and $\delta$ is the thickness of each layer.

The elements of row $i$ represent rates of material leaving layer $i$ while the elements of the $i^{th}$ column represent material entering layer $i$. Clearly the sum of the elements of layer $i$ must be equal to the sum of the elements of the $i^{th}$ column. It should be noted the different layers of the fixed bed have different throughput rates.

In the case of the core the sum of the elements of the corresponding row represents the rate at which material passes into the core, this is of course equal to the sum of the column representing particles leaving the core.

The sum of all the elements of the mixing matrix is the total rate at which solids emerge onto the cascade layer from the fixed bed.

Examination of such mixing matrices gives useful quantitative information but can also provide accessible qualitative information on the bed behaviour, especially if the matrices are displayed in the form of 3-dimensional histograms. In these histograms a marked peaking of the histogram along the diagonal represents a mixing pattern where the solids tend to return to the layer from which they emerged. A relatively flat histogram represents a well mixed bed. Histograms representing the two extreme patterns are shown on figures 3.1 and 3.2. In these histograms the the rows of the matrices are represented by the lines of blocks sloping upwards from left to right and the columns are represented by the lines of blocks sloping downwards from left to right, as shown on figure 3.1. Thus the shaded block on figure 3.1 corresponds to solids leaving layer 5 of the bed and entering layer 3 because it is in the 5^{th} row and the 3^{rd} column.

Mixing matrices have been used in the present work in developing models of residence time distribution and of heat transfer from the wall to the solids.
Figure 3.1 Histogram from 'Well Mixed' matrix

Figure 3.2 Histogram from 'Segregated' matrix
As explained before information of this detail on the transverse mixing in drums has not been produced before and allows a completely new approach to the modelling of heat transfer and residence time distribution.

A second matrix can be defined which is related to bed mixing. This is the 'transition matrix', $T$, which records the transitions of a single particle from one layer of the bed to another. Thus the element $t_{ij}$ of $T$ is the number of times that the particle makes the transition from layer $i$ to layer $j$ in some period of time. If the number of transitions observed is large this transition matrix can be used to produce the mixing matrix of the bed.

The mixing matrix as defined here is made up of the rates at which material passes between the layers of the bed. The transition matrix is made up of the number of times that particle is observed to make each transition during the experimental period. If the total number of transitions is large enough then the ratios of the elements of the transition matrix and the corresponding elements of the mixing matrix should be the same for all elements. The mixing matrix is then obtained by multiplying the elements of this transition matrix by the appropriate constant of proportionality.

For a particle transition matrix $T$, with elements $t_{ij}$

$$m_{ij} = kt_{ij} \quad (3.2)$$
$$\text{and} \quad \sum_j m_{ij} = k \sum_j t_{ij} \quad (3.3)$$

The constant $k$ can be obtained from calculation of the rate at which solids leave the fixed bed and the sum of all the elements of the transition matrix.

The path of a single particle should ideally be followed in a three-dimensional bed with solids passing through it. However it would be very difficult to follow a particle in such experiments. It would not be possible to follow it visually since the particle would spend a large majority of its time in the interior of the bed. The most feasible way to study the bed behaviour is in a two-dimensional model. This is effectively a slice of the drum one particle deep.
A vertical slice of a three dimensional drum would differ from the two dimensional slice used here as there would inevitably be particles overlapping the boundaries of the slice. These particles overlapping the boundary would interlock with the particles in the slice. This interlocking would probably help to maintain the fixed nature of the fixed bed.

On the cascade layer the voidage is much higher than in the fixed bed region. Here the interlocking effect would be much less significant in this part of the bed. There would however be scope for longitudinal mixing due to random collisions between adjacent particles, which cannot be accounted for in two dimensional experiments.

There could also be longitudinal mixing in the core region. The behaviour of the core could be different in a three dimensional bed. These factors should certainly be the subject of investigation in the future, possibly by using a bed say one and a half times the particle diameter.

There have been previous experimental studies in which the behaviour of batch drums, where there is no throughput of solids or axial advance on the cascade layer, has been taken to behave in the same way as drums with solids flow [L3,W1,R2].

Taking into account the extreme difficulty in following the path of a particle in a 3-dimensional bed it was decided to use a 2-dimensional drum in the present experimentation.

3.2. Experimental Apparatus

The design of the experimental drums is shown on figures 3.3 and 3.4. The solids were contained between the two walls of the drum. The back wall was painted matt black and the front wall was of transparent perspex. The space between the walls was slightly wider than the diameter of the particles used, to allow the solids to move freely in the plane of the drum. The front wall of the drum was removable to allow the solids to be introduced. The experimental drums used had diameters 300mm and 450mm.

The drum was rotated on two rubber coated rollers, one of which was driven by a motor through a variable gear box. The drive mechanism is described in more detail by Scott [S7]. The drum drive table on which the rollers were mounted could be inclined but all the experiments in the present work were carried out with the drum horizontal.

The particles used were spherical ceramic catalyst support pellets. These had diameters between 6.5 and 7.5 mm.
Figure 3.3 The experimental drum

Figure 3.4 Section through Drum
The obvious method of identifying a single particle in a bed of similar particles is to colour it differently from the remainder of the bed. Both a single white particle in a bed of black solids and a single black particle among white were tested. The single white particle was eventually chosen because of difficulty in distinguishing the one black particle from interparticle shadows in the white bed. The back of the drum was painted matt black to eliminate reflections from this surface which would have made identification of the white ball difficult.

The catalyst pellets in their raw state were an off white colour so that the bulk of these had to be coloured black. Matt black enamel spray paint was used for this purpose. The pellets to be painted were placed in a shallow tray, so that there was only one layer of particles in the tray. The spray paint was then applied, agitating the tray so that the particles were completely coated. The paint was then allowed to dry naturally with occasional agitation of the tray to prevent the solids sticking to it as they dried. It was found that this coating was tough enough to withstand the wear from tumbling in the bed over the length of an experimental run satisfactorily. There was some wear of the paint but this was acceptable so long as the black particles remained easily distinguishable from the white one.

Coating the white particle was more difficult. Although the pellets were light in colour in their natural state it was desirable to have as wide a contrast between the black and white particles, so that a brilliant white coating was needed. A major problem with the white particle was its tendency to pick up fragments of black paint from the black solids as they wore during the experiments. This made identification of the white particle very difficult after a period of time in the drum. There was also a tendency for the white coating to chip off, adding to the difficulty of identifying it. The coating which was found to minimize both the adherence of black fragments and the chipping of the white coating was white nail polish. This was very resistant to chipping, since it contained nylon precisely to resist chipping and cracking. It also provided a high gloss finish which tended not to pick up black paint fragments.

In some of the experiments the particles' surface characteristics were altered by applying a different coating. Several experiments were carried out with PTFE coated particles, these were produced by spraying the painted particles with PTFE from an aerosol spray. In other experiments the particles
were roughened by coating them with fine sand. This was achieved by first coating the pellets thinly with glue and then placing them in a beaker of the sand which was shaken to ensure that each particle was coated with the sand. A water soluble glue was used so that the sand could later be removed and the particles recoated. The sand coated particles were then removed from the loose sand by sieving and allowed to dry completely before being painted as normal. There were some problems of wear of this sand coating but this was alleviated by replacing the worn particles with fresh solids part of the way through the experimental run.

3.3. Video Analysis System

It was necessary to record the position of the white particle in the drum described above over at least 2000 of its passages down the cascade layer. It was thus not feasible therefore to record these positions manually. The experimental runs were recorded on video tape and this tape was analysed afterwards by means of a BBC microcomputer and a video digitizing system. A video digitizer is a device which converts a standard video signal into digital information which can be stored and analysed by the computer.

The aim of the experimental procedure was to detect and the record the position of the white particle in the fixed bed each time it had passed down the cascade layer.

The system for analysing the video recording of an experimental run had then to include three features:

1. The ability to detect and record the position of a white particle in a digitized image.
2. The ability to record the particle's position exactly once on each of its cycles through the bed.
3. The ability to ignore the particle when it was on the cascade layer. This was important since the position of the particle in the cascade provides no information about its eventual path radius in the fixed bed.

All these elements were successfully incorporated into a system based on a computer program which controlled the video playback, analysed images and
recorded the white particle's positions. This system proceeded as follows:

1. Replay of video recording was stopped by the computer and an image of part of the fixed bed (see figure 3.5) was digitized and analysed to find if the white particle was visible in that part of the bed.

2. If the particle was visible its position was recorded, otherwise the recording was allowed to move forward for a short time and paused again. Steps 1 and 2 were repeated until the white particle was successfully located in the part of the bed being examined.

3. Once the particle had been found the recording was allowed to run forward for just under the time taken for the particle to go through one cycle through the fixed bed and the cascade and the process began again from step 1.

In this way the path of the particle was recorded over the required number of cycles.

It was originally intended to take the video signal directly from the video recorder output to the digitizer. However this was not possible because of the mode of operation of the recorder. The image had to be collected while the recorder was 'paused' because the digitizer required 5 seconds to generate the digitized image. The video image produced by the recorder while paused was produced by rapidly alternating between two frames on the tape. This switching between frames caused a large amount of 'noise' on the digitized image which made this image impossible to analyse. This problem was solved by collecting the images for digitizing indirectly from a video camera pointed at the video monitor. This effectively filtered the noise produced by the alternating images and gave a clear digitized image. Figure 3.6 shows the arrangement of the monitor and the camera. The distance from the screen to the camera could be altered by sliding the plate to which the camera stand was fixed to bring the desired area of the bed into view. During analysis of the experimental tapes the monitor and camera were enclosed in a lightproof box to eliminate reflections from the screen.

The problem of detecting the particle only when it was in the fixed bed was solved by masking the unwanted areas of the screen with black paper. In this way the cascade layer could be obscured. This also allowed irrelevant
Figure 3.5 Digitized area of screen

Figure 3.6 Arrangement of monitor and camera
areas of the video image, such as the drum rollers and bearings to be eliminated from the digitized images (see figure 3.6).

The image was displayed on the screen of the computer, that is to say the digitized information was stored in the screen memory of the machine. The information stored in screen memory may be examined and manipulated in the same way as any other computer memory. The way in which the screen memory is arranged is complicated (see Appendix IV)

Locating the white particle's position meant essentially finding an area of the screen with a high proportion of bright pixels. To do this a program was written to examine each memory location in turn. The number of '1's in the binary number stored in each byte of memory is counted and compared to the limit chosen for defining a 'bright byte' (usually 7 lit pixels were required).

Once a bright byte is found other memory locations adjacent to it are examined to test whether a relatively large white area has been identified. If four adjacent memory locations were found to contain a total of 28 or more '1's then it is assumed that the white particle has been found. The program then found the centre point of the white area by identifying the upper and lower-most bright bytes and those on the extreme right and left of the bright area. Converting the memory location to x-y coordinates for further calculation was a simple arithmetic exercise.

The analysis of each image by the computer using BASIC was prohibitively slow, each image taking around ten minutes. The program was therefore translated into assembly code to speed it up, giving a processing time for each image of less than 15s. The program used assembler up to the point where the block of four bright bytes was found and then reverted to BASIC to locate the centre point. This analysis program is described in more detail in Appendix IV.

The system described above produced the x-y coordinates of the white particle each time it passed through the fixed bed. In order to be useful these coordinates had to be converted to give the radius of the particle's position in the bed. To do this the coordinates of the centre point of the drum were needed. These were found at the beginning of each experimental run by marking a point on the circumference of the drum and a point at the mid-point of the radius to the rim mark. The coordinates of these points as determined
from their digitized images were then used to calculate the coordinates of the centre point of the drum. The scale of the digitized image was also calculated from the coordinates of these points. The centre point coordinates were used as an input to the analysis program so that the recorded positions of the white particle were in terms of its radius. These radii were in units of pixels on the computer screen, which could be converted to mm in the original bed by using the scale calculated from the coordinates of the reference points. The analysis of a complete experiment took between 40 and 100 hours of continuous operation of the computer and video system.

3.4. Experimental Procedure

For each experiment the required number of particles was prepared with the appropriate coating and placed in the drum. A point on the circumference of the drum and one on the mid-point of the radius to the rim point were marked by means of small circular self-adhesive labels. These provided the reference points for the calculation of the coordinates of the centre of the drum and of the scale of the digitized image. The camera was turned on and the drum was turned by hand so that three different positions of the reference points were recorded. This allowed the centre point coordinates to be calculated from three sets of reference point coordinates for confirmation of its position.

The reference point labels were then removed and the drum motor was turned on and adjusted to the correct speed. The motion of the bed was recorded for three hours. The particles were checked occasionally to ensure that the contrast between the white particle and the black solids remained clear enough for the analysis system.

The analysis of the tape proceeded as described above. The centrepoint coordinates were found and used as an input to the analysis program. The time for the particle to pass through the bed once was estimated and adjusted so that the white particle was recorded each time it passed through the bed. The output of the analysis system was a series of the radius of the particle as it passed from one layer of the bed to another. To form the particle transition matrix from the data produced from the analysis system, which is in the form of a series of particle radii, the bed must be divided into discrete layers. Since the particles in the fixed bed tended to arrange themselves in well defined
layers it was convenient to use these layers in the transition matrix. The number of fixed bed layers was determined from observation of the bed.

The processing of the crude particle radius series into a transition matrix was done in two stages:

1. The radii were converted into layer numbers
2. The layer number series was used to form the transition matrix.

The layer number corresponding to each radius was determined by setting upper and lower limits on the radius of each layer and determining which of these layers the observed particle fell into.

The particle transition matrix was constructed from the layer number series by looking at each pair of adjacent layer numbers, representing the particle's transition from one layer to the other and incrementing the corresponding element of the transition matrix by 1.

By means of the method described above the path of a single particle in a 2-dimensional bed could be followed with no disturbance to the motion of the bed. The transition matrix produced can be used to derive a mixing matrix required for modelling residence time and heat transfer in beds. As stated previously the aim of the experimental work was to investigate the effect of drum speed, filling fraction, particle coating and drum diameter on the mixing behaviour of the bed. To this end experiments were carried out in two drums of different diameter (300 and 450mm) at different speeds and fillings with the particles described in the previous section. The rotational speeds used were in the range 8.7% to 23% of the critical speed (the speed where centrifuging begins). All the experiments were carried out under either rolling or cascading bed motion (see section 2.1). The three filling fractions used were 0.16, 0.20 and 0.24. At low speeds the lowest filling gave some problems of slip at the wall so that this filling was not used at the lowest rotational speed. The highest rotational speed produced an increase in the wear suffered by the painted particles and was not used with the sand and PTFE coated solids for this reason. The sand and PTFE coated particles were not used in the larger drum because the wear was more serious in this drum. A summary of the experiments carried out appears in Appendix VII.
No experiments with a range of particle sizes were carried out, although segregation effects would probably affect the mixing behaviour of the bed. It is well known that relatively large lumps of material in an agitated bed tend to rise to the surface and if this were to occur on the cascade layer of a bed in a rotating drum this could lead to segregation in the fixed bed. If different mixing patterns for different sizes of particles in a bed were expected then separate experiments would have to be performed, following the path of a particle of each different size. This would be time consuming if there were several sizes of particle. The space between the drum walls would have to accommodate the largest size of particle which would lead to there being more than one thickness of the smaller particles. This would mean that these smaller particles would not always be visible from the front of the drum, making following the path of a small particle difficult. For these reasons no experiments of this type were attempted.

3.5. Results and Discussion

The transition matrices produced by these experiments were found not to have row totals in the proportions predicted from the calculated throughput rates for each row of the bed. There is no obvious explanation of this. Simulated generation of transition matrices using mixing matrices showed that the agreement of transition matrices with predicted row totals would be expected to improve rapidly with increasing number of observed transitions up to about 1750 transitions and then to improve less dramatically with further increase in the number of recorded transitions. This simulation was the basis on which a figure of 2000 transitions was chosen as the minimum number of transitions to be recorded for each experiment. The further improvement in agreement with predicted rates for larger numbers of transitions was not sufficient to justify the additional time needed to analyse longer experiments.

Simulations of transition matrices produced from a given mixing matrix where the particle was periodically either not detected or assigned to the wrong layer of the bed were also carried out. The conclusion drawn from these simulations was that errors of this type should not produce the disparity between the predicted and experimental matrix row totals (see Appendix V).

In order for the transition matrices to be used to derive mixing matrices the row totals had to be in the correct ratios. It was assumed that the
explanation for the transition matrices being incorrect could only be that the analysis of the tape was assigning the wrong layer number to a proportion of the detected particles. Observation of the system in operation showed that the white particle sometimes appeared in a position between the layers of particles, in these cases the assignment of the layer number was somewhat arbitrary as the centre point of the particle lay close to the boundary of the two layers. The maximum error in the assignment of the layer number was taken to be 1; observation of the analysis process showed that the particle was never assigned to a layer more than one layer removed from its actual position. The calculation of the position of the centre point of the particle was not precise enough to give this position to better than within half a particle diameter either way, ie the point recorded could only be guaranteed to be somewhere within the area of the particle, although its position was usually close to the centre. The frequency with which the particle was assigned to the wrong layer when it clearly lay within the boundaries of a layer was judged to be less than 1 in 20. Where the particle lay across the boundary of two layers there appeared to be approximately equal probability of it being assigned to either layer.

Using the assumption that any error in the assignment of the layer number was not greater than 1 a computer program was written which adjusted each row of the transition matrix to the correct total by adding or subtracting its elements from the elements of adjacent rows, and subtracting of adding to these adjacent elements to maintain the overall sum of all the elements of the matrix. (see Appendix X)

3.6. Summary of Results

Examination of the histograms of the transition matrices produced from these experiments shows that there is no dramatic difference in the mixing matrices for different experimental conditions.

In general the histograms can be seen to have relatively high blocks on the diagonal; showing a tendency towards segregation. It is obvious, however, that the motion in these experiments was not completely segregated just as it was not completely well mixed.

The motion appears to become more segregated as the rotational speed is increased. This is in agreement with Wachters and Kremers findings, they
reported that the tendency for wall particles to return preferentially to the wall increased with increasing speed. It also appears that the amount of interchange between the core and the fixed bed decreases with increasing speed (see figure 3.7).

If the matrices for PTFE coated, sand coated and painted particles are compared there are differences in the mixing patterns between experiments carried out at the same filling and speed with differently coated particles. At the lowest speed used the rate at which solids entered the core increased with the roughness of the particles, the PTFE coated solids had the lowest rate of core mixing, the sand coated particles had the highest rate. The PTFE coating seemed to promote segregation of the wall layer. At a higher speed the PTFE gave the maximum core mixing rate, with the other two coatings producing very similar histograms. There is less contrast between the matrices at the highest speed, all show a decrease in core mixing and an increase in segregation (see figures 3.8, 3.9 and 3.10).

Experiments in the two different drums showed that the large drum tended to produce more segregation in the wall layer at the two lower speeds and more core mixing at low speed than in the corresponding small drum experiments (see figure 3.11). At the highest speed the mixing matrices for the two drums are very similar.

Increasing the filling in drums produced an increase in the core mixing shown in the resulting matrix. The increase in interchange between core and fixed bed particles is most marked at lower speeds (figure 3.12).

Histograms not shown here appear in Appendix VI.

The overall conclusion from these experiments is that the bed is neither well mixed nor segregated under any of the operation conditions used here. It is clear that drum speed, particle coating and drum diameter do affect the mixing behaviour of the bed, but more work is necessary to quantify and explain these effects.
Figure 3.7 Histograms showing mixing patterns at different speeds for $R_d = 0.15m$, $f_c = 0.20$.
$\omega = 18.2 \text{ rpm}$
Figure 3.8 Histograms showing mixing patterns for different particle coatings. $R_d = 0.15m$, $\omega = 6.75rpm$, $f_c = 0.20$
Figure 3.9 Histograms showing mixing patterns for different particle coatings. $R_d = 0.15m$, $\omega = 9.37\text{rpm}$, $f_c = 0.20$
Figure 3.10 Histograms showing mixing patterns for different particle coatings. $R_d = 0.15\text{m}$, $\omega = 14.8\text{rpm}$, $f_c = 0.20$
Figure 3.11 Histograms showing mixing patterns in the two drums

$f_c = 0.20$, $\omega = 8.7\% \omega_c$
Figure 3.12 Histograms showing the effect of increased filling at low speed
$R_d = 0.225m, \omega = 8.7\% \omega_c$
Previous models of residence time distribution in rotating drums have differed in the extent to which the motion of solids within the bed was considered. The axial dispersion models made no attempt to model the actual bed motion, but assumed it to be equivalent to plug flow with superimposed mixing, without accounting for the source of this mixing. The finite stage method was based on the main features of the bed; the fixed and cascade layers, and the flow regimes in these regions. The variance of the residence time distribution was accounted for by a backmixing term. Rogers and Gardner's Monte Carlo simulation was closely based on the mixing behaviour, since it was used to predict the path of single particles through the bed. This is also the technique used in the present work. However Rogers and Gardner assumed that the whole bed was well mixed with no core. The residence time variance produced by their simulation was largely due to an axial mixing element.

The present work aims to predict the residence time distribution in a drum from a mixing matrix as described in the previous chapter.

4.1. The Residence Time Model

The residence time model used a mixing matrix along with the dimensions and operating parameters of a drum and calculated the length of time spent in the drum by 1000 individual particles. This set of times then formed the residence time distribution of the drum.

Beginning with the mixing matrix, \( M \), a probability matrix, \( P \), was derived. The probability that a particle leaving layer \( i \) and re-entering the bed at layer \( j \) was given by:

\[
P_{ij} = \frac{m_{ij}}{\sum_{k=1}^{n} m_{ik}}
\]

(4.1)

where \( m_{ij} \) was the rate at which particles leave layer \( i \) and re-enter at row \( j \) and \( n \) was the total number of rows, including the core.
The calculation of the probability of a particle coming from the fixed bed and entering the core (layer n) was simply $P(i,n)$ as in (4.1). The probability of a particle leaving the core was not determinable from the mixing matrix alone. In the fixed bed layers (i.e., layers 1 to $n-1$) the particles passed through the bed in plug flow; all particles in one layer spent the same length of time in that layer. In the core however, there was a distribution of times spent in this area of the bed. The probability of a particle escaping from the core was given by

$$P_{\text{core}} = \frac{\text{volume of exit from core}}{\text{volume of core}}$$

$$= \frac{\sum_{i=1}^{n-1} m_{ni}}{V_{\text{core}}} \quad (4.2)$$

This was the probability that a particle in the core escaped from it in the time interval defined by the rate used in (4.2).

The volume of the core, for a bed with uniform bed height, is given by:

$$A_{\text{core}} = r_c^2(\phi_{\text{core}} - \sin^2\phi_{\text{core}}/2) \quad (4.3)$$

where $\phi_{\text{core}}$ is the core filling angle;

$$\phi_{\text{core}} = \arccos\left(\frac{R_c \cos \phi + \Delta}{r_c}\right) \quad (4.4)$$

$\Delta$ is the thickness of the cascade layer.

The probability of the particle, once it had escaped the core, moving into layer $i$ was

$$P_{ni} = \frac{m_{ni}}{\sum_{i=1}^{n-1} m_{ni}} \quad (4.5)$$

similar to (4.1) above. From the definition of the mixing matrix $M(n,n)=0$. 

This probability matrix was used to predict the series of transitions from one layer to another. Cumulative probabilities for each transition were derived:

\[ P_{c(i,j)} = \sum_{k=1}^{L_k} P_{i,k} \]  \hspace{1cm} (4.6)

These cumulative probabilities were used to predict the re-entry layer for a given exit layer. A uniformly distributed random number between 0 and 1 was generated and compared to the cumulative probability series for the exit layer. The particle re-entry layer was that for which the random number satisfied the following expression:

\[ P_{c(i,j-1)} < R_n < P_{c(i,j)} \]

where \( R_n \) was the random variable and \( j \) was the re-entry layer. In the case of the core the re-entry layer was found in the same way, once the particle was deemed to have escaped the core.

The core escape modelling was similar, again a random number was generated and here compared to \( P_{core} \). If the random number was greater than \( P_{core} \) then the particle had escaped. If the random number was lower than \( P_{core} \) then the total time spent in the core was incremented by the appropriate interval and another random number generated, until the particle escaped.

For each transition between layers it was possible to calculate the time spent in the fixed bed, time spent on the cascade layer and axial advance along the drum. Obviously the accuracy with which these terms can be calculated was crucial to the accuracy of the predicted residence time distribution.

The time spent in the fixed bed in layer \( i \) (\( i<n \)) depended on the filling of the drum, the rotational speed, the radius of the layer and the thickness of the cascade layer; and was given by

\[ t_f(i) = \frac{1}{\omega} \arccos(R_d \cos \phi + \Delta) \]  \hspace{1cm} (4.7)

where \( \Delta \) is the thickness of the cascade layer, \( r(i) \) is the radius of the layer, \( R_d \) is the drum radius and \( \phi \) is the overall filling angle of the bed.
Time spent on the cascade layer was less easily determined. It had been sometimes assumed in the past that the time spent in the cascade layer was negligible in comparison to the fixed bed time [V1,C3,P2]. This assumption on its own would have led to underprediction of the time for each bed turn and hence underestimate the mean residence time in a bed. However it was also implicitly assumed in these cases that the cascade thickness was negligible, which would have increased the fixed bed time and so reduce the error introduced by the first assumption.

Other workers have taken the cascade time to be proportional to the fixed bed time [H5,A6] ie

$$t_f = B(t_f + t_c)$$ (4.8)

Scott [S7] derived an expression for cascade time based on a model of the motion on the bed surface proposed by Mu and Perlmutter [M5]:

$$t_c = \sqrt{\frac{2l_c}{g(\sin \alpha_d - \tan \alpha_s \cos \alpha_d / 3.5)}}$$ (4.9)

where \(l_c\) is the cascade path length, \(\alpha_s\) is the static angle of repose of the material and \(\alpha_d\) is its dynamic angle of repose.

The cascade path length has been shown to be [S7,H5]

$$l_c = (r(i)\sin(i) + r(j)\sin(j))\cos\theta$$ (4.10)

where \(\phi(i)\) is the filling angle for layer \(i\) and \(\theta\) is the angle at which the particle travels down the surface.

In the model the angles \(\phi(i)\) used in (4.10) did not allow for the cascade thickness. In reality the cascade path length would have been less than predicted by (4.9) because the particle would, on average, travel through the middle of the cascade layer. The error thus introduced would have been less than 5% of the cascade time and less than 0.5% of the total time.

The cascade time for particles in the two dimensional drum could be
estimated from the video recording of the experimental runs. Cascade times calculated from (4.9) were compared with these experimental cascade times. It was found that the predicted times were generally within 10% of the mean experimental cascade times. The experimental velocities in the cascade varied widely; some particles passed down the surface very quickly, virtually without touching the other particles, while others were impeded greatly by collisions. The former particles had velocities well above the average while the slower particles spent much longer than average times on the cascade. The idealized motion described by equation (4.9); where the particles were assumed to slide uniformly down the surface of the bed, did not correspond well to the observed motion. The experimental estimates of mean bed velocity were used here to calculate cascade times, using the path length from (4.10). However equation (4.9) could have been used without introducing serious inaccuracies. The main error would have been that stemming from the measurement of the angle of repose needed to calculate the cascade time from (4.9).

The axial advance for each transition was calculated from

\[ l_a(i,j) = (r(i)\sin\phi(i) + r(j)\sin\phi(j))\tan\theta\cos\beta \]  

(4.11)

The process of predicting transitions and calculating the corresponding time and axial advance continued until the total axial distance moved by the particle was greater than or equal to the drum length. The total time spent in the drum by that particle was recorded and the whole process repeated for 1000 particles to form a simulated residence time distribution.

4.1.1. Effect of Cascade Thickness

The effect of allowing for the cascade thickness was to reduce the time spent in the fixed bed. This effect increased with increasing cascade thickness and with decreasing filling. The difference between the fixed bed times with and without the cascade layer increased with decreasing path radius.

Henein \cite{H4} measured the maximum thickness of the cascade layer (referred to by him as the ‘active layer’) for various materials, drum speeds and fillings. He found thicknesses ranging from 3% to 25% of the bed height. The
fraction of the bed occupied by the cascade layer decreased with increasing filling and with decreasing rotational speed. The figure of 8% of bed area occupied by the cascade layer was given as typical for beds where the bed height was greater than 50 times the particle diameter. For beds with lower ratios of bed height to particle diameter the fraction of the bed occupied by the cascade was found to be as large as 33%. Many experiments on rotating drums have been reported where the bed height was less than 50 times the particle size, suggesting that the cascade layer thickness would have been significant [A1,A2,A3,S7,H5,S1].

It is the change in the core area and fixed bed filling angles associated with cascade thickness which affects the bed turn times. The thickness shown in figure 4.1, that which could be measured in an actual bed may not be the thickness which must be used in calculations. This is because the bulk density of the cascade layer is much lower than in the fixed and core areas. The thickness used in the model was therefore an 'effective thickness'; the thickness of the cascade layer if it had had the same density as the rest of the bed. The effective thickness was estimated experimentally by measuring the fixed bed thickness and subtracting this from the stationary bed depth.

4.2. Residence Time Distributions from Experimental Matrices

To test the suitability of the model the experimental mixing matrices were used to simulate the behaviour of solids in notional drums with the same filling, rotational speed and drum diameter as the drums used in the experiments. The length of the smaller diameter drum (Rd = 0.15m) was chosen to be 3.6m, for the larger drum (Rd = 0.225m) the length was 5.4m. In both cases the length was 12 times the diameter. The drums were both considered to be inclined at 2° to the horizontal. The mixing matrices had shown that the mixing pattern depended on the filling, as well as on the rotational speed, so that only beds with constant bed height could be modelled. This meant that only inclined drums could be simulated since horizontal drums must have a decreasing bed height along their length if there is to be any flow of solids. However it is likely that in horizontal and inclined drums where the variation of filling along the bed is not great the model may be applicable.

The throughput needed to produce the correct uniform bed height at the
Figure 4.1 Cross-section of bed
rotational speed specified was estimated using the model developed by Scott [S7]. The mean residence time was calculated from the known values of bed volume and this throughput.

In order to produce the residence time distribution the cascade layer thickness had to be determined. This was needed to calculate the core area and the fixed bed times, as discussed previously. It was impossible to accurately determine the cascade thickness experimentally because the boundary between the cascade and the other bed areas was not well defined. An error in this velocity when calculating the cascade time did not introduce serious errors to the overall model, using it to calculate the cascade thickness would have caused much large errors. The cascade layer thickness was therefore found indirectly, using the residence time model itself. The thickness was found by trial and error, the correct value being taken as that which gave the correct mean residence time for the appropriate operating conditions.

As a check the thicknesses determined from the residence time model were compared with those estimated from experimental measurement of the fixed bed depth on the video image. As mentioned previously it was very difficult to identify the boundary between the cascade layer and the rest of the bed, so that the experimental estimates could only be taken to be accurate to within 25% at best. In general the experimental values of the cascade thicknesses were lower than those produced by the model. The model value was frequently just within the upper limit of the experimental range. However in several cases, notably for the lowest filling in the smaller drum the experimental value was very much lower than the value produced by the model. This consistent overestimation of the cascade thickness could be due to the overestimation of the throughput by the model use to predict it.

The mixing matrices produced experimentally give mixing patterns for drums with 15%, 20% and 24% filling at a range of experimental speeds for the two drums. The simulated RTD's produced from these matrices show the effect of varying the rotational speed and feedrate at constant uniform bed height and of varying the filling and feedrate at constant speed. Figure 4.2 shows how both the mean and the variance of the residence time distribution decrease with increasing drum speed at constant filling for a typical set of experimental matrices. This is true of all the sets of experimental matrices, except those for the sand coated particles where the variance showed a maximum at the
intermediate speed used. The E curves for the other simulations are shown in appendix VI.

The most important element of the mixing pattern in determining the variance was the rate at which particles enter and leave the core. The general trend is for the probability of a core particle escaping (Pcore in the previous discussion) to increase as the speed increases. This is largely due to the decrease in core area with increased cascade thickness at higher speed.

The importance of the core area in determining the variance is illustrated when the variances of the residence time distributions produced under the two extreme mixing patterns; the segregated and well mixed models are examined. Here the core is assumed to be completely stagnant; there is no exchange of particles with the fixed bed. For a 0.15m diameter drum, length 3.6m with a filling angle of $60^\circ$ and speed 9.37rpm the variances for the two ideal mixing mechanisms are shown below, along with that for the experimental mixing matrix corresponding to these operating conditions.

<table>
<thead>
<tr>
<th></th>
<th>Variance (hr$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>well mixed</td>
<td>$2.0 \times 10^{-7}$</td>
</tr>
<tr>
<td>segregated</td>
<td>$6.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>experimental</td>
<td>$27.5 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

There is a significant difference between the variances for the well mixed and segregated models, the well mixed model producing the lower variance as was expected. There is also a considerable difference between the segregated model and the experimental simulation which give a variance some four times that of the segregated model. Since it would be true to say that the experimental matrices were probably closer to the well mixed matrix than to the segregated model it is clear that the core behaviour is a very important factor in the determination of the residence time distribution.

The shape of the simulated distributions is very similar to those produced by actual kilns [A4,C3,G1,G7,H6,L1,M7,O1,R3,S6,Z2,K7]. The simulated distributions are clearly slightly skewed, with a distinct 'tailing off' at the upper ends of the residence time ranges. This tailing off is more obvious in the simulations with lower rotational speeds and correspondingly higher variances. Residence time distributions from experimentation on full size kilns
Figure 4.2 RTD at different rotational speeds
demonstrated both skewness and tailing off [A4,C3,G1,G7,H6 etc]. Experimental data from small scale drums [A1,A2,A3,H9,K2,R4] showed these features to a lesser extent, probably because the length to diameter ratio in these small drums was too small to allow them to develop. The relationship between the length of drum and the residence time distribution is illustrated in figure 4.3, where the variance can be seen to decrease with decreasing simulated drum length. The estimation of the cascade layer thickness using the mean bed residence time predicted by Scott's model [S7] limits the accuracy of the residence time distribution modelling. The assumptions used in that model differ significantly from those used in the present work, which may affect the results produced by the present model and is discussed in the following section.

4.3. Consideration of Previous Bed Motion Models

In these models solids are assumed to emerge from the bed onto the cascade from the full length of the upper half of the bed surface. This would give a flowrate into the cascade layer which was higher than if the core were considered.

Expressions have been derived which give the solids throughput (q) of a bed for a given fractional filling, material properties and drum operating conditions; most recently by Scott [S7]:

\[ q = 4\omega \tan \beta \cos \beta \int_b^{R_d} r^2 (\sin \phi) (r) \phi (r) \, dr \]

(4.12)

where b is the height from the bed surface to the drum axis.

Looking at the effect of the core on (4.12) first, it is clear that introducing the core will increase b to r_c, the radius of the core. This would reduce the value of the throughput of the drum. Including the cascade layer thickness in the calculation of the filling angle would affect (4.12) by reducing the filling angle. The denominator of the integral term is the time for a transition with equal exit and entry radii, multiplied by the rotational speed. This is reduced by increased cascade layer thickness, tending to increase q. However the numerator of the integral is also dependent on the filling angle, decreasing with
Figure 4.3 RTD in different drum lengths

- \( L = 1.8 \text{m} \)
- \( L = 3.6 \text{m} \)
- \( L = 5.4 \text{m} \)
- \( L = 0.9 \text{m} \)

\[ R_d = 0.15 \text{m} \]
\[ F_c = 0.200 \]
\[ \omega = 9.37 \text{r.p.m.} \]
increasing cascade thickness. Overall the complete integral term tends to
decrease with increasing cascade thickness or decreasing filling angle.

It is interesting that the models used to predict bed behaviour have
assumed the segregated bed type of motion. This pattern of mixing has been
rejected by those studying mixing and heat transfer in rotary drums. A model
using a mixed bed assumption has been considered [S7] but was rejected as
predicting fractional fillings which were too high in comparison to experimental
fillings for given flowrates. The core has never been considered by workers
modelling throughput although it has been identified in other contexts [R4,12].

In general the effect of neglecting the core is to underpredict the
residence time in a given drum with a given throughput by underpredicting the
filling needed to produce that flowrate. This is partly offset by the effect of
neglecting the cascade thickness, which would give overestimates of the fixed
bed time. Some of Scott's comparisons of experimental and predicted feedrates
did show predicted rates higher than experimental. However there were other
examples in his experiments, notably at higher feedrates where the predicted
filling was higher than the experimental value so there must be other factors
which are important. There are many variables which could affect this type of
model, and further work is necessary to explore this field more completely.

4.3.1. Simulation of Experimental RTD's

The model has also been used to simulate some of Abouzied's
experiments. Obviously mixing matrices for these experiments were not
available. The mixing matrix which was produced for the mixing experiment
with the rotational speed (expressed as a fraction of the critical speed) and
filling most similar to the speed and average filling in Abouzied's experiments
was used in each case. The experimental conditions and the variances
predicted from the simulations and found experimentally by Abouzied are
shown below:

<table>
<thead>
<tr>
<th>φ</th>
<th>feedrate</th>
<th>f₀</th>
<th>ω/ω₀</th>
<th>variances (s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rpm</td>
<td>g/s</td>
<td>%</td>
<td>%</td>
<td>experimental</td>
</tr>
<tr>
<td>42</td>
<td>2.4</td>
<td>21</td>
<td>28</td>
<td>200</td>
</tr>
<tr>
<td>20</td>
<td>2.4</td>
<td>19</td>
<td>13</td>
<td>120</td>
</tr>
<tr>
<td>10</td>
<td>2.4</td>
<td>23</td>
<td>6.5</td>
<td>130</td>
</tr>
</tbody>
</table>
The main source of uncertainty in these simulations, apart from the lack of data on the mixing, was the estimation of the density of the sand used in the experiments. Abouzied did not give the density of his sand so this had to be estimated from values given elsewhere for similar materials. This introduced an error in the determination of the filling angle of the beds, and also in the angle at which particles tumbled down the bed surface ($\theta$) since this depends on the bed profile. Scott's model [S7] was used to estimate the bed profile and hence the angle $\theta$. Taking these uncertainties into account the variances produced by the simulations compare well with the original experimental values. The variation of bed height along the drum in these experiments were small; the angle between the bed surface and the drum axis was less than 3° in each case so that a single mixing matrix may be used to describe the behaviour of the bed along its full length. This would be true in many drums with a constricted discharge.

4.3.2. Comparison of Simulations and Previous Experimental Work

There were two main problems encountered in comparing the results from the model simulations to previous experimental work. Firstly almost all the previous experimental work was carried out in horizontal drums although, as Abouzied [A3] acknowledged many of the drums used in industrial processes are inclined. Abouzied also indicated that the residence time behaviour in inclined drums was 'substantially different' from that in horizontal drums. The second problem stems from the type of experiment which has been carried out. The effect of rotational speed has been studied experimentally by varying the speed with a constant feedrate. Here the effect of speed cannot be isolated from the effect of the change in bed profile caused by the speed variation. Similarly, examination of the effect of feedrate on the residence time distribution at constant feedrate and of the effect of drum inclination at constant feedrate and speed also involved changes in bed profile.

The results produced by the model gave the variance of the residence time distribution for three values of constant bed height at various speeds and corresponding throughputs. It was also possible to simulate the effect of inclination on the residence time distribution, again at fixed filling angles. Although the model results did not simulate conditions which were directly comparable to the experiments reported in previous work it was possible to interpolate between the various sets of model variances to be compared with
literature relationships.

The most important difference between the horizontal and inclined drums is the difference in the particle cascade path angle $\theta$, in the horizontal case this is much smaller than in an inclined drum. This would mean that a particle in a horizontal drum would have to pass through the bed more often than one in the bed of an inclined drum with the same filling and speed. This should indicate that the residence time in horizontal drums would be much longer than in inclined drums for similar operating conditions, as is in fact the case [A3,H5]. The more often a particle passes over the cascade the more chance it has of entering and becoming trapped in the core, this suggests that the variance of the residence time in horizontal drums should be higher than in corresponding inclined drums.

Abouzied reported a distinct discontinuity between the variances of horizontal and slightly inclined drums, with the horizontal drum variance being found to be much higher than that of the inclined drums.

The reason for the lower path angles found in horizontal drums is explained by examining the equation from which this angle is calculated. This equation was derived from consideration of the geometry of the bed by various workers [H5]. The path angle $\theta$ is given by:

$$\tan \theta = \frac{\cos \alpha_d \sin \beta + \tan \psi \cos \beta}{\sin \alpha_d} \tag{4.13}$$

$\psi$ is the angle of inclination of the drum, $\beta$ is the angle between the drum axis and the bed surface. In a horizontal drum, with $\psi = 0$ the path angle is given by:

$$\tan \theta = \frac{\sin \beta}{\tan \alpha_d} \tag{4.14}$$

While in an inclined drum with uniform bed height such as those simulated previously:
\[ \tan \theta = \frac{\tan \psi}{\sin \alpha_d} \] (4.15)

Since both \( \psi \) and \( \beta \) are small \( \tan \psi \) and \( \sin \beta \) may be taken to be approximately equal to \( \psi \) and \( \beta \) respectively. Hence, since \( \tan \alpha_d \) is always greater than \( \sin \alpha_d \), and \( \psi \) and \( \beta \) are of similar magnitude \( \theta \) will usually be greater for the inclined drum. Where both inclination and the angle between the axis and the bed are non-zero, as in Abouzied's inclined drum experiments \( \theta \) will clearly be greater in the inclined drums.

The path angle and its relationship to the angle between the bed surface and the axis is also important in drums with constricted discharge and very low throughput. Here the bed angle and therefore the path angle is very small. The variance in this case would be expected to be high, due to the large number of passes through the bed each particle must take. In fact Abouzied did observe high variances for low flowrates in his experiments.

Four main variables have been studied in the past; drum speed, feedrate, particle size and drum inclination.

Abouzied [A3] found that the variance of the residence time distribution decreased with increasing rotational speed at low speeds (less than 20% of the critical speed) and then increased with further increase in rotational speed above that. Hehl et al [H9] also reported a decrease in residence time variance with increasing rotational speed, where the maximum speed used was 12% of the critical.

Results for the smaller drum using the model show the RTD variance decreasing with increasing rotational speed, at different constant feedrates, at speeds ranging from 8% to 22% of the critical speed (see figure 4.4). The larger drum simulations show decreasing variances with increased speed at the lowest feedrate used. At higher feedrates and correspondingly higher speeds the variance passes through a minimum, increasing with speed at the highest rotational speeds, as Abouzied found in his experiments (see figure 4.5).

Both Hehl and Abouzied found that the RTD variance decreased with increasing feedrate, at constant speed. This trend was shown by the results for the larger drum (see figure 4.6). In the smaller drum the variances at 20% and
24% filling do decrease with increased throughput but the lowest filling \( f_c = 15.4\% \) consistently produces lower variances than the higher fillings, although the lower filling represents a lower feedrate (see figure 4.6). This may be due to the suspected underestimate of residence time discussed in the consideration of cascade thickness.

The RTD variance had been found to decrease linearly with increasing drum inclination, at constant feedrate and rotational speed, by Abouzied. The mean residence time and therefore the holdup also decreased linearly with increased inclination. In the residence time model simulations neither the mean nor the variance displayed a linear relationship with drum inclination, although both did decrease with increasing inclination. This difference between the experimental and simulated RTD’s could be because the experimental bed profiles were different from those simulated by the model. In the experiments the bed height varied along the length of the bed, while in the simulations the bed height is constant along the full drum length, so that the bed surface is always parallel to the drum axis. In the experimental work it is likely that the angle between the bed surface and the drum axis varied with the drum inclination.

The magnitudes of the the variances produced by the model agree with experimental variances reported by several workers for similar mean residence times. The model variances are in the range \( 10^{-5} \) to \( 10^{-3} \) hr\(^2\). Much of the experimental work produced variances in the same range \([A3,K2,R4,R2]\), although Hehl et al \([H9]\) found variances above \( 10^{-3} \) hr\(^2\). Apart from Hehl most of the experimental variances reported have been between \( 10^{-5} \) and \( 10^{-4} \) hr\(^2\), lower than many of the simulated values. This was due to the low ratios of drum length to diameter used in the experiments. The drums used have had lengths between 2.4 and 3 times their diameters. If drums of this aspect ratio are simulated the variances produced were in the same range as the experimental variances. Figure 4.3 shows the effect of drum length, at constant feedrate and drum speed, on the residence time distribution. The variance decreases with decreasing drum length. This is as expected for a drum where the dispersion coefficient is independent of the drum length since the Peclet number is then proportional to the length.
Figure 4.6 Variance vs feedrate for small drum

Figure 4.7 Variance vs feedrate for large drum
Figure 4.4 Variance vs Rotational speed for small drum

Figure 4.5 Variance vs Rotational speed for large drum
4.4. Summary of Residence Time Modelling

The Monte Carlo simulation of residence time distribution in rotary drums developed here uses the mixing matrices produced by the 2-dimensional mixing experiments discussed previously to predict the residence time distribution in drums with uniform filling. The simulation requires the mean residence time of the drum which can be found using an established model of bed motion.

The model has been used to simulate the residence time distribution in various notional drums. The results from these simulations follow the same trends in mean and variance of the residence time distributions as shown in previous experimental work. The magnitudes of the simulated variances produced are similar to those determined experimentally for drums with similar diameter to length ratios.

Simulated residence time distributions using data from previous residence time experiments in small scale drums compare well with the original experimental results.

The influence of the core on the variance of the residence time distribution has been highlighted. The effect of the neglect of the core in existing models of bed motion has been discussed and identified as a possible source of the discrepancies between the predictions of these models and experimental data.

The role of the angle at which particles roll down the bed surface in accounting for the contrast in residence time behaviour between horizontal and inclined drums, which was previously unexplained, has been discussed.
5.1. Introduction

The importance of mixing within the bulk solids in a rotating drum has been identified in previous work as an important element in the overall heat transfer behaviour in the drum. Brimacombe and Watkinson [B4] showed that the transition from slumping to rolling bed motion was reflected in an accompanying increase in the convective heat transfer coefficient between the gas and the solids. They attributed this effect to the more efficient bulk mixing in the rolling regime.

If less dramatic changes in bed behaviour are considered, for example different rotational speeds with no change in the type of bed motion, the effect on gas-solid convective heat transfer is much less marked. Once rolling motion has been achieved the majority of the solids in the bed will be tumbling down the bed surface in the course of their passage down the drum and will therefore be directly exposed to the heating effect. The bulk mixing pattern is therefore a minor consideration in heat transfer for rolling or cascading beds.

The effect on heat transfer between the wall and the underside of the bed of changes in bed mixing behaviour is more significant. In this case the heat transfer to the bulk is controlled by the distribution of particles originally in contact with the wall in the bulk after mixing on the cascade. Because of the poor effective conductivity of particulate materials only a very thin layer, relative to the total bed depth, is heated directly by contact with the hot wall. The bulk of the bed is heated indirectly via the mixing of the wall layer into the inner bed.

The importance of mixing in this mechanism of heat transfer was emphasized by Wachters and Kremers [W1] and Lehmberg et al [L3], although they did not agree on the actual mixing pattern existing in rotary drums. Lehmberg stated that the whole bed was well mixed, irrespective of the operating conditions. Wachters identified a layer at the wall which tended not to mix with the bulk of the solids, the thickness of this layer depending on the rotational speed of the drum. The solids outwith this thin layer were assumed to be well mixed. No detailed assessment of the bulk mixing pattern and its
effect on the heat transfer within the bed has been reported.

The effect of the behaviour of the core on the heat transfer within the bed is another topic which has not been explored.

There are therefore two main areas of interest which the present work aims to address; the effect of radial mixing patterns on temperature distribution and total heat flux in the bed and the effect of the presence of the core on heat transfer. The mixing behaviour information provided by the 2-dimensional mixing experiments, which is more detailed than has been available before is used to investigate these effects. The rate at which solids pass in and out of the core region, which is available for the first time from these experiments is used to model the effect of the core on heat transfer in the bed.

There are two important factors to be considered in the development of the heat transfer model;

- The curvature of the bed – if the bed can be approximated as having a rectangular cross-section, as has been done in the past [11] the model might be simplified. This was not found to be a completely acceptable approximation.

- The resolution of the mixing model – how is the resulting heat transfer model affected by the size of section used in the mixing model? This was found to be an important factor under some mixing regimes.

The bed was modelled as being made up of thin axial slices. The heat transfer in each slice is modelled and the temperature distribution of material leaving one slice is used to calculate the temperature distribution entering the next.

The thickness of each axial slice is equal to average axial advance of the particles from the fixed bed for one passage down the cascade layer. Solids leaving the bed travel a distance along the bed which is dependent on the point from which they leave the fixed bed, as well as the drum inclination, filling fraction and bed angle. The difference between path lengths for different transitions is not large however. For the bed modelled here, which has a diameter of 3.6m, the outermost layer has an axial advance of 0.03m for each passage down the cascade, while the innermost fixed bed layer has an advance of 0.02m. These are calculated for segregated motion using equation 4.15. Where there is mixing between layers the variance of the axial advances is
lessened so that the assumption of constant axial advance should not introduce serious errors. It has already been demonstrated that the spreading of residence time in the bed is mainly caused by the presence of the core rather than the variation of axial advance rate.

The bed was initially modelled with a rectangular cross-section and then with the segment shaped cross-section of a real bed. The rectangular approximation to the bed geometry simplified the formulation of mixing matrices. In all cases the bed is considered to be at steady state so that the temperature at any position in the bed would not vary with time. This is necessary for the calculation of the core temperature as will be seen.

5.2. Rectangular Approximation

The first approximation to the thermal behaviour of the bed was to model a thin rectangular slab of material, heated along one edge. In the actual curved bed the depth of the solid bed varies with the angle \( \phi(r) \) as shown on figure 5.1. The rectangular slab will be considered to have depth \( D_b \), the maximum bed depth, where \( \phi(r) = 0 \).

\[
D_b = Rd(1 - \cos \phi) \tag{5.1}
\]

The heated edge of the slab is maintained at the wall temperature \( T_w \). The slab is divided into two parts, representing the fixed and core areas of the bed (see figure 5.2). To model the circulating behaviour of the real bed a narrow slice of the bed perpendicular to the heated edge is considered. This narrow slice may be considered to approximate the behaviour of the element \( pq \) (figure 5.1) as it moves from A to B, passes down the cascade layer and reforms at point A on the next slice for the next cycle. It is more difficult to picture the motion in the rectangular bed, but the slice may be thought of as moving from A' to B', emerging at B', going through a mixing process and reforming at A', this cycle is repeated to simulate the circulation in the real segment shaped bed.

The temperature distribution in the fixed part of a slice is determined by the initial conditions at A' at the beginning of each cycle and the conduction from the wall during the contact time. Heat transfer is considered to take place only in the direction perpendicular to the heated edge of the slab. No heat
Figure 5.1 Cross-section of bed

Figure 5.2 Rectangular approximation to bed geometry
transfer takes place between the solids on the cascade layer, i.e. during the mixing phase of the cycle. The 'core' part of the slice is isothermal, since it is assumed to be well mixed at all times.

The 'contact time' $t_c$ is the time that a particle in the curved bed, in contact with the wall, would take to move from A to B in figure 5.1 i.e.:

$$t_c = \frac{2\phi}{\omega} \quad (5.2)$$

The temperature of the core may be deduced by examining the conditions needed to maintain steady state in the bed. If it is assumed that:

- There is no axial movement in the core
- There is no radial temperature gradient in the core at any axial position

then a heat balance can be constructed to describe the thermal behaviour of the core region. Heat transfer to and from the core can take place at two boundaries; the core-fixed bed interface and the core-cascade layer interface. In both cases the transfer may be by conduction at the boundary or by the passage of particles into or out of the core. Solids only enter from and leave the core into the cascade layer; there is no direct particle movement over the core-fixed bed interface.

Solids leaving the core are assumed to be at the core temperature, $T_{core}$. Particles entering are at a temperature which depends on the radius from which they left the fixed bed.

The heat transfer rate to the core due to particles moving across its boundaries is therefore:

$$q_{pc} = F_c \rho C_p (T_{pc} - T_{core}) \quad (5.3)$$

where $T_{pc}$ is the mean temperature of particles entering the core, $F_c$ is the volumetric flowrate of solids into the core and $T_{pc}$ is the mean temperature of particles entering the core. To maintain the steady state the rate at which
solids enter the core and the rate at which they leave must obviously be equal.

For steady state the overall heat transfer rate into the core must be zero so the heat balance is:

\[ h_{cc}A_{cc}(T_{\text{core}} - T_{cc}) + h_{cf}A_{cf}(T_{\text{core}} - T_{cf}) + \\
F_{c} \rho C_{p}(T_{\text{core}} - T_{pc}) = 0 \]  

(5.4)

where \( h_{cc} \) and \( h_{cf} \) are the heat transfer coefficients between the core and cascade layer, and core and fixed bed respectively, and \( A_{cc} \) and \( A_{cf} \) are the corresponding heat transfer areas. \( T_{cc} \) is the temperature of the cascade layer adjacent to the core and \( T_{cf} \) is the temperature of the upper surface of the fixed bed.

5.2.1. The Well Mixed and Segregated Regimes

The two simplest modes of bed mixing to simulate are the two extreme conditions - the well mixed and segregated regimes. Comparison of the heat transfer patterns produced by these mixing mechanisms provides an estimate of the maximum and minimum heat transfer rates achievable, subject to consideration of the core behaviour.

In the well mixed case the fixed section will be isothermal after each mixing phase. It is assumed that the particles in the bed are small enough to allow mean temperatures of adjacent particles to be used as the temperature of all those particles for modelling purposes.

In this ideal case there is no interchange of particles between the fixed and core regions. The core temperature is then fixed by the conductive transfer at its boundaries.

In the segregated model the slice is assumed to reform at \( A' \) in the next slice, after each mixing phase, in exactly the same arrangement as before mixing. The temperature distributions of the solids leaving one slice and entering the next are therefore identical. In both the well mixed and segregated mixing modes there is no heat transfer amongst solids on the cascade layer. In the segregated model there is again no exchange of fixed bed and core solids so that the core temperature is completely defined by the conductive heat
transfer.

In both cases the temperature distribution within the fixed section may be found from the basic equations of unsteady state heat conduction:

\[ \frac{\partial^{2} \theta}{\partial t} = \alpha \frac{\partial^{2} \theta}{\partial y^{2}} \quad (5.5) \]

where \( \theta = (T - T_{o}) / (T_{w} - T_{o}) \) and \( \alpha = k / \rho C_{p} \) the thermal diffusivity of the bed, \( T_{o} \) is the initial solids temperature.

If the initial and boundary conditions are applied i.e.;

\[ \theta = 0 \quad t < 0 , \quad \text{for all} \ y \]
\[ \theta = 1 \quad y = 0 , \quad \text{all} \ t > 0 \]
\[ \theta = 0 \quad y = D_{f} , \quad \text{all} \ t > 0 \]

The solution to (5.5) is

\[ \theta = 1 - \text{erf} \ \frac{y}{\sqrt{4\alpha t}} \quad (5.6) \]

and the heat flux at the wall is

\[ q_{w} = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} = \frac{k}{\sqrt{\pi \alpha t}} (T_{w} - T_{o}) \quad (5.7) \]

The total heat transferred over the contact time \( t_{c} \) is then

\[ Q_{w} = \int_{0}^{t_{c}} q_{w} \ dt = \frac{2}{\sqrt{\pi}} t_{c} k \rho C_{p} (T_{w} - T_{o}) \quad (5.8) \]

per unit contact area.

In the well mixed case each slice has a uniform initial temperature, \( T_{o}(s+1) \) for the \((s+1)\)th axial slice. \( T_{o}(s + 1) \) is given by
\[ T_o(s + 1) = T_o(s) + \frac{Q_w(s)}{D_f \rho C_p} \] (5.9)

where \( Q_w(s) \) is the heat transferred to the strip as it passes through the \( s \)th slice of the bed, \( D_f \) is the fixed bed depth and \( \rho \) and \( C_p \) are the density and specific heat of the particulate material. Transfer to the core is neglected here.

In the segregated case the temperature distribution of the solids entering the \( (s+1) \)th slice of the bed may be calculated from (5.6) with \( t \) replaced by \( s \). In this case the mean temperature of the fixed bed at the start of the \( (s + 1) \)th cycle will be given by

\[ T_o(s + 1) = T_o + \frac{Q_w(s)}{D_f \rho C_p} \] (5.10)

where in this case

\[ Q_w(s) = 2\sqrt{\frac{st_{kD}C_p}{\pi}}(T_w - T_o) \] (5.11)

\( T_o \) is the original uniform bed temperature.

In order to model the core conduction the behaviour of the real bed under the two idealized regimes is considered. In both cases the rate of conduction into and out of the core is governed by the temperatures of the upper surface of the fixed bed and the lower surface of the cascade layer. The dependence of the core heat transfer on these temperatures is shown in equation (5.4). The fixed bed temperature distribution can be obtained from the analysis above, neglecting transfer from the core. This temperature can then be used to estimate the rates of heat transfer to and from the core. For simplicity the upper surface of the core is considered only to be in contact with solids leaving the same slice, so that the cascade temperature is directly related to the temperature distribution of the solids leaving that slice.

The required cascade temperature is less easily estimated. Because axial movement on the cascade layer is neglected in this 2-dimensional model, particles on the cascade layer will have temperatures directly relating to those...
of the slab.

In the well mixed case the cascade layer is also well mixed, so that the average temperature of particles in contact with the core will be the mean bed temperature of the solids leaving the slice \( T_o(s+1) \) for the \( s^{th} \) cycle through the bed. In a segregated bed there is no mixing on the cascade layer, implying that the underside of the cascade layer would consist of particles from only the uppermost part of the fixed bed. Here there would be no conduction into or out of the core because its temperature is identical to that of both the surfaces with which it is in contact.

In a well mixed bed there will be some transfer of heat at the core boundaries. The cascade temperature will be higher than that of the upper fixed bed \( T_o(s+1) \) as opposed to \( T_o(s) \) so that heat will be transferred from the core to the fixed bed and from the cascade layer to the core. Conduction from the core to the fixed bed may be approximated by (5.7), substituting \( T_{core} \) for \( T_w \) assuming that the flux at the boundary is small. The transfer from the cascade layer is more difficult to account for, because the motion of this layer is not well known. The contact between the cascade layer and the core is much poorer than that between the core and the fixed bed, which would reduce the heat transfer coefficient at the core-cascade layer interface. The temperature rise in the bed over one cycle is not large so the fluxes over the core interfaces would be very small and may be neglected. The core temperature will therefore be taken to be that of the upper fixed bed \( T_o(s) \).

Figure 5.3 shows the average fixed bed temperatures for up to 1000 cycles under the two regimes. The data is taken from an actual kiln:

\[
\begin{align*}
 k &= 7 \times 10^{-4} \text{ kW/m°C} \\
 C_p &= 1 \text{ kJ/kg°C} \\
 \rho &= 2170 \text{ kg/m}^3 \\
 D_f &= 0.507 \text{ m} \\
 D_b &= 0.667 \text{ m} \\
 t_c &= 11 \text{ s} \\
 T_w &= 300 \text{ °C} \\
 T_o &= 100 \text{ °C}
\end{align*}
\]

The fixed bed depth \( D_f \) was estimated. The difference between the two cases is very clear, heat transfer is much better in the well mixed case, as expected. Figure 5.4 shows the temperature distribution within the bed for the
Figure 5.3 Mean temperatures for well mixed and segregated beds

Figure 5.4 Temperature distribution within a segregated bed
segregated model.

It is unlikely that either of these ideal mixing patterns would be followed by a real bed. A general model should simulate mixing pattern between the two extremes, as well as the interchange of material between the core and the fixed bed.

5.2.2. The General Model

There are two factors to be considered in a general model of the heat transfer in the 2-dimensional slab:

- the motion and mixing of the bed
- the thermal behaviour of the bed.

The mixing behaviour is described by a mixing matrix $M$, similar to those described in the account of the experimental work. The slab is divided into layers, parallel to its heated edge. The core is undivided, and is treated in the mixing matrix as a single layer. These layers correspond to the concentric layers in the curved bed used in the mixing model. The mixing matrix for the slab is slightly different from those developed for the curved bed. In the curved bed the rate at which solids emerge from a layer depends on the radius of that layer. In the rectangular model this rate is independent of the exit position. The row totals of the mixing matrix for the curved bed are related to the rate at which solids leave each layer, so that these totals are different for each layer. In the rectangular model the row totals will be identical for every layer, because the rate is the same for each layer. In the matrices the elements $m_{ij}$ represent the rate at which solids leave layer $i$ and re-enter the bed at layer $j$. For the slice $p'q'$ used in the rectangular model the fraction of the solids in layer $i$ of the $s^{th}$ slice which is in layer $j$ of the $(s + 1)^{th}$ slice is given by:

$$m_{ij} / \sum_j m_{ij}$$

The temperature of each layer entering each slice is taken to be the mean temperature of the particles of which it is made up. Each layer is assumed to be well mixed so that there is no temperature gradient within any layer.
entering each slice of the bed, although all the layers may not be at the same temperature. The temperature of a layer entering a slice is given by:

\[ T_{ij}(s + 1) = \frac{\sum_{i=1}^{n} m_{ij} T_{ij}(s)}{\sum_{i=1}^{n} m_{ij}} \quad (5.12) \]

where \( T_{ij}(s + 1) \) is the initial temperature of layer \( j \) entering the \((s + 1)\)th slice; \( T_{ij}(s) \) is the final mean temperature of the \( i \)th layer leaving the \( s \)th slice and \( n \) is the total number of layers, including the core.

Although the core region behaves differently from the fixed bed it must be included as a layer in the mixing model to account for the interchange of core and fixed bed particles.

5.2.3. The Effect of Layer Depth

The depth of the layers in the mixing model, which is determined by the order of the mixing matrix, can have an effect on the temperature distribution produced by the model. The depth of the layer adjacent to the hot wall is especially important. The effect of layer size is most easily demonstrated and explained by means of examples. In these examples the conductive transfer between the different layers, and between the fixed bed and the core is neglected. There is no exchange of fixed and core section solids, so the core area is omitted from the mixing matrices in this special case.

Example (1)

In a bed with \( D_f = 1 \) and 10 fixed bed layers, each with depth 0.1, half of the wall layer (layer 1) is mixed uniformly through the remainder of the bed (layers 2 to 9), while the remaining 50% returns to the wall layer after each mixing phase. The constants used in the calculations are:

- \( t_c = 10 \)
- \( \alpha = 3.185 \times 10^{-5} \)
- \( k = 0.03125 \)
- \( Cp = 1 \)
- \( T_o = 100 \)
- \( T_w = 200 \)
- \( \rho = 1000 \)

the mixing matrix is
and the resulting temperatures of the strip $p'q'$ and heat flows are:

<table>
<thead>
<tr>
<th>Cycle</th>
<th>$T_{01}(s)$</th>
<th>$T_{11}(s)$</th>
<th>$T_{02}(s)$</th>
<th>$T_{03}-T_{01}(s)$</th>
<th>$Q_w$</th>
<th>$T(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>119.95</td>
<td>100</td>
<td>1995</td>
<td>100.00</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>110.0</td>
<td>129.01</td>
<td>101.1</td>
<td>1894</td>
<td>101.99</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>115.2</td>
<td>133.62</td>
<td>102.6</td>
<td>1843</td>
<td>103.88</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>118.3</td>
<td>136.40</td>
<td>104.3</td>
<td>1812</td>
<td>105.73</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>120.5</td>
<td>138.40</td>
<td>106.1</td>
<td>1790</td>
<td>107.54</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>122.4</td>
<td>140.14</td>
<td>107.9</td>
<td>1771</td>
<td>109.33</td>
<td></td>
</tr>
</tbody>
</table>

Each cycle is composed of one passage through the fixed bed and one passage down the cascade for the solids in the strip $p'q'$ ie one slice of the bed.

Example (2)

If the number of layers in the bed described in example (1) is reduced to 5 (and the layer depth increased to 0.2 accordingly) the mixing matrix becomes:

$$M = \begin{bmatrix} 5.556 & 5.556 & 5.556 & 5.556 & \ldots & 5.556 \\ 5.556 \\ 5.556 \\ 10.5 \\ 5.556 \\ \vdots \\ 5.556 \end{bmatrix}$$

giving the following results:

<table>
<thead>
<tr>
<th>Cycle</th>
<th>$T_{01}(s)$</th>
<th>$T_{11}(s)$</th>
<th>$T_{03}-T_{01}(s)$</th>
<th>$Q_w$</th>
<th>$T(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.0</td>
<td>109.9</td>
<td>100.0</td>
<td>1995</td>
<td>100.00</td>
</tr>
<tr>
<td>2</td>
<td>103.6</td>
<td>110.9</td>
<td>101.6</td>
<td>1958</td>
<td>102.00</td>
</tr>
<tr>
<td>3</td>
<td>105.8</td>
<td>111.5</td>
<td>102.5</td>
<td>1936</td>
<td>103.96</td>
</tr>
<tr>
<td>4</td>
<td>107.8</td>
<td>112.4</td>
<td>103.5</td>
<td>1917</td>
<td>105.88</td>
</tr>
<tr>
<td>5</td>
<td>109.7</td>
<td>119.2</td>
<td>107.3</td>
<td>1898</td>
<td>107.78</td>
</tr>
<tr>
<td>6</td>
<td>111.6</td>
<td>121.0</td>
<td>109.2</td>
<td>1879</td>
<td>109.68</td>
</tr>
</tbody>
</table>

In the second example, with five layers, the wall layer temperature is
always lower than in example (1). This lower temperature causes an increase in the flux at the wall, and hence in average bed temperature. Two factors contribute to the lowering of the wall layer temperature in deeper layers. The mean wall layer temperature at the end of a fixed period is decreased, despite the higher flux, because of the dilution effect of the larger mass of cooler solids in the deeper layer. The flux for any initial wall layer temperature is independent of the layer thickness, so that it is obvious that the same input of heat will cause a smaller temperature rise in a larger mass of material.

The change in layer depth also changes the mixing pattern. The fraction of the wall layer returning to the wall was reduced in the five layer model, from 0.5 to 0.32. This reduction means that more of the heat transferred into the wall layer is carried into the bulk by the mixing, increasing the inner layer temperatures and decreasing the wall layer temperature, relative to the ten layer model.

The effect of the layer depth on the resulting is not great over a small number of cycles, but over the large number of cycles which occur in a real bed it may be significant.

The dependence of temperature distribution and heat flux on the layer size is an effect of the limitations of the mixing model, not the heat transfer model. The resolution of the mixing pattern is crucial to the accuracy of the heat transfer model. This is illustrated by a further example. The five layer matrix used above was that which would have been constructed from the same experiment as the ten layer model in example (1). The elements of the five row matrix were calculated by summing the appropriate elements of the ten row matrix. There is only one unique five row matrix which can be derived in this way from a specific ten row matrix. There are however an infinite number of ten row matrices which can be constructed from the five row matrix.

Example (3)

A matrix with order 10 can be constructed to give exactly the same mixing pattern as the five row matrix in example (2), by dividing each element in the 5x5 matrix by 4. This matrix is:
Figure 5.5 Typical bed temperature distribution at beginning of contact time
Finer transition matrices could be generated from coarser ones by using an interpolation scheme to produce the new matrix. This would give a different temperature distribution to that produced from the original matrix, which may or may not be a more accurate representation of a real bed. Simply using an arbitrary scheme to expand the mixing matrix would not necessarily model the behaviour of the bed more closely. The temperature modelling would only be more accurate if the mixing modelling were more accurate.

5.2.4 Modelling Conduction between Layers

The assumption that each layer is well mixed gives an initial temperature distribution at the beginning of each fixed period similar to that shown in figure 5.5. This distribution will change during the fixed period due to the conduction of heat between the adjacent layers, tending to smooth the temperature differences between the layers.

The heat flux at each boundary can be calculated from unsteady state conduction relationships, in a similar way to the calculation of the wall flux. The boundary temperature is assumed to be the mean of the two adjacent layer temperatures. This boundary temperature will remain constant during the conduction period. The heat transferred at the boundary is given by:
\[ Q_{i,i+1}(s) = 2 \sqrt{\frac{t_c k \rho C_p}{\pi}} \left( \frac{T_{oi}(s) - T_{oi+1}(s)}{2} \right) \]

\[ = \sqrt{\frac{t_c k \rho C_p}{\pi}} \left( T_{oi}(s) - T_{oi+1}(s) \right) \]  

The net transfer into layer \( i \) during the \( s^{th} \) fixed period is:

\[ Q_i(s) = Q_{i-1}(s) - Q_{i+1}(s) \]

\[ = \sqrt{\frac{t_c k \rho C_p}{\pi}} \left( T_{o(i-1)}(s) - T_{o(i+1)}(s) - 2T_o(s) \right) \]  

(5.14)

and the mean layer temperature at the end of the fixed period is:

\[ T_{1i}(s) = T_o(s) + \frac{Q_i(s)}{\rho C_p d} \]  

(5.15)

where \( d \) is the layer depth.

In this way the temperature distribution in the bed may be calculated at the end of each fixed period. It is assumed that the depth of each layer is such that the heat transfer in one side of the layer is unaffected by that at the other boundary. The effective thermal conductivity of particulate materials is small and the contact times short so this assumption is acceptable. The model incorporating conduction between the layers was used to simulate the well mixed and segregated regimes, using the data listed in section 2.2.

In the well mixed case the temperature of the bed is independent of the layer depth used. The heated particles from the wall layer are evenly distributed through the entire bed, giving a uniform bed temperature which is unaffected by the number of layers used. There is no conduction between layers in this case since all the layers are at the same temperature.

In the segregated regime the heat transfer was modelled over a range of layer sizes. The mean bed temperature, produced over 1000 cycles at each layer size are shown on figure 5.3. This shows that the pure segregated regime is not well modelled by the layer mixing model. This is because the form of the mixing model is such that it cannot exactly describe the segregated mode of
motion in the bed. In the mixing model it is assumed that the particles within one layer are well mixed. In true segregated motion there is no mixing at all so to model this type of motion the layers would have to be infinitesimally small. It is clear that there is an improvement in the modelling of the segregated regime as the number of layers is increased, although there is still a significant deviation between the analytical and modelled results, even with 50 layers. This mechanism is unlikely to occur in a real bed so that the model's failure to simulate it well is not too important. Even if it were to occur the analytical solution would be easier to apply then the model in any case.

5.2.5. Modelling Intermediate mixing

The heat transfer patterns produced by various regimes other than the two extremes are illustrated by example. Mixing matrices showing intermediate behaviour have been formulated to illustrate the gross effect of different types of mixing. It is not anticipated that a real bed would exhibit any of these mixing modes in its pure form but experimental matrices do show these features to a lesser extent and in combinations of more than one mode.

Example(4) – Segregated wall layer

The interior of the bed (layers 2 to 19) are well mixed, the wall layer does not mix with any of the other layers. This is similar to the mixing mechanism proposed by Wachters [W1]. Over 1000 cycles the resulting mean bed temperatures are:

<table>
<thead>
<tr>
<th>Cycles</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>T'C</td>
<td>135.1</td>
<td>161.7</td>
<td>183.9</td>
<td>202.6</td>
<td>218.3</td>
<td>231.4</td>
<td>242.4</td>
<td>251.7</td>
<td>259.7</td>
<td>265.9</td>
</tr>
</tbody>
</table>

These are compared with the corresponding bed temperatures under the well mixed regime (fig 5.6). The insulating effect of the segregated wall layer is clear. Because heat is only transferred from the wall section by conduction, at a rate which is limited by the poor conductivity of the the material, the wall layer heats up rapidly to virtually the wall temperature. The wall flux is then controlled by the rate at which heat is conducted through the wall layer to the bulk of the bed. The temperature within the bed is uniform apart from the wall layer.
Figure 5.6 Mean bed temperatures example (5)a

Figure 5.7 Bed temperature distributions for example (5)a
**Example(5) Mixing in Part of the Bed**

The second mixing pattern considered models the situation where particles can re-enter the bed within a certain distance from their exit position. Within this range the re-entry point is random. This is represented by a mixing matrix with mixing between a set number of adjacent layers only.

(5)a Mixing between 3 layers

In this example mixing takes place between layers i, i-1 and i+1, these layers being well mixed. The mixing matrix is:

\[
M = \begin{bmatrix}
16.67 & 8.33 & 0 & 0 & 0 & 0 \\
8.33 & 8.33 & 8.33 & 0 & 0 & 0 \\
0 & 8.33 & 8.33 & 8.33 & 0 & 0 \\
0 & 0 & 8.33 & 8.33 & 8.33 & 0 \\
0 & 0 & 0 & 8.33 & 8.33 & 8.33 \\
: & : & : & : & : & :
\end{bmatrix}
\]

Elements \( m_{11} \) and \( m_{20} \) are double the value of the other non-zero elements, to maintain the continuity of the mixing. The physical significance of this is that since solids from layer 1 cannot move into a lower layer they simply return to layer 1, the wall layer. Similarly the upper layer solids cannot move into a higher layer. The mean bed temperatures produced by this mixing regime are shown on figure 5.7 along with the corresponding well mixed model temperatures.

(5)b Mixing between 5 layers

Here the solids may return to a layer within 2 layers either side of their original position.

\[
M = \begin{bmatrix}
15 & 5 & 5 & 0 & 0 & 0 & 0 & 0 \\
5 & 10 & 5 & 5 & 0 & 0 & 0 & 0 \\
5 & 5 & 5 & 5 & 5 & 0 & 0 & 0 \\
0 & 5 & 5 & 5 & 5 & 5 & 0 & 0 \\
0 & 0 & 5 & 5 & 5 & 5 & 5 & 0 \\
: & : & : & : & : & : & : & : 
\end{bmatrix}
\]
and the resulting mean bed temperatures are shown on figure 5.7 with those for the well mixed case.

Clearly as the amount of mixing increases so does the overall rate of heat transfer, as would be expected since the increased mixing of the wall layer causes a decrease in the temperature of that layer at the beginning of each fixed period.

The contrast between the internal temperature distribution in the bed for the well mixed regime and for the limited mixing cases is more significant than the difference in the mean bed temperatures. The purpose of most rotary kilns is to heat the solids to their reacting temperature and to supply the heat needed for an endothermic reaction. It is desirable to have as uniform a temperature as possible in the bed to promote uniform reaction. If all the solids react at the same rate at the optimum temperature the size of the kiln may be optimized and the heat supplied minimized. In the beds with limited mixing there is a definite temperature gradient within the bed which would retard the reaction in the inner, cooler layers. The internal temperature distributions are shown in figures 5.8 and 5.9 for examples (5)a and (5)b.

Example (5)c

Here particles re-enter the bed at a random position within 5 layers either side of their original layer. Thus 11 adjacent layers are completely mixed.

The resulting mean bed temperatures are similar to those for the well mixed bed:

<table>
<thead>
<tr>
<th>Cycles</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (well mixed)</td>
<td>168.6</td>
<td>213.7</td>
<td>243.3</td>
<td>262.7</td>
<td>275.4</td>
</tr>
<tr>
<td>T (11 layers mixed)</td>
<td>162.0</td>
<td>204.5</td>
<td>234.0</td>
<td>254.3</td>
<td>268.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (well mixed)</td>
<td>283.8</td>
<td>289.3</td>
<td>292.9</td>
<td>295.3</td>
<td>296.9</td>
</tr>
<tr>
<td>T (11 layers mixed)</td>
<td>278.2</td>
<td>284.9</td>
<td>289.6</td>
<td>292.8</td>
<td>295.0</td>
</tr>
</tbody>
</table>

The internal temperature distributions (figure 5.10) are much flatter than those for the previous two examples, and therefore closer to the well mixed model. On figure 5.10 there is a drop in temperature in the bed at $y=137\text{mm}$, corresponding to the limit of mixing of the wall layer.
Figure 5.8 Bed temperature distributions for example (5)b

Figure 5.9 Bed temperature distributions for example (5)c
Figure 5.10 Bed temperature distributions for example (6)a

Figure 5.11 Bed temperature distributions for example (6)b
Example (6) Partial Mixing in part of the bed

These examples are similar to (5) but the adjacent layers are not completely mixed. The probability of a particle re-entering a layer decreases as the distance from it original layer increases. This is possibly a more realistic pattern than that of example (5).

Example(6)a Mixing between 5 layers (c.f. ex. (5)b)

The mixing matrix is:

$$ M = \begin{pmatrix} 17 & 6 & 2 & 0 & 0 & 0 \\ 6 & 11 & 6 & 2 & 0 & 0 \\ 2 & 6 & 9 & 6 & 2 & 0 \\ 0 & 2 & 6 & 9 & 6 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} $$

The temperatures for the partially mixed case are lower than those for the case where the mixing between the layers was complete. The gradient of the temperature distribution within the bed was also greater in the case with partial mixing between the adjacent layers (figure 5.11). these differences arise because in the partially mixed case the wall layer is less well distributed over the inner layers than in the example where the mixing was complete.

Example (6)b Partial mixing between 11 adjacent layers

The mixing matrix is:

$$ M = \begin{pmatrix} 15.0 & 4.0 & 3.0 & 1.6 & 1.0 & 0.4 & 0 & 0 & 0 & \vdots \\ 4.0 & 11.0 & 4.0 & 3.0 & 1.6 & 1.0 & 0.4 & 0 & 0 & \vdots \\ 3.0 & 4.0 & 8.0 & 4.0 & 3.0 & 1.6 & 1.0 & 0.4 & 0 & \vdots \\ 1.6 & 3.0 & 4.0 & 6.4 & 4.0 & 3.0 & 1.6 & 1.0 & 0.4 & \vdots \\ 1.0 & 1.6 & 3.0 & 4.0 & 5.4 & 4.0 & 3.0 & 1.6 & 1.0 & \vdots \\ 0.4 & 1.0 & 1.6 & 3.0 & 4.0 & 5.0 & 4.0 & 3.0 & 1.6 & \vdots \\ 0 & 0.4 & 1.0 & 1.6 & 3.0 & 4.0 & 5.0 & 4.0 & 0 & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} $$

The resulting mean bed temperatures are below, along with those for
Figure 5.12 Bed temperature distributions for example (9)
example (5)c where each set of 11 layers was uniformly mixed.

<table>
<thead>
<tr>
<th>Cycles</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 layers well mixed</td>
<td>162.0</td>
<td>204.5</td>
<td>234.0</td>
<td>254.3</td>
<td>268.4</td>
<td>278.2</td>
</tr>
<tr>
<td>11 layers part mixed</td>
<td>158.2</td>
<td>198.5</td>
<td>227.3</td>
<td>248.0</td>
<td>262.8</td>
<td>273.2</td>
</tr>
<tr>
<td>700</td>
<td>800</td>
<td>900</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>280.9</td>
<td>286.3</td>
<td>290.2</td>
<td>292.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>284.9</td>
<td>289.6</td>
<td>292.8</td>
<td>295.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Again there is a decrease in the mean bed temperatures in the case where the mixing between the layers is not complete. There is also a decrease in the temperature gradients within the bed, as in the examples with mixing between 5 layers. The drop at 137mm has been smoothed in the partially mixed case (figure 5.12).

Example(7) Wall layer evenly distributed

If the wall layer is evenly distributed over the bulk of the bed after each mixing phase the bed is isothermal at the corresponding temperature of the well mixed bed, irrespective of the mixing pattern between the inner layers.

5.2.6. Core Mixing

Where there is interchange between the fixed and core regions a modification to the heat transfer model is needed. It has been shown that where there is no core-fixed bed mixing the temperature of the core may be assumed to be the same as that of the upper fixed bed layer. This is not necessarily true where heat may be carried into and out of the core by particles moving into and out of it. The core temperature must then be found from the heat balance (equation (5.4)) viz

\[ h_{cc}A_{cc}(T_{core} - T_{cc}) + h_{cf}A_{cf}(T_{core} - T_{cf}) + F_{c}\rho Cp(T_{core} - T_{pc}) = 0 \]  

(5.4)

It can be assumed that that core-cascade conduction term is negligible, the contact between the two bodies of solids is poor and the voidage of the cascade layer is high, reducing its conductivity.
The temperature $T_{cf}$ is taken to be the initial upper fixed bed surface temperature as before.

The mean temperature of the particles entering the core can be found from the known temperature distribution and the mixing matrix viz:

$$T_{pc} = \frac{\Sigma m_{core} T_i}{F_c}$$ (5.16)

The core temperature is found by using the above assumptions and the heat balance. The final upper fixed layer is also determined using the heat flow between the fixed bed and the core.

The effect of core mixing on heat transfer is illustrated by examples. The data is that from section 5.2.1, as in the previous examples. The bed is divided into 21 layers; 20 fixed bed layers and the core.

Example (8) Well mixed bed with equal mixing between all layers and the core

1/21st of each layer is transferred to the core at each mixing phase and the whole fixed bed is completely mixed.

$m_{core} = m_{corei}$ for all layers.

The mean fixed bed temperatures produced along with those for the well mixed bed without core mixing are:

<table>
<thead>
<tr>
<th>Cycle</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well mixed no core</td>
<td>168.6</td>
<td>213.7</td>
<td>243.3</td>
<td>262.7</td>
<td>275.4</td>
<td>283.8</td>
</tr>
<tr>
<td>Well mixed with core</td>
<td>167.2</td>
<td>211.8</td>
<td>241.5</td>
<td>261.1</td>
<td>274.2</td>
<td>282.9</td>
</tr>
<tr>
<td></td>
<td>700</td>
<td>800</td>
<td>900</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>289.3</td>
<td>292.9</td>
<td>295.3</td>
<td>296.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>288.6</td>
<td>292.4</td>
<td>294.9</td>
<td>296.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The entire bed, including the core is isothermal. The mean bed temperatures are slightly lower than for the well mixed bed without core.
mixing. This is because the fraction of the hot wall layer mixing into each layer is reduced by the core mixing. This effect is partly counteracted by the increase in wall flux due to the decrease in wall layer initial temperatures for each cycle.

Example (9) Complete mixing over part of the bed

Here the mixing pattern is identical to that of example (5)b but extended to include the core (ie 21 layers rather than 20). Particles may re-enter the bed at any point up to two layers either side of their original position, with equal probability. The average fixed bed temperatures are:

<table>
<thead>
<tr>
<th>Cycle</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without core</td>
<td>162.0</td>
<td>204.5</td>
<td>234.0</td>
<td>254.3</td>
<td>268.4</td>
<td>278.2</td>
<td>284.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With core</td>
<td>152.3</td>
<td>187.4</td>
<td>214.0</td>
<td>234.3</td>
<td>249.8</td>
<td>261.7</td>
<td>270.7</td>
<td>289.6</td>
<td>292.8</td>
<td>295.0</td>
</tr>
</tbody>
</table>

Again where core interchange is included the mean bed temperatures are slightly lower than for the similar mixing pattern without core mixing. The internal temperature distributions for the two cases are otherwise very alike (see figure 5.13).

Example (10) Mixing over part of the bed with core-wall interchange

The mixing matrix for this example is similar to that of the previous example, but interchange between the wall layer and the core is superimposed.

\[ m_{1\text{core}} = m_{\text{core1}} = 1 \]

The mean bed temperatures are:
Figure 5.13 Bed temperature distributions for example (10)

Figure 5.14 Bed temperature distributions for example (11)
These temperatures are higher than those for the same mixing pattern without core-wall interchange. The flux at the wall is increased by the extra mixing of the wall layer which reduces the temperature at the wall. Conduction from the core also tends to increase the mean fixed bed temperature. Mixing between the core and the upper layers of the fixed bed also increases the bed temperature over that for the case with no core mixing. The temperature distributions within the bed are shown on figure 5.14.

Example (11) Mixing over part of the bed, core only mixing with wall layer.

This is similar to example (10) above, but with no mixing between the upper fixed bed layers and the core. The core only interchanges particles with the wall layer. The mean bed temperatures are:

<table>
<thead>
<tr>
<th>Cycles</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Temp</td>
<td>151.4</td>
<td>187.1</td>
<td>214.1</td>
<td>234.1</td>
<td>250.3</td>
<td>262.2</td>
<td>271.2</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>900</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>281.1</td>
<td>283.3</td>
<td>287.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The bed temperatures here are slightly lower than for example (10). This is due to the decrease in heat transfer between the upper fixed bed and the core. In figure 5.15 the core can be seen to be at a higher temperature than the upper fixed bed but heat transfer is limited by being confined to conduction only.

Example (12) Well mixed interior and core, segregated wall layer

The inner 19 fixed bed layers are well mixed, with 1/20\textsuperscript{th} of each of these layers being exchanged with the core at each mixing phase. The wall layer is segregated. This resembles example (4). The resulting temperatures are:
Figure 5.15 Bed temperature distributions for example (12)

Figure 5.16 Bed temperature distributions for example (13)
Again these are much lower than for the completely well mixed case (example (4)), because of the insulating effect of the wall layer.

Example (13) Segregated bed with superimposed wall-core mixing

The inner 19 fixed bed layers are segregated, 1/5th of the wall layer is interchanged with the core. The bed temperature distributions are shown on figure 5.16. Mean fixed bed temperatures are:

<table>
<thead>
<tr>
<th>Cycles</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Temp</td>
<td>120.0</td>
<td>131.6</td>
<td>141.3</td>
<td>149.7</td>
<td>157.4</td>
<td>164.6</td>
<td>171.2</td>
</tr>
<tr>
<td>Ideal segregated</td>
<td>108.4</td>
<td>111.9</td>
<td>114.5</td>
<td>116.8</td>
<td>118.8</td>
<td>120.5</td>
<td>122.2</td>
</tr>
</tbody>
</table>

These mean temperatures are significantly higher than those for the completely segregated model because conduction into the fixed bed takes place from both the wall layer and the core. The overall flux from the wall is increased by the lowering of the wall layer temperature caused by the mixing with the core. The temperature of the particles leaving the core and re-entering the wall layer is significantly lower than the temperature of those leaving the wall layer and entering the core, reducing the mean temperature of the wall layer.

The temperature distributions produced by these examples are very unusual and would not be expected to be produced in any real bed. However the effects of various mixing regimes on temperature distribution are important.

5.3. The Segment Model

The next step in developing the full bed model is to move to a more realistic bed shape. The rectangular slab is not a very close approximation to the true bed shape, which is a segment of the circular cross section of the
drum. Both phases of heat transfer in the bed; conduction and mixing, need slightly different approaches in the curved bed.

5.3.1. Conductive Transfer

The bed is divided into concentric layers as shown in figure 5.17. In the rectangular analogue of the bed all particles spend the same length of time in the fixed bed, regardless of the layer they are in. This is not true in the segment bed, where the time spent in the bed depends on the radius of the path of the particle in the bed. The smaller the path radius, the shorter the time in the bed will be. The time spent by a particle in the fixed bed at radius \( r \) is

\[
t_c(r) = \frac{2\phi(r)}{\omega}
\]

(5.17)

where \( \phi(r) = \cos^{-1}(\frac{R - D_b}{r}) \) (see fig 5.1)

These different residence times in the layers affect the conductive heat transfer within the bed by reducing the contact times between the layers, as compared to the constant contact time used in the rectangular model.

5.3.2. Mixing in the Segment Model

The mixing matrix in the 2-dimensional segment model must be interpreted in a slightly different way from its meaning in the rectangular slab model. For the rectangular slab a strip of the slab was taken as the basis of the model and the mixing matrix predicted the configuration of the particles making up the strip as it entered each axial slice of the bed. Material could not pass into or out of the strip. In the segment bed it is not possible to identify an equivalent body of material, spanning all the layers of the bed, which remains intact during a cycle of the bed. The mixing matrix in the segment model represents the rates at which particles leave one row and re-enter at another. The basis of this matrix is therefore not a volume or mass of solids, as in the rectangular model, but a period of time. The elements \( m_{ij} \) represent the mass of material leaving layer \( i \) and entering row \( j \) over a certain period of time.

The rate at which material leaves a layer of depth \( d \) at radius \( r \) (assuming that \( d \) is sufficiently small to allow \( r \) to be considered constant over the whole
Figure 5.17 Layers used in heat transfer model
layer) is:

\[ F(r) = \rho rdr_1 \omega \]  

(5.18)

\( r_1 \) is the axial thickness of the slice of the bed.

The sum of the rates for all of the layers gives the total rate at which particles enter the cascade layer:

\[ \Sigma_i F(r_i) = \rho \omega r_1 \Sigma_i r_i d_i \]  

(5.19)

The initial temperature of each layer is given by:

\[ T_i = \Sigma_j m_{ji} T_j / F(r_i) \]  

(5.20)

\[ F(r_i) = \Sigma_j m_{ji} \]  

(5.21)

Examples similar to those used for the rectangular bed will be considered, along with the well mixed and segregated patterns. The temperature distributions and mean fixed bed temperatures can then be compared to those produced by the rectangular bed approximation. The data used is the same as for the rectangular bed, from section 2.2. In addition the radius of the wall is 1.83m, the rotational speed is 1.67 r.p.m. and the filling angle is 55°.

5.3.3. Well Mixed Bed

The mixing matrix consists of elements \( m_{ij} \) where

\[ m_{ij} = F(r_i) \times \frac{F(r_j)}{\Sigma_k F(r_k)} \]  

(5.22)

\( m_{ij} \) is therefore proportional to both the rate at which solids leave layer \( i \) and the rate at which they enter layer \( j \). This produces a well mixed isothermal bed. Since the ideal case is considered initially no core-fixed bed interchange is included in the matrix.

The bed temperatures produced are:
<table>
<thead>
<tr>
<th>Cycles</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment model</td>
<td>177.4</td>
<td>224.8</td>
<td>253.9</td>
<td>271.7</td>
<td>282.6</td>
<td>289.3</td>
</tr>
<tr>
<td>Rectangular model</td>
<td>168.6</td>
<td>213.7</td>
<td>243.3</td>
<td>262.7</td>
<td>275.4</td>
<td>283.8</td>
</tr>
<tr>
<td></td>
<td>700</td>
<td>800</td>
<td>900</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>293.4</td>
<td>295.9</td>
<td>297.5</td>
<td>298.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>289.3</td>
<td>292.9</td>
<td>295.3</td>
<td>296.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The mean temperatures for the well mixed segment model are higher than those for the corresponding rectangular model. The cross sectional area is of course smaller than the rectangular slab, suggesting that higher bed temperatures should be expected. The difference in the bed temperatures between the two beds decreases with an increasing number of bed cycles, because the effect of the higher bed temperature on the wall flux becomes relatively greater as the temperature difference between the bed and the wall decreases. Where the difference between the wall and bed temperatures is large, at the beginning of the simulations, the small increase in the bed temperature in the segment bed produces a relatively small change in the flux. In this case the temperature difference between the rectangular and segment beds is maintained by the effect of the smaller cross-sectional area of the curved bed. After a large number of cycles (ie far from the feed end of the drum) the temperature difference at the wall is small and the effect of a change in the bed temperature has a relatively large effect on the wall flux. Here the change in flux more than compensates for the different bed areas and the bed temperatures tend to converge.

5.3.4. Segregated Motion

In the segregated bed

\[ m_{ij} = F(r_i) \]

and \( m_{ij} = 0 \) \( i \neq j \)

The resulting bed temperatures, and those for the segregated rectangular bed are:
Cycles | 100 | 200 | 300 | 400 | 500 | 600  
Segment bed | 125.8 | 136.4 | 144.3 | 150.8 | 156.4 | 161.5  
Rectangular bed | 120.0 | 130.5 | 138.7 | 145.6 | 151.8 | 157.4  
700 | 800 | 900 | 1000  
166.1 | 170.7 | 174.2 | 177.9  
162.5 | 167.3 | 171.8 | 176.0  

Here the segment model temperatures are higher than those for the rectangular bed. This is due again to the difference in cross-sectional areas for the two beds. The difference in temperatures between the two bed shapes decreases with time as the temperature difference at the wall decreases more rapidly in the segment bed because of the constriction on the conduction into the inner layers imposed by the shorter contact times. The difference in temperature between the two beds is smaller than the difference in the well mixed case because the restricted conduction counteracts the decrease in bed size.

5.3.5. Intermediate Mixing

Example (14) Well mixed bed including core mixing

The matrix is similar to that for the well mixed case but with elements $m_{\text{core}}$ and $m_{\text{corei}}$ set to $1/21^{\text{st}}$ of the appropriate rate $F(r_i)$ and the other elements reduced to compensate. The mean fixed bed temperatures are:

Cycles | 100 | 200 | 300 | 400 | 500 | 600  
T | 174.5 | 221.2 | 250.5 | 269.0 | 280.5 | 287.7  
700 | 800 | 900 | 1000  
297.3 | 295.1 | 269.9 | 298.0  

These are slightly lower than those for the well mixed segment bed with no core interchange, as was the case for the rectangular bed. The temperatures for the segment bed are higher than those for the corresponding mixing pattern in the rectangular bed, as in the examples with no core mixing.
Example (15) Complete mixing over a limited transition length

This is similar to example (9) with particles re-entering the bed at a random position up to 2 layers either side of their original position.

Mean bed temperatures are:

<table>
<thead>
<tr>
<th>Cycles</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (°C)</td>
<td>158.5</td>
<td>196.0</td>
<td>223.5</td>
<td>243.7</td>
<td>258.6</td>
<td>269.5</td>
<td>277.5</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>900</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>283.4</td>
<td>287.8</td>
<td>291.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Again the resulting temperatures are higher than those for the corresponding rectangular bed model. The distributions are shown on figure 5.18 and are very similar to those on figure 5.13, for the rectangular bed. The main heat transfer mechanism is mixing so that the effect of the reduction of the bed cross-sectional area in the segment bed is more important than the restriction of the conduction between the layers.

Example (16) Complete mixing over a limited transition length with wall-core interchange

The mixing matrix is identical to that of the previous example but with some interchange between the wall layer and the core superimposed, i.e

\[ m_{\text{core}} = m_{\text{core1}} = \frac{F(r_i)}{25} \]  

(5.23)

corresponding to the value used in example (10).

The mean bed temperatures are:

<table>
<thead>
<tr>
<th>Cycles</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (°C)</td>
<td>162.7</td>
<td>204.2</td>
<td>233.1</td>
<td>253.3</td>
<td>267.4</td>
<td>277.2</td>
<td>284.1</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>900</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>288.8</td>
<td>292.2</td>
<td>294.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The increase in the mean bed temperature caused by the wall-core
Figure 5.18 Bed temperature distributions for example (15)

Figure 5.19 Bed temperature distributions for example (16)
mixing in the segment model is much smaller than in the rectangular bed model. This is because the upper layers of the fixed bed which are heated via the core are relatively small compared to the total bed area so that the rise in temperature in these layers has only a limited effect on the mean bed temperature (see figure 5.19).

**Example (17) Complete mixing over a limited transition length, core only mixing with the wall layer**

The mixing is as in the previous example except that the only interchange with the core is from the wall layer, as in example (11).

The resulting mean fixed bed temperatures are:

<table>
<thead>
<tr>
<th>Cycles</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (°C)</td>
<td>161.4</td>
<td>201.7</td>
<td>230.3</td>
<td>250.6</td>
<td>264.9</td>
<td>275.1</td>
<td>282.3</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>900</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>287.4</td>
<td>291.1</td>
<td>193.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Again the pattern established in the rectangular bed model is followed; these temperatures are slightly lower than the previous example where mixing heat transfer from the core to the upper fixed layers was included. This effect is less marked here than in the rectangular bed, again this is because the innermost layers of the segment bed form a smaller fraction of the total bed area than the inner layers of the rectangular bed. Again these temperatures are higher than those for the corresponding rectangular bed, due to the smaller bed cross-sectional area. The temperature distribution in the bed is shown on figure 5.20.

**Example (18) Well mixed inner bed with segregated wall layer**

The smaller bed cross-sectional area in the segment bed produces higher mean bed temperatures in this case. Because the heat transfer within the inner layers is by mixing alone the restriction on conduction of the short contact times between these layers has no effect. The conduction through the wall layer is the controlling heat transfer step here. The contact time between the wall layer and the layer adjacent to it is only slightly lower than the contact time for the rectangular bed so that the effect of the smaller bed size on
Figure 5.20 Bed temperature distributions for example (17)

Figure 5.21 Bed temperature distributions for example (19)
the mean temperature more than compensates for the restricted conduction into the bulk of the bed. The temperatures of the segment and rectangular beds diverge with time as the wall layer temperature rises faster in the segment bed, reducing the wall flux.

<table>
<thead>
<tr>
<th>Cycle</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
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<tr>
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<td>179.7</td>
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<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>188.9</td>
<td>203.5</td>
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</table>

Example (19) Segregated bed with wall-core mixing

The dominant heat transfer step is the conduction through the wall layer, since the contact time between the wall layer and the layer adjacent to it is only slightly smaller than the rectangular bed contact time. The size difference between the beds more than compensates for the constriction of conduction so that the mean fixed bed temperatures in the segment bed are higher than those for the corresponding rectangular bed (see figure 5.21).

<table>
<thead>
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<th>200</th>
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<td>210.6</td>
<td>217.6</td>
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<td>177.5</td>
<td>183.5</td>
<td>189.1</td>
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</tbody>
</table>

5.4. Summary of Rectangular and Segment Models

The purpose of examining the two bed geometries was to test whether the rectangular bed approximation produced results close enough to those produced by the segment model to allow it to be used as a satisfactory model of the heat transfer in the bed of an externally heat rotary kiln. This approximation has been used before [11].

It has been shown that the rectangular model does not provide an analogue of the decreasing contact time between the layers in the fixed bed as the layer radius decreases. This effect is most important where conduction from the core is significant. Where the innermost layers of the fixed bed are
relatively hot or cold the effect on the mean bed temperature of the segment bed is less significant than in the rectangular bed because the inner layers in the segment bed form a smaller fraction of the whole bed than do the corresponding layers in the rectangular bed.

Where the majority of the heat transport in the bed is by mixing the segment cross-section temperatures are higher than those for the corresponding rectangular bed, although the difference in mean temperature decreases for large numbers of cycles. This increase in temperature is due to the smaller cross-section of the segment bed. The convergence of the temperatures in the two geometries with increasing number of cycles is caused by the reduction in wall flux with higher bed temperatures in the segment bed.

The effect of the geometry on the heat transfer depends on the mixing pattern as well as on the geometry itself so that the rectangular approximation is not suitable for this application in all cases. However where the mixing is always the dominant heat transfer mechanism, as in configurations where the heat is initially transferred to the cascade layer, a rectangular approximation would possibly be acceptable.

Although the relationship between the heat transfer in the two different beds is complicated and depends on the mixing pattern as well as on the geometry the trends in heat transfer for each geometry are the same. The rectangular bed model is therefore useful for an initial exploration of the heat transfer behaviour of a bed. It is much simpler to formulate mixing matrices for the rectangular bed where all the row totals of the matrix are equal.

The use of simple mixing examples allows the principal features of the relationship between the heat transfer to and within the bed and the mixing pattern to be identified. The behaviour of the wall layer and the core are the most important factors in the transport of heat to and through the bed.

Where the wall layer is distributed evenly over the rest of the bed the whole bed is isothermal, irrespective of the mixing between the inner layers of the bed. If the wall layer is segregated the heat transfer is restricted by the rate of conduction through the wall layer. The bed temperature distribution does depend on the mixing between the inner layers in this case. With a segregated wall layer the wall flux depends on the temperature of the inner bed in contact with the wall layer and therefore on the inner layer mixing
pattern. The dominant factor is the segregated wall layer however since this limits the heat transfer rate which can be achieved with any mixing pattern.

The effect of interchange between the core and the fixed bed is also significant. Although the net heat transfer to the core over any cycle is zero its behaviour affects both the mean bed temperature and the temperature distribution within the fixed bed. This affect is most clearly seen where there is interchange between the core and the wall layer, increasing the mean bed temperature through heat transfer between the core and the upper fixed bed layers.

In conclusion the rectangular bed approximation, which is easier to manipulate and construct matrices for, is useful for examining trends in heat transfer for different mixing patterns. However for predicting the heat transfer in a real bed the rectangular bed model is not appropriate because of the dependence of the conduction within the bed on the geometry of the bed.

If the the mixing matrices are derived from experiments in real bed the segment model must be used since the data provided by those matrices is in the form needed for this model rather than for the rectangular bed.

If there were a reaction taking place in the bed the heat absorbed by the reaction this would be affected by the temperature distribution in the bed. Generally reaction rate and therefore the rate at which heat is absorbed by the reaction increases with temperature.

In a well mixed bed the reaction rate would be uniform over the whole bed and the temperature of the bed would likewise be uniform. The reduction in temperature from the temperature in a non-reacting bed due to the heat absorbed by the reaction would be somewhat offset by the increase in the flux from the wall caused by the lowered temperature.

In the segregated bed the reaction in the wall layer would initially retard the conduction into the bulk of the bed. Absorption of heat by the reaction would reduce the temperature of the solids near the wall below that found in a non-reacting bed. This process would continue as inner layers heated and reacted so that the overall effect would be to delay the heat transfer to the inner bed. This would produce a very slow reaction in the inner layers of the bed. Once all of the material in a layer of the bed has reacted it then behaves
in the same way as the non-reacting bed. In this case reaction rate and conversion would depend strongly on the position in the bed.

The reaction rates in beds under other mixing regimes would also be affected by the mixing patterns, with relatively hot regions reacting faster than the material in cooler parts of the bed. The more uniform the temperature in the non-reacting bed for a given mixing regime the more uniform the reaction will be. The heat transfer model for the non-reacting bed is therefore useful for predicting areas of the bed where reaction rate will be slow. The elimination of such areas is important in optimising the operation of reactors of this type.

5.5. Heat Transfer Model with Experimental Matrices

The mixing matrices produced by the experiments described in chapter 3 were used with the data from section 5.2.1 to produce temperature distributions and mean bed temperatures in the bed over 1000 cycles or slices of the bed. The same contact time and filling were used with all the matrices, even though the original experiments had been carried out with different rotational speeds and fillings. This was to make the resulting temperatures directly comparable with each other and with the earlier examples. If different speeds and fillings had been used in the modelling the resulting variation in the contact times might have obscured the effect of the differences in mixing pattern. The resulting temperatures are tabulated in appendix VII.

For the most part the resulting temperature distributions show only a very small gradient within the bed with the layers near the wall slightly hotter than those towards the core. After 100 cycles the maximum temperature difference between the wall layer and the upper fixed layer was 2.7 C. Some of the modelled beds showed a marginal temperature difference between these layers even after 1000 cycles.

The mean bed temperatures produced are lower than those for the ideal well mixed bed (section 5.3.3) and those for the well mixed bed with uniform core mixing (example 14). Typical values of the mean bed temperatures produced from the experimental matrices and the well mixed bed temperatures are shown on figure 5.22. The extent by which the fixed bed temperature is lowered by core mixing depends on the rate at which material is interchanged between the fixed and core areas of the bed. In example 14 1/21st of each layer
Figure 5.22 Well mixed and experimental mean bed temperatures
enters the core from each slice of the bed. In the experimental matrices the amount of the layers near the wall exchanging with the core is generally less than $1/21^{st}$, while the inner layers have more than $1/21^{st}$ of their solids interchanging with the core. Overall more than $1/21^{st}$ of the fixed bed is interchanged with the core in the experimental matrices, so that the temperatures would be expected to be lower than those in example 14.

The matrices appear to be similar to those of example 6 for the rectangular bed, where there was partial mixing between a number of adjacent layers, with the amount of material transferring between layers decreasing as the distance between the two layers increased. This partial segregation also lowers the bed temperature and is the source of the temperature gradients within the beds.

The modelling of beds using the experimental matrices is somewhat limited by the small number of bed layers in these matrices. The depth of the layers has been shown to be important in modelling segregated beds and it is possible that some of the effects of the tendency towards segregation shown in the experimental results have been obscured by the relatively large layer size used. However the experimental matrices did not approach pure segregation so that the mixing between layers should have reduced the effect of layer size. Comparison of the temperatures for those matrices with 6 rows and those with 12 rows shows that while the mean temperatures are very similar the temperature gradients in the beds are larger in the 12 layer bed. This may indicate that there is more segregation in beds with relatively small particles or that the effect of layer depth is significant.

These simulations have shown that the mixing pattern in the bed can reduce the mean bed temperature below that expected for a well mixed bed.

It is not possible to model the heat transfer experiments carried out by Wachters and Kremers and Lehmberg et al. This is because these experiments were carried out under unsteady state conditions and the model requires a steady state condition for the calculation of the core temperature. There are also no mixing matrices for these experiments. Comparison of modelled bed temperatures, had it been possible to produce them, and experimental temperatures might not have been productive in any case since the uncertainties in recording bed temperatures with thermocouples under
unsteady state conditions would be considerable.
CHAPTER 6

DISCUSSION AND CONCLUSIONS

The purpose of this work was to investigate the heat transfer to the solid bed in an externally heated rotating drum. Consideration of previous work indicated that knowledge of the radial mixing of the bed was fundamental to the understanding of the heat transfer process in the solids. The limited amount of previously reported work on radial mixing was contradictory and inconclusive, mainly due to experimental difficulties. Mixing is of particular importance in externally heated drums where the majority of the heat transfer takes place initially to a thin layer of solids which subsequently mixes with the bulk of the bed. It was also clear that information on the mixing patterns in beds, would allow a new approach to the modelling of residence time distribution.

The three main sections of the work, experimental investigation of mixing, residence time modelling and heat transfer modelling are summarized below.

6.1. Experimental Work

The bed mixing behaviour was studied through analysis of the behaviour of a 2-dimensional drum. A system of image acquisition and analysis was developed to locate and record the position of a single white particle in a bed of black solids from a video recording of the bed in motion. A mixing matrix, describing the mixing pattern of the bed, was derived from a transition matrix which recorded the path of the single particle as it moved through the bed.

An important observation made during the experimental work was the presence of the core area of the bed, a part of the bed which tended not to mix with the rest of the solids. The core had been identified previously, but had not been considered in any model of heat transfer or residence time distribution.

The differences between the mixing matrices produced in experiments with different rotational speeds, fillings, drum size and particle coatings were not dramatic. However several trends were observed in the bed behaviour. The behaviour of the wall layer and the core were most easily identified as changing with varying operating conditions.
In general the beds were neither well mixed or completely segregated, but did tend to show some segregation. The variables studied were rotational speed, fractional filling, interparticle friction and drum size. The effect of varying these parameters is outlined below.

- Rotational Speed. As rotational speed increased segregation increased and core interchange with the fixed bed decreased.

- Filling. Increasing the filling gave an increase in core mixing at low speed, but had no noticeable effect at higher speeds.

- Interparticle friction. At low speeds the rate at which solids entered the core was increased for rough particles and smooth particles promoted segregation of the wall layer. At an intermediate speed the smoothest particles gave the maximum rate of core interchange, while at a higher speed the matrices for the three coatings were similar in all respects.

- Drum Size. At low and intermediate rotational speeds the larger drum produced more segregation of the wall layer and more core interchange. At the highest speed there was no clear difference between the matrices from the two drums. The increasing segregation and decreasing core mixing with increasing rotational speed may be due to the increasing velocity of particles on the cascade layer at higher speeds. The dynamic angle of repose of the solids increases with increasing speed so that the particles are tumbling down a steeper slope. The particles also emerge onto the cascade layer with more energy. These factors could lead to increased segregation on the cascade layer and hence a less well mixed bed. The higher speed of particles on the cascade layer may make it more difficult for solids to escape from the core into the cascade layer.

The increase in core mixing at low speeds for higher fillings could be due to the increase in the contact area between the core and the cascade layer. At higher speeds the higher velocity of particles on the cascade layer could suppress core mixing, as in the variation of core mixing with drum speed, and counteract the increase due to increased contact area.

The low rate of core mixing with smooth particles at low speed may be caused by the ease with which the particles on the cascade layer could slip over the core surface without dragging core solids into the cascade. The
increased wall layer segregation could have been caused by a reduction in mixing on the cascade layer due to the reduced friction between the particles. The increased core mixing in the smooth particles at the intermediate speed has no obvious explanation, especially since the core mixing is similar for all three coatings at the highest speed. Wall segregation is similar for the three coatings at the intermediate and high speeds.

The increased core interchange in the larger drum at low speeds may be explained by the increased core surface area, as in the variation of core mixing with filling. The suppression of this at higher rotational speed is also similar to that shown in the matrices for different degrees of filling. The increased segregation of the wall layer in the larger drum may be due to the increased velocity of particles on the cascade. The wall layer solids have a higher linear velocity than those in the small drum for the corresponding fraction of the critical speed.

In conclusion it appears that interchange between the fixed bed and the core is promoted by low particle velocities on the cascade layer and by a large core upper surface area. Core mixing is suppressed by high rotational speeds, which lead to high cascade velocities, and by a small core surface area. Segregation is increased by high rotational speeds, while wall layer segregation increases in larger drums. The effect of particle surface coating is complicated and requires further work define the relationship between the nature of the particles' surface and the mixing of the bed.

Confining the bed movement to two dimensions in these experiments should not introduce serious errors. In previous work batch drums, where there is no axial movement of the solids, have been used to provide data used in modelling drums with solids flow. The rate of axial advance in 3-dimensional beds is generally low, due to the small bed surface inclination angles found in most drums, so that eliminating axial movement should not greatly affect transverse motion.

The video analysis system developed in this work provides a method of gathering detailed data on the position of a coloured particle in situations where real time data collection is difficult because of the speed of movement in the process being studied, which could be useful in other experimental work.
6.2. Residence Time Distribution

A Monte Carlo simulation, based on the mixing matrix of a bed, was used to predict the residence time of a single particle in the bed. The main source of variation in the times spent by different particles in the drum was the time spent trapped in the core.

Simulations of notional drums using the mixing matrices obtained in the experimental work followed the main trends identified by previous experimental work. The model was also used to predict the residence time distributions for previous experiments and the simulated residence time variances agreed well with the reported experimentally derived variances, taking into account the limited data available.

The simulation required the throughput of the drum, which was obtained from an existing model of the bed motion. This model did not allow for the existence of the core layer and assumed the cascade layer thickness to be negligible. It has been shown that these factors could affect the accuracy of the predicted mean residence times, and the resulting residence time distribution. Comparison of modelled residence time distributions and previous experimental results was made difficult by the limited amount of work carried out in inclined drums, even though most industrial drums are inclined and it has been shown that the behaviour of the bed in inclined drums differs significantly from that in horizontal drums. The model as derived here is strictly only applicable to inclined drums with constant bed height. However, the change in bed height along many drums is small so that one mixing matrix could be used to cover the complete bed. In cases where the change in bed height along the drum is larger it would be possible to interpolate between two or more matrices to produce a simulation of the bed.

6.3. Heat Transfer

Whereas previous heat transfer modelling has been based on the assumption that the bulk of the bed was well mixed, the present model is based on the mixing behaviour of the bed as described by its mixing matrix. The model calculates the temperature distribution in the bed in successive axial slices in the drum by considering the slices to be made up of concentric well mixed layers and determining the initial and final temperature of each layer in each slice of the bed.
The behaviour of the core and of the layer of material adjacent to the wall were found to be the most important factors in the heat transfer model. If the wall layer is evenly distributed through the remainder of the solids the bed is isothermal, regardless of the mixing pattern in the inner layers of the bed and thus heat transfer and mean bed temperature are maximized. If the wall layer tends to be segregated from the other layers an adverse temperature gradient will exist in the bed, reducing the heat transfer to the bed and the mean bed temperature.

Mixing between the wall layer and the core tends to increase the temperature of the bed over that for a similar mixing pattern with no core to wall layer mixing.

The resolution of the mixing pattern used in the heat transfer model, ie the order of the mixing matrix describing the bed's behaviour does affect the temperature distributions produced. The model is based on the assumption that the solids in any layer are well mixed so that no temperature gradient exists within a layer as it is formed by solids leaving the cascade.

When experimentally derived mixing matrices were used in the heat transfer model the mean bed temperatures produced were lower than the predicted well mixed bed temperatures, confirming that the bed mixing pattern could affect the heat transfer into the bed significantly. The temperature gradients within the simulated beds were small but were more significant for experiments with a larger ratio of drum to particle diameter. The mixing matrices had shown an increase in wall layer segregation in the larger drum, which would be the source of this increase in bed temperature gradient.

Radiative and convective transfer were not considered in the model, these being much less important in externally heated drums than the conduction from the wall.

There was no suitable experimental data for comparison with the heat transfer model. Previous experimental work has been carried out under unsteady state conditions using thermocouples in the bed to measure solids temperatures. This is not an ideal method of temperature measurement in particulate beds at unsteady state since the relationship between the temperature recorded by the thermocouple and the temperature of the solids is unclear, due to the poor contact between solids and thermocouple. The
presence of thermocouples in the bed would also disturb the mixing pattern within the solids and thereby disturb the heat transfer behaviour of the bed. For these reasons this type of experimentation was rejected. We considered experimentation with liquid crystal coated solids, which would have indicated their temperature by changing colour. This would have permitted the bed temperature distribution to be studied without the need to introduce measurement devices into the bed. Initial exploratory work was carried out on the feasibility of this type of experiment. However considerable problems were experienced in calibrating the colour to temperature relationship and in producing reproducible results and the method was not used (see appendix IX).
6.4. Conclusions

1. The effect of varying rotational speed, filling, particle coating and drum diameter on the mixing behaviour of beds in rotating drums was investigated. All of these variables were found to have some effect on the mixing pattern, particularly on the core and the wall layer mixing.

2. A Monte Carlo simulation of residence time in drums was developed which produced residence time distributions which agreed well with previous experimental work.

3. The importance of the core in residence time distribution in rotation drums has been identified. The time spent trapped in the core by particles is the main source of residence time spreading.

4. The mixing in the bed has been shown to produce a reduction in heat transfer from that predicted by a well mixed model, although more heat transfer experimentation is needed to provide a quantitative comparison.

5. The behaviour of the layer of the bed adjacent to the wall and that of the core have been identified as having the greatest effect on heat transfer.
6.5. Recommendations for Further Work

Further experimental work of the type carried out in the present work is needed to account more completely for the effects of the various parameters on mixing in the bed. More experimental data is required to confirm and quantify the effects of rotational speed, filling and inter-particle friction on the bed motion. The study of motion in a three dimensional drum would also be useful, although this would be very difficult. The effect of particle size distribution should certainly be investigated.

A study of residence time distribution in small scale inclined drums would be useful to complement the existing work on horizontal drums, and for comparison with the simulations produced by the model. Residence time distribution experiments carried out under conditions for which the mixing matrices are were known would be ideal for assessing the model in more detail.

Experimental study of heat transfer in an externally heated drum would provide useful information to confirm the results of the heat transfer model developed here. The method should be carefully chosen so that the mixing of the bed is not disturbed and that the temperatures measured are those of the solids. Again these experiments should ideally be carried out in beds with known mixing matrices.
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<th>Symbol</th>
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<tr>
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<tr>
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<tr>
<td>δ</td>
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References

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I. Derivation of Finite Stage Transport Model of Mu and Perlmutter [M5]

Referring to figure 2.6 The transfer function for the plug flow region is

\[ G_1(s) = \frac{C_2(s)}{C_1(s)} = \exp(-\tau_1 s) \]  

(1)

where the constant \( \tau_1 \) is

\[ \tau_1 = \frac{V(1 - P)}{M(1 + Q)} \]  

(2)

where \( V \) is the total volume of the solid bed, \( P \) is the fraction of the bed in the cascade layer and \( Q \) is the recycle ratio.

For the cascade layer

\[ G_2(s) = \frac{C_4(s)}{C_2(s)} = \frac{1}{\tau_2 s + 1} \]  

(3)

where

\[ \tau_2 = \frac{VP}{M[(1 - a)(1 + Q)]} \]  

(4)

\( a \) is the bypass fraction on the cascade.

The ratio of input to output tracer concentration for the block is given by

\[ G_3(s) = \frac{C_5(s)}{C_0(s)} = \frac{(\tau_3 s + 1) \exp(-\tau_1 s)}{(Q + 1)(\tau_2 s + 1) - Q \exp(-\tau_1 s)} \]  

(5)

and

\[ \tau_3 = \frac{VP[a]}{M[1 - a]} \]  

(6)

It is assumed that \( P \) is independent of axial position, so
\[
C_F(s) = \frac{(\tau_3 s + 1) \exp(-\tau_1 Ns)}{C_0(s) \left[ (Q + 1)(\tau_2 s + 1) - Q \exp(-\tau_2 s) \right]^N} \tag{7}
\]

which can be reduced to

\[
\frac{C_F(s)}{C_0(s)} = \sum_{m=0}^{\infty} \sum_{k=0}^{N} B(m, k) \exp(-\tau_1 (m + N)s) \left( s + 1/\tau_2 \right)^{-m-N+k} \tag{8}
\]

where \( C_F \) is the tracer concentration in the efflux from the drum. By using a polynomial expansion formula

\[
B(m, k) = \frac{(N + m - 1)! \ N^{m+1}!}{(N-k)! \ k! \ m! \ (1 + Q)^{N+m}} \left\{ \frac{Q^m}{(1 + Q)^{N+m}} \right\} \cdot \left[ \tau_2^{-m+k-2N} \ \tau_3^k (\tau_2 - \tau_3)^{N-k} \right] \tag{9}
\]

For a pulse input of tracer of magnitude \( C_0 \)

\[
C_F(s) = \sum_{m=0}^{\infty} \sum_{k=0}^{N} C_0 B(m, k) \exp(-\tau_1 (m + N)s) \left( s + 1/\tau_2 \right)^{-m-N+k} \tag{10}
\]

The inverse of this gives the overall RTD as:

\[
E(t) = \frac{C_F(t)}{C_0} = \sum_{m=0}^{\infty} \sum_{k=0}^{N} \frac{B(m, k)}{(m+n-k-1)} \exp\left(-\frac{t-\tau_1 (m+N)}{\tau_2}\right) \cdot \left[ t - \tau_1 (m + N) \right]^{m+N-k-1} \tag{11}
\]
II. The Monte Carlo Method of Rogers and Gardner [R5]

II.I. Simulation of Radial Position of Re-entry

For an axial element $dx$ at a distance $x$ from the feed end of the drum $H_b$ as shown in figure 2.3. $\rho$ is the solids bulk density. The total mass leaving the free surface, from the upper half of the bed surface, is

$$M = 2\rho\int_0^r \phi(r) r \, dr \, dx$$

which is simply the product of the cross-sectional area at $x$, the element thickness $dx$ and the density.

so $M = (\rho \, R_d^2)[\phi - 0.5\sin 2\phi] \, dx$ (13)

The fraction of $M$ per cycle entering the surface in a radial element from $r$ to $r + dr$ is

$$f(r) = 2\rho \, \phi(r) \, r \, dr \, dx$$

The probability of material entering the bed at any radial position between $r$ and $R_d$ is

$$P(r) = \rho/M \int_r^{R_d} 2u \, \phi(u) \, du \, dx$$

$$= (\rho r^2/M) \left[\phi(r) - 0.5\sin 2\phi(r)\right] - r^2\left[\phi(r) - 0.5\sin 2\phi(r)\right]$$ (15)

which is the fraction of the bed with radius between $H_b$ and $r$. $P(r)$ was expressed in terms of $r$, $H_b$ and $R_d$.

$$P(r) = \frac{r^2 \left(\cos^{-1}\left[H_b/r\right] - \left[H_b/r\right] \sqrt{1 - \left[H_b/r\right]^2}\right)}{R_d^2 \left(\cos^{-1}\left[H_b/Rd\right] - \left[H_b/Rd\right] \sqrt{1 - \left[H_b/Rd\right]^2}\right)}$$ (16)

The method proceeded by setting the uniform random variable $y$ equal to $P(r)$ and solving for $r$. Equations (4) and (5) did not have explicit solutions so Newton's
method was used to find \( r \) from

\[ y = P(r) \] (17)

A function

\[
g(r) = r^2\left[\cos^{-1}\left(\frac{H_b}{r}\right) - \frac{H_b}{r}\sqrt{1 - \left(\frac{H_b}{r}\right)^2}\right] - yRd^2\left[\cos^{-1}\left(\frac{H_b}{Rd}\right) - \frac{H_b}{Rd}\sqrt{1 - \left(\frac{H_b}{Rd}\right)^2}\right]
\] (18)

was defined and the root of this equation found using Newton's iterative method

\[
r_{i+1} = r_i - \frac{g(r_i)}{g'(r_i)}
\] (19)

The result was a random value of \( r \) corresponding to the chosen random variable \( y \), distributed according to \( P(r) \).

II.II. Simulation of Path Angle

Axial mixing in a batch horizontal cylinder may be described by a one-dimensional dispersion equation:

\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}
\] (20)

where \( C \) is the tracer concentration at time \( t \) and axial position \( x \) and \( D \) is the axial dispersion coefficient.

The solution to this equation was

\[
\frac{C}{C_0} = L(4\pi Dt)^{-0.5} \exp\left\{-\left(\frac{\varepsilon - \delta}{4(4Dt)}\right)^2\right\}
\] (21)

\( \varepsilon \) was the position at which the tracer was initially injected. \( L \) is the drum length.

The cumulative distribution of tracer along the length of the drum was
found by integrating (21) to give

\[ G = \left( \frac{G_0}{2} \right) \left[ 1 - \text{erf} \left( \frac{(e - x)}{\sqrt{4Dt}} \right) \right] \] (22)

\( G_0 \) was the total mass of tracer present.

Equation (21) was transformed to the standard normal distribution with zero mean and standard deviation 1 using the transformation

\[ z = \Delta x/\sqrt{4Dt} \] (23)
\[ \Delta x = x - \epsilon \]

replacing \( \Delta x \) by \((2\sin\phi \tan\theta \cos\beta)\) and \( t \) by the sum of the cascade and fixed bed times \((t_f + t_{\text{casc}})\) equation (23) could be rewritten as a function of \( r \)

\[ z(r) = \frac{rsin\phi . tan\theta \cos\beta}{D (\phi/\omega + rsin\phi/g \sin\alpha)^{0.5}} \] (24)

Averaging \( z(r) \) for all \( r \)

\[ k = \int \frac{rsin\phi (\phi + rsin\phi)^{0.5} f(r) \ dr}{(\omega/sin\alpha)} \] (25)

which was a constant

For known mean path angle \( \theta \) and standard deviation \( \sigma_\theta \) random values of \( \theta \) were obtained by setting

\[ y = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp \left\{ -(1/2)[(u - \theta)/\sigma_\theta]^2 \right\} du \] (26)
The dimensionless mean residence time was defined as the ratio of the fractional filling and a dimensionless feedrate function $F^*$, which had been derived previously [1] for horizontal drums; where

$$F^* = \frac{F}{B\rho D^3} \cdot \frac{L}{D} \quad (27)$$

where $F$ is the feedrate, $\omega$ is the rotational speed and $B$ is a material constant related to the angle of repose of the solids:

$$B = \pi \cot \alpha$$

Expressing the filling fraction in terms of $H$, the mass holdup in the drum gives an expression for fractional filling:

$$f_c = \frac{4H}{\pi \rho D^2 L} \quad (28)$$

and the ratio of the two was

$$\bar{\tau}^* = K\omega \left(\frac{D}{L}\right)^2 \quad (29)$$

where $K = 4 \cot \alpha$

The characteristic time $\tau$ was defined as

$$\tau = \frac{\bar{\tau}}{\bar{\tau}^*} = \frac{1}{K\omega} \left(\frac{L}{D}\right)^2 \quad (30)$$
The dimensionless scale up variance of the RTD was then given by:

\[
\sigma_{\theta^*}^2 = \frac{\sigma_t^2}{\bar{t}^2} = \frac{\sigma_l^2}{\bar{t}^2} \tau^2 \]

(31)
IV. Video Processing

The purpose of the image processing programs was to control the production of digitized images by the digitizer with the video system and to process these images to find the position of the white ball each time it passed through the bed.

The software consisted of two programs; one which produced the digitized image on the screen and one which analysed the images and controlled the playback of the video system. The first of these programs (called BLK1) was supplied with the digitizer. The other ("BLOB") was developed specifically for this work.

The way in which the two programs were used to follow the path of the white ball in the bed is summarized in figure 1. An image of the bed was digitized using BLK1, this image was then searched by BLOB for the presence of the white ball. If the white ball was visible in the image its position was recorded and the video recorder was allowed to run on for just under the time taken for a particle to pass through the bed. If it was not visible in the digitized section of the bed the video was moved forward for a fraction of the time for one bed turn and a new image was produced and analysed. In this way the position of the ball was recorded each time it passed through the fixed bed.

Four items of data had to be supplied to the analysis system; the coordinates of the centrepoint of the drum and the two timesteps needed to control the video playback.

Screen Layout

Before describing how BLOB works it is useful to describe how the BBC computer screen is laid out in a high resolution monochrome mode. The screen memory occupies 16K of memory, between locations &3000 and &7FFF. The image displayed on the screen is a visual representation of binary numbers stored in these 20480 memory locations.

Each memory location is represented on the screen by eight pixels, one pixel per bit in the binary number stored in that memory location. A zero is represented by an unlit pixel, a one by a lit pixel. The eight pixels representing a location are arranged in a horizontal row. Thus if the number stored is zero the
Figure 1  Operation of Video Analysis System
eight pixels will all be unlit, if the number is 255 all the pixels will be lit and the screen will show a bright horizontal line. If the number lies between 0 and 255 some of the pixels will be lit. For example the pixels representing the number 173 are shown on figure 2. the binary equivalent of 173 is 10101101. The screen consists of 80 columns and 256 rows of these sets of eight pixels, each representing one byte of data. For convenience the set of eight pixels representing a byte will be referred to as the byte from now on. These bytes are arranged on the screen as shown in figure 3; in horizontal rows of vertical groups of eight bytes.

Locating the White Particle

The digitized image of the bed of particles was predominantly black with a white area representing the white particle. If there had been no lit particles on the screen other than those associated with the white particle finding its location would have been simple. The screen could have been searched byte by byte until a non-zero byte was found. However there were always smaller bright areas produced by reflections from particles or by 'noise' from the video system. The problem was to differentiate between the relatively large bright area being sought and these smaller bright areas.

The white area was identified by a procedure in BLOB (see figure 4). The screen was examined byte by byte and the number of '1's or lit pixels in each byte counted. This procedure was written in assembler for speed. The number of 1's in a byte was counted by successively dividing the binary number by 2 (ie shifting each bit to the right by one position) and counting the times the remainder of this division was 1. If the byte was found to have fewer than seven 1's it was assumed not to be part of the white area sought and the next byte was examined.

When a byte with seven or eight bright pixels was found three adjacent bytes were examined to find if the original bright byte formed part of a larger area of bright bytes (a bright byte being one with seven or more lit pixels). If the total number of 1's in the four adjacent bytes was greater than 28 it was assumed that the particle had been found. The bytes considered were in the relative positions shown in figure 5. Byte A was the original bright byte found. From figures 3 and 5 it is clear that the location C will have an address 8 higher than A. B and D will be at (A + 1) and (C + 1) respectively.
Figure 2  Pixel arrangement

Figure 3  Screen layout
A C
B D

Figure 5 Configuration of adjacent bytes investigated

Figure 6 Diagram showing first bright byte found.
Find top bright byte

Find furthest left and right bright bytes and max no. of bright bytes in a group

Any bright bytes in row?
Value of the addresses of B and D would be more difficult to obtain if A were the bottom byte in a vertical group of eight in which case B would be \((A + 633)\). However the extra complication involved in programming for this special case is not necessary. At this stage it does not matter which block of four bright bytes in the white area is found, only that a block is found. Figure 6 illustrates this; the first bright byte found would be byte A. However looking at \((A + 1)\), \((A + 8)\) and \((A + 9)\) would not show a block of four bright bytes because the bytes examined would not be those adjacent to A. The program would then move on, examining following bytes until byte B was reached when a bright block of 4 adjacent bytes would be found, indicating that the white particle had been detected.

**Finding the Centre of the White Particle**

Once the location of the white ball had been found by the process described above the program proceeded to find the centre of the white area, which was used to calculate the radius of the white particle in the bed. This was done by finding the upper and lowermost and farthest left and farthest right bright bytes in the white area. A relatively small area of the screen was examined at this stage so BASIC was used since the speed advantage of Assembly code was less significant that when the whole screen was to be searched.

In order to calculate the radius from the memory location of the centre of the bright area, and also to facilitate calculating the centre the memory locations were converted to coordinates. The digitized image covered a section of the screen which was 64 bytes wide and 256 bytes deep. The X and Y coordinates of a location A were given by:

\[
X(A) = \frac{(A - &3000 \mod 640) \div 8}{8}
\]

\[
Y(A) = \frac{(A - &3000 \mod 8) + ((A - &3000 \div 640) \times 8}{8}
\]

where \( R \mod S \) = remainder of \( R/S \)
\( R \div S \) = integer part of \( R/S \)

X-coordinates on the image ran from 0 to 63, increasing from left to right. Y-coordinates ranged from 0 to 255 increasing down the screen.

The way in which the extremes of the white area were found is shown on
figure7, which is the flowsheet for the procedure which locates the furthest right bright byte in a row of groups of bytes and records its x-coordinate. The procedures which find the top and leftmost bright bytes are similar. Once the bright block of four bytes has been found the program proceeded by searching along the row of groups of bytes in which the original bright byte was found to identify the uppermost bright byte of the ball’s image. The top of the bright area must be either in this group of bytes, or possibly on the lowest byte of the row above, if a bright block was missed because of the position of the original bright byte, as described above. The top bright byte was found by examining each group of bytes in turn. Each byte in the group was examined, beginning with the lowest byte until a non-bright byte was found. The number of bright bytes below this non-bright byte was recorded, and a record was kept of the largest number of bright bytes in any group. This process stopped when a group was found which contained no bright bytes. Because it could be assumed that the original bright byte was at the left hand side of the top row containing bright bytes this procedure only needed to examine groups of bytes to the right of the original byte. The X-coordinate of the rightmost bright byte in this row was recorded, giving, with the X-coordinate or the original byte, the lateral extent of the white area in this row. The maximum number of bright bytes in a group in this row gave the vertical extent of the white area in this row, this along with the Y-coordinate of the uppermost bright byte in the row, was recorded.

The program then used two procedures, to examine the rows below the first row for the extent of the white area. Both procedures recorded the maximum number of bright bytes in any one group and the x-coordinate of the farthest left or right bright byte. Each row was examined until a row was found with no bright bytes. A running total was kept of the maximum number of bright bytes in a group in each row and the most extreme lateral bright byte coordinates were recorded. In the case of the maximum number of bright bytes in a vertical group of bytes all rows except the upper and lower rows returned a figure of 8.

Once the extremes of the white area had been identified the averages of the extreme X and Y-coordinates were calculated and taken to be the X and Y-coordinates of the centrepoint of the particle. The coordinates of the centre of the drum were then used to calculate the radius of the particle.
**Figure 7** Procedure for finding furthest right bright byte in a row
V. Simulation of Transition Matrices

In order to estimate the number of particle transitions which had to be recorded in each experiment transition matrices were simulated from mixing matrices. A mixing matrix was constructed which had the correct row and column totals. This was then used to calculate the probability of each possible transition from one layer of the bed to another. These probabilities were used to calculate cumulative probabilities that the particle would re-enter the bed in any layer, given the layer from which it left. A simple Monte Carlo simulation was then used to generate a series of particle transitions using the cumulative probabilities. For each transition a random number between 0 and 1 was generated compared with the cumulative probability series for the exit layer of the particle. The re-entry layer is that for which the random number \( R_n \) satisfies:

\[
P_{ij} < R_n < P[j(j+1)]
\]

where \( P_{ij} \) is the probability that the particle will re-enter the bed in a layer with radius greater than the outside radius of layer \( j \), having left layer \( i \), and \( P[j(j+1)] \) is the probability that it will re-enter at a radius smaller than or equal to the inner radius of layer \( j \).

This simulation procedure was repeated for the required number of transitions and the particle transition matrix was constructed. This was carried out for a between 100 and 10000 transitions. The row totals were calculated and compared with the expected totals. The standard deviation between the simulated row totals and those calculated from the rate at which material would emerge onto the cascade layer from each layer was used as a measure of the correspondence between the simulated transition matrices and the original mixing matrix. The resulting standard deviations are shown on figures 1 and 2. The mixing matrices used are shown below. The first matrix is biased towards segregation, but mixing takes place between all layers. The second matrix tends towards segregation more, mixing generally only occurs between adjacent layers.
Matrix 1
\[
\begin{bmatrix}
47 & 30 & 30 & 15 & 8 & 4 & 4 & 16 \\
30 & 38 & 30 & 10 & 5 & 4 & 2 & 18 \\
20 & 25 & 28 & 15 & 8 & 6 & 4 & 25 \\
11 & 20 & 25 & 37 & 7 & 8 & 6 & 12 \\
14 & 15 & 18 & 20 & 27 & 6 & 3 & 13 \\
8 & 3 & 6 & 6 & 20 & 34 & 15 & 17 \\
2 & 2 & 0 & 5 & 19 & 25 & 12 & 17 \\
12 & 4 & 4 & 18 & 22 & 22 & 57 & 0 \\
\end{bmatrix}
\]

Matrix 2
\[
\begin{bmatrix}
119 & 19 & 6 & 0 & 0 & 0 & 0 & 0 \\
12 & 100 & 16 & 9 & 0 & 0 & 0 & 0 \\
13 & 18 & 85 & 10 & 5 & 0 & 0 & 0 \\
0 & 10 & 14 & 80 & 20 & 0 & 0 & 0 \\
0 & 0 & 0 & 20 & 75 & 10 & 9 & 3 \\
0 & 0 & 0 & 4 & 10 & 65 & 25 & 9 \\
0 & 0 & 0 & 0 & 4 & 20 & 50 & 30 \\
0 & 0 & 0 & 1 & 3 & 16 & 20 & 0 \\
\end{bmatrix}
\]

The deviation between simulated and expected row totals decreased rapidly with increasing number of transitions up to around 1750 transitions in both cases. The rate of decrease in the deviation beyond this point was low so that 2000 transitions was chosen as the minimum number to be recorded for each experiment in a bed with 8 layers of particles. For beds with larger numbers of layers the number of transitions was increased.
Figure 1 Standard Deviation between Calculated and Simulated Row Totals for Matrix 1

Figure 2. Standard Deviation between Calculated and Simulated row Totals for Matrix 2
VI. Graphical Results

Figure 1 Residence Time Distribution for \( Rd = 150 \text{mm}, f_c = 0.154 \)

![Graph 1]

Figure 2 Residence Time Distribution for \( Rd = 150 \text{mm}, f_c = 0.200 \)

![Graph 2]
Figure 3 Residence Time Distribution for Rd = 150mm, $f_c = 0.2$, PTFE coated

Figure 4 Residence Time Distribution for Rd = 150mm, $f_c = 0.24$, PTFE coated
Figure 5 Residence Time Distribution for $R_d = 225\text{mm}$, $f_c = 0.154$

Figure 6 Residence Time Distribution for $R_d = 225\text{mm}$, $f_c = 0.2$
Figure 7 Residence Time Distribution for Rd = 225mm, $f_c = 0.24$

Figure 8 Residence Time Distribution for Rd = 150mm, $f_c = 0.20$, sand coated
Figure 9 Mixing Histograms for Rd = 150mm, $f_c = .24$
Figure 10 Mixing Histograms for $R_d = 150\text{mm}$, $f_c = 0.154$
Figure 11 Mixing Histograms for Rd = 150mm, \( f_c = 0.2 \), PTFE coated
Figure 12 Mixing Histograms for Rd = 150mm, f_c = 0.24, PTFE coated
### VII. Tables and Results

#### Mixing Matrices for Experimental Runs

**Run 6**

\[
\begin{array}{cccccccc}
R_d &=& 0.15 m & f_c &=& 0.20 & w &=& 18.2 rpm & \text{uncoated} \\
398.0 & 283.2 & 260.1 & 132.0 & 74.9 & 13.0 & 19.6 & 5.2 \\
282.7 & 302.2 & 230.5 & 165.7 & 109.2 & 47.5 & 18.7 & 13.4 \\
203.9 & 236.1 & 179.1 & 137.8 & 90.0 & 59.5 & 12.5 \\
139.1 & 154.5 & 164.3 & 201.2 & 182.2 & 126.5 & 135.3 & 34.9 \\
92.5 & 92.2 & 125.0 & 193.9 & 238.6 & 156.3 & 158.4 & 65.2 \\
42.6 & 60.2 & 73.9 & 161.6 & 196.6 & 242.8 & 238.6 & 89.7 \\
27.2 & 36.5 & 48.2 & 93.0 & 163.7 & 297.0 & 256.4 & 168.1 \\
& 0.0 & 5.0 & 16.9 & 11.5 & 19.1 & 132.9 & 203.6 & 0.0 \\
\end{array}
\]

**Run 7**

\[
\begin{array}{cccccccc}
R_d &=& 0.15 m & f_c &=& 0.20 & w &=& 6.75 rpm & \text{uncoated} \\
93.6 & 88.6 & 67.7 & 56.6 & 42.0 & 39.0 & 29.3 & 23.2 \\
91.5 & 75.6 & 68.4 & 67.5 & 45.9 & 30.0 & 29.5 & 25.7 \\
72.7 & 69.9 & 65.2 & 60.7 & 50.7 & 47.8 & 36.7 & 24.4 \\
61.4 & 60.3 & 59.6 & 63.7 & 52.2 & 44.8 & 43.9 & 36.1 \\
48.7 & 52.4 & 48.2 & 51.0 & 51.2 & 59.3 & 40.4 & 64.9 \\
36.3 & 41.9 & 50.6 & 40.0 & 60.5 & 55.3 & 50.7 & 74.6 \\
28.3 & 31.9 & 36.2 & 41.0 & 57.9 & 67.3 & 61.3 & 80.2 \\
\end{array}
\]

**Run 8**

\[
\begin{array}{cccccccc}
R_d &=& 0.15 m & f_c &=& 0.20 & w &=& 9.37 rpm & \text{uncoated} \\
152.3 & 135.6 & 95.1 & 87.5 & 68.4 & 38.2 & 21.7 & 11.3 \\
129.1 & 107.0 & 98.0 & 82.4 & 81.3 & 53.3 & 29.1 & 21.8 \\
111.6 & 102.1 & 116.5 & 94.3 & 58.2 & 52.1 & 34.7 & 24.5 \\
83.0 & 81.9 & 105.7 & 83.1 & 75.2 & 84.0 & 47.7 & 25.3 \\
58.4 & 75.6 & 81.3 & 85.0 & 81.0 & 85.7 & 73.0 & 38.1 \\
38.2 & 52.8 & 48.1 & 62.3 & 82.2 & 94.6 & 118.0 & 72.8 \\
21.4 & 30.9 & 32.1 & 44.9 & 85.7 & 117.4 & 125.6 & 103.1 \\
\end{array}
\]

**Run 11**

\[
\begin{array}{cccccccc}
R_d &=& 0.15 m & f_c &=& 0.20 & w &=& 14.8 rpm & \text{uncoated} \\
271.3 & 256.4 & 183.8 & 111.6 & 91.0 & 46.9 & 0.0 & 3.1 \\
231.0 & 233.5 & 174.0 & 128.1 & 100.4 & 42.8 & 21.5 & 19.8 \\
168.2 & 161.9 & 176.2 & 154.8 & 129.3 & 67.4 & 46.3 & 34.0 \\
124.7 & 118.8 & 144.1 & 166.4 & 138.3 & 121.8 & 88.1 & 22.8 \\
85.9 & 90.9 & 117.1 & 149.0 & 123.3 & 171.9 & 124.4 & 49.5 \\
48.9 & 54.1 & 76.8 & 115.4 & 150.3 & 191.2 & 179.0 & 84.2 \\
23.3 & 25.2 & 51.4 & 75.5 & 139.4 & 180.8 & 261.8 & 129.7 \\
10.8 & 10.3 & 14.7 & 24.2 & 40.0 & 77.1 & 166.0 & 0.0 \\
\end{array}
\]
Figure 13 Mixing Histograms for Rd = 225mm, $f_c = 0.2$
Figure 14 Mixing Histograms for Rd = 225mm, $E_c = 0.154$
Figure 15 Mixing Histograms for \( R_d = 225 \text{mm} \), \( f_c = 0.24 \)

\[ \omega = 5.66 \text{rpm} \]

\[ \omega = 7.65 \text{rpm} \]
$\omega = 12.1 \text{ rpm}$
Figure 16 Mixing Histograms for Rd = 150mm, fc = 0.2, sand coated
Run 12
\( R_d = 0.15 \text{m} \quad f_c = 0.24 \quad \omega = 14.8\text{rpm} \quad \text{uncoated} \)

<table>
<thead>
<tr>
<th>( R_d )</th>
<th>( f_c )</th>
<th>( \omega )</th>
<th>( \text{Data} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>265.0</td>
<td>0.24</td>
<td>14.8</td>
<td>76.8 47.6 17.4 3.7</td>
</tr>
<tr>
<td>218.5</td>
<td>0.24</td>
<td>14.8</td>
<td>71.0 21.8 24.3</td>
</tr>
<tr>
<td>166.2</td>
<td>0.24</td>
<td>14.8</td>
<td>82.7 73.8 35.7</td>
</tr>
<tr>
<td>124.8</td>
<td>0.24</td>
<td>14.8</td>
<td>120.7 85.4 38.4</td>
</tr>
<tr>
<td>84.1</td>
<td>0.24</td>
<td>14.8</td>
<td>131.1 148.5 60.2</td>
</tr>
<tr>
<td>54.5</td>
<td>0.24</td>
<td>14.8</td>
<td>154.8 167.4 98.8</td>
</tr>
<tr>
<td>37.7</td>
<td>0.24</td>
<td>14.8</td>
<td>191.6 182.8 162.0</td>
</tr>
<tr>
<td>7.2</td>
<td>0.24</td>
<td>14.8</td>
<td>81.2 93.4 183.8</td>
</tr>
</tbody>
</table>

Run 13
\( R_d = 0.15 \text{m} \quad f_c = 0.24 \quad \omega = 6.78\text{rpm} \quad \text{uncoated} \)

<table>
<thead>
<tr>
<th>( R_d )</th>
<th>( f_c )</th>
<th>( \omega )</th>
<th>( \text{Data} )</th>
</tr>
</thead>
<tbody>
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<td>91.0</td>
<td>0.24</td>
<td>6.78</td>
<td>45.0 24.7 20.4 34.0</td>
</tr>
<tr>
<td>84.1</td>
<td>0.24</td>
<td>6.78</td>
<td>44.9 34.8 37.0 33.1</td>
</tr>
<tr>
<td>78.5</td>
<td>0.24</td>
<td>6.78</td>
<td>50.3 37.4 30.6 35.3</td>
</tr>
<tr>
<td>67.0</td>
<td>0.24</td>
<td>6.78</td>
<td>58.8 50.3 34.5 42.2</td>
</tr>
<tr>
<td>44.3</td>
<td>0.24</td>
<td>6.78</td>
<td>57.9 60.9 44.2 55.4</td>
</tr>
<tr>
<td>42.2</td>
<td>0.24</td>
<td>6.78</td>
<td>49.3 63.3 60.4 72.7</td>
</tr>
<tr>
<td>21.9</td>
<td>0.24</td>
<td>6.78</td>
<td>45.0 64.2 75.5 90.4</td>
</tr>
<tr>
<td>11.9</td>
<td>0.24</td>
<td>6.78</td>
<td>66.9 76.3 103.4 0</td>
</tr>
</tbody>
</table>

Run 14
\( R_d = 0.15 \text{m} \quad f_c = 0.24 \quad \omega = 9.30\text{rpm} \quad \text{uncoated} \)

<table>
<thead>
<tr>
<th>( R_d )</th>
<th>( f_c )</th>
<th>( \omega )</th>
<th>( \text{Data} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>140.9</td>
<td>0.24</td>
<td>9.30</td>
<td>82.2 69.9 39.7 23.8 15.3</td>
</tr>
<tr>
<td>127.3</td>
<td>0.24</td>
<td>9.30</td>
<td>71.8 60.8 36.7 34.9 28.7</td>
</tr>
<tr>
<td>100.5</td>
<td>0.24</td>
<td>9.30</td>
<td>76.0 70.1 66.8 43.6 31.8</td>
</tr>
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<td>84.4</td>
<td>0.24</td>
<td>9.30</td>
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<td>9.30</td>
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</tr>
<tr>
<td>44.2</td>
<td>0.24</td>
<td>9.30</td>
<td>38.8 62.6 85.9 73.8 84.4 131.3</td>
</tr>
<tr>
<td>23.5</td>
<td>0.24</td>
<td>9.30</td>
<td>68.1 77.3 71.1 70.7 188.8</td>
</tr>
<tr>
<td>17.7</td>
<td>0.24</td>
<td>9.30</td>
<td>66.7 55.9 132.4 159.9 0</td>
</tr>
</tbody>
</table>

Run 15
\( R_d = 0.15 \text{m} \quad f_c = 0.156 \quad \omega = 14.9\text{rpm} \quad \text{uncoated} \)

<table>
<thead>
<tr>
<th>( R_d )</th>
<th>( f_c )</th>
<th>( \omega )</th>
<th>( \text{Data} )</th>
</tr>
</thead>
<tbody>
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<td>81.7 32.3</td>
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<tr>
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<td>14.9</td>
<td>200.6 157.3 101.2 30.6</td>
</tr>
<tr>
<td>217.6</td>
<td>0.156</td>
<td>14.9</td>
<td>204.8 152.6 119.9 45.6</td>
</tr>
<tr>
<td>141.0</td>
<td>0.156</td>
<td>14.9</td>
<td>197.0 183.6 160.6 88.9</td>
</tr>
<tr>
<td>72.7</td>
<td>0.156</td>
<td>14.9</td>
<td>128.0 199.6 250.8 172.7</td>
</tr>
<tr>
<td>22.4</td>
<td>0.156</td>
<td>14.9</td>
<td>34.1 84.0 204.7 0</td>
</tr>
</tbody>
</table>

Run 16
\( R_d = 0.15 \text{m} \quad f_c = 0.156 \quad \omega = 10.0\text{rpm} \quad \text{uncoated} \)

<table>
<thead>
<tr>
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<th>( f_c )</th>
<th>( \omega )</th>
<th>( \text{Data} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>170.7</td>
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<td>10.0</td>
<td>107.4 117.0 100.8 64.6 90.5</td>
</tr>
<tr>
<td>137.4</td>
<td>0.156</td>
<td>10.0</td>
<td>141.8 109.2 96.6 66.7 91.3</td>
</tr>
<tr>
<td>124.4</td>
<td>0.156</td>
<td>10.0</td>
<td>127.5 121.5 95.9 70.6 94.1</td>
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<tr>
<td>101.6</td>
<td>0.156</td>
<td>10.0</td>
<td>115.6 107.2 98.2 106.1</td>
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<td>64.4</td>
<td>0.156</td>
<td>10.0</td>
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<td>52.5</td>
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Run 17

<table>
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<th>PTFE</th>
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</tr>
<tr>
<td>141.5</td>
<td>130.0</td>
<td>92.4</td>
<td>90.9</td>
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<tr>
<td>127.2</td>
<td>119.1</td>
<td>91.2</td>
<td>88.2</td>
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<tr>
<td>116.0</td>
<td>95.0</td>
<td>96.8</td>
<td>88.3</td>
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<td>90.1</td>
<td>87.6</td>
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<td>51.0</td>
<td>66.1</td>
<td>92.1</td>
<td>63.4</td>
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<td>30.9</td>
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<tr>
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<td>44.6</td>
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Run 18

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<th>$\omega$</th>
<th>PTFE</th>
</tr>
</thead>
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<td>0.20</td>
<td>14.7rpm</td>
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</tr>
<tr>
<td>298.8</td>
<td>256.1</td>
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<td>104.0</td>
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<td>224.5</td>
<td>190.8</td>
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<td>20.0</td>
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<td>94.9</td>
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Run 19

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<td>0.20</td>
<td>6.98rpm</td>
<td></td>
</tr>
<tr>
<td>117.4</td>
<td>93.0</td>
<td>79.0</td>
<td>58.7</td>
</tr>
<tr>
<td>82.9</td>
<td>97.5</td>
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<td>6.9</td>
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</table>

Run 20

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<th>PTFE</th>
</tr>
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<td>0.15</td>
<td>0.24</td>
<td>6.98rpm</td>
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</tr>
<tr>
<td>104.8</td>
<td>116.9</td>
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<td>106.2</td>
<td>79.3</td>
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<td>29.2</td>
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<tr>
<td>69.4</td>
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Run 21

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<td>Run 23</td>
<td>Run 25</td>
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<td>( R_d = 0.15m \quad f_c = 0.156 \quad \omega = 18.0 \text{rpm} \quad \text{uncoated} )</td>
<td>( R_d = 0.225m \quad f_c = 0.20 \quad \omega = 5.66 \text{rpm} \quad \text{uncoated} )</td>
<td>( R_d = 0.225m \quad f_c = 0.20 \quad \omega = 7.65 \text{rpm} \quad \text{uncoated} )</td>
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<td>( 341.0 \quad 325.4 \quad 265.3 \quad 118.2 \quad 101.8 \quad 21.3 )</td>
<td>( 152.0 \quad 60.5 \quad 73.9 \quad 79.6 \quad 63.6 \quad 45.2 \quad 33.8 \quad 12.9 )</td>
<td>( 186.4 \quad 146.5 \quad 97.4 \quad 86.2 \quad 64.4 \quad 58.8 \quad 27.7 \quad 41.3 )</td>
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### Run 28

**Rd** = 0.225m \( f_c = 0.156 \) \( \omega = 5.66 \text{rpm uncoated} \)

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### Run 29

**Rd** = 0.225m \( f_c = 0.156 \) \( \omega = 7.65 \text{rpm uncoated} \)

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### Run 30

**Rd** = 0.225m \( f_c = 0.156 \) \( \omega = 12.1 \text{rpm uncoated} \)

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### Run 31

**Rd** = 0.225m \( f_c = 0.24 \) \( \omega = 5.66 \text{rpm uncoated} \)

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Run 36
\( R_d = 0.15m \) \( f_c = 0.20 \) \( \omega = 14.7 \text{rpm uncoated} \)

\[
\begin{array}{c|cccccc}
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162.9 & 190.8 & 179.9 & 142.8 & 102.5 & 57.9 & 40.1 & 55.0 \\
130.6 & 123.3 & 161.3 & 110.5 & 146.2 & 94.2 & 52.4 & 100.5 \\
95.5 & 95.0 & 107.8 & 103.3 & 97.2 & 139.0 & 128.6 & 139.6 \\
59.4 & 78.7 & 80.0 & 125.1 & 148.5 & 122.4 & 141.2 & 137.6 \\
17.0 & 43.7 & 50.2 & 75.2 & 141.4 & 199.7 & 149.2 & 204.7 \\
49.8 & 51.4 & 76.5 & 86.0 & 60.8 & 133.1 & 290.6 & 0 \\
\end{array}
\]

Summary of Residence Time Simulation Results

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<th>( t )</th>
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Data for Figure 4.4

Varying rotational speed at constant feedrates, $R_d=0.15m$, uncoated particles.

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<th>filling angle deg</th>
<th>variance $hr^2x10^{-5}$</th>
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Data for Figure 4.5

Varying rotational speed at constant feedrates, $R_d=0.225$, uncoated particles.

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Data for Figure 4.7

Varying feedrate at constant rotational speed, \( R_d = 0.150 \text{m}, f_c = 0.2 \) uncoated particles.

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Data for Figure 4.6

Varying feedrate at constant rotational speed \( R_d = 0.15 \text{m}, f_c = 0.20 \) uncoated particles.

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VIII. Packed Bed Heat Transfer

Heat transfer from the wall to the solid bed involves conduction between the wall and the solids and conduction within the packed bed itself. In the present work it was assumed that the fluid phase in the bed was stagnant, i.e., that there was no bulk fluid flow through the bed. This is not unrealistic since only externally heated kilns are considered, in which case the only gas present is likely to be that produced in the reaction taking place. The flow of gas through the kiln is therefore likely to be less than in types where the gas is the heating medium. The area of interest in the bed, adjacent to the wall, would have the minimum flow of gas. The amount of gas flowing through the bed will increase from the wall to the bed surface. The flow would be directed vertically through the bed. It will be assumed here that there is no flow of gas near the wall. In any case it would be fair to assume that the temperature of the product gas would be identical to that of the solids from which it is produced, so that no heat transfer would take place between the two phases.

The two areas of interest are therefore the heat transfer between a wall and a packed particulate bed and conduction through a packed bed with a stagnant fluid.

There have been many studies of packed bed heat transfer, of great interest in the design of equipment such as packed bed reactors, which are very common in the chemical industry. Much of this has been concerned with non-stagnant beds, as is appropriate to reactors. Less has been reported on stagnant beds but there is still a considerable body of work to be examined.

Melanson and Dixon [M2] produced an excellent review of the literature available on prediction of stagnant bed thermal conductivity which summarized the various theoretical approaches which have been made to this problem. Crane and Vachon [C7] also reviewed the basis of the different models in more mathematical detail than is appropriate here.

The aim of the models is to predict the effective thermal conductivity of packed beds from the solid and fluid transport properties and the characteristics of the bed; voidage, particle size and shape etc. Although these models are intended to predict an effective conductivity for use in expressions for heat transfer by conduction the actual mechanism of transfer through the bed is a combination of conduction, both through solid contacts and through the fluid,
radiation and convection in the fluid. It is convenient to lump all these resistances together and represent them in a single conductivity. The relative importance of the various elements depends on the conditions in the bed. At high temperatures radiation makes a considerable contribution to the total flux. This has been studied in some detail [S5,W2]. At very low gas pressure the conduction between the solid particles over their contact area is important.

There are two main types of model used to predict effective bed conductivities; the Fourier’s Law models and the Ohm’s Law models.

The Fourier’s Law type [W2,W3,W5,P1] develops an analytical solution for the temperature field in a bed with a regular, ideal geometry. The effective thermal conductivity is then found from Fourier’s Law. The extension of results from these models to the less ideal packing structures and particle shapes of a real bed is problematic.

Ohm’s Law models are more widely applicable and simpler [K5,Y1,Y2,Z3]. The heat transfer is modelled in a simplified, repetitive geometry which represents the features of the more complicated real bed. A network of thermal resistances in series and parallel is then derived which can be solved for the effective thermal conductivity. There are two extreme conditions of heat transfer which provide bounds for the results derived from this type of model: [C7]

- Infinite lateral conductivity (i.e. normal to the main direction of heat flux). This would give straight isotherms normal to the primary direction of flux.
- Zero lateral conductivity which would give a constant heat flux in the main direction.

Kunii and his co-workers [K5,Y1,Y2] developed a well known model using an Ohm’s Law technique. They based much of their work on the use of an equivalent film thickness between the particles which accounted for the transfer at the particle contact points which could not be explained by the conduction over the solid to solid contact area alone. This equivalent film thickness could be related to the bed voidage. Kunii and Smith [K5] reported agreement within 30% for stagnant bed experiments with this type of model.

Schlunder [Z3,S8] and co-workers attempted to expand the applicability of
Ohm's Law models to less ideal conditions by using a variable particle shape in their cell model. In order to use the models produced in this way an experimental constant had to be introduced to account for the more complex geometry and solids contact pattern [Z3].

Experimental work has been carried out under both unsteady and steady state conditions. The earlier work investigated the transient behaviour of cylindrical stagnant beds by heating at the wall and measuring the resulting temperature distributions. More recent work has used two different types of steady state experiment to obtain values of the stagnant bed conductivity.

The first method used cylindrical beds, heated at the wall with a flow of fluid through the bed. The stagnant bed radial conductivity was found by plotting the measured conductivities against Reynolds number and extrapolating back to zero flow.

In the second type of steady state experiment the conductivity of an annular stagnant bed was measured. Fluid flow was maintained over the outside and inside walls of the annulus, allowing a steady temperature distribution to become established. The average thermal conductivity of the bed could then be found from the heat balance [W5,P1,K5].

The second problem of interest, the heat transfer from the wall to the bed is of some importance in annular bed experiments. The resistance at the wall has been found to be most significant in beds with a high ratio of particle to bed diameter. This could be significant at the inner wall of an annular bed. If the wall resistance is not negligible then it must be eliminated from the experimental value of the thermal conductivity in order to produce the `bed-centre' value of the conductivity.
IX. Liquid Crystal Experiments

In order to perform successful unsteady state heat transfer experiments it is essential to have some means of quickly and accurately determining local temperatures. This is difficult in cases where solid particles are involved since it is virtually impossible to ensure intimate contact between the measuring device and the material. It is therefore uncertain whether the temperature recorded is that of the solids or of the surrounding fluid. In previous work in rotary kilns it has been usual to measure solids temperature by means of thermocouples in the bed. Apart from contacting problems this method also has the drawback that that regions to be studied must be pre-selected so that thermocouples may be implanted at the required positions. It would then be possible to miss important phenomena if they were to occur in a region where no measurements were being taken. It would also be possible that relatively small scale phenomena could be overlooked because of the practical problems of having a large concentration of thermocouples.

The ideal system of temperature distribution determination would be one where the temperature of the whole bed could be determined and recorded continuously, without the use of localized measurements. A possible method is the use of temperature indicating liquid crystal ink. This ink changes colour according to its temperature, progressing through the spectrum from red to violet as its temperature increases. Thus by coating the particles with this ink it would be possible to monitor the temperature distribution in a section of the bed in a way which would show the complete behaviour of the bed.

Particle selection and coating

For the proposed method to be useful it was necessary to confirm that it would be possible to coat particles satisfactorily with the ink. The coating should be of uniform thickness and should resist attrition from the action of the particles. The ink is extremely expensive and only a small sample was available for initial experimentation. It was decided that an alternative material should be used for these experiments and poster paint was chosen as it appeared to have similar properties to the ink, both being water based suspensions.

The particles themselves were to be of uniform size and shape, it was also felt that the particles should be somewhat porous to aid adhesion of the coating.
They should also respond quickly to temperature changes and have minimum internal temperature gradient, so that the temperature indicated by the ink would be representative of the complete particle. Rice was initially chosen since it was uniform in size and shape and readily available. However it was found to crumble after being wetted and dried in the coating process. Pearl barley was chosen to replace the rice and was found to be satisfactory.

The coating method which was found to be most effective was simply to dump the barley into the paint; stir to ensure complete coverage and then to pour the mixture onto a mesh which allowed the excess paint to drain away. The barley was then allowed to dry naturally. It was found to be essential that the individual grains should be separated on the mesh while they were wet. It was also helpful to move the grains around on the mesh during drying to prevent them adhering to it. In this way it was possible to achieve an even coating. The problem with this method is that when coating larger quantities of barley it would be extremely time consuming and tedious.

A small sample of barley was coated with the ink, in order to assess its adhesive qualities and resistance to wear. The barley was first blackened with drawing ink, to provide the black background needed for the liquid crystal ink. It was found that the liquid crystal ink gave a smooth uniform coating with no signs of chipping or cracking except under severe treatment.

Initial Experiments

Unsteady state heat transfer to a moving particulate bed is an extremely complicated situation. It would therefore be unwise to attempt to study it using a novel technique without first establishing the benefits of the technique under a much simpler regime.

Rather than working with particulate materials a flat sheet coated with the ink was used. Initially black paper was used. The ink's colour is due to selective reflection of certain wavelengths of light and only appears coloured when applied to a black background material. The paper was later replaced by a sheet of semi-rigid pvc. This was to ensure a constant value of the specific heat capacity, in the case of paper this would vary with humidity. It is assumed that the barley would also be affected by humidity, but in order to keep these preliminary experiments as simple as possible it was desirable to eliminate any variation in
the heat capacity.

The pvc sheet was less easy to coat than the paper, being non-porous. The ink tended to for drops on the surface. This was eliminated to a certain extent by rubbing the sheet to create a static electric charge on the surface. The ink was then painted on to the sheet as evenly as possible and then further smoothed by drawing the edge of a ruler over it. The application of a second coat of ink further smoothed the surface. The reverse side of the transparent sheet was painted with black drawing ink.

Calibration Experiments

The range of temperatures over which the ink is sensitive is obtainable from the maker, however the actual temperature to colour correspondence must be calibrated by the user. To avoid any difficulty in determining the heat capacity of a calibration system the calibrating experiments were carried out at steady state.

The obvious method of calibration was to enclose a test piece, coated with ink, in an environment in which the temperature could be closely controlled and to identify the temperature at which colour changes took place. However this would have been difficult, the temperature range of the ink was small (5°C) and precise air temperature control would have been problematic. It then seemed that a method which would show the full temperature range simultaneously with some means of determining the temperature at the positions where the colours changed would be ideal. It was therefore decided to use a method which had been used to determine heat transfer coefficients from known temperatures in reverse, ie determining temperatures from known heat transfer coefficients. The experiment used involved heat transfer from a horizontal flat surface to a vertical laminar jet.

In this experiment a flat surface was heated from below by an electric heating pad. At steady state the temperature of the sheet was such that the heat supplied by the pad was equal to that lost by the sheet to the surroundings. The heating pad rested on foam which was assumed to perfectly insulate the underside of the sheet. Once this steady state was achieved the air jet was applied to the sheet. The cooling effect of this jet caused coloured rings to appear on the liquid crystal coated sheet, the area directly below the jet being coolest with the temperature increasing radially outwards. Steady state was
established when each coloured ring reached a steady diameter. The radius at which colour change took place was recorded.

The heat transfer coefficient due to such an impinging jet, at a particular radius may be calculated from:

$$h(r)d = 0.159\ Re^{0.75}\ Pr^{3.3}\ (d/r)^{1.25}$$

(32)

And, knowing the ambient temperature, the temperature at that radius may be found using:

$$h(r) = \frac{W}{A(T_w(r) - T_a)}$$

(33)

Several problems were identified during this calibration process:

- Because the range of the ink was only just above room temperature the experiments were very sensitive to external heat sources. Even sunlight could affect results drastically.

- It emerged that carrying the experiments out under different lighting conditions affected the appearance of the colours.

- The identification of the colour change boundaries was extremely subjective. It was very difficult to be consistent from day to day. The colours did not change sharply but went through a gradual transition from one colour to the next, making the choice of the point of change quite arbitrary.

However the experiment was carried out three times and reproducible results were produced. This calibration was then used in unsteady state experiments.

**Unsteady State Experiments**

The unsteady state experiments were carried out with the aim of determining the specific heat capacity of the plastic sheet and comparing this to the value obtained from literature sources. The results of these experiments were to be used in assessing the usefulness of this method of temperature determination.

The experiment was basically similar to the steady state calibration but the
radii of the coloured rings were to be measured as they developed from the time the jet was first turned on to the time when steady state became established. In this way cooling curves for the sheet at various radii could be produced.

The heat transfer coefficient and the power supplied at each radius could be calculated. It was then possible to calculate the desired heat capacity from:

\[ h \int (T_w(t) - T_A) dt = (q/a)t + (mC_p/A) (T_w(t) - T_{w0}) \] (34)

The experiment consisted of recording the the diameters of the four most easily identified colour changes at intervals of 10 to 15 seconds. The diameters were measured by means of a pair of crossed scales marked on the sheet by scraping the layer of ink off. These scales then showed black against the coloured ink. The scale division was 1mm. Using this method it was then necessary to record 16 pieces of data (the intersection of each ring with each scale) at each time interval. This was clearly impossible since each reading required a subjective decision about the position of the colour boundary. It was therefore necessary to carry out separate experiments for each colour change ensuring that all operating conditions; ie power input, air flow rate and ambient temperature were constant. This was difficult, especially the ambient temperature constraint. In order to eliminate the problem it was decided to record the progress of the experiment on video tape. This would also assess the usefulness of video recording in future experimentation.

The use of video would have eliminated the need for repeated experiments. The use of 'freeze frame' would also have allowed all measurements to be made at exactly the same time, eliminating the errors caused by the time taken to identify the ring boundaries. However it was found that the colours on video were very poorly defined and were also not true to the original. The second problem could have been overcome by recalibrating the colours on the video. However the lack of differentiation between the colours was so serious as to make this solution impractical.

In conclusion it was decided not to continue further with liquid crystal temperature measurements since the difficulties encountered in a simple system seemed insurmountable without a great deal of work. Translating the method to measurements in packed and moving beds could only have added to the problems. However if the difficulties described here could be overcome this could be a very useful technique.
APPENDIX X Correction of Experimental Matrices

In order to give matrices which satisfy continuity for the bed the row and column totals for each bed layer must be equal. The row totals for different layers must also be in exactly the same proportion as the rates at which material leaves the corresponding bed layers. This rate can be simply calculated:

\[ \text{Rate} = \frac{\sum \text{rate}}{\sum \text{row total}} \]

The experimental matrices always produced equal corresponding rows and columns. The proportions of the row totals were not exactly correct however. A computer program was developed to correct this discrepancy to allow the experimental matrices to be used in the residence time and heat transfer models. This program proceeded as follows:

1. Each column is multiplied by the ratio of the experimental column total and the calculated rate. The row totals are now not equal to the corresponding column totals.

2. The new row totals are calculated and the difference between these and the column totals are calculated.

3. Taking each row in turn the elements of the row are increased by

\[ \frac{\text{element}}{\text{row total}} \times (\text{column total} - \text{row total}) \]

and the corresponding element on the next row is decreased by the same amount so that the column total is unaffected. This corresponds to correcting the situation where a particle has been observed in the wrong layer.

In general the correction necessary was less than 20% of the row total, which is an acceptable experimental accuracy.